

Probability Exercises Lecture 3 I

1. (a) $A = \{g | st\}$ $B = \{g | st\}$

$$P(A) = 0.5, P(B) = 0.5$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A) \cdot P(B)}{P(A)} = 0.5$$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.5 - 0.5 \times 0.5 = 0.75$

$$P(A \cap B | A \cup B) = \frac{P(A \cap B)}{P(A \cup B)} = \frac{0.5 \times 0.5}{0.75} = \frac{1}{3}$$

2. (a) $T = \{2, \dots, 12\}$

$$A = \left\{ \overset{2}{3}, \overset{4}{5}, \overset{6}{7}, \overset{8}{9}, \overset{10}{11} \right\}, P = \frac{P(\{3, 5, 7\})}{P(A)} = \frac{12/36}{18/36} = \frac{2}{3}$$

(b) $P(A) = \frac{1}{36} \times 6 = \frac{1}{6}$, $P(B) = P(\{ \overset{5}{8}, \overset{4}{9}, \overset{3}{10}, \overset{2}{11}, \overset{1}{12} \}) = \frac{15}{36} = \frac{5}{12}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/36}{15/36} = \frac{1}{5}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{3/36}{1/6} = \frac{1}{2}$$

3. $P(A) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{\frac{4!}{2!2!}}{\frac{52!}{2!50!}} = \frac{4!50!}{2!52!} = \frac{\overset{1}{4} \times \overset{1}{3} \times 2}{\underset{13}{2} \times \underset{17}{52} \times 51} = \frac{1}{221}$ 52张牌