

$$2.1.3 \text{ a. } 3A - 2B = 3 \cdot \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} - 2 \cdot \begin{pmatrix} 3 & -1 & 2 \\ 0 & 1 & 4 \end{pmatrix} \text{ DNE}$$

$$\text{b. } 5C = 5 \cdot \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 15 & -5 \\ 10 & 0 \end{pmatrix}$$

$$\text{c. } 3E^T = 3 \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}^T = 3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \\ 3 & 0 \end{pmatrix}$$

$$\text{d. } B + D = \begin{pmatrix} 3 & -1 & 2 \\ 0 & 1 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ -1 & 0 \\ 1 & 4 \end{pmatrix} \text{ DNE}$$

$$\text{e. } 4A^T - 3C = 4 \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}^T - 3 \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix} = 4 \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix} - 3 \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ -2 & -4 \end{pmatrix}$$

$$\text{f. } (A+C)^T = \left[\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix} \right]^T = \begin{pmatrix} 5 & 0 \\ 2 & -1 \end{pmatrix}^T = \begin{pmatrix} 5 & 2 \\ 0 & -1 \end{pmatrix}$$

$$\text{g. } 2B - 3E = 2 \cdot \begin{pmatrix} 3 & -1 & 2 \\ 0 & 1 & 4 \end{pmatrix} - 3 \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 1 \\ 0 & -1 & 8 \end{pmatrix}$$

$$\text{h. } A - D = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ -1 & 0 \\ 1 & 4 \end{pmatrix} \text{ DNE}$$

$$\text{i. } (B - 2E)^T = \left[\begin{pmatrix} 3 & -1 & 2 \\ 0 & 1 & 4 \end{pmatrix} - 2 \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \right]^T = \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 4 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ 0 & 4 \end{pmatrix}$$

$$2.1.5 \text{ a. } A + B = 3A + 2B \Rightarrow -2A = B \Rightarrow A = -\frac{1}{2}B$$

$$\text{b. } 2A - B = 5(A + 2B) \Rightarrow -3A = 11B \Rightarrow A = -\frac{11}{3}B$$

$$2.3.1 \text{ a. } \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix} = \left[\begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right] = \left[\begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \end{pmatrix} \right] = \begin{pmatrix} 2 & 2 \\ 0 & -2 \end{pmatrix}$$

$$\text{c. } \begin{pmatrix} 5 & 0 & -7 \\ 1 & 5 & 9 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 22 \\ -1 \end{pmatrix}$$

$$\text{d. } \begin{pmatrix} 1 & 3 & -3 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ -2 & 1 \\ 0 & 6 \end{pmatrix} = \left[\begin{pmatrix} 1 & 3 & -3 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 3 & -3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix} \right] = (-3, -15)$$

$$\text{f. } \begin{pmatrix} 1 & -1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix} = -23$$

$$\text{g. } \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix} \begin{pmatrix} 1 & -1 & 3 \end{pmatrix} = \left[\begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix} \cdot 1, \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix} \cdot (-1), \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix} \cdot 3 \right] = \begin{pmatrix} 2 & -2 & 6 \\ 1 & -1 & 3 \\ -7 & 7 & -21 \end{pmatrix}$$

$$2.3.2 \text{ b. } A^2 = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix} \text{ DNE}$$

$$AB = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix}^{2 \times 3} \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix}^{2 \times 2} \text{ DNE}$$

$$AC = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix}^{2 \times 3} \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix}^{3 \times 2} = \left[\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right] = \left[\begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 10 \\ -1 \\ -1 \end{pmatrix} \right] = \begin{pmatrix} 4 & 10 \\ -2 & -1 \end{pmatrix}$$

$$BA = \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix}^{2 \times 2} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix}^{2 \times 3} = \left[\begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} \right] = \left[\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -10 \\ -4 \\ -10 \end{pmatrix} \right] = \begin{pmatrix} -1 & -4 & -10 \\ 1 & 2 & 4 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix}^{2 \times 2} \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix}^{2 \times 2} = \left[\begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 6 \\ 0 \end{pmatrix} \right] = \left[\begin{pmatrix} 7 \\ -1 \end{pmatrix}, \begin{pmatrix} -6 \\ 6 \end{pmatrix} \right] = \begin{pmatrix} 7 & -6 \\ -1 & 6 \end{pmatrix}$$

$$BC = \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix}^{2 \times 2} \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix}^{3 \times 2} \text{ DNE}$$

$$CA = \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix}^{3 \times 2} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix}^{2 \times 3} = \left[\begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} \right] = \left[\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 8 \\ -5 \\ 2 \end{pmatrix} \right] = \begin{pmatrix} 2 & 4 & 8 \\ -1 & -1 & -5 \\ 1 & 4 & 2 \end{pmatrix}$$

$$CB = \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix}^{3 \times 2} \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix}^{2 \times 2} = \left[\begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 0 \end{pmatrix} \right] = \left[\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} \right] = \begin{pmatrix} -2 & 12 \\ 2 & -6 \\ 1 & 6 \end{pmatrix}$$

$$C^2 = \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix}^{3 \times 2} \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix}^{3 \times 2} \text{ DNE}$$

$$2.3.4 \text{ a. } \begin{pmatrix} 3 & -1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 0 & -2 \end{pmatrix} - \begin{pmatrix} 3 & -1 \\ 0 & -2 \end{pmatrix} - 6 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 9 & -1 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 3 & -1 \\ 0 & -2 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\text{b. } \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix} - 6 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 2 \\ 2 & 5 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$2.3.5 \text{ c. } (CD)^T = \left[\begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 5 & 8 \end{pmatrix} \begin{pmatrix} 3 & -1 & 2 \\ 1 & 0 & 5 \end{pmatrix} \right]^T = \begin{pmatrix} 3 & -1 & 2 \\ 7 & -2 & 9 \\ 23 & -5 & 50 \end{pmatrix}^T = \begin{pmatrix} 3 & 7 & 23 \\ -1 & -2 & -5 \\ 2 & 9 & 50 \end{pmatrix}$$

$$D^T C^T = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 0 & 5 \end{pmatrix}^T \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 5 & 8 \end{pmatrix}^T = \begin{pmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 8 \end{pmatrix} = \begin{pmatrix} 3 & 7 & 23 \\ -1 & -2 & -5 \\ 2 & 9 & 50 \end{pmatrix}$$

$$(CD)^T = D^T C^T$$

$$1.1.1 \text{ a. } \begin{cases} 2x + 3y + z = 5 \\ 5x + 7y - 4z = 0 \end{cases}$$

$$(A|b) = \left(\begin{array}{ccc|c} 2 & 3 & 1 & 5 \\ 5 & 7 & -4 & 0 \end{array} \right) \xrightarrow{R_1 \times \frac{1}{2}} \left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{1}{2} & \frac{5}{2} \\ 5 & 7 & -4 & 0 \end{array} \right) \xrightarrow{R_2 - 5R_1} \left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & -\frac{1}{2} & -\frac{13}{2} & -\frac{25}{2} \end{array} \right)$$

$$\xrightarrow{-2R_2} \left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & 1 & 13 & 25 \end{array} \right) \xrightarrow{R_1 - \frac{3}{2}R_2} \left(\begin{array}{ccc|c} 1 & 0 & -19 & -35 \\ 0 & 1 & 13 & 25 \end{array} \right)$$

$$\begin{cases} x - 19z = -35 \\ y + 13z = 25 \end{cases} \Rightarrow \begin{cases} x = 19t - 35 \\ y = 25 - 13t \\ z = t \end{cases}$$

b. $\begin{cases} 2x_1 + 5x_2 + 9x_3 + 3x_4 = -1 \\ x_1 + 2x_2 + 4x_3 = 1 \end{cases}$

$$\begin{cases} x_1 = 2s + 12t + 13 \\ x_2 = s \\ x_3 = -5 - 3t - 3 \\ x_4 = t \end{cases} \Rightarrow \begin{cases} 2(2s + 12t + 13) + 5s + 9(-5 - 3t - 3) + 3t = 26 - 27 = -1 \\ 2s + 12t + 13 + 2s + 4(-5 - 3t - 3) = 13 - 12 = 1 \end{cases}$$

1.1.4 $(4 \ -2 \ 0 \ | \ 3) \xrightarrow{\frac{1}{4}R_1} (1 \ -\frac{1}{2} \ 0 \ | \ \frac{3}{4}) \Rightarrow x - \frac{1}{2}y = \frac{3}{4} \Rightarrow \begin{cases} x = \frac{1}{2}t + \frac{3}{4} \\ y = t \\ z = s \end{cases}$

1.1.7 c. $\left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 1 & 0 & -1 & 1 \\ 2 & 1 & 0 & 0 \end{array} \right)$ d. $\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & 2 \end{array} \right)$

1.1.8 a. $\begin{cases} x - y + 6z = 0 \\ y = 3 \\ 2x - y = 1 \end{cases}$ b. $\begin{cases} 2x - y = -1 \\ -3x + 2y + z = 0 \\ y + z = 3 \end{cases}$

1.2.1 reduced row-echelon:

row-echelon: c.d.e

1.2.3 a. $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 3 & -1 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow \begin{cases} x_1 + 2x_2 + 3x_4 + x_5 = -1 \\ x_3 - x_4 + x_5 = 2 \\ x_6 = 3 \end{cases} \Rightarrow \begin{cases} x_1 = -2u - 3s - t - 1 \\ x_2 = u \\ x_3 = 5 - t + 2 \\ x_4 = s \\ x_5 = t \\ x_6 = 3 \end{cases}$

1.2.3 b. $\left(\begin{array}{cccc|c} 1 & -2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 5 & -3 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow \begin{cases} x_1 - 2x_2 + 2x_4 + x_6 = 1 \\ x_3 + 5x_4 - 3x_6 = -1 \\ x_5 + 6x_6 = 1 \end{cases} \Rightarrow \begin{cases} x_1 = 2u - 2s - t + 1 \\ x_2 = u \\ x_3 = -5s + 3t - 1 \\ x_4 = s \\ x_5 = 1 - 6t \\ x_6 = t \end{cases}$

1.2.4 a. $\begin{cases} x - 2y = 1 \\ -x + 4y = -2 \end{cases} (A|b) = \left(\begin{array}{cc|c} 1 & -2 & 1 \\ -1 & 4 & -2 \end{array} \right) \xrightarrow{R_2 + R_1} \left(\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 2 & -1 \end{array} \right) \xrightarrow{\frac{1}{2}R_2} \left(\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & -1/2 \end{array} \right) \xrightarrow{R_1 + 2R_2} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -1/2 \end{array} \right) \Rightarrow \begin{cases} x = 0 \\ y = -1/2 \end{cases}$

d. $\begin{cases} 3x - y = 2 \\ -6x + 2y = -4 \end{cases} (A|b) = \left(\begin{array}{cc|c} 3 & -1 & 2 \\ -6 & 2 & -4 \end{array} \right) \xrightarrow{\frac{1}{3}R_1} \left(\begin{array}{cc|c} 1 & -1/3 & 2/3 \\ -6 & 2 & -4 \end{array} \right) \xrightarrow{R_1 + 6R_2} \left(\begin{array}{cc|c} 1 & -1/3 & 2/3 \\ 0 & 0 & 0 \end{array} \right) \Rightarrow$

$$x - \frac{1}{2}y = \frac{2}{3} \Rightarrow \begin{cases} x = \frac{1}{2}t + \frac{2}{3} \\ y = t \end{cases}$$

$$f. \begin{cases} 2x - 3y = 5 \\ -2x + 3y = 2 \end{cases} \quad (A|b) = \begin{pmatrix} 2 & -3 & 5 \\ -2 & 3 & 2 \end{pmatrix} \xrightarrow{\pm R_1} \begin{pmatrix} 1 & -3/2 & 5/2 \\ -2 & 3 & 2 \end{pmatrix} \xrightarrow{R_2 + 2R_1} \begin{pmatrix} 1 & -3/2 & 5/2 \\ 0 & 0 & 7 \end{pmatrix} \Rightarrow$$

$$\begin{cases} x - \frac{1}{2}y = \frac{2}{3} \\ 0 = 7 \end{cases} \Rightarrow \text{no solution}$$

$$1.2.5 \ a. \begin{pmatrix} 1 & 1 & 2 & 8 \\ 3 & -1 & 1 & 0 \\ -1 & 3 & 4 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 21 \\ 0 & 0 & 1 & -12 \end{pmatrix} \Rightarrow \begin{cases} x = 11 \\ y = 21 \\ z = -12 \end{cases}$$

$$b. \begin{pmatrix} -2 & 3 & 3 & -9 \\ 3 & -4 & 1 & 5 \\ -5 & 7 & 2 & -14 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 15 & -21 \\ 0 & 1 & 11 & -17 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x + 15z = -21 \\ y + 11z = -17 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} x = -15t - 21 \\ y = -11t - 17 \\ z = t \end{cases}$$

$$d. \begin{pmatrix} 1 & 2 & -1 & 2 \\ 2 & 5 & -3 & 1 \\ 1 & 4 & -3 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & -11/3 \\ 0 & 1 & -1 & 5/3 \\ 0 & 0 & 0 & 2/3 \end{pmatrix} \Rightarrow \begin{cases} x + z = -11/3 \\ y - z = 5/3 \\ 0 = 2/3 \end{cases} \Rightarrow \text{no solution}$$

$$f. \begin{pmatrix} 3 & -2 & 1 & -2 \\ 1 & -1 & 3 & 5 \\ -1 & 1 & 1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 1 \end{pmatrix} \Rightarrow \begin{cases} x = -7 \\ y = -9 \\ z = 1 \end{cases}$$

$$1.2.7 \ a. \begin{pmatrix} 3 & 8 & -3 & -14 & 2 \\ 2 & 3 & -1 & -2 & 1 \\ 1 & -2 & 1 & 10 & 0 \\ 1 & 5 & -2 & -12 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0.14 & 3.71 & 0.29 \\ 0 & 1 & -0.43 & -3.14 & 0.14 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x_1 + 0.14x_3 + 3.71x_4 = 0.29 \\ x_2 - 0.43x_3 - 3.14x_4 = 0.14 \\ 0 = 0 \\ 0 = 0 \end{cases}$$

$$\begin{cases} x_1 = -0.14s - 3.71t + 0.29 \\ x_2 = 0.43s + 3.14t + 0.14 \\ x_3 = s \\ x_4 = t \end{cases}$$

$$b. \begin{pmatrix} 1 & -1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 + x_4 = 0 \\ x_3 = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = -t \\ x_3 = 0 \\ x_4 = t \end{cases}$$

$$c. \begin{pmatrix} 1 & -1 & 1 & -2 & 1 \\ -1 & 1 & 1 & 1 & -1 \\ -1 & 2 & 3 & -1 & 2 \\ 1 & -1 & 2 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$