Homework 4: Due Tuesday, September 20

Reading: Chapters 5, 10.

Problem 1: Find all the generators of $\mathbb{Z}_8$ and $\mathbb{Z}_{20}$.

Problem 2: Give an example of a non-cyclic group all of whose subgroups are cyclic.

Problem 3: Suppose $(G, *, e)$ is an abelian group of order 35, and that every element $x \in G$ satisfies the equation $x^{35} = e$. Show that G is cyclic.

Problem 4: List all elements of $\mathbb{Z}_{40}$ that have order 10. Let $x \in \mathbb{Z}_{40}$ such that $|x| = 40$. List all elements of order 10 in $< x >$.

Problem 5: Prove that a group of order 3 must be cyclic.

Problem 6: Let H and K denote two subgroups of a groups G. Show that $H \cap K$ is also a subgroup of G.

Problem 7: Let G be a cyclic group of order n and let r be an integer dividing n. Prove that G has exactly one subgroup of order r.

Problem 8: Prove that in any group the orders of ab and ba are equal.