University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Tutorials 2019 — Assignment 2 (40 marks)

Question 1 (8 marks)

Use Doolittle's method to find **LU** decomposition of the matrix

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & -1 & 0 \\ 9 & 1 & -3 & 4 \\ -6 & 0 & -2 & 2 \\ 0 & 2 & 4 & 8 \end{bmatrix}$$

Question 2 (8 marks)

Use the **LU** decomposition obtained in Question 1 to solve the system of equations

$$\begin{aligned} 3x_1 - x_3 &= 0 \\ 9x_1 + x_2 - 3x_3 + 4x_4 &= 18 \\ -6x_1 - 2x_3 + 2x_4 &= -4 \\ 2x_2 + 4x_3 + 8x_4 &= 48 \end{aligned}$$

Question 3 (10 marks)

Let

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 3 & -1 & -2 \\ 2 & -4 & 5 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} -2 \\ 10 \\ 3 \end{bmatrix}.$$

(a) Use Crout's method to find **LU** decomposition of $\mathbf{A}$.

(b) Use the decomposition obtained in part (a) to solve the system $\mathbf{Ax} = \mathbf{b}$.

(see next page)

Question 4 (10 marks)

Let

$$\mathbf{A} = \begin{bmatrix} 4 & 6 & -2 \\ 6 & 10 & -1 \\ -2 & -1 & 14 \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ -10 \end{bmatrix}.$$

(a) Use Cholesky's method to find $\mathbf{U}^{\mathsf{T}}\mathbf{U}$ decomposition of $\mathbf{A}$.

(b) Use the decomposition obtained in part (a) to solve the system $\mathbf{A}\mathbf{x} = \mathbf{b}$.

Question 5 (4 marks)

For each of the matrices specified in Questions 1, 3, and 4, please state (and explain) whether one could be certain that the requested decompositions exist prior to an attempt to construct a factorisation.