Calculus Exercise Week 3

Section 3.3

107~117

107. $f(x)=5x^3-x+1$

$f'(x)=5\cdot 3x^2-1=15x^2-1$

109. $f(x)=8x^4+9x^2-1$

$f'(x)=8\cdot 4x^3+9\cdot 2x=32x^3+18x$

111. $f(x)=3x(18x^4+\frac{13}{x+1})=54x^5+\frac{39x}{x+1}$

$f'(x)=54\cdot 5x^4+\frac{39(x+1)-39x}{(x+1)^2}=270x^4+\frac{39}{(x+1)^2}$

113. $f(x)=x^2(\frac{2}{x^2}+\frac{5}{x^3})=2+\frac{5}{x}$

$f'(x)=5\cdot(-\frac{1}{x^2})=-\frac{5}{x^2}$

115. $f(x)=\frac{4x^3-2x+1}{x^2}$

$$f'(x)=\frac{(4\cdot 3x^2-2)\cdot x^2-(4x^3-2x+1)\cdot 2x}{x^4}$$

$$=\frac{12x^4-2x^2-8x^4+4x^2-2x}{x^4}$$

$$=\frac{4x^4+2x^2-2x}{x^4}$$

$$=\frac{4x^3+2x-2}{x^3}$$

117. $f(x)=\frac{x+9}{x^2-7x+1}$

$$f'(x)=\frac{1\cdot(x^2-7x+1)-(2x-7)\cdot(x+9)}{(x^2-7x+1)^2}$$

$$=\frac{x^2-7x+1-(2x^2+11x-63)}{(x^2-7x+1)^2}$$

$$=\frac{-x^2-18x+64}{(x^2-7x+1)^2}$$

Section 3.6

229, 231, 239

229. $y=(5-2x)^{-2}$

$u=5-2x,\ y=u^{-2}$

$\frac{du}{dx}=-2,\ \frac{dy}{du}=-2u^{-3}$

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=-2u^{-3}\cdot -2=-2(5-2x)^{-3}\cdot(-2)=4(5-2x)^{-3}$

231. $y=(2x^3-x^2+6x+1)^3$

$u=2x^3-x^2+6x+1,\ y=u^3$

$\frac{du}{dx}=6x^2-2x+6,\ \frac{dy}{du}=3u^2$

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=3u^2\cdot(6x^2-2x+6)=3(2x^3-x^2+6x+1)^2(6x^2-2x+6)$

239. $y=[f(x)+5x^2]^4$

$u=f(x)+5x^2,\ y=u^4$

$\frac{du}{dx}=f'(x)+10x,\ \frac{dy}{du}=4u^3$

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=4u^3\cdot[f'(x)+10x]=4(f(x)+5x^2)(f'(x)+10x)$

When $x=-1$, $\frac{dy}{dx}=4[f(-1)+5(-1)^2][f'(-1)+10\times(-1)]$

$=4(-4+5)[f'(-1)-10]$

$=4(f'(-1)-10)$

$=3$

$f'(-1)=\frac{43}{4}$

Section 3.9

331-341, 345

331. $f(x)=x^2e^x$

$f'(x)=2x\cdot e^x+x^2e^x=(2x+x^2)\cdot e^x$

333. $f(x)=e^{x^3\ln x}$

$f'(x)=e^{x^3\ln x}(3x^2\cdot\ln x+x^3\frac{1}{x})=(3x^2\ln x+x^2)e^{x^3\ln x}$

335. $f(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}$

$f'(x)=\frac{[e^x-(-e^{-x})](e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$

$=\frac{(e^x+e^{-x})^2-(e^x-e^{-x})^2}{(e^x+e^{-x})^2}$

$=\frac{4}{(e^x+e^{-x})^2}$

337. $f(x)=2^{4x}+4x^2$

$f'(x)=2^{4x}\ln 2\cdot 4+8x=2^{4x+2}\ln 2+8x$

339. $f(x)=x^\pi\cdot\pi^x$

$f'(x)=\pi x^{\pi-1}\cdot\pi^x+x^\pi\cdot\pi^x\ln\pi=\pi^x x^{\pi-1}(\pi+x\ln\pi)$

341. $f(x)=\ln\sqrt{5x-7}$

$f'(x)=\frac{1}{\sqrt{5x-7}}\cdot\frac{1}{2}\frac{1}{\sqrt{5x-7}}\cdot 5=\frac{5}{10x-14}$

345. $f(x)=2^x\cdot\log_3 7^{x^2-4}$

$f'(x)=2^x\ln 2\cdot\log_3 7^{x^2-4}+2^x\frac{1}{7^{x^2-4}\cdot\ln 3}\cdot 7^{x^2-4}\cdot\ln 7\cdot 2x$

$=2^x\ln 2\cdot\log_3 7^{x^2-4}+\frac{2x\cdot 2^x\ln 7}{\ln 3}$

Volume II Section 6.3

117.121.123,135

117. $f(x)=1+x+x^2,\ a=-1$ $\quad f(x)=\frac{f^{(0)}(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+\dots$

$f(-1)=1$

$f'(x)=1+2x,\ f'(-1)=-1$

$f''(x)=2,\ f''(-1)=2$

$p_2(x)=f(-1)+f'(-1)(x+1)+\frac{f''(-1)}{2}(x+1)^2=1-1\times(x+1)+(x+1)^2=x^2+x+1$

121. $f(x)=\ln x,\ a=1$

$f(1)=0$

$f'(x)=\frac{1}{x},\ f'(1)=1$

$f''(x)=-\frac{1}{x^2},\ f''(1)=-1$

$p_2(x)=f(1)+f'(1)(x-1)+\frac{f''(1)}{2}(x-1)^2=0+1\times(x-1)-\frac{1}{2}(x-1)^2$

$=-\frac{1}{2}x^2+2x-\frac{3}{2}$

123. $f(x)=e^x,\ a=1$

$f(1)=e$

$f'(x)=e^x,\ f'(1)=e$

$f''(x)=e^x,\ f''(1)=e$

$p_2(x)=f(1)+f'(1)(x-1)+\frac{f''(1)}{2}(x-1)^2=e+e(x-1)+\frac{e}{2}(x-1)^2$

$=\frac{e}{2}x^2+\frac{e}{2}$

135. $f(x)=e^{-x}$ on $[-3,3],\ a=0$

$f'(x)=-e^{-x},\ f''(x)=(-1)^2e^{-x},\ f^{(n)}(x)=(-1)^n e^{-x}$

$$R_n = \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1} = \frac{(-1)^{n+1}e^{-c}}{(n+1)!}x^{n+1}, \quad c \in (0, x)$$

$$|R_n| = \frac{e^{-c}}{(n+1)!}|x|^{n+1}$$

$$c \in [-3, 3], \quad e^{-c} \leq e^{-(-3)} = e^3$$

$$|R_n| \leq \frac{e^3}{(n+1)!}|x|^{n+1} \leq \frac{e^3}{(n+1)!}3^{n+1} \leq \frac{1}{1000} \quad n=14$$