Prima Facie Norms, Normative Conflicts, and Dilemmas

by L. Goble

Lecturer: X. Parent

Chapter summary

- Normative conflicts
    - OA, OB but $\neg\Diamond(A \wedge B)$ ($\neg$: not / $\Diamond$:possible)
- Problem:
    - Horn1: they are common-place
    - Horn 2: ... and ruled out by core principles of deontic logic
- Aim of chapter
    - Roadmap of all the solutions available from literature
    - Assess them

Lecture layout

Lecture layout (with required reading)

1. How come conflicts are ruled out by deontic logic?
    - Section 1 "The dilemma of normative conflicts"
2. Acommodating conflicts
    - Sections 5.1 to 5.3
    - the problem will be introduced, but not resolved
3. Conflicts resolution mechanisms
    - Introduction to Section 4.3 on *prima facie* oughts
        - last paragraph may be skipped
    - Section 4.3.2 "Defeasible inference"

Examples from text

Ada's predicament (opening paragraph)

> Ada promised to take her son to the circus Friday afternoon, and so presumably she ought to spend that afternoon with him at the circus. It also happens, however, that there is an important meeting of her committee that same afternoon, and she ought to be present for that. She cannot do both. Ada seems stuck; whatever she does, it seems she will not do something she ought to do.

Logical representation: p, m, Oc, Oa, $\neg\Diamond(c \wedge a)$

$\Diamond$: possible

Examples from text

Ross's example (p. 13)

tions. When Ross [1930] introduced this distinction, or this terminology for it, he illustrated it (p. 18) with the well-known example of a person who has promised to meet a friend for some trivial purpose, but who could also help the victims of a serious accident, though only by breaking the promise. Because of the promise, presumably the person ought to meet the friend. Because of the need of the accident's victims, presumably the person ought to help them. Each is possible, but they are not jointly possible. Because

Logical representation: p, a, Om, Oh, $\neg\Diamond(m \wedge h)$

Why are conflicts ruled out by deontic logic?
Argument 1

Pausible laws

(P)	$OA \to \Diamond A$	($\Diamond$: possible)	(ought-can)
(And)	$(OA \wedge OB) \to O(A \wedge B)$		(aggregation/agglomeration)

NB: Goble uses the label C for the second.

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Assume OA and OB hold, but $\neg\Diamond(A \wedge B)$

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Problem

Assume OA and OB hold, but $\neg\Diamond(A \wedge B)$

$$\dfrac{\dfrac{OA \wedge OB}{O(A \wedge B)}\ \text{And}}{\Diamond(A \wedge B)}\ \text{P}$$

Why are conflicts ruled out by deontic logic?
Argument 2

Pausible laws

D) $OA \rightarrow \neg O \neg A$ (principle of seriality)

W) $\Box(A \rightarrow B) \rightarrow (OA \rightarrow OB)$ ($\Box$: necessary)

Why are conflicts ruled out by deontic logic?

Argument 2

Pausible laws

D) $OA \rightarrow \neg O \neg A$ (principle of seriality)
W) $\Box(A \rightarrow B) \rightarrow (OA \rightarrow OB)$ ($\Box$: necessary)

About W:

- W is a shorthand for *Weakening* (my own terminology)
- Goble uses the label NM
- antecedent is very strong: B necessary for A

$\Box(\mathrm{dentist} \rightarrow \mathrm{bus})$ - I have no car. etc

Why are conflicts ruled out by deontic logic?
Argument 2

Pausible laws

D) $OA \rightarrow \neg O \neg A$ (principle of seriality)
W) $\Box(A \rightarrow B) \rightarrow (OA \rightarrow OB)$ ($\Box$: necessary)

About W:

- W is a shorthand for *Weakening* (my own terminology)
- Goble uses the label NM
- Compare with $O(A \rightarrow B) \rightarrow (OA \rightarrow OB)$

  A=pay; B= receipt

Why are conflicts ruled out by deontic logic?
Argument 2

Pausible laws

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Problem

Assume OA and OB hold, but $\neg\Diamond(A \wedge B)$

Why are conflicts ruled out by deontic logic?

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Pausible laws

D) $OA \rightarrow \neg O \neg A$ (principle of seriality)

W) $\Box(A \rightarrow B) \rightarrow (OA \rightarrow OB)$ ($\Box$: necessary)

Problem

$$\mathrm{PL}\,\dfrac{\mathrm{D}\,\dfrac{\mathrm{W}\,\dfrac{\dfrac{\neg\Diamond(A \wedge B)}{\Box(A \rightarrow \neg B)} \qquad OA}{O\neg B}}{\neg O \neg\neg B}}{\neg OB}$$

Why are conflicts ruled out by deontic logic?

Summary

In a nutshell

The logic makes the following set inconsistent:

$$\neg\Diamond(A \wedge B)$$
$$OA$$
$$OB$$

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Methodological remark

- you can make the same points using $\vdash$ ('prove') and $\perp$ (contradiction) instead $\Box$ and $\Diamond$:

$\Box$-idiom	$\vdash$-idiom
$\Box(A \to B)$	$A \vdash B$
$\neg\Diamond(A \wedge B)$	$A, B \vdash \perp$
$\Diamond(A \wedge B)$	$A, B \nvdash \perp$

- During the lecture, I will occasionnally switch idiom

Why are conflicts ruled out by deontic logic?

Summary

In a nutshell

The logic makes the following set inconsistent:

$$\neg\Diamond(A \wedge B)$$
$$OA$$
$$OB$$

Historical remark

- Problem initially raised for DSL - it has all the above laws
- The author states the problem in its full generality

Part I
How to accommodate the existence of conflicts

Conflict: Revisionist strategies

Revisionist strategies (sections 6 and 7)

At least one of [(P), (And)] and [(W), (D)] must go

Systems	Law rejected
Non-kantian	P
Non-aggregative	And
Non-distributive	W
?	D

Given And/W, D is equivalent to P

Desiderata

Desideratum 1 (conflicts are logically consistent)

$\neg\Diamond(A \wedge B), OA, OB \nvdash \bot \qquad (\bot : \text{contradiction})$

Arguments 1 and 2 must be blocked.

Desideratum 2 (Avoid deontic explosion)

The logic should not contain (DEX), or anything like it.

DEX) $\neg\Diamond(A \wedge B), OA, OB \vdash OC$

Desideratum 3

The logic should explain in a plausible way the apparent validity of several paradigm arguments, including the Smith Argument.

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Smith argument (Horty)

i) Smith ought to fight in the army or perform alternative national service. — $O(f \vee s)$

ii) Smith ought not to fight in the army. — $O\neg f$

$\therefore$ iii) Smith ought to perform alternative national service. — Os

Non-kantian systems

Lemmon (1962)

(P) $OA \rightarrow \Diamond A$ rejected
Argument 1 blocked, because P directly involved in the derivation.
Argument 2 blocked, because P $\Leftrightarrow$ D

Deontic explosion

DEX) $\neg\Diamond(A \wedge B), OA, OB \vdash OC$ holds

Non-kantian systems

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Argument 1 blocked, because P directly involved in the derivation.
Argument 2 blocked, because P $\Leftrightarrow$ D

Deontic explosion

DEX) $\neg\Diamond(A \wedge B), OA, OB \vdash OC$ holds

Exercise: show DEX using And and W
Hint: $\neg\Diamond(A \wedge B) \leftrightarrow \Box((A \wedge B) \to C)$

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Argument 2 blocked, because P $\Leftrightarrow$ D

Deontic explosion

DEX) $\neg\Diamond(A \wedge B), OA, OB \vdash OC$ holds

$$\dfrac{\dfrac{\neg\Diamond(A \wedge B)}{\Box((A \wedge B) \rightarrow C)} \qquad \dfrac{OA \qquad OB}{O(A \wedge B)}\,(\text{And})}{OC}\,(\text{W})$$

Non-aggregative systems

Non-aggregative systems

(And) $(OA \wedge OB) \rightarrow O(A \wedge B)$ rejected

Non-aggregative systems

Non-aggregative systems

(And) $(OA \wedge OB) \rightarrow O(A \wedge B)$ rejected

Implementation - multi-relational semantics

Intuition: conflicts result from clashes between normative systems

$\mathcal{R}$: a set of serial binary deontic alternativeness relations $R_1, R_2, \ldots$

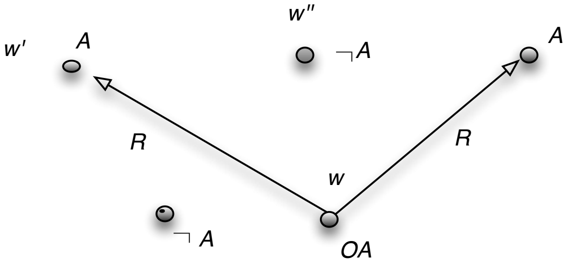
$$w \models OA \text{ iff there is an } R_i \in \mathcal{R} \text{ such that: } w' \models A \text{ for all } w' \text{ with } wR_iw'$$

Non-aggregative systems

Non-aggregative systems

(And) $(OA \wedge OB) \rightarrow O(A \wedge B)$ rejected

SDL is mono-relational



- we write $w \rightarrow w'$ to express that wRw'
- intuitively: w' is a ‘good’ successor of w

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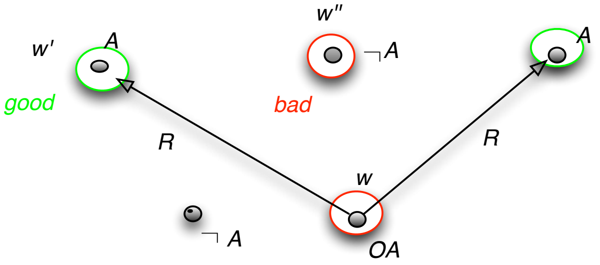

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Non-aggregative systems

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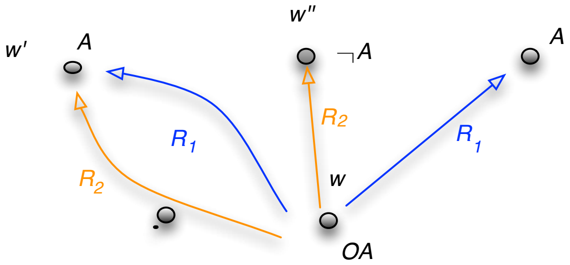
Going multi-relational:

- familiity of accessibility relations $R_1, R_2, \ldots$ (one per normative system)



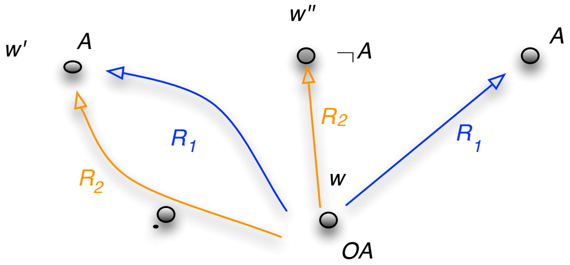
Non-aggregative systems

Non-aggregative systems

(And) $(OA \wedge OB) \rightarrow O(A \wedge B)$ rejected

Going multi-relational:

$$w \models OA \text{ iff there is an } R_i \in \mathcal{R} \text{ such that:}$$
$$w' \models A \text{ for all } w' \text{ with } w \xrightarrow{R_i} w'$$



Non-aggregative systems

Desideratum 1 (conflicts are logically consistent)

$\neg\Diamond(A \wedge B), OA, OB \nvdash \bot$

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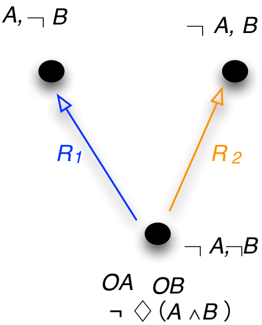


Non-aggregative systems

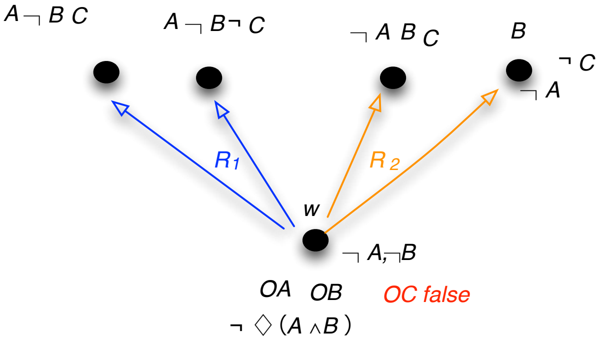
Desideratum 2 (no deontic explosion)

$\neg\Diamond(A \wedge B), OA, OB \nvdash OC$

Non-aggregative systems

Desideratum 2 (no deontic explosion)

$\neg\Diamond(A \wedge B), OA, OB \nvdash OC$



Non-aggregative systems

Desideratum 3 (Smith argument)

We want $O(A \vee B), O\neg A \vdash OB$

Non-aggregative systems

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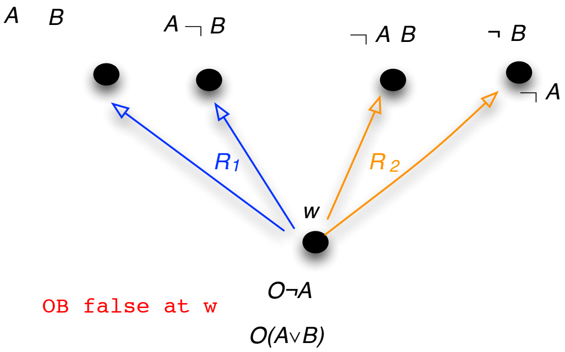

Non-distributive systems

Non-distributive systems

(W) $\quad \Box(A \to B) \to (OA \to OB)$ rejected - along with (P).

Non-distributive systems

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Why call them "distributive"?

(W) implies O distributes over $\wedge$:

$$O(A \wedge B) \to OA$$

Non-distributive systems

Non-distributive systems

(W) $\quad \Box(A \to B) \to (OA \to OB)$ rejected - along with (P).

Desideratum 3 - Smith Argument

Below: a proof of $O(A \vee B), OA \vdash OB$

$$\dfrac{\dfrac{O(A \vee B) \qquad O\neg A}{O((A \vee B) \wedge \neg A)}\text{(And)} \qquad \Box(((A \vee B) \wedge \neg A) \to B)}{OB}\text{(W)}$$

Non-distributive systems

Non-distributive systems

(W) $\quad \Box(A \rightarrow B) \rightarrow (OA \rightarrow OB)$ rejected - along with (P).

Desideratum 3 - Smith Argument

$$\dfrac{\dfrac{O(A \vee B) \qquad O\neg A}{O((A \vee B) \wedge \neg A)}\,(\text{And}) \qquad \Box(((A \vee B) \wedge \neg A) \rightarrow B)}{OB}\,(\text{W})$$

Enough to disqualify the approach.

Let's take stock!

Summary

By rejecting one of the rules, the logic is either too strong (desideratum 2 not met) or too weak (desideratum 3 not met)

What do we do?

Let's take stock!

Summary

By rejecting one of the rules, the logic is either too strong (desideratum 2 not met) or too weak (desideratum 3 not met)

What do we do?

Mainstream solution

Keep all the rules, but restrict their application

Explains why more and more have been moving away from possible-worlds semantics (SDL, DSDL).

Let's take stock!

Summary

By rejecting one of the rules, the logic is either too strong (desideratum 2 not met) or too weak (desideratum 3 not met)

What do we do?

Main research challenge

Find the best way to do it

- consistency proviso "A is consistent with B" or $\Diamond(A \wedge B)$
- permissions

Explains why more and more have been moving away from possible-worlds semantics (SDL, DSDL).

Restricted And

Restricted aggregation:

$$\frac{\bigcirc A \qquad \bigcirc B \qquad A \text{ consistent with } B}{\bigcirc (A \wedge B)}$$

Restricted And

Restricted aggregation:

$$\frac{\bigcirc A \qquad \bigcirc B \qquad A \text{ consistent with } B}{\bigcirc(A \wedge B)}$$

Deontic explosion

$$\frac{(\text{And})\,\dfrac{OA \qquad O\neg A}{O(A \wedge \neg A)} \qquad A \wedge \neg A \vdash B}{OB}\,(\text{W})$$

Restricted And

Restricted aggregation:

$$\frac{\bigcirc A \qquad \bigcirc B \qquad A \text{ consistent with } B}{\bigcirc(A \wedge B)}$$

Deontic explosion not avoided

$$\dfrac{\text{(And)}\ \dfrac{\text{(W)}\ \dfrac{OA}{O(A \vee B)} \qquad O\neg A}{O((A \vee B) \wedge \neg A)}}{OB}\ \text{(W)}$$

Part II
Conflict resolution mecanisms for obligations

Overall picture

2 types of obligations (Ross, 1930)

Ceteris paribus principle: a hedged generalization

- Other things being equal, lying is wrong
- Conditional
  - what to do in the absence of further complications
- May conflict with each others

Prima facie obligation: *as if* I have that duty

- May conflict
- Still binding

All-things-considered obligation: what I must do after balancing all the conflicting prima facie obligations

- Duty proper

Overall picture

2 types of obligations (Ross, 1930)

Hybrid approach

- Prima facie obligations can conflict; all-things-considered obligations cannot
- Core principles of deontic logic
  - revised for prima facie obligations
  - maintained for all-things-considered obligations

Prima facie obligations

Preliminary step

In a given scenario, what are the *prima facie* obligations?

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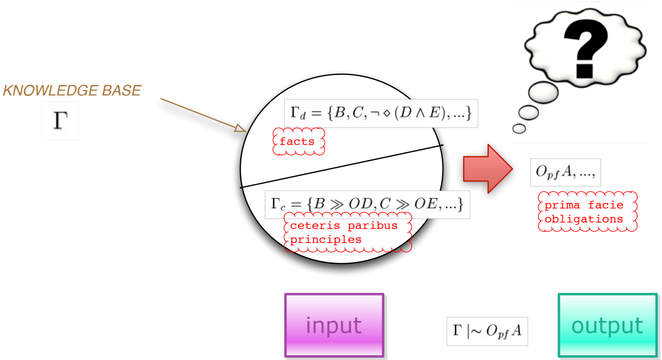

Prima facie obligations

Preliminary step

In a given scenario, what are the *prima facie* obligations?

We write $\Gamma \mid\!\sim A$ to say that Γ outputs A

- $\mid\!\sim$: a defeasible consequence relation (called "snake")
- On top of a deontic logic with all the core principles
  - $\vdash_d$: consequence relation of the base logic
- Output A is a sentence of the form $O_{pf} B$

To avoid clusters of symbols, we say

- $O_{pf} B$ is 'in', when $\Gamma \mid\!\sim O_{pf} B$
- $O_{pf} B$ is 'out', when $\Gamma \not\mid\!\sim O_{pf} B$

Question

How to define in/out (*aka* $\mid\!\sim$) using $\vdash_d$?

Prima facie obligations

Preliminary step

In a given scenario, what are the *prima facie* obligations?

Faithfull maximal consistent subset

> - A set Γ^* is a faithful maximal consistent subset of Γ^{d+c} if and only if (i) $\Gamma^d \subseteq \Gamma^*$ and (ii) $\Gamma^* \subseteq \Gamma^{d+c}$, and (iii) Γ^* is consistent (as defined by $\vdash_d$) and (iv) there is no $B \in \Gamma^{d+c}$ such that $B \notin \Gamma^*$ and $\Gamma^* \cup \{B\}$ is consistent.

Figure : Page 41

$$\Gamma^{d+c} = \Gamma^d \cup \mathcal{C}(\Gamma^c)$$
$$\mathcal{C}(\Delta) = \{C : \exists B(B \gg C \in \Delta)\}$$

Prima facie obligations

Preliminary step

In a given scenario, what are the *prima facie* obligations?

Credulous approach

Def 6) $\Gamma \mathrel{|\!\!\sim} A$ if and only if $\Gamma^* \vdash_d A$, for ~~each~~ Γ^* that is a faithful maximal consistent subset of Γ^{d+c}. some

Figure : Page 41

Prima facie obligations

Preliminary step

In a given scenario, what are the *prima facie* obligations?

Credulous approach

$$O_{pf}A \text{ is } \begin{cases} \text{in} & \text{if } \exists \Gamma^* s.t. \Gamma^* \vdash_d A \\ \text{out} & \text{otherwise.} \end{cases}$$

N.B.: Γ^* must be a faithfull maximal consistent subset of Γ^{d+c}

Prima facie obligations

Preliminary step

In a given scenario, what are the *prima facie* obligations?

Promise example
$\Gamma^d = \{p, a, \neg\Diamond(m \wedge h)\}$
$\Gamma^c = \{p \gg Om, a \gg Oh\}$

$$\Gamma^{d+c} = \{p, a, \neg\Diamond(m \wedge h), Om, Oh\} \vdash_d \bot$$

$\Gamma^\star_1 = \{p, a, \neg\Diamond(m \wedge h), Om\} \nvdash_d \bot$, and maximally so

$\Gamma^\star_2 = \{p, a, \neg\Diamond(m \wedge h), Oh\} \nvdash_d \bot$, and maximally so

Prima facie obligations

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$$\Gamma^{d+c} = \{p, a, \neg\Diamond(m \wedge h), Om, Oh\} \vdash_d \bot$$

$\Gamma_1^\star = \{p, a, \neg\Diamond(m \wedge h), Om\} \nvdash_d \bot$, and maximally so

- $O_{pf}\, m$ is 'in' , since $\Gamma_1^\star \vdash_d Om$

$\Gamma_2^\star = \{p, a, \neg\Diamond(m \wedge h), Oh\} \nvdash_d \bot$, and maximally so

- $O_{pf}\, h$ is 'in' , since $\Gamma_2^\star \vdash_d Oh$

Conflicts resolution mechanisms

Incremental procedure - A standard in AI

Incremental procedure, p. 45

For a complete strict order, $\succ$, over Γ^c that respects $\sqsupset_\Gamma$, let the ordering of Γ^c by $\succ$ be $\langle c_1, \ldots, c_i, \ldots \rangle$, so that $c_i < c_j$ in the sequence just in case $c_i \succ c_j$.[47] Then define $\Phi_\succ$ from a sequence of subsets, $\Phi^i_\succ$, of Γ^{d+c}, thus:

- $\Phi^0_\succ = \Gamma^d$;
- for $0 \leq i < j = i+1$, $\Phi^j_\succ = \Phi^i_\succ \cup \{\mathcal{C}(c_j)\}$ if that is consistent, otherwise $\Phi^j_\succ = \Phi^i_\succ$;
- $\Phi_\succ = \bigcup \Phi^i_\succ$.

Each such set, $\Phi_\succ$, is a faithful maximal consistent subset of Γ^{d+c}, and so is appropriate to apply to define $\Gamma \mathrel{|\!\!\sim} A$.

Def 7) $\Gamma \mathrel{|\!\!\sim} A$ iff $\Phi_\succ \vdash_d A$, for each complete strict order, $\succ$, that respects $\sqsupset_\Gamma$.

Conflicts resolution mechanisms

Incremental procedure - A standard in AI

Question

In a given scenario, what are the all-things-considered (ATC) obligations?

Basic idea

Priority relation on the obligations
Pick up a "preferred" faithfull maximal consistent subset
NB: priority relation is given

Preferred faithfull maximal consistent subset

Let $\Gamma^c = \{c_1, c_2, ..., \}$ be totally ordered by $\sqsupset$

- $c_i \sqsupset c_j$: c_i is stronger than (overrides, etc) c_j

Rearrange the c_i's in Γ^c by decreasing strenght

Preferred faithfull maximal consistent subset

Φ: 'preferred' faithfull maximal consistent subset

Φ: defined from a sequence Φ^0, Φ^1,..., of subsets of Γ^{d+c}

$$\Phi^0 = \Gamma^d$$

$$\text{For } 1 \leq i, \Phi^i = \begin{cases} \Phi^{i-1} \cup \{\mathcal{C}(c_i)\} & \text{if that is consistent,} \\ \Phi^{i-1} & \text{otherwise.} \end{cases}$$

Put $\Phi = \bigcup \Phi^i$

$$O_{atc}A \text{ is } \begin{cases} \text{in} & \text{if } \Phi = \bigcup \Phi^i \vdash_d OA \\ \text{out} & \text{otherwise.} \end{cases}$$

Preferred faithfull maximal consistent subset

Promise example

$\Gamma^d = \{p, a, \neg\Diamond(m \wedge h)\}$
$\Gamma^c = \{\underbrace{p \gg Om}_{c_1}, \underbrace{a \gg Oh}_{c_2}$ with $c_2 \sqsupset c_1$

Here $\Gamma^{d+c} = \{p, a, \neg\Diamond(m \wedge h), Om, Oh\} \vdash_d \bot$

Preferred faithfull maximal consistent subset

Promise example

$\Gamma^d = \{p, a, \neg\Diamond(m \wedge h)\}$
$\Gamma^c = \{a \gg Oh, p \gg Om\}$ ① Commute

Here $\Gamma^{d+c} = \{p, a, \neg\Diamond(m \wedge h), Om, Oh\}$

② Calculate Φ (preferred faithfull maximal consistent subset)

$$\Phi^0 = \Gamma^d = \{p, a, \neg\Diamond(m \wedge h)\}$$
$$\Phi^1 = \{\underbrace{p, a, \neg\Diamond(m \wedge h)}_{\Phi^0}, Oh\}$$
$$\Phi^2 = \Phi^1$$
$$\vdots$$

$\Phi = \Phi^1$

③ATC obligations

$O_{atc}h$ in, since $\Phi \vdash_d Oh$
$O_{atc}m$ out, since $\Phi \not\vdash_d Om$

Completion of an order

Back to the def from text

For a complete strict order, $\succ$, over Γ^c that respects $\sqsupset_\Gamma$, let the ordering of Γ^c by $\succ$ be $\langle c_1, \ldots, c_i, \ldots \rangle$, so that $c_i < c_j$ in the sequence just in case $c_i \succ c_j$.[47] Then define $\Phi_\succ$ from a sequence of subsets, $\Phi^i_\succ$, of Γ^{d+c}, thus:

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Def 7) $\Gamma \mathrel{|\!\!\sim} A$ iff $\Phi_\succ \vdash_d A$, for each complete strict order, $\succ$, that respects $\sqsupset_\Gamma$.

Completion of an order

What if gaps in the ranking?

Consider

$$
\begin{aligned}
\Gamma^d &= \emptyset \\
\Gamma^c &= \{c_1, c_2, c_3, c_4\}
\end{aligned}
$$

$$
\begin{array}{ll}
Captain(c_1) & \top \gg OA \\
Major(c_2) & \top \gg O\neg A \\
Priest(c_3) & \top \gg OA \\
Bishop(c_4) & \top \gg O\neg A
\end{array}
$$

$$c_2 \sqsupset c_1 \qquad c_4 \sqsupset c_3$$

Completion of an order

Completion

Let $\sqsupset$ be a partial order. $\succ$ is said a completion of $\sqsupset$ if

- $\succ$ extends ("respects") $\sqsupset$, viz $\sqsupset \subseteq \succ$
- $\succ$ is a total order

$\succ$ just fills in the gaps left in $\sqsupset$

There might be many such completions

Completion of an order

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Let $\sqsupset$ be a partial order. $\succ$ is said a completion of $\sqsupset$ if

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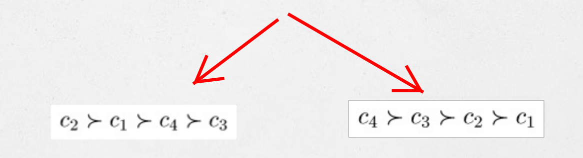

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There might be many such completions $\succ$.

Each $\succ$ gives its own faithfull maximal consistent subset.

Subscript notation: $\Phi_\succ$

$$O_{atc}A \text{ is } \begin{cases} \text{in} & \text{if } \Phi_\succ \vdash_d OA \text{ for all completion } \succ \\ \text{out} & \text{otherwise.} \end{cases}$$

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There might be many such completions $\succ$.

Captain/Priest example: the procedure yields correct result

- $O_{atc}\neg A$ in

Non-triggerred obligations

Consider

$$\begin{aligned}\Gamma^d &= \{\neg(A \wedge B)\}\\ \Gamma^c &= \{c_1, c_2, c_3\}\end{aligned}$$

$$\begin{array}{ll} Priest(c_1) & \top \gg OA \\ Bishop(c_2) & \top \gg OB \\ Cardinal(c_3) & A \gg O\neg B \end{array}$$

A: heating on
B: window open

$c_3 \sqsupset c_2 \sqsupset c_1$

Non-triggerred obligations

Consider

$$\begin{aligned} \Gamma^d &= \{\neg(A \wedge B)\} \\ \Gamma^c &= \{c_1, c_2, c_3\} \end{aligned}$$

$Priest(c_1)$	$\top \gg OA$	
$Bishop(c_2)$	$\top \gg OB$	A: heating on
$Cardinal(c_3)$	$A \gg O\neg B$	B: window open

$c_3 \sqsupset c_2 \sqsupset c_1$

$O_{atc}A$ and $O_{atc}\neg B$ both in
$O_{atc}B$ out
Is it as it should be?