Probability Practice Exam 2023

1. (a) $1-0.1\times3-0.2-0.15=0.35$ (iii)

(b) $P(X_1\le1\cap X_2<1)=0.1+0.1=0.2$ (iii)

0, 1, 2, 4

(c) ~~(iii)~~ (iv) $E(X_1X_2)=0\times0.3+1\times0.2+2\times0.35+4\times0.15=1.5$

(d) (iii) $E(X_1)=0\times0.1+1\times0.3+2\times0.6=1.5$

(e) (ii) $E(X_2)=0\times0.3+1\times0.55+2\times0.15=0.85$

(f) $0\times\frac{1}{3}+1\times\frac{1}{3}+2\times\frac{1}{3}=1$ (ii) $Cov(X_1,X_2)=1.5-1.5\times0.85=0.225$

2. (a) $X\sim Poi(4)$ $P(X=k)=\frac{4^k}{k!}e^{-4}$

$P(X=5\cup X=6\cup X=7)=\frac{4^5}{5!}e^{-4}+\frac{4^6}{6!}e^{-4}+\frac{4^7}{7!}e^{-4}$

(b) $P(X=0)=e^{-4}$

$[P(X=0)]^{10}=(e^{-4})^{10}=e^{-40}$

(c) $E(10X)=10\times4=40$ $E(Y)=10\lambda=40$

~~$Var(10X)=100\times4=400$~~ $Var(Y)=10\lambda=40$

~~$\sqrt{Var(10X)}=20$~~ $\sqrt{Var(Y)}=\sqrt{40}$

(d) $Y\sim Exp(4)$ $1-e^{-4\times\frac{1}{3}}$

$P(Y>\frac{1}{3})=1-P(Y\le\frac{1}{3})=1-[F_X(\frac{1}{3})-F_X(0)]=1-[(1-e^{-\frac{4}{3}})-0]=$ ~~$\frac{1}{e}$~~ $e^{-\frac{4}{3}}$ 0.26

3. (a) X_i $E(X_i)=p=0.5$, $Var(X_i)=p(1-p)=0.25$

$Y=\sum_{i}^{900}X_i\sim N(n\mu,n\sigma^2)=N(450,225)$

$P(Y\ge495)=P(\frac{Y-450}{15}\ge3)=1-\Phi(3)=0.00135$

(b) $P(440\le Y\le m)=P(\frac{440-450}{15}\le\frac{Y-450}{15}\le\frac{m-450}{15})=\Phi(\frac{m-450}{15})-\Phi(-\frac{2}{3})\approx0.5$

$\Phi(\frac{m-450}{15})-[1-\Phi(0.67)]=\Phi(\frac{m-450}{15})+\Phi(0.67)-1=0.5$

$\Phi(\frac{m-450}{15})=0.75143$

$\frac{m-450}{15}=0.68$

$m=460.2$

4. (a) $P(A)=0.8$, $P(B)=0.2$

$P(1|A)=0.15,\ P(1|B)=0.9$

$P(1) = P(1|A)\times P(A) + P(1|B)\times P(B)$

$= 0.15\times 0.8 + 0.9\times 0.2$

$= 0.3$

(b) $P(B|1) = \frac{P(1.B)}{P(1)} = \frac{P(1|B)\cdot P(B)}{P(1)} = \frac{0.9\times 0.2}{0.3} = \frac{3}{5}$