Chapter 1

Problem 1

Evaluate Prey-Pradator Model $\frac{dx}{dt} = \alpha x + \beta xy + c$ using the quadrature rule with $\alpha = 0.01,\ \beta = 0.5,\ c = 0.4$ and $x(0) = 1$, taking $n = 2$ and $n = 4$

Solution

Given $\frac{dx}{dt} = \alpha x - \beta xy + c$

This can be re-written as votterral equation of the second order

$\phi(x) = \mathrm{f}(x) + \displaystyle\int_{x_0}^{x} k(x,y)\phi(y)\ \mathrm{dy}$

$\phi(x) = 0.01x + 0.4 + \displaystyle\int_0^1 (0.5)(xy)\phi(y)\ \mathrm{dy}$

$\phi(x) - \dfrac{1}{2}\displaystyle\int_0^1 (xy)\phi(y)\ \mathrm{dy} = \dfrac{1}{100}x + \dfrac{2}{5}$

When $n = 2$

$h = \dfrac{b-a}{n} = \dfrac{1-0}{2} = \dfrac{1}{2}$

$\Rightarrow y_0 = 0,\ y_1 = \dfrac{1}{2},\ y_2 = 1$

By Trapezoidal Rule

$$\phi(x) - \frac{1}{2} \cdot \frac{h}{2} \left[(xy_0)\phi(y_0) + 2(xy_1)\phi(y_1) + (xy_2)\phi(y_2) \right] = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - \frac{1}{8} \left[(xy_0)\phi(y_0) + 2(xy_1)\phi(y_1) + (xy_2)\phi(y_2) \right] = \frac{1}{100}x + \frac{2}{5} \tag{1}$$

Changing x terms to y_i

$$\phi(y_i) - \frac{1}{8} \left[(y_i y 0)\phi y_0 + 2y_i y_1 \phi y_1 + y_i y_2 \phi y_2 \right] = \frac{1}{100}y_i + \frac{2}{5}$$

When $i = 0$

$$\phi_0 - \frac{1}{8} \left[y_0^2 \phi_0 + 2y_0 y_1 \phi_1 + y_0 y_2 \phi_2 \right] = \frac{1}{100}y_0 + \frac{2}{5}$$

But $y_0 = 0,\ y_1 = \frac{1}{2},\ y_2 = 1,\ \phi_0 = \frac{2}{5}$

$$\Rightarrow \quad \phi_0 = 0.4 \tag{i}$$

When $i = 1$

$$\phi_1 - \frac{1}{8} \left[y_0 y_1 \phi_0 + 2y_1^2 \phi_1 + y_1 y_2 \phi_2 \right] = \frac{1}{100}y_1 + \frac{2}{5}$$

$$\phi_1 - \frac{1}{8} \left[0 + 2\left(\frac{1}{2}\right)^2 \phi_1 + \frac{1}{2}\phi_2 \right] = \frac{1}{100}\left(\frac{1}{2}\right) + \frac{2}{5}$$

$$\phi_1 - \frac{1}{8} \left[\frac{1}{2}\phi_1 + \frac{1}{2}\phi_2 \right] = \frac{81}{200}$$

$$\frac{15}{16}\phi_1 + \frac{1}{16}\phi_2 = \frac{81}{200} \tag{ii}$$

When $i = 2$

$$\phi_2 - \frac{1}{8} \left[y_0 y_2 \phi_0 + 2y_1 y_2 \phi_1 + y_2^2 \phi_2 \right] = \frac{1}{100}y_2 + \frac{2}{5}$$

$$\phi_2 - \frac{1}{8} \left[0 + 2\left(\frac{1}{2}\right) \phi_1 + \phi_2 \right] = \frac{1}{100} + \frac{2}{5}$$

$$\phi_2 - \frac{1}{8}\phi_1 + \frac{1}{8}\phi_2 = \frac{41}{100}$$

$$-\frac{1}{8}\phi_1 + \frac{7}{8}\phi_2 = \frac{41}{100} \tag{iii}$$

Bringing together the system of equation

$$\phi_0 = \frac{2}{5} \tag{i}$$

$$\frac{15}{16}\phi_1 - \frac{1}{16}\phi_2 = \frac{81}{200} \tag{ii}$$

$$-\frac{1}{8}\phi_1 - \frac{7}{8}\phi_2 = \frac{41}{100} \tag{iii}$$

Solving the system of equations we obtain

$$\phi_0 = \frac{2}{5}, \quad \phi_1 = \frac{152}{325}, \quad \phi_2 = \frac{174}{325}$$

From equation **(1)** we obtain

$$\phi(x) - \frac{1}{8}\left[0 + 2x\left(\frac{1}{2}\right)\left(\frac{152}{325}\right) + \frac{174}{325}x\right] = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - \frac{19}{325}x - \frac{87}{1300}x = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x)) = \frac{19}{325}x + \frac{87}{1300}x + \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) = \frac{44}{325}x + \frac{2}{5}$$

$$\phi(\mathbf{x}) = \mathbf{0.13538x + 0.4}$$

SIMPSON'S 1/3 RULE

$$\phi(x) - \frac{1}{2}\frac{h}{3}\left[(xy_0)\phi y_0 + 4(y_i y_1)\phi y_1 + (xy_2)\phi y_2\right] = \frac{1}{100}x + \frac{2}{5} \tag{2}$$

Changing x terms to y_i

$$\phi(y_i)\frac{1}{12}\left[(y_i y_0)\phi y_0 + 4(y_i y_1)\phi y_1 + (y_i y_2)\phi y_2\right] = \frac{1}{100}x + \frac{2}{5}$$

When $i = 0,\ y_0 = 0,\ y_1 = \frac{1}{2},\ y_2 = 1$

$$\phi_0 - \frac{1}{12}\left[y_0^2\phi_0 + 4y_0y_1\phi_1 + y_0y_2\phi_2\right] = \frac{1}{100}y_0 + \frac{2}{5}$$

$$\phi_0 - \frac{1}{12}[0] = \frac{2}{5}$$

$$\phi_0 = \frac{2}{5} \tag{i}$$

When $i = 1$

$$\phi_1 - \frac{1}{12}\left[y_0y_1\phi_0 + 4y_1^2\phi_1 + y_1y_2\phi_2\right] = \frac{1}{100}y_1 + \frac{2}{5}$$

$$\phi_1 - \frac{1}{12}\left[0 + 4\left(\frac{1}{2}\right)^2\phi_1 + \left(\frac{1}{2}\right)\phi_2\right] = \frac{1}{100}\left(\frac{1}{2}\right) + \frac{2}{5}$$

$$\phi_1 - \frac{1}{12}\phi_1 - \frac{1}{24}\phi_2 = \frac{81}{200}$$

$$\frac{11}{12}\phi_1 - \frac{1}{24}\phi_2 = \frac{81}{200} \tag{ii}$$

When $i = 2$

$$\phi_2 - \frac{1}{12}\left[y_0y_2\phi_0 + 4y_1y_2\phi_1 + y_2^2\phi_2\right] = \frac{1}{100}y_2 + \frac{2}{5}$$

$$\phi_2 - \frac{1}{12}\left[0 + 4\left(\frac{1}{2}\right)\phi_1 + \phi_2\right] = \frac{1}{100} + \frac{2}{5}$$

$$\phi_2 - \frac{1}{6}\phi_1 - \frac{1}{12}\phi_2 = \frac{41}{100}$$

$$-\frac{1}{6}\phi_1 + \frac{11}{12}\phi_2 = \frac{41}{100} \tag{iii}$$

The System of equations are

$$\phi_0 = \frac{2}{5} \tag{i}$$

$$\frac{11}{12}\phi_1 - \frac{1}{24}\phi_2 = \frac{81}{200} \quad \text{(ii)}$$

$$-\frac{1}{6}\phi_1 + \frac{11}{12}\phi_2 = \frac{41}{100} \quad \text{(iii)}$$

Solving the System of equation we obtain

$$\phi_0 = \frac{2}{5}, \quad \phi_1 = \frac{233}{500}, \quad \phi_2 = \frac{133}{250}$$

Then, equation **(2)** becomes

$$\phi(x) - \frac{1}{12}\left[0 + 4x\left(\frac{1}{2}\right)\left(\frac{233}{500}\right) + x\,(1)\,\frac{133}{250}\right] = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - \frac{233}{3000}x - \frac{133}{250}x = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x)) = \frac{233}{3000}x + \frac{133}{250}x + \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) = \frac{1859}{3000}x + \frac{2}{5}$$

$\boldsymbol{\phi(\mathbf{x}) = 0.61967\mathbf{x} + 0.4}$

SIMPSON'S 3/8 RULE

$$\phi(x) - \frac{1}{2}\cdot\frac{3h}{8}\left[(xy_0)\phi y_0 + 3(xy_1)\phi y_1 + (xy_2)\phi y_2\right] = \frac{1}{100}x + \frac{2}{5} \quad (3)$$

$$\phi(x) - \frac{3}{32}\left[(xy_0)\phi y_0 + 3(xy_1)\phi y_1 + (xy_2)\phi y_2\right] = \frac{1}{100}x + \frac{2}{5}$$

Changing x terms to y_i

$$\phi(y_i) - \frac{3}{32}\left[y_i y_0 \phi y_0 + 3y_i y_1 \phi y_1 + y_i y_2 \phi y_2\right] = \frac{1}{100}y_i + \frac{2}{5}$$

When $i = 0,\ y_0 = 0,\ y_1 = \dfrac{1}{2},\ y_2 = 2$

$$\phi_0 - \frac{3}{32}\left[y_0^2\phi_0 + 3y_0y_1\phi_1 + y_iy_2\phi y_2\right] = \frac{1}{100}y_0 + \frac{2}{5}$$

$$\phi_0 - \frac{3}{32}(0) = \frac{2}{5}$$

$$\phi_0 = \frac{2}{5} \tag{i}$$

When $i = 1$

$$\phi_1 - \frac{3}{32}\left[y_0y_1\phi_0 + 3y_1^2\phi_1 + y_1y_2\phi_2\right] = \frac{1}{100}y_1 + \frac{2}{5}$$

$$\phi_1 - \frac{3}{32}\left[0 + 3\left(\frac{1}{2}\right)^2\phi_1 + \left(\frac{1}{2}\right)\phi_2\right] = \frac{1}{100}\left(\frac{1}{2}\right) + \frac{2}{5}$$

$$\phi_1 - \frac{3}{32}\left[\frac{3}{4}\phi_1 + \frac{1}{2}\phi_2\right] = \frac{81}{200}$$

$$\phi_1 - \frac{9}{128}\phi_1 - \frac{3}{64}\phi_2 = \frac{81}{200}$$

$$\frac{119}{128}\phi_1 - \frac{3}{64}\phi_2 = \frac{81}{200} \tag{ii}$$

When $i = 2$

$$\phi_2 - \frac{3}{32}\left[y_0y_2\phi_0 + 3y_1y_2\phi_1 + y_2^2\phi_2\right] = \frac{1}{100}y_2 + \frac{2}{5}$$

$$\phi_2 - \frac{3}{32}\left[0 + 3\left(\frac{1}{2}\right)\phi_1 + \phi_2\right] = \frac{1}{100} + \frac{2}{5}$$

$$\phi_2 - \frac{9}{64}\phi_1 - \frac{3}{32}\phi_2 = \frac{41}{100}$$

$$-\frac{9}{64}\phi_1 + \frac{29}{32}\phi_2 = \frac{41}{100} \tag{iii}$$

The System of equations are

$$\phi_0 = \frac{2}{5} \tag{i}$$

$$\frac{119}{128}\phi_1 - \frac{3}{64}\phi_2 = \frac{81}{200} \tag{ii}$$

$$-\frac{9}{64}\phi_1 + \frac{29}{32}\phi_2 = \frac{41}{100} \tag{iii}$$

Solving the System of equation we obtain
$\phi_0 = \frac{2}{5},\ \phi_1 = \frac{1236}{2675},\ \phi_2 = \frac{1402}{2675}$

Equation**(3)** becomes

$$\phi(x) - \frac{3}{32}\left[0 + 3x\left(\frac{1}{2}\right)\left(\frac{1236}{2675}\right) + x(1)\left(\frac{1402}{2675}\right)\right] = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - \frac{3}{32}\left[\frac{1854}{2675}x + \frac{1402}{2675}x\right] = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - \frac{2781}{42800}x - \frac{2103}{42800}x = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) = \frac{2781}{42800}x + \frac{2103}{42800}x + \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) = \frac{332}{2675}x + \frac{2}{5}$$

$$\phi(\mathbf{x}) = \mathbf{0.12411x + 0.4}$$

$$\phi(x) - \frac{1}{2}\int_0^1 (xy)\phi(y)\ \mathrm{dy} = \frac{1}{100}x + \frac{2}{5}$$

When $n = 4$

$$h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$$

$$y_0 = 0,\ y_1 = \frac{1}{4},\ y_2 = \frac{1}{2},\ y_3 = \frac{3}{4},\ y_4 = 1$$

TRAPEZOIDAL RULE

$$\phi(x) - \frac{1}{2} \cdot \frac{h}{2} \left[(xy_0)\phi(y_0) + 2(xy_1)\phi(y_1) + 2(xy_2)\phi(y_2) + 2(xy_3)\phi(y_3) + (xy_4)\phi(y_4) \right]$$
$$= \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - \frac{1}{16} \left[(xy_0)\phi y_0 + 2(xy_1)\phi y_1 + 2(xy_2)\phi y_2 + 2(xy_3)\phi y_3 + (xy_4)\phi y_4 \right]$$
$$= \frac{1}{100}x + \frac{2}{5} \qquad (4)$$

Changing x terms to y_i

$$\phi(x) - \frac{1}{16} \left[(y_i y_0)\phi y_0 + 2(y_i y_1)\phi y_1 + 2(y_i y_2)\phi y_2 + 2(y_i y_3)\phi y_3 + (y_i y_4)\phi y_4 \right]$$
$$= \frac{1}{100}y_i + \frac{2}{5}$$

When $i = 0$

$$\phi_0 - \frac{1}{16} \left[y_0^2\phi_0 + 2y_0y_1\phi_1 + 2y_0y_2\phi_2 + 2y_0y_3\phi_3 + y_0y_4\phi_4 \right] = \frac{1}{100}y_0 + \frac{2}{5}$$

$$\phi_0 - \frac{1}{6}[0] = \frac{2}{5}$$
$$\phi_0 = \frac{2}{5} \qquad \text{(i)}$$

When $i = 1$

$$\phi_1 - \frac{1}{16} \left[y_0y_1\phi_0 + 2y_1^2\phi_1 + 2y_1y_2\phi_2 + 2y_1y_3\phi_3 + y_1y_4\phi_4 \right] = \frac{1}{100}y_1 + \frac{2}{5}$$

$$\phi_1 - \frac{1}{16} \left[0 + 2\left(\frac{1}{4}\right)^2 \phi_1 + 2\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\phi_2 + 2\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\phi_3 + \left(\frac{1}{4}\right)(1)\phi_4 \right]$$
$$= \frac{1}{100}\left(\frac{1}{4}\right) + \frac{2}{5}$$

$$\phi_1 - \frac{1}{16}\left[\frac{1}{8}\phi_1 + \frac{1}{4}\phi_2 + \frac{3}{8}\phi_3 + \frac{1}{4}\phi_4\right] = \frac{1}{400} + \frac{2}{5}$$

$$\phi_1 - \frac{1}{128}\phi_1 - \frac{1}{64}\phi_2 - \frac{3}{128}\phi_3 - \frac{1}{64}\phi_4 = \frac{161}{400}$$

$$\frac{127}{128}\phi_1 - \frac{1}{64}\phi_2 - \frac{3}{128}\phi_3 - \frac{1}{64}\phi_4 = \frac{161}{400} \tag{ii}$$

When $i = 2$

$$\phi_2 - \frac{1}{16}\left[y_0y_2\phi_0 + 2y_1y_2\phi_1 + 2y_2^2\phi_2 + 2y_2y_3\phi_3 + y_2y_4\phi_4\right] = \frac{1}{100}y_2 + \frac{2}{5}$$

$$\phi_2 - \frac{1}{16}\left[0 + 2\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\phi_1 + 2\left(\frac{1}{4}\right)^2\phi_2 + 2\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\phi_3 + \frac{1}{2}\phi_4\right] = \frac{1}{100}\left(\frac{1}{2}\right) + \frac{2}{5}$$

$$\phi_2 - \frac{1}{16}\left[\frac{1}{4}\phi_1 + \frac{1}{2}\phi_2 + \frac{3}{4}\phi_3 + \frac{1}{2}\phi_4\right] = \frac{1}{200} + \frac{2}{5}$$

$$\phi_2 - \frac{1}{64}\phi_1 - \frac{1}{32}\phi_2 - \frac{3}{64}\phi_3 - \frac{1}{32}\phi_4 = \frac{81}{200}$$

$$-\frac{1}{64}\phi_1 - \frac{31}{32}\phi_2 - \frac{3}{64}\phi_3 - \frac{1}{32}\phi_4 = \frac{81}{200} \tag{iii}$$

When $i = 3$

$$\phi_3 - \frac{1}{16}\left[y_0y_3\phi_0 + 2y_3y_1\phi_1 + 2y_2y_3\phi_2 + 2y_3^2\phi_3 + y_3y_4\phi_4\right] = \frac{1}{100}y_3 + \frac{2}{5}$$

$$\phi_3 - \frac{1}{16}\left[0 + 2\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\phi_1 + 2\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\phi_2 + 2\left(\frac{3}{4}\right)^2\phi_3 + \left(\frac{3}{4}\right)(1)\,\phi_4\right]$$
$$= \frac{1}{100}\left(\frac{3}{4}\right) + \frac{2}{5}$$

$$\phi_3 - \frac{3}{128}\phi_1 - \frac{3}{64}\phi_2 - \frac{9}{128}\phi_3 - \frac{3}{64}\phi_4 = \frac{163}{400}$$

$$-\frac{3}{128}\phi_1 - \frac{3}{64}\phi_2 + \frac{119}{128}\phi_3 - \frac{3}{64}\phi_4 = \frac{163}{400} \qquad \text{(iv)}$$

When $i = 4$

$$\phi_4 - \frac{1}{16}\left[y_0y_4\phi_0 + 2y_4y_1\phi_1 + 2y_2y_4\phi_2 + 2y_3y_4\phi_3 + y_4^2\phi_4\right] = \frac{1}{100}y_4 + \frac{2}{5}$$

$$\phi_4 - \frac{1}{16}\left[0 + 2\left(\frac{1}{4}\right)(1)\,\phi_1 + 2\left(\frac{1}{2}\right)(1)\,\phi_2 + 2\left(\frac{3}{4}\right)(1)\phi_3 + (1)^2\,\phi_4\right]$$
$$= \frac{1}{100}(1) + \frac{2}{5}$$

$$\phi_4 - \frac{1}{32}\phi_1 - \frac{1}{16}\phi_2 - \frac{3}{32}\phi_3 - \frac{1}{16}\phi_4 = \frac{41}{100}$$

$$-\frac{1}{32}\phi_1 - \frac{1}{16}\phi_2 - \frac{3}{32}\phi_3 + \frac{15}{16}\phi_4 = \frac{41}{100} \qquad \text{(v)}$$

The System of equations are

$$\phi_0 = \frac{2}{5} \qquad \text{(i)}$$

$$\frac{127}{128}\phi_1 - \frac{1}{64}\phi_2 - \frac{3}{128}\phi_3 - \frac{1}{64}\phi_4 = \frac{161}{400} \qquad \text{(ii)}$$

$$-\frac{1}{64}\phi_1 - \frac{31}{32}\phi_2 - \frac{3}{64}\phi_3 - \frac{1}{32}\phi_4 = \frac{81}{200} \qquad \text{(iii)}$$

$$-\frac{3}{128}\phi_1 - \frac{3}{64}\phi_2 + \frac{119}{128}\phi_3 - \frac{3}{64}\phi_4 = \frac{163}{400} \qquad \text{(iv)}$$

$$-\frac{1}{32}\phi_1 - \frac{1}{16}\phi_2 - \frac{3}{32}\phi_3 + \frac{15}{16}\phi_4 = \frac{41}{100} \qquad \text{(v)}$$

Solving the System of equation we obtain

$$\phi_0 = \frac{2}{5},\ \phi_1 = 0.43321,\ \phi_2 = 0.46642,\ \phi_3 = 0.49962,\ \phi_4 = 0.53283$$

Then equation**(4)** implies

$$\phi(x) - \frac{1}{16}\left[0 + 2x\left(\frac{1}{4}\right)(0.43321) + 2x\left(\frac{1}{2}\right)(0.46642) + 2x\left(\frac{3}{4}\right)(0.49962) + x(1)(0.53283)\right] = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - \frac{1}{16}\left[0.216605x + 0.46642x + 0.74943x + 0.53283x\right] = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - 0.013537x - 0.029151x - 0.046339x - 0.033302x = 0.01x + 0.4$$

$$\phi(x) = 0.122829x + 0.01x + 0.4$$

$$\boldsymbol{\phi(\mathbf{x}) = \mathbf{0.132829x + 0.4}}$$

SIMPSON'S 1/3 RULE

$$\phi(x) - \frac{1}{2} \cdot \frac{h}{3}\left[(xy_0)\phi(y_0) + 4(xy_1)\phi(y_1) + 2(xy_2)\phi(y_2) + 4(xy_3)\phi(y_3) + (xy_4)\phi(y_4)\right]$$
$$= \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - \frac{1}{24}\left[(xy_0)\phi y_0 + 4(xy_1)\phi y_1 + 2(xy_2)\phi y_2 + 4(xy_3)\phi y_3 + (xy_4)\phi y_4\right]$$
$$= \frac{1}{100}x + \frac{2}{5} \qquad (5)$$

Changing x terms to y_i

$$\phi(x) - \frac{1}{24}\left[(y_iy_0)\phi y_0 + 4(y_iy_1)\phi y_1 + 2(y_iy_2)\phi y_2 + 4(y_iy_3)\phi y_3 + (y_iy_4)\phi y_4\right]$$
$$= \frac{1}{100}y_i + \frac{2}{5}$$

When $i = 0$

$$\phi_0 - \frac{1}{24}\left[y_0^2\phi_0 + 4y_0y_1\phi_1 + 2y_0y_2\phi_2 + 4y_0y_3\phi_3 + y_0y_4\phi_4\right] = \frac{1}{100}y_0 + \frac{2}{5}$$

$$\phi_0 - \frac{1}{24}[0] = \frac{2}{5}$$

$$\phi_0 = \frac{2}{5} \qquad \text{(i)}$$

When $i = 1$

$$\phi_1 - \frac{1}{24}\left[y_0y_1\phi_0 + 4y_1^2\phi_1 + 2y_1y_2\phi_2 + 4y_1y_3\phi_3 + y_1y_4\phi_4\right] = \frac{1}{100}y_1 + \frac{2}{5}$$

$$\phi_1 - \frac{1}{24}\left[0 + 4\left(\frac{1}{4}\right)^2\phi_1 + 2\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\phi_2 + 4\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\phi_3 + \left(\frac{1}{4}\right)\phi_4\right] = \frac{1}{100}\left(\frac{1}{4}\right) + \frac{2}{5}$$

$$\phi_1 - \frac{1}{24}\left[\frac{1}{4}\phi_1 + \frac{1}{4}\phi_2 + \frac{3}{4}\phi_3 + \frac{1}{4}\phi_4\right] = \frac{1}{400} + \frac{2}{5}$$

$$\phi_1 - \frac{1}{96}\phi_1 - \frac{1}{96}\phi_2 - \frac{1}{32}\phi_3 - \frac{1}{96}\phi_4 = \frac{161}{400}$$

$$\frac{95}{96}\phi_1 - \frac{1}{96}\phi_2 - \frac{1}{32}\phi_3 - \frac{1}{96}\phi_4 = \frac{161}{400} \qquad \text{(ii)}$$

When $i = 2$

$$\phi_2 - \frac{1}{24}\left[y_2 y_1 \phi_0 + 4y_1 y_2 \phi_1 + 2y_2^2 \phi_2 + 4y_2 y_3 \phi_3 + y_2 y_4 \phi_4\right] = \frac{1}{100} y_2 + \frac{2}{5}$$

$$\phi_2 - \frac{1}{24}\left[0 + 4\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\phi_1 + 2\left(\frac{1}{2}\right)^2 \phi_2 + 4\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\phi_3 + \left(\frac{1}{2}\right)(1)\phi_4\right] = \frac{1}{100}\left(\frac{1}{2}\right) + \frac{2}{5}$$

$$\phi_2 - \frac{1}{24}\left[\frac{1}{2}\phi_1 + \frac{1}{2}\phi_2 + \frac{3}{2}\phi_3 + \frac{1}{2}\phi_4\right] = \frac{1}{200} + \frac{2}{5}$$

$$\phi_2 - \frac{1}{48}\phi_1 - \frac{1}{48}\phi_2 - \frac{1}{16}\phi_3 - \frac{1}{48}\phi_4 = \frac{81}{200}$$

$$-\frac{1}{48}\phi_1 + \frac{47}{48}\phi_2 - \frac{1}{16}\phi_3 - \frac{1}{48}\phi_4 = \frac{161}{200} \qquad \text{(iii)}$$

When $i = 3$

$$\phi_3 - \frac{1}{24}\left[y_3 y_1 \phi_0 + 4y_1 y_3 \phi_1 + 2y_2 y_3 \phi_2 + 4y_3^2 \phi_3 + y_3 y_4 \phi_4\right] = \frac{1}{100} y_3 + \frac{2}{5}$$

$$\phi_3 - \frac{1}{24}\left[0 + 4\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\phi_1 + 2\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\phi_2 + 4\left(\frac{3}{4}\right)^2 \phi_3 + \left(\frac{3}{4}\right)(1)\phi_4\right] = \frac{1}{100}\left(\frac{3}{4}\right) + \frac{2}{5}$$

$$\phi_3 - \frac{1}{24}\left[\frac{3}{4}\phi_1 + \frac{3}{4}\phi_2 + \frac{9}{4}\phi_3 + \frac{3}{4}\phi_4\right] = \frac{3}{400} + \frac{2}{5}$$

$$\phi_3 - \frac{1}{32}\phi_1 - \frac{1}{32}\phi_2 - \frac{3}{32}\phi_3 - \frac{1}{32}\phi_4 = \frac{163}{400}$$

$$-\frac{1}{32}\phi_1 - \frac{1}{32}\phi_2 + \frac{29}{32}\phi_3 - \frac{1}{32}\phi_4 = \frac{163}{400} \qquad \text{(iv)}$$

When $i = 4$

$$\phi_4 - \frac{1}{24}\left[y_4 y_1 \phi_0 + 4y_1 y_4 \phi_1 + 2y_2 y_4 \phi_2 + 4y_3 y_4 \phi_3 + y_4^2 \phi_4\right] = \frac{1}{100} y_4 + \frac{2}{5}$$

$$\phi_4 - \frac{1}{24}\left[0 + 4\left(\frac{1}{4}\right)\phi_1 + 2\left(\frac{1}{2}\right)\phi_2 + 4\left(\frac{3}{4}\right)\phi_3 + \phi_4\right] = \frac{1}{100} + \frac{2}{5}$$

$$\phi_4 - \frac{1}{24}\left[\phi_1 + \phi_2 + 3\phi_3 + \phi_4\right] = \frac{41}{100}$$

$$\phi_4 - \frac{1}{24}\phi_1 - \frac{1}{24}\phi_2 - \frac{1}{8}\phi_3 - \frac{1}{24}\phi_4 = \frac{41}{100}$$

$$-\frac{1}{24}\phi_1 - \frac{1}{24}\phi_2 - \frac{1}{8}\phi_3 + \frac{23}{24}\phi_4 = \frac{41}{100} \qquad \text{(v)}$$

The System of equations are

$$\phi_0 = \frac{2}{5} \qquad \text{(i)}$$

$$\frac{95}{96}\phi_1 - \frac{1}{96}\phi_2 - \frac{1}{32}\phi_3 - \frac{1}{96}\phi_4 = \frac{161}{400} \qquad \text{(ii)}$$

$$-\frac{1}{48}\phi_1 + \frac{47}{48}\phi_2 - \frac{1}{16}\phi_3 - \frac{1}{48}\phi_4 = \frac{161}{200} \qquad \text{(iii)}$$

$$-\frac{1}{32}\phi_1 - \frac{1}{32}\phi_2 + \frac{29}{32}\phi_3 - \frac{1}{32}\phi_4 = \frac{163}{400} \qquad \text{(iv)}$$

$$-\frac{1}{24}\phi_1 - \frac{1}{24}\phi_2 - \frac{1}{8}\phi_3 + \frac{23}{24}\phi_4 = \frac{41}{100} \qquad \text{(v)}$$

Solving the system of equations, we have

$$\phi_0 = \frac{2}{5}, \quad \phi_1 = 0.43300, \quad \phi_2 = 0.4660, \quad \phi_3 = 0.49989, \quad \phi_4 = 0.5320$$

The Equation**(5)** implies

$$\phi(x) - \frac{1}{24}\left[0 + 4x\left(\frac{1}{4}\right)(0.4330) + 2x\left(\frac{1}{2}\right)(0.4660) + 4x\left(\frac{3}{4}\right)(0.49989) + x(1)(0.5320)\right]$$

$$= \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - \frac{433}{24000}x - \frac{233}{12000}x - 0.06249 - \frac{133}{6000}x = \frac{x}{100} + \frac{2}{5}$$

$$\phi(x) - 0.01804x - 0.01942x - 0.06249 - 0.02216x = \frac{x}{100} + \frac{2}{5}$$

$$\phi(x) - 0.12211x = 0.01x + 0.4$$

$$\boldsymbol{\phi(\mathbf{x}) = 0.13211\mathbf{x} + 0.4}$$

SIMPSON'S 3/8 RULE

$$\phi(x)-\frac{1}{2}\cdot\frac{3h}{8}\left[(xy_0)\phi(y_0)+3(xy_1)\phi(y_1)+3(xy_2)\phi(y_2)+2(xy_3)\phi(y_3)+(xy_4)\phi(y_4)\right]$$
$$=\frac{1}{100}x+\frac{2}{5}$$

$$\phi(x)-\frac{3}{64}\left[(xy_0)\phi(y_0)+3(xy_1)\phi(y_1)+3(xy_2)\phi(y_2)+2(xy_3)\phi(y_3)+(xy_4)\phi(y_4)\right]$$
$$=\frac{1}{100}x+\frac{2}{5} \tag{6}$$

Changing x terms to y_i

$$\phi(y_i)-\frac{3}{64}\left[(y_iy_0)\phi(y_0)+3(y_iy_1)\phi(y_1)+3(y_iy_2)\phi(y_2)+2(y_iy_3)\phi(y_3)+(y_iy_4)\phi(y_4)\right]$$
$$=\frac{1}{100}y_i+\frac{2}{5}$$

When $i=0$

$$\phi_0-\frac{3}{64}\left[y_0^2\phi_0+3y_0y_1\phi_1+3y_0y_2\phi_2+2y_0y_3\phi_3+y_0y_4\phi_4\right]=\frac{1}{100}y_0+\frac{2}{5}$$
$$\phi_0=\frac{2}{5} \tag{i}$$

When $i=1$

$$\phi_1-\frac{3}{64}\left[y_0y_1\phi_0+3y_1^2\phi_1+3y_1y_2\phi_2+2y_1y_3\phi_3+y_1y_4\phi_4\right]=\frac{1}{100}y_1+\frac{2}{5}$$
$$\phi_1-\frac{3}{64}\left[3\left(\frac{1}{4}\right)^2\phi_1+3\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\phi_2+2\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\phi_3+\left(\frac{1}{4}\right)\phi_4\right]=\frac{1}{100}\left(\frac{1}{4}\right)+\frac{2}{5}$$
$$\phi_1-\frac{3}{64}\left[\frac{1}{16}\phi_1+\frac{3}{8}\phi_2+\frac{3}{8}\phi_3+\frac{1}{4}\phi_4\right]=\frac{1}{400}+\frac{2}{5}$$
$$\phi_1-\frac{3}{1024}\phi_1-\frac{9}{512}\phi_2-\frac{9}{512}\phi_3-\frac{3}{256}\phi_4=\frac{161}{400}$$

$$\frac{1021}{1024}\phi_1 - \frac{9}{512}\phi_2 - \frac{9}{512}\phi_3 - \frac{3}{256}\phi_4 = \frac{161}{400} \tag{ii}$$

When $i = 2$

$$\phi_2 - \frac{3}{64}\left[y_0y_2\phi_0 + 3y_1y_2\phi_1 + 3y_2^2\phi_2 + 2y_2y_3\phi_3 + y_2y_4\phi_4\right] = \frac{1}{100}y_2 + \frac{2}{5}$$

$$\phi_2 - \frac{3}{64}\left[3\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\phi_1 + 3\left(\frac{1}{2}\right)^2\phi_2 + 2\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\phi_3 + \left(\frac{1}{2}\right)\phi_4\right] = \frac{1}{100}\left(\frac{1}{2}\right) + \frac{2}{5}$$

$$\phi_2 - \frac{3}{64}\left[\frac{3}{8}\phi_1 + \frac{3}{4}\phi_2 + \frac{3}{4}\phi_3 + \frac{1}{2}\phi_4\right] = \frac{1}{200} + \frac{2}{5}$$

$$\phi_2 - \frac{9}{512}\phi_1 - \frac{9}{256}\phi_2 - \frac{9}{256}\phi_3 - \frac{3}{128}\phi_4 = \frac{81}{200}$$

$$-\frac{9}{512}\phi_1 + \frac{247}{256}\phi_2 - \frac{9}{256}\phi_3 - \frac{3}{128}\phi_4 = \frac{81}{200} \tag{iii}$$

When $i = 3$

$$\phi_3 - \frac{3}{64}\left[y_0y_3\phi_0 + 3y_1y_3\phi_1 + 3y_2y_3\phi_2 + 2y_3^2\phi_3 + y_3y_4\phi_4\right] = \frac{1}{100}y_3 + \frac{2}{5}$$

$$\phi_3 - \frac{3}{64}\left[3\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\phi_1 + 3\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\phi_2 + 2\left(\frac{3}{4}\right)^2\phi_3 + \left(\frac{3}{4}\right)\phi_4\right] = \frac{1}{100}\left(\frac{3}{4}\right) + \frac{2}{5}$$

$$\phi_3 - \frac{3}{64}\left[\frac{9}{16}\phi_1 + \frac{9}{8}\phi_2 + \frac{9}{8}\phi_3 + \frac{3}{4}\phi_4\right] = \frac{3}{400} + \frac{2}{5}$$

$$\phi_3 - \frac{27}{1024}\phi_1 - \frac{27}{512}\phi_2 - \frac{27}{512}\phi_3 - \frac{9}{256}\phi_4 = \frac{163}{400}$$

$$-\frac{27}{1024}\phi_1 - \frac{27}{512}\phi_2 + \frac{485}{512}\phi_3 - \frac{9}{256}\phi_4 = \frac{163}{400} \tag{iv}$$

When $i = 4$

$$\phi_4 - \frac{3}{64}\left[y_0y_4\phi_0 + 3y_1y_4\phi_1 + 3y_2y_4\phi_2 + 2y_3y_4\phi_3 + y_4^2\phi_4\right] = \frac{1}{100}y_4 + \frac{2}{5}$$

$$\phi_4 - \frac{3}{64}\left[3\left(\frac{1}{4}\right)\phi_1 + 3\left(\frac{1}{2}\right)\phi_2 + 2\left(\frac{3}{4}\right)\phi_3 + \phi_4\right] = \frac{1}{100} + \frac{2}{5}$$

$$\phi_4 - \frac{3}{64}\left[\frac{3}{4}\phi_1 + \frac{3}{2}\phi_2 + \frac{3}{2}\phi_3 + \phi_4\right] = \frac{41}{100}$$

$$\phi_4 - \frac{9}{256}\phi_1 - \frac{9}{128}\phi_2 - \frac{9}{128}\phi_3 - \frac{3}{64}\phi_4 = \frac{41}{100}$$

$$-\frac{9}{256}\phi_1 - \frac{9}{128}\phi_2 - \frac{9}{128}\phi_3 + \frac{61}{64}\phi_4 = \frac{41}{100} \qquad \text{(v)}$$

The System of equation are

$$\phi_0 = \frac{2}{5} \qquad \text{(i)}$$

$$\frac{1021}{1024}\phi_1 - \frac{9}{512}\phi_2 - \frac{9}{512}\phi_3 - \frac{3}{256}\phi_4 = \frac{161}{400} \qquad \text{(ii)}$$

$$-\frac{9}{512}\phi_1 + \frac{247}{256}\phi_2 - \frac{9}{256}\phi_3 - \frac{3}{128}\phi_4 = \frac{81}{200} \qquad \text{(iii)}$$

$$-\frac{27}{1024}\phi_1 - \frac{27}{512}\phi_2 + \frac{485}{512}\phi_3 - \frac{9}{256}\phi_4 = \frac{163}{400} \qquad \text{(iv)}$$

$$-\frac{9}{256}\phi_1 - \frac{9}{128}\phi_2 - \frac{9}{128}\phi_3 + \frac{61}{64}\phi_4 = \frac{41}{100} \qquad \text{(v)}$$

Solving the System of equations we have

$$\phi_0 = \frac{2}{5}, \quad \phi_1 = 0.42639, \quad \phi_2 = 0.45778, \quad \phi_3 = 0.48667, \quad \phi_4 = 0.51556$$

Equation**(6)** implies

$$\phi(x) - \frac{3}{64}\left[0 + 3x\left(\frac{1}{4}\right)(0.42639) + 3x\left(\frac{1}{2}\right)(0.45778) + 2x\left(\frac{3}{4}\right)(0.48667) + x(1)(0.51556)\right] = \frac{1}{100}x + \frac{2}{5}$$

$$\phi(x) - 0.011499x - 0.03218x - 0.03421x - 0.02416x = 0.01x + 0.4$$

$$\phi(x) - 0.10554x = 0.01x + 0.4$$

$$\phi(x) = 0.01x + 0.10554x + 0.4$$

$$\boldsymbol{\phi(\mathbf{x}) = \mathbf{0.11554x + 0.4}}$$