$$\Lambda^\mu{}_\nu(\bar v) = \begin{pmatrix} \gamma & -\frac{v_x}{c}\gamma & \dots & -\gamma\frac{v_z}{c} \\ \vdots & & & \\ -\gamma\frac{v_z}{c} & & \delta_{ij} + (\gamma-1)\frac{v_i v_j}{v^2} & \end{pmatrix}$$

$$\Lambda \cdot \begin{pmatrix} ct \\ v_x t \\ v_y t \\ v_z t \end{pmatrix} = \begin{pmatrix} \# \\ 0 \\ 0 \\ 0 \end{pmatrix}$$ — проверить, что верное р-во

$$\begin{pmatrix} ch\varphi & sh\varphi & 0 & 0 \\ sh\varphi & ch\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ vt \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \# \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Преобразования Лоренца

$\Lambda^T \eta \Lambda = \eta$

- переход в подв. с/о
  Бусты
  $\Lambda^T = \Lambda$
  не явл. подгруппу
- повороты
  $\Lambda^T = \Lambda^{-1}$
  подгруппа
- смешанные

$$\Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma \eta_{\mu\nu} = \eta_{\rho\sigma}$$

$$\Lambda^\mu{}_\rho X^\rho{}_\nu = \delta^\mu{}_\nu$$

$$\Lambda^\mu{}_\rho \underbrace{\Lambda^\nu{}_\sigma \eta_{\mu\nu} \eta^{\sigma\gamma}}_{X^\gamma{}_\rho} = \underbrace{\eta_{\rho\sigma}\eta^{\sigma\gamma}}_{\delta^\gamma{}_\rho}$$

$$(\Lambda^{-1})^\gamma{}_\rho = \Lambda^\nu{}_\sigma \eta_{\mu\nu} \eta^{\sigma\gamma}$$

$$\Lambda^T \eta \Lambda = \eta$$

$$(\eta^{-1}\Lambda^T\eta)\Lambda = E_{4\times4}$$

$B(u, v)$ $\qquad B(u, Av) = B(A^T u, v)$

$$B(u, v) = \eta_{\mu\nu} u^\mu v^\nu$$

$$B(u, Av) = \eta_{\mu\nu} u^\mu \underbrace{A^\nu{}_\rho v^\rho}_{(Av)^\nu} = B(A^T u, v) = \eta_{\mu\nu} \underbrace{\delta^\mu{}_{\mu'} u^{\mu'}}_{u^\mu} A^\nu{}_\rho v^\rho =$$

$$= \eta_{\mu\nu}(\quad)^\mu v^\nu$$

$$= \eta_{\mu\nu}\eta_{\mu'\sigma}\eta^{\mu\sigma} u^{\mu'} A^{\nu}{}_{\rho} v^{\rho} = \eta_{\mu\nu} u^{\mu} A^{\nu}{}_{\rho} \overset{v^{\rho}}{\delta^{\rho}{}_{\rho'} v^{\rho'}} = \eta_{\mu\nu} u^{\mu} A^{\nu}{}_{\rho}\, \eta^{\rho\sigma}\eta_{\rho'\sigma} v^{\rho'} =$$

$$= \eta_{\sigma\rho'} \underbrace{(\eta_{\mu\nu} u^{\mu} A^{\nu}{}_{\rho}\, \eta^{\rho\sigma})}_{(A^T u)^{\sigma} = (\eta^{\rho\sigma} A^{\nu}{}_{\rho}\eta_{\mu\nu}) u^{\mu} = (A^T)^{\sigma}{}_{\mu}} v^{\rho'}$$

№3

$$p^{\mu} p^{\nu} \eta_{\mu\nu} = m^2 c^2$$
$$p^{\mu} p_{\mu} = m^2 c^2$$

$$p^{\mu} = mc u^{\mu}$$

$$p_1^{\mu} = \begin{pmatrix} \varepsilon/c \\ 0 \\ 0 \\ \sqrt{\frac{\varepsilon^2}{c^2} - m^2c^2} \end{pmatrix} \qquad p_2^{\mu} = \begin{pmatrix} \varepsilon/c \\ 0 \\ 0 \\ -p \end{pmatrix}$$

$$\frac{\varepsilon^2}{c^2} - p^2 = m^2 c^2$$

$$p_1'^{\mu} = \Lambda^{\mu}{}_{\nu} p_1^{\nu} = \begin{pmatrix} mc \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u^{\mu} = \begin{pmatrix} \gamma \\ \frac{v_x}{c}\gamma \\ \frac{v_y}{c}\gamma \\ \frac{v_z}{c}\gamma \end{pmatrix}$$

$$u^{\mu}u_{\mu} = \gamma^2 - \gamma^2\frac{v_x^2}{c^2} - \gamma^2\frac{v_y^2}{c^2} - \gamma^2\frac{v_z^2}{c^2} =$$
$$= \gamma^2\left(1 - \frac{v^2}{c^2}\right) = 1$$

$$p^0 = \frac{\varepsilon}{c} = mc\gamma$$

$$\Lambda = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$$p_2'^{\mu} = \Lambda^{\mu}{}_{\nu} p^{\nu} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} mc\gamma \\ 0 \\ 0 \\ -mc\beta\gamma \end{pmatrix} = \begin{pmatrix} mc\gamma^2 + mc\beta^2\gamma^2 \\ 0 \\ 0 \\ \# \end{pmatrix} = \quad mc\gamma' \quad = \quad \gamma' = \gamma'(v_{\text{отн}})$$

$$= (1+\beta^2)\gamma^2 = \gamma'(v_{\text{отн}})$$

$$p_1^{\mu} + \dots + p_N^{\mu} = \begin{pmatrix} p^0 \\ p^1 \\ 0 \\ 0 \end{pmatrix}$$

$$\Lambda^{\mu}{}_{\nu}(p_1^{\nu} + \dots + p_N^{\nu}) = \begin{pmatrix} \# \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p^0 \\ p^1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \# \\ -\beta\gamma p_0 + \gamma p_1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$-\beta p_0 + p_1 = 0 \qquad \beta = \frac{p_1}{p_0}$$

№4.

$$\sum_{j=1,2,3} \sum_{i=1,2,3} \delta_{ij} \delta^{ij} = 3$$

$$\delta_{ij} \delta^{jk} \delta_{km} = \delta_{im}$$

$$\epsilon_{ijk} = -\epsilon_{jik} = -\epsilon_{kji} = \dots$$

$$\epsilon_{112} = -\epsilon_{112} = 0$$

$$\epsilon_{123} \neq 0 \quad := 1$$ символ Леви-Чевиты

$$\sum_{k=1,2,3} \sum_{j=1,2,3} \epsilon_{ijk} \epsilon_{mjk} =$$

$$\epsilon_{ijk} \epsilon_{mjk} = 2 \delta_{im}$$

$$\sum \epsilon_{ijk} A_{1i} A_{2j} A_{3k} =$$