

CS 381: Assignment #6

Due on Tuesday, Dec 11th, 2014

Prof. Grigorescu 12:00pm

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Problem 1

1. As shown in the textbook and the slides. In order to prove Longest path is NP. We need to show that

- it is apparently easy to verify if the length is greater than t
- the edge should be simple-path
- the edge should be on the graph

All of them can be verified in polynomial time.

2. We need to prove that

$$3 - SAT \leq_p \text{Longest} - \text{Path}$$

Since 3-SAT is NP-complete, so I have to reduce the longest-path from 3-sat

As we see in the slides

$$3 - SAT \leq_p \text{Hamilton} - \text{path}$$

So we construct a Hamilton path with all the edges with weight=1. The problem becomes can I find a hamilton path with the length $k-1$.

So

$$3 - SAT \leq_p \text{Hamilton} - \text{path} \leq_p \text{Longest} - \text{Path}$$

3. Use binary search($0, \sum^e \text{length}$), for each result, verify if that longest path exists. If so, binary search(upperhalf), else binary search(lowerhalf), the complexity becomes $O(\log(\sum^e \text{length}))$

Problem 2

1. To show it if it is NP just need to verify if all the sports is covered and the number of teacher is less than k .

2. Construct a graph with

$$G(V, E), \text{ where } E = \text{sports}, V = \text{instructor}$$

if an instructor can teach a sport set that instructor on the vertex of the edge. So this problem becomes vertex cover question.

So

$$\text{Vertex} - \text{cover} \leq_p \text{Instructor}$$