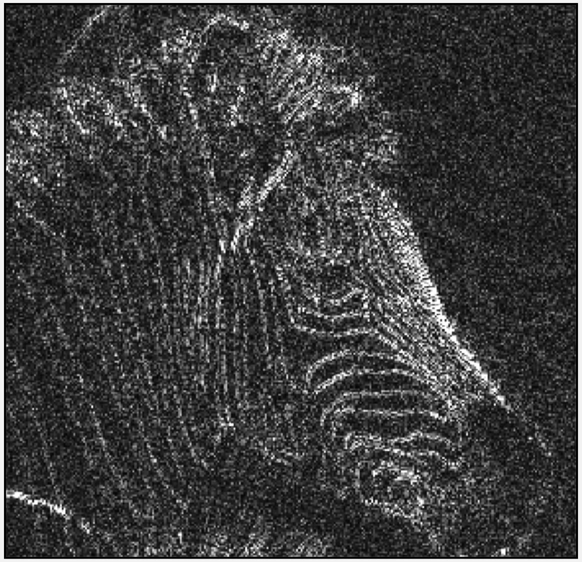
Department of Computer Science
University of Bristol

COMS30030 - Image Processing and Computer Vision



Week 02

Frequency Domain & Transforms II

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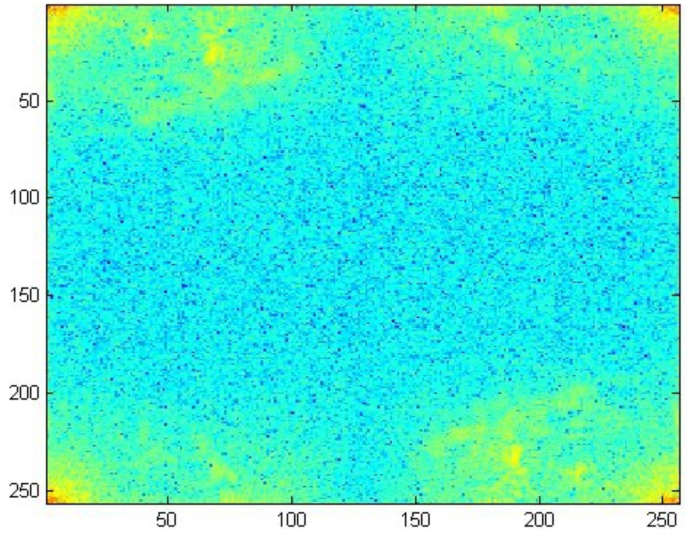
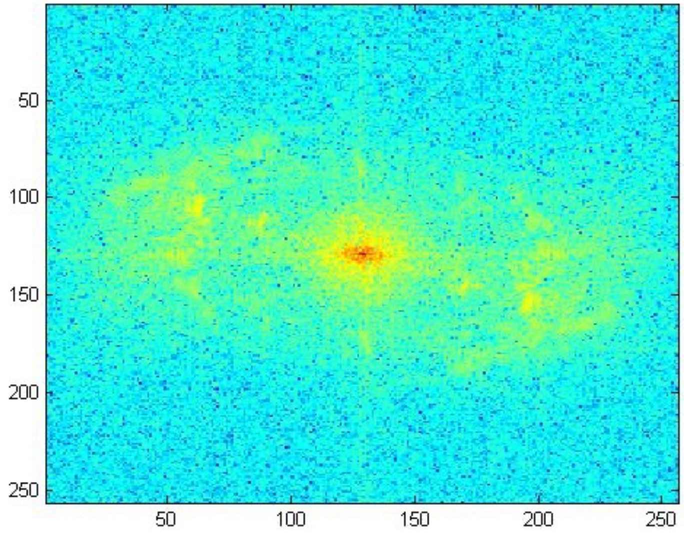
Conjugate Symmetry

- Important property of the FT
- The FT of a real function f(x,y) gives:

$$F(u,v) = F^*(-u,-v) \quad \longrightarrow \quad |F(u,v)| = |F^*(-u,-v)|$$

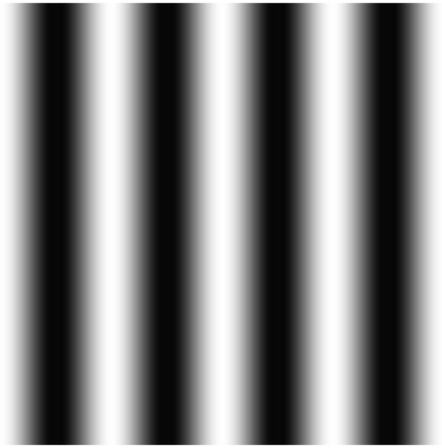


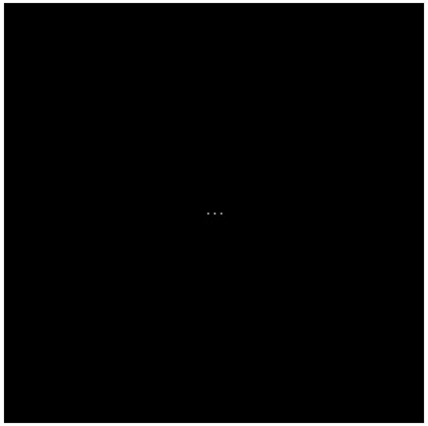
Before shift

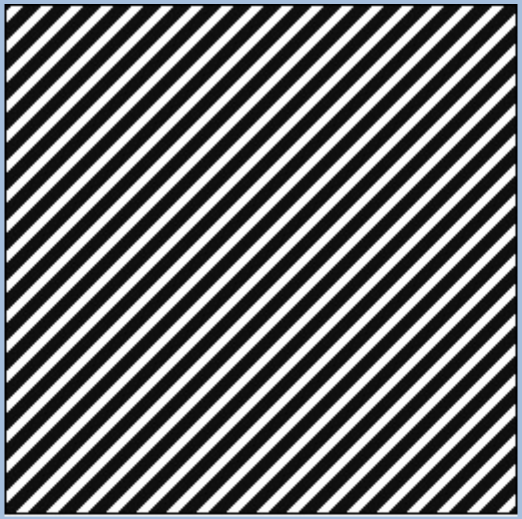


After shift

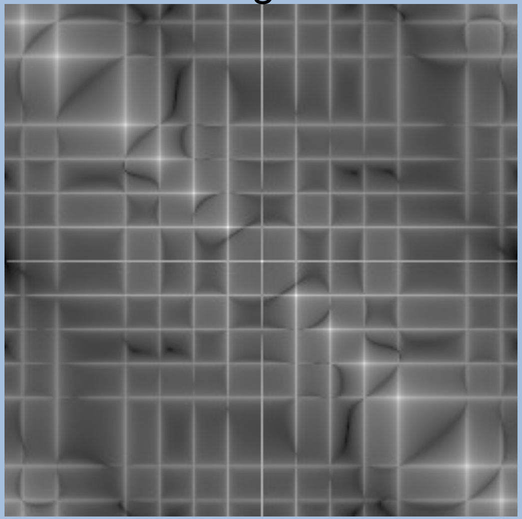
Spatial Domain ⟷ Frequency Domain

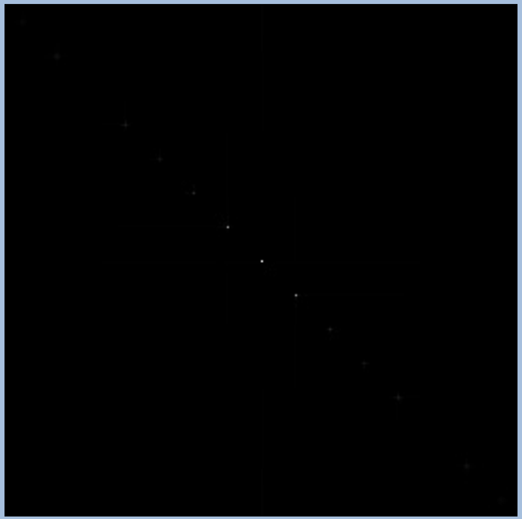




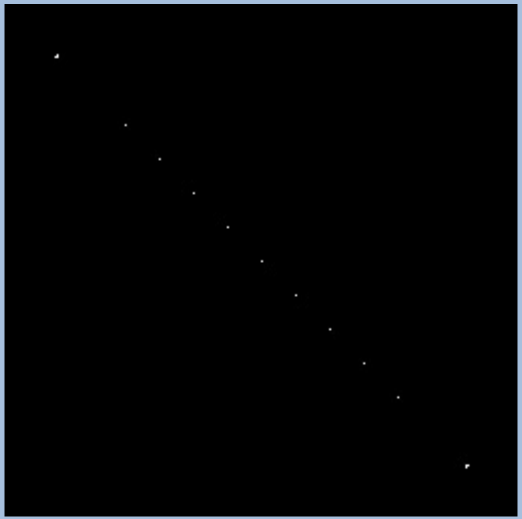


log of FT + 1



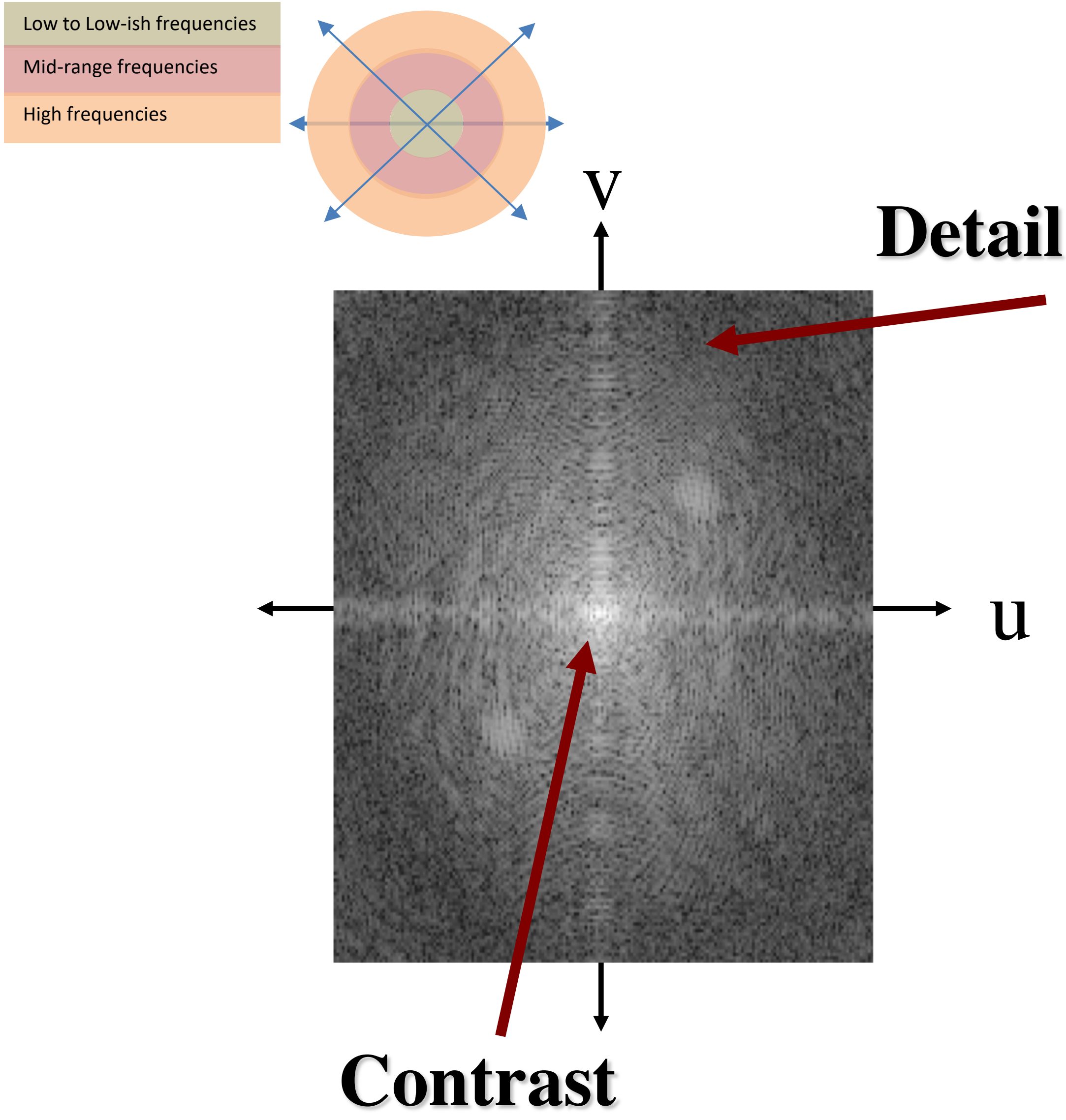


FT

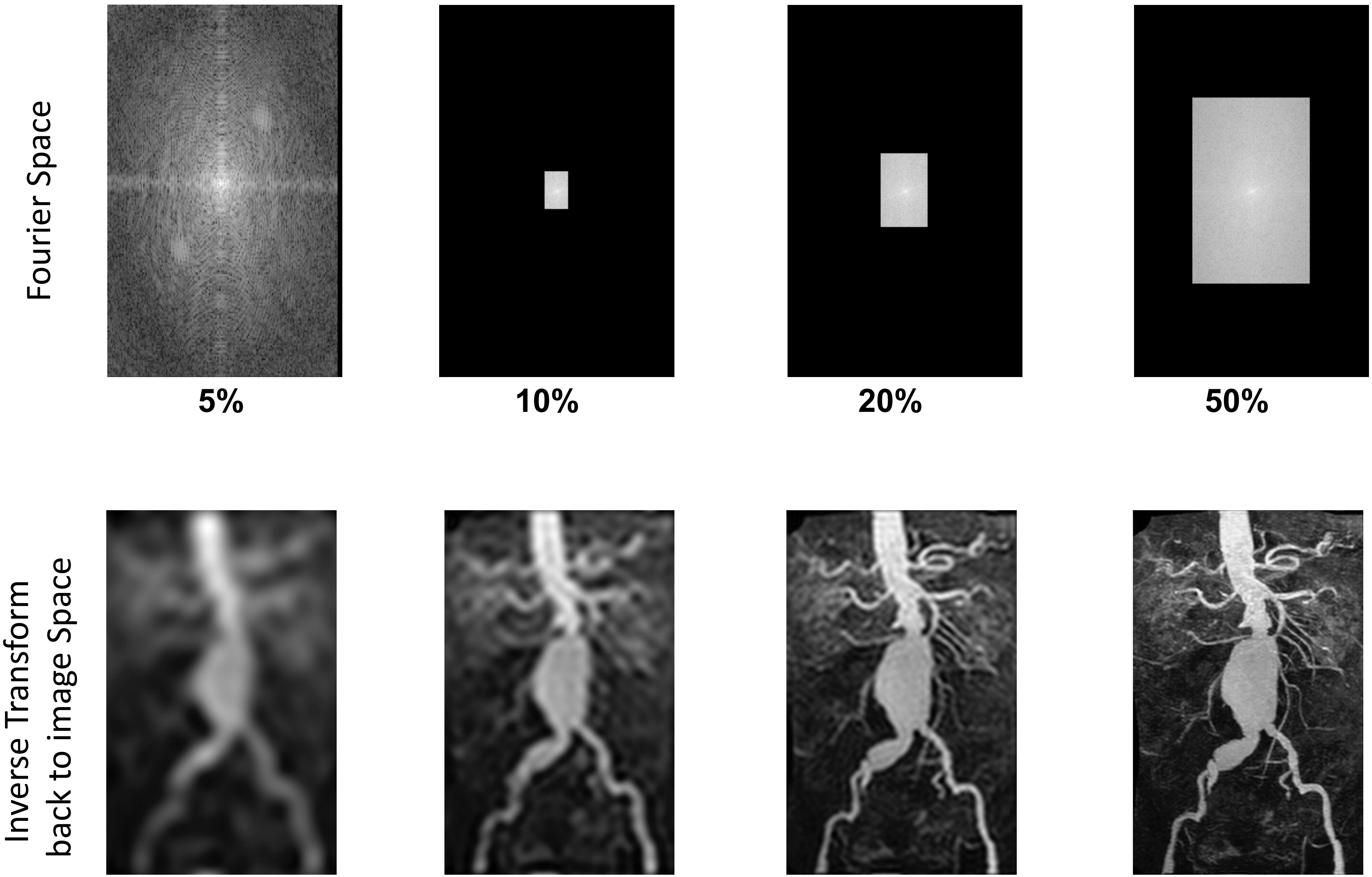


Thresholded log of FT+1

Relating Frequencies to Images

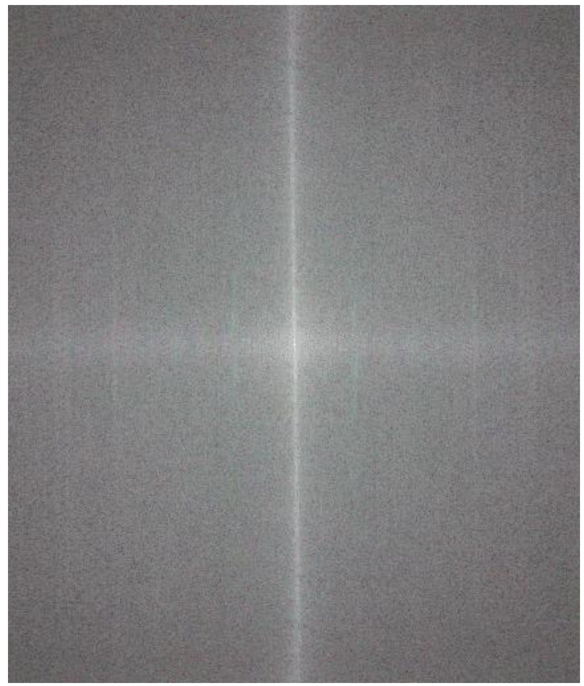


Example: Relating Frequencies to Images



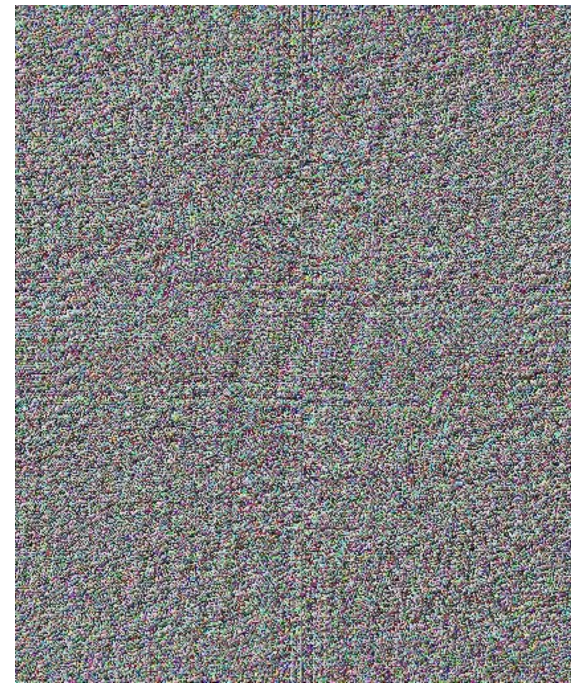
Example: Relating Frequencies to Images

Magnitude



$|F(u,v)|$

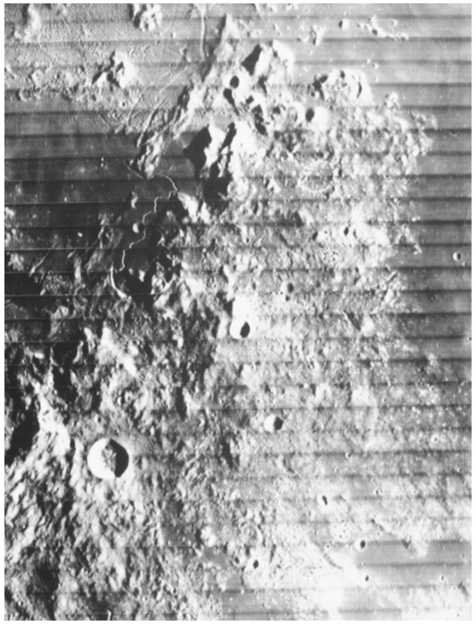
Phase

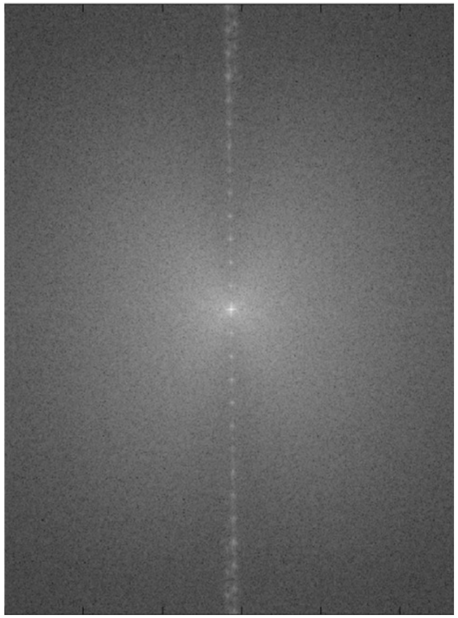


$\angle F(I)$

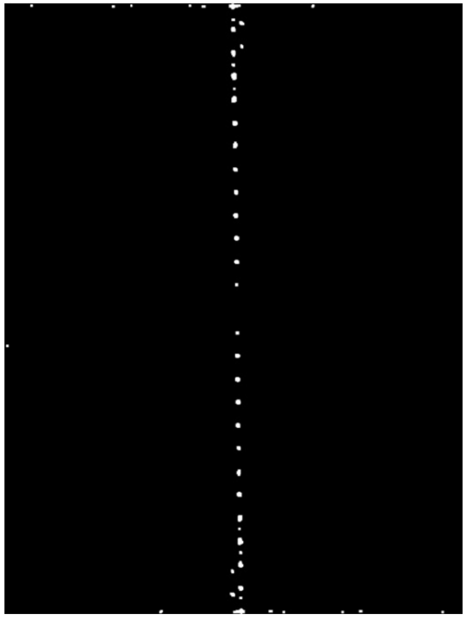
Example: Manipulating the FS

Lunar orbital image (1966)





$|F(u,v)|$



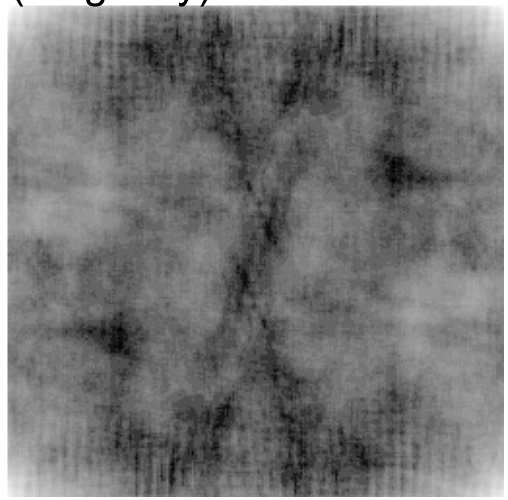
Remove peaks

$\mathrm{i}FFT(F(u,v))$

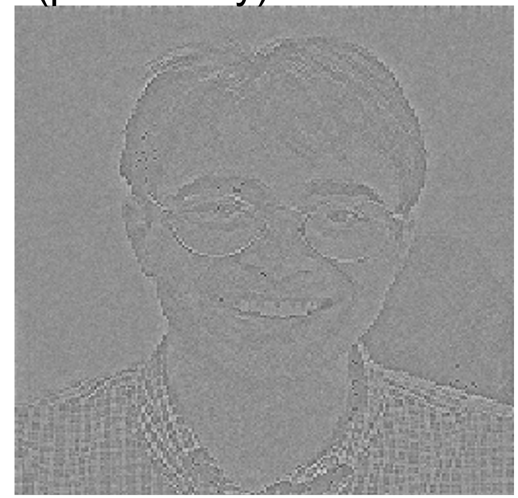
Slide by A. Zisserman

Importance of Phase

ifft(mag only)



ifft(phase only)



ifft(mag(Peter) and Phase(Andrew))

ifft(mag(Andrew) and Phase(Peter))

Recall: 2D Discrete Convolution

- The discrete version of 2D convolution is defined as:

$$g(x,y)=\sum_m\sum_n f(x-m,y-n)h(m,n)$$

$$\underbrace{\begin{array}{c|ccccc} & & y-1 & y & y+1 & \\ \hline & & & & & \\ x-1 & & 43 & 12 & 61 & \\ x & & 44 & \boxed{45} & 60 & \\ x+1 & & 43 & 50 & 61 & \\ & & & & & \end{array}}_{f} \quad * \quad \underbrace{\begin{array}{c|ccc} & -1 & 0 & 1 \\ \hline -1 & -1 & 0 & 1 \\ 0 & -2 & 0 & 2 \\ 1 & -1 & 0 & 1 \end{array}}_{h} \quad = \quad -68 \quad = \quad \begin{aligned} & f(x+1,y+1)h(-1,-1) \\ & + f(x+1,y)h(-1,0) \\ & + f(x+1,y-1)h(-1,1) \\ & + f(x,y+1)h(0,-1) \\ & + f(x,y)h(0,0) \\ & + f(x,y-1)h(0,1) \\ & + f(x-1,y+1)h(1,-1) \\ & + f(x-1,y)h(1,0) \\ & + f(x-1,y-1)h(1,1) \end{aligned}$$

Convolution in the Spatial/Frequency Domain

Convolution Theorem:

Convolution in spatial domain
is equivalent to
multiplication in frequency domain
(and vice versa)

$h = f * g$ ***implies*** $H = FG$

$h = fg$ ***implies*** $H = F * G$

Deriving the Convolution Theorem

$$h(x) = f(x) * g(x) = \sum_y f(x-y)g(y)$$

$$H(u) = \sum_x \left(\sum_y f(x-y)g(y) \right) e^{(-iux2\pi/N)}$$

$$H(u) = \sum_y g(y) \left(\sum_x f(x-y)\, e^{(-iux2\pi/N)} \right)$$

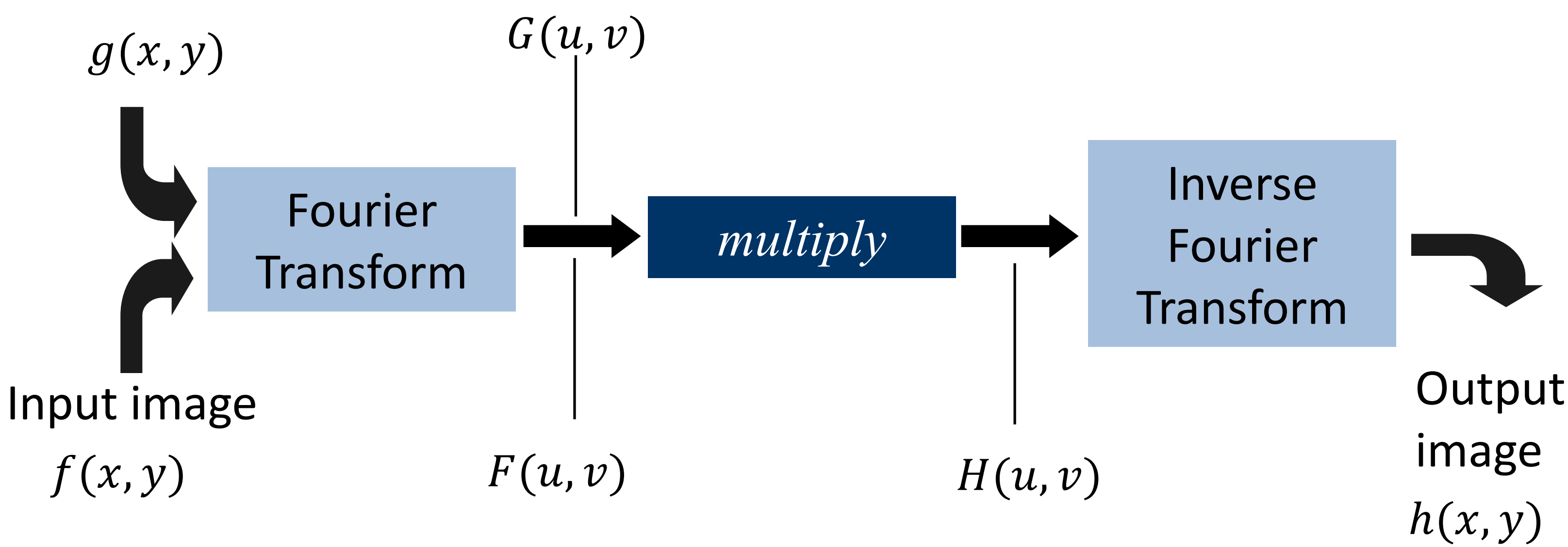
$$H(u) = \sum_y g(y) \left(F(u) e^{(-iuy2\pi/N)} \right)$$

$$H(u) = \sum_y g(y) e^{(-iuy2\pi/N)} F(u) = G(u) \cdot F(u) = F(u) \cdot G(u)$$

Fast Filtering using the Convolution Theorem

1D: $H(u) = F(u)G(u)$

2D: $H(u,v) = F(u,v)G(u,v)$



Recall: Spatial Low/High Pass Filtering

f

Low Pass

h=

1	1	1
1	1	1
1	1	1

High Pass

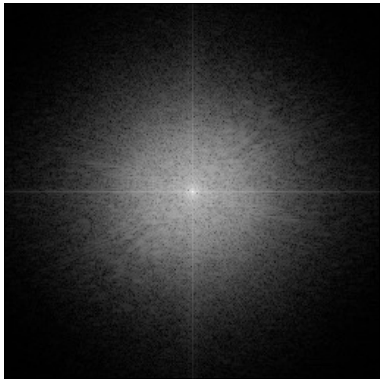
h=

-1	-1	-1
-1	8	-1
-1	-1	-1

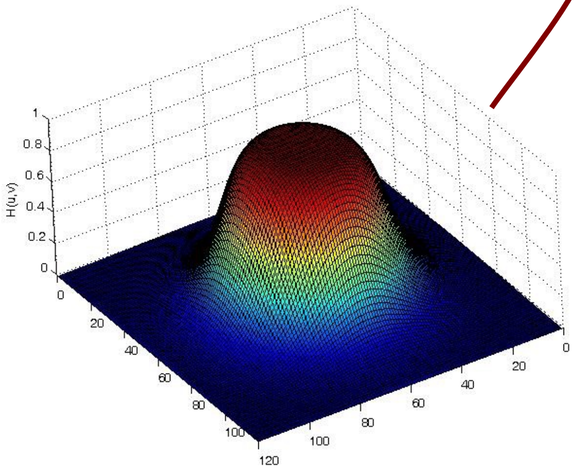
f * h

Butterworth's Low Pass Filter

Input image



After applying to freq. domain

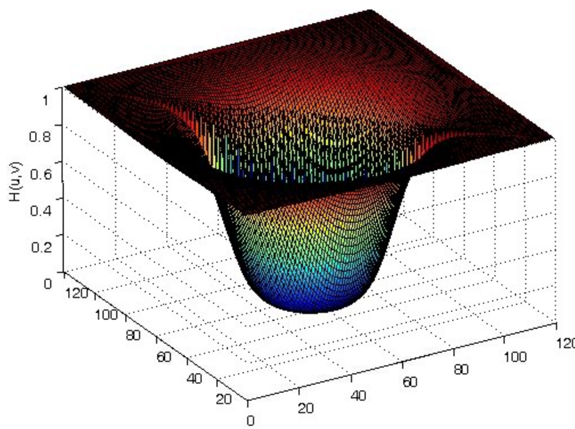


After filtering

$$H(u,v) = \frac{1}{1 + [r(u,v) / r_0]^{2n}}$$ of order n

Butterworth’s High Pass Filter

Input image

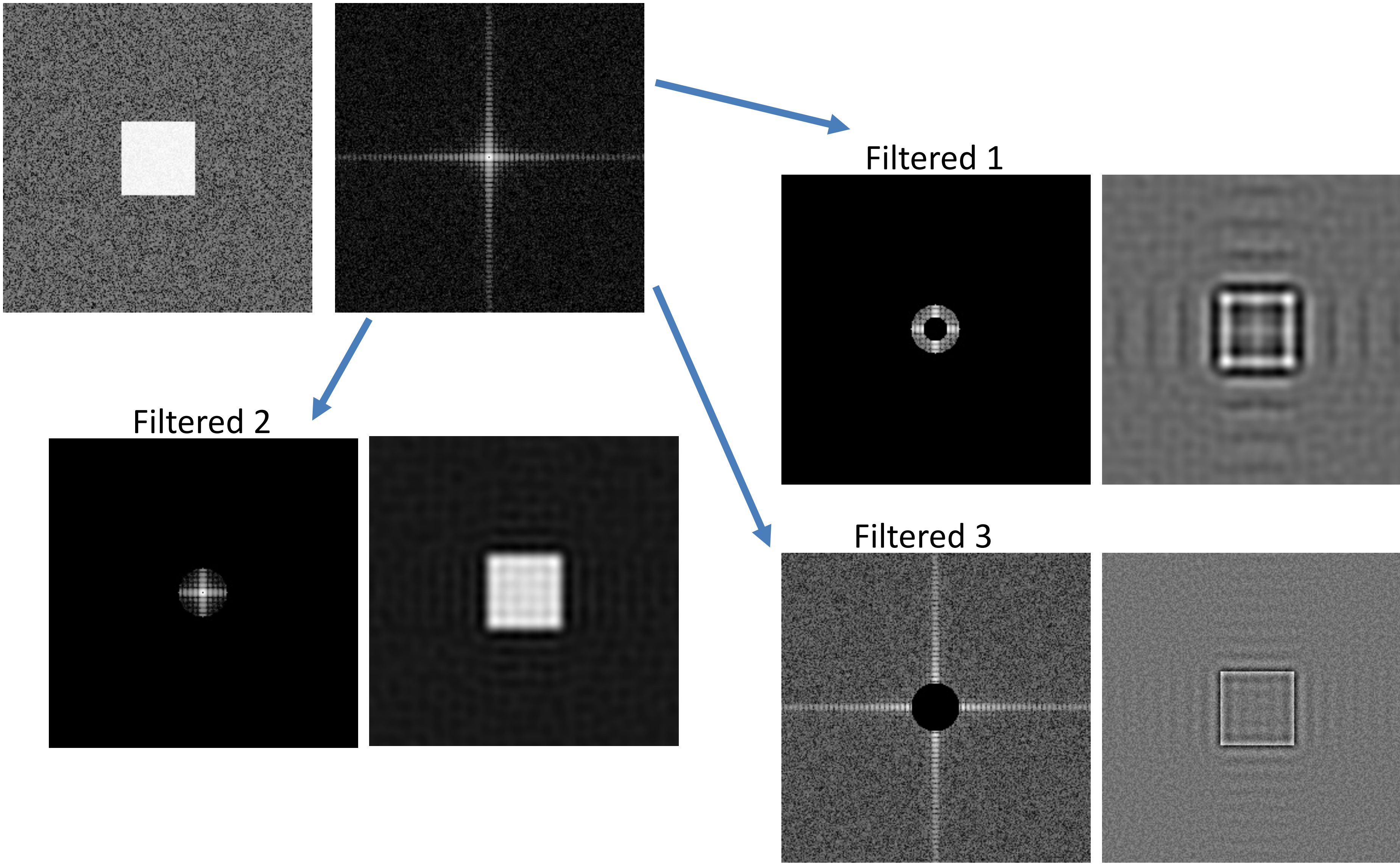


After filtering

Order of n=3

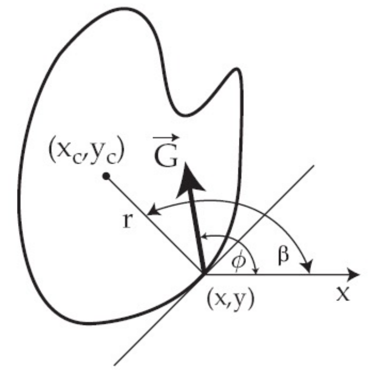
$$H(u,v) = \frac{1}{1 + [r_0 / r(u,v)]^{2n}}$$ of order n

Example filters



Next Week

Edge Detection



$\phi_1 = 0$	$(r,\beta)_{1_1}$	$(r,\beta)_{1_2}$	$\cdots$	$(r,\beta)_{1_{n_1}}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
ϕ_j	$(r,\beta)_{j_1}$	$(r,\beta)_{j_2}$	$\cdots$	$(r,\beta)_{j_{n_1}}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\phi_k = \pi$	$(r,\beta)_{k_1}$	$(r,\beta)_{k_2}$	$\cdots$	$(r,\beta)_{k_{n_1}}$

Hough Transform