Elements of Algebra I: Homework #3

Due on January 31st, 2017

Professor Deepam Patel Section 161

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Problem 1

1)

1. Not commutative, when $x = 1$ $y = 0$, $x * y = 1$ $y * x = 0$
2. Not associative, when

$$(x + 2y + xy) + 2z + (x + 2y + xy)z = x + 2y + xy + 2z + xz + 2yz + xyz$$
$$x + 2(y + 2z + yz) + x(y + 2z + yz) = x + 2y + z4 + 2yz + xy + 2xz + xyz$$

3. There's no identity due to 0*y $\neq$ x*0 and 2y $\neq$ y
4. There's no inverse due to 3

2)

1. It's commutative, since when $x > y$ $|x - y| = x - y$ $|y - x| = -(y - x) = x - y$ when $y > x$ $|y - x| = y - x$ $|x - y| = -(x - y) = y - x$
2. It's not associative, when x=2,y=1,z=1

$$||2 - 1| - 1| = 0$$

while

$$|2 - |1 - 1|| = 2$$

3. There's no identity since when $x < 0$, the result must be positive so there would be no e satisfied
4. There's no inverse due to 3

3)

1. It's commutative since $maxx, y = maxy, x$
2. It's associative since $max(x, max(y, z)) = max(x, y, z) = max(max(x, y), z)$
3. There's no identity, If $e \in \mathbb{R}$ were an identity, then we can choose an integer $x < e$ and for this integer and we could get $max(x, e) = e \neq\ x$. So there's no identity
4. There's no inverse due to 3

Problem 2

$$\begin{aligned} g \in G& \\ e &= g^{-1} * g \\ (g^{-1})^{-1} * e &= (g^{-1})^{-1} * (g^{-1} * g) \\ &= ((g^{-1})^{-1} * g^{-1}) * g) \end{aligned}$$

Since $g^{-1} * g = e$ Let $g' = g^{-1}$ $g'^{-1} * g' = e$ So the formula above becomes

$$\begin{aligned} (g^{-1})^{-1} * e &= (e) * g) \\ (g^{-1})^{-1} &= g \end{aligned}$$

In order to show $(gh)^{-1} = h^{-1}g^{-1}$ We just need to show the right hand side is the inverse of gh.

$$\begin{aligned} gh(h^{-1}g^{-1}) &= g(h^{-1}h)g^{-1} \\ &= geg^{-1} \\ &= gg^{-1} \\ &= e \end{aligned}$$

So $(h^{-1}g^{-1})$ is the inverse of gh, as a result, $gh^{-1} = h^{-1}g^{-1}$

Problem 3

Using the theory of Problem 2, we could get

$$\begin{aligned} g^{-1}h^{-1} &= (hg)^{-1} \\ &= (gh)^{-1} \\ &= h^{-1}g^{-1} \end{aligned}$$

So its inverse commutes as well.

Problem 4

By Problem 2, we have

$$(ab)^{-1} = b^{-1}a^{-1} = a^{-1}b^{-1}$$

If it commutes
So we could get

$$(ab)^n = ((ab)^{-1})^{-n} = (a^{-1}b^{-1})^{-n} = a^n b^n$$

Problem 5

1. Closure. By the definition, If $x, y \in G$

   $$\exists x^{-1}, y^{-1} \in S$$

   From problem 2, we know $y^{-1} * x^{-1} = (xy)^{-1}$. If xy has an inverse, so it must be in G. We proved that $x * y \in G$

2. Associative, since G is a subset of S and S , it must hold

3. Identity, since $e \in S$, e has an inverse e, $e \in G$ must hold

4. Inverse, let $x \in G$, there must be an inverse $x^{-1} \in S$ since $x \in S$. We know that $(x^{-1})^{-1} = x$ and $x \in G$. There must be an element $x^{-1} \in G$

Problem 6

	id	r1	r2	r3	f	fr	fr2	fr3
id	id	r1	r2	r3	f	fr	fr2	fr3
r1	r1	r2	r3	id	fr	fr2	fr3	f
r2	r2	r3	id	r1	fr2	fr3	f	fr
r3	r3	id	r1	r2	fr3	f	fr	fr2
f	f	fr3	fr2	fr	id	r3	r2	r1
fr	fr	f	fr3	fr2	r1	id	r3	r2
fr2	fr2	fr	f	fr3	r2	r1	id	r3
fr3	fr3	fr2	fr	f	r3	r2	r1	id

Problem 7

We assume set $\mathbb{Z}_n$, which 2 has an inverse. So $\exists k, l \in \mathbb{Z}_n$ such that $2k + 1 = nl$ so $2k = nl - 1$, nl-1 is even, nl must be odd, which means n and l must be odd. n is odd.

Problem 8

(12)(13) doesn't commute. (12)(13)=(132). (13)(12)=(123)