Emergence of the Matching Function in Multi Agent Based Simulations of the Labor Market

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November 12, 2007

Abstract

Multi Agent Based Simulations (MABS) is a promising research tool with a great potential within social science. There is numerous applications, e.g., researchers can test hypotheses about social interaction and the emergence of conventions.

We model and simulate the labor market as a typical example for interactions of extremely heterogeneous agents [1]. Unemployed people and vacant jobs co-exist, i.e., imperfect and incomplete information causes frictions and delays in the matching process in the labor market. For example, job searchers may not know about a vacancy because it is not in their range of information gathering or because there is a mismatch between their abilities and the expertise required to fill these vacancies. A so called matching function captures phenomena of the like. It is, therefore, a very useful mechanism in classical economic models. In the labor market, the rate of encounters (hires) between vacancies and unemployed persons per time (interval) is computed.

We implement a model of the labor market using a multi agent based simulation, wherein the matching function emerges from simple interactions between heterogeneous agents.

1 Introduction

Economic models are usually constructed using a typical representative agent capturing behaviors, decision making, and the characteristics of a large set of heterogeneous agents. This approach is called *top-down*, i.e., the system is described in a central way and attributes are aggregated in order to describe one agent. Various tools are used in this process to capture the diversity of all the agents. Frictions and differences are introduced artificially. When adopting the *bottom-up* approach, one has to seggregate and capture all the heterogenities of the agents participating. This change of approach forces the reasercher to completely change his point of view. We present a simple model of the labor market using the bottom-up approach. Agents have specific characteristics,

[1]By "extremely heterogeneous" we mean that the agents have a number of characteristics significantly greater than the number used in previous works, which is usually one.

resulting in a highly diverse set. In the top-down approach, a *matching function* (see Appendix A for a brief introduction) is used to encapsulate these differences and to introduce frictions and delays. In our approach, the matching function emerges in the simulation and can be 'measured' from the results obtained.

2 Labor market

The term 'labor market' captures interactions between employers (demand) and employees (supply). The patterns and norms of these interactions and the outcome of the resulting transactions constitute the labor market. Norms and patters can be, e.g., wages set in the market, minimum wages, unemployment rate, participation rate, or taxes paid. Results can be studied and analyzed to gain an understanding of the market and, thus, to predict the influence of political measures. Important questions ask for the reasons of high unemployment or low participation rates. The effects of a tax increase or a change of the minimum wage are also of high interest. Both individuals and companies depend on the labor market to a considerable extent, which makes these questions and problems very important for governments.

Definition 1 Labor force L *is the sum of the number of people that are working (having a job) plus the number of people that are unemployed (not having a job, but actively looking for one, denoted by* U*).*

Definition 2 *The* unemployment level *is the size of the labor force minus the number of employed people, i.e.,* U*.*

Definition 3 *The* unemployment rate *is the unemployment level divided by the labor force* ($u := \frac{U}{L}$) *and the* employment rate *is the number of people having a job divided by the number of people in the working age.*

Definition 4 *The* vacancy rate *is the number of vacant jobs* V *divided by the labor force, i.e.,* $v := \frac{V}{L}$*.*

Definition 5 *The* participation rate *is the labor force divided by the size of the adult civilian population in the working age.*

2.1 Unemployment

Unemployment describes the state of a person who is looking for a job without getting hired. Lack of vacancies lists among possible reasons, however, there is various other factors influencing unemployment:

- *Frictional unemployment* represents the delays taking place when a person looks for a job or when he changes his job. Job hunting is time-consuming, activities like answering to different ads or going to interviews are represented by this notion. Professional trainings required for a new job result in additional delays, often as a consequence of structural reasons.

- *Structural unemployment* represents frictions in the labor market such as a mismatch between skills requested by companies and the endowments that unemployed persons actually have or any other attributes like different location of job and job-seeker.

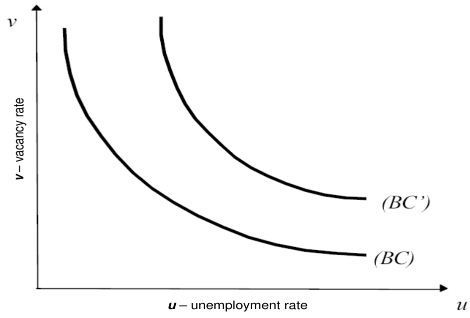

Figure 1: The Beveridge curve

- *Natural unemployment* is the sum of frictional and structural unemployment. Both types are unavoidable, thus, natural unemployment is the lowest rate a stable economy can achieve.

2.2 The Beveridge curve

The Beveridge curve illustrates the relationship between the unemployment rate u and the vacancy rate v. It is often used to analyze the efficiency of the matching mechanism in the labor market. Few open jobs most probably result in a high unemployment rate. Contrary, if many vacancies exist, it is much easier for job-seekers, therefore, the unemployment rate will be rather low. Refer to Figure 1 for two examples of matching mechanisms in labor markets. The ***BC*** curve is close to the origin, which means that, compared to ***BC'***, for a certain rate of vacancies there is a lower rate of unemployment. We say that the matching mechanism represented by ***BC*** is more efficient then the one of ***BC'***.

3 Agent based simulation

In the past, economists used to artificially introduce frictions using a matching function (see Appendix A), which aggregates different factors chosen to be taken under consideration in a particular simulation. One agent represents the whole population and averages the interactions, behaviors, and decisions of all agents. Simulation parameters include the agent's features and the matching function (often probabilistic). One important parameter is the rate (probability) of job matches in the labor market per time unit. The classical model proposes an equation to recompute this rate in every round. Usually, complete and perfect information for all agents is assumed.

Constructing a market model using a multi agent based simulation (MABS) is a completely different approach. The problem is modelled in an ascending manner, now, researchers have to concentrate on the characteristics and properties of individual agents (or group of individuals). As each agent decides about his future actions and interacts with the environment and with other agents more or less independently, the match or mismatch rates can be observed and measured, i.e., we say that a matching function emerges from the simulation.

3.1 Agents

We keep our model as simple as possible in order to catch the most fundamental aspects of the labor market only. Taking into account more parameters might lead to a more accurate model, which we leave open for future work. Additional parameters, however, are to be selected carefully in order not to bias results.

Three types of agents are used: PERSON agents, COMPANY agents, and a MATCHING agent. A PERSON agent can be in one of these two states:

- Employed: The agent currently has a job, works, and produces a profit.
- Unemployed: The agent is not occupied but he is looking for a job.

Note that, in our model, a worker agent can hold only one job. A COMPANY agent can offer a job. In our model each COMPANY agent can offer exactly one job (multiple jobs per company remain future work). Jobs are represented as an object belonging to companies. A job can be in one of these two states:

- Filled: A PERSON agent is currently occupying this job.
- Vacant: The job is not filled; the COMPANY agent is looking for an appropriate PERSON agent to fill the job.

Please observe that while a job is filled none of the parties is looking for better opportunities. PERSON and COMPANY agent interact by sending messages to the MATCHING agent, which takes the role of the environment (it is the only mean of communication) and the role of employment agency (it is where the matchings are made).

Both PERSON and COMPANY agents are described by a tuple of values representing their characteristics. For all agents the same features are defined, thus, we can compare any two agents. The features and similarities defined are the following:

name	type	comparison	comment
skills	sequence of 5 boolean values	number of equal entries	skills of a worker resp. skills needed for a job
location	some discrete value	equality	
salary	numerical	difference	minimum salary resp. effective salary

Similarity measures (comparisons) are then normalized to $[0, 1]$, 1 means very similar/close, 0 means completely different.

An agent's rationality is represented by differents means of aggregating the characteristic features. This can be implemented using different similarity functions for features of the same type or for tuples of features. Therefore, similarity of two agents is in general not symmetric and needs to be computed from both perspectives. However, in our simulation, all agents use the same aggregation function.

3.2 Interaction

Interaction is performed using the well-known message-passing model. The MATCHING agent represents the labor market's environment that intoduces delays and frictions between PERSON and COMPANY agents' matchings. The following protocol is executed: A PERSON agent sends a "looking for a job" message to the MATCHING agent, analogeously, a COMPANY agent sends a "looking for an employee" message. Depending on the scenario, the MATCHING agent runs a specific algorithm and replies with messages ("employee/job found") to match employees and jobs. After the matching, the job is filled and in every round the PERSON agent reports its productivity level allowing the COMPANY agent to evaluate the employee's performance. As long as the PERSON's productivity [2] exceeds a certain threshold, this relationship is kept. If the threshold is unmet, the COMPANY 'fires' its worker and both agents re-contact the MATCHING agent to search for a different partner. Job/employee search is executed in every round, i.e., the MATCHING agent tries to connect people with jobs.

3.3 Matching procedure

The MATCHING agent holds complete lists of both vacant jobs and unemployed workers. In each round, a limited number of random pairs is considered, somehow representing the frictions resulting from incomplete knowledge of the agents. Assuming perfect knowledge, the matching function would need to compute the quality of all possible pairs and then perform matchings according to the order of these similarities. A more accurate matching function still preserving realistic frictions is left open for interesting future work.

Both a job seeker and a job opening are chosen at random and the similarity function is computed from both perspectives (recall that similarity in general is not symmetric, as it highly depends on the agent). Then, the mean value of these two similarity measures is computed. If the mean value is bigger than the matching quality threshold, an unemployed worker and a company looking for an employee have successfully found each other and the position gets filled.

Two agents' profiles are compared as follows:

1. The similarity between corresponding features is computed.

2. These similarity values are aggregated.

For the distance between tuples we consider the agregation of the different distance measure. This is done by doing a normalized sum of the distance between individual characteristics.

3.4 Experiment

As we decided to keep the model relatively simple, we also assume that the number of agents does not vary over time. Modeling an open system is an interesting point for futur works.

[2] This takes into account real life factors like motivation, the ability to solve a new task, or sympathy. We hereby introduce natural dynamics of the labor market.

The initial configuration is defined by U, V and the following parameters:

matching quality treshold	0.5	least necessary similarity
firing quality treshold	0.5	least productivity necessary
unexpected firing	0.1	
max. productivity fluctuation	0.3	
unexpected company motivation	0.1	execeptional motivation for a company to find an employee, increases the similarity value of a matching
unexpected worker motivation	0.1	exceptional motivation for a worker to find a job, increases the similarity value of a matching
exceptional matching		treshold below which the matching is considered to hold

What we were intersted in the experiments is trying to emerge the matching function. This function is defined by only two parameters U the number of unemployed workers and V the number of vacant jobs. For a fixed configuration of the parametes of the systems, we run differents times the models for distinct initial values of U and V. After convergence, we consider the market tension defined as the ratio of vacant jobs over the labor force.

In order to test and analyse the emergence of a matching function from interactions that take place in a micro level, we built a basic model with several extensions and simulated it on the Repast platform [2].

3.5 Result

Experiments for $U, V \in \{100, 200, 300, 400\}$ result in the curve are shown in Figure 2. We obtain a curve similar to the Beveridge curve, which is often

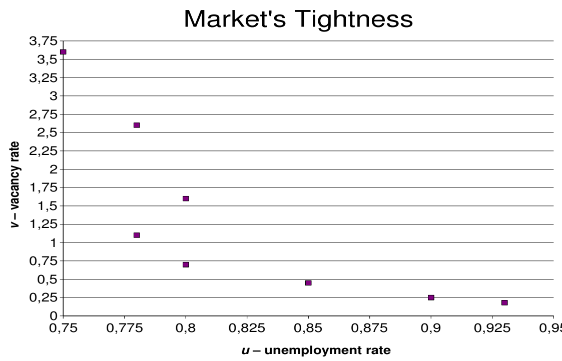

Figure 2: Simulation results

observed in real labor markets. Therefore, we conclude that frictions and delays are captured successfully using multiple agents and that a valid matching function emerges.

4 Conclusion

We model deterministic behavior of employees and companies in the labor market with a simple agent. The objective – simulating frictions in the labor market with a bottom up approach – was met successfully. A curve similar to the Beveridge curve of real scenarios is obtained.

Future work includes refining the model and interpreting the emerging behavior of the market. A realistic scenario should also take into account the agent's history. Variables changing over time might include wealth or learning. In addition, both similarity measures and matching procedure are to be made more sophisticated. Agents may then act more rational by reasoning about past actions and possible outcomes.

References

[1] Barbara Petrongolo and Christopher A. Pissarides. Looking into the black box: A survey of the matching function. *Journal of Economic Literature*, 39(2):390–431, June 2001. available at http://ideas.repec.org/a/aea/jeclit/v39y2001i2p390-431.html.

[2] Repast. Recursive porus agent simulation toolkit.

A Matching function

Frictions and delays are used to explain unemployment in the labor market. The use of a matching function when modeling the labor market is very appealing because it enables to introduce these frictions and delays with minimum complexity. The origins of frictions could be imperfect or incomplete information, heterogeneities, slow mobility of agents, congestion or several other reasons. When using classical economic models, modeling each one of these frictions explicitly will entail an increase in complexity. The matching function aggregates all these frictions to one output, while using a small number of variables, usually without an explicit reference to the origin of the friction.

Most of our review on the matching function is based on the well cited paper: Looking into the black box: a survey of the matching function by Barbara Petrongolo and Christopher Pissarides [1]. As they focus in their survey on microfoundations underlying the matching function and on its empirical success, they do not deal with its modeling effectiveness.

The matching function aggregates agents' interactions when they go to look for a job in employment agencies, when they follow a professional training in purpose of changing their profession or when they have to move from one city (or country) to another in the search of a new position. It also takes under consideration the companies' investments of time and money while searching for an employee to fill a vacancy: the advertising, interviews, internships, evaluations and sorting of employees.

The simplest form of the matching function is:

$$M = m(U, V) \tag{1}$$

where M is the number of jobs created (by matching) during a given time interval, U is the number of unemployed workers, who are actively looking for a job, and V the number of vacant jobs in the market. The matching function is assumed to possess these characteristics:

1. Increasing in both its arguments.
2. Concave.
3. Homogeneous of degree 1 (constant returns to scale).
4. $m(0; V) = m(U; 0) = 0$.

The first matching function model is inspired from coordination failure: randomly placing balls in urns, where firms play the role of urns and workers the role of balls. A job becomes productive when it has a ball in it. Because of coordination failure between those who place randomly the balls in the urns (job searchers), even if there is the same number of balls as the number of urns, there will be some urns that stay empty while others have more than one ball in them. In the case of the labor market, if only one worker can occupy each job, the market will have some jobs that are over popular between workers (the ones that have several applications), while other jobs will stay with no applications at all. This situation is due to the lack of information the workers have about other workers' actions.

In the simplest version, U workers know exactly the location of V vacancies and send one application to each. If a vacancy receives more than one application, it selects an applicant randomly and forms a match. The other applications are rejected and returned to the unemployed workers to apply again. The matching function in this case is constructed by the expression for the number of vacancies that do not receive any application. Given that each vacancy receives a worker's application with probability $\frac{1}{V}$ and there are U applicants, there is a probability $(1-\frac{1}{V})^U$ that a given vacancy will not receive any application. Therefore the number of matches that take place at each application round is:

$$M = V(1-(1-\frac{1}{V})^U) \tag{2}$$

For a large V a good approximation to $(1-\frac{1}{V})^U$ is the exponential $e^{-\frac{U}{V}}$, giving the matching function:

$$M = V(1-e^{-\frac{U}{V}}) \tag{3}$$

This matching function satisfies the properties stated by the general function 1, but it is too naive to be empirically a good approximation to matching in the real labor market. Several extensions are possible to this basic function. First of all we'll consider that workers do not have information about the firms' vacancies, so they choose at random one firm to apply. If N is the level of employment, then the probability that a vacancy receives no applications is: $(1-\frac{1}{N+V})^U$. If in addition the labor force size is L, $N = L-U$, the matching function becomes:

$$M = V(1-e^{-\frac{U}{L-U+V}}) \tag{4}$$

This matching function exhibits increasing returns to scale in U and in V. In the second extension not all the workers are suitable to the vacancies available, but they do not know which vacancies are suitable for them. Let K be the fraction of workers who have the suitable qualities for a randomly picked vacancy. The probability that a vacancy will not be contacted by a worker is still $1-\frac{1}{V}$, but only KU workers are suitable for the job. The matching function, therefore takes the form:

$$M = V(1-e^{-\frac{KU}{V}}) \tag{5}$$

Where $\frac{1}{K}$ is the index of mismatch between the available jobs and workers. A third extension takes under consideration a new parameter that is associated with search intensity of workers. Each round a fraction $1-s$ of the unemployed persons do not apply for a job. This fraction rotates, so each unemployed person misses one application round out of every $\frac{1}{1-s}$ rounds. Now, the probability that a given vacancy receives no applications during a given application round is $(1-\frac{1}{V})^{sU}$, giving the matching function:

$$M = V(1-e^{-\frac{sU}{V}}) \tag{6}$$

Both equations 5 and 6 not only satisfy all the properties of equation 1 for a given K and s, but also permit to integrate mismatch and the frequency of applications and thus bringing the simple form of equation 1 closer to reality.