UNIVERSITY OF ILORIN, KWARA STATE NIGERIA

Department of Mathematics

Project Proposal

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TOPIC:

Solutions of Systems of First Order Ordinary Differential Equations and Applications to Real Life Problems

INTRODUCTION

Background of the study

A system of n linear first order differential equations in n unknowns has the general form

$$\begin{array}{ccccccccccc} x_1' & = & a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & + & g_1 \\ x_2' & = & a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & + & g_2 \\ x_3' & = & a_{31}x_1 & + & a_{32}x_2 & + & \cdots & + & a_{3n}x_n & + & g_3 \\ & & \vdots & & \vdots & & & & \vdots & & \vdots \\ x_n' & = & a_{n1}x_1 & + & a_{n2}x_2 & + & \cdots & + & a_{nn}x_n & + & g_n \end{array} \qquad (*)$$

where a_{ij}'s and g_i's are arbitrary function of t.

If ever term g_i is constant zero, then the system is staid to be **homogeneous**. Otherwise, it is a **non-homogeneous** system if even one of the g's is non-zero.

The System (*) is most often given in a shorthand format as a matrix vector equation in the form

$$x' = Ax + g$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ \vdots \\ x_n' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ \vdots \\ g_n \end{bmatrix}$$

For a homogeneous system, g is the zero vector. Hence it has the form

$$x' = Ax$$

AIMS AND OBJECTIVES OF THE PROJECT

The main aim of the research work is to examine the solutions of systems of first order ordinary differential equations and its applications to real life problems.

The objectives of the study includes

1. To reduce higher order system value problem to the equivalent first system.
2. to use matrix methods involving eigenvalues and eigenvectors
3. to rewrite linear D.E into a system of two equations
4. to use Laplace Transform method to solve system of linear equations.
5. to solve application problems using first order linear D.E

METHODOLOGY

1.1 Reduction of higher order to system of first order O.D.E

Every n-th order linear equation is equivalent to a system of n first order linear equation.

Example 1: Reduce $y''' - 3y'' + 2y' - 6y = 0$ From the equation above is equivalent to:

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= x_3 \\ x_3' &= 6x_1 - 2x_2 + 3x_3 \end{aligned}$$

Example 2:

$y'' + 5y' - 6y = 0$

Solution

$y'' + 5y' - 6y = 0$

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= -5y_2 + 6y_1 \end{aligned}$$

1.2 Using Matrix method involving Eigenvalues and Eigenvectors

This method involves using determinant of the form:

$$\det(A - \lambda I)$$

where λ is the eigenvalues and A is a matrix, I is the identity element.

Example 3: Find te eigen values and corresponding eigenvectors.

Let

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

Solution:

Recall, $A - \lambda I$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix}$$

$$\begin{aligned} \det(A - \lambda I) &= (1-\lambda)(1-\lambda) - 2 \times 2 = 0 \\ & (\lambda - 1)^2 - 4 = 0 \\ & (\lambda - 1)^2 = 4 \\ \lambda &= 1 \pm 2 \\ & \lambda_+ = 3 \quad \text{or} \quad \lambda_- = -1 \end{aligned}$$

Therefore the eigenvalues are $\lambda_1 = 3$, and $\lambda_2 = -1$.

To find the eigenvectors

When $\lambda = 3$

$$
\begin{aligned}
(A - 3I) &= 0 \\
\vec{V_1} &= \begin{bmatrix} 1-3 & 2 \\ 2 & 1-(-1) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} -2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{aligned}
$$

$$-2x_1 + 2x_2 = 0$$

$$x_1 = x_2$$

$$\vec{V_1} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

When $\lambda = -1$

$$
\begin{aligned}
(A + I) &= 0 \\
\vec{V_2} &= \begin{bmatrix} 1-(-1) & 2 \\ 2 & 1-(-1) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{aligned}
$$

$$2x_1 + 2x_2 = 0$$

$$2x_1 = -x_2 \implies x_1 = -x_2$$

$$\vec{V_2} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

1.3 Laplace Transform methods of solving system of linear equations

To solve a differential equation of the form:

$$af'(t) + bf(t) = g(t)$$

given than $f(0) = k$ where a, b and k are constants and $g(t)$ is a known expression in y.

REFERENCES

Gabriel Nagy, January 18, 2021, Background of the study, Example 3.

Prof. Gbamigbola: Lecture notes (2019) example 2 eqn (B)

K.A Stroud (2001) 7th Edition

TOPIC:

Laplace Transform: Theory, Problems and Solutions

INTRODUCTION

Background of the study

Laplace transform is yet another operational tool for solving constant coefficients linear differential equations.

The process of solution consists of three main steps:

- The given "hard" problem is transformed into a "simple" equation
- This simple equation is solved by purely algebraic manipulations
- The solution is transformed back to obtain the solution of te given problem

AIMS AND OBJECTIVES

The main aim of the research work is to examine the Laplace Transform theory, problems and solutions.

The objective of the study are:

1. To determine Laplace Transform
2. To solve te algebraic equation
3. To determine the inverse transform

4. To determine the Laplace Transform a polynomial
5. Solving initial value problem using Laplace Transform
6. To examine the Laplace Transform and of partial fractions

Definition

The Laplace Transform is defined in the following way:

Let $f(t)$ be defined for $t \geq 0$. Then the Laplace transform of f, which is denoted $\mathcal{L}[f(t)]$ or $F(s)$, is defined by te following equation

$$\begin{aligned} \mathcal{L}[f(t)] = F(s) &= \lim_{T \to \infty} \int_0^T f(t)e^{-st}\mathrm{d}t \\ &= \int_0^\infty f(t)e^{-st}\mathrm{d}t \\ &\quad \text{for } s > 0 \end{aligned}$$

METHODOLOGY

1.0 Finding Laplace Transform

We have methods to find $F(s)$ for a given $f(t)$:

From the definition:

$$\mathcal{L}[f(t)] = F(s) \int_0^\infty f(t)e^{-st}\mathrm{d}t$$

For

$$\begin{aligned} f(t) = 1 \implies \mathcal{L}[1] &= \int_0^\infty e^{-st}\mathrm{d}t \\ &= \left[-\frac{1}{s}e^{-st}\right]_0^\infty = \frac{1}{s} \end{aligned}$$

For

$$\begin{aligned} f(t) = t, \mathcal{L}[t] &= \int_0^\infty e^{-st} t \mathrm{d}t \\ &= \left[-\frac{1}{s} e^{-st} \right]_0^\infty + \int_0^\infty \frac{1}{s} e^{-st} \mathrm{d}t = \frac{1}{s} \end{aligned}$$

1.1 Finding the Inverse Transform Using Partial Fractions

Given a function $f(t)$, we denote its Laplace Transform by $\mathcal{L}[f] = F$, the Inverse process is written as

$$\mathcal{L}^{-1}[F] = f$$

1.2 Solving ODEs and ODE Systems

The application of Laplace Transform method is effective for linear ODEs with constant coefficients and for Systems of such ODEs

Example 1.0

Solve $f'(t) - f(t) = 2 - - - - - - (a)$ where $f(0) = 0$

$2\frac{\mathrm{d}y}{\mathrm{d}x} - y = \sin t - - - - - - (b), \quad y(0) = 1$

REFERENCES

C.T.J Dodson, School of Mathematics Manchester University

Marcel B. Finan Arkansas Tech University

K.A. Stroud, Dexter J. Booth (2001) 7th Edition(example 1.0 (a))