

## AMP Sheet 7

### Tips

Holomorphic  $\equiv$  Analytic  $\equiv$  Differentiable

### Cauchy-Riemann Equations to test for Differentiability

For a complex function defined as:

$$f(z) = u(x, y) + jv(x, y)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

1. Find the values of  $a$  and  $b$  such that  $f$  is analytic

$$f(z) = 3x - 2y^2 + j(2ax + 5by)$$

2. Prove that  $\sin^2(z) + \cos^2(z) = 1$  holds for the complex variable  $z$

3. Find the following to 4 decimal places and represent in cartesian form.

$$(a) \sin(2\pi - j3) \quad (b) \cos^2(j2) \quad (c) j^{-j2}$$

4. For the complex number  $z = x + jy$  evaluate the function  $f(z)$  with the given points:

$$f(z) = 3x^2 + 2x^2 - 2xy + y^2$$

$$(a) -j3 \quad (b) 1 + j2 \quad (c) 2 + j\sqrt{2}$$