Set1

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. (a) A function is defined by $f(x) = \begin{cases} 1-x, & if \;\; x \le -1 \\ x^2 & if \;\; x > -1 \end{cases}$ [5]

Evaluate f(-3); f(-1) and f(0) and sketch the graph.

(b) Prove that the limit $\lim_{x \to 0} \frac{|x|}{x}$ does not exist. [5]

2. (a) Sketch the curve : $\frac{2x^2}{x^2-1}$ [5]

(b) Estimate the area between the curve $y = e^x$ and the lines x = 0 and x = 1, using rectangle method. [5]

3. (a) Show that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$. [4]

(b) Define initial value problem. Solve: $x^2y''+xy'=1, \;\; x>0 \;\; , y'(1)=0 \; and \;\; y(1)=0.$ [6]

4. (a) Find the torsion, normal and curvature of helix curve: $(2\cos t)\vec{i} + (2\sin t)\vec{j} + 3t\vec{k}$. [6]

(b) Show that the curvature of a circle of radius a is 1/a. [4]

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $y = x^5 + x$ (b) $y = 1 - x^4$ (c) $y = 2x - x^2$

6. Define continuity of a function at a point x = a. Show that the function $f(x) = 1 - \sqrt{1-x^2}$ is continuous on the interval [-1, 1].

7. Verify the Rolle's theorem for $f(x) = x^3 - x^2 - 6x + 2$ in [0, 3].

8. Find the third approximation x_3 to the root of the equation $f(x) = x^3 - 6x - 5$ setting $x_0 = 2$.

9. Evaluate: $\int_{-\infty}^{0} x\, e^x dx.$

10. Find the volume of the solid obtained by about y - axis the region between y = x and y = x^2.

11. Solve: $y''+y'-6y = 0, \;\; x>0 \;\; , y'(0)=0 \; and \;\; y(0)=1.$

12. Find the curvature of the twisted cubic $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ at a general point and at (0, 0, 0)

13. Show that the series $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$ converges.

14. Calculate: $\iint_R f(x,y)\,dA$ for r = $f(x,y)=x^2y-2xy, \quad R:-2 \le x \le 0, \;\; 0 \le y \le 3.$

15. Find the partial derivatives of $f(x,y) = x^3 + x^2y^3 - 2y^2$ at (2, 1).

Diwash Sapkota

Set 1

Group-'A'

1.a) solution,

$$f(x) = \begin{cases} 1-x & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

so,

$f(-3) = 1-(-3)$ $[\because -3 < -1]$

$= 1+3$

$\therefore \boxed{f(-3) = 4}$

And,

$f(-1) = 1-(-1)$ $[\because -1 = -1]$

$= 1+1$

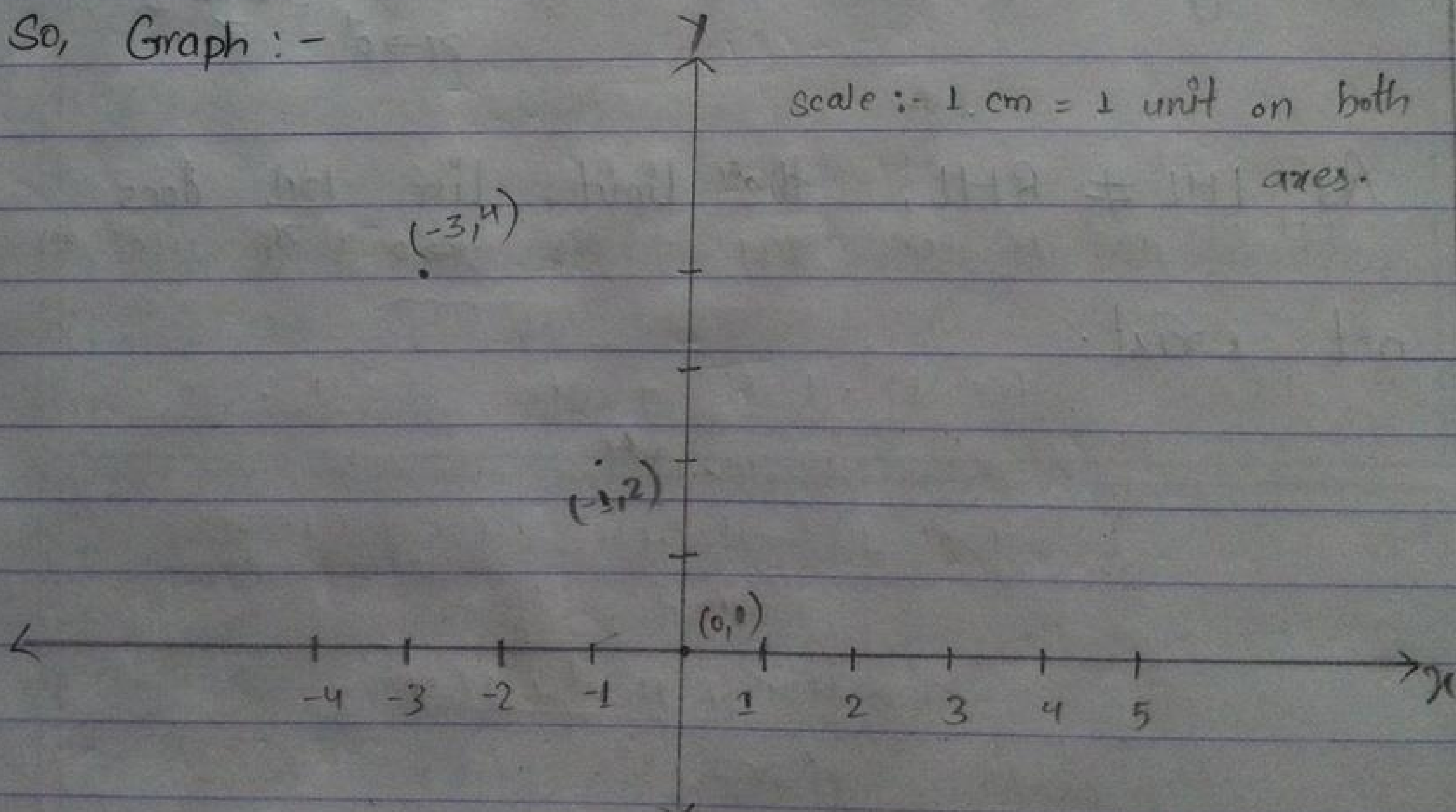
$\therefore \boxed{f(-1) = 2}$

Then,

$f(0) = 0^2$ $[\because 0 > -1]$

$\therefore \boxed{f(0) = 0}$

So, Graph :-



(Q.N.1.b)

solution,

To prove:

$$\lim_{x \to 0} \frac{|x|}{x} \text{ does not exist.}$$

we know;

$$\frac{|x|}{x} = \begin{cases} x/x & \text{if } x \geq 0 \\ -\frac{x}{x} & \text{if } x < 0 \end{cases} = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

since,

Left Hand limit (LHL) $= \lim_{x \to 0^-} \frac{|x|}{x} = -1$

and Right Hand limit (RHL) $= \lim_{x \to 0^+} \frac{|x|}{x} = 1$

As LHL $\neq$ RHL, the limit: $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

(Q.N.2.a)

= solution,

here,

$$Y = f(x) = \frac{2x^2}{(x^2-1)}$$

(i) When $x = 0$; $Y = 0$. So, the curve passes through origin and thus does not have intercepts.

(ii) Domain = $(-\infty,-1) \cup (-1, 1) \cup (1, \infty)$

(iii) When x is replaced by $-x$; $f(x) = f(-x)$. So, the curve is an even function, symmetric about the Y-axis.

(iv) As, $\lim\limits_{x\to\infty} \frac{2x^2}{x^2-1}$

$$= \lim_{x\to\infty} \frac{2}{1-1/x^2}$$

$$= 2$$

$\therefore$ $Y = 2$ is a horizontal asymptote.

And, the denominator of $f(x)$ becomes infinite whenever $x = 1$ or $x = -1$. As:-

$\lim\limits_{x\to 1^+} f(x) = \infty$, $\lim\limits_{x\to 1^-} f(x) = -\infty$, $\lim\limits_{x\to -1^+} f(x) = -\infty$

$\lim\limits_{x\to -1^-} f(x) = \infty$

$\therefore$ $x = 1$ and $x = -1$ are the vertical asymptotes.

(v) $f'(x) = \dfrac{(x^2-1)\times(4x) - 2x^2\cdot(2x)}{(x^2-1)^2}$

$= \dfrac{4x^3 - 4x - 4x^3}{(x^2-1)^2}$

$= \dfrac{-4x}{(x^2-1)^2}$

When $x=0$ and $x=\pm 1$; $f'(x)$ is zero and infinity respectively. But; since $x=\pm 1$ does not lie in the domain, $x=0$ is the critical point here,

Table I : (Rise and Fall) :-

$-4x$	+	+	−	−
$(x^2-1)^2$	+	+	+	+
$f'(x)$	+	+	−	−
	Rise ↑	↑ Rise	↓ Fall	↓ Fall

(points on the line: -1, 0, 1)

So, the curve is increasing in the interval $(-\infty,-1) \cup (-1,0)$ and is decreasing in the interval $(0,1)\cup(1,\infty)$.

(vi) $f''(x) = \dfrac{(x^2-1)^2\cdot(-4) + (4x)\,2(x^2-1)\cdot 2x}{(x^2-1)^4}$

$= \dfrac{(x^2-1)(-4x^2+4) + (x^2-1)(16x^2)}{(x^2-1)^4}$

$= \dfrac{4 - 4x^2 + \cancel{16x^4} + 16x^2}{(x^2-1)^3}$

$$= \frac{(4+12x^2)}{(x^2-1)^3}$$

$$\therefore f''(x) = \frac{4(3x^2+1)}{(x^2-1)^3}$$

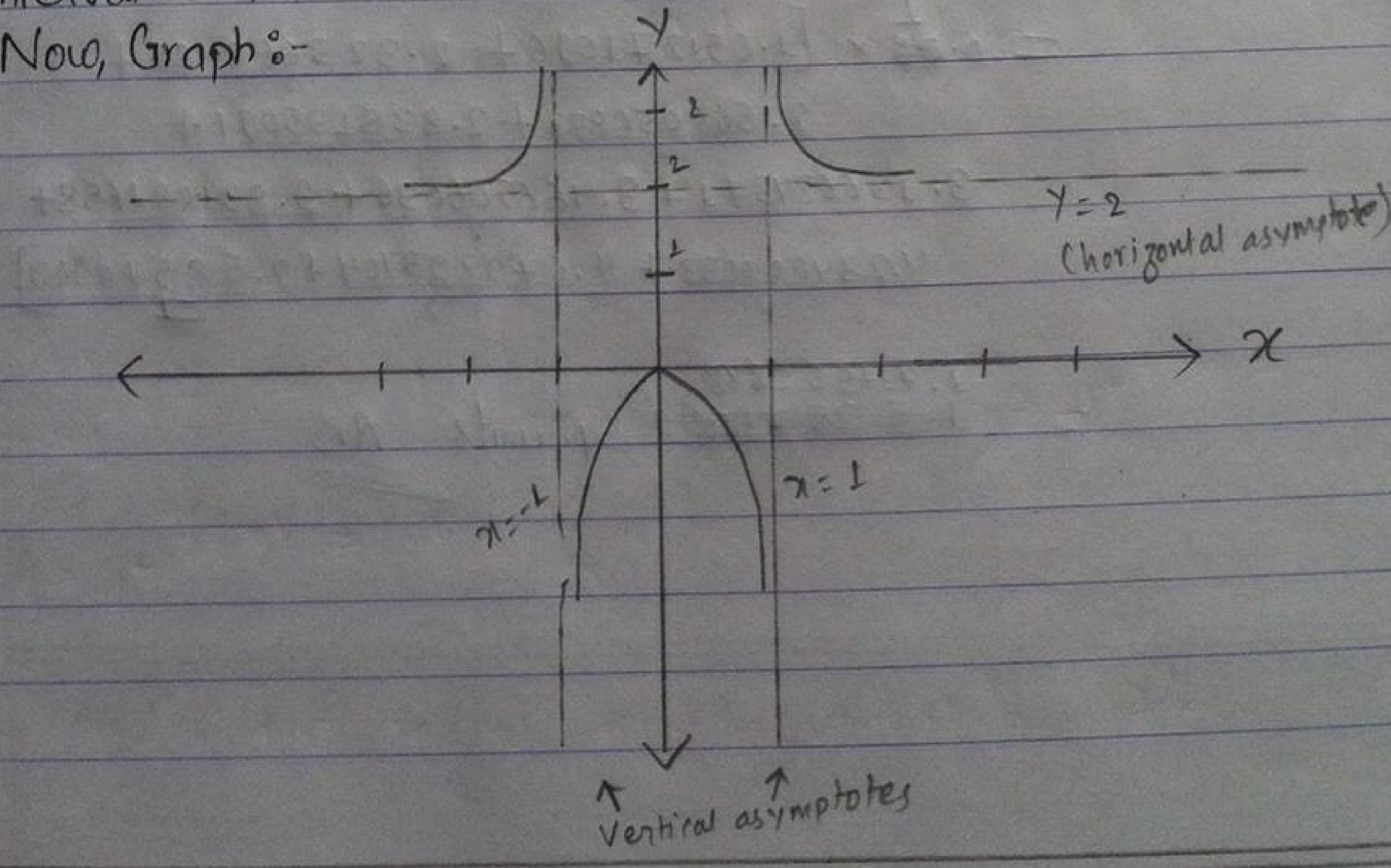
$f''(x)$ does not exist when $x = \pm 1$. Now,

Table II: Concavity test:-

	$(-\infty, -1)$	-1	$(-1, 1)$	1	$(1, \infty)$
$4\times(3x^2+1)$	+		+		+
$(x^2-1)^3$	+		−		+
$f''(x)$	+		−		+
	Concave upward		Concave downward		Concave upward

So, the curve of the function is concave upward in the interval $(-\infty, -1) \cup (1, \infty)$ and is concave downward in the interval $(-1, 1)$.

Now, Graph:-



(Q.N.2b)

solution,

$$y = e^x$$

$$\therefore f(x) = e^x$$

endpoints :- $a = 0$, $b = 1$

let 'n' be the number of mid-points ; so $n = 10$ (let).

And,

$$\Delta x = \left(\frac{b-a}{n}\right) = \frac{1-0}{10} = 0.1$$

$\bar{x}_1 = 0.05$, $\bar{x}_2 = 0.15$, $\bar{x}_3 = 0.25$, $\bar{x}_4 = 0.35$,
$\bar{x}_5 = 0.45$, $\bar{x}_6 = 0.55$, $\bar{x}_7 = 0.65$, $\bar{x}_8 = 0.75$,
$\bar{x}_9 = 0.85$, $\bar{x}_{10} = 0.95$

So,

Area enclosed $\approx \frac{\Delta x}{2} \left[f(\bar{x}_0) + 2f(\bar{x}_1) + 2f(\bar{x}_2) + \cdots + 2f(\bar{x}_8) + 2f(\bar{x}_9) + f(\bar{x}_{10}) \right]$

$$\approx \frac{0.1}{2} \left[f(0.05) + 2f(0.15) + 2f(0.25) + 2f(0.35) + \cdots + 2f(0.85) + 2f(0.95) \right]$$

$$= 0.1 \times \left[1.051271096 + \frac{2.323668485}{2} + \frac{2.568050833}{2} + \frac{2.838135097}{2} + \frac{3.136624371}{2} + \frac{3.466506036}{2} + \frac{3.831081658}{2} + \frac{4.234000033}{2} + \frac{4.679293704}{2} + \frac{2.585709659}{2} \right]$$

$= 1.717552848$ ~~1.535717049~~ sq. units Ans

Set 1 Continued

(Q.N. 4.a)

Solution,

here,

Consider:

$$\overrightarrow{r(t)} = (2\cos t)\vec{i} + (2\sin t)\vec{j} + (3t)\vec{k}$$

$$\therefore \overrightarrow{r'(t)} = (-2\sin t)\vec{i} + (2\cos t)\vec{j} + 3\vec{k}$$

$$\therefore \overrightarrow{r''(t)} = (-2\cos t)\vec{i} + (-2\sin t)\vec{j} + 0\vec{k}$$

$$\therefore \overrightarrow{r'''(t)} = (2\sin t)\vec{i} + (-2\cos t)\vec{j} + 0\vec{k}$$

here,

~~Principal~~ Unit Tangent Vector $(\vec{T}) = \dfrac{\overrightarrow{r'(t)}}{|\overrightarrow{r'(t)}|}$

$$= \frac{(-2\sin t)\vec{i} + (2\cos t)\vec{j} + 3\vec{k}}{\sqrt{4+9}}$$

$$= \left(\frac{-2}{\sqrt{13}}\sin t\,\vec{i} + \frac{2}{\sqrt{13}}\cos t\,\vec{j} + \frac{3}{\sqrt{13}}\vec{k}\right)$$

$$\overrightarrow{T'(t)} = \left(-\frac{2}{\sqrt{13}}\cos t\,\vec{i} - \frac{2}{\sqrt{13}}\sin t\,\vec{j} + 0\vec{k}\right)$$

So,

Principal unit normal vector $(\vec{N}) = \dfrac{\overrightarrow{T'(t)}}{|\overrightarrow{T'(t)}|}$

$$= \frac{(-2/\sqrt{13})(\cos t\,\vec{i} + \sin t\,\vec{j} + 0\vec{k})}{\sqrt{4/13}}$$

$$\vec{N} = (\cos t\,\vec{i} + \sin t\,\vec{j} + 0\vec{k})$$

Ans

Now,

For Curvature,

$$\vec{r}'(t) \times \vec{r}''(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2\sin t & 2\cos t & 3 \\ -2\cos t & -2\sin t & 0 \end{vmatrix}$$

$$= (6\sin t)\vec{i} + (-6\cos t)\vec{j} + 4\vec{k}$$

And, $|\overrightarrow{r'(t)} \times \overrightarrow{r''(t)}| = \sqrt{36+16}$

$$= \sqrt{52}$$

$$= 2\sqrt{13} \text{ units}$$

Then,

Curvature $(k) = \dfrac{|\overrightarrow{r'(t)} \times \overrightarrow{r''(t)}|}{|r'(t)|^3}$

$$= \frac{2\sqrt{\cancel{13}}}{(\sqrt{13})^{\cancel{3}\,2}}$$

$$\therefore k = \left(\frac{2}{13}\right) \quad \underline{\underline{\text{Ans}}}$$

Now,

For Torsion (τ);

$$\tau = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{|\overrightarrow{r'(t)} \times \overrightarrow{r''(t)}|^2}$$

$$= \frac{\begin{vmatrix} -2\sin t & 2\cos t & 3 \\ -2\cos t & -2\sin t & 0 \\ 2\sin t & -2\cos t & 0 \end{vmatrix}}{(2\sqrt{13})^2}$$

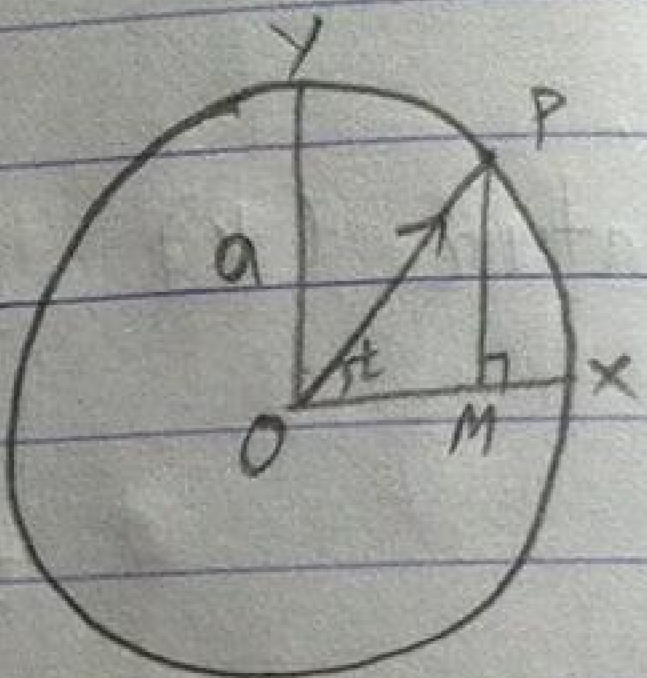
$$= \frac{(-2\sin t)(0) - 2\cos t(0) + 3(4\cos^2 t + 4\sin^2 t)}{4 \times 13}$$

$$= \frac{12}{4 \times 13}$$

$$\tau = \left(\frac{3}{13}\right) \quad \underline{\underline{\text{Ans}}}$$

(Q.N. 4.b)

= solution:



Consider a circle with center at O, and radius 'a' units. let P be any point in the circumference of circle. OX and OY be the coordinate axes. Then, in order to reach P; we have to travel straight along OM horizontally and MP vertically. So;

$$\overrightarrow{OP} = OM\,\vec{i} + MP\,\vec{j} \quad \text{——— (i)}$$

If t be the $\angle POM$ and $\vec{r}$ be $\overrightarrow{OP}$. Then,

$$\vec{r} = OM\,\vec{i} + MP\,\vec{j}$$

or, $\vec{r} = |\overrightarrow{OP}|\cos t\,\vec{i} + |\overrightarrow{OP}|\sin t\,\vec{j}$

$\Rightarrow \vec{r} = (a\cos t\,\vec{i} + a\sin t\,\vec{j})$ ——— (ii)

So,

$$\overrightarrow{r'(t)} = (-a\sin t)\vec{i} + (a\cos t)\vec{j}$$

$$\overrightarrow{r''(t)} = (-a\cos t)\vec{i} + (-a\sin t)\vec{j} + \cancel{0\vec{k}}$$

so,

$$\overrightarrow{r'(t)} \times \overrightarrow{r''(t)} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a\sin t & a\cos t & 0 \\ -a\cos t & -a\sin t & 0 \end{vmatrix}$$

$$= 0\vec{i} + 0\vec{j} + a^2\vec{k}$$

so $$|\overrightarrow{r'(t)} \times \overrightarrow{r''(t)}| = a^2$$

Now,

$$\text{Curvature } (k) = \frac{|\overrightarrow{r'(t)} \times \overrightarrow{r''(t)}|}{|r'(t)|^3}$$

$$= \frac{a^2}{(\sqrt{a^2})^3}$$

$$= \frac{a^2}{a^3}$$

$$= (1/a)$$

So, the curvature of a circle of radius 'a' units is $(1/a)$. Ans

Group-'B'

(Q.N. 5)

= solution,

(a) $Y = x^5 + x$

$\Rightarrow f(x) = (x^5 + x)$

$\therefore f(-x) = (-x)^5 + (-x) = -x^5 - x = -(x^5 + x)$

$\Rightarrow f(-x) = -f(x)$

So, $Y = (x^5 + x)$ is an odd function.

(b) $Y = 1 - x^4$

$\Rightarrow f(x) = (1 - x^4)$

$\therefore f(-x) = 1 - (-x)^4 = (1 - x^4) = f(x)$

$\Rightarrow f(-x) = f(x)$

So, $Y = (1 - x^4)$ is an even function

(c) $Y = 2x - x^2$

$\Rightarrow f(x) = 2x - x^2$

$\therefore f(-x) = 2 \times (-x) - (-x)^2 = (-2x - x^2)$

$\Rightarrow f(-x) \neq f(x)$

So, $Y = (2x - x^2)$ is neither odd nor even function.

(Q.N. 6)

A function $f(x)$ is said to be continuous at a point $x = a$ if and only if :

$$\lim_{x \to a} f(x) = f(a)$$

This definition implicitly requires following conditions for $f(x)$ to be continuous at a :-

(i) $f(a)$ is defined. (i.e. 'a' is in the domain of f)

(ii) $\lim_{x \to a} f(x)$ exists.

(iii) $\lim_{x \to a} f(x) = f(a)$

A function is continuous on an interval if it is continuous at every point in the interval.

= solution,

If $-1 < a < 1$, then using the limit laws, we have --

$$\lim_{x \to a} f(x) = \lim_{x \to a} \left(1 - \sqrt{1 - x^2}\right)$$

$$= 1 - \lim_{x \to a} \sqrt{1 - x^2}$$

$$= 1 - \sqrt{\lim_{x \to a} (1 - x^2)}$$

$$= \left(1 - \sqrt{1 - a^2}\right) = f(a)$$

Similarly,

$\lim_{x \to -1^+} f(x) = 1$ and, $\lim_{x \to 1^-} f(x) = 1$

So, $f(x)$ is continuous in the interval $[-1, 1]$.

(Q.N.7)

= solution,

here,

$$f(x) = (x^3 - x^2 - 6x + 2) \text{ in } [0, 3]$$

(i) Since $f(x)$ is a polynomial function, it is continuous in the closed interval $[0, 3]$.

And,

(ii) $f'(x) = (3x^2 - 2x - 6)$; which exists in the open interval $(0, 3)$.

Now,

(iii) for equality of end points,

$$f(0) = 0 - 0 - 0 + 2 = 2$$

$$f(3) = 3^3 - 3^2 - 6 \times 3 + 2$$

$$= -9 + 27 - 18 + 2$$

$$= 2$$

$\therefore f(0) = f(3)$

So, the conditions for Rolle's theorem are verified in the interval $[0, 3]$ for the function $f(x) = (x^3 - x^2 - 6x + 2)$. Therefore, there exists at least one point in $[0,3]$, where:

$$f'(c) = 0$$

$$\Rightarrow 3x^2 - 2x - 6 = 0$$

or, $3c^2 - 2c - 6 = 0$

$$\Rightarrow c = \frac{2 \pm \sqrt{4+72}}{6}$$

$$c = \frac{2 \pm \sqrt{76}}{6}$$; where

$c \in [0,3]$.

(Q.N.8)

= solution,

$$f(x) = (x^3 - 6x - 5)$$

$$x_0 = 2$$

$$x_3 = ?$$

Now,

The general method for Newton's method is :-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x^3 - 6x - 5}{(3x^2 - 6)}$$

So,

First Approximation $\Rightarrow$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\text{or, } x_1 = 2 - \frac{f(2)}{f'(2)}$$

$$\Rightarrow x_1 = 2 - \frac{8-12-5}{12-6}$$

$$\therefore x_1 = 3.5$$

Now,

Second Approximation :-

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\text{or, } x_2 = 3.5 - \frac{f(3.5)}{f'(3.5)}$$

$$\text{or, } x_2 = 3.5 - \frac{16.875}{30.75}$$

$$\therefore x_2 = 2.951219512$$

Finally,

Third Approximation :-

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.951219512 - \frac{f(2.951219512)}{f'(2.951219512)}$$

$$= 2.951219512 - \frac{2.996909501}{20.12908982}$$

$$\therefore x_3 = 2.802335011$$

Ans

(Q.N.9)

= solution,

Consider $I = \int_{-\infty}^{0} x e^{x} \cdot dx$

The ~~toto~~ integral is improper, since the lower limit yields infinity. So, we consider:-

$$I = \lim_{b \to -\infty^+} \int_{b}^{0} x e^{x} \cdot dx$$

$$= \lim_{b \to -\infty^+} \left[x \int e^{x} \cdot dx - \int \frac{dx}{dx} \left(\int e^{x} \cdot dx \right) \cdot dx \right]_{b}^{0}$$

$$= \lim_{b \to -\infty^+} \left[x e^{x} - \int 1 \cdot e^{x} \cdot dx \right]_{b}^{0}$$

$$= \lim_{b \to -\infty^+} \left[x e^{x} - e^{x} \right]_{b}^{0}$$

$$= \lim_{b \to -\infty^+} \left[0 - b e^{b} - e^{0} + e^{b} \right]$$

$$= \lim_{b \to -\infty^+} \left[e^{b} - b e^{b} - 1 \right]$$

$$= 0 - \lim_{b \to -\infty^+} b e^{b} - 1$$

$$= -1 - \lim_{b \to -\infty^+} b e^{b} \qquad \left[\because \frac{\infty}{\infty} \text{ form} \right]$$

$$= -1 - \lim_{b \to -\infty^{+}} \frac{b}{e^{-b}}$$

Using L-Hospital's rule,

$$I = -1 - \lim_{b \to -\infty^{+}} \frac{1}{-e^{-b}}$$

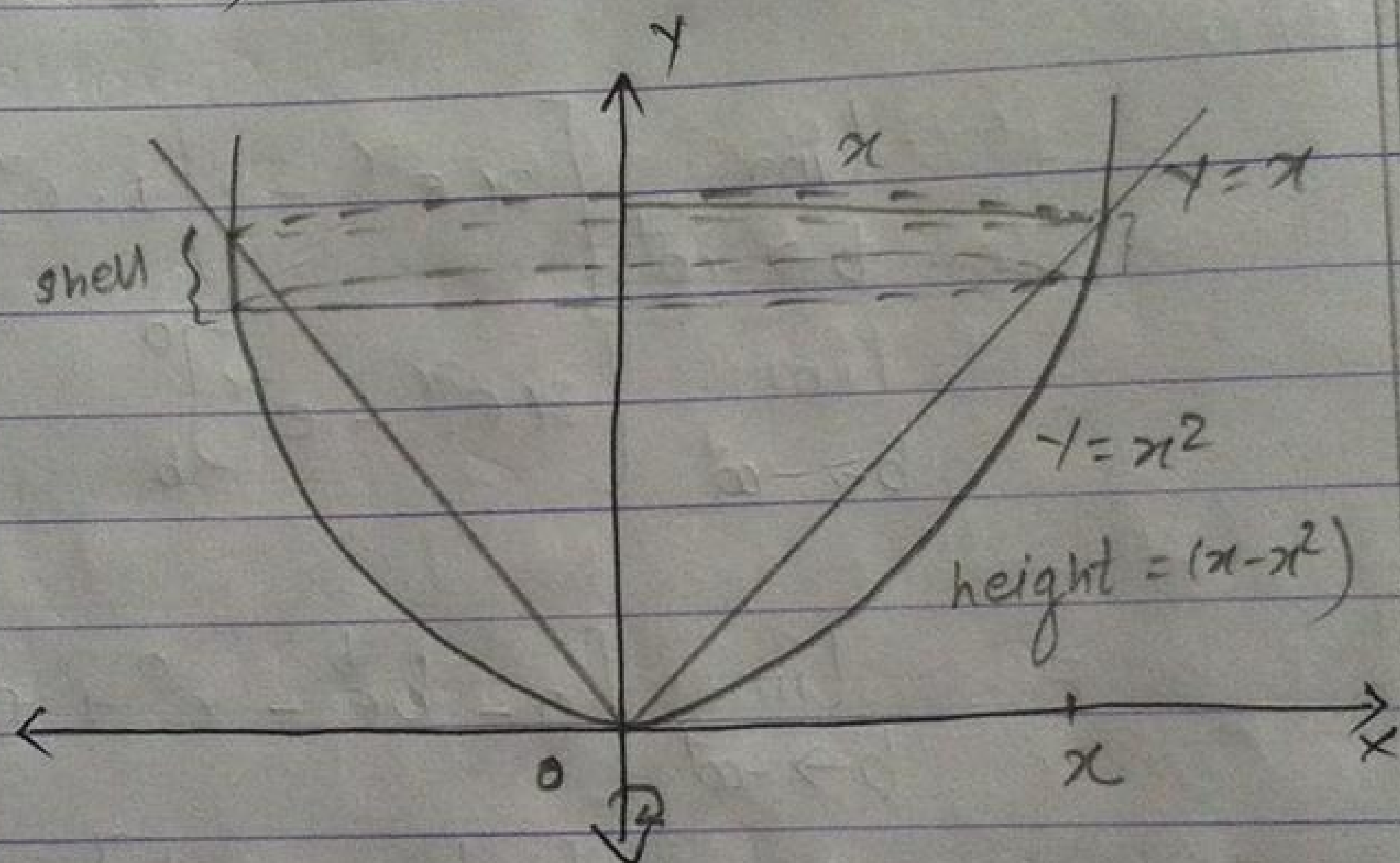
$$= -1 - \frac{1}{-\infty}$$

$$\therefore I = -1$$ Ans

(Q.N. 10)

= solution,

here,



From the sketch above, we see that the typical shell has a radius of 'x' units, circumference of $2\pi x$ units and a height of $f(x) = (x - x^2)$.

Now, solving the equations $Y = x$ and $Y = x^2$, we get -

$$x = x^2$$

or, $x^2 - x = 0$

or, $x(x-1) = 0$

$\Rightarrow \quad x = 0, 1$

So, the two curves meet at $(0,0)$ and $(1,1)$. The limits of integration are $x=0$ and $x=1$ respectively. Then, by shell method, the volume is :-

$$V = \int_0^1 2\pi x \cdot f(x) \cdot dx$$

$$= \int_0^1 2\pi x \cdot (x - x^2) \cdot dx$$

$$= 2\pi \int_0^1 (x^2 - x^3) \cdot dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1$$

$$= 2\pi \left[1/3 - 0 - 1/4 + 0 \right]$$

$$= 2\pi \left[1/3 - 1/4 \right]$$

$$= 2\pi \times 1/12$$

$$= \left(\frac{\pi}{6} \right) \text{ cubic units}$$

Ans

(Q.N. 11)

= solution,

$$y'' + y' - 6y = 0, \quad x > 0, \; y'(0) = 0 \text{ and } y(0) = 1.$$

The given differential equation is a homogeneous second order differential equation. The general solution is it's complementary function y_c.

We know,

For complementary function, y_c;

We take an auxiliary equation of the form $am^2 + bm + c = 0$; as:-

$$m^2 + m - 6 = 0$$

or, $(m+3)(m-2) = 0$

$\Rightarrow m = -3, 2$

$\Rightarrow m_1 = -3, \quad m_2 = 2$

Hence,

Complementary function $\Rightarrow$

$$y_c = (c_1 e^{-3x} + c_2 e^{2x}) \text{ ——— (i)}$$

Where, c_1 and c_2 are arbitrary constants. Then,

As $y(0) = 1$; we have;

$$1 = c_1 e^{-3\times 0} + c_2 e^{2\times 0}$$

or, $1 = (c_1 + c_2)$

$\Rightarrow c_1 + c_2 = 1$ ——— (ii)

Also, $y'(0) = 0$

so, $$\frac{d(c_1 e^{-3x} + c_2 e^{2x})}{dx} = 0$$

or, $$-3c_1 e^{-3x} + 2c_2 e^{2x} = 0$$

$$\Rightarrow -3 \times c_1 e^{0} + 2c_2 e^{0} = 0$$

or, $$2c_2 = 3c_1$$

$$\Rightarrow c_2 = \left(\frac{3c_1}{2}\right)$$

Then, eq? (ii) yields --

$$c_1 + \frac{3c_1}{2} = 1$$

or, $$5c_1 = 2$$

$$\Rightarrow c_1 = \left(\frac{2}{5}\right)$$

$$\therefore c_2 = \frac{3}{2} \times 2/5 = (3/5)$$

Therefore, eq? (i) can be written as:-

$$y = \left(\frac{2}{5} e^{-3x} + \frac{3}{5} e^{2x}\right)$$

or, $$y = \frac{1}{5}(2e^{-3x} + 3e^{2x});$$

which is the required solution;

(Q.N.12)

= solution,

we have:

$\overrightarrow{r(t)} = \cancel{(1+2t)}\ (t, t^2, t^3)$

$\therefore \overrightarrow{r'(t)} = (1, 2t, 3t^2)$

$\therefore \overrightarrow{r''(t)} = (0, 2, 6t)$

Now,

$$\left(\overrightarrow{r'(t)} \times \overrightarrow{r''(t)}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix}$$

$$= (12t^2 - 6t^2)\vec{i} + (-6t)\vec{j} + 2\vec{k}$$

$$= \left((6t^2)\vec{i} - (6t)\vec{j} + 2\vec{k}\right)$$

$$\therefore \left|\overrightarrow{r'(t)} \times \overrightarrow{r''(t)}\right| = \sqrt{36t^4 + 36t^2 + 4}$$

$$= \left(2\sqrt{9t^4 + 9t^2 + 1}\right)$$

$$\left|\overrightarrow{r'(t)}\right| = \left(\sqrt{1 + 4t^2 + 9t^4}\right)$$

Now,

$$\text{Curvature } (k) = \frac{\left|\overrightarrow{r'(t)} \times \overrightarrow{r''(t)}\right|}{\left|\overrightarrow{r'(t)}\right|^3}$$

$$\therefore k = \frac{2\sqrt{9t^4 + 9t^2 + 1}}{(1 + 4t^2 + 9t^4)^{3/2}}$$ at general point.

Now, at origin, i.e. (0,0,0);

$$\text{Curvature } (k) = \frac{2 \times \sqrt{0+0+1}}{(1+0+0)^{3/2}}$$

$$\therefore k(0) = 2$$

Ans

(Q.N.13)

= solution,

here;

$$S_n = \sum_{n=0}^{\infty} \frac{1}{1+n^2}$$

here,

$$a_n = \frac{1}{(1+n^2)}$$

Let, the auxiliary series be represented by b_n. Then,

$$b_n = \frac{1}{(n^2)} \qquad \text{so, } a_n < b_n$$

Now,

$b_n = \left(\frac{1}{n^2}\right)$ is convergent by p-series test; since the degree of n is greater than 1.

Hence, by direct comparison test, a_n is also convergent.

(Q.N. 14)

solution,

$$\iint_R f(x,y) \cdot dA \text{ for } r = f(x,y) = (x^2y - 2xy),$$

$$R: -2 \leq x \leq 0, 0 \leq y \leq 3.$$

Now, by Fubini's theorem;

$$\iint_R f(x,y) \cdot dA = \int_0^3 \int_{-2}^0 (x^2y - 2xy) \cdot dx \cdot dy$$

$$= \int_0^3 \left[\frac{yx^3}{3} - 2y\frac{x^2}{2} \right]_{-2}^0 \cdot dy$$

$$= \int_0^3 \left[0 + \frac{8y}{3} - 0 + 4y \right] \cdot dy$$

$$= \int_0^3 \left[\frac{20y}{3} \right] \cdot dy$$

$$= \frac{20}{3} \times \left[\frac{y^2}{2} \right]_0^3$$

$$= \frac{\cancel{20}^{10}}{\cancel{3}_{1}} \times \left[\frac{\cancel{9}^{3}}{\cancel{2}_{1}} \right]$$

$= 30$ ~~cubic~~ sq. units. Ans

(Q.N.15)

= solution,

$$f(x,y) = (x^3 + x^2y^3 - 2y^2) \quad \text{at } (2,1)$$

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} = ?$$

we know,

Partial derivative of f with respect to x is :-

$$\frac{\partial f}{\partial x} = (3x^2 + 2xy^3 - 0)$$

At (2,1), $\frac{\partial f}{\partial x} = 3\times 2^2 + 2\times 2\times 1 = (12+4) = 16$

Partial derivative of f with respect to y is :-

$$\frac{\partial f}{\partial y} = (3x^2y^2 - 4y)$$

At (2,1), $\frac{\partial f}{\partial y} = (3\times 4\times 1 - 4\times 1) = 8$ Ans