University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Tutorials 2019 — Assignment 4 (40 marks)

Question 1 (8 marks)

Let

$$\mathbf{a}_1 = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \qquad \mathbf{a}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad \mathbf{b}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad \mathbf{b}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad \mathbf{b}_3 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Calculate by hand where possible, or identify those calculations which are *not* possible:

$$\mathbf{a}_1 - 4\mathbf{a}_2, \qquad \mathbf{b}_1 + 2\mathbf{b}_3, \qquad \mathbf{a}_2 + 3\mathbf{b}_2, \qquad \mathbf{b}_1 + \mathbf{b}_2 - 3\mathbf{b}_3$$

Question 2 (8 marks)

Find the conditions onto b_1, b_2 and b_3 such that the following system is consistent:

$$\begin{bmatrix} 2 & -2 \\ 3 & 3 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Provide a geometrical interpretation of the result.

Question 3 (8 marks)

Determine whether the following vectors span $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \qquad \mathbf{v}_4 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

Question 4 (8 marks)

Find a set of vectors in $\mathbb{R}^4$ that spans the solution space of the homogeneous system

$$\begin{bmatrix} 1 & 1 & 4 & 1 \\ 1 & 2 & 9 & 0 \\ 2 & 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Question 5 (8 marks)

Write the solution to the following linear system by finding a particular solution to the inhomogeneous system and a general solution to the corresponding homogeneous system.

$$\begin{bmatrix} 1 & 1 & -1 & -2 \\ 2 & -1 & -3 & -5 \\ 1 & 2 & -6 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ -6 \end{bmatrix}.$$

In this way, express the result via the particular solution $\mathbf{p}$ and the general solution $\mathbf{v}$.