37233 Linear Algebra

Problem Sheet 8 Solutions - Part b

Question 7

Orthogonal basis :

In[8]:=
```
w1 = v1 = {0, 4, 2}
```
Out[8]=
$$\{0, 4, 2\}$$

In[9]:=
```
v2 = {5, 6, 7}
```
Out[9]=
$$\{5, 6, 7\}$$

In[10]:=
```
w2 = v2 - (v2.w1) / (w1.w1) w1
```
Out[10]=
$$\left\{5, -\frac{8}{5}, \frac{16}{5}\right\}$$

Check orthogonality :

In[11]:=
```
w1.w2
```
Out[11]=
$$0$$

Orthonormal basis :

In[12]:=
```
u1 = w1 / Norm[w1]
```
Out[12]=
$$\left\{0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}$$

In[15]:=
```
u2 = w2 / Norm[w2]
```
Out[15]=
$$\left\{\frac{5\sqrt{\frac{5}{21}}}{3}, -\frac{8}{3\sqrt{105}}, \frac{16}{3\sqrt{105}}\right\}$$

Question 8

Part a

```
w1 = v1 = {1, 0, 2}
```
$$\{1, 0, 2\}$$

```
v2 = {0, -2, 1}
```
$$\{0, -2, 1\}$$

```
v3 = {1, 1, 0}
```

$$\{1, 1, 0\}$$

```
w2 = v2 - (v2.w1) / (w1.w1) w1
```

$$\left\{-\frac{2}{5}, -2, \frac{1}{5}\right\}$$

```
w3 = v3 - (v3.w1) / (w1.w1) w1 - (v3.w2) / (w2.w2) w2
```

$$\left\{\frac{4}{7}, -\frac{1}{7}, -\frac{2}{7}\right\}$$

```
q1 = w1 / Norm[w1]
```

$$\left\{\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right\}$$

```
q2 = w2 / Norm[w2]
```

$$\left\{-\frac{2}{\sqrt{105}}, -2\sqrt{\frac{5}{21}}, \frac{1}{\sqrt{105}}\right\}$$

```
q3 = w3 / Norm[w3]
```

$$\left\{\frac{4}{\sqrt{21}}, -\frac{1}{\sqrt{21}}, -\frac{2}{\sqrt{21}}\right\}$$

Part b

```
MatrixForm[qtranspose = {q1, q2, q3}]
```

$$\begin{pmatrix} \frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{105}} & -2\sqrt{\frac{5}{21}} & \frac{1}{\sqrt{105}} \\ \frac{4}{\sqrt{21}} & -\frac{1}{\sqrt{21}} & -\frac{2}{\sqrt{21}} \end{pmatrix}$$

```
q = Transpose[qtranspose]
```

$$\left\{\left\{\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{105}}, \frac{4}{\sqrt{21}}\right\}, \left\{0, -2\sqrt{\frac{5}{21}}, -\frac{1}{\sqrt{21}}\right\}, \left\{\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{105}}, -\frac{2}{\sqrt{21}}\right\}\right\}$$

```
v = Transpose[{v1, v2, v3}]
```

$$\{\{1, 0, 1\}, \{0, -2, 1\}, \{2, 1, 0\}\}$$

```
r = qtranspose.v
```

$$\left\{\left\{\sqrt{5}, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}, \left\{0, 4\sqrt{\frac{5}{21}} + \frac{1}{\sqrt{105}}, -2\sqrt{\frac{5}{21}} - \frac{2}{\sqrt{105}}\right\}, \left\{0, 0, \sqrt{\frac{3}{7}}\right\}\right\}$$

```
r // Simplify // MatrixForm
```

$$\begin{pmatrix} \sqrt{5} & \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ 0 & \sqrt{\frac{21}{5}} & -4\sqrt{\frac{3}{35}} \\ 0 & 0 & \sqrt{\frac{3}{7}} \end{pmatrix}$$

Check that QR = A

```
q.r == v
```

```
True
```

Question 9

Part a

```
w1 = v1 = {3, 1, -1, 3}
```

```
{3, 1, -1, 3}
```

```
v2 = {-5, 1, 5, -7}
```

```
{-5, 1, 5, -7}
```

```
v3 = {1, 1, -2, 8}
```

```
{1, 1, -2, 8}
```

```
w2 = v2 - (v2.w1) / (w1.w1) w1
```

```
{1, 3, 3, -1}
```

```
w3 = v3 - (v3.w1) / (w1.w1) w1 - (v3.w2) / (w2.w2) w2
```

```
{-3, 1, 1, 3}
```

```
q1 = w1 / Norm[w1]
```

$$\left\{\frac{3}{2\sqrt{5}}, \frac{1}{2\sqrt{5}}, -\frac{1}{2\sqrt{5}}, \frac{3}{2\sqrt{5}}\right\}$$

```
q2 = w2 / Norm[w2]
```

$$\left\{\frac{1}{2\sqrt{5}}, \frac{3}{2\sqrt{5}}, \frac{3}{2\sqrt{5}}, -\frac{1}{2\sqrt{5}}\right\}$$

```
q3 = w3 / Norm[w3]
```

$$\left\{-\frac{3}{2\sqrt{5}}, \frac{1}{2\sqrt{5}}, \frac{1}{2\sqrt{5}}, \frac{3}{2\sqrt{5}}\right\}$$

Part b

```
MatrixForm[qtranspose = {q1, q2, q3}]
```

$$\begin{pmatrix} \frac{3}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & \frac{3}{2\sqrt{5}} \\ \frac{1}{2\sqrt{5}} & \frac{3}{2\sqrt{5}} & \frac{3}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} \\ -\frac{3}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{3}{2\sqrt{5}} \end{pmatrix}$$

```
q = Transpose[qtranspose]
```

$$\left\{\left\{\frac{3}{2\sqrt{5}}, \frac{1}{2\sqrt{5}}, -\frac{3}{2\sqrt{5}}\right\}, \left\{\frac{1}{2\sqrt{5}}, \frac{3}{2\sqrt{5}}, \frac{1}{2\sqrt{5}}\right\}, \left\{-\frac{1}{2\sqrt{5}}, \frac{3}{2\sqrt{5}}, \frac{1}{2\sqrt{5}}\right\}, \left\{\frac{3}{2\sqrt{5}}, -\frac{1}{2\sqrt{5}}, \frac{3}{2\sqrt{5}}\right\}\right\}$$

```
v = Transpose[{v1, v2, v3}]
```

```
{{3, -5, 1}, {1, 1, 1}, {-1, 5, -2}, {3, -7, 8}}
```

```
r = qtranspose.v
```

$$\left\{\left\{2\sqrt{5}, -4\sqrt{5}, 3\sqrt{5}\right\}, \left\{0, 2\sqrt{5}, -\sqrt{5}\right\}, \left\{0, 0, 2\sqrt{5}\right\}\right\}$$

```
r // Simplify // MatrixForm
```

$$\begin{pmatrix} 2\sqrt{5} & -4\sqrt{5} & 3\sqrt{5} \\ 0 & 2\sqrt{5} & -\sqrt{5} \\ 0 & 0 & 2\sqrt{5} \end{pmatrix}$$

Check that QR = A

```
q.r == v
```

```
True
```