Final Exam Revision for Advanced Mathematics and Physics

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Mathematics Topics

- Line Integrals
- Green's Theorem
- Stoke's Theorem
- Divergence Theorem
- Gradient
- Divergence
- Curl
- Complex Roots
- Maclaurin Series
- Laurent Series
- Residues
- Contour Integrals
- Inverse Laplace Transforms

Physics Topics

- Partial Differential Equations
- Shrodinger's Wave Equation
- Particle in a box
- Heisenberg's Uncertainty Principle
- Snell's Law
- Diffraction and Interference – Young's Double Slit Experiment
- De Broglie Waves
- Energy-Momentum Relationship
- Derive an Interesting Equation
- Compton Scattering
- Bohr Model of the atom – be able to prove the diameter of an atom is approximately an amstrong (Å$= 10^{-10}$m)
- Solving the heat equation with boundary conditions

Recommended Resources

Split by Physics/Mathematics and then listed in order of importance:

Physics

- Revision lecture
- Lecture Notes' Additional Notes – learn the derivations mentioned in the revision lecture.
- Learn the rest of the derivations.

Mathematics

- Examples in the lecture notes
- Tutorial problems
- *Vector Calculus* by Susan J. Colley
  (there are solutions/instructor's manuals for this, which gives a good feedback loop).
- *Problems and Solutions for Complex Analysis* by Rami Shakarchi

Radioactive Decay

Derive the formula for the number of atoms present during radioactive decay, $N(t)$, given that:

$$-\frac{dN}{dt} \propto N \tag{1}$$

$$-\frac{dN}{dt} = \lambda dt \tag{2}$$

Where:

$N \equiv$ the number of atoms present

$\lambda \equiv$ rate of radioactive decay (specific to the element)

$N_0 \equiv$ is the number of atoms present at $t = 0$

Gauss'/Divergence Theorem

Consider the field:

$$\mathbf{F} = e^y \cos(z)\mathbf{i} + \sqrt{x^3+1}\sin(z)\mathbf{j} + (x^2+y^2+3)\mathbf{k} \tag{3}$$

And the surface:

$$z = (1-x^2-y^2)e^{1-x^2-3y^2} \text{ for } z \geq 0 \tag{4}$$

Parameterised Surface

Consider the field:

$$\mathbf{X}(s,t) = \begin{bmatrix} s \\ s+t \\ t \end{bmatrix} \tag{5}$$

Where $0 \leq s \leq 1$ and $0 \leq t \leq 2$

$$\int\int_{\mathbf{X}} \left(x^2 + y^2 + z^2\right) dS \tag{6}$$

Ask yourself, what is visually happening here?

Computing the Roots of Complex Numbers

Find the expressions for all unique roots of the following complex numbers:

$$\sqrt{z_a} = -i \tag{7}$$

$$\left(82,3543\ e^{i\frac{\pi}{3}}\right)^{\frac{1}{7}} \tag{8}$$

Contour Integration

Find:

$$\int_C \left(x^2 + iy^2\right) dz \tag{9}$$

Where C is the parabola $y = x^2$ from $(0, 0)$ to $(2, 2)$

Then:
Determine if $x^2 + iy^2$ is analytic and calculate:

$$\oint_C \left(x^2 + iy^2\right) dz \tag{10}$$

Where C has now changed to a circle with radius 1 around the origin of the complex plane.

Compton Scattering

Compton scattering is described by:

$$\lambda^{'} - \lambda = \frac{h}{m_e c} \left(1 - \cos\theta\right) \tag{11}$$

(i) If a photon hits a stationary electron and refracts with an angle of 85^o, what is the change in wavelength of the photon?

(ii) Draw a diagram of this.

(iii) Would the colour change be visible to the human eye?

Schrodinger's Wave Equation

Prove that the following is a solution to Schrodinger's wave equation.

$$\Psi(x,t) = Ae^{i(kx-wt)} \tag{12}$$

Where we're working with a single dimensional version of the wave equation with no potential field, which has the form:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} \tag{13}$$

Hint: Think about how "proving this is a solution" is done, otherwise you might get to the end and not realise it.

Partial Differential Equations

The heat equation is given by:

$$\frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2} \tag{14}$$

With boundary conditions given by:

$$u(x, 0) = f(x) \qquad u(0, t) = 0 \qquad u(L, t) = 0 \tag{15}$$

There exists a rod of length L.
At $t = 0$ it has a heat distribution of:

$$f(x) = 12 \sin\left(\frac{9\pi x}{L}\right) - 9 \sin\left(\frac{4\pi x}{L}\right) \tag{16}$$

By inspection (or by the *Lecture 5 Extra Notes*) what is the final form of $u(x, t)$?

Note: for the final form, assume κ is yet to be known and t is a variable.

Stoke's Theorem

Stoke's Theorem is:

$$\int\int_{\mathbf{S}} \nabla \times \mathbf{F} \cdot d\mathbf{S} = \oint_{\partial S} \mathbf{F} \cdot d\mathbf{s} \tag{17}$$

For a surface:

$$S: \qquad z = 9 - x^2 - y^2 \qquad \text{for } z \geq 0 \tag{18}$$

And a field:

$$\mathbf{F} = (2z - y)\,\mathbf{i} + (x + z)\,\mathbf{j} + (3x - 2y)\,\mathbf{k} \tag{19}$$

Calculate:

$$\int\int_{\mathbf{S}} \nabla \times \mathbf{F} \cdot d\mathbf{S} \tag{20}$$