Hw1

Wednesday, January 22, 2025 12:04 PM

1)

A	B	C	$P(A,B,C)$
0	0	1	0.15
0	1	0	0.05
0	1	1	0.01
1	0	0	0.14
1	0	1	0.18
1	1	0	0.27
1	1	1	0.06
0	0	0	0.14

a) $P(A=0, B=0, C=0)$

$= 1 - (0.15 + 0.05 + 0.01 + 0.14 + 0.18 \ldots$
$+ 0.27 + 0.06)$

$= 0.14$

$P(A=0, B=0, C=0) = 0.14$

b) Find $P(A)$

$$P(A=0) = \sum_{B,C} P(A=0, B, C) = 0.14 + 0.15 + 0.05 + 0.01 = 0.35$$

$$P(A=1) = \sum_{B,C} P(A=1, B, C) = 0.14 + 0.18 + 0.27 + 0.06 = 0.65$$

$P(A) \rightarrow$

A	$P(A)$
0	0.35
1	0.65

c) $P(A \mid B=0, C=1)$

$A=0 \rightarrow \dfrac{0.15}{0.15+0.18} = 0.45\overline{45} \rightarrow 0.45$

$A = 1 \rightarrow \frac{0.18}{0.15+0.18} = 0.5454 \rightarrow 0.55$

A	P(A)
0	0.45
1	0.55

2) $P(B) = 0.02$ $\quad B$ = breast cancer

$P(!B) = 0.98$ $\quad P$ = positive mammogram

$P(P|B) = 0.91$

$P(P|B=0) = 0.08$

Want $P(B|P)$

$$P(B|P) = \frac{P(P|B)\,P(B)}{P(P)}$$

$$P(P) = P(P|B)\cdot P(B) + P(P|B=0)\,P(B=0)$$

$$= 0.91 \cdot 0.02 + 0.08 \cdot 0.98$$

$$P(P) = 0.0966$$

$$P(B|P) = \frac{0.91 \cdot 0.02}{0.0966} = 0.1884$$

$$P(B|P) = \frac{0.91 \cdot 0.02}{0.0966} = 0.1884$$

Chance of actually having breast cancer: 18.84%

3) Want $P(B|L)$

$$P(B|L) = \frac{P(L|B)\,P(B)}{P(L)}$$

$$P(L) = P(L|B)\,P(B) + P(L|\neg B)\,P(\neg B)$$

$$P(B) = P(B|A)\,P(A) + P(B|\neg A)\,P(\neg A)$$

$$= 0.8 \cdot 0.5 + 0.2 \cdot 0.5$$

$$= 0.5$$

$$P(L) = 0.9 \cdot 0.5 + 0.3 \cdot 0.5$$

$$= 0.6$$

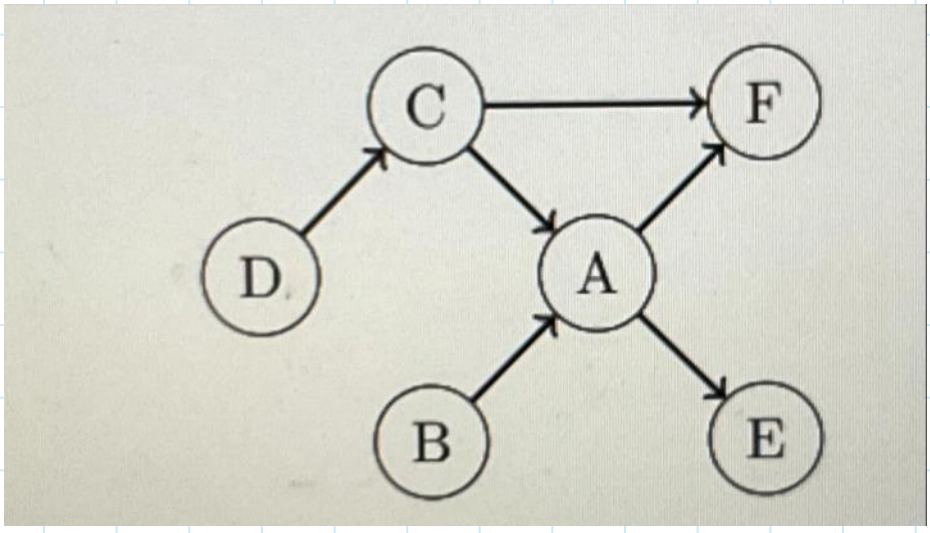
$$P(B|L) = \frac{P(L|B)\,P(B)}{P(L)}$$

$$\rightarrow P(B|L) = \frac{P(L|B)P(B)}{P(L)}$$

$$= \frac{0.9 \cdot 0.5}{0.6}$$

$$P(B|L) = 0.75$$

4)



a) $B \perp F \mid L$???

$B \rightarrow A \rightarrow F$ ✗ $A \notin G$

$B \rightarrow A \leftarrow C \rightarrow F$ ✓ $A \leftarrow C \rightarrow F$. $L \in G$

So, these are not d-separated. This means that

We can not conclude that $B \perp F$

b) $B \perp F \mid A$??

$B \rightarrow A \rightarrow F$ ✓ $A \in G$

$B \rightarrow A \leftarrow C \rightarrow F$ ✗ $A \in G$, $C \notin G$

So, we can not conclude that $B \perp F$ because path 2 fails rule 2 and 3.

c) $B \perp E \mid A$??

$B \rightarrow A \rightarrow E$ $A \in C$, so yes

Yes! These <u>are</u> d-separated