Calculus Exercise Week 4

Section 4.5

225, 227, 231-245

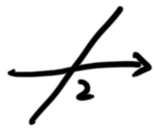
225. $f(x)=x^3-6x^2$

a. $f'(x)=3x^2-12x$. $f'(x)=0 \Rightarrow x=0$ or 4

$(-\infty,0)\cup(4,+\infty)\uparrow$, $(0,4)\downarrow$

b. minima $x=4$, maxima $x=0$

c. $f''(x)=6x-12$, $f''(x)=0 \Rightarrow x=2$

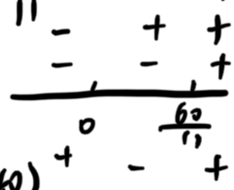


$(-\infty,2)$ concave down, $(2,+\infty)$ concave up

d. $x=2$

227. $f(x)=x^{11}-6x^{10}$ $\qquad x^9(11x-60)$

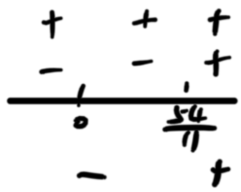
a. $f'(x)=11x^{10}-60x^9$, $f'(x)=0 \Rightarrow x=0$ or $\frac{60}{11}$



$(-\infty,0)\cup(\frac{60}{11},+\infty)\uparrow$, $(0,\frac{60}{11})\downarrow$

b. min $x=\frac{60}{11}$??, max $x=0$ $\quad x^8(110x-540)$

c. $f''(x)=110x^9-540x^8$, $f''(x)=0 \Rightarrow x=0$ or $\frac{54}{11}$



$(-\infty,\frac{54}{11})$ concave down, $(\frac{54}{11},+\infty)$ concave up

d. $x=\frac{54}{11}$

231-245

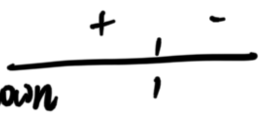
235. $f(x)=\frac{1}{1-x}$, $x\neq 1$

a. $f'(x)=-\frac{-1}{(1-x)^2}=\frac{1}{(1-x)^2}>0$

$(-\infty,1)\cup(1,+\infty)\uparrow$

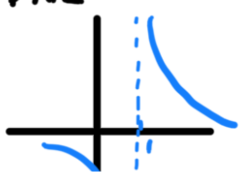
b. DNE

c. $f''(x)=-2\frac{1}{(1-x)^3}(-1)=\frac{2}{(1-x)^3}$



$(-\infty,1)$ concave up, $(1,+\infty)$ concave down

d. DNE



$\sqrt[3]{16}$ $\qquad 16^{-\frac{1}{2}}\cdot 16^{\frac{1}{6}}$

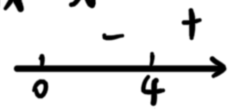
$\frac{1}{4}\sqrt{16^{\frac{1}{3}}} + \frac{1}{16^{\frac{1}{3}}} = \frac{1}{4}16^{\frac{1}{6}} + 16^{-\frac{1}{3}}$

$\frac{1}{2}+\frac{1}{4} = \frac{3}{4}$

239. $f(x) = \frac{1}{4}\sqrt{x} + \frac{1}{x},\ x>0$

a. $f'(x) = \frac{1}{4}\cdot\frac{1}{2}x^{-\frac{1}{2}} - x^{-2} = \frac{1}{8\sqrt{x}} - \frac{1}{x^2}$

$f'(x) = 0 \Rightarrow x = 4$



$(0,4)\downarrow,\ (4,+\infty)\uparrow$

$\frac{1}{8\sqrt{x}} - \frac{1}{x^2} = 0$

$8\sqrt{x} = x^2$

$8 = x^{\frac{3}{2}}$

$x = 8^{\frac{2}{3}}$

$\sqrt[3]{64}$

b. min $x=4$

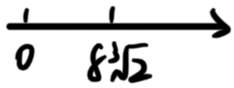
$\frac{1}{4}\sqrt{x}+\frac{1}{x}=0$

$x^{\frac{1}{2}}\ \ x^{-1}$

$x^{-\frac{3}{2}} = -\frac{1}{4}$

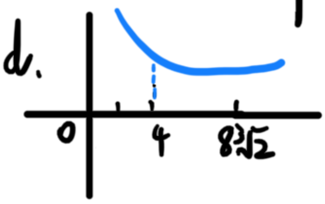
c. $f''(x) = -\frac{1}{16}x^{-\frac{3}{2}} + 2x^{-3}$

$f''(x) = 0 \Rightarrow x = 8\sqrt[3]{2}$



$(0, 8\sqrt[3]{2})$ concave up, $(8\sqrt[3]{2}, +\infty)$ concave down

d.



$2\sqrt{3}$

$x = (-\frac{1}{4})^{-\frac{2}{3}}$

$\sqrt{(-4)^2}$

$\sqrt[3]{16}$

$2\sqrt[3]{2}$

231-245

241. $f\uparrow,\ f'>0,\ f''<0$

243. $f\downarrow,\ f'<0,\ f''<0$

243. $f\uparrow,\ f'>0,\ f''>0$

Section 4.7

315 317 319 321

335 337

315. $\max_h V(h) = h(31-\frac{1}{2}h)^2 = \frac{1}{4}h^3 - 31h^2 + 961h$

s.t. $\begin{cases} h>0 \\ 31-\frac{1}{2}h>0 \end{cases} \Rightarrow 0<h<62$

$V'(h) = \frac{3}{4}h^2 - 62h + 961$

$V'(h) = 0 \Rightarrow h = \frac{62}{3}$ or $62 \xRightarrow{s.t.} h = \frac{62}{3}$

$x^{-\frac{1}{2}}$

$\frac{1}{4\sqrt{x}} = -\frac{1}{x}$

$-\frac{1}{4} = x^{-1}\cdot x^{\frac{1}{2}}$

$-\frac{1}{4} = x^{-\frac{3}{2}}$

$(-\frac{1}{4})^{-\frac{2}{3}} = x$

$[(-\frac{1}{4})^{-2}]^{\frac{1}{3}}$

317. $f(x) = x + \frac{1}{x},\ x\in\{1,2,\ldots\}$

$f'(x) = 1 - \frac{1}{x^2},\ f'(x)=0 \Rightarrow x=\pm 1 \xRightarrow{D(f)} x=1$

319, 321, 335, 337

319. width x, length $\frac{400}{2} - x = 200-x$

$A(x) = x(200-x) = -x^2+200x$

$$\text{s.t.} \begin{cases} x > 0 \\ 200 - x > 0 \end{cases} \Rightarrow 0 < x < 200$$

$$\max_{x} A(x)$$

$A'(x) = -2x + 200$, $A'(x) = 0 \Rightarrow x = 100$

width $= 100$, length $= 100$

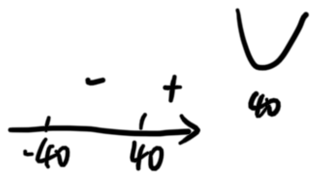
321. width $= x$, length $= \frac{1600}{x}$

$$f(x) = \left(x + \frac{1600}{x}\right) \times 2 = 2x + \frac{3200}{x}$$

$$\text{s.t.} \begin{cases} x > 0 \\ \frac{1600}{x} > 0 \end{cases} \Rightarrow x > 0$$

$$f'(x) = 2 + 3200\left(-\frac{1}{x^2}\right) = 2 - \frac{3200}{x^2}$$

$f'(x) = 0 \Rightarrow x = \pm 40 \xRightarrow{\text{s.t.}} x = 40$



width $= 40$, length $= 40$

335. $2\pi r + 4x = 4 \Rightarrow r = \frac{2 - 2x}{\pi}$

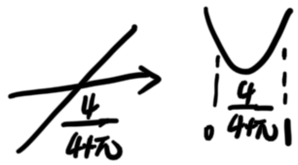
$$A(x) = \pi r^2 + x^2 = \pi\left(\frac{2-2x}{\pi}\right)^2 + x^2 = \frac{4+\pi}{\pi}x^2 - \frac{8}{\pi}x + \frac{4}{\pi}$$

$$\max_{x} A(x)$$

$\frac{4+\pi}{\pi} - \frac{8}{\pi} + \frac{4}{\pi}$

$$\text{s.t.} \begin{cases} 0 \le 2\pi r \le 4 \\ 0 \le 4x \le 4 \\ r = \frac{2-2x}{\pi} \end{cases} \Rightarrow 0 \le x \le 1$$

$$A'(x) = \frac{8 + 2\pi}{\pi}x - \frac{8}{\pi}$$



$A'(x) = 0 \Rightarrow x = \frac{4}{4+\pi}$

$\left(0, \frac{4}{4+\pi}\right) \downarrow, \left(\frac{4}{4+\pi}, 1\right) \uparrow$

$A(0) = \frac{4}{\pi}$, $A(1) = 1 \Rightarrow A_{max} = A(0)$

$x = 0$, $r = \frac{2}{\pi}$

337. $x + y = 10 \Rightarrow y = 10 - x$

$$f = xy = x(10 - x) = -x^2 + 10x$$

$$\text{s.t.} \begin{cases} x_1 \ge 0 \\ 10 - x_1 \ge 0 \end{cases} \Rightarrow 0 \le x_1 \le 10$$

$$f' = -2x_1 + 10, \quad f' = 0 \Rightarrow x_1 = 5$$

$$f_{max} = f(5) = 25$$

$$f(0) = 0, \; f(10) = 0 \Rightarrow f_{min} = f(0) = f(10) = 0$$