4.1.1 a. $\sqrt{2^2+(-1)^2+2^2}=3$.

b. $\sqrt{1^2+(-1)^2+2^2}=\sqrt{6}$

c. $\sqrt{1^2+0^2+(-1)^2}=\sqrt{2}$

d. $\sqrt{(-1)^2+0^2+2^2}=\sqrt{5}$

e. $2\sqrt{1^2+(-1)^2+2^2}=2\sqrt{6}$

$49+1+25$ $\quad$ 75

f. $|-3|\sqrt{1^2+1^2+2^2}=3\sqrt{6}$

4.1.2 a. $\|\vec{v}\|=\sqrt{7^2+(-1)^2+5^2}=5\sqrt{3}$, $\frac{\vec{v}}{\|\vec{v}\|}=\begin{pmatrix}7/5\sqrt{3}\\-1/5\sqrt{3}\\1/\sqrt{3}\end{pmatrix}$

b. $\|\vec{v}\|=\sqrt{(-2)^2+(-1)^2+2^2}=3$, $\frac{\vec{v}}{\|\vec{v}\|}=\begin{pmatrix}-2/3\\-1/3\\2/3\end{pmatrix}$

4.1.21 a. $\vec{v}=\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$, $\|\vec{v}\|=\sqrt{v_1^2+v_2^2+v_3^2}\geq 0$, $\|\vec{v}\|=0$ only if $v_1=v_2=v_3=0$

b. use a, $\|\vec{v}-\vec{w}\|=0\Rightarrow\vec{v}-\vec{w}=\vec{0}\Rightarrow\vec{v}=\vec{w}$

c. $\vec{v}=-\vec{v}\Rightarrow 2\vec{v}=\vec{0}\Rightarrow\vec{v}=\vec{0}$

d. False, $\vec{v}=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\vec{w}=\begin{pmatrix}0\\1\\0\end{pmatrix}$, $\|\vec{v}\|=\|\vec{w}\|=1$, but $\vec{v}\neq\vec{w}$

e. False, $\vec{v}=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\vec{w}=\begin{pmatrix}0\\1\\0\end{pmatrix}$, $\|\vec{v}\|=\|\vec{w}\|=1$, but $\vec{v}\neq\pm\vec{w}$

f. False, $\vec{v}\cdot\vec{w}=\vec{v}\cdot t\vec{v}=t\|\vec{v}\|^2$. if $t\leq 0$, $\vec{v}\perp\vec{w}$ or $\vec{v}$ and $\vec{w}$ are in opposite direction.

g. $\vec{v}$ and $\vec{v}+\vec{w}$ are parallel $\Rightarrow\vec{v}=d(\vec{v}+\vec{w}), d\neq 0,1\Rightarrow\vec{v}=\frac{d}{1-d}\vec{w}\Rightarrow\vec{v}$ and $\vec{w}$ are parallel

h. False, $\vec{v}=\begin{pmatrix}0\\1\end{pmatrix}$, $\|-5\vec{v}\|=5\neq-5\|v\|$

i. $\|\vec{v}\|=\|2\vec{v}\|=2\|\vec{v}\|\Rightarrow\|\vec{v}\|=\frac{1}{2}\Rightarrow\vec{v}\neq\vec{0}$

j. False, $\vec{v}=\begin{pmatrix}0\\1\end{pmatrix}$, $\vec{w}=\begin{pmatrix}1\\0\end{pmatrix}$, $\|\vec{v}+\vec{w}\|=\sqrt{1^2+1^2}=\sqrt{2}$, $\|\vec{v}\|+\|\vec{w}\|=2$

4.2.1 a. $\vec{u}\cdot\vec{v}=2\times(-1)+(-1)\times 1+3\times 1=0$

$1^2+2^2+(-1)^2=6$

b. $\vec{u}\cdot\vec{v}=\|\vec{u}\|^2=1^2\cancel{+1^2+(-3)^2=11}$

c. $\vec{u}\cdot\vec{v} = 1\times2+1\times(-1)+(-3)\times1 = -2$ $\quad 18+7-25 \quad (1,0)-(0,-1) \quad (1,1)$

d. $\vec{u}\cdot\vec{v} = 3\times6+(-1)\times(-7)+5\times(-5) = 0$ ✓

e. $\vec{u}\cdot\vec{v} = ax+by+cz$ $\quad -18-2 \quad -20$

f. $\vec{u}\cdot\vec{v} = 0$ ✓

4.2.2 b. $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \dfrac{3\times(-6)+(-1)\times2+0}{\sqrt{10}\sqrt{40}} = -1,\ \theta=\pi$ ✓

d. $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \dfrac{2\times3+1\times6+(-1)\times3}{\sqrt{6}\sqrt{54}} = \dfrac{1}{2},\ \theta = \dfrac{1}{3}\pi$ ✓

$\dfrac{60}{180}$

4.2.3 a. $\vec{v}\cdot\vec{u}=0 \Rightarrow 2x+2+3=0 \Rightarrow x=-\dfrac{5}{2}$

4.2.10 a. $\mathrm{proj}_{\vec{v}}\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \dfrac{5\times2+7\times(-1)+1\times3}{2^2+(-1)^2+3^2}\begin{pmatrix}2\\-1\\3\end{pmatrix} = \begin{pmatrix}6/7\\-3/7\\9/7\end{pmatrix}$

b. $\mathrm{proj}_{\vec{v}}\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \dfrac{3\times4+(-2)\times1+1\times1}{4^2+1^2+1^2}\begin{pmatrix}4\\1\\1\end{pmatrix} = \dfrac{11}{18}\begin{pmatrix}4\\1\\1\end{pmatrix}$ ✓ $\quad \dfrac{6}{14} \quad \dfrac{3}{7}$

4.2.11 a. $\vec{u}_2 = \mathrm{proj}_{\vec{v}}\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \dfrac{2\times1+(-1)\times(-1)+1\times3}{1^2+(-1)^2+3^2}\begin{pmatrix}1\\-1\\3\end{pmatrix} = \dfrac{6}{11}\begin{pmatrix}1\\-1\\3\end{pmatrix}$

$\vec{u}_1 = \vec{u}-\vec{u}_2 = \begin{pmatrix}2\\-1\\1\end{pmatrix} - \dfrac{6}{11}\begin{pmatrix}1\\-1\\3\end{pmatrix} = \dfrac{1}{11}\begin{pmatrix}16\\-5\\-7\end{pmatrix}$

b. $\vec{u}_2 = \mathrm{proj}_{\vec{v}}\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \dfrac{3\times(-2)+1\times1+0\times4}{(-2)^2+1^2+4^2}\begin{pmatrix}-2\\1\\4\end{pmatrix} = -\dfrac{5}{21}\begin{pmatrix}-2\\1\\4\end{pmatrix}$ ✓

$\vec{u}_1 = \vec{u}-\vec{u}_2 = \begin{pmatrix}3\\1\\0\end{pmatrix} - \left(-\dfrac{5}{21}\right)\begin{pmatrix}-2\\1\\4\end{pmatrix} = \dfrac{1}{21}\begin{pmatrix}53\\26\\20\end{pmatrix}$ ✓

2.3.32 a. $0_{n\times n}^2 = 0_{n\times n},\ I^2 = I\cdot I = I$

b. $\begin{pmatrix}1&1\\0&0\end{pmatrix}\begin{pmatrix}1&1\\0&0\end{pmatrix} = \begin{pmatrix}1&1\\0&0\end{pmatrix},\ \begin{pmatrix}1&0\\1&0\end{pmatrix}\begin{pmatrix}1&0\\1&0\end{pmatrix} = \begin{pmatrix}1&0\\1&0\end{pmatrix},\ \dfrac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}\dfrac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix} = \dfrac{1}{4}\begin{pmatrix}2&2\\2&2\end{pmatrix} = \dfrac{1}{2}(1$

c. $(I-P)(I-P) = I\cdot I - I\cdot P - P\cdot I + P^2 = I - 2P + P = I - P$

$P(I-P) = P - P^2 = P - P = 0_{n\times n}$

d. $P^T\cdot P^T = (P\cdot P)^T = P^T$

e. $Q\cdot Q = (P+AP-PAP)(P+AP-PAP) = P^2+PAP-P^2AP+AP^2+APAP-AP^2AP-PAP^2-PAPAP$ $+PAP^2A$

$$= P + PAP - PAP + AP + APAP - APAP - PAP - PAPAP + PAPAP$$

$$= P + AP - PAP = Q$$

f. $BA \cdot BA = BI_nA = BA$