onnx_operator_cost

March 10, 2022

1 Infer operator computation cost

This notebooks explores a way to predict the cost of operator Transpose based on some features.

[1]:
```
from jyquickhelper import add_notebook_menu
add_notebook_menu()
```

[1]:
```
<IPython.core.display.HTML object>
```

[2]:
```
%matplotlib inline
```

[3]:
```
%load_ext mlprodict
```

1.1 ONNX graph and measures

[4]:
```
import numpy
from skl2onnx.common.data_types import FloatTensorType
from skl2onnx.algebra.onnx_ops import OnnxTranspose


def create_onnx_graph(perm=(0, 1, 2, 3), target_opset=14):
    tr = OnnxTranspose('X', perm=perm, output_names=['Y'], op_version=target_opset)
    return tr.to_onnx({'X': FloatTensorType([None] * len(perm))})


onx = create_onnx_graph()

%onnxview onx
```

[4]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x258ef832760>
```

[5]:
```
from mlprodict.onnxrt import OnnxInference

onx = create_onnx_graph(perm=(1, 0, 3, 2))
oinf = OnnxInference(onx)
inputs = {'X': numpy.full((5, 6, 7, 8), 1, dtype=numpy.float32)}
res = oinf.run(inputs)['Y']
res.shape
```

[5]:
```
(6, 5, 8, 7)
```

[6]:
```
from onnxruntime import InferenceSession
sess = InferenceSession(onx.SerializeToString())
res = sess.run(None, inputs)[0]
res.shape
```

[6]:
```
(6, 5, 8, 7)
```

[7]:
```
from cpyquickhelper.numbers.speed_measure import measure_time

def measure_time_onnx(sess, X, number=50, repeat=30):
    inputs = {'X': X}
    return measure_time(lambda: sess.run(None, inputs), context=dict(sess=sess,␣
 ↪inputs=inputs),
                        div_by_number=True, number=number, repeat=repeat)

X = numpy.random.random((3, 224, 224, 4)).astype(numpy.float32)
measure_time_onnx(sess, X)
```

[7]:
```
{'average': 0.0024677738666666646,
 'deviation': 0.00022911153911864325,
 'min_exec': 0.0022292380000000023,
 'max_exec': 0.003265080000000005,
 'repeat': 30,
 'number': 50,
 'context_size': 232}
```

1.2 Simulation to build a database

1.2.1 Many dimensions, many permutations

[8]:
```
from itertools import permutations
from tqdm import tqdm
from pandas import DataFrame


def process_shape(shape, rnd=False, number=50, repeat=30, bar=True):
    X = numpy.random.random(shape).astype(numpy.float32)
    obs = []
    perms = list(permutations(list(range(len(X.shape)))))
    baseline = None
    itergen = perms if (rnd or not bar) else tqdm(perms)
    for perm in itergen:
        if baseline is not None and rnd:
            if random.randint(0, 4) != 0:
                continue
        onx = create_onnx_graph(perm=perm)
        sess = InferenceSession(onx.SerializeToString())
        res = measure_time_onnx(sess, X, number=number, repeat=repeat)
        res['perm'] = perm
        res['shape'] = shape
        if baseline is None:
            baseline = res
        res["ratio"] = res["average"] / baseline["average"]
```

```
        res['dim'] = len(shape)
        obs.append(res)
    return DataFrame(obs).sort_values('average')

dfs = []
df = process_shape((12, 13, 15, 18))
dfs.append(df)
df
```

```
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```

[8]:

	average	deviation	min_exec	max_exec	repeat	number	context_size	perm	shape	ratio	dim
3	0.000044	0.000006	0.000039	0.000057	30	50	232	(0, 2, 3, 1)	(12, 13, 15, 18)	0.750316	4
1	0.000048	0.000003	0.000045	0.000058	30	50	232	(0, 1, 3, 2)	(12, 13, 15, 18)	0.820821	4
18	0.000049	0.000003	0.000045	0.000062	30	50	232	(3, 0, 1, 2)	(12, 13, 15, 18)	0.823070	4
9	0.000049	0.000001	0.000048	0.000053	30	50	232	(1, 2, 3, 0)	(12, 13, 15, 18)	0.830604	4
12	0.000051	0.000004	0.000039	0.000062	30	50	232	(2, 0, 1, 3)	(12, 13, 15, 18)	0.861994	4
4	0.000052	0.000005	0.000047	0.000073	30	50	232	(0, 3, 1, 2)	(12, 13, 15, 18)	0.889753	4
8	0.000054	0.000006	0.000044	0.000067	30	50	232	(1, 2, 0, 3)	(12, 13, 15, 18)	0.909477	4
2	0.000054	0.000007	0.000049	0.000081	30	50	232	(0, 2, 1, 3)	(12, 13, 15, 18)	0.922354	4
14	0.000057	0.000006	0.000046	0.000064	30	50	232	(2, 1, 0, 3)	(12, 13, 15, 18)	0.972198	4
0	0.000059	0.000019	0.000034	0.000093	30	50	232	(0, 1, 2, 3)	(12, 13, 15, 18)	1.000000	4
6	0.000092	0.000019	0.000053	0.000139	30	50	232	(1, 0, 2, 3)	(12, 13, 15, 18)	1.557903	4
11	0.000136	0.000020	0.000119	0.000186	30	50	232	(1, 3, 2, 0)	(12, 13, 15, 18)	2.301556	4
13	0.000138	0.000023	0.000121	0.000181	30	50	232	(2, 0, 3, 1)	(12, 13, 15, 18)	2.336826	4
10	0.000138	0.000018	0.000118	0.000176	30	50	232	(1, 3, 0, 2)	(12, 13, 15, 18)	2.346118	4
16	0.000140	0.000015	0.000124	0.000193	30	50	232	(2, 3, 0, 1)	(12, 13, 15, 18)	2.379168	4
15	0.000144	0.000019	0.000119	0.000196	30	50	232				
17	0.000145	0.000022	0.000123	0.000199	30	50	232				
23	0.000145	0.000017	0.000125	0.000196	30	50	232				
20	0.000146	0.000015	0.000128	0.000184	30	50	232				
22	0.000150	0.000017	0.000127	0.000170	30	50	232				
19	0.000158	0.000021	0.000127	0.000192	30	50	232				
21	0.000164	0.000045	0.000124	0.000231	30	50	232				
7	0.000214	0.000060	0.000136	0.000295	30	50	232				
5	0.000215	0.000071	0.000143	0.000340	30	50	232				

```
15  (2, 1, 3, 0)  (12, 13, 15, 18)  2.443392    4
17  (2, 3, 1, 0)  (12, 13, 15, 18)  2.455098    4
23  (3, 2, 1, 0)  (12, 13, 15, 18)  2.456431    4
20  (3, 1, 0, 2)  (12, 13, 15, 18)  2.473250    4
22  (3, 2, 0, 1)  (12, 13, 15, 18)  2.539817    4
19  (3, 0, 2, 1)  (12, 13, 15, 18)  2.684876    4
21  (3, 1, 2, 0)  (12, 13, 15, 18)  2.778193    4
7   (1, 0, 3, 2)  (12, 13, 15, 18)  3.627240    4
5   (0, 3, 2, 1)  (12, 13, 15, 18)  3.640132    4
```

[9]:
```
df = process_shape((43, 44, 45))
dfs.append(df)
df
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 6/6 [00:01<00:00,  4.70it/s]
```

[9]:
```
    average  deviation  min_exec  max_exec  repeat  number  context_size  \
3  0.000073   0.000009  0.000062  0.000094      30      50           232
0  0.000074   0.000009  0.000065  0.000109      30      50           232
1  0.000077   0.000008  0.000069  0.000101      30      50           232
4  0.000097   0.000004  0.000083  0.000110      30      50           232
2  0.000113   0.000029  0.000061  0.000141      30      50           232
5  0.000375   0.000121  0.000292  0.000750      30      50           232

        perm         shape     ratio  dim
3  (1, 2, 0)  (43, 44, 45)  0.985513    3
0  (0, 1, 2)  (43, 44, 45)  1.000000    3
1  (0, 2, 1)  (43, 44, 45)  1.032759    3
4  (2, 0, 1)  (43, 44, 45)  1.300915    3
2  (1, 0, 2)  (43, 44, 45)  1.515711    3
5  (2, 1, 0)  (43, 44, 45)  5.054301    3
```

[10]:
```
df = process_shape((3, 244, 244))
dfs.append(df)
df
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 6/6 [00:01<00:00,  3.05it/s]
```

[10]:
```
    average  deviation  min_exec  max_exec  repeat  number  context_size  \
2  0.000100   0.000009  0.000090  0.000125      30      50           232
0  0.000105   0.000016  0.000078  0.000138      30      50           232
1  0.000123   0.000013  0.000108  0.000161      30      50           232
4  0.000124   0.000017  0.000108  0.000171      30      50           232
3  0.000151   0.000016  0.000136  0.000197      30      50           232
5  0.000672   0.000083  0.000626  0.001030      30      50           232

        perm           shape     ratio  dim
2  (1, 0, 2)  (3, 244, 244)  0.955203    3
0  (0, 1, 2)  (3, 244, 244)  1.000000    3
1  (0, 2, 1)  (3, 244, 244)  1.178827    3
4  (2, 0, 1)  (3, 244, 244)  1.185666    3
3  (1, 2, 0)  (3, 244, 244)  1.438446    3
```

```
5  (2, 1, 0)  (3, 244, 244)  6.418195    3
```

[11]:
```
df = process_shape((3, 244, 244, 1))
dfs.append(df)
df
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 24/24 [00:19<00:00,  1.26it/s]
```

[11]:

	average	deviation	min_exec	max_exec	repeat	number	context_size	\
4	0.000092	0.000008	0.000078	0.000107	30	50	232	
0	0.000107	0.000018	0.000084	0.000157	30	50	232	
6	0.000124	0.000068	0.000088	0.000323	30	50	232	
12	0.000126	0.000017	0.000107	0.000185	30	50	232	
3	0.000130	0.000009	0.000120	0.000163	30	50	232	
18	0.000137	0.000047	0.000090	0.000250	30	50	232	
1	0.000147	0.000017	0.000106	0.000175	30	50	232	
8	0.000185	0.000017	0.000164	0.000246	30	50	232	
9	0.000189	0.000044	0.000142	0.000265	30	50	232	
2	0.000201	0.000054	0.000121	0.000289	30	50	232	
7	0.000522	0.000061	0.000457	0.000733	30	50	232	
10	0.000533	0.000157	0.000456	0.001128	30	50	232	
13	0.000640	0.000189	0.000477	0.001289	30	50	232	
16	0.000660	0.000106	0.000503	0.000860	30	50	232	
5	0.000692	0.000136	0.000529	0.001021	30	50	232	
19	0.000749	0.000206	0.000508	0.001324	30	50	232	
14	0.000754	0.000105	0.000633	0.000994	30	50	232	
11	0.000791	0.000264	0.000561	0.001386	30	50	232	
15	0.000818	0.000278	0.000625	0.001522	30	50	232	
17	0.000893	0.000212	0.000646	0.001477	30	50	232	
21	0.000944	0.000293	0.000581	0.001626	30	50	232	
20	0.000976	0.000347	0.000584	0.001742	30	50	232	
22	0.001011	0.000337	0.000544	0.001810	30	50	232	
23	0.001128	0.000322	0.000629	0.001737	30	50	232	

	perm	shape	ratio	dim
4	(0, 3, 1, 2)	(3, 244, 244, 1)	0.859903	4
0	(0, 1, 2, 3)	(3, 244, 244, 1)	1.000000	4
6	(1, 0, 2, 3)	(3, 244, 244, 1)	1.162456	4
12	(2, 0, 1, 3)	(3, 244, 244, 1)	1.180996	4
3	(0, 2, 3, 1)	(3, 244, 244, 1)	1.210077	4
18	(3, 0, 1, 2)	(3, 244, 244, 1)	1.276642	4
1	(0, 1, 3, 2)	(3, 244, 244, 1)	1.369978	4
8	(1, 2, 0, 3)	(3, 244, 244, 1)	1.725391	4
9	(1, 2, 3, 0)	(3, 244, 244, 1)	1.766905	4
2	(0, 2, 1, 3)	(3, 244, 244, 1)	1.878802	4
7	(1, 0, 3, 2)	(3, 244, 244, 1)	4.874009	4
10	(1, 3, 0, 2)	(3, 244, 244, 1)	4.973916	4
13	(2, 0, 3, 1)	(3, 244, 244, 1)	5.980796	4
16	(2, 3, 0, 1)	(3, 244, 244, 1)	6.167703	4
5	(0, 3, 2, 1)	(3, 244, 244, 1)	6.460759	4
19	(3, 0, 2, 1)	(3, 244, 244, 1)	6.996362	4
14	(2, 1, 0, 3)	(3, 244, 244, 1)	7.041007	4
11	(1, 3, 2, 0)	(3, 244, 244, 1)	7.389431	4

```
15  (2, 1, 3, 0)  (3, 244, 244, 1)   7.634646    4
17  (2, 3, 1, 0)  (3, 244, 244, 1)   8.339926    4
21  (3, 1, 2, 0)  (3, 244, 244, 1)   8.814785    4
20  (3, 1, 0, 2)  (3, 244, 244, 1)   9.112243    4
22  (3, 2, 0, 1)  (3, 244, 244, 1)   9.437403    4
23  (3, 2, 1, 0)  (3, 244, 244, 1)  10.530182    4
```

[12]:
```
df = process_shape((1, 244, 244, 3))
dfs.append(df)
df
```

```
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```

[12]:
```
      average  deviation  min_exec  max_exec  repeat  number  context_size  \
8    0.000092   0.000014  0.000078  0.000132      30      50           232
6    0.000098   0.000013  0.000083  0.000142      30      50           232
9    0.000107   0.000018  0.000075  0.000137      30      50           232
3    0.000115   0.000015  0.000092  0.000147      30      50           232
0    0.000122   0.000028  0.000094  0.000201      30      50           232
1    0.000194   0.000036  0.000160  0.000311      30      50           232
4    0.000195   0.000019  0.000163  0.000258      30      50           232
18   0.000235   0.000058  0.000172  0.000345      30      50           232
2    0.000408   0.000156  0.000229  0.000718      30      50           232
12   0.000513   0.000215  0.000300  0.001430      30      50           232
10   0.000558   0.000131  0.000458  0.001023      30      50           232
7    0.000604   0.000188  0.000471  0.001065      30      50           232
14   0.000620   0.000142  0.000410  0.001121      30      50           232
23   0.000679   0.000097  0.000590  0.000928      30      50           232
22   0.000710   0.000161  0.000620  0.001390      30      50           232
17   0.000737   0.000240  0.000493  0.001174      30      50           232
11   0.000824   0.000288  0.000515  0.001879      30      50           232
21   0.000913   0.000216  0.000613  0.001410      30      50           232
20   0.000918   0.000328  0.000572  0.002079      30      50           232
16   0.001057   0.000609  0.000502  0.002702      30      50           232
5    0.001061   0.000612  0.000539  0.003790      30      50           232
19   0.001212   0.000417  0.000719  0.002561      30      50           232
15   0.001311   0.000505  0.000856  0.003377      30      50           232
13   0.001433   0.000505  0.000721  0.002335      30      50           232

            perm             shape      ratio  dim
8   (1, 2, 0, 3)  (1, 244, 244, 3)   0.753009    4
6   (1, 0, 2, 3)  (1, 244, 244, 3)   0.802808    4
9   (1, 2, 3, 0)  (1, 244, 244, 3)   0.873932    4
3   (0, 2, 3, 1)  (1, 244, 244, 3)   0.940606    4
0   (0, 1, 2, 3)  (1, 244, 244, 3)   1.000000    4
1   (0, 1, 3, 2)  (1, 244, 244, 3)   1.585479    4
4   (0, 3, 1, 2)  (1, 244, 244, 3)   1.598770    4
18  (3, 0, 1, 2)  (1, 244, 244, 3)   1.923654    4
2   (0, 2, 1, 3)  (1, 244, 244, 3)   3.345406    4
12  (2, 0, 1, 3)  (1, 244, 244, 3)   4.205477    4
10  (1, 3, 0, 2)  (1, 244, 244, 3)   4.572658    4
7   (1, 0, 3, 2)  (1, 244, 244, 3)   4.947937    4
14  (2, 1, 0, 3)  (1, 244, 244, 3)   5.078387    4
```

```
23  (3, 2, 1, 0)  (1, 244, 244, 3)   5.561888    4
22  (3, 2, 0, 1)  (1, 244, 244, 3)   5.818089    4
17  (2, 3, 1, 0)  (1, 244, 244, 3)   6.040189    4
11  (1, 3, 2, 0)  (1, 244, 244, 3)   6.752663    4
21  (3, 1, 2, 0)  (1, 244, 244, 3)   7.476378    4
20  (3, 1, 0, 2)  (1, 244, 244, 3)   7.521481    4
16  (2, 3, 0, 1)  (1, 244, 244, 3)   8.657076    4
5   (0, 3, 2, 1)  (1, 244, 244, 3)   8.693870    4
19  (3, 0, 2, 1)  (1, 244, 244, 3)   9.929308    4
15  (2, 1, 3, 0)  (1, 244, 244, 3)  10.739398    4
13  (2, 0, 3, 1)  (1, 244, 244, 3)  11.740772    4
```

[13]:
```
df = process_shape((3, 244, 244, 3), number=15, repeat=15)
dfs.append(df)
df
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 24/24 [00:14<00:00,  1.62it/s]
```

[13]:

	average	deviation	min_exec	max_exec	repeat	number	context_size	perm	shape	ratio	dim
0	0.001088	0.000085	0.000986	0.001291	15	15	232	(0, 1, 2, 3)	(3, 244, 244, 3)	1.000000	4
4	0.001227	0.000088	0.001152	0.001474	15	15	232	(0, 3, 1, 2)	(3, 244, 244, 3)	1.128126	4
18	0.001277	0.000118	0.001079	0.001490	15	15	232	(3, 0, 1, 2)	(3, 244, 244, 3)	1.173721	4
6	0.001311	0.000320	0.001007	0.001925	15	15	232	(1, 0, 2, 3)	(3, 244, 244, 3)	1.205182	4
1	0.001415	0.000307	0.001200	0.002498	15	15	232	(0, 1, 3, 2)	(3, 244, 244, 3)	1.300901	4
3	0.001426	0.000221	0.001191	0.001863	15	15	232	(0, 2, 3, 1)	(3, 244, 244, 3)	1.311361	4
9	0.001510	0.000432	0.001132	0.002417	15	15	232	(1, 2, 3, 0)	(3, 244, 244, 3)	1.388068	4
8	0.001552	0.000030	0.001500	0.001602	15	15	232	(1, 2, 0, 3)	(3, 244, 244, 3)	1.427105	4
12	0.001724	0.000193	0.001470	0.002142	15	15	232				
2	0.001790	0.000191	0.001566	0.002238	15	15	232				
7	0.002528	0.000154	0.002327	0.002983	15	15	232				
19	0.002571	0.000186	0.002383	0.002922	15	15	232				
21	0.002591	0.000253	0.002431	0.003403	15	15	232				
22	0.002698	0.000412	0.002346	0.003689	15	15	232				
20	0.002806	0.000783	0.002147	0.004296	15	15	232				
16	0.003212	0.000304	0.002773	0.003851	15	15	232				
14	0.003228	0.000796	0.002071	0.004791	15	15	232				
11	0.003257	0.000287	0.002912	0.003739	15	15	232				
17	0.003574	0.000479	0.003028	0.005042	15	15	232				
10	0.003942	0.001860	0.002446	0.008241	15	15	232				
15	0.004249	0.001217	0.003175	0.008041	15	15	232				
5	0.004685	0.001343	0.002827	0.006868	15	15	232				
13	0.005539	0.002180	0.002991	0.009602	15	15	232				
23	0.005575	0.001930	0.002876	0.008157	15	15	232				

```
12  (2, 0, 1, 3)  (3, 244, 244, 3)  1.585155    4
2   (0, 2, 1, 3)  (3, 244, 244, 3)  1.645717    4
7   (1, 0, 3, 2)  (3, 244, 244, 3)  2.324384    4
19  (3, 0, 2, 1)  (3, 244, 244, 3)  2.363443    4
21  (3, 1, 2, 0)  (3, 244, 244, 3)  2.381860    4
22  (3, 2, 0, 1)  (3, 244, 244, 3)  2.480308    4
20  (3, 1, 0, 2)  (3, 244, 244, 3)  2.579517    4
16  (2, 3, 0, 1)  (3, 244, 244, 3)  2.953032    4
14  (2, 1, 0, 3)  (3, 244, 244, 3)  2.967523    4
11  (1, 3, 2, 0)  (3, 244, 244, 3)  2.994043    4
17  (2, 3, 1, 0)  (3, 244, 244, 3)  3.285842    4
10  (1, 3, 0, 2)  (3, 244, 244, 3)  3.624145    4
15  (2, 1, 3, 0)  (3, 244, 244, 3)  3.906361    4
5   (0, 3, 2, 1)  (3, 244, 244, 3)  4.307072    4
13  (2, 0, 3, 1)  (3, 244, 244, 3)  5.092422    4
23  (3, 2, 1, 0)  (3, 244, 244, 3)  5.125597    4
```

[14]:
```
df = process_shape((3, 244, 244, 6), number=15, repeat=15)
dfs.append(df)
df
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 24/24 [00:34<00:00,  1.43s/it]
```

[14]:
```
      average  deviation  min_exec  max_exec  repeat  number  context_size  \
1    0.002249   0.000144  0.002067  0.002627      15      15           232
3    0.002711   0.000171  0.002458  0.002995      15      15           232
12   0.002773   0.000683  0.002260  0.004103      15      15           232
4    0.002953   0.000677  0.002187  0.004132      15      15           232
2    0.003232   0.000963  0.002303  0.005088      15      15           232
6    0.003363   0.000372  0.002883  0.004025      15      15           232
8    0.003397   0.000237  0.002886  0.003846      15      15           232
9    0.003653   0.000874  0.002567  0.005244      15      15           232
14   0.003697   0.000186  0.003495  0.004150      15      15           232
0    0.003705   0.000797  0.002111  0.005164      15      15           232
18   0.003780   0.000882  0.002701  0.005402      15      15           232
10   0.004938   0.000367  0.004532  0.005844      15      15           232
7    0.005918   0.001085  0.004598  0.008312      15      15           232
13   0.006106   0.000556  0.005619  0.007305      15      15           232
11   0.006722   0.001807  0.005067  0.011245      15      15           232
20   0.007071   0.000982  0.005454  0.008559      15      15           232
21   0.007441   0.001732  0.006199  0.012169      15      15           232
15   0.007815   0.001757  0.005932  0.010779      15      15           232
16   0.008546   0.001384  0.005878  0.010614      15      15           232
5    0.010339   0.002789  0.005878  0.018301      15      15           232
17   0.010677   0.001457  0.008504  0.014070      15      15           232
23   0.012421   0.003052  0.007818  0.018106      15      15           232
22   0.013432   0.004496  0.006536  0.021250      15      15           232
19   0.014579   0.004026  0.007144  0.020739      15      15           232

            perm             shape     ratio  dim
1    (0, 1, 3, 2)  (3, 244, 244, 6)  0.606961    4
3    (0, 2, 3, 1)  (3, 244, 244, 6)  0.731795    4
12   (2, 0, 1, 3)  (3, 244, 244, 6)  0.748578    4
```

```
4   (0, 3, 1, 2)  (3, 244, 244, 6)  0.797062    4
2   (0, 2, 1, 3)  (3, 244, 244, 6)  0.872427    4
6   (1, 0, 2, 3)  (3, 244, 244, 6)  0.907834    4
8   (1, 2, 0, 3)  (3, 244, 244, 6)  0.917011    4
9   (1, 2, 3, 0)  (3, 244, 244, 6)  0.986071    4
14  (2, 1, 0, 3)  (3, 244, 244, 6)  0.997901    4
0   (0, 1, 2, 3)  (3, 244, 244, 6)  1.000000    4
18  (3, 0, 1, 2)  (3, 244, 244, 6)  1.020432    4
10  (1, 3, 0, 2)  (3, 244, 244, 6)  1.333061    4
7   (1, 0, 3, 2)  (3, 244, 244, 6)  1.597357    4
13  (2, 0, 3, 1)  (3, 244, 244, 6)  1.648325    4
11  (1, 3, 2, 0)  (3, 244, 244, 6)  1.814552    4
20  (3, 1, 0, 2)  (3, 244, 244, 6)  1.908667    4
21  (3, 1, 2, 0)  (3, 244, 244, 6)  2.008635    4
15  (2, 1, 3, 0)  (3, 244, 244, 6)  2.109489    4
16  (2, 3, 0, 1)  (3, 244, 244, 6)  2.306951    4
5   (0, 3, 2, 1)  (3, 244, 244, 6)  2.790823    4
17  (2, 3, 1, 0)  (3, 244, 244, 6)  2.882191    4
23  (3, 2, 1, 0)  (3, 244, 244, 6)  3.352770    4
22  (3, 2, 0, 1)  (3, 244, 244, 6)  3.625680    4
19  (3, 0, 2, 1)  (3, 244, 244, 6)  3.935483    4
```

1.2.2 Random cases

[15]:
```python
import random

if False:  # comment out for more training data
    for i in tqdm(range(0, 30)):
        dim = random.randint(3, 5)
        total = 1e8
        while total > 1e6 or total < 0:
            if dim == 3:
                shape = [random.randint(3, 64), random.randint(3, 224), random.
 ↪randint(3, 64)]
            elif dim == 4:
                shape = (
                    [random.randint(3, 8)] +
                    [random.randint(16, 224) for d in range(2)] +
                    [random.randint(16, 64)])
            elif dim == 5:
                shape = (
                    [random.randint(3, 8)] +
                    [random.randint(16, 32) for d in range(3)] +
                    [random.randint(16, 64)])
            else:
                raise NotImplementedError()
            ashape = numpy.array(shape, dtype=numpy.float64)
            total = numpy.prod(ashape)

        if total > 1000000:
            number, repeat = 2, 2
        elif total > 800000:
            number, repeat = 3, 3
```

```
        elif total > 500000:
            number, repeat = 5, 5
        elif total > 200000:
            number, repeat = 7, 7
        else:
            number, repeat = 10, 10

        df = process_shape(tuple(shape), number=number, repeat=repeat, bar=False)
        dfs.append(df)

        for i in range(len(shape)):
            shape2 = shape.copy()
            shape2[i] = 1
            df = process_shape(tuple(shape), number=number, repeat=repeat, bar=False)
            dfs.append(df)

len(dfs)
```

[15]:
```
7
```

[16]:
```
import pandas

data = pandas.concat(dfs, axis=0).reset_index(drop=True)
data.tail()
```

[16]:

	average	deviation	min_exec	max_exec	repeat	number	context_size	perm	shape	ratio	dim
127	0.010339	0.002789	0.005878	0.018301	15	15	232	(0, 3, 2, 1)	(3, 244, 244, 6)	2.790823	4
128	0.010677	0.001457	0.008504	0.014070	15	15	232	(2, 3, 1, 0)	(3, 244, 244, 6)	2.882191	4
129	0.012421	0.003052	0.007818	0.018106	15	15	232	(3, 2, 1, 0)	(3, 244, 244, 6)	3.352770	4
130	0.013432	0.004496	0.006536	0.021250	15	15	232	(3, 2, 0, 1)	(3, 244, 244, 6)	3.625680	4
131	0.014579	0.004026	0.007144	0.020739	15	15	232	(3, 0, 2, 1)	(3, 244, 244, 6)	3.935483	4

[17]:
```
data.shape
```

[17]:
```
(132, 11)
```

[18]:
```
data[['dim', 'shape', 'ratio']].groupby(['dim', 'shape']).agg({'ratio': [min, max,␣
↪numpy.mean, numpy.median]})
```

[18]:

		ratio			
		min	max	mean	median
dim	shape				
3	(3, 244, 244)	0.955203	6.418195	2.029389	1.182247
	(43, 44, 45)	0.985513	5.054301	1.814867	1.166837
4	(1, 244, 244, 3)	0.753009	11.740772	5.023301	5.013162
	(3, 244, 244, 1)	0.859903	10.530182	4.882680	5.477356
	(3, 244, 244, 3)	1.000000	5.125597	2.481287	2.372651

```
(3, 244, 244, 6)  0.606961   3.935483  1.704169  1.465209
(12, 13, 15, 18)  0.750316   3.640132  1.866691  2.319191
```

1.3 features

1.3.1 Computing the features

[19]:
```
def _edit_distance(mot1, mot2):
    dist = {(-1, -1): 0}
    pred = {(-1, -1): None}
    if len(mot1) == 0:
        for j, d in enumerate(mot2):
            dist[-1, j] = dist[-1, j - 1] + 1
            pred[-1, j] = (-1, j - 1)
            dist[j, -1] = dist[j - 1, -1] + 1
            pred[j, -1] = (j - 1, -1)
    for i, c in enumerate(mot1):
        dist[i, -1] = dist[i - 1, -1] + 1
        pred[i, -1] = (i - 1, -1)
        dist[-1, i] = dist[-1, i - 1] + 1
        pred[-1, i] = (-1, i - 1)
        for j, d in enumerate(mot2):
            opt = []
            if (i - 1, j) in dist:
                x = dist[i - 1, j] + 1
                opt.append((x, (i - 1, j)))
            if (i, j - 1) in dist:
                x = dist[i, j - 1] + 1
                opt.append((x, (i, j - 1)))
            if (i - 1, j - 1) in dist:
                x = dist[i - 1, j - 1] + (1 if c != d else 0)
                opt.append((x, (i - 1, j - 1)))
            mi = min(opt)
            dist[i, j] = mi[0]
            pred[i, j] = mi[1]

    return dist[len(mot1) - 1, len(mot2) - 1]

_edit_distance("abdc", "cbda")
```

[19]: 2

[20]:
```
_edit_distance((0, 1, 2, 3), (0, 2, 1, 3))
```

[20]: 2

[21]:
```
from math import log


def _is_rotation(perm):
    t = tuple(perm)
    c = list(range(len(perm)))
    for i in range(len(c)):
```

```
        for k in range(len(c)):
            c[k] = (k + i) % len(c)
        if t == tuple(c):
            return True
    return False


def _relu(x, origin=0):
    return origin if x < origin else x


def compute_features(shape, perm):
    total = numpy.prod(numpy.array(shape, dtype=numpy.int64))

    begin = 1
    dbegin = 0
    for i, p in enumerate(perm):
        if p != i:
            break
        dbegin += 1
        begin *= shape[i]

    end = 1
    dend = 0
    for i in range(len(perm)-1, -1, -1):
        if perm[i] != i:
            break
        dend += 1
        end *= shape[i]

    dis_cont = 0
    for i in range(1, len(shape)):
        if perm[i] != perm[i-1] + 1:
            dis_cont += 1

    middle = max(1, int(total / (end * begin)))
    feat = dict(size=total, begin=begin, end=end, middle=middle,
                dim=len(shape), discont=dis_cont)

    for c in [16, 32]:
        feat["end%d" % c] = _relu(end, c)

    keys = list(feat)
    for k in keys:
        if k in {'dim', 'cpu', 'size'}:
            continue
        feat['r%s' % k] = float(feat[k] / total)

    for c in [2, 4, 8, 16, 32, 64]:
        feat["iend%d" % c] = float(end >= c)
        feat["ibegin%d" % c] = float(begin >= c)

    # feat['CST'] = 1
```

```
    feat['CST_'] = -1
    feat['dbegin'] = - dbegin
    feat['dend'] = - dend

    keys = list(feat)
    for k in keys:
        if k.startswith('end') or k.startswith('begin'):
            feat[k] = - feat[k]
        elif k.startswith('rend') or k.startswith('rbegin'):
            feat[k] = - feat[k]
        elif k.startswith('iend') or k.startswith('ibegin'):
            feat[k] = - feat[k]
        elif k == "rdiscont":
            feat[k] = - feat[k]

    idp = list(range(len(perm)))
    feat["rot"] = -1 if _is_rotation(perm) else 0
    feat["rev"] = 1 if perm == tuple(idp[::-1]) else 0
    feat["edit"] = _edit_distance(idp, perm)
    feat["redit"] = feat["edit"] / len(idp)
    return feat


compute_features((3, 5, 7), (0, 1, 2))
```

[21]:
```
{'size': 105,
 'begin': -105,
 'end': -105,
 'middle': 1,
 'dim': 3,
 'discont': 0,
 'end16': -105,
 'end32': -105,
 'rbegin': -1.0,
 'rend': -1.0,
 'rmiddle': 0.009523809523809525,
 'rdiscont': -0.0,
 'rend16': -1.0,
 'rend32': -1.0,
 'iend2': -1.0,
 'ibegin2': -1.0,
 'iend4': -1.0,
 'ibegin4': -1.0,
 'iend8': -1.0,
 'ibegin8': -1.0,
 'iend16': -1.0,
 'ibegin16': -1.0,
 'iend32': -1.0,
 'ibegin32': -1.0,
 'iend64': -1.0,
 'ibegin64': -1.0,
 'CST_': -1,
 'dbegin': -3,
```

```
 'dend': -3,
 'rot': -1,
 'rev': 0,
 'edit': 0,
 'redit': 0.0}
```

[22]:
```
compute_features((3, 5, 7), (2, 1, 0))
```

[22]:
```
{'size': 105,
 'begin': -1,
 'end': -1,
 'middle': 105,
 'dim': 3,
 'discont': 2,
 'end16': -16,
 'end32': -32,
 'rbegin': -0.009523809523809525,
 'rend': -0.009523809523809525,
 'rmiddle': 1.0,
 'rdiscont': -0.01904761904761905,
 'rend16': -0.1523809523809524,
 'rend32': -0.3047619047619048,
 'iend2': -0.0,
 'ibegin2': -0.0,
 'iend4': -0.0,
 'ibegin4': -0.0,
 'iend8': -0.0,
 'ibegin8': -0.0,
 'iend16': -0.0,
 'ibegin16': -0.0,
 'iend32': -0.0,
 'ibegin32': -0.0,
 'iend64': -0.0,
 'ibegin64': -0.0,
 'CST_': -1,
 'dbegin': 0,
 'dend': 0,
 'rot': 0,
 'rev': 1,
 'edit': 2,
 'redit': 0.6666666666666666}
```

[23]:
```
compute_features((3, 5, 7), (1, 2, 0))
```

[23]:
```
{'size': 105,
 'begin': -1,
 'end': -1,
 'middle': 105,
 'dim': 3,
 'discont': 1,
 'end16': -16,
 'end32': -32,
 'rbegin': -0.009523809523809525,
 'rend': -0.009523809523809525,
```

```
 'rmiddle': 1.0,
 'rdiscont': -0.009523809523809525,
 'rend16': -0.1523809523809524,
 'rend32': -0.3047619047619048,
 'iend2': -0.0,
 'ibegin2': -0.0,
 'iend4': -0.0,
 'ibegin4': -0.0,
 'iend8': -0.0,
 'ibegin8': -0.0,
 'iend16': -0.0,
 'ibegin16': -0.0,
 'iend32': -0.0,
 'ibegin32': -0.0,
 'iend64': -0.0,
 'ibegin64': -0.0,
 'CST_': -1,
 'dbegin': 0,
 'dend': 0,
 'rot': -1,
 'rev': 0,
 'edit': 2,
 'redit': 0.6666666666666666}
```

1.3.2 Computing the features for all simulations

[24]:
```
def compute_features_dataframe(df):

    def merge(row):
        feat = compute_features(row['shape'], row['perm'])
        feat['yt'] = row['average']
        feat['yr'] = row['ratio']
        return feat

    rows = []
    for i in tqdm(range(df.shape[0])):
        rows.append(dict(shape=df.loc[i, "shape"], perm=df.loc[i, "perm"],
                         average=df.loc[i, "average"], ratio=df.loc[i, "ratio"]))
    obs = []
    for row in tqdm(rows):
        obs.append(merge(row))
    return DataFrame(obs)

fdata = compute_features_dataframe(data)
col_sort = list(sorted(fdata.columns))
fdata = fdata[col_sort]
fdata.tail()
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 132/132 [00:00<00:00, 9459.22it/s]
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 132/132 [00:00<00:00, 3601.95it/s]
```

[24]:
```
      CST_  begin  dbegin  dend  dim  discont  edit  end  end16  end32  …  \
127    -1     -3      -1     0    4        3     2   -1    -16    -32  …
128    -1     -1       0     0    4        2     4   -1    -16    -32  …
129    -1     -1       0     0    4        3     4   -1    -16    -32  …
130    -1     -1       0     0    4        2     4   -1    -16    -32  …
131    -1     -1       0     0    4        3     3   -1    -16    -32  …

     redit          rend    rend16   rend32  rev   rmiddle  rot     size  \
127   0.50 -9.331422e-07 -0.000015 -0.00003    0  0.333333    0  1071648
128   1.00 -9.331422e-07 -0.000015 -0.00003    0  1.000000    0  1071648
129   1.00 -9.331422e-07 -0.000015 -0.00003    1  1.000000    0  1071648
130   1.00 -9.331422e-07 -0.000015 -0.00003    0  1.000000    0  1071648
131   0.75 -9.331422e-07 -0.000015 -0.00003    0  1.000000    0  1071648

           yr        yt
127  2.790823  0.010339
128  2.882191  0.010677
129  3.352770  0.012421
130  3.625680  0.013432
131  3.935483  0.014579

[5 rows x 35 columns]
```

1.3.3 correlations

[25]:
```
fdata.corr()
```

[25]:
```
           CST_     begin    dbegin      dend       dim   discont      edit  \
CST_        NaN       NaN       NaN       NaN       NaN       NaN       NaN
begin       NaN  1.000000  0.596816  0.596414  0.014118  0.404952  0.405175
dbegin      NaN  0.596816  1.000000  0.676899  0.077162  0.486887  0.669598
dend        NaN  0.596414  0.676899  1.000000  0.077162  0.486887  0.669598
dim         NaN  0.014118  0.077162  0.077162  1.000000  0.305320  0.272614
discont     NaN  0.404952  0.486887  0.486887  0.305320  1.000000  0.531254
edit        NaN  0.405175  0.669598  0.669598  0.272614  0.531254  1.000000
end         NaN  0.999998  0.596384  0.596936  0.014153  0.404971  0.405223
end16       NaN  0.999998  0.596374  0.596907  0.014145  0.404961  0.405204
end32       NaN  0.999998  0.596363  0.596881  0.014135  0.404948  0.405189
ibegin16    NaN  0.488586  0.854056  0.553792  0.160800  0.476225  0.528132
ibegin2     NaN  0.297779  0.792225  0.326418  0.080033  0.230393  0.539082
ibegin32    NaN  0.488586  0.854056  0.553792  0.160800  0.476225  0.528132
ibegin4     NaN  0.420023  0.814178  0.474232  0.114432  0.388689  0.517951
ibegin64    NaN  0.510659  0.869357  0.586430  0.083333  0.488512  0.533376
ibegin8     NaN  0.420023  0.814178  0.474232  0.114432  0.388689  0.517951
iend16      NaN  0.405858  0.452474  0.807989  0.181503  0.383654  0.517311
iend2       NaN  0.297323  0.326418  0.792225  0.080033  0.230393  0.539082
iend32      NaN  0.468224  0.524298  0.841277  0.233408  0.465593  0.524167
iend4       NaN  0.360597  0.400119  0.792963  0.141421  0.321878  0.519634
iend64      NaN  0.487959  0.553792  0.854056  0.160800  0.476225  0.528132
iend8       NaN  0.405858  0.452474  0.807989  0.181503  0.383654  0.517311
middle      NaN  0.126896  0.303868  0.319057  0.178317  0.152095  0.377874
rbegin      NaN  0.681576  0.832794  0.831933  0.160296  0.594171  0.594568
rdiscont    NaN -0.132163 -0.158903 -0.158903 -0.077379 -0.320270 -0.168278
```

redit	NaN	0.418022	0.690333	0.690333	0.115902	0.504206	0.984655
rend	NaN	0.681573	0.831887	0.833059	0.160407	0.594219	0.594688
rend16	NaN	0.681573	0.831895	0.832975	0.160417	0.594226	0.594639
rend32	NaN	0.681573	0.831903	0.832924	0.160414	0.594223	0.594619
rev	NaN	0.038216	0.111636	0.111636	-0.160357	0.150144	0.208568
rmiddle	NaN	0.256349	0.605990	0.623582	0.106693	0.225854	0.652532
rot	NaN	0.325594	0.298090	0.298090	0.240946	0.823937	0.338994
size	NaN	-0.133581	0.016411	0.016411	0.212685	0.064937	0.057981
yr	NaN	0.127658	0.291318	0.305489	0.138961	0.388140	0.464225
yt	NaN	-0.008816	0.139951	0.155098	0.192305	0.203342	0.283262

	end	end16	end32	…	redit	rend	rend16	\
CST_	NaN	NaN	NaN	…	NaN	NaN	NaN	
begin	0.999998	0.999998	0.999998	…	0.418022	0.681573	0.681573	
dbegin	0.596384	0.596374	0.596363	…	0.690333	0.831887	0.831895	
dend	0.596936	0.596907	0.596881	…	0.690333	0.833059	0.832975	
dim	0.014153	0.014145	0.014135	…	0.115902	0.160407	0.160417	
discont	0.404971	0.404961	0.404948	…	0.504206	0.594219	0.594226	
edit	0.405223	0.405204	0.405189	…	0.984655	0.594688	0.594639	
end	1.000000	1.000000	1.000000	…	0.418062	0.681565	0.681565	
end16	1.000000	1.000000	1.000000	…	0.418044	0.681550	0.681550	
end32	1.000000	1.000000	1.000000	…	0.418029	0.681533	0.681533	
ibegin16	0.487938	0.487930	0.487919	…	0.533981	0.715870	0.715889	
ibegin2	0.297285	0.297281	0.297277	…	0.548605	0.436111	0.436126	
ibegin32	0.487938	0.487930	0.487919	…	0.533981	0.715870	0.715889	
ibegin4	0.419433	0.419424	0.419415	…	0.528284	0.615385	0.615471	
ibegin64	0.509999	0.509990	0.509979	…	0.555529	0.748243	0.748253	
ibegin8	0.419433	0.419424	0.419415	…	0.528284	0.615385	0.615471	
iend16	0.406614	0.406575	0.406542	…	0.513917	0.597165	0.597061	
iend2	0.297930	0.297895	0.297872	…	0.548605	0.437541	0.437398	
iend32	0.469061	0.469029	0.468993	…	0.514805	0.688714	0.688640	
iend4	0.361290	0.361251	0.361222	…	0.521120	0.530594	0.530467	
iend64	0.488816	0.488786	0.488752	…	0.533981	0.717677	0.717612	
iend8	0.406614	0.406575	0.406542	…	0.513917	0.597165	0.597061	
middle	0.126960	0.126947	0.126937	…	0.355980	0.186472	0.186669	
rbegin	0.681564	0.681549	0.681532	…	0.613417	0.999992	0.999993	
rdiscont	-0.132191	-0.132195	-0.132192	…	-0.163660	-0.193464	-0.193106	
redit	0.418062	0.418044	0.418029	…	1.000000	0.613517	0.613466	
rend	0.681565	0.681550	0.681533	…	0.613517	1.000000	1.000000	
rend16	0.681565	0.681550	0.681533	…	0.613466	1.000000	1.000000	
rend32	0.681565	0.681550	0.681533	…	0.613446	0.999999	1.000000	
rev	0.038236	0.038231	0.038228	…	0.244134	0.056153	0.056129	
rmiddle	0.256479	0.256451	0.256430	…	0.655658	0.376658	0.376574	
rot	0.325559	0.325557	0.325552	…	0.317106	0.477579	0.477613	
size	-0.133665	-0.133671	-0.133677	…	0.024651	0.034412	0.034679	
yr	0.127730	0.127716	0.127707	…	0.450928	0.187551	0.187559	
yt	-0.008844	-0.008852	-0.008859	…	0.256097	0.095557	0.095755	

	rend32	rev	rmiddle	rot	size	yr	yt
CST_	NaN	NaN	NaN	NaN	NaN	NaN	NaN
begin	0.681573	0.038216	0.256349	0.325594	-0.133581	0.127658	-0.008816
dbegin	0.831903	0.111636	0.605990	0.298090	0.016411	0.291318	0.139951
dend	0.832924	0.111636	0.623582	0.298090	0.016411	0.305489	0.155098

```
dim       0.160414 -0.160357  0.106693  0.240946  0.212685  0.138961  0.192305
discont   0.594223  0.150144  0.225854  0.823937  0.064937  0.388140  0.203342
edit      0.594619  0.208568  0.652532  0.338994  0.057981  0.464225  0.283262
end       0.681565  0.038236  0.256479  0.325559 -0.133665  0.127730 -0.008844
end16     0.681550  0.038231  0.256451  0.325557 -0.133671  0.127716 -0.008852
end32     0.681533  0.038228  0.256430  0.325552 -0.133677  0.127707 -0.008859
ibegin16  0.715901  0.078215  0.522399  0.283800  0.037462  0.254352  0.136416
ibegin2   0.436148  0.128338  0.685605  0.049586 -0.027198  0.324851  0.109929
ibegin32  0.715901  0.078215  0.522399  0.283800  0.037462  0.254352  0.136416
ibegin4   0.615584  0.090985  0.594774  0.207651  0.116460  0.278810  0.179014
ibegin64  0.748257  0.074833  0.501069  0.307207  0.017724  0.243128  0.124459
ibegin8   0.615584  0.090985  0.594774  0.207651  0.116460  0.278810  0.179014
iend16    0.597069  0.094032  0.619928  0.191751  0.125689  0.307633  0.182267
iend2     0.437338  0.128338  0.724562  0.049586 -0.027198  0.337071  0.146321
iend32    0.688553  0.081511  0.544599  0.262448  0.049643  0.261930  0.139099
iend4     0.530439  0.105830  0.673384  0.136300 -0.039355  0.351151  0.127468
iend64    0.717534  0.078215  0.523746  0.283800  0.030938  0.255567  0.127728
iend8     0.597069  0.094032  0.619928  0.191751  0.125689  0.307633  0.182267
middle    0.186918  0.052991  0.467981  0.000903  0.728990 -0.008120  0.821357
rbegin    0.999993  0.056108  0.376357  0.477649  0.034328  0.187411  0.095471
rdiscont -0.192632 -0.054527 -0.001602 -0.265672  0.551880  0.004517  0.386893
redit     0.613446  0.244134  0.655658  0.317106  0.024651  0.450928  0.256097
rend      0.999999  0.056153  0.376658  0.477579  0.034412  0.187551  0.095557
rend16    1.000000  0.056129  0.376574  0.477613  0.034679  0.187559  0.095755
rend32    1.000000  0.056116  0.376557  0.477630  0.035005  0.187607  0.095996
rev       0.056116  1.000000  0.180470  0.117200 -0.034106  0.218387  0.094260
rmiddle   0.376557  0.180470  1.000000 -0.064351 -0.013771  0.468925  0.195497
rot       0.477630  0.117200 -0.064351  1.000000  0.051246  0.243294  0.126195
size      0.035005 -0.034106 -0.013771  0.051246  1.000000 -0.236289  0.805926
yr        0.187607  0.218387  0.468925  0.243294 -0.236289  1.000000 -0.013907
yt        0.095996  0.094260  0.195497  0.126195  0.805926 -0.013907  1.000000

[35 rows x 35 columns]
```

[26]:
```
fdata.corr()['yt']
```

[26]:
```
CST_            NaN
begin      -0.008816
dbegin      0.139951
dend        0.155098
dim         0.192305
discont     0.203342
edit        0.283262
end        -0.008844
end16      -0.008852
end32      -0.008859
ibegin16    0.136416
ibegin2     0.109929
ibegin32    0.136416
ibegin4     0.179014
ibegin64    0.124459
ibegin8     0.179014
iend16      0.182267
```

```
iend2       0.146321
iend32      0.139099
iend4       0.127468
iend64      0.127728
iend8       0.182267
middle      0.821357
rbegin      0.095471
rdiscont    0.386893
redit       0.256097
rend        0.095557
rend16      0.095755
rend32      0.095996
rev         0.094260
rmiddle     0.195497
rot         0.126195
size        0.805926
yr         -0.013907
yt          1.000000
Name: yt, dtype: float64
```

We check the sign of the correlations of all features with *yt*. If it is positive, increasing the feature increases the processing time. We try to get only positive correlations. *end* is the flattened last dimensions left unchanged by the permutation. The bigger it is, the faster the transposition is. That's why the function computing all features multiplies this number by `-1` to get a feature positively correlated to the processing time. *end16* is equal to *end* when `end<-16` and `-16` when `end>=-16`. This is a simplification of the cost of moving data from memory to cache L1. This cost is linear when the data to move is big enough, but almost constant for small chunks.

1.4 Linear regression

We choose a linear regression because the prediction are not limited. The training set does not include all configuration and surely does not include all possible high value the model may have to predict.
The goal is not necessarily to predict the fastest permutation but to predict the processing time as the goal is to find the best combination of transpositions in a ONNX graph (einsum). The final goal is to predict which graphs optimizes a series of transpositions.
The target could be the processing time or the logarithm of this time. However, making mistakes on small times is not an issue but errors on high processing time is not a good thing.
We could also try to predict a ratio *transposition time /copy time* but it still gives more important to small matrix size.
Many variables are correlated. Variables should be selected.

1.4.1 Dataset

[27]:
```
X = fdata.drop(["yt", "yr"], axis=1)
x_names = list(X.columns)
yt = fdata['yt'] * 1000
```

[28]:
```
numpy.mean(yt)
```

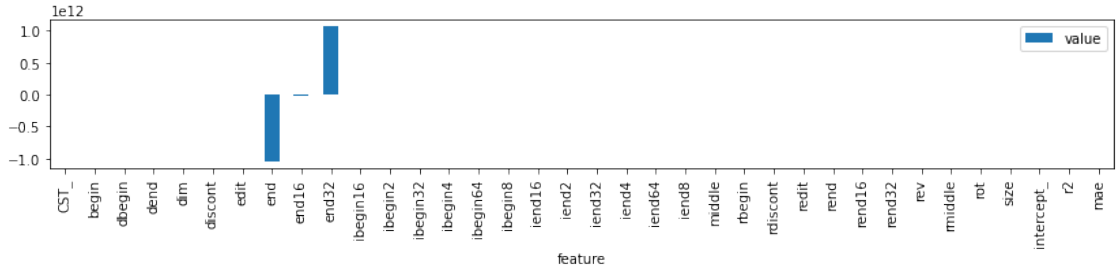
[28]:
```
1.8809171132996723
```

1.4.2 Simple model

[29]:
```
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import StandardScaler
from sklearn.pipeline import make_pipeline
from sklearn.metrics import r2_score, mean_absolute_error

pipe = make_pipeline(StandardScaler(with_mean=False),␣
 ↪LinearRegression(fit_intercept=False))
pipe.fit(X, yt)
model = pipe.steps[1][1]
coef = {k: v for k, v in zip(X.columns, model.coef_)}
coef['name'] = 'reg'
coef['intercept_'] = model.intercept_
pred = numpy.maximum(pipe.predict(X), 0)
coef['r2'] = r2_score(yt, pred)
coef['mae'] = mean_absolute_error(yt, pred)
coef['model'] = pipe
coefs = [coef]
coef["r2"], coef['mae']
```

[29]:
```
(0.8157414076410756, 0.6368865305095469)
```

[30]:
```
df = DataFrame([(k, v) for k, v in coef.items() if k not in {'name', 'model'}],
               columns=["feature", "value"]).set_index("feature")
df.plot(kind="bar", figsize=(14, 2));
```



[31]:
```
df
```

[31]:

feature	value
CST_	-3.076618e+08
begin	-2.941725e+01
dbegin	-1.854147e-01
dend	-9.638954e-02
dim	-1.037599e-01
discont	5.204404e-01
edit	3.582481e-01
end	-1.046584e+12
end16	-2.278042e+10
end32	1.069321e+12
ibegin16	-3.713466e+00

```
ibegin2     1.439716e-02
ibegin32    3.784367e+00
ibegin4    -6.813416e+00
ibegin64   -7.576102e-02
ibegin8     6.927856e+00
iend16      2.028144e+07
iend2       8.225773e+06
iend32      4.322857e+07
iend4       1.097274e+07
iend64      1.996315e-01
iend8       2.028143e+07
middle      1.541218e+00
rbegin      4.940619e+01
rdiscont    7.614642e-01
redit       8.622710e-02
rend        6.615750e+02
rend16      3.459172e+02
rend32     -1.057057e+03
rev         1.537206e-01
rmiddle    -4.563712e-01
rot         7.771901e-02
size        1.295707e+00
intercept_  0.000000e+00
r2          8.157414e-01
mae         6.368865e-01
```

Coefficients associated to features *end*, *end16* are almost opposed and it would better to get a model which keeps only one.

1.4.3 Quantile Regression

[32]:
```
from mlinsights.mlmodel import QuantileLinearRegression
pipe = make_pipeline(StandardScaler(with_mean=False),␣
  ↪QuantileLinearRegression(fit_intercept=False))
pipe.fit(X, yt)
model = pipe.steps[1][1]
coef = {k: v for k, v in zip(X.columns, model.coef_)}
coef['name'] = 'med'
coef['intercept_'] = model.intercept_
pred = numpy.maximum(pipe.predict(X), 0)
coef['r2'] = r2_score(yt, pred)
coef['mae'] = mean_absolute_error(yt, pred)
coef['model'] = pipe
coefs.append(coef)
coef["r2"], coef['mae']
```

[32]:
```
(0.7924498414927943, 0.5679387557069854)
```

[33]:
```
DataFrame(coef.items(), columns=["feature", "value"]).set_index("feature")
```

[33]:

feature	value
CST_	1433409.249051
begin	27.13405

```
dbegin                                                    0.07931
dend                                                     0.087576
dim                                                      0.006919
discont                                                  0.413378
edit                                                     0.186032
end                                           4876069525.422424
end16                                          106134745.367844
end32                                         -4982003112.711292
ibegin16                                                 0.129918
ibegin2                                                 -0.069604
ibegin32                                                -0.221099
ibegin4                                                 -0.045585
ibegin64                                                  -0.1085
ibegin8                                                  0.073031
iend16                                                -94492.918693
iend2                                                  -38324.37475
iend32                                               -201401.795017
iend4                                                 -51122.392443
iend64                                                     0.15928
iend8                                                 -94492.881923
middle                                                    1.588707
rbegin                                                   36.958438
rdiscont                                                  0.375421
redit                                                     0.071189
rend                                                    4424.263222
rend16                                                 -7664.018684
rend32                                                  3202.681647
rev                                                       0.08288
rmiddle                                                 -0.207068
rot                                                     -0.095643
size                                                     0.938597
name                                                          med
intercept_                                                      0
r2                                                        0.79245
mae                                                       0.567939
model       (StandardScaler(with_mean=False), QuantileLine…
```

1.4.4 Lasso

To select features.

[34]:
```
from sklearn.linear_model import Lasso

scores = []
models = []
for a in tqdm([0.001, 0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1., 2.]):
    alpha = a * 1.
    pipe = make_pipeline(
        StandardScaler(with_mean=False),
        Lasso(alpha=alpha, fit_intercept=False, max_iter=5000))
    pipe.fit(X, yt)
    pred = numpy.maximum(pipe.predict(X), 0)
    model = pipe.steps[1][1]
    scores.append(dict(r2=r2_score(yt, pred), mae=mean_absolute_error(yt, pred),
```

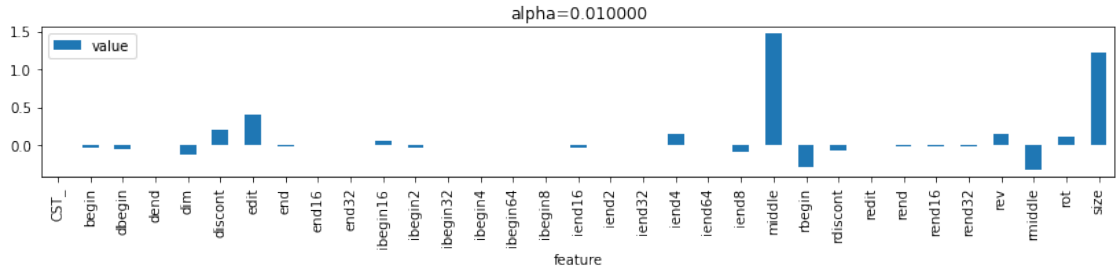
```
                        alpha=alpha, null=(numpy.abs(model.coef_) < 1e-6).sum(),
                        n=len(model.coef_)))
    models.append(pipe)
    if alpha >= 0.01 and alpha <= 0.2:
        coef = {k: v for k, v in zip(X.columns, pipe.steps[1][1].coef_)}
        coef['name'] = "Lasso-%f" % alpha
        coef['model'] = pipe
        coef['r2'] = r2_score(yt, pred)
        coef['mae'] = mean_absolute_error(yt, pred)
        coefs.append(coef)

DataFrame(scores)
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 13/13 [00:00<00:00, 69.97it/s]
```

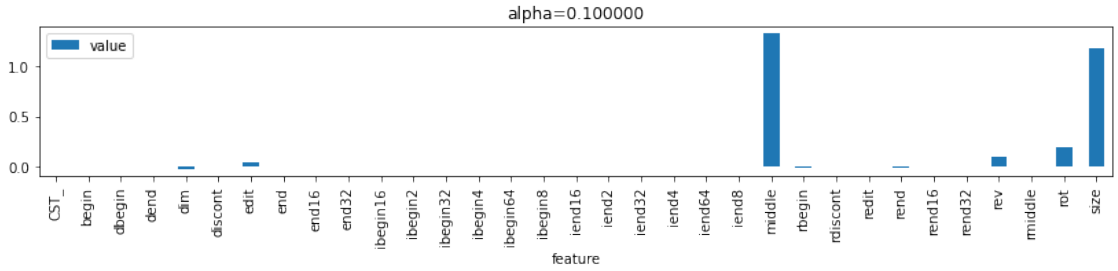
[34]:

	r2	mae	alpha	null	n
0	0.809704	0.629480	0.001	4	33
1	0.807546	0.629886	0.010	10	33
2	0.782541	0.676499	0.100	23	33
3	0.766911	0.680344	0.200	28	33
4	0.751546	0.703684	0.300	29	33
5	0.738223	0.742962	0.400	30	33
6	0.730937	0.735958	0.500	31	33
7	0.718437	0.758143	0.600	30	33
8	0.701329	0.800503	0.700	30	33
9	0.681590	0.848549	0.800	30	33
10	0.659218	0.898770	0.900	30	33
11	0.634218	0.949493	1.000	30	33
12	0.239413	1.600542	2.000	30	33

[35]:
```
coef = {k: v for k, v in zip(X.columns, models[1].steps[1][1].coef_)}
df = DataFrame(coef.items(), columns=["feature", "value"]).set_index("feature")
df.plot(kind="bar", figsize=(14, 2), title="alpha=%f" % scores[1]["alpha"]);
```



[36]:
```
coef = {k: v for k, v in zip(X.columns, models[2].steps[1][1].coef_)}
df = DataFrame(coef.items(), columns=["feature", "value"]).set_index("feature")
df.plot(kind="bar", figsize=(14, 2), title="alpha=%f" % scores[2]["alpha"]);
```

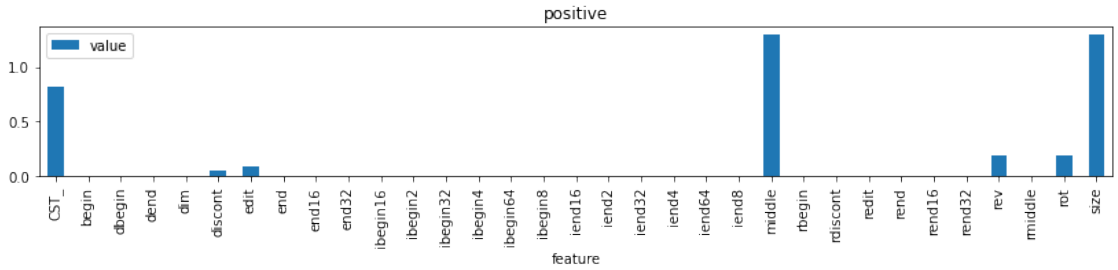


1.4.5 Linear regression with positive weights

[37]:
```
pipe = make_pipeline(StandardScaler(with_mean=False), LinearRegression(positive=True,␣
↪fit_intercept=False))
pipe.fit(X, yt)
model = pipe.steps[1][1]
coef = {k: v for k, v in zip(X.columns, model.coef_)}
coef['name'] = 'pos'
coef['intercept_'] = model.intercept_
pred = numpy.maximum(pipe.predict(X), 0)
coef['r2'] = r2_score(yt, pred)
coef['mae'] = mean_absolute_error(yt, pred)
coef['model'] = pipe
coefs.append(coef)
coef["r2"], coef['mae']
```

[37]:
```
(0.7905447080626958, 0.6768663007518693)
```

[38]:
```
coef = {k: v for k, v in zip(X.columns, pipe.steps[1][1].coef_)}
df = DataFrame(coef.items(), columns=["feature", "value"]).set_index("feature")
df.plot(kind="bar", figsize=(14, 2), title="positive");
```



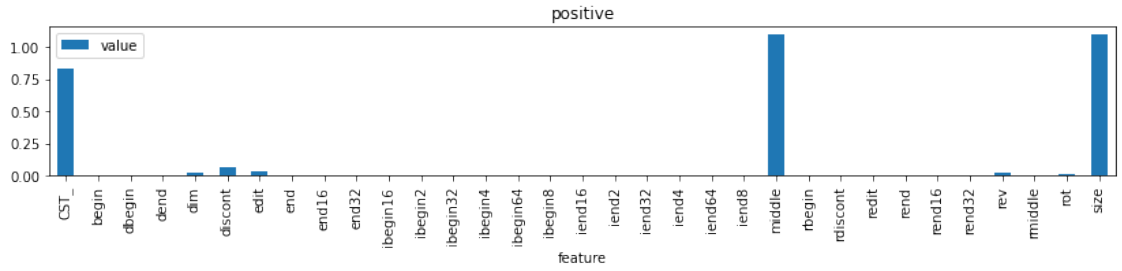
1.4.6 Quantile regression with positive weights

[39]:
```
pipe = make_pipeline(StandardScaler(with_mean=False),␣
↪QuantileLinearRegression(positive=True, fit_intercept=False))
pipe.fit(X, yt)
model = pipe.steps[1][1]
```

```
coef = {k: v for k, v in zip(X.columns, model.coef_)}
coef['name'] = 'medpos'
coef['intercept_'] = model.intercept_
pred = numpy.maximum(pipe.predict(X), 0)
coef['r2'] = r2_score(yt, pred)
coef['mae'] = mean_absolute_error(yt, pred)
coef['model'] = pipe
coefs.append(coef)
coef["r2"], coef['mae']
```

[39]: (0.752689515971656, 0.6468340444504788)

[40]:
```
coef = {k: v for k, v in zip(X.columns, pipe.steps[1][1].coef_)}
df = DataFrame(coef.items(), columns=["feature", "value"]).set_index("feature")
df.plot(kind="bar", figsize=(14, 2), title="positive");
```



1.4.7 Summary

[41]:
```
dfcoef = DataFrame(coefs)
dfcoef[::-1].T
```

[41]:

	6
CST_	0.829482
begin	0.0
dbegin	0.0
dend	0.0
dim	0.023846
discont	0.060636
edit	0.03823
end	0.0
end16	0.0
end32	0.0
ibegin16	0.0
ibegin2	0.0
ibegin32	0.0
ibegin4	0.0
ibegin64	0.0
ibegin8	0.0
iend16	0.0
iend2	0.0
iend32	0.0

```
iend4                                                    0.0
iend64                                                   0.0
iend8                                                    0.0
middle                                                1.101543
rbegin                                                   0.0
rdiscont                                                 0.0
redit                                                    0.0
rend                                                     0.0
rend16                                                   0.0
rend32                                                   0.0
rev                                                   0.026757
rmiddle                                                  0.0
rot                                                   0.009222
size                                                  1.100532
name                                                    medpos
intercept_                                               0.0
r2                                                     0.75269
mae                                                   0.646834
model       (StandardScaler(with_mean=False), QuantileLine…

                                                           5  \
CST_                                                  0.821048
begin                                                    0.0
dbegin                                                   0.0
dend                                                     0.0
dim                                                      0.0
discont                                               0.056297
edit                                                  0.094856
end                                                      0.0
end16                                                    0.0
end32                                                    0.0
ibegin16                                                 0.0
ibegin2                                                  0.0
ibegin32                                                 0.0
ibegin4                                                  0.0
ibegin64                                                 0.0
ibegin8                                                  0.0
iend16                                                   0.0
iend2                                                    0.0
iend32                                                   0.0
iend4                                                    0.0
iend64                                                   0.0
iend8                                                    0.0
middle                                                1.30347
rbegin                                                   0.0
rdiscont                                                 0.0
redit                                                    0.0
rend                                                     0.0
rend16                                                   0.0
rend32                                                   0.0
rev                                                   0.189909
rmiddle                                                  0.0
rot                                                   0.185687
```

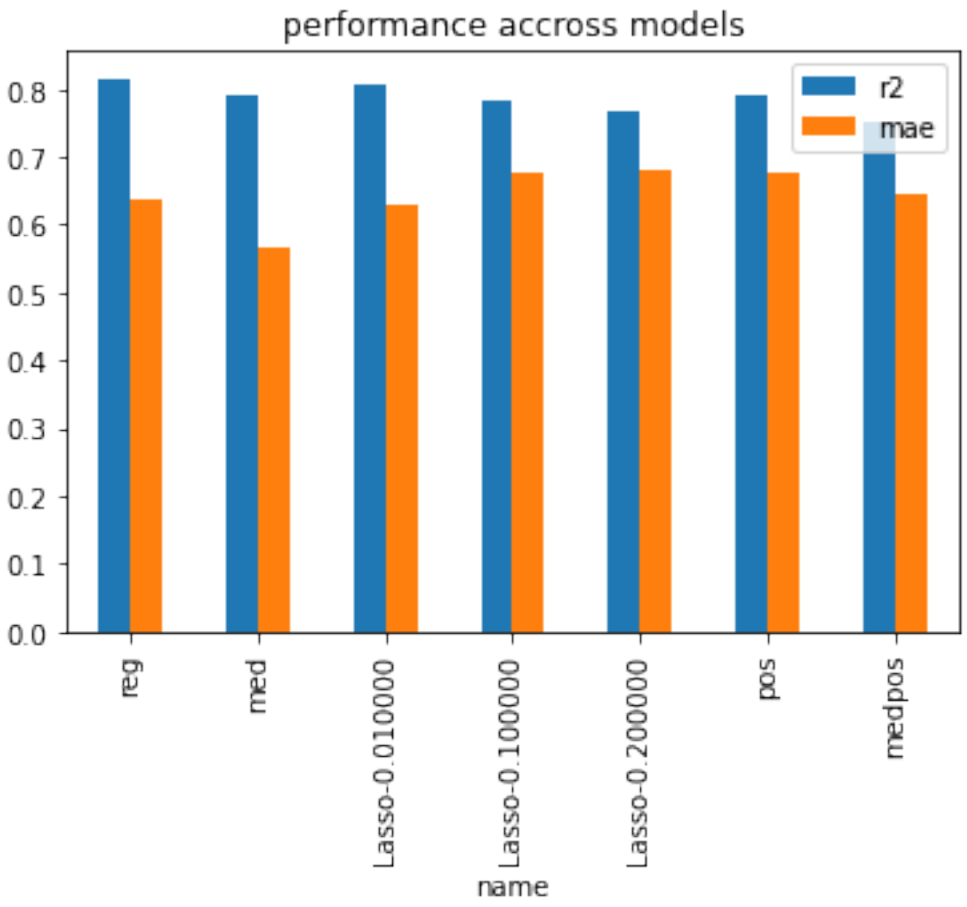
```
size                                                 1.300222
name                                                      pos
intercept_                                                0.0
r2                                                   0.790545
mae                                                  0.676866
model       (StandardScaler(with_mean=False), LinearRegres…

                                                            4  \
CST_                                                      0.0
begin                                                    -0.0
dbegin                                                   -0.0
dend                                                     -0.0
dim                                                 -0.014763
discont                                                   0.0
edit                                                      0.0
end                                                      -0.0
end16                                                    -0.0
end32                                                    -0.0
ibegin16                                                 -0.0
ibegin2                                                  -0.0
ibegin32                                                 -0.0
ibegin4                                                  -0.0
ibegin64                                                 -0.0
ibegin8                                                  -0.0
iend16                                                   -0.0
iend2                                                     0.0
iend32                                                   -0.0
iend4                                                    -0.0
iend64                                                   -0.0
iend8                                                    -0.0
middle                                               1.290699
rbegin                                                   -0.0
rdiscont                                                  0.0
redit                                                     0.0
rend                                                     -0.0
rend16                                                   -0.0
rend32                                                   -0.0
rev                                                  0.013992
rmiddle                                                  -0.0
rot                                                  0.108468
size                                                 1.099463
name                                           Lasso-0.200000
intercept_                                                NaN
r2                                                   0.766911
mae                                                  0.680344
model       (StandardScaler(with_mean=False), Lasso(alpha=…

                                                            3  \
CST_                                                      0.0
begin                                                    -0.0
dbegin                                                   -0.0
dend                                                     -0.0
dim                                                 -0.030446
```

```
discont                                                      0.0
edit                                                      0.0418
end                                                         -0.0
end16                                                       -0.0
end32                                                       -0.0
ibegin16                                                    -0.0
ibegin2                                                     -0.0
ibegin32                                                    -0.0
ibegin4                                                     -0.0
ibegin64                                                    -0.0
ibegin8                                                     -0.0
iend16                                                      -0.0
iend2                                                       -0.0
iend32                                                      -0.0
iend4                                                       -0.0
iend64                                                      -0.0
iend8                                                       -0.0
middle                                                  1.325916
rbegin                                                 -0.020369
rdiscont                                                    -0.0
redit                                                        0.0
rend                                                   -0.007655
rend16                                                 -0.003393
rend32                                                 -0.005349
rev                                                     0.097585
rmiddle                                                     -0.0
rot                                                     0.197021
size                                                    1.183553
name                                                Lasso-0.100000
intercept_                                                     NaN
r2                                                      0.782541
mae                                                     0.676499
model       (StandardScaler(with_mean=False), Lasso(alpha=…

                                                               2  \
CST_                                                         0.0
begin                                                    -0.03443
dbegin                                                 -0.044705
dend                                                        -0.0
dim                                                    -0.120949
discont                                                 0.210421
edit                                                    0.396052
end                                                    -0.007053
end16                                                  -0.000036
end32                                                   -0.00004
ibegin16                                                0.066669
ibegin2                                                 -0.02181
ibegin32                                                     0.0
ibegin4                                                      0.0
ibegin64                                                     0.0
ibegin8                                                      0.0
iend16                                                 -0.022416
iend2                                                        0.0
```

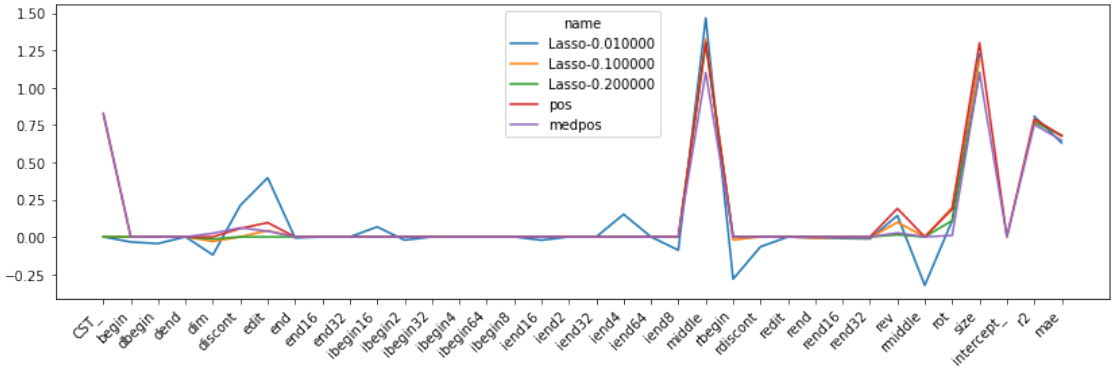
```
iend32                                                    -0.0
iend4                                                 0.151081
iend64                                                    -0.0
iend8                                                 -0.08907
middle                                                1.466733
rbegin                                                -0.28295
rdiscont                                             -0.066385
redit                                                       0.0
rend                                                 -0.007593
rend16                                               -0.010514
rend32                                               -0.013172
rev                                                   0.142791
rmiddle                                              -0.324716
rot                                                   0.108146
size                                                   1.22329
name                                              Lasso-0.010000
intercept_                                                 NaN
r2                                                    0.807546
mae                                                   0.629886
model       (StandardScaler(with_mean=False), Lasso(alpha=…

                                                             1  \
CST_                                            1433409.249051
begin                                                 27.13405
dbegin                                                 0.07931
dend                                                  0.087576
dim                                                   0.006919
discont                                               0.413378
edit                                                  0.186032
end                                           4876069525.422424
end16                                          106134745.367844
end32                                        -4982003112.711292
ibegin16                                              0.129918
ibegin2                                              -0.069604
ibegin32                                             -0.221099
ibegin4                                              -0.045585
ibegin64                                               -0.1085
ibegin8                                               0.073031
iend16                                             -94492.918693
iend2                                               -38324.37475
iend32                                             -201401.795017
iend4                                              -51122.392443
iend64                                                 0.15928
iend8                                              -94492.881923
middle                                                 1.588707
rbegin                                                36.958438
rdiscont                                               0.375421
redit                                                  0.071189
rend                                                4424.263222
rend16                                             -7664.018684
rend32                                              3202.681647
rev                                                    0.08288
rmiddle                                              -0.207068
```

```
rot                                                  -0.095643
size                                                  0.938597
name                                                       med
intercept_                                                 0.0
r2                                                     0.79245
mae                                                   0.567939
model       (StandardScaler(with_mean=False), QuantileLine…

                                                             0
CST_                                        -307661768.128088
begin                                              -29.417247
dbegin                                              -0.185415
dend                                                 -0.09639
dim                                                 -0.10376
discont                                              0.52044
edit                                                0.358248
end                                     -1046583604803.358887
end16                                     -22780416305.902706
end32                                    1069320839370.567505
ibegin16                                            -3.713466
ibegin2                                              0.014397
ibegin32                                             3.784367
ibegin4                                             -6.813416
ibegin64                                            -0.075761
ibegin8                                              6.927856
iend16                                           20281439.108194
iend2                                             8225773.255917
iend32                                           43228573.054944
iend4                                            10972737.091606
iend64                                                0.199631
iend8                                            20281426.580972
middle                                                1.541218
rbegin                                               49.406192
rdiscont                                              0.761464
redit                                                 0.086227
rend                                                661.575013
rend16                                              345.917179
rend32                                            -1057.05651
rev                                                   0.153721
rmiddle                                              -0.456371
rot                                                   0.077719
size                                                  1.295707
name                                                       reg
intercept_                                                 0.0
r2                                                    0.815741
mae                                                   0.636887
model       (StandardScaler(with_mean=False), LinearRegres…
```

[42]:
```
dfcoef[["name", "r2", "mae"]].set_index('name').plot(kind="bar", title="performance␣
↪accross models");
```



[43]:
```
import matplotlib.pyplot as plt

dfp = dfcoef.drop(['name', 'model'], axis=1).T.drop([0, 1], axis=1).copy()
dfp.columns = dfcoef['name'][2:]
ax = dfp.plot(figsize=(14, 4), kind="line")
ax.set_xticks(numpy.arange(0, dfp.shape[0]))
ax.set_xticklabels(dfp.index)
plt.setp(ax.get_xticklabels(), rotation=45, horizontalalignment='right');
```



1.5 Investigation

[44]:
```
data_err = data.drop(["context_size", "repeat"], axis=1).copy()
data_err['predict'] = numpy.maximum(coefs[0]['model'].predict(X), 0) / 1000
data_err['err'] = (data_err['predict'] - data_err['average'])
data_err['abserr'] = numpy.abs(data_err['predict'] - data_err['average'])
data_err['rel'] = (data_err['predict'] - data_err['average']) / data_err['average']
s = data_err.sort_values('abserr')
pandas.concat([s.head(n=10), s.tail(n=10)])
```

[44]:

	average	deviation	min_exec	max_exec	number	perm	\
28	0.000113	0.000029	0.000061	0.000141	50	(1, 0, 2)	
55	0.000893	0.000212	0.000646	0.001477	50	(2, 3, 1, 0)	
26	0.000077	0.000008	0.000069	0.000101	50	(0, 2, 1)	
39	0.000126	0.000017	0.000107	0.000185	50	(2, 0, 1, 3)	
66	0.000195	0.000019	0.000163	0.000258	50	(0, 3, 1, 2)	
50	0.000692	0.000136	0.000529	0.001021	50	(0, 3, 2, 1)	
76	0.000824	0.000288	0.000515	0.001879	50	(1, 3, 2, 0)	
54	0.000818	0.000278	0.000625	0.001522	50	(2, 1, 3, 0)	
1	0.000048	0.000003	0.000045	0.000058	50	(0, 1, 3, 2)	
2	0.000049	0.000003	0.000045	0.000062	50	(3, 0, 1, 2)	
120	0.005918	0.001085	0.004598	0.008312	15	(1, 0, 3, 2)	
128	0.010677	0.001457	0.008504	0.014070	15	(2, 3, 1, 0)	
121	0.006106	0.000556	0.005619	0.007305	15	(2, 0, 3, 1)	
118	0.003780	0.000882	0.002701	0.005402	15	(3, 0, 1, 2)	
115	0.003653	0.000874	0.002567	0.005244	15	(1, 2, 3, 0)	
129	0.012421	0.003052	0.007818	0.018106	15	(3, 2, 1, 0)	
119	0.004938	0.000367	0.004532	0.005844	15	(1, 3, 0, 2)	
127	0.010339	0.002789	0.005878	0.018301	15	(0, 3, 2, 1)	
130	0.013432	0.004496	0.006536	0.021250	15	(3, 2, 0, 1)	
131	0.014579	0.004026	0.007144	0.020739	15	(3, 0, 2, 1)	

	shape	ratio	dim	predict	err	abserr	\
28	(43, 44, 45)	1.515711	3	0.000113	1.251063e-07	1.251063e-07	
55	(3, 244, 244, 1)	8.339926	4	0.000893	-2.410649e-07	2.410649e-07	
26	(43, 44, 45)	1.032759	3	0.000077	4.172780e-07	4.172780e-07	
39	(3, 244, 244, 1)	1.180996	4	0.000115	-1.179187e-05	1.179187e-05	
66	(1, 244, 244, 3)	1.598770	4	0.000210	1.510728e-05	1.510728e-05	
50	(3, 244, 244, 1)	6.460759	4	0.000709	1.714180e-05	1.714180e-05	
76	(1, 244, 244, 3)	6.752663	4	0.000843	1.902846e-05	1.902846e-05	
54	(3, 244, 244, 1)	7.634646	4	0.000843	2.572773e-05	2.572773e-05	
1	(12, 13, 15, 18)	0.820821	4	0.000000	-4.837787e-05	4.837787e-05	
2	(12, 13, 15, 18)	0.823070	4	0.000000	-4.851040e-05	4.851040e-05	
120	(3, 244, 244, 6)	1.597357	4	0.008259	2.341673e-03	2.341673e-03	
128	(3, 244, 244, 6)	2.882191	4	0.008132	-2.545011e-03	2.545011e-03	
121	(3, 244, 244, 6)	1.648325	4	0.008700	2.593662e-03	2.593662e-03	
118	(3, 244, 244, 6)	1.020432	4	0.006488	2.707333e-03	2.707333e-03	
115	(3, 244, 244, 6)	0.986071	4	0.006488	2.834624e-03	2.834624e-03	
129	(3, 244, 244, 6)	3.352770	4	0.009386	-3.034652e-03	3.034652e-03	
119	(3, 244, 244, 6)	1.333061	4	0.008700	3.761588e-03	3.761588e-03	

```
127  (3, 244, 244, 6)  2.790823    4  0.005271 -5.068171e-03  5.068171e-03
130  (3, 244, 244, 6)  3.625680    4  0.008132 -5.299336e-03  5.299336e-03
131  (3, 244, 244, 6)  3.935483    4  0.008259 -6.320138e-03  6.320138e-03

          rel
28   0.001111
55  -0.000270
26   0.005440
39  -0.093246
66   0.077417
50   0.024778
76   0.023087
54   0.031471
1   -1.000000
2   -1.000000
120  0.395716
128 -0.238356
121  0.424746
118  0.716171
115  0.775972
129 -0.244323
119  0.761694
127 -0.490205
130 -0.394540
131 -0.433499
```

All big errors are negative. The model seems to give a lower value for all big errors. These errors may be outliers, the processor was busy doing something else at that time.

[45]:
```
s = data_err.sort_values('predict')
pandas.concat([s.head(n=10), s.tail(n=10)])
```

[45]:
```
      average  deviation  min_exec  max_exec  number          perm  \
20   0.000158   0.000021  0.000127  0.000192      50  (3, 0, 2, 1)
42   0.000147   0.000017  0.000106  0.000175      50  (0, 1, 3, 2)
34   0.000151   0.000016  0.000136  0.000197      50     (1, 2, 0)
33   0.000124   0.000017  0.000108  0.000171      50     (2, 0, 1)
44   0.000189   0.000044  0.000142  0.000265      50  (1, 2, 3, 0)
27   0.000097   0.000004  0.000083  0.000110      50     (2, 0, 1)
25   0.000074   0.000009  0.000065  0.000109      50     (0, 1, 2)
24   0.000073   0.000009  0.000062  0.000094      50     (1, 2, 0)
22   0.000214   0.000060  0.000136  0.000295      50  (1, 0, 3, 2)
21   0.000164   0.000045  0.000124  0.000231      50  (3, 1, 2, 0)
128  0.010677   0.001457  0.008504  0.014070      15  (2, 3, 1, 0)
130  0.013432   0.004496  0.006536  0.021250      15  (3, 2, 0, 1)
122  0.006722   0.001807  0.005067  0.011245      15  (1, 3, 2, 0)
125  0.007815   0.001757  0.005932  0.010779      15  (2, 1, 3, 0)
120  0.005918   0.001085  0.004598  0.008312      15  (1, 0, 3, 2)
123  0.007071   0.000982  0.005454  0.008559      15  (3, 1, 0, 2)
131  0.014579   0.004026  0.007144  0.020739      15  (3, 0, 2, 1)
121  0.006106   0.000556  0.005619  0.007305      15  (2, 0, 3, 1)
119  0.004938   0.000367  0.004532  0.005844      15  (1, 3, 0, 2)
129  0.012421   0.003052  0.007818  0.018106      15  (3, 2, 1, 0)
```

	shape	ratio	dim	predict	err	abserr	rel
20	(12, 13, 15, 18)	2.684876	4	0.000000	-0.000158	0.000158	-1.000000
42	(3, 244, 244, 1)	1.369978	4	0.000000	-0.000147	0.000147	-1.000000
34	(3, 244, 244)	1.438446	3	0.000000	-0.000151	0.000151	-1.000000
33	(3, 244, 244)	1.185666	3	0.000000	-0.000124	0.000124	-1.000000
44	(3, 244, 244, 1)	1.766905	4	0.000000	-0.000189	0.000189	-1.000000
27	(43, 44, 45)	1.300915	3	0.000000	-0.000097	0.000097	-1.000000
25	(43, 44, 45)	1.000000	3	0.000000	-0.000074	0.000074	-1.000000
24	(43, 44, 45)	0.985513	3	0.000000	-0.000073	0.000073	-1.000000
22	(12, 13, 15, 18)	3.627240	4	0.000000	-0.000214	0.000214	-1.000000
21	(12, 13, 15, 18)	2.778193	4	0.000000	-0.000164	0.000164	-1.000000
128	(3, 244, 244, 6)	2.882191	4	0.008132	-0.002545	0.002545	-0.238356
130	(3, 244, 244, 6)	3.625680	4	0.008132	-0.005299	0.005299	-0.394540
122	(3, 244, 244, 6)	1.814552	4	0.008259	0.001537	0.001537	0.228654
125	(3, 244, 244, 6)	2.109489	4	0.008259	0.000444	0.000444	0.056871
120	(3, 244, 244, 6)	1.597357	4	0.008259	0.002342	0.002342	0.395716
123	(3, 244, 244, 6)	1.908667	4	0.008259	0.001188	0.001188	0.168070
131	(3, 244, 244, 6)	3.935483	4	0.008259	-0.006320	0.006320	-0.433499
121	(3, 244, 244, 6)	1.648325	4	0.008700	0.002594	0.002594	0.424746
119	(3, 244, 244, 6)	1.333061	4	0.008700	0.003762	0.003762	0.761694
129	(3, 244, 244, 6)	3.352770	4	0.009386	-0.003035	0.003035	-0.244323

1.5.1 Correlation between predictors

[46]:
```
cc = DataFrame(dict([(c['name'], numpy.maximum(c['model'].predict(X), 0)) for c in␣
 ↪coefs]))
cc['yt'] = yt
cc
```

[46]:

	reg	med	Lasso-0.010000	Lasso-0.100000	Lasso-0.200000	\
0	0.298789	0.052436	0.000000	0.000000	0.000000	
1	0.000000	0.071575	0.000000	0.000000	0.000000	
2	0.000000	0.048393	0.000000	0.000000	0.000000	
3	0.000000	0.048393	0.000000	0.000000	0.000000	
4	0.248089	0.050781	0.000000	0.000000	0.000000	
..	…	…	…	…	…	
127	5.270700	4.177012	4.917105	4.615490	4.464429	
128	8.132342	7.354799	8.107191	7.646966	7.334861	
129	9.386005	8.186190	8.991256	8.082431	7.397300	
130	8.132342	7.354799	8.107191	7.646966	7.334861	
131	8.259236	7.561004	7.962160	7.605605	7.334861	

	pos	medpos	yt
0	0.000000	0.000000	0.044222
1	0.000000	0.000000	0.048378
2	0.000000	0.000000	0.048510
3	0.000000	0.000000	0.048954
4	0.000000	0.000000	0.050805
..	…	…	…
127	4.837032	4.251381	10.338870
128	7.858363	6.706548	10.677354
129	8.771040	6.896204	12.420657
130	7.858363	6.706548	13.431679

```
131  7.829728  6.738972  14.579374

[132 rows x 8 columns]
```

[47]:
```
cc.corr()
```

[47]:
```
                    reg       med  Lasso-0.010000  Lasso-0.100000  \
reg            1.000000  0.994124        0.996922        0.985715
med            0.994124  1.000000        0.995863        0.989990
Lasso-0.010000 0.996922  0.995863        1.000000        0.992689
Lasso-0.100000 0.985715  0.989990        0.992689        1.000000
Lasso-0.200000 0.979826  0.987374        0.987930        0.998564
pos            0.988323  0.990341        0.994420        0.998756
medpos         0.980433  0.988401        0.988358        0.997985
yt             0.903528  0.894833        0.899384        0.886902

                Lasso-0.200000       pos    medpos        yt
reg                   0.979826  0.988323  0.980433  0.903528
med                   0.987374  0.990341  0.988401  0.894833
Lasso-0.010000        0.987930  0.994420  0.988358  0.899384
Lasso-0.100000        0.998564  0.998756  0.997985  0.886902
Lasso-0.200000        1.000000  0.995092  0.999385  0.880614
pos                   0.995092  1.000000  0.995169  0.890093
medpos                0.999385  0.995169  1.000000  0.881208
yt                    0.880614  0.890093  0.881208  1.000000
```

1.6 Standalone predictions

[48]:
```
def get_coef(pipe, names):
    c1 = pipe.steps[0][-1].scale_
    c2 = pipe.steps[1][-1].coef_
    return dict(zip(names, c2 / c1))


get_coef(coefs[-1]["model"], X.columns)
```

[48]:
```
{'CST_': 0.829481835464256,
 'begin': 0.0,
 'dbegin': 0.0,
 'dend': 0.0,
 'dim': 0.08294721851224843,
 'discont': 0.07025394222472751,
 'edit': 0.03782977428195987,
 'end': 0.0,
 'end16': 0.0,
 'end32': 0.0,
 'ibegin16': 0.0,
 'ibegin2': 0.0,
 'ibegin32': 0.0,
 'ibegin4': 0.0,
 'ibegin64': 0.0,
 'ibegin8': 0.0,
 'iend16': 0.0,
```

```
 'iend2': 0.0,
 'iend32': 0.0,
 'iend4': 0.0,
 'iend64': 0.0,
 'iend8': 0.0,
 'middle': 3.42896339670081e-06,
 'rbegin': 0.0,
 'rdiscont': 0.0,
 'redit': 0.0,
 'rend': 0.0,
 'rend16': 0.0,
 'rend32': 0.0,
 'rev': 0.11940214295823245,
 'rmiddle': 0.0,
 'rot': 0.023189032947793925,
 'size': 3.021302183272755e-06}
```

[49]:
```
def predict(coefs, shape, perm):
    feat = compute_features(shape, perm)
    res = 0
    for k, v in feat.items():
        res += v * coefs[k]
    return res / 1000


def predict_model(model, shape, perm, names):
    feat = compute_features(shape, perm)
    a = numpy.zeros((1, len(names)), dtype=numpy.float64)
    for i, n in enumerate(names):
        a[0, i] = feat[n]
    return model.predict(a) / 1000


coef = get_coef(coefs[-1]["model"], X.columns)
(predict(coef, (3, 224, 224, 6), (3, 0, 1, 2)),
 predict_model(coefs[-1]["model"], (3, 224, 224, 6), (3, 0, 1, 2), X.columns))
```

[49]:
```
(0.005450704959759156, array([0.0054507]))
```

[50]:
```

```