Homework 1: Due Tuesday, August 30

Reading: Chapter 1, Appendix A, Appendix C

Problem 1: Given two set A and B, prove that $A \cap B = B \cap A$.

Problem 2: Given sets A, B, C, prove that $A \cap (B \setminus C) = (A \cap B) \setminus C$.

Problem 3: Let S be a fixed set and A, B denote two subsets of S. Show that $(A \cup B)^c = A^c \cap B^c$.

Problem 4: Show that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

Problem 5: Suppose we are given two statements p and q. Show that

$$(\sim (p \Rightarrow q)) \Leftrightarrow (p \land \sim q).$$

Problem 6: Prove the following statement:

If x and y are odd integers, then x + y is an even integer.

Problem 7: Show that for every integer x, x + 4 is odd if and only if x + 7 is even.

Problem 8: Show that if x is a real number such that $\frac{x^2-1}{x+2} > 0$, then either x > 1 or -2 < x < -1.

Hint: Factor the numerator of the given fraction. Then note that you have 4 cases to consider depending on the 'signs' (i.e. whether it is positive of negative) of each of the linear factors appearing in the numerator and denominator.