1.1

Linear system:

Page 7 definition.1

(from Note EE Circuit - Basic & System Theory(Zao).pdf)

or

$T\{X_1(t) + X_2(t)\} = T\{X_2(t)\} + T\{X_2(t)\} = y_1(t) + y_2(t)$

$T\{A*X_1(t)\} = A* y_1(t)$

Superposition:

In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone.

1.2

Open circuit, short circuit, parallel connections, serial connections:

Page 6, 5. electric circuit, operation

(from Note EE Circuit - Basic & System Theory(Zao).pdf)

1.3

Electric power p(t), voltage v(t), current i(t):

Page 1, 公式.2 + 公式.4 推導出公式.6

(from Note EE Circuit - Basic & System Theory(Zao).pdf)

1.4

Root-mean-square voltage value, Vrms:

Page 2, 公式 9

(from Note EE Circuit - Basic & System Theory(Zao).pdf)

1.5 **Kirchhoff's voltage and current (KVL/KCL):**

Page 7, 6 circuit laws

(from Note EE Circuit - Basic & System Theory(Zao).pdf)

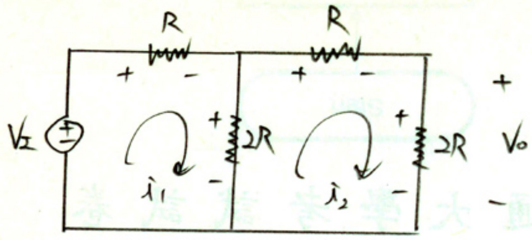
1.6 **Thevenin and Norton equivalent circuits:**

(a) Page 9, theorem 4 & 5

(b) superposition

2.1

2.1



$$\begin{cases} V_I - R i_1 - 2R(i_1 - i_2) = 0 \\ 2R(i_1 - i_2) - R i_2 - 2R i_2 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} V_I - 3R i_1 + 2R i_2 = 0 \quad \longrightarrow \quad V_I - \frac{15}{2} R i_2 + 2R i_2 = 0 \\ 2R i_1 - 5R i_2 = 0 \quad \longrightarrow \quad \underline{i_1 = \frac{5}{2} i_2} \end{cases}$$

$$\therefore V_I = \frac{11}{2} R i_2 \quad \longrightarrow \quad \underline{i_2 = \frac{2}{11} \cdot \frac{V_I}{R}}$$

$$\therefore V_o = \frac{2}{11} \times \frac{V_I}{R} \times 2R = \underline{\frac{4}{11} V_I}$$

$$V_o = \frac{\frac{6}{5}R}{R + \frac{6}{5}R} \times \frac{2R}{2R + R} \times V_I = \underline{\frac{4}{11} V_I}$$

2.2

2.2 #

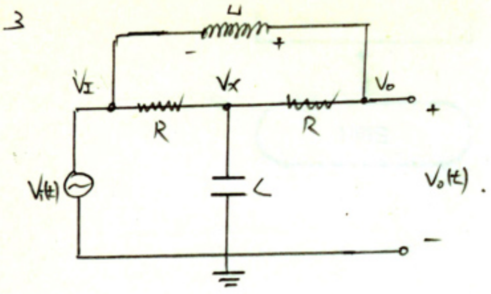
① $V_{th} = \frac{4}{11} V_I$

② R_{out}

$$R_{out} = [(R /\!/ 2R) + R] /\!/ 2R = \frac{10}{11} R$$

2.3

2.3



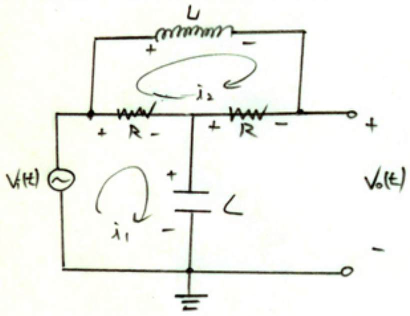
$V_L = L\frac{di_L}{dt} \rightarrow i_L = \frac{1}{L}\int_0^t V_L\,dz + i_L(0)$

$i_c = C\frac{dV_c}{dt}$

KCL:

for V_x : $\frac{V_x - V_I}{R} + C\frac{dV_x}{dt} + \frac{V_x - V_o}{R} = 0.$

for V_o : $\frac{V_o - V_x}{R} + \frac{1}{L}\int_0^t (V_o - V_I)\,dz + i_L(0) = 0$



KVL:

for i_1 : $V_{in} - R(i_1 - i_2) - \left[\frac{1}{C}\int_0^t i_1\,dz + V_c(0)\right] = 0.$

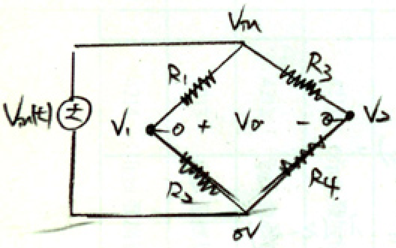
for i_2 : $R(i_1 - i_2) + R(-i_2) - L\frac{di_2}{dt} = 0.$

2.4

2.4 Wheatstone bridge.

(a) $V_o = 0$.

KCL



令 V_1 ; V_2.

$$\begin{cases} \text{for } V_1 : \dfrac{V_1 - V_{in}}{R_1} + \dfrac{V_1}{R_2} = 0. & -① \\ \text{for } V_2 : \dfrac{V_2 - V_{in}}{R_3} + \dfrac{V_2}{R_4} = 0 & -② \end{cases}$$

整理 ①

$$V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V_{in}}{R_1}$$

$$V_1 = \frac{R_1 R_2}{R_1 + R_2} \times \frac{V_{in}}{R_1} = \frac{R_2}{R_1 + R_2} V_{in}.$$

整理 ②

$$V_2 = \frac{R_4}{R_3 + R_4} V_{in}.$$

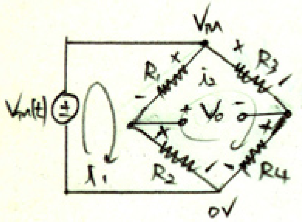
$\because V_o = V_1 - V_2 = 0.$

$$\therefore \frac{R_2}{R_1 + R_2} = \frac{R_4}{R_3 + R_4}.$$

$$\frac{1}{\frac{R_1}{R_2} + 1} = \frac{1}{\frac{R_3}{R_4} + 1}$$

$$\therefore \frac{R_1}{R_2} = \frac{R_3}{R_4}$$ 得證.

KVL



假設 i_1 , i_2

$$\begin{cases} i_1: \ V_{in} - R_1(i_1-i_2) - R_2(i_1-i_2) = 0 & -① \\ i_2: \ R_2(i_1-i_2) + R_1(i_1-i_2) - R_3 i_2 - R_4 i_2 = 0 & -② \end{cases}$$

from ①

$$V_{in} - (R_1+R_2)(i_1-i_2) = 0$$

$$V_{in} + i_2(R_1+R_2) = i_1(R_1+R_2)$$

$$\therefore i_1 = \frac{V_{in}}{R_1+R_2} + i_2 \quad -③$$

整理②

$$(R_1+R_2)(i_1-i_2) - (R_3+R_4)i_2 = 0$$

$$(R_1+R_2)i_1 - (R_1+R_2+R_3+R_4)i_2 = 0$$

③代入②

$$V_{in} + (R_1+R_2)i_2 - (R_1+R_2+R_3+R_4)i_2 = 0$$

$$\therefore V_{in} - (R_3+R_4)i_2 = 0$$

$$i_2 = \frac{V_{in}}{R_3+R_4} \quad -④$$

④代入③

$$i_1 = \frac{V_{in}}{R_1+R_2} + \frac{V_{in}}{R_3+R_4} \quad -⑤$$

$$\therefore V_0 = [V_{in} - R_1(i_1-i_2)] - [V_{in} - R_3 i_2] = 0$$

$$\Rightarrow \cancel{V_{in}}\left(1 - \frac{R_1}{R_1+R_2}\right) = \cancel{V_{in}}\left(1 - \frac{R_3}{R_3+R_4}\right)$$

$$\frac{R_2}{R_1+R_2} = \frac{R_4}{R_3+R_4} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

2.4

(b)

$$\begin{cases} R_1 = R_3 = R \;/\!/ \\ V_o = V_1 - V_2 = \frac{1}{2} V_I . \;/\!/ \end{cases}$$

$$\begin{cases} V_1 = \frac{R_2}{R_1+R_2} V_I \\ V_2 = \frac{R_4}{R_3+R_4} V_I . \end{cases}$$

$$\therefore\ V_o = \left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4} \right) \cancel{V_I} = \frac{\cancel{V_I}}{2}$$

$$= \frac{R_2(R_3+R_4) - R_4(R_1+R_2)}{(R_1+R_2)(R_3+R_4)} = \frac{1}{2}$$

$$= \frac{R_2(R+R_4) - R_4(R+R_2)}{(R+R_2)(R+R_4)} = \frac{1}{2}$$

$$= \frac{RR_2 + \cancel{R_2R_4} - RR_4 - \cancel{R_2R_4}}{R^2 + RR_2 + RR_4 + R_2R_4} = \frac{1}{2}$$

$$= \frac{RR_2 - RR_4}{R^2 + RR_2 + RR_4 + R_2R_4} = \frac{1}{2}$$

$$2RR_2 - 2RR_4 = R^2 + RR_2 + RR_4 + R_2R_4 .$$

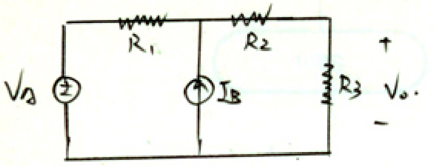
$$RR_2 = R^2 + 3RR_4 + R_2R_4$$

$$R(R_2 - R) = R_4(3R + R_2)$$

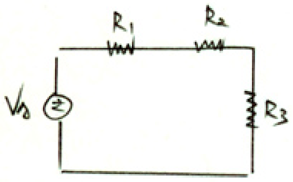
$$R_4 = \frac{R(R_2-R)}{(R_2+3R)} \;/\!/$$ 且 $R_2 > R$ // 解才正確

2.5

2.5

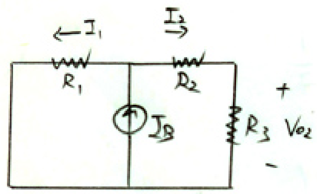


for V_s :



$$V_{o1} = \frac{R_3}{R_1+R_2+R_3} V_s$$

for I_B :



$$\begin{cases} R_1 I_1 = I_2 (R_2+R_3) \\ I_1 + I_2 = I_B \end{cases}$$

$\rightarrow I_1 = I_B - I_2.$

$$R_1 (I_B - I_2) = I_2 (R_2+R_3)$$

$$R_1 I_B - R_1 I_2 = I_2 (R_2+R_3)$$

$$\therefore\ I_2 = \frac{R_1 I_B}{R_1+R_2+R_3}$$

$$\therefore\ V_{o2} = I_2 R_3 = \frac{R_1 R_3}{R_1+R_2+R_3} I_B$$

$$\therefore\ V_o = V_{o1} + V_{o2}$$

$$= \frac{R_3}{R_1+R_2+R_3} V_s + \frac{R_1 R_3}{R_1+R_2+R_3} I_B$$