

# L2b: Magnetic Force

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Perhaps a general what and why

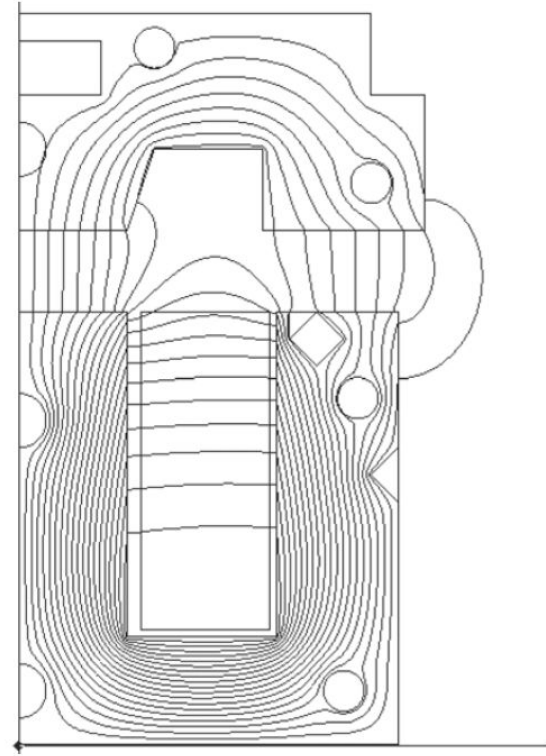
Show the equations and then discuss that we will be going through them one by one

# Magnetic Flux Line Plots

# What

Flux lines show contour lines of constant  $A_z$

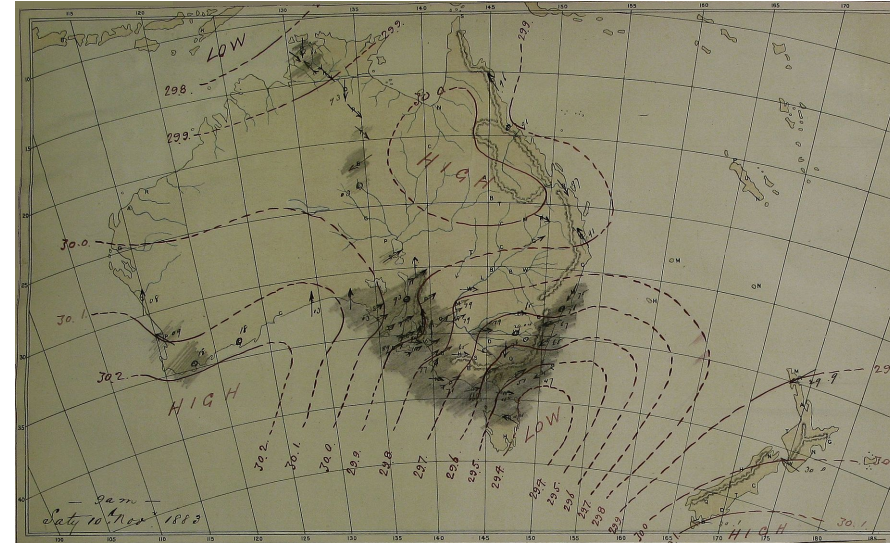
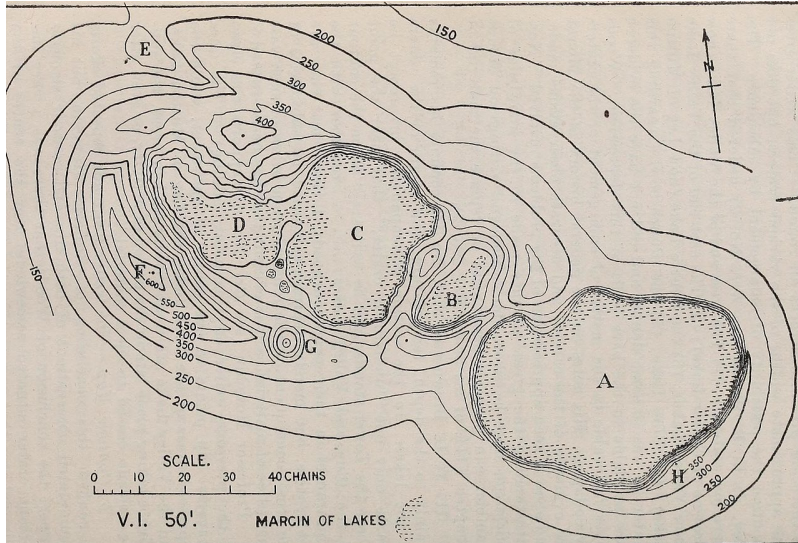
The closer the lines are together the greater the flux density



**FIGURE 5.1** Computer display in black and white showing flux line plot for 2D planar Eaton actuator model in the preceding chapter.

# Review - Examples of other contour plots

- What are the contour lines in these drawings?
- What does it mean when you have the lines closer together in each drawing?



# Magnetic flux lines

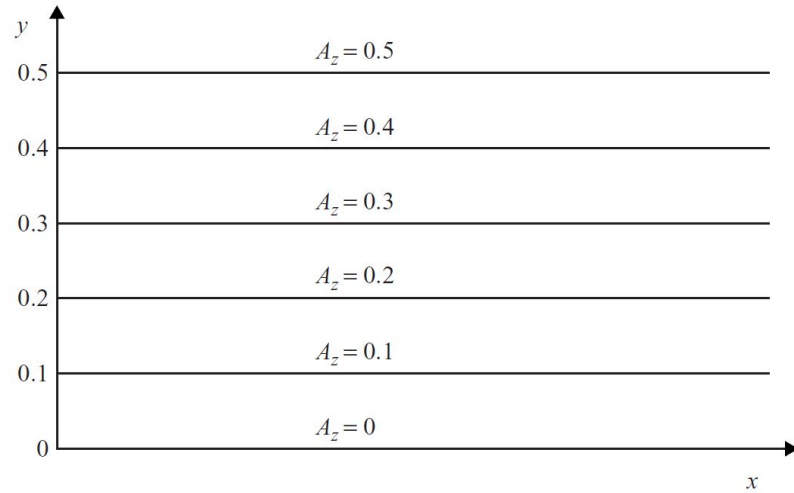
Image of magnet on “green” paper.

Preferably with the same situation modelled in FEA

# Why

- Flux lines can give a good “view” of what is going on magnetically
- They are better than “iron filings” as you can see what is happening inside magnetic materials
- Errors in analysis can be found
  - Wrong material selection
  - Wrong boundary conditions
  - Wrong units
  - Wrong input excitation (current or voltage)
  - Wrong number of turns

## Example 5.1



**FIGURE E5.1.1** Flux line plot of Example 5.1, shown over a finite region of the  $xy$  plane.

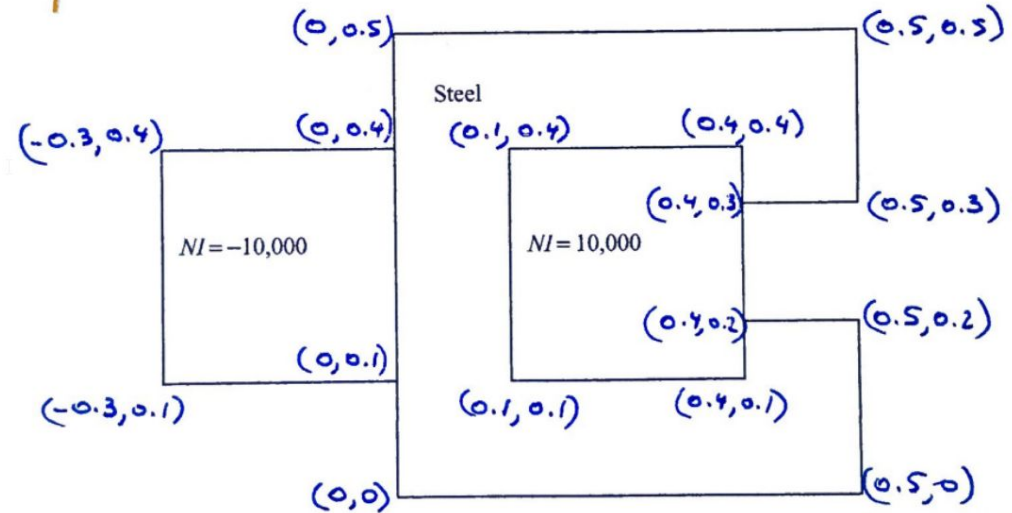
**Example 5.1 Relation Between A and B for 2D Planar Problem** Given a 2D planar finite-element solution of  $A_z = y$ , find the corresponding magnetic flux density **B** and describe the flux line plot.



# Example 5.2

Using the annotated figure  
(E3.1.1):

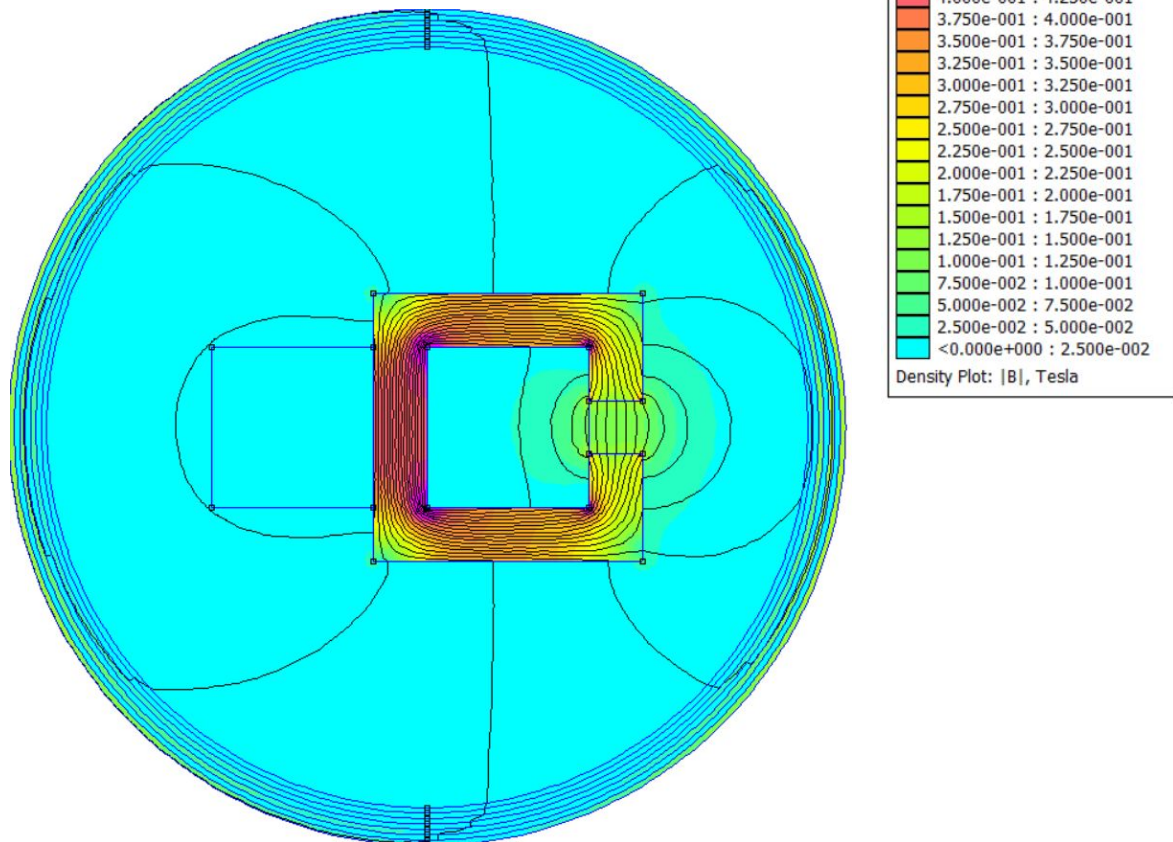
1. Geometry:
  - Enter the points into FEMM
  - Join the lines appropriately
2. Materials:
  - Define Materials for Steel, Copper and Air.
3. Excitations:
  - Define circuit properties.
4. Boundary conditions.
5. Solve



**FIGURE E3.1.1** Steel magnetic structure with a single airgap. Each side of the steel “C” core is 0.4 m long and the steel thickness and depth are 0.1 m.

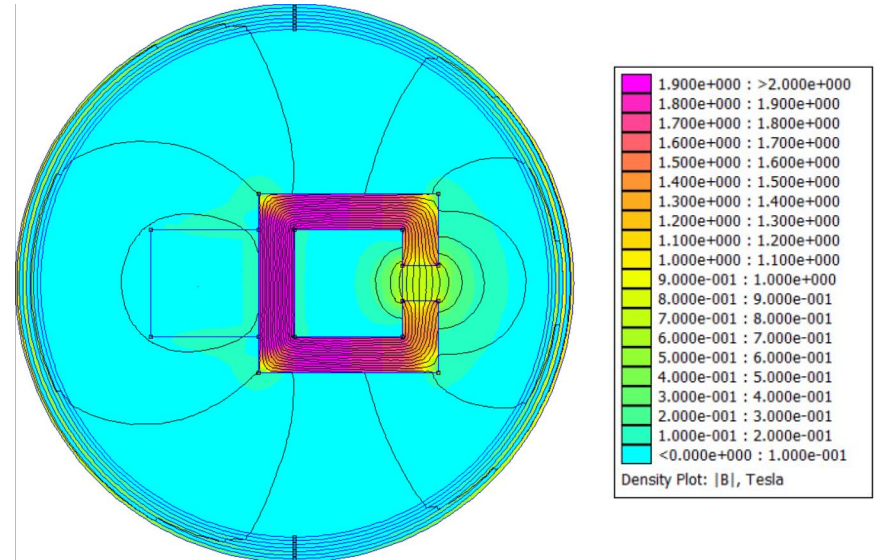
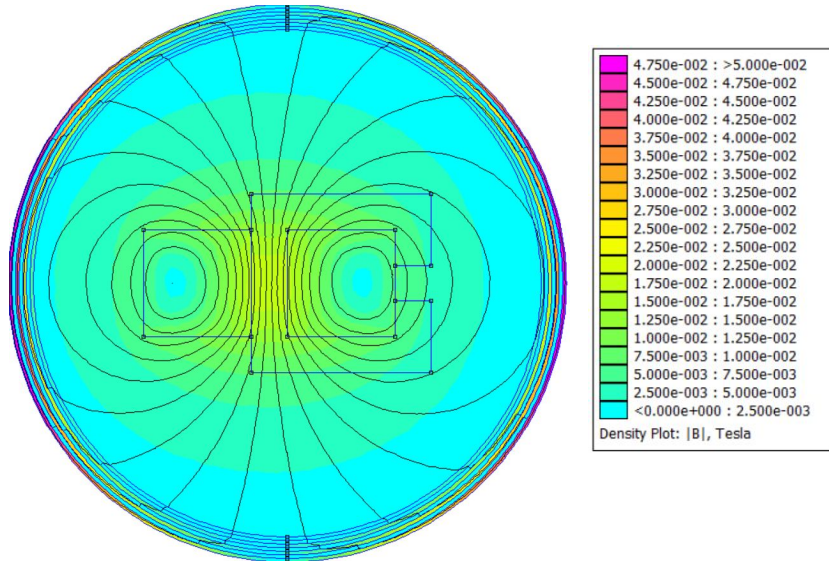
## Example 5.2 debrief

- Flux lines/ Flux density
- Fringing
- Boundary conditions



# Test for understanding

What problem may be wrong with the following two figures?



# Magnetic Energy

# Magnetic Energy

- Magnetic energy is the integral of  $\mathbf{H}$  w.r.t.  $\mathbf{B}$  (area to left of curve)
- Magnetic coenergy is the integral of  $\mathbf{B}$  w.r.t.  $\mathbf{H}$  (area under the curve)
- The sum of the energy and the is the product of  $\mathbf{H}$  and  $\mathbf{B}$
- For linear materials (constant permeability) the energy and coenergy are equal.

$$w_{\text{mag}} = \int \mathbf{H} \cdot d\mathbf{B}$$

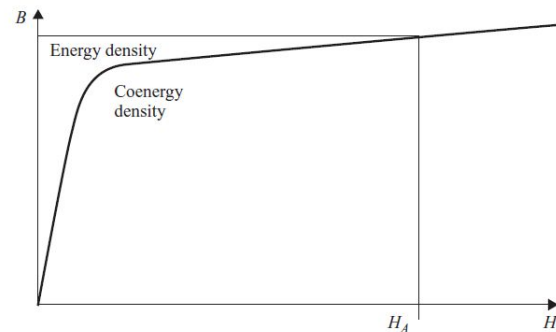


FIGURE 5.4 Energy density and coenergy density for a typical operating point on a typical nonlinear  $B$ - $H$  curve.

$$w_{\text{mag}} + w_{\text{co}} = \mathbf{H}\mathbf{B}$$

$$w_{\text{mag}} = \frac{B^2}{2\mu} = \frac{1}{2}BH$$

# Why

Using magnetic energy sometimes allows simpler analysis

Energy methods do not require knowledge of the path taken, on the end points.

# Magnetic Force on Steel

# Why

This was one of the main goals of this course.

Force on solenoids, torque on motors....



# How

$$F_y = \frac{\Delta W}{\Delta y}$$

$$W_{\text{mag}} = [B^2/(2\mu_o)]v$$

$$F_y = \frac{\Delta W}{\Delta y} = \frac{S(y + \Delta y)B^2/(2\mu_o) - SyB^2/(2\mu_o)}{\Delta y}$$

$$F_{\text{mag}} = SB^2/(2\mu_o)$$

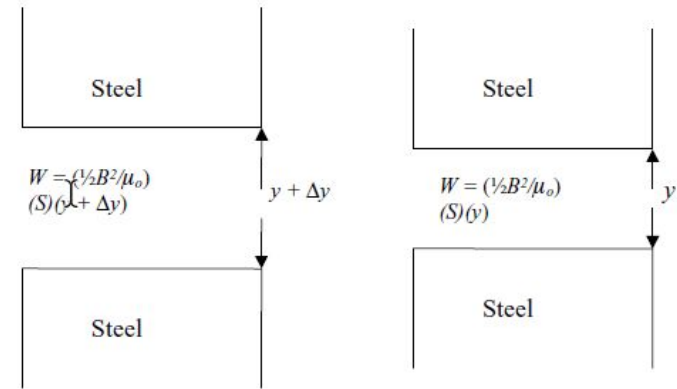


FIGURE 5.5 Typical magnetic actuator or sensor with steel poles and the energy stored in the airgap between them for two vertical positions.

$$F_{\text{mag}} = S[B^2/(2\mu_o) - B^2/(2\mu_s)]$$

## Example 5.3 - Reluctance method

# Example 5.3 - FEA

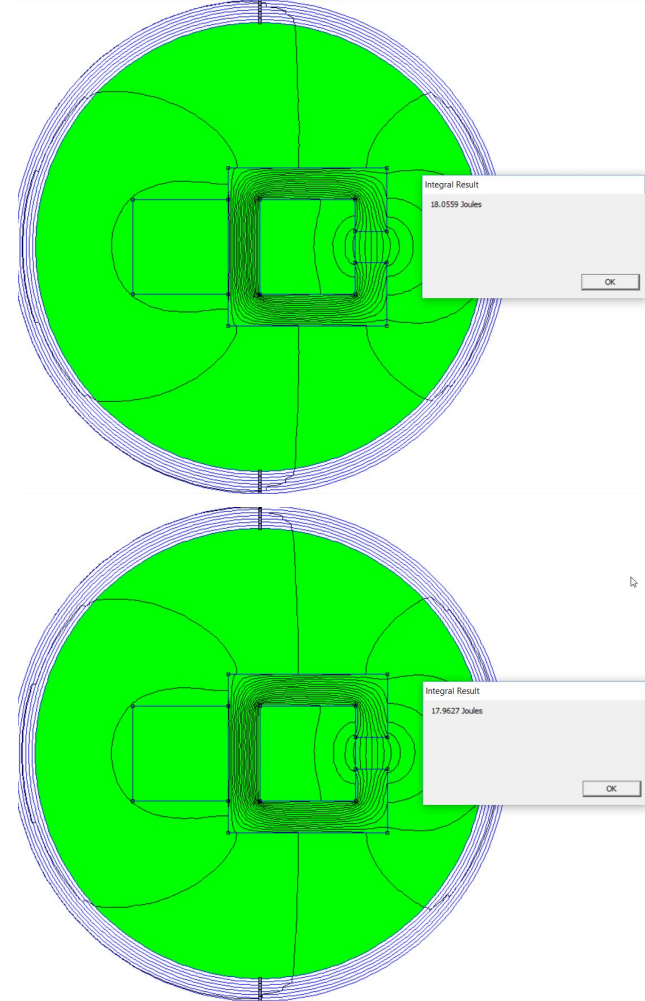
Before move 18.0559 Joules

After 0.001mm vertical move 17.9627 Joules

$$F_y = \frac{\Delta W}{\Delta y}$$

$$F_y = (17.9627 - 18.0559)/0.001$$
$$=-93.2N$$

	Position 1	Position 2	Delta Y	
Energy (1 turn)	18.0559	17.9627	0.001	-93.2
Co-energy (1 turn)	18.0628	17.9695	0.001	-93.3



# Test for understanding

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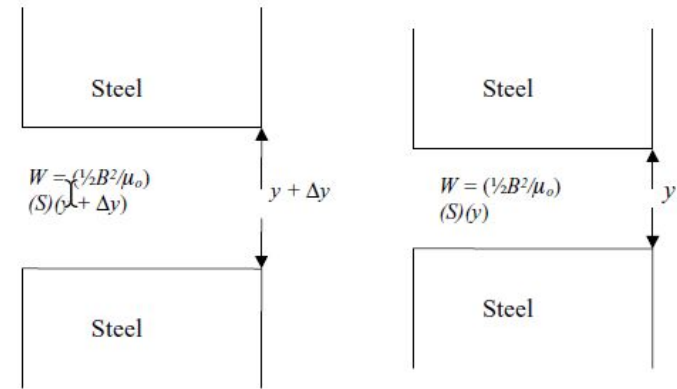


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# Example 5.3 - FEA

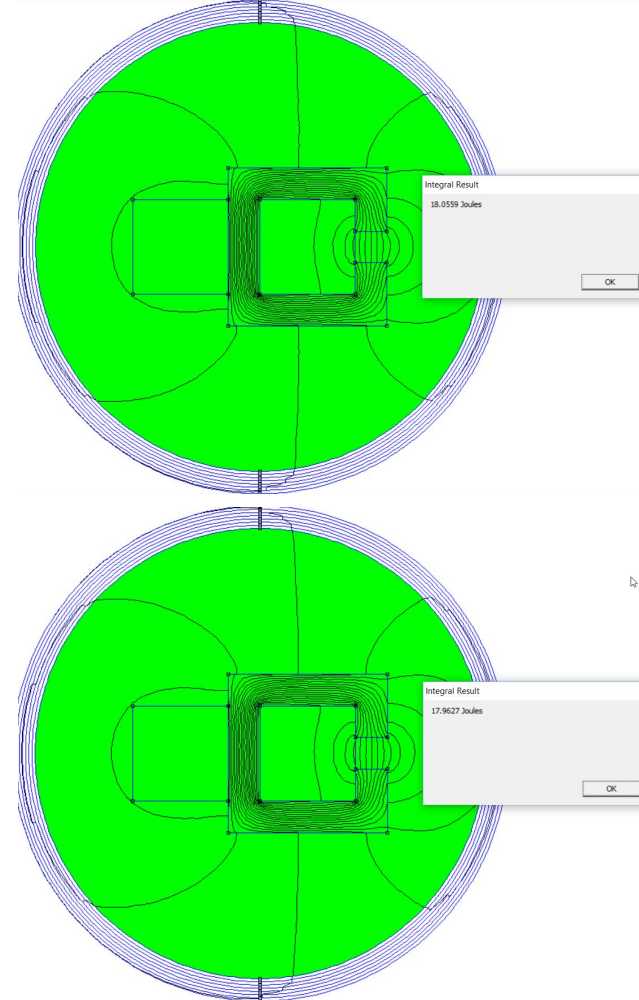
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# Test for understanding

# Lorenz Force

# Why

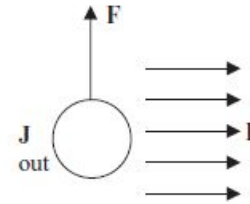
- Remember BIL?
- Using the Lorenz force is another way to determine the force
- It is really only relevant for analysing a conductor that is in a magnetic field
- Not really appropriate for analysis of motors where the flux is travelling through the teeth rather than the slots.

# How

$$\mathbf{F}_L = \int (\mathbf{J}ds) \times (\mathbf{B}dl)$$

$$\mathbf{F}_L = \int (\mathbf{J}) \times (\mathbf{B})dv$$

$$F = BIl$$



**FIGURE 5.8** Lorentz force vectors. If the right hand is laid flat with its fingers in the direction of flux density  $\mathbf{B}$  and the thumb in the direction of current density  $\mathbf{J}$ , then the force is in the direction of the palm of the right hand.

We will look at the analysis of actuators using the Lorentz force in a later section

# Permanent Magnets

# Why

To create force you need flux

To do work, you need changing flux

But..... not all the flux needs to change

Using permanent magnets gives you some flux for “free” and you can use current to change some of the flux.

(eg. PM vs wound field)

# Permanent magnet properties

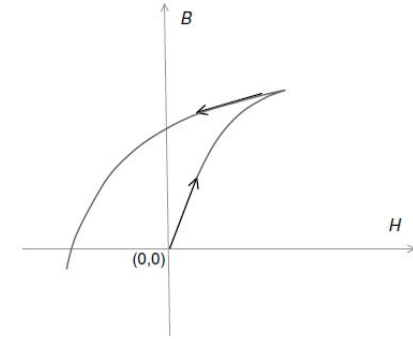


FIGURE 5.9  $B$ - $H$  relation of typical material that can exhibit permanent magnetization.

TABLE 5.1 Permanent Magnet Types and Typical Properties

Type	Approximate Year	$B_r$ [T (teslas)]	$H_c$ (A/m)	$(BH)_{\max}$ (T-A/m)
Carbon steel (quenched)	1600	0.9	4.E3	1.6E3
Cobalt steel (quenched)	1916	0.96	18.E3	7.2E3
Alnico3	1931	0.68	39.E3	11.1E3
Alnico5	1948	1.03	49.E3	51.7E3
Ceramic5 (ferrite)	1952	0.38	192.E3	24.E3
Ceramic8 (ferrite)	1955	0.39	240.E3	28.E3
Samarium cobalt	1970	0.88	640.E3	150.E3
Neodymium iron (early)	1983	1.08	800.E3	223.E3
Neodymium iron (latest)	2004	1.43	1080.E3	400.E3

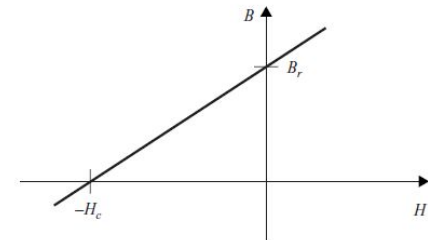
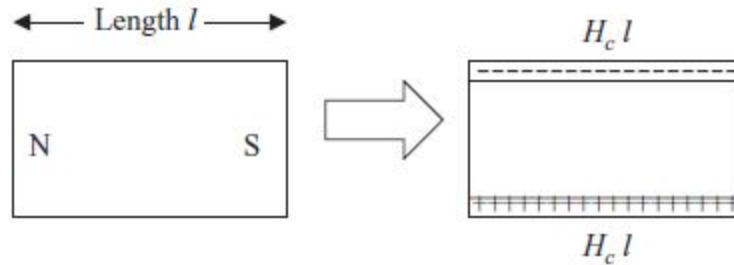


FIGURE 5.10 Linearized second quadrant of Figure 5.9 with definitions of  $B_r$  and  $H_c$ .



# How

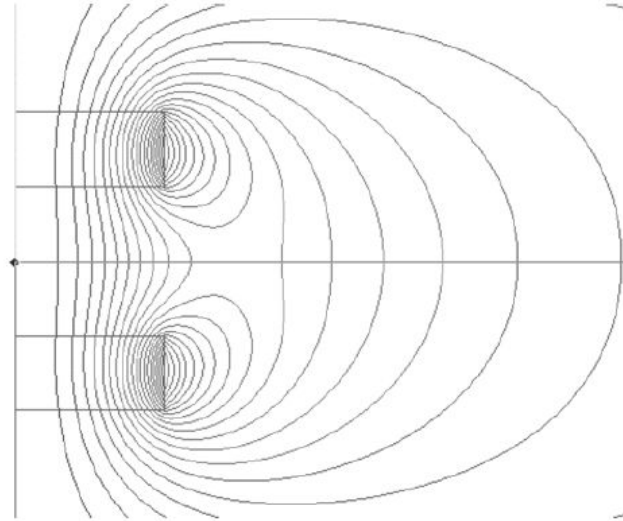


**FIGURE 5.11** Simple permanent magnet and its equivalent coil.

$$NI = H_c l$$

# Example

**Example 5.4 Force between Two Permanent Magnets** Given two identical cylindrical neodymium iron permanent magnets with a straight line  $B$ - $H$  curve with  $B_r = 1.23$  T and  $H_c = 8.9\text{E}5$  A/m. They both have radius 20 mm and height 10 mm, and are spaced 20 mm apart as shown in Figure E5.4.1. If both magnets have their north poles on their top, obtain the magnetic force on the lower magnet using Maxwell SV or version 16.



**FIGURE E5.4.1** Computer display of two cylindrical permanent magnets and their flux lines.

# Magnetic Torque

# Why

Often we are interested in torque rather than force

Most motor analysis is usually focussed on torque.

# How

Virtual work

$$T = \frac{\partial W_{\text{co}}}{\partial \theta}$$

Stress tensor or Lorenz force can be used with appropriate post calculation for radius to axis of rotation (ie shaft)

# Summary

In Chapter 2 we determined relationships between input current and flux density. In this section we looked at the relationship between flux density and force.

The choice of method you use to determine force will change depending on what the object is.

- If it carries current then Lorenz's law is probably easiest.
- If you want an overall force then virtual work is a good choice.
- If you want a deeper understanding of the force as it changes over the surface of an object then you should use "Maxwell's stress tensor" method.