Regular expressions for polynomial coalgebras

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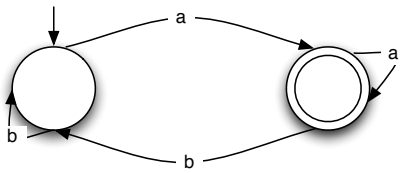
Regular expressions are a syntactic means to describe exactly the class of languages accepted by deterministic finite state automata, i.e. the class of regular languages. Ignoring the initial state for the moment, a deterministic automata is a set of states equipped with function determining for each state whether or not it is final, and assigning for each input symbol a next state.

There is a well-known correspondence between regular expressions and deterministic automata. For instance the automata



accepts the same language as described by the expression $b^*a(a + bb^*a)^*$. There are methods to obtain a regular expression that accepts the same language as a given automaton and vice-versa.

Deterministic automata can be generalized to coalgebras for an endofunctor G on the category Set. A coalgebra is pair (S, g) consisting of a set of states S and a transition function $g : S \to GS$, where the functor G determines the type of the dynamic system under consideration.

For polynomial set functors G, we generalize the notion of regular expressions and introduce a language of expressions for describing elements of the final G-coalgebra. We show that every state of a finite G-coalgebra corresponds to an expression in the language, in the sense that they both have the same semantics. Conversely, we give a compositional synthesis algorithm which transforms every expression into a finite G-coalgebra. The language of expressions is equipped with an equational system that is sound, complete and expressive with respect to G-bisimulation.