HW3 solution

1 a)

$$\dfrac{\dfrac{\dfrac{c : t4 \qquad \dfrac{f : t1 \quad a : t2}{f\ a : t3}}{c\ (f\ a) : t5} \qquad \dfrac{f : t1 \quad b : t6}{f\ b : t7}}{c\ (f\ a)\ (f\ b) : t8}}{\lambda f.\lambda a.\lambda b.\lambda c.c\ (f\ a)\ (f\ b) : t9}$$

We have the following equations:

$t1 = t2 \rightarrow t3$ (1)

$t4 = t3 \rightarrow t5$ (2)

$t1 = t6 \rightarrow t7$ (3)

$t5 = t7 \rightarrow t8$ (4)

$t9 = t1 \rightarrow t2 \rightarrow t6 \rightarrow t4 \rightarrow t8$

Form (1) and (3), t2 = t6, and t3 = t7.

From (2) and (4), $t4 = t3 \rightarrow t7 \rightarrow t8 = t3 \rightarrow t3 \rightarrow t8$

Thus, the solution for t9 is

$t9 = (t2 \rightarrow t3) \rightarrow t2 \rightarrow t2 \rightarrow (t3 \rightarrow t3 \rightarrow t8) \rightarrow t8$

b)

$$\dfrac{\dfrac{\dfrac{b : t4 \quad f : t1}{b\ f : t5} \qquad \dfrac{\dfrac{f : t1 \quad a : t2}{f\ a : t3} \quad b : t4}{f\ a\ b : t6}}{b\ f\ (\ f\ a\ b) : t7}}{\lambda f.\lambda a.\lambda b.b\ f\ (f\ a\ b) : t8}$$

We have the following equations:

$t1 = t2 \rightarrow t3$

$t3 = t4 \rightarrow t6$

$t4 = t1 \rightarrow t5$

$t5 = t6 \rightarrow t7$

$t8 = t1 \rightarrow t2 \rightarrow t4 \rightarrow t7$

This system of equations has no solutions, due to circularity:

$t1 = t2 \rightarrow t3 = t2 \rightarrow t4 \rightarrow t6 = t2 \rightarrow (t1 \rightarrow t5) \rightarrow t6$

It follows that t1 is an infinite type.

c) $\underline{\text{fix} : t1 \quad f : t2}$

$\underline{\text{fix } f : t3 \qquad x : t4}$

$\underline{\text{fix } f\ x : t5}$

$\underline{f : t2 \qquad\quad \lambda x.\text{fix } f\ x : t6}$

$\underline{f\ (\lambda x.\text{fix } f\ x) : t7}$

$\text{fix} = \lambda f.\ f\ (\lambda x.\text{fix } f\ x) : t1$

We have the following equations:

$t1 = t2 \rightarrow t3$ (*)

$t3 = t4 \rightarrow t5$

$t6 = t4 \rightarrow t5$

$t2 = t6 \rightarrow t7$

$t1 = t2 \rightarrow t7$ (*)

It follows from the two starred lines that t3 = t7.

Hence, $t1 = t2 \rightarrow t7 = (t6 \rightarrow t7) \rightarrow t7 = ((t4 \rightarrow t5) \rightarrow t7) \rightarrow t7$

$= ((t4 \rightarrow t5) \rightarrow t3) \rightarrow t3$

$= ((t4 \rightarrow t5) \rightarrow t4 \rightarrow t5\ ) \rightarrow t4 \rightarrow t5$

2 a) Basis : []

$y \oplus (\text{foldl} \otimes z\ [])$

$= y \oplus z$ (foldl.1)

$= \text{foldl} \otimes (y \oplus z)\ []$ (foldl.1)

Induction step: x :: xs

$y \oplus (\text{foldl} \otimes z\ (x :: xs))$

$= y \oplus (\text{foldl} \otimes (z \otimes x)\ xs)$ (foldl.2)

$= \text{foldl} \otimes (y \oplus (z \otimes x))\ xs$ (induction hypothesis)

$= \text{foldl} \otimes ((y \oplus z) \otimes x)\ xs$ (assumption)

$= \text{foldl} \otimes (y \oplus z)\ (x :: xs)$ (foldl.2)

b) Basis: []

$\text{foldr} \oplus a\ []$

$= a$ (foldr.1)

$= \text{foldl} \otimes a\ []$ (foldl.1)

Induction step: x :: xs

$\text{foldr} \oplus a\ (x :: xs)$

$= x \oplus (\text{foldr} \oplus a\ xs)$ (foldr.2)

$= x \oplus (\text{foldl} \otimes a\ xs)$ (induction hypothesis)

$= \text{foldl} \otimes (x \oplus a)\ xs$ (lemma)

$= \text{foldl} \otimes (a \otimes x)\ xs$ (assumption)

$= \text{foldl} \otimes a\ (x :: xs)$ (foldl.2)

c) Since

$x \oplus (y \oplus z) = (x \oplus y) \oplus z$ ($\oplus$ is associative)

and

$x \oplus a = a \oplus x$ (a is the identity of $\oplus$)

It follows from the theorem of part b) that, for any list xs,

foldr $\oplus$ a xs = foldl $\oplus$ a xs

d) Since + is associative and 0 is the identity of +, it follows from the corollary that, for any list xs,

foldr op+ 0 xs = foldl op+ 0 xs

$\Rightarrow$ foldr op+ 0 = foldl op+ 0 (f = g iff f x = g x for all x)

$\Rightarrow$ sumr = suml (def. of sumr and suml)

e) Let $\oplus$ = fn (x,xs) => xs @ [x]

and $\otimes$ = fn (xs,x) => x :: xs

Then,

x $\oplus$ [] = [] @ [x] = [x] = x :: [] = [] $\otimes$ x

and

x $\oplus$ (y $\otimes$ z)

= x $\oplus$ (z :: y) (def. of $\otimes$)

= (z :: y) @ [x] (def. of $\oplus$)

= ([z] @ y) @ [x] (z :: y = [z] @ y, for any list y. Prove it!)

= [z] @ (y @ [x]) (@ is associative. Prove it!)

= z :: (y @ [x])

= z :: (x $\oplus$ y) (def. of $\oplus$)

= (x $\oplus$ y) $\otimes$ z (def. of $\otimes$)

Thus, by the theorem, we have, for any list xs,

foldr $\oplus$ [] xs = foldl $\otimes$ [] xs

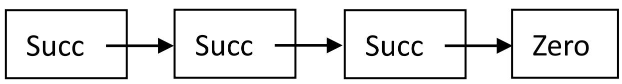
$\Rightarrow$ revr xs = revl xs (def. of revr and revl)

$\Rightarrow$ revr = revl (f = g iff f x = g x for all x)

f) See file hw3.ml

3 See file hw3.ml

4 a)



b~g) See file hw3.ml

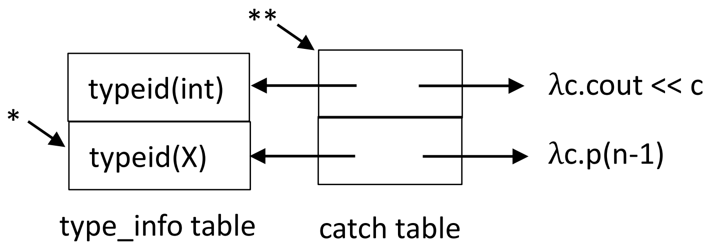
5 See file hw3.ml

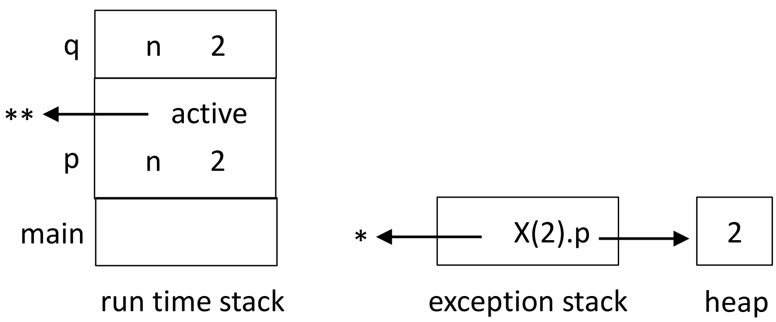
6 a) X(2) constructed
X(1) constructed
X(1) destructed
X(2) destructed
Comment
By return-value optimization (RVO), the exception objects X(2) and X(1) are directly constructed on the exception stack. Thus, the copy ctor and move ctor aren't involved.

Q: What would happen if function q is written as
void q(int n) { if (n>0) { X a(n); throw a; } }

A: In this case, the X object a of function q is expiring and so its resource is stolen by the move ctor for the exception object.
The output becomes:
X(2) constructed
X(2) moved
Moved X object: Nothing to destruct
X(1) constructed
X(1) moved
Moved X object: Nothing to destruct
X(1) destructed
X(2) destructed

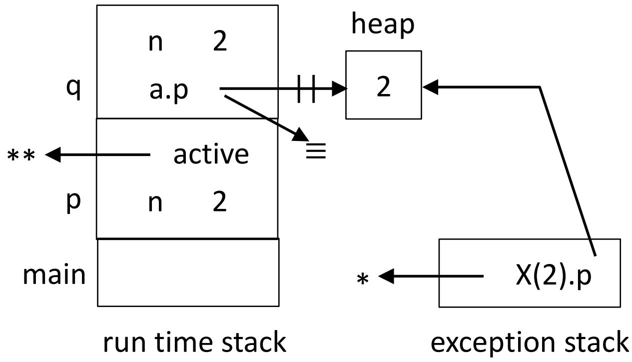
b)





4 b) Comment (Continuing part a) comment)

Resource stealing illustrated



c)