# 37233 Linear Algebra
# Problem set 2 Solutions

## Question 1 (a)
### i)

```
A = MatrixForm[{{2, 1, 3}, {-6, -6, -5}, {10, 11, 6}}]
```

$$\begin{pmatrix} 2 & 1 & 3 \\ -6 & -6 & -5 \\ 10 & 11 & 6 \end{pmatrix}$$

```
L = MatrixForm[{{1, 0, 0}, {-3, 1, 0}, {5, -2, 1}}]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 5 & -2 & 1 \end{pmatrix}$$

```
U = MatrixForm[{{2, 1, 3}, {0, -3, 4}, {0, 0, -1}}]
```

$$\begin{pmatrix} 2 & 1 & 3 \\ 0 & -3 & 4 \\ 0 & 0 & -1 \end{pmatrix}$$

$Ax = LUx = Ly = b; Ux = y$

First solve Ly=b using forward substitution.

$$\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 5 & -2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

So $y_1=-1$; $-3y_1+y_2=2 \Rightarrow y_2=-1$; $5y_1-2y_2+y_3=-1 \Rightarrow y_3=2$

$$y = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

Now we solve Ux=y

$$\begin{pmatrix} 2 & 1 & 3 \\ 0 & -3 & 4 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

So $x_3=-2$; $-3x_2+4x_3=-1 \Rightarrow x_2=(1+4x_3)/3=-7/3$; $x_1=11/3$

$$\text{So } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 11/3 \\ -7/3 \\ -2 \end{pmatrix}$$

## Question 1(a) (ii)

```
L = MatrixForm[{{2, 0, 0}, {-6, -3, 0}, {10, 6, -1}}]
```

$$\begin{pmatrix} 2 & 0 & 0 \\ -6 & -3 & 0 \\ 10 & 6 & -1 \end{pmatrix}$$

```
U = MatrixForm[{{1, 1/2, 3/2}, {0, 1, -4/3}, {0, 0, 1}}]
```

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{4}{3} \\ 0 & 0 & 1 \end{pmatrix}$$

**Setting Ax = LUx = Ly = b; Ux = y we first solve Ly=b using forward substitution.**

$$\begin{pmatrix} 2 & 0 & 0 \\ -6 & -3 & 0 \\ 10 & 6 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

**So** $y_1=-1/2$; $-6y_1-3y_2=2 \Rightarrow y_2=1/3$; $10y_1+6y_2-y_3=-1 \Rightarrow y_3=-2$

$$y = \begin{pmatrix} -1/2 \\ 1/3 \\ -2 \end{pmatrix}$$

**Now we solve Ux=y**

$$\begin{pmatrix} 1 & 1/2 & 3/2 \\ 0 & 1 & 4/3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 1/3 \\ -2 \end{pmatrix}$$

**So** $x_3=-2$; $x_2+4x_3/3=1/3 \Rightarrow x_2=(1+4x_3)/3=-7/3$; $x_1=11/3$

**So** $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 11/3 \\ -7/3 \\ -2 \end{pmatrix}$

**(b)** $A=LDU=\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 5 & -2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1/2 & 3/2 \\ 0 & 1 & 4/3 \\ 0 & 0 & 1 \end{pmatrix}$

# Question 2

$U=\begin{pmatrix} 2 & 1 & -1 & 0 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 2 & -5 \\ 0 & 0 & 0 & 4 \end{pmatrix}$ **; So** $U^T=\begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & -3 & 2 & 0 \\ 0 & 1 & -5 & 4 \end{pmatrix}$

**Setting Ax** $=U^T Ux=U^T y=b$, **Ux=y. First solve** $U^T y=b$ **using forward substitution we obtain** $y=\begin{pmatrix} 7 \\ 10 \\ -9 \\ 4 \end{pmatrix}$**; Then we use backward substitution in Ux=y to obtain** $x=\begin{pmatrix} 1 \\ 3 \\ -2 \\ 1 \end{pmatrix}$

# Question 3

**No, the matrix is not strictly diagonally dominant because of the last row $|6|=2+4$.**

# Question 4

**Yes, because the matrix is diagonaly dominanant**

# Question 5

(a)

$$E_1=\begin{pmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ 0 & 0 & 1\end{pmatrix},\qquad E_2=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -3 & 0 & 1\end{pmatrix};\qquad E_3=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & -6/7 & 1\end{pmatrix};$$

(b)

$E_3\, E_2 E_1 Ax = E_3\, E_2\, E_1 b$

$U = E_3\, E_2 E_1 A$, $\qquad y = E_3\, E_2\, E_1 b$, $\qquad$ So $Ux=y$.

$$U=\begin{pmatrix}1 & 3 & 3\\ 0 & -7 & -5\\ 0 & 0 & 2/7\end{pmatrix}$$

**Using backward substituton we obtain**

$$\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}-15\\ -12\\ 17\end{pmatrix}$$

(c) $\quad L=(E_3\, E_2\, E_1)^{-1}=\begin{pmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 6/7 & 1\end{pmatrix};$