University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Tutorials 2019 — Assignment 3 (40 marks)

Question 1 (10 marks)

For a linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$ with

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} -4 \\ 4 \\ 18 \end{bmatrix}.$$

compute by hand the first <u>four</u> iterations with the Jacobi method, using $\mathbf{x}^{(0)} = \mathbf{0}$.

Hint: for the ease of calculation, keep to rational fractions rather than decimals.

Question 2 (10 marks)

For the same linear system as in Question 1, compute by hand the first <u>three</u> iterations with the Gauss–Seidel method, using $\mathbf{x}^{(0)} = \mathbf{0}$.

Hint: for the ease of calculation, keep to rational fractions rather than decimals.

Question 3 (5 marks)

For the same linear system as in Question 1, compute by hand the <u>first only</u> iteration with the relaxation method, choosing the relaxation parameter $\omega = 0.5$ and using $\mathbf{x}^{(0)} = \mathbf{0}$.

Hint: for the answer, retain rational fractions rather than decimals.

Question 4 (5 marks)

By analysing the matrix $\mathbf{A}$ used for Questions 1–3, explain whether it is possible to determine, prior to running iterations, if the iterative schemes will converge.

Question 5 (10 marks)

Using any programming language, implement the algorithms for Jacobi, Gauss-Seidel and relaxation methods (do not submit the code) and use it for the following questions:

(a) With each method, solve the system used in Questions 1–3 to a relative precision for supremum norm $\frac{\|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\|}{\|\mathbf{x}^{(k+1)}\|} < 10^{-4}$, and find out how many iteration steps were required with each method.

(b) With a restriction to one-digit (0.x) precision for ω, figure out its optimal value in the range from 0.1 to 0.9 for the problem in Question 3.