Coalgebra Course – Studiegids

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Course title: Coalgebra: a unifying approach to system behaviour

Introduction (Inleiding)

Coalgebra is a unifying framework for studying the behaviour of state-based systems, many of which are of fundamental interest in computer science. Examples include all kinds of infinite data types, such as streams (infinite sequences) and trees. Other examples are deterministic and nondeterministic automata, labelled and probabilistic transition systems, etc.

Coalgebra is a rather recent field of research, existing for mere two decades, and it is attracting an enthusiastic, ever-growing community. Being relatively young, it still has many elementary and exciting research questions to offer.

The general theory of *universal coalgebra* provides general notions of observable behaviour and bisimilarity, as well as a powerful and fascinating reasoning principle called *coinduction* (a notion that is dual to the well known induction principle).

In this course, we will start by studying how basic examples of datatypes, such as infinite streams and trees, can be modeled as coalgebras. Next, we will show how fundamental notions and constructions, such as language equivalence of automata, bisimilarity of processes, and determinisation of nondeterministic automata, are instances of more general concepts from coalgebra. We will also explain how to generalise these constructions to other types of systems. In the last part of the course, we will learn how to derive, in a general fashion, (modal) logics that are suitable to reason about the behaviour of systems. The course will also have a practical component: we will show how to prove various equivalences fully automatically by means of a theorem prover.

Objectives (Leerdoelen)

After successful completion of the course, the participants:

- can explain how streams and various types of automata are modelled as coalgebras;
- can apply coinduction as a proof- and definition principle for streams and automata;
- have knowledge of the general coalgebraic definitions of behavior and equivalence;
- can explain the basic syntax and semantics of coalgebraic modal logic;
- can prove basic facts in coalgebraic modal logic;

Subjects (Onderwerpen)

Infinite streams; automata and conduction; (generalized) power set construction; automated coinductive reasoning; coalgebraic language equivalence; coalgebraic modal logic; behavioural equivalence; universal coalgebra.

Study investment (Studielastverdeling)

lectures:	32 hours
tutorials (werkcolleges):	32 hours
self study and homework:	104 hours

Teaching methods (Toelichting werkvormen)

The course will consist of lectures, tutorials (exercise classes), homework assignments, and practical exercises with the tool CIRC.

Examination (Toetsingsvorm)

The participants will be graded based on homework assignments and a final exam. The final grade will be the maximum of (H+E)/2 and E, where H is the grade given for the homework assignments and E is the grade given for the final exam.

Prerequisites (Vereiste voorkennis)

We only assume basic knowledge of automata, formal languages and propositional logic, for example, as covered in the courses Talen en Automaten, Discrete Wiskunde, en Logica. With respect to category theory and modal logic, the course will be self-contained: Only basic definitions will be needed, and these will be introduced as part of the course.

Literature (Literatuur)

The course material will consist of slides, hand-outs, and research papers on coalgebra. All material will be made available electronically via the course website, or distributed during the course.

[BRV01] P. Blackburn, M. de Rijke, and Y. Venema. *Modal Logic*. Cambridge University Press, 2001.

[CKP+11] C. Cîrstea, A. Kurz, D. Pattinson, L. Schröder, and Y. Venema. Modal logics are coalgebraic. *The Computer Journal*, 54(1):31–41, 2011.

[JR97] Bart Jacobs and Jan J. M. M. Rutten. A tutorial on (co)algebras and (co)induction. *Bull. Eur. Assoc. Theor. Comput. Sci. EATCS*, 62:222–259, 1997.

[KP11] C. Kupke and D. Pattinson. Coalgebraic semantics of modal logics: An overview. *Theoretical Computer Science*, 412:5070–5094, 2011.

[Rut98] J.J.M.M. Rutten. Automata and coinduction (an exercise in coalgebra). In D. Sangiorgi and R. de Simone, editors, *Proceedings of 9th International Conference on Concurrency Theory (CONCUR 1998)*, volume 1466 of *Lecture Notes in Computer Science*, pages 194–218. Springer, 1998.

[Rut00] Jan J. M. M. Rutten. Universal coalgebra: a theory of systems. *Theoret. Comput. Sci.*, 249(1):3–80, 2000.

[Rut03] Jan J. M. M. Rutten. Behavioural differential equations: a coinductive calculus of streams, automata, and power series. *Theoret. Comput. Sci.*, 308:1–53, 2003.

[Rut05a] Jan J. M. M. Rutten. A coinductive calculus of streams. *Math. Structures Comput. Sci.*, 15(1):93–147, 2005.

[Rut05b] Jan J. M. M. Rutten. A tutorial on coinductive stream calculus and signal flow graphs. *Theor. Comput. Sci.*, 343(3):443–481, 2005.

[SBBR10] A. Silva, F. Bonchi, M. Bonsangue, and J. Rutten. Generalizing the powerset construction, coalgebraically. In *Proc. FSTTCS 2010*, volume 8 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 272–283, 2010.

[Ven06] Y. Venema. Algebras and coalgebras. In P. Blackburn, J. van Benthem, and F. Wolter, editors, *Handbook of Modal Logic*, volume 3 of *Studies in Logic and Practical Reasoning*, pages 331–426. Elsevier, 2006.

Website

`http://www.ru.nl/is/education/coalgebra2012`

Extra information (Bijzonderheden)

This course fits well in a program that includes one or more of the following: Category Theory, Semantiek en Correctheid, Semantics and Domain Theory, Universal Algebra, Model Theory, Proof assistants, Analysis of Embedded Systems.

Docenten

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