Differential Equation: Homework #6

Due on October 5th, 2015 at 3:10pm

Professor Heather Lee Section 061

Yao Xiao

Problem 1

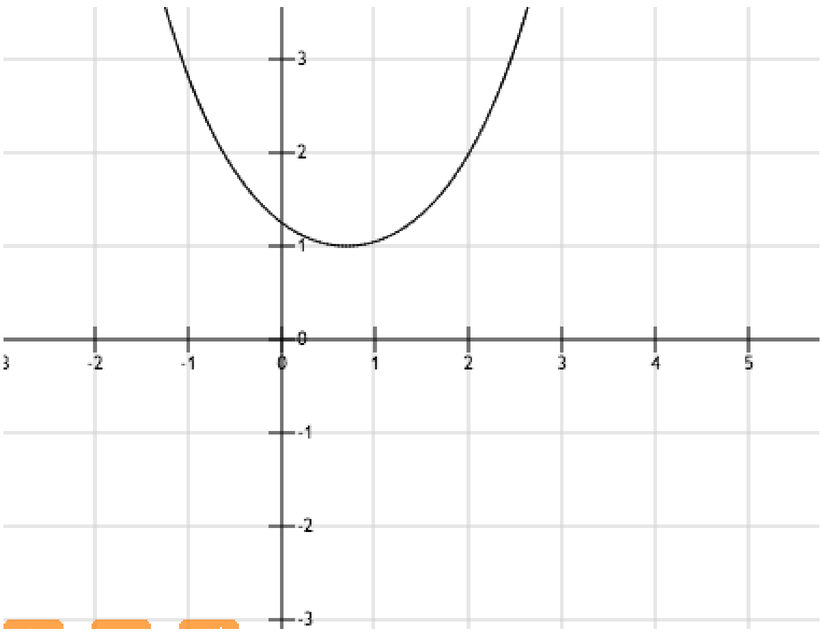
$$y'' - 1 = 0$$

We could get $r = -1$ and $r = 1$ So

$$y = Ae^t + Be^{-t}$$

We plug in the IV, get

$$y = 1/4e^t + e^{-t}$$



When $t = ln2$ $y = 1$

Problem 2

$$4y'' - y = 0$$

$$y = Ae^{1/2t} + Be^{-1/2t}$$

Plug in the IV, we get

$$A = 1 + \beta$$

$$B = 1 - \beta$$

As $t \to \infty$ $y \to 0$ So

$$\beta = -1$$

Problem 3

$$y_1' = 2t y_2' = -t^{-2}$$

Plug it in, it equals to 0, so it's the solution

Since $wronskian(x^2, x^{-1}) \neq 0$ It is all the solution

Problem 4

Plug it in, we get y1,y2 equals to 0, so it's the solution

But

$$y = c1 + c2t^{1/2}$$

is not the solution.

It doesn't contradict the theorem, because $yy'' + y'^2 = 0$ is not a linear equation.

Problem 5

Since y1 is the solution (verified by plug it in) y2 is also the solution (verified by plug it in) and $wronskian(y1, y2) \neq 0$ So it forms a fundamental set of solutions

Problem 6

Since $r = \pm 2i$ The equation should be

$$y = Acos(2t) + Bsin(2t)$$

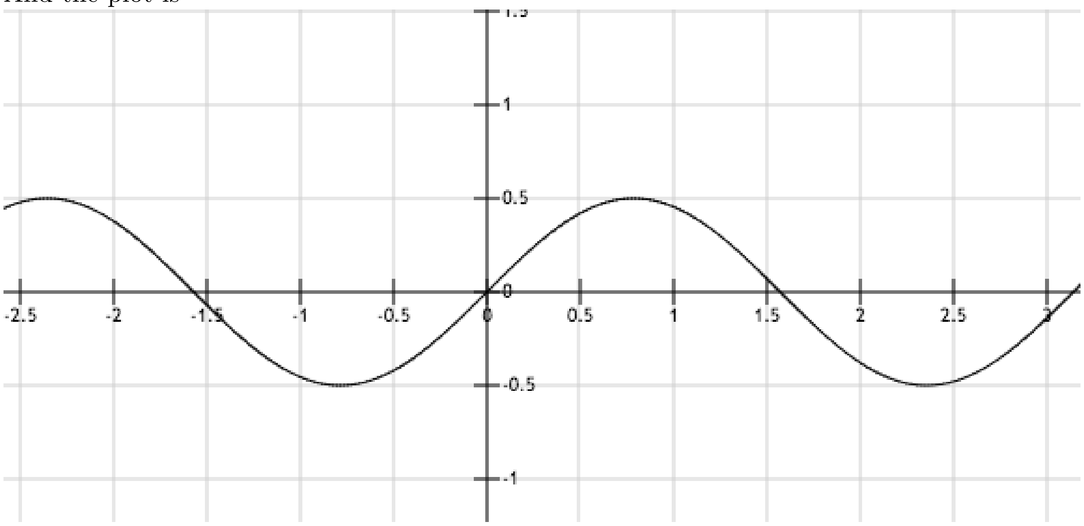
Plug in the IV, we get

$$A = 0 B = 1/2$$

So the solution is

$$y = \frac{1}{2} sin(2t)$$

And the plot is



As

$$t \to \infty y \to [-0.5, 0.5]$$

Problem 7

$$r^2 - \frac{1}{3}r + \frac{2}{3} = 0$$

Solve it, we get

$$r = \frac{1 \pm \sqrt{23}}{6}$$

We plug it in with the initial value. We get

$$u(x) = -2/23e^{x/6} * [\sqrt{23}sin((\sqrt{23}x)/6) - 23cos((\sqrt{23}x)/6)]$$

2. plug in u=10, we get $t = 10.76$

Problem 8

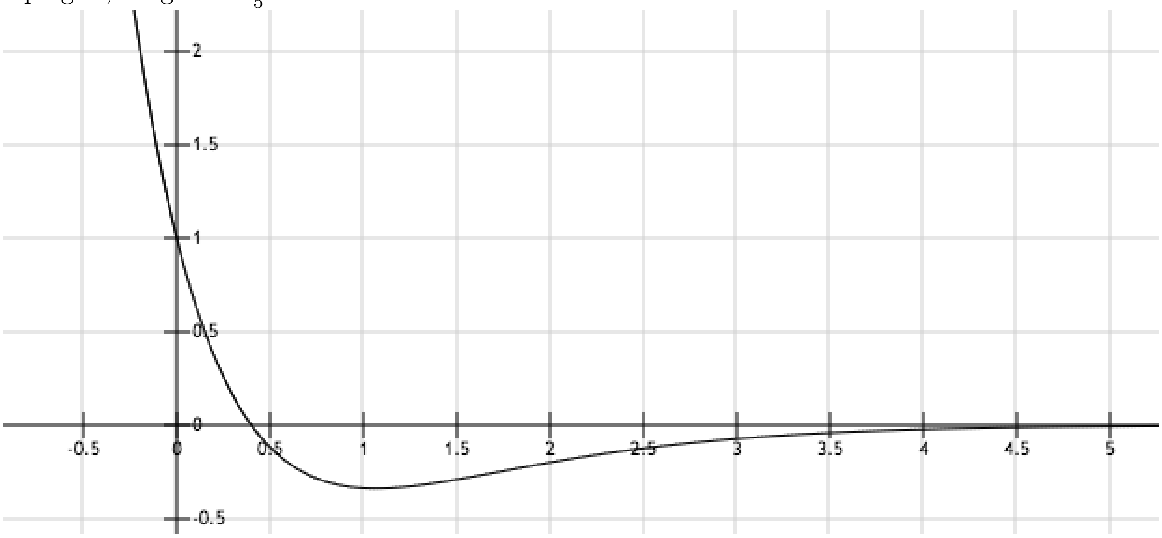
1.We could get

$$r = \frac{3}{2}$$

Plug it in with IV. We get

$$y(t) = e^{\frac{-3t}{2}} - \frac{5}{2}e^{\frac{-3t}{2}}$$

2.plug in, we get $t = \frac{2}{5}$



3.Let y'=0, we get

$$y = \frac{-5}{3e^{8/5}} \ at \ x = \frac{16}{15}$$

4.We plug in with the new IV. We get

$$y(t) = e^{\frac{-3t}{2}} - (b + \frac{3}{2})e^{\frac{-3t}{2}}$$

When $b = -\frac{3}{2}$ The solution will change

Problem 9

$$t^2y'' + 3ty' + y = 0$$

Problem 9 continued on next page. . .

Let $y_2 = v(t)y_1$

$$y_2 = vt^{-1}$$

$$y_2' = v't^{-1} - vt^{-2}$$

$$y_2'' = v''t^{-1} - 2v't^{-2} + 2vt^{-3}$$

Plug it in the old formula, we get

$$tv'' + v' = 0$$

Let $r = v'$ Solve the formula above, we get

$$r = At^{-1}$$

We let A=1 so

$$r = t^{-1}$$

$$v = ln(t)$$

So the solution will be

$$y = ln(t)t^{-1}$$