CMPSC 464 Introduction to the Theory of Computation, Fall 2020

Homework 3

Out: Sep. 10, **Due: Sep. 17**

Instructions: *Problems are to be turned in on Gradescope. Start a new page for each problem and when uploading, select the appropriate pages for each problem. Your assignments may be handwritten, use latex, etc. Write your name, "CMPSC 464' on your assignments. Write the names of up to three collaborators, or "Collaborators: none". Please review the homework policy on the syllabus.*

1. *Use the pumping lemma to show that the language* $A = \{www | w \in \{a, b\}^*\}$ *is not regular.*

2. *Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection, and complement.*

   (a) $\{0^n 1^m 0^n | m, n \geq 0\}$

   (b) $\{w | w \in \{0, 1\}^*$ *is not a palindrome*$\}$. *A palindrome is a string that reads the same forward and backward.*

3. (a) *Let* $B = \{1^k y | y \in \{0, 1\}^*$ *and* y *contains at least* k *1s, for* $k \geq 1\}$. *Show that* B *is a regular language.*

   (b) *Let* $C = \{1^k y | y \in \{0, 1\}^*$ *and* y *contains at most* k *1s, for* $k \geq 1\}$. *Show that* C *is not a regular language.*

4. *Book 1.55 (f) and (g).*

5. *Give context-free grammars that generate the following languages. In all parts the alphabet* Σ *is* $\{0, 1\}$.

   (a) $\{w |$ *w starts and ends with the same symbol*$\}$

   (b) $\{w |$ *the length of* w *is odd* $\}$

   (c) $\emptyset$

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1. Use the pumping lemma to show that $A = \{www \mid w \in \{a,b\}^*\}$ is not regular.

Proof:

Assume A is regular, let p be the pumping length by the PL.

Let's consider a string $s = a^p b a^p b a^p b$ where $s \in A$, $|s| \geq p$

S can be divided into xyz such that $\underset{(1)}{|y| > 0}$, $\underset{(2)}{|xy| \leq p}$, $\underset{(3)}{\forall i > 0,\ xy^iz \in A}$

$x = a^l$ $y = a^m$ $z = a^n b a^p b a^p b$ where $l+m+n = p$ by condition (2)

let $s' = xyyz$ where $s' \in A$, by condition (3)

$\boxed{\underbrace{a \cdots\cdots a}_{p\text{ 'a's}}b \mid a \cdots\cdots ab \mid a \cdots\cdots ab}$

xy being all 'a's.

xyyz:

no b in the left block

$\notin A$

we have

$s' = a^{l+2m+n} b a^p b a^p b$

since $l+m+n = p$, $|y| > 0$ and $s' \in A$

$l+m+n = l+2m+n = p$, $m > 0$

contradict

Therefore, A is not regular $\square$

2.

(a) Prove that $A = \{0^n 1^m 0^n \mid m, n \geq 0\}$ is not regular.

Proof.

Assume A is regular, let p be the pumping length by the PL.

let's consider a string $S = 0^p 1 0^p$ where $S \in A$, $|S| \geq p$

S can be divided into such that $\underset{(1)}{|y| > 0}$, $\underset{(2)}{|xy| \leq p}$, $\underset{(3)}{\forall i, xy^iz \in A}$

$x = 0^k$ $y = 0^{p-k}$ $z = 10^p$ where $p-k>0$, k is Integer by condition (1)

let $s' = xy^0z = xz$ where $s' \in A$. by condition (3)

we have

$s' = xz = 0^k 1 0^p$

Since $s' \in A$

$k = p \Rightarrow p-k=0$ contradict

Therefore, A is not regular. $\square$

(b) Prove that $B = \{w \mid w \in \{0,1\}^* \text{ is not a palindrome}\}$ is not regular

Proof.

Assume B is regular, let p be the pumping length by the PL.

let's consider a string $S = 0^p 1 0^p$ where $S \in B$, $|S| \geq p$

S can be divided into xyz such that $\underset{(1)}{|y| > 0}$, $\underset{(2)}{|xy| \leq p}$, $\underset{(3)}{\forall i > 0, xy^iz \in B}$

$x = 0^k$ $y = 0^{p-k}$ $z = 10^p$ where $p-k>0$, k is Integer by condition (1)

let $s' = xy^0z = xz$ where $s' \in B$. by condition (3)

we have

$s' = xz = 0^k 1 0^p$

Since $s' \in B$

$k = p \Rightarrow p-k=0$ contradict

Therefore, B is not regular. $\square$

4.

Book 1.55 (f) Σ, find minimum pumping length

minimum pumping length $p=1$

p must be ≥ 1, even language can't pump

Book 1.55 (g) $1^*01^*01^*$

minimum pumping length $p=3$

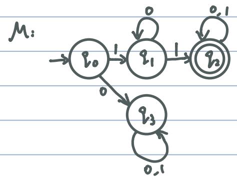
010 is the shortest string that can pump

3.

(a) Show that $B=\{1^k y \mid y\in\{0,1\}^*$ and y contains at least k 1s, for $k\geq 1\}$ is regular.

Construct a DFA M to accept B

M:



$M=(Q,\Sigma,\delta,q_0,F)$

$Q=\{q_0,q_1,q_2,q_3\}$

$\Sigma=\{0,1\}$

$F=\{q_2\}$

Therefore, since we can construct a DFA M where $L(M)=B$, we can say B is regular. □

(b) Show $C=\{1^k y \mid y\in\{0,1\}^*$ and y contains at most k 1s, for $k\geq 1\}$ is not regular

Proof:

Assume C is regular, let p be the pumping length by the PL.

Let's consider a string $S=1^p01^p$ where $S\in C$, $|S|\geq p$

S can be divided into xyz such that $\underset{(1)}{|y|>0}$, $\underset{(2)}{|xy|\leq p}$, $\underset{(3)}{\forall i\geq 0,\ xy^iz\in C}$

$x=1^k$ $y=1^{p-k}$ $z=01^p$ where $p-k>0$ by condition (2)

let $s'=xy^0z=xz=1^k01^p$ where $s'\in C$ by condition (3)

thus, we get $k\geq p$ contradict

however, $p-k>0 \Rightarrow p>k$

Therefore, C is not regular.

5. CFG, $\Sigma = \{0,1\}$

(a) $\{w \mid w$ starts and ends with same symbol$\}$

$w \to 1A1 \mid 0A0$

$A \to A1 \mid A0 \mid \varepsilon$

(b) $\{w \mid$ the length of w is odd$\}$

$w \to 1w1 \mid 1w0 \mid 0w0 \mid 0w1 \mid A$

$A \to 0 \mid 1$

(C) ϕ

$w \to w$