Simple Games

- Games: a mathematical formalism for rational interaction

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- Games: a mathematical formalism for rational interaction
- What is the most popular solution concept? (Nash Equilibrium)

Types of Uncertainty

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Alleatory

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Markov Decision Process

Types of Uncertainty

Alleatory

Epistemic (Static)

Markov Decision Process

Types of Uncertainty

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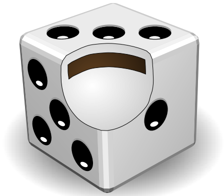
Markov Decision Process

Epistemic (Static)



Reinforcement Learning

Types of Uncertainty

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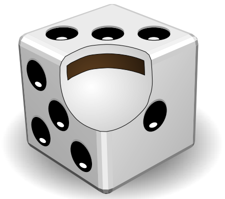
Markov Decision Process

Epistemic (Static)



Reinforcement Learning

Epistemic (Dynamic)

POMDP

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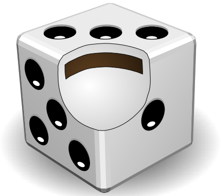

Reinforcement Learning

Epistemic (Dynamic)

POMDP

Interaction

Types of Uncertainty

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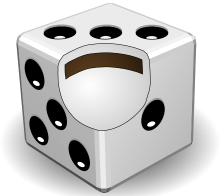
Epistemic (Static)		**Reinforcement Learning**
Epistemic (Dynamic)		**POMDP**
Interaction		**Game**

Normal Form Games

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Alice's Payoffs

B

A

	S	**W**
S	4	2
W	3	1

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Called a **Normal Form**, **Simple**, or **Bimatrix** Game

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Question for today: What **solution concept** should we use for games?

Dominant Strategies

Dominant Strategies

Bob

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	W	3, 2	1, 1

Dominant Strategies

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Alice

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Definitions

- Action $a^i \in A^i$
- Joint Action $a = (a^1, \ldots, a^k)$
- All Other Actions
  $a^{-i} = (a^1, \ldots, a^{i-1}, a^{i+1}, \ldots, a^k)$
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Action a^i is a deterministic best response to a^{-i} if

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Dominant Strategies

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- **Dominant (Pure) Strategy**: Action a is a dominant strategy if it is a best response to every action taken by the other player.

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Is the dominant strategy equilibrium always the best outcome for the players?

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The Prisoner's Dilemma

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Player 2

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T	0, -4	-3, -3

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Do all simple games have a dominant strategy equilibrium?

Collision Avoidance Game

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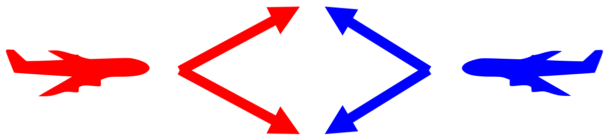
Example: Airborne Collision Avoidance



Player 2

Player 1

		Player 2: Up	Player 2: Down
Player 1	Up	Collision -5, -5	-1, 0
	Down	0, -1	Collision -4, -4

Collision Avoidance Game

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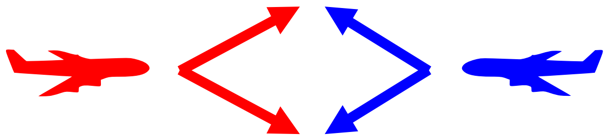

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Pure Nash Equilibrium: All players play a deterministic best response.

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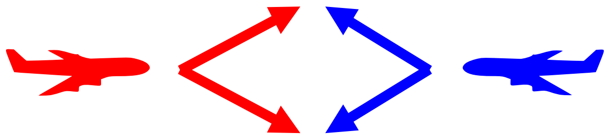


Player 2

Player 1	Up	Down
Up	Collision -5, -5	-1, 0
Down	0, -1	Collision -4, -4

Pure Nash Equilibrium: All players play a deterministic best response.

Which equilibrium is better?

Do all simple games have a pure Nash equilibrium?

Practice: Find Pure Nash Equilibria

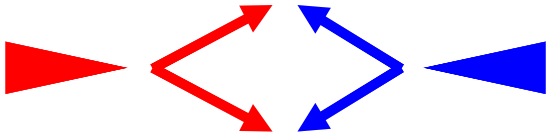
Player 2

Player 1		a	b	c
	a	4,4	2,5	0,0
	b	5,2	3,3	0,0
	c	0,0	0,0	10,10

Missile Defense Game

Missile Defense Game

Missile Defense (simplified)



Defender

Attacker

		Defender: Up	Defender: Down
Attacker	Up	Collision -1, 1	1, -1
	Down	1, -1	Collision -1, 1

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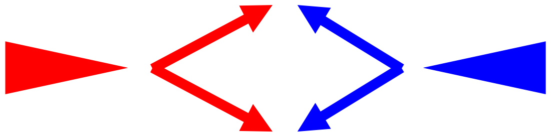

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Attacker \ Defender	Up	Down
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No Pure Nash Equilibrium!

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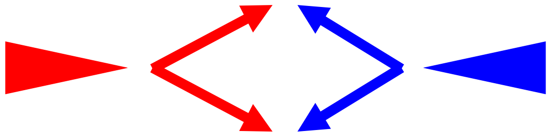

		Defender	
		Up	Down
Attacker	Up	Collision -1, 1	1, -1
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No Pure Nash Equilibrium!

Need a broader solution concept: Mixed Nash equilibrium.

Vocabulary and Notation for Mixed Strategies

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Single Player	Joint

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Two Player Zero Sum:

$$R^1(a) + R^2(a) = 0 \quad \forall a$$

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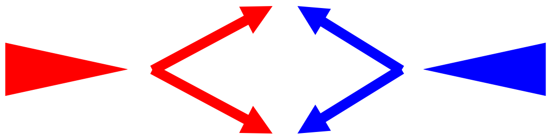
Best Response: Given a joint policy of all other agents, π^{-i}, a best response is a policy π^i that satisfies

$$U^i\left(\pi^i, \pi^{-i}\right) \geq U^i\left(\pi^{i\prime}, \pi^{-i}\right)$$

for all other $\pi^{i\prime}$.

Missile Defense Game

Missile Defense (simplified)



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Missile Defense Game

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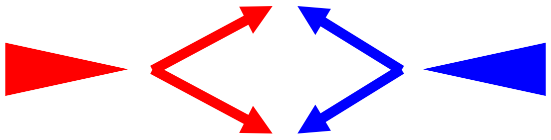

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- A ***Nash equilibrium*** is a joint policy in which all agents are following a best response

Rock-paper scissors

1. Guess the Nash Equilibrium argument
2. Make a qualitative argument that this is an NE based on best responses

		agent 2		
		rock	paper	scissors
agent 1	rock	0, 0	−1, 1	1, −1
	paper	1, −1	0, 0	−1, 1
	scissors	−1, 1	1, −1	0, 0

NE = $\pi^1(r) = \frac{1}{3}$, $\pi^1(p) = \frac{1}{3}$, $\pi^1(s) = \frac{1}{3}$

$\pi^1(r) = 0.34$ $\quad \pi^1(p) = 0.33$ $\quad \pi^1(s) = 0.33$

player 2 BR: $\pi^2(p) = 1$

Rock-paper scissors

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Do all simple games have at least one Nash equilibrium?

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Do all simple games have at least one Nash equilibrium? Yes!! (might be mixed)

Every finite game has a Nash Equilibrium

Every finite game has a Nash Equilibrium

EQUILIBRIUM POINTS IN N-PERSON GAMES

By John F. Nash, Jr.*

Princeton University

Communicated by S. Lefschetz, November 16, 1949

One may define a concept of an n-person game in which each player has a finite set of pure strategies and in which a definite set of payments to the n players corresponds to each n-tuple of pure strategies, one strategy being taken for each player. For mixed strategies, which are probability distributions over the pure strategies, the pay-off functions are the expectations of the players, thus becoming polylinear forms in the probabilities with which the various players play their various pure strategies.

Any n-tuple of strategies, one for each player, may be regarded as a point in the product space obtained by multiplying the n strategy spaces of the players. One such n-tuple counters another if the strategy of each player in the countering n-tuple yields the highest obtainable expectation for its player against the $n - 1$ strategies of the other players in the countered n-tuple. A self-countering n-tuple is called an equilibrium point.

The correspondence of each n-tuple with its set of countering n-tuples gives a one-to-many mapping of the product space into itself. From the definition of countering we see that the set of countering points of a point is convex. By using the continuity of the pay-off functions we see that the graph of the mapping is closed. The closedness is equivalent to saying: if $P_1, P_2, \ldots$ and $Q_1, Q_2, \ldots, Q_n, \ldots$ are sequences of points in the product space where $Q_n \to Q$, $P_n \to P$ and Q_n counters P_n then Q counters P.

Since the graph is closed and since the image of each point under the mapping is convex, we infer from Kakutani's theorem[1] that the mapping has a fixed point (i.e., point contained in its image). Hence there is an equilibrium point.

In the two-person zero-sum case the "main theorem"[2] and the existence of an equilibrium point are equivalent. In this case any two equilibrium points lead to the same expectations for the players, but this need not occur in general.

* The author is indebted to Dr. David Gale for suggesting the use of Kakutani's theorem to simplify the proof and to the A. E. C. for financial support.

[1] Kakutani, S., *Duke Math. J.*, 8, 457–459 (1941).

[2] Von Neumann, J., and Morgenstern, O., *The Theory of Games and Economic Behaviour*, Chap. 3, Princeton University Press, Princeton, 1947.

Every finite game has a Nash Equilibrium

Kakutani's fixed-point theorem

A correspondence f: X → X has a fixed point (i.e., $\mathbf{x} \in f(\mathbf{x})$ for some $\mathbf{x} \in X$) if all of the following conditions hold.

(1) X is a non-empty, closed, bounded, and convex set.

(2) $f(\mathbf{x})$ is non-empty for any $\mathbf{x}$.

(3) $f(\mathbf{x})$ is convex for any $\mathbf{x}$.

(4) The set $\{ (\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in f(\mathbf{x}) \}$ is closed.

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- A fixed point of BR is a Nash Equilibrium
- The BR operator and policy space for finite games meet the conditions above

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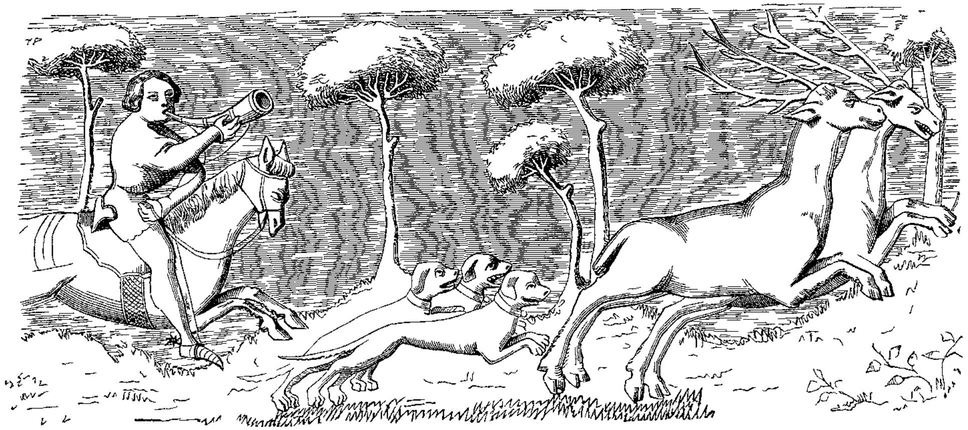
(4) The set $\{ (\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in f(\mathbf{x}) \}$ is closed.

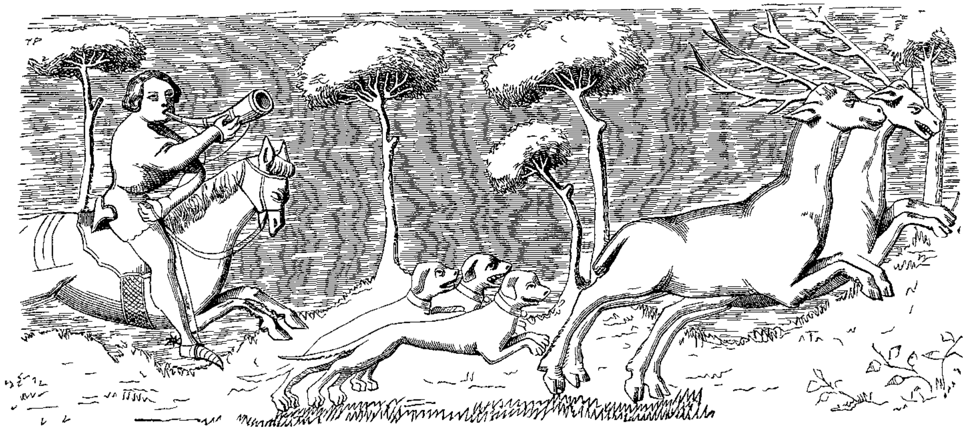
- Let x be a strategy profile, π.
- Let f be BR, that is, the best response operator
- A fixed point of BR is a Nash Equilibrium
- The BR operator and policy space for finite games meet the conditions above
- BR has a fixed point for every finite game, i.e. every finite game has a Nash Equilibrium

Calculating Mixed Nash



- In a Mixed Nash Equilibrium, players must be *indifferent* between two or more actions
- (In large games, finding the support of the mixed strategies is the hard part)

Calculating Mixed Nash



	Stag	Hare
Stag	4, 4	1, 3
Hare	3, 1	2, 2

- In a Mixed Nash Equilibrium, players must be *indifferent* between two or more actions
- (In large games, finding the support of the mixed strategies is the hard part)

→ P1 indifferent

player i's utility

P1:stag $4\pi^2(s) + 1\pi^2(h) = u$

P1:hare $3\pi^2(s) + 2\pi^2(h) = u$

$\pi^2(s) + \pi^2(h) = 1$

$\pi^2(s) = 0.5$
$\pi^2(h) = 0.5$
$u = 2.5$

$$\begin{bmatrix} 4 & 1 & -1 \\ 3 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \pi^2(s) \\ \pi^2(h) \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

General approach to find Nash Equilibria

General approach to find Nash Equilibria

$$\underset{\pi, U}{\text{minimize}} \quad \sum_i \left(U^i - U^i(\pi) \right)$$

$$\text{subject to} \quad U^i \geq U^i(a^i, \pi^{-i}) \text{ for all } i, a^i$$

Each player plays a best response

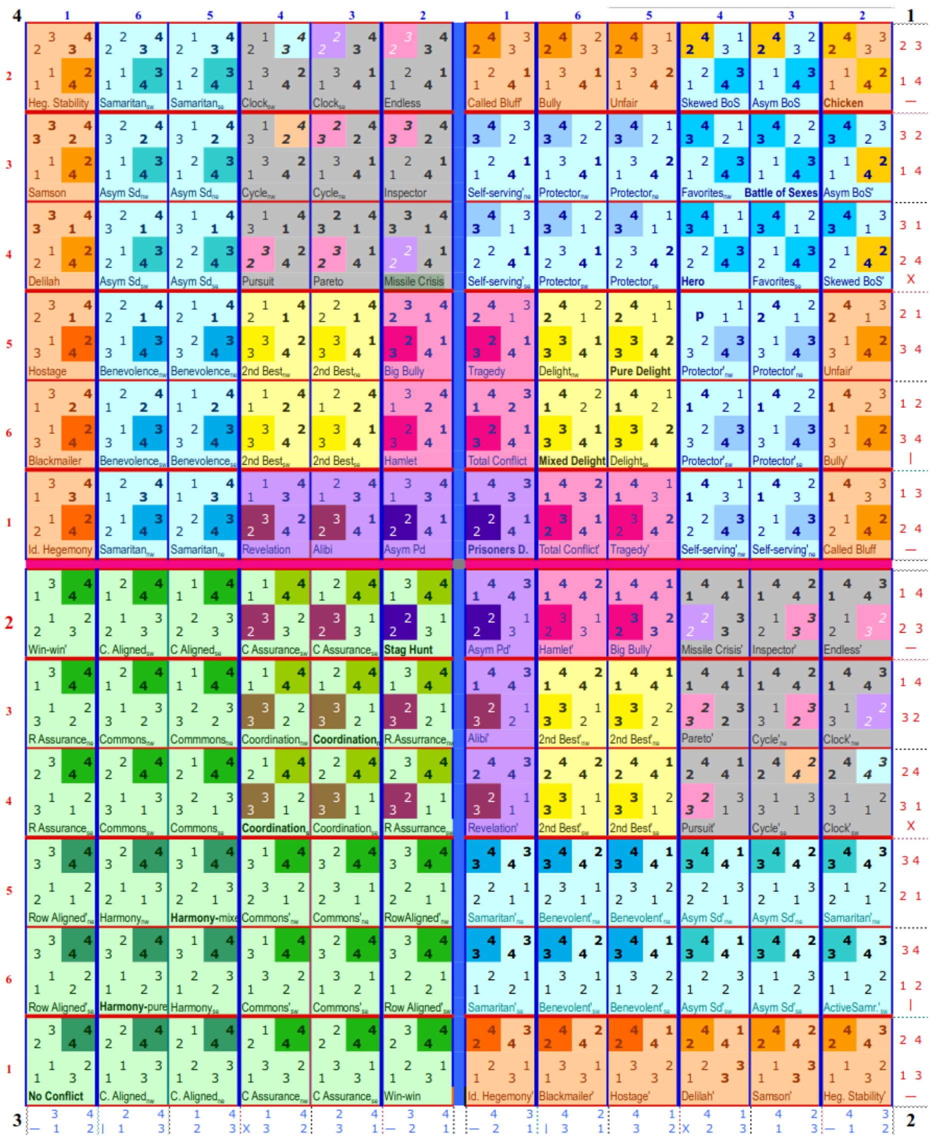
$$\sum_{a^i} \pi^i(a^i) = 1 \text{ for all } i$$

$$\pi^i(a^i) \geq 0 \text{ for all } i, a^i$$

π are probability distributions

General approach to find Nash Equilibria

$$\begin{aligned}
\underset{\pi,U}{\text{minimize}} \quad & \sum_i \left(U^i - U^i(\pi) \right) \\
\text{subject to} \quad & U^i \geq U^i(a^i, \pi^{-i}) \text{ for all } i, a^i \\
& \sum_{a^i} \pi^i(a^i) = 1 \text{ for all } i \\
& \pi^i(a^i) \geq 0 \text{ for all } i, a^i
\end{aligned}$$



Topology of bimatrix games:

Algorithms that use best response

Iterated Best Response: randomly cycle between agents who play the best response for the current policy (converges to Nash for certain narrow classes of games)

Fictitious Play:

1. Estimate maximum likelihood policies for opponents:

$$\pi^j(a^j) \propto N(j, a^j)$$

2. Play best response to estimated policy

does not necessarily converge

(converges to Nash for wider class of games, notably zero-sum)

average policy over time

~~Battle of the Sexes~~
Bach or Stravinsky

- Two people want to go to a concert
- P1 prefers Bach, P2 Stravinsky

~~Battle of the Sexes~~
Bach or Stravinsky

- Two people want to go to a concert
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	B	**S**
B	2, 1	0,0
S	0, 0	1, 2

~~Battle of the Sexes~~
Bach or Stravinsky

- Two people want to go to a concert
- P1 prefers Bach, P2 Stravinsky

	B	**S**
B	2, 1	0,0
S	0, 0	1, 2

Correlated Equilibrium

- A *correlated joint policy* is a single distribution over the joint actions of all agents.
- A *correlated equilibrium* is a correlated joint policy where no agent *i* can increase their expected utility by deviating from their current action to another.
- Easier to find than Nash equilibrium (Linear Program)

https://youtube.com/shorts/w3q77ZZIqwA
?si=J8H6L6W5kTRs-mUx

Recap

https://youtube.com/shorts/w3q77ZZIqwA
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Recap

- Games provide a mathematical framework for analyzing interaction between rational agents https://youtube.com/shorts/w3q77ZZIqwA?si=J8H6L6W5kTRs-mUx

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Recap

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- If every player is playing a best response, that joint policy is a Nash Equilibrium

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?si=J8H6L6W5kTRs-mUx

Recap

- Games provide a mathematical framework for analyzing interaction between rational agents
- Games may not have a single "optimal" solution; instead there are equilibria
- If every player is playing a best response, that joint policy is a Nash Equilibrium
- Every finite game has at least one Nash Equilibrium (pure or mixed)
- Mixed Nash equillibria occur when players are indifferent between two outcomes