Solutions for Assignment 5

Question 1

In[1]:=
```
A = {{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {0, 0, 0, 0}}
```

Out[1]=
```
{{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {0, 0, 0, 0}}
```

In[2]:=
```
A // MatrixForm
```

Out[2]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[4]:=
```
MatrixForm[A.A]
```

Out[4]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[5]:=
```
MatrixForm[A.A.A]
```

Out[5]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[6]:=
```
MatrixForm[A.A.A.A]
```

Out[6]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Question 2

In[9]:= $A = \{\{\lambda_1, 1, 0, 0\}, \{0, \lambda_1, 1, 0\}, \{0, 0, \lambda_1, 0\}, \{0, 0, 0, \lambda_2\}\}$

Out[9]= $\{\{\lambda_1, 1, 0, 0\}, \{0, \lambda_1, 1, 0\}, \{0, 0, \lambda_1, 0\}, \{0, 0, 0, \lambda_2\}\}$

In[10]:=
```
MatrixForm[A]
```

Out[10]//MatrixForm=

$$\begin{pmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{pmatrix}$$

In[11]:= `MatrixForm[A.A]`

Out[11]//MatrixForm=

$$\begin{pmatrix} \lambda_1^2 & 2\lambda_1 & 1 & 0 \\ 0 & \lambda_1^2 & 2\lambda_1 & 0 \\ 0 & 0 & \lambda_1^2 & 0 \\ 0 & 0 & 0 & \lambda_2^2 \end{pmatrix}$$

In[12]:= `MatrixForm[A.A.A]`

Out[12]//MatrixForm=

$$\begin{pmatrix} \lambda_1^3 & 3\lambda_1^2 & 3\lambda_1 & 0 \\ 0 & \lambda_1^3 & 3\lambda_1^2 & 0 \\ 0 & 0 & \lambda_1^3 & 0 \\ 0 & 0 & 0 & \lambda_2^3 \end{pmatrix}$$

In[13]:= `MatrixForm[A.A.A.A]`

Out[13]//MatrixForm=

$$\begin{pmatrix} \lambda_1^4 & 4\lambda_1^3 & 6\lambda_1^2 & 0 \\ 0 & \lambda_1^4 & 4\lambda_1^3 & 0 \\ 0 & 0 & \lambda_1^4 & 0 \\ 0 & 0 & 0 & \lambda_2^4 \end{pmatrix}$$

Question 3

In[36]:= `A = {{1, 2, -1}, {-2, -5, 3}, {-1, -3, 0}}`

Out[36]= `{{1, 2, -1}, {-2, -5, 3}, {-1, -3, 0}}`

In[37]:= `A // MatrixForm`

Out[37]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & -1 \\ -2 & -5 & 3 \\ -1 & -3 & 0 \end{pmatrix}$$

In[54]:= $L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$

Out[54]= `{{1, 0, 0}, {-2, 1, 0}, {-1, 1, 1}}`

In[55]:= $U = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

Out[55]= `{{1, 2, -1}, {0, 1, -1}, {0, 0, 1}}`

In[56]:= $DD = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

Out[56]= `{{1, 0, 0}, {0, -1, 0}, {0, 0, -2}}`

In[57]:= `L.DD.U == A`

Out[57]= `True`

Question 4

In[15]:=
```
A = {{0, 0, 0, 1}, {0, 0, 2, 0}, {0, 3, 0, 0}, {4, 0, 0, 0}};
```

In[17]:=
```
A // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{pmatrix}$$

In[16]:=
```
MatrixForm[Inverse[A]]
```

Out[16]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

In[18]:=
```
B = {{0, 0, 1}, {1, 0, 0}, {0, 1, 0}};
```

In[19]:=
```
B // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

In[20]:=
```
MatrixForm[Inverse[B]]
```

Out[20]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

In[25]:=
```
c = {{1, 0, 0}, {0, 1, 0}, {0, -2, 1}};
MatrixForm[c]
```

Out[26]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

In[27]:=
```
MatrixForm[Inverse[c]]
```

Out[27]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

In[30]:=
```
d = {{-1/4, 0, 0}, {0, 1, 0}, {0, 0, 1}};
MatrixForm[d]
```

Out[31]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Question 5

In[32]:=
```
A = {{1, 1, 1, 2}, {2, 1, 4, 3}, {0, 1, -2, 2}}
```

Out[32]=
```
{{1, 1, 1, 2}, {2, 1, 4, 3}, {0, 1, -2, 2}}
```

In[34]:= `MatrixForm[A]`

Out[34]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & 4 & 3 \\ 0 & 1 & -2 & 2 \end{pmatrix}$$

In[35]:= `MatrixForm[RowReduce[A]]`

Out[35]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Each row has a pivot so these vectors span R^3.