Homework 4

Out: Sep. 17, Due: Sep. 24

Instructions: Problems are to be turned in on Gradescope. Start a new page for each problem and when uploading, select the appropriate pages for each problem. Your assignments may be handwritten, use latex, etc. Write your name, "CMPSC 464' on your assignments. Write the names of up to three collaborators, or "Collaborators: none". Please review the homework policy on the syllabus.

- 1. Give context-free grammars generating the following languages.
 - The complement of the language $\{a^nb^n|n\geq 0\}$.
 - $\{x_1 \# x_2 \# \dots \# x_k | k \ge 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j, x_i = x_j^R\}.$
- 2. Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k | i = j \text{ or } j = k \text{ where } i, j, k \ge 0\}.$$

Is your grammar ambiguous? Why or why not?

- 3. Book 2.12.
- 4. Let B be the language of all palindromes over $\{0,1\}$ containing an equal numbers of 0s and 1s. Show that B is not context free.
- 5. Let $\Sigma = \{1, 2, 3, 4\}$ and $C = \{w \in \Sigma^* | in w$, the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s}. Show that C is not context free.

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1. Give CFGs generating layuages
1) Complement of A= fa^b^ n > 03
(ases:
a^5 n≠m: a, aab, b, abb
(aub)*ba(aub)* : ba, aba, bab
E GA, so no empty stray generate by CFG
let G=CfR.R.R.A.B.MI, fa, by, R, S) be the CFG of A, where R is delivered by
$\mathbb{R} \to \mathbb{R}_1 / \mathbb{R}_2$
$R_1 \rightarrow \alpha R_1 b A B$
A > a k a '
B⇒ LB L
$\mathbb{R}_{z} \rightarrow M \delta_{a} M$ $M \rightarrow M a M b \in \mathbb{R}_{z}$
M-2 Wa We 5
2) {x, #x2 # #xk k>1, each x; efa, b}*, and for some i and j, x;=xj*
Case:
O i=j, then X; is palindrome for some i
@i*j. Hen k>1, x::xjR for some i ad j
lot G=Cf 3, fa, b, #1, R, s) when R is delived by Q#5
R-> R. R2 # is a character life a and b in the layring about another
Ri > P Ri #X X#Ri Ri makes sure a paltulrome in string aba
$\chi \rightarrow \alpha \chi [k \chi] \in [P]$
$P \rightarrow aPa bPb a b \epsilon$
$P_2 \rightarrow T \mid P_2 + Y \mid Y + P_2$
ا ا داد اما ک در ا

2 Gire a CHa s	Generales A: Paihick is in or int here is here	
lot G= (PS, S, Se, I, K	generates A= faibick i=j or j=k where i,j,k>0? 1, fa,b,c?,R,S) be the CFG of A, where R is delivered by	
$R \rightarrow R_1/R_2$,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	
R -> R1 R2 S1 -> aR1 K		
K→ bKc E		
R2→R2c I		
I→aIb E		
This grammer is and	biguous because it will have 2 parsity trees for some string when	ù=j=k
Take 'abe' as a ex	Nample D	
<u> </u>		
Rı		
(k)		
/ k		
a b & c	'abc'	
'abc'	usc	

3. Book 2.12
G: R→XRX S
S-> aTLILTa
S→aTb bTa T→XTX X E X→a b
λ ⇒ α Δ
$(\xi_1,\xi_2) = (\xi_1,\xi_2) + (\xi_1,\xi_2) + (\xi_2,\xi_2) + (\xi_1,\xi_2) + (\xi_1,\xi_2) + (\xi_2,\xi_2) + (\xi_1,\xi_2) + (\xi_2,\xi_2) + (\xi_1,\xi_2) + (\xi_2,\xi_2) + (\xi_2$
$start \rightarrow (p_0) \underbrace{\xi, \xi \rightarrow \ddagger}_{\xi_1} (p_1) \underbrace{\xi, \xi \rightarrow \xi}_{\xi_2} (p_2) \underbrace{\xi, \xi \rightarrow \xi}_{\xi_2} (p_2)$
£,R→XPX
4.2→5
€,5⇒αTb €,5⇒bTα €,7→XTX
€, S⇒bTa
£,T→XTX
τ.T→X
€, X→a €, X→b €, T→€
り, スプロ ら T みら
2,1→2 0,2→a
b, €->b
♥, ▼

A. Let B be the layuage of all palindromes over fo, is where # of 0 equals # of 1
Power.
lat R. Le a contract five lawrent the it must catify the 1. I CTI
for to on a contract five forgraph most in most small me samply sounds of Lites.
lat B be a contest free layunge that it sout satisfy the pumping bounds of CFLs. Lat p be the pumping legath and consider a string S=0 ^{PlaP} 0 ^P where S >P Split S Ma wxxy2 Such that vxy = P and vy >0
Spin > Mo LLYXY & Such That IVYY = P and IVYY = P
Case .
10 viry is only in the Os, thus no matter we pump up or pump down, the legish of Os doesn't equal => not a palindrome
also 과 바 한 产 제가 l
So there is a controdiction.
② vxy in the middle part: Is. thus if he pump up or pump down that 0 ≠# at 1
So contradiction
Theorefor, since there are contradition in all cases, we say B is not context free.

S. Let Z = {1,2,3,4} ad C= fwes* h w, # of 1 = # of 2 and # of 3 = # of 4}
Prod:
Tooks
let C be a context free layunge than it must satisfy the pumping bound of CFLs. Let p be the pumping legisle and consider a string $S=2^{3}3^{5}1^{2}3^{5}$ where $ S \ge p$
lat p be the pumping legth and consider a string S=2'3'1'a' where S≥p
splic s man uvxyz such that [vxy = p and vy >0
Case:
(1) UXY in any of parts where contains only 1 numbers: 20, 30, 15, or 45.
thus, when he pump, why is & C 4000 Sine #of 1 # # of 2 , or # of 3 # # of 4
Discourse of bands, and 3 & c coss Size and 17 and c Jose and c J
② Vity in ≥ consecutive pures, 2 ^k 3 ^k or 3 ^k 1 ^k or 1 ^k 4 ^k thus, when pumping, uvixg ² 3 ∉ C for not equal length of 1 and 2, or 3 and 4 Therefore, there are construdiction in all cases. C is not content free.
thus, when pumping, uving it all control equal legges at 1 and 2, or 3 and 4
Therefore, there are construdiction in all cases. C is not content free.