# Probability Exercise Lecture 5

1. (a) $P(X \le \frac{5}{3}) = P(0 < X \le \frac{5}{3}) = 1 - e^{-\frac{1}{2}(\frac{5}{3})^2} - 0 = 1 - e^{-\frac{25}{18}}$

(b) $P(X > \frac{3}{2}) = 1 - P(X \le \frac{3}{2}) = 1 - [1 - e^{-\frac{1}{2}(\frac{3}{2})^2}] = e^{-\frac{9}{8}}$

(c) $P(\frac{4}{3} < X \le \frac{5}{3}) = P(X \le \frac{5}{3}) - P(X \le \frac{4}{3}) = [1 - e^{-\frac{1}{2}(\frac{5}{3})^2}] - [1 - e^{-\frac{1}{2}(\frac{4}{3})^2}]$

$= e^{-\frac{16}{18}} - e^{-\frac{25}{18}}$

2. (a) $P(X > 10) = P(\frac{X-5}{10} > 0.5) = 1 - \Phi(0.5) = 1 - 0.69146 = 0.30854$

(b) $P(-20 < X < 15) = P(-2.5 < \frac{X-5}{10} < 1) = \Phi(1) - \Phi(-2.5) = \Phi(1) + \Phi(2.5) - 1 = 0.84134 + 0.99379 - 1 = 0.83513$

(c) $P(X > x_1) = 0.05 \Rightarrow P(X \le x_1) = 0.95 \Rightarrow y = 1.65$ $X = 10 \times 1.65 + 5 = 21.5$

3. (a) $P(Z > 2.64) = 1 - P(Z \le 2.64) = 1 - 0.99585 = 0.00415$

(b) $P(0 \le Z < 0.87) = \Phi(0.87) - \Phi(0) = 0.80785 - 0.5 = 0.30785$

4. $F_X(t) = 1 - e^{-\lambda t} = 1 - e^{-0.1t}$

(a) $P(X < 10) = F_X(10) = 1 - e^{-1}$

(b) $P(5 < X < 15) = F_X(15) - F_X(5) = e^{-0.5} - e^{-1.5}$

(c) $P(X > t) = 0.01 \Rightarrow 1 - F_X(t) = 1 - (1 - e^{-0.1t}) = e^{-0.1t} = 0.01 \Rightarrow t = -10 \ln 0.01$

$100 e^{-0.1t} = 1$

$-0.1t = -\ln 100$

$e^{-0.1t} = 0.99$

$10 \ln 100$

5. (a) $P(X > 60 \times 12) = P(\frac{X-70}{3} > \frac{2}{3}) = 1 - \Phi(0.67) = 0.25143$

(b) $X \sim N(70, 3^2)$, $2.54X \sim N(177.8, 7.62^2)$,

$0.0254X \sim N(1.778, 0.0762^2)$

6. $X \sim \chi^2(20)$

(a) $P(X > x_0) = 0.95$, $x_0 = 10.851$

(b) $P(X \le 12.443) = 1 - 0.9 = 0.1$