University of Technology Sydney
Department of Mathematical and Physical Sciences

37233 Linear Algebra
Problem Set 2

Question 1.

(a) Find the solution of linear system

$$\mathbf{Ax} = \begin{pmatrix} 2 & 1 & 3 \\ -6 & -6 & -5 \\ 10 & 11 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix};$$

using:

(i) the Doolittle **LU** decomposition of the matrix **A**

(ii) the Crout **LU** decomposition of the matrix **A**.

(b) Construct **LDU** decomposition of matrix **A**.

Question 2.

(a) Use Choleski's algorithm to find an **LU** factorisation for

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & -2 & 0 \\ 2 & 2 & -4 & 1 \\ -2 & -4 & 14 & -13 \\ 0 & 1 & -13 & 42 \end{pmatrix}.$$

(b) Hence solve the equation $A\mathbf{x} = (14, 17, -55, 71)^T$.

Question 3.

Is the matrix

$$\mathbf{A} = \begin{pmatrix} 3 & -1 & 1 \\ 3 & -7 & 3 \\ 2 & 4 & 6 \end{pmatrix}$$

strictly diagonally dominant?

.../Over

Question 4.

Can you guarantee that the matrix

$$\mathbf{A} = \begin{pmatrix} -4 & 1 & 2 \\ 3 & 4 & 0 \\ -5 & 3 & 9 \end{pmatrix}$$

has an **LU** factorisation **without** attempting to construct a factorisation?

Question 5.

Consider the system $A\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 3 \\ 2 & -1 & 1 \\ 3 & 3 & 5 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}.$$

(a) Construct elementary row operation matrices $\mathbf{E_{ij}(a)}$ which will reduce the matrix $\mathbf{A}$ above to low triangular form.

$$\mathbf{E_{ij}(a_r)} \ldots \mathbf{E_{ij}(a_1)A} = \mathbf{U}$$

(b) Find the solution using backward substitution

(c) Find a lower triangular matrix $\mathbf{L}$ such that $\mathbf{A} = \mathbf{LU}$ using the constructed elementary matrices.

(d) Verify, by putting $\mathbf{y} = U\mathbf{x}$ then solving $L\mathbf{y} = \mathbf{b}$ for $\mathbf{y}$ and then $U\mathbf{x} = \mathbf{y}$ for $\mathbf{x}$, that you obtain the same solution as in part

(e) Solve the system by hand using Gaussian elimination and back substitution.