APPENDIX C

Final Answers for Odd-Numbered Problems

(2.1) $\nabla T = 10\mathbf{u}_x + 60y^2\mathbf{u}_y + 30\mathbf{u}_z\,^\circ/\text{m}$

(2.3) $\nabla \cdot \mathbf{A} = 32(1)^3 + 10 = 42$, $\nabla \times \mathbf{A} = (-11)\mathbf{u}_x + (-4)\mathbf{u}_y + (-15)\mathbf{u}_z$

(2.5) $\mathbf{J} = -12\mathbf{u}_x - 4\mathbf{u}_y - 279\mathbf{u}_z$ A/m^2

(2.7) **(a)** $\mathbf{B} = (1.2\text{E–}3\ \mathbf{u}_x + 1.6\text{E-}3\mathbf{u}_y)$T
(b) $\mathbf{B} = (3\mathbf{u}_x + 4\mathbf{u}_y)$T
(c) $\mathbf{B} = (0.93\mathbf{u}_x + 1.24\mathbf{u}_y)$T

(2.9) $V = -4524\cos(2\pi 60t)$ V, $I = -1131\cos(2\pi 60t)$A,

$E = -514\cos(2\pi 60t)$ V/m, $J = -18.2\text{E}9\cos(2\pi 60t)$ A/m^2

(2.11) $\mathbf{E} = -4\mathbf{u}_y - 75.4x\cos(2\pi 60t)\mathbf{u}_z$ V/m

(2.13) $\mathbf{J}_{\text{disp}} = 12.07\text{E–}6\cos(2\pi 50t)\mathbf{u}_y$ A/m^2,

$I_C = 42.25\text{E–}9\cos 314.16t$ A

(3.1) $\phi = 10.93\text{E} - 4$ Wb, $B = 0.102$ T

(3.3) $B_{\text{align}} = 0.6285$ T, $B_{\text{misalign}} = 0.06285$ T

(4.1)

$$\begin{pmatrix} 1352 & 1750 & 2148 & 0 & 0 \\ 1750 & 2307 & 2864 & 0 & 0 \\ 2148 & 2864 & (3580 + 2.704) & 3.500 & 4.296 \\ 0 & 0 & 3.500 & 4.614 & 5.728 \\ 0 & 0 & 4.296 & 5.728 & 7.160 \end{pmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} = \begin{bmatrix} 1.333 \\ 1.333 \\ 1.333 \\ 0 \\ 0 \end{bmatrix}$$

(4.3) Energy stored = 137.28 J with energy error = 0.41%

Magnetic Actuators and Sensors, Second Edition. John R. Brauer.

(5.1) $\mathbf{B} = -1\mathbf{u}_y$

(5.3) $\mathbf{B} = 2y\mathbf{u}_x$

(5.5) $P_{mag} = 480{,}984$ Pa

(5.7) $F_{mag} = 62.03$ N

(5.9) $\mathbf{F} = 80\mathbf{u}_z$ N/m

(5.11) The computed force on the lower magnet is 3.43 N upward.

(5.13) The computed force on the lower magnet is 68.45 N upward.

(5.15) This proof uses the differential volume of a cylinder.

(6.1) The finite-element value of flux for 0.1 m depth is 0.00314 Wb for one turn. The reluctance method obtains flux = 0.00125 Wb.

(6.3) Proof. Evaluation gives magnetic force = −113 N, agreeing exactly with corrected result of Example 6.2.

(6.5) $L_{11} = L_{22} = 6.06\text{E–}8$ H and $L_{12} = L_{21} = 1.799\text{E–}8$ H, all for one turn.

(6.7) $L = 12.477\text{E–}4$ H

(6.9) $Z = (-565.5 + j377)\ \Omega$. Note: for realistic positive R, given λ should have negative imaginary part.

(6.11) For 20 turns in the lower coil, the matrices for one turn have $L_{11} = L_{22} = 8.90\text{E–}8$ H, $L_{12} = L_{21} = 1.785\text{E–}8$ H, $R_{11} = R_{22} = 8.9\text{E–}8\ \Omega$, and $R_{12} = R_{21} = 6.23\text{E–}8\ \Omega$.

(7.1) **(a)** force on the left gap = 529 N in the $-y$ direction for 1 m depth.
(b) same force on right gap, so total force = 1058 N/m depth.
(c) 1431 N/m depth.
(d) 1431.6 N/m depth
(e) plot shows force as high as 2200 N.

(7.3) **(a)** force = 1244 N for 1 m depth.
(b) force = 1533 N for 1 m depth (for 1600 A).
(c) force = 1566 N for 1 m depth (for 1600 A).

(7.5) 15.1 N

(7.7) Plot shows total force as high as 1000 N.

(8.1) Matrix for one turn has $L_{11} = L_{22} = 6.058\text{E–}8$ H, $L_{12} = L_{21} = 1.795\text{E–}8$ H, $R_{11} = R_{22} = 5.6\text{E–}9\ \Omega$ and $R_{12} = R_{21} = 4.05\text{E–}9\ \Omega$

(8.3) (a) 64.97 m, (b) 8.53 mm, (c) 325 μm

(8.5) $L_p = 1.7$ mH, $R_p = 9.612\ \Omega$

(8.7) Force has time-average value of 8.566 N and an "AC fluctuation" of 8.528 N

(8.9) Power loss = 0.198 W/m depth

(9.1) $a = 3613$ m/s^2, for $s = 5.\text{E–}3$ m $t = 1.664$ ms

(9.3) (a) 581.4 ms, (b) energy = 0.658 J, (c) 581.4 ms

(9.5) (a) 0.0581 ms, (b) energy = 0.653 J, (c) 0.0581 ms

(9.7) Finite-element diffusion times are 28 and 14 ms

(10.1) (a) $\sigma_o = 6.4$ S/m, (b)

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} 6.1761 & -1.1759 \\ 1.1759 & 6.1761 \end{pmatrix}$$

(10.3) Power loss = 5 W without Al, 5.224 W with Al

(10.5) $V_y = k_H J_x d_y (0.03143 + 0.2828 \sin n_T \theta)$V

(11.1) $V = -\Omega N n_T (283\text{E–}6) \cos(n_T \Omega t)$V

(11.3) **(a)** $Z(0.002) = 228\text{E–}5 + j664\text{E–}5\ \Omega$, $Z(0.004) = 2055\text{E–}5 + j757\text{E–}5\ \Omega$

(b) $Z(0.002) = 318.7\text{E–}5 + j1206\text{E–}5\ \Omega$, $Z(0.004) = 280\text{E–}5 + j1408\text{E–}5\ \Omega$.

(c) $Z(0.002) = 489\text{E–}5 + j2215\text{E–}5\ \Omega$, $Z(0.004) = 426\text{E–}5 + j2652\text{E–}5\ \Omega$.

(11.4) 15.55 V

(12.1) $N = 826$, $I = 1.211$ A

(12.3) $N = 1077$, $I = 0.929$ A

(12.5) $R = 0.5726\text{E–}4\ \Omega$/m, J plot is as high as almost 350,000 A/m^2

(12.8) $P = 0.365$ W, J plot shows differing densities.

(12.9) Maximum temperature is 48.1°C.

(12.11) Maximum temperature is 71.3°C.

(13.1) **(a)** plot with skin depth = 0.6 mm

(b) plot shows aperture has 0.038 T inside and 0.020 T outside

(c) plot with skin depth = 0.08 mm

(d) outside location has 104 mT

(13.3) Characteristic impedance = 67.7 Ω

(13.5) Characteristic impedance = 65.55 Ω

(14.1) Reluctance $\mathcal{R} = 95.47$, $|I_1| = 738.5$ A, $|I_2| = 184.6$ A

(15.1) Plot shows $I(t = 1) = 0.79987$ A

(15.3) The current minimum at 0.8 s is now 0.721 A

(15.5) The maximum (negative) force is now 51.22 N

(15.7) Comparison of responses of actuator models with mass 0.06 kg

Time domain Specification	Third-order system Simulation results	Reduced-order Simulation results	Reduced-order Using (E15.4.2)
P.O.	14.7%	15.0%	15.04%
t_p, s	0.745	0.704	0.7104
t_s, s	1.56	1.52	1.5

(15.9) The peak magnitude is now at a higher frequency, at approximately $\omega = 5.164$ rad/s $= (1.6/.06)^{1/2}$. At $\omega = 1000$ rad/s, the second-order magnitude is approximately -90 dB and the third-order magnitude is approximately -120 dB. At the same frequency, the second-order phase angle is close to -180° and the third-order phase angle is close to -270°. At lower frequencies the two curves become quite close.

(16.1) The flow rates are 2.5E–3 m^3/s in R_1 and 1.25E–3 m^3/s in R_2

(16.3) $Q = 2.70$ m^3/s

(16.5) $Q = 2.31$ m^3/s

(16.7) At $t = 30$ ms, the pressure is 499.755 Pa