

Exercises week 4.

1. Use the following data

	X	C	Y	n
1	0	0	0	80
2	0	0	1	20
3	0	1	0	20
4	0	1	1	10
5	1	0	0	80
6	1	0	1	20
7	1	1	0	80
8	1	1	1	40

a. Calculate the observed risk on  $Y=1$  for those with  $X=1$  versus those with  $X=0$ . Also calculate the difference in risk. Why can you not interpret this risk as a causal effect of  $X$  on  $Y$ ?

There are in total  $80+20+80+40=220$  people with  $X=1$ , of which  $20+40=60$  with  $Y=1$ . Therefore  $P(Y=1|X=1) = 60/220 = 0.273$

$P(Y=1|X=0) = 30/130 = 0.231$ .

The risk difference is 0.042. This is not a causal effect, because there is confounding. People with  $C=1$  more often receive  $X=1$ , and have a worse outcome.

b. Calculate using standardization via outcome modeling:

- $E(C = c, X = x)$ , for  $x=0,1, c=0,1$

$$E(C = 0, X = 0) = 20/100 = 0.2$$

$$E(C = 0, X = 1) = 20/100 = 0.2$$

$$E(C = 1, X = 0) = 10/30 = 0.33$$

$$E(C = 1, X = 1) = 40/120 = 0.33$$

- $P(C = 1)$  and  $P(C = 0)$

$$P(C = 1) = 150/350 = 0.43$$

$$P(C = 0) = 200/350 = 0.57$$

- Estimate  $E(Y(x))$ , for  $X=0,1$

Under consistency and conditional exchangeability:

$$E(Y(x)) = E_C E(C, X=x) = E(Y | C=0, X=x)P(C=0) + E(C=1, X=x)P(C=1)$$

Therefore

$$E(Y(0)) = E(Y | C=0, X=0)P(C=0) + E(C=1, X=0)P(C=1) = 0.2*0.57 + 0.33*0.43 = 0.26$$

$$E(Y(1)) = E(Y | C=0, X=1)P(C=0) + E(C=1, X=1)P(C=1) = 0.2*0.57 + 0.33*0.43 = 0.26$$

- Estimate the ATE (risk difference,) and the ATT (effect in the treated)  
The ATE = 0.26-0.26 = 0

For the ATT, we average only over those in the treated (exposed) group. That means that

$$E(Y(t) | t=1) = E_{C|X=1} E(C, X=x) =$$

$$E(Y | C=0, X=x)P(C=0|X=1) + E(C=1, X=x)P(C=1|X=1)$$

Therefore instead of using P(C=0) and P(C=1) we use P(C=0|X=1) and P(C=1|X=1)

$$P(C=1|X=1) = 120/220 = 0.55$$

$$P(C=0|X=1) = 100/220 = 0.45$$

$$E(Y(0)|X=1) = E(Y | X=0, C=0)P(C=0|X=1) + E(C=1, X=0)P(C=1|X=1) = 0.2*0.45 + 0.33*0.55 = 0.2715$$

$$E(Y(1)|X=1) = E(Y | C=0, X=1)P(C=0|X=1) + E(C=1, X=1)P(C=1|X=1) = 0.2*0.45 + 0.33*0.55 = 0.2715$$

The ATT is also equal to 0.