Homework 3: Due Tuesday, September 13

Reading: Chapters 3,4

Problem 1: For each of the following operations on $\mathbb{R}$, determine whether it is commutative, associative, has an identity, and inverses. Justify your answer (i.e. if it's true, prove it; otherwise justify your answer.)

1. $x * y = x + 2y - xy$
2. $x * y = |x - y|$
3. $x * y = \max\{x, y\}$ (i.e. the maximum of x and y).

Problem 2: Let G be a group, and $g \in G$. Show that $(g^{-1})^{-1} = g$. If $g, h \in G$, show that $(gh)^{-1} = h^{-1}g^{-1}$.

Problem 3: Let G be a group. Two elements $g, h \in G$ are said to commute if $gh = hg$. Show that if g and h commute, then so do their inverses.

Problem 4: If two elements a and b commute, show that $(ab)^n = a^n b^n$.

Problem 5: Let S be a set with an associative law of composition and with an identity element. Let G be the subset of S consisting of invertible elements (i.e. those $s \in S$ for which there is an inverse under the given law of composition). Show that G is a group.

Problem 6: Determine all integers n such that 2 has an inverse (under multiplication) modulo n.

Problem 7: Let a, b be elements of a group G. Suppose that a has order 5 and that $a^3 b = ba^3$. Prove that $ab = ba$.

Problem 8: Prove that any non-empty subset of a group G is a subgroup if for all $x, y \in H$, the element $xy^{-1} \in H$.