University of Technology Sydney
Department of Mathematical and Physical Sciences

37233 Linear Algebra
Tutorial Assignment 3

Question 1.

Solve the following system using 3-digits precision calculations and compare the solution with the exact solution.

$$A = \begin{pmatrix} 0.03 & 58.9 & 59.2 \\ 5.31 & -6.1 & 47.0 \end{pmatrix}.$$

Question 2.

Let

$$A = \begin{pmatrix} 3 & -1 & 0 & 3 \\ -1 & 3 & 1 & 2 \\ 0 & 2 & 4 & 1 \\ 1 & 0 & 3 & -3 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ -2 \\ 2 \end{pmatrix}.$$

Compute by hand the first three iterations of the Jacobi method, for solving $\mathbf{Ax} = \mathbf{b}$, using $\mathbf{x}^{(0)} = \mathbf{0}$.

Question 3.

Show that the Jacobi iterations in Question 2, can be written as $\mathbf{x^{(k+1)}} = \mathbf{T}\mathbf{x^{(k)}} + \mathbf{c}$, where $\mathbf{T}$ is a 4×4 matrix and $\mathbf{c}$ is a column vector. Wright explicit expression for matrix $\mathbf{T}$ and vector $\mathbf{c}$.

Question 4.

Let

$$A = \begin{pmatrix} 3 & -1 & 0 & 3 \\ -1 & 3 & 1 & 2 \\ 0 & 2 & 4 & 1 \\ 1 & 0 & 3 & -3 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ -2 \\ 2 \end{pmatrix}.$$

Compute by hand the first three iterations of the Gauss–Seidel method for solving $A\mathbf{x} = \mathbf{b}$, using $\mathbf{x}^{(0)} = \mathbf{0}$.

Question 5.

What can you conclude about convergence of the Jacobi method for the system of the previous question, on the basis of the theorems presented in lectures? Justify your answer.