# Elements of Algebra I: Homework #2

Due on January 24th, 2017

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# Problem 1

Proof by induction
When n=1,

$$1 = \frac{n(n+1)(2n+1)}{6} = \frac{1*2*3}{6}$$

Suppose n=k

$$1 + 2^2 + 3^2 + ... + k^2 = \frac{k(k+1)(2k+1)}{6}$$

$$\begin{aligned}
1 + 2^2 + 3^2 + ... + k^2 + (k+1)^2 &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\
&= \frac{6(k+1)^2 + k(k+1)(2k+1)}{6} \\
&= \frac{(k+1)(6(k+1) + (2k+1)k)}{6} \\
&= \frac{(k+1)(6k+6+2k^2+k)}{6} \\
&= \frac{(k+1)(2k^2+7k+6)}{6} \\
&= \frac{(k+1)(2k+3)(k+2)}{6} \\
&= \frac{(k+1)(2(k+1)+1)((k+1)+1)}{6}
\end{aligned}$$

So the equation holds for n=k+1. Therefore, the inequalities hold for all positive integers n ¿ 1 by induction

# Problem 2

Proof by induction
When n=2
We have $1 < 8/3 < 5$, so the inequalities hold when n=2
When n is greater than 2,

$$\begin{aligned}
1 + 2^2 + 3^2 + ... + (n-1)^2 + n^2 &< \frac{n^3}{3} + n^2 \\
&= \frac{n^3 + 3n^2}{3} \\
&< \frac{n^3 + 3n^2 + 3n + 1}{3} \\
&= \frac{(n+1)^3}{3}
\end{aligned}$$

Problem 2 continued on next page...

So the left inequality holds

$$\begin{aligned} 1 + 2^2 + 3^2 + ...n^2 + (n+1)^2 &> \frac{n^3}{3} + (n+1)^2 \\ &= \frac{n^3 + 3(n+1)^2}{3} \\ &= \frac{n^3 + 3n^2 + 6n + 3}{3} \\ &> \frac{n^3 + 3n^2 + 3n + 1}{3} \\ &= \frac{(n+1)^3}{3} \end{aligned}$$

So the right inequality holds as well

## Problem 3

1. Let a be the integer, (a-a)=0. 0 mod anything is 0. So it holds
2. Let a and b be integers. Suppose that a $\equiv$ b (mod n), aka a-b = kn. b-a=-(a-b)=(-k)n. So it holds
3. Let a,b,c be integers. a-b=nk, b-c=nj, let the first one plus the second one a-b+b-c=nk+nj, a-c=n(k+j) so a-c=nk' as well

## Problem 4

$a \to b$ is true iff $\neg b \to \neg a$ is true. So we assume a is even, and the product of the two even number is $2k * 2k = 2(2k)$ and it's even too.

## Problem 5

Let rotate be function R, so that

$$R(1) = 2 \; R(2) = 3 \; R(3) = 4 \; R(4) = 1$$

Let flip be function F, so that

$$F(1) = 2 \; F(2) = 1 \; F(3) = 4 \; F(4) = 3$$

When you rotate the square once, twice and three times. It will become (2,3,4,1),(3,4,1,2),(4,1,2,3), respectively. So when you do $R^3 * R$. it will become (1,2,3,4) which is I. So any rotation could be one of the I,$R, R^2, R^3$

## Problem 6

Assume we are starting from (1,2,3,4)

$$\begin{aligned} F &= (2,1,4,3) \\ FR = R(2,1,4,3) &= (3,2,1,4) \\ FR^2 = R^2(2,1,4,3) &= (4,3,2,1) \\ FR^3 = R^3(2,1,4,3) &= (1,4,3,2) \end{aligned}$$

## Problem 7

Since $F^2 = I$ and $R^4 = I$, there will be all the symmetric flips.

$$RF = F(2, 3, 4, 1) = (3, 2, 1, 4) = FR$$

## Problem 8

The inverse of R is $R^3$
The inverse of $R^2$ is $R^2$
The inverse of $R^3$ is $R$