

Master's Thesis
Optimizing Neural Networks for Mobile
Devices

Radboud University, Nijmegen

Erdi Çallı

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Abstract

Recently, Convolutional Neural Networks became the state of the art method in Image Processing. However, there exists a gap between their potential and real life applications. This gap is caused by the computational requirements of Convolutional Neural Networks. State of the art Convolutional Neural Networks require heavy computations. We investigate methods to reduce the computational requirements and preserve the accuracy. Then we combine these methods to create a new model that is comparable with state of the art. We benchmark our model on mobile devices and compare its performance with others.

Contents

1	Introduction	3
1.1	Neural Networks	4
1.1.1	Fully Connected Layers	5
1.1.2	Activation Function and Nonlinearity	6
1.1.3	Loss	7
1.1.4	Stochastic Gradient Descent	8
1.1.5	Convolutional Layer	8
1.1.6	Pooling	9
1.1.7	Deconvolution	10
1.2	Efficient Operations	10
1.3	Factorization	10
1.3.1	SVD	11
1.3.2	Weight Sharing	11
1.4	Convolution Operation Alternatives	12
1.4.1	Kernel Composition	12
1.4.2	Separable Convolutions	12
1.5	Pruning	13
1.5.1	Pruning Weights	14
1.5.2	Pruning Nodes	14
1.6	Quantization	15
1.7	Improving Network Efficiency	15
1.7.1	Residual Connections	15
1.7.2	Batch Normalization	15
1.7.3	Regularization	16
1.8	Efficient Structures	16
1.8.1	Residual Blocks	17
1.8.2	Residual Bottleneck Blocks	17

1.9	Datasets	18
1.9.1	MNIST	18
1.9.2	CIFAR10	18
1.10	Tools	18
1.10.1	Tensorflow	18
2	Methods	19
2.1	Pruning	19
2.1.1	Fully Connected Summation	20
2.1.2	MNIST Autoencoder	20
2.1.3	Regularization	21
2.1.4	Distortion	21
2.1.5	Activations	21
2.2	Convolution Operation Alternatives	22
2.3	Separable Resnet	23
2.3.1	Design Choices	23
2.3.2	Training	25
2.3.3	Pruning Nodes	25
2.3.4	Pruning Weights	26
2.3.5	Factorization	26
2.3.6	Quantization	26
3	Results	27
4	Discussion	28
5	Conclusion	29

Chapter 1

Introduction

The state of the art in Image Processing has changed when graphics processing units (GPU) were used to train neural networks. GPUs contain many cores, they have very large data bandwidth and they are optimized for efficient matrix operations. In 2012, [KSH12] won the ImageNet Large Scale Visual Recognition Competition (ILSVRC) classification task ([DBS⁺12]). They used two GPUs to train an 8 layer convolutional neural network (CNN). Their model has improved the previous (top-5) classification accuracy record from $\sim 74\%$ to $\sim 84\%$. This caused a big trend shift in Computer Vision.

As the years pass, GPUs got more and more powerful. In 2012, [KSH12] used GPUs that had 3 GB memory. Today there are GPUs with up to 12 GB memory. The number of floating point operations per second (FLOPs) has also increased from 2.5 tera FLOPs (TFLOPs) to 12 TFLOPs. This gradual but steep change has allowed the use of more layers and more parameters. For example, [SZ14] introduced a model called VGGNet. Their model used up to 19 layers and shown that adding more layers affects the accuracy. [HZRS15] introduced a new method called residual connections, that allowed the use of up to 200 layers. Building up on such models, in 2016 ILSVRC winning (top-5) classification accuracy is increased to $\sim 97\%$.

In contrast, [SLJ⁺14] have shown that having a good harmony within the network worked better than having more parameters. It has been supported by [CPC16]. They have shown the relation between number of parameters of a model and its top-1 classification accuracy in ILSVRC dataset. According to their report, 48 layer Inception-v3 ([SVI⁺16]) provides better top-1 classification accuracy than 152 layer ResNet ([HZRS15]). They also show that Inception-v3 requires fewer number of floating point operations to compute

results. Therefore, revealing that of providing more layers and parameters would not yield better results.

ILSVRC is one of the most famous competitions in Image Processing. Every year, the winners of this competition are driving the research on the field. But this competition is not considering the competitive value of limiting number of operations. If we look at the models of 2016 competitors, we see that they use ensembles of models¹. These ensembles are far from being usable in real life because they require a great amount of operations per inference. Not mentioning the number of operations from the result is misleading for the AI community and the public. It creates an unreal expectation that these models are applicable in real life. In this thesis, we want to come up with a state of the art solution that requires a low number of floating point operations per inference. Therefore, bridging the gap between expectations and reality. We will answer,

How can we reduce the inference complexity of Neural Networks?

How do these modifications effect accuracy?

First, we will briefly describe neural networks and some underlying concepts. We will mention the complexities of necessary operations. Then, we will provide known solutions to reduce these complexities.

1.1 Neural Networks

In this chapter, we will try to describe neural networks briefly. We will provide some terminology and give some examples.

Neural networks are *weighted graphs*. They consist of an ordered set of *layers*, where every layer is a set of *nodes*. The first layer of the neural network is called the *input layer*, and the last one is called the *output layer*. The layers in between are called *hidden layers*. In our case, nodes belonging to one layer are connected to the nodes in the following and/or the previous layers. These connections are weighted edges, and they are mostly called as *weights*.

Given an input, neural network nodes have *outputs*, which are real numbers. The output of a node is calculated by applying a function (ψ) the outputs of the nodes belonging to previous layers. Preceding that, the output of the input layer is calculated using the input data (see Eq. 1.8.2).

¹<http://image-net.org/challenges/LSVRC/2016/results#team>

By calculating the layer outputs consecutively we calculate the output of the output layer. This process is called *inference*. We use the following notations to denote the concepts that we just explained.

$$\begin{aligned}
l_k &: \text{a column vector of nodes for layer } k \\
m_k &: \text{the number of nodes in } l_k \\
l_{k,i} &: \text{node } i \text{ in } l_k \\
o_k &: \text{the output vector representing the outputs of nodes in } l_k \\
o_{k,i} &: \text{the output of } l_{k,i} \\
\mathbf{w}^{(k)} &: \text{weight matrix connecting nodes in } l_{k-1} \text{ to nodes in } l_k \\
w_{i,j}^{(k)} &: \text{the weight connecting nodes } l_{(k-1),i} \text{ and } l_{k,j} \\
\mathbf{b}^{(k)} &: \text{the bias term for } l_k \\
\psi_k &: \text{function to determine } o_k \text{ given } o_{k-1} \\
\sigma &: \text{activation functions} \\
\mathbf{x} &: \text{all inputs of the dataset, consisting of } N \text{ data points} \\
\mathbf{y} &: \text{all outputs of the dataset} \\
\hat{\mathbf{y}} &: \text{approximation of the output} \\
x_n &: \text{nth input data } (0 < n \leq N) \\
y_n &: \text{nth output data } (0 < n \leq N) \\
\hat{y}_n &: \text{approximation of } y_n \text{ given } x_n \text{ } (0 < n \leq N) \\
\text{FC} &: \text{stands for Fully Connected (e.g. } \psi^{(FC)})
\end{aligned} \tag{1.1}$$

Therefore the structure of a neural network is determined by the number of layers and the functions that determine the outputs of layers.

$$o_k = \begin{cases} \psi(o_{k-1}), & \text{if } k \geq 1 \\ \mathbf{x}, & k = 0 \end{cases} \tag{1.2}$$

1.1.1 Fully Connected Layers

As the name suggests, for two consecutive layers to be *fully connected*, all nodes in the previous layer must be connected to all nodes in the following layer.

Let's assume two consecutive layers, $l_{k-1} \in \mathbb{R}^{m_{k-1} \times 1}$ and $l_k \in \mathbb{R}^{m_k \times 1}$. For these layers to be fully connected, the weight matrix connecting them would

be defined as $\mathbf{w}^{(k)} \in \mathbb{R}^{m_{k-1} \times m_k}$. Most fully connected layers also include a bias term for every node in l_k ($\mathbf{b}^{(k)} \in \mathbb{R}^{m_k}$). In fully connected layers, o_k would simply be calculated using layer function $\psi^{(FC)}$.

$$\psi_k^{(FC)}(o_{k-1}) = o_{k-1}^T \mathbf{w}^{(k)} + \mathbf{b}^{(k)}$$

Therefore the computational complexity of $\psi^{(FC)}$ would become

$$\mathcal{O}(\psi_k^{(FC)}) = \mathcal{O}(m_{k-1}m_k)$$

1.1.2 Activation Function and Nonlinearity

By stacking fully connected layers, we can increase the depth of a neural network. By doing so we want to increase approximation quality of the neural network. However, the $\psi^{(FC)}$ we have defined is a linear function. Therefore if we stack multiple fully connected layers we would end up with a linear model.

To achieve non-linearity, we apply *activation functions* to the results of ψ . There are many activation functions (such as *tanh* or *sigmoid*) but one very commonly used activation function is *ReLU* [NH10].

$$ReLU(x) = \begin{cases} x, & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (1.3)$$

Therefore we will redefine the fully connected $\psi^{(FC)}$ as;

$$\psi_k^{(FC)}(o) = \sigma(o^T \mathbf{w}^{(k)} + \mathbf{b}^{(k)})$$

The activation functions does not strictly belong to the definition of fully connected layers. But for simplicity, we are going to include them in the layer functions (ψ).

$\psi^{(FC)}$ is one of the most basic building blocks of any Neural Network. Stacking \mathcal{K} of them after the input, we can try to approximate an output given an input. To do that we will calculate the outputs of every layer, starting from the input.

$$\mathbf{o} = \{\psi_k(o_{k-1}) | k \in [1, \dots, \mathcal{K}]\}$$

1.1.3 Loss

To represent the quality of an approximation, we are going to use a loss (or cost) function. A good example would be the loss of a salesman. Assuming a customer who would pay at most \$10 for a given product, if the salesman sells this product for \$4, the salesman would face a loss of \$6 from his potential profit. Or if the salesman tries to sell this product for \$14, the customer will not purchase it and he will face a loss of \$10. In this example, the salesman would want to minimize the loss to earn as much as possible. There are two common properties of loss functions. First, loss is never negative. Second, if we compare two approximations, the one with a smaller loss is better.

Root Mean Square Error

A commonly used loss function is Root Mean Square Error (RMSE). Given an approximation ($\hat{y} \in \mathbb{R}^N$) and the expected output ($y \in \mathbb{R}^N$), RMSE can be calculated as,

$$\mathcal{L} = RMSE(\hat{y}, y) = \sqrt{\frac{\sum_{n=1}^N (\hat{y}_n - y_n)^2}{N}}$$

Softmax Cross Entropy

Another commonly used loss function is Softmax cross entropy (SCE). Softmax cross entropy is used with classification tasks where we are trying to find the class that our input belongs to. Softmax cross entropy first calculates the class probabilities given the input.

Given an approximation ($\hat{y} \in \mathbb{R}^N$) it first calculates the class probabilities as,

$$p(i|\hat{y}_n) = \frac{e^{\hat{y}_i}}{\sum_{n=1}^N e^{\hat{y}_n}}$$

Then comparing it with the the expected output ($y \in \mathbb{R}^N$), SCE loss can be calculated as,

$$\mathcal{L} = CE(\hat{y}, y) = - \sum_{n=1}^N y_n \cdot \log(p(i|\hat{y}_n))$$

this definition could be better

1.1.4 Stochastic Gradient Descent

To provide better approximations, we will try to optimize the neural network parameters. One common way to optimize these parameters is to use Stochastic Gradient Descent (SGD). SGD is an iterative learning method that starts with some initial (random) parameters. Given $\theta \in (\mathbf{w} \cup \mathbf{b})$ to be a parameter that we want to optimize. The learning rule updating theta would be;

$$\theta = \theta - \eta \nabla_{\theta} \mathcal{L}(f(x), y)$$

where η is the learning rate, and $\nabla_{\theta} \mathcal{L}(f(x), y)$ is the partial derivative of the loss in terms of given parameter, θ . One iteration is completed when we update every parameter for given example(s). By performing many iterations, SGD aims to find a global minimum for the loss function, given data and initial parameters.

There are several other optimizers that work in different ways. We will be using Adam Optimizer ([KB14]), Momentum Optimizer ([Qia99]) and SGD.

1.1.5 Convolutional Layer

So far we have seen the key elements we can use to create and train fully connected neural networks. To be able to apply neural networks to image inputs (or an at least 2 dimensional data), we can use convolutional layers and convolution operation. Please note that we are assuming one or two dimensional convolutions with same padding.

Let's assume a 3 dimensional layer output $o_{k-1} \in \mathbb{R}^{H_{k-1} \times W_{k-1} \times m_{k-1}}$ where the dimensions W_{k-1} representing the width, H_{k-1} representing height and m_{k-1} representing number of nodes. Convolution operation first creates a sliding window that goes through width and height. The contents of this sliding window would be patches ($p_{k-1,(I,J)} \in \mathbb{R}^{K \times K \times m_{k-1}}$). By multiplying a weight matrix $\mathbf{w}^{(k)} \in \mathbb{R}^{K \times K \times m_{k-1} \times m_k}$ with every patch, we create a set of output nodes $o_{(k+1),(I,J)} \in \mathbb{R}^{1 \times m_k}$. Those output nodes represent the features at location (I, J) . By performing this operation for every patch, we calculate the outputs of a convolutional layer. While calculating the patches, we also make use of a parameter called stride, $s_k \in \mathbb{N}^+$. s defines the number of vertical and horizontal indexes between each patch.

$$\begin{aligned}
W_k &= \left\lfloor \frac{W_{k-1}}{s_k} \right\rfloor \\
H_k &= \left\lfloor \frac{H_{k-1}}{s_k} \right\rfloor \\
\psi_k^{(Conv)} : \mathbb{R}^{H_{k-1} \times W_{k-1} \times m_{k-1}} &\rightarrow \mathbb{R}^{H_k \times W_k \times m_k} \\
p_{k-1,(I,J)} &\in \mathbb{R}^{K \times K \times m_{k-1}} \\
p_{k-1,(I,J)} &\subset o_{k-1} \\
p_{k-1,(I,J)} &= (p_{k-1,(I,J),(i,j)}) \\
p_{k-1,(I,J),(i,j)} &\in \mathbb{R}^{m_{k-1}} \\
p_{k-1,(I,J),(i,j)} &= o_{k-1,(a,b)}
\end{aligned}$$

where

$$\begin{aligned}
o_{k-1} &= (o_{k-1,(a,b)}) \\
a &= Is + (i - \lfloor K/2 \rfloor) \\
b &= Js + (j - \lfloor K/2 \rfloor)
\end{aligned}$$

and

$$\begin{aligned}
0 &< I \leq H_k \\
0 &< J \leq W_k
\end{aligned}$$

$$\psi_k^{(Conv)}(o_{k-1}) = (\sigma(p_{k-1,(I,J)} \mathbf{w}^{(k)} + \mathbf{b}^{(k)}))$$

this definition can be corrected/better.

1.1.6 Pooling

Pooling is a way of reducing the dimensionality of an output. Depending on the task, one may choose from different pooling methods. Similar to convolution operation, pooling methods also work with patches $p_{k-1,(I,J)} \in \mathbb{R}^{K \times K \times m_{k-1}}$ and strides s . But this time, instead of applying a weight, bias and activation function, they apply functions. The function that we will make use of is $M : \mathbb{R}^{K \times K \times m} \rightarrow \mathbb{R}^m$. By defining different variations of M, we will define *max pooling* and *average pooling*.

Max Pooling

Max pooling takes the maximum value in a channel within the patch.

$$M^{(max)}(p) = \left\{ \max(\{p_{i,j,l} \mid i \in [1, \dots, K], j \in [1, \dots, K]\}) \mid l \in [1, \dots, m] \right\}$$

Average Pooling

Average pooling averages the values within the patch per channel.

$$M^{(avg)}(p) = \left\{ \sum_{i=1}^K \sum_{j=1}^K \frac{p_{i,j,m}}{K^2} \mid l \in [1, \dots, m] \right\}$$

1.1.7 Deconvolution

1.2 Efficient Operations

In this section we are going to look at some ways to reduce the computational complexities of fully connected layers and convolutional layers.

1.3 Factorization

Factorization is approximating a weight matrix using smaller matrices. This has interesting uses with Neural Networks. Assume that we have a fully connected layer k . Using factorization, we can approximate $\mathbf{w}^{(k)} \in \mathbb{R}^{m_{k-1} \times m_k}$ using two smaller matrices, $U_{\mathbf{w}^{(k)}} \in \mathbb{R}^{m_{k-1} \times n}$ and $V_{\mathbf{w}^{(k)}} \in \mathbb{R}^{n \times m_k}$. If we can find matrices such that $U_{\mathbf{w}^{(k)}} V_{\mathbf{w}^{(k)}} \approx \mathbf{w}^{(k)}$, we can rewrite,

$$\psi_k^{(FC)}(o) \approx \psi_k'^{(FC)}(o) = \sigma(o^T U_{\mathbf{w}^{(k)}} V_{\mathbf{w}^{(k)}} + \mathbf{b}^{(k)})$$

Therefore, we can reduce the complexity of layer k by setting n . As we have mentioned before, $\mathcal{O}(\psi_k^{(FC)}) = \mathcal{O}(m_{k-1} m_k)$. When we approximate this operation, the complexity becomes,

$$\mathcal{O}(\psi_k'^{(FC)}) = \mathcal{O}(n(m_{k-1} + m_k))$$

Therefore, if there is a good enough approximation, satisfying $n < \frac{m_{k-1} m_k}{m_{k-1} + m_k}$, we can reduce the complexity of a fully connected layer without effecting the

results. One thing that's similar between a convolutional layer and a fully connected layer is that both are performing matrix multiplication to calculate results. The only difference is, a convolutional layer is possibly performing this matrix multiplication many times. Therefore the same technique can be used with convolutional layers.

Luc says: If XW is not exactly equal XOP , how will you deal with this?

1.3.1 SVD

Singular Value Decomposition (SVD) ([GR70]), is a factorization method that we can use to calculate this approximation. SVD decomposes the weight matrix $\mathbf{w}^{(k)} \in \mathbb{R}^{m_{k-1} \times m_k}$ into 3 parts.

$$\mathbf{w}^{(k)} = USV^T$$

Where, $U \in \mathbb{R}^{m_{k-1} \times m_{k-1}}$ and $V \in \mathbb{R}^{m_k \times m_k}$. And $S \in \mathbb{R}^{m_{k-1} \times m_k}$ is a rectangular diagonal matrix. The diagonal values of S are called as the singular values of M . Selecting the n highest values from S and corresponding columns and rows from U and V , respectively, lets us create a *low-rank decomposition* of $\mathbf{w}^{(k)}$.

$$\mathbf{w}^{(k)} \approx U' S' V'^T$$

where $U' \in \mathbb{R}^{m_{k-1} \times n}$, $V' \in \mathbb{R}^{n \times m_k}$, and $S' \in \mathbb{R}^{n \times m_k}$. By choosing a sufficiently small n value and setting $U_{\mathbf{w}^{(k)}} = U' S'$ and $V_{\mathbf{w}^{(k)}} = V'^T$, we can approximate the weights, and reduce the complexity of a layer. [ZZHS16] applies this method to reduce the execution time of a network by 4 times and increase accuracy by 0.5%.

1.3.2 Weight Sharing

Introduced by [NH92], weight sharing starts with regular weight matrices, $\mathbf{w}^{(k)} \in \mathbb{R}^{m_{k-1} \times m_k}$ where $k \in [1, \dots, K]$. Once the weights are learned, they use clustering to find a set of weights, $\mathbf{W} \in \mathbb{R}^a$. Then they store the cluster index per weight in $d^{(k)} \in \mathbb{N}^{m_{k-1} \times m_k}$. By redefining $\mathbf{w}_{i,j}^{(k)} = \mathbf{W}_{d_{i,j}^{(k)}}$, they perform weight sharing. Please note that this method does not necessarily reduce model complexity by itself. It reduces the model size by storing indices using less bits. In theory, such a method when applied before factorization should provide a lower rank decomposition.

1.4 Convolution Operation Alternatives

1.4.1 Kernel Composition

As [AP16] explains, a convolution operation with a weight matrix $\mathbf{w}^{(k)} \in \mathbb{R}^{K \times K \times m_{k-1} \times m_k}$, could be composed using two convolution operations with kernels $\mathbf{w}^{(k,1)} \in \mathbb{R}^{1 \times K \times m_{k-1} \times n}$ and $\mathbf{w}^{(k,2)} \in \mathbb{R}^{K \times 1 \times n \times m_k}$. Their technique, instead of factorizing learned weight matrices, aims to learn the factorized kernels. They also aim to increase non-linearity by adding bias and activation function in between. Therefore defining;

$$\psi_{(k)}^{(ConvCompose)}(o) = \psi_{(k,2)}^{(Conv)}(\psi_{(k,1)}^{(Conv)}(o))$$

This method forces the separability of the weight matrix as a hard constraint. By performing such an operation, they convert the computational complexity of a convolution operation from $\mathcal{O}(KKm_{k-1}m_k)$ to $\mathcal{O}(Kn(m_{k-1} + m_k))$. Suggesting that, if we can find an n satisfying $\frac{\mathcal{O}(KKm_{k-1}m_k)}{\mathcal{O}(Kn(m_{k-1} + m_k))} > 1$, we can reduce the complexity of this layer. This equation can be rewritten as;

$$\frac{Km_{k-1}m_k}{m_{k-1} + m_k} > n$$

1.4.2 Separable Convolutions

Separable convolutions separate the standard convolution operation into two parts. These parts are called depthwise convolutions and pointwise convolutions. Depthwise convolution applies a given number of filters on every input channel, one by one therefore results with output channels equal to input channel times number of filters.

Depthwise Convolution

Given a patch $p \in \mathbb{R}^{K \times K \times m}$, depthwise convolution has a weight matrix $\mathbf{w}^{(k, Depthwise)} \in \mathbb{R}^{K \times K \times m}$. For easiness, let's assume variants of p and $\mathbf{w}^{(k, Depthwise)}$, described as $p' \in \mathbb{R}^{K \times K}$ and $\mathbf{w}'^{(k, Depthwise)} \in \mathbb{R}^{K \times K}$,

$$\mathbf{w}'_m{}^{(k, Depthwise)} = \{\mathbf{w}_{i,j,m}^{(k, Depthwise)} \mid i \in [1, \dots, K], j \in [1, \dots, K]\}$$

$$p'_m = \{p_{i,j,m} \mid i \in [1, \dots, K], j \in [1, \dots, K]\}$$

Therefore, depthwise convolution operation $\psi_k^{(Depthwise)} : \mathbb{R}^{W_{k-1} \times H_{k-1} \times m_{k-1}} \rightarrow \mathbb{R}^{W_k \times H_k \times m_{k-1}}$ and it's complexity can be defined as,

$$\psi_k^{(Depthwise)}(p) = \{p'_i \mathbf{w}_i^{(k, Depthwise)} \mid i \in [1, \dots, m]\}$$

$$\mathcal{O}(\psi_k^{(Depthwise)}) = \mathcal{O}(H_k W_k K^2 m_{k-1})$$

Pointwise Convolution

Pointwise convolution ($\psi_k^{(Pointwise)} : \mathbb{R}^{H_k \times W_k \times m_{k-1}} \rightarrow \mathbb{R}^{H_k \times W_k \times m_k}$) is a regular convolution operation with kernel size 1 ($K = 1$). The weight matrix that we'll use for this operation is $\mathbf{w}^{(k, Pointwise)} \in \mathbb{R}^{m_{k-1} \times m_k}$. As you can see, $\mathbf{w}^{(k, Pointwise)}$ is the same shape as a fully connected layer weight matrix. The pointwise convolution operation can be defined as,

$$o_{k-1, (I, J)} \in \mathbb{R}^{1 \times m_{k-1}} \text{ where } 0 < I \leq H_k \text{ and } 0 < J \leq W_k$$

$$\psi_k^{(Pointwise)}(o_{k-1}) = \{o_{k-1, (I, J)} \mathbf{w}^{(k, Pointwise)} \mid I \in [1, \dots, H_k], J \in [1, \dots, W_k]\}$$

$$\mathcal{O}(\psi_k^{(Pointwise)}) = \mathcal{O}(H_k W_k m_{k-1} m_k)$$

Therefore we can describe $\psi_k^{(Separable)} : \mathbb{R}^{W_{k-1} \times H_{k-1} \times m_{k-1}} \rightarrow \mathbb{R}^{W_k \times H_k \times m_k}$ as,

$$\psi_k^{(Separable)}(o_{k-1}) = \psi_k^{(Pointwise)}(\psi_k^{(Depthwise)}(o_{k-1}))$$

The complexity of this operation is,

$$\begin{aligned} \mathcal{O}(\psi_k^{(Separable)}) &= \mathcal{O}(\psi_k^{(Pointwise)}) + \mathcal{O}(\psi_k^{(Depthwise)}) \\ &= \mathcal{O}(H_k W_k m_{k-1} m_k + H_k W_k K^2 m_{k-1}) \\ &= \mathcal{O}(H_k W_k m_{k-1} (m_k + K^2)) \end{aligned}$$

1.5 Pruning

Pruning aims to reduce the model complexity by *deleting* the parameters that has low or no impact in the result. [LDS⁺89] has shown that using second order derivative of a parameter, we can estimate the effect it will have on the training loss. By removing the parameters that have low effect, they have reduced the network complexity and increased accuracy. [HPTT16] has shown that there may be some neurons that are not being activated

by the activation function (i.e. ReLU in their case). Therefore, they count the activations in neurons and remove the ones that have are not getting activated. After that they retrain their network and achieve better accuracy than non-pruned network. [HPTD15] shows that we can prune the weights that are very close to 0. By doing that they reduce the number of parameters in some networks about 10 times with no loss in accuracy. To do that, they train the network, prune the unnecessary weights, and train the remaining network again. [TBCS16] shows that using Fisher Information Metric we can determine the importance of a weight. Using this information they prune the unimportant weights. They also use Fisher Information Metric to determine the number of bits to represent a weight. Also, [Ree93] compiled many pruning algorithms.

1.5.1 Pruning Weights

This subcategory of pruning algorithms try to optimize the number of floating point operations by removing some individual values. In theory, it should benefit the computational complexity to remove individual scalars from $\mathbf{w}^{(k)}$. However, we are defining our layer operations using matrix multiplications. To our knowledge, most of the matrix multiplication implementations require dense matrices. Another option would be to use sparse matrix multiplication. But to be able to gain *any* speed up using sparse matrices, we need to prune about 90% of the weights. Which doesn't seem feasible.

1.5.2 Pruning Nodes

Since it is not possible to remove individual weights and reduce the computational complexity, we are going to look at another case of pruning. This case focuses on pruning a node and all the weights connected to it. Let's assume two fully connected layers, k and $k + 1$. The computational complexity of computing the outputs of these two layers would be $\mathcal{O}(\psi_{k+1}^{(FC)}(\psi_k^{(FC)}(o_{k-1})) = \mathcal{O}(m_k(m_{k-1} + m_{k+1}))$. Assuming that we have removed a single node from layer k , this complexity would drop by $\mathcal{O}(m_{k-1} + m_{k+1})$.

Similar to the fully connected layer, a convolutional layer k also contains m_k nodes. The only difference is, in a convolutional layer, these nodes are repeated in dimensions H_k and W_k . Therefore, it is possible to apply this technique to convolutional layers.

1.6 Quantization

A floating point variable can not represent all decimal numbers perfectly. An n -bit floating point variable can only represent 2^n decimals. The decimals that can not be represented perfectly using 32-bits are going to be represented with some error. Quantization is the process used to represent values using less bits.

In a higher level, the computational complexity doesn't depend on number of bits. But if we dive deeper in the computer architecture, using less bits to represent variables provide some major advantages. It takes less cpu-cycles to perform an operation, reduces the cost of transferring data from memory to cpu and finally increases the amount of data that can fit into cache. One major disadvantage is, most architectures implement optimizations that speed up 16/32/64-bit floating point operations. By using less bits, we are giving up on these optimizations.

1.7 Improving Network Efficiency

1.7.1 Residual Connections

[HZRS15] introduced a method called residual connections. Assuming groups of consecutive layers in our network, we create a residual connection when we add the input of a block to the output of the block to calculate the input of the next block. Let's assume a block b with input $o_{b-1} \in \mathbb{R}^{m_{b-1}}$ and output $o_b \in \mathbb{R}^{m_b}$. We call these two blocks residually connected if we perform $o'_b = o_b + o_{b-1}$ and set o'_b as the input of the next block.

Residual connections allow us to train deeper networks by preventing the vanishing gradient problem. As we increase the number of layers in a neural network, the gradient values for weights in the former layers start getting smaller and smaller. They get so small that they become irrelevant and don't change anything. Residual connections increase the effect of deeper layers for calculating the output. Therefore, their gradients stay do not vanish because they have significant contribution to the result.

1.7.2 Batch Normalization

[IS15] introduced a method called batch normalization. Batch normalization aims to normalize the output distribution of a every node in a layer. By

doing so it allows the network to be more stable.

Assume the layer k with $o_k \in \mathbb{R}^{m_k}$ where m_k is the number of nodes. Batch normalization has four parameters. Mean is $\mu_k \in \mathbb{R}^{m_k}$, variance is $\sigma_k \in \mathbb{R}^{m_k}$, scale is $\gamma_k \in \mathbb{R}^{m_k}$ and offset is $\beta_k \in \mathbb{R}^{m_k}$.

Since we are interested in normalizing the nodes, even if k was a convolutional layer, the shape of these parameters would not change.

$$BN(o_k) = \frac{\gamma_k(o_k - \mu_k)}{\sigma_k} + \beta_k$$

1.7.3 Regularization

Regularization methods aim to prevent overfitting in neural networks. Overfitting is the case where the weights of a neural network converge for the training dataset. Meaning that the network performs very very good for the training dataset, while it is not generalized to work with any other data. Regularization methods try to prevent this.

One common regularization method is to add a new term to the loss, which punishes high weight values. We also add a term λ which determines the effect of this regularization. This parameter, if too high would prevent the network from learning, if too low would have no effect.

L1 Regularization

$$L1 = \lambda \sum_{w \in \mathbf{W}} |w|$$

L2 Regularization

$$L2 = \lambda \sum_{w \in \mathbf{W}} w^2$$

1.8 Efficient Structures

Some structures help neural networks represent more information using less parameters. We are going to look at some structures that are known to work well with convolutional neural networks.

1.8.1 Residual Blocks

[HZRS15] introduced two types of residually connected blocks. First is called a residual block, consisting of two convolution operations and a residual connection between the input and the output of the block. They have trained networks with and without residual connections on ImageNet dataset. Their results show that introduction of the residual connections reduce the top-1 error rate of the 34 layer network from %28.54 to %25.3.

explain this as you explained residual bottleneck blocks.

1.8.2 Residual Bottleneck Blocks

[HZRS15] have used residual blocks to train networks up to 34 layers. For networks greater than 34 layers, they have introduced a second block called as residual bottleneck block. Residual bottleneck blocks consist of three convolution operations with different kernel sizes and number of output nodes.

Before explaining how residual bottleneck blocks are configured, we need more notations to define the layers inside the same block. Let's assume the block b with input $o_{b-1} \in \mathbb{R}^{H_{b-1} \times W_{b-1} \times m_{b-1}}$. We will index the layers inside the block b with a pair (b, k) where k stands for the index of convolutional layer inside the block. For example if we're talking about the second convolutional layer in block b , the input of this convolutional layer would be $o_{(b,1)} \in \mathbb{R}^{H_{(b,1)} \times W_{(b,1)} \times m_{(b,1)}}$ and the output would be $o_{(b,2)} \in \mathbb{R}^{H_{(b,2)} \times W_{(b,2)} \times m_{(b,2)}}$. also since we are talking about layers with different kernel sizes, we need to define them with indexes as well. Therefore the kernel size of convolutional layer (b, k) would be $K_{(b,k)} \in \mathbb{R}$. Therefore, we can define the residual bottleneck block as;

$$\psi_b^{(ResidualBottleneck)} : \mathbb{R}^{H_{b-1} \times W_{b-1} \times m_{b-1}} \rightarrow \mathbb{R}^{H_b \times W_b \times m_b}$$

with the helper function S

$$\psi_b^{(ResidualBottleneck)}(o) = S(o) + \psi_{(b,3)}^{(Conv)}(\psi_{(b,2)}^{(Conv)}(\psi_{(b,1)}^{(Conv)}(o)))$$

$$S_b(o) = \begin{cases} o, & \text{if } (W_b, H_b, m_b) = (W_{b-1}, H_{b-1}, m_{b-1}) \\ P_b(M_b^{(avg)}(o)), & \text{otherwise} \end{cases}$$

where P is a padding function that equalizes the number of nodes in the outputs and $M_b^{(avg)}$ is the average pooling function that equalizes the width and

height dimensions of the input and output of the block. To their definition the first convolution operation in the block reduces the number of nodes with kernel size $K_{(b,1)} = 1$. The number of nodes is defined with a dependency to the stride of block ($s_b \in \{1, 2\}$) as $m_{(b,1)} = m_b/(4/s_b)$. By that logic, if a residual block is reducing the width and height by half, it is doubling the number of nodes to represent more information. Second convolution operation kernel size $K_{(b,2)} = 3$ and has the same number of nodes as the previous layer $m_{(b,2)} = m_{(b,1)}$. The third convolutional layer quadruples the number of nodes ($m_{(b,3)} = 4m_{(b,1)}$) with kernel size $K_{(b,3)} = 1$.

Their 50-layer network using residual bottleneck blocks achieves %22.85 top-1 error rate on ImageNet dataset.

1.9 Datasets

1.9.1 MNIST

MNIST dataset [LCB98] consists of 60.000 training and 10.000 test samples. Each sample is a 28×28 black and white image of a handwritten digit (0 to 9). To our knowledge, best model trained on MNIST achieve almost zero (%0.23, [CMS12]) error rate.

1.9.2 CIFAR10

CIFAR10 dataset [KH09] consists of 50.000 training and 10.000 test samples. Each sample is a 32×32 colored image belonging to one of 10 classes. The classes are airplane, automobile, bird, cat, deer, dog, frog, horse, ship and truck. To our knowledge, best models trained on CIFAR10 achieve %3.47 ([Gra14]) error rate.

1.10 Tools

1.10.1 Tensorflow

To develop and train our neural networks, we are going to be using tensorflow [AAB⁺16]. Tensorflow provides us the necessary tools to deploy our trained models on mobile devices.

Chapter 2

Methods

So far we have explained some neural network building blocks and some techniques for reduced complexity. We have run some experiments to better understand some of these techniques. In this chapter, we are going to talk about these experiments and tools we had to built to run these experiments. First we identified some methods and to understand them better, we ran experiments on them. Then we have combined these methods to come up with a model.

2.1 Pruning

Since pruning individual weights do not effect complexity directly, we are going to focus on pruning nodes from layers. To do that, as [HPTT16] explained, we are going to use *training cycles*. First, we will define a neural network and some data. Then we will train this network using the training dataset. After the training is done, we will run inference for the training dataset and collect ReLU output statistics. Using these statistics, we will try to understand which nodes have minimal effect on accuracy. We will prune these nodes by removing the relevant dimensions from weight matrices. Then we will go back to the training step. We will keep iterating over this list until there are no nodes to be pruned.

2.1.1 Fully Connected Summation

For the sake of simplicity, we started with a very easy problem. We have implemented a neural network consisting of 2 input dimensions ($x_n \in \mathbb{R}^2$), one fully connected layer with 1000 nodes and a one fully connected output with a single node ($y_n \in \mathbb{R}$). We have defined the output as the summation of two inputs, ($y_n = x_{n,1} + x_{n,2}$). So that we precisely know the optimum neural network structure that would be able to perform this calculation. Which is a neural network with one fully connected layer with one node and an output layer fully connected to that. All weights equal to 1 and all biases equal to 0.

We have calculated the loss using mean squared error, and used Momentum Optimizer (learning rate 0.01 and momentum 0.9) to learn the weights. We have generated 1.000.000 samples, we trained the network with batch size 1000.

2.1.2 MNIST Autoencoder

To expand our pruning experiments to convolutional neural networks, we have implemented an autoencoder for MNIST Dataset. An autoencoder consists of two parts. First is the encoder, which aims to reduce the dimensionality of input. The other is decoder, which aims to convert the encoded data back to it's original form. Therefore an autoencoder reduces the dimensions of an input data and then tries to recreate that data from those reduced dimensions.

We have defined the auto encoder with two encoder layers followed by two decoder layers. Each encoding layer running a convolution with kernel size 3 and stride of 2. Following each of these, we are applying batch normalization and ReLU activation. Each decoding layer is running deconvolutions with kernel size 3 and stride of 2. Followed by adding bias, batch normalization and ReLU activations.

confirm the configuration above

The information contained in one input image $x_n \in \mathbb{R}^{28 \times 28 \times 1}$ is represented with 784 floating points. Therefore, a good auto-encoder should be reducing this number with each encoding layer. Similarly, converting encoded image back to it's original form with minimal loss while decoding.

Since we will try to find and prune unused nodes, we will start with a

configuration that is larger than necessary.

$$\begin{aligned}
x_n &\in \mathbb{R}^{28 \times 28 \times 1} \\
\psi_1^{(Conv)} &: \mathbb{R}^{28 \times 28 \times 1} \rightarrow \mathbb{R}^{14 \times 14 \times 32} \\
\psi_2^{(Conv)} &: \mathbb{R}^{14 \times 14 \times 32} \rightarrow \mathbb{R}^{7 \times 7 \times 64} \\
\psi_3^{(Deconv)} &: \mathbb{R}^{7 \times 7 \times 64} \rightarrow \mathbb{R}^{14 \times 14 \times 32} \\
\psi_4^{(Deconv)} &: \mathbb{R}^{14 \times 14 \times 32} \rightarrow \mathbb{R}^{28 \times 28 \times 1} \\
L &= RMSE(x_n, \hat{y}_n)
\end{aligned}$$

2.1.3 Regularization

In both experiments, we test no regularization, L1 regularization and L2 regularization to see if they effect the activation statistics and pruning.

2.1.4 Distortion

In case of two nodes in one layer, if the weights connecting to them are proportional, it may not be possible to prune one of them. To prevent that, we introduce a little bit distortion, forcing the differentiation of features represented by nodes.

2.1.5 Activations

We test 3 pruning configurations for activation counts, activation values and activation correlations.

we need proper mathematical notations to be able to describe these properly. We need to be able to define all training samples as a set and as individuals.

Activation Counts

We count the ReLU activations per node. Using this information we determine which nodes are necessary.

Activation Values

We collect statistics about activation values per node. Using this information we determine which nodes are necessary.

Activation Correlation

We collect statistics about activation correlations to see which nodes are activated together most of the times. Using this information we determine which nodes represent similar features.

2.2 Convolution Operation Alternatives

In general, convolution operations are expensive. In this section, we experiment with *kernel composition* and *separable convolutions* to see which one is the better alternative to regular convolutions. To see the differences between both operations, we ran two experiments for each. One classifying MNIST dataset, the other classifying CIFAR10 dataset.

We have defined a network with three convolutional layers followed by one fully connected layer. Each convolutional layer has kernel size 5. Convolutional layers are followed by batch normalization, then ReLU activations. First two convolution layers are followed by an average pooling layer with kernel size 2 and strides of 2. The third convolutional layer is followed by a global average pooling layer, where we reduce the width and height dimensions to the average of all values in those dimensions. We using SCE loss and momentum optimizer (learning rate 10^{-4} , and momentum 0.9), we train this network for 20000 steps with batch size 32.

$$\begin{aligned}x_n &\in \mathbb{R}^{32 \times 32 \times 1} \\ \psi_1^{(Conv)} : \mathbb{R}^{32 \times 32 \times 3} &\rightarrow \mathbb{R}^{32 \times 32 \times 32} \\ \psi_2^{(Conv)} : \mathbb{R}^{16 \times 16 \times 32} &\rightarrow \mathbb{R}^{16 \times 16 \times 64} \\ \psi_3^{(Conv)} : \mathbb{R}^{8 \times 8 \times 64} &\rightarrow \mathbb{R}^{8 \times 8 \times 128} \\ \psi_4^{(FC)} : \mathbb{R}^{1 \times 1 \times 128} &\rightarrow \mathbb{R}^{10} \\ L &= SCE(x_n, \hat{y}_n)\end{aligned}$$

this is missing the pooling layers and global average pooling

By changing $\psi_{(k)}^{(Conv)}$ with $\psi_{(k)}^{(ConvCompose)}$ and $\psi_{(k)}^{(Separable)}$ we obtain 3 versions of this model. Please remember that $\psi_{(k)}^{(ConvCompose)}$ requires an additional parameter for number of intermediate output channels. We use the number of output channels for that. We train each configuration for both MNIST and CIFAR-10 for a fair comparison.

2.3 Separable Resnet

Inspired by ResNet ([HZRS15],), we have created a convolutional neural network with residual connections. In Figure 2.1 we show the architecture of our neural network.

2.3.1 Design Choices

- Except for the first convolutional operation, we have replaced every convolution layer with a separable convolution layer. The first layer has 3 input and 16 output channels. Therefore, for this layer the number of FLOP for a convolution operation ($K * K * m_{k-1} * m_k = 3 * 3 * 3 * 16 = 432$) is not sufficiently higher than a separable convolution ($K * K * m_{k-1} + m_{k-1} m_k = 3 * 3 * 3 + 3 * 16 = 75$). But for this layer, separable convolution comes with a disadvantage. By definition, it applies a depthwise convolution for color channels separately. We believe that the feature representations of the input are highly dependent on the color channels, therefore we argue that applying a depthwise convolution without mixing the colors is inefficient.
- ResNet-34 starts with a 7×7 convolutional kernel with strides of 2. It is followed by a pooling layer with strides of 2. In our model we chose not to reduce the image size that early. We think that these two layers with strides of 2 are used to speed up the network and kernel size 7 is used to minimize the loss of information. We argue that this early application of strides will lead to loss of information.
- As [HZRS16] proposed, we are using full pre-activation residual connections. See Figure 2.2.

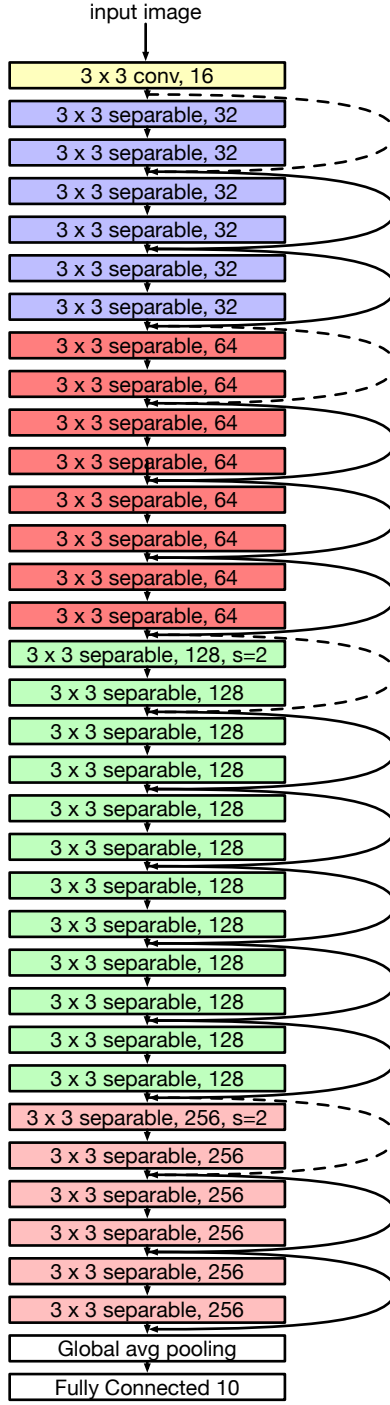


Figure 2.1: Separable Resnet-34. Branching lines represent residual connections, dashed ones are padded with zeros to match the number of channels. If $s=2$, depthwise convolution is ran with strides of two and residual branch is average pooled with strides 2 and kernel size 2. ReLU and Batch Normalization operations are hidden.

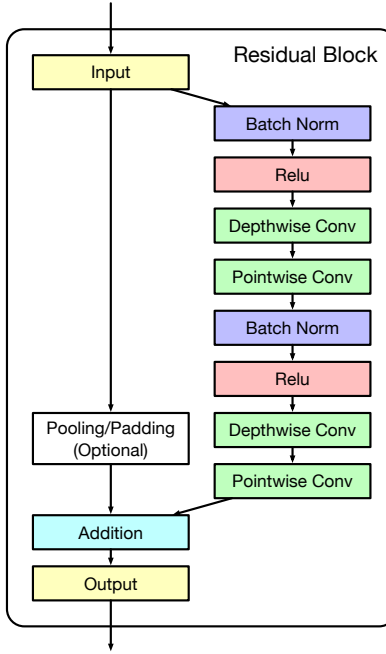


Figure 2.2: Full pre-activation residual connections.

2.3.2 Training

We train our network using CIFAR-10 dataset. We divided the dataset for 50,000 training images and 10,000 validation images. We use momentum optimizer with momentum 0.9 and learning rates 0.1, 0.01, 0.001 for steps 0 to 40,000, 40,000 to 60,000 and 60,000 to 80,000 respectively. We have defined the loss as SCE of the truth and prediction with an addition of L2 norm of weights multiplied by 0.0002. We trained our model using the training images for 80,000 steps with batch size 128.

2.3.3 Pruning Nodes

Pruning Residual Connections

The addition operation in the residual block creates a one-to-one relationship between different blocks. With the existence of such a relationship, it is not possible to prune a residual block's output nodes. Therefore we first group the residual blocks that are directly connected to each other. We consider the residual blocks that don't apply pooling/padding operation to be directly connected to the previous residual block. Then we calculate the indexes of nodes to keep in the output of every residual block. We union these indexes and prune the remaining nodes from the outputs of every directly connected

residual block. We also prune the outputs of first pointwise convolution in the residual blocks separately.

Pruning with Adam Optimizer

We have trained our network using Adam Optimizer ([KB14]) as well. Adam optimizer keeps two moment matrices for each weight matrix. Therefore, if we are pruning the weights, we prune these matrices.

2.3.4 Pruning Weights

We set weight values that are very close to 0 to 0. By doing so, we see the difference in accuracy. Despite the fact that this operation does not change computational complexity, when combined with other methods (i.e. factorization), it will be useful.

2.3.5 Factorization

We factorize the trained and pruned model. Using SVD we try to decompose the convolution, pointwise convolution and fully connected weights. To do that, we calculate U , S and V for each of the weight matrices. From S we pick the most important values that are bigger than threshold value (ϵ). If the decomposed weight matrix would reduce the number of operations, we replace the layer with two new layers, applying the decomposed weights consecutively. We experiment with two versions of this threshold value. One for determining values that are sufficiently small ($\epsilon = 10^{-3}$) and other for finding outliers ($\epsilon = E[S] - \text{Var}(S)$).

2.3.6 Quantization

We quantize the weights from 32-bits to 8-bit and run performance and accuracy benchmarks.

Chapter 3

Results

Chapter 4

Discussion

Chapter 5

Conclusion

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