$$E(T) = \int_0^{\infty} t f(t)\, dt = \int_0^{\infty} \left[ \int_0^{t} du \right] f(t) dt$$

$$= \int_0^{\infty} \left[ \int_0^{\infty} I(t > u)\, du \right] f(t)\, dt$$

$$I(t) = \begin{cases} 0 & \text{if } t < u \\ 1 & \text{otherwise} \end{cases}$$

$$= \int_0^{\infty} \left[ \int_0^{\infty} I(t > u)\, f(t)\, dt \right] du$$

$$= \int_0^{\infty} \left[ \int_0^{\infty} f(t)\, dt \right] du$$

$$= \int_0^{\infty} S(u)\, du$$

Or by using the fact that

$$dS(u) = d[1 - F(u)] = -f(u)\, du$$

and integrating by parts and using the fact that

$$S(0) = 1 \quad , S(\infty) = 0$$

$$\underbrace{-t\, S(u) \Big|_0^{\infty}}_{=0} + \int_0^{\infty} S(t)\, dt$$

$$h(t) = \lim_{\delta \to 0} \frac{P(t \le T < t+\delta \mid T \geq t)}{\delta}$$

$$= \lim_{\delta \to 0} \frac{P(t \le T < t+\delta)}{P(T \geq t) \cdot \delta}$$

$$= \lim_{\delta \to 0} \frac{S(t) - S(t+\delta)}{\delta} \cdot \frac{1}{S(t)}$$

$$= f(t) \cdot \frac{1}{S(t)} = \frac{f(t)}{S(t)} = -\frac{d \ln S(t)}{dt}$$

$$H(t) = \int_0^t h(u)\,du = -\int_0^t \frac{1}{S(u)}\,dS(u)$$

$$= -\ln[S(t)] + \ln[S(0)]$$

$$= -\ln[S(t)] + \ln(1) = -\ln S(t)$$

$$\Rightarrow S(t) = e^{-H(t)}$$

$$h(x) = \frac{f(x)}{S(x)}$$

per slide 40

Memoryless property:

$$P(X>s+t \mid X>s) = P(X>t)$$

$$= \frac{P(X>s+t \wedge X>t)}{P(X>s)}$$

$$= \frac{P(X>s+t)}{P(X>s)} = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} = e^{-\lambda t}$$

The past does not influence the future, every time is like the beginning of a new random period, which has the same distribution regardless how much time has already passed

$X \sim Exp(\lambda)$

we know that $x > s$, $P(X > s+t) = ?$

This type of problems occurs in queueing systems where we are interested in time between events.

$$f(x) = \frac{1}{\theta} e^{-x/\theta} \qquad x \geq 0, \ \theta > 0$$

$$EX = \theta$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x/\theta} & x \geq 0 \end{cases}$$

$X \sim exp$ with a mean of 40

$$f(x) = \frac{1}{40} e^{-x/40}$$

$$P(x < 36) = \int_0^{36} \frac{1}{40} e^{-x/40} dx$$

$$= 1 - e^{-36/40}$$

$$= pexp(36, 1/40) = 0.593$$

median : $F(m) = 0.5$

$$1 - e^{-m/\theta} = 0.5$$

$qexp(0.5, 1/40)$

N.B. in R: $f(x) = \lambda e^{-\lambda x}$

plot the p.d.f.

Ex: the life of a certain type of electronic component has an exponential distr. with a mean life of 500 hours. If $X$ denote the lifetime of this component (or the time to failure of this component) then

$$P(X > x) = \int_x^{\infty} \frac{1}{500} e^{-t/500} dt$$

$$= e^{-x/500}$$

Suppose that the component has been operating for 300 hours, what is the conditional pr. that it will last for another 600 hours?

$$P(X > 900 \mid X > 600) = \frac{P(X > 900)}{P(X > 600)}$$

$$= \frac{e^{-900/500}}{e^{-600/500}} = e^{-6/5} = P(X > 600)$$

for each component an old component is as good as the new one

$$EX = \int_0^\infty x \lambda e^{-\lambda x} \, dx$$

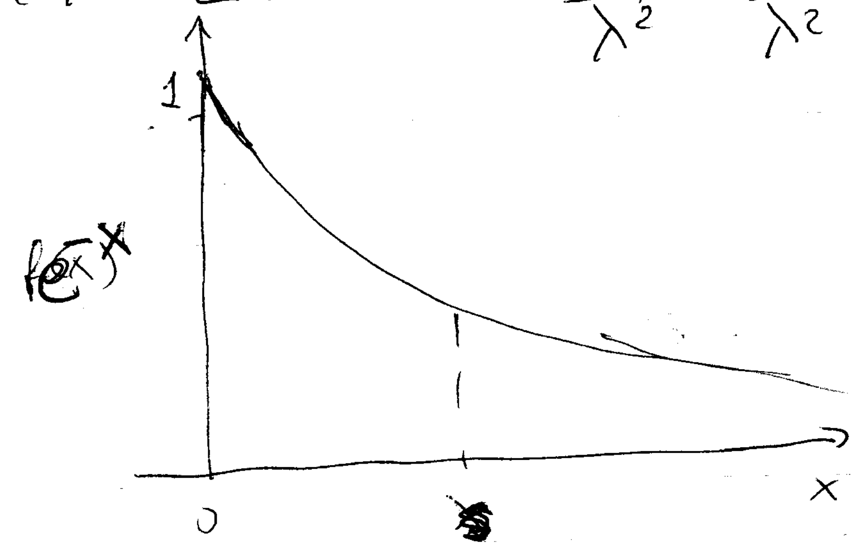
$$= \lambda \left[ \frac{-x e^{-\lambda x}}{\lambda} \right]_0^\infty + \frac{1}{\lambda} \int_0^\infty e^{-\lambda x} \, dx$$

$$= \lambda \left[ 0 + \frac{1}{\lambda} \frac{-e^{-\lambda x}}{\lambda} \right|_0^\infty = \lambda \cdot \frac{1}{\lambda^2} = \frac{1}{\lambda}$$

$$EX^2 = \int_0^\infty x^2 \lambda e^{-\lambda x} = \dots \frac{2}{\lambda^2}$$

$$\mathrm{Var}\, X = EX^2 - EX = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

$\lambda = 1$




Memory less: the graph after s is ~~exactly~~ an exact copy of the original. $\Rightarrow$ the distribution of X conditioned on $\{X > s\}$ is again exponential!