37233 Linear Algebra

Tutorial Assignment 9 Solutions

Question 1

In[1]:= `y = {2, 6}`

Out[1]= $\{2, 6\}$

In[2]:= `u = {7, 1}`

Out[2]= $\{7, 1\}$

In[7]:= `Projyu = (y.u) / (u.u) u`

Out[7]= $\left\{\frac{14}{5}, \frac{2}{5}\right\}$

In[8]:= `z = y - Projyu`

Out[8]= $\left\{-\frac{4}{5}, \frac{28}{5}\right\}$

Check orthogonality :

In[9]:= `z.u`

Out[9]= 0

Question 2

In[10]:= `u1 = {1, 1, 1}`

Out[10]= $\{1, 1, 1\}$

In[11]:= `u2 = {-1, 3, -2}`

Out[11]= $\{-1, 3, -2\}$

In[12]:= `y = {-1, 4, 3}`

Out[12]= $\{-1, 4, 3\}$

In[14]:= `ProjyW = (y.u1) / (u1.u1) u1 + (y.u2) / (u2.u2) u2`

Out[14]= $\left\{\frac{3}{2}, \frac{7}{2}, 1\right\}$

In[15]:= y - ProjyW

Out[15]= $\left\{-\frac{5}{2}, \frac{1}{2}, 2\right\}$

$$\text{Proj } y_W = \begin{pmatrix} 3/2 \\ 7/2 \\ 1 \end{pmatrix}$$

$$y - \text{Proj } y_W \begin{pmatrix} -5/2 \\ 1/2 \\ 2 \end{pmatrix} \text{ is in complement of } W$$

Question 3

In[16]:= x1 = {1, -1, -1, 1}

Out[16]= {1, -1, -1, 1}

In[17]:= x2 = {2, 1, 0, 1}

Out[17]= {2, 1, 0, 1}

In[18]:= x3 = {2, 2, 1, 2}

Out[18]= {2, 2, 1, 2}

Gram - Smidth process

In[19]:= v1 = x1

Out[19]= {1, -1, -1, 1}

In[20]:= v2 = x2 - (x2.v1) / (v1.v1) v1

Out[20]= $\left\{\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right\}$

In[21]:= v3 = x3 - (x3.v1) / (v1.v1) v1 - (x3.v2) / (v2.v2) v2

Out[21]= $\left\{-\frac{1}{2}, 0, \frac{1}{2}, 1\right\}$

Checking orthogonality

In[22]:= v1.v2

Out[22]= 0

In[23]:= v1.v3

Out[23]= 0

In[25]:= v2.v3

Out[25]= 0

Question 4

Normalization

In[26]:= `u1 = v1 / Norm[v1]`

Out[26]= $\left\{\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$

In[27]:= `u2 = v2 / Norm[v2]`

Out[27]= $\left\{\frac{3}{2\sqrt{5}}, \frac{3}{2\sqrt{5}}, \frac{1}{2\sqrt{5}}, \frac{1}{2\sqrt{5}}\right\}$

In[28]:= `u3 = v3 / Norm[v3]`

Out[28]= $\left\{-\frac{1}{\sqrt{6}}, 0, \frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}\right\}$

Question 5

In[29]:= `MatrixForm[qtranspose = {u1, u2, u3}]`

Out[29]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{3}{2\sqrt{5}} & \frac{3}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} \\ -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} \end{pmatrix}$$

In[37]:= `Q = Transpose[qtranspose]`

Out[37]= $\left\{\left\{\frac{1}{2}, \frac{3}{2\sqrt{5}}, -\frac{1}{\sqrt{6}}\right\}, \left\{-\frac{1}{2}, \frac{3}{2\sqrt{5}}, 0\right\}, \left\{-\frac{1}{2}, \frac{1}{2\sqrt{5}}, \frac{1}{\sqrt{6}}\right\}, \left\{\frac{1}{2}, \frac{1}{2\sqrt{5}}, \sqrt{\frac{2}{3}}\right\}\right\}$

In[38]:= `Q // MatrixForm`

Out[38]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & \frac{3}{2\sqrt{5}} & -\frac{1}{\sqrt{6}} \\ -\frac{1}{2} & \frac{3}{2\sqrt{5}} & 0 \\ -\frac{1}{2} & \frac{1}{2\sqrt{5}} & \frac{1}{\sqrt{6}} \\ \frac{1}{2} & \frac{1}{2\sqrt{5}} & \sqrt{\frac{2}{3}} \end{pmatrix}$$

In[39]:= `X = Transpose[{x1, x2, x3}]`

Out[39]= {{1, 2, 2}, {-1, 1, 2}, {-1, 0, 1}, {1, 1, 2}}

In[40]:= `X // MatrixForm`

Out[40]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

In[41]:=
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R = qtranspose.X
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Out[41]= $\left\{\left\{2, 1, \frac{1}{2}\right\}, \left\{0, \sqrt{5}, \frac{3\sqrt{5}}{2}\right\}, \left\{0, 0, \sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}}\right\}\right\}$

In[42]:=
```
R // Simplify // MatrixForm
```

Out[42]//MatrixForm=

$$\begin{pmatrix} 2 & 1 & \frac{1}{2} \\ 0 & \sqrt{5} & \frac{3\sqrt{5}}{2} \\ 0 & 0 & \sqrt{\frac{3}{2}} \end{pmatrix}$$

Check that QR = X

In[43]:=
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Q.R == X
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Out[43]= True

In[45]:=
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Q.R // MatrixForm // Simplify
```

Out[45]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$