Useful Formulae:

Residue Theorem:

$$2\pi i \sum_{k=1}^{m} \frac{1}{(n-1)!} \frac{d^{n-1}}{ds^{n-1}} \lim_{s \to s_k} (s - s_k)^n F(s) \tag{1}$$

Using Residue Theorem for Inverse Laplace Transforms:

$$\sum_{k=1}^{m} \frac{1}{(n-1)!} \frac{d^{n-1}}{ds^{n-1}} \lim_{s \to s_k} (s - s_k)^n e^{st} F(s) \tag{2}$$

Useful Theorems:

For a simply connected domain:

$$\oint_C f(z)dz = 0 \tag{3}$$

Cauchy-Gorsat Theorem:
For a domain with discontinuities and C_1 and C_2 both loop around those discontinuities:

$$\oint_{C_1} f(z)dz = \oint_{C_2} f(z)dz \tag{4}$$

More general Cauchy-Gorsat Theorem:

$$\oint_{C_1} f(z)dz = \sum_{n=2}^{k} \oint_{C_n} f(z)dz \tag{5}$$

Questions:

1. Evaluate

$$\oint_C \frac{dz}{z-i} \tag{6}$$

Where C is the contour [1,1] $\rightarrow [-1,1] \rightarrow [-1,-1] \rightarrow [1,-1] \rightarrow [1,1]$

2. Using residue theorem, evaluate:

$$\mathcal{L}^{-1}\left\{\frac{\omega}{s^2+\omega^2}\right\} \tag{7}$$

3. Using residue theorem, evaluate:

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+\omega^2}\right\} \tag{8}$$

Challenge question:

4. Using frequency domain techniques and Residue Theorem, solve the following question:

A switch closes on an RL circuit at t=0 that now connects it to an input of $v(t) = sin(\omega t + \phi)$

Find the function $i(t)$ for $t > 0$

Questions: