L1d: Reluctance Method

gregheins@ieee.org

Simplifying Ampere’s Law

Why - Simplifying Ampere's Law

Often electromagnetic problems start with a number of turns (N) carrying a current (I) and you need to find the flux.

Once you can find that flux then you are able to find force

Simplifying Ampere's law allows us to relate NI to B.

How

Converting the integral version of Ampere's law to a summation

$$\oint \mathbf{H} \cdot \mathbf{dl} = NI \ (3.1)$$

$$\sum_k H_k l_k = NI \ (3.2)$$

Using the relationships between H and B and B and ϕ we get: (S is Area)

$$\sum_k [(\varphi_k / (\mu_k S_k)] l_k = NI \ (3.6)$$

If we define reluctance

$$\mathcal{R} = l/(\mu S) \ (3.9)$$

We can write:

$$\varphi \sum_k \mathcal{R}_k = NI \ (3.8)$$

Analogy to electric circuits

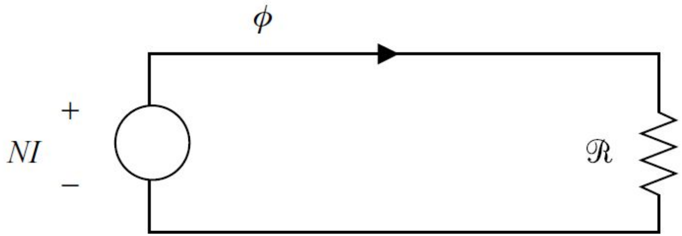


FIGURE 3.1 Basic magnetic circuit parameters.

TABLE 3.1 Basic Analogous Parameters of Electric Circuits and Magnetic Circuits

Parameter	Electric Circuit	Magnetic Circuit
Flow	Current I in amperes (A)	Flux ϕ in webers (Wb)
Potential	EMF in volts (V)	MMF in ampere-turns
Potential/flow	Resistance R in ohms (Ω)	Reluctance $\mathcal{R}$ in A/Wb
Flow/potential	Conductance G in siemens (S)	Permeance $\mathcal{P}$ in Wb/A
Flow density	Current density $\boldsymbol{J}$ in A/m^2	Flux density $\boldsymbol{B}$ in teslas (T)

$$IR = V \quad (3.13)$$

$$\varphi\mathcal{R} = NI \quad (3.12)$$

(1) Find the closed flux path (or paths) that "circles" the given ampere-turns NI. This path is usually through high permeability materials such as steel, but may also contain air segments, which should be as short as possible. The path direction must follow the right-hand rule, where your thumb is in the direction of NI.

(2) Find the lengths and cross-sectional areas of all path segments, assuming average values of lengths and the inverses of areas. Also note the permeability of each path segment. For nonlinear materials, assume initially that their permeability is the constant value below their "knee."

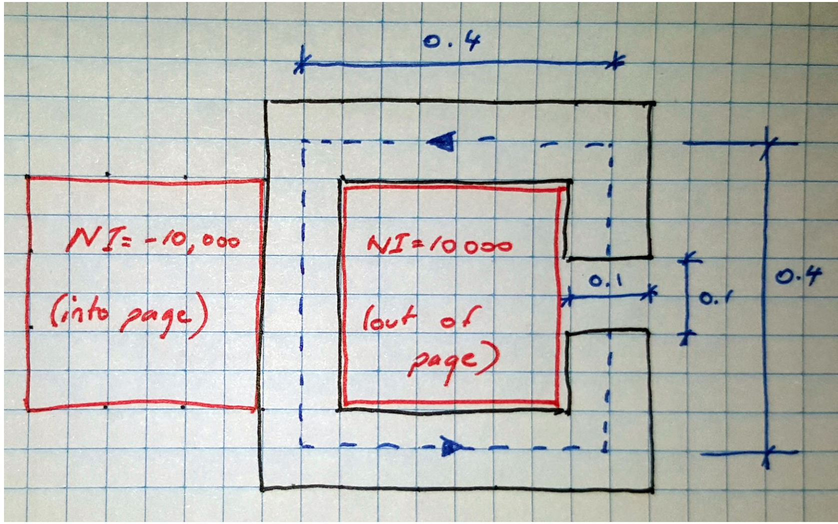
(3) Find the reluctance of each path segment using (3.9).

(4) Combine the reluctances and use (3.10) or (3.8) to find the flux. Reluctances in series add directly. Reluctances in parallel combine just like resistances in parallel; the combined value is the reciprocal of the sum of reciprocals.

(5) Find the flux density of each path segment using (3.11).

(6) If the flux density in any nonlinear material is beyond the knee, then make a new (lower) assumption of its permeability and repeat steps (3), (4), and (5) until the calculated flux densities match the assumed nonlinear permeabilities. For details and various examples consult books such as that by Roters [1].

Example 3.1 Reluctance Method for "C" Steel Path with Airgap The first example of the reluctance method is shown in Figure E3.1.1. A "C"-shaped piece of steel of uniform thickness 0.1 m lies in the plane of the paper. The steel is of depth 0.1 m into the page. The opening of the "C" is an airgap of length 0.1 m between the steel poles. (A magnetic *pole* is a surface where magnetic flux leaves to form a North pole or enters to form a South pole.) The object is to find magnetic flux density **B** throughout the steel and airgap produced by the 10,000 ampere-turns directed out of the page in a coil inside the "C" and returning to its left. The steel is assumed to have a relative permeability of 2000.



Example 3.2 Reluctance Method for Sensor with Variable Airgap The second example of the reluctance method is shown in Figure E3.2.1. It shows a simplified magnetic sensor with stationary stator and movable armature. Both stator and armature are made of steel, with the stator also having a coil made of copper or aluminum. The armature is shown in two positions. In the first position the armature and stator teeth are aligned. In the second position the armature teeth are moved halfway between the stator teeth, so they are as misaligned as possible.

The stator coil in Figure E3.2.1 has 1000 ampere-turns. The stator and armature teeth are shown to have width equal to 1 cm. The airgap between the stator and rotor teeth is 1 mm when they are aligned. In the misaligned position, the airgap is assumed to increase to 1 cm. The depth into the page of the sensor of Figure E3.2.1 is given as 10 cm.

The steel permeability is given as infinitely high, and thus only airgap reluctances are needed. Hence dimensions of the steel are not given.

The airgap flux densities are to be found for both armature positions in Figure E3.2.1.

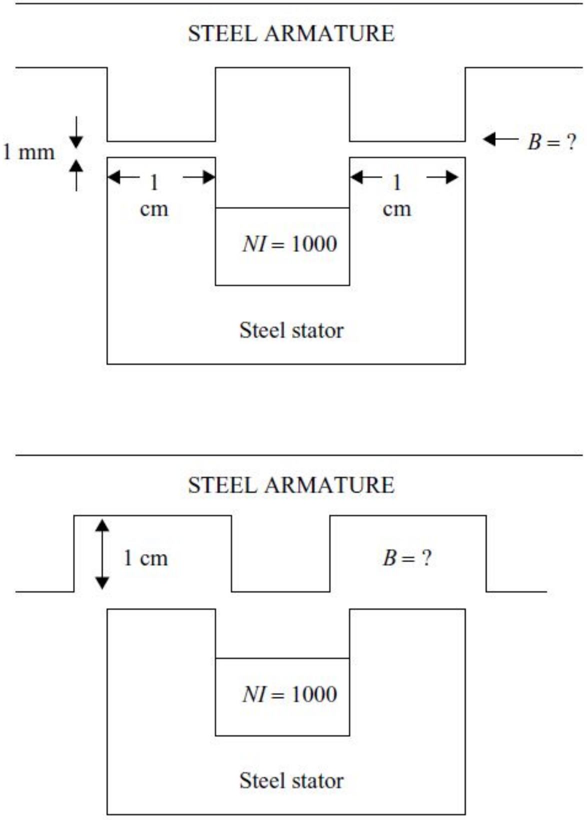


FIGURE E3.2.1 Portion of simple magnetic sensor.

Test for understanding

If the 1mm air gap was changed to 2mm how would the circuit reluctance change?

If the width of the “teeth” was increased to 20mm, how would the circuit reluctance change?

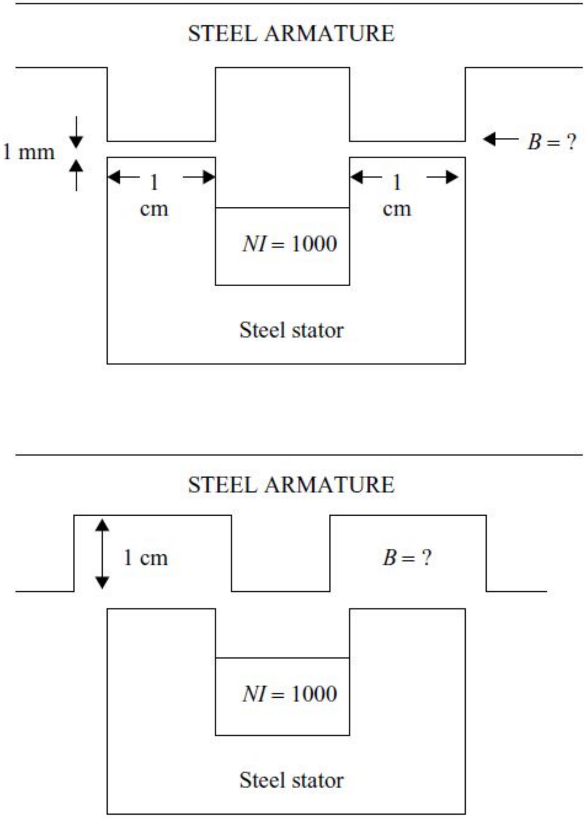
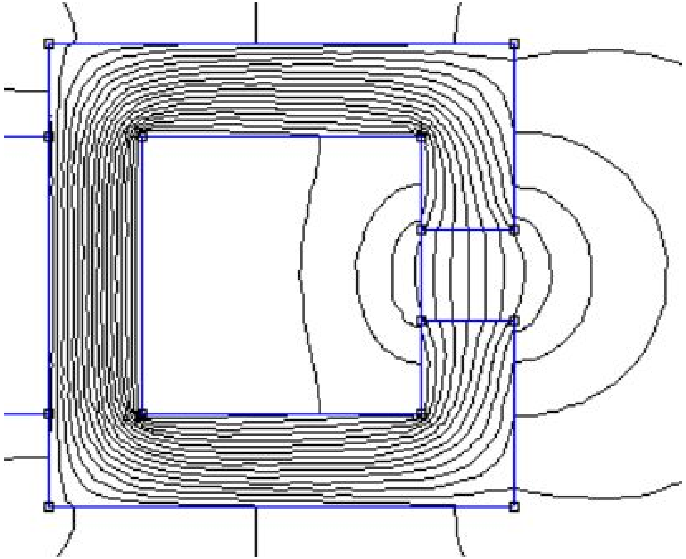


FIGURE E3.2.1 Portion of simple magnetic sensor.

Fringing Flux

Why - Fringing flux

The reluctance method assumes that the flux goes directly from one part of steel to another.



How - Fringing Flux

One way to account for fringing flux is to use a “fringing factor”

$$K_{\text{fringe}} = \mathcal{R}_{\text{nofringe}} / \mathcal{R}_{\text{fringe}} \quad (3.15)$$

A more complex but more accurate way is to use the “Carter’s Coefficient”

$$g' = g c_s \quad (\text{B-1})$$

$$c_s = \frac{w_{ss} + w_{st}}{w_{st} + \frac{4g}{\pi} \ln\left(1 + \frac{\pi w_{ss}}{4g}\right)} \quad (\text{B-10})$$

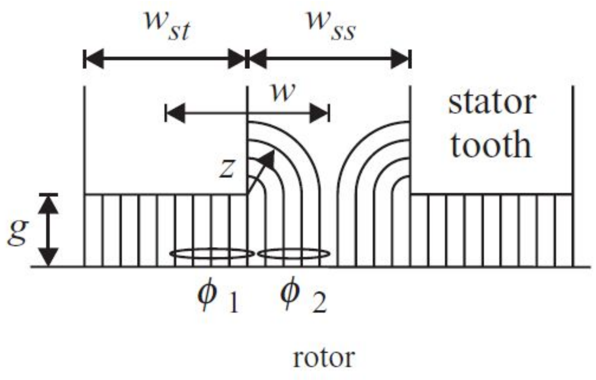


Figure B-1. Carter’s coefficient.

Analysis of Electric Machinery and Drive Systems, Third Edition
http://onlinelibrary.wiley.com/book/10.1002/9781118524336

Complex Reactance

Complex Reluctance

When the fields are not time varying, the reluctance will be "real"

The reluctance in the air gap will always be real

When time varying fields exist and eddy current effects are significant then there is an imaginary component in the steel reluctance.

We will deal with this more in Chapter 8

$$\phi = NI/(\mathscr{R}_{\mathrm{Fe}} + \mathscr{R}_{\mathrm{gap}}) \ (3.16)$$

$$\mathscr{R}_{\mathrm{Fe}} = \mathscr{R}_{\mathrm{RE}} + \mathrm{j}\mathscr{R}_{\mathrm{IM}} = |\mathscr{R}_{\mathrm{Fe}}|\angle\theta \ \ (3.17)$$

Limitations

(1) Fringing flux in air is either ignored or approximated with fringing factors, or derived assuming the shape of the flux path.

(2) The path area of each part is often assumed to be average path area. A more accurate method is to use the reciprocal of the average reciprocal area, since reluctance is proportional to the reciprocal of area.

(3) Since reluctance is inversely proportional to permeability, predicting nonlinear *B–H* effects in steel is difficult. However, *B–H* curves can be used in the reluctance method [1, 3].

(4) As discussed in the preceding section, losses in steel can be represented by the use of complex reluctance. However, obtaining values of complex reluctance is often very difficult.

(5) Most real world devices have complicated geometries with multiple flux paths that are difficult to analyze. However, magnetic circuits can be constructed with series and parallel reluctances to more accurately model such devices.

(6) For many magnetic devices, however, the necessity of assuming flux paths makes the reluctance method inaccurate. Thus the reluctance method cannot be accurately applied to many devices.

Summary

- The reluctance method gives us a way to determine fluxes based on current and number of turns.
- The reluctance method is analogous to determining the total resistance in a circuit.
- Due to the underlying assumptions, the reluctance method is only an approximation.
- Care must be taken with:
  - Changing cross sections
  - Parallel paths
  - Non-linear materials
  - Fringing flux