Differential Equation: Homework #6

Due on October 16th, 2015 at 3:10pm

Professor Heather Lee Section 061

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Problem 1

$$y'' + y' - 2y = 2t$$

$$(r+2)(r-1) = 0$$

So the characteristic equation is

$$y = c_1 e^{-2t} + c_2 e^t + Y(t)$$

Let $Y(t) = At + B$

$$y'(t) = A$$

$$y'' = 0$$

$$A - 2(At + B) = 2t$$

$$A - 2B = 0$$

$$-2At = 2t$$

$$A = -1$$

$$B = -\frac{1}{2}$$

so the formula is

$$y = c_1 e^{-2t} + c_2 e^t - t - \frac{1}{2}$$

Plug in the IV, we get

$$y = -\frac{1}{2} e^{-2t} + e^t - t - \frac{1}{2}$$

Problem 2

The Solution of the system should be

$$y(t) = c_1 + c_2 e^{-3t} + Y(t)$$

From $2t^4$ we could get

$$Y_1(t) = A_1 t^4 + A_2 t^3 + A_3 t^2 + A_4 t + A_5$$

Problem 2 continued on next page...

But two formula has the same constant So

$$Y_1(t) = A_1t^5 + A_2t^4 + A_3t^3 + A_4t^2 + A_5t$$

For t^2e^{-3t} We could get

$$Y_2(t) = (B_1t^2 + B_2t + B_3)e^{-3t}$$

But B_3e^{-3t} is duplicated So the solution shall be

$$Y_2(t) = (B_1t^3 + B_2t^2 + B_3t)e^{-3t}$$

For $sin(3t)$ We have

$$Y_3(t) = C_1sin(3t) + C_2cos(3t)$$

There is no duplication So the solution is gonna be

$$Y(t) = A_1t^5 + A_2t^4 + A_3t^3 + A_4t^2 + A_5t + (B_1t^3 + B_2t^2 + B_3t)e^{-3t} + C_1sin(3t) + C_2cos(3t)$$

Problem 3

The original solution is

$$y(t) = c_1cos(t) + c_2sin(t)$$

For t we have $y = A_1t + A_2$ There is no duplication For $tsin(t)$ we have

$$Y_0 = (B_1t + B_2)sin(t) + (C_1t + C_2)cos(t)$$

$B_2sin(t)i$ is the duplication Hence

$$Y_1 = (B_1t^2 + B_2t)sin(t) + (C_1t^2 + C_2t)cos(t)$$

$$Y(t) = A_1t + A_2 + (B_1t^2 + B_2t)sin(t) + (C_1t^2 + C_2t)cos(t)$$

Problem 4

$$y_1'(t) = 1$$

$$y_1''(t) = 0$$

Plug it in, we get $-(1+t) + 1 + t = 0$

$$y_2'(t) = e^t$$

$$y_2''(t) = e^t$$

Plug it in, we get $e^t t - (1+t)e^t + e^t = 0$ So y_1 and y_2 are both the solution of the given equation

$$y'' - \frac{1+t}{t}y' + \frac{1}{t}y = te^{2t}$$

And the Wronskian of the equation is te^t

So the general solution will be

$$\begin{aligned} Y(t) &= -(1+t)\int \frac{e^t te^{2t}}{te^t} + e^t \int \frac{(1+t)te^{2t}}{te^t} \\ &= -(1+t)\int e^{2t} + e^t \int (1+t)e^t \\ &= -\frac{(1+t)}{2}e^{2t} + e^t(e^t + \int te^t) \end{aligned}$$

Integrate by part, we get

$$\int te^t dt = te^t - \int e^t dt = te^t - e^t$$

Hence

$$\begin{aligned} -\frac{(1+t)}{2}e^{2t} + e^t(e^t + \int te^t) \\ = -\frac{(1+t)}{2}e^{2t} + te^{2t} \\ = \frac{1}{2}te^{2t} - \frac{1}{2}e^{2t} \end{aligned}$$

Problem 5

Since $y_1(t) = t$ $y_2(t) = t^4$

Problem 5 continued on next page...

Plug it in, we get

$$-4t + 4t = 0$$

$$12t^2 * t^2 - 4t * 4t^3 + 4t^4 = 0$$

So they are both the solution of the homogeneous equation, the Wronskian of the equation is $4t^4 - t^4 = 3t^4$

$$y'' - \frac{4}{t}y' + \frac{4}{t^2}y = -2$$

$$Y = -t\int \frac{-2t^4}{3t^4} + t^4 \int \frac{-2t}{3t^4} = t^2$$

Hence the solution will be

$$y = c_1 t + c_2 t^4 + t^2$$

Plug it in with IV, we get

$$y = \frac{10}{3}t - \frac{4}{3}t^4 + t^2$$