$$\Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma \eta_{\mu\nu} = \eta_{\rho\sigma}$$

$$\det \Lambda = +1$$

$$\Lambda^0{}_0 \geq 1$$

$$\Lambda = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Многообразие

т. Уитни

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Вектор. поле $\quad \forall p \in M \quad p \Rightarrow v \in T_p M$

$V^\mu(x) \quad V \Rightarrow V^* \quad \forall p, \quad p \Rightarrow v^* \in T_p M^*$

$$f(x,y,z) \Rightarrow f(r\sin\theta\cdot\cos\varphi,\ r\sin\theta\sin\varphi,\ r\cos\theta)$$

$$x, y, z \longrightarrow r, \theta, \varphi$$

$$f(x) \cong f(x^0, x^1, x^2, x^3)$$

$$f'(x') = f(x(x'))$$

$$V'^\mu(x') = \sum_{\nu=0,1,2,3} \frac{\partial x'^\mu}{\partial x^\nu} V^\nu(x(x'))$$

Победря
Тензорный анализ

$V \ni v \qquad V^* \ni u \qquad u(v) \in R$

$\{e_i\} \qquad \{f^j\}$ e^j $\qquad u = \sum_j \mu^j f^j$

$$u\left(\sum \lambda_i e_i\right) = \sum \lambda_i u(e_i) = \sum_i \sum_j \lambda_i \mu^j \overbrace{f^j(e_i)}^{\delta^j_i}$$

$$u = \sum_{j=1}^{d} \mu_j e^j = \sum_{j=1}^{d} \mu_j O^j{}_m e'^m$$

$$v = \sum_{i=1}^{d} \lambda^i e_i = \sum_{i=1}^{d} \lambda_i \sum_{k=1}^{d} U_i{}^k e'_k$$

$$u(v) = \sum \mu_j O^j{}_m e'^m (\lambda^i U_i{}^k e'_k) = \sum \mu_j O^j{}_m \lambda^i U_i{}^k \overbrace{e'^m(e'_k)}^{\delta^m_k} \Rightarrow O = U^{-1}$$

$$W'_\mu(x') = \sum_{\nu=0,1,2,3} \frac{\partial x^\nu}{\partial x'^\mu} W_\nu(x(x'))$$

$$x'^\nu = \Lambda^\mu{}_\nu x^\nu \qquad x^\nu = (\Lambda^{-1})^\nu{}_\mu \, x'^\mu$$

$$V'^\mu(x') = \Lambda^\mu{}_\nu V^\nu(x(x'))$$

$$W'_\mu(x') = (\Lambda^{-1})^\nu{}_\mu W_\nu(x(x'))$$

V $\qquad$ V^*

$B(\cdot\cdot,\cdot\cdot)$

$v \in V$ $\qquad$ $B(v,\cdot)$

$$T^{\mu'_1\dots\mu'_p}{}_{\nu'_1\dots\nu'_q}(x') = \frac{\partial x'^{\mu_1}}{\partial x^{\mu_1}} \cdots \frac{\partial x'^{\mu_p}}{\partial x} \frac{\partial x^{\nu_1}}{\partial x'^{\nu_1}} \cdots \frac{\partial x^{\nu_q}}{\partial x'^{\nu_q}} = T^{\mu_1\dots\mu_p}{}_{\nu_1\dots\nu_q}(x(x'))$$

$$g_{\mu\nu}(x)$$

$$g_{\mu'\nu'}(x') = \frac{\partial x^\mu}{\partial x'^{\mu'}} \frac{\partial x^\nu}{\partial x'^{\nu'}} g_{\mu\nu}(x(x'))$$

$$(g_{\mu\nu}(x) V^\nu(x))' = \frac{\partial x^\sigma}{\partial x'^\mu} \frac{\partial x^\lambda}{\partial x'^\nu} g_{\sigma\lambda}(x(x')) \frac{\partial x'^\nu}{\partial x^\rho} V^\rho(x(x')) = \frac{\partial x^\sigma}{\partial x'^\mu} g_{\sigma\lambda}(x(x')) \cdot$$

$$\cdot \underbrace{\boxed{\frac{\partial x^\lambda}{\partial x'^\nu} \frac{\partial x'^\nu}{\partial x^\rho}}}_{\delta^\lambda_\rho} \cdot V^\rho(x(x')) = \frac{\partial x^\sigma}{\partial x'^\mu} g_{\sigma\lambda}(x(x')) \cdot \underbrace{\delta^\lambda_\rho V^\rho(x(x'))}_{= V^\lambda(x(x'))}$$

V	$B(\cdot\cdot,\cdot\cdot\cdot)$	V^*	$B^*(\cdot\cdot,\cdot\cdot)$
T_pM	$g_{\mu\nu}(x)$	T_p^*M	$g^{\mu\nu}(x)$

$V^\mu \rightarrow$ $g_{\mu\nu} V^\nu = V_\mu$

$W_\mu \Rightarrow$ $g^{\mu\nu} W_\nu = W^\mu$

$$p^\mu = mcu^\mu = \begin{pmatrix} \varepsilon/c \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} \qquad p'^\mu = \Lambda^\mu{}_\nu p^\nu \qquad x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

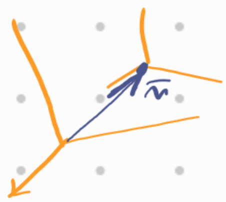
$$p_\mu = \eta_{\mu\nu} p^\nu = \begin{pmatrix} \varepsilon/c \\ -p_x \\ -p_y \\ -p_z \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \Lambda^{-1} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$p^\mu = \begin{pmatrix} \varepsilon/c \\ p_x \\ p_y \\ p_z \end{pmatrix} \Rightarrow \quad p'^\mu = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \varepsilon/c \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \gamma\,\varepsilon/c - \beta\gamma p_x \\ -\beta\gamma\,\varepsilon/c + \gamma p_x \\ p_y \\ p_z \end{pmatrix}$$

$$p_\mu = \begin{pmatrix} \varepsilon/c \\ -p_x \\ -p_y \\ -p_z \end{pmatrix} \Rightarrow \quad p'_\mu = (\Lambda^{-1})^\nu{}_\mu\, p_\nu = \begin{pmatrix} \gamma & \beta\gamma & & \\ \beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \varepsilon/c \\ -p_x \\ -p_y \\ -p_z \end{pmatrix} = \begin{pmatrix} \gamma\,\varepsilon/c - \beta\gamma p_x \\ \beta\gamma\,\varepsilon/c - \gamma p_x \\ -p_y \\ -p_z \end{pmatrix}$$

$$T^\mu{}_\nu\, \eta_{\mu\lambda} = T'_{\lambda\nu}$$
$$T'_{\nu\lambda}$$



$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \bar{r} = \begin{pmatrix} \gamma ct - \beta\gamma x \\ -\beta\gamma ct + \gamma x \\ y \\ z \end{pmatrix}$$

$\bar{v}$

$$\bar{r} = \bar{r}_\parallel + \bar{r}_\perp = \frac{(\bar{r},\bar{v})}{v^2}\cdot\bar{v} + \left(\bar{r} - \frac{(\bar{r},\bar{v})}{v^2}\cdot\bar{v}\right)$$

$$\begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} ct \\ r_\parallel \\ r_{\perp 1} \\ r_{\perp 2} \end{pmatrix} = \begin{aligned} ct' &= \gamma ct - \beta\gamma r_\parallel \\ r'_\parallel &= -\beta\gamma ct + \gamma r_\parallel \\ r'_{\perp 1} &= r_{\perp 1} \\ r'_{\perp 2} &= r_{\perp 2} \end{aligned}$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\frac{v_x}{c}\gamma & \dots & -\frac{v_z}{c}\gamma \\ -\frac{v_x}{c}\gamma & & & \\ \vdots & & & \\ -\frac{v_z}{c}\gamma & & & \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$A\bar{r} = \bar{r} + (\gamma-1)\frac{(\bar{v},\bar{r})}{v^2}\,\bar{v}$$
$$\sum_{j=1}^{3} A_{ij} r_j = \sum_{j=1}^{3}\left(\delta_{ij} + (\gamma-1)\frac{v_i v_j}{v^2}\right) r_j$$

$$\bar{r}' = \frac{\bar{v}}{|v|} r'_\parallel + \left(\bar{r} - \frac{(\bar{r},\bar{v})}{v^2}\cdot\bar{v}\right)$$

$$\bar{r}' = \frac{\bar{v}}{|v|}\left(-\beta\gamma ct + \gamma\frac{(\bar{r},\bar{v})}{|v|} + \bar{r} - \frac{(\bar{r},\bar{v})}{|v|^2}\,\bar{v}\right)$$

Переход в ПСО $\qquad \Lambda^T = \Lambda$

$$\Lambda_x \Lambda_y \overset{?}{=} \Lambda_{\bar{v}}$$