LFDS - 17/11/2015
UCL, London

# Path Logics for Querying Graphs

## combining expressiveness and efficiency

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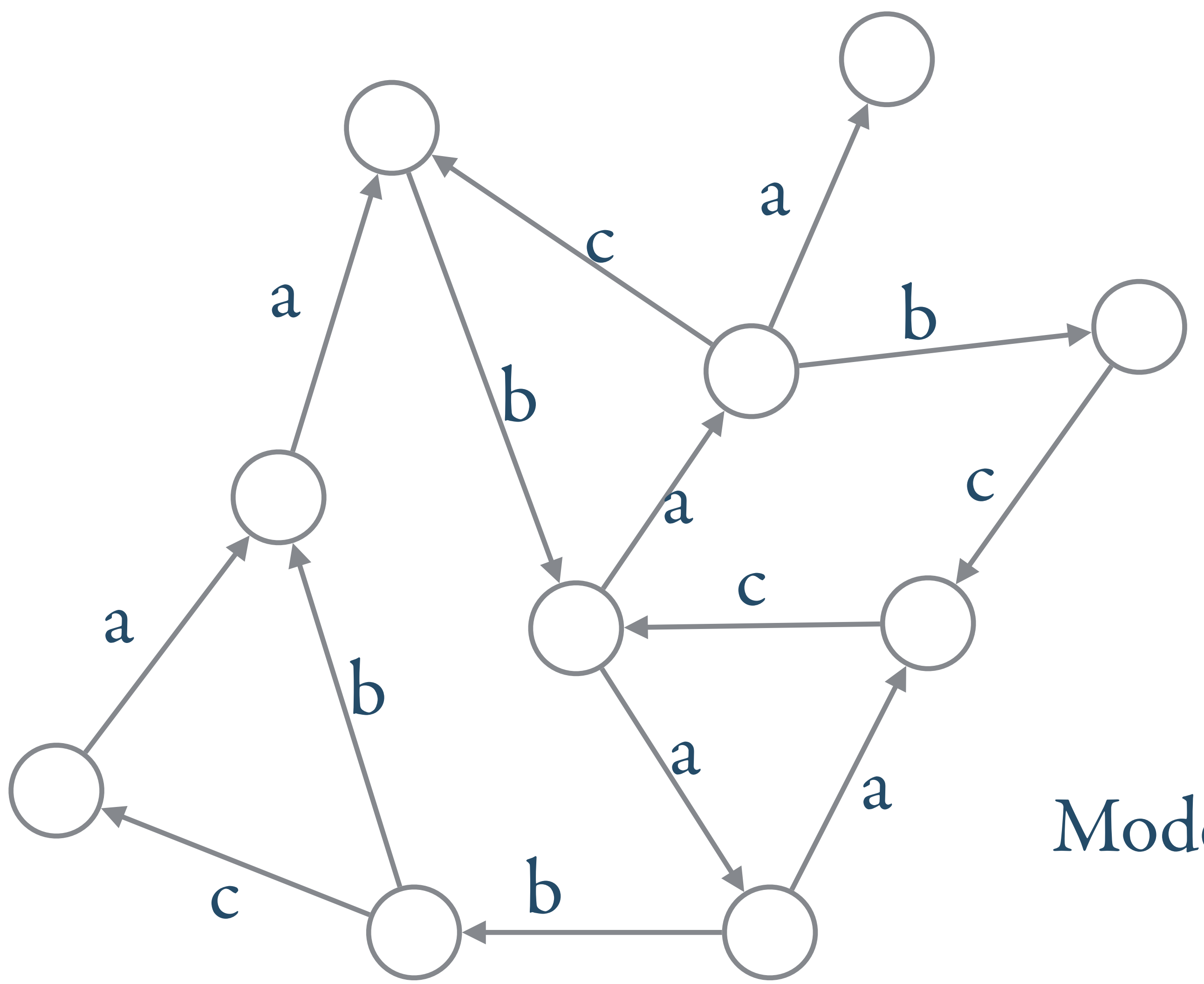
# Graph *databases*

Semantic web / RDF / social networks / . . .

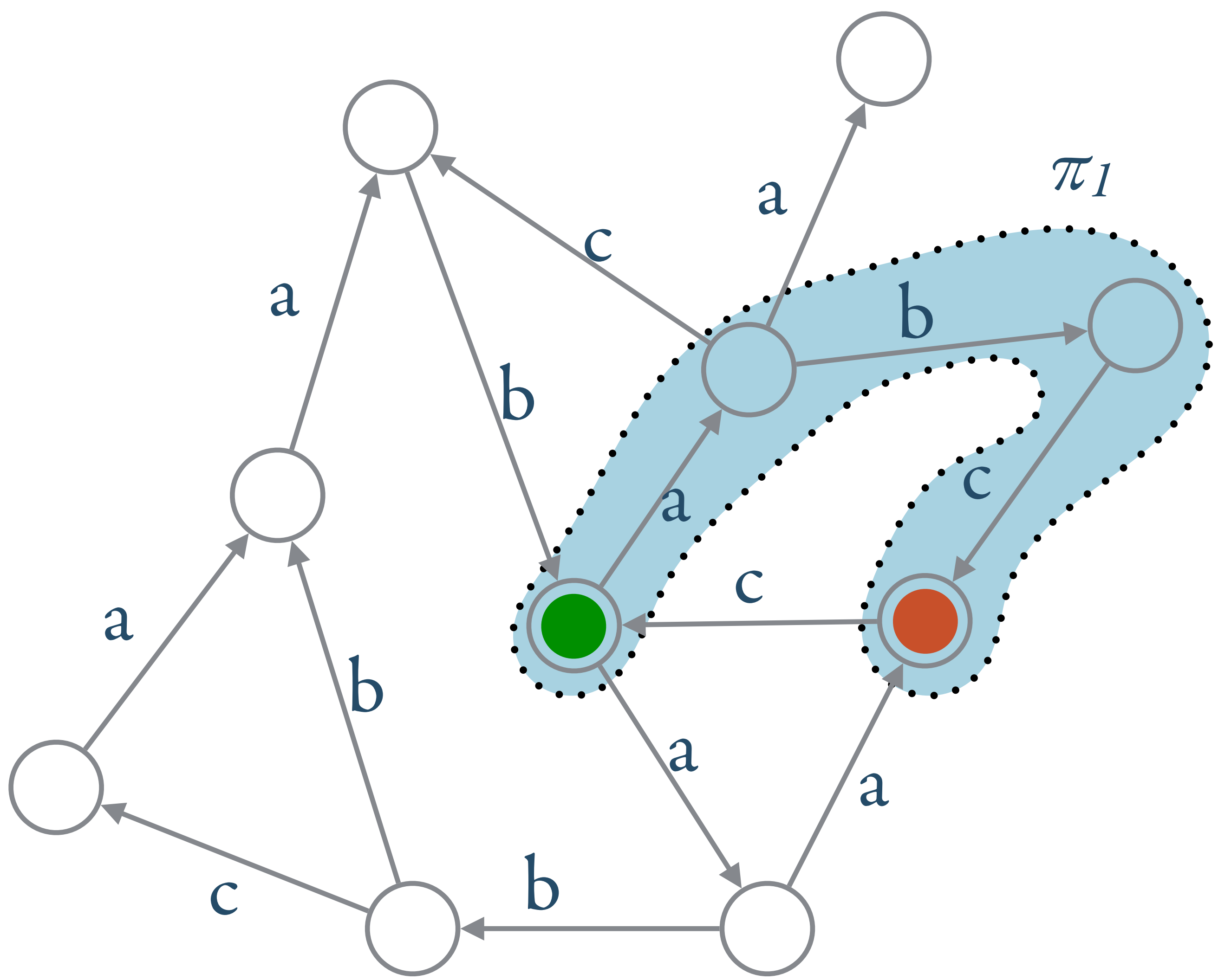


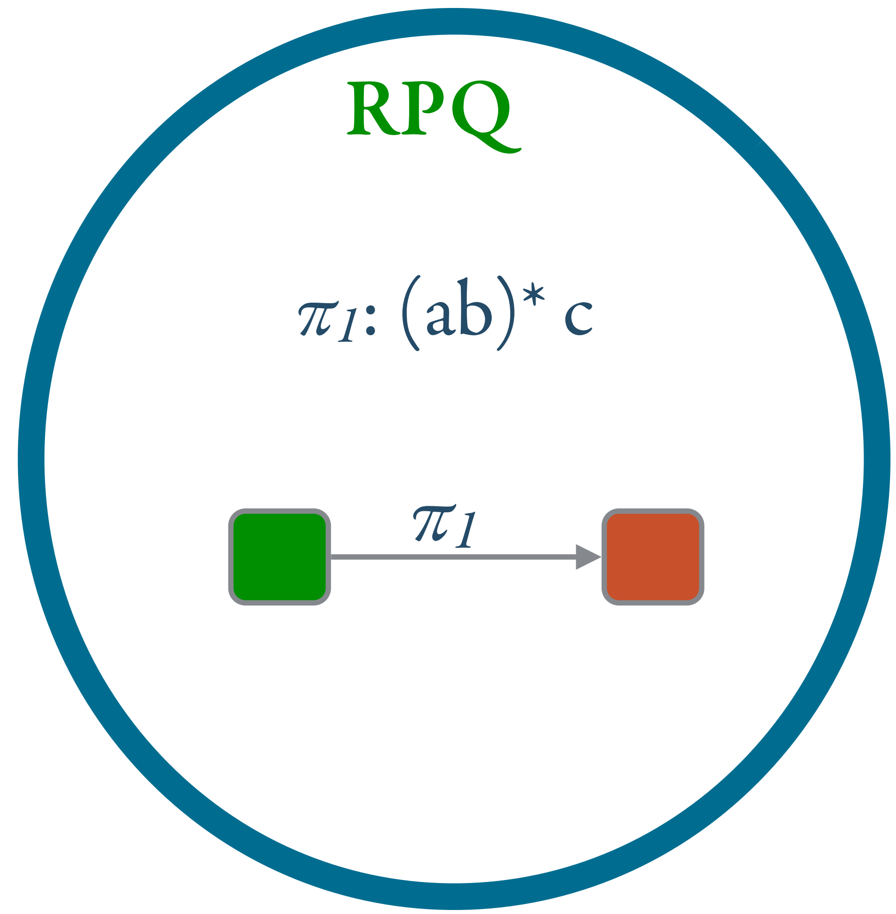

"Entities + Relations"

Modelled as: edge-labelled directed graphs

Notion of ***path*** of central importance

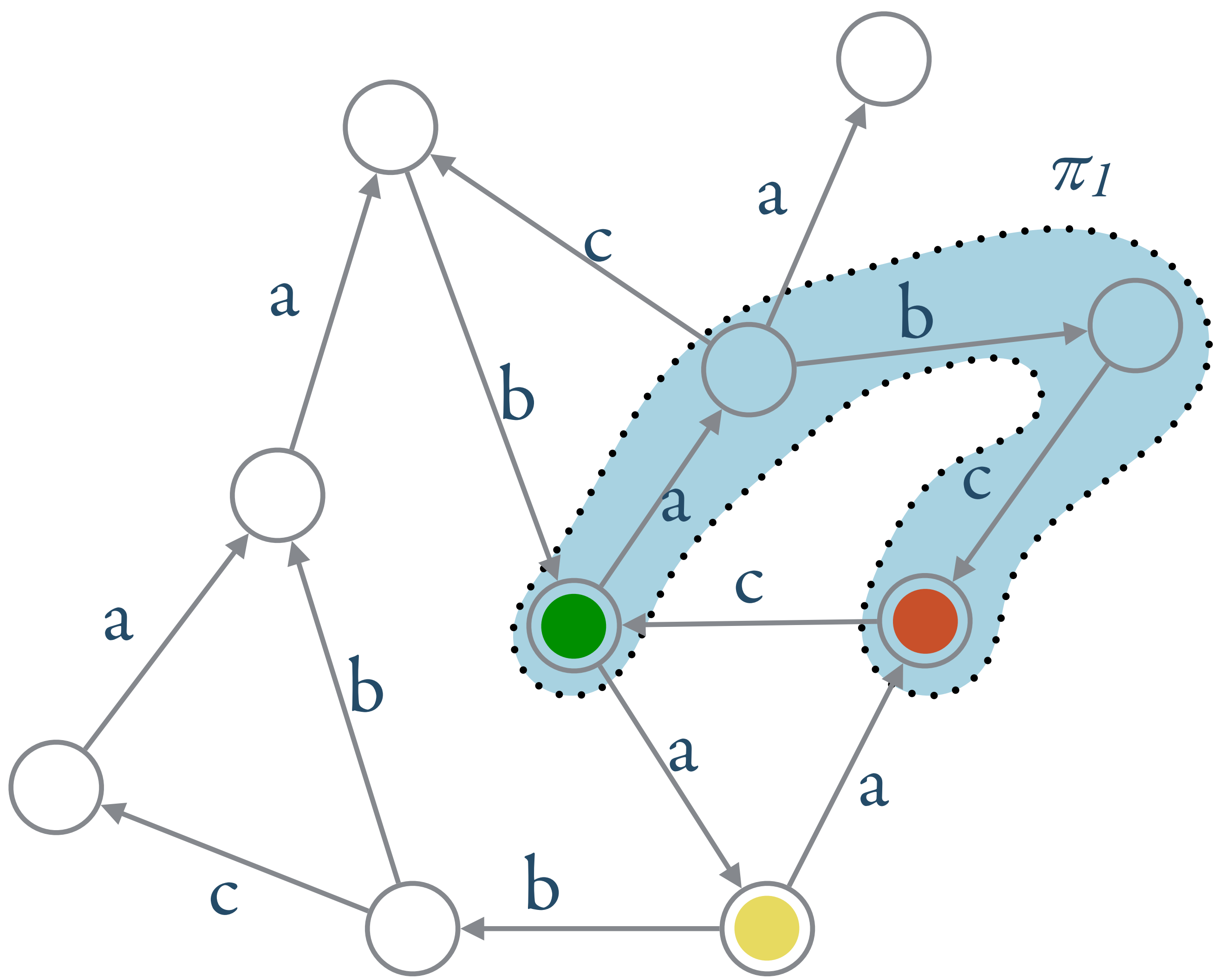
# Graph*databases*

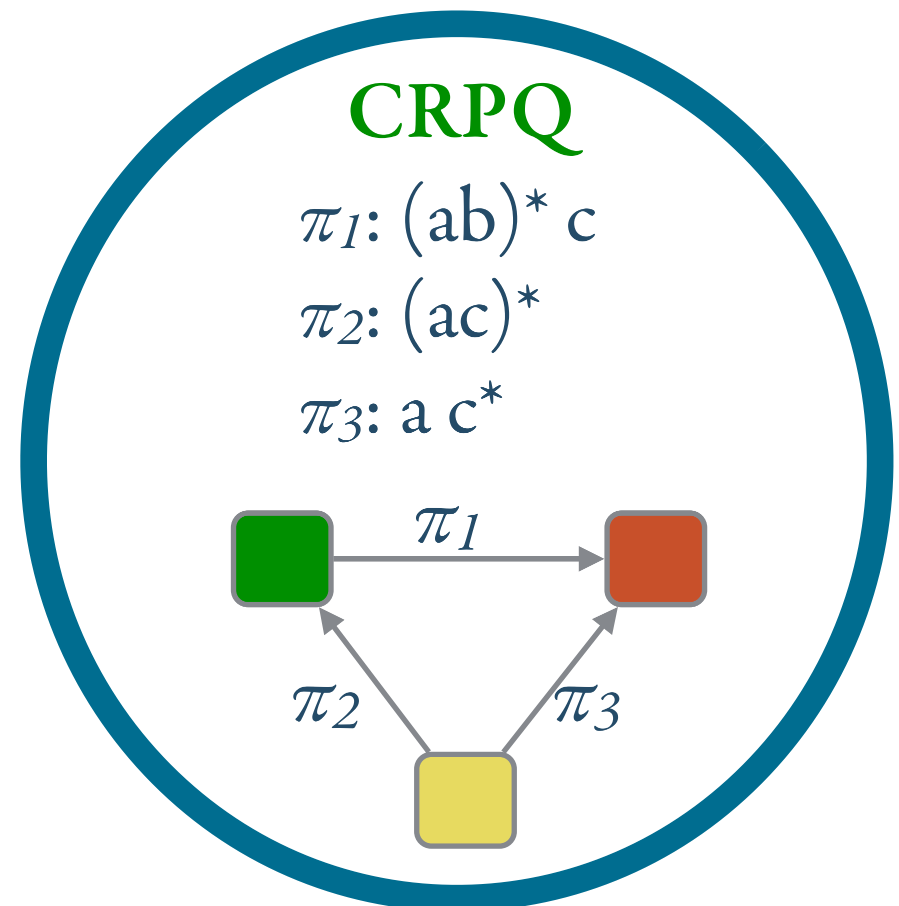



Evaluation: P (combined)
NL (data)

# Graph *databases*



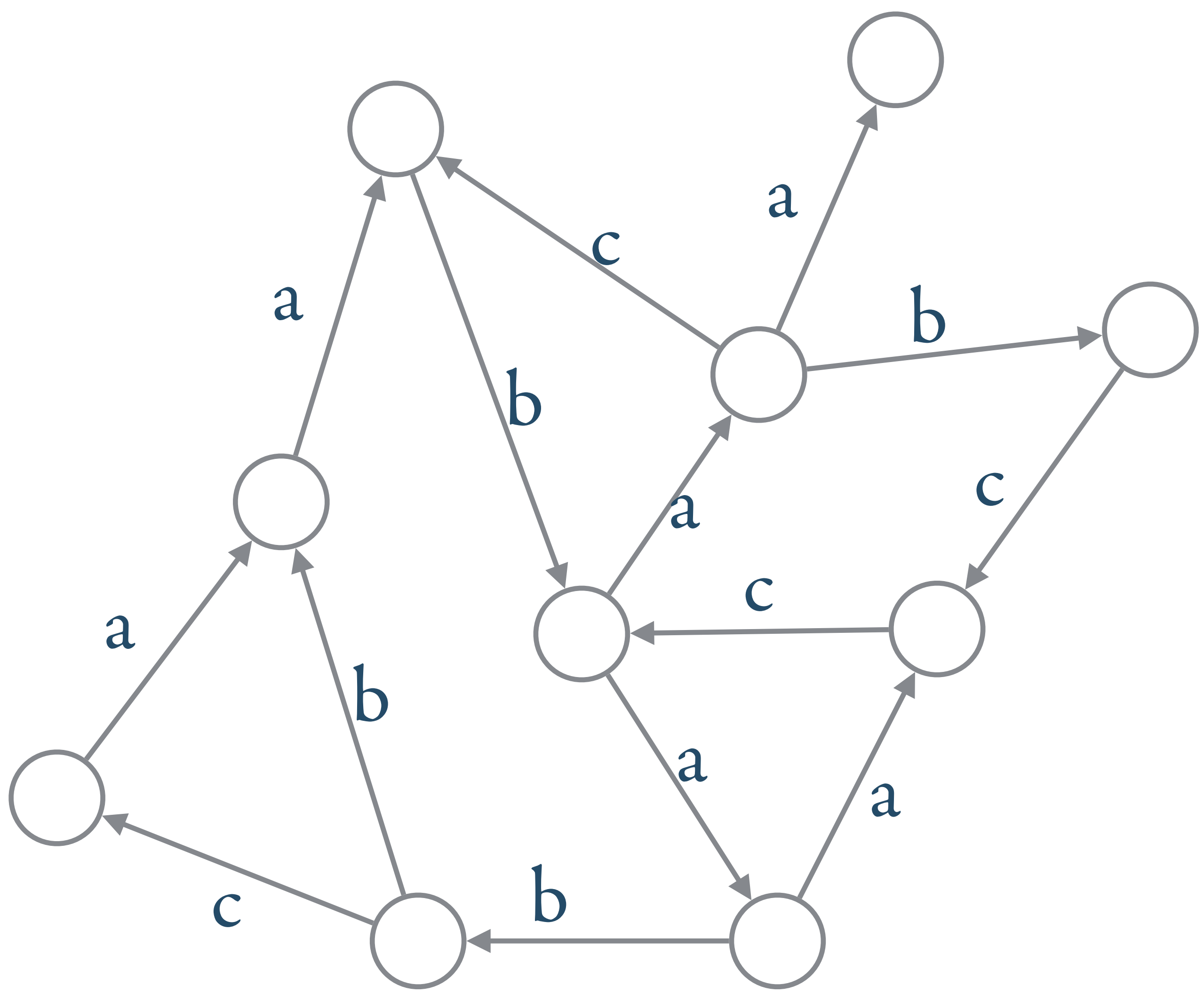

**Acyclic**

Evaluation: ~~NP~~ **P** (combined)

NL (data)

Unions, inverse

# Graph *databases*

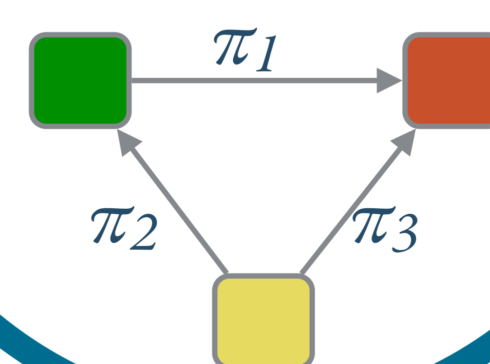



**CRPQ**

$\pi_1$: (ab)* c

$\pi_2$: (ac)*

$\pi_3$: a c*



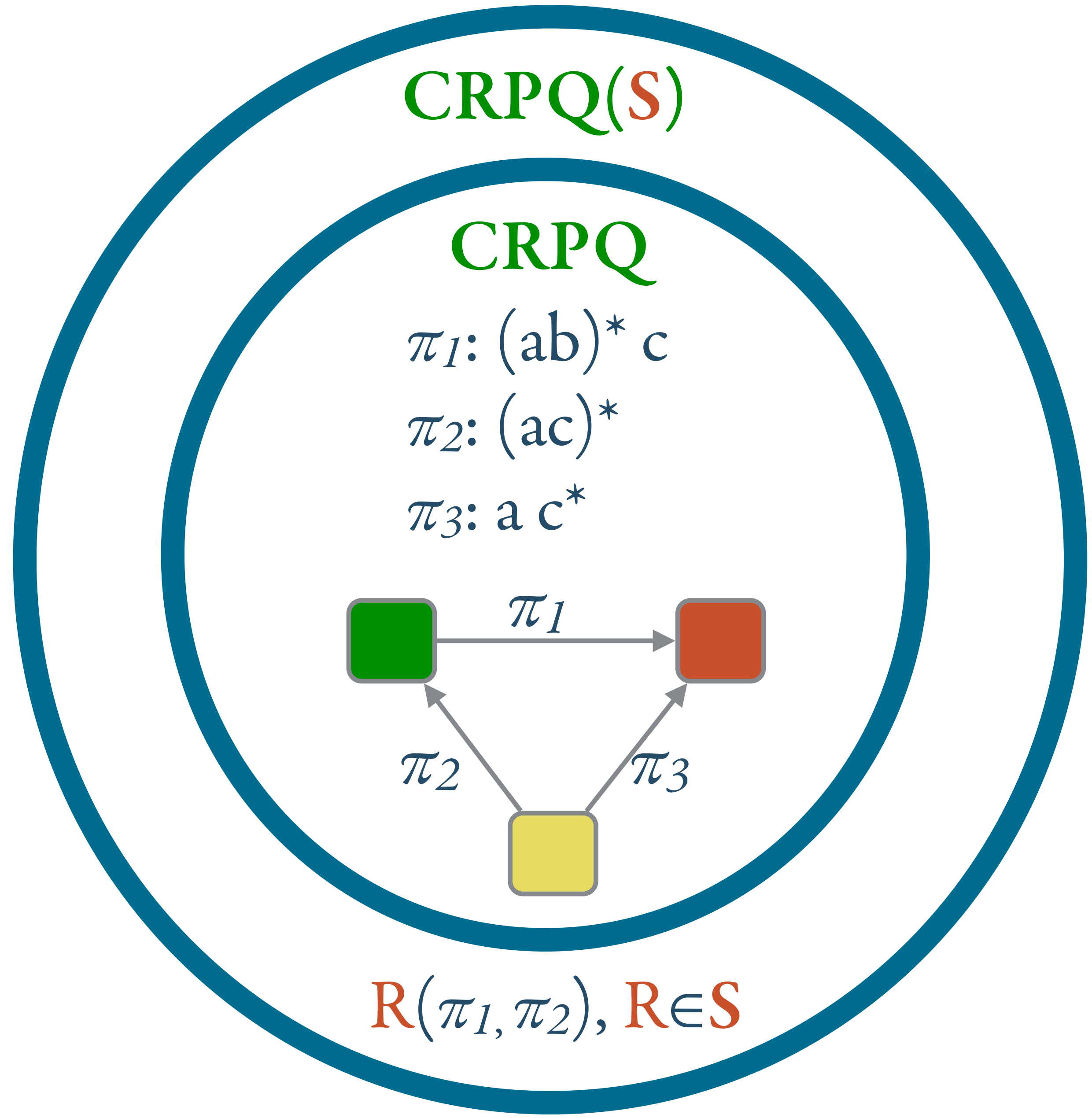

What about...

*"All the pairs (**u**,**v**) that can reach some node **z** in the same number of steps"*

# Graph *databases*

What about testing for **relations** on the paths?

- $|\pi_i| = |\pi_j|$
- $\pi_i$ is a **prefix** of $\pi_j$
- $\pi_i$ is a **subsequence** of $\pi_j$
- $\pi_i$ is a **factor** of $\pi_j$
- $\pi_i = \pi_j$ projected onto A



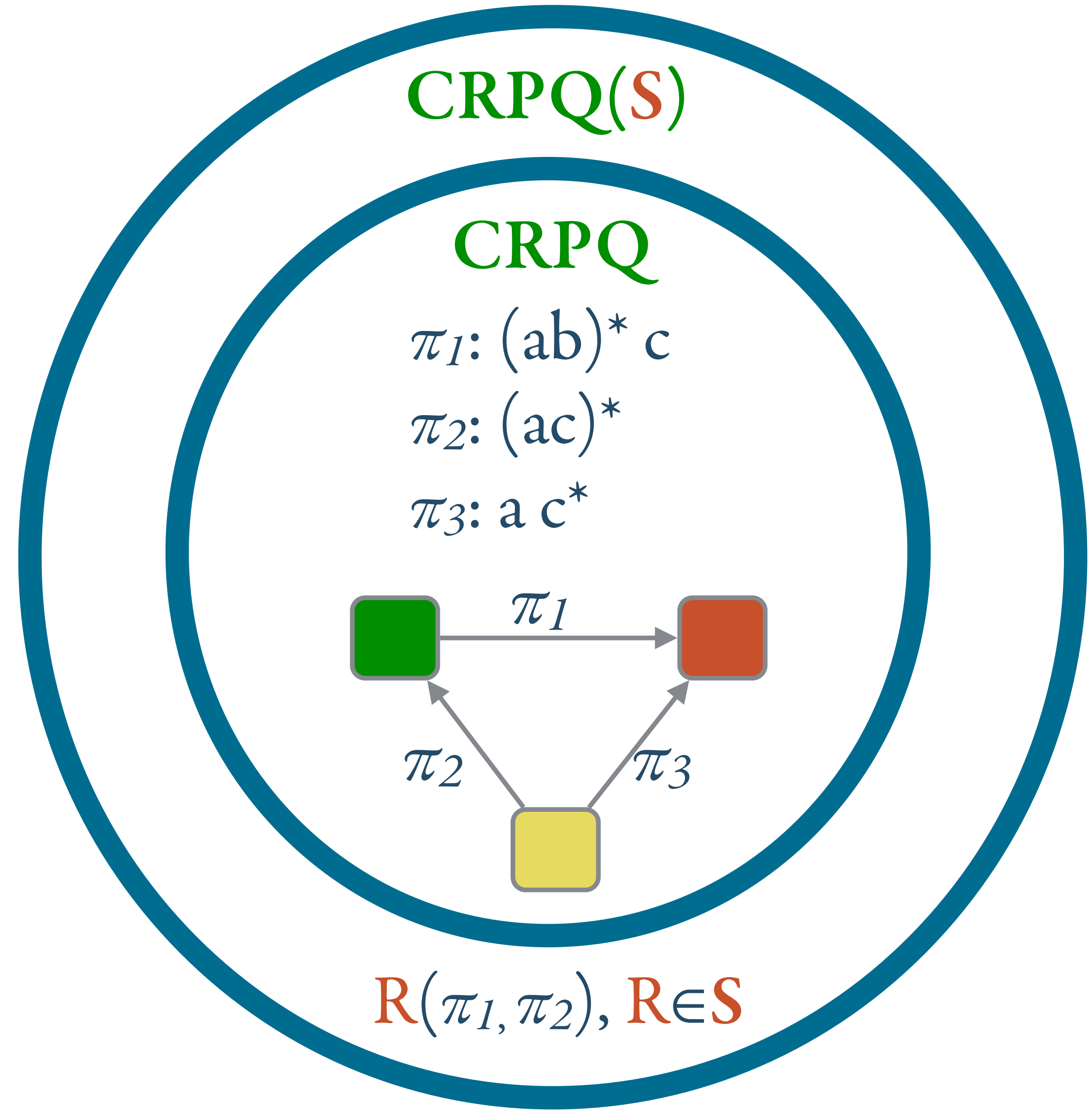

Motivations from: entity resolution, semantic associations, crime detection,...

# Graph *databases*

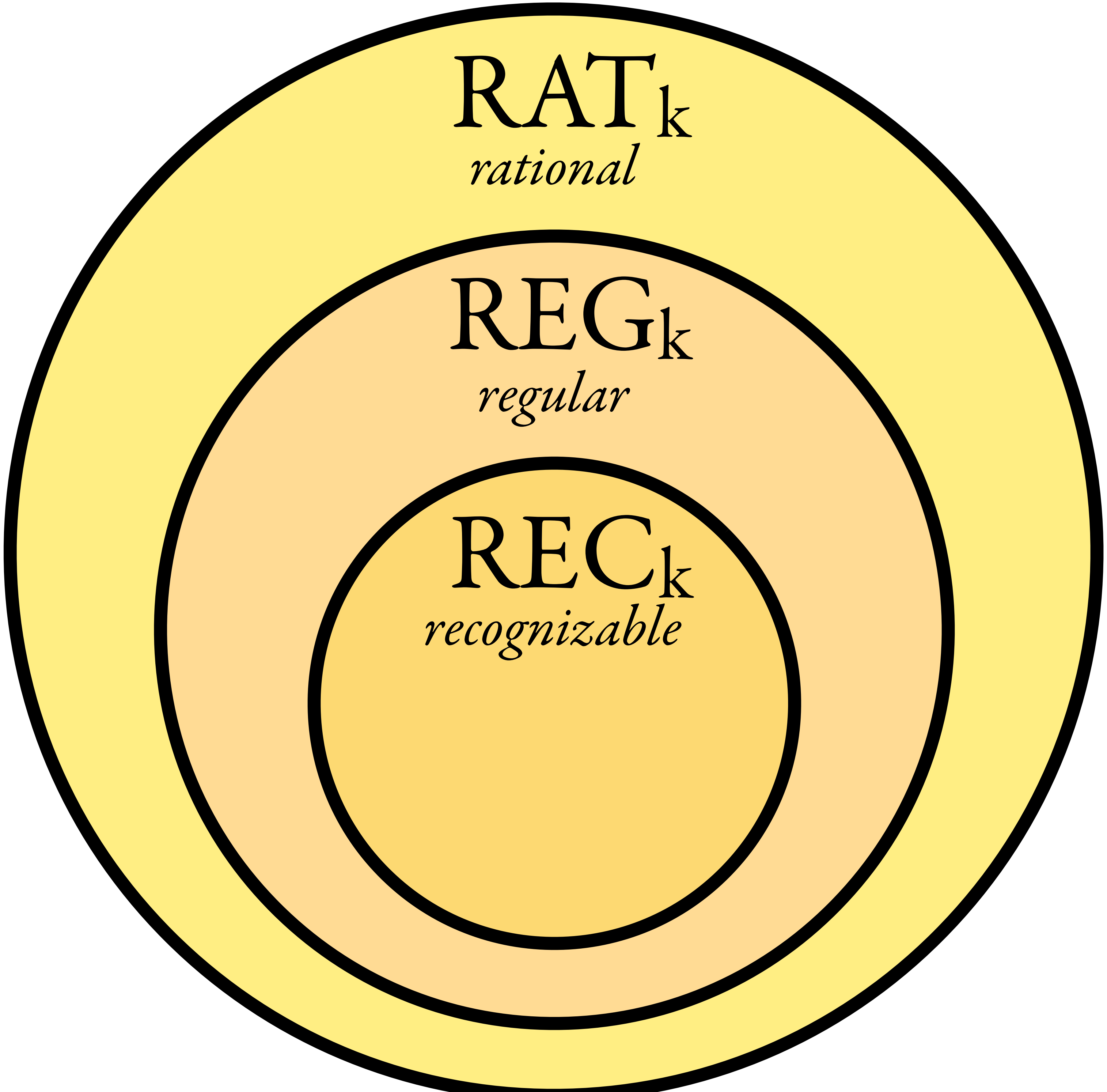
What about testing for **relations** on the paths?

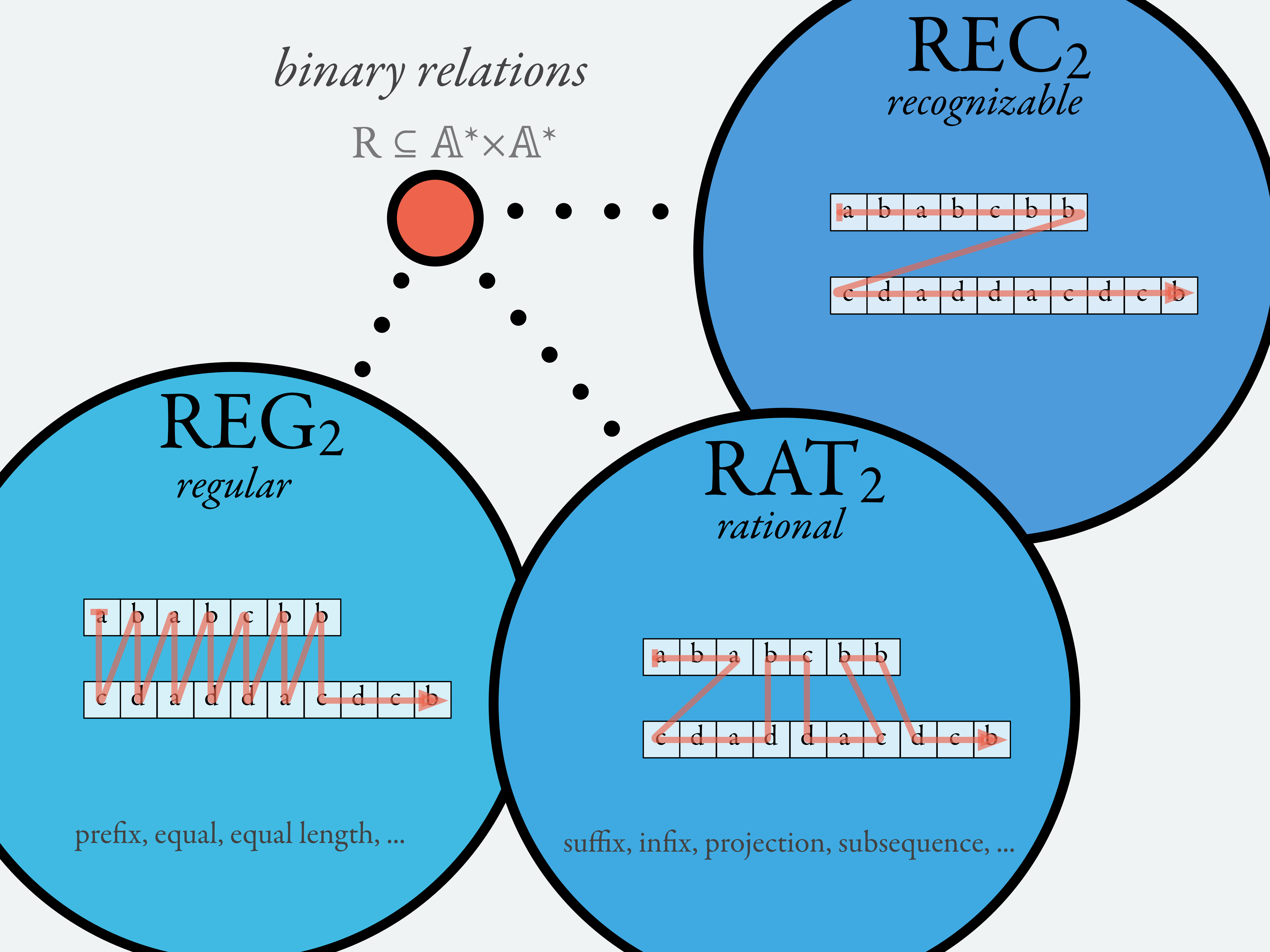
CRPQ(S) = CRPQ + tests $R(\pi_{i_1},\dots,\pi_{i_n})$, $R \in S$

S: Class of well-behaved word relations...



# Word relations




binary relations
R ⊆ 𝔸*×𝔸*
REC2
recognizable
a b a b c b b
c d a d d a c d c b
REG2
regular
a b a b c b b
c d a d d a c d c b
prefix, equal, equal length, ...
RAT2
rational
a b a b c b b
c d a d d a c d c b
suffix, infix, projection, subsequence, ...

# Graph *databases*

**CRPQ(S)** = **CRPQ** +
tests $R(\pi_{i_1},\ldots,\pi_{i_n})$, $R \in \mathbf{S}$

**CRPQ(REC)** **NP/NL complexity**

**Can this be extended?**

**CRPQ(REG)** **PSPACE/NL complexity**

**CRPQ(RAT)** ***undecidable***

**Related to the Intersection Problem:**
Given relations $R_1,\ldots,R_n$, whether $R_1 \cap \cdots \cap R_n \neq \varnothing$

# intersection *problem*

$R \cap S = \varnothing$ ?

$R, S$ : classes of binary relations

it has been studied...

*input:* $R \in \boldsymbol{R}$, $S \in \boldsymbol{S}$
*output:* $R \cap S = \varnothing$ ?

REG ∩ RAT = ∅ ?
already undecidable

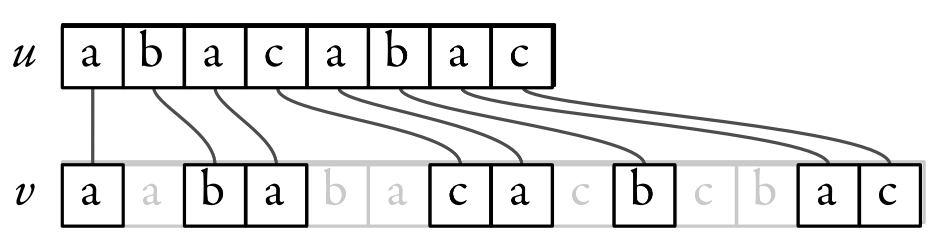
$(u_{i_1} \ldots u_{i_n}, v_{i_1} \ldots v_{i_n})$

PCP

...but what about
real world relations?

like

$u \sqsubseteq v$




subsequence

**subsequence**...?

**subword**...?

**suffix**...?

# Can we extend CRPQ beyond REG relations?

| Language | Data complexity | Combined complexity |
|---|---|---|
| CRPQ($REG_k$) | NL | PSPACE |
| CRPQ($RAT_k$) | Undecidable | Undecidable |
| CRPQ($REG_k$ + suffix) | Undecidable | Undecidable |
| CRPQ($REG_k$ + factor) | Undecidable | Undecidable |
| CRPQ($REG_k$ + subsequence) | non-elementary | non-PR |
| CRPQ(suffix) | NL | PSPACE |
| CRPQ(factor) | PSPACE | PSPACE |
| CRPQ(subsequene) | PSPACE | NEXPTIME |

$\forall\, k>1$

# Can we extend CRPQ beyond REG relations?

Proposed alternative: **approximate** RAT through
REG + counters

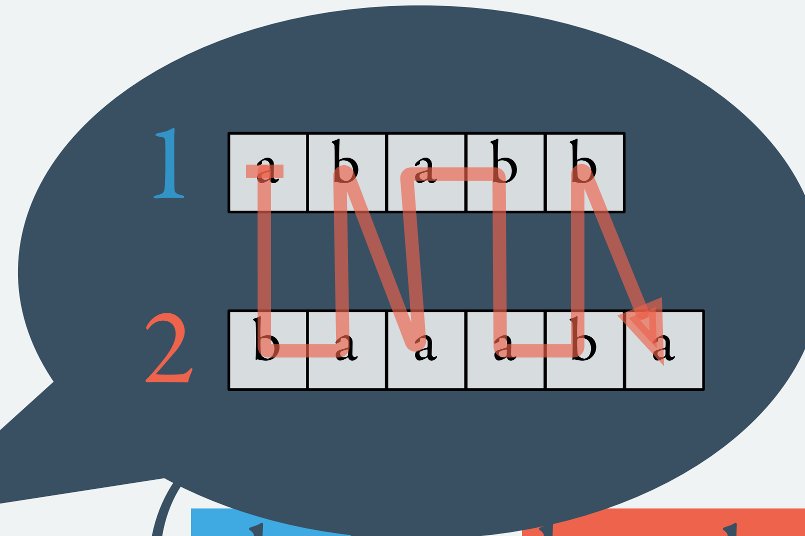
How?

1) take a an NFA
2) add counters
3) use it to read $k$-tuples of words

# 2 tapes over $\mathbb{A}$ $\approx$ 1 tape over $\mathbb{A}\times\{1,2\}$

control word

$$\left[\!\left[ \begin{array}{ccccccccccc} 1 & 2 & 2 & 1 & 2 & 1 & 1 & 2 & 2 & 1 & 2 \\ a & b & a & b & a & a & b & a & b & b & a \end{array} \right]\!\right] = (\text{ababb}, \text{baaaba})$$




$\cap$ $\cap$

$(\mathbb{A}\times\{1,2\})^*$ $\mathbb{A}^*\times\mathbb{A}^*$

$(1|2)^*$-controlled

$[\![(\mathbb{A}\times\{1,2\})^*]\!] = \mathbb{A}^*\times\mathbb{A}^*$

$[\![((a,1)(a,2)|(b,1)(b,2))^*]\!]$ = equality

$(12)^*$-controlled

$L\subseteq\{1,2\}^*$

**Rel**$(L)$= { ⟦S⟧| S∈REG($\mathbb{A}\times\{1,2\}$) is $L$-controlled }

# Eg:

$$\mathbf{Rel}((1|2)^*) = \mathrm{RAT}_2$$

$$\mathbf{Rel}((12)^*(1^*|2^*)) = \mathrm{REG}_2$$

$$\mathbf{Rel}(1^*2^*) = \mathrm{REC}_2$$

$$\mathbf{Rel}((12)^*) = \text{length-preserving } \mathrm{REG}_2$$

$$\mathbf{Rel}((1^*|2^*)(12)^*) = \mathrm{REG}_2^{rev}$$

Approximate with regular relations that can **count patterns**

$$R = \left\{ (u,v) \;\middle|\; \begin{array}{c} \text{\# of times } (ab)^*c \text{ appears in } u \\ = \\ 2 \cdot \text{\# of times } c^*b \text{ appears in } v \end{array} \right\}$$

More than just counting letters

Idea

Instead of **regular languages**...

$$\mathbf{Rel}(L) = \{ [\![S]\!] \mid S \in \mathbf{REG}(\mathbb{A} \times \{1,2\}) \text{ is } L\text{-controlled} \}$$

...use **automata with counting**

Idea

**Evaluation of CRPQ with counting is feasible**

**PSPACE** in combined complexity

**NL** in data complexity

# Parikh Automata*

[Klaedtke & Rueß]

dimension

NFA with **n** counters $c_1,\ldots,c_n$ and a semilinear set $S \subseteq \mathbb{N}^{\mathbf{n}}$

$$(\mathbb{A},Q,q_0,\delta,F,\mathbf{n},\mathbf{S})$$

Transitions of $\delta$: $(q,a,(x_1,\ldots,x_n),q') \in Q \times \mathbb{A} \times \mathbb{N}^{\mathbf{n}} \times Q$

Run:

counters can only be **incremented**

- Initial configuration: $(q_0,(0,\ldots,0)) \in Q \times \mathbb{N}^{\mathbf{n}}$
- $(q,\mathbf{x}) \xrightarrow{(q,a,y,p) \,\in \delta} (p,(\mathbf{x}+\mathbf{y}))$
- Acceptance: last configuration in $F \times \mathbf{S}$

* Many equivalent definitions (eg. reversal-bounded counter systems)

# Parikh Automata

Eg: $L_{ba=ca} = \left\{ w \;\middle|\; \begin{array}{c} \text{number of } a\text{'s after a } b \\ = \\ \text{number of } a\text{'s after a } c \end{array} \right\}$

| | | $c_1$++ | | | $c_2$++ | | $c_1$++ | | $c_2$++ | | $c_2$++ | | $c_1$++ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *a* | *b* | *a* | *a* | *c* | *a* | *b* | *a* | *c* | *a* | *c* | *a* | *b* | *a* |

Parikh Automaton $\mathcal{A} = (\mathbb{A}, Q, q_0, \delta, F, 2, \{(k,k) \mid k \in \mathbb{N}\})$

- dimension 2 (2 counters)
- increment $c_1$ whenever we see "**ba**"
- increment $c_2$ whenever we see "**ca**"
- F=Q
- Semilinear set assures that counters must be equal to accept a word

# Parikh Automata

## Decidable

non-emptiness,
membership

## Closed under

intersection,
union,
(inverse) homomorphisms,
concatenation

(not complementation/iteration)

$$\mathbf{Rel}^{\mathrm{PA}}(L) = \{\, [\![S]\!] \mid S \in \mathbf{PA}(\mathbb{A}_{\times}\{1,2\}) \text{ is } L\text{-controlled} \,\}$$

PA relations

# Eg:

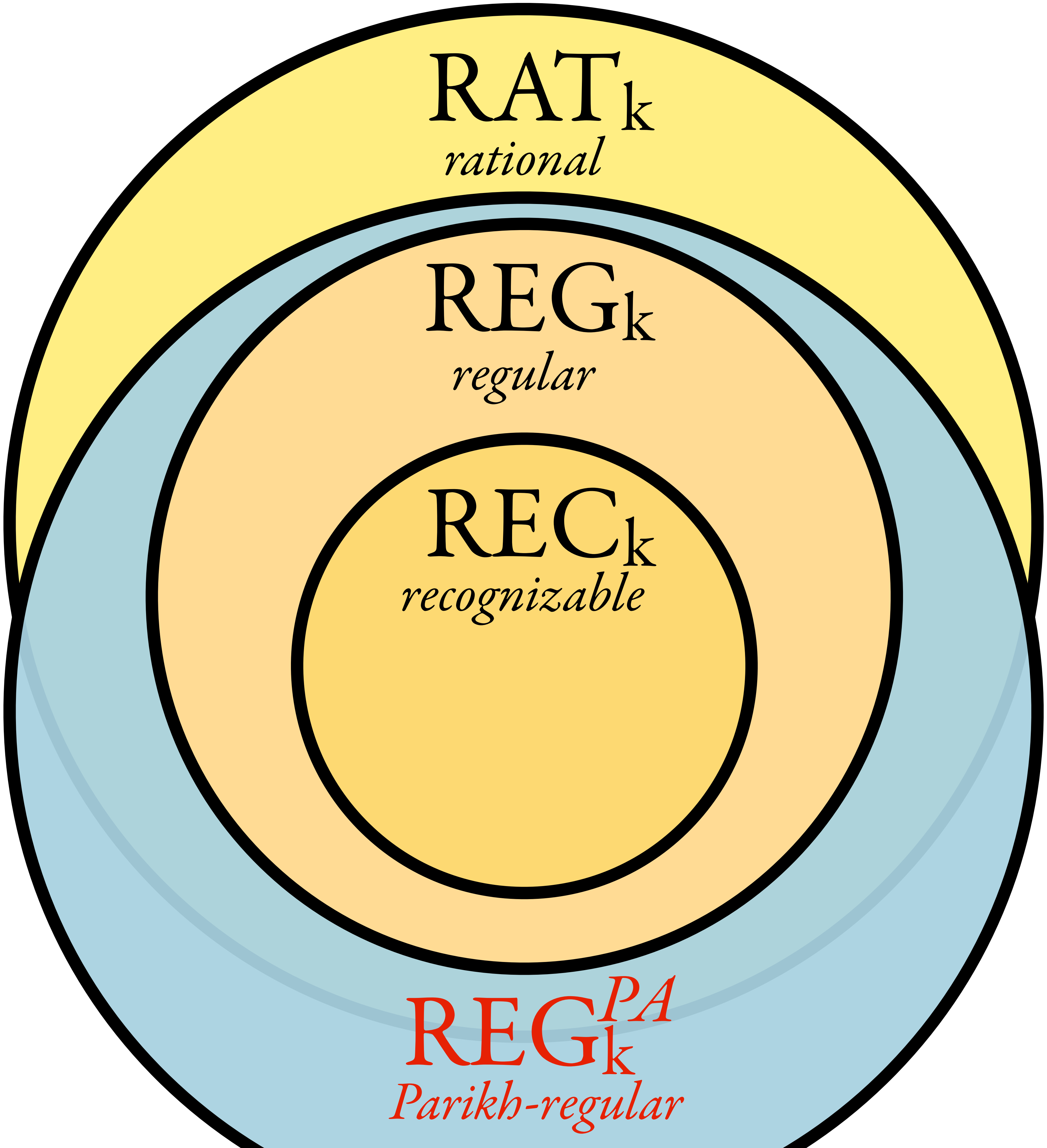
$$\mathrm{REG}_2^{\mathbf{PA}} = \mathbf{Rel}^{\mathbf{PA}}\left((12)^*(1^*|2^*)\right)$$

$$\mathrm{REG}_{2\,rev}^{\mathbf{PA}} = \mathbf{Rel}^{\mathbf{PA}}\left((1^*|2^*)(12)^*\right)$$

$$\mathrm{RAT}_2^{\mathbf{PA}} = \mathbf{Rel}^{\mathbf{PA}}\left((1|2)^*\right)$$

$$\vdots$$

# Word relations



Theorem: Evaluation of CRPQ(**REG**$^{PA}$) is
**PSPACE** in combined complexity
**NL** in data complexity

Proof ingredients:

- **Intersection problem** for Parikh Automata

  Given PA's $A_1,\dots,A_n$, is $L(A_1) \cap \cdots \cap L(A_n) \neq \varnothing$ ?

  is PSPACE-complete

- **Intersection closure** for REG$^{PA}$

  For all $R,S \in$ REG$^{PA}$, $R \cap S \in$ REG$^{PA}$

  it suffices to intersect the automata representing them

- **Closure under product** of REG$^{PA}$

Theorem: Evaluation of CRPQ$^{PA}$ (no relations) is
**NP** in combined complexity
**NL** in data complexity

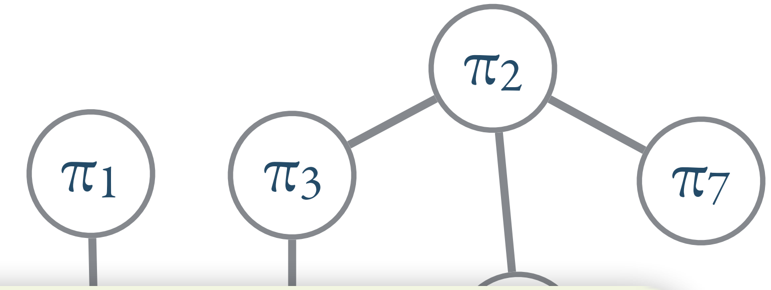
# Approximating rational relations

$u \sim_k v$ are **k-similar** iff for all $w$ with $|w| \leq k$, they have the same number of appearances of $w$ ~~(as factor)~~ (as subsequence)

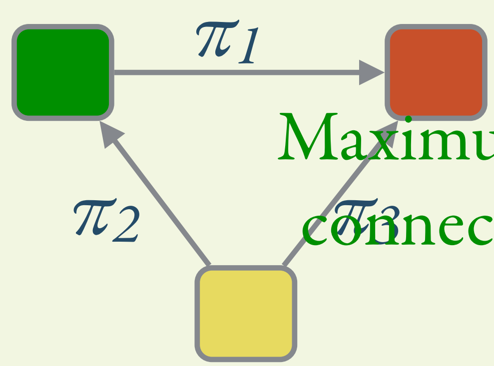
Given $R \in \mathrm{RAT}$,

$$R_k = \{(u,v) \mid u \sim_k u', v \sim_k v', (u', v') \in R\} \in \mathrm{REG}^{PA}$$

# Alternative: Syntactic restrictions




E.g.

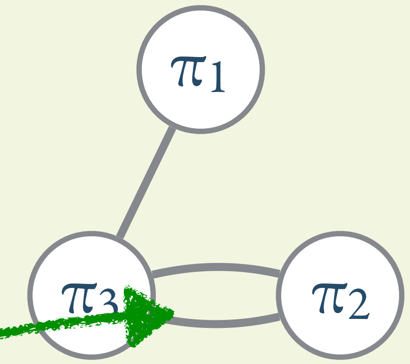



$\pi_1$: (ab)* c R($\pi_1$,$\pi_3$)

$\pi_2$: (ac)* S($\pi_3$,$\pi_2$)

$\pi_3$: a c* R($\pi_3$,$\pi_2$)

Maximum cardinality of connected component




**cyclic**

Theorem: Evaluation of **acyclic**-CRPQ(**RAT**$^{\text{PA}}$) is
**PSPACE** in combined complexity
**NL** in data complexity

If also fixed **join size**: **NP** combined complexity

E.g.

$\pi_2$: (ac)* R($\pi_1$,$\pi_3$)

$\pi_2$

**acyclic**

If also fixed **PA dimension** and **unary representation**:
**PTIME** combined complexity

# Conclusion

Counting does not increase complexity

Avoid the curse of of rational relations

Approximating by regular relations with counting

Or staying away from cycles in path relations

Thank you