Dynamic Programming Approach to Behavioral Operations

Park Sinchaisri

OIDD 934 Spring 2018

What is *Behavioral Operations*?

Operations Management	+	Human Behavior /Emotions
"People are fully rational"		Biases, limited cognitive ability, ...

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Operations Management + Human Behavior /Emotions

Supply Chain

Capacity Allocation

Adelman & Mersereau
Management Science '13

Memory

Revenue Management

Dynamic Pricing

Nasiry & Popescu
Operations Research '11

Loss Aversion

Dynamic Capacity Allocation to Customers Who Remember Past Service

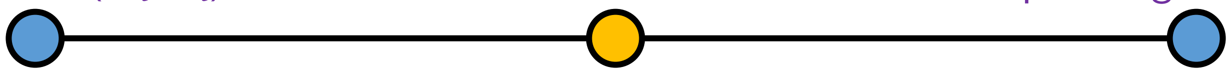
Adelman & Mersereau, Management Science '13

- Supplier sells a single good to a finite set of customers: 1, 2, ..., n
- Capacity constraint of Xbar > 0 units per period
- **Goal:** Maximize supplier's long-run average profit
- Model **"goodwill" toward the supplier / customer's memory**
- Trade-off **margin, demand volatility,** and **memory**
- Show that **ADP policy** outperforms Greedy

MDP Formulation

Customer places an order $y^i(G^i_t, D^i_t)$

Receiving x^i_t, customer updates goodwill G^i_{t+1}



Supplier chooses vector of shipments $\mathbf{x_t}$
makes profit $r^i > 0$ for each unit of x^i_t

- **Goodwill**: based on quality of past service $\boldsymbol{G^i_t}$ entering period t, modeled as exponential smoothing of utilities from past fill rates.

$$G^i_{t+1} = \begin{cases} \beta^i G^i_t + u^i(x^i_t / y^i(G^i_t, D^i_t)) & \text{if } y^i(G^i_t, D^i_t) > 0, \\ \beta^i G^i_t & \text{if } y^i(G^i_t, D^i_t) = 0, \end{cases}$$

B^i: (0,1): customer-specific memory parameter
u^i: [0, 1] -> R+: nondecreasing c-specific utility function
D_t: demands follow stationary dist, dependent across i, but independent across t

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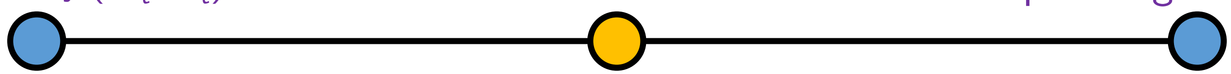

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- **State space:** $\mathcal{G} = \{\mathbf{G} \in \mathbb{R}^n_+ : u^i(0)/\ (1-\beta^i) \le G^i \le u^i(1)/(1-\beta^i)\, \forall\, i \in \mathcal{I}\}$
- **Action space:** allocations provided under each demand scenario

$$\mathcal{X}(\mathbf{G}, \mathbf{D}) = \left\{ \mathbf{x} \in \mathbb{R}^n_+ : \sum_{i \in \mathcal{I}} x^i \le \bar{X}, 0 \le x^i \le y^i(G^i, D^i)\ \forall i \in \mathcal{I} \right\}$$

Challenges from MDP

Let $\boldsymbol{\beta}\mathbf{G} + \mathbf{u}(\mathbf{x}/\mathbf{y}(\mathbf{G}, \mathbf{D}))$ be a vectorized representation of the expressions $\beta^i G^i + u^i(x^i/y^i(G^i, D^i))$ for all i, which denote the next state. We can write the unichain optimality equations in two equivalent ways:

$$h(\mathbf{G}) = \mathsf{E}_{\mathbf{D}}\left[\max_{\mathbf{x}\in\mathscr{X}(\mathbf{G},\mathbf{D})}\left\{\sum_i r^i x^i - \rho + h\left(\boldsymbol{\beta}\mathbf{G} + \mathbf{u}\left(\frac{\mathbf{x}}{\mathbf{y}(\mathbf{G},\mathbf{D})}\right)\right)\right\}\right] \quad \forall \mathbf{G} \in \mathscr{G}, \qquad (1)$$

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- Stochastic demands lead to stochastic dynamics -> steady states needs to be characterized by complex joint probability distribution
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- **Solution**: Switch = in (2) into >= and get infinite-d LP

$$(\mathrm{D}_0) \quad \inf_{\rho,h} \rho \tag{3a}$$

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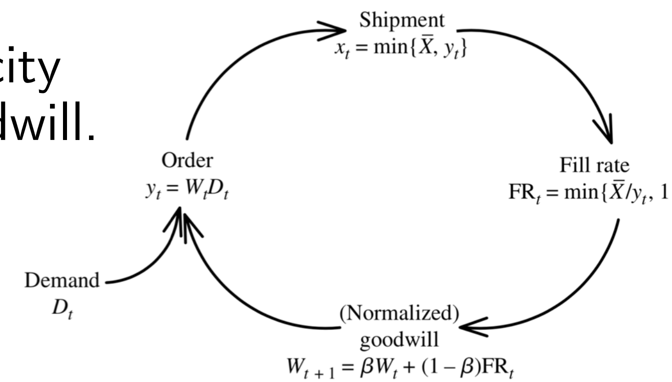
Greedy Policy Can Be Bad

Myopic margin-greedy policy = allocates capacity based on margins alone without regard to goodwill.

$$\max \sum_i r^i x_t^i \quad \text{s.t.} \ \sum_i x_t^i \le \bar{X}, \quad 0 \le x_t^i \le y_t^i \quad \forall i \in \mathcal{I}.$$



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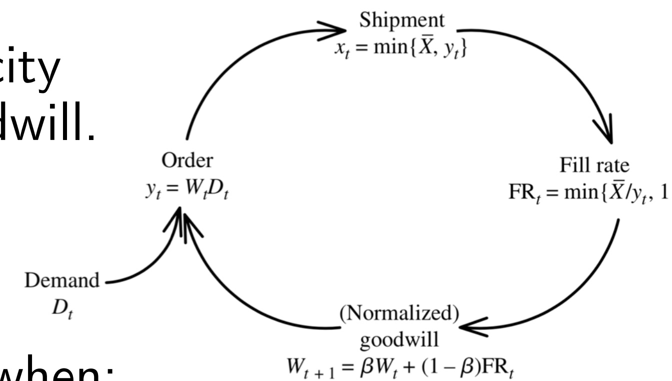

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- Little tolerance for lapses in service
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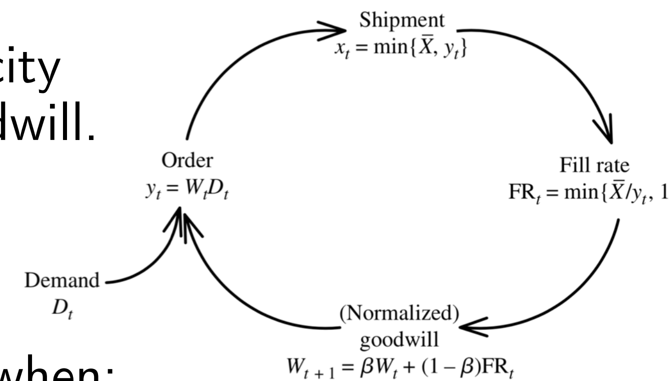

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Better policies strategically withhold capacity to keep customer goodwills within a manageable range, keep multiple customers ordering actively, and pooling demand variances across customers.

Approximate DP Approach

Based on a polynomial approximation of the optimal bias function *h* of D_0

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$$\forall (\mathbf{G}, \mathbf{x}) \in \mathcal{M}, \tag{3b}$$

$$h \in \mathbb{B}(\mathcal{G}), \rho \in \mathbb{R}. \tag{3c}$$

Now suppose we are given a collection $\mathcal{K} = \{1, 2, \ldots, K\}$ of basis functions $\{\phi_k(\cdot)\}_{k \in \mathcal{K}}$, where $\phi_k \in \mathbb{B}(\mathcal{G})$. For a weight vector $\mathbf{w} = (w_1, w_2, \ldots, w_K) \in \mathbb{R}^K$, we can approximate the bias function as

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Dual

$$(\mathrm{P}_\phi) \quad \sup_{\mu} \sum_{(\mathbf{G}, \mathbf{x}) \in M} \mathsf{E}_{\mathbf{D}}\left[\sum_{i \in \mathcal{I}} r^i x^i(\mathbf{D})\right] \mu(\mathbf{G}, \mathbf{x}), \tag{11a}$$

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$$\sum_{(\mathbf{G}, \mathbf{x}) \in M} \mu(\mathbf{G}, \mathbf{x}) = 1, \tag{11c}$$

$$\mu \geq 0, M = \mathrm{supp}(\mu) \subset \mathcal{M}, |M| < \infty, \tag{11d}$$

where $\mathrm{supp}(\cdot)$ denotes the support set.

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Solved using column generation for a given dual price vector and price

THEOREM 2 (DUALITY). *Suppose $\{\phi_k\}_{k \in \mathcal{K}}$ are continuous functions on $\mathcal{G}$. Then (P_ϕ) is solvable, and* $\sup (P_\phi) = \inf (D_\phi)$.

Optimal ADP Policy

- Given demand realization **D**, customers are in goodwill state **G**, optimal dual variables **w*** from solving (P_{phi}), the supplier solves

$$\begin{aligned} \max_{\mathbf{x}} \quad & \sum_{i \in \mathcal{I}} \left(r^i x^i + \sum_{k \in \mathcal{K}} w_k^* \phi_k \left(\boldsymbol{\beta} \mathbf{G} + \mathbf{u} \left(\frac{\mathbf{x}}{\mathbf{y}(\mathbf{G}, \mathbf{D})} \right) \right) \right) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{I}} x^i \le \bar{X}, \quad 0 \le x^i \le y^i(G^i, D^i) \quad \forall i \in \mathcal{I}, \end{aligned}$$

or possibly a multiperiod lookahead version of this problem

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or possibly a multiperiod lookahead version of this problem

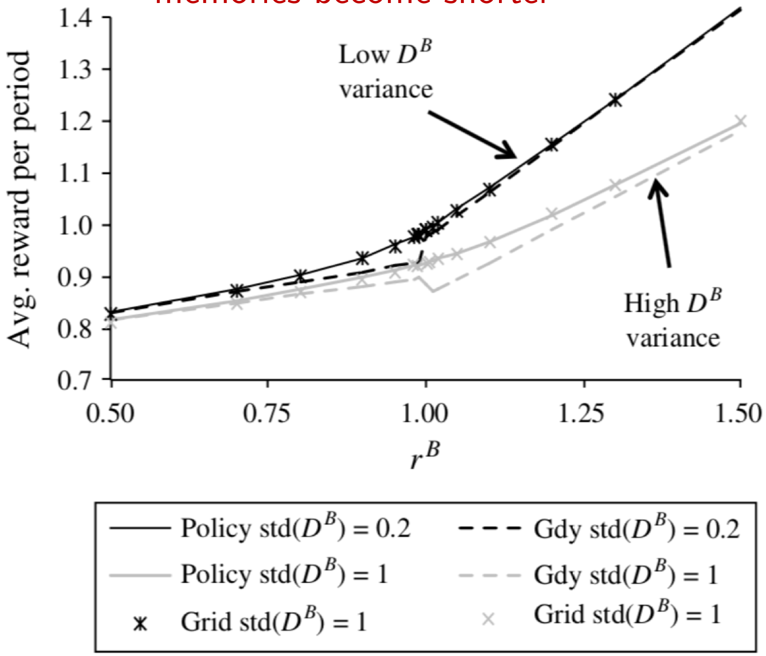
- Linear Approx. $u^i(f) = f$ and linear basis function $phi_i(G^i) = G^i$ Optimization becomes simple LP with an index solution: allocate capacity to customers in decreasing order of the customer indices

$$r^i + \frac{w_i^*}{y^i(G^i, D^i)}$$

until capacity is exhausted.

Conclusions

Goodwills are important when
D variance increases and
memories become shorter



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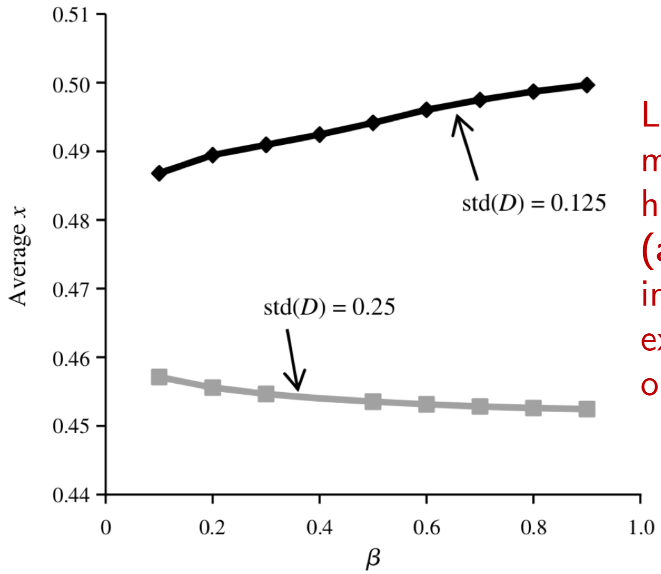

Longer memory can have + or − **(ambiguous)** impact on expected orders

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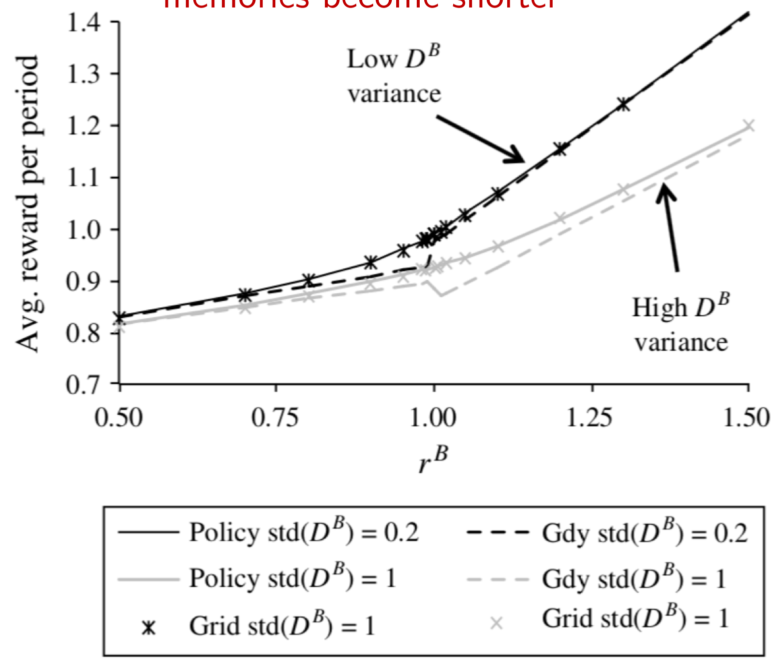

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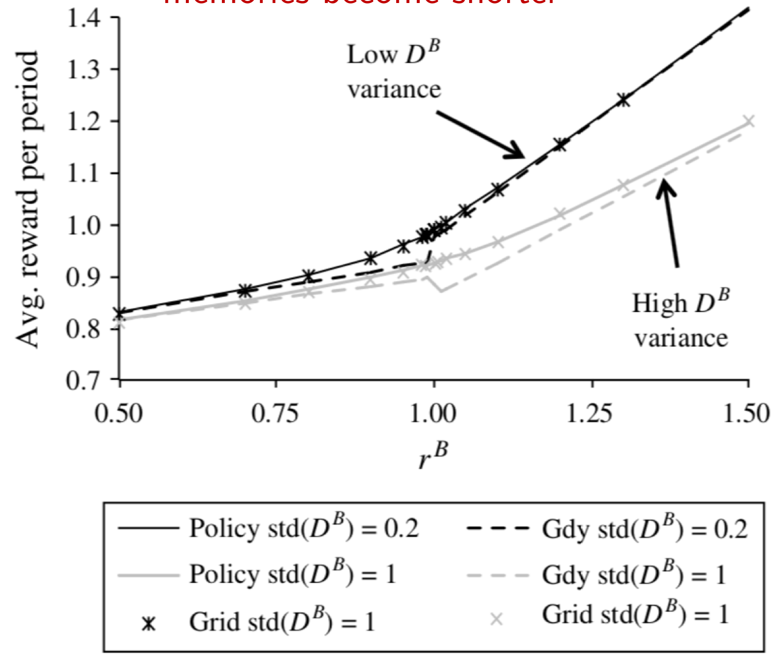

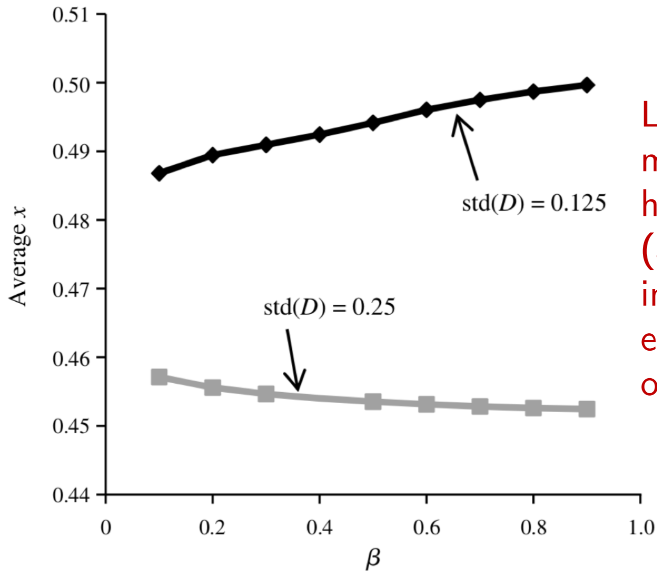

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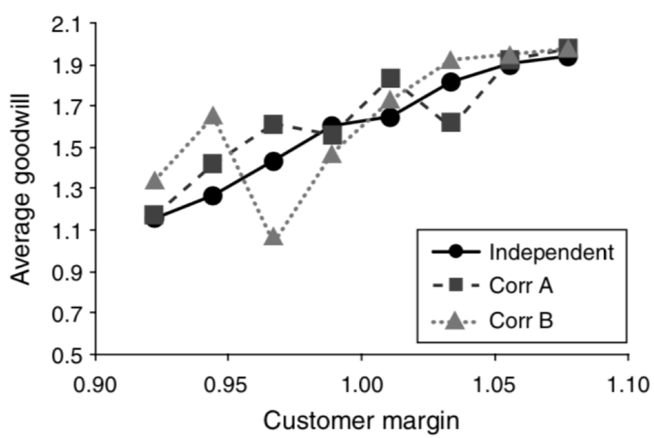
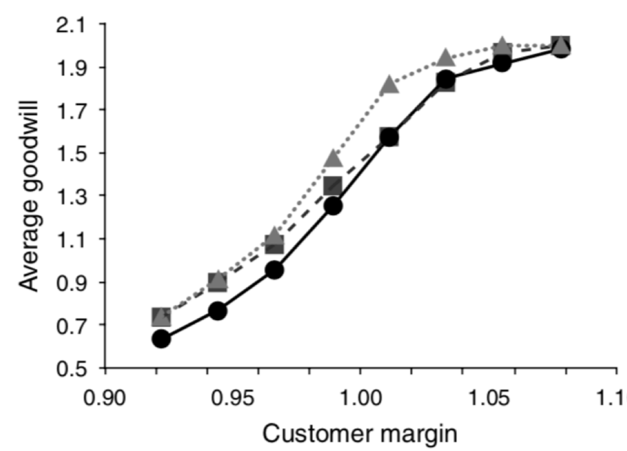
ADP policy maintains more balanced goodwills
Goodwill-sensitive: prioritize customers by "adjusted margins"





▲	Name		Artist	Price	
1	Boom Boom Pow		Black Eyed Peas	$1.29	BUY SONG
2	Poker Face		Lady GaGa	$1.29	BUY SONG
3	Right Round		Flo Rida	$1.29	BUY SONG
4	The Climb		Miley Cyrus	$0.99	BUY SONG
5	Day 'n' Nite	EXPLICIT	Kid Cudi	$0.99	BUY SONG
6	Kiss Me Thru the Phone (feat. ...		Soulja Boy Tell 'Em	$1.29	BUY SONG
7	Gives You Hell		The All-America...	$1.29	BUY SONG
8	You Found Me		The Fray	$0.99	BUY SONG
9	Blame It (feat. T-Pain)	EXPLICIT	Jamie Foxx	$0.99	BUY SONG
10	Love Sex Magic (feat. Justin Ti...		Ciara	$1.29	BUY SONG
11	Just Dance		Lady GaGa & Col...	$1.29	BUY SONG
12	I Love College	EXPLICIT	Asher Roth	$0.99	BUY SONG
13	Halo		Beyoncé	$0.99	BUY SONG
14	Sugar (feat. Wynter)		Flo Rida	$1.29	BUY SONG
15	If U Seek Amy		Britney Spears	$0.99	BUY SONG
16	My Life Would Suck Without You		Kelly Clarkson	$1.29	BUY SONG
17	Dead and Gone (feat. J...	EXPLICIT	T.I.	$0.99	BUY SONG
18	Don't Trust Me	EXPLICIT	3OH!3	$0.99	BUY SONG
19	Jai Ho! (You Are My Destiny) [f...		A. R. Rahman & ...	$1.29	BUY SONG
20	Second Chance		Shinedown	$0.99	BUY SONG

Dynamic Pricing with Loss-Averse Consumers and Peak-End Anchoring

Nasiry & Popescu, Operations Research '11

▲	Name	Artist	Price	
1	Boom Boom Pow	Black Eyed Peas	$1.29	BUY SONG
2	Poker Face	Lady GaGa	$1.29	BUY SONG
3	Right Round	Flo Rida	$1.29	BUY SONG
4	The Climb	Miley Cyrus	$0.99	BUY SONG
5	Day 'n' Nite EXPLICIT	Kid Cudi	$0.99	BUY SONG
6	Kiss Me Thru the Phone (feat. ...	Soulja Boy Tell 'Em	$1.29	BUY SONG
7	Gives You Hell	The All-America...	$1.29	BUY SONG
8	You Found Me	The Fray	$0.99	BUY SONG
9	Blame It (feat. T-Pain) EXPLICIT	Jamie Foxx	$0.99	BUY SONG
10	Love Sex Magic (feat. Justin Ti...	Ciara	$1.29	BUY SONG
11	Just Dance	Lady GaGa & Col...	$1.29	BUY SONG
12	I Love College EXPLICIT	Asher Roth	$0.99	BUY SONG
13	Halo	Beyoncé	$0.99	BUY SONG
14	Sugar (feat. Wynter)	Flo Rida	$1.29	BUY SONG
15	If U Seek Amy	Britney Spears	$0.99	BUY SONG
16	My Life Would Suck Without You	Kelly Clarkson	$1.29	BUY SONG
17	Dead and Gone (feat. J... EXPLICIT	T.I.	$0.99	BUY SONG
18	Don't Trust Me EXPLICIT	3OH!3	$0.99	BUY SONG
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- In repeated purchases, consumers form price expectations.
- Operationalize RF in a dynamic pricing context and study how memory and anchoring process influences pricing strategies.
- Solves DP with nonsmooth reward and transition functions.

Modeling Reference Prices

- Utility = acquisition + transaction utilities **(mental accounting)**
- D increases in the reference gap $r - p$ **(prospect theory)**

$$d(p,r) = d_0(p) - \lambda(p-r)^+ + \gamma(r-p)^+$$
$$= \begin{cases} d_0(p) + \lambda(r-p), & \text{if } p \geqslant r; \\ d_0(p) + \gamma(r-p), & \text{if } p \leqslant r. \end{cases}$$

Loss aversion is captured by $\lambda \geqslant \gamma > 0$. We assume that the base demand, $d_0(p)$, is nonnegative, bounded, continuously differentiable, and decreasing in price, and the base profit $\pi_0(p) = p\, d_0(p)$ is nonmonotone and strictly concave. The firm's short-term profit is denoted $\pi(p,r) = p\, d(p,r)$. All our results extend for a nonzero marginal cost c.

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- **Peak-end rule**: Consumers anchor on a reference price = weighted average of lowest + most recent prices

Reference price

(0,1]: how much relying on low prices

$$r_t = \theta m_{t-1} + (1-\theta) p_{t-1}, \quad m_{t-1} = \min(m_{t-2}, p_{t-1})$$

Minimum price

Most recent price

The Model

- Firm maximizes infinite horizon B-discounted revenues:

$$J(m_0, p_0) = \max_{p_t \in \mathbf{P}} \sum_{t=1}^{\infty} \beta^{t-1} \pi(p_t, \theta m_{t-1} + (1-\theta) p_{t-1}),$$

$$\text{where } m_t = \min(m_{t-1}, p_t),$$

- Prices are bounded to [0, pbar] and at price pbar there is no demand.
- Implicitly assume that lowest prices can be remembered indefinitely. Insights remain valid even when consumers forget/update.
- **Bellman equation:**

$$J(m_{t-1}, p_{t-1}) = \max_{p_t \in \mathbf{P}} \Big\{ \pi(p_t, \theta m_{t-1} + (1-\theta) p_{t-1}) + \beta J(\min(p_t, m_{t-1}), p_t) \Big\}$$

Thanks to Loss Aversion…

- J(m,p) is increasing in both arguments.
  - Transitions and profit per stage are increasing. Also, intuitive.
- Loss aversion allows us to rewrite kinked profit pi(p,r) as the minimum of two smooth profit functions.

$$\pi_k(p, r) = [d_0(p) + k(r - p)]p = \pi_0(p) + k(r - p)p, \quad \text{for } k \in \{\lambda, \gamma\}$$

LEMMA 1. *The short-term profit is*

$$\pi(p, r) = \min(\pi_\lambda(p, r), \pi_\gamma(p, r))$$

and it is supermodular in (p, r).

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LEMMA 1. *The short-term profit is*

$$\pi(p, r) = \min(\pi_\lambda(p, r), \pi_\gamma(p, r))$$

and it is supermodular in (p, r).

- We can re-write the Bellman equation as:

$$J(m_{t-1}, p_{t-1}) = \max_{p_t \in \mathbf{P}} \{\min(\pi_\lambda, \pi_\gamma)(p_t, r_t) + \beta J(\min(p_t, m_{t-1}), p_t)\};$$
$$r_t = \theta m_{t-1} + (1 - \theta) p_{t-1}$$

  - Optimal pricing policy $p^*(m_{t-1},p_{t-1})$ solves the above equation.
  - (m,p) is a steady-state if p*(m,p) = p / optimal state path is constant.

Solution Approach

1. Characterize the **steady states**
   - Identify smooth upper-bound problems & characterize their steady states

$$J_m^\nu(p_{t-1}) = \max_{p_t \in \mathbf{P}} \{(1-\nu)\pi_\lambda(p_t, \theta m + (1-\theta)p_{t-1}) + \nu\pi_\gamma(p_t, p_{t-1}) + \beta J_m^\nu(p_t)\} \geq J(m,p)$$

   - Match steady states of original problem with those of these smooth relaxations and show that no other steady states exists

2. Characterize the **transient properties** of the optimal policy
   - Starting at any initial state (m_0,p_0), price paths remain in the same region as initial m_0
   - Within each region, the optimal price paths are monotone, so the prices converge monotonically to a steady state in the corresponding region.

Optimal Policy

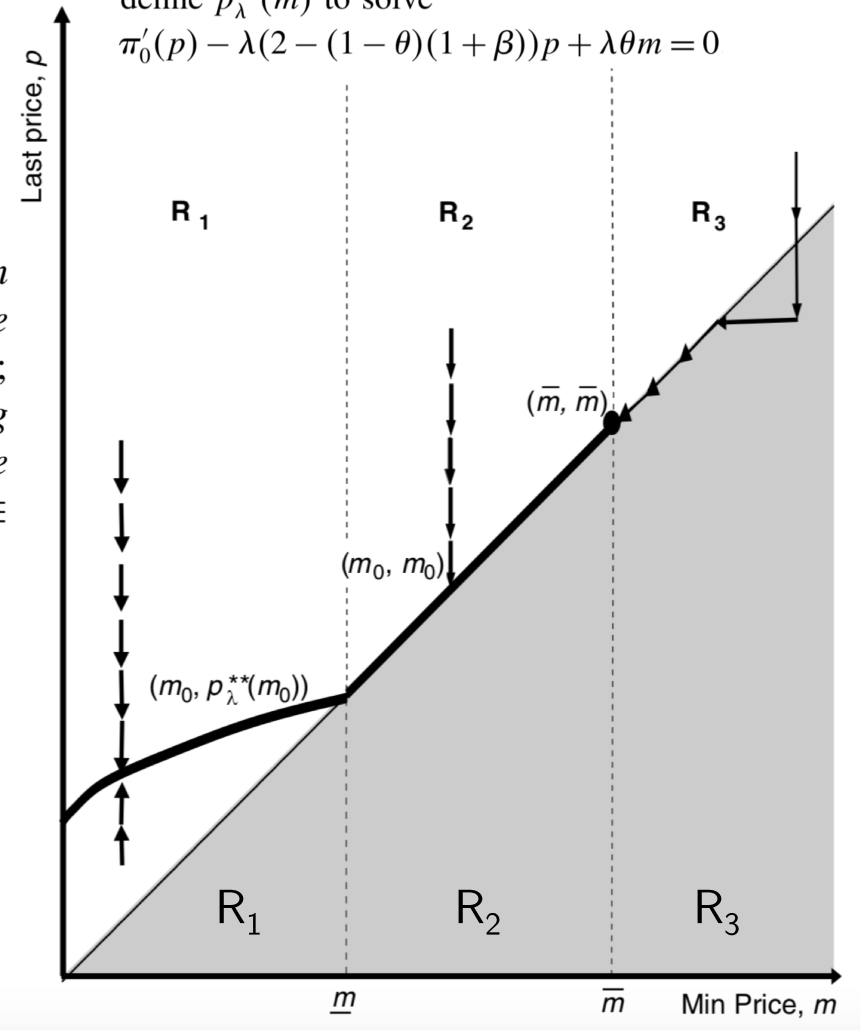
Optimal price path converges monotonically to a steady-state price (or is constant).

PROPOSITION 1. *Given any* (m_0, p_0), *the optimal price path of Problem* (3) *converges monotonically to a steady-state price, which is* (a) $p_\lambda^{**}(m_0)$, *if* $m_0 \in \mathbf{R}_1$; (b) m_0, *if* $m_0 \in \mathbf{R}_2$; *and* (c) $\overline{m}$, *if* $m_0 \in \mathbf{R}_3$. *In particular, the optimal pricing policy,* $p^*(m, p)$, *is increasing in both* m *and* p, *and the set of steady states of Problem* (3) *is* $\{(m, p_\lambda^{**}(m)) \mid m \in \mathbf{R}_1\} \cup \{(m, m) \mid m \in \mathbf{R}_2\}$.

Figure 1. Steady states (bold line) and generic optimal price paths (arrows) for each region of Problem (3).

define $p_\lambda^{**}(m)$ to solve

$\pi_0'(p) - \lambda(2-(1-\theta)(1+\beta))p + \lambda\theta m = 0$



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- Oscillating prices not LR optimal
- Steady-state price depends only on m_0
- Firm ignoring reference effects forgoes profit by overcharging customers.

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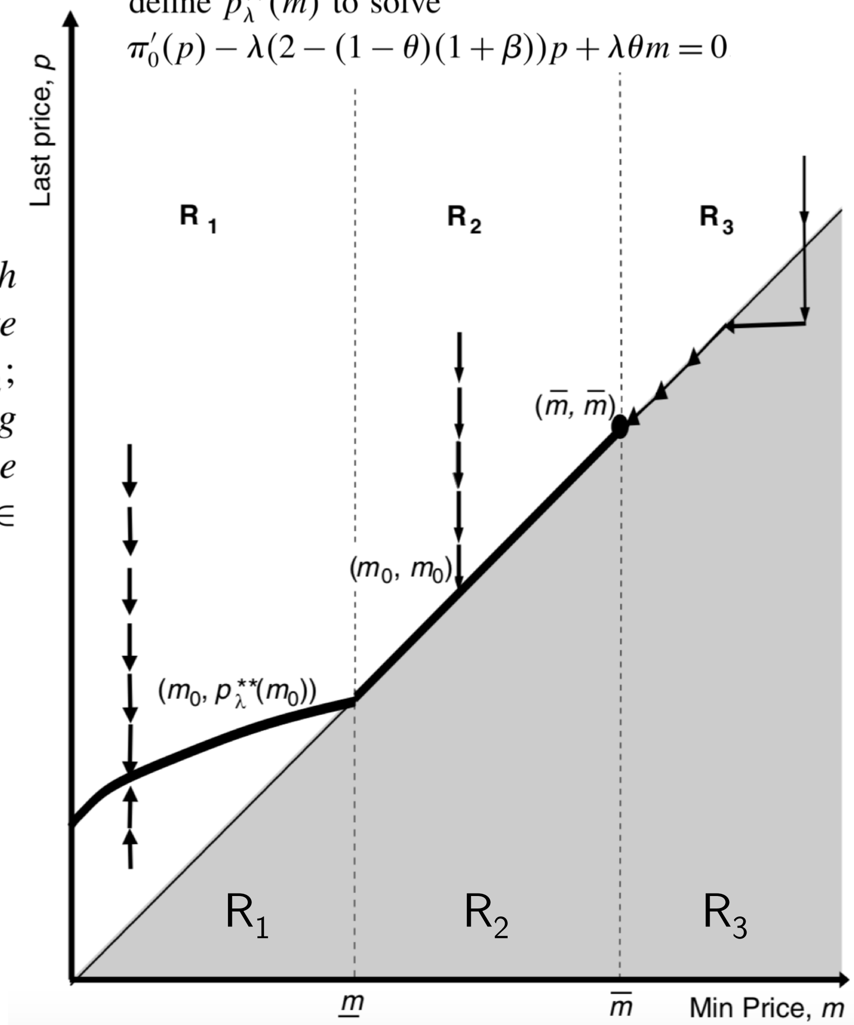

Summary

Supply Chain

Capacity Allocation

Adelman & Mersereau
Management Science '13

Memory

- Memories matter particularly when # customers is small-moderate, high demand volatility, short memories.
- ADP policy dominates greedy policy by managing expectations, pooling demand volatilities
- When goodwill matters, mechanisms that link goodwill to profits.

Revenue Management

Dynamic Pricing

Nasiry & Popescu
Operations Research '11

Loss Aversion

- Operationalize peak-end rule in a dynamic pricing context.
- Constant pricing policy is optimal.
  - Similar to Su 2007, Gallego & van Ryzin 1994, and Celik et al 2009
- Behavioral regularities ensure prices will converge to steady-state (initial low expectation affects SS price)