Decoding the Curve: A Quantitative Research Journey into Interest Rate Dynamics and Forecasting

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Abstract

I carried out this project to bridge the gap between textbook quant finance and real-world trading decisions. Interest rate models are everywhere in pricing and risk management, but a lot of academic treatments skip the messy details of implementation. I wanted to see how these models actually perform when applied to market data—and whether they could be used to generate actionable insights.Focusing on the U.S. Treasury market, I develop a complete pipeline from data acquisition to risk analysis, including:

- Parametric yield curve fitting using Nelson-Siegel methodology
- Dynamic term structure modeling via the Hull-White one-factor model
- Model calibration to both yield curves and interest rate cap volatilities
- Risk factor decomposition through Principal Component Analysis

The implementation evaluates key risk metrics including duration, convexity, Value-at-Risk, and Expected Shortfall, while incorporating stress testing across various market scenarios. A backtesting framework compares the model's forecasting performance against naive benchmarks.

While Hull-White is theoretically elegant, the real value came from seeing its limitations first hand, like its struggle with extreme volatility regimes. More importantly, the project helped me develop intuition for how traders use these models: not as perfect predictors, but as tools for quantifying risk and spotting relative value. After peeking behind the curtain of the quantitative side of rates trading, my interest in building models that interact with real markets has been reinforced.

Contents

1 Introduction

The fixed income market, particularly the segment dealing with interest rates, forms the bedrock of the global financial system. Understanding, modelling, and predicting the behavior of interest rates are paramount for financial institutions involved in trading, investment, and risk management. Quantitative researchers and traders on rates and FX desks dedicate significant effort to developing and implementing models that can capture the complex dynamics of yield curves and associated derivatives.

This project aims to replicate and explore core components of such quantitative work. It involves a multi-faceted approach:

- **Data Management**: Sourcing and preparing reliable yield curve data.
- **Static Modelling**: Fitting observed yield curves using the Nelson-Siegel model to extract meaningful parameters representing level, slope, and curvature.
- **Dynamic Modelling**: Implementing and calibrating the Hull-White one-factor model to describe the stochastic evolution of the short-term interest rate and price interest rate contingent claims.
- **Factor Analysis**: Employing Principal Component Analysis (PCA) to deconstruct yield curve movements into their most significant underlying drivers.
- **Risk Assessment**: Quantifying interest rate risk through metrics like duration, convexity, VaR, and ES, and evaluating model behavior under stressed market conditions.
- **Model Validation**: Backtesting the predictive capabilities of the dynamic model.

The overarching goal is to build an end-to-end analytical framework in Python, providing insights that are both theoretically sound and practically relevant to the tasks faced by quantitative finance professionals. This report details the theoretical underpinnings, implementation strategies, and an extensive analysis of the results generated by this framework.

2 Methodology and Implementation

The project is implemented in Python, leveraging libraries such as NumPy, Pandas, SciPy, Matplotlib, and Scikit-learn. The code is structured into several classes, each responsible for a specific analytical task.

2.1 Data Acquisition and Preparation

Accurate and consistent data is the foundation of any quantitative analysis.

- **Real Data Sourcing**: The primary data source is the Federal Reserve Economic Data (FRED) database, accessed via the `pandas_datareader` library. Daily U.S. Treasury yield data for various maturities (e.g., 1-month, 3-month, 6-month, 1-year, 2-year, 5-year, 10-year, 30-year) are fetched. The raw data, typically in percentage points, is converted to decimal form.

- **Synthetic Data Generation**: As a fallback and for testing purposes, a synthetic data generator (`YieldCurveData` class) is implemented. This creates plausible yield curve data incorporating random walks, mean reversion, and seasonality.
- **Data Cleaning**: Initial data cleaning involves handling missing values (e.g., using forward-fill) and ensuring data consistency. Further cleaning specific to PCA (handling outliers, infinities) is performed within the `PCAAnalyzer` class. The analysis period for this report spans from January 2020 to June 2025.

The `RealYieldCurveData` class encapsulates the logic for fetching and pre-processing FRED data, while `YieldCurveData` provides the synthetic alternative.

2.2 Yield Curve Fitting: The Nelson-Siegel Model

The Nelson-Siegel model [3] is a widely used parametric model to fit the term structure of interest rates.

2.2.1 Theory

The yield $y(T)$ for a given maturity T is expressed as:

$$y(T) = \beta_0 + \beta_1 \left(\frac{1 - e^{-T/\lambda}}{T/\lambda} \right) + \beta_2 \left(\frac{1 - e^{-T/\lambda}}{T/\lambda} - e^{-T/\lambda} \right) \tag{1}$$

where:

- β_0: Represents the long-term interest rate (level factor).
- β_1: Represents the short-term spread (slope factor); typically negative for an upward sloping curve.
- β_2: Represents the medium-term component (curvature factor).
- λ: Is a time constant that determines the maturity at which the loading on β_2 achieves its maximum and influences the decay rate of the other factor loadings.

2.2.2 Implementation (`NelsonSiegelFitter` Class)

The `NelsonSiegelFitter` class implements the fitting procedure.

- **Objective Function**: The parameters $(\beta_0, \beta_1, \beta_2, \lambda)$ are estimated by minimizing the sum of squared errors (SSE) between the model-implied yields (from Equation 1) and the observed market yields for a given set of maturities.
- **Optimization**: The `scipy.optimize.minimize` function with the L-BFGS-B algorithm is used for optimization, subject to bounds on the parameters to ensure economic sensibility and numerical stability.
- **Output Functions**: The class also provides methods to derive the zero-coupon yield curve function $Z(0,T)$ and the instantaneous forward curve function $f(0,T)$ from the fitted parameters. The forward rate is approximated numerically as $f(0,T) = Z(0,T) + T\frac{dZ(0,T)}{dT}$.

2.3 Interest Rate Dynamics: The Hull-White One-Factor Model

The Hull-White model [1] is an affine term structure model that extends the Vasicek model by allowing the mean-reversion level to be time-dependent, ensuring a perfect fit to the initial term structure.

2.3.1 Theory

The instantaneous short rate $r(t)$ follows the stochastic differential equation (SDE):

$$dr(t) = (\theta(t) - a \cdot r(t))dt + \sigma dW(t) \tag{2}$$

where:

- $a > 0$: Is the constant speed of mean reversion.
- $\sigma > 0$: Is the constant instantaneous volatility of the short rate.
- $\theta(t)$: Is a time-dependent function calibrated to match the model's theoretical forward rates to the market's initial forward rate curve $f^M(0,t)$. It is given by:

$$\theta(t) = \frac{\partial f^M(0,t)}{\partial t} + a \cdot f^M(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at}) \tag{3}$$

- $dW(t)$: Is a standard Wiener process under the risk-neutral measure $\mathbb{Q}$.

The price $P(t,T)$ of a zero-coupon bond maturing at time T, seen from time t, is given by:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \tag{4}$$

where $B(t,T) = \frac{1}{a}(1 - e^{-a(T-t)})$ and $A(t,T)$ is a more complex term ensuring $P(0,T)$ matches the market discount factor derived from $f^M(0,t)$. Specifically,

$$\ln A(t,T) = \ln \frac{P^M(0,T)}{P^M(0,t)} - B(t,T)f^M(0,t) - \frac{\sigma^2}{4a^3}(e^{-aT} - e^{-at})^2(e^{2at} - 1) \tag{5}$$

Alternatively, for calibration at $t = 0$, $A(0,T)$ can be found by integrating $\theta(u)$ terms.

2.3.2 Implementation (`EnhancedHullWhiteModel` Class)

- **Parameter Storage**: Stores a and σ.
- **Theta Function Construction**: `build_theta_grid` numerically computes $\theta(t)$ using the market forward curve (derived from Nelson-Siegel) and Equation 3.
- **Analytical Bond Pricing**: `bond_price_analytical(`r_0`, T)` calculates $P(0,T)$ using the current short rate r_0 and the derived $A(0,T)$ and $B(0,T)$ functions, where $A(0,T)$ is computed via numerical integration involving $\theta(u)$.

- **Calibration**: `calibrate_to_market_data(maturities, market_yields,` r_0`, vol_data)` is a key method. It finds optimal (a, σ) by minimizing an objective function:

$$\text{Error} = \sum_i (y_i^{\text{model}}(a,\sigma) - y_i^{\text{market}})^2 + w \sum_j (\text{vol}_j^{\text{model}}(a,\sigma) - \text{vol}_j^{\text{market}})^2 \tag{6}$$

  The first term is the squared error between model-implied yields and market yields. The second term (optional, weighted by w) penalizes deviations between model-implied cap volatilities (approximated, e.g., $\sigma_{\text{caplet}}(T) \approx \sigma\sqrt{T}$) and market-observed cap volatilities. This helps anchor σ to market expectations of volatility. Optimization is performed using `scipy.optimize.minimize`.

- **Path Simulation**: `simulate_paths_enhanced(`r_0`, T, n_steps, n_paths)` simulates future paths of $r(t)$ using an Euler-Maruyama discretization of the SDE (Equation 2).

2.4 Principal Component Analysis (PCA)

PCA is a statistical technique used to reduce the dimensionality of a dataset while retaining most of its variance. In finance, it's often applied to yield curve changes to identify the dominant systematic factors driving their movements.

2.4.1 Theory

PCA finds a new set of orthogonal variables (principal components, PCs) that are linear combinations of the original variables (yield changes at different tenors). The PCs are ordered such that PC1 explains the largest possible variance, PC2 explains the largest remaining variance, and so on. For yield curves, these components often have economic interpretations:

- **PC1 (Level)**: A parallel shift in the yield curve.
- **PC2 (Slope)**: A steepening or flattening of the curve.
- **PC3 (Curvature)**: Changes in the convexity or concavity of the curve.

2.4.2 Implementation (`PCAAnalyzer` Class)

- **Input Data**: Daily percentage changes in yields are used.
- **Data Preprocessing**: Data is cleaned (handling NaNs, infinities, extreme outliers using IQR) and then standardized (mean zero, unit variance) before PCA is applied, as PCA is sensitive to variable scaling.
- **PCA Execution**: `sklearn.decomposition.PCA` is used to extract the principal components, their explained variance ratios, and the component loadings (eigenvectors).
- **Factor Process Fitting**: Optionally, an Ornstein-Uhlenbeck (OU) process, $dX_t = \kappa(\mu - X_t)dt + \sigma_{OU} dW_t$, can be fitted to the time series of the principal component scores (factor loadings over time) to model their dynamics. This is approximated by fitting an AR(1) process.

2.5 Risk Metrics

The `AdvancedAnalytics` class implements calculation of several standard risk metrics.

- **Duration and Convexity**:
  - Modified Duration: $D_{\mathrm{mod}} = -\frac{1}{P}\frac{dP}{dy}$. Measures the percentage price sensitivity of a bond to a unit change in its yield.
  - Convexity: $C = \frac{1}{P}\frac{d^2P}{dy^2}$. Measures the rate of change of duration.

  These are calculated numerically using finite differences on bond prices derived from a zero-coupon yield curve function (e.g., from Nelson-Siegel). For a continuously compounded zero-coupon bond of maturity T and yield y, $P = e^{-yT}$, $D_{\mathrm{mod}} = T$, and $C = T^2$.

- **Value at Risk (VaR)**: Estimates the maximum potential loss over a specific time horizon at a given confidence level (e.g., 95%). Calculated using the historical simulation method on a series of returns (e.g., percentage changes in 10Y yield).

- **Expected Shortfall (ES)**: Also known as Conditional VaR (CVaR). Calculates the expected loss given that the loss exceeds the VaR threshold. It provides a measure of tail risk.

2.6 Stress Testing

Stress testing involves evaluating the impact of extreme but plausible market scenarios on portfolio values or model parameters.

2.6.1 Implementation (`AdvancedAnalytics` Class)

The `stress_test_scenarios` method takes the calibrated Hull-White model and applies pre-defined shocks to its parameters (a, σ) or the initial short rate r_0. For each scenario:

1. The model parameters are temporarily adjusted (e.g., `a_multiplier`, `sigma_multiplier`).
2. The Hull-White analytical bond pricing formula is used to re-calculate bond prices across a range of maturities under these stressed parameters.
3. The impact on prices is recorded.

This helps understand the model's sensitivity and potential losses under adverse conditions.

2.7 Backtesting Engine

The `BacktestEngine` class is designed to evaluate the forecasting performance of the Hull-White model.

2.7.1 Implementation

- **Procedure**: Iterates through historical yield data. At each rebalance period (e.g., every 30 days), it uses a lookback window of data (e.g., 252 days) to calibrate the Hull-White model.
- **Forecasting**: The calibrated model is then used to forecast yields for various tenors one period ahead (e.g., 1-month). This is done by calculating the model-implied bond price $P(0, T)$ using the calibrated parameters and r_0 from the calibration date, and then deriving the yield.
- **Evaluation Metrics**: The forecasted yields are compared against the actual realized yields. Performance is measured using Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE).
- **Benchmark**: A naive "carry" benchmark (forecasted yield = current yield) is also computed for comparison.
- **Parameter History**: The calibrated Hull-White parameters (a, σ) are stored at each re-calibration point to track their stability over time.

2.8 Orchestration (`ComprehensiveIRAnalyzer` Class)

This main class integrates all components:

- Manages data loading and cleaning.
- Executes the analysis pipeline: Nelson-Siegel fitting, Hull-White calibration, PCA, risk metric calculation, stress testing, Monte Carlo simulation, and backtesting.
- Stores all results.
- Generates a consolidated text report and graphical visualizations.

3 Results and Analysis

The analysis was performed on U.S. Treasury yield data from January 2, 2020, to May 30, 2025. The report was generated on June 2, 2025.

3.1 Nelson-Siegel Curve Fitting

On the latest analysis date (2025-05-30), the Nelson-Siegel model was fitted to the observed market yields. The parameters are summarized in Table 1.

Table 1: Nelson-Siegel Fitted Parameters (2025-05-30)

Parameter	Value
β_0 (Level)	0.052438
β_1 (Slope)	-0.007805
β_2 (Curvature)	-0.031727
λ	2.355248
Fitting Error (SSE)	0.00000259

Analysis:

- The long-term yield level (β_0) is estimated at 5.24%.
- The negative β_1 (-0.78%) indicates an upward-sloping yield curve, where short-term rates are lower than long-term rates.
- The negative β_2 (-3.17%) suggests a typical humped shape in the medium-term segment of the curve.
- The λ value of approximately 2.36 years influences the position and scale of the slope and curvature effects.
- The extremely low fitting error (Sum of Squared Errors) signifies an excellent fit of the Nelson-Siegel model to the observed market yields on this specific date. This is visually confirmed in Figure 3 (top-left panel).

3.2 Hull-White Model Calibration

The Hull-White one-factor model was calibrated to the term structure on 2025-05-30, potentially also incorporating cap volatility targets. The calibrated parameters are shown in Table 2.

Table 2: Hull-White Calibrated Parameters (2025-05-30)

Parameter	Value
Mean Reversion (a)	0.037505
Volatility (σ)	0.039641
Calibration RMSE	0.026563

Analysis:

- The mean reversion speed (a) of 0.0375 is relatively low, implying that the short rate reverts slowly towards its time-dependent mean $\theta(t)$. This suggests that deviations in the short rate can be persistent.
- The short-rate volatility (σ) is calibrated at 3.96% per annum.
- The calibration Root Mean Squared Error (RMSE) of 0.026563 (or 2.66 percentage points if yields are in decimals) indicates the average deviation between the model-implied yields and the market yields used for calibration. While the model aims to fit the term structure,

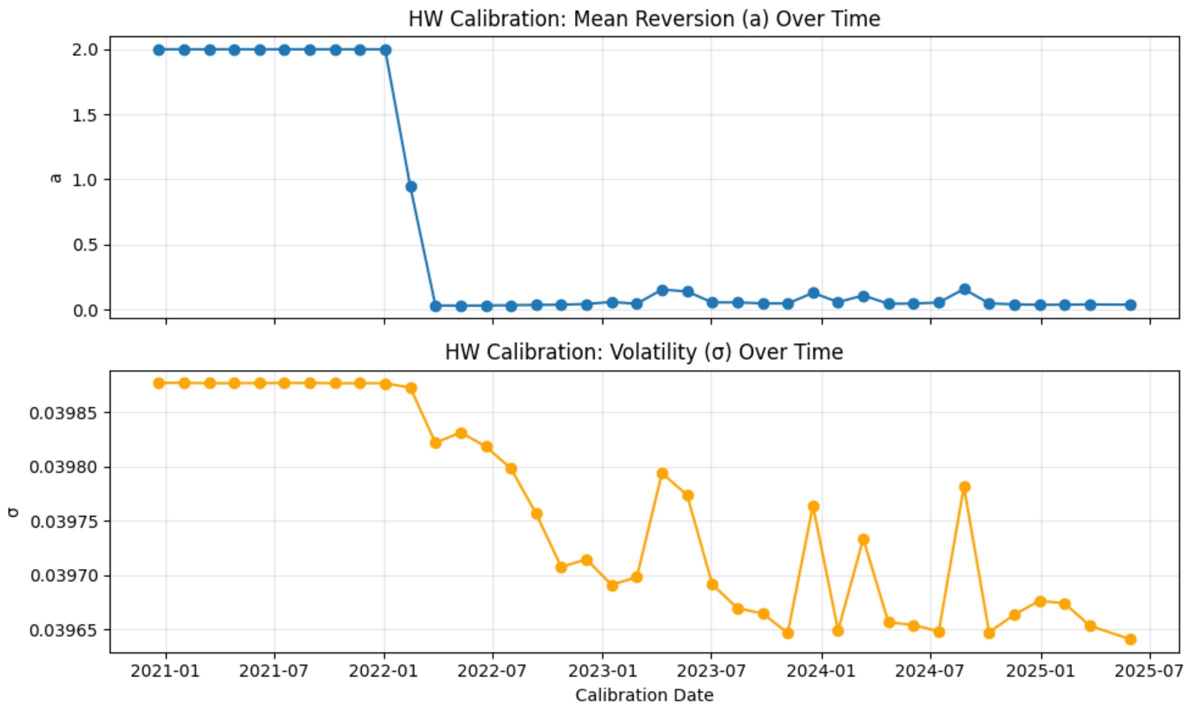

Figure 1: Hull-White Calibration Parameters (a and σ) Over Time

a non-zero RMSE is expected for a one-factor model attempting to capture a multi-dimensional yield curve, especially if cap volatilities also influence the calibration. The quality of this fit is visualized in Figure 3 (bottom-middle panel), where some dispersion around the perfect fit line is visible.

3.2.1 Calibration Parameter Stability

Figure 1 shows the evolution of the calibrated Hull-White parameters (a and σ) over the back-testing period (approximately early 2021 to mid-2025).

Analysis of Parameter Stability:

- **Mean Reversion (a)**: The top panel shows that a started at a high value (around 2.0) in early 2021, then dropped sharply to near zero around late 2022. Subsequently, it remained very low, often fluctuating between 0.0 and 0.2. This instability, particularly the initial high values followed by a regime of very low mean reversion, suggests that the model's characterization of short-rate dynamics changed significantly over the period. Low a implies the short rate behaves more like a random walk, with its drift primarily dictated by $\theta(t)$ to match the evolving term structure. This behavior is common during periods of strong directional rate movements, such as the hiking cycle observed post-2022.

- **Volatility (σ)**: The bottom panel shows that σ was more stable, generally fluctuating between 0.0396 (3.96%) and 0.0399 (3.99%). There were some periods of increased or decreased calibrated volatility, but no dramatic regime shifts comparable to a. This suggests a relatively consistent market-implied short-rate volatility as captured by the model and calibration targets.

3.3 Principal Component Analysis (PCA)

PCA was applied to daily percentage changes of yields across available tenors over the entire data period. The variance explained by the first three principal components is detailed in Table 3.

Table 3: Principal Component Analysis Results

Component	Variance Explained	Cumulative Variance
PC1 (Level)	0.4867 (48.67%)	48.67%
PC2 (Slope)	0.1957 (19.57%)	68.24%
PC3 (Curvature)	0.1150 (11.50%)	79.74%

Analysis:

- The first three principal components collectively explain approximately 79.74% of the total variance in daily yield curve percentage changes.
- **Level**: The first component accounts for 48.67
- **Slope**: The second component explains 19.57
- **Curvature**: The third component accounts for 11.50
- The loadings of these components across maturities are visualized in Figure 3 (top-right panel), exhibiting the typical shapes associated with level, slope, and curvature factors.

3.4 Risk Metrics

Key risk metrics were calculated based on the latest yield curve (fitted by Nelson-Siegel) and historical 10-year yield data, as of 2025-05-30 (Table 4).

Table 4: Calculated Risk Metrics (for a 10-Year Zero-Coupon Bond / 10Y Yield)

Metric	Value
10Y Modified Duration	10.0000
10Y Convexity	100.0000
95% VaR (Daily, 10Y Returns)	-0.043036
95% ES (Daily, 10Y Returns)	-0.076501

Analysis:

- **Duration and Convexity**: For a 10-year zero-coupon bond under continuous compounding, the modified duration is indeed 10.0 and convexity is $10^2 = 100.0$. These values indicate high sensitivity to interest rate changes for such an instrument.
- **VaR**: The 95% daily VaR of -4.30% (based on historical percentage changes of the 10Y yield) suggests that there is a 5% probability of experiencing a loss of at least this magnitude on a position sensitive to 10Y yield changes over a single day.

- **Expected Shortfall**: The 95% daily ES of -7.65% indicates that if a loss exceeding the VaR occurs (i.e., in the worst 5% of scenarios), the average magnitude of that loss is expected to be 7.65%. ES provides a more comprehensive measure of tail risk than VaR.

3.5 Stress Test Results

The Hull-White model, calibrated as of 2025-05-30, was subjected to several stress scenarios by shocking its parameters (a, σ). Table 5 summarizes the scenarios and their impact on an average of bond prices across maturities [0.25, 0.5, 1, 2, 5, 10, 30] years. The detailed stressed prices for each maturity were also computed. Figure 2 visualizes these average price changes.

Table 5: Summary of Stress Test Scenario Impacts on Average Bond Prices

Scenario	Stressed a Multiplier	Stressed σ Multiplier	Approx. Avg. Price Change (%)
Rate Shock Up	1.0	1.5	$\approx -39\%$
Rate Shock Down	1.0	0.5	$\approx +130\%$
Volatility Spike	0.5	2.0	$\approx -41\%$
Mean Reversion Slow	0.3	1.0	$\approx -38\%$

Note: Average price changes are approximate, inferred from Figure 2. The "Rate Shock Down" scenario showed anomalous behavior for the 30Y bond price in the raw output, significantly affecting its average.

Analysis:

- **Rate Shock Up** (Increased σ): Leads to a significant average price decrease ($\approx -39\%$). Higher volatility increases uncertainty and discount rates, lowering bond prices.

- **Rate Shock Down** (Decreased σ): The output showed an average price increase of $\approx +130\%$. However, the raw data indicated an anomalous price for the 30-year bond (10.546), which is unrealistic for a bond price unless r_0 is deeply negative. This suggests a potential numerical instability or edge case in the bond pricing formula under these specific stressed parameters for very long maturities. This result needs careful review and potential correction in the pricing logic.

- **Volatility Spike** (Lower a, higher σ): This severe scenario also results in a substantial average price decrease ($\approx -41\%$), reflecting the combined impact of slower mean reversion and much higher volatility.

- **Mean Reversion Slow** (Lower a): Decreasing the speed of mean reversion leads to an average price decrease ($\approx -38\%$). If rates revert more slowly, they can drift to more extreme levels for longer periods, increasing risk.

Excluding the anomalous "Rate Shock Down" result, the stress tests generally show that scenarios increasing risk (higher σ, lower a) lead to lower bond prices, which is economically intuitive.

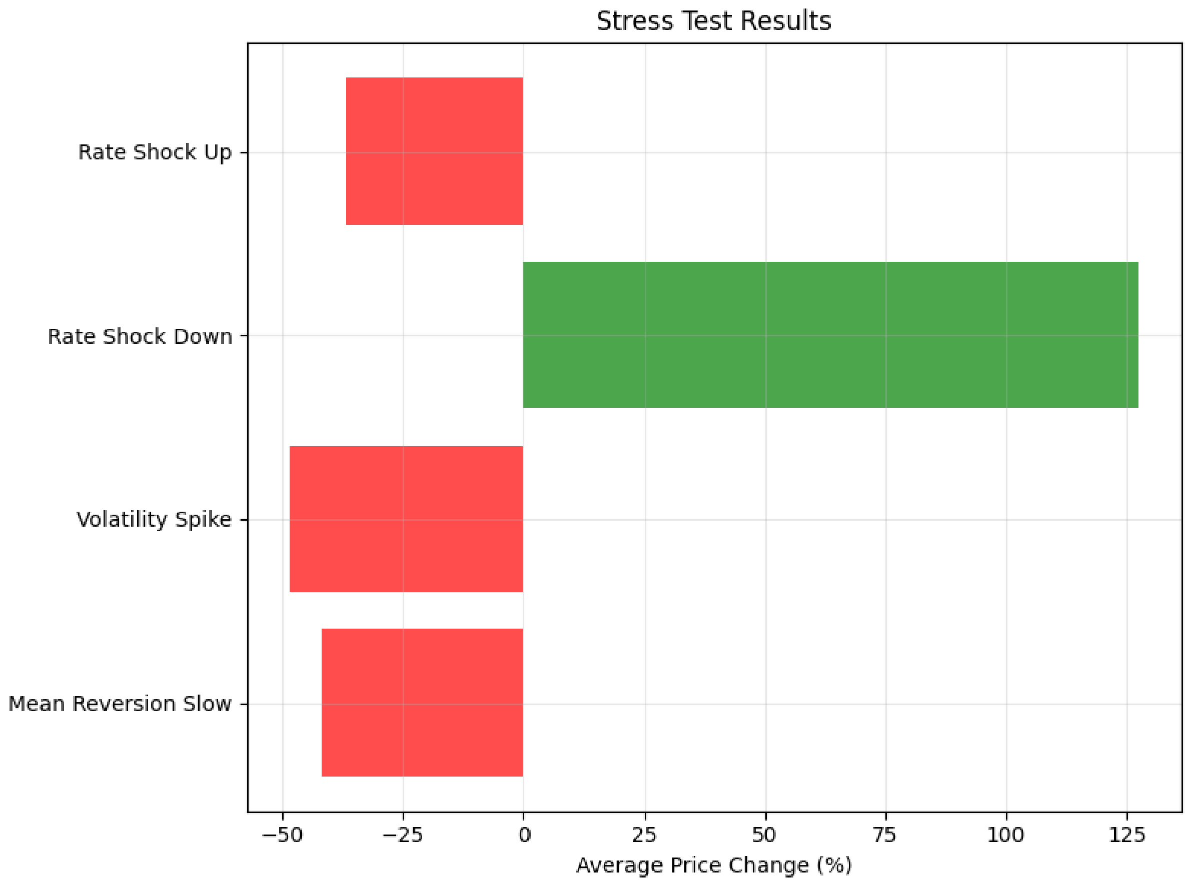

Figure 2: Stress Test Results: Average Price Change (%)

3.6 Monte Carlo Simulation (1-Year Horizon)

Using the Hull-White model calibrated on 2025-05-30, 500 paths of the short rate were simulated over a 1-year horizon (252 steps). Statistics of the final simulated short rates are in Table 6. Sample paths are shown in Figure 3 (bottom-left panel).

Table 6: Monte Carlo Simulation of Short Rate (1-Year Horizon)

Statistic	Value
Mean Final Rate	0.0387
Standard Deviation	0.0386
5th Percentile Final Rate	-0.0262
95th Percentile Final Rate	0.0989

Analysis:

- The mean simulated short rate after one year is 3.87%, with a standard deviation of 3.86%.
- The 90% confidence interval for the short rate at the 1-year horizon is wide, ranging from -2.62% to 9.89%. This reflects the calibrated volatility and relatively slow mean reversion, allowing for significant divergence in rate paths. The possibility of negative rates is consistent with the Hull-White model's Gaussian nature.

3.7 Backtesting Results

The Hull-White model's 1-month ahead yield forecasting ability was backtested against a naive "carry" benchmark over the period 2020-2025. The results are summarized in Table 7.

Table 7: Backtesting Results (1-Month Ahead Yield Forecasts)

Metric	Value
Number of Predictions	38
HW Model Average MAE	0.019796
Naive Carry Average MAE	0.002709
HW Model Average RMSE	0.025742

Analysis:

- The backtest involved 38 prediction periods.
- The Hull-White model had an average Mean Absolute Error (MAE) of approximately 1.98 percentage points in forecasting 1-month ahead yields. This is a substantial error.
- The naive carry strategy (forecasting next month's yield to be the current yield) had a significantly lower MAE of 0.27 percentage points.
- The Root Mean Squared Error (RMSE) for the Hull-White model was 2.57 percentage points, also indicating large forecast errors.

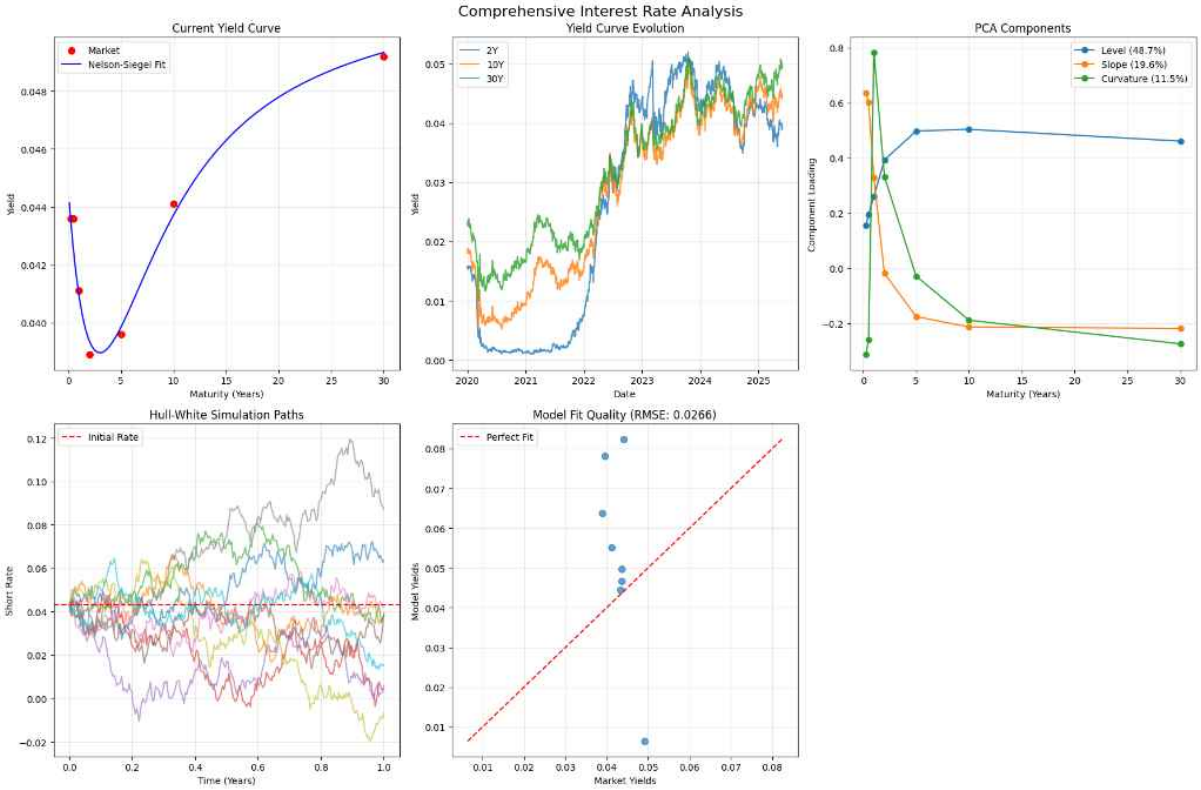

Figure 3: Comprehensive Interest Rate Analysis Dashboard (as of 2025-05-30)

Top-Left: Current Yield Curve and Nelson-Siegel Fit. Top-Middle: Yield Curve Evolution (2Y, 10Y, 30Y). Top-Right: PCA Component Loadings. Bottom-Left: Hull-White Simulation Paths (1Y Horizon). Bottom-Middle: Hull-White Model Fit Quality (Market vs. Model Yields).

- **Crucially, the naive benchmark substantially outperformed the Hull-White model in terms of direct, short-term yield level forecasting.** This is a common finding in academic literature; while term structure models like Hull-White are valuable for pricing derivatives and understanding relative value, they are not necessarily superior for point forecasting of future yield levels, especially over short horizons where yields can exhibit near random-walk behavior.

3.8 Comprehensive Visualization Dashboard

Figure 3 presents a consolidated view of several key analyses.

Analysis of Dashboard Components:

- **Current Yield Curve & NS Fit (Top-Left)**: Shows an upward sloping yield curve on 2025-05-30, well-fitted by the Nelson-Siegel model.
- **Yield Curve Evolution (Top-Middle)**: Illustrates the significant rise in 2Y, 10Y, and 30Y yields from 2020 to 2025, particularly the sharp increases starting in 2022, reflecting changes in monetary policy and inflation expectations.
- **PCA Components (Top-Right)**: Displays the characteristic shapes of the Level, Slope, and Curvature factors derived from PCA, confirming their standard interpretations.

- **Hull-White Simulation Paths (Bottom-Left)**: Visualizes the uncertainty in future short rates as predicted by the calibrated model, highlighting a wide range of potential outcomes.
- **Model Fit Quality (Bottom-Middle)**: The scatter plot of market yields versus Hull-White model-implied yields shows points clustering around the perfect fit line but with noticeable deviations. The RMSE of 0.0266 quantifies this imperfect fit, which is expected for a one-factor model.

4 Relevance to Quant Roles

This project directly mirrors many tasks and required skills for a quantitative researcher or trader on a rates or FX desk.

- **Data Handling and Time Series Analysis**: Sourcing, cleaning, and analyzing financial time series (yields) is a fundamental daily activity.
- **Yield Curve Modelling**:
  - **Rates Desks**: Nelson-Siegel is used for smoothing observed yields, deriving forward curves, and as an input for relative value analysis. Understanding its parameters is key.
  - **FX Desks**: Accurately modelled domestic and foreign yield curves are essential for pricing FX forwards and swaps via interest rate parity.
- **Dynamic Interest Rate Modelling (Hull-White)**:
  - **Rates Desks**: Models like Hull-White (and its multi-factor extensions) are the backbone for pricing and hedging interest rate derivatives (swaps, options, caps, floors, swaptions). Calibration to market instruments (including bonds and vol products like caps) is a critical skill. The simulation capabilities are used for valuing path-dependent options or for risk management.
  - **FX Desks**: While less direct, understanding stochastic interest rate models is important for pricing FX options that may have interest rate components or for understanding the dynamics of interest rate differentials that drive FX.
- **Principal Component Analysis**:
  - **Rates Desks**: PCA helps in understanding the primary drivers of yield curve risk, constructing factor models for risk management (e.g., hedging against level, slope, curvature risks), and identifying relative value opportunities.
  - **FX Desks**: Similar techniques can be applied to FX forward curves or volatility surfaces to identify common factors.
- **Risk Management**:

  - Calculating duration, convexity, VaR, and ES are standard risk management practices across all trading desks to quantify market risk.
  - Stress testing, as performed here, is crucial for understanding potential losses under extreme market conditions and is often a regulatory requirement. Desk quants design and implement these stress tests.

- **Model Validation and Backtesting**:
  - Any model used for pricing or trading must be validated. Backtesting, as done for the Hull-White model's forecasting ability, is a core part of this. Recognizing a model's limitations (e.g., poor forecasting of yield levels by Hull-White) is as important as identifying its strengths. Quants are responsible for assessing model risk.

- **Programming and Technical Skills**: Proficiency in Python and its scientific stack (NumPy, Pandas, SciPy, Matplotlib, Scikit-learn) is a prerequisite for most quantitative roles today. The ability to implement models, run analyses, and visualize results is essential.

- **Problem Solving and Analytical Thinking**: This project's scope of activity spans approaching a complex financial problem, breaking it down into manageable components, applying appropriate mathematical techniques, and impartial interpretation of results. This analytical mindset is highly valued.

The project's focus on calibration, simulation, risk assessment, and model evaluation was my shot at aligning myself with the day-to-day responsibilities of a quantitative investment professional. The attempt to calibrate to cap volatilities, (simplified in this case), shows an understanding of linking models to derivatives markets.

5 Conclusion and Future Work

This project successfully developed and implemented a comprehensive framework for analyzing interest rate dynamics. Key achievements include fitting the Nelson-Siegel model, calibrating and simulating the Hull-White one-factor model, performing PCA on yield curve movements, calculating various risk metrics, and conducting stress tests and backtesting.

Key Findings:

- The Nelson-Siegel model provides an excellent static fit to the observed yield curve.
- The Hull-White model can be calibrated to the term structure, but its one-factor nature limits its ability to perfectly match all market yields simultaneously (RMSE of 2.66%).
- The Hull-White mean-reversion parameter (a) exhibited instability over the backtesting period, suggesting changes in underlying market dynamics or challenges in uniquely identifying it from market data alone.
- PCA confirmed that level, slope, and curvature are the dominant drivers of yield curve changes.

- The backtesting results indicated that the Hull-White model, in its current form, is not a strong predictor of 1-month ahead yield levels, performing worse than a naive carry benchmark. This underscores its primary utility as a model for pricing and relative value rather than directional forecasting.
- Stress tests highlighted significant model sensitivity to parameter shocks, with a notable anomaly in bond pricing for long maturities under one scenario that warrants further investigation.

Limitations:

- The Hull-White model is a one-factor model, which may not fully capture the complex dynamics of the yield curve driven by multiple factors.
- The calibration to cap volatilities was based on a simplified approximation; a more rigorous approach would involve pricing caplets directly using the Hull-White model.
- The backtest focused on yield level forecasting; other applications, such as hedging effectiveness or relative value signal generation, were not explored.

Future Work: Potential extensions to this project could include:

- **Multi-Factor Models**: Implementing and calibrating two-factor (e.g., G2++) or three-factor affine models to potentially achieve better fits and more realistic dynamics.
- **Enhanced Calibration**: Incorporating a broader set of market instruments in the Hull-White calibration, such as swaptions, using more precise pricing formulas.
- **Stochastic Volatility**: Exploring models that allow for stochastic volatility (e.g., Hull-White with stochastic volatility, or SABR for specific options).
- **Macro-Finance Models**: Integrating macroeconomic variables more directly into the term structure model, for instance, by making $\theta(t)$ or the PCA factors dependent on macro forecasts.
- **Trading Strategy Development**: Designing and backtesting specific relative value trading strategies based on mispricings identified by the calibrated models.
- **Numerical Stability**: Rigorously investigating and addressing the numerical instability observed in the bond pricing formula under certain stressed, long-maturity scenarios.
- **Machine Learning Applications**: Exploring machine learning techniques for yield curve forecasting or for identifying non-linear relationships between driving factors.

Overall, this project provides a solid foundation in quantitative interest rate modelling and serves as a strong starting point for more advanced research and practical applications in the field.

References

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