2.1.3 a. $3A-2B = 3\cdot\begin{pmatrix}2 & 1\\ 0 & -1\end{pmatrix} - 2\cdot\begin{pmatrix}3 & -1 & 2\\ 0 & 1 & 4\end{pmatrix}$ DNE

b. $5C = 5\cdot\begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix} = \begin{pmatrix}15 & -5\\ 10 & 0\end{pmatrix}$ ✓

c. $3E^T = 3\cdot\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}^T = 3\begin{pmatrix}1 & 0\\ 0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}3 & 0\\ 0 & 3\\ 3 & 0\end{pmatrix}$

d. $B+D = \begin{pmatrix}3 & -1 & 2\\ 0 & 1 & 4\end{pmatrix} + \begin{pmatrix}1 & 3\\ -1 & 0\\ 1 & 4\end{pmatrix}$ DNE ✓

e. $4A^T-3C = 4\begin{pmatrix}2 & 1\\ 0 & -1\end{pmatrix}^T - 3\begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix} = 4\cdot\begin{pmatrix}2 & 0\\ 1 & -1\end{pmatrix} - 3\cdot\begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix} = \begin{pmatrix}-1 & 3\\ -2 & -4\end{pmatrix}$

f. $(A+C)^T = \left[\begin{pmatrix}2 & 1\\ 0 & -1\end{pmatrix} + \begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix}\right]^T = \begin{pmatrix}5 & 0\\ 2 & -1\end{pmatrix}^T = \begin{pmatrix}5 & 2\\ 0 & -1\end{pmatrix}$ ✓

g. $2B-3E = 2\cdot\begin{pmatrix}3 & -1 & 2\\ 0 & 1 & 4\end{pmatrix} - 3\cdot\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix} = \begin{pmatrix}3 & -2 & 1\\ 0 & -1 & 8\end{pmatrix}$

h. $A-D = \begin{pmatrix}2 & 1\\ 0 & -1\end{pmatrix} - \begin{pmatrix}1 & 3\\ -1 & 0\\ 1 & 4\end{pmatrix}$ DNE ✓

i. $(B-2E)^T = \left[\begin{pmatrix}3 & -1 & 2\\ 0 & 1 & 4\end{pmatrix} - 2\cdot\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\right]^T = \begin{pmatrix}1 & -1 & 0\\ 0 & -1 & 4\end{pmatrix}^T = \begin{pmatrix}1 & 0\\ -1 & -1\\ 0 & 4\end{pmatrix}$

2.1.5 a. $A+B = 3A+2B \Rightarrow -2A = B \Rightarrow A = -\frac{1}{2}B$

b. $2A-B = 5(A+2B) \Rightarrow -3A = 11B \Rightarrow A = -\frac{11}{3}B$ ✓

2.3.1 a. $\begin{pmatrix}1 & 3\\ 0 & -2\end{pmatrix}\begin{pmatrix}2 & -1\\ 0 & 1\end{pmatrix} = \left[\begin{pmatrix}1 & 3\\ 0 & -2\end{pmatrix}\begin{pmatrix}2\\ 0\end{pmatrix}, \begin{pmatrix}1 & 3\\ 0 & -2\end{pmatrix}\begin{pmatrix}-1\\ 1\end{pmatrix}\right] = \left[\begin{pmatrix}2\\ 0\end{pmatrix}, \begin{pmatrix}2\\ -2\end{pmatrix}\right] = \begin{pmatrix}2 & 2\\ 0 & -2\end{pmatrix}$

c. $\begin{pmatrix}5 & 0 & -7\\ 1 & 5 & 9\end{pmatrix}\begin{pmatrix}3\\ 1\\ -1\end{pmatrix} = \begin{pmatrix}22\\ -1\end{pmatrix}$

d. $\overset{1\times3}{\begin{pmatrix}1 & 3 & -3\end{pmatrix}}\overset{3\times2}{\begin{pmatrix}3 & 0\\ -2 & 1\\ 0 & 6\end{pmatrix}} = \left[\begin{pmatrix}1 & 3 & -3\end{pmatrix}\begin{pmatrix}3\\ -2\\ 0\end{pmatrix}, \begin{pmatrix}1 & 3 & -3\end{pmatrix}\begin{pmatrix}0\\ 1\\ 6\end{pmatrix}\right] = (-3, -15)$ ✓

f. $\begin{pmatrix}1 & -1 & 3\end{pmatrix}\begin{pmatrix}2\\ 1\\ -8\end{pmatrix} = -23$ ✓

g. $\begin{pmatrix}2\\ 1\\ -7\end{pmatrix}\begin{pmatrix}1 & -1 & 3\end{pmatrix} = \left[\begin{pmatrix}2\\ 1\\ -7\end{pmatrix}\cdot 1, \begin{pmatrix}2\\ 1\\ -7\end{pmatrix}\cdot(-1), \begin{pmatrix}2\\ 1\\ -7\end{pmatrix}\cdot 3\right] = \begin{pmatrix}2 & -2 & 6\\ 1 & -1 & 3\\ -7 & 7 & -21\end{pmatrix}$

2.3.2 b. $A^2 = \overset{2\times3}{\begin{pmatrix}1 & 2 & 4\\ 0 & 1 & -1\end{pmatrix}}\cdot\overset{2\times3}{\begin{pmatrix}1 & 2 & 4\\ 0 & 1 & -1\end{pmatrix}}$ DNE ✓

$AB = \begin{pmatrix}1&2&4\\0&1&-1\end{pmatrix}^{2\times3}\begin{pmatrix}-1&6\\1&0\end{pmatrix}^{2\times2}$ DNE ✓

$AC = \begin{pmatrix}1&2&4\\0&1&-1\end{pmatrix}^{2\times3}\begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}^{3\times2} = \left[\begin{pmatrix}1&2&4\\0&1&-1\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix}, \begin{pmatrix}1&2&4\\0&1&-1\end{pmatrix}\begin{pmatrix}0\\1\\2\end{pmatrix}\right] = \left[\begin{pmatrix}4\\-2\end{pmatrix}, \begin{pmatrix}10\\-1\end{pmatrix}\right]$

$= \begin{pmatrix}4&10\\-2&-1\end{pmatrix}$ ✓

$BA = \begin{pmatrix}-1&6\\1&0\end{pmatrix}^{2\times2}\begin{pmatrix}1&2&4\\0&1&-1\end{pmatrix}^{2\times3} = \left[\begin{pmatrix}-1&6\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}-1&6\\1&0\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}-1&6\\1&0\end{pmatrix}\begin{pmatrix}4\\-1\end{pmatrix}\right]$

$= \left[\begin{pmatrix}-1\\1\end{pmatrix}, \begin{pmatrix}4\\2\end{pmatrix}, \begin{pmatrix}10\\4\end{pmatrix}\right] = \begin{pmatrix}-1&4&10\\1&2&4\end{pmatrix}$

$B^2 = \begin{pmatrix}-1&6\\1&0\end{pmatrix}\begin{pmatrix}-1&6\\1&0\end{pmatrix} = \left[\begin{pmatrix}-1&6\\1&0\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}, \begin{pmatrix}-1&6\\1&0\end{pmatrix}\begin{pmatrix}6\\0\end{pmatrix}\right] = \left[\begin{pmatrix}7\\-1\end{pmatrix}, \begin{pmatrix}-6\\6\end{pmatrix}\right] = \begin{pmatrix}7&-6\\-1&6\end{pmatrix}$ ✓

$BC = \begin{pmatrix}-1&6\\1&0\end{pmatrix}^{2\times2}\begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}^{3\times2}$ DNE ✓

$CA = \begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}^{3\times2}\begin{pmatrix}1&2&4\\0&1&-1\end{pmatrix}^{2\times3} = \left[\begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}\begin{pmatrix}4\\-1\end{pmatrix}\right]$

$= \left[\begin{pmatrix}2\\-1\\1\end{pmatrix}, \begin{pmatrix}4\\-1\\4\end{pmatrix}, \begin{pmatrix}8\\-5\\2\end{pmatrix}\right] = \begin{pmatrix}2&4&8\\-1&-1&-5\\1&4&2\end{pmatrix}$ ✓

$CB = \begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}^{3\times2}\begin{pmatrix}-1&6\\1&0\end{pmatrix}^{2\times2} = \left[\begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}, \begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}\begin{pmatrix}6\\0\end{pmatrix}\right] = \left[\begin{pmatrix}-2\\2\\1\end{pmatrix}, \begin{pmatrix}12\\-6\\6\end{pmatrix}\right] = \begin{pmatrix}-2&12\\2&-6\\1&6\end{pmatrix}$ ✓

$C^2 = \begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}^{3\times2}\begin{pmatrix}2&0\\-1&1\\1&2\end{pmatrix}^{3\times2}$ DNE ✓

2.3.4 a. $\begin{pmatrix}3&-1\\0&-2\end{pmatrix}\begin{pmatrix}3&-1\\0&-2\end{pmatrix} - \begin{pmatrix}3&-1\\0&-2\end{pmatrix} - 6\cdot\begin{pmatrix}1&0\\0&1\end{pmatrix} = \begin{pmatrix}9&-1\\0&4\end{pmatrix} - \begin{pmatrix}3&-1\\0&-2\end{pmatrix} - \begin{pmatrix}6&0\\0&6\end{pmatrix} = \begin{pmatrix}0&0\\0&0\end{pmatrix}$

b. $\begin{pmatrix}2&2\\2&-1\end{pmatrix}\begin{pmatrix}2&2\\2&-1\end{pmatrix} - \begin{pmatrix}2&2\\2&-1\end{pmatrix} - 6\cdot\begin{pmatrix}1&0\\0&1\end{pmatrix} = \begin{pmatrix}8&2\\2&5\end{pmatrix} - \begin{pmatrix}2&2\\2&-1\end{pmatrix} - \begin{pmatrix}6&0\\0&6\end{pmatrix} = \begin{pmatrix}0&0\\0&0\end{pmatrix}$ ✓

2.3.5 c. $(CD)^T = \left[\begin{pmatrix}1&0\\2&1\\5&8\end{pmatrix}\begin{pmatrix}3&-1&2\\1&0&5\end{pmatrix}\right]^T = \begin{pmatrix}3&-1&2\\7&-2&9\\23&-5&50\end{pmatrix}^T = \begin{pmatrix}3&7&23\\-1&-2&-5\\2&9&50\end{pmatrix}$

$D^TC^T = \begin{pmatrix}3&-1&2\\1&0&5\end{pmatrix}^T\begin{pmatrix}1&0\\2&1\\5&8\end{pmatrix}^T = \begin{pmatrix}3&1\\-1&0\\2&5\end{pmatrix}\begin{pmatrix}1&2&5\\0&1&8\end{pmatrix} = \begin{pmatrix}3&7&23\\-1&-2&-5\\2&9&50\end{pmatrix}$

$(CD)^T = D^TC^T$

1.1.1 a. $\begin{cases}2x+3y+z=5\\5x+7y-4z=0\end{cases}$

$(A|b) = \left(\begin{array}{ccc|c} 2 & 3 & 1 & 5 \\ 5 & 7 & -4 & 0 \end{array}\right) \xrightarrow{R_1 \times \frac{1}{2}} \left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{1}{2} & \frac{5}{2} \\ 5 & 7 & -4 & 0 \end{array}\right) \xrightarrow{R_2 - 5R_1} \left(\begin{array}{ccc|c} 1 & 3/2 & 1/2 & 5/2 \\ 0 & -1/2 & -13/2 & -25/2 \end{array}\right)$

$\xrightarrow{-2R_2} \left(\begin{array}{ccc|c} 1 & 3/2 & 1/2 & 5/2 \\ 0 & 1 & 13 & 25 \end{array}\right) \xrightarrow{R_1 - \frac{3}{2}R_2} \left(\begin{array}{ccc|c} 1 & 0 & -19 & -35 \\ 0 & 1 & 13 & 25 \end{array}\right)$

$\begin{cases} x - 19z = -35 \\ y + 13z = 25 \end{cases} \Rightarrow \begin{cases} x = 19t - 35 \\ y = 25 - 13t \\ z = t \end{cases}$

b. $\begin{cases} 2x_1 + 5x_2 + 9x_3 + 3x_4 = -1 \\ x_1 + 2x_2 + 4x_3 = 1 \end{cases}$

$\begin{cases} x_1 = 2s + 12t + 13 \\ x_2 = s \\ x_3 = -s - 3t - 3 \\ x_4 = t \end{cases} \Rightarrow \begin{cases} 2(2s + 12t + 13) + 5s + 9(-s - 3t - 3) + 3t = 26 - 27 = -1 \\ 2s + 12t + 13 + 2s + 4(-s - 3t - 3) = 13 - 12 = 1 \end{cases}$ ✓

1.1.4 $(4 \quad -2 \quad 0 \mid 3) \xrightarrow{\frac{1}{4}R_1} (1 \quad -\frac{1}{2} \quad 0 \mid \frac{3}{4}) \Rightarrow x - \frac{1}{2}y = \frac{3}{4} \Rightarrow \begin{cases} x = \frac{1}{2}t + \frac{3}{4} \\ y = t \\ z = s \end{cases}$ ✓

1.1.7 c. $\left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 1 & 0 & -1 & 1 \\ 2 & 1 & 0 & 0 \end{array}\right)$ d. $\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & 2 \end{array}\right)$ ✓

1.1.8 a. $\begin{cases} x - y + 6z = 0 \\ y = 3 \\ 2x - y = 1 \end{cases}$ b. $\begin{cases} 2x - y = -1 \\ -3x + 2y + z = 0 \\ y + z = 3 \end{cases}$ ✓

1.2.1 reduced row-echelon:

row-echelon: c. d. e

1.2.3 a. $\left(\begin{array}{cccccc|c} 1 & 2 & 0 & 3 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 + 2x_2 + 3x_4 + x_5 = -1 \\ x_3 - x_4 + x_5 = 2 \\ x_6 = 3 \end{cases} \Rightarrow \begin{cases} x_1 = -2u - 3s - t - 1 \\ x_2 = u \\ x_3 = s - t + 2 \\ x_4 = s \\ x_5 = t \\ x_6 = 3 \end{cases}$

1.2.3 b. $\left(\begin{array}{cccccc|c} 1 & -2 & 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 5 & 0 & -3 & -1 \\ 0 & 0 & 0 & 0 & 1 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 - 2x_2 + 2x_4 + x_6 = 1 \\ x_3 + 5x_4 - 3x_6 = -1 \\ x_5 + 6x_6 = 1 \end{cases} \Rightarrow \begin{cases} x_1 = 2u - 2s - t + 1 \\ x_2 = u \\ x_3 = -5s + 3t - 1 \\ x_4 = s \\ x_5 = 1 - 6t \\ x_6 = t \end{cases}$ ✓

1.2.4 a. $\begin{cases} x - 2y = 1 \\ -x + 4y = -2 \end{cases}$ $(A|b) = \left(\begin{array}{cc|c} 1 & -2 & 1 \\ -1 & 4 & -2 \end{array}\right) \xrightarrow{R_2 + R_1} \left(\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 2 & -1 \end{array}\right) \xrightarrow{\frac{1}{2}R_2} \left(\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & -1/2 \end{array}\right) \xrightarrow{R_1 + 2R_2}$

$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -1/2 \end{array}\right) \Rightarrow \begin{cases} x = 0 \\ y = -\frac{1}{2} \end{cases}$

d. $\begin{cases} 3x - y = 2 \\ -6x + 2y = -4 \end{cases}$ $(A|b) = \left(\begin{array}{cc|c} 3 & -1 & 2 \\ -6 & 2 & -4 \end{array}\right) \xrightarrow{\frac{1}{3}R_1} \left(\begin{array}{cc|c} 1 & -1/3 & 2/3 \\ -6 & 2 & -4 \end{array}\right) \xrightarrow{R_2 + 6R_1} \left(\begin{array}{cc|c} 1 & -1/3 & 2/3 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow$

$x - \frac{1}{3}y = \frac{2}{3} \Rightarrow \begin{cases} x = \frac{1}{3}t + \frac{2}{3} \\ y = t \end{cases}$ ✓

f. $\begin{cases} 2x - 3y = 5 \\ -2x + 3y = 2 \end{cases}$ $(A|b) = \left(\begin{array}{cc|c} 2 & -3 & 5 \\ -2 & 3 & 2 \end{array}\right) \xrightarrow{\frac{1}{2}R_1} \left(\begin{array}{cc|c} 1 & -3/2 & 5/2 \\ -2 & 3 & 2 \end{array}\right) \xrightarrow{R_2 + 2R_1} \left(\begin{array}{cc|c} 1 & -3/2 & 5/2 \\ 0 & 0 & 7 \end{array}\right) \Rightarrow$

$\begin{cases} x - \frac{3}{2}y = \frac{5}{2} \\ 0 = 7 \end{cases} \Rightarrow$ no solution ✓

1.2.5 a $\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 3 & -1 & 1 & 0 \\ -1 & 3 & 4 & 4 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 21 \\ 0 & 0 & 1 & -12 \end{array}\right) \Rightarrow \begin{cases} x = 11 \\ y = 21 \\ z = -12 \end{cases}$

b. $\left(\begin{array}{ccc|c} -2 & 3 & 3 & -9 \\ 3 & -4 & 1 & 5 \\ -5 & 7 & 2 & -14 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 15 & -21 \\ 0 & 1 & 11 & -17 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x + 15z = -21 \\ y + 11z = -17 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} x = -15t - 21 \\ y = -11t - 17 \\ z = t \end{cases}$ ✓

d. $\left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & 5 & -3 & 1 \\ 1 & 4 & -3 & 3 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & -\frac{11}{3} \\ 0 & 1 & -1 & \frac{5}{3} \\ 0 & 0 & 0 & \frac{2}{3} \end{array}\right) \Rightarrow \begin{cases} x + z = -\frac{11}{3} \\ y - z = \frac{5}{3} \\ 0 = \frac{2}{3} \end{cases} \Rightarrow$ no solution ✓

f. $\left(\begin{array}{ccc|c} 3 & -2 & 1 & -2 \\ 1 & -1 & 3 & 5 \\ -1 & 1 & 1 & -1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow \begin{cases} x = -7 \\ y = -9 \\ z = 1 \end{cases}$ ✓

1.2.7 a. $\left(\begin{array}{cccc|c} 3 & 8 & -3 & -14 & 2 \\ 2 & 3 & -1 & -2 & 1 \\ 1 & -2 & 1 & 10 & 0 \\ 1 & 5 & -2 & -12 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0.14 & 3.71 & 0.29 \\ 0 & 1 & -0.43 & -3.14 & 0.14 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 + 0.14x_3 + 3.71x_4 = 0.29 \\ x_2 - 0.43x_3 - 3.14x_4 = 0.14 \\ 0 = 0 \\ 0 = 0 \end{cases} \Rightarrow$

$\begin{cases} x_1 = -0.14s - 3.71t + 0.29 \\ x_2 = 0.43s + 3.14t + 0.14 \\ x_3 = s \\ x_4 = t \end{cases}$

b. $\left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 = 0 \\ x_2 + x_4 = 0 \\ x_3 = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = -t \\ x_3 = 0 \\ x_4 = t \end{cases}$ ✓

c. $\left(\begin{array}{cccc|c} 1 & -1 & 1 & -2 & 1 \\ -1 & 1 & 1 & 1 & -1 \\ -1 & 2 & 3 & -1 & 2 \\ 1 & -1 & 2 & 1 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & \end{array}\right.$