1. EQ_{CFG} is co-Turing-recognizable

EQ_{CFG} is co-Turing-recognizable IFF $\overline{EQ_{CFG}}$ is Turing-recognizable

thus we consider a TM M as following:

M = on input $\langle G_1, G_2 \rangle$ where G_1 and G_2 are CFGs over Σ

i) For each string w generated from Σ^* in lexicographical order, where $w \in \Sigma^*$

ii) using algorithm showing A_{CFG} is decidable, convert G_1, G_2 into equivalent CFG $G_1{}'$ $G_2{}'$ in Chomsky normal form.

iii) check whether $w \in L(G_1)$ and $w \in L(G_2)$

iv) if one of $G_1{}'$ and $G_2{}'$ generate w, which is ① $w \in L(G_1)$ and $w \notin L(G_2)$ or

② $w \notin L(G_1)$ and $w \in L(G_2)$

if ① or ② is positive, accept, else, continue the loop and go to step (i)

M will loop when $L(G_1) = L(G_2)$, hence M accepts IFF $L(G_1) \neq L(G_2)$

thus M is a recognizer of $\overline{EQ_{CFG}}$, thus we say EQ_{CFG} is co-Turing-recognizable.

2. $T = \{\langle M \rangle \mid M$ is TM that accepts w^R whenever it accepts $w\}$

a) assum T is decidable, and reduce A_{TM} to T, where A_{TM} is undecidable by theorem.

$A_{TM} = \{\langle M, w \rangle \mid M$ is a TM that accepts $w\}$

we use T to construct a decider A for A_{TM} as followng:

A = on input $\langle M, w \rangle$, where M is a TM. w is a string

i) construct a TM M' as followng

M' = on input $\langle s \rangle$, where s is a string

a) if s has form '01' or '10', accept

b) if s doesn't have the form, run M on input $\langle w \rangle$

c) if M accepts, accept, if M reject, reject.

ii) Run T on input $\langle M' \rangle$

iii) if T accept, accept. else, reject.

since, we can decide A_{TM} using T, there is a contradiction

Therefore, proof by contradiction T is undecidable.

(B) We can use AI consider 2 languages, L_1 and L_2:

L_1 has the property that if $w \in L_1$ then $w^R \notin L_1$, hence if a TM M recognizes L_1 then the input $< M > \notin T$

L_2 has the property that is $w \in L_2$ then $w^R \in L_2$, hence if a TM M recognizes L_2 then the input $< M > \in T$

Then for the language that recognized by M', we have 2 cases:

- Case 1

  Input <M> accepts string w, then since M′ will always accepts string s where $s \in L_1$, when M accepts w, M′ will also accepts strings str that $str \notin L_1$, hence $L(M') = L_2$

- Case 2

  Input <M> rejects string w, thus M′ will only accepts string s from L_1, hence $L(M') = L_1$

So, we can conclude that $M' \in T$ *IFF* $< M, w > \in A_{TM}$. *However,* A_{TM} *is never decidable, so we can't construct a TM A showing that* $< M, w > \in A_{TM}$. *Therefore, there is a contradiction, and we conclude that we can't show T is decidable.*

Even if we can use artificial intelligence to define everything that we need, we cannot use AI to get over the basic property of a TM.

3. A is Turing-recognizable IFF $A \leq_m A_{TM}$

let $A \leq_m A_{TM}$, according to the book. A_{TM} is Turing-recognizable
thus if $A \leq_m A_{TM}$, A is Turing-recognizable, by theorem.
then, we say there is some TM M that recognize A.
let $f(x) = \langle M, x \rangle$, f is computable.
$\Rightarrow$ $x \in A$ IFF M accepts x IFF $\langle M, x \rangle \in A_{TM}$
thus f is a mapping function from A to A_{TM}, hence $A \leq_m A_{TM}$
Therefore, we can say A is Turing-recognizable IFF $A \leq_m A_{TM}$

4.

a) $2n = O(n)$ T

let $c=2$, thus $2n \le cn = 2n$, for all $n \ge 1$

b) $3^n = 2^{O(n)}$ T

$3^n = 2^{n \log_2 3} \Rightarrow 2^{\log_2 3 \cdot n} = 2^{O(n)}$ for some c

c) $n = o(2n)$ F

$o(2n) = o(n)$, since $f(n) \neq o(f(n)) \Rightarrow$ False.

d) $n = o(\log n)$ F

$\lim_{n\to\infty} \frac{n}{\log n} \neq 0 \Rightarrow n \neq o(\log n)$

5. TRIANGLE $= \{\langle G\rangle \mid G$ contains a triangle$\}$. TRIANGLE $\in P$

let $G = (V, E)$ where V is a set of vertices, E is set of edges.

For triples of vertices from V, enumerate them can check if 3 pairs of edges is included in E.

that is $x, y, z \in V$. IS (x,y) (y,z) (x,z) exist in E?

To enumerate all tripes of vertices, it take $O(V^3)$

To check if 3 pairs of edges in E, it takes $O(E)$

Thus, total time is $O(V^3 E)$

is polynomial in the length of input graphs $\langle G\rangle$.

Therefore, we say TRIANGLE $\in P$