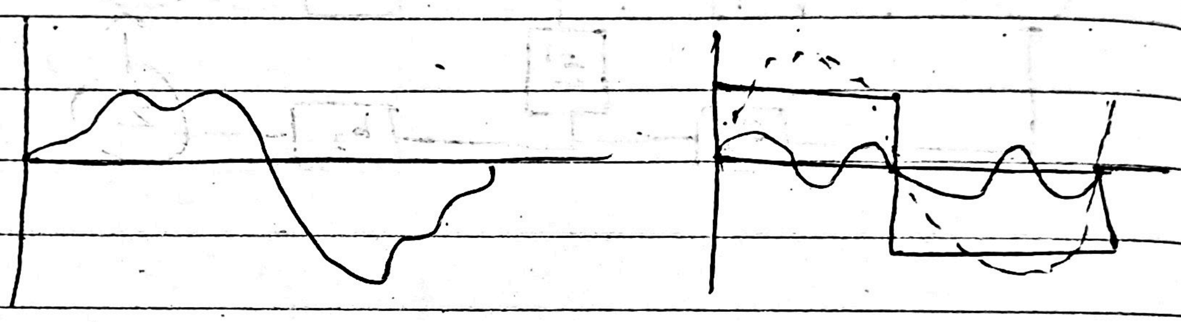
CTFS

Fouriers Series :-

(I) CTFS

(II) DTFS

A Fourier series can be used to represent a periodic signal in terms of harmonicly related complex exponentials or sinusoidal signals.



CTFS ⇒

(1) Trignomateric fourier series ⇒

" Periodic signal $x(t)$ can be represented as,

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

Where $\boxed{a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt}$ ⇒ DC or avg. value of $x(t)$

Sin → odd
Cos → Even

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$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos n\omega t \, dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin n\omega t \, dt$$

Case I → Signals with even symmetry

For even signal, $\boxed{x(t) = x(-t)}$

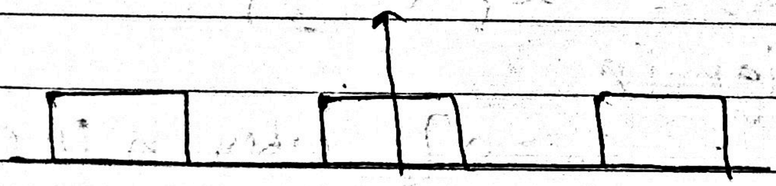
For such signals having even symmetry,

$$\boxed{b_n = 0}$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, dt$$

$$a_n = \frac{4}{T}\int_{0}^{T/2} x(t)\cos n\omega t \, dt$$

even →



Case II → Signals having Odd Symmetry: (B)

For odd signal $x(t) = -x(-t)$

For such signals having odd symmetry

$$\boxed{a_n = 0} \quad \boxed{a_0 = 0}$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt = 0$$

$$\boxed{b_n = \frac{4}{T}\int_{0}^{T/2} x(t)\sin n\omega t\,dt}$$

Case III → Signals having half wave symmetry

(A) Signals with half wave odd symmetry:

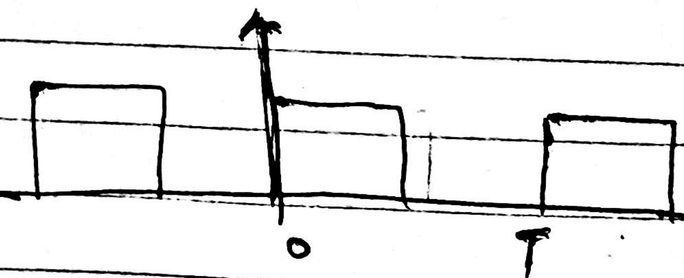
$$x(t) = -x\left(t \pm \frac{T}{2}\right)$$

For such signals

$$\left.\begin{array}{l} a_n = 0 \\ b_n = 0 \end{array}\right\} \text{ when } n \text{ is even}$$

For signals having half wave odd symmetry only odd harmonics are present.

i-Ex →



(6) Half wave even Symmetry

$$\left.\begin{aligned} a_n &= 0 \\ b_n &= 0 \end{aligned}\right\} \text{when } n \text{ is odd}$$

Alternate Representation of trignomateric Fourier series :—

$$\boxed{x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega t + b_n \sin n\omega t]$$

let $a_n = A_n \cos \phi_n$

$b_n = A_n \sin \phi_n$

$$a_n^2 + b_n^2 = A_n^2$$

$$\boxed{A_n = \sqrt{a_n^2 + b_n^2}}$$

$$\boxed{\phi_n = \tan^{-1} \frac{b_n}{a_n}}$$

$$\therefore x(t) = a_0 + \sum_{n=1}^{\infty} A_n [\cos \phi_n \cos n\omega t + \sin \phi_n \sin n\omega t]$$

$$= a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t - \phi_n)$$

Fourier Spectrum →

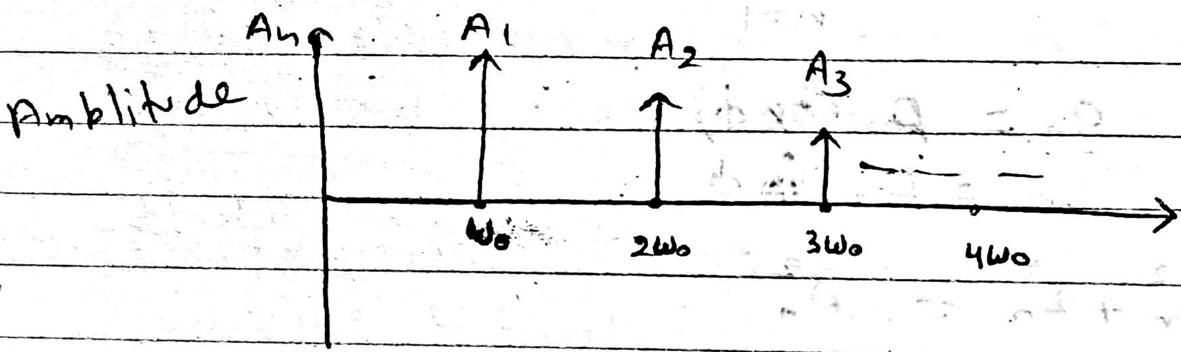
Above reprerepresentation of fourier series gives the fourier spectrum of the signal which consist of amplitude as well as phase spectrum.

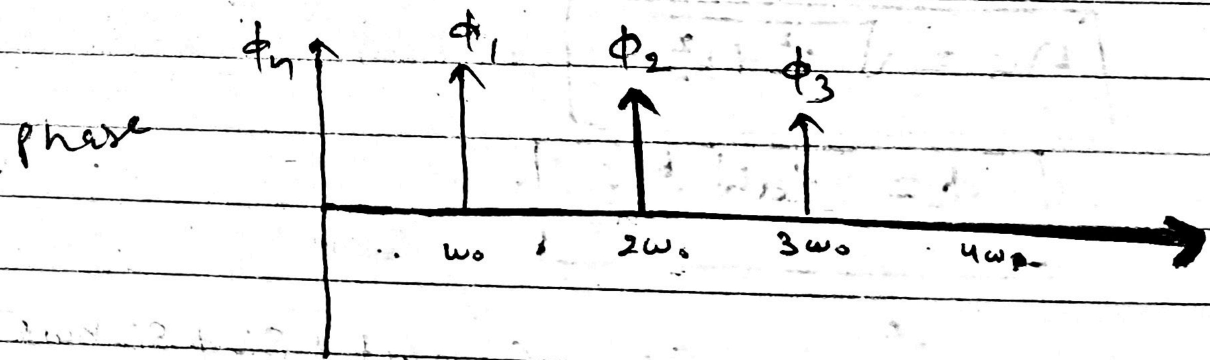
In the Above representation

$A_n \rightarrow$ Amplitude

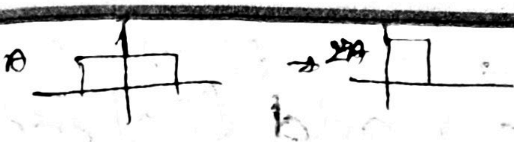
$\phi_n \rightarrow$ phase angle of the nth harmonic of the signal

· One sided amplitude & phase spectrum of such periodic signal are impulses at discrete frequencies as shown below





Fourier spectrum is freq. domain representation of a signal



A continuous and periodic function of time becomes aperiodic and discrete function of frequency in frequency domain.

Exponential Fourier series:—

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

where $$C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt$$

$e^{jn\omega_0 t}$ → Complex exponential signal

Complex exponential fourier series has both sided spectrum which include both +ve as well as -ve frequency. Where as sinusoidal fourier series has one sided spectrum which includes only +ve frequency.

If $x(t)$ is a real signal

$$C_n^* = \left[\frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt\right]^*$$

$$C_n^* = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{+jn\omega_0 t}\, dt$$

Amp → even f^n of freq.
Phase → odd f^n of freq.

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j(-n)\omega_0 t} = C_{-n}$$

$$\boxed{C_n^* = C_{-n}}$$

$$|C_n^*| = |C_{-n}|$$

Amplitude spectrum is always an even function of frequency. Whereas phase spectrum is always odd function of frequency.

Relationship between trignometric & exponential fourier ~~signal~~ series ⇒

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$\boxed{\cos n\omega_0 t = \frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}}$$

$$\boxed{\sin n\omega_0 t = \frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}}$$

$$\therefore \quad x(t) = a_0 + \sum_{n=1}^{\infty} a_n \left[\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}\right]$$

$$+ \sum_{n=1}^{\infty} b_n \left[\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}\right]$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n - jb_n}{2}\right) e^{jn\omega_0 t}$$

$$+ \sum_{n=1}^{\infty} \left(\frac{a_n + jb_n}{2}\right) e^{-jn\omega_0 t}$$

let $\dfrac{a_n - jb_n}{2} = C_n$

$$\frac{a_n + jb_n}{2} = C_n^{*} = C_{-n}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega_0 t} + \sum_{n=1}^{\infty} C_{-n} e^{-jn\omega_0 t}$$

- Replacing $-n$ by n in second term

$$= a_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega_0 t} + \sum_{n=-\infty}^{-1} C_n e^{jn\omega_0 t}$$

$$x(t) = \sum_{n=0}^{\infty} C_n e^{jn\omega_0 t} + \sum_{n=-\infty}^{-1} C_n e^{jn\omega_0 t}$$

where $C_0 = a_0$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$$

Drichlet Condition of convergence of fourier series:-

① The signal must be absolutely integrable over a finite period.

$$\left| \int_T x(t)\, dt \right| < \infty$$

② In any finite interval $x(t)$ must have finite number of minimas & maximas.

③ In any finite interval $x(t)$ should have finite number of discontinuities and each dis-continuity should be finite.

Condition ② & ③ are called strong drichlet condition.

Drichlet Condition are sufficient conditions for existence of fourier series of a signal but these are not necessary conditions