

# 1 Functions

## 1.1 Functions and The Analysis of Graphical Information

## 1.2 Properties of Functions

## 1.3 Graphic Functions on Calculators and Computers; Computer Algebra Systems

## 1.4 New Functions from Old

## 1.5 Mathematical Models; Linear Models

## 1.6 Families of Functions

## 1.7 Parametric Equations

# 2 Limits and Continuity

## 2.1 Limits (An Intuitive Introduction)

The Tangent Line, Area, and Velocity Problems	Tangent Lines and Limits	Instantaneous Velocity and Limits	Limits	Numerical Pitfalls
One-Sided Limits	The Relationship Between One-Sided and Two-Sided Limits	A First Look at Continuity	Infinite Limits and Vertical Asymptotes	
Limits at Infinity and Horizontal Asymptotes	How Limits at Infinity Can Fail to Exist			

## 2.2 Limits (Computational Techniques)

Some Basic Limits	Limits of Polynomials as $x \rightarrow a$	Limits of $x^n$ as $x \rightarrow +\infty$ or $x \rightarrow -\infty$	Limits of Polynomials as $x \rightarrow +\infty$ or $x \rightarrow -\infty$
Limits of Rational Functions as $x \rightarrow a$	Limits of Rational Functions as $x \rightarrow +\infty$ or $x \rightarrow -\infty$	A Quick Method for Finding Limits of Rational Functions	

### • Limits Involving Radicals

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$$\lim_{x \rightarrow +\infty} \sqrt[3]{\frac{3x+5}{6x-8}} = \sqrt[3]{\lim_{x \rightarrow +\infty} \frac{3x+5}{6x-8}} = \frac{1}{\sqrt[3]{2}}$$

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$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+2}}{3x-6} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+2}/|x|}{(3x-6)/|x|} \stackrel{x \geq 0}{=} \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+2}/\sqrt{x^2}}{(3x-6)/x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{1+2/x^2}}{3-6/x} = \frac{\sqrt{\lim_{x \rightarrow +\infty} 1+2/x^2}}{\lim_{x \rightarrow +\infty} 3-6/x} = \frac{1}{3}$$

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$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+2}}{3x-6} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+2}/|x|}{(3x-6)/|x|} \stackrel{x \leq 0}{=} \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+2}/\sqrt{x^2}}{(3x-6)/(-x)} = \lim_{x \rightarrow -\infty} \frac{\sqrt{1+2/x^2}}{-3+6/x} = \frac{\sqrt{\lim_{x \rightarrow -\infty} 1+2/x^2}}{\lim_{x \rightarrow -\infty} -3+6/x} = -\frac{1}{3}$$

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Limits of Functions Defined Piecewise
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## 2.3 Limits (Discussed More Rigorously)

Definition of a Limit	The Value of $\delta$ Is Not Unique	Limits as $x \rightarrow +\infty$ or $x \rightarrow -\infty$	Infinite Limits
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## 2.4 Continuity

Definition of Continuity	Continuity in Applications	Continuity of Polynomials	Some Properties of Continuous Functions	Continuity of Rational Functions
Continuity of Compositions	Continuity from the Left and from the Right	The Intermediate-Value Theorem	Approximating Roots Using the Intermediate-Value Theorem	
Approximating Roots by Zooming with a Graphic Utility				

## 2.5 Limits and Continuity of Trigonometric Functions

Continuity of Trigonometric Functions	Obtaining Limits by Squeezing
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# 3 The Derivative

Slope of a Tangent Line	Average Versus Instantaneous Velocity	Average and Instantaneous Rate of Change
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### 3.1 Tangent Lines and Rates of Change

Tangent Lines Defined Precisely Slopes of Tangent Lines by Zooming

### 3.2 The Derivative

The Derivative Differentiability Relationship Between Differentiability and Continuity Derivative Notation Other Notations Derivatives of Functions

### 3.3 Techniques of Differentiation

Derivative of a Constant Derivative of  $x$  to a Power Derivative of a Constant Times a Function Derivatives of Sums and Differences  
Derivative of a Product Derivative of a Quotient Derivative of a Reciprocal The Power Rule for Integer Exponents Higher Derivatives

### 3.4 Derivatives of Trigonometric Functions

Derivatives of the Trigonometric Functions

### 3.5 The Chain Rule

Derivatives of Compositions Generalized Derivative Formulas An Alternative Approach to Using the Chain Rule Differentiating Using Composites

### 3.6 Local Linear Approximation; Differentials

Increments Differentials Local Linear Approximation Error in Local Linear Approximations Error Propagation in Applications Differentials

## 4 Logarithmic and Exponential Functions

### 4.1 Inverse Functions

Inverse Functions Domain and Range of Inverse Functions A Method for Finding Inverses Existence of Inverse Functions Graphs of Inverse Functions  
Increasing or Decreasing Functions Have Inverses Restricting Domains to Make Functions Invertible Continuity of Inverse Functions  
Differentiability of Inverse Functions Graphing Inverse Functions with Graphing Utilities

### 4.2 Logarithmic and Exponential Functions

Irrational Exponents The Family of Exponential Functions Logarithms Logarithmic Functions Solving Equations Involving Exponentials and Logarithms  
Change of Base Formula for Logarithms Logarithmic Scales in Science and Engineering Exponential and Logarithmic Growth

### 4.3 Implicit Differentiation

Functions Defined Explicitly and Implicitly Graphs of Equations in  $x$  and  $y$  Implicit Differentiation Differentiability of Functions Defined Implicitly  
Derivatives of Rational Powers of  $x$  Derivatives of Inverse Functions

### 4.4 Derivatives of Logarithmic and Exponential Functions

Derivatives of Logarithmic Functions Logarithmic Differentiation Derivatives of Irrational Powers of  $x$  Derivatives of Exponential Functions

### 4.5 Derivatives of Inverse Trigonometric Functions

Inverse Trigonometric Functions Evaluating Inverse Trigonometric Functions Identities for Inverse Trigonometric Functions Derivatives of Inverse Trigonometric Functions  
Differentiability of the Inverse Trigonometric Functions

### 4.6 Related Rates

Rates of Changes Using the Chain Rule

### 4.7 L'Hôpital's Rule ; Inderterminate Forms

Indeterminates Forms of Type  $0/0$  L'Hôpital's Rule Indeterminate Forms of Type  $\infty/\infty$  Analyzing the Growth of Exponential Functions and Logarithmic Functions  
Indeterminate Forms of Type  $0 \cdot \infty$  Indeterminate Forms of Type  $\infty - \infty$  Indeterminate Forms of Type  $0^0, \infty^0, 1^\infty$

## 5 Analysis of Functions and Their Graphs

### 5.1 Analysis of Functions I: Increase, Decrease, and Concavity

### 5.2 Analysis of Functions II: Relative Extrema; First and Second Derivative Tests

### 5.3 Analysis of Functions III: Applying Technology and the Tools of Calculus

## 6 Applications of the Derivative

### 6.1 Absolute Maxima and Minima

### 6.2 Applied Maximum and Minimum Problems

### 6.3 Rectilinear Motion (Motion Along a Line)

### 6.4 Newton's Method

### 6.5 Rolle's Theorem; Mean-Value Theorem

## 7 Integration

### 7.1 An Overview of the Area Problem

Defining Area

The Rectangle Method for Finding Areas

The Antiderivative Method for Finding Areas

### 7.2 The Indefinite Integral; Integral Curves and Direction Fields

The Indefinite Integral

#### • Integration Formulas

$$\int dx = x + C$$

$$\int x^r dx = \frac{x^{r+1}}{r+1} + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int (\sec x)^2 dx = \tan x + C$$

$$\int (\csc x)^2 dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int b^x dx = \frac{b^x}{\ln b} + C$$

$$\int \frac{1}{x} dx = \ln |x| + C$$

#### • Properties of the Indefinite Integral

$$\frac{d}{dx} \left( \int f(x) dx \right) = f(x)$$

Integral Curves

Integration from the Viewpoint of Differential Equations

Direction Fields

## 7.3 Integration by Substitution

[u-Substitution](#) [Integration Using Computer Algebra Systems](#)

## 7.4 Sigma Notation

[Sigma Notation](#) [Changing the Index of Summation](#) [Properties of Sigma Notation](#) [Summation Formulas](#)

## 7.5 The Definite Integral

[A Definition of Area](#) [The Definite Integral of a Continuous Function](#) [The Riemann Integral](#) [Integrability](#) [Properties of the Definite Integral](#)  
[Conditions for Integrability](#)

## 7.6 The Fundamental Theorem of Calculus

[The Fundamental Theorem of Calculus](#) [The Relationship Between Definite and Indefinite Integrals](#) [Dummy Variables](#) [The Mean-Value Theorem](#)  
[Part 2 of the Fundamental Theorem of Calculus](#) [Differentiation and Integration are Inverse Processes](#)

## 7.7 Rectilinear Motion Revisited; Average Value

[Finding Position and Velocity by Integration](#) [Uniformly Accelerated Motion](#) [The Free-Fall Model](#) [Integrating Rates of Change](#) [Displacement](#)  
[Distance Traveled in Rectilinear Motion](#) [Analyzing the Velocity Versus Time Curve](#) [Average Value of a Continuous Function](#) [Average Velocity](#)

## 7.8 Evaluating Definite Integrals by Substitution

[Two Methods for Making Substitutions in Definite Integrals](#)

## 7.9 Logarithmic Functions from the Integral Point of View

[The Link Between Natural Logarithms and Integrals](#) [Approximating  \$\ln x\$  Numerically](#) [Differentiability and Continuity of  \$\ln x\$  and  \$e^x\$](#)   
[The Definition of  \$e\$  revisited](#) [Functions Defined by Integrals](#) [Evaluating and Graphing Functions Defined by Integrals](#) [Integrals with Functions](#)

# 8 Applications of the Definite Integral in Geometry, Science, and Engineering

## 8.1 Area Between Two Curves

## 8.2 Volumes by Slicing; Disks and Washers

## 8.3 Volumes by Cylindrical Shells

## 8.4 Length of a Plane Curve

## 8.5 Area of a Surface of Revolution

## 8.6 Work

## 8.7 Fluid Pressure and Force

## 8.8 Hyperbolic Functions and Hanging Cables

# 9 Principles of Integral Evaluation

## 9.1 An Overview of Integration Methods

[Methods for Approaching Integration Problems](#) [A Review of Familiar Integration Formulas](#)

## 9.2 Integration by Parts

[Derivation of the Formula for Integration by Parts](#) [Integration by Parts for Definite Integrals](#) [Reduction Formulas](#)

## 9.3 Trigonometric Integrals

Integrating Powers of Sine and Cosine	Integrating Products of Sines and Cosines	Integrating Powers of Tangent and Secant	Integrating Products of Tangent and Secant
An Alternative Method for Integrating Powers of Sine, Cosine, Tangent and Secant	Mercator's Map of the World		

## 9.4 Trigonometric Substitutions

The Method of Trigonometric Substitution	Integrals Involving $ax^2 + bx + c$
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## 9.5 Integrating Rational Functions by Partial Fractions

Partial Fractions	Finding the Form of a Partial Fraction Decomposition	Linear Factors	Quadratic Factors	Integrating Improper Rational Functions
Concluding Remarks				

## 9.6 Using Tables of Integrals and Computer Algebra Systems

Integral Tables	Perfect Matches	Matches Requiring Substitutions	Matches Requiring Reduction Formulas	Matches Requiring Special Substitutions
Integrating with Computer Algebra Systems	Computer Algebra Systems can Fail			

## 9.7 Numerical Integration; Simpson's Rule

A Review of Riemann Sum Approximations	Trapezoidal Approximation	Comparison of the Midpoint and Trapezoidal Approximations
Simpson's Rule	Error Estimates	A Comparison of the Three Methods

## 9.8 Improper Integrals

Improper Integrals	Integrals over Infinite Intervals	Integrals whose Integrands have Infinite Discontinuities	The Application of Improper Integrals
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# 10 Mathematical Modeling with Differential Equations

## 10.1 First-Order Differential Equations and Applications

## 10.2 Direction Fields; Euler's Method

## 10.3 Modeling with Differential Equations

# 11 Infinite Series

## 11.1 Sequences

Definition of a Sequence	Graphs of Sequences	Limit of a Sequence	The Squeezing Theorem for Sequences	Sequences Defined Recursively
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## 11.2 Monotone Sequences

Terminology	Testing for Monotonicity	Properties that Hold Eventually	An Intuitive View of Convergence	Convergence of Monotone Sequences
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## 11.3 Infinite Series

Sum of Infinite Series	Geometric Series	Harmonic Series
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## 11.4 Convergence Tests

The Divergence Test	Algebraic Properties of Infinite Series	The Integral Test	$p$ -Series	Proof of the Integral Test
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## 11.5 Taylor and Maclaurin Series

Local Quadratic Approximations	Maclaurin Polynomials	Taylor Polynomials	Sigma Notation for Taylor and Maclaurin Polynomials
Taylor and Maclaurin Series			

## 11.6 The Comparison, Ratio, and Root Tests

The Comparison Test	Using the Comparison Test	The Limit Comparison Test	The Ratio Test	The Root Test
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11.7 Alternating Series; Conditional Convergence

Alternating Series	Approximating Sums of Alternating Series	Absolute Convergence	Conditional Convergence	The Ratio Test for Absolute
Summary of Convergence Tests				

11.8 Power Series

Power Series in $x$	Radius and Interval of Convergence	Finding the Interval of Convergence	Power Series in $x - x_0$	Functions Defined by P
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11.9 Convergence of Taylor Series; Computational Methods

The $n$ th remainder	Estimating the $n$ th remainder	Approximating Trigonometric Functions	Roundoff and Truncation Error	Approximating
Approximating Logarithms	Approximating $\pi$	Binomial Series		

11.10 Differentiating and Integrating Power Series; Modeling with Taylor Series

Differentiating Power Series	Integrating Power Series	Power Series Representations Must Be Taylor Series	Some Practical Ways to Find Tay
Finding MacLaurin Series by Multiplication and Division		Modeling Physical Laws with Taylor Series	

## 12 Analytic Geometry in Calculus

### 12.1 Polar Coordinates

### 12.2 Tangent Lines and Arc Length for Parametric and Polar Curves

### 12.3 Area in Polar Coordinates

### 12.4 Conic Sections in Calculus

### 12.5 Conic Sections in Polar Coordinates

## 13 Three-Dimensional Space; Vectors

### 13.1 Rectangular Coordinates in 3-Space; Spheres; Cylindrical Surfaces

### 13.2 Vectors

### 13.3 Dot Product; Projections

### 13.4 Cross Product

### 13.5 Parametric Equations of Lines

### 13.6 Planes in 3-Space

### 13.7 Quadric Surfaces

### 13.8 Cylindrical and Spherical Coordinates

## 14 Vector-Valued Functions

### 14.1 Introduction to Vector-Valued Functions

### 14.2 Calculus of Vector-Valued Functions

### 14.3 Change of Parameters; Arc Length

### 14.4 Unit Tangent, Normal, and Binormal Vectors

### 14.5 Curvature

### 14.6 Motion Along a Curve

### 14.7 Kepler's Laws of Planetary Motion

## 15 Partial Derivatives

### 15.1 Functions of Two or More Variables

Notation and Terminology	Graphs of Functions of Two Variables	Graphs of Functions of Two Variables Using Technology	Level Curves
Contour Plots Using Technology	Level Surfaces	Graphing Functions of Two Variables Using Technology	

### 15.2 Limits and Continuity

Open and Closed Sets	Bounded Sets	Limits Along Curves	General Limits of Functions of Two Variables	Properties of Limits	Relationships
Continuity	Limits at Points of Discontinuity	Extension to Three Variables			

### 15.3 Partial Derivatives

Partial Derivatives of Functions of Two Variables	Partial Derivatives Viewed as Rates of Change and Slopes	Partial Derivative Notation	
Implicit Partial Differentiation	Higher-Order Partial Derivatives	The Wave Equation	Partial Derivatives of Functions With More Than Two Variables

### 15.4 Differentiability and Chain Rules

Differentiability of Functions of Two Variables	Sufficient Conditions for Differentiability	Equality of Mixed Partial Derivatives	Chain Rules	Related Topics
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## 15.5 Tangent Planes; Total Differentials for Functions of Two Variables

Tangent Planes   The Geometric Significance of Differentiability   Total Differentials   Local Linear Approximation   Approximations Using Total Differentials

## 15.6 Directional Derivatives and Gradients for Functions of Two Variables

Directional Derivatives   The Relationship Between Directional Derivatives and Partial Derivatives   The Effect of Reversing Direction  
The Gradient   Properties of the Gradient   Gradients Are Normal to Level Curves   An Application of Gradients

## 15.7 Differentiability, Directional Derivatives, and Gradients for Functions of Three or More Variables

Differentiability   Directional Derivatives and Gradients   Gradient are Normal to Level Surfaces   Using Gradients to Find Tangent Planes  
Using Gradients to Find Tangent Lines to Intersections of Surfaces   Total Differentials   Approximations Using Total Differentials   Extensions  
Total Differentials   Chain Rules

## 15.8 Maxima and Minima of Functions of Two Variables

Extrema   The Extreme-Value Theorem   Finding Relative Extrema   The Second Partial Test   Finding Absolute Extrema on Closed and Bounded Regions

## 15.9 Lagrange Multipliers

Extremum Problems with Constraints   Lagrange Multipliers   Three Variables and One Constraint

# 16 Multiple Integrals

## 16.1 Double Integrals

Volume   Definition of a Double Integral   Properties of Double Integrals   Evaluating Double Integrals

## 16.2 Double Integrals over Nonrectangular Region

Iterated Integrals with Nonconstant Limits of Integration   Double Integrals over Nonrectangular Regions   Setting up Limits of Integration for Double Integrals  
Reversing the Order of Integration   Area Calculated as a Double Integral

## 16.3 Double Integrals in Polar Coordinates

Simple Polar Regions   Double Integrals in Polar Coordinates   Evaluating Polar Double Integrals   Finding Areas Using Polar Double Integrals  
Converting Double Integral from Rectangular to Polar Coordinates

## 16.4 Parametric Surfaces; Surface Area

Parametric Representation of Surfaces   Representing Surfaces of Revolution Parametrically   Vector-Valued Functions of Two Variables  
Partial Derivatives of Vector-Valued Functions   Tangent Planes to Parametric Surfaces   Surface Area of Parametric Surfaces   Surface Area of Surfaces of Revolution

## 16.5 Triple Integrals

Definition of a Triple Integral   Properties of Triple Integrals   Evaluating Triple Integrals over Rectangular Boxes   Evaluating Triple Integrals over General Regions  
Volume Calculated as a Triple Integral   Integration in Other Orders

## 16.6 Centroid, Center of Gravity, Theorem of Pappus

Density of Lamina   Mass of a Lamina   Center of Gravity of a Lamina   Centroids   Center of Gravity and Centroid of a Solid

## 16.7 Triple Integrals in Cylindrical and Spherical Coordinates

Triple Integrals in Cylindrical Coordinates   Converting Triple Integrals from Rectangular to Cylindrical Coordinates   Triple Integral in Spherical Coordinates  
Converting Triple Integral from Rectangular to Spherical Coordinates



## 16.8 Change of Variables in Multiple Integrals; Jacobians

Change of Variable in a Single Integral	Transformations of the Plane	Jacobians in Two Variables	Change of Variables in Double Integrals
Change of Variables in Triple Integrals			

## 17 Topics in Vector Calculus

### 17.1 Vector Fields

Vector Fields Graphical Representations of Vector Fields A Compact Notation for Vector Fields Inverse-Square Fields Gradient Fields Conservative Fields and Potential Functions Divergence and Curl The  $\nabla$  Operator The Laplacian  $\nabla^2$

### 17.2 Line Integrals

Line Integrals Evaluating Line Integrals Line Integrals in 3-Space Mass of a Wire as a Line Integral Arc Length as a Line Integral Line Integrals with Respect to  $x$ ,  $y$  and  $z$  Line Integrals along Piecewise Smooth Curves Change of Parameter in Line Integrals Reversing the Direction of Integration Work as a Line Integral A Method for Calculating Work Work Expressed in Scalar Form

### 17.3 Independence of Path; Conservative Vector Fields

Work Integrals Independence of Path The Fundamental Theorem of Work Integrals Work Integrals Along Closed Paths A Test for Conservative Vector Fields Conservative Vector Fields in 3-Space Conservative of Energy

### 17.4 Green's Theorem

Green's Theorem A Notation for Line Integrals Around Simple Closed Curves Finding Work Using Green's Theorem Finding Areas Using Green's Theorem Green's Theorem for Multiply Connected Regions

### 17.5 Surface Integral

Definition of a Surface Integral Evaluating Surface Integrals Surface Integrals over  $z = g(x, y)$ ,  $y = g(x, z)$  and  $x = g(y, z)$  Mass of a Curved Lamina as a Surface Integral Surface Area as a Surface Integral

### 17.6 Application of Surface Integrals; Flux

Flow Fields Oriented Surfaces Orientation of a Smooth Parametric Surface Evaluating Flux Integrals Orientation of Nonparametric Surfaces

### 17.7 The Divergence Theorem

Orientation of Piecewise Smooth Closed Surfaces The Divergence Theorem Using the Divergence Theorem to Find Flux Divergence Viewed as Flux Density Sources and Sinks Gauss's Law for Inverse-Square Fields Gauss's Law in Electrostatics

### 17.8 Stokes' Theorem

Relative Orientation of Curves and Surfaces Stoke's Theorem Using Stoke's Theorem to Calculate Work Relationship Between Green's Theorem and Stoke's Theorem Curl Viewed as Circulation