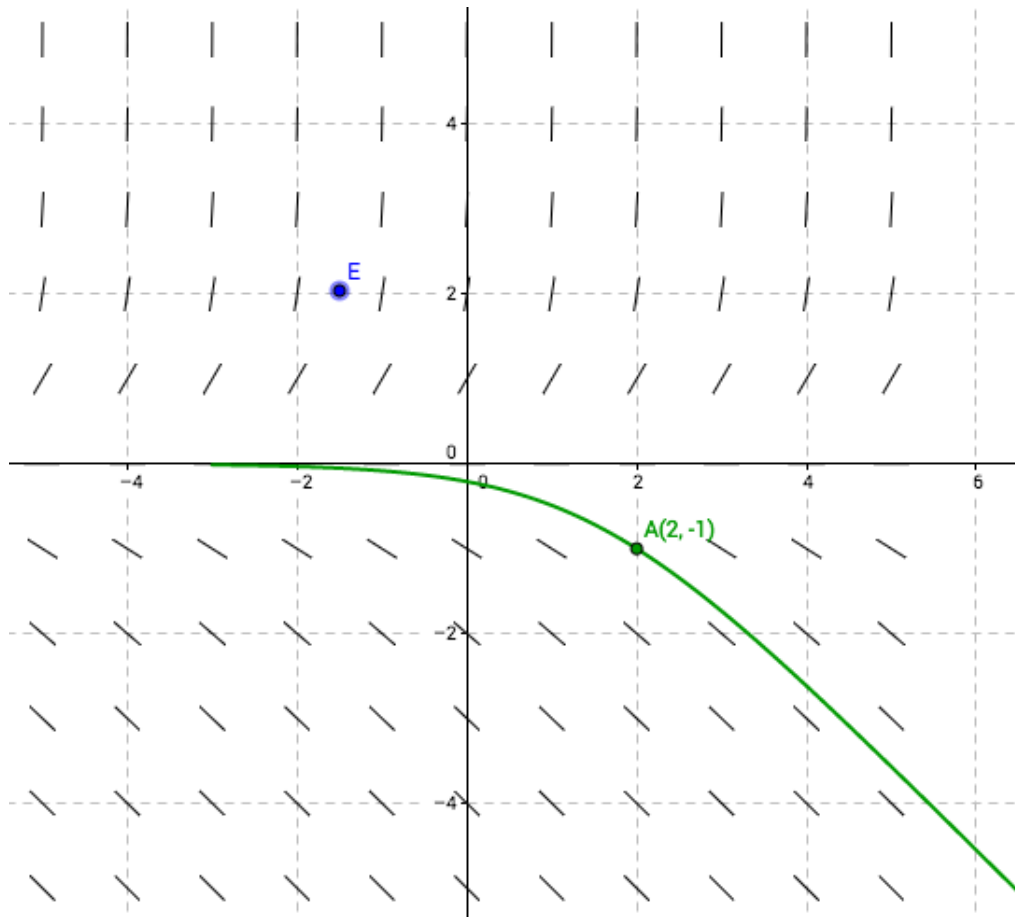# Differential Equation: Homework #5

Due on September 25th, 2015 at 3:10pm

Professor Heather Lee Section 061
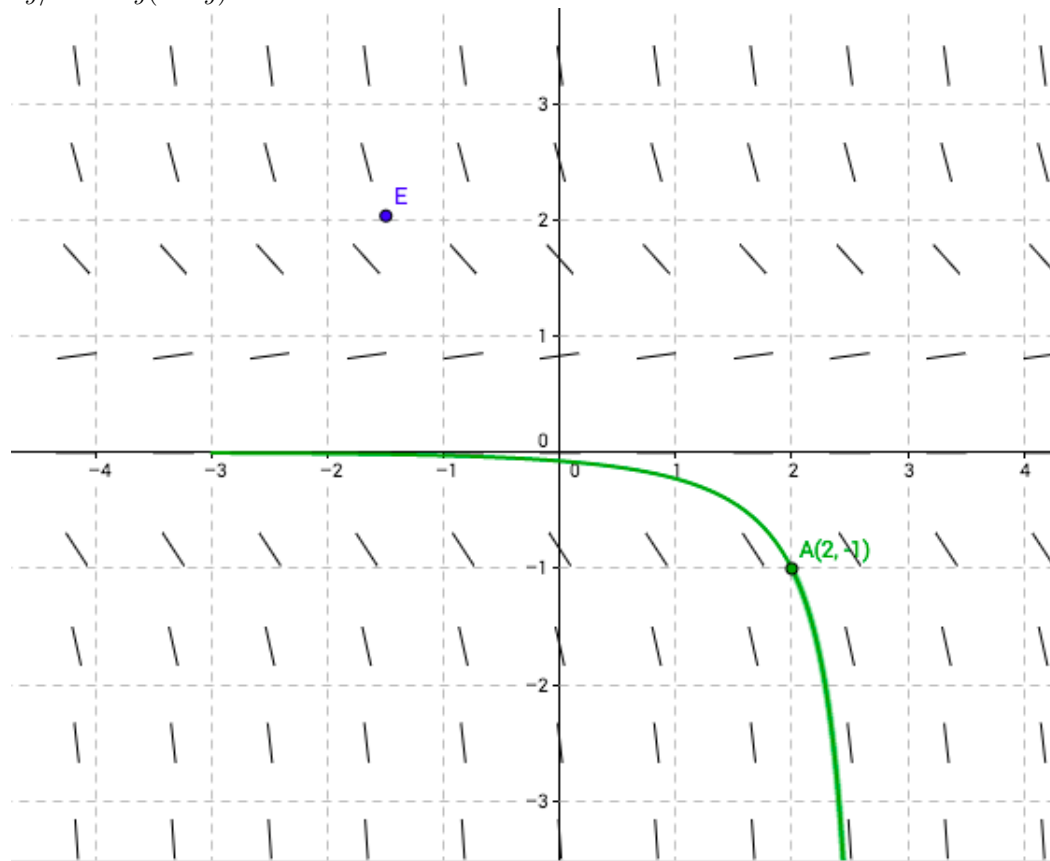
Yao Xiao

Problem 1

From this sketch it appears that solutions that start near y = 0 all move away from it as t increases, so it's

unstable.

Problem 2

$dy/dx = \alpha y(1 - y)$

From this sketch it appears that solutions that start near y = 0 all move away from it as t increases, so it's unstable. solutions that start near y = 0 all move towards it as t increases, so it's asymptotically stable

2.

$$\int \frac{1}{y(1-y)} dy = \int \alpha dt$$

$$ln(y) - ln(1 - y) = \alpha t + C$$

$$\frac{y}{1 - y} = Ae^{\alpha t}$$

$$A = \frac{y_0}{1 - y_0}$$

$$\frac{y}{1 - y} = \frac{y_0}{1 - y_0} e^{\alpha t}$$

$$y = \frac{y_0 e^{\alpha t}}{y_0 e^{\alpha t} + 1 - y_0}$$

When $t \to \infty, y \to 1$

Problem 3

1.The equilibrium solutions should be -2,4,8

2. When y=-2, it's asymptotic stable When y=4, it's semi-stable When y=8, it's not stable

Problem 4

Since

$$\frac{\partial M(x)}{\partial y} = \frac{\partial N(y)}{\partial x} = 0$$

So it's exact equation

Problem 5

$$\frac{dw}{dt} = \frac{2tw}{w^2 - t^2}$$

Let w=at, dw/dt=a+t(da/dt)

$(2tw)/(w^2 - t^2) = (2at^2)/((at)^2 - t^2) = 2a/(a^2 - 1)$

$$a + t\frac{da}{dt} = \frac{2a}{a^2 - 1}$$

$$t\frac{da}{dt} = \frac{2a - a(a^2 - 1)}{a^2 - 1}$$

$$t\frac{da}{dt} = \frac{2a - a(a^2 - 1)}{a^2 - 1}$$

$$\frac{3a - a^3}{a^2 - 1} = t\frac{da}{dt}$$

$$\frac{a^2 - 1}{3a - a^3}da = \frac{1}{tdt}$$

$$t(a) = \frac{C}{\sqrt[3]{a(3 - a^2)}}$$

$$t^3 a(3 - a^2) = C$$

$$a = w/t$$

$$wt^2(3 - w^2/t^2) = C$$

$$w = \frac{C}{3t^2 - w^2}$$

4

Problem 6

a)

$$y' = 1 - t + y$$

$$y(t_0) = y_0$$

$$(e^{-t}y)' = (1 - t)e^{-t}$$

Integrate both side, we get $e^{-t}y = te^{-t} + C$ So

$$y(t) = t + (y_0 - t_0)e^{t-t_0}$$

b)

$$y_k = y_{k-1} + hf(k - 1)$$

$$f(k - 1) = 1 - t + y$$

$$y_k = (1 + h)y_{k-1} + h - ht_{k-1}$$

$$y_k = y_{k1} + h(1t_{k1} + y_{k1}) = (1 + h)y_{k1} + hht_{k1}$$

d) when $h = \frac{t-t_0}{n}$

$$y_n = (1 + h)^n(y_0 - t_0) + t = (1 + \frac{t - t_0}{n})^n(y_0 - t_0) + t$$

since $n \to \infty$, the upper formula becomes $y_n = (1 + \frac{t-t_0}{n})^n(y_0 - t_0) + t$

$$= e^{t-t_0}(y_0 - t_0) + t$$

Problem 7

$$y' = 2y - 3e^{-t}$$

$$(e^{2t}y)' = [e^2 t * e^- t] + C$$

$$y = e^- t + cC = 0$$

$$y(t) = e^{-t}$$

Problem 8

$$y' = 2y - 3e^{-t}$$

$$y' - 2y = -3e^{-t}$$

$$(e^{-2t}y)' = -3e^{-3t}$$

$$e^{-2t}y = e^{-3t} + C$$

$$C = 0$$

$$y(t) = e^{-t}$$

So $y(1) = \frac{1}{e}$

Problem 9