Q3 i)$T(n) = T\left(\frac{n}{3}\right) 2T\left(\frac{2n}{3}\right) + \log n$

By drawing the recursion tree: focusing on $\log\left(\frac{2n}{3}\right)$ *node as it will give the highest depth*

Depth 0 : log n

Depth 1 (3 node): $\log\left(\frac{n}{3}\right); \log\left(\frac{2n}{3}\right); \log\left(\frac{2n}{3}\right);$

Depth 2 (9 node):

$\log\left(\frac{\mathrm{n}}{3}\right)$ child node: $\mathbf{\log\left(\frac{n}{3^2}\right); \log\left(\frac{2n}{3^2}\right); \log\left(\frac{2n}{3^2}\right)}$

$\log\left(\frac{2\mathrm{n}}{3}\right)$ child node : $\boldsymbol{\log\left(\frac{2n}{3^2}\right); \log\left(\frac{2^2 n}{3^2}\right); \log\left(\frac{2^2 n}{3^2}\right)};$

Depth k : $\log\left(\frac{2^k n}{3^k}\right)$ … ..

$Therefore, k = \log_{3/2} n$

Following the $\log\left(\frac{2n}{3}\right)$ *branch , we would have*:

$$\log\left(\frac{2n}{3}\right) + \log\left(\frac{2^2 n}{3^2}\right) + \cdots .. + \log\left(\frac{2^k n}{3^k}\right)$$

$$= \log\left(\frac{2n}{3} * \frac{2^2 n}{3^2} * \ldots .* \frac{2^k n}{3^k}\right)$$

$$= \log\left(\frac{2}{3}^{\frac{k(k+1)}{2}} n^k\right) = \frac{k(k+1)}{2} \log\frac{2}{3} + k \log n$$

$$= \frac{\log_{\frac{3}{2}} n \left(\log_{\frac{3}{2}} n + 1\right)}{2} \log\frac{2}{3} + \log_{\frac{3}{2}} n * \log n$$

$$= \log_{\frac{3}{2}} n \left(\log_{\frac{3}{2}} n + 1\right) + \log_{\frac{3}{2}} n * \log n \ \ => ignore\ the\ constant$$

Thus, $\boldsymbol{T(n) = O(\log_{\frac{3}{2}} n * log_{\frac{3}{2}} n)}$

Q3 ii) $T(n) = 3T\left(\frac{3n}{4}\right) + logn$

$a = 3; b = \frac{4}{3}; f(n) = logn; Since\ a \geq 1; b > 1\ , can\ use\ Master\ Theorem$

$\log_b a = \log_{\frac{4}{3}} 3 \approx 3.82; Since\ f(n) = O\left(n^{\log_b a - \varepsilon}\right) holds\ when\ \varepsilon > 0$

$It\ fulfill\ the\ first\ case\ of\ the\ master\ therorem. Therefore\ \boldsymbol{T(n) = \theta(n^{\log_{\frac{4}{3}} 3})}$