实验报告

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实验结果

本实验选用了$f(x)=(x-1)^3-x^2+x$ 和 $f(x)=sin^3(x)+cos^3(x)$两个函数。

二分法

二分法直接用求根区间作为输入。

实验输出如下图。

```
Below: f(x) = (x-1)^3-x^2+x,  Bisection
iteration 1 : f(2.500000000000) = -0.375000000000
iteration 2 : f(2.750000000000) = 0.546875000000
iteration 3 : f(2.625000000000) = 0.025390625000
iteration 4 : f(2.562500000000) = -0.189208984375
iteration 5 : f(2.593750000000) = -0.085601806641
iteration 6 : f(2.609375000000) = -0.031040191650
iteration 7 : f(2.617187500000) = -0.003059864044
iteration 8 : f(2.621093750000) = 0.011106431484
iteration 9 : f(2.619140625000) = 0.004008568823
iteration 10 : f(2.618164062500) = 0.000470676459
iteration 11 : f(2.617675781250) = -0.001295512426
iteration 12 : f(2.617919921875) = -0.000412647685
iteration 13 : f(2.618041992188) = 0.000028956956
iteration 14 : f(2.617980957031) = -0.000191859722
iteration 15 : f(2.618011474609) = -0.000081454972
iteration 16 : f(2.618026733398) = -0.000026249905
iteration 17 : f(2.618034362793) = 0.000001353301
End, x = 2.618034362793

Below: f(x) = (sin(x))^3+(cos(x))^3,  Bisection
iteration 1 : f(2.500000000000) = -0.299844768317
iteration 2 : f(2.250000000000) = 0.223165248351
iteration 3 : f(2.375000000000) = -0.039880755668
iteration 4 : f(2.312500000000) = 0.092542655874
iteration 5 : f(2.343750000000) = 0.026395343550
iteration 6 : f(2.359375000000) = -0.006746823284
iteration 7 : f(2.351562500000) = 0.009825759345
iteration 8 : f(2.355468750000) = 0.001539526758
iteration 9 : f(2.357421875000) = -0.002603673093
iteration 10 : f(2.356445312500) = -0.000532074436
iteration 11 : f(2.355957031250) = 0.000503726461
iteration 12 : f(2.356201171875) = -0.000014173989
iteration 13 : f(2.356079101562) = 0.000244776245
iteration 14 : f(2.356140136719) = 0.000115301129
iteration 15 : f(2.356170654297) = 0.000050563570
iteration 16 : f(2.356185913086) = 0.000018194790
iteration 17 : f(2.356193542480) = 0.000002010400
End, x = 2.356193542480
```

牛顿法

Newton法尝试了三个初始值，2.2、2.5和2.8作为输入 。

实验输出如下图。

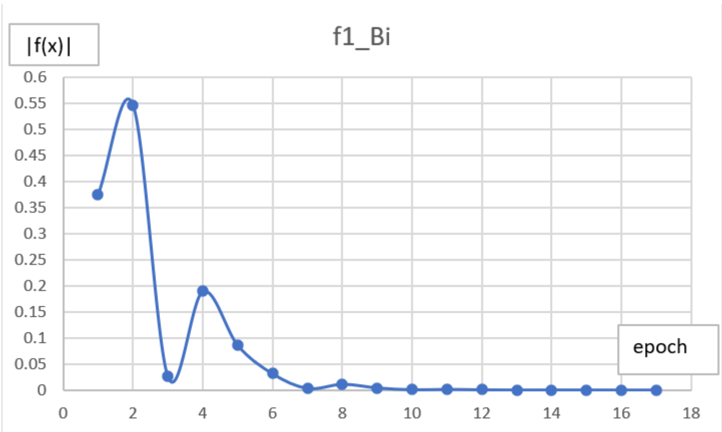
可以看出，不同的初始值作为输入，输出的解直到小数点后六位都是一样的。

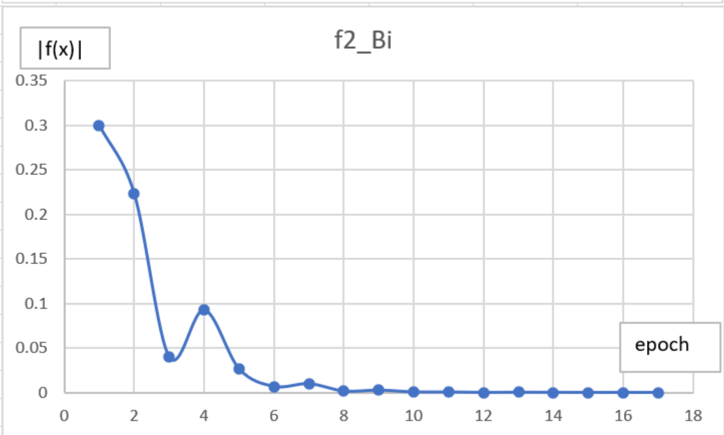
```
Below: f(x) = (x-1)^3-x^2+x,
Newton, Origin = 2.20000000000
iteration 1 : f(3.190661567815) = 3.523321039472
iteration 2 : f(2.799833308690) = 0.791146662748
iteration 3 : f(2.645322995787) = 0.101622987655
iteration 4 : f(2.618800670258) = 0.002776145647
iteration 5 : f(2.618034576599) = 0.000002126861
End, x = 2.618034576599

Below: f(x) = (x-1)^3-x^2+x,
Newton, Origin = 2.50000000000
iteration 1 : f(2.636351514635) = 0.067572750947
iteration 2 : f(2.618382654456) = 0.001261952952
iteration 3 : f(2.618034240353) = 0.000000910309
End, x = 2.618034240353

Below: f(x) = (x-1)^3-x^2+x,
Newton, Origin = 2.80000000000
iteration 1 : f(2.645323031011) = 0.101623122586
iteration 2 : f(2.618800670267) = 0.002776145680
iteration 3 : f(2.618034764321) = 0.000002806046
End, x = 2.618034764321

Below: f(x) = (sin(x))^3+(cos(x))^3,
Newton, Origin = 2.20000000000
iteration 1 : f(2.362803664585) = -0.014019665754
iteration 2 : f(2.356194677149) = -0.000000396596
End, x = 2.356194677149

Below: f(x) = (sin(x))^3+(cos(x))^3,
Newton, Origin = 2.50000000000
iteration 1 : f(2.351074630619) = 0.010860625025
iteration 2 : f(2.356194151952) = 0.000000717516
End, x = 2.356194151952

Below: f(x) = (sin(x))^3+(cos(x))^3,
Newton, Origin = 2.80000000000
iteration 1 : f(2.139529209015) = 0.441978972083
iteration 2 : f(2.374548553012) = -0.038923918319
iteration 3 : f(2.356187000269) = 0.000015888526
iteration 4 : f(2.356194488881) = 0.000000002781
End, x = 2.356194488881
```

结果分析

二分法

开始时稍有波动，但迅速收敛。



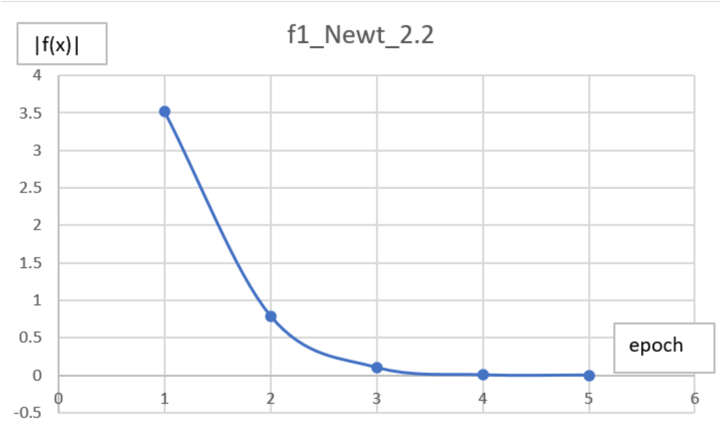


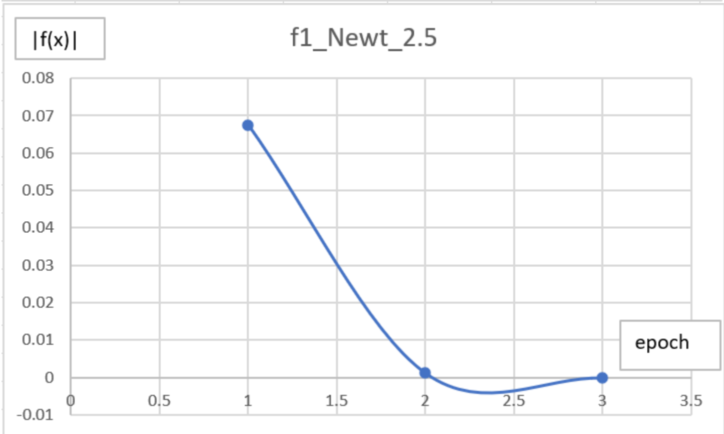
牛顿法

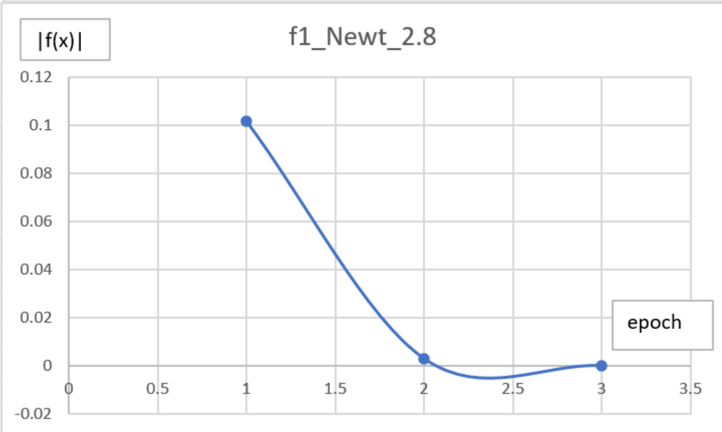
与二分法相比，整体收敛趋势几乎没有波动。

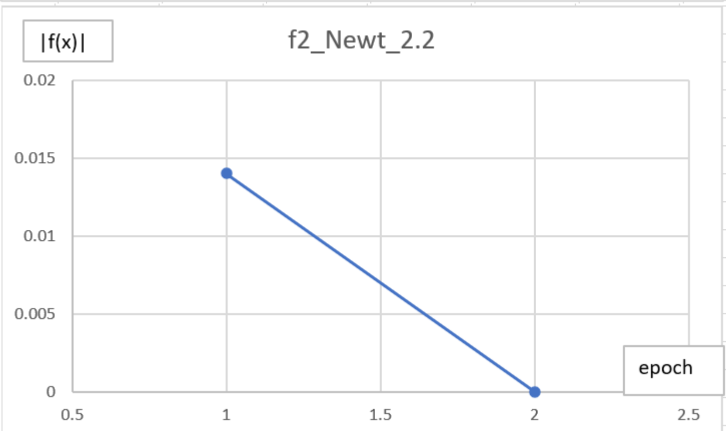
不同的初始值确实会影响收敛的速度快慢。

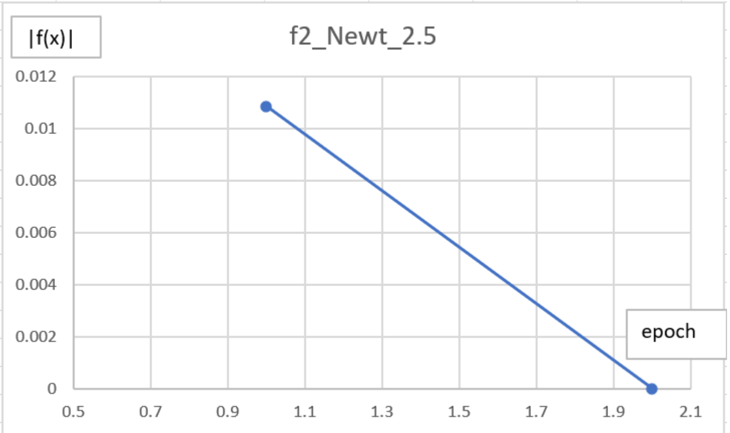
如，$f_1(x)$在$x=2.2$时需要5轮迭代才能收敛，而在$x=2.5$和$x=2.8$时都只需要3轮迭代即可收敛。$f_2(x)$在$x=2.8$时需要4轮迭代才能收敛，而在$x=2.2$和$x=2.5$都只需要2轮迭代即可收敛。

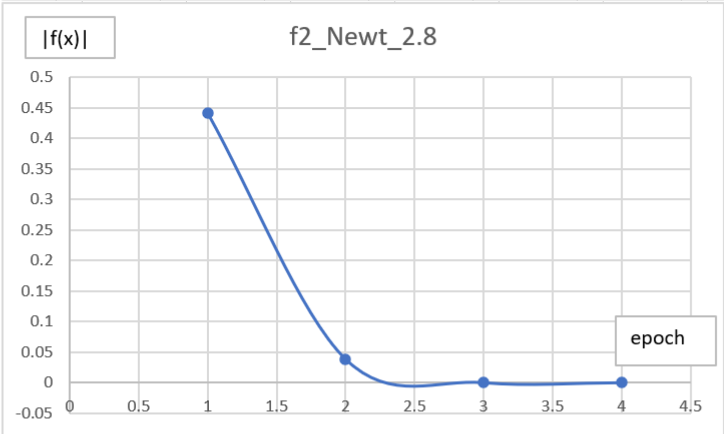









|f(x)|
f2_Newt_2.5
0.012
0.01
0.008
0.006
0.004
0.002
0
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
epoch


|f(x)|
f2_Newt_2.8
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
epoch