Math 453

Selected Solutions to Assignment 1

Problem 3: Let S be a fixed set and A, B denote two subsets of S. Show that $(A \cup B)^c = A^c \cap B^c$.

Solution: We have:

$$\begin{aligned}
(A \cup B)^c &= \{x \in S : x \notin (A \cup B)\} && \text{definition of relative complement} \\
&= \{x \in S : x \notin A \ \text{ and } \ x \notin B\} && \text{de Morgan's law} \\
&= \{x \in S : x \notin A\} \cap \{x \in S : x \notin B\} && \text{definition of set intersection} \\
&= \{x \in S : x \in A^c\} \cap \{x \in S : x \in B^c\} && \text{definition of relative complement} \\
&= A^c \cap B^c
\end{aligned}$$

Problem 4: Show that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

Solution: We have:

$$\begin{aligned}
(A \times B) \cap (C \times D) &= \{(x, y) : x \in A,\ y \in B)\} \cap \{(x, y) : x \in C,\ y \in D\} \\
&\qquad \text{definition of Cartesian product} \\
&= \{(x, y) : x \in A,\ y \in B,\ x \in C,\ y \in D\} \\
&\qquad \text{definition of set intersection} \\
&= \{(x, y) : x \in (A \cap C),\ y \in (B \cap D)\} \\
&\qquad \text{definition of set intersection} \\
&= (A \cap C) \times (B \cap D) \\
&\qquad \text{definition of Cartesian product}
\end{aligned}$$

Problem 7: Show that for every integer x, $x + 4$ is odd if and only if $x + 7$ is even.

Solution: First, suppose $x + 4$ is odd. Then by definition of odd, we have $x + 4 = 2m + 1$ for some $m \in \mathbb{Z}$. Then $x + 7 = 2m + 4 = 2(m + 2)$, and since $m + 2 \in \mathbb{Z}$, $x + 7$ is even by definition of even.

On the other hand, suppose $x + 7$ is even. Then by definition of even, $x + 7 = 2n$ for some $n \in \mathbb{Z}$. Then $x + 4 = 2n - 3 = 2(n - 2) + 1$, and since $n - 2 \in \mathbb{Z}$, $x + 4$ is odd by definition of odd.