Updated Unit Commitment Model

Sets

$\mathcal{G}$	Set of all generators
$\mathcal{G}_{\text{solar}}$	Set of solar generators
$\mathcal{G}_{\text{wind}}$	Set of wind generators
$\mathcal{G}_{\text{hydro}}$	Set of hydro generators
$\mathcal{G}_{\text{batt}}$	Set of battery storage units
$\mathcal{G}_{\text{renew}}$	Set of renewable generators ($\mathcal{G}_{\text{solar}} \cup \mathcal{G}_{\text{wind}}$)
$\mathcal{G}_{\text{foss}}$	Set of fossil fuel generators
$\mathcal{T}$	Set of time periods
$\mathcal{S}$	Set of scenarios
$\mathcal{N}$	Set of nodes
$\mathcal{L}$	Set of transmission lines
Ω	Set of uncertainty realizations
$\mathcal{D}$	Set of demands

Parameters

CapEx_g	Capacity cost (fixed) of generator g
OpEx_g	Operating cost (variable) of generator g
C_g^{SU}	Start-up cost of generator g
C_g^{SD}	Shut-down cost of generator g
π_s	Probability of scenario s
$\underline{P}_g, \overline{P}_g$	Minimum and maximum power output of generator g
R_g	Ramping rate of generator g
D_t	Demand curve function at time t
$\overline{P}_g^{\text{chg}}$	Maximum charging power for battery g
$\overline{P}_g^{\text{dchg}}$	Maximum discharging power for battery g
η_g^{chg}	Charging efficiency of battery g
η_g^{dchg}	Discharging efficiency of battery g
$\underline{SoC}_g, \overline{SoC}_g$	Minimum and maximum state of charge for battery g
$\overline{F}_l$	Maximum power flow on line l
$\text{PTDF}_{n,l}$	Power Transfer Distribution Factor for node n and line l
ξ_{st}^{ω}	Realized solar power availability for scenario ω at time t
ξ_{wt}^{ω}	Realized wind power availability for scenario ω at time t
ξ_{dt}^{ω}	Realized electric demand for scenario ω at time t
$\overline{P}_g(\xi_{st}^{\omega})$	Maximum available power from solar generator g based on realized solar availability
γ_g	Wind distribution factor for wind generator g
SU_g, SD_g	Start-up and shut-down rates of generator g
UT_g, DT_g	Minimum up and down times of generator g
r_t^{up}, r_t^{dn}	System-wide upward and downward reserve requirements at time t
$\Lambda_{gi}, \Lambda_{li}, \Lambda_{di}$	Incidence matrices for generators, lines, and loads
X_l	Reactance of line l
δ_d	Load distribution factor for demand d
Δt	Time step duration
E_g^{cap}	Energy capacity of battery g
Inflow_{gts}	Water inflow for hydro generator g at time t in scenario s
$\underline{e}_g, \overline{e}_g$	Minimum and maximum reservoir levels for hydro generator g
$\underline{q}_g, \overline{q}_g$	Minimum and maximum water discharge for hydro generator g
$\overline{s}_g$	Maximum water spillage for hydro generator g
h_g^{base}	Base head for hydro generator g
e_g^{base}	Base reservoir level for hydro generator g
α_g, β_g	Head-storage and head-discharge coefficients for hydro generator g
$e_g^{\text{init}}, e_g^{\text{final}}$	Initial and final reservoir levels for hydro generator g
η_g	Efficiency of hydro generator g

Variables

u_{gts}	Binary variable: 1 if generator g is on at time t in scenario s, 0 otherwise
y_{gts}	Binary variable: 1 if generator g starts up at time t in scenario s, 0 otherwise
z_{gts}	Binary variable: 1 if generator g shuts down at time t in scenario s, 0 otherwise
p_{gts}	Power output of generator g at time t in scenario s
r_{gts}	Reserve provided by generator g at time t in scenario s
SoC_{gts}	State of charge for battery g at time t in scenario s
$p_{gts}^{\text{chg}}, p_{gts}^{\text{dchg}}$	Charging and discharging power for battery g at time t in scenario s
$u_{gts}^{\text{chg}}, u_{gts}^{\text{dchg}}$	Binary variables for charging and discharging of battery g at time t in scenario s
θ_{nts}	Voltage angle at node n at time t in scenario s
f_{lts}	Power flow on line l at time t in scenario s
p_{gts}^{ω}	Second-stage adjustment of generator g at time t in scenario s and realization ω
$\overline{P}_{gts}$	Maximum available capacity of generator g at time t in scenario s
r_{gts}^{U}, r_{gts}^{D}	Upward and downward reserves provided by generator g at time t in scenario s
UD_{dts}^{ω}	Unserved demand at bus d at time t in scenario s and realization ω
f_{lts}^{ω}	Power flow on line l at time t in scenario s and realization ω
θ_{its}^{ω}	Voltage angle at bus i at time t in scenario s and realization ω
e_{gts}	Reservoir level of hydro generator g at time t in scenario s
q_{gts}	Water discharge of hydro generator g at time t in scenario s
s_{gts}	Water spillage of hydro generator g at time t in scenario s
H_{gts}	Net head of hydro generator g at time t in scenario s

Objective Function

$$\min \sum_{s\in\mathcal{S}} \pi_s \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}} \left(\mathrm{CapEx}_g u_{gts} + \mathrm{OpEx}_g p_{gts} + C_g^{SU} y_{gts} + C_g^{SD} z_{gts}\right) \tag{1}$$

Constraints

Renewable Generation Constraints (Solar and Wind)

$$\begin{aligned}
&p_{gts} + p_{gts}^{\omega} \le \overline{P}_g(\xi_{st}^{\omega}), && \forall g \in \mathcal{G}_{\text{solar}}, t\in\mathcal{T}, s\in\mathcal{S}, \omega\in\Omega && (2)\\
&p_{gts} - p_{gts}^{\omega} \ge 0, && \forall g \in \mathcal{G}_{\text{solar}}, t\in\mathcal{T}, s\in\mathcal{S}, \omega\in\Omega && (3)\\
&p_{gts} + p_{gts}^{\omega} \le \gamma_g \cdot \xi_{wt}^{\omega}, && \forall g \in \mathcal{G}_{\text{wind}}, t\in\mathcal{T}, s\in\mathcal{S}, \omega\in\Omega && (4)\\
&p_{gts} - p_{gts}^{\omega} \ge 0, && \forall g \in \mathcal{G}_{\text{wind}}, t\in\mathcal{T}, s\in\mathcal{S}, \omega\in\Omega && (5)
\end{aligned}$$

$p_{gts} \le \overline{P}_g(\xi_{st}^{\omega}), \quad \forall g \in \mathcal{G}_{\text{solar}}, t\in\mathcal{T}, s\in\mathcal{S}, \omega\in\Omega p_{gts} \le \gamma_g \cdot \xi_{wt}^{\omega}, \quad \forall g \in \mathcal{G}_{\text{wind}}, t\in\mathcal{T}, s\in\mathcal{S}, \omega\in\Omega$

Hydro Constraints

$$\begin{aligned}
&e_{gts} = e_{g,t-1,s} + \mathrm{Inflow}_{gts} - q_{gts} - s_{gts}, && \forall g\in\mathcal{G}_{\text{hydro}}, t\in\mathcal{T}, s\in\mathcal{S} && (6)\\
&p_{gts} = \eta_g \cdot q_{gts} \cdot H_{gts}, && \forall g\in\mathcal{G}_{\text{hydro}}, t\in\mathcal{T}, s\in\mathcal{S} && (7)\\
&\underline{e}_g \le e_{gts} \le \overline{e}_g, && \forall g\in\mathcal{G}_{\text{hydro}}, t\in\mathcal{T}, s\in\mathcal{S} && (8)\\
&\underline{q}_g \le q_{gts} \le \overline{q}_g, && \forall g\in\mathcal{G}_{\text{hydro}}, t\in\mathcal{T}, s\in\mathcal{S} && (9)\\
&0 \le s_{gts} \le \overline{s}_g, && \forall g\in\mathcal{G}_{\text{hydro}}, t\in\mathcal{T}, s\in\mathcal{S} && (10)\\
&H_{gts} = h_g^{\text{base}} + \alpha_g \cdot (e_{gts} - e_g^{\text{base}}) - \beta_g \cdot q_{gts}^2, && \forall g\in\mathcal{G}_{\text{hydro}}, t\in\mathcal{T}, s\in\mathcal{S} && (11)\\
&e_{g,0,s} = e_g^{\text{init}}, \quad e_{g,|\mathcal{T}|,s} = e_g^{\text{final}}, && \forall g\in\mathcal{G}_{\text{hydro}}, s\in\mathcal{S} && (12)
\end{aligned}$$

Battery Constraints

$$\begin{aligned}
&\underline{SoC}_g \le SoC_{gts} \le \overline{SoC}_g, && \forall g\in\mathcal{G}_{\text{batt}}, t\in\mathcal{T}, s\in\mathcal{S} && (13)\\
&0 \le p_{gts}^{chg} \le \overline{P}_g^{chg} u_{gts}^{chg}, && \forall g\in\mathcal{G}_{\text{batt}}, t\in\mathcal{T}, s\in\mathcal{S} && (14)\\
&0 \le p_{gts}^{dchg} \le \overline{P}_g^{dchg} u_{gts}^{dchg}, && \forall g\in\mathcal{G}_{\text{batt}}, t\in\mathcal{T}, s\in\mathcal{S} && (15)\\
&SoC_{gts} = SoC_{g,t-1,s} + (\eta_g^{chg} p_{gts}^{chg} - \frac{1}{\eta_g^{dchg}} p_{gts}^{dchg}) \cdot \frac{\Delta t}{E_g^{cap}}, && \forall g\in\mathcal{G}_{\text{batt}}, t\in\mathcal{T}, s\in\mathcal{S} && (16)\\
&u_{gts}^{chg} + u_{gts}^{dchg} \le 1, && \forall g\in\mathcal{G}_{\text{batt}}, t\in\mathcal{T}, s\in\mathcal{S} && (17)
\end{aligned}$$

Power Balance and DCPF Constraints

$$\sum_{g\in\mathcal{G}} \Lambda_{gi}(p_{gts} + p_{gts}^{\omega}) + \sum_{g\in\mathcal{G}_{\text{batt}}} \Lambda_{gi}(p_{gts}^{dchg} - p_{gts}^{chg}) - \sum_{l\in\mathcal{L}} \Lambda_{li} \cdot f_{lts}^{\omega} = \sum_{d\in\mathcal{D}} \Lambda_{di}(\xi_{dt}^{\omega} - UD_{dts}^{\omega}), \quad \forall t\in\mathcal{T}, s\in\mathcal{S}, \omega\in\Omega, i\in\mathcal{N} \tag{18}$$

$$f_{lts}^{\omega} = \sum_{i\in\mathcal{N}} \Lambda_{li} \cdot \theta_{its}^{\omega} / X_l, \qquad \forall l\in\mathcal{L}, t\in\mathcal{T}, s\in\mathcal{S}, \omega\in\Omega \tag{19}$$

$$-\overline{F}_l \le f_{lts}^{\omega} \le \overline{F}_l, \qquad \forall l\in\mathcal{L}, t\in\mathcal{T}, s\in\mathcal{S}, \omega\in\Omega \tag{20}$$

$$\sum_{g\in\mathcal{G}} p_{gts} + \sum_{g\in\mathcal{G}_{\text{batt}}} (p_{gts}^{dchg} - p_{gts}^{chg}) = D_t, \qquad \forall t\in\mathcal{T}, s\in\mathcal{S} \tag{21}$$

Reserve Constraints

$$\sum_{g\in\mathcal{G}} r^U_{gts} \geq r^{up}_t, \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{22}$$

$$\sum_{g\in\mathcal{G}} r^D_{gts} \geq r^{dn}_t, \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{23}$$

$$p_{gts} + r^U_{gts} + p^\omega_{gts} \leq \overline{P}_{gts}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}, \omega \in \Omega \tag{24}$$

$$p_{gts} - r^D_{gts} - p^\omega_{gts} \geq \underline{P}_{gt} \cdot u_{gts}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}, \omega \in \Omega \tag{25}$$

Thermal Generator Constraints

$$p_{gts} + p^\omega_{gts} - (p_{g,t-1,s} - p^\omega_{g,t-1,s}) \leq R^U_g(u_{gts} - y_{gts}) + SU_g \cdot y_{gts}, \qquad \forall g \in \mathcal{G}_{\text{foss}}, t \in \mathcal{T}, s \in \mathcal{S}, \omega \in \Omega \tag{26}$$

$$p_{gts} + p^\omega_{gts} - (p_{g,t+1,s} - p^\omega_{g,t+1,s}) \leq R^D_g(u_{gts} - z_{g,t+1,s}) + SD_g \cdot z_{g,t+1,s}, \qquad \forall g \in \mathcal{G}_{\text{foss}}, t \in \mathcal{T}, s \in \mathcal{S}, \omega \in \Omega \tag{27}$$

$$y_{gts} \leq u_{g\tau s}, \qquad \forall g \in \mathcal{G}_{\text{foss}}, t \in \mathcal{T}, s \in \mathcal{S}, \tau \in [t+1, \min\{t + UT_g - 1, |\mathcal{T}|\}] \tag{28}$$

$$z_{gts} \leq 1 - u_{g\tau s}, \qquad \forall g \in \mathcal{G}_{\text{foss}}, t \in \mathcal{T}, s \in \mathcal{S}, \tau \in [t+1, \min\{|\mathcal{T}|, t + DT_g - 1\}] \tag{29}$$

$$y_{gts} - z_{gts} = u_{gts} - u_{g,t-1,s}, \qquad \forall g \in \mathcal{G}_{\text{foss}}, t \in \mathcal{T}, s \in \mathcal{S} \tag{30}$$

$$y_{gts} + z_{gts} \leq 1, \qquad \forall g \in \mathcal{G}_{\text{foss}}, t \in \mathcal{T}, s \in \mathcal{S} \tag{31}$$

$$\underline{P}_g u_{gts} \leq p_{gts} \leq \overline{P}_g u_{gts}, \qquad \forall g \in \mathcal{G}_{\text{foss}}, t \in \mathcal{T}, s \in \mathcal{S} \tag{32}$$

$$p_{gts} - p_{g,t-1,s} \leq R^U_g, \qquad \forall g \in \mathcal{G}_{\text{foss}}, t \in \mathcal{T}, s \in \mathcal{S} \tag{33}$$

$$p_{g,t-1,s} - p_{gts} \leq R^D_g, \qquad \forall g \in \mathcal{G}_{\text{foss}}, t \in \mathcal{T}, s \in \mathcal{S} \tag{34}$$

Binary Variables

$$u_{gts}, y_{gts}, z_{gts} \in 0, 1, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S} \tag{35}$$

$$u^{chg}_{gts}, u^{dchg}_{gts} \in 0, 1, \qquad \forall g \in \mathcal{G}_{\text{batt}}, t \in \mathcal{T}, s \in \mathcal{S} \tag{36}$$

Explanation of Constraints

Renewable Generation Constraints

- **Limits solar and wind power output based on available renewable resources:** Ensures that the generation from solar and wind units does not exceed the maximum potential generation given the realized resource availability (e.g., sunlight for solar, wind speed for wind).
- **Ensures renewable generation doesn't exceed forecasted availability:** Ensures that the generation is bounded by the forecasted or observed availability of the renewable resource under different scenarios.
- **Allows for adjustments in real-time operation:** The model allows for adjusting the generation from renewable sources in real-time to reflect changes in availability due to uncertainty.

Hydro Constraints

- **Water balance:** Tracks the inflow, discharge, and spillage of water in the reservoir, ensuring conservation of mass in the water balance equation.
- **Power output:** Relates the amount of water discharged to the power generated, accounting for the generator's efficiency and the net head, which depends on the reservoir level.
- **Reservoir limits:** Ensures the reservoir level stays within specified minimum and maximum limits to avoid overfilling or draining the reservoir.
- **Discharge limits:** Restricts the discharge of water to stay within operational limits, ensuring that the hydro generator operates safely and efficiently.
- **Spillage limits:** Sets an upper bound on water spillage to minimize wastage and optimize the use of water resources.
- **Head calculation:** Computes the effective water head, which is a critical factor in determining the hydroelectric power output, based on the current reservoir level and discharge rate.
- **Initial/final conditions:** Sets the initial and final reservoir levels to predefined values, ensuring that the system starts and ends within a specified operational state.

Battery Constraints

- **State of charge limits:** Ensures the battery's state of charge remains within safe operating limits, preventing overcharging or deep discharging that could damage the battery.
- **Charging power limit:** Caps the charging rate of the battery to prevent overloading the charging mechanism and to respect the battery's maximum charging capacity.
- **Discharging power limit:** Limits the discharging rate of the battery, ensuring it does not exceed the battery's maximum discharging capacity.
- **State of charge update:** Updates the state of charge over time, considering the amount of energy charged or discharged and the efficiency of these processes.
- **Charging/discharging mutual exclusivity:** Prevents the battery from charging and discharging simultaneously, which would be physically impossible and could cause operational issues.

Power Balance and DCPF Constraints

- **Nodal power balance:** Ensures that the sum of generation, load, and power flows at each node satisfies the power balance equation, ensuring supply meets demand at each node.
- **DC power flow:** Uses a linear approximation (DC power flow) to model power flows on transmission lines, which simplifies the complexity of power flow calculations while maintaining accuracy for optimization.
- **Transmission capacity:** Limits the power flow on each transmission line to prevent overloading and potential line failures, ensuring the reliability of the power system.
- **System-wide power balance:** Ensures that the total generation across the system meets the total demand, maintaining the overall balance between supply and demand.

Reserve Constraints

- **Upward reserve requirement:** Ensures that there is enough reserve capacity available to increase generation if needed, providing flexibility to respond to unexpected demand increases or generator outages.
- **Downward reserve requirement:** Ensures there is capacity to decrease generation if necessary, allowing the system to respond to reductions in demand or sudden increases in renewable generation.
- **Generator reserve limits:** Restricts the amount of reserve that each generator can provide, based on its operational capabilities, ensuring reserves are within feasible limits.

Thermal Generator Constraints

- **Ramp-up limit:** Limits the rate at which a thermal generator can increase its output, reflecting physical constraints and avoiding operational stress on the equipment.
- **Ramp-down limit:** Limits the rate at which a thermal generator can decrease its output, ensuring smooth transitions and preventing sudden drops that could destabilize the system.
- **Minimum up-time:** Requires generators to remain on for a minimum period after starting, reflecting operational constraints and preventing frequent cycling, which could increase wear and tear.
- **Minimum down-time:** Requires generators to remain off for a minimum period after shutting down, preventing frequent start-stop cycles that could damage the equipment.
- **Start-up/shut-down logic:** Tracks the status of generators to determine whether they are starting up, shutting down, or remaining in their current state, ensuring the correct application of start-up and shut-down costs and ramp constraints.
- **Generation limits:** Ensures that the power output of each generator remains within its minimum and maximum operating limits, maintaining operational safety and efficiency.