Week 8
AMP Sheet 06

Useful Formulae

Conjugate

$$z = x + jy \Rightarrow \bar{z} = x - jy$$

Real part

$$\Re\{z\} = \frac{1}{2}\big(z + \bar{z}\big)$$

Imaginary part

$$\Im\{z\} = \frac{1}{j2}\big(z - \bar{z}\big)$$

Squaring a complex number

$$z\bar{z} = |z|^2$$

Simplifying complex numbers in the denominator

$$z = x + jy \Rightarrow \frac{1}{z} = \frac{1}{x + jy} = \frac{1}{x + jy}\frac{(x - jy)}{(x - jy)} = \frac{x}{x^2 + y^2} + j\ \frac{-y}{x^2 + y^2}$$

Drills

1. Express $(-1 + j3)^{-1}$ in polar form.

2. You have:

$$\frac{2 + j}{2 - j}$$

and need to express it as $x + jy$ to enter it into a MATLAB function – what is x and what is y?

3. You have a function to describe the output of your modelled system as:

$$e^{st}$$

$$\big(\text{where } s = \sigma + j\omega\big)$$

You must find the amplitude of the output to ensure it doesn't exceed the ratings of your electrical components. What is the mathematical expression for the amplitude, A?

De Moivre's Theorem

4. Using de Moivre's theorem, find:

$$z^{10} = 2 \angle \frac{\pi}{2}$$

5. Using de Moivre's theorem, find:

$$\sqrt[3]{e^{j\frac{\pi}{4}}}$$

Satisfying Proofs

6. Use the Euler's formula, $\Re\{z\}$ and $\Im\{z\}$ to prove the double-angle formulae of $\sin(\theta + \phi)$ and $\cos(\theta + \phi)$

7. You only have the value of $\tan(\theta)$ and want to find the tangent of triple the angle. Using de Moivre's Theorem and $z = 1 + \tan(\theta)$, find the expression for:

$$\tan(3\theta)$$