Differential Equation: Homework #4

Due on September 19th, 2015 at 3:10pm

Professor Heather Lee Section 061

Yao Xiao

Problem 1

16. The statement of the problem implies

$$\frac{dT}{dt} = k(T - 70)$$

We also know that $T(0) = 200$ and $T(1) = 190$

So we can get

$$T = \frac{\int -70ke^{-kt}dt}{e^{-kt}} = 70 + Ce^{kt}$$

Since $T(0) = 200$ so $C = 130$

$T = 70 + 130e^{kt}$, when $t = 1$ we plug it in, we could get $70 + 130e^{k} = 190$ $k = ln\sqrt{12/13}$

$70 + 130e^{ln(12/13)t} = 150$

$t \approx 6.065$

Problem 2

21.

$$F = ma$$

$$m\frac{dv}{dt} = -mg - v/30$$
$$\frac{dv}{dt} + \frac{v}{4.5} = -g$$

Solve it, plug in t=0, v=20 get: $v = 64.1e^{-t/4.5} - 44.1$ So distance

$x = -44.1t - 299.45 * e^{-0.222t} + C$ plug in t=0, x=30, we get C=318.5.

so $x = -44.1t - 299.45e^{-0.22t} + 318.5$

when $v = 0$ it reaches the max height. we get $t = 1.683s$

$$x = 45.783m$$

b) when the ball is dropping. We use similar method which could get(don't wanna type the entire procedure in latex = =)

$$x = 4.5g(t + 4.5e^{-0.22t}) - 4.5^2$$

we could get t=3.446, and therefore the entire time is 1.683+3.446=5.1129s

Problem 3

2.4, 17:

$$y' = ty(3-y)$$

$$\partial f/\partial = 3t - 2ty$$

$$dy/dt = t(3y - y^2)$$

$$1/3(ln(y) - ln(3-y)) = \frac{t^2}{2} + C$$

$$y(t) = \frac{3e^{3t^2/2}}{3e/y_0 - 3 + 3e^{3t^2/2}}$$

Problem 4

Supplementary problem D:

$$\frac{dy}{dt} = y^2 - 4y, y(0) = 8$$

$$\frac{dy}{y^2 - 4y} = dt$$

$$\frac{ln(4-y) - ln(y)}{4} = t + C$$

$$\frac{(ln(-4) - ln(8))}{4} = C$$

$$\frac{ln(4-y) - ln(y)}{4} = t + \frac{(ln(-4) - ln(8))}{4}$$

$$\ln(4-y) - ln(y) = 4t + ln(-2)$$

$$y = \frac{4}{1 - (e^{4t}/2)}$$

since we want the solution to be continuous, we could get $y \neq \frac{ln(2)}{4}$, but the solution need to be containing 0, so the answer is

$$(-\infty, \frac{ln(2)}{4})$$