[1st Semester]

[Bsc.CSIT Physics Numerical Solution]

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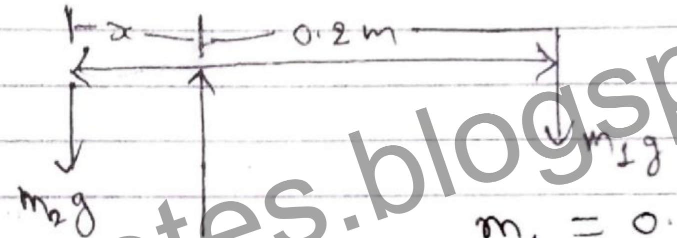
Unit 1: Rotational Dynamics and oscillatory motion.

Q.1) A balance scale consisting of a weightless rod has a mass of 0.1 kg on the right side 0.2 m from the pivot point. (a) How far from the pivot point on the left must 0.4 kg be placed so that a balance is acheived? (b) If the 0.4 kg mass is suddenly removed, what is the instantaneous rotational acceleration of the rod? (c) what is the instantaneous tangential acceleration of the mass 0.1 kg when the 0.4 kg mass is removed?

$\Rightarrow$



$m_1 = 0.1$ kg
$m_2 = 0.4$ kg

(a) when a balance is achieved $\alpha = 0$

So,

$$\sum \tau = 0$$

$$\therefore m_2 g \times x \times \sin 90 - m_1 g \times 0.2 \times \sin 90 = 0$$

$$\therefore x = 0.05 \text{ m}$$

(b)

$$\alpha = \frac{\tau}{I} = \frac{m_1 g \times 0.2 \times \sin 90}{m_1 \cdot (0.2)^2}$$

$$\alpha = 4g \text{ rad/s}^2 \text{ (clockwise)}$$

2

(c) $a_T = r\alpha$

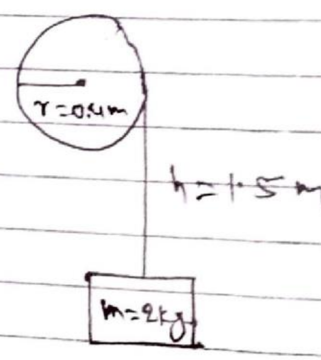
$= 0.2 \times 49$

$= 9.8\ m/s^2$

8.2 > A large wheel of radius 0.4 m and moment of inertia 1.2 kg-m², pivoted at the center, is free to rotate without friction. A rope is wound around it and a 2 kg weight is attached to the rope. When the weight descends 1.5 m from its starting position. (a) what is its downward velocity? (b) what is the rotational velocity?

=>

from conservation of energy



P.E of weight = K.E of weight + K.E of wheel

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 \quad \because v = r\omega$$

on solving

$$v = \sqrt{\frac{2mgh}{\left(m + \frac{I}{r^2}\right)}} = \sqrt{\frac{2\times2\times9.8\times1.5}{\left(2+\frac{1.2}{0.4^2}\right)}}$$

$$v = 2.5\ m/s$$

(b) $v = r\omega$

$$\omega = \frac{v}{r}$$

$$\omega = \frac{2.5}{0.4} = 6.2 \text{ rad/sec}$$

8.4) suppose the body of an ice skater has a moment of inertia $I = 4 \text{ kg m}^2$ and her arms have a mass of 5 kg each with the center of mass at 0.4 m from her body. She starts to turn at 0.5 rev/sec on the point of her skate with her arms outstretched. She then pulls her arms inward so that their center of mass is at the axis of her body, r = 0. what will be her speed of rotation?

⇒

$$I_0 \omega_0 = I_f \omega_f$$

$$(I_{body} + I_{arms}) \omega_0 = I_{body} \omega_f$$

$$(I_{body} + 2mr^2) \omega_0 = I_{body} \omega_f$$

$$\therefore \omega_f = \frac{(I_{body} + 2mr^2) \omega_0}{I_{body}}$$

$$= \frac{[4 + 2 \times 5 \times (0.4)^2]\ 0.5}{4}$$

$$= 0.7 \text{ rev/sec}$$

10.2) A given spring stretches 0.1 m when a force of 20N pulls on it. A 2-kg block attached to it on a frictionless surface ~~as in~~ is pulled to the right 0.2m and released. (a) what is the frequency of oscillation of the block? (b) what is its velocity at the midpoint? (c) what is its acceleration at either end (d) what is the velocity and acceleration when $x = 0.12m$ on the block's first passing this point?

=)

$$\text{spring constant } (k) = \frac{F}{x}$$
$$= \frac{20}{0.1}$$
$$= 200 \text{ N/m}$$

(a)

$$\omega = 2\pi f$$
$$\sqrt{\frac{k}{m}} = 2\pi f$$
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{200}{2}} = 1.6$$

(b) amplitude (a) = 0.2m

$$V_{max} = \pm a\omega = \pm 0.2 \times \sqrt{\frac{k}{m}}$$
$$= \pm 0.2 \times \sqrt{\frac{200}{2}}$$
$$= \pm 0.2 \times 10 = \pm 2 \text{ m/s}$$

(c) $a_{max} = \pm A\omega^2$

$= \pm 0.2 \times (10)^2$

$= \pm 20 m/sec^2$

(d) $v = \omega\sqrt{a^2 - x^2} = 2\pi \times 1.6 \times \sqrt{0.2^2 - 0.12^2}$

$= 1.6 m/s$

$a = \omega^2 x = (2\pi \times 1.6)^2 \times 0.12 = 12.12 m/sec^2$

10.3> A spring mass system consist of a block of mass 2 kg and a spring with spring constant 200 N/m. The block is released from rest at a position of $x_1 = 0.2m$ (a) what is the velocity at $x_2 = 0.1m$ on other side of mean position? (b) what is its acceleration at this point?

⇒ (a) Here, $x_1 = 0.2m$ $v_1 = 0$ $x_2 = 0.1m$

$v_2 = ?$

from conservation of energy

$\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}kx_2^2 + \frac{1}{2}mv_2^2$

on solving

$v_2 = \sqrt{\frac{k(x_1^2 - x_2^2)}{m}} = \sqrt{\frac{200(0.2^2 - 0.1^2)}{2}}$

$= 1.73 m/sec$

(b) from Newton's second law

$$F = ma$$

$$kx = ma$$

$$a = \frac{kx}{m} = \frac{200 \times 0.1}{2} = 10 \, m/sec^2$$

8.1) A bicycle wheel of mass 2 kg and radius 0.32 m is spinning freely on its axle at 2 rev/sec. when you place your hand against the tire the wheel decelerat uniformly and comes to a stop in 8 sec. what was th torque of your hand against the wheel?

=>

Here,
mass (m) = 2kg
radius (r) = 0.32m
~~frequency (f_0) = 2rev/sec~~ X ω_0 = 2rev/sec.
time (t) = 8
Torque (τ) = ?

$$\omega_0 = 2\pi f_0 = 2\pi \cdot 2 = 4\pi \, rad/sec$$

$$I = mr^2 = 2 \cdot (0.32)^2 = 0.18 \, kg m^2$$

Now,

$$\omega = \omega_0 - \alpha t$$

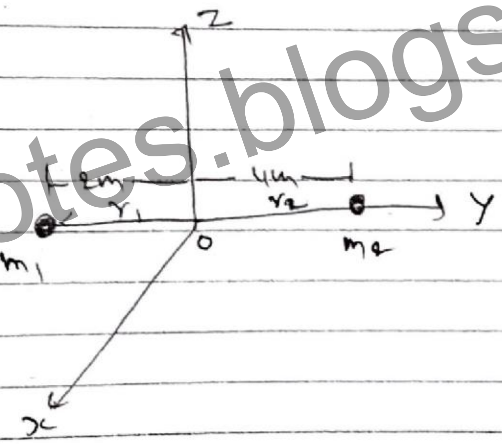
or $0 = 4\pi - \frac{\tau}{I} t$ $\quad \because \omega = 0$

or $\frac{\tau}{I} t = 4\pi$

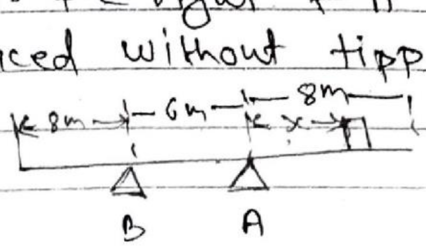
or $\frac{\tau}{0.18} \times 8 = 4T$

$\therefore \tau = 0.28$ Nm

Q.2) Two masses, $m_1 = 1$ kg and $m_2 = 5$ kg, are connected by a rigid rod of negligible weight. The system pivoted about O. the gravitational forces act in negative z direction. (a) Express the position vectors and the forces on the masses in terms of unit vector and calculate the torque on the system. (b) what is the angular acceleration of the system at the instant.



8.7) A uniform wooden board of mass 20kg rests on two supports as shown in fig. A 30 kg steel block is placed to the right of support A. How far to the right of A can the steel block be placed without tipping the board?



8.18) A children's merry-go-round of radius 4 m and mass 100 kg has an 80 kg man standing at the rim. The merry-go-round coasts on a frictionless bearing at 0.2 rev/sec. The man walks inward 2m toward the center. What is the new rotational speed of merry-go-round? What is the source of this energy? (The moment of inertia of a solid disk is $I = \frac{1}{2}mr^2$)

⇒ By the conservation of angular momentum

$I_i \omega_i = I_f \omega_f$

$I_i \cdot 2\pi f_i = I_f \cdot 2\pi f_f$ — (1)

Now,

I_i = moment of inertia of merry-go-round + moment of inertia of man at rim

$I_i = \frac{1}{2} \cdot 100 \cdot 4^2 + 80 \cdot 4^2$

$I_i = 50 \times 16 + 80 \times 16$

$I_i = 800 + 1280$

$\therefore I_i = 2080 \ kg\,m^2$

Again,

I_f = moment of inertia of merry-go-round + moment of Inertia of man at 2cm from center.

$I_f = 800 + 80 \times 2^2$

$I_f = 800 + 320$

$I_f = 1120 \ kg\,m^2$

from eqⁿ (I)

$$2080 \times \cancel{2\pi} \times 0.2 = 1120 \times \cancel{2\pi} \times f_f$$

$\therefore f_f = 0.37$ rev/sec

<u>10.5)</u> An oscillating block of mass 250 g takes 0.15 sec to move between the end points of the motion, which are 40 cm apart. (a) what is the frequency of the motion? (b) what is the amplitude of the motion? (c) what is force constant of the spring?

⇒

Given

mass (m) = 250 g = 250×10^{-3} kg

Time period (T) = 0.15 sec

distance between end point = 40 cm = 0.4 m

(a) frequency

$$f = \frac{1}{T} = \frac{1}{0.15} = 6.67 \text{ Hz}$$

(b) Amplitude = 0.4 m

(c) $T = 2\pi\sqrt{\frac{m}{k}}$

$0.15 = 2\pi\sqrt{\frac{250 \times 10^{-3}}{k}}$ ⇒ $k = 438.65$ N/m

Q.13) A block is oscillating with an amplitude of 20 cm. The spring constant is 150 N/m. (a) What is the energy of the system? (b) When the displacement is 5 cm, what is the kinetic energy of the block and the potential energy of the spring?

⇒ Given,

amplitude (A) = 20 cm = 0.2 m

spring constant (K) = 150 N/m

(a) Total energy (E) $= \frac{1}{2} K A^2$

$= \frac{1}{2} \cdot 150 \cdot (0.2)^2$

$= 3\ J$

(b) displacement (x) = 5 cm

let, $x = A \sin(\omega t + \phi)$

$5 = 20 \sin(\omega t + \phi)$

$\sin(\omega t + \phi) = \frac{5}{20}$

$\sin^2(\omega t + \phi) = \left(\frac{5}{20}\right)^2$ and $\cos^2(\omega t + \phi) = 1 - \left(\frac{5}{20}\right)^2$

$= \frac{1}{16}$ $\qquad = 1 - \frac{25}{400}$

$= \frac{15}{16}$

Now,

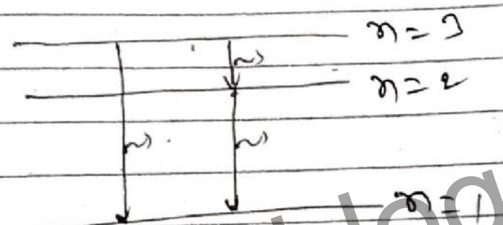
$P.E = \frac{1}{2} K A^2 \sin^2(\omega t + \phi) = 3 \times \frac{1}{16} = 0.1875\ J$

$K.E = \frac{1}{2} K A^2 \cos^2(\omega t + \phi) = 3 \times \frac{15}{16} = 2.8125$

Fundamental of Atomic Theory

Q.2> After being excited, the electron of hydrogen atom eventually falls back to the ground state. This can take place in one jump or in a series of jumps, the electron falling into lower excited states before it ends up in the ground state. Consider a hydrogen atom that has been raised to the second excited state, that is $n=3$. Calculate the different photon energies that may be emitted as the atom returns to the ground state.

Soln.



for $n=3$ to $n=1$

$$h\nu_{31} = E_3 - E_1 = E_0\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 13.56\left(1 - \frac{1}{9}\right)$$

$$= 12.05\,eV$$

for $n=3$ to $n=2$

$$h\nu_{32} = E_0\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.56\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$= 1.88\,eV$$

for $n = 2$ to $n = 1$

$$h\nu_{21} = E_0\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= 13.56\left(1 - \frac{1}{4}\right)$$

$$= 10.17 \text{ eV}$$

19.1) A beam of monochromatic neutron is incident on a KCl crystal with lattice spacing of 3.14 Å. The first order diffraction maximum is observed when the angle θ between the incident beam and the atomic planes is 37°. What is the kinetic energy of the neutrons?

⇒ $d = 3.14$ Å

$n = 1$

$\theta = 37°$

from Bragg's condition.

$$2d\sin\theta = n\lambda$$

or, $2 \times 3.14 \times \sin 37 = \lambda$

$\therefore \lambda = 3.78$ Å

from de Broglie's hypothesis

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{3.78 \times 10^{-10}} = 1.75 \times 10^{-24} \text{ kg m/sec}$$

$$K.E = \frac{P^2}{2m} = \frac{(1.75 \times 10^{-24})^2}{2 \times 1.67 \times 10^{-27}} = 9.21 \times 10^{-22}\ J$$

$$\therefore K.E = 5.75 \times 10^{-3}\ eV$$

<u>18.1)</u> calculate the shortest and the longest wavelength of the Balmer series of hydrogen.

$\Rightarrow$

for Balmer series

$n_1 = 2$

<u>longest wavelength</u>

$n_2 = 3$

$$\frac{1}{\lambda_L} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\text{or, } \frac{1}{\lambda_L} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\therefore \lambda_L = 6.56 \times 10^{-7}\ m$$

<u>shortest wavelength</u>

$n_2 = \infty$

$$\text{or, } \frac{1}{\lambda_S} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$\text{or, } \frac{1}{\lambda_S} = 1.097 \times 10^7 \cdot \frac{1}{4}$$

8.2) What are (a) the energy (b) the momentum and (c) the wavelength of the photon that is emitted when a hydrogen atom undergoes a transition from the state $n = 3$ to $n = 1$?

⇒

Now, $$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^{7}\left(\frac{1}{1} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^{7}\left(1 - \frac{1}{9}\right)$$

$$\lambda = 1.0255 \times 10^{-7} \text{ m}$$

(c) $$\lambda = 1026 \, Å$$

(a) $$\text{energy} = \frac{hc}{\lambda} = \frac{6.624 \times 10^{-34} \times 3 \times 10^{8}}{1.0255 \times 10^{-7}}$$

$$= 1.9378 \times 10^{-18} \text{ J}$$

$$= 12.1 \text{ eV}$$

(b) $$\text{momentum}(P) = \frac{h}{\lambda} = \frac{6.624 \times 10^{-34}}{1.0255 \times 10^{-7}}$$

$$= 6.45 \times 10^{-27} \text{ kg m/sec}$$

18.3) The shortest wavelength of the paschen series from hydrogen is 8204 Å. From this fact, calculate the Rydberg constant.

⟹

for paschen series

$n_1 = 3$

for shortest wavelength

$n_2 = \infty$

$\lambda_s = 8204 \text{Å} = 8204 \times 10^{-10} m$

$$\frac{1}{\lambda_s} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)$$

or, $\frac{1}{8204 \times 10^{-10}} = R \cdot \frac{1}{9}$

or, $R = \frac{9}{8204 \times 10^{-10}}$

$R = 10970258.41$

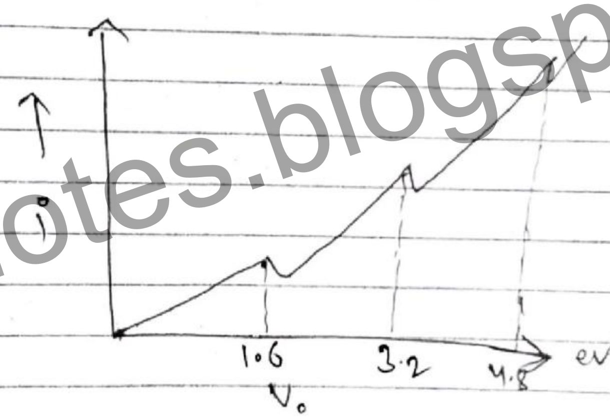
$R = 1.097 \times 10^{7} m^{-1}$

18.19) The ground and the first excited-state energies of potassium atoms are −4.3eV and −2.7eV, respectively. If we use potassium vapour in the the Frank-Hertz experiment, at what voltages would we see drops in the plot of current versus voltage?

⇒)

1^{st} excited state energy = −2.7eV
ground state energy = −4.3eV

The energy difference between 1^{st} excited State and ground state is = 1.6eV



19.2) The de Broglie wavelength of a proton is 10^{-13}m
(i) what is the speed of proton? Through what potential difference must the proton be accelerated to acquire such a speed?

⇒)

=)

$$\lambda = \frac{h}{mv}$$

$$v = \frac{h}{m_p \lambda} = \frac{6.62 \times 10^{-34}}{1.67 \times 10^{-27} \times 10^{-13}}$$

$$= 3.957 \times 10^{6} \ m/sec$$

(b)

$$E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 1.67 \times 10^{-27} \times (3.957 \times 10^{6})^2$$

$$= 81749.87 \ ev$$

$$= 81.74987 \ Kev$$

19.7) An α particle is emitted from a nucleus with an energy of 5Mev (5×10^6 ev). calculate the wavelength of α particle with such energy and compare it with the size of the emitting nucleus that has a radius of 8×10^{-15} m.

=)

mass of α particle (m) = 6.6×10^{-27} kg

energy (E) = 5Mev = 5×10^6 ev

Now,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 6.6 \times 10^{-27} \times 1.6 \times 10^{-19} \times 5 \times 10^{6}}}$$

$$\lambda = 6.44 \times 10^{-15} \, m$$

19.11) In neutron spectroscopy a beam of mono-energetic neutrons is obtain by reflecting ~~by~~ reactor neutrons from a beryllium crystal. If the separation between the atomic planes of the beryllium crystal is 0.732 Å, what is the angle between the incident neutron beam and the ~~will~~ atomic planes that will yield a monochromatic beam of neutrons of wavelength 0.1 Å?

→)

$d = 0.732$ Å

$\lambda = 0.1$ Å

$\theta = ?$

from Bragg's condition for $n = 1$

$$2d \sin\theta = n\lambda$$

$$\sin\theta = \frac{\lambda}{2d}$$

$$\sin\theta = \frac{0.1}{2\times 0.732}$$

$$\theta = 3.9^{\circ}$$

19.16) The x-component of velocity for a small particle of mass 10^{-6}g is measured with an uncertainty of 10^{-6} m/s. (a) what is the uncertainty in the x-coordinate of the particle? (b) Repeat the calculation for an electron assuming that the uncertainty in its velocity is also 10^{-6} m/sec.

⟹

(a) $m = 10^{-6} = 10^{-6}\times 10^{-3} = 10^{-9}$ kg

$\Delta v = 10^{-6}$ m/s

$\Delta x = ?$

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$$

$$\Delta x \cdot m \cdot \Delta v \geq \frac{h}{2\pi}$$

$$\Delta x \times 10^{-9} \times 10^{-6} \geq \frac{6.62\times 10^{-34}}{2\pi}$$

$$\Delta x \geq \frac{6.62\times 10^{-34}}{2\pi\times 10^{-9}\times 10^{-6}} = 1.05\times 10^{-19}\ \text{m}$$

$$\Delta x = 1.05\times 10^{-19}\ \text{m}$$

(b) $m = 9.1 \times 10^{-31}$ kg

$\Delta v = 10^{-6}$ m/sec

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$$

$$\Delta x \cdot m\,\Delta v \geq \frac{h}{2\pi}$$

$$\Delta x \times 9.1 \times 10^{-31} \times 10^{-6} \geq \frac{6.6 \times 10^{-34}}{2\pi}$$

$$\Delta x \geq \frac{6.6 \times 10^{-34}}{2\pi \times 9.1 \times 10^{-31} \times 10^{-6}}$$

$$\Delta x = 115\,m$$

19.19) The uncertainty in the position of a particle is equal to the de-Broglie wavelength of the particle. Calculate the ~~velocity of~~ uncertainty in the velocity of the particle in terms of the velocity of the de Broglie wave associated with the particle.

=> According to question

$$\Delta x = \frac{h}{mv}$$

Now, by uncertainty relation

$$\Delta p \cdot \Delta x \geq \frac{h}{2\pi}$$

$$m\Delta V \cdot \Delta x \sim \frac{h}{2\pi}$$

$$\cancel{m}\Delta V \cdot \frac{h}{\cancel{m}V} \sim \frac{h}{2\pi}$$

$$\Delta V = \frac{V}{2\pi}$$

unit 4 :- Methods of quantum mechanics.

20.1) consider a particle in the ground state, that is, one represented by $\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)e^{\frac{-iE_0 t}{\hbar}}$

what is (a) average position
(b) average momentum
(c) average Energy.

$\Rightarrow$

$$\psi = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)e^{\frac{-iE_0 t}{\hbar}}$$

$$\psi^* = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)e^{\frac{iE_0 t}{\hbar}}$$

(a) The average value of position

$$\langle x \rangle = \int_0^a \psi^* x \psi \, dx$$

$$= \frac{2}{a} \int_0^a \sin^2\left(\frac{\pi}{a}x\right) x \, dx$$

$$= \frac{2}{a} \frac{a^2}{\pi^2} \int_0^a \sin^2\left(\frac{\pi x}{a}\right)\left(\frac{\pi x}{a}\right)\left(\frac{\pi dx}{a}\right)$$

$$= \frac{2a}{\pi^2}\left[\frac{1}{4}\left(\frac{\pi x}{a}\right)^2 - \frac{\pi x \sin\left(\frac{2\pi x}{a}\right)}{4a} - \frac{\cos\left(\frac{2\pi x}{a}\right)}{8}\right]_0^a$$

$$= \frac{2a}{\pi^2}\left[\left\{\left(\frac{\pi^2}{4} - 0 - \frac{1}{8}\right) - \left(0 - 0 - \frac{1}{8}\right)\right\}\right]$$

$$= \frac{a}{2}$$

(b) The average value of momentum

$$\langle p \rangle = \int_0^a \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \psi \, dx$$

$$= \frac{2}{a}\int_0^a \left(\sin\frac{\pi x}{a}\right)\left(-i\hbar\frac{\partial}{\partial x}\right)\left(\sin\frac{\pi x}{a}\right) dx$$

$$= \frac{2}{a}\int_0^a \left(\sin\frac{\pi x}{a}\right)\left(-i\hbar\frac{\pi}{a}\cos\frac{\pi x}{a}\right) dx$$

$$= \frac{2}{a}(-i\hbar)\int_0^a \left(\sin\frac{\pi x}{a}\right)\left(\cos\frac{\pi x}{a}\right)\left(\frac{\pi}{a}\, dx\right)$$

$$= \frac{2}{a}(-i\hbar)\left.\frac{\sin^2\frac{\pi x}{a}}{2}\right|_0^a$$

$$\langle p\rangle = 0$$

(c) average value of the energy ~~can be~~ is

$$\langle E\rangle = \int_0^a \psi^*\left(-i\hbar\frac{\partial}{\partial t}\right)\psi\, dx$$

$$\psi = \chi(x)\, e^{-\frac{iE_0 t}{\hbar}}$$

$$\psi^* = \chi(x)\, e^{\frac{iE_0 t}{\hbar}}$$

where $\chi(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{\pi x}{a}\right)$

$$\langle E \rangle = \int_0^a \psi^* \left(-i\hbar \frac{\partial}{\partial t}\right) \chi(x)\, e^{\frac{-iE_0 t}{\hbar}}\, dx$$

$$= \int_0^a \psi^* (i\hbar)\left(-i\frac{E_0}{\hbar}\right) \chi(x)\, e^{\frac{-iE_0 t}{\hbar}}\, dx$$

$$= \int_0^a E_0\, \psi^* \psi\, dx$$

$$\langle E \rangle = E_0$$

21.2) A beam of hydrogen atoms is used in a Stern - Gerlach experiment. The atoms emerge from the oven with a velocity $v = 10^4$ m/sec. They enter a region 20 cm long where there is a magnetic field gradient $\frac{dB}{dz} = 3\times10^4$ T/m. The field gradient is perpendicular to the incident velocity of atoms. The mass of the hydrogen atom is 1.67×10^{-27} kg. What is the separation of the two components of the beam as they emerg from the magnet?

=) we have,

$$F_z = \pm \frac{|e|\hbar}{2m} \frac{dB}{dz}$$

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$$m_{atom} a_z = \frac{|e| \hbar \frac{dB}{dz}}{2m}$$

$$a_z = \frac{|e| \hbar \frac{dB}{dz}}{2m \cdot m_{atom}}$$

$$= \frac{1.60 \times 10^{-19} \times 1.05 \times 10^{-34} \times 3 \times 10^{4}}{2 \times 9.1 \times 10^{-31} \times 1.67 \times 10^{-27}}$$

$$= 1.65 \times 10^{8} \, m/sec^2$$

The deflection of each component in the direction of the force will be

$$\Delta z = \frac{1}{2} a_z t^2 \quad — (1)$$

$$\cancel{\Delta z = \frac{1}{2} \cdot 1.65 \times 10^{8} \times t^2}$$

where t is the time that the atoms spend in the magnet

$$t = \frac{\text{distance of magnet}}{\text{incident velocity of the atoms}}$$

$$t = \frac{0.2}{10^4} = 2 \times 10^{-5} \text{ sec}$$

$\therefore$ from (1)

$$\Delta z = \frac{1}{2} \times 1.65 \times 10^{8} \times 4 \times 10^{-10}$$

$$= 3.3 \times 10^{-2} m$$

$$= 3.3 \text{ cm}$$

20.1) Show by direct substitution into the time-dependent Schrodinger equation for the free particle; that $\psi(x,t) = A\cos(kx - \omega t)$ is not a solution.

$\Rightarrow$

Time dependent Schrodinger wave equation for free particle.

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar\frac{\partial \psi}{\partial t} \longrightarrow (1)$$

Now,

$$\frac{\partial \psi}{\partial x} = -Ak\sin(kx - \omega t)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -Ak^2\cos(kx - \omega t) \quad — (2)$$

and $$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial t} A\cos(kx - \omega t) = +A\omega\sin(kx - \omega t)$$

— (3)

substituting eqⁿ ② and ③ in ①

$$\frac{\hbar^2 A k^2}{2m} \cos(kx-\omega t) = i\hbar A\omega \sin(kx-\omega t)$$

Because the sine and cosine functions are equal only for certain angles (ie 45°) i.e. this cannot be satisfied for all x's and t's.

<u>20.2</u>) show by direct substitution that the wavefunction $\psi(x,t) = A\cos kx\, e^{-i\omega t}$ satisfies the time-dependent schrodinger eqⁿ for the free particle.

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar \frac{\partial \psi}{\partial t} \quad \text{— ①}$$

Now,

$$\frac{\partial \psi}{\partial x} = \frac{\partial A\cos kx \cdot e^{-i\omega t}}{\partial x}$$

$$= -kA e^{-i\omega t} \cdot \sin kx$$

$$\frac{\partial^2 \psi}{\partial x^2} = -k^2 A e^{-i\omega t} \cos kx \quad \text{— ②}$$

$$\frac{\partial \psi}{\partial t} = A \cos kx \frac{\partial e^{-i\omega t}}{\partial t}$$

$$= A \cos kx \cdot e^{-i\omega t} \times -i\omega$$

$$\frac{\partial \psi}{\partial t} = -i\omega A \cos kx \, e^{-i\omega t} \quad \text{--- (3)}$$

put eqn (2) and (3) in (1)

$$\frac{\hbar^2 k^2 \cancel{A}}{2m} \cancel{e^{-i\omega t}} \cancel{\cos kx} = i\hbar \times -i\omega \cancel{A \cos kx} \cancel{e^{-i\omega t}}$$

$$\frac{\hbar^2 k^2}{2m} = \hbar\omega \quad \text{--- (4)}$$

$$\cancel{\frac{\hbar^2 k^2}{2m} = \hbar\omega}$$

It means that $\psi(x,t) = A \cos kx \, e^{-i\omega t}$ is a solution of eqn (1) ~~for~~ this relation of constant.

let's check eqn (4) is consistent with de Broglie hypothesis.

$$E = \frac{p^2}{2m} \qquad E = \hbar\omega$$

$$p = \hbar k$$

So, $\frac{\hbar^2 k^2}{2m} = \hbar\omega$

which is same.

Q.2) Explain why the following eigenfunctions are not acceptable solutions of the Schrodinger equation.

(a) $\chi(x) = 0$ for $x \le 0$

$\chi(x) = A\cos kx$ for $x \ge 0$

(b) $\chi(x) = A\frac{e^{ikx}}{x}$

(c) $\chi(x) = A\tan kx$

$\Rightarrow$

20.12) What is the probability of finding a particle in a well of width a at a position $\frac{a}{4}$ from the wall if $n=1$, if $n=2$, if $n=3$. Use the normalized wave function $\psi(x,t) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} e^{-\frac{iEt}{\hbar}}$

$\Rightarrow$ Find $P\left(\frac{a}{4}\right) = \int_0^{a/4} \psi^* \psi \, dx$ and put $n=1$, $n=2$ and $n=3$

21.3) ~~In the Bohr model of the hydrogen atom, the electron is assumed to move in circular orbits around the proton, that is, the motion takes place in a plane that we can call the x-y plain.~~

21.6 How many atomic ~~states~~ are there in Hydrogen with $n=3$? (b) How they are distributed among the subshells?

label each state with the appropriate set of quantum numbers n, l, m_l, m_s.

(c) Show that the number of state in a shell, that is, states having the same number(n), ~~of states~~ is given by $2n^2$.

(a) Total number of atomic states $= n^2 = 9$ (when

electron spin is ignored)

when spin is included then $= 2n^2 = \underline{18}$ 34

(b)

$n = 3$

$l = 0, 1, 2$

$m_l = 0, \pm 1, \pm 2$

$m_s = +\frac{1}{2}$ or $-\frac{1}{2}$

(c) for a given n, $0 \le l \le (n-1)$. Since for each l the number of possible state is $2(2l+1)$, the number of possible states for each n is

$$\sum_{l=0}^{n-1} 2(2l+1) = 4\sum_{l=0}^{n-1} l + \sum_{l=0}^{n-1}$$

$$= 4\left(\frac{n(n-1)}{2}\right) + 2n$$

$$= 2n^2 - \cancel{2n} + \cancel{2n}$$

$$= 2n^2$$

Numerical problems and their solutions for B.Sc. CSIT first year

Unit-2 Electric and magnetic fields

Example 14.1 A charge $q_1 = 3\times10^{-6}$ C is located at the origin the x-axis. A second charge $q_2 = -5\times10^{-6}$ C is also on the x- 4m from the origin in the +ve x-direction. (a) Calculate the ele field at the midpoint P of the line joining the two charges. (b) At what point P' on that line is the resultant field zero?

Soln:- Given, $q_1 = 3\times10^{-6}$ C, $q_2 = -5\times10^{-6}$ C and $r = 4$ m

(a) Net electric field at the midpoint (E) = ?

In this case, E = Electric field due to q_1 + Electric field due to q_2

$$\text{or, } E = E_1 + E_2$$

$$\text{or, } E = \frac{q_1}{4\pi\epsilon_0}\frac{1}{(r/2)^2} + \frac{q_2}{4\pi\epsilon_0}\frac{1}{(r/2)^2}$$

$$\text{or, } E = \frac{1}{4\pi\epsilon_0}\frac{1}{(r/2)^2}(q_1+q_2)$$

$$\text{or, } E = 9\times10^{9}\times\frac{1}{4}\left(3\times10^{-6} + 5\times1\right.$$

$$\therefore E = 18\times10^{3}\ \text{N/C}.$$

(b) Let x be distance of a point from charge q_1 in -ve x-direction. Th the resultant electric field is zero i.e

$$E_1 = E_2$$

$$\text{or, } \frac{1}{4\pi\epsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{(r+x)^2}$$

$$\text{or, } \frac{3\times10^{-6}}{x^2} = \frac{5\times10^{-6}}{(4+x)^2}$$

$$\text{or, } 3(4+x)^2 = 5x^2$$

$$\text{or, } 3(x^2+8x+16) =$$

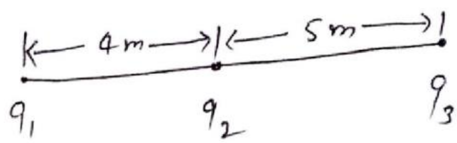
$$\text{or, } 3x^2+24x+48 =$$

$$\text{or, } 2x^2-24x-48 =$$

By solving, we get

Either $x = 13.75$ cm or $x = -1.75$ cm

Example 14.2 Three charges $q_1 = 3\times10^{-6}\,C$, $q_2 = -5\times10^{-6}\,C$ and $q_3 = -8\times10^{-6}\,C$ are positioned on a straight line as shown in figure. Find the potential energy of the charges.



Sol^n:- Given, $q_1 = 3\times10^{-6}\,C$, $q_2 = -5\times10^{-6}\,C$ and $q_3 = -8\times10^{-6}\,C$

potential energy = ?

we know that

$$\text{Potential energy } (E_p) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$\therefore E_p = 9\times10^{9}\left[\frac{3\times10^{-6}\times(-5\times10^{-6})}{4} + \frac{3\times10^{-6}\times(-8\times10^{-6})}{9} + \frac{(-5\times10^{-6})(}{5}\right.$$

$$= 4.43\times10^{-2}\,J$$

Example 14.3 A potential difference of 100V is established between the two plates A and B, where B is at high potential. A proton of charge $q = 1.6\times10^{-19}\,C$ is released from plate B. What will be the velocity of the proton when it reaches plate A? The mass of the proton is $1.67\times10^{-27}\,kg$.

Sol^n:- Give, $V = 100$ volts, $q = 1.6\times10^{-19}\,C$ and $m = 1.67\times1$

velocity (v) = ?

In this case

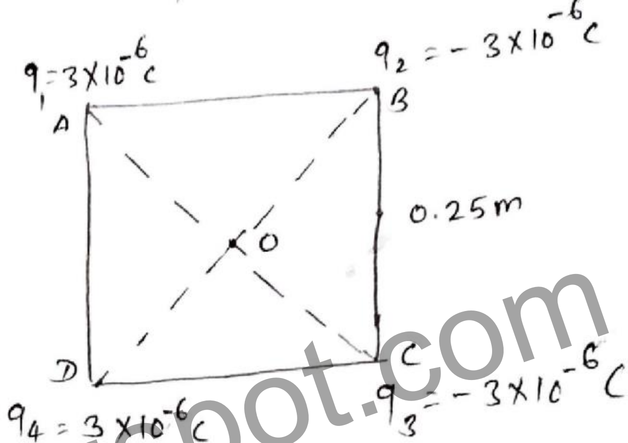
Kinetic Energy = potential Energy

$$\frac{1}{2}mv^2 = qV$$

$$\therefore\ v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2\times 1.6\times 10^{-19}\times 100}{1.67\times 10^{-27}}}$$

$$= 1.38\times 10^{5}\ m/sec$$

<u>Problem 14.6</u> Four charges of equal magnitude are placed at the corners of a square of side 0.25m. What is the electric field at the center of the square? Given figure is



<u>Sol^n</u>: Given; $q_1 = q_4 = 3\times10^{-6}$ C, $q_2 = q_3 = -3\times10^{-6}$ C, and $l = 0.25$ m,

Electric field at the centre of the given square = ?

In this case, $AO = OB = OC = OD = \frac{1}{2}\sqrt{(0.25)^2+(0.25)^2}$

$= 0.1767$ m

electric field due to q_1 and q_3 at 'O' $= 2\times\frac{1}{4\pi\epsilon_0}\frac{q_1}{(OA)^2}$

$$E_1 = 2\times 9\times10^{9}\times\frac{3\times10^{-6}}{0.031}$$

$$= 1742\times10^{3}\ N/C$$

Along OC

And electric field due to q_2 and q_4 at 'O' is given by

$$E_2 = 2\times\frac{1}{4\pi\epsilon_0}\frac{q_2}{(OB)^2}$$

$$E_2 = 2 \times 9 \times 10^9 \times \frac{3 \times 10^{-6}}{0.031} = 1742 \times 10^3 \text{ N/C along OE}$$

Then, the resultant electric field at the Centre 'O' is given by

$$\therefore E = \sqrt{E_1^2 + E_2^2}$$

$$= \sqrt{(1742 \times 10^3)^2 + (1742 \times 10^3)^2}$$

$$= 1742 \times 10^3 \times \sqrt{2}$$

$$= 2463.56 \times 10^3 \text{ N/C}$$

Problem 14.8 :- Two large parallel plates are separated by a distance of 5cm. The plates have equal but opposite charges that create an electric field in the region between the plates. An α-particle ($q = 3.2 \times 10^{-19}$ C and $m = 6.68 \times 10^{-27}$ kg) is released from the +vely charged plate and it strikes the -vely charged plate 2×10^{-6} sec later. Assuming the electric field between the plates is uniform and perpendicular to the plates, what is the strength of the field?

Sol^n :- Given; $d = 5\text{cm} = 5 \times 10^{-2}$ m, $q = 3.2 \times 10^{-19}$ C, $m = 6.68 \times 10^{-27}$ kg and $t = 2 \times 10^{-6}$ sec

$E = ?$

Now, $\text{velocity}(v) = \frac{d}{t} = \frac{5 \times 10^{-2}}{2 \times 10^{-6}} = 2.5 \times 10^4$ m/s

In this Case, potential energy = kinetic Energy

$$qV = \frac{1}{2} mv^2$$

$$\therefore V = \frac{mv^2}{2q} = \frac{6.68 \times 10^{-27} \times (2.5 \times 10^4)^2}{2 \times 3.2 \times 10^{-19}}$$

$\therefore V = 6.523$ Volts

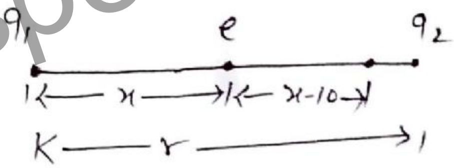
Then the electric field between the plates is given by

$$E = \frac{V}{d} = \frac{6.523}{5\times10^{-2}} = 1.30\times10^{2} \text{ Volts/m}$$

Problem 14.21 An electron is placed midway between two fixed charges $q_1 = 2.5\times10^{-10}$ C and $q_2 = 5\times10^{-10}$ C. If the charges are 1m apart, what is the velocity of the electron when it reaches a point 10 cm from q_2?

Soln:- Given; $q_1 = 2.5\times10^{-10}$ C, $q_2 = 5\times10^{-10}$ C, $e = -1.6\times10^{-19}$ C, $r = 1\text{m} = 100$ cm and $x = \frac{r}{2} = 50$ cm

Now, the net electric force on the electron is given by



$$F = \frac{1}{4\pi\epsilon_0}\frac{e\times q_2}{x^2} - \frac{1}{4\pi\epsilon_0}\frac{e\times q_1}{x^2}$$

or, $F = \frac{1}{4\pi\epsilon_0}\frac{e}{x^2}(q_2 - q_1)$ along q_2

or $\frac{F}{e} = 9\times10^{9}\times\frac{1}{(0.5)^2}\left[5\times10^{-10} - 2.5\times10^{-10}\right]$

$\therefore E = \frac{F}{e} = 9$ N/C.

Then, potential difference $V = Ed = E\times(0.5-0.1)$
$= 9\times0.4$
$= 3.6$ Volts.

In this case, Kinetic Energy = Potential Energy

$$\frac{1}{2}mv^2 = eV$$

$$\therefore v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 3.6}{9.1 \times 10^{-31}}} =$$

$$= 1.125 \times 10^{6} \ m/s$$

Example 16.1 Assume that the electron in a hydrogen atom is essentially in a circular orbit of radius 0.5×10^{-10} m and rotates about the nucleus at the rate of 10^{14} times per second. What is the magnetic moment of the H-ato due to the orbital motion of the electron?

Sol^n :- Given; $r = 0.5 \times 10^{-10}$ m , $e = 1.6 \times 10^{-19}$ C

$f = 10^{14}$ revolutions / second

$\mu = ?$

we know that

$$\mu = I \cdot A = \frac{e}{T} \times \pi r^2 = ef \times \pi r^2$$

$$= 1.6 \times 10^{-19} \times 10^{14} \times 3.14 \times (0.5 \times 10^{-10})^2$$

$$= 1.26 \times 10^{-25} \ Am^2$$

Example 16.2 A current of 50 A is established in a slab of copper 0.5 cm thick and 2 cm wide. The slab is placed in a magnetic field B of 1.5 T. The magnetic field is perpendicular to the plane of the slab and to the current. The free electron concentration in copper is 8.4×10^{28} electrons/m^3. What will be the magnitude of the Hall voltage across the width of the slab?

Sol^n :- Given; $t = 0.5$ cm $= 0.5 \times 10^{-2}$ m, $d = 2$ cm $= 2 \times 10^{-2}$ m,

$I = 50$ A, $B = 1.5$ T and $n = 8.4 \times 10^{28}$ electrons/m^3

$V_H = ?$

we Know that

$$V_H = \frac{IBd}{neA} = \frac{50 \times 1.5 \times 2 \times 10^{-2}}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-4}}$$

$$= 1.12 \times 10^{-6}\ V$$

Problem 16.1 What force is experienced by a wire of length $l = 0.08m$ at an angle of $20°$ to the magnetic field direction carrying a current of 2A in a magnetic field of 1.4 T?

Sol^n:- Given, $l = 0.08m$, $\theta = 20°$, $I = 2A$ and $B = 1.4$

$F = ?$

we Know that

$$F = BIl\sin\theta$$
$$= 1.4 \times 2 \times 0.08 \times \sin 20°$$
$$= 7.66 \times 10^{-2}\ N$$

Problem 16.2 The earth's magnetic field at the equator is 4×10^{-5} T and is parallel to the surface of the earth in the South-north direction. (Note that the earth's geographic north pole is the magnetic south pole). A wire 2m long of mass $m = 9$ gm is suspended by a string. The wire is also parallel to the earth's surface and carries a current of 150A in the east-West direction. (a) What is the tension on the string? (b) What would be the tension if the current was in the West-east direction?

Sol^n:- Given, $B = 4 \times 10^{-5}$ T, $m = 9$ gm $= 9 \times 10^{-3}$ kg,

$l = 2m$, $I = 150A$

(a) when the Current is in the east-west direction, then the tension in the string is given by

$$T = mg + BIl\sin\theta \qquad (\because \theta = 90^\circ)$$

$$= 9\times10^{-3}\times 9.8 + 4\times10^{-5}\times 150\times 2\times 1$$

$$= 10.02\times10^{-2}\ N$$

(b) when the Current is in the west-east direction, then the tension in the string is given by

$$T = mg - BIl\sin\theta$$

$$= 9\times10^{-3}\times 9.8 - 4\times10^{-5}\times 150\times 2\times 1$$

$$= 7.62\times10^{-2}\ N$$

Problem 16.12 A proton is moving with a velocity $\vec{v} = (3\times10^5\,\vec{i} + 7\times10^5\,\vec{k})$ m/sec in a region where there is a magnetic field $\vec{B} = 0.4\,\vec{j}$ T. What is the force experienced by the proton?

Soln: Given, $q = e = 1.6\times10^{-19}$ C, $\vec{v} = (3\times10^5\,\vec{i} + 7\times10^5\,\vec{k})$ m/s.

$\vec{B} = 0.4\,\vec{j}$ T $\qquad \vec{F} = ?$

we know that

$$\vec{F} = Bqv\sin\theta$$

$$= q(\vec{v}\times\vec{B}) = 1.6\times10^{-19}[(3\times10^5\,\vec{i} + 7\times10^5\,\vec{k})\times 0.4$$

$$= 1.6\times10^{-14}\,[3\times0.4(\vec{i}\times\vec{j}) + 7\times0.4\,(\vec{k}\times\vec{j})]$$

$$= 10^{-14}\,[1.6\times3\times0.4\,\vec{k} - 1.6\times7\times0.4\,\vec{i}]$$

$$= [1.92\,\vec{k} - 4.48\,\vec{i}]\times10^{-14}\ N$$

Note:- $\vec{i}\times\vec{j} = \vec{k}$, $\vec{k}\times\vec{j} = -(\vec{j}\times\vec{k}) = -\vec{i}$

Problem 16.13 A proton is accelerated through a p.d. of 200V It then enters a region where there is a magnetic field $B = 0.5T$. The magnetic field is perpendicular to the direction of motion of the proton. What is the force experienced by the proton?

Sol^n :- Given, $V = 200$ Volts, $B = 0.5T$, $q = e = 1.6\times10^{-19}C$

In this Case,

$$\frac{1}{2}mv^2 = eV$$

$$\therefore \quad v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times200}{1.67\times10^{-27}}}$$

$$= 19.576\times10^4 \text{ m/sec}$$

Then,

$$F = Bev\sin\theta$$

$$= 0.5\times1.6\times10^{-19}\times19.576\times10^4\times1 \quad (\theta=90^\circ)$$

$$= 15.66\times10^{-15} \text{ N}$$

Unit - 5

Fundamentals of Solid State physics

Example 23.1 Consider a copper wire of cross-section area $A = 1mm^2$ carrying a current $i = 1A$. What is the drift velocity of the electron? Cu is monovalent, that is, there is one free electron per atom. The density and the molecular weight of Cu are $9\ gm/cm^3$ and 64 gm/mole respectively.

Sol^n :- Given, $I = 1A$, $A = 1mm^2 = 1\times10^{-6}m^2$, $\rho = 9\ gm/cm^3$

$M = 64$ gm/mole

we Know that

$$J = \frac{I}{A} = \frac{1}{10^{-6}} = 10^{6}\ A/m^2$$

In this case, no. of free electrons per unit volume is equal to the no. of atoms per unit volume. Therefore,

n = no. of atoms per unit volume

= no. of mole per unit volume × Avogadro's number

$$= \frac{\rho}{M} \times N_A = \frac{9}{64} \times 6.02 \times 10^{23} = 8.4 \times 10^{22}\ \text{atoms/cm}^3$$

$$\therefore n = 8.4 \times 10^{28}\ \text{atoms}/m^3$$

we have,

$$\text{Drift velocity } (V_d) = \frac{J}{en} = \frac{10^{6}}{1.6 \times 10^{-19} \times 8.4 \times 10^{28}}$$

$$= 7 \times 10^{-5}\ m/sec$$

Example 23.2 We showed in example 23.1 that the no. of free electrons in Copper is 8.4×10^{28} electrons/m^3. (a) Calculate the Fermi energy for Cu. (b) At what temperature T_F will the average thermal energy K_BT of a gas is equal to that energy?

sol^n :- Given, $n = 8.4 \times 10^{28}$ electrons/m^3

$$\left(\because \hbar = \frac{h}{2\pi}\right)$$

(a) Fermi energy $= \frac{\hbar^2}{2m}(3n\pi^2)^{2/3}$

$$E_F(0) = \frac{(1.05 \times 10^{-34})^2 \left[3 \times 8.4 \times 10^{28} \times (3.14)^2\right]^{2/3}}{2 \times 9.1 \times 10^{-31}}$$

$$E_F(0) = 11.1 \times 10^{-19}\ J$$

$$\therefore E_F(0) = 6.95\ eV$$

(b) $k_B T_f = E_F(0)$

$$\therefore T_f = \frac{E_F(0)}{k_B} = \frac{11.1 \times 10^{-19}}{1.38 \times 10^{-23}} = 80,500K$$

Note :- Boltzmann's Constant $k_B = 1.38 \times 10^{-23}$ J/K is given.

Problem 22.1 :- Copper has a FCC structure with a one atom basis. The density of Copper is 8.96 g/cm³ and it's atomic weight is 63.5 g/mole. What is the length of the unit Cube of the Structure?

Solⁿ :- Given, $\rho = 8.96$ gm/cm³

$M_{at} = 63.5$ gm/mole

$a = ?$

For FCC Structure, the no. of atoms per unit cell is 4. we know that

$$\text{Density} = \frac{\text{No. of atoms per unit cell} \times \text{Mass of one atom}}{\text{Volume of unit cell}}$$

$$\rho = \frac{N \times \left(\frac{M_{at}}{N_A}\right)}{a^3} = \frac{4 \times \left(\frac{63.5}{6.023 \times 10^{23}}\right)}{a^3}$$

$$\text{or, } a = \left[\frac{4 \times 63.5}{8.96 \times 6.023 \times 10^{23}}\right]^{1/3}$$

$$\text{or, } a = 3.608 \times 10^{-8} \text{ cm}$$

$$\therefore a = 3.608 \times 10^{-10} \text{ m}$$

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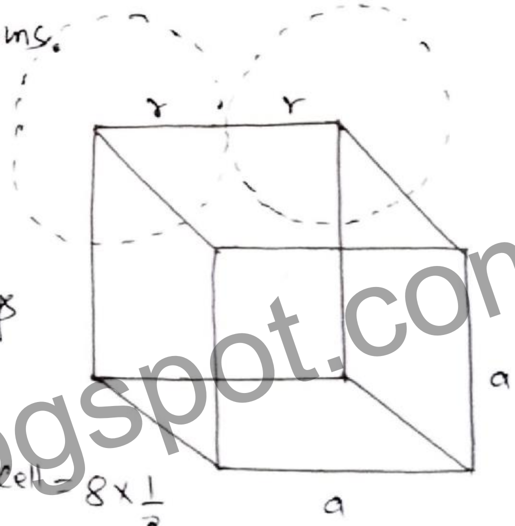
Problem 22.3 Assuming that atoms in a crystal structure a arranged as close-packed spheres, what is the ratio of the volume of the atoms to the volume available for the SC structure? Assume a one-atom basis.

Solⁿ :- An unit cell having same dimension in all directio and containing only one corner atoms is called simple cubic unit cell. In this case, the side of the cube gives distance between two atoms.



In this structure, each corner atom is shared by 8 unit cell. So, only $(1/8)^{th}$ of each corner atom belongs to an unit cell.

∴ The no. of atoms per unit cell $= 8 \times \frac{1}{8}$

$= 1$

Let r is the radius of an atom and a be the side of the unit cell. Then, $a = 2r$.

From the definition,

$$\text{Packing density or packing fraction} = \frac{\text{Volume of atoms in unit cell}}{\text{Volume of an unit cell}}$$

$$= \frac{\frac{4}{3}\pi r^3 \times 1}{a^3} = \frac{\frac{4}{3}\pi r^3}{(2r)^3}$$

$$= \pi/6 = 0.52 = 52\%$$

Therefore, 52% space of an unit cell is occupied by the ato

In this case, the coordination no. for each corner atom is 6.

Therefore, the atomic concentration for SC unit cell is $\frac{1}{a^3}$

Problem 22.4 Assuming that atoms in a crystalline structure are arranged as close-packed spheres, what is the ratio of the volume of the atoms to the volume available for the BCC structure? Assume a one-atom basis.

Soln: A cubic unit cell with corner atoms and an atom at body centred position is called BCC unit cell. In this case, each corner atom is sheared by 8 unit cells. So, only $\left(\frac{1}{8}\right)^{th}$ of each corner atom belongs to an unit cell. The atom at body centred position can't be shared by any other unit cell.

Therefore, no. of atoms per unit cell $= 8 \times \frac{1}{8} + 1 = 2$

In this structure, we can write

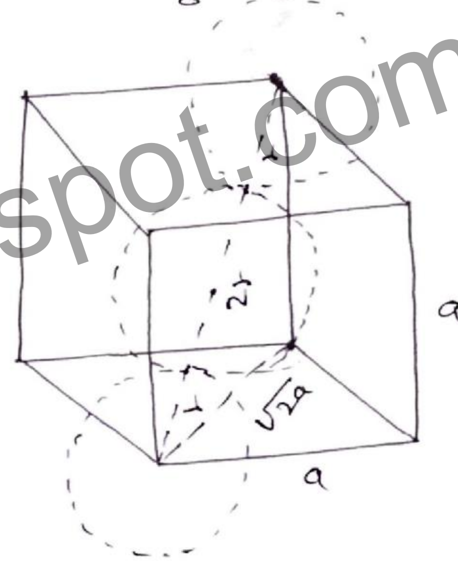
$$\sqrt{(\sqrt{2}a)^2 + a^2} = r + 2r + r$$

or. $\sqrt{2a^2 + a^2} = 4r$

or. $\sqrt{3a^2} = 4r$

or, $\sqrt{3}\,a = 4r$

$\therefore a = \frac{4r}{\sqrt{3}}$



Then, packing fraction $= \frac{\text{volume of atoms in an unit cell}}{\text{Volume of an unit cell}}$

$$= \frac{\frac{4}{3}\pi r^3 \times 2}{a^3} = \frac{4/3\,\pi r^3 \times 2}{\left(\frac{4r}{\sqrt{3}}\right)^3}$$

$$= \sqrt{3}\,\pi/8$$

$$= 0.68 = 68\%$$

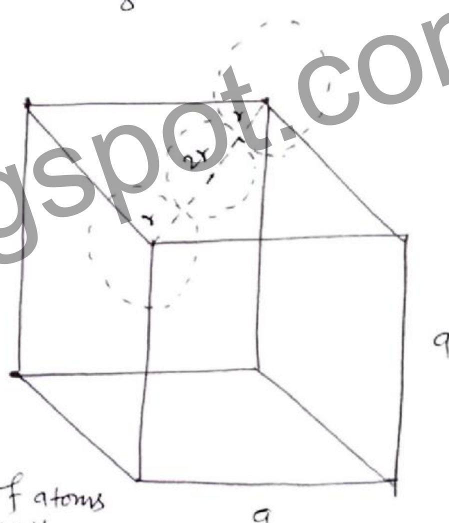
This means that 68% of space of the unit cell is occupied by the atoms. In this case, the coordination no. for interior atom is 8. Then, the atomic concentration for BCC unit is $\frac{2}{a^3}$.

Problem 22.5 Assuming that atoms in a crystalline structure are arranged as close-packed spheres, what is the ratio of the volume of the atoms to the volume available for the FCC structure. Assume a one-atom basis.

Soln :- A cubic unit cell formed by corner atoms along with atom in each face centered position is called FCC unit cell. In this case, each corner atom is shared by 8 unit cells. So, only $\left(\frac{1}{8}\right)^{th}$ of each corner atom belongs to unit cell. Each atom at face centered position is shared by two unit cell. So, $\left(\frac{1}{2}\right)^{th}$ of each face centered atom belongs to unit cell. Therefore,

$$\text{Number of atoms per unit cell} = 8\times\frac{1}{8} + 6\times\frac{1}{2} = 4$$

Let r be the radius of an atom and a be the side of the unit cell. Then, we can write



$$\sqrt{2}\,a = r + 2r + r$$

$$\therefore a = \frac{4r}{\sqrt{2}} = 2\sqrt{2}\,r$$

From the definition,

$$\text{packing Fraction} = \frac{\text{Volume of atoms in unit cell}}{\text{Volume of an unit cell}}$$

$$= \frac{4\times\frac{4}{3}\pi r^3}{a^3} = \frac{16\pi r^3}{3(2\sqrt{2}\,r)^3}$$

$$= \frac{\pi}{3\sqrt{2}} = 0.74$$

$$= 74\%$$

This means that 74% space of FCC unit cell is occupied by atoms. In this case, the coordination number is $4\times3=12$. Therefore, the atomic concentration of FCC unit cell is $\frac{4}{a^3}$.

Problem 22.9 The dissociation energy of the KF molecule is 5.12 eV. The ionization energy for K is 4.34 eV and the electron affinity of F is 4.07 eV. What is the equilibrium separation constant for the KF molecule?

Sol^n :- Given; here to ionize K-atom, an energy of 4.07 eV is provided i.e

$$K + 4.34\ eV \longrightarrow K^+ + e^-$$

On the other hand, when F capture one electron, the energy released is 4.07 eV i.e

$$F + e^- \longrightarrow 4.07\ eV + F^-$$

When K^+ and F^- are brought together at equilibrium separation (r), then Coulomb attractive potential energy is given by

$$E = -\frac{e^2}{4\pi\epsilon_0 r}$$

Or $$5.12 \times 1.6 \times 10^{-19} = -\frac{(1.6\times10^{-19})^2 \times 9\times10^9}{r}$$

$$\therefore r = -\frac{(1.6\times10^{-19})^2 \times 9\times10^9}{5.12\times1.6\times10^{-19}}$$

$$= 2.81 \times 10^{-10}\ m$$

Problem 24.6 The energy gaps of some alkali halides are KCl = 7.6 eV, KBr = 6.3 eV, KI = 5.6 eV. Which of these are transparent to visible light? At what wavelength does each become opaque?

24.8 The density of aluminum is 2.70 g/cm^3 and it's molecular weight is 26.98 gm/mole. (a) Calculate the Fermi energy. (b) If the experimental value of E_F is 12 eV, what is the electron effective mass in aluminum? (Aluminum is trivalent).

Sol^n: Given, $\rho = 2.70\ gm/cm^3$, $M = 26.98\ gm/mole$

The electron concentration is given by

$$n = \frac{3 \times \rho \times N_A}{M} = \frac{3 \times 2.7 \times 6.02 \times 10^{23}}{26.98} = 1.81 \times 10^{23}\ electrons/cm^3$$

$\therefore n = 1.81 \times 10^{29}\ electrons/m^3$

(a) we know that

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} = \frac{(1.05 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31}} \times \left[3 \times (3.14)^2 \times 1.81 \times 10^{29}\right]^{2/3}$$

$$= 1.85 \times 10^{-18}\ J$$

$\left[\because \hbar = \frac{h}{2\pi}\right]$

(b) we have, $E_F = 12\ eV = 12 \times 1.6 \times 10^{-19}\ J$, $m^* = ?$

We know that

$$E_F = \frac{\hbar^2}{2m^*}(3\pi^2 n)^{2/3}$$

$$\therefore m^* = \frac{\hbar^2}{2 \times E_F}(3\pi^2 n)^{2/3} = \frac{(1.05 \times 10^{-34})^2}{2 \times 12 \times 1.6 \times 10^{-19}} \times \left[3 \times (3.14)^2 \times 1.81 \times 10^{29}\right]^{2/3}$$

$\therefore m^* = 8.78 \times 10^{-31}$ kg $= 0.97\, m$

where $m = 9.1 \times 10^{-31}$ kg is the mass of free electron.

Unit - 6

Semiconductor and Semiconductor devices

Example 25.2 :- A sample of Si is doped with phosphorous. The donor impurity level lies 0.045 eV below the bottom of the conduction band. At $T = 300$ K, E_F is 0.010 eV above the donor level. Calculate (a) the impurity concentration, (b) the number of ionized impurities, (c) the free electron concentration and (d) the hole concentration. [For Si, $E_g = 1.100$ eV, $m_e^* = 0.31m$, $m_h^* = 0.38m$].

sol^n: (a) $N_D = ?$

$$N_c = \frac{1}{4}\left[\frac{2 m_e^* k_B T}{\hbar^2 \pi}\right]^{3/2}$$

$$= \frac{1}{4}\left[\frac{2 \times 0.31 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(1.05)^2 \times 10^{-68} \times 3.14}\right]^{3/2}$$

$$= 4.39 \times 10^{24}\ m^{-3}$$

$$\hbar = \frac{h}{2\pi}$$

and

$$N_v = \frac{1}{4}\left[\frac{2 m_h^* k_B T}{\hbar^2 \pi}\right]^{3/2}$$

$$= \frac{1}{4}\left[\frac{2 \times 0.38 \times 9.1 \times 10^{-23} \times 300}{(1.05)^2 \times 10^{-68} \times 3.14}\right]^{3/2}$$

$$= 5.95 \times 10^{24}\ m^{-3}$$

We know that

$$N_c\, e^{-(E_g - E_F)/k_BT} = N_v\, e^{-E_F/k_BT} + N_D\left[1 - \frac{1}{e^{\frac{E_D - E_F}{k_BT}} + 1}\right]$$

or. $$4.39\times10^{24}\, e^{-\frac{(1.1-1.065)\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}} = 5.95\times10^{24}\times e^{-\frac{1.065\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}}$$

$$+ N_D\left[1 - \frac{1}{e^{-\frac{0.01\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}} + 1}\right]$$

or. $1.08\times10^{24} = 1.88\times10^{6} + N_D(0.4)$ ——— (1)

$\therefore N_D = 2.7\times10^{24}\ m^{-3}$ //

(b)

$$N_D^+ = N_D\left[1 - \frac{1}{e^{\frac{E_D - E_F}{k_BT}} + 1}\right]$$

$$= 2.7\times10^{24}\left[1 - \frac{1}{e^{-\frac{0.01\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}} + 1}\right]$$

$= 1.08\times10^{24}\ m^{-3}$ //

(c) $N_e = N_c\, e^{-(E_g - E_F)/k_BT} = 1.08\times10^{24}\ m^{-3}$ // (From eqn (1))

(d) $N_h = N_v\, e^{-E_F/k_BT} = 1.88\times10^{6}\ m^{-3}$ //

Note :- $E_F - E_D = 0.01\,ev$, $E_D = E_g - 0.045 = 1.055$ and

$E_F = E_D + 0.01 = 1.055 + 0.01 = 1.065\,ev$

Problem 25.1 The band gap in pure germenium is $E_g = 0.67$ (a) Calculate the number of electrons per unit volume in th conduction band at 250 K, 300 K and at 350 K. (b) Do th same for silicon assuming $E_g = 1.1\,eV$. [$m_e^* = 0.12\,m$ f Ge and $m^* = 0.31\,m$ for S

Sol^n :- (a) For Ge,

$$E_g = 0.67\,eV = 0.67 \times 1.6 \times 10^{-19}\,J \;;\; E_F = \frac{E_g}{2}$$

$$m_e^* = 0.12\,m$$

$$m = 9.1 \times 10^{-31}\,kg \qquad \left(\because \hbar = \frac{h}{2\pi}\right)$$

$$T = 250\,K$$

$$h = 6.62 \times 10^{-34}\,JS$$

$$K = k_B = 1.38 \times 10^{-23}\,J/K$$

We know that

$$\text{electron Concentration}\,(N_e) = N_e\, e^{-(E_g - E_F)/k_B T}$$

$$= \frac{1}{4}\left(\frac{2\, m_e^* k_B T}{\hbar^2 \pi}\right)^{3/2} e^{-\left(E_g - \frac{E_g}{2}\right)/k_B T}$$

$$= \frac{1}{4}\left(\frac{2 \times 0.12 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 250}{(1.05)^2 \times 10^{-68} \times 3.14}\right)^{3/2} \times$$

$$e^{-\frac{(0.67 - 0.335) \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 250}}$$

$$= 1.489 \times 10^{17}\,m^{-3}$$

Similarly, we can find at 300 K, 350 K.

(b) For Silicon,

$$E_g = 1.1\,eV = 1.1 \times 1.6 \times 10^{-19}\,J \;;\; E_F = E_g/2$$

$$m_e^* = 0.31\,m$$

$$m = 9.1 \times 10^{-31}\,kg$$

we have,
$$N_e = N_c \, e^{-(E_g - E_F)/kT}$$
$$= \frac{1}{4}\left[\frac{2 m_e^* kT}{\hbar^2 \pi}\right]^{3/2} e^{-\left[E_g - \frac{E_g}{2}\right)/kT}$$
$$= \frac{1}{4}\left[\frac{2 \times 0.31 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(1.05)^2 \times 10^{-68} \times 3.14}\right]^{3/2} \cdot$$
$$e^{-\frac{(1.1 - 0.55) \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}}$$
$$= 2.6 \times 10^{15} \ m^{-3}$$

Similarly, we can find at 250k and 350k also.

Problem 25.2 : Suppose that the effective mass of holes in a material is four time that of electrons. At what temperature would the fermi level be shifted by 10% from the middle of the forbidden energy gap? Let $E_g = 1ev$

Soln: Given,
$$m_h^* = 4 m_e^* \qquad E_F = \left[\left(E_g/2 = 0.5ev\right) + 10\% \text{ of } 0.\right.$$
$$E_g = 1 ev \qquad = 0.55 \ ev$$

we know that
$$E_F = \frac{E_g}{2} + \frac{3}{4} kT \ln \frac{m_h^*}{m_e^*}$$

$$\text{or, } 0.55 \times 1.6 \times 10^{-19} = 0.5 \times 1.6 \times 10^{-19} + \frac{3}{4} \times 1.38 \times 10^{-23} \times T \times \ln(4)$$

$$\therefore T = 557 K$$

Problem 25.3 The energy gap in germenium is 0.67 ev. electron and the hole effective masses are 0.12m and 0.23 respectively, where m is the free electron mass. Calculate (a) the Fermi energy (b) the electron density and (c) the hole density at T = 300K.

Soln: Given,

$$E_g = 0.67\,ev = 0.67 \times 1.6 \times 10^{-19}\,J$$
$$m_e^* = 0.12\,m$$
$$m_h^* = 0.23\,m$$
$$m = 9.1 \times 10^{-31}\,kg$$
$$T = 300K$$
$$K_B = K = 1.38 \times 10^{-23}\,J/K$$

$(\because \ln = \log_e)$

(a) we know that

$$E_F = \frac{E_g}{2} + \frac{3}{4} KT \ln\left(\frac{m_h^*}{m_e^*}\right)$$

$$= \frac{0.67}{2} + \frac{3}{4}\,\frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln\left(\frac{0.23}{0.12}\right)$$

$$= 0.3476\,ev$$

$$= 0.348\,ev$$

(b) we know that electron density is given by

$$N_e = N_c\, e^{-(E_g - E_F)/KT}$$

$$= \frac{1}{4}\left[\frac{2 m_e^* KT}{\pi \hbar^2}\right]^{3/2} e^{-(E_g - E_F)/KT}$$

$$= \frac{1}{4}\left[\frac{2 \times 0.12 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{3.14 \times (1.05)^2 \times 10^{-68}}\right]^{3/2} \times$$

$$e^{-\frac{(0.67 - 0.348) \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}}$$

(c) we know that the hole concentration (Hole density) is given by

$$N_h = N_v \, e^{-E_F/kT}$$

$$= \frac{1}{4}\left[\frac{2 m_h^* kT}{\pi \hbar^2}\right]^{3/2} e^{-E_F/kT}$$

$$= \frac{1}{4}\left[\frac{2 \times 0.23 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{3.14 \times (1.05)^2 \times 10^{-68}}\right]$$

$$e^{-\frac{0.348 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}}$$

$$= 4.1 \times 10^{18} \; m^{-3}$$

P25.13 A certain intrinsic semiconductor has a band gap $E_g = 0.2\,eV$. Measurement shows that it has a resistiv at room temperature 300K of $0.3\,\Omega m$. What would you predit it's resistivity to be at 350K?

Sol^n: Given, $E_g = 0.2\,eV = 0.2 \times 1.6 \times 10^{-19} J$, $T_1 = 300K$, $T_2 = 350K$, $\rho_1 = 0.3\,\Omega m$ and $\rho_2 = ?$

$$\therefore \frac{\rho_2}{0.3} = e^{\frac{0.2 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23}}\left(\frac{1}{350} - \frac{1}{300}\right)}$$

$$\therefore \rho_2 = 0.17 \ \Omega m$$

Problem 25.16 The energy gap in silicon is 1.1 ev, whereas in diamond it is 6 ev. What conclusion can you draw about the transparency of the two materials to visible light (4000 A° to 7000 A°)?

Sol^n: Given:

For Si, $E_g = 1.1$ ev

For diamond, $E_g = 6$ ev

Energy of visible light $(E) = hf$

$$= \frac{hc}{\lambda}$$

For $\lambda = 4000 A°$,

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{4000 \times 10^{-10}}$$

$$= 3.1 \text{ ev}$$

For $\lambda = 700 A°$,

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{7000 \times 10^{-10}}$$

$$= 1.77 \text{ ev}$$

Since the energy of visible light is greater that $E_g = 1.1$ ev for Si, the absorption takes place in the Silicon. So, it is opaque for visible light. But the band gap energy $E_g = 6$ ev for diamon is greater that the light energy. So, the light gets transmitted.

Problem 26.1 The Current through a p-n junction is 1×10^{-8} A when a reverse bias Voltage of 10V is applied across the Junction at T = 300K. What will be the Current through the diode when a forward bias Voltage of (a) 0.1 V (b) 0.3 V and (c) 0.5 V is applied

Sol^n:- Given, $I = 1\times10^{-8}$ A $\qquad K = 1.38\times10^{-23}$ J/K

Reverse Voltage $V_0 = -10$ V

$T = 300$K

we know that

$$I = I_0\left[e^{\frac{|e|V_0}{KT}} - 1\right]$$

or $$1\times10^{-8} = I_0\left[e^{\frac{-1.6\times10^{-19}\times10}{1.38\times10^{-23}\times300}} - 1\right]$$

or $$1\times10^{-8} \simeq -I_0$$

$$\therefore I_0 = -1\times10^{-8} \text{ A}$$

(a) Now, For forward bias, $V_0 = 0.1$ V, $I = ?$

we have,

$$I = I_0\left(e^{\frac{|e|V_0}{KT}} - 1\right)$$

$$= 1\times10^{8}\left[e^{\frac{0.1\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}} - 1\right]$$

$$= 46.46\times10^{-8} \text{ A}$$

$$= 46.46\, I_0$$

Similarly, we can do for $V_0 = 0.3$ V and $V_0 = 0.5$ V.

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Problem 26.2 In the ideal diode, the reverse saturation current should be as small as possible. Considering the fact that E_g for Si is 1.1 eV and E_g for Ge is 0.67 eV, which material is better suited for the fabrication of p-n junction diode?

Solⁿ:-

Problem 26.3 The reverse saturation current of a silicon diode is $I_o = 5 \times 10^{-9}$ A. The voltage across that diode when forward biased is 0.45 V. (a) What is the current through the diode at 27°C? (b) If the voltage across the diode is held constant and we assume that I_o does not change with temperature, what is the current through the diode at 47°C?

Solⁿ:- Given, $I_o = 5 \times 10^{-9}$ A, $T = 27 + 273 = 300K$

Forward biased voltage $(V_c) = 0.45V$

(a) $I = ?$

we know that

$$I = I_0\left[e^{|e|V_0/KT} - 1\right]$$

$$= 5\times10^{-9}\left[e^{\frac{1.6\times10^{-19}\times0.45}{1.38\times10^{-23}\times300}} - 1\right]$$

$$= 0.18A$$

(b) Similarly, we can do for $T = 47^\circ C = 320K$.

$$I = 5\times10^{-9}\left[e^{\frac{1.6\times10^{-19}\times0.45}{1.38\times10^{-23}\times320}} - 1\right]$$

$$= 0.06A$$

∴ Remaining Problems 26.4 and 26.5

Unit - 7

Universal Gates and physics of integrated Circuit

Problem 27.1 Make the appropriate truth tables to prove the following distributive law of boolean algebra.

$$A(B+C) = AB + AC$$

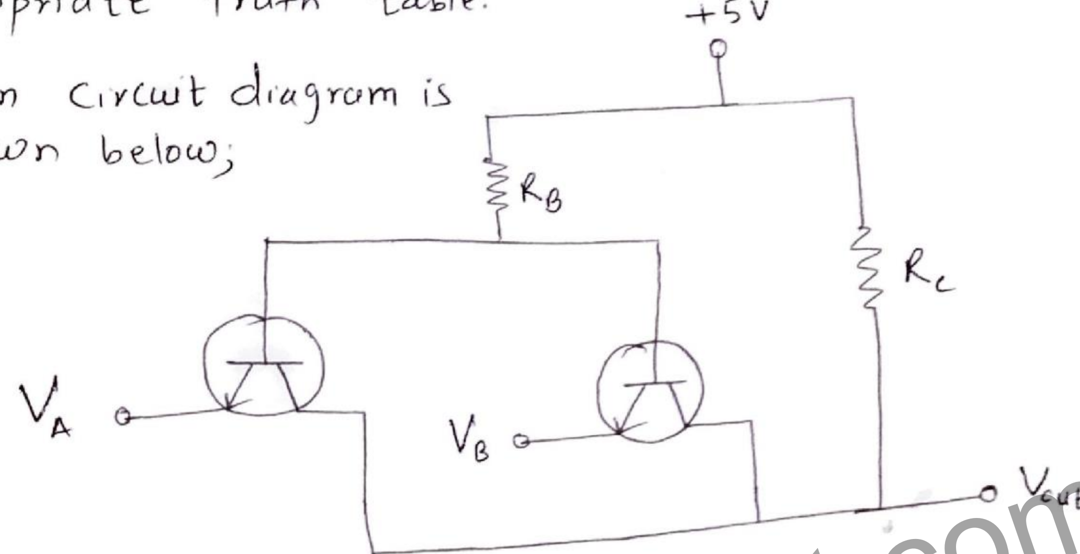
Solution :- Let A, B and C can have the value either 0 or 1

A	B	C	AB	AC	B+C	A(B+C)	AB+AC
0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0
0	1	0	0	0	1	0	0
0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	0
1	0	1	0	1	1	1	1
1	1	0	1	0	1	1	1
1	1	1	1	1	1	1	1

From the truth table $A(B+C) = AB + AC$.

Problem 27.6 Analyze the given Circuit. Determine the logic function performed by the circuit by making and justifying the appropriate truth table.

Soln:- Given circuit diagram is shown below;



In this Circuit diagram, two inputs V_A and V_B are used from the emitters of two transistors A and B. The common output is taken out across the load resistance R_C.

(i) If $V_A = 5V$ and $V_B = 5V$, then the base-emitter junction of both transistors become reverse biased and their collector base junctions become forward biased. As a result, current flows through R_C. Therefore, V_{out} is high.

(ii) If $V_A = 5V$ and $V_B = 0V$, then the collector-base junction of first transistor is forward biased while that of second transistor becomes reversed biased. As a result, no current flows through R_C. Therefore, V_{out} is low. Similarly, if $V_A = 0V$ and $V_B = 5V$, then V_{out} is low.

(iii) If $V_A = 0V$ and $V_B = 0V$, then both base-emitter junctions of transistors become forward biased while both collector-base junctions become reverse biased. As a result, no current flows through R_C. Therefore, V_{out} is low.

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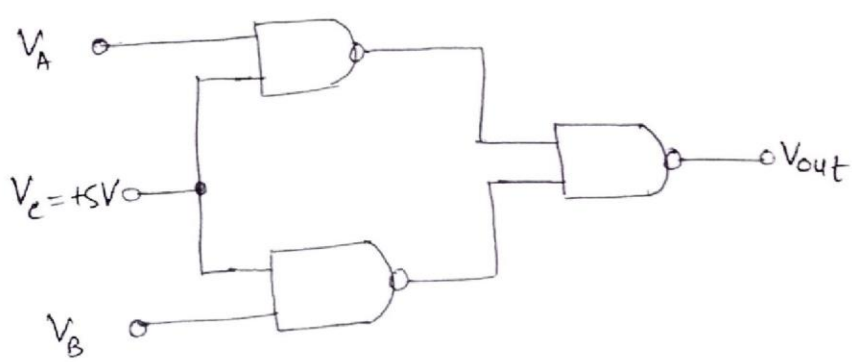
Thus, this is AND gate logic function. It's truth table is shown below;

A	B	$Y = A.B$
0	0	0
1	0	0
0	1	0
1	1	1

Problem 27.9 (a) Find the truth table for the circuit given below. What logic function does the circuit perform?
(b) what logic function will the circuit perform if the constant +5V input to the first two gates is changed to ground potential?

Sol^n:- Given circuit diagram is shown below;



Let A and B can have the value either 0 or 1. But, C = 1. Then, truth table becomes as shown below;

A	B	C	AC	$Y_1 = \overline{AC}$	BC	$Y_2 = \overline{BC}$	$Y_1 Y_2$	$Y = \overline{Y_1 \cdot Y_2}$
0	0	1	0	1	0	1	1	0
1	0	1	1	0	0	1	0	1
0	1	1	0	1	1	0	0	1
1	1	1	1	0	1	0	0	1

This is OR gate logic function

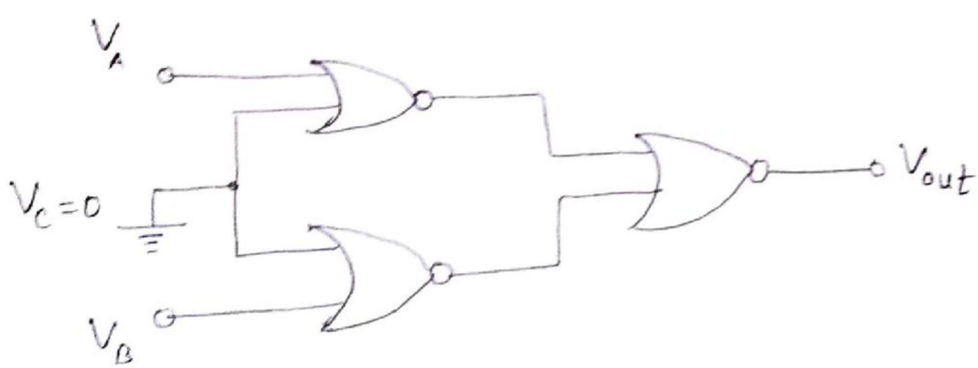
(b) If $V_c = 0$, then

A	B	C	AC	$Y_1 = \overline{AC}$	BC	$Y_2 = \overline{BC}$	$Y_1 \cdot Y_2$	$Y = \overline{Y_1 \cdot Y_2}$
0	0	0	0	1	0	1	1	0
1	0	0	0	1	0	1	1	0
0	1	0	0	1	0	1	1	0
1	1	0	0	1	0	1	1	0

This is none logic function.

Problem 27.10 (a) Find the truth table for the given circuit. What logic function does the circuit perform? (b) What logic function will the circuit perform if the common grounded input to the first two NOR gate is changed to +5V?

Sol^n:- Given circuit diagram is shown below;



Let A and B can have value either 0 or 1. But, C = 0. Then, the truth table becomes as shown in figure.

A	B	C	A+C	$Y_1=\overline{A+C}$	B+C	$Y_2=\overline{B+C}$	Y_1+Y_2	$Y=\overline{Y_1+Y_2}$
0	0	0	0	1	0	1	1	0
1	0	0	1	0	0	1	1	0
0	1	0	0	1	1	0	1	0
1	1	0	1	0	1	0	0	1

This is AND gate logic function

(b) If $V_c = +5V$, then

A	B	C	A+C	$Y=\overline{A+C}$	B+C	$Y_2=\overline{B+C}$	Y_1+Y_2	$Y=\overline{Y_1+Y_2}$
0	0	1	1	0	1	0	0	1
1	0	1	1	0	1	0	0	1
0	1	1	1	0	1	0	0	1
1	1	1	1	0	1	0	0	1

This is none logic function

<u>ex for</u>

(9) A grindstone with $I = 240\,kg\cdot m^2$ rotates with a speed of 2 rev/sec. A knife blade is pressed against it, and the wheel coasts to a stop with constant deceleration in 12 sec. What torque did the knife exert on the wheel.

=>

$I = 240\ kg\cdot m^2$

$f_0 = 2\ rev/sec$

$t = 12\ sec$

$\omega = \omega_0 - \alpha t$

$0 = 2\pi f_0 - \alpha t$

or, $\alpha t = 2\pi f_0$

or, $\frac{\Gamma}{I} t = 2\pi f_0$

$\therefore \Gamma = \frac{2\pi f_0 I}{t} = \frac{2\pi \times 2 \times \cancel{240}^{20}}{\cancel{12}}$

$= 80\pi\ N\text{-}m$