Differential Equation: Homework #1

Due on September 2nd, 2014 at 3:10pm

Professor Heather Lee Section 061

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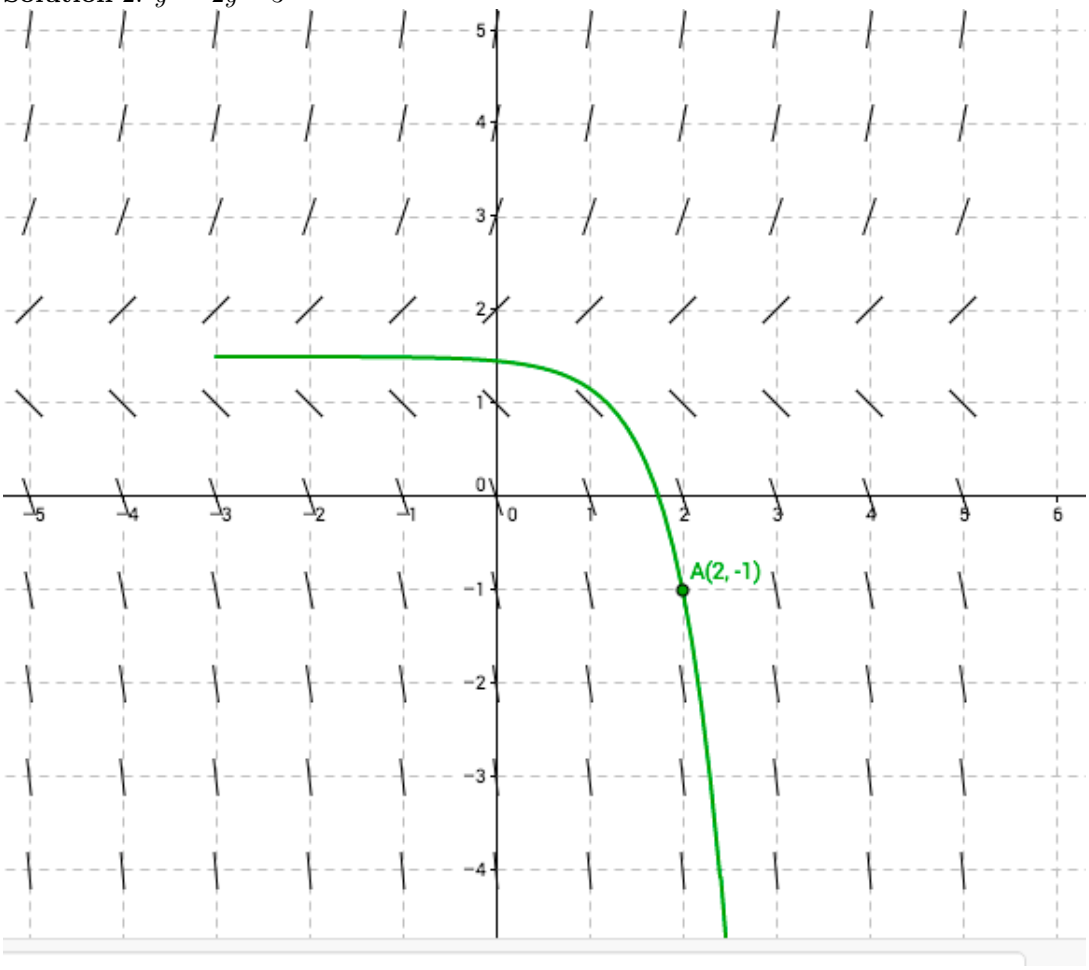
Problem 1

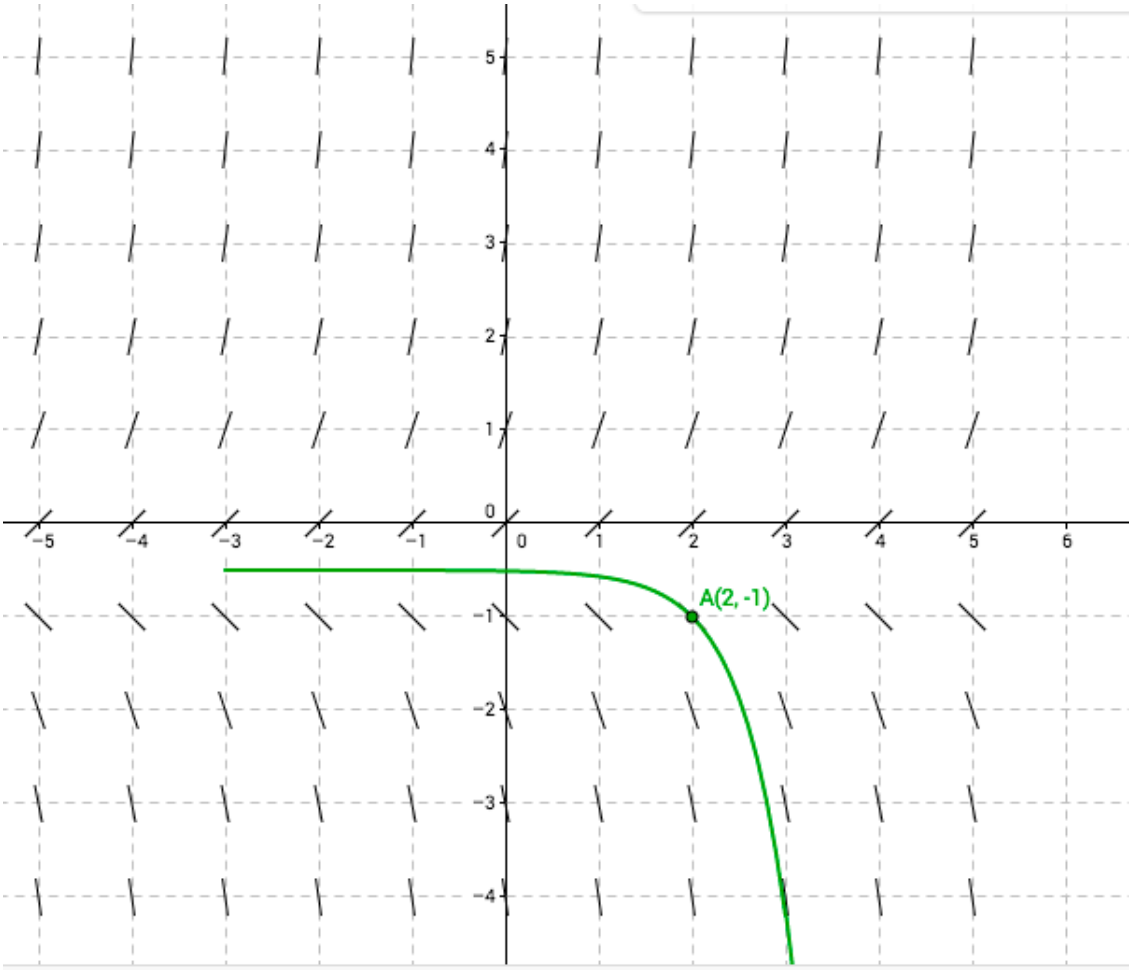
Draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe the dependency.

Solution 2. $y' = 2y - 3$



when $t \to \infty$ the slope stays as -3

5. $y' = 1 + 2y$



when $t \to \infty$ the slope stays as 1

Problem 2

Verify that each given function is a solution of the differ- ential equation.
8. $y'' + 2y' - 3y = 0; y_1(t) = e^{-3t}; y_2(t) = e^t$
Solution
For y1:

$$\begin{aligned} y1'(t) &= -3e^{-3t} \\ y1''(t) &= 9e^{-3t} \\ 9e^{-3t} + 2(-3e^{-3t}) - 3e^{3t} &= 9 - 6 - 3e^{3t} \\ &= 0e^{-3t} \\ &= 0 \end{aligned}$$

For y2:

$$\begin{aligned} y2'(t) &= e^t \\ y2''(t) &= e^t \\ e^t + 2e^t - 3e^t &= 0 \end{aligned}$$

So both of the solution are legit.
11. $2t^2y'' + 3ty' - y = 0t > 0y_1(t) = t^{1/2}y_2(t) = t^{-1}$

$$\begin{aligned} y1'(t) &= \frac{1}{2\sqrt{t}} \\ y1''(t) &= \frac{-1}{(4t^{3/2})} \\ 2t^2 * \frac{-1}{(4t^{3/2})} + 3t * \frac{1}{2\sqrt{t}} - t^{1/2} &= -1/2\sqrt{t} + 2/3\sqrt{t} - \sqrt{t} \\ &= 0 \end{aligned}$$

$$\begin{aligned} y2'(t) &= \frac{-1}{t^2} \\ y2''(t) &= \frac{2}{t^3} \\ \frac{2t^2 * 2}{t^3} - 3t * t^{-2} - t^{-1} &= 4/t - 3/t - 1/t \\ &= 0 \end{aligned}$$

So both solutions are legit

Problem 3

$$\begin{aligned} y' &= A(e^{-2t}(1-2t)) \\ 2A(e^{-2t}(1-2t)) + 4Ate^{-2t} &= 3e^{-2t} \\ LHS &= 2Ae^{-2t} - 4Ate^{-2t} + 4Ate^{-2t} = 2Ae^{-2t} \\ &= 3e^{-2t}A = 1.5 \end{aligned}$$

So A=1.5

$$\begin{aligned} y' &= Be^{-2t} \\ 2 * -2Be^{-2t} + 4Be^{-2t} &= 3e^{-2t} \\ 0 &= 3e^{-2t} \end{aligned}$$

So there is no solution for this