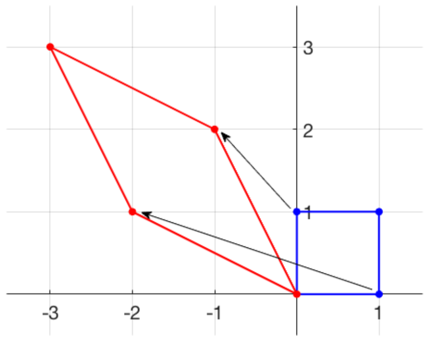
University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Tutorials 2019 — Assignment 6 (40 marks)

Question 1 (10 marks)



(a) Find the standard matrix $\mathbf{T}$ for a mapping which action (shown in red) on a unit square (blue) is depicted in the picture (the corners are mapped as shown by the arrows).

(b) Use $\mathbf{T}$ to verify the image of the top-right corner of the unit square is where expected.

(c) Apply $\mathbf{T}$ to the image of the original square, and depict the resulting secondary image.

(d) Describe the result obtained in (c) in terms of a linear transformation from the unit square, finding the corresponding standard matrix.

Question 2 (10 marks)

Consider vectors

$$\mathbf{b}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 7 \\ -2 \end{bmatrix}; \qquad \mathbf{c}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad \mathbf{c}_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}.$$

(a) Show that $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ are bases for $\mathbb{R}^2$.

(b) Find the $\mathcal{B}$-coordinates of $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(c) Find the change of coordinates matrix $\mathbf{P}_{\mathcal{C}\leftarrow\mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$ and use it to find $[\mathbf{x}]_{\mathcal{C}}$.

(d) Find the $\mathcal{C}$-coordinates of $\mathbf{y} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$.

(e) Find the change of coordinates matrix $\mathbf{P}_{\mathcal{B}\leftarrow\mathcal{C}}$ from $\mathcal{C}$ to $\mathcal{B}$ and use it to find $[\mathbf{y}]_{\mathcal{B}}$.

Question 3 (10 marks)

Suppose $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ are bases for a vector space V, even though we do not know the coordinates of all those vectors relative to the standard basis.

However, we know that $\mathbf{b}_1 = -\mathbf{c}_1 + 3\mathbf{c}_2$ and $\mathbf{b}_2 = 2\mathbf{c}_1 - 4\mathbf{c}_2$.

(a) Show that if $\mathcal{C}$ is a basis, then $\mathcal{B}$ is also a basis.

(b) Find $[\mathbf{x}]_{\mathcal{C}}$ given that $\mathbf{x} = 5\mathbf{b}_1 + 3\mathbf{b}_2$.

(c) Find $[\mathbf{y}]_{\mathcal{B}}$ given that $\mathbf{y} = 3\mathbf{c}_1 - 5\mathbf{c}_2$.

Question 4 (10 marks)

The first four of the Hermite polynomials are:

$h_1 = 1, \quad h_2 = 2t, \quad h_3 = 4t^2 - 2, \quad \text{and} \quad h_4 = 8t^3 - 12t.$

As it was shown at the tutorials, the above set forms a basis $\mathcal{H}$ for $\mathbb{P}^3$.

Consider another set of polynomials in $\mathbb{P}^3$:

$q_1 = 1, \quad q_2 = 1 + t, \quad q_3 = 1 + t + t^2, \quad \text{and} \quad q_4 = 1 + t + t^2 + t^3.$

(a) Check whether or not the set $\{q_i\}$ forms a basis $\mathcal{Q}$ in $\mathbb{P}^3$.

(b) Find the change of coordinates matrix from basis $\mathcal{H}$ to basis $\mathcal{Q}$.

(c) Find the coordinates of $r = 3t^3 + 2t^2 + t$ relative to $\mathcal{H}$.

(d) Use the $\mathbf{P}_{\mathcal{Q}\leftarrow\mathcal{H}}$ matrix found in (b) to find $[r]_{\mathcal{Q}}$.