University of Technology Sydney
Department of Mathematical and Physical Sciences

37233 Linear Algebra
Tutorial Assignment 9

Question 1.

Let

$$\mathbf{y} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \ \mathbf{u} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}.$$

Write $\mathbf{y}$ as the sum of a vector in $Span\{\mathbf{u}\}$ and a vector orthogonal to $\mathbf{u}$. Compute the distance from $\mathbf{y}$ to the line through $\mathbf{u}$ and the origin.

Question 2.

Let W be the subspace spanned by the $\mathbf{u}$'s. Write $\mathbf{y}$ as the sum of a vector in W and a vector orthogonal to W, where

$$\mathbf{u_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \ \mathbf{u_2} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}, \ \mathbf{y} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}.$$

Question 3.

Let

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}, \ \mathbf{x}_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \ \mathbf{x}_3 = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 2 \end{pmatrix}$$

and let $\mathbf{X}$ be the matrix with columns $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$. Apply the Gram–Schmidt process to construct orthogonal basis vectors $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ for $W = \text{Span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$.

Question 4.

Construct an orthonormal basis $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ from the orthogonal set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ obtained in Question 3.

Question 5.

Find the **QR** factorisation of matrix $\mathbf{X}$ from Question 3.