CHAPTER 1 PRELIMINARIES

1.1 REAL NUMBERS AND THE REAL LINE

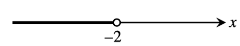
1. Executing long division, $\frac{1}{9} = 0.\overline{1}$, $\frac{2}{9} = 0.\overline{2}$, $\frac{3}{9} = 0.\overline{3}$, $\frac{8}{9} = 0.\overline{8}$, $\frac{9}{9} = 0.\overline{9}$

2. Executing long division, $\frac{1}{11} = 0.\overline{09}$, $\frac{2}{11} = 0.\overline{18}$, $\frac{3}{11} = 0.\overline{27}$, $\frac{9}{11} = 0.\overline{81}$, $\frac{11}{11} = 0.\overline{99}$

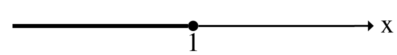
3. NT = necessarily true, NNT = Not necessarily true. Given: $2 < x < 6$.
   a) NNT. 5 is a counter example.
   b) NT. $2 < x < 6 \Rightarrow 2 - 2 < x - 2 < 6 - 2 \Rightarrow 0 < x - 2 < 2$.
   c) NT. $2 < x < 6 \Rightarrow 2/2 < x/2 < 6/2 \Rightarrow 1 < x < 3$.
   d) NT. $2 < x < 6 \Rightarrow 1/2 > 1/x > 1/6 \Rightarrow 1/6 < 1/x < 1/2$.
   e) NT. $2 < x < 6 \Rightarrow 1/2 > 1/x > 1/6 \Rightarrow 1/6 < 1/x < 1/2 \Rightarrow 6(1/6) < 6(1/x) < 6(1/2) \Rightarrow 1 < 6/x < 3$.
   f) NT. $2 < x < 6 \Rightarrow x < 6 \Rightarrow (x - 4) < 2$ and $2 < x < 6 \Rightarrow x > 2 \Rightarrow -x < -2 \Rightarrow -x + 4 < 2 \Rightarrow -(x - 4) < 2$.
   The pair of inequalities $(x - 4) < 2$ and $-(x - 4) < 2 \Rightarrow |x - 4| < 2$.
   g) NT. $2 < x < 6 \Rightarrow -2 > -x > -6 \Rightarrow -6 < -x < -2$. But $-2 < 2$. So $-6 < -x < -2 < 2$ or $-6 < -x < 2$.
   h) NT. $2 < x < 6 \Rightarrow -1(2) > -1(x) < -1(6) \Rightarrow -6 < -x < -2$

4. NT = necessarily true, NNT = Not necessarily true. Given: $-1 < y - 5 < 1$.
   a) NT. $-1 < y - 5 < 1 \Rightarrow -1 + 5 < y - 5 + 5 < 1 + 5 \Rightarrow 4 < y < 6$.
   b) NNT. $y = 5$ is a counter example. (Actually, never true given that $4 < y < 6$)
   c) NT. From a), $-1 < y - 5 < 1, \Rightarrow 4 < y < 6 \Rightarrow y > 4$.
   d) NT. From a), $-1 < y - 5 < 1, \Rightarrow 4 < y < 6 \Rightarrow y < 6$.
   e) NT. $-1 < y - 5 < 1 \Rightarrow -1 + 1 < y - 5 + 1 < 1 + 1 \Rightarrow 0 < y - 4 < 2$.
   f) NT. $-1 < y - 5 < 1 \Rightarrow (1/2)(-1 + 5) < (1/2)(y - 5 + 5) < (1/2)(1 + 5) \Rightarrow 2 < y/2 < 3$.
   g) NT. From a), $4 < y < 6 \Rightarrow 1/4 > 1/y > 1/6 \Rightarrow 1/6 < 1/y < 1/4$.
   h) NT. $-1 < y - 5 < 1 \Rightarrow y - 5 > -1 \Rightarrow y > 4 \Rightarrow -y < -4 \Rightarrow -y + 5 < 1 \Rightarrow -(y - 5) < 1$.
   Also, $-1 < y - 5 < 1 \Rightarrow y - 5 < 1$. The pair of inequalities $-(y - 5) < 1$ and $(y - 5) < 1 \Rightarrow |y - 5| < 1$.

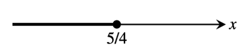
5. $-2x > 4 \Rightarrow x < -2$



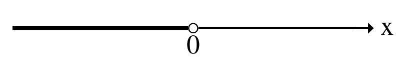
6. $8 - 3x \geq 5 \Rightarrow -3x \geq -3 \Rightarrow x \leq 1$



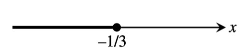
7. $5x - 3 \leq 7 - 3x \Rightarrow 8x \leq 10 \Rightarrow x \leq \frac{5}{4}$



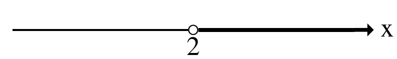
8. $3(2 - x) > 2(3 + x) \Rightarrow 6 - 3x > 6 + 2x$
   $\Rightarrow 0 > 5x \Rightarrow 0 > x$



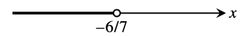
9. $2x - \frac{1}{2} \geq 7x + \frac{7}{6} \Rightarrow -\frac{1}{2} - \frac{7}{6} \geq 5x$
   $\Rightarrow \frac{1}{5}\left(-\frac{10}{6}\right) \geq x$ or $-\frac{1}{3} \geq x$



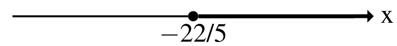
10. $\frac{6-x}{4} < \frac{3x-4}{2} \Rightarrow 12 - 2x < 12x - 16$
    $\Rightarrow 28 < 14x \Rightarrow 2 < x$



11. $\frac{4}{5}(x-2) < \frac{1}{3}(x-6) \Rightarrow 12(x-2) < 5(x-6)$
$\Rightarrow 12x - 24 < 5x - 30 \Rightarrow 7x < -6$ or $x < -\frac{6}{7}$



12. $-\frac{x+5}{2} \le \frac{12+3x}{4} \Rightarrow -(4x+20) \le 24 + 6x$
$\Rightarrow -44 \le 10x \Rightarrow -\frac{22}{5} \le x$



13. $y = 3$ or $y = -3$

14. $y - 3 = 7$ or $y - 3 = -7 \Rightarrow y = 10$ or $y = -4$

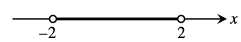
15. $2t + 5 = 4$ or $2t + 5 = -4 \Rightarrow 2t = -1$ or $2t = -9 \Rightarrow t = -\frac{1}{2}$ or $t = -\frac{9}{2}$

16. $1 - t = 1$ or $1 - t = -1 \Rightarrow -t = 0$ or $-t = -2 \Rightarrow t = 0$ or $t = 2$

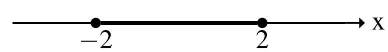
17. $8 - 3s = \frac{9}{2}$ or $8 - 3s = -\frac{9}{2} \Rightarrow -3s = -\frac{7}{2}$ or $-3s = -\frac{25}{2} \Rightarrow s = \frac{7}{6}$ or $s = \frac{25}{6}$

18. $\frac{s}{2} - 1 = 1$ or $\frac{s}{2} - 1 = -1 \Rightarrow \frac{s}{2} = 2$ or $\frac{s}{2} = 0 \Rightarrow s = 4$ or $s = 0$

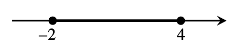
19. $-2 < x < 2$; solution interval $(-2, 2)$



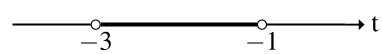
20. $-2 \le x \le 2$; solution interval $[-2, 2]$



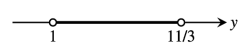
21. $-3 \le t - 1 \le 3 \Rightarrow -2 \le t \le 4$; solution interval $[-2, 4]$



22. $-1 < t + 2 < 1 \Rightarrow -3 < t < -1$;
solution interval $(-3, -1)$

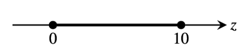


23. $-4 < 3y - 7 < 4 \Rightarrow 3 < 3y < 11 \Rightarrow 1 < y < \frac{11}{3}$;
solution interval $\left(1, \frac{11}{3}\right)$



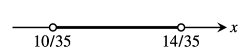
24. $-1 < 2y + 5 < 1 \Rightarrow -6 < 2y < -4 \Rightarrow -3 < y < -2$;
solution interval $(-3, -2)$

25. $-1 \le \frac{z}{5} - 1 \le 1 \Rightarrow 0 \le \frac{z}{5} \le 2 \Rightarrow 0 \le z \le 10$;
solution interval $[0, 10]$



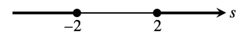
26. $-2 \le \frac{3z}{2} - 1 \le 2 \Rightarrow -1 \le \frac{3z}{2} \le 3 \Rightarrow -\frac{2}{3} \le z \le 2$;
solution interval $\left[-\frac{2}{3}, 2\right]$

27. $-\frac{1}{2} < 3 - \frac{1}{x} < \frac{1}{2} \Rightarrow -\frac{7}{2} < -\frac{1}{x} < -\frac{5}{2} \Rightarrow \frac{7}{2} > \frac{1}{x} > \frac{5}{2}$
$\Rightarrow \frac{2}{7} < x < \frac{2}{5}$; solution interval $\left(\frac{2}{7}, \frac{2}{5}\right)$

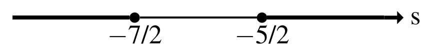


28. $-3 < \frac{2}{x} - 4 < 3 \Rightarrow 1 < \frac{2}{x} < 7 \Rightarrow 1 > \frac{x}{2} > \frac{1}{7}$
$\Rightarrow 2 > x > \frac{2}{7} \Rightarrow \frac{2}{7} < x < 2$; solution interval $\left(\frac{2}{7}, 2\right)$

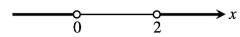
29. $2s \geq 4$ or $-2s \geq 4 \Rightarrow s \geq 2$ or $s \leq -2$;
solution intervals $(-\infty, -2] \cup [2, \infty)$



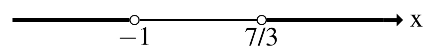
30. $s + 3 \geq \frac{1}{2}$ or $-(s + 3) \geq \frac{1}{2} \Rightarrow s \geq -\frac{5}{2}$ or $-s \geq \frac{7}{2}$
$\Rightarrow s \geq -\frac{5}{2}$ or $s \leq -\frac{7}{2}$;
solution intervals $\left(-\infty, -\frac{7}{2}\right] \cup \left[-\frac{5}{2}, \infty\right)$



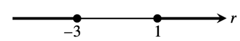
31. $1 - x > 1$ or $-(1 - x) > 1 \Rightarrow -x > 0$ or $x > 2$
$\Rightarrow x < 0$ or $x > 2$; solution intervals $(-\infty, 0) \cup (2, \infty)$



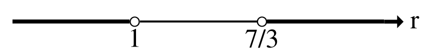
32. $2 - 3x > 5$ or $-(2 - 3x) > 5 \Rightarrow -3x > 3$ or $3x > 7$
$\Rightarrow x < -1$ or $x > \frac{7}{3}$;
solution intervals $(-\infty, -1) \cup \left(\frac{7}{3}, \infty\right)$



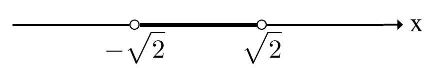
33. $\frac{r+1}{2} \geq 1$ or $-\left(\frac{r+1}{2}\right) \geq 1 \Rightarrow r + 1 \geq 2$ or $r + 1 \leq -2$
$\Rightarrow r \geq 1$ or $r \leq -3$; solution intervals $(-\infty, -3] \cup [1, \infty)$



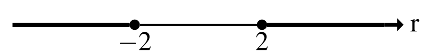
34. $\frac{3r}{5} - 1 > \frac{2}{5}$ or $-\left(\frac{3r}{5} - 1\right) > \frac{2}{5}$
$\Rightarrow \frac{3r}{5} > \frac{7}{5}$ or $-\frac{3r}{5} > -\frac{3}{5} \Rightarrow r > \frac{7}{3}$ or $r < 1$
solution intervals $(-\infty, 1) \cup \left(\frac{7}{3}, \infty\right)$



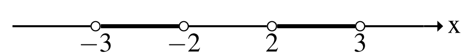
35. $x^2 < 2 \Rightarrow |x| < \sqrt{2} \Rightarrow -\sqrt{2} < x < \sqrt{2}$;
solution interval $\left(-\sqrt{2}, \sqrt{2}\right)$



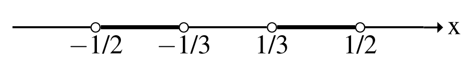
36. $4 \leq x^2 \Rightarrow 2 \leq |x| \Rightarrow x \geq 2$ or $x \leq -2$;
solution interval $(-\infty, -2] \cup [2, \infty)$



37. $4 < x^2 < 9 \Rightarrow 2 < |x| < 3 \Rightarrow 2 < x < 3$ or $2 < -x < 3$
$\Rightarrow 2 < x < 3$ or $-3 < x < -2$;
solution intervals $(-3, -2) \cup (2, 3)$

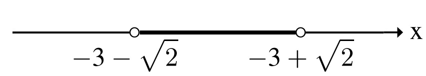


38. $\frac{1}{9} < x^2 < \frac{1}{4} \Rightarrow \frac{1}{3} < |x| < \frac{1}{2} \Rightarrow \frac{1}{3} < x < \frac{1}{2}$ or $\frac{1}{3} < -x < \frac{1}{2}$
$\Rightarrow \frac{1}{3} < x < \frac{1}{2}$ or $-\frac{1}{2} < x < -\frac{1}{3}$;
solution intervals $\left(-\frac{1}{2}, -\frac{1}{3}\right) \cup \left(\frac{1}{3}, \frac{1}{2}\right)$



39. $(x - 1)^2 < 4 \Rightarrow |x - 1| < 2 \Rightarrow -2 < x - 1 < 2$
$\Rightarrow -1 < x < 3$; solution interval $(-1, 3)$

40. $(x + 3)^2 < 2 \Rightarrow |x + 3| < \sqrt{2}$
$\Rightarrow -\sqrt{2} < x + 3 < \sqrt{2}$ or $-3 - \sqrt{2} < x < -3 + \sqrt{2}$;
solution interval $\left(-3 - \sqrt{2}, -3 + \sqrt{2}\right)$



41. $x^2 - x < 0 \Rightarrow x^2 - x + \frac{1}{4} < \frac{1}{4} \Rightarrow \left(x - \frac{1}{2}\right)^2 < \frac{1}{4} \Rightarrow \left|x - \frac{1}{2}\right| < \frac{1}{2} \Rightarrow -\frac{1}{2} < x - \frac{1}{2} < \frac{1}{2} \Rightarrow 0 < x < 1$.
So the solution is the interval $(0, 1)$

42. $x^2 - x - 2 \ge 0 \Rightarrow x^2 - x + \frac{1}{4} \ge \frac{9}{4} \Rightarrow \left|x - \frac{1}{2}\right| \ge \frac{3}{2} \Rightarrow x - \frac{1}{2} \ge \frac{3}{2}$ or $-\left(x - \frac{1}{2}\right) \ge \frac{3}{2} \Rightarrow x \ge 2$ or $x \le -1$.
The solution interval is $(-\infty, -1] \cup [2, \infty)$

43. True if $a \ge 0$; False if $a < 0$.

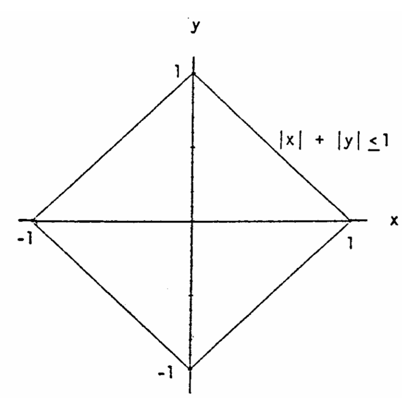
44. $|x - 1| = 1 - x \Leftrightarrow |-(x - 1)| = 1 - x \Leftrightarrow 1 - x \ge 0 \Leftrightarrow x \le 1$

45. (1) $|a + b| = (a + b)$ or $|a + b| = -(a + b)$;
both squared equal $(a + b)^2$
(2) $ab \le |ab| = |a|\,|b|$
(3) $|a| = a$ or $|a| = -a$, so $|a|^2 = a^2$; likewise, $|b|^2 = b^2$
(4) $x^2 \le y^2$ implies $\sqrt{x^2} \le \sqrt{y^2}$ or $x \le y$ for all nonnegative real numbers x and y. Let $x = |a + b|$ and $y = |a| + |b|$ so that $|a + b|^2 \le (|a| + |b|)^2 \Rightarrow |a + b| \le |a| + |b|$.

46. If $a \ge 0$ and $b \ge 0$, then $ab \ge 0$ and $|ab| = ab = |a|\,|b|$.
If $a < 0$ and $b < 0$, then $ab > 0$ and $|ab| = ab = (-a)(-b) = |a|\,|b|$.
If $a \ge 0$ and $b < 0$, then $ab \le 0$ and $|ab| = -(ab) = (a)(-b) = |a|\,|b|$.
If $a < 0$ and $b \ge 0$, then $ab \le 0$ and $|ab| = -(ab) = (-a)(b) = |a|\,|b|$.

47. $-3 \le x \le 3$ and $x > -\frac{1}{2} \Rightarrow -\frac{1}{2} < x \le 3$.

48. Graph of $|x| + |y| \le 1$ is the interior of “diamond-shaped" region.



49. Let δ be a real number > 0 and $f(x) = 2x + 1$. Suppose that $|x-1| < \delta$. Then $|x-1| < \delta \Rightarrow 2|x-1| < 2\delta \Rightarrow |2x - 2| < 2\delta \Rightarrow |(2x + 1) - 3| < 2\delta \Rightarrow |f(x) - f(1)| < 2\delta$

50. Let $\epsilon > 0$ be any positive number and $f(x) = 2x + 3$. Suppose that $|x - 0| < \epsilon/2$. Then $2|x - 0| < \epsilon$ and $|2x + 3 - 3| < \epsilon$. But $f(x) = 2x + 3$ and $f(0) = 3$. Thus $|f(x) - f(0)| < \epsilon$.

51. Consider: i) $a > 0$; ii) $a < 0$; iii) $a = 0$.
i) For $a > 0$, $|a| = a$ by definition. Now, $a > 0 \Rightarrow -a < 0$. Let $-a = b$. By definition, $|b| = -b$. Since $b = -a$, $|-a| = -(-a) = a$ and $|a| = |-a| = a$.
ii) For $a < 0$, $|a| = -a$. Now, $a < 0 \Rightarrow -a > 0$. Let $-a = b$. By definition, $|b| = b$ and thus $|-a| = -a$. So again $|a| = |-a|$.
iii) By definition $|0| = 0$ and since $-0 = 0$, $|-0| = 0$. Thus, by i), ii), and iii) $|a| = |-a|$ for any real number.

52. i) Prove $|x| > 0 \Rightarrow x > a$ or $x < -a$ for any positive number, a.
For $x \geq 0$, $|x| = x$. $|x| > a \Rightarrow x > a$.
For $x < 0$, $|x| = -x$. $|x| > a \Rightarrow -x > a \Rightarrow x < -a$.
ii) Prove $x > a$ or $x < -a \Rightarrow |x| > 0$ for any positive number, a.
$a > 0$ and $x > a \Rightarrow |x| = x$. So $x > a \Rightarrow |x| > a$.
For $a > 0$, $-a < 0$ and $x < -a \Rightarrow x < 0 \Rightarrow |x| = -x$. So $x < -a \Rightarrow -x > a \Rightarrow |x| > a$.

53. a) $1 = 1 \Rightarrow |1| = 1 \Rightarrow \left|b \cdot \frac{1}{b}\right| = \frac{|b|}{|b|} \Rightarrow |b| \cdot \left|\frac{1}{b}\right| = \frac{|b|}{|b|} \Rightarrow \frac{|b| \cdot \left|\frac{1}{b}\right|}{|b|} = \frac{|b|}{|b| \cdot |b|} \Rightarrow \left|\frac{1}{b}\right| = \frac{1}{|b|}$

b) $\frac{|a|}{|b|} = \left|a \cdot \frac{1}{b}\right| = |a| \cdot \left|\frac{1}{b}\right| = |a| \cdot \frac{1}{|b|} = \frac{|a|}{|b|}$

54. Prove $S_n = |a^n| = |a|^n$ for any real number a and any positive integer n.
$|a^1| = |a|^1 = a$, so S_1 is true. Now, assume that $S_k = |a^k| = |a|^k$ is true form some positive integer k.
Since $|a^1| = |a|^1$ and $|a^k| = |a|^k$, we have $|a^{k+1}| = |a^k \cdot a^1| = |a^k||a^1| = |a|^k\,|a|^1 = |a|^{k+1}$. Thus, $S_{k+1} = |a^{k+1}| = |a|^{k+1}$ is also true. Thus by the Principle of Mathematical Induction, $S_n = |a^n| = |a|^n$ is true for all n positive integers.

1.2 LINES, CIRCLES, AND PARABOLAS

1. $\Delta x = -1 - (-3) = 2$, $\Delta y = -2 - 2 = -4$; $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{4 + 16} = 2\sqrt{5}$

2. $\Delta x = -3 - (-1) = -2$, $\Delta y = 2 - (-2) = 4$; $d = \sqrt{(-2)^2 + 4^2} = 2\sqrt{5}$

3. $\Delta x = -8.1 - (-3.2) = -4.9$, $\Delta y = -2 - (-2) = 0$; $d = \sqrt{(-4.9)^2 + 0^2} = 4.9$

4. $\Delta x = 0 - \sqrt{2} = -\sqrt{2}$, $\Delta y = 1.5 - 4 = -2.5$; $d = \sqrt{\left(-\sqrt{2}\right)^2 + (-2.5)^2} = \sqrt{8.25}$

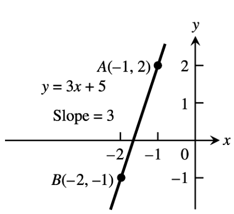
5. Circle with center $(0, 0)$ and radius 1.

6. Circle with center $(0, 0)$ and radius $\sqrt{2}$.

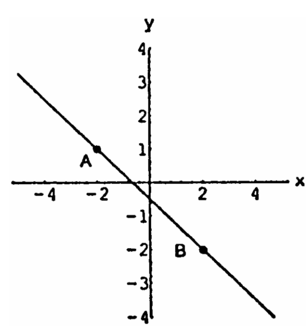
7. Disk (i.e., circle together with its interior points) with center $(0, 0)$ and radius $\sqrt{3}$.

8. The origin (a single point).

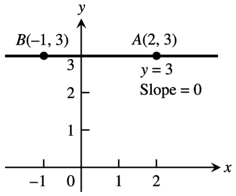
9. $m = \frac{\Delta y}{\Delta x} = \frac{-1-2}{-2-(-1)} = 3$
perpendicular slope $= -\frac{1}{3}$



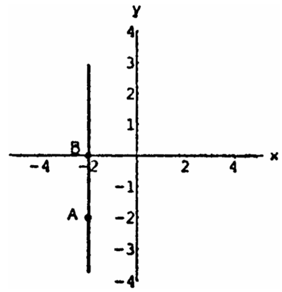
10. $m = \frac{\Delta y}{\Delta x} = \frac{-2-1}{2-(-2)} = -\frac{3}{4}$
perpendicular slope $= \frac{4}{3}$



11. $m = \frac{\Delta y}{\Delta x} = \frac{3-3}{-1-2} = 0$
perpendicular slope does not exist



12. $m = \frac{\Delta y}{\Delta x} = \frac{-2-0}{-2-(-2)}$; no slope
perpendicular slope = 0



13. (a) $x = -1$
(b) $y = \frac{4}{3}$

14. (a) $x = \sqrt{2}$
(b) $y = -1.3$

15. (a) $x = 0$
(b) $y = -\sqrt{2}$

16. (a) $x = -\pi$
(b) $y = 0$

17. $P(-1, 1), m = -1 \Rightarrow y - 1 = -1(x - (-1)) \Rightarrow y = -x$

18. $P(2, -3), m = \frac{1}{2} \Rightarrow y - (-3) = \frac{1}{2}(x - 2) \Rightarrow y = \frac{1}{2}x - 4$

19. $P(3, 4), Q(-2, 5) \Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{5-4}{-2-3} = -\frac{1}{5} \Rightarrow y - 4 = -\frac{1}{5}(x - 3) \Rightarrow y = -\frac{1}{5}x + \frac{23}{5}$

20. $P(-8, 0), Q(-1, 3) \Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{3-0}{-1-(-8)} = \frac{3}{7} \Rightarrow y - 0 = \frac{3}{7}(x - (-8)) \Rightarrow y = \frac{3}{7}x + \frac{24}{7}$

21. $m = -\frac{5}{4}, b = 6 \Rightarrow y = -\frac{5}{4}x + 6$

22. $m = \frac{1}{2}, b = -3 \Rightarrow y = \frac{1}{2}x - 3$

23. $m = 0, P(-12, -9) \Rightarrow y = -9$

24. No slope, $P\left(\frac{1}{3}, 4\right) \Rightarrow x = \frac{1}{3}$

25. $a = -1, b = 4 \Rightarrow (0, 4)$ and $(-1, 0)$ are on the line $\Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{0-4}{-1-0} = 4 \Rightarrow y = 4x + 4$

26. $a = 2, b = -6 \Rightarrow (2, 0)$ and $(0, -6)$ are on the line $\Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{-6-0}{0-2} = 3 \Rightarrow y = 3x - 6$

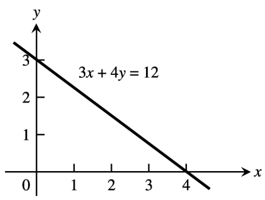
27. $P(5, -1)$, L: $2x + 5y = 15 \Rightarrow m_L = -\frac{2}{5} \Rightarrow$ parallel line is $y - (-1) = -\frac{2}{5}(x - 5) \Rightarrow y = -\frac{2}{5}x + 1$

28. $P\left(-\sqrt{2}, 2\right)$, L: $\sqrt{2}x + 5y = \sqrt{3} \Rightarrow m_L = -\frac{\sqrt{2}}{5} \Rightarrow$ parallel line is $y - 2 = -\frac{\sqrt{2}}{5}\left(x - \left(-\sqrt{2}\right)\right) \Rightarrow y = -\frac{\sqrt{2}}{5}x + \frac{8}{5}$

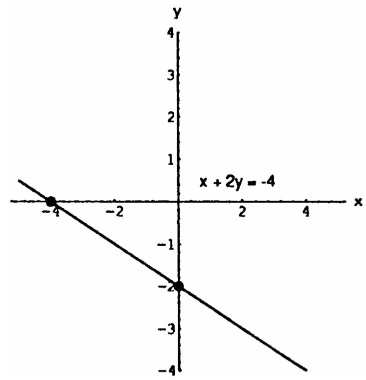
29. $P(4, 10)$, L: $6x - 3y = 5 \Rightarrow m_L = 2 \Rightarrow m_\perp = -\frac{1}{2} \Rightarrow$ perpendicular line is $y - 10 = -\frac{1}{2}(x - 4) \Rightarrow y = -\frac{1}{2}x + 12$

30. $P(0, 1)$, L: $8x - 13y = 13 \Rightarrow m_L = \frac{8}{13} \Rightarrow m_\perp = -\frac{13}{8} \Rightarrow$ perpendicular line is $y = -\frac{13}{8}x + 1$

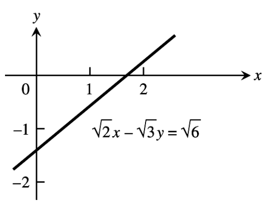
31. x-intercept $= 4$, y-intercept $= 3$



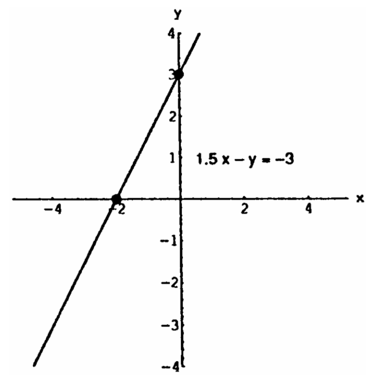
32. x-intercept $= -4$, y-intercept $= -2$



33. x-intercept $= \sqrt{3}$, y-intercept $= -\sqrt{2}$



34. x-intercept $= -2$, y-intercept $= 3$



35. $Ax + By = C_1 \;\Leftrightarrow\; y = -\frac{A}{B}x + \frac{C_1}{B}$ and $Bx - Ay = C_2 \;\Leftrightarrow\; y = \frac{B}{A}x - \frac{C_2}{A}$. Since $\left(-\frac{A}{B}\right)\left(\frac{B}{A}\right) = -1$ is the product of the slopes, the lines are perpendicular.

36. $Ax + By = C_1 \;\Leftrightarrow\; y = -\frac{A}{B}x + \frac{C_1}{B}$ and $Ax + By = C_2 \;\Leftrightarrow\; y = -\frac{A}{B}x + \frac{C_2}{B}$. Since the lines have the same slope $-\frac{A}{B}$, they are parallel.

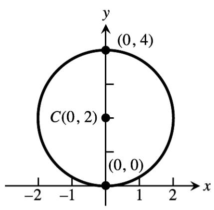
37. New position $= (x_{old} + \Delta x, y_{old} + \Delta y) = (-2 + 5, 3 + (-6)) = (3, -3)$.

38. New position $= (x_{old} + \Delta x, y_{old} + \Delta y) = (6 + (-6), 0 + 0) = (0, 0)$.

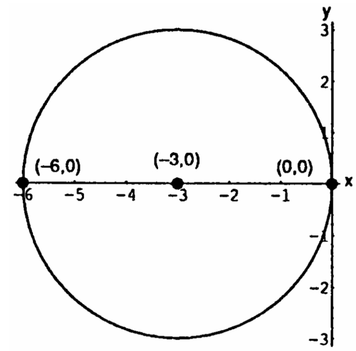
39. $\Delta x = 5$, $\Delta y = 6$, $B(3, -3)$. Let $A = (x, y)$. Then $\Delta x = x_2 - x_1 \;\Rightarrow\; 5 = 3 - x \;\Rightarrow\; x = -2$ and $\Delta y = y_2 - y_1 \;\Rightarrow\; 6 = -3 - y \;\Rightarrow\; y = -9$. Therefore, $A = (-2, -9)$.

40. $\Delta x = 1 - 1 = 0$, $\Delta y = 0 - 0 = 0$

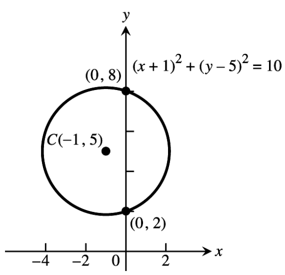
41. $C(0, 2)$, $a = 2 \Rightarrow x^2 + (y - 2)^2 = 4$



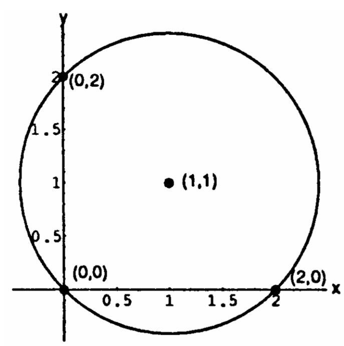
42. $C(-3, 0)$, $a = 3 \Rightarrow (x + 3)^2 + y^2 = 9$



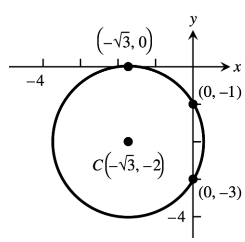
43. $C(-1, 5)$, $a = \sqrt{10} \Rightarrow (x + 1)^2 + (y - 5)^2 = 10$



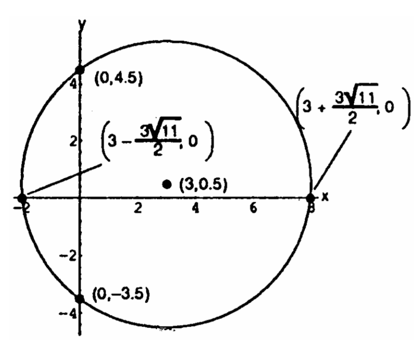
44. $C(1, 1)$, $a = \sqrt{2} \Rightarrow (x - 1)^2 + (y - 1)^2 = 2$
$x = 0 \Rightarrow (0 - 1)^2 + (y - 1)^2 = 2 \Rightarrow (y - 1)^2 = 1$
$\Rightarrow y - 1 = \pm 1 \Rightarrow y = 0$ or $y = 2$.
Similarly, $y = 0 \Rightarrow x = 0$ or $x = 2$



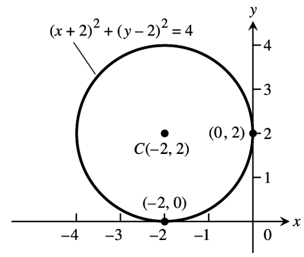
45. $C\left(-\sqrt{3}, -2\right)$, $a = 2 \Rightarrow \left(x + \sqrt{3}\right)^2 + (y + 2)^2 = 4$,
$x = 0 \Rightarrow \left(0 + \sqrt{3}\right)^2 + (y + 2)^2 = 4 \Rightarrow (y + 2)^2 = 1$
$\Rightarrow y + 2 = \pm 1 \Rightarrow y = -1$ or $y = -3$. Also, $y = 0$
$\Rightarrow \left(x + \sqrt{3}\right)^2 + (0 + 2)^2 = 4 \Rightarrow \left(x + \sqrt{3}\right)^2 = 0$
$\Rightarrow x = -\sqrt{3}$



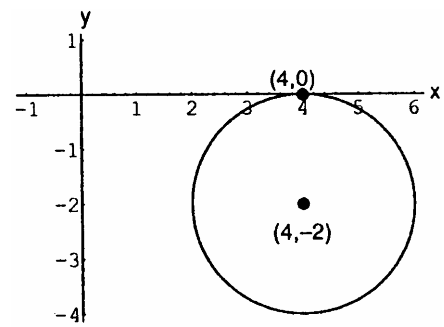
46. $C\left(3, \frac{1}{2}\right)$, $a = 5 \Rightarrow (x - 3)^2 + \left(y - \frac{1}{2}\right)^2 = 25$, so
$x = 0 \Rightarrow (0 - 3)^2 + \left(y - \frac{1}{2}\right)^2 = 25$
$\Rightarrow \left(y - \frac{1}{2}\right)^2 = 16 \Rightarrow y - \frac{1}{2} = \pm 4 \Rightarrow y = \frac{9}{2}$
or $y = -\frac{7}{2}$. Also, $y = 0 \Rightarrow (x - 3)^2 + \left(0 - \frac{1}{2}\right)^2 = 25$
$\Rightarrow (x - 3)^2 = \frac{99}{4} \Rightarrow x - 3 = \pm \frac{3\sqrt{11}}{2}$
$\Rightarrow x = 3 \pm \frac{3\sqrt{11}}{2}$



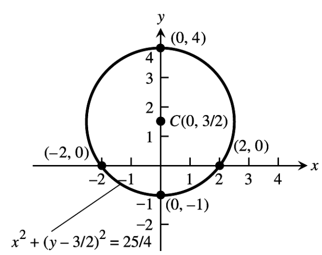
47. $x^2 + y^2 + 4x - 4y + 4 = 0$
$\Rightarrow\ x^2 + 4x + y^2 - 4y = -4$
$\Rightarrow\ x^2 + 4x + 4 + y^2 - 4y + 4 = 4$
$\Rightarrow\ (x+2)^2 + (y-2)^2 = 4\ \Rightarrow\ C = (-2, 2),\ a = 2.$



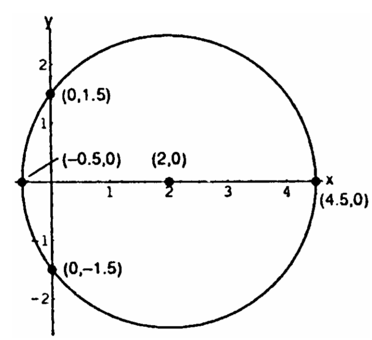
48. $x^2 + y^2 - 8x + 4y + 16 = 0$
$\Rightarrow\ x^2 - 8x + y^2 + 4y = -16$
$\Rightarrow\ x^2 - 8x + 16 + y^2 + 4y + 4 = 4$
$\Rightarrow\ (x-4)^2 + (y+2)^2 = 4$
$\Rightarrow\ C = (4, -2),\ a = 2.$



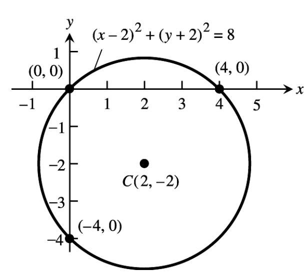
49. $x^2 + y^2 - 3y - 4 = 0\ \Rightarrow\ x^2 + y^2 - 3y = 4$
$\Rightarrow\ x^2 + y^2 - 3y + \frac{9}{4} = \frac{25}{4}$
$\Rightarrow x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{25}{4}\ \Rightarrow\ C = \left(0, \frac{3}{2}\right),$
$a = \frac{5}{2}.$



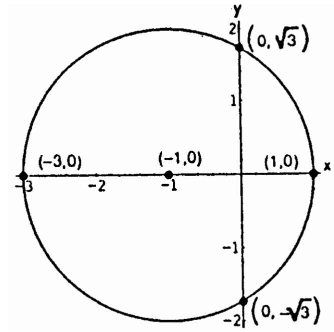
50. $x^2 + y^2 - 4x - \frac{9}{4} = 0$
$\Rightarrow\ x^2 - 4x + y^2 = \frac{9}{4}$
$\Rightarrow\ x^2 - 4x + 4 + y^2 = \frac{25}{4}$
$\Rightarrow\ (x-2)^2 + y^2 = \frac{25}{4}$
$\Rightarrow\ C = (2, 0),\ a = \frac{5}{2}.$



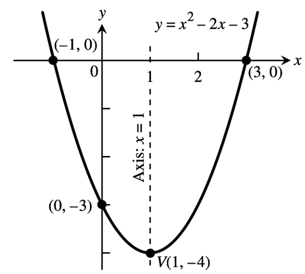
51. $x^2 + y^2 - 4x + 4y = 0$
$\Rightarrow\ x^2 - 4x + y^2 + 4y = 0$
$\Rightarrow\ x^2 - 4x + 4 + y^2 + 4y + 4 = 8$
$\Rightarrow (x-2)^2 + (y+2)^2 = 8$
$\Rightarrow\ C(2, -2),\ a = \sqrt{8}.$



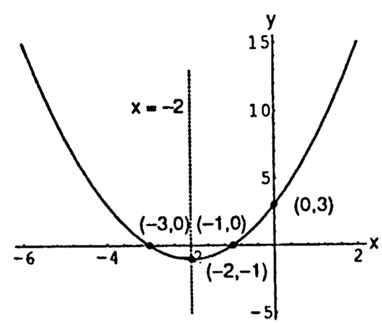
52. $x^2 + y^2 + 2x = 3$
$\Rightarrow\ x^2 + 2x + 1 + y^2 = 4$
$\Rightarrow\ (x+1)^2 + y^2 = 4$
$\Rightarrow\ C = (-1, 0),\ a = 2.$



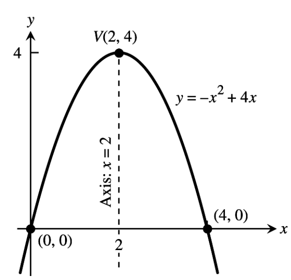
53. $x = -\frac{b}{2a} = -\frac{-2}{2(1)} = 1$
$\Rightarrow\ y = (1)^2 - 2(1) - 3 = -4$
$\Rightarrow\ V = (1, -4)$. If $x = 0$ then $y = -3$.
Also, $y = 0\ \Rightarrow\ x^2 - 2x - 3 = 0$
$\Rightarrow\ (x-3)(x+1) = 0\ \Rightarrow\ x = 3$ or
$x = -1$. Axis of parabola is $x = 1$.



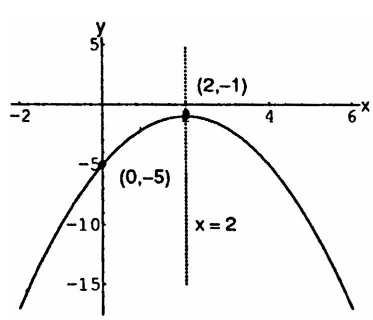
54. $x = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$
$\Rightarrow\ y = (-2)^2 + 4(-2) + 3 = -1$
$\Rightarrow\ V = (-2, -1)$. If $x = 0$ then $y = 3$.
Also, $y = 0\ \Rightarrow\ x^2 + 4x + 3 = 0$
$\Rightarrow\ (x+1)(x+3) = 0\ \Rightarrow\ x = -1$ or
$x = -3$. Axis of parabola is $x = -2$.



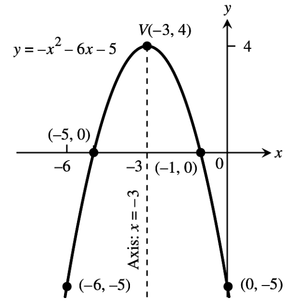
55. $x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$
$\Rightarrow\ y = -(2)^2 + 4(2) = 4$
$\Rightarrow\ V = (2, 4)$. If $x = 0$ then $y = 0$.
Also, $y = 0\ \Rightarrow\ -x^2 + 4x = 0$
$\Rightarrow\ -x(x-4) = 0\ \Rightarrow\ x = 4$ or $x = 0$.
Axis of parabola is $x = 2$.



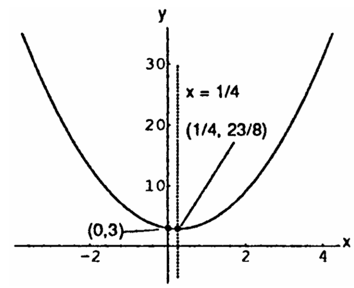
56. $x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$
$\Rightarrow\ y = -(2)^2 + 4(2) - 5 = -1$
$\Rightarrow\ V = (2, -1)$. If $x = 0$ then $y = -5$.
Also, $y = 0\ \Rightarrow\ -x^2 + 4x - 5 = 0$
$\Rightarrow\ x^2 - 4x + 5 = 0\ \Rightarrow\ x = \frac{4 \pm \sqrt{-4}}{2}$
$\Rightarrow$ no x intercepts. Axis of parabola is $x = 2$.



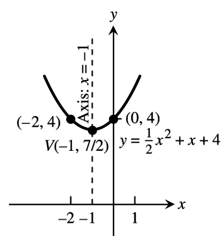
57. $x = -\frac{b}{2a} = -\frac{-6}{2(-1)} = -3$
$\Rightarrow\ y = -(-3)^2 - 6(-3) - 5 = 4$
$\Rightarrow\ V = (-3, 4)$. If $x = 0$ then $y = -5$.
Also, $y = 0\ \Rightarrow\ -x^2 - 6x - 5 = 0$
$\Rightarrow\ (x+5)(x+1) = 0\ \Rightarrow\ x = -5$ or
$x = -1$. Axis of parabola is $x = -3$.



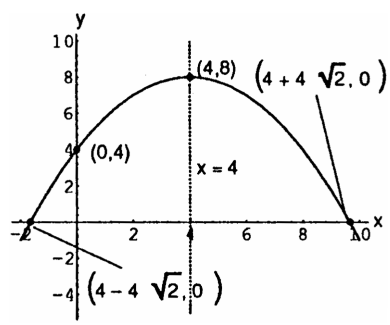
58. $x = -\frac{b}{2a} = -\frac{-1}{2(2)} = \frac{1}{4}$
$\Rightarrow\ y = 2\left(\frac{1}{4}\right)^2 - \frac{1}{4} + 3 = \frac{23}{8}$
$\Rightarrow\ V = \left(\frac{1}{4}, \frac{23}{8}\right)$. If $x = 0$ then $y = 3$.
Also, $y = 0\ \Rightarrow\ 2x^2 - x + 3 = 0$
$\Rightarrow\ x = \frac{1 \pm \sqrt{-23}}{4}\ \Rightarrow$ no x intercepts.
Axis of parabola is $x = \frac{1}{4}$.



59. $x = -\frac{b}{2a} = -\frac{1}{2(1/2)} = -1$
$\Rightarrow\ y = \frac{1}{2}(-1)^2 + (-1) + 4 = \frac{7}{2}$
$\Rightarrow\ V = \left(-1, \frac{7}{2}\right)$. If $x = 0$ then $y = 4$.
Also, $y = 0\ \Rightarrow\ \frac{1}{2}x^2 + x + 4 = 0$
$\Rightarrow\ x = \frac{-1 \pm \sqrt{-7}}{1}\ \Rightarrow$ no x intercepts.
Axis of parabola is $x = -1$.



60. $x = -\frac{b}{2a} = -\frac{2}{2(-1/4)} = 4$
$\Rightarrow\ y = -\frac{1}{4}(4)^2 + 2(4) + 4 = 8$
$\Rightarrow\ V = (4, 8)$. If $x = 0$ then $y = 4$.
Also, $y = 0\ \Rightarrow\ -\frac{1}{4}x^2 + 2x + 4 = 0$
$\Rightarrow\ x = \frac{-2 \pm \sqrt{8}}{-1/2} = 4 \pm 4\sqrt{2}$.
Axis of parabola is $x = 4$.



61. The points that lie outside the circle with center $(0, 0)$ and radius $\sqrt{7}$.

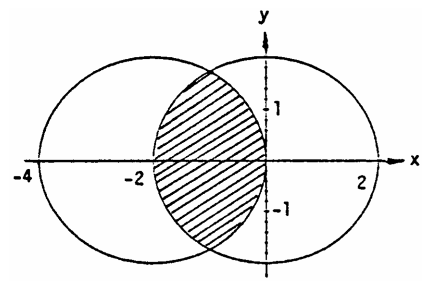
62. The points that lie inside the circle with center $(0, 0)$ and radius $\sqrt{5}$.

63. The points that lie on or inside the circle with center $(1, 0)$ and radius 2.

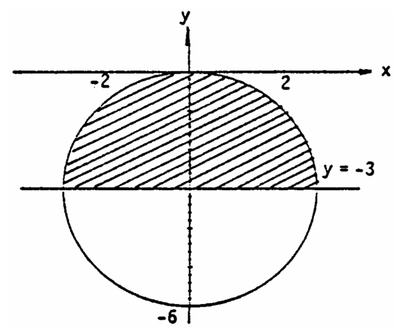
64. The points lying on or outside the circle with center $(0, 2)$ and radius 2.

65. The points lying outside the circle with center $(0, 0)$ and radius 1, but inside the circle with center $(0, 0)$, and radius 2 (i.e., a washer).

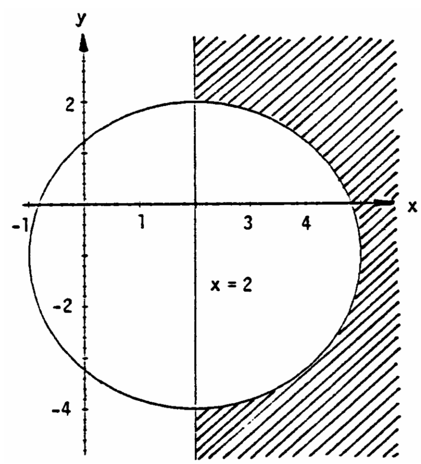
66. The points on or inside the circle centered at $(0, 0)$ with radius 2 and on or inside the circle centered at $(-2, 0)$ with radius 2.



67. $x^2 + y^2 + 6y < 0 \Rightarrow x^2 + (y + 3)^2 < 9$.
The interior points of the circle centered at $(0, -3)$ with radius 3, but above the line $y = -3$.



68. $x^2 + y^2 - 4x + 2y > 4 \Rightarrow (x - 2)^2 + (y + 1)^2 > 9$.
The points exterior to the circle centered at $(2, -1)$ with radius 3 and to the right of the line $x = 2$.



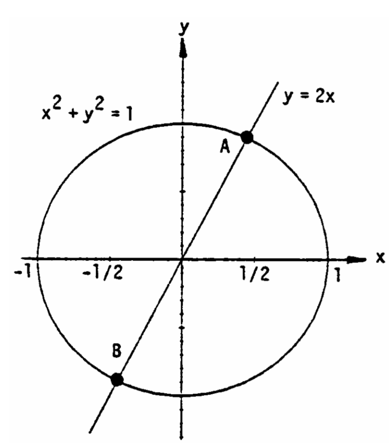
69. $(x + 2)^2 + (y - 1)^2 < 6$

70. $(x + 4)^2 + (y - 2)^2 > 16$

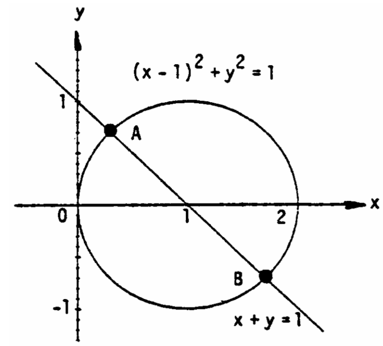
71. $x^2 + y^2 \le 2, x \ge 1$

72. $x^2 + y^2 > 4, (x - 1)^2 + (y - 3)^2 < 10$

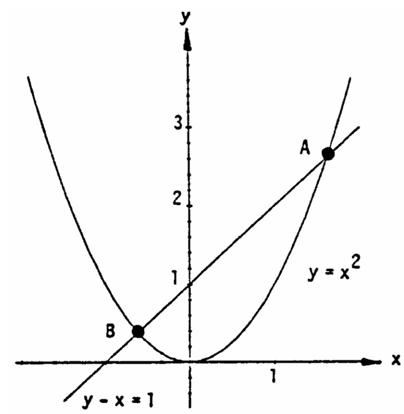
73. $x^2 + y^2 = 1$ and $y = 2x \Rightarrow 1 = x^2 + 4x^2 = 5x^2$
$\Rightarrow \left(x = \frac{1}{\sqrt{5}} \text{ and } y = \frac{2}{\sqrt{5}}\right)$ or $\left(x = -\frac{1}{\sqrt{5}} \text{ and } y = -\frac{2}{\sqrt{5}}\right)$.
Thus, $A\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$, $B\left(-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$ are the points of intersection.



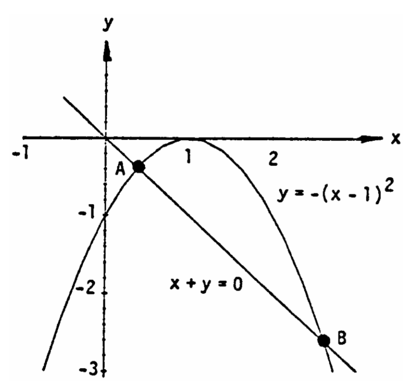
74. $x + y = 1$ and $(x - 1)^2 + y^2 = 1$
$\Rightarrow 1 = (-y)^2 + y^2 = 2y^2$
$\Rightarrow \left(y = \frac{1}{\sqrt{2}} \text{ and } x = 1 - \frac{1}{\sqrt{2}}\right)$ or $\left(y = -\frac{1}{\sqrt{2}} \text{ and } x = 1 + \frac{1}{\sqrt{2}}\right)$. Thus, $A\left(1 - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $B\left(1 + \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ are intersection points.



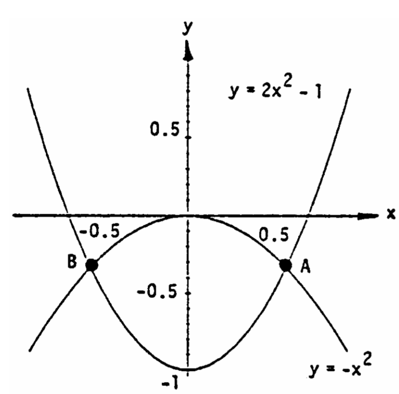
75. $y - x = 1$ and $y = x^2 \Rightarrow x^2 - x = 1$
$\Rightarrow x^2 - x - 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{5}}{2}$.
If $x = \frac{1+\sqrt{5}}{2}$, then $y = x + 1 = \frac{3+\sqrt{5}}{2}$.
If $x = \frac{1-\sqrt{5}}{2}$, then $y = x + 1 = \frac{3-\sqrt{5}}{2}$.
Thus, $A\left(\frac{1+\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right)$ and $B\left(\frac{1-\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right)$ are the intersection points.



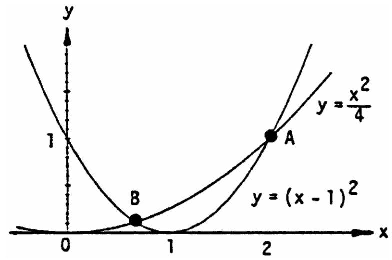
76. $y = -x$ and $y = -(x - 1)^2 \Rightarrow (x - 1)^2 = x$
$\Rightarrow x^2 - 3x + 1 = 0 \Rightarrow x = \frac{3 \pm \sqrt{5}}{2}$. If $x = \frac{3-\sqrt{5}}{2}$, then $y = -x = \frac{\sqrt{5}-3}{2}$. If $x = \frac{3+\sqrt{5}}{2}$, then $y = -x = -\frac{3+\sqrt{5}}{2}$.
Thus, $A\left(\frac{3-\sqrt{5}}{2}, \frac{\sqrt{5}-3}{2}\right)$ and $B\left(\frac{3+\sqrt{5}}{2}, -\frac{3+\sqrt{5}}{2}\right)$ are the intersection points.



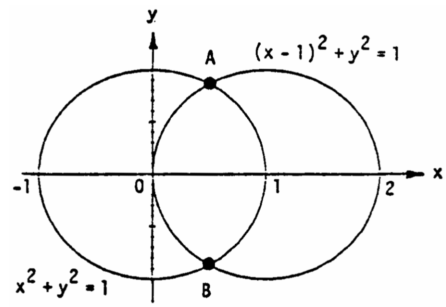
77. $y = 2x^2 - 1 = -x^2 \Rightarrow 3x^2 = 1$
$\Rightarrow x = \frac{1}{\sqrt{3}}$ and $y = -\frac{1}{3}$ or $x = -\frac{1}{\sqrt{3}}$ and $y = -\frac{1}{3}$.
Thus, $A\left(\frac{1}{\sqrt{3}}, -\frac{1}{3}\right)$ and $B\left(-\frac{1}{\sqrt{3}}, -\frac{1}{3}\right)$ are the intersection points.



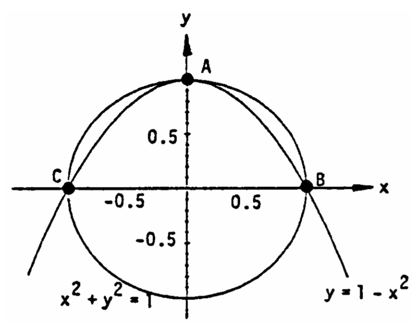
78. $y = \frac{x^2}{4} = (x-1)^2 \Rightarrow 0 = \frac{3x^2}{4} - 2x + 1$
$\Rightarrow 0 = 3x^2 - 8x + 4 = (3x-2)(x-2)$
$\Rightarrow x = 2$ and $y = \frac{x^2}{4} = 1$, or $x = \frac{2}{3}$ and
$y = \frac{x^2}{4} = \frac{1}{9}$. Thus, $A(2, 1)$ and $B\left(\frac{2}{3}, \frac{1}{9}\right)$
are the intersection points.



79. $x^2 + y^2 = 1 = (x-1)^2 + y^2$
$\Rightarrow x^2 = (x-1)^2 = x^2 - 2x + 1$
$\Rightarrow 0 = -2x + 1 \Rightarrow x = \frac{1}{2}$. Hence
$y^2 = 1 - x^2 = \frac{3}{4}$ or $y = \pm\frac{\sqrt{3}}{2}$. Thus,
$A\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $B\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ are the
intersection points.



80. $x^2 + y^2 = 1 = x^2 + y \Rightarrow y^2 = y$
$\Rightarrow y(y-1) = 0 \Rightarrow y = 0$ or $y = 1$.
If $y = 1$, then $x^2 = 1 - y^2 = 0$ or $x = 0$.
If $y = 0$, then $x^2 = 1 - y^2 = 1$ or $x = \pm 1$.
Thus, $A(0, 1)$, $B(1, 0)$, and $C(-1, 0)$ are the
intersection points.



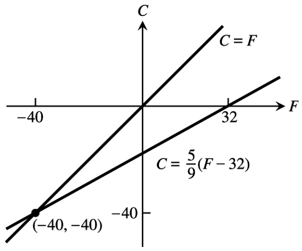
81. (a) $A \approx (69°, 0 \text{ in})$, $B \approx (68°, .4 \text{ in}) \Rightarrow m = \frac{68° - 69°}{.4 - 0} \approx -2.5°/\text{in}$.
(b) $A \approx (68°, .4 \text{ in})$, $B \approx (10°, 4 \text{ in}) \Rightarrow m = \frac{10° - 68°}{4 - .4} \approx -16.1°/\text{in}$.
(c) $A \approx (10°, 4 \text{ in})$, $B \approx (5°, 4.6 \text{ in}) \Rightarrow m = \frac{5° - 10°}{4.6 - 4} \approx -8.3°/\text{in}$.

82. The time rate of heat transfer across a material, $\frac{\Delta Q}{\Delta t}$, is directly proportional to the cross-sectional area, A, of the material, to the temperature gradient across the material, $\frac{\Delta T}{\Delta x}$ (the slopes from the previous problem), and to a constant characteristic of the material. $\frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x} \Rightarrow k = -\frac{\frac{\Delta Q}{\Delta t}/A}{\frac{\Delta T}{\Delta x}}$. Note that $\frac{\Delta Q}{\Delta t}$ and $\frac{\Delta T}{\Delta x}$ are of opposite sign because heat flow is toward lower temperature. So a small value of k corresponds to low heat flow through the material and thus the material is a good insulator.Since all three materials have the same cross section and the heat flow across each is the same (temperatures are not changing), we may define another constant, K, characteristics of the material: $K = -\frac{1}{\frac{\Delta T}{\Delta x}}$. Using the values of $\frac{\Delta T}{\Delta x}$ from the prevous problem, fiberglass has the smallest K at 0.06 and thus is the best insulator. Likewise, the wallboard is the poorest insulator, with $K = 0.4$.

83. $p = kd + 1$ and $p = 10.94$ at $d = 100 \Rightarrow k = \frac{10.94 - 1}{100} = 0.0994$. Then $p = 0.0994d + 1$ is the diver's pressure equation so that $d = 50 \Rightarrow p = (0.0994)(50) + 1 = 5.97$ atmospheres.

84. The line of incidence passes through $(0, 1)$ and $(1, 0) \Rightarrow$ The line of reflection passes through $(1, 0)$ and $(2, 1)$
$\Rightarrow m = \frac{1-0}{2-1} = 1 \Rightarrow y - 0 = 1(x-1) \Rightarrow y = x - 1$ is the line of reflection.

85. $C = \frac{5}{9}(F - 32)$ and $C = F \Rightarrow F = \frac{5}{9}F - \frac{160}{9} \Rightarrow \frac{4}{9}F = -\frac{160}{9}$ or $F = -40°$ gives the same numerical reading.



86. $m = \frac{37.1}{100} = \frac{14}{\Delta x} \Rightarrow \Delta x = \frac{14}{.371}$. Therefore, distance between first and last rows is $\sqrt{(14)^2 + \left(\frac{14}{.371}\right)^2} \approx 40.25$ ft.

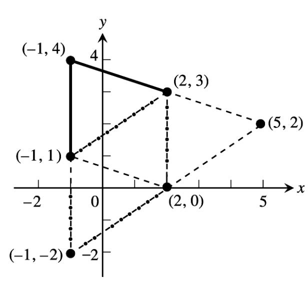
87. length $AB = \sqrt{(5-1)^2 + (5-2)^2} = \sqrt{16+9} = 5$
length $AC = \sqrt{(4-1)^2 + (-2-2)^2} = \sqrt{9+16} = 5$
length $BC = \sqrt{(4-5)^2 + (-2-5)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2} \neq 5$

88. length $AB = \sqrt{(1-0)^2 + \left(\sqrt{3}-0\right)^2} = \sqrt{1+3} = 2$
length $AC = \sqrt{(2-0)^2 + (0-0)^2} = \sqrt{4+0} = 2$
length $BC = \sqrt{(2-1)^2 + \left(0-\sqrt{3}\right)^2} = \sqrt{1+3} = 2$

89. Length $AB = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{1^2 + 4^2} = \sqrt{17}$ and length $BC = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{4^2 + 1^2} = \sqrt{17}$. Also, slope $AB = \frac{4}{-1}$ and slope $BC = \frac{1}{4}$, so $AB \perp BC$. Thus, the points are vertices of a square. The coordinate increments from the fourth vertex $D(x, y)$ to A must equal the increments from C to B $\Rightarrow 2 - x = \Delta x = 4$ and $-1 - y = \Delta y = 1 \Rightarrow x = -2$ and $y = -2$. Thus $D(-2, -2)$ is the fourth vertex.

90. Let $A = (x, 2)$ and $C = (9, y) \Rightarrow B = (x, y)$. Then $9 - x = |AD|$ and $2 - y = |DC| \Rightarrow 2(9 - x) + 2(2 - y) = 56$ and $9 - x = 3(2 - y) \Rightarrow 2(3(2 - y)) + 2(2 - y) = 56 \Rightarrow y = -5 \Rightarrow 9 - x = 3(2 - (-5)) \Rightarrow x = -12$. Therefore, $A = (-12, 2)$, $C = (9, -5)$, and $B = (-12, -5)$.

91. Let $A(-1, 1)$, $B(2, 3)$, and $C(2, 0)$ denote the points. Since BC is vertical and has length $|BC| = 3$, let $D_1(-1, 4)$ be located vertically upward from A and $D_2(-1, -2)$ be located vertically downward from A so that $|BC| = |AD_1| = |AD_2| = 3$. Denote the point $D_3(x, y)$. Since the slope of AB equals the slope of CD_3 we have $\frac{y-3}{x-2} = -\frac{1}{3} \Rightarrow 3y - 9 = -x + 2$ or $x + 3y = 11$. Likewise, the slope of AC equals the slope of BD_3 so that $\frac{y-0}{x-2} = \frac{2}{3} \Rightarrow 3y = 2x - 4$ or $2x - 3y = 4$.



Solving the system of equations $\left.\begin{aligned} x + 3y &= 11 \\ 2x - 3y &= 4 \end{aligned}\right\}$ we find $x = 5$ and $y = 2$ yielding the vertex $D_3(5, 2)$.

92. Let (x, y), $x \neq 0$ and/or $y \neq 0$ be a point on the coordinate plane. The slope, m, of the segment $(0, 0)$ to (x, y) is $\frac{y}{x}$. A 90° rotation gives a segment with slope $m' = -\frac{1}{m} = -\frac{x}{y}$. If this segment has length equal to the original segment, its endpoint will be $(-y, x)$ or $(y, -x)$, the first of these corresponds to a counter-clockwise rotation, the latter to a clockwise rotation.

(a) $(-1, 4)$; (b) $(3, -2)$; (c) $(5, 2)$; (d) $(0, x)$;

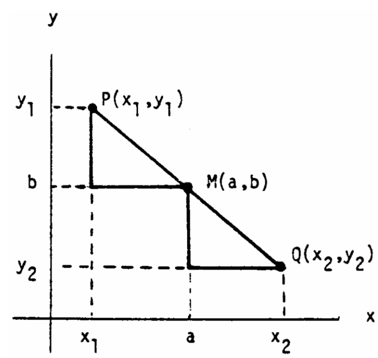
(e) $(-y, 0)$; (f) $(-y, x)$; (g) $(3, -10)$

93. $2x + ky = 3$ has slope $-\frac{2}{k}$ and $4x + y = 1$ has slope -4. The lines are perpendicular when $-\frac{2}{k}(-4) = -1$ or $k = -8$ and parallel when $-\frac{2}{k} = -4$ or $k = \frac{1}{2}$.

94. At the point of intersection, $2x + 4y = 6$ and $2x - 3y = -1$. Subtracting these equations we find $7y = 7$ or $y = 1$. Substitution into either equation gives $x = 1 \Rightarrow (1, 1)$ is the intersection point. The line through $(1, 1)$ and $(1, 2)$ is vertical with equation $x = 1$.

95. Let $M(a, b)$ be the midpoint. Since the two triangles shown in the figure are congruent, the value a must lie midway between x_1 and x_2, so $a = \frac{x_1+x_2}{2}$.

Similarly, $b = \frac{y_1+y_2}{2}$.



96. (a) L has slope 1 so M is the line through $P(2, 1)$ with slope -1; or the line $y = -x + 3$. At the intersection point, Q, we have equal y-values, $y = x + 2 = -x + 3$. Thus, $2x = 1$ or $x = \frac{1}{2}$. Hence Q has coordinates $\left(\frac{1}{2}, \frac{5}{2}\right)$. The distance from P to L = the distance from P to Q = $\sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{3}{2}\right)^2} = \sqrt{\frac{18}{4}} = \frac{3\sqrt{2}}{2}$.

(b) L has slope $-\frac{4}{3}$ so M has slope $\frac{3}{4}$ and M has the equation $4y - 3x = 12$. We can rewrite the equations of the lines as L: $x + \frac{3}{4}y = 3$ and M: $-x + \frac{4}{3}y = 4$. Adding these we get $\frac{25}{12}y = 7$ so $y = \frac{84}{25}$. Substitution into either equation gives $x = \frac{4}{3}\left(\frac{84}{25}\right) - 4 = \frac{12}{25}$ so that $Q\left(\frac{12}{25}, \frac{84}{25}\right)$ is the point of intersection. The distance from P to L = $\sqrt{\left(4 - \frac{12}{25}\right)^2 + \left(6 - \frac{84}{25}\right)^2} = \frac{22}{5}$.

(c) M is a horizontal line with equation $y = b$. The intersection point of L and M is $Q(-1, b)$. Thus, the distance from P to L is $\sqrt{(a+1)^2 + 0^2} = |a + 1|$.

(d) If $B = 0$ and $A \neq 0$, then the distance from P to L is $\left|\frac{C}{A} - x_0\right|$ as in (c). Similarly, if $A = 0$ and $B \neq 0$, the distance is $\left|\frac{C}{B} - y_0\right|$. If both A and B are $\neq 0$ then L has slope $-\frac{A}{B}$ so M has slope $\frac{B}{A}$. Thus, L: $Ax + By = C$ and M: $-Bx + Ay = -Bx_0 + Ay_0$. Solving these equations simultaneously we find the point of intersection $Q(x, y)$ with $x = \frac{AC - B(Ay_0 - Bx_0)}{A^2+B^2}$ and $y = \frac{BC + A(Ay_0 - Bx_0)}{A^2+B^2}$. The distance from P to Q equals $\sqrt{(\Delta x)^2 + (\Delta y)^2}$, where $(\Delta x)^2 = \left(\frac{x_0(A^2+B^2) - AC + ABy_0 - B^2x_0}{A^2+B^2}\right)^2$ $= \frac{A^2(Ax_0+By_0+C)^2}{(A^2+B^2)^2}$, and $(\Delta y)^2 = \left(\frac{y_0(A^2+B^2) - BC - A^2y_0 + ABx_0}{A^2+B^2}\right)^2 = \frac{B^2(Ax_0+By_0+C)^2}{(A^2+B^2)^2}$.
Thus, $\sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{\frac{(Ax_0+By_0+C)^2}{A^2+B^2}} = \frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$.

1.3 FUNCTIONS AND THEIR GRAPHS

1. domain $= (-\infty, \infty)$; range $= [1, \infty)$

2. domain $= [0, \infty)$; range $= (-\infty, 1]$

3. domain $= (0, \infty)$; y in range $\Rightarrow y = \frac{1}{\sqrt{t}}, t > 0 \Rightarrow y^2 = \frac{1}{t}$ and $y > 0 \Rightarrow$ y can be any positive real number $\Rightarrow$ range $= (0, \infty)$.

4. domain $= [0, \infty)$; y in range $\Rightarrow$ $y = \frac{1}{1+\sqrt{t}}$, $t > 0$. If $t = 0$, then $y = 1$ and as t increases, y becomes a smaller and smaller positive real number $\Rightarrow$ range $= (0, 1]$.

5. $4 - z^2 = (2 - z)(2 + z) \geq 0 \Leftrightarrow z \in [-2, 2] =$ domain. Largest value is $g(0) = \sqrt{4} = 2$ and smallest value is $g(-2) = g(2) = \sqrt{0} = 0 \Rightarrow$ range $= [0, 2]$.

6. domain $= (-2, 2)$ from Exercise 5; smallest value is $g(0) = \frac{1}{2}$ and as $0 < z$ increases to 2, $g(z)$ gets larger and larger (also true as $z < 0$ decreases to -2) $\Rightarrow$ range $= \left[\frac{1}{2}, \infty\right)$.

7. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Is the graph of a function of x since any vertical line intersects the graph at most once.

8. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Not the graph of a function of x since it fails the vertical line test.

9. $y = \sqrt{\left(\frac{1}{x}\right) - 1} \Rightarrow \frac{1}{x} - 1 \geq 0 \Rightarrow x \leq 1$ and $x > 0$. So,
   (a) No $(x > 0)$;
   (b) No; division by 0 undefined;
   (c) No; if $x \geq 1$, $\frac{1}{x} < 1 \Rightarrow \frac{1}{x} - 1 < 0$;
   (d) $(0, 1]$

10. $y = \sqrt{2 - \sqrt{x}} \Rightarrow 2 - \sqrt{x} \geq 0 \Rightarrow \sqrt{x} \geq 0$ and $\sqrt{x} \leq 2$. $\sqrt{x} \geq 0 \Rightarrow x \geq 0$ and $\sqrt{x} \leq 2 \Rightarrow x \leq 4$. So, $0 \leq x \leq 4$.
    (a) No; (b) No; (c) $[0, 4]$

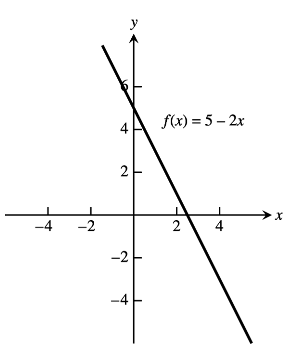
11. base $= x$; $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow$ height $= \frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$; perimeter is $p(x) = x + x + x = 3x$.

12. $s =$ side length $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$

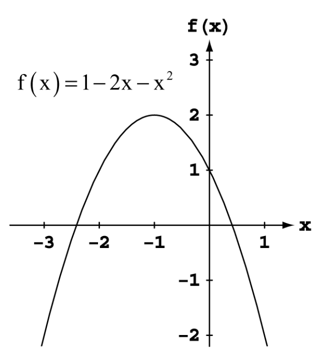
13. Let D = diagonal of a face of the cube and ℓ = the length of an edge. Then $\ell^2 + D^2 = d^2$ and (by Exercise 10) $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d^2}{3}\right)^{3/2} = \frac{d^3}{3\sqrt{3}}$.

14. The coordinates of P are $\left(x, \sqrt{x}\right)$ so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$ $(x > 0)$. Thus, $\left(x, \sqrt{x}\right) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$.

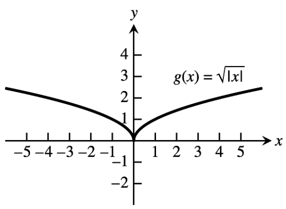
15. The domain is $(-\infty, \infty)$.



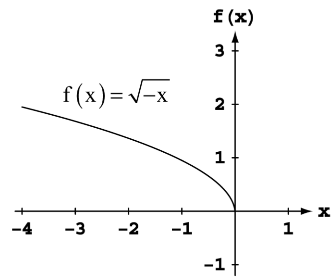
16. The domain is $(-\infty, \infty)$.



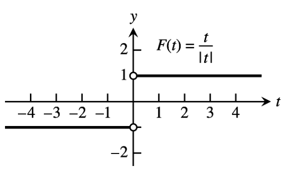
17. The domain is $(-\infty, \infty)$.



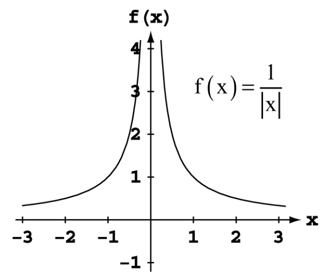
18. The domain is $(-\infty, 0]$.



19. The domain is $(-\infty, 0) \cup (0, \infty)$.

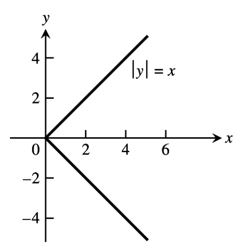


20. The domain is $(-\infty, 0) \cup (0, \infty)$.

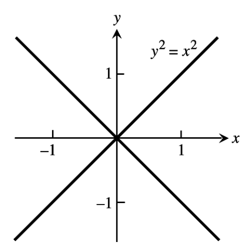


21. Neither graph passes the vertical line test

(a)

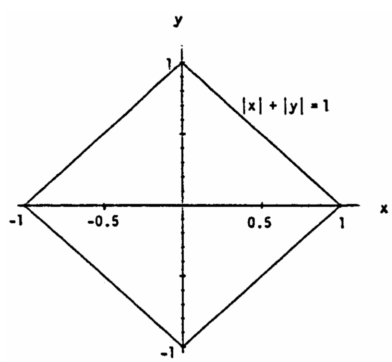


(b)

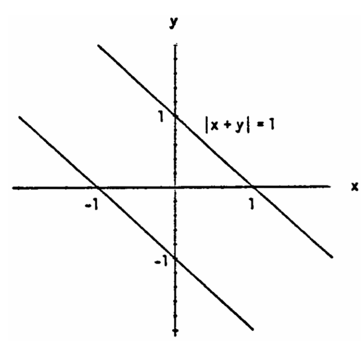


22. Neither graph passes the vertical line test

(a)



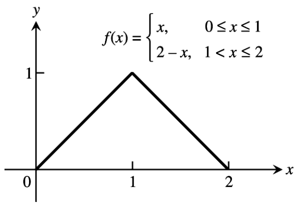
(b)



$$|x + y| = 1 \Leftrightarrow \left\{ \begin{array}{c} x + y = 1 \\ \text{or} \\ x + y = -1 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} y = 1 - x \\ \text{or} \\ y = -1 - x \end{array} \right\}$$

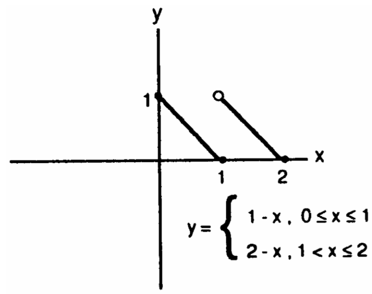
23.

x	0	1	2
y	0	1	0

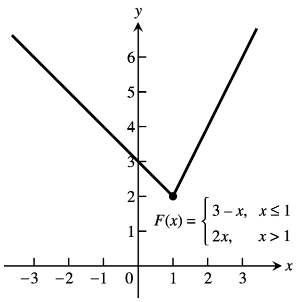


24.

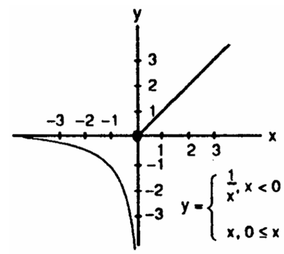
x	0	1	2
y	1	0	0



25. $y = \begin{cases} 3-x, & x \le 1 \\ 2x, & 1 < x \end{cases}$



26. $y = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & 0 \le x \end{cases}$



27. (a) Line through $(0, 0)$ and $(1, 1)$: $y = x$

Line through $(1, 1)$ and $(2, 0)$: $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x + 2, & 1 < x \le 2 \end{cases}$$

(b) $$f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$$

28. (a) Line through $(0, 2)$ and $(2, 0)$: $y = -x + 2$

Line through $(2, 1)$ and $(5, 0)$: $m = \frac{0-1}{5-2} = \frac{-1}{3} = -\frac{1}{3}$, so $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x + 2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$$

(b) Line through $(-1, 0)$ and $(0, -3)$: $m = \frac{-3-0}{0-(-1)} = -3$, so $y = -3x - 3$

Line through $(0, 3)$ and $(2, -1)$: $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x - 3, & -1 < x \le 0 \\ -2x + 3, & 0 < x \le 2 \end{cases}$$

29. (a) Line through $(-1, 1)$ and $(0, 0)$: $y = -x$

Line through $(0, 1)$ and $(1, 1)$: $y = 1$

Line through $(1, 1)$ and $(3, 0)$: $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \le x < 0 \\ 1 & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

(b) Line through $(-2, -1)$ and $(0, 0)$: $y = \frac{1}{2}x$

Line through $(0, 2)$ and $(1, 0)$: $y = -2x + 2$

Line through $(1, -1)$ and $(3, -1)$: $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \le x \le 0 \\ -2x + 2 & 0 < x \le 1 \\ -1 & 1 < x \le 3 \end{cases}$$

30. (a) Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$: $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

(b) $$f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$$

31. (a) From the graph, $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow x \in (-2, 0) \cup (4, \infty)$

(b) $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow \frac{x}{2} - 1 - \frac{4}{x} > 0$

$x > 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2-2x-8}{2x} > 0 \Rightarrow \frac{(x-4)(x+2)}{2x} > 0$

$\Rightarrow x > 4$ since x is positive;

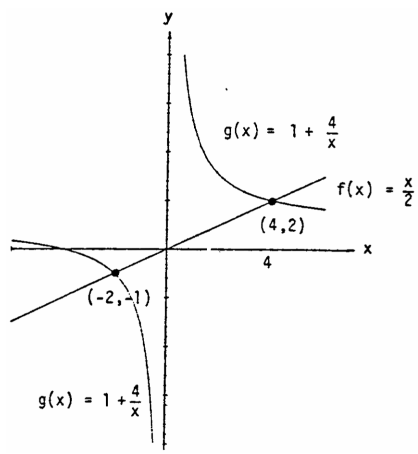
$x < 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2-2x-8}{2x} < 0 \Rightarrow \frac{(x-4)(x+2)}{2x} < 0$

$\Rightarrow x < -2$ since x is negative;

sign of $(x-4)(x+2)$

$+ \quad -2 \quad - \quad 4 \quad +$

Solution interval: $(-2, 0) \cup (4, \infty)$



32. (a) From the graph, $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$

(b) <u>Case</u> $x < -1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$

$\Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5$.

Thus, $x \in (-\infty, -5)$ solves the inequality.

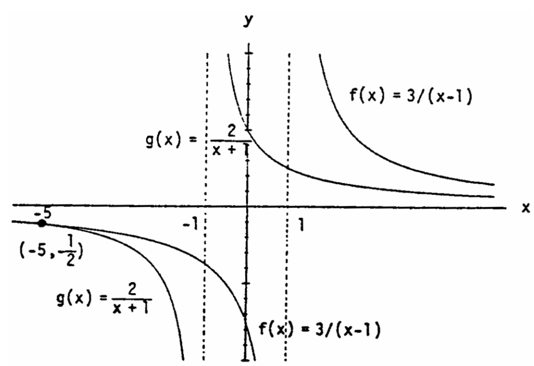
<u>Case</u> $-1 < x < 1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$

$\Rightarrow 3x + 3 > 2x - 2 \Rightarrow x > -5$ which is true if $x > -1$. Thus, $x \in (-1, 1)$ solves the inequality.

<u>Case</u> $1 < x$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5$

which is never true if $1 < x$, so no solution here.

In conclusion, $x \in (-\infty, -5) \cup (-1, 1)$.

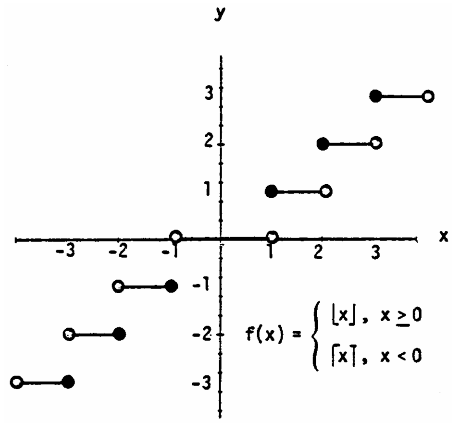


33. (a) $\lfloor x \rfloor = 0$ for $x \in [0, 1)$ (b) $\lceil x \rceil = 0$ for $x \in (-1, 0]$

34. $\lfloor x \rfloor = \lceil x \rceil$ only when x is an integer.

35. For any real number x, $n \le x \le n + 1$, where n is an integer. Now: $n \le x \le n + 1 \Rightarrow -(n+1) \le -x \le -n$. By definition: $\lceil -x \rceil = -n$ and $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$. So $\lceil -x \rceil = -\lfloor x \rfloor$ for all $x \in \Re$.

36. To find f(x) you delete the decimal or fractional portion of x, leaving only the integer part.



37. $v = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x;\ 0 < x < 7.$

38. (a) Let h = height of the triangle. Since the triangle is isosceles, $\overline{AB}^2 + \overline{AB}^2 = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$. So, $h^2 + 1^2 = \left(\sqrt{2}\right)^2 \Rightarrow h = 1 \Rightarrow$ B is at $(0, 1) \Rightarrow$ slope of AB $= -1 \Rightarrow$ The equation of AB is $y = f(x) = -x + 1;\ x \in [0, 1]$.

(b) $A(x) = 2x\,y = 2x(-x + 1) = -2x^2 + 2x;\ x \in [0, 1]$.

39. (a) Because the circumference of the original circle was 8π and a piece of length x was removed.

(b) $r = \frac{8\pi - x}{2\pi} = 4 - \frac{x}{2\pi}$

(c) $h = \sqrt{16 - r^2} = \sqrt{16 - \left(4 - \frac{x}{2\pi}\right)^2} = \sqrt{16 - \left(16 - \frac{4x}{\pi} + \frac{x^2}{4\pi^2}\right)} = \sqrt{\frac{4x}{\pi} - \frac{x^2}{4\pi^2}} = \sqrt{\frac{16\pi x}{4\pi^2} - \frac{x^2}{4\pi^2}} = \frac{\sqrt{16\pi x - x^2}}{2\pi}$

(d) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{8\pi - x}{2\pi}\right)^2 \cdot \frac{\sqrt{16\pi x - x^2}}{2\pi} = \frac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2}$

40. (a) Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and $(10,560 - x)$ feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10,560 - x)$.

(b) $C(0) = \$1,200,000$
$C(500) \approx \$1,175,812$
$C(1000) \approx \$1,186,512$
$C(1500) \approx \$1,212,000$
$C(2000) \approx \$1,243,732$
$C(2500) \approx \$1,278,479$
$C(3000) \approx \$1,314,870$
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from the point P.

41. A curve symmetric about the x-axis will not pass the vertical line test because the points (x, y) and (x, −y) lie on the same vertical line. The graph of the function $y = f(x) = 0$ is the x-axis, a horizontal line for which there is a single y-value, 0, for any x.

42. Pick 11, for example: $11 + 5 = 16 \to 2 \cdot 16 = 32 \to 32 - 6 = 26 \to \frac{26}{2} = 13 \to 13 - 2 = 11$, the original number.
$f(x) = \frac{2(x+5)-6}{2} - 2 = x$, the number you started with.

1.4 IDENTIFYING FUNCTIONS; MATHEMATICAL MODELS

1. (a) linear, polynomial of degree 1, algebraic. (b) power, algebraic.
   (c) rational, algebraic. (d) exponential.

2. (a) polynomial of degree 4, algebraic. (b) exponential.
   (c) algebraic. (d) power, algebraic.

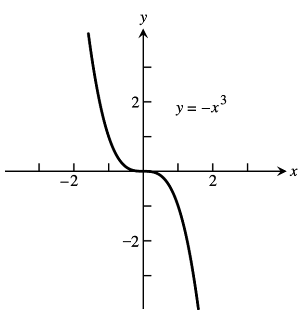
3. (a) rational, algebraic. (b) algebraic.
   (c) trigonometric. (d) logarithmic.

4. (a) logarithmic. (b) algebraic.
   (c) exponential. (d) trigonometric.

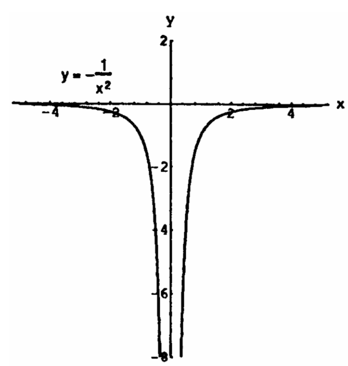
5. (a) Graph h because it is an even function and rises less rapidly than does Graph g.
   (b) Graph f because it is an odd function.
   (c) Graph g because it is an even function and rises more rapidly than does Graph h.

6. (a) Graph f because it is linear.
   (b) Graph g because it contains $(0, 1)$.
   (c) Graph h because it is a nonlinear odd function.

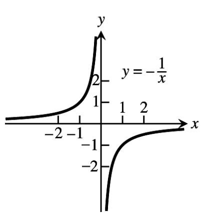
7. Symmetric about the origin
   Dec: $-\infty < x < \infty$
   Inc: nowhere



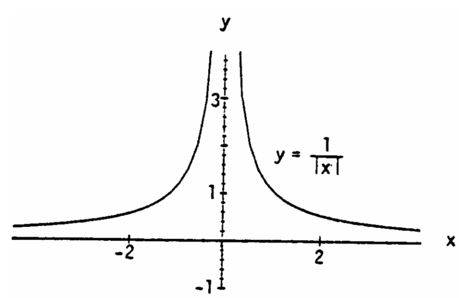
8. Symmetric about the y-axis
   Dec: $-\infty < x < 0$
   Inc: $0 < x < \infty$



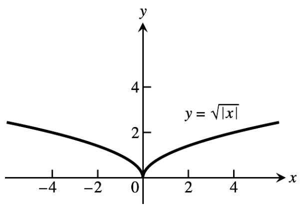
9. Symmetric about the origin
   Dec: nowhere
   Inc: $-\infty < x < 0$
   $0 < x < \infty$



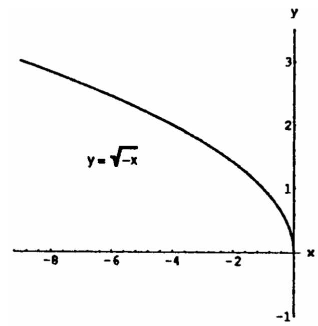
10. Symmetric about the y-axis
    Dec: $0 < x < \infty$
    Inc: $-\infty < x < 0$



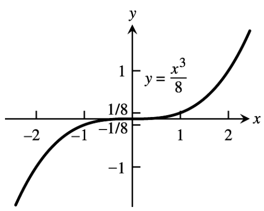
11. Symmetric about the y-axis
Dec: $-\infty < x \leq 0$
Inc: $0 < x < \infty$



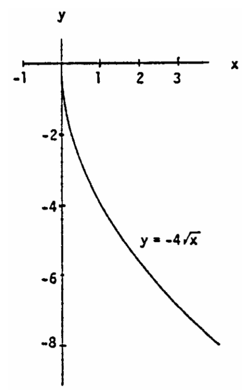
12. No symmetry
Dec: $-\infty < x \leq 0$
Inc: nowhere



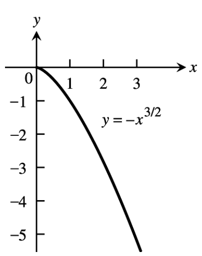
13. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < \infty$



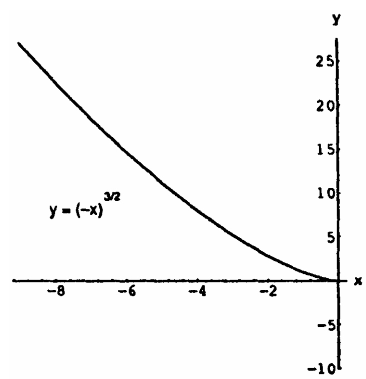
14. No symmetry
Dec: $0 \leq x < \infty$
Inc: nowhere



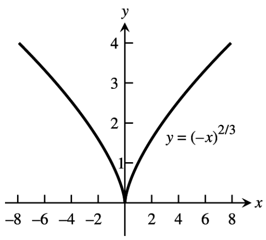
15. No symmetry
Dec: $0 \leq x < \infty$
Inc: nowhere



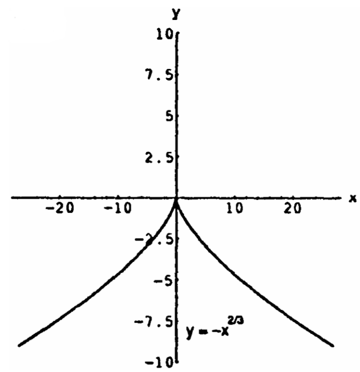
16. No symmetry
Dec: $-\infty < x \leq 0$
Inc: nowhere



17. Symmetric about the y-axis
Dec: $-\infty < x \le 0$
Inc: $0 < x < \infty$



18. Symmetric about the y-axis
Dec: $0 \le x < \infty$
Inc: $-\infty < x < 0$



19. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

20. $f(x) = x^{-5} = \frac{1}{x^5}$ and $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$. Thus the function is odd.

21. Since $f(x) = x^2 + 1 = (-x)^2 + 1 = -f(x)$. The function is even.

22. Since $[f(x) = x^2 + x] \neq [f(-x) = (-x)^2 - x]$ and $[f(x) = x^2 + x] \neq [-f(x) = -(x)^2 - x]$ the function is neither even nor odd.

23. Since $g(x) = x^3 + x$, $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$. So the function is odd.

24. $g(x) = x^4 + 3x^2 + 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$, thus the function is even.

25. $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$. Thus the function is even.

26. $g(x) = \frac{x}{x^2 - 1}$; $g(-x) = -\frac{x}{x^2 - 1} = g(-x)$. So the function is odd.

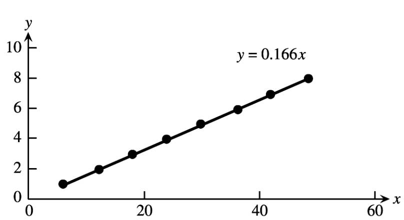
27. $h(t) = \frac{1}{t - 1}$; $h(-t) = \frac{1}{-t - 1}$; $-h(t) = \frac{1}{1 - t}$. Since $h(t) \neq -h(t)$ and $h(t) \neq h(-t)$, the function is neither even nor odd.

28. Since $|t^3| = |(-t)^3|$, $h(t) = h(-t)$ and the function is even.

29. $h(t) = 2t + 1$, $h(-t) = -2t + 1$. So $h(t) \neq h(-t)$. $-h(t) = -2t - 1$, so $h(t) \neq -h(t)$. The function is neither even nor odd.

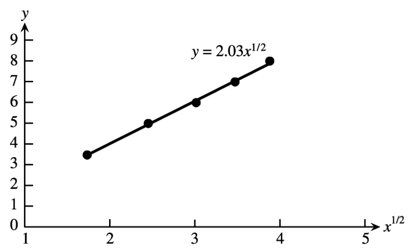
30. $h(t) = 2|t| + 1$ and $h(-t) = 2|-t| + 1 = 2|t| + 1$. So $h(t) = h(-t)$ and the function is even.

31. (a)



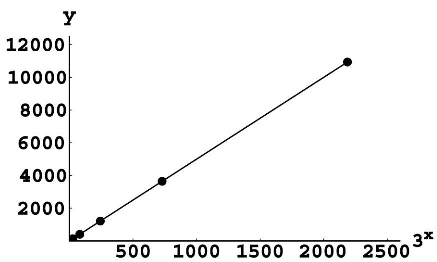
The graph supports the assumption that y is proportional to x. The constant of proportionality is estimated from the slope of the regression line, which is 0.166.

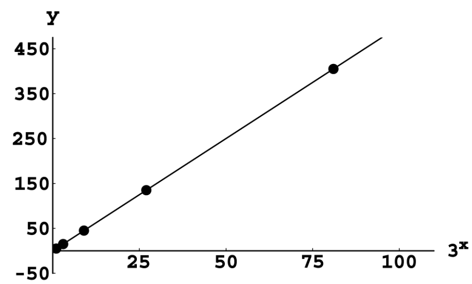
(b)



The graph supports the assumption that y is proportional to $x^{1/2}$. The constant of proportionality is estimated from the slope of the regression line, which is 2.03.

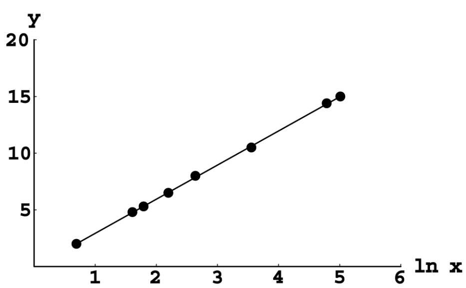
32. (a) Because of the wide range of values of the data, two graphs are needed to observe all of the points in relation to the regression line.



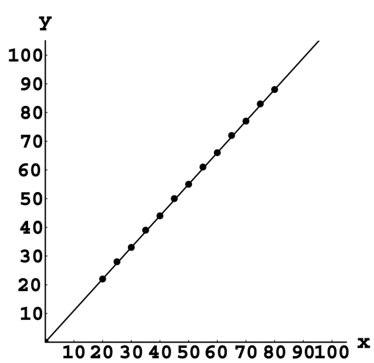


The graphs support the assumption that y is proportional to 3^x. The constant of proportionality is estimated from the slope of the regression line, which is 5.00.

(b) The graph supports the assumption that y is proportional to ln x. The constant of proportionality is extimated from the slope of the regression line, which is 2.99.

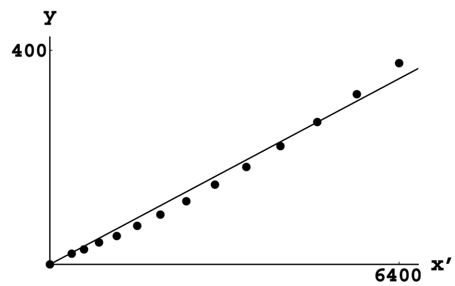


33. (a) The scatterplot of y = reaction distance versus x = speed is



Answers for the constant of proportionality may vary. The constant of proportionality is the slope of the line, which is approximately 1.1.

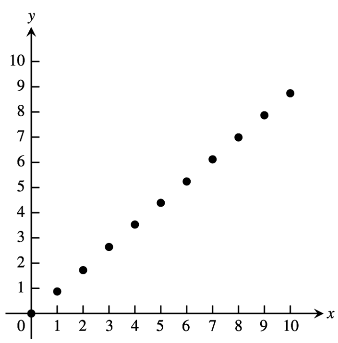
(b) Calculate $x' =$ speed squared. The scatterplot of x' versus $y =$ braking distance is:



Answers for the constant of proportionality may vary. The constant of proportionality is the slope of the line, which is approximately 0.059.

34. Kepler's 3rd Law is $T(\text{days}) = 0.41R^{3/2}$, R in millions of miles. "Quaoar" is 4×10^9 miles from Earth, or about $4 \times 10^9 + 93 \times 10^6 \approx 4 \times 10^9$ miles from the sun. Let $R = 4000$ (millions of miles) and $T = (0.41)(4000)^{3/2}$ days $\approx 103,723$ days.

35. (a)

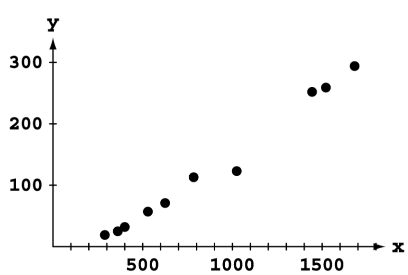


The hypothesis is reasonable.

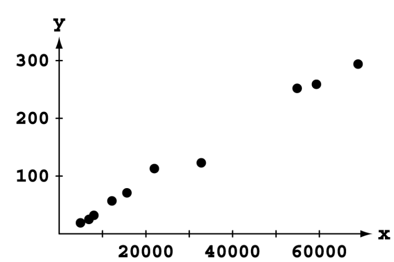
(b) The constant of proportionality is the slope of the line $\approx \frac{8.741 - 0}{10 - 0}$ in./unit mass $= 0.874$ in./unit mass.

(c) $y(\text{in.}) = (0.87\text{ in./unit mass})(13\text{ unit mass}) = 11.31$ in.

36. (a)



(b)



Graph (b) suggests that $y = kx^3$ is the better model. This graph is more linear than is graph (a).

1.5 COMBINING FUNCTIONS; SHIFTING AND SCALING GRAPHS

1. $D_f: -\infty < x < \infty$, $D_g: x \ge 1 \Rightarrow D_{f+g} = D_{fg}: x \ge 1$. $R_f: -\infty < y < \infty$, $R_g: y \ge 0$, $R_{f+g}: y \ge 1$, $R_{fg}: y \ge 0$

2. $D_f: x + 1 \ge 0 \Rightarrow x \ge -1$, $D_g: x - 1 \ge 0 \Rightarrow x \ge 1$. Therefore $D_{f+g} = D_{fg}: x \ge 1$. $R_f = R_g: y \ge 0$, $R_{f+g}: y \ge \sqrt{2}$, $R_{fg}: y \ge 0$

3. $D_f: -\infty < x < \infty$, $D_g: -\infty < x < \infty \Rightarrow D_{f/g}: -\infty < x < \infty$ since $g(x) \neq 0$ for any x; $D_{g/f}: -\infty < x < \infty$ since $f(x) \neq 0$ for any x. $R_f: y = 2$, $R_g: y \geq 1$, $R_{f/g}: 0 < y \leq 2$, $R_{g/f}: y \geq \frac{1}{2}$

4. $D_f: -\infty < x < \infty$, $D_g: x \geq 0 \Rightarrow D_{f/g}: x \geq 0$ since $g(x) \neq 0$ for any $x \geq 0$; $D_{g/f}: x \geq 0$ since $f(x) \neq 0$ for any $x \geq 0$. $R_f: y = 1$, $R_g: y \geq 1$, $R_{f/g}: 0 < y \leq 1$, $R_{g/f}: y \geq 1$

5. (a) $f(g(0)) = f(-3) = 2$
   (b) $g(f(0)) = g(5) = 22$
   (c) $f(g(x)) = f(x^2 - 3) = x^2 - 3 + 5 = x^2 + 2$
   (d) $g(f(x)) = g(x + 5) = (x + 5)^2 - 3 = x^2 + 10x + 22$
   (e) $f(f(-5)) = f(0) = 5$
   (f) $g(g(2)) = g(1) = -2$
   (g) $f(f(x)) = f(x + 5) = (x + 5) + 5 = x + 10$
   (h) $g(g(x)) = g(x^2 - 3) = (x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$

6. (a) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(\frac{2}{3}\right) = -\frac{1}{3}$
   (b) $g\left(f\left(\frac{1}{2}\right)\right) = g\left(-\frac{1}{2}\right) = 2$
   (c) $f(g(x)) = f\left(\frac{1}{x+1}\right) = \frac{1}{x+1} - 1 = \frac{-x}{x+1}$
   (d) $g(f(x)) = g(x - 1) = \frac{1}{(x-1)+1} = \frac{1}{x}$
   (e) $f(f(2)) = f(1) = 0$
   (f) $g(g(2)) = g\left(\frac{1}{3}\right) = \frac{1}{\frac{4}{3}} = \frac{3}{4}$
   (g) $f(f(x)) = f(x - 1) = (x - 1) - 1 = x - 2$
   (h) $g(g(x)) = g\left(\frac{1}{x+1}\right) = \frac{1}{\frac{1}{x+1}+1} = \frac{x+1}{x+2}$ $(x \neq -1$ and $x \neq -2)$

7. (a) $u(v(f(x))) = u\left(v\left(\frac{1}{x}\right)\right) = u\left(\frac{1}{x^2}\right) = 4\left(\frac{1}{x}\right)^2 - 5 = \frac{4}{x^2} - 5$
   (b) $u(f(v(x))) = u\left(f\left(x^2\right)\right) = u\left(\frac{1}{x^2}\right) = 4\left(\frac{1}{x^2}\right) - 5 = \frac{4}{x^2} - 5$
   (c) $v(u(f(x))) = v\left(u\left(\frac{1}{x}\right)\right) = v\left(4\left(\frac{1}{x}\right) - 5\right) = \left(\frac{4}{x} - 5\right)^2$
   (d) $v(f(u(x))) = v(f(4x - 5)) = v\left(\frac{1}{4x-5}\right) = \left(\frac{1}{4x-5}\right)^2$
   (e) $f(u(v(x))) = f\left(u\left(x^2\right)\right) = f\left(4\left(x^2\right) - 5\right) = \frac{1}{4x^2-5}$
   (f) $f(v(u(x))) = f(v(4x - 5)) = f\left((4x - 5)^2\right) = \frac{1}{(4x-5)^2}$

8. (a) $h(g(f(x))) = h\left(g\left(\sqrt{x}\right)\right) = h\left(\frac{\sqrt{x}}{4}\right) = 4\left(\frac{\sqrt{x}}{4}\right) - 8 = \sqrt{x} - 8$
   (b) $h(f(g(x))) = h\left(f\left(\frac{x}{4}\right)\right) = h\left(\sqrt{\frac{x}{4}}\right) = 4\sqrt{\frac{x}{4}} - 8 = 2\sqrt{x} - 8$
   (c) $g(h(f(x))) = g\left(h\left(\sqrt{x}\right)\right) = g\left(4\sqrt{x} - 8\right) = \frac{4\sqrt{x}-8}{4} = \sqrt{x} - 2$
   (d) $g(f(h(x))) = g(f(4x - 8)) = g\left(\sqrt{4x-8}\right) = \frac{\sqrt{4x-8}}{4} = \frac{\sqrt{x-2}}{2}$
   (e) $f(g(h(x))) = f(g(4x - 8)) = f\left(\frac{4x-8}{4}\right) = f(x - 2) = \sqrt{x-2}$
   (f) $f(h(g(x))) = f\left(h\left(\frac{x}{4}\right)\right) = f\left(4\left(\frac{x}{4}\right) - 8\right) = f(x - 8) = \sqrt{x-8}$

9. (a) $y = f(g(x))$ (b) $y = j(g(x))$
   (c) $y = g(g(x))$ (d) $y = j(j(x))$
   (e) $y = g(h(f(x)))$ (f) $y = h(j(f(x)))$

10. (a) $y = f(j(x))$ (b) $y = h(g(x)) = g(h(x))$
    (c) $y = h(h(x))$ (d) $y = f(f(x))$
    (e) $y = j(g(f(x)))$ (f) $y = g(f(h(x)))$

11.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
(a)	$x - 7$	$\sqrt{x}$	$\sqrt{x-7}$
(b)	$x + 2$	$3x$	$3(x+2) = 3x + 6$
(c)	x^2	$\sqrt{x-5}$	$\sqrt{x^2 - 5}$
(d)	$\frac{x}{x-1}$	$\frac{x}{x-1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{x}{x-(x-1)} = x$
(e)	$\frac{1}{x-1}$	$1 + \frac{1}{x}$	x
(f)	$\frac{1}{x}$	$\frac{1}{x}$	x

12. (a) $(f \circ g)(x) = \lvert g(x) \rvert = \frac{1}{\lvert x-1 \rvert}$.

(b) $(f \circ g)(x) = \frac{g(x)-1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{x}{x+1} = \frac{1}{g(x)} \Rightarrow \frac{1}{x+1} = \frac{1}{g(x)}$, so $g(x) = x + 1$.

(c) Since $(f \circ g)(x) = \sqrt{g(x)} = \lvert x \rvert$, $g(x) = x^2$.

(d) Since $(f \circ g)(x) = f\left(\sqrt{x}\right) = \lvert x \rvert$, $f(x) = x^2$. (Note that the domain of the composite is $[0, \infty)$.)

The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f \circ g)(x)$
$\frac{1}{x-1}$	$\lvert x \rvert$	$\frac{1}{\lvert x-1 \rvert}$
$x + 1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
x^2	$\sqrt{x}$	$\lvert x \rvert$
$\sqrt{x}$	x^2	$\lvert x \rvert$

13. (a) $f(g(x)) = \sqrt{\frac{1}{x} + 1} = \sqrt{\frac{1+x}{x}}$

$g(f(x)) = \frac{1}{\sqrt{x+1}}$

(b) Domain $(f \circ g)$: $(0, \infty)$, domain $(g \circ f)$: $(-1, \infty)$

(c) Range $(f \circ g)$: $(1, \infty)$, range $(g \circ f)$: $(0, \infty)$

14. (a) $f(g(x)) = 1 - 2\sqrt{x} + x$

$g(f(x)) = 1 - \lvert x \rvert$

(b) Domain $(f \circ g)$: $(0, \infty)$, domain $(g \circ f)$: $(0, \infty)$

(c) Range $(f \circ g)$: $(0, \infty)$, range $(g \circ f)$: $(-\infty, 1)$

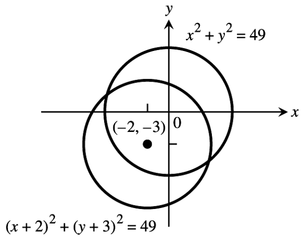
15. (a) $y = -(x+7)^2$ (b) $y = -(x-4)^2$

16. (a) $y = x^2 + 3$ (b) $y = x^2 - 5$

17. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3

18. (a) $y = -(x-1)^2 + 4$ (b) $y = -(x+2)^2 + 3$ (c) $y = -(x+4)^2 - 1$ (d) $y = -(x-2)^2$

19.



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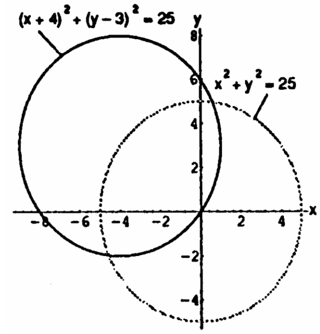

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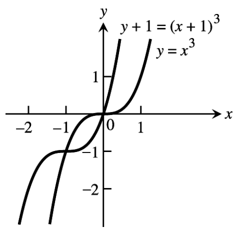

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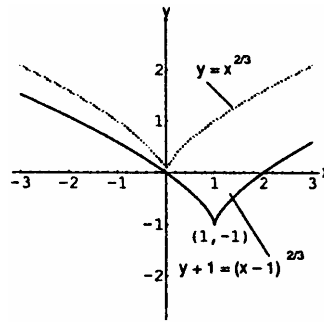

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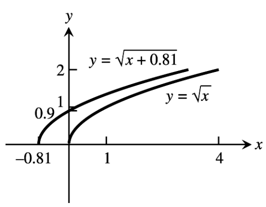

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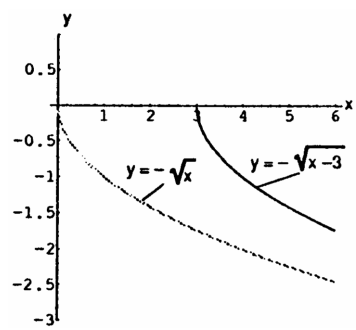

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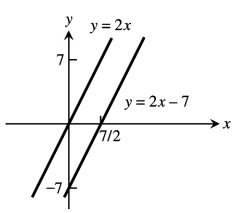

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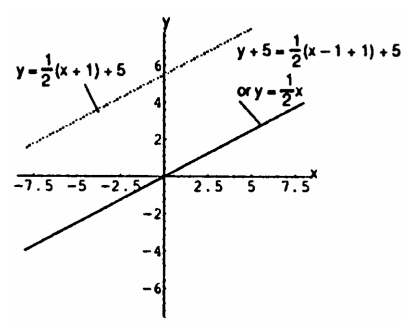

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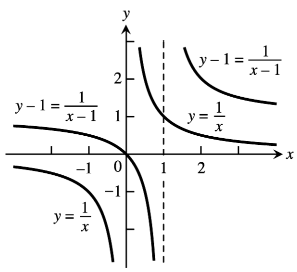

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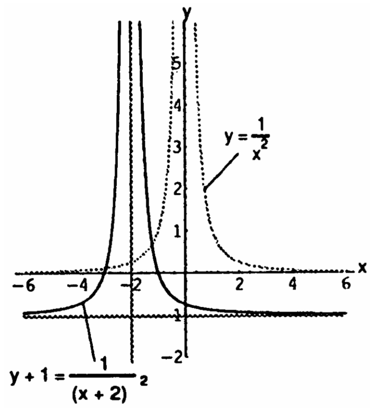

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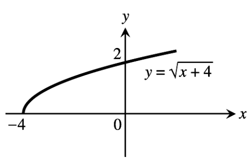

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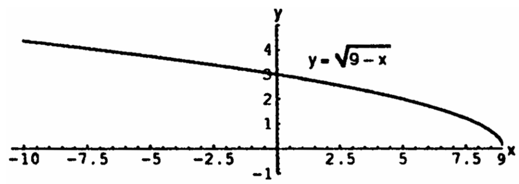

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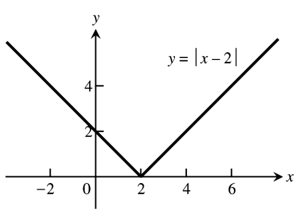

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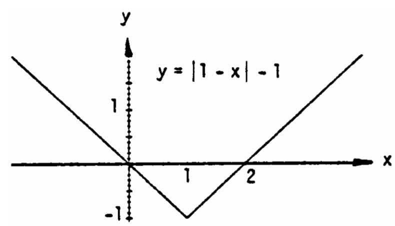
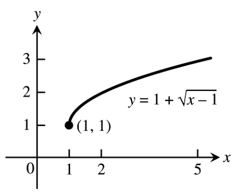


33.



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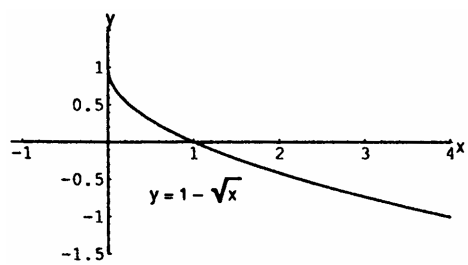
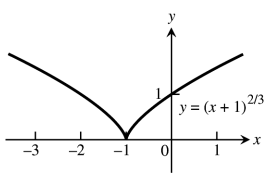


35.



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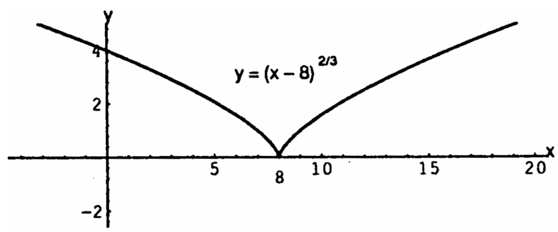

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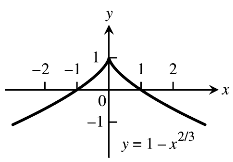

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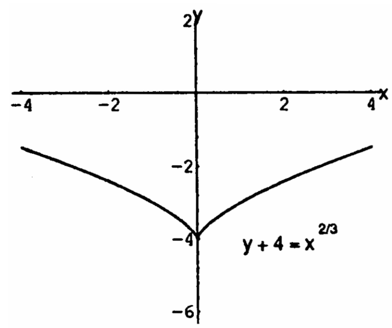

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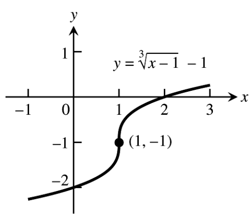

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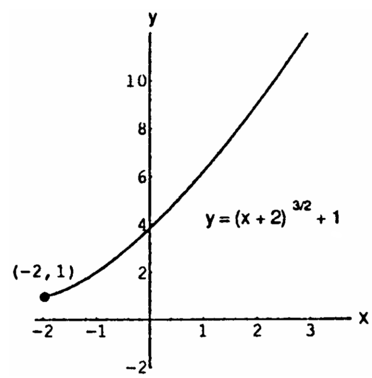

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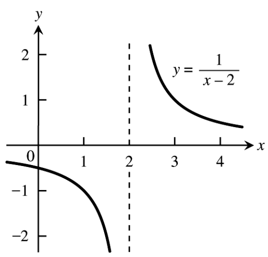

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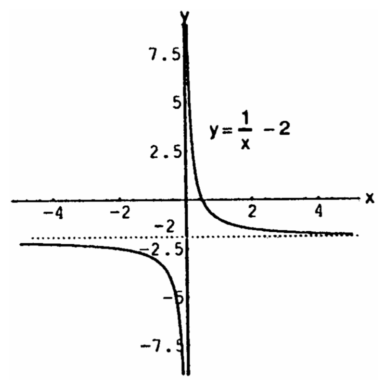

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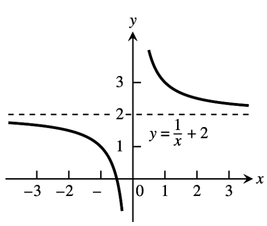

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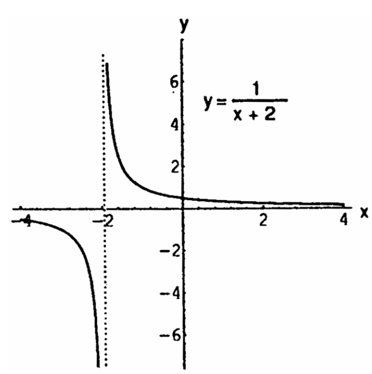

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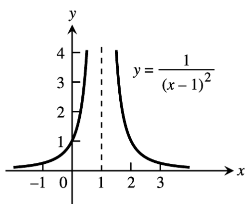

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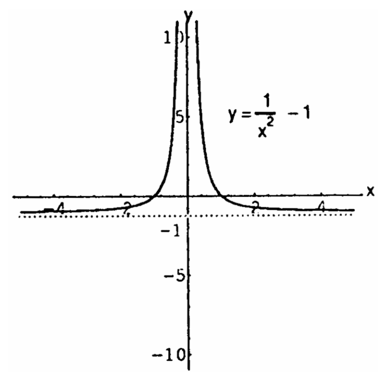
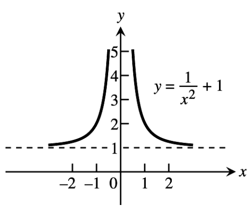
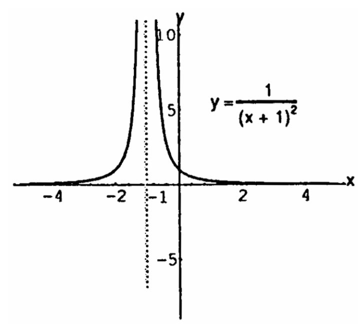


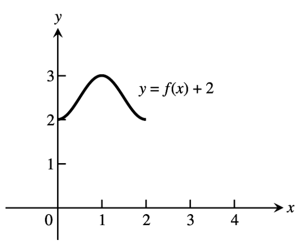
47.



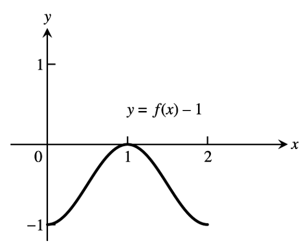
48.



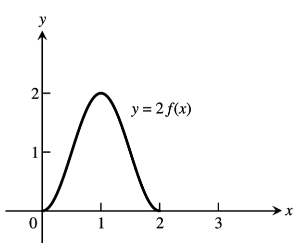
49. (a) domain: $[0, 2]$; range: $[2, 3]$



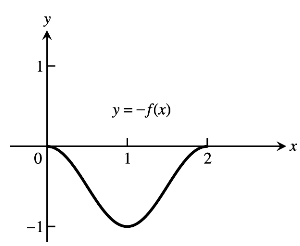
(b) domain: $[0, 2]$; range: $[-1, 0]$



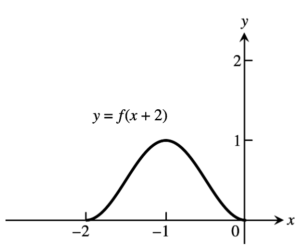
(c) domain: $[0, 2]$; range: $[0, 2]$



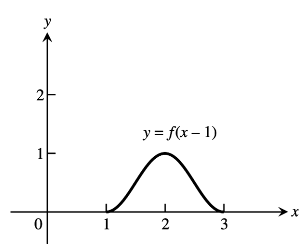
(d) domain: $[0, 2]$; range: $[-1, 0]$



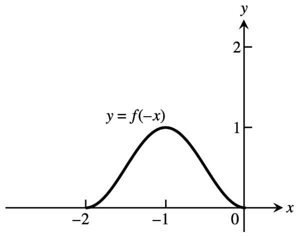
(e) domain: $[-2, 0]$; range: $[0, 1]$



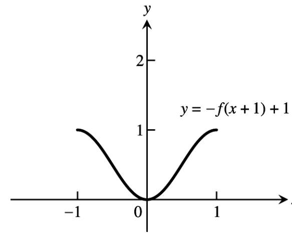
(f) domain: $[1, 3]$; range: $[0, 1]$



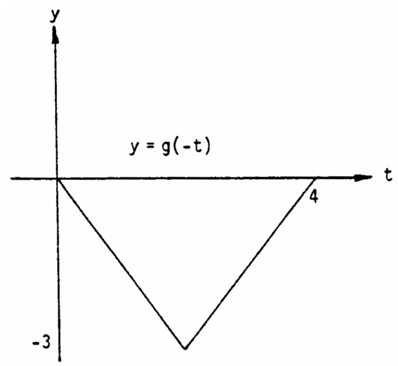
(g) domain: $[-2,0]$; range: $[0,1]$



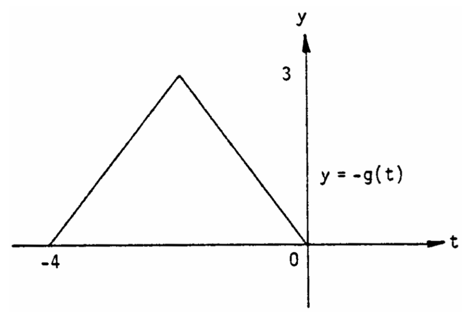
(h) domain: $[-1,1]$; range: $[0,1]$



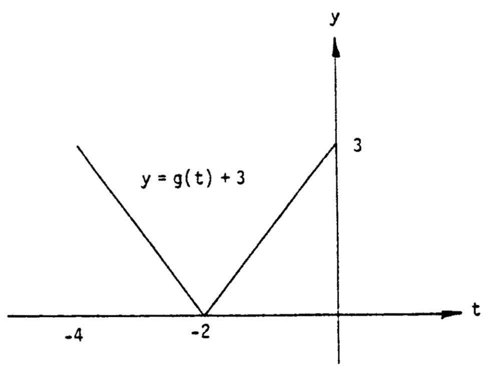
50. (a) domain: $[0,4]$; range: $[-3,0]$



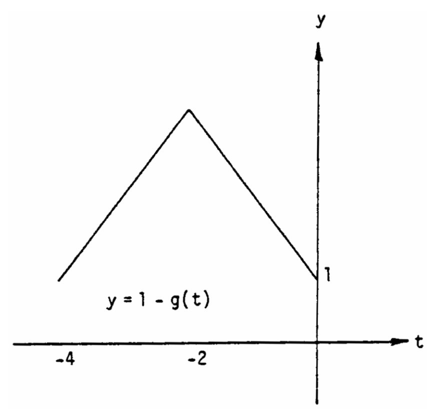
(b) domain: $[-4,0]$; range: $[0,3]$



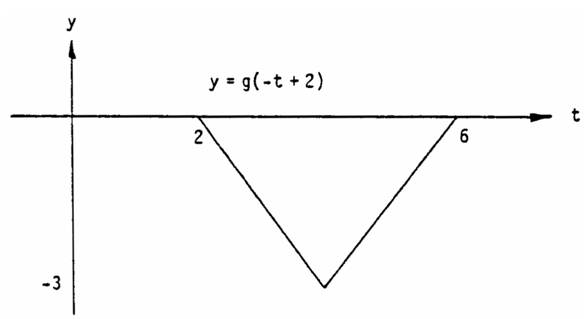
(c) domain: $[-4,0]$; range: $[0,3]$



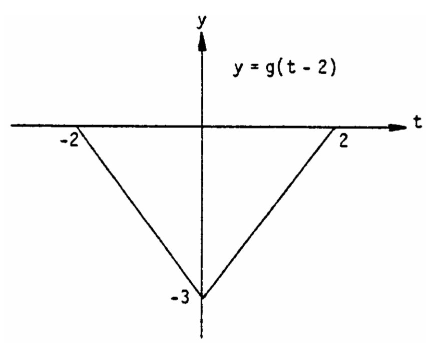
(d) domain: $[-4,0]$; range: $[1,4]$



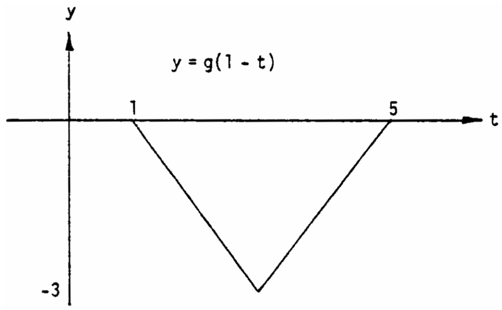
(e) domain: $[2,4]$; range: $[-3,0]$



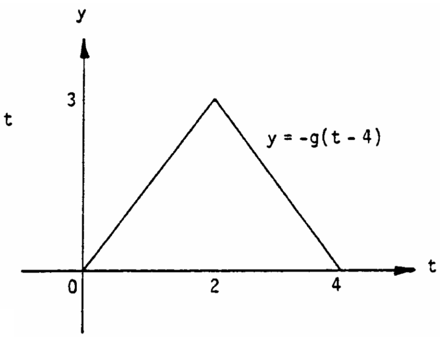
(f) domain: $[-2,2]$; range: $[-3,0]$



(g) domain: $[1, 5]$; range: $[-3, 0]$



(h) domain: $[0, 4]$; range: $[0, 3]$



51. $y = 3x^2 - 3$

52. $y = (2x)^2 - 1 = 4x^2 - 1$

53. $y = \frac{1}{2}\left(1 + \frac{1}{x^2}\right) = \frac{1}{2} + \frac{1}{2x^2}$

54. $y = 1 + \frac{1}{(x/3)^2} = 1 + \frac{9}{x^2}$

55. $y = \sqrt{4x + 1}$

56. $y = 3\sqrt{x + 1}$

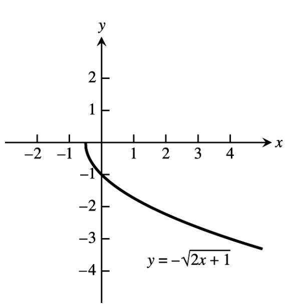
57. $y = \sqrt{4 - \left(\frac{x}{2}\right)^2} = \frac{1}{2}\sqrt{16 - x^2}$

58. $y = \frac{1}{3}\sqrt{4 - x^2}$

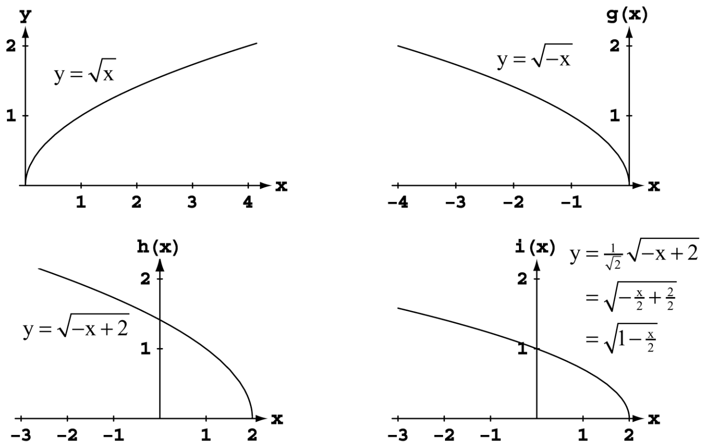
59. $y = 1 - (3x)^3 = 1 - 27x^3$

60. $y = 1 - \left(\frac{x}{2}\right)^3 = 1 - \frac{x^3}{8}$

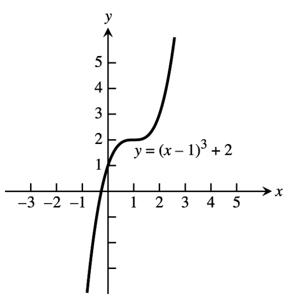
61. Let $y = -\sqrt{2x + 1} = f(x)$ and let $g(x) = x^{1/2}$, $h(x) = \left(x + \frac{1}{2}\right)^{1/2}$, $i(x) = \sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}$, and $j(x) = -\left[\sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}\right] = f(x)$. The graph of $h(x)$ is the graph of $g(x)$ shifted left $\frac{1}{2}$ unit; the graph of $i(x)$ is the graph of $h(x)$ stretched vertically by a factor of $\sqrt{2}$; and the graph of $j(x) = f(x)$ is the graph of $i(x)$ reflected across the x-axis.



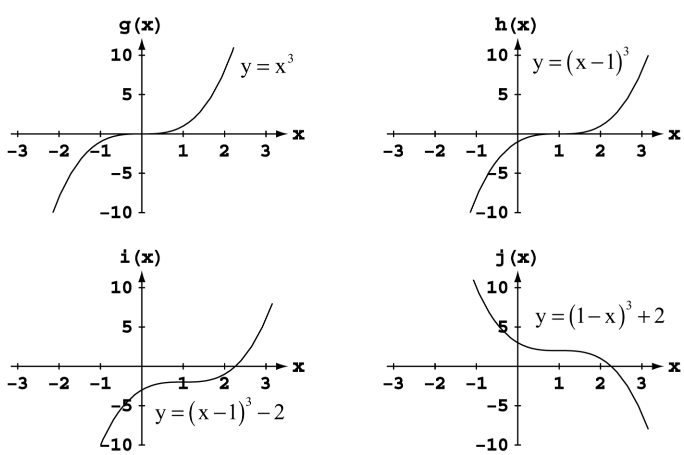
62. Let $y = \sqrt{1 - \frac{x}{2}} = f(x)$. Let $g(x) = (-x)^{1/2}$, $h(x) = (-x + 2)^{1/2}$, and $i(x) = \frac{1}{\sqrt{2}}(-x + 2)^{1/2} = \sqrt{1 - \frac{x}{2}} = f(x)$. The graph of $g(x)$ is the graph of $y = \sqrt{x}$ reflected across the x-axis. The graph of $h(x)$ is the graph of $g(x)$ shifted right two units. And the graph of $i(x)$ is the graph of $h(x)$ compressed vertically by a factor of $\sqrt{2}$.



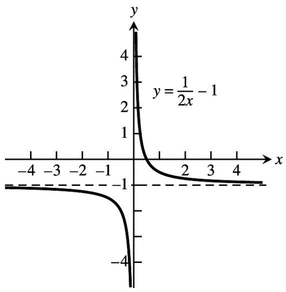
63. $y = f(x) = x^3$. Shift $f(x)$ one unit right followed by a shift two units up to get $g(x) = (x - 1)^3 + 2$.



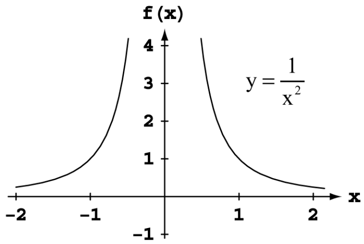
64. $y = (1 - x)^3 + 2 = -[(x - 1)^3 + (-2)] = f(x)$. Let $g(x) = x^3$, $h(x) = (x - 1)^3$, $i(x) = (x - 1)^3 + (-2)$, and $j(x) = -[(x - 1)^3 + (-2)]$. The graph of $h(x)$ is the graph of $g(x)$ shifted right one unit; the graph of $i(x)$ is the graph of $h(x)$ shifted down two units; and the graph of $f(x)$ is the graph of $i(x)$ reflected across the x-axis.

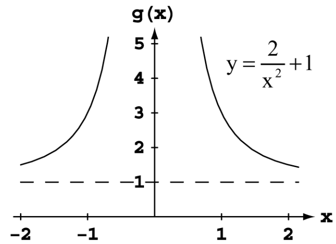


65. Compress the graph of $f(x) = \frac{1}{x}$ horizontally by a factor of 2 to get $g(x) = \frac{1}{2x}$. Then shift $g(x)$ vertically down 1 unit to get $h(x) = \frac{1}{2x} - 1$.

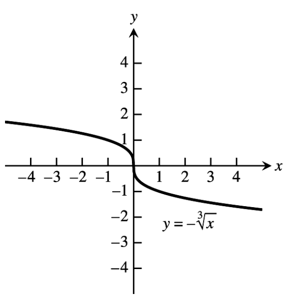


66. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{2}{x^2} + 1 = \frac{1}{\left(\frac{x^2}{2}\right)} + 1 = \frac{1}{\left(x/\sqrt{2}\right)^2} + 1 = \frac{1}{\left[\left(1/\sqrt{2}\right)x\right]^2} + 1$. Since $\sqrt{2} \approx 1.4$, we see that the graph of $f(x)$ stretched horizontally by a factor of 1.4 and shifted up 1 unit is the graph of $g(x)$.

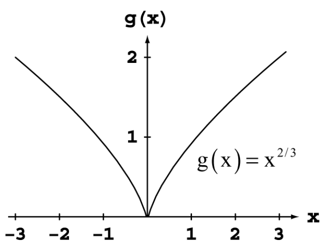


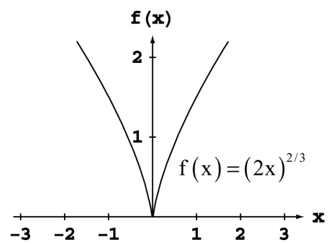


67. Reflect the graph of $y = f(x) = \sqrt[3]{x}$ across the x-axis to get $g(x) = -\sqrt[3]{x}$.

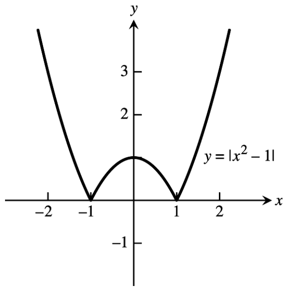


68. $y = f(x) = (-2x)^{2/3} = [(-1)(2)x]^{2/3} = (-1)^{2/3}(2x)^{2/3} = (2x)^{2/3}$. So the graph of $f(x)$ is the graph of $g(x) = x^{2/3}$ compressed horizontally by a factor of 2.

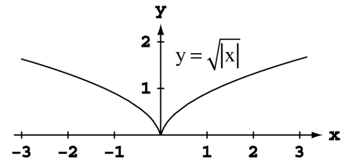




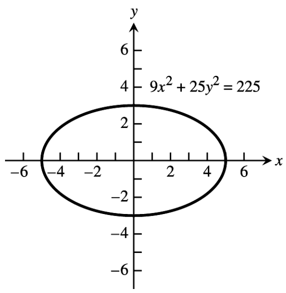
69.



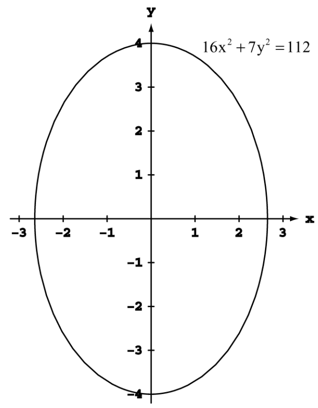
70.



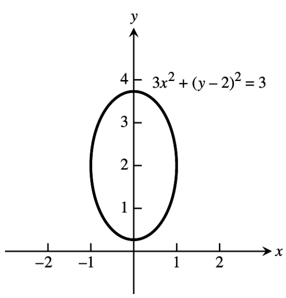
71. $9x^2 + 25y^2 = 225 \Rightarrow \frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$



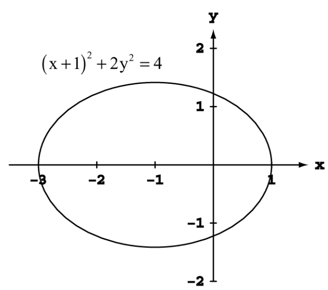
72. $16x^2 + 7y^2 = 112 \Rightarrow \frac{x^2}{\left(\sqrt{7}\right)^2} + \frac{y^2}{4^2} = 1$



73. $3x^2 + (y-2)^2 = 3 \Rightarrow \frac{x^2}{1^2} + \frac{(y-2)^2}{\left(\sqrt{3}\right)^2} = 1$

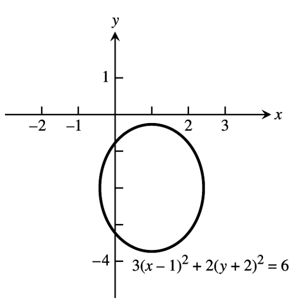


74. $(x+1)^2 + 2y^2 = 4 \Rightarrow \frac{[x-(-1)]^2}{2^2} + \frac{y^2}{\left(\sqrt{2}\right)^2} = 1$



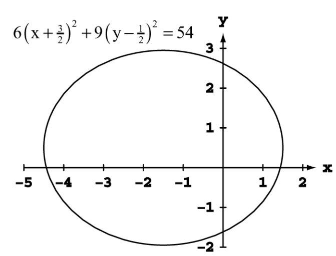
75. $3(x-1)^2 + 2(y+2)^2 = 6$

$\Rightarrow \frac{(x-1)^2}{\left(\sqrt{2}\right)^2} + \frac{[y-(-2)]^2}{\left(\sqrt{3}\right)^2} = 1$

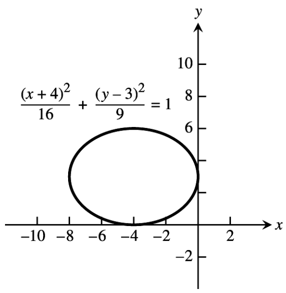


76. $6\left(x + \frac{3}{2}\right)^2 + 9\left(y - \frac{1}{2}\right)^2 = 54$

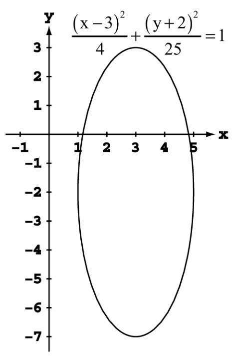
$\Rightarrow \frac{\left[x-\left(-\frac{3}{2}\right)\right]^2}{3^2} + \frac{\left(y-\frac{1}{2}\right)^2}{\left(\sqrt{6}\right)^2} = 1$



77. $\frac{x^2}{16} + \frac{y^2}{9} = 1$ has its center at $(0,\ 0)$. Shiftinig 4 units left and 3 units up gives the center at $(h, k) = (-4,\ 3)$. So the equation is $\frac{[x-(-4)]^2}{4^2} + \frac{(y-3)^2}{3^2} = 1 \Rightarrow \frac{(x+4)^2}{4^2} + \frac{(y-3)^2}{3^2} = 1$. Center, C, is $(-4,\ 3)$, and major axis, $\overline{AB}$, is the segment from $(-8,\ 3)$ to $(0, 3)$.



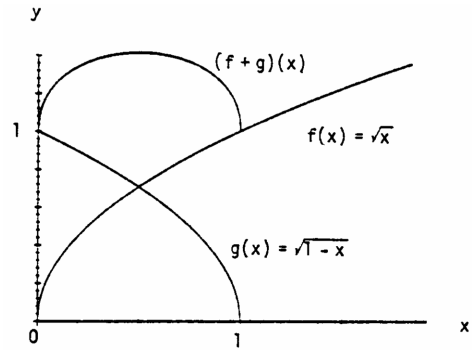
78. The ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$ has center $(h, k) = (0,\ 0)$. Shifting the ellipse 3 units right and 2 units down produces an ellipse with center at $(h, k) = (3,\ -2)$ and an equation $\frac{(x-3)^2}{4} + \frac{[y-(-2)]^2}{25} = 1$. Center, C, is $(3,\ -2)$, and $\overline{AB}$, the segment from $(3,\ 3)$ to $(3, -7)$ is the major axis.



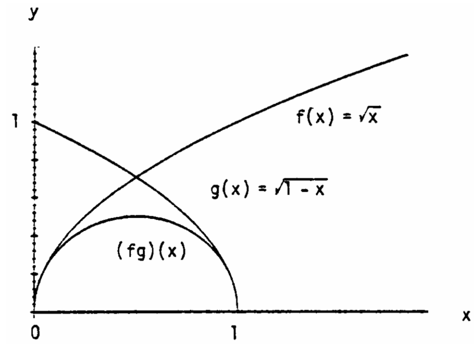
79. (a) $(fg)(-x) = f(-x)g(-x) = f(x)(-g(x)) = -(fg)(x)$, odd
    (b) $\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f}{g}\right)(x)$, odd
    (c) $\left(\frac{g}{f}\right)(-x) = \frac{g(-x)}{f(-x)} = \frac{-g(x)}{f(x)} = -\left(\frac{g}{f}\right)(x)$, odd
    (d) $f^2(-x) = f(-x)f(-x) = f(x)f(x) = f^2(x)$, even
    (e) $g^2(-x) = (g(-x))^2 = (-g(x))^2 = g^2(x)$, even
    (f) $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$, even
    (g) $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$, even
    (h) $(f \circ f)(-x) = f(f(-x)) = f(f(x)) = (f \circ f)(x)$, even
    (i) $(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -(g \circ g)(x)$, odd

80. Yes, $f(x) = 0$ is both even and odd since $f(-x) = 0 = f(x)$ and $f(-x) = 0 = -f(x)$.

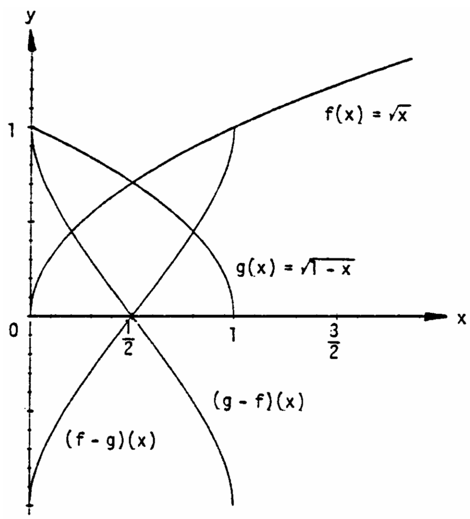
81. (a)



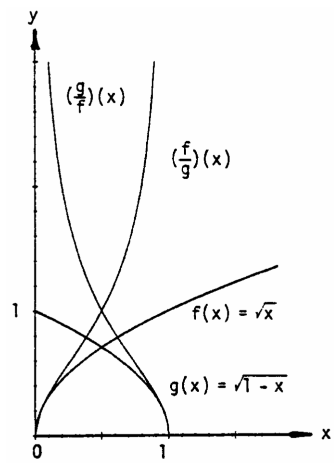
(b)



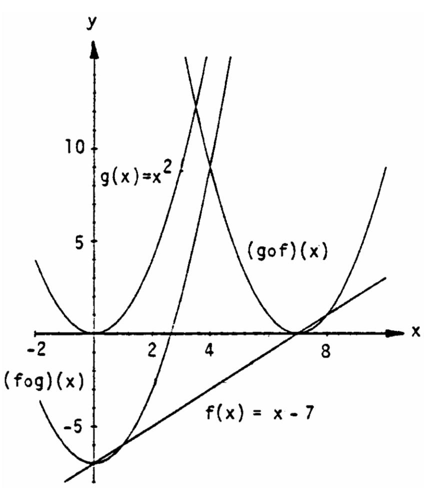
(c)



(d)



82.



1.6 TRIGONOMETRIC FUNCTIONS

1. (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m (b) $s = r\theta = (10)(110°)\left(\frac{\pi}{180°}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m

2. $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$

3. $\theta = 80° \Rightarrow \theta = 80°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$ in. (since the diameter $= 12$ in. $\Rightarrow$ radius $= 6$ in.)

4. $d = 1$ meter $\Rightarrow$ $r = 50$ cm $\Rightarrow$ $\theta = \frac{s}{r} = \frac{30}{50} = 0.6$ rad or $0.6\left(\frac{180^\circ}{\pi}\right) \approx 34^\circ$

5.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin\theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos\theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	und.	-1
$\cot\theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec\theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc\theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin\theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot\theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec\theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc\theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

7. $\cos x = -\frac{4}{5}$, $\tan x = -\frac{3}{4}$

8. $\sin x = \frac{2}{\sqrt{5}}$, $\cos x = \frac{1}{\sqrt{5}}$

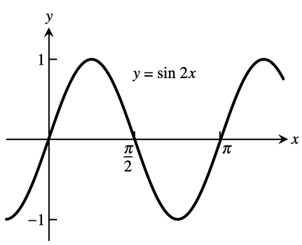
9. $\sin x = -\frac{\sqrt{8}}{3}$, $\tan x = -\sqrt{8}$

10. $\sin x = \frac{12}{13}$, $\tan x = -\frac{12}{5}$

11. $\sin x = -\frac{1}{\sqrt{5}}$, $\cos x = -\frac{2}{\sqrt{5}}$

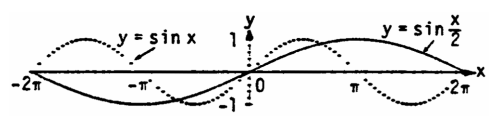
12. $\cos x = -\frac{\sqrt{3}}{2}$, $\tan x = \frac{1}{\sqrt{3}}$

13.



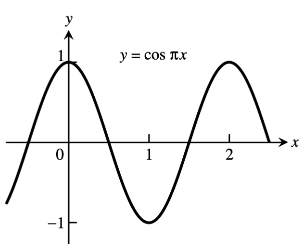
period $= \pi$

14.



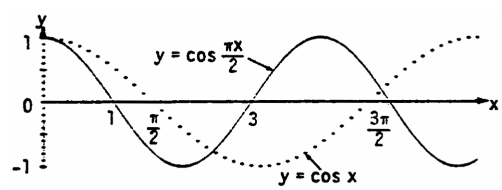
period $= 4\pi$

15.



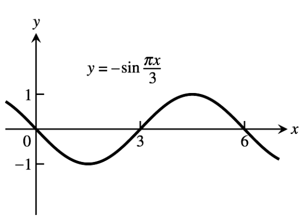
period $= 2$

16.



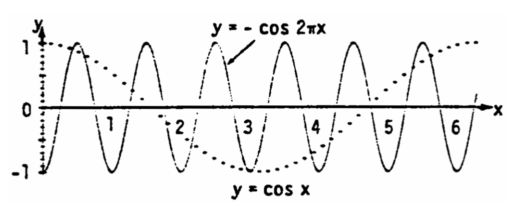
period $= 4$

17.



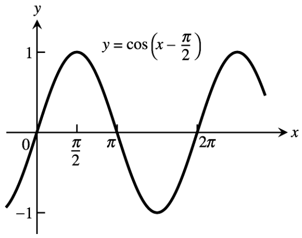
period $= 6$

18.



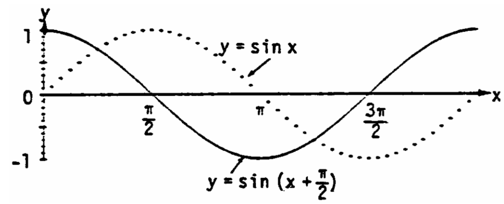
period $= 1$

19.



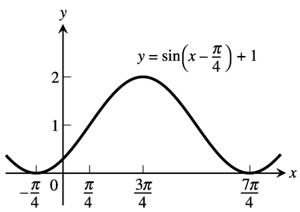
period $= 2\pi$

20.



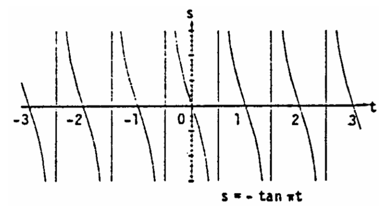
period $= 2\pi$

21.



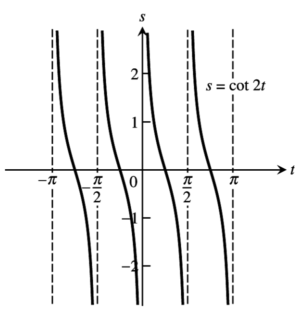
period $= 2\pi$

22.

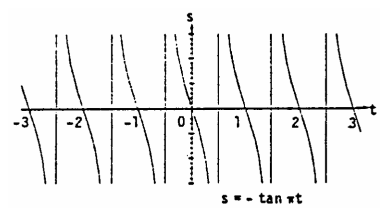


period $= 2\pi$

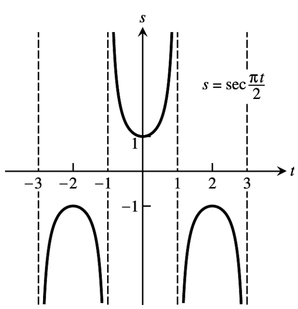
23. period $= \frac{\pi}{2}$, symmetric about the origin



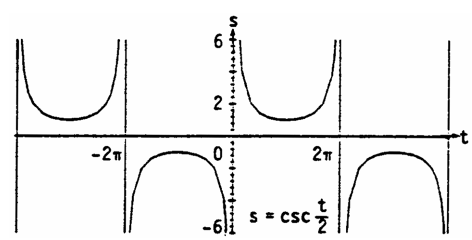
24. period $= 1$, symmetric about the origin



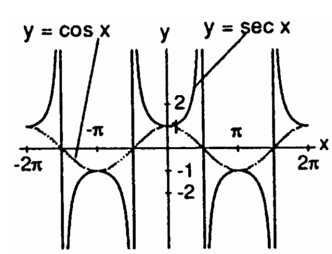
25. period $= 4$, symmetric about the y-axis



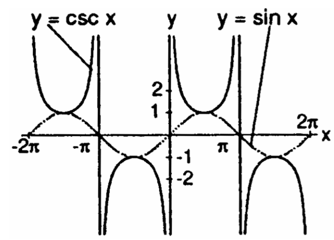
26. period $= 4\pi$, symmetric about the origin



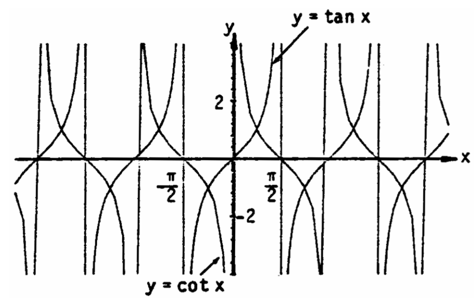
27. (a) Cos x and sec x are positive in QI and QIV and negative in QII and QIII. Sec x is undefined when cos x is 0. The range of sec x is $(-\infty, -1] \cup [1, \infty)$; the range of cos x is $[-1, 1]$.



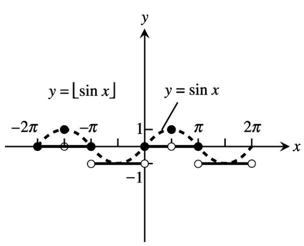
(b) Sin x and csc x are positive in QI and QII and negative in QIII and QIV. Csc x is undefined when sin x is 0. The range of csc x is $(-\infty, -1] \cup [1, \infty)$; the range of sin x is $[-1, 1]$.



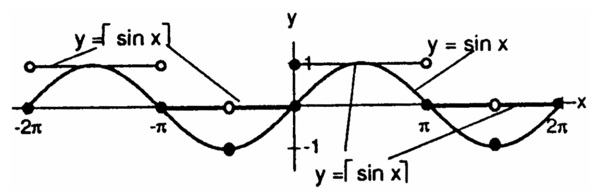
28. Since cot x $= \frac{1}{\tan x}$, cot x is undefined when tan x = 0 and is zero when tan x is undefined. As tan x approaches zero through positive values, cot x approaches infinity. Also, cot x approaches negative infinity as tan x approaches zero through negative values.



29. D: $-\infty < x < \infty$; R: y = −1, 0, 1



30. D: $-\infty < x < \infty$; R: y = −1, 0, 1



31. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$

32. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$

33. $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$

34. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$

35. $\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$
$= \cos A \cos B + \sin A \sin B$

36. $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
$= \sin A \cos B - \cos A \sin B$

37. If B = A, A − B = 0 $\Rightarrow$ $\cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$ $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.

38. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π.

39. $\cos(\pi + x) = \cos\pi \cos x - \sin\pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$

40. $\sin(2\pi - x) = \sin 2\pi \cos(-x) + \cos(2\pi)\sin(-x) = (0)(\cos(-x)) + (1)(\sin(-x)) = -\sin x$

41. $\sin\left(\frac{3\pi}{2} - x\right) = \sin\left(\frac{3\pi}{2}\right)\cos(-x) + \cos\left(\frac{3\pi}{2}\right)\sin(-x) = (-1)(\cos x) + (0)(\sin(-x)) = -\cos x$

42. $\cos\left(\frac{3\pi}{2} + x\right) = \cos\left(\frac{3\pi}{2}\right)\cos x - \sin\left(\frac{3\pi}{2}\right)\sin x = (0)(\cos x) - (-1)(\sin x) = \sin x$

43. $\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$

44. $\cos\frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{\pi}{4}\sin\frac{2\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}+\sqrt{6}}{4}$

45. $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right) - \sin\frac{\pi}{3}\sin\left(-\frac{\pi}{4}\right) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

46. $\sin\frac{5\pi}{12} = \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

47. $\cos^2\frac{\pi}{8} = \frac{1+\cos\left(\frac{2\pi}{8}\right)}{2} = \frac{1+\frac{\sqrt{2}}{2}}{2} = \frac{2+\sqrt{2}}{4}$

48. $\cos^2\frac{\pi}{12} = \frac{1+\cos\left(\frac{2\pi}{12}\right)}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4}$

49. $\sin^2\frac{\pi}{12} = \frac{1-\cos\left(\frac{2\pi}{12}\right)}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4}$

50. $\sin^2\frac{\pi}{8} = \frac{1-\cos\left(\frac{2\pi}{8}\right)}{2} = \frac{1-\frac{\sqrt{2}}{2}}{2} = \frac{2-\sqrt{2}}{4}$

51. $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \cos A\cos B}{\cos A\cos B - \sin A\sin B} = \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

52. $\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A\cos B - \cos A\cos B}{\cos A\cos B + \sin A\sin B} = \frac{\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

53. According to the figure in the text, we have the following: By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos\theta$ $= 1^2 + 1^2 - 2\cos(A-B) = 2 - 2\cos(A-B)$. By distance formula, $c^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$ $= \cos^2 A - 2\cos A\cos B + \cos^2 B + \sin^2 A - 2\sin A\sin B + \sin^2 B = 2 - 2(\cos A\cos B + \sin A\sin B)$. Thus $c^2 = 2 - 2\cos(A-B) = 2 - 2(\cos A\cos B + \sin A\sin B) \Rightarrow \cos(A-B) = \cos A\cos B + \sin A\sin B$.

54. (a) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ and $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

Let $\theta = A + B$

$\sin(A+B) = \cos\left[\frac{\pi}{2} - (A+B)\right] = \cos\left[\left(\frac{\pi}{2} - A\right) - B\right] = \cos\left(\frac{\pi}{2} - A\right)\cos B + \sin\left(\frac{\pi}{2} - A\right)\sin B$

$= \sin A\cos B + \cos A\sin B$

(b) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\cos(A-(-B)) = \cos A\cos(-B) + \sin A\sin(-B)$

$\Rightarrow \cos(A+B) = \cos A\cos(-B) + \sin A\sin(-B) = \cos A\cos B + \sin A(-\sin B)$

$= \cos A\cos B - \sin A\sin B$

Because the cosine function is even and the sine functions is odd.

55. $c^2 = a^2 + b^2 - 2ab\cos C = 2^2 + 3^2 - 2(2)(3)\cos(60°) = 4 + 9 - 12\cos(60°) = 13 - 12\left(\frac{1}{2}\right) = 7$.

Thus, $c = \sqrt{7} \approx 2.65$.

56. $c^2 = a^2 + b^2 - 2ab\cos C = 2^2 + 3^2 - 2(2)(3)\cos(40°) = 13 - 12\cos(40°)$. Thus, $c = \sqrt{13 - 12\cos 40°} \approx 1.951$.

57. From the figures in the text, we see that $\sin B = \frac{h}{c}$. If C is an acute angle, then $\sin C = \frac{h}{b}$. On the other hand, if C is obtuse (as in the figure on the right), then $\sin C = \sin(\pi - C) = \frac{h}{b}$. Thus, in either case, $h = b \sin C = c \sin B \Rightarrow ah = ab \sin C = ac \sin B$.

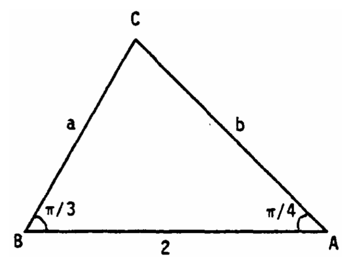
By the law of cosines, $\cos C = \frac{a^2+b^2-c^2}{2ab}$ and $\cos B = \frac{a^2+c^2-b^2}{2ac}$. Moreover, since the sum of the interior angles of a triangle is π, we have $\sin A = \sin(\pi - (B+C)) = \sin(B+C) = \sin B \cos C + \cos B \sin C$
$= \left(\frac{h}{c}\right)\left[\frac{a^2+b^2-c^2}{2ab}\right] + \left[\frac{a^2+c^2-b^2}{2ac}\right]\left(\frac{h}{b}\right) = \left(\frac{h}{2abc}\right)(2a^2+b^2-c^2+c^2-b^2) = \frac{ah}{bc} \Rightarrow ah = bc \sin A$.

Combining our results we have $ah = ab \sin C$, $ah = ac \sin B$, and $ah = bc \sin A$. Dividing by abc gives

$$\frac{h}{bc} = \underbrace{\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}}_{\text{law of sines}}.$$

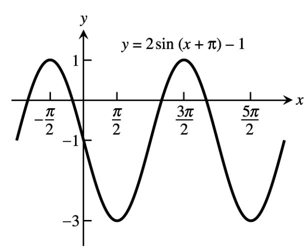
58. By the law of sines, $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sqrt{3}/2}{c}$. By Exercise 55 we know that $c = \sqrt{7}$.
Thus $\sin B = \frac{3\sqrt{3}}{2\sqrt{7}} \simeq 0.982$.

59. From the figure at the right and the law of cosines,
$b^2 = a^2 + 2^2 - 2(2a)\cos B$
$= a^2 + 4 - 4a\left(\frac{1}{2}\right) = a^2 - 2a + 4$.
Applying the law of sines to the figure, $\frac{\sin A}{a} = \frac{\sin B}{b}$
$\Rightarrow \frac{\sqrt{2}/2}{a} = \frac{\sqrt{3}/2}{b} \Rightarrow b = \sqrt{\frac{3}{2}}\,a$. Thus, combining results,
$a^2 - 2a + 4 = b^2 = \frac{3}{2}a^2 \Rightarrow 0 = \frac{1}{2}a^2 + 2a - 4$
$\Rightarrow 0 = a^2 + 4a - 8$. From the quadratic formula and the fact that $a > 0$, we have
$a = \frac{-4+\sqrt{4^2-4(1)(-8)}}{2} = \frac{4\sqrt{3}-4}{2} \simeq 1.464$.

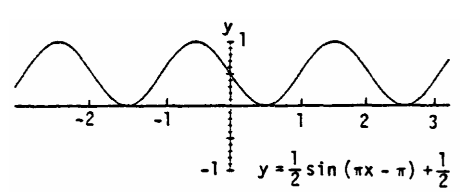


60. (a) The graphs of $y = \sin x$ and $y = x$ nearly coincide when x is near the origin (when the calculator is in radians mode).
(b) In degree mode, when x is near zero degrees the sine of x is much closer to zero than x itself. The curves look like intersecting straight lines near the origin when the calculator is in degree mode.

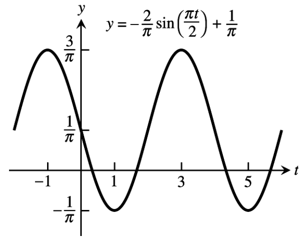
61. $A = 2, B = 2\pi, C = -\pi, D = -1$



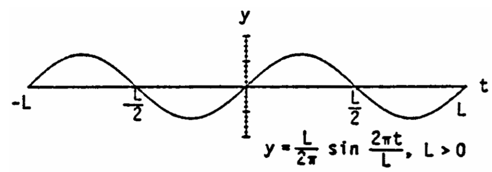
62. $A = \frac{1}{2}, B = 2, C = 1, D = \frac{1}{2}$



63. $A = -\frac{2}{\pi}$, $B = 4$, $C = 0$, $D = \frac{1}{\pi}$



64. $A = \frac{L}{2\pi}$, $B = L$, $C = 0$, $D = 0$



65. (a) amplitude $= |A| = 37$
(b) period $= |B| = 365$
(c) right horizontal shift $= C = 101$
(d) upward vertical shift $= D = 25$

66. (a) It is highest when the value of the sine is 1 at $f(101) = 37\sin(0) + 25 = 62°$ F.
The lowest mean daily temp is $37(-1) + 25 = -12°$ F.
(b) The average of the highest and lowest mean daily temperatures $= \frac{62° + (-12)°}{2} = 25°$ F.
The average of the sine function is its horizontal axis, $y = 25$.

67-70. Example CAS commands:

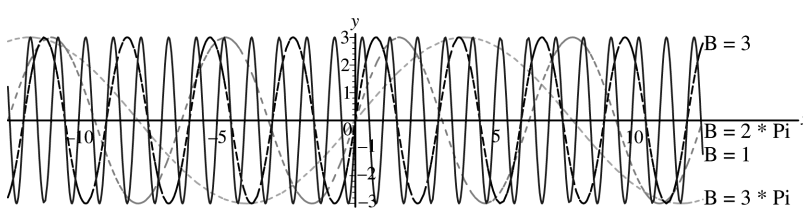
<u>Maple</u>

```
f := x -> A*sin((2*Pi/B)*(x-C))+D1;
A:=3; C:=0; D1:=0;
f_list := [seq( f(x), B=[1,3,2*Pi,5*Pi] )];
plot( f_list, x=-4*Pi..4*Pi, scaling=constrained,
      color=[red,blue,green,cyan], linestyle=[1,3,4,7],
      legend=["B=1","B=3","B=2*Pi","B=3*Pi"],
      title="#67 (Section 1.6)" );
```

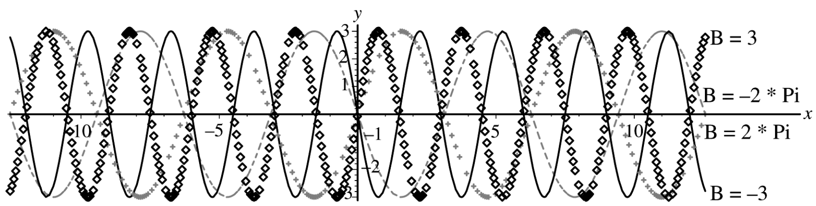
<u>Mathematica</u>

```
Clear[a, b, c, d, f, x]
f[x_]:=a  Sin[2π/b (x − c)] + d
Plot[f[x]/.{a → 3, b → 1, c → 0, d → 0}, {x, −4π, 4π }]
```

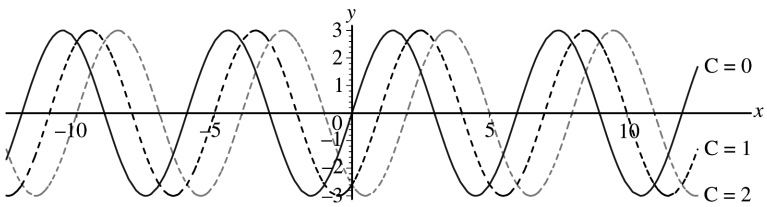
67. (a) The graph stretches horizontally.



(b) The period remains the same: period $= |\,B\,|$. The graph has a horizontal shift of $\frac{1}{2}$ period.



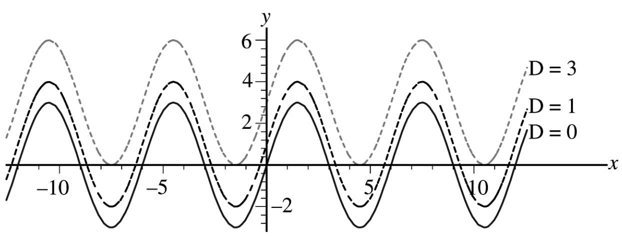
68. (a) The graph is shifted right C units.



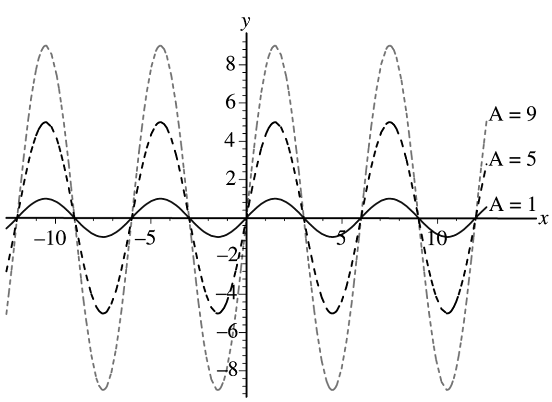
(b) The graph is shifted left C units.

(c) A shift of $\pm$ one period will produce no apparent shift. $|\,C\,| = 6$

69. The graph shifts upwards $|\,D\,|$units for $D > 0$ and down $|\,D\,|$units for $D < 0$.



70. (a) The graph stretches $|\,A\,|$ units.

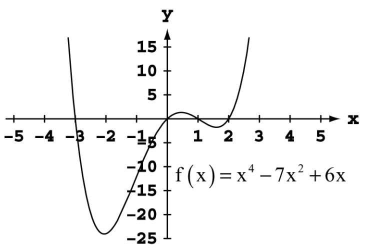


(b) For $A < 0$, the graph is inverted.

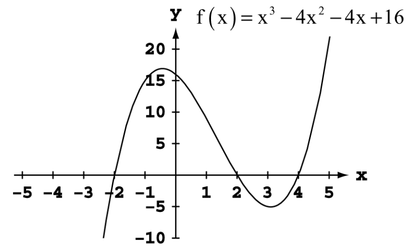
1.7 GRAPHING WITH CALCULATORS AND COMPUTERS

1-4. The most appropriate viewing window displays the maxima, minima, intercepts, and end behavior of the graphs and has little unused space.

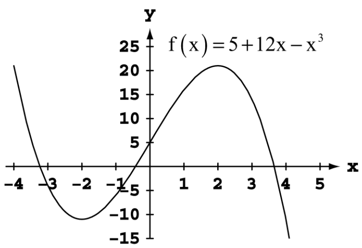
1. d.



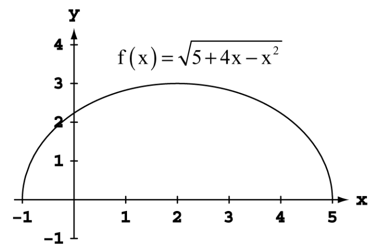
2. c.



3. d.

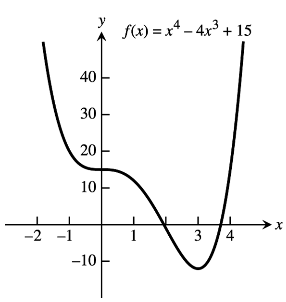


4. b.

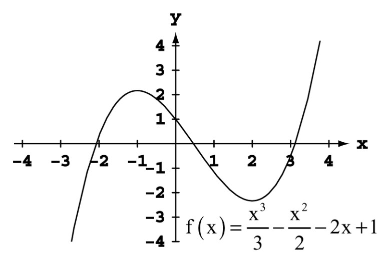


5-30. For any display there are many appropriate display widows. The graphs given as answers in Exercises 5–30 are not unique in appearance.

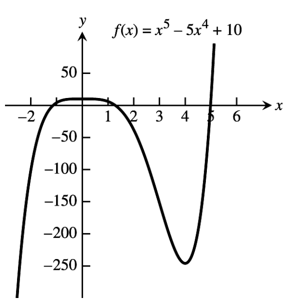
5. $[-2, 5]$ by $[-15, 40]$



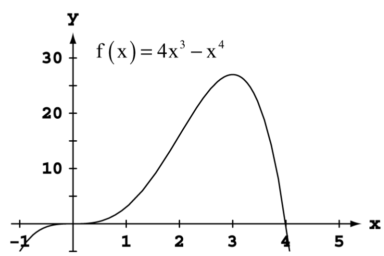
6. $[-4, 4]$ by $[-4, 4]$



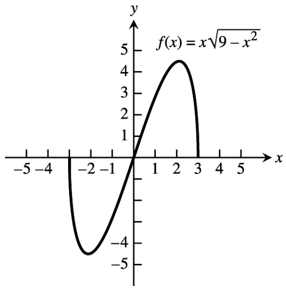
7. $[-2, 6]$ by $[-250, 50]$



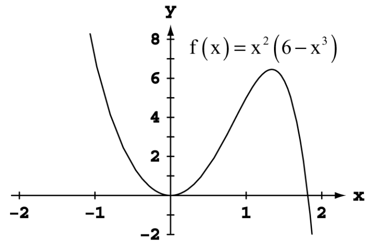
8. $[-1, 5]$ by $[-5, 30]$



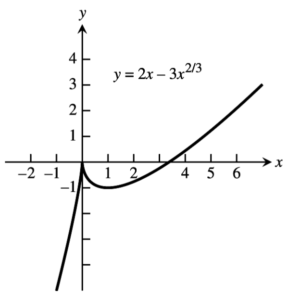
9. $[-4, 4]$ by $[-5, 5]$



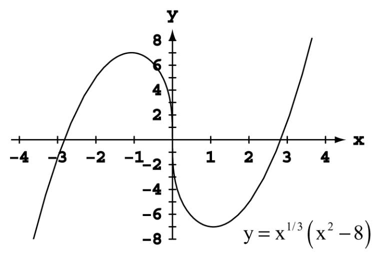
10. $[-2, 2]$ by $[-2, 8]$



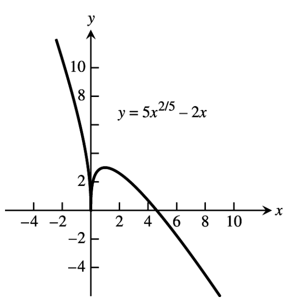
11. $[-2, 6]$ by $[-5, 4]$



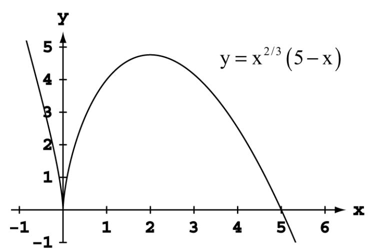
12. $[-4, 4]$ by $[-8, 8]$



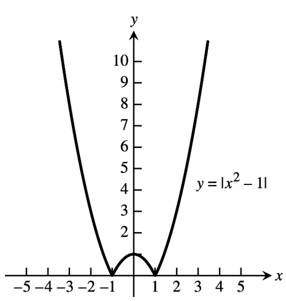
13. $[-1, 6]$ by $[-1, 4]$



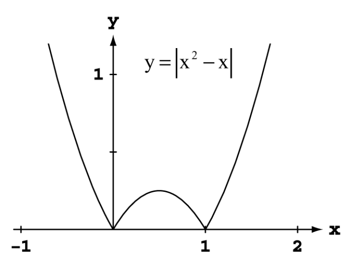
14. $[-1, 6]$ by $[-1, 5]$



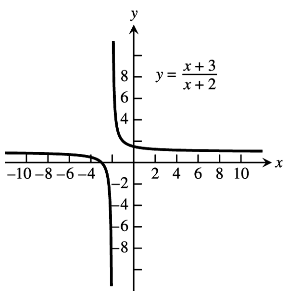
15. $[-3, 3]$ by $[0, 10]$



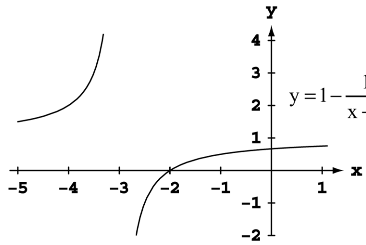
16. $[-1, 2]$ by $[0, 1]$



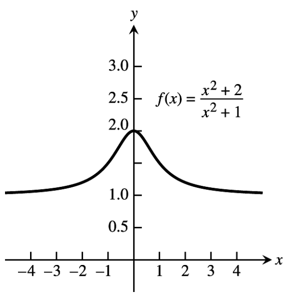
17. $[-5, 1]$ by $[-5, 5]$



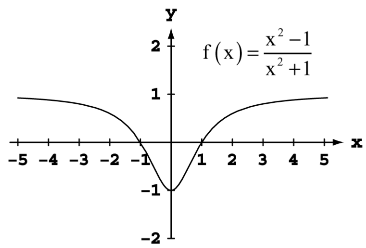
18. $[-5, 1]$ by $[-2, 4]$



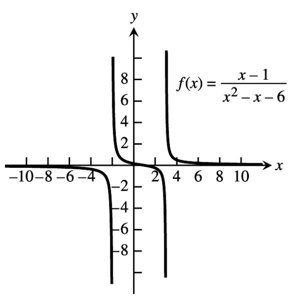
19. $[-4, 4]$ by $[0, 3]$



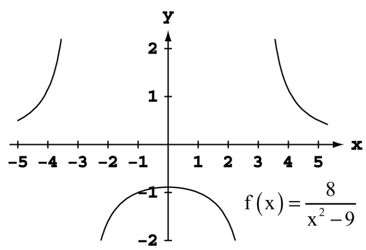
20. $[-5, 5]$ by $[-2, 2]$



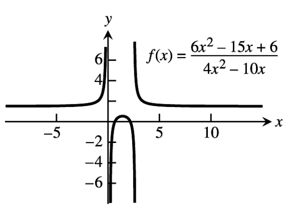
21. $[-10, 10]$ by $[-6, 6]$



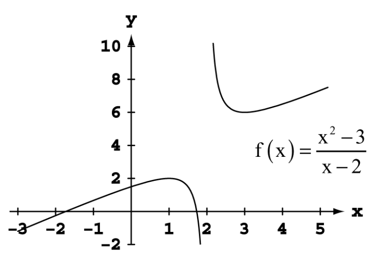
22. $[-5, 5]$ by $[-2, 2]$



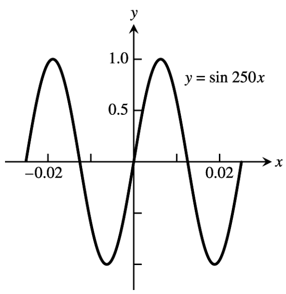
23. $[-6, 10]$ by $[-6, 6]$



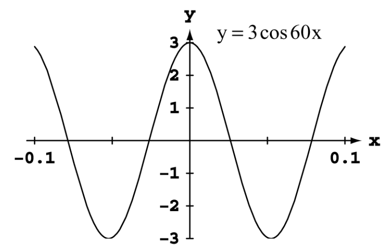
24. $[-3, 5]$ by $[-2, 10]$



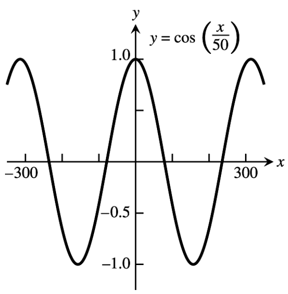
25. $[-0.03,\ 0.03]$ by $[-1.25,\ 1.25]$



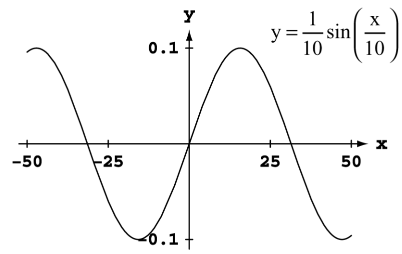
26. $[-0.1,\ 0.1]$ by $[-3,\ 3]$



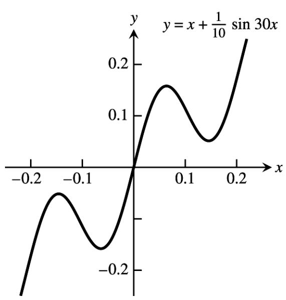
27. $[-300,\ 300]$ by $[-1.25,\ 1.25]$



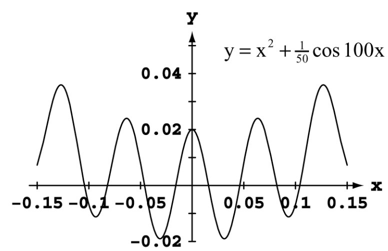
28. $[-50,\ 50]$ by $[-0.1,\ 0.1]$



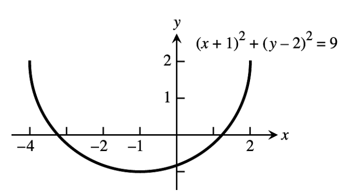
29. $[-0.25,\ 0.25]$ by $[-0.3,\ 0.3]$



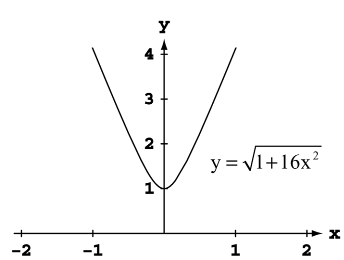
30. $[-0.15,\ 0.15]$ by $[-0.02,\ 0.05]$



31. $x^2 + 2x = 4 + 4y - y^2 \Rightarrow y = 2 \pm \sqrt{-x^2 - 2x + 8}$. The lower half is produced by graphing $y = 2 - \sqrt{-x^2 - 2x + 8}$.



32. $y^2 - 16x^2 = 1 \Rightarrow y = \pm\sqrt{1 + 16x^2}$. The upper branch is produced by graphing $y = \sqrt{1 + 16x^2}$.



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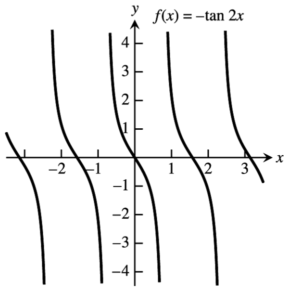

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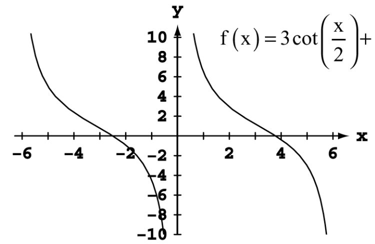

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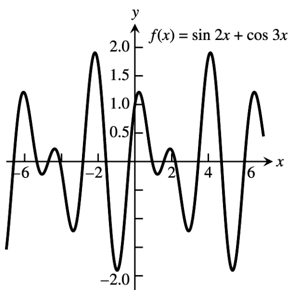

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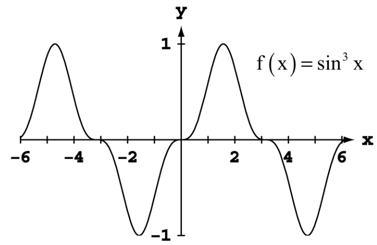

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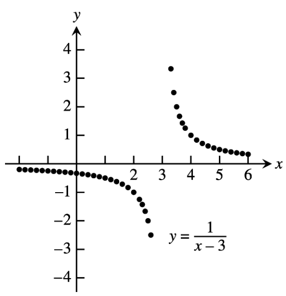

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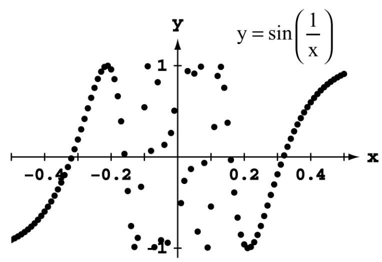
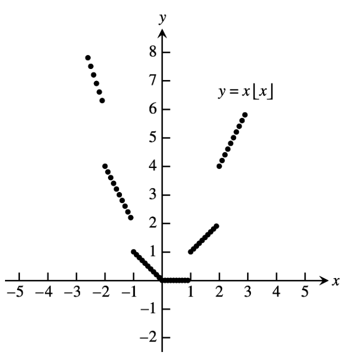
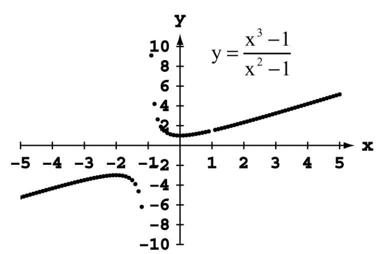


39.



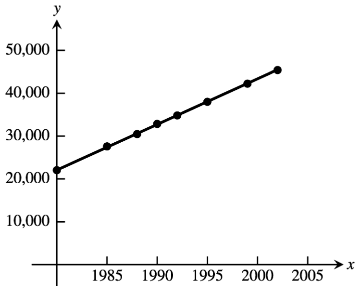
40.



41. (a) $y = 1059.14x - 2074972$

(b) $m = 1059.14$ dollars/year, which is the yearly increase in compensation.

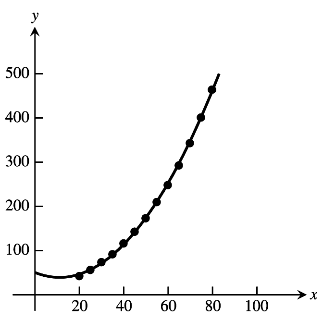
(c)



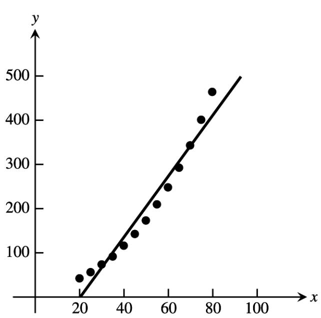
(d) Answers may vary slightly. $y = (1059.14)(2010) - 2074972 = \$53,899$

42. (a) Let C = cost and x = year.
$C = (7960.71)x - 1.6 \times 10^7$
(b) Slope represents increase in cost per year
(c) $C = (2637.14)x - 5.2 \times 10^6$
(d) The median price is rising faster in the northeast (the slope is larger).

43. (a) Let x represent the speed in miles per hour and d the stopping distance in feet. The quadratic regression function is $d = 0.0866x^2 - 1.97x + 50.1$.
(b)

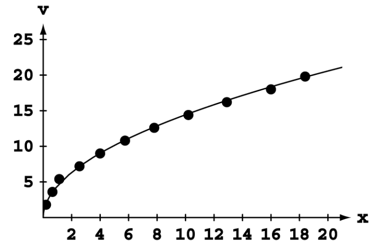


(c) From the graph in part (b), the stopping distance is about 370 feet when the vehicle is 72 mph and it is about 525 feet when the speed is 85 mph.
Algebraically: $d_{\text{quadratic}}(72) = 0.0866(72)^2 - 1.97(72) + 50.1 = 367.6$ ft.
$d_{\text{quadratic}}(85) = 0.0866(85)^2 - 1.97(85) + 50.1 = 522.8$ ft.
(d) The linear regression function is $d = 6.89x - 140.4 \Rightarrow d_{\text{linear}}(72) = 6.89(72) - 140.4 = 355.7$ ft and $d_{\text{linear}}(85) = 6.89(85) - 140.4 = 445.2$ ft. The linear regression line is shown on the graph in part (b). The quadratic regression curve clearly gives the better fit.



44. (a) The power regression function is $y = 4.44647x^{0.511414}$.

(b)

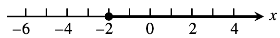


(c) 15.2 km/h

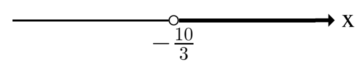
(d) The linear regression function is $y = 0.913675x + 4.189976$ and it is shown on the graph in part (b). The linear regession function gives a speed of 14.2 km/h when $y = 11$ m. The power regression curve in part (a) better fits the data.

CHAPTER 1 PRACTICE EXERCISES

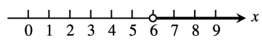
1. $7 + 2x \geq 3 \Rightarrow 2x \geq -4 \Rightarrow x \geq -2$



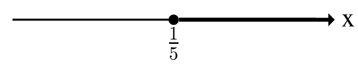
2. $-3x < 10 \Rightarrow x > -\frac{10}{3}$



3. $\frac{1}{5}(x-1) < \frac{1}{4}(x-2) \Rightarrow 4(x-1) < 5(x-2)$
$\Rightarrow 4x - 4 < 5x - 10 \Rightarrow 6 < x$



4. $\frac{x-3}{2} \geq -\frac{4+x}{3} \Rightarrow 3(x-3) \geq -2(4+x)$
$\Rightarrow 3x - 9 \geq -8 - 2x \Rightarrow 5x \geq 1 \Rightarrow x \geq \frac{1}{5}$



5. $|x+1| = 7 \Rightarrow x + 1 = 7$ or $-(x+1) = 7 \Rightarrow x = 6$ or $x = -8$

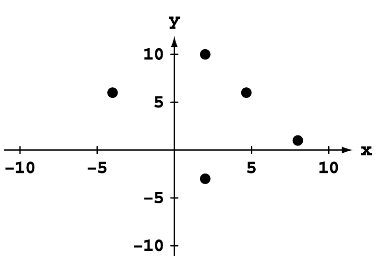
6. $|y-3| < 4 \Rightarrow -4 < y - 3 < 4 \Rightarrow -1 < y < 7$

7. $\left|1 - \frac{x}{2}\right| > \frac{3}{2} \Rightarrow 1 - \frac{x}{2} < -\frac{3}{2}$ or $1 - \frac{x}{2} > \frac{3}{2} \Rightarrow -\frac{x}{2} < -\frac{5}{2}$ or $-\frac{x}{2} > \frac{1}{2} \Rightarrow -x < -5$ or $-x > 1$
$\Rightarrow x > 5$ or $x < -1$

8. $\left|\frac{2x+7}{3}\right| \leq 5 \Rightarrow -5 \leq \frac{2x+7}{3} \leq 5 \Rightarrow -15 \leq 2x + 7 \leq 15 \Rightarrow -22 \leq 2x \leq 8 \Rightarrow -11 \leq x \leq 4$

9. Since the particle moved to the y-axis, $-2 + \Delta x = 0 \Rightarrow \Delta x = 2$. Since $\Delta y = 3\Delta x = 6$, the new coordinates are $(x + \Delta x, y + \Delta y) = (-2 + 2, 5 + 6) = (0, 11)$.

10. (a)



(b)

line	slope
AB	$\frac{10-1}{2-8} = \frac{9}{-6} = -\frac{3}{2}$
BC	$\frac{10-6}{2-(-4)} = \frac{4}{6} = \frac{2}{3}$
CD	$\frac{6-(-3)}{-4-2} = \frac{9}{-6} = -\frac{3}{2}$
DA	$\frac{1-(-3)}{8-2} = \frac{4}{6} = \frac{2}{3}$
CE	$\frac{6-6}{-4-\frac{14}{3}} = 0$
BD	is vertical and has no slope

(c) Yes; A, B, C and D form a parallelogram.

(d) Yes. The line AB has equation $y - 1 = -\frac{3}{2}(x - 8)$. Replacing x by $\frac{14}{3}$ gives $y = -\frac{3}{2}\left(\frac{14}{3} - 8\right) + 1$ $= -\frac{3}{2}\left(-\frac{10}{3}\right) + 1 = 5 + 1 = 6$. Thus, $E\left(\frac{14}{3}, 6\right)$ lies on the line AB and the points A, B and E are collinear.

(e) The line CD has equation $y + 3 = -\frac{3}{2}(x - 2)$ or $y = -\frac{3}{2}x$. Thus the line passes through the origin.

11. The triangle ABC is neither an isosceles triangle nor is it a right triangle. The lengths of AB, BC and AC are $\sqrt{53}$, $\sqrt{72}$ and $\sqrt{65}$, respectively. The slopes of AB, BC and AC are $\frac{7}{2}$, -1 and $\frac{1}{8}$, respectively.

12. $P(x, 3x + 1)$ is a point on the line $y = 3x + 1$. If the distance from P to $(0, 0)$ equals the distance from P to $(-3, 4)$, then $x^2 + (3x + 1)^2 = (x + 3)^2 + (3 - 3x)^2 \Rightarrow x^2 + 9x^2 + 6x + 1 = x^2 + 6x + 9 + 9 - 18x + 9x^2$ $\Rightarrow 18x = 17$ or $x = \frac{17}{18} \Rightarrow y = 3x + 1 = 3\left(\frac{17}{18}\right) + 1 = \frac{23}{6}$. Thus the point is $P\left(\frac{17}{18}, \frac{23}{6}\right)$.

13. $y = 3(x - 1) + (-6) \Rightarrow y = 3x - 9$

14. $y = -\frac{1}{2}(x + 1) + 2 \Rightarrow y = -\frac{1}{2}x + \frac{3}{2}$

15. $x = 0$

16. $m = \frac{-2-6}{1-(-3)} = \frac{-8}{4} = -2 \Rightarrow y = -2(x + 3) + 6 \Rightarrow y = -2x$

17. $y = 2$

18. $m = \frac{5-3}{-2-3} = \frac{2}{-5} = -\frac{2}{5} \Rightarrow y = -\frac{2}{5}(x - 3) + 3 \Rightarrow y = -\frac{2}{5}x + \frac{21}{5}$

19. $y = -3x + 3$

20. Since $2x - y = -2$ is equivalent to $y = 2x + 2$, the slope of the given line (and hence the slope of the desired line) is 2. $y = 2(x - 1) + 1 \Rightarrow y = 2x - 5$

21. Since $4x + 3y = 12$ is equivalent to $y = -\frac{4}{3}x + 4$, the slope of the given line (and hence the slope of the desired line) is $-\frac{4}{3}$. $y = -\frac{4}{3}(x - 4) - 12 \Rightarrow y = -\frac{4}{3}x - \frac{20}{3}$

22. Since $3x - 5y = 1$ is equivalent to $y = \frac{3}{5}x - \frac{1}{5}$, the slope of the given line is $\frac{3}{5}$ and the slope of the perpendicular line is $-\frac{5}{3}$. $y = -\frac{5}{3}(x + 2) - 3 \Rightarrow y = -\frac{5}{3}x - \frac{19}{3}$

23. Since $\frac{1}{2}x + \frac{1}{3}y = 1$ is equivalent to $y = -\frac{3}{2}x + 3$, the slope of the given line is $-\frac{3}{2}$ and the slope of the perpendicular line is $\frac{2}{3}$. $y = \frac{2}{3}(x + 1) + 2 \Rightarrow y = \frac{2}{3}x + \frac{8}{3}$

24. The line passes through $(0, -5)$ and $(3, 0)$. $m = \frac{0-(-5)}{3-0} = \frac{5}{3} \Rightarrow y = \frac{5}{3}x - 5$

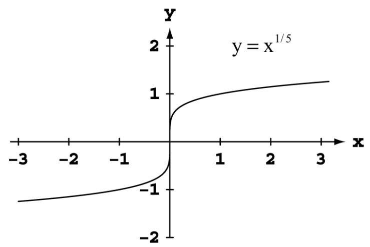
25. The area is $A = \pi r^2$ and the circumference is $C = 2\pi r$. Thus, $r = \frac{C}{2\pi} \Rightarrow A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$.

26. The surface area is $S = 4\pi r^2 \Rightarrow r = \left(\frac{S}{4\pi}\right)^{1/2}$. The volume is $V = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$. Substitution into the formula for surface area gives $S = 4\pi r^2 = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$.

27. The coordinates of a point on the parabola are (x, x^2). The angle of inclination θ joining this point to the origin satisfies the equation $\tan\theta = \frac{x^2}{x} = x$. Thus the point has coordinates $(x, x^2) = (\tan\theta, \tan^2\theta)$.

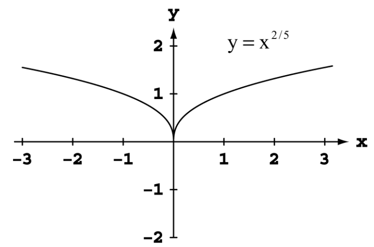
28. $\tan\theta = \frac{\text{rise}}{\text{run}} = \frac{h}{500} \Rightarrow h = 500\tan\theta$ ft.

29.



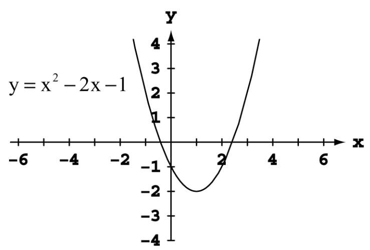
Symmetric about the origin.

30.



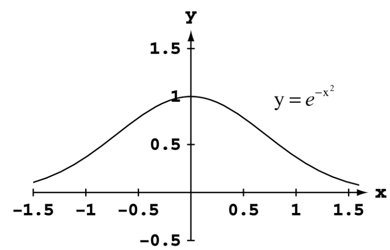
Symmetric about the y-axis.

31.



Neither

32.



Symmetric about the y-axis.

33. $y(-x) = (-x)^2 + 1 = x^2 + 1 = y(x)$. Even.

34. $y(-x) = (-x)^5 - (-x)^3 - (-x) = -x^5 + x^3 + x = -y(x)$. Odd.

35. $y(-x) = 1 - \cos(-x) = 1 - \cos x = y(x)$. Even.

36. $y(-x) = \sec(-x)\tan(-x) = \frac{\sin(-x)}{\cos^2(-x)} = \frac{-\sin x}{\cos^2 x} = -\sec x \tan x = -y(x)$. Odd.

37. $y(-x) = \frac{(-x)^4+1}{(-x)^3-2(-x)} = \frac{x^4+1}{-x^3+2x} = -\frac{x^4+1}{x^3-2x} = -y(x)$. Odd.

38. $y(-x) = 1 - \sin(-x) = 1 + \sin x$. Neither even nor odd.

39. $y(-x) = -x + \cos(-x) = -x + \cos x$. Neither even nor odd.

40. $y(-x) = \sqrt{(-x)^4 - 1} = \sqrt{x^4 - 1} = y(x)$. Even.

41. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) Since $|\,x\,|$ attains all nonnegative values, the range is $[-2, \infty)$.

42. (a) Since the square root requires $1 - x \geq 0$, the domain is $(-\infty, 1]$.
    (b) Since $\sqrt{1-x}$ attains all nonnegative values, the range is $[-2, \infty)$.

43. (a) Since the square root requires $16 - x^2 \geq 0$, the domain is $[-4, 4]$.
    (b) For values of x in the domain, $0 \leq 16 - x^2 \leq 16$, so $0 \leq \sqrt{16 - x^2} \leq 4$. The range is $[0, 4]$.

44. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) Since 3^{2-x} attains all positive values, the range is $(1, \infty)$.

45. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) Since $2e^{-x}$ attains all positive values, the range is $(-3, \infty)$.

46. (a) The function is equivalent to $y = \tan 2x$, so we require $2x \neq \frac{k\pi}{2}$ for odd integers k. The domain is given by $x \neq \frac{k\pi}{4}$ for odd integers k.
    (b) Since the tangent function attains all values, the range is $(-\infty, \infty)$.

47. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) The sine function attains values from -1 to 1, so $-2 \leq 2\sin(3x + \pi) \leq 2$ and hence $-3 \leq 2\sin(3x + \pi) - 1 \leq 1$. The range is $[-3, 1]$.

48. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) The function is equivalent to $y = \sqrt[5]{x^2}$, which attains all nonnegative values. The range is $[0, \infty)$.

49. (a) The logarithm requires $x - 3 > 0$, so the domain is $(3, \infty)$.
    (b) The logarithm attains all real values, so the range is $(-\infty, \infty)$.

50. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) The cube root attains all real values, so the range is $(-\infty, \infty)$.

51. (a) The function is defined for $-4 \leq x \leq 4$, so the domain is $[-4, 4]$.
    (b) The function is equivalent to $y = \sqrt{|\,x\,|}$, $-4 \leq x \leq 4$, which attains values from 0 to 2 for x in the domain. The range is $[0, 2]$.

52. (a) The function is defined for $-2 \leq x \leq 2$, so the domain is $[-2, 2]$.
    (b) The range is $[-1, 1]$.

53. First piece: Line through $(0, 1)$ and $(1, 0)$. $m = \frac{0-1}{1-0} = \frac{-1}{1} = -1 \Rightarrow y = -x + 1 = 1 - x$
    Second piece: Line through $(1, 1)$ and $(2, 0)$. $m = \frac{0-1}{2-1} = \frac{-1}{1} = -1 \Rightarrow y = -(x-1) + 1 = -x + 2 = 2 - x$
    $$f(x) = \begin{cases} 1 - x, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x \leq 2 \end{cases}$$

54. First piece: Line through $(0, 0)$ and $(2, 5)$. $m = \frac{5-0}{2-0} = \frac{5}{2} \Rightarrow y = \frac{5}{2}x$
    Second piece: Line through $(2, 5)$ and $(4, 0)$. $m = \frac{0-5}{4-2} = \frac{-5}{2} = -\frac{5}{2} \Rightarrow y = -\frac{5}{2}(x-2) + 5 = -\frac{5}{2}x + 10 = 10 - \frac{5x}{2}$
    $$f(x) = \begin{cases} \frac{5}{2}x, & 0 \leq x < 2 \\ 10 - \frac{5x}{2}, & 2 \leq x \leq 4 \end{cases}$$ (Note: $x = 2$ can be included on either piece.)

55. (a) $(f\circ g)(-1) = f(g(-1)) = f\left(\frac{1}{\sqrt{-1+2}}\right) = f(1) = \frac{1}{1} = 1$

(b) $(g\circ f)(2) = g(f(2)) = g\left(\frac{1}{2}\right) = \frac{1}{\sqrt{\frac{1}{2}+2}} = \frac{1}{\sqrt{2.5}}$ or $\sqrt{\frac{2}{5}}$

(c) $(f\circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x,\ x \neq 0$

(d) $(g\circ g)(x) = g(g(x)) = g\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{\sqrt{\frac{1}{\sqrt{x+2}}+2}} = \frac{\sqrt[4]{x+2}}{\sqrt{1+2\sqrt{x+2}}}$

56. (a) $(f\circ g)(-1) = f(g(-1)) = f\left(\sqrt[3]{-1+1}\right) = f(0) = 2 - 0 = 2$

(b) $(g\circ f)(2) = f(g(2)) = g(2-2) = g(0) = \sqrt[3]{0+1} = 1$

(c) $(f\circ f)(x) = f(f(x)) = f(2-x) = 2-(2-x) = x$

(d) $(g\circ g)(x) = g(g(x)) = g\left(\sqrt[3]{x+1}\right) = \sqrt[3]{\sqrt[3]{x+1}+1}$

57. (a) $(f\circ g)(x) = f(g(x)) = f\left(\sqrt{x+2}\right) = 2 - \left(\sqrt{x+2}\right)^2 = -x,\ x \geq -2.$

$(g\circ f)(x) = f(g(x)) = g(2-x^2) = \sqrt{(2-x^2)+2} = \sqrt{4-x^2}$

(b) Domain of $f\circ g$: $[-2, \infty)$.

Domain of $g\circ f$: $[-2, 2]$.

(c) Range of $f\circ g$: $(-\infty, 2]$.

Range of $g\circ f$: $[0, 2]$.

58. (a) $(f\circ g)(x) = f(g(x)) = f\left(\sqrt{1-x}\right) = \sqrt{\sqrt{1-x}} = \sqrt[4]{1-x}.$

$(g\circ f)(x) = f(g(x)) = g\left(\sqrt{x}\right) = \sqrt{1-\sqrt{x}}$

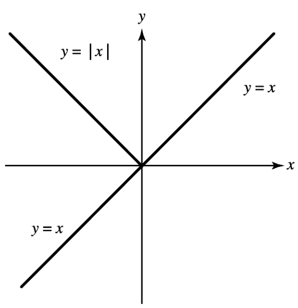
(b) Domain of $f\circ g$: $(-\infty, 1]$.

Domain of $g\circ f$: $[0, 1]$.

(c) Range of $f\circ g$: $[0, \infty)$.

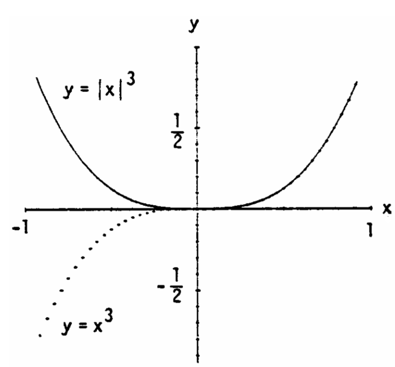
Range of $g\circ f$: $[0, 1]$.

59.



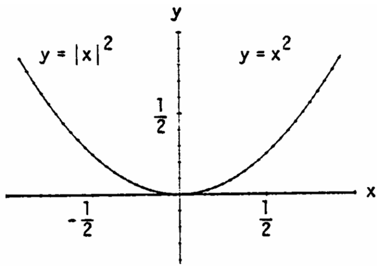
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x),\ x \geq 0$ across the y-axis.

60.



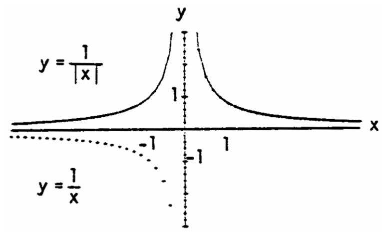
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x),\ x \geq 0$ across the y-axis.

61.



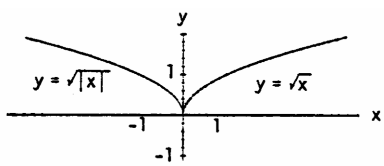
It does not change the graph.

62.



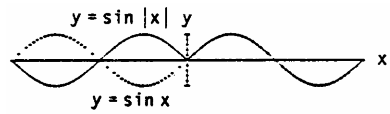
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

63.



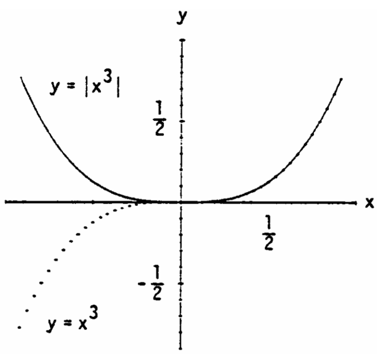
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

64.



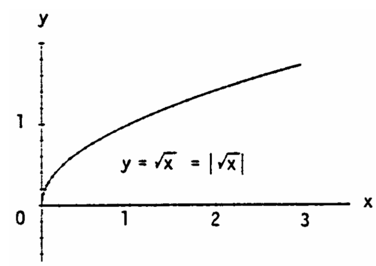
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

65.



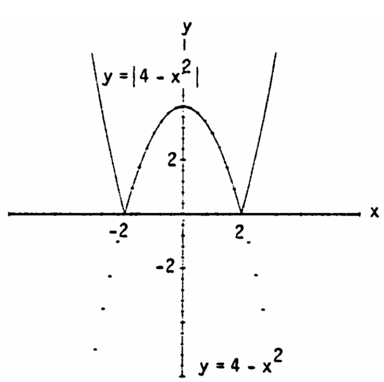
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

66.

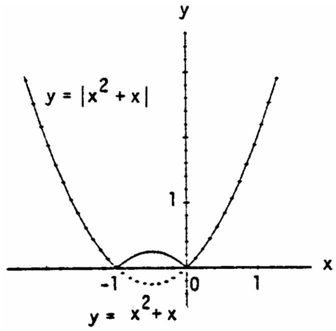


It does not change the graph.

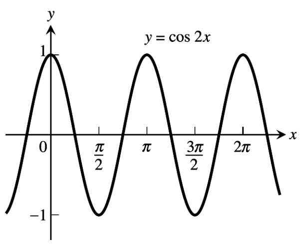
67.



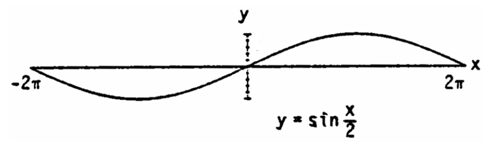
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

68. 

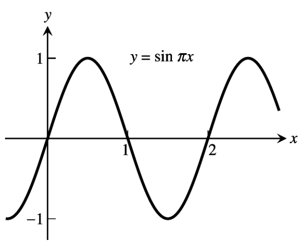
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

69. 

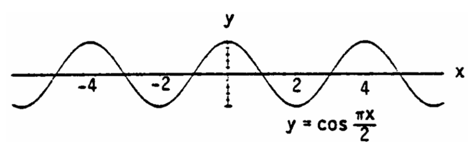
period $= \pi$

70. 

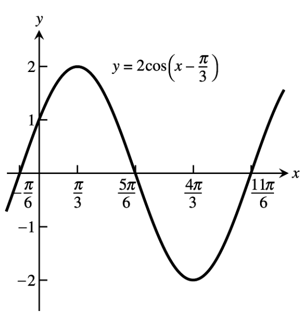
period $= 4\pi$

71. 

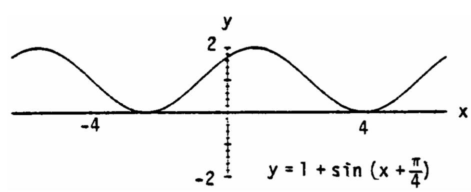
period $= 2$

72. 

period $= 4$

73. 

period $= 2\pi$

74. 

period $= 2\pi$

75. (a) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{b}{2} \Rightarrow b = 2 \sin \frac{\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$. By the theorem of Pythagoras, $a^2 + b^2 = c^2 \Rightarrow a = \sqrt{c^2 - b^2} = \sqrt{4 - 3} = 1$.

(b) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{2}{c} \Rightarrow c = \frac{2}{\sin \frac{\pi}{3}} = \frac{2}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{4}{\sqrt{3}}$. Thus, $a = \sqrt{c^2 - b^2} = \sqrt{\left(\frac{4}{\sqrt{3}}\right)^2 - (2)^2} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$.

76. (a) $\sin A = \frac{a}{c} \Rightarrow a = c \sin A$

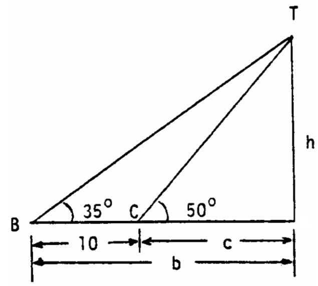
(b) $\tan A = \frac{a}{b} \Rightarrow a = b \tan A$

77. (a) $\tan B = \frac{b}{a} \Rightarrow a = \frac{b}{\tan B}$

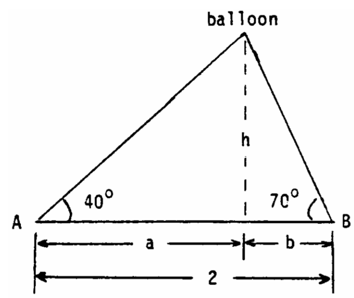
(b) $\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A}$

78. (a) $\sin A = \frac{a}{c}$ (c) $\sin A = \frac{a}{c} = \frac{\sqrt{c^2 - b^2}}{c}$

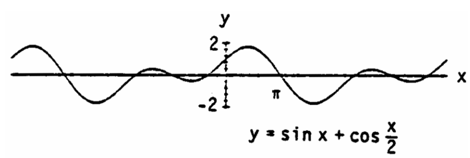
79. Let h = height of vertical pole, and let b and c denote the distances of points B and C from the base of the pole, measured along the flatground, respectively. Then, $\tan 50° = \frac{h}{c}$, $\tan 35° = \frac{h}{b}$, and $b - c = 10$.
Thus, $h = c \tan 50°$ and $h = b \tan 35° = (c + 10) \tan 35°$
$\Rightarrow c \tan 50° = (c + 10) \tan 35°$
$\Rightarrow c (\tan 50° - \tan 35°) = 10 \tan 35°$
$\Rightarrow c = \frac{10 \tan 35°}{\tan 50° - \tan 35°} \Rightarrow h = c \tan 50°$
$= \frac{10 \tan 35° \tan 50°}{\tan 50° - \tan 35°} \approx 16.98$ m.



80. Let h = height of balloon above ground. From the figure at the right, $\tan 40° = \frac{h}{a}$, $\tan 70° = \frac{h}{b}$, and $a + b = 2$. Thus, $h = b \tan 70° \Rightarrow h = (2 - a) \tan 70°$ and $h = a \tan 40°$
$\Rightarrow (2 - a) \tan 70° = a \tan 40° \Rightarrow a(\tan 40° + \tan 70°)$
$= 2 \tan 70° \Rightarrow a = \frac{2 \tan 70°}{\tan 40° + \tan 70°} \Rightarrow h = a \tan 40°$
$= \frac{2 \tan 70° \tan 40°}{\tan 40° + \tan 70°} \approx 1.3$ km.

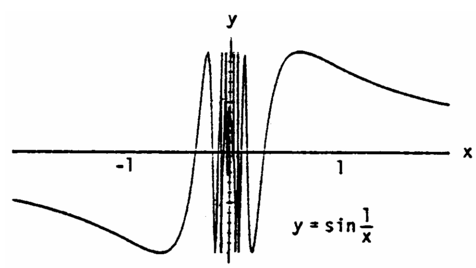


81. (a)



(b) The period appears to be 4π.
(c) $f(x + 4\pi) = \sin(x + 4\pi) + \cos\left(\frac{x + 4\pi}{2}\right) = \sin(x + 2\pi) + \cos\left(\frac{x}{2} + 2\pi\right) = \sin x + \cos \frac{x}{2}$ since the period of sine and cosine is 2π. Thus, f(x) has period 4π.

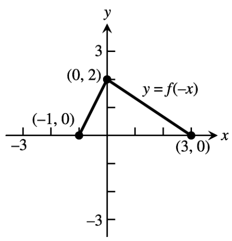
82. (a)



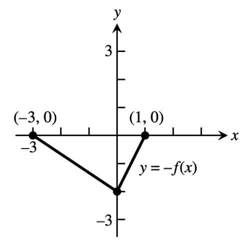
(b) $D = (-\infty, 0) \cup (0, \infty)$; $R = [-1, 1]$
(c) f is not periodic. For suppose f has period p. Then $f\left(\frac{1}{2\pi} + kp\right) = f\left(\frac{1}{2\pi}\right) = \sin 2\pi = 0$ for all integers k. Choose k so large that $\frac{1}{2\pi} + kp > \frac{1}{\pi} \Rightarrow 0 < \frac{1}{(1/2\pi) + kp} < \pi$. But then $f\left(\frac{1}{2\pi} + kp\right) = \sin\left(\frac{1}{(1/2\pi) + kp}\right) > 0$ which is a contradiction. Thus f has no period, as claimed.

CHAPTER 1 ADDITIONAL AND ADVANCED EXERCISES

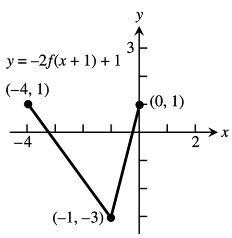
1. (a) The given graph is reflected about the y-axis.



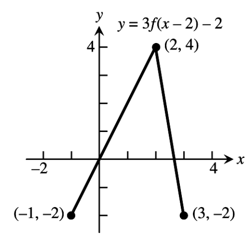
(b) The given graph is reflected about the x-axis.



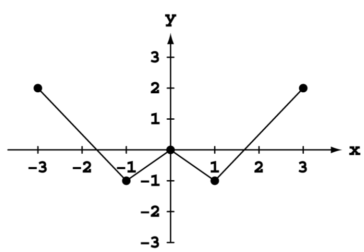
(c) The given graph is shifted left 1 unit, stretched vertically by a factor of 2, reflected about the x-axis, and then shifted upward 1 unit.



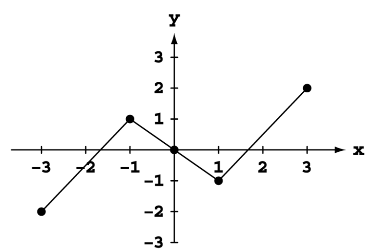
(d) The given graph is shifted right 2 units, stretched vertically by a factor of 3, and then shifted downward 2 units.



2. (a)



(b)



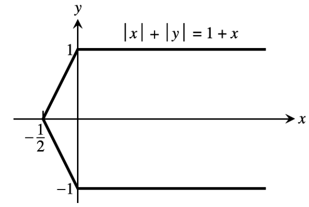
3. There are (infinitely) many such function pairs. For example, $f(x) = 3x$ and $g(x) = 4x$ satisfy $f(g(x)) = f(4x) = 3(4x) = 12x = 4(3x) = g(3x) = g(f(x))$.

4. Yes, there are many such function pairs. For example, if $g(x) = (2x+3)^3$ and $f(x) = x^{1/3}$, then $(f \circ g)(x) = f(g(x)) = f\left((2x+3)^3\right) = \left((2x+3)^3\right)^{1/3} = 2x + 3$.

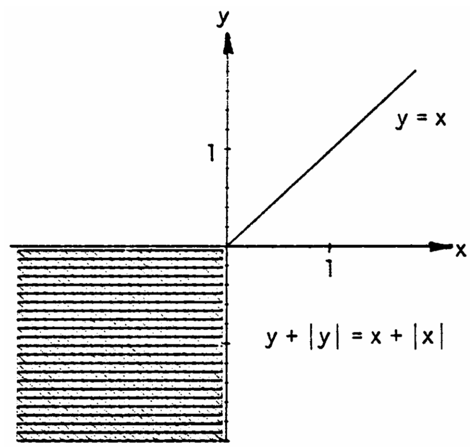
5. If f is odd and defined at x, then $f(-x) = -f(x)$. Thus $g(-x) = f(-x) - 2 = -f(x) - 2$ whereas $-g(x) = -(f(x) - 2) = -f(x) + 2$. Then g cannot be odd because $g(-x) = -g(x) \Rightarrow -f(x) - 2 = -f(x) + 2$ $\Rightarrow 4 = 0$, which is a contradiction. Also, g(x) is not even unless $f(x) = 0$ for all x. On the other hand, if f is even, then $g(x) = f(x) - 2$ is also even: $g(-x) = f(-x) - 2 = f(x) - 2 = g(x)$.

6. If g is odd and g(0) is defined, then $g(0) = g(-0) = -g(0)$. Therefore, $2g(0) = 0 \Rightarrow g(0) = 0$.

7. For (x, y) in the 1st quadrant, $|x| + |y| = 1 + x$ $\Leftrightarrow$ $x + y = 1 + x$ $\Leftrightarrow$ $y = 1$. For (x, y) in the 2nd quadrant, $|x| + |y| = x + 1$ $\Leftrightarrow$ $-x + y = x + 1$ $\Leftrightarrow$ $y = 2x + 1$. In the 3rd quadrant, $|x| + |y| = x + 1$ $\Leftrightarrow$ $-x - y = x + 1$ $\Leftrightarrow$ $y = -2x - 1$. In the 4th quadrant, $|x| + |y| = x + 1$ $\Leftrightarrow$ $x + (-y) = x + 1$ $\Leftrightarrow$ $y = -1$. The graph is given at the right.



8. We use reasoning similar to Exercise 7.
   (1) 1st quadrant: $y + |y| = x + |x|$ $\Leftrightarrow$ $2y = 2x$ $\Leftrightarrow$ $y = x$.
   (2) 2nd quadrant: $y + |y| = x + |x|$ $\Leftrightarrow$ $2y = x + (-x) = 0$ $\Leftrightarrow$ $y = 0$.
   (3) 3rd quadrant: $y + |y| = x + |x|$ $\Leftrightarrow$ $y + (-y) = x + (-x)$ $\Leftrightarrow$ $0 = 0$ $\Rightarrow$ all points in the 3rd quadrant satisfy the equation.
   (4) 4th quadrant: $y + |y| = x + |x|$ $\Leftrightarrow$ $y + (-y) = 2x$ $\Leftrightarrow$ $0 = x$. Combining these results we have the graph given at the right:



9. By the law of sines, $\frac{\sin\frac{\pi}{3}}{\sqrt{3}} = \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin\frac{\pi}{4}}{b}$ $\Rightarrow$ $b = \frac{\sqrt{3}\sin(\pi/4)}{\sin(\pi/3)} = \frac{\sqrt{3}\left(\frac{\sqrt{2}}{2}\right)}{\frac{\sqrt{3}}{2}} = \sqrt{2}$.

10. By the law of sines, $\frac{\sin\frac{\pi}{4}}{4} = \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin B}{3}$ $\Rightarrow$ $\sin B = \frac{3}{4}\sin\frac{\pi}{4} = \frac{3}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{3\sqrt{2}}{8}$.

11. By the law of cosines, $a^2 = b^2 + c^2 - 2bc\cos A$ $\Rightarrow$ $\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{2^2+3^2-2^2}{2(2)(3)} = \frac{3}{4}$.

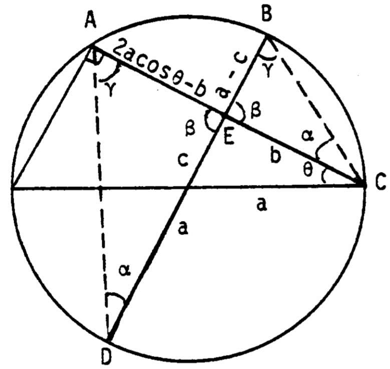
12. By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos C = 2^2 + 3^2 - (2)(2)(3)\cos\frac{\pi}{4} = 4 + 9 - 12\left(\frac{\sqrt{2}}{2}\right)$ $= 13 - 6\sqrt{2}$ $\Rightarrow$ $c = \sqrt{13 - 6\sqrt{2}}$, since $c > 0$.

13. By the law of cosines, $b^2 = a^2 + c^2 - 2ac\cos B$ $\Rightarrow$ $\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{2^2+4^2-3^2}{(2)(2)(4)} = \frac{4+16-9}{16}$ $= \frac{11}{16}$. Since $0 < B < \pi$, $\sin B = \sqrt{1-\cos^2 B} = \sqrt{1-\frac{121}{256}} = \frac{\sqrt{135}}{16} = \frac{3\sqrt{15}}{16}$.

14. By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos C$ $\Rightarrow$ $\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{2^2+4^2-5^2}{(2)(2)(4)} = \frac{4+16-25}{16}$ $= -\frac{5}{16}$. Since $0 < C < \pi$, $\sin C = \sqrt{1-\cos^2 C} = \sqrt{1-\frac{25}{256}} = \frac{\sqrt{231}}{16}$.

15. (a) $\sin^2 x + \cos^2 x = 1$ $\Rightarrow$ $\sin^2 x = 1 - \cos^2 x = (1-\cos x)(1+\cos x)$ $\Rightarrow$ $(1-\cos x) = \frac{\sin^2 x}{1+\cos x}$ $\Rightarrow$ $\frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$
   (b) Using the definition of the tangent function and the double angle formulas, we have
   $\tan^2\left(\frac{x}{2}\right) = \frac{\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} = \frac{\frac{1-\cos\left(2\left(\frac{x}{2}\right)\right)}{2}}{\frac{1+\cos\left(2\left(\frac{x}{2}\right)\right)}{2}} = \frac{1-\cos x}{1+\cos x}$.

16. The angles labeled γ in the accompanying figure are equal since both angles subtend arc CD. Similarly, the two angles labeled α are equal since they both subtend arc AB. Thus, triangles AED and BEC are similar which implies $\frac{a-c}{b} = \frac{2a\cos\theta - b}{a+c}$
$\Rightarrow (a-c)(a+c) = b(2a\cos\theta - b)$
$\Rightarrow a^2 - c^2 = 2ab\cos\theta - b^2$
$\Rightarrow c^2 = a^2 + b^2 - 2ab\cos\theta$.



17. As in the proof of the law of sines of Section P.5, Exercise 57, $ah = bc\sin A = ab\sin C = ac\sin B$
$\Rightarrow$ the area of ABC $= \frac{1}{2}$ (base)(height) $= \frac{1}{2}ah = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$.

18. As in Section P.5, Exercise 57, $(\text{Area of ABC})^2 = \frac{1}{4}(\text{base})^2(\text{height})^2 = \frac{1}{4}a^2h^2 = \frac{1}{4}a^2b^2\sin^2 C$
$= \frac{1}{4}a^2b^2(1-\cos^2 C)$. By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow \cos C = \frac{a^2+b^2-c^2}{2ab}$.
Thus, $(\text{area of ABC})^2 = \frac{1}{4}a^2b^2(1-\cos^2 C) = \frac{1}{4}a^2b^2\left(1 - \left(\frac{a^2+b^2-c^2}{2ab}\right)^2\right) = \frac{a^2b^2}{4}\left(1 - \frac{(a^2+b^2-c^2)^2}{4a^2b^2}\right)$
$= \frac{1}{16}\left(4a^2b^2 - (a^2+b^2-c^2)^2\right) = \frac{1}{16}\left[(2ab + (a^2+b^2-c^2))(2ab - (a^2+b^2-c^2))\right]$
$= \frac{1}{16}\left[((a+b)^2 - c^2)(c^2 - (a-b)^2)\right] = \frac{1}{16}\left[((a+b)+c)((a+b)-c)(c+(a-b))(c-(a-b))\right]$
$= \left[\left(\frac{a+b+c}{2}\right)\left(\frac{-a+b+c}{2}\right)\left(\frac{a-b+c}{2}\right)\left(\frac{a+b-c}{2}\right)\right] = s(s-a)(s-b)(s-c)$, where $s = \frac{a+b+c}{2}$.
Therefore, the area of ABC equals $\sqrt{s(s-a)(s-b)(s-c)}$.

19. 1. $b + c - (a+c) = b - a$, which is positive since $a < b$. Thus, $a + c < b + c$.
2. $b - c - (a-c) = b - a$, which is positive since $a < b$. Thus, $a - c < b - c$.
3. $c > 0$ and $a < b \Rightarrow c - 0 = c$ and $b - a$ are positive $\Rightarrow (b-a)c = bc - ac$ is positive $\Rightarrow ac < bc$.
4. $a < b$ and $c < 0 \Rightarrow b - a$ and $-c$ are positive $\Rightarrow (b-a)(-c) = ac - bc$ is positive $\Rightarrow bc < ac$.
5. Since $a > 0$, a and $\frac{1}{a}$ are positive $\Rightarrow \frac{1}{a} > 0$.
6. Since $0 < a < b$, both $\frac{1}{a}$ and $\frac{1}{b}$ are positive. By (3), $a < b$ and $\frac{1}{a} > 0 \Rightarrow a\left(\frac{1}{a}\right) < b\left(\frac{1}{a}\right)$ or $1 < \frac{b}{a}$
$\Rightarrow 1\left(\frac{1}{b}\right) < \frac{b}{a}\left(\frac{1}{b}\right)$ by (3) since $\frac{1}{b} > 0 \Rightarrow \frac{1}{b} < \frac{1}{a}$.
7. $a < b < 0 \Rightarrow \frac{1}{a}$ and $\frac{1}{b}$ are both negative, i.e., $\frac{1}{a} < 0$ and $\frac{1}{b} < 0$. By (4), $a < b$ and $\frac{1}{a} < 0 \Rightarrow b\left(\frac{1}{a}\right) < a\left(\frac{1}{a}\right)$
$\Rightarrow \frac{b}{a} < 1 \Rightarrow 1\left(\frac{1}{b}\right) < \frac{b}{a}\left(\frac{1}{b}\right)$ by (4) since $\frac{1}{b} < 0 \Rightarrow \frac{1}{b} < \frac{1}{a}$.

20. (a) If $a = 0$, then $0 = |a| < |b| \Leftrightarrow b \neq 0 \Leftrightarrow 0 = |a|^2 < |b|^2$. Since $|a|^2 = |a|\,|a| = |a^2| = a^2$ and $|b|^2 = b^2$ we obtain $a^2 < b^2$. If $a \neq 0$ then $|a| > 0$ and $|a| < |b| \Rightarrow a^2 < b^2$. On the other hand, if $a^2 < b^2$ then $a^2 = |a|^2 < |b|^2 = b^2 \Rightarrow 0 < |b|^2 - |a|^2 = (|b| - |a|)(|b| + |a|)$. Since $(|b| + |a|) > 0$ and the product $(|b| - |a|)(|b| + |a|)$ is positive, we must have $(|b| - |a|) > 0 \Rightarrow |b| > |a|$. Thus $|a| < |b| \Leftrightarrow a^2 < b^2$.
(b) $ab \le |ab| \Rightarrow -ab \ge -2|ab|$ by Exercise 19(4) above $\Rightarrow a^2 - 2ab + b^2 \ge |a|^2 - 2|a|\,|b| + |b|^2$, since $|a|^2 = a^2$ and $|b|^2 = b^2$. Factoring both sides, $(a-b)^2 \ge (|a| - |b|)^2 \Rightarrow |a - b| \ge ||a| - |b||$, by part (a).

21. The fact that $|a_1 + a_2 + \ldots + a_n| \le |a_1| + |a_2| + \ldots + |a_n|$ holds for $n = 1$ is obvious. It also holds for $n = 2$ by the triangle inequality. We now show it holds for all positive integers n, by induction.
Suppose it holds for $n = k \ge 1$: $|a_1 + a_2 + \ldots + a_k| \le |a_1| + |a_2| + \ldots + |a_k|$ (this is the induction hypothesis). Then $|a_1 + a_2 + \ldots + a_k + a_{k+1}| = |(a_1 + a_2 + \ldots + a_k) + a_{k+1}| \le |a_1 + a_2 + \ldots + a_k| + |a_{k+1}|$ (by the triangle inequality) $\le |a_1| + |a_2| + \ldots + |a_k| + |a_{k+1}|$ (by the induction hypothesis) and the inequality holds for $n = k + 1$. Hence it holds for all n by induction.

22. The fact that $|a_1 + a_2 + \ldots + a_n| \geq |a_1| - |a_2| - \ldots - |a_n|$ holds for $n = 1$ is obvious. It holds for $n = 2$ by Exercise 21(b), since $|a_1 + a_2| = |a_1 - (-a_2)| \geq ||a_1| - |-a_2|| = ||a_1| - |a_2|| \geq |a_1| - |a_2|$. We now show it holds for all positive integers n by induction.

    Suppose the inequality holds for $n = k \geq 1$. Then $|a_1 + a_2 + \ldots + a_k| \geq |a_1| - |a_2| - \ldots - |a_k|$ (this is the induction hypothesis). Thus $|a_1 + \ldots + a_k + a_{k+1}| = |(a_1 + \ldots + a_k) - (-a_{k+1})|$ $\geq ||(a_1 + \ldots + a_k)| - |-a_{k+1}||$ (by Exercise 21(b)) $= ||a_1 + \ldots + a_k| - |a_{k+1}|| \geq |a_1 + \ldots + a_k| - |a_{k+1}|$ $\geq |a_1| - |a_2| - \ldots - |a_k| - |a_{k+1}|$ (by the induction hypothesis). Hence the inequality holds for all n by induction.

23. If f is even and odd, then $f(-x) = -f(x)$ and $f(-x) = f(x) \Rightarrow f(x) = -f(x)$ for all x in the domain of f. Thus $2f(x) = 0 \Rightarrow f(x) = 0$.

24. (a) As suggested, let $E(x) = \frac{f(x)+f(-x)}{2} \Rightarrow E(-x) = \frac{f(-x)+f(-(-x))}{2} = \frac{f(x)+f(-x)}{2} = E(x) \Rightarrow$ E is an even function. Define $O(x) = f(x) - E(x) = f(x) - \frac{f(x)+f(-x)}{2} = \frac{f(x)-f(-x)}{2}$. Then $O(-x) = \frac{f(-x)-f(-(-x))}{2} = \frac{f(-x)-f(x)}{2} = -\left(\frac{f(x)-f(-x)}{2}\right) = -O(x) \Rightarrow$ O is an odd function $\Rightarrow f(x) = E(x) + O(x)$ is the sum of an even and an odd function.

    (b) Part (a) shows that $f(x) = E(x) + O(x)$ is the sum of an even and an odd function. If also $f(x) = E_1(x) + O_1(x)$, where E_1 is even and O_1 is odd, then $f(x) - f(x) = 0 = (E_1(x) + O_1(x))$ $- (E(x) + O(x))$. Thus, $E(x) - E_1(x) = O_1(x) - O(x)$ for all x in the domain of f (which is the same as the domain of $E - E_1$ and $O - O_1$). Now $(E - E_1)(-x) = E(-x) - E_1(-x) = E(x) - E_1(x)$ (since E and E_1 are even) $= (E - E_1)(x) \Rightarrow E - E_1$ is even. Likewise, $(O_1 - O)(-x) = O_1(-x) - O(-x) = -O_1(x) - (-O(x))$ (since O and O_1 are odd) $= -(O_1(x) - O(x)) = -(O_1 - O)(x) \Rightarrow O_1 - O$ is odd. Therefore, $E - E_1$ and $O_1 - O$ are both even and odd so they must be zero at each x in the domain of f by Exercise 23. That is, $E_1 = E$ and $O_1 = O$, so the decomposition of f found in part (a) is unique.

25. $y = ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

    (a) If $a > 0$ the graph is a parabola that opens upward. Increasing a causes a vertical stretching and a shift of the vertex toward the y-axis and upward. If $a < 0$ the graph is a parabola that opens downward. Decreasing a causes a vertical stretching and a shift of the vertex toward the y-axis and downward.

    (b) If $a > 0$ the graph is a parabola that opens upward. If also $b > 0$, then increasing b causes a shift of the graph downward to the left; if $b < 0$, then decreasing b causes a shift of the graph downward and to the right.

    If $a < 0$ the graph is a parabola that opens downward. If $b > 0$, increasing b shifts the graph upward to the right. If $b < 0$, decreasing b shifts the graph upward to the left.

    (c) Changing c (for fixed a and b) by Δc shifts the graph upward Δc units if $\Delta c > 0$, and downward $-\Delta c$ units if $\Delta c < 0$.

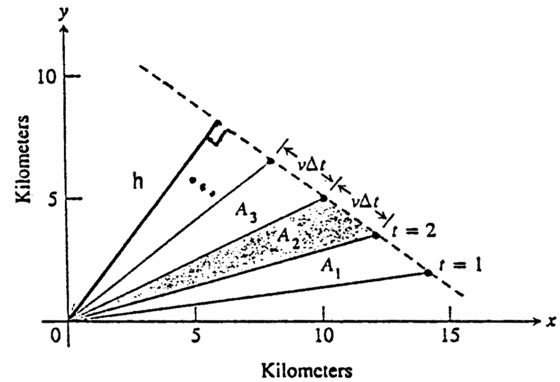
26. (a) If $a > 0$, the graph rises to the right of the vertical line $x = -b$ and falls to the left. If $a < 0$, the graph falls to the right of the line $x = -b$ and rises to the left. If $a = 0$, the graph reduces to the horizontal line $y = c$. As $|a|$ increases, the slope at any given point $x = x_0$ increases in magnitude and the graph becomes steeper. As $|a|$ decreases, the slope at x_0 decreases in magnitude and the graph rises or falls more gradually.

    (b) Increasing b shifts the graph to the left; decreasing b shifts it to the right.

    (c) Increasing c shifts the graph upward; decreasing c shifts it downward.

27. If $m > 0$, the x-intercept of $y = mx + 2$ must be negative. If $m < 0$, then the x-intercept exceeds $\frac{1}{2}$ $\Rightarrow 0 = mx + 2$ and $x > \frac{1}{2} \Rightarrow x = -\frac{2}{m} > \frac{1}{2} \Rightarrow 0 > m > -4$.

28. Each of the triangles pictured has the same base $b = v\Delta t = v(1 \text{ sec})$. Moreover, the height of each triangle is the same value h. Thus $\frac{1}{2}$ (base)(height) $= \frac{1}{2}\, bh = A_1 = A_2 = A_3 = \ldots$. In conclusion, the object sweeps out equal areas in each one second interval.



29. (a) By Exercise #95 of Section 1.2, the coordinates of P are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$. Thus the slope of OP $= \frac{\Delta y}{\Delta x} = \frac{b/2}{a/2} = \frac{b}{a}$.

(b) The slope of AB $= \frac{b-0}{0-a} = -\frac{b}{a}$. The line segments AB and OP are perpendicular when the product of their slopes is $-1 = \left(\frac{b}{a}\right)\left(-\frac{b}{a}\right) = -\frac{b^2}{a^2}$. Thus, $b^2 = a^2 \Rightarrow a = b$ (since both are positive). Therefore, AB is perpendicular to OP when $a = b$.

NOTES: