Properties of the unilateral Laplace transform

	Time domain	Frequency domain	Comment
Linearity	$af(t)+bg(t)$	$aF(s)+bG(s)$	Can be proved using basic rules of integration.
Frequency differentiation	$tf(t)$	$-F'(s)$	
Frequency differentiation	$t^n f(t)$	$(-1)^n F^{(n)}(s)$	More general form
Differentiation	$f'(t)$	$sF(s)-f(0^-)$	Obtained by integration by parts
Second Differentiation	$f''(t)$	$s^2F(s)-sf(0^-)-f'(0^-)$	Apply the Differentiation property to $f'(t)$.
General Differentiation	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0^-)-\cdots-f^{(n-1)}(0^-)$	Follow the process briefed for the Second Differentiation.
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma)\,d\sigma$	
Integration	$\int_0^t f(\tau)\,d\tau = u(t)*f(t)$	$\frac{1}{s}F(s)$	$u(t)$ is the Heaviside step function. Note $u(t)*f(t)$ is the convolution of $u(t)$ and $f(t)$, not multiplication.
Scaling	$f(at)$	$\frac{1}{\|a\|}F\left(\frac{s}{a}\right)$	
Frequency shifting	$e^{at}f(t)$	$F(s-a)$	
Time shifting	$f(t-a)u(t-a)$	$e^{-as}F(s)$	$u(t)$ is the Heaviside step function
Convolution	$(f*g)(t)$	$F(s)\cdot G(s)$	
Periodic Function	$f(t)$	$\frac{1}{1-e^{-Ts}}\int_0^T e^{-st}f(t)\,dt$	$f(t)$ is a periodic function of period T so that $f(t)=f(t+T),\ \forall t\geq 0$. This is the result of the time shifting property and the geometric series.

- **Initial value theorem**:

$$f(0^+)=\lim_{s\to\infty} sF(s)$$

- **Final value theorem**:

The integral form of the Borel transform is identical to the Laplace transform; indeed, these are sometimes mistakenly assumed to be synonyms. The generalized Borel transform generalizes the Laplace transform for functions not of exponential type.

Fundamental relationships

Since an ordinary Laplace transform can be written as a special case of a two-sided transform, and since the two-sided transform can be written as the sum of two one-sided transforms, the theory of the Laplace-, Fourier-, Mellin-, and Z-transforms are at bottom the same subject. However, a different point of view and different characteristic problems are associated with each of these four major integral transforms.

Table of selected Laplace transforms

The following table provides Laplace transforms for many common functions of a single variable. For definitions and explanations, see the *Explanatory Notes* at the end of the table.

Because the Laplace transform is a linear operator:

- The Laplace transform of a sum is the sum of Laplace transforms of each term.

$$\mathcal{L}\left\{f(t)+g(t)\right\}=\mathcal{L}\left\{f(t)\right\}+\mathcal{L}\left\{g(t)\right\}$$

- The Laplace transform of a multiple of a function, is that multiple times the Laplace transformation of that function.

$$\mathcal{L}\left\{af(t)\right\}=a\mathcal{L}\left\{f(t)\right\}$$

The unilateral Laplace transform is only valid when t is non-negative, which is why all of the time domain functions in the table below are multiples of the Heaviside step function, u(*t*).

ID	Function	Time domain $x(t)=\mathcal{L}^{-1}\left\{X(s)\right\}$	Laplace s-domain $X(s)=\mathcal{L}\left\{x(t)\right\}$	Region of convergence *for causal systems*
1	ideal delay	$\delta(t-\tau)$	$e^{-\tau s}$	
1a	unit impulse	$\delta(t)$	1	$\mathrm{all}\ s$
2	delayed *n*th power with frequency shift	$\frac{(t-\tau)^n}{n!}e^{-\alpha(t-\tau)}\cdot u(t-\tau)$	$\frac{e^{-\tau s}}{(s+\alpha)^{n+1}}$	$\mathrm{Re}\{s\}>0$
2a	*n*th power (for integer *n*)	$\frac{t^n}{n!}\cdot u(t)$	$\frac{1}{s^{n+1}}$	$\mathrm{Re}\{s\}>0$

2a.1	*q*th power (for real *q*)	$\frac{t^q}{\Gamma(q+1)} \cdot u(t)$	$\frac{1}{s^{q+1}}$	$\mathrm{Re}\{s\} > 0$
2a.2	unit step	$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} > 0$
2b	delayed unit step	$u(t-\tau)$	$\frac{e^{-\tau s}}{s}$	$\mathrm{Re}\{s\} > 0$
2c	ramp	$t \cdot u(t)$	$\frac{1}{s^2}$	$\mathrm{Re}\{s\} > 0$
2d	*n*th power with frequency shift	$\frac{t^n}{n!}e^{-\alpha t} \cdot u(t)$	$\frac{1}{(s+\alpha)^{n+1}}$	$\mathrm{Re}\{s\} > -\alpha$
2d.1	exponential decay	$e^{-\alpha t} \cdot u(t)$	$\frac{1}{s+\alpha}$	$\mathrm{Re}\{s\} > -\alpha$
3	exponential approach	$(1-e^{-\alpha t}) \cdot u(t)$	$\frac{\alpha}{s(s+\alpha)}$	$\mathrm{Re}\{s\} > 0$
4	sine	$\sin(\omega t) \cdot u(t)$	$\frac{\omega}{s^2+\omega^2}$	$\mathrm{Re}\{s\} > 0$
5	cosine	$\cos(\omega t) \cdot u(t)$	$\frac{s}{s^2+\omega^2}$	$\mathrm{Re}\{s\} > 0$
6	hyperbolic sine	$\sinh(\alpha t) \cdot u(t)$	$\frac{\alpha}{s^2-\alpha^2}$	$\mathrm{Re}\{s\} > \|\alpha\|$
7	hyperbolic cosine	$\cosh(\alpha t) \cdot u(t)$	$\frac{s}{s^2-\alpha^2}$	$\mathrm{Re}\{s\} > \|\alpha\|$
8	Exponentially-decaying sine wave	$e^{\alpha t}\sin(\omega t) \cdot u(t)$	$\frac{\omega}{(s-\alpha)^2+\omega^2}$	$\mathrm{Re}\{s\} > \alpha$
9	Exponentially-decaying cosine wave	$e^{\alpha t}\cos(\omega t) \cdot u(t)$	$\frac{s-\alpha}{(s-\alpha)^2+\omega^2}$	$\mathrm{Re}\{s\} > \alpha$
10	*n*th root	$\sqrt[n]{t} \cdot u(t)$	$s^{-(n+1)/n} \cdot \Gamma\left(1+\frac{1}{n}\right)$	$\mathrm{Re}\{s\} > 0$

11	natural logarithm	$\ln\left(\frac{t}{t_0}\right)\cdot u(t)$	$-\frac{t_0}{s}\ [\ \ln(t_0 s)+\gamma\ ]$	$\mathrm{Re}\{s\}>0$
12	Bessel function of the first kind, of order *n*	$J_n(\omega t)\cdot u(t)$	$\frac{\omega^n\left(s+\sqrt{s^2+\omega^2}\right)^{-n}}{\sqrt{s^2+\omega^2}}$	$\mathrm{Re}\{s\}>0$ $(n>-1)$
13	Modified Bessel function of the first kind, of order *n*	$I_n(\omega t)\cdot u(t)$	$\frac{\omega^n\left(s+\sqrt{s^2-\omega^2}\right)^{-n}}{\sqrt{s^2-\omega^2}}$	$\mathrm{Re}\{s\}>\lvert\omega\rvert$
14	Bessel function of the second kind, of order 0	$Y_0(\alpha t)\cdot u(t)$	$-\frac{2\sinh^{-1}(s/\alpha)}{\pi\sqrt{s^2+\alpha^2}}$	$\mathrm{Re}\{s\}>0$
15	Modified Bessel function of the second kind, of order 0	$K_0(\alpha t)\cdot u(t)$		
16	Error function	$\mathrm{erf}(t)\cdot u(t)$	$\frac{e^{s^2/4}\left(1-\mathrm{erf}\left(s/2\right)\right)}{s}$	$\mathrm{Re}\{s\}>0$

Explanatory notes:

- $u(t)$ represents the Heaviside step function.
- $\delta(t)$ represents the Dirac delta function.
- $\Gamma(z)$ represents the Gamma function.
- γ is the Euler-Mascheroni constant.

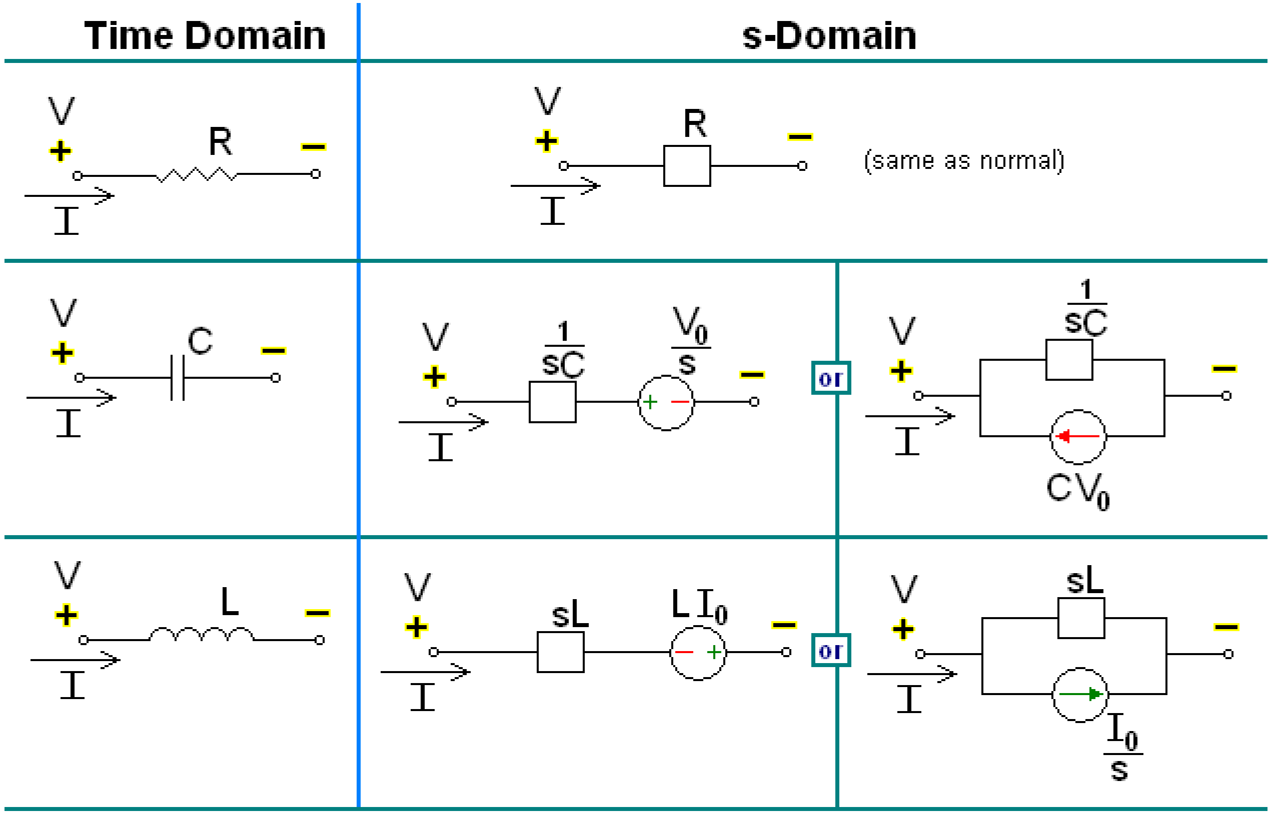
- t, a real number, typically represents *time*, although it can represent *any* independent dimension.
- s is the complex angular frequency, and Re{*s*} is its real part.
- α, β, τ, and ω are real numbers.
- n, is an integer.

- A causal system is a system where the impulse response *h*(*t*) is zero for all time *t* prior to *t* = 0. In general, the ROC for causal systems is not the same as the ROC for anticausal systems. See also causality.

s-Domain equivalent circuits and impedances

The Laplace transform is often used in circuit analysis, and simple conversions to the s-Domain of circuit elements can be made. Circuit elements can be transformed into impedances, very similar to phasor impedances.

Here is a summary of equivalents:



Note that the resistor is exactly the same in the time domain and the s-Domain. The sources are put in if there are initial conditions on the circuit elements. For example, if a capacitor has an initial voltage across it, or if the inductor has an initial current through it, the sources inserted in the s-Domain account for that.

The equivalents for current and voltage sources are simply derived from the transformations in the table above.

Examples: How to apply the properties and theorems

The Laplace transform is used frequently in engineering and physics; the output of a linear time invariant system can be calculated by convolving its unit impulse response with the input signal. Performing this calculation in Laplace space turns the convolution into a multiplication; the latter being easier to solve because of its algebraic form. For more information, see control theory.

The Laplace transform can also be used to solve differential equations and is used extensively in electrical engineering. The method of using the Laplace Transform to solve differential equations was developed by the English electrical engineer Oliver Heaviside.