1 Functions

1.1 Functions and The Analysis of Graphical Information

1.2 Properties of Functions

1.3 Graphic Functions on Calculators and Computers; Computer Algebra Systems

1.4 New Functions from Old

1.5 Mathematical Models; Linear Models

1.6 Families of Functions

1.7 Parametric Equations

2 Limits and Continuity

2.1 Limits (An Intuitive Introduction)

The Tangent Line, Area, and Velocity Problems | Tangent Lines and Limits | Instantenous Velocity and Limits | Limits | Numerical Pitfalls | One-Sided Limits | The Relationship Between One-Sided and Two-Sided Limits | A First Look at Continuity | Infinite Limits and Vertical Asy | Limits at Infinity and Horizontal Asymptotes | How Limits at Infinity Can Fail to Exist

2.2 Limits (Computational Techniques)

Some Basic Limits | Limits of Polynomials as $x \to a$ | Limits of x^n as $x \to +\infty$ or $x \to -\infty$ | Limits of Polynomials as $x \to +\infty$ or $x \to -\infty$ | Limits of Rational Functios as $x \to a$ | Limits of Rational Functions as $x \to +\infty$ or $x \to -\infty$ | A Quick Method for Finding Limits of Rational

- **Limits Involving Radicals**

$$\lim_{x\to+\infty} \sqrt[3]{\frac{3x+5}{6x-8}} = \sqrt[3]{\lim_{x\to+\infty} \frac{3x+5}{6x-8}} = \frac{1}{\sqrt[3]{2}}$$

$$\lim_{x\to+\infty} \frac{\sqrt{x^2+2}}{3x-6} = \lim_{x\to+\infty} \frac{\sqrt{x^2+2}/|x|}{(3x-6)/|x|} \overset{x\geq 0}{=} \lim_{x\to+\infty} \frac{\sqrt{x^2+2}/\sqrt{x^2}}{(3x-6)/x} = \lim_{x\to+\infty} \frac{\sqrt{1+2/x^2}}{3-6/x} = \frac{\sqrt{\lim\limits_{x\to+\infty} 1+2/x^2}}{\lim\limits_{x\to+\infty} 3-6/x} = \frac{1}{3}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+2}}{3x-6} = \lim_{x\to-\infty} \frac{\sqrt{x^2+2}/|x|}{(3x-6)/|x|} \overset{x\leq 0}{=} \lim_{x\to-\infty} \frac{\sqrt{x^2+2}/\sqrt{x^2}}{(3x-6)/(-x)} = \lim_{x\to-\infty} \frac{\sqrt{1+2/x^2}}{-3+6/x} = \frac{\sqrt{\lim\limits_{x\to-\infty} 1+2/x^2}}{\lim\limits_{x\to-\infty} -3+6/x} = -\frac{1}{3}$$

Limits of Functions Defined Piecewise

2.3 Limits (Discussed More Rigorously)

Definition of a Limit | The Value of δ Is Not Unique | Limits as $x \to +\infty$ or $x \to -\infty$ | Infinite Limits

2.4 Continuity

Definition of Continuity | Continuity in Applications | Continuity of Polynomials | Some Properties of Continuous Functions | Continuity of Ra | Continuity of Compositions | Continuity from the Left and from the Right | The Intermediate-Value Theorem | Approximating Roots Using th | Approximating Roots by Zooming with a Graphic Utility

2.5 Limits and Continuity of Trigonometric Functions

Continuty of Trigonometric Functions | Obtaning Limits by Squeezing

3 The Derivative

Slope of a Tangent Line | Average Versus Instantaneous Velocity | Average and Instantaneous Rate of Change

3.1 Tangent Lines and Rates of Change

Tangent Lines Defined Precisely | Slopes of Tangent Lines by Zooming

3.2 The Derivative

The Derivative | Differentiability | Relationship Between Differentiability and Continuity | Derivative Notation | Other Notations | Derivatives a

3.3 Techniques of Differentiation

Derivative of a Constant | Derivative of x to a Power | Derivative of a Constant Times a Function | Derivatives of Sums and Differences
Derivative of a Product | Derivative of a Quotient | Derivative of a Reciprocal | The Power Rule for Integer Exponents | Higher Derivatives

3.4 Derivatives of Trigonometric Functions

Derivatives of the Trigonometric Functions

3.5 The Chain Rule

Derivatives of Compositions | Generalized Derivative Formulas | An Alternative Approach to Using the Chain Rule | Differentiating Using Com

3.6 Local Linear Approximation; Diffferentials

Increments | Differentials | Local Linear Approximation | Error in Local Linear Approximations | Error Propagation in Applications | Differenti

4 Logarithmic and Exponential Functions

4.1 Inverse Functions

Inverse Functions | Domain and Range of Inverse Functions | A Method for Finding Inverses | Existence of Inverse Functions | Graphs of Invers
Increasing or Decreasing Functions Have Inverses | Restricting Domains to Make Functions Invertible | Continuity of Inverse Functions
Differentiability of Inverse Functions | Graphing Inverse Functions with Graphing Utilities

4.2 Logarithmic and Exponential Functions

Irrational Exponents | The Family of Exponential Functions | Logarithms | Logarithmic Functions | Solving Equations Involving Exponentials a
Change of Base Formula for Logarithms | Logarithmic Scales in Science and Engineering | Exponential and Logarithmic Growth

4.3 Implicit Differentiation

Functions Defined Explicitly and Implicitly | Graphs of Equations in x and y | Implicit Differentiation | Differentiability of Functions Defined I
Derivatives of Rational Powers of x | Derivatives of Inverse Functions

4.4 Derivatives of Logarithmic and Exponential Functions

Derivatives of Logarithmic Functions | Logarithmic Differentiation | Derivatives of Irrational Powers of x | Derivatives of Exponential Functions

4.5 Derivatives of Inverse Trigonometric Functions

Inverse Trigonometric Functions | Evaluating Inverse Trigonometric Functions | Identities for Inverse Trigonometric Functions | Derivatives of t
Differentiability of the Inverse Trigonometric Functions

4.6 Related Rates

Rates of Changes Using the Chain Rule

4.7 L'Hôpital's Rule ; Inderterminate Forms

Indeterminates Forms of Type $0/0$ | L'Hôpital's Rule | Indeterminate Forms of Type ∞/∞ | Analyzing the Growth of Exponential Functions U
Indeterminate Forms of Type $0 \cdot \infty$ | Indeterminate Forms of Type $\infty - \infty$ | Indeterminate Forms of Type 0^0, ∞^0, 1^∞

5 Analysis of Functions and Their Graphs

5.1 Analysis of Functions I: Increase, Decrease, and Concavity

5.2 Analysis of Functions II: Relative Extrema; First and Second Derivative Tests

5.3 Analysis of Functions III: Applying Technology and the Tools of Calculus

6 Applications of the Derivative

6.1 Absolute Maxima and Minima

6.2 Applied Maximum and Minimum Problems

6.3 Rectilinear Motion (Motion Along a Line)

6.4 Newton's Method

6.5 Rolle's Theorem; Mean-Value Theorem

7 Integration

7.1 An Overview of the Area Problem

Defining Area | The Rectangle Method for Finding Areas | The Antiderative Method for Finding Areas

7.2 The Indefinite Integral; Integral Curves and Direction Fields

The Indefinite Integral

• **Integration Formulas**

$$\int \mathrm{d}x = x + C$$

$$\int x^r \,\mathrm{d}x = \frac{x^{r+1}}{r+1} + C$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int (\sec x)^2 \,\mathrm{d}x = \tan x + C$$

$$\int (\csc x)^2 \,\mathrm{d}x = -\cot x + C$$

$$\int \sec x \tan x \,\mathrm{d}x = \sec x + C$$

$$\int \csc x \cot x \,\mathrm{d}x = -\csc x + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int b^x \,\mathrm{d}x = \frac{b^x}{\ln b} + C$$

$$\int \frac{1}{x} \,\mathrm{d}x = \ln |x| + C$$

• **Properties of the Indefinite Integral**

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\int f(x)\,\mathrm{d}x\right) = f(x)$$

Integral Curves | Integration from the Viewpoint of Differential Equations | Direction Fields

7.3 Integration by Substitution

u-Substitution | Integration Using Computer Algebra Systems

7.4 Sigma Notation

Sigma Notation | Changing the Index of Summation | Properties of Sigma Notation | Summation Formulas

7.5 The Definite Integral

A Definition of Area | The Definite Integral of a Continuous Function | The Riemann Integral | Integrability | Properties of the Definite Integra
Conditions for Integrability

7.6 The Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus | The Relationship Between Definite and Indefinite Integrals | Dummy Variables | The Mean-Value The
Part 2 of the Fundamental Theorem of Calculus | Differentiation and Integration are Inverse Processes

7.7 Rectilinear Motion Revisited; Average Value

Finding Position and Velocity by Integration | Uniformly Accelerated Motion | The Free-Fall Model | Integrating Rates of Change | Displaceme
Distance Traveled in Rectilinear Motion | Analyzing the Velocity Versus Time Curve | Average Value of a Continuous Function | Average Veloc

7.8 Evaluating Definite Integrals by Substitution

Two Methods for Making Substitutions in Definite Integrals

7.9 Logarithmic Functions from the Integral Point of View

The Link Between Natural Logarithms and Integrals | Approximating $\ln x$ Numerically | Differentiability and Continuity of $\ln x$ and e^x
The Definition of e revisited | Functions Defined by Integrals | Evaluating and Graphing Functions Defined by Integrals | Integrals with Functio

8 Applications of the Definite Integral in Geometry, Science, and Engineering

8.1 Area Between Two Curves

8.2 Volumes by Slicing; Disks and Washers

8.3 Volumes by Cylindrical Shells

8.4 Length of a Plane Curve

8.5 Area of a Surface of Revolution

8.6 Work

8.7 Fluid Pressure and Force

8.8 Hyperbolic Functions and Hanging Cables

9 Principles of Integral Evaluation

9.1 An Overview of Integration Methods

Methods for Approaching Integration Problems | A Review of Familiar Integration Formulas

9.2 Integration by Parts

Derivation of the Formula for Integration by Parts | Integration by Parts for Definite Integrals | Reduction Formulas

9.3 Trigonometric Integrals

Integrating Powers of Sine and Cosine | Integrating Products of Sines and Cosines | Integrating Powers of Tangent and Secant | Integrating Pro
An Alternative Method for Integrating Powers of Sine, Cosine, Tangent and Secant | Mercator's Map of the World

9.4 Trigonometric Substitutions

The Method of Trigonometric Substitution | Integrals Involving $ax^2 + bx + c$

9.5 Integrating Reational Functions by Partial Fractions

Partial Fractions | Finding the Form of a Partial Fraction Decomposition | Linear Factors | Quadratic Factors | Integrating Improper Rational
Concluding Remarks

9.6 Using Tables of Integrals and Computer Algebra Systems

Integral Tables | Perfect Matches | Matches Requiring Substitutions | Matches Requiring Reduction Formulas | Matches Requiring Special Subs
Integrating with Computer Algebra Systems | Computer Algebra Systems can Fail

9.7 Numerical Integration; Simpson's Rule

A Review of Riemann Sum Approximations | Trapezoidal Approximation | Comparison of the Midpoint and Trapezoidal Approximations
Simpson's Rule | Error Estimates | A Comparison of the Three Methods

9.8 Improper Integrals

Improper Integrals | Integrals over Infinite Intervals | Integrals whose Integrands have Infinite Discontinuities | The Application of Improper Int

10 Mathematical Modeling with Differential Equations

10.1 First-Order Differential Equations and Applications

10.2 Direction Fields; Euler's Method

10.3 Modeling with Differential Equations

11 Infinite Series

11.1 Sequences

Definition of a Sequence | Graphs of Sequences | Limit of a Sequence | The Squeezing Theorem for Sequences | Sequences Defined Recursively

11.2 Monotone Sequences

Terminology | Testing for Monotonicity | Properties that Hold Eventually | An Intuitive View of Convergence | Convergence of Monotone Seque

11.3 Infinite Series

Sum of Infinite Series | Geometric Series | Harmonic Series

11.4 Convergence Tests

The Divergence Test | Algebraic Properties of Infinite Series | The Integral Test | p-Series | Proof of the Integral Test

11.5 Taylor and Maclaurin Series

Local Quadratic Approximations | MacLaurin Polynomials | Taylor Polynomials | Sigma Notation for Taylor and MacLaurin Polynomials
Taylor and MacLaurin Series

11.6 The Comparison, Ratio, and Root Tests

The Comparison Test | Using the Comparison Test | The Limit Comparison Test | The Ratio Test | The Root Test

11.7 Alternating Series; Conditional Convergence

Alternating Series | Approximating Sums of Alternating Series | Absolute Convergence | Conditional Convergence | The Ratio Test for Absolut
Summary of Convergence Tests

11.8 Power Series

Power Series in x | Radius and Interval of Convergence | Finding the Interval of Convergence | Power Series in $x - x_0$ | Functions Defined by Po

11.9 Convergence of Taylor Series; Computational Methods

The nth remainder | Estimating the nth remainder | Approximating Trigonometric Functions | Roundoff and Truncation Error | Approximating
Approximating Logarithms | Approximating π | Binomial Series

11.10 Differentiating and Integrating Power Series; Modeling with Taylor Series

Differentiating Power Series | Integrating Power Series | Power Series Representations Must Be Taylor Series | Some Practical Ways to Find Ta
Finding MacLaurin Series by Multiplication and Division | Modeling Physical Laws with Taylor Series

12 Analytic Geometry in Calculus

12.1 Polar Coordinates

12.2 Tangent Lines and Arc Length for Parametric and Polar Curves

12.3 Area in Polar Coordinates

12.4 Conic Sections in Calculus

12.5 Conic Sections in Polar Coordinates

13 Three-Dimensional Space; Vectors

13.1 Rectangular Coordinates in 3-Space; Spheres; Cylindrical Surfaces

13.2 Vectors

13.3 Dot Product; Projections

13.4 Cross Product

13.5 Parametric Equations of Lines

13.6 Planes in 3-Space

13.7 Quadric Surfaces

13.8 Cylindrical and Spherical Coordinates

14 Vector-Valued Functions

14.1 Introduction to Vector-Valued Functions

14.2 Calculus of Vector-Valued Functions

14.3 Change of Parameters; Arc Length

14.4 Unit Tangent, Normal, and Binormal Vectors

14.5 Curvature

14.6 Motion Along a Curve

14.7 Kepler's Laws of Planetary Motion

15 Partial Derivatives

15.1 Functions of Two or More Variables

Notation and Terminology | Graphs of Functions of Two Variables | Graphs of Functions of Two Variables Using Technology | Level Curves | Contour Plots Using Technology | Level Surfaces | Graphing Functions of Two Variables Using Technology

15.2 Limits and Continuity

Open and Closed Sets | Bounded Sets | Limits Along Curves | General Limits of Functions of Two Variables | Properties of Limits | Relationsh | Continuity | Limits at Points of Discontinuity | Extension to Three Variables

15.3 Partial Derivatives

Partial Derivatives of Functions of Two Variables | Partial Derivatives Viewed as Rates of Change and Slopes | Partial Derivative Notation | Implicit Partial Differentiation | Higher-Order Partial Derivatives | The Wave Equation | Partial Derivatives of Functions With More Than Two

15.4 Differentiability and Chain Rules

Differentiability of Functions of Two Variables | Sufficient Conditions for Differentiability | Equality of Mixed Partials | Chain Rules | Related R

15.5 Tangent Planes; Total Differentials for Functions of Two Variabls

Tangent Planes | The Geometric Signifiance of Differentiability | Total Differentials | Local Linear Approximation | Approximations Using Tota

15.6 Directional Derivatives and Gradients for Functions of Two Variables

Directional Derivatives | The Relationship Between Directional Derivatives and Partial Derivatives | The Effect of Reversing Direction | The Gradient | Properties of the Gradient | Gradients Are Normal to Level Curves | An Application of Gradients

15.7 Differentiability, Directional Derivatives, and Gradients for Functions of Three or More Variables

Differentiability | Directional Derivatives and Gradients | Gradient are Normal to Level Surfaces | Using Gradients to Find Tangent Planes | Using Gradients to Find Tangent Lines to Intersections of Surfaces | Total Differentials | Approximations Using Total Differentials | Extensions | Total Differentials | Chain Rules

15.8 Maxima and Minima of Functions of Two Variables

Extrema | The Extreme-Value Theorem | Finding Relative Extrema | The Second Partials Test | Finding Absolute Extrema on Closed and Bou

15.9 Lagrange Multipliers

Extremum Problems with Constraints | Lagrange Multipliers | Three Variables and One Constraint

16 Multiple Integrals

16.1 Double Integrals

Volume Double integrals can be used to compute volumes.

Definition of a Double Integral

$$\iint_R f(x,y)\,\mathrm{d}A = \lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*, y_k^*)\Delta A_k$$

Properties of Double Integrals Similar rules for sums, differences, and products with constants.

Similar rules for subdivisions of areas.

Evaluating Double Integrals Over a rectangle,

$$\iint_R f(x,y)\,\mathrm{d}A = \int_{y_{\min}}^{y_{\max}} \left(\int_{x_{\min}}^{x_{\max}} f(x,y)\,\mathrm{d}x\right)\mathrm{d}y$$

16.2 Double Integrals over Nonrectangular Region

Iterated Integrals with Nonconstant Limits of Integration

$$\int_a^b \left(\int_{g_1(x)}^{g_2(x)} f(x,y)\,\mathrm{d}y\right)\mathrm{d}x$$

Double Integrals over Nonrectangular Regions Type I regions and Type II regions

Setting up Limits of Integration for Evaluating Double Integrals

Example:

$$\iint_R xy\,\mathrm{d}A$$

between $y=\frac{x}{2}$ and $y=\sqrt{x}$, and $x=2$ and $x=4$

Reversing the Order of Integration

Example:

$$\int_0^2 \int_{y/2}^1 e^{x^2}\,\mathrm{d}x$$

Area Calculated as a Double Integral

16.3 Double Integrals in Polar Coordinates

Simple Polar Regions | Double Integrals in Polar Coordinates | Evaluating Polar Double Integrals | Finding Areas Using Polar Double Integrals
Converting Double Integral from Rectangular to Polar Coordinates

16.4 Parametric Surfaces; Surface Area

Parametric Representation of Surfaces | Representing Surfaces of Revolution Parametrically | Vector-Valued Functions of Two Variables
Partial Derivatives of Vector-Valued Functions | Tangent Planes to Parametric Surfaces | Surface Area of Parametric Surfaces | Surface Area of

16.5 Triple Integrals

Definition of a Triple Integral | Properties of Triple Integrals | Evaluating Triple Integrals over Rectangular Boxes | Evaluating Triple Integrals
Volume Calculated as a Triple Integral | Integration in Other Orders

16.6 Centroid, Center of Gravity, Theorem of Pappus

Density of Lamina | Mass of a Lamina | Center of Gravity of a Lamina | Centroids | Center of Gravity and Centroid of a Solid

16.7 Triple Integrals in Cylindrical and Spherical Coordinates

Triple Integrals in Cylindrical Coordinates | Converting Triple Integrals from Rectangular to Cylindrical Coordinates | Triple Integral in Spheri
Converting Triple Integral from Rectangular to Spherical Coordinates

16.8 Change of Variables in Multiple Integrals; Jacobians

Change of Variable in a Single Integral | Transformations of the Plane | Jacobians in Two Variables | Change of Variables in Double Integrals
Change of Variables in Triple Integrals

17 Topics in Vector Calculus

17.1 Vector Fields

Vector Fields | Graphical Representations of Vector Fields | A Compact Notation for Vector Fields | Inverse-Square Fields | Gradient Fields
Conservative Fields and Potential Functions | Divergence and Curl | The ∇ Operator | The Laplacian ∇^2

17.2 Line Integrals

Line Integrals | Evaluating Line Integrals | Line Integrals in 3-Space | Mass of a Wire as a Line Integral | Arc Length as a Line Integral
Line Integrals with Respect to x, y and z | Line Integrals along Piecewise Smooth Curves | Change of Parameter in Line Integrals | Reversing t
Work as a Line Integral | A Method for Calculating Work | Work Expressed in Scalar Form

17.3 Independence of Path; Conservative Vector Fields

Work Integrals | Independence of Path | The Fundamental Theorem of Work Integrals | Work Integrals Along Closed Paths | A Test for Conser
Conservative Vector Fields in 3-Space | Conservative of Energy

17.4 Green's Theorem

Green's Theorem | A Notation for Line Integrals Around Simple Closed Curves | Finding Work Using Green's Theorem | Finding Areas Using G
Green's Theorem for Multiply Connected Regions

17.5 Surface Integral

Definition of a Surface Integral | Evaluating Surface Integrals | Surface Integrals over $z = g(x, y)$, $y = g(x, z)$ and $x = g(y, z)$ | Mass of a Curved

Surface Area as a Surface Integral

17.6 Application of Surface Integras; Flux

Flow Fields | Oriented Surfaces | Orientation of a Smooth Parametric Surface | Evaluating Flux Integrals | Orientation of Nonparametric Surfa

17.7 The Divergence Theorem

Orientation of Piecewise Smooth Closed Surfaces | The Divergence Theorem | Using the Divergence Theorem to Find Flux | Divergence Viewed

Sources and Sinks | Gauss's Law for Inverse-Square Fields | Gauss's Law in Eletrostatics

17.8 Stokes' Theorem

Relative Orientation of Curves and Surfaces | Stoke's Theorem | Using Stoke's Theorem to Calculate Work | Relationship Between Green's The

Curl Viewed as Circulation