Calculus Exercise Week 5

Section 4.10

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513

465. $F(x)=5x^3+2x^2+3x+1$

$F'(x)=5\cdot 3x^2+2\cdot 2x+3=15x^2+4x+3=f(x)$

467. $F(x)=x^2e^x$

$F'(x)=2xe^x+x^2e^x=e^x(2x+x^2)=f(x)$

469. $F(x)=e^x,\ F'(x)=e^x=f(x)$

477. $f(x)=(\sqrt{x})^3$

$\int f(x)\,dx=\int(\sqrt{x})^3\,dx=\int x^{\frac{3}{2}}\,dx=\frac{2}{5}x^{\frac{5}{2}}+C$

499. $f'(x)=x^{-3},\ f(1)=1$

$f(x)=\int f'(x)\,dx=\int x^{-3}\,dx=-\frac{1}{2}x^{-2}+C$

$\because f(1)=-\frac{1}{2}+C=1 \quad \therefore C=\frac{3}{2}$

$\therefore f(x)=-\frac{1}{2}x^{-2}+\frac{3}{2}$

500. $f'(x)=\sqrt{x}+x^2,\ f(0)=2$

$f(x)=\int f'(x)\,dx=\int \sqrt{x}+x^2\,dx=\frac{2}{3}x^{\frac{3}{2}}+\frac{1}{3}x^3+C$

$\because f(0)=C=2 \quad \therefore C=2$

$\therefore f(x)=\frac{2}{3}x^{\frac{3}{2}}+\frac{1}{3}x^3+2$

502. $f'(x)=x^3-8x^2+16x+1,\ f(0)=0$

$f(x)=\int f'(x)\,dx=\int x^3-8x^2+16x+1\,dx=\frac{1}{4}x^4-\frac{8}{3}x^3+8x^2+x+C$

$\because f(0)=C=0 \quad \therefore C=0$

$\therefore f(x)=\frac{1}{4}x^4-\frac{8}{3}x^3+8x^2+x$

503. $f'(x)=\frac{2}{x^2}-\frac{x^2}{2},\ f(1)=0$

$f(x)=\int f(x)\,dx=\int \frac{2}{x^2}-\frac{x^2}{2}\,dx=-\frac{2}{x}-\frac{1}{6}x^3+C$

$\because f(1)=-2-\frac{1}{6}+C=0 \quad \therefore C=\frac{13}{6}$

$\therefore f(x)=-\frac{2}{x}-\frac{1}{6}x^3+\frac{13}{6}$

153

157

159

171~181

509. $V(t) = 40 - 10t$ $\quad 40\times 1.47 - 10t$

$1 \text{ mile} = 5280 \text{ feet}$

$1 \text{ m/h} = 1.47 \text{ f/s}$

$V(t) = 0 \Rightarrow t = 4$ $\quad t = 5.88$

$\cancel{\int_0^4 V(t)\,dt = \int_0^4 40 - 10t\,dt = (40t - 5t^2)\Big|_0^4 = 80}$

511. $V(t) = 12t$

$V(t) = 60 \Rightarrow t = 5$ $\quad 60\times 1.47 \quad t = 7.35$

$\cancel{\int_0^5 V(t)\,dt = \int_0^5 12t\,dt = (6t^2)\Big|_0^5 = 150}$

513. dece $= a$

$V(t) = 25 - at$ $\quad 25\times 1.47$

$V(8) = 0 \Rightarrow a = \cancel{\frac{25}{8}}$ $\quad 13.78$

$25 - 8a = 0$

$a = \frac{25}{8} \times 1.47$

Section 5.3

153. $\frac{d}{dx}\int_0^{\sqrt{x}} t\,dt$

$= \sqrt{x} \cdot \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{1}{2}$

157. $\frac{d}{dx}\int_1^{x^2} \frac{\sqrt{t}}{1+t}\,dt$

$= \frac{|x|}{1+x^2} \cdot 2x$

$= \cancel{\frac{2x^2}{1+x^2}}$ $\quad \frac{2x|x|}{1+x^2}$

159. $\frac{d}{dx}\int_1^{e^x} \ln u^2\,du$

$= \ln e^{2x} \cdot e^x$

$= 2xe^x$

171 $\int_{-2}^{3} (x^2 + 3x - 5)\,dx$

$= \left(\frac{1}{3}x^3 + \frac{3}{2}x^2 - 5x\right)\Big|_{-2}^{3}$

$= \left(9 + \frac{27}{2} - 15\right) - \left(-\frac{8}{3} + 6 + 10\right)$

$= -\frac{35}{6}$

173 $\int_2^3 (t^2 - 9)(4 - t^2)\,dt$

$= \int_2^3 -t^4 + 13t^2 - 36\,dt$

$= \left(-\frac{1}{5}t^5 + \frac{13}{3}t^3 - 36t\right)\Big|_2^3$

$= -\frac{1}{5}\cdot 3^5 + \frac{13}{3}3^3 - 36\cdot 3 - \left(-\frac{1}{5}\cdot 2^5 + \frac{13}{3}\cdot 2^3 - 36\cdot 2\right)$

$= \frac{62}{15}$

175 $\int_0^1 x^{99}\,dx$

$= \frac{1}{100}x^{100}\Big|_0^1$

$= \frac{1}{100}$

177 $\int_{\frac{1}{4}}^4 \left(x^2 - \frac{1}{x^2}\right) dx$ $\qquad \frac{602 - 36\times 15}{15}$

$= \left[\frac{1}{3}x^3 - \left(-\frac{1}{x}\right)\right]\Big|_{\frac{1}{4}}^4$

$= \frac{1}{3}\cdot 4^3 + \frac{1}{4} - \left[\frac{1}{3}\cdot\left(\frac{1}{4}\right)^3 + 4\right]$

$= \frac{1125}{64}$

179 $\int_1^4 \frac{1}{2\sqrt{x}}\,dx$

$= x^{\frac{1}{2}}\Big|_1^4$

$= \sqrt{4} - \sqrt{1}$

$= 1$ $\qquad \frac{1}{2}x^{-\frac{1}{2}}$

181 $\int_1^{16} \frac{1}{t^{1/4}}\,dt$ $\qquad x^{\frac{1}{2}} \qquad \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{4}{3}t^{\frac{3}{4}}\Big|_1^{16}$

$= \frac{4}{3}\cdot 16^{\frac{3}{4}} - \frac{4}{3}$ $\qquad t^{-\frac{1}{4}}$

$= \frac{28}{3}$ $\qquad \left(t^{\frac{3}{4}}\right)' = \frac{3}{4}t^{-\frac{1}{4}}$

Section 5.4 $\qquad 2^{4\cdot 3}$

223-227

223 $\int_0^t 4 - 2t\,dt$ $\qquad (16^3)^{\frac{1}{4}}$ $\qquad 2^3 \qquad 8$

$= (4t - t^2)\Big|_0^t$

$= -t^2 + 4t$

$t = 2 \Rightarrow \int_0^2 4 - 2t\,dt = -2^2 + 4\cdot 2 = 4$

225 $\int_0^t |2t-6|\,dt = \begin{cases} \int_0^t -(2t-6)\,dt, & t \le 3 \\ \int_0^3 -(2t-6)\,dt + \int_3^t 2t - 6\,dt, & t > 3 \end{cases}$

$$= \begin{cases} -t^2+6t, & t\le 3 \\ t^2-6t+18, & t>3 \end{cases}$$

$$t=6 \Rightarrow \int_0^6 (2t-6)\,dt = 6^2-6\times6+18=18$$

227. $V(t)=40-9.8t$

$$h(t) = 1.5 + \int_0^t V(t)\,dt$$

$$= 1.5 + \int_0^t 40-9.8t\,dt$$

$$= 1.5 + \left(40t - \frac{9.8}{2}t^2\right)\Big|_0^t$$

$$= -4.9t^2+40t+1.5$$