Homework #3

Due date: 11/3

Prime factorization and divisor

Given an integer $n \geq 2$, factor it into primes and use the factorization to determine the number and sum of divisors of n.

Let $n = {p_1}^{e_1}{p_2}^{e_2} \cdots {p_k}^{e_k}$, where $p_1 < p_2 < \cdots < p_k$ are primes and $e_i > 0$, be the prime factorization of n. Then,

$$\text{the number of divisors of } n = \prod_{i=1}^{k}(1 + e_i)$$

and

$$\text{the sum of divisors of } n = \prod_{i=1}^{k}\sum_{j=0}^{e_i} p_i^j$$

For example, $20 = 2^2 \cdot 5$ has 6 divisors, namely, $1, 2, 4, 5, 10$ and 20.
By the formulas, we see that
the number of divisors of 20 is $(1 + 2)(1 + 1) = 6$, and
the sum of divisors of 20 is $(2^0 + 2^1 + 2^2)(5^0 + 5^1) = 42$,
as desired.

Requirements

1 You shall write the following function

```
void factorization(int n);
```

to factor n and compute the number and sum of its divisors. The kernel of this function is the following loop:

while (*not finish yet*) {
    Let $p =$ the next prime
    Let $e =$ the largest integer such that p^e divides n
    Let $n = n/p^e$
}

For example, let $n = 20 = 2^2 \cdot 5$, the values of $p, e,$ and n at the end of each iteration are shown below:

1st iteration	$p = 2$	$e = 2$	$n = 5$
2nd iteration	$p = 3$	$e = 0$	$n = 5$
3rd iteration	$p = 5$	$e = 1$	$n = 1$

In this case, the loop terminates when $n = 1$. In other cases, we don't need to wait until $n = 1$ to terminate the loop.
For example, let $n = 84 = 2^2 \cdot 3 \cdot 7$, then

1st iteration	$p = 2$	$e = 2$	$n = 21$	
2nd iteration	$p = 3$	$e = 1$	$n = 7$	
3rd iteration	$p = 5$	$e = 0$	$n = 7$	(redundant)
4th iteration	$p = 7$	$e = 1$	$n = 1$	(redundant)

The last two iterations are redundant, because n is already a prime at the end of the 2nd iteration
Figure out a good termination condition for the loop.

2 Compute the value of p_i^j incrementally. That is, do not compute p_i^j from scratch. Instead, use the value of p_i^{j-1} to compute p_i^j.

3 Refer to the sample run below for the required output format

Sample run

```
Enter an integer >= 2: 20
Prime factorization of 20 = pow(2,2)pow(5,1)
Number of divisors = 6
Sum of divisors = 42

Enter an integer >= 2: 84
Prime factorization of 84 = pow(2,2)pow(3,1)pow(7,1)
Number of divisors = 12
Sum of divisors = 224
```

```
Enter an integer >= 2: 427309124
Prime factorization of 427309124 = pow(2,2)pow(11,1)pow(9711571,1)
Number of divisors = 12
Sum of divisors = 815772048

Enter an integer >= 2: 291347131
Prime factorization of 291347131 = pow(291347131,1)
Number of divisors = 2
Sum of divisors = 291347132

Enter an integer >= 2: ^Z
```