$$f_X(x) = \frac{1}{b-a}$$

$$Var(X) = \int_a^b x^2 \frac{1}{b-a}\,dx - \left(\int_a^b x \frac{1}{b-a}\,dx\right)^2$$

$$= \frac{1}{b-a}\frac{1}{3}x^3\Big|_a^b - \left(\frac{1}{b-a}\frac{1}{2}x^2\Big|_a^b\right)^2$$

$$= \frac{b^3-a^3}{3(b-a)} - \left(\frac{a+b}{2}\right)^2 \qquad \frac{b^2-a^2}{2(b-a)}$$

$$= \frac{a^2+ab+b^2}{3} - \frac{a^2+2ab+b^2}{4}$$

$$= \frac{(a-b)^2}{12}$$

$4a^2+4ab+4b^2-3a^2-6ab-3b^2$.

$a^2-2ab+b^2$

$$\begin{array}{r} b^2+ab+a^2 \\ b-a\,\overline{)\,b^3-a^3} \\ b^3-ab^2 \\ \hline ab^2-a^3 \\ ab^2-a^2b \\ \hline a^2b-a^3 \\ a^2b-a^3 \\ \hline 0 \end{array}$$