University of Technology Sydney
Department of Mathematical and Physical Sciences

37233 Linear Algebra—Autumn 2017
Problem Set 6 – Solutions – Part B

Note: you may use *Mathematica* to carry out any calculations you feel may be of use.

Question 9.

How many vectors are required in a basis of the subspace H spanned by the vectors: $\begin{pmatrix}1\\0\\2\end{pmatrix}$, $\begin{pmatrix}3\\1\\1\end{pmatrix}$, $\begin{pmatrix}9\\4\\-2\end{pmatrix}$, $\begin{pmatrix}-7\\-3\\1\end{pmatrix}$? Write down a basis for H.

Solution: We have

$$\begin{pmatrix}1&3&9&-7\\0&1&4&-3\\2&1&-2&1\end{pmatrix}\sim\begin{pmatrix}1&3&9&-7\\0&1&4&-3\\0&-5&-20&15\end{pmatrix}\sim\begin{pmatrix}1&3&9&-7\\0&1&4&-3\\0&0&0&0\end{pmatrix}\sim\begin{pmatrix}1&0&-3&2\\0&1&4&-3\\0&0&0&0\end{pmatrix}.$$

Columns 1 and 2 are pivot columns so two vectors are required, and $\left\{(1,0,2)^T,\ (3,1,1)^T\right\}$ is a basis for H.