

Differential Equation: Homework #6

Due on October 5th, 2015 at 3:10pm

Professor Heather Lee Section 061

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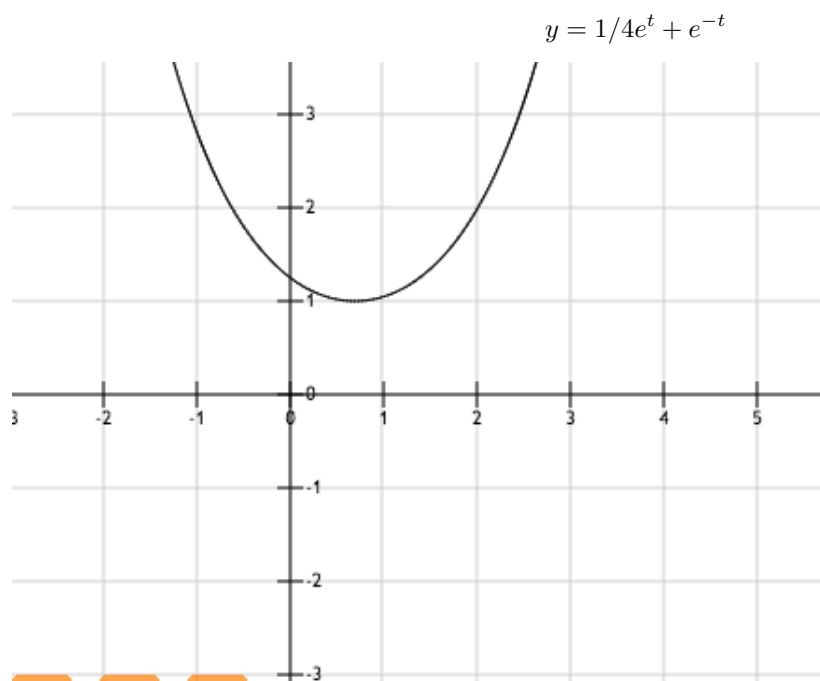
Problem 1

$$y'' - 1 = 0$$

We could get $r = -1$ and $r = 1$ So

$$y = Ae^t + Be^{-t}$$

We plug in the IV, get



When $t = \ln 2$ $y = 1$

Problem 2

$$4y'' - y = 0$$

$$y = Ae^{1/2t} + Be^{-1/2t}$$

Plug in the IV, we get

$$A = 1 + \beta$$

$$B = 1 - \beta$$

As $t \rightarrow \infty$ $y \rightarrow 0$ So

$$\beta = -1$$

Problem 3

$$y_1' = 2ty_2' = -t^{-2}$$

Plug it in, it equals to 0, so it's the solution

Since $wronskian(x^2, x^{-1}) \neq 0$ It is all the solution

Problem 4

Plug it in, we get y_1, y_2 equals to 0, so it's the solution

But

$$y = c_1 + c_2 t^{1/2}$$

is not the solution.

It doesn't contradict the theorem, because $yy'' + y'^2 = 0$ is not a linear equation.

Problem 5

Since y_1 is the solution (verified by plug it in) y_2 is also the solution (verified by plug it in) and $wronskian(y_1, y_2) \neq$

0 So it forms a fundamental set of solutions

Problem 6

Since $r = \pm 2i$ The equation should be

$$y = A\cos(2t) + B\sin(2t)$$

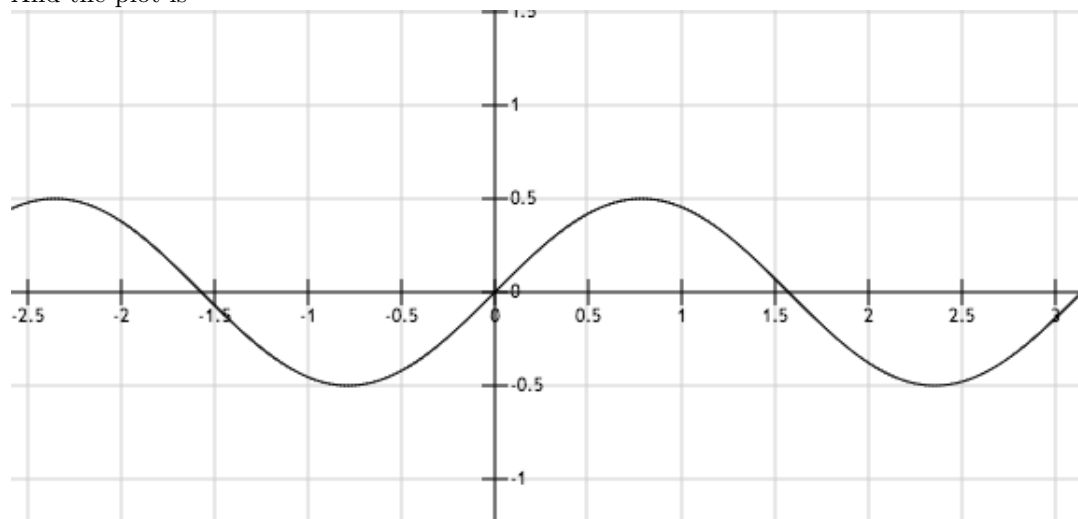
Plug in the IV, we get

$$A = 0, B = 1/2$$

So the solution is

$$y = \frac{1}{2}\sin(2t)$$

And the plot is



As

$$t \rightarrow \infty y \rightarrow [-0.5, 0.5]$$

Problem 7

$$r^2 - \frac{1}{3}r + \frac{2}{3} = 0$$

Solve it, we get

$$r = \frac{1 \pm \sqrt{23}}{6}$$

We plug it in with the initial value. We get

$$u(x) = -2/23e^{x/6} * [\sqrt{23}\sin((\sqrt{23}x)/6) - 23\cos((\sqrt{23}x)/6)]$$

2. plug in $u=10$, we get $t = 10.76$

Problem 8

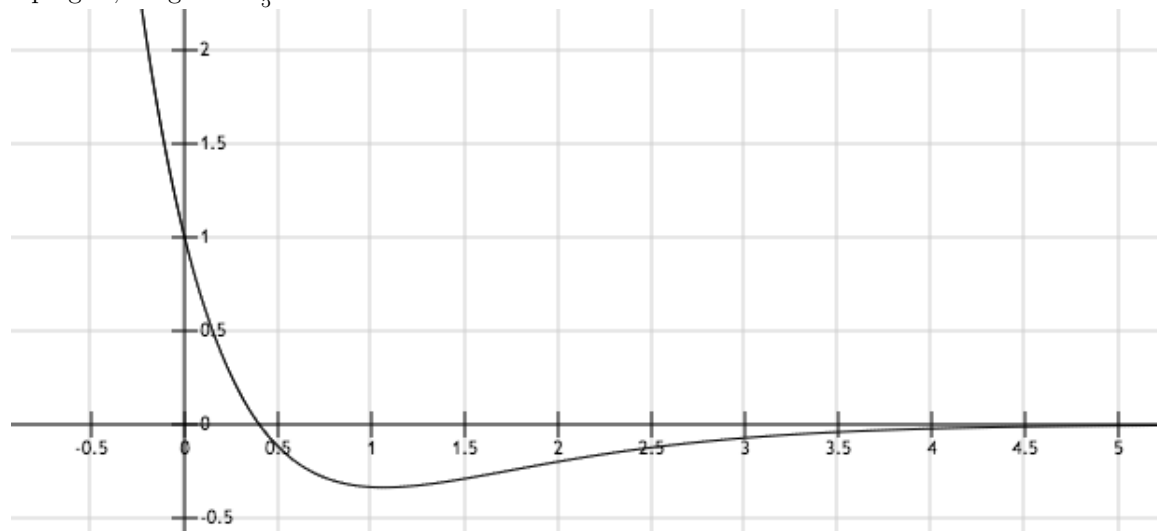
1. We could get

$$r = \frac{3}{2}$$

Plug it in with IV. We get

$$y(t) = e^{\frac{-3t}{2}} - \frac{5}{2}e^{\frac{-3t}{2}}$$

2. plug in, we get $t = \frac{2}{5}$



3. Let $y'=0$, we get

$$y = \frac{-5}{3e^{8/5}} \text{ at } x = \frac{16}{15}$$

4. We plug in with the new IV. We get

$$y(t) = e^{\frac{-3t}{2}} - (b + \frac{3}{2})e^{\frac{-3t}{2}}$$

When $b = -\frac{3}{2}$ The solution will change

Problem 9

$$t^2 y'' + 3t y' + y = 0$$

Let $y_2 = v(t)y_1$

$$y_2 = vt^{-1}$$

$$y_2' = v't^{-1} - vt^{-2}$$

$$y_2'' = v''t^{-1} - 2v't^{-2} + 2vt^{-3}$$

Plug it in the old formula, we get

$$tv'' + v' = 0$$

Let $r = v'$ Solve the formula above, we get

$$r = At^{-1}$$

We let $A=1$ so

$$r = t^{-1}$$

$$v = \ln(t)$$

So the solution will be

$$y = \ln(t)t^{-1}$$