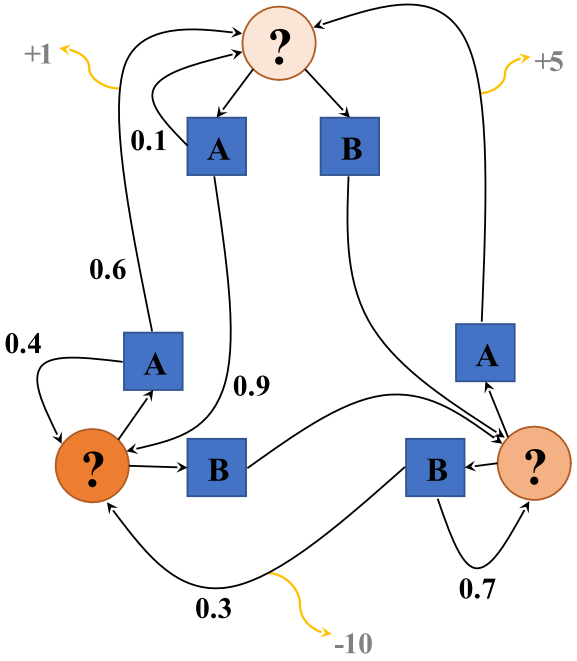
Incomplete Information Dynamic Games

Incomplete Information

Partially Observable Markov Decision Process (POMDP)



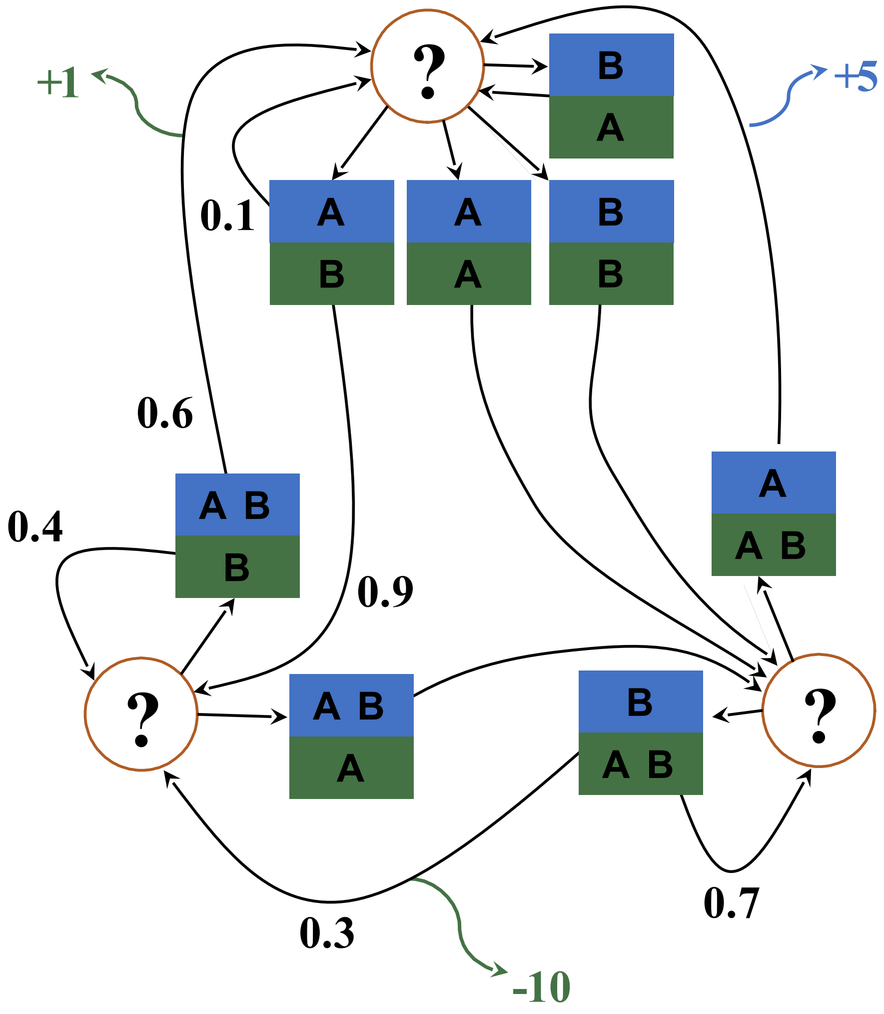
- $\mathcal{S}$ - State space
- $T : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}$ - Transition probability distribution
- $\mathcal{A}$ - Action space
- $R : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ - Reward
- $\mathcal{O}$ - Observation space
- $Z : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \times \mathcal{O} \rightarrow \mathbb{R}$ - Observation probability distribution

Alleatory

Epistemic (Static)

Epistemic (Dynamic)

Partially Observable Markov Game



- $\mathcal{S}$ - State space
- $T(s' \mid s, \boldsymbol{a})$ - Transition probability distribution
- $\mathcal{A}^i,\ i \in 1..k$ - Action spaces
- $R^i(s, \boldsymbol{a})$ - Reward function
- $\mathcal{O}^i,\ i \in 1..k$ - Observation space
- $Z(o^i \mid \boldsymbol{a}, s')$ - Observation probability distribution

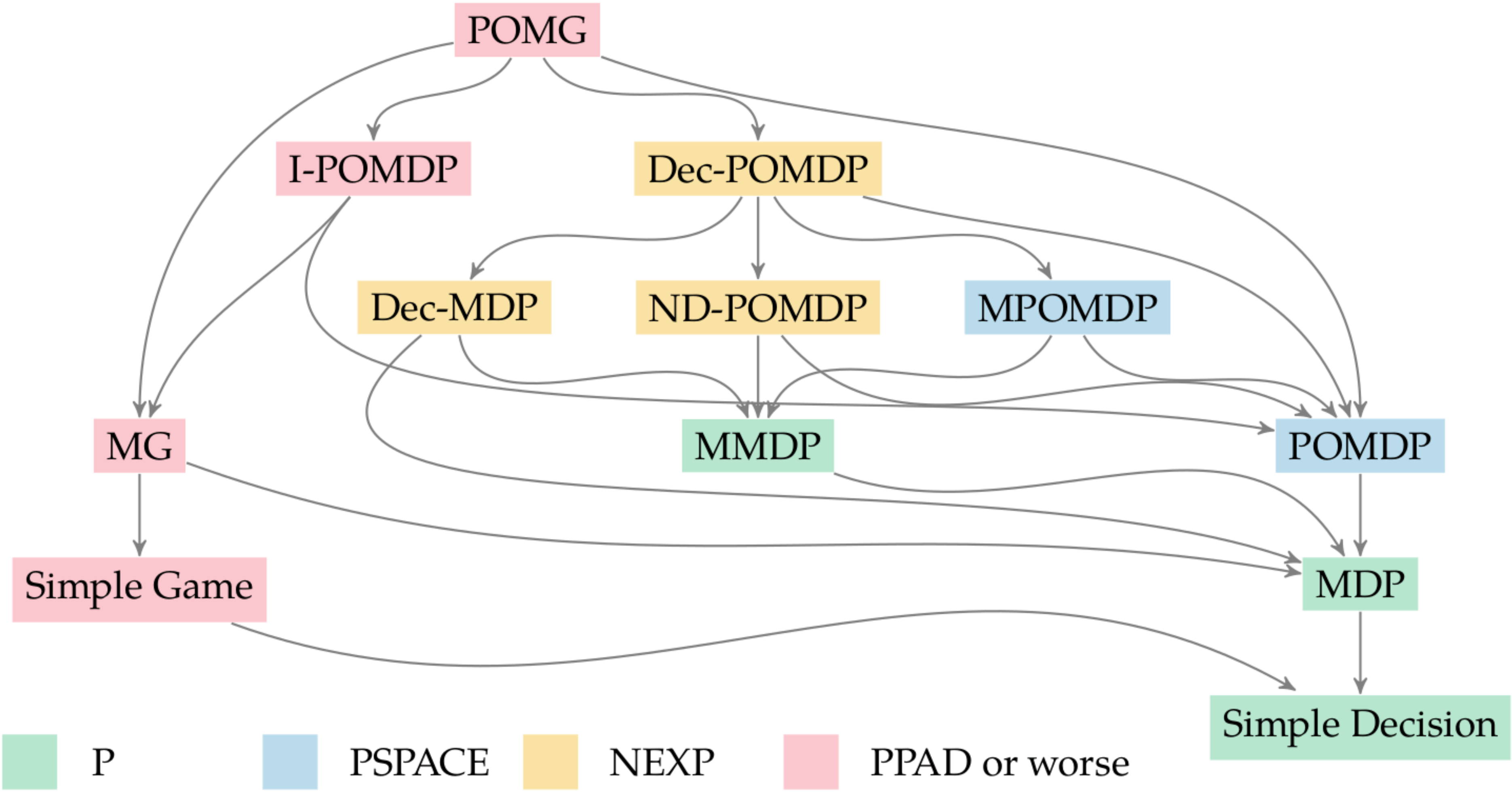
Alleatory

Epistemic (Static)

Epistemic (Dynamic)

Interaction

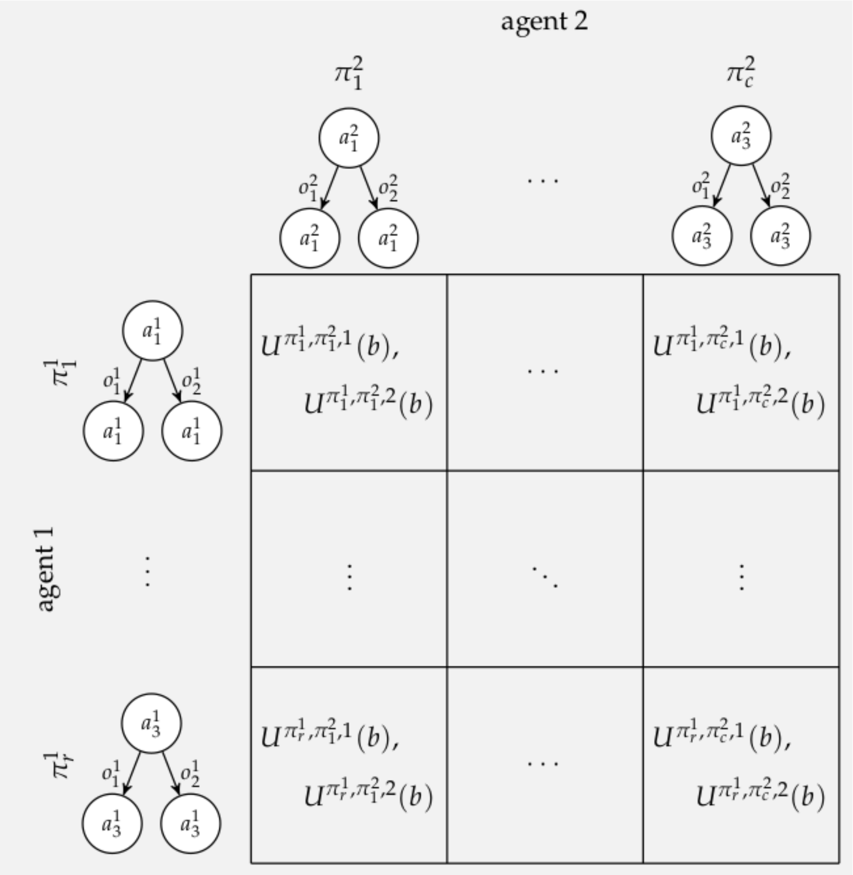
Hierarchy of Problems



Belief updates?

Reduction to Simple Game

Reduction to Simple Game



Pruning in Dynamic Programming

$$\sum_{\boldsymbol{\pi}^{-i}} \sum_{s} b(\boldsymbol{\pi}^{-i}, s) U^{\pi^{i\prime}, \boldsymbol{\pi}^{-i}, i}(s) \geq \sum_{\boldsymbol{\pi}^{-i}} \sum_{s} b(\boldsymbol{\pi}^{-i}, s) U^{\pi^{i}, \boldsymbol{\pi}^{-i}, i}(s)$$

$$\begin{aligned}
\underset{\delta, b}{\text{maximize}} \quad & \delta \\
\text{subject to} \quad & b(\boldsymbol{\pi}^{-i}, s) \geq 0 \text{ for all } \boldsymbol{\pi}^{-i}, s \\
& \sum_{\boldsymbol{\pi}^{-i}} \sum_{s} b(\boldsymbol{\pi}^{-i}, s) = 1 \\
& \sum_{\boldsymbol{\pi}^{-i}} \sum_{s} b(\boldsymbol{\pi}^{-i}, s) \left(U^{\pi^{i\prime}, \boldsymbol{\pi}^{-i}, i}(s) - U^{\pi^{i}, \boldsymbol{\pi}^{-i}, i}(s) \right) \geq \delta \text{ for all } \pi^{i\prime}
\end{aligned}$$

Extensive Form Game

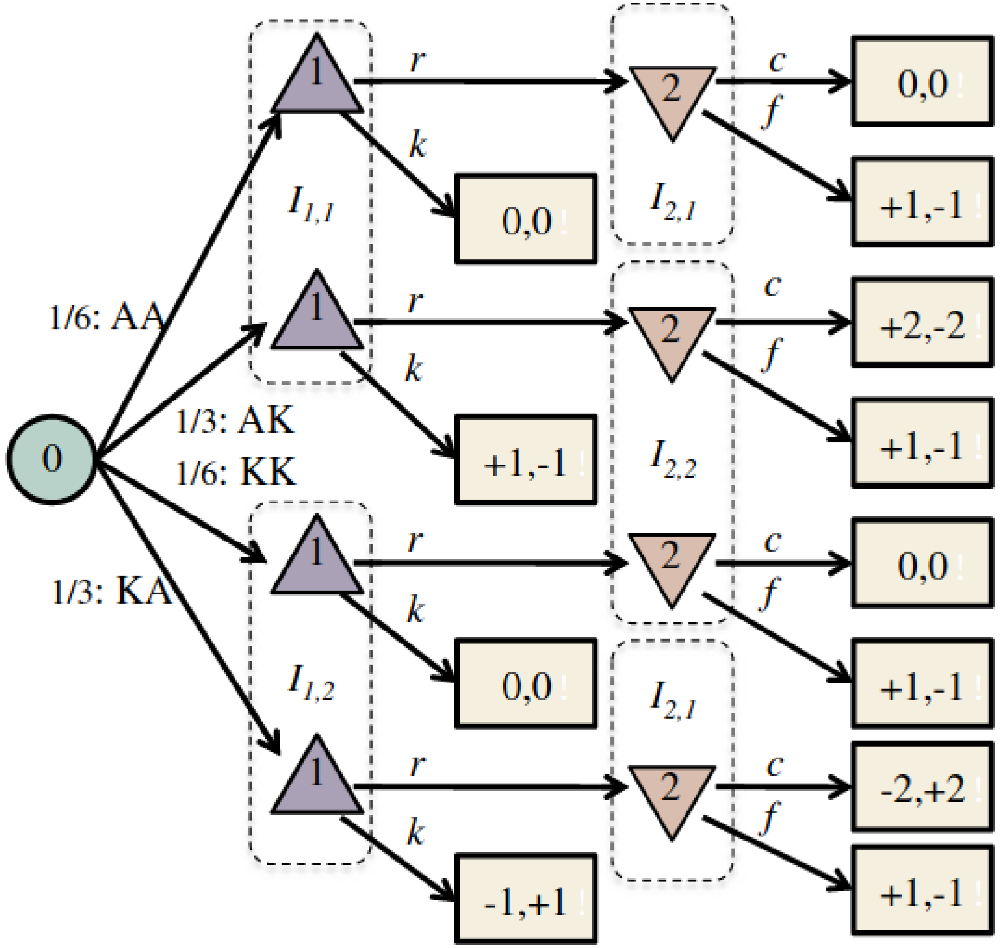
(Alternative to POMGs that is more common in the literature)

- Similar to a minimax tree for a turn-taking game
- Chance nodes
- Information sets

King-Ace Poker Example

- 4 Cards: 2 Aces, 2 Kings
- Each player is dealt a card
- P1 can either *raise* (r) the payoff to 2 points or *check* (k) the payoff at 1 point
- If P1 raises, P2 can either *call* (c) Player 1's bet, or *fold* (f) the payoff back to 1 point
- The highest card wins

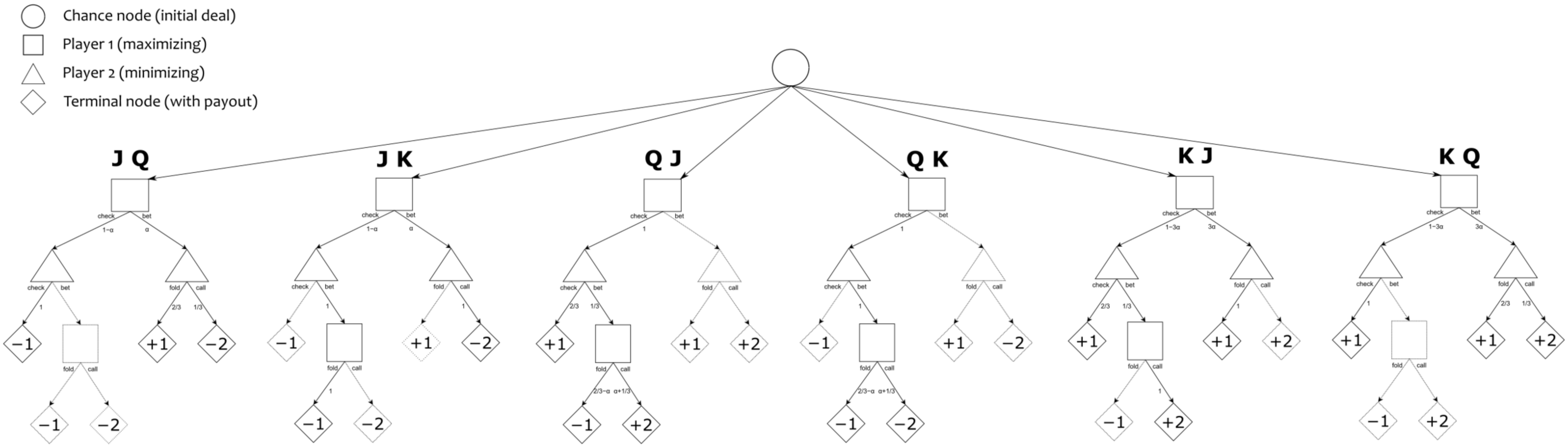
Extensive to Matrix Form



	2:*cc*	2:*cf*	2:*ff*	2:*fc*
1:*rr*	0	-1/6	1	7/6
1:*kr*	-1/3	-1/6	5/6	2/3
1:*rk*	1/3	0	1/6	1/2
1:*kk*	0	0	0	0

Exponential in number of info states!

A more interesting example: Kuhn Poker

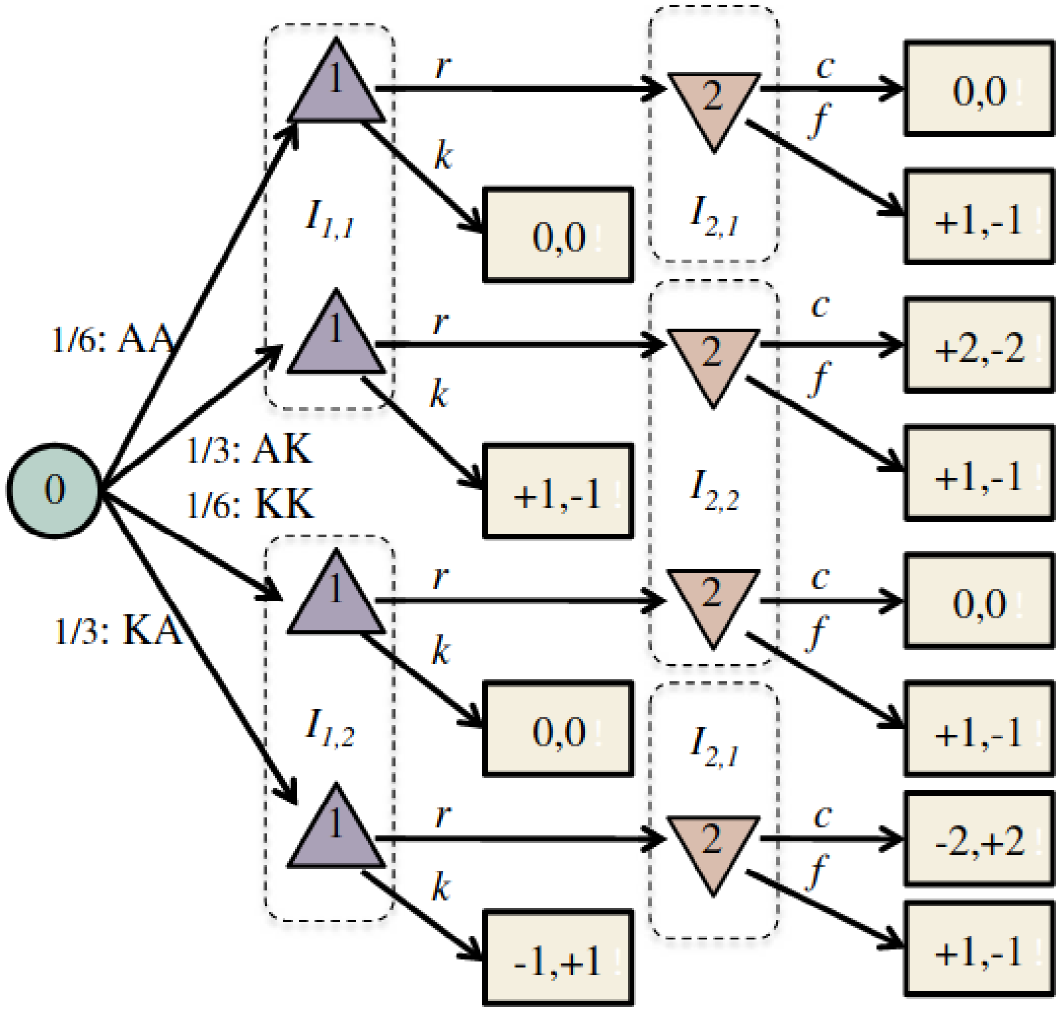


Fictitious Play in Extensive Form Games

Algorithm 2 General Fictitious Self-Play

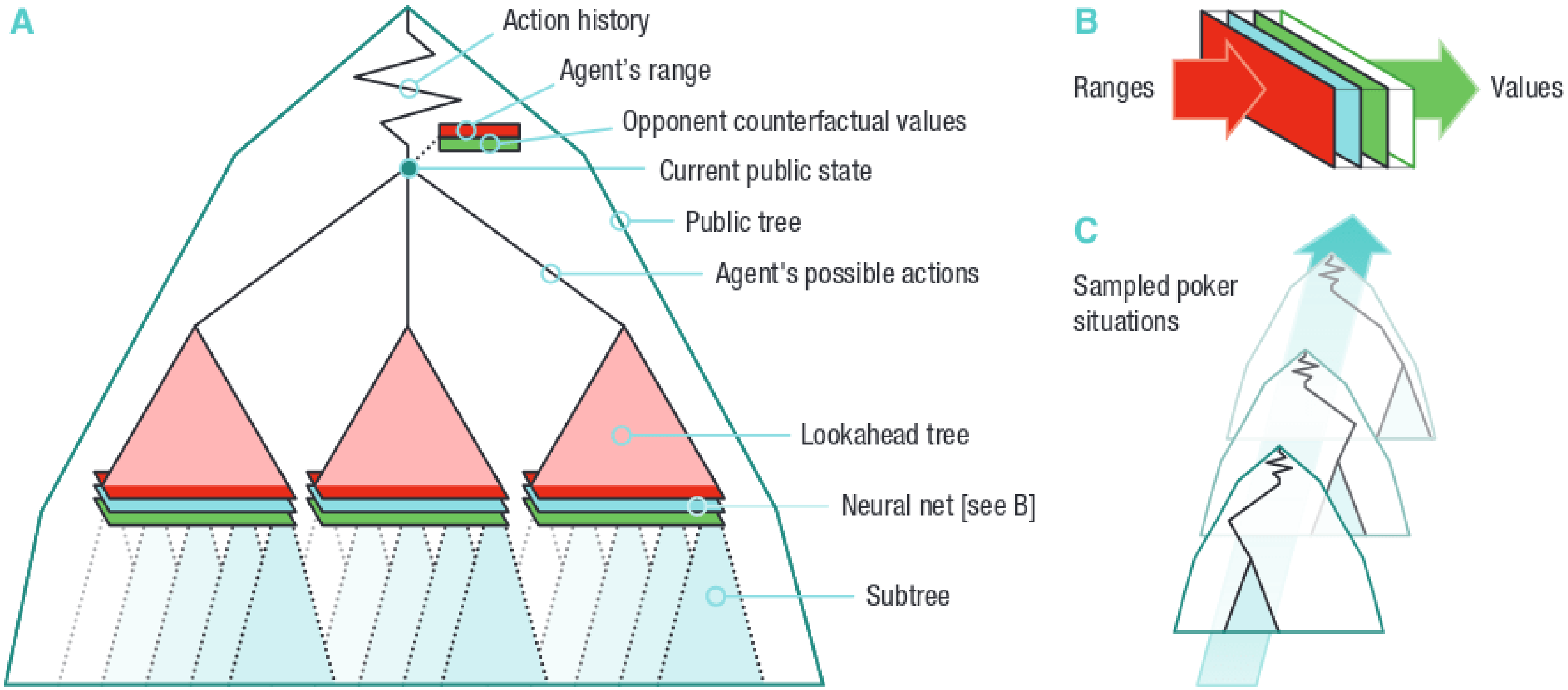
function FICTITIOUSSELFPLAY(Γ, n, m)
  Initialize completely mixed π_1
  $\beta_2 \leftarrow \pi_1$
  $j \leftarrow 2$
  while within computational budget **do**
    $\eta_j \leftarrow$ MIXINGPARAMETER(j)
    $\mathcal{D} \leftarrow$ GENERATEDATA$(\pi_{j-1}, \beta_j, n, m, \eta_j)$
    for each player $i \in \mathcal{N}$ **do**
      $\mathcal{M}^i_{RL} \leftarrow$ UPDATERLMEMORY$(\mathcal{M}^i_{RL}, \mathcal{D}^i)$
      $\mathcal{M}^i_{SL} \leftarrow$ UPDATESLMEMORY$(\mathcal{M}^i_{SL}, \mathcal{D}^i)$
      $\beta^i_{j+1} \leftarrow$ REINFORCEMENTLEARNING$(\mathcal{M}^i_{RL})$
      $\pi^i_j \leftarrow$ SUPERVISEDLEARNING$(\mathcal{M}^i_{SL})$
    end for
    $j \leftarrow j + 1$
  end while
  return π_{j-1}
end function

function GENERATEDATA(π, β, n, m, η)
  $\sigma \leftarrow (1 - \eta)\pi + \eta\beta$
  $\mathcal{D} \leftarrow n$ episodes $\{t_k\}_{1 \le k \le n}$, sampled from strategy profile σ
  for each player $i \in \mathcal{N}$ **do**
    $\mathcal{D}^i \leftarrow m$ episodes $\{t^i_k\}_{1 \le k \le m}$, sampled from strategy profile (β^i, σ^{-i})
    $\mathcal{D}^i \leftarrow \mathcal{D}^i \cup \mathcal{D}$
  end for
  return $\{\mathcal{D}^k\}_{1 \le k \le N}$
end function



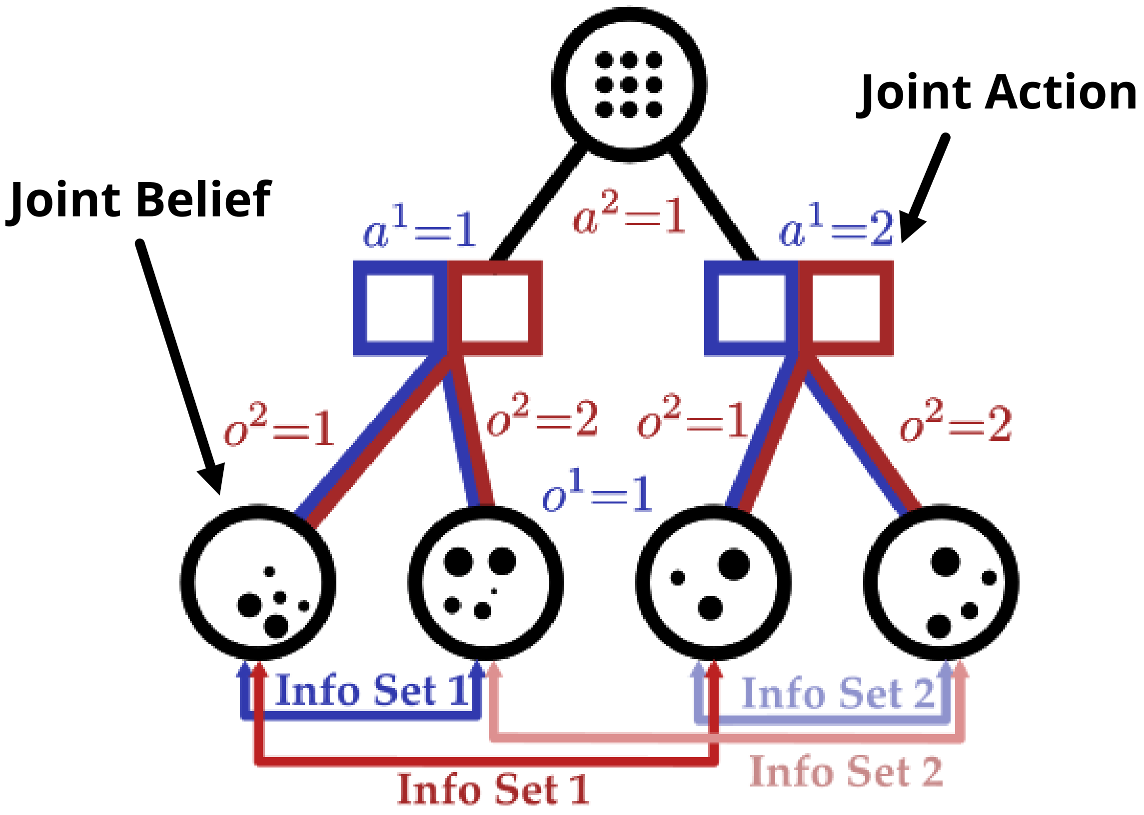
Heinrich et al. 2015 "Fictitious Self Play in Extensive-Form Games"

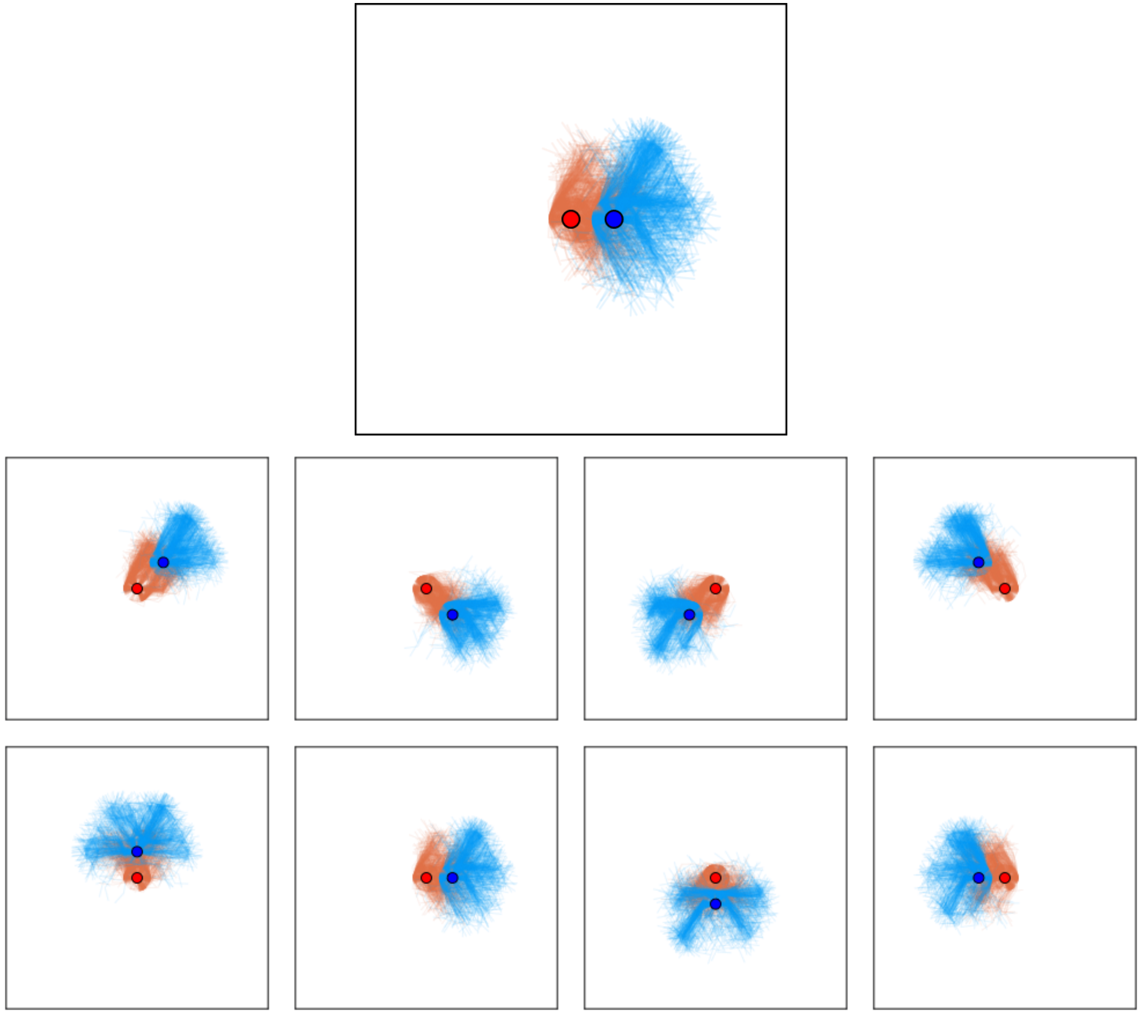
Deep Stack: Scaling to Heads Up No Limit Texas Hold 'Em



Tree-Based Planning in POSGs

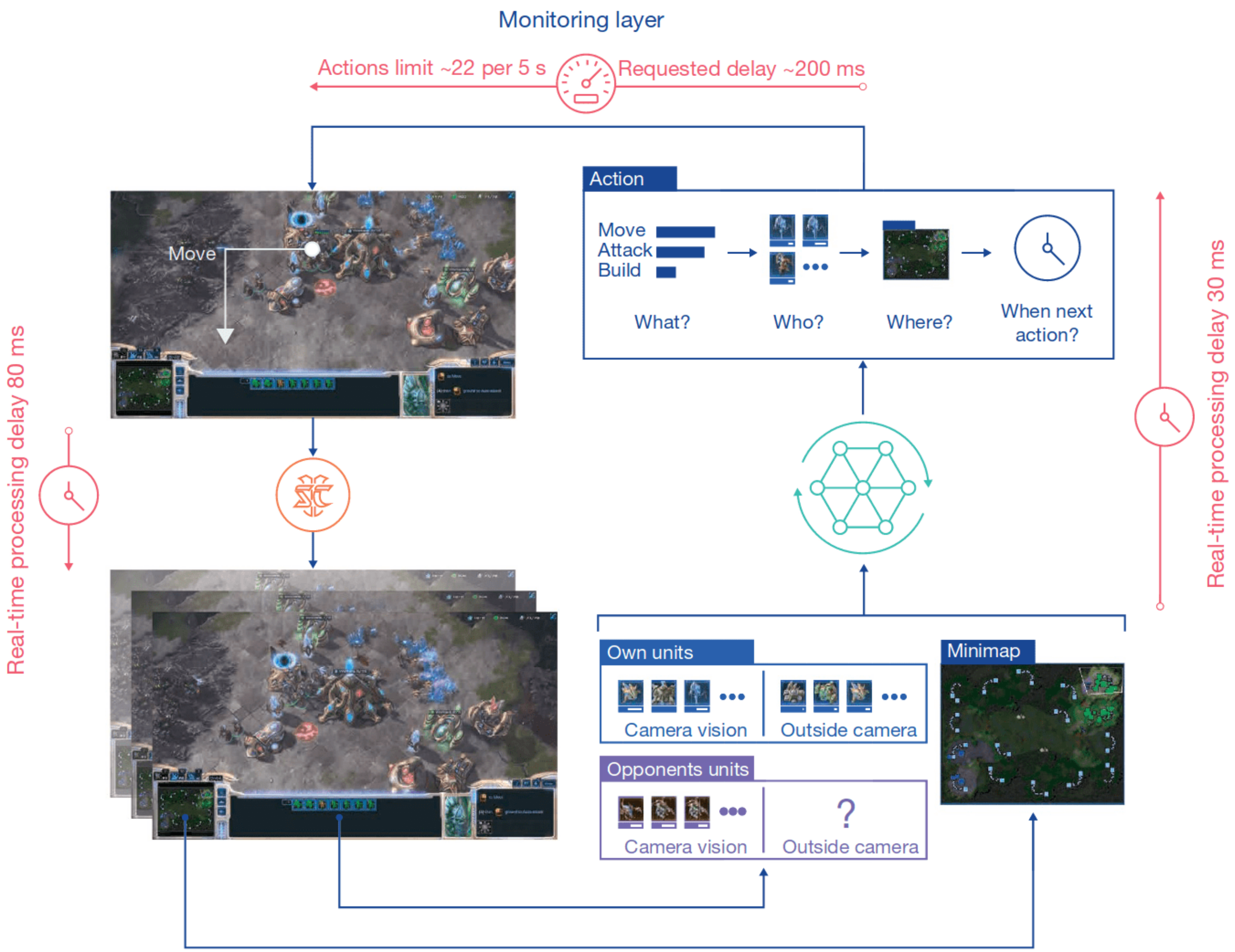
Our approach: combine particle filtering and information sets

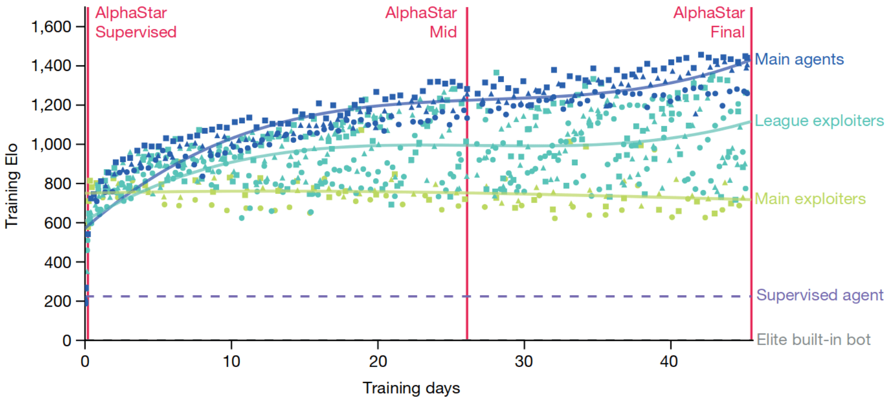


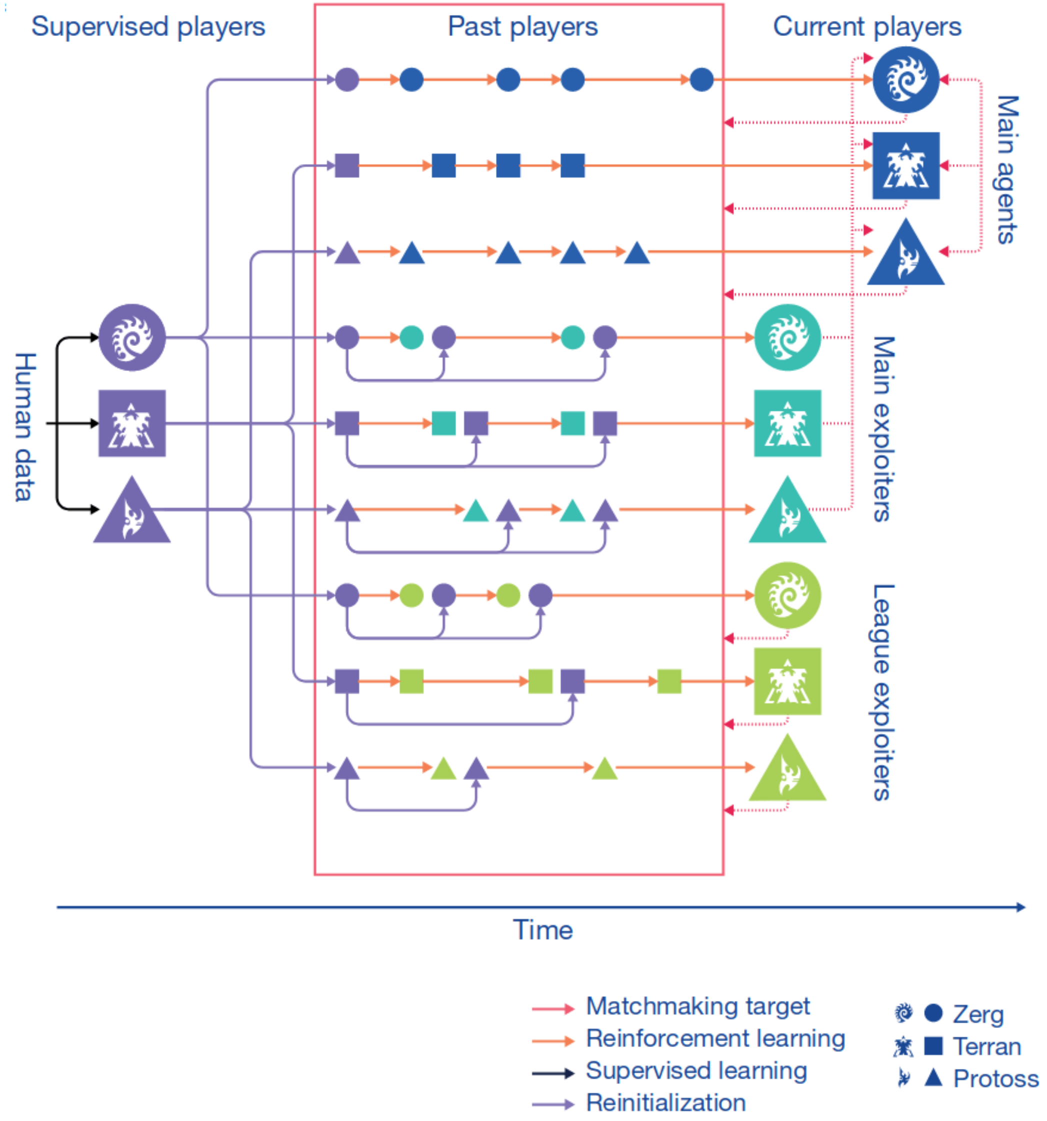


[Becker & Sunberg, AAMAS 2025]

Alpha Star


Monitoring layer
Actions limit ~22 per 5 s
Requested delay ~200 ms
Action
Move
Attack
Build
What?
Who?
Where?
When next action?
Move
Real-time processing delay 80 ms
Real-time processing delay 30 ms
Own units
Camera vision
Outside camera
Opponents units
Camera vision
?
Outside camera
Minimap


AlphaStar Supervised
AlphaStar Mid
AlphaStar Final
1,600
1,400
1,200
1,000
800
600
400
200
0
Training Elo
0
10
20
30
40
Training days
Main agents
League exploiters
Main exploiters
Supervised agent
Elite built-in bot


Supervised players
Past players
Current players
Main agents
Main exploiters
League exploiters
Human data
Time
Matchmaking target
Reinforcement learning
Supervised learning
Reinitialization
Zerg
Terran
Protoss

Deep Nash

Player 2

Player 1		Head: H	Tail: T
	Head: H	1	-1
	Tail: T	-1	1

(a) Matching pennies

R-NaD Iteration

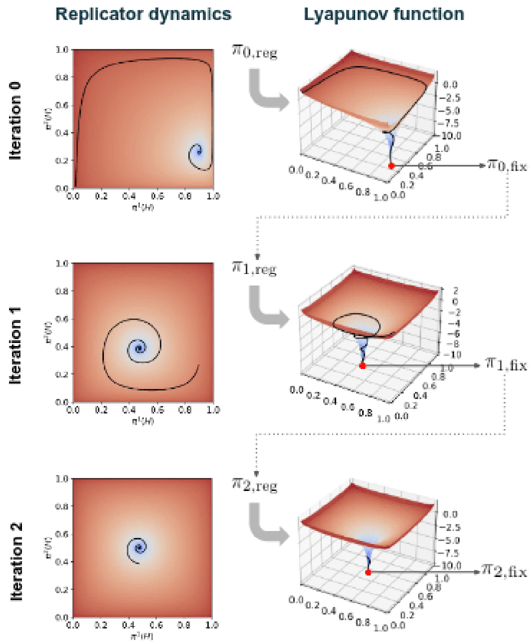
Start with an arbitrary regularization policy: $\pi_{0,\text{reg}}$

1. Reward transformation: Construct the transformed game with: $\pi_{n,\text{reg}}$
2. Dynamics: Run the replicator dynamics until convergence to: $\pi_{n,\text{fix}}$
3. Update: Set the regularization policy:

$$\pi_{n+1,\text{reg}} = \pi_{n,\text{fix}}$$

Repeat steps until convergence

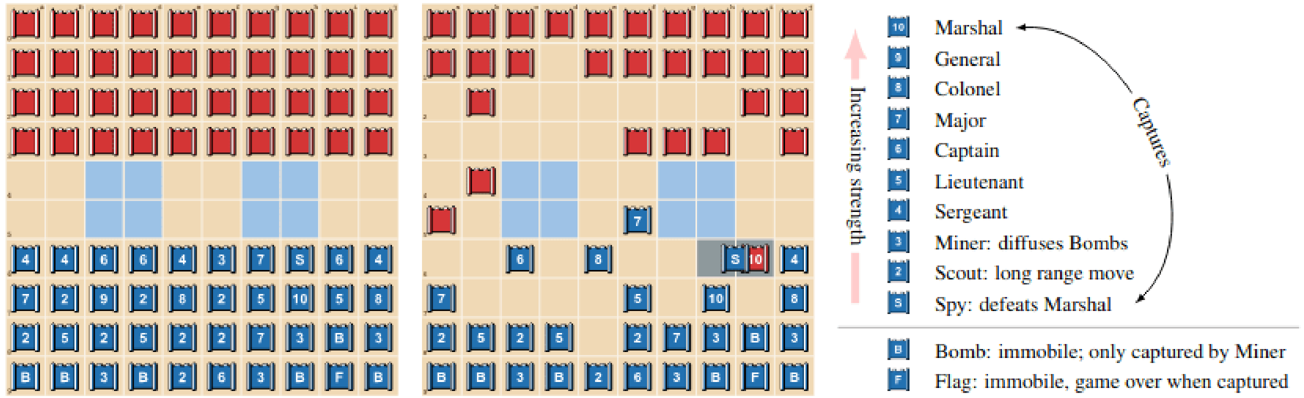
(b) Algorithmic steps



(c) Dynamics and Lyapunov function

Figure 2: The R-NaD learning algorithm illustrated with the matching pennies game

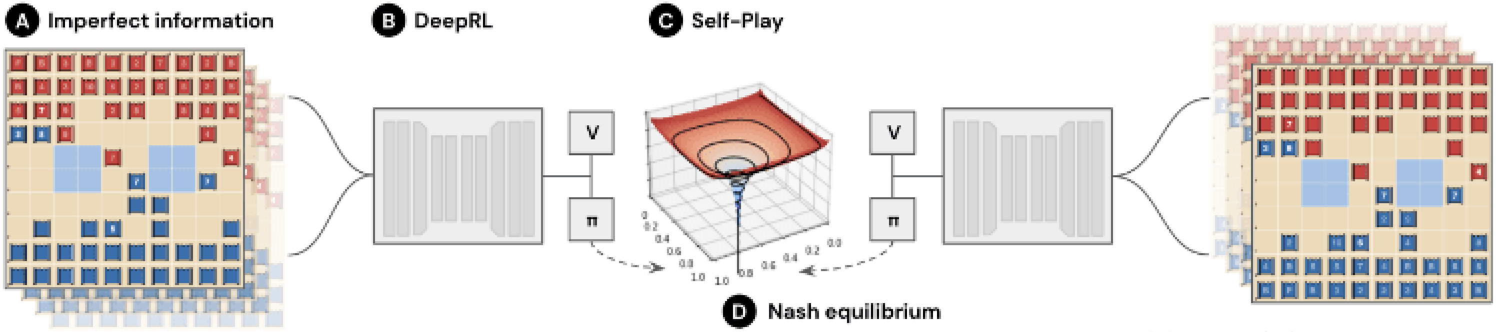
Stratego



Phase 1: Private deployment

Phase 2: Game play

Piece types



Replicator dynamics: $\frac{d}{d\tau}\pi^i_\tau(a^i) = \pi^i_\tau(a^i)\left[Q^i_{\pi_\tau}(a^i) - \sum_{b^i}\pi^i_\tau(b^i)Q^i_{\pi_\tau}(b^i)\right]$

Reward transformation: $r^i(\pi^i,\pi^{-i},a^i,a^{-i}) = r^i(a^i,a^{-i}) - \eta\log\left(\frac{\pi^i(a^i)}{\pi^i_{\text{reg}}(a^i)}\right) + \eta\log\left(\frac{\pi^{-i}(a^{-i})}{\pi^{-i}_{\text{reg}}(a^{-i})}\right)$