CHAPTER 6

Other Magnetic Performance Parameters

The preceding chapter discussed force and related performance parameters of magnetic devices. This chapter discusses other magnetic performance parameters. After defining these key parameters, they are evaluated using reluctance and finite-element methods. Their relations with energy and force are also explained.

6.1 MAGNETIC FLUX AND FLUX LINKAGE

6.1.1 Definition and Evaluation

Magnetic flux through any surface **S** can be determined by integrating magnetic flux density **B**, using the surface integral from Chapter 2:

$$\phi = \int \mathbf{B} \cdot \mathbf{dS} \tag{6.1}$$

If magnetic vector potential **A** is used as in Chapter 4, then the flux can be expressed as:

$$\phi = \int (\nabla \times \mathbf{A}) \cdot \mathbf{dS} \tag{6.2}$$

Using Stokes' vector identity of Chapter 2, the surface integral can be changed to a closed line integral around the surface:

$$\phi = \oint A \cdot dl \tag{6.3}$$

The closed line integral of (6.3) is easily evaluated from finite-element solutions of **A**. For 2D planar solutions of depth d and vector potential component A_z obtained as described in Chapter 4, (6.3) becomes:

$$\phi_{12} = (A_{z1} - A_{z2})d \tag{6.4}$$

Magnetic Actuators and Sensors, Second Edition. John R. Brauer.

where the flux is found that flows between any points one and two on the planar model.

Recall from Chapter 2 [Eq. (2.35)] that flux linkage is defined as flux times the number of turns of a coil:

$$\lambda = N\phi \tag{6.5}$$

Thus if the flux of (6.1)–(6.4) is found through a coil, the flux linkage is easily evaluated.

Example 6.1 Finding Flux in Example 5.3 using Maxwell Given the one-half model of the "C" steel path with airgap of Example 5.3, find the flux passing through the steel pole face using Maxwell finite-element software and compare it with the reluctance solution of Example 3.1.

Solution If you are using Maxwell SV, in its main menu click on "Setup Executive Parameters" and then select "Flux Lines." Enter the points on the left corner and right corner of the pole face, separated 0.1 m apart in the x direction, thereby forming "Line1." Then return to the main menu and click on "Solve." You may have to request a smaller error under "Setup Solution Options" in order to execute a new finite-element solution. When the solution is completed, click on the upper "Solutions" tab and then on "Flux Lines." It displays "Flux Linkage" = −0.014607 Wb.

If instead you are using Maxwell version 16, its model has a Polyline object, Line1 created on the steel pole face. To create the flux calculation, open the Fields Calculator, enter Quantity -> B, Geometry -> Line, select Line1, Vector -> Normal, Scalar -> Integrate, Output -> Eval. This is also added as a Named Expression, "Flux," for later plotting. You may choose to request a smaller error in the Solve Setup dialog box. "Analyze" the model and find the Data Table result for Flux under Results. It displays −0.014611 Wb.

The Maxwell value is obtained using (6.4) for 1 m depth and for one turn. Thus the finite-element value for 0.1 m depth is −0.00146 Wb for one turn. One would have to multiply by the number of turns to obtain the flux linkage of (6.5).

The reluctance method solution of Example 3.1 obtains a flux density of 0.125 T. Multiplying by the pole area of 0.1 m times 0.1 m gives a flux of 0.00125 Wb, or 125/146th the Maxwell flux. Thus, as before, the reluctance method is considerably less accurate than the finite-element method.

6.1.2 Relation to Force and Other Parameters

Flux linkage is useful in several ways. One way, mentioned in Chapter 2, is to determine voltage induced by Faraday's law (2.34):

$$V = -\frac{\partial \lambda}{\partial t} \tag{6.6}$$

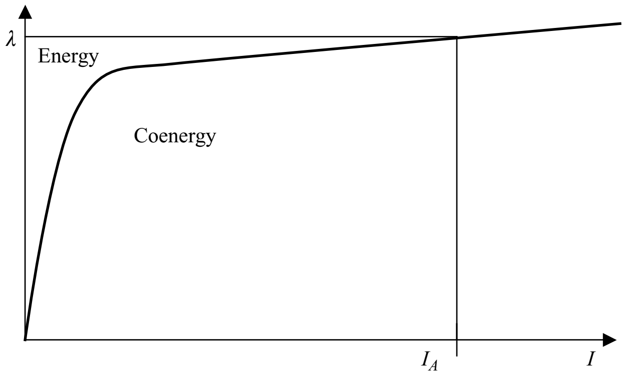

FIGURE 6.1 Magnetic energy and coenergy at a typical current value on a typical λ–I curve of a magnetic device.

Another use of flux linkage is to obtain alternative expressions for the energy, coenergy, and force of the preceding chapter. Energy input is power (voltage times current) integrated over time, giving [1]:

$$W = \int VIdt = \int Id\lambda \tag{6.7}$$

where in general I versus λ is a nonlinear relation shown in Figure 6.1. Figure 6.1 has a shape similar to the nonlinear relation of B–H in a magnetic device graphed in Figure 5.4. From the relations ϕ equals B times area, λ equals N times ϕ, and H equals N times I, Figure 6.1 is a scaled version of Figure 5.4. From (6.7), the energy stored is the area to the left of the curve:

$$W_{\text{mag}} = \int Id\lambda \tag{6.8}$$

Similarly to Figure 5.4, the coenergy is the area below the curve, which is [1, 2]:

$$W_{\text{co}} = \int \lambda dI \tag{6.9}$$

With the coenergy and energy known, the magnetic force may be obtained using techniques of the preceding chapter. As shown in Figure 6.2, and in agreement with (5.13), the force in the x direction is:

$$F_x = \frac{\partial W_{\text{co}}}{\partial x}|_{I=\text{const}} \tag{6.10}$$

Similarly, torque can be found by the derivative with respect to angle as described previously in Section 5.7.

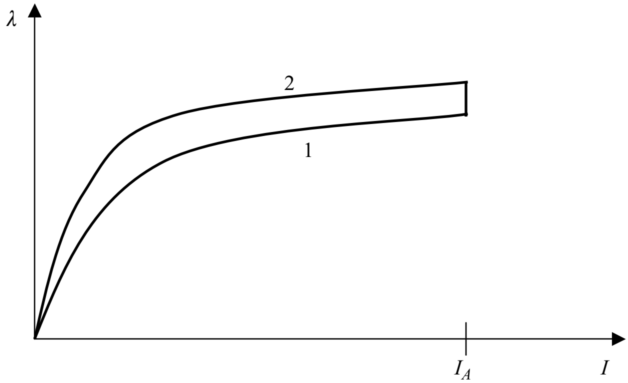

FIGURE 6.2 Nonlinear λ–*I* relation of a typical magnetic device at positions one and two. The applied current is assumed to be held constant.

Example 6.2 Finding Force Given Flux Linkage Versus Current and Position
Given the following relation for a magnetic actuator, find its magnetic force for $I = 2$ A and position $x = 0.05$ m:

$$\lambda = 0.15\sqrt{I}/x \tag{E6.2.1}$$

Solution The coenergy of the actuator is found using (6.9):

$$W_{co} = \int \lambda dI = \int 0.15 I^{0.5} x^{-1} dI = (0.15/1.5) I^{1.5} x^{-1} = 0.1 I^{1.5} x^{-1} \tag{E6.2.2}$$

The magnetic force is then found using (6.10):

$$F_{mag} = \frac{\partial W_{co}}{\partial x}|_{I=\text{const}} = 0.1 I^{1.5} \frac{\partial}{\partial x} x^{-1} = -0.1 I^{1.5} x^{-2} \tag{E6.2.3}$$

which, evaluated for the given current and position, gives:

$$F_{mag} = -0.1(2)^{1.5}/(0.05)^2 = -113\,\mathrm{N} \tag{E6.2.4}$$

6.2 INDUCTANCE

6.2.1 Definition and Evaluation

The basic definition of *inductance* is:

$$L = \lambda/I \tag{6.11}$$

In the case of devices with purely linear *B–H* materials (of constant permeability), Ampere's law gives flux and flux linkage proportional to current. Hence in linear devices, inductance is a constant, independent of current. Inductance units are henrys (H). All coils have inductance and can be called *inductors.*

For a coil with the flux linkage and current of (6.11), the inductance obtained is the *self inductance.* For multiple coils, the ratios of different flux linkages and currents may be taken, expressed as:

$$L_{jk} = \lambda_j / I_k \tag{6.12}$$

If $j \neq k$, then L_{jk} is called *mutual inductance.* Self inductance is for $j = k$. An inductance matrix is a way to express (6.12), where diagonal elements are self inductances and off-diagonal elements are mutual inductances. The matrix is usually symmetric, that is,

$$L_{kj} = L_{jk}$$

For devices with nonlinear *B–H* materials, flux linkage is not proportional to current and thus inductance is not uniquely defined. Figure 6.3 shows two inductances equal to two slopes. *Secant inductance* (L_{sec}), sometimes also called *apparent inductance*, is the term often given to the ratio λ/I at a particular current. *Incremental inductance* (also called differential inductance) is the slope of the λ–I curve:

$$L_{\mathrm{inc}} = \frac{\partial \lambda}{\partial I} \tag{6.13}$$

Both secant and incremental inductances vary with current. Only if the current is low enough that no saturation occurs does a unique inductance apply, for then secant and incremental inductances are the same constant values.

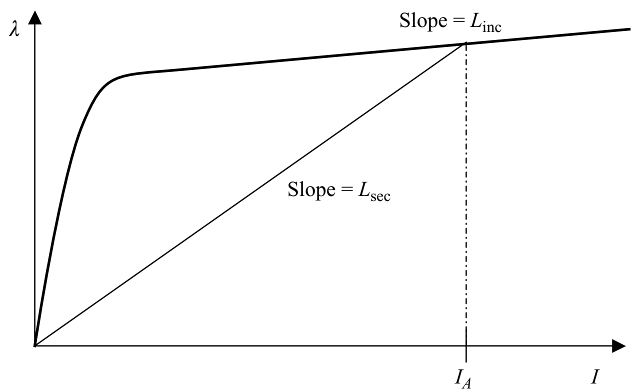

FIGURE 6.3 Inductances L_{sec} (secant) and L_{inc} (incremental) shown as slopes on a typical λ–I curve of a magnetic device.

Example 6.3 Finding an Inductance Matrix using Maxwell Two coils are wound on a steel cylinder of length 20 mm and radius 10 mm. Both have cross section 4 mm by 4 mm and are placed flush with both ends of the cylinder as shown in Figure E6.3.1. The steel has relative permeability of 2000. Each coil has 10 turns and carries 1 A. Make an axisymmetric finite-element model in Maxwell, and use it to find the inductance matrix.

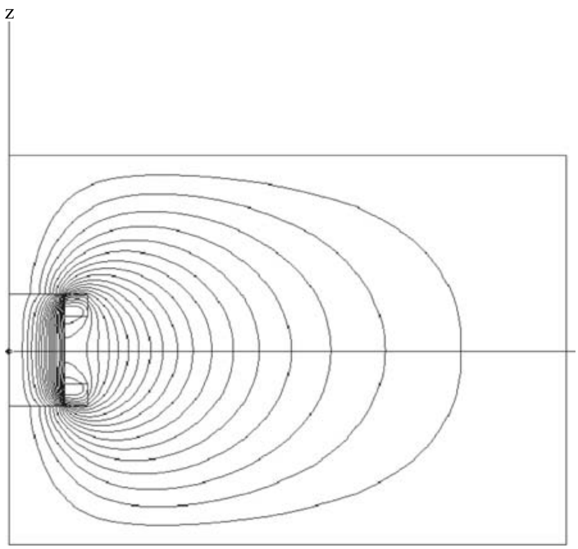

FIGURE E6.3.1 Computer display of two coils on a steel cylinder and the computed flux line plot for DC magnetostatics.

Solution If you are using Maxwell SV, in its main menu click on "Solver" to make it "Magnetostatic" and click on "Drawing" to make it "RZ Plane." Then use "Define Model" to enter the geometry, followed by "Setup Materials" to enter the linear steel for the cylinder, and copper or vacuum for the coils. Next, under "Setup Boundaries/Sources" give both coils 10 ampere-turns and select zero potential for the outer boundary. Under "Setup Executive Parameters" click on "Matrix/Flux." Using its new window, select each coil region and (with the default return path) assign it to the matrix. After allowing 13 passes under "Setup Solution Options," click on "Solve." Obtain the flux line plot shown in Figure E6.3.1 using 21 divisions.

If instead you are using Maxwell version 10, change the solution type to "Magnetostatic," with Geometry Mode as "Cylindrical about Z." In the Maxwell drawing window, create the geometry and Assign Materials for linear steel for the cylinder and copper for the coils. Under Excitations, assign both coils with 10 ampere-turns, and zero Vector Potential for the outer boundary. Under "Parameters," assign a Matrix and "Include" both of the coils (with default return path). Optionally, specify the number of turns for each coil in the Post Processing tab. After allowing 13 passes in the Solve Setup, click on "Analyze."

Finally (for either version of Maxwell), click on the upper "Solutions" tab to obtain "Matrix." It shows an inductance matrix with $L_{11} = L_{22} = 6.063\text{E}{-8}$ H and with $L_{12} = L_{21} = 1.799\text{E}{-8}$ H. However, these inductances are for one turn in each coil. Each L_{jk} must be multiplied by $N_j N_k$, which is 100 in this problem, to obtain actual inductances. Note that the mutual inductances are much less than self inductances, which is often the case. Note also that due to the symmetry of this problem, both self inductances are the same, and both mutual inductances are the same.

6.2.2 Relation to Force and Other Parameters

The relation between inductance and reluctance is obtained as follows. Rewriting (6.11) as:

$$L = N\phi/I \tag{6.14}$$

and using the reluctance definition:

$$\mathcal{R} = NI/\phi \tag{6.15}$$

to solve for N and substitute it into (6.14), giving:

$$L = N^2/\mathcal{R} \tag{6.16}$$

The magnetic energy stored in a constant (linear) inductor is [3]:

$$W_{\text{mag}} = W_{\text{co}} = \int I d\lambda = \int \lambda dI = \int LIdI = \frac{1}{2}LI^2 \tag{6.17}$$

When inductance is constant over a current range but varies with position, it can be used to obtain magnetic force as follows. From (6.10), we obtain:

$$F_x = \frac{\partial W_{\text{co}}}{\partial x}|_{I=\text{const}} = \frac{\partial}{\partial x}\left[\frac{1}{2}L(x)I^2\right]|_{I=\text{const}} \tag{6.18}$$

$$F_x = \frac{1}{2}I^2\frac{dL(x)}{dx} \tag{6.19}$$

The above force equation is valid for a magnetic device with a single coil and single self inductance. When multiple coils are energized, energies and forces are added due to the other self inductances and mutual inductances. For example, for two coils coupled through mutual inductance L_{12}, magnetic energy is:

$$W_{\text{mag}} = \frac{1}{2}L_{11}I_1^2 + \frac{1}{2}L_{22}I_2^2 + L_{12}I_1I_2 \tag{6.20}$$

and the related force is:

$$F_x = \frac{1}{2} I_1^2 \frac{dL_{11}(x)}{dx} + \frac{1}{2} I_2^2 \frac{dL_{22}(x)}{dx} + I_1 I_2 \frac{dL_{12}(x)}{dx} \tag{6.21}$$

Similar expressions can be derived for torque as a function of angular position.

6.3 CAPACITANCE

6.3.1 Definition

Coils and other parts of magnetic devices possess *capacitance*. The basic definition of capacitance is:

$$C = Q/V \tag{6.22}$$

where Q is the charge in coulombs and V is the voltage. The unit of capacitance is the farad (F), and a device possessing capacitance is called a capacitor.

A typical capacitor has been shown in Figure 2.7. As discussed in Chapter 2, capacitors are electric field devices, not magnetic field devices. Thus this book does not discuss capacitors in detail. However, because capacitance plays a role in the performance of magnetic actuators and sensors, especially at high frequencies, a few basics of capacitance are needed here.

The capacitance of (6.22) is called *self capacitance* if the voltage is across two conducting plates (as in Figure 2.7) and the charge is the magnitude of the charge on each plate. A capacitance matrix, analogous to an inductance matrix, is important if there are more than two plates.

Capacitance can be evaluated using the finite-element method for electrostatic fields, as will be shown in the next subsection.

6.3.2 Relation to Energy and Force

The electric field energy stored in a capacitor is:

$$W_{\text{el}} = \frac{1}{2} \int \mathbf{D} \cdot \mathbf{E} dv = \frac{1}{2} \int \varepsilon E^2 dv = \frac{1}{2} C V^2 \tag{6.23}$$

where the various parameters, including permittivity ε, have been defined in Chapter 2.

When capacitance varies with position, an electric field force is created. The virtual work method can again be used to obtain the force as the derivative of energy:

$$F_x = \frac{\partial W_{\text{el}}}{\partial x} |_{V=\text{const}} = \frac{\partial}{\partial x} \left[\frac{1}{2} C(x) V^2 \right] |_{V=\text{const}} \tag{6.24}$$

$$F_x = \frac{1}{2} V^2 \frac{dC(x)}{dx} \tag{6.25}$$

If the force is desired in terms of fields rather than capacitance, then the change in stored electric field energy in Figure 2.7 can be found for a virtual displacement Δy of either plate. By observing the change in stored energy as was done for magnetic fields in Figure 5.5, the pressure due to an electric field E in air is found to be:

$$P_{\mathrm{el}} = \frac{1}{2}\varepsilon_o E^2 \tag{6.26}$$

Because electric field E is usually limited due to material breakdown (arcing and similar destructive behavior), electric field forces and pressures are usually much smaller than magnetic field forces and pressures. For example, in air the breakdown E is approximately 3 MV/m, varying somewhat with humidity, temperature, and pressure. Substituting that E into (6.26) gives an electric field pressure of 39.8 Pa, far below the maximum 20.E5 Pa produced by magnetic fields on steel poles in air.

Example 6.4 Finding Capacitance using Maxwell Two aluminum plates 2 m wide are separated by 1 m as shown originally in Figure 2.7 and also in Figure E6.4.1. The lower plate is at 0-V DC and the upper at 1 V DC. The region between the two plates is assumed filled with polystyrene, which has a relative permittivity of 2.6. Find the voltage contours, electric field, energy stored, and capacitance using Maxwell. Validate the energy stored using (6.23).

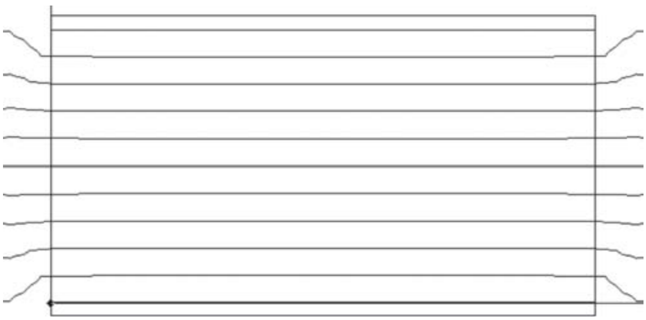

FIGURE E6.4.1 Computer display of capacitor with computed voltage contours.

Solution In the main Maxwell SV menu make sure that "Solver" is set to "Electrostatic" and "Drawing" to "XY Plane." If using Maxwell version 16, set "Solution Type" to "Electrostatic" and "Geometry Mode" to "Cartesian XY." Then for either version, the geometry of Figure E6.4.1 is entered using three "Box" commands. The material properties of aluminum and polystyrene are selected from the material list. The boundary conditions are then entered, consisting solely of setting the lower plate to 0 V and the upper plate to 1 V. The solution, obtained in one pass, has an energy stored of 2.41E−11 J. Postprocessing gives contours of constant phi (voltage) shown in Figure E6.4.1. A color plot of the magnitude of E shows it to be 1.0 V/m between the plates, as expected from Figure 2.7. The expected electric energy stored from (6.23) is:

$$W_{\mathrm{el}} = \frac{1}{2}\int \varepsilon E^2 dv = (0.5)(2.6)(8.854\mathrm{E}{-}12)(1)^2(1)(2)(1) = 2.3\mathrm{E}{-}11\ \mathrm{J} \tag{E6.4.1}$$

which is in reasonable agreement with the 2.41E−11 J output by Maxwell. Using this Maxwell energy value in (6.23) gives:

$$W_{\mathrm{el}} = 2.41\mathrm{E}{-}11 = \frac{1}{2}CV^2 = 0.5\ \mathrm{C} \tag{E6.4.2}$$

$$C = 2(2.41\mathrm{E}{-}11) = 48.2\mathrm{pF} \tag{E6.4.3}$$

6.4 IMPEDANCE

Many magnetic devices carry sinusoidal alternating current, often of frequency 50 Hz or 60 Hz. For AC devices and systems, *impedance* is a complex number (in ohms) defined as:

$$Z = V/I \tag{6.27}$$

where V is complex phasor voltage and I is complex phasor current.

In magnetic devices, voltage obeys Faraday's law (6.6), which becomes for AC frequency f (in Hz):

$$V = -j2\pi f\lambda \tag{6.28}$$

where the time derivative of (6.6) has been replaced by $j2\pi f$, where j is the square root of minus one [4]. The imaginary number j causes the voltage V to be 90° out of phase from the flux linkage λ. In Chapter 2, such a 90° phase shift was shown in Example 2.7, in which a time derivative caused a sine to become a cosine.

In AC analyses, it is common to call $2\pi f$ the angular frequency ω (in radians/second), and thus (6.28) becomes:

$$V = -j\omega\lambda \tag{6.29}$$

Substituting (6.3) and (6.5) gives:

$$V = -j\omega N \oint A \cdot dl \tag{6.30}$$

and hence the impedance (with the sign reversed to follow normal sign conventions) of (6.26) is

$$Z = j\omega N \oint A \cdot dl/I = j\omega\lambda/I \tag{6.31}$$

Z is in general a complex number [4], and can be split into real and imaginary components:

$$Z = R + jX \tag{6.32}$$

X is called *reactance* and is shown using (6.14) and Faraday's law to be proportional to inductance:

$$X = j\,\omega\,L \tag{6.33}$$

As discussed in Chapter 2, the resistance R can account for power losses due to induced eddy currents in conducting materials. Maxwell has a solution type "Eddy Current" which solves for AC fields and impedance. Similar to the inductance and capacitance matrices discussed above, impedance matrices can be obtained for multiple coils. Further discussions of AC fields will appear in Chapter 8.

Example 6.5 Finding an Impedance Matrix using Maxwell As in Example 6.3, two coils are wound on a steel cylinder of length 20 mm and radius 10 mm. Both have cross section 4 mm by 4 mm and are placed flush with both ends of the cylinder as shown in Figures E6.3.1 and E6.5.1. The steel has relative permeability of 2000 and conductivity 2.E6 S/m. Each coil has 10 turns of stranded wire (the stranding eliminates eddy currents in the wire) and carries 1 A real of AC current of frequency 60 Hz. Make an axisymmetric finite-element model in Maxwell, and use it to find the impedance matrix.

Solution If you are using Maxwell SV and have already solved Example 6.3, copy it to a new project. In the main Maxwell menu click on "Solver" to make it "Eddy Current" and click on "Drawing" to make it "RZ Plane." If you have not previously defined the geometry, use "Define Model" to enter the geometry. Next use "Setup Materials" to enter the linear steel for the cylinder, and copper or vacuum for the coils. Make sure that the steel has the conductivity 2.E6 S/m. Next, under "Setup Boundaries/Sources" give both coils 10 ampere-turns (be sure to select stranded and phase angle 0) and select zero potential for the outer boundary. Under "Setup Executive Parameters" click on "Matrix/Flux." Using its new window, select each coil region and (with the default return path) assign it to the matrix. After allowing 14 passes under "Setup Solution Options," click on "Solve."

If instead you are using Maxwell version 16 and have already solved Example 6.3, copy the Design to a new Project. Change the Solution Type to "Eddy Current" and select the Geometry Mode as "Cylindrical about Z." If you have not previously defined the geometry, create the objects and Assign Materials as linear steel for the

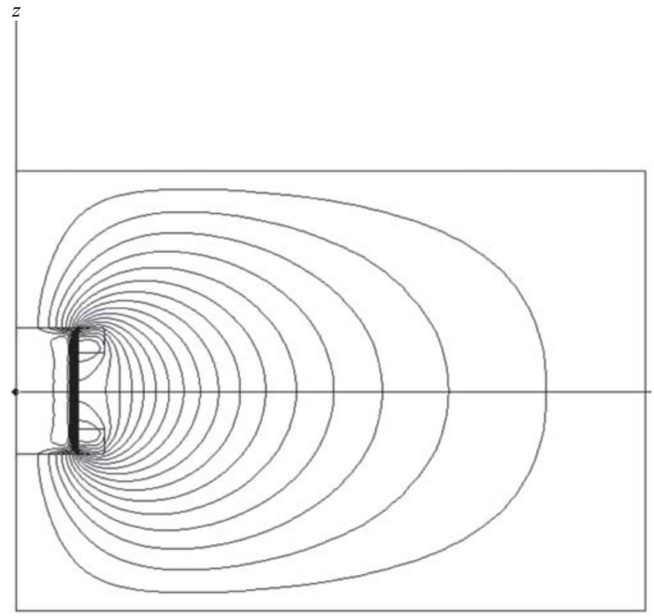

FIGURE E6.5.1 Computer display of two coils on a conducting steel cylinder and the computed 60-Hz flux line plot.

cylinder and copper for the coils. Make sure the steel has the conductivity 2.E6 S/m. Under Excitations, Assign both coils 10 ampere-turns (be sure to select stranded and phase angle 0) and select zero Vector Potential for the out boundary. Under "Parameters," Assign a Matrix and "Include" both coils. After allowing 14 passes under "Solve Setup" dialog box, click on "Analyze."

Either version of Maxwell obtains the flux line plot shown in Figure E6.5.1 using 21 divisions. Note that it is very different from the DC flux line plot of Figure E6.3.1, especially in the steel, evidently due to its eddy currents. Chapter 8 will explain that the eddy currents force the flux to flow near the skin of the conducting steel.

Finally, click on the upper "Solutions" tab to obtain "Matrix." It shows a matrix with $L_{11} = L_{22} = 6.046\text{E}{-8}$ H and with $L_{12} = L_{21} = 1.785\text{E}{-8}$ H. However, these values are for one turn in each coil. Each L_{jk} must be multiplied by $N_j N_k$, which is 100 in this problem, to obtain actual inductances. Note that the mutual inductances are much less than self inductances, and are only slightly different from the DC case of Example 6.3. Note also that due to the symmetry of this problem, both self inductances are the same, and both mutual inductances are the same. To obtain reactance components of impedance, all inductances must be multiplied by $\omega = 2\pi f = 2\pi 60 = 377$.

The matrix also contains resistances $R_{11} = R_{22} = 8.916\text{E}{-8}\ \Omega$ and $R_{12} = R_{21} = 6.246\text{E}{-8}\ \Omega$. Again, these values are for one turn in each coil. Each R_{jk} must be multiplied by $N_j N_k$, which is 100 in this problem, to obtain the actual resistances. These resistances account for eddy current power loss in the conducting steel.

PROBLEMS

6.1 Redo the flux computation of Example 6.1 but find the flux in the steel along the horizontal symmetry plane.

6.2 Recalculate the force of Example 6.2 at the same current and voltage but with a new relation:

$$\lambda = 0.3I^{0.7}/x \tag{P6.2.1}$$

6.3 Prove that an alternative formula for magnetic force is:

$$F_{\text{mag}} = \frac{\partial W_{\text{mag}}}{\partial x} \,|\lambda = \text{const.} \tag{P6.3.1}$$

Then use it to obtain the same force value found in Example 6.2.

6.4 Redo Example 6.3 with the steel changed to the nonlinear "steel_1010."

6.5 Redo Example 6.3 with the number of turns of the lower coil changed to 20.

6.6 Redo Example 6.3 with the upper coil moved up by 4 mm.

6.7 From the results of Example 3.1 using the reluctance method, find the inductance of the coil assuming its number of turns is 100.

6.8 From the results of Example 3.2 using the reluctance method, find the inductance of the coil for both positions assuming the number of turns is 100.

6.9 Given the complex flux linkage $\lambda = 2 + j3$ weber-turns and current $I = 0.5$ A, find the impedance, resistance, reactance, and inductance (for eight turns and 60 Hz).

6.10 Redo Example 6.5 with the steel changed to the nonlinear "steel_1010."

6.11 Redo Example 6.5 with the number of turns of the lower coil changed to 20.

6.12 Redo Example 6.5 with the upper coil moved up by 4 mm.

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