Probability Exercises Lecture 3 II

1. a. $P(A)\times\frac{3}{5}+P(B)\times\frac{2}{7}=\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{2}{7}=\frac{31}{70}$

b. C = red drawn $P(A|C)=\frac{P(AC)}{P(C)}=\frac{3/10}{31/70}=\frac{3\times70}{31\times10}=\frac{21}{31}$

2. $P(\text{male})=0.7$, $P(\text{female})=0.3$,

$P(A)=P(A\cap\text{male})+P(A\cap\text{female})=P(\text{male})P(A|\text{male})+P(\text{female})P(A|\text{female})$

$=0.7\times0.05+0.3\times0.10$

$=0.065$

$P(\text{male}|A)=\frac{P(\text{male}\cap A)}{P(A)}=\frac{0.7\times0.05}{0.065}=\frac{0.035}{0.065}=\frac{7}{13}$

3. $P(2H)=\frac{1}{3}$, $P(2T)=\frac{1}{3}$, $P(H+T)=\frac{1}{3}$

(a) $P=\frac{P(2H)\times1}{P(2H)\times1+P(2T)\times0+P(H+T)\times\frac{1}{2}}=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}}=\frac{2}{3}$

(b) $P=\frac{2}{3}\times1+\frac{1}{3}\times\frac{1}{2}=\frac{5}{6}$

3. H: coin 2H. T: coin 2T, F: coin H+T

R: result H

$P(H)=P(T)=P(F)=\frac{1}{3}$

$P(R|H)=1$, $P(R|T)=0$, $P(R|F)=\frac{1}{2}$

(a) $P(H|R)=\frac{P(H\cap R)}{P(R)}=\frac{P(R|H)\cdot P(H)}{P(R|H)\cdot P(H)+P(R|T)\cdot P(T)+P(R|F)\cdot P(F)}$

$=\frac{1\times\frac{1}{3}}{1\times\frac{1}{3}+0\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{3}}=\frac{2}{3}$

(b) R_1: first head, R_2: second head

$P(R_1\cap R_2)=P(R_1\cap R_2|H)P(H)+P(R_1\cap R_2|T)P(T)+P(R_1\cap R_2|F)P(F)$

$=1\times\frac{1}{3}+0\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{3}=\frac{5}{12}$

$P(R_1)=P(R_1|H)\cdot P(H)+P(R_1|T)\cdot P(T)+P(R_1|F)\cdot P(F)$

$=1\times\frac{1}{3}+0\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{3}=\frac{1}{2}$

$P(R_2|R_1)=\frac{\frac{5}{12}}{\frac{1}{2}}=\frac{5}{6}$

4. $P(H\geq1)=1-P(H=0)=1-(\frac{1}{2})^{10}=\frac{2^{10}-1}{2^{10}}$