Probability Exercises Lecture 6

1.(a) $P(Y=2 \text{ or } 3 \cap X=1)$

$= P(Y=2 \cap X=1) + P(Y=3 \cap X=1)$

$= 0.166 + 0.107$

$= 0.273$

(b) $P(X=0)$

$= \sum_{i=1}^{4} P(X=0 \cap Y=i)$

$= 0.120 + 0.087 + 0.146 + 0.160$

$= 0.513$

2.(a) $P(X=1) = 0.1 + 0.05 + 0.02 + 0.02 = 0.19$

$P(X=2) = 0.05 + 0.2 + 0.05 + 0.02 = 0.32$

$P(X=3) = 0.02 + 0.05 + 0.2 + 0.04 = 0.31$

$P(X=4) = 0.02 + 0.02 + 0.04 + 0.1 = 0.18$

$P(Y=1) = 0.1 + 0.05 + 0.02 + 0.02 = 0.19$

$P(Y=2) = 0.05 + 0.2 + 0.05 + 0.02 = 0.32$

$P(Y=3) = 0.02 + 0.05 + 0.2 + 0.04 = 0.31$

$P(Y=4) = 0.02 + 0.02 + 0.04 + 0.1 = 0.18$

(b) $P(X<Y) = P(X=1 \cap Y=2) + P(X=1 \cap Y=3) + P(X=2 \cap Y=3)$
$+ P(X=1 \cap Y=4) + P(X=2 \cap Y=4) + P(X=3 \cap Y=4)$

$= 0.05 + 0.02 + 0.05 + 0.02 + 0.02 + 0.04$

$= 0.2$

3.(a) $P(X=3, Y=6) = P(X=3)P(Y=6) = 0.2 \times 0.3 = 0.06$

(b) $P(X \leq 3, Y \leq 6) = P(X \leq 3)P(Y \leq 6) = 0.3 \times 0.6 = 0.18$

4.

$y \backslash x$	0	1	2	$f_Y(y)$
0	0.03	0.15	0.12	0.3

1	0.04	0.2	0.16	0.4
2	0.03	0.15	0.12	0.3
$f_X(x)$	0.1	0.5	0.4	1

5. $E(XY)=E(X)E(Y)=\frac{0+1}{2}\times\frac{5+9}{2}=\frac{7}{2}$

6. $Cov(X+Y, X-Y)$

$=E[(X+Y)(X-Y)]-E(X+Y)E(X-Y)$

$=E(X^2-Y^2)-[E^2(X)-E^2(Y)]$

$=E(X^2)-E^2(X)-[E(Y^2)-E^2(Y)]$

$=D(X)-D(Y)$

$=0$

7.

X \ Y	0	1	2
0	0.3	0.1	0.01
1	0.1	0.3	0.06
2	0.01	0.05	0.07

$E(X)=0\times0.41+1\times0.46+2\times0.13=0.72$

$E(Y)=0\times0.41+1\times0.45+2\times0.14=0.73$

$E(XY)=0\times0.52+1\times0.3+2\times0.11+4\times0.07=0.8$

$Cov(X,Y)=E(XY)-E(X)E(Y)=0.8-0.72\times0.73=0.2744$