TRANSPORTATION PROBLEM

A Case Study Of Unilorin Water Enterprise(UWE)

BY

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17/56EB094

A PROJECT SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, FACULTY OF PHYSICAL SCIENCES, UNIVERSITY OF ILORIN, ILORIN, NIGERIA,

IN PARTIAL FULFILMENT OF REQUIREMENTS FOR THE AWARD OF BACHELOR OF SCIENCE (B.Sc.) DEGREE IN MATHEMATICS.

NOVEMBER, 2022

CERTIFICATION

This is to certify that this project work was carried out by **David, Udo Uduak** with matriculation number **17/56EB094** and approved as meeting the requirement for the award of the Bachelor of Science (B. Sc.) degree of the Department of Mathematics, Faculty of Physical Sciences, University of Ilorin, Ilorin, Nigeria.

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Prof. K. Rauf Date

Head of Department

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Prof. Date

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DEDICATION

This is dedicated to God Almighty, my creator, my strong pillar, my source of inspiration, wisdom, knowledge, and comprehension. He has been the source of my strength throughout this program, and I have soared only on his wings.

I also dedicate this work to my father, Apostle David Udosen, who has always encouraged me and ensured that I give it everything I have to finish what I started. Aduragbemi Olorunyomi; my buddy turned sister, thank you for everything you do for me. And to my friends, course-mate, family, and well-wishers; my love for you all is immeasurable. God's blessings on you all.

ACKNOWLEDGMENTS

All praises, adoration and glorification are for Almighty God.

My profound gratitude and appreciation go to my protean and persisting supervisor, Prof. M. O. Ibrahim, for his kind- heartedness, and for his candid advice, encouragement and useful guidelines towards the success of this project. I pray that the Almighty God be with him and his family.

I acknowledge my level adviser Dr. K. A. Bello for his support and help when required, I'm really grateful.

I also extend my sincere gratitude to the Head of Department, Prof. K. Rauf and also to all my esteem Lecturers: Prof. J. A. Gbadeyan, Prof. T. O. Opoola, Prof. O. M. Bamigbola, Prof. O. A. Taiwo, Prof. M. O. Ibrahim, Prof. R. B. Adeniyi, Prof. S. O. Makanjuola, Prof. M. S. Dada, Prof. A. S. Idowu, Prof. E. O. Titiloye, Prof. O. A. Fadipe-Joseph, Dr. Yidiat O. Aderinto, Dr. Catherine N. Ejieji, Dr. B. M. Yisa, Dr J. U. Abubakar, Dr. Gata N. Bakare, Dr T. O. Olotu, Dr. B. M. Ahmed, Dr Idayat F. Usamot, Dr O. A. Uwaheren, O. Odetunde and all other members of staff of the department of mathematics, who contributed greatly to my academic excellence, obtained during my period of study in the department. May God bless them all.

I also want to thank the Manager of Unilorin Water Enterprise (UWE) for providing the data for the study.

ABSTRACT

This study takes into account the proposed transportation model of manufacturing goods to consumers (key distributors).

This transportation model will be useful for Unilorin Water Enterprise logistics managers in making strategic decisions regarding the optimal allocation of products from four storehouses; factory, consultancy, business school, and UITH, to various customers (key distributors) at the lowest transportation cost.

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Chapter 1

GENERAL INTRODUCTION

1.1 Introduction

The **Transportation Problem (TP)** is the overall term for a wide range of issues in which transportation is required. The following are the general parameters of Transportation Problem:

(A) **Resources:** These are the elements that can be moved from one location to another. Goods, machinery, tools, people, and cargo are examples of discrete resources; continuous resources include energy, liquids, and money..

(B) **Locations:** Point of delivery, recollection depots, nodes, railway stations, bus terminals, loading ports, seaports, airports, refueling depots, and schools are all examples of locations.

(C) **Transportation Modes:** Transportation modes are methods of delivering resources to certain areas. Water, space, air, road, train, and cable are used as forms of transportation. The infrastructure, capacity, schedules, activities, and rules varies depending on the mode of transportation. Ships, planes, trucks, trains, pipelines, motorcycles, and other forms of movement are examples.

1.2 Types of Transportation Problem

There are basically two (2) types of transportation problem:

1. Balanced Transportation Problem
2. Unbalanced Transportation Problem

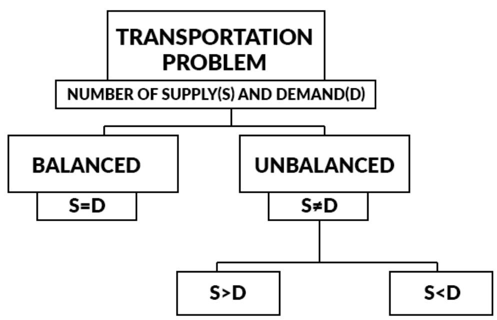

Figure 1.1: Types of Transportation Problem

1.3 Model of A Transportation Problem

The transportation problem model is defined by

$$\text{Minimize } Z = \sum_{i=1}^{m} X_{ij}C_{ij} \tag{1.1}$$

$$\sum_{j=1}^{n} X_{ij} \leq a_i \quad , \quad i = 1, 2, 3, \ldots, m \;\; \text{(Demand Constraint} \tag{1.2}$$

$$\sum_{i=1}^{m} X_{ij} \geq b_j \quad , \quad j = 1, 2, 3, \ldots, n \;\; \text{(Supply constraint)} \tag{1.3}$$

$$X_{ij} \geq 0, 1, 2, 3, \ldots, n \tag{1.4}$$

This is a Linear Program with $m \cdot n$ decision variables, $m + n$ functional constants, and $m \cdot n$ non-negative constraints. Where

n is the number of destination

m is the number of resources

a_i is the capacity of i source

b_j is the demand of jth destination

C_{ij} is the unit transportation cost between ith source and jth destination (in naira or as a distance in Kilometers, miles, etc.). While X_{ij} is the size of material transported between ith source and jth destination (in tons, pounds, liters etc.).

A transportation problem is said to be unbalanced if and only if

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \tag{1.5}$$

There are two cases:

Case (1)

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j \tag{1.6}$$

Case (2)

$$\sum_{i=1}^{m} a_i \leq \sum_{j=1}^{n} b_j \tag{1.7}$$

To balance the Transportation Problem, introduce a dummy origin or source in the transportation table with a zero cost. The availability at the origin is

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = 0 \tag{1.8}$$

1.4 Tableau And Network Representation

The transportation problem is illustrated with the model of a linear program and it appears in a network and tableau form

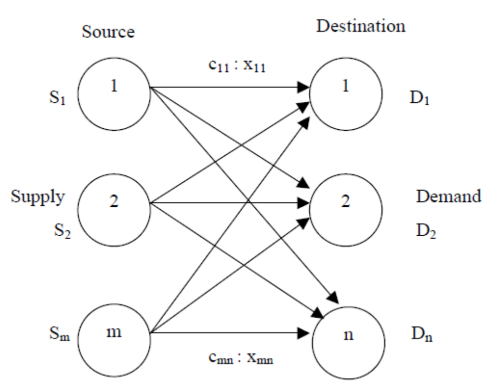

Figure 1.2: The Transportation Network

Destination / Plants	D_1	D_2	D_3		D_{n-1}	D_n	Supply quantity
S_1	x_{11}	x_{12}	x_{13}	…	$x_{1,m-1}$	$x_{1,n}$	s_1
S_2	x_{21}	x_{22}	x_{23}	…	$x_{2,m-1}$	$x_{2,n}$	s_2
S_3	x_{31}	x_{32}	x_{33}	…	$x_{3,m-1}$	$x_{3,n}$	s_3
	.	.	.		.	.	.
	.	.	.		.	.	.
	.	.	.		.	.	.
S_{m-1}	$x_{m-1,1}$	$x_{m-1,2}$	$x_{m-1,3}$	…	$x_{m-1,n-1}$	$x_{m-1,n}$	s_{m-1}
S_m	$x_{n,1}$	$x_{n,2}$	$x_{n,3}$	…	$x_{m,n-1}$	$x_{m,n}$	s_m
Demand quantity	d_1	d_2	d_3	…	d_{n-1}	d_n	

Figure 1.3: The Transportation Tableau

1.5 Flowchart Solution of the Transportation Problem

- the problem is formulated as a transportation model
- is the transportation model balanced?
- if yes, got next step, add dummy to the rows or column
- determine initial basic solution
- go to next step if the solution is optimized else go to fourth step
- using the optimal solution, calculate the total transportation cost

1.6 Background of Study

1.6.1 Company Profile

The Unilorin Water Enterprise began operations on September 23, 2013, with the promise of producing sparkling clean water for the use of members of the

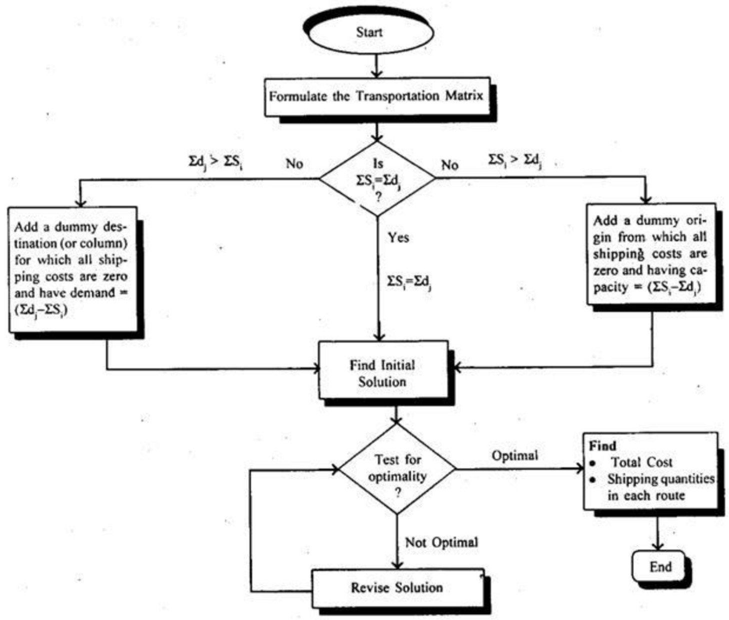

Figure 1.4: Flowchart of Transportation Solution

university community and beyond. The Unilorin Water Enterprise is open from Mondays through Saturdays.

1.6.2 Company Brand

On March 4, 2014, the National Agency for Food and Drug Administration and Control (NAFDAC) approved Unilorin Water Enterprise (UWE) by assigning product registration numbers to the two water brands. Unilorin table water 50cl and 75cl bottles with registration number (1-2090L) and Unilorin Pure water with registration number (1-2049L).

1.6.3 Process of Production

The factory manufactures and packages the product in a calm and sanitary atmosphere; the company has five sedimentation tanks connected to a particular borehole, huwa-san, sand filter, carbon filter, and four treated water tanks. The business has a micro filter reserve osmosis, treated water tank, ultra violet sterilizer, and washing, filling, and capping equipment for sachet water, and an automatic packaging machine for bottle water.

1.6.4 Storehouses

The warehouse in the senior staff quarters store house stores raw materials, semi-finished items, and completed goods. This final goods and services are provided on time and at a low cost. There are only three registered transporters in charge of loading, packing, unloading, and moving finished items from the manufacturing warehouse to the distributors.

1.6.5 Distributions

Finished goods are sold directly to registered distributors. The distributors are the primary agents who distribute to retailers; the university of Ilorin water company produces on a huge scale and has over 70 distributors on and off campus. The firm produces about 4000 bags of sachet water, 500 packs of 50cl Unilorin table water, and 1000 packs of 75cl table water.

1.7 Problem Statement

The project will attempt to resolve the challenge of determining the ideal transportation plan that will reduce the overall cost of transporting products from the major production locations to the many important distributors in Ilorin who are geographically dispersed.

1.8 Aim and Objectives

1.8.1 Aim

The Aim of the study is to develop a transportation model of Unilorin Water Enterprise .

1.8.2 Objectives

The study intended:

(i) To develop a model of distribution of Unilorin Water Enterprise (UWE) products as a transportation problem .

(ii) To minimize the transportation cost.

(iii) To maximize profit.

Chapter 2

LITERATURE REVIEW

The transportation problem (TP) is an important Linear Programming (LP) model that arises in several context and has deservingly received much attention in literature.

The transportation problem is probably the most important special linear transportation problem in terms of relative frequency with which it appears in the applications and also in the simplicity of the procedure developed for its solution. The following features of the transportation problem are considered to be most important.

The transportation problem were the earliest class of Linear Programs discovered to have totally unimodular matrices and integrand extreme points resulting in considerable simplification of the Simplex method.

The study of the transportation problems laid the foundation for further theoretical and algorithmic development of the minimal cost network flow problems.

The transportation problem was formalized by the French mathematician Monges (1781). Major advances were made in the field during World War II by the Soviet/Russian mathematician and Economist Leomd Kantorovich. Consequently, the problem as it is now stated is sometimes known as the Monge-Kantorovich transportation problem . Kantorovich(1942) published a paper on continuous version of the problem and later with Gavurian, and applied study of the capacitated transportation problem Kantorovich et al (1949).

Many scientific disciplines have contributed toward analysing problems associated with the transportation problem, including operation research, Economics, Engineering, Geographic Information Science and Geography. It is explored extensively in the Mathematical Programming and Engineering literatures. Sometimes referred to as the facility location and allocation problem, the transportation optimization problem can be modelled as a large-scale mixed integer linear programming problem.

The origin of transportation was first presented by Hitchcock, (1941) also presented a study entitled *'The Distribution of a product from several sources to numerous locations'*, this presentation is considered to be transportation problems. Kropmans, (1947), presented an independent study, not related to Hitchcock's and called *"Optimum utilization of the transportation sys-*

tem". These two contributions helped in the development of transportation methods which involve a number of shopping sources and a number of destination. The transportation problem, received this named because many of its applications involve determining how to optimally transport goods.

Chapter 3

METHODOLOGY

3.1 Introduction

This chapter reviews the proposed solution methodology and approach for handling transportation problem in Unilorin Water Enterprise . The transportation problem seeks to minimize the total shipping cost of transporting goods from m origins (each with a supply s_i) to n destinations (each with a demand d_j) when the unit shipping cost from an origin i, to a destination j, is C_{ij}.

3.2 Mathematical Formulation

Supposed a company has m warehouses and n retail outlets. A single product is to be shipped from the warehouse to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation cost between every pair of warehouse and outlet, and these cost are assumed to be linear. More explicitly, the assumptions are

- the total supply of products from warehouse $i = a$, where $i = 1, 2, 3, \ldots, m$
- the total demand of the products at the outlet $j = b$, where $j = 1, 2, 3, \ldots, n$
- the cost of sending one unit of the product from warehouse i to outlet j is equal to C_{ij}, where $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$. The total cost of a shipment is linear in size of shipment.

3.3 Solution of A Transportation Problem

Solving the Transportation Problem:- There are three popular methods to finding an initial basic feasible solution and they include:

(1) Northwest Corner Rule

(2) Least Cost Method

(3) Vogel Approximation Method

Northwest Corner Rule(NCR)

In this method, allocation of quantities being transported from source to some destination must start from the upper most left hand cell that is the Northwest Corner of the table. The steps include:

(a) Make allocation in the northwest (upper left) corner of the transportation problem table. Compare the supply of plant 1 say S_1 with the demand at the warehouse or destination 1 say d_1. Then,

   (i) If $d_1 < S_1$ i.e If the amount required at d_1 is less than the number of units available at S_1, set x_{11} equal to d_1, find the balance supply and demand and proceed horizontally.

   (ii) If $d_1 = S_1$, set x_{11} equal to d_1, balance supply and demand and proceed diagonally. Remember to make a zero allocation to the least cost cell in S_1/d_1.

   (iii) If $d_1 > S_1$, set x_{11} equal to S_1, balance demand and supply and proceed vertically.

(b) Continue with *itoiii*, step by step away from the upper left corner until you reach a value in the South-East corner.

(c) calculate the total transportation cost.

This method does not take into account the transportation cost and hence may not yield a good initial basic feasible solution.

Least Cost Method (LCM)

The Least Cost Method is also called the Matrix Minimum Method, is a method of finding an initial basic feasible solution where allocation of resources begins from the least cost. The steps includes:

1. Determine the cell having the least transportation cost (C_{ij})

2. Allocate as much as possible to this least cost
3. If there's a tie in least cost, select the cell having the greatest least cost.
4. Delete the row or column which has been exhausted
5. Select the next least cost and allocate as much as possible
6. Continue this manner till all row and column requirements are met.

The Vogel Approximation Method(VAM)

This procedure is an iterative method of finding an initial basic feasible solution. It is an improved version of the least cost method. The steps include:

1. Find the difference between the least cost and next least cost of each row and column(This difference is the row or column penalty).
2. Select the row or column with the biggest penalty
3. In case of a tie in penalty, select the row or column with the greatest least cost
4. Make allocation as much as possible to the cell in that row/column
5. Delete the column or row that has been completely exhausted.
6. Repeat steps 1 to 5 until all allocation are made

3.3.1 Numerical Illustration

Consider A company which has 3 production facilities S_1, S_2 and S_3 with production capacity of $7, 9$ and 18 units(in 100's) per week of a product, respectively. These units are to be shipped to 4 warehouses D_1, D_2, D_3 and D_4 with requirement of $5, 8, 7$ and 14 units (in 100's) per week, respectively. The transportation costs (in rupees) per units between factories to warehouse are given in the table below

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

To find an initial basic feasible solution for the given transportation problem. Using the three method.

3.3.2 METHOD 1: Using North West Corner

TOTAL number of supply constraints: 3

TOTAL number of demand constraints: 4

Problem Table is

	D_1	D_2	D_3	D_4	Capacity

S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

The rim values for $S_1 = 7$ and $D_1 = 5$ are compared

The smaller of the two i.e $\min(7, 5) = 5$ is assigned to S_1D_1.

This meets the complete demand of D_1 and leaves $7 - 5 = 2$ units with S_1.

Table-1

	D_1	D_2	D_3	D_4	Capacity
S_1	19(5)	30	50	10	2
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	8	7	14	

The rim values for $S_1 = 2$ and $D_2 = 8$ are compared.

The smaller of the two i.e $\min(2, 8) = 2$ is assigned to S_1D_2

Table-2

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19(5)~~	~~30(2)~~	~~50~~	~~10~~	0
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	6	7	14	

The rim value for $S_2 = 9$ and $D_2 = 6$ are compared

The smaller of the two i.e $\min(9, 6) = 6$ is assigned to S_2D_2.

This exhausts the capacity of S_1 and leaves $8 - 2 = 6$ units with D_2

Table-3

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19(5)~~	~~30(2)~~	~~50~~	~~10~~	0
S_2	70	30(6)	40	60	3
S_3	40	8	70	20	18
Demand	0	0	7	14	

The rim values for $S_2 = 3$ and $D_3 = 7$ are compared.

The smaller of the two i.e $\min(3, 7) = 3$ is assigned to S_2D_3. This exhausts the capacity of S_2 and leaves $7 - 3 = 4$ units with D_3

Table-4

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19(5)~~	~~30(2)~~	~~50~~	~~10~~	0
~~S_2~~	~~70~~	~~30(6)~~	~~40(3)~~	~~60~~	0
S_3	40	8	70	20	18
Demand	0	0	4	14	

The rim values for $S_3 = 18$ and $D_3 = 4$ are compared

This smaller of the two i.e $\min(18, 4) = 4$ is assigned to S_3D_3

This meets the complete demands of D_3 and leaves $18 - 4 = 14$ units with S_3.

Table-5

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19(5)~~	~~30(2)~~	~~50~~	~~10~~	0
~~S_2~~	~~70~~	~~30(6)~~	~~40(3)~~	~~60~~	0
S_3	40	8	70(4)	20	14
Demand	0	0	0	14	

The rim values for $S_3 = 14$ and $D_4 = 14$ are compared.

The smaller of the two i.e $\min(14, 14) = 14$ is assigned to S_3D_4

Table-6

	D_1	D_2	D_3	D_4	Capacity
S_1	19(5)	30(2)	50	10	0
S_2	70	30(6)	40(3)	60	0
S_3	40	8	70(4)	20(14)	0
Demand	0	0	0	0	

Initial feasible solution is

	D_1	D_2	D_3	D_4	Capacity
S_1	19(5)	30(2)	50	10	7
S_2	70	30(6)	40(3)	60	9
S_3	40	8	70(4)	20(14)	18
Demand	5	8	7	14	

The minimum total transportation cost

$$19 \times 5 + 30 \times 2 + 30 \times 6 + +40 \times 3 + 70 \times 4 + 20 \times 14 = 1015$$

Here, the number of allocated cells = 6 is equal to

$$m + n - 1 = 3 + 4 - 1 = 6$$

$\therefore$ This solution is non-degenerate

3.3.3 METHOD 2: Using Least Cost Method to find solution

TOTAL number of supply constraints: 3

TOTAL number of demand constraints: 4

Problem Table is

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

The smallest transportation cost is 8 in cell S_3D_2

The allocation to this cell is $\min(18, 8) = 8$

This satisfies the entire demand of D_2 and leaves $18 - 8 = 10$ units with S_3

Table-1

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8(8)	70	20	10
Demand	5	0	7	14	

The smallest transportation cost is 10 in cell S_1D_4

The allocation to this is $\min(7, 14) = 7$

The exhausts the capacity of S_1 and leaves $14 - 7 = 7$ units with D_4

Table-2

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
S_2	70	30	40	60	9
S_3	40	8(8)	70	20	10
Demand	5	0	7	7	

The smallest transportation cost is 20 in cell S_3D_4

The allocation of the cell is $\min(10, 7) = 7$

This satisfies the entire demand of D_4 and leaves $10 - 7 = 3$ units with S_3

Table-3

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
S_2	70	30	40	60	9
S_3	40	8(8)	70	20(7)	3
Demand	5	0	7	0	

The smallest transportation cost is 40 in cell S_2D_3

The allocation to this cell is $\min(9, 7) = 7$

This satisfies the entire demand of D_3 and leaves $9 - 7 = 2$ units with S_2

Table-4

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
S_2	70	30	40(7)	60	2
S_3	40	8(8)	70	20(7)	3
Demand	5	0	0	0	

The smallest transportation cost is 40 in cell S_3D_1

The allocation to this cell is $\min(3, 5) = 3$

This exhausts the capacity of S_3 and leaves $5 - 2 = 2$ units with D_1

Table-5

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
S_2	70	30	40(7)	60	2
~~S_3~~	~~40(3)~~	~~8(8)~~	~~70~~	~~20(7)~~	0
Demand	2	0	0	0	

The smallest transportation cost is 70 in cell S_2D_1

The allocation to this cell is $\min(2, 2) = 2$

Table-6

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
~~S_2~~	~~70(2)~~	~~30~~	~~40(7)~~	~~60~~	0
~~S_3~~	~~40(3)~~	~~8(8)~~	~~70~~	~~20(7)~~	0
Demand	0	0	0	0	

Initial feasible solution is

	D_1	D_2	d4 D_3	D_4	Capacity
S_1	19	30	50	10(7)	7
S_2	70(2)	30	40(7)	60	9
S_3	40(3)	8(8)	70	20(7)	18
Demand	5	8	7	14	

The minimum total transportation cost

$$10 \times 7 + 70 \times 2 + 40 \times 3 + 8 \times 8 + 20 \times 7 = 814$$

Here, the number of allocated cells = 6 is equal to

$$m + n - 1 = 3 + 4 - 1 = 6$$

$\therefore$ This solution is non-degenerate

3.3.4 METHOD 3: Using Vogel's Approximation Method to find solution

TOTAL number of supply constraints: 3

TOTAL number of demand constraints: 4

Problem Table is

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

Table-1

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19	30	50	10	7	9=19-10
S_2	70	30	40	60	9	10 = 40-30
S_3	40	8	70	20	18	12 = 20-8
Demand	5	8	7	14		
Column Penalty	21 = 40-19	22=30-8	10=50-40	10=20-10		

The maximum penalty, 22 occur in column D_2

The minimum C_{ij} in this column is $C_{32} = 8$

The maximum allocation in this cell is $\min(18, 8) = 8$

It satisfy demand of D_2 and adjust the supply of S_3 from 18 to 10 $(18 - 8 = 10)$.

Table-2

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19	30	50	10	7	9=19-10
S_2	70	30	40	60	9	20 = 60-40
S_3	40	8(8)	70	20	10	20 = 40-20
Demand	5	0	7	14		
Column Penalty	21 = 40-19	–	10=50-40	10=20-10		

The maximum penalty, 21, occur in column D_1

The minimum C_{ij} in this column is $C_{11} = 19$

The maximum allocation in this cell is $\min(7, 5) = 5$

It satisfy demand of D_1 and adjust the supply of S_1 from 7 to 2 $(7 - 5 = 2)$

Table-3

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19(5)	30	50	10	2	40=50-10
S_2	70	30	40	60	9	20 = 60-40
S_3	40	8(8)	70	20	10	50 = 70-20
Demand	0	0	7	14		
Column Penalty	–	–	10=50-40	10=20-10		

The maximum penalty, 50 occurs in row S_3

The minimum C_{ij} in this row is $C_{34} = 20$

The maximum allocation in this cell is $\min(10, 14) = 10$

It satisfy supply of S_3 and adjust the demand of D_4 from 14 to 4 $(14 - 10 = 4)$.

Table-4

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19(5)	30	50	10	2	40=50-10
S_2	70	30	40	60	9	20 = 60-40
S_3	40	8(8)	70	20(10)	0	–
Demand	0	0	7	4		
Column Penalty	–	–	10=50-40	50=60-10		

The maximum penalty, 50, occurs in column D_4

The minimum C_{ij} in this column is $C_{14} = 10$

The maximum allocation in this cell is $\min(2, 4) = 2$

It satisfy supply of S_1 and adjust the demand of D_4 from 4 to 2 $(4 - 2 = 2)$.

Table-5

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19(5)	30	50	10(2)	0	–
S_2	70	30	40	60	9	20 = 60-40
S_3	40	8(8)	70	20(10)	0	–
Demand	0	0	7	2		
Column Penalty	–	–	40	60		

The maximum penalty, 60, occurs in column D_4

the minimum C_{ij} in this column is $C_{24} = 60$

The maximum allocation in this cell is $\min(9, 2) = 2$

It satisfy demand of D_4 and adjust the supply of S_2 from 9 to 7 $(9 - 2 = 7)$

Table-6

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
$\cancel{S_1}$	19(5)	30	50	10(2)	0	–
S_2	70	30	40	60(2)	7	40
$\cancel{S_3}$	40	8(8)	70	20(10)	0	–
Demand	0	0	7	0		
Column Penalty	–	–	40	–		

The maximum penalty, 40, occurs in row S_2

The minimum C_{ij} in this row is $C_{23} = 40$

The maximum allocation in this cell is $\min(7, 7) = 7$

It satisfy supply of S_2 and demand of D_3

Initial feasible solution is

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19(5)	30	50	10(2)	7	9|9|40|40|-|-|
S_2	70	30	40	60(2)	9	10|20|20|20|20|40|
S_3	40	8(8)	70	20(10)	18	12|20|50|-|-|-|
Demand	5	8	7	14		
	21	22	10	10		
	21	–	10	10		
	–	–	10	10		
	–	–	10	50		
	–	–	40	60		
	–	–	40	–		

The minimum total transportation cost

$$19 \times 5 + 10 \times 2 + 40 \times 7 + 60 \times 2 + 8 \times 8 + 20 \times 10 = 779$$

Here, the number of allocated cells = 6 is equal to

$$m + n - 1 = 3 + 4 - 1 = 6$$

$\therefore$ This solution is non-degenerate

Chapter 4

DATA COLLECTION AND ANALYSIS

4.1 Data Collection

The table below displays a weekly unbalanced transportation problem data from Unilorin Water Enterprise's storehouses: Factory, Consultancy, Business School, and UITH, each. This problem's demand and supply capacity are provided as 400, 150, 100, 200 and 350, 300, 250, 200 (in packs per week), respectively. The transportation costs (in thousand naira) between factories to warehouses are given per week.
This study covers data collection from the month of September - October, 2021.

4.2 Analysis

	W_1	W_2	W_3	W_4	Supply Capacity|Availability
Factory	5	6	5	7	400
Consultancy	2	2	3	3	150
Business School	5	4	4	3	100
Uith	13	12	15	12	200
Demand	250	200	150	250	

To find an initial basic feasible solution for the given transportation problem using the three method. Representing the stores as S_1, S_2, S_3, S_4.

4.2.1 METHOD 1: Using North West Corner

TOTAL number of supply constraints: 4

TOTAL number of demand constraints: 4

	W_1	W_2	W_3	W_4	Supply
S_1	5	6	5	7	400
S_2	2	2	3	3	150
S_3	5	4	4	3	100
S_4	13	12	15	12	200
Demand	250	200	150	250	

The rim values for $S_1 = 400$ and $W_1 = 250$ are compared

The smaller of the two i.e $\min(400, 250) = 250$ is assigned to S_1W_1

This meets the complete demand of W_1 and leaves $400 - 250 = 150$ units with S_1

Table-1

	W_1	W_2	W_3	W_4	Supply
S_1	5(250)	6	5	7	150
S_2	2	2	3	3	150
S_3	5	4	4	3	100
S_4	13	12	15	12	200
Demand	0	200	150	250	

The rim values for $S_1 = 150$ and $W_2 = 200$ are compared

The smaller of the two i.e $\min(150, 200) = 150$ is assigned to S_1W_2

This exhausts the capacity of S_1 and leaves $200 - 150 = 50$ units with W_2

Table-2

	W_1	W_2	W_3	W_4	Supply
S_1	5(250)	6(150)	5	7	0
S_2	2	2	3	3	150
S_3	5	4	4	3	100
S_4	13	12	15	12	200
Demand	0	50	150	250	

The rim values for $S_2 = 150$ and $W_2 = 50$ are compared

The smaller of the two i.e $\min(150, 50) = 50$ is assigned to S_2W_2

This meets the complete demand of W_2 and leaves $150 - 50 = 100$ units with S_2

Table-3

	W_1	W_2	W_3	W_4	Supply
S_1	5(250)	6(150)	5	7	0
S_2	2	2(50)	3	3	100
S_3	5	4	4	3	100
S_4	13	12	15	12	200
Demand	0	0	150	250	

The rim values for $S_2 = 100$ and $W_3 = 150$ are compared

The smaller of the two i.e $\min(100, 150) = 100$ is assigned to S_2W_3

This exhausts the capacity of S_2 and leaves $150 - 100 = 50$ units with W_3

Table-4

	W_1	W_2	W_3	W_4	Supply
S_1	5(250)	6(150)	5	7	0
S_2	2	2(50)	3(100)	3	0
S_3	5	4	4	3	100
S_4	13	12	15	12	200
Demand	0	0	50	250	

The rim values for $S_3 = 100$ and $W_3 = 50$ are compared

The smaller of the two i.e $\min(100, 50) = 50$ is assigned to S_3W_3

This meets the complete demand of W_3 and leaves $100 - 50 = 50$ units with S_3

Table-5

	W_1	W_2	W_3	W_4	Supply
S_1	5(250)	6(150)	5	7	0
S_2	2	2(50)	3(100)	3	0
S_3	5	4	4(50)	3	50
S_4	13	12	15	12	200
Demand	0	0	0	250	

The rim values for $S_3 = 50$ and $W_4 = 250$ are compared

The smaller of the two i.e $\min(50, 250) = 50$ is assigned to S_3W_4

This exhausts the capacity of S_3 and leaves $250 - 50 = 200$ units with W_4

Table-6

	W_1	W_2	W_3	W_4	Supply
S_1	5(250)	6(150)	5	7	0
S_2	2	2(50)	3(100)	3	0
S_3	5	4	4(50)	3(50)	0
S_4	13	12	15	12	200
Demand	0	0	0	200	

The rim values for $S_4 = 200$ and $W_4 = 200$ are compared

The smaller of the two i.e $\min(200, 200) = 200$ is assigned to S_4W_4

Table-7

	W_1	W_2	W_3	W_4	Supply
~~S_1~~	5(250)	6(150)	5	7	0
~~S_2~~	2	2(50)	3(100)	3	0
~~S_3~~	5	4	4(50)	3(50)	0
~~S_4~~	13	12	15	12(200)	0
Demand	0	0	0	0	

Initial feasible solution is

	W_1	W_2	W_3	W_4	Supply
S_1	5(250)	6(150)	5	7	400
S_2	2	2(50)	3(100)	3	150
S_3	5	4	4(50)	3(50)	100
S_4	13	12	15	12(200)	200
Demand	250	200	150	250	

The minimum total transportation cost

$$5 \times 250 + 6 \times 150 + 2 \times 50 + 3 \times 100 + 4 \times 50 + 3 \times 50 + 12 \times 200 = 5300$$

Here, the number of allocated cells = 7 is equal to $m + n - 1 = 4 + 4 - 1 = 7$

$\therefore$ the solution is non-degenerate.

4.3 METHOD 2: Using Least Cost

TOTAL number of supply constraints: 4

TOTAL number of demand constraints: 4

Problem Table is

	W_1	W_2	W_3	W_4	Supply
S_1	5	6	5	7	400
S_2	2	2	3	3	150
S_3	5	4	4	3	100
S_4	13	12	15	12	200
Demand	250	200	150	250	

The smallest transportation cost is 2 in cell S_2W_1

The allocation to this cell is $\min(150, 250) = 150$

This exhausts the capacity of S_2 and leaves $250 - 150 = 100$ units with W_1

Table-1

	W_1	W_2	W_3	W_4	Supply
S_1	5	6	5	7	400
~~S_2~~	~~2(150)~~	~~2~~	~~3~~	~~3~~	0
S_3	5	4	4	3	100
S_4	13	12	15	12	200
Demand	100	200	150	250	

The smallest transportation cost is 3 in cell S_3W_4

The allocation to this cell is $\min(100, 250) = 100$

This exhausts the capacity of S_3 and leaves $250 - 100 = 150$ units with W_4

Table-2

	W_1	W_2	W_3	W_4	Supply
S_1	5	6	5	7	400
~~S_2~~	~~2(150)~~	~~2~~	~~3~~	~~3~~	0
~~S_3~~	~~5~~	~~4~~	~~4~~	~~3(100)~~	0
S_4	13	12	15	12	200
Demand	100	200	150	150	

The smallest transportation cost is 5 in cell S_1W_3

The allocation to this cell is $\min(400, 150) = 150$

This satisfies the entire demand of W_3 and leaves $400 - 150 = 250$ units with S_1

Table-3

	W_1	W_2	W_3	W_4	Supply
S_1	5	6	5(150)	7	250
S_2	2(150)	2	3	3	0
S_3	5	4	4	3(100)	0
S_4	13	12	15	12	200
Demand	100	200	0	150	

The smallest transportation cost is 5 in cell S_1W_1

The allocation to this cell is $\min(250, 100) = 100$

This satisfies the entire demand of W_1 and leaves $250 - 100 = 150$ units with S_1

Table-4

	~~W_1~~	W_2	~~W_3~~	W_4	Supply
S_1	~~5(100)~~	6	~~5(150)~~	7	150
~~S_2~~	~~2(150)~~	~~2~~	~~3~~	~~3~~	0
~~S_3~~	~~5~~	~~4~~	~~4~~	~~3(100)~~	0
S_4	~~13~~	12	~~15~~	12	200
Demand	0	200	0	150	

The smallest transportation cost is 5 in cell S_1W_1

The allocation to this cell is $\min(250, 100) = 100$

This satisfies the entire demand of W_1 and leaves $250 - 100 = 150$ units with S_1

Initial Feasible solution is

	D_1	D_2	D_3	D_4	Supply
S_1	100(100)	50	130	70(100)	0
S_2	90	60	80(100)	100	0
S_3	150(100)	20(150)	300(50)	100	0
S_4	15	12	24	10(30)	0
Demand	0	0	0	0	

The minimum total transportation cost

$100 \times 100 + 70 \times 100 + 80 \times 100 + 150 \times 100 + 20 \times 150 + 300 \times 50 + 10 \times 30 = 58300$

Here, the number of allocated cells = 7 which is one less than to

$$m + n - 1 = 4 + 4 - 1 = 7$$

$\therefore$ this solution is non-degenerate.

4.4 METHOD 3: Finding Solution using Voggel's Approximation(VAM)

TOTAL number of supply constraints: 4

TOTAL number of demand constraints: 4

Problem Table is

	D_1	D_2	D_3	D_4	Supply
S_1	100	50	130	70	200
S_2	90	60	80	100	100
S_3	150	20	300	100	300
S_4	15	12	24	10	30
Demand	200	150	150	130	

Table-1

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100	50	130	70	200	20=70-50
S_2	90	60	80	100	100	20=80-60
S_3	150	20	300	100	300	80=100-20
S_4	15	12	24	10	30	2=12-10
Demand	200	150	150	130		
Column Penalty	75=90-15	8=20-12	56=80-24	60=70-10		

The maximum penalty, 80 occur in column S_3

The minimum C_{ij} in this column is $C_{32} = 20$

The maximum allocation in this cell is $\min(300, 150) = 150$

It satisfy demand of D_2 and adjust the supply of S_3 from 300 to 150 $(300 - 150 = 150)$.

Table-2

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100	50	130	70	200	30=100-70
S_2	90	60	80	100	100	10=90-80
S_3	150	20(150)	300	100	150	50=150-100
S_4	15	12	24	10	30	5=15-10

Demand	200	0	150	130		
Column Penalty	75=90-15	-	56=80-24	60=70-10		

The maximum penalty, 75 occur in column D_1

The minimum C_{ij} in this column is $C_{41} = 15$

The maximum allocation in this cell is $\min(30, 200) = 30$

It satisfy supply of S_4 and adjust the demand of D_1 from 200 to 170 $(200 - 30 = 170)$.

Table-3

	D_1	~~D_2~~	D_3	D_4	Capacity	Row Penalty
S_1	100	~~50~~	130	70	200	30=100-70
S_2	90	~~60~~	80	100	100	10=90-80
S_3	150	~~20(150)~~	300	100	150	50=150-100
~~S_4~~	~~15(30)~~	~~12~~	~~24~~	~~10~~	0	-
Demand	170	0	150	130		
Column Penalty	10=100-90	-	50=130-80	30=100-70		

The maximum penalty, 50 occur in column D_3

The minimum C_{ij} in this column is $C_{23} = 80$

The maximum allocation in this cell is $\min(100, 150) = 100$

It satisfy supply of S_2 and adjust the demand of D_3 from 150 to 50 $(150 - 100 = 50)$.

Table-4

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100	50	130	70	200	30=100-70
~~S_2~~	~~90~~	~~60~~	~~80(100)~~	~~100~~	0	-
S_3	150	20(150)	300	100	150	50=150-100
~~S_4~~	~~15(30)~~	~~12~~	~~24~~	~~10~~	0	-
Demand	170	0	50	130		
Column Penalty	50=150-100	-	170=300-130	30=100-70		

The maximum penalty, 170 occur in column D_3

The minimum C_{ij} in this column is $C_{13} = 130$

The maximum allocation in this cell is $\min(200, 50) = 50$

It satisfy demand of D_3 and adjust the supply of S_1 from 200 to 150 $(200 - 50 = 150)$.

Table-5

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100	50	130(50)	70	150	30=100-70
~~S_2~~	~~90~~	~~60~~	~~80(100)~~	~~100~~	0	-
S_3	150	20(150)	300	100	150	50=150-100
~~S_4~~	~~15(30)~~	~~12~~	~~24~~	~~10~~	0	-
Demand	170	0	0	130		
Column Penalty	50=150-100	-	-	30=100-70		

The maximum penalty, 50 occur in column D_1

The minimum C_{ij} in this column is $C_{11} = 100$

The maximum allocation in this cell is $\min(150, 170) = 150$

It satisfy supply of S_1 and adjust the demand of D_1 from 170 to 20 $(170 - 150 = 20)$.

Table-6

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100(150)	50	130(50)	70	0	-
S_2	90	60	80(100)	100	0	-
S_3	150	20(150)	300	100	150	50=150-100
S_4	15(30)	12	24	10	0	-
Demand	20	0	0	130		
Column Penalty	150	-	-	100		

The maximum penalty, 50 occur in column D_1

The minimum C_{ij} in this column is $C_{31} = 150$

The maximum allocation in this cell is $\min(150, 20) = 20$

It satisfy demand of D_1 and adjust the supply of S_3 from 150 to 130 $(150 - 20 = 130)$.

Table-7

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100(150)	50	130(50)	70	0	-
S_2	90	60	80(100)	100	0	-
S_3	150(20)	20(150)	300	100	130	100
S_4	15(30)	12	24	10	0	-
Demand	0	0	0	130		
Column Penalty	-	-	-	100		

The maximum penalty, 100 occur in column S_3

The minimum C_{ij} in this column is $C_{34} = 100$

The maximum allocation in this cell is $\min(130, 130) = 130$

It satisfy supply of S_3 and demand of D_4.

Initial feasible solution is

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100(150)	50	130(50)	70	200	20|30|30|30|30|-|-
S_2	90	60	80(100)	100	100	20|10|10|-|-|-|-
S_3	150(20)	20(150)	300	100(130)	300	80|50|50|50|50|50|100
S_4	15(30)	12	24	10	30	2|5|-|-|-|-|-
Demand	200	150	150	130		
	75	8	56	60		
	75	-	56	60		
	10	-	50	30		
	50	-	170	30		
	50	-	-	30		
	150	-	-	100		
	-	-	-	100		

The minimum total transportation cost

$100 \times 150 + 130 \times 50 + 80 \times 100 + 150 \times 20 + 20 \times 150 + 100 \times 130 + 150 \times 30 = 61450$

Here the number of allocated cells = 7 is equal to

$$m + n - 1 = 4 + 4 - 1 = 7$$

$\therefore$ this solution is non-degenerate.

Chapter 5

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 SUMMARY

The transportation cost is an important element of total cost structure for any business.

Through the use of this mathematical model (Transportation Model), the business (Unilorin Water Enterprise) can identify easily and efficiently plan out its transportation. So that it can not only minimize the cost of transporting goods and services but also create time utility by reading the goods and services at the right place and right time.

5.2 CONCLUSION

Northwest Corner Cell Method, Least Cost Method and Vogel's Approximation methods are used in finding the initial basic feasible solution of a transportation problem. Vogel's approximation method is an improvement on Least Cost Cell method that generates a better solution. Northwest Corner Cell method is the simplest but most inefficient as it has the largest total cost of transportation compare to the other method because it does not take into account the cost of transportation for all possible alternative routes. Least Cost method focuses on allocation for the cheaper route, it is a better method compare to Northwest Corner Cell method because costs are considered for allocation.

5.3 RECOMMENDATION

Based on the result of the study, I recommend to the management of Unilorin Water Enterprise to seek the application of mathematical model or theories into their operation as a necessary tool when it comes to decision making, not only in the area of logistics (the transportation problem) but in the production as well as administration.

This study employed mathematical technique to solve management problems and make timely optimal decision. If the University of Ilorin Water Enterprise are to employ the proposed transportation model, it will assist them to efficiently plan out its transportation schedule at a minimum cost. This proposed method is applicable to any transportation problem.

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