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1. $A = \{\langle R \rangle \mid R$ is a regular expression describing a language containing at least one string w that has 111 as a substring$\}$

first define a language $B = \{w \in \Sigma^* \mid w$ contains 111 as substring$\}$, B is a regular language with regular expression $\Sigma^* 111 \Sigma^*$

thus we can construct a DFA_B that recognize B.

if $L(R) \cap B \neq \emptyset$, then $\langle R \rangle \in A$, therefore we construct a DFA D that recognize $L(R) \cap B$ since $L(R) \cap B$ is a regular language.

So, we give the following TM M to decide A.

T = On input $\langle R \rangle$, where R is a regular expression

(i) Construct DFA_B that accept $\Sigma^* 111 \Sigma^*$

(ii) construct DFA D such that it recognize $L(R) \cap B$

(iii) Run TM T on input $\langle B \rangle$, where T decides E_{DFA} by Theorem

(iv) If T accepts, rejects, if T accepts, accepts.

2. B is the set of all infinite sequence over $\{0,1\}$

we get every element in B is infinite seq $(b_1, b_2, b_3, \dots)$ when $b_i \in \{0,1\}$

Assume B is countable

then we can have a correspondence f between B and N when $N = \{1, 2, 3, \dots\}$

let $f(n) = (b_{n1}, b_{n2}, b_{n3})$ for $n \in N$, and we can get

n	$f(n)$
1	$(b_{11}, b_{12}, b_{13}, \dots)$
2	$(b_{21}, b_{22}, b_{23}, \dots)$
3	$(b_{31}, b_{32}, b_{33}, \dots)$
$\vdots$	$\vdots$

then define an infinite seq $S = (S_1, S_2, S_3, \dots)$

$S \in B$ and $S_i = \overline{b_{ii}}$ for $i \in N$

with all that been set, for any n, S differs each b_{ii}

hence S cannot occur in the enumeration

Therefore, by Contradiction, the set B is uncountable.

3. $T = \{(i, j, k) \mid i, j, k \in N\}$

4. EQ_{CFG} is undecidable

idea: using reduction of All_{CFG} to EQ_{CFG}

$All_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$

Assume EQ_{CFG} is decidable, and we have a decider T for EQ_{CFG}

construct a TM M to decide All_{CFG} using TM T.

define a CFG $G_1 = (V, \Sigma, R, S)$ where $L(G_1) = \Sigma^*$

TM M follows, M = on input $\langle G \rangle$, where G is a CFG.

(i) Run TM T on input $\langle G, G_1 \rangle$

(ii) if T accept, accept, else reject.

Since M decides All_{CFG}, and by Theorem All_{CFG} is undecidable,

Therefore, by contradiction, EQ_{CFG} is undecidable

5. if $A \leq_m B$ and B is a regular language.

A is not a regular language.

for example. let $A = \{a^n b^n \mid n \geq 0\}$ $B = \{b\}$ $\Sigma = \{a, b\}$ and

define function $f: \Sigma^* \to \Sigma^*$ as

$$f(w) = \begin{cases} b & \text{if } w \in A \\ a & \text{if } w \notin A \end{cases}$$

we can see that A is decidable by theorem, but it's also a context free language. f is computable.

also. $w \in A$ IFF $f(w) = b$ or let's say IFF $f(w) \in B$

Therefore if $A \leq_m B$ and B is regular, A is not a regular language.