1 Functions

1.1 Functions and The Analysis of Graphical Information

1.2 Properties of Functions

1.3 Graphic Functions on Calculators and Computers; Computer Algebra Systems

1.4 New Functions from Old

1.5 Mathematical Models; Linear Models

1.6 Families of Functions

1.7 Parametric Equations

2 Limits and Continuity

2.1 Limits (An Intuitive Introduction)

The Tangent Line, Area, and Velocity Problems | Tangent Lines and Limits | Instantenous Velocity and Limits | Limits | Numerical Pitfalls | One-Sided Limits | The Relationship Between One-Sided and Two-Sided Limits | A First Look at Continuity | Infinite Limits and Vertical Asy | Limits at Infinity and Horizontal Asymptotes | How Limits at Infinity Can Fail to Exist

2.2 Limits (Computational Techniques)

Some Basic Limits | Limits of Polynomials as $x \to a$ | Limits of x^n as $x \to +\infty$ or $x \to -\infty$ | Limits of Polynomials as $x \to +\infty$ or $x \to -\infty$ | Limits of Rational Functios as $x \to a$ | Limits of Rational Functions as $x \to +\infty$ or $x \to -\infty$ | A Quick Method for Finding Limits of Rational

• **Limits Involving Radicals**

$$\lim_{x\to+\infty} \sqrt[3]{\frac{3x+5}{6x-8}} = \sqrt[3]{\lim_{x\to+\infty} \frac{3x+5}{6x-8}} = \frac{1}{\sqrt[3]{2}}$$

$$\lim_{x\to+\infty} \frac{\sqrt{x^2+2}}{3x-6} = \lim_{x\to+\infty} \frac{\sqrt{x^2+2}/|x|}{(3x-6)/|x|} \overset{x\geq 0}{=} \lim_{x\to+\infty} \frac{\sqrt{x^2+2}/\sqrt{x^2}}{(3x-6)/x} = \lim_{x\to+\infty} \frac{\sqrt{1+2/x^2}}{3-6/x} = \frac{\sqrt{\lim\limits_{x\to+\infty} 1+2/x^2}}{\lim\limits_{x\to+\infty} 3-6/x} = \frac{1}{3}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+2}}{3x-6} = \lim_{x\to-\infty} \frac{\sqrt{x^2+2}/|x|}{(3x-6)/|x|} \overset{x\leq 0}{=} \lim_{x\to-\infty} \frac{\sqrt{x^2+2}/\sqrt{x^2}}{(3x-6)/(-x)} = \lim_{x\to-\infty} \frac{\sqrt{1+2/x^2}}{-3+6/x} = \frac{\sqrt{\lim\limits_{x\to-\infty} 1+2/x^2}}{\lim\limits_{x\to-\infty} -3+6/x} = -\frac{1}{3}$$

Limits of Functions Defined Piecewise

2.3 Limits (Discussed More Rigorously)

Definition of a Limit | The Value of δ Is Not Unique | Limits as $x \to +\infty$ or $x \to -\infty$ | Infinite Limits

2.4 Continuity

Definition of Continuity | Continuity in Applications | Continuity of Polynomials | Some Properties of Continuous Functions | Continuity of Ra | Continuity of Compositions | Continuity from the Left and from the Right | The Intermediate-Value Theorem | Approximating Roots Using th | Approximating Roots by Zooming with a Graphic Utility

2.5 Limits and Continuity of Trigonometric Functions

Continuty of Trigonometric Functions | Obtaning Limits by Squeezing

3 The Derivative

Slope of a Tangent Line | Average Versus Instantaneous Velocity | Average and Instantaneous Rate of Change

3.1 Tangent Lines and Rates of Change

Tangent Lines Defined Precisely | Slopes of Tangent Lines by Zooming

3.2 The Derivative

The Derivative | Differentiability | Relationship Between Differentiability and Continuity | Derivative Notation | Other Notations | Derivatives a

3.3 Techniques of Differentiation

Derivative of a Constant | Derivative of x to a Power | Derivative of a Constant Times a Function | Derivatives of Sums and Differences | Derivative of a Product | Derivative of a Quotient | Derivative of a Reciprocal | The Power Rule for Integer Exponents | Higher Derivatives

3.4 Derivatives of Trigonometric Functions

Derivatives of the Trigonometric Functions

3.5 The Chain Rule

Derivatives of Compositions | Generalized Derivative Formulas | An Alternative Approach to Using the Chain Rule | Differentiating Using Com

3.6 Local Linear Approximation; Diffferentials

Increments | Differentials | Local Linear Approximation | Error in Local Linear Approximations | Error Propagation in Applications | Differenti

4 Logarithmic and Exponential Functions

4.1 Inverse Functions

Inverse Functions | Domain and Range of Inverse Functions | A Method for Finding Inverses | Existence of Inverse Functions | Graphs of Invers | Increasing or Decreasing Functions Have Inverses | Restricting Domains to Make Functions Invertible | Continuity of Inverse Functions | Differentiability of Inverse Functions | Graphing Inverse Functions with Graphing Utilities

4.2 Logarithmic and Exponential Functions

Irrational Exponents | The Family of Exponential Functions | Logarithms | Logarithmic Functions | Solving Equations Involving Exponentials a | Change of Base Formula for Logarithms | Logarithmic Scales in Science and Engineering | Exponential and Logarithmic Growth

4.3 Implicit Differentiation

Functions Defined Explicitly and Implicitly | Graphs of Equations in x and y | Implicit Differentiation | Differentiability of Functions Defined I | Derivatives of Rational Powers of x | Derivatives of Inverse Functions

4.4 Derivatives of Logarithmic and Exponential Functions

Derivatives of Logarithmic Functions | Logarithmic Differentiation | Derivatives of Irrational Powers of x | Derivatives of Exponential Functions

4.5 Derivatives of Inverse Trigonometric Functions

Inverse Trigonometric Functions | Evaluating Inverse Trigonometric Functions | Identities for Inverse Trigonometric Functions | Derivatives of t | Differentiability of the Inverse Trigonometric Functions

4.6 Related Rates

Rates of Changes Using the Chain Rule

4.7 L'Hôpital's Rule ; Inderterminate Forms

Indeterminates Forms of Type $0/0$ | L'Hôpital's Rule | Indeterminate Forms of Type ∞/∞ | Analyzing the Growth of Exponential Functions U | Indeterminate Forms of Type $0 \cdot \infty$ | Indeterminate Forms of Type $\infty - \infty$ | Indeterminate Forms of Type 0^0, ∞^0, 1^∞

5 Analysis of Functions and Their Graphs

5.1 Analysis of Functions I: Increase, Decrease, and Concavity

5.2 Analysis of Functions II: Relative Extrema; First and Second Derivative Tests

5.3 Analysis of Functions III: Applying Technology and the Tools of Calculus

6 Applications of the Derivative

6.1 Absolute Maxima and Minima

6.2 Applied Maximum and Minimum Problems

6.3 Rectilinear Motion (Motion Along a Line)

6.4 Newton's Method

6.5 Rolle's Theorem; Mean-Value Theorem

7 Integration

7.1 An Overview of the Area Problem

Defining Area | The Rectangle Method for Finding Areas | The Antiderative Method for Finding Areas

7.2 The Indefinite Integral; Integral Curves and Direction Fields

The Indefinite Integral | Integration Formulas | Properties of the Indefinite Integral | Integral Curves | Integration from the Viewpoint of Differ
Direction Fields

7.3 Integration by Substitution

u-Substitution | Integration Using Computer Algebra Systems

7.4 Sigma Notation

Sigma Notation | Changing the Index of Summation | Properties of Sigma Notation | Summation Formulas

7.5 The Definite Integral

A Definition of Area | The Definite Integral of a Continuous Function | The Riemann Integral | Integrability | Properties of the Definite Integra
Conditions for Integrability

7.6 The Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus | The Relationship Between Definite and Indefinite Integrals | Dummy Variables | The Mean-Value The
Part 2 of the Fundamental Theorem of Calculus | Differentiation and Integration are Inverse Processes

7.7 Rectilinear Motion Revisited; Average Value

Finding Position and Velocity by Integration | Uniformly Accelerated Motion | The Free-Fall Model | Integrating Rates of Change | Displaceme
Distance Traveled in Rectilinear Motion | Analyzing the Velocity Versus Time Curve | Average Value of a Continuous Function | Average Veloc

7.8 Evaluating Definite Integrals by Substitution

Two Methods for Making Substitutions in Definite Integrals

7.9 Logarithmic Functions from the Integral Point of View

The Link Between Natural Logarithms and Integrals | Approximating $\ln x$ Numerically | Differentiability and Continuity of $\ln x$ and e^x
The Definition of e revisited | Functions Defined by Integrals | Evaluating and Graphing Functions Defined by Integrals | Integrals with Functio

8 Applications of the Definite Integral in Geometry, Science, and Engineering

8.1 Area Between Two Curves

8.2 Volumes by Slicing; Disks and Washers

8.3 Volumes by Cylindrical Shells

8.4 Length of a Plane Curve

8.5 Area of a Surface of Revolution

8.6 Work

8.7 Fluid Pressure and Force

8.8 Hyperbolic Functions and Hanging Cables

9 Principles of Integral Evaluation

9.1 An Overview of Integration Methods

Methods for Approaching Integration Problems | A Review of Familiar Integration Formulas

9.2 Integration by Parts

Derivation of the Formula for Integration by Parts | Integration by Parts for Definite Integrals | Reduction Formulas

9.3 Trigonometric Integrals

Integrating Powers of Sine and Cosine | Integrating Products of Sines and Cosines | Integrating Powers of Tangent and Secant | Integrating Pro
An Alternative Method for Integrating Powers of Sine, Cosine, Tangent and Secant | Mercator's Map of the World

9.4 Trigonometric Substitutions

The Method of Trigonometric Substitution | Integrals Involving $ax^2 + bx + c$

9.5 Integrating Reational Functions by Partial Fractions

Partial Fractions | Finding the Form of a Partial Fraction Decomposition | Linear Factors | Quadratic Factors | Integrating Improper Rational
Concluding Remarks

9.6 Using Tables of Integrals and Computer Algebra Systems

Integral Tables | Perfect Matches | Matches Requiring Substitutions | Matches Requiring Reduction Formulas | Matches Requiring Special Subs
Integrating with Computer Algebra Systems | Computer Algebra Systems can Fail

9.7 Numerical Integration; Simpson's Rule

A Review of Riemann Sum Approximations | Trapezoidal Approximation | Comparison of the Midpoint and Trapezoidal Approximations
Simpson's Rule | Error Estimates | A Comparison of the Three Methods

9.8 Improper Integrals

Improper Integrals | Integrals over Infinite Intervals | Integrals whose Integrands have Infinite Discontinuities | The Application of Improper Int

10 Mathematical Modeling with Differential Equations

10.1 First-Order Differential Equations and Applications

10.2 Direction Fields; Euler's Method

10.3 Modeling with Differential Equations

11 Infinite Series

11.1 Sequences

Definition of a Sequence | Graphs of Sequences | Limit of a Sequence | The Squeezing Theorem for Sequences | Sequences Defined Recursively

11.2 Monotone Sequences

Terminology | Testing for Monotonicity | Properties that Hold Eventually | An Intuitive View of Convergence | Convergence of Monotone Seque

11.3 Infinite Series

Sum of Infinite Series | Geometric Series | Harmonic Series

11.4 Convergence Tests

The Divergence Test | Algebraic Properties of Infinite Series | The Integral Test | p-Series | Proof of the Integral Test

11.5 Taylor and Maclaurin Series

Local Quadratic Approximations | MacLaurin Polynomials | Taylor Polynomials | Sigma Notation for Taylor and MacLaurin Polynomials | Taylor and MacLaurin Series

11.6 The Comparison, Ratio, and Root Tests

The Comparison Test | Using the Comparison Test | The Limit Comparison Test | The Ratio Test | The Root Test

11.7 Alternating Series; Conditional Convergence

Alternating Series | Approximating Sums of Alternating Series | Absolute Convergence | Conditional Convergence | The Ratio Test for Absolut | Summary of Convergence Tests

11.8 Power Series

Power Series in x | Radius and Interval of Convergence | Finding the Interval of Convergence | Power Series in $x - x_0$ | Functions Defined by P

11.9 Convergence of Taylor Series; Computational Methods

The nth remainder | Estimating the nth remainder | Approximating Trigonometric Functions | Roundoff and Truncation Error | Approximatin | Approximating Logarithms | Approximating π | Binomial Series

11.10 Differentiating and Integrating Power Series; Modeling with Taylor Series

Differentiating Power Series | Integrating Power Series | Power Series Representations Must Be Taylor Series | Some Practical Ways to Find Ta | Finding MacLaurin Series by Multiplication and Division | Modeling Physical Laws with Taylor Series