University of Technology Sydney
Department of Mathematical and Physical Sciences

37233 Linear Algebra
Problem Set 6

Note: you may use *Mathematica* to carry out any calculations you feel may be of use.

Question 1.

Illustrate the effect of the linear transformation T with standard matrix $\mathbf{A} = \begin{pmatrix} 3 & 3 \\ 0 & 1 \end{pmatrix}$ by mapping its action on the unit square in $\mathbb{R}^2$.

Question 2.

Illustrate the effect of the linear transformation T with standard matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ by mapping its action on the triangle with vertices $(0,0)$, $(1,0)$ and $(1,1)$.

Question 3.

Let $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{y}_1 = \begin{pmatrix} 3 \\ 5 \\ -7 \end{pmatrix}$, $\mathbf{y}_2 = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$, $\mathbf{y}_3 = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation that maps $\mathbf{e}_k$ to $\mathbf{y}_k$ for $k = 1, 2, 3$.

(a) Find the image of $\begin{pmatrix} -1 & 2 & 1 \end{pmatrix}^T$.

(b) Write down the standard matrix representation of T.

Question 4.

Let

$$\mathbf{v}_1 = \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 3 \\ 12 \\ 7 \end{pmatrix}$$

and $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$.

(a) Show that $\mathcal{B}$ is a basis for $H = Span\{\mathbf{v}_1, \mathbf{v}_2\}$.

(b) Determine whether $\mathbf{x}$ is in H and, if it is, find the coordinate vector of $\mathbf{x}$ relative to $\mathcal{B}$.

Question 5.

Let

$$A = \begin{pmatrix} 30 & 20 & 18 \\ 50 & 0 & -20 \\ 70 & 30 & 17 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 5 \\ 15 \\ 15 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 15 \\ 5 \\ -10 \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} -8 \\ 30 \\ -20 \end{pmatrix}.$$

For each of the vectors **b**, **c**, **d** determine whether the vector is in

(a) Nul A,

(b) Col A.

You may use *Mathematica*.

Question 6.

Find spanning sets for Nul A and Col A where

$$A = \begin{pmatrix} 1 & -6 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & 5 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Question 7.

Determine the numbers of vectors required in bases of Nul A and Col A where

$$A = \begin{pmatrix} 1 & -6 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & 5 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Question 8.

Find basis sets for Nul A and Col A spaces where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{pmatrix}.$$

Question 9.

How many vectors are required in a basis of the subspace H spanned by the vectors: $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 9 \\ 4 \\ -2 \end{pmatrix}$, $\begin{pmatrix} -7 \\ -3 \\ 1 \end{pmatrix}$? Write down a basis for H.