

Advanced Mathematics Physics - Sheet 10

Formulae

Gauss/Divergence Theorem

$$\oint_{\partial D} \mathbf{F} \cdot d\mathbf{S} = \iiint_D \nabla \cdot \mathbf{F} \, dV \quad (1)$$

Question 1

Consider the field:

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \quad (2)$$

Find the flux of this field through the surface:

$$0 \leq z \leq 9 - x^2 - y^2 \quad (3)$$

Question 2

Consider the field:

$$\mathbf{F} = x^2\mathbf{i} + y\mathbf{j} + z\mathbf{k} \quad (4)$$

Find the flux of this field through the surface:

$$x^2 + y^2 + 1 \leq z \leq 5 \quad (5)$$

Challenge Question

Consider the field:

$$\frac{\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{\sqrt{x^2 + y^2 + z^2}} \quad (6)$$

Find the flux through the surface:

$$a^2 \leq x^2 + y^2 + z^2 \leq b^2 \quad (7)$$