1.1.14 a. False, suppose $n=3, m=2$, $\begin{cases} x_1+x_2+3x_3=5 \\ x_1-6x_2+x_3=1 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 3 & 5 \\ 1 & -6 & 1 & 1 \end{array}\right) \Rightarrow$ 2 rows $\neq n$ rows ✓

b. False, if r(augmented matrix) $= n$, only 1 solution. ✓

c. True, $r(A|b)=r(A)$, row operations don't change $r(A|b)$, thus $r[(A|b)']=r(A)$

d. True $r[(A|b)'] > r(A) \Rightarrow r(A|b) > r(A)$

1.2.12 $\underset{m\times n}{A} = C|b$

a. False, if $r(A)=m<n \Rightarrow$ more than 1 solution but no zero row.

$\begin{cases} x_1 \quad\quad +x_4=2 \\ x_2 \quad -3x_4=0 \\ x_3-x_4=1 \end{cases} \Rightarrow \vec{x}=\begin{pmatrix} 2-t \\ 3t \\ 1+t \\ t \end{pmatrix}, A=\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right)$ ✓

b. False, $\begin{cases} x_1+2x_2=2 \\ x_2=1 \\ 0=0 \end{cases} \Rightarrow \vec{x}=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $r(A)=n<m$ A has a zero row, but only 1 solution. ✓

c. True, no solution $r(C)<r(A)$ ✓

d. False, $\begin{cases} x_1+2x_2=2 \\ x_2=1 \\ 0=0 \end{cases} \Rightarrow C=\begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, b=\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, A=\left(\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right)$, C has zero row, but 1 solution ✓

e. True, use $\vec{0}$ as constant $\Rightarrow$ homogeneous system. ✓

f. False, $\begin{cases} x_1+2x_2=2 \\ x_2=1 \\ 0=0 \end{cases}$ is consistent, $\begin{cases} x_1+2x_2=2 \\ x_2=1 \\ 0=3 \end{cases}$ is inconsistent. ✓

1.3.1. a. False, $C_{m\times n}$ $x_1-x_2=0 \Rightarrow \vec{x}=t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ homogenous but nontrivial solution ✓

b. False, $x_1-x_2=0 \Rightarrow \vec{x}=t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, nontrivial solution but homogenous. ✓

c. True. $C\vec{x}=\vec{b}$, $\vec{x}=\vec{0} \Rightarrow C\cdot\vec{x}=C\cdot\vec{0}=\vec{0} \Rightarrow \vec{b}=\vec{0}$ ✓

d. False, $x_1-x_2=1 \Rightarrow \vec{x}=\begin{pmatrix} 1+t \\ t \end{pmatrix} \Rightarrow$ consistent but not homogeneous. ✓

e. $A=C|\vec{0}$, False, $\vec{x}=\vec{0}$ is always a solution ✓

f. False, $\begin{cases} x_1 \quad =0 \\ x_2=0 \end{cases} \Rightarrow \vec{x}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is the only 1 solution ✓

g. nontrivial solutions $\Rightarrow r(A)<n$, False, $\begin{cases} x_1 \quad +x_3=0 \\ x_2-x_3=0 \end{cases} \Rightarrow \vec{x}=\begin{pmatrix} -t \\ t \\ t \end{pmatrix}=t\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow$

no zero row ✓

h. False, $\begin{cases} x_1 \quad =0 \\ x_2=0 \end{cases} \Rightarrow \vec{x}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow$ zero row but only trivial solution ✓

$1 \quad 0=0$

i. True, row operation to $\vec{0}$ generates $\vec{0}$. ✓

2.1.19. a. True, $A+B=A+C \Rightarrow B=C$ ✓

b. False, $A=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $B=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$, $A+B=O_{2\times2}$, but $B\neq O_{2\times2}$ ✓

c. False, $A=[a_{ij}]$, $A^T=[b_{ij}]$, $b_{ij}=a_{ji}$, $b_{13}=a_{31}=5$ ✓

d. True, $A_{m\times n}=[a_{ij}]$, main diagonal of $A=\{a_{11},a_{22},\cdots,a_{kk}\}$ $k=\min(m,n)$

$A^T{}_{n\times m}=[b_{ij}]=[a_{ji}]$, main diagonal of $A^T=\{b_{11},b_{22},\cdots,b_{kk}\}$, $k=\min(m,n)$

$=\{a_{11},a_{22},\cdots,a_{kk}\}$ ✓

e. True, $A=(A^T)^T=(3B)^T=3B^T=3B$ ✓

f. True, $(kA+mB)^T=kA^T+mB^T=kA+mB$ $\therefore kA+mB$ is symmetric. ✓

2.2.10 a. True, $\begin{pmatrix}3\\2\end{pmatrix}=3\begin{pmatrix}1\\0\end{pmatrix}+2\begin{pmatrix}0\\1\end{pmatrix}$ ✓

b. False, $\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix} \Rightarrow A\vec{x}$ has zero entry, but A doesn't have zero row. ✓

c. False, $\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix} \Rightarrow A\vec{x}=\vec{0}$ but $A\neq O_{2\times2}$ ✓

d. True, $a_1\vec{u}_1+a_2\vec{u}_2+\cdots+a_k\vec{u}_k=\underbrace{(\vec{u}_1,\vec{u}_2,\cdots,\vec{u}_k)}_{A_{11}}\underbrace{\begin{pmatrix}a_1\\a_2\\\cdots\\a_k\end{pmatrix}}_{\vec{x}}=A_{n\times k}\vec{x}$ ✓

e. True, $A\vec{x}=(\vec{a}_1,\vec{a}_2,\vec{a}_3)\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=x_1\vec{a}_1+x_2\vec{a}_2+x_3\vec{a}_3=\vec{b}=3\vec{a}_1-2\vec{a}_2 \Rightarrow x_1=3, x_2=-2, x_3=0$

$\Rightarrow \vec{x}=\begin{pmatrix}3\\-2\\0\end{pmatrix}$ ✓

f. False, the solution $\vec{x}=\begin{pmatrix}1\\1\\1\end{pmatrix}$, $\vec{b}=A\vec{x}=\vec{a}_1+\vec{a}_2+\vec{a}_3\neq s\vec{a}_1+t\vec{a}_2$ ✓

$r(A)\le m$ $\quad A=\begin{pmatrix}1&2&3\\0&0&0\end{pmatrix}$, $\vec{b}=\begin{pmatrix}1\\2\end{pmatrix}$ no solution

False

g. ~~True~~, $r([A|b]_{m\times(n+1)})\le m<n \Rightarrow$ indefinite solutions

h. False, $A=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{pmatrix}$, $\vec{b}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$, $A\vec{x}=\vec{b}\Rightarrow\vec{x}=\begin{pmatrix}0\\0\\0\end{pmatrix}$, but $\vec{b}_1=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$, $A\vec{x}=\vec{b}_1\Rightarrow$ no solution ✓

i. True, $A(\vec{x}_1-\vec{x}_2)=A\vec{x}_1-A\vec{x}_2=\vec{b}-\vec{b}=\vec{0}$ ✓

j. True, $A\vec{x}=(\vec{a}_1,\vec{a}_2,\vec{a}_3)\begin{pmatrix}s\\t\\-1\end{pmatrix}=s\vec{a}_1+t\vec{a}_2-\vec{a}_3=\vec{0}$ ✓

(-1)

2.3.27 a. False, $A=\begin{pmatrix}2&-1\\3&-2\end{pmatrix}$, $A\cdot A=I$ ✓ $A=\begin{pmatrix}1&2\\1&2\end{pmatrix}$ $J=\begin{pmatrix}-2&-2\\1&1\end{pmatrix}$

b. True, False $AJ=A\Rightarrow AJ-A=0\Rightarrow A(J-I)=0\Rightarrow J-I=0\Rightarrow J=I$

c. True, $(A^T)^3=A^TA^TA^T=(AAA)^T=(A^3)^T$ ✓ $AB=0 \nRightarrow A=0$ or $B=0$

d. True, $(I+A)^T=I^T+A^T=I+A$ ✓ $A=\begin{pmatrix}1&2\\1&2\end{pmatrix}$ $C=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $B=\begin{pmatrix}-1&-2\\1&2\end{pmatrix}$

e. True, False $AB=AC\Rightarrow AB-AC=0\Rightarrow A(B-C)=0,\ A\neq 0\Rightarrow B-C=0\Rightarrow B=C$

f. False, $A=\begin{pmatrix}-1&1\\-1&1\end{pmatrix}$ $A\cdot A=\begin{pmatrix}0&0\\0&0\end{pmatrix}$ ✓

g. False, $A=\begin{pmatrix}1&2\\0&0\end{pmatrix}$, $B=\begin{pmatrix}1&1\\1&1\end{pmatrix}$, $BA=\begin{pmatrix}1&2\\1&2\end{pmatrix}$ no zero row. ✓

h. True, $A(A+B)=(A+B)A\Rightarrow A^2+AB=A^2+BA\Rightarrow AB=BA$ ✓

i. True, $B=(\vec{b}_1,\vec{b}_2,\vec{0})$ $AB=A(\vec{b}_1,\vec{b}_2,\vec{0})=(A\vec{b}_1,A\vec{b}_2,\vec{0})$ ✓

j. False, $A=(1\quad 1)$, $B=\begin{pmatrix}1&1\\1&-1\end{pmatrix}$, $AB=(2\quad 0)$. AB has zero col, but B doesn't ✓

k. True, $A=\begin{pmatrix}\vec{a}_1\\\vec{0}\end{pmatrix}\Rightarrow AB=\begin{pmatrix}\vec{a}_1B\\\vec{0}B\end{pmatrix}=\begin{pmatrix}\vec{a}_1B\\\vec{0}\end{pmatrix}$ ✓

l. False, $A=\begin{pmatrix}1&1\\1&-1\end{pmatrix}$, $B=\begin{pmatrix}1\\1\end{pmatrix}$, $AB=\begin{pmatrix}2\\0\end{pmatrix}$ ✓

2.4.9 e. False, $A^2=A\Rightarrow |A^2|=|A|^2=|A|\Rightarrow |A|(|A|-1)=0\Rightarrow |A|=0$, $A=\begin{pmatrix}1&0\\0&0\end{pmatrix}$ ✓

f. False $|AB|=|A||B|=|B|\Rightarrow |B|(|A|-1)=0\Rightarrow |B|=0$ or $|A|=1$

$A=\begin{pmatrix}1&0\\0&0\end{pmatrix}$ $B=\begin{pmatrix}1&0\\0&0\end{pmatrix}$ $AB=B$ ✓

g. True, $(A^{-1})^T=(A^T)^{-1}=(-A)^{-1}=-A^{-1}$ ✓

h. True, $|A^2|=|A|^2\neq 0\Rightarrow |A|\neq 0\Rightarrow A$ inv ✓

i. ~~False~~, True. $AB=I\Rightarrow BA=I$

3.1.9 a. False, $A=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $B=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$, $A+B=\begin{pmatrix}0&0\\0&0\end{pmatrix}$. $|A+B|\neq |A|+|B|$

b. False, $A=\begin{pmatrix}1&2\\0&0\end{pmatrix}$, $|A|=0$

c. True

d. True

e. True, $A=\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}$, $7A=\begin{pmatrix}7a_{11}&7a_{12}\\7a_{21}&7a_{22}\end{pmatrix}$ $|7A|=49a_{11}a_{22}-49a_{12}a_{21}=49|A|$

f. False, $A=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $|A|=1$, $|A^T|=1\neq -|A|$

g. False, $A=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $|A|=1$, $-A=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$, $|-A|=1\neq-|A|$

h. False, $A=\begin{pmatrix}1&1\\0&1\end{pmatrix}$, $B=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $|A|=|B|=1$, $A\neq B$