Homework 4

Out: Sep. 17, **Due: Sep. 24**

Instructions: *Problems are to be turned in on Gradescope. Start a new page for each problem and when uploading, select the appropriate pages for each problem. Your assignments may be handwritten, use latex, etc. Write your name, "CMPSC 464' on your assignments. Write the names of up to three collaborators, or "Collaborators: none". Please review the homework policy on the syllabus.*

1. *Give context-free grammars generating the following languages.*
   - *The complement of the language* $\{a^n b^n | n \geq 0\}$.
   - $\{x_1 \# x_2 \# \ldots \# x_k | k \geq 1$, *each* $x_i \in \{a, b\}^*$, *and for some* i *and* j, $x_i = x_j^R\}$.
2. *Give a context-free grammar that generates the language*

   $$A = \{a^i b^j c^k | i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

   Is your grammar ambiguous? Why or why not?
3. *Book 2.12.*
4. *Let* B *be the language of all palindromes over* $\{0, 1\}$ *containing an equal numbers of 0s and 1s. Show that* B *is not context free.*
5. *Let* $\Sigma = \{1, 2, 3, 4\}$ *and* $C = \{w \in \Sigma^* |$ *in* w, *the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s*$\}$. *Show that* C *is not context free.*

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1. Give CFGs generating languages

1) Complement of $A = \{a^n b^n \mid n \geq 0\}$

Cases:

$a^n b^m \mid n \neq m$: a, aab, b, abb...

$(a \cup b)^* ba (a \cup b)^*$: ba, aba, bab...

$\varepsilon \in A$, so no empty string generate by CFG

Let $G = (\{R, R_1, R_2, A, B, M\}, \{a, b\}, R, S)$ be the CFG of $\bar{A}$, where R is delivered by

$R \to R_1 \mid R_2$

$R_1 \to aR_1b \mid A \mid B$

$A \to aA \mid a$

$B \to bB \mid b$

$R_2 \to MbaM$

$M \to Ma \mid Mb \mid \varepsilon$

2) $\{x_1 \# x_2 \# \ldots \# x_k \mid k \geq 1$, each $x_i \in \{a, b\}^*$, and for some i and j, $x_i = x_j^R\}$

Case:

① $i = j$, then x_i is palindrome for some i

② $i \neq j$, then $k > 1$, $x_i = x_j^R$ for some i and j

Let $G = (\{$ $\}, \{a, b, \#\}, R, S)$ where R is delivered by

a#b

ab#a#ba

aba

$R \to R_1 \mid R_2$

\# is a character like a and b in the language.

$R_1 \to P \mid R_1 \# X \mid X \# R_1$

R_1 makes sure a palindrome in string

$X \to aX \mid bX \mid \varepsilon \mid P$

$P \to aPa \mid bPb \mid a \mid b \mid \varepsilon$

$R_2 \to T \mid R_2 \# Y \mid Y \# R_2$

$Y \to X \mid Y \# X \mid X \# Y$

$T \to aTa \mid bTb \mid \# Y \#$

2. Give a CFG generates $A = \{a^i b^j c^k \mid i=j \text{ or } j=k \text{ where } i,j,k \geq 0\}$

let $G = (\{S, S_1, S_2, I, K\}, \{a,b,c\}, R, S)$ be the CFG of A, where R is delivered by

$R \to R_1 \mid R_2$

$S_1 \to aR_1 \mid K$

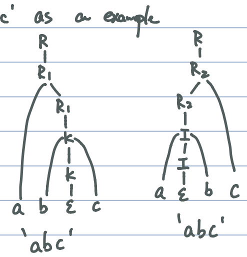
$K \to bKc \mid \varepsilon$

$R_2 \to R_2c \mid I$

$I \to aIb \mid \varepsilon$

This grammar is ambiguous because it will have 2 parsing trees for same string when $i=j=k$

Take 'abc' as an example



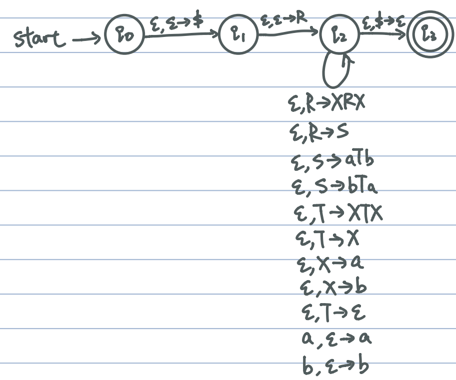
3. Book 2.12

G: $R \to XRX \mid S$

$S \to aTb \mid bTa$

$T \to XTX \mid X \mid \varepsilon$

$X \to a \mid b$



4. Let B be the language of all palindromes over $\{0,1\}$ where # of 0 equals # of 1

Proof:

let B be a context free language then it must satisfy the pumping lemma of CFLs.

let p be the pumping length and consider a string $S = 0^p 1^{2p} 0^p$ where $|S| \geq p$

split S into $uvxyz$ such that $|vxy| \leq p$ and $|vy| > 0$

Case:

① vxy is only in the 0s, thus no matter we pump up or pump down, the length of 0s doesn't equal ⇒ not a palindrome

also, # of 0 ≠ # of 1

So there is a contradiction.

② vxy in the middle part: 1s. thus if we pump up or pump down, # of 0 ≠ # of 1

So contradiction

Therefore since there are contradiction in all cases, we say B is not context free.

5. Let $\Sigma = \{1, 2, 3, 4\}$ and $C = \{w \in \Sigma^* \mid$ in w, # of 1 = # of 2 and # of 3 = # of 4$\}$

Proof:

let C be a context free language then it must satisfy the pumping lemma of CFLs.

let p be the pumping length and consider a string $S = 2^p 3^p 1^p 4^p$ where $|S| \geq p$

split S into $uvxyz$ such that $|vxy| \leq p$ and $|vy| > 0$

Case:

① vxy in any of parts where contains only 1 numbers: 2s, 3s, 1s, or 4s.

thus, when we pump, $uv^ixy^iz \notin C$ $\forall i \geq 0$ since # of 1 $\neq$ # of 2, or # of 3 $\neq$ # of 4

② vxy in 2 consecutive parts: 2^p3^p or 3^p1^p or 1^p4^p

thus, when pumping, $uv^ixy^iz \notin C$ for not equal length of 1 and 2, or 3 and 4

Therefore, there are contradiction in all cases. C is not context free.