

对线性映射 \mathcal{A} , $\mathcal{A} \underline{\varphi}_\alpha, A$.

$$\star \quad \mathcal{A}\alpha = \lambda\alpha \Rightarrow \mathcal{A}(\alpha_1, \alpha_2, \alpha_3) = (\lambda\alpha_1, \dots, \lambda\alpha_3).$$

$$A(\alpha_1, \alpha_2, \alpha_3) = (\alpha_1, \alpha_2, \alpha_3) \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix}.$$

$$A \begin{pmatrix} \vdots \\ \vdots \\ \vdots \end{pmatrix} = \lambda \begin{pmatrix} \vdots \\ \vdots \\ \vdots \end{pmatrix}.$$

$$\Rightarrow \quad A(\lambda_1, \lambda_2, \dots, \lambda_n) = (\lambda_1, \dots, \lambda_n) \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \\ & & & \lambda_n \end{pmatrix}.$$

$$A(\xi_1, \xi_2, \dots, \xi_n) = (\xi_1, \dots, \xi_n) \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \\ & & & \lambda_n \end{pmatrix}.$$

$$A = (\xi_1, \dots, \xi_n) \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \\ & & & \lambda_n \end{pmatrix} (\xi_1, \dots, \xi_n)^{-1}.$$

由此可得, (ξ_1, \dots, ξ_n) 可逆即可得到 A 可对角化.

$\xi = (\xi_1, \dots, \xi_n)$ 为 $n \times n$.

\therefore 需要满秩, 对 $\lambda: \dim V = n$.

故 $\{\xi_1, \dots, \xi_n\}$ 可作为 V 的一组基.