Homework #1

Due date: 10/20

Farey sequence

The ascending sequence of all reduced fractions between 0 and 1 which have denominators $\leq n$ is called the Farey sequence of order n.
For example, the Farey sequence of order 7 is

$$\frac{0}{1},\frac{1}{7},\frac{1}{6},\frac{1}{5},\frac{1}{4},\frac{2}{7},\frac{1}{3},\frac{2}{5},\frac{3}{7},\frac{1}{2},\frac{4}{7},\frac{3}{5},\frac{2}{3},\frac{5}{7},\frac{3}{4},\frac{4}{5},\frac{5}{6},\frac{6}{7},\frac{1}{1}$$

Note that there are 19 terms in this sequence.

Let x_i/y_i , $i = 0,1,2,3,\ldots$ be the Farey sequence of order n.
It can be shown that

$x_0 = 0, x_1 = 1$

$y_0 = 1, y_1 = n$

$x_k = \lfloor (y_{k-2} + n)/y_{k-1} \rfloor \, x_{k-1} - x_{k-2}$

$y_k = \lfloor (y_{k-2} + n)/y_{k-1} \rfloor \, y_{k-1} - y_{k-2} \quad k \geq 2$

Note: In mathematics, the floor function $\lfloor x \rfloor$ maps the real number x to the largest integer that is less than or equal to x, e.g. $\lfloor 3.5 \rfloor = 3$, $\lfloor 3.0 \rfloor = 3$. The floor functions in the preceding formulas may be omitted when coding in C/C++, since y_i and n are integers and the integer division for quotient yields the same result as the floor function. For example,

$\lfloor 10/4 \rfloor = \lfloor 2.5 \rfloor = 2$ where / is the division in math

and

$10/4 = 2$ where / is the integer division in C/C++

Given an integer $n \geq 1$, generate the Farey sequence of order n and count the number of terms in it.

Hint

This problem is analog to the computation of the Fibonacci numbers as discussed in class.

Requirements

1 Write an interactive C program that is capable of handling multiple inputs. You may assume that the inputs are correct.

2 See the sample run below for the required output format.

3 Your program shall consist of two functions: function **`main`** handles multiple inputs and function **`Farey`** computes the Farey sequences.

4 For the sake of fairness, DO NOT use any technique, e.g. recursion, that hasn't been taught so far.

Sample run

```
Enter an integer >= 1: 1
0/1 1/1
The Farey sequence of order 1 has 2 terms.

Enter an integer >= 1: 5
0/1 1/5 1/4 1/3 2/5 1/2 3/5 2/3 3/4 4/5 1/1
The Farey sequence of order 5 has 11 terms.

Enter an integer >= 1: 7
0/1 1/7 1/6 1/5 1/4 2/7 1/3 2/5 3/7 1/2 4/7 3/5 2/3 5/7 3/4 4/5 5/6 6/7 1/1
The Farey sequence of order 7 has 19 terms.

Enter an integer >= 1: 15
0/1 1/15 1/14 1/13 1/12 1/11 1/10 1/9 1/8 2/15 1/7 2/13 1/6 2/11 1/5 3/14 2/9 3/
13 1/4 4/15 3/11 2/7 3/10 4/13 1/3 5/14 4/11 3/8 5/13 2/5 5/12 3/7 4/9 5/11 6/13
 7/15 1/2 8/15 7/13 6/11 5/9 4/7 7/12 3/5 8/13 5/8 7/11 9/14 2/3 9/13 7/10 5/7 8
/11 11/15 3/4 10/13 7/9 11/14 4/5 9/11 5/6 11/13 6/7 13/15 7/8 8/9 9/10 10/11 11
/12 12/13 13/14 14/15 1/1
The Farey sequence of order 15 has 73 terms.

Enter an integer >= 1: ^Z
```