TRANSPORTATION PROBLEM
A Case Study Of Unilorin Water Enterprise(UWE)

BY

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17/56EB094

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CERTIFICATION

This is to certify that this project work was carried out by **David, Udo Uduak** with matriculation number **17/56EB094** and approved as meeting the requirement for the award of the Bachelor of Science (B. Sc.) degree of the Department of Mathematics, Faculty of Physical Sciences, University of Ilorin, Ilorin, Nigeria.

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DEDICATION

I would like to dedicate the project to God, for the grace and faithfulness of God thus far. For His mercies, guidance and protection throughout my years of study.

ACKNOWLEDGMENTS

All praises, adoration and glorification are for Almighty God.

My profound gratitude and appreciation goes to my versatile and persevering supervisor, Prof. A.S. Idowu for his kind-hearted, and for his candid advice, encouragement and useful guidelines towards the success of this work. I pray Almighty God be with him and his family.

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ABSTRACT

This project work is concerned with

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Chapter 1

GENERAL INTRODUCTION

1.1 Introduction

The **Transportation Problem (TP)** is the generic name given to a whole class of problems in which the transportation is necessary. The general parameters of Transportation Problem are as follows:

(A) **Resources:** the resources are those elements that can be transported from sources to destination. Examples of discrete resources are goods, machines, tools, people, cargo; continuous resources include energy, liquids, and money.

(B) **Locations:** The locations are points of supply, recollection depot, nodes, railway station, bus stations, loading port, seaports, airports, refuelling depots or school.

(C) **Transportation Modes:** The transportation modes are the form of transporting some resources to locations. The transportation modes use water, space, air, road, rail, and cable. The form of transport has different infrastructure, capacity, times, activities, and regulations. Example of transportation modes are ship, aircraft, truck, train, pipeline, motorcycle and others.

1.2 Mathematical Model of Transportation Problem

In a typical Transportation Problem , a homogeneous product is to be transported from each of m sources to any of n destinations and their capacities are $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ respectively. In addition there is a penalty c_{ij} associated with transporting a unit of the product from source i to destination j. The penalty could represent transportation cost, delivery time, quantity of good delivered, safety delivery and many others. A variable X_{ij} represents the unknown quantity to be transported from source i to destination j. In the real life, all transportation problems are not single objective. Thus in general, the objective will also be controversial. In this paper, those transportation problems are considered, which are described by multiple objective functions.

The mathematical model of the multi-objective transportation problem is written as follows

$$s_1 : \text{Minimize } Z_k(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{k} x_{ij}, \quad k = 1, 2, \ldots, k \tag{1.1}$$

subject to

$$\begin{aligned}\sum_{i=1}^{m} x_{ij} &= a_i \qquad\qquad i = 1, 2, \ldots, m \\ \sum_{i=1}^{n} x_{ij} &= b_j \qquad\qquad j = 1, 2, \ldots, n \\ x_{ij} \geq 0, &\qquad \forall \;\; i \text{ and } j\end{aligned}$$

Where $Z_k(x) = \{Z_1(x), z_2(x), \ldots, Z_k(x)\}$ is a vector of k objective functions, the subscripts on $Z_k(x)$ and superscript on C_{ij}^k are used to identify the number of objective functions $(k = 1, 2, \ldots, k)$ without loss of generality it will be assumed in the whole paper that $a :> 0 \;\; \forall \;\; i, \;\; b_j > 0 \;\; \forall \;\; j, \; C_{ij}^k \geq 0 \;\; \forall \;\; (i, j)$ and $\sum_i a_i = \sum_j b_j$

1.3 Types of Transportation Problem

There are basically two (2) types of transportation problem:

1. Balanced Transportation Problem
2. Unbalanced Transportation Problem

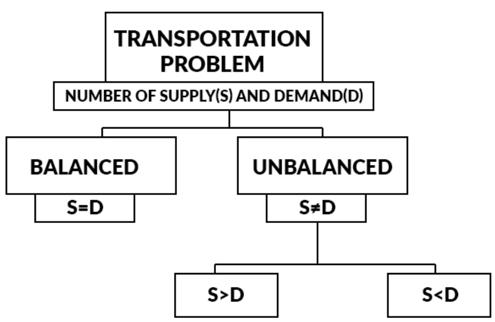

Figure 1.1: Types of Transportation Problem

1.4 Background of Study

1.4.1 Company Profile

The Unilorin Water Enterprise started its operation on the 23rd September 2013 with promises to produce sparkling clean water for the consumption of the member of the university community and beyond. The Unilorin Water Enterprise days of operation is Mondays to Saturdays.

1.4.2 Company Brand

The National Agency for Food and Drug Administration and Control (NAFDAC) on March 4, 2014 gave its seal of approval for Unilorin Water Enterprise(UWE) with the allocation of product registration number to the two water brands. Unilorin table water 50cl and 75cl bottles with registration number (1-2090L) and Unilorin Pure water with registration number (1-2049L).

1.4.3 Process of Production

The factory produces and package the product within a serene and hygiene environment, the company have five sedimentation tanks connected to special borehole, huwa-san, sand filter, carbon filter and four treated water tanks. For sachet water, the factory has filter micros ultra violet sterilize and automatic packaging machine while for the bottle water, there is a micro filter reserve osmosis, treated water tank, ultra violet sterilizer and washing, filling and capping machine.

1.4.4 Storehouses

Raw material, semi-finished goods, and finished goods are kept at the warehouse at the senior staff quarters store house. This finished goods and services are delivered at the right time to the right place in accordance with the planning schedule and at a minimum cost. There are few registered transporters(3 registered trucks) that are responsible for loading, packaging, offloading and movements of finished products from production warehouse to distributors.

1.4.5 Distributions

Finished products are sold directly to registered distributors. The distributors are the main agents who sells to retailer, the university of Ilorin water enterprise produce in large scale and it has over 70 distributors in and outside the university campus. The company produces over 4000 bags of sachet water, 500 packs of 50cl Unilorin table water and 1000 packs of 75cl table water respectively.

1.5 Problem Statement

The project to seek to address the problem of determine the optimal transportation schedule that will minimize the total cost of transporting products from the two production sites to the various key distributors geographically scatter in Ilorin.

1.6 Aims and Objectives

The study intended:

1. To model the distribution of Unilorin Water Enterprise (UWE) products as a transportation problem .

2. To minimize the transportation cost

Chapter 2

LITERATURE REVIEW

The transportation problem (TP) is an important Linear Programming (LP) model that arises in several context and has deservingly received much attention in literature.

The transportation problem is probably the most important special linear transportation problem in terms of relative frequency with which it appears in the applications and also in the simplicity of the procedure developed for its solution. The following features of the transportation problem are considered to be most important.

The transportation problem were the earliest class of Linear Programs discovered to have totally unimodular matrices and integrand extreme points resulting in considerable simplification of the Simplex method.

The study of the transportation problems laid the foundation for further theoretical and algorithmic development of the minimal cost network flow problems.

The transportation problem was formalized by the French mathematician (Monges 1781). Major advances were made in the field during World War II by the Soviet/Russian mathematician and economist Leomd Kantorovich. Consequently, the problem as it is now stated is sometimes known as the Monge-Kantorovich transportation problem . Kantorovich(1942) published a paper on continuous version of the problem and later with Gavurian, and applied study of the capacitated transportation problem (Kantorovich and Gavarian 1949).

Many scientific disciplines have contributed toward analysing problems associated with the transportation problem, including operation research, economics, engineering, Geographic information science and geography. It is explored extensively in the mathematical programming and engineering literatures. Sometimes referred to as the facility location and allocation problem, the transportation optimization problem can be modelled as a large-scale mixed integer linear programming problem.

The origin of transportation was first presented by Hitchcock, (1941) also presented a study entitled *'The Distribution of a product from several sources to numerous locations'*, this presentation is considered to be transportation problems. Kropmans, (1947), presented an independent study, not related to Hitchcock's and called *"Optimum utilization of the transportation sys-*

tem". These two contributions helped in the development of transportation methods which involve a number of shopping sources and a number of destination. The transportation problem, received this named because many of its applications involve determining how to optimally transport goods.

Chapter 3

METHODOLOGY

3.1 Introduction

This chapter reviews the proposed solution methodology and approach for handling transportation problem in Unilorin Water Enterprise . The transportation problem seeks to minimize the total shipping cost of transporting goods from m origins (each with a supply s_i) to n destinations (each with a demand d_j) when the unit shipping cost from an origin i, to a destination j, is C_{ij}.

3.2 Mathematical Formulation

Supposed a company has m warehouses and n retail outlets. A single product is to be shipped from the warehouse to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation cost between every pair of warehouse and outlet, and these cost are assumed to be linear. More explicitly, the assumptions are

- the total supply of products from warehouse $i = a$, where $i = 1, 2, 3, \ldots, m$
- the total demand of the products at the outlet $j = b$, where $j = 1, 2, 3, \ldots, n$
- the cost of sending one unit of the product from warehouse i to outlet j is equal to C_{ij}, where $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$. The total cost of a shipment is linear in size of shipment.

3.3 Model of A Transportation Problem

The transportation problem model is defined by

$$\text{Minimize } Z = \sum_{i=1}^{m} X_{ij} C_{ij} \tag{3.1}$$

$$\sum_{j=1}^{n} X_{ij} \leq a_i \quad , \quad i = 1, 2, 3, \ldots, m \quad \text{(Demand Constraint} \tag{3.2}$$

$$\sum_{i=1}^{m} X_{ij} \geq b_j \quad , \quad j = 1, 2, 3, \ldots, n \quad \text{(Supply constraint)} \tag{3.3}$$

$$X_{ij} \geq 0, 1, 2, 3, \ldots, n \tag{3.4}$$

This is a Linear Program with $m \cdot n$ decision variables, $m + n$ functional constants, and $m \cdot n$ non-negative constraints. Where

n is the number of destination

m is the number of resources

a_i is the capacity of i source

b_j is the demand of jth destination

C_{ij} is the unit transportation cost between ith source and jth destination (in naira or as a distance in Kilometers, miles, etc.). While X_{ij} is the size of material transported between ith source and jth destination (in tons, pounds, liters etc.).

A transportation problem is said to be unbalanced if and only if

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \tag{3.5}$$

There are two cases:

Case (1)

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j \tag{3.6}$$

Case (2)

$$\sum_{i=1}^{m} a_i \leq \sum_{j=1}^{n} b_j \tag{3.7}$$

To balance the Transportation Problem, introduce a dummy origin or source in the transportation table with a zero cost. The availability at the origin is

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = 0 \tag{3.8}$$

3.4 Solution of A Transportation Problem

3.4.1 Tableau And Network Representation

The transportation problem is illustrated with the model of a linear program and it appears in a network and tableau form

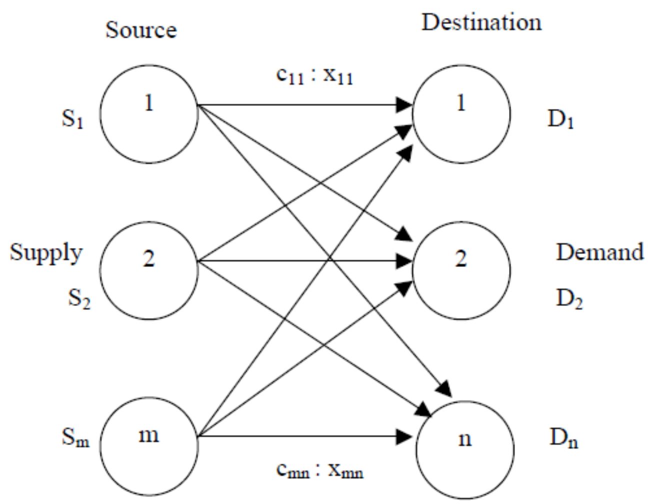

Figure 3.1: The Transportation Network

Destination / Plants	D_1	D_2	D_3		D_{n-1}	D_n	Supply quantity
S_1	x_{11}	x_{12}	x_{13}	…	$x_{1,m-1}$	$x_{1,n}$	s_1
S_2	x_{21}	x_{22}	x_{23}	…	$x_{2,m-1}$	$x_{2,n}$	s_2
S_3	x_{31}	x_{32}	x_{33}	…	$x_{3,m-1}$	$x_{3,n}$	s_3
	.	.	.		.	.	.
	.	.	.		.	.	.
	.	.	.		.	.	.
S_{m-1}	$x_{m-1,1}$	$x_{m-1,2}$	$x_{m-1,3}$	…	$x_{m-1,n-1}$	$x_{m-1,n}$	s_{m-1}
S_m	$x_{n,1}$	$x_{n,2}$	$x_{n,3}$	…	$x_{m,n-1}$	$x_{m,n}$	s_m
Demand quantity	d_1	d_2	d_3	…	d_{n-1}	d_n	

Figure 3.2: The Transportation Tableau

3.4.2 Flowchart Solution of the Transportation Problem

- the problem is formulated as a transportation model
- is the transportation model balanced?
- if yes, got next step, add dummy to the rows or column
- determine initial basic solution
- go to next step if the solution is optimized else go to fourth step
- using the optimal solution, calculate the total transportation cost

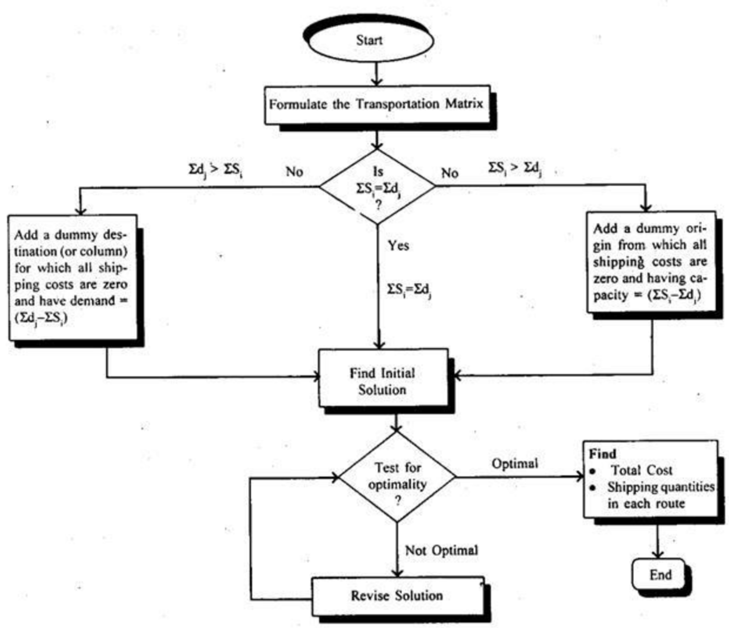

Figure 3.3: Flowchart of Transportation Solution

Solving the Transportation Problem:- There are three popular methods to finding an initial basic feasible solution and they include:

(1) Northwest Corner Rule

(2) Least Cost Method

(3) Vogel Approximation Method

Northwest Corner Rule(NCR)

In this method, allocation of quantities being transported from source to some destination must start from the upper most left hand cell that is the Northwest Corner of the table. The steps include:

(a) Make allocation in the northwest (upper left) corner of the transportation problem table. Compare the supply of plant 1 say S_1 with the demand at the warehouse or destination 1 say d_1. Then,

(i) If $d_1 < S_1$ i.e If the amount required at d_1 is less than the number of units available at S_1, set x_{11} equal to d_1, find the balance supply and demand and proceed horizontally.

(ii) If $d_1 = S_1$, set x_{11} equal to d_1, balance supply and demand and proceed diagonally. Remember to make a zero allocation to the least cost cell in S_1/d_1.

(iii) If $d_1 > S_1$, set x_{11} equal to S_1, balance demand and supply and proceed vertically.

(b) Continue with $itoiii$, step by step away from the upper left corner until you reach a value in the South-East corner.

(c) calculate the total transportation cost.

This method does not take into account the transportation cost and hence may not yield a good initial basic feasible solution.

Least Cost Method (LCM)

The Least Cost Method is also called the Matrix Minimum Method, is a method of finding an initial basic feasible solution where allocation of resources begins from the least cost. The steps includes:

1. Determine the cell having the least transportation cost (C_{ij})
2. Allocate as much as possible to this least cost
3. If there's a tie in least cost, select the cell having the greatest least cost.
4. Delete the row or column which has been exhausted
5. Select the next least cost and allocate as much as possible
6. Continue this manner till all row and column requirements are met.

The Vogel Approximation Method(VAM)

This procedure is an iterative method of finding an initial basic feasible solution. It is an improved version of the least cost method. The steps include:

1. Find the difference between the least cost and next least cost of each row and column(This difference is the row or column penalty).

2. Select the row or column with the biggest penalty
3. In case of a tie in penalty, select the row or column with the greatest least cost
4. Make allocation as much as possible to the cell in that row/column
5. Delete the column or row that has been completely exhausted.
6. Repeat steps 1 to 5 until all allocation are made

3.4.3 Numerical Illustration

Consider A company which has 3 production facilities S_1, S_2 and S_3 with production capacity of $7, 9$ and 18 units(in 100's) per week of a product, respectively. These units are to be shipped to 4 warehouses D_1, D_2, D_3 and D_4 with requirement of $5, 8, 7$ and 14 units (in 100's) per week, respectively. The transportation costs (in rupees) per units between factories to warehouse are given in the table below

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

To find an initial basic feasible solution for the given transportation problem. Using the three method.

METHOD 1: Using North West Corner

TOTAL number of supply constraints: 3

TOTAL number of demand constraints: 4

Problem Table is

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

The rim values for $S_1 = 7$ and $D_1 = 5$ are compared

The smaller of the two i.e $\min(7, 5) = 5$ is assigned to S_1D_1.

This meets the complete demand of D_1 and leaves $7 - 5 = 2$ units with S_1.

Table-1

	D_1	D_2	D_3	D_4	Capacity
S_1	19(5)	30	50	10	2
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	8	7	14	

The rim values for $S_1 = 2$ and $D_2 = 8$ are compared.

The smaller of the two i.e $\min(2, 8) = 2$ is assigned to S_1D_2

Table-2

	D_1	D_2	D_3	D_4	Capacity
S_1	19(5)	30(2)	50	10	0
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	6	7	14	

The rim value for $S_2 = 9$ and $D_2 = 6$ are compared

The smaller of the two i.e $\min(9, 6) = 6$ is assigned to S_2D_2.

This exhausts the capacity of S_1 and leaves $8 - 2 = 6$ units with D_2

Table-3

	D_1	D_2	D_3	D_4	Capacity
S_1	19(5)	30(2)	50	10	0
S_2	70	30(6)	40	60	3
S_3	40	8	70	20	18
Demand	0	0	7	14	

The rim values for $S_2 = 3$ and $D_3 = 7$ are compared.

The smaller of the two i.e $\min(3, 7) = 3$ is assigned to S_2D_3. This exhausts the capacity of S_2 and leaves $7 - 3 = 4$ units with D_3

Table-4

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19(5)~~	~~30(2)~~	~~50~~	~~10~~	0
~~S_2~~	~~70~~	~~30(6)~~	~~40(3)~~	~~60~~	0
S_3	40	8	70	20	18
Demand	0	0	4	14	

The rim values for $S_3 = 18$ and $D_3 = 4$ are compared

This smaller of the two i.e $\min(18, 4) = 4$ is assigned to S_3D_3

This meets the complete demands of D_3 and leaves $18 - 4 = 14$ units with S_3.

Table-5

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19(5)~~	~~30(2)~~	~~50~~	~~10~~	0
~~S_2~~	~~70~~	~~30(6)~~	~~40(3)~~	~~60~~	0
S_3	40	8	70(4)	20	14
Demand	0	0	0	14	

The rim values for $S_3 = 14$ and $D_4 = 14$ are compared.

The smaller of the two i.e $\min(14, 14) = 14$ is assigned to S_3D_4

Table-6

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	19(5)	30(2)	50	10	0
~~S_2~~	70	30(6)	40(3)	60	0
~~S_3~~	40	8	70(4)	20(14)	0
Demand	0	0	0	0	

Initial feasible solution is

	D_1	D_2	D_3	D_4	Capacity
S_1	19(5)	30(2)	50	10	7
S_2	70	30(6)	40(3)	60	9
S_3	40	8	70(4)	20(14)	18
Demand	5	8	7	14	

The minimum total transportation cost

$$19 \times 5 + 30 \times 2 + 30 \times 6 + +40 \times 3 + 70 \times 4 + 20 \times 14 = 1015$$

Here, the number of allocated cells = 6 is equal to

$$m + n - 1 = 3 + 4 - 1 = 6$$

$\therefore$ This solution is non-degenerate

METHOD 2: Using Least Cost Method to find solution

TOTAL number of supply constraints: 3

TOTAL number of demand constraints: 4

Problem Table is

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

The smallest transportation cost is 8 in cell S_3D_2

The allocation to this cell is $\min(18, 8) = 8$

This satisfies the entire demand of D_2 and leaves $18 - 8 = 10$ units with S_3

Table-1

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8(8)	70	20	10
Demand	5	0	7	14	

The smallest transportation cost is 10 in cell S_1D_4

The allocation to this is $\min(7, 14) = 7$

The exhausts the capacity of S_1 and leaves $14 - 7 = 7$ units with D_4

Table-2

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10(7)	0
S_2	70	30	40	60	9
S_3	40	8(8)	70	20	10
Demand	5	0	7	7	

The smallest transportation cost is 20 in cell S_3D_4

The allocation of the cell is $\min(10, 7) = 7$

This satisfies the entire demand of D_4 and leaves $10 - 7 = 3$ units with S_3

Table-3

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10(7)	0
S_2	70	30	40	60	9
S_3	40	8(8)	70	20(7)	3
Demand	5	0	7	0	

The smallest transportation cost is 40 in cell S_2D_3

The allocation to this cell is $\min(9, 7) = 7$

This satisfies the entire demand of D_3 and leaves $9 - 7 = 2$ units with S_2

Table-4

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
S_2	70	30	40(7)	60	2
S_3	40	8(8)	70	20(7)	3
Demand	5	0	0	0	

The smallest transportation cost is 40 in cell S_3D_1

The allocation to this cell is $\min(3, 5) = 3$

This exhausts the capacity of S_3 and leaves $5 - 2 = 2$ units with D_1

Table-5

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
S_2	70	30	40(7)	60	2
~~S_3~~	~~40(3)~~	~~8(8)~~	~~70~~	~~20(7)~~	0
Demand	2	0	0	0	

The smallest transportation cost is 70 in cell S_2D_1

The allocation to this cell is $\min(2, 2) = 2$

Table-6

	D_1	D_2	D_3	D_4	Capacity
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
~~S_2~~	~~70(2)~~	~~30~~	~~40(7)~~	~~60~~	0
~~S_3~~	~~40(3)~~	~~8(8)~~	~~70~~	~~20(7)~~	0
Demand	0	0	0	0	

Initial feasible solution is

	D_1	D_2	d4 D_3	D_4	Capacity
S_1	19	30	50	10(7)	7
S_2	70(2)	30	40(7)	60	9
S_3	40(3)	8(8)	70	20(7)	18
Demand	5	8	7	14	

The minimum total transportation cost

$$10 \times 7 + 70 \times 2 + 40 \times 3 + 8 \times 8 + 20 \times 7 = 814$$

Here, the number of allocated cells = 6 is equal to

$$m + n - 1 = 3 + 4 - 1 = 6$$

$\therefore$ This solution is non-degenerate

METHOD 3: Using Vogel's Approximation Method to find solution

TOTAL number of supply constraints: 3

TOTAL number of demand constraints: 4

Problem Table is

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

Table-1

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19	30	50	10	7	9=19-10
S_2	70	30	40	60	9	10 = 40-30
S_3	40	8	70	20	18	12 = 20-8
Demand	5	8	7	14		
Column Penalty	21 = 40-19	22=30-8	10=50-40	10=20-10		

The maximum penalty, 22 occur in column D_2

The minimum C_{ij} in this column is $C_{32} = 8$

The maximum allocation in this cell is $\min(18, 8) = 8$

It satisfy demand of D_2 and adjust the supply of S_3 from 18 to 10 $(18 - 8 = 10)$.

Table-2

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19	30	50	10	7	9=19-10
S_2	70	30	40	60	9	20 = 60-40
S_3	40	8(8)	70	20	10	20 = 40-20
Demand	5	0	7	14		
Column Penalty	21 = 40-19	–	10=50-40	10=20-10		

The maximum penalty, 21, occur in column D_1

The minimum C_{ij} in this column is $C_{11} = 19$

The maximum allocation in this cell is $\min(7, 5) = 5$

It satisfy demand of D_1 and adjust the supply of S_1 from 7 to 2 $(7 - 5 = 2)$

Table-3

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19(5)	30	50	10	2	40=50-10
S_2	70	30	40	60	9	20 = 60-40
S_3	40	8(8)	70	20	10	50 = 70-20
Demand	0	0	7	14		
Column Penalty	–	–	10=50-40	10=20-10		

The maximum penalty, 50 occurs in row S_3

The minimum C_{ij} in this row is $C_{34} = 20$

The maximum allocation in this cell is $\min(10, 14) = 10$

It satisfy supply of S_3 and adjust the demand of D_4 from 14 to 4 ($14-10 = 4$).

Table-4

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19(5)	30	50	10	2	40=50-10
S_2	70	30	40	60	9	20 = 60-40
~~S_3~~	~~40~~	~~8(8)~~	~~70~~	~~20(10)~~	0	–
Demand	0	0	7	4		
Column Penalty	–	–	10=50-40	50=60-10		

The maximum penalty, 50, occurs in column D_4

The minimum C_{ij} in this column is $C_{14} = 10$

The maximum allocation in this cell is $\min(2, 4) = 2$

It satisfy supply of S_1 and adjust the demand of D_4 from 4 to 2 $(4 - 2 = 2)$.

Table-5

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
~~S_1~~	~~19(5)~~	~~30~~	~~50~~	~~10(2)~~	0	–
S_2	70	30	40	60	9	20 = 60-40
~~S_3~~	~~40~~	~~8(8)~~	~~70~~	~~20(10)~~	0	–
Demand	0	0	7	2		
Column Penalty	–	–	40	60		

The maximum penalty, 60, occurs in column D_4

the minimum C_{ij} in this column is $C_{24} = 60$

The maximum allocation in this cell is $\min(9, 2) = 2$

It satisfy demand of D_4 and adjust the supply of S_2 from 9 to 7 $(9 - 2 = 7)$

Table-6

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19(5)	30	50	10(2)	0	–
S_2	70	30	40	60(2)	7	40
S_3	40	8(8)	70	20(10)	0	–
Demand	0	0	7	0		
Column Penalty	–	–	40	–		

The maximum penalty, 40, occurs in row S_2

The minimum C_{ij} in this row is $C_{23} = 40$

The maximum allocation in this cell is $\min(7, 7) = 7$

It satisfy supply of S_2 and demand of D_3

Initial feasible solution is

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	19(5)	30	50	10(2)	7	9|9|40|40|-|-|
S_2	70	30	40	60(2)	9	10|20|20|20|20|40|
S_3	40	8(8)	70	20(10)	18	12|20|50|-|-|-|
Demand	5	8	7	14		
Column Penalty	–	–	40	–		
	21	22	10	10		
	21	–	10	10		
	–	–	10	10		
	–	–	10	50		
	–	–	40	60		
	–	–	40	–		

The minimum total transportation cost

$$19 \times 5 + 10 \times 2 + 40 \times 7 + 60 \times 2 + 8 \times 8 + 20 \times 10 = 779$$

Here, the number of allocated cells = 6 is equal to

$$m + n - 1 = 3 + 4 - 1 = 6$$

$\therefore$ This solution is non-degenerate

Chapter 4

	D_1	D_2	D_3	D_4	Supply Capacity/Availability
Factory	100	50	130	70	200
Consultancy	90	60	80	100	100
Business School	150	20	300	100	300
UITH	15	12	24	10	30
Demand	200	150	150	130	630

To find an initial basic feasible solution for the given transportation problem using the three method.

Representing the stores as $S_1,\ S_2,\ S_3,\ S_4$.

METHOD 1 Using North West Corner

TOTAL number of supply constraints: 4

TOTAL number of demand constraints: 4

	D_1	D_2	D_3	D_4	Supply
S_1	100	50	130	70	200
S_2	90	60	80	100	100
S_3	150	20	300	100	300
S_3	15	12	24	10	30
Demand	200	150	150	130	

The rim values for $S_1 = 200$ and $D_1 = 200$ are compared

The smaller of the two i.e $\min(200, 200) = 200$ is assigned to S_1D_1

This exhausts the capacity of S_1 and leaves $200 - 200 = 0$ units with D_1

Table-1

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~100(200)~~	~~50~~	~~130~~	~~70~~	0
S_2	90	60	80	100	100
S_3	150	20	300	100	300
S_3	15	12	24	10	30
Demand	0	150	150	130	

The rim values for $S_2 = 100$ and $D_1 = 0$ are compared.

The smaller of the two i.e $\min(100, 0) = 0$ is assigned to S_2D_1

This meets the complete demand of D_1 and leaves $100 - 0 = 100$ units with S_2

Table-2

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~100(200)~~	~~50~~	~~130~~	~~70~~	0
S_2	90	60	80	100	100
S_3	150	20	300	100	300
S_3	15	12	24	10	30
Demand	0	150	150	130	

The rim values for $S_2 = 100$ and $D_2 = 150$ are compared

The smaller of the two i.e $\min(100, 150) = 100$ is assigned to S_2D_2

This exhaust the capacity of S_2 and leaves $150 - 100 = 50$ units wit D_2

Table-3

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~100(200)~~	~~50~~	~~130~~	~~70~~	0
~~S_2~~	~~90~~	~~60(100)~~	~~80~~	~~100~~	0
S_3	150	20	300	100	300
S_3	15	12	24	10	30
Demand	0	50	150	130	

The rim values for $S_3 = 300$ and $D_2 = 50$ are compared

The smaller of the two i.e $\min(300, 50) = 50$ is assigned to S_3D_2

This meets the complete demand of D_2 and leaves $300 - 50 = 250$ units with S_3.

Table-4

	D_1	D_2	D_3	D_4	Supply
S_1	100(200)	50	130	70	0
S_2	90	60(100)	80	100	0
S_3	150	20(50)	300	100	250
S_3	15	12	24	10	30
Demand	0	0	150	130	

The rim values for $S_3 = 250$ and $D_3 = 150$ are compared.

The smaller of the two i.e $\min(250, 150) = 150$ is assigned to S_3D_3

This meets the complete demand of D_3 and leaves $250 - 150 = 100$ units with S_3

Table-5

	D_1	D_2	D_3	D_4	Supply
S_1	100(200)	50	130	70	0
S_2	90	60(100)	80	100	0
S_3	150	20(50)	300(150)	100	100
S_3	15	12	24	10	30
Demand	0	0	0	130	

The rim values for $S_3 = 100$ and $D_4 = 130$ are compared.

The smaller of the two i.e $\min(100, 130) = 100$ is assigned to S_3D_4

This exhausts the capacity of S_3 and leaves $130 - 100 = 30$ units with D_4

Table-6

	D_1	D_2	D_3	D_4	Supply
S_1	100(200)	50	130	70	0
S_2	90	60(100)	80	100	0
S_3	150	20(50)	300(150)	100(100)	0
S_3	15	12	24	10	30
Demand	0	0	0	30	

The rim values for $S_4 = 30$ and $D_4 = 30$ are compared.

The smaller of the two i.e $\min(30, 30) = 30$ is assigned to S_4D_4

Table-7

	D_1	D_2	D_3	D_4	Supply
S_1	100(200)	50	130	70	0
S_2	90	60(100)	80	100	0
S_3	150	20(50)	300(150)	100(100)	0
S_3	15	12	24	10(30)	0
Demand	0	0	0	0	

Initial feasible solution is

	D_1	D_2	D_3	D_4	Supply
S_1	100(200)	50	130	70	200
S_2	90	60(100)	80	100	100
S_3	150	20(50)	300(150)	100(100)	300
S_3	15	12	24	10(30)	30
Demand	200	150	150	130	

The minimum total transportation cost is given by

$100 \times 200 + 60 \times 100 + 20 \times 50 + 300 \times 150 + 100 \times 100 + 10 \times 30 = 82300$

Here, the number of allocated cells = 6 which is one less than to $m + n - 1 = 4 + 4 - 1 = 7$

$\therefore$ The solution is degenerate.

METHOD 2 Using Lease Cost Method

TOTAL number of supply constraints: 4

TOTAL number of demand constraints: 4

Problem Table is

	D_1	D_2	D_3	D_4	Supply
S_1	100	50	130	70	200
S_2	90	60	80	100	100
S_3	150	20	300	100	300
S_3	15	12	24	10	30
Demand	200	150	150	130	

The smallest transportation cost is 10 in cell S_4D_4

The allocation to this cell is $\min(30, 130) = 30$

This exhausts the capacity of S_4 and leaves $130 - 30 = 100$ units with D_4

Table-1

	D_1	D_2	D_3	D_4	Supply
S_1	100	50	130	70	200
S_2	90	60	80	100	100
S_3	150	20	300	100	300
~~S_3~~	~~15~~	~~12~~	~~24~~	~~10(30)~~	0
Demand	200	150	150	100	

The smallest transportation cost is 20 in cell S_3D_2

The allocation to this cell is $\min(300, 150) = 150$

This satisfies the entire demand of D_2 and leaves $300 - 150 = 150$ units with S_3.

Table-2

	D_1	~~D_2~~	D_3	D_4	Supply
S_1	100	~~50~~	130	70	200
S_2	90	~~60~~	80	100	100
S_3	150	~~20(150)~~	300	100	150
~~S_3~~	~~15~~	~~12~~	~~24~~	~~10(30~~)	0
Demand	200	0	150	100	

The smallest transportation cost is 70 in cell S_1D_4

The allocation to this cell is $\min(200, 100) = 100$

This satisfies the entire demand of D_4 and leaves $200 - 100 = 100$ units with S_1

Table-3

	D_1	~~D_2~~	D_3	~~D_4~~	Supply
S_1	100	~~50~~	130	~~70(100)~~	100
S_2	90	~~60~~	80	~~100~~	100
S_3	150	~~20(150)~~	300	~~100~~	150
~~S_3~~	~~15~~	~~12~~	~~24~~	~~10(30)~~	0
Demand	200	0	150	0	

The smallest transportation cost is 80 in cell S_2D_3

The allocation to this cell is $\min(100, 150) = 100$

This exhaust the capacity of S_2 and leaves $150 - 100 = 50$ units with D_3

Table-4

	D_1	~~D_2~~	D_3	~~D_4~~	Supply
S_1	100	~~50~~	130	~~70(100)~~	100
~~S_2~~	~~90~~	~~60~~	~~80(100)~~	~~100~~	0
S_3	150	~~20(150)~~	300	~~100~~	150
~~S_3~~	~~15~~	~~12~~	~~24~~	~~10(30)~~	0
Demand	200	0	50	0	

The smallest transportation cost is 100 in cell S_1D_1

The allocation to this cell is $\min(100, 200) = 100$

This exhausts the capacity of S_1 and leaves $200 - 100 = 100$ units with D_1

Table-5

	D_1	D_2	D_3	D_4	Supply
S_1	100(100)	50	130	70(100)	0
S_2	90	60	80(100)	100	0
S_3	150	20(150)	300	100	150
S_3	15	12	24	10(30)	0
Demand	100	0	50	0	

The smallest transportation cost is 150 in cell S_3D_1

The allocation to this cell is $\min(150, 100) = 100$

This satisfies the entire demand of D_1 and leaves $150 - 100 = 50$ units with S_3

Table-6

	D_1	D_2	D_3	D_4	Supply
S_1	100(100)	50	130	70(100)	0
S_2	90	60	80(100)	100	0
S_3	150(100)	20(150)	300	100	50
S_3	15	12	24	10(30)	0
Demand	0	0	50	0	

The smallest transportation cost is 300 in cell S_3D_3

The allocation to this cell is $\min(50, 50) = 50$

Table-7

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~100(100)~~	~~50~~	~~130~~	~~70(100)~~	0
~~S_2~~	~~90~~	~~60~~	~~80(100)~~	~~100~~	0
~~S_3~~	~~150(100)~~	~~20(150)~~	~~300(50)~~	~~100~~	0
~~S_3~~	~~15~~	~~12~~	~~24~~	~~10(30)~~	0
Demand	0	0	0	0	

Initial feasible solution is given by

	D_1	D_2	D_3	D_4	Supply
S_1	100(100)	50	130	70(100)	200
S_2	90	60	80(100)	100	100
S_3	150(100)	20(150)	300(50)	100	300
S_3	15	12	24	10(30)	30
Demand	200	150	150	130	

The minimum total transportation cost is given by

$100 \times 100 + 70 \times 100 + 80 \times 100 + 150 \times 100 + 20 \times 150 + 300 \times 50 + 10 \times 30 = 58300$

Here, the number of allocated cells $= 7$ is equal to $m + n - 1 = 4 + 4 - 1 = 7$

$\therefore$ The solution is non-degenerate.

METHOD 3 Finding Solution Using Vogel's Approximation Method (VAM)

TOTAL number of supply constraints: 4

TOTAL number of demand constraints: 4

Problem Table is

	D_1	D_2	D_3	D_4	Supply
S_1	100	50	130	70	200
S_2	90	60	80	100	100
S_3	150	20	300	100	300
S_4	15	12	24	10	30
Demand	200	150	150	130	

Table-1

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100	50	130	70	200	20=70-50
S_2	90	60	80	100	100	20=80-60
S_3	150	20	300	100	300	80=100-20
S_4	15	12	24	10	30	2=12-10
Demand	200	150	150	130		
Column Penalty	75=90-15	8=20-12	56=80-24	60=70-10		

The maximum penalty, 80 occurs in row S_3

The minimum C_{ij} in this row is $C_{32} = 20$

The maximum allocation in this cell is $\min(300, 150) = 150$

It satisfy demand of D_2 and adjust the supply of S_3 from 300 to 150 ($300 - 150 = 150$)

Table-2

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100	50	130	70	200	30=100-70
S_2	90	60	80	100	100	10=90-80
S_3	150	20(150)	300	100	150	50=150-100
S_4	15	12	24	10	30	5-15-10
Demand	200	0	150	130		
Column Penalty	75=90-15	-	56=80-24	60=70-10		

The maximum penalty, 75 occurs in column D_1

The minimum C_{ij} in this column is $C_{41} = 15$

The maximum allocation in this cell is $\min(30, 200) = 30$

It satisfy supply of S_4 and adjust the demand of D_1 from 200 to 170 ($200 - 30 = 170$)

Table-3

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100	50	130	70	200	30=100-70
S_2	90	60	80	100	100	10=90-80
S_3	150	20(150)	300	100	150	50=150-100
~~S_4~~	~~15(30)~~	~~12~~	~~24~~	~~10~~	0	-
Demand	170	0	150	130		
Column Penalty	10=100-90	-	50=130-80	30=100-70		

The maximum penalty, 50 occurs in column D_3

The minimum C_{ij} in this column is $C_{23} = 80$

The maximum allocation in this cell is $\min(100, 50) = 50$

It satisfy supply of S_2 and adjust the demand of D_3 from 150 to 50 $(150 - 100 = 50)$

Table-4

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100	50	130	70	200	30=100-70
~~S_2~~	~~90~~	~~60~~	~~80(100)~~	~~100~~	0	-
S_3	150	20(150)	300	100	150	50=150-100
~~S_4~~	~~15(30)~~	~~12~~	~~24~~	~~10~~	0	-
Demand	170	0	50	130		
Column Penalty	50=150-100	-	170=300-130	30=100-70		

The maximum penalty, 170 occurs in column D_3

The minimum C_{ij} in this column is $C_{13} = 130$

The maximum allocation in this cell is $\min(200, 50) = 50$

It satisfy demand of D_3 and adjust the supply of S_1 from 200 to 150 $(200 - 50 = 150)$

Table-5

	D_1	~~D_2~~	~~D_3~~	D_4	Capacity	Row Penalty
S_1	100	~~50~~	~~130(50)~~	70	150	30=100-70
~~S_2~~	~~90~~	~~60~~	~~80(100)~~	~~100~~	0	-
S_3	150	~~20(150)~~	~~300~~	100	150	50=150-100
~~S_4~~	~~15(30)~~	~~12~~	~~24~~	~~10~~	0	-
Demand	170	0	0	130		
Column Penalty	50=150-100	-	-	30=100-70		

The maximum penalty, 50 occurs in column D_1

The minimum C_{ij} in this column is $C_{11} = 100$

The maximum allocation in this cell is $\min(150, 170) = 150$

It satisfy supply of S_1 and adjust the demand of D_1 from 170 to 20 $(170 - 150 = 20)$

Table-6

	D_1	~~D_2~~	~~D_3~~	D_4	Capacity	Row Penalty
~~S_1~~	~~100(150)~~	~~50~~	~~130(50)~~	~~70~~	0	-
~~S_2~~	~~90~~	~~60~~	~~80(100)~~	~~100~~	0	-
S_3	150	~~20(150)~~	~~300~~	100	150	50=150-100
~~S_4~~	~~15(30)~~	~~12~~	~~24~~	~~10~~	0	-
Demand	20	0	0	130		
Column Penalty	150	-	-	100		

The maximum penalty, 150 occurs in column D_1

The minimum C_{ij} in this column is $C_{31} = 150$

The maximum allocation in this cell is $\min(150, 20) = 20$

It satisfy demand of D_1 and adjust the supply of S_3 from 150 to 130 $(150 - 20 = 130)$

Table-7

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100(150)	50	130(50)	70	0	-
S_2	90	60	80(100)	100	0	-
S_3	150(20)	20(150)	300	100	130	100
S_4	15(30)	12	24	10	0	-
Demand	0	0	0	130		
Column Penalty	-	-	-	100		

The maximum penalty, 150 occurs in row S_3

The minimum C_{ij} in this column is $C_{34} = 100$

The maximum allocation in this cell is $\min(130, 130) = 130$

It satisfy supply of S_3 and demand of D_4

Initial feasible solution is

	D_1	D_2	D_3	D_4	Capacity	Row Penalty
S_1	100(150)	50	130(50)	70	200	20|30|30|30|30|-|-|
S_2	90	60	80(100)	100	100	20|10|10|-|-|-|-|
S_3	150(20)	20(150)	300	100(130)	300	80|50|50|50|50|50|100
S_4	15(30)	12	24	10	30	2|5|-|-|-|-|-|
Demand	200	150	150	130		
Column Penalty	75 75 10 50 50 – 150	8 – – – – – –	56 56 50 170 – – –	60 60 30 30 30 100 100		

The minimum total transportation cost

$100 \times 150 + 130 \times 50 + 80 \times 100 + 150 \times 20 + 20 \times 150 + 100 \times 130 + 15 \times 30$

Here, the number of allocated cells $= 7$ is equal to $m + n - 1 = 3 + 4 - 1 = 7$

$\therefore$ This solution is non-degenerate

Chapter 5

SUMMARY AND CONCLUSION

REFERENCES

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