Differential Equation: Homework #3

Due on September 11th, 2015 at 3:10pm

Professor Heather Lee Section 061

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Problem 1

1.

$xdx + ye^{-x}dy = 0$

$xe^{x}dx = -ydy$

$e^{x}(x-1) = -\frac{1}{2}y^2 + C$

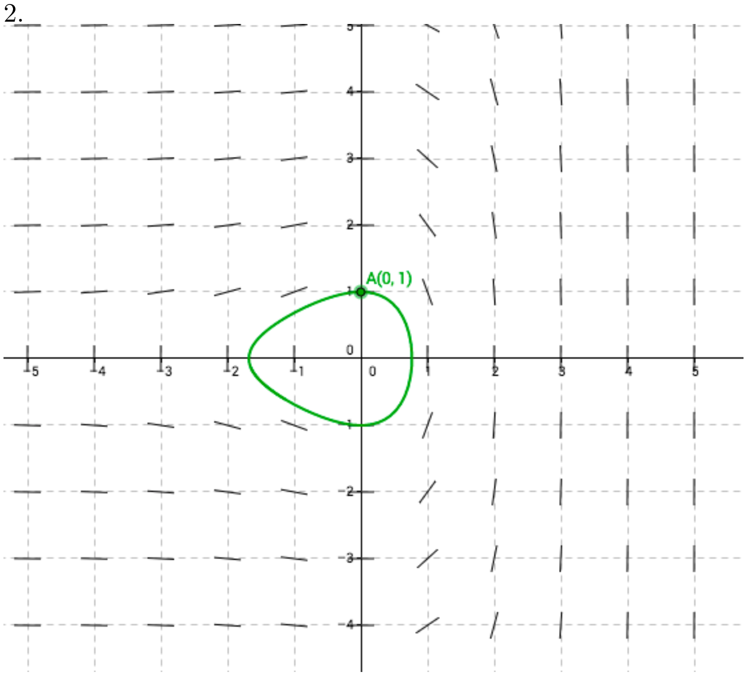
$-1 = -\frac{1}{2} + C$

$C = -\frac{1}{2}$

$2e^{x}(1-x) = y^2 + 1$

$y = \sqrt{2e^{x}(1-x) - 1}$

2.



3. Since the function will become vertical at around 0.7 and -1.7, so it would be valid when $-1.7 < x < 0.7$

Problem 2

$y' = xy^3(1+x^2)^{-1/2}$

1.

$$\frac{dy}{dx} = xy^3(1+x^2)^{-1/2}$$

$$y^{-3}dy = x(1+x^2)^{-1/2}dx$$

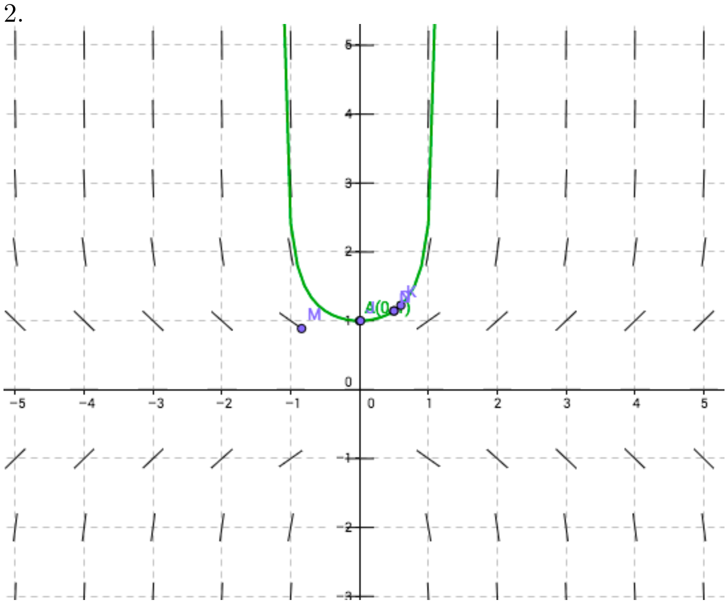
$$-\frac{1}{2}y^{-2} = \sqrt{x^2+1} + C$$

When x=0, y=1 $1 + C = -1/2,\ C = -3/2$

$y^{-2} = -2\sqrt{x^2+1} + 3$

$y = (-2\sqrt{x^2+1} + 3)^{-1/2}$

2.



3.

We need to make sure $-2\sqrt{x^2+1} + 3 \neq 0$ So $x \neq \frac{1}{2}\sqrt{5}$, since x=0 should be in the interval, the answer would be $-\frac{1}{2}\sqrt{5} < x < \frac{1}{2}\sqrt{5}$

Problem 3

1. When $t \to \infty$, $\frac{t}{1+t} \to 1$, $y' = y(4-y) = 0$, also $y \neq 0$ so $y \to 4$

2.

$$dy/(y*(4-y)) = t/(1+t)dt$$

$$ln(y) - ln(4-y) = 4t - 4ln(1+t) + C$$

$$ln\frac{4}{4-y} = 4t - 4ln(1+t) + C$$

$$\frac{y}{4-y} = \frac{Ce^{4t}}{(1+t)^4}$$

If $y_0 = 2$ $C = 1$ and we plug in $y = 2$ we get $3.99/(4-3.99) = e^{4t}/(1+t)^4$, $t = 2.84$

3. I don't know...

Problem 4

1.

$(x^2 + 3xy + y^2)dx - x^2dy = 0$ is equal to $v = y/x, (v^2 + 3v + 1)dx - 1dy = 0, v^2 + 3v + 1 = \frac{dy}{dx}$ So it's homogeneous.

2.

$$v + x\frac{dv}{dx} = v^2 + 3v + 1$$

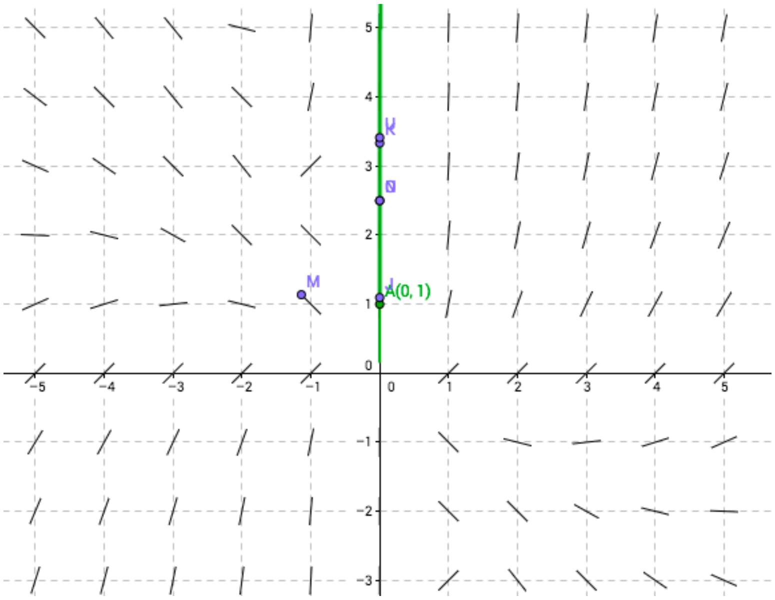
$$\frac{dv}{dx} = (v+1)^2$$

$$ln(x) + (v+1)^{-1} + C = 0$$

$$ln(x) + (y/x + 1^{-1}) + C = 0$$

3.

Problem 4 continued on next page. . .



It's not symmentric

Problem 5

$$u = x + y$$

$$u' = x' + y' = 1 + y'$$

$$u' = u^2 + 1$$

$$x = tan^{-1}(u) + C$$

$$x = tan^{-1}(x + y) + C$$

Problem 6

$$\frac{du}{dx} = 3y^2 dy/dx$$

$$1/3 * du/dx + u(x)/x = 2/x^2$$

$$x^3 * u = 3x^2 + C$$

$$(xy)^3 = 3x^2 + C$$

Problem 7

Q(t) means the amount of dye in the tank at time t, the system lost 2 L/min *(Q(t)/200) g/L = Q(t)/100 g/min

$$\frac{dQ}{dt} = -\frac{Q}{100}$$

Also Q(0)=200g So $Q(t) = 200e^{-0.01t}$

when $200e^{-0.01t} = 2$, t=460.5min

Problem 8

$rate\ in = \gamma g/L * 2L/min = 2\gamma g/min$

$rate\ out = Q(t)/120g/L * 2L/min = \frac{Q(t)}{60} g/min.$

So the equation is

$$\frac{dQ}{dt} = \frac{120\gamma - Q(t)}{60}$$

Solve it with Q(0)=0, we get

$$Q(t) = 120\gamma - 120\gamma * e^{\frac{-t}{60}}$$

when $t \to \infty$, $Q(t) \to 120\gamma$