Homework 6: Due Thursday, October 6

Reading: Chapters 8,9,12,13.

Problem1 Express the following as a product of transpositions:
(1) (416)(8235)
(2) (123)(456)(1574)

Problem 2 Let $\alpha := (\alpha_1 \cdots \alpha_s) \in S_n$ be a cycle, and $\pi \in S_n$. Show that $\pi\alpha\pi^{-1}$ is the cycle $(\pi(\alpha_1) \cdots \pi(\alpha_s))$. Here we think of π as a permutation of $\{1, \ldots, n\}$. Therefore, it makes sense to consider the elements $\pi(\alpha_i) \in \{1, \ldots, n\}$.

Problem 3 Given α and π as in the previous exercise, the cycle $\pi\alpha\pi^{-1}$ is called a conjugate of α. We say that a cycle β is a conjugate of α if there is an element π such that $\beta = \pi\alpha\pi^{-1}$. Note that this already implies that there exists a π' such that $\beta = \pi'\alpha\pi'^{-1}$. Therefore, α is a conjugate of β if and only if β is a conjugate of α. In this case, we simply say that α and β are conjugates (or conjugate to one another). Use the last exercise to show that any two cycles of the same length are conjugates of each other.

Problem 4: Let $\alpha, \beta \in S_n$. Show that $\alpha\beta$ is even if an only if α and β are both even or both odd.

Problem 5: Recall, given a group G, its center $Z(G) := \{g \in G | gh = hg \forall h \in G\}$. You saw on the midterm that this is a subgroup. Show that $Z(S_n) = \{e\}$ (i.e. it consists only of the identity permutation).
Problem 6: Let $H = \{(1), (12)(34), (13)(24), (14)(23)\}$. Find the left cosets H in A_4.

Problem 7: Let K be a proper subgroup of H, and H a proper subgroup of G. If $|K| = 42$ and $|G| = 420$, what are the possible orders of H?

Problem 8: Let $|a| = 30$. How many left cosets of $< a^4 >$ in $< a >$ are there?