① LMM with random intercepts and heterogeneous errors:

$$y_{ij} = \beta_0 + \beta_1 t_{ij} + b_i + \varepsilon_{ij},\ t_{ij} \in \{0, 1\},\ b_i \sim N(0, \sigma_b^2),\ \varepsilon_j \sim N_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\sigma_1^2 & 0\\ 0 & \sigma_2^2\end{pmatrix}\right)$$

$$V_i = Z_i D Z_i^T + \Sigma_i = \begin{pmatrix}1\\1\end{pmatrix}\sigma_b^2(1 \quad 1) + \begin{pmatrix}\sigma_1^2 & 0\\ 0 & \sigma_2^2\end{pmatrix} = \begin{pmatrix}\sigma_b^2+\sigma_1^2 & \sigma_b^2\\ \sigma_b^2 & \sigma_b^2+\sigma_2^2\end{pmatrix}$$

② LMM with uncorrelated random intercepts and slopes and homogeneous errors:

$$y_{ij} = \beta_0 + \beta_1 t_{ij} + b_{i0} + b_{i1} t_{ij} + \varepsilon_{ij},\ t_{ij} \in \{0, 1\},\ \vec{b_i} = \begin{pmatrix}b_{i0}\\ b_{i1}\end{pmatrix} \sim N_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\sigma_{b_1}^2 & 0\\ 0 & \sigma_{b_2}^2\end{pmatrix}\right)$$

$$\varepsilon_j \sim N_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\sigma^2 & 0\\ 0 & \sigma^2\end{pmatrix}\right)$$

$$V_i = Z_i D Z_i^T + \Sigma_i = \begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\begin{pmatrix}\sigma_{b_1}^2 & 0\\ 0 & \sigma_{b_2}^2\end{pmatrix}\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} + \begin{pmatrix}\sigma^2 & 0\\ 0 & \sigma^2\end{pmatrix} = \begin{pmatrix}\sigma_{b_1}^2+\sigma^2 & \sigma_{b_1}^2\\ \sigma_{b_1}^2 & \sigma_{b_1}^2+\sigma_{b_2}^2+\sigma^2\end{pmatrix}$$