einsum_decomposition

March 10, 2022

1 Einsum decomposition

This notebook shows a way to decompose einsum into a subset of operations (expand_dims, squeeze, transpose, extended matrix multiplication).

[1]:
```
from jyquickhelper import add_notebook_menu
add_notebook_menu()
```

[1]:
```
<IPython.core.display.HTML object>
```

[2]:
```
%load_ext mlprodict
```

1.1 Operator explanation with equation `bac,cd,def=ebc`

The operator einsum takes an equation and some inputs. Every letter involved in the equation is a loop. Let's see on one example.

[3]:
```
import numpy

m1 = numpy.arange(0, 8).astype(numpy.float32).reshape((2, 2, 2)) + 10
m2 = numpy.arange(0, 4).astype(numpy.float32).reshape((2, 2)) + 100
m3 = numpy.arange(0, 8).astype(numpy.float32).reshape((2, 2, 2)) + 1000

equation = "bac,cd,def->ebc"
truth = numpy.einsum(equation, m1, m2, m3)
truth
```

[3]:
```
array([[[ 8866198.,  9864696.],
        [12090270., 13152928.]],

       [[ 8883886.,  9884376.],
        [12114390., 13179168.]]], dtype=float32)
```

This summation is equalent to:

[4]:
```
res = numpy.zeros((2, 2, 2))
for a in range(0, 2):
    for b in range(0, 2):
        for c in range(0, 2):
            for d in range(0, 2):
                for e in range(0, 2):
                    for f in range(0, 2):
                        res[e, b, c] += m1[b, a, c] * m2[c, d] * m3[d, e, f]
```

```
res
```

[4]:
```
array([[[ 8866198.,  9864696.],
        [12090270., 13152928.]],

       [[ 8883886.,  9884376.],
        [12114390., 13179168.]]])
```

Theoritically, this summation is in this case has a cost of $O(N^6)$. However this simple computation is usually much longer than using matrix multiplications along the path. $O(N^4)$ is the cost of the heaviest matrix multiplication in this case). But to do that, the equation needs to be decomposed into a sequence of matrix multiplications.

1.1.1 Decomposition of `bac,cd,def=ebc`

[5]:
```
import numpy
from mlprodict.testing.einsum import (
    decompose_einsum_equation, apply_einsum_sequence)
```

[6]:
```
m1 = numpy.arange(0, 8).astype(numpy.float32).reshape((2, 2, 2)) + 10
m2 = numpy.arange(0, 4).astype(numpy.float32).reshape((2, 2)) + 100
m3 = numpy.arange(0, 8).astype(numpy.float32).reshape((2, 2, 2)) + 1000
```

[7]:
```
seq = decompose_einsum_equation("bac,cd,def->ebc")
```

[8]:
```
from jyquickhelper import RenderJsDot
RenderJsDot(seq.to_dot(size=7))
```

[8]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x1c6d45f6250>
```

Then the result can be obtained as follows:

[9]:
```
apply_einsum_sequence(seq, m1, m2, m3)
```

[9]:
```
array([[[ 8866198.,  9864696.],
        [12090270., 13152928.]],

       [[ 8883886.,  9884376.],
        [12114390., 13179168.]]], dtype=float32)
```

1.1.2 operator matmul

This operator can be used to represent either a multiplication, either a matrix multiplication but it applies only on arrays with the same number of dimensions. It can be broken into multiplication of matrix multiplication.

[10]:
```
seq_clean = decompose_einsum_equation("bac,cd,def->ebc", strategy='numpy', clean=True)
RenderJsDot(seq_clean.to_dot(size=7))
```

[10]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x1c6d45f6910>
```

Operator *transpose_mm* is a regular transposition, it takes two inputs but only tranposes the first input before returning it. Operator *batch_dot* is a matrix multiplication. It is left that way on purpose as it may be implemented with function dot or gemm. The operator distinguishes between 3 kind of axes: batch axes, kept axes, sum(mation) axes. It then reshapes both input matrices with 3D tensors, batch axis, row axis, column axis to use function numpy.dot.

1.1.3 ONNX

The previous graph can be converted into ONNX.

[11]:
```
onx = seq_clean.to_onnx("Y", "X1", "X2", "X3", dtype=numpy.float32)
# with open("einsum.onnx", "wb") as f:
#      f.write(onx.SerializeToString())
%onnxview onx
```

[11]: <jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x1c6d4631640>

[12]:
```
from onnxruntime import InferenceSession
sess = InferenceSession(onx.SerializeToString())
sess.run(None, {'X1': m1.astype(numpy.float32),
                'X2': m2.astype(numpy.float32),
                'X3': m3.astype(numpy.float32)})[0]
```

[12]:
```
array([[[ 8866198.,  9864696.],
        [12090270., 13152928.]],

       [[ 8883886.,  9884376.],
        [12114390., 13179168.]]], dtype=float32)
```

1.1.4 onnxruntime

[13]:
```
import onnx
from onnx import helper, numpy_helper
from onnxruntime import InferenceSession


def make_model1(equation):
    model = helper.make_model(
        opset_imports=[helper.make_operatorsetid('', 13)],
        graph=helper.make_graph(
            name='einsum_test',
            inputs=[helper.make_tensor_value_info("X", onnx.TensorProto.FLOAT, None),
                    helper.make_tensor_value_info("Y", onnx.TensorProto.FLOAT, None),
                    helper.make_tensor_value_info("Z", onnx.TensorProto.FLOAT, None)],
            outputs=[helper.make_tensor_value_info("A", onnx.TensorProto.FLOAT, None)],
            nodes=[
                helper.make_node("Einsum", ["X", "Y", "Z"], ["A"], equation=equation)
            ]
        )
    )
    return model


model = make_model1("bac,cd,def->ebc")
sess = InferenceSession(model.SerializeToString())
```

[14]:
```
sess.run(None, {'X': m1.astype(numpy.float32),
                'Y': m2.astype(numpy.float32),
                'Z': m3.astype(numpy.float32)})[0]
```

[14]: array([[[8866198., 9864696.],
        [12090270., 13152928.]],

       [[8883886., 9884376.],
        [12114390., 13179168.]]], dtype=float32)

1.1.5 Benchmark

It clearly shows the summation done with the basic algorithm is the slowest.

[15]:
```python
from mlprodict.onnxrt.validate.validate_helper import measure_time
from tqdm import tqdm
from pandas import DataFrame


def raw_product(m1, m2, m3):
    N = m1.shape[0]
    res = numpy.zeros((N, N, N))
    for a in range(0, N):
        for b in range(0, N):
            for c in range(0, N):
                for d in range(0, N):
                    for e in range(0, N):
                        for f in range(0, N):
                            res[e, b, c] += m1[b, a, c] * m2[c, d] * m3[d, e, f]
    return res


def benchmark0(equation):
    sess = None
    sess2 = None
    seq = None
    seq2 = None

    results = []
    for N in tqdm([2, 3, 4, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60]):
        m1 = numpy.random.randn(N, N, N)
        m2 = numpy.random.randn(N, N)
        m3 = numpy.random.randn(N, N, N)

        if seq is None:
            seq = decompose_einsum_equation(equation, clean=True)
        if seq2 is None:
            seq2 = decompose_einsum_equation(equation, clean=True, strategy='numpy')
        if sess is None:
            model = make_model1(equation)
            sess = InferenceSession(model.SerializeToString())
        if sess2 is None:
            onx = seq2.to_onnx("Y", "X1", "X2", "X3", dtype=numpy.float32)
            sess2 = InferenceSession(onx.SerializeToString())

        res = measure_time(lambda x: numpy.einsum(equation, *x, optimize=True),
                           [m1, m2, m3],
                           repeat=10, number=10)
```

```
        res['name'] = "numpy.einsum"
        res["N"] = N
        results.append(res)

        if N <= 4:
            res = measure_time(lambda x: raw_product(*x),
                               [m1, m2, m3],
                               repeat=10, number=10)
            res['name'] = "raw_product"
            res["N"] = N
            results.append(res)

        res = measure_time(lambda x: apply_einsum_sequence(seq, *x),
                           [m1, m2, m3],
                           repeat=10, number=10)

        res['name'] = "custom_einsum"
        res["N"] = N
        results.append(res)

        res = measure_time(lambda x: apply_einsum_sequence(seq, *x, matmul_impl="pyf"),
                           [m1, m2, m3],
                           repeat=10, number=10)
        res['name'] = "dec-matmul"
        res["N"] = N
        results.append(res)

        res = measure_time(lambda x: apply_einsum_sequence(seq2, *x,␣
↪matmul_impl="pyf"),
                           [m1, m2, m3],
                           repeat=10, number=10)
        res['name'] = "dec-batch_dot"
        res["N"] = N
        results.append(res)

        res = measure_time(lambda x: sess.run(None, {'X': x[0], 'Y': x[1], 'Z': x[2]}),
                           [m1.astype(numpy.float32), m2.astype(numpy.float32),
                            m3.astype(numpy.float32)],
                           repeat=10, number=10)
        res['name'] = "ort-einsum"
        res["N"] = N
        results.append(res)

        res = measure_time(lambda x: sess2.run(None, {'X1': x[0], 'X2': x[1], 'X3':␣
↪x[2]}),
                           [m1.astype(numpy.float32), m2.astype(numpy.float32),
                            m3.astype(numpy.float32)],
                           repeat=10, number=10)
        res['name'] = "ort-matmul"
        res["N"] = N
        results.append(res)
    return DataFrame(results)
```

```python
df = benchmark0("bac,cd,def->ebc")
df.tail()
```

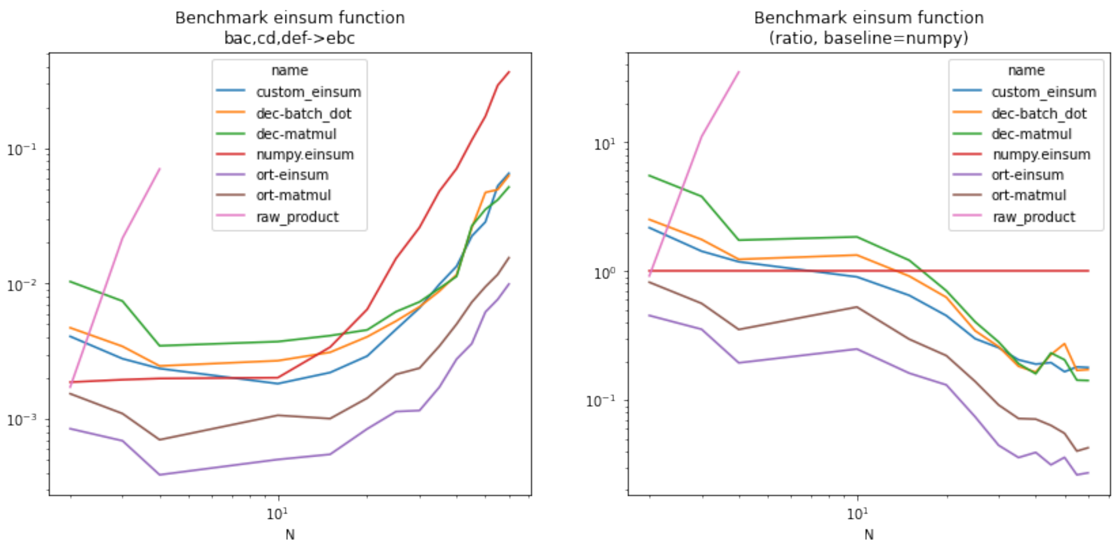
```
C:\xavierdupre\__home_\github_fork\scikit-
learn\sklearn\experimental\enable_hist_gradient_boosting.py:16: UserWarning:
Since version 1.0, it is not needed to import enable_hist_gradient_boosting
anymore. HistGradientBoostingClassifier and HistGradientBoostingRegressor are
now stable and can be normally imported from sklearn.ensemble.
  warnings.warn(
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 14/14 [00:20<00:00,  1.47s/it]
```

[15]:

```
     average  deviation  min_exec  max_exec  repeat  number     total  \
82  0.065132   0.001338  0.063801  0.068927      10      10  0.651318
83  0.051615   0.001206  0.049987  0.053465      10      10  0.516154
84  0.062689   0.003658  0.058949  0.073073      10      10  0.626888
85  0.009917   0.000274  0.009737  0.010686      10      10  0.099166
86  0.015518   0.001107  0.014413  0.018179      10      10  0.155178

              name   N
82  custom_einsum  60
83     dec-matmul  60
84  dec-batch_dot  60
85     ort-einsum  60
86     ort-matmul  60
```

[16]:

```python
import matplotlib.pyplot as plt

piv = df.pivot("N", "name", "average")
piv2 = piv.copy()
np = piv["numpy.einsum"]
for c in piv2.columns:
    piv2[c] /= np

fig, ax = plt.subplots(1, 2, figsize=(14, 6))
piv.plot(logy=True, logx=True, ax=ax[0])
ax[0].set_title("Benchmark einsum function\nbac,cd,def->ebc")
piv2.plot(logy=True, logx=True, ax=ax[1])
ax[1].set_title("Benchmark einsum function\n(ratio, baseline=numpy)");
```



Version `dec-matmul` is an implementation based on the decomposition of a simplified einsum into a sequence of transpose, reshape, (batch_)dot or mul operations. This decomposition is converted into ONNX and executed with *onnxruntime*, version `ort-matmul`. Both versions are faster than the numpy optimized version.

1.2 Another example with `bsnh,btnh=bnts`

Another case, more frequent in deep learning.

1.2.1 Decomposition of `bsnh,btnh=bnts`

[17]:
```
seq2 = decompose_einsum_equation("bsnh,btnh->bnts", strategy='numpy', clean=True)
RenderJsDot(seq2.to_dot(size=7))
```

[17]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x1c6d8eae100>
```

1.2.2 ONNX version

[18]:
```
onx2 = seq2.to_onnx("Y", "X1", "X2", dtype=numpy.float32)
%onnxview onx2
```

[18]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x1c6d91382b0>
```

1.2.3 Benchmark

[19]:
```
def make_model2(equation):
    model = helper.make_model(
        opset_imports=[helper.make_operatorsetid('', 13)],
        graph=helper.make_graph(
            name='einsum_test',
            inputs=[helper.make_tensor_value_info("X", onnx.TensorProto.FLOAT, None),
                    helper.make_tensor_value_info("Y", onnx.TensorProto.FLOAT, None)],
            outputs=[helper.make_tensor_value_info("A", onnx.TensorProto.FLOAT, None)],
            nodes=[
```

```
                helper.make_node("Einsum", ["X", "Y"], ["A"], equation=equation)
            ]
        )
    )
    return model


def benchmark(equation, second_input_size=4):
    sess = None
    sess2 = None
    seq = None
    seq2 = None


    results = []
    for N in tqdm([2, 3, 4, 10, 20, 30, 40]):
        m1 = numpy.random.randn(10, N, N, N)
        m2 = numpy.random.randn(10 * N ** (second_input_size-1)).reshape((10, ) + (N,␣
↪) * (second_input_size-1))


        if seq is None:
            seq = decompose_einsum_equation(equation, clean=True)
        if seq2 is None:
            seq2 = decompose_einsum_equation(equation, clean=True, strategy='numpy')
        if sess is None:
            model = make_model2(equation)
            sess = InferenceSession(model.SerializeToString())
        if sess2 is None:
            onx = seq2.to_onnx("Y", "X1", "X2", dtype=numpy.float32)
            sess2 = InferenceSession(onx.SerializeToString())

        res = measure_time(lambda x: numpy.einsum(equation, *x, optimize=True),
                           [m1, m2],
                           repeat=10, number=10)

        res['name'] = "numpy.einsum"
        res["N"] = N
        results.append(res)

        res = measure_time(lambda x: apply_einsum_sequence(seq, *x),
                           [m1, m2],
                           repeat=10, number=10)
        res['name'] = "custom_einsum"
        res["N"] = N
        results.append(res)

        res = measure_time(lambda x: apply_einsum_sequence(seq, *x, matmul_impl="pyf"),
                           [m1, m2],
                           repeat=10, number=10)
        res['name'] = "dec-matmul"
        res["N"] = N
        results.append(res)
```

```
        res = measure_time(lambda x: apply_einsum_sequence(seq2, *x,␣
 ↪matmul_impl="pyf"),
                           [m1, m2],
                           repeat=10, number=10)
        res['name'] = "dec-batch_dot"
        res["N"] = N
        results.append(res)

        res = measure_time(lambda x: sess.run(None, {'X': x[0], 'Y': x[1]}),
                           [m1.astype(numpy.float32), m2.astype(numpy.float32),
                            m3.astype(numpy.float32)],
                           repeat=10, number=10)
        res['name'] = "ort-einsum"
        res["N"] = N
        results.append(res)

        res = measure_time(lambda x: sess2.run(None, {'X1': x[0], 'X2': x[1]}),
                           [m1.astype(numpy.float32), m2.astype(numpy.float32),
                            m3.astype(numpy.float32)],
                           repeat=10, number=10)
        res['name'] = "ort-matmul"
        res["N"] = N
        results.append(res)
    return DataFrame(results)


df = benchmark("bsnh,btnh->bnts")
df.tail()
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 7/7 [00:13<00:00,  1.93s/it]
```

[19]:

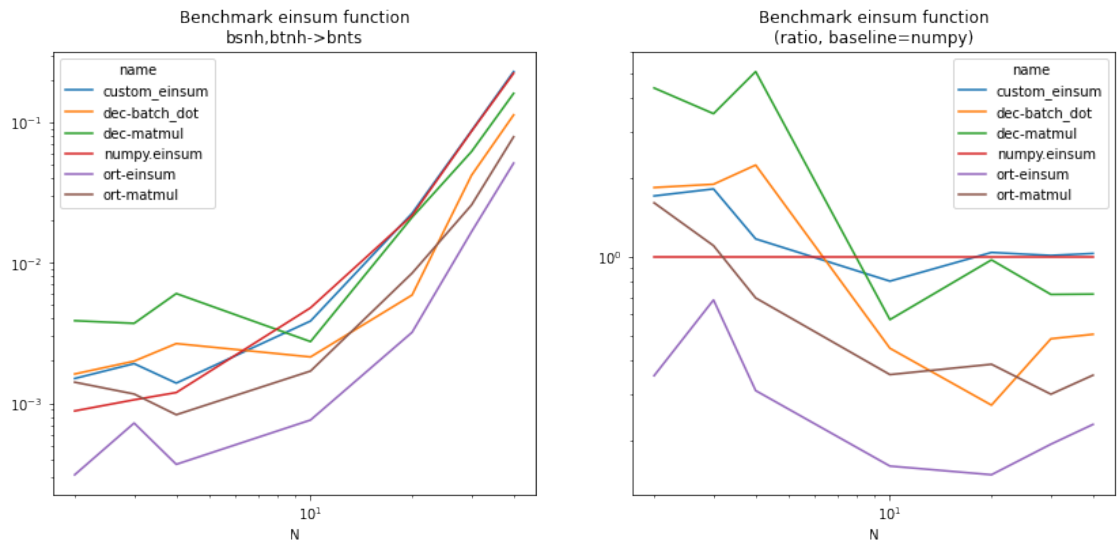
	average	deviation	min_exec	max_exec	repeat	number	total	\
37	0.229418	0.020792	0.217997	0.291032	10	10	2.294175	
38	0.160575	0.005435	0.150772	0.167411	10	10	1.605746	
39	0.112844	0.011305	0.102173	0.141890	10	10	1.128436	
40	0.051181	0.003533	0.047244	0.057054	10	10	0.511815	
41	0.078827	0.008735	0.067893	0.099156	10	10	0.788271	

	name	N
37	custom_einsum	40
38	dec-matmul	40
39	dec-batch_dot	40
40	ort-einsum	40
41	ort-matmul	40

[20]:

```
piv = df.pivot("N", "name", "average")
piv2 = piv.copy()
np = piv["numpy.einsum"]
for c in piv2.columns:
    piv2[c] /= np

fig, ax = plt.subplots(1, 2, figsize=(14, 6))
```

```
piv.plot(logy=True, logx=True, ax=ax[0])
ax[0].set_title("Benchmark einsum function\nbsnh,btnh->bnts")
piv2.plot(logy=True, logx=True, ax=ax[1])
ax[1].set_title("Benchmark einsum function\n(ratio, baseline=numpy)");
```



1.2.4 Permutation

Einsum's algorithm started by aligning all matrices involved in the computation to the same dimension in the same order. But which order is the best, that's the question.

[21]:
```
equation = "bsnh,btnh->bnts"
letters = list(sorted(set([c for c in equation if "a" <= c < "z"])))
letters
```

[21]: ['b', 'h', 'n', 's', 't']

[22]:
```
from itertools import permutations


def benchmark_perm(equation, number=5, second_input_size=4, repeat=3, N=15):

    def n_operator(seq, name):
        n = 0
        for op in seq:
            if op.name == name:
                n += 1
        return n


    def n_onnx_op(onx, name):
        n = 0
        for op in onx.graph.node:
            if op.op_type == name:
```

```
            n += 1
    return n


def get_kind(seq):
    n = 0
    for op in seq:
        if op.name == 'batch_dot':
            return op.get_dot_kind()
    return None


m1 = numpy.random.randn(N, N, N, N)
m2 = numpy.random.randn(N ** second_input_size).reshape((N, ) * second_input_size)

results = []
for perm in tqdm(list(permutations(letters))):
    replace = {d: c for c, d in zip(letters, perm)}
    eq = equation
    for k, v in replace.items():
        eq = eq.replace(k, v.upper())
    eq = eq.lower()

    seq = decompose_einsum_equation(eq, clean=True)
    seq2 = decompose_einsum_equation(eq, clean=True, strategy='numpy')
    model = make_model2(eq)
    sess = InferenceSession(model.SerializeToString())
    onx = seq2.to_onnx("Y", "X1", "X2", dtype=numpy.float32)
    sess2 = InferenceSession(onx.SerializeToString())

    n_tra = n_operator(seq2, 'transpose')
    n_tra_onnx = n_onnx_op(onx, 'Transpose')
    n_gemm_onnx = n_onnx_op(onx, 'Gemm')
    kind = get_kind(seq2)

    res = measure_time(lambda x: numpy.einsum(eq, *x, optimize=True),
                       [m1, m2],
                       repeat=repeat, number=number)

    res['name'] = "numpy.einsum"
    res["N"] = N
    res["eq"] = eq
    results.append(res)

    res = measure_time(lambda x: apply_einsum_sequence(seq, *x),
                       [m1, m2],
                       repeat=repeat, number=number)
    res['name'] = "custom_einsum"
    res["N"] = N
    res["eq"] = eq
    res['transpose'] = n_tra
    res['kind'] = kind
    results.append(res)
```

```
        res = measure_time(lambda x: apply_einsum_sequence(seq, *x, matmul_impl="pyf"),
                           [m1, m2],
                           repeat=repeat, number=number)
        res['name'] = "dec-matmul"
        res["N"] = N
        res["eq"] = eq
        res['transpose'] = n_tra
        res['kind'] = kind
        results.append(res)

        res = measure_time(lambda x: apply_einsum_sequence(seq2, *x,␣
↪matmul_impl="pyf"),
                           [m1, m2],
                           repeat=repeat, number=number)
        res['name'] = "dec-batch_dot"
        res["N"] = N
        res["eq"] = eq
        res['transpose'] = n_tra
        res['kind'] = kind
        results.append(res)

        res = measure_time(lambda x: sess.run(None, {'X': x[0], 'Y': x[1]}),
                           [m1.astype(numpy.float32), m2.astype(numpy.float32),
                            m3.astype(numpy.float32)],
                           repeat=repeat, number=number)
        res['name'] = "ort-einsum"
        res["N"] = N
        res["eq"] = eq
        res['transpose'] = n_tra_onnx
        res['gemm'] = n_gemm_onnx
        results.append(res)

        res = measure_time(lambda x: sess2.run(None, {'X1': x[0], 'X2': x[1]}),
                           [m1.astype(numpy.float32), m2.astype(numpy.float32),
                            m3.astype(numpy.float32)],
                           repeat=repeat, number=number)
        res['name'] = "ort-matmul"
        res["N"] = N
        res["eq"] = eq
        res['transpose'] = n_tra_onnx
        res['gemm'] = n_gemm_onnx
        results.append(res)
    return DataFrame(results)


df = benchmark_perm("bsnh,btnh->bnts", number=4)
df.tail()
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 120/120 [00:11<00:00, 10.23it/s]
```

[22]:

```
      average  deviation  min_exec  max_exec  repeat  number     total  \
715  0.006162   0.000038  0.006128  0.006216       3       4  0.018485
```

```
716  0.002343   0.000046  0.002294  0.002405       3       4  0.007029
717  0.001645   0.000035  0.001610  0.001694       3       4  0.004934
718  0.000833   0.000015  0.000820  0.000853       3       4  0.002498
719  0.001251   0.000012  0.001238  0.001268       3       4  0.003753

              name   N               eq  transpose kind  gemm
715  custom_einsum  15  thns,tbns->tnbh        3.0   NN   NaN
716     dec-matmul  15  thns,tbns->tnbh        3.0   NN   NaN
717  dec-batch_dot  15  thns,tbns->tnbh        3.0   NN   NaN
718     ort-einsum  15  thns,tbns->tnbh        4.0  NaN   0.0
719     ort-matmul  15  thns,tbns->tnbh        4.0  NaN   0.0
```

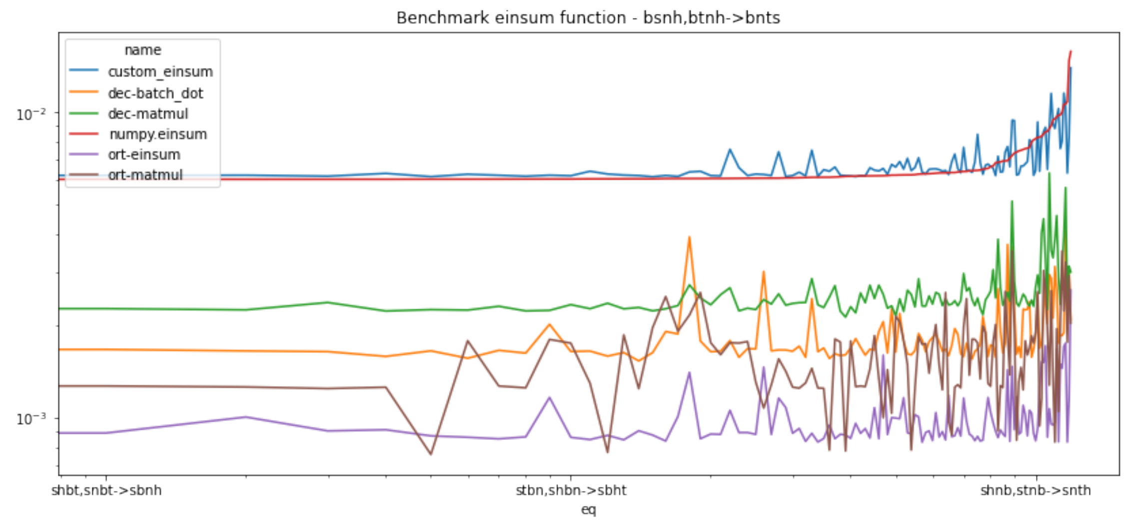
[23]:
```python
df = df.sort_values("average").reset_index(drop=True)
df.head()
```

[23]:
```
    average  deviation  min_exec  max_exec  repeat  number     total  \
0  0.000758   0.000015  0.000738  0.000771       3       4  0.002275
1  0.000770   0.000023  0.000739  0.000793       3       4  0.002310
2  0.000778   0.000020  0.000758  0.000806       3       4  0.002334
3  0.000783   0.000021  0.000760  0.000812       3       4  0.002350
4  0.000784   0.000011  0.000774  0.000799       3       4  0.002351

         name   N               eq  transpose kind  gemm
0  ort-matmul  15  hsnt,hbnt->hnbs        4.0  NaN   0.0
1  ort-matmul  15  hnts,hbts->htbn        4.0  NaN   0.0
2  ort-matmul  15  bnst,bhst->bshn        4.0  NaN   0.0
3  ort-matmul  15  bnht,bsht->bhsn        4.0  NaN   0.0
4  ort-matmul  15  hnst,hbst->hsbn        4.0  NaN   0.0
```

[24]:
```python
df.tail()
```

[24]:
```
      average  deviation  min_exec  max_exec  repeat  number     total  \
715  0.011529   0.000882  0.010456  0.012617       3       4  0.034587
716  0.011548   0.000422  0.010967  0.011953       3       4  0.034644
717  0.013971   0.001984  0.012279  0.016754       3       4  0.041912
718  0.014765   0.001483  0.013366  0.016818       3       4  0.044295
719  0.015813   0.002921  0.012546  0.019636       3       4  0.047438

              name   N               eq  transpose kind  gemm
715  custom_einsum  15  sbnt,shnt->snhb        3.0   NN   NaN
716  custom_einsum  15  htsb,hnsb->hsnt        3.0   NN   NaN
717  custom_einsum  15  nbsh,ntsh->nstb        3.0   NN   NaN
718   numpy.einsum  15  bnsh,btsh->bstn        NaN  NaN   NaN
719   numpy.einsum  15  nbsh,ntsh->nstb        NaN  NaN   NaN
```

[25]:
```python
piv = df.pivot("eq", "name", "average").sort_values("numpy.einsum")

fig, ax = plt.subplots(1, 1, figsize=(14, 6))
piv.plot(logy=True, logx=True, ax=ax)
ax.set_title("Benchmark einsum function - bsnh,btnh->bnts");
```




```
[26]: set(df['transpose'].dropna()), set(df['gemm'].dropna()), set(df['kind'].dropna())
```

```
[26]: ({3.0, 4.0}, {0.0}, {'NN'})
```

1.3 Decomposition of bsnh,ctnh=nts

```
[27]: seq3 = decompose_einsum_equation("bsnh,ctnh->nts", strategy='numpy', clean=True)
      RenderJsDot(seq3.to_dot(size=7))
```

```
[27]: <jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x1c6d8f4da90>
```

```
[28]: onx3 = seq3.to_onnx("Y", "X1", "X2", dtype=numpy.float32)
      %onnxview onx3
```

```
[28]: <jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x1c6d8ee15e0>
```

1.3.1 Benchmark size

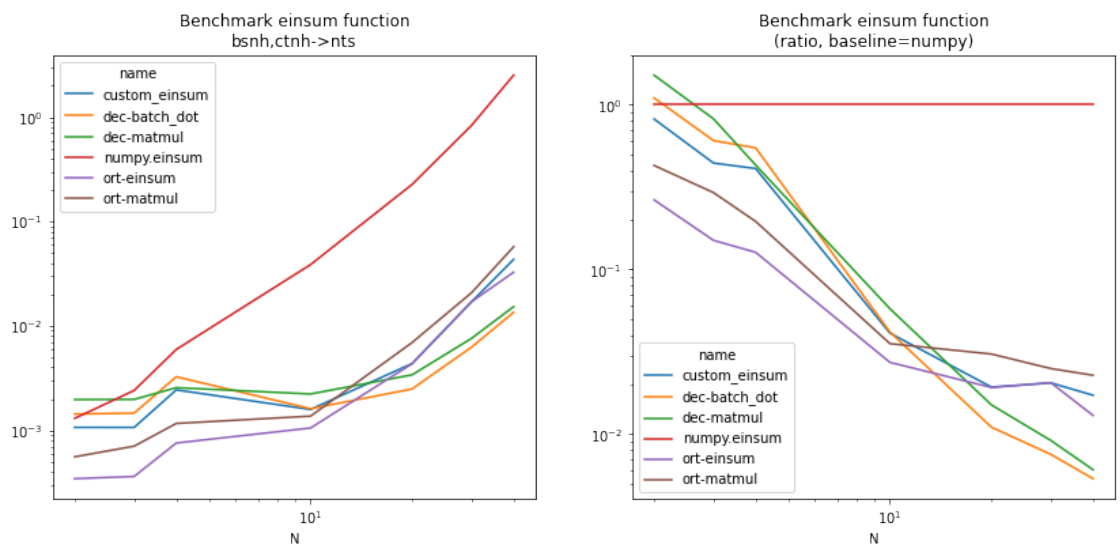
```
[29]: df = benchmark("bsnh,ctnh->nts")
      df.tail()
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 7/7 [00:39<00:00,  5.71s/it]
```

```
[29]:     average  deviation  min_exec  max_exec  repeat  number     total  \
      37  0.043389   0.016879  0.030195  0.077480      10      10  0.433885
      38  0.015310   0.000222  0.014909  0.015622      10      10  0.153098
      39  0.013508   0.000425  0.013148  0.014576      10      10  0.135085
      40  0.032725   0.000266  0.032409  0.033212      10      10  0.327254
      41  0.057384   0.002703  0.053734  0.062845      10      10  0.573841

                   name   N
      37  custom_einsum  40
```

```
38     dec-matmul  40
39  dec-batch_dot  40
40     ort-einsum  40
41     ort-matmul  40
```

[30]:
```
piv = df.pivot("N", "name", "average")
piv2 = piv.copy()
np = piv["numpy.einsum"]
for c in piv2.columns:
    piv2[c] /= np

fig, ax = plt.subplots(1, 2, figsize=(14, 6))
piv.plot(logy=True, logx=True, ax=ax[0])
ax[0].set_title("Benchmark einsum function\nbsnh,ctnh->nts")
piv2.plot(logy=True, logx=True, ax=ax[1])
ax[1].set_title("Benchmark einsum function\n(ratio, baseline=numpy)");
```



1.3.2 Benchmark permutation

[31]:
```
df = benchmark_perm("bsnh,ctnh->nts", number=2, repeat=3, N=10)
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 120/120 [00:06<00:00, 17.41it/s]
```

[32]:
```
df = df.sort_values("average").reset_index(drop=True)
df.head()
```

[32]:

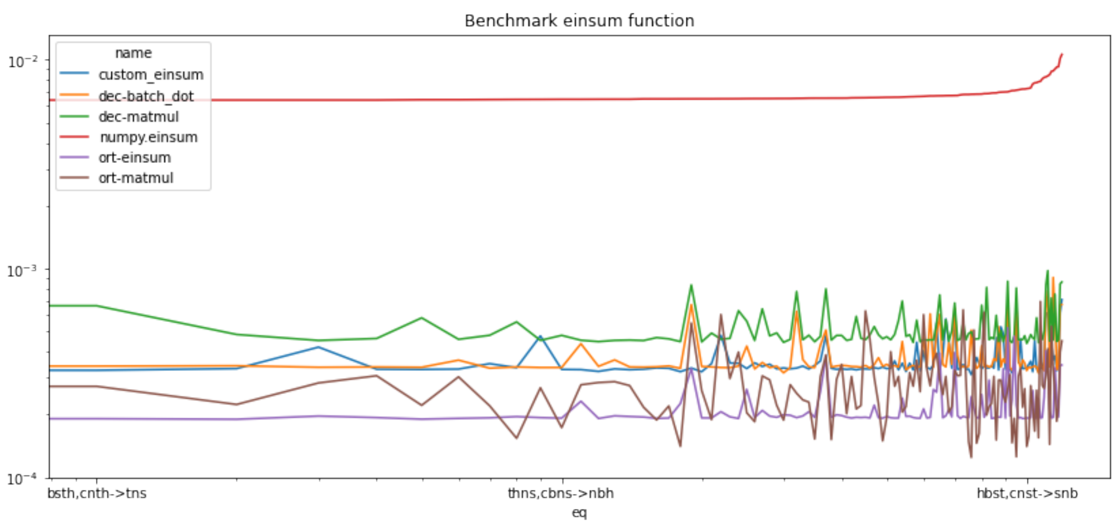
	average	deviation	min_exec	max_exec	repeat	number	total	\
0	0.000125	0.000008	0.000118	0.000136	3	2	0.000374	
1	0.000126	0.000007	0.000119	0.000136	3	2	0.000377	
2	0.000141	0.000006	0.000136	0.000150	3	2	0.000422	
3	0.000141	0.000007	0.000135	0.000151	3	2	0.000423	

```
4  0.000144   0.000007  0.000138  0.000154      3       2  0.000432

         name   N              eq  transpose kind  gemm
0  ort-matmul  10  bnst,chst->shn        4.0  NaN   0.0
1  ort-matmul  10  bhst,cnst->snh        4.0  NaN   0.0
2  ort-matmul  10  hbst,cnst->snb        5.0  NaN   0.0
3  ort-matmul  10  nbst,chst->shb        5.0  NaN   0.0
4  ort-matmul  10  btns,chns->nht        5.0  NaN   0.0
```

[33]:
```python
set(df['transpose'].dropna()), set(df['gemm'].dropna()), set(df['kind'].dropna())
```

[33]:
```
({3.0, 4.0, 5.0, 6.0}, {0.0}, {'NN'})
```

[34]:
```python
piv = df.pivot("eq", "name", "average").sort_values("numpy.einsum")

fig, ax = plt.subplots(1, 1, figsize=(14, 6))
piv.plot(logy=True, logx=True, ax=ax)
ax.set_title("Benchmark einsum function");
```



1.3.3 Best permutation

One of the best permutation is `bnst,chst->shn`.

[35]:
```python
seq4 = decompose_einsum_equation("bnst,chst->shn", strategy='numpy', clean=True)
RenderJsDot(seq4.to_dot(size=7))
```

[35]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x1c6d8e73640>
```

[36]:
```python
onx4 = seq4.to_onnx("Y", "X1", "X2", dtype=numpy.float32)
%onnxview onx4
```

[36]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x1c6d9428d90>
```

```
[37]:
```