

UNIVERSITY OF TECHNOLOGY SYDNEY
SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES
37233 LINEAR ALGEBRA

Tutorials 2019 — Assignment 7 (40 marks)

Question 1

(30 marks)

For each of the two examples below:

1. (15 marks)

$$\mathbf{A} = \begin{bmatrix} 1 & 7 & -6 & -1 & -3 \\ 1 & -8 & 9 & -3 & 10 \\ -2 & -9 & 7 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} 2 \\ -9 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

2. (15 marks)

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 7 & 8 \\ 0 & -3 & 3 \\ -1 & 3 & -8 \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} 0 \\ 2 \\ -6 \\ 10 \end{bmatrix}$$

- (a) Obtain explicit descriptions for $\text{Col } \mathbf{A}$, $\text{Nul } \mathbf{A}$, $\text{Row } \mathbf{A}$. (3)
- (b) Find bases for $\text{Col } \mathbf{A}$, $\text{Nul } \mathbf{A}$, $\text{Row } \mathbf{A}$. (3)
- (c) Determine $\dim(\text{Col } \mathbf{A})$, $\dim(\text{Nul } \mathbf{A})$, $\dim(\text{Row } \mathbf{A})$. (3)
- (d) Check if \mathbf{x} or \mathbf{y} belong to $\text{Col } \mathbf{A}$, to $\text{Nul } \mathbf{A}$, or to $\text{Row } \mathbf{A}$. (6)

Question 2

(10 marks)

For a homogeneous system of 7 linear equations with 8 unknowns, it has been realised that 2 (and only 2) of the equations are linear combinations of the other equations.

- (a) Specify the rank of the matrix \mathbf{A} of the system.
- (b) Find $\dim(\text{Nul } \mathbf{A})$.
- (c) Determine if $\mathbf{Ax} = \mathbf{b}$ has a solution for any \mathbf{b} .
- (d) Specify the domain and codomain of the linear transformation $\mathcal{T} : \mathbf{x} \mapsto \mathbf{Ax}$.
- (e) Find the dimension of the range of \mathcal{T} .