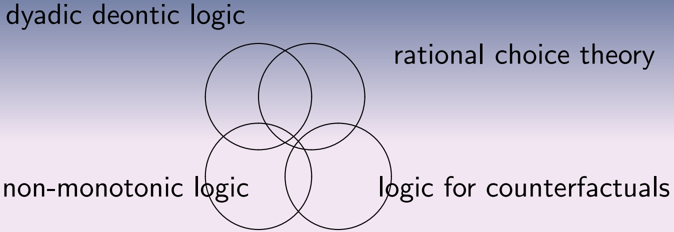
Lecture layout

DSDL: Dyadic Standard Deontic Logic

1. Introduction
2. Syntax
3. Semantics
4. Validities/Invalidities
5. Meta-theory

Bird's eye view



1969: Hansson's paper
1973: **Counterfactuals** by D. Lewis
Early 90's: KLM non-monotonic systems

For more information on the interplay between these areas, see Makinson (1993)

Language

$$A ::= p \mid \neg A \mid A \wedge B \mid \Box A \mid \bigcirc(B/A) \mid P(B/A)$$

New building blocks

- $\bigcirc(B/A) = B$ is obligatory, given A
- $P(B/A) = B$ is permitted, given A

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Typically, A and B are propositional letters

Context-dependent approach to norms

- Truth of a norm usually depends on context
- Dyadic: two arguments

For an unconditional norm, use $\top$ for the condition

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Here iterations of $\bigcirc(/)$ allowed.
What ought to be the case should be done:
$\bigcirc(\bigcirc A \to A)$

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Here iterations of $\bigcirc(/)$ allowed.
What ought to be the case should be done:
$\bigcirc(B/\bigcirc(B/A) \wedge A)$

Model

Model

$M = (W, \geq, V)$, with

- W: a set of possible worlds $\{x, y, ...\}$
- $\geq$: a binary relation ranking all the possible worlds in terms of betterness
  - $x \geq y$: x is al least as good as y
- V is as usual

Note: the ranking can be made world-relative too.

Evaluation rules

Evaluation rules

- $\bigcirc(B/A)$ true at x iff, in all the best (according to $\geq$) A-worlds, B is true
- Similarly for $P(B/A)$ (but with $\forall$ replaced by $\exists$).

P dual of $\bigcirc$, i.e., $P(B/A) = \neg \bigcirc (\neg B/A)$

Evaluation rules

Evaluation rules

$M, x \models \bigcirc(B/A)$ iff $\mathrm{opt}_{\succeq}(\|A\|) \subseteq \|B\|$

$M, x \models P(B/A)$ iff $\mathrm{opt}_{\succeq}(\|A\|) \cap \|B\| \neq \emptyset$

where

- $\|A\| = \{x \in M : x \models A\}$
- $\mathrm{opt}_{\succeq}(X) = \{x \in X : (\forall y \in X) x \geq y\}$

P dual of $\bigcirc$, i.e., $P(B/A) = \neg \bigcirc (\neg B/A)$

Example

$n_1 : \bigcirc A$ =Primary obligation

$n_2 : \bigcirc(B/\neg A)$ ($\neg$:not) =CTD obligation

Citizens ought to pay taxes

If the Tax Office collects evidence about tax evasion by one citizen, then it ought to pursue him according to the law

Example

$n_1 : \bigcirc A$ $\qquad$ =Primary obligation

$n_2 : \bigcirc(B/\neg A)$ $\qquad$ ($\neg$:not) $\qquad$ =CTD obligation

$x_1 \bullet\ A, B$

$x_2 \bullet\ \neg A, B$

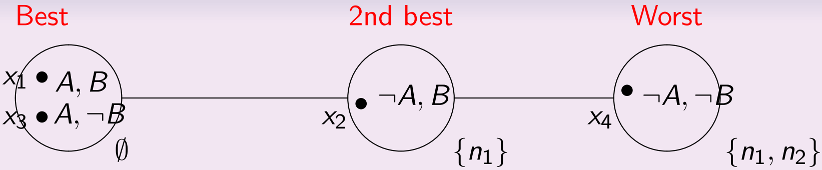
$x_3 \bullet\ A, \neg B$

$x_4 \bullet\ \neg A, \neg B$

Example

$n_1 : \bigcirc A$ $=$Primary obligation

$n_2 : \bigcirc(B/\neg A)$ ($\neg$:not) $=$CTD obligation



Meaning of $\bigcirc A, \bigcirc(B/\neg A)$

Violation set V of state x $=$ set of norms that are violated in x

Put $x \succ y$ iff $V(x) \subset V(y)$

Example

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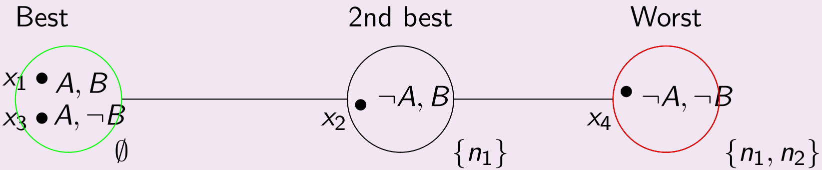

Various levels of ideality
= a generalization of the SDL-ish binary classification of states into good/bad (green/red) ones

Invalidities

Strengthening of the Antecedent invalid:

$$\bigcirc(B/A) \to \bigcirc(B/A \wedge C)$$

Intuitively: obligations are defeasible (cf. non-monotonic logic)

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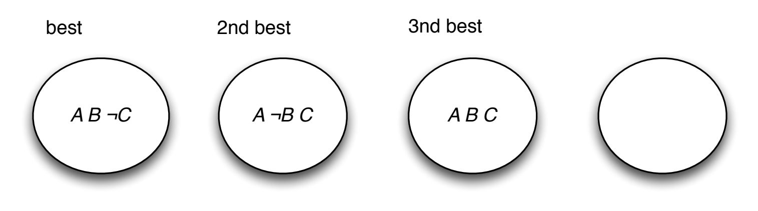

Invalidities

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$$\bigcirc(B/A) \rightarrow \bigcirc(B/A \wedge C)$$

Intuitively: obligations are defeasible (cf. non-monotonic logic)

Other characteristic laws that are rejected:

$$\bigcirc(B/A) \wedge \bigcirc(C/B) \rightarrow \bigcirc(C/A) \qquad \text{(Full transitivity)}$$

$$\bigcirc(B/A) \rightarrow \bigcirc(\neg A/\neg B) \qquad \text{(Contraposition)}$$

New validity

(Strong) Factual Detachment

$$\not\models \bigcirc(B/A) \wedge A \to \bigcirc B \tag{FD}$$

$$\models \bigcirc(B/A) \wedge \underbrace{\Box A}_{\textit{settled}} \to \bigcirc B \tag{SFD}$$

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CTD problem

- When detach $\bigcirc B$ from $\bigcirc(B/\neg A)$? ($\bigcirc A$ in the backbround)
- Answer: when the violation is unavoidable, $\Box\neg A$.

Intuition:

- Obligations from a ‘better’ context dominate
- ‘Ought’ implies ‘can’

New validity

(Strong) Factual Detachment

$$\not\models \bigcirc(B/A) \wedge A \rightarrow \bigcirc B \qquad \text{(FD)}$$

$$\models \bigcirc(B/A) \wedge \underbrace{\Box A}_{\textit{settled}} \rightarrow \bigcirc B \qquad \text{(SFD)}$$

CTD problem

- When detach $\bigcirc B$ from $\bigcirc(B/\neg A)$? ($\bigcirc A$ in the backbround)

Ex: Get instant formula only if you <u>can't</u> breath-feed

Classes of structures

Constraints on $\geq$

Reflexivity: $x \geq x$
Transitivity: $x \geq y$ and $y \geq z$ implies $x \geq z$
Totalness: $x \geq y$ or $y \geq x$
Limit assumption: no infinite sequence of strictly better worlds

$$\|A\| \neq \emptyset \to \mathrm{opt}_{\succeq}(\|A\|) \neq \emptyset$$

	constraints on $\geq$
DSDL1	reflexivity
DSDL2	reflexivity, and limit assumption
DSDL3	reflexivity, transitivity, totalness, and limit assumption

Table : Hansson's systems

Total order case

$\Gamma \vdash A$ iff $\Gamma \models A$

- Axiomatization problem
  - Weak completeness result ✓
    - Spohn (1975): flat (non-nested) fragment of the language
    - Åqvist (1987): full language; system **G**; canonical model
  - Strong or full completeness ✓
    - Hansen (1998): failure of the 1987 proof w.r.t. strong completeness
    - Parent (2008): strong completeness restored
- Consistency ✓
- Decidability ✓
  - Spohn (1975)

Proof-theory for DSDL3

Page 123 of the chapter:

$$A \geq B =_{\mathrm{df}} \neg \bigcirc (\neg A / A \vee B) \qquad (\mathrm{Df}{\geq})$$

Proof-theory

$$\bigcirc (B \to C/A) \to (\bigcirc(B/A) \to \bigcirc(C/A)) \qquad (\mathrm{CKD})$$

$$\bigcirc (B/A) \to \neg \bigcirc (\neg B/A) \qquad (\mathrm{COD})$$

$$\bigcirc (\top/\top) \qquad (\mathrm{CON})$$

$$\bigcirc (B/A) \to \bigcirc(B \wedge A/A) \qquad (\mathrm{AND})$$

$$(A \geq B \wedge B \geq C) \to A \geq C \qquad (\mathrm{TRANS})$$

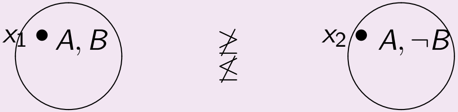
$$\text{If } \vdash A \leftrightarrow B \text{ then } \vdash \bigcirc(C/A) \leftrightarrow \bigcirc(C/B) \qquad (\mathrm{CRED})$$

$$\text{If } \vdash B \to C \text{ then } \vdash \bigcirc(B/A) \to \bigcirc(C/A) \qquad (\mathrm{CRMD})$$

Partial order case

Conflicts between obligations allowed

Below: A model for $\{\bigcirc(B/A), \bigcirc(\neg B/A)\}$

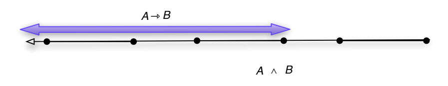


Syntactical counterpart of the total order assumption:

$$\Diamond A \rightarrow \neg(\bigcirc(B/A) \wedge \bigcirc(\neg B/A)) \qquad (\Diamond : \text{ ‘possible’})$$

Partial order case

- Axiomatization problem
  - Strong & weak completeness: ✓
    - Goble (2003): system DP. Detour via multiplex semantics
    - Danielson-type evaluation rule:



    - Equivalent to Best-antecedent account only if $\succeq$ total
- Consistency ✓
- Decidability?

Letting transitivity go

Recall

Transitivity: $x \geq y$ and $y \geq z$ implies $x \geq z$

Viewed with suspicion by philosophers and economists.

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Parent (2010): Strongly complete axiomatization using an optimality language for dyadic deontic logic

1987 Åqvist: conjectured axiomatization

- New building block: QA ("ideally A")
- $\bigcirc(B/A) =_{def} \Box(QA \to B)$ and $P(B/A) =_{def} \Diamond(QA \wedge B)$

2006 Ardeshir and Nabavi: conjecture settled in the negative

- $QA \wedge QB \to Q(A \vee B)$ not provable

Letting transitivity go

Recall

Transitivity: $x \geq y$ and $y \geq z$ implies $x \geq z$

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Parent (2010) strongly complete axiomatization using Q

Open problems

- Axiomatize the logic using $\bigcirc(-/-)$ and $P(-/-)$ as primitive constructs
  - Åqvist's conjectured axiomatization: system **F**
- Show decidability

Transitivity redundant?

Question asked at the end of the lecture: transitivity redundant?

Conjecture

If $\succeq$ is reflexive, total and limited, then $\succeq$ is transitive

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Counter-example

Put $W = \{x, y, z\}$, with $\succeq$ the reflexive closure of:

- $x \succeq z$, $z \succeq x$, $z \succeq y$, $y \succeq z$, $y \succeq x$
- NB: when $\succeq$ holds in both directions, the items are equally good
- intuitively: all the worlds are equally good, except for y being strictly better than x

$\succeq$ is reflexive, total and limited.
But $\succeq$ is not transitive (witness: $x \not\succeq y$)

Transitivity redundant?

Question asked at the end of the lecture: transitivity redundant?

Conjecture

If $\succeq$ is reflexive, total and limited, then $\succeq$ is transitive

So the conjecture is false.
Transitivity isn't redundant.

Selected bibliography (1)

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