Differential Equation: Homework #9

Due on November 6th, 2015 at 3:10pm

Professor Heather Lee Section 061

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Problem 1

6.1-5b

From the table, we get

$$f(t) = t^2$$
$$\mathcal{L}(t^2) = \frac{2!}{s^3}$$

Problem 2

6.1-8

From the table, we get

$$\mathcal{L}(sinhat) = \frac{a}{s^2 - a^2}$$
$$\mathcal{L}(sinhbt) = \frac{b}{s^2 - b^2}$$

Problem 3

6.1-15

From the table, we get

$$\mathcal{L}(t^n e^{at}) = \frac{n!}{(s-a)^{n+1}}$$

So

$$\mathcal{L}(te^{at}) = \frac{1}{(s-a)^2}$$

Problem 4

6.2-10

$$\begin{aligned} F(s) &= \frac{2s-3}{s^2+2s+10} \\ &= \frac{2s-3}{(s+1)^2+9} \\ &= \frac{2(s+1)-5}{(s+1)^2+9} \\ &= 2[\frac{s+1}{(s+1)^2+3}] - \frac{5}{3}[\frac{3}{(s+1)^2+3^2}] \end{aligned}$$

Looking up the table, plug it in, we get.

$$\mathcal{L}^{-1}(F(s)) = 2e^{-t}cos(3t) - \frac{5}{3}e^{-t}sin(3t)$$

Problem 5

6.2-21

$$y'' - 2y' + 2y = cos(t)$$

$$y(0) = 1$$

$$y'(0) = 0$$

$$\mathcal{L}(y'') - 2\mathcal{L}(y') + 2\mathcal{L}(y) = \mathcal{L}(cost)$$

$$\mathcal{L}(y'') = s^2\mathcal{L}(y) - s$$

$$\mathcal{L}(y') = s\mathcal{L}(y) - 1$$

$$\mathcal{L}(cos(t)) = \frac{s}{s^2+1}$$

Plug it in, we get

$$\mathcal{L}(y) = \frac{s^3-2s^2+2s-2}{(s^2-2s+2)(s^2+1)}$$

We dispose the equation above, we could get

Problem 5 continued on next page. . .

$$\mathcal{L}(y) = \frac{4}{5}\frac{s-1}{(s-1)^2+1} - \frac{2}{5}[\frac{1}{(s-1)^2+1}] + 1/5\frac{s}{s^2+1} - 2/5\frac{1}{s^2+1}$$

Plug it in with the table, we get the solution

$$y(t) = \frac{4}{5}e^t cost - \frac{2}{5}e^t sin(t) + \frac{1}{5}cos(t) - \frac{2}{5}sin(t)$$

Problem 6

6.3-1

$$g(t) = \begin{cases} 0 & 0 <= t < 1 \\ 1 & 1 <= t < 3 \\ 3 & 3 <= t < 4 \\ -3 & 4 <= t < \infty \end{cases}$$

Problem 7

6.3-2

$$g(t) = \begin{cases} 0 & 0 <= t < 2 \\ t-3 & 2 <= t < 3 \\ -1 & 3 <= t < \infty \end{cases}$$

Problem 8

6.3-18

$$f(t) = t - u_1(t)(t-1)$$

$$\begin{aligned}\mathcal{L}(f(t)) &= \mathcal{L}(t) - \mathcal{L}(t-1)u_1(t) \\ &= \frac{1}{s^2} - e^{-s}\frac{1}{s^2} \\ &= \frac{(1-e^{-s})}{s^2}\end{aligned}$$

Problem 9

6.3-21

$$\begin{aligned}F(s) &= \frac{2(s-1)e^{-2s}}{s^2-2s+2} \\ &= 2\mathcal{L}(\frac{(s-1)e^{-2s}}{(s-1)^2+1}\end{aligned}$$

So we could transfer it back using the formula on the table. We get

$$\mathcal{L}^{-1}(F(s)) = 2u_2(t)e^{t-2}cos(t-2)$$

Problem 10

6.3-23

$$F(s) = \frac{(s-2)e^{-s}}{(s-2)^2-1}$$

Let

$$g(s) = \frac{s-2}{(s-2)^2-1}$$

Since

$$\mathcal{L}(e^{2t}cosht = \frac{s-2}{(s-2)^2-1}$$

Problem 10 continued on next page...

So the result is going to be

$$\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}(e^{-s}g(s)) = f(t-1)u(t-1)$$

which is equal to

$$e^{2t-2}cosh(t-1)u(t-1)$$