L1c: Basic Electromagnetics

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Overview

This lecture will explore three different areas:

1. The vector calculus techniques later work is based on.
2. Introduction to electromagnetic variables.
3. Mathematical relationship between electromagnetic components.

Vector calculations: Gradient, Divergence and Curl

Why

Most of electromagnetic theory is based on Maxwell's equations (see right)

To use these equations requires knowledge of vector calculus (grad, div and curl operators).

Vector calculus simplifies and abstracts complex relationships into a form that is more easily c

$$\nabla \cdot \mathbf{B} = 0 \qquad (2.48)$$

$$\nabla \cdot \mathbf{D} = \rho_v \qquad (2.49)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (2.50)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad (2.51)$$

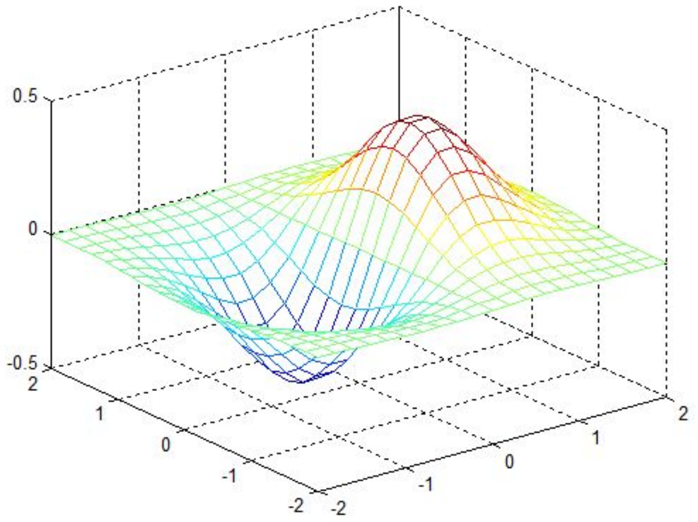
How - Gradient

$$\nabla T = \frac{\partial T}{\partial x}\mathbf{u}_x + \frac{\partial T}{\partial y}\mathbf{u}_y + \frac{\partial T}{\partial z}\mathbf{u}_z \quad (2.2)$$

Equation 2.2 shows the expression for the gradient.

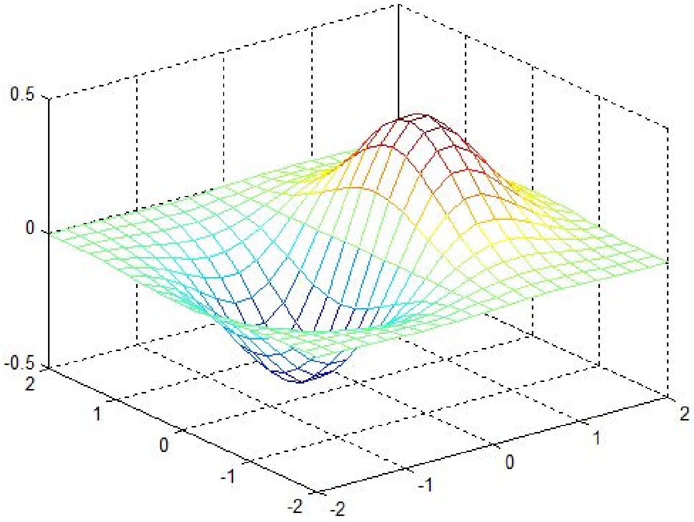
Note $\mathbf{u}_x$, $\mathbf{u}_x$ and $\mathbf{u}_x$ are unit vectors in the x, y and z directions respectively.

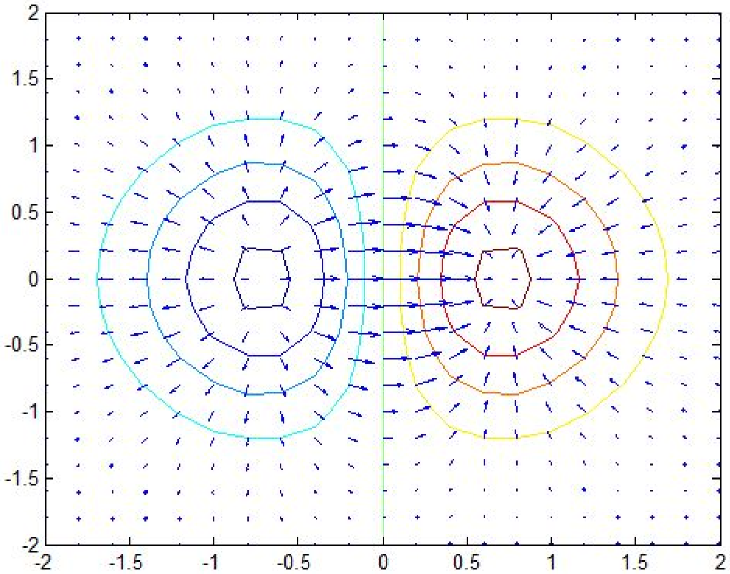
Scalar in > vector out



https://au.mathworks.com/help/matlab/ref/gradient.html

Gradient visualisation





Example 2.1

Example 2.1 Gradient Calculations Find the gradient of the following temperature distribution at locations $(x,y,z) = (1,2,3)$ and $(4,-2,5)$:

$$T = 5x + 8y^2 + 3z \qquad \text{(E2.1.1)}$$

$$\nabla T = \frac{\partial T}{\partial x}\mathbf{u}_x + \frac{\partial T}{\partial y}\mathbf{u}_y + \frac{\partial T}{\partial z}\mathbf{u}_z$$

1. Find the expression for the gradient by taking partial derivatives in each direction
2. Evaluate the gradient at the specified locations

How - Divergence

Vector in, Scalar out

$$\nabla \cdot \mathbf{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} \quad (2.5)$$

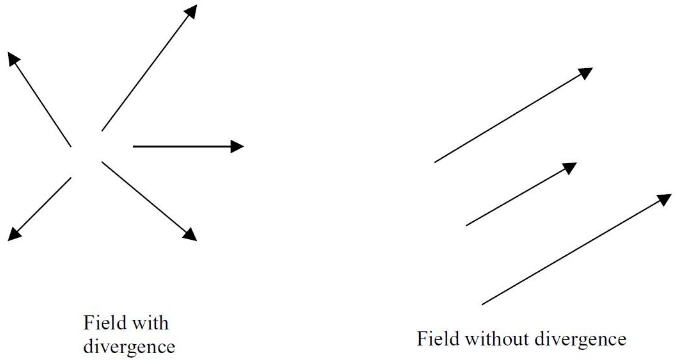


FIGURE 2.2 Field with and without divergence.

$$\nabla \cdot \mathbf{J} = 0 \quad (2.6)$$

Example 2.2: Divergence

Find the Divergence at location (3,2,-1) of the vector:

$$\mathbf{A} = [8x^4 + 6(y^2 - 2)]\mathbf{u_x} + [9x + 10y + 11z]\mathbf{u_y} + [4x]\,\mathbf{u_z}$$

$$\nabla \cdot \mathbf{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}$$

1. Find the expression for the divergence by taking the sum of the partial derivatives in each direction
2. Evaluate the divergence at the specified location

How - Curl

Vector in, Vector out

$$\nabla \times \mathbf{A} = \begin{pmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \\ \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \end{pmatrix} \quad (2.7)$$

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\mathbf{u}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\mathbf{u}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\mathbf{u}_z \qquad (2.8)$$

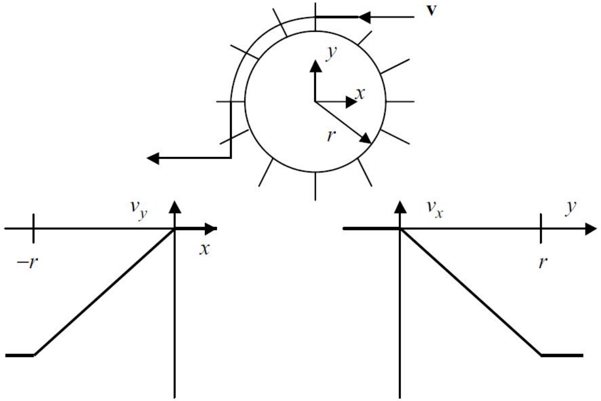


FIGURE 2.3 A rotating wheel is analogous to curl.

Curl visualisation - exercise

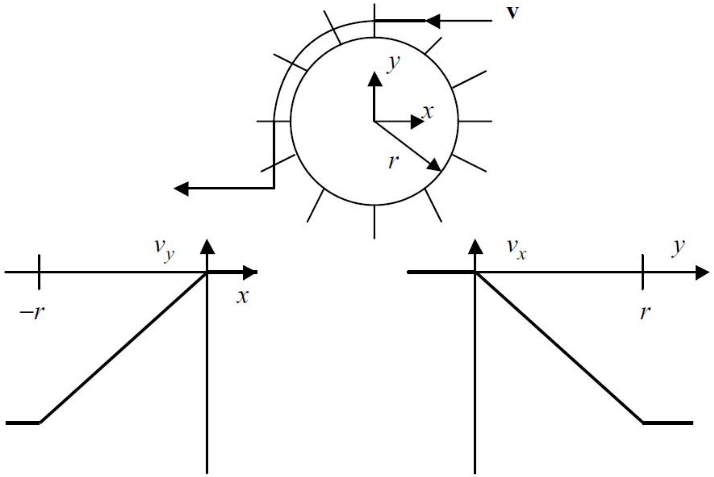
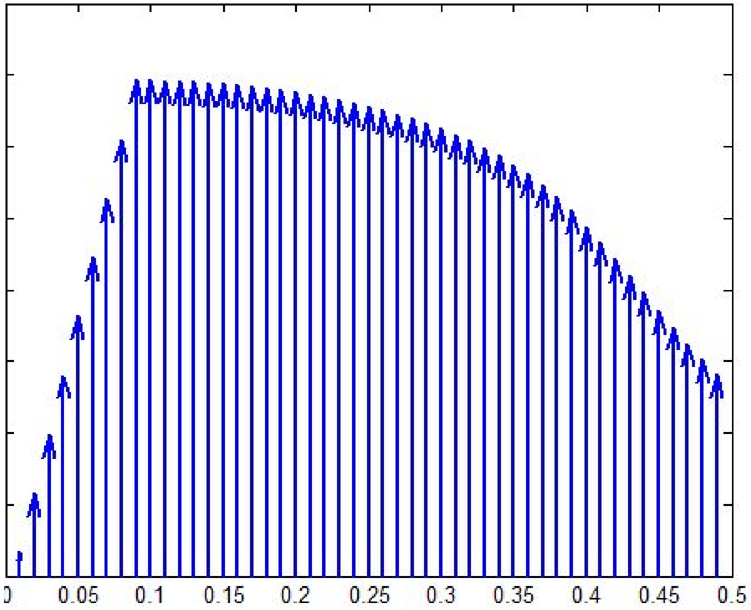
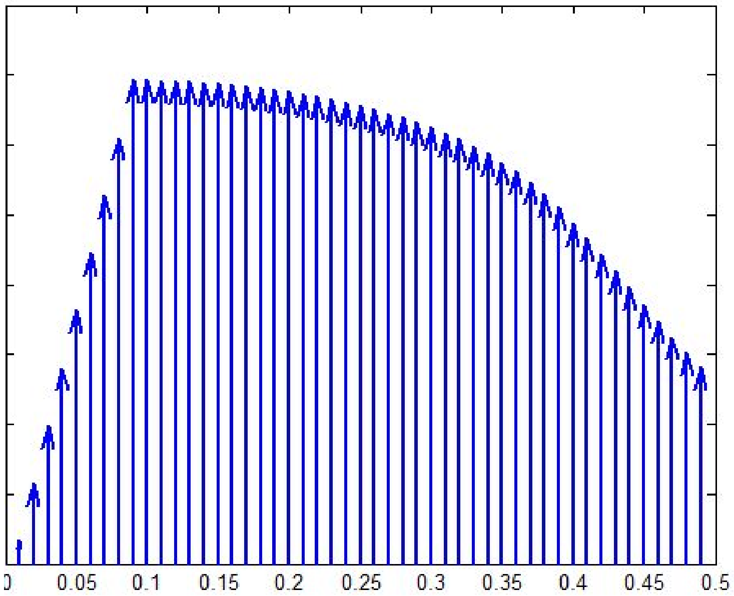


FIGURE 2.3 A rotating wheel is analogous to curl.

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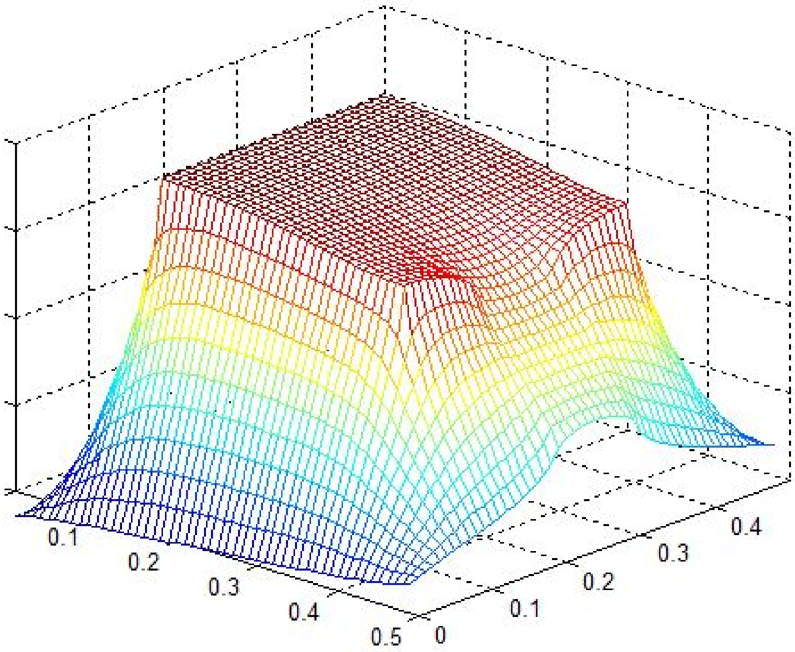
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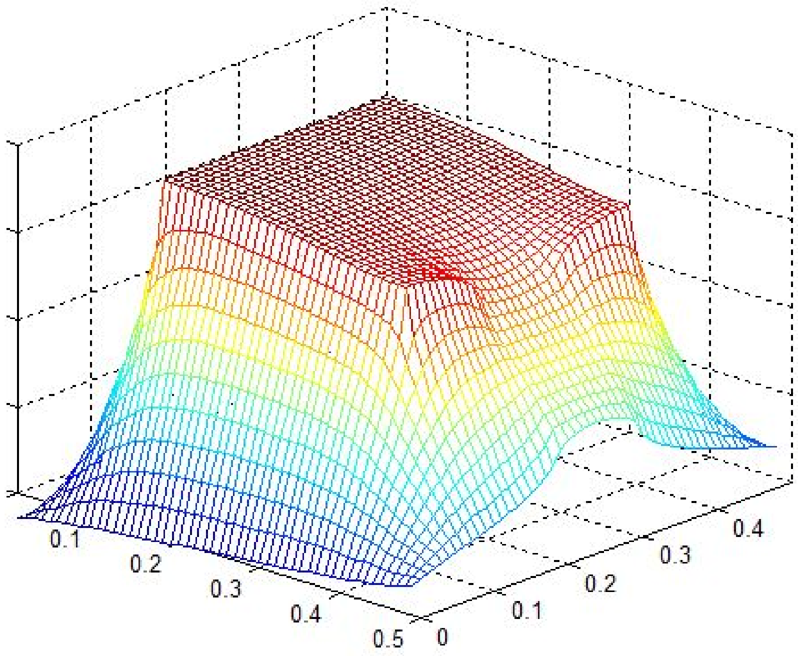
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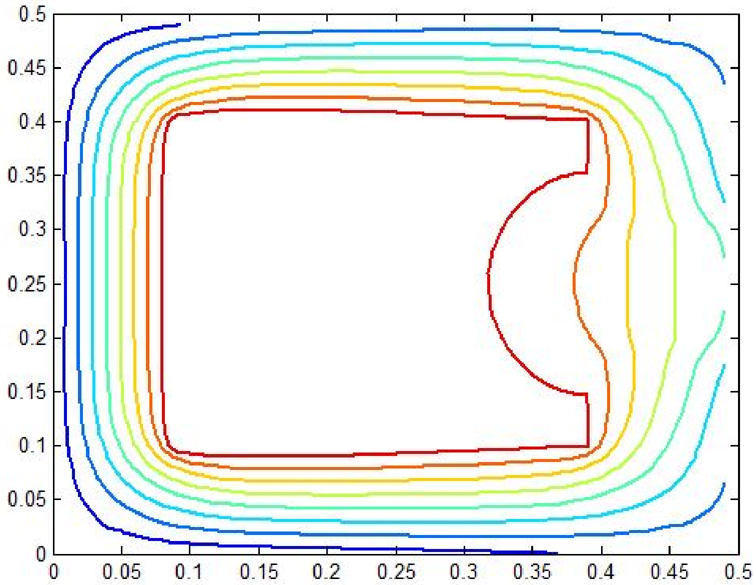
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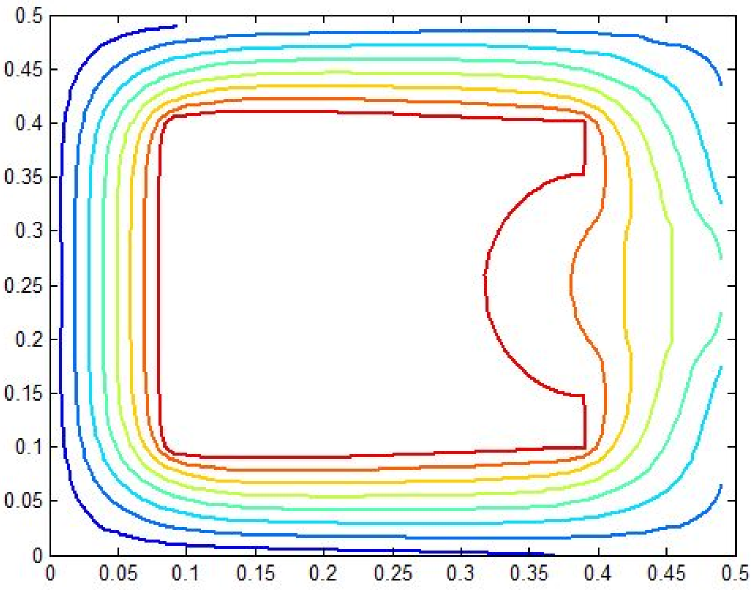
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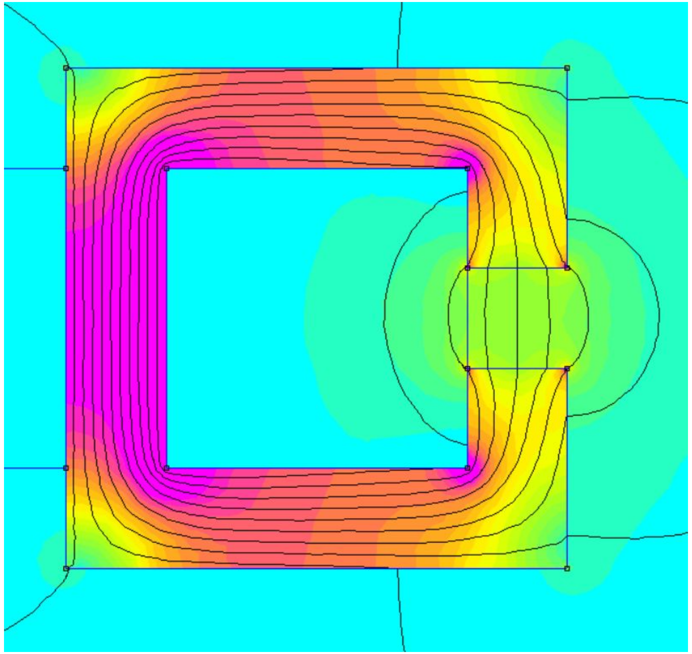


Example 2.2: Curl

Find the Curl at location (3,2,-1) of the vector:

$$\mathbf{A} = [8x^4 + 6(y^2 - 2)]\mathbf{u_x} + [9x + 10y + 11z]\mathbf{u_y} + [4x]\,\mathbf{u_z}$$

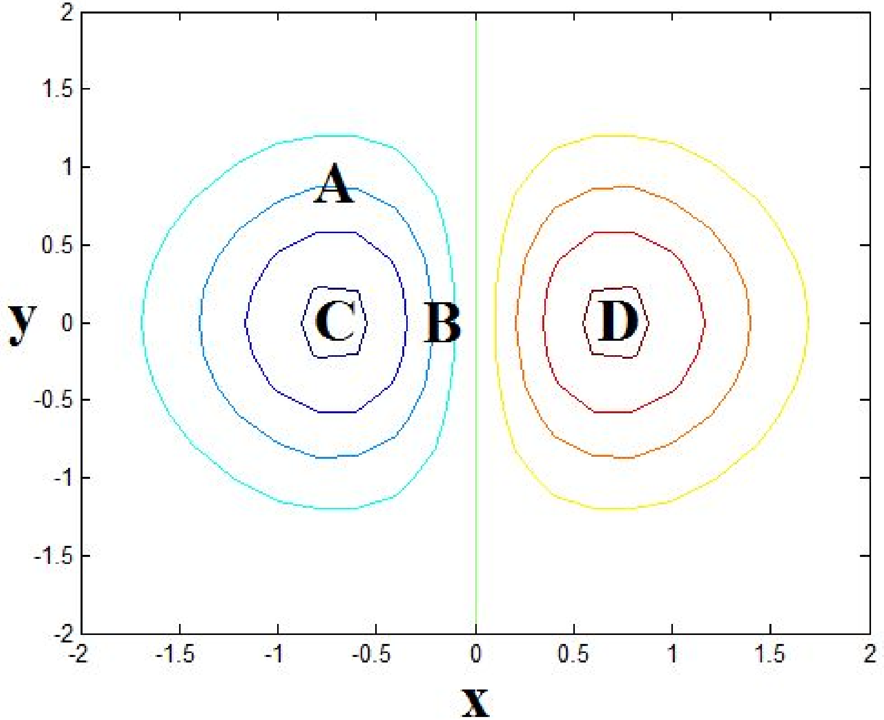
$$\nabla \times \mathbf{A} = \begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \\ \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \end{pmatrix}$$

1. Find the expression for the curl by taking the cross product of the partial derivatives in each direction
2. Remember how to evaluate the cross product using the determinant
3. Evaluate the curl at the specified location

Test for understanding

For this contour plot showing a function in x and y, where is the greatest gradient (A,B,C,D)?

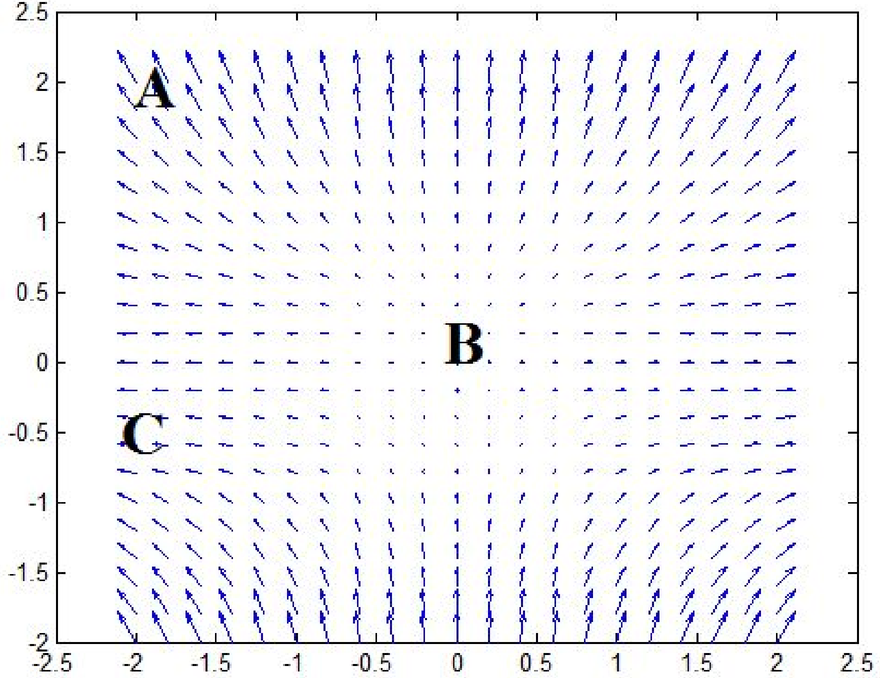
The direction of the greatest gradient is parallel to the x axis (True/ False)?



Test for understanding

For this contour plot showing a function ($F = x\mathbf{u}_x + y^2\mathbf{u}_y$), where is the greatest divergence (A, B, C)?

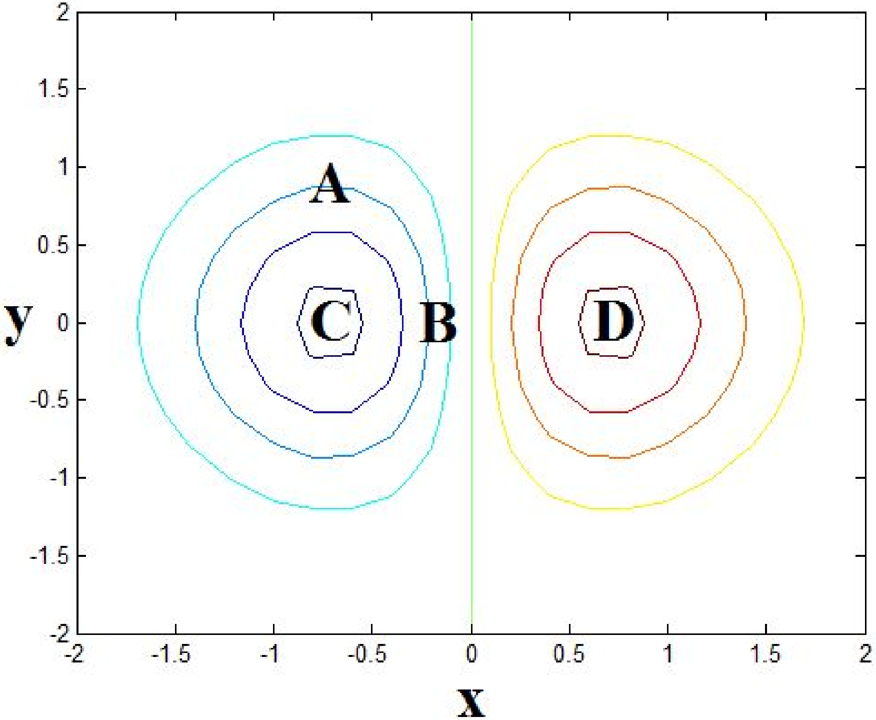
Where does the divergence equal zero? (A, B, C)?



Test for understanding

For this contour plot showing a function in x and y, where is the greatest curl (A,B,C,D)?

The direction of the greatest curl is parallel to the x axis (True/ False)?



Ampere’s Law

Why - Ampere's Law

Ampere's law relates the field intensity to the current density.

A version of Ampere's law is critical to the reluctance method presented in the next lecture.

Electromagnetic quantities

Quantity	Symbol	Units	Comment
Magnetic Field Intensity	**H**	Ampere-Turn/ metre (A/m)	Independent of Material
Magnetic Flux Density	**B**	Tesla (T)	Dependant on Material
Magnetic Vector Potential	**A**	Volt seconds / metre (Vs/m)	
Electric Flux Density	**D**	Coulomb/ square meter (C/m^2)	
Electric Field Intensity	**E**	Volt/ metre (V/m)	
Current density	**J**	Current Density	
Permeability	μ	Tesla-metre/Ampere-Turn (Tm/A)	Material property
Flux	φ	Weber (Wb)	
Flux linkage	λ	Weber (Wb)	
Reluctance	$\mathfrak{R}$	Ampere-Turn/Weber (A/Wb)	Material and geometrical property

How - Ampere's Law

Ampere's law at any point in space states:

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (2.9)$$

Using stokes theorem this can be modified to be:

$$\oint \mathbf{H} \cdot \mathbf{dl} = \int \mathbf{J} \cdot \mathbf{dS} = NI \quad (2.12)$$

Example 2.3 a)

(a) Given the magnetic field intensity expression (in A/m):

$$\mathbf{H} = [8x^4 + 6(y^2 - 2)]\,\mathbf{u_x} + [9x + 10y + 11z]\,\mathbf{u_y} + [4x]\mathbf{u_z}$$

Find the expression for current density **J** at location (2,4,6).

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Ampere’s law states that J (current density) is the curl of H

Example 2.3 b)

(b) Given a region of four conductors, each carrying current $I = 5$ A outward. As shown in Figure E2.3.1, two closed paths are defined, $\mathbf{l}_1$ and $\mathbf{l}_2$. Find the integral of **H** along each of the closed paths.

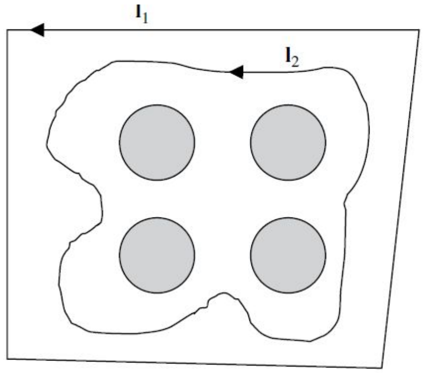


FIGURE E2.3.1 Two closed paths adjacent to a current-carrying region.

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int \mathbf{J} \cdot d\mathbf{S} = NI$$

The integral of H over a closed path is NI

Flux and Flux Density

Why - Flux and Flux density

Flux density is a critical variable for calculating force which is one of the main goals of this course.

Electromagnetic quantities

Quantity	Symbol	Units	Comment
Magnetic Field Intensity	**H**	Ampere-Turn/ metre (A/m)	Independent of Material
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How - Flux and Flux density

Flux is the surface integral of flux density

$$\phi = \int \mathbf{B} \cdot \mathbf{dS} \quad (2.13)$$

Flux is divergenceless (no magnetic monopoles)

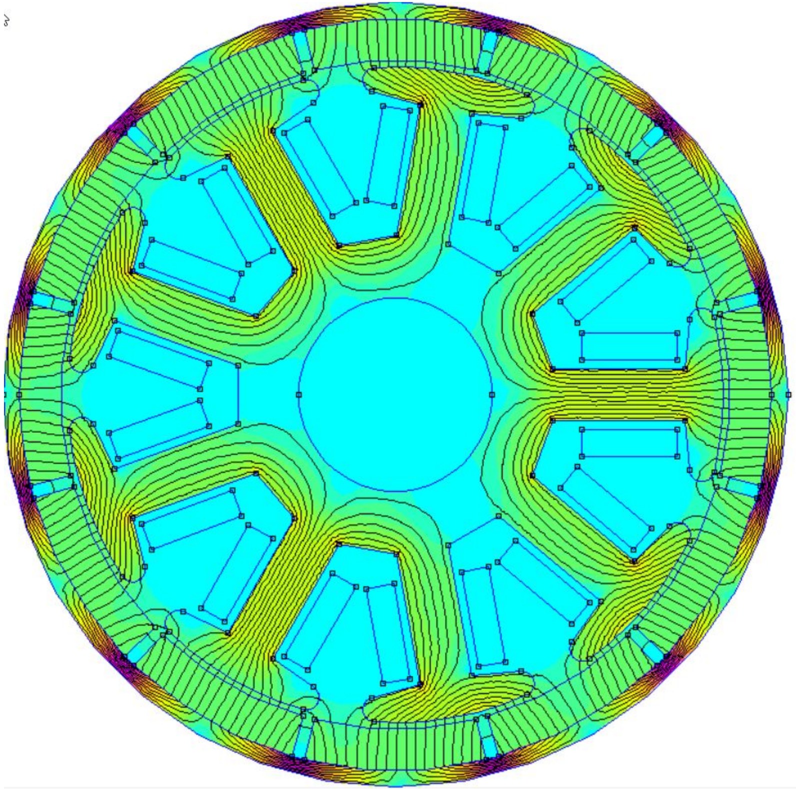
$$\nabla \cdot \mathbf{B} = 0 \quad (2.15)$$

Magnetic Materials

Why - Magnetic Materials

Magnetic fields will change depending on the material.

Material properties define the relationship between B and H



Electromagnetic quantities

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Electric Flux Density	**D**	Coulomb/ square meter (C/m^2)	
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How - Magnetic Materials

In air (vacuum) flux density is linearly proportional to magnetic field intensity

$$\mathbf{B} = \mu_o \mathbf{H} \quad (2.10)$$

μ_0=1.257e-6

Permeability is often defined relative to permeability of free space.

$$\mu = \mu_r \mu_o \quad (2.16)$$

Ampere's law can be used for a closed path:

$$\oint \mathbf{H} \cdot \mathbf{dl} = 2\pi r H = I \quad (2.20)$$

The flux density around a conductor is

$$B = \mu_r \mu_o H = 2000 \mu_o I / (2\pi r) \quad (2.23)$$

How - Magnetic materials (cont.)

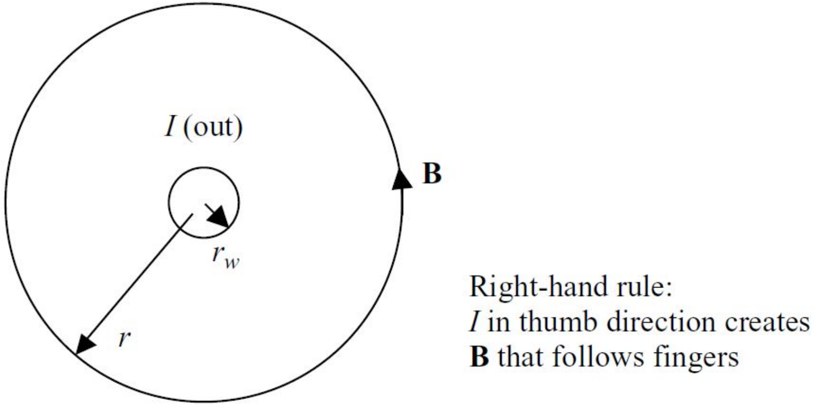


FIGURE 2.4 Magnetic field of a wire found using Ampere's law.

$$B = \mu_r \mu_o H = 2000 \mu_o I / (2\pi r) \quad (2.23)$$

Non- linear materials

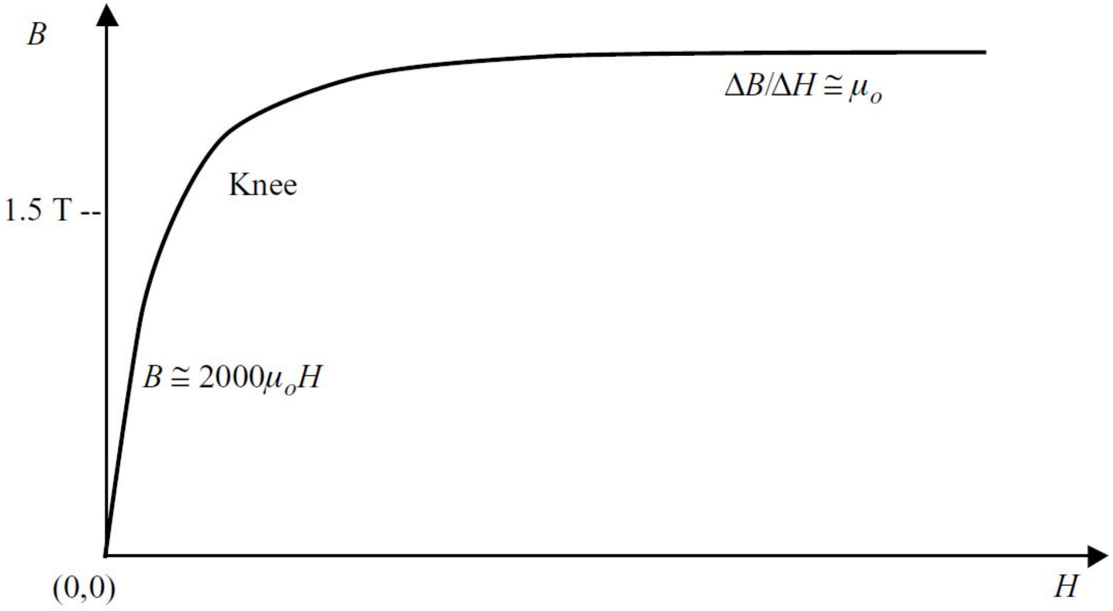
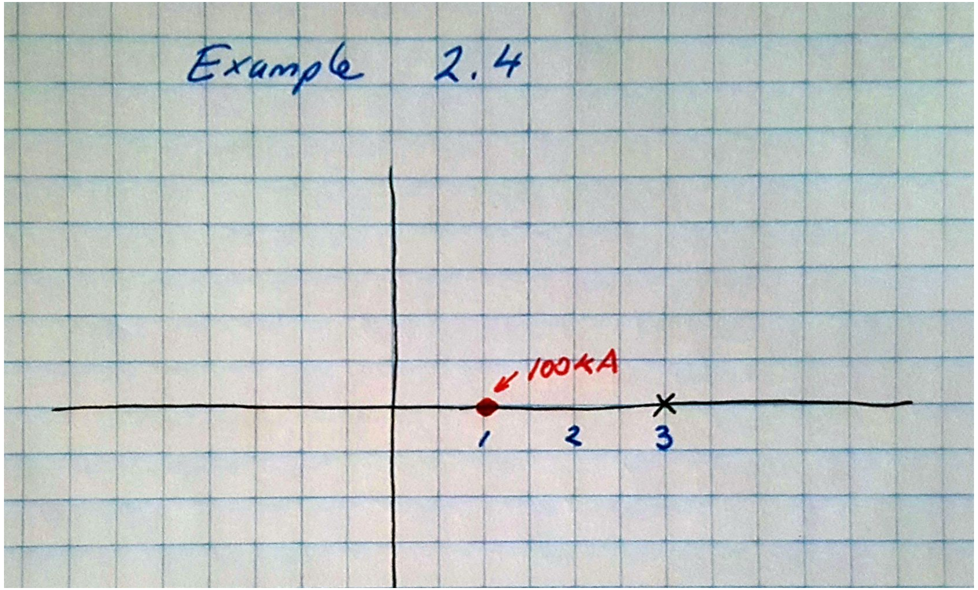


FIGURE 2.5 Typical nonlinear magnetization curve of steel.

Example 2.4 Magnetic Flux Density in Various Materials Surrounding a Wire

(a) A copper wire of radius 1 mm is placed at location (1,0,0) m, carrying current of 100 kA in the $+z$ direction. Find the vector **B** at location (3,0,0) for the wire embedded in the following materials that extend infinitely far in all three directions: (1) air, (2) steel with constant (assuming no saturation) relative permeability = 2500, (3) steel with a B–H curve with the following (B,H) values: (0,0), (1.5,1000), (1.8,7958), . . .



Test for understanding - Magnetic materials

Will there be more or less B in a steel plate 10mm away from a current carrying wire than in air 10mm away from the wire?

What is the “rule of thumb” flux density above which saturation occurs? (0.8, 1., 1.5, 2.0T)?

Faraday’s Law

Why - Faraday's Law

While Ampere's Law deals with current, Faraday's law deals with voltage

Useful for back EMF calculations in electric motors

Electromagnetic quantities

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Current density	**J**	Current Density	
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Flux	**φ**	Weber (Wb)	
Flux linkage	**λ**	Weber (Wb)	
Reluctance	$\mathfrak{R}$	Ampere-Turn/Weber (A/Wb)	Material and geometrical property

How - Faraday's Law

Faraday's law states that at any point in space:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.27)$$

Using stokes law and integrating gives:

$$V = -N\frac{\partial \varphi}{\partial t} \quad (2.33)$$

Using the definition of flux linkage

$$\lambda = N\phi \quad (2.35)$$

$$V = -\frac{\partial \lambda}{\partial t} \quad (2.34)$$

This voltage can be converted using ohms law

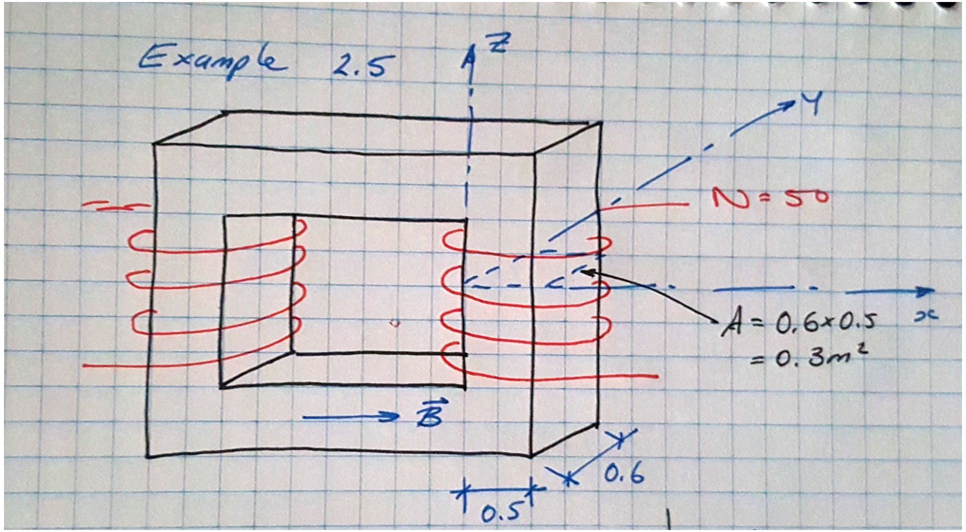
$$I = V/R \quad (2.37)$$

$$\mathbf{J} = \sigma \mathbf{E} \quad (2.38)$$

Example 2.5 Induced Voltage and Current

(a) A coil called the primary coil establishes the magnetic flux density $\mathbf{B} = 1.1\sin(2\pi ft)\ \mathbf{u}_z$ T. The frequency $f = 60$ Hz. A secondary coil is of thin uniform copper (conductivity 5.8E6 S/m) wire and has resistance $R = 2\ \Omega$. Assume $I = V/R$ and that the magnetic field is not changed by the secondary current. The secondary coil is square in shape, connecting the points $(x,y,z) = (0,0,0)$, (0.5 m,0,0), (0.5 m,0.6 m,0), (0,0.6 m,0) and back to the origin, with a total of 50 turns.

Find the voltage and the current induced in the secondary, including their polarities (directions). Also find both **E** and **J** in the wire.



Test for understanding - Faraday's Law

If you spin a generator twice as fast (double the rate of change of flux linkage) how much will the induced voltage increase?

If you double the number of turns (N) in a generator, how much will the induced voltage increase?

Potentials

Why

Using potentials can be useful for calculations.

For example, energy is a potential and using energy methods can be useful in dynamics.

The magnetic vector potential is critical in FEA.

Electromagnetic quantities

Quantity	Symbol	Units	Comment
Magnetic Field Intensity	**H**	Ampere-Turn/ metre (A/m)	Independent of Material
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Magnetic Vector Potential	**A**	Volt seconds / metre (Vs/m)	
Electric Flux Density	**D**	Coulomb/ square meter (C/m^2)	
Electric Field Intensity	**E**	Volt/ metre (V/m)	
Electrostatic scalar potential	φ_v	Joule / Coulomb (J/C)	
Current density	**J**	Current Density	
Permeability	μ	Tesla-metre/Ampere-Turn (Tm/A)	Material property
Flux	φ	Weber (Wb)	
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Reluctance	$\mathcal{R}$	Ampere-Turn/Weber (A/Wb)	Material and geometrical property

How

Electric field - electrostatic scalar potential

$$\mathbf{E} = -\nabla \phi_v \ (2.39)$$

For time varying fields another term is added

$$\mathbf{E} = -\nabla \phi_v - \frac{\partial \mathbf{A}}{\partial t} \ (2.47)$$

For magnetostatics the magnetic vector potential is used

$$\mathbf{B} = \nabla \times \mathbf{A} \ (2.44)$$

Example 2.6 Fields from Potentials Given the following potentials in a region, find its fields **B** and **E**:

$$\phi_v = 2x, \qquad \mathbf{A} = 0.2y \;\sin(2\pi 60t)\,\mathbf{u}_z \tag{E2.6.1}$$

Maxwell's Equations

Why

Maxwell’s equations summarised electromagnetics

Includes Ampere’s Law, Faraday’s Law

How - Maxwell’s equations

No magnetic monopoles

$$\nabla \cdot \mathbf{B} = 0 \qquad (2.48)$$

Not used in this course

$$\nabla \cdot \mathbf{D} = \rho_v \qquad (2.49)$$

Faraday’s Law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (2.50)$$

Ampere’s Law (+ a displacement current term that only applies to high frequencies (1MHz) so will not be used in this course

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad (2.51)$$

Summary

We have covered the relationships between some fundamental electromagnetic variables

We are now able to start doing some calculations to analyse electromagnetic devices.