Homework 9: Due Thursday, November 2

Reading: Chapter 16.

Problem 1: Let $f : G \to G'$ be a group homomorphism , and $H \subset G$ be a subgroup. Show that $f(H) \subset G'$ is a subgroup. Show that if $|H| = n$, then $|f(H)|$ divides n. Finally, show that if H is a normal subgroup, then $f(H)$ is a normal subgroup.

Problem 2: Prove that $(\mathbb{Z} \times \mathbb{Z})/ < (n, m) >$ is isomorphic to $\mathbb{Z}_n \times \mathbb{Z}_m$.

Problem 3: Suppose $\phi : \mathbb{Z}_{30} \to \mathbb{Z}_{30}$ is a homomorphism such that $ker(\phi) = \{0, 10, 20\}$. If $\phi(23) = 9$, determine all elements that map to 9.

Problem 4: Are there any (non-trivial) homomorphisms from $\mathbb{Z}_8 \times \mathbb{Z}_2$ to $\mathbb{Z}_4 \times \mathbb{Z}_4$? Explain your answer.

Problem 5: Determine all homomorphisms from $\mathbb{Z}_{12} \to \mathbb{Z}_{30}$.

Problem 6: Compute the number of elements of order 2 and order 4 in each of the following groups: $\mathbb{Z}_{16}, \mathbb{Z}_8 \times \mathbb{Z}_2, \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$.

Problem 7: Find all abelian groups (up to isomorphism) of order 360.

Problem 8: Give an example of an abelian group whose order is divisible by 4 but has not have a cyclic subgroup of order 4.