Differential Equation: Homework #2

Due on September 4th, 2014 at 3:10pm

Professor Heather Lee Section 061

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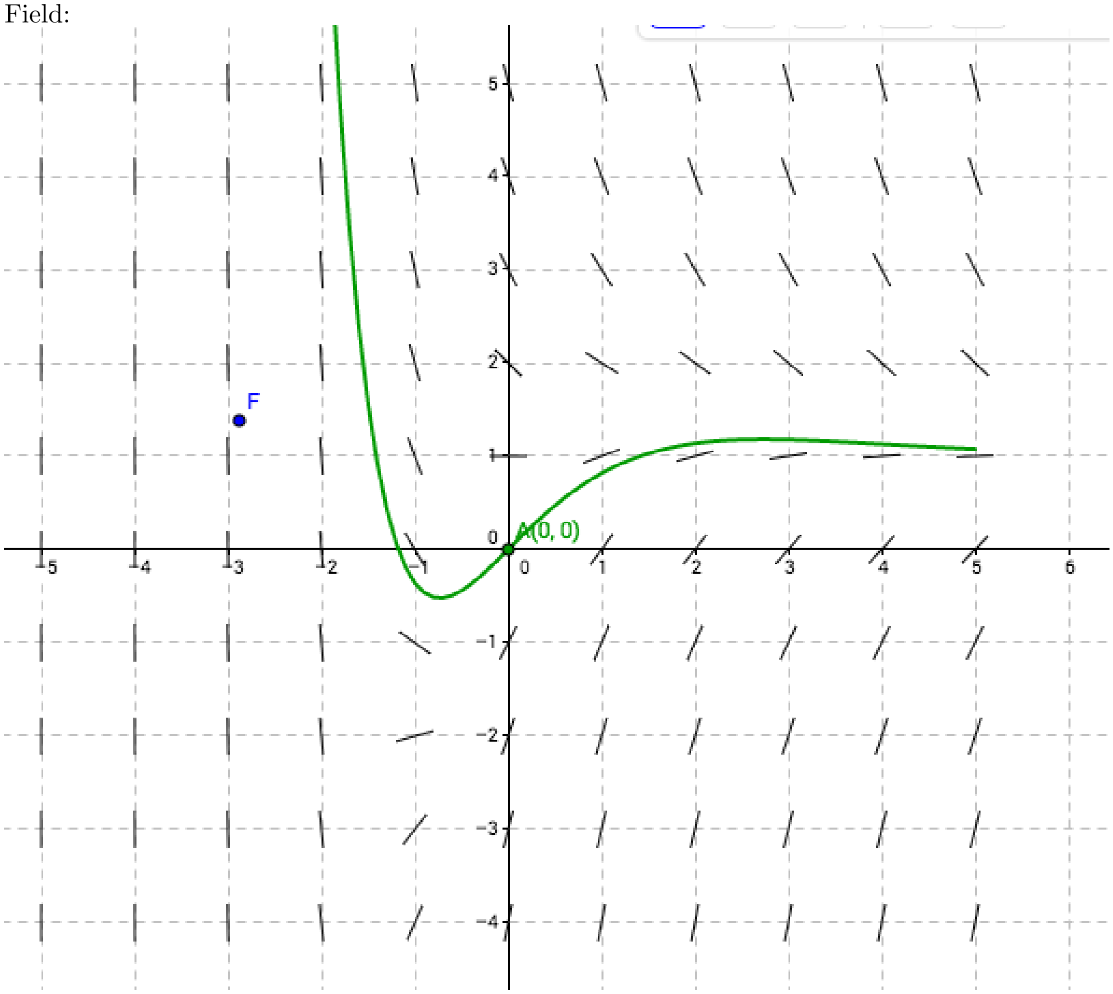
Problem 1

$y' + y = te^{-t} + 1$

Solution

1. Field:



2. When $t > 0$, since $e^{-t} \to 0$ so $te^{-t} + 1 \to 1$ the solution will go towards $1 - y$

Problem 1 continued on next page. . .

3.

$$\begin{aligned}
y' + y &= te^{-t} + 1 \\
\mu(t) &= e^t \\
e^t y' + e^t y &= t + e^t \\
\frac{d(e^t y)}{dt} &= t + e^t \\
\int \frac{d(e^t y)}{dt} &= \int t + e^t \\
e^t y &= \frac{1}{2}t^2 + e^t + C \\
y &= \frac{1}{2}t^2 e^{-t} + 1 + Ce^{-t}
\end{aligned}$$

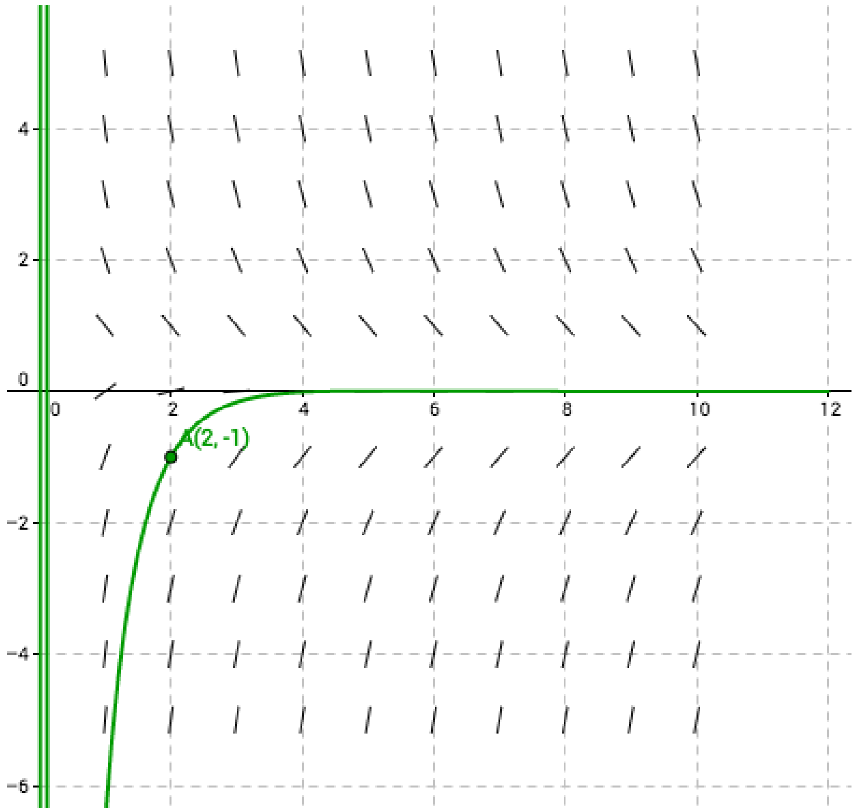
When $t \to \infty$ $t^2 e^{-t} \to 0$ $e^{-t} \to 0$ $y \to 1$

Problem 2

$ty' + (t+1)y = 2te^{-t}$

1. Field:

Problem 2 continued on next page...



As $t \to 0$ the solution will be either ∞ or $-\infty$ so the initial value do affect the result, a_0 should close to 0

2.

$$ty' + (t+1)y = 2te^{-t}$$

$$y' + \frac{t+1}{t}y = 2e^{-t}$$

$$\frac{d\mu}{dt} = \frac{\mu(t+1)}{t}$$

$$\frac{d\mu}{\mu} = \frac{dt(t+1)}{dt}$$

$$ln(\mu) = t + ln(t)$$

$$\mu(t) = e^t t$$

$$e^t ty' + e^t t\frac{t+1}{t}y = 2e^{-t}e^t t$$

$$e^t ty' + e^t(t+1)y = 2t$$

$$\frac{de^t ty}{dt} = 2t$$

$$e^t ty = \int 2t$$

$$e^t ty = t^2 + C$$

$$y = \frac{t}{e^t} + \frac{C}{e^t t}$$

When $y(1) = a$, $\frac{1}{e} + \frac{C}{e} = \frac{1+C}{e} = a$ Hence $C = ae - 1$

$y = \frac{t}{e^t} + \frac{ae-1}{e^t t} = \frac{t^2+ae-1}{e^t t}$ As $t \to 0$, $e^t t \to \infty$

So $ae - 1$ changes the behavior, $a_0 = \frac{1}{e}$

3. When $a < \frac{1}{e}$, As $t \to 0$, $y \to -\infty$

When $a = \frac{1}{e}$, As $t \to 0$, $y = 0$

When $a > \frac{1}{e}$, As $t \to 0$, $y \to \infty$