Universiteit Leiden
University College The Hague

Leiden University

2023

Block 1

Mathematics for Statisticians

Level:	100
Credits:	5 ECTS
Class meeting times:	Thursday 13.15-16.45
Room:	Snellius 312
Course sections/instructors:	Garnet Akeyr

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Office hours, location: Tuesday 13.00-15.00 Snellius 213 (Please only use this time for course-related questions – for study advisor questions, please schedule a separate appointment!)

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INTRODUCTION

In this course we will learn the foundational calculus that is needed for any statistician. The material taught will come up repeatedly in future courses in the Statistics and Data Science programme.

After a brief review of basic functions, sets, and important mathematical notation, students will learn about continuity of functions and then what it means for a function to be differentiable. We'll continue to discuss differentiation and how it's used to explain important properties of functions such as maxima/minima and the shape of graphs. We'll conclude this first part by looking at approximations of functions and briefly discuss Taylor's theorem.

The second part is focused on integration. We'll learn what an integral is and techniques of integration, and relate the integral to derivatives via the Fundamental Theorem of Calculus.

We will conclude by briefly looking at multivariate calculus, i.e. calculus of functions with more than one variable. This is needed in the most elementary applications in statistics, such as finding the line of best fit of a data set. We'll cover partial derivatives and how they can inform us about local maxima/minima.

COURSE LEARNING OUTCOMES

1. Students will be able to use elementary functions and mathematical notation in applied problems, where they directly model a scenario using mathematics and use the equations to understand more about the scenario.

2. Students will understand the meaning of the derivative and be able to compute derivatives of diverse functions. Students can use the derivative to find maxima and minima of functions, including in applied scenarios, and can use the derivative to draw functions.

3. Students will be able to approximate functions using Taylor polynomials and use Taylor's theorem to interpret how good an approximation is.

4. Students will understand the meaning of the integral and be able to compute integrals using various techniques. Students can use the integral in applied scenarios to find the total rate of change of something via the Net Change Theorem.

5. Students will be able to compute partial derivatives of multivariate functions. They will be able to find the local minima/maxima of multivariate functions and use this to be able to compute a line of best fit of a small data set.

TEACHING METHODS

This course is taught in-person. Students will have 3 hours of lectures and 1 hour of tutorials each week. It is expected that students go through a weekly reading before each lecture and that they are active in their learning process via trying practice problems and keeping up with the readings.

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LITERATURE AND COURSE MATERIALS

Course notes, plus additional optional readings from:

Openstax calculus book Volume 1 (and later Volume 2)

ASSESSMENT

The final grade is the best of the following marking schemes:

Scheme 1: Exam: 100%

Scheme 2: Exam: 70%, Homework: 30%

Scheme 3: Exam: 50%, Midterm: 30%, Homework: 20%

WEEKLY OVERVIEW

Each session has an associated chapter in the notes, along with practice problems. There are also additional recommended practice problems from the Openstax calculus book.

WEEK 1 (September 7th)

Session 1 *Introduction, sets, review of elementary functions*

We spend some time reviewing basic functions – polynomials, exponential, and logarithmic functions. We also discuss sets and introduce important notation.

Optional reading: Chapter 1 of the Openstax calculus book – just review the parts you feel are necessary

Additional exercises: Chapter 1 Openstax calculus book review : 315, 317, 319, 325, 329

Exercises:

Openstax calculus section 2.2: 46 – 54,

Openstax calculus section 2.3 : 93, 95, 97, 103, 105, 107, 109

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Week 2 (September 14th)

Session 2 *Limits and Continuity, Derivatives*

We discuss the concept of a limit in broad strokes, which naturally leads into continuity. Armed with these definitions, we look at slopes of lines and tangent lines, and the proper definition of a derivative.

Optional reading: Openstax calculus book 2.2-2.4, 3.1

Exercises:

Openstax calculus section 2.2: 46 – 54,

Openstax calculus section 2.3 : 93, 95, 97, 103, 105, 107, 109

Openstax calculus section 2.4 : 131, 133, 139, 141, 145

Openstax calculus section 3.1 : 21, 23, 25

Openstax calculus section 3.2: 64-67, 78-80 (these are purely visual problems and can help you understand how continuity and differentiability are related

Week 3 (September 21st)

Session 5 *Derivative rules, Taylor approximations*

We study the rules of differentiation, which is how most derivatives are found analytically (as opposed to the definition of the derivative). We then expand on the notion of the tangent line as an approximation of a function, to look at higher-order approximations, viz. Taylor polynomials.

Optional reading: Openstax calculus book 3.3, 3.6, 3.9

Openstax calculus Volume II, section 6.3 (only read 'Taylor's Theorem with Remainder')

Additional exercises: Openstax calculus section 3.3 : 107-117 (odd only)

Openstax calculus section 3.6 : 229, 231, 239

Openstax calculus section 3.9 : 331-341 (odd only), 345
Openstax calculus Volume II, section 6.3 : 117, 121, 123, 135

Week 4 (September 28th)

Session 4 *Drawing graphs using derivatives, optimisation problems*

Having covered the basics of derivatives, we now turn to applications thereof. Namely, how can we use the derivative of a function to study the graph of a function, and, perhaps more importantly, how can we use derivatives to solve optimization problems.

Optional reading: Openstax calculus book 4.5

Optional reading: Openstax calculus book 4.7

Additional exercises: Openstax calculus section 4.5 : 225, 227, 231-245 (odd only)

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Openstax calculus section 4.7 : 315, 317, 319, 321, 335, 337

Week 5 (October 5th)

Session 5 *Definite and Indefinite integrals*

We turn now to the other half of calculus, namely integration. We shall study how we can find the area under a graph and see how this is fundamentally related to differentiation! Integration forms a fundamental part of probability theory, for example in how any probability density function must have an area of 1 underneath it over all of its domain.

Optional reading: Openstax calculus book 4.10

Openstax calculus book 5.2, 5.3, 5.4

Additional exercises: Openstax calculus section 4.10 : 465, 467, 469, 477, 499, 500, 502, 503, 509, 511, 513

Openstax calculus section 5.3 : 153, 157, 159, 171-181 (odd only)

Openstax calculus section 5.4 : 223-227 (odd only)

Week 6 (October 12th)

Session 6 *Integration by substitution, multivariate functions*

This session has two topics. First, we'll conclude our discussion on integrals by looking at integration by substitution. Integrals have a *lot* of different techniques, but substitution is by far the most common. After this, in the second half we'll look at multivariate functions. Inevitably, real-world problems have multiple variables – a study of the health of a population might look at things like height, weight, age, etc. We'll introduce the necessary terminology, then introduce partial derivatives and tangent planes, which are higher-dimensional analogues of derivatives and tangent lines.

Optional reading: Openstax calculus book 5.5

Additional exercises: Openstax calculus section 5.5 : 261-267 (odd only), 271, 273, 293

Week 7 (October 19th)

Session 7 *Gradients and extreme points*

We can finally study two topics of key importance in statistics, namely gradients and extreme points. The applications of the gradient will be covered in later courses, but that of extreme points has a very crucial application – that of the line of best fit!

Reading: Chapter 11 of the notes

Optional reading: Openstax calculus Volume III section 4.6, 4.7

Additional exercises: Openstax calculus Volume III section 4.6 : 263, 265, 267, 271, 281

Openstax calculus Volume III section 4.7 : 311, 323, 325, 333, 349