Homework 10: Due Thursday, November 10

Note: Two of the problems on Midterm 2 will be *very* similar to two of the problems below.

Problem 1: Let $H = \{\beta \in S_5 | \beta(1) = 1 \text{ and } \beta(3) = 3\}$. Prove that H is a subgroup. What is the order of H?

Problem 2: Suppose β is a 10-cycle. For which i such that $1 < i < 10$ is β^i also a 10-cycle?

Problem 3: Suppose $\phi : \mathbb{Z}_{50} \to \mathbb{Z}_{50}$ is an automorphism with $\phi(11) = 11$. Find a formula for ϕ. Explain your answer.

Problem 4: Let H and K be subgroups of a finite group G such that $H \subset K$. Show that $[G : H] = [G : K][K : H]$.

Problem 5: How many elements of order p are there in $\mathbb{Z}_{p^2} \times \mathbb{Z}_{p^2}$? Here p is a prime.

Problem 6: Suppose $|G| = pq$ where p and q are not necessarily distinct primes. Show that $|Z(G)| = 1$ or pq.

Problem 7: Find a homomorphism $\phi : \mathbb{Z}_{30}^{\times} \to \mathbb{Z}_{30}^{\times}$ with kernel $\{1, 11\}$ and such that $\phi(7) = 7$.

Problem 8: Determine the group $Aut(\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5)$ up to isomorphism.