

University of Technology Sydney
Department of Mathematical and Physical Sciences

37233 Linear Algebra
Tutorial Assignment 4

Question 1.

Find all values of h such that

$$\mathbf{A} = \begin{pmatrix} 2 & 4 & 2 \\ 1 & h & 3 \\ 1 & 2 & 1 \end{pmatrix}.$$

is invertible.

Question 2.

Solve the following linear system

$$\begin{pmatrix} 1 & -2 & 1 & 1 \\ 2 & 1 & -3 & -1 \\ 1 & -7 & -6 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 6 \end{pmatrix}.$$

and express the solution as a sum of a particular \mathbf{p} solution and general solutions \mathbf{v} of corresponding homogeneous system $\mathbf{Ax} = \mathbf{0}$.

Question 3.

Find a set of vectors in R^4 that spans the solution space of the homogeneous system

$$\begin{pmatrix} 1 & 1 & 3 & 1 \\ 2 & 3 & 1 & 1 \\ 1 & 0 & 8 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Question 4.

What conditions must b_1 , b_2 and b_3 satisfy for the following system to be consistent?

$$\begin{pmatrix} 4 & -6 \\ 8 & 12 \\ 6 & -9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

Question 5.

Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 & -1/2 \end{pmatrix}.$$

Using row operations, and showing your working, show that

$$\text{Det } \mathbf{A} = |\mathbf{A}| = 1.$$