Value-Based Model Free RL

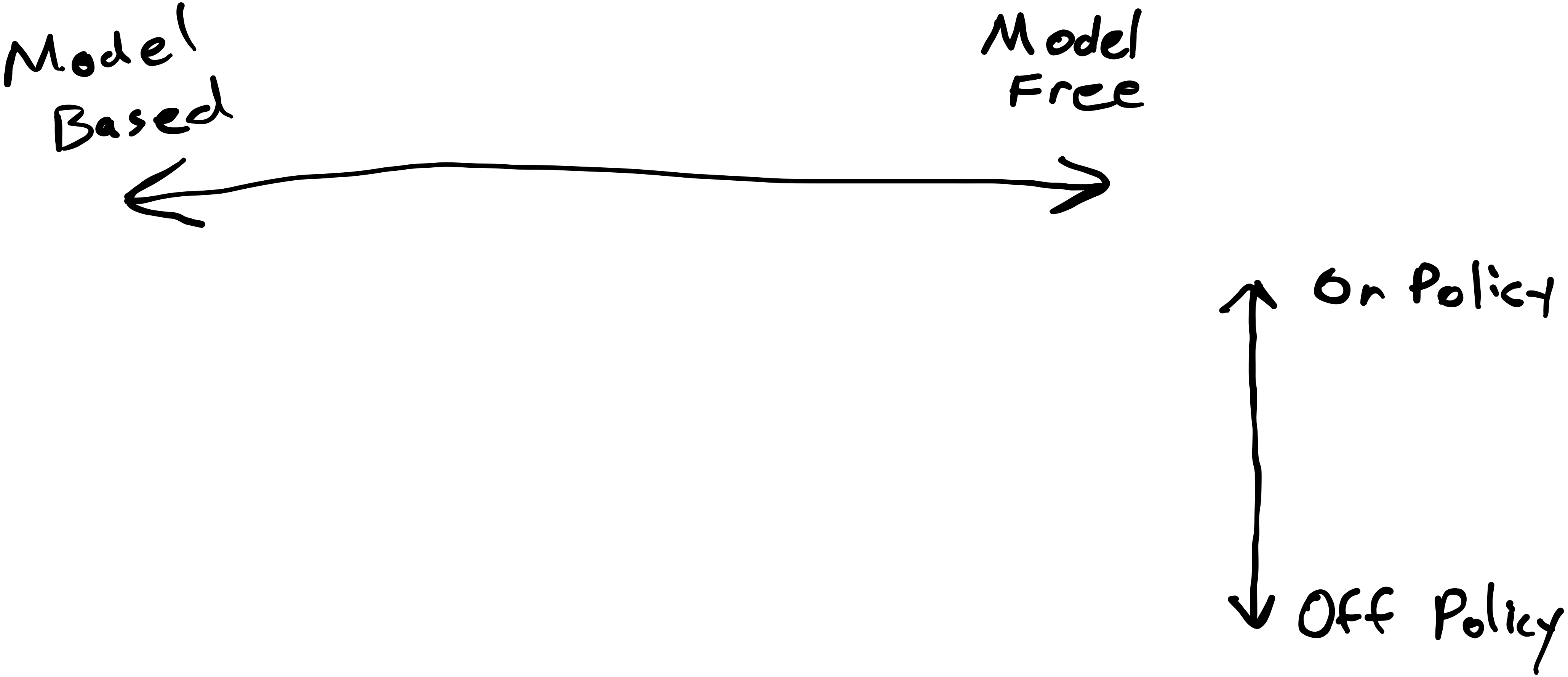
Last Time

- Policy Optimization ← Cross Entropy
- Policy Gradient
- Tricks for Policy Gradient
  - log-derivative
  - Causality
  - baseline subtraction

Map

Map

Map



Map

Model Based

Model Free

On Policy

Off Policy

MLMBTRL
(learn T,R)

Map

Model Based

Model Free

learn Q

learn π

On Policy

Off Policy

MLMBTRL
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Policy Gradient

On Policy

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MLMBTRL (learn T,R)

Map

Model Based

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SARSA

learn π

Policy Gradient

On Policy

Off Policy

MLMBTRL
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Map

Model Based

Model Free

learn Q

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Policy Gradient

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MLMBTRL (learn T,R)

Q-learning

Off Policy

Today

Today

- Basic On- and Off-Policy **value based** model free RL algorithms

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- Basic On- and Off-Policy **value based** model free RL algorithms
- Tricks for tabular value based RL algorithms

Today

- Basic On- and Off-Policy **value based** model free RL algorithms
- Tricks for tabular value based RL algorithms
- Understanding of On- vs Off-Policy

Why learn Q?

T, R $\qquad$ π $\qquad$ U, Q

$$\pi(s) = \underset{a}{\text{argmax}} \left(\underline{\underline{R}}(s,a) + \gamma E\, \underline{U}(s') \right)$$

$$\pi(s) = \underset{a}{\text{argmax}} \left(Q(s,a) \right)$$

Incremental Mean Estimation

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$$\hat{x}_m = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$$

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$$\hat{x}_m = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$$

$$= \frac{1}{m}\left(x^{(m)} + \sum_{i=1}^{m-1} x^{(i)}\right)$$

Incremental Mean Estimation

$$\begin{aligned}\hat{x}_m &= \frac{1}{m}\sum_{i=1}^{m} x^{(i)} \\ &= \frac{1}{m}\left(x^{(m)} + \sum_{i=1}^{m-1} x^{(i)}\right) \\ &= \frac{1}{m}\left(x^{(m)} + (m-1)\,\hat{x}_{m-1}\right)\end{aligned}$$

Incremental Mean Estimation

$$\begin{aligned}
\hat{x}_m &= \frac{1}{m}\sum_{i=1}^{m} x^{(i)} \\
&= \frac{1}{m}\left(x^{(m)} + \sum_{i=1}^{m-1} x^{(i)}\right) \\
&= \frac{1}{m}\left(x^{(m)} + (m-1)\,\hat{x}_{m-1}\right) \\
&= \hat{x}_{m-1} + \frac{1}{m}\left(\underbrace{x^{(m)}}_{\text{new data}} - \hat{x}_{m-1}\right)
\end{aligned}$$

Incremental Mean Estimation

$$
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&= \frac{1}{m} \left(x^{(m)} + (m-1)\, \hat{x}_{m-1} \right) \\
&= \hat{x}_{m-1} + \frac{1}{m} \left(x^{(m)} - \hat{x}_{m-1} \right)
\end{aligned}
$$

```
function simulate!(π::MonteCarloTreeSearch, s, d=π.d)
    if d ≤ 0
        return π.U(s)
    end
    𝒫, N, Q, c = π.𝒫, π.N, π.Q, π.c
    𝒜, TR, γ = 𝒫.𝒜, 𝒫.TR, 𝒫.γ
    if !haskey(N, (s, first(𝒜)))
        for a in 𝒜
            N[(s,a)] = 0
            Q[(s,a)] = 0.0
        end
        return π.U(s)
    end
    a = explore(π, s)
    s′, r = TR(s,a)
    q = r + γ*simulate!(π, s′, d-1)
    N[(s,a)] += 1
    Q[(s,a)] += (q-Q[(s,a)])/N[(s,a)]
    return q
end
```

Incremental Mean Estimation

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&= \hat{x}_{m-1} + \frac{1}{m} \left(x^{(m)} - \hat{x}_{m-1} \right)
\end{aligned}
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    Q[(s,a)] += (q-Q[(s,a)])/N[(s,a)]
end
```

loop

$$\hat{x} \leftarrow \hat{x} + \alpha\,(x - \hat{x})$$

learning

Incremental Mean Estimation

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\begin{aligned}
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\end{aligned}
$$

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end
```

loop

$$\hat{x} \leftarrow \hat{x} + \alpha\,(x - \hat{x})$$

"Temporal Difference (TD) Error"

Q Learning

Q learning and SARSA

Have a table $Q(s,a)$

Want $\quad Q(s,a) \leftarrow Q(s,a) + \alpha\left(q(s,a,r,s') - Q(s,a)\right)$

$$Q(s,a) = R(s,a) + \gamma E\left(U(s')\right)$$

$$= R(s,a) + \gamma E\left[\max_{a'} Q(s',a')\right]$$

(s,a,r,s')

$$q(s,a,r,s') = r + \gamma \max_{a'} Q(s',a')$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\left(r + \gamma \max_{a'} Q(s',a') - Q(s,a)\right)$$

(s, a, r, s') (s, a, r, s', a')

Q learning and SARSA

Off Policy On-Policy

Q-Learning

$Q(s,a) \leftarrow 0 \quad \forall s,a$

$s \leftarrow s_0$

< a ← rand(A)

loop

$a \leftarrow \operatorname{argmax} Q(s,a)$ w.p. $1-\epsilon$, rand(A) o.w.

ε-greedy

X

$r \leftarrow \text{act!}(\text{env}, a)$

$s' \leftarrow \text{observe}(\text{env})$

< a' ← epsilon greedy policy

$Q(s,a) \leftarrow Q(s,a) + \alpha\,(r + \gamma \max_{a'} Q(s',a') - Q(s,a))$

$Q(s,a) \leftarrow Q(s,a) + \alpha(r + \gamma Q(s',a') - Q(s,a))$

$s \leftarrow s'$

< a ← a'

Q learning and SARSA

Q-Learning

$Q(s, a) \leftarrow 0$

$s \leftarrow s_0$

loop

$a \leftarrow \text{argmax}\, Q(s, a) \text{ w.p. } 1 - \epsilon, \quad \text{rand}(A) \text{ o.w.}$

$r \leftarrow \text{act!}(\text{env}, a)$

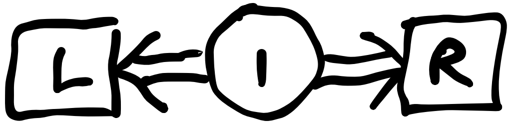
$s' \leftarrow \text{observe}(\text{env})$

$Q(s, a) \leftarrow Q(s, a) + \alpha \left(\underbrace{r + \gamma \max_{a'} Q(s', a') - Q(s, a)}_{TD}\right)$

$s \leftarrow s'$

Illustrative Problem

Illustrative Problem



Illustrative Problem

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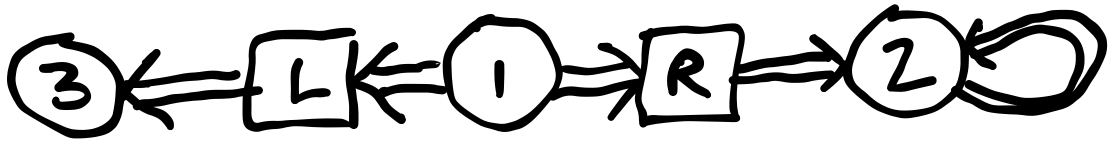
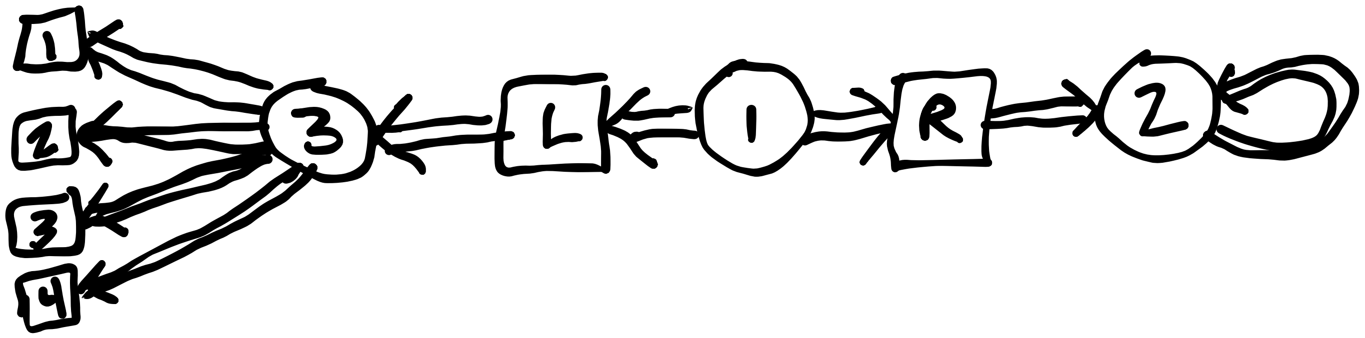


Illustrative Problem



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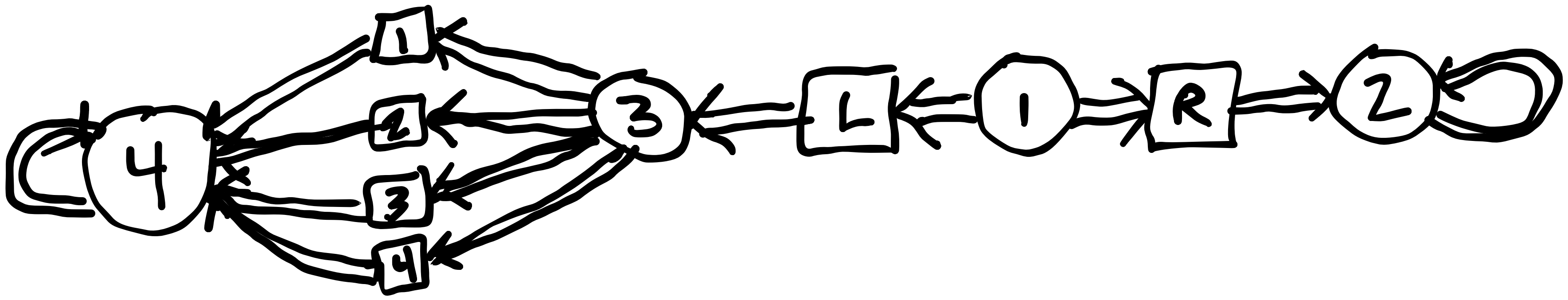

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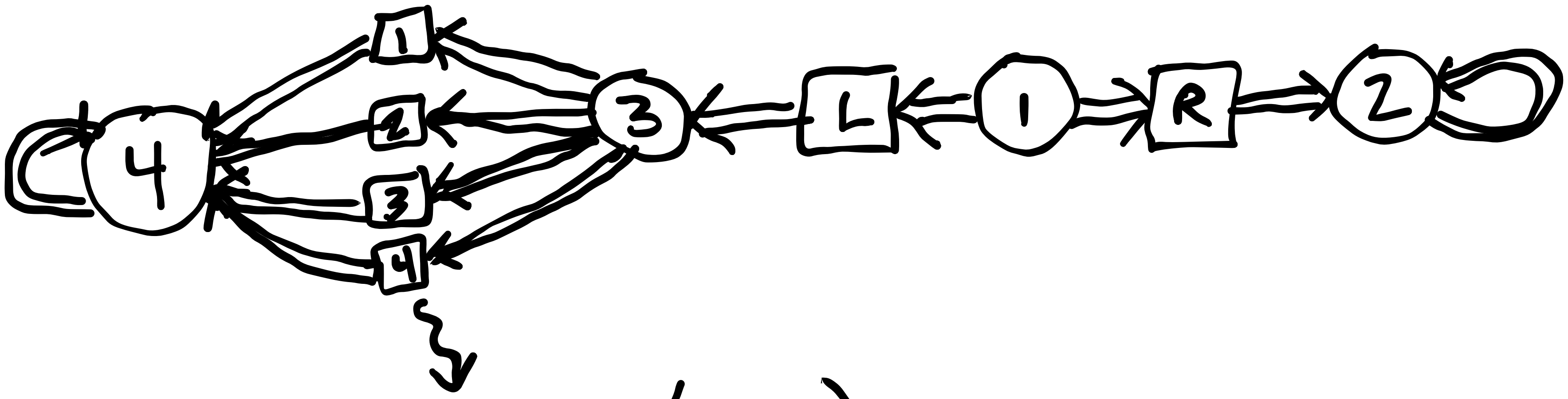
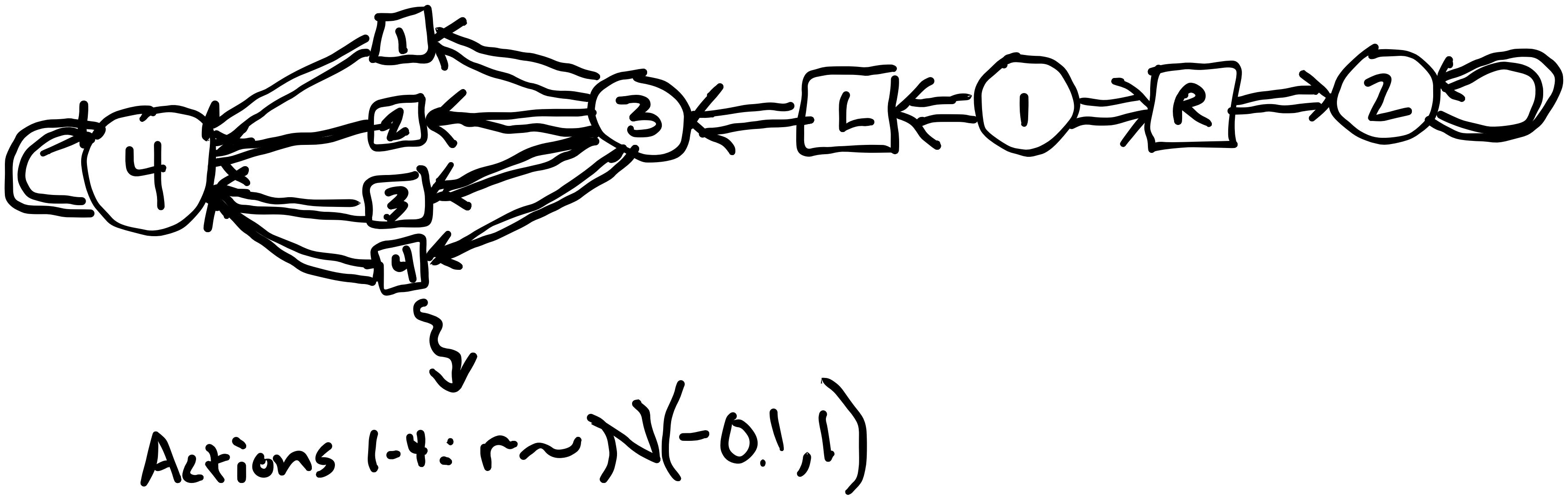


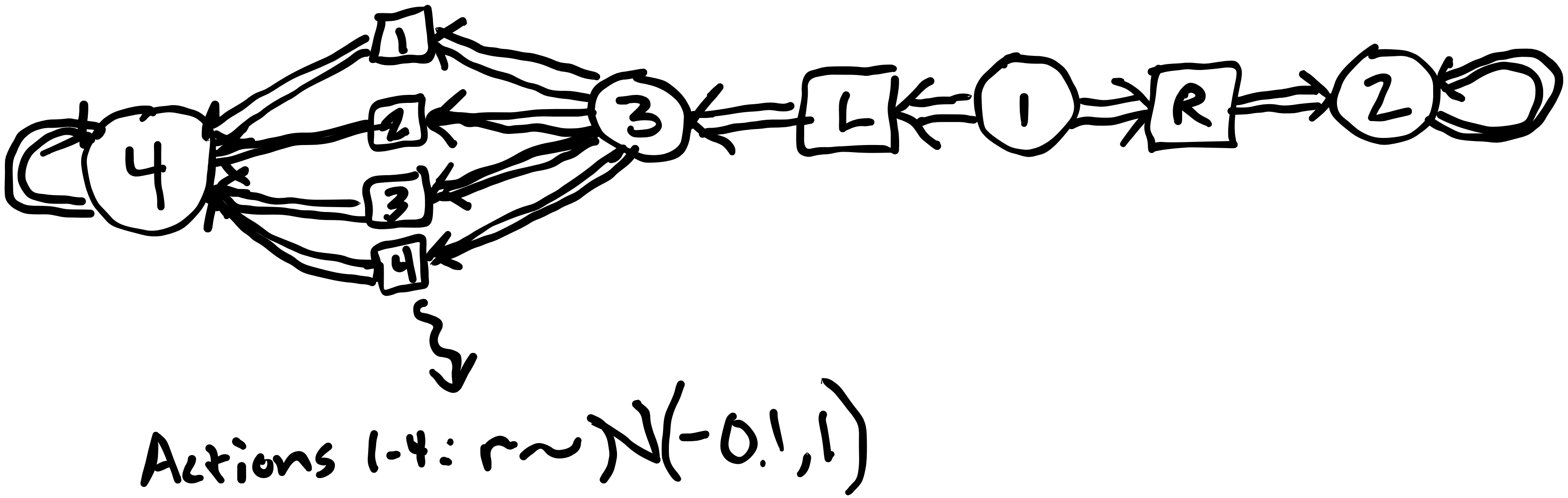
Actions 1-4: $r \sim \mathcal{N}(-0.1, 1)$

Illustrative Problem



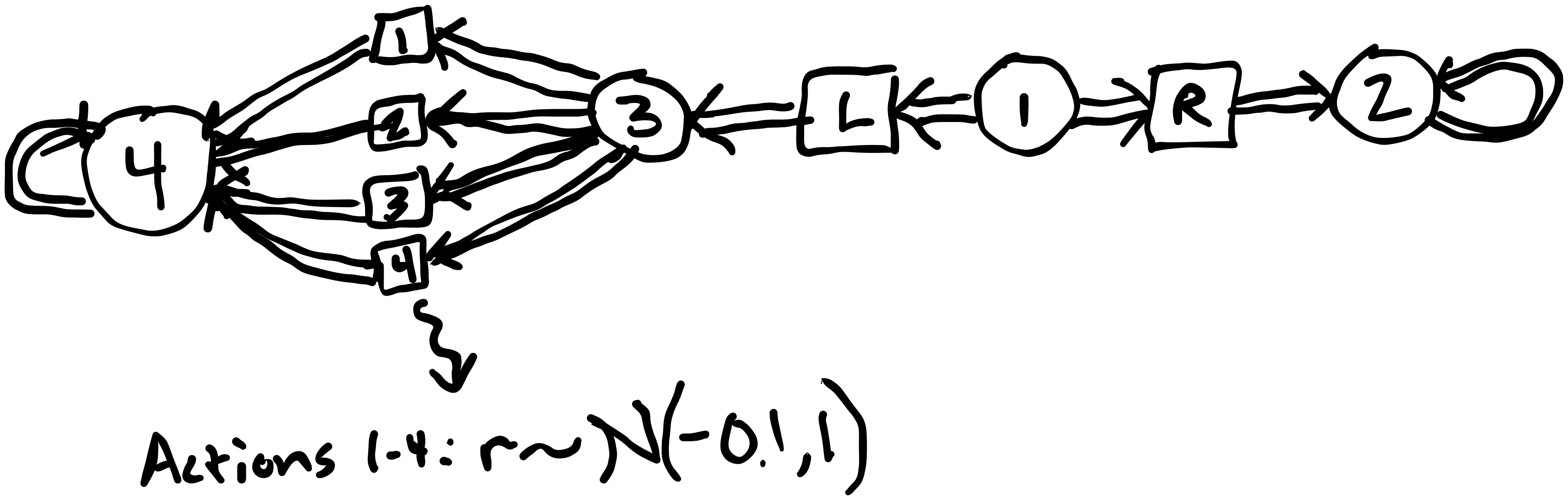
1. After a few episodes, what is Q(3, a) for a in 1-4?

Illustrative Problem



1. After a few episodes, what is Q(3, a) for a in 1-4?
2. After a few episodes, what is Q(1, L)?

Illustrative Problem



1. After a few episodes, what is Q(3, a) for a in 1-4?
2. After a few episodes, what is Q(1, L)?
3. Why is this a problem and what are some possible solutions?

$Q(3,a) \approx -0.1 \pm 1$

$Q(1,L) \approx \max_a Q(3,a)$

Big Problem: Maximization Bias

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Even if all $Q(s', a')$ unbiased, $\max_{a'} Q(s', a')$ is biased!

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Solution: Double Q Learning

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Solution: Double Q Learning Q_1, Q_2

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Solution: Double Q Learning Q_1, Q_2

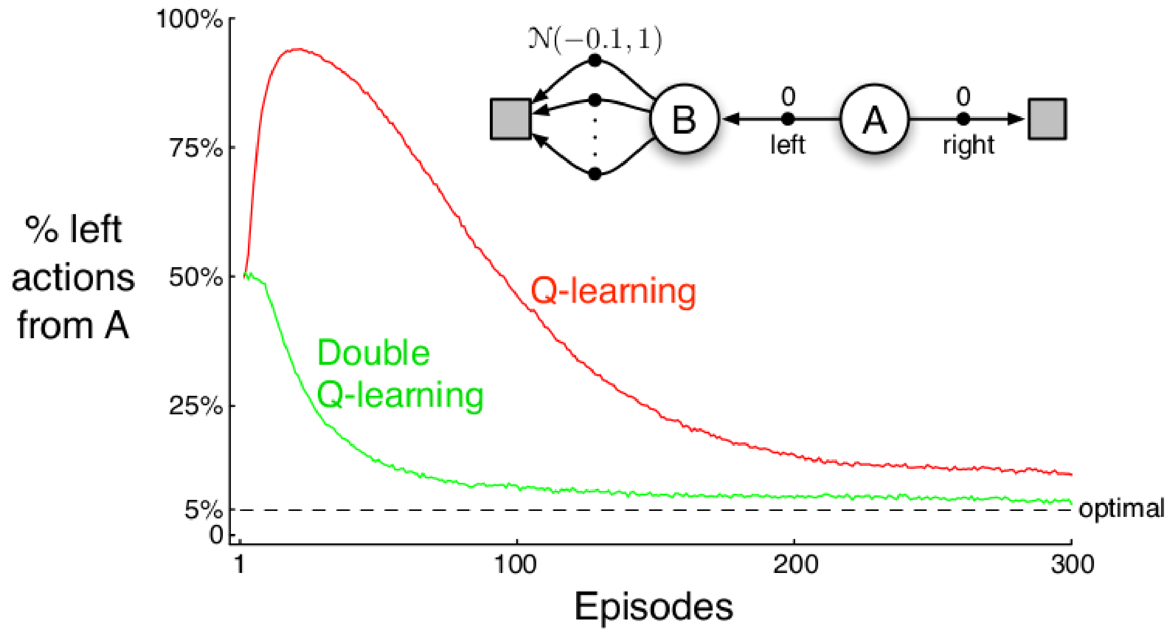
$$Q_1(s,a) \leftarrow Q_1(s,a) + \alpha \left(r + \gamma \, Q_2 \left(s', \underset{a'}{\mathrm{argmax}} Q_1(s',a') \right) - Q_1(s,a) \right)$$

Big Problem: Maximization Bias

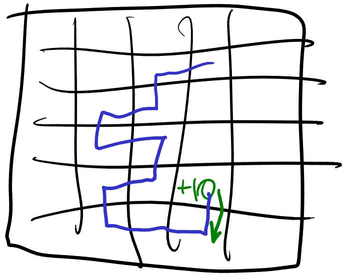
Even if all $Q(s', a')$ unbiased, $\max_{a'} Q(s', a')$ is biased!

Solution: Double Q Learning $\quad Q_1, Q_2$

$$Q_1(s,a) \leftarrow Q_1(s,a) + \alpha \left(r + \gamma \underbrace{Q_2\left(s', \underbrace{\operatorname*{argmax}_{a'} Q_1(s', a')}\right)} - Q_1(s,a) \right)$$



Eligibility Traces



SARSA-λ

SARSA-λ

$Q(s,a), N(s,a) \leftarrow 0 \quad \forall(s,a)$

N = decaying count of visits to (s,a)

initialize s, a, r, s'

loop

$a' \leftarrow \operatorname{argmax} Q(s', a)$ w.p. $1-\epsilon$, $\quad$ rand(A) o.w.

$N(s,a) \leftarrow N(s,a) + 1$

$\delta \leftarrow r + \gamma Q(s', a') - Q(s,a)$

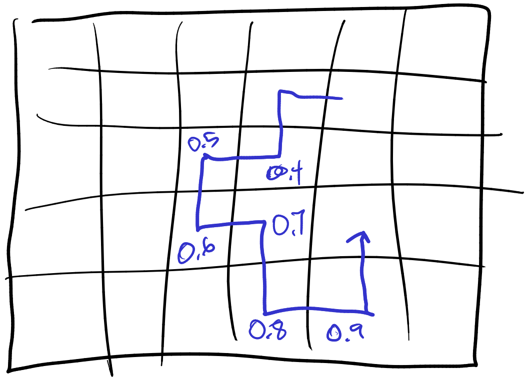
$Q(s,a) \leftarrow Q(s,a) + \alpha\delta\, N(s,a) \quad \forall s, a$

$N(s,a) \leftarrow \gamma\lambda N(s,a) \quad \forall s, a$

$s \leftarrow s', \quad a \leftarrow a'$

$r \leftarrow \text{act!}(\text{env}, a)$

$s' \leftarrow \text{observe}(\text{env})$



Convergence

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- Q learning converges to optimal Q-values w.p. 1
  (Sutton and Barto, p. 131)

Convergence

- Q learning converges to optimal Q-values w.p. 1
  (Sutton and Barto, p. 131)
- SARSA converges to optimal Q-values w.p. 1 ***provided that***

  $\pi \to \mathrm{greedy}$

  (Sutton and Barto, p. 129)

On vs Off-Policy

On vs Off-Policy

On Policy

On vs Off-Policy

On Policy

Off Policy

On vs Off-Policy

On Policy

SARSA:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \ (r + \gamma Q(s', a') - Q(s,a))$$

Off Policy

On vs Off-Policy

On Policy

SARSA:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma Q(s',a') - Q(s,a))$$

Off Policy

Q-learning:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

On vs Off-Policy

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Will eligibility traces work with Q-learning?

On vs Off-Policy

On Policy

SARSA:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \ (r + \gamma Q(s',a') - Q(s,a))$$

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Will eligibility traces work with Q-learning?

Not easily

On vs Off-Policy

On Policy

SARSA:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma Q(s',a') - Q(s,a))$$

Policy Gradient:

$$\theta \leftarrow \theta + \alpha \sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) R(\tau)$$

Off Policy

Q-learning:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

Will eligibility traces work with Q-learning?

Not easily

Today

- Basic On- and Off-Policy **value based** model free RL algorithms
- Tricks for tabular value based RL algorithms ← Double Q-learning
- Understanding of On- vs Off-Policy

Eligibility Traces