Unit 2 1-7

Booleam Algebra

An English mathematician George Boole developed the concepts of the mathematics of logic and thought. The algebra of the logic is known as the boolean algebra. The main purpose of this algebra is to simplify the logical statements and to solve the logical problems.

The basic definition of the ~~booleam~~ algebra is as follow - Boolean algebra is defined as a set of elements, a set of operators and number of postulates. (Postulates are not proved but as assumed to be true.)

Consider S is a set a and b are two elements then $a \in S$ denotes that a is a member of the set S and notation $a \notin S$ denotes that a is not member of the set S. For example $A = \{1, 2, 3, 4, 5\}$ is a set and elements of this set are 1, 2, 3, 4 & 5. A binary operator defined on a set S takes two elements of set S & result also belongs to set S. eg

$c = a * b$ for every $a, b \in S$

Here $*$ is a binary operator and hence c also belongs to S.

Posteulates of Booleam Algebra:-

These are the true assumptions from which it is possible to derive the rules, properties & some theorems of the systems. Here are the main posteulates -

1. Closure Property:-
for every $a, b \in S$
$a+b \in S$
$a \cdot b \in S$

2. Existance of identity
There exists unique element 0 & 1 in S such that for every $a \in S$
$a+0=a$
$a \cdot 1 = a$

3. Cummutative laws.
For every a and b in S
$a+b=b+a$
$a \cdot b = b \cdot a$

4. Distributive law
for all $a, b, c \in S$
$a \cdot (b+c) = (a \cdot b)+(a \cdot c)$
$a+(b \cdot c) = (a+b) \cdot (a+c)$

5. Law of complementarity (inverse)
for every $a \in S$ there exist $\bar{a} \in S$ such that
$a+\bar{a} = 1$
$a \cdot \bar{a} = 0$

6. Associative law
for any binary operator $*$ & $+$ on a set S to be associative whenever
$(x*y)*z = x*(y*z) \quad \forall\ x, y, z \in S$
$(a+b)+c = a+(b+c) \quad \forall\ a, b, c \in S$

7. There are atleast two elements $a, b \in S$ such that $a \neq b$.

Basic theorems of Boolean algebra

Theorem 1 a. $x+x=x$
b. $x \cdot x = x$

Theorem 2. a. $x+1=1$
b. $x \cdot 0 = 0$

Theorem 3. $(x')' = x$
Involution

Theorem 4. $x+(y+z)=(x+y)+z$
Associative $x(yz) = (xy)z$

$x+x = (x+x) \cdot 1$ by 2b
$= (x+x)(x+x')$ by 5b
$= x+xx'$ by 4b
$= x+0$ by 5b
$= x$ by 2a

$x \cdot x = x$

$x \cdot x = xx+0$ by 2a
$= xx+xx'$ 5b
$= x(x+x')$ 4a
$= x \cdot 1$ 5a
$= x$ 2b

Theorem 5. Demorgan's law
$(x+y)' = x'y'$
$(xy)' = x'+y'$

Theorem 6. $x+xy=x$
Absorption $x(x+y)=x$

Principle of duality:

The duality principle states that if a theorem or result can be established for a Boolean algebra then theorem derived from the given one by
→ interchange of '+' and '.'
→ interchange of the elements 0 & 1
also holds true & need not be proved separately.

Canonical form of boolean logic (express...

When each term of a logic expression contains all the variables, it is said to be in the canonical form. There are two types of canonical forms.

→ Standard sum of product (SOP) form

→ Standard Product of sum (POS) form.

SOP form

As we know any boolean variable may appear either in its normal form (x) or in its complement form ($\bar{x}$). If we combine two variables x & y with AND operation then there are four possible combinations $\bar{x}\bar{y}$ $\bar{x}y$ $x\bar{y}$ & xy. Each of these four AND terms are known as minterms or standard product. In case of 3 variables the possible combination is $2^3 = 8$ minterms as shown below.

x	y	z	Dec. equt.	(AND) Terms	minterms
0	0	0	0	$x'y'z'$	m_0
0	0	1	1	$x'y'z$	m_1
0	1	0	2	$x'yz'$	m_2
0	1	1	3	$x'yz$	m_3
1	0	0	4	$xy'z'$	m_4
1	0	1	5	$xy'z$	m_5
1	1	0	6	xyz'	m_6
1	1	1	7	xyz	m_7

Any function expressed in terms of sum of minterms are called minterm canonical form or Standard sum of products form.

Example

$$F = \bar{x}\bar{y}z + x\bar{y}\bar{z} + \bar{x}\bar{y}\bar{z}$$

$$= m_1 + m_4 + m_0$$

$$= \Sigma(m_0, m_1, m_4)$$

$$F = \Sigma(0,1,4)$$

POS form

If the boolean variables are operated by the OR operator, we obtains the combinations of sums. Each of the terms are then called as maxterms or standard sum. In case of 3 variables the possible combination is $2^3 = 8$ maxterms as shown below.

x	y	z	Decimal equt.	(OR) term	Maxterms
0	0	0	0	$x+y+z$	M_0
0	0	1	1	$x+y+z'$	M_1
0	1	0	2	$x+y'+z$	M_2
0	1	1	3	$x+y'+z'$	M_3
1	0	0	4	$x'+y+z$	M_4
1	0	1	5	$x'+y+z'$	M_5
1	1	0	6	$x'+y'+z$	M_6
1	1	1	7	$x'+y'+z'$	M_7

Any function expressed in terms of product of maxterms are called maxterm canonical form or Standard product of sums form.

<u>example</u>

$$F = (x+y+z)(x+\bar{y}+z)(\bar{x}+y+z)$$

$$= M_0 M_2 M_4$$

$$= \prod (M_0 M_2 M_4)$$

$$= \prod (0,2,4)$$

Boolean Function:-

Boolean function is an expression having the binary variables, the binary operators AND and OR, the unary operator NOT. Also for given value of the variables the function can be either 0 or 1. Consider the following boolean function

$$F = xyz$$

This function F is equal to 1 if $x=1, y=1 \& z=1$ otherwise $F=0$. Simillarly we can write different boolean functions as

$$F_1 = x+yz$$

$$F_2 = xyz + xy \quad \text{etc etc.}$$

Complement of a function:-

The complement of any function is represented by using bar() sign or complement (') sign after the name of the function. And is obtained by nterchanging 0's by 1's and 1's by 0's in the value of F. Also this can be obtained algebraically through De Morgan's theorem. Consider the following example.

$$F = x'yz' + x'y'z$$

Here $F' = (x'yz' + x'y'z)'$

Using De Morgan's theorem

$$= (x'yz')'(x'y'z)'$$

$$= (x+y'+z)(x+y+z')$$

We can also find the complement of any function by using the principle of duality and after that complementing each literals. Consider the following example

$$F = x'yz' + x'y'z$$

Here the dual of F is $(x'+y+z')(x'+y'+z)$

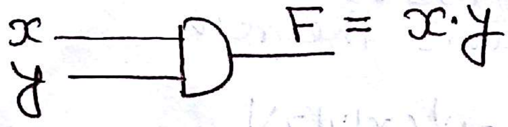
Now the complementing of each literal

$$(x+y'+z)(x+y+z')$$

$$= F'$$

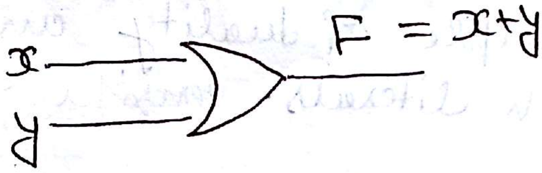
Logic Gates:- The digital computer generally uses the binary numbers for its operations. Also in the binary number system there are only two digits 0 & 1. For the manipulation of those digits the logic circuits are used those logic circuits are called logic gates. We can also define the logic gates as the building blocks of the digital electronics. These circuits are used for designing other circuits like as the combinational circuits, sequential circuits as well as the integrated circuits. The different circuits performs different operations. Starting from the basic operations of the logic, here are the different circuits along with the diagrams as well as their operation with the truth table.

AND gate



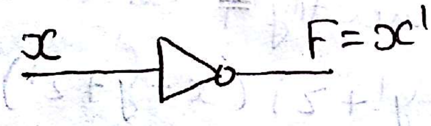
x	y	F=x·y
0	0	0
0	1	0
1	0	0
1	1	1

OR gate



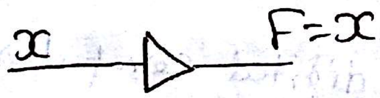
x	y	F=x+y
0	0	0
0	1	1
1	0	1
1	1	1

NOT
(Inverter)



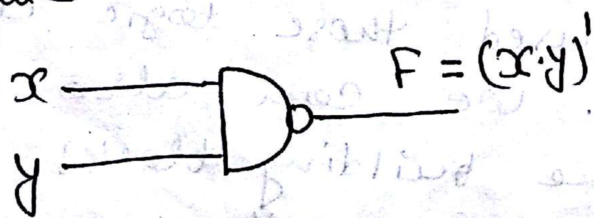
x	F=x'
0	1
1	0

Buffer



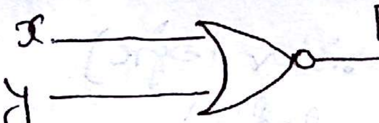
x	F=x
0	0
1	1

NAND gate



x	y	F=(x·y)'
0	0	1
0	1	1
1	0	1
1	1	0

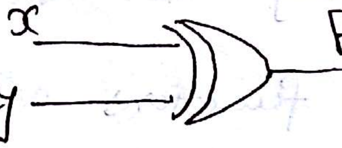
NOR gate



$F = (x+y)'$

x	y	$F=(x+y)'$
0	0	1
0	1	0
1	0	0
1	1	0

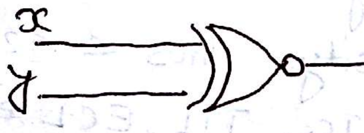
XOR gate
Exclusive-OR



$F = x'y + xy'$
$= x \oplus y$

x	y	$F = x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

XNOR gate
Exclusive-NOR



$F = xy + x'y'$
$= x \odot y$

x	y	$F = x \odot y$
0	0	1
0	1	0
1	0	0
1	1	1

IC digital Logic Families:-

As we know that the gates acts as the building blocks of the digital electronics. The gates are used for constructing the integrated circuits. The ICs are recognized according as the operations performed by the basic circuits as well as the specific logic circuit family to which they belongs. The commercially developed ICs follows the basic working principle of NAND and NOR gates. Most of the popular ICs are given as follows-

TTL (Transistor transistor logic)
ECL (Emitter coupled logic)
MOS (Metal oxide semiconductor)
CMOS (Complementary MOS)
I^2L (Integreated injection logic)

The most of the digital functions are performed by the TTL family ICs. High speed operations are performed by the ECL family ICs. Similarly MOS & I^2L family circuits are used as high density mechanism where as the CMOS family is used as the IC consuming low electric power.
due to the high density MOS & I^2L are used in the LSI scheme of ICs. TTL, ECL & CMOS ~~can~~ also can be used in both LSI as well as SSI scheme.

TTL comes under the numerical designation as 5400 & 7400 series
Some vendors make TTL also available " " 9000 or 8000 "
ECL comes under the numerical designation 10,000 series
Where as CMOS comes under the " 4000 series.

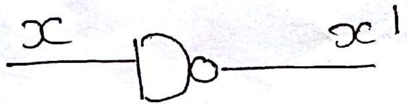
Universal Gates

As we know that AND, OR and NOT gates are the basic gates and the other gates are generally designed by the help of these basic gates. out of other gates the NAND and NOR gates are known as the universal gates because we can derive all the logical function that can be generated by the basic gates. Means all the functions of AND, OR, NOT gates can be done by these two gates and hence these gates are known as universal gates.

We can verify that both of these gates perform the basic gate function as follows-

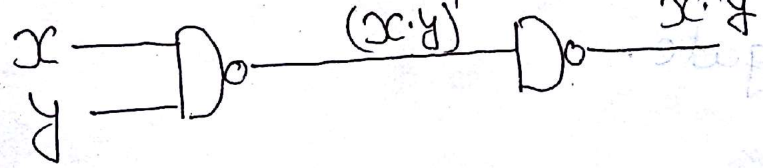
NAND gate as universal gate

(I)



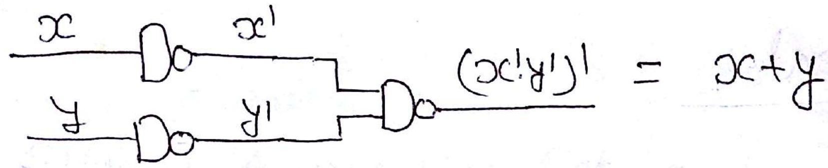
Here this gate generate the complement value of the input and acts as the NOT gate.

(II)



Here the NAND gate takes two inputs and generate the output simillar as the AND gate. Here we use two NAND gates.

(III)



Here we use three NAND gates and two inputs by the Sequential operations finally the output is as similler of the OR gate.

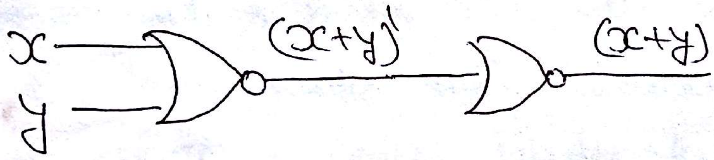
Finally we can conclude that all the basic gate's functions can be easily obtained by using NAND gate only and hence NAND gate is ~~the~~ universal gate.

NOR gate as universal gate

(I)

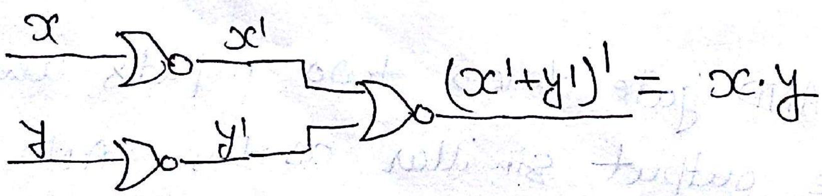
This gate also generate the complement value of the input and hence acts as the NOT gate.

(II)



Here we takes two NOR gates. First of all we take two inputs. The output generated by the 1st NOR gate is taken as the input for the 2nd NOR gate and the result of this acts as the OR gate.

(III)



Here, we take three NOR Gates through which we supply the inputs as shown and finally the result obtained is as similler as of the AND gate.

And hence here also all the functions of the basic Gates are done by the NOR gate and we can easily say that NOR gate is also the universal gate.