MFS Practice Exam 2

4) Suppose the distance between point B and point C is x km. and $f(x)$ is the total energy cost.

$$f(x) = \sqrt{5^2 + x^2} \times 1.4 + (13 - x)$$
$$= 1.4\sqrt{x^2+25} - x + 13$$

According to the problem, $x \in [0, 13]$.

To minimize $f(x)$, we need to calculate when $f'(x) = 0$.

$$f'(x) = 1.4 \times \frac{1}{2} \times \frac{1}{\sqrt{x^2+25}} \times 2x - 1 = 1.4\frac{x}{\sqrt{x^2+25}} - 1$$

$$f'(x) = 0 \Rightarrow x = \pm\frac{25\sqrt{6}}{12},\ \because x \in [0, 13]\ \therefore x = \frac{25\sqrt{6}}{12}$$

$$\because f''(x) = \frac{1.4\sqrt{x^2+25} + \frac{x}{\sqrt{x^2+25}} \times 1.4x}{x^2+25} = \frac{2.8x^2+35}{(x^2+25)^{\frac{3}{2}}}$$

$$\therefore f''\left(\frac{25\sqrt{6}}{12}\right) = 0.296 > 0$$

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$$\therefore f\left(\frac{25\sqrt{6}}{12}\right) \text{ is minimum.}$$

$\therefore$ The birds need to choose point C where the distance BC is equal to $\frac{25\sqrt{6}}{12}$ km.

5) $g(x) = \sqrt{x},\ x = 1$

$$\because g'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

$\frac{1}{2}x^{-\frac{1}{2}}$

$$g''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$$

$$\frac{2.8 \times \frac{625}{24} + 35}{\left(\frac{625}{24}+25\right)^{\frac{3}{2}}}$$

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$$g'''(x) = \frac{3}{8}x^{-\frac{5}{2}}$$

$\frac{1}{2}\left(\frac{1}{2}\right)x^{-\frac{3}{2}}$

$$g^{(4)}(x) = -\frac{15}{16}x^{-\frac{7}{2}}$$

$$\therefore g(1) = 1,\ g'(1) = \frac{1}{2},\ g''(1) = -\frac{1}{4},\ g'''(1) = \frac{3}{8},\ g^{(4)}(1) = -\frac{15}{16}$$

$$\therefore T_4(x) = \frac{g(1)}{0!}(x-1)^0 + \frac{g'(1)}{1!}(x-1) + \frac{g''(1)}{2!}(x-1)^2 + \frac{g'''(1)}{3!}(x-1)^3 + \frac{g^{(4)}(1)}{4!}(x-1)^4$$
$$= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4$$

6) Choose $u = 4x$, $du = 4dx$

$$\int e^{4x}dx = \int e^u \frac{1}{4}du = \frac{1}{4}e^u + C = \frac{1}{4}e^{4x} + C$$

$$\therefore R = \int_0^1 f(x)dx = \int_0^1 e^{4x}dx = \left(\frac{1}{4}e^{4x}\right)\Big|_0^1 = \frac{1}{4}e^4 - \frac{1}{4}$$

$$S=\left(\frac{5}{4}-1\right)\times f\left(\frac{5}{4}\right)-\int_1^{\frac{5}{4}} f(x)\,dx$$
$$=\frac{1}{4}\times e^5-\int_1^{\frac{5}{4}} e^{4x}\,dx$$
$$=\frac{1}{4}e^5-\left(\frac{1}{4}e^{4x}\right)\Big|_1^{\frac{5}{4}}$$
$$=\frac{1}{4}e^4$$

$$\therefore R+S=\frac{1}{4}e^4-\frac{1}{4}+\frac{1}{4}e^4=\frac{1}{2}e^4-\frac{1}{4}$$

7) (a) $\int_0^2 r(t)\,dt$ represents the total oil that consumed by world from Jan 1, 2000 to Jan 1, 2002.

(b) $\int_0^2 r(t)\,dt$
$$=\int_0^2 (t+1)^3 \ln[(t+1)^2]\,dt$$
$$=\int_0^2 \ln[(t+1)^2]\,d\left[\frac{1}{4}(t+1)^4\right]$$
$$=\frac{1}{4}(t+1)^4\ln[(t+1)^2]\Big|_0^2-\int_0^2\frac{1}{4}(t+1)^4\,d\{\ln[(t+1)^2]\}$$
$$=\frac{81}{4}\ln 9-\int_0^2\frac{1}{4}(t+1)^4\cdot\frac{1}{(t+1)^2}\cdot 2(t+1)\,dt$$
$$=\frac{81}{4}\ln 9-\int_0^2\frac{1}{2}(t+1)^3\,dt$$
$$=\frac{81}{4}\ln 9-\left[\frac{1}{8}(t+1)^4\right]\Big|_0^2$$
$$=\frac{81}{2}\ln(3)-10$$