1 Problem

Seven pirates wash ashore on a deserted island after their ship sinks. In order to survive, they gather as many coconuts as they can find and throw them into a central pile. As the sun sets, they all go to sleep.

One pirate wakes up in the middle of the night. Being the greedy person he is, this pirate decides to take some coconuts from the pile and hide them for himself. As he approaches the pile, though, he notices a monkey watching him. To keep the monkey quiet, the pirate tosses it one coconut from the pile. He then divides the rest of the pile into seven equally sized bunches and hides one of the bunches in the bushes. Finally, he recombines the remaining coconuts into a single pile and goes back to sleep. (Note that individual coconuts are very hard, and therefore indivisible.)

Later that night, a second pirate wakes up with the same idea. She tosses the monkey one coconut from the central pile, divides the pile into seven bunches, hides her bunch, recombines the rest, and goes back to sleep. After that, a third pirate wakes up and does the same thing. Then a fourth. Then a fifth, and so on until all seven pirates have hidden a share of the coconuts.

In the morning, the pirates look at the remaining central pile and notice that it has gotten quite small. They decide to split the pile into seven equal bunches and take one bunch each. (Note: The monkey does not get one this time.)

If there were N coconuts in the pile originally, what is the smallest possible value of N?

2 Solution

To begin, let's let N_7 be the number of coconuts left after the last pirate has taken their share and let N_6 be the number of coconuts after the 6th pirate has taken their share. From here we can express N_6 as follows.

$$N_6 = \frac{7N_7}{6} + 1 \tag{1}$$

To verify this, we can rearrange Equation 1

$$N_7 = \frac{6(N_6 - 1)}{7} \tag{2}$$

This makes sense. The Pirate first gave one cocunut to the monkey (i.e. $N_6 - 1$) then divided the remaining pile into seven parts and took their share (i.e. 6/7). This formula can now be applied to each pirate.

$$N_5 = \frac{7N_6}{6} + 1 = \frac{7(\frac{7N_7}{6}) + 1}{6} + 1 = \frac{7^2 N_7}{6^2} + \frac{7^1}{6^1} + \frac{7^0}{6^0} \tag{3}$$

Repeating until we obtain the original number of coconuts in the pile (i.e. before the first pirate took any), we get

$$N_0 = \frac{7^7 N_7}{6^7} + \frac{7^6}{6^6} + \cdots + \frac{7^0}{6^0} \tag{4}$$

It is important to note that N_7 and N_0 must be whole numbers. Furthermore, N_7 must also be divisible by 7. Because I'm not very good with discrete mathematics, I will use a computer program to find solutions that satisfy Equation 4 and the previously mentioned conditions. The program ends up finding a solution of

$$N_0 = 93060353 \tag{5}$$

The code used to find this number is shown below.

```
def func(m, n, give):
    # Generates the function that relates the inital pile of coconuts
    # to the pile once each pirate has stolen from it.

    tot = m
    for i in range(n):
        tot *= n/(n-1)
        tot += give
    return tot

pirates = 7
give_to_monkey = 1

# Check through a huge number of values
# because I don't know how to find it a better way.
```

```
for i in range(100000000):
    ans = func(i,pirates,give_to_monkey)
    if ans.is_integer() and i % 7 == 0:
        print(ans)
```