37233 Linear Algebra

Assignment 3 - Solutions

Question 1

```
a = {{0.03, 58.9}, {5.31, -6.10}}
{{0.03, 58.9}, {5.31, -6.1}}

b = {59.2, 47.0}
{59.2, 47.}

LinearSolve[a, b]
{10., 1.}
```

Now carry out the row reduction step by step, arraying p significant figures at each step. First form the augmented matrix:

```
aug // MatrixForm
```
$$\begin{pmatrix} 0.03 & 58.9 & 59.2 \\ 5.31 & -6.1 & 47. \end{pmatrix}$$

Now row reduce

```
aug // MatrixForm
```
$$\begin{pmatrix} 0.03 & 58.9 & 59.2 \\ 5.31 & -6.1 & 47. \end{pmatrix}$$

$$\begin{pmatrix} 1.00 & 1.96 \times 10^3 & 1.97 \times 10^3 \\ 5.31 & -6.10 & 47.0 \end{pmatrix}$$

$$\begin{pmatrix} 1.00 & 1.96 \times 10^3 & 1.97 \times 10^3 \\ 0 & -1.04 \times 10^4 & -1.04 \times 10^4 \end{pmatrix}$$

$$\begin{pmatrix} 1.00 & 1.96 \times 10^3 & 1.97 \times 10^3 \\ 0 & 1.0 & 1.0 \end{pmatrix}$$

$$\begin{pmatrix} 1.00 & 0 & 10.0 \\ 0 & 1.0 & 1.0 \end{pmatrix}$$

The solution should be the last column

```
solution = {10.0., 1.00}
```

In this case, three - digit precision is sufficient for the calculation

Question 2

Jacobi iterations

```
A = {{3, -1, 0, 3}, {-1, 3, 1, 2}, {0, 2, 4, 1}, {1, 0, 3, -3}};
b = {1, 3, -2, 2};
```

	x1	x2	x3	x4
0	0.	0.	0.	0.
1	0.333333	1.	−0.5	−0.666667
2	1.33333	1.72222	−0.833333	−1.05556
3	1.96296	2.42593	−1.09722	−1.05556

Question 3

$$
\begin{pmatrix} x_1^{(k+1)} \\ x_2^{(k+1)} \\ x_3^{(k+1)} \\ x_4^{(k+1)} \end{pmatrix} = \begin{pmatrix} 0 & 1/3 & 0 & -1 \\ 1/3 & 0 & -1/3 & -2/3 \\ 0 & -1/2 & 0 & -1/4 \\ 1/3 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1^{(k)} \\ x_2^{(k)} \\ x_3^{(k)} \\ x_4^{(k)} \end{pmatrix} + \begin{pmatrix} 1/3 \\ 1 \\ -1/2 \\ -2/3 \end{pmatrix}
$$

$$
T = \begin{pmatrix} 0 & 1/3 & 0 & -1 \\ 1/3 & 0 & -1/3 & -2/3 \\ 0 & -1/2 & 0 & -1/4 \\ 1/3 & 0 & 1 & 0 \end{pmatrix}; \qquad C = \begin{pmatrix} 1/3 \\ 1 \\ -1/2 \\ -2/3 \end{pmatrix}
$$

$$
\mathbf{x}^{(k+1)} = T\mathbf{x}^{(k)} + C
$$

Question 4

Gauss-Seidel iterations

	x1	x2	x3	x4
0	0.	0.	0.	0.
1	0.333333	1.11111	−1.05556	−1.61111
2	2.31481	3.19753	−1.69599	−1.59105
3	2.99023	3.62277	−1.91362	−1.58355

Question 5

The Matrix is not diagonally dominant, so the iterations may converge or diverge.