3.3.1 a. $P=(\vec{x}_1,\vec{x}_2)=\begin{pmatrix}2&-1\\3&1\end{pmatrix}$

b. $P=(\vec{x}_1,\vec{x}_2)=\begin{pmatrix}1&-4\\1&1\end{pmatrix}$

c. $P=(\vec{x}_1,\vec{x}_2,\vec{x}_3)=\begin{pmatrix}0&1&4\\1&0&0\\0&1&5\end{pmatrix}$

d. P DNE ??

e. P DNE ??

f. P DNE

3.3.8 a. $P^{-1}=\frac{1}{3}\begin{pmatrix}-2&5\\1&-1\end{pmatrix}$ $P^{-1}AP=\frac{1}{3}\begin{pmatrix}-2&5\\1&-1\end{pmatrix}\begin{pmatrix}6&-5\\2&-1\end{pmatrix}\begin{pmatrix}1&5\\1&2\end{pmatrix}=\begin{pmatrix}1&0\\0&4\end{pmatrix}$

$\therefore A^n=P\begin{pmatrix}1&0\\0&4\end{pmatrix}P^{-1}=\begin{pmatrix}1&5\\1&2\end{pmatrix}\begin{pmatrix}1&0\\0&4^n\end{pmatrix}\begin{pmatrix}-2/3&5/3\\1/3&-1/3\end{pmatrix}=\begin{pmatrix}1&5\times4^n\\1&2\times4^n\end{pmatrix}\frac{1}{3}\begin{pmatrix}-2&5\\1&-1\end{pmatrix}=$

$\frac{1}{3}\begin{pmatrix}5\times4^n-2&-5\times4^n+5\\2\times4^n-2&-2\times4^n+5\end{pmatrix}$

b. $P^{-1}=\begin{pmatrix}-3&-4\\-2&-3\end{pmatrix}$ $P^{-1}AP=\begin{pmatrix}-3&-4\\-2&-3\end{pmatrix}\begin{pmatrix}-7&-12\\6&10\end{pmatrix}\begin{pmatrix}-3&4\\2&-3\end{pmatrix}=\cancel{\begin{pmatrix}161&-204\\120&-178\end{pmatrix}}$ $\begin{pmatrix}1&0\\0&2\end{pmatrix}$??

$A^n=P\begin{pmatrix}161&-204\\120&-178\end{pmatrix}^nP^{-1}$ $=\begin{pmatrix}-3&4\\2&-3\end{pmatrix}\begin{pmatrix}1&0\\0&2^n\end{pmatrix}\begin{pmatrix}-3&-4\\-2&-3\end{pmatrix}=\begin{pmatrix}-3&4\times2^n\\2&-3\times2^n\end{pmatrix}\begin{pmatrix}-3&-4\\-2&-3\end{pmatrix}=\begin{pmatrix}9-8\times2^n&12-12\times2^n\\-6+6\times2^n&-8+9\times2^n\end{pmatrix}$

3.3.9 a. $C_x(A)=\begin{vmatrix}x-1&-3\\0&x-2\end{vmatrix}=(x-1)(x-2)=0\Rightarrow x=1,2\Rightarrow\lambda_1=1,\lambda_2=2$

$A_{2\times2}$ has 2 distinct eigenvalues, so $A_{2\times2}$ is diagonalizable.

$C_x(B)=\begin{vmatrix}x-2&0\\0&x-1\end{vmatrix}=(x-2)(x-1)=0\Rightarrow x=2\text{ or }1\Rightarrow\lambda_1=1,\lambda_2=2$

$B_{2\times2}$ has 2 distinct eigenvalues, so $B_{2\times2}$ is diagonalizable.

$AB=\begin{pmatrix}2&3\\0&2\end{pmatrix}$, $C_x(AB)=\begin{vmatrix}x-2&-3\\0&x-2\end{vmatrix}=(x-2)^2=0\Rightarrow x=2\Rightarrow\lambda_1=\lambda_2=2$

$(\lambda_1I-AB)\vec{x}=\vec{0}\Rightarrow\begin{pmatrix}0&-3\\0&0\end{pmatrix}\vec{x}=0\Rightarrow\vec{x}=t\begin{pmatrix}1\\0\end{pmatrix}\Rightarrow\vec{x}_1=\begin{pmatrix}1\\0\end{pmatrix}$

The eigenvalue 2 of multiplicity 2 doesn't yeild 2 eigenvectors, so AB is not diagonalizable.

5.5.1 a. $|A|=-3$, $|B|=2$, $|A|\neq|B|\Rightarrow A\sim B$ is false

b. $|A|=-5$, $|B|=-1$. $|A|\neq|B|\Rightarrow A\sim B$ is false.

c. $|A|=-3$, $|B|=-3$, $tr(A)=1$, $tr(B)=2$, $tr(A)\neq tr(B)\Rightarrow A\sim B$ is false.

d. $|A|=7$, $|B|=7$, $tr(A)=5$, $tr(B)=4$, $tr(A)\neq tr(B)\Rightarrow A\sim B$ is false.