Homework 2

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CS 402: Advanced Studies II

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Exercise 1.5.

Proof. There are three parts to this problem:

a. Taking the following equation:

$$\text{CPU time} = \text{Instruction Count} \cdot \text{CPI} \cdot \text{Clock Cycle Time} = \frac{\text{Instruction Count} \cdot \text{CPI}}{\text{Clock Rate}} \tag{1}$$

Using equation (1) we get the following *"CPU times"* for each of the processors.

$$t_1 = \frac{1 \cdot 1.5}{3 \cdot 10^9} = 5 \cdot 10^{-10}$$

$$t_2 = \frac{1 \cdot 1}{2.5 \cdot 10^9} = 4 \cdot 10^{-10}$$

$$t_3 = \frac{1 \cdot 2.2}{4.0 \cdot 10^9} = 5.5 \cdot 10^{-10}$$

Then taking the inverse of the *"CPU times"* we will get the overall performance, so for each processors we get the following performances expressed in instructions per second.

$$P_1 = \frac{1}{t_1} = 2 \cdot 10^9$$

$$P_2 = \frac{1}{t_2} = 2.5 \cdot 10^9$$

$$P_3 = \frac{1}{t_3} \approx 1.82 \cdot 10^9$$

$\boxed{\text{Hence, } P_2 \text{ has the greatest performance out of the three.}}$

b. Here we are asked to find the number of cycles and the number of instructions. We will have to use the two equations below:

$$P_{cycles} = \text{Clock Rate} \cdot t \tag{2}$$

$$P_{ins} = \frac{P_{cycles}}{\text{CPI}} \tag{3}$$

Using equation (2) we get the following for each processor.

$$P_{1_{cycles}} = 3 \cdot 10^9 \cdot 10 = 3 \cdot 10^{10}$$

$$P_{2_{cycles}} = 2.5 \cdot 10^9 \cdot 10 = 2.5 \cdot 10^{10}$$

$$P_{3_{cycles}} = 4.0 \cdot 10^9 \cdot 10 = 4.0 \cdot 10^{10}$$

Knowing these values we can find the number instructions for each processor.

$$P_{1_{ins}} = \frac{3 \cdot 10^{10}}{1.5} = 2 \cdot 10^{10}$$

$$P_{2_{ins}} = \frac{2.5 \cdot 10^{10}}{1} = 2.5 \cdot 10^{10}$$

$$P_{3_{ins}} = \frac{4.0 \cdot 10^{10}}{2.2} \approx 1.82 \cdot 10^{10}$$

As we can see the following is our answers for part b.

$$\boxed{\begin{array}{l} P_{1_{cycles}} = 3 \cdot 10^{10} \\ P_{2_{cycles}} = 2.5 \cdot 10^{10} \\ P_{3_{cycles}} = 4.0 \cdot 10^{10} \\ P_{1_{ins}} = 2 \cdot 10^{10} \\ P_{2_{ins}} = 2.5 \cdot 10^{10} \\ P_{3_{ins}} \approx 1.82 \cdot 10^{10} \end{array}}$$

c. For the execution time we want to get a 30% increase in execution time but this will result in a 20% increase in CPI. So our general formula will be below.

$$P_{CR} = \frac{P_{ins} \cdot 1.2 \cdot \text{CPI}}{0.7 \cdot t} \tag{4}$$

For each processor we get:

$$P_{1_{CR}} = \frac{2 \cdot 10^{10} \cdot 1.2 \cdot 1.5}{0.7 \cdot 10} \approx 5.14 \cdot 10^9$$

$$P_{2_{CR}} = \frac{2.5 \cdot 10^{10} \cdot 1.2 \cdot 1}{0.7 \cdot 10} \approx 4.29 \cdot 10^9$$

$$P_{3_{CR}} = \frac{1.82 \cdot 10^{10} \cdot 1.2 \cdot 2.2}{0.7 \cdot 10} \approx 6.86 \cdot 10^9$$

Hence the clock rate we should get to achieve this reduction is below:

$$\boxed{\begin{array}{l} P_{1_{CR}} = 5.14 \text{ GHz} \\ P_{2_{CR}} = 4.29 \text{ GHz} \\ P_{3_{CR}} = 6.86 \text{ GHz} \end{array}}$$

□

Exercise 1.6.

Proof. This exercise has two parts

Before getting into the problem, we must proportion the instructions with their respective percentages.

$$I_A = (0.1) \cdot (1 \cdot 10^6) = 1 \cdot 10^5$$
$$I_B = (0.2) \cdot (1 \cdot 10^6) = 2 \cdot 10^5$$
$$I_C = (0.5) \cdot (1 \cdot 10^6) = 5 \cdot 10^5$$
$$I_D = (0.2) \cdot (1 \cdot 10^6) = 2 \cdot 10^5$$

a. We are asked to find the global CPI for each processor. To do this we will need to find the time it takes and then we will be able to find the global CPI. Using the following equation.

$$t_N = \frac{I_N \cdot CPI_N}{CR_n} \tag{5}$$

Where *"N"* is the various class and *"n"* is the processor number. To find the global CPI for each implementation, we can utilize:

$$\text{CPI}_{Global} = \frac{CR_n \cdot \sum_{i=0}^{N} t_i}{I_{Total}} \tag{6}$$

Using equation (5) and (6) let's find the global CPI for the first implementation and then the second.

$$t_A = \frac{1 \cdot 10^5 \cdot 1}{2.5 \cdot 10^9} = 4 \cdot 10^{-5} s$$
$$t_B = \frac{2 \cdot 10^5 \cdot 2}{2.5 \cdot 10^9} = 1.6 \cdot 10^{-4} s$$
$$t_C = \frac{5 \cdot 10^5 \cdot 3}{2.5 \cdot 10^9} = 6 \cdot 10^{-4} s$$
$$t_D = \frac{2 \cdot 10^5 \cdot 3}{2.5 \cdot 10^9} = 2.4 \cdot 10^{-4} s$$
$$CPI_{1_{Global}} = \frac{(0.4 + 1.6 + 6 + 2.4) \cdot 10^{-4} \cdot 2.5 \cdot 10^9}{1 \cdot 10^6} = 2.6$$
$$t_A = \frac{1 \cdot 10^5 \cdot 2}{3 \cdot 10^9} = \frac{2}{3} \cdot 10^{-4} s$$
$$t_B = \frac{2 \cdot 10^5 \cdot 2}{3 \cdot 10^9} = \frac{4}{3} \cdot 10^{-4} s$$
$$t_C = \frac{5 \cdot 10^5 \cdot 2}{3 \cdot 10^9} = \frac{10}{3} \cdot 10^{-4} s$$
$$t_D = \frac{2 \cdot 10^5 \cdot 2}{3 \cdot 10^9} = \frac{4}{3} \cdot 10^{-4} s$$

$$CPI_{2_{Global}} = \frac{(\frac{2}{3} + \frac{4}{3} + \frac{10}{3} + \frac{4}{3}) \cdot 10^{-4} \cdot 3 \cdot 10^9}{1 \cdot 10^6} = 2$$

Hence, we get that the following global CPI's for each implementation.

$$\boxed{\begin{array}{l} CPI_{1_{Global}} = 2.6 \\ CPI_{2_{Global}} = 2 \end{array}}$$

b. To find the clock cycles of each implementation, we will have to use the equation below:

$$P(\text{Clock Cycles}) = \sum_{i=0}^{N} I_N \cdot CPI_N \tag{7}$$

Using equation (7) we get the following for each implementation.

$$P_1(\text{Clock Cycles}) = (1 \cdot 10^5) \cdot (1) + (2 \cdot 10^5) \cdot (2) + (5 \cdot 10^5) \cdot (3) + (2 \cdot 10^5) \cdot (3) = 2.6 \cdot 10^6$$

$$P_2(\text{Clock Cycles}) = (1 \cdot 10^5) \cdot (2) + (2 \cdot 10^5) \cdot (2) + (5 \cdot 10^5) \cdot (2) + (2 \cdot 10^5) \cdot (2) = 2 \cdot 10^6$$

Hence, we get the following clock cycles.

$$\boxed{\begin{array}{l} P_1(\text{Clock Cycles}) = 2.6 \cdot 10^6 \\ P_2(\text{Clock Cycles}) = 2 \cdot 10^6 \end{array}}$$

□

Exercise 1.7.

Proof. There are three parts to this problem:

a. Using the equation below:

$$CPI = t_{exec} \cdot \frac{1}{\text{Clock Cycles} \cdot \text{Instruction Count}} \tag{8}$$

Using (8) we get the following for each compiler

$$CPI_A = 1.1 \cdot \frac{1}{10^{-9} \cdot 10^9} = 1.1$$

$$CPI_B = 1.5 \cdot \frac{1}{1.2 \cdot 10^{-9} \cdot 10^9} = 1.25$$

Hence, we get

$$\boxed{\begin{array}{l} CPI_A = 1.1 \\ CPI_B = 1.25 \end{array}}$$

b. We are asked to compare how fast respectively of the two compilers.

$$\frac{1.2 \cdot 10^9 \cdot 1.25}{10^9 \cdot 1.1} \approx 1.37$$

$$\boxed{\text{Hence, we get that compiler B is approximately 1.37 times faster than compiler A.}}$$

c. Comparison with a new compiler we get the following:

$$\frac{T_A}{T_{new}} = \frac{1.1 \cdot 1 \cdot 10^9}{1.1 \cdot 6 \cdot 10^8} \approx 1.67$$

$$\frac{T_B}{T_{new}} = \frac{1.25 \cdot 1.2 \cdot 10^9}{1.1 \cdot 6 \cdot 10^8} \approx 2.27$$

Hence, the speed up with regards to compiler A is approximately 1.67 and for compiler B is approximately 2.27

□

Exercise 1.9.1.

Proof. We can utilize equation (5) for this part of 1.9. So we get that:

$$t_1 = \frac{(\frac{2.56 \cdot 10^9}{0.7} \cdot 1) + (\frac{1.28 \cdot 10^9}{0.7} \cdot 12) + (2.56 \cdot 10^8 \cdot 5)}{2 \cdot 10^9} = 13.44s$$

$$t_2 = \frac{(\frac{2.56 \cdot 10^9}{0.7 \cdot 2} \cdot 1) + (\frac{1.28 \cdot 10^9}{0.7 \cdot 2} \cdot 12) + (2.56 \cdot 10^8 \cdot 5)}{2 \cdot 10^9} = 7.04s$$

$$t_4 = \frac{(\frac{2.56 \cdot 10^9}{0.7 \cdot 4} \cdot 1) + (\frac{1.28 \cdot 10^9}{0.7 \cdot 4} \cdot 12) + (2.56 \cdot 10^8 \cdot 5)}{2 \cdot 10^9} = 3.84s$$

$$t_8 = \frac{(\frac{2.56 \cdot 10^9}{0.7 \cdot 8} \cdot 1) + (\frac{1.28 \cdot 10^9}{0.7 \cdot 8} \cdot 12) + (2.56 \cdot 10^8 \cdot 5)}{2 \cdot 10^9} = 2.24s$$

Hence we get the following execution time with their relative speed ups below:

$t_1 = 13.44s$	$S_1 = 1$
$t_2 = 7.04s$	$S_2 = 1.91$
$t_4 = 3.84s$	$S_4 = 3.5$
$t_8 = 2.24s$	$S_8 = 6$

□

Exercise 1.9.2.

Proof. Doubling the CPI for the arithmetic value we get the following results:

$t_1 = 15.27s$
$t_2 = 7.95s$
$t_4 = 4.30s$
$t_8 = 2.47s$

□

Exercise 1.9.3.

Proof. $\boxed{\text{It should be reduced to 3}}$ $\square$

Exercise 1.11.1.

Proof. First we have to get the clock rate which is found below from the cycle time:

$$\text{Clock Rate} = \frac{1}{\text{Cycle Time}} = \frac{1}{0.333 \cdot 10{-}9} \approx 3 \cdot 10^9$$

Knowing this we can get the CPI:

$$CPI(biz2) = 3 \cdot 10^9 \cdot \frac{750}{2389 \cdot 10^9} \approx 0.94$$

$\boxed{\text{Hence, we get that the CPI to be approximately 0.94.}}$ $\square$

Exercise 1.11.2.

Proof. To find the SPECratio, we calculated below:

$$\text{SPECratio} = \frac{t_r}{I_N} = \frac{9650}{750} \approx 12.87$$

$\boxed{\text{Hence, the SPECratio is approximately 12.87.}}$ $\square$

Exercise 1.11.3.

Proof. $\boxed{\text{Taking equation 1, assuming the clock rate and CPI stay constant then there should be a 10\% increase in CPU time.}}$ $\square$

Exercise 1.11.4.

Proof. Taking equation (1) we can formulate this using proportions again as shown:

$$\frac{\text{CPU Time(after)}}{\text{CPU Time(before)}} = \frac{1.1 \cdot \text{Instruction Count} \cdot 1.05 \cdot \text{CPI}}{\text{Clock Rate}} \cdot \frac{\text{Clock Rate}}{\text{Instruction Count} \cdot \text{CPI}} = 1.1 \cdot 1.05 = 1.155$$

$\boxed{\text{Hence, the CPU time increases by 15.5\%}}$ $\square$

Exercise 1.11.5.

Proof. We can using proportions here as well as shown below:

$$\frac{\text{SPECratio(after)}}{\text{SPECratio(before)}} = \frac{t_r}{\text{CPU Time(after)}} \cdot \frac{\text{CPU Time(before)}}{t_r} = \frac{\text{CPU Time(before)}}{\text{CPU Timeafter)}} = \frac{1}{1.155} \approx 0.87$$

$\boxed{\text{Hence, the SPECratio decreased by 13\%}}$

□

Exercise 1.11.6.

Proof. Taking equation (1), we get the following for CPI.

$$\text{CPI} = \frac{700 \cdot 4 \cdot 10^9}{0.85 \cdot 2389 \cdot 10^9} \approx 1.38$$

$\boxed{\text{Hence, the CPI is approximately 1.38}}$

□

Exercise 1.11.7.

Proof. Taking the ratios as below we get that.

$$\frac{4}{3} \approx 1.33$$

$$\frac{1.38}{0.94} \approx 1.47$$

These two values are different due to the fact that although the number of instructions were reduced by 15%, the CPU time was reduced by a lower percentage.

□

Exercise 1.11.8.

Proof. Looking at execution time, we can get the following ratio.

$$\frac{700}{750} \approx 0.933$$

$\boxed{\text{Hence, knowing this we can conclude that the CPU time reduced by 6.7\%}}$

□

Exercise 1.11.9.

Proof. Rearranging (1) we get that:

$$\text{Instruction Count} = \frac{\text{CPU Time} \cdot \text{Clock Rate}}{\text{CPI}}$$

Plugging in the number we get that:

$$\text{Instruction Count} = \frac{960 \cdot 0.9 \cdot 4 \cdot 10^9}{1.61} \approx 2.147 \cdot 10^{12}$$

$\boxed{\text{Hence, the instruction count is approximately } 2.147 \cdot 10^{12}}$

□

Exercise 1.11.10.

Proof. We can use equation (1) again and get the clock rate below:

$$\text{Clock Rate}_{new} = \frac{\text{Clock Rate}_{old}}{0.9} \approx 3.33 \cdot 10^9$$

$\boxed{\text{Hence, the new clock rate is approximately 3.33 GHz}}$

□

Exercise 1.11.11.

Proof. We can use equation (1) again:

$$\text{Clock Rate}_{new} = \frac{0.85}{0.80} \cdot \text{Clock Rate}_{old} \approx 3.19 \cdot 10^9$$

$\boxed{\text{Hence, the clock rate would be approximately 3.19 GHz}}$

□

Exercise 1.12.1.

Proof. Utilizing equation (1) again:

$$t_1 = \frac{5 \cdot 10^9 \cdot 0.9}{4 \cdot 10^9} = 1.125s$$

$$t_2 = \frac{1 \cdot 10^9 \cdot 0.75}{3 \cdot 10^9} = 0.25s$$

Even though the clock rate for processor 1 is larger than processor 2, processor 2 has better performance than processor 1, hence the statement isn't true.

□

Exercise 1.12.2.

Proof. We know that $t_1 = t_2$ in this case so we have to find the value of t_1 first by:

$$t_1 = \frac{1 \cdot 10^9 \cdot 0.9}{4 \cdot 10^9} = 0.225$$

Taking this value we can rearrange to solve for the number of instructions executed in this time frame from processor 2.

$$N_2 = \frac{0.225 \cdot 3 \cdot 10^9}{0.75} = 9 \cdot 10^8$$

$\boxed{\text{Hence, the number of instructions processor 2 can execute in that time is } 9 \cdot 10^8}$

□

Exercise 1.12.3.

Proof. Take the equation for MIPS below:

$$\text{MIPS} = \frac{\text{Clock Rate} \cdot 10^{-6}}{\text{CPI}} \tag{9}$$

Taking equation (9) we can calculate both MIPS values of each processor:

$$\text{MIPS}_1 = \frac{4 \cdot 10^9 \cdot 10^{-6}}{0.9} \approx 4.44 \cdot 10^3$$

$$\text{MIPS}_2 = \frac{3 \cdot 10^9 \cdot 10^{-6}}{0.75} = 4 \cdot 10^3$$

Hence, the MIPS value is greater in processor 1 yet we still have the performance to be better for processor 2, which we see in 1.12.1

□

Exercise 1.12.4.

Proof. Utilizing the MFLOPS which can be expressed as below:

$$\text{MFLOPS} = \frac{\text{No. FP Operations}}{t_{exec} \cdot 1 \cdot 10^6} \tag{10}$$

We can see that for each processor we get the following values:

$$\text{MFLOPS}_1 = \frac{0.4 \cdot 5 \cdot 10^9}{1.125 \cdot 10^6} \approx 1.78 \cdot 10^3$$

$$\text{MFLOPS}_2 = \frac{0.4 \cdot 1 \cdot 10^9}{0.25 \cdot 10^6} = 1.6 \cdot 10^3$$

Hence, we get the same trend where the MFLOPS value is greater in processor 1 yet we still have the performance to be better for processor 2, which we see in 1.12.1

□

Exercise 1.13.1.

Proof. Reducing FP operations by 20% we get:

$$t_{FP} = 70 \cdot 0.8 = 56s$$

So we get a new total time for the program to be.

$$t_{total} = 56 + 85 + 40 + 55 = 236s$$

Hence, we get a 5.6% reduction from reducing the FP operations by 20%.

□

Exercise 1.13.2.

Proof. Taking the total time and reducing it by 20% we get:

$$t_{new} = 250 \cdot 0.8 = 200s$$

Then we take this new total and subtract from all time except INT instructions.

$$t_{new} - (t_{FP} + t_{L/S} + t_{branch}) = 200 - 165 = 35s$$

Hence, we get a 58.8% reduction for the INT operations.

□

Exercise 1.13.3.

Proof. Using the value from the last problem we have that $t_{new} = 200s$. So setting $t_{branch} = 0$ we get that:

$$t_{FP} + t_{L/S} + t_{INT} = 210s > 200s$$

Hence, since this is greater than the threshold this is not possible.

□

Exercise 1.14.1.

Proof. Taking our general equation (1) and expanding it to:

$$\text{Clock Cycles} = CPI_{FP} \cdot N_{FP} + CPI_{INT} \cdot N_{INT} + CPI_{L/S} \cdot N_{L/S} + CPI_{branch} \cdot N_{branch}$$

$$t_{cpu} = \frac{512 \cdot 10^6}{2 \cdot 10^9} = 0.256s$$

So we want to run the program twice as fast with only changing the CPI of the FP instructions. We would end up with:

$$\text{CPI}_{newFP} = \frac{256 - 462}{50} = -4.12$$

$\boxed{\text{Hence, since the number would be negative, this is impossible.}}$

□

Exercise 1.14.2.

Proof. Utilizing the same methodology from the previous problem we would end up with:

$$\text{CPI}_{newL/S} = \frac{256 - 198}{80} = 0.725$$

$\boxed{\text{Hence, we would need the L/S CPI to be 0.725 to run the program twice as fast.}}$

□

Exercise 1.14.3.

Proof. Taking each of these we get the following values:

$$\text{CPI}_{FP} = \text{CPI}_{INT} = 0.6 \cdot 1 = 0.6$$

$$\text{CPI}_{L/S} = 0.7 \cdot 4 = 2.8$$

$$\text{CPI}_{branch} = 0.7 \cdot 2 = 1.4$$

Plugging these values to our equation we get that our t_{cpu} value to be:

$$t_{cpu} = 0.171s$$

$\boxed{\text{Hence, we get a 1.5 times speed up from the CPI improvements.}}$

□

Exercise 1.15.

Proof. The data should be below, where primes are the ideal number:

$$\begin{aligned} t_1 &= 100s & t_1^{'} &= 100s \\ t_2 &= 54s & t_2^{'} &= 50s \\ t_4 &= 29s & t_4^{'} &= 25s \\ t_8 &= 16.5s & t_8^{'} &= 12.5s \\ t_{16} &= 10.25s & t_{16}^{'} &= 6.25s \\ t_{32} &= 7.125s & t_{32}^{'} &= 3.125s \\ t_{64} &= 5.5625s & t_{64}^{'} &= 1.5625s \\ t_{128} &= 4.78125s & t_{128}^{'} &= 0.78125s \end{aligned}$$

Taking these values we get the following speed ups:

$$\begin{aligned} S_1 &= 1 & S_1^{'} &= 1 \\ S_2 &= 1 + 0.85 & S_2^{'} &= 1 + 1 \\ S_4 &= 1 + 2.45 & S_4^{'} &= 1 + 3 \\ S_8 &= 1 + 5.06 & S_8^{'} &= 1 + 7 \\ S_{16} &= 1 + 8.76 & S_{16}^{'} &= 1 + 15 \\ S_{32} &= 1 + 13.04 & S_{32}^{'} &= 1 + 32 \\ S_{64} &= 1 + 16.98 & S_{64}^{'} &= 1 + 63 \\ S_{128} &= 1 + 19.92 & S_{128}^{'} &= 1 + 127 \end{aligned}$$

Taking these values we can finally get our ratios which are:

$$\frac{S_1}{S_1'} = 1$$
$$\frac{S_2}{S_2'} = \frac{25}{27}$$
$$\frac{S_4}{S_4'} = \frac{25}{29}$$
$$\frac{S_8}{S_8'} = \frac{25}{33}$$
$$\frac{S_{16}}{S_{16}'} = \frac{25}{41}$$
$$\frac{S_{32}}{S_{32}'} = \frac{25}{57}$$
$$\frac{S_{64}}{S_{64}'} = \frac{25}{89}$$
$$\frac{S_{128}}{S_{128}'} = \frac{25}{153}$$

$\square$