Calculus Exercise week7

Section 4.6

263 263. $f(x,y)=xy$, $P(0,-2)$, $\vec{v}=(\frac{1}{2},\frac{\sqrt{3}}{2})$

265 $\because f_x(x,y)=y,\ f_y(x,y)=x$

$\therefore \nabla f(x,y)=(f_x(x,y),f_y(x,y))=(y,x)$

$\therefore \nabla f(0,-2)=(-2,0)$

$\therefore D_{\vec{v}}f(0,-2)=\nabla f(0,-2)\cdot\vec{v}=(-2,0)\cdot(\frac{1}{2},\frac{\sqrt{3}}{2})=-2\times\frac{1}{2}+0\times\frac{\sqrt{3}}{2}=-1$

265. $h(x,y,z)=xyz$, $P(2,1,1)$, $\vec{v}=(2,1,-1)$

$\because \|\vec{v}\|=\sqrt{2^2+1^2+(-1)^2}=\sqrt{6}\neq 1$ $\therefore$ we need to normalize $\vec{v}$

$\vec{v}_n=\frac{\vec{v}}{\|\vec{v}\|}=\frac{(2,1,-1)}{\sqrt{6}}=(\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{6})$

$\because h_x(x,y,z)=yz,\ h_y(x,y,z)=xz,\ h_z(x,y,z)=xy$

$\therefore \nabla h(x,y,z)=(h_x(x,y,z),h_y(x,y,z),h_z(x,y,z))=(yz,xz,xy)$

$\therefore \nabla h(2,1,1)=(1,2,2)$

$\therefore D_{\vec{v}}h(2,1,1)=\nabla h(2,1,1)\cdot\vec{v}_n=(1,2,2)\cdot(\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{6})=\frac{\sqrt{6}}{3}$

267. $f(x,y)=x^2-y^2$, $\vec{u}=(\frac{\sqrt{3}}{2},\frac{1}{2})$, $P(1,0)$

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271 $\because f_x(x,y)=2x,\ f_y(x,y)=-2y$

281 $\therefore \nabla f(x,y)=(f_x(x,y),f_y(x,y))=(2x,-2y)$

$\therefore \nabla f(1,0)=(2,0)$

$\therefore D_{\vec{u}}f(1,0)=\nabla f(1,0)\cdot\vec{u}=(2,0)\cdot(\frac{\sqrt{3}}{2},\frac{1}{2})=\sqrt{3}$

271. $f(x,y)=\ln(x^2+y^2)$, $\vec{u}=(\frac{3}{5},\frac{4}{5})$, $P=(1,2)$

$\because f_x(x,y)=\frac{2x}{x^2+y^2},\ f_y(x,y)=\frac{2y}{x^2+y^2}$

$\therefore \nabla f(x,y)=(f_x(x,y),f_y(x,y))=(\frac{2x}{x^2+y^2},\frac{2y}{x^2+y^2})$

$\therefore \nabla f(1,2)=(\frac{2}{5},\frac{4}{5})$

$\therefore D_{\vec{u}}f(1,2)=\nabla f(1,2)\cdot\vec{u}=(\frac{2}{5},\frac{4}{5})\cdot(\frac{3}{5},\frac{4}{5})=\frac{22}{25}$

281. $f(x,y,z)=xy+yz+xz$, $P(1,2,3)$

$\because f_x(x,y,z) = y+z,\ f_y(x,y,z) = x+z,\ f_z(x,y,z) = y+x$

$\therefore \nabla f(x,y,z) = (f_x(x,y,z), f_y(x,y,z), f_z(x,y,z))$

$= (y+z, x+z, y+x)$

$\therefore \nabla f(1,2,3) = (5,4,3)$

Section 4.7

311 311. $f(x,y) = (3x-2)^2 + (y-4)^2$

323 $f_x(x,y) = 2(3x-2)\cdot 3 = 18x-12$

$f_y(x,y) = 2(y-4) = 2y-8$

$f_x(x,y) = f_y(x,y) = 0 \Rightarrow \begin{cases} 18x-12=0 \\ 2y-8=0 \end{cases} \Rightarrow \begin{cases} x=\frac{2}{3} \\ y=4 \end{cases}$

critical points: $(\frac{2}{3}, 4)$

323. $f(x,y) = x^2+4xy+y^2$

$\because f_x(x,y) = 2x+4y,\ f_y(x,y) = 4x+2y$

$f_x(x,y) = f_y(x,y) = 0 \Rightarrow \begin{cases} 2x+4y=0 \\ 4x+2y=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases}$

$\therefore$ critical points are $(0,0)$

$\because f_{xx}(x,y) = 2,\ f_{xy}(x,y) = 4,\ f_{yy}(x,y) = 2$

$\therefore D = f_{xx}(x,y) f_{yy}(x,y) - [f_{xy}(x,y)]^2 = 2\cdot 2 - 4^2 = -12$

$\therefore D(0,0) = -12 < 0$

$\therefore (0,0)$ is saddle point. No maximum, minimum.

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325. $f(x,y) = 9 - x^4y^4$

$\because f_x(x,y) = -4x^3y^4,\ f_y(x,y) = -4x^4y^3$

$f_x(x,y) = f_y(x,y) = 0 = \begin{cases} -4x^3y^4=0 \\ -4x^4y^3=0 \end{cases} \Rightarrow x=0 \text{ or } y=0$

$\therefore$ critical points are $x=0$ or $y=0$

$\because f_{xx}(x,y)=-12x^2y^4,\ f_{xy}(x,y)=-16x^3y^3,\ f_{yy}(x,y)=-12x^4y^2$

$\therefore D=f_{xx}(x,y)f_{yy}(x,y)-[f_{xy}(x,y)]^2=-112x^6y^6$

① For points at $x=0$:

$D(0,y)=0$

So there is no conclusion.

② For points at $y=0$:

$D(x,0)=0$

So there is no conclusion.

333. $f(x,y)=e^{-(x^2+y^2+2x)}$

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$\because f_x(x,y)=e^{-(x^2+y^2+2x)}\cdot[-(2x+2)]=(-2x-2)e^{-(x^2+y^2+2x)}$

$f_y(x,y)=e^{-(x^2+y^2+2x)}\cdot(-2y)=-2ye^{-(x^2+y^2+2x)}$

$$f_x(x,y)=f_y(x,y)=0\Rightarrow\begin{cases}(-2x-2)e^{-(x^2+y^2+2x)}=0\\ -2ye^{-(x^2+y^2+2x)}=0\end{cases}\Rightarrow\begin{cases}x=-1\\ y=0\end{cases}$$

$\therefore$ critical point is $(-1,0)$

$\because f_{xx}(x,y)=-2e^{-(x^2+y^2+2x)}+(-2x-2)e^{-(x^2+y^2+2x)}[-(2x+2)]$

$=(4x^2+8x+2)e^{-(x^2+y^2+2x)}$

$f_{xy}(x,y)=(-2x-2)e^{-(x^2+y^2+2x)}(-2y)=(4xy+4y)e^{-(x^2+y^2+2x)}$

$f_{yy}(x,y)=-2e^{-(x^2+y^2+2x)}+(-2y)e^{-(x^2+y^2+2x)}(-2y)$

$=(4y^2-2)e^{-(x^2+y^2+2x)}$