vineetloomba.com

Class XI

PREPARED BY:
Er. Vineet Loomba
(B.Tech. IIT Roorkee)

TARGET : JEE Main/Adv

Theory

MATHEMATICS

Trigonometry

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Trigonometrical Ratios & Identities

1. Basic Trigonometric Identities:

(A) $\sin^2\theta + \cos^2\theta = 1$; $-1 \le \sin\theta \le 1$; $-1 \le \cos\theta \le 1 \ \forall \ \theta \in R$

(B) $\sec^2\theta - \tan^2\theta = 1$; $|\sec\theta| \ge 1 \ \forall \ \theta \in R - \left\{(2n+1)\frac{\pi}{2}, n \in I\right\}$

(C) $\text{cosec}^2\theta - \cot^2\theta = 1$; $|\text{cosec}\,\theta| \ge 1 \ \forall \ \theta \in R - \{n\pi, n \in I\}$

2. Circular Definition Of Trigonometric Functions:

$\sin\theta = \frac{PM}{OP}$

$\cos\theta = \frac{OM}{OP}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cos\theta \ne 0$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$, $\sin\theta \ne 0$

$\sec\theta = \frac{1}{\cos\theta}$, $\cos\theta \ne 0$ $\text{cosec}\,\theta = \frac{1}{\sin\theta}$, $\sin\theta \ne 0$

3. Trigonometric Functions Of Allied Angles:

If θ is any angle, then $-\theta$, $90 \pm \theta$, $180 \pm \theta$, $270 \pm \theta$, $360 \pm \theta$ etc. are called Allied Angles.

(A) $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$

(B) $\sin(90° - \theta) = \cos\theta$; $\cos(90° - \theta) = \sin\theta$

(C) $\sin(90° + \theta) = \cos\theta$; $\cos(90° + \theta) = -\sin\theta$

(D) $\sin(180° - \theta) = \sin\theta$; $\cos(180° - \theta) = -\cos\theta$

(e) $\sin(180° + \theta) = -\sin\theta$; $\cos(180° + \theta) = -\cos\theta$

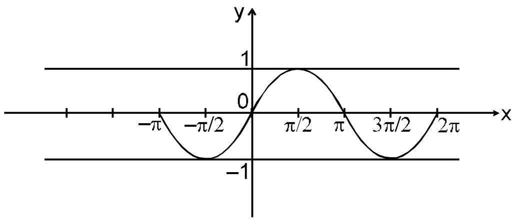
(f) $\sin(270° - \theta) = -\cos\theta$; $\cos(270° - \theta) = -\sin\theta$

(g) $\sin(270° + \theta) = -\cos\theta$; $\cos(270° + \theta) = \sin\theta$

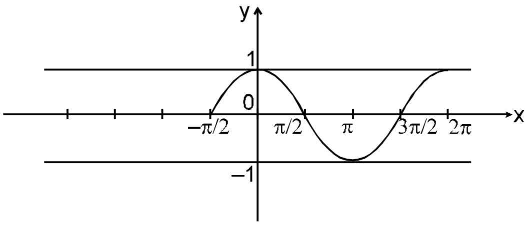
(h) $\tan(90° - \theta) = \cot\theta$; $\cot(90° - \theta) = \tan\theta$

4. Graphs of Trigonometric functions:

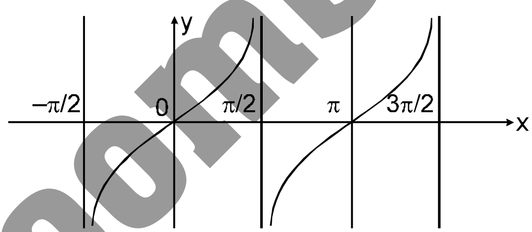
(A) $y = \sin x \quad x \in R;\ y \in [-1, 1]$



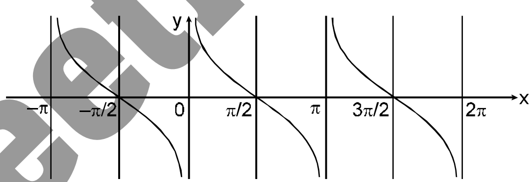
(B) $y = \cos x \quad x \in R;\ y \in [-1, 1]$



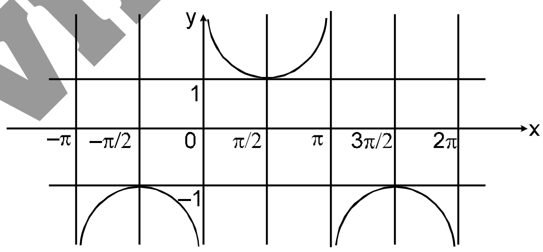
(C) $y = \tan x \quad x \in R - (2n+1)\pi/2,\ n \in I;\ y \in R$



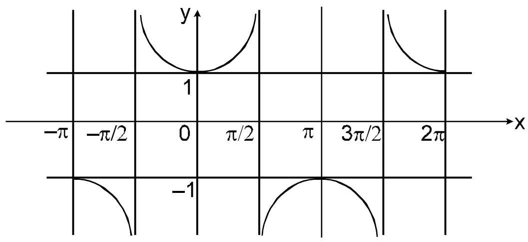
(D) $y = \cot x \quad x \in R - n\pi,\ n \in I;\ y \in R$



(e) $y = \text{cosec}\, x x \in R - n\pi,\ n \in I;\ y \in (-\infty, -1] \cup [1, \infty)$



(f) $y = \sec x$ $\quad x \in R - (2n+1)\pi/2, n \in I$; $y \in (-\infty, -1] \cup [1, \infty)$



5. Trigonometric Functions of Sum or Difference of Two Angles:

(A) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(B) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(C) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B).\sin(A-B)$

(D) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B).\cos(A-B)$

(e) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(f) $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

(g) $\tan(A + B + C)$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}.$$

6. Factorisation of the Sum or Difference of Two Sines or Cosines:

(A) $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(B) $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(C) $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(D) $\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

7. Transformation of Products into Sum or Difference of Sines & Cosines:

(A) $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

(B) $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

(C) $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

(D) $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

8. Multiple and Sub-multiple Angles :

(A) $\sin 2A = 2\sin A \cos A$; $\sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

(B) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$;

$2\cos^2\frac{\theta}{2} = 1 + \cos\theta$, $2\sin^2\frac{\theta}{2} = 1 - \cos\theta$.

(C) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$; $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

(D) $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$, $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

(e) $\sin 3A = 3\sin A - 4\sin^3 A$

(f) $\cos 3A = 4\cos^3 A - 3\cos A$

(g) $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

9. Important Trigonometric Ratios:

(A) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$, where $n \in I$

(B) $\sin 15°$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75°$ or $\cos\frac{5\pi}{12}$;

$\cos 15°$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75°$ or $\sin\frac{5\pi}{12}$;

$\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75°$; $\tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15°$

(C) $\sin\frac{\pi}{10}$ or $\sin 18° = \frac{\sqrt{5}-1}{4}$ & $\cos 36°$ or $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$

10. Conditional Identities:

If $A + B + C = \pi$ then :

(i) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

(ii) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(iii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$

(iv) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(v) $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

(vi) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

(vii) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}$

(viii) $\cot A\cot B + \cot B\cot C + \cot C\cot A = 1$

(ix) $A + B + C = \frac{\pi}{2}$ then $\tan A\tan B + \tan B\tan C + \tan C\tan A = 1$

11. Range of Trigonometric Expression:

$E = a \sin\theta + b \cos\theta$

$E = \sqrt{a^2+b^2} \sin(\theta + \alpha)$, where $\tan\alpha = \frac{b}{a}$

$= \sqrt{a^2+b^2} \cos(\theta - \beta)$, where $\tan\beta = \frac{a}{b}$

Hence for any real value of θ, $-\sqrt{a^2+b^2} \le E \le \sqrt{a^2+b^2}$

12. Sine and Cosine Series:

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots\ldots + \sin\left(\alpha + \overline{n-1}\beta\right) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sin\left(\alpha + \frac{n-1}{2}\beta\right)$$

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots\ldots + \cos\left(\alpha + \overline{n-1}\beta\right) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cos\left(\alpha + \frac{n-1}{2}\beta\right)$$

Trigonometrical Equations

1. DEFINITION

The equations involving trigonometric function of unknown angles are known as Trigonometric equations

e.g. $\cos\theta = 0$, $\cos^2\theta - 4\cos\theta = 1$, $\sin^2\theta + \sin\theta = 2$, $\cos^2\theta - 4\sin\theta = 1$

A solution of a trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g., $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$ or $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$.........

2. PERIODIC FUNCTION

A function f(x) is said to be periodic if there exists T > 0 such that f(x + T) = f(x) for all x in the domain of definitions of f(x). If T is the smallest positive real numbers such that f(x + T) = f(x) , then it is called the period of f(x)

Since $\sin(2n\pi + x) = \sin x$, $\cos(2n\pi + x) = \cos x$; $\tan(n\pi + x) = \tan x$ for all $n \in Z$

Therefore sinx , cosx and tanx are perodic function, the period of sinx and cos x is 2π and that of tanx is π .

Function	Period
sin (ax + b) , cos (ax +b), sec (ax + b) , cosec (ax +b)	$2\pi/a$
tan (ax + b), cot (ax +b)	π/a
| sin (ax + b)| , | cos (ax +b) | , | sec (ax +b) | , | cosec (ax +b) |	π/a
| tan (ax + b) | , | cot (ax +b) |	$\pi/2a$

3. TRIGONOMETRICAL EQUATIONS WITH THEIR GENERAL SOLUTION

Trigonometrical equation		General solution
If $\sin\theta = 0$	then	$\theta = n\pi$
If $\cos\theta = 0$	then	$\theta = (n\pi + \pi/2) = (2n+1)\pi/2$
If $\tan\theta = 0$	then	$\theta = n\pi$
If $\sin\theta = 1$	then	$\theta = 2n\pi + \pi/2 = (4n+1)\pi/2$
If $\cos\theta = 1$	then	$\theta = 2n\pi$
If $\sin\theta = \sin\alpha$	then	$\theta = n\pi + (-1)^n\alpha$ where $\alpha \in [-\pi/2, \pi/2]$
If $\cos\theta = \cos\alpha$	then	$\theta = 2n\pi \pm \alpha$ where $\alpha \in [0,\pi]$
If $\tan\theta = \tan\alpha$	then	$\theta = n\pi + \alpha$ where $\alpha \in (-\pi/2, \pi/2)$
If $\sin^2\theta = \sin^2\alpha$	then	$\theta = n\pi \pm \alpha$
If $\cos^2\theta = \cos^2\alpha$	then	$\theta = n\pi \pm \alpha$
If $\tan^2\theta = \tan^2\alpha$	then	$\theta = n\pi \pm \alpha$
If $\left.\begin{matrix}\sin\theta = \sin\alpha \\ \cos\theta = \cos\alpha\end{matrix}\right\|^{*}$	then	$\theta = 2n\pi + \alpha$
If $\left.\begin{matrix}\sin\theta = \sin\alpha \\ \tan\theta = \tan\alpha\end{matrix}\right\|^{*}$	then	$\theta = 2n\pi + \alpha$
If $\left.\begin{matrix}\tan\theta = \tan\alpha \\ \cos\theta = \cos\alpha\end{matrix}\right\|^{*}$	then	$\theta = 2n\pi + \alpha$

* Every where in this chapter "n" is taken as an integer.

* If α be the least positive value of θ which statisfy two given trigonometrical equations , then the general value of θ will be $2n\pi + \alpha$

4. GENERAL SOLUTION OF STANDARD TRIGONOMETRICAL EQUATIONS

Since, trigonometric functions are periodic, The solution consisting of all possible solutions of a trigonometric equation is called its general solution.

We use of following results for solving the trigonometric equations ;

Result 1 : $\sin\theta = 0 \Leftrightarrow \theta = n\pi, n \in I$.

We know that $\sin\theta = 0$ for all integral multiples of π. (by graphical approach)

$\therefore$ $\sin\theta = 0 \Leftrightarrow \theta = 0, \pm\pi, \pm 2\pi, \pm 3\pi, ...$

$\Leftrightarrow$ $\theta = n\pi, n \in I$.

$\therefore$ $\sin\theta = 0$

$\Leftrightarrow$ $\theta = n\pi, n \in I$.

Result 2 : $\cos\theta = 0 \Leftrightarrow \theta = (2n+1)\frac{\pi}{2}$, $n \in I$.

We know that $\cos\theta = 0$ for all odd multiples of $\frac{\pi}{2}$ (by graphical approch)

$\therefore$ $\quad \cos\theta = 0 \Leftrightarrow \theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots\ldots$

$\Leftrightarrow$ $\quad \theta = (2n+1)\frac{\pi}{2}$, $n \in I$.

$\therefore$ $\quad \cos\theta = 0$

$\Leftrightarrow$ $\quad \theta = (2n+1)\frac{\pi}{2}$, $n \in I$.

Result 3 : $\tan\theta = 0 \Leftrightarrow \theta = n\pi$, $n \in I$.

We know that $\tan\theta = 0$ for all integral multiple of π .

$\therefore$ $\quad \tan\theta = 0 \Leftrightarrow \theta = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\ldots$

$\Leftrightarrow$ $\quad \theta = n\pi$, $n \in I$.

$\therefore$ $\quad \tan\theta = 0$

$\Leftrightarrow$ $\quad \theta = n\pi$, $n \in I$.

Result 4 : $\sin\theta = \sin\alpha \Leftrightarrow \theta = n\pi + (-1)^n\alpha$, where $n \in I$ and $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

We have ,

$\quad \sin\theta = \sin\alpha$, $\quad$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Leftrightarrow$ $\quad \sin\theta - \sin\alpha = 0$

$\Leftrightarrow$ $\quad 2\cos\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\Leftrightarrow$ $\quad \cos\left(\frac{\theta+\alpha}{2}\right) = 0$ or $\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\Leftrightarrow$ $\quad \left(\frac{\theta+\alpha}{2}\right) = (2m+1)\frac{\pi}{2}$, $m \in I$ or $\left(\frac{\theta-\alpha}{2}\right) = m\pi$, $m \in I$.

$\Leftrightarrow$ $\quad (\theta+\alpha) = (2m+1)\pi$, $m \in I$ or $(\theta-\alpha) = 2m\pi$, $m \in I$

$\Leftrightarrow$ $\quad \theta = (2m+1)\pi - \alpha$, $m \in I$ or $\theta = (2m\pi) + \alpha$, $m \in I$

$\Leftrightarrow$ $\quad \theta =$ (any odd multiple of π) $-\alpha$

or $\quad \theta =$ (any even multiple of π) $+ \alpha$

$\Leftrightarrow$ $\quad \theta = n\pi + (-1)^n\alpha$, where $n \in I$

$\therefore$ $\quad \sin\theta = \sin\alpha$

$\Leftrightarrow$ $\quad \theta = n\pi + (-1)^n\alpha$, where $n \in I$ and $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Result 5 : $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi \pm \alpha,\ n \in I$ and $\alpha \in [0, \pi]$

We have,

$$\cos\theta = \cos\alpha, \qquad \text{where } \alpha \in [0, \pi]$$

$$\Leftrightarrow \quad \cos\theta - \cos\alpha = 0$$

$$\Leftrightarrow \quad -2\sin\left(\frac{\theta+\alpha}{2}\right).\sin\left(\frac{\theta-\alpha}{2}\right) = 0$$

$$\Leftrightarrow \quad \sin\left(\frac{\theta+\alpha}{2}\right) = 0 \text{ or } \sin\left(\frac{\theta-\alpha}{2}\right) = 0$$

$$\Leftrightarrow \quad \frac{\theta+\alpha}{2} = n\pi \text{ or } \frac{\theta-\alpha}{2} = n\pi,\ n \in I.$$

$$\Leftrightarrow \quad \theta + \alpha = 2n\pi \text{ or } \theta - \alpha = 2n\pi,\ n \in I$$

$$\Leftrightarrow \quad \theta = 2n\pi - \alpha \text{ or } \theta = 2n\pi + \alpha,\ n \in I$$

$$\Leftrightarrow \quad \theta = 2n\pi \pm \alpha,\ n \in I$$

$$\therefore \quad \cos\theta = \cos\alpha$$

$$\Leftrightarrow \quad \theta = 2n\pi \pm \alpha,\ n \in I, \text{ where } \alpha \in [0, \pi]$$

Result 6 : $\tan\theta = \tan\alpha \Leftrightarrow \theta = n\pi + \alpha,\ n \in I$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

we have

$\tan\theta = \tan\alpha$, where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\Leftrightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha}{\cos\alpha}$$

$$\Leftrightarrow \quad \sin\theta\cos\alpha - \cos\theta\sin\alpha = 0$$

$$\Leftrightarrow \quad \sin(\theta - \alpha) = 0$$

$$\Leftrightarrow \quad \theta - \alpha = n\pi,\ n \in I$$

$$\Leftrightarrow \quad \theta = n\pi + \alpha,\ n \in I$$

$$\therefore \quad \tan\theta = \tan\alpha$$

$$\Leftrightarrow \quad \theta = n\pi + \alpha \qquad \text{where } \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

5. GENERAL SOLUTION OF THE TRIGONOMETRICAL EQUATION

Result 7 : $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$, $\tan^2\theta = \tan^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$

(i) $\sin^2\theta = \sin^2\alpha$

$$\Leftrightarrow \quad \frac{1-\cos 2\theta}{2} = \frac{1-\cos 2\alpha}{2}$$

$$\Leftrightarrow \quad \cos 2\theta = \cos 2\alpha$$

$$\Leftrightarrow \quad 2\theta = 2n\pi \pm 2\alpha,\ n \in I$$

$$\Leftrightarrow \quad \theta = n\pi \pm \alpha,\ n \in I$$

(ii) $\cos^2\theta = \cos^2\alpha$

$\Leftrightarrow \quad \dfrac{1+\cos 2\theta}{2} = \dfrac{1+\cos 2\alpha}{2}$

$\Leftrightarrow \quad \cos 2\theta = \cos 2\alpha$

$\Leftrightarrow \quad 2\theta = 2n\pi \pm 2\alpha,\ n \in I$

$\Leftrightarrow \quad \theta = n\pi \pm \alpha,\ n \in I$

(iii) $\tan^2\theta = \tan^2\alpha$

$\Leftrightarrow \quad \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$

(applying componendo and dividendo)

$\Leftrightarrow \quad \cos 2\theta = \cos 2\alpha$

$\Leftrightarrow \quad 2\theta = 2n\pi \pm \backslash 2\alpha,\ n \in I$

$\Leftrightarrow \quad \theta = n\pi \pm \alpha,\ n \in I$

6. GENERAL SOLUTION OF TRIGONOMETRICAL EQUATION $a\cos\theta + b\sin\theta = c$

To solve the equation $a\cos\theta + b\sin\theta = c$, put $a = r\cos\phi$, $b = r\sin\phi$ such that

$$r = \sqrt{a^2+b^2},\ \phi = \tan^{-1}\frac{b}{a}$$

Substituting these values in the equation we have $r\cos\phi\cos\theta + r\sin\phi\sin\theta = c$

$$\cos(\theta-\phi) = \frac{c}{r} \quad \Rightarrow \quad \cos(\theta-\phi) = \frac{c}{\sqrt{a^2+b^2}}$$

If $|c| > \sqrt{a^2+b^2}$, then the equation ;

$a\cos\theta + b\sin\theta = c$ has no solution

If $|c| \le \sqrt{a^2+b^2}$, then put ;

$\dfrac{c}{\sqrt{a^2+b^2}} = \cos\alpha$, so that

$\cos(\theta-\phi) = \cos\alpha$

$\Rightarrow \quad (\theta-\phi) = 2n\pi \pm \alpha$

$\Rightarrow \quad \theta = 2n\pi \pm \alpha + \phi$

7. SOLUTIONS IN THE CASE OF TWO EQUATIONS ARE GIVEN

Two equations are given and we have to find the values of variable θ which may satisfy both the given equations, like

$\cos\theta = \cos\alpha$ and $\sin\theta = \sin\alpha$

so the common solution is $\theta = 2n\pi + \alpha$, $n \in I$

Similarly, $\sin\theta = \sin\alpha$ and $\tan\theta = \tan\alpha$

so the common solution is , $\theta = 2n\pi + \alpha$, $n \in I$

Rule : Find the common values of θ between 0 and 2π and then add $2\pi n$ to this common value

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