# MATHEMATICAL MODELLING HOMEWORK 3

[33/33] Rough grading scheme for HW3 - mathematical modelling spring 2021.

1) [9/9]
   (a) [6/6] Evaluate the integral $\int \frac{1}{x+x\sqrt{x}}dx$. You must show any substitutions, etc. to receive full points.
   (b) [3/3] Evaluate the integral $\int \frac{e^{5x}}{1+e^{5x}}dx$. You must show any substitutions, etc. to receive full points.

2) [8/8]
   (a) [3/3] Find a number $a$ such that the line $x = a$ bisects the area under the curve $y = \frac{1}{x^2}$ between $1 \leq x \leq 4$.
   (b) [5/5] Find a number $b$ such that the line $y = b$ bisects the area in part $a$).

3) [6/6] Effective temperature is used in cold regions to give an indication of the effect of the wind on how cold it feels outside. Suppose the effective temperature outside depends on both the actual temperature $T$ in $°C$ and the wind speed $v$ in $km/h$, and is given by the equation

$$W = 13.12 + 0.6215T - 11.37v^{0.16} + 0.3965Tv^{0.16}.$$

Using the partial derivatives, if $T = -15°$ and $v = 25km/h$, by how much would you expect the effective temperature to drop if the actual temperature $T$ decreased by $1°$? What if the wind speed *dropped* by $1km/h$? Compare the answers you found with the actual values for $W(-16, 25)$ and $W(-15, 24)$. Explain how you arrived at your conclusions to receive full points.

4) [10/10]

Many events are random, and a proper model requires taking into account probability. Unlike random variables with discrete probabilities, such as flipping a coin, many random variables are continuous, such as the height of a randomly selected person. Such random variables have associated probability density functions $f(x)$, which must satisfy the following:
   (a) $f(x) > 0$, and
   (b) $\int_{-\infty}^{\infty} f(x)dx = 1$.

A famous probability density function is the Gaussian, or bell-curve. We'll study another example. Suppose

$$f(t) = \begin{cases} \frac{1}{4}te^{-\frac{t}{2}} \text{ if } t \geq 0 \\ 0 \text{ if } t < 0 \end{cases}$$

is the probability density function for the lifespan of the battery in a new electric vehicle, given in years.
   (a) [3/3] Show that $f(t)$ is in fact a probability density function.

(b) [2/2] What is the probability that a battery will have a life expectancy of half a year or less?
(c) [1/1] What is the probability that a battery will have a life expectancy between 2 and 4 years?
(d) [1/1] What is the probability that a battery will have a life expectancy larger than 5 years?
(e) [3/3] What is the average expected life expectancy? Note that the average value of a random variable is found via $\int_{-\infty}^{\infty} xf(x)dx$.