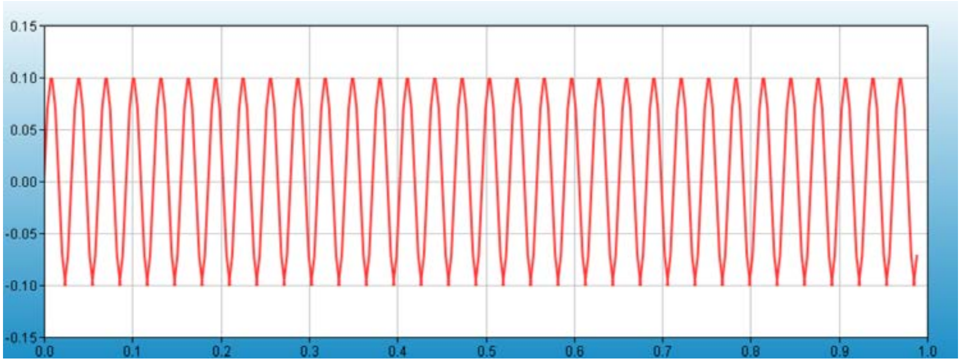
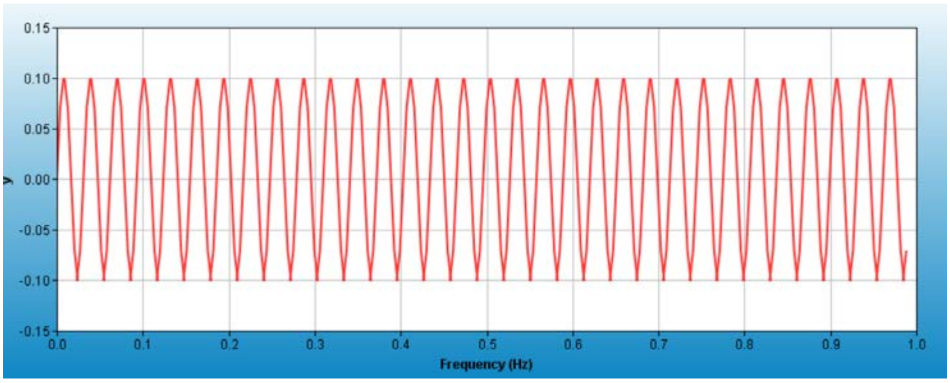
B. Implementation of digital anti-aliasing filters on a DSP

a) Plot of the sample signal x(n) = 0.1 * sin(0.25 * 1.0 * n);

The Time domain representation of the signal has been ploteed in CCES as the figure below, where its x axis is time(s) and y axis is amplitude.



The frequency domain representation of the signal is shown in the figure below, where the x axis is frequency(Hz), y axis is amplitude.



b) Write a c-program which generates two sequences of 256 samples of x(t), using the sampling frequencies F1 = 1.2Hz and F2 = 4.8Hz.

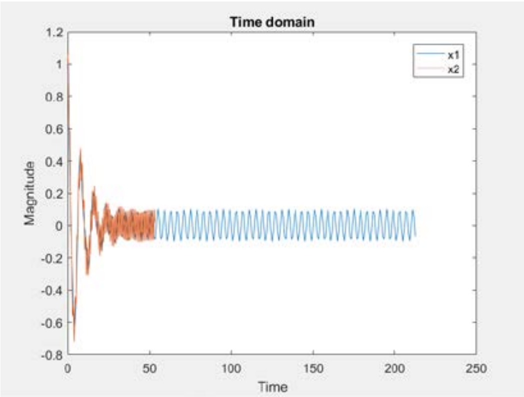
The code used to generate the 2 signals are attached below.

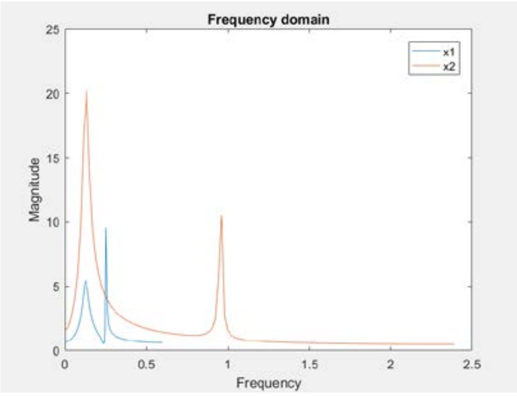
```
#include <stdio.h>
#include <math.h>

// Globals
#define N       256
#definePI       3.1415

float x1[N]; // Sample at T1
float x2[N]; // Sample at T2
int main(void)
{
```

```
        int i;
        float omega1 = 0.25 * PI, omega2 = 1.9 * PI;
        float T2 = 1/4.8;
        float T1 = 1/1.2;
        float a=0.12;
        float alpha1 = 0.593, alpha2 = 0.464;

        x1[0]  = exp(-a*0*T1)*cos(omega1*0*T1) + 0.1*sin(omega2*0*T1);
        x2[0]  = exp(-a*0*T2)*cos(omega1*0*T2) + 0.1*sin(omega2*0*T2);

        for (i = 0; i < N; i++)
        {
                x1[i] = exp(-a*i*T1)*cos(omega1*i*T1) + 0.1*sin(omega2*i*T1);
                x2[i] = exp(-a*i*T2)*cos(omega1*i*T2) + 0.1*sin(omega2*i*T2);

        }
        printf("Done.\n");
        return 0;
}
```

c) **Use the plot facility within CCES to plot the sampled signals in the time domain and in the frequency domain. Comment on the results.**

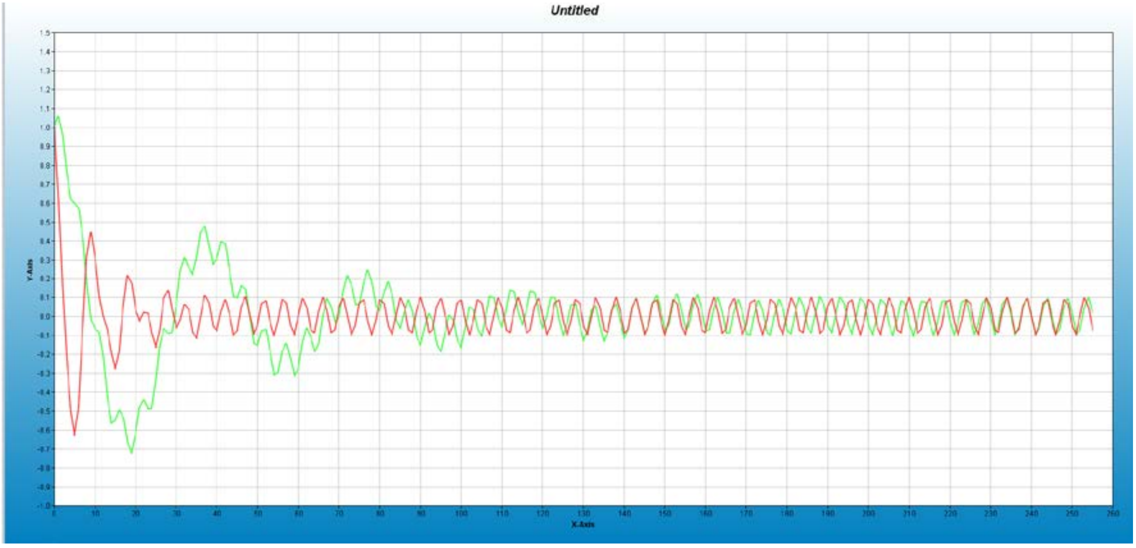
To predict how the signal is going to behave, Matlab was used to plot x1 and x2 in time and frequency domain. (Code attached in appendix).



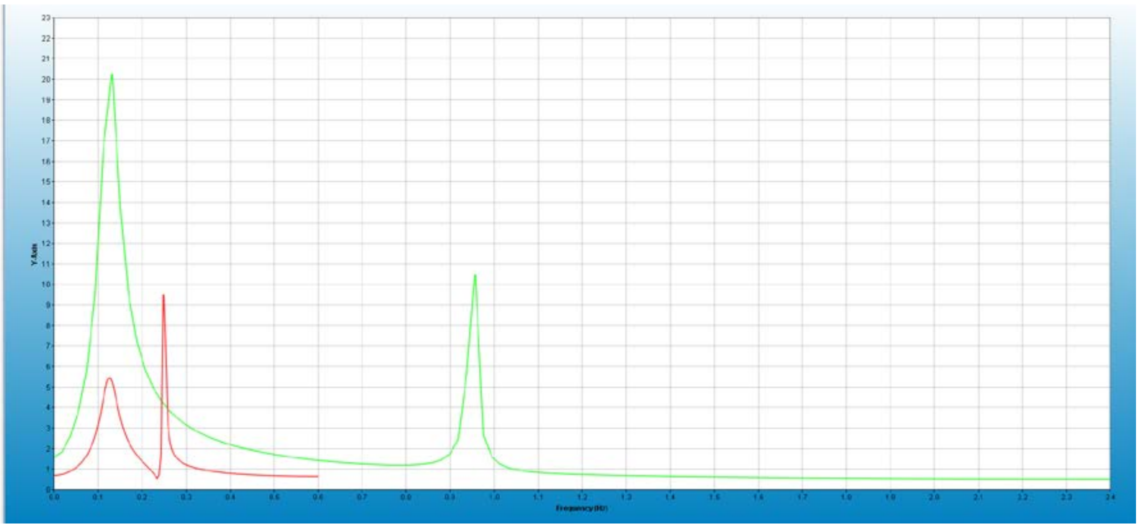


The result generated by CESS are shown as below

Time domain plot. x axis: sample time, y axis: magnitude(linear scale)



Frequency domain plot. x axis: frequency(Hz), y axis: magnitude(linear scale)



As the plots above show, the signal generated in the DSP board are identical to that simulated in Matlab.

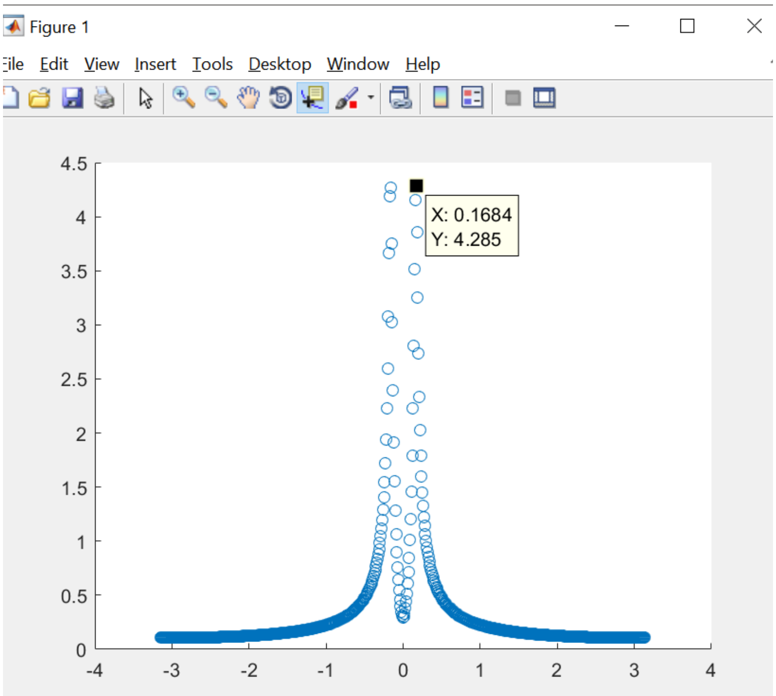
Comment on the result:

Theoretically speaking, the signal should generate two peaks at $\boldsymbol{f1} = \frac{\Omega_1}{2*pi} = \frac{0.25*pi}{2*pi} = 0.125Hz$, and $\boldsymbol{f}_2 = \frac{\Omega_2}{2*pi} = \frac{1.9*pi}{2*pi} = 0.95Hz$. Which is matching up with what signal x2(the green trace) in the CCES FFT plot.

However, there are two facts about the signal x1's FFT plot were noticed in the FFT plot: 1. The 0.95Hz peak was missing, 2. There is an "unpredicted" peak between the region of 0.2-0.3Hz. Fact 1 might due to the sampling angular velocity $\Omega_{sampling1} = 1.2 * 2\mathrm{pi} < 2\Omega_2 = 2 * 1.9 * 2pi$. When the sampling frequency is less than $2\Omega_{max}$ of the signal, high frequency component of the signal will be distorted while sampling. Fact 2 is due to the fact that the sampling frequency is too low and it cause the folding effect around $\frac{f_{samling}}{2} = \frac{1.2}{2} = 0.6Hz$. The frequency component from $\boldsymbol{f}_2 = \frac{\Omega_2}{2*pi} = \frac{1.9*pi}{2*pi} = 0.95Hz$ is distorted and folded back to 0.6-(0.95-0.6) = 0.25Hz, which fell into the 0.2-0.3Hz region we mentioned in the last paragraph.

d) Analysis of the first order and second order filter

After DTFT, the signal $\boldsymbol{x_1} = e^{-at}\cos(\Omega_1 t)\,, where\ a = 0.12, \Omega_1 = 0.25pi$, has been plotted in frequency domain in question part A(c), and $\omega_{max} = 0.1684\ rad/s$ can be obtained from the plot.



(i) Find $\boldsymbol{\alpha}$ of the first order filter

It has been proven that

$$|H_{LP}(e^{j\omega})|^2 = \frac{(1-\alpha)^2(1+\cos\omega)}{2(1+\alpha^2-2\alpha\cos\omega)}$$

The gain(magnitude)=0.95 of this filter at $\omega_{max} = 0.1684\ rad/s$,

can be calculated by

$$|H_{LP}(e^{j\omega})| = \sqrt{|H_{LP}(e^{j\omega})|^2} = \frac{1-\alpha}{\sqrt{2}} * \frac{\sqrt{1+\cos(w_{max})}}{\sqrt{1+\alpha^2-2\alpha\cos(w\_max)}} = 0.95$$

Solving this equation at Wolfram Alpha gives us $\alpha = 0.593$.

As the filter is designed to filter out 5% of the gain of x1, now we need to evaluate the gain of the signal after the filter.

Recall in discrete frequency domain, there will be a peak at $w_2 = \Omega_2 * T_{4.8Hz} = 1.9pi * \frac{1}{4.8} = 1.2435$ rad/s.

$$\left|H_{LP}\left(e^{j\omega_2}\right)\right| = \frac{1-\alpha}{\sqrt{2}} * \frac{\sqrt{1+\cos(w_2)}}{\sqrt{1+\alpha^2-2\alpha\cos(w_2)}}$$
$$= \frac{1-0.593}{\sqrt{2}} * \frac{\sqrt{1+\cos(1.2435)}}{\sqrt{1+0.593^2-2*0.593*\cos(1.2435)}}$$
$$= 0.3385$$

As the requirement of the filter is to get a gain less than 0.25 at ω_2, the calculation above has shown that this filter won't satisfy the design requirement.

(ii) Find $\boldsymbol{\alpha}$ of the second order filter

Since $H_2\left(e^{j\omega}\right) = H_{LP}\left(e^{j\omega}\right)^2$, $H_2\left(e^{j\omega}\right) = |H_{LP}(e^{j\omega})|^2 = \dfrac{(1-\alpha)^2(1+\cos\omega)}{2(1+\alpha^2-2\alpha\cos\omega)}$

The gain(magnitude)=0.95 of this filter at $\omega_{max} = 0.1684\ rad/s$,

can be calculated by

$$\left|H_2\left(e^{j\omega}\right)\right| = \frac{(1-\alpha)^2}{2} * \frac{1+\cos(w_{max})}{1+\alpha^2-2\alpha\cos(w\_max)} = 0.95$$

Solving this equation at Wolfram Alpha gives us $\alpha = 0.464$.

As the filter is designed to filter out 5% of the gain of x1, now we need to evaluate the gain of the signal after the filter.

$$\left|H_2\left(e^{j\omega_2}\right)\right| = \frac{(1-\alpha)^2}{2} * \frac{1+\cos(w_2)}{1+\alpha^2-2\alpha\cos(w_2)}$$
$$= \frac{(1-0.464)^2}{2} * \frac{1+\cos(1.2435)}{1+0.464^2-2*0.464*\cos(1.2435)} = 0.207$$

Gain = 0.207 is less than 0.25 therefore this filter will satisfy the design specification.