$\boxed{\text{Sets}}$

Definition A set is a collection of numbers, either finite and infinite

Exemple:

rational number

Remark The real numbers came up all the time and we donate them by $\mathbb{R}$

Definition A special kind of set are intervals, which are all the real numbers between 2 numbers.

For example

· an open bracket (or) indicates an endpoint is not included

$(0,1)$ 不含0和1

$[\ ,\ ]$ 闭区间

use $\pm\infty$ to donate +'ve or -'ve with open bracket

$(-\infty,+\infty)\setminus\{0\}$ R不含0

all elements of set can be written between set containing brackets

set $\{1,2,3\}$. $[0,1]$ equals to $\{x\in\mathbb{R}: 0\le x\le 1\}$ ":" such that

"$\in$" means "in" or "an elements of"

$[-20,\pi] \Rightarrow \{x\in\mathbb{R}: -20\le x<\pi\}$

$(-\infty,-4) \Rightarrow \{x\in\mathbb{R}: -\infty<x<-4\} \Rightarrow \{x\in\mathbb{R}: x<-4\}$ $x\in\mathbb{R}$可省略

$\boxed{\text{Fuctions}}$

Def: A function is a map between 2 sets that assigns a unique output to each input

The domain of a fuction is the set of all valid inputs

The range of a fuction is the set of all valid outputs

Notation we often call names like f or g

$f: x \to x^2$ sends x to x^2, $f(x) = x^2$

Domain $D(f)$, Range $R(f) = \{\ \}$

$f(x) = x^2$, $D(f) = \mathbb{R}$, $R(f) = \{x \in \mathbb{R} : x \geq 0\}$ 

$f(x) = \sqrt{x}$, $D(f) = \{x \geq 0\}$, $R(f) = \{x \geq 0\}$ 

$g(t) = \frac{1}{t}$, $D(g) = \{t \neq 0\}$, $R(g) = \{t \neq 0\}$ 

$h(t) = \sqrt{t-1}$ $D(h) = \{t \geq 1\}$, $R(h) = \{t \geq 0\}$

Defination Let $f(x)$ and $g(x)$ be 2 func : the range of g is contained in domain of f

$(f \circ g)(x) = f(g(x))$

$f(x) = x^2 + 1$, $g(x) = x - 2$

$(f \circ g)(x) = (x-2)^2 + 1$

$(g \circ f)(x) = x^2 + 1 - 2 = x^2 - 1$

Defination: If f and g are two fuctions, such that $f(g(x)) = x$ and $g(f(x)) = x$, then we say f and g are inverse func, and write $g = f^{-1}$

$f(x) = x^2$, $g(x) = \sqrt{x}$

$f(g(x)) = (\sqrt{x})^2 = x$, $g(f(x)) = \sqrt{x^2} = |x|$

Function Review

Polynomial: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

$D(f) = \mathbb{R}$, $a_n \neq 0$ highest power = degree

A polynomial of deg 1 is a line

$f(x) = mx + b$

m: slope of line, b: intercept

A polynomial of deg 2 is a quadratic (2项式)

$f(x_i)=ax_i^2+bx_i+c$

parabola goes up if $a>0$

goes down if $a<0$

roots/zeroes: $f(x)=0$ 的 x_i

$x_i=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $b^2-4ac=0$, 1 root

$b^2-4ac>0$, 2 roots

$b^2-4ac<0$, 0 root

$f(x)=x_i^2+x_i-6$ $\frac{-1\pm\sqrt{1^2-4\times1\times(-6)}}{2\times1}=\frac{-1\pm5}{2}=2/-3$

$f(x)=-x^2-1$ $\frac{\pm\sqrt{0-4\times(-1)\times(-1)}}{2\times(-1)}$ 0 root

线性的

Definition A func f is one-to-one if $f(a)=f(b)$ implies $a=b$

A func is onto or surjective if the func achives every value in the range 满射的 值域是R

Exponential Function

Let $b>0$, $b\neq1$, be a real number

$f(x)=b^x$, $b=3$, 

$b=\frac{1}{3}$ 

In general, $b^0=1$, $\forall\, b>0$

$b^x\neq0$, $\forall\, b>0$

$b^x>0$, $\forall\, b>0$

$R(b^x)>0$, $D(b^x)=\mathbb{R}$

if $b>1$, $x\to\infty\Rightarrow f(x)\to\infty$, $x\to-\infty\Rightarrow f(x)\to0$

if $0<b<1$, $x\to\infty\Rightarrow f(x)\to0$, $x\to-\infty\Rightarrow f(x)\to\infty$

$b^{x_1}b^{x_2}=b^{x_1+x_2}$

$(b^{x_1})^{x_2} = b^{x_1 x_2}$

$b \approx e \approx 2.718$, e^x: natural exponential func

Logarithms: The inverses of b^x is the logarithms base b. written $\log_b(x)$

If $b = e$, $\log_b(x) = \ln(x) = \log(x)$

If $b > 1$, $\log_b(x)$ is always increasing and only

$b > 1$, $D(\log_b(x)) = \{x > 0\}$, $R = \mathbb{R}$

$\log_b x$ and b^x are inverses

$\log_b(b^x) = x$, $b^{\log_b x} = x$

$\log_b(1) = 0$

$\log_b(xy) = \log_b(x) + \log_b(y)$

$\log_b(x^r) = r \log_b(x)$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$