Logics for Artificial Intelligence Homework 3/10 Sebastian Berns March 5th 2017

Skolemization

\mathbf{A}

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\begin{array}{lll}
& \neg \exists_{x}(P_{x} \wedge \neg Q_{x} \wedge \forall_{y}(R_{yy} \rightarrow S_{xy})) \\
& \vee \neg (P_{x} \wedge \neg Q_{x} \wedge \forall_{y}(\neg R_{yy} \vee S_{xy})) \\
& \vee \forall_{x}(\neg P_{x} \vee Q_{x} \vee \neg \forall_{y}(\neg R_{yy} \vee S_{xy})) \\
& \vee \forall_{x}(\neg P_{x} \vee Q_{x} \vee \exists_{y} \neg (\neg R_{yy} \vee S_{xy})) \\
& \vee \forall_{x} \exists_{y}(\neg P_{x} \vee Q_{x} \vee (R_{yy} \wedge \neg S_{xy})) \\
& \vee \forall_{x} \exists_{y}((\neg P_{x} \vee Q_{x} \vee R_{yy}) \wedge (\neg P_{x} \vee Q_{x} \vee \neg S_{xy})) \quad \text{PCNF } \{f_{x}/y\} \\
& \vee \forall_{x} ((\neg P_{x} \vee Q_{x} \vee R_{f_{x}f_{x}}) \wedge (\neg P_{x} \vee Q_{x} \vee \neg S_{xf_{x}})) \quad \text{SNF}
\end{array}
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\mathbf{B}

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\exists_{x}(P_{x} \wedge ((\forall_{y}(Q_{y} \rightarrow \neg R_{xy})) \rightarrow \neg \exists_{z}R_{zx})) 

\exists_{x}(P_{x} \wedge (\neg(\forall_{y}(\neg Q_{y} \vee \neg R_{xy})) \vee \neg \exists_{z}R_{zx})) 

\exists_{x}(P_{x} \wedge ((\exists_{y} \neg (\neg Q_{y} \vee \neg R_{xy})) \vee \forall_{z} \neg R_{zx})) 

\exists_{x}(P_{x} \wedge (\exists_{y}(Q_{y} \wedge R_{xy}) \vee \forall_{z} \neg R_{zx})) 

\exists_{x}\exists_{y}\forall_{z}(P_{x} \wedge ((Q_{y} \wedge R_{xy}) \vee \neg R_{zx})) 

\vdots \exists_{x}\exists_{y}\forall_{z}(P_{x} \wedge (Q_{y} \vee \neg R_{zx}) \wedge (R_{xy} \vee \neg R_{zx})) \quad \text{PCNF } \{^{a}/_{x}, \, ^{b}/_{y}\} 

\forall_{z}(P_{a} \wedge (Q_{b} \vee \neg R_{za}) \wedge (R_{ab} \vee \neg R_{za})) \quad \text{SNF}
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\mathbf{C}

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\begin{array}{lll}
1 & \left(\exists_{x}(P_{x} \wedge \forall_{y}(R_{xy} \rightarrow \neg Q_{y}))\right) \rightarrow \neg \forall_{x} \forall_{y} T_{xy} \\
2 & \neg (\exists_{x}(P_{x} \wedge \forall_{y}(\neg R_{xy} \vee \neg Q_{y}))) \vee \neg \forall_{x'} \forall_{y'} T_{x'y'} \\
3 & \left(\forall_{x} \neg (P_{x} \wedge \forall_{y}(\neg R_{xy} \vee \neg Q_{y}))\right) \vee \exists_{x'} \exists_{y'} \neg T_{x'y'} \\
4 & \left(\forall_{x}(\neg P_{x} \vee \neg \forall_{y}(\neg R_{xy} \vee \neg Q_{y}))\right) \vee \exists_{x'} \exists_{y'} \neg T_{x'y'} \\
5 & \left(\forall_{x}(\neg P_{x} \vee \exists_{y} \neg (\neg R_{xy} \vee \neg Q_{y}))\right) \vee \exists_{x'} \exists_{y'} \neg T_{x'y'} \\
6 & \left(\forall_{x}(\neg P_{x} \vee \exists_{y}(R_{xy} \wedge Q_{y}))\right) \vee \exists_{x'} \exists_{y'} \neg T_{x'y'} \\
7 & \left(\forall_{x} \exists_{y}(\neg P_{x} \vee (R_{xy} \wedge Q_{y}))\right) \vee \exists_{x'} \exists_{y'} \neg T_{x'y'} \\
8 & \left(\forall_{x} \exists_{y}((\neg P_{x} \vee R_{xy} \wedge (\neg P_{x} \vee Q_{y}))) \vee \exists_{x'} \exists_{y'} \neg T_{x'y'} \right) \\
9 & \forall_{x} \exists_{y} \exists_{x'} \exists_{y'} (((\neg P_{x} \vee R_{xy} \vee \neg T_{x'y'}) \wedge (\neg P_{x} \vee Q_{y} \vee \neg T_{x'y'})) & \text{PCNF } \{f_{x}/y, f_{x}/x', f_{x}/y'\} \\
10 & \forall_{x} \exists_{y} \exists_{x'} \exists_{y'} ((\neg P_{x} \vee R_{xy} \vee \neg T_{x'y'}) \wedge (\neg P_{x} \vee Q_{y} \vee \neg T_{x'y'})) & \text{PCNF } \{f_{x}/y, f_{x}/x', f_{x}/y'\} \\
11 & \forall_{x} ((\neg P_{x} \vee R_{xx} \vee \neg T_{qx} f_{xx}) \wedge (\neg P_{x} \vee Q_{f_{x}} \vee \neg T_{qx} f_{xx})) & \text{SNF}
\end{array}
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 \mathbf{D}

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\begin{array}{ll}
1 & \exists_{x}\forall_{y}(\neg\exists_{z}P_{z} \rightarrow \forall_{w}(T_{wx} \rightarrow (Q_{x} \vee R_{wy}))) \\
2 & \exists_{x}\forall_{y}(\neg\neg\exists_{z}P_{z} \vee \forall_{w}(\neg T_{wx} \vee (Q_{x} \vee R_{wy}))) \\
3 & \exists_{x}\forall_{y}\exists_{z}\forall_{w}(P_{z} \vee (\neg T_{wx} \vee (Q_{x} \vee R_{wy}))) \\
4 & \exists_{x}\forall_{y}\exists_{z}\forall_{w}(P_{z} \vee \neg T_{wx} \vee Q_{x} \vee R_{wy}) & \text{PCNF } \{^{a}/_{x}, \, ^{f_{y}}/_{z}\} \\
5 & \forall_{y}\forall_{w}(P_{f_{y}} \vee \neg T_{wa} \vee Q_{a} \vee R_{wy}) & \text{SNF}
\end{array}
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