Calculus Exercise Week6

Section 5.5

261-267

261 $\int (x+1)^4 dx$, $u = x+1$

$= \int u^4 du$

$= \frac{1}{5}u^5 + C$

$= \frac{1}{5}(x+1)^5 + C$

263 $\int (2x-3)^{-7} dx$, $u = 2x-3$ $\quad x = \frac{u+3}{2}$

$= \int u^{-7} \frac{1}{2} du$

$= \frac{1}{2}(-\frac{1}{6}u^{-6}) + C$

$= -\frac{1}{12}(2x-3)^{-6} + C$

265 $\int \frac{x}{\sqrt{x^2+1}} dx$, $u = x^2+1$

$= \int \frac{\sqrt{u-1}}{\sqrt{u}} \cdot \frac{1}{2}\frac{1}{\sqrt{u-1}} du$ $\quad x = \sqrt{u-1}$

$= \int \frac{1}{2}\frac{1}{\sqrt{u}} du$ $\quad \frac{1}{2}(u-1)^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot 2u^{\frac{1}{2}} + C$

$= \sqrt{x^2+1} + C$ $\quad x = 1 - \sqrt{1+u}$

267 $\int (x-1)(x^2-2x)^3 dx$, $u = x^2-2x$ $\quad x = 1 + \sqrt{1+u}$

$= \int \pm\sqrt{1+u} \cdot u^3 \cdot \pm\frac{1}{2\sqrt{1+u}} du$ $\quad \pm\frac{1}{2}(1+u)^{-\frac{1}{2}}$

$= \frac{1}{2}\frac{1}{4}u^4 + C$ $\quad x^2 - 2x - u = 0$

$= \frac{1}{8}(x^2-2x)^4 + C$ $\quad \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-u)}}{2 \times 1}$ $\quad \sqrt{4+4u}$

271

273

293

271 $\int x(1-x)^{99} dx$ $\quad \frac{2 \pm 2\sqrt{1+u}}{2}$

$u = 1-x$, $x = 1-u$ $\quad \pm\sqrt{1+u}$

$\int x(1-x)^{99} dx$

$= \int (1-u)u^{99} \cdot -1 \, du$

$= \int u^{100} - u^{99} du$

$= \frac{1}{101}u^{101} - \frac{1}{100}u^{100} + C$

$$= \frac{1}{101}(1-x)^{101} - \frac{1}{100}(1-x)^{100} + C$$

273 $\int (11x-7)^{-3}\,dx$

$u = 11x-7,\ x = \frac{u+7}{11}$

$\int (11x-7)^{-3}\,dx$

$= \int u^{-3}\frac{1}{11}\,du$

$= \frac{1}{11}(-\frac{1}{2})u^{-2} + C$

$= -\frac{1}{22}(11x-7)^{-2} + C$

293. $\int_0^1 \frac{x}{\sqrt{1+x^2}}\,dx$

$u = 1+x^2,\ x = \pm\sqrt{u-1}$

$\int \frac{x}{\sqrt{1+x^2}}\,dx$

$= \int \frac{\pm\sqrt{u-1}}{\sqrt{u}} \cdot \frac{1}{2}\frac{1}{\pm\sqrt{u-1}}\,du$

$= \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

$= \frac{1}{2}\cdot 2\sqrt{u} + C$

$= \sqrt{1+x^2} + C$

$\int_0^1 \frac{x}{\sqrt{1+x^2}}\,dx = (\sqrt{1+x^2}+C)\Big|_0^1 = \sqrt{2}-1$