Differential Equation: Homework #7

Due on October 26th, 2015 at 3:10pm

Professor Heather Lee Section 061

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Problem 1

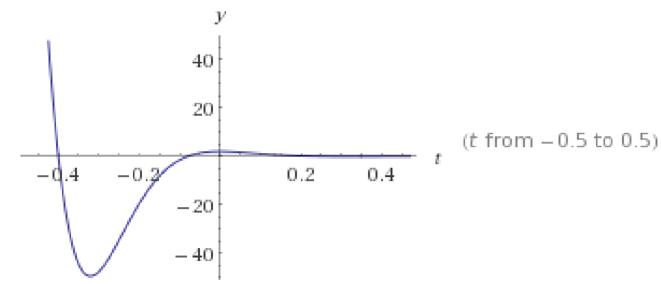
$$mu'' + \gamma u' + ku = 0$$

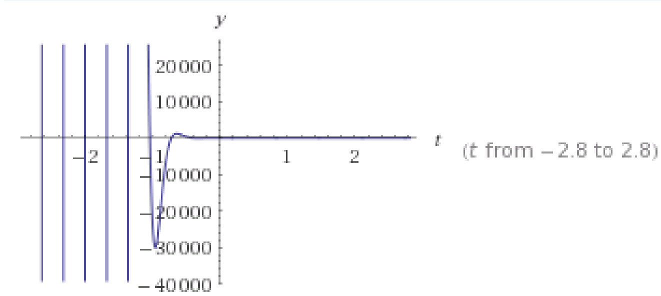
where $m = 20g$ $\gamma = 400$ $k = \frac{mg}{L} = 3920$ Plug it in, we get

$$u'' + 20u' + 196u = 0$$

Solve it with the initial value, we get

$$u(t) = e^{-10t}(2cos4\sqrt{6}t + \frac{5}{\sqrt{6}}sin4\sqrt{6}t)$$





The quasi-frequency is $\mu = 4\sqrt{6}$

The quasi-period is $T_d = \frac{\pi}{2\sqrt{6}}$

The ratio is $T_d/T = \frac{7}{2\sqrt{6}}$

Problem 2

$$u'' + 2u = 0$$

$$u(0) = 0$$

$$u'0) = 2$$

Problem 2 continued on next page...

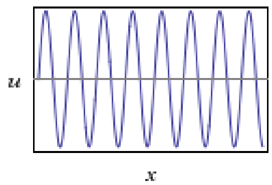
We can get

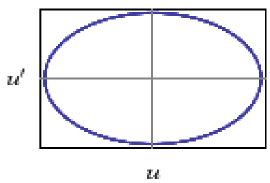
$$u = Acos\sqrt{2}t + Bsin\sqrt{2}t$$

Plug it in, we get

$$u = \sqrt{2}sin\sqrt{2}t$$

We get the plot of the graph





Problem 3

(a)We can get

$$u'' + 256u = 0$$

So

$$y = Acos(16t) + Bsin(16t)$$

Comparing the equation and the $mu'' + ku = F_0coswt$ We get $w_0 = 16$ So the equation becomes

$$u = Acosw_0t + Bsinw_0t + \frac{F_0}{m(w_0^2 - w^2)}coswt$$

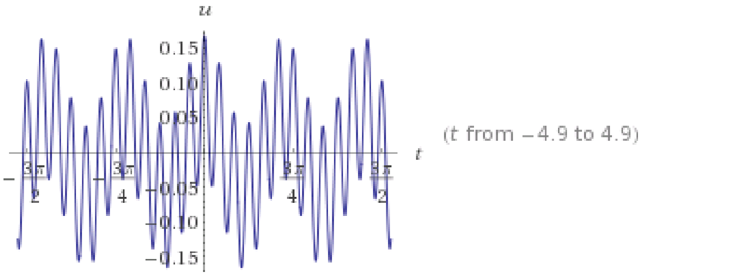
$$= Acos(16t) + Bsin(16t) + \frac{16}{247}cos(3t)$$

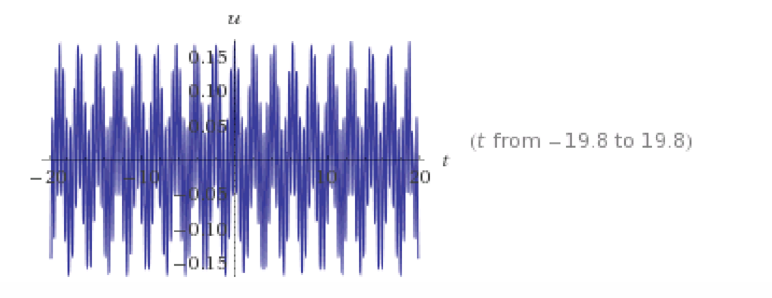
Plug it in with initial condition, we get

$$u = \frac{151}{1482}cos16t + \frac{16}{247}cos3t$$

Problem 3 continued on next page. . .

(b) Also we get the plot





(c) The equation becomes

$$mu'' + ku = 4sinwt$$

And we can get

$$u(t) = Acos(16t) + Bsin(16t) + U(t)$$

Since

$$U(t) = \frac{32}{256 - w^2} sinwt$$

$$w = w_0 = 16$$

Problem 4

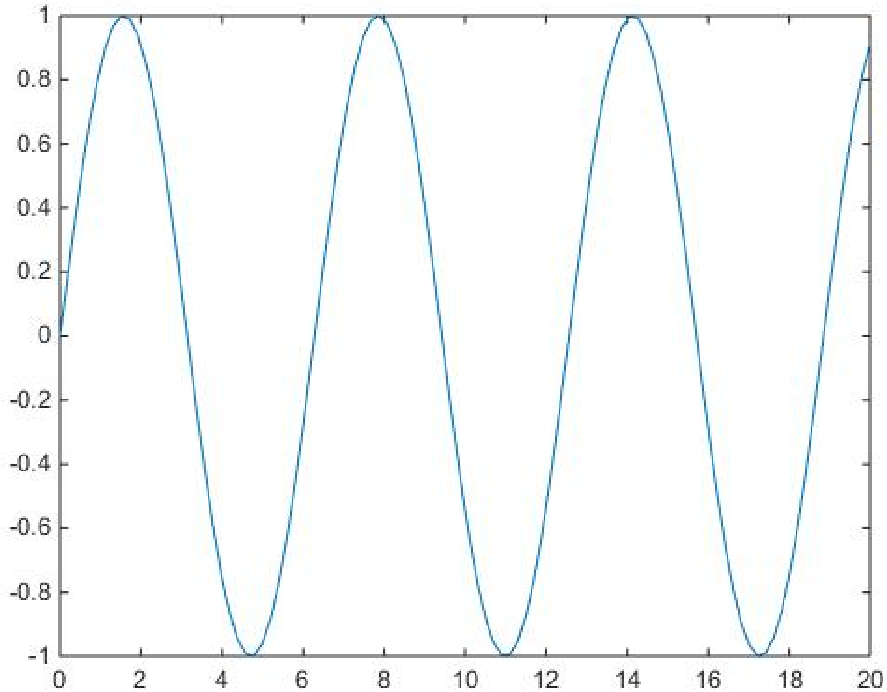
Project

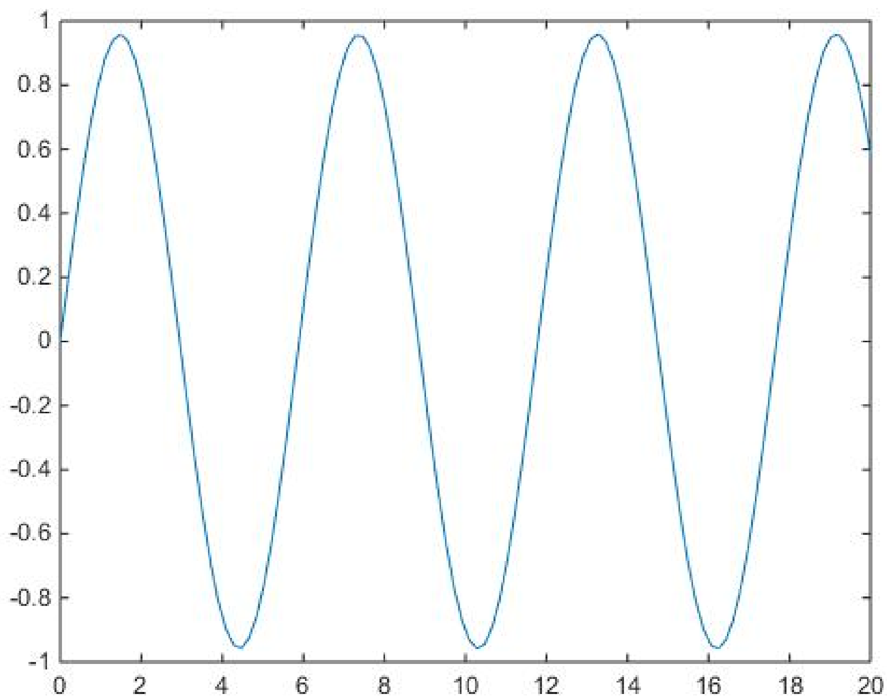
```
function xp=F(t,x)

xp=zeros(2,1); % since output must be a column vector

e=-0.3;

xp(1)=x(2);

xp(2)=-e*x(1)^3-x(1);
```

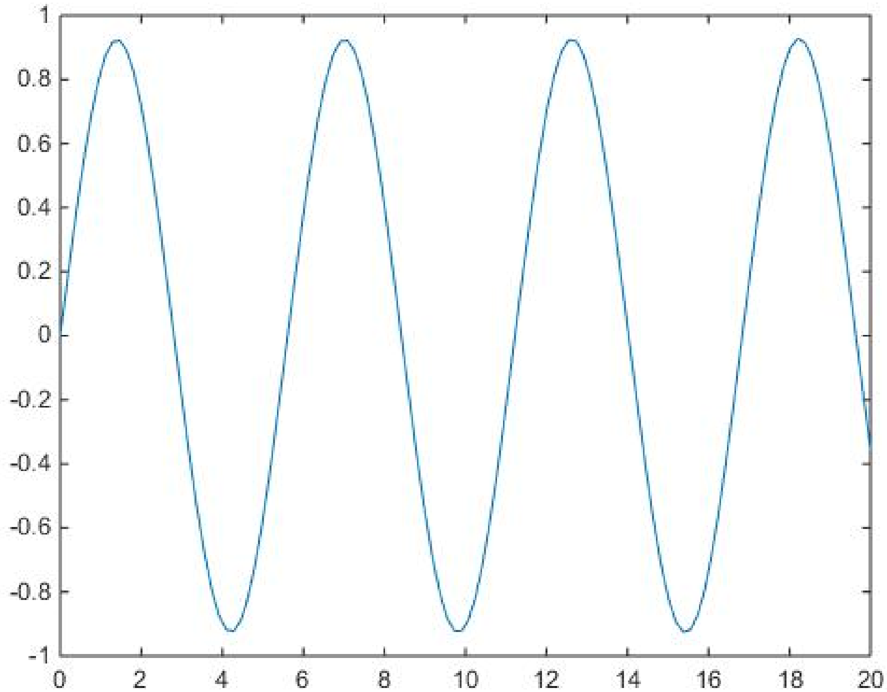
Problem 4 continued on next page...

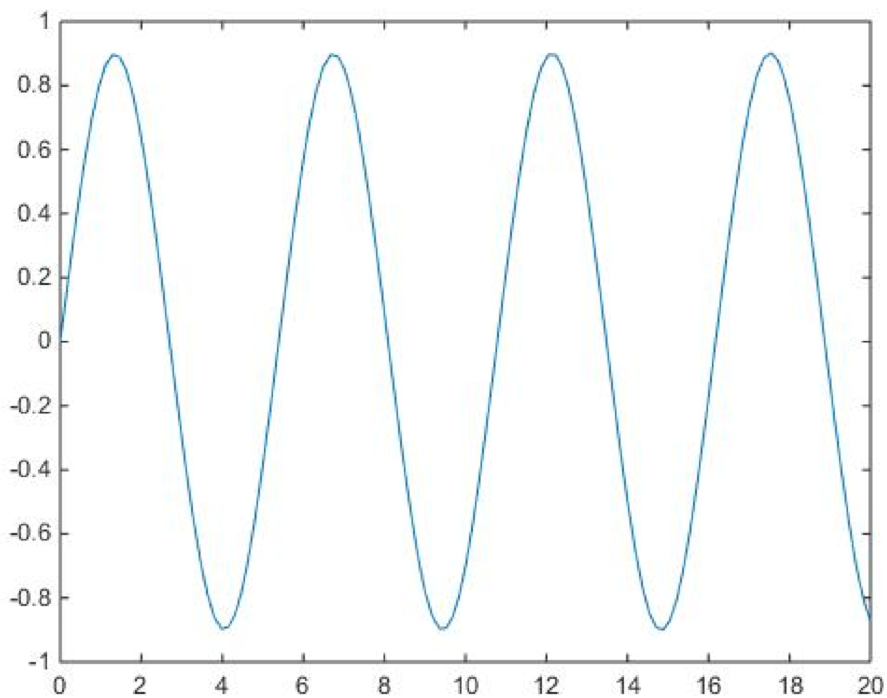
Question 1

We get the plot with e=0,0.2,0.4,0.6,0.8,1









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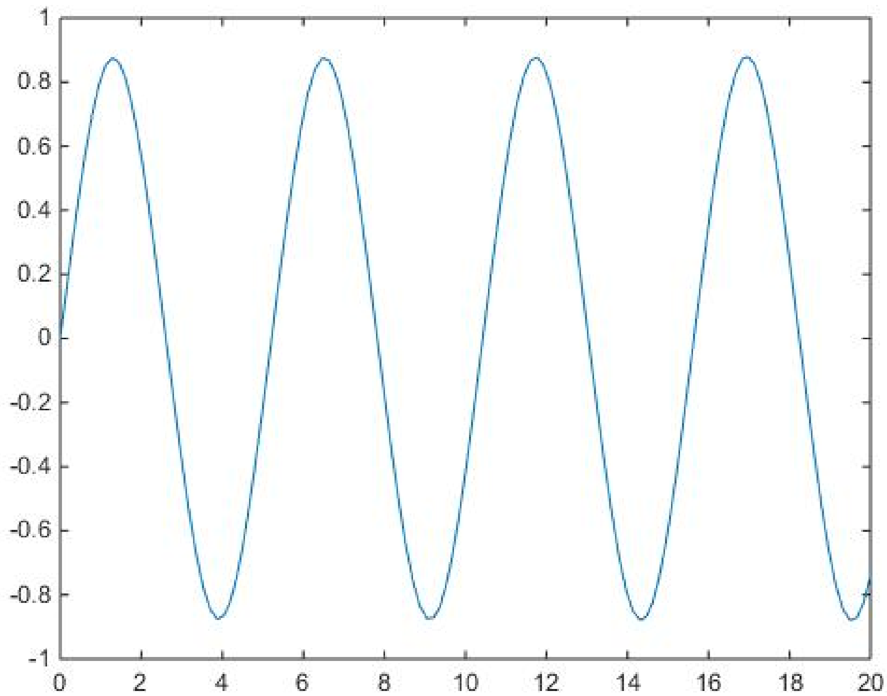
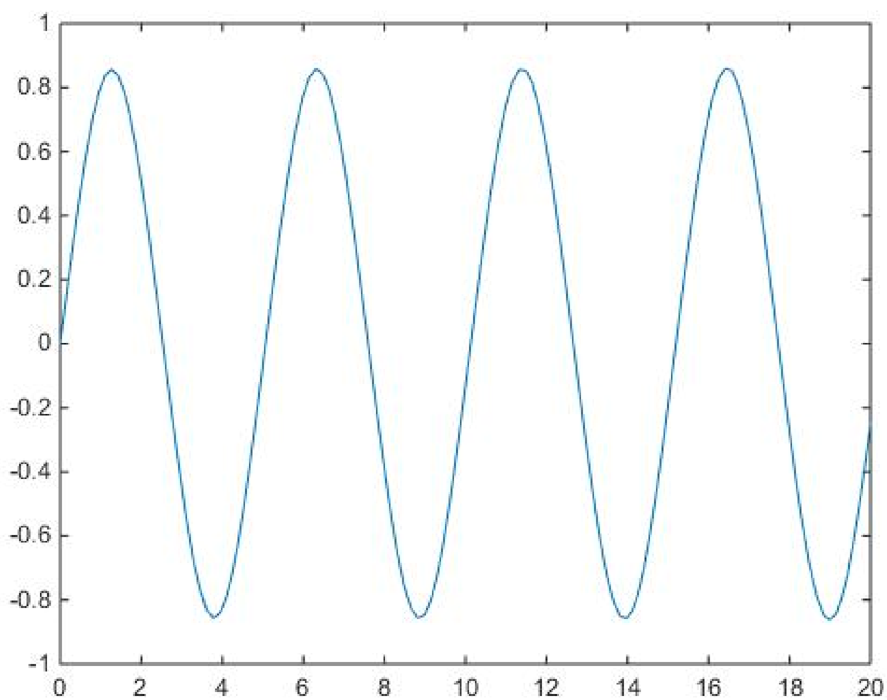




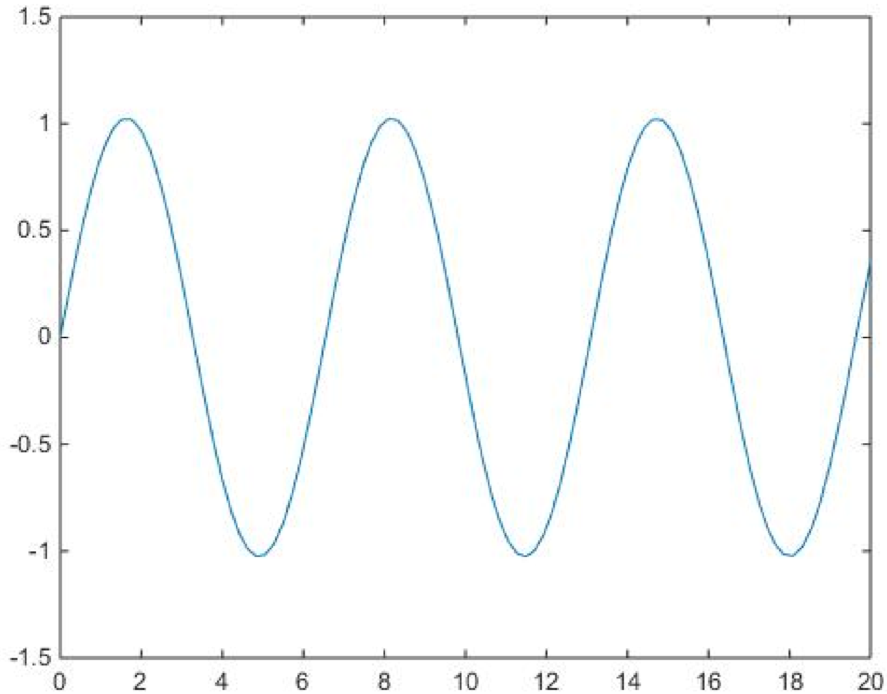
As I can see, as e goes toward infinity, the period is smaller. And the equilibrium equals to 3,2.8,2.6,2.4,2.2,2

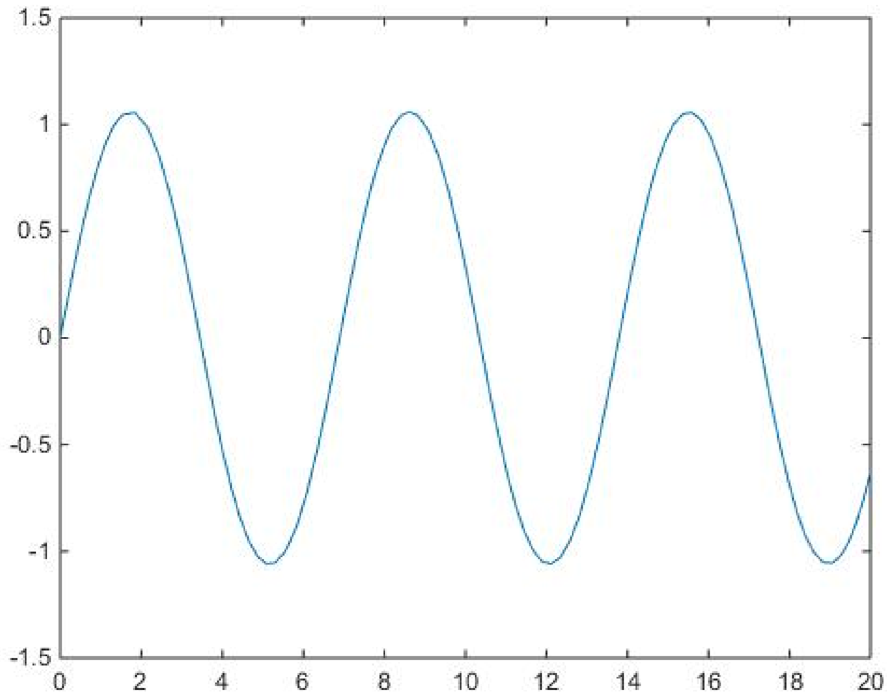
So the u^+ is smaller

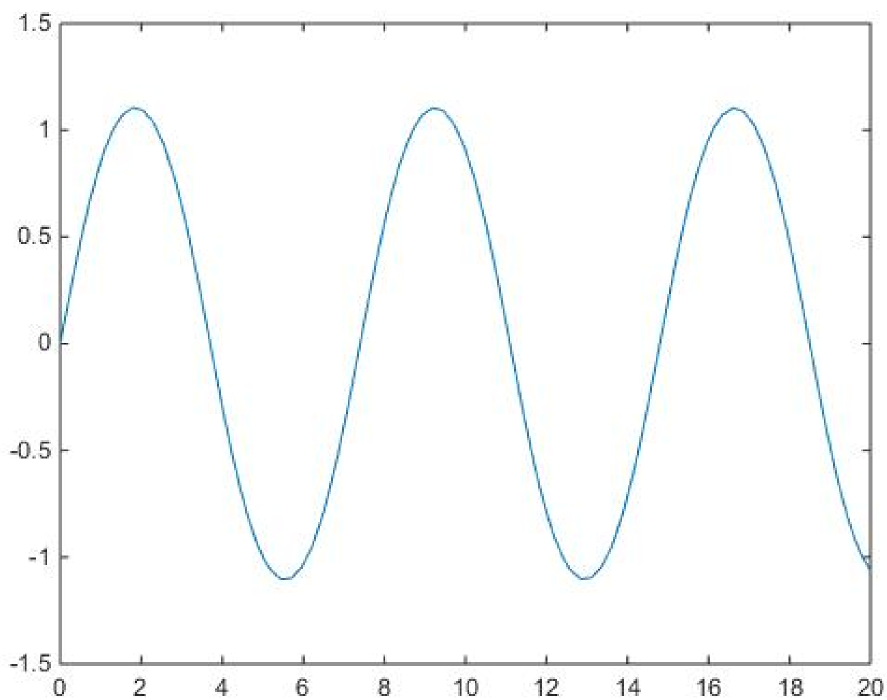
Problem 4 continued on next page. . .

Question 2

We get the plot with e=-0.1,-0.2,-0.3,-0.4







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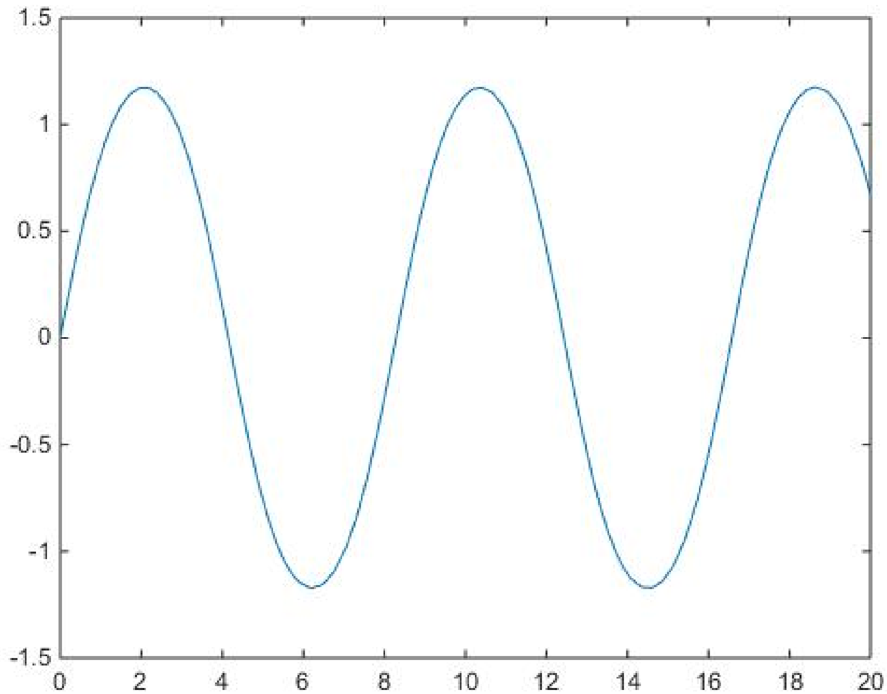


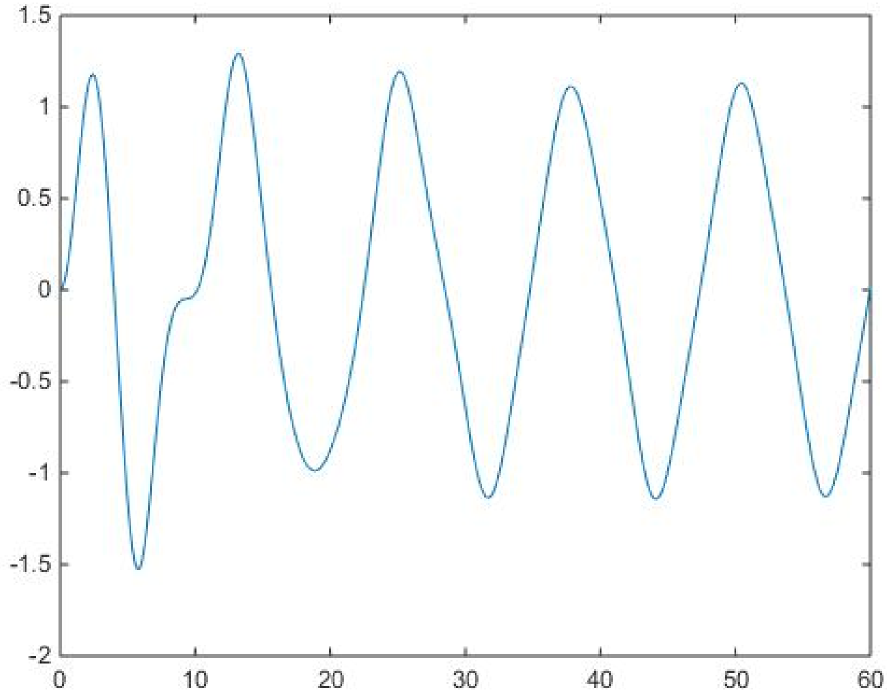
As I can see, as e goes toward negative infinity, the period is longer.

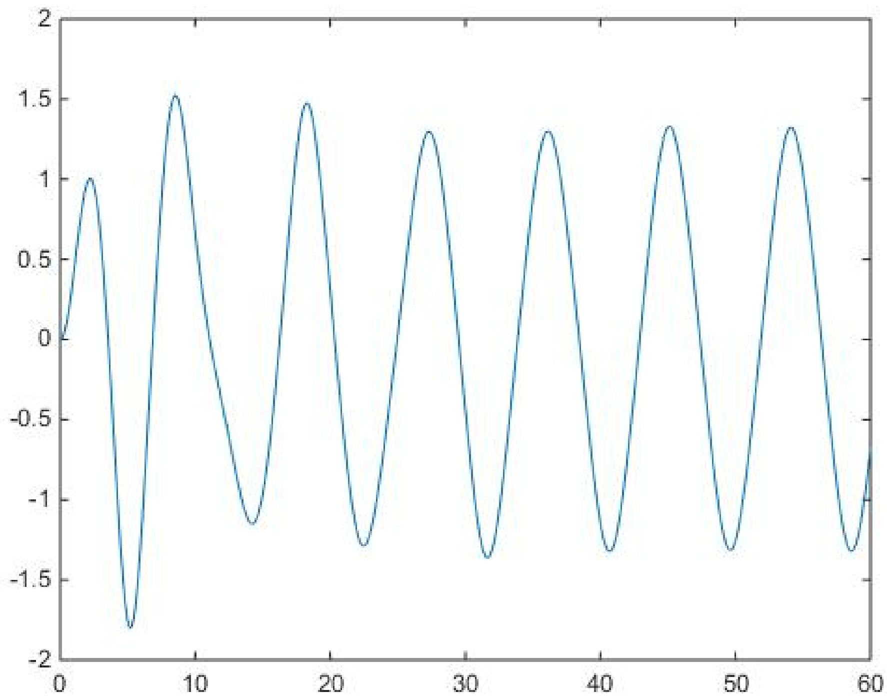
So the u^+ is bigger

And the equilibrium equals to 3,3.2,3.4,3.6

Question 3

```
function xp=F(t,x)

xp=zeros(2,1); % since output must be a column vector

w=0.5;

xp(1)=x(2);

xp(2)=cos(w*t)-(1/5)*x(2)-x(1)-1/5*x(1)^3;
```





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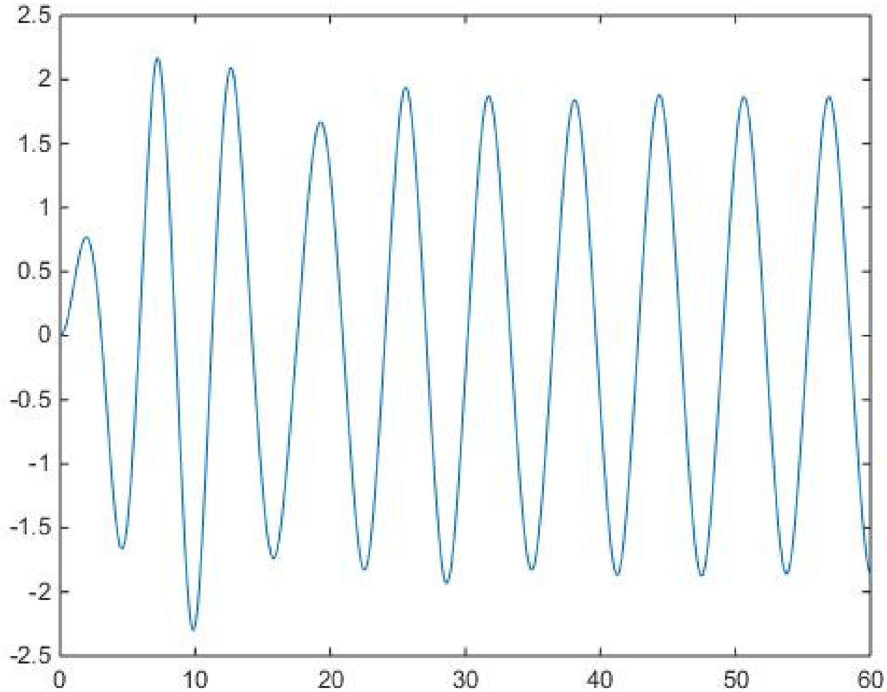

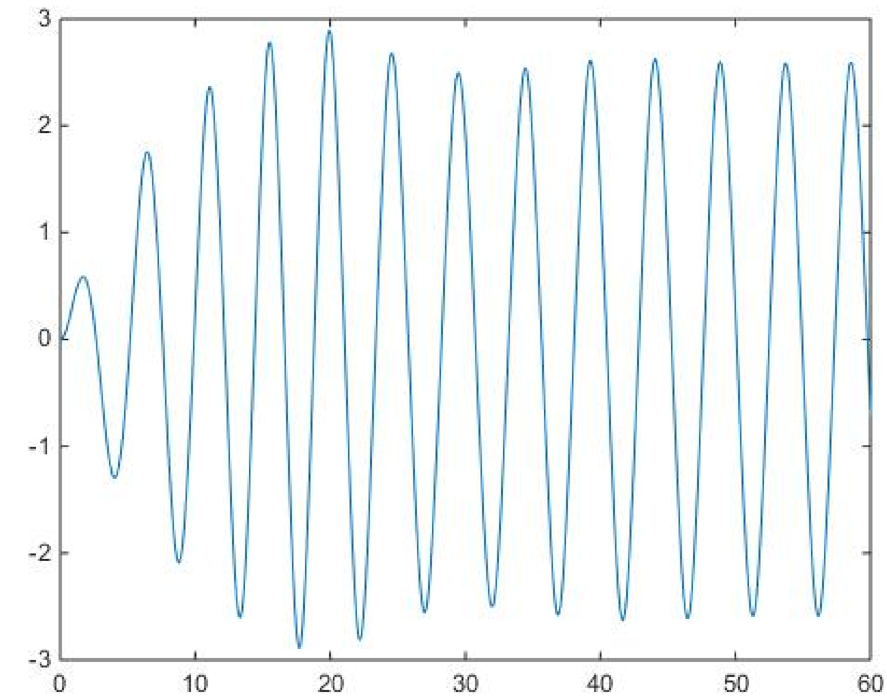

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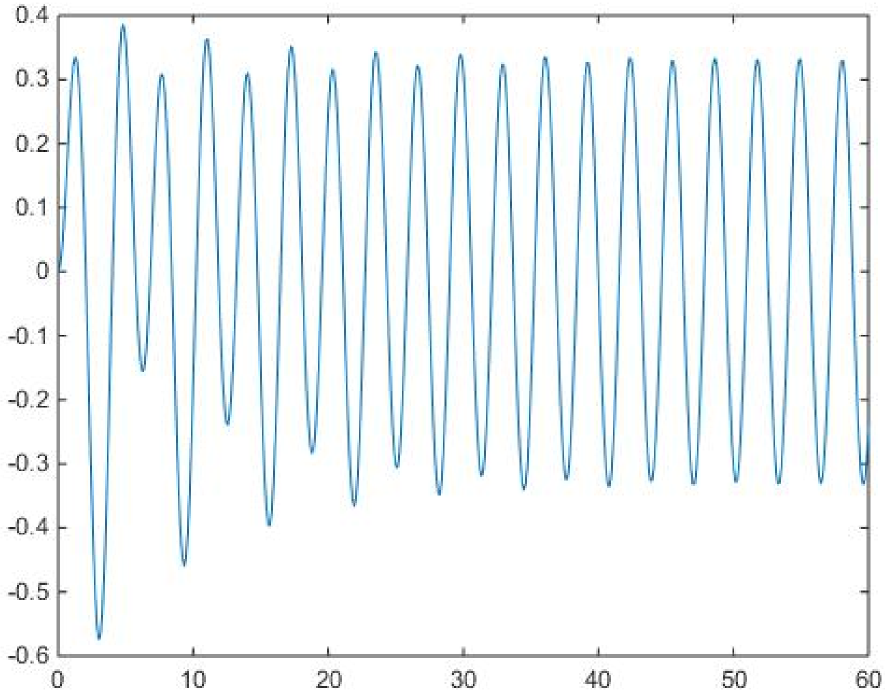

Based on my eyes, the w should be close to 1.4 which $|u(t)|$ is largest over