Differential Equation: Homework #5

Due on September 25th, 2015 at 3:10pm

Professor Heather Lee Section 061

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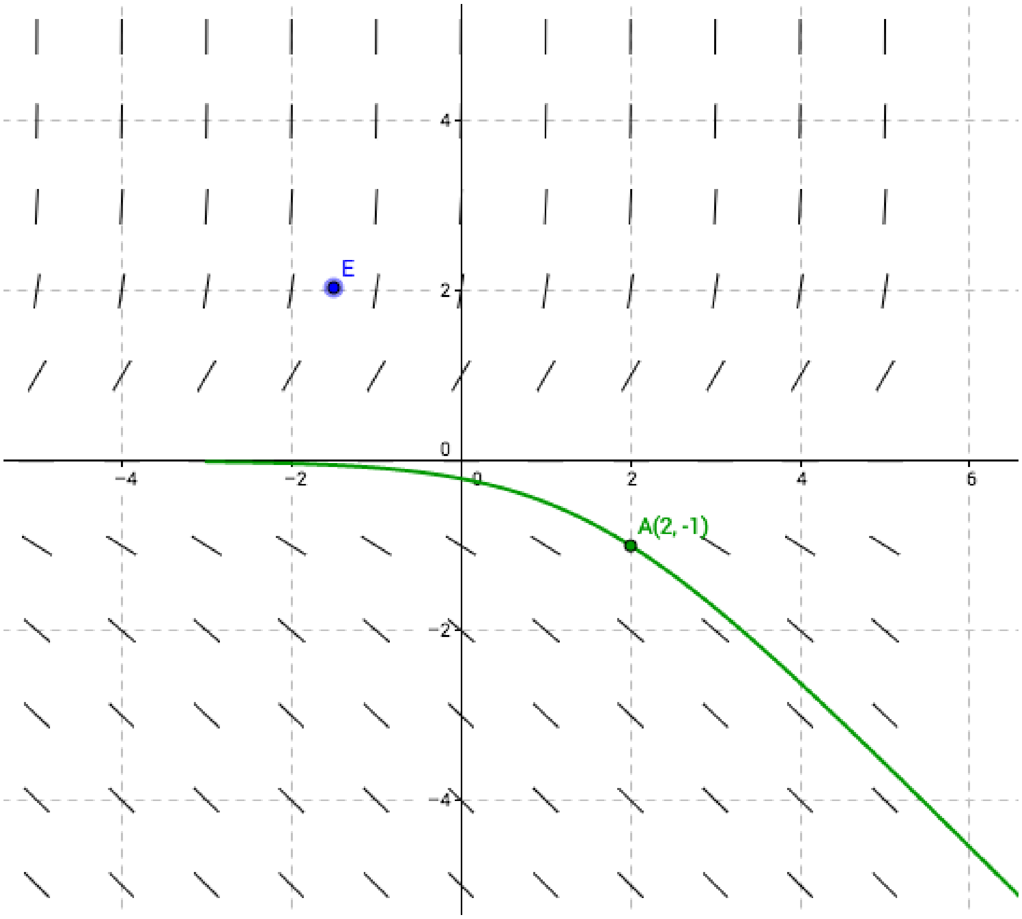
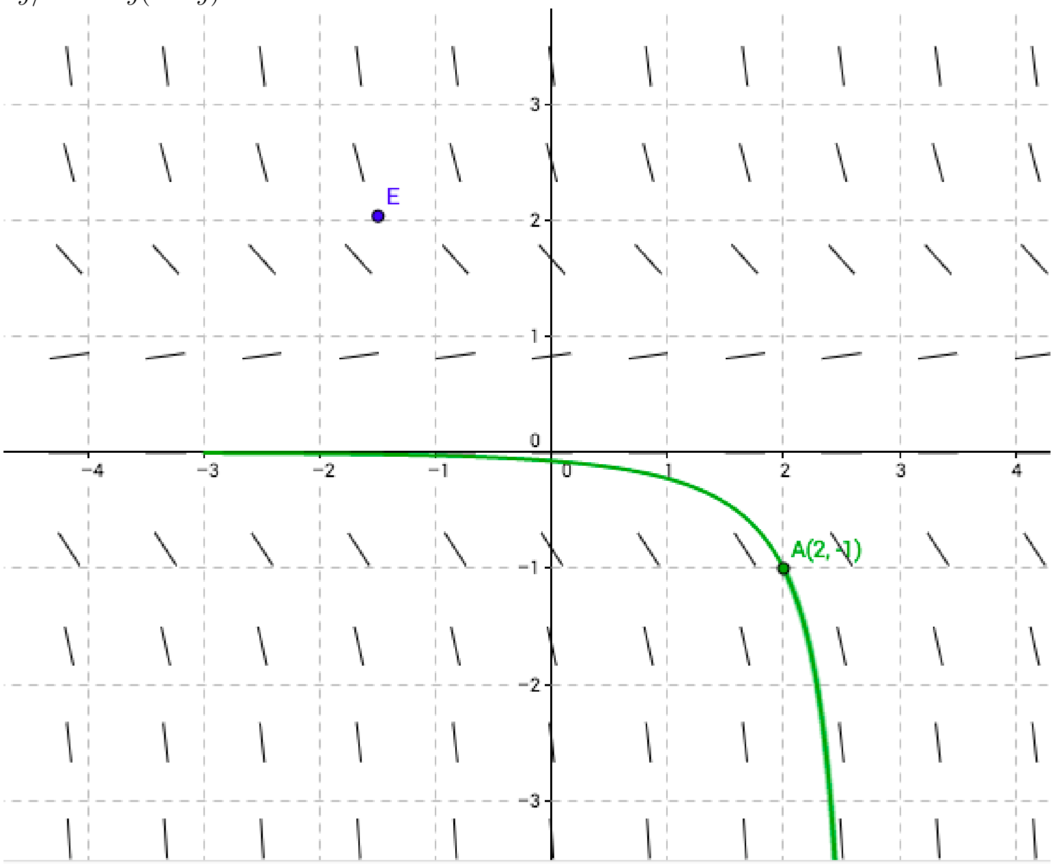
Problem 1



From this sketch it appears that solutions that start near y = 0 all move away from it as t increases, so it's unstable.

Problem 2

$dy/dx = \alpha y(1-y)$



From this sketch it appears that solutions that start near y = 0 all move away from it as t increases, so it's unstable. solutions that start near y = 0 all move towards it as t increases, so it's asymptotically stable

2.

$$\int \frac{1}{y(1-y)} dy = \int \alpha dt$$

$$ln(y) - ln(1-y) = \alpha t + C$$

$$\frac{y}{1-y} = Ae^{\alpha t}$$

$$A = \frac{y_0}{1-y_0}$$

$$\frac{y}{1-y} = \frac{y_0}{1-y_0} e^{\alpha t}$$

$$y = \frac{y_0 e^{\alpha t}}{y_0 e^{\alpha t} + 1 - y_0}$$

When $t \to \infty, y \to 1$

Problem 3

1.The equilibrium solutions should be -2,4,8

2. When y=-2, it's asymptotic stable When y=4, it's semi-stable When y=8, it's not stable

Problem 4

Since

$$\frac{\partial M(x)}{\partial y} = \frac{\partial N(y)}{\partial x} = 0$$

So it's exact equation

Problem 5

$$\frac{dw}{dt} = \frac{2tw}{w^2 - t^2}$$

Let w=at, dw/dt=a+t(da/dt)

$(2tw)/(w^2 - t^2) = (2at^2)/((at)^2 - t^2) = 2a/(a^2 - 1)$

$$\begin{aligned}
a + t\frac{da}{dt} &= \frac{2a}{a^2 - 1} \\
t\frac{da}{dt} &= \frac{2a - a(a^2 - 1)}{a^2 - 1} \\
t\frac{da}{dt} &= \frac{2a - a(a^2 - 1)}{a^2 - 1} \\
\frac{3a - a^3}{a^2 - 1} &= t\frac{da}{dt} \\
\frac{a^2 - 1}{3a - a^3} da &= \frac{1}{tdt} \\
t(a) &= \frac{C}{\sqrt[3]{a(3 - a^2)}}
\end{aligned}$$

$$t^3 a(3 - a^2) = C$$

$$a = w/t$$

$$wt^2(3 - w^2/t^2) = C$$

$$w = \frac{C}{3t^2 - w^2}$$

Problem 5 continued on next page...

Problem 6

a)

$$y' = 1 - t + y$$

$$y(t_0) = y_0$$

$$(e^{-t}y)' = (1-t)e^{-t}$$

Integrate both side, we get $e^{-t}y = te^{-t} + C$ So

$$y(t) = t + (y_0 - t_0)e^{t-t_0}$$

b)

$$y_k = y_{k-1} + hf(k-1)$$

$$f(k-1) = 1 - t + y$$

$$y_k = (1+h)y_{k-1} + h - ht_{k-1}$$

$$y_k = y_{k1} + h(1t_{k1} + y_{k1}) = (1+h)y_{k1} + hht_{k1}$$

d) when $h = \frac{t-t_0}{n}$

$$y_n = (1+h)^n(y_0 - t_0) + t = (1 + \frac{t-t_0}{n})^n(y_0 - t_0) + t$$

since $n \to \infty$, the upper formula becomes $y_n = (1 + \frac{t-t_0}{n})^n(y_0 - t_0) + t$

$$= e^{t-t_0}(y_0 - t_0) + t$$

Problem 7

$$y' = -2y + e^{-t}$$

$$y = e^{-}t + cC = 0$$

$$y(t) = e^{-t}$$

Problem 7 continued on next page...

So $y(1) = \frac{1}{e}$

2. Using the following code:

```
n=1;
while (1)
[x,y]=eul('fcn1',[0,1],1,1/n);
if abs(y(end)-1/exp(1))<0.05
     disp(n);
     break;
end
n=n+1;
end
```

We get the result:

```
x =

     0
     1


y =

     1
     0


x =
```

```
         0

    0.5000

    1.0000



y =

    1.0000

    0.5000

    0.3033



x =

         0

    0.3333

    0.6667

    1.0000



y =

    1.0000

    0.6667

    0.4611

    0.3248
```

so n=3

Problem 8

$$y' = 2y - 3e^{-t}$$

$$y' - 2y = -3e^{-t}$$

$$(e^{-2t}y)' = -3e^{-3t}$$

$$e^{-2t}y = e^{-3t} + C$$

$$C = 0$$

$$y(t) = e^{-t}$$

So $y(1) = \frac{1}{e}$

2. Using the same code as above, we get $n = 22$

Problem 9

1. Since

$$\frac{dy}{dt} = \frac{d\int_a^t e^{-u^2}}{dt}$$

Using fundamental theory of calculus. We get

$$\frac{dy}{dt} = f(t) = e^{-t^2}$$

2. plug in using the same code for problem 7

```
ans =

         0         0
    0.5000    0.5000
    1.0000    0.8894
    1.5000    1.0733
    2.0000    1.1260
```

So f(2)=1.1260