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CERTIFICATION

This is to certify that this project was carried out by **XXXXXXXXXXXXXXXXXXXXXXX** with Matriculation Number 17/56EB0XX in the Department of Mathematics, Faculty of Physical Sciences, University of Ilorin, Ilorin, Nigeria, for the award of Bachelor of Science (B.Sc.) degree in Mathematics.

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ACKNOWLEDGMENTS

All praises

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DEDICATION

I would like to dedicate the project to God

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Example 1

$y(x) = x + \int_0^x y^4(t)dt$

Solution

$$\sum_{n=0}^{\infty} y_n(x) = x + \int_0^x \sum_{n=0}^{\infty} A_n(t)dt$$

The Adomian polynomial for y^4

$A_0 = y_0^4, \quad A_1 = 4y_1y_0^3, \quad A_2 = 4y_2y_0^3 + 6y_1^2y_0^2, \quad A_3 = 4y_3y_0^3 + 12y_2y_1y_0^2 + 4y_1^3y_0$

where

$$y_0(x) = x, \quad y_{k+1}(x) = \int_0^x A_k(t)dt, k \geq 0$$

$$
\begin{aligned}
y_1(x) &= \int_0^x y_0^4(t)dt = \int_0^x t^4 dt = \left.\frac{t^5}{5}\right|_0^x = \frac{1}{5}x^5 \\
y_2(x) &= \int_0^x (4y_1y_0^3)dt = \int_0^x \left(\frac{4}{5}t^5 \cdot t^3\right) dt \\
&= \frac{4}{5}\int_0^x t^8 dt = \left.\frac{4}{5}\frac{t^9}{9}\right|_0^x = \frac{4}{45}x^9 \\
y_3(x) &= \int_0^x \left(4y_2y_0^3 + 6y_1^2y_0^2\right) dt \\
&= \int_0^x 4\left(\frac{4}{45}t^9\right)\left(t^3\right) + 6\left(\frac{1}{5}t^5\right)^2 \left(t^2\right) dt \\
&= \int_0^x \left(\frac{16}{45}t^{12} + \frac{6}{5}t^{12}\right) dt \\
&= \frac{14}{9}\int_0^x t^{12} dt = \left.\frac{14}{9}\frac{t^{13}}{13}\right|_0^x = \frac{14}{117}x^{13} \\
\therefore y(x) &= x + \frac{1}{5}x^5 + \frac{4}{45}x^9 + \frac{14}{117}x^{13} + \cdots
\end{aligned}
$$

Example 2

$y(x) = x + \int_0^x (t-x)y^2(t)dt$

Solution

$$\sum_{n=0}^{\infty} y_n(x) = x + \int_0^x (t-x)\sum_{n=0}^{\infty} A_n(t)dt$$

the Adomian polynomial of y^2

$$A_0 = y_0^2, \quad A_1 = 2y_0y_1, \quad A_2 = 2y_2y_0 + y_1^2, \quad A_3 = 2y_0y_3 + 2y_1y_2,$$

$$y_0(x) = x, \quad y_{k+1}(x) = \int_0^x (t-x)A_k(t)dt$$

$$
\begin{aligned}
y_1(x) &= \int_0^x (t-x)t^2 dt = \int_0^x (t^3 - xt^2) dt = \left.\frac{t^4}{4} - \frac{xt^3}{3}\right|_0^x \\
&= \frac{x^4}{4} - \frac{x^4}{3} = -\frac{1}{12}x^4 \\
y_2(x) &= \int_0^\infty (t-x)(2t)\left(-\frac{1}{12}t^4\right) dt = \int_0^x (t-x)\left(-\frac{1}{6}t^5\right) dt \\
&= -\frac{1}{6}\int_0^x (t^6 - xt^5) dt = -\frac{1}{6}\left[\frac{t^7}{7} - \frac{xt^6}{6}\right]_0^x \\
&= -\frac{1}{6}\left[\frac{x^7}{7} - \frac{x^7}{6}\right] = -\frac{1}{6}\left[-\frac{1}{42}x^7\right] = \frac{1}{180}x^7 \\
y_3(x) &= \int_0^x (t-x)\left[\left(\frac{2}{180}t^7\right)(t) + \left(-\frac{1}{12}t^4\right)^2\right] dt \\
&= \int_0^x (t-x)\left(\frac{1}{90}t^8 + \frac{1}{12}t^8\right) dt = \int_0^x (t-x)\left(\frac{17}{180}t^8\right) dt \\
&= \frac{17}{180}\int_0^x (t^9 - xt^8) dt = \frac{17}{180}\left[\frac{t^10}{10} - \frac{xt^9}{9}\right]_0^x \\
&= \frac{17}{180}\left[\frac{x^{10}}{10} - \frac{x^{10}}{9}\right] = \frac{17}{180}\left[-\frac{1}{90}x^{10}\right] = -\frac{17}{16200}x^{10} \\
\therefore y(x) &= x - \frac{1}{12}x^4 + \frac{1}{180}x^7 - \frac{17}{16200}x^{10}
\end{aligned}
$$

Example 3

$y(x) = x + \int_0^x (x^2 t - xt^2) y^2(t) dt$

Solution

$$
\sum_{n=0}^{\infty} y_n(x) = x + \int_0^x (x^2 t - xt^2) \sum_{n=0}^{\infty} A_n(t) dt
$$

$$y_0(x) = x, \quad y_{k+1}(x) = \int_0^{\infty} (x^2 t - x t^2) A_k(t) dt$$

$$\begin{aligned}
y_1(x) &= \int_0^x (x^2 t - x t^2)(t^2) dt = \int_0^x (x^2 t^3 - x t^4) dt \\
&= \left. \frac{x^2 t^4}{4} - \frac{x t^5}{5} \right|_0^x = \frac{x^6}{4} - \frac{x^6}{5} = \frac{1}{20} x^6 \\
y_2(x) &= \int_0^x (x^2 t - x t^2)(2 y_0 y_1) dt \\
&= \int_0^x (x^2 t - x t^2) \left(\frac{2}{10} t^7 \right) dt = \frac{1}{10} \int_0^x (x^2 t^8 - x t^9) dt \\
&= \frac{1}{10} \left[\frac{x^2 t^9}{9} - \frac{x t^{10}}{10} \right]_0^x = \frac{1}{10} \left[\frac{x^{11}}{9} - \frac{x^{11}}{10} \right] = \frac{1}{900} x^{11} \\
y_3(x) &= \int_0^x (x^2 t - x t^2)(2 y_2 y_0 + y_1^2) dt \\
&= \int_0^x (x^2 t - x t^2) \left[\left(\frac{2}{900} t^{11} \right) (t) + \left(\frac{1}{20} t^6 \right)^2 \right] dt \\
&= \int_0^x \left(x^2 t - x t^2 \right) \left(\frac{1}{450} t^{12} + \frac{1}{400} t^{12} \right) dt \\
&= \int_0^x (x^2 t - x t^2) \left(\frac{17}{3600} t^{12} \right) dt \\
&= \frac{17}{3600} \int_0^x (x^2 t^{13} - x t^{14}) dt \\
&= \frac{17}{3600} \left[\frac{x^2 t^{14}}{14} - \frac{x t^{15}}{15} \right]_0^x = \frac{17}{3600} \left[\frac{x^{16}}{14} - \frac{x^{16}}{15} \right] \\
&= \frac{17}{3600} \left[\frac{1}{210} x^{16} \right] = \frac{17}{756000} x^{16} \\
y(x) &= x + \frac{1}{20} x^6 + \frac{1}{900} x^{11} + \frac{17}{756000} x^{16} + \cdots
\end{aligned}$$

4.2 Non-linear Fredholm Integral Equation using the Adomian Decomposition Method

Example 1

$y(x) = 3 + \lambda \int_0^1 ty^2(t)dt$

Solution

$$y_0(x) = 3, \quad y_{k+1}(x) = \lambda \int_0^1 y^2(t)dt$$

$$\begin{aligned}
y_1(x) &= \lambda \int_0^1 ty_0^2(t)dt \\
&= \lambda \int_0^1 9tdt = \left.\frac{9\lambda t^2}{2}\right|_0^1 = \frac{9\lambda}{2} \\
y_2(x) &= \lambda \int_0^1 (2y_0y_1)tdt \\
&= \lambda \int_0^1 (6)\left(\frac{9\lambda}{2}\right) tdt = \left.\frac{27\lambda^2 t^2}{2}\right|_0^1 = \frac{27\lambda^2}{2} \\
y_3(x) &= \lambda \int_0^1 \left(81\lambda^2 + \frac{81\lambda^2}{2}\right) tdt = \lambda \int_0^1 \left(\frac{243\lambda^2}{2}\right) tdt \\
&= \left.\frac{243\lambda^3 t^2}{4}\right|_0^1 = \frac{243\lambda^3}{4} \\
y_4(x) &= \lambda \int_0^1 (2y_0y_3 + 2y_1y_2)tdt \\
&= \lambda \int_0^1 \left(\frac{729}{2}\lambda^3 + \frac{243}{2}\lambda^3\right) tdt
\end{aligned}$$

$$
\begin{aligned}
&= \lambda \int_0^1 (486\lambda^3) t dt = \left.\frac{486\lambda^4}{2}\right|_0^1 \\
&= 243\lambda^4 \\
y(x) &= y_0(x) + y_1(x) + y_2(x) + y_3(x) + y_4(x) + \cdots \\
y(x) &= 3 + \frac{9\lambda}{2} + \frac{27\lambda^2}{2} + \frac{243\lambda^3}{4} + 243\lambda^4 + \cdots
\end{aligned}
$$

Example 2

$y(x) = 4 + \lambda \int_0^1 t^2 y^2(t) dt$

Solution

The Adomian polynomial for $y^2(x)$

$$A_0(x) = y_0^2, \quad A_1(x) = 2y_0 y_1,$$

$$A_2(x) = 2y_0 y_2 + y_1^2, \quad A_3 = 2y_0 y_3 + 2y_1 y_2$$

$$
\begin{aligned}
y_0(x) &= 4 \\
y_1(x) &= \lambda \int_0^1 y^2(t) t^2 dt = \lambda \int_0^1 16t^2 dt = \left.\frac{16}{3} t^3 \lambda\right|_0^1 = \frac{16}{3}\lambda \\
y_2(x) &= \lambda \int_0^1 (2y_0 y_1) t^2 dt = \lambda \int_0^1 \frac{128}{3}\lambda t^2 dt = \left.\frac{\lambda^2 128}{9} t^3\right|_0^1 = \frac{128}{9}\lambda^2 \\
y_3(x) &= \lambda \int_0^1 (2y_0 y_2 + y_1^2) t^2 dt = \lambda \int_0^1 \left(\frac{1024\lambda^2}{9} + \frac{256\lambda^2}{9}\right) t^2 dt \\
&= \lambda \int_0^1 \frac{1280\lambda^2}{9} t^2 dt = \left.\frac{1280}{27}\lambda^3 t^3\right|_0^1 = \frac{1280}{27}\lambda^3
\end{aligned}
$$

$$
\begin{aligned}
y_4(x) &= \lambda \int_0^1 (2y_0y_3 + 2y_1y_2)t^2 dt = \lambda \int_0^1 \left(\frac{5120}{27}\lambda^3 + \frac{4096}{27}\lambda^3 \right) t^2 dt \\
&= \lambda \int_0^1 \left(\frac{9216}{27}\lambda^3 \right) t^3 dt = \frac{9216}{81}\lambda^4 t^4 \Big|_0^1 = \frac{9216}{81}\lambda^4 \\
y(x) &= 4 + \frac{16}{3}\lambda + \frac{128}{9}\lambda^2 + \frac{1280}{27}\lambda^3 + \frac{9216}{81}\lambda^4 + \cdots
\end{aligned}
$$

Example 3

$$y(x) = 1 + \lambda \int_0^1 y^4(t) dt$$

Solution

The Adomian polynomials of $y^4(t)$ are

$$A_0(x) = y_0^4, \quad A_1(x) = 4y_1y_0^3, \quad A_2(x) = 4y_2y_0^3 + 6y_1^2y_0^2$$

$$A_3(x) = 4y_3y_0^3 + 12y_2y_1y_0^2 + 4y_1^3y_0$$

$$
\begin{aligned}
y_0(x) &= 1 \\
y_1(x) &= \lambda \int_0^1 y_0^4(t) dt = \lambda \int_0^1 (1^4) dt = \lambda t \Big|_0^1 = \lambda \\
y_2(x) &= \lambda \int_0^1 4y_1y_0^3(t) dt = \lambda \int_0^1 4\lambda dt = 4\lambda^2 t \Big|_0^1 = 4\lambda^2 \\
y_3(x) &= \lambda \int_0^1 4y_2y_0^3 + 6y_1^2y_0^2(t) dt = \lambda \int_0^1 (16\lambda^2 + 6\lambda^2) dt \\
&= \lambda \int_0^1 22\lambda^2 dt = 22\lambda^3 \Big|_0^1 = 22\lambda^3 \\
y_4(x) &= \lambda \int_0^1 (4y_3y_0^3 + 12y_2y_1y_0^2 + 4y_1^3y_0) dt
\end{aligned}
$$

$$
\begin{aligned}
&= \lambda \int_0^1 (88\lambda^3 + 48\lambda^3 + 4\lambda^3)dt = \lambda \int_0^1 140\lambda^3 dt \\
&= 140\lambda^4 t \Big|_0^1 = 140\lambda^4 \\
y(x) &= 1 + \lambda + 4\lambda^2 + 22\lambda^3 + 140\lambda^4 + \cdots
\end{aligned}
$$

Chapter 5

SUMMARY AND CONCLUSION

5.1 Summary

5.2 Conclusion

REFERENCES