Bayesian Network Recap

Basic Distributions

Binary R.V. $\Rightarrow$ support is $\{0, 1\}$

Bernoulli(θ) Coin Flip

If $X \sim \text{Bernoulli}(\theta)$

$$P(X=1) = \theta$$
$$P(X=0) = 1-\theta$$

X	$P(X)$
0	$1-\theta$
1	θ

Categorical(ϕ) Cat(ϕ)

vector

If $X \sim \text{Cat}(\phi)$ then

$$\text{support}(X) = \{1, 2, \ldots, \dim(\phi)\}$$
$$P(X=1) = \phi_1$$
$$P(X=2) = \phi_2$$
$$\vdots$$
$$P(X=n) = \phi_n$$

X	$P(X)$
1	ϕ_1
2	ϕ_2
$\vdots$	$\vdots$
n	ϕ_n

Causal Bayesian Networks

$A \to B$

Each Node: R.V.

Each Edge: Causal relationship

Informally, B is a function of A with some additional uncertainty

Example:

$A \to B$

$$B \sim \begin{cases} \text{Bernoulli}(0.2) & \text{if } A=0 \\ \text{Bernoulli}(0.7) & \text{if } A=1 \end{cases}$$

$$P(B=1 \mid A=1) = 0.7$$
$$P(B=1 \mid A=0) = 0.2$$

If we have a BN with nodes $X_1 \ldots X_n$

$$P(X_1, X_2, \ldots X_n) = P(X_1 \mid pa(X_1))\, P(X_2 \mid pa(X_2)) \ldots P(X_n \mid pa(X_n))$$
$$= \prod_{i \in 1..n} P(X_i \mid pa(X_i))$$

X_1	X_2	X_3	$P(X_1, X_2, X_3)$
0	0	0	
0	0	1	

A	B	C	P(A,B,C)
0	0	0	θ_1
0	0	1	θ_2
⋮			

$\Big\}\ 2^3$

$2^3 - 1$ indep parameters

$\boxed{7 \text{ parameters}}$

$P(A \mid pa(A)) = P(A)$ $\boxed{1 \text{ param}}$
Bernoulli( )

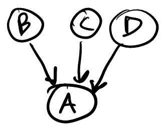
$P(B \mid pa(B)) = P(B \mid A)$
Bernoulli(·) for each possible value of A $\boxed{2 \text{ param}}$

$P(C \mid pa(C)) = P(C \mid B)$ $\boxed{2 \text{ param}}$

Since $P(A,B,C)$ is defined by *
Just need $P(X \mid pa(X))$

$\boxed{5 \text{ total parameters}}$

A, B, C, D binary RV



$$P(A \mid pa(A)) = P(A \mid B, C, D)$$

$2^3 = 8$ possibilities

8 parameters

Independence

$A \perp B$

$$P(A \mid B) = P(A)$$

$$\frac{P(A,B)}{P(B)} = P(A)$$

$$P(A,B) = P(A)\,P(B)$$

$$P(A,B) = P(A)\,P(B)$$

Conditional Independence

$A \perp B \mid C$

$$P(A \mid B, C) = P(A \mid C)$$

$$\frac{P(A,B \mid C)}{P(B \mid C)} = P(A \mid C)$$

$$P(A,B \mid C) = P(A \mid C)\,P(B \mid C)$$

$$P(A,B \mid C) = P(A \mid C)\,P(B \mid C)$$

$A \perp C \mid B$?

Yes

need to prove that

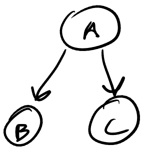
$$P(A,C \mid B) = P(A \mid B)\,P(C \mid B)$$

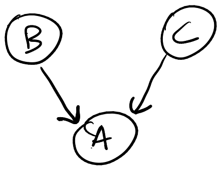
$$P(A,B,C) = P(A \mid pa(A))\,P(B \mid pa(B))\,P(C \mid pa(C))$$
$$= P(A)\,P(B \mid A)\,P(C \mid B)$$

$$\frac{P(A,B,C)}{P(B)} = \frac{P(A)\,P(B \mid A)\,P(C \mid B)}{P(B)}$$

$$P(A,C \mid B) = P(A \mid B)\,P(C \mid B)$$

Similarly can prove $B \perp C \mid A$





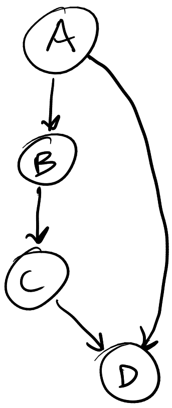
$B \perp C \mid A$? cannot prove from structure

but you could pick particular parameters where $B \perp C \mid A$

example: $A \sim$ Bernoulli(0.5) for any values of B, C

$B \sim$ " "

$C \sim$ " "



$A \perp C \mid B$? True

paths

$A \to B \to C$ rule 1: d-sep

$A \to D \leftarrow C$ rule 3: d-sep

all paths are d-sep

$A \perp C \mid B, D$? Inconclusive

paths

$A \to B \to C$ rule 1: d-sep

$A \to D \leftarrow C$ rule 3: not d-sep

not d-sep