Homework 3

Out: Sep. 10, **Due: Sep. 17**

Instructions: Problems are to be turned in on Gradescope. Start a new page for each problem and when uploading, select the appropriate pages for each problem. Your assignments may be handwritten, use latex, etc. Write your name, "CMPSC 464' on your assignments. Write the names of up to three collaborators, or "Collaborators: none". Please review the homework policy on the syllabus.

- 1. Use the pumping lemma to show that the language $A = \{www|w \in \{a,b\}^*\}$ is not regular.
- 2. Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection, and complement.
 - (a) $\{0^n 1^m 0^n | m, n \ge 0\}$
 - (b) $\{w|w\in\{0,1\}^*\ is\ not\ a\ palindrome\}$. A palindrome is a string that reads the same forward and backward.
- 3. (a) Let $B = \{1^k y | y \in \{0,1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1\}$. Show that B is a regular language.
 - (b) Let $C = \{1^k y | y \in \{0,1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\}$. Show that C is not a regular language.
- 4. Book 1.55 (f) and (g).
- 5. Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0,1\}$.
 - (a) $\{w \mid w \text{ starts and ends with the same symbol}\}$
 - (b) $\{w \mid \text{ the length of } w \text{ is odd }\}$
 - *(c)* ∅

Xiangyn Pen CMPSC 464

Collaborator. None

| [. Use the pumping lemma to show that A: fum | w ∈ {a,b} } is not regular. |
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| Proof: | • |
| Assume A is regular, but p be the pumping let's consider a string S= abababb where S & S and be divided into xy & such that y >0, xy | earth by the Pl. |
| let's consider a strong S=abababab where Se | A, s 3P |
| S an be divided it xy & such that y >0, xy | ≤p, ∀;>0, xy'2 ∈ A |
| (I) (| ۵) (٤) ^٧ |
| X=a y=a z=a ba ba'b where (fmfn=p | by Condition (2) |
| (OL 5 = X9/2 LANCE 5 ERG , by Cardition (3) | Wasses of the second of the se |
| s'= a babab | xy len all 'a'c. |
| since Umtn=P, 19170 and s'EA | እ _ከ ነ የ |
| L+m+n=L+2m+n=p , m>0 | no b in the left block |
| X=a y=a ^m z=a ^h ba ^p b where (tmtn=p) [et s'= xyyz where s'6A, by condition (3) we have s'= a bababb since stantn=p y > 0 and s' eA Ltentn= stantn=p m>0 contradict Therefore, A is not regular | ∉ Ø |
| Therefore, A is not regular | |
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| 2. |
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| (a) Prove that A=f0n/mon/m,n>07 is not regular. |
| Proof. |
| Assume A is regular, last p he the pumping length by the PL. |
| Proof: Assume A is regular, but P we the pumping length by the PL. let's consider a string $S=0^P \cdot 0^P$ where $S=6A$, $\{s\} \ge P$ S can be divided into such that $\{y\} \ge 0$, $\{x,y\} \le P$, $\{y\} \ge 0$. |
| S can be divided into cuch that 14/20, 124/5 P to 24/2 EA |
| (1) (3) (7) |
| X=0k y=0P-k z=10P where P-k>0, k is Integer by condition (1) |
| V= 0 3=0 3=10 Mac 6-k20 K R Tutche Pil Condution (1) |
| let s'=xy°z=xz when s'&A. by condition (3) |
| we have |
| 5'=x2=0k10k |
| sina s'ea |
| k=P >> P-k=0 contradict |
| |
| Therefore A is not regular. |
| |
| (b) Prove that B= fw w ∈ fe,13° is not a paladrone? is not replan |
| |
| Assume B is regular, but P he the pumping length by the PL. |
| lety consider a strong C-PLP here SER 15 2P |
| S a la divided it will all the helps bales Hissonia B |
| Assume B is regular, but p we the pumping length by the PL. Let's consider a string $S=o^{p_1}o^{p}$ where $S\in B$, $ s \geq p$ S can be divided into $xy\xi$ such that $ y >o$, $ xy \leq p$, $\forall i>o$, $ xy \leq B$ |
| k. Pk P. L L |
| X=0k y=0k z=10k where p-k>0, k is Integer by condition (1) |
| let s'=xy°z=xz when s'&B. by condition (3) |
| rive house |
| 5'=x2+0k10k |
| sina s'eB |
| k=P => P-k=0 contradict |
| |
| Therefor, B is not regular. |
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| A. Book 125 (f) E, fed minimum pumply length |
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| Book 125 (f) E. fed minimum transphy length |
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| minimum pumping length p= |
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| P must be > 1, even laguage count pump |
| Je strip |
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| Book 1.22 Ch 1,01,01, |
| Dark 1:22 ct. (01 of |
| minimum pumpty leigh p=3 |
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| 010 is the shortest striy that can pump |
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| 3. |
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| (a) Show that B= f1ky y ∈ f0,11 and y contains at least k 15, for k>13 is regular. |
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| construct a DFA M to accept B |
| |
| M: (Q,Σ,δ,β,Ţ) |
| |
| Q: {Po, 21, 22, 23} |
| $\Sigma = \{0,1\}$ $\Gamma = \{21\}$ |
| F= { 2-1} |
| o, i |
| Therefore, since we are construct a DFD M when LCM=B, we are say B & regular. |
| france to the position of the part of the |
| |
| (b) Show C=fityly & fo, 13° and y contains at most k is, for k>13 is not regular |
| |
| Trung have a second of the second of the DI |
| assume C S regular, the C has bumping length by the IC. |
| Let's consider a string S=1'01' where SEC, ISI 3P |
| 5 an be divided now xy ? such other ly/>0, lxy/≤p, ti>0, xy's ∈ C |
| Proof: Assume C is regular, lost P be the pumping length by the PL. Let's consider a string $S=I^POI^P$ where $S\in C$, $ S \geq P$ S can be divided at XYZ such that $ Y >0$, $ XY \leq P$, $Y:>0$, $ XY \leq C$ |
| $X=1^k$ $Y=1^{p-k}$ $Z=01^p$ where $P-k>0$ by condition (2) |
| Le de vile von l'en l'an electric de vile es |
| let S'= 74 = 1 to 1 P where S' & C by condition (3) |
| thus, he get k ? P contradict however, p-k>0 => p>k |
| hower, p-k>0 => p>k |
| Therefore, C is not regular. |
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| (a) full starts and ends with some symbol? |
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| 18-18 1 80 C |
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| (6) fw the length of w is add? w-> \ w \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ |
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