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Interrelated Factor Demand Functions

By M. Ishag Nadiri and Sherwin Rosen*

The purpose of this paper is to integrate empirical investment and employment functions and to link these with capacity considerations, i.e., hours of work per man and utilization of capital equipment. Existing time-series employment models[1] assume fixed capital stock, yet estimated labor stock adjustments are so long as to place this assumption in serious doubt. On the other hand, most investment models treat labor as a completely variable factor,[2] though the employment models would appear to indicate otherwise. Furthermore, few of these models make adequate allowance for variations in utilization rates of labor and capital, and this sometimes makes parameter estimates difficult to interpret.[3] Theoretical considerations suggests mutually interdependent time-series demand functions for labor and capital. We specify and estimate a dynamic model for all input demand functions, which allows interactions among these variables over time. The model presents a unified framework for interpreting and estimating input stock demand fuctions, the role of utilization rates in these functions and variations in utilization rates themselves.

The paper is outlined as follows: Section I discusses the problem. The model is specified in Section II, and estimates are presented in Section III. These results are compared with some alternative models in Section IV and conclusions are contained in Section V.

I. General Considerations

A. Setting of the Problem

Many writers have recently suggested integrating dynamic adjustment costs into the neoclassical theory of the firm [8], [11], [19], [29]. However, their approach reveals considerable differences. Robert Eisner and Robert H. Strotz [8] assume quadratic profit and adjustment cost functions and derive a Koyck flexible accelerator distributed lag as an approximation to the optimal accumulation path of capital stock. Robert E. Lucas [19] and Arthur Treadway [29] generalize this model by using less restrictive functions and derive distributed lag models as linear approximations to optimum paths near long run equilibrium. The only empirical analysis known to us along these lines is that of R. Schramm [27]. He assumes quadratic profit and cost functions and specifies rates of accumulation of some inputs to be linear functions of deflated input prices and all initial values. All these models employ the restrictive assumption of nonprobabilistic and stationary (or static) price expectations. J. P. Gould [11] has shown that when price expectations are nonstationary, optimal input paths depend on the entire *future* course of prices. In this case, characterization of optimum paths by flexible accelerator approximations is more difficult.

[1] Two of the best known references are [4], [17]. Also, see various papers cited there and in [21].

[2] See [13] and a number of subsequent papers.

[3] Some employment models use man-hours rather than employment stock, but do not determine the division between them. An exception is Kuh [18], but his estimates do not have a clear interpretation. Nadiri [20] explicitly takes into account capital utilization in his investment function, but it is exogenous to the model.

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In this paper, we attempt to specify and estimate an economically meaningful set of input demand functions. We recognize that the dynamic input and output paths are jointly determined, contingent on future product price expectations. But their joint estimation requires a full market theory not yet available. Therefore, we set the limited goal of estimating optimum input paths consistent with an optimum and given output path. This allows us to concentrate on interactions among changes in inputs and on factor substitution.

B. An Example

To illustrate the nature of dynamic interactions among time paths of inputs, consider a simple two factor example. Suppose the production function is $x = f(y_1, y_2)$, where x is output, y_1 and y_2 are inputs and f has the usual continuity properties. Two isoquants are illustrated in Figure 1. The dotted line, AB, is the locus of efficient expansion points along which total costs are minimized and is derived in the conventional way. Though this may be an adequate description of long run behavior, there is plenty of evidence to suggest that firms do not remain along AB at every

Figure 1

moment of time and several explanations for this divergence have been offered. Most important, in addition to direct rental charges of factors, there are costs involved in changing their level. That is, there are substantial transactions costs, and these must be viewed as additional investment costs if they are to be undertaken. There are search, hiring, training, and layoff costs and associated morale problems among workers [24], [25]. Similarly, there are searching, waiting, and installation costs in purchasing new capital goods and there are poorly organized secondary markets in these goods. If initial input values deviate from their long run equilibrium levels, existence of these costs implies that optimum adjustment paths to equilibrium are not instantaneous. Since exogenous variables are generally subject to change and uncertainty, these costs often make it profitable for firms to engage in hoarding of input stocks [17], [28].

The conventional way of incorporating adjustment costs is the well-known partial adjustment model:

$$(1) \quad y_{1t} - y_{1t-1} = \beta(y_{1t}^* - y_{1t-1}), \quad 0 < \beta < 1$$

where y_1^* is the desired level of y_1 as defined by AB and β is the adjustment coefficient. Suppose the firm wants to increase output to x_2, given initial condition A in Figure 1. Equation (1) implies an immediate move from A to (say) C, with convergence along the new isoquant to the new equilibrium point B. The diagram indicates a corresponding and implied adjustment path for y_2.[4] In general, two independent adjustments imply additional hypotheses concerning the role of the production function during the adjustment
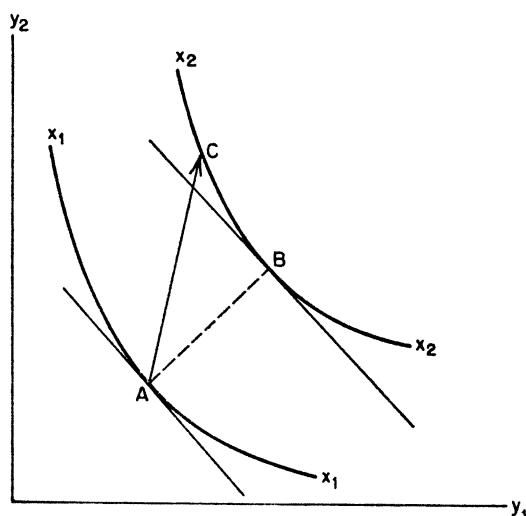
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[4] Assume f is Cobb-Douglas, $x = (y_1)^a (y_2)^b$. The demand for y_2 may be derived from a logarithmic form of equation (1) and from $y_2 = (x)^{1/b}(y_1)^{-a/b}$. It is given by

$$y_{2t} = (x_t)^{1/b} [(y_{1t}^*)^\beta (y_{1t-1})^{1-\beta}]^{-a/b}$$

period. There are two possibilities: (i) If the production function constraint always holds as an equality, independent adjustments imply an output decision function, which may not be optimum. (ii) If output is taken as exogenous two independent specifications mean that firms must be *off* their production functions, i.e., they must be capable of producing more than they actually do during the adjustment period.[5]

We adopt an intermediate position: Output is assumed exogenous, but input adjustments are specified to be interrelated. For example, a generalization of equation (1) is

$$
(2) \quad
\begin{bmatrix} y_{1t} - y_{1t-1} \\ y_{2t} - y_{2t-1} \end{bmatrix}
= \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}
\begin{bmatrix} y_{1t}^* - y_{1t-1} \\ y_{2t}^* - y_{2t-1} \end{bmatrix}
$$

This incorporates exogenous output; and allows firms to remain on their production functions during the adjustment process, since input adjustments are not independent.[6] Refer again to Figure 1. If the true adjusted path is described by ACB, β_{21} and β_{22} must be sufficiently positive to initially push y_2 above its ultimate value. This overshooting sets up forces that ultimately decrease y_2 to its equilibrium value at B. The net values of β_{11} and β_{12} must be positive for y_1 to increase monotonically to its equilibrium at B. Obviously, there must be restrictions on the β_{ij} to insure that the firm remains along isoquant $x_2 x_2$.

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[5] Nerlove [23] adopts the second approach. In his model, firms react not to observed values of output and relative prices, but to forecasts of unobserved (trend-cycle) components. Desired and actual output are identical, but firms may be off their production functions.

[6] It is interesting to note that a similar hypothesis has been proposed by Brainard and Tobin [3] in the related context of portfolio adjustment among assets. These authors have assumed the wealth path to be exogenous and have addressed themselves to determining optimum adjustments of various assets consistent with that path.

II. *The Model*

A. *Input Demand Functions*

Our model can be viewed as a first approximation to the solution of an optimal control problem in which the firm maximizes its net worth when there are adjustment costs associated with changes in factors of production. First we investigate the stationary long run solution. At that point, the firm must be minimizing long run total cost. Therefore, long run input demand functions are generated. Secondly optimum input paths are found by minimizing discounted costs, including costs of adjustment, subject to this *optimum* output path and the production function. These are described by a set of simultaneous differential equations in $y_1, \ldots, y_4$, factor prices, and output. The model approximates these differential equations by a corresponding set of difference equations.

To elaborate this procedure, consider the following:

Step I: We assume a Cobb-Douglas production function for simplicity. This function is the first-order logarithmic approximation to any production function and at least long run scale parameters may be approximated reasonably well. The production function is:

$$
(3) \quad x_t \leq A \prod_{i=1}^{4} (y_{it})^{\alpha_i} = F(\ ).
$$

where x is output, y_1 is the stock of labor, y_2 is the rate of labor services per unit of stock (hours per man), y_3 is capital stock, y_4 is rate of capital services per unit of stock, and A is an exogenous technological shift parameter.[7] Notice that equation (3) includes both stock and flow dimensions to inputs in a fairly general way. We see no reason a priori to restrict it, as is usually done, to two dimensions—total flows of

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[7] The production function can be made stochastic by addition of a random term. In this case the discussion must be cast in terms of expected values.

labor and capital services or manhours and machine hours.[8] Therefore, direct objects of choice by the firm are both stocks and their intensity of use.

Given static price expectations, solutions for output and inputs, x^*, y_1^*, $\cdots$, y_4^* can be found. Given x^*, the y^*'s are defined by the following:

Minimize costs

$$C = y_1 y_2 w + y_1 s + p_k(r + \delta) y_3$$
$$+ \lambda[x^* - F(\ )]$$

where λ is an undetermined multiplier and the firm is on its production function. The variable y_4 does not explicitly appear in this function, but implicitly through δ, the rate of depreciation of capital stock. Depreciation depends on the rate of utilization (y_4) as well as time, i.e., $\delta = \delta\ (y_4, t)$. The variable p_k denotes the price of capital goods and r the cost of capital. Therefore, $p_k\ (r+\delta) = c$ is the user cost of capital. The variable w is the hourly wage rate ($y_1 y_2$ is total man-hours) and may depend on y_2 through overtime wage premiums. To account for this possibility, define $\epsilon = (y_2/w)\ (dw/dy_2) \geq 0$. Finally, s is the user cost of labor. It is analogous to c and includes hiring, training and related costs discounted at the relevant rate of interest and adjusted for quits, retirements, etc. See S. Rosen [26].

The solutions (equations 4.1–4.4) are optimum long run demand functions (neglecting constants) where R is a vector of relative factor prices, $\rho = \alpha_1 + \alpha_3$ and δ' is the derivative of δ with respect to y_4. There are several interesting properties of these solutions.

(i) As noted previously, all long run scale phenomena are embedded in stock demand functions and not in their service flows per unit of stock: Output enters demand functions for y_1 and y_3, but not for y_2 and y_4. For example, if output doubles and there are constant returns to scale, employment and capital stock will double, but hours per man and the capital utilization will remain unchanged. There are increasing, constant, or decreasing returns to scale in the conventional sense when $\rho = (\alpha_1 + \alpha_3)$ is greater, equal or less than unity.

(ii) Factor prices affect long run input demands in various ways, which differ from the familiar solutions. An increase in the wage rate unambiguously decreases y_2 (hours per man) and may or may not decrease y_1, the stock of labor. Increased w induces substitution of employment for hours per man—a positive effect, but also raises total cost of labor services relative to capital stock, leading to substitution of capital for both labor inputs—a negative effect.[9] Changes in w and s affect the stock of capital but not its utilization. Similarly, an increase in c induces substitution of capital utilization for capital stock and increases employment without affecting hours per man.

(iii) Long run utilization rates are independent of cross price effects; that is, c does not enter y_2^* and w and s do not effect y_4^*.

Step II: The approach to long run equilibrium is approximated by the following set of difference equations,

[9] For the Cobb-Douglas production function

$$\frac{w}{y_1^*} \frac{\partial y_1^*}{\partial w} = \frac{\alpha_2}{\rho} > 0$$

(4.1) Employment $y_1^* = (x^*)^{1/\rho}(w/c)^{\alpha_2/\rho}(c/s)^{(\alpha_3+\alpha_2)/\rho}[(r + \delta)/\delta']^{-\alpha_4/\rho} = g_1(x^*, R)$

(4.2) Hours/man $y_2^* = (s/w)[(1 + \epsilon)(\alpha_1/\alpha_2) - 1]$ $= g_2(R)$

(4.3) Captial $y_3^* = (x^*)^{1/\rho}(w/c)^{\alpha_1/\rho}(s/w)^{(\alpha_1-\alpha_2)/\rho}[(r + \delta)/\delta']^{-\alpha_4/\rho} = g_3(x^*, R)$

(4.4) Services/capital $y_4^* = [(r + \delta)/\delta'][\alpha_4/\alpha_5]$ $= g_4(R)$

$$Y_{it} - Y_{it-1}$$

$$(5) \qquad = \sum_{j=1}^{4} \beta_{ij}[G_j(X_t, R_t) - Y_{jt-1}] + V_{it}$$

$$(i = 1, \cdots, 4)$$

which differ from (2) because capital letters denote logs, where $G_i = log\ g_i$, and $V_1, \ldots, V_4$ are random terms with zero means and variance-covariance matrix Ω.[10] Equation (5) traces moving equilibrium paths of inputs to output and relative factor price paths.[11]

Viewing this solution as a control problem clearly requires that firms are on their production functions at every moment of time. Therefore, certain relationships among the β_{ij}'s are implied by the model, as illustrated by Figure 1. These restrictions are obtained by substituting the (mathematical) expected input demand functions (5) into the production function and equating coefficients. In matrix notation the model to be estimated is

$$Y_t = AX_t + BR_t + (I - \beta)Y_{t-1}$$

where A is a 4×1 vector of regression coefficients with respect to output and B is a 4×3 matrix of regression coefficients on the three factor price ratios in R (i.e., W/S, C/S and $(r+\delta)/\delta'$). The production function is $X_t = \alpha' Y_t$, where α is a vector of Cobb-Douglas exponents. Substituting the regression equations into this expression yields

$$X_t = \alpha' A X_t + \alpha' BR_t + \alpha'(I - \beta)Y_{t-1}$$

[10] This formulation is a proportional adjustment rather than an adjustment of first differences as in (2). It can be further generalized by thinking of the β_{ij} as functions of the lag operator Z, β_{ij} (Z), so that less restrictive forms are implied. In fact, estimates indicate that more general forms are required for some of these equations. A further generalization could be achieved by making the lag adjustments β_{ij} vary with relative prices or rate of change of output, though we do not pursue it here.

[11] Note that the model describes more general situations when price expectations change. For each price expectation there are optimum paths of output and input. When price expectations change, observed paths trace loci among these equilibrium paths over time.

Evidently this is satisfied when

$$\alpha' A = 1 \qquad \text{(one equation)}$$
$$\alpha' B = 0 \qquad \text{(three equations)}$$
$$\alpha'(I - \beta) = 0 \qquad \text{(four equations)}$$

yielding a system of 8 equations in 4 unknowns (the α_j's). If $\alpha'(I-\beta) \equiv 0$ the α_j's are exactly identified.[12] These are the restrictions we seek. Since each α_j is nonzero, this implies

$$|I - \beta| = 0$$

In principle, this provides a test of this restriction.[13] Evidently, if these restrictions do not hold, the production function is over identified, for there is more than one possible estimate of each α_j. Further tests are derivable by virtue of the fact that dynamic stability of this model requires all characteristic roots of $|I-\beta|$ to have modules less than unity.[14] Notice that there is nothing in the analysis to preclude negative adjustment coefficients. Certainly, we expect positive own adjustment coefficients; but individual cross effects could be negative or positive so long as their weighted sum across equations is approximately zero.

[12] The restriction is equivalent to

$$\sum_i \alpha_i \beta_{ij} = \alpha_j \qquad i = 1, \cdots, 4$$

[13] We emphasize "in principle" because the sampling distribution of $|I-\hat{\beta}'|$ is not available. Moreover, in our data there are some measurement and other difficulties, and we cannot expect this condition to hold precisely.

[14] This requirement can be shown as follows: Consider constant values of X and R and let $K = AX + BR$. Then

$$Y_t = K + (I - \beta)Y_{t-1}$$

given Y_0, successive iteration yields

$$Y_t = [I + (I - \beta) + (I - \beta)^2 + \cdots + (I - \beta)^{t-1}]K + (I - \beta)^t Y_0$$

which converges if $(I-\beta)^n$ approaches 0 as n gets large. $(I-\beta) = M'\lambda M$, where M is a matrix of characteristic vectors of $(I-\beta)$ and λ is a diagonal matrix of characteristic roots. Then $(I-\beta)^n = M'\lambda^n M$, which approaches zero if each element of λ is less than unity.

B. Interpretation of Adjustment Coefficients

Our a priori view is that variations in utilization rates largely serve residual functions of maintaining output and reducing its variability in the face of changing demand and slowly adjusting input stocks. When desired stocks are higher than actual stocks, utilization rates rise to temporarily fill in the gaps. On this argument it would be expected that at least β_{21} and β_{43} are positive. y_2 and y_4 would be completely variable factors in the usual sense if there were no costs associated with changing their levels, but only with the levels themselves. If this were true, it may suggest instantaneous own adjustment for these variables (i.e., $\beta_{22} = \beta_{44} = 1.0$). Certainly there is some intuitive sense to this view, but we see no reason for restricting ourselves to it. There are likely to be nontrivial set-up and other costs from temporary changes in utilization rates, so we chose to treat all inputs symmetrically. If there is any truth to the above argument, however, we expect utilization rates to have own adjustment coefficients greater than those of stock inputs, and also expect their stock cross-adjustments to be relatively high. Moreover, we anticipate much slower own and cross-adjustment of capital stock than of labor, because transactions costs are surely higher in the capital goods market. This also tends to be borne out by existing studies [21].

C. Comparison with Other Models

Existing investment and employment demand functions, though apparently very different from this model, are essentially special cases of it. This can be shown by examining the reduced form of equation (5), in which each input is expressed as a function of current and lagged output, relative prices and its own lagged values. To simplify the algebra, let Z be the lag operator, i.e.,

$$ZY_t = Y_{t-1}, \qquad Z^2 Y_t = Y_{t-2}, \cdots.$$

In this notation, equation (5) may be written as (6) below.

In matrix form (6) is

$$[I - (I - \beta)Z]Y_t = \beta G(X_t, R_t) + V_t$$

and the reduced form is:

(7) $\quad Y_t = [I - (I - \beta)Z]^{-1} [\beta G(X_t, R_t) + V_t]$

Each element of the matrix $[I-(I-\beta)Z]^{-1}$ is a rational function, i.e, a ratio of two polynomial functions of the operator Z. The denominator of each of these functions is $|I-(I-\beta)\bar{Z}|$, which is a polynomial function of Z, $\theta(Z)$.

(8) $\quad \theta[Z] = b_0(1 - b_1 Z - b_2 Z^2 + \cdots)$

where coefficients $b_0, b_1, \ldots$ are functions of β_{ij}. Similarly, the numerator is another polynomial in Z, $\theta_{ij}(Z)$, with[15]

[15] The order of polynomial $\theta(Z)$ is 3 if the restrictions apply. Otherwise, it is 4. The order of $\theta_{ij}(Z)$ is 3 in

(6)

$$\begin{bmatrix} 1 - (1 - \beta_{11})Z & \beta_{12}Z & \beta_{13}Z & \beta_{14}Z \\ \beta_{21}Z & 1 - (1 - \beta_{22})Z & \beta_{23}Z & \beta_{24}Z \\ \beta_{31}Z & \beta_{32}Z & 1 - (1 - \beta_{33})Z & \beta_{34}Z \\ \beta_{41}Z & \beta_{42}Z & \beta_{43}Z & 1 - (1 - \beta_{44})Z \end{bmatrix} \begin{bmatrix} Y_{1t} \\ Y_{2t} \\ Y_{3t} \\ Y_{4t} \end{bmatrix}$$

$$= \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} \end{bmatrix} \begin{bmatrix} G_1(X_t, R_t) \\ G_2(X_t, R_t) \\ G_3(X, R_t) \\ G_4(X_t, R_t) \end{bmatrix} + \begin{bmatrix} V_{1t} \\ V_{2t} \\ V_{3t} \\ V_{4t} \end{bmatrix}$$

(9) $\quad \theta_{ij}(Z) = a_{0j} + a_{1j}Z + a_{2j}Z^2 + \cdots$

where $a_{0j}, a_{1j} \ldots$, are also functions of β_{ij}.

Carrying out the multiplication we obtain

(10) $\quad Y_{it} = \dfrac{\sum_j \theta_{ij}(Z)\beta_{ij}G_j + \sum_j \theta_{ij}(Z)V_{jt}}{\theta(Z)}$

Multiplying both sides by $\theta(Z)$ and operating with Z, an equivalent expression is

$$
\begin{aligned}
Y_{it} = \; & H(X_t, X_{t-1}, \cdots, R_t, R_{t-1} \cdots) \\
& + b_1 Y_{it-1} + b_2 Y_{it-2} + \cdots \\
& + \sum \theta_{ij}(Z)V_{jt}, \quad i = 1, \cdots, 4
\end{aligned}
$$
(10')

where H is a linear function of all its arguments.[16] Equation (10') could be further reduced to an infinite distributed lag function of current and past values of X and R.

As an example, the systematic part of Y_3 in system (10') is identical in form to neoclassical investment functions [2], [14]. Short run employment functions [5], [17], can be considered as special cases of Y_1 in (10'). Thus, the model integrates these two branches of the literature and we are able to obtain alternative estimates of these functions. Note, however, that we have taken into account the *cross* and own

---

either case. In a two-factor case θ_{ij} is the ijth cofactor of the matrix

$$
\begin{bmatrix}
1 - (1 - \beta_{11})Z & \beta_{12}Z \\
\beta_{21}Z & 1 - (1 - \beta_{22})Z
\end{bmatrix}
$$

and $\theta(Z)$ is its determinant, $1 - (2 - \beta_{11} - \beta_{22})Z + [(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}]Z^2$. If restrictions hold, the bracketed term is zero.

[16] H is proportional to

$(a_{01} + a_{11}Z + \cdots)\beta_{i1}G_1(X_t, R_t)$
$\quad + (a_{02} + a_{21}Z + \cdots)\beta_{i2}G_2(X_t, R_t) + \cdots$,

by (8), (9) and (10)

or to

$[a_{01}\beta_{i1}G_1(X_t, R_t) + a_{02}\beta_{i2}G_2(X_t, R_t) + \cdots]$
$\quad + a_{11}\beta_{i1}G_1(X_{t-1}, R_{t-1}) + a_{21}\beta_{i2}G_2(X_{t-1}, R_{t-1}) + \cdots]$
$\quad + \cdots$ after operating with Z.

---

adjustments in each equation of (10') which suggests a substantially different interpretation of the adjustment process than is provided in the literature. Many distributed lag investment and employment functions display distributions with complex roots which are considered unacceptable on economic grounds [10]. It is indeed difficult to accept the economic interpretation of such results in a single equation model. However, (10') shows that the adjustment hypotheses embedded in equation (5) have smooth and sensible economic interpretations, yet there is nothing whatsoever to preclude the possibility that $(I - \beta)$ has complex roots. Such models will generate a complex distributed lag weighting pattern for each input separately.[17]

III. *Estimation of the Model*

The model specified in Section II has been estimated using seasonally adjusted quarterly data for total manufacturing for the period 1947-I to 1962-IV. Construction of variables and data sources are discussed in the Appendix. The empirical specification slightly differs from (5). The variable s, user cost of labor, has been omitted due to lack of data. Measurement of capital stock and its depreciation rate do not take into account changes in capital utilization rates, for the same reason.

The model is estimated by ordinary least squares. We purposely have not imposed the a priori restrictions on the adjustment coefficients due to difficulties in measuring the variables accurately.[18] The estimates are presented in Table 1 and are generally consistent with the a priori specification of the model. Statistical

[17] The main difference between (10') and the existing employment and investment models lies in the disturbance term. In (10') it is a complicated weighted average of contemporaneous and past original disturbances. If the original disturbances are serially independent, those in (10') are not and the estimation techniques must be different.

[18] See Data Appendix.

properties of the estimates, as reflected by $\bar{R}^2$ and Durbin-Watson statistics, are satisfactory. Estimated feedback and own adjustment coefficients are strong and statistically significant. Since X_t is not truly exogenous, we attempted using various instrumental variables for it in estimating the model. These included "predicted" values for X_t from regressions of X on new orders, trend and past values of X. The estimates were similar to those reported in Table 1.

1. Structural Estimates

The structural form of the model is given by estimated parameters of equations (5). These estimates show the immediate response of $y_1 \ldots, y_4$ to changes in output and relative prices, their own adjustment lags and cross adjustment effects.

Current output, X_t, has its strongest effect on the rate of capital utilization, y_4. It has less powerful effects on labor stock (y_1) and hours per man (y_2) and has the least influence on capital stock (y_3). This ranking almost corresponds to the ranking of own adjustment coefficients and is consistent with the hypothesis that capital utilization is most easily adjusted, followed by hours per man and employment and, lastly, by capital stock.

Initial period responses of inputs to relative prices are much smaller in comparison to current output responses. The impact of relative prices ranks much the same as output, and this is also consistent with the relative adjustment hypothesis. The trend variable enters utilization equations as well as stock equations, because desired stocks enter utilization demand functions and technical change appears in the latter. Trend is statistically significant in all equations, with negative signs in all except for capital stock (y_3). A second order lag term has been included in capital stock equation to eliminate serial correlation of the residuals. However, the es-

TABLE 1—ESTIMATED STRUCTURE AND IMPLIED LONG RUN COEFFICIENTS[a]

Independent Variables	Dependent Variables			
	y_{1t}	y_{2t}	y_{3t}	y_{4t}
X_t	.3804	.2093	.0038	.8014
	(.0401)	(.0196)	(.0057)	(.0590)
$(w/c)_t$	.0251	.0273	.0037	.0433
	(.0152)	(.0074)	(.0024)	(.0233)
t	−.0025	−.0007	.0002	−.0027
	(.0005)	(.0002)	(.0001)	(.0007)
y_{1t-1}	.6504	−.0991	.0349	−.0700
	(.0744)	(.0364)	(.0180)	(.1096)
y_{2t-1}	.7845	.3842	.0000[b]	.3725
	(.2591)	(.1268)	—	(.3817)
y_{3t-1}	−.1636	−.1873	1.6578	−.7646
	(.0799)	(.0391)	(.0759)	(.1177)
y_{3t-2}			−.6955	
			(.0692)	
y_{4t-1}	−.2759	−.0584	−.0022	.0098
	(.0849)	(.0415)	(.0142)	(.1251)
$\bar{R}^2$	.9686	.9119	.9999	.9696
D.W.	1.406	2.1510	2.1768	1.1108
	LONG RUN ELASTICITIES			
X	.6116	.0177	.6513	.2680
w/c	.0568	−.0025	.1471	−.0752
t	−.0055	−.0001	.0004	−.0027

[a] Standard errors in parentheses. $\bar{R}^2$ is coefficient of determination adjusted for degrees of freedom. D.W. is Durbin-Watson statistic.
[b] Denotes coefficient constrained to be zero.

timates are not greatly affected by this modification.

Own lag coefficients indicate extremely fast adjustment for capital utilization, approximately instantaneous $(1.0 - .0098)$. The other rate of utilization, hours per man, also displays rapid adjustment but not quite as high as capital utilization, its coefficient being .6158, i.e. $(1 - .3842)$. Own stock adjustments are considerably longer, as expected. For labor this is .3496 and for capital it is about .05.

There are significant cross-adjustment effects in each demand equation, though of varying magnitude. These are calculated

as $-\hat{\beta}_{ij}$, $i \neq j$. For example, β_{3j}, $j = 1, 2, 4$ measures the effect of disequilibrium in the market for capital stocks on the demand for employment, hours per man and capital utilization. The effect of disequilibrium employment in each demand function is shown by regression coefficients in row y_{1t-1}. It has positive effects on both utilization rates, as expected, though its influence on capital stock is negative. That is, excess demand for labor increases utilization rates, but reduces investment. This may indicate some complementarity among stocks: if previous labor stock is high, there is a tendency for capital stock to increase. The cross-effect of hours disequilibrium is negative in each equation, with its highest effect on labor and a nonsignificant effect on utilization of capital. The former partly reflects the fact that hours per man lead other inputs by about one quarter during the sample period. This in itself implies that unanticipated increases in output are first met by hours, which signals increases in labor stocks. The absence of cross effects among utilization rates reflects the fact that both respond jointly to changes in stock and output variations. Lagged hours were always small and insignificant in the capital stock equation and therefore were deleted from it. This reflects slow initial adjustment of capital stock.

Excess demand for capital stock has a positive and significant effect on all other inputs, with the largest effects on capital utilization and approximately equal effects on both labor and hours per man. This is due to a slow initial adjustment of capital and faster adjustments of other inputs. Disequilibrium in capital utilization rates, y_4, has positive effects on other inputs, with a large impact on labor stock, but insignificant effects on hours and capital stock. Apparently, one way of increasing capital utilization is to increase the number of employees.

The restrictions on β_{ij} discussed earlier

do not hold, which means the production function is over-identified. The explanation for this is: (a) a second-order lag has been used in the y_3 equation, but not in the others; (b) the identity of firms changes greatly over time, which cannot be captured at the aggregate level of total manufacturing; (c) certain prices are unavailable and could not be included; (d) nonproduction workers do not appear in y_1; and (e) there are measurement difficulties with y_3 and y_4 (see Appendix).

2. Distributed Lag Relations

To fully understand the meaning of these structural estimates, it is necessary to explore implied distributed lag response of each input. According to equations (7) or (10'), standard distributed lag responses are obtained by computing $[I - (I - \hat{\beta})Z]^{-1} \beta[G(X_t, R_t)]$. The lag patterns or transient responses of each input to output changes for the estimates are shown in Figure 2. These are expressed as fractions of total long run responses. Figure 3 presents cumulated input response patterns to changes in output. These are obtained by adding up successive values on the corresponding diagrams in Figure 2.

The response patterns are remarkably consistent with our a priori expectations. Consider the conceptual experiment of a once and for all unit change in output, as indicated in Figure 3. y_2 (hours per man) and y_4 (capital utilization) react immediately and strongly to this change. On the other hand y_1 (labor stock) reacts moderately, while y_3 (capital stock) hardly changes at all. Utilization rates, y_2 and y_4, immediately overshoot their ultimate values in the first or second period and monotonically decline to their equilibrium values as the stock adjustments proceed. The response of employment is somewhat similar: it slowly overshoots its long run value, then gradually declines to it. These differences in response support the view of employment as a quasi-fixed input. The
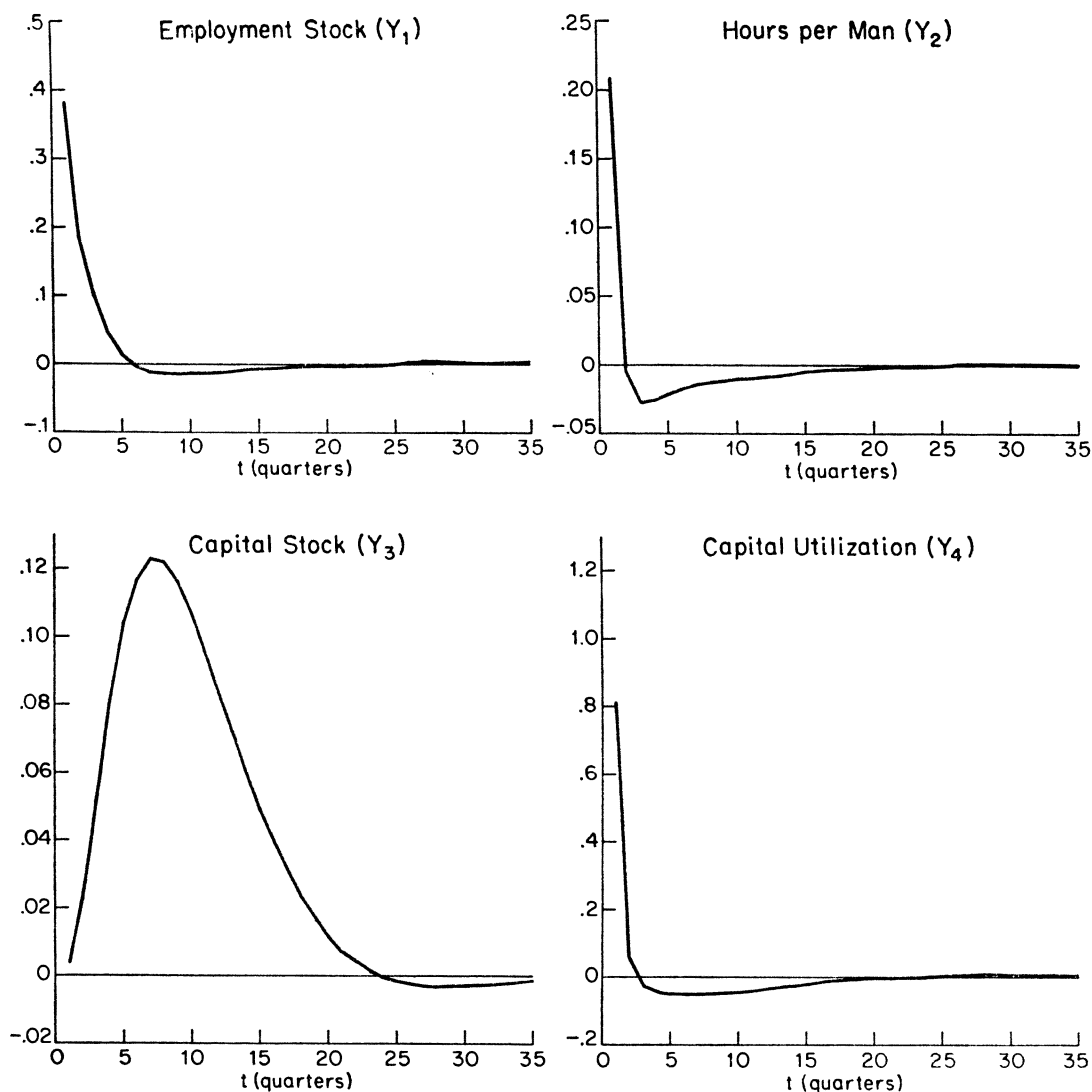
FIGURE 2

capital stock y_3 grows according to a logistic path with considerable sluggish response.

These comparisons show that: (a) physical capital is relatively "fixed" compared with the other variables; and (b) the primary roles of variations in utilization rates, and to a lesser extent employment variations, serve to maintain output levels while capital stock is slowly adjusting. Note that the distributed lag weights for capital stock in Figure 3 trace a humped

pattern similar to the results of Dale Jorgenson [13]. However, it is important to realize that our result has been achieved *without* imposing any a priori form on the lag structure. Also, note that about 20 percent of the total response takes place in the first 4 or 5 quarters, contrary to Jorgenson's results.[19]

[19] The characteristic roots of $[I-\hat{\beta}]$ are as follows:

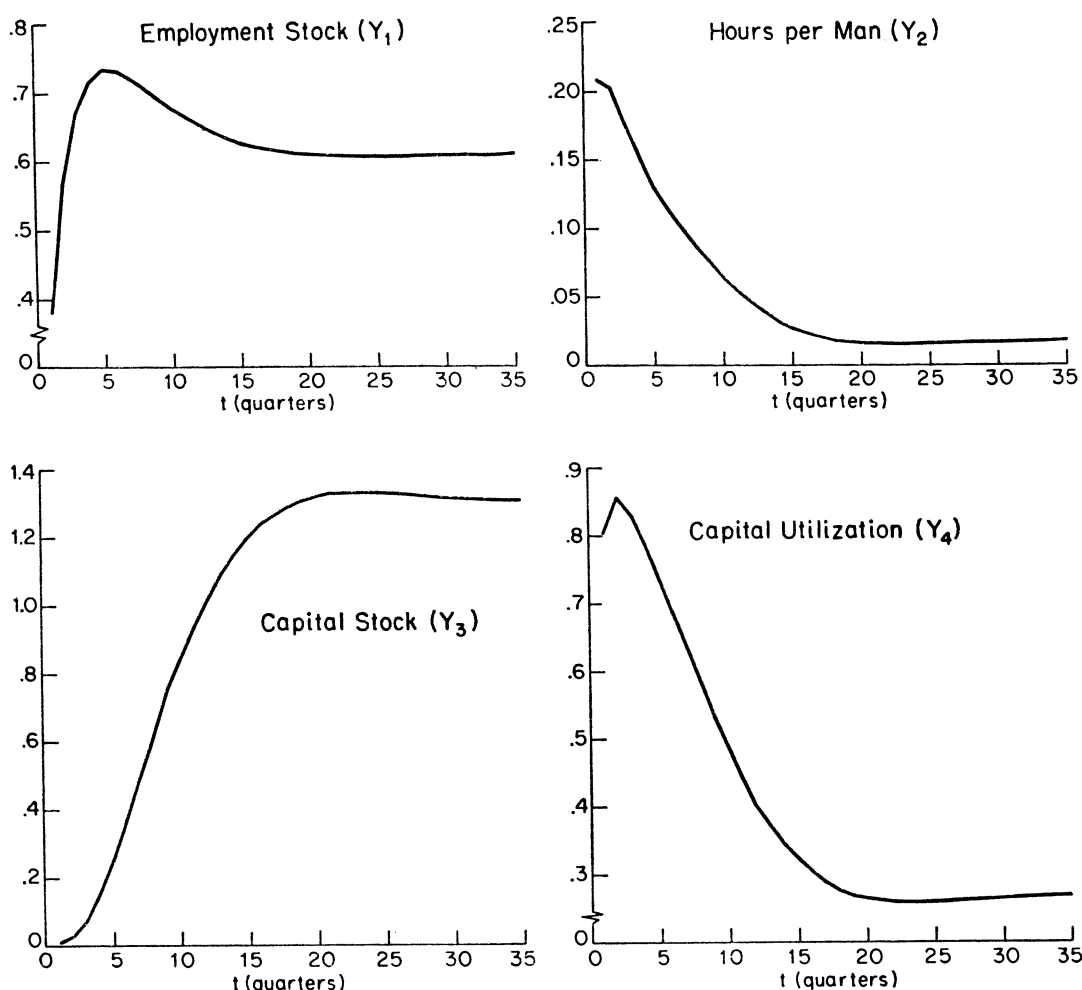$$.83249I \pm .12503i$$
$$.47252I \pm .15853i$$
$$.09218$$

FIGURE 3

3. Long Run Estimates

Implied long run elasticities are shown in the last three rows of Table 1. These are identical in meaning to the coefficients of equations (4) (in log form) and are computed from the stationary solutions of structural equations (5). Long run output elasticities of labor stock and capital stock demand functions estimate the inverse of returns to scale, $1/\rho = [1/(\alpha_1 + \alpha_3)]$. These are .6116 and .6513 respectively and not significantly different from each other. Note that there was nothing in our estimation procedure to guarantee correspondence of these two estimates. This result was repeated in all experiments of the model. The implied value of ρ is high, but a marked improvement over many other time-series estimates. As expected, the long run output elasticity of hours per man is approximately zero. But that of capital utilization is positive,

These roots are complex and less than one in absolute value. This explains the humped capital stock lag and overshooting of inputs in the short run. All of the response patterns in Figures 2 and 3 display damped oscillations after twenty quarters which are at most on the order of magnitude 10^{-3}.

though small. This may be a manifestation of the residual nature of the measurement of this variable (see Appendix).

There are essentially no long run relative factor price effects on utilization rates, their coefficients being negative, but very close to zero. On the other hand, there is a positive relative factor price effect on capital stock, as expected. Note that in the structural estimates for the capital stock equation, current output and current prices have approximately equal and very small initial effects. However, long run output effects on capital stock are four times as great as that of relative prices.

The positive price elasticity for labor is unexpected. However, this may be due to the following two factors: the production function is over-identified; and data for user cost of labor, s, and implicit prices of capital utilization are not available. Similarly, we note another difficulty with our estimates of trend. If (i) the Cobb-Douglas assumption truly held, (ii) the production was exactly identified, and (iii) the variables were measured correctly; long run coefficients of t in the stock equations, y_1 and y_3, would have been identical. Unfortunately, not all of these conditions hold and these results are inconclusive. Therefore we are unable to make reasonable inferences about the true rate of technical progress.

IV. Comparison With Other Models

In this section we compare implied reduced form results of this model with results obtained from the single equation estimates of employment and investment functions reported in the literature.

A. Distributed Lag Investment Functions

It was noted above that the distributed lag capital stock response computed from our model is similar to the results of Jorgenson and others. However, our long run estimates for capital are more consistent with the results of R. Eisner and

M. I. Nadiri [7]. They treat investment as a distributed lag function of current and past output and relative prices, and obtain estimates of 1.3 for the scale parameter and a price elasticity of about .15. These are essentially identical to ours, though they are obtained by different methods. Most other investment functions [14], [13], have restricted the scale parameter to be unity, a priori. Those studies [12] that report separate output and price elasticities obtain small values such as ours. Furthermore, our estimated scale parameter is consistent with time-series production function estimates of R. K. Diwan [6], L. R. Klein-A. S. Goldberger [15] and Klein et al. [16]. Our low price elasticities are consistent with low estimates of capital labor substitution in time-series production function studies.

B. Increasing or Decreasing Returns to Labor?

Studies of short run employment functions invariably estimate increasing returns to labor alone, sometimes of very large magnitudes. No acceptable explanation of such results has yet been provided. Our model offers fresh evidence on this important question. By performing certain conceptual experiments with the model, all possible short term input demand functions can be generated, depending on what factors are considered fixed.

In most time-series employment studies, capital is assumed fixed in the short run. Since these studies do not distinguish between capital stock and its rate of utilization, one interpretation of this assumption is that both dimensions of capital are assumed fixed. Generally, man-hours or employment are regressed on output and trend with a Koyck lag adjustment mechanism. Such functions can be arbitrarily approximated by conceptually fixing y_3 and y_4 in our model. We emphasise the conjectural nature of this exercise, since our model stresses the dynamic interrela-

tionship of all factors. In any event, for example, taking the estimates of the first and second equations and ignoring the third and fourth equations, stationary solutions for y_1 and y_2 may be derived as functions of X, W/C, t, y_3, and y_4. The estimated output elasticities under this procedure are 1.36 for y_1 (employment) and 0.12 for y_2 (hours per man). Estimated return to scale for employment is .735 ($=1/1.36$), which suggests substantial decreasing returns, in contrast to the usual estimates which range from 1.3 to 2.0. To approximate the returns to man hours, output coefficients of y_1 and y_2 may be added, and the short run returns to scale of total labor input is 0.68.

If capital services are allowed to vary by holding only y_3 (capital stock) fixed, we get entirely different results. These estimates are obtained by solving the first, second and fourth equations for equilibrium values of y_1, y_2, and y_4 as functions of X, W/C, t, and y_3. The output elasticities are 0.77 for y_1, 0.14 for y_2 and 0.81 for y_4. Estimated returns to scale are 1.30 for y_1 and 1.10 for total labor input.[20]

Comparison of these two experiments suggests the following important conclusion: The reason for large returns to labor estimated from short run employment functions is due to omission of the rate of utilization of capital. These high estimates should not be considered as returns to labor alone, as most writers have done, but are more properly interpreted as short run returns to both labor and capital utilization.

V. Conclusions

1. It has been shown that two divergent branches of economic literature, employment and investment functions, can be

[20] We have intentionally chosen these procedures as most consistent with the spirit of the employment function literature. However, there are several ways of obtaining these estimates, due to the fact that the restrictions on β_{ij} are not met, and that the production function is not exactly identified.

successfully integrated into a unified structure. The most important result of this study is the estimation of a consistent set of response patterns of all inputs. They clearly show that the order of response to output changes runs from utilization rates to employment and finally to capital stock.

2. By specifying a completely interrelated demand structure, the model provides an alternative rationale for existing distributed lag investment and employment functions.

3. Through various conceptual experiments we can explain apparently low employment-output elasticities found in short term employment functions. In these studies, large short run returns to scale to labor seem to be due to omission of input utilization rates, particularly that of capital.

4. The model provides an explanation of two neglected dimensions of time-series production functions; that is, the determination of time-series variations in hours per man and rate of utilization of capital stock. It also provides evidence on interactions in dynamic adjustment paths of input stocks and their rates of utilization.

Empirical studies of input demand functions will never be complete until they are linked directly with empirical production functions. The converse is also true. Our model represents one step in this direction, though much work remains to be done. There are two major requirements that will have to be met before the problem can be solved. One is the production of more conceptually adequate data, particularly on capital utilization rates. The other is a comprehensive theoretical treatment of the market and the role of expectations.

DATA APPENDIX

y_1 is number of production workers in total manufacturing; [32].

y_2 is hours worked per man (total man-hours ÷ employment), [32].

y_3 is capital stock at beginning of the pe-

riod. This series is taken from [14] and is calculated as $K_{t+1}=I_t+(1-\delta)K_t$ where K_{t+1} is the capital stock at the beginning of period $t+1$, I_t is deflated gross investment in period t, and δ is calculated by linear interpolation between the initial and terminal stock of capital at beginning of years 1948 and 1960. The deflated investment series is from OBE investment survey [31].

y_1 is the capacity utilization rate [30].

w is average hourly earnings calculated as total compensation divided by total man-hours (not adjusted for overtime) [32].

c is the user cost of capital, defined as

$$c = \frac{q}{1-\mu}\left[(1-\mu v)\delta + (1-\mu m)r\right].$$

The data for constructing c are chiefly from the *Survey of Current Business* [31] and *Statistics of Income* [33]. The income tax rate, μ is the ratio of corporate tax payments to gross corporate profits. The proportion of current replacement cost allowable for tax purposes, v, is the ratio of tax depreciation to replacement in constant prices. r is the cost of capital, computed by using the earnings price ratio and the rate of interest on government bonds. m, the proportion of the total cost of capital allowable for tax purposes, is the ratio of net monetary interest to total cost of capital.

x is seasonally adjusted quarterly income originating in manufacturing deflated by the seasonally adjusted GNP price deflator [31].

t time trend (exponential).

A number of important qualifications and comments are necessary:

(i) The most serious difficulty with data lies in measurement of y_4, utilization rate of capital stock. An ideal measure would approach the capital stock analogue of hours per man. For example, relative amounts of multiple shift working might be a good proxy for it, but such data do not exist. The indicies of "capacity" prepared by the Federal Reserve Board and others are really residual measures and reflect variations in other inputs. They are computed by essentially dividing peak-to-peak trend output by actual output. We use this measure because no better one exists.

(ii) The employment measure is not quite a true stock variable, since it is a count of all names appearing on payrolls during a week near the middle of the month. It also excludes nonproduction workers. Similarly, hours per man are averages for that week. Moreover, wage rates reflect overtime to a small extent, since overtime data were not systematically collected until 1956.

(iii) There are no direct measures of s, user cost of labor, available. We used the quit rate as a proxy, but the results were not satisfactory. In any case, the quit rate is a conceptually inadequate measure of s. Another possibility would be the ratio of skilled to unskilled employment but it is not available on a quarterly basis.

(iv) There are slight difficulties with respect to dating of inputs. y_1 (labor stock) refers to mid-quarter, whereas y_3 (capital stock) refers to beginning of quarter. Experimentation with end of period capital and an average of end and beginning quarters yielded no essential differences from results presented in Section III.

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