MATHEMATICS FOR STATISTICIANS HOMEWORK 1

[35/35] Please upload a typewritten or (legible!) scanned copy of your homework on Brightspace. Collaboration is encouraged, but please ensure you state at the start of your homework with whom you've collaborated. For full marks you must provide a justification of your answers, i.e. only a (correct) answer without any work shown will not be considered sufficient.

1) [14/14]
   (a) [4/4] Rewrite the following expression into a single logarithm with a coefficient of 1:
   $$2\log_3(x+y) + 6\log_3(x) - \frac{1}{3}.$$
   (b) A quantity of a radioactive substance is modelled by the equation $Q(t) = Q_0 e^{kt}$, where t is time and Q_0 is the initial quantity. The constant k depends on the substance in question. A given radioactive substance begins with 2kg, and after 1750 years, there is 1.5kg remaining.
      (i) [4/4] Determine the exponential decay equation for this substance. i.e. What are Q_0 and k? Leave your answer in exact form.
      (ii) [3/3] How long will it take for half of the substance to decay? Leave your answer in exact form.
      (iii) [3/3] How long will it take until there is only 250g remaining? Leave your answer in exact form.
2) [15/15] Determine the following limits. You do *not* have to use the epsilon-delta definition.
   (a) [5/5] $\lim\limits_{x\to\infty} \frac{3x^4+6x^2+3x+12}{-2x^4-6x^3+\pi x-27} + 2^{-x} + 3$
   (b) [5/5] $\lim\limits_{x\to 3} \frac{1}{(x-3)^3}$
   (c) [5/5] $\lim\limits_{x\to 2} \frac{(x^2-x-2)(x^2-5x+6)}{x^2-4x+4}$
3) [6/6] Using the definition of the derivative as a limit, compute the derivative of
(a) [3/3] $f(x) = \frac{1}{x+1}$
(a) [3/3] $h(t) = 2t^2 + 2t$ at the point $t = 7$.