Redundant Robot
Xavier Waller
EECE/ME 271
December 04,2010

Forward Kinematics

Link Transformation Matrices:

In[1]:= $$T01 = \begin{pmatrix} \mathrm{Cos}[th1] & -\mathrm{Sin}[th1] & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \mathrm{Sin}[th1] & \mathrm{Cos}[th1] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$T12 = \begin{pmatrix} \mathrm{Cos}[th2] & -\mathrm{Sin}[th2] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{Sin}[th2] & -\mathrm{Cos}[th2] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$T23 = \begin{pmatrix} \mathrm{Cos}[th3] & -\mathrm{Sin}[th3] & 0 & 0 \\ 0 & 0 & -1 & -32 \\ \mathrm{Sin}[th3] & \mathrm{Cos}[th3] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$T34 = \begin{pmatrix} \mathrm{Cos}[th4] & -\mathrm{Sin}[th4] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{Sin}[th4] & -\mathrm{Cos}[th4] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$T45 = \begin{pmatrix} \mathrm{Cos}[th5] & -\mathrm{Sin}[th5] & 0 & 0 \\ 0 & 0 & -1 & -16 \\ \mathrm{Sin}[th5] & \mathrm{Cos}[th5] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$T56 = \begin{pmatrix} \mathrm{Cos}[th6] & -\mathrm{Sin}[th6] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{Sin}[th6] & -\mathrm{Cos}[th6] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$T67 = \begin{pmatrix} \mathrm{Cos}[th7] & -\mathrm{Sin}[th7] & 0 & 0 \\ 0 & 0 & -1 & -8 \\ \mathrm{Sin}[th7] & \mathrm{Cos}[th7] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$T78 = \begin{pmatrix} \mathrm{Cos}[th8] & -\mathrm{Sin}[th8] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{Sin}[th8] & -\mathrm{Cos}[th8] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

General Overall Transformation

In[9]:=
```
T = T01.T12.T23.T34.T45.T56.T67.T78;
Do[Print["T[", i, ",", j, "] = ", T[[i, j]]], {i, 1, 4}, {j, 4}]
```

```
T[4,4] = 1

T[4,3] = 0

T[4,2] = 0

T[4,1] = 0

T[3,4] = 32 Sin[th1] Sin[th2] -
  16 (-Cos[th4] Sin[th1] Sin[th2] - (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) Sin[th4]) -
  8 (-Cos[th6]
      (Cos[th4] Sin[th1] Sin[th2] + (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) Sin[th4]) -
     (Cos[th5] (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1] Sin[th2]
           Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) Sin[th6])

T[3,3] = Cos[th7] (Cos[th5] (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) -
     (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1] Sin[th2] Sin[th4])
      Sin[th5]) -
  (Cos[th6] (Cos[th5] (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1]
           Sin[th2] Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) -
     (Cos[th4] Sin[th1] Sin[th2] + (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) Sin[th4])
      Sin[th6]) Sin[th7]

T[3,2] = -Cos[th8]
   (Cos[th6] (Cos[th4] Sin[th1] Sin[th2] + (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) Sin[th4]) +
     (Cos[th5] (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1] Sin[th2]
           Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) Sin[th6]) -
  (Cos[th7] (Cos[th6] (Cos[th5] (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[
                th1] Sin[th2] Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) -
        (Cos[th4] Sin[th1] Sin[th2] + (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) Sin[th4])
         Sin[th6]) + (Cos[th5] (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) -
        (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1] Sin[th2] Sin[th4])
         Sin[th5]) Sin[th7]) Sin[th8]

T[3,1] = Cos[th8]
   (Cos[th7] (Cos[th6] (Cos[th5] (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1]
              Sin[th2] Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) -
        (Cos[th4] Sin[th1] Sin[th2] + (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) Sin[th4])
         Sin[th6]) + (Cos[th5] (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) -
        (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1] Sin[th2] Sin[th4])
         Sin[th5]) Sin[th7]) -
  (Cos[th6] (Cos[th4] Sin[th1] Sin[th2] + (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) Sin[th4]) +
     (Cos[th5] (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1] Sin[th2]
           Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) Sin[th6]) Sin[th8]

T[2,4] = -32 Cos[th2] - 16 (Cos[th2] Cos[th4] - Cos[th3] Sin[th2] Sin[th4]) -
  8 (-Cos[th6] (-Cos[th2] Cos[th4] + Cos[th3] Sin[th2] Sin[th4]) -
     (Cos[th5] (Cos[th3] Cos[th4] Sin[th2] + Cos[th2] Sin[th4]) - Sin[th2] Sin[th3] Sin[th5]) Sin[th6])

T[2,3] =
 Cos[th7] (-Cos[th5] Sin[th2] Sin[th3] - (Cos[th3] Cos[th4] Sin[th2] + Cos[th2] Sin[th4]) Sin[th5]) -
  (Cos[th6] (Cos[th5] (Cos[th3] Cos[th4] Sin[th2] + Cos[th2] Sin[th4]) - Sin[th2] Sin[th3] Sin[th5]) -
     (-Cos[th2] Cos[th4] + Cos[th3] Sin[th2] Sin[th4]) Sin[th6]) Sin[th7]
```

```
T[2,2] = -Cos[th8] (Cos[th6] (-Cos[th2] Cos[th4] + Cos[th3] Sin[th2] Sin[th4]) +
      (Cos[th5] (Cos[th3] Cos[th4] Sin[th2] + Cos[th2] Sin[th4]) - Sin[th2] Sin[th3] Sin[th5])
       Sin[th6]) -
   (Cos[th7] (Cos[th6] (Cos[th5] (Cos[th3] Cos[th4] Sin[th2] + Cos[th2] Sin[th4]) - Sin[th2]
              Sin[th3] Sin[th5]) - (-Cos[th2] Cos[th4] + Cos[th3] Sin[th2] Sin[th4]) Sin[th6]) +
      (-Cos[th5] Sin[th2] Sin[th3] - (Cos[th3] Cos[th4] Sin[th2] + Cos[th2] Sin[th4]) Sin[th5])
       Sin[th7]) Sin[th8]

T[2,1] =
 Cos[th8] (Cos[th7] (Cos[th6] (Cos[th5] (Cos[th3] Cos[th4] Sin[th2] + Cos[th2] Sin[th4]) - Sin[th2]
              Sin[th3] Sin[th5]) - (-Cos[th2] Cos[th4] + Cos[th3] Sin[th2] Sin[th4]) Sin[th6]) +
      (-Cos[th5] Sin[th2] Sin[th3] - (Cos[th3] Cos[th4] Sin[th2] + Cos[th2] Sin[th4]) Sin[th5])
       Sin[th7]) -
   (Cos[th6] (-Cos[th2] Cos[th4] + Cos[th3] Sin[th2] Sin[th4]) +
      (Cos[th5] (Cos[th3] Cos[th4] Sin[th2] + Cos[th2] Sin[th4]) - Sin[th2] Sin[th3] Sin[th5])
       Sin[th6]) Sin[th8]

T[1,4] = 32 Cos[th1] Sin[th2] -
   16 (-Cos[th1] Cos[th4] Sin[th2] - (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) Sin[th4]) -
   8 (-Cos[th6]
       (Cos[th1] Cos[th4] Sin[th2] + (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) Sin[th4]) -
      (Cos[th5] (Cos[th4] (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) - Cos[th1] Sin[th2]
              Sin[th4]) + (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) Sin[th6])

T[1,3] = Cos[th7] (Cos[th5] (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) -
      (Cos[th4] (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) - Cos[th1] Sin[th2] Sin[th4])
       Sin[th5]) -
   (Cos[th6] (Cos[th5] (Cos[th4] (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) - Cos[th1]
              Sin[th2] Sin[th4]) + (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) -
      (Cos[th1] Cos[th4] Sin[th2] + (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) Sin[th4])
       Sin[th6]) Sin[th7]

T[1,2] = -Cos[th8]
    (Cos[th6] (Cos[th1] Cos[th4] Sin[th2] + (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) Sin[th4]) +
      (Cos[th5] (Cos[th4] (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) - Cos[th1] Sin[th2]
              Sin[th4]) + (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) Sin[th6]) -
   (Cos[th7] (Cos[th6] (Cos[th5] (Cos[th4] (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) - Cos[th1]
                 Sin[th2] Sin[th4]) + (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) -
         (Cos[th1] Cos[th4] Sin[th2] + (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) Sin[th4])
          Sin[th6]) + (Cos[th5] (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) -
         (Cos[th4] (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) - Cos[th1] Sin[th2] Sin[th4])
          Sin[th5]) Sin[th7]) Sin[th8]

T[1,1] = Cos[th8]
    (Cos[th7] (Cos[th6] (Cos[th5] (Cos[th4] (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) - Cos[th1]
                 Sin[th2] Sin[th4]) + (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) -
         (Cos[th1] Cos[th4] Sin[th2] + (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) Sin[th4])
          Sin[th6]) + (Cos[th5] (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) -
         (Cos[th4] (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) - Cos[th1] Sin[th2] Sin[th4])
          Sin[th5]) Sin[th7]) -
   (Cos[th6] (Cos[th1] Cos[th4] Sin[th2] + (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) Sin[th4]) +
      (Cos[th5]
          (Cos[th4] (Cos[th1] Cos[th2] Cos[th3] - Sin[th1] Sin[th3]) - Cos[th1] Sin[th2] Sin[th4]) +
         (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) Sin[th6]) Sin[th8]
```

```
T[1,1] =
 Cos[θ8] (Cos[θ7] (Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[
                θ2] Sin[θ4]) + (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ5]) -
          (Cos[θ1] Cos[θ4] Sin[θ2] + (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6]) +
       (Cos[θ5] (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) - (Cos[θ4]
              (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4]) Sin[θ5]) Sin[θ7]) -
   (Cos[θ6] (Cos[θ1] Cos[θ4] Sin[θ2] + (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) +
       (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4]) +
          (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ5]) Sin[θ6]) Sin[θ8]

T[1,2] =
 -Cos[θ8] (Cos[θ6] (Cos[θ1] Cos[θ4] Sin[θ2] + (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) +
       (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4]) +
          (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ5]) Sin[θ6]) -
   (Cos[θ7] (Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) -
                Cos[θ1] Sin[θ2] Sin[θ4]) + (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ5]) -
          (Cos[θ1] Cos[θ4] Sin[θ2] + (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6]) +
       (Cos[θ5] (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) -
          (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4])
           Sin[θ5]) Sin[θ7]) Sin[θ8]

T[1,3] = Cos[θ7] (Cos[θ5] (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) -
       (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4]) Sin[θ5]) -
   (Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4]) +
          (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ5]) -
       (Cos[θ1] Cos[θ4] Sin[θ2] + (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6]) Sin[θ7]

T[1,4] = 32 Cos[θ1] Sin[θ2] -
   16 (-Cos[θ1] Cos[θ4] Sin[θ2] - (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) -
   8 (-Cos[θ6] (Cos[θ1] Cos[θ4] Sin[θ2] + (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) -
       (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4]) +
          (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ5]) Sin[θ6])

T[2,1] = Cos[θ8]
    (Cos[θ7] (Cos[θ6] (Cos[θ5] (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) - Sin[θ2] Sin[θ3] Sin[θ5]) -
          (-Cos[θ2] Cos[θ4] + Cos[θ3] Sin[θ2] Sin[θ4]) Sin[θ6]) +
       (-Cos[θ5] Sin[θ2] Sin[θ3] - (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) Sin[θ5]) Sin[θ7]) -
   (Cos[θ6] (-Cos[θ2] Cos[θ4] + Cos[θ3] Sin[θ2] Sin[θ4]) +
       (Cos[θ5] (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) - Sin[θ2] Sin[θ3] Sin[θ5]) Sin[θ6]) Sin[θ8]

T[2,2] = -Cos[θ8] (Cos[θ6] (-Cos[θ2] Cos[θ4] + Cos[θ3] Sin[θ2] Sin[θ4]) +
       (Cos[θ5] (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) - Sin[θ2] Sin[θ3] Sin[θ5]) Sin[θ6]) -
   (Cos[θ7] (Cos[θ6] (Cos[θ5] (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) - Sin[θ2] Sin[θ3] Sin[θ5]) -
          (-Cos[θ2] Cos[θ4] + Cos[θ3] Sin[θ2] Sin[θ4]) Sin[θ6]) +
       (-Cos[θ5] Sin[θ2] Sin[θ3] - (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) Sin[θ5]) Sin[θ7]) Sin[θ8]

T[2,3] = Cos[θ7] (-Cos[θ5] Sin[θ2] Sin[θ3] - (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) Sin[θ5]) -
   (Cos[θ6] (Cos[θ5] (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) - Sin[θ2] Sin[θ3] Sin[θ5]) -
       (-Cos[θ2] Cos[θ4] + Cos[θ3] Sin[θ2] Sin[θ4]) Sin[θ6]) Sin[θ7]

T[2,4] = -32 Cos[θ2] - 16 (Cos[θ2] Cos[θ4] - Cos[θ3] Sin[θ2] Sin[θ4]) -
   8 (-Cos[θ6] (-Cos[θ2] Cos[θ4] + Cos[θ3] Sin[θ2] Sin[θ4]) -
       (Cos[θ5] (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) - Sin[θ2] Sin[θ3] Sin[θ5]) Sin[θ6])
```

```
T[3,1] =
 Cos[θ8] (Cos[θ7] (Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[
                    θ2] Sin[θ4]) + (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ5]) -
          (Cos[θ4] Sin[θ1] Sin[θ2] + (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6]) +
       (Cos[θ5] (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) - (Cos[θ4]
                (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4]) Sin[θ5]) Sin[θ7]) -
   (Cos[θ6] (Cos[θ4] Sin[θ1] Sin[θ2] + (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) +
       (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4]) +
          (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ5]) Sin[θ6]) Sin[θ8]

T[3,2] =
 -Cos[θ8] (Cos[θ6] (Cos[θ4] Sin[θ1] Sin[θ2] + (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) +
       (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4]) +
          (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ5]) Sin[θ6]) -
   (Cos[θ7] (Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) -
                 Sin[θ1] Sin[θ2] Sin[θ4]) + (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ5]) -
          (Cos[θ4] Sin[θ1] Sin[θ2] + (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6]) +
       (Cos[θ5] (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) - (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] +
                 Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4]) Sin[θ5]) Sin[θ7]) Sin[θ8]

T[3,3] = Cos[θ7] (Cos[θ5] (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) -
       (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4]) Sin[θ5]) -
   (Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4]) +
          (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ5]) -
       (Cos[θ4] Sin[θ1] Sin[θ2] + (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6]) Sin[θ7]

T[3,4] = 32 Sin[θ1] Sin[θ2] -
   16 (-Cos[θ4] Sin[θ1] Sin[θ2] - (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) -
   8 (-Cos[θ6] (Cos[θ4] Sin[θ1] Sin[θ2] + (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) -
       (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4]) +
          (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ5]) Sin[θ6])

T[4,1] = 0

T[4,2] = 0

T[4,3] = 0

T[4,4] = 1
```

Formation of Jacobian

The first row of the Jacobian is the T[1,4] function differentiated with respect to (θ1,θ2,...θ8).

```
f1 = 32 Cos[θ1] Sin[θ2] -
   16 (-Cos[θ1] Cos[θ4] Sin[θ2] - (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) -
   8 (-Cos[θ6] (Cos[θ1] Cos[θ4] Sin[θ2] + (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) -
       (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4]) +
          (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ5]) Sin[θ6]);
```

```
df1dθ1 = D[f1, θ1]
df1dθ2 = D[f1, θ2]
df1dθ3 = D[f1, θ3]
df1dθ4 = D[f1, θ4]
df1dθ5 = D[f1, θ5]
df1dθ6 = D[f1, θ6]
df1dθ7 = D[f1, θ7]
df1dθ8 = D[f1, θ8]
```

```
-32 Sin[θ1] Sin[θ2] -
 16 (Cos[θ4] Sin[θ1] Sin[θ2] - (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ1] Sin[θ3]) Sin[θ4]) -
 8 (-Cos[θ6] (-Cos[θ4] Sin[θ1] Sin[θ2] + (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ1] Sin[θ3]) Sin[θ4]) -
    (Cos[θ5] (Cos[θ4] (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ1] Sin[θ3]) + Sin[θ1] Sin[θ2] Sin[θ4]) +
       (-Cos[θ1] Cos[θ3] + Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ5]) Sin[θ6])
```

```
32 Cos[θ1] Cos[θ2] - 16 (-Cos[θ1] Cos[θ2] Cos[θ4] + Cos[θ1] Cos[θ3] Sin[θ2] Sin[θ4]) -
 8 (-Cos[θ6] (Cos[θ1] Cos[θ2] Cos[θ4] - Cos[θ1] Cos[θ3] Sin[θ2] Sin[θ4]) -
    (Cos[θ5] (-Cos[θ1] Cos[θ3] Cos[θ4] Sin[θ2] - Cos[θ1] Cos[θ2] Sin[θ4]) +
       Cos[θ1] Sin[θ2] Sin[θ3] Sin[θ5]) Sin[θ6])
```

```
16 (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ4] -
 8 (-Cos[θ6] (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ4] -
    (Cos[θ4] Cos[θ5] (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) +
       (-Cos[θ1] Cos[θ2] Cos[θ3] + Sin[θ1] Sin[θ3]) Sin[θ5]) Sin[θ6])
```

```
-16 (-Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) + Cos[θ1] Sin[θ2] Sin[θ4]) -
 8 (-Cos[θ6] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4]) -
    Cos[θ5]
     (-Cos[θ1] Cos[θ4] Sin[θ2] - (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6])
```

```
8 (Cos[θ5] (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) -
   (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4])
    Sin[θ5]) Sin[θ6]
```

```
-8 (-Cos[θ6]
     (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4]) +
       (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ5]) +
   (Cos[θ1] Cos[θ4] Sin[θ2] + (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6])
```

```
0
```

```
0
```

- The second row of the Jacobian is the T[2,4] function differentiated with respect to (θ1,θ2,...θ8).

```
f2 = -32 Cos[θ2] - 16 (Cos[θ2] Cos[θ4] - Cos[θ3] Sin[θ2] Sin[θ4]) -
   8 (-Cos[θ6] (-Cos[θ2] Cos[θ4] + Cos[θ3] Sin[θ2] Sin[θ4]) -
      (Cos[θ5] (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) - Sin[θ2] Sin[θ3] Sin[θ5]) Sin[θ6]);
```

```
df2dθ1 = D[f2, θ1]
df2dθ2 = D[f2, θ2]
df2dθ3 = D[f2, θ3]
df2dθ4 = D[f2, θ4]
df2dθ5 = D[f2, θ5]
df2dθ6 = D[f2, θ6]
df2dθ7 = D[f2, θ7]
df2dθ8 = D[f2, θ8]
```

```
0

32 Sin[θ2] - 16 (-Cos[θ4] Sin[θ2] - Cos[θ2] Cos[θ3] Sin[θ4]) -
 8 (-Cos[θ6] (Cos[θ4] Sin[θ2] + Cos[θ2] Cos[θ3] Sin[θ4]) -
    (Cos[θ5] (Cos[θ2] Cos[θ3] Cos[θ4] - Sin[θ2] Sin[θ4]) - Cos[θ2] Sin[θ3] Sin[θ5]) Sin[θ6])

-16 Sin[θ2] Sin[θ3] Sin[θ4] - 8 (Cos[θ6] Sin[θ2] Sin[θ3] Sin[θ4] -
    (-Cos[θ4] Cos[θ5] Sin[θ2] Sin[θ3] - Cos[θ3] Sin[θ2] Sin[θ5]) Sin[θ6])

-16 (-Cos[θ3] Cos[θ4] Sin[θ2] - Cos[θ2] Sin[θ4]) -
 8 (-Cos[θ6] (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) -
    Cos[θ5] (Cos[θ2] Cos[θ4] - Cos[θ3] Sin[θ2] Sin[θ4]) Sin[θ6])

8 (-Cos[θ5] Sin[θ2] Sin[θ3] - (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) Sin[θ5]) Sin[θ6]

-8 (-Cos[θ6] (Cos[θ5] (Cos[θ3] Cos[θ4] Sin[θ2] + Cos[θ2] Sin[θ4]) - Sin[θ2] Sin[θ3] Sin[θ5]) +
   (-Cos[θ2] Cos[θ4] + Cos[θ3] Sin[θ2] Sin[θ4]) Sin[θ6])

0

0
```

- The third row of the Jacobian is the T[2,4] function differentiated with respect to (θ1,θ2,...θ8).

```
f3 = 32 Sin[θ1] Sin[θ2] -
   16 (-Cos[θ4] Sin[θ1] Sin[θ2] - (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) -
   8 (-Cos[θ6] (Cos[θ4] Sin[θ1] Sin[θ2] + (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) -
      (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4]) +
         (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ5]) Sin[θ6]);
```

```
df3dθ1 = D[f3, θ1]
df3dθ2 = D[f3, θ2]
df3dθ3 = D[f3, θ3]
df3dθ4 = D[f3, θ4]
df3dθ5 = D[f3, θ5]
df3dθ6 = D[f3, θ6]
df3dθ7 = D[f3, θ7]
df3dθ8 = D[f3, θ8]
```

```
32 Cos[θ1] Sin[θ2] -
 16 (-Cos[θ1] Cos[θ4] Sin[θ2] - (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) -
 8 (-Cos[θ6] (Cos[θ1] Cos[θ4] Sin[θ2] + (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4]) -
    (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) - Cos[θ1] Sin[θ2] Sin[θ4]) +
       (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ5]) Sin[θ6])
```

```
32 Cos[θ2] Sin[θ1] - 16 (-Cos[θ2] Cos[θ4] Sin[θ1] + Cos[θ3] Sin[θ1] Sin[θ2] Sin[θ4]) -
 8 (-Cos[θ6] (Cos[θ2] Cos[θ4] Sin[θ1] - Cos[θ3] Sin[θ1] Sin[θ2] Sin[θ4]) -
    (Cos[θ5] (-Cos[θ3] Cos[θ4] Sin[θ1] Sin[θ2] - Cos[θ2] Sin[θ1] Sin[θ4]) +
       Sin[θ1] Sin[θ2] Sin[θ3] Sin[θ5]) Sin[θ6])
```

```
16 (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ4] -
 8 (-Cos[θ6] (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ4] -
    (Cos[θ4] Cos[θ5] (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) +
       (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ1] Sin[θ3]) Sin[θ5]) Sin[θ6])
```

```
-16 (-Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) + Sin[θ1] Sin[θ2] Sin[θ4]) -
 8 (-Cos[θ6] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4]) -
    Cos[θ5]
     (-Cos[θ4] Sin[θ1] Sin[θ2] - (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6])
```

```
8 (Cos[θ5] (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) -
   (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4])
    Sin[θ5]) Sin[θ6]
```

```
-8 (-Cos[θ6]
    (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) - Sin[θ1] Sin[θ2] Sin[θ4]) +
      (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ5]) +
   (Cos[θ4] Sin[θ1] Sin[θ2] + (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6])
```

```
0
```

```
0
```

- The Z Vectors of the Jacobian are found by transforming the Z vectors from each frame to the base.

$$Z = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix};$$

```
Z01 = T01.Z // MatrixForm
Z02 = T01.T12.Z // MatrixForm
Z03 = T01.T12.T23.Z // MatrixForm
Z04 = T01.T12.T23.T34.Z // MatrixForm
Z05 = T01.T12.T23.T34.T45.Z // MatrixForm
Z06 = T01.T12.T23.T34.T45.T56.Z // MatrixForm
Z07 = T01.T12.T23.T34.T45.T56.T67.Z // MatrixForm
Z08 = T01.T12.T23.T34.T45.T56.T67.T78.Z // MatrixForm
```

$$\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -\text{Sin}[\theta 1] \\ 0 \\ \text{Cos}[\theta 1] \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 33\,\text{Cos}[\theta 1]\,\text{Sin}[\theta 2] \\ -33\,\text{Cos}[\theta 2] \\ 33\,\text{Sin}[\theta 1]\,\text{Sin}[\theta 2] \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -\text{Cos}[\theta 3]\,\text{Sin}[\theta 1] + 32\,\text{Cos}[\theta 1]\,\text{Sin}[\theta 2] - \text{Cos}[\theta 1]\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3] \\ -32\,\text{Cos}[\theta 2] - \text{Sin}[\theta 2]\,\text{Sin}[\theta 3] \\ \text{Cos}[\theta 1]\,\text{Cos}[\theta 3] + 32\,\text{Sin}[\theta 1]\,\text{Sin}[\theta 2] - \text{Cos}[\theta 2]\,\text{Sin}[\theta 1]\,\text{Sin}[\theta 3] \\ 1 \end{pmatrix}$$

```
( 32 Cos[θ1] Sin[θ2] + Cos[θ1] Cos[θ4] Sin[θ2] + (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[θ4] -
                                      -32 Cos[θ2] - Cos[θ2] Cos[θ4] + Cos[θ3] Sin[θ2] Sin[θ4] - 16
  32 Sin[θ1] Sin[θ2] + Cos[θ4] Sin[θ1] Sin[θ2] + (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sin[θ4] -
                                                                                              1

( 32 Cos[θ1] Sin[θ2] + Cos[θ5] (-Cos[θ3] Sin[θ1] - Cos[θ1] Cos[θ2] Sin[θ3]) - 16 (-Cos[θ1] Cos[θ4] Sir
                                                  -32 Cos[θ2] - Cos[θ5] Sin[θ2] Sin[θ3] - 16 (Co
  32 Sin[θ1] Sin[θ2] + Cos[θ5] (Cos[θ1] Cos[θ3] - Cos[θ2] Sin[θ1] Sin[θ3]) - 16 (-Cos[θ4] Sin[θ1] Sin

( 32 Cos[θ1] Sin[θ2] - 16 (-Cos[θ1] Cos[θ4] Sin[θ2] - (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[

  32 Sin[θ1] Sin[θ2] - 16 (-Cos[θ4] Sin[θ1] Sin[θ2] - (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Sir

( 32 Cos[θ1] Sin[θ2] - 16 (-Cos[θ1] Cos[θ4] Sin[θ2] - (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) Sin[

  32 Sin[θ1] Sin[θ2] - 16 (-Cos[θ4] Sin[θ1] Sin[θ2] - (Cos[θ2] Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3]) Si
```

The Jacobian

$$\text{Jac} = \begin{pmatrix} \text{df1d}\theta 1 & \text{df1d}\theta 2 & \text{df1d}\theta 3 & \text{df1d}\theta 4 & \text{df1d}\theta 5 & \text{df1d}\theta 6 & \text{df1d}\theta 7 & \text{df1d}\theta 8 \\ \text{df2d}\theta 1 & \text{df2d}\theta 2 & \text{df2d}\theta 3 & \text{df2d}\theta 4 & \text{df2d}\theta 5 & \text{df2d}\theta 6 & \text{df2d}\theta 7 & \text{df2d}\theta 8 \\ \text{df3d}\theta 1 & \text{df3d}\theta 2 & \text{df3d}\theta 3 & \text{df3d}\theta 4 & \text{df3d}\theta 5 & \text{df3d}\theta 6 & \text{df3d}\theta 7 & \text{df3d}\theta 8 \\ \text{Z01} & \text{Z02} & \text{Z03} & \text{Z04} & \text{Z05} & \text{Z06} & \text{Z07} & \text{Z08} \end{pmatrix} \text{ // MatrixForm}$$

```
( -32 Sin[θ1] Sin[θ2] - 16 (Cos[θ4] Sin[θ1] Sin[θ2] - (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ1] Sin[θ3]) Sin
     32 Cos[θ1] Sin[θ2] - 16 (-Cos[θ1] Cos[θ4] Sin[θ2] - (Cos[θ1] Cos[θ2] Cos[θ3] - Sin[θ1] Sin[θ3]) S:
```

- ## Velocity Vector

Now I need a velocity vector in order to solve for rate of change of the joint values.

The relationship between manipulator velocity and position is given by :

$$\begin{pmatrix} \text{xdot\_desired} \\ \text{ydot\_desired} \\ \text{zdot\_desired} \end{pmatrix} = \alpha \left(\begin{pmatrix} \text{xdesired} \\ \text{ydesired} \\ \text{zdesired} \end{pmatrix} - \begin{pmatrix} \text{xcurrent} \\ \text{ycurrent} \\ \text{zcurrent} \end{pmatrix} \right)$$