# Exercises Lecture 2: Probability - Part I

1. The weather forecast says that the probability of rain on Saturday is 25% and the probability of rain on Sunday is 25%. Is the probability of rain during the weekend 50% or is it $> 50\%$ or $< 50\%$? Why or why not?

   *[Hint: Rain during the weekend can happen in 3 ways: either it rains only on Saturday, either it rains only on Sunday either it rains on both days.]*

   **Solution:**
   Rain during weekend can happen in 3 ways: either it rains on Saturday $(A)$, either it rains on Sunday $(B)$ either it rains both days $(C)$. Thus, by applying the addition law: $P(\text{ rain on weekend}) = P(A) + P(B) - P(C) \leq 0.5$ because $P(C) \geq 0$.

2. Simpson's Paradox

   - A black urn contains 5 red and 6 green balls, and a white urn contains 3 red and 4 green balls. You are allowed to choose an urn and then choose a ball at random from the urn. If you choose a red ball, you get a prize. Which urn should you choose to draw from?

     **Solution:**
     If you draw from the black urn, the probability of choosing a red ball is $5/11 = 0.455$ (the number of ways you can draw a red ball divided by the total number of outcomes). If you choose to draw from the white urn, the probability of choosing a red ball is $3/7 = 0.429$, so you should choose to draw from the black urn.

   - Now consider another game in which a second black urn has 6 red and 3 green balls, and a second white urn has 9 red and 5 green balls. If you choose a red ball, you get a prize. Which urn should you choose to draw from?

     **Solution:**
     If you draw from the black urn, the probability of a red ball is $6/9 = 0.667$, whereas if you choose to draw from the white urn, the probabil-

ity is $9/14 = 0.643$. So, again you should choose to draw from the black urn.

- In the final game, the contents of the second black urn are added to the first black urn, and the contents of the second white urn are added to the first white urn. Again, you can choose which urn to draw from. Which should you choose?

**Solution:**

The black urn now contains 11 red and 9 green balls, so the probability of drawing a red ball from it is $11/20 = 0.55$. The white urn now contains 12 red and 9 green balls, so the probability of drawing a red ball from it is $12/21 = 0.571$. So, you should choose the white urn. This seems a counter intuitive result but it's an example of Simpson's paradox, which states that a trend that appears in different groups of data disappears when these groups are combined, and the reverse trend appears for the aggregate data.