

# Moral Particularism and Deontic Logic

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**Abstract.** The aim of this paper is to strengthen the point made by Horty about the relationship between reason holism and moral particularism. In the literature *prima facie* obligations have been considered as the only source of reason holism. I strengthen Horty's point in two ways. First, I show that contrary-to-duties provide another independent support for reason holism. Next I outline a formal theory that is able to capture these two sources of holism. While in simple settings the proposed account coincides with Horty's one, this is not true in more complicated or "realistic" settings in which more than two norms collide. My chosen formalism is so-called input/output logic.

**Keywords:** Moral particularism; *prima facie* obligation; conflict resolution; norm violation; contrary-to-duties; default; input/output logic.

## 1 Introduction

In this paper I present a variant to Horty's theory for reasoning with prioritized (deontic or epistemic) defaults presented in [1] and [2]. There the framework is used to shed light on the on-going debate on so-called moral particularism, launched in moral philosophy by the British philosopher Dancy. Neither Dancy nor Horty see the connection with contrary-to-duties and norm violations. I will argue that these are highly relevant to the latter debate, and investigate how Horty's framework cope with them. Next I will present an alternative way to handle prioritized defaults in the setting of input-output logic, and show that while in simple settings both accounts coincide, this is not true in more complicated or "realistic" settings in which more than two norms collide.

## 2 Moral Particularism

Not long ago there was a debate running on Brian Leiter's Legal Philosophy blog<sup>1</sup> about why there is so little interest in deontic logic (and in the formal semantics

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<sup>1</sup> <http://leiterlegalphilosophy.typepad.com/leiter/2007/10/green-v-shapiro.html>

of normative concepts) on the part of other academic disciplines, like moral philosophy. To get them more interested in deontic logic, we need to show them how the latter one can contribute to the issues they are discussing. The paper [1] by Harty can be seen as an attempt to do it. There the focus is on so-called moral particularism, a justly popular “cutting-edge” topic in contemporary ethics that has been most notably defended by the British moral philosopher Dancy [3,4].<sup>2</sup> Moral particularism – or situationism, as it has sometimes been called – seems to present an especially radical objection to the enterprise of moral theory, and hence of deontic logic. To put it simply, particularism is the view that there are no moral principles, only moral intuitions. In his paper Harty examines and criticizes the main argument Dancy gives in support of his view.

First, Dancy’s argument. Particularism is a negative thesis on how moral reasoning works. Dancy calls “generalism” the view that “the very possibility of moral thought and judgment depends on the provision of a suitable supply of moral principles” [4, p. 73]. This is the traditional approach in moral theory. Particularism can be viewed as the negation of the latter view. It is the claim that “the possibility of moral thought and judgement does not depend on the provision of a suitable supply of moral principles” [4, p. 73]. There is, then, the obvious issue of explaining what moral reasoning is. Particularists do not say much about it, and here no attempt will be made to discuss this issue.

In this paper I shall focus on another point of controversy. It is that Dancy grounds his claim on a form of reason holism, holding that what is a reason in one case need not be any reason in another. The main support he provides in support of holism in the theory of reasons comes in the form of examples, in which the notion of *prima facie* obligation takes centre stage. The latter notion was introduced in moral philosophy by Ross [5]. A *prima facie* obligation is one that binds unless overridden by another stronger obligation, and so it is defeasible: it leaves room for exceptions.

It is natural to ask if moral particularism is any different from moral relativism, which is often regarded as flawed. Moral relativism is the view that moral truths vary between individuals and cultures. The relativists are pluralist. They believe that on any moral question there can be more than one correct answer. Particularists still maintain that each particular (‘token’) action is either objectively obligatory or not, depending on the precise details of the situation in question.

Leaving this issue aside, I get back to Harty’s paper. What he does in there can be described as an attack on attack. He builds a formal theory of reasons, which challenges the connection made between reason holism and particularism. The proposed framework supports reason holism: a reason need not retain its supporting value across contexts. Yet, it is based on a system of principles. These are thought of as defeasible generalizations. So the framework provides a counter-example to the claim that reason holism implies particularism.

I agree with the main point made by Harty. However what is missing from his discussion is the notion of contrary-to-duty (CTD) obligation – a highly

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<sup>2</sup> Dancy’s views were historically developed in response to Hare.

problematic topic in deontic logic. Neither Dancy nor Horty see the relevance of CTDs to the topic under discussion. But let us have a look of the kind of examples used by the former to justify reason holism:<sup>3</sup>

The book

*Case 1.* My borrowing a book from you is a reason to return it to you

*Case 2.* My borrowing a book from you is not a reason to return it to you, since you've stolen it from the library

This example has the form:

*Case 1.*  $A$  is a reason for  $B$

*Case 2.*  $A$  is not a reason for  $B$ , since  $C$

The example originally used by Chisholm to make his point about contrary-to-duties (see [6]) can also be given this form. The example consists of the following sentences: i) it ought to be that I go to the assistance of my neighbours ii) it ought to be that if I go I tell them I am coming iii) it ought to be that if I do not go I do not tell them I am coming iv) I do not go. From i)-iv), we can extract:

The neighbours

*Case 1.* My being obliged to go is a reason for telling

*Case 2.* My being obliged to go is not a reason for telling, since it turns out that I do not go.

In a violation context, the primary obligation to go to the assistance remains in force. Is it to say that it remains a reason for telling? Obviously, not. For iii) provides iv) as a (stronger) reason not to tell.

The above considerations show that some CTD scenarios give another independent support for reason holism. Horty's framework was not devised to deal with the latter scenarios. In the next section, I investigate how the theory copes with them. The hope is to show that a stronger statement about the relationship between reason holism and particularism can be made by showing that the framework can deal with both sources of holism.

### 3 Horty's Framework

First, I give an outline of Horty's framework, focusing for the purposes of the present analysis on the special case where all priority relations among defaults are fixed in advance. I will incorporate the amendments to the framework proposed in [2].

A (fixed priority) default theory is noted  $(W, D, <)$ .  $W$  is a set of propositional formulae,  $D$  a set of defaults subject to a strict partial ordering  $<$ . Defaults are of the form  $A \rightarrow B$ , where  $\rightarrow$  stands for material implication. The conclusion or head  $B$  denotes the content of an obligation.  $\delta < \delta'$  means that  $\delta'$  has a higher priority than  $\delta$ .  $Conclusion(D)$  is  $\{conclusion(\delta) : \delta \in D\}$ .

<sup>3</sup> This example can be found in [3, p.60].

Horty speaks in terms of scenario. A scenario is a particular subset  $S$  of  $D$ , which represent the “set of defaults that have been accepted by the agent, at some stage of the reasoning process, as providing sufficient reasons for their conclusions” [1, p. 5]. The goal is to determine what Horty calls the “proper scenarios” [1, p. 8] based on a default theory. Intuitively, the defaults in a proper scenario tell us what counts as a (good, satisfactory, etc. ) reason for what. Thus, if  $A \rightarrow B$  is in the proper scenario  $S$  based on a given default theory, then  $S$  is said to provide  $A$  as a reason for  $B$ .

Formally, the notion of proper scenario is defined using four other notions.

The first one is the notion of a default being triggered in scenario  $S$ . Let  $Triggered_{(W,D,<)}(S)$  denote the set of all such defaults. The definition runs as follows, where  $\vdash$  denotes classical consequence:

$$Triggered_{(W,D,<)}(S) = \{\delta \in D : W \cup Conclusion(S) \vdash Premise(\delta)\}$$

The second notion is that of a default being conflicted in  $S$ . Let  $Conflicted_{(W,D,<)}(S)$  denote the set of all such defaults. The definition reads:

$$Conflicted_{(W,D,<)}(S) = \{\delta \in D : W \cup Conclusion(S) \vdash \neg conclusion(\delta)\}$$

The third notion is that of a default being defeated in  $S$ . Let  $Defeated_{(W,D,<)}(S)$  denote the set of all such defaults. For  $D, D' \subseteq S$ , put  $D < D'$  if  $\delta < \delta'$  for all  $\delta$  in  $D$  and  $\delta'$  in  $D'$ . Let  $S^{S'/D'}$  denote the result of replacing  $S'$  by  $D'$  in  $S$ . The definition of defeat reads:

$$\begin{aligned} Defeated_{(W,D,<)}(S) = \{ & \delta \in D : \exists D' \in Triggered_{(W,D,<)}(S) \text{ s.t.} \\ & a) \delta < D' \text{ and} \\ & b) \exists S' \subseteq S \text{ with } S' < D' \text{ such that} \\ & \quad W \cup Conclusion(S^{S'/D'}) \text{ is consistent} \\ & \quad W \cup Conclusion(S^{S'/D'}) \vdash \neg Conclusion(\delta) \} \end{aligned}$$

The fourth notion is that of a default being binding in  $S$ . Let  $Binding_{(W,D,<)}(S)$  denote the set of all such defaults. The definition reads:

$$\begin{aligned} Binding_{(W,D,<)}(S) = \{ & \delta \in D : \delta \in Triggered_{(W,D,<)}(S) \\ & \delta \notin Conflicted_{(W,D,<)}(S) \\ & \delta \notin Defeated_{(W,D,<)}(S) \} \end{aligned}$$

The notion of proper scenario is given a quasi-inductive definition:

**Definition 1 (Proper scenario).** *Let  $S$  be a scenario based on the ordered default theory  $(W, D, <)$ . Then  $S$  is a proper scenario based on  $(W, D, <)$  just in case  $S = \bigcup_{i \geq 0} S_i$  where*

$$\begin{aligned}
S_0 &= \emptyset \\
S_{i+1} &= \{ \delta \in D : \delta \in \text{Triggered}_{(W,D,<)}(S_i), \\
&\quad \delta \notin \text{Conflicted}_{(W,D,<)}(S), \\
&\quad \delta \notin \text{Defeated}_{(W,D,<)}(S) \}
\end{aligned}$$

The notion of proper scenario can be used to define the analogue of the notion of extension in Reiter's theory. Intuitively, an extension gathers all the agent's obligations that follow from what it knows about the world. The idea is to assume that the agent derives its obligations from justifications or reasons for those obligations: in particular, that the agent is bounded by an obligation if it possesses a satisfactory reason for that obligation. The extension of a default theory can, thus, be defined as the set of obligations generated by a proper scenario. Given a set  $X$  of formulae, let  $Cn(X)$  denote the closure of  $X$  under classical consequence. The definition runs:

**Definition 2 (Extension).** *Let  $(W, D, <)$  be an ordered default theory and  $\mathcal{E}$  a set of formulae. Then  $\mathcal{E}$  is an extension of  $(W, D, <)$  just in case  $\mathcal{E} = Cn(W \cup S)$ , where  $S$  is a proper scenario based on this default theory.*

Below I apply the construction to the two examples from section 2 used to justify reason holism, and show that the construction yields the correct outcomes.

**Example 3 (The book).** *Let  $b$ ,  $s$ ,  $y$  and  $l$  represent the respective propositions that I borrowed the book from you, that you stole it from the library, that I return the book to you, and that I return it to the library. Put  $D = \{b \rightarrow y, s \rightarrow l\}$  with  $s \rightarrow l > b \rightarrow y$ . Assume  $W = \{y \rightarrow \neg l\}$  – I cannot simultaneously return the book to you and the library.*

**Case 1.** *Suppose that  $W$  contains, in addition, the sole formula  $b$ . In this case, the ordered default theory  $(W, D, <)$  yields  $S_1 = \{b \rightarrow y\}$  as its unique proper scenario, providing  $b$  as a reason for  $y$ .*

**Case 2.** *Suppose that  $W$  contains, in addition, the two formulae  $b$  and  $s$ . In this case, the ordered default theory  $(W, D, <)$  yields  $S_2 = \{s \rightarrow l\}$  as its unique proper scenario, and thereby provides  $s$  as a reason for  $l$ , rather than providing  $b$  as a reason for  $y$ .*

**Example 4 (The neighbours).** *Let  $a$  and  $t$  represent the respective propositions that I go the assistance of my neighbours, and that I tell them I am coming. Put  $D = \{\top \rightarrow a, a \rightarrow t, \neg a \rightarrow \neg t\}$  with  $<$  the empty relation. Note that in the case of an empty priority relation the notion of defeat plays no role, because  $\text{Defeated}_{(W,D,\emptyset)}(S) = \emptyset$ .*

**Case 1.** *Suppose that  $W$  contains no formula,  $W = \emptyset$ . In this case, the ordered default theory  $(W, D, <)$  yields  $S_3 = \{\top \rightarrow a, a \rightarrow t\}$  as its unique proper scenario, providing  $a$  as a reason for  $t$ .*

**Case 2.** Suppose that  $W$  contains the formula  $\neg a$ . In this case, the ordered default theory  $(W, D, <)$  yields  $S_4 = \{\neg a \rightarrow \neg t\}$  as its unique proper scenario, and thereby provides  $\neg a$  as a reason for  $\neg t$ , rather than  $a$  as a reason for  $t$ .

Examples 3 and 4 are two simple cases of reason holism. Harty's theory appears to handle them relatively well. In the next section I will show that the theory is not adequate for a more complex case of reason holism. Before showing it, I need to introduce an alternative framework based on so-called input/output logic with constraints (constrained input/output logic, or "cIOL" for short), which will allow me to handle this more complex case of reason holism in – I believe – a more satisfactory way.

## 4 Constrained IOL with Priorities

For clarity's sake, I proceed in three steps. I first give an outline of unconstrained IOL, on top of which constrained IOL is built. Next, I introduce constraints. Finally I add priorities to the framework.

To keep things simple, I only consider obligations – permissions are put to one side. In input/output logic, a normative code is a set  $G$  of conditional norms, which is a set of ordered pairs  $(a, x)$ . Here  $a$  and  $x$  are two formulae of propositional logic. Each such pair will be referred to as a generator. The body  $a$  is thought of as an input, representing some condition or situation, and the head  $x$  is thought of as an output, representing what the norm tells us to be obligatory in that situation.

Some notation.  $L$  is the set of all formulae of propositional logic. Given an input  $A \subset L$ , and a set of generators  $G$ ,  $G(A)$  denotes the image of  $G$  under  $A$ , i.e.,  $G(A) = \{x : (a, x) \in G \text{ for some } a \in A\}$ .

**Definition 5 (Output operations).** Let  $A$  be an input set, and let  $G$  be a set of generators. The following input/output operations can be defined, where a complete set is one that is either maximal consistent<sup>4</sup> or equal to  $L$ :

$$\begin{aligned} out_1(G, A) &= Cn(G(Cn(A))) \\ out_2(G, A) &= \cap \{Cn(G(V)) : A \subseteq V, V \text{ complete}\} \\ out_3(G, A) &= \cap \{Cn(G(B)) : A \subseteq B \supseteq Cn(B) \supseteq G(B)\} \\ out_4(G, A) &= \cap \{Cn(G(V)) : A \subseteq V \supseteq G(V), V \text{ complete}\} \end{aligned}$$

$out_1(G, A)$ ,  $out_2(G, A)$ ,  $out_3(G, A)$  and  $out_4(G, A)$  are called *simple-minded* output, *basic* output, *reusable simple-minded* output and *reusable basic* output, respectively.

When analysing examples, I will occasionally choose to instantiate *out* into either  $out_3$  or  $out_4$ . This is because these two operations satisfy plain transitivity

<sup>4</sup> The set is consistent, and none of its proper extensions is consistent.

(“from  $(a, x)$  and  $(x, y)$  infer  $(a, y)$ ”). With such a property it is much easier to get an intuitive feeling of what is going on.

Now we can turn to IOL with constraints. The motivation is best explained by considering the case of a conflict between two obligations. Put  $G = \{(a, x), (a, \neg x)\}$ , and consider input  $a$ . The reader may easily verify that, for all output operations,  $out(G, a) = L$ . This shows that none of the operations considered so far are conflict-tolerant. The idea is to cut back the set of norms to just below the threshold of yielding excess, and consider the resulting output. To do that, we look at the maximal non-excessive subsets, i.e. the maximal  $G' \subseteq G$  such that  $out(G', A)$  is consistent. The family of all such  $G'$  may be called the maxfamily of  $A$ , and the family of outputs  $out(G', A)$  for  $G'$  in the maxfamily, may be called the outfamily of  $A$ . Below this maximal subsets strategy is referred to as the “threshold idea”. Such a strategy is familiar from the literature on belief revision. A key difference is that the latter strategy is here applied to the norms used to generate the output. In the dominant belief revision theory, the so-called AGM model, the maximal subsets strategy is applied to the output set directly. Formulated in such general terms, this may seem a rather nebulous and inconsequential distinction to make. In fact, this makes a real difference, as we will see in due course.<sup>5</sup>

The example I have just used to motivate the approach involves what might be called a “strict” or logical conflict: the head of one rule contradicts the head of the other. The point that has been made about such conflicts also applies to “natural” (or non-strict) conflicts. These are of the form:  $(a, x)$  and  $(a, y)$ , with  $x$  incompatible with  $y$  in the sense of natural or physical necessity, broadly conceived. The treatment of this type of conflict necessitates the use of a set  $C$  of “integrity constraints” (as they are sometimes called) that the output is never allowed to contradict. In this case,  $C = \{x \rightarrow \neg y\}$ . This is meant to indicate that  $x$  and  $y$  cannot be simultaneously true, given the agent’s present physical and psychical capabilities, etc. Compared to the treatment of a strict conflict, the main difference is that we take the maximal  $G' \subseteq G$  such that  $out(G', A)$  is consistent with  $C$ . Example 7 below provides an illustration.

For contrary-to-duties, the idea is similar. Typically we take the maximal  $G' \subseteq G$  such that  $out(G', A)$  is consistent with  $A$ . It is easy to check that not doing so would create the same problems as those encountered in Standard Deontic Logic (SDL). This is illustrated clearly by example 12.

The formal definition below is general, covering as special case both inconsistency of output and its inconsistency with input.

**Definition 6 (Threshold).** *Let  $G$  be a set of generators and  $out$  be an input/output operation. Let  $C$  be an arbitrary set of formulae, which we may call “consistency constraints”. We define:*

- *maxfamily( $G, A, C$ ) is the set of  $\subseteq$ -maximal subsets  $G'$  of  $G$  such that  $out(G', A)$  is consistent with  $C$ .*
- *outfamily( $G, A, C$ ) =  $\{out(G', A) \mid G' \in \text{maxfamily}(G, A, C)\}$ .*

<sup>5</sup> The contrast between the two approaches is more fully discussed in [7, p. 107-108].

The cases  $C = \emptyset$  and  $C = A$  express consistency of output, and its consistency with input, respectively. In practice, there is no general rule for choosing the specific instantiation. It all depends on the nature of the particular example we are analysing. The instantiation  $C = A$  is expedient for those cases where the input describes a state of affairs that is “settled” as true, and cannot be changed afterwards.<sup>6</sup> In such cases, it makes sense to require that the output be consistent with the input: “ought” implies “can”.

Notably, a set of generators and an input do not have a set of propositions as output, but a set of sets of propositions. So, like in the logics of belief change and nonmonotonic inference, we can infer a set of propositions by taking either a credulous or a sceptical approach. In [9], the two resulting operations are called full meet and full join constrained output, and they are noted  $\cap outfamily(G, A, C)$  and  $\cup outfamily(G, A, C)$ , respectively.

Example 7 below illustrates the notions of maxfamily and outfamily.

**Example 7 (Natural conflict).** Put  $G = \{(a, x), (a, y)\}$  with  $C = \{x \rightarrow \neg y\}$  and  $A = \{a\}$ . For all the output operations,  $out(G, A) = Cn(x, y)$ , which is inconsistent with  $C$ . The maxfamily has two elements  $\{(a, x)\}$  and  $\{(a, y)\}$ , and thus the outfamily has two elements  $Cn(x)$  and  $Cn(y)$ . Let the final output be calculated using the full meet operation. We have

$$\cap outfamily(G, A, C) = Cn(x) \cap Cn(y) = Cn(x \vee y)$$

Since no rule has priority over the other, the most that comes is that the disjunction of  $x$  and  $y$  is obligatory.

Now I turn to the task of adding priorities. Here I follow the most natural approach. I assume that conflicts are resolved based on a priority ordering on the powerset of generators, such that only a suitably chosen subset of the maxfamily is used to generate the output. This is implemented using a relation on set of rules. However, in practical applications, one uses a relation on rules, not a relation on sets of rules. So the question is: given a relation on rules, how can it be lifted to a relation on sets of rules? The definition of lifting given below is taken from Brass [10].<sup>7</sup> The relation  $\geq$  is read “at least as strong as”, and the superscript  $s$  (mnemonic for “set”) is used to distinguish between the two relations.

I assume that  $\geq$  is a pre-order, i.e., the relation is reflexive and transitive.  $>$  denotes the strengthened complement of  $\geq$ , defined by putting  $a > b$  whenever  $a \geq b$  and  $b \not\geq a$ .  $\sim$  is the equivalence relation generated by  $\geq$ , defined by putting  $a \sim b$  whenever  $a \geq b$  and  $b \geq a$ . Each of the latter two notions has a counterpart in terms of ordering on sets – the definition is analogous. And  $a \in S$  is called a  $\geq^s$ -maximal element of  $S$  if, for all  $b \in S$ ,  $b \geq^s a$  implies  $a \geq^s b$ .

<sup>6</sup> This reading is related to Hansson’s interpretation of circumstances in so-called dyadic deontic logic (see [8]).

<sup>7</sup> One reviewer pointed out that the same proposal was repeated in a perhaps wider known paper by Sakama and Inoue [11], and that some equivalents to this definition are discussed in Hansen [12, p. 9]. The definition has been widely used in the literature. For a list of references, see Halpern [13, p. 4].



**Definition 8 (Lifting).** Let  $G$  be a set of generators equipped with a pre-order  $\geq$ . For any  $G_1, G_2 \subseteq G$ , we define  $G_1 \geq^s G_2$  to hold if for every  $\delta_2 \in G_2 \setminus G_1$  there is  $\delta_1 \in G_1 \setminus G_2$  with  $\delta_1 \geq \delta_2$ .

Now comes the main construction.

**Definition 9 (Outfamily with priorities).** Let  $A$  and  $G$  be an input set and a set of generators equipped with a pre-order  $\geq$ , respectively. Let  $\geq^s$  be defined as in definition 8. We put

- $\text{maxfamily}(G, A, C)$  is the set of  $\subseteq$ -maximal subsets  $G'$  of  $G$  such that  $\text{out}(G', A)$  is consistent with  $C$
- $\text{filterfamily}(G, A, C)$  is the set of  $G' \in \text{maxfamily}(G, A, C)$  that ‘maximize’ the output, i.e., that are such that  $\text{out}(G', A) \subset \text{out}(G'', A)$  for no  $G'' \in \text{maxfamily}(G, A, C)$
- $\text{preffamily}(G, A, C)$  is the set of  $\geq^s$ -maximal elements of  $\text{filterfamily}(G, A, C)$
- $\text{preffamily}_d(G, A, C)$  is the set of elements  $G'$  of  $\text{preffamily}(G, A, C)$  stripped of all the pairs  $(a, x)$  that are “inactive” in  $G'$ , in the sense that  $\text{out}(G', A) = \text{out}(G' - \{(a, x)\}, A)$ .

The subscript “ $d$ ” is short for “distilled”.

Some further comments on definition 9 are in order. I suggest viewing each member of the distilled preffamily as the analogue of a proper scenario in Horty’s theory. There are four steps involved in their construction. We start by determining the maxfamily. It gathers all the maximal subsets of  $G$  whose output remains consistent with the constraints  $C$ . This step is mandatory to guard against possible contradictions when applying rules to the input set. The filterfamily, then, selects the elements of the maxfamily with the most informative output, i.e., the elements of the maxfamily that “maximize” the number of conclusions that can be drawn. This is needed to avoid an unwanted loss of information created by the threshold idea as applied to rules. Example 13 below provides a good illustration of this. The preffamily, then, determines the most preferred elements in the filterfamily by use of the given priorities between the norms. This step is needed to resolve conflicts between norms. As will become clear in the treatment of the examples, each element in the preffamily may contain norms that are in force, but are not triggered. The last step consists in removing them from the preffamily, because they have no effect on the output. In [14], such norms are called “redundant”. In the present context I prefer to call them “inactive”. Like in Horty’s framework, the rules in the distilled preffamily tell us what counts as a reason for what.

Although it is not essential for present purposes, I introduce below the associated output operation. It can be viewed as the analogue of the notion of extension in Horty’s account. The subscript  $p$  is short for “preferred”.

**Definition 10 (Preferred output).** Let  $G$ ,  $A$  and  $C$  be a pre-ordered set of generators, an input set, and a set of integrity constraints, respectively. We define

$$x \in \text{out}_p(G, A) \text{ iff } x \in \cap \{ \text{out}(G', A) \mid G' \in \text{preffamily}_d(G, A, C) \}$$

Below I apply the account to the two examples from section 3, and I show that it yields the same results as Horty's.

**Example 11 (The book).** Let  $b$ ,  $s$ ,  $y$  and  $l$  be instantiated as in example 3. Put  $C = \{y \rightarrow \neg l\}$ . Let  $G = \{(b, y), (s, l)\}$  with  $(s, l) > (b, y)$ .

**Case 1.** Assume  $A = \{b\}$ . For all output operations,  $\text{out}(G, A)$  equates  $Cn(y)$ , which is consistent with  $C$ . In this case, the outfamily/filterfamily/preffamily has one element  $\{(b, y), (s, l)\}$ . And the distilled preffamily has one element  $\{(b, y)\}$ , providing  $b$  as a reason for  $y$ .

**Case 2.** Assume  $A = \{b, s\}$ . For all output operations,  $\text{out}(G, A)$  equates  $Cn(y, l)$ , and thus it is inconsistent with  $C$ . The maxfamily has two elements,  $\{(b, y)\}$  and  $\{(s, l)\}$ , and so has the filterfamily. Furthermore,  $\{(s, l)\} >^s \{(b, y)\}$  since  $(s, l) > (b, y)$ . So the preffamily/distilled preffamily has only one element  $\{(s, l)\}$ , providing  $s$  as a reason for  $l$ .

**Example 12 (The neighbours).** Put  $G = \{(\top, a), (a, t), (\neg a, \neg t)\}$ , where  $a$  and  $t$  are instantiated as in example 4. Let  $\geq$  be the empty relation, and  $\text{out} \in \{\text{out}_3, \text{out}_4\}$ . Put  $C = A$

**Case 1.**  $W = \emptyset$ . In this case,  $\text{out}(G, A) = Cn(a, t)$  which is consistent. So the outfamily/filterfamily/preffamily has one element  $G$ . And the distilled preffamily has one element  $\{(\top, a), (a, t)\}$ , providing  $a$  as a reason for  $t$ .

**Case 2.**  $W = \{\neg a\}$ .  $\text{out}(G, A)$  equates  $Cn(a, t, \neg t)$ , which is inconsistent. The outfamily/filterfamily/preffamily has one element  $\{(a, t), (\neg a, \neg t)\}$ . The distilled preffamily has one element  $\{(\neg a, \neg t)\}$ , providing  $\neg a$  as a reason for  $\neg t$ .

Example 13 below shows why the filterfamily step is needed in definition 9.

**Example 13 (The birthday).** Assume  $G = \{(\top, b), (b, t)\}$ , where  $b$  denotes the proposition that I go to a birthday party, and  $t$  the proposition that I tell their organizers that I am coming. Assume the organizers need to know the exact number of persons who will come. In this respect,  $(b, t) > (\top, b)$ . Let  $\text{out} \in \{\text{out}_3, \text{out}_4\}$ . Put  $A = \{\neg t\}$  and  $C = A$ . This means that  $\neg t$  is settled as true. The outfamily has two elements,  $\{(\top, b)\}$  and  $\{(b, t)\}$ . The filterfamily has one element  $\{(\top, b)\}$ , because  $\text{out}(\{(b, t)\}, A) = Cn(\emptyset) \subset \text{out}(\{(\top, b)\}, A) = Cn(b)$ . So the preffamily and the distilled preffamily have one element  $\{(\top, b)\}$ , and thus  $b \in \text{out}_p(G, A)$ .

Now take definition 9. Assume the filterfamily step is removed from it, and the notion of preffamily is amended accordingly:  $\text{preffamily}(G, A, C)$  is the set of  $\geq^s$ -maximal elements of  $\text{maxfamily}(G, A, C)$ . (The notion of distilled preffamily remains the same.) Since  $(b, t) > (\top, b)$ , the preffamily and the distilled preffamily have  $\{(b, t)\}$  as unique element, and thus  $\text{out}_p(G, A) = Cn(\emptyset)$ , which might be considered to be counter-intuitive.

The reader is invited to verify that Horty's account yields the same outcome in example 13. That is, the ordered default theory  $(W, D, <)$ , with  $W = \{\neg t\}$ ,  $D = \{\top \rightarrow b, b \rightarrow t\}$ , and  $b \rightarrow t > \top \rightarrow b$ , yields  $S_5 = \{\top \rightarrow b\}$  as its unique proper scenario, and thus  $b \in \mathcal{E}$

Another interesting feature of the filterfamily step is that it clarifies how the present account connects with Reiter's one. We can read the elements of  $G$  as normal reiter rules  $a : b/b$ . So let  $(G, A)$  be a Reiter normal default system. Let  $extfamily(G, A)$  denote the family of all the extensions of  $(G, A)$  in the sense of Reiter. Using an observation from [15] it is straightforward to show that the account proposed here is a conservative generalization of the Reiter account in the following sense.

**Remark 14.** *Let  $out$  be  $out_3^+$ .<sup>8</sup> If  $\geq$  is the empty relation, then*

$$extfamily(G, A) = \{out(G', A) \mid G' \in preffamily(G, A, C)\}$$

where  $C$  is either  $A$  or  $\emptyset$ .

*Proof.* The proof is straightforward, combining the observation from [15, section 4] that (for  $out = out_3^+$ , and  $C = A$  or  $\emptyset$ )  $extfamily(G, A)$  consists of exactly the maximal (w.r.t. set-inclusion) elements of  $outfamily(G, A, C)$

$$extfamily(G, A) = max(outfamily(G, A, C))$$

with the observation that

$$max(outfamily(G, A, C)) = \{out(G', A) \mid G' \in preffamily(G, A, C)\}$$

when  $\geq$  is the empty relation. □

Now I present a bottom line example showing why the IOL account is different from Horty's one. The specificity of the threshold approach as applied to rules (rather than extension) is brought to the surface in this example.

**Example 15 (Bottom line example, with Horty's solution).** Put  $D = \{a \rightarrow \neg b, a \rightarrow b, b \rightarrow c\}$  with  $a \rightarrow \neg b < a \rightarrow b < b \rightarrow c$ .

**Case 1.** Assume  $W = \{a\}$ . In this case, the ordered default theory  $(W, D, <)$  yields  $S_6 = \{a \rightarrow b, b \rightarrow c\}$  as its unique proper scenario, providing  $a$  as a reason for  $b$ , and  $b$  as a reason for  $c$ .

**Case 2.** Assume  $W = \{a, \neg c\}$ . The ordered default theory  $(W, D, <)$  yields  $S_7 = \{a \rightarrow b\}$  as its unique proper scenario, hence providing  $a$  as a reason for  $b$ .

**Example 16 (Bottom line example, with the IOL solution).** Assume we have  $G = \{(a, \neg b), (a, b), (b, c)\}$  with  $(a, \neg b) < (a, b) < (b, c)$ . Let  $out \in \{out_3, out_4\}$ . Put  $C = A$ .

**Case 1.** Assume  $A = \{a\}$ . The outfamily and the filterfamily are the same; they have two elements  $\{(a, b), (b, c)\}$  and  $\{(a, \neg b), (b, c)\}$ . We have  $\{(a, b), (b, c)\} >^s \{(a, \neg b), (b, c)\}$ . So the preffamily/distilled preffamily has one element  $\{(a, b), (b, c)\}$ , providing  $a$  as a reason for  $b$  and  $b$  as a reason for  $c$ .

<sup>8</sup>  $out_3^+$  is the throughput version of  $out_3$  that allows inputs to reappear as outputs.

**Case 2.** Assume  $A = \{a, \neg c\}$ . The outfamly and the filterfamily are the same; they have two elements  $\{(a, b)\}$  and  $\{(a, \neg b), (b, c)\}$ . We have  $\{(a, \neg b), (b, c)\} >^s \{(a, b)\}$ . So the preffamly has one element  $\{(a, \neg b), (b, c)\}$ , and the distilled preffamly has one element  $\{(a, \neg b)\}$ , hence providing  $a$  as a reason for  $\neg b$ .

In case 2, Horty's construction keeps the default  $a \rightarrow b$ , but discards the default  $b \rightarrow c$  because it is conflicted, given that  $\neg c$  is in  $W$ . In the IOL framework the corresponding pair  $(b, c)$  is kept in the (only) set in preffamly, excluding further addition of  $(a, b)$ , given that  $\neg c$  is in  $A$ , but allowing further addition of  $(a, \neg b)$ .

There is a way to look at the example that makes the IOL solution more intuitive than Horty's solution. The question is: shall I do  $b$  or not? The ordering  $(a, \neg b) < (a, b)$  says that  $b$  has priority over  $\neg b$ . So it would seem to follow that I should do  $b$ . But, in reply, it can be said that the ordering  $(a, b) < (b, c)$  says that compliance with the stronger of the two conflicting norms triggers an obligation of even higher rank, namely the obligation to do  $c$ . Furthermore,  $c$  is already (settled as) false. Hence if I go for  $b$  I will put myself in a violation state with respect to a norm with an even higher rank. I might wish to avoid the violation of the most important norms. And so I shall not do  $b$ . This is the outcome that definition 9 predicts.

Here is a natural language version of the above example that might be of help to check intuitions. Take the case of a Professor at the University, who is worried that he has not heard from his PhD student for months. Let  $a$  stand for the proposition that the Professor has not heard from his PhD student for months, and let  $b$  denote the proposition that the Professor meets his student at an agreed time and place (so he can check on the progress he has made). A typical duty expected of a PhD supervisor is that he maintains regular contact with his or her students so he can provide support and guidance. We have  $(a, \neg b), (a, b) \in G$  and  $(a, \neg b) < (a, b)$ , meaning that, should there be a conflict w.r.t.  $b$ ,  $(a, b)$  will normally take precedence over  $(a, \neg b)$ . Now suppose the Dean of the University requests (via his secretary) a meeting with the Professor to discuss a programme the latter is in charge of. Suppose also there is a substantial overlap between the meeting requested by the Dean and the appointment scheduled with the student. Suppose, finally, that the Dean leaves the Professor the option to reschedule his meeting with him in the event he cannot make it. Due to this flexibility, the request from the Dean does not create a clash of obligations yet. We have  $(b, c) \in G$ , where  $c$  denotes the proposition that the Professor reschedules his meeting with the Dean. We also have  $(a, b) < (b, c)$ . This priority ordering just mirrors the organization hierarchy. Now suppose that, for one reason or the other, the meeting with the Dean cannot be rescheduled, viz.  $A = \{a, \neg c\}$ . Intuitively, it seems clear that the Professor should skip his meeting with the student.

This is also an example of polarity reversal. In case 1, the consideration  $a$  ("not having heard from the student") is a reason for  $b$  ("meeting the student"), and  $b$  is a reason for  $c$  ("rescheduling the meeting with the Dean"). In case 2,  $a$  is a reason for  $\neg b$ , and  $b$  is no longer a reason for  $c$ . Hence Horty's theory solves simple cases of reason holism, but appears not to be well-suited for slightly more complex cases like this one, where more than two norms collide – viz. where two

obligations conflict, and disregard for the more important one can be excused by obedience to an even more important obligation.

Clearly there is a disagreement between the two accounts. How should it be diagnosed? Here is a possible explanation – there might be more convincing ones. Dancy believes reasons behave the same way whether they are epistemic (reasons for belief) or practical (reasons for action). It is even this very belief that led him to formulate the moral particularism thesis.<sup>9</sup> Horty seems to follow this line, when he suggests using the same notation for both kinds of reason, and when he warns the reader that throughout the discussion he will switch back and forth, rather casually, between the two readings (see [1, p. 4]). It is natural to ask what of the bottom line example for the case of epistemic reasons. Can we make sense of the Horty outcome in this case? Consider the following variant. Let  $a$ ,  $b$ ,  $c$  and  $d$  be for “being a bird”, “flying”, “being a penguin”, and “having wings”, respectively. Put  $a \rightarrow b$ ,  $c \rightarrow \neg b$ ,  $\neg b \rightarrow \neg d$ . The ordering between the first two is  $a \rightarrow b < c \rightarrow \neg b$ . The ordering between the last two is less clear-cut, but let us suppose (for the argument’s sake) that it is  $c \rightarrow \neg b < \neg b \rightarrow \neg d$ . Put  $W = \{a, c, d\}$ . Here it makes more sense to keep the default  $c \rightarrow \neg b$ , and disregard the default  $\neg b \rightarrow \neg d$  for the sole reason its head is conflicted with the facts. Indeed, these provide an exception to the normality claim  $\neg b \rightarrow \neg d$ . In the deontic domain, such a disregard looks more problematic, because a violation is not the same as an exception. Here lies perhaps the explanation. To say that holism is a general phenomenon that applies to both practical and epistemic reasons is one thing. To say that reasons always behave the same way whether epistemic or practical is another thing.

## 5 Conclusion

The aim of this paper was to strengthen the point made by Horty about the relationship between reason holism and particularism. In the literature only the notion of *prima facie* obligation has been used to motivate reason holism. I strengthened his point, by first showing that contrary-to-duties provide another independent support for reason holism, and then outlining a formal theory that is able to capture these two sources of holism. While in simple settings the proposed account coincides with Horty’s one, this is not true in more complicated or “realistic” settings in which more than two norms collide.

Needless to say, much work remains to be done to get a full-blooded formal account. On the one hand, I have been concentrated on two different approaches and discussed only two or three examples of reason holism. Future studies should include alternative approaches available from literature, like the prioritized imperative semantics of Hansen [12,16]. Also a wider set of examples should be used to test and compare them. On the other hand, it may be felt that the proposed account is not sophisticated enough, and that there is more to default reasoning than what the present study has suggested. For instance, there are situations

<sup>9</sup> In [4, chapter 4.2], he points out that holism is uncontested for epistemic reasons, and therefore it would be surprising if practical reasons behaved differently.

where the priority relation amongst reasons, or defaults, are established through default reasoning. This is a topic for future research. The expected output is a better understanding of reason holism, and some of its intricacies.

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