一、填空、选择(每空 3 分，共 30 分)

1. $\underline{\ 0.3\ }$
2. $\underline{\ 1/9\ }$
3. $\underline{\ 8\ }$
4. $\underline{\ (-0.196,\ \ 0.196)\ }$
5. $\underline{\ 1\ }$
6. $\underline{\ D\ }$
7. $\underline{\ B\ }$
8. $\underline{\ D\ }$
9. $\underline{\ C\ }$
10. $\underline{\ D\ }$

二、计算题

1.解:(1)设A表示“取出的产品为次品”,A_i分别表示“元件来自甲乙公厂”,$i=1,2$.则利用全概率公式

$$\begin{aligned}P(A) &= \Sigma_{i=1}^{2}P(A_i)P(A|A_i)\\ &= 0.4\times 0.06+0.6\times 0.04=0.048.\end{aligned}$$ .... (4分)

(2)利用贝叶斯公式可知

$$P(A_1|A)=\frac{P(A_1)P(A|A_1)}{P(A)}=0.5.$$ .... (3分)

$$P(A_2|A)=\frac{P(A_2)P(A|A_2)}{P(A)}=0.5.$$ .... (3分)

2.解:(1)利用归一性,可知

$$1=\int_0^{\frac{\pi}{2}}Acosxdx=A.$$ ....(2分)

(2)

$$P(0<X<\frac{\pi}{4})=\int_0^{\frac{\pi}{4}}cosxdx=\frac{\sqrt{2}}{2}.$$ ....(2分)

(3)

$$EX=\int_0^{\frac{\pi}{2}}xcosxdx=\frac{\pi}{2}-1.$$ ....(2 分)

$$EX^2=\int_0^{\frac{\pi}{2}}x^2cosxdx=\frac{\pi^2-8}{4}$$

$$DX=\pi-3.$$ ....(2分)

(4)

$$F(x)=\begin{cases}0, & x<0\\ sinx, & 0\le x\le\frac{\pi}{2}\\ 1, & x>\frac{\pi}{2}.\end{cases}$$ ....(2分)

三、(14分)

解:(1)

$Y \setminus X$	0	1	$p_{\cdot j}$
0	2/15	4/15	2/5
1	4/15	1/3	3/5
$p_{i\cdot}$	2/5	3/5	1

....(8分)

(2)边缘如上.

(3)$EX = EY = 3/5$, ...(2分)

$DX = DY = 6/25$, ...(2分)

$EXY = 1/3$, ...(1分)

$\rho_{XY} = -\frac{1}{9}$. ....(1分)

四、(10分) 解:(1)

Y_1	-4	0	4
p_k	0.3	0.4	0.3

....(3分)

(2)

Y_2	0	4
p_k	0.4	0.6

....(2分)

(3)

$E(Y_1) = 2EX = 2 \times \big((-2) \times 0.3 + 0 \times 0.4 + 2 \times 0.3\big) = 0$ ....(2分)
$D(Y_1) = 4DX = 9.6$ ....(1分)
$E(Y_2) = 2.4$ ....(1分)
$D(Y_2) = 3.84$ ....(1分)

2.解:(1)利用分布函数法或者定理,可得

$$f_Y(y) = \begin{cases} e^{1-y}, y > 1 \\ 0, y \leq 1. \end{cases}$$ ....(4分)

(2)

$EY = 2$ ....(3分)
$DY = 1$ ....(3分)

五、(10分)

(1)利用已知条件,可知

$$\mu_1 = EX = \int_0^{+\infty} x\frac{1}{\theta}e^{-\frac{1}{\theta}x}dx = \theta, \quad \text{..................................(2 分)}$$

$$\theta = EX, \quad \text{..(2分)}$$

故

$$\hat{\theta} = \bar{X}, \quad \text{..(1分)}$$

(2)设$x_1, \cdots, x_n$为观察值,则

$$\mathcal{L}(\theta) = \theta^{-n}e^{-\frac{1}{\theta}\Sigma_{i=1}^n x_i}, \quad \text{..(2 分)}$$

则

$$\ln\mathcal{L}(\theta) = -n\ln\theta - \frac{1}{\theta}\Sigma_{i=1}^n x_i,$$

由

$$\frac{d}{d\theta}\ln\mathcal{L}(\theta) = \frac{-n}{\theta} + \frac{1}{\theta^2}\Sigma_{i=1}^n x_i = 0, \quad \text{..(2 分)}$$

知

$$\theta = \bar{X}. \quad \text{..(1分)}$$

六、(10分)

解:(1)利用题意知

$$f(x,y)=\begin{cases}1,(x,y)\in G\\0,\text{其它}.\end{cases}\qquad \text{.................(2分)}$$

(2)关于X的边缘为

$$f_X(x)=\begin{cases}1,x\in(0,1)\\0,\text{其它}.\end{cases}\qquad \text{.................(3分)}$$

关于Y的边缘为

$$f_Y(y)=\begin{cases}1,y\in(0,1)\\0,\text{其它}.\end{cases}\qquad \text{.................(3分)}$$

(3)利用卷积公式$f_Z(z)=\int_{-\infty}^{\infty}f_X(x)\times f_Y(z-x)dx$, 则

$$f_Z(z)=\begin{cases}z,z\in(0,1)\\2-z,z\in(1,2)\\0,\text{其它}.\end{cases}\qquad \text{.................(2分)}$$

(4)利用卷积公式$f_Z(z)=\int_{-\infty}^{\infty}f_X(x)\times f_Y(z-x)dx$, 则

$$f_Z(z)=\begin{cases}\frac{z}{2}, & z\in(0,1)\\\frac{1}{2}, & z\in(1,2)\\\frac{3-z}{2}, & z\in(2,3)\\0, & \text{其它}.\end{cases}\qquad \text{.................(2分)}$$

七、(10分)

1. 解:$EX^2 = DX + (EX)^2 = 1$, ...(3分)
因$X^2 \sim \chi^2(1)$, 故$DX^2 = 2$...............(3分)

2.解:利用题意知

$$\begin{aligned} F_Z(z) &= P(Z \le z) \\ &= P(Y=1)P(X+Y \le z|Y=1) + P(Y=-1)P(X+Y \le z|Y=-1).......(3分) \\ &= 0.5 \times \Phi(z-1) + 0.5 \times \Phi(z+1).......(2分) \\ \Rightarrow f_Z(z) &= 0.5\varphi(z-1) + 0.5\varphi(z+1).................(1分) \end{aligned}$$