Exercises Lecture 1: Sample spaces

1. We toss a coin repeatedly until the *first* head turns up. The outcome of the experiment is the number of tosses it takes to have this first occurrence of a head. What is the sample space of this experiment?

   Solution:
   The sample space is the space of all positive integers, i.e., $\Omega = \{1, 2, 3, \ldots\}$.

2. We toss a coin repeatedly until a head occurs for the *second* time. The outcome of the experiment is the number of tosses it takes to have a head for the second time. What is the sample space of this experiment?

   Solution:
   The sample space is, $\Omega = \{2, 3, 4, \ldots\}$.

3. MediaMarkt has two service desks to help its customers. If you are interested in the *total* number of customers that will be queued in front of the two help desks, what would be the sample space?

   Solution:
   If N is the capacity of MediaMarkt, the sample space is, $\Omega = \{0, 1, 2, 3, \ldots, N\}$.

4. We toss a coin three times. Let $\Omega = \{HHH, THH, HTH, HHT, TTH, THT, HTT, TTT\}$ be the sample space of this experiment, where T stands for tails and H for heads.

   (i) Write down the set of outcomes corresponding to each of the following events:
   A : "we throw tails exactly two times."
   B : "we throw tails at least two times."
   C : "tails did not appear before a head appeared."
   D : "the first throw results in tails."

   (ii) Write down the set of outcomes corresponding to each of the following events: A^C, $A \cup (C \cap D)$, and $A \cap D^C$.

   Solution:

(i) $A = \{TTH, THT, HTT\}, B = \{TTH, THT, HTT, TTT\}, C = \{HHH, HHT, HTT, TTT\}$, $D = \{THH, TTH, THT, TTT\}$.

(ii) $A^C = \{HHH, THH, HTH, HHT, TTT\}$, $A \cup (C \cap D) = \{TTH, THT, HTT, TTT\}$, $A \cap D^C = \{HTT\}$.

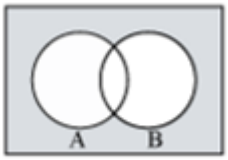
5. Let the events A and B in the sample space Ω.

(i) Draw in the Venn diagram the events $(A \cup B)^C$ and $A^C \cap B^C$. What do you observe?

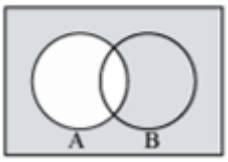
(ii) Draw in the Venn diagram the events $(A \cap B)^C$ and $A^C \cup B^C$. What do you observe?

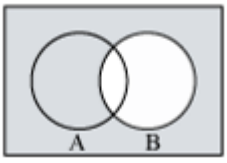
Solution:

(i) The shaded area in the figure below represents the Venn diagram for $(A \cup B)^C$.

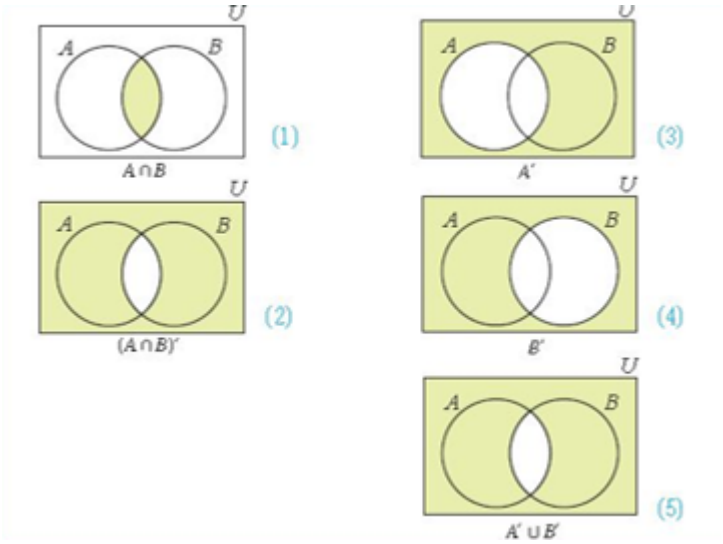


A^C and B^C are shown respectively by the shaded areas in the figures below:





(ii) Note that A^C is denoted by A' in the figures below. $(A \cap B)^C$ is shown using Figures (1) and (2) and $A^C \cup B^C$ is depicted using Figures (3)-(5).



6. Let A and B be arbitrary events. Let C be the event that either A occurs or B occurs, but not both. Express C in terms of A and B using any of the basic operations of union, intersection and complement.

   Solution: We can express A occurs (and not B) as: $A \cap B^C$. We express B occurs (and not A) as: $B \cap A^C$.
   Thus, $C = \{A$ or B occurs but not both$\}$ is $(A \cap B^C) \cup (B \cap A^C)$.

   Remember, that with "and" we use $\cap$ and with "or" we use $\cup$. We can visualize this by the shaded area in the following diagram: