HiddenBasket: The Most Undervalued NBA Players

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Introduction

The purpose of this project is to create a model that will determine which basketball players in the NBA are the most undervalued based on their salaries (as of the 2019-2020 season) and performance statistics (e.g. average points, rebounds, assists, and blocks per game).

This concept of finding undervalued athletes comes from the book and movie *Moneyball*, based on a true story about the Oakland Athletics and their general manager Billy Beane who was tasked with assembling a competitive baseball team with a limited salary budget. In 2002, the Oakland Athletics had one of the lowest team payrolls in Major League Baseball which made it difficult to pay high salaries required to attract star baseball players. Thus, Beane had to come up with a creative way to form a team given these salary constraints. He decided to use statistical analysis to find and acquire undervalued baseball players. Ultimately, this method of scouting was intended to help small-market Major League Baseball teams, like Oakland, compete with larger-market teams.

The main objective of this current project is to explore whether the concept of "Moneyball" can be applied to basketball. Statistical analyses performed in this project include multiple logistic regression and discriminant analysis. Prior to the analysis, each player will be assigned to a salary tier based on their salary. Then, logistic regression will be performed to analyze which combination of performance statistics significantly predict player salaries. After determining these variables, they will be incorporated into a discriminant analysis to predict which salary tier each player should belong to based on their performance statistics. Ideally, the most undervalued players will be those who are predicted to be in a high salary tier but are actually in a low salary tier

Packages

```
library(tidyverse)
## -- Attaching packages ------ tidyverse 1.3.0 --
## v ggplot2 3.3.0 v purrr 0.3.3
## v tibble 2.1.3 v dplyr 0.8.5
## v tidyr 1.0.2 v stringr 1.4.0
## v readr 1.3.1 v forcats 0.5.0
## -- Conflicts ----- tidyverse conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()
                   masks stats::lag()
library(readxl)
library(MASS)
##
## Attaching package: 'MASS'
## The following object is masked from 'package:dplyr':
##
##
      select
library(conflicted)
library(plyr)
## You have loaded plyr after dplyr - this is likely to cause problems.
## If you need functions from both plyr and dplyr, please load plyr first, then dplyr:
## library(plyr); library(dplyr)
## -----
options(scipen=10000)
conflict_prefer("select", "dplyr")
## [conflicted] Will prefer dplyr::select over any other package
conflict_prefer("filter", "dplyr")
## [conflicted] Will prefer dplyr::filter over any other package
conflict_prefer("mutate", "plyr")
## [conflicted] Will prefer plyr::mutate over any other package
conflict_prefer("count", "dplyr")
## [conflicted] Will prefer dplyr::count over any other package
conflict_prefer("summarize", "plyr")
## [conflicted] Will prefer plyr::summarize over any other package
```

Data Cleansing

There are three data sets used in this project collected from https://www.basketball-reference.com/:

- The Player Performance Statistics data set presents information and performance statistics about each player such as their position (Pos), age (Age), team (Tm), games played (G), rebounds per game (REB), assists per game (AST), blocks per game (BLK), and points per game (PTS).
- The Player Efficiency Rating data set presents the player efficiency rating (PER) of each player.
 - PER is an advanced metric used in basketball to measure the overall rating of a player's perminute statistical production. The formula for PER is quite complex but, in simple terms, it sums up all the good things a player does and subtracts negative things a player does relative to their team's style of play. To learn more about how PER is calculated: https://www.basketball-reference.com/about/per.html
- The Player Salaries data set presents the salaries of all NBA players for the 2019-2020 season.

Players who have played less than 10 games in the 2019-2020 season will be removed from the data set due to playing too few games. I have decided to set the cutoff at 10 games because any less than that, players would have played less than 15% of the season which is too small of a sample.

After removing these players from the data set, the three tables will be joined together to show each player's performance statistics and their corresponding salary.

Please note that since there are over 400 players in the NBA, only the first 10 entries of each table will be presented throughout the data cleansing portion of this report. However, a count of entries will be taken after each phase of the data cleansing to monitor changes to the number of entries.

Player Performance Statistics

```
gamePerformance <- read_excel("nba_data.xlsx", sheet = "GamePerformance")
gamePerformance %>% slice(1:10)
```

```
## # A tibble: 10 x 9
##
      Player
                                  Pos
                                           Age Tm
                                                          G
                                                              REB
                                                                     AST
                                                                            BLK
                                                                                  PTS
##
      <chr>
                                  <chr> <dbl>
                                               <chr> <dbl>
                                                            <dbl> <dbl>
                                                                         <dbl> <dbl>
                                            26 OKC
                                                                     2.4
                                                                                 10.9
##
    1 Steven Adams
                                  С
                                                         58
                                                              9.4
                                                                            1.1
                                                         65
    2 Bam Adebayo
                                  PF
                                            22 MIA
                                                             10.5
                                                                     5.1
                                                                            1.3
                                                                                 16.2
##
                                                              7.4
##
    3 LaMarcus Aldridge
                                  C
                                            34 SAS
                                                         53
                                                                     2.4
                                                                            1.6
                                                                                 18.9
                                                                            0.2
##
    4 Nickeil Alexander-Walker SG
                                            21 NOP
                                                         41
                                                              2
                                                                     1.8
                                                                                  5.1
##
    5 Grayson Allen
                                  SG
                                            24 MEM
                                                         30
                                                              2.2
                                                                     1.4
                                                                                  7.4
##
    6 Jarrett Allen
                                  С
                                            21 BRK
                                                              9.5
                                                                     1.3
                                                                            1.3
                                                                                 10.6
                                                         64
##
    7 Kadeem Allen
                                  SG
                                            27 NYK
                                                         10
                                                              0.9
                                                                     2.1
                                                                            0.2
                                                                                  5
                                  PF
                                                              4.8
                                                                            0.4
                                                                                  4.3
##
    8 Al-Farouq Aminu
                                            29 ORL
                                                         18
                                                                     1.2
    9 Justin Anderson
                                  SF
                                            26 BRK
                                                          3
                                                              0.7
                                                                     0
                                                                            0.3
                                                                                  1
## 10 Kyle Anderson
                                  PF
                                            26 MEM
                                                         59
                                                              4.4
                                                                     2.2
                                                                            0.5
                                                                                  5.7
```

count(gamePerformance)

There are a total of 514 players in this data set.

Now let's take a look at the summary statistics of the number of games played by the players in the data set.

```
gamePerformance %>% select(G) %>% summary()
```

```
##
##
            : 1.00
    Min.
    1st Qu.:22.00
##
   Median :47.00
##
##
    Mean
            :39.89
##
    3rd Qu.:58.00
##
    Max.
            :66.00
```

As you can see, there is quite a lot of variability in the number of games played as the most games played by a player was 66 games and the least number of games played by a player was 1.

Now, I will remove the players who have played less than 10 games.

```
gamePerformance %>% filter(G > 9) -> gamePerformance_tidy
gamePerformance_tidy %>% slice(1:10)
```

```
## # A tibble: 10 x 9
##
      Player
                                  Pos
                                           Age Tm
                                                          G
                                                              REB
                                                                     AST
                                                                            BLK
                                                                                  PTS
                                                                          <dbl>
##
      <chr>
                                  <chr> <dbl>
                                               <chr> <dbl>
                                                            <dbl>
                                                                   <dbl>
                                                                                <dbl>
    1 Steven Adams
                                  C
                                            26 OKC
                                                         58
                                                              9.4
                                                                     2.4
                                                                                  10.9
##
                                                                            1.1
                                  PF
                                            22 MIA
                                                         65
                                                              10.5
                                                                            1.3
                                                                                 16.2
##
    2 Bam Adebayo
                                                                     5.1
    3 LaMarcus Aldridge
                                  С
                                            34 SAS
                                                         53
                                                              7.4
                                                                     2.4
                                                                            1.6
                                                                                 18.9
    4 Nickeil Alexander-Walker SG
                                                              2
                                            21 NOP
                                                         41
                                                                     1.8
                                                                            0.2
                                                                                  5.1
##
    5 Grayson Allen
                                  SG
                                            24 MEM
                                                         30
                                                              2.2
                                                                     1.4
                                                                            0
                                                                                  7.4
```

```
## 6 Jarrett Allen
                                С
                                         21 BRK
                                                     64
                                                          9.5
                                                                 1.3
                                                                       1.3 10.6
## 7 Kadeem Allen
                                         27 NYK
                                                     10
                                                          0.9
                                                                 2.1
                                                                       0.2
                                                                             5
                                SG
## 8 Al-Faroug Aminu
                               PF
                                         29 ORL
                                                     18
                                                          4.8
                                                                 1.2
                                                                       0.4
                                                                             4.3
## 9 Kyle Anderson
                                                          4.4
                                                                 2.2
                               PF
                                         26 MEM
                                                     59
                                                                       0.5
                                                                             5.7
## 10 Giannis Antetokounmpo
                               PF
                                         25 MIL
                                                     57 13.7
                                                                 5.8
                                                                            29.6
count(gamePerformance_tidy)
```

After removing 68 players who played less than 10 games, 446 players remained.

Player Efficiency Rating (PER)

```
efficiency <- read_excel("nba_data.xlsx", sheet = "EfficiencyRating")
efficiency %>% slice(1:10)
```

```
## # A tibble: 10 x 2
##
     Player
                                 PER
      <chr>
##
                               <dbl>
   1 Steven Adams
                                20.8
##
##
   2 Bam Adebayo
                                20.6
## 3 LaMarcus Aldridge
                                19.8
## 4 Nickeil Alexander-Walker
                                7.6
## 5 Grayson Allen
                                11.4
## 6 Jarrett Allen
                                20.3
## 7 Kadeem Allen
                               14
## 8 Al-Farouq Aminu
                                7.6
## 9 Justin Anderson
                                -3.8
## 10 Kyle Anderson
                                13
```

Player Salaries

```
salaries <- read_excel("nba_data.xlsx", sheet = "Salaries")
salaries %>% slice(1:10)
```

```
## # A tibble: 10 x 2
##
      Player
                          Salary
##
      <chr>
                           <dbl>
##
   1 Stephen Curry
                        40231758
   2 Chris Paul
                        38506482
  3 Russell Westbrook 38178000
##
##
   4 James Harden
                        37800000
                        37800000
## 5 John Wall
## 6 LeBron James
                        37436858
## 7 Kevin Durant
                        37199000
## 8 Blake Griffin
                        34234964
## 9 Kyle Lowry
                        33296296
## 10 Paul George
                        33005556
```

PER will now be added to the data set containing all the other variables.

```
all_stats <- inner_join(gamePerformance_tidy, efficiency, by="Player")
all_stats %>% slice(1:10)
## # A tibble: 10 x 10
##
      Player
                                                         G
                                                             REB
                                                                    AST
                                                                          BLK
                                                                                 PTS
                                                                                       PER
                                Pos
                                         Age Tm
##
      <chr>
                                 <chr> <dbl> <chr> <dbl>
                                                           <dbl> <dbl>
                                                                        <dbl> <dbl>
                                                                                     <dbl>
##
    1 Steven Adams
                                С
                                          26 OKC
                                                        58
                                                             9.4
                                                                    2.4
                                                                          1.1
                                                                                10.9
                                                                                      20.8
##
    2 Bam Adebayo
                                PF
                                          22 MIA
                                                        65
                                                            10.5
                                                                    5.1
                                                                          1.3
                                                                                16.2
                                                                                      20.6
    3 LaMarcus Aldridge
##
                                С
                                          34 SAS
                                                        53
                                                             7.4
                                                                    2.4
                                                                          1.6
                                                                               18.9
                                                                                      19.8
##
    4 Nickeil Alexander-Walk~ SG
                                          21 NOP
                                                        41
                                                             2
                                                                    1.8
                                                                          0.2
                                                                                 5.1
                                                                                       7.6
                                                                                 7.4
##
    5 Grayson Allen
                                SG
                                          24 MEM
                                                        30
                                                             2.2
                                                                    1.4
                                                                          0
                                                                                      11.4
##
    6 Jarrett Allen
                                С
                                          21 BRK
                                                        64
                                                             9.5
                                                                    1.3
                                                                          1.3
                                                                                10.6
                                                                                      20.3
##
    7 Kadeem Allen
                                SG
                                          27 NYK
                                                        10
                                                             0.9
                                                                    2.1
                                                                          0.2
                                                                                 5
                                                                                      14
    8 Al-Farouq Aminu
                                                                                 4.3
                                                                                       7.6
##
                                PF
                                          29 ORL
                                                        18
                                                             4.8
                                                                    1.2
                                                                          0.4
    9 Kyle Anderson
                                PF
                                          26 MEM
                                                        59
                                                             4.4
                                                                    2.2
                                                                          0.5
                                                                                 5.7
                                                                                      13
##
## 10 Giannis Antetokounmpo
                                PF
                                          25 MIL
                                                        57
                                                            13.7
                                                                    5.8
                                                                          1
                                                                                29.6
                                                                                      31.6
```

Now that PER has been added to the **Player Performance Statistics** data set, we must attach the player salaries to each corresponding player.

```
nba <- inner_join(all_stats, salaries, by="Player")</pre>
nba %>% slice(1:10)
## # A tibble: 10 x 11
                                 Age Tm
                                                G
                                                    REB
                                                           AST
                                                                        PTS
##
      Player
                                                                  BLK
                                                                               PER
                                                                                    Salary
##
                                                  <dbl> <dbl>
                                                                      <dbl> <dbl>
                                                                                      <dbl>
      <chr>>
                        <chr> <dbl> <chr> <dbl>
                                                               <dbl>
                                                                              20.8
##
    1 Steven Adams
                        C
                                  26 OKC
                                               58
                                                    9.4
                                                           2.4
                                                                  1.1
                                                                       10.9
                                                                                    2.58e7
##
    2 Bam Adebayo
                        PF
                                  22 MIA
                                               65
                                                   10.5
                                                           5.1
                                                                  1.3
                                                                       16.2
                                                                              20.6
                                                                                    3.45e6
##
    3 LaMarcus Aldri~
                                  34 SAS
                                               53
                                                    7.4
                                                                  1.6
                                                                       18.9
                                                                              19.8
                                                           2.4
                                                                                    2.60e7
    4 Nickeil Alexan~
                       SG
                                  21 NOP
                                               41
                                                    2
                                                           1.8
                                                                  0.2
                                                                        5.1
                                                                               7.6
                                                                                    2.96e6
##
    5 Grayson Allen
                        SG
                                  24 MEM
                                               30
                                                    2.2
                                                                  0
                                                                        7.4
                                                                                    2.43e6
                                                           1.4
                                                                              11.4
##
    6 Jarrett Allen
                        С
                                  21 BRK
                                               64
                                                    9.5
                                                           1.3
                                                                  1.3
                                                                       10.6
                                                                              20.3
                                                                                    2.38e6
                                                                  0.4
##
    7 Al-Farouq Aminu PF
                                  29 ORL
                                               18
                                                    4.8
                                                           1.2
                                                                        4.3
                                                                               7.6
                                                                                    9.26e6
    8 Kyle Anderson
                                  26 MEM
                                               59
                                                    4.4
                                                           2.2
                                                                  0.5
                                                                        5.7
                                                                              13
                                                                                     9.07e6
    9 Giannis Anteto~ PF
                                                                       29.6
                                                                              31.6
                                                                                    2.58e7
                                  25 MIL
                                               57
                                                   13.7
                                                           5.8
                                                                  1
## 10 Thanasis Antet~ SF
                                  27 MIL
                                               18
                                                    1.1
                                                           0.5
                                                                  0.1
                                                                        2.5
                                                                              14.2 1.45e6
count (nba)
## # A tibble: 1 x 1
##
         n
```

After joining the three tables by player name, 410 players remain. Only players who appeared in all three data sets were included into this final compiled data set.

##

1

<int>

410

Defining Salary Tiers

In order to perform a logistic regression and discriminant analysis later on, an ordinal response needs to be defined. Thus, salary tiers will be created to group the players.

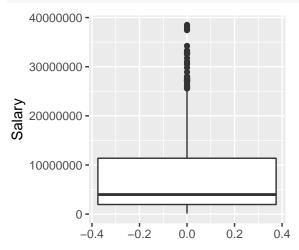
As you can see below from the summary statistics and boxplot of the player salaries:

- The 25th percentile of salaries lie approximately below \$2 million
- The middle of the interquartile range (IQR) is at approximately \$5 million
- The 75th percentile of salaries appears to be slightly above \$10 million
- The upper outliers appear to be salaries over \$25 million

nba %>% select(Salary) %>% summary()

```
Salary
##
##
           : 101504
    Min.
    1st Qu.: 1951650
##
    Median: 3976460
##
##
           : 8117626
    Mean
##
    3rd Qu.:11369948
    Max.
           :38506482
```

ggplot(nba, aes(y = Salary)) + geom_boxplot()



Based on the inferences made from the summary statistics and boxplot of the player salaries, I will split up the players into the following salary tiers:

- Tier 1: less than \$2 million
- **Tier 2**: \$2 to 5 million
- **Tier 3**: \$5 to 10 million
- Tier 4: \$10 to 25 million
- Tier 5: more than \$25 million

```
nba %>% mutate(Tier=ifelse(Salary < 2000000, "tier1",</pre>
                    ifelse(Salary < 5000000, "tier2",</pre>
                    ifelse(Salary < 10000000, "tier3",</pre>
                    ifelse(Salary < 25000000, "tier4", "tier5"))))</pre>
               ) -> nba_tiers
nba_tiers <- nba_tiers %>% mutate(Tier = factor(Tier))
nba_tiers %>% slice(1:10)
## # A tibble: 10 x 12
##
                                                             PTS
     Player
                Pos
                         Age Tm
                                       G
                                           REB
                                                 AST
                                                       BLK
                                                                    PER Salary Tier
##
      <chr>
                 <chr> <dbl> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <fct>
##
  1 Steven Ad~ C
                          26 OKC
                                      58
                                           9.4
                                                 2.4
                                                        1.1 10.9 20.8 2.58e7 tier5
   2 Bam Adeba~ PF
                          22 MIA
                                      65 10.5
                                                 5.1
                                                       1.3 16.2 20.6 3.45e6 tier2
## 3 LaMarcus ~ C
                          34 SAS
                                           7.4
                                                 2.4
                                                        1.6 18.9 19.8 2.60e7 tier5
                                      53
## 4 Nickeil A~ SG
                          21 NOP
                                      41
                                           2
                                                 1.8
                                                       0.2
                                                             5.1
                                                                   7.6 2.96e6 tier2
## 5 Grayson A~ SG
                          24 MEM
                                      30
                                           2.2
                                                 1.4
                                                       0
                                                             7.4 11.4 2.43e6 tier2
## 6 Jarrett A~ C
                          21 BRK
                                      64
                                           9.5
                                                 1.3
                                                       1.3 10.6 20.3 2.38e6 tier2
##
   7 Al-Farouq~ PF
                          29 ORL
                                      18
                                           4.8
                                                 1.2
                                                       0.4
                                                             4.3
                                                                   7.6 9.26e6 tier3
## 8 Kyle Ande~ PF
                          26 MEM
                                      59
                                           4.4
                                                 2.2
                                                       0.5
                                                             5.7 13
                                                                      9.07e6 tier3
## 9 Giannis A~ PF
                          25 MIL
                                      57 13.7
                                                 5.8
                                                       1
                                                            29.6 31.6 2.58e7 tier5
                                                       0.1
## 10 Thanasis ~ SF
                          27 MIL
                                                 0.5
                                                             2.5 14.2 1.45e6 tier1
                                      18
                                           1.1
```

Now each row of this data set consists of each player and their performance statistics, salary, and salary tier.

Data Analysis

Correlation Between Variables

Before creating a regression model, let's first take a look at the correlation between salary (response variable) and the possible predictor (independent) variables.

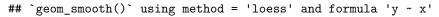
```
nba_tiers %>% select(Salary, REB, AST, BLK, PTS, PER) %>% cor() -> COR
COR[1,]
##
       Salary
                       REB
                                   AST
                                               BLK
                                                           PTS
                                                                       PER
## 1.0000000 0.4844740 0.5328877 0.2606822 0.6312167 0.4667826
nba_tiers %>%
  pivot_longer((c(REB, AST, BLK, PTS, PER)), names_to = "xname", values_to = "x") %>%
  ggplot(aes(x = x, y = Salary, colour=Tier)) + geom_point() +
  facet_wrap(~xname, scales = "free") -> g1
g1
                                                                                      PER
  40000000
                                                                    40000000
                                   40000000
  30000000
                                   30000000
                                                                    30000000
  20000000
                                   20000000
                                                                    20000000
  10000000
                                   10000000
                                                                    10000000
                                                                                                      Tier
                              10.0
                                                                                                          tier2
                                                     REB
  40000000
                                   40000000 -
                                                                                                         tier4
                                                                                                         tier5
  30000000 -
                                   30000000 -
  20000000
                                   20000000 -
  10000000
                                   100000000
       0 -
```

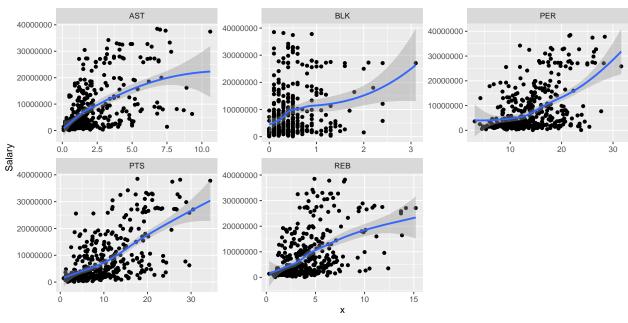
As you can see from the correlations and scatter plots, the direction of all correlations are positive. This means a higher salary is associated with scoring many points and having a high player efficiency rating. In addition, a higher salary is associated with getting a lot of assists, blocks, and rebounds.

However, it is important to note that these relationships vary in terms of correlation strength:

- Salary is strongly correlated with Points Per Game (PTS)
- Salary is moderately correlated with Rebounds Per Game (REB), Assists Per Game (AST), and Player Efficiency Rating (PER)
- Salary is weakly associated with Blocks Per Game (BLK)

```
nba_tiers %>%
  pivot_longer(c(PTS, REB, AST, BLK, PER), names_to = "xname", values_to = "x") %>%
  ggplot(aes(x = x, y = Salary)) + geom_point() + geom_smooth() +
  facet_wrap(~xname, scales = "free") -> g2
g2
```





Logistic Regression

Multiple logistic regression uses predictor variables to model the probability of a certain outcome occurring or a subject belonging to a specific group. The multiple logistic regression model below predicts a player's salary tier based on their average rebounds, assists, blocks, and points per game, along with player efficiency rating (PER). Since the salary tier is an ordered categorical variable (tier 1 to tier 5), an ordered logistic model will be created using *polr*.

```
nba.1 <- polr(Tier ~ REB + AST + BLK + PTS + PER , data = nba_tiers)</pre>
drop1(nba.1, test="Chisq")
## Single term deletions
## Model:
## Tier ~ REB + AST + BLK + PTS + PER
                AIC
##
          Df
                       LRT
                                 Pr(>Chi)
             1056.5
## <none>
           1 1066.6 12.176
## REB
                                  0.000484 ***
## AST
           1 1070.1 15.683 0.000074875131 ***
## BLK
           1 1060.0 5.574
                                 0.018229 *
           1 1088.3 33.881 0.000000005859 ***
## PTS
## PER
           1 1062.7
                     8.274
                                  0.004022 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

The results of the regression model show that REB, AST, BLK, PTS, and PER are all significant as they have p-values of less than 0.05. Thus, all the explanatory variables have some impact on the salary tier, and none of them should be removed from the model.

```
nba.1$coefficients

## REB AST BLK PTS PER

## 0.22406618 0.29930215 0.73864764 0.15879231 -0.08871348
```

With the exception of PER, all of the coefficients are positive which means the model predicts an increase in REB, AST, BLK, or PTS will result in an increase in salary. The model also predicts an increase in PER will result in a decrease in salary.

Now these variables will be incorporated into a discriminant analysis.

Discriminant Analysis

A discriminant analysis predicts group membership based on numerous factors (measured variables), assuming the groups are known. This type of analysis can be performed to predict a player's salary tier based on their performance statistics.

```
salaries.1 <- lda(Tier ~ REB + AST + BLK + PTS + PER, data = nba_tiers)</pre>
salaries.1
## Call:
## lda(Tier ~ REB + AST + BLK + PTS + PER, data = nba_tiers)
##
  Prior probabilities of groups:
##
      tier1
                tier2
                          tier3
                                    tier4
                                              tier5
  0.2634146 0.3048780 0.1585366 0.1731707 0.1000000
##
##
## Group means:
##
             REB
                      AST
                                BI.K
                                          PTS
                                                   PER
## tier1 2.692593 1.275000 0.3240741
                                     5.810185 12.17500
## tier2 3.649600 1.592000 0.4176000
                                     8.046400 12.65520
## tier3 4.550769 2.867692 0.4769231 11.926154 15.10923
  tier4 4.642254 2.594366 0.5197183 11.587324 13.99577
  tier5 7.260976 4.587805 0.8000000 20.297561 20.74634
##
## Coefficients of linear discriminants:
##
              LD1
                          LD2
                                      LD3
                                                  LD4
## REB
       0.15550187 -0.26570594
                               0.09212061
                                          0.49804037
       0.56530753 -0.41081322 -0.06268459 -3.26444824
       0.12120380 -0.10393985
                               0.09459629 -0.04657632
## PER -0.03129032 0.31306455
                               0.07317913 0.04133825
##
## Proportion of trace:
##
     LD1
            LD2
                   LD3
                          I.D4
## 0.9482 0.0420 0.0087 0.0011
```

As you can see from looking at the group means, the higher the salary tier, the greater the REB, AST, BLK, PTS, and PER. However, there are a few exceptions to this general trend, especially when looking at tier 4.

The number of linear discriminants is either the number of variables or number of groups - 1, depending on which value is smaller. Since there are 5 variables (REB, AST, BLK, PTS, and PER) and 5 groups (tier 1 to 5), there are 4 linear discriminants (LD1 to LD4). Each linear discriminant is a linear combination of features (REB, AST, BLK, PTS, and PER) that characterizes a certain group of player.

Now focusing on the proportion of trace, LD1 makes up most of the proportion of trace (0.9482). Thus, we should primarily be focused on LD1.

Moving onto the coefficients of the linear discriminants, LD1 is positive when REB, AST, BLK, and PTS are high since their coefficients under LD1 are positive.

Finally, since the LD1 coefficient of PER (-0.03129032) is close to 0, PER will have close to no impact on the model.

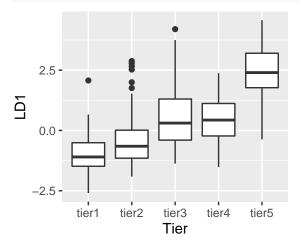
Now, this model will be used to predict which salary tier each player belongs to.

```
salaries.pred <- predict(salaries.1)</pre>
pp <- round(salaries.pred$posterior, 3)</pre>
predictions <- cbind(nba_tiers, pp)</pre>
predictions %>% slice(1:10)
##
                         Player Pos Age
                                         Tm
                                             G
                                                 REB AST BLK
                                                              PTS
                                                                    PER
                                                                          Salary
## 1
                  Steven Adams
                                     26 OKC 58
                                                 9.4 2.4 1.1 10.9 20.8 25842697
                                  C
## 2
                    Bam Adebayo
                                 PF
                                     22 MIA 65 10.5 5.1 1.3 16.2 20.6
                                                                         3454080
## 3
             LaMarcus Aldridge
                                                 7.4 2.4 1.6 18.9 19.8 26000000
                                  C
                                     34 SAS 53
## 4
      Nickeil Alexander-Walker
                                 SG
                                     21 NOP 41
                                                 2.0 1.8 0.2
                                                              5.1
                                                                   7.6
                                                                         2964840
## 5
                 Grayson Allen
                                 SG
                                     24 MEM
                                            30
                                                 2.2 1.4 0.0
                                                              7.4 11.4
                                                                         2429400
## 6
                  Jarrett Allen
                                  C
                                     21 BRK 64
                                                 9.5 1.3 1.3 10.6 20.3
                                                                         2376840
                                 PF
## 7
               Al-Farouq Aminu
                                     29 ORL 18
                                                 4.8 1.2 0.4
                                                              4.3
                                                                         9258000
## 8
                 Kyle Anderson
                                 PF
                                     26 MEM 59
                                                 4.4 2.2 0.5
                                                              5.7 13.0
                                                                         9073050
## 9
         Giannis Antetokounmpo
                                 PF
                                     25 MIL 57 13.7 5.8 1.0 29.6 31.6 25842697
## 10
                                                1.1 0.5 0.1 2.5 14.2
        Thanasis Antetokounmpo
                                 SF
                                     27 MIL 18
                                                                         1445697
##
       Tier tier1 tier2 tier3 tier4 tier5
## 1
      tier5 0.073 0.232 0.293 0.261 0.141
      tier2 0.002 0.017 0.109 0.118 0.754
##
  3
      tier5 0.011 0.070 0.148 0.237 0.534
      tier2 0.341 0.425 0.095 0.139 0.000
      tier2 0.408 0.410 0.091 0.092 0.001
      tier2 0.085 0.293 0.247 0.275 0.099
      tier3 0.221 0.475 0.105 0.198 0.001
      tier3 0.297 0.386 0.155 0.158 0.003
      tier5 0.000 0.000 0.003 0.002 0.996
## 10 tier1 0.716 0.240 0.027 0.017 0.000
```

I have combined player statistics, actual player salary tier, and tier predictions into one table.

Now let's take a look at side-by-side boxplots to see the relationship between LD1 and the salary tiers.

```
tierLD <- cbind(nba_tiers, salaries.pred$x, pp)
ggplot(tierLD, aes(x = Tier, y = LD1)) + geom_boxplot()</pre>
```



Since LD1 is positive when REB, AST, BLK, and PTS are high and these stats are associated with a greater salary tier, higher tiers have a greater LD1 score.

Statistical Inferences

Below is a frequency table comparing each player's actual salary tier (obs) to their predicted salary tier (pred).

```
table(obs = nba_tiers$Tier, pred = salaries.pred$class)
##
          pred
## obs
            tier1 tier2 tier3 tier4 tier5
##
               60
                     43
     tier1
                             1
                                    7
                                          6
##
     tier2
               39
                     69
                             4
                             9
                                          9
##
     tier3
               11
                     32
                                    4
##
     tier4
                2
                     39
                            10
                                   15
                                          5
                0
                       3
                             3
                                    4
##
     tier5
                                          31
```

The most undervalued player would be located in the cell with an observed (actual) tier of tier 1 and a predicted tier of tier 5. This player makes less than \$2 million but are predicted to play at the level of someone who makes more than \$25 million.

```
data.frame(nba$Player, obs = nba_tiers$Tier, pred = salaries.pred$class) %>%
    filter(obs == "tier1", pred == "tier5") -> pool1
left_join(pool1, nba, by = c("nba.Player" = "Player")) -> Pool1

## Warning: Column `nba.Player`/`Player` joining factor and character vector,
## coercing into character vector
Pool1

## nba.Player obs pred Pos Age Tm G REB AST BLK PTS PER Salary
## 1 Devonte' Graham tier1 tier5 PG 24 CHO 63 3.4 7.5 0.2 18.2 15.8 1416852
```

Devonte' Graham is the player in tier 1 who was predicted to be in tier 5. Let's compare his performance statistics to the league average.

League Average

```
## nba.Player obs pred Pos Age Tm G REB AST BLK PTS PER Salary
## 1 Devonte' Graham tier1 tier5 PG 24 CHO 63 3.4 7.5 0.2 18.2 15.8 1416852
## 2 NBA Average <NA> <NA> <NA> NA <NA> NA 4.1 2.2 0.5 9.9 14.0 8117626
```

Based on the model, it makes sense that Devonte' Graham was selected since compared to the league average, he scores a lot of points (PTS) and gets a lot of assists (AST). He is also close to the league average in rebounds (REB), blocks (BLK), and player efficiency rating (PER).

Players who are in salary tier 2 but were predicted to be in salary tier 5 are also heavily undervalued and may be considered the next-most undervalued group of players.

```
data.frame(nba$Player, obs = nba_tiers$Tier, pred = salaries.pred$class) %>%
  filter(obs == "tier2", pred == "tier5") -> pool2
left_join(pool2, nba, by = c("nba.Player" = "Player")) -> Pool2
## Warning: Column `nba.Player`/`Player` joining factor and character vector,
## coercing into character vector
comparison2 <- rbind.fill(Pool2, league avg)</pre>
comparison2
##
                  nba.Player
                               obs pred
                                          Pos Age
                                                         G REB AST BLK PTS
## 1
                 Bam Adebayo tier2 tier5
                                           PF
                                               22
                                                    MIA 65 10.5 5.1 1.3 16.2 20.6
## 2
                John Collins tier2 tier5
                                                    ATL 41 10.1 1.5 1.6 21.6 23.5
                                           PF
                                                22
## 3 Shai Gilgeous-Alexander tier2 tier5
                                           SG
                                               21
                                                    OKC 63
                                                           6.1 3.3 0.7 19.3 17.8
            Donovan Mitchell tier2 tier5
                                           SG
                                               23
                                                            4.4 4.2 0.2 24.2 19.1
                                                    UTA 63
                                             С
                                               23
                                                    IND 62 12.4 5.0 0.5 18.5 20.7
## 5
            Domantas Sabonis tier2 tier5
## 6
               Pascal Siakam tier2 tier5
                                           PF
                                               25
                                                    TOR 53
                                                           7.5 3.6 0.9 23.6 18.7
## 7
                 NBA Average <NA>
                                               NA <NA> NA 4.1 2.2 0.5 9.9 14.0
                                    <NA> <NA>
##
      Salary
## 1 3454080
## 2 2686560
## 3 3952920
## 4 3635760
## 5 3529555
## 6 2351839
## 7 8117626
```

This is the pool of players in tier 2 but who play as if they are in tier 5. Based on the model, it makes sense that these players were selected since compared to the league average, they score a lot of points (PTS), get a lot of rebounds (REB), assists (AST), and blocks (BLK). They also have high player efficiency ratings (PER).

Finally, to further show how undervalued each of these players are, let's take a look at the (posterior) probability of each of these players being in each tier according to the model.

```
tier_predictions <- predictions %>% select(c(Player, tier1, tier2, tier3, tier4, tier5))
left_join(pool2, tier_predictions, by = c("nba.Player" = "Player"))
```

```
## coercing into character vector

## nba.Player obs pred tier1 tier2 tier3 tier4 tier5

## 1 Bam Adebayo tier2 tier5 0.002 0.017 0.109 0.118 0.754

## 2 John Collins tier2 tier5 0.003 0.034 0.076 0.114 0.773

## 3 Shai Gilgeous-Alexander tier2 tier5 0.023 0.122 0.243 0.293 0.319

## 4 Donovan Mitchell tier2 tier5 0.016 0.076 0.226 0.201 0.480

## 5 Domantas Sabonis tier2 tier5 0.001 0.018 0.109 0.103 0.768

## 6 Pascal Siakam tier2 tier5 0.003 0.028 0.099 0.151 0.719
```

Warning: Column `nba.Player`/`Player` joining factor and character vector,

According to the model predictions, there was over a 70% chance that Bam Adebayo, John Collins, Domantas Sabonis, and Pascal Siakam would belong to tier 5. This further supports the notion that these players are severely undervalued based on the parameters of the model.

Conclusion

Correlation Between Variables

Salary has a:

- strong positive correlation with Points Per Game
- moderate positive correlation with Rebounds, Assists, and Steals Per Game
- moderate positive correlation with Player Efficiency Rating (PER)
- weak positive correlation with Blocks Per Game.

It intuitively makes sense that points per game is strongly correlated with salary since the primary objective of basketball is to score, so players who score a lot of points should be paid the most. Also, it intuitively makes sense that defensive statistics such as blocks are not as strongly correlated to salary as defense is often overlooked by many teams.

Logistic Regression

The results of the regression model show that REB, AST, BLK, PTS, and PER all have some impact on salary tier. In addition, with the exception of PER, all of the coefficients are positive which means the model predicts an increase in REB, AST, BLK, or PTS will result in an increase in salary.

One may assume that elite players who get paid a lot should be efficient and thus the coefficient for player efficiency rating (PER) should be positive rather than negative. However, since elite players possess the ball more, they have more opportunities to make "mistakes" resulting in inefficiencies. Thus, it is plausible that the coefficient of PER is negative.

Discriminant Analysis

Based on my model, there are numerous players who are undervalued. Specifically, Devonte' Graham, Bam Adebayo, John Collins, Shai Gilgeous-Alexander, Donovan Mitchell, Domantas Sabonis, and Pascal Siakam. The interesting thing is 4 of these 7 players have been named to the 2019-2020 All-Star Team which supports the notion that these players are playing at an elite level but are being underpaid/undervalued.

The model could have also been used to predict the most overvalued players by looking at players in a high salary tier who are predicted to be in a low salary tier. However, I wanted to focus on determining the most undervalued players because I believe it is more important for NBA teams to consider. In conclusion, this model can be used by teams to generate a short list of players to target in free agency.