Homework 7

Out: Oct. 8, **Due: Oct. 15**

Instructions: *Problems are to be turned in on Gradescope. Start a new page for each problem and when uploading, select the appropriate pages for each problem. Your assignments may be handwritten, use latex, etc. Write your name, "CMPSC 464' on your assignments. Write the names of up to three collaborators, or "Collaborators: none". Please review the homework policy on the syllabus.*

1. *Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable.*

2. *Let* $A\varepsilon_{CFG} = \{\langle G\rangle | G \text{ is a CFG that generates } \varepsilon\}$. *Show that* $A\varepsilon_{CFG}$ *is decidable.*

3. *Let* $INFINITE_{PDA} = \{\langle M\rangle | M \text{ is a PDA and } L(M) \text{ is an infinite language}\}$. *Show that* $INFINITE_{PDA}$ *is decidable.*

4. *A* **useless state** *in a pushdown automata is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.*

5. *(10 points) Fill out the ETFE survey on Canvas.*
   Please be sure to answer the bottom four, 24-27. I ask those every semester.

   It's anonymous, so for the class to get credit for this problem we need
   (# surveys submitted) $\geq$ *(# HW's turned in).*

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1. We define the language $C = \{\langle M, R\rangle \mid M$ is a DFA and R is a regular expression and $L(M) = L(R)\}$

The TM F decides C: on input $\langle M, R\rangle$, where M is a DFA, R is a regular expression

a) Construct a DFA D_R for R using the Kleene's Theorem

b) Run TM T from Theorem 4.5 on input $\langle M, D_R\rangle$

c) If T accepts, accept. If T reject, reject.

2. $A_{\varepsilon CFG} = \{ \langle G \rangle \mid G$ is a CFG that generate $\varepsilon \}$

in the book, they list '$2n-1$ steps' derivation to check G generate w is sufficient.

So, to find it sufficient, Convert G into G' where $G' = (V', \Sigma, R', S')$, and $L(G) = L(G')$

IFF G' includes $S' \to \varepsilon$, $\varepsilon \in L(G)$, thus

The TM S decides $A_{\varepsilon CFG}$ = On input $\langle G \rangle$, where G is a CFG

a) Convert G to an equivalent grammar in Chomsky normal form

b) If G' includes $S' \to \varepsilon$, accept. else, reject

3.

4. To show the language $A = \{\langle M\rangle \mid M \in S$ & M has a useless state$\}$ where S is the set of pushdown automata.

we can get from book that a PDA has an empty language is decidable

thus if q is the accepting state, and results in the PDA has an empty language, q is useless state.

The TM U decide A follows, on the input $\langle M\rangle$

a) iterate all the states in PDA by setting them to the accepting state

b) if it results in an empty language, accept, else, reject.