Probability Practice Exam 2022

1.
$$X = \text{which coin}$$
 $Y = \text{head count}$

$$P(X = \text{ht}) = 0.5, P(X = \text{hh}) = 0.5$$

$$P(X = \text{ht}) Y = 3) = \frac{P(X = \text{ht}, Y = 3)}{P(X = \text{ht})} = \frac{P(Y = 3 \mid X = \text{ht})}{P(X = \text{ht})} P(X = \text{ht}) P(X = \text{ht}) P(X = \text{ht})} = \frac{0.5^3 \times 0.5}{0.5^3 \times 0.5 + 1 \times 0.5} = \frac{1}{9}$$

$$= \frac{A^4 - 5}{0.5^3 \times 0.5 + 1 \times 0.5} = \frac{1}{9}$$

2.0.
$$P = \frac{A_{6}^{4}}{6^{4}} = \frac{5}{18}$$

b. $P = \frac{C_{10}^{10}}{C_{10}^{10}} = \frac{33}{1865}$
 $C.P = \frac{A_{10}^{10}/(A_{3}^{2}A_{7}^{2})}{A_{12}^{12}/(A_{3}^{2}.A_{7}^{2})} = \frac{A_{10}^{10}}{A_{3}^{2}A_{7}^{2}} \times \frac{A_{5}^{2}A_{7}^{2}}{A_{12}^{2}} = \frac{5}{33}$

4.
$$X = Positive/N, Y = have the disease$$

$$P(X = P | Y = T) = 0.9, P(X = P | Y = F) = 0.000)$$

$$P(Y = T | X = P) = \frac{P(X = P, Y = T)}{P(X = P)} = \frac{P(X = P | Y = T)}{P(X = P)} P(Y = T) P(Y = T) P(Y = T) P(Y = F) P(Y = F$$

$$= \frac{0.9 \times 0.000}{0.9 \times 0.000} + 0.1 \times 0.9999$$

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5. 2~N(0.1)

(a)
$$P(2|
 $\phi(c)=\frac{2-a}{2}$, $c=\phi^{-1}(\frac{2-a}{2})$$$

(b)
$$\alpha = 0.05$$
, $C = \phi^{-1}(\frac{2-0.05}{2}) = \phi^{-1}(0.975) = 1.96$

6. X~N(68,22)

(a)
$$P(66 \le X \le 72) = P(\frac{66 - 68}{2} \le \frac{X - 68}{2} \le \frac{72 - 68}{2}) = \phi(2) - \phi(-1) = \phi(2) + \phi(1) - 1$$

$$= 0.819$$

(b)
$$X_{0.75}: P(X \le X_{0.75}) = P(\frac{2}{X-68} \le \frac{2}{X_{0.75}-68}) = \Phi(\frac{2}{X_{0.75}-68}) = 0.75$$

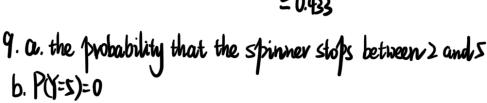
$$\frac{X_{0.75}-68}{2}=\phi^{-1}(0.75), X_{0.75}=2\phi^{-1}(0.75)+68=2\times0.68+68=69.36$$

 $X_{0.25}=2\phi^{-1}(0.25)+68=2\times(-0.68)+68=66.64$

7.
$$X \sim \exp(0.01) \lambda = \frac{1}{100}$$

 $P(x_0 \le X \le 200) = F_x(x_0) - F_x(x_0) = (1 - e^{-0.01 \times 200}) - (1 - e^{-0.01 \times 200})$
 $= e^{-0.5} - e^{-2} = 0.471$

(a)
$$P(x=0) = \frac{2^{\circ}}{\circ!} e^{-2} = e^{-2} = 0.135$$



C. Yallni(0,12)

$$P(5$$

