Differential Equation: Homework #10

Due on November 6th, 2015 at 3:10pm

Professor Heather Lee Section 061

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Problem 1

6.5 - 14

a

$$y'' + \gamma y' + y = \delta(t-1)$$

$$y'' + \frac{1}{2}y' + y = \delta(t-1)$$

$$s^2\mathcal{L}(y) - sy(0) - y'(0) + \frac{1}{2}[s\mathcal{L}(y) - y(0)) + \mathcal{L}(y) = e^{-s}$$

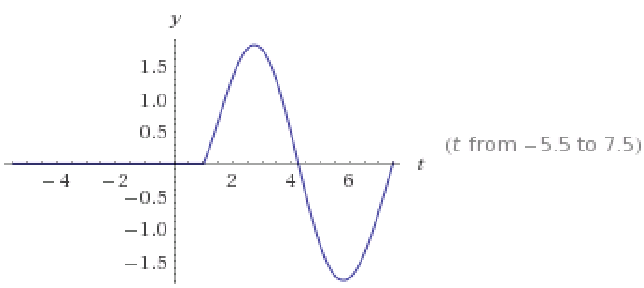
Plug it in, we get

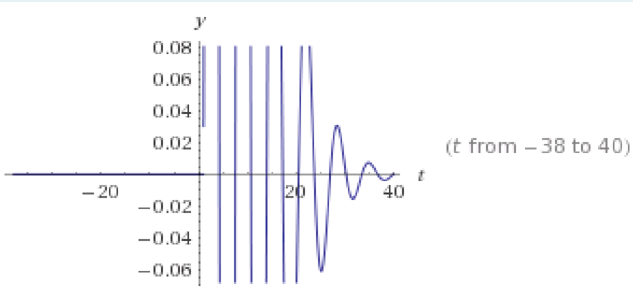
$$Y(s) = \frac{e^{-s}}{s^2 + \frac{1}{2}s + 1}$$

Inverse the Laplace Transform. We get

$$\begin{aligned} y(t) &= \mathcal{L}^{-1}\left(\frac{e^{-s}}{(s+\frac{1}{4})^2 + (\frac{\sqrt{15}}{4})^2}\right) \\ &= \frac{4}{\sqrt{15}}u_1(t)e^{-\frac{1}{4}(t-1)}sin\left(\frac{\sqrt{15}}{4}(t-1)\right) \end{aligned}$$

Plots:





b

For maximum, let $y'(t1) = 0$ We get $t = 2.36134$ Plug it in, we get $y = 0.711531$

Problem 2

L

$$y'' + 4y = 20e^t$$

$$y'(0) = 0 \ y(0) =$$

$$s^2y(s) - sy(0) - y'(0) + 4y(s) = \frac{20}{s-1}$$

$$y(s) = \frac{20}{(s-1)(s^2+4)}$$

Find the inverse of $\mathcal{L}Y(s)$

$$\begin{aligned} y(s) &= \frac{A}{s-1} + \frac{Bs+C}{s^2+4} \\ &= \frac{4}{s-1} + \frac{-4s-4}{s^2+4} \\ &= \frac{4}{s-1} - \frac{4s}{s^2+4} - \frac{4}{s^2+4} \end{aligned}$$

Looking up the table , we get

$$y(t) = 4e^t - 4cos(2t) - 2sin(2t)$$