# University of Technology Sydney
## Department of Mathematical and Physical Sciences

## 37233 Linear Algebra
## Problem Set 3

**Question 1.**

Find $\|\mathbf{x}\|_1$, $\|\mathbf{x}\|_2$ and $\|\mathbf{x}\|_\infty$ for

(a) $\mathbf{x} = (3, -4, 0, 3/2)$;

(b) $\mathbf{x} = (\sin k, \cos k, 2^k)$, for $k > 1$;

**Question 2.**

Find the limit of the sequence $\{\mathbf{x}^{(k)}\}_{k=1}^{\infty}$ defined by

$$\mathbf{x}^{(k)} = \left(\frac{1}{k}, 1 - e^{1-k}, \frac{-2}{k^2}\right)$$

**Question 3.**

Compute by hand the first two iterations ($\mathbf{x}^{(\mathbf{1})}$ and $\mathbf{x}^{(\mathbf{2})}$) of the Jacobi method for the following linear system, using $\mathbf{x}^{(0)} = \mathbf{0}$.

$$\begin{array}{rcrcrcl} 3x_1 & - & x_2 & + & x_3 & = & 1, \\ 3x_1 & + & 6x_2 & + & 2x_3 & = & 0, \\ 3x_1 & + & 3x_2 & + & 7x_3 & = & 4. \end{array}$$

**Question 4.**

Compute by hand the first two iterations ($\mathbf{x}^{(\mathbf{1})}$ and $\mathbf{x}^{(\mathbf{2})}$) of the Gauss-Seidel method for the following linear system, using $\mathbf{x}^{(0)} = \mathbf{0}$.

$$\begin{array}{rcrcrcl} 10x_1 & - & x_2 & & & = & 9, \\ -x_1 & + & 10x_2 & - & 2x_3 & = & 7, \\ & & -2x_2 & + & 10x_3 & = & 6. \end{array}$$

**Question 5.**

Use the theorems from the lectures to try to determine whether Jacobi's method converges for the system

$$\begin{pmatrix} 3 & 1 & -1 \\ 0 & 2 & 1 \\ 1 & 2 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

.../Over

**Question 6.**

Show that, for the system in Question 3 the Jacobi iteration equation may be written in the form

$$\mathbf{x}^{(k+1)} = T\mathbf{x}^{(k)} + \mathbf{c}$$

where

$$T = \begin{pmatrix} 0 & \frac{1}{3} & \frac{-1}{3} \\ \frac{-1}{2} & 0 & \frac{-1}{3} \\ \frac{-3}{7} & \frac{-3}{7} & 0 \end{pmatrix}, \qquad \mathbf{c} = \begin{pmatrix} \frac{1}{3} \\ 0 \\ \frac{4}{7} \end{pmatrix}.$$

**Question 7.**

Show that, the system in Question 4 the Gauss-Seidel iteration equation may be written in the form,

$$\mathbf{x}^{(k+1)} = L\mathbf{x}^{(k+1)} + U\mathbf{x}^{(k)} + \hat{\mathbf{c}},$$

where $\mathbf{L}$ is lower triangular matrix, and $\mathbf{U}$ is upper triangular matrix

Hence find a matrix $\mathbf{T}$ (in terms of $\mathbf{L}$ and $\mathbf{U}$) and a vector $\mathbf{c}$ such that the Gauss-Seidel iteration equation can be written in the form

$$\mathbf{x}^{(k+1)} = T\mathbf{x}^{(k)} + \mathbf{c}.$$

Above representation can be done for an arbitrary $n \times n$ system of equations $A\mathbf{x} = \mathbf{b}$, find a lower triangular matrix $L$, an upper triangular matrix $U$ and a vector $\hat{\mathbf{c}}$ such that the Gauss-Seidel iteration equation can be written in the form

**Question 8.**

Solve (by hand)

$$\begin{aligned} \frac{1}{3}x_1 + \frac{1}{2}x_2 + \frac{1}{4}x_3 &= -1 \\ \frac{1}{4}x_1 + \frac{1}{3}x_2 + \frac{1}{5}x_3 &= 0 \\ \frac{1}{2}x_1 + \phantom{\frac{1}{2}}x_2 + \frac{1}{3}x_3 &= 2 \end{aligned}$$

using partial pivoting.