4.1.1 a. $\sqrt{2^2+(-1)^2+2^2}=3$

b. $\sqrt{1^2+(-1)^2+2^2}=\sqrt{6}$

c. $\sqrt{1^2+0^2+(-1)^2}=\sqrt{2}$

d. $\sqrt{(-1)^2+0^2+2^2}=\sqrt{5}$

e. $2\sqrt{1^2+(-1)^2+2^2}=2\sqrt{6}$

$49+1+25 \quad 75$

f. $|-3|\sqrt{1^2+1^2+2^2}=3\sqrt{6}$

4.1.2 a. $\|\vec{v}\|=\sqrt{7^2+(-1)^2+5^2}=5\sqrt{3}$, $\frac{\vec{v}}{\|\vec{v}\|}=\begin{pmatrix}7/5\sqrt{3}\\-1/5\sqrt{3}\\1/\sqrt{3}\end{pmatrix}$

b. $\|\vec{v}\|=\sqrt{(-2)^2+(-1)^2+2^2}=3$, $\frac{\vec{v}}{\|\vec{v}\|}=\begin{pmatrix}-2/3\\-1/3\\2/3\end{pmatrix}$

4.1.21 a. $\vec{v}=\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$, $\|\vec{v}\|=\sqrt{v_1^2+v_2^2+v_3^2}\geq 0$, $\|\vec{v}\|=0$ only if $v_1=v_2=v_3=0$

b. use a, $\|\vec{v}-\vec{w}\|=0 \Rightarrow \vec{v}-\vec{w}=\vec{0} \Rightarrow \vec{v}=\vec{w}$

c. $\vec{v}=-\vec{v} \Rightarrow 2\vec{v}=\vec{0} \Rightarrow \vec{v}=\vec{0}$

d. False, $\vec{v}=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\vec{w}=\begin{pmatrix}0\\1\\0\end{pmatrix}$, $\|\vec{v}\|=\|\vec{w}\|=1$, but $\vec{v}\neq\vec{w}$

e. False, $\vec{v}=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\vec{w}=\begin{pmatrix}0\\1\\0\end{pmatrix}$, $\|\vec{v}\|=\|\vec{w}\|=1$, but $\vec{v}\neq\pm\vec{w}$

f. False, $\vec{v}\cdot\vec{w}=\vec{v}\cdot t\vec{v}=t\|\vec{v}\|^2$. if $t\leq 0$, $\vec{v}\perp\vec{w}$ or $\vec{v}$ and $\vec{w}$ are in opposite direction.

g. $\vec{v}$ and $\vec{v}+\vec{w}$ are parallel $\Rightarrow \vec{v}=d(\vec{v}+\vec{w}), d\neq 0,1 \Rightarrow \vec{v}=\frac{d}{1-d}\vec{w} \Rightarrow \vec{v}$ and $\vec{w}$ are parallel

h. False, $\vec{v}=\begin{pmatrix}0\\1\end{pmatrix}$, $\|-5\vec{v}\|=5\neq -5\|v\|$

i. $\|\vec{v}\|=\|2\vec{v}\|=2\|\vec{v}\| \Rightarrow \|\vec{v}\|=\frac{1}{2} \Rightarrow \vec{v}\neq\vec{0}$

j. False, $\vec{v}=\begin{pmatrix}0\\1\end{pmatrix}$, $\vec{w}=\begin{pmatrix}1\\0\end{pmatrix}$, $\|\vec{v}+\vec{w}\|=\sqrt{1^2+1^2}=\sqrt{2}$, $\|\vec{v}\|+\|\vec{w}\|=2$

4.2.1 a. $\vec{u}\cdot\vec{v}=2\times(-1)+(-1)\times 1+3\times 1=0$

$1^2+2^2+(-1)^2=6$

b. $\vec{u}\cdot\vec{v}=\|\vec{u}\|^2=\cancel{1^2+1^2+(-3)^2=11}$

c. $\vec{u}\cdot\vec{v} = 1\times2+1\times(-1)+(-3)\times1 = -2$ ✓    $18+7-25$    $(1,0)-(0,-1)$  $(1,1)$

d. $\vec{u}\cdot\vec{v} = 3\times6+(-1)\times(-7)+5\times(-5) = 0$ ✓

e. $\vec{u}\cdot\vec{v} = ax+by+cz$ ✓

$-18-2$

$-20$

f. $\vec{u}\cdot\vec{v} = 0$ ✓

4.2.2 b. $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{3\times(-6)+(-1)\times2+0}{\sqrt{10}\sqrt{40}} = -1,\ \theta = \pi$ ✓

d. $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{2\times3+1\times6+(-1)\times3}{\sqrt{6}\sqrt{54}} = \frac{1}{2},\ \theta = \frac{1}{3}\pi$ ✓

$\frac{60}{180}$

4.2.3 a. $\vec{v}\cdot\vec{u} = 0 \Rightarrow 2x+2+3=0 \Rightarrow x = -\frac{5}{2}$ ✓

4.2.10 a. $\mathrm{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{5\times2+7\times(-1)+1\times3}{2^2+(-1)^2+3^2}\begin{pmatrix}2\\-1\\3\end{pmatrix} = \begin{pmatrix}6/7\\-3/7\\9/7\end{pmatrix}$ ??

b. $\mathrm{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{3\times4+(-2)\times1+1\times1}{4^2+1^2+1^2}\begin{pmatrix}4\\1\\1\end{pmatrix} = \frac{11}{18}\begin{pmatrix}4\\1\\1\end{pmatrix}$ ✓    $\frac{6}{14}$    $\frac{3}{7}$

4.2.11 a. $\vec{u}_2 = \mathrm{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{2\times1+(-1)\times(-1)+1\times3}{1^2+(-1)^2+3^2}\begin{pmatrix}1\\-1\\3\end{pmatrix} = \frac{6}{11}\begin{pmatrix}1\\-1\\3\end{pmatrix}$ ✓

$\vec{u}_1 = \vec{u}-\vec{u}_2 = \begin{pmatrix}2\\-1\\1\end{pmatrix} - \frac{6}{11}\begin{pmatrix}1\\-1\\3\end{pmatrix} = \frac{1}{11}\begin{pmatrix}16\\-5\\-7\end{pmatrix}$ ✓

b. $\vec{u}_2 = \mathrm{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{3\times(-2)+1\times1+0\times4}{(-2)^2+1^2+4^2}\begin{pmatrix}-2\\1\\4\end{pmatrix} = -\frac{5}{21}\begin{pmatrix}-2\\1\\4\end{pmatrix}$ ✓

$\vec{u}_1 = \vec{u}-\vec{u}_2 = \begin{pmatrix}3\\1\\0\end{pmatrix} - \left(-\frac{5}{21}\right)\begin{pmatrix}-2\\1\\4\end{pmatrix} = \frac{1}{21}\begin{pmatrix}53\\26\\20\end{pmatrix}$ ✓

2.3.32 a. $0_{n\times n}^2 = 0_{n\times n},\ I^2 = I\cdot I = I$ ✓

b. $\begin{pmatrix}1&1\\0&0\end{pmatrix}\begin{pmatrix}1&1\\0&0\end{pmatrix} = \begin{pmatrix}1&1\\0&0\end{pmatrix}$ ✓, $\begin{pmatrix}1&0\\1&0\end{pmatrix}\begin{pmatrix}1&0\\1&0\end{pmatrix} = \begin{pmatrix}1&0\\1&0\end{pmatrix}$ ✓, $\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix} = \frac{1}{4}\begin{pmatrix}2&2\\2&2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}$ ✓

c. $(I-P)(I-P) = I\cdot I - I\cdot P - P\cdot I + P^2 = I-2P+P = I-P$ ✓

$P(I-P) = P-P^2 = P-P = 0_{n\times n}$ ✓

d. $P^T\cdot P^T = (P\cdot P)^T = P^T$ ✓

e. $Q\cdot Q = (P+AP-PAP)(P+AP-PAP) = P^2+PAP-P^2AP+AP^2+APAP-AP^2AP-PAP^2-PAPAP+PAP^2A$

$$= P + PAP - PAP + AP + APAP - APAP - PAP - PAPAP + PAPAP$$

$$= P + AP - PAP = Q$$

f. $BA \cdot BA = BI_nA = BA$