

$$2.2.3 \text{ a. (i) } A = \begin{pmatrix} 3 & -2 & 0 \\ 5 & -4 & 1 \end{pmatrix}^{2 \times 3}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^{3 \times 1}$$

$$AX = x_1 \begin{pmatrix} 3 \\ 5 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ -4 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3x_1 - 2x_2 \\ 5x_1 - 4x_2 + x_3 \end{pmatrix}$$

$$(ii) AX = \begin{pmatrix} (3 \ -2 \ 0) X \\ (5 \ -4 \ 1) X \end{pmatrix} = \begin{pmatrix} 3x_1 - 2x_2 \\ 5x_1 - 4x_2 + x_3 \end{pmatrix}$$

$$b. (i) AX = x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ -4 \end{pmatrix} + x_3 \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} x_1 + 2x_2 + 3x_3 \\ -4x_2 + 5x_3 \end{pmatrix} \checkmark$$

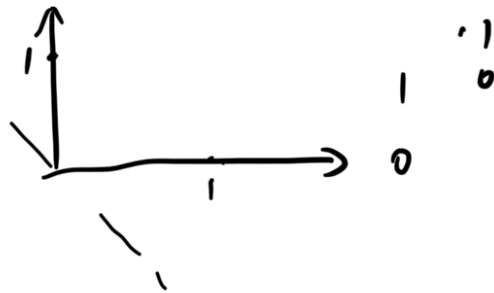
$$(ii) AX = \begin{pmatrix} (1 \ 2 \ 3) X \\ (0 \ -4 \ 5) X \end{pmatrix} = \begin{pmatrix} x_1 + 2x_2 + 3x_3 \\ -4x_2 + 5x_3 \end{pmatrix} \checkmark$$

$$2.2.11 \text{ a. } \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$b. \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \checkmark$$

$$c. \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$d. \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \checkmark$$



$$2.2.12 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \quad X = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$2.2.3 \text{ c. (i) } AX = \begin{pmatrix} -2 \\ 1 \\ -5 \end{pmatrix} \cdot 0 + \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix} \cdot 1 + \begin{pmatrix} 5 \\ 0 \\ -7 \end{pmatrix} \cdot 0 + \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix} \cdot -1$$

$$= \begin{pmatrix} -4 \\ -1 \\ -2 \end{pmatrix}$$

$$(ii) AX = \begin{pmatrix} (-2 \ 0 \ 5 \ 4) X \\ (1 \ 2 \ 0 \ 3) X \\ (-5 \ 6 \ -7 \ 8) X \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \\ -2 \end{pmatrix}$$

$$d. (i) AX = \begin{pmatrix} 3 \\ 0 \\ -8 \end{pmatrix} \cdot 1 + \begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix} \cdot 0 + \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} \cdot 0 + \begin{pmatrix} 6 \\ 5 \\ 0 \end{pmatrix} \cdot 0 = \begin{pmatrix} 3 \\ 0 \\ -8 \end{pmatrix} \checkmark$$

$$(ii) AX = \begin{pmatrix} (3 \ -4 \ 1 \ 6) \vec{e}_1 \\ (0 \ 2 \ 1 \ 5) \vec{e}_1 \\ (-8 \ 7 \ -3 \ 0) \vec{e}_1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -8 \end{pmatrix} \checkmark$$

$$d(ii) Ax = \begin{pmatrix} 3 \\ 0 \\ -8 \end{pmatrix} \cdot 0 + \begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix} \cdot 1 + \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} \cdot 0 + \begin{pmatrix} 6 \\ 5 \\ 0 \end{pmatrix} \cdot 0 = \begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix} \checkmark$$

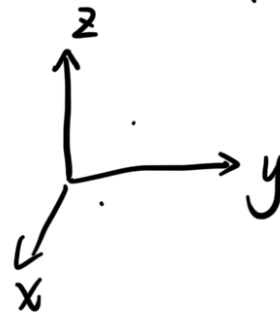
$$(ii) Ax = \begin{pmatrix} (3 \ -4 \ 1 \ 6) \vec{e}_1 \\ (0 \ 2 \ 1 \ 5) \vec{e}_2 \\ (-8 \ 7 \ -3 \ 0) \vec{e}_3 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix} \checkmark$$

...

$$2.6.3 \ b. \ A = [T(\vec{e}_1), T(\vec{e}_2)] \\ = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot (-1), \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot (1) \right) \\ = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \checkmark \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$2.6.4 \ a. \ A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$b. \ A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \checkmark$$



$$2.6.5 \ a. \ T(a\vec{x}_1 + b\vec{x}_2) = aT(\vec{x}_1) + bT(\vec{x}_2) = a\vec{0} + b\vec{0} = \vec{0}$$

$\therefore a\vec{x}_1 + b\vec{x}_2$ is in the kernel.

$$b. \ \vec{y}_1 = T(\vec{x}_1), \ \vec{y}_2 = T(\vec{x}_2)$$

$$a\vec{y}_1 + b\vec{y}_2 = aT(\vec{x}_1) + bT(\vec{x}_2) = T(a\vec{x}_1 + b\vec{x}_2)$$

$\therefore a\vec{y}_1 + b\vec{y}_2$ is in the image of T . \checkmark

$$2.6.7. a. \ T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xy \\ 0 \end{pmatrix}$$

$$T(\vec{e}_1) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$T(\vec{e}_2) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$A = [T(\vec{e}_1), T(\vec{e}_2)] = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \neq \begin{pmatrix} xy \\ 0 \end{pmatrix}$$

$$b. \ T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ y \end{pmatrix}$$

$$T(\vec{e}_1) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ T(\vec{e}_2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$T\left(2\begin{pmatrix} 0 \\ 2 \end{pmatrix}\right) = T\begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 16 \end{pmatrix}$$

$$2T\begin{pmatrix} 0 \\ 2 \end{pmatrix} = 2 \cdot \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

$$A = [\vec{e}_1, \vec{e}_2] = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad T(2\vec{x}) \neq 2T(\vec{x})$$

$$A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ y \end{pmatrix}$$