Homework 11: Due Thursday, November 17

Reading: Chapters 17, 18, 19

Problem 1: Let R be a ring. Show that the following hold in R:
(1) If $a \in R$, then $0 \cdot a = 0$
(2) For all $a \in R, (-1) \cdot a = -a$.
(3) The multiplicative identity 1 is unique.

Problem 2: Show that a finite integral domain is a field.

Problem 3: Let $f : R \to R'$ denote a ring homomorphism. Show that $ker(f)$ is an ideal.

Problem 4: List all the ideals of $\mathbb{Z}_{12}$.

Problem 5: Let R be a ring and $I \subset R$ an ideal. An element $a \in R$ is invertible if a has a multiplicative inverse. Show that if an element $a \in I$ is invertible, then $I = R$.

Problem 6: Let $f : R \to R'$ be a homomorphism. Show that the induced map $\phi : R[x] \to R'[x]$, where $\phi(a_n x^n + \cdots + a_0) := f(a_n)x^n + \cdots + f(a_0)$, is a ring homomorphism.

Problem 7: Let R be a ring. Show that there is exactly one homomorphism $\phi : \mathbb{Z} \to R$.

Problem 8: Describe the kernel of the map $\mathbb{Z}[x] \to \mathbb{R}$ defined by sending a polynomial $f(x)$ to $f(1 + \sqrt{2})$.