Linear Algebra, Assignment 7

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Question 1

Question 1: Part 1 (a)

$$\mathbf{A} = \begin{bmatrix} 1 & 7 & -6 & -1 & -3 \\ 1 & -8 & 9 & -3 & 10 \\ -2 & -9 & 7 & 1 & 0 \end{bmatrix}$$

$$\text{ref}(\mathbf{A}) = \begin{bmatrix} 2 & \frac{9}{2} & -\frac{7}{2} & -\frac{1}{2} & 0 \\ 0 & 5 & -5 & 1 & -4 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\text{rref}(\mathbf{A}) = \begin{bmatrix} 1 & 0 & 1 & 0 & 5 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Col(A)

$$\text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ -8 \\ -9 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} \right\} = \text{Span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\} \Rightarrow \text{Col}(\mathbf{A}) = \{\mathbf{0} = a \cdot \mathbf{u} + b \cdot \mathbf{v} + c \cdot \mathbf{w} \mid \{a, b, c\} \in \mathbb{R}\}$$

Nul(A)

Row(A)

$$\text{Span}\left\{ \begin{bmatrix} 2 \\ \frac{9}{2} \\ -\frac{7}{2} \\ -\frac{1}{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -5 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} = \text{Span}\{\mathbf{u_r}, \mathbf{v_r}, \mathbf{w_r},\} \Rightarrow \text{Row}(\mathbf{A}) = \{\mathbf{0} = a \cdot \mathbf{u_r} + b \cdot \mathbf{v_r} + c \cdot \mathbf{w_r} \mid \{a, b, c\} \in \mathbb{R}\}$$

Question 1: Part 1 (b)

Basis for Col($\mathbf{A}$)

Basis for $\text{Nul}(\mathbf{A})$
Basis for $\text{Row}(\mathbf{A})$

Question 1: Part 1 (c)

$\dim(\text{Col}(\mathbf{A}))$
$\dim(\text{Nul}(\mathbf{A}))$
$\dim(\text{Row}(\mathbf{A}))$

Question 1: Part 1 (d)

$x \in \text{Col}(\mathbf{A})$
$x \in \text{Nul}(\mathbf{A})$
$x \in \text{Row}(\mathbf{A})$

$y \in \text{Col}(\mathbf{A})$
$y \in \text{Nul}(\mathbf{A})$
$y \in \text{Row}(\mathbf{A})$

Question 1: Part 2 (a)

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 7 & 8 \\ 0 & -3 & 3 \\ -1 & 3 & -8 \end{bmatrix}$$

$$\text{ref}(\mathbf{A}) = \begin{bmatrix} 3 & 7 & 8 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{rref}(\mathbf{A}) = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Col(A)

Row(A)

Nul(A)

Question 1: Part 2 (b)

Basis for $\text{Col}(\mathbf{A})$
Basis for $\text{Nul}(\mathbf{A})$
Basis for $\text{Row}(\mathbf{A})$

Question 1: Part 2 (c)

$\dim(\text{Col}(\mathbf{A}))$
$\dim(\text{Nul}(\mathbf{A}))$
$\dim(\text{Row}(\mathbf{A}))$

Question 1: Part 2 (d)

$x \in \text{Col}(\mathbf{A})$
$x \in \text{Nul}(\mathbf{A})$
$x \in \text{Row}(\mathbf{A})$

$y \in \text{Col}(\mathbf{A})$
$y \in \text{Nul}(\mathbf{A})$
$y \in \text{Row}(\mathbf{A})$

Question 2

Part (a)

Matrix is 7 equations (rows) by 8 variables (columns), meaning the size is (7×8).

$$\begin{bmatrix} * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * \end{bmatrix}$$

If two of the rows are linearly dependent, then there must be rank 6.
We can also prove this with the rank theorem:

$$\text{Rank}(\mathbf{A}) + \dim(\text{Nul}(\mathbf{A})) = n$$

Where $n :=$ number of columns

$$\therefore n = 8$$

and with there being two dependent equations:

$$\dim(\text{Nul}(\mathbf{A})) = 2$$

so

$$\text{Rank}(\mathbf{A}) = n - \dim(\text{Nul}(\mathbf{A})) = 8 - 2$$
$$\therefore \text{Rank}(\mathbf{A}) = 6$$

Part (b)

$$\dim(\text{Nul}(\mathbf{A})) = 2$$

Part (c)

Part (d)

Domain = $\mathbb{R}^8$
Codomain = $\mathbb{R}^7$

Part (e)

Becuase of the two independent rows, can only span up to $\mathbb{R}^5$.