

Differential Equation: Homework #8

Due on October 26th, 2015 at 3:10pm

Professor Heather Lee Section 061

Yao Xiao

Problem 1

3.8

(a) We can get

$$u'' + 256u = 0$$

So

$$y = A\cos(16t) + B\sin(16t)$$

Comparing the equation and the $mu'' + ku = F_0\cos wt$ We get $w_0 = 16$ So the equation becomes

$$u = A\cos w_0 t + B\sin w_0 t + \frac{F_0}{m(w_0^2 - w^2)}\cos wt$$

$$= A\cos(16t) + B\sin(16t) + \frac{16}{247}\cos(3t)$$

Plug it in with initial condition, we get

$$u = \frac{151}{1482}\cos 16t + \frac{16}{247}\cos 3t$$

(b) Also we get the plot

(c) The equation becomes

$$mu'' + ku = 4\sin wt$$

And we can get

$$u(t) = A\cos(16t) + B\sin(16t) + U(t)$$

Since

$$U(t) = \frac{32}{256 - w^2}\sin wt$$

$$w = w_0 = 16$$

Problem 2

Problem k

(a)

$$y'''' = -24$$

$$y(0) = y(4) = 0$$

$$y'(0) = y'(4) = 0$$

Hence,

$$y'''' = -24$$

$$y''' = -24x + Y(x) \dots$$

$$y'' = -12x^2 + Y(x) \dots$$

$$y' = -4x^3 + Y(x) \dots$$

$$y = -x^4 + C_1x^3 + C_2x^2 + C_3x + C_4 \dots$$

Plug it in with the IV, we get

$$y = -x^4 + 8x^3 - 16x^2$$

(b)

$$y' = (-(-4 + x)^2 x^2)'$$

$$= -4x(x^2 - 6x + 8)$$

$$= -4x(x - 2)(x - 4)$$

So $x=2$ or $x=4$, but when $x=4$ $y'' < 0$ and when $x=2$ $y'' > 0$. The result should be $x=2$, which is

$$x = \frac{L}{2}$$

Problem 3

Project B

I use the code below

```
function xp=F(t,x)

xp=zeros(2,1); % since output must be a column vector

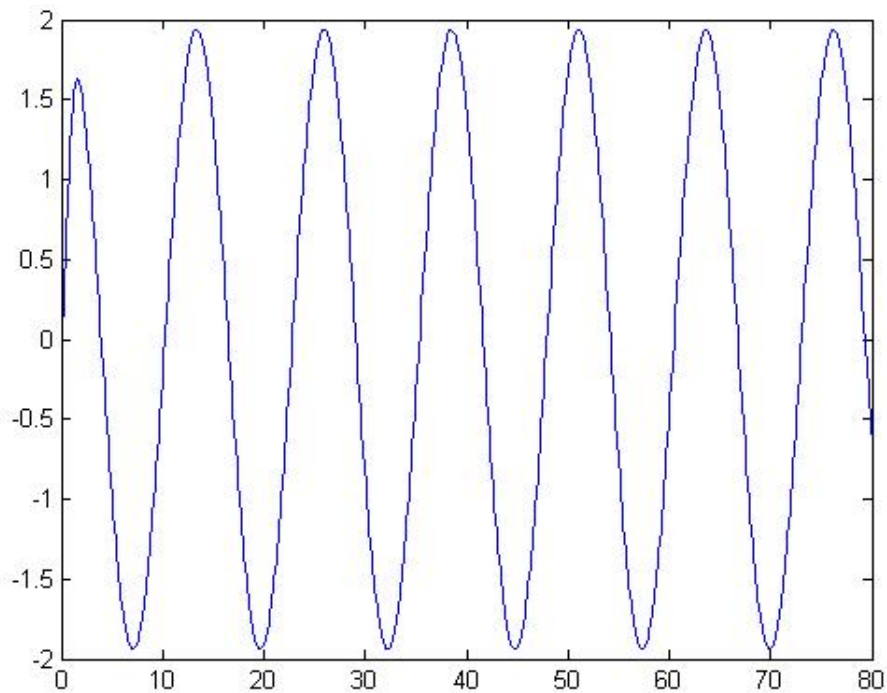
w=0.1;

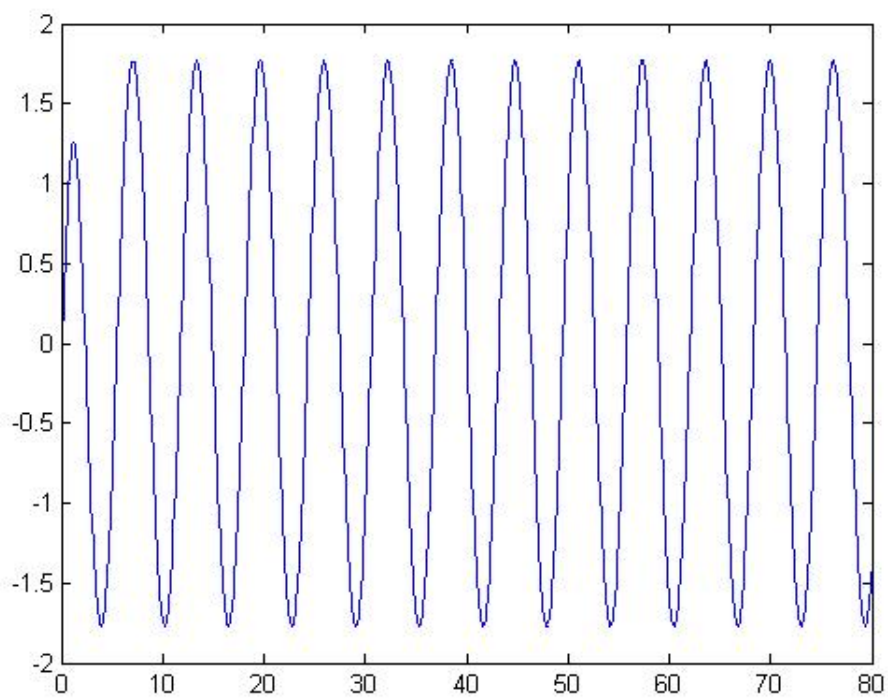
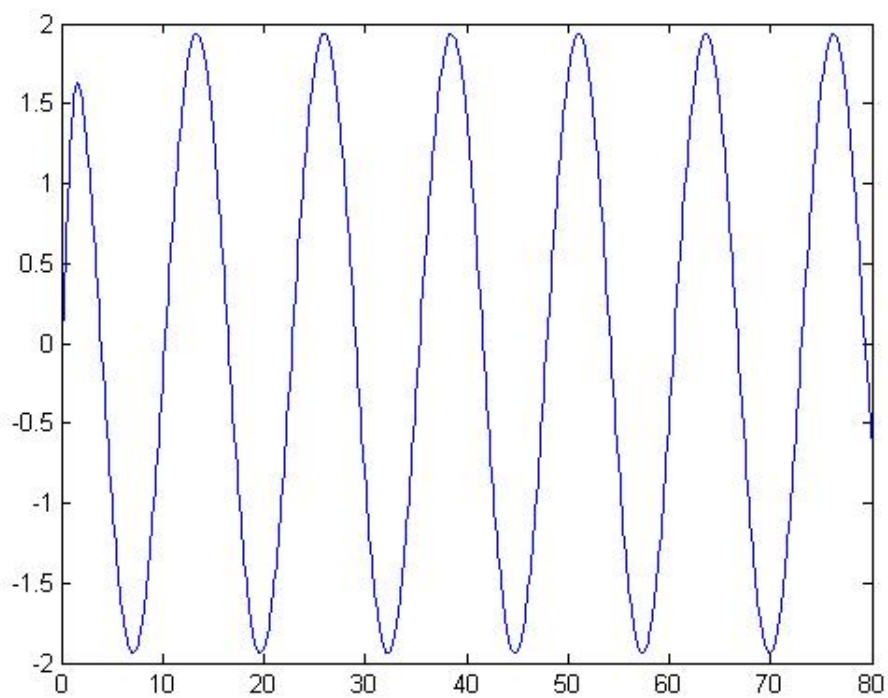
xp(1)=x(2);

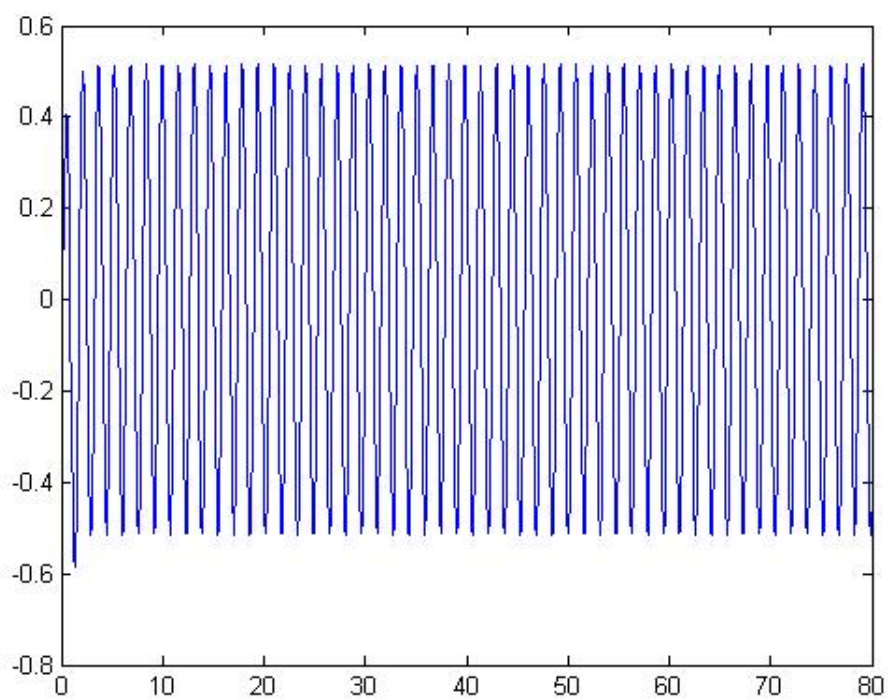
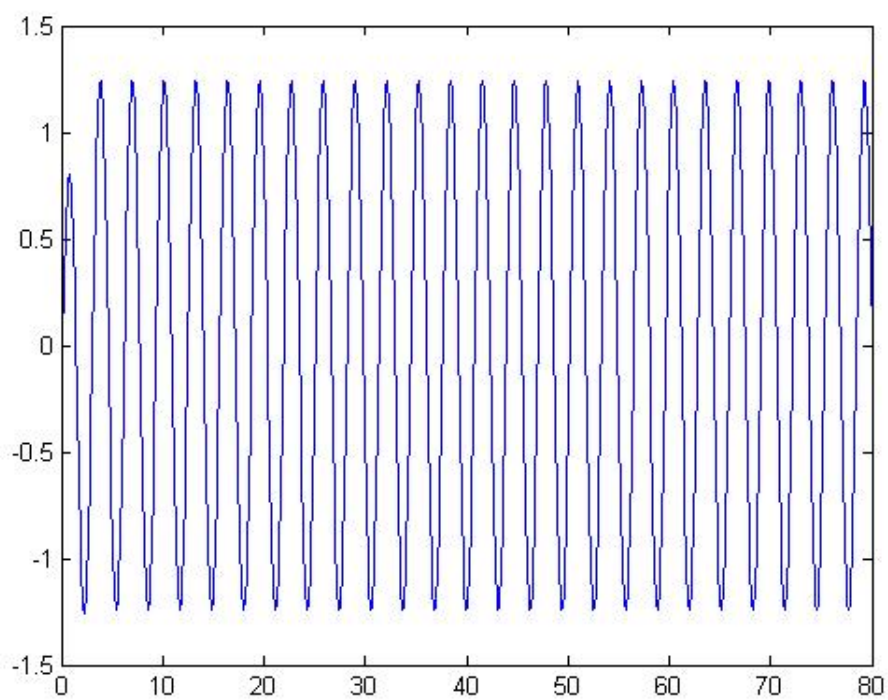
xp(2)=10*cos(w*t)-4*x(2)-5*x(1);
```

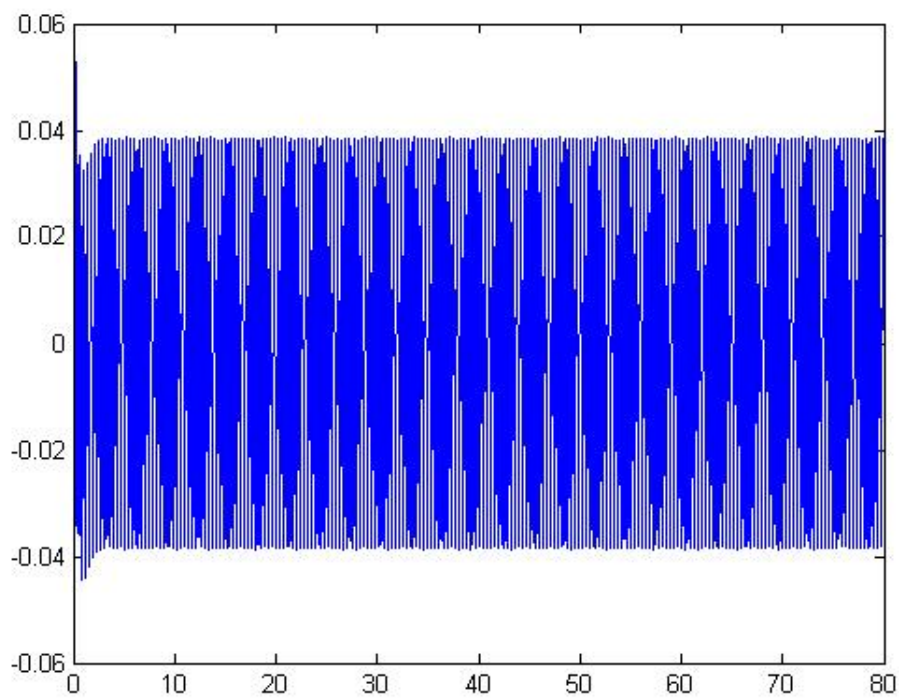
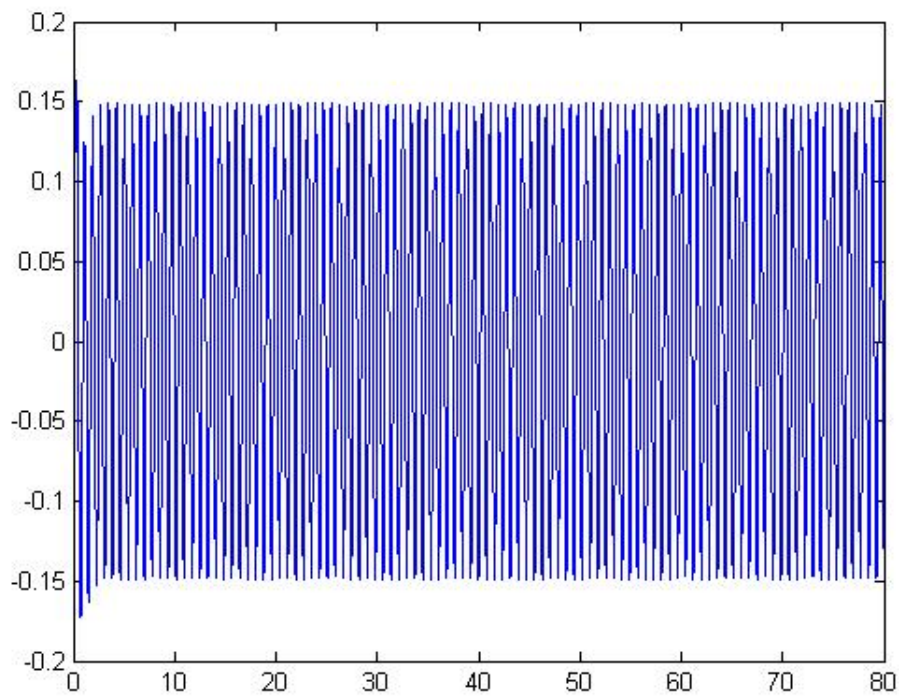
```
[t,x]=ode45('F',[0,80],[0,0]); plot(t,x(:,1))
```

The plot below shows the result of $w = 0, 0.5, 1, 2, 4, 8, 16$ accordingly.

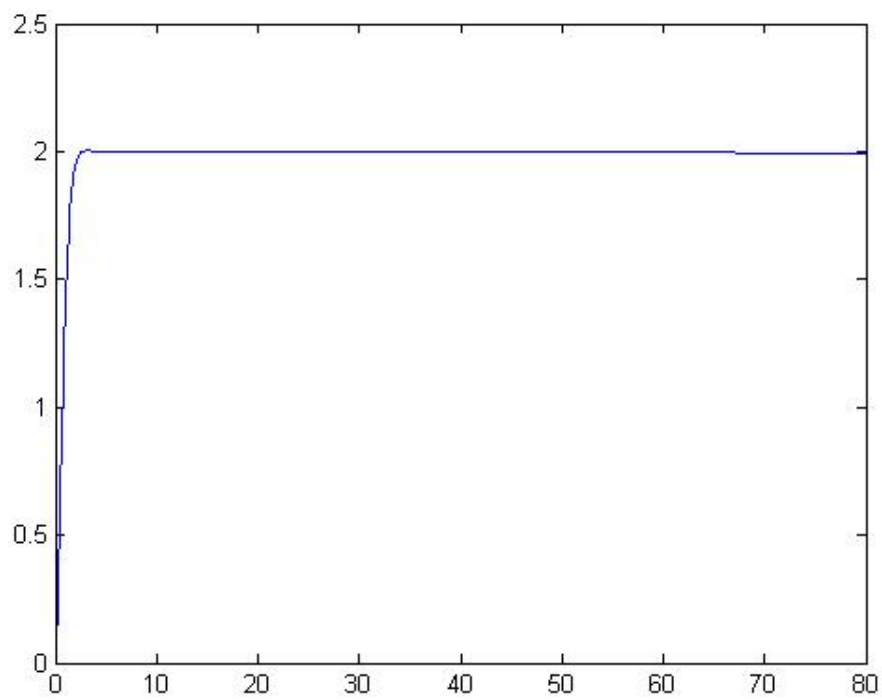
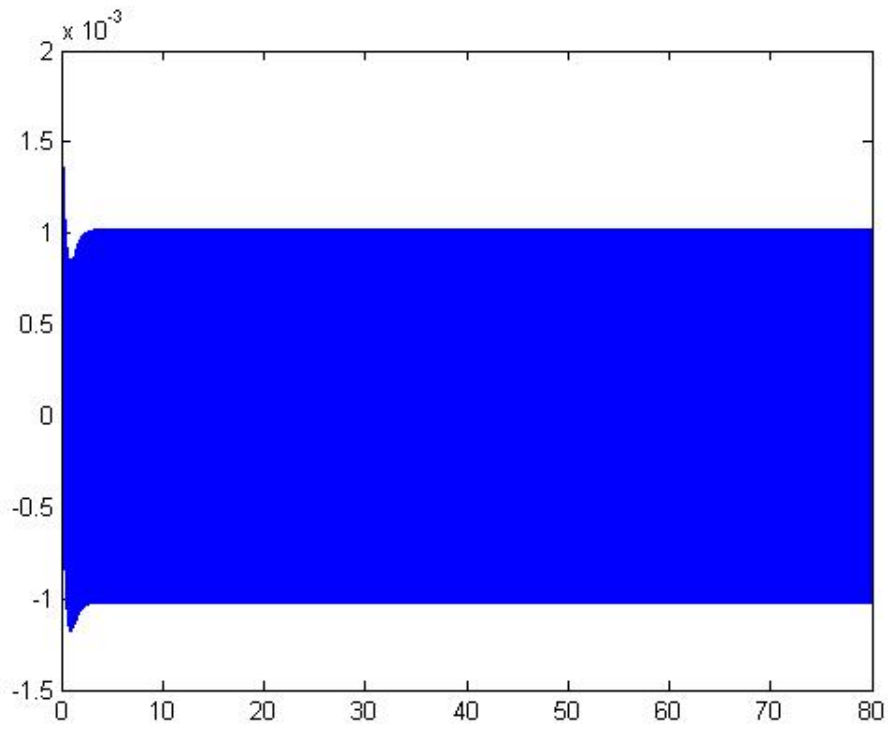








(b) As state above. I tried $w = 999$ and $w = 0.01$



(The matlab actually broke several times) From the graph we could draw to the conclusion that as $w \rightarrow \infty$

the $A(w)$ (which is maximum Displacement) is getting smaller

As $w \rightarrow 0$ the $A(w)$ (which is maximum Displacement) is getting bigger.