Differential Equation: Homework #13

Due on Dec 11th, 2015 at 3:10pm

Professor Heather Lee Section 061

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Problem 1

O

$$x' = x + y$$

$$y' = 4x + y$$

Apply Laplace on the both sides.

The first equation becomes

$$sX(s) - 0 = X(s) + Y(s)$$

$$(s-1)X(s) = Y(s)$$

The second one becomes

$$sY(s) - 2 = 4X(s) + Y(s)$$

$$(s-1)Y(s) = 4X(s) + 2$$

We combine these to equations, we get

$$X(s) = \frac{1}{2}[\frac{1}{s-3} - \frac{1}{s+1}]$$

Find the inverse of laplace, we get

$$x(t) = \frac{1}{2}(e^{3t} - e^{-t})$$

Also

$$x'(t) = \frac{1}{2}(3e^{3t} - e^{-t})$$

$$y = x' - x = e^{3t} + e^{-t}$$

Problem 2

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(a)

$$\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$$

We find that the eigenvalue are

$$\gamma_1 = 1 \quad \gamma_2 = -3$$

and the corresponding eigenvectors are

$$v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

and

$$v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

So

$$x = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-3t}$$

The particular solution will be

$$X_p = \begin{bmatrix} a_1 e^{2t} + a_2 \\ b_1 e^{2t} + b_2 \end{bmatrix}$$

And

$$X_p' = \begin{bmatrix} 2a_1 e^{2t} \\ 2b_1 e^{2t} \end{bmatrix}$$

Plug it in the X,we get

$$\begin{bmatrix} 2a_1 e^{2t} \\ 2b_1 e^{2t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} a_1 e^{2t} + a_2 \\ b_1 e^{2t} + b_2 \end{bmatrix} + \begin{bmatrix} 5e^{2t} \\ 3 \end{bmatrix}$$

Problem 2 continued on next page...

Solve it, plug it in, we get

$$a_1 = 5$$

$$a_2 = 0$$

$$b_1 = 2$$

$$b_2 = 1$$

So

$$x_p = \begin{bmatrix} 5e^{2t} \\ 5e^{2t} + 1 \end{bmatrix}$$

$$x = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-3t} + \begin{bmatrix} 5e^{2t} \\ 5e^{2t} + 1 \end{bmatrix}$$

(b)

We get

$$x = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-3t} + x_p(t)$$

And

$$x_p(t) = \begin{bmatrix} Acos(t) + Bsin(t) \\ Ccos(t) + Dsin(t) \end{bmatrix}$$

and

$$x_p'(t) = \begin{bmatrix} -Asin(t) + Bcos(t) \\ -Csin(t) + Dcos(t) \end{bmatrix}$$

Plug it in to the original equation, solve it. We get

$$x_p(t) = \begin{bmatrix} -5cos(t) + 5sin(t) \\ -4cos(t) + 2sin(t) \end{bmatrix}$$

$$x = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-3t} + \begin{bmatrix} -5cos(t) + 5sin(t) \\ -4cos(t) + 2sin(t) \end{bmatrix}$$