Step 1:

$\frac{ds}{dt} = 32t - 2$

$ds = (32t - 2)dt$

$\int ds = \int (32t - 2)dt$

$S = \frac{32t^2}{2} - 2t + c$

Step 2: When $t = \frac{1}{2}, S = 4$

$4 = 16(\frac{1}{2})^2 - 2(\frac{1}{2}) + C$

$\Rightarrow 4 = 4 - 1 + C \Rightarrow C = 1$

SOLUTION: $S = 16t^2 - 2t + 1$

Example VIX: Given the aceleration $a = \frac{d^2s}{dt^2} = -4\sin 2t$. Initial Velocity $V(0) = 2$, and the Initial position of the body as $S(0) - 3$. Find the body's position at time t

<u>Solution</u>

Step 1:

$V = \int -4\sin 2t dt \Rightarrow V = \frac{4\cos 2t}{2} + C$

Step 2: When $t = 0,\ V = 2$

$\Rightarrow 2 = 2\cos(0) + C \Rightarrow 2 = 2 + C\ \Rightarrow C = 0$

$\therefore V = 2\cos 2t$

Step 1:

$S = \int 2\cos 2t dt \Rightarrow S = \frac{2\sin 2t}{2} + C$

Step 2: When $t = 0,\ S = -3$

$-3 = \sin(0) + C \Rightarrow C = -3$

SOLUTION: $S = \sin 2t - 3$

REDUCING N-ORDER ODEs INTO SYSTEM OF FIRST-ORDER ORDINARY DIFFERENTIAL

In this section we show how to reduce higher-order Ordinary Differential Equationinto systems of first order Ordinary Differential Equation(O.D.Es).

Example I: Reduce the differential equation into its equivalent system of first-order O.D.Es
$y''' + 6y'' + 11y' + 6y = 0 - - - - - - - - - - (1.0)$
$y(0) = 1 - - - - - - - - - - - - - - - - - - - (1.1)$
$y'(0) = 0 - - - - - - - - - - - - - - - - - - (1.2)$
$y''(0) = 0 - - - - - - - - - - - - - - - - - - (1.3)$

Let $y = y_1$
(1.0) becomes, $y_1''' + 6y_1'' + 11y_1' + 6y_1 = 0$

$y_1' = y_2 \Rightarrow y_1'' = y_2,\ y_1(0) = 1$

$y_1'' = y_2' = y_3 \Rightarrow y_2' = y_3,\ y_2(0) = 0$

$y_1''' = y_2'' = y_3'$

$\Rightarrow y_3' + 6y_3 + 11y_2 + 6y_1 = 0$

$\Rightarrow y_3' = -6y_3 - 11y_2 - 6y_1, y_3(0) = 0$

$\therefore$ The third-Order Ordinary Differential Equationin its equivalent system of first Order ODEs is:

$y_1' = y_2, y_1(0) = 1$
$y_2' = y_3, y_2(0) = 0$
$y_3' = -6y_1 - 11y_2 - 6y_3, y_3(0) = 0$

which matrice form gives,

$$y' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Example II: Reduce the differential equation into its equivalent System of First-Order O.D.Es

$y'' + 2y' + 6y = 0 - - - - - - - - - (1.0)$

$y(0) = 0,\ y'(0) = 1$

Let $y = y_1$
(1.0) becomes

$y_1'' + 2y_1' + 6y_1 = 0$
$y_1' = y_2 \Rightarrow y_1' = y_2,\ y_1(0) = 0$
$y_1'' = y_2'$
$\Rightarrow y_2' + 2y_2 + 6y_1 = 0 \Rightarrow y_2' = -6y_1 - 2y_2, y_2(0) = 1$