

37233 Linear Algebra

Assignment 8 - Solutions

Question 1

$A = \{\{1, 2, -3, 1, 1\}, \{-3, 1, 7, -1, 1\}, \{-2, 3, 4, 0, 2\}\};$

`MatrixForm[A]`

$$\begin{pmatrix} 1 & 2 & -3 & 1 & 1 \\ -3 & 1 & 7 & -1 & 1 \\ -2 & 3 & 4 & 0 & 2 \end{pmatrix}$$

`MatrixForm[RowReduce[A]]`

$$\begin{pmatrix} 1 & 0 & -\frac{17}{7} & \frac{3}{7} & -\frac{1}{7} \\ 0 & 1 & -\frac{2}{7} & \frac{2}{7} & \frac{4}{7} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The number of the pivot columns is 2 therefore rank $A = 2$.

Question 2

Dimension of the Row space is 2; $\dim \text{Row } A = 2$

The basis of the row space is given by the vectors

$$\begin{pmatrix} 1 \\ 0 \\ -17/7 \\ 3/7 \\ -1/7 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ -2/7 \\ 2/7 \\ 4/7 \end{pmatrix};$$

Question 3

Dimension of the null space is 2 therefore the rank of the matrix is 7 and the dimension of the Column space of matrix A is 7. The column space of matrix A is a subspace of \mathbb{R}^7 therefore the columns of A span all \mathbb{R}^7 (given the dimension of col A is 7) and the linear system $Ax=b$ will always have a solution for any b.

Question 4

$x=5b_1+3b_2 = 5*(-c_1+4c_2)+3*(5c_1-3c_2) = -5c_1+20c_2+15c_1-9c_2 = 10c_1+11c_2$. So $[x]_C = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$.

Question 5

$P_{bc} = \{\{4, 5, 7, 2\}, \{1, 2, -2, -1\}\}$

$\{\{4, 5, 7, 2\}, \{1, 2, -2, -1\}\}$

`MatrixForm[%]`

$$\begin{pmatrix} 4 & 5 & 7 & 2 \\ 1 & 2 & -2 & -1 \end{pmatrix}$$

`RowReduce[Pbc]`

$\{\{1, 0, 8, 3\}, \{0, 1, -5, -2\}\}$

`MatrixForm[%]`

$$\begin{pmatrix} 1 & 0 & 8 & 3 \\ 0 & 1 & -5 & -2 \end{pmatrix}$$

The Change of coordinate matrix from B to C is:

$$P = \begin{pmatrix} 8 & 3 \\ -5 & -2 \end{pmatrix}$$