## Exercises

**1.**

Verify mathematically:

(a) $\nabla \times \nabla \phi = 0$ for any scalar field.

(b) $\nabla \cdot \nabla \times \mathbf{F} = 0$ for any vector field.

(c) Show that $\nabla^2 \mathbf{F}$ must in general be the sum of 9 terms.

(d) $\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$

(e) $\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$

(f) $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{C} \cdot \mathbf{A} \times \mathbf{B} = -\mathbf{B} \cdot \mathbf{A} \times \mathbf{C}$ (use determinants).

(g) $\nabla \cdot \mathbf{B} \times \mathbf{C} = \mathbf{C} \cdot \nabla \times \mathbf{B} - \mathbf{B} \cdot \nabla \times \mathbf{C}$

(h) $\nabla \cdot [\phi \mathbf{F}] = \phi[\nabla \cdot \mathbf{F}] + \nabla \phi \cdot \mathbf{F}$

(i) $\nabla\{\phi \psi\} = \{\nabla \phi\}\psi + \phi \nabla \psi$

(j) $\nabla \psi \times \mathbf{F} = \nabla \times (\psi \mathbf{F}) - \psi(\nabla \times \mathbf{F})$

(k) $\nabla(1/\mathbf{r}) = -\hat{\mathbf{r}}/r^2$

(l) $\nabla \cdot (\psi \nabla \phi) = \nabla \psi \cdot \nabla \phi + \psi \nabla^2 \phi$

Do (a), (b), (h), (i) and (j) tally with your knowledge of fields, Helmholtz' Theorem and operator behaviour? Why is (f) different from (g)?

**2.**

Derive the expression for $\nabla \cdot \mathbf{A}$ in cylindrical coordinates.

**3.**

Derive the expression for $\nabla \cdot \mathbf{A}$ in spherical coordinates.

**4.**

Derive the expression for $\nabla \times \mathbf{A}$ in cylindrical coordinates.

**5.**

Derive the expression for $\nabla \times \mathbf{A}$ in spherical coordinates.

**6.**

A $\phi$-directed electric field in some region is given by:

$$E = a_0 K \rho^2 z \quad \text{Vm}^{-1}$$

where $a_0$ and $K$ are constants.

(a) Find curl **E** at any point. Is **E** conservative?

(b) Evaluate the line integral of $\mathbf{E} \cdot \mathbf{dl}$ taken about a closed path $l = l_1 + l_2 + l_3 + l_4$ on a circular cylinder of radius 2 and height 3 as illustrated: