2.2.3 a.(i) $A=\begin{pmatrix}3&-2&0\\5&-4&1\end{pmatrix}^{2\times3}$, $x=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}^{3\times1}$

$$Ax = x_1\begin{pmatrix}3\\5\end{pmatrix}+x_2\begin{pmatrix}-2\\-4\end{pmatrix}+x_3\begin{pmatrix}0\\1\end{pmatrix}$$

$$=\begin{pmatrix}3x_1-2x_2\\5x_1-4x_2+x_3\end{pmatrix}$$ ✓

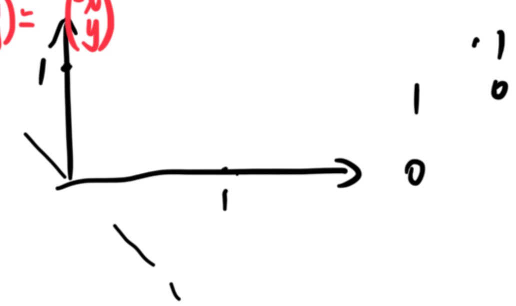
(ii) $$Ax=\begin{pmatrix}(3\ \ -2\ \ 0)x\\(5\ \ -4\ \ 1)x\end{pmatrix}=\begin{pmatrix}3x_1-2x_2\\5x_1-4x_2+x_3\end{pmatrix}$$ ✓

b.(i) $$Ax = x_1\begin{pmatrix}1\\0\end{pmatrix}+x_2\begin{pmatrix}2\\-4\end{pmatrix}+x_3\begin{pmatrix}3\\5\end{pmatrix}$$

$$=\begin{pmatrix}x_1+2x_2+3x_3\\-4x_2+5x_3\end{pmatrix}$$ ✓

(ii) $$Ax=\begin{pmatrix}(1\ \ 2\ \ 3)x\\(0\ \ -4\ \ 5)x\end{pmatrix}=\begin{pmatrix}x_1+2x_2+3x_3\\-4x_2+5x_3\end{pmatrix}$$ ✓

2.2.11 a. $\begin{pmatrix}-1&0\\0&1\end{pmatrix}$ $T\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-x\\y\end{pmatrix}$



b. $\begin{pmatrix}0&1\\1&0\end{pmatrix}$ ✓

c. $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$ ✓

d. $\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ ✓

2.2.12 $$\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$$ ✓

2×1

$\begin{matrix}1\\0\end{matrix}$

$$\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$$

$x=\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$

2.2.3 c.(i) $$Ax=\begin{pmatrix}-2\\1\\-5\end{pmatrix}\cdot0+\begin{pmatrix}0\\2\\6\end{pmatrix}\cdot1+\begin{pmatrix}5\\0\\-7\end{pmatrix}\cdot0+\begin{pmatrix}4\\3\\8\end{pmatrix}\cdot-1$$

$$=\begin{pmatrix}-4\\-1\\-2\end{pmatrix}$$ ✓

(ii) $$Ax=\begin{pmatrix}(-2\ \ 0\ \ 5\ \ 4)x\\(1\ \ 2\ \ 0\ \ 3)x\\(-5\ \ 6\ \ -7\ \ 8)x\end{pmatrix}=\begin{pmatrix}-4\\-1\\-2\end{pmatrix}$$ ✓

d(i) $$Ax=\begin{pmatrix}3\\0\\-8\end{pmatrix}\cdot1+\begin{pmatrix}-4\\2\\7\end{pmatrix}\cdot0+\begin{pmatrix}1\\1\\-3\end{pmatrix}\cdot0+\begin{pmatrix}6\\5\\0\end{pmatrix}\cdot0=\begin{pmatrix}3\\0\\-8\end{pmatrix}$$ ✓

(ii) $$Ax=\begin{pmatrix}(3\ \ -4\ \ 1\ \ 6)\vec{e}_1\\(0\ \ 2\ \ 1\ \ 5)\vec{e}_1\\(-8\ \ 7\ \ -3\ \ 0)\vec{e}_1\end{pmatrix}=\begin{pmatrix}3\\0\\-8\end{pmatrix}$$ ✓

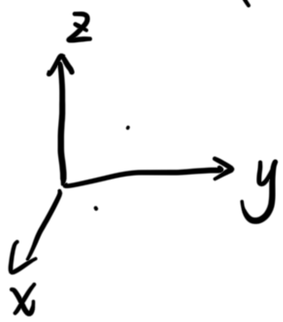
d(i) $Ax = \begin{pmatrix}3\\0\\-8\end{pmatrix}\cdot 0 + \begin{pmatrix}-4\\2\\7\end{pmatrix}\cdot 1 + \begin{pmatrix}1\\1\\-3\end{pmatrix}\cdot 0 + \begin{pmatrix}6\\5\\0\end{pmatrix}\cdot 0 = \begin{pmatrix}-4\\2\\7\end{pmatrix}$

(ii) $Ax = \begin{pmatrix}(3 \; -4 \; 1 \; 6)\vec{e_2}\\(0 \; 2 \; 1.5)\vec{e_2}\\(-8 \; 7 \; -3 \; 0)\vec{e_2}\end{pmatrix} = \begin{pmatrix}-4\\2\\7\end{pmatrix}$

...

2.6.3 b. $A = [T(\vec{e_1}), T(\vec{e_2})]$
$= \left(\begin{pmatrix}1\\0\end{pmatrix}\cdot(-1), \begin{pmatrix}0\\1\end{pmatrix}\cdot(-1)\right)$
$= \begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$

$\begin{pmatrix}1\\0\\0\end{pmatrix}$



2.6.4 a. $A = \begin{pmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{pmatrix}$

b. $A = \begin{pmatrix}-1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}$

2.6.5 a. $T(a\vec{x_1}+b\vec{x_2}) = aT(\vec{x_1})+bT(\vec{x_2}) = a\cdot\vec{0}+b\cdot\vec{0} = \vec{0}$

$\therefore a\vec{x_1}+b\vec{x_2}$ is in the kernel.

b. $\vec{y_1} = T(\vec{x_1}),\ \vec{y_2} = T(\vec{x_2})$

$a\vec{y_1}+b\vec{y_2} = aT(\vec{x_1})+bT(\vec{x_2}) = T(a\vec{x_1}+b\vec{x_2})$

$\therefore a\vec{y_1}+b\vec{y_2}$ is in the image of T.

2.6.7.a. $T\begin{pmatrix}x\\y\end{pmatrix}^{2\times1} = \begin{pmatrix}xy\\0\end{pmatrix}^{2\times1}$

$T\begin{pmatrix}x_1+x_2\\y_1+y_2\end{pmatrix} = \begin{pmatrix}(x_1+x_2)(y_1+y_2)\\0\end{pmatrix}$

$T\begin{pmatrix}x_1\\y_1\end{pmatrix}+T\begin{pmatrix}x_2\\y_2\end{pmatrix} = \begin{pmatrix}x_1y_1\\0\end{pmatrix}+\begin{pmatrix}x_2y_2\\0\end{pmatrix} = \begin{pmatrix}x_1y_1+x_2y_2\\0\end{pmatrix} \neq T\begin{pmatrix}x_1+x_2\\y_1+y_2\end{pmatrix}$

$T(\vec{e_1}) = \begin{pmatrix}0\\0\end{pmatrix}$

$T(\vec{e_2}) = \begin{pmatrix}0\\0\end{pmatrix}$

$A = [T(\vec{e_1}), T(\vec{e_2})] = \begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$

$A\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \neq \begin{pmatrix}xy\\0\end{pmatrix}$

b. $T\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}0\\y^2\end{pmatrix}$

$T(\vec{e_1}) = \begin{pmatrix}0\\0\end{pmatrix},\ T(\vec{e_2}) = \begin{pmatrix}0\\1\end{pmatrix}$

$T\left(2\begin{pmatrix}0\\2\end{pmatrix}\right) = T\begin{pmatrix}0\\4\end{pmatrix} = \begin{pmatrix}0\\16\end{pmatrix}$

$2T\begin{pmatrix}0\\2\end{pmatrix} = 2\cdot\begin{pmatrix}0\\4\end{pmatrix} = \begin{pmatrix}0\\8\end{pmatrix}$

$$A = [T(\vec{e}_1), T(\vec{e}_2)] = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$ $T(2\vec{x}) \neq 2T(\vec{x})$

$$A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ y^2 \end{pmatrix}$$