Differential Equation: Homework #8

Due on October 30th, 2015 at 3:10pm

Professor Heather Lee Section 061

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Problem 1

3.8

(a)We can get

$$u'' + 256u = 0$$

So

$$y = Acos(16t) + Bsin(16t)$$

Comparing the equation and the $mu'' + ku = F_0 coswt$ We get $w_0 = 16$ So the equation becomes

$$u = Acosw_0 t + Bsinw_0 t + \frac{F_0}{m(w_0^2 - w^2)} coswt$$

$$= Acos(16t) + Bsin(16t) + \frac{16}{247} cos(3t)$$

Plug it in with initial condition, we get

$$u = \frac{151}{1482} cos16t + \frac{16}{247} cos3t$$

(b) Also we get the plot

(c) The equation becomes

$$mu'' + ku = 4sinwt$$

And we can get

$$u(t) = Acos(16t) + Bsin(16t) + U(t)$$

Since

$$U(t) = \frac{32}{256 - w^2} sinwt$$

$$w = w_0 = 16$$

Problem 2

Problem k

(a)

$$y'''' = -24$$

$$y(0) = y(4) = 0$$

$$y'(0) = y'(4) = 0$$

Hence,

$$y'''' = -24$$

$$y''' = -24x + Y(x)...$$

$$y'' = -12x^2 + Y(x)...$$

$$y' = -4x^3 + Y(x)...$$

$$y = -x^4 + C_1x^3 + C_2x^2 + C_3x + C_4...$$

Plug it in with the IV, we get

$$y = -x^4 + 8x^3 - 16x^2$$

(b)

$$y' = (-(-4+x)^2x^2)'$$

$$= -4x(x^2 - 6x + 8)$$

$$= -4x(x-2)(x-4)$$

So x=2 or x=4, but when x=4 $y'' < 0$ and when x=2 $y'' > 0$. The result should be x=2, which is

$$x = \frac{L}{2}$$

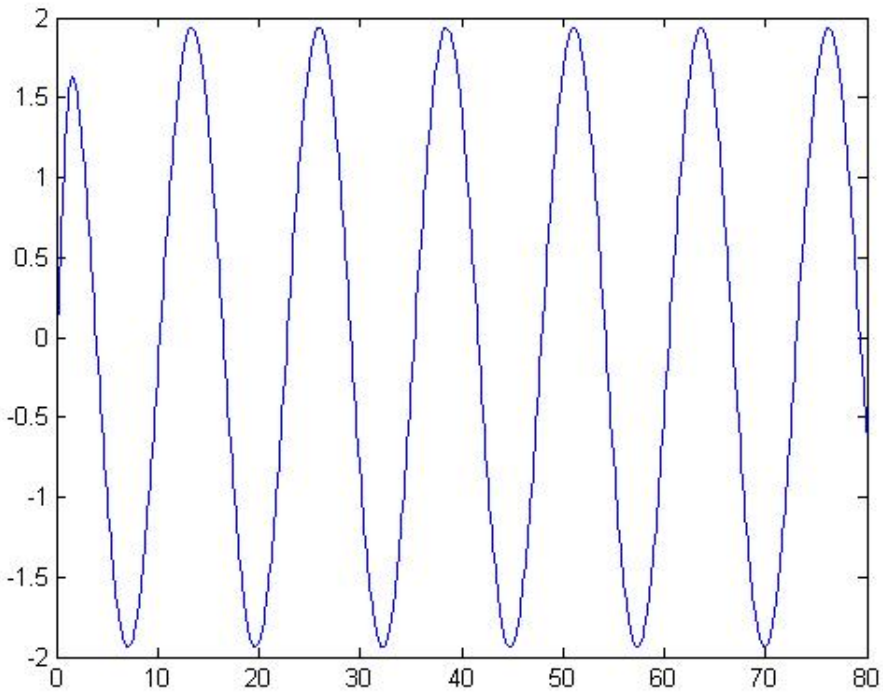
Problem 3

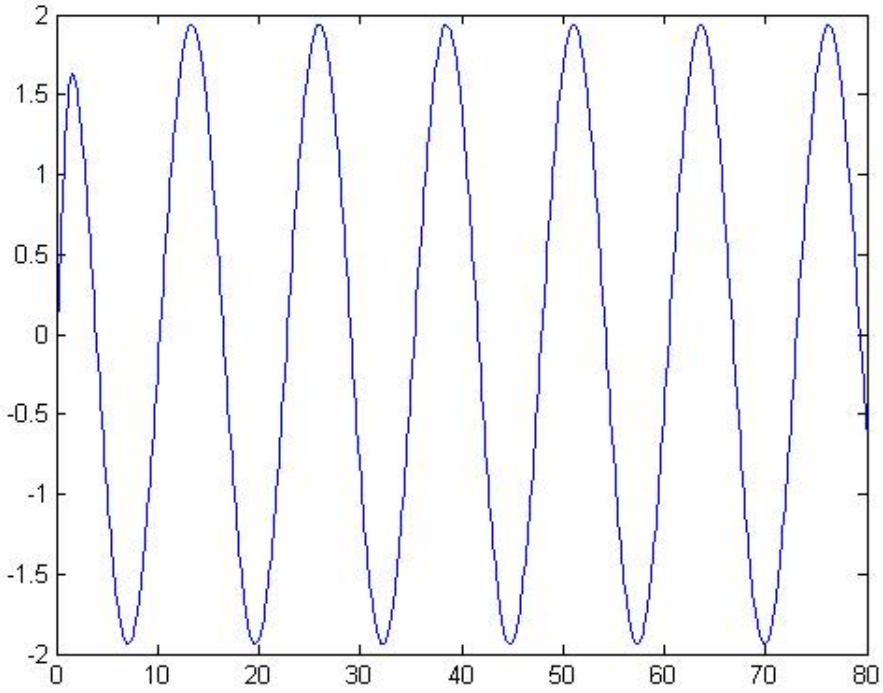
Project B

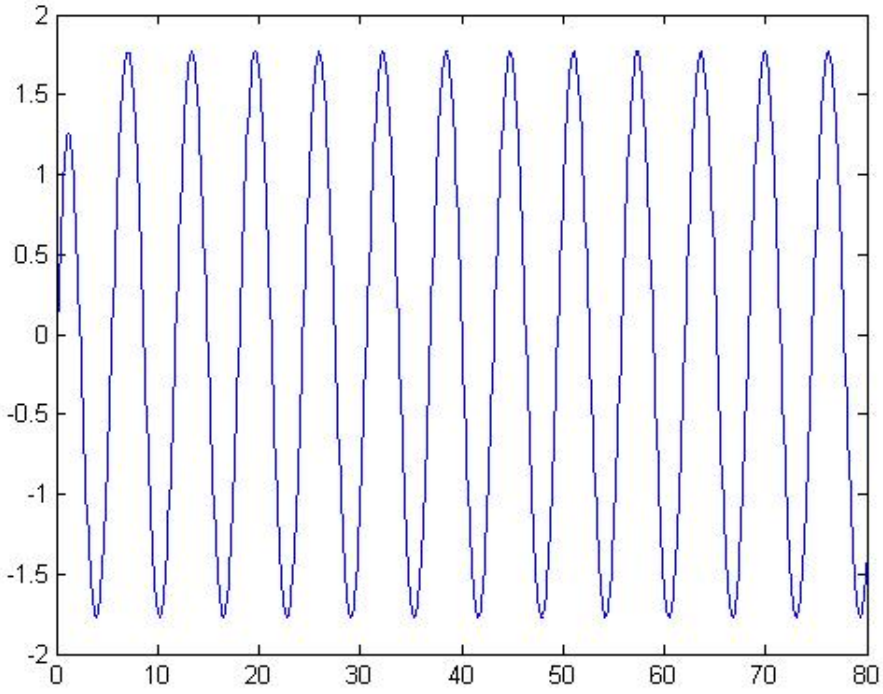
I use the code below

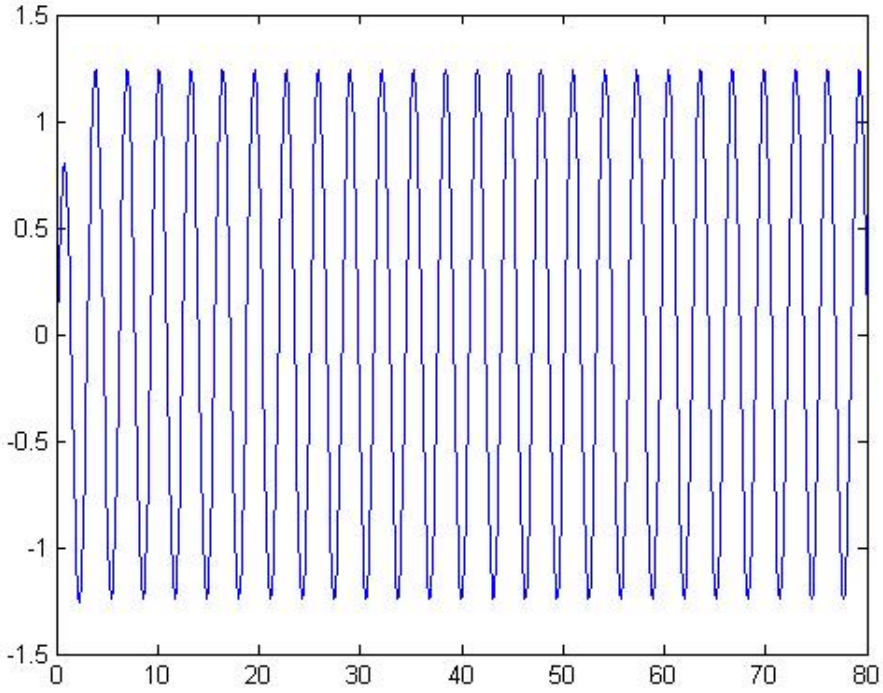
```
function xp=F(t,x)

xp=zeros(2,1); % since output must be a column vector

w=0.1;

xp(1)=x(2);

xp(2)=10*cos(w*t)-4*x(2)-5*x(1);


[t,x]=ode45('F',[0,80],[0,0]); plot(t,x(:,1))
```

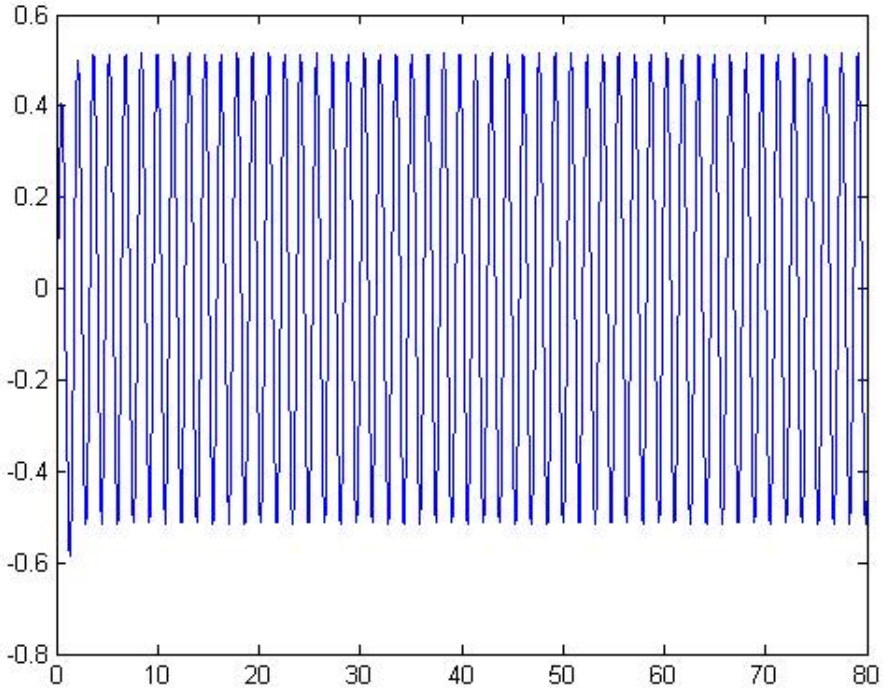
The plot below shows the result of $w = 0, 0.5, 1, 2, 4, 8, 16$ accordingly.

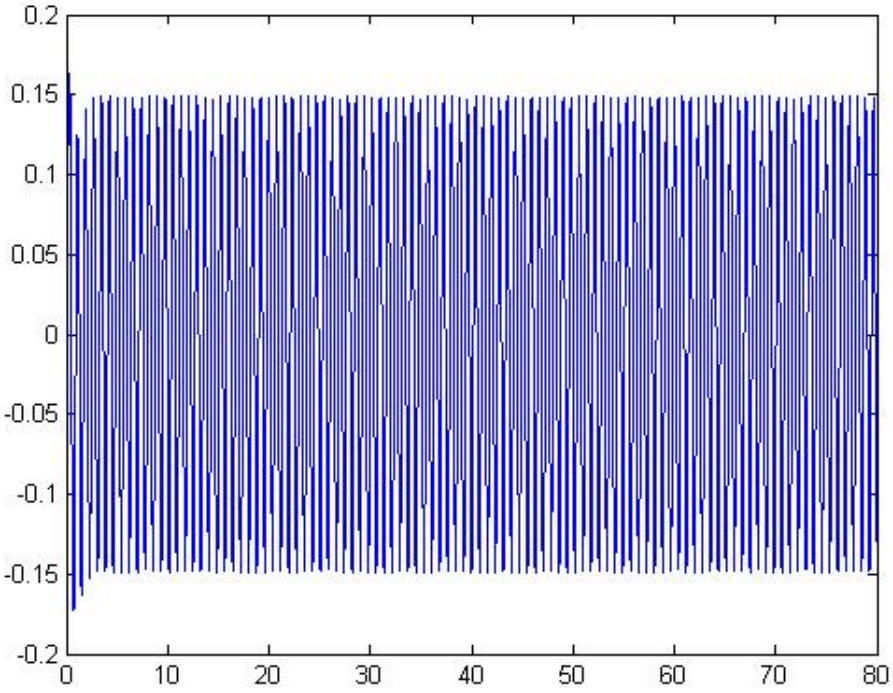


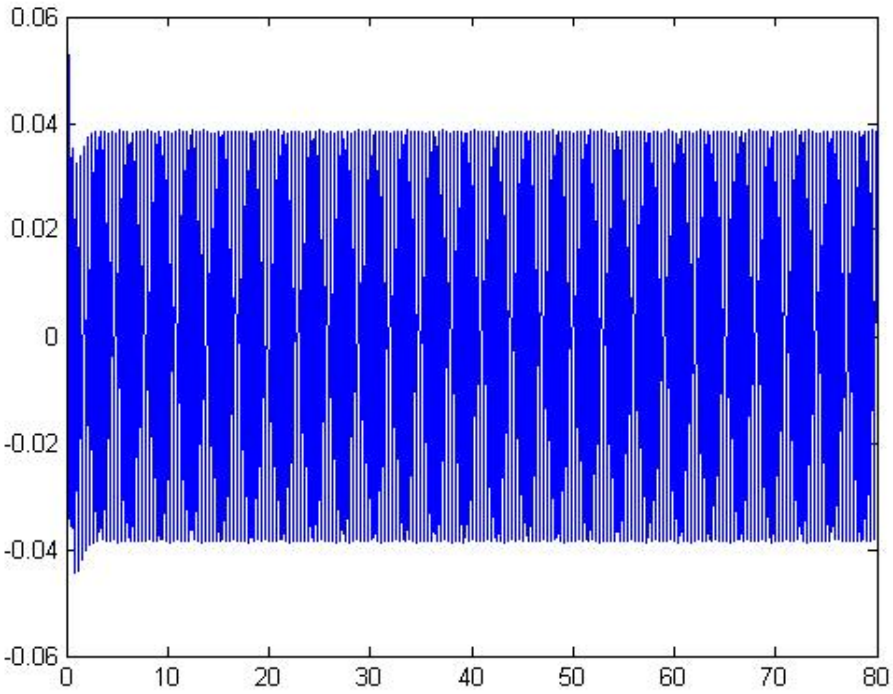

2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
10
20
30
40
50
60
70
80


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
10
20
30
40
50
60
70
80

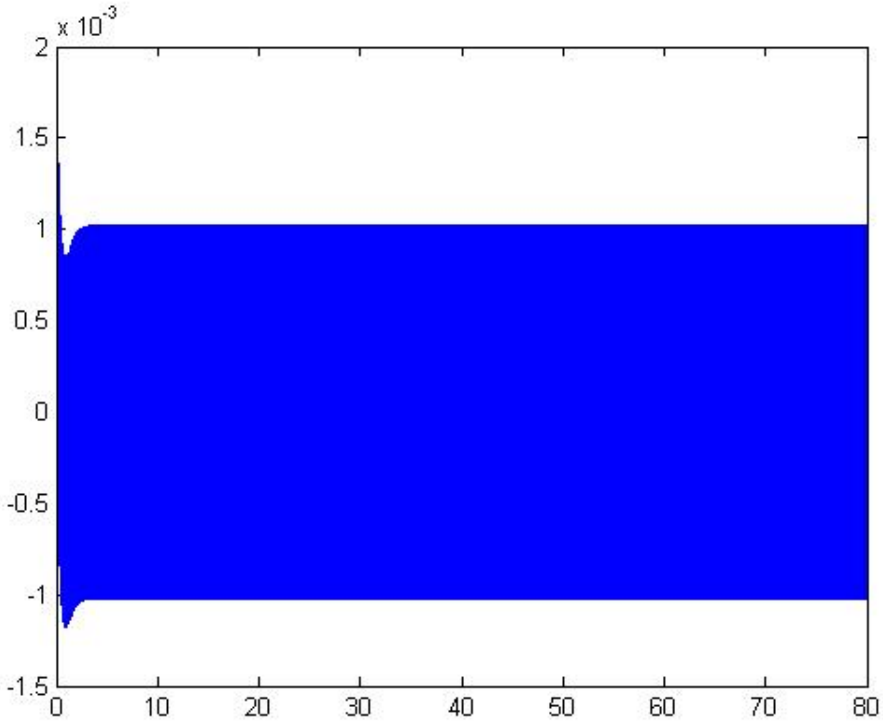

1.5
1
0.5
0
-0.5
-1
-1.5
0
10
20
30
40
50
60
70
80

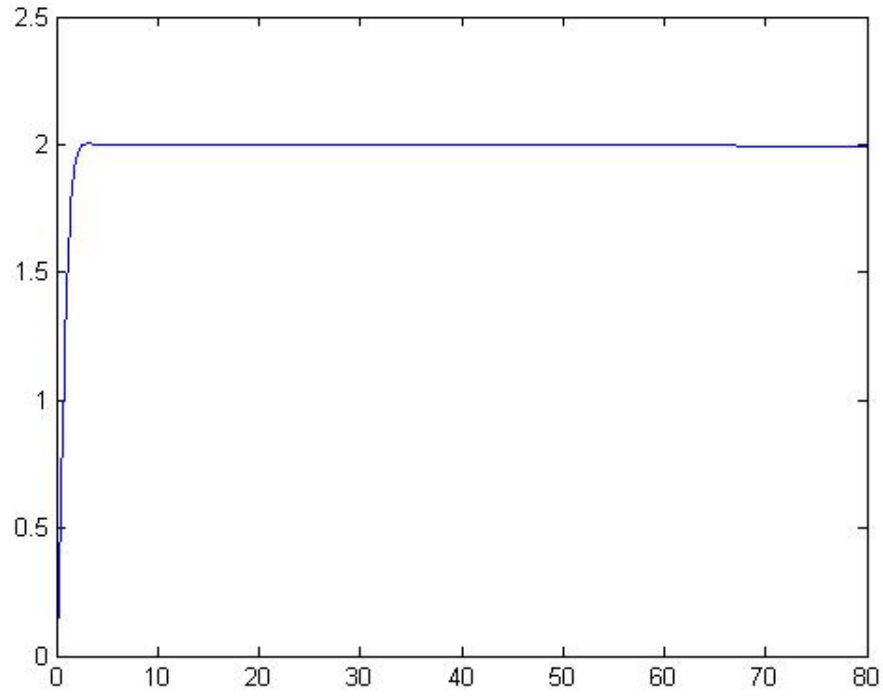

0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
10
20
30
40
50
60
70
80





(b) As state above. I tried $w = 999$ and $w = 0.01$





(The matlab actually broke several times) From the graph we could draw to the conclusion that as $w \to \infty$

the $A(w)$ (which is maximum Displacement) is getting smaller

As $w \to 0$ the $A(w)$ (which is maximum Displacement) is getting bigger.