

$$8.6.2 \ A = P \Sigma_A Q^T \Rightarrow \Sigma_A = P^T A Q \Rightarrow \Sigma_A^T = (P^T A Q)^T = Q^T A^T P = \Sigma_{A^T} \text{ for } A^T, \text{ use } Q \text{ as } P, P \text{ as } Q$$

$$8.6.3 \ \text{rank}(A) = m = n$$

$$8.6.7. \because A = P \Sigma_A Q^T \therefore A^{-1} = (P \Sigma_A Q^T)^{-1} = (Q^T)^{-1} \Sigma_A^{-1} P^{-1} = Q \Sigma_A^{-1} P^T \therefore \Sigma_{A^{-1}} = \Sigma_A^{-1} \checkmark$$

$$8.6.11 \because A = U \Sigma V^T \therefore A^T = (U \Sigma V^T)^T = V \Sigma^T U^T$$

$$8.6.4 \ a. \ A^T \text{ single values: } b_1, \dots, b_r$$

$$b. \ t b_1, \dots, t b_r \checkmark$$

$$c. \ \frac{1}{b_1}, \dots, \frac{1}{b_r}$$

$$\text{Example. 1. } \lambda_1 = 10, \lambda_2 = 3, \lambda_3 = 2, \lambda_4 = 1$$

$$b_1 = 10, b_2 = 9.15, b_3 = 2.28, b_4 = 0.29 \text{ . not same}$$

$$2. \ \Sigma_{A^T} = \begin{pmatrix} 10 & & & \\ & 9.15 & & \\ & & 2.28 & \\ & & & 0.29 \end{pmatrix}, \ \Sigma_{A^T} = (\Sigma_A)^T = \Sigma_A$$

$$3. \ \Sigma_{A^{-1}} = \begin{pmatrix} 3.48 & & & \\ & 0.44 & & \\ & & 0.11 & \\ & & & 0.1 \end{pmatrix}, \ \text{diag}(\Sigma_{A^{-1}}) = \frac{1}{\text{diag}(\Sigma_A)}$$