week10 exercise

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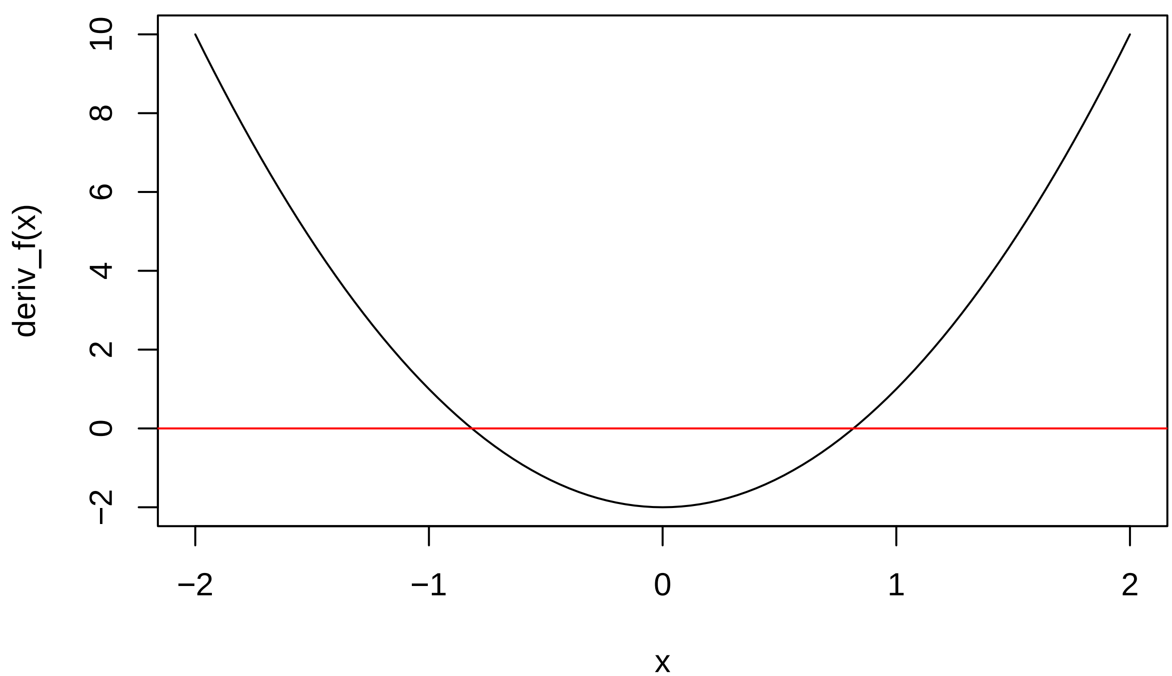
Exercise 1

1

```r
f = function (x){
  x**3-2*x+5
}
deriv_f = function (x){
  3*x**2-2
}
curve(expr = deriv_f(x), from = -2, to = 2)
abline(h=0, col='red')
```



The curve of $f'(x)$ is above. On interval $(-\infty, -\sqrt{\frac{2}{3}}) \bigcup (\sqrt{\frac{2}{3}}, +\infty)$, $f'(x)$ is positive and $f(x)$ is increasing; on interval $(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}})$, $f'(x)$ is negative and $f(x)$ is decreasing.

2

No. Because based on the sign of $f'(x)$, the global minimum of $f(x)$ is on $x \to -\infty$ and $\lim_{x\to-\infty} f(x) = -\infty$ and global maximum of $f(x)$ is on $x \to +\infty$ and $\lim_{x\to+\infty} f(x) = +\infty$.

3

Yes. Based on the sign of $f'(x)$, the local minima and maxima are points where $f'(x) = 0$. Thus, $x = -\sqrt{\frac{2}{3}}$ is local maxima and $x = \sqrt{\frac{2}{3}}$ is local minima.

```
c(-(2/3)**0.5, (2/3)**0.5)
```

```
## [1] -0.8164966  0.8164966
```

4

```
optimize(f = f, interval = c(-5, 0), maximum = TRUE)
```

```
## $maximum
## [1] -0.8165118
##
## $objective
## [1] 6.088662
```

```
optimize(f = f, interval = c(-5, 0), maximum = FALSE)
```

```
## $minimum
## [1] -4.999922
##
## $objective
## [1] -109.9943
```

```
optimize(f = f, interval = c(0, 5), maximum = TRUE)
```

```
## $maximum
## [1] 4.999922
##
## $objective
## [1] 119.9943
```

```
optimize(f = f, interval = c(0, 5), maximum = FALSE)
```

```
## $minimum
## [1] 0.8165118
##
## $objective
## [1] 3.911338
```

The results show that my answers to (2) and (3) are correct.

Exercise 2

1

```
lik = function (x, lamb){
  result = prod(dpois(x, lambda = lamb))
  return(result)
}
```

2

```
loglik = function (x, lamb){
  result = sum(dpois(x, lambda = lamb, log = TRUE))
  return(result)
}
```

3

$\lambda = \bar{x}$

4

```
loglik = function (x, lamb){
  result = sum(dpois(x, lambda = lamb, log = TRUE))
  return(result)
}
```

5

```
x1 = c(9, 7, 7, 8, 10, 5, 8, 4, 3, 5, 7, 7, 9, 6)
optimize(loglik, c(3, 10), x = x1, maximum =TRUE)
```

```
## $maximum
## [1] 6.785719
##
## $objective
## [1] -30.23411
```

```r
set.seed(10)
x2 = rpois(300, lambda = 3)
optimize(loglik, c(0, 4), x = x2, maximum =TRUE)
```

```
## $maximum
## [1] 3.030011
##
## $objective
## [1] -573.0289
```

6

```r
mean(x1)
```

```
## [1] 6.785714
```

```r
mean(x2)
```

```
## [1] 3.03
```

The results almost match.

Exercise 3

1

$l(\alpha, \beta) = n\alpha ln(\beta) + (\alpha - 1)\sum_{i=1}^{n} ln(x_i) - \beta \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} ln((\alpha - 1)!)$

2

```r
loglik = function (para, x){
  alpha = para[1]
  beta = para[2]
  result = -sum(dgamma(x, shape = alpha, scale = beta, log = TRUE))
  return(result)
}
```

3

```r
x = iris$Petal.Width
```

Nelder-Mead algorithm:

```r
optim(c(1, 1), loglik, x = x)
```

```
## $par
## [1] 1.5580323 0.7697392
##
## $value
## [1] 169.4108
##
## $counts
## function gradient
##       57       NA
##
## $convergence
## [1] 0
##
## $message
## NULL
```

L-BFGS-B algorithms:

```r
optim(c(1, 1), loglik, x = x, method = 'L-BFGS-B', lower = c(0, 0))
```

```
## $par
## [1] 1.5578168 0.7698817
##
## $value
## [1] 169.4108
##
## $counts
## function gradient
##       11       11
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"
```

4

```r
loglik1 = function (para, x){
  alpha = exp(para[1])
  beta = exp(para[2])
  result = -sum(dgamma(x, shape = alpha, scale = beta, log = TRUE))
  return(result)
}
opt_result = optim(c(1, 1), loglik1, x = x)
exp(opt_result$par)
```

```
## [1] 1.5580290 0.7697708
```

Exercise 4

1

```r
iris_sp = split(iris, iris$Species)
```

2

The 95% confidence interval of μ_X:

```r
mu_x = t.test(iris_sp$setosa$Sepal.Length)
mu_x$conf.int
```

```
## [1] 4.905824 5.106176
## attr(,"conf.level")
## [1] 0.95
```

The 95% confidence interval of μ_Y:

```r
mu_y = t.test(iris_sp$versicolor$Sepal.Length)
mu_y$conf.int
```

```
## [1] 5.789306 6.082694
## attr(,"conf.level")
## [1] 0.95
```

The 95% confidence interval of μ_Z:

```r
mu_z = t.test(iris_sp$virginica$Sepal.Length)
mu_z$conf.int
```

```
## [1] 6.407285 6.768715
## attr(,"conf.level")
## [1] 0.95
```

3

```r
t.test(iris_sp$setosa$Sepal.Length, mu = 5, conf.level = 0.99)
```

```
##
##  One Sample t-test
##
## data:  iris_sp$setosa$Sepal.Length
## t = 0.12036, df = 49, p-value = 0.9047
## alternative hypothesis: true mean is not equal to 5
## 99 percent confidence interval:
##  4.872406 5.139594
## sample estimates:
## mean of x
##     5.006
```

The conclusion is that not reject the null hypothesis $\mu_X = 5$.

4

```
t.test(iris_sp$versicolor$Sepal.Length, mu = 5, conf.level = 0.95)
```

```
##
##  One Sample t-test
##
## data:  iris_sp$versicolor$Sepal.Length
## t = 12.822, df = 49, p-value < 2.2e-16
## alternative hypothesis: true mean is not equal to 5
## 95 percent confidence interval:
##  5.789306 6.082694
## sample estimates:
## mean of x
##     5.936
```

The conclusion is that reject the null hypothesis $\mu_Y = 5$.

5

```
t.test(x = iris_sp$versicolor$Sepal.Length, y = iris_sp$virginica$Sepal.Length, conf.level = 0.99)
```

```
##
##  Welch Two Sample t-test
##
## data:  iris_sp$versicolor$Sepal.Length and iris_sp$virginica$Sepal.Length
## t = -5.6292, df = 94.025, p-value = 1.866e-07
## alternative hypothesis: true difference in means is not equal to 0
## 99 percent confidence interval:
##  -0.9565202 -0.3474798
## sample estimates:
## mean of x mean of y
##     5.936     6.588
```

The conclusion is that reject the null hypothesis $\mu_Y = \mu_Z$.