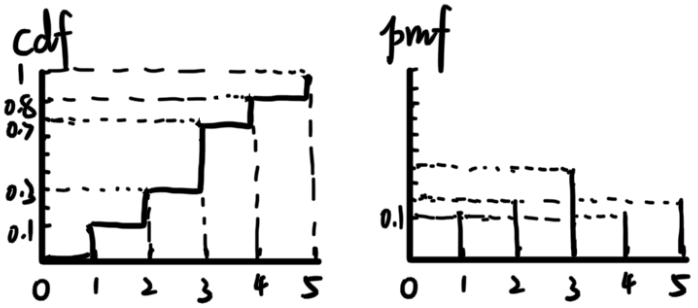
Probability Exercises Lecture 4

1. $\sum_{i=1}^{5} P(X=i) = P(X=1)+\cdots+P(X=5) = C+\cdots+5C = 15C = 1,\ C = \frac{1}{15}$

2.



3. (a) $P(X=9) = 1-\frac{1}{3}-\frac{1}{4}-\frac{1}{6}-\frac{1}{6} = \frac{1}{12}$

(b) $F_X(5) = P(X\le 5) = P(X=3)+P(X=4) = \frac{1}{3}+\frac{1}{4} = \frac{7}{12}$

(c) $P(4\le X\le 8) = P(X=4)+P(X=7)+P(X=8) = \frac{1}{4}+\frac{1}{6}+\frac{1}{6} = \frac{7}{12}$

(d) $P(X\ge 8) = P(X=8)+P(X=9) = \frac{1}{6}+\frac{1}{12} = \frac{1}{4}$?? a25

4. $E(\text{Age}) = 18\times\frac{20}{50}+19\times\frac{22}{50}+20\times\frac{4}{50}+21\times\frac{3}{50}+25\times\frac{1}{50}$

$= 18.92$

5. $E(X) = 0\times 0+1\times 0.1+2\times 0.2+3\times 0.4+4\times 0.1+5\times 0.2 = 3.1$

6. (a) $E(X) = 0\times\frac{1}{2}+1\times\frac{3}{8}+2\times\frac{1}{8} = \frac{5}{8}$

(b) $E(Y) = 0^2\times\frac{1}{2}+1^2\times\frac{3}{8}+2^2\times\frac{1}{8} = \frac{7}{8}$

7. $E(Z) = E\left(\frac{X-\mu}{\sigma}\right) = \frac{E(X)-\mu}{\sigma} = 0$

$Var(Z) = Var\left(\frac{X-\mu}{\sigma}\right) = \frac{Var(X)}{\sigma^2} = 1$

8. $cdf(x) = \begin{cases} \frac{1}{2}, X=0 \\ 1, X=1 \end{cases} = \begin{cases} 0, x<0 \\ 1-p, 0\le x<1 \\ p, x\ge 1 \end{cases}$

9. (a) $P(X=3) = \binom{2000}{3}0.001^3\,0.999^{1997} \approx \frac{2^3}{3!}e^{-2} = 0.18$ Poisson(2000×0.001) = Poisson(2)

(b) $P(X>2) = 1-P(X=0)-P(X=1) = 1-0.999^{2000}-\binom{2000}{1}0.001^1\,0.999^{1999}$ $-P(X=2)$

$1-e^{-2}-2e^{-2}-2e^{-2} = 1-5e^{-2}$

10. (a) $0.99^4\times 0.01 = 0.0096$

(b) 几何分布 $\frac{1}{p} = \frac{1}{0.01} = 100$

11. $\binom{10^5}{k}\times 0.001^k\times 0.999^{10^5-k} \approx \frac{100^k}{k!}e^{-100}$

$k=0,\ \binom{10^5}{0}\times 0.001^0\times 0.999^{10^5} \approx e^{-100}$

$k=1,\ \binom{10^5}{1}\times 0.001^1\times 0.999^{10^5-1} \approx 100e^{-100}$

$k=2,\ \binom{10^5}{2}\times 0.001^2\times 0.999^{10^5-2} \approx 5000e^{-100}$

12. $P(X\le 4) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 0.2+0.8\times 0.2+0.8^2\times 0.2+0.8^3\times 0.2+0.8^4\times 0.2$

$= 0.672$ 0.5904