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Chapter-3

Fundamentals of Atomic theory

Q. What is blackbody and black body radiation? Explain the characteristics of black body radiation.

→ First part:-

A perfectly black body is one which absorbs heat radiation of wavelengths that fall upon it. It has absorption coefficient = 1. Such a black body neither reflects nor transmits the incident heat radiation. Hence it appears black. Nobody is perfectly black body but the surface coated with lamp black absorbs 96% to 98% of incident radiation.

When black body is heated it emits radiation called black body radiation. The wavelength range of emitted radiation is independent of material of the body but depends on temperature of blackbody.

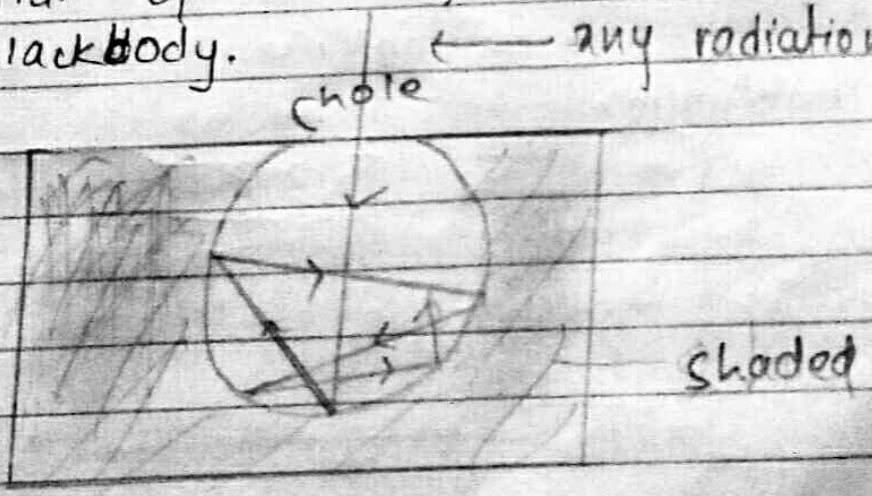

fig:- A metallic cavity with a small hole.

A perfectly black body is a metalic cavity with a small hole as shown in figure. Any

radiation entering the hole has a very small probability of being reflected out. Hence the object is black. After multiple reflection by the inner wall of the cavity the radiation is eventually absorbed by the atoms in the walls of the cavity. These atoms intorn will re-radiate electromagnetic waves into the cavity and some of it will leak out through the hole.

The radiation coming out from such an enclosure through an opening in it at any temperature of its walls is the blackbody radiation at that temperature.

Characteristics of blackbody radiation:-

1) The band of frequencies of blackbody radiation (spectrum) is continuous with a broad maximum.

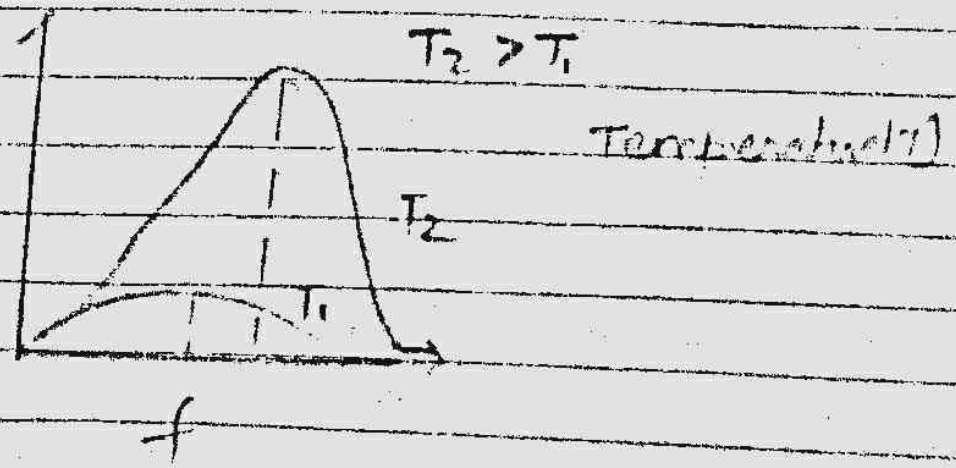

fig:- Intensity of blackbody radiation frequency

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This fact is shown in figure (ii) which is a plot of spectral radiance at each frequency $E(f)$ versus frequency of the radiation(f). Spectral radiance is energy per frequency emitted per unit time per unit area of the black body. The two curves correspond to two different temperature of the object.

2) The integral of $E(f)$ represents the energy emitted per unit time per unit area is found to increase with the fourth power of the temperature. This emperical fact is known as Stefans Rose man law.

i.e $E_T = \int_0^\infty E(f)\,df = \sigma T^4$

where,

σ = stefan's constant $= 5.67 \times 10^{-8}\, W m^{-2} K^{-4}$

3) figure (ii) also shows that the spectrum shifts towards higher frequencies as the temperature increases. Infact it is found that maximum frequency at which $E(f)$ is maximum increases linearly with the temperature of the blackbody i.e $f_{max} \propto T$

$$\lambda_{min} \propto \frac{1}{T}$$

$$\lambda_m T = \text{constant}$$

This is wiens displacement law.

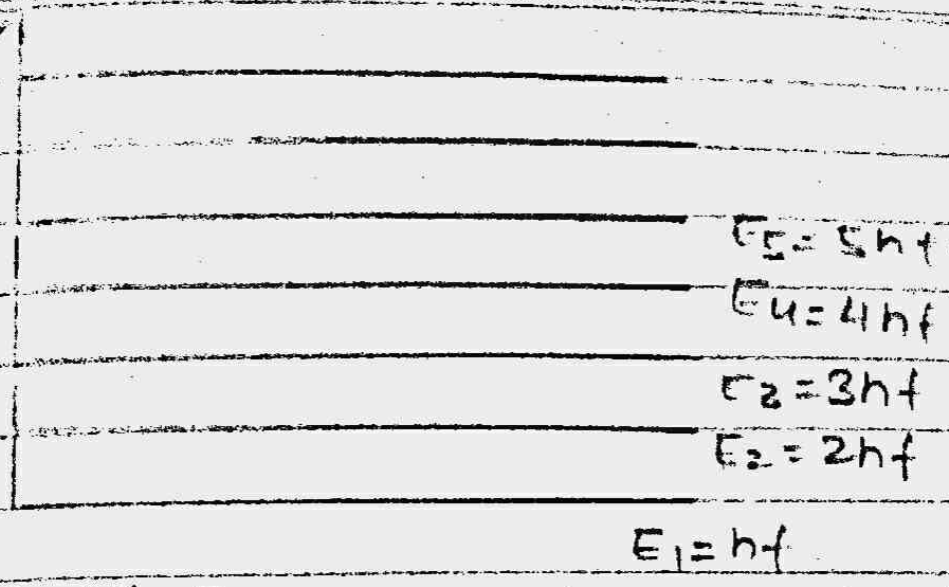

fig (P) Energy spectrum of atomic oscillator.

2) As long as oscillator has energy equal to one of the allowed values of energy given by relation $E = nhf$, it cannot ~~or~~ emit or absorb energy. Therefore, the oscillator is said to be in stationary energy state. The emission and absorption of energy ocurs only when the oscillator jumps from one energy state to another. If the oscillator jumps from higher energy state to lower energy state then when energy is emitted but it absorbs energy when it is climbing from lower energy state to upper excited energy state. The energy emitted or absorbed is given by

$$\Delta E = E_2 - E_1 = n_2 hf - n_1 hf$$

$$\therefore \Delta E = (n_2 - n_1) hf$$

Planck's derived an expression for $E(f)$ that matches the experimental data is

$$E(f) = \frac{2\pi h f^3}{c^2} \cdot \frac{1}{e^{\frac{hf}{kT}} - 1}$$

where c = velocity of light.
k = Boltzman constant
f = frequency of electromagnetic wave
T = absolute temperature of blackbody.

The plot of energy of atomic oscillation with amplitude is shown in fig below:-

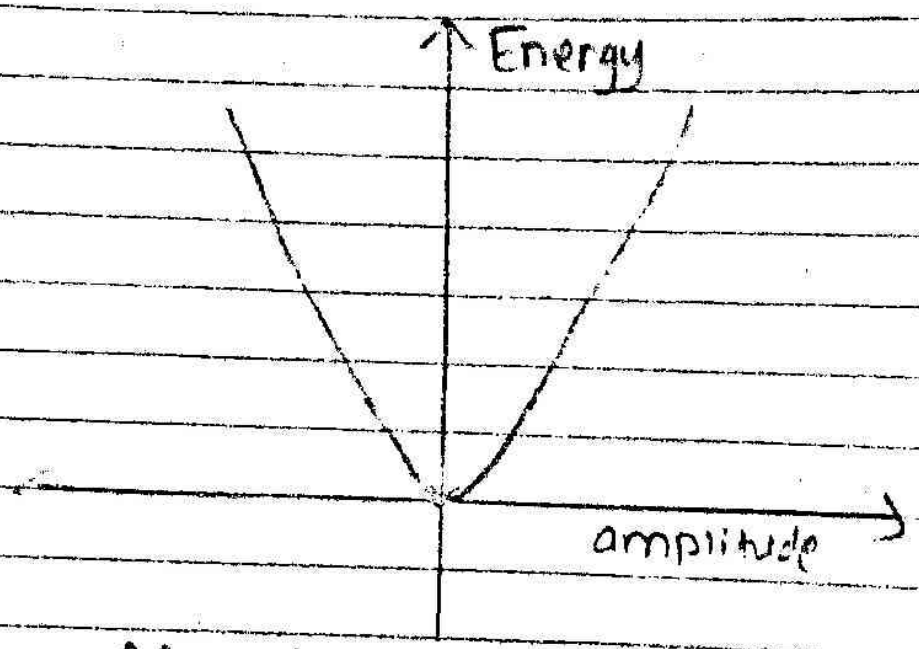

fig (ii) energy vs amplitude

Q3 Explain how Franck - Hartz experiment demonstrate the existence of discrete energy level in atoms.

→ (fig) →

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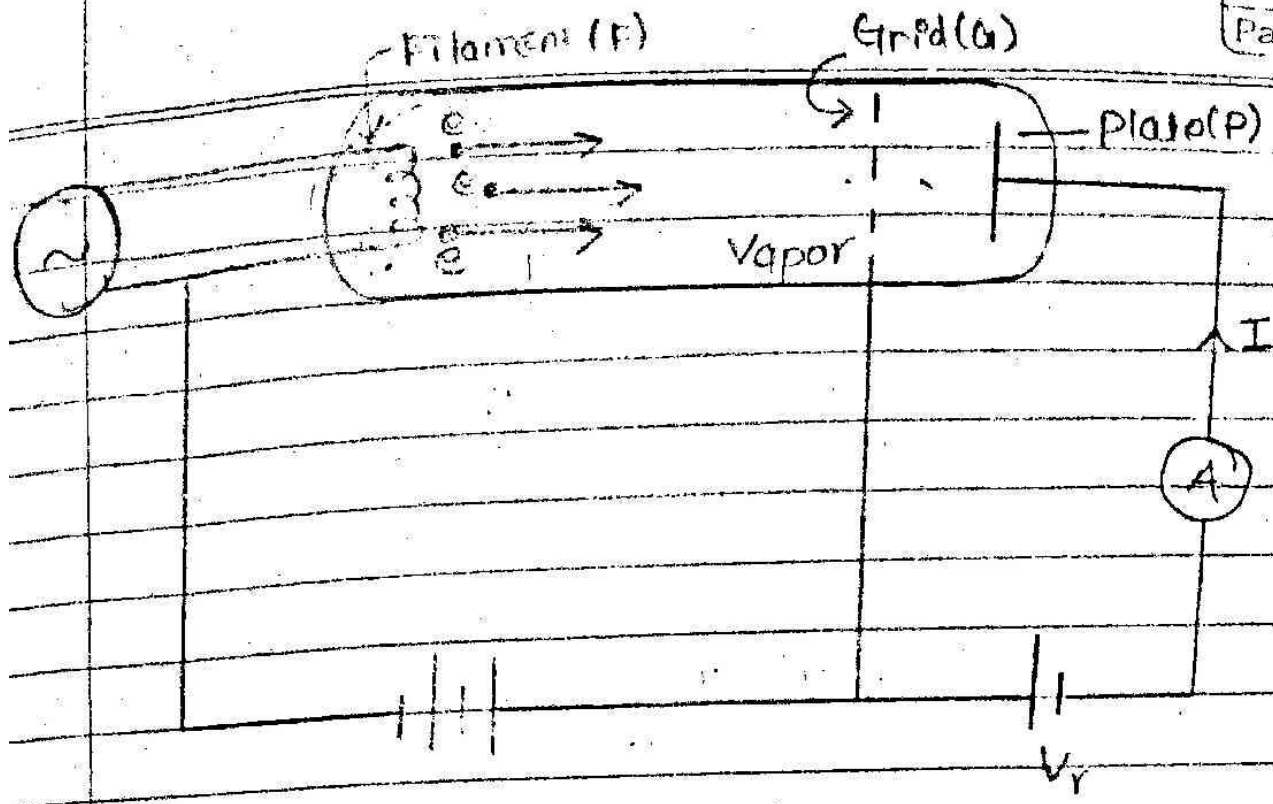


Fig(F) Franck Hertz Experiment.

James Franck and Gustav Hertz conducted an experiment in 1914 which confirm the prediction of quantum theory that electrons occupy only discrete quantized states. This experiment support Bhor's model of atom.

The apparatus used for experiment consists of tube containing low pressure mercury vapor, fitted with 3 electrode i-e cathode for electron emission, a mesh grid for acceleration of electrons and a collective plate.

With the help of thermionic emmission electrons are emitted by a heated cathode and then accelerated foward a grid maintained at positive potential relative to cathode. The collecting plate is at lower potential and is negative with respect to grid.

If electrons have sufficient energy on reaching grid some will pass through grid and reach collecting plate and current is measured through ammeter.

As V_0 increases, the current also increases. |But as ~~the accelerating potential~~ (V_0)| But as the accelerating potential (V_0) reaches a particular value (4.9 eV, for mercury) each electron passes that much potential and now collision becomes inelastic with mercury atoms. The energy level of electron bound to atom is raised. Now, electron bound to become almost loses ~~the~~ energy and measured drops abruptly. This sudden onset suggest that mercury atoms (electrons) cannot accept energy ~~atoms~~ until it reaches the threshold to elevate them to an excited state.

At ~~g~~ 9.8 ev each electron gets sufficient energy to participate in two elastic collision collected currents drops at multiple of 4.9 ev.

Thus this experiment shows that there is discrete energy levels, in mercury atoms.

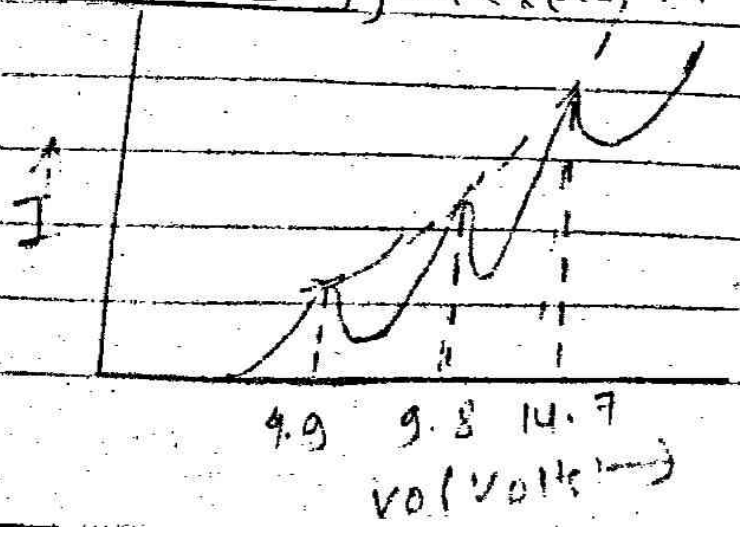


fig: Dependence of plate current with V_0.

Q4) What is de-Broglie hypothesis? Explain Davission-Germer experiment to show the electron behaves like a wave?

→ De Broglie hypothesis states that "Any particle can show wave like nature!! It is called wave particle dualism. The waves associated with matter is called matter waves.

From Planck's theory of quantum radiation,

$$E = hf = \frac{hc}{\lambda} \quad \text{—— (i)}$$

Again,

From Einstine mass energy relationship.

$$E = mc^2 \quad \text{—— (ii)}$$

Equating (i) & (ii),

$$\frac{hc}{\lambda} = mc^2$$

$$\text{or, } \frac{h}{\lambda} = mc$$

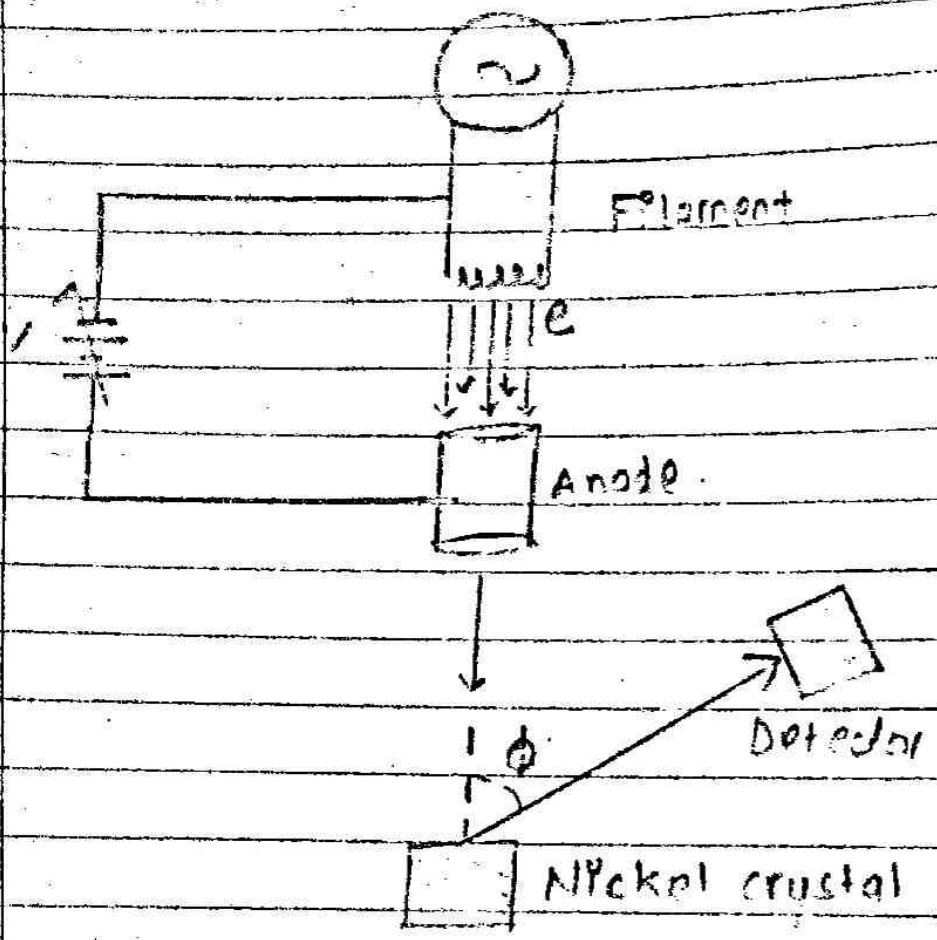
$$\text{or, } \lambda = \frac{h}{mc}$$

$$\therefore \lambda = \frac{h}{p} \quad \text{—— (iii)}$$

Eqn (iii) shows any particle with momentum p has wavelength 'λ' associated with it is called de-Broglie wavelength.

$$\boxed{\text{i.e } \lambda = \frac{h}{p} = \frac{h}{mv}}$$

Second part:-

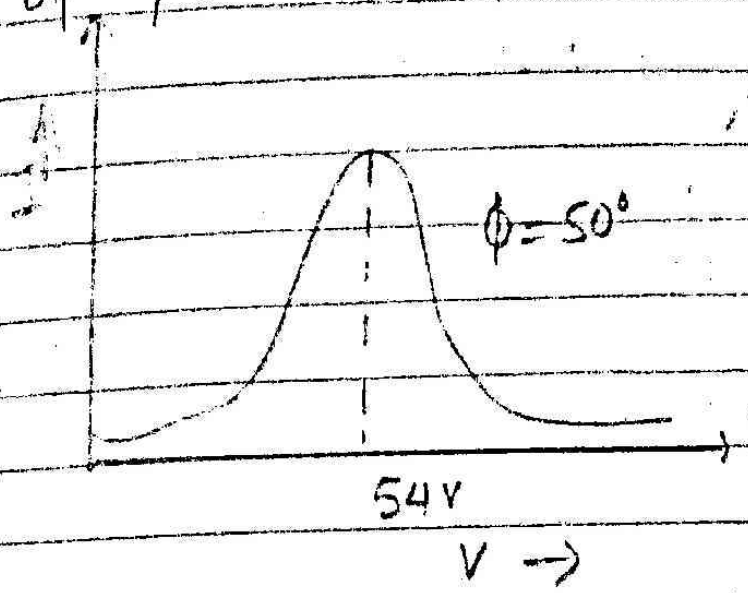


fig(10) :- Schematic of apparatus used in Davission-Germal experiment.

This experiment conducted by Davission Germer supported De Broglies Hypothesis. It demostrate diffraction of electron beams similar to light wave.

Electron from a heated filament are accelerated with variable voltage 'V'. The electron emerged from cylindrical shield kept at fixed known high positive voltage as find beam. This beam is made to fall on surface of nickel crystal. The intensity

of scattered beam can be measured by detector for different angles(ϕ) and various voltage (V). The detector observed maximum intensity at a certain value of ϕ (50°) and V (54 V) as shown in fig(2).



fig(ii) Intensity of scattered beam as a fn of the accelerating voltage V for the fixed value of $\phi = 50°$.

If electrons were particles only then their intensity must be same for all values of ϕ. Here, intensity varies with ϕ, so electrons behave as wave.

For mathematical verification

K.E of electron = Workdone by potential on electron

or, $\frac{1}{2}mv^2 = eV$

or, $v = \frac{2eV}{m}$

we have,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\lambda = \frac{h}{m\sqrt{\frac{2ev}{m}}}$$

$$= \frac{h}{\sqrt{\frac{2m^2ev}{m}}}$$

$$\therefore \lambda = \frac{h}{\sqrt{2mev}}$$

For $V = 54V$

$$\lambda = \frac{6.62 \times 10^{-3}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 54}}$$

$$= 1.67 \times 10^{-10} m$$

$$= 1.67 A^{\circ}$$

If x ray is made to incident on that plane.

From Bragg's law,

$$2d\sin\theta = n\lambda$$

For $n = 1$,

$$\therefore 2d\sin\theta = \lambda.$$

Atomic spacing for nickel crystal, $(d) = 0.91 A^{\circ}$

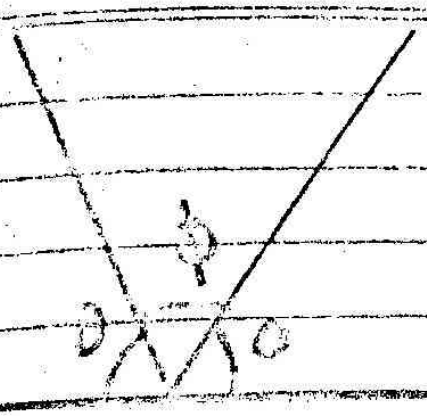


fig (i) X-ray diffraction mechanism.

i.e $\theta + \phi + \theta = 180^\circ$

or, $2\theta = 180^\circ - \phi$

or, $2\theta = 180^\circ - 50^\circ$

or, $\theta = \frac{130^\circ}{2}$

or, $\theta = 65^\circ$

$\therefore \theta = 65^\circ$

Now, $2d\sin\theta = \lambda$

$2 \times 0.9 \times 10^{-10} \sin 65^\circ = \lambda$

$\therefore \lambda = 1.65 A^\circ$

It is nearly equal to wavelength of electron beam. X-ray is electromagnetic wave. Thus electron behaved as wave.

Thus this experiment suggest De-broglie hypothesis that particles have wave properties

Q. State Uncertainity Principle. Justify "The measuring process itself introduces uncertainity".

→ First part

It is impossible to determine precisely and simultaneously the values of both the members of a pair of physical variable which describe motion of a atomic system. Such pair of variables are called canonically confugate variables. For example the position and momentum, energy & time angular momentum(J) and angular co-ordinate.

Let Δx be error in determing position, $\Delta p(x)$ be error in determining momentum position, then,

$$\boxed{\Delta x . \Delta P_x \geq \hbar} \quad \text{—(1)}$$

where $\hbar = \frac{h}{2\pi}$

Similarly,

$$\Delta E . \Delta t \geq \hbar$$

$$\Delta J . \Delta \theta \geq \hbar$$

Where,

ΔE = uncertainity in energy

Δt = uncertainity in time

ΔJ = uncertainity in anglar momentum

$\Delta \theta$ = uncertainity in angular position

~~Second part~~

From ①, If we want to minimize error in position, ie $\Delta x = 0$.

then,

$$\Delta P_x \geq \frac{\hbar}{\Delta x}$$

$$\text{i.e } \Delta P_x \geq \frac{\hbar}{0}$$

$$\therefore \Delta P_x \geq \infty$$

The error in determination of momentum becomes infinite.

Let us consider example of hypothetical example of microscope in Bhor's experiment.

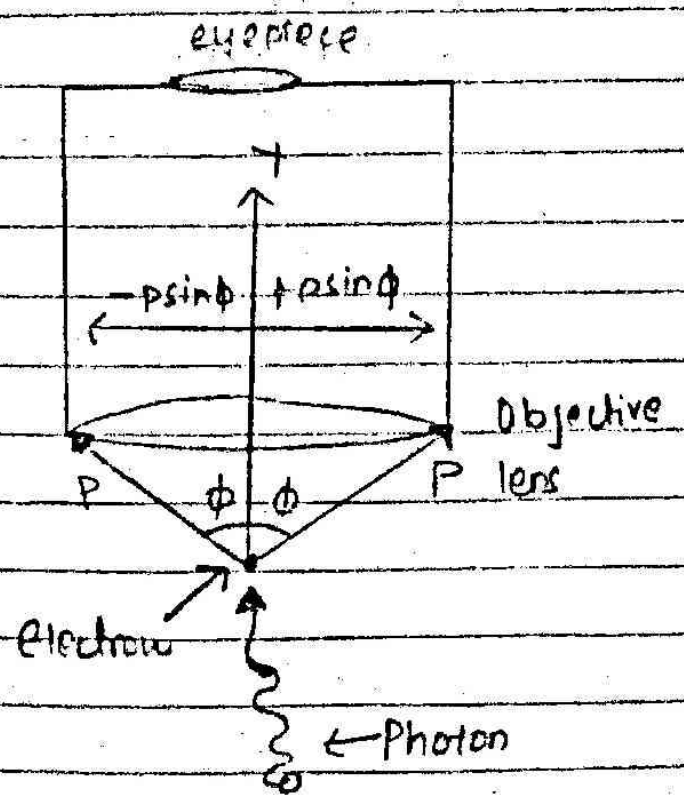


fig (i):- "looking" at an electron with a hypothetical microscope in Bhor's experiment.

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(a) For larger wavelength (λ), uncertainty in momentum decreases.

By reducing ϕ, ΔP_x decreases. (reducing apertain of microscope)

But resembling power of microscope

$$\Delta x = \frac{\lambda}{2\mu \sin\phi}$$

which depends inversly on λ & ϕ. → directly of λ.

∴ If we try to reduce uncertainty in position it leads to increase uncertainty in momentum.

Thus the measuring process itself ~~reduc~~ introduces uncertainity.

Q. What is group velocity. Show that group velocity is equal to particle velocity.

Sol: The group velocity is the velocity of transferenie of energy of the wave packet from one point to another point. It is denoted by V_g.

$$V_g = \frac{d\omega}{dk}.$$

This is also called transmission velocity of a wave packet.

(OR)

The velocity of group of wave are wave

packet ~~with~~ which is quantitatively equal to mechanical velocity of particle is called group velocity.

Second part:-

$$\text{Velocity of wave } (v) = f\lambda$$
$$= f\frac{h}{p}$$
$$= \frac{hf}{p}$$
$$\therefore v_{wave} = \frac{E}{p}$$

$$\text{Energy } (E) = \frac{1}{2} m v^2_{particle}$$
$$\text{or, } E = \frac{1}{2}(m v_{particle})\, v_{particle}$$
$$\text{or, } 2E = p\,(v_{particle})$$
$$\text{or, } v_{particle} = \frac{2E}{p}$$
$$\text{or, } v_{particle} = 2 \times v_{wave}$$

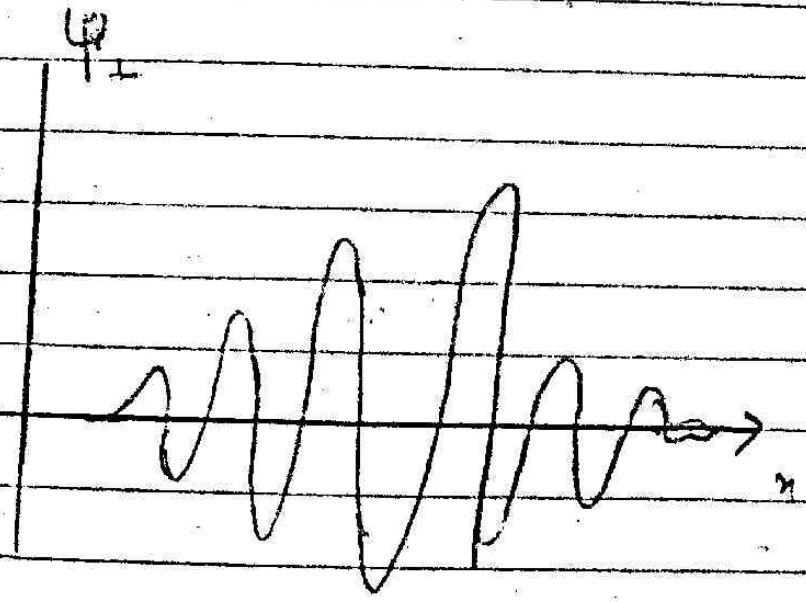
$$\boxed{\text{i.e. } v_{wave} = \frac{v_{particle}}{2}}$$

This shows that velocity of wave is not same as velocity of particle that it guides. Schrodinger thus said a

material particle in motion is equivalent to wave packet. rather than single wave trend.

Thus a wave packet is formed by taking a sum of larger no. of plane ~~waves~~ with different wavelength and frequency. The velocity by which the amplitude of such wave travel is called group velocity. The group velocity is velocity with which energy in the group is transmitted.

The individual wave travel inside group with phase velocities.



wave packet

Now the wave packet tells us where the particle may be formed. According to De Broglie each particle of matter in motion is associated with group of wave differ in slightly in wavelength and frequency called wavepacket. The velocity of this wavepacket which is equal to mechanical velocity of a particle is group velocity. Let us consider two waves that

differ slightly in wavelength and in frequency.

$$y_1 = A\sin(\omega_1 t - k_1 x) \quad \text{—(i)}$$

$$\cancel{y_2 = A\sin[k + \Delta\omega x - (\omega + \Delta\omega)t]}$$

$$y_2 = A\sin(\omega_2 t - k_2 x) \quad \text{—(ii)}$$

Since the simple position of wave.

$$y = y_1 + y_2$$

$$= A\sin(\omega_1 t - k_1 x) + A\sin(\omega_2 t \pm k_2 x)$$

$$\cancel{= 2A\sin\ \omega_1 t + \omega_2 t + (-k_1 + k_2)}$$

$$= A\, 2\sin\left[\frac{(\omega_1+\omega_2)t - (k_1+k_2)x}{2}\right]$$

$$\cos\left[\frac{(\omega_1-\omega_2)t - (k_1-k_2)x}{2}\right]$$

$$= 2A\cos\left[\frac{(\omega_1-\omega_2)t - (k_1-k_2)x}{2}\right] \cdot$$

$$\sin\left[\frac{(\omega_1+\omega_2)t - (k_1+k_2)x}{2}\right]$$

—(iii)

The resultant wave has angular frequency $\left(\frac{\omega_1+\omega_2}{2}\right)$ and propagation constant $\left(\frac{k_1+k_2}{2}\right)$. From eqn (iii) it is also seen that amplitude of wave varies periodically with time. The velocity of amplitude

time is called group velocity

$$V_g = \frac{\omega}{k}$$

$$= \frac{\frac{\omega_1 - \omega_2}{2}}{\frac{k_1 - k_2}{2}}$$

$$= \frac{\omega_1 - \omega_2}{k_1 - k_2}$$

$$= \frac{\Delta\omega}{\Delta k}$$

If $\Delta k \to 0$, then

$$V_g = \frac{d\omega}{dk}$$

$$= \frac{d(\hbar\omega)}{d(\hbar k)}$$

$$\boxed{\therefore V_g = \frac{dE}{dp}}$$

Rough

$\lambda = \frac{h}{p}$

$p = \frac{h}{\lambda}$

$p = \frac{2\pi h}{2\pi\lambda}$

$= \hbar\left(\frac{2\pi}{\lambda}\right)$

$= \hbar k$

$E = hf$

$= \frac{2\pi f h}{2\pi}$

$= \hbar f 2\pi$

$= \hbar\omega$

Again,

For particle, (mechanical)

$$K.\ Energy = \frac{1}{2}mv^2$$

$$= \cancel{\frac{1}{2}(mv)v} = \frac{1}{2}\frac{m^2v^2}{m}$$

$$= \frac{1}{2}pv = \frac{1}{2}\frac{p^2}{m}$$

$$\frac{dE}{dp} = \frac{2p}{2m} = \frac{p}{m} = \frac{mv}{m}$$

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$$\therefore \frac{dE}{dP} = v_{particle} \quad \text{—(v)}$$

From eqn (iv) & (v) ; it is seen that,

group velocity = particle velocity

From this result it can be concluded that the wave packet moves along with the particle.