

Solutions for Assignment 6

Question 1

The columns of matrix A are linearly dependent because the number of columns are greater than the number of rows.

Question 2

```
A = {{1, 1, 0, 0}, {1, 0, 0, 1}, {0, 1, 1, 0}, {0, 0, 1, 1}}  
{{1, 1, 0, 0}, {1, 0, 0, 1}, {0, 1, 1, 0}, {0, 0, 1, 1}}
```

```
A // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

```
MatrixForm[RowReduce[A]]
```

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

There are free variables so the columns are linearly dependent.

Question 3

```
A = {{1, 1, 2, -2}, {2, 3, 1, 3},  
      {1, -1, 0, 0}, {2, 2, 1, 2}}  
{{1, 1, 2, -2}, {2, 3, 1, 3}, {1, -1, 0, 0}, {2, 2, 1, 2}}
```

```
MatrixForm[A]
```

$$\begin{pmatrix} 1 & 1 & 2 & -2 \\ 2 & 3 & 1 & 3 \\ 1 & -1 & 0 & 0 \\ 2 & 2 & 1 & 2 \end{pmatrix}$$

`MatrixForm[RowReduce[A]]`

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

If the columns of matrix A are v_1, v_2, v_3, v_4 then $-v_1 - v_2 + 2v_3 + v_4 = 0$

Question 4

The columns of this matrix are inevitably linearly dependent. There are more columns than rows.

Basis vectors must be linearly independent.

Question 5

$$u_1 = \{1, 2, 3, 4\}$$

$$u_2 = \{1, 2, 3, 0\}$$

$$u_3 = \{1, 2, 0, 0\}$$

$$u_4 = \{1, 0, 0, 0\}$$

$$0 \cdot u_1 - 2 \cdot u_2 + u_3 + 2 \cdot u_4$$

$$\{1, -2, -6, 0\}$$

$$x = \begin{pmatrix} 1 \\ -2 \\ -6 \\ 0 \end{pmatrix}$$