Differential Equation: Homework #12

Due on Dec 7th, 2015 at 3:10pm

Professor Heather Lee Section 061

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Problem 1

7.3 - 17

$$\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$$

$$(3-\gamma)(-1-\gamma)-(-2)4=0$$

$$(\gamma-1)^2=-4$$

$$\gamma_1=1+2i$$

$$\gamma_2=1-2i$$

Plug it in to the original matrix, for 1+2i we get

$$(2-2i)v_1-2v_2=04v_1+(-2-2i)v_2=0$$

Solve it

$$v_1=\begin{bmatrix} 1 \\ 1-i \end{bmatrix}$$

Same for v_2

$$v_2=\begin{bmatrix} 1 \\ 1+i \end{bmatrix}$$

Problem 2

7.3 -20

$$(1-\gamma)(-1-\gamma)-3=0$$

$$\gamma=\pm 2$$

when $\gamma=2$

$$v_1=\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$$

Problem 2 continued on next page. . .

when $\gamma = -2$

$$v_2 = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$$

Problem 3

7.5-1

$$\begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$$

We found that the eigenvalue for the matrix is

$$\gamma = -1 \quad \gamma = 2$$

and the corresponding eigenvectors are

$$v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and

$$v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

So the solution is

$$x(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$$

Problem 4

7.5-4

$$\begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix}$$

Problem 4 continued on next page. . .

We found that the eigenvalue for the matrix is

$$\gamma = 2 \quad \gamma = -3$$

and the corresponding eigenvectors are

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and

$$v_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

So the solution is

$$x(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ -4 \end{bmatrix} e^{-3t}$$

Problem 5

N

$$\begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix}$$

We found that the eigenvalue for the matrix is

$$\gamma = 0 \quad \gamma = -2$$

and the corresponding eigenvectors are

$$v_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

and

$$v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

So the corresponding fundamental matrix will be

$$\begin{bmatrix} 3e^0 & 1e^{-2t} \\ 4e^0 & 2e^{-2t} \end{bmatrix}$$

Problem 6

7.6-2

$$\begin{bmatrix} -1 & -4 \\ 1 & -1 \end{bmatrix}$$

We found that the eigenvalue for the matrix is

$$\gamma = -1 + 2i \quad \gamma = -1 - 2i$$

and the corresponding eigenvectors are

$$v_1 = \begin{bmatrix} 2i \\ 1 \end{bmatrix}$$

and

$$v_2 = \begin{bmatrix} -2i \\ 1 \end{bmatrix}$$

So the fundamental set of solution is

$$x(t) = \begin{bmatrix} 2i \\ 1 \end{bmatrix} e^{(-1+2i)t} + \begin{bmatrix} -2i \\ 1 \end{bmatrix} e^{(-1-2i)t}$$

Since $re^{i\theta} = rcos(\theta) + irsin(\theta)$

$$x(t) = c_1 e^{-t} \begin{bmatrix} 2cos(2t) \\ sin(2t) \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} -2sin(2t) \\ cos(2t) \end{bmatrix}$$

Problem 7

7.6-6

$$\begin{bmatrix} 1 & 2 \\ -5 & -1 \end{bmatrix}$$

We found that the eigenvalue for the matrix is

$$\gamma = 3i \quad \gamma = -3i$$

Problem 7 continued on next page. . .

and the corresponding eigenvectors are

$$v_1 = \begin{bmatrix} 1-3i \\ \\ -5 \end{bmatrix}$$

and

$$v_2 = \begin{bmatrix} 1+3i \\ \\ 5 \end{bmatrix}$$

So the fundamental set of solution is

$$x(t) = \begin{bmatrix} 2i \\ \\ 1 \end{bmatrix} e^{(-1+2i)t} + \begin{bmatrix} -2i \\ \\ 1 \end{bmatrix} e^{(-1-2i)t}$$

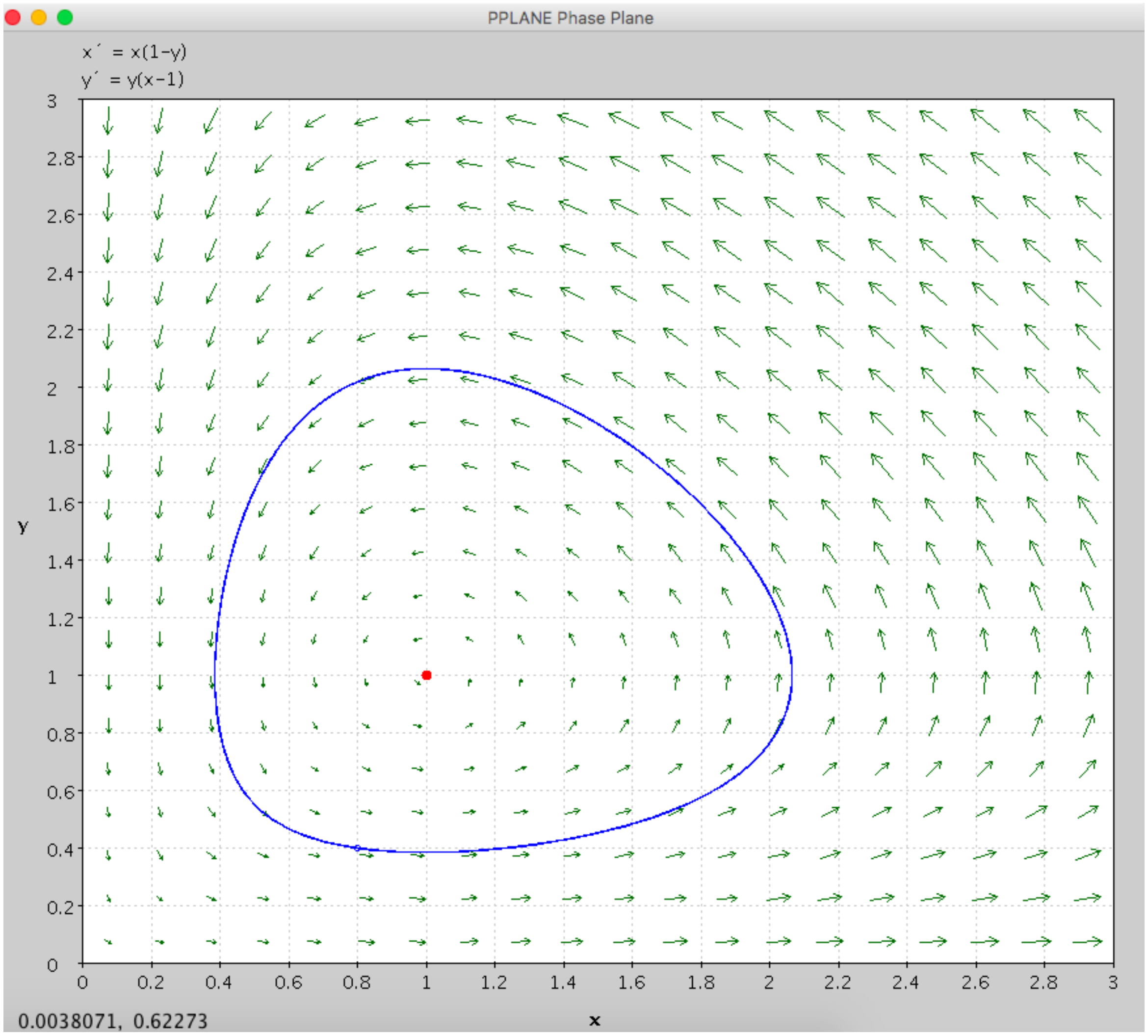
Since $re^{i\theta} = rcos(\theta) + irsin(\theta)$

$$x(t) = c_1 \begin{bmatrix} -5cos(3t) \\ \\ cos(3t) + sin(3t) \end{bmatrix} + c_2 \begin{bmatrix} -5sin(3t) \\ \\ sin(3t) - 3cos(3t) \end{bmatrix}$$

Problem 8

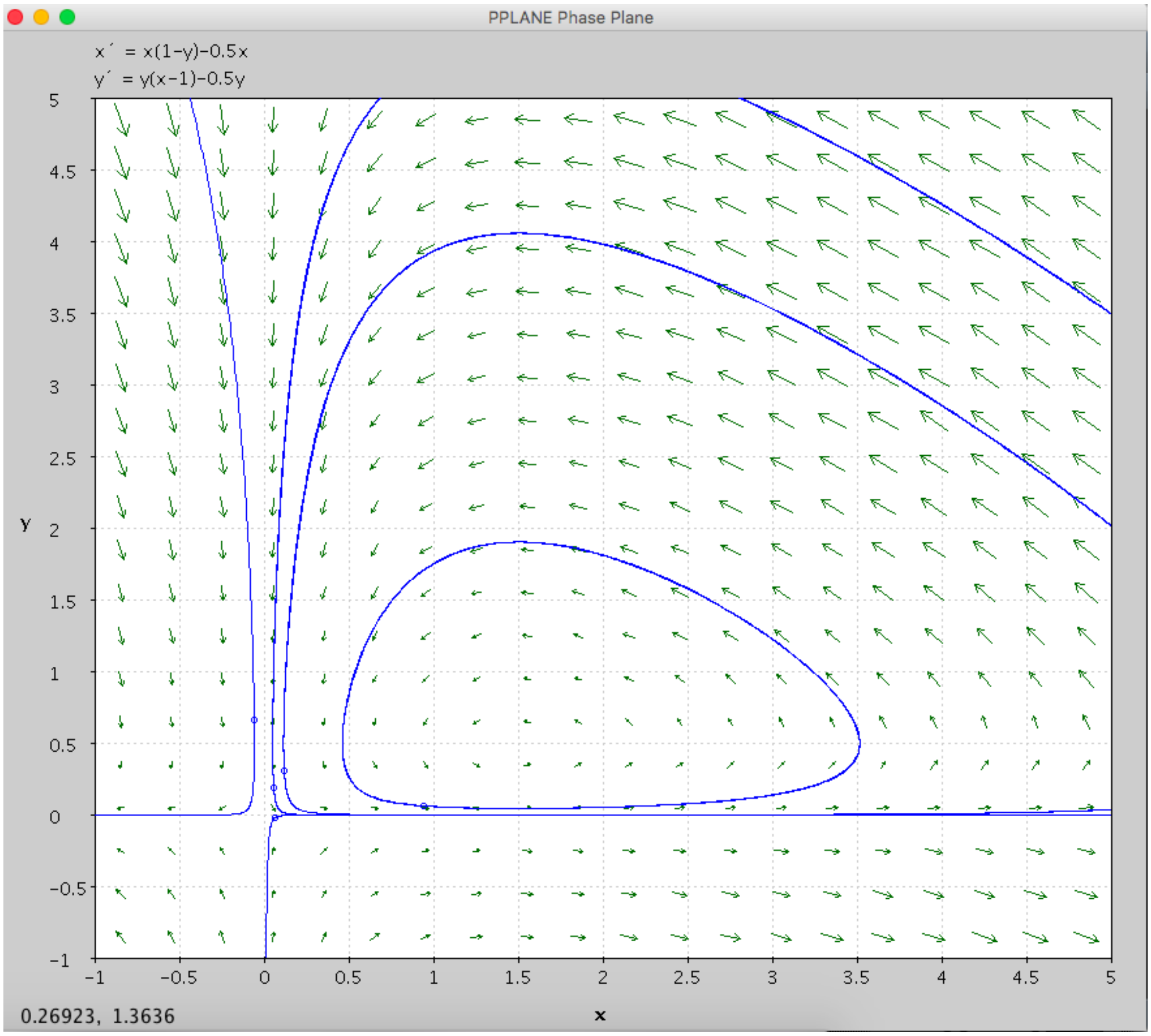
Project 3

1

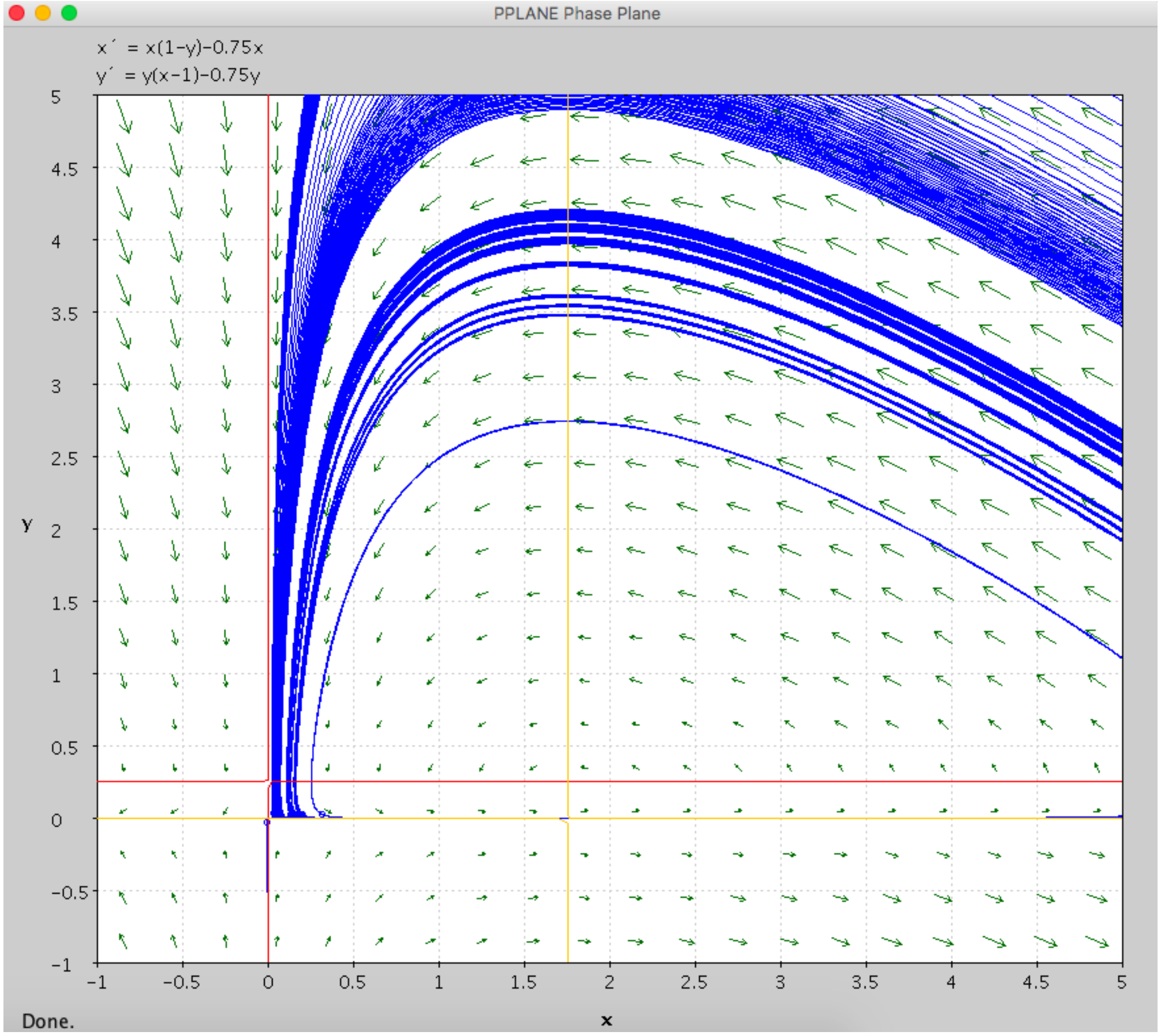


From the graph we could see the x is never less than 0.3million(300,000) so it's not eradicated as well. And the ladybug (y axis) is greater than 2 so it will exceed 2million

2

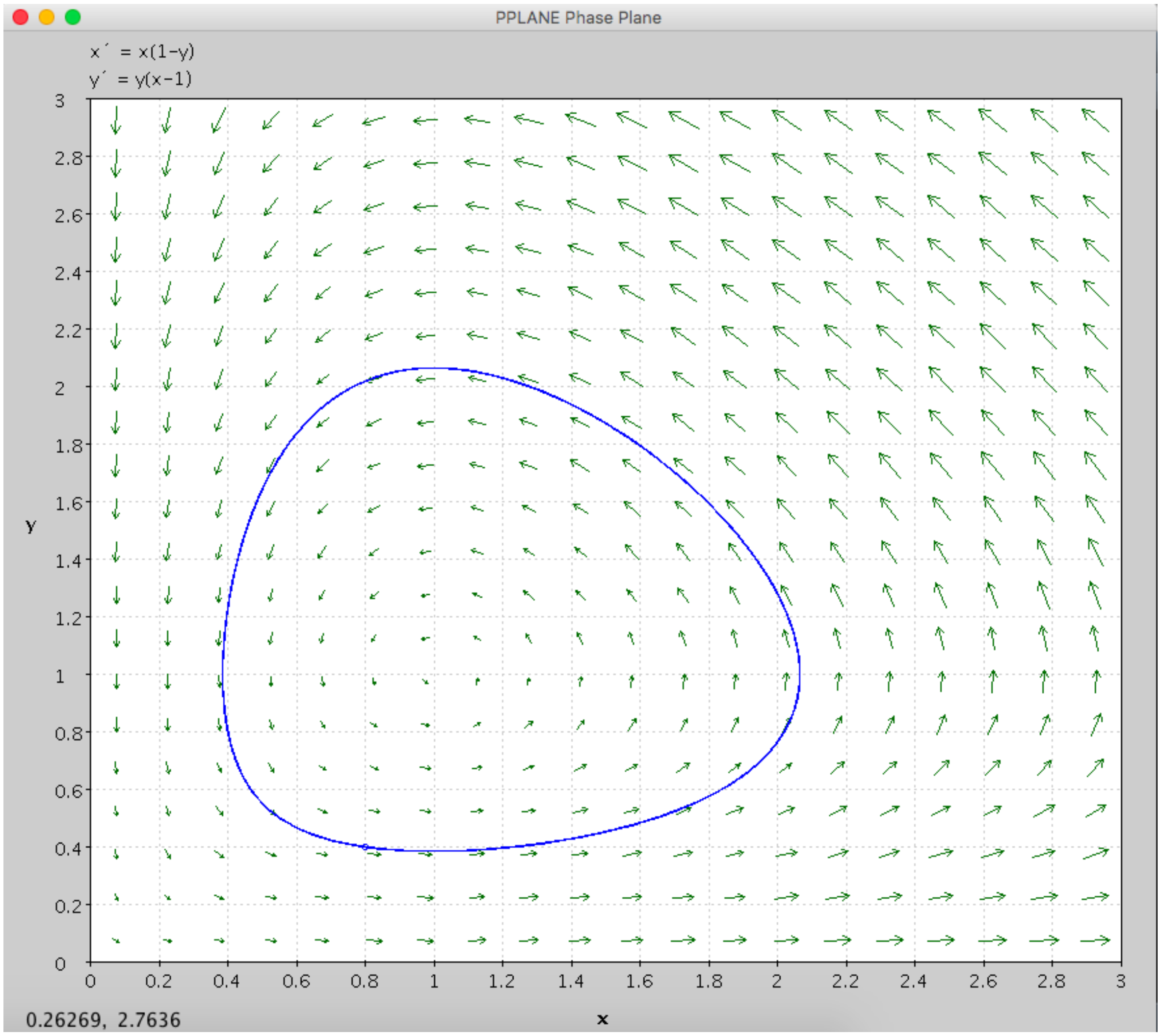


From the graph we could see that the x is never becoming 0 so it won't be eradicated for s =0.5

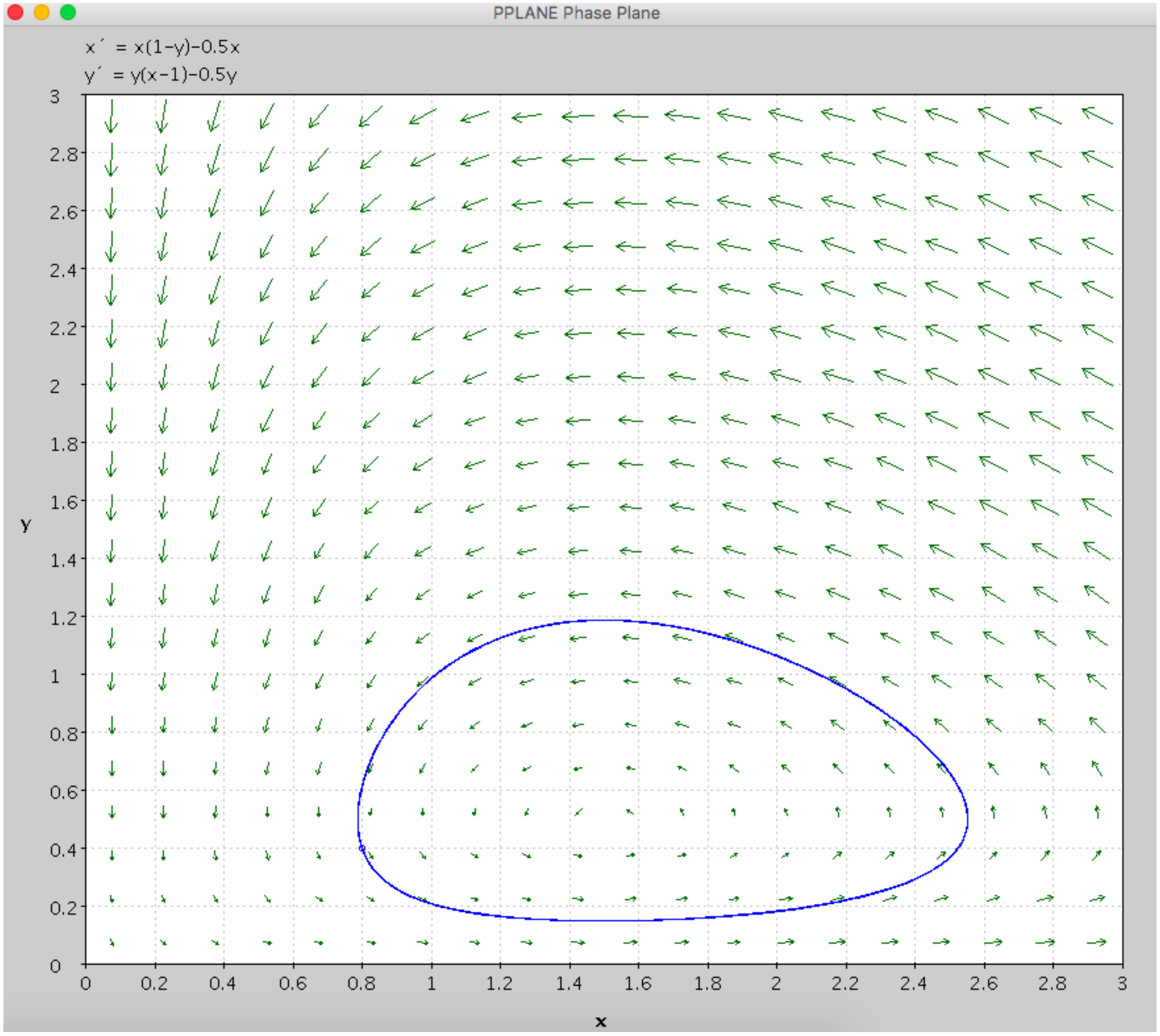


From the graph we could see that the x is never becoming 0 so it won't be eradicated. as well for s=0.75

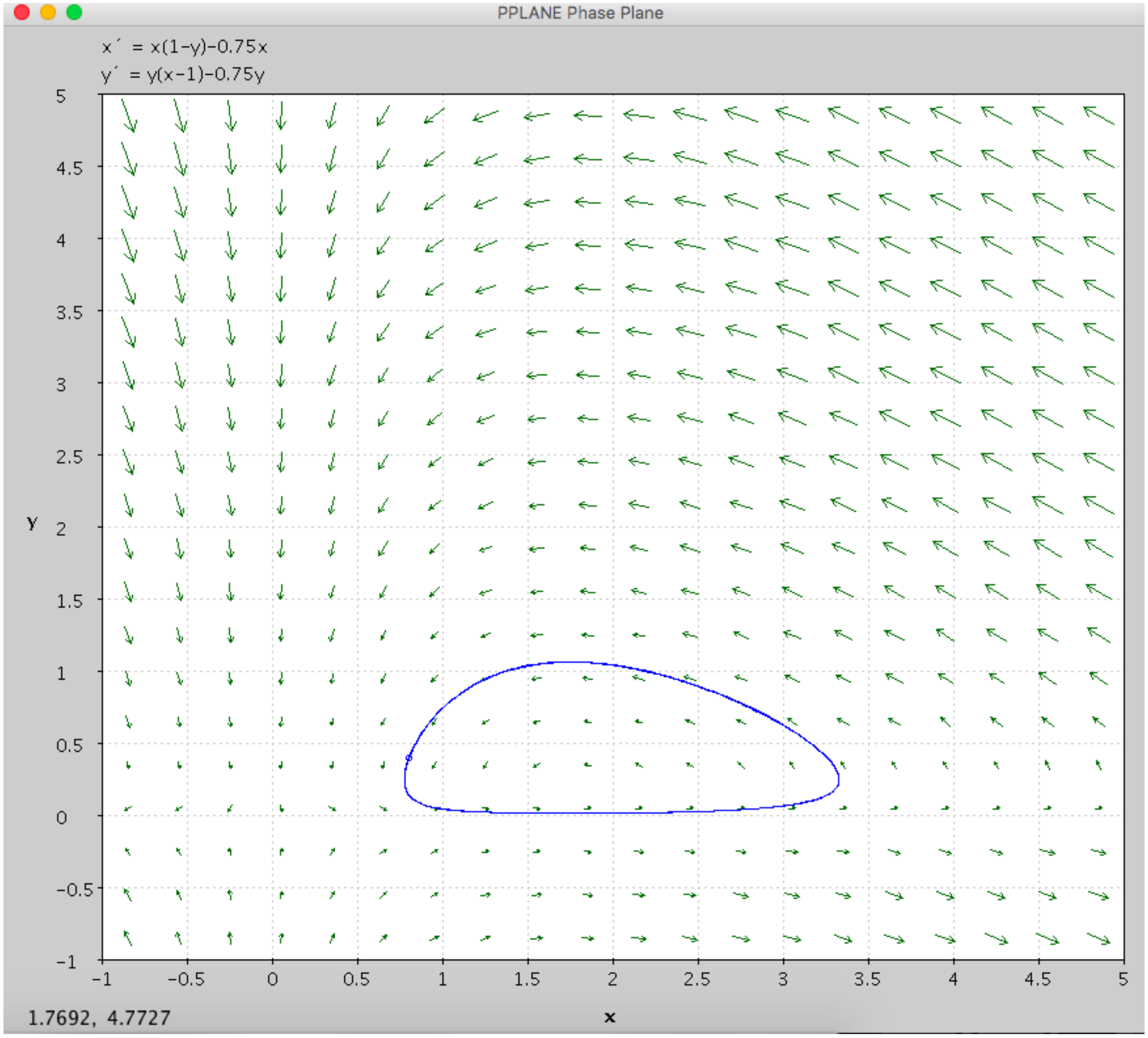
3



Problem 8 continued on next page. . .

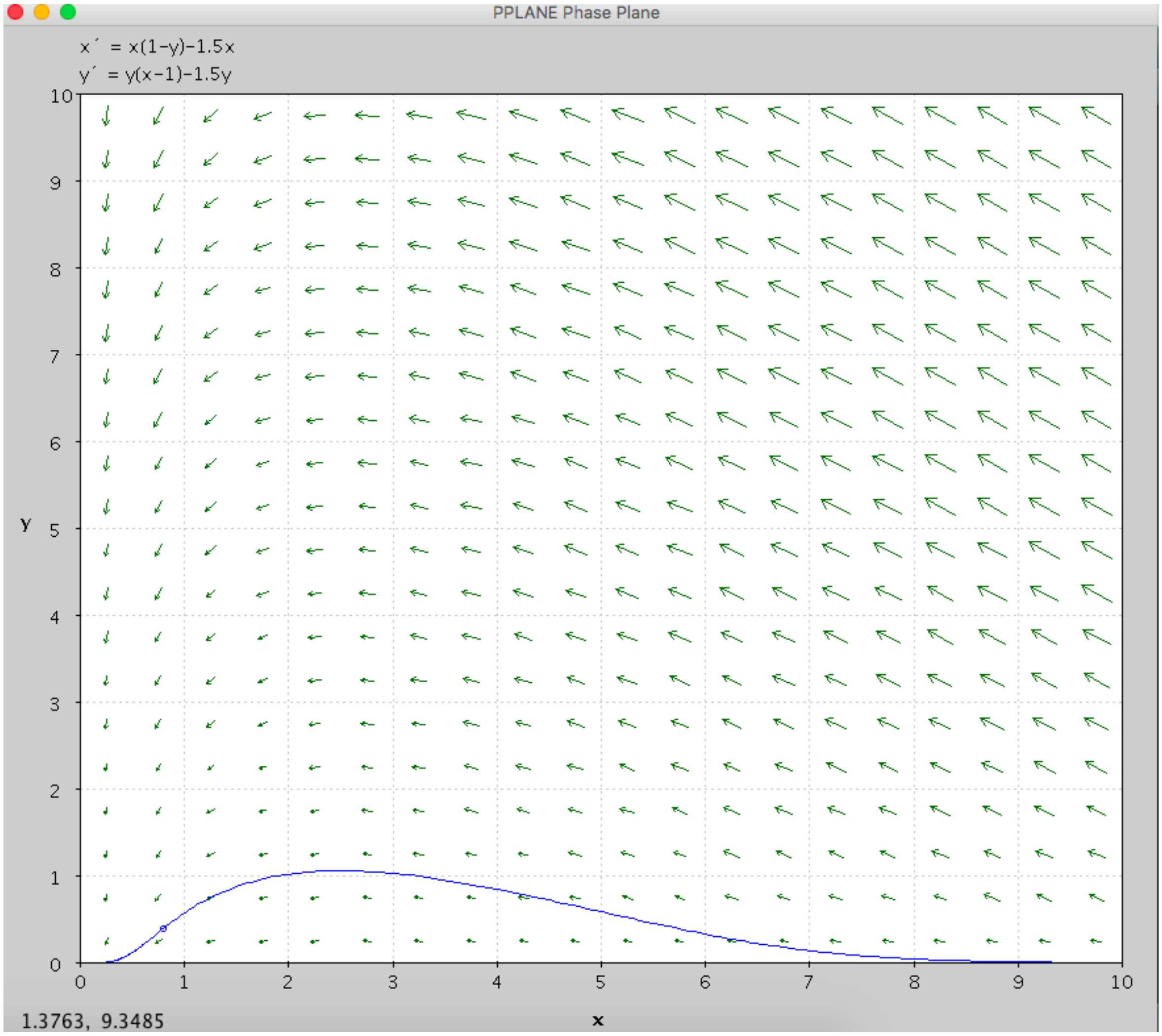


Problem 8 continued on next page. . .



From the plot above we could see it would work for s=0 and s=0.5

4



As you can see on the graph, both bugs will be eradicated