5.3.3 a. $\vec{b}_1\cdot\vec{b}_2 = 1\times(-2)+(-1)\times 1+3\times 1 = 0$, $\vec{b}_1\cdot\vec{b}_3 = 1\times 4+(-1)\times 7+3\times 1 = 0$, $\vec{b}_2\cdot\vec{b}_3 = -2\times 4+1\times 7+1\times 1 = 0$

$\text{proj}_{\vec{b}_1}\vec{x} = \frac{\vec{x}\cdot\vec{b}_1}{\|\vec{b}_1\|^2}\vec{b}_1 = \frac{a-b+3c}{1^2+(-1)^2+3^2} = \frac{a-b+3c}{11}\begin{pmatrix}1\\-1\\3\end{pmatrix}$, $\text{proj}_{\vec{b}_2}\vec{x} = \frac{\vec{x}\cdot\vec{b}_2}{\|\vec{b}_2\|^2}\vec{b}_2 = \frac{-2a+b+c}{6}\begin{pmatrix}-2\\1\\1\end{pmatrix}$

$\text{proj}_{\vec{b}_3}\vec{x} = \frac{\vec{x}\cdot\vec{b}_3}{\|\vec{b}_3\|^2}\vec{b}_3 = \frac{4a+7b+c}{66}\begin{pmatrix}4\\7\\1\end{pmatrix}$

b. $\vec{b}_1\cdot\vec{b}_2 = 1\times 1+0\times 4+(-1)\times 1 = 0$, $\vec{b}_1\cdot\vec{b}_3 = 1\times 2+0\times(-1)+(-1)\times 2 = 0$, $\vec{b}_2\cdot\vec{b}_3 = 1\times 2+4\times(-1)+1\times 2 = 0$

$\text{proj}_{\vec{b}_1}\vec{x} = \frac{\vec{x}\cdot\vec{b}_1}{\|\vec{b}_1\|^2}\vec{b}_1 = \frac{a-c}{2}\begin{pmatrix}1\\0\\-1\end{pmatrix}$, $\text{proj}_{\vec{b}_2}\vec{x} = \frac{\vec{x}\cdot\vec{b}_2}{\|\vec{b}_2\|^2}\vec{b}_2 = \frac{a+4b+c}{18}\begin{pmatrix}1\\4\\1\end{pmatrix}$

$\text{proj}_{\vec{b}_3}\vec{x} = \frac{\vec{x}\cdot\vec{b}_3}{\|\vec{b}_3\|^2}\vec{b}_3 = \frac{2a-b+2c}{9}\begin{pmatrix}2\\-1\\2\end{pmatrix}$

5.3.4 a. $\text{proj}_{\vec{u}_1}\vec{x} = \frac{\vec{x}\cdot\vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1 = \frac{13\times 1+(-20)\times(-2)+15\times 3}{1^2+(-2)^2+3^2}\begin{pmatrix}1\\-2\\3\end{pmatrix} = 7\begin{pmatrix}1\\-2\\3\end{pmatrix}$

$\text{proj}_{\vec{u}_2}\vec{x} = \frac{\vec{x}\cdot\vec{u}_2}{\|\vec{u}_2\|^2}\vec{u}_2 = \frac{13\times(-1)+(-20)\times 1+15\times 1}{(-1)^2+1^2+1^2}\begin{pmatrix}-1\\1\\1\end{pmatrix} = -6\begin{pmatrix}-1\\1\\1\end{pmatrix}$

$\vec{x} = 7\vec{u}_1 - 6\vec{u}_2$

b. $\text{proj}_{\vec{u}_1}\vec{x} = \frac{\vec{x}\cdot\vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1 = \frac{14\times 2+1\times(-1)+(-8)\times 0+5\times 3}{2^2+(-1)^2+0^2+3^2}\vec{u}_1 = 3\vec{u}_1$

$\text{proj}_{\vec{u}_2}\vec{x} = \frac{\vec{x}\cdot\vec{u}_2}{\|\vec{u}_2\|^2}\vec{u}_2 = \frac{14\times 2+1\times 1+(-8)\times(-2)+5\times(-1)}{2^2+1^2+(-2)^2+(-1)^2}\vec{u}_2 = 4\vec{u}_2$

$\vec{x} = 3\vec{u}_1+4\vec{u}_2$ ✓

8.1.2a, $\text{proj}_U\vec{x} = \text{proj}_{\vec{u}_1}\vec{x}+\text{proj}_{\vec{u}_2}\vec{x} = \frac{\vec{x}\cdot\vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1+\frac{\vec{x}\cdot\vec{u}_2}{\|\vec{u}_2\|^2}\vec{u}_2 = \frac{1\times 1+5\times(-2)+7\times 3}{1^2+(-2)^2+3^2}\vec{u}_1+\frac{1\times(-1)+5\times 1+7\times 1}{(-1)^2+1^2+1^2}\vec{u}_2$

$= \frac{6}{7}\vec{u}_1+\frac{11}{3}\vec{u}_2$

$\vec{x} = \text{proj}_U\vec{x}+(\vec{x}-\text{proj}_U\vec{x}) = \frac{6}{7}\vec{u}_1+\frac{11}{3}\vec{u}_2+\begin{pmatrix}1\\5\\7\end{pmatrix}-\frac{6}{7}\begin{pmatrix}1\\-2\\3\end{pmatrix}-\frac{11}{3}\begin{pmatrix}-1\\1\\1\end{pmatrix} = \frac{6}{7}\vec{u}_1+\frac{11}{3}\vec{u}_2+\begin{pmatrix}80/21\\64/21\\16/21\end{pmatrix}$

b. $\text{proj}_U\vec{x} = \text{proj}_{\vec{u}_1}\vec{x}+\text{proj}_{\vec{u}_2}\vec{x} = \frac{\vec{x}\cdot\vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1+\frac{\vec{x}\cdot\vec{u}_2}{\|\vec{u}_2\|^2}\vec{u}_2 = \frac{2\times 3+1\times(-1)+6\times 2}{3^2+(-1)^2+2^2}\vec{u}_1+\frac{2\times 2+1\times 0+6\times(-3)}{2^2+0^2+(-3)^2}\vec{u}_2$

$= \frac{17}{14}\vec{u}_1-\frac{14}{13}\vec{u}_2$??

$\vec{x} = \text{proj}_U\vec{x}+(\vec{x}-\text{proj}_U\vec{x}) = \frac{17}{14}\vec{u}_1-\frac{14}{13}\vec{u}_2+\begin{pmatrix}2\\1\\6\end{pmatrix}-\frac{17}{14}\begin{pmatrix}3\\-1\\2\end{pmatrix}+\frac{14}{13}\begin{pmatrix}2\\0\\-3\end{pmatrix} = \frac{17}{14}\vec{u}_1-\frac{14}{13}\vec{u}_2+\begin{pmatrix}341/182\\31/14\\508/91\end{pmatrix}$

8.1.4. a. $A^T = \begin{pmatrix} \vec{u}_1^T \\ \vec{u}_2^T \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $\vec{p} = \text{proj}_U \vec{x} \in U$

$\left.\begin{array}{l} A^T(\vec{x}-\vec{p}) = \vec{0} \\ \vec{p} = A\vec{\alpha} \end{array}\right\} \Rightarrow A^T(\vec{x} - A\vec{\alpha}) = \vec{0} \Rightarrow \vec{\alpha} = (A^TA)^{-1}A^T\vec{x} = \begin{pmatrix} -1 \\ 2.5 \end{pmatrix} \Rightarrow \vec{p} = A\vec{\alpha} = \begin{pmatrix} -1 \\ 1.5 \\ 1.5 \end{pmatrix}$

b. $A = \begin{pmatrix} 1 & -1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix}$, $\vec{x} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $\vec{p} = \text{proj}_U \vec{x} \in U$

$\frac{2\times14\times13 - 17\times3 + 14\times2}{14\times13}$ $\quad \frac{31}{14}$ $\quad \frac{\sqrt{308}}{91}$

$\vec{\alpha} = (A^TA)^{-1}A^T\vec{x} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$, $\vec{p} = A\vec{\alpha} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ ✓

c. $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\vec{x} = \begin{pmatrix} 2 \\ 0 \\ -1 \\ 3 \end{pmatrix}$, $\vec{p} = \text{proj}_U \vec{x} \in U$

$\vec{\alpha} = (A^TA)^{-1}A^T\vec{x} = \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}$, $\vec{p} = A\vec{\alpha} = \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

d. $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, $\vec{x} = \begin{pmatrix} 2 \\ 0 \\ 3 \\ 1 \end{pmatrix}$, $\vec{p} = \text{proj}_U \vec{x} \in U$

$\vec{\alpha} = (A^TA)^{-1}A^T\vec{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\vec{p} = A\vec{\alpha} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ ✓

5.6.2 a. $M = \begin{pmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{pmatrix}$, $\vec{y} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$, $M^TM = \begin{pmatrix} 4 & 14 \\ 14 & 62 \end{pmatrix}$, $M^T\vec{y} = \begin{pmatrix} 10 \\ 43 \end{pmatrix}$

$M^TM\vec{z} = M^T\vec{y} \Rightarrow \begin{pmatrix} 4 & 14 \\ 14 & 62 \end{pmatrix}\begin{pmatrix} z_0 \\ z_1 \end{pmatrix} = \begin{pmatrix} 10 \\ 43 \end{pmatrix} \Rightarrow \begin{cases} z_0 = 0.35 \\ z_1 = 0.62 \end{cases} \Rightarrow y = 0.35 + 0.62x$

b. $M = \begin{pmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 7 \\ 1 & 8 \end{pmatrix}$, $\vec{y} = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$, $M^TM = \begin{pmatrix} 4 & 21 \\ 21 & 133 \end{pmatrix}$, $M^T\vec{y} = \begin{pmatrix} 10 \\ 42 \end{pmatrix}$

$M^TM\vec{z} = M^T\vec{y} \Rightarrow \begin{pmatrix} 4 & 21 \\ 21 & 133 \end{pmatrix}\begin{pmatrix} z_0 \\ z_1 \end{pmatrix} = \begin{pmatrix} 10 \\ 42 \end{pmatrix} \Rightarrow \begin{cases} z_0 = 4.92 \\ z_1 = -0.46 \end{cases} \Rightarrow y = 4.92 - 0.46x$ ✓

c. $M = \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}$, $\vec{y} = \begin{pmatrix} -1 \\ 1 \\ 2 \\ 4 \\ 6 \end{pmatrix}$, $M^TM = \begin{pmatrix} 5 & 5 \\ 5 & 15 \end{pmatrix}$, $M^T\vec{y} = \begin{pmatrix} 12 \\ 29 \end{pmatrix}$

$M^TM\vec{z} = M^T\vec{y} \Rightarrow \begin{pmatrix} 5 & 5 \\ 5 & 15 \end{pmatrix}\begin{pmatrix} z_0 \\ z_1 \end{pmatrix} = \begin{pmatrix} 12 \\ 29 \end{pmatrix} \Rightarrow \begin{cases} z_0 = 0.7 \\ z_1 = 1.7 \end{cases} \Rightarrow y = 0.7 + 1.7x$

d. $M=\begin{pmatrix}1&-2\\1&-1\\1&0\\1&1\\1&2\end{pmatrix}$, $\vec{y}=\begin{pmatrix}3\\1\\0\\-2\\-4\end{pmatrix}$, $M^TM=\begin{pmatrix}5&0\\0&10\end{pmatrix}$, $M^T\vec{y}=\begin{pmatrix}-2\\-17\end{pmatrix}$

$M^TM\vec{z}=M^T\vec{y} \Rightarrow \begin{pmatrix}5&0\\0&10\end{pmatrix}\begin{pmatrix}z_0\\z_1\end{pmatrix}=\begin{pmatrix}-2\\-17\end{pmatrix} \Rightarrow \begin{cases}z_0=-0.4\\z_1=-1.7\end{cases} \Rightarrow y=-0.4-1.7x$ ✓

5.6.9. $M=\begin{pmatrix}1&50&18&10\\1&40&20&16\\1&35&14&10\\1&40&12&12\\1&30&16&14\end{pmatrix}$, $\vec{y}=\begin{pmatrix}28\\30\\21\\23\\23\end{pmatrix}$, $M^TM=\begin{pmatrix}5&195&80&62\\195&7825&3150&2390\\80&3150&1320&1008\\62&2390&1008&796\end{pmatrix}$, $M^T\vec{y}=\begin{pmatrix}125\\4945\\2042\\1568\end{pmatrix}$

$M^TM\vec{z}=M^T\vec{y} \Rightarrow \begin{cases}z_0=-5.19\\z_1=0.34\\z_2=0.51\\z_3=0.71\end{cases} \Rightarrow y=-5.19+0.34x_1+0.51x_2+0.71x_3$ ✓