Elements of Algebra I: Homework #1

Due on January 17th, 2017

Professor Deepam Patel Section 161

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Problem 1

1. The elements of the set are all numbers between 1 and 3, excluding 1 and 3
2. An empty set

Problem 2

$$\begin{aligned} A \cap (B \setminus C) &:= \{x : x \in A \; and(\; x \in B \; and \; x \notin C)\} \\ &= \{x : x \in A \; and \;\; x \in B \; and \; x \notin C)\} \\ &= \{x : (x \in A \; and \;\; x \in B) \; and \; x \notin C)\} \\ &= \{x : x \in A \cap B \; and \; x \notin C)\} \\ &= (A \cap B) \setminus C \end{aligned}$$

Problem 3

$$\begin{aligned} (A \cap B)^c : &= \{x \in S \; and \; x \notin (A \cap B)\} \\ &= \{x \in S \; and \; x \notin A \; and \; x \notin B)\} \\ &= \{x \in S \; and \; x \notin A\} \cap \{x \in S \; and \; x \notin B)\} \\ &= \{x \in S \; and \; x \in A^c\} \cap \{x \in S \; and \; x \in B^c)\} \\ &= A^c \cap B^c \end{aligned}$$

Problem 4

$$\begin{aligned} (A \times B) \cap (C \times D) &= \{(x, y) : x \in A \; and \; y \in B\} \cap \{(x, y) : x \in C \; and \; y \in D\} \\ &= \{(x, y) : x \in A \; and \; y \in B \; and \; x \in C \; and \; y \in D\} \\ &= \{(x, y) : x \in A \; and \; x \in C \; and \; y \in B \; and \; y \in D\} \\ &= \{(x, y) : x \in A \cap C \; and \; y \in B \cap D\} \\ &= (A \cap C) \times (B \cap D) \end{aligned}$$

Problem 5

$$\begin{aligned} LHS = (\neg(p \to q)) = \neg(\neg p \vee (p \wedge q)) = p \wedge \neg(p \wedge q) = p \wedge (\neg p \vee \neg q) = (p \wedge \neg p) \vee (p \wedge \neg q) \\ = False \vee p \wedge \neg q = p \wedge \neg q = RHS \end{aligned}$$

Problem 6

Suppose the two numbers are a and b

$$\begin{aligned} \exists x, y \in \mathbb{Z} \; a = 2x + 1, b = 2y + 1 \\ a + b = 2x + 2y + 2 = 2(x + y + 1) \end{aligned}$$

So the sum of the two odd number is even

Problem 7

Suppose x+4 is odd, then $\exists y\ x+4=2y+1\ x+7=x+4+3=2y+1+3=2(y+2)$
So x+7 is even.
On the other hand, suppose x+7 is even then $\exists y\ x+7=2y\ x+4=x+7-3=2y-3=2(y-2)+1$
So x+4 is odd

Problem 8

$$A=\{1,2\}$$
$$B=\{2,3\}$$
$$A\setminus B=\{1\}$$
$$B\setminus A=\{3\}$$