University of Technology Sydney
Department of Mathematical and Physical Sciences

37233 Linear Algebra Problem Set 10

Note: you may use *Mathematica* to carry out any calculations you feel may be of use.

Question 1.

Compute the quadratic form $\mathbf{x}^T A\mathbf{x}$ for

$$A = \begin{pmatrix} 4 & 3 & 0 \\ 3 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad \text{where} \quad \text{(a)} \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \text{(b)} \quad \mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}.$$

Question 2.

Find the matrices of the quadratic forms

$$\begin{aligned} (a)\ Q &= 8x_1^2 + 7x_2^2 - 3x_3^2 - 6x_1x_2 + 4x_1x_3 - 2x_2x_3 \\ (b)\ Q &= 4x_1x_2 + 6x_1x_3 - 8x_2x_3 \end{aligned}$$

Question 3.

Classify the quadratic form and make a change of variable $\mathbf{x} = P\mathbf{y}$ that transforms the quadratic form into one with no cross-product term.

$$3x_1^2 - 4x_1x_2 + 6x_2^2$$

Question 4.

Let $Q(\mathbf{x}) = 5x_1^2 + 5x_2^2 - 4x_1x_2$. Find

(a) the maximum value of $Q(\mathbf{x})$ subject to the constraint $\mathbf{x}^T\mathbf{x} = 1$;

(b) a unit vector $\mathbf{u}$ where this maximum is attained;

(c) the maximum of $Q(\mathbf{x})$ subject to the constraints $\mathbf{x}^T\mathbf{x} = 1$, $\mathbf{x}^T\mathbf{u} = 0$

Question 5.

Find the singular values of A and a unit vector $\mathbf{x}$ at which $A\mathbf{x}$ has maximum length, given

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}$$

.../Over

Question 6.

Find the singular value decomposition of the matrix in the previous question.

Question 7.

Suppose the factorization below is an SVD of a matrix A:

$$A = \begin{pmatrix} 0.40 & -0.78 & 0.47 \\ 0.37 & -0.33 & -0.87 \\ -0.84 & -0.52 & -0.16 \end{pmatrix} \begin{pmatrix} 7.10 & 0 & 0 \\ 0 & 3.1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.30 & -0.51 & -0.81 \\ 0.76 & 0.64 & -0.12 \\ 0.58 & -0.58 & 0.58 \end{pmatrix}.$$

(a) What is the rank of A?

(b) What are bases for $Col\,A$ and $Nul\,A$?