Introduction

Topic of the lecture

Reasoning about norm violation

- Contrary-to-duties, CTDs

- Importance for NMAS
  - Sanctions; norm enforcement
- Importance for deontic logic
- Notion hard to formalize

Aim of the lecture

Explain the problem(s)

Layout

- Introduction
- The CTD problem
    - how to detach an obligation in a violation context
- Solution
- Groundwork for next lecture on 2-D modal logic
- Chapter in the handbook

A bit of terminology (1)

Unconditional obligation:

- $\bigcirc A$
- A: a fact; $\bigcirc$: a 'monadic' modal operator

Conditional obligation:

- $\bigcirc(B/A)$
- $\bigcirc(-/-)$ is 'dyadic'

A bit of terminology (1)

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Conditional obligation:

- $\bigcirc(B/A)$
- $\bigcirc(-/-)$ is 'dyadic'

$\bigcirc(-/-)$ can be defined in terms of $\bigcirc$, or the other way around. Here: the second; $\bigcirc A$ abbreviates $\bigcirc(A/\top)$.

A bit of terminology (2)

A so-called **primary** obligation:

$\boxed{\bigcirc A}$ Ex : breastfeeding

A **contrary-to-duty** (or secondary, CTD) obligation

$\boxed{\bigcirc(B/\neg A)}$ getting a nanny

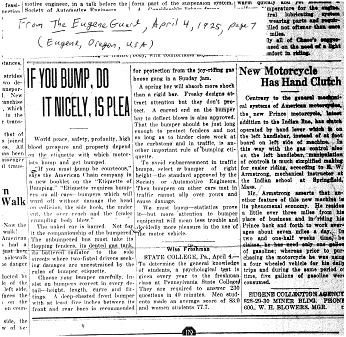
An **according-to-duty** (ATD) obligation

$\boxed{\bigcirc(B/A)}$ 2 glasses of wine per week

A **contrary-to-contrary-to-duty** obligation

$\boxed{\bigcirc(C/\neg A \wedge \neg B)}$ instant formula

Bump nicely



From The Eugene Guard, April 4, 1925, page 7
(Eugene, Oregon, USA)

IF YOU BUMP, DO IT NICELY, IS PLEA

World peace, safety, profanity, high blood pressure and property depend on the etiquette with which motorists bump and get bumped.

"If you must bump be courteous," says the American Chain company in a new booklet on the "Etiquette of Bumping." "Etiquette requires bumpers on all cars—bumpers which will ward off without damage the head on collision, the side hook, the under cut, the over reach and the fender crumpling body blow."

The naked car is barred. Not for it the companionship of the bumpered. The unbumpered bus must take its flopping fenders, its dented gas tank, its battered radiator to the side streets where two-fisted drivers seeking damages are unrestrained by the rules of bumper etiquette.

Choose your bumper carefully. Insist on bumpers correct in every detail—height, length, curve and fittings. A deep-chested front bumper with at least five inches between its front and rear bars is recommended for protection from the joy-riding gas house gang in a Sunday jam.

A spring bar will absorb more shock than a rigid bar. Freaky designs attract attention but they don't protect. A curved end on the bumper bar to deflect blows is also approved. That the bumper should be just long enough to protect fenders and not so long as to hinder close work at the curbstone and in traffic, is another important rule of bumping etiquette.

To avoid embarrassment in traffic bumps, select a bumper of right height—the standard approved by the Society or Automotive Engineers. Then bumpers on other cars met in traffic cannot slip over yours and cause damage.

We must bump—statistics prove it—but more attention to bumper equipment will mean less trouble and decidedly more pleasure in the use of the motor vehicle.

Wise Freshman

STATE COLLEGE, Pa., April 4.—To determine the general knowledge of students, a psychological test is given every year to the freshman class at Pennsylvania State College. They are required to answer 230 questions in 40 minutes. Men students made an average score of 83.9 and women students 77.7.

New Motorcycle Has Hand Clutch

Contrary to the general mechanical systems of American motorcycles, the new Prince motorcycle, latest addition to the Indian line, has clutch operated by hand lever which is on the left handlebar, instead of at foot board on left side of machine. In this way with the gas control also on the left handlebar, manipulation of controls is much simplified making for easier riding, according to H. W. Armstrong, mechanical instructor at the Indian school at Springfield, Mass.

Mr. Armstrong asserts that another feature of this new machine is its phenomenal economy. He resides a little over three miles from his place of business and in riding his Prince back and forth to work averages about seven miles a day. In two and one-half weeks time, he claims, he has used only one gallon of gasoline; whereas prior to purchasing the motorcycle he was using a four wheeled vehicle for his daily trips and during the same period of time, five gallons of gasoline were consumed.

EUGENE COLLECTION AGENCY 828-29-30 MINER BLDG. PHONE 600. W. H. BLOWERS, MGR.

Examples from literature

Chisholm, 1963

- It is obligatory that you go the assistance of your neighbours
- It is obligatory that, if you go, you tell them that you are coming
- It is obligatory that, if you do not go, you do not tell them that you are coming
- You do not go

Forrester, 1984

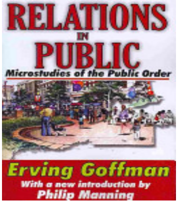
- It is obligatory not to kill
- It is obligatory that, if you kill, you do it gently

Prakken and Sergot, 1996

- There should be no dog
- If there is a dog, there should be a warning sign
- If there is a dog and no warning sign, there should be high fence

Remedial interchange

Chap. 3 of *Relations in Public* (1971)



- Social life as norm-governed
- "Territories of the self"
  - Personal space
  - Possessionnal territory
  - Etc
- Remedial interchange
  - Offensive $\rightarrow$ acceptable
  - Social order

Remedial interchange

A: “Can I use your phone to make a local call?"
B: “Sure, go ahead"
A: “That’s very good of you"
B: “It’s okay"

Remedial interchange

remedy	*A: “Can I use your phone to make a local call?"*
relief	*B: “Sure, go ahead"*
appreciation	*A: “That’s very good of you"*
minimization	*B: “It’s okay"*

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Goffman's suggestion

> *By making a move, I put the other participant under the obligation to make the next one.*

Dialogue game rules are soft rather than hard constraints
More flexible than the standard ACL approach in MAS.

Logical representation

	Normative premisses (α)	Integrity constraints (β)
(I)	$\bigcirc\neg o$	
(II)	$\bigcirc(r_1/o)$	$\Box(r_1 \rightarrow o)$
	$\bigcirc(r_2/o \wedge r_1)$	$\Box(r_2 \rightarrow (o \wedge r_1))$
(III)	$\bigcirc(a/o \wedge r_1 \wedge r_2)$	$\Box(a \rightarrow (o \wedge r_1 \wedge r_2))$
	$\bigcirc(m/o \wedge r_1 \wedge r_2 \wedge a)$	$\Box(m \rightarrow (o \wedge r_1 \wedge r_2 \wedge a))$

with o= offence, r_2= relief, m= minimization

r_1= remedy, a= appreciation

The ‘Strengthening of the antecedent’ problem

Strengthening of the Antecedent

$$\frac{\bigcirc(B/A) \qquad \Box(C \rightarrow A)}{\bigcirc(B/C)}\ (SA)$$

Deontic Detachment

$$\frac{\bigcirc(B/A) \qquad \bigcirc(C/B)}{\bigcirc(C/A)}\ (DD)$$

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Chisholm set

$$\bigcirc a, \bigcirc(t/a), \bigcirc(\neg t/\neg a)$$

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$$\dfrac{(SA)\ \dfrac{(DD)\ \dfrac{\bigcirc a \qquad \bigcirc(t/a)}{\bigcirc t}}{\bigcirc(t/\neg a)} \qquad \bigcirc(\neg t/\neg a)}{\bot}$$

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Intuitively, the set is consistent!

The ‘Strengthening of the antecedent’ problem

Strengthening of the Antecedent *Deontic Detachment*

$$\frac{\bigcirc(B/A) \qquad \Box(C \rightarrow A)}{\bigcirc(B/C)}\ (SA) \qquad \frac{\bigcirc(B/A) \qquad \bigcirc(C/B)}{\bigcirc(C/A)}\ (DD)$$

Chisholm set

$$\bigcirc a, \bigcirc(t/a), \bigcirc(\neg t/\neg a)$$

In this lecture: focus on another similar, albeit slightly different, problem

Factual Detachment

$$\frac{\bigcirc(B/A) \qquad A}{\bigcirc B}\ (FD) \qquad\qquad \frac{A \rightarrow B \qquad A}{B}\ (MP)$$

Factual Detachment

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" How can we take seriously a conditional obligation if it cannot, by way of detachment, lead to an unconditional obligation?" (van Eck, 1982, p. 263).

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" How can we take seriously a conditional obligation if it cannot, by way of detachment, lead to an unconditional obligation?" (van Eck, 1982, p. 263).

Detachment: constitutive of the concept of the notion of conditional (Boghossian, 2000)

The ‘factual detachment’ problem (1)

Weakening the Consequent

$$\frac{\bigcirc(B/A) \qquad \Box(B \to C)}{\bigcirc(C/A)}\ (WC)$$

Factual Detachment

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Forrester set

$$\bigcirc\neg k, \bigcirc(k \wedge g/k), k$$

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$$\bigcirc\neg k, \bigcirc(k \wedge g/k), k$$

$$\frac{\bigcirc\neg k}{\bigcirc\neg(k \wedge g)}\ (WC)$$

$$\frac{\bigcirc(k \wedge g/k) \qquad k}{\bigcirc(k \wedge g)}\ (FD)$$

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$$\bigcirc\neg k, \bigcirc(k \wedge g/k), k$$

$$\frac{\bigcirc\neg k}{\bigcirc\neg(k \wedge g)}\ (WC) \qquad \frac{\bigcirc(k \wedge g/k) \qquad k}{\bigcirc(k \wedge g)}\ (FD)$$

'Gentle murderer' scenario

$$\{\bigcirc\neg k, \bigcirc(k \wedge g/k), k\} \vdash \bot$$

The ‘factual detachment’ problem (2)

Deontic Detachment

$$\frac{\bigcirc(B/A) \qquad \bigcirc(C/B)}{\bigcirc(C/A)}\ (DD)$$

Chisholm set

$$\bigcirc a, \bigcirc(t/a), \bigcirc(\neg t/\neg a), \neg a$$

$$\frac{\bigcirc a \qquad \bigcirc(t/a)}{\bigcirc t}\ (DD) \qquad \frac{\bigcirc(\neg t/\neg a) \qquad \neg a}{\bigcirc \neg t}\ (FD)$$

Beyond consistency (1)

Inside: the CTD problem in SDL

Standard Deontic Logic (SDL)

von Wright 1951, Kripke 1963
The first deontic logic
Unable to deal with CTDs, and thus very quickly rejected:
Chisholm - 1963

Beyond consistency (2)

Chisholm set: $\{\bigcirc a, \bigcirc(t/a), \bigcirc(\neg t/\neg a), \neg a\}$

Below $\rightarrow$ is material implication, and O is like $\Box$

Two candidate definitions

Option 1	$\bigcirc(B/A) =_{def} O(A \rightarrow B)$
Option 2	$\bigcirc(B/A) =_{def} A \rightarrow OB$

Usual representation of Chisholm set:

$$\{\bigcirc a, O(a \rightarrow t), \neg a \rightarrow O\neg t, \neg a\} \vdash \bot$$

Why option 1 + option 2?

Beyond consistency (2)

$$\{\bigcirc a, \bigcirc(t/a), \bigcirc(\neg t/\neg a), \neg a\}$$

option 1 + option 1:

$$\{\bigcirc a, O(a \rightarrow t), O(\neg a \rightarrow \neg t), \neg a\}$$

option 2 + option 2:

$$\{\bigcirc a, a \rightarrow Ot, \neg a \rightarrow O\neg t, \neg a\}$$

option 2 + option 1:

$$\{\bigcirc a, a \rightarrow Ot, O(\neg a \rightarrow \neg t), \neg a\}$$

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Why 1+2? Answer: independance requirement

Ways out

$$\frac{\bigcirc(B/A) \qquad A}{\bigcirc B}\ (FD) \qquad\qquad \frac{\bigcirc A \qquad \bigcirc(B/A)}{\bigcirc B}\ (DD)$$

$$\frac{\bigcirc(B/A) \qquad \Box(B \rightarrow C)}{\bigcirc(C/A)}\ (WC)$$

Keep (*FD*) - perhaps in a qualified form - but

Easy way: give up (*DD*)/(*WC*)

- So-called non-normal modal logic (Forrester)

Hard way: make (*FD*) coexist with (*DD*)/(*WC*)

- Both rules allowed
- In case of conflict, (FD) overrides (WC)
  - Next lecture: 2-D modal logic (D. Gabbay)
  - This lecture: another way to do it

Adding time

Basic idea

before the violation $\neq$ after the violation

Åqvist's system DARB: DDL supplemented with $\oplus$ ("next") and $\boxed{s}$ ("settledness")

Semantics of DARB (1)

Frame

- $(Tree, <)$: a tree-like structure, where
  - *Tree*: a set of moments, m_1, m_2, ...
  - $<$: temporal relation (says which moment is next to which)
    - h : a history (a maximal chain of moments)
    - H_m : the set of histories passing through moment m.
- $\geq$: a ranking of histories in terms of betterness

NB: Truth-value of formulae made relative to pairs m/h

$\mathfrak{M}, m/h \models A$: in model $\mathfrak{M}$, formula A is true at moment m in history h

$[A]_m$: the set of histories making A true at m

Semantics of DARB (2)

Evaluation rules

$$\mathfrak{M}, m/h \models \oplus A \Leftrightarrow \mathfrak{M}, m+1/h \models A$$
$$\mathfrak{M}, m/h \models \boxed{\mathrm{s}} A \Leftrightarrow (\forall h' \in H_m)(\mathfrak{M}, m/h' \models A)$$
$$\mathfrak{M}, m/h \models \bigcirc(B/A) \Leftrightarrow \mathrm{best}_{\geq}([A]_m) \subseteq [B]_m.$$

Semantics of DARB (2)

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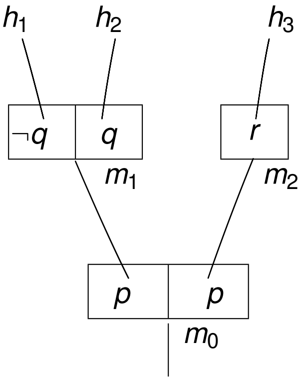
To evaluate $\bigcirc(B/A)$ at m/h:

1. determine set of histories passing through m where A holds
2. zoom into the subset of those that are best
3. check whether B holds in all of them at m

Example



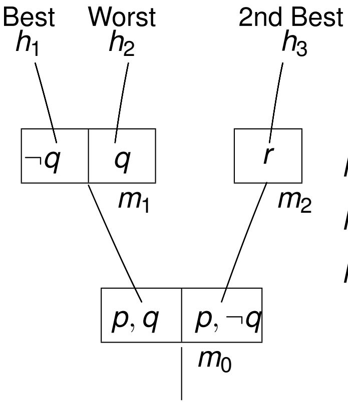
$$h_1 = \{m_0, m_1\}$$

$$h_2 = \{m_0, m_1\}$$

$$h_3 = \{m_0, m_2\}$$

$$m_0/h_1 \models ?$$

Example



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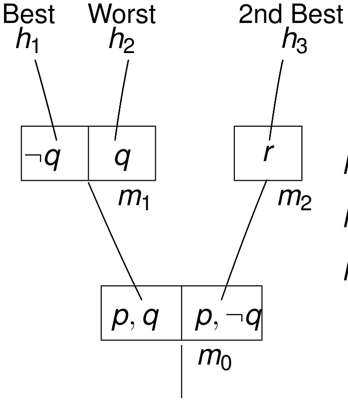

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$$m_0/h_1 \models \bigcirc(q/p)$$
$$m_0/h_1 \models \bigcirc(\oplus\neg q/p)$$

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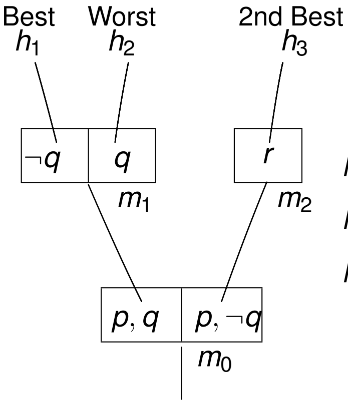

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$$m_0/h_1 \models \bigcirc(?/\neg q)$$

Chisholm set

Case where A and B (will) occur simultaneously

Old representation	New representation
OA	$\bigcirc \oplus A$
$O(A \rightarrow B)$	$\bigcirc(\oplus B / \oplus A)$
$\neg A \rightarrow O\neg B$	$\oplus \bigcirc (\neg B / \neg A)$
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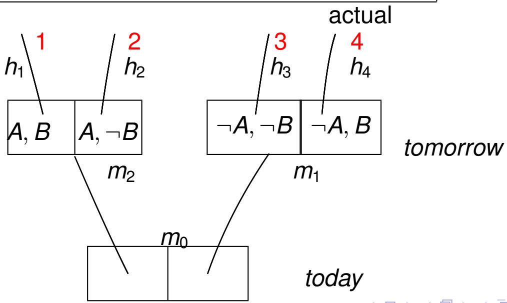

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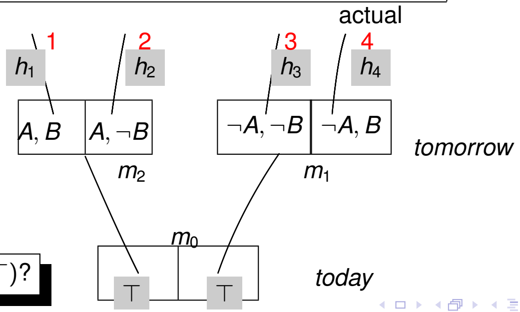

in $h_4/m_0 : \bigcirc(\oplus A/\top)$?

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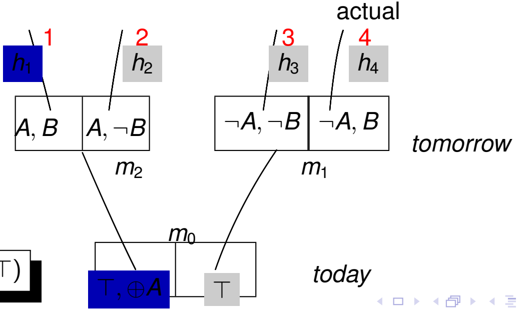

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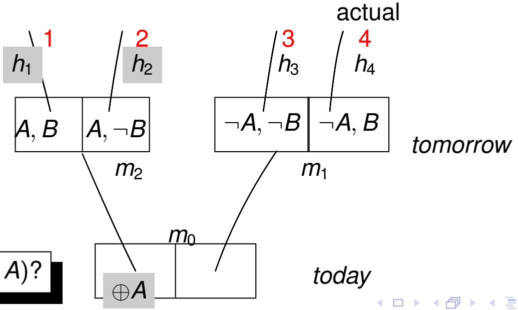

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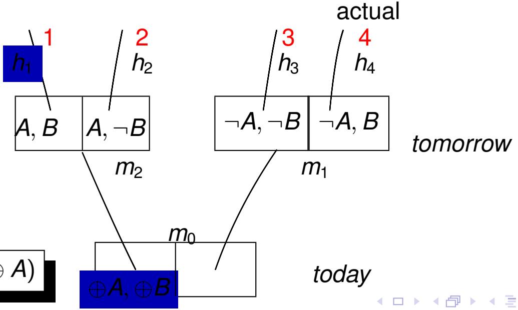

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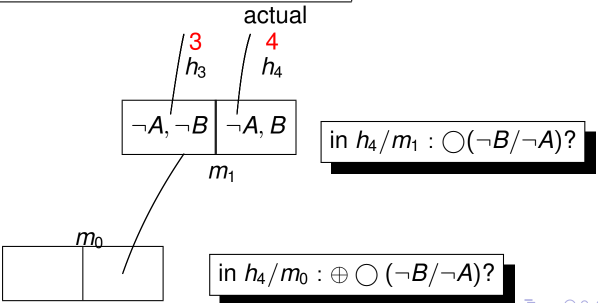


Conclusion on ‘FD vs DD’

Basic idea

before the violation $\neq$ after the violation

In a nutshell

Before the violation: DD gives $\bigcirc \oplus B$
When the violation occurs: FD supports $\oplus \bigcirc \neg B$

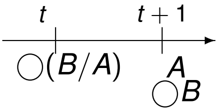
Main message

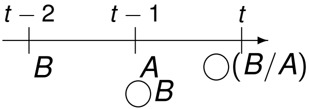
Not an abstract solution
Obligations may change over time!
Ex: Picking up the kids from school

Consequent-before-the-antecedent

Chisholm example

In DARB: qualified version of the factual detachment rule: move from $\bigcirc(B/A)$ and A to $\bigcirc B$ warranted only if A occurs no later than the time of being in force of $\bigcirc(B/A)$.





Can the past be undone?

Basics

evaluation

reference

$$\langle t, s \rangle \Vdash A$$

Two senses of “ought"

- actual
- ideal or prima facie

Y modality: goes backwards in time

Material reading: *Synthese* paper on Moodle.

Chisholm example

$ObA =_{df} YO(r \rightarrow A) \qquad Ob(B/A) =_{df} YO(A \rightarrow ObB)$

Premisses are evaluated at $< t, t >$
t: violation context / ‘ideal’ u immediately before t

Chisholm example

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$$\frac{\begin{array}{c} < t, t > \models Ob(B/A) \\ < t, t > \models ObA \end{array}}{< u, t > \models r \rightarrow ObB} \qquad \frac{\begin{array}{c} < t, t > \models Ob(\neg B/\neg A) \\ < t, t > \models \neg A \end{array}}{< t, t > \models Ob\neg B}$$

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$\Downarrow$ ideal (prima facie) “ought"

$\Downarrow$ actual “ought"

Chisholm example

$ObA =_{df} YO(r \rightarrow A) \qquad Ob(B/A) =_{df} YO(A \rightarrow ObB)$

Premisses are evaluated at $< t, t >$
t: violation context / ‘ideal’ u immediately before t

$$\frac{\begin{array}{c} < t, t > \models Ob(B/A) \\ < t, t > \models ObA \end{array}}{< u, t > \models r \rightarrow ObB} \qquad \frac{\begin{array}{c} < t, t > \models Ob(\neg B/\neg A) \\ < t, t > \models \neg A \end{array}}{< t, t > \models Ob\neg B}$$

$\Downarrow$

ideal (prima facie) “ought"

$< \boxed{u}, t >$

$\Downarrow$

actual “ought"

$< \boxed{t}, t >$