

$$2.4.1 a. \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

$$b. \frac{1}{6} \begin{pmatrix} 3 & 0 \\ 1 & -4 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 4 & 0 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

$$2.4.2 a. \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ -1 & 3 & 0 & 1 \end{array} \right) \xrightarrow{R_2+R_1} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 1 \end{array} \right) \xrightarrow{\frac{1}{2}R_2} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \end{array} \right) \xrightarrow{R_1+R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \end{array} \right)$$

$$b. \left(\begin{array}{cc|cc} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array} \right) \xrightarrow{\frac{1}{4}R_1} \left(\begin{array}{cc|cc} 1 & \frac{1}{4} & \frac{1}{4} & 0 \\ 3 & 2 & 0 & 1 \end{array} \right) \xrightarrow{R_2-3R_1} \left(\begin{array}{cc|cc} 1 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{5}{4} & -\frac{3}{4} & 1 \end{array} \right) \xrightarrow{\frac{4}{5}R_2} \left(\begin{array}{cc|cc} 1 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 1 & -\frac{3}{5} & \frac{4}{5} \end{array} \right) \xrightarrow{R_1-\frac{1}{4}R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{2}{5} & -\frac{1}{5} \\ 0 & 1 & -\frac{3}{5} & \frac{4}{5} \end{array} \right) \checkmark$$

$$c. \begin{pmatrix} 0 & 1 & 2 \\ 0 & -1 & -3 \\ -1 & 1 & 2 \end{pmatrix} d. \begin{pmatrix} 2 & -1 & 3 \\ 3 & 1 & -1 \\ 1 & 1 & -2 \end{pmatrix}$$

$$0 - \frac{1}{4} \times \frac{4}{5}$$

$$2.4.4. a. \vec{x} = A^{-1}b = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 5 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 11 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ 17 \\ -2 \end{pmatrix} \checkmark$$

$$b. B = A^{-1}AB = A^{-1} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 5 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 4 & -2 & 1 \\ 7 & -2 & 4 \\ -1 & 2 & -1 \end{pmatrix} \checkmark$$

$$c. C = CAA^{-1} = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 5 \\ -1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 6 & -2 & 13 \\ 4 & -2 & 14 \end{pmatrix} \checkmark$$

$$2.4.9 a. \text{False, } A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \checkmark$$

$$b. \text{False, } A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

$$c. (A^{-1}B)^T = B^T(A^{-1})^T = B^T(A^T)^{-1} \quad B^T \text{ inv, } (A^T)^{-1} \text{ inv} \checkmark$$

$$d. A^4 = 3I \Rightarrow |A^4| = 3^3 = 27 \Rightarrow |A| = 27^{\frac{1}{4}} \neq 0 \Rightarrow A \text{ inv} \checkmark \quad A^4 = 3I \Rightarrow A(\frac{1}{3}A^3) = I \Rightarrow A^{-1} = \frac{1}{3}A^3$$

$$3.1.1 a. \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 4 - (-3) = 7 \checkmark$$

$$b. \begin{vmatrix} 6 & 9 \\ 8 & 12 \end{vmatrix} = 6 \times 12 - 9 \times 8 = 0 \checkmark$$

$$f. -39 \checkmark$$

$$g. 0, \{x \mid x = \begin{pmatrix} 5 \\ -3 \end{pmatrix}, s \in \mathbb{R}\} \checkmark$$

$$k. -33 \checkmark$$

$$l. 0, \{x \mid x = \begin{pmatrix} -35 \\ 0 \\ 5 \end{pmatrix}, s \in \mathbb{R}\} \checkmark$$

$$3.2.3 a. \det(A^3 B C^T B^{-1}) = |A|^3 |B| |C| \frac{1}{|B|} = |A|^3 |C| = (-1)^3 3 = -3 \checkmark$$

$$b. |B^2 C^{-1} A B^{-1} C^T| = |B|^2 \frac{1}{|C|} |A| \frac{1}{|B|} |C| = |B| |A| = 2 \cdot (-1) = -2 \checkmark$$

3.2.4 a. $|B^{-1}AB| = \frac{1}{|B|} |A| |B| = |A|$ ✓

b. $|A^{-1}B^{-1}AB| = \frac{1}{|A|} \frac{1}{|B|} |A| |B| = 1$ ✓