University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Tutorials 2019 — Assignment 8 (40 marks)

Question 1 (10 marks)

Let $\mathbf{y} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 8 \\ -6 \end{bmatrix}$.

(a) Write an orthogonal decomposition of $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$ where $\hat{\mathbf{y}} = \operatorname{proj}_{\mathbf{u}} \mathbf{y}$ and $\mathbf{z} \perp \mathbf{u}$.

(b) Compute the distance from $\mathbf{y}$ to the line through $\mathbf{u}$ and the origin.

Question 2 (10 marks)

Let $\mathbf{u}_1 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -15 \\ 2 \\ 4 \end{bmatrix}$, $\mathbf{y}_1 = \begin{bmatrix} 5 \\ 3 \\ 5 \end{bmatrix}$, $\mathbf{y}_2 = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$.

Consider $W = \operatorname{Span}\{\mathbf{u}_1,\ \mathbf{u}_2\}$. For each $\mathbf{y}_i$:

(a) obtain an orthogonal decomposition $\mathbf{y}_i = \hat{\mathbf{y}}_i + \mathbf{z}_i$ with $\hat{\mathbf{y}}_i = \operatorname{proj}_W \mathbf{y}_i$ and $\mathbf{z}_i \in W^{\perp}$;

(b) compute the distance from $\mathbf{y}_i$ to W.

Question 3 (10 marks)

Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 4 \\ 2 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ be a basis for $W = \operatorname{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

(a) Construct an orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for W using the Gram–Schmidt process.

(b) Obtain an orthonormal basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ from the orthogonal set found in (a).

Question 4 (10 marks)

Is it possible to construct a square matrix $\mathbf{A}$ such that certain $\mathbf{x}$ may at once belong to:

(a) $\operatorname{Col}\mathbf{A}$ and $\operatorname{Row}\mathbf{A}$; (b) $\operatorname{Row}\mathbf{A}$ and $\operatorname{Nul}\mathbf{A}$; (c) $\operatorname{Nul}\mathbf{A}$ and $\operatorname{Col}\mathbf{A}$.

Justify the answer for each case with a proof or with an example.