University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Tutorial 6

Question 1.

Illustrate the effect of a linear transformation $\mathcal{T}$ with the standard matrix $\mathbf{T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ by mapping its action on a triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.

Question 2.

Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{y}_1 = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{y}_2 = \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix}$, $\mathbf{y}_3 = \begin{bmatrix} -6 \\ 3 \\ 5 \end{bmatrix}$.

Let $\mathcal{T} : \mathbb{R}^3 \mapsto \mathbb{R}^3$ be a linear transformation that maps $\mathbf{e}_i$ to $\mathbf{y}_i$.

(a) Write the standard matrix representation of $\mathcal{T}$.

(b) Find the image of $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

(c) Find the standard matrix of the reverse transformation from $\mathbf{y}_i$ to $\mathbf{e}_i$.

(d) Using the reverse transformation, obtain the image of $[\mathbf{u}]_{\mathcal{Y}} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$.

Question 3.

Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \dots \mathbf{b}_n\}$ be a basis for a vector space V. Explain why the $\mathcal{B}$-coordinates of vectors $\mathbf{b}_1, \dots \mathbf{b}_n$ are the columns $\mathbf{e}_1, \dots \mathbf{e}_n$ of the $n \times n$ identity matrix.

Question 4.

Let $\mathcal{B} = \{\mathbf{b}_1,\ \mathbf{b}_2,\ \mathbf{b}_3\}$ and $\mathcal{C} = \{\mathbf{c}_1,\ \mathbf{c}_2,\ \mathbf{c}_3\}$ be bases for a vector space V, and suppose $\mathbf{b}_1 = 2\mathbf{c}_1 - \mathbf{c}_2$, $\mathbf{b}_2 = 2\mathbf{c}_2 + 7\mathbf{c}_3$, and $\mathbf{b}_3 = \mathbf{c}_1 + 2\mathbf{c}_3$.

(a) Find the change-of-coordinate matrices from $\mathcal{B}$ to $\mathcal{C}$, and from $\mathcal{C}$ to $\mathcal{B}$.

(b) Find $[\mathbf{x}]_{\mathcal{C}}$ for $[\mathbf{x}]_{\mathcal{B}} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$, and find $[\mathbf{x}]_{\mathcal{B}}$ for $[\mathbf{x}]_{\mathcal{C}} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$.

Question 5.

Let

$$\mathbf{B} = \begin{bmatrix} 6 & 0 & 0 & 4 \\ 4 & 2 & 0 & 0 \\ 0 & -8 & 4 & 0 \\ 0 & 0 & 2 & -6 \end{bmatrix} \qquad \text{and} \qquad \mathbf{C} = \begin{bmatrix} 1 & 5 & -5 & 5 \\ 1 & 3 & -1 & 1 \\ 6 & -6 & -2 & 6 \\ 4 & -4 & 4 & -2 \end{bmatrix}$$

(a) Check if the columns of $\mathbf{B}$ form a basis $\mathcal{B}$ for $\mathbb{R}^4$, and if the columns of $\mathbf{C}$ form another basis $\mathcal{C}$ for $\mathbb{R}^4$.

(b) Find $\mathcal{B}$-coordinates of $\mathbf{x} = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}$.

(c) Find the change-of-coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$.

(d) Using the above matrix, find $[\mathbf{x}]_{\mathcal{C}}$.

It would be helpful to use Mathematica to facilitate the required calculations.

Question 6.

The Chebyshev polynomials are widely used in calculus and mathematical methods. The first five Chebyshev polynomials (of the first kind) are given by

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_2(x) &= 2x^2 - 1 \\ T_3(x) &= 4x^3 - 3x \\ T_4(x) &= 8x^4 - 8x^2 + 1 \end{aligned}$$

(a) Confirm that the set of the above Chebyshev polynomials is a basis Θ for $\mathbb{P}^4$.

(b) Find the change of coordinates matrix from the standard basis $\mathcal{P}$ in $\mathbb{P}^4$, $\{x^0, x^1, x^2, x^3, x^4\}$, to the Chebyshev basis.

(c) Define f as a polynomial of Maclaurin series for $\cos(4x)$, keeping the terms up to the fourth power in x, and write $[f]_{\mathcal{P}}$.

(d) Using the change of coordinate matrix found in (b), find $[f]_{\Theta}$.

It may be helpful to use Mathematica to facilitate the required calculations.