Report - Mid-price estimation for European corporate bonds: a particle filtering approach

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1 Introduction

This paper from Olivier Guéant and Jiang Pu presents a Bayesian method for estimating the mid-price of corporate bonds using the real-time information which is available to a market-maker. The paper implements methods from the particle filtering / sequential Monte Carlo literature.
Market makers have several problems with static and dynamic components. The static problem is how to give a spread for the quotation : we know that a tight spread can generate numerous trades while a large spread can generate few trades. The dynamic problem is the adaptation of the bid-ask spread to the inventory to limit the risks of possibly adversely moves for the assets they have in their inventory.
Some articles are already tackling the problem of the estimation of the bid-ask spread in the financial literature [2], [3], [4]. But those articles provide bid-ask spreads for quite liquid assets which is not the case for corporate bonds. They can give good estimation of the bid ask spread compared to a benchmark but it requires to have enough data to have a good approximation of the spread. While, as we will study later, with the SMC approach we can have good approximations for illiquid assets. In this report, we will resume some sections of the main paper in the part 2 and 3 to describe the algorithm, the scenarios and the simulation of the trajectories. We will then think of the calibration of the parameters. And to conclude we will present our results and the potential improvements to give.

2 The modeling framework and the different scenarios

Here, we consider a set of d corporate bonds. Instead of working with the bond prices directly, we work with the yields to benchmark (YtB). Working with this quantity allows to remove the risk-free interest rate component of bond prices.

With $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\mathbb{R}_+}, \mathbb{P})$ a filtered probability space, the model for the mid-YtB of the d corporate bonds is the following d-dimensional process $(y_t)_t$:

$$\forall i \in \{1, \cdots, d\}, dy_t^i = \sigma^i dW_t^i \tag{1}$$

with $(W_t)_t$ a d-dimensional Brownian motion adapted to $(\mathcal{F}_t)_{t\in\mathbb{R}_+}$ and $d\langle W_t^i, W_t^j\rangle = \rho^{i,j}dt$.
For the half bid-ask spread process of asset i, we first introduce a d-dimensional Ornstein-Ulhenbeck process $(x_t)_t$:

$$dx_t = -Ax_t dt + V dB_t \tag{2}$$

with A and V which are $d \times d$ matrices, and $(B_t)_t$ is a d-dimensional Brownian motion adapted to $(\mathcal{F}_t)_{t\in\mathbb{R}_+}$ and independent from the process $(W_t)_t$. Then, the half bid-ask spread process $(\psi_t^i)_t$ of asset i is defined for $i \in \{1, \cdots, d\}$ by :

$$\psi_t^i = \Psi^i \exp(x_t^i) \tag{3}$$

This means that for the point of view of the market maker, $y_t^i + \psi_t^i$ and $y_t^i - \psi_t^i$ are respectively the bid-YtB and the ask-YtB.
There are 5 different scenarios for the market maker depending on the information available to him :

- $J_t = (i, 1)$: A client buys bond i from the market maker at time t. The YtB of the transaction is $Y_t^i = y_t^i - \psi_t^i + \epsilon_t^i$ and the observation of the market maker is then $O_t = Y_t^i$.

- $J_t = (i, 2)$: A client sells bond i to the market maker at time t. The YtB of the transaction is $Y_t^i = y_t^i + \psi_t^i + \epsilon_t^i$ and the observation of the market maker is then $O_t = Y_t^i$.

- $J_t = (i, 3)$: A client buys bond i from another market maker at time t. The market maker we study has proposed a YtB Z_t^i but a better price was proposed by the other market maker. We assumed that the YtB of the transaction is $Y_t^i = y_t^i - \psi_t^i + \epsilon_t^i$. The observation of the market maker is the $O_t = 1_{Y_t^i \geq Z_t^i}$.

- $J_t = (i, 4)$: A client sells bond i from another market maker at time t. The market maker we study has proposed a YtB Z_t^i but a better price was proposed by the other market maker. We assumed that the YtB of the transaction is $Y_t^i = y_t^i + \psi_t^i + \epsilon_t^i$. The observation of the market maker is the $O_t = 1_{Y_t^i \leq Z_t^i}$.

- $J_t = (i, 5)$: It is a transaction on the inter-dealer broker market. The YtB of the transaction (Y_t^i) is in the range $[y_t^i - \alpha_t^i + \epsilon_t^i, y_t^i + \alpha_t^i + \epsilon_t^i]$ with α_t^i chosen proportional to ψ_t^i. The observation of the market maker is $O_t = Y_t^i$.

3 Simulation of trajectories by induction

This part is the same as the 2.3 in the article. But we will give some details about the implementations in the notebook.
First, we want to mention that the K-sample $((y^m_{n,k}, x^m_{n,k}, \psi^m_{n,k})_{0\leq n\leq m})_{1\leq k\leq K}$ is represented as three matrices. As an example, the matrix for $((y^m_{n,k})_{0\leq n\leq m})_{1\leq k\leq K}$ is a matrix with m rows and K columns where the element of the matrix are vectors representing the bonds. For the first 2 steps, we follow the article as it is mentioned. The *remark 5* can be useful and it is a remark to keep in mind in case of a bad simulation of the algorithm.
For the resampling step, we use a systematic resampling as it is mentioned in [1] in the part 9.7. Then we combine all the steps in a single function called *simulate* to simulate the different steps. For the moment, we could only run the algorithm with simulated data because the CBBT are coming from Bloomberg and we do not have access to Bloomberg to download this data.

4 Reflexion on the calibration of the parameters

For the simulations, we work with 3 bonds. Because we did not have access to the CBBT prices of the bonds, we took the value provided in the paper. We then have the following correlation matrix for the mid-price of the 3 bonds :

$$\begin{pmatrix} \sigma^1 & \rho^{1,2} & \rho^{1,3} \\ \rho^{1,2} & \sigma^2 & \rho^{2,3} \\ \rho^{1,3} & \rho^{2,3} & \sigma^3 \end{pmatrix} \tag{4}$$

with :

- $\sigma^1 = 0.50\text{bp.day}^{-1/2}$
- $\sigma^2 = 0.62\text{bp.day}^{-1/2}$
- $\sigma^3 = 0.69\text{bp.day}^{-1/2}$
- $\rho^{1,2} = 0.843$, $\rho^{1,3} = 0.835$, $\rho^{2,3} = 0.887$

We made the simulations for $K = 1.000$ particles, $V = 10^{-6} I_3$ and $A = I_3$ and Γ as mentioned in part 2.1 with : $\Gamma^{i,j}(\tau) = \frac{1}{a^i+a^j}(1 - \exp(-(a^i + a^j)\tau))(VV')^{i,j}$. Also for the sake of simplicity (and because it is not clearly mentioned how to estimate this quantity with simulated data), we decided to take $\Psi^i = 1$ for all $i \in \{1, \cdots, d\}$. We also took $\tau_{m+1} - \tau_m = 1$ because we are working on simulated data with a price each day (as an example a closing price) and the simulated data are constructed such that $\tau_{m+1} - \tau_m = 1$. In the case of real data, $\tau_{m+1} - \tau_m$ can be different because it depends on the demand of the clients and on the context there exists in the real market. This is visible in the graphics depicted in the article, as an example for the Bond 1 there was a lot of transactions in the first six days compared to the last six days. Also we introduced a variable *action* in the creation of the data which allows us to determine the action the market maker is doing at time τ_{m+1}. We could discuss with Jiang Pu about the implementation of this part and he told us that we have the possibility to determine the action while working with real data. But it is not our case so we implemented a random decision case to choose the $J_{\tau_{m+1}}$.

5 Results and discussions

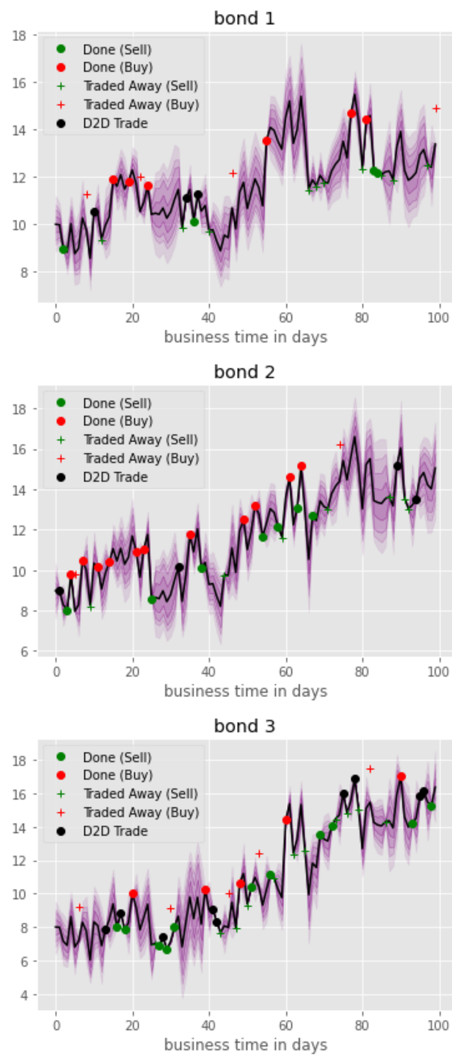
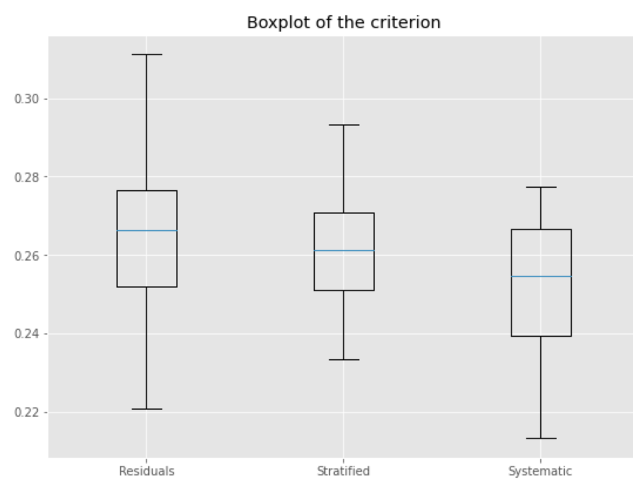


Figure 1: Estimations

We made a simulation for 100 business days. We have represented the confidence intervals corresponding to quantile envelopes (25%-75%, 10%-90%, 5%-95%, 1%-99%). Compared to the figures in the article we can see that the estimation have more volatility (if we look at the median for example). But this is because the data we simulated also have more volatility. If we change the parameters (A, V, Γ) and change some ratios in the function *data next step* for the action 3,4 and 5, we can maybe have less volatility in the YtB.

We mentioned in part 3 that we do a systematic resampling if $ESS \leq ESS_{min} = \frac{K}{2}$ because it is recommended by practitioners to use systematic resampling. We are going to verify that by running the algorithm 30 times with $T = 100$ and doing the resampling with the residuals method, the stratified method and the systematic method. We use the criterion $\frac{1}{2K}\sum_{k=1}^{K}|KW^k - \tilde{W}^k|$ where $\tilde{W}^k$ is the number of times particle k is resampled to evaluates how much resampling degrades the approximation brought by importance sampling. In the algorithm, we compute this criterion every time a resampling step is done. We compute the mean for each running of the algorithm and we do the following boxplot :



We can see that the mean of the criterion is smaller for the systematic resampling and then it is interesting to use this resampling to limit the degradation of the approximation.

In the figure 1, we can also observe something quite interesting but also reassuring, it is the fact that when we do not have observations on the bond i, the distribution of the K particles is more dispersed. As an example, for the bond 1, we do not have observations between day 22 and day 33 but because of the observations on the other bonds we can have the distribution of the mid-price but the confidence intervals are way more dispersed than when we have an observation (see day 40 for a Done (Sell) observation).

References

[1] N. Chopin and O. Papaspiliopoulos, *An Introduction to Sequential Monte Carlo*, Springer, 2020.

[2] Corwin, S. A., and P. Schultz. 2012. A simple way to estimate bid-ask spreads from daily high and low prices. *Journal of Finance* 67:719–60.

[3] Abdi, F., and A. Ranaldo. 2017. A simple estimation of bid-ask spreads from daily close, high, and low prices. *Review of Financial Studies* 30:4437–80.

[4] David Ardia, Emanuele Guidotti, and Tim A. Kroencke, Efficient Estimation of Bid-Ask Spreads from Open, High, Low, and Close Prices, 2022