L2a: Other Magnetic Performance Parameters

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Why

Some derived parameters are frequently used for electromagnetic analysis.

In particular, we will consider:

a) Flux Linkage
b) Inductance
c) Capacitance

An understanding of these parameters is important when considering the electronic circuit driving the electromagnetic device

Magnetic Flux and Flux Linkage

Why

Some analysis, particularly in electric motors (Week 3 and 4) relies on the concept of Flux Linkage.

It is important to understand the relationship between Flux, Flux Density, Flux Linkage and Magnetic Vector Potential

How

By replacing B in the integral expression for Flux and then using Stoke's vector identity we can find a closed line integral for Flux.

For FEA, the flux can be found flowing between two points.

Flux Linkage is defined as the product of the number of turns that "Link" the flux and the magnitude of that Flux

$$\phi = \int \mathbf{B} \cdot \mathbf{dS} \; (6.1)$$

$$\phi = \int (\nabla \times \mathbf{A}) \cdot \mathbf{dS} \; (6.2)$$

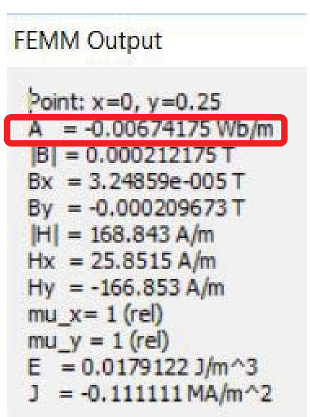
$$\phi = \oint A \cdot dl \; (6.3)$$

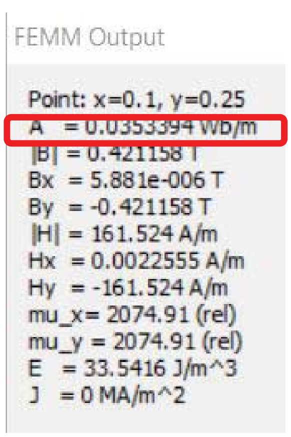
$$\phi_{12} = (A_{z1} - A_{z2})d \; (6.4)$$

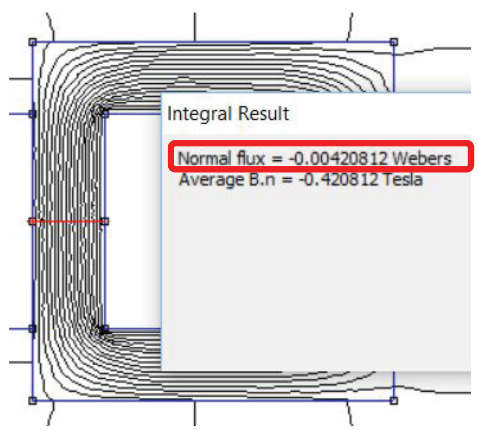
$$\lambda = N\phi \; (6.5)$$

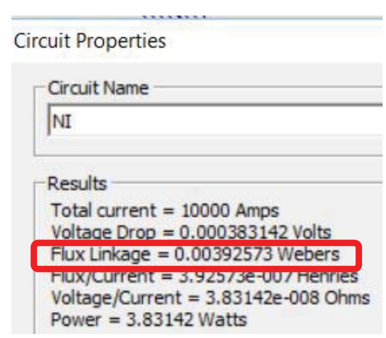
Example 6.1

Example 6.1 Finding Flux in Example 5.3 using Maxwell Given the one-half model of the "C" steel path with airgap of Example 5.3, find the flux passing through the steel pole face using Maxwell finite-element software and compare it with the reluctance solution of Example 3.1.





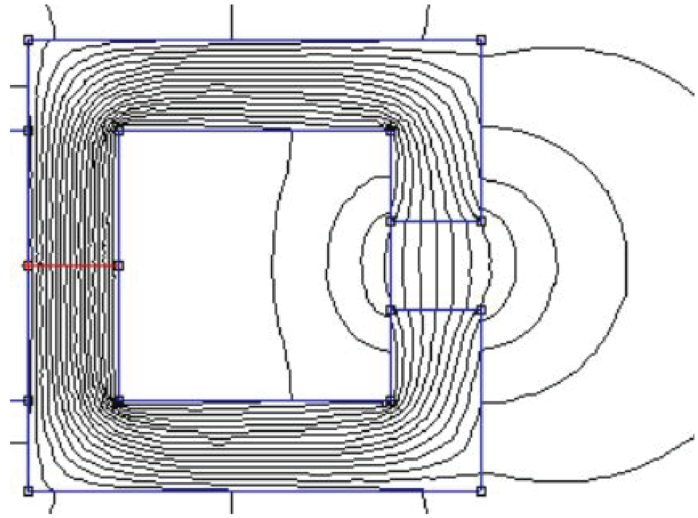




A1\`(Wb/m)	A2 (Wb/m)	d (m)	Flux 1-2 (Wb)	Flux - FEMM (Wb)	Flux Linkage - Coil Properties (Wb)
-6.74E-03	3.53E-02	0.1	-4.21E-03	-4.21E-03	3.93E-03

Test for understanding

Why is the Flux Linkage from the coil properties lower then the other calculated flux linkages?



A1`(Wb/m)	A2 (Wb/m)	d (m)	Flux 1-2 (Wb)	Flux - FEMM (Wb)	Flux Linkage - Coil Properties (Wb)
-6.74E-03	3.53E-02	0.1	-4.21E-03	-4.21E-03	3.93E-03

Inductance

Why

Inductance is an important parameter, particularly when considering the external circuit that may control an actuator or motor.

It is a nice way of taking all this "electromagnetic stuff" and creating a simplified model.

How - Inductance

- Inductance is defined as the Flux Linkage created per unit of current
- This can either be the flux linkage created in the same coil (self) or in another coil (mutual)
- If non-linear behaviour needs to be linearised then the two options are:
    - Secant Inductance
    - Incremental inductance

$$L = \lambda / I \quad (6.11)$$

$$L_{jk} = \lambda_j / I_k \quad (6.12)$$

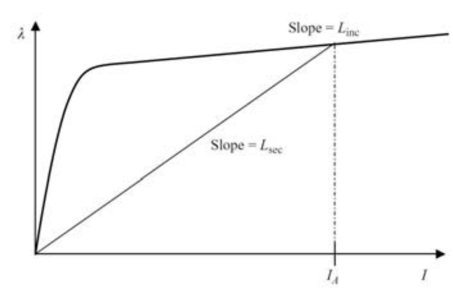


FIGURE 6.3 Inductances L_{sec} (secant) and L_{inc} (incremental) shown as slopes on a typical λ–I curve of a magnetic device.

How - Inductance

Using the definitions for inductance and reluctance, inductance can be shown to be proportional to $\mathbf{N}^2$

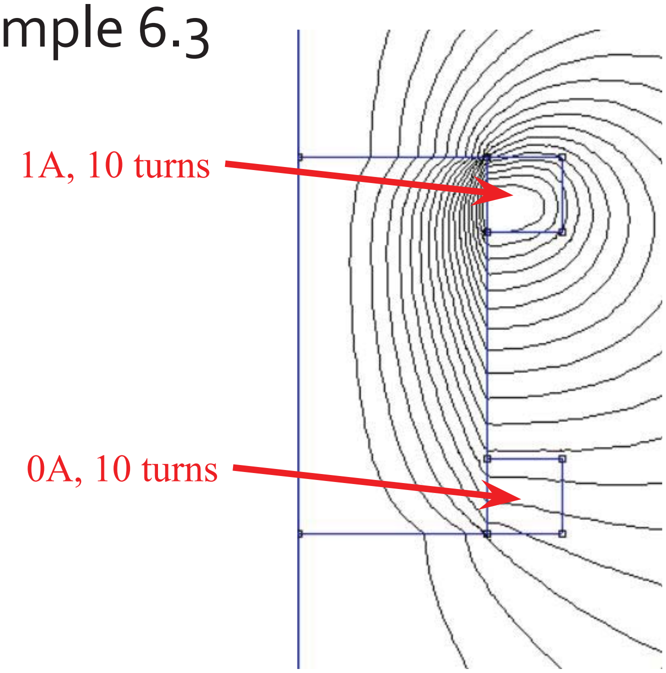
$$L = \lambda / I \quad (6.11)$$

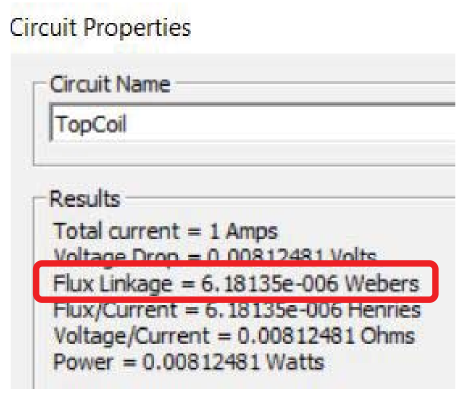
$$L = N\phi / I \quad (6.14)$$

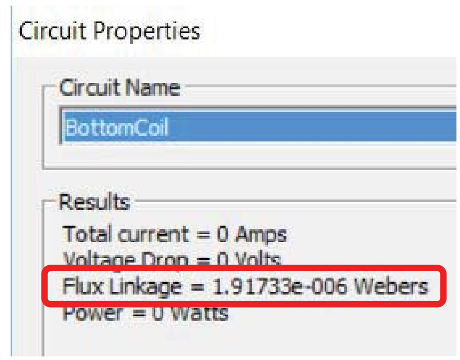
$$\mathscr{R} = NI/\phi \quad (6.15)$$

$$L = N^2/\mathscr{R} \quad (6.16)$$

Example 6.3







Test for understanding

What does it indicate if the value for inductance changes with the amount of current?

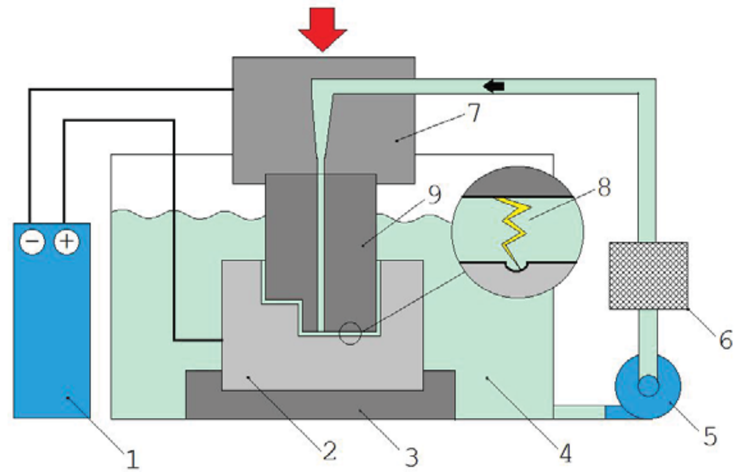
Would you expect this to happen with an “air-core” inductor?

Capacitance

Why

Capacitance effects usually occur at higher frequencies.

We will cover it briefly here for awareness, particularly of bearing currents.



https://commons.wikimedia.org/wiki/File%3AEDM_scheme.png

How

Rather than using the **B, H** and **A** fields we need to use the **D** and **E** fields

$$C = Q/V \quad (6.22)$$

$$W_{el} = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}dv = \frac{1}{2}\int \varepsilon E^2 dv = \frac{1}{2}CV^2 \quad (6.23)$$

Example 6.4

Example 6.4 Finding Capacitance using Maxwell Two aluminum plates 2 m wide are separated by 1 m as shown originally in Figure 2.7 and also in Figure E6.4.1. The lower plate is at 0-V DC and the upper at 1 V DC. The region between the two plates is assumed filled with polystyrene, which has a relative permittivity of 2.6. Find the voltage contours, electric field, energy stored, and capacitance using Maxwell. Validate the energy stored using (6.23).

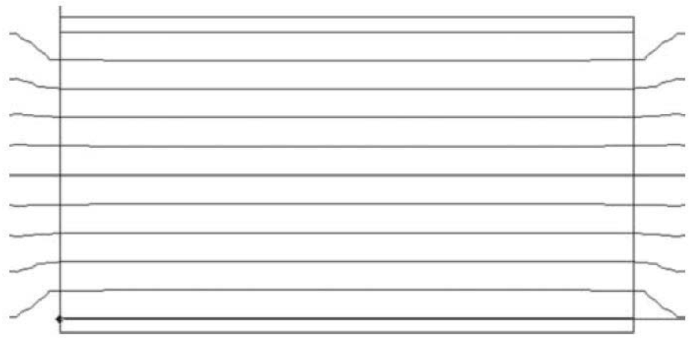


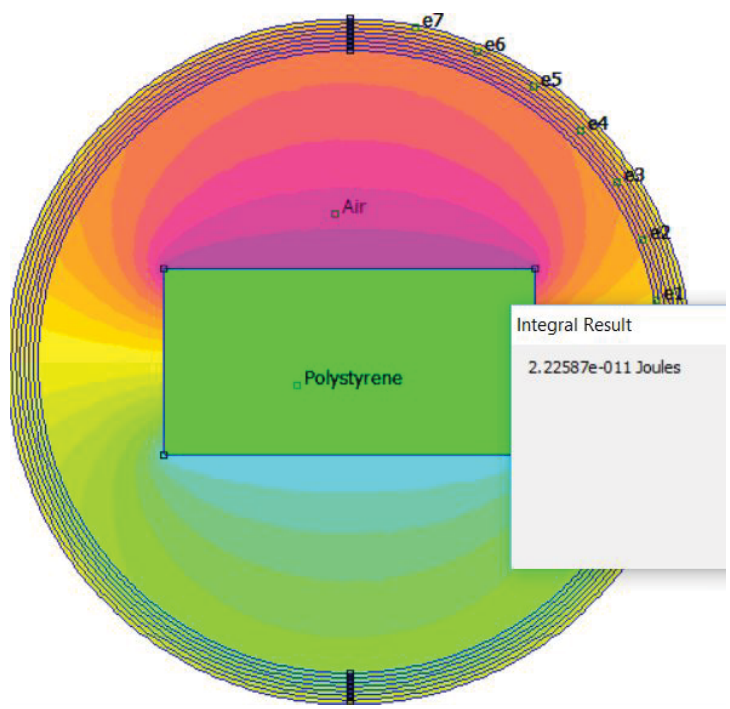
FIGURE E6.4.1 Computer display of capacitor with computed voltage contours.

Example

If the capacitance between two different components can be found then the entire motor can be modelled using an equivalent "spice" network

$$W_{el} = 2.41E{-}11 = \frac{1}{2}CV^2 = 0.5\ C \quad (E6.4.2)$$

$$C = 2(2.41E{-}11) = 48.2pF \qquad (E6.4.3)$$



Summary

Some derived parameters are frequently used for electromagnetic analysis.

In particular, we will consider:

a) Flux Linkage
b) Inductance
c) Capacitance

An understanding of these parameters is important when considering the electronic circuit driving the electromagnetic device

L3b: Magnetic Actuators Operated by AC

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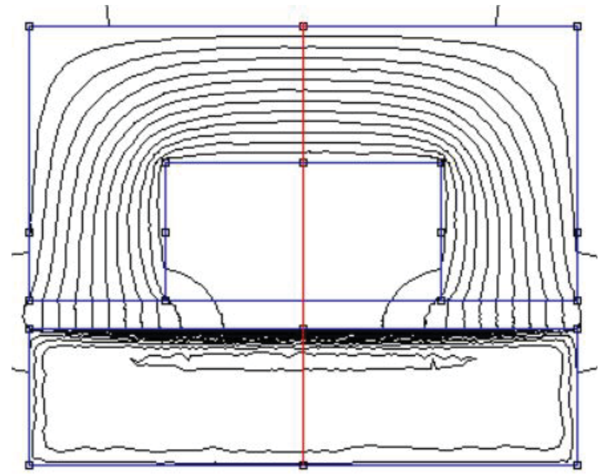
Why

- So far we have mostly focussed on DC operation.
- Many solenoids and motors operate on AC.
- We need to know what design changes are required.
- This lecture will focus on the impact of AC operation on steel cores. The impact on current carrying conductors will be addressed in L3c



Skin Depth

Why

Skin depth is one of the critical issues that arises in electromagnetics when we begin to deal with AC frequencies.

It is one of the main reasons laminated cores are required.

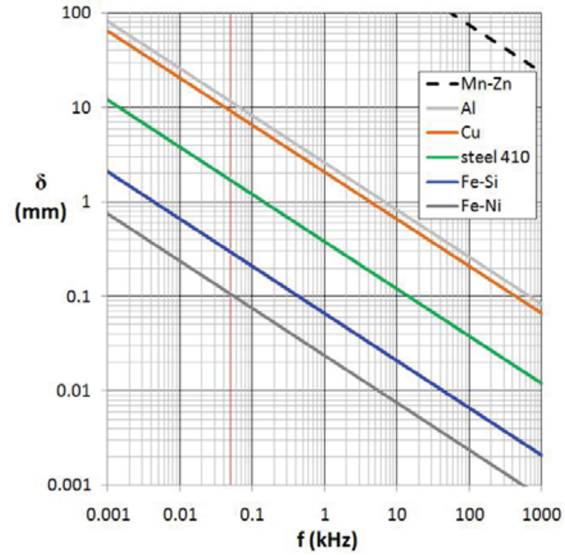
How

The skin depth is the depth at which and eddy currents have decayed to 1/e or 36.8% of their surface value.

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad (8.1)$$

Skin depth for different materials

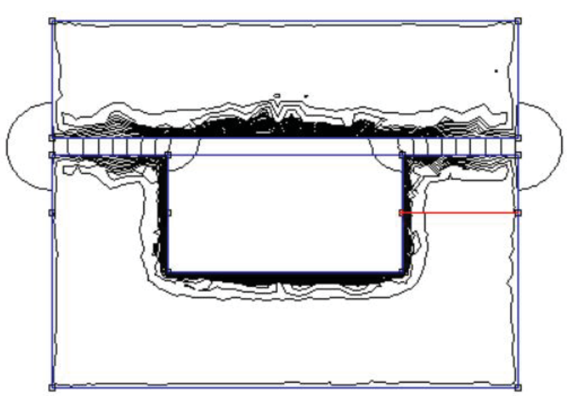
	Frequency (Hz)	Relative Permeability	conductivity (S/m)	Skin Depth (mm)
Carbon steel (1010)	50	2000	6.99E+06	0.6
Silicon Steel (M15)	50	2000	1.90E+06	1.2
Copper	50	1	5.96E+07	9.2
Carbon steel (1010)	500	2000	6.99E+06	0.2
Silicon Steel (M15)	500	2000	1.90E+06	0.4
Copper	500	1	5.96E+07	2.9

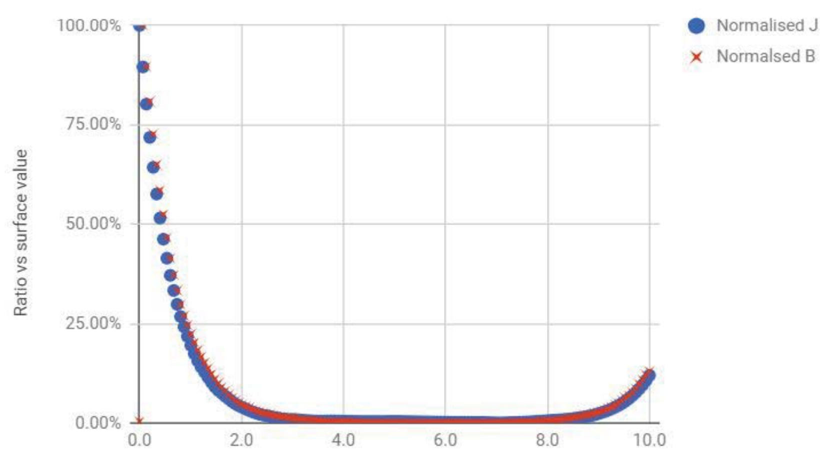


https://commons.wikimedia.org/wiki/File%3ASkin_depth_by_Zureks.png

Example 8.0 (7.1 at 50Hz)

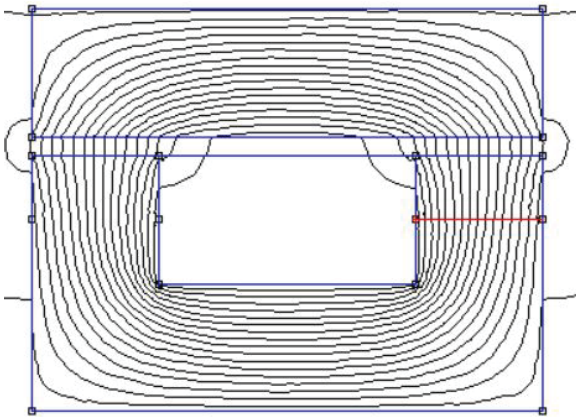
Solid Steel

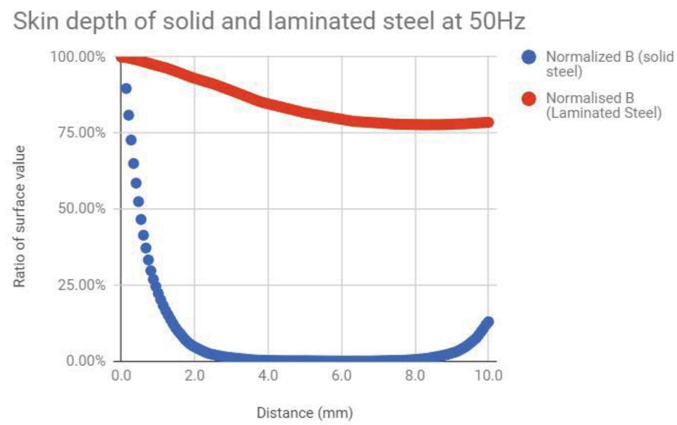




Example 8.0 (7.1 at 50Hz)

0.5 mm laminations





Lamination methods

- *For planar actuators with clapper armatures*, the stator is easily laminated, but often the armature is made of solid steel, especially if the armature is steel plate (or steel scrap) to be lifted by a *lifting magnet*.
- *For axisymmetric actuators with clapper armatures,* lamination is difficult and expensive, and thus both stator and armature steels are usually solid.
- *For planar actuators with plunger armatures,* both stator and armature are usually laminated, but a solid armature is sometimes required due to its greater mechanical rigidity.
- *For axisymmetric actuators with clapper armatures,* lamination is difficult and expensive, and thus both stator and armature steels are usually solid.

Test for understanding

For the same frequency, is the skin depth larger in:

a) Steel
b) Aluminium

A smaller skin depth allows:

a) More flux to flow
b) Less flux to flow

Power Losses in Steel: Laminated Steel

Why

One of the main losses in electric motors and actuators is “core loss”

Any high efficiency motor or AC actuator design will need to carefully consider core loss.

How

$$\frac{P_e}{v} = \frac{t^2 \omega^2 B^2 \sigma}{24} \quad (8.2)$$

$$\frac{P_h}{V} = K_h f B^n \quad (8.3)$$

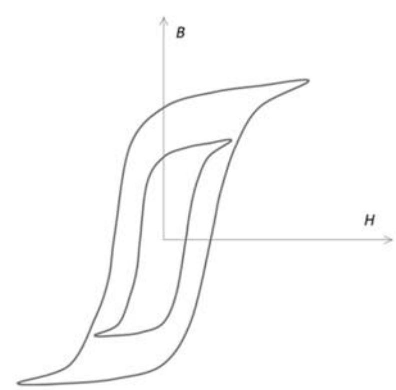


FIGURE 8.1 *B–H* hysteresis loops for AC *H* due to AC. The area enclosed is the energy lost per cycle. Two loops are shown for two different peak AC values.

How

The value of n in 8.3 is defined by fitting curves to loss data.

In the FEMM wiki (http://www.femm.info/wiki/SPMLoss) a reasonable starting point is n = 2.

$$\frac{P_e}{v} = \frac{t^2\omega^2 B^2\sigma}{24} \quad (8.2)$$

$$\frac{P_h}{V} = K_h f B^n \quad (8.3)$$

$$\frac{P_{core}}{m} = \frac{P_h}{m} + \frac{P_e}{m} = C_h\omega B^2 + C_e\omega^2 B^2$$

Example

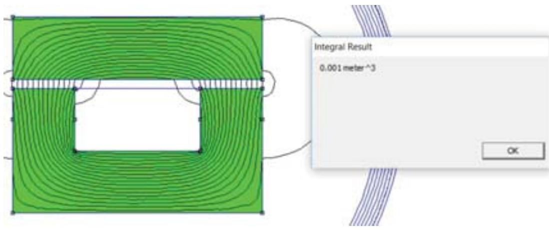
Modifying the coefficients from http://www.femm.info/wiki/SPMLoss for M19 steel 29 gauge (0.36mm) gives:

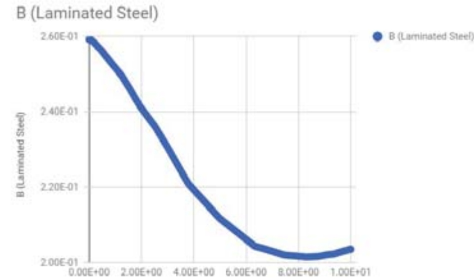
$$C_h = 0.0186\frac{W}{kg.T^2.Hz}$$

$$C_e = 68.8e^{-6}\frac{W}{kg.T^2.Hz^2}$$

		Frequency (Hz)				
		50	100	200	400	1000
B (T)	0.3	0.1	0.2	0.6	1.7	7.9
	0.6	0.4	0.9	2.3	6.6	31
	0.9	0.9	2.1	5.2	15	71
	1.2	1.6	3.7	9.3	27	126
	1.5	2.5	5.7	15	42	197

Example





$$\frac{P_{core}}{m}(50Hz) = 0.04\frac{W}{kg}$$

$$V = 0.001m^3$$

$$\rho = 7800 \times 0.96 = 7488\frac{kg}{m^3}$$

$$m = 7488 \times 0.001 = 7.488kg$$

$$P_{core} = 0.04 \times 7.488 = 0.3W$$

		Frequency (Hz)				
		50	100	200	400	1000
B (T)	0.2	0.04	0.1	0.3	0.7	3.5
	0.3	0.1	0.2	0.6	1.7	7.9
	0.6	0.4	0.9	2.3	6.6	31
	0.9	0.9	2.1	5.2	15	71
	1.2	1.6	3.7	9.3	27	126
	1.5	2.5	5.7	15	42	197

Industry core loss data

Traditionally core loss data has been published only at 50 and 60Hz

The coefficients from the FEMM wiki were extracted from data on the Protolam site http://www.protolam.com/page5.html

Thinner laminations (higher gauge number) will be better for eddy current losses

Semi processed steel is popular in the United States. Fully processed steel is standard elsewhere.

Table 5
Core Loss Limits for Fully Processed Material at 60 Hz* (ASTM A 677) (Watts/lb)

	15 kG		
AK Steel Grade	**0.014" (29 gauge)**	**0.0185" (26 gauge)**	**0.025" (24 gauge)**
DI-MAX M-15	1.45	1.60	—
DI-MAX M-19	1.55	1.65	2.00
DI-MAX M-22	1.60	1.80	2.10
DI-MAX M-27	1.75	1.90	2.25
DI-MAX M-36	1.85	2.00	2.35
DI-MAX M-43	1.95	2.10	2.50
DI-MAX M-45	2.05	2.40	2.75
DI-MAX M-47	—	2.80	3.20

*As-Sheared, 50/50.

Table 6
Core Loss Limits for Semi-Processed Material at 60 Hz* (ASTM A 683) (Watts/lb)

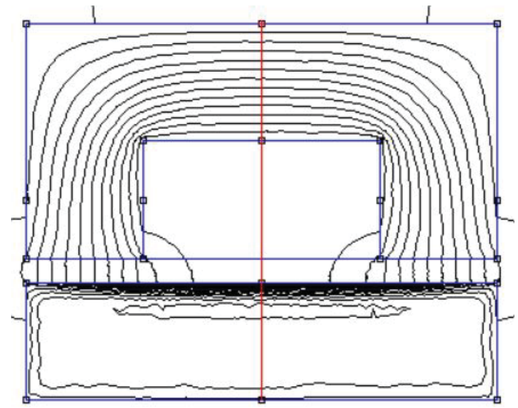
	15 kG	
AK Steel Grade	**0.0185" (26 gauge)**	**0.025" (24 gauge)**
DI-MAX M-43	1.55	2.00
DI-MAX M-47	1.65	2.10

http://www.aksteel.com/pdf/markets_products/electrical/non_oriented_bulletin.pdf

Test for understanding

Example 8.1 AC Flux Linkage and Equivalent Circuits of Solenoid of Example 7.1 with a Solid Steel Clapper Figure E8.1.1 shows the solenoid of Example 7.1 turned upside down to act as an AC lifting magnet. The stator is laminated but the clapper being lifted is solid steel. The stator winding shown has 200 turns carrying 2-A rms AC 60-Hz current. The dimensions are $w = 10$ mm, $Al_1 = 5$ mm, $Al_2 = 30$ mm, $Al_3 = 5$ mm, $Sl_1 = 15$ mm, $Sl_2 = 30$ mm, $Sl_3 = 15$ mm, $g = 2$ mm. Assuming all steel has relative permeability of 2000, and the clapper has conductivity 2.E6 S/m

Assume that the stator has a depth into the page of 1 m in the z direction



Plot the flux density along the red line shown in the figure. Compare the impact of skin depth shown in impact with the skin depth calculated from equation 8.1

Summary

For AC operation, laminations are required to avoid the impact of skin depth.

$$\frac{P_{core}}{m} = \frac{P_h}{m} + \frac{P_e}{m} = C_h \omega B^2 + C_e \omega^2 B^2$$

L3b: Coil Design and Temperature Calculations

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Why

So far in this course we have only modelled a coil as a rectangle.

In reality we need to design a coil with multiple turns.

The coil design needs to:

- Create the desired electromagnetic force
- Match the voltage of the power supply
- Not exceed the maximum temperature rise

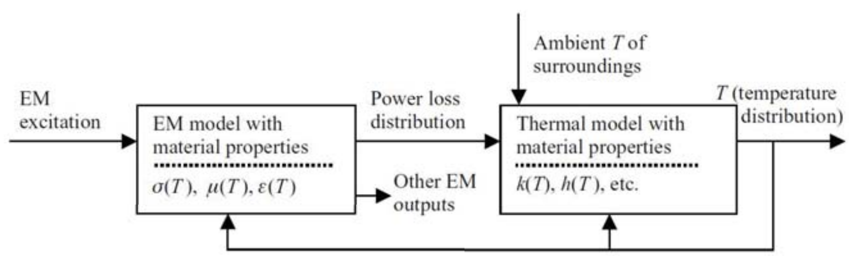


FIGURE 12.1 Modeling temperature and its effects in magnetic devices, to be discussed in this chapter.

Wire Size Determination for DC Currents

Why - Wire Size Determination for DC Currents

DC is a good place to start!

How - Wire Size Determination for DC Currents

DC current is determined by Ohm's law

$$I = V/R \quad (12.1)$$

Resistance is defined by the length, conductivity and cross sectional area.

$$R = l/(\sigma S_c) \quad (12.2)$$

For a circular conductor:

$$S_c = \pi r^2 = \pi d^2/4 \quad (12.4)$$

Conductivity is affected by temperature

$$\sigma = \frac{5.8\text{E}7\ \text{S/m}}{1 + 0.00393(T - 20°\text{C})} \quad (12.3)$$

Commercially available circular wire sizes

$$d = (0.00826)(1.123^{-\text{AWG}})\ \text{m} \quad (12.5)$$

AWG SIZE	NEMA Nominal Diameter (Inches)	NEMA Nominal Diameter (mm)	IEC R-20 Series (mm)
4/0	0.46	11.684	-
3/0	0.4096	10.404	-
2/0	0.3648	9.266	-
1/0	0.3249	8.252	-
1	0.2893	7.348	-
2	0.2576	6.543	-
3	0.2294	5.827	-
4	0.2043	5.189	5
5	0.1819	4.62	4.5
6	0.162	4.115	4
7	0.1443	3.665	3.55
8	0.1285	3.264	3.15
9	0.1144	2.906	2.8
10	0.1019	2.588	2.5
11	0.0907	2.304	2.24

AWG SIZE	NEMA Nominal Diameter (Inches)	NEMA Nominal Diameter (mm)	IEC R-20 Series (mm)
12	0.0808	2.052	2
13	0.072	1.829	1.8
14	0.0641	1.628	1.6
15	0.0571	1.45	1.4
16	0.0508	1.29	1.25
17	0.0453	1.151	1.12
18	0.0403	1.024	1
19	0.0359	0.912	0.9
20	0.032	0.813	0.8
21	0.0285	0.724	0.71
22	0.0253	0.643	0.63
23	0.0226	0.574	0.56
24	0.0201	0.511	0.5
25	0.0179	0.455	0.45
26	0.0159	0.404	0.4

AWG SIZE	NEMA Nominal Diameter (Inches)	NEMA Nominal Diameter (mm)	IEC R-20 Series (mm)
27	0.0142	0.361	0.355
28	0.0126	0.32	0.315
29	0.0113	0.287	0.28
30	0.01	0.254	0.25
31	0.0089	0.226	0.224
32	0.008	0.203	0.2
33	0.0071	0.18	0.18
34	0.0063	0.16	0.16
35	0.0056	0.142	0.14
36	0.005	0.127	0.125
37	0.0045	0.114	0.112
38	0.004	0.102	0.1
39	0.0035	0.089	0.09
40	0.0031	0.079	0.08
41	0.0028	0.071	0.071

AWG SIZE	NEMA Nominal Diameter (Inches)	NEMA Nominal Diameter (mm)	IEC R-20 Series (mm)
42	0.0025	0.064	0.063
43	0.0022	0.056	0.056
44	0.002	0.051	0.05
45	0.00176	0.0447	0.045
46	0.00157	0.0399	0.04
47	0.0014	0.0356	0.035
48	0.00124	0.0315	0.031
49	0.00111	0.0282	0.028
50	0.00099	0.0251	0.025
51	0.00088	0.0224	0.022
52	0.00078	0.0198	0.02
53	0.0007	0.0178	0.0187
54	0.00062	0.0157	0.0157
55	0.00055	0.014	0.014
56	0.00049	0.0124	0.0124

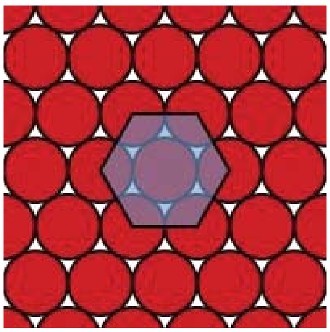
Adapted from data: http://www.litz-wire.com/wirediminsions.php

Theoretical packing

The relationship between the conductor cross sectional area and the available area is the ***Packing Factor***

$$F_p = NS_c / S_w \quad (12.6)$$

Maximum theoretical circle packing 91%



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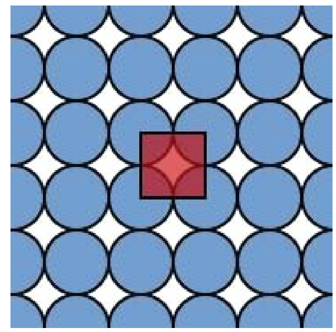

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Practical packing

Wire enamel

Bobbin or insulator

Nesting or not

Wire insulation

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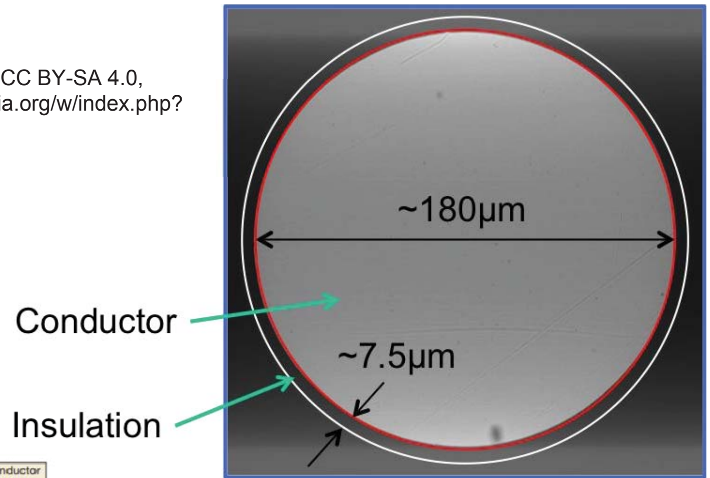

Dimensional table of magnet wire

The dimensional tables of magnet wire are as shown in Tables 12 to 18.

Table 12: Diamensional table of enamelled wire

Conductor			Class 0			Class 1			Class 2			Maximum conductor resistance 20℃ (Ω/km)	
Diameter (mm)	Tolerance (mm) Class 0, 1	Class 2	Minimum film thickness (mm)	Maximum overall diameter (mm)	Estimated mass (kg/km)	Minimum film thickness (mm)	Maximum overall diameter (mm)	Estimated mass (kg/km)	Minimum film thickness (mm)	Maximum overall diameter (mm)	Estimated mass (kg/km)	Class 0, 1	Class 2
3.20	±0.04	–	0.049	3.388	72.4	0.034	3.338	72.2	–	–	–	2.198	–
3.00	±0.03	–	0.049	3.178	63.7	0.034	3.128	63.4	–	–	–	2.489	–
2.90	±0.03	–	0.049	3.078	59.5	0.034	3.028	59.3	–	–	–	2.665	–
2.80	±0.03	–	0.049	2.978	55.5	0.034	2.928	55.3	–	–	–	2.861	–
2.70	±0.03	–	0.049	2.878	51.7	0.034	2.828	51.4	–	–	–	3.079	–
2.60	±0.03	–	0.049	2.778	47.9	0.034	2.728	47.7	–	–	–	3.324	–
2.50	±0.03	–	0.049	2.678	44.3	0.034	2.628	44.1	–	–	–	3.598	–
2.40	±0.03	–	0.048	2.574	40.9	0.033	2.526	40.7	–	–	–	3.908	–
2.30	±0.03	–	0.046	2.468	37.6	0.032	2.422	37.4	–	–	–	4.260	–
2.20	±0.03	–	0.046	2.368	34.4	0.032	2.322	34.2	–	–	–	4.662	–
2.10	±0.03	–	0.045	2.266	31.3	0.031	2.220	31.2	–	–	–	5.123	–
2.00	±0.03	–	0.044	2.162	28.4	0.030	2.118	28.3	–	–	–	5.656	–
1.90	±0.03	–	0.044	2.062	25.7	0.030	2.018	25.6	–	–	–	6.278	–
1.80	±0.03	–	0.042	1.956	23.1	0.029	1.914	22.9	–	–	–	7.007	–
1.70	±0.03	–	0.042	1.856	20.6	0.029	1.814	20.5	–	–	–	7.871	–

https://www.hitachi-metals.co.jp/e/products/auto/el/pdf/MagnetWire_en.pdf

Example 12.1

Example 12.1 Simple DC Coil Design at a Given Temperature An axisymmetric copper coil is to be designed to operate at a maximum temperature of 60°C. The available winding area S_w is 1.E−3 m^2 and the average coil radius is 5 cm. Assume a packing factor of 70% and a DC voltage of 12 V. If $NI = 1000$ ampere-turns, find the bare wire diameter, the number of turns N, and the current density.

Test for understanding

List 3 components that will need to be present in the coil space that will reduce the conductor cross sectional area.

Coil Time Constant and Impedance

Why

Often the dynamic operation of an actuator is important

This will depend on the:

- Electric time constant
- And the mechanical time constant (***J/b***)

$$I = \frac{V}{R}(1 - e^{-t/\tau_e}) \quad (12.7)$$

How/ Example

The force required for an actuator will define the product of N and I.

Will the choice of N affect the time constant or the power loss?

$$I = \frac{V}{R}(1 - e^{-t/\tau_e}) \quad (12.7)$$

Test for understanding

Which of the following will be affected by the choice of N

a) Power loss
b) Voltage required
c) Time constant
d) a) and b)
e) b) and c)

Skin Effects and Proximity Effects for AC Currents

Why

In L3b we looked at how AC excitation effects the flux and power loss in steel cores.

In this section we will consider how AC excitation effects current carrying conductors.

How

The skin depth equation is the same.

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad (12.17)$$

When the skin depth is much greater than the wire radius:

$$\frac{P_e}{v} = \frac{\omega^2 \sigma}{24} \left(w_y^2 B_x^2 + w_x^2 B_y^2\right) (12.18)$$

When the skin depth is less than the wire radius then skin effects losses occur:

$$P = \int \frac{J^2}{\sigma} dv \, (12.19)$$

Litz wire

Litz wire can be used to reduce the skin effects by reducing the diameter of each conductor.

It also reduces "proximity" effects by transposing the wires to cancel eddy current producing fields.

Example

Example 12.2 Skin Effect in an Isolated Conductor A circular copper wire carrying 100 A peak is placed in air, far from any other materials. If its radius is 10 mm, find its current density and magnetic flux density distributions at 400 Hz and 1 Hz using Maxwell. Also find the power loss and resistance per meter.

Test for understanding

Use FEMM to find:

- Power loss
- Flux line plot
- Eddy current distribution

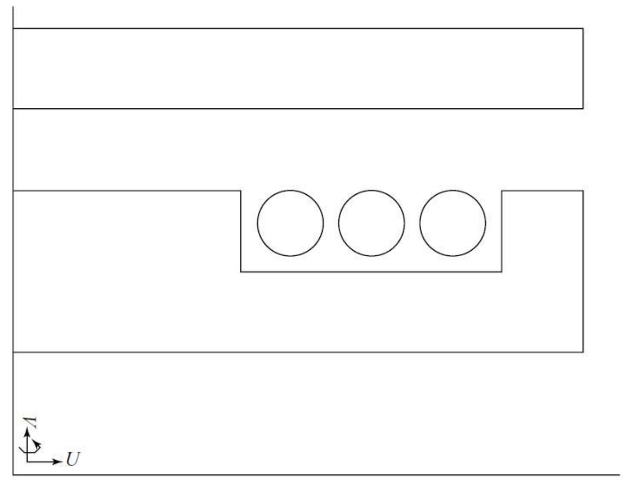

FIGURE E12.3.1 Three-turn winding in ferrite cup core inductor, as drawn in Maxwell's default window. The wire radius is 4 mm.

Example 12.3 Skin and Proximity Effects in Stator Coil with Clapper An axisymmetric copper coil is placed in a ferrite "cup core" inductor shown in Figure E12.3.1 as created in Maxwell's default drawing area. The area is assumed to confine the flux. The ferrite in the lower stator and upper clapper armature has a relative permeability of 1000 and conductivity of 0.01 S/m. The coil has three copper conductors, each of radius 4 mm and carrying 50 A 400 Hz. Use Maxwell to find the power loss, flux line plot, and current density distribution, showing skin and proximity effects in the three wires.

Thermal Conduction

Why - Conduction

Heat is usually the limiting factor that defines how much force can be obtained from a given actuator.

How - Conduction

Governing equation:

$$\nabla \cdot k \nabla T = -\frac{P}{v} \quad (12.20)$$

$\boldsymbol{k}$ - Thermal conductivity (material property)

$\boldsymbol{T}$ - Temperature

$\boldsymbol{P}$ - Power

$\boldsymbol{v}$ - volume

Example 12.4

Compute the temperature distribution

Example 12.4 Steady Thermal Conduction Computation Using Analogy to Electrostatics An axisymmetric copper coil is surrounded by glastic insulation and a cylindrical steel core as shown in Figure E12.4.1. The thermal conductivities are

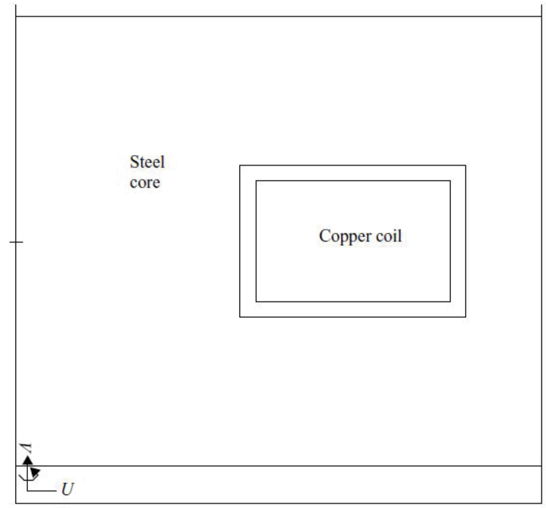

FIGURE E12.4.1 Axisymmetric inductor of radius 70 mm and height 60 mm. The temperature distribution is to be computed when there are power losses in the steel and in the copper coil of size 26 mm radially and 16 mm high, surrounded by glastic insulation 2 mm thick on all sides.

Summary

While in FEA it is fine to have a rectangle with NI, in reality this needs to be a coil that is designed with real wire (What N, what d)?

- Choose N for available voltage
- The choice of N will not affect the time constant or the power loss

Fill factor is affected by:

- Wire diameter (hence possible packing)
- Enamel thickness
- Bobbin or core insulation

FEMM can model conduction heat transfer