3.4

get data: library(KMsurv)

data(drug6mp)

Exact event times and right-censored times present in data:

$$L = \prod_{i=1}^{n} h(t_i)^{\delta_i} S(t_i) = \prod_{i=1}^{n} \lambda^{\delta_i} \exp(-\lambda t_i)$$

$$= \lambda^{\Sigma \delta_i} \cdot \exp(-\lambda \Sigma t_i)$$

$$\log(L) = \sum_{i=1}^{n} \delta_i \log(\lambda) - \lambda \sum_{i=1}^{n} t_i$$

$$\frac{\partial}{\partial \lambda} \log(L) = \frac{\Sigma \delta_i}{\lambda} - \sum_{i=1}^{n} t_i$$

$$\frac{\partial}{\partial \lambda} \log(L) = 0$$

$$\Leftrightarrow \frac{\Sigma \delta_i}{\lambda} - \sum_{i=1}^{n} t_i = 0$$

$$\Leftrightarrow \frac{\Sigma \delta_i}{\lambda} = \sum_{i=1}^{n} t_i$$

$$\Leftrightarrow \lambda = \frac{\Sigma \delta_i}{\sum^{n} t_i}$$

- Use "t2" as time variable and "relapse" as status variable.

MLE:

```
λ <- sum(drug6mp$relapse)/sum(drug6mp$t2)
```

Scanned by CamScanner

3.7

a) · left truncation at 50 years
· interval censoring (1 year intervals)
· right censoring at end of study

b)
$$f(x) = \lambda \alpha x^{\alpha-1} \exp(-\lambda x^{\alpha})$$
$$S(x) = \exp(-\lambda x^{\alpha})$$

$$L = \prod_{i \in I} \frac{S(L_i) - S(R_i)}{S(Y_i)} \cdot \prod_{r \in R} \frac{S(t_r)}{S(Y_r)}$$

[factor $\frac{1}{S(\alpha)}$ because of left truncation]

interval censored patients

right censored patients

$$= \prod_{i \in I} \frac{\exp(-\lambda L_i^{\alpha}) - \exp(-\lambda R_i^{\alpha})}{\exp(-\lambda Y_i^{\alpha})} \prod_{r \in R} \frac{\exp(-\lambda t_r^{\alpha})}{\exp(-\lambda Y_r^{\alpha})}$$

$$= \frac{S(55)-S(56)}{S(50)} \cdot \frac{S(58)-S(59)}{S(50)} \cdot \frac{S(52)-S(53)}{S(50)} \cdot \frac{S(59)-S(60)}{S(50)}$$

interval censored contributions

$$\cdot \left(\frac{S(60)}{S(50)}\right)^4$$ } right censored contribution

3.5

Use parametrization of log logistic distribution of wikipedia

$$f(x) = \frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{(1+(x/\alpha)^{\beta})^2}$$

$$S(x) = 1 - \frac{x^\beta}{\alpha^\beta + x^\beta} = \frac{\alpha^\beta}{\alpha^\beta + x^\beta} = \frac{1}{1+\left(\frac{x}{\alpha}\right)^\beta}$$

$$L = \underbrace{\prod_{i\in D}}_{\text{exact event times}} f(x_i) \cdot \underbrace{\prod_{l\in L}}_{\text{left censored}} (1-S(C_l))$$

$$= \prod_{i\in D} \frac{(\beta/\alpha)(x_i/\alpha)^{\beta-1}}{(1+(x_i/\alpha)^{\beta})^2} \cdot \prod_{l \in L} \left(1 - \frac{1}{1+\left(\frac{C_l}{\alpha}\right)^\beta}\right)$$

$$= \frac{(\beta/\alpha)(0.5/\alpha)^{\beta-1}}{(1+(0.5/\alpha)^\beta)^2} \cdot \frac{(\beta/\alpha)(1/\alpha)^{\beta-1}}{(1+(1/\alpha)^\beta)^2} \cdot \frac{(\beta/\alpha)(0.75/\alpha)^{\beta-1}}{(1+(1/\alpha)^\beta)^2}$$

$$\cdot \left(1 - \frac{1}{1+(0.25/\alpha)^\beta}\right) \cdot \left(1 - \frac{1}{1+(1.25/\alpha)^\beta}\right)$$