Calculus Exercise Week 2

Section 2.4

131. $f(x) = \frac{1}{\sqrt{x}}$

   $D(f) = \{x > 0\}$

133. $f(x) = \frac{x}{x^2 - x}$

   $D(f) = \{x \neq 0 \text{ and } x \neq 1\}$

   $f(0)$ undefined

   $\lim\limits_{x \to 0} f(x) = -1 \Rightarrow x = 0$ is removable discontinuity

   $f(1)$ undefined

   $\lim\limits_{x \to 1^-} f(x) = -\infty, \lim\limits_{x \to 1^+} f(x) = +\infty \Rightarrow x = 1$ is infinite discontinuity

139. $f(1)$ is undefined

   $\lim\limits_{x \to 1} f(x) = \lim\limits_{x \to 1} \frac{2x^2 - 5x + 3}{x - 1} = \lim\limits_{x \to 1} \frac{(2x - 3)(x - 1)}{x - 1} = -1$

   $x = 1$ is removable continuity

141. $\lim\limits_{u \to \frac{1}{2}} g(u) = \lim\limits_{u \to \frac{1}{2}} \frac{6u^2 + u - 2}{2u - 1} = \lim\limits_{u \to \frac{1}{2}} \frac{(3u + 2)(2u - 1)}{2u - 1} = \frac{7}{2} = g(\frac{1}{2})$

   continuous

145. $\lim\limits_{x \to k^{-1}} f(x) = f(k)$

   $3k + 2 = 2k - 3 \Rightarrow k = -5$

定义
$\Downarrow$
$\lim\limits_{x \to a} f(x)$
$\Downarrow$
$\lim\limits_{x \to a} f(x) = f(a)$

Section 3.1

$f'(a) = \lim\limits_{x \to a} \frac{f(x) - f(a)}{x - a}$

21. $f'(-1) = \lim\limits_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)} = \lim\limits_{x \to -1} \frac{5x + 4 - (-1)}{x + 1} = \lim\limits_{x \to -1} \frac{5(x + 1)}{x + 1} = 5$

23. $f(x) = x^2 + 9x$, $a = 2$

   $f'(2) = \lim\limits_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim\limits_{x \to 2} \frac{x^2 + 9x - 22}{x - 2} = \lim\limits_{x \to 2} \frac{(x + 11)(x - 2)}{x - 2} = 13$

25. $f(x) = \sqrt{x}$, $a = 4$

   $f'(4) = \lim\limits_{x \to 4} \frac{f(x) - f(4)}{x - 4} = \lim\limits_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} = \lim\limits_{x \to 4} \frac{(\sqrt{x} - 2)}{(\sqrt{x} - 2)(\sqrt{x} + 2)} = \frac{1}{4}$

Section 3.2

64. $f'(x) = \begin{cases} <0, & -2<x<\frac{1}{2} \text{ 或 } x>\frac{5}{2} \\ 0, & x=\frac{1}{2} \text{ 或 } \frac{5}{2} \\ >0, & \frac{1}{2}<x<\frac{5}{2} \end{cases}$

65. $f'(x) = \begin{cases} <0, & x<0 \text{ 或 } 0<x<3 \\ 0, & x=0 \text{ 或 } 3 \\ >0, & x>3 \end{cases}$

66. $f'(x) = \begin{cases} <0, & -\frac{3}{2}<x<0 \text{ or } x>\frac{3}{2} \\ 0, & x=\pm\frac{3}{2} \\ >0, & x<-\frac{3}{2} \text{ or } 0<x<\frac{3}{2} \end{cases}$

67. $f'(x) = \begin{cases} <0, & x>0 \\ 0, & x=0 \\ >0, & x<0 \end{cases}$ $kx \, (k<0)$

78. a. $x=4$ a. 不连续有极限

b. None   b. 连续不可微

79. a. $x=1$

b. $x=2$

80. $f'(-0.5) = \frac{0-3}{-3-0} = 1$

$f'(0)$ DNE

$f'(1) = \frac{3-1}{0-2} = -1$

$f(2)$ DNE