Model Checking Modulo Theories
Model Checking of a Fault-tolerant System
Sally
A new old input language: Sal
Parametrization

Model Checking of Fault-tolerant Systems

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1. Model Checking Modulo Theories

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Model Checking Modulo Theories

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Model Checking

- a state type is a list of variables: $\mathbf{x}$
- a state is a valuation for these variables
- a transition is a formula over the current state variables and the next state variables
  (usually represented as a guard $H(\mathbf{x})$ and a (partial) assignment $V(\mathbf{x}'))$
- $(H_1(\mathbf{x}) \wedge V_1(\mathbf{x}')) \vee \ldots \vee (H_n(\mathbf{x}) \wedge V_n(\mathbf{x}'))$

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Modulo Theories

- the formulae can be in any theory
- example:
    - if the state type is a variable x of type int
    - transition: $x < 0 \wedge x' = x \vee x \geq 0 \wedge x' = x + 1$
    - describes a system which has a variable x which keeps increasing unless it is lower than 0.

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The Byzantine General Problem

- One general wants to give an order to $n - 1$ lieutenants.
- Some of them may be faulty (including the general).
- In the end, they have to decide on the same order (at least for all the non faulty lieutenants/general)

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Pseudo-Code

```
% message[i][j] is the message sent by lieutenant j to i
message: array of (lieutenants * lieutenants) to message

source: the general id

% value[i] is the order that lieutenant i will follow
value: array of lieutenants to message

% good[i] is true if lieutenant i is non faulty
good: array of lieutenants to boolean
```

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Pseudo-Code

```
propagate():
  forall i, j do
    if good[i]
      message[j][i] <- message[i][source]

agree():
  forall i do
    if good[i]
      value[i] <- majority(message[i])

propagate();
propagate();
agree();
```

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The Byzantine General Problem

- $\exists v\ \forall i\ good[i] \Rightarrow value[i] = v$
- How many lieutenants can be faulty? Does that work for a third of the lieutenants?
- $\#\{i \mid good[i]\} > 2N/3$

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Sally

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A model checker for infinite-state systems

- sri-csl.github.io/sally
- a symbolic model checker
- several engines: bmc, kind, ic3
- works with various smt solvers: mathsat, yices2, z3

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Input language

- lisp-like language
- low level
- easy to parse and work with

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Input language

- state type

```
(define-state-type my_state_type
  ((x Real) (y Real))
)
```

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Input language

- state type

```
(define-state-type my_state_type
  ((x Real) (y Real))
)
```

- state formula

```
(define-states x_is_zero my_state_type
  (= x 0)
)
```

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Input language

- transition: a first order formula over state variables and next state variables

```
(define-transition my_transition my_state_type
  (or
    (= next.x (+ state.x 1))
    next.x_is_zero
  )
)
```

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Input language

- queries: check that a property always holds

```
(query my_system
  (>= x 0)
)
```

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Sal

- an older model checker, developed at SRI
- developed actively until 2006, minor versions until 2013
- finite state systems

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Input language

- already used
- supports modules, composition

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Input language

```
my_module: MODULE
BEGIN
  OUTPUT
    x: REAL,
    y: REAL
  INITIALIZATION
    x = 0;
  TRANSITION
    ...
END;
```

```
TRANSITION
  [x >= 0 -->
     x' = x + 1;
     y' IN { i: REAL | TRUE }
  []
   TRUE -->
     x' = 0;
     y' IN { i: REAL | TRUE }
  ]
```

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Input language

- a lemma is translated to a Sally query
- multiple lemma
- syntax for temporal logic (not available in Sally)

```
my_context: CONTEXT =
BEGIN
  my_module: MODULE
        ...

  always_positive: LEMMA
    my_module |- G(x >= 0);

  wrong_lemma: LEMMA
    my_module |- G(x > 0 -> x = 1)


END
```

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Arrays

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Quantifiers

- for most examples, they can be avoided in transitions
- works only with z3
- example:

  ```
  (forall (i Int) (select a i))
  ```

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Counting in SMT

- $\phi(y) \wedge y = \# \{x | \psi(x)\}$
- $\psi(.)$: first order formula of Presburger arithmetic, then with arrays too
- how is it solved?

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State of the art

- Bradley, Manna, and Sipma (2006): a decision procedure for a fragment of arrays, with distinct theories for elements and indexes
- Alberti, Ghilardi, and Pagani (2016): a decision procedure for counting on arithmetic and arrays, via various rewriting and quantifier eliminations, mix index and elements theories, but quantify only over one element at a time
- Bjørner, Gleissenthall, and Rybalchenko (n.d.): a model checking oriented way to deal with arrays (one update at a time for every arrays)

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Counting over Presburger arithmetic

- given a model, one can compute the value of $\# \{x | \psi(x)\}$
- given an ordering on the integer variables, one can compute the symbolic value of $\# \{x | \psi(x)\}$
- $\Rightarrow$ symbolic computation of cardinalities, for the ordering of a given model

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Counting over Presburger arithmetic, with an ordering

- the ordering is called an oracle: when asked wether $a > b$, it looks in the model the value of a and b and answers accordingly.
- when the cardinality is computed symbolically, it is equal to a formula which holds under some assumptions, and the oracle can say what they are

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- example: $y = \#\{x | 0 \leq x < z \land 0 \leq x < u\}$
- if the oracle says $z > u$, then y can be computed and $y = z$.
- under the assumption $z > u$, $y = z$

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Counting over Presburger arithmetic, with an ordering

- compute a symbolic interval list in which the formula is satisfiable i.e. $\exists I \in V_x(\psi) \; x \in I \Leftrightarrow \psi(x)$
- if members of $V_x(\psi)$ are disjoint
  $card(V_x(\psi)) = \sum_{[v,v'] \in \#_x(\psi))}(v - v') = \#\{x | \psi(x)\}$

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  $card(V_x(\psi)) = \sum_{[v,v'] \in \#_x(\psi))} (v - v') = \#\{x | \psi(x)\}$

- $V_x(y < x) = \{[y, +\infty)\}$

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Counting over Presburger arithmetic, with an ordering

- $V_x(\psi \wedge \phi) = V_x(\psi) \sqcap V_x(\phi)$
  where $A \sqcap B$ is the set of every intersection of an interval of A with an interval of B (of course some are empty and must be deleted)

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Counting over Presburger arithmetic, with an ordering

- $V_x(\psi \wedge \phi) = V_x(\psi) \sqcap V_x(\phi)$
  where $A \sqcap B$ is the set of every intersection of an interval of A with an interval of B (of course some are empty and must be deleted)

- Intersection between two intervals: $[a, b)$ and $[c, d)$ can be computed: there is an oracle giving the ordering on the variables, so $max(a, c)$ and $min(b, d)$ are computable given the assumptions that the oracle makes.
  Then the intersection is $[max(a, c), min(b, d))$ if $max(a, c) \leq min(b, d)$

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Counting over Presburger arithmetic, with an ordering

- if $V_x(\psi) = \{[a_1, b_1), \dots [a_n, b_n)\}$ (with $a_1 \leq b_1 \leq \dots \leq b_n$, $a_1 \neq -\infty$ and $b_n = +\infty$)
- $V_x(\neg\psi) = \{(-\infty, a_1), [b_1, a_2), \dots, [b_n, +\infty)\}$
- other cases are easy too, disjunction on $a_1 = -\infty$ and $b_n = +\infty$

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Counting with multiplication

- to deal with constant multiplications, a modulo information can be added to every intervals (such as $([5, 10], = 1[3])$) are the integers x between 5 and 10 and such that $3|x - 1$)
- intersection, negation of these intervals can be done in an analog way

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Future Work

- Counting over arrays
- IC3 with arrays and counting quantifiers

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References

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