Homework 1

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CS 530: Theory of Computation

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Exercise 1. Give DFA accepting each of the following languages over the alphabet $\{0, 1\}$

a. The set of all strings such that every block of five consecutive symbols contains at least two 0's (strings of length less than 5 do not have this condition)

b. The set of all string beginning with a 1 which, interpreted as the binary representation of an integer, is congruent to zero modulo 5.

Proof. The two graphs are the two separate things for each parts.

a.

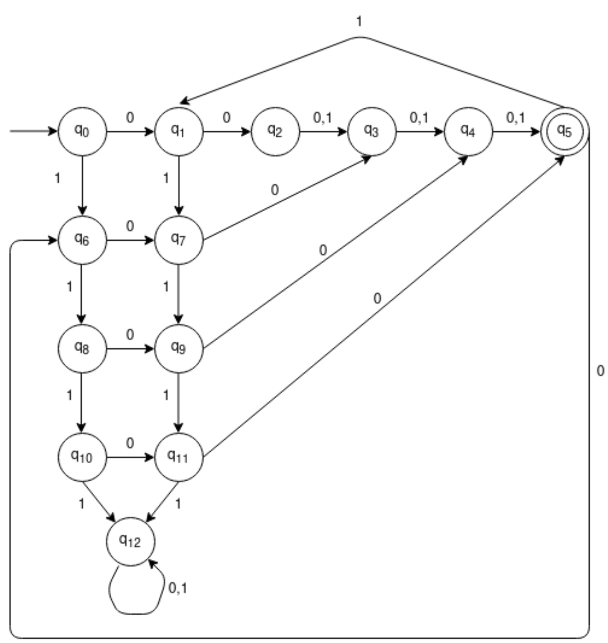

Figure 1: DFA of Exercise 1a

b.

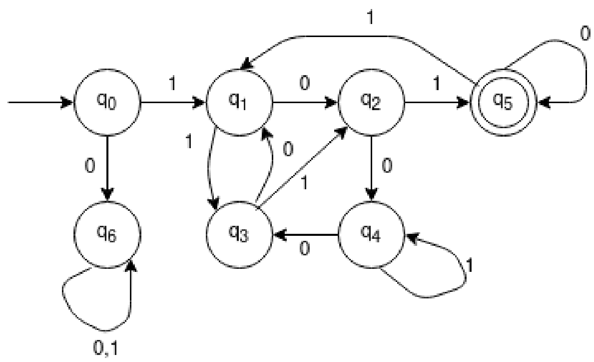

Figure 2: DFA of Exercise 1b

□

Exercise 2. What language is recognized by Figure 1 and Figure 2 in the homework.

Proof. Both answers are in separate parts.

a. The set of all strings beginning with a and alternating between a, b where a should also be the last element. This is represented by $(ab)^*a$. e.g. $ababa...a$

b. The set of all strings under $(ab + ba)^*$ or Λ, also known as the empty string.

□

Exercise 3. Prove the correctness of the construction of the cross-product automaton, discussed in class, that recognizes the union of two regular languages.

Proof. Let's take that we have two regular languages A_1 and A_2. We want to show that $A_1 \cup A_2$, is recognized some finite automation, M

Let M_1 recognize A_1, where $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$

Now, we will use proof by construction by constructing M that recognizes $A_1 \cup A_2$, where $M = (Q, \Sigma, \delta, q_0, F)$.

Taking $Q = \{(r_1, r_2 | r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$, this set is the Cartesian product of the sets Q_1 and Q_2. The Σ will be the same as it is in M_1 and M_2. The transition function, δ for each $(r_1, r_2) \in Q$ and each $a \in Sigma$, which letting $\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$. This shows that δ gets a state M which is the pair of states from M_1 and M_2 together that returns M's next state. q_0 is the pair (q_1, q_2). Lastly, taking F is the set of pairs in which either member is an accept state of M_1 or M_2, which is $F = \{(r_1, r_2), r_1 \in F_1 \text{ or } r_2 \in F_2\}$. Taking note that $F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$

Hence this is the construction of the finite automaton M that recognizes the union of two regular languages A_1 and A_2.

$\square$

Exercise 4. Prove the correctness of the construction of a DFA from a NFA.

Proof. We must show the construction of a DFA from an NFA in turn let's take that an NFA recognizing some language A, $N = (Q, \Sigma, \delta, q_0, F)$, then we will construct a DFA recognizing A $M = (Q', \Sigma, \delta', q_0', F')$.

Taking in account that $Q' = \mathcal{P}(Q)$, meaning that every state of M is a set of states of N. For $R \in Q'$ and $a \in \Sigma$, let $\delta'(R, a) = \{q \in Q | q \in \delta(r, a) \text{ for some } r \in R\}$, where R is state of M and is also a set of states of N. $q_0' = \{q_0\}$, this means that M starts in the state corresponding to the collection containing just the start state of N. Lastly, $F' = \{R \in Q' | R \text{ contains an accept state of } N\}$, meaning that the machine M accepts if one of the possible states that N could be in at this point is an accept state. By taking the ε arrows into account for the DFA, we will have to note that for any state R of M, we must define $E(R)$ to be the collection of states that can be reached from members of R by going only along ε, which include the members of R themselves. This can be seen when $R \subseteq Q$, then $E(R) = \{q | q \text{ can be reached from } R \text{ by travelling alone 0 or more } \varepsilon \text{ arrows}\}$. By modifying the transition function of M, by replacing the original $\delta(r, a)$ by $E(\delta(r, a))$, thus $\delta'(R, a) = \{q \in Q | q \in E(\delta(r, a)) \text{ for some } r \in R\}$. Now to Finally we have to change the q_0' to be $E(\{q_0\})$

Hence from above we have achieved that by proof of construction the DFA M from the NFA of N. This works due to the face that at every step in the computation of M on an input, we can see that the enters a state that corresponds to the subset of states that N could be in at that specified point.

$\square$

Exercise 5. Give a NFA accepting each of the following languages over the alphabet $\{a, b\}$

a. The set of string in $(a + b)^*$ such that some two a's are separated by a string whose length is $4i$ for some $i \geq 0$.

b. $((a^*b^*a^*)^*b)^*$

c. $(ba \cup b)^* \cup (bb \cup a)^*$

Proof. Below is all the corresponding NFAs in figure 3, 4, and 5.

$\square$

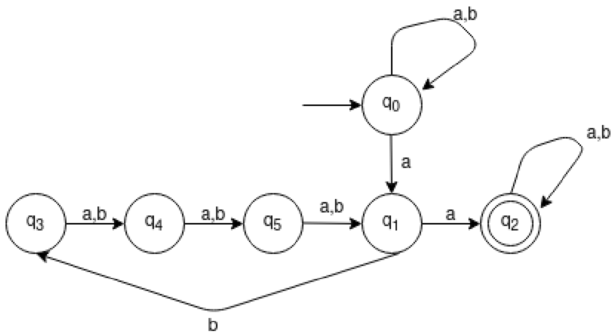

Figure 3: NFA of Exercise 5a

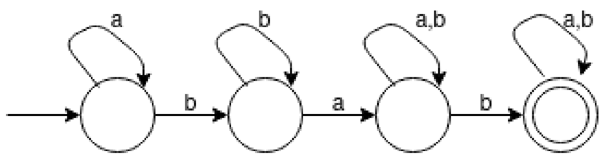

Figure 4: NFA of Exercise 5b

Exercise 6. Let $L \subseteq \Sigma^*$. Show that if L is accepted by some finite automation so is each of the following

a. $SUBSEQ(L) = \{w_1 w_2 ... w_k \mid k \in N, w_j \in \Sigma^*$ for $j = 1, ..., k$, and there is a string $x = x_0 w_1 x_1 w_2 ... w_k x_k$ in $L\}$

b. $MAX(L) = \{w \in L \mid |x| \neq 0 \Rightarrow wx \notin L\}$

Proof. Below are the two parts

a. We will using proof by induction, so here taking the base case where $k = 1$ we get that the w_1 and that the string value for $x = x_0 w_1 x_1$ is indeed in L by the definition. Now let's take a look at $k = k + 1$, here we get that, $\{w_1 w_2 ... w_k w_{k+1} \mid k \in N, w_j \in \Sigma^*$ for $j = 1, ..., k, k+1$, and there is a string $x = x_0 w_1 x_1 w_2 ... w_k x_k w_{k+1} x_{k+1}$ in $L\}$ is indeed true as well as all the previous subscript values are also true. Hence result.

b. We will utilize proof by contradiction, so take that $|x| = 0$, then we can see that we $wx = w(0) \notin L$ since, the absolute value of 0 can never be the MAX. Hence $wx \notin L$, we can conclude that $|x| \neq 0$.

□

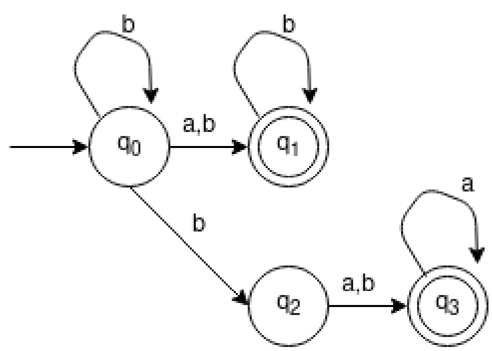

Figure 5: NFA of Exercise 5c