Probability Exercises Lecture 8

1. (a) $\bar{X} \sim N(37, \frac{4^2}{25}) = N(37, 9^2)$

(b) $P(\bar{X} > 43.1) = P(\frac{\bar{X}-37}{9} > 0.67) = 1 - \Phi(0.68) = 0.24825$

2. (a) $\bar{X} = \frac{1}{30}\sum_{i=1}^{30} X_i$

$E(\bar{X}) = \mu = 0.3$

$Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{0.05^2}{30} = 0.000083$ $\sqrt{Var(\bar{X})} = \frac{\sigma}{\sqrt{n}} = \frac{0.05}{\sqrt{30}} = 0.00913$

(b) $P(\bar{X} > 0.31) = P(\frac{\bar{X}-0.3}{0.05/\sqrt{30}} > 1.10) = 1 - \Phi(1.10) = 0.13567$

3. $P(X > 10200) = 1 - P(X \le 10200)$

$= 1 - \sum_{i=0}^{10200} P(X = i)$

?? $P(X > 10200) = P(\frac{X-10000}{100} > 2)$

$= 1 - \Phi(2) = 0.02275$

10000个 Poisson(1) 相加 $X \sim Poisson(10000) \sim N(10000, 10000)$

4. (1) $E(X_i) = E(U_i - 0.5) = E(U_i) - 0.5 = \frac{1}{2} - 0.5 = 0$

$Var(X_i) = Var(U_i - 0.5) = Var(U_i) = \frac{1}{12}$

(2) $\bar{X} = \frac{1}{12}\sum_{i=1}^{12} X_i$

$E(\bar{X}) = \mu = 0$

$Var(\bar{X}) = \frac{\sigma^2}{n} = 1$

$\times 12^5$ $60 \times \frac{25}{\frac{12\times12}{2}}$ $\frac{5\times25}{12}$

5. $Y = \sum_{i=1}^{100} X_i$, $Y \sim N(100\times15, 100\times10^2) = N(1500, 10000)$

$P(Y \ge 1700) = P(\frac{Y-1500}{100} \ge 2) = 1 - \Phi(2) = 0.02275$

0.97725 $(5/12)^2$ $\frac{125}{12}$

6. $Y = \sum_{i=1}^{60} X_i$, $Y \sim N(60\times4, 60\times5^2) = N(240, 1500)$

$P(Y \ge 250) = P(\frac{Y-240}{10\sqrt{15}} \ge \frac{1}{\sqrt{15}}) = 1 - \Phi(0.26) = 0.39743$

$\frac{3.10}{\sqrt{125}}$ 3.10 0.00097

$\frac{2}{12}\frac{\sqrt{12}}{\sqrt{125}}$ $\frac{5\sqrt{5}}{24}$ $\frac{2\sqrt{n}}{\sqrt{5}}$

7. $X_i = \begin{cases} 1, & \text{suburb} \\ 0, & \text{city} \end{cases}$, $E(X_i) = 1\times0.25 + 0\times0.75 = 0.25$

$Var(X_i) = E(X_i^2) - E^2(X_i) = \frac{3}{16}$

$Y = \sum_{i=1}^{1200} X_i \sim N(1200\times0.25, 1200\times\frac{3}{16}) = N(300, 15^2)$

$P(Y < 270) = P(\frac{Y-300}{15} < -2) = 1 - \Phi(2) = 0.02275$

8. $\bar{X} = \frac{1}{125}\sum_{i=1}^{125} X_i \sim N(5, \frac{1}{25})$

$P(\bar{X} < 5.5) = P(\frac{\bar{X}-5}{0.2} < 2.5) = \Phi(2.5) = 0.99379$

9. $E(X_i)=3,\ Var(X_i)=9$

$Y=\sum_{i=1}^{16} X_i \sim N(16\times3,\ 16\times9)=N(48,12^2)$

$P(Y>60)=P(\frac{Y-48}{12}>1)=1-\Phi(1)=0.15866$

10. $X_i \sim Poisson(0.01)$

$E(X_i)=0.01,\ Var(X_i)=0.01$

$Y=\sum_{i=1}^{2000} X_i \sim N(2000\times0.01,\ 2000\times0.01)=N(20,20)$

$P(Y\geq15)=P(\frac{Y-20}{\sqrt{20}}\geq-1.12)=\Phi(1.12)=0.86864$