3.3.1 a. $P=(\vec{x}_1,\vec{x}_2)=\begin{pmatrix}2 & -1\\ 3 & 1\end{pmatrix}$

b. $P=(\vec{x}_1,\vec{x}_2)=\begin{pmatrix}1 & -4\\ 1 & 1\end{pmatrix}$ ✓

c. $P=(\vec{x}_1,\vec{x}_2,\vec{x}_3)=\begin{pmatrix}0 & 1 & 4\\ 1 & 0 & 0\\ 0 & 1 & 5\end{pmatrix}$

d. P DNE ✓

e. P DNE

f. P DNE ✓

3.3.8 a. $P^{-1}=\frac{1}{-3}\begin{pmatrix}-2 & 5\\ 1 & -1\end{pmatrix}$ $P^{-1}AP=\frac{1}{3}\begin{pmatrix}-2 & 5\\ 1 & -1\end{pmatrix}\begin{pmatrix}6 & -5\\ 2 & -1\end{pmatrix}\begin{pmatrix}1 & 5\\ 1 & 2\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 4\end{pmatrix}$

$\therefore A^n=P\begin{pmatrix}1 & 0\\ 0 & 4\end{pmatrix}P^{-1}=\begin{pmatrix}1 & 5\\ 1 & 2\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 4^n\end{pmatrix}\begin{pmatrix}-2/3 & 5/3\\ 1/3 & -1/3\end{pmatrix}=\begin{pmatrix}1 & 5\times4^n\\ 1 & 2\times4^n\end{pmatrix}\frac{1}{3}\begin{pmatrix}-2 & 5\\ 1 & -1\end{pmatrix}=$

$\frac{1}{3}\begin{pmatrix}5\times4^n-2 & -5\times4^n+5\\ 2\times4^n-2 & -2\times4^n+5\end{pmatrix}$

b. $P^{-1}=\begin{pmatrix}-3 & -4\\ -2 & -3\end{pmatrix}$, $P^{-1}AP=\begin{pmatrix}-3 & -4\\ -2 & -3\end{pmatrix}\begin{pmatrix}-7 & -12\\ 6 & -10\end{pmatrix}\begin{pmatrix}-3 & 4\\ 2 & -3\end{pmatrix}=\begin{pmatrix}161 & -204\\ 120 & -178\end{pmatrix}$??

$A^n=P\begin{pmatrix}161 & -204\\ 120 & -178\end{pmatrix}^n P^{-1}$

3.3.9 a. $C_x(A)=\begin{vmatrix}x-1 & -3\\ 0 & x-2\end{vmatrix}=(x-1)(x-2)=0\Rightarrow x=1,2\Rightarrow\lambda_1=1,\lambda_2=2$

$A_{2\times2}$ has 2 distinct eigenvalues, so $A_{2\times2}$ is diagonalizable.

$C_x(B)=\begin{vmatrix}x-2 & 0\\ 0 & x-1\end{vmatrix}=(x-2)(x-1)=0\Rightarrow x=2\text{ or }1\Rightarrow\lambda_1=1,\lambda_2=2$

$B_{2\times2}$ has 2 distinct eigenvalues, so $B_{2\times2}$ is diagonalizable.

$AB=\begin{pmatrix}2 & 3\\ 0 & 2\end{pmatrix}$, $C_x(AB)=\begin{vmatrix}x-2 & -3\\ 0 & x-2\end{vmatrix}=(x-2)^2=0\Rightarrow x=2\Rightarrow\lambda_1=\lambda_2=2$

$(\lambda_1 I-AB)\vec{x}=\vec{0}\Rightarrow\begin{pmatrix}0 & -3\\ 0 & 0\end{pmatrix}\vec{x}=0\Rightarrow\vec{x}=t\begin{pmatrix}1\\ 0\end{pmatrix}\Rightarrow\vec{x}_1=\begin{pmatrix}1\\ 0\end{pmatrix}$

The eigenvalue 2 of multiplicity 2 doesn't yeild 2 eigenvectors, so AB is not diagonalizable.

5.5.1 a. $|A|=-3$, $|B|=2$, $|A|\neq|B|\Rightarrow A\sim B$ is false

b. $|A|=-5$, $|B|=-1$. $|A|\neq|B|\Rightarrow A\sim B$ is false. ✓

c. $|A|=-3$, $|B|=-3$, $tr(A)=1$, $tr(B)=2$, $tr(A)\neq tr(B)\Rightarrow A\sim B$ is false.

d. $|A|=7$, $|B|=7$, $tr(A)=5$, $tr(B)=4$, $tr(A)\neq tr(B)\Rightarrow A\sim B$ is false. ✓