Homework 8: Due Thursday, October 27

Reading: Chapters 14, 15.

Problem 1: Let $H = \{(1), (12)\} \subset S_3$. Is H a normal subgroup?

Problem 2: Let $f : G \to H$ be a homomorphism of groups. Show that $ker(f)$ is a normal subgroup.

Problem 3: Show the A_n is a normal subgroup of S_n.
Hint: Show that the map $f : S_n \to \mathbb{Z}_2$ which sends an odd permutation to 1 and an even permutation to 0 is a homomorphism.

Problem 4: Suppose H is a subgroup of G with index 2. Show that H is a normal subgroup.

Problem 5: What is the order of the element $14+ < 8 >$ in the quotient group $\mathbb{Z}_{24}/ < 8 >$.

Problem 6: What is the order of the quotient group $\mathbb{Z}_{10} \times \mathbb{Z}_{10}^{\times}/ < (2, 9) >$?

Problem 7: Show that a homomorphism $f : G \to G'$ is injective if and only if $ker(f) := \{e\}$.

Problem 8: Show that $A_4 \times \mathbb{Z}_3$ has no subgroup of order 18.