Exercises Survival Analysis Lecture 1

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1 Introduction

Discuss the following study protocols. Is survival analysis needed for their analysis? If yes, determine the outcome and relevant time scale.

Note that these examples are only meant to acquire some intuition, a more formal approach will be given later in the course.

1. In a randomized clinical trial, one group of patients receives a new treatment and the other group placebo. We want to know if the treatment prolongs survival. Yes
2. The performance of several hospitals is compared. One of the indicators is the proportion of patients dying due to short-term surgery-related complications. No
3. Survival after diagnosis of breast cancer is studied. Patients who have a distant metastasis are considered as censored. Yes
4. A study is designed to investigate the relationship between the duration of breast-feeding and several explanatory variables. Yes
5. A cohort of children is followed to study when they can identify at least 10 letters correctly. They are followed between their 5th and 7th birthday. During this period they are tested every 3 months. For practical reasons, the children are only followed as long as they attend one of the participating schools (all normal primary schools). Yes
6. Consider the figure (appeared in a published article). It compares the survival of larynx carcinoma patients with and without a secondary tumor. Time is measured since detection of the first tumor. Do you see anything contra-intuitive in the graph? Yes

2 Parametric models

$f(x) = \lambda e^{-\lambda x}$ $F(x) = 1 - e^{-\lambda x}$, $S(x) = 1 - F(x) = e^{-\lambda x}$

1. The lifetime of rats follows an exponential distribution with a hazard rate of 1/3 per week. Write down a formula for the survival function from 0 to 6 weeks, sketch it and determine the mean survival.

   $h(x) = \frac{f(x)}{S(x)} = \lambda = \frac{1}{3}$ $S(x) = e^{-\frac{1}{3}x}$ $f(x) = \lambda e^{-\lambda x} = \frac{1}{3} e^{-\frac{1}{3}x}$ $E(x) = \frac{1}{\lambda} = 3$

2. The time in days to development of a tumor follows a Weibull distribution with $\gamma = 1.1$ and $\lambda = 0.05$. Calculate the hazard and survival functions at 1 day and 1 week.

   $S(x) = e^{-0.05x^{1.1}}$

   $h(x) = 1.1 \times 0.05 x^{1.1-1} = 0.055x^{0.1}$

   $S(1) = e^{-0.05 \times 1^{1.1}} = 0.9512$

   $h(1) = 0.055 \times 1^{0.1} = 0.055$

   $S(7) = e^{-0.05 \times 7^{1.1}} = 0.6536$

   $h(7) = 0.055 \times 7^{0.1} = 0.067$

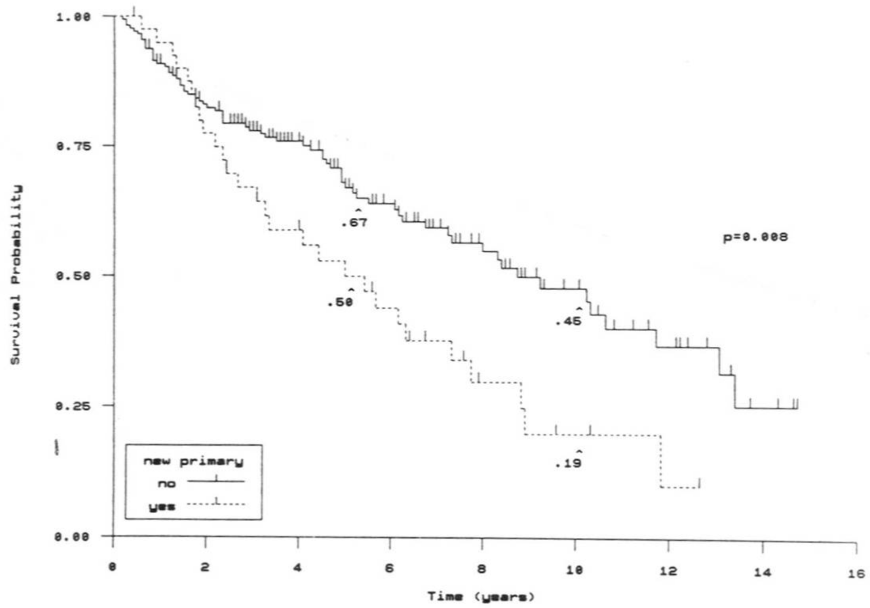

Fig. 3. The actuarial survival for patients who did (41) or did not (177) develop an SMN.

3 Exercises from Klein & Moeschberger

1. Solve exercises 2.1, 2.2, 2.6, 2.7 and 2.9.

   (a) For exercise 2.1: compute the mean lifetime, the median and the probability that a light bulb will still function after 2.000 hours. Check your results with R.

   (b) For the remaining exercises use only R commands.

   (c) Make graphs of all relevant hazard and survival functions.

2.1 (a) $E(X) = \frac{1}{0.001} = 1000$

(b) $Median(X) = 693.1472$

(c) $P(X > 2000) = 0.1353$

2.2 (a) $S(x) = e^{-\lambda x^{\alpha}}$

$F(x) = 1 - e^{-\lambda x^{\alpha}}$

$f(x) = -e^{-\lambda x^{\alpha}}(-\lambda \alpha x^{\alpha-1}) = \lambda \alpha x^{\alpha-1} e^{-\lambda x^{\alpha}}$

$f(x) = \begin{cases} \frac{\alpha}{\sigma}\left(\frac{x}{\sigma}\right)^{\alpha-1} e^{-\left(\frac{x}{\sigma}\right)^{\alpha}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ $\quad \alpha\left(\frac{1}{\sigma}\right)^{\alpha} x^{\alpha-1} e^{-\left(\frac{x}{\sigma}\right)^{\alpha}}$ $\quad \left(\frac{1}{\sigma}\right)^{\alpha} = \lambda$

$\sigma = \lambda^{-\frac{1}{\alpha}}$

$S(30) = 0.4066$

$S(45) = 0.1320$

$S(60) = 0.0273$

(b) $E(X) = 28.03$

(c) $h(x) = \frac{f(x)}{S(x)} = \frac{\lambda \alpha x^{\alpha-1} e^{-\lambda x^{\alpha}}}{e^{-\lambda x^{\alpha}}} = \lambda \alpha x^{\alpha-1}$

$h(30) = 0.06$, $h(45) = 0.09$, $h(60) = 0.12$

(d) $Median(X) = 26.32$

2.6 (a) $P(X>12)=S(12)=0.4661$

(b) $P(X\le 6)=1-S(6)=0.1300$

(c) $\text{Median}(X)=11.6339$

2.7 (a) $P(X>18)=S(18)=0.3027$

(b) $P(X\le 12)=F(12)=0.4303$

(c) $E(X)=15$

2.9 (a) $\ln(X)=\begin{cases} 2.5+2W, & Z=1 \\ 2+2W, & Z=0 \end{cases}$

$P(X>60\mid Z=1)=P(\ln(X)>\ln 60\mid Z=1)=P(2.5+2W>\ln 60)=P(W>\frac{\ln 60-2.5}{2})=0.213$

$P(X>60\mid Z=0)=P(\ln(X)>\ln 60\mid Z=0)=P(2+2W>\ln 60)=P(W>\frac{\ln 60-2}{2})=0.148$

(b) $P(X>60\mid Z=1)=P(W>\frac{\ln 60-2.5}{2})=0.311$

$P(X>60\mid Z=0)=P(W>\frac{\ln 60-2}{2})=0.260$