# University of Technology Sydney
# Department of Mathematical and Physical Sciences

## 37233 Linear Algebra Problem Set 9

**Note:** you may use *Mathematica* to carry out any calculations you feel may be of use.

**Question 1.**

Find the best least squares solution to the following (inconsistent) set of equations.

$$\begin{aligned} x + 3y &= 2 \\ 4x + \phantom{3}y &= 1 \\ 2x - \phantom{3}y &= 0 \\ 3x + \phantom{3}y &= -2 \end{aligned}$$

**Question 2.**

Find the line of best fit through the following data points $(x, y)$.

| $x$ | $y$ |
|---|---|
| -0.291996 | 3.13651 |
| 0.664258 | 9.06364 |
| 2.08586 | 12.1178 |
| 2.64251 | 13.9248 |
| 4.08646 | 18.0404 |
| 5.41087 | 20.9689 |

**Question 3.**

By hand, find the eigenvalues and corresponding eigenvectors of each of the matrices below. In each case, determine whether or not it is possible to find a matrix $P$ and a diagonal matrix $D$ such that $A = PDP^{-1}$. If it is possible, find the matrices $P$ and $D$. If it is not possible, explain why this is so.

(a) $$A = \begin{pmatrix} 3 & -1 & -1 \\ 0 & 2 & 0 \\ 2 & -2 & 0 \end{pmatrix}.$$

(b) $$A = \begin{pmatrix} 3 & -1 & -1 \\ -2 & 3 & 1 \\ 4 & -3 & -1 \end{pmatrix}.$$

**Question 4.**

Orthogonally diagonalize the matrix $A$ given below and construct its spectral decomposition.

$$A = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}.$$

.../Over

**Question 5.**

Suppose $\mathbf{A}$ is a symmetric $n \times n$ matrix and $\mathbf{B}$ is any $n \times m$ matrix. Show that $\mathbf{B^T A B}$, $\mathbf{B^T B}$ and $\mathbf{B B^T}$ are symmetric matrices.

**Question 6.**

Show that if $\mathbf{A}$ is an $n \times n$ symmetric matrix then $(\mathbf{Ax}) \cdot \mathbf{y} = \mathbf{x} \cdot \mathbf{Ay}$ for all $\mathbf{x}, \mathbf{y}$ in $\mathbb{R}^n$.