Probability Practice Exam 2022

1. X = which coin Y = head count

$P(X=ht)=0.5,\ P(X=hh)=0.5$

$$P(X=ht\mid Y=3)=\frac{P(X=ht,\ Y=3)}{P(Y=3)}=\frac{P(Y=3\mid X=ht)P(X=ht)}{P(Y=3\mid X=ht)P(X=ht)+P(Y=3\mid X=hh)P(X=hh)}$$

$$=\frac{0.5^3\times0.5}{0.5^3\times0.5+1\times0.5}=\frac{1}{9}$$

2.a. $P=\dfrac{A_6^4}{6^4}=\dfrac{5}{18}$

b. $P=\dfrac{\cancel{C_{90}^{8}}\ C_{98}^{10}}{C_{100}^{12}}=\cancel{\dfrac{33}{18620}}$

c. $P=\dfrac{A_{10}^{10}/(A_3^3A_7^7)}{A_{12}^{12}/(A_5^5\cdot A_7^7)}=\dfrac{A_{10}^{10}}{A_3^3A_7^7}\times\dfrac{A_5^5A_7^7}{A_{12}^{12}}=\dfrac{5}{33}$

3. $P(A)=0.5,\ P(B)=0.2,\ P(AB)=0.1$

$P(A\cup B)=P(A)+P(B)-P(AB)=0.6$

4. X = Positive/N, Y = have the disease

$P(X=P\mid Y=T)=0.9,\ P(X=P\mid Y=F)=0.1,\ P(Y=T)=0.0001$

$$P(Y=T\mid X=P)=\frac{P(X=P,\ Y=T)}{P(X=P)}=\frac{P(X=P\mid Y=T)P(Y=T)}{P(X=P\mid Y=T)P(Y=T)+P(X=P\mid Y=F)P(Y=F)}$$

$$=\frac{0.9\times0.0001}{0.9\times0.0001+0.1\times0.9999}$$

$$=\frac{9}{10008}$$

5. $Z\sim N(0,1)$

(a) $P(|Z|<c)=P(-c<Z<c)=\phi(c)-\phi(-c)=2\phi(c)-1=1-a$

$\phi(c)=\dfrac{2-a}{2},\ c=\phi^{-1}\left(\dfrac{2-a}{2}\right)$

(b) $a=0.05,\ c=\phi^{-1}\left(\dfrac{2-0.05}{2}\right)=\phi^{-1}(0.975)=1.96$

6. $X\sim N(68,2^2)$

(a) $P(66\le X\le 72)=P\left(\dfrac{66-68}{2}\le\dfrac{X-68}{2}\le\dfrac{72-68}{2}\right)=\phi(2)-\phi(-1)=\phi(2)+\phi(1)-1$

$=0.819$

(b) $X_{0.75}$: $P(X\le X_{0.75})=P\left(\dfrac{X-68}{2}\le\dfrac{X_{0.75}-68}{2}\right)=\phi\left(\dfrac{X_{0.75}-68}{2}\right)=0.75$

$\frac{x_{0.75}-68}{2} = \phi^{-1}(0.75)$, $x_{0.75} = 2\phi^{-1}(0.75)+68 = 2\times0.68+68 = 69.36$

$x_{0.25} = 2\phi^{-1}(0.25)+68 = 2\times(-0.68)+68 = 66.64$

$x_{0.5} = 2\phi^{-1}(0.5)+68 = 2\times0+68 = 68$

7. $X \sim exp(0.01)$ $\lambda = \frac{1}{100}$

$P(50 \le X \le 200) = F_X(200) - F_X(50) = (1-e^{-0.01\times200}) - (1-e^{-0.01\times50})$

$= e^{-0.5} - e^{-2} = 0.471$

8. $X \sim Pois(2)$, $\lambda = 2$

(a) $P(X=0) = \frac{2^0}{0!}e^{-2} = e^{-2} = 0.135$

(b) $Y \sim Pois(4)$, $\lambda = 4$

$P(Y\le3) = \sum_{i=0}^{3} P(Y=i) = \sum_{i=0}^{3} \frac{4^i}{i!}e^{-4} = e^{-4} + 4e^{-4} + 8e^{-4} + \frac{32}{3}e^{-4}$

$= 0.433$

9. a. the probability that the spinner stops between 2 and 5

b. $P(Y=5) = 0$

c. $Y \sim Uni(0,12)$

$P(5<Y<6) = F_Y(6) - F_Y(5) = \frac{6-0}{12-0} - \frac{5-0}{12-0} = \frac{1}{12}$

$P(5\le Y<6) = F(5<Y<6) = \frac{1}{12}$

10. a. $P(X=9) = 1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{6} - \frac{1}{6} = \frac{1}{12}$

b. $E(X) = \frac{1}{3}\times3 + \frac{1}{4}\times4 + \frac{1}{6}\times7 + \frac{1}{6}\times8 + \frac{1}{12}\times9 = 5.25$

$E(X^2) = \frac{1}{3}\times9 + \frac{1}{4}\times16 + \frac{1}{6}\times49 + \frac{1}{6}\times64 + \frac{1}{12}\times81 = 32.583$

$Var(X) = E(X^2) - [E(X)]^2 = 5.02$