Redundant Robot Xavier Waller EECE/ME 271 December 04,2010

Forward Kinematics

Link Transformation Matrices:

$$I_{\text{In[1]:=}} \ \ \text{T01} \ = \left(\begin{array}{cccc} \text{Cos[th1]} & -\text{Sin[th1]} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \text{Sin[th1]} & \text{Cos[th1]} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right);$$

$$T12 = \begin{pmatrix} Cos[th2] & -sin[th2] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin[th2] & -Cos[th2] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$T23 = \begin{pmatrix} \cos[th3] & -\sin[th3] & 0 & 0 \\ 0 & 0 & -1 & -32 \\ \sin[th3] & \cos[th3] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$T34 = \begin{pmatrix} Cos[th4] & -Sin[th4] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -Sin[th4] & -Cos[th4] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$T45 = \begin{pmatrix} Cos[th5] & -Sin[th5] & 0 & 0 \\ 0 & 0 & -1 & -16 \\ Sin[th5] & Cos[th5] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} ;$$

$$T56 = \begin{pmatrix} Cos[th6] & -Sin[th6] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -Sin[th6] & -Cos[th6] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{T67} = \begin{pmatrix} \mathbf{Cos[th7]} & -\mathbf{Sin[th7]} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{1} & -\mathbf{8} \\ \mathbf{Sin[th7]} & \mathbf{Cos[th7]} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix};$$

$$T78 = \left(\begin{array}{cccc} Cos[th8] & -Sin[th8] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -Sin[th8] & -Cos[th8] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right);$$

General Overall Transformation

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ln[9]:= T = T01.T12.T23.T34.T45.T56.T67.T78;
                                                   Do[Print["T[", i, ",", j, "] = ", T[[i, j]]], {i, 1, 4}, {j, 4}]
T[4,4] = 1
T[4,3] = 0
T[4,2] = 0
T[4,1] = 0
T[3,4] = 32 \sin[th1] \sin[th2] -
                    16 \left(-\cos[th4] \sin[th1] \sin[th2] - \left(\cos[th2] \cos[th3] \sin[th1] + \cos[th1] \sin[th3]\right) \sin[th4]\right) - \left(\cos[th4] \sin[th4]\right) - \left(\cos
                    8 (-Cos[th6]
                                                               (\cos[th4]\,\sin[th1]\,\sin[th2] + (\cos[th2]\,\cos[th3]\,\sin[th1] + \cos[th1]\,\sin[th3])\,\sin[th4]) - \cos[th2]\,\sin[th3]) + \cos[th3]\,\sin[th3]) + \cos[th3]\,\sin[th3] + \cos[th3]\,\sin[th3]) + \cos[th3]\,\sin[th3] + \cos[th3] 
                                                      (Cos[th5] (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1] Sin[th2]
                                                                                                                         Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) Sin[th6])
T[3,3] = Cos[th7] (Cos[th5] (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) - Cos[th2] Sin[th3] - Cos[th3] - Cos[th3] Sin[th3] - Cos[th3] - Cos[th
                                                   (\cos[\th 4]\ (\cos[\th 2]\ \cos[\th 3]\ \sin[\th 1]\ +\ \cos[\th 1]\ \sin[\th 3])\ -\ \sin[\th 1]\ \sin[\th 2]\ \sin[\th 4])
                                                           Sin[th5]) -
                        (\cos[th6] \ (\cos[th5] \ (\cos[th4] \ (\cos[th4] \ \cos[th3] \ \sin[th1] + \cos[th1] \ \sin[th3]) - \sin[th1] 
                                                                                                                         Sin[th2] Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) - Cos[th2] Sin[th4] Sin[th4]) + (Cos[th4] Cos[th4] Cos[th4] Sin[th4] Sin[th4] Sin[th4]) + (Cos[th4] Cos[th4] Cos[th4] Sin[th4] S
                                                      (\cos[th4] \sin[th1] \sin[th2] + (\cos[th2] \cos[th3] \sin[th1] + \cos[th1] \sin[th3]) \sin[th4])
                                                           Sin[th6]) Sin[th7]
T[3,2] = -Cos[th8]
                               (\cos[\th 6] (\cos[\th 4] \sin[\th 1] \sin[\th 2] + (\cos[\th 2] \cos[\th 3] \sin[\th 1] + \cos[\th 1] \sin[\th 3]) \sin[\th 4]) + (\cos[\th 6] \sin[\th 6] \sin[\th 6]) + (\cos[\th 6] \sin[\th 6]) \sin[\th 6] \sin[\th 6]) + (\cos[\th 6] \sin[\th 6]) \sin[\th 6] \sin[\th 6]) + (\cos[\th 6] \sin[\th 6]) \sin[\th 6]) \sin[\th 6]
                                                    (\cos[th5](\cos[th4](\cos[th2]\cos[th3]\sin[th1]+\cos[th1]\sin[th3])-\sin[th1]\sin[th2]
                                                                                                                        Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) Sin[th6]) -
                       (\cos[th7] (\cos[th6] (\cos[th5] (\cos[th4] (\cos[th4]) \cos[th3] \sin[th1] + \cos[th1] \sin[th3]) - \sin[th4] \cos[th7] \cos[t
                                                                                                                                                                  th1] Sin[th2] Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) -
                                                                                   (\cos[th4] \sin[th1] \sin[th2] + (\cos[th2] \cos[th3] \sin[th1] + \cos[th1] \sin[th3]) \sin[th4])
                                                                                         Sin[th6]) + (Cos[th5] (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) -
                                                                                     (\cos[\th4]\ (\cos[\th2]\ \cos[\th3]\ \sin[\th1]\ +\cos[\th1]\ \sin[\th3])\ -\sin[\th1]\ \sin[\th2]\ \sin[\th4])
                                                                                         Sin[th5]) Sin[th7]) Sin[th8]
T[3,1] = Cos[th8]
                                 (Cos[th7] (Cos[th6] (Cos[th5] (Cos[th4] (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1] + Cos[th1] Sin[th3]) - Sin[th1] + Cos[th3] Sin[th3] + Cos[th3] 
                                                                                                                                                       Sin[th2] Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) - Cos[th2] Sin[th4] Sin[th4]) + (Cos[th4] Cos[th4] Cos[th4] Sin[th4] Sin[th4] Sin[th4]) + (Cos[th4] Cos[th4] Cos[th4] Sin[th4] S
                                                                                     (\mathsf{Cos}[\mathsf{th4}] \; \mathsf{Sin}[\mathsf{th1}] \; \mathsf{Sin}[\mathsf{th2}] \; + \; (\mathsf{Cos}[\mathsf{th2}] \; \mathsf{Cos}[\mathsf{th3}] \; \mathsf{Sin}[\mathsf{th1}] \; + \; \mathsf{Cos}[\mathsf{th1}] \; \mathsf{Sin}[\mathsf{th3}]) \; \mathsf{Sin}[\mathsf{th4}])
                                                                                         Sin[th6]) + (Cos[th5] (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) -
                                                                                   (\cos[\th4]\ (\cos[\th2]\ \cos[\th3]\ \sin[\th1]\ + \cos[\th1]\ \sin[\th3])\ - \sin[\th1]\ \sin[\th2]\ \sin[\th4])
                                                                                         Sin[th5]) Sin[th7]) -
                       (Cos[th6] (Cos[th4] Sin[th1] Sin[th2] + (Cos[th2] Cos[th3] Sin[th1] + Cos[th1] Sin[th3]) Sin[th4]) + (Cos[th6] Cos[th6] Cos[th6
                                                    (\cos[th5])(\cos[th4])(\cos[th2])\cos[th3])\sin[th1] + \cos[th1]\sin[th3]) - \sin[th1]\sin[th2]
                                                                                                                        Sin[th4]) + (Cos[th1] Cos[th3] - Cos[th2] Sin[th1] Sin[th3]) Sin[th5]) Sin[th6]) Sin[th8]
T[2,4] = -32 \cos[th2] - 16 (\cos[th2] \cos[th4] - \cos[th3] \sin[th2] \sin[th4]) -
                      8 \left( -\cos[\th 6] \left( -\cos[\th 2] \cos[\th 4] + \cos[\th 3] \sin[\th 2] \sin[\th 4] \right) - 
                                                   (\cos[\th 5] (\cos[\th 3] \cos[\th 4] \sin[\th 2] + \cos[\th 2] \sin[\th 4]) - \sin[\th 2] \sin[\th 3] \sin[\th 5]) \sin[\th 6])
T[2,3] =
         \begin{aligned} & \cos[\th 7] \ (-\cos[\th 5] \ Sin[\th 2] \ Sin[\th 3] \ - \ (\cos[\th 3] \ Cos[\th 4] \ Sin[\th 2] \ + \ Cos[\th 2] \ Sin[\th 4]) \ Sin[\th 5]) \ - \ & \end{aligned} 
                       (\mathsf{Cos}[\mathsf{th6}]\ (\mathsf{Cos}[\mathsf{th5}]\ (\mathsf{Cos}[\mathsf{th3}]\ \mathsf{Cos}[\mathsf{th4}]\ \mathsf{Sin}[\mathsf{th2}] + \mathsf{Cos}[\mathsf{th2}]\ \mathsf{Sin}[\mathsf{th4}]) - \mathsf{Sin}[\mathsf{th2}]\ \mathsf{Sin}[\mathsf{th3}]\ \mathsf{Sin}[\mathsf{th5}]) - \mathsf{Sin}[\mathsf{th3}]\ \mathsf{Sin}[\mathsf{th3}]\ \mathsf{Sin}[\mathsf{th3}] + \mathsf{Sin}[\mathsf{th3}]\ \mathsf{Sin}[\mathsf{th3}]\ \mathsf{Sin}[\mathsf{th3}] + \mathsf{Sin}[\mathsf{th3}]\ \mathsf{Sin}[\mathsf{th3}] + \mathsf{Sin}[\mathsf{th3}]\ \mathsf{Sin}[\mathsf{th3}] + \mathsf{Sin}[\mathsf{th3}]\ \mathsf{Sin}[\mathsf{th3}] + \mathsf{Sin}[\mathsf{th3}] + \mathsf{Sin}[\mathsf{th3}]\ \mathsf{Sin}[\mathsf{th3}] + \mathsf{Sin}[\mathsf{th3}]\ \mathsf{Sin}[\mathsf{th3}] + \mathsf{Sin}[\mathsf{th3}]
                                                    (-\cos[th2]\cos[th4] + \cos[th3]\sin[th2]\sin[th4])\sin[th6])\sin[th7]
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T[2,2] = -\cos[th8] (\cos[th6] (-\cos[th2] \cos[th4] + \cos[th3] \sin[th2] \sin[th4]) +
                                                          (Cos[th3] \ Cos[th4] \ Sin[th2] + Cos[th2] \ Sin[th4]) - Sin[th2] \ Sin[th3] \ Sin[th5])
                                                                 Sin[th6]) -
                         (\cos[th7](\cos[th6](\cos[th5])(\cos[th3])\cos[th4]\sin[th2] + \cos[th2]\sin[th4]) - \sin[th2]
                                                                                                                                    Sin[th3] Sin[th5]) - (-Cos[th2] Cos[th4] + Cos[th3] Sin[th2] Sin[th4]) Sin[th6]) +
                                                           (-\cos[\th5]\,\sin[\th2]\,\sin[\th3]\,-\,(\cos[\th3]\,\cos[\th4]\,\sin[\th2]\,+\,\cos[\th2]\,\sin[\th4])\,\sin[\th5])
                                                                 Sin[th7]) Sin[th8]
T[2,1] =
          \texttt{Cos[th8]} \hspace{0.1cm} (\texttt{Cos[th7]} \hspace{0.1cm} (\texttt{Cos[th6]} \hspace{0.1cm} (\texttt{Cos[th5]} \hspace{0.1cm} (\texttt{Cos[th4]} \hspace{0.1cm} \texttt{Sin[th2]} \hspace{0.1cm} + \hspace{0.1cm} \texttt{Cos[th2]} \hspace{0.1cm} \texttt{Sin[th4]}) \hspace{0.1cm} - \hspace{0.1cm} \texttt{Sin[th2]} \hspace{0.1cm} + \hspace{0.1cm} \texttt{Cos[th8]} \hspace{0.1cm} (\texttt{Cos[th8]} \hspace{0.1cm} + \hspace{0.1cm} \texttt{Cos[th8]} \hspace{0.1cm} + \hspace{0.1cm} \texttt{C
                                                                                                                                      Sin[th3] Sin[th5]) - (-Cos[th2] Cos[th4] + Cos[th3] Sin[th2] Sin[th4]) Sin[th6]) + Cos[th3] Sin[th5] + Cos[th5] Sin[th5]) + Cos[th5] Sin[th5]) + Cos[th5] Sin[th5] Sin[th5] Sin[th5]) + Cos[th5] Sin[th5] Sin[th
                                                          (-\cos[th5] \sin[th2] \sin[th3] - (\cos[th3] \cos[th4] \sin[th2] + \cos[th2] \sin[th4]) \sin[th5])
                                                                 Sin[th7]) -
                         (Cos[th6] (-Cos[th2] Cos[th4] + Cos[th3] Sin[th2] Sin[th4]) +
                                                          (Cos[th3] \ Cos[th4] \ Sin[th2] + Cos[th2] \ Sin[th4]) - Sin[th2] \ Sin[th3] \ Sin[th5])
                                                                 Sin[th6]) Sin[th8]
T[1,4] = 32 \cos[th1] \sin[th2] -
                      16 \left(-\cos[\th 1] \cos[\th 4] \sin[\th 2] - \left(\cos[\th 1] \cos[\th 2] \cos[\th 3] - \sin[\th 1] \sin[\th 3]\right) \sin[\th 4]\right) - \cos[\th 3] \cos[\th 4] \sin[\th 4]
                       8 (-Cos[th6]
                                                                      (\cos[\th1]\cos[\th4]\sin[\th2] + (\cos[\th1]\cos[\th2]\cos[\th3] - \sin[\th1]\sin[\th3])\sin[\th4]) - \sin[\th4] + \cos[\th4]\sin[\th4] + \cos[\th4] + \sin[\th4] + \sin
                                                           (\cos[th5]) (\cos[th4]) (\cos[th1]) \cos[th2]) \cos[th3] - \sin[th1]) \sin[th3]) - \cos[th1]) \sin[th2]
                                                                                                                                      Sin[th4]) + (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) Sin[th6])
T[1,3] = Cos[th7] (Cos[th5] (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) -
                                                           (\cos[\th 4] (\cos[\th 1] \cos[\th 2] \cos[\th 3] - \sin[\th 1] \sin[\th 3]) - \cos[\th 1] \sin[\th 2] \sin[\th 4])
                                                                 Sin[th5]) -
                         (\cos[th6]\ (\cos[th5]\ (\cos[th4]\ (\cos[th1]\ \cos[th2]\ \cos[th3]\ -\sin[th1]\ \sin[th3])\ -\cos[th1]
                                                                                                                                      Sin[th2] Sin[th4]) + (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) - Cos[th2] Sin[th4] + (-Cos[th4] Sin[th5]) - (-Cos[th4] Sin[th4]) + (-Cos[th4
                                                          (\mathsf{Cos}[\mathsf{th1}] \; \mathsf{Cos}[\mathsf{th4}] \; \mathsf{Sin}[\mathsf{th2}] \; + \; (\mathsf{Cos}[\mathsf{th1}] \; \mathsf{Cos}[\mathsf{th2}] \; \mathsf{Cos}[\mathsf{th3}] \; - \; \mathsf{Sin}[\mathsf{th1}] \; \mathsf{Sin}[\mathsf{th3}]) \; \mathsf{Sin}[\mathsf{th4}])
                                                                 Sin[th6]) Sin[th7]
T[1,2] = -Cos[th8]
                                   (\texttt{Cos[th6]} \ (\texttt{Cos[th1]} \ \texttt{Cos[th4]} \ \texttt{Sin[th2]} + (\texttt{Cos[th1]} \ \texttt{Cos[th2]} \ \texttt{Cos[th3]} - \texttt{Sin[th1]} \ \texttt{Sin[th3]}) \ \texttt{Sin[th4]}) + \\ (\texttt{Cos[th6]} \ (\texttt{Cos[th6]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Sin[th3]}) + \\ (\texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Sin[th3]}) + \\ (\texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Sin[th3]}) + \\ (\texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]}) + \\ (\texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]}) + \\ (\texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]}) + \\ (\texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]}) + \\ (\texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]}) + \\ (\texttt{Cos[th1]} \ \texttt{Cos[th1]} \ \texttt{Cos[th1]}) + \\ (\texttt{Cos[th1]} \ \texttt{Cos[th1]}) + \\ 
                                                          (Cos[th5] \ (Cos[th4] \ (Cos[th1] \ Cos[th2] \ Cos[th3] - Sin[th1] \ Sin[th3]) - Cos[th1] \ Sin[th2]
                                                                                                                                      Sin[th4]) + (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) Sin[th6]) - Cos[th2] Sin[th6]) - Cos[th8] Sin[th6]) - Cos[th8] Sin[th8]) - Cos[th8] Sin[th8] Sin[th8]) - Cos[th8] Sin[th8] Sin[th8]) - Cos[th8] Sin[th8] Sin[th8] Sin[th8]) - Cos[th8] Sin[th8] S
                         (\mathsf{Cos}[\mathsf{th7}]\ (\mathsf{Cos}[\mathsf{th6}]\ (\mathsf{Cos}[\mathsf{th5}]\ (\mathsf{Cos}[\mathsf{th4}]\ (\mathsf{Cos}[\mathsf{th1}]\ \mathsf{Cos}[\mathsf{th2}]\ \mathsf{Cos}[\mathsf{th3}]\ -\ \mathsf{Sin}[\mathsf{th1}]\ \mathsf{Sin}[\mathsf{th3}])\ -\ \mathsf{Cos}[\mathsf{th1}]
                                                                                                                                                                       Sin[th2] Sin[th4]) + (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) - Cos[th2] Sin[th3]) Sin[th5]) - Cos[th2] Sin[th4] Sin[th4]) Sin[th4] Sin
                                                                                             (\cos[th1] \cos[th4] \sin[th2] + (\cos[th1] \cos[th2] \cos[th3] - \sin[th1] \sin[th3]) \sin[th4])
                                                                                                   Sin[th6]) + (Cos[th5] (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) -
                                                                                             (\cos[th4] (\cos[th1] \cos[th2] \cos[th3] - \sin[th1] \sin[th3]) - \cos[th1] \sin[th2] \sin[th4])
                                                                                                  Sin[th5]) Sin[th7]) Sin[th8]
T[1,1] = Cos[th8]
                                   (\cos[\th7]\ (\cos[\th6]\ (\cos[\th6]\ (\cos[\th4]\ (\cos[\th4]\ (\cos[\th1]\ \cos[\th2]\ \cos[\th3]\ -\sin[\th1]\ \sin[\th3])\ -\cos[\th1]\ \cos[\th3]\ -\sin[\th3])\ -\cos[\th3]\ \cos[\th3]\ \sin[\th3]\ \sin[\th3]\ \cos[\th3]\ \cos[\th3]\ \cos[\th3]\ \sin[\th3]\ \sin[
                                                                                                                                                                       Sin[th2] Sin[th4]) + (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) - Cos[th2] Sin[th3] + (-Cos[th3] Sin[th3]) - Cos[th3] Sin[th3] + (-Cos[th3] Sin[th3]) Sin[th3] + (-Cos[th3]
                                                                                             (\cos[\th1] \cos[\th4] \sin[\th2] + (\cos[\th1] \cos[\th2] \cos[\th3] - \sin[\th1] \sin[\th3]) \sin[\th4])
                                                                                                   Sin[th6]) + (Cos[th5] (-Cos[th3] Sin[th1] - Cos[th1] Cos[th2] Sin[th3]) -
                                                                                             (\cos[th4] (\cos[th1] \cos[th2] \cos[th3] - \sin[th1] \sin[th3]) - \cos[th1] \sin[th2] \sin[th4])
                                                                                                   Sin[th5]) Sin[th7]) -
                         (\texttt{Cos[th6]} \ (\texttt{Cos[th1]} \ \texttt{Cos[th4]} \ \texttt{Sin[th2]} + (\texttt{Cos[th1]} \ \texttt{Cos[th2]} \ \texttt{Cos[th3]} - \texttt{Sin[th1]} \ \texttt{Sin[th3]}) \ \texttt{Sin[th4]}) + (\texttt{Sin[th4]} \ \texttt{Sin[th4]}) + (\texttt{Sin[th4]}) + (\texttt{Sin[th4]} \ \texttt{Sin[th4]}) + (\texttt{Sin[th4]} \ \texttt{Sin[th4]}) + (\texttt{Sin[th4]}) + (\texttt{Sin[th4]} \ \texttt{Sin[th4]}) + (\texttt{Sin[th4]}) + (\texttt{
                                                          (Cos[th5]
                                                                                                      (Cos[th4] \ (Cos[th1] \ Cos[th2] \ Cos[th3] \ - \ Sin[th1] \ Sin[th3]) \ - \ Cos[th1] \ Sin[th2] \ Sin[th4]) \ + \ (Cos[th4] \ Sin[th2] \ Sin[th4]) \ + \ (Cos[th4] \ Si
                                                                                             (-Cos[th3] Sin[th1] -Cos[th1] Cos[th2] Sin[th3]) Sin[th5]) Sin[th6]) Sin[th8]
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T[1,1] =
                      \texttt{Cos}[\theta 8] \ (\texttt{Cos}[\theta 7] \ (\texttt{Cos}[\theta 6] \ (\texttt{Cos}[\theta 5] \ (\texttt{Cos}[\theta 4] \ (\texttt{Cos}[\theta 1] \ \texttt{Cos}[\theta 2] \ \texttt{Cos}[\theta 3] \ - \ \texttt{Sin}[\theta 1] \ \texttt{Sin}[\theta 3]) \ - \ \texttt{Cos}[\theta 1] \ \texttt{Sin}[\theta 1] \ \texttt{Sin}[\theta 3]) \ - \ \texttt{Cos}[\theta 1] \ \texttt{Sin}[\theta 1] \ \texttt{Sin}[\theta 1] \ \texttt{Sin}[\theta 3] \ + \ \texttt{Sin}[\theta 1] \ \texttt{Sin}[\theta 3] \ + \ \texttt{Sin
                                                                                                                                                                                                                                                                                                                                                                                             \theta 2] \sin[\theta 4]) + (-\cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \cos[\theta 2] \sin[\theta 3]) \sin[\theta 5]) - \cos[\theta 1] \sin[\theta 3] \sin[\theta 5]) - \cos[\theta 1] \sin[\theta 5] \sin[
                                                                                                                                                                                                      (\cos[\theta 1]\cos[\theta 4]\sin[\theta 2]+(\cos[\theta 1]\cos[\theta 2]\cos[\theta 3]-\sin[\theta 1]\sin[\theta 3])\sin[\theta 4])\sin[\theta 6])+(\cos[\theta 1]\cos[\theta 4]\sin[\theta 6])+(\cos[\theta 1]\cos[\theta 6])\sin[\theta 6])+(\cos[\theta 1]\cos[\theta 6])\sin[\theta 6])
                                                                                                                         (\texttt{Cos}[\theta 5] \ (-\texttt{Cos}[\theta 3] \ \texttt{Sin}[\theta 1] \ - \texttt{Cos}[\theta 1] \ \texttt{Cos}[\theta 2] \ \texttt{Sin}[\theta 3]) \ - \ (\texttt{Cos}[\theta 4]
                                                                                                                                                                                                                                                                                                      (\cos[\theta 1] \cos[\theta 2] \cos[\theta 3] - \sin[\theta 1] \sin[\theta 3]) - \cos[\theta 1] \sin[\theta 2] \sin[\theta 4]) \sin[\theta 5]) \sin[\theta 7]) - \cos[\theta 7] \sin[\theta 7] \cos[\theta 7] \cos[\theta 7] \cos[\theta 7] \cos[\theta 7] \sin[\theta 7] \cos[\theta 7] \cos[\theta
                                                      (\cos[\theta 6] \ (\cos[\theta 1] \ \cos[\theta 4] \ \sin[\theta 2] \ + \ (\cos[\theta 1] \ \cos[\theta 2] \ \cos[\theta 3] \ - \ \sin[\theta 1] \ \sin[\theta 3]) \ \sin[\theta 4]) \ + \ (\cos[\theta 1] \ \cos[\theta 2] \ \cos[\theta 3] \ - \ \sin[\theta 1] \ \sin[\theta 3]) \ \sin[\theta 4]) \ + \ (\cos[\theta 1] \ \cos[\theta 1] \ \cos[\theta 1] \ \cos[\theta 1] \ \sin[\theta 3]) \ \sin[\theta 4]) \ + \ (\cos[\theta 1] \ \cos[\theta 1] \ \cos[\theta 1] \ \cos[\theta 1] \ \sin[\theta 1]) \ \sin[\theta 1]) \ \sin[\theta 1] \ \sin[\theta 1]) \ + \ (\cos[\theta 1] \ \cos[\theta 1] \ \cos[\theta 1] \ \sin[\theta 1]) \ \sin[\theta 1]) \ \sin[\theta 1]) \ \sin[\theta 1]
                                                                                                                             (\cos[\theta 5]\ (\cos[\theta 4]\ (\cos[\theta 1]\ \cos[\theta 2]\ \cos[\theta 3]\ -\sin[\theta 1]\ \sin[\theta 3])\ -\cos[\theta 1]\ \sin[\theta 2]\ \sin[\theta 4])\ +
                                                                                                                                                                                                   (-\cos[\theta 3]\,\sin[\theta 1]\,-\cos[\theta 1]\,\cos[\theta 2]\,\sin[\theta 3])\,\sin[\theta 5])\,\sin[\theta 6])\,\sin[\theta 8]
T[1,2] =
                            -\cos\left[\theta8\right]\left(\cos\left[\theta6\right]\left(\cos\left[\theta1\right]\cos\left[\theta4\right]\sin\left[\theta2\right]+\left(\cos\left[\theta1\right]\cos\left[\theta2\right]\cos\left[\theta3\right]-\sin\left[\theta1\right]\sin\left[\theta3\right]\right)\sin\left[\theta4\right]\right)+\left(\cos\left[\theta1\right]\cos\left[\theta1\right]\sin\left[\theta1\right]\sin\left[\theta1\right]\sin\left[\theta1\right]\sin\left[\theta1\right]
                                                                                                                         (\cos[\theta 5]\ (\cos[\theta 4]\ (\cos[\theta 1]\ \cos[\theta 2]\ \cos[\theta 3]\ -\sin[\theta 1]\ \sin[\theta 3])\ -\cos[\theta 1]\ \sin[\theta 2]\ \sin[\theta 4])\ +
                                                                                                                                                                                                         (-\cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \cos[\theta 2] \sin[\theta 3]) \sin[\theta 5]) \sin[\theta 6]) -
                                                      (\cos[\theta 7] \ (\cos[\theta 6] \ (\cos[\theta 5] \ (\cos[\theta 4] \ (\cos[\theta 1] \ \cos[\theta 2] \ \cos[\theta 3] \ - \sin[\theta 1] \ \sin[\theta 3]) \ - \cos[\theta 6] \ (\cos[\theta 6] \ (\cos[\theta
                                                                                                                                                                                                                                                                                                                                             \texttt{Cos}\left[\theta 1\right] \; \texttt{Sin}\left[\theta 2\right] \; \texttt{Sin}\left[\theta 4\right]) \; + \; \left(-\texttt{Cos}\left[\theta 3\right] \; \texttt{Sin}\left[\theta 1\right] \; - \; \texttt{Cos}\left[\theta 1\right] \; \texttt{Cos}\left[\theta 2\right] \; \texttt{Sin}\left[\theta 3\right]\right) \; \texttt{Sin}\left[\theta 5\right]\right) \; - \; \texttt{Cos}\left[\theta 1\right] \; \texttt{Sin}\left[\theta 3\right] \; + \; \texttt{Si
                                                                                                                                                                                                      (\cos[\theta 1]\cos[\theta 4]\sin[\theta 2]+(\cos[\theta 1]\cos[\theta 2]\cos[\theta 3]-\sin[\theta 1]\sin[\theta 3])\sin[\theta 4])\sin[\theta 6])+(\cos[\theta 1]\cos[\theta 4]\sin[\theta 6])+(\cos[\theta 1]\cos[\theta 6])\sin[\theta 6])+(\cos[\theta 1]\cos[\theta 6])\sin[\theta 6])
                                                                                                                             (\cos[\theta 5] (-\cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \cos[\theta 2] \sin[\theta 3]) -
                                                                                                                                                                                                      (\cos[\theta 4] (\cos[\theta 1] \cos[\theta 2] \cos[\theta 3] - \sin[\theta 1] \sin[\theta 3]) - \cos[\theta 1] \sin[\theta 2] \sin[\theta 4])
                                                                                                                                                                                                                   Sin[\theta 5]) Sin[\theta 7]) Sin[\theta 8]
T[1,3] = Cos[\theta 7] (Cos[\theta 5] (-Cos[\theta 3] Sin[\theta 1] - Cos[\theta 1] Cos[\theta 2] Sin[\theta 3]) - Cos[\theta 3] Co
                                                                                                                             (\cos[\theta 4] (\cos[\theta 1] \cos[\theta 2] \cos[\theta 3] - \sin[\theta 1] \sin[\theta 3]) - \cos[\theta 1] \sin[\theta 2] \sin[\theta 4]) \sin[\theta 5]) - \cos[\theta 1] \sin[\theta 2] \sin[\theta 3]
                                                      (\cos[\theta 6] \ (\cos[\theta 5] \ (\cos[\theta 4] \ (\cos[\theta 1] \ \cos[\theta 2] \ \cos[\theta 3] \ - \ \sin[\theta 1] \ \sin[\theta 3]) \ - \ \cos[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4]) \ + \ \cos[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4]) \ + \ \cos[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4]) \ + \ \cos[\theta 1] \ \sin[\theta 2] \ \sin[\theta 3] \ + \ \cos[\theta 1] \ \sin[\theta 3] \ \sin[\theta 4]) \ + \ \cos[\theta 1] \ \sin[\theta 1] \ \sin[\theta 3] \ \sin[\theta 4]) \ + \ \cos[\theta 1] \ \sin[\theta 1] \ \sin[\theta 1] \ \sin[\theta 3]) \ + \ \cos[\theta 1] \ \sin[\theta 1] \
                                                                                                                                                                                                      (-\cos\left[\theta 3\right]\,\sin\left[\theta 1\right]\,-\cos\left[\theta 1\right]\,\cos\left[\theta 2\right]\,\sin\left[\theta 3\right])\,\sin\left[\theta 5\right])\,-
                                                                                                                             (\cos[\theta 1] \cos[\theta 4] \sin[\theta 2] + (\cos[\theta 1] \cos[\theta 2] \cos[\theta 3] - \sin[\theta 1] \sin[\theta 3]) \sin[\theta 4]) \sin[\theta 6]) \sin[\theta 7]
   T[1,4] = 32 \cos[\theta 1] \sin[\theta 2] -
                                                \textbf{16} \; \left( -\cos\left[\theta\mathbf{1}\right] \; \cos\left[\theta\mathbf{4}\right] \; \sin\left[\theta\mathbf{2}\right] \; - \; \left(\cos\left[\theta\mathbf{1}\right] \; \cos\left[\theta\mathbf{2}\right] \; \cos\left[\theta\mathbf{3}\right] \; - \; \sin\left[\theta\mathbf{1}\right] \; \sin\left[\theta\mathbf{3}\right] \right) \; \sin\left[\theta\mathbf{4}\right] \right) \; - \; \sin\left[\theta\mathbf{3}\right] \; \cos\left[\theta\mathbf{3}\right] \; \sin\left[\theta\mathbf{3}\right] \; \sin\left[\theta\mathbf{3}\right] \; \cos\left[\theta\mathbf{3}\right] \; \cos\left[\theta\mathbf{3}\right]
                                                   8 \; (-\cos[\theta 6] \; (\cos[\theta 1] \; \cos[\theta 4] \; \sin[\theta 2] \; + \; (\cos[\theta 1] \; \cos[\theta 2] \; \cos[\theta 3] \; - \; \sin[\theta 1] \; \sin[\theta 3]) \; \sin[\theta 4]) \; - \; \sin[\theta 4] \; - \; \cos[\theta 4] \; - \; \sin[\theta 4] \; - \; \cos[\theta 4] \;
                                                                                                                         (\cos[\theta 5] \ (\cos[\theta 4] \ (\cos[\theta 1] \ \cos[\theta 2] \ \cos[\theta 3] \ - \ \sin[\theta 1] \ \sin[\theta 3]) \ - \ \cos[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4]) \ + \ \cos[\theta 1] \ \sin[\theta 3] \ + \ \cos[\theta 1] \ \sin[\theta 3]) \ + \ \cos[\theta 1] \ \sin[\theta 3] \ + \ \cos[\theta 1] \ \sin[\theta 1] \
                                                                                                                                                                                                (-\cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \cos[\theta 2] \sin[\theta 3]) \sin[\theta 5]) \sin[\theta 6])
T[2,1] = Cos[\theta 8]
                                                                            (\cos[\theta 7] \ (\cos[\theta 6] \ (\cos[\theta 5] \ (\cos[\theta 3] \ \cos[\theta 4] \ \sin[\theta 2] \ + \ \cos[\theta 2] \ \sin[\theta 4]) \ - \ \sin[\theta 2] \ \sin[\theta 3] \ \sin[\theta 5]) \ - \ \cos[\theta 3] \ \sin[\theta 3] \ \sin[\theta 5]) \ - \ \cos[\theta 3] \ \sin[\theta 3
                                                                                                                                                                                                      (-\cos[\theta 2]\cos[\theta 4] + \cos[\theta 3]\sin[\theta 2]\sin[\theta 4])\sin[\theta 6]) +
                                                                                                                         (-\cos[\theta 5]\,\sin[\theta 2]\,\sin[\theta 3]\,-\,(\cos[\theta 3]\,\cos[\theta 4]\,\sin[\theta 2]\,+\,\cos[\theta 2]\,\sin[\theta 4])\,\sin[\theta 5])\,\sin[\theta 7])\,-\,\cos[\theta 7]
                                                      (\cos\left[\theta6\right]\ (-\cos\left[\theta2\right]\ \cos\left[\theta4\right]\ + \cos\left[\theta3\right]\ \sin\left[\theta2\right]\ \sin\left[\theta4\right])\ +
                                                                                                                             (\cos[\theta 5] (\cos[\theta 3] \cos[\theta 4] \sin[\theta 2] + \cos[\theta 2] \sin[\theta 4]) - \sin[\theta 2] \sin[\theta 3] \sin[\theta 5]) \sin[\theta 6]) \sin[\theta 8]
T[2,2] = -\cos[\theta 8] (\cos[\theta 6] (-\cos[\theta 2] \cos[\theta 4] + \cos[\theta 3] \sin[\theta 2] \sin[\theta 4]) + \cos[\theta 3] \sin[\theta 4]) + \cos[\theta 3] \sin[\theta 4] \cos[\theta 4] 
                                                                                                                             (\cos[\theta 5] \ (\cos[\theta 3] \ \cos[\theta 4] \ \sin[\theta 2] + \cos[\theta 2] \ \sin[\theta 4]) - \sin[\theta 2] \ \sin[\theta 3] \ \sin[\theta 5]) \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6] \ \sin[\theta 6] \ \sin[
                                                      (\cos[\theta 7] (\cos[\theta 6] (\cos[\theta 5] (\cos[\theta 3] \cos[\theta 4] \sin[\theta 2] + \cos[\theta 2] \sin[\theta 4]) - \sin[\theta 2] \sin[\theta 3] \sin[\theta 5]) - \cos[\theta 6] \cos[
                                                                                                                                                                                                      (-\cos[\theta 2]\cos[\theta 4] + \cos[\theta 3]\sin[\theta 2]\sin[\theta 4])\sin[\theta 6]) +
                                                                                                                             (-\cos[\theta 5] \sin[\theta 2] \sin[\theta 2] \sin[\theta 3] - (\cos[\theta 3] \cos[\theta 4] \sin[\theta 2] + \cos[\theta 2] \sin[\theta 4]) \sin[\theta 5]) \sin[\theta 7]) \sin[\theta 8]
\texttt{T}[\texttt{2},\texttt{3}] = \texttt{Cos}[\theta \texttt{7}] \; (-\texttt{Cos}[\theta \texttt{5}] \; \texttt{Sin}[\theta \texttt{2}] \; \texttt{Sin}[\theta \texttt{3}] \; - \; (\texttt{Cos}[\theta \texttt{3}] \; \texttt{Cos}[\theta \texttt{4}] \; \texttt{Sin}[\theta \texttt{2}] \; + \; \texttt{Cos}[\theta \texttt{2}] \; \texttt{Sin}[\theta \texttt{4}]) \; \texttt{Sin}[\theta \texttt{5}]) \; - \; \texttt{Cos}[\theta \texttt{3}] \; \texttt{Cos}[\theta \texttt{3}] \; \texttt{Cos}[\theta \texttt{4}] \; \texttt{Sin}[\theta \texttt{2}] \; + \; \texttt{Cos}[\theta \texttt{3}] \; \texttt{Cos}[\theta \texttt{3}] \; \texttt{Cos}[\theta \texttt{4}] \; \texttt{Sin}[\theta \texttt{3}] \; + \; \texttt{Cos}[\theta \texttt{3}] \; \texttt{Sin}[\theta \texttt{4}]) \; \texttt{Sin}[\theta \texttt{3}] \; + \; \texttt{Cos}[\theta \texttt{3}] \texttt{3}] \;
                                                      (\cos[\theta 6] \ (\cos[\theta 5] \ (\cos[\theta 3] \ \cos[\theta 4] \ \sin[\theta 2] + \cos[\theta 2] \ \sin[\theta 4]) - \sin[\theta 2] \ \sin[\theta 3] \ \sin[\theta 5]) - \cos[\theta 3] \ \sin[\theta 5]) - \cos[\theta 4] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5] \ \sin[\theta 5]) - \cos[\theta 5] \ \sin[\theta 5] \
                                                                                                                             (-\cos[\theta 2]\cos[\theta 4]+\cos[\theta 3]\sin[\theta 2]\sin[\theta 4])\sin[\theta 6])\sin[\theta 7]
T[2,4] = -32 \cos[\theta 2] - 16 (\cos[\theta 2] \cos[\theta 4] - \cos[\theta 3] \sin[\theta 2] \sin[\theta 4]) - \cos[\theta 3] \sin[\theta 4]
                                                8 (-\cos[\theta 6] (-\cos[\theta 2] \cos[\theta 4] + \cos[\theta 3] \sin[\theta 2] \sin[\theta 4]) -
                                                                                                                         (\cos[\theta 5] \ (\cos[\theta 3] \ \cos[\theta 4] \ \sin[\theta 2] \ + \cos[\theta 2] \ \sin[\theta 4]) \ - \sin[\theta 2] \ \sin[\theta 3] \ \sin[\theta 5]) \ \sin[\theta 6])
```

```
T[3,1] =
                          \texttt{Cos}[\theta 8] \ (\texttt{Cos}[\theta 7] \ (\texttt{Cos}[\theta 6] \ (\texttt{Cos}[\theta 5] \ (\texttt{Cos}[\theta 4] \ (\texttt{Cos}[\theta 2] \ \texttt{Cos}[\theta 3] \ \texttt{Sin}[\theta 1] \ + \ \texttt{Cos}[\theta 1] \ \texttt{Sin}[\theta 3]) \ - \ \texttt{Sin}[\theta 1] \ \texttt{Sin}[\theta 1] \ + \ \texttt{Cos}[\theta 3] \ \texttt{Sin}[\theta 3] \ + \ \texttt{Sin}[\theta
                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \theta 2] \, \operatorname{Sin}[\theta 4]) \, + \, (\operatorname{Cos}[\theta 1] \, \operatorname{Cos}[\theta 3] \, - \, \operatorname{Cos}[\theta 2] \, \operatorname{Sin}[\theta 1] \, \operatorname{Sin}[\theta 3]) \, \operatorname{Sin}[\theta 5]) \, - \, \operatorname{Sin}[\theta 3]) \, + \, \operatorname{Sin}[\theta 3] \, 
                                                                                                                                                                                                                                             (\cos[\theta 4] \sin[\theta 1] \sin[\theta 2] + (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]) \sin[\theta 4]) \sin[\theta 6]) + \cos[\theta 1] \sin[\theta 1] \sin[\theta
                                                                                                                                                 (\texttt{Cos}[\theta 5] \ (\texttt{Cos}[\theta 1] \ \texttt{Cos}[\theta 3] \ - \ \texttt{Cos}[\theta 2] \ \texttt{Sin}[\theta 1] \ \texttt{Sin}[\theta 3]) \ - \ (\texttt{Cos}[\theta 4]
                                                                                                                                                                                                                                                                                                                                                                 (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]) - \sin[\theta 1] \sin[\theta 2] \sin[\theta 4]) \sin[\theta 5]) \sin[\theta 7]) - \sin[\theta 7]
                                                                 (\cos[\theta 6] (\cos[\theta 4] \sin[\theta 1] \sin[\theta 2] + (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]) \sin[\theta 4]) + (\cos[\theta 6] \cos[\theta 6] \sin[\theta 6] \sin[\theta 6] \sin[\theta 6]) + (\cos[\theta 6] \cos[\theta 6] \sin[\theta 6] \sin[\theta 6]) + (\cos[\theta 6] \cos[\theta 6]) +
                                                                                                                                                     (\cos[\theta 5] (\cos[\theta 4] (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]) - \sin[\theta 1] \sin[\theta 2] \sin[\theta 4]) + \cos[\theta 1] \sin[\theta 2] \sin[\theta 4]) + \cos[\theta 1] \sin[\theta 2] \sin[\theta 3] \sin[\theta 4]) + \cos[\theta 1] \sin[\theta 3] \sin[\theta 4] \sin[\theta 
                                                                                                                                                                                                                                             (\texttt{Cos}[\theta 1] \; \texttt{Cos}[\theta 3] \; - \; \texttt{Cos}[\theta 2] \; \texttt{Sin}[\theta 1] \; \texttt{Sin}[\theta 3]) \; \texttt{Sin}[\theta 5]) \; \texttt{Sin}[\theta 6]) \; \texttt{Sin}[\theta 8]
T[3,2] =
                                  -\cos\left[\theta8\right]\left(\cos\left[\theta6\right]\left(\cos\left[\theta4\right]\sin\left[\theta1\right]\sin\left[\theta2\right]+\left(\cos\left[\theta2\right]\cos\left[\theta3\right]\sin\left[\theta1\right]+\cos\left[\theta1\right]\sin\left[\theta3\right]\right)\sin\left[\theta4\right]\right)+\cos\left[\theta1\right]\sin\left[\theta3\right]\sin\left[\theta3\right]\sin\left[\theta4\right]
                                                                                                                                                 (\cos[\theta 5]\ (\cos[\theta 4]\ (\cos[\theta 2]\ \cos[\theta 3]\ \sin[\theta 1]\ +\cos[\theta 1]\ \sin[\theta 3])\ -\sin[\theta 1]\ \sin[\theta 2]\ \sin[\theta 4])\ +\cos[\theta 4])\ +\cos[\theta 4]
                                                                                                                                                                                                                                             (\cos[\theta 1] \cos[\theta 3] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 5]) \sin[\theta 6]) - \cos[\theta 1] \cos[\theta 3] - \cos[\theta 3] \sin[\theta 4] \sin[\theta 5]) \sin[\theta 5]
                                                                 (\cos[\theta 7] \ (\cos[\theta 6] \ (\cos[\theta 5] \ (\cos[\theta 4] \ (\cos[\theta 2] \ \cos[\theta 3] \ \sin[\theta 1] + \cos[\theta 1] \ \sin[\theta 3]) - \cos[\theta 6] \ \sin[\theta 6] \ \sin[\theta 7] \ \sin[\theta 
                                                                                                                                                                                                                                                                                                                                                                                                                  \sin[\theta 1] \sin[\theta 2] \sin[\theta 4]) + (\cos[\theta 1] \cos[\theta 3] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 5]) - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 5]) - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 5]
                                                                                                                                                                                                                                             (\cos[\theta 4] \sin[\theta 1] \sin[\theta 2] + (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]) \sin[\theta 4]) \sin[\theta 6]) + \cos[\theta 1] \sin[\theta 1] \sin[\theta 2] + \cos[\theta 1] \sin[\theta 3]) \sin[\theta 6]) + \cos[\theta 1] \sin[\theta 
                                                                                                                                                     (\cos[\theta 5] (\cos[\theta 1] \cos[\theta 3] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) - (\cos[\theta 4] (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 4] \cos[\theta 4]) + \cos[\theta 4] \cos[\theta 3] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 4] \cos[\theta 4] \cos[\theta 4] \cos[\theta 3] \cos[\theta 3] \cos[\theta 4] \cos[\theta 4] \cos[\theta 4] \cos[\theta 3] \cos[\theta 4] \cos[\theta 4
                                                                                                                                                                                                                                                                                                                                                                                                               \cos[\theta 1] \sin[\theta 3] - \sin[\theta 1] \sin[\theta 2] \sin[\theta 4] \sin[\theta 5] \sin[\theta 7] \sin[\theta 8]
T[3,3] = Cos[\theta 7] (Cos[\theta 5] (Cos[\theta 1] Cos[\theta 3] - Cos[\theta 2] Sin[\theta 1] Sin[\theta 3]) -
                                                                                                                                                 (\cos [\theta 4] (\cos [\theta 2] \cos [\theta 3] \sin [\theta 1] + \cos [\theta 1] \sin [\theta 3]) - \sin [\theta 1] \sin [\theta 2] \sin [\theta 4]) \sin [\theta 5]) - \sin [\theta 5] \cos [\theta 6] \sin [\theta 7] \sin
                                                                 (\cos[\theta 6] \ (\cos[\theta 5] \ (\cos[\theta 4] \ (\cos[\theta 2] \ \cos[\theta 3] \ \sin[\theta 1] + \cos[\theta 1] \ \sin[\theta 3]) - \sin[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4]) + \cos[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4]) + \cos[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4] \ \sin[\theta 4]) + \cos[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4] \ \sin[\theta 4]) + \cos[\theta 1] \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) + \cos[\theta 1] \ \sin[\theta 4] \ \sin[\theta 4]) + \cos[\theta 1] \ \sin[\theta 4] \ \sin[\theta 4]) + \cos[\theta 1] \ \sin[\theta 4] \ \sin[\theta 4]) + \cos[\theta 1] \ \sin[\theta 4] \ \sin[\theta 4]) + \cos[\theta 1] \ \sin[\theta 4])
                                                                                                                                                                                                                                                 (\cos[\theta 1] \cos[\theta 3] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 5]) -
                                                                                                                                                     (\cos[\theta 4] \sin[\theta 1] \sin[\theta 2] + (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]) \sin[\theta 4]) \sin[\theta 6]) \sin[\theta 7]
T[3,4] = 32 \sin[\theta 1] \sin[\theta 2] -
                                                         16 \ (-\cos\left[\theta 4\right] \ \sin\left[\theta 1\right] \ \sin\left[\theta 2\right] \ - \ (\cos\left[\theta 2\right] \ \cos\left[\theta 3\right] \ \sin\left[\theta 1\right] \ + \ \cos\left[\theta 1\right] \ \sin\left[\theta 3\right]) \ \sin\left[\theta 4\right]) \ - \ \cos\left[\theta 1\right] \ \sin\left[\theta 3\right]) \ \sin\left[\theta 4\right]) \ - \ \cos\left[\theta 1\right] \ \sin\left[\theta 3\right]) \ \sin\left[\theta 4\right]
                                                             8 \; (-\cos\left[\theta 6\right] \; (\cos\left[\theta 4\right] \; \sin\left[\theta 1\right] \; \sin\left[\theta 2\right] \; + \; (\cos\left[\theta 2\right] \; \cos\left[\theta 3\right] \; \sin\left[\theta 1\right] \; + \; \cos\left[\theta 1\right] \; \sin\left[\theta 3\right]) \; \sin\left[\theta 4\right]) \; - \; \cos\left[\theta 1\right] \; \sin\left[\theta 3\right] \; \sin\left[\theta 3\right] \; \sin\left[\theta 4\right] \; + \; \cos\left[\theta 1\right] \; \sin\left[\theta 3\right] \; \cos\left[\theta 3\right] \; \sin\left[\theta 3\right] \; \cos\left[\theta 3\right] \; \sin\left[\theta 3\right] \; \cos\left[\theta 3\right] \; \cos\left[\theta 3\right] \; \sin\left[\theta 3\right] \; \sin\left[\theta 3\right] \; \cos\left[\theta 
                                                                                                                                                 (\cos[\theta 5] \ (\cos[\theta 4] \ (\cos[\theta 2] \ \cos[\theta 3] \ \sin[\theta 1] \ + \ \cos[\theta 1] \ \sin[\theta 3]) \ - \ \sin[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4]) \ + \ \cos[\theta 1] \ \sin[\theta 3] \ - \ \sin[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4]) \ + \ \cos[\theta 1] \ \sin[\theta 3] \ - \ \sin[\theta 1] \ \sin[\theta 3] \ \sin[\theta 3] \ - \ \sin[\theta 1] \ \sin[\theta 3] \ \sin[\theta 3] \ + \ \cos[\theta 1] \ \sin[\theta 3] \ - \ \sin[\theta 1] \ \sin[\theta 3] \ \sin[\theta 3] \ - \ \sin[\theta 1] \ -
                                                                                                                                                                                                                                      (\cos[\theta 1] \cos[\theta 3] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 5]) \sin[\theta 6])
T[4,1] = 0
T[4,2] = 0
T[4,3] = 0
T[4,4] = 1
```

Formation of Jacobian

The first row of the Jacobian is the T[1,4] function differentiated with respect to $(\theta 1, \theta 2, ... \theta 8)$.

```
f1 = 32 \cos[\theta 1] \sin[\theta 2] -
                                                                                                         16 \; (-\cos[\theta 1] \; \cos[\theta 4] \; \sin[\theta 2] \; - \; (\cos[\theta 1] \; \cos[\theta 2] \; \cos[\theta 3] \; - \; \sin[\theta 1] \; \sin[\theta 3]) \; \sin[\theta 4]) \; - \; \cos[\theta 1] \; \cos[\theta 1] \; \sin[\theta 2] \; - \; \cos[\theta 1] \; \sin[\theta 1] \; \sin[\theta 2] \; - \; \cos[\theta 1] \; \cos[\theta 1] \; \sin[\theta 1] \; \cos[\theta 1] 
                                                                                                              8 \left( -\cos\left[\theta 6\right] \left(\cos\left[\theta 1\right] \cos\left[\theta 4\right] \sin\left[\theta 2\right] + \left(\cos\left[\theta 1\right] \cos\left[\theta 2\right] \cos\left[\theta 3\right] - \sin\left[\theta 1\right] \sin\left[\theta 3\right] \right) \sin\left[\theta 4\right] \right) - \cos\left[\theta 6\right] \left(\cos\left[\theta 1\right] \sin\left[\theta 4\right] + \cos\left[\theta 1\right] \sin\left[\theta 4\right] \right) - \cos\left[\theta 1\right] \sin\left[\theta 
                                                                                                                                                                                                                           (\cos[\theta 5] (\cos[\theta 4] (\cos[\theta 1] \cos[\theta 2] \cos[\theta 3] - \sin[\theta 1] \sin[\theta 3]) - \cos[\theta 1] \sin[\theta 2] \sin[\theta 4]) +
                                                                                                                                                                                                                                                                                                                                (-\cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \cos[\theta 2] \sin[\theta 3]) \sin[\theta 5]) \sin[\theta 6]);
```

```
df1d\theta1 = D[f1, \theta1]
df1d\theta2 = D[f1, \theta2]
   df1d\theta3 = D[f1, \theta3]
df1d\theta 4 = D[f1, \theta 4]
df1d\theta5 = D[f1, \theta5]
df1d\theta6 = D[f1, \theta6]
   df1d\theta7 = D[f1, \theta7]
df1d\theta8 = D[f1, \theta8]
   -32 \sin[\theta 1] \sin[\theta 2] -
                        16 \left( \cos \left[ \theta 4 \right] \sin \left[ \theta 1 \right] \sin \left[ \theta 2 \right] - \left( -\cos \left[ \theta 2 \right] \cos \left[ \theta 3 \right] \sin \left[ \theta 1 \right] - \cos \left[ \theta 1 \right] \sin \left[ \theta 3 \right] \right) \sin \left[ \theta 4 \right] \right) - \cos \left[ \theta 1 \right] \sin \left[ \theta 3 \right] \sin \left[ \theta 4 \right] 
                        8 \; (-\cos[\theta 6] \; (-\cos[\theta 4] \; \sin[\theta 1] \; \sin[\theta 2] \; + \; (-\cos[\theta 2] \; \cos[\theta 3] \; \sin[\theta 1] \; - \; \cos[\theta 1] \; \sin[\theta 3]) \; \sin[\theta 4]) \; - \; \cos[\theta 1] \; \sin[\theta 2] \; + \; (-\cos[\theta 2] \; \cos[\theta 3] \; \sin[\theta 1] \; - \; \cos[\theta 1] \; \sin[\theta 3]) \; \sin[\theta 4]) \; - \; \cos[\theta 1] \; \sin[\theta 2] \; + \; (-\cos[\theta 2] \; \cos[\theta 3] \; \sin[\theta 1] \; - \; \cos[\theta 1] \; \sin[\theta 3]) \; \sin[\theta 4]) \; - \; \cos[\theta 1] \; \sin[\theta 3] \; - \; \cos[\theta 1] \; \sin[\theta 3]) \; \sin[\theta 1] \; - \; \cos[\theta 1] \; \sin[\theta 3]) \; \sin[\theta 1] \; - \; \cos[\theta 1] \; \cos[\theta 1] \; - \; \cos[\theta 1] \; - \; \cos[\theta 1] \; \cos[\theta 1] \; - \; \cos[\theta 1] \;
                                                                                                   (\cos[\theta 5] (\cos[\theta 4] (-\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \sin[\theta 3]) + \sin[\theta 1] \sin[\theta 2] \sin[\theta 4]) + \sin[\theta 4] \sin[\theta 4]) + \sin[\theta 4] \sin[\theta 4] \sin[\theta 4] \sin[\theta 4] \sin[\theta 4] \sin[\theta 4]) + \sin[\theta 4] \sin[\theta
                                                                                                                                                                  (-\cos[\theta 1] \cos[\theta 3] + \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 5]) \sin[\theta 6]
   32\cos\left[\theta1\right]\cos\left[\theta2\right]-16\left(-\cos\left[\theta1\right]\cos\left[\theta2\right]\cos\left[\theta4\right]+\cos\left[\theta1\right]\cos\left[\theta3\right]\sin\left[\theta2\right]\sin\left[\theta4\right]\right)-16\left(-\cos\left[\theta1\right]\cos\left[\theta3\right]\sin\left[\theta4\right]\right)-16\left(-\cos\left[\theta1\right]\cos\left[\theta3\right]\cos\left[\theta4\right]\right)
                        8 \; (-\cos[\theta 6] \; (\cos[\theta 1] \; \cos[\theta 2] \; \cos[\theta 4] \; -\cos[\theta 1] \; \cos[\theta 3] \; \sin[\theta 2] \; \sin[\theta 4]) \; -\cos[\theta 3] \; \sin[\theta 4] \; -\cos[\theta 4] \; \cos[\theta 4] \;
                                                                                                   (\cos [\theta 5] (-\cos [\theta 1] \cos [\theta 3] \cos [\theta 4] \sin [\theta 2] -\cos [\theta 1] \cos [\theta 2] \sin [\theta 4]) + (\cos [\theta 5] \cos [\theta
                                                                                                                                                               Cos[\theta 1] Sin[\theta 2] Sin[\theta 3] Sin[\theta 5]) Sin[\theta 6])
16 (-\cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \cos[\theta 2] \sin[\theta 3]) \sin[\theta 4] -
                        8 (-\cos[\theta 6] (-\cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \cos[\theta 2] \sin[\theta 3]) \sin[\theta 4] -
                                                                                                (\cos[\theta 4]\cos[\theta 5](-\cos[\theta 3]\sin[\theta 1]-\cos[\theta 1]\cos[\theta 2]\sin[\theta 3])+
                                                                                                                                                                  (-\cos[\theta 1] \cos[\theta 2] \cos[\theta 3] + \sin[\theta 1] \sin[\theta 3]) \sin[\theta 5]) \sin[\theta 6])
   -16 \; (-\cos[\theta 4] \; (\cos[\theta 1] \; \cos[\theta 2] \; \cos[\theta 3] \; - \\ \sin[\theta 1] \; \sin[\theta 3]) \; + \\ \cos[\theta 1] \; \sin[\theta 2] \; \sin[\theta 4]) \; - \\ \sin[\theta 1] \; \cos[\theta 1] \; \cos[\theta 2] \; \sin[\theta 3] \; - \\ \sin[\theta 1] \; \sin[\theta 3]) \; + \\ \cos[\theta 1] \; \sin[\theta 2] \; \sin[\theta 3] \; - \\ \sin[\theta 1] \; \sin[\theta 3]) \; + \\ \cos[\theta 1] \; \sin[\theta 3] \; - \\ \sin[\theta 1] \; \sin[\theta 3]) \; + \\ \cos[\theta 1] \; \sin[\theta 3] \; - \\ \sin[\theta 1] \; \sin[\theta 3]) \; + \\ \cos[\theta 1] \; \sin[\theta 3] \; - \\ \sin[\theta 1] \; \cos[\theta 3] \; - \\ \sin[\theta 1] \; - \\ \sin[\theta 1] \; \cos[\theta 1] \; - \\ \sin[\theta 1
                        8 \; (-\cos\left[\theta 6\right] \; (\cos\left[\theta 4\right] \; (\cos\left[\theta 1\right] \; \cos\left[\theta 2\right] \; \cos\left[\theta 3\right] \; - \\ \sin\left[\theta 1\right] \; \sin\left[\theta 3\right]) \; - \; \cos\left[\theta 1\right] \; \sin\left[\theta 2\right] \; \sin\left[\theta 4\right]) \; - \; \cos\left[\theta 1\right] \; \sin\left[\theta 2\right] \; \sin\left[\theta 4\right]) \; - \; \cos\left[\theta 1\right] \; \sin\left[\theta 2\right] \; \sin\left[\theta 4\right] \; - \; \cos\left[\theta 1\right] \; \sin\left[\theta 2\right] \; \sin\left[\theta 4\right] \; - \; \cos\left[\theta 1\right] \; \sin\left[\theta 2\right] \; \sin\left[\theta 4\right] \; - \; \cos\left[\theta 1\right] \; \cos\left[
                                                                                                                        (-\cos[\theta 1]\cos[\theta 4]\sin[\theta 2]-(\cos[\theta 1]\cos[\theta 2]\cos[\theta 3]-\sin[\theta 1]\sin[\theta 3])\sin[\theta 4])\sin[\theta 6])
   8 (\cos[\theta 5] (-\cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \cos[\theta 2] \sin[\theta 3]) -
                                                                           (\cos [\theta 4] \ (\cos [\theta 1] \ \cos [\theta 2] \ \cos [\theta 3] \ - \ \sin [\theta 1] \ \sin [\theta 3]) \ - \ \cos [\theta 1] \ \sin [\theta 2] \ \sin [\theta 4])
                                                                                       Sin[\theta 5]) Sin[\theta 6]
   -8 (-\cos[\theta 6]
                                                                                             (\cos[\theta 5] (\cos[\theta 4] (\cos[\theta 1] \cos[\theta 2] \cos[\theta 3] - \sin[\theta 1] \sin[\theta 3]) - \cos[\theta 1] \sin[\theta 2] \sin[\theta 4]) + (\cos[\theta 5] \cos[\theta 5] \cos[\theta 6] \cos[
                                                                                                                                             (-\cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \cos[\theta 2] \sin[\theta 3]) \sin[\theta 5]) +
                                                                           (\cos[\theta 1] \cos[\theta 4] \sin[\theta 2] + (\cos[\theta 1] \cos[\theta 2] \cos[\theta 3] - \sin[\theta 1] \sin[\theta 3]) \sin[\theta 4]) \sin[\theta 6])
   0
   0
```

• The second row of the Jacobian is the T[2,4] function differentiated with respect to $(\theta 1, \theta 2, ... \theta 8)$.

```
f2 = -32 \cos[\theta 2] - 16 (\cos[\theta 2] \cos[\theta 4] - \cos[\theta 3] \sin[\theta 2] \sin[\theta 4]) -
      8 (-\cos[\theta 6] (-\cos[\theta 2] \cos[\theta 4] + \cos[\theta 3] \sin[\theta 2] \sin[\theta 4]) -
             (\cos[\theta 5] (\cos[\theta 3] \cos[\theta 4] \sin[\theta 2] + \cos[\theta 2] \sin[\theta 4]) - \sin[\theta 2] \sin[\theta 3] \sin[\theta 5]) \sin[\theta 6]);
```

```
df2d\theta1 = D[f2, \theta1]
df2d\theta2 = D[f2, \theta2]
df2d\theta3 = D[f2, \theta3]
df2d\theta4 = D[f2, \theta4]
df2d\theta 5 = D[f2, \theta 5]
df2d\theta6 = D[f2, \theta6]
df2d\theta7 = D[f2, \theta7]
df2d\theta8 = D[f2, \theta8]
32 \sin[\theta 2] - 16 (-\cos[\theta 4] \sin[\theta 2] - \cos[\theta 2] \cos[\theta 3] \sin[\theta 4]) -
       8 (-\cos[\theta 6] (\cos[\theta 4] \sin[\theta 2] + \cos[\theta 2] \cos[\theta 3] \sin[\theta 4]) -
                            (\cos[\theta 5] \ (\cos[\theta 2] \ \cos[\theta 3] \ \cos[\theta 4] \ - \sin[\theta 2] \ \sin[\theta 4]) \ - \cos[\theta 2] \ \sin[\theta 3] \ \sin[\theta 5]) \ \sin[\theta 6])
-16 \sin[\theta 2] \sin[\theta 3] \sin[\theta 4] - 8 (\cos[\theta 6] \sin[\theta 2] \sin[\theta 3] \sin[\theta 4] -
                            (-\cos[\theta 4]\cos[\theta 5]\sin[\theta 2]\sin[\theta 3]-\cos[\theta 3]\sin[\theta 2]\sin[\theta 5])\sin[\theta 6]
-16 \left(-\cos\left[\theta 3\right] \cos\left[\theta 4\right] \sin\left[\theta 2\right] - \cos\left[\theta 2\right] \sin\left[\theta 4\right]\right) -
       8 (-\cos[\theta 6] (\cos[\theta 3] \cos[\theta 4] \sin[\theta 2] + \cos[\theta 2] \sin[\theta 4]) -
                            Cos[\theta 5] (Cos[\theta 2] Cos[\theta 4] - Cos[\theta 3] Sin[\theta 2] Sin[\theta 4]) Sin[\theta 6])
8 \ (-\cos[\theta 5] \ \sin[\theta 2] \ \sin[\theta 3] \ - \ (\cos[\theta 3] \ \cos[\theta 4] \ \sin[\theta 2] \ + \ \cos[\theta 2] \ \sin[\theta 4]) \ \sin[\theta 5]) \ \sin[\theta 6]
-8 \; (-\cos[\theta 6] \; (\cos[\theta 5] \; (\cos[\theta 3] \; \cos[\theta 4] \; \sin[\theta 2] \; + \\ \cos[\theta 2] \; \sin[\theta 4]) \; - \\ \sin[\theta 2] \; \sin[\theta 3] \; \sin[\theta 5]) \; + \\ \cos[\theta 3] \; \sin[\theta 4] \; - \\ \sin[\theta 3] \; \sin[\theta 3] \; \sin[\theta 5]) \; + \\ \cos[\theta 3] \; \sin[\theta 4] \; - \\ \sin[\theta 3] \; \sin[\theta 3] \; \sin[\theta 4] \; - \\ \sin[\theta 3] \; \sin[\theta 3] \; - \\ \sin[\theta 3] \; - \\ \sin[\theta 3] \; \sin[\theta 3] \; - \\ \sin[\theta 3] \; - \\
                      (-\cos[\theta 2]\cos[\theta 4] + \cos[\theta 3]\sin[\theta 2]\sin[\theta 4])\sin[\theta 6])
0
0
```

• The third row of the Jacobian is the T[2,4] function differentiated with respect to $(\theta 1, \theta 2, ... \theta 8)$.

```
f3 = 32 \sin[\theta 1] \sin[\theta 2] -
                                                                                                                                                  16 \ (-\cos[\theta 4] \ \sin[\theta 1] \ \sin[\theta 2] \ - \ (\cos[\theta 2] \ \cos[\theta 3] \ \sin[\theta 1] \ + \ \cos[\theta 1] \ \sin[\theta 3]) \ \sin[\theta 4]) \ - \ \cos[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) \ - \ \cos[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) \ - \ \cos[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[\theta 4] \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[\theta 4]) \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[\theta 4]) \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[\theta 4]) \ \sin[\theta 4]) \ \sin[\theta 4] \ \sin[\theta 4]) \ \sin[
                                                                                                                                                   8 \left( -\cos\left[\theta 6\right] \left(\cos\left[\theta 4\right] \sin\left[\theta 1\right] \sin\left[\theta 2\right] + \left(\cos\left[\theta 2\right] \cos\left[\theta 3\right] \sin\left[\theta 1\right] + \cos\left[\theta 1\right] \sin\left[\theta 3\right] \right) \sin\left[\theta 4\right] \right) - \cos\left[\theta 6\right] \left(\cos\left[\theta 4\right] \sin\left[\theta 1\right] \sin\left[\theta 4\right] \right) - \cos\left[\theta 6\right] \left(\cos\left[\theta 4\right] \sin\left[\theta 1\right] \sin\left[\theta 4\right] \right) - \cos\left[\theta 6\right] \left(\cos\left[\theta 4\right] \sin\left[\theta 1\right] \sin\left[\theta 4\right] \right) - \cos\left[\theta 6\right] \cos\left
                                                                                                                                                                                                                                                                                                    (\cos[\theta 5] \ (\cos[\theta 4] \ (\cos[\theta 2] \ \cos[\theta 3] \ \sin[\theta 1] + \cos[\theta 1] \ \sin[\theta 3]) - \sin[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4]) + \cos[\theta 1] \ \sin[\theta 3] \ \sin[\theta 4] + \cos[\theta 1] \ \sin[\theta 4] + \cos[\theta 4] + \cos[\theta 1] \ \sin[\theta 4] + \cos[\theta 1] \ \sin[\theta 4] + \cos[\theta 4
                                                                                                                                                                                                                                                                                                                                                                                                                                          (\cos[\theta 1] \cos[\theta 3] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 5]) \sin[\theta 6]);
```

```
df3d\theta1 = D[f3, \theta1]
df3d\theta2 = D[f3, \theta2]
   df3d\theta3 = D[f3, \theta3]
   df3d\theta4 = D[f3, \theta4]
   df3d\theta5 = D[f3, \theta5]
   df3d\theta6 = D[f3, \theta6]
   df3d\theta7 = D[f3, \theta7]
df3d\theta8 = D[f3, \theta8]
   32 \cos[\theta 1] \sin[\theta 2] -
                        16 \; (-\cos[\theta 1] \; \cos[\theta 4] \; \sin[\theta 2] \; - \; (\cos[\theta 1] \; \cos[\theta 2] \; \cos[\theta 3] \; - \; \sin[\theta 1] \; \sin[\theta 3]) \; \sin[\theta 4]) \; - \; \cos[\theta 1] \; \cos[\theta 4] \; \sin[\theta 4]) \; - \; \cos[\theta 1] \; \cos[\theta 4] \; \sin[\theta 4] \; - \; \cos[\theta 4] \; \cos[\theta 4] \; \cos[\theta 4] \; - \; 
                        8 \left(-\cos\left[\theta 6\right] \left(\cos\left[\theta 4\right] \sin\left[\theta 4\right] + \left(\cos\left[\theta 1\right] \cos\left[\theta 2\right] \cos\left[\theta 3\right] - \sin\left[\theta 1\right] \sin\left[\theta 3\right]\right) \sin\left[\theta 4\right]\right) - \cos\left[\theta 6\right] 
                                                                                                (\cos[\theta 5] (\cos[\theta 4] (\cos[\theta 1] \cos[\theta 2] \cos[\theta 3] - \sin[\theta 1] \sin[\theta 3]) - \cos[\theta 1] \sin[\theta 2] \sin[\theta 4]) + (\cos[\theta 1] \cos[\theta 1] \cos[\theta 2] \cos[\theta 3] - \sin[\theta 1] \sin[\theta 3]) + (\cos[\theta 1] \sin[\theta 3]) + (\cos[\theta 1] \cos[\theta 1] \cos[\theta 3]) + (\cos[\theta 1] \cos[\theta 1]) + (\cos[\theta 1
                                                                                                                                                                   (-\cos[\theta 3] \sin[\theta 1] - \cos[\theta 1] \cos[\theta 2] \sin[\theta 3]) \sin[\theta 5]) \sin[\theta 6])
   32\cos\left[\theta2\right]\,\sin\left[\theta1\right]\,-\,16\,\left(-\cos\left[\theta2\right]\,\cos\left[\theta4\right]\,\sin\left[\theta1\right]\,+\,\cos\left[\theta3\right]\,\sin\left[\theta1\right]\,\sin\left[\theta2\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta2\right]\,\cos\left[\theta4\right]\,\sin\left[\theta1\right]\,+\,\cos\left[\theta3\right]\,\sin\left[\theta1\right]\,\sin\left[\theta2\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta2\right]\,\cos\left[\theta4\right]\,\sin\left[\theta1\right]\,+\,\cos\left[\theta3\right]\,\sin\left[\theta1\right]\,\sin\left[\theta2\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta2\right]\,\cos\left[\theta4\right]\,\sin\left[\theta1\right]\,+\,\cos\left[\theta3\right]\,\sin\left[\theta1\right]\,\sin\left[\theta2\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta2\right]\,\cos\left[\theta4\right]\,\sin\left[\theta1\right]\,+\,\cos\left[\theta3\right]\,\sin\left[\theta1\right]\,\sin\left[\theta2\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta2\right]\,\cos\left[\theta4\right]\,\sin\left[\theta1\right]\,+\,\cos\left[\theta3\right]\,\sin\left[\theta1\right]\,\sin\left[\theta2\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\cos\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\cos\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\,\sin\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\cos\left[\theta4\right]\,\cos\left[\theta4\right]\right)\,-\,16\,\left(-\cos\left[\theta4\right]\,\sin\left[\theta4\right]\,\cos\left[\theta4\right]\,\cos\left[\theta4\right]\,\cos\left[\theta4\right]\,\cos\left[\theta4\right]\,\cos\left[\theta4\right]
                        8 (-Cos[\theta6] (Cos[\theta2] Cos[\theta4] Sin[\theta1] -Cos[\theta3] Sin[\theta1] Sin[\theta2] Sin[\theta4]) -
                                                                                                (\cos [\theta 5] \ (-\cos [\theta 3] \ \cos [\theta 4] \ \sin [\theta 1] \ \sin [\theta 2] \ -\cos [\theta 2] \ \sin [\theta 1] \ \sin [\theta 4]) \ +
                                                                                                                                                                Sin[\theta 1] Sin[\theta 2] Sin[\theta 3] Sin[\theta 5]) Sin[\theta 6])
16 (\cos[\theta 1] \cos[\theta 3] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 4] -
                        8 (-\cos[\theta 6] (\cos[\theta 1] \cos[\theta 3] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 4] -
                                                                                                (\cos[\theta 4] \cos[\theta 5] (\cos[\theta 1] \cos[\theta 3] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) +
                                                                                                                                                                   (-\cos[\theta 2]\cos[\theta 3]\sin[\theta 1]-\cos[\theta 1]\sin[\theta 3])\sin[\theta 5])\sin[\theta 6]
   -16 \; (-\cos[\theta 4] \; (\cos[\theta 2] \; \cos[\theta 3] \; \sin[\theta 1] \; + \; \cos[\theta 1] \; \sin[\theta 3]) \; + \; \sin[\theta 1] \; \sin[\theta 2] \; \sin[\theta 4]) \; - \; \cos[\theta 4] \; \sin[\theta 4] \; + \; \cos[\theta 4] \; \cos[\theta 4] \; + \; \cos[\theta 4] \; + \; \cos[\theta 4] \; \cos[\theta 4] \; + \; \cos[\theta 
                         8 \; (-\cos\left[\theta 6\right] \; (\cos\left[\theta 4\right] \; (\cos\left[\theta 2\right] \; \cos\left[\theta 3\right] \; \sin\left[\theta 1\right] \; + \; \cos\left[\theta 1\right] \; \sin\left[\theta 3\right]) \; - \; \sin\left[\theta 1\right] \; \sin\left[\theta 2\right] \; \sin\left[\theta 4\right]) \; - \; \sin\left[\theta 1\right] \; \sin\left[\theta 3\right] \; \cos\left[\theta 3\right] \; \cos\left[\theta 3\right] \; \sin\left[\theta 3\right] \; \cos\left[\theta 3\right]
                                                                                                                      (-\cos[\theta 4]\,\sin[\theta 1]\,\sin[\theta 2]\,-\,(\cos[\theta 2]\,\cos[\theta 3]\,\sin[\theta 1]\,+\,\cos[\theta 1]\,\sin[\theta 3])\,\sin[\theta 4])\,\sin[\theta 6])
   8 (Cos[\theta 5] (Cos[\theta 1] Cos[\theta 3] - Cos[\theta 2] Sin[\theta 1] Sin[\theta 3]) -
                                                                           (\cos[\theta 4] \ (\cos[\theta 2] \ \cos[\theta 3] \ \sin[\theta 1] + \cos[\theta 1] \ \sin[\theta 3]) - \sin[\theta 1] \ \sin[\theta 2] \ \sin[\theta 4])
                                                                                       Sin[\theta 5]) Sin[\theta 6]
   -8 (-Cos[\theta 6]
                                                                                             (\cos[\theta 5] (\cos[\theta 4] (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]) - \sin[\theta 1] \sin[\theta 2] \sin[\theta 4]) + \cos[\theta 1] \sin[\theta 2] \sin[\theta 4]) + \cos[\theta 1] \sin[\theta 3] \sin[\theta 4] 
                                                                                                                                              (\cos[\theta 1] \cos[\theta 3] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]) \sin[\theta 5]) +
                                                                           (\cos[\theta 4] \sin[\theta 1] \sin[\theta 2] + (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]) \sin[\theta 4]) \sin[\theta 6])
0
   0
```

• The Z Vectors of the Jacobian are found by transforming the Z vectors from each frame to the base.

```
Z01 = T01.Z // MatrixForm
  Z02 = T01.T12.Z // MatrixForm
  Z03 = T01.T12.T23.Z // MatrixForm
Z04 = T01.T12.T23.T34.Z // MatrixForm
  Z05 = T01.T12.T23.T34.T45.Z // MatrixForm
Z06 = T01.T12.T23.T34.T45.T56.Z // MatrixForm
Z07 = T01.T12.T23.T34.T45.T56.T67.Z // MatrixForm
Z08 = T01.T12.T23.T34.T45.T56.T67.T78.Z // MatrixForm
                                0
                         - 1
                                0
                             1
                           -\sin[\theta 1]
                                                                              0
                                Cos[\theta 1]
                                                                              1
                         33 Cos[\theta 1] Sin[\theta 2]
                                                                       -33 \cos [\theta 2]
                       33 Sin[\theta 1] Sin[\theta 2]
                           -\cos[\theta 3] \sin[\theta 1] + 32\cos[\theta 1] \sin[\theta 2] - \cos[\theta 1]\cos[\theta 2] \sin[\theta 3]
                                                                                                                                                                                                                                                                        -32 \cos [\theta 2] - \sin [\theta 2] \sin [\theta 3]
                                \cos[\theta 1] \cos[\theta 3] + 32 \sin[\theta 1] \sin[\theta 2] - \cos[\theta 2] \sin[\theta 1] \sin[\theta 3]
                       32\cos\left[\theta1\right]\sin\left[\theta2\right] + \cos\left[\theta1\right]\cos\left[\theta4\right]\sin\left[\theta2\right] + (\cos\left[\theta1\right]\cos\left[\theta2\right]\cos\left[\theta3\right] - \sin\left[\theta1\right]\sin\left[\theta3\right])\sin\left[\theta4\right] - \sin\left[\theta4\right]\sin\left[\theta4\right] - \sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right] - \sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right] - \sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[\theta4\right]\sin\left[
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      -32 \cos[\theta 2] - \cos[\theta 2] \cos[\theta 4] + \cos[\theta 3] \sin[\theta 2] \sin[\theta 4] - 16
                       32 \sin[\theta 1] \sin[\theta 2] + \cos[\theta 4] \sin[\theta 1] \sin[\theta 2] + (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]) \sin[\theta 4] - \cos[\theta 1] \sin[\theta 2] + \cos[\theta 1] \sin[\theta 3] \sin[\theta 4] - \cos[\theta 1] \sin[\theta 3] \sin[\theta 4] - \cos[\theta 4] \sin[\theta 4] \sin[\theta 4] \sin[\theta 4] + \cos[\theta 4] \sin[\theta 4] 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  1
                       32\cos\left[\theta1\right]\sin\left[\theta2\right]+\cos\left[\theta5\right]\left(-\cos\left[\theta3\right]\sin\left[\theta1\right]-\cos\left[\theta1\right]\cos\left[\theta2\right]\sin\left[\theta3\right]\right)-16\left(-\cos\left[\theta1\right]\cos\left[\theta4\right]\sin\left[\theta3\right]\right)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            -32\cos[\theta 2] - \cos[\theta 5]\sin[\theta 2]\sin[\theta 3] - 16 (Co
                                32\sin[\theta 1]\sin[\theta 2] + \cos[\theta 5](\cos[\theta 1]\cos[\theta 3] - \cos[\theta 2]\sin[\theta 1]\sin[\theta 3]) - 16(-\cos[\theta 4]\sin[\theta 1]\sin[\theta 3]) - 16(-\cos[\theta 4]\sin[\theta 1]\sin[\theta 3]) - 16(-\cos[\theta 4]\sin[\theta 4]) - 16(-\cos[\theta 4]) - 1
                       32\cos\left[\theta1\right]\sin\left[\theta2\right]-16\left(-\cos\left[\theta1\right]\cos\left[\theta4\right]\sin\left[\theta2\right]-\left(\cos\left[\theta1\right]\cos\left[\theta2\right]\cos\left[\theta3\right]-\sin\left[\theta1\right]\sin\left[\theta3\right]\right)\sin\left[\theta3\right]
                                         32 \sin[\theta 1] \sin[\theta 2] - 16 \left(-\cos[\theta 4] \sin[\theta 1] \sin[\theta 2] - (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]\right) \sin[\theta 2] - \cos[\theta 2] \sin[\theta 3] \sin
                       32\cos\left[\theta1\right]\sin\left[\theta2\right] - 16\left(-\cos\left[\theta1\right]\cos\left[\theta4\right]\sin\left[\theta2\right] - \left(\cos\left[\theta1\right]\cos\left[\theta2\right]\cos\left[\theta3\right] - \sin\left[\theta1\right]\sin\left[\theta3\right]\right)\sin\left[\theta3\right]
                                                  32 \sin[\theta 1] \sin[\theta 2] - 16 \left(-\cos[\theta 4] \sin[\theta 1] \sin[\theta 2] - (\cos[\theta 2] \cos[\theta 3] \sin[\theta 1] + \cos[\theta 1] \sin[\theta 3]\right) \sin[\theta 3]
```

• The Jacobian

```
-32\sin[\theta 1]\sin[\theta 2]-16\left(\cos[\theta 4]\sin[\theta 1]\sin[\theta 2]-\left(-\cos[\theta 2]\cos[\theta 3]\sin[\theta 1]-\cos[\theta 1]\sin[\theta 3]\right)\sin[\theta 3]\right)
    32\cos[\theta 1]\sin[\theta 2] - 16(-\cos[\theta 1]\cos[\theta 4]\sin[\theta 2] - (\cos[\theta 1]\cos[\theta 2]\cos[\theta 3] - \sin[\theta 1]\sin[\theta 3]) S:
```

Velocity Vector

Now I need a velocity vector in order to solve for rate of change of the joint values.

The relationship between manipulator velocity and position is given by:

$$\left(egin{array}{c} {\sf xdot_desired} \\ {\sf ydot_desired} \\ {\sf zdot_desired} \end{array} \right) = oldsymbol{lpha} \left(\left(egin{array}{c} {\sf xdesired} \\ {\sf ydesired} \\ {\sf zdesired} \end{array} \right) - \left(egin{array}{c} {\sf xcurrent} \\ {\sf ycurrent} \\ {\sf zcurrent} \end{array} \right) \right)$$