

Mathematics For Statisticians Homework 1

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$$\begin{aligned}
 1) (a) \quad & 2\log_3(x+y) + 6\log_3(x) - \frac{1}{3} \\
 &= \log_3(x+y)^2 + \log_3 x^6 - \log_3 3^{\frac{1}{3}} \\
 &= \log_3(x+y)^2 x^6 \log_3 \frac{1}{\sqrt[3]{3}} \\
 &= \log_3 \frac{x^6 + 2x^2y + x^6y^2}{\sqrt[3]{3}}
 \end{aligned}$$

$$(b)(i) \because Q_0 = 2$$

$$\therefore Q(t) = 2e^{kt}$$

$$\therefore Q(1750) = 2e^{1750k} = 1.5$$

$$\therefore e^{1750k} = 0.75$$

$$\therefore k = \frac{1}{1750} \ln 0.75 \approx -1.64 \times 10^{-4}$$

$$\therefore Q(t) = 2e^{\frac{\ln 0.75}{1750}t}$$

$$(ii) \because Q(t) = 2e^{\frac{\ln 0.75}{1750}t} = 2 \times \frac{1}{2} = 1$$

$$\therefore t \approx 4216$$

$$(iii) \because Q(t) = 2e^{\frac{\ln 0.75}{1750}t} = 0.25$$

$$\therefore t \approx 12649$$

$$\begin{aligned}
 2) (a) \quad & \lim_{x \rightarrow \infty} \frac{3x^4 + 6x^3 + 3x + 12}{-2x^4 - 6x^3 + 70x - 27} + 2^{-x} + 3 \\
 &= \lim_{x \rightarrow \infty} \frac{3 + 6/x^2 + 3/x^3 + 12/x^4}{-2 - 6/x + 70/x^2 - 27/x^4} + 0 + 3 \\
 &= -\frac{3}{2} + 0 + 3
 \end{aligned}$$

$$= \frac{3}{2}$$

$$(b) \lim_{x \rightarrow 3} \frac{1}{(x-3)^3}$$

$$\therefore \lim_{x \rightarrow 3^-} \frac{1}{(x-3)^3} = -\infty$$

$$\lim_{x \rightarrow 3^+} \frac{1}{(x-3)^3} = +\infty$$

$$\therefore \lim_{x \rightarrow 3^-} \frac{1}{(x-3)^3} \neq \lim_{x \rightarrow 3^+} \frac{1}{(x-3)^3}$$

$$\therefore \lim_{x \rightarrow 3} \frac{1}{(x-3)^3} \text{ DNE}$$

$$\begin{aligned} (c) \lim_{x \rightarrow 2} \frac{(x^2 - x - 2)(x^2 - 5x + 6)}{x^2 - 4x + 4} \\ = \lim_{x \rightarrow 2} \frac{(x-2)(x+1)(x-2)(x-3)}{(x-2)^2} \\ = \lim_{x \rightarrow 2} (x+1)(x-3) \\ = -3 \end{aligned}$$

$$\begin{aligned} 3 > (a) f'(x) &= \lim_{a \rightarrow 0} \frac{f(x+a) - f(x)}{a} \\ &= \lim_{a \rightarrow 0} \frac{\frac{1}{x+a+1} - \frac{1}{x+1}}{a} \\ &= \lim_{a \rightarrow 0} -\frac{1}{(x+a+1)(x+1)} \\ &= -\frac{1}{(x+1)^2} \end{aligned}$$

$$\begin{aligned} (a) h'(7) &= \lim_{a \rightarrow 0} \frac{h(7+a) - h(7)}{a} \\ &= \lim_{a \rightarrow 0} \frac{2(7+a)^2 + 2(7+a) - (2 \cdot 7^2 + 2 \cdot 7)}{a} \\ &= \lim_{a \rightarrow 0} \frac{2a^2 + 30a}{a} \\ &= \lim_{a \rightarrow 0} 2a + 30 \\ &= 30 \end{aligned}$$