CS 381: Assignment #1

Due on Monday, September 12th, 2012

Prof. Grigorescu 12:00pm

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Problem 1

$(\log n)^{0.3}$, $(\log n)^6$, $\sqrt{n}$, $\{n \log n, \log n^n\}$, $n(\log n)^4$, n^2, n^{21}, 2^n, $n!$

Problem 2

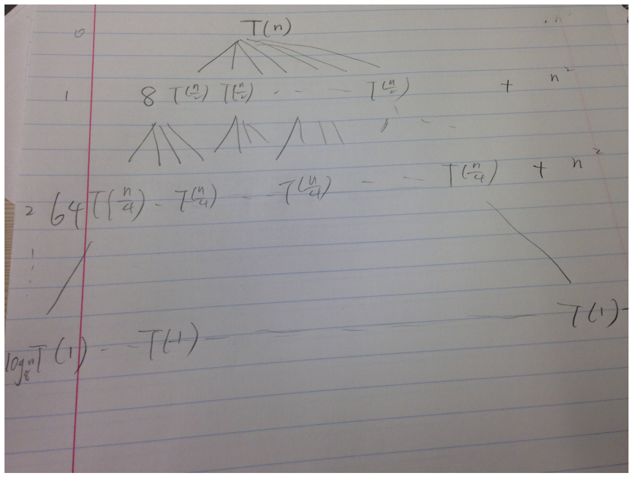

Figure 1. Recursion Tree

What we can get from Figure 1 is :

$T(n) = 8T(\frac{n}{2}) + n^2$ Level 0
$8T(\frac{n}{2}) = 64T(\frac{n}{4}) + n^2$ Level 1
$64T(\frac{n}{4}) = 512T(\frac{n}{8}) + n^2$ Level 2
...
$T(n) = 8^{log_2 n}T(1) + log_2 n n^2$

So we can conclude that $T(n) = \Theta(n^3)$
From Master theorem: $T(n) = \Theta(n^{log_2 8}) = \Theta(n^3)$ since $a = 8$ $b = 2$ $f(n) = n^2$ $where$ $c = 2$ $c < log_b a$

Proof:
Basis: $T(1) = 1; T(1) = 1^3 + log_2 1 * 1 = 1 + 0 = 1$

Induction: $Assume$ $T(n) = n^3 + log_2 n * n^2$ is $true$
From the original definition:
$T(n+1) = 8(T(\frac{n+1}{2})) + (n+1)^2$
$= 8[(\frac{n+1}{2})^3 + log_2 \frac{n+1}{2} * (\frac{n+1}{2})^2] + (n+1)^2$
$= 8(n+1)^3 + 2log_2 \frac{n+1}{2} * (n+1)^2 + (n+1)^2$
$....Some\ Magic...$
$= (n+1)^3 + log_2(n+1) * (n+1)^2$

So by the basis and induction. We conclude that $T(n) = n^3 + log_2 n * n^2$

Problem 3

Apparently $F(n) = F(n-1) + F(n-2); F(0) = 1; F(1) = 1; F(2) = 2$

Problem 3 continued on next page...

So We can get
F(0)=1
F(1)=1
F(2)=2
F(3)=3
F(4)=5
F(5)=8

It Fibonacci sequence.
We keep going on(I wrote a small program) And get the result F(12)=233

Problem 4

Create a boolean array from [0,1000000], and a hash function.

```
For i from first number to last number
if arr[hash(i+k)]=true or arr[hash(i-k)]=true
the element exists.
print it out && break

If not exists, print out not exists
```

The complexity is $O(n)$

Problem 5

1. Scan the array, find the minimum one, set the element as ∞. Repeat it 3 times.

2. Use $O(nlogn)$ sorting method. return the $\sqrt{n}$ element.

3. Create a Min-Heap ,$O(n)$, Delete a node every time, $O(\sqrt{n})$. So the total complexity is $O(N)$