University of Technology Sydney
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37233 Linear Algebra Problem Set 4

Note: you may use *Mathematica* to carry out any calculations you feel may be of use.

Question 1.

Let $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} h \\ -3 \\ -5 \end{pmatrix}$. For what values of h vector $\mathbf{y}$ can be written as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$?

Question 2.

Describe and compare the solution sets of $x_1 + 9x_2 - 4x_3 = 0$ and $x_1 + 9x_2 - 4x_3 = 4$.

Question 3.

Wright the general solution to

$$\mathbf{Ax} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}.$$

as the sum of a particular solution to $\mathbf{Ax} = \mathbf{b}$ and the general solution to $\mathbf{Ax} = \mathbf{0}$.

Question 4.

Find all solutions of the linear system $\mathbf{Ax} = \mathbf{0}$ and write the solution in a vector form.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 & 5 \\ 3 & 3 & 6 & 1 & 14 \\ 0 & -1 & 0 & -2 & -9 \end{pmatrix}$$

What is the geometric representation of the solution?

Question 5.

Answer true or false for each of the following, justifying your answers.

(a)   Any linear combination of vectors in $\mathbb{R}^n$ can always be written in the form $A\mathbf{x}$.

(b)   The solution set of a linear system whose augmented matrix has the form $(\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3 \ \mathbf{b})$ is the same as the solution set of $A\mathbf{x} = \mathbf{b}$ if $A = (\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3)$.

(c)   If $A\mathbf{x} = \mathbf{b}$ is inconsistent, then $\mathbf{b}$ is not in the space spanned by the columns of A.

(d)   If the augmented matrix $(A \mid \mathbf{b})$ has a pivot in every row, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent.

(e)   If A is an $n \times m$ matrix whose columns do not span $\mathbb{R}^n$ then $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b} \in \mathbb{R}^n$.

Question 6.

Let

$$\mathbf{v_1} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}, \mathbf{v_2} = \begin{pmatrix} 0 \\ -3 \\ 8 \end{pmatrix}, \mathbf{v_3} = \begin{pmatrix} 4 \\ -1 \\ -5 \end{pmatrix}.$$

Does $[\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}]$ span $\mathbb{R}^3$?

Question 7.

Are the columns of

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 0 & -3 \\ 0 & 12 & -2 \\ 3 & 3 & 4 \end{pmatrix}$$

span $\mathbb{R}^4$? What about the columns of A^T?

Question 8.

Find the value for h, which makes the following system consistent

$$\mathbf{Ax} = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & -1 \\ 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ h \end{pmatrix}.$$