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# **CME 213**

**SPRING 2012-2013**

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The background of the slide features a large, light gray watermark of the Stanford University seal. The seal is circular and contains a redwood tree in the center, with the words "LELAND STANFORD JUNIOR UNIVERSITY" around the top and "1891" at the bottom. The text "DIE LUFT DER FREIHEIT" is also visible within the seal's border.

# **OPENMP MASTER AND SYNCHRONIZATION CONSTRUCTS**

## SYNCHRONIZATION CONSTRUCTS

- Several constructs are available.
- We won't go into all the details.

## MASTER

```
#pragma omp master
```

Definition: structured block is executed by the master thread of the team (i.e., thread ID 0).

## CRITICAL

```
#pragma omp critical
```

Definition: the execution of the associated structured block is restricted to a single thread at a time.

## BARRIER

```
#pragma omp barrier
```

Definition: explicit barrier. Threads wait until all threads reach the construct and then all can resume execution.

## ATOMIC

```
#pragma omp atomic [read | write | update | capture]
```

Definition: ensures that a specific storage location is accessed atomically, rather than exposing it to the possibility of multiple, simultaneous reading and writing threads that may result in indeterminate values.

Typical application:

```
int main() {  
    int count = 0;  
    #pragma omp parallel  
    {  
        #pragma omp atomic  
        count++;  
        // update is implied when no clause is present  
    }  
    printf("Number of threads: %d\n", count);  
}
```



## **DATA SHARING ATTRIBUTES**



## THE SIMPLE VERSION

- I am explaining the “simple” version.
- Data sharing attributes. There are several options. We focus on:
  - **shared**: variable refers to the same block of memory for all threads
  - **private**: variable refers to a different block of memory for each thread
  - **firstprivate**: private variable but each copy is initialized with the value that the corresponding original item has when the construct is encountered
- There are three cases:
  1. pre-determined
  2. explicitly determined
  3. implicitly determined

## PRE-DETERMINED

- Pre-determined sharing attributes (i.e., these cannot be modified):
  - the user cannot changed the sharing attribute.
  - the attribute is determined by a rule.
- Rules:
  - Variables declared inside the parallel region are **private** (“variables with automatic storage duration declared inside the construct”).
  - Static data members are **shared**.
  - A loop variable in a **parallel** loop is **private**.
  - A **const** variable is **shared**.

## EXPLICITLY/IMPLICITLY

- **Explicitly determined** attributes: use `shared()` or `private()`.
- `default(none)` or `default(shared)` can be used to explicitly specify a default.
- **Implicitly determined** attributes: basically the fall back case if not pre-determined or explicitly determined:
  - In a parallel construct, variable is **shared**.
  - In a task construct, variable is **firstprivate**.

## SPECIAL DATA-SHARING CLAUSE: REDUCTION

```
#include <omp.h>
#include <stdio.h>

int main () {

    const int n = 100;
    float a[100], b[100], result;

    /* Some initializations */
    result = 0.0;
    for (int i=0; i < n; i++) {
        a[i] = i * 1.0;
        b[i] = i * 2.0;
    }

    #pragma omp parallel for reduction(+:result)
        for (int i=0; i < n; i++)
            result += a[i] * b[i];

    printf("Final result= %f\n",result);
}
```

## TECHNICAL DEFINITION

**reduction (op: list)**

- **op** can be: {+, -, \*, &, ^, |, &&, ||}
- For each of the variables in **list**, a private copy is created for each thread.
- The private copies are initialized to the neutral element (zero) of the operation **op** and can be updated by the owning thread.
- At the end of the region, the local values of the reduction variables are combined according to the operator **op** and the result of the reduction is written into the original shared variable.

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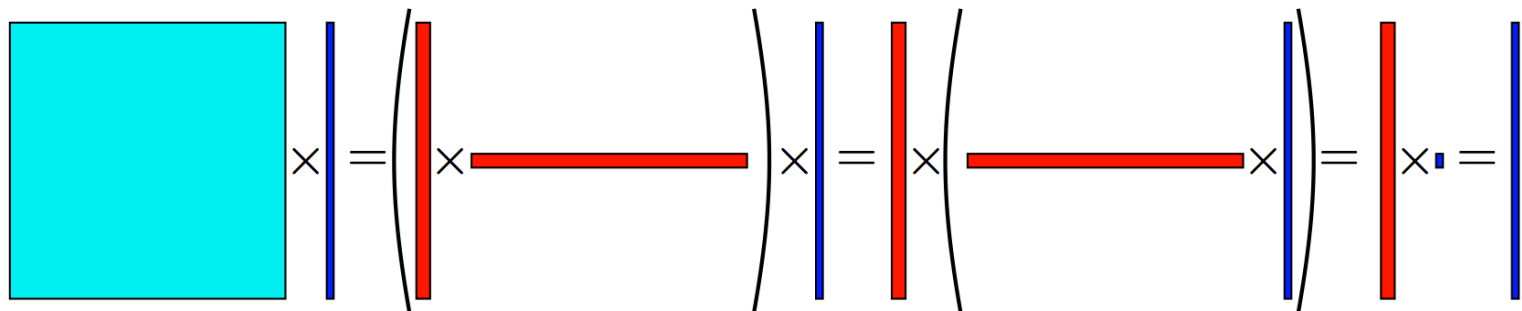
# **FAST MULTIPOLE METHOD**

## FAST MULTIPOLE METHOD

- A technique to calculate matrix vector products with  $O(N)$  computational cost:

$$\phi_i = \sum_{j=1}^N K(\vec{r}_i, \vec{r}_j) q_j \quad j \in \{1, 2, \dots, N\}$$

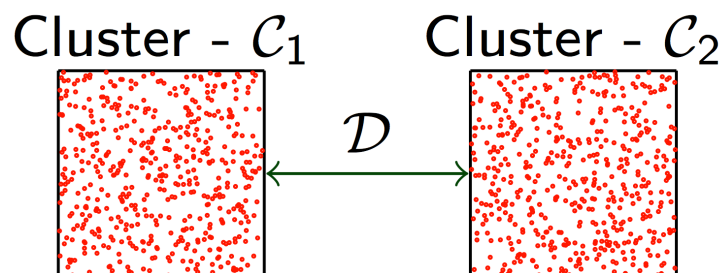
- Method is based on approximating blocks in the matrix  $K(r_i, r_j)$  **low-rank matrix**:



Example of matrix-vector product with low-rank matrix

## FINDING LOW-RANK BLOCKS

- For most kernels  $K$  in engineering applications, if we consider interacting points that belong to well-separated clusters, the interaction is approximately low-rank:



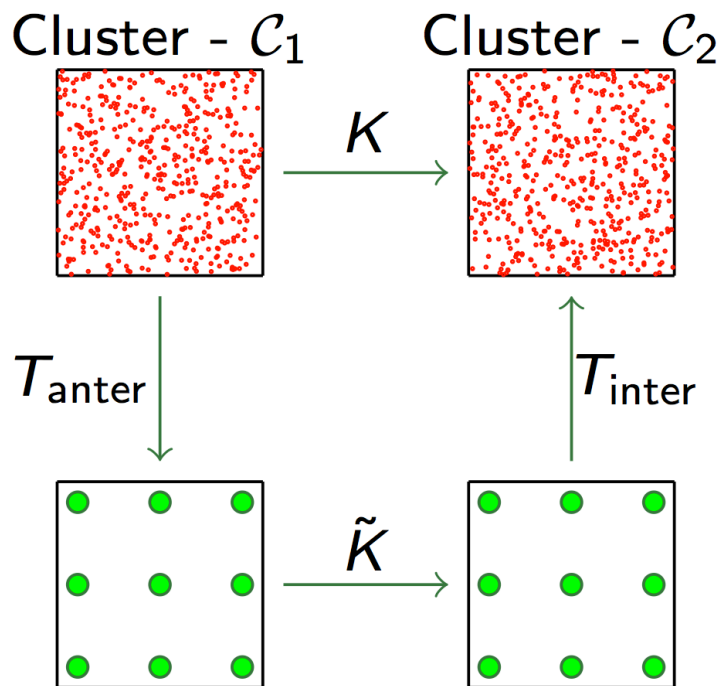
- Well-separated criterion:** interaction is low-rank if the clusters are separated by at least  $aR$ ,  $R$  radius of clusters, with  $0 < a$ , e.g.,  $a=0.87$ .



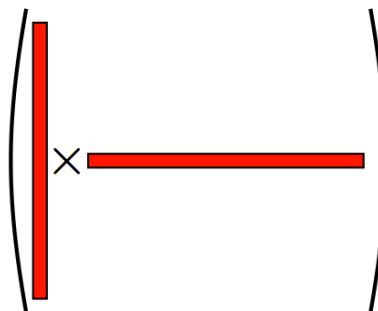
## INTERPOLATORY LOW-RANK APPROXIMATIONS

An efficient method to construct low-rank approximation is to use interpolation formulas:

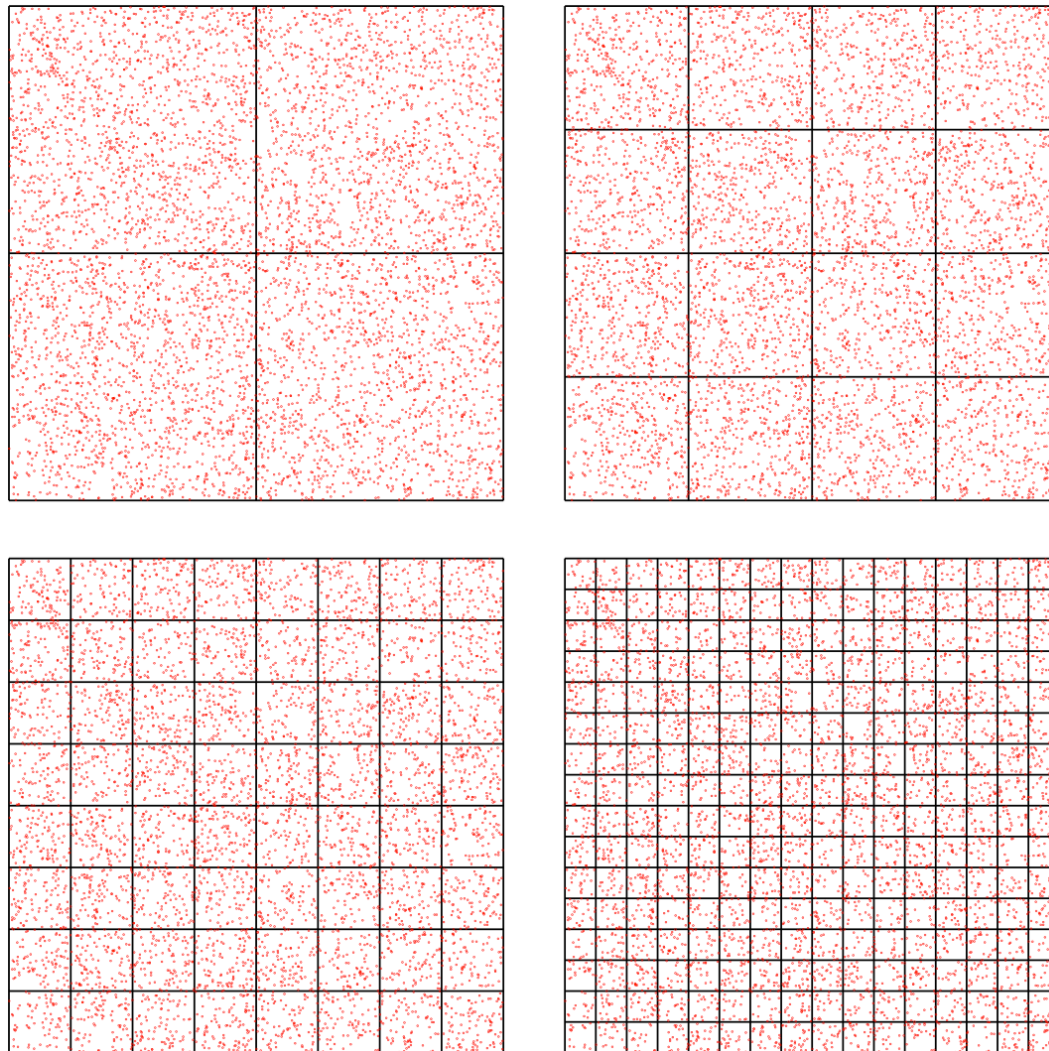
$$K(x, y) \approx \sum_{k=1}^r w_k(x) K(\bar{x}_k, y) \approx \sum_{k=1}^r w_k(x) \left( \sum_{j=1}^r w_j(y) K(\bar{x}_k, \bar{y}_j) \right)$$



$$K_{n \times n} = \underbrace{(T_{\text{inter}})_{n \times r} \tilde{K}_{r \times r} (T_{\text{anter}})_{r \times n}}_{\text{Low rank}} + \epsilon$$

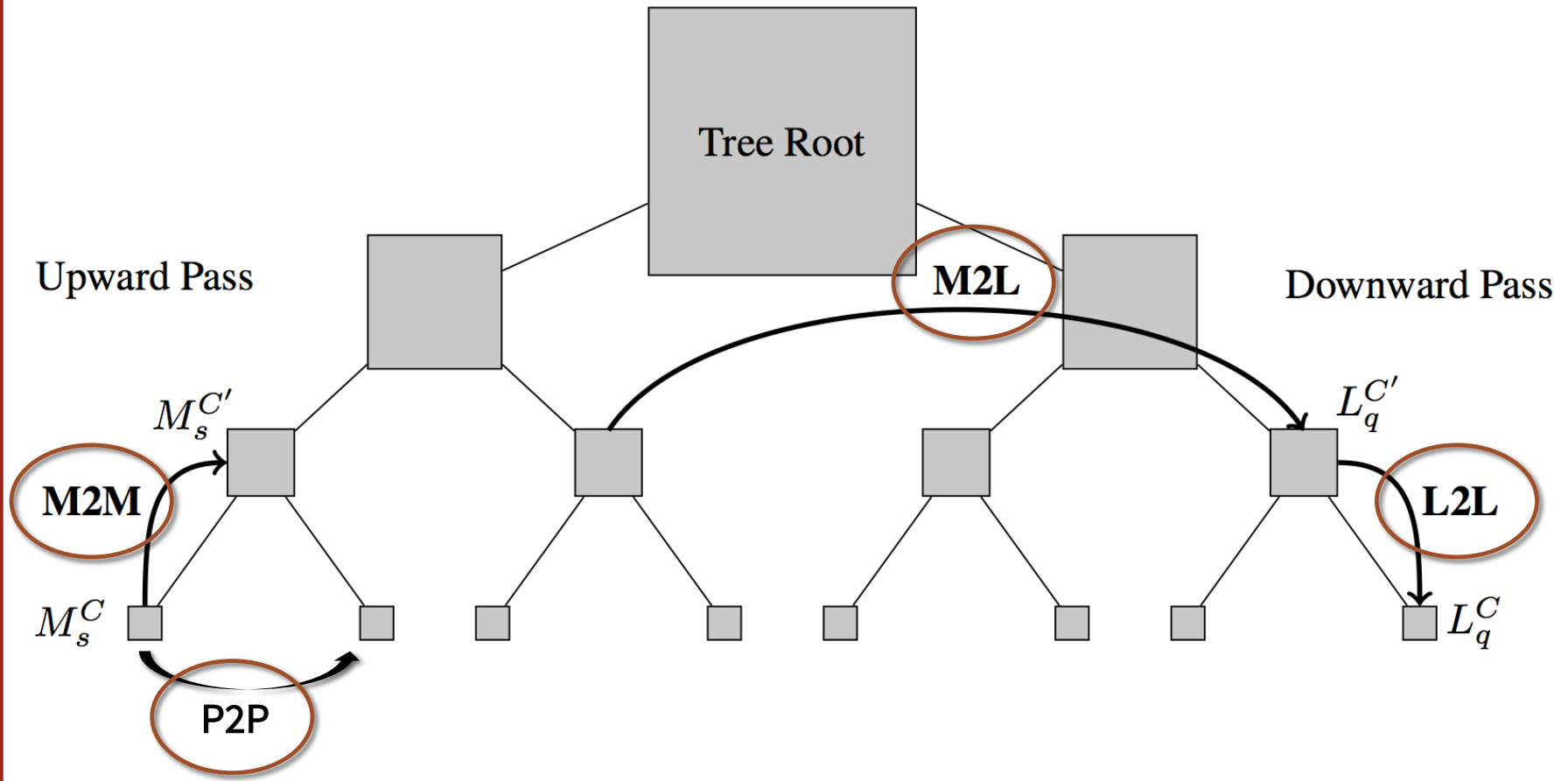


# FMM TREE



2D quad tree

# FMM OPERATORS



## M2L AND L2L

|    |    |    |    |    |    |  |  |
|----|----|----|----|----|----|--|--|
|    |    |    |    |    |    |  |  |
|    |    |    |    |    |    |  |  |
| 1  | 3  | 5  | 7  | 9  | 11 |  |  |
| 2  | 4  | 6  | 8  | 10 | 12 |  |  |
| 27 |    |    |    | 13 | 15 |  |  |
| 26 |    |    |    | 14 | 16 |  |  |
| 25 |    |    |    | 17 | 19 |  |  |
| 24 | 23 | 22 | 21 | 18 | 20 |  |  |

### L2L at level 3

Interpolate from parent the far-field effect.

### M2L at level 3

Compute the interaction from well-separated boxes.

Do this for all boxes. Repeat at all levels.

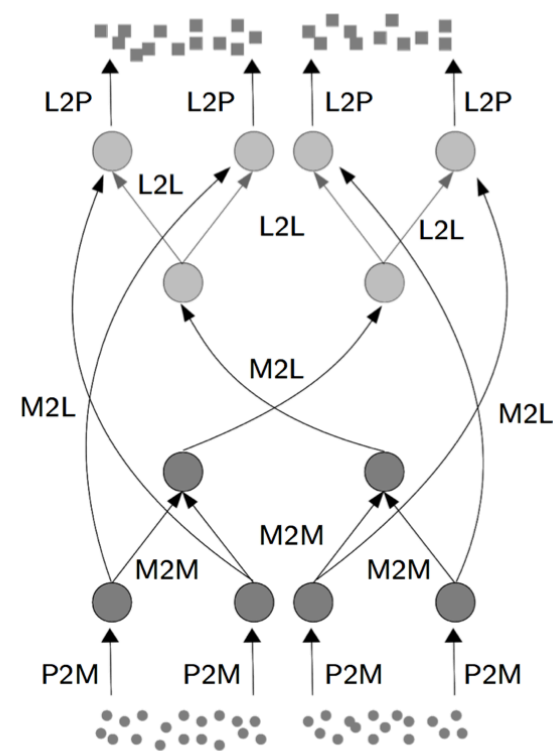
The total computational cost for all these operations is  $\mathcal{O}(rN)$

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# **PARALLEL IMPLEMENTATION**

## PARALLELIZATION OF THE FMM

| Task               | Concurrency | Load-balancing       |
|--------------------|-------------|----------------------|
| M2L (lower level)  | High        | Homogeneous          |
| M2L (higher level) | <b>Low</b>  | Homogeneous          |
| P2P                | High        | <b>Heterogeneous</b> |
| M2M/L2L            | <b>Low</b>  | Homogeneous          |

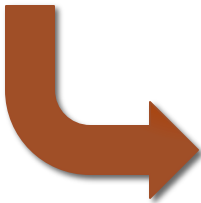


Directed acyclic graph of tasks

# CLASSICAL OPENMP PARALLELIZATION

```
function FMM(tree)
  // Near-field
  P2P(tree.levels[tree.height-1]);
  // Far-field
  P2M(tree.levels[tree.height-1]);
  forall the level l from tree.height-2 to 2 do
    M2M(tree.levels[l]);
  forall the level l from 2 to tree.height-2 do
    M2L(tree.levels[l]);
    L2L(tree.levels[l]);
  M2L(tree.levels[tree.height-1]);
  L2P(tree.levels[tree.height-1]);
```

The loop over cells at a given level are parallelized:



```
function M2L(level)
  #pragma omp parallel for
  foreach cell c1 in level.cells do
    kernel.m2l(c1.local,
              c1.far_field.multipole);
```

## IMPROVING THE SCALABILITY

- When running on a large number of cores or if the computational problem is too small, one may run into performance issues, e.g.:
  - some cores may become idle because there is not enough concurrent work or
  - many cores are done and only a few are still executing.
- The top of the tree represents a parallel bottleneck: limited amount of work is available.
- The P2P tasks can be highly heterogeneous: long tasks are mixed up with short tasks.
- Solution:
  - Mix parallel sections with more sequential ones.
  - Try to execute long tasks first and finish with short tasks.



# INTERLEAVING FAR FIELD AND NEAR FIELD CALCULATIONS

```
#pragma omp parallel
```

```
  #pragma omp sections
```

```
    #pragma omp section
```

```
      P2Ptask(tree.levels[tree.height-1])
```

```
    #pragma omp section
```

```
      P2Mtask(tree.levels[tree.height-1])
```

```
      forall the level l from tree.height-2 to 2 do
```

```
        M2Mtask(tree.levels[l])
```

```
      forall the level l from 2 to tree.height-2 do
```

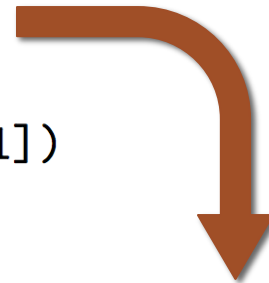
```
        M2Ltask(tree.levels[l])
```

```
        L2Ltask(tree.levels[l])
```

```
      M2Ltask(tree.levels[tree.height-1])
```

```
#pragma omp single
```

```
  L2Ptask(tree.levels[tree.height-1])
```



```
function M2Ltask(level)
```

```
  foreach block of cells bl in level.blocks do
```

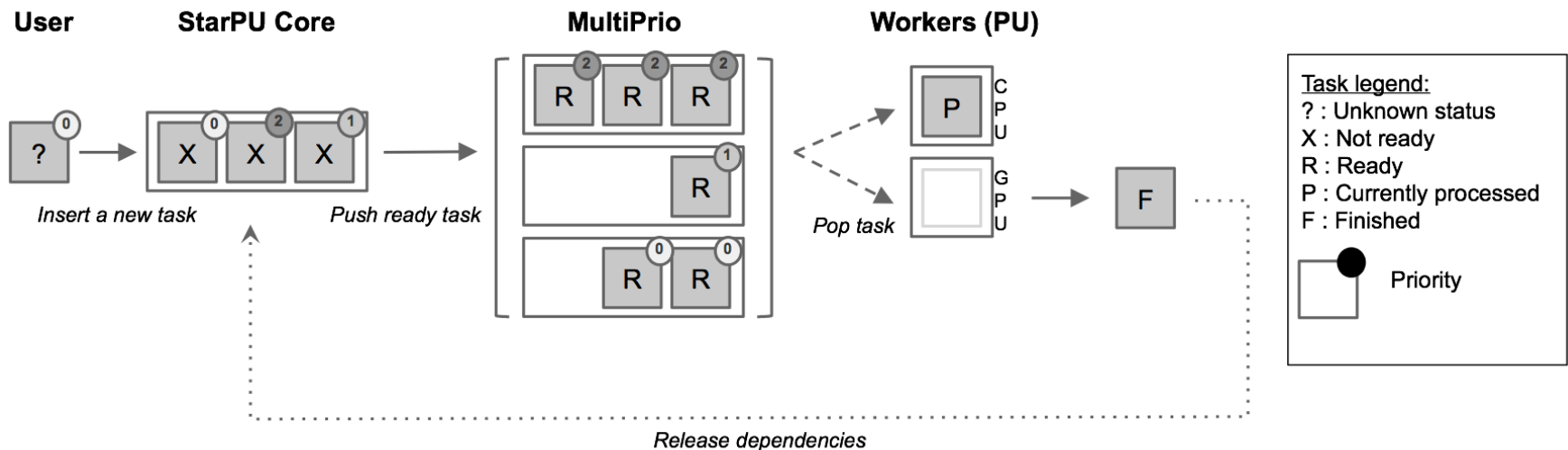
```
    #pragma omp task
```

```
      m2l(bl.local, bl.far_field.multipole);
```

```
    #pragma omp taskwait;
```

## BETTER YET: PTHREADS WITH RUNTIME SCHEDULER

Example of scheduling (implemented using Pthreads) based on priority lists:



**P2M > M2M > L2L(l) > P2P (large) > M2L(l) > P2P (small) > L2P**

Task priority rule

- Task insertion and retrieval.
- Use mutexes and condition variables in the implementation.