Concurrent Algorithms and Data Structures – Theory Assignment 1

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Please, submit your solutions in .pdf format.

Problem 1 (20p) Consider the Register abstract data type that we have studied in the lectures. For each of the concurrent histories in Fig.1: check whether it is linearizable wrt. to Register? If *yes*, redraw the history and put the linearization points in the correct places (no explanation is required in this case). If *no* explain in no more than five lines the reason. Assume that all three histories are initiated from the register value 0.

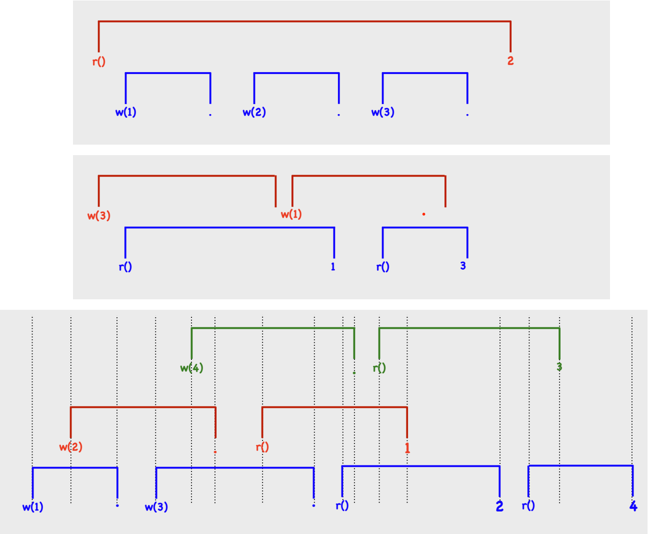

Figure 1: A set of histories.

Problem 2 (80p) We use the linearizable register of the previous problem to design a concurrent counter that is shared by n threads. The counter library allows two operations, namely `inc`(i) that increments the value of the counter of thread i by one, and `rd`$()$ that returns the current value of the counter. The modules for `inc`(i) and `rd`$()$ are depicted in Algorithm 1 and Algorithm 2 respectively

Algorithm 1: `inc`(i)

```
$b := b + 1$;
write($R_i$)($b$)
```

Algorithm 2: `rd`$()$

```
$r := 0$;
for $1 \leq i \leq n$ do
    $a :=$ read($R_i$);
    $r := r + a$
return($r$)
```

The threads share n registers $R_1, \ldots, R_n$. Each R_i behaves like a register, as discussed in the lectures. The `inc`(i) module uses a local variable b, initialized to 0. Each time it is called, it increments the value of b and stores the new value in R_i. The `rd`$()$ module iterates over all the registers and returns their sum.

- Give the abstract data type for the counter.
- Assume that each R_i is linearizable wrt. the register data type. Is the

given library linearizable wrt. the counter data type. If *yes*, describe the linearization policy, and justify it in no more than **5 lines**. If *no*, give a library history that is not linearizable wrt. the counter data type.

- Assume that each $\texttt{R}_i$ is atomic, i.e., calls to $\texttt{R}_i$ cannot overlap. Is the given library linearizable wrt. the counter data type. If *yes*, describe the linearization policy, and justify it in no more than **five lines**. If *no*, give a library history that is not linearizable wrt. the counter data type.