MSE 223: Introduction to Fluid Mechanics

Run-of-the-River Final Project

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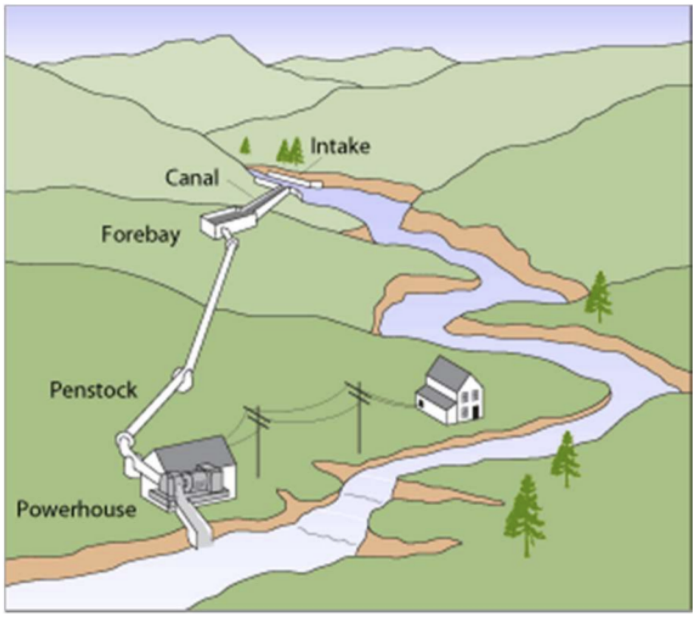
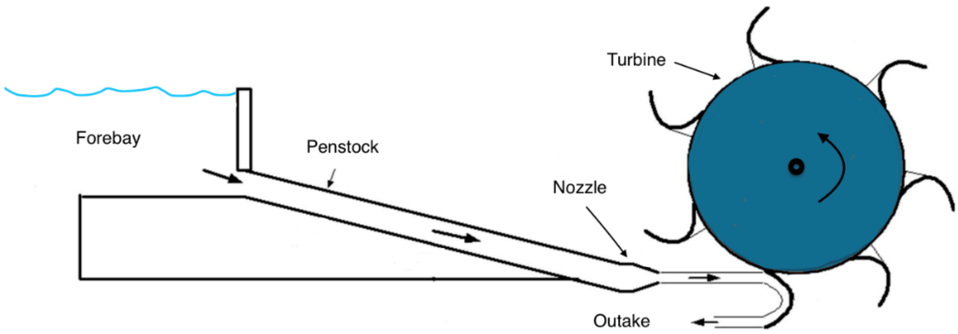
1. Introduction

Run-of-the-River (RoR) is a type of hydroelectric power generating facility which uses the natural elevation change and flow of a river to generate electricity. A portion of the flow is pulled through an intake and stored in the forebay, which prevents variation in flow rate, and is fed into a long tube known as a penstock. The water accelerates due to gravity, gaining kinetic energy and is projected through a nozzle towards the turbine blade. The blade redirects the flow, which creates a moment about the turbine, causing it to rotate, producing electricity. From there the flow is returned to the river as to not affect the water levels.





This type of energy production facility has the potential to address environmental, economic and social issues that larger facilities cannot.

<u>Environmental Impact</u>

Ecological disruption is one of the main environmental concerns associated with hydroelectric dam facilities [1]. Dams are an engineering marvel that are capable of efficiently supplying a large amount of renewable energy, controlling stream regimes to prevent flooding and supplying water to cities and farms. They have an insignificant carbon footprint, mostly created during their inception, and require but gravity and the flow of water to operate. These presumptions were what drove British Columbia to produce 91% of its energy from hydroelectric dams [2]. However, as more research occurs, long-term negative impacts of dams are becoming a concern.

RoR facilities are looking to avoid some of the major impacts that dams can have on the natural ecosystems that rely on the river's flow. Some concerns are fish breeding grounds, sediment retention, salt and oxygen water content, animal territory disruption, increased water evaporation, microclimate change and more. Most of these issues arise from the immense amount of stagnant water a dam requires to operate. RoR facilities only store enough water to maintain constant generation for the day, keeping the river flow constant.

Although less impactful compared to hydroelectric dams, especially when it comes to carbon footprint, RoR are not entirely eco-friendly [3]. Any disruption of the river has the potential to affect fish populations and the overall health of the river, as well as increased turbidity as a side effect of the higher water speeds exiting the system, and this all comes at a cost.

Efficiency and Cost

Hydroelectric dams are incredibly efficient due to their ability to maintain constant flow through the turbines. This flow can be modified, known as hydropeaking, to increase or decrease the amount of energy produced during peak energy consumption hours. By having plenty of stored water, dams are capable of operating during the summer, at near full power, even after most of the glaciers melt. RoR facilities are subject to the river's flow and therefore can only be built in locations where there is constant flow all year round. Dams are incredibly efficient, operating between 85-89% year-round whereas RoR facilities vary from 40-80% [4]. This power output inefficiency is not the only con of RoR.

Cost is a huge factor in what allows any energy generating sector to function efficiently. Hydroelectric dams tend to be very expensive upfront, whereas RoR is significantly cheaper. However, in the long term, dams cost less per kWh produced; 3 cents for large hydropower versus 4 to 10 cents for RoR. This may not seem like much but, in B.C. alone, 43000 GWh are produced annually [5], which could cost 4.3 to 30.1 million extra dollars per year. While dams are more economical on such a large-scale, RoR has the opportunity to build in secondary locations where dams are not suitable.

In B.C. there are many large rivers which are suitable for hydroelectric dams, many of which have already been built upon, yet the need for electricity is ever growing. Since large hydroelectric facilities are limited to locations with heavy flow, RoR facilities can be built in smaller locations. The only limitations for RoR is that the flow must be relatively steady throughout the year. Having greater choice in build location is incredibly beneficial for less accessible communities.

Socioeconomics

The cost of electricity in Canada varies from 7.3¢/kWh in Quebec to up to 38.2¢/kWh in the Northwest Territories [6]. This discrepancy is largely due to accessibility, the further away from nearby ports or large cities the greater the cost of electricity. Without ease of access to supplies, building a large dam is nearly impossible, making conditions harsher. What is found is that the majority of these communities are made up of Indigenous Peoples.

Due to RoR's lesser material cost, it is easier to build in remote locations which could help those who are disproportionately affected by energy poverty. Many indigenous communities use wood or natural gas to heat their homes. With gas prices rising and the damage burning wood does to the environment, many people are turning to electricity. However, electric energy heaters are not as efficient at heating homes, especially in impoverished areas due to their homes not being properly fit for cold weather [7]. RoR has the potential to reach these communities and help deal with the energy crisis affecting them.

"As engineers, we were going to be in a position to change the world – not just study it." -Henry Petroski

In this project, the main objective will be to better understand how run-of-the-river facilities operate. Using the principles of fluid properties, fluid flow, energy/mass conservation and some fundamental fluid mechanic equations, the potential power output of such a system will be analyzed and discussed.

2. Theory

Fluid analysis is broken up into two categories: fluid statics and fluid dynamics.

Hydrostatics deals with the properties of a fluid at rest. Since fluids are not a rigid body, their mass is difficult to describe exactly and typically will be described as mass per unit volume or density (ρ).

Although the river is supplying the fluid, and therefore the fluid must be in motion, it is assumed to be static. This assumption is due to the relatively small diameter of the penstock relative to the total flow of the river. Only a small portion of the flow may enter, causing the water to pool and become stagnant.

The specific weight of a material is the product of the density of a material and the gravitational acceleration (g) acting upon it or weight per unit volume (γ)

$$\gamma = \rho g$$

In hydrostatics, the pressure created by the weight of the fluid increases linearly according to depth (h)

$$P = \gamma h$$

This pressure is what pushes the fluid into the penstock.

Once the fluid enters the penstock, the analysis switches to fluid dynamics. Here, flow rate (Q) is described as the total volume per second passing through an area (A) at a velocity (V)

$$Q = VA$$

For steady flow, due to the conservation of mass, the intake flow (Q_1) must equal the outtake flow (Q_2)

$$Q_1 = Q_2$$

and therefore

$$V_1A_1 = V_2A_2$$

Fluid flow is described primarily with two equations: Bernoulli's equation and the energy equation.

Bernoulli's equation states that the external pressure (P), velocity (V) and elevation change (z) of a system remains constant

$$P + \frac{1}{2}\rho V^2 + \rho g z = constant$$

For this equation to hold true, the fluid must have a steady flow rate, be incompressible, inviscid and along a streamline. Bernoulli's equation states that if the velocity and pressure of a steady system are inversely related; as one increases, the other decreases.

The energy equation is similar to the Bernoulli equation, except has the potential to include a greater number of influential factors

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p - h_t = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f$$

The energy equation, also known as the extended Bernoulli equation, is described in terms of 'heads', each of which conveniently has units in meters. Each head of the energy equation would cause a pressure gauge to rise/fall accordingly. The pressure head (P/γ), velocity head (V2/2g) and elevation head (z) are derived from Bernoulli's equation. The additional terms, pump head (hp), turbine head (ht) and head loss (hf), all contribute to the energy balance of the system from one point to another.

Another important equation is the dimensionless Reynolds number (Re) which is generally used to determine if fluid flow is turbulent or laminar, as well as in the calculation of the head loss due to friction

$$Re = \frac{\rho V L}{\mu}$$

where the characteristic length (L) and viscosity of the fluid (μ) are determined on a case-by-case basis.

The head loss due to friction is a function of the length (L) and diameter (D) of the pipe and the velocity (V) of the fluid

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

where the friction factor (f) can be obtained analytically or by using the Moody chart.

Fluid flow, like any mass, has momentum which requires force (F) for a change in velocity

$$F = Q\rho(V2 - V1)$$

This change in velocity can be considered in one-dimension. Applying Newton's third law, the force acting on the fluid will have an equal and opposite reaction on the turbine blade, causing it to rotate as the fluid is redirected.

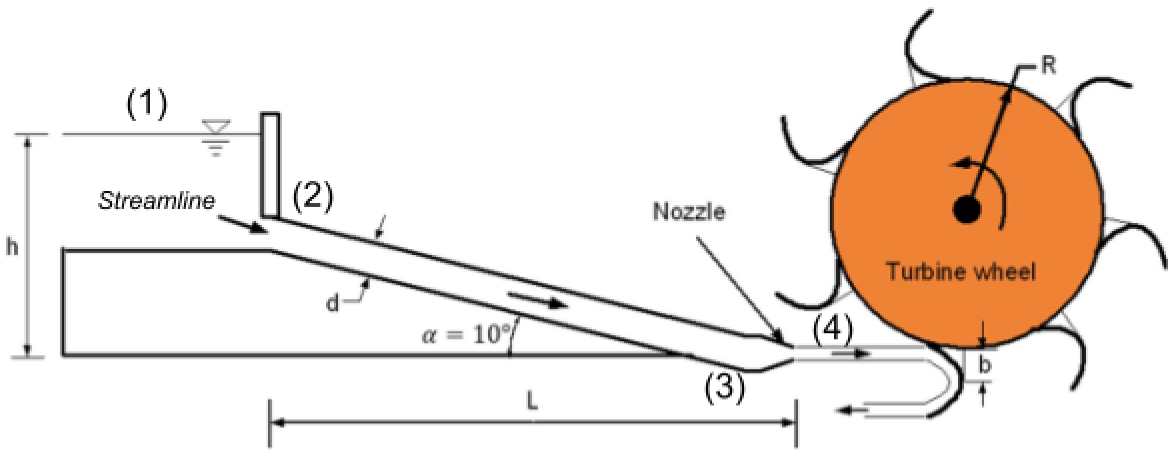
The total power (P) the turbine, with a certain radius (r), can produce is dependent on the torque (τ) created by the change in momentum

$$\tau = Fr$$
$$P = \tau\omega$$
$$P = Fr\omega$$
$$P = Q\rho(V2 - V1)r\omega$$

and the turbines' angular velocity (ω).

3. Results and Discussion

We conducted our analysis based on the following parameters for a typical RoR hydroelectric facility:



Parameter	Value
$h\ [m]$	200
$L\ [m]$	900
$R\ [m]$	0.8
$b\ [m]$	0.2
$Q_{river}\ [m^3/s]$	27.1

$Q\ [m^3/s]$	8.13
$T\ [^\circ C]$	10
$\rho\ [kg/m^3]$	1000
$\mu\ [kg/ms]$	1.307E-3

Table 1. Known Values

Note that Q_{river} is the typical volumetric flow rate of the river. We have taken the volumetric flow rate which we will use in the analysis, Q, to be 30% of the volumetric flow rate of the river. We chose a lower proportion of the river's volumetric flow rate to not cause too much ecological impact but a value that we deemed still high enough to generate a decent power output that is consistent with the typical power output of existing RoR facilities.

<u>Max Energy per Unit Mass Flow Rate</u>

Assuming steady, frictionless, incompressible flow along a streamline from the surface of the forebay to the outlet of the penstock nozzle, we can use the Bernoulli equation to determine the maximum amount of energy per unit mass flow rate of water that can be transformed into electricity.

$$\frac{P_f}{\gamma} + \frac{V_f^2}{2g} + z_f = \frac{P_n}{\gamma} + \frac{V_n^2}{2g} + z_n$$
$$(f\ denotes\ forebay,\ n\ denotes\ nozzle)$$

Since we have a standing free surface at point (1) in figure X, $V \approx 0$. Since both ends of the streamline are exposed to the same atmosphere, $P_f = P_n = P_{atm}$. Rearranging such that $z_f - z_n = h$,

$$=> h = \frac{1}{2g} V_n^2;\ g = 9.81\ [m/s^2]$$

By extension of this knowledge, we can determine the maximum speed of water at the nozzle outlet as well as the necessary diameter of the nozzle to convey the desired flow. Also note that by conservation of mass, $Q_{intake} = Q_{penstock} = Q$.

$$=> V_n = \sqrt{2gh} = 62.64\ [m/s]\ \star$$
$$Q = V_n \cdot A_n;\ A_n = \pi(d_n/2)^2$$
$$=> d_n = 2Q/\pi V_n = 8.263\ [cm]\ \star$$

From the principle of kinetic energy, we can derive the max power output per unit mass flow rate.

$$P_{th} = \frac{1}{2}\dot{m}V_n^{\ 2} = \rho Q g h = 15.95\ [MW]\ \star$$

Head Loss Derivation

Considering friction, and given that friction head, $h_f = CQ^2$ where C depends on penstock dimensions and properties of water, we will use the energy equation to show that $P_{max} = 2\rho g h Q/3$ @ $Q = \sqrt{h/3c}$. Without loss of prior assumptions, we write the energy equation for a streamline from the forebay to the turbine as

$$\frac{P_f}{\gamma} + \frac{V_f^{\ 2}}{2g} + z_f = \frac{P_t}{\gamma} + \frac{V_t^{\ 2}}{2g} + z_t + h_f + h_t$$

Note that for the same reasons as above, $P_f = P_t = P_{atm}$. We also note that since all kinetic energy is absorbed by the turbine. $V_f \approx V_t \approx 0$.

$$=> \ z_f = z_t + h_f + h_t$$
$$=> \ h_t = h - h_f = h - CQ^2$$
$$=> \ P = \rho g(h - CQ^2)Q$$
$$(P\ is\ the\ dependent\ variable,\ Q\ is\ the\ independent\ variable)$$

$$=> \ P = \rho g h Q - \rho g C Q^3$$

Now take the derivative and set to 0 to find Q that maximizes P,

$$\frac{dP}{dQ} = \rho g h - 3\rho g C Q^2 = 0$$
$$=> \ Q = \sqrt{h/3C}\ \star$$

Now plugging back in for P_{max},

$$P_{max} = \rho g h Q - \rho g C Q(\sqrt{h/3C})^2 = \rho g h Q(1 - \tfrac{1}{3})$$
$$=> \ P_{max} = 2\rho g h Q/3\ \star$$

Flow Properties Depending on Pipe Angle

Using the aforementioned theory and equations that have been derived, we can Find the flow rate, velocity at the nozzle outlet, and friction head in the penstock if the nozzle diameter is 8 cm and penstock diameter is 0.5 m (both fixed) at 10℃. For variations in the angle α in Figure X, we can obtain the following results for the flow in the penstock and nozzle:

See flowWizard.cpp for calculation details.

α [∘]	h_f [m]	V_n [m/s]	Q [m^3/s]	$(Re_d)_{penstock}$	$(Re_d)_{nozzle}$
5	33.51148	57.15335	0.287284	18659632	3498292
6	31.59482	57.48139	0.288933	17397105	3518371
7	29.49703	57.8383	0.290727	16030262	3540217
8	27.15875	58.23354	0.292714	14528699	3564410
9	24.48518	58.6822	0.294969	12844112	3591872
10	21.30593	59.2113	0.297628	10890530	3624257

Table 2. Headloss, Velocity, Flow Rate, Penstock and Nozzle Reynolds Numbers vs. Angle

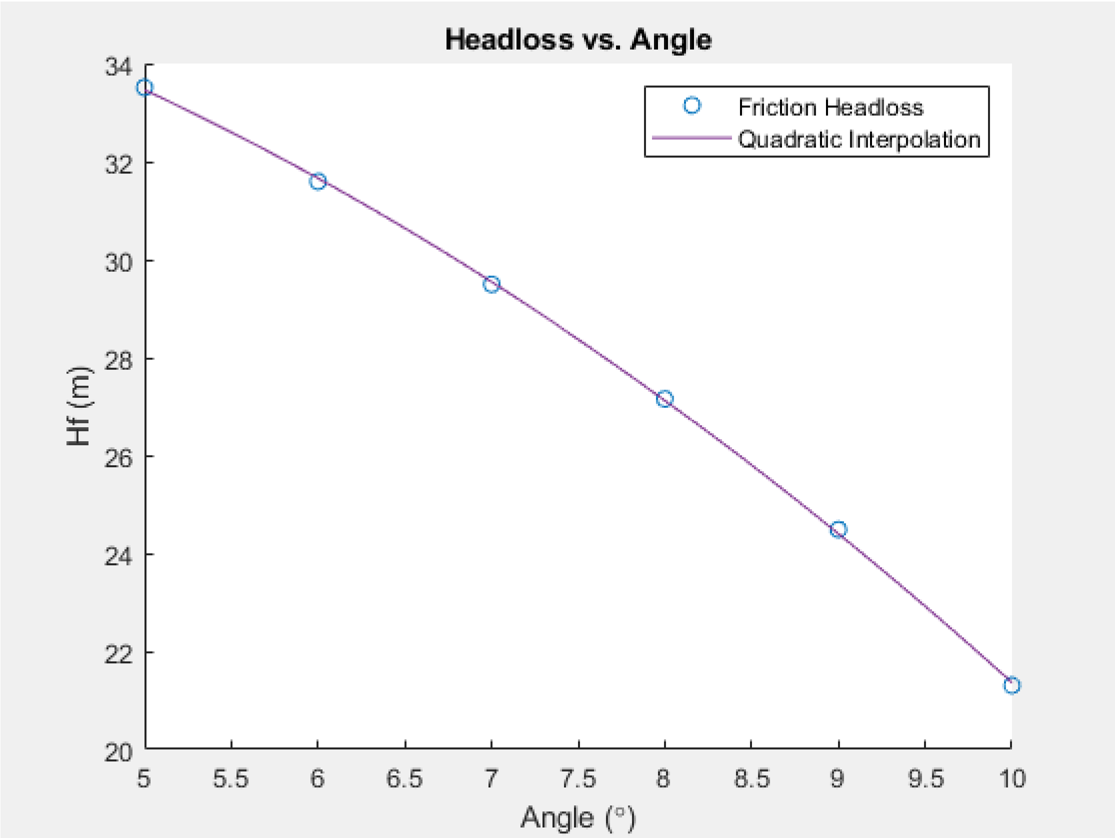


Fig 1. Headloss vs. Angle

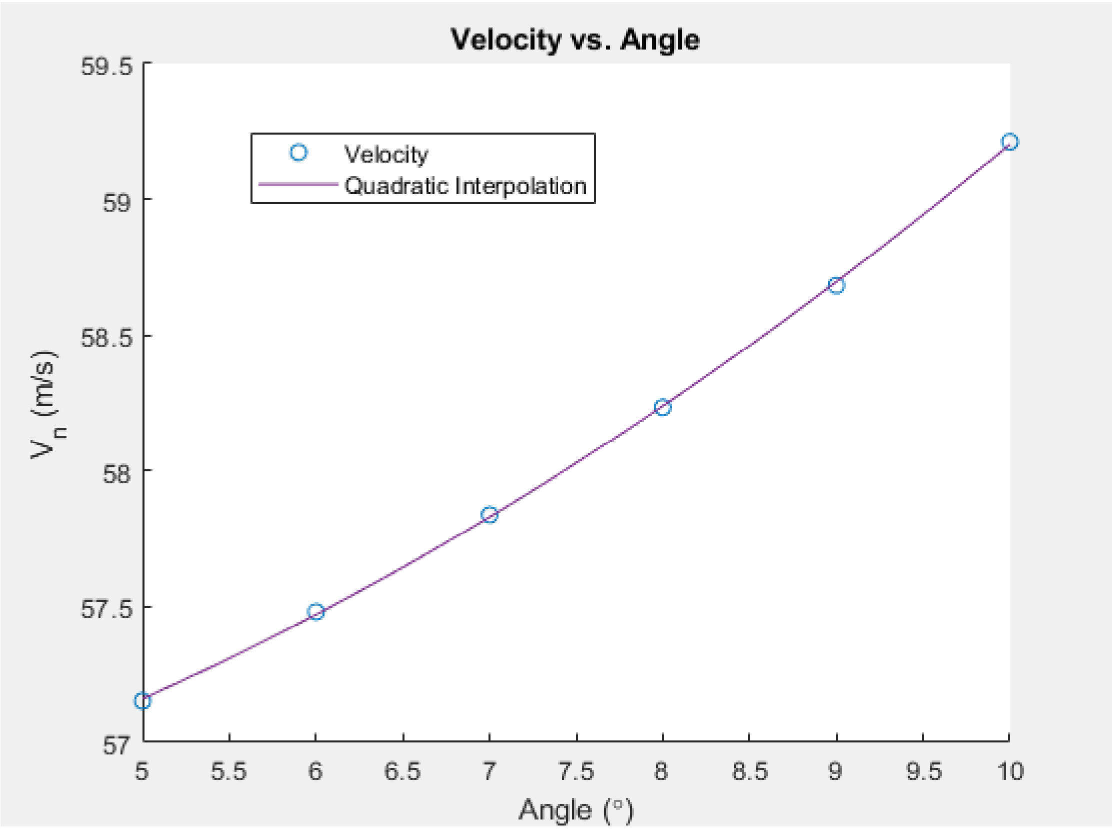


Fig 2. Velocity vs. Angle

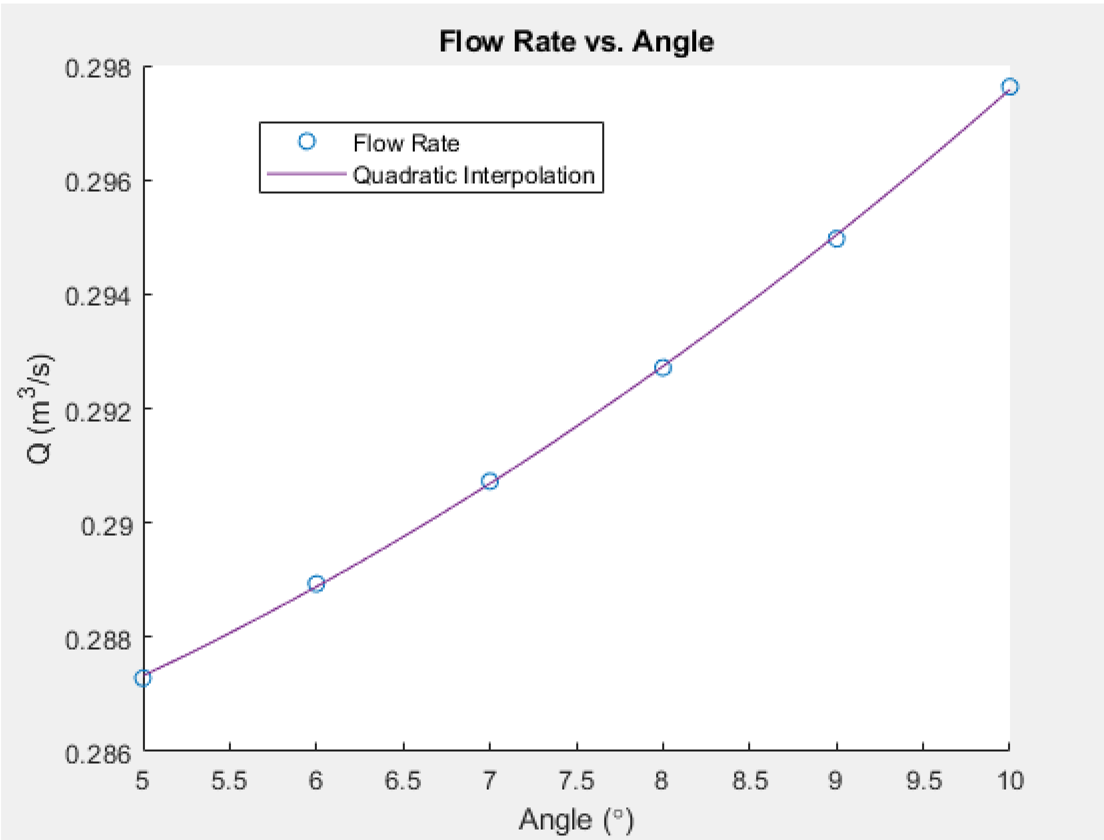


Fig 3. Flow Rate vs. Angle

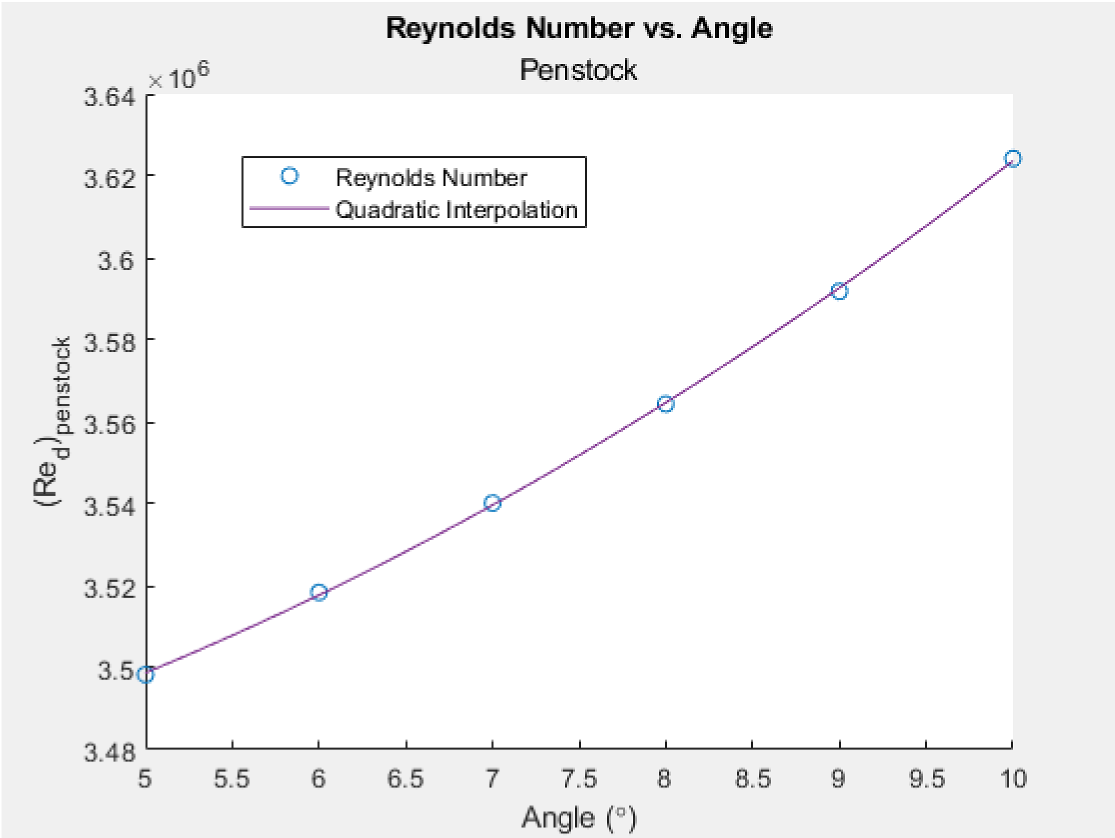


Fig 4. Penstock Reynolds Number vs. Angle

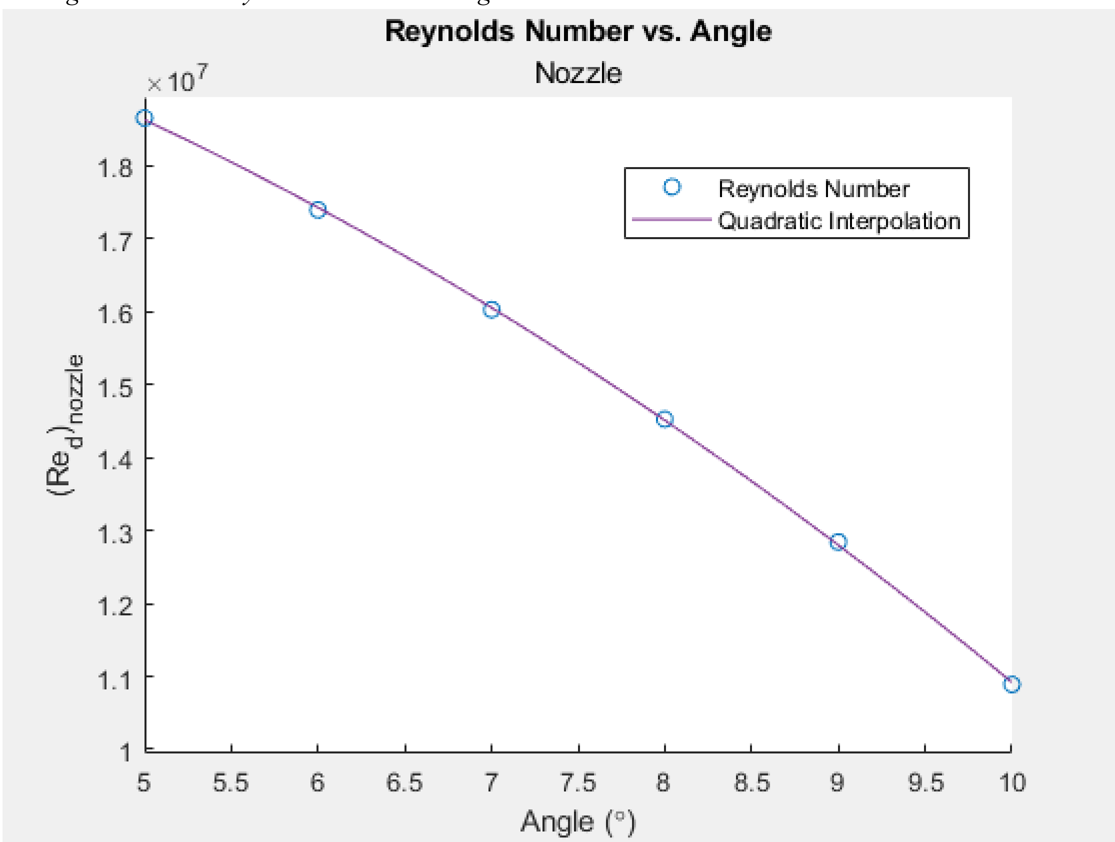


Fig 5. Nozzle Reynolds Number vs. Angle

We neglected the kinetic energy correction factor since this is an elementary turbulent flow regime as shown by the exceptionally high Reynolds number in the penstock and nozzle. The flow rate and velocity at the nozzle increase with increasing angle of the pipe. The reason for this is that we see a decrease in friction head with increasing angle. The reason for the decrease in friction head is due to the fact the friction head is dependent on the Reynolds number in the penstock which is proportional to the velocity in the penstock, which decreases as the penstock is raised.

Power Generated for Horizontal Turbine Blade

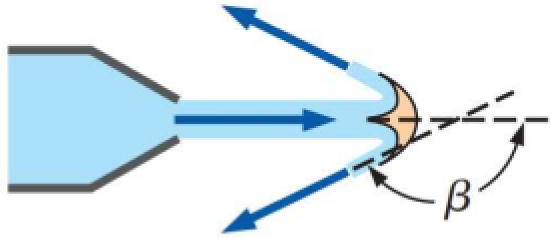


Fig 6. Momentum Change

We can establish a relationship for power generated in the turbine as a function of the turbine speed (revolutions per minute, rpm) and nozzle velocity, assuming that the water reflects with the angle of β=180°. Using this relationship, calculate the power generated by the turbine if it rotates at 300 rpm. Assume now the pipe angle (α) is fixed at 10°.

$$V_n = 59.21\ [m/s]$$
$$Q = V_n \cdot \pi(\frac{d_n}{2})^2$$

From the principle of conservation of linear momentum:

$$F_{th} = \dot{m}(V_n - V_t) = \rho Q(V_n - V_t)$$
$$V_t = V_n cos\beta$$

For $\beta = 180^{\circ}$,

$$F_{th} = 2\rho Q V_n$$
$$=> P = 4\rho Q V_n \pi R\Omega;\ \Omega = 5\ Hz\ \star$$

For pipe diameters ranging from 0.2 - 1.5 m, we obtain:

See flowWizard.cpp for calculation details.

$d\ [m]$	$P\ [MW]$	$d_{nozzle}\ [m]$
0.2	21.35461	0.099047
0.3	23.02532	0.09186
0.4	23.77514	0.088963
0.5	24.19719	0.087411
0.6	24.46625	0.08645
0.7	24.65199	0.085798
0.8	24.78751	0.085329
0.9	24.89053	0.084976
1	24.97134	0.084701
1.1	25.03634	0.084481
1.2	25.08968	0.084302
1.3	25.13421	0.084152
1.4	25.1719	0.084026
1.5	25.20421	0.083919

Table 3. Power vs. Penstock Diameter

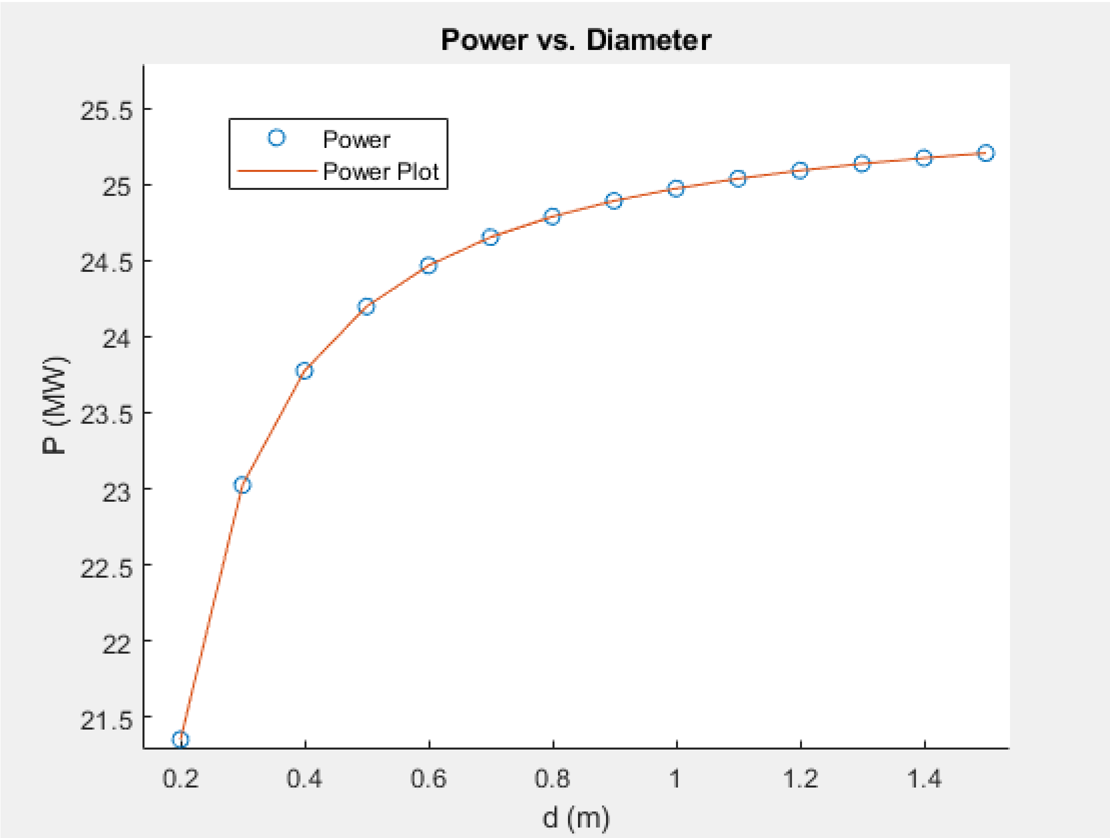


Fig 7. Power vs. Penstock Diameter

It turns out that it is actually not possible for the turbine to spin at 300 rpm as our power output is exceeding the max power output found in the *max energy per unit mass flow rate*. In reality, we should be calculating the optimum turbine speed that will achieve 100% of the possible power output. In such a case, all of the available energy in part one would be converted into electricity by the turbine if the turbine is sized appropriately and we neglect energy losses in the turbine mechanism.

Tangential Velocity and Max Power Generation

Using the relationship above, we can find the tangential velocity that leads to maximum power generation and the corresponding angular velocity. We can also use the relationship to determine how much power is generated if the turbine cannot achieve the optimal velocity.

Tangential velocity $U = 2\pi R\Omega$,

$$\Omega = \frac{P}{4\rho Q\pi R V_n} = \frac{gh}{4\pi R V_n}$$

We acquire the following results:

See flowWizard.cpp for calculation details.

$d\ [m]$	$U\ [m/s]$	$\Omega_{max}\ [rpm]$	$P_{max}\ [MW]$	$d_{nozzle} [m]$
0.2	18.77317	224.0883	15.95106	0.099047
0.3	17.411	207.8285	15.95106	0.09186
0.4	16.86189	201.274	15.95106	0.088963
0.5	16.56778	197.7634	15.95106	0.087411
0.6	16.38559	195.5885	15.95106	0.08645
0.7	16.26213	194.1149	15.95106	0.085798
0.8	16.17322	193.0536	15.95106	0.085329
0.9	16.10628	192.2546	15.95106	0.084976
1	16.05416	191.6324	15.95106	0.084701
1.1	16.01248	191.1349	15.95106	0.084481
1.2	15.97844	190.7285	15.95106	0.084302
1.3	15.95013	190.3907	15.95106	0.084152
1.4	15.92624	190.1055	15.95106	0.084026
1.5	15.90583	189.8619	15.95106	0.083919

Table 4. Velocity, Angular Velocity, Max Power vs. Penstock Diameter

We can also calculate the power output for varying angular velocities for three different pipe diameters:

See flowWizard.cpp for calculation details.

$d\ [m]$	$\Omega_{max}\ [rpm]$	$P\ [MW]$
0.2	224.1	15.9519
0.2	214.1	15.24008
0.2	204.1	14.52826

0.2	194.1	13.81643
0.2	184.1	13.10461
0.2	174.1	12.39279
0.2	164.1	11.68097
0.2	154.1	10.96915
0.2	144.1	10.25733
0.2	134.1	9.545512
0.2	124.1	8.833691
0.2	114.1	8.121871
0.2	104.1	7.410051
0.2	94.1	6.69823
0.2	84.1	5.98641
0.2	74.1	5.274589
0.2	64.1	4.562769
1	191.6	15.94836
1	181.6	15.11599
1	171.6	14.28361
1	161.6	13.45123
1	151.6	12.61885
1	141.6	11.78647
1	131.6	10.9541
1	121.6	10.12172
1	111.6	9.289339
1	101.6	8.456961

1	91.6	7.624583
1	81.6	6.792205
1	71.6	5.959827
1	61.6	5.127449
1.5	189.9	15.95426
1.5	179.9	15.11412
1.5	169.9	14.27398
1.5	159.9	13.43384
1.5	149.9	12.5937
1.5	139.9	11.75356
1.5	129.9	10.91342
1.5	119.9	10.07328
1.5	109.9	9.233141
1.5	99.9	8.393001
1.5	89.9	7.55286
1.5	79.9	6.71272
1.5	69.9	5.87258

Table 5. Power vs. Angular Velocity

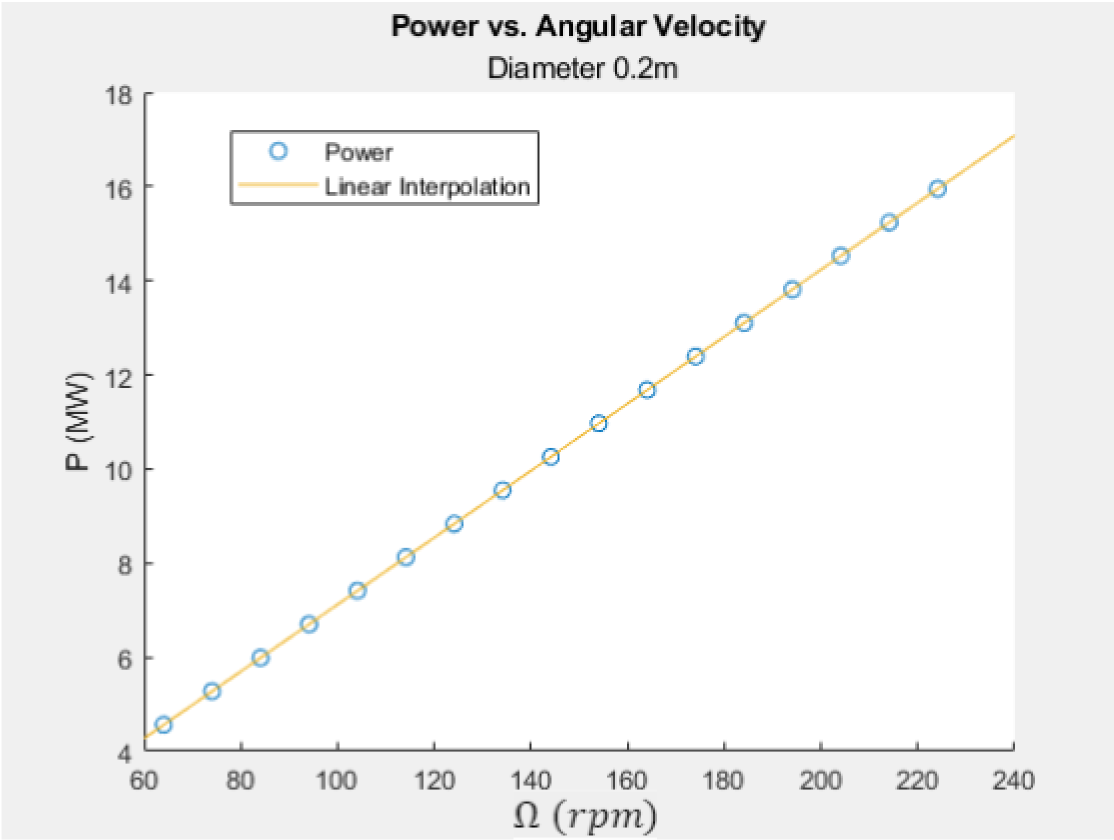


Fig. 8 Power vs. Angular Velocity, Diameter 0.2m

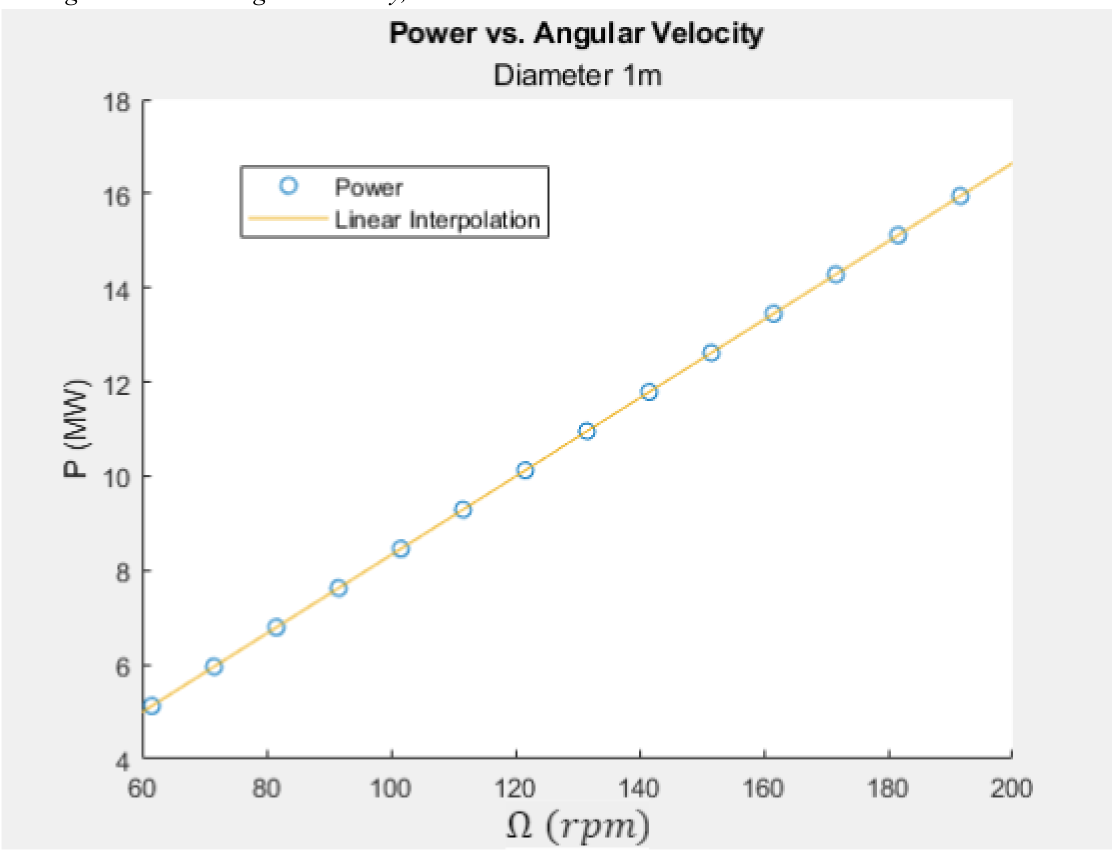


Fig. 9 Power vs. Angular Velocity, Diameter 1m

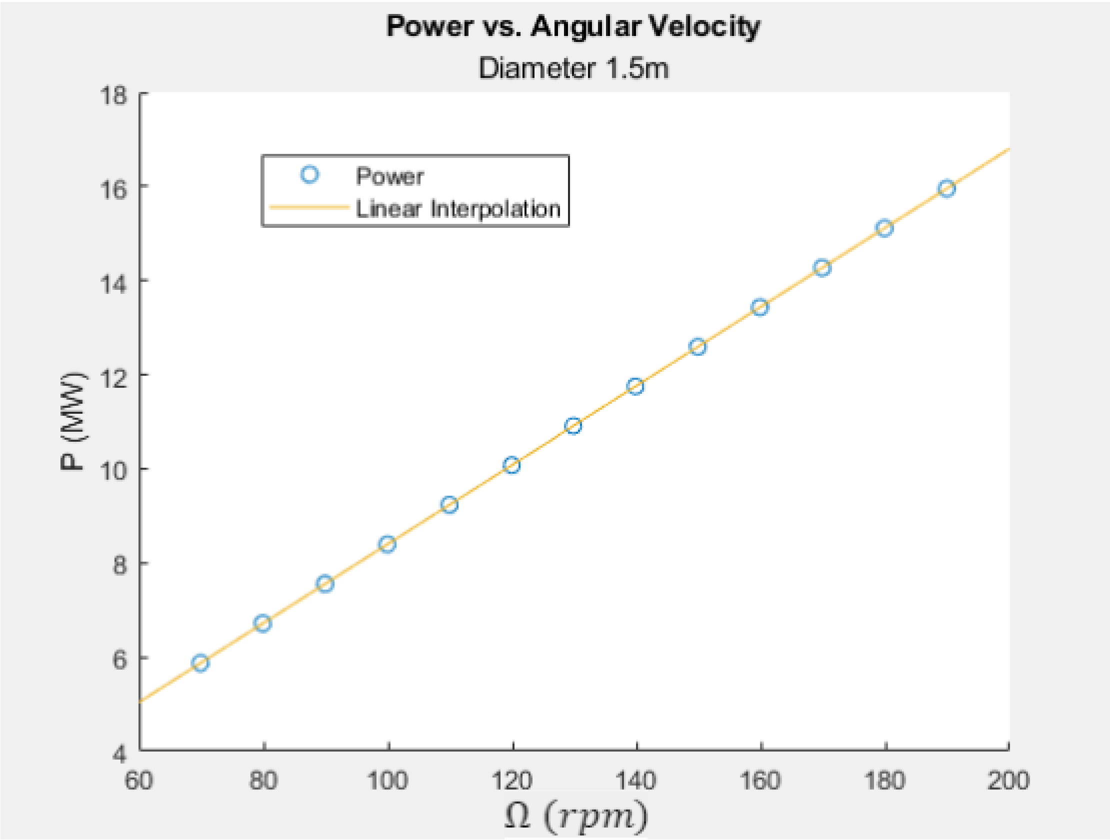


Fig. 10 Power vs. Angular Velocity, Diameter 1.5m

Efficiency of Varying Turbine Blade Angles

Using the aforementioned equations and theory, we can see that the turbine efficiency vs blade angle has a sinusoid relationship:

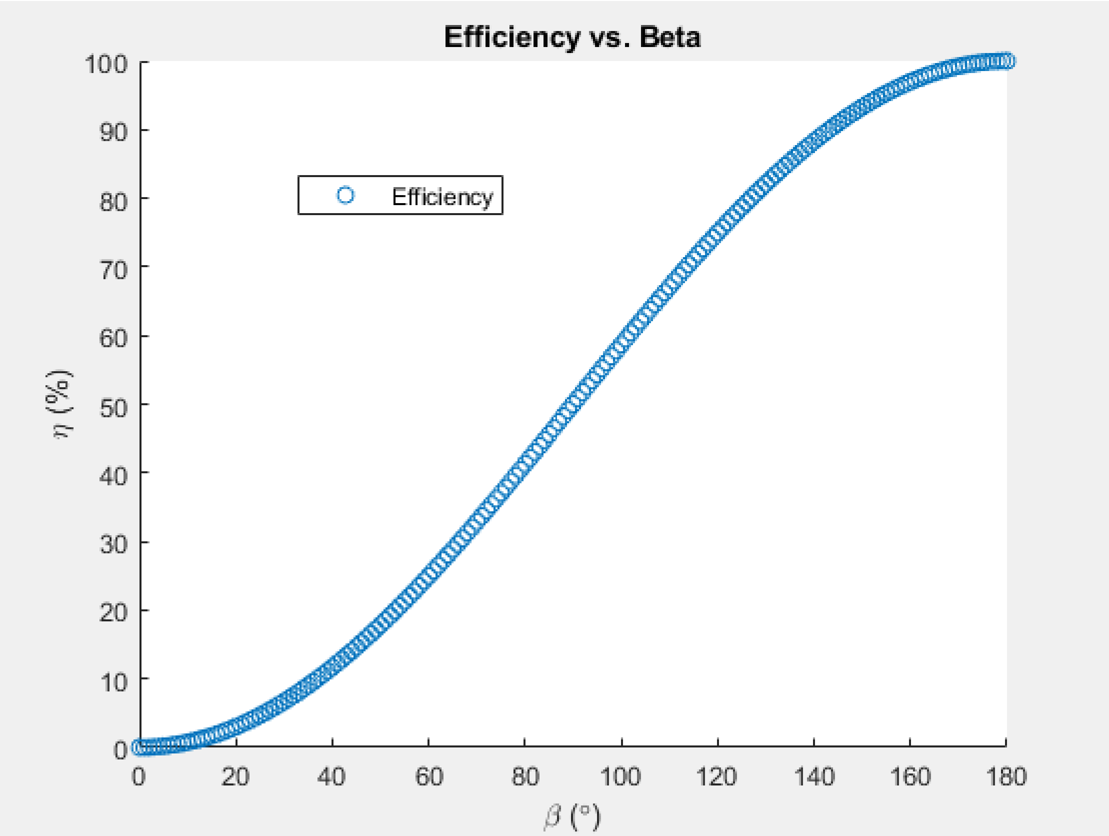


Fig 11. Turbine Efficiency vs. Angle Beta

4. Conclusions and Recommendations

Run-of-the-river facilities use the river's natural flow and elevation drop to produce energy. The potential energy per unit mass flow rate of an RoR facility can be estimated using Bernoulli's equation.

$$P_{ideal} = 15.95\ MWh$$

Once the flow is no longer considered inviscid, friction, as a function of flow rate, absorbs energy from the system and is found to limit power output to the turbine.

$$P_{max} = 2\rho g h Q/3 \quad \text{where } Q = \sqrt{h/3C}$$

To find the optimal angle of descent for the penstock, the flow rate, velocity at the nozzle and friction head loss were analyzed at five different positions. For an increase in angle of the penstock, the plotted data reveals that the headloss due to friction decreases while the velocity and flow rate both increase. This observation was to be expected as there is more potential energy to accelerate the fluid velocity. Since flow rate is a function of velocity, it would also increase. The friction head decreases as the angle increases due to it being a function of the Reynolds number; a greater velocity creates a greater Reynolds number which in turn produces a smaller friction headloss.

When considering the angular velocity of the turbine, the assigned value of 300 rpm is unrealistic for the pipe diameters given. We recommend that the flow rate and desirable power output be determined, then calculate the velocity of the turbine to achieve the desired power output while accounting for energy losses in the turbine. The turbine can then be designed appropriately.

In this analysis, we found that the power output is highly dependent on the geometry of the penstock and turbine. Furthermore, no single turbine will be optimal across all real world conditions due to fluctuations in the river's flow. We recommend conducting further studies on designing an RoR system with adjustable factors according to flow, i.e. allowing switching between a smaller turbine for lesser flows and a larger turbine for greater flows to optimize efficiency.

References

[1] M. Sait Tahmiscioğlu, Nermin Anul, Fatih Ekmekçi, Nurcan Durmuş, *Positive and Negative Impacts of Dams on the Environment*
[2] (2021, March 17) *Provincial and Territorial Energy Profiles – British Columbia* [website]. Available:
https://www.cer-rec.gc.ca/en/data-analysis/energy-markets/provincial-territorial-energy-profiles/provincial-territorial-energy-profiles-british-columbia.html#:~:text=About%2091%25%20of%20electricity%20in,northeastern%20B.C%20(Figure%203)
[3] M. Whiticar. (2016) *Run of River Power* [website]. Available:
http://www.energybc.ca/runofriver.html#footnote-11
[4] (1997) *Transition and Turbulence* [website]. Available:
https://www.princeton.edu/~asmits/Bicycle_web/transition.html#:~:text=Thus%20the%20dynamic%20viscosity%20of,any%20small%20disturbances%20are%20present
[5] White, F. Introduction to Fluid Mechanics [9th ed.], McGraw Hill, 2021
[6] J. L. Gordon. (1998). *Small Hydro Conduit Optimization with Differential Calculus* [article]. Available:
https://www.ieahydro.org/media/c01e7531/Small-hydro-counduit-optimization-with-differental-calculus.pdf
[7] A. Lewis, K. Healy. *Managing flow ramping operations at run-of-river hydroelectric projects* [article]. Available:
https://www.ecofishresearch.com/wp-content/uploads/2020/10/Lewis-Managing-flow-ramping-operations-at-run-of-river-hydroelectric-projects_final.pdf