Differential Equation: Homework #11

Due on November 23th, 2015 at 3:10pm

Professor Heather Lee Section 061

Yao Xiao

Problem 1

6.6 - 4

$$\int_0^t (t-\tau)^2 cos(2\tau)d\tau$$

So we could let $g(t) = t^2 \ h(t) = cos(2t)$

So $G(s) = \frac{2}{s^3}$

$H(s) = \frac{s}{s^2+4}$

$$G(s)H(s) = \frac{2s}{s^3(s^2+4)}$$

Problem 2

6.6 -5

$$\int_0^t e^{-(t-\tau)}sin(\tau)d\tau$$

$$g(t) = e^{-t}$$

$$h(t) = sin(t)$$

$$G(s) = \frac{1}{s+1}$$

$$H(s) = \frac{1}{s^2+1}$$

So the result should be

$$G(s)H(s) = \frac{1}{(s+1)(s^2+1)}$$

Problem 3

7.2-22

$$\begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} * \begin{bmatrix} 4e^{2t} \\ 2e^{2t} \end{bmatrix} = \begin{bmatrix} 8e^{2t} \\ 4e^{2t} \end{bmatrix} = \begin{bmatrix} (4e^{2t})' \\ (2e^{2t})' \end{bmatrix}$$

So it satisfies the condition

Problem 4

7.2-23

Plug x into formula 1, we get

$$x' = \begin{bmatrix} e^t(3+2t) \\ 2e^t(1+t) \end{bmatrix}$$

which is equal to

$$x' = \begin{bmatrix} 3e^t + 2te^t \\ 2e^t + 2te^t \end{bmatrix}$$

So it satisfies the condition

Problem 5

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For Tank 1: RateIn = 2oz/gal * 5gal/min = 10oz/min

RateOut = x1oz/50gal*5gal/min=x1/10 oz/min

Hence, the solution is

$$x1'(t) = 10 - \frac{x1(t)}{10}$$

For Tank 2:

3

RateIn = x1/10

RateOut = x2/20 * 55= x2/4

Hence, the solution is

$$x2'(t) = \frac{x1}{10} - \frac{x2}{4}$$