INSTRUCTOR'S SOLUTIONS MANUAL

SEARS & ZEMANSKY'S

UNIVERSITY PHYSICS

15TH EDITION

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ISBN 10: 0-135-27588-1
ISBN 13: 978-0-135-27588-7

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Preface

This Instructor's Solutions Manual contains the solutions to all the problems and exercises in *University Physics*, Fifteenth Edition, by Hugh Young and Roger Freedman.

In preparing this manual, we assumed that its primary users would be college professors; thus the solutions are condensed, and some steps are not shown. Some calculations were carried out to more significant figures than demanded by the input data in order to allow for differences in calculator rounding. In many cases answers were then rounded off. Therefore, you may obtain slightly different results, especially when powers or trig functions are involved.

This edition was constructed from the previous editions authored by Craig Watkins and Mark Hollabaugh, and much of what is here is due to them.

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UNITS, PHYSICAL QUANTITIES, AND VECTORS

VP1.7.1. **IDENTIFY:** We know that the sum of three known vectors and a fourth unknown vector is zero. We want to find the magnitude and direction of the unknown vector.

SET UP: The sum of their x-components and the sum of their y-components must both be zero.

$A_x + B_x + C_x + D_x = 0$

$A_y + B_y + C_y + D_y = 0$

The magnitude of a vector is w $A = \sqrt{A_x^2 + A_y^2}$ and the angle θ it makes with the $+x$-axis is

$\theta = \arctan\dfrac{A_y}{A_x}$.

EXECUTE: We use the results of Ex. 1.7. See Fig. 1.23 in the text.

$A_x = 38.37$ m, $B_x = -46.36$ m, $C_x = 0.00$ m, $A_y = 61.40$ m, $B_y = -33.68$ m, $C_y = -17.80$ m

Adding the x-components gives

38.37 m + (–46.36 m) + 0.00 m + $D_x = 0$ $\quad\rightarrow\quad$ $D_x = 7.99$ m

Adding the y-components gives

61.40 m + (–33.68 m) + (–17.80 m) + $D_y = 0$ $\quad\rightarrow\quad$ $D_y = -9.92$ m

$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(7.99\text{ m})^2 + (-9.92\text{ m})^2} = 12.7$ m

glo $\quad \theta = \arctan\dfrac{D_y}{D_x} = \arctan[(-9.92\text{ m})/(7.99\text{ m})] = -51°$

Since $\vec{D}$ has a positive x-component and a negative y-component, it points into the fourth quadrant making an angle of 51° below the $+x$-axis and an angle of 360° – 51° = 309° counterclockwise with the $+x$-axis.

EVALUATE: The vector $\vec{D}$ has the same magnitude as the resultant in Ex. 1.7 but points in the opposite direction. This is reasonable because $\vec{D}$ must be opposite to the resultant of the three vectors in Ex. 1.7 to make the resultant of all four vectors equal to zero.

VP1.7.2. **IDENTIFY:** We know three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ and we want to find the sum $\vec{S}$ where $\vec{S} = \vec{A} - \vec{B} + \vec{C}$. The components of $-\vec{B}$ are the negatives of the components of $\vec{B}$.

SET UP: The components of $\vec{S}$ are

$S_x = A_x - B_x + C_x$

$S_y = A_y - B_y + C_y$

The magnitude A of a vector $\vec{A}$ is $A = \sqrt{A_x^2 + A_y^2}$ and the angle θ it makes with the $+x$-axis is

$\theta = \arctan\dfrac{A_y}{A_x}$.

EXECUTE: Using the components from Ex. 1.7 we have

S_x = 38.37 m – (–46.36 m) + 0.00 m = 84.73 m

S_y = 61.40 m – (–33.68 m) + (–17.80 m) = 77.28 m

$$S = \sqrt{S_x^2 + S_y^2} = \sqrt{(84.73\text{ m})^2 + (77.28\text{ m})^2} = 115\text{ m}$$

$$\theta = \arctan\frac{S_y}{S_x} = \arctan[(77.28\text{ m})/(84.73\text{ m})] = 42°$$

Since both components of $\vec{S}$ are positive, $\vec{S}$ points into the first quadrant. Therefore it makes an angle of 42° with the +x-axis.

EVALUATE:

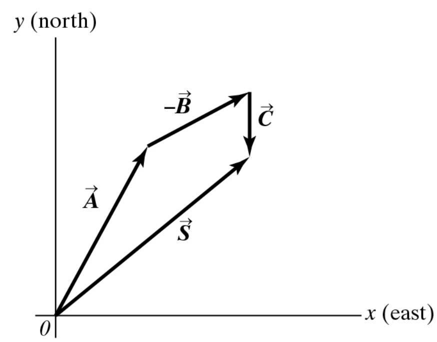


Figure VP1.7.2

The graphical solution shown in Fig. VP1.7.2 shows that our results are reasonable.

VP1.7.3. **IDENTIFY:** We know three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ and we want to find the sum $\vec{T}$ where $\vec{T} = \vec{A} + \vec{B} + 2\vec{C}$.

SET UP: Find the components of vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ and use them to find the magnitude and direction of $\vec{T}$. The components of $2\vec{C}$ are twice those of $\vec{C}$.

EXECUTE: $S_x = A_x + B_x + 2C_x$ and $S_y = A_y + B_y + 2C_y$

(a) Using the components from Ex. 1.7 gives

T_x = 38.37 m + (–46.36 m) + 2(0.00 m) = –7.99 m

T_y = 61.40 m + (–33.68 m) + 2(–17.80 m) = –7.88 m

(b) $T = \sqrt{T_x^2 + T_y^2} = \sqrt{(-7.99\text{ m})^2 + (-7.88\text{ m})^2} = 11.2\text{ m}$

$$\theta = \arctan\frac{T_y}{T_x} = \arctan[(-7.88\text{ m})/(-7.99\text{ m})] = 45°$$

Both components of $\vec{T}$ are negative, so it points into the third quadrant, making an angle of 45° below the –x-axis or 45° + 180° = 225° counterclockwise with the +x-axis, in the third quadrant.

EVALUATE:

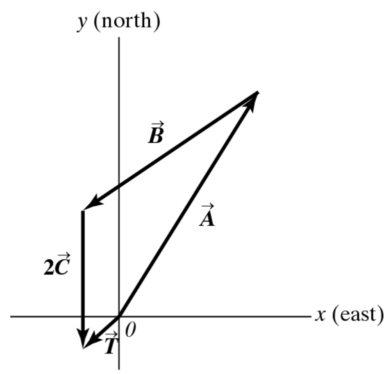


Figure VP1.7.3

The graphical solution shown in Fig. VP1.7.3 shows that this result is reasonable.

VP1.7.4. **IDENTIFY:** The hiker makes two displacements. We know the first one and their resultant, and we want to find the second displacement.

SET UP: Calling $\vec{A}$ the known displacement, $\vec{R}$ the known resultant, and $\vec{D}$ the unknown vector, we know that $\vec{A} + \vec{D} = \vec{R}$. We also know that $R = 38.0$ m and $\vec{R}$ makes an angle $\theta_R = 37.0° + 90° = 127°$ with the $+x$-axis. Fig. VP1.7.4 shows a sketch of these vectors.

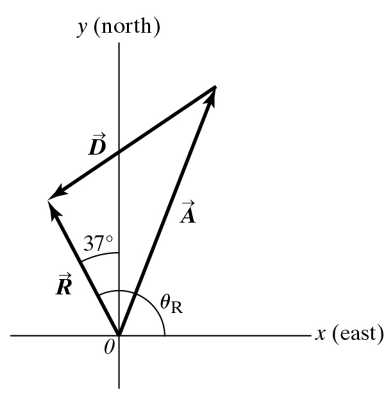


Figure VP1.7.4

EXECUTE: From Ex. 1.7 we have $A_x = 38.37$ m and $A_y = 61.40$ m. The components of $\vec{R}$ are
$R_x = R \cos 127.0° = (38.0 \text{ m}) \cos 127.0° = –22.87$ m
$R_y = R \sin 38.0° = (38.0 \text{ m}) \sin 127.0° = 30.35$ m
$R_x = A_x + D_x$ and $R_y = A_y + D_y$

Using these components, we find the components of $\vec{D}$.
$38.37 \text{ m} + D_x = –22.87 \text{ m} \quad \rightarrow \quad D_x = –22.87 \text{ m}$
$61.40 \text{ m} + D_y = –31.05 \text{ m} \quad \rightarrow \quad D_y = –31.05 \text{ m}$

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(–61.24 \text{ m})^2 + (–31.05 \text{ m})^2} = 68.7 \text{ m}$$

$$\theta = \arctan\frac{D_y}{D} = \arctan[(–31.05 \text{ m})/(–61.24 \text{ m})] = 27°$$

Both components of $\vec{D}$ are negative, so it points into the third quadrant, making an angle of 27° + 180° = 207° with the +x-axis.

EVALUATE: A graphical solution will confirm these results.

VP1.10.1. **IDENTIFY:** We know the magnitude and direction of two vectors. We want to use these to find their components and their scalar product.

SET UP: $A_x = A\cos\theta_A$, $A_y = A\sin\theta_A$, $B_x = B\cos\theta_B$, $B_y = B\sin\theta_B$. We can find the scalar product using the vector components or using their magnitudes and the angle between them. $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$ and $\vec{A}\cdot\vec{B} = AB\cos\phi$. Which form you use depends on the information you have.

EXECUTE: (a) $A_x = A\cos\theta_A = (5.00)\cos(360° - 36.9°) = 4.00$

$A_y = A\sin\theta_A = (5.00)\sin(360° - 36.9°) = -3.00$

$B_x = (6.40)\cos(90° + 20.0°) = -2.19$

$B_y = (6.40)\sin(90° + 20.0°) = 6.01$

(b) Using components gives

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = (4.00)(-2.19) + (-3.00)(6.01) = -26.8$

EVALUATE: We check by using $\vec{A}\cdot\vec{B} = AB\cos\phi$.

$\vec{A}\cdot\vec{B} = AB\cos\phi = (5.00)(6.40)\cos(20.0° + 90° + 36.9°) = -26.8$

This agrees with our result in part (b).

VP1.10.2. **IDENTIFY:** We know the magnitude and direction of one vector and the components of another vector. We want to use these to find their scalar product and the angle between them.

SET UP: The scalar product can be expressed as $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$ and $\vec{A}\cdot\vec{B} = AB\cos\phi$. Which form you use depends on the information you have.

EXECUTE: (a) $C_x = C\cos\theta_C = (6.50)\cos 55.0° = 3.728$

$C_y = C\sin\theta_C = (6.50)\sin 55.0° = 5.324$

$D_x = 4.80$ and $D_y = -8.40$

Using components gives $\vec{C}\cdot\vec{D} = C_xD_x + C_yD_y = (3.728)(4.80) + (5.324)(-8.40) = -26.8$

(b) $D = \sqrt{D_x^2 + D_y^2} = \sqrt{(4.80)^2 + (-8.40)^2} = 9.675$

$\vec{C}\cdot\vec{D} = CD\cos\phi$, so $\cos\phi = \vec{C}\cdot\vec{D}/CD = (-26.8)/[(6.50)(9.67)] = -0.426$. $\phi = 115°$.

EVALUATE: Find the angle that $\vec{D}$ makes with the +x-axis.

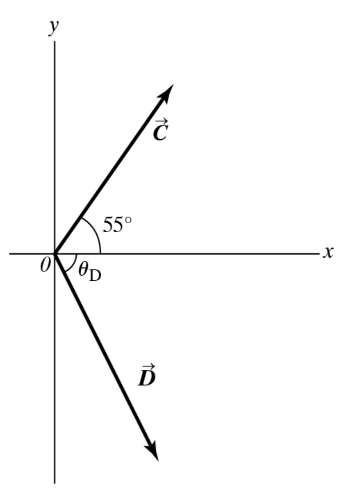


Figure VP1.10.2

$\theta_D = \arctan\dfrac{D_y}{D_x}$ = arctan[8.40/(–4.80] = –60.3°, which is 60.3° below the +x-axis. From Fig. VP1.10.2, we can easily seed that the angle between $\vec{C}$ and $\vec{D}$ is ϕ = 60.3° + 55.0° = 115°, as we found in (b).

VP1.10.3. **IDENTIFY:** We know the components of two vectors and want to find the angle between them.
SET UP: The scalar product $\vec{A}\cdot\vec{B} = AB\cos\phi$ involves the angle between two vectors. We can find this product using components from $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$. From this result we can find the angle ϕ.
EXECUTE: First find the magnitudes of the two vectors.
$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(-5.00)^2 + (3.00)^2 + 0^2} = 5.83$
$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(2.50)^2 + (4.00)^2 + (-1.50)^2} = 4.95$
Now use $\vec{A}\cdot\vec{B} = AB\cos\phi = A_xB_x + A_yB_y$ and solve for ϕ.
$(5.83)(4.95)\cos\phi = (-5.00)(2.50) + (3.00)(4.00) + (0)(-1.50) \quad\rightarrow\quad \phi = 91°$.
EVALUATE: The scalar product is positive, so ϕ must be between 90° and 180°, which agrees with our result.

VP1.10.4. **IDENTIFY:** We know the scalar product of two vectors. We also know both components of one of them and the x-component of the other one. We want to find the y-component of the other one and the angle between the two vectors. The scalar product involves the angle between two vectors.
SET UP: We use $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$ and $\vec{A}\cdot\vec{B} = AB\cos\phi$.
EXECUTE: (a) Use $\vec{F}\cdot\vec{s} = F_xs_x + F_ys_y$ to find F_y.
$26.0\ \text{N}\cdot\text{m} = (-12.0\ \text{N})(4.00\ \text{m}) + F_y(5.00\ \text{m}) \quad\rightarrow\quad F_y = 14.8\ \text{N}$.
(b) Use $\vec{F}\cdot\vec{s} = Fs\cos\phi$ and $A = \sqrt{A_x^2 + A_y^2}$ to find the magnitudes of the two vectors.
$\sqrt{(-12.0\ \text{N})^2 + (14.8\ \text{N})^2}\ \sqrt{(4.00\ \text{m})^2 + (5.00\ \text{m})^2}\ \cos\phi = 26.0\ \text{N}\cdot\text{m} \quad\rightarrow\quad \phi = 77.7°$.
EVALUATE: The work is positive, so the angle between $\vec{F}$ and $\vec{s}$ must be between 0° and 90°, which agrees with our result in part (b).

1.1. **IDENTIFY:** Convert units from mi to km and from km to ft.
SET UP: 1 in. = 2.54 cm, 1 km = 1000 m, 12 in. = 1 ft, 1 mi = 5280 ft.
EXECUTE: **(a)** $1.00\ \text{mi} = (1.00\ \text{mi})\left(\dfrac{5280\ \text{ft}}{1\ \text{mi}}\right)\left(\dfrac{12\ \text{in.}}{1\ \text{ft}}\right)\left(\dfrac{2.54\ \text{cm}}{1\ \text{in.}}\right)\left(\dfrac{1\ \text{m}}{10^2\ \text{cm}}\right)\left(\dfrac{1\ \text{km}}{10^3\ \text{m}}\right) = 1.61\ \text{km}$
(b) $1.00\ \text{km} = (1.00\ \text{km})\left(\dfrac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\dfrac{10^2\ \text{cm}}{1\ \text{m}}\right)\left(\dfrac{1\ \text{in.}}{2.54\ \text{cm}}\right)\left(\dfrac{1\ \text{ft}}{12\ \text{in.}}\right) = 3.28\times10^3\ \text{ft}$
EVALUATE: A mile is a greater distance than a kilometer. There are 5280 ft in a mile but only 3280 ft in a km.

1.2. **IDENTIFY:** Convert volume units from L to in.^{3}.
SET UP: $1\ \text{L} = 1000\ \text{cm}^3$. 1 in. = 2.54 cm
EXECUTE: $0.473\ \text{L}\times\left(\dfrac{1000\ \text{cm}^3}{1\ \text{L}}\right)\times\left(\dfrac{1\ \text{in.}}{2.54\ \text{cm}}\right)^3 = 28.9\ \text{in.}^3$.
EVALUATE: 1 in.^{3} is greater than 1 cm^3, so the volume in in.^{3} is a smaller number than the volume in cm^3, which is 473 cm^3.

1.3. **IDENTIFY:** We know the speed of light in m/s. $t = d/v$. Convert 1.00 ft to m and t from s to ns.
SET UP: The speed of light is $v = 3.00\times10^8$ m/s. 1 ft = 0.3048 m. 1 s = 10^9 ns.

EXECUTE: $t = \dfrac{0.3048\text{ m}}{3.00\times10^8\text{ m/s}} = 1.02\times10^{-9}\text{ s} = 1.02\text{ ns}$

EVALUATE: In 1.00 s light travels $3.00\times10^8\text{ m} = 3.00\times10^5\text{ km} = 1.86\times10^5\text{ mi}$.

1.4. IDENTIFY: Convert the units from g to kg and from cm^3 to m^3.
SET UP: $1\text{ kg} = 1000\text{ g}$. $1\text{ m} = 100\text{ cm}$.

EXECUTE: $19.3\dfrac{\text{g}}{\text{cm}^3}\times\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)\times\left(\dfrac{100\text{ cm}}{1\text{ m}}\right)^3 = 1.93\times10^4\dfrac{\text{kg}}{\text{m}^3}$

EVALUATE: The ratio that converts cm to m is cubed, because we need to convert cm^3 to m^3.

1.5. IDENTIFY: Convert seconds to years. 1 gigasecond is a billion seconds.
SET UP: $1\text{ gigasecond} = 1\times10^9\text{ s}$. $1\text{ day} = 24\text{ h}$. $1\text{ h} = 3600\text{ s}$.

EXECUTE: $1.00\text{ gigasecond} = (1.00\times10^9\text{ s})\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)\left(\dfrac{1\text{ day}}{24\text{ h}}\right)\left(\dfrac{1\text{ y}}{365\text{ days}}\right) = 31.7\text{ y}.$

EVALUATE: The conversion $1\text{ y} = 3.156\times10^7\text{ s}$ assumes $1\text{ y} = 365.24\text{ d}$, which is the average for one extra day every four years, in leap years. The problem says instead to assume a 365-day year.

1.6. IDENTIFY: Convert units.
SET UP: Use the unit conversions given in the problem. Also, $100\text{ cm} = 1\text{ m}$ and $1000\text{ g} = 1\text{ kg}$.

EXECUTE: **(a)** $\left(60\dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)\left(\dfrac{5280\text{ ft}}{1\text{ mi}}\right) = 88\dfrac{\text{ft}}{\text{s}}$

(b) $\left(32\dfrac{\text{ft}}{\text{s}^2}\right)\left(\dfrac{30.48\text{ cm}}{1\text{ ft}}\right)\left(\dfrac{1\text{ m}}{100\text{ cm}}\right) = 9.8\dfrac{\text{m}}{\text{s}^2}$

(c) $\left(1.0\dfrac{\text{g}}{\text{cm}^3}\right)\left(\dfrac{100\text{ cm}}{1\text{ m}}\right)^3\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right) = 10^3\dfrac{\text{kg}}{\text{m}^3}$

EVALUATE: The relations $60\text{ mi/h} = 88\text{ ft/s}$ and $1\text{ g/cm}^3 = 10^3\text{ kg/m}^3$ are exact. The relation $32\text{ ft/s}^2 = 9.8\text{ m/s}^2$ is accurate to only two significant figures.

1.7. IDENTIFY: Convert miles/gallon to km/L.
SET UP: $1\text{ mi} = 1.609\text{ km}$. $1\text{ gallon} = 3.788\text{ L}$.

EXECUTE: **(a)** $55.0\text{ miles/gallon} = (55.0\text{ miles/gallon})\left(\dfrac{1.609\text{ km}}{1\text{ mi}}\right)\left(\dfrac{1\text{ gallon}}{3.788\text{ L}}\right) = 23.4\text{ km/L}.$

(b) The volume of gas required is $\dfrac{1500\text{ km}}{23.4\text{ km/L}} = 64.1\text{ L}$. $\dfrac{64.1\text{ L}}{45\text{ L/tank}} = 1.4\text{ tanks}.$

EVALUATE: $1\text{ mi/gal} = 0.425\text{ km/L}$. A km is very roughly half a mile and there are roughly 4 liters in a gallon, so $1\text{ mi/gal} \sim \frac{2}{4}\text{ km/L}$, which is roughly our result.

1.8. IDENTIFY: Convert units.
SET UP: We know the equalities $1\text{ mg} = 10^{-3}\text{ g}$, $1\ \mu\text{g}\ 10^{-6}\text{ g}$, and $1\text{ kg} = 10^3\text{ g}$.

EXECUTE: **(a)** $(410\text{ mg/day})\left(\dfrac{10^{-3}\text{ g}}{1\text{ mg}}\right)\left(\dfrac{1\ \mu\text{g}}{10^{-6}\text{ g}}\right) = 4.10\times10^5\ \mu\text{g/day}.$

(b) $(12\text{ mg/kg})(75\text{ kg}) = (900\text{ mg})\left(\dfrac{10^{-3}\text{ g}}{1\text{ mg}}\right) = 0.900\text{ g}.$

(c) The mass of each tablet is $(2.0\text{ mg})\left(\dfrac{10^{-3}\text{ g}}{1\text{ mg}}\right) = 2.0\times10^{-3}\text{ g}$. The number of tablets required each day is the number of grams recommended per day divided by the number of grams per tablet:
$\dfrac{0.0030\text{ g/day}}{2.0\times10^{-3}\text{ g/tablet}} = 1.5$ tablet/day. Take 2 tablets each day.

(d) $(0.000070\text{ g/day})\left(\dfrac{1\text{ mg}}{10^{-3}\text{ g}}\right) = 0.070\text{ mg/day}$.

EVALUATE: Quantities in medicine and nutrition are frequently expressed in a wide variety of units.

1.9. **IDENTIFY:** We know the density and mass; thus we can find the volume using the relation density = mass/volume = m/V. The radius is then found from the volume equation for a sphere and the result for the volume.

SET UP: Density $= 19.5\text{ g/cm}^3$ and $m_{\text{critical}} = 60.0\text{ kg}$. For a sphere $V = \frac{4}{3}\pi r^3$.

EXECUTE: $V = m_{\text{critical}}/\text{density} = \left(\dfrac{60.0\text{ kg}}{19.5\text{ g/cm}^3}\right)\left(\dfrac{1000\text{ g}}{1.0\text{ kg}}\right) = 3080\text{ cm}^3$.

$r = \sqrt[3]{\dfrac{3V}{4\pi}} = \sqrt[3]{\dfrac{3}{4\pi}(3080\text{ cm}^3)} = 9.0\text{ cm}$.

EVALUATE: The density is very large, so the 130-pound sphere is small in size.

1.10. **IDENTIFY:** Model the bacteria as spheres. Use the diameter to find the radius, then find the volume and surface area using the radius.

SET UP: From Appendix B, the volume V of a sphere in terms of its radius is $V = \frac{4}{3}\pi r^3$ while its surface area A is $A = 4\pi r^2$. The radius is one-half the diameter or $r = d/2 = 1.0\ \mu\text{m}$. Finally, the necessary equalities for this problem are: $1\ \mu\text{m} = 10^{-6}\text{ m}$; $1\text{ cm} = 10^{-2}\text{ m}$; and $1\text{ mm} = 10^{-3}\text{ m}$.

EXECUTE: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0\ \mu\text{m})^3\left(\dfrac{10^{-6}\text{ m}}{1\ \mu\text{m}}\right)^3\left(\dfrac{1\text{ cm}}{10^{-2}\text{ m}}\right)^3 = 4.2\times10^{-12}\text{ cm}^3$ and

$A = 4\pi r^2 = 4\pi(1.0\ \mu\text{m})^2\left(\dfrac{10^{-6}\text{ m}}{1\ \mu\text{m}}\right)^2\left(\dfrac{1\text{ mm}}{10^{-3}\text{ m}}\right)^2 = 1.3\times10^{-5}\text{ mm}^2$

EVALUATE: On a human scale, the results are extremely small. This is reasonable because bacteria are not visible without a microscope.

1.11. **IDENTIFY:** When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. When we add or subtract numbers it is the location of the decimal that matters.

SET UP: 12 mm has two significant figures and 5.98 mm has three significant figures.

EXECUTE: **(a)** $(12\text{ mm})\times(5.98\text{ mm}) = 72\text{ mm}^2$ (two significant figures)

(b) s $\dfrac{5.98\text{ mm}}{12\text{ mm}} = 0.50$ (also two significant figures)

(c) 36 mm (to the nearest millimeter)

(d) 6 mm

(e) 2.0 (two significant figures)

EVALUATE: The length of the rectangle is known only to the nearest mm, so the answers in parts **(c)** and (d) are known only to the nearest mm.

1.12. **IDENTIFY:** This is a problem in conversion of units.
SET UP: 10 mm = 1 cm, $V = \pi r^2 h$.
EXECUTE: $V = \pi(0.036\text{ cm})^2(12.1\text{ cm}) = 0.049\text{ cm}^3$. Now convert to mm^3.

$$0.049\text{ cm}^3\left(\frac{10\text{ mm}}{1\text{ cm}}\right)^3 = 49\text{ mm}^3.$$

EVALUATE: The answer has only 2 significant figures. Even though π and h have more than that, r has only 2 which limits the answer.

1.13. **IDENTIFY:** Use your calculator to display $\pi \times 10^7$. Compare that number to the number of seconds in a year.
SET UP: $1\text{ yr} = 365.24\text{ days}$, $1\text{ day} = 24\text{ h}$, and $1\text{ h} = 3600\text{ s}$.
EXECUTE: $(365.24\text{ days/1 yr})\left(\frac{24\text{ h}}{1\text{ day}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 3.15567\ldots\times 10^7\text{ s}$; $\pi\times 10^7\text{ s} = 3.14159\ldots\times 10^7\text{ s}$

The approximate expression is accurate to two significant figures. The percent error is 0.45%.
EVALUATE: The close agreement is a numerical accident.

1.14. **IDENTIFY:** To asses the accuracy of the approximations, we must convert them to decimals.
SET UP: Use a calculator to calculate the decimal equivalent of each fraction and then round the numeral to the specified number of significant figures. Compare to π rounded to the same number of significant figures.
EXECUTE: **(a)** 22/7 = 3.14286 **(b)** 355/113 = 3.14159 **(c)** The exact value of π rounded to six significant figures is 3.14159.
EVALUATE: We see that 355/113 is a much better approximation to π than is 22/7.

1.15. **IDENTIFY:** Express 200 kg in pounds. Express each of 200 m, 200 cm and 200 mm in inches. Express 200 months in years.
SET UP: A mass of 1 kg is equivalent to a weight of about 2.2 lbs. $1\text{ in.} = 2.54\text{ cm}$. $1\text{ y} = 12\text{ months}$.
EXECUTE: **(a)** 200 kg is a weight of 440 lb. This is much larger than the typical weight of a man.
(b) $200\text{ m} = (2.00\times 10^4\text{ cm})\left(\frac{1\text{ in.}}{2.54\text{ cm}}\right) = 7.9\times 10^3$ inches. This is much greater than the height of a person.
(c) $200\text{ cm} = 2.00\text{ m} = 79\text{ inches} = 6.6\text{ ft}$. Some people are this tall, but not an ordinary man.
(d) $200\text{ mm} = 0.200\text{ m} = 7.9\text{ inches}$. This is much too short.
(e) $200\text{ months} = (200\text{ mon})\left(\frac{1\text{ y}}{12\text{ mon}}\right) = 17\text{ y}$. This is the age of a teenager; a middle-aged man is much older than this.
EVALUATE: None are plausible. When specifying the value of a measured quantity it is essential to give the units in which it is being expressed.

1.16. **IDENTIFY:** Estimate the number of people and then use the estimates given in the problem to calculate the number of gallons.
SET UP: Estimate 3×10^8 people, so 2×10^8 cars.
EXECUTE: (Number of cars × miles/car day)/(mi/gal) = gallons/day

$$(2\times 10^8\text{ cars}\times 10000\text{ mi/yr/car}\times 1\text{ yr/365 days})/(20\text{ mi/gal}) = 3\times 10^8\text{ gal/day}$$

EVALUATE: The number of gallons of gas used each day approximately equals the population of the U.S.

1.17. IDENTIFY: Estimation problem.

SET UP: Estimate that the pile is $18\text{ in.}\times 18\text{ in.}\times 5\text{ ft }8\text{ in.}$. Use the density of gold to calculate the mass of gold in the pile and from this calculate the dollar value.

EXECUTE: The volume of gold in the pile is $V = 18\text{ in.}\times 18\text{ in.}\times 68\text{ in.} = 22{,}000\text{ in.}^3$. First convert to cm^3:

$$V = 22{,}000\text{ in.}^3(1000\text{ cm}^3/61.02\text{ in.}^3) = 3.6\times 10^5\text{ cm}^3.$$

The density of gold is 19.3 g/cm^3, so the mass of this volume of gold is

$$m = (19.3\text{ g/cm}^3)(3.6\times 10^5\text{ cm}^3) = 6.95\times 10^6\text{ g}.$$

The monetary value of one gram is \$40, so the gold has a value of

$$(\$40\text{ /gram})\ (6.95\times 10^6\text{ grams}) = \$2.8\times 10^8$$

or about $\$300\times 10^6$ (three hundred million dollars).

EVALUATE: This is quite a large pile of gold, so such a large monetary value is reasonable.

1.18. IDENTIFY: Approximate the number of breaths per minute. Convert minutes to years and cm^3 to m^3 to find the volume in m^3 breathed in a year.

SET UP: Assume 10 breaths/min. $1\text{ y} = (365\text{ d})\left(\dfrac{24\text{ h}}{1\text{ d}}\right)\left(\dfrac{60\text{ min}}{1\text{ h}}\right) = 5.3\times 10^5\text{ min}$. $10^2\text{ cm} = 1\text{ m}$ so $10^6\text{ cm}^3 = 1\text{ m}^3$. The volume of a sphere is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$, where r is the radius and d is the diameter. Don't forget to account for four astronauts.

EXECUTE: **(a)** The volume is $(4)(10\text{ breaths/min})(500\times 10^{-6}\text{ m}^3)\left(\dfrac{5.3\times 10^5\text{ min}}{1\text{ y}}\right) = 1\times 10^4\text{ m}^3\text{/yr}$.

(b) $d = \left(\dfrac{6V}{\pi}\right)^{1/3} = \left(\dfrac{6[1\times 10^4\text{ m}^3]}{\pi}\right)^{1/3} = 27\text{ m}$

EVALUATE: Our estimate assumes that each cm^3 of air is breathed in only once, where in reality not all the oxygen is absorbed from the air in each breath. Therefore, a somewhat smaller volume would actually be required.

1.19. IDENTIFY: Estimate the diameter of a drop and from that calculate the volume of a drop, in m^3. Convert m^3 to L.

SET UP: Estimate the diameter of a drop to be $d = 2\text{ mm}$. The volume of a spherical drop is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$. $10^3\text{ cm}^3 = 1\text{ L}$.

EXECUTE: $V = \frac{1}{6}\pi(0.2\text{ cm})^3 = 4\times 10^{-3}\text{ cm}^3$. The number of drops in 1.0 L is $\dfrac{1000\text{ cm}^3}{4\times 10^{-3}\text{ cm}^3} = 2\times 10^5$

EVALUATE: Since $V \sim d^3$, if our estimate of the diameter of a drop is off by a factor of 2 then our estimate of the number of drops is off by a factor of 8.

1.20. IDENTIFY: Estimate the number of beats per minute and the duration of a lifetime. The volume of blood pumped during this interval is then the volume per beat multiplied by the total beats.

SET UP: An average middle-aged (40 year-old) adult at rest has a heart rate of roughly 75 beats per minute. To calculate the number of beats in a lifetime, use the current average lifespan of 80 years.

EXECUTE: $N_{\text{beats}} = (75\text{ beats/min})\left(\dfrac{60\text{ min}}{1\text{ h}}\right)\left(\dfrac{24\text{ h}}{1\text{ day}}\right)\left(\dfrac{365\text{ days}}{\text{yr}}\right)\left(\dfrac{80\text{ yr}}{\text{lifespan}}\right) = 3\times 10^9\text{ beats/lifespan}$

$$V_{\text{blood}} = (50\text{ cm}^3\text{/beat})\left(\frac{1\text{ L}}{1000\text{ cm}^3}\right)\left(\frac{1\text{ gal}}{3.788\text{ L}}\right)\left(\frac{3\times 10^9\text{ beats}}{\text{lifespan}}\right) = 4\times 10^7\text{ gal/lifespan}$$

EVALUATE: This is a very large volume.

1.21. **IDENTIFY:** Draw each subsequent displacement tail to head with the previous displacement. The resultant displacement is the single vector that points from the starting point to the stopping point.
SET UP: Call the three displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$. The resultant displacement $\vec{R}$ is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.
EXECUTE: The vector addition diagram is given in Figure 1.21. Careful measurement gives that $\vec{R}$ is 7.8 km, 38° north of east.
EVALUATE: The magnitude of the resultant displacement, 7.8 km, is less than the sum of the magnitudes of the individual displacements, $2.6\text{ km} + 4.0\text{ km} + 3.1\text{ km}$.

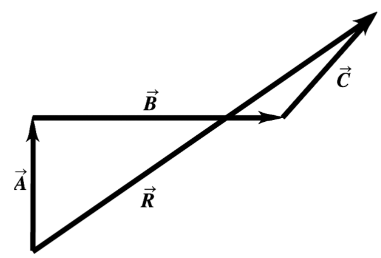


Figure 1.21

1.22. **IDENTIFY:** Draw the vector addition diagram to scale.
SET UP: The two vectors $\vec{A}$ and $\vec{B}$ are specified in the figure that accompanies the problem.
EXECUTE: **(a)** The diagram for $\vec{R} = \vec{A} + \vec{B}$ is given in Figure 1.22a. Measuring the length and angle of $\vec{R}$ gives $R = 9.0$ m and an angle of $\theta = 34°$.
(b) The diagram for $\vec{E} = \vec{A} - \vec{B}$ is given in Figure 1.22b. Measuring the length and angle of $\vec{E}$ gives $D = 22$ m and an angle of $\theta = 250°$.
(c) $-\vec{A} - \vec{B} = -(A + B)$, so $-\vec{A} - \vec{B}$ has a magnitude of 9.0 m (the same as $\vec{A} + \vec{B}$) and an angle with the $+x$ axis of $214°$ (opposite to the direction of $\vec{A} + \vec{B}$).
(d) $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has a magnitude of 22 m and an angle with the $+x$ axis of $70°$ (opposite to the direction of $\vec{A} - \vec{B}$).
EVALUATE: The vector $-\vec{A}$ is equal in magnitude and opposite in direction to the vector $\vec{A}$.

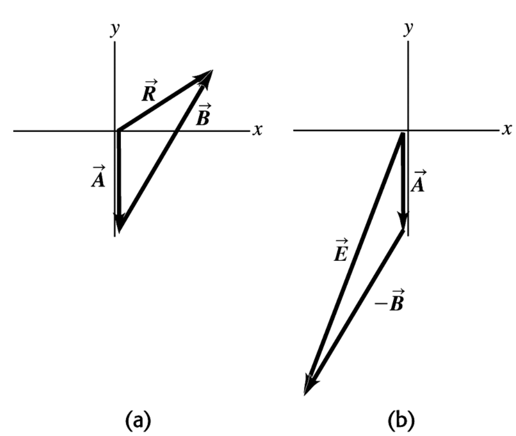


Figure 1.22

1.23. **IDENTIFY:** Since she returns to the starting point, the vector sum of the four displacements must be zero.

SET UP: Call the three given displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$, and call the fourth displacement $\vec{D}$. $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0.$

EXECUTE: The vector addition diagram is sketched in Figure 1.23. Careful measurement gives that $\vec{D}$ is 144 m, 41° south of west.

EVALUATE: $\vec{D}$ is equal in magnitude and opposite in direction to the sum $\vec{A}+\vec{B}+\vec{C}$.

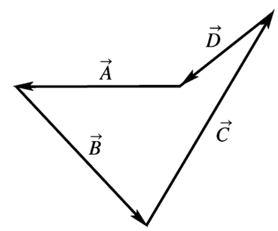


Figure 1.23

1.24. **IDENTIFY:** $\tan\theta = \dfrac{A_y}{A_x}$, for θ measured counterclockwise from the $+x$-axis.

SET UP: A sketch of A_x, A_y and $\vec{A}$ tells us the quadrant in which $\vec{A}$ lies.

EXECUTE:

(a) $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00 \text{ m}}{2.00 \text{ m}} = -0.500.\ \theta = \tan^{-1}(-0.500) = 360° - 26.6° = 333°.$

(b) $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{1.00 \text{ m}}{2.00 \text{ m}} = 0.500.\ \theta = \tan^{-1}(0.500) = 26.6°.$

(c) $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{1.00 \text{ m}}{-2.00 \text{ m}} = -0.500.\ \theta = \tan^{-1}(-0.500) = 180° - 26.6° = 153°.$

(d) $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00 \text{ m}}{-2.00 \text{ m}} = 0.500.\ \theta = \tan^{-1}(0.500) = 180° + 26.6° = 207°$

EVALUATE: The angles 26.6° and 207° have the same tangent. Our sketch tells us which is the correct value of θ.

1.25. **IDENTIFY:** For each vector $\vec{V}$, use that $V_x = V\cos\theta$ and $V_y = V\sin\theta$, when θ is the angle $\vec{V}$ makes with the $+x$ axis, measured counterclockwise from the axis.

SET UP: For $\vec{A}$, $\theta = 270.0°$. For $\vec{B}$, $\theta = 60.0°$. For $\vec{C}$, $\theta = 205.0°$. For $\vec{D}$, $\theta = 143.0°$.

EXECUTE: $A_x = 0$, $A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m. $D_x = -7.99$ m, $D_y = 6.02$ m.

EVALUATE: The signs of the components correspond to the quadrant in which the vector lies.

1.26. **IDENTIFY:** Given the direction and one component of a vector, find the other component and the magnitude.

SET UP: Use the tangent of the given angle and the definition of vector magnitude.

EXECUTE: **(a)** $\tan 34.0° = \dfrac{|A_x|}{|A_y|}$

$$|A_y| = \frac{|A_x|}{\tan 34.0°} = \frac{16.0 \text{ m}}{\tan 34.0°} = 23.72 \text{ m}$$

$A_y = -23.7$ m.

(b) $A = \sqrt{A_x^2 + A_y^2} = 28.6$ m.

EVALUATE: The magnitude is greater than either of the components.

1.27. **IDENTIFY:** Given the direction and one component of a vector, find the other component and the magnitude.

SET UP: Use the tangent of the given angle and the definition of vector magnitude.

EXECUTE: **(a)** $\tan 32.0° = \dfrac{|A_x|}{|A_y|}$

$|A_x| = (9.60 \text{ m})\tan 32.0° = 6.00$ m. $A_x = -6.00$ m.

(b) $A = \sqrt{A_x^2 + A_y^2} = 11.3$ m.

EVALUATE: The magnitude is greater than either of the components.

1.28. **IDENTIFY:** Find the vector sum of the three given displacements.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The driver's vector displacements are:

$\vec{A} = 2.6$ km, 0° of north; $\vec{B} = 4.0$ km, 0° of east; $\vec{C} = 3.1$ km, 45° north of east.

EXECUTE: $R_x = A_x + B_x + C_x = 0 + 4.0 \text{ km} + (3.1 \text{ km})\cos(45°) = 6.2$ km; $R_y = A_y + B_y + C_y =$

$2.6 \text{ km} + 0 + (3.1 \text{ km})(\sin 45°) = 4.8$ km; $R = \sqrt{R_x^2 + R_y^2} = 7.8$ km; $\theta = \tan^{-1}[(4.8 \text{ km})/(6.2 \text{ km})] = 38°$;

$\vec{R} = 7.8$ km, 38° north of east. This result is confirmed by the sketch in Figure 1.28.

EVALUATE: Both R_x and R_y are positive and $\vec{R}$ is in the first quadrant.

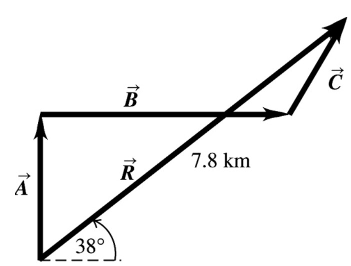


Figure 1.28

1.29. **IDENTIFY:** If $\vec{C} = \vec{A} + \vec{B}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. Use C_x and C_y to find the magnitude and direction of $\vec{C}$.

SET UP: From Figure E1.30 in the textbook, $A_x = 0$, $A_y = -8.00$ m and $B_x = +B\sin 30.0° = 7.50$ m, $B_y = +B\cos 30.0° = 13.0$ m.

EXECUTE: **(a)** $\vec{C} = \vec{A} + \vec{B}$ so $C_x = A_x + B_x = 7.50$ m and $C_y = A_y + B_y = +5.00$ m. $C = 9.01$ m.

$\tan\theta = \dfrac{C_y}{C_x} = \dfrac{5.00 \text{ m}}{7.50 \text{ m}}$ and $\theta = 33.7°$.

(b) $\vec{B} + \vec{A} = \vec{A} + \vec{B}$, so $\vec{B} + \vec{A}$ has magnitude 9.01 m and direction specified by 33.7°.

(c) $\vec{\boldsymbol{D}} = \vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}$ so $D_x = A_x - B_x = -7.50 \text{ m}$ and $D_y = A_y - B_y = -21.0 \text{ m}$. $D = 22.3 \text{ m}$.

$\tan\phi = \dfrac{D_y}{D_x} = \dfrac{-21.0 \text{ m}}{-7.50 \text{ m}}$ and $\phi = 70.3°$. $\vec{\boldsymbol{D}}$ is in the 3rd quadrant and the angle θ counterclockwise from the $+x$ axis is $180° + 70.3° = 250.3°$.

(d) $\vec{\boldsymbol{B}} - \vec{\boldsymbol{A}} = -(\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}})$, so $\vec{\boldsymbol{B}} - \vec{\boldsymbol{A}}$ has magnitude 22.3 m and direction specified by $\theta = 70.3°$.

EVALUATE: These results agree with those calculated from a scale drawing in Problem 1.22.

1.30. **IDENTIFY:** Use $A = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = \dfrac{A_y}{A_x}$ to calculate the magnitude and direction of each of the given vectors.

SET UP: A sketch of A_x, A_y and $\vec{\boldsymbol{A}}$ tells us the quadrant in which $\vec{\boldsymbol{A}}$ lies.

EXECUTE: **(a)** $\sqrt{(-8.60 \text{ cm})^2 + (5.20 \text{ cm})^2} = 10.0 \text{ cm}$, $\arctan\left(\dfrac{5.20}{-8.60}\right) = 148.8°$ (which is $180° - 31.2°$).

(b) $\sqrt{(-9.7 \text{ m})^2 + (-2.45 \text{ m})^2} = 10.0 \text{ m}$, $\arctan\left(\dfrac{-2.45}{-9.7}\right) = 14° + 180° = 194°$.

(c) $\sqrt{(7.75 \text{ km})^2 + (-2.70 \text{ km})^2} = 8.21 \text{ km}$, $\arctan\left(\dfrac{-2.7}{7.75}\right) = 340.8°$ (which is $360° - 19.2°$).

EVALUATE: In each case the angle is measured counterclockwise from the $+x$ axis. Our results for θ agree with our sketches.

1.31. **IDENTIFY:** Vector addition problem. We are given the magnitude and direction of three vectors and are asked to find their sum.

SET UP:

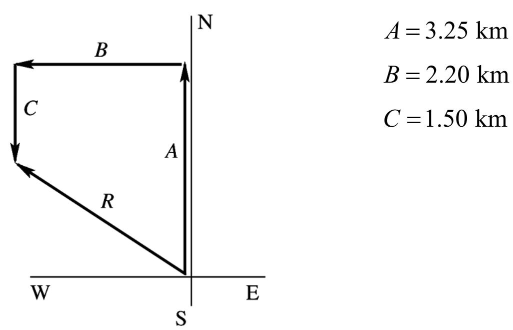
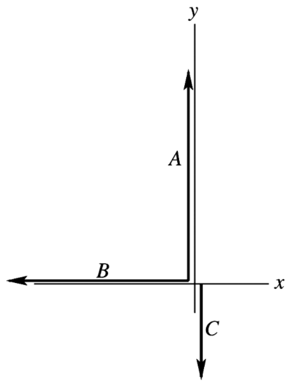


Figure 1.31a

Select a coordinate system where $+x$ is east and $+y$ is north. Let $\vec{\boldsymbol{A}}$, $\vec{\boldsymbol{B}}$, and $\vec{\boldsymbol{C}}$ be the three displacements of the professor. Then the resultant displacement $\vec{\boldsymbol{R}}$ is given by $\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} + \vec{\boldsymbol{C}}$. By the method of components, $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$. Find the x and y components of each vector; add them to find the components of the resultant. Then the magnitude and direction of the resultant can be found from its x and y components that we have calculated. As always it is essential to draw a sketch.

EXECUTE:



$A_x = 0,\ A_y = +3.25\ \text{km}$

$B_x = -2.20\ \text{km},\ B_y = 0$

$C_x = 0,\ C_y = -1.50\ \text{km}$

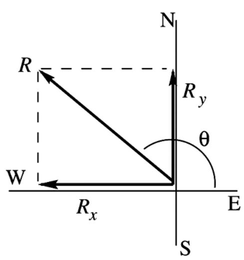
$R_x = A_x + B_x + C_x$

$R_x = 0 - 2.20\ \text{km} + 0 = -2.20\ \text{km}$

$R_y = A_y + B_y + C_y$

$R_y = 3.25\ \text{km} + 0 - 1.50\ \text{km} = 1.75\ \text{km}$

Figure 1.31b



$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.20\ \text{km})^2 + (1.75\ \text{km})^2}$

$R = 2.81\ \text{km}$

$\tan\theta = \dfrac{R_y}{R_x} = \dfrac{1.75\ \text{km}}{-2.20\ \text{km}} = -0.800$

$\theta = 141.5°$

Figure 1.31c

The angle θ measured counterclockwise from the $+x$-axis. In terms of compass directions, the resultant displacement is $38.5°$ N of W.

Evaluate: $R_x < 0$ and $R_y > 0$, so $\vec{R}$ is in the 2nd quadrant. This agrees with the vector addition diagram.

1.32. **Identify:** This problem involves vector addition. We know one vector and the resultant of that vector with a second vector, and we want to find the magnitude and direction of the second vector.

Set Up: $\vec{A} + \vec{B} = \vec{R}$. We know $\vec{A}$ and $\vec{R}$ and want to find $\vec{B}$. Use $A_x + B_x = R_x$ and $A_y + B_y = R_y$ to find the components of $\vec{B}$, then use $B = \sqrt{B_x^2 + B_y^2}$ to find B and $\theta = \arctan\dfrac{B_y}{B_x}$ to find its direction. First do a graphical sum, as shown in Fig. 1.32.

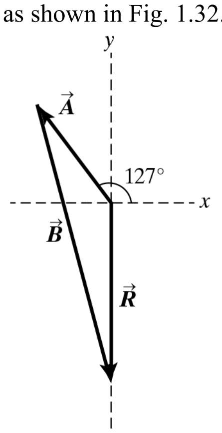


Figure 1.32

EXECUTE: First find the components. $A_x = A\cos 127° = (8.00 \text{ m})\cos 127° = -4.185$ m, $R_x = 0$, $A_y = A \sin 127° = (8.00 \text{ m})\sin 127° = 6.389$ m, $R_y = -12.0$ m. Now use $A_x + B_x = R_x$ and $A_y + B_y = R_y$ to find the components of $\vec{B}$.

$-4.185 \text{ m} + B_x = 0 \quad \rightarrow \quad B_x = 4.185$ m. $6.389 \text{ m} + B_y = -12.0 \text{ m} \quad \rightarrow \quad B_y = -18.39$ m.

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(4.185 \text{ m})^2 + (-18.39 \text{ m})^2} = 19.0 \text{ m}.$$

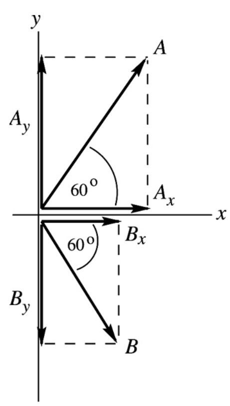
$\theta = \arctan\dfrac{B_y}{B_x} = \arctan\left(\dfrac{-18.39 \text{ m}}{4.185 \text{ m}}\right) = -75.3°$. From Fig. 1.32 we can see that $\vec{B}$ must point below the x-axis. This tells us that $\vec{B}$ makes an angle of 75.3° clockwise below the $+x$-axis, which we can also express as $360° - 75.3° = 284.7°$ counterclockwise with the $+x$-axis.

EVALUATE: Our vector sum in Fig. 1.32 agrees with our calculations.

1.33. **IDENTIFY:** Vector addition problem. $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.

SET UP: Find the x- and y-components of $\vec{A}$ and $\vec{B}$. Then the x- and y-components of the vector sum are calculated from the x- and y-components of $\vec{A}$ and $\vec{B}$.

EXECUTE:



$$A_x = A\cos(60.0°)$$
$$A_x = (2.80 \text{ cm})\cos(60.0°) = +1.40 \text{ cm}$$
$$A_y = A\sin(60.0°)$$
$$A_y = (2.80 \text{ cm})\sin(60.0°) = +2.425 \text{ cm}$$
$$B_x = B\cos(-60.0°)$$
$$B_x = (1.90 \text{ cm})\cos(-60.0°) = +0.95 \text{ cm}$$
$$B_y = B\sin(-60.0°)$$
$$B_y = (1.90 \text{ cm})\sin(-60.0°) = -1.645 \text{ cm}$$

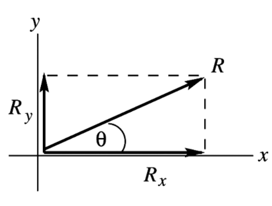
Note that the signs of the components correspond to the directions of the component vectors.

Figure 1.33a

(a) Now let $\vec{R} = \vec{A} + \vec{B}$.

$R_x = A_x + B_x = +1.40 \text{ cm} + 0.95 \text{ cm} = +2.35 \text{ cm}.$

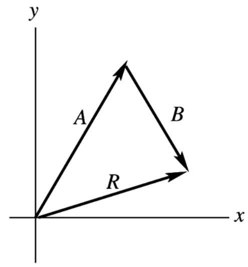
$R_y = A_y + B_y = +2.425 \text{ cm} - 1.645 \text{ cm} = +0.78 \text{ cm}.$



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.35 \text{ cm})^2 + (0.78 \text{ cm})^2}$$
$$R = 2.48 \text{ cm}$$
$$\tan\theta = \frac{R_y}{R_x} = \frac{+0.78 \text{ cm}}{+2.35 \text{ cm}} = +0.3319$$
$$\theta = 18.4°$$

Figure 1.33b

EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + \vec{B}$ is



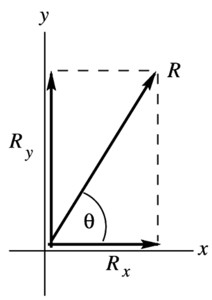
$\vec{R}$ is in the 1st quadrant, with $|R_y| < |R_x|$, in agreement with our calculation.

Figure 1.33c

(b) EXECUTE: Now let $\vec{R} = \vec{A} - \vec{B}$.

$R_x = A_x - B_x = +1.40 \text{ cm} - 0.95 \text{ cm} = +0.45 \text{ cm}.$

$R_y = A_y - B_y = +2.425 \text{ cm} + 1.645 \text{ cm} = +4.070 \text{ cm}.$



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.45 \text{ cm})^2 + (4.070 \text{ cm})^2}$$

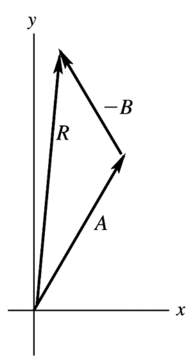
$$R = 4.09 \text{ cm}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{4.070 \text{ cm}}{0.45 \text{ cm}} = +9.044$$

$$\theta = 83.7°$$

Figure 1.33d

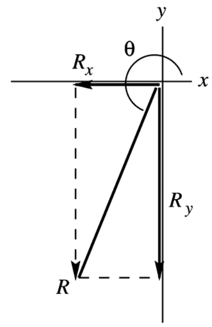
EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + (-\vec{B})$ is



$\vec{R}$ is in the 1st quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

Figure 1.33e

(c) EXECUTE:



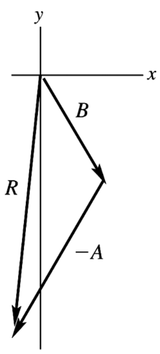
$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$

$\vec{B} - \vec{A}$ and $\vec{A} - \vec{B}$ are equal in magnitude and opposite in direction.

$R = 4.09$ cm and $\theta = 83.7° + 180° = 264°$

Figure 1.33f

EVALUATE: The vector addition diagram for $\vec{R} = \vec{B} + (-\vec{A})$ is



$\vec{R}$ is in the 3rd quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

Figure 1.33g

1.34. **IDENTIFY:** The general expression for a vector written in terms of components and unit vectors is $\vec{A} = A_x\hat{i} + A_y\hat{j}$.

SET UP: $5.0\vec{B} = 5.0(4\hat{i} - 6\hat{j}) = 20\vec{i} - 30\vec{j}$

EXECUTE: **(a)** $A_x = 5.0,\ A_y = -6.3$ **(b)** $A_x = 11.2,\ A_y = -9.91$ **(c)** $A_x = -15.0,\ A_y = 22.4$

(d) $A_x = 20,\ A_y = -30$

EVALUATE: The components are signed scalars.

1.35. **IDENTIFY:** Find the components of each vector and then use the general equation $\vec{A} = A_x\hat{i} + A_y\hat{j}$ for a vector in terms of its components and unit vectors.

SET UP: $A_x = 0,\ A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m. $D_x = -7.99$ m, $D_y = 6.02$ m.

EXECUTE: $\vec{A} = (-8.00\text{ m})\hat{j}$; $\vec{B} = (7.50\text{ m})\hat{i} + (13.0\text{ m})\hat{j}$; $\vec{C} = (-10.9\text{ m})\hat{i} + (-5.07\text{ m})\hat{j}$; $\vec{D} = (-7.99\text{ m})\hat{i} + (6.02\text{ m})\hat{j}$.

EVALUATE: All these vectors lie in the *xy*-plane and have no *z*-component.

1.36. **IDENTIFY:** Find A and B. Find the vector difference using components.

SET UP: Identify the *x*- and *y*-components and use $A = \sqrt{A_x^2 + A_y^2}$.

EXECUTE: **(a)** $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$; $A_x = +4.00$; $A_y = +7.00$.

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(4.00)^2 + (7.00)^2} = 8.06$. $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $B_x = +5.00$; $B_y = -2.00$;

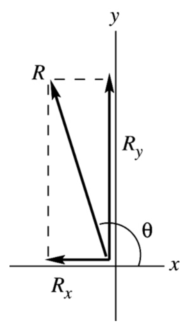
$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.00)^2 + (-2.00)^2} = 5.39$.

EVALUATE: Note that the magnitudes of $\vec{A}$ and $\vec{B}$ are each larger than either of their components.

EXECUTE: **(b)** $\vec{A} - \vec{B} = 4.00\hat{i} + 7.00\hat{j} - (5.00\hat{i} - 2.00\hat{j}) = (4.00 - 5.00)\hat{i} + (7.00 + 2.00)\hat{j}$.

$\vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$

(c) Let $\vec{R} = \vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$. Then $R_x = -1.00$, $R_y = 9.00$.



$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{(-1.00)^2 + (9.00)^2} = 9.06$.

$\tan\theta = \frac{R_y}{R_x} = \frac{9.00}{-1.00} = -9.00$

$\theta = -83.6° + 180° = 96.3°$.

Figure 1.36

EVALUATE: $R_x < 0$ and $R_y > 0$, so $\vec{R}$ is in the 2nd quadrant.

1.37. **IDENTIFY:** Use trigonometry to find the components of each vector. Use $R_x = A_x + B_x + \cdots$ and $R_y = A_y + B_y + \cdots$ to find the components of the vector sum. The equation $\vec{A} = A_x\hat{i} + A_y\hat{j}$ expresses a vector in terms of its components.

SET UP: Use the coordinates in the figure that accompanies the problem.

EXECUTE: **(a)** $\vec{A} = (3.60\text{ m})\cos 70.0°\hat{i} + (3.60\text{ m})\sin 70.0°\hat{j} = (1.23\text{ m})\hat{i} + (3.38\text{ m})\hat{j}$

$\vec{B} = -(2.40\text{ m})\cos 30.0°\hat{i} - (2.40\text{ m})\sin 30.0°\hat{j} = (-2.08\text{ m})\hat{i} + (-1.20\text{ m})\hat{j}$

(b) $\vec{C} = (3.00)\vec{A} - (4.00)\vec{B} = (3.00)(1.23\text{ m})\hat{i} + (3.00)(3.38\text{ m})\hat{j} - (4.00)(-2.08\text{ m})\hat{i} - (4.00)(-1.20\text{ m})\hat{j}$

$\vec{C} = (12.01\text{ m})\hat{i} + (14.94\text{ m})\hat{j}$

(c) From $A = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = \frac{A_y}{A_x}$,

$C = \sqrt{(12.01\text{ m})^2 + (14.94\text{ m})^2} = 19.17\text{ m}$, $\arctan\left(\frac{14.94\text{ m}}{12.01\text{ m}}\right) = 51.2°$

EVALUATE: C_x and C_y are both positive, so θ is in the first quadrant.

1.38. **IDENTIFY:** We use the vector components and trigonometry to find the angles.

SET UP: Use the fact that $\tan\theta = A_y / A_x$.

EXECUTE: **(a)** $\tan\theta = A_y / A_x = \frac{6.00}{-3.00}$. $\theta = 117°$ with the $+x$-axis.

(b) $\tan\theta = B_y / B_x = \frac{2.00}{7.00}$. $\theta = 15.9°$.

(c) First find the components of $\vec{C}$. $C_x = A_x + B_x = -3.00 + 7.00 = 4.00,$

$C_y = A_y + B_y = 6.00 + 2.00 = 8.00$

$\tan\theta = C_y / C_x = \dfrac{8.00}{4.00} = 2.00$. $\theta = 63.4°$

EVALUATE: Sketching each of the three vectors to scale will show that the answers are reasonable.

1.39. **IDENTIFY:** $\vec{A}$ and $\vec{B}$ are given in unit vector form. Find A, B and the vector difference $\vec{A}-\vec{B}$.

SET UP: $\vec{A} = -2.00\hat{i} + 3.00\hat{j} + 4.00\hat{k}$, $\vec{B} = 3.00\hat{i} + 1.00\hat{j} - 3.00\hat{k}$

Use $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ to find the magnitudes of the vectors.

EXECUTE: **(a)** $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(-2.00)^2 + (3.00)^2 + (4.00)^2} = 5.38$

$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(3.00)^2 + (1.00)^2 + (-3.00)^2} = 4.36$

(b) $\vec{A}-\vec{B} = (-2.00\hat{i} + 3.00\hat{j} + 4.00\hat{k}) - (3.00\hat{i} + 1.00\hat{j} - 3.00\hat{k})$

$\vec{A}-\vec{B} = (-2.00-3.00)\hat{i} + (3.00-1.00)\hat{j} + (4.00-(-3.00))\hat{k} = -5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k}.$

(c) Let $\vec{C} = \vec{A}-\vec{B}$, so $C_x = -5.00,\ C_y = +2.00,\ C_z = +7.00$

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-5.00)^2 + (2.00)^2 + (7.00)^2} = 8.83$$

$\vec{B}-\vec{A} = -(\vec{A}-\vec{B})$, so $\vec{A}-\vec{B}$ and $\vec{B}-\vec{A}$ have the same magnitude but opposite directions.

EVALUATE: A, B, and C are each larger than any of their components.

1.40. **IDENTIFY:** Target variables are $\vec{A}\cdot\vec{B}$ and the angle ϕ between the two vectors.

SET UP: We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the scalar product using $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$. The angle ϕ can then be found from $\vec{A}\cdot\vec{B} = AB\cos\phi$.

EXECUTE: **(a)** $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $A = 8.06$, $B = 5.39$.

$\vec{A}\cdot\vec{B} = (4.00\hat{i} + 7.00\hat{j})\cdot(5.00\hat{i} - 2.00\hat{j}) = (4.00)(5.00) + (7.00)(-2.00) = 20.0 - 14.0 = +6.00.$

(b) $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{6.00}{(8.06)(5.39)} = 0.1382;\ \phi = 82.1°.$

EVALUATE: The component of $\vec{B}$ along $\vec{A}$ is in the same direction as $\vec{A}$, so the scalar product is positive and the angle ϕ is less than $90°$.

1.41. **IDENTIFY:** $\vec{A}\cdot\vec{B} = AB\cos\phi$

SET UP: For $\vec{A}$ and $\vec{B}$, $\phi = 150.0°$. For $\vec{B}$ and $\vec{C}$, $\phi = 145.0°$. For $\vec{A}$ and $\vec{C}$, $\phi = 65.0°$.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = (8.00\text{ m})(15.0\text{ m})\cos 150.0° = -104\text{ m}^2$

(b) $\vec{B}\cdot\vec{C} = (15.0\text{ m})(12.0\text{ m})\cos 145.0° = -148\text{ m}^2$

(c) $\vec{A}\cdot\vec{C} = (8.00\text{ m})(12.0\text{ m})\cos 65.0° = 40.6\text{ m}^2$

EVALUATE: When $\phi < 90°$ the scalar product is positive and when $\phi > 90°$ the scalar product is negative.

1.42. **IDENTIFY:** Target variable is the vector $\vec{A}\times\vec{B}$ expressed in terms of unit vectors.

SET UP: We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the vector product using $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = 0$, $\hat{i}\times\hat{j} = \hat{k}$, and $\hat{j}\times\hat{i} = -\hat{k}$.

EXECUTE: $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$.

$\vec{A}\times\vec{B} = (4.00\hat{i} + 7.00\hat{j})\times(5.00\hat{i} - 2.00\hat{j}) = 20.0\hat{i}\times\hat{i} - 8.00\hat{i}\times\hat{j} + 35.0\hat{j}\times\hat{i} - 14.0\hat{j}\times\hat{j}$. But $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = 0$ and $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{i} = -\hat{k}$, so $\vec{A}\times\vec{B} = -8.00\hat{k} + 35.0(-\hat{k}) = -43.0\hat{k}$. The magnitude of $\vec{A}\times\vec{B}$ is 43.0.

EVALUATE: Sketch the vectors $\vec{A}$ and $\vec{B}$ in a coordinate system where the *xy*-plane is in the plane of the paper and the *z*-axis is directed out toward you. By the right-hand rule $\vec{A}\times\vec{B}$ is directed into the plane of the paper, in the $-z$-direction. This agrees with the above calculation that used unit vectors.

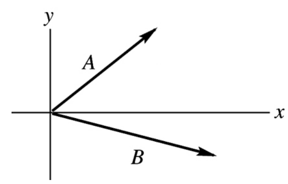


Figure 1.42

1.43. **IDENTIFY:** For all of these pairs of vectors, the angle is found from combining $\vec{A}\cdot\vec{B} = AB\cos\phi$ and $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$, to give the angle ϕ as $\phi = \arccos\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right) = \arccos\left(\dfrac{A_xB_x + A_yB_y}{AB}\right)$.

SET UP: $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$ shows how to obtain the components for a vector written in terms of unit vectors.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = -22$, $A = \sqrt{40}$, $B = \sqrt{13}$, and so $\phi = \arccos\left(\dfrac{-22}{\sqrt{40}\sqrt{13}}\right) = 165°$.

(b) $\vec{A}\cdot\vec{B} = 60$, $A = \sqrt{34}$, $B = \sqrt{136}$, $\phi = \arccos\left(\dfrac{60}{\sqrt{34}\sqrt{136}}\right) = 28°$.

(c) $\vec{A}\cdot\vec{B} = 0$ and $\phi = 90°$.

EVALUATE: If $\vec{A}\cdot\vec{B} > 0$, $0 \le \phi < 90°$. If $\vec{A}\cdot\vec{B} < 0$, $90° < \phi \le 180°$. If $\vec{A}\cdot\vec{B} = 0$, $\phi = 90°$ and the two vectors are perpendicular.

1.44. **IDENTIFY:** The right-hand rule gives the direction and $|\vec{A}\times\vec{B}| = AB\sin\phi$ gives the magnitude.

SET UP: $\phi = 120.0°$.

EXECUTE: **(a)** The direction of $\vec{A}\times\vec{B}$ is into the page (the $-z$-direction). The magnitude of the vector product is $AB\sin\phi = (2.80\text{ cm})(1.90\text{ cm})\sin 120° = 4.61\text{ cm}^2$.

(b) Rather than repeat the calculations, $\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$ may be used to see that $\vec{B}\times\vec{A}$ has magnitude 4.61 cm^2 and is in the $+z$-direction (out of the page).

EVALUATE: For part (a) we could use the components of the cross product and note that the only non-vanishing component is $C_z = A_xB_y - A_yB_x = (2.80\text{ cm})\cos 60.0°(-1.90\text{ cm})\sin 60°$

$$-(2.80\text{ cm})\sin 60.0°(1.90\text{ cm})\cos 60.0° = -4.61\text{ cm}^2.$$

This gives the same result.

1.45. **IDENTIFY:** $\vec{A}\times\vec{D}$ has magnitude $AD\sin\phi$. Its direction is given by the right-hand rule.

SET UP: $\phi = 180° - 53° = 127°$

EXECUTE: **(a)** $|\vec{A}\times\vec{D}| = (8.00\text{ m})(10.0\text{ m})\sin 127° = 63.9\text{ m}^2$. The right-hand rule says $\vec{A}\times\vec{D}$ is in the $-z$-direction (into the page).

(b) $\vec{D}\times\vec{A}$ has the same magnitude as $\vec{A}\times\vec{D}$ and is in the opposite direction.

EVALUATE: The component of $\vec{D}$ perpendicular to $\vec{A}$ is $D_{\perp} = D\sin 53.0° = 7.99$ m.
$|\vec{A}\times\vec{D}| = AD_{\perp} = 63.9\text{ m}^2$, which agrees with our previous result.

1.46. **IDENTIFY:** Apply Eqs. (1.16) and (1.20).
SET UP: The angle between the vectors is $20° + 90° + 30° = 140°$.
EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = AB\cos\phi$ gives $\vec{A}\cdot\vec{B} = (3.60\text{ m})(2.40\text{ m})\cos 140° = -6.62\text{ m}^2$.
(b) From $|\vec{A}\times\vec{B}| = AB\sin\phi$, the magnitude of the cross product is $(3.60\text{ m})(2.40\text{ m})\sin 140° = 5.55\text{ m}^2$ and the direction, from the right-hand rule, is out of the page (the $+z$-direction).
EVALUATE: We could also use $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$ and the cross product, with the components of $\vec{A}$ and $\vec{B}$.

1.47. **IDENTIFY:** This problem involves the vector product of two vectors.
SET UP: The magnitude is $\left|\vec{A}\times\vec{B}\right| = AB\sin\phi$ and the right-hand rule gives the direction. Since $\vec{A}\times\vec{B}$ is in the $+z$ direction, both $\vec{A}$ and $\vec{B}$ must lie in the xy-plane.
EXECUTE: $\vec{A}$ has no y-component and $\vec{B}$ has no x-component, so they must be perpendicular to each other. Since $\vec{A}\times\vec{B}$ is in the $+z$ direction, the right-hand rule tells us that $\vec{B}$ must point in the $-y$ direction. $\left|\vec{A}\times\vec{B}\right| = AB\sin\phi = (8.0\text{ m})B\sin 90° = 16.0\text{ m}^2$, so $B = 2.0$ m.
EVALUATE: In unit vector notation, $\vec{B} = -2.0\text{ m }\hat{j}$.

1.48. **IDENTIFY:** This problem involves the vector product and the scalar of two vectors.
SET UP: The scalar product is $\vec{A}\cdot\vec{B} = AB\cos\phi$ and the magnitude of the vector product is $\left|\vec{A}\times\vec{B}\right| = AB\sin\phi$.
EXECUTE: **(a)** Calculate both products. $\left|\vec{A}\times\vec{B}\right| = AB\sin\phi = AB\sin 30.0° = 0.500\,AB$ and $\vec{A}\cdot\vec{B} = AB\cos\phi = AB\cos 30.0° = 0.866\,AB$. Therefore the scalar product has the greater magnitude.
(b) Equate the magnitudes. $AB\sin\phi = AB\cos\phi \rightarrow \tan\phi = 1 \rightarrow \phi = 45°$ or $135°$. At 45° both products are positive, but at 135° the scalar product is negative. However in both cases the *magnitudes* are the same.
EVALUATE: Note that the problem says that the *magnitudes* of the products are equal. We *cannot* say that the products are equal because $\vec{A}\cdot\vec{B}$ is a scalar but $\vec{A}\times\vec{B}$ is a vector.

1.49. **IDENTIFY:** We model the earth, white dwarf, and neutron star as spheres. Density is mass divided by volume.
SET UP: We know that density = mass/volume = m/V where $V = \frac{4}{3}\pi r^3$ for a sphere. From Appendix B, the earth has mass of $m = 5.97\times10^{24}$ kg and a radius of $r = 6.37\times10^6$ m whereas for the sun at the end of its lifetime, $m = 1.99\times10^{30}$ kg and $r = 7500\text{ km} = 7.5\times10^6$ m. The star possesses a radius of $r = 10\text{ km} = 1.0\times10^4$ m and a mass of $m = 1.99\times10^{30}$ kg.
EXECUTE: **(a)** The earth has volume $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6.37\times10^6\text{ m})^3 = 1.0827\times10^{21}\text{ m}^3$. Its density is

$$\text{density} = \frac{m}{V} = \frac{5.97\times10^{24}\text{ kg}}{1.0827\times10^{21}\text{ m}^3} = (5.51\times10^3\text{ kg/m}^3)\left(\frac{10^3\text{ g}}{1\text{ kg}}\right)\left(\frac{1\text{ m}}{10^2\text{ cm}}\right)^3 = 5.51\text{ g/cm}^3$$

(b) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7.5\times10^6\text{ m})^3 = 1.77\times10^{21}\text{ m}^3$

$$\text{density} = \frac{m}{V} = \frac{1.99\times10^{30}\text{ kg}}{1.77\times10^{21}\text{ m}^3} = (1.1\times10^9\text{ kg/m}^3)\left(\frac{1\text{ g/cm}^3}{1000\text{ kg/m}^3}\right) = 1.1\times10^6\text{ g/cm}^3$$

(c) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0\times10^4 \text{ m})^3 = 4.19\times10^{12} \text{ m}^3$

$$\text{density} = \frac{m}{V} = \frac{1.99\times10^{30} \text{ kg}}{4.19\times10^{12} \text{ m}^3} = (4.7\times10^{17} \text{ kg/m}^3)\left(\frac{1 \text{ g/cm}^3}{1000 \text{ kg/m}^3}\right) = 4.7\times10^{14} \text{ g/cm}^3$$

EVALUATE: For a fixed mass, the density scales as $1/r^3$. Thus, the answer to (c) can also be obtained from (b) as

$$(1.1\times10^6 \text{ g/cm}^3)\left(\frac{7.50\times10^6 \text{ m}}{1.0\times10^4 \text{ m}}\right)^3 = 4.7\times10^{14} \text{ g/cm}^3.$$

1.50. **IDENTIFY** and **SET UP:** Unit conversion.

EXECUTE: **(a)** $f = 1.420\times10^9$ cycles/s, so $\dfrac{1}{1.420\times10^9}$ s $= 7.04\times10^{-10}$ s for one cycle.

(b) $\dfrac{3600 \text{ s/h}}{7.04\times10^{-10} \text{ s/cycle}} = 5.11\times10^{12}$ cycles/h

(c) Calculate the number of seconds in 4600 million years $= 4.6\times10^9$ y and divide by the time for 1 cycle:

$$\frac{(4.6\times10^9 \text{ y})(3.156\times10^7 \text{ s/y})}{7.04\times10^{-10} \text{ s/cycle}} = 2.1\times10^{26} \text{ cycles}$$

(d) The clock is off by 1 s in $100{,}000 \text{ y} = 1\times10^5$ y, so in 4.60×10^9 y it is off by

$(1 \text{ s})\left(\dfrac{4.60\times10^9}{1\times10^5}\right) = 4.6\times10^4$ s (about 13 h).

EVALUATE: In each case the units in the calculation combine algebraically to give the correct units for the answer.

1.51. **IDENTIFY:** The density relates mass and volume. Use the given mass and density to find the volume and from this the radius.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg and radius $r_E = 6.37\times10^6$ m. The volume of a sphere is $V = \frac{4}{3}\pi r^3$. $\rho = 1.76 \text{ g/cm}^3 = 1760 \text{ km/m}^3$.

EXECUTE: **(a)** The planet has mass $m = 5.5m_E = 3.28\times10^{25}$ kg.

$V = \dfrac{m}{\rho} = \dfrac{3.28\times10^{25} \text{ kg}}{1760 \text{ kg/m}^3} = 1.86\times10^{22} \text{ m}^3.$

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3[1.86\times10^{22} \text{ m}^3]}{4\pi}\right)^{1/3} = 1.64\times10^7 \text{ m} = 1.64\times10^4 \text{ km}$$

(b) $r = 2.57r_E$

EVALUATE: Volume V is proportional to mass and radius r is proportional to $V^{1/3}$, so r is proportional to $m^{1/3}$. If the planet and earth had the same density its radius would be $(5.5)^{1/3}r_E = 1.8r_E$. The radius of the planet is greater than this, so its density must be less than that of the earth.

1.52. **IDENTIFY:** Use the extreme values in the piece's length and width to find the uncertainty in the area.

SET UP: The length could be as large as 7.61 cm and the width could be as large as 1.91 cm.

EXECUTE: **(a)** The area is $14.44 \pm 0.095 \text{ cm}^2$.

(b) The fractional uncertainty in the area is $\dfrac{0.095\ \text{cm}^2}{14.44\ \text{cm}^2} = 0.66\%,$ and the fractional uncertainties in the length and width are $\dfrac{0.01\ \text{cm}}{7.61\ \text{cm}} = 0.13\%$ and $\dfrac{0.01\ \text{cm}}{1.9\ \text{cm}} = 0.53\%.$ The sum of these fractional uncertainties is $0.13\% + 0.53\% = 0.66\%,$ in agreement with the fractional uncertainty in the area.

EVALUATE: The fractional uncertainty in a product of numbers is greater than the fractional uncertainty in any of the individual numbers.

1.53. **IDENTIFY:** The number of atoms is your mass divided by the mass of one atom.
SET UP: Assume a 70-kg person and that the human body is mostly water. Use Appendix D to find the mass of one H_2O molecule: $18.015\ \text{u} \times 1.661 \times 10^{-27}\ \text{kg/u} = 2.992 \times 10^{-26}\ \text{kg/molecule}.$

EXECUTE: $(70\ \text{kg})/(2.992 \times 10^{-26}\ \text{kg/molecule}) = 2.34 \times 10^{27}$ molecules. Each H_2O molecule has 3 atoms, so there are about 6×10^{27} atoms.

EVALUATE: Assuming carbon to be the most common atom gives 3×10^{27} molecules, which is a result of the same order of magnitude.

1.54. **IDENTIFY:** Estimate the volume of each object. The mass m is the density times the volume.
SET UP: The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3.$ The volume of a cylinder of radius r and length l is $V = \pi r^2 l.$ The density of water is $1000\ \text{kg/m}^3.$

EXECUTE: **(a)** Estimate the volume as that of a sphere of diameter 10 cm: $V = 5.2 \times 10^{-4}\ \text{m}^3.$
$m = (0.98)(1000\ \text{kg / m}^3)(5.2 \times 10^{-4}\ \text{m}^3) = 0.5\ \text{kg}.$

(b) Approximate as a sphere of radius $r = 0.25\,\mu\text{m}$ (probably an overestimate): $V = 6.5 \times 10^{-20}\ \text{m}^3.$
$m = (0.98)(1000\ \text{kg / m}^3)(6.5 \times 10^{-20}\ \text{m}^3) = 6 \times 10^{-17}\ \text{kg} = 6 \times 10^{-14}\ \text{g}.$

(c) Estimate the volume as that of a cylinder of length 1 cm and radius 3 mm: $V = \pi r^2 l = 2.8 \times 10^{-7}\ \text{m}^3.$
$m = (0.98)(1000\ \text{kg/m}^3)(2.8 \times 10^{-7}\ \text{m}^3) = 3 \times 10^{-4}\ \text{kg} = 0.3\ \text{g}.$

EVALUATE: The mass is directly proportional to the volume.

1.55. **IDENTIFY:** We are dealing with unit vectors, which must have magnitude 1. We will need to use the scalar product and to express vectors using the unit vectors.
SET UP: $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$, $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$. If two vectors are perpendicular, their scalar product is zero.
EXECUTE: **(a)** If we divide a vector by its magnitude, the result will have magnitude 1 but still point in the same direction as the original vector, so it will be a unit vector. First find the magnitude of the given vector. $A = \sqrt{A_x^2 + A_z^2} = \sqrt{(3.0)^2 + (-4.0)^2} = 5.0$. Therefore $\dfrac{3.0\hat{\boldsymbol{i}} - 4.0\hat{\boldsymbol{k}}}{5} = 0.60\,\hat{\boldsymbol{i}} - 0.80\,\hat{\boldsymbol{k}}$ is a unit vector that is parallel to $\vec{A}$.
(b) Reversing the direction of the unit vector in (a) will make it antiparallel to $\vec{A}$, so the unit vector is $-0.60\,\hat{\boldsymbol{i}} + 0.80\,\hat{\boldsymbol{k}}$.
(c) Call $\vec{B}$ the unknown unit vector. Since it has no y-component, we can express it as $\vec{B} = B_x\hat{\boldsymbol{i}} + B_z\hat{\boldsymbol{k}}$. Since $\vec{A}$ and $\vec{B}$ are perpendicular, $\vec{A} \cdot \vec{B} = 0$, so $A_x B_x + A_y B_y + A_z B_z = 0$. This gives $(3.0)B_x - (4.0)B_z = 0 \ \rightarrow\ B_z = 0.75\,B_x$. Since $\vec{B}$ is a unit vector, we have $B_x^2 + B_z^2 = B_x^2 + \left(0.75 B_x\right)^2 = 1$. Solving gives $B_x = \pm 0.80$. Therefore $B_z = \pm(0.75\,B_x) = \pm(0.75)(0.80) = \pm 0.60$. Therefore $\vec{B} = \pm(0.80\,\hat{\boldsymbol{i}} + 0.60\,\hat{\boldsymbol{k}}$), so the two unit vectors are

$\vec{B}_+ = 0.80\,\hat{i} + 0.60\,\hat{k}$ and $\vec{B}_- = -0.80\,\hat{i} - 0.60\,\hat{k}$.

EVALUATE: $\vec{A}\Box\vec{B}_+ = (3.0)(0.80) + (-4.0)(6.0) = 0$ and $\vec{A}\cdot\vec{B}_- = (3.0)(-0.80) + (-4.0)(-6.0) = 0$, so the two vectors are perpendicular to $\vec{A}$. Their magnitudes are $\sqrt{(\pm 0.80)^2 + (\pm 0.60)^2} = 1$, so they are unit vectors.

1.56. **IDENTIFY:** Let $\vec{D}$ be the fourth force. Find $\vec{D}$ such that $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$, so $\vec{D}=-(\vec{A}+\vec{B}+\vec{C})$.

SET UP: Use components and solve for the components D_x and D_y of $\vec{D}$.

EXECUTE: $A_x = +A\cos 30.0° = +86.6\text{ N}$, $A_y = +A\sin 30.0° = +50.00\text{ N}$.

$B_x = -B\sin 30.0° = -40.00\text{ N}$, $B_y = +B\cos 30.0° = +69.28\text{ N}$.

$C_x = -C\cos 53.0° = -24.07\text{ N}$, $C_y = -C\sin 53.0° = -31.90\text{ N}$.

Then $D_x = -22.53$ N, $D_y = -87.34$ N and $D = \sqrt{D_x^2 + D_y^2} = 90.2$ N. $\tan\alpha = |D_y/D_x| = 87.34/22.53$.

$\alpha = 75.54°$. $\phi = 180° + \alpha = 256°$, counterclockwise from the $+x$-axis.

EVALUATE: As shown in Figure 1.56, since D_x and D_y are both negative, $\vec{D}$ must lie in the third quadrant.

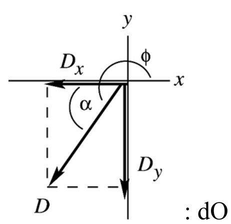


Figure 1.56

1.57. **IDENTIFY:** Vector addition. Target variable is the 4th displacement.

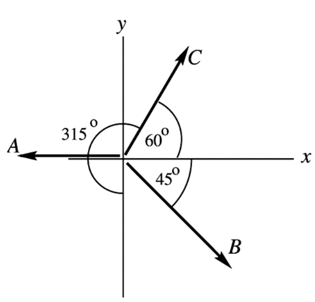
SET UP: Use a coordinate system where east is in the $+x$-direction and north is in the $+y$-direction.

Let $\vec{A}$, $\vec{B}$, and $\vec{C}$ be the three displacements that are given and let $\vec{D}$ be the fourth unmeasured displacement. Then the resultant displacement is $\vec{R} = \vec{A}+\vec{B}+\vec{C}+\vec{D}$. And since she ends up back where she started, $\vec{R} = 0$.

$0 = \vec{A}+\vec{B}+\vec{C}+\vec{D}$, so $\vec{D} = -(\vec{A}+\vec{B}+\vec{C})$

$D_x = -(A_x + B_x + C_x)$ and $D_y = -(A_y + B_y + C_y)$

EXECUTE:



$A_x = -180$ m, $A_y = 0$

$B_x = B\cos 315° = (210\text{ m})\cos 315° = +148.5$ m

$B_y = B\sin 315° = (210\text{ m})\sin 315° = -148.5$ m

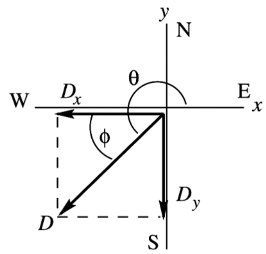
$C_x = C\cos 60° = (280\text{ m})\cos 60° = +140$ m

$C_y = C\sin 60° = (280\text{ m})\sin 60° = +242.5$ m

Figure 1.57a

$D_x = -(A_x + B_x + C_x) = -(-180\text{ m} + 148.5\text{ m} + 140\text{ m}) = -108.5$ m

$D_y = -(A_y + B_y + C_y) = -(0 - 148.5\text{ m} + 242.5\text{ m}) = -94.0$ m



$$D = \sqrt{D_x^2 + D_y^2}$$

$$D = \sqrt{(-108.5\text{ m})^2 + (-94.0\text{ m})^2} = 144\text{ m}$$

$$\tan\theta = \frac{D_y}{D_x} = \frac{-94.0\text{ m}}{-108.5\text{ m}} = 0.8664$$

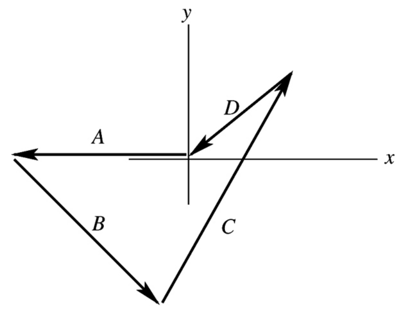
$$\theta = 180° + 40.9° = 220.9°$$

($\vec{D}$ is in the third quadrant since both D_x and D_y are negative.)

Figure 1.57b

The direction of $\vec{D}$ can also be specified in terms of $\phi = \theta - 180° = 40.9°$; $\vec{D}$ is $41°$ south of west.

EVALUATE: The vector addition diagram, approximately to scale, is



Vector $\vec{D}$ in this diagram agrees qualitatively with our calculation using components.

Figure 1.57c

1.58. **IDENTIFY:** Find the vector sum of the two displacements.

SET UP: Call the two displacements $\vec{A}$ and $\vec{B}$, where $A = 170$ km and $B = 230$ km. $\vec{A} + \vec{B} = \vec{R}$. $\vec{A}$ and $\vec{B}$ are as shown in Figure 1.58.

EXECUTE: $R_x = A_x + B_x = (170\text{ km})\sin 68° + (230\text{ km})\cos 36° = 343.7$ km.

$R_y = A_y + B_y = (170\text{ km})\cos 68° - (230\text{ km})\sin 36° = -71.5$ km.

a $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(343.7\text{ km})^2 + (-71.5\text{ km})^2} = 351$ km. $\tan\theta_R = \left|\frac{R_y}{R_x}\right| = \frac{71.5\text{ km}}{343.7\text{ km}} = 0.208$.

$\theta_R = 11.8°$ south of east.

EVALUATE: Our calculation using components agrees with $\vec{R}$ shown in the vector addition diagram, Figure 1.58.

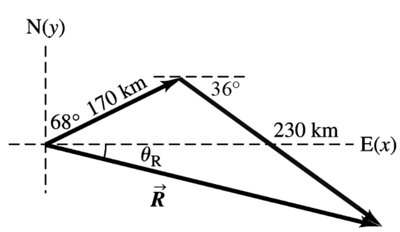


Figure 1.58

1.59. **IDENTIFY:** This problem requires vector addition. We can find the components of the given vectors and then use them to find the magnitude and direction of the resultant vector.

SET UP: $A_x = A\cos\theta$, $A_y = A\sin\theta$, $\theta = \arctan\dfrac{A_y}{A_x}$, $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$, $R_x = A_x + B_x$, and $R_y = A_y + B_y$. Sketch the given vectors to help find the components (see Fig. 1.59).

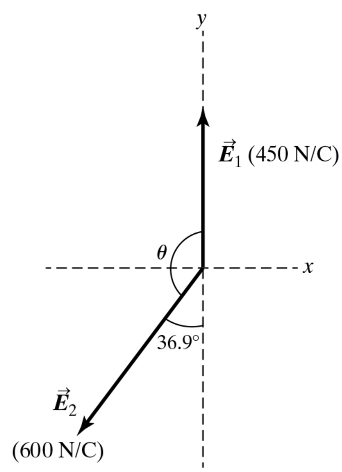


Figure 1.59

EXECUTE: From Fig. 1.59 we can see that the component s are

$E_{1x} = 0$ and $E_{1y} = 450$ N/C

$E_{2x} = E_2\cos\theta = (600\text{ N/C})\cos 233.1° = -360.25$ N/C

$E_{2y} = E_2\sin\theta = (600\text{ N/C})\sin 233.1° = -479.81$ N/C.

Now find the components of the resultant field:

$E_x = E_{1x} + E_{2x} = 0 + (-360.25\text{ N/C}) = -360.25$ N/C

$E_y = E_{1y} + E_{2y} = 450\text{ N/C} + (-479.81\text{ N/C}) = -29.81$ N/C

Now find the magnitude and direction of $\vec{E}$:

$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-360.25\text{ N/C})^2 + (-29.81\text{ N/C})^2} = 361$ N/C.

$\theta = \arctan\dfrac{A_y}{A_x} = \theta = \arctan\left(\dfrac{-29.81\text{ N/C}}{-360.25\text{ N/C}}\right) = 4.73°$. Both components of $\vec{E}$ are negative, so it must point into the third quadrant. Therefore the angle below the $-x$-axis is 4.73°. The angle with the $+x$-axis is 180° + 4.73° = 184.73°.

EVALUATE: Make a careful graphical sum to check your answer.

1.60. **IDENTIFY:** Solve for one of the vectors in the vector sum. Use components.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are:

$\vec{A} = 2.00$ km, 0°of east; $\vec{B} = 3.50$ m, 45° south of east; and $\vec{R} = 5.80$ m, 0° east

EXECUTE: $C_x = R_x - A_x - B_x = 5.80\text{ km} - (2.00\text{ km}) - (3.50\text{ km})(\cos 45°) = 1.33$ km;

$C_y = R_y - A_y - B_y$

$= 0\text{ km} - 0\text{ km} - (-3.50\text{ km})(\sin 45°) = 2.47$ km; $C = \sqrt{(1.33\text{ km})^2 + (2.47\text{ km})^2} = 2.81$ km;

$\theta = \tan^{-1}[(2.47\text{ km})/(1.33\text{ km})] = 61.7°$ north of east. The vector addition diagram in Figure 1.60 shows good qualitative agreement with these values.

EVALUATE: The third leg lies in the first quadrant since its x and y components are both positive.

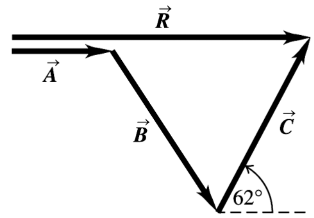


Figure 1.60

1.61. **IDENTIFY:** We know the resultant of two forces of known equal magnitudes and want to find that magnitude (the target variable).

SET UP: Use coordinates having a horizontal $+x$ axis and an upward $+y$ axis. Then $A_x + B_x = R_x$ and $R_x = 12.8\text{ N}$.

SOLVE: $A_x + B_x = R_x$ and $A\cos 32° + B\sin 32° = R_x$. Since $A = B$,

$2A\cos 32° = R_x$, so $A = \dfrac{R_x}{(2)(\cos 32°)} = 7.55\text{ N}$.

EVALUATE: The magnitude of the x component of each pull is 6.40 N, so the magnitude of each pull (7.55 N) is greater than its x component, as it should be.

1.62. **IDENTIFY:** The four displacements return her to her starting point, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$, where $\vec{A}$, $\vec{B}$, and $\vec{C}$ are in the three given displacements and $\vec{D}$ is the displacement for her return.

SET UP: Let $+x$ be east and $+y$ be north.

EXECUTE: **(a)** $D_x = -[(147\text{ km})\sin 85° + (106\text{ km})\sin 167° + (166\text{ km})\sin 235°] = -34.3\text{ km}$.

$D_y = -[(147\text{ km})\cos 85° + (106\text{ km})\cos 167° + (166\text{ km})\cos 235°] = +185.7\text{ km}$.

$D = \sqrt{(-34.3\text{ km})^2 + (185.7\text{ km})^2} = 189\text{ km}$.

(b) The direction relative to north is $\phi = \arctan\left(\dfrac{34.3\text{ km}}{185.7\text{ km}}\right) = 10.5°$. Since $D_x < 0$ and $D_y > 0$, the direction of $\vec{D}$ is $10.5°$ west of north.

EVALUATE: The four displacements add to zero.

1.63. **IDENTIFY:** We have two known vectors and a third unknown vector, and we know the resultant of these three vectors.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are:

$\vec{A} = 23.0\text{ km}$ at 34.0° south of east; $\vec{B} = 46.0\text{ km}$ due north; $\vec{R} = 32.0\text{ km}$ due west ; $\vec{C}$ is unknown.

EXECUTE: $C_x = R_x - A_x - B_x = -32.0\text{ km} - (23.0\text{ km})\cos 34.0° - 0 = -51.07\text{ km}$;

$C_y = R_y - A_y - B_y = 0 - (-23.0\text{ km})\sin 34.0° - 46.0\text{ km} = -33.14\text{ km}$;

$C = \sqrt{C_x^2 + C_y^2} = 60.9\text{ km}$

Calling θ the angle that $\vec{C}$ makes with the $-x$-axis (the westward direction), we have

$\tan\theta = C_y / C_x = \dfrac{33.14}{51.07}$; $\theta = 33.0°$ south of west.

EVALUATE: A graphical vector sum will confirm this result.

1.64. **IDENTIFY:** Let the three given displacements be $\vec{A}$, $\vec{B}$ and $\vec{C}$, where $A = 40$ steps, $B = 80$ steps and $C = 50$ steps. $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. The displacement $\vec{C}$ that will return him to his hut is $-\vec{R}$.

SET UP: Let the east direction be the $+x$-direction and the north direction be the $+y$-direction.

EXECUTE: **(a)** The three displacements and their resultant are sketched in Figure 1.64.

(b) $R_x = (40)\cos 45° - (80)\cos 60° = -11.7$ and $R_y = (40)\sin 45° + (80)\sin 60° - 50 = 47.6$.

The magnitude and direction of the resultant are $\sqrt{(-11.7)^2 + (47.6)^2} = 49$, acrtan$\left(\frac{47.6}{11.7}\right) = 76°$, north of west. We know that $\vec{R}$ is in the second quadrant because $R_x < 0$, $R_y > 0$. To return to the hut, the explorer must take 49 steps in a direction $76°$ south of east, which is $14°$ east of south.

EVALUATE: It is useful to show R_x, R_y, and $\vec{R}$ on a sketch, so we can specify what angle we are computing.

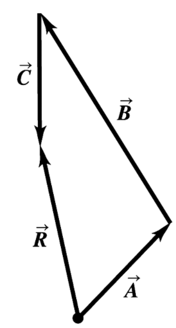


Figure 1.64

1.65. **IDENTIFY:** We want to find the resultant of three known displacement vectors: $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

SET UP: Let $+x$ be east and $+y$ be north and find the components of the vectors.

EXECUTE: The magnitudes are A = 20.8 m, B = 38.0 m, C = 18.0 m. The components are
$A_x = 0$, A_y = 28.0 m, B_x = 38.0 m, $B_y = 0$,
C_x = –(18.0 m)(sin33.0°) = –9.804 m, C_y = –(18.0 m)(cos33.0°) = –15.10 m
$R_x = A_x + B_x + C_x$ = 0 + 38.0 m + (–9.80 m) = 28.2 m
$R_y = A_y + B_y + C_y$ = 20.8 m + 0 + (–15.10 m) = 5.70 m
$R = \sqrt{R_x^2 + R_y^2}$ = 28.8 m is the distance you must run. Calling θ_R the angle the resultant makes with the $+x$-axis (the easterly direction), we have
tan $\theta_R = R_y/R_x$ = (5.70 km)/(28.2 km); θ_R = 11.4° north of east.

EVALUATE: A graphical sketch will confirm this result.

1.66. **IDENTIFY:** This is a problem in vector addition. We want the magnitude of the resultant of four known displacement vectors in three dimensions. We can use components to do this.

SET UP: $R_x = A_x + B_x + C_x + D_x$, and likewise for the other components. The magnitude is $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$. Call the $+x$-axis toward the east, the $+y$-axis toward the north, and the $+z$-axis vertically upward. First find the components of the four displacements, then use them to find the magnitude of the resultant displacement.

EXECUTE: Finding the components in the order in which the displacements are listed in the problem, the components of the four displacement vectors are A_x = –14.0 m, B_z = +22.0 m, C_y = +12.0 m, and D_x = 6.0 m. All the other components are zero. Now find the components of $\vec{R}$.

$R_x = -14.0\text{ m} + 6.0\text{ m} = -8.0\text{ m}$, $R_y = 12.0\text{ m}$, $R_z = 22.0\text{ m}$. Now find the resultant displacement using $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$. $R = \sqrt{(-8.0\text{ m})^2 + (12.0\text{ m})^2 + (22.0\text{ m})^2} = 26\text{ m}$.

EVALUATE: This is one case where a graphical solution would not be useful as a check since three-dimensional drawings are very difficult to visualize. Note that the answer has only 2 significant figures even though all the given numbers have 3 significant figures. The reason for this is that in the subtraction to find R_x we lost one significant figure because $-14.0 + 6.0 = 8.0$, which has only 2 significant figures.

1.67. **IDENTIFY:** We know the resultant of two vectors and one of the vectors, and we want to find the second vector.

SET UP: Let the westerly direction be the $+x$-direction and the northerly direction be the $+y$-direction. We also know that $\vec{R} = \vec{A} + \vec{B}$ where $\vec{R}$ is the vector from you to the truck. Your GPS tells you that you are 122.0 m from the truck in a direction of 58.0° east of south, so a vector from the truck to you is 122.0 m at 58.0° east of south. Therefore the vector from you to the truck is 122.0 m at 58.0° west of north. Thus $\vec{R}$ = 122.0 m at 58.0° west of north and $\vec{A}$ is 72.0 m due west. We want to find the magnitude and direction of vector $\vec{B}$.

EXECUTE: $B_x = R_x - A_x = (122.0\text{ m})(\sin 58.0°) - 72.0\text{ m} = 31.462\text{ m}$

$B_y = R_y - A_y = (122.0\text{ m})(\cos 58.0°) - 0 = 64.450\text{ m}$; $B = \sqrt{B_x^2 + B_y^2} = 71.9\text{ m}$.

$\tan\theta_B = B_y / B_x = \dfrac{64.650\text{ m}}{31.462\text{ m}} = 2.05486$; $\theta_B = 64.1°$ north of west.

EVALUATE: A graphical sum will show that the results are reasonable.

1.68. **IDENTIFY:** We are dealing with vector addition. We know the resultant vector is the same for both trips, but we take different displacements on two different days. It is best to use components.

SET UP: First make a clear sketch showing the displacement vectors on the two different days (see Fig. 1.68). Use $R_x = A_x + B_x$ and $R_y = A_y + B_y$ and $A = \sqrt{A_x^2 + A_y^2}$ for the magnitude of a vector. Let the x-axis be eastward the y-axis be toward the north.

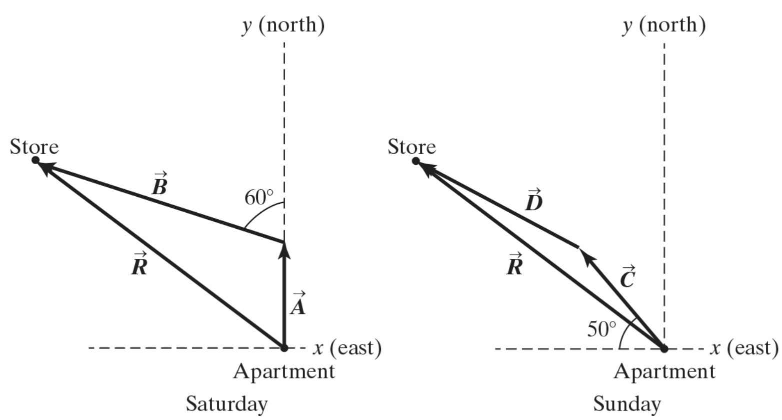


Figure 1.68

EXECUTE: **(a)** The magnitude R of the resultant is the distance to the store from your apartment. Using the Saturday trip, let $\vec{A}$ be the first drive and $\vec{B}$ be the second drive, so $\vec{A} + \vec{B} = \vec{R}$. First find the components of $\vec{R}$.

$R_x = A_x + B_x = 0 + (1.40 \text{ km}) \cos 150.0° = -1.212$ km

$R_y = A_y + B_y = 0.600 \text{ km} + (1.40 \text{ km}) \sin 150.0° = 1.30$ km.

Now find R: $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-1.212 \text{ km})^2 + (1.30 \text{ km})^2} = 1.78$ km.

(b) The distance traveled each day is *not* the magnitude of the resultant. Rather, it is the sum of the magnitudes of both displacement vectors for each trip. We know all of them except the second drive on the Sunday trip. Call $\vec{C}$ the first drive on Sunday and $\vec{D}$ the second Sunday drive. The resultant of these is the same as for the Saturday trip, so we can find the components of $\vec{D}$.

$C_x + D_x = R_x$: $(0.80 \text{ km}) \cos 130° + D_x = -1.212$ km $\rightarrow$ $D_x = -0.6978$ km

$C_y + D_y = R_y$: $(0.80 \text{ km}) \sin 130° + D_y = 1.30$ km $\rightarrow$ $D_y = 0.6872$ km.

Now find the magnitude of $\vec{D}$: $D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-0.6978 \text{ km})^2 + (0.6872 \text{ km})^2} = 0.9793$ km.

The distances driven on the two days are

Saturday: 0.60 km + 1.40 km = 2.00 km

Sunday: 0.80 km + 0.9793 km = 1.7793 km

The difference in distance is 2.00 km – 1.7793 km = 0.22 km. On Saturday you drove 0.22 km farther than on Sunday.

EVALUATE: Even though the resultant displacement was the same on both days, you drove different distances on the two days because you took different paths.

1.69. **IDENTIFY:** The sum of the four displacements must be zero. Use components.

SET UP: Call the displacements $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$, where $\vec{D}$ is the final unknown displacement for the return from the treasure to the oak tree. Vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are sketched in Figure 1.69a. $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$ says $A_x + B_x + C_x + D_x = 0$ and $A_y + B_y + C_y + D_y = 0$. $A = 825$ m, $B = 1250$ m, and $C = 1000$ m. Let $+x$ be eastward and $+y$ be north.

EXECUTE: (a) $A_x + B_x + C_x + D_x = 0$ gives $D_x = -(A_x + B_x + C_x) = -[0 - (1250 \text{ m})\sin 30.0° + (1000 \text{ m})\cos 32.0°] = -223.0$ m. $A_y + B_y + C_y + D_y = 0$ gives $D_y = -(A_y + B_y + C_y) = -[-825 \text{ m} + (1250 \text{ m})\cos 30.0° + (1000 \text{ m})\sin 32.0°] = -787.4$ m. The fourth displacement $\vec{D}$ and its components are sketched in Figure 1.69b. $D = \sqrt{D_x^2 + D_y^2} = 818.4$ m.

$\tan\phi = \dfrac{|D_x|}{|D_y|} = \dfrac{223.0 \text{ m}}{787.4 \text{ m}}$ and $\phi = 15.8°$. You should head 15.8° west of south and must walk 818 m.

(b) The vector diagram is sketched in Figure 1.69c. The final displacement $\vec{D}$ from this diagram agrees with the vector $\vec{D}$ calculated in part **(a)** using components.

EVALUATE: Note that $\vec{D}$ is the negative of the sum of $\vec{A}$, $\vec{B}$, and $\vec{C}$, as it should be.

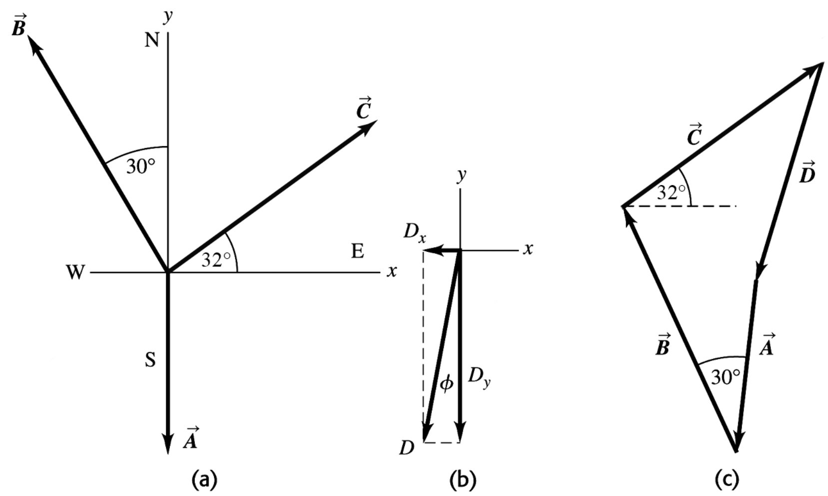


Figure 1.69

1.70. **IDENTIFY:** The displacements are vectors in which we want to find the magnitude of the resultant and know the other vectors.

SET UP: Calling $\vec{A}$ the vector from you to the first post, $\vec{B}$ the vector from you to the second post, and $\vec{C}$ the vector from the first post to the second post, we have $\vec{A}+\vec{C}=\vec{B}$. We want to find the magnitude of vector $\vec{B}$. We use components and the magnitude of $\vec{C}$. Let $+x$ be toward the east and $+y$ be toward the north.

EXECUTE: $B_x = 0$ and B_y is unknown. $C_x = -A_x = -(52.0\text{ m})(\cos 37.0°) = -41.529\text{ m}$ $A_x = 41.53\text{ m}$ $C = 68.0\text{ m}$, so $C_y = \pm\sqrt{C^2 - C_x^2} = -53.8455\text{ m}$. We use the minus sign because the second post is south of the first post.

$B_y = A_y + C_y = (52.0\text{ m})(\sin 37°) + (-53.8455\text{ m}) = -22.551\text{ m}$.

Therefore you are 22.6 m from the second post.

EVALUATE: B_y is negative since post is south of you (in the negative y direction), but the distance to you is positive.

1.71. **IDENTIFY:** We are given the resultant of three vectors, two of which we know, and want to find the magnitude and direction of the third vector.

SET UP: Calling $\vec{C}$ the unknown vector and $\vec{A}$ and $\vec{B}$ the known vectors, we have $\vec{A}+\vec{B}+\vec{C}=\vec{R}$. The components are $A_x + B_x + C_x = R_x$ and $A_y + B_y + C_y = R_y$.

EXECUTE: The components of the known vectors are $A_x = 12.0\text{ m}$, $A_y = 0$, $B_x = -B\sin 50.0° = -21.45\text{ m}$, $B_y = B\cos 50.0° = +18.00\text{ m}$, $R_x = 0$, and $R_y = -10.0\text{ m}$. Therefore the components of $\vec{C}$ are $C_x = R_x - A_x - B_x = 0 - 12.0\text{ m} - (-21.45\text{ m}) = 9.45\text{ m}$ and $C_y = R_y - A_y - B_y = -10.0\text{ m} - 0 - 18.0\text{ m} = -28.0\text{ m}$.

Using these components to find the magnitude and direction of $\vec{C}$ gives $C = 29.6\text{ m}$ and $\tan\theta = \dfrac{9.45}{28.0}$ and $\theta = 18.6°$ east of south.

EVALUATE: A graphical sketch shows that this answer is reasonable.

1.72. **IDENTIFY:** The displacements are vectors in which we know the magnitude of the resultant and want to find the magnitude of one of the other vectors.

SET UP: Calling $\vec{A}$ the vector of Ricardo's displacement from the tree, $\vec{B}$ the vector of Jane's displacement from the tree, and $\vec{C}$ the vector from Ricardo to Jane, we have $\vec{A}+\vec{C}=\vec{B}$. Let the $+x$-axis be to the east and the $+y$-axis be to the north. Solving using components we have $A_x + C_x = B_x$ and $A_y + C_y = B_y$.

EXECUTE: **(a)** The components of $\vec{A}$ and $\vec{B}$ are $A_x = -(26.0\text{ m})\sin 60.0° = -22.52\text{ m}$,
$A_y = (26.0\text{ m})\cos 60.0° = +13.0\text{ m}$, $B_x = -(16.0\text{ m})\cos 30.0° = -13.86\text{ m}$,
$B_y = -(16.0\text{ m})\sin 30.0° = -8.00\text{ m}$, $C_x = B_x - A_x = -13.86\text{ m} - (-22.52\text{ m}) = +8.66\text{ m}$,
$C_y = B_y - A_y = -8.00\text{ m} - (13.0\text{ m}) = -21.0\text{ m}$

Finding the magnitude from the components gives $C = 22.7\text{ m}$.

(b) Finding the direction from the components gives $\tan\theta = \dfrac{8.66}{21.0}$ and $\theta = 22.4°$, east of south.

EVALUATE: A graphical sketch confirms that this answer is reasonable.

1.73. **IDENTIFY:** If the vector from your tent to Joe's is $\vec{A}$ and from your tent to Karl's is $\vec{B}$, then the vector from Karl's tent to Joe's tent is $\vec{A}-\vec{B}$.

SET UP: Take your tent's position as the origin. Let $+x$ be east and $+y$ be north.

EXECUTE: The position vector for Joe's tent is
$[(21.0\text{ m})\cos 23°]\hat{i} - [(21.0\text{ m})\sin 23°]\hat{j} = (19.33\text{ m})\hat{i} - (8.205\text{ m})\hat{j}$.

The position vector for Karl's tent is $[(32.0\text{ m})\cos 37°]\hat{i} + [(32.0\text{ m})\sin 37°]\hat{j} = (25.56\text{ m})\hat{i} + (19.26\text{ m})\hat{j}$.

The difference between the two positions is
$(19.33\text{ m} - 25.56\text{ m})\hat{i} + (-8.205\text{ m} - 19.25\text{ m})\hat{j} = -(6.23\text{ m})\hat{i} - (27.46\text{ m})\hat{j}$. The magnitude of this vector is the distance between the two tents: $D = \sqrt{(-6.23\text{ m})^2 + (-27.46\text{ m})^2} = 28.2\text{ m}$.

EVALUATE: If both tents were due east of yours, the distance between them would be $32.0\text{ m} - 21.0\text{ m} = 11.0\text{ m}$. If Joe's was due north of yours and Karl's was due south of yours, then the distance between them would be $32.0\text{ m} + 21.0\text{ m} = 53.0\text{ m}$. The actual distance between them lies between these limiting values.

1.74. **IDENTIFY:** Calculate the scalar product and use Eq. (1.16) to determine ϕ.

SET UP: The unit vectors are perpendicular to each other.

EXECUTE: The direction vectors each have magnitude $\sqrt{3}$, and their scalar product is $(1)(1)+(1)(-1)+(1)(-1) = -1$, so from Eq. (1.16) the angle between the bonds is
$\arccos\left(\dfrac{-1}{\sqrt{3}\sqrt{3}}\right) = \arccos\left(-\dfrac{1}{3}\right) = 109°$.

EVALUATE: The angle between the two vectors in the bond directions is greater than $90°$.

1.75. **IDENTIFY:** This problem involves the scalar product of two vectors.

SET UP: $W = \vec{F}\cdot\vec{s} = Fs\cos\phi = F_x s_x + F_y s_y$.

EXECUTE: Since $\vec{F}$ is at 60° above the $-x$-axis and $\vec{s}$ is along the $+x$-axis, the angle between them is 120°. The work is $W = Fs\cos\phi = (5.00\text{ N})(0.800\text{ m})\cos 120° = -2.00\text{ J}$.

EVALUATE: Use $W = F_x s_x + F_y s_y$ to check.

$W = (5.00\text{ N}\cos 120°)(0.800\text{ m}) + (5.00\text{ N}\sin 120°)(0) = -2.00\text{ J}$, which agrees with our result.

1.76. **IDENTIFY:** This problem involves the vector product of two vectors.

SET UP: The magnetic force is $\vec{F} = q\vec{v}\times\vec{B}$, $F = qvB\sin\phi$.

EXECUTE: $F = qvB \sin\phi$ gives the *magnitude* of a vector, so it must be positive. Therefore we only need to use the sign of q, so $F = (8.00\times 10^{-6}\ \text{C})(3.00\times 10^{4}\ \text{m/s})(5.00\ \text{T}) \sin 90° = 1.20\ \text{N}$.
Since $\vec{v}$ is in the $+x$ direction and $\vec{B}$ is in the $-y$ direction, $\vec{v}\times\vec{B}$ is in the $-z$ direction. But $q\vec{v}\times\vec{B}$ is in the $+z$ direction because q is negative, so the force is in the $-z$ direction.
EVALUATE: Careful! The quantity $qvB\sin\phi$ cannot be negative since it is the magnitude of a vector. Both v and B are vector magnitudes, so they are always positive, and $\sin\phi$ is positive because $0 \le \phi \le 120°$. Only q could be negative, but when using $qvB\sin\phi$, we must use only the *magnitude* of q. When using $\vec{F} = q\vec{v}\times\vec{B}$, we *do* use the minus sign for q because it affects the direction of the force.

1.77. **IDENTIFY:** We know the scalar product and the magnitude of the vector product of two vectors and want to know the angle between them.
SET UP: The scalar product is $\vec{A}\cdot\vec{B} = AB\cos\theta$ and the vector product is $|\vec{A}\times\vec{B}| = AB\sin\theta$.
EXECUTE: $\vec{A}\cdot\vec{B} = AB\cos\theta = -6.00$ and $|\vec{A}\times\vec{B}| = AB\sin\theta = +9.00$. Taking the ratio gives

$\tan\theta = \dfrac{9.00}{-6.00}$, so $\theta = 124°$.

EVALUATE: Since the scalar product is negative, the angle must be between 90° and 180°.

1.78. **IDENTIFY:** This problem involves the vector product of two vectors.
SET UP: The torque is $\vec{r}\times\vec{F}$, so its magnitude is $rF\sin\phi$.
EXECUTE: We know that $\vec{r}$ makes a 36° angle counterclockwise from the $+y$-axis and $\vec{F}$ points in the $-y$ direction. Therefore the angle between these two vectors is 180° – 36° = 144°. So the magnitude of the torque is $|\vec{r}\times\vec{F}| = rF\sin\phi = (4.0\ \text{m})(22.0\ \text{N})\sin 144° = 52\ \text{N}\cdot\text{m}$. The direction of the torque is in the direction of $\vec{r}\times\vec{F}$. By the right-hand rule, this is in the $+z$ direction.
EVALUATE: The torque vector could point along a negative axis (such as –z), but it would still always have a positive *magnitude*.

1.79. **IDENTIFY:** This problem involves the vector product and the scalar product of two vectors. It is best to use components.
SET UP: $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$, the components of $\vec{A}\times\vec{B}$ are shown in Eq. 1.25 in the text.
EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = a(0) + (0)(-c) + (-b)(d) = -bd$.

Realizing that $A_y = 0$ and $B_x = 0$, Eq. 1.25 gives the components of $\vec{A}\times\vec{B}$.
$(\vec{A}\times\vec{B})_x = -A_zB_y = -(-b)(-c) = -bc$
$(\vec{A}\times\vec{B})_y = -A_xB_z = -(a)(d) = -ad$
$(\vec{A}\times\vec{B})_z = A_xB_y = (a)(-c) = -ac$
$\vec{A}\times\vec{B} = -bc\,\hat{i} - ad\,\hat{j} - ac\,\hat{k}$.

(b) If $c = 0$, $\vec{A}\cdot\vec{B} = -bd$ and $\vec{A}\times\vec{B} = -ad\,\hat{j}$. The magnitude of $\vec{A}\times\vec{B}$ is ad and its direction is $-\hat{j}$ (which is in the $-y$ direction). Figure 1.79 shows a sketch of $\vec{A}$ and $\vec{B}$ in the xy plane. In this figure, the $+y$ axis would point into the paper. By the right-hand rule, $\vec{A}\times\vec{B}$ points out of the paper, which is in the $-y$ direction (or the $-\hat{j}$ direction), which agrees with our results.

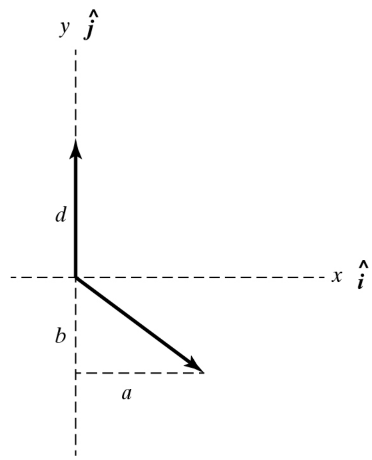


Figure 1.79

From Fig. 1.79 we see that the component of $\vec{A}$ that is parallel to $\vec{B}$ is $-b$. So the product of B with the component of $\vec{A}$ that is parallel to $\vec{B}$ is $d(-b) = -bd$, which agrees with our result. From the same figure we see that the component of $\vec{A}$ that is perpendicular to $\vec{B}$ is a. So the product of B and the component of $\vec{A}$ that is perpendicular to $\vec{B}$ is da, which is the magnitude of the vector product we found above.

EVALUATE: The geometric interpretations of $\vec{A}\cdot\vec{B}$ and $\vec{A}\times\vec{B}$ can be reversed in the sense that $\vec{A}\cdot\vec{B}$ equals A times the component of $\vec{B}$ that is parallel to $\vec{A}$ and $|\vec{A}\times\vec{B}|$ equals A times the component of $\vec{B}$ that is perpendicular to $\vec{A}$.

1.80. IDENTIFY: We are dealing with the scalar product and the vector product of two vectors.

SET UP: $\vec{A}\cdot\vec{B} = AB\cos\phi$ and $|\vec{A}\times\vec{B}| = AB\sin\phi$.

EXECUTE: **(a)** For the notation in the problem, $AB\cos\theta$ has its maximum value when $\theta = 0°$. In that case $|\vec{A}\times\vec{B}| = 0$ because $\sin 0° = 0$.

(b) $|\vec{A}\times\vec{B}| = AB\sin\theta$, so its value occurs when $\theta = 90°$. The scalar product is zero at that angle because $\cos 90° = 0$.

(c) $\vec{A}\cdot\vec{B} = 2|\vec{A}\times\vec{B}|$, so $AB\cos\theta = 2AB\sin\theta \rightarrow \tan\theta = ½ \rightarrow \theta = 26.6°$.

EVALUATE: It might appear that a second solution is $\theta = 180° - 26.6° = 153.4°$, but that is not true because in that case $\vec{A}\cdot\vec{B} = -2|\vec{A}\times\vec{B}|$.

1.81. IDENTIFY: We know the magnitude of two vectors and their scalar product and want to find the magnitude of their vector product.

SET UP: The scalar product is $\vec{A}\cdot\vec{B} = AB\cos\phi$ and the vector product is $|\vec{A}\times\vec{B}| = AB\sin\phi$.

EXECUTE: $\vec{A}\cdot\vec{B} = AB\cos\phi = 90.0\ \text{m}^2$, which gives $\cos\phi = \dfrac{112.0\ \text{m}^2}{AB} = \dfrac{112.0\ \text{m}^2}{(12.0\ \text{m})(16.0\ \text{m})} = 0.5833$, so $\phi = 54.31°$. Therefore $|\vec{A}\times\vec{B}| = AB\sin\phi = (12.0\ \text{m})(16.0\ \text{m})(\sin 54.31°) = 156\ \text{m}^2$.

EVALUATE: The magnitude of the vector product is greater than the scalar product because the angle between the vectors is greater than 45º.

1.82. IDENTIFY: We are dealing with the scalar product and the vector product of two vectors.

SET UP: $\vec{A}\cdot\vec{B} = AB\cos\phi$ and $|\vec{A}\times\vec{B}| = AB\sin\phi$.

EXECUTE: **(a)** In order to have the maximum positive z-component, $\vec{A}\times\vec{B}$ should have its maximum magnitude (which is AB) and it should all point in the $+z$ direction. Thus $\vec{B}$ should be perpendicular to $\vec{A}$ and have a direction so that $\vec{A}\times\vec{B}$ points in the $+z$ direction, as shown in Fig. 1.82. As you can see in this figure, the direction of $\vec{B}$ is at an angle of 53.0° + 90° = 143.0° with the $+x$-axis.

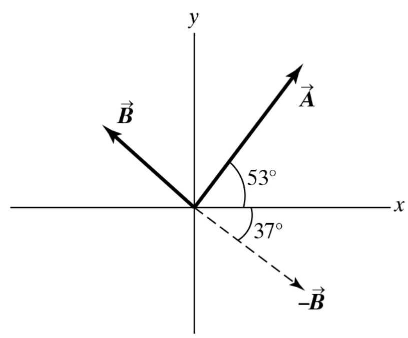


Figure 1.82

(b) In this case, $\vec{A}\times\vec{B}$ must point in the $-z$ direction, so $\vec{B}$ must be the reverse of what we found in part (a). Therefore its angle with the $+x$-axis is 90° – 53.0° = 37.0° clockwise as shown in Fig. 1.82. This angle is 323° counterclockwise with the $+x$-axis.

(c) $\vec{A}\times\vec{B} = 0$ when the angle ϕ between $\vec{A}$ and $\vec{B}$ is 0° or 180°. When ϕ = 0°, the vectors point in the same direction, so they are parallel. When ϕ = 180°, they point in opposite directions, so they are antiparallel. When $\vec{B}$ is parallel to $\vec{A}$, $\vec{B}$ makes an angle of 53.0° counterclockwise from the $+x$-axis. When $\vec{B}$ is antiparallel to $\vec{A}$, $\vec{B}$ makes an angle of 53° + 180° = 233° with the $+x$-axis.

EVALUATE: When calculating the work done by a force, we frequently encounter parallel and antiparallel vectors.

1.83. **IDENTIFY:** We know the scalar product of two vectors, both their directions, and the magnitude of one of them, and we want to find the magnitude of the other vector.

SET UP: $\vec{A}\cdot\vec{B} = AB\cos\phi$. Since we know the direction of each vector, we can find the angle between them.

EXECUTE: The angle between the vectors is $\theta = 79.0°$. Since $\vec{A}\cdot\vec{B} = AB\cos\phi$, we have

$$B = \frac{\vec{A}\cdot\vec{B}}{A\cos\phi} = \frac{48.0\ \text{m}^2}{(9.00\ \text{m})\cos 79.0°} = 28.0\ \text{m}.$$

EVALUATE: Vector $\vec{B}$ has the same units as vector $\vec{A}$.

1.84. **IDENTIFY:** The cross product $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

SET UP: Use Eq. (1.23) to calculate the components of $\vec{A}\times\vec{B}$.

EXECUTE: The cross product is

$(-13.00)\hat{\boldsymbol{i}} + (6.00)\hat{\boldsymbol{j}} + (-11.00)\hat{\boldsymbol{k}} = 13\left[-(1.00)\hat{\boldsymbol{i}} + \left(\frac{6.00}{13.00}\right)\hat{\boldsymbol{j}} - \frac{11.00}{13.00}\hat{\boldsymbol{k}}\right]$. The magnitude of the vector in square brackets is $\sqrt{1.93}$, and so a unit vector in this direction is

$$\left[\frac{-(1.00)\hat{\boldsymbol{i}} + (6.00/13.00)\hat{\boldsymbol{j}} - (11.00/13.00)\hat{\boldsymbol{k}}}{\sqrt{1.93}}\right].$$

The negative of this vector,

$$\left[\frac{(1.00)\hat{\boldsymbol{i}}-(6.00/13.00)\hat{\boldsymbol{j}}+(11.00/13.00)\hat{\boldsymbol{k}}}{\sqrt{1.93}}\right],$$

is also a unit vector perpendicular to $\vec{A}$ and $\vec{B}$.

EVALUATE: Any two vectors that are not parallel or antiparallel form a plane and a vector perpendicular to both vectors is perpendicular to this plane.

1.85. **IDENTIFY** and **SET UP:** The target variables are the components of $\vec{C}$. We are given $\vec{A}$ and $\vec{B}$. We also know $\vec{A}\cdot\vec{C}$ and $\vec{B}\cdot\vec{C}$, and this gives us two equations in the two unknowns C_x and C_y.

EXECUTE: $\vec{A}$ and $\vec{C}$ are perpendicular, so $\vec{A}\cdot\vec{C}=0$. $A_xC_x+A_yC_y=0$, which gives $5.0C_x-6.5C_y=0$.

$\vec{B}\cdot\vec{C}=15.0$, so $3.5C_x-7.0C_y=15.0$

We have two equations in two unknowns C_x and C_y. Solving gives $C_x=-8.0$ and $C_y=-6.1$.

EVALUATE: We can check that our result does give us a vector $\vec{C}$ that satisfies the two equations $\vec{A}\cdot\vec{C}=0$ and $\vec{B}\cdot\vec{C}=15.0$.

1.86. **IDENTIFY:** Calculate the magnitude of the vector product and then use $|\vec{A}\times\vec{B}|=AB\sin\theta$.

SET UP: The magnitude of a vector is related to its components by $A=\sqrt{A_x^2+A_y^2}$.

EXECUTE: $|\vec{A}\times\vec{B}|=AB\sin\theta$. $\sin\theta=\dfrac{|\vec{A}\times\vec{B}|}{AB}=\dfrac{\sqrt{(-5.00)^2+(2.00)^2}}{(3.00)(3.00)}=0.5984$ and

$\theta=\sin^{-1}(0.5984)=36.8°$.

EVALUATE: We haven't found $\vec{A}$ and $\vec{B}$, just the angle between them.

1.87. **IDENTIFY:** Express all the densities in the same units to make a comparison.

SET UP: Density ρ is mass divided by volume. Use the numbers given in the table in the problem and convert all the densities to kg/m³.

EXECUTE: Sample A: $\rho_A=\dfrac{8.00\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{1.67\times10^{-6}\text{ m}^3}=4790\text{ kg/m}^3$

Sample B: $\rho_B=\dfrac{6.00\times10^{-6}\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{9.38\times10^{6}\ \mu\text{m}^3\left(\dfrac{10^{-6}\text{ m}}{1\ \mu\text{m}}\right)^3}=640\text{ kg/m}^3$

Sample C: $\rho_C=\dfrac{8.00\times10^{-3}\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{2.50\times10^{-3}\text{ cm}^3\left(\dfrac{1\text{ m}}{100\text{ cm}}\right)^3}=3200\text{ kg/m}^3$

Sample D: $\rho_D=\dfrac{9.00\times10^{-4}\text{ kg}}{2.81\times10^{3}\text{ mm}^3\left(\dfrac{1\text{ m}}{1000\text{ mm}}\right)^3}=320\text{ kg/m}^3$

Sample E: $\rho_E=\dfrac{9.00\times10^{4}\text{ ng}\left(\dfrac{1\text{ g}}{10^9\text{ ng}}\right)\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{1.41\times10^{-2}\text{ mm}^3\left(\dfrac{1\text{ m}}{1000\text{ mm}}\right)^3}=6380\text{ kg/m}^3$

Sample F: $\rho_F = \dfrac{6.00\times 10^{-5}\ \text{g}\left(\dfrac{1\ \text{kg}}{1000\ \text{g}}\right)}{1.25\ \times\ 10^{8}\ \mu\text{m}^3\left(\dfrac{1\ \text{m}}{10^6\ \mu\text{m}}\right)^3} = 480\ \text{kg/m}^3$

EVALUATE: In order of increasing density, the samples are D, F, B, C, A, E.

1.88. **IDENTIFY:** We know the magnitude of the resultant of two vectors at four known angles between them, and we want to find out the magnitude of each of these two vectors.

SET UP: Use the information in the table in the problem for $\theta = 0.0°$ and 90.0°. Call A and B the magnitudes of the vectors.

EXECUTE: **(a)** At 0°: The vectors point in the same direction, so $A + B = 8.00$ N.

At 90.0°: The vectors are perpendicular to each other, so $A^2 + B^2 = R^2 = (5.83\ \text{N})^2 = 33.99\ \text{N}^2$.

Solving these two equations simultaneously gives

$B = 8.00\ \text{N} - A$

$A^2 + (8.00\ \text{N} - A)^2 = 33.99\ \text{N}^2$

$A^2 + 64.00\ \text{N}^2 - 16.00\ \text{N}\ A + A^2 = 33.99\ \text{N}^2$

The quadratic formula gives two solutions: $A = 5.00$ N and $B = 3.00$ N or $A = 3.00$ N and $B = 5.00$ N. In either case, the larger force has magnitude 5.00 N.

(b) Let $A = 5.00$ N and $B = 3.00$ N, with the larger vector along the x-axis and the smaller one making an angle of +30.0° with the +x-axis in the first quadrant. The components of the resultant are

$R_x = A_x + B_x = 5.00\ \text{N} + (3.00\ \text{N})(\cos 30.0°) = 7.598\ \text{N}$

$R_y = A_y + B_y = 0 + (3.00\ \text{N})(\sin 30.0°) = 1.500\ \text{N}$

$R = \sqrt{R_x^2 + R_y^2} = 7.74\ \text{N}$

EVALUATE: To check our answer, we could use the other resultants and angles given in the table with the problem.

1.89. **IDENTIFY:** Use the x and y coordinates for each object to find the vector from one object to the other; the distance between two objects is the magnitude of this vector. Use the scalar product to find the angle between two vectors.

SET UP: If object A has coordinates (x_A, y_A) and object B has coordinates (x_B, y_B), the vector $\vec{r}_{AB}$ from A to B has x-component $x_B - x_A$ and y-component $y_B - y_A$.

EXECUTE: **(a)** The diagram is sketched in Figure 1.89.

(b) (i) In AU, $\sqrt{(0.3182)^2 + (0.9329)^2} = 0.9857$.

(ii) In AU, $\sqrt{(1.3087)^2 + (-0.4423)^2 + (-0.0414)^2} = 1.3820$.

(iii) In AU, $\sqrt{(0.3182 - 1.3087)^2 + (0.9329 - (-0.4423))^2 + (0.0414)^2} = 1.695$.

(c) The angle between the directions from the Earth to the Sun and to Mars is obtained from the dot product. Combining Eqs. (1.16) and (1.19),

$$\phi = \arccos\left(\frac{(-0.3182)(1.3087 - 0.3182) + (-0.9329)(-0.4423 - 0.9329) + (0)}{(0.9857)(1.695)}\right) = 54.6°.$$

(d) Mars could not have been visible at midnight, because the Sun-Mars angle is less than 90°.

EVALUATE: Our calculations correctly give that Mars is farther from the Sun than the earth is. Note that on this date Mars was farther from the earth than it is from the Sun.

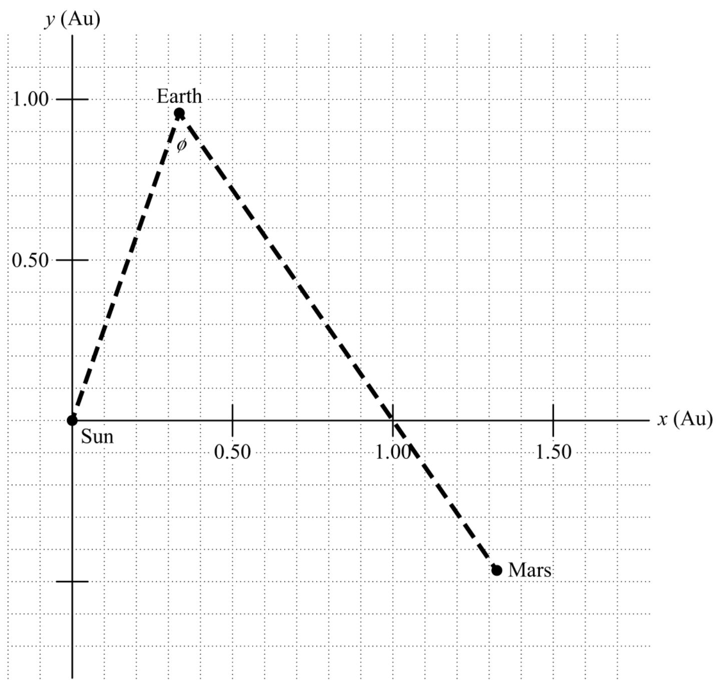


Figure 1.89

1.90. **IDENTIFY:** Add the vector displacements of the receiver and then find the vector from the quarterback to the receiver.

SET UP: Add the x-components and the y-components.

EXECUTE: The receiver's position is

$[(+1.0+9.0-6.0+12.0)\text{yd}]\hat{\boldsymbol{i}}+[(-5.0+11.0+4.0+18.0)\,\text{yd}]\,\hat{\boldsymbol{j}}=(16.0\text{ yd})\hat{\boldsymbol{i}}+(28.0\text{ yd})\hat{\boldsymbol{j}}.$

The vector from the quarterback to the receiver is the receiver's position minus the quarterback's position, or $(16.0\text{ yd})\hat{\boldsymbol{i}}+(35.0\text{ yd})\hat{\boldsymbol{j}},$ a vector with magnitude $\sqrt{(16.0\text{ yd})^2+(35.0\text{ yd})^2}=38.5\text{ yd}.$ The angle is $\arctan\left(\dfrac{16.0}{35.0}\right)=24.6°$ to the right of downfield.

EVALUATE: The vector from the quarterback to receiver has positive x-component and positive y-component.

1.91. **IDENTIFY:** Draw the vector addition diagram for the position vectors.

SET UP: Use coordinates in which the Sun to Merak line lies along the x-axis. Let $\vec{\boldsymbol{A}}$ be the position vector of Alkaid relative to the Sun, $\vec{\boldsymbol{M}}$ is the position vector of Merak relative to the Sun, and $\vec{\boldsymbol{R}}$ is the position vector for Alkaid relative to Merak. $A=138$ ly and $M=77$ ly.

EXECUTE: The relative positions are shown in Figure 1.91. $\vec{\boldsymbol{M}}+\vec{\boldsymbol{R}}=\vec{\boldsymbol{A}}.$ $A_x=M_x+R_x$ so

$R_x=A_x-M_x=(138\text{ ly})\cos 25.6°-77\text{ ly}=47.5\text{ ly}.$ $R_y=A_y-M_y=(138\text{ ly})\sin 25.6°-0=59.6\text{ ly}.$

$R=76.2$ ly is the distance between Alkaid and Merak.

(b) The angle is angle ϕ in Figure 1.91. $\cos\theta=\dfrac{R_x}{R}=\dfrac{47.5\text{ ly}}{76.2\text{ ly}}$ and $\theta=51.4°.$ Then $\phi=180°-\theta=129°.$

EVALUATE: The concepts of vector addition and components make these calculations very simple.

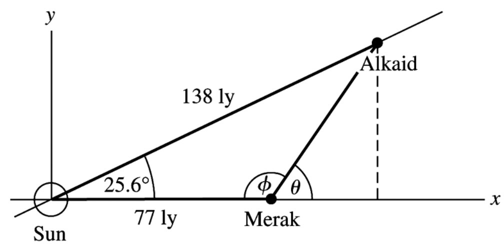


Figure 1.91

1.92. **IDENTIFY:** The total volume of the gas-exchanging region of the lungs must be at least as great as the total volume of all the alveoli, which is the product of the volume per alveoli times the number of alveoli.

SET UP: $V = NV_{alv}$, and we use the numbers given in the introduction to the problem.

EXECUTE: $V = NV_{alv} = (480 \times 10^6)(4.2 \times 10^6\ \mu\text{m}^3) = 2.02 \times 10^{15}\ \mu\text{m}^3$. Converting to liters gives

$V = 2.02\times10^{15}\ \text{m}^3\left(\dfrac{1\ \text{m}}{10^6\ \mu\text{m}}\right)^3 = 2.02\ \text{L} \approx 2.0\ \text{L}$. Therefore choice (c) is correct.

EVALUATE: A volume of 2 L is reasonable for the lungs.

1.93. **IDENTIFY:** We know the volume and want to find the diameter of a typical alveolus, assuming it to be a sphere.

SET UP: The volume of a sphere of radius r is $V = 4/3\ \pi r^3$ and its diameter is $D = 2r$.

EXECUTE: Solving for the radius in terms of the volume gives $r = (3V/4\pi)^{1/3}$, so the diameter is

$D = 2r = 2(3V/4\pi)^{1/3} = 2\left[\dfrac{3\left(4.2\times10^6\ \mu\text{m}^3\right)}{4\pi}\right]^{1/3} = 200\ \mu\text{m}$. Converting to mm gives

$D = (200\ \mu\text{m})[(1\ \text{mm})/(1000\ \mu\text{m})] = 0.20\ \text{mm}$, so choice (a) is correct.

EVALUATE: A sphere that is 0.20 mm in diameter should be visible to the naked eye for someone with good eyesight.

1.94. **IDENTIFY:** Draw conclusions from a given graph.

SET UP: The dots lie more-or-less along a horizontal line, which means that the average alveolar volume does not vary significantly as the lung volume increases.

EXECUTE: The volume of individual alveoli does not vary (as stated in the introduction). The graph shows that the volume occupied by alveoli stays constant for higher and higher lung volumes, so there must be more of them, which makes choice (c) the correct one.

EVALUATE: It is reasonable that a large lung would need more alveoli than a small lung because a large lung probably belongs to a larger person than a small lung.

2

MOTION ALONG A STRAIGHT LINE

VP2.5.1. **IDENTIFY:** The bus and the car leave the same point at the same time. The bus has constant velocity, but the car starts from rest with constant acceleration. So the constant-acceleration formulas apply. We want to know how long it takes for the car to catch up to the bus and how far they both travel during that time.

SET UP: When they meet, x is the same for both of them and they have traveled for the same time. The formulas $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ and $v_x = v_{0x} + a_x t$ both apply.

EXECUTE: (a) When the car and bus meet, they have traveled the same distance in the same time. We apply the formula $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to each of them, with the origin at their starting point, which makes $x_0 = 0$ for both of them. The bus has no acceleration and the car has no initial velocity. The equation reduces to $\frac{1}{2}a_{\text{car}}t^2 = v_{\text{bus}}t \quad \rightarrow \quad t = 2v_{\text{bus}}/a_{\text{car}}$.

$t = 2(18\text{ m/s})/(8.0\text{ m/s}^2) = 4.5\text{ s}$.

(b) The bus has zero acceleration, so $v_x = v_{0x} + a_x t$ reduces to $x_{\text{bus}} = v_{\text{bus}}t$

$x_{\text{bus}} = (18\text{ m/s})(4.5\text{ s}) = 81\text{ m}$.

EVALUATE: To check, use the car's motion to find the distance.

$x_{\text{car}} = \frac{1}{2}a_{\text{car}}t^2 = \frac{1}{2}(8.0\text{ m/s}^2)(4.5\text{ s})^2 = 81\text{ m}$, which agrees with our result in part (b).

VP2.5.2. **IDENTIFY:** This is very similar to VP2.5.1 and VP2.5.2. The motorcycle and the SUV leave the same point at the same time. The motorcycle has a constant velocity, but the SUV has an initial velocity and a constant acceleration. So the constant-acceleration formulas apply.

SET UP: When they meet, x is the same for both of them and they have traveled for the same time. The formulas $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ and $v_x = v_{0x} + a_x t$ both apply.

EXECUTE: (a) When the SUV and motorcycle meet, they have traveled the same distance in the same time. We apply the formula $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to each of them, with the origin at their starting point, which makes $x_0 = 0$ for both of them. The motorcycle has no acceleration and the SUV has an initial velocity and an acceleration. The acceleration is *opposite* to the velocity of the SUV. If we take the x-axis to be in the direction of motion, a_{SUV} is negative. The equation reduces to

$v_{\text{motorcycle}}t = v_{\text{SUV}}t + \frac{1}{2}a_{\text{SUV}}t^2$. We want the time. Putting in the numbers gives

$(20.0\text{ m/s})t = (30.0\text{ m/s})t + \frac{1}{2}(-1.80\text{ m/s}^2)t^2$

$t = 0$ s and $t = 11.1$ s. The $t = 0$ s solution is when they both of them leave the same point, and the $t =$ 11.1 s is the time when the motorcycle passes the SUV.

(b) Both have traveled the same distance when they meet. For the motorcycle this gives $x_{\text{motorcycle}} = v_{\text{motorcycle}}t = (20.0\text{ m/s})(11.1\text{ s}) = 222$ m.

(c) The equation $v_x = v_{0x} + a_x t$ gives the speed of the SUV when they meet in 11.1 s.

$v_x = 30.0\text{ m/s} + (-1.80\text{ m/s}^2)(11.1\text{ s}) = 10.0$ s.

EVALUATE: Use $x = v_{\text{av}}t$ to find the distance the SUV has traveled in 11.1 s. For constant acceleration, the average velocity is $v_{\text{av}} = (v_1 + v_2)/2$, which gives us

$x = [(30.0\text{ m/s} + 10.0\text{ m/s})/2](11.1\text{ s}) = 222$ m, which agrees with our previous result.

VP2.5.4. **IDENTIFY:** The truck and car have constant (but different) accelerations and the car has an initial velocity but the truck starts from rest. They leave from the same place at the same time and the truck eventually passes the car. The constant-acceleration equation $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ applies.

SET UP: (a) The truck and car have traveled the same distance in the same time when the truck reaches the car to pass. They start at the same place so x_0 is the same for both and the truck has no initial velocity.

EXECUTE: Use the equation $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ for each of them, which gives

$$\frac{1}{2}a_{\text{T}}t^2 = v_{\text{C}}t - \frac{1}{2}a_{\text{C}}t^2 \qquad \rightarrow \qquad t = \frac{2v_{\text{C}}}{a_{\text{T}} + a_{\text{C}}}.$$

(b) Looking at the truck gives $x_{\text{T}} = \frac{1}{2}a_{\text{T}}t^2 = \frac{1}{2}a_{\text{T}}\left(\frac{2v_{\text{C}}}{a_{\text{T}} + a_{\text{C}}}\right)^2 = \frac{2a_{\text{T}}v_{\text{C}}^2}{(a_{\text{T}} + a_{\text{C}})^2}$.

EVALUATE: We can calculate the distance the car travels using $x_{\text{C}} = v_{\text{C}}t - \frac{1}{2}a_{\text{C}}t^2$ and the value of t we found in part (a). Doing this and simplifying the result gives the same answer as in part (b).

VP2.7.1. **IDENTIFY:** The ball is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertical, the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y = v_{0y} + a_y t$ apply to the motion of the ball. We know that $a_y = 9.80\text{ m/s}^2$ downward and $v_{0y} = 12.0$ m/s upward.

EXECUTE: (a) At time $t = 0.300$ s, the vertical coordinate of the ball is given by $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, where $y_0 = 0$ at the location of the hand.

$y = 0 + (12.0\text{ m/s})(0.300\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(0.300\text{ s})^2 = 3.16$ m. Since y is positive, the ball is above the hand. The vertical velocity is given by $v_y = v_{0y} + a_y t$.

$v_y = 12.0\text{ m/s} + (-9.80\text{ m/s}^2)(0.300\text{ s}) = 9.06$ m/s. Since v_y is positive, the ball is moving upward.

(b) At $t = 2.60$ s, $y = (12.0\text{ m/s})(2.60\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(2.60\text{ s})^2 = -1.92$ m. Since y is negative, the ball is now below the hand. The ball must be moving downward since it is now below the hand.

EVALUATE: Check with v_y: $v_y = v_{0y} + a_y t = 12.0\text{ m/s} + (-9.80\text{ m/s}^2)(2.60\text{ s}) = -13.5$ m/s. Since v_y is negative, the ball is moving downward, as we saw above.

VP2.7.2. **IDENTIFY:** The stone is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertical, the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply to the motion of the stone. We know that a_y = 9.80 m/s^2 downward, v_{0y} = 8.00 m/s downward, and y_0 = 0.

EXECUTE: (a) The equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives

y = 0 + (–8.00 m/s)(1.50 s) + $\frac{1}{2}$(–9.80 m/s^2)(1.50 s)^2 = –23.0 m. The minus sign means that the stone is below your hand. The velocity of the stone is given by

$v_y = v_{0y} + a_y t$ = –8.00 m/s + (–9.80 m/s^2)(1.50 s) = –22.7 m/s. The minus sign tells us it is moving downward.

(b) We know the stone's position and acceleration and want its velocity. The equation

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

v_y^2 = (–8.00 m/s)^2 + 2(–9.80 m/s^2)(–8.00 m)

v_y = ±14.9 m/s. The stone must be moving downward, so v_y = –14.9 m/s.

EVALUATE: When the stone returned to the level of your hand, its speed was the same as its initial speed of 8.00 m/s. But as the stone has continued to accelerate downward since then, its speed must be greater than its initial speed, which is what we found.

VP2.7.3. **IDENTIFY:** The football is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertical, the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. We know that a_y = 9.80 m/s^2 downward, v_y = 0.500 m/s upward when y = 4.00 m, and y_0 = 0.

EXECUTE: (a) We know the speed, acceleration, and position of the ball and want its initial speed, so we use the equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find its initial speed v_{0y}.

(0.500 m/s)^2 = v_{0y}^2 + 2(–9.80 m/s^2)(4.00 m) → v_{0y} = 8.87 m/s.

(b) Use the result from (a) in the equation $v_y = v_{0y} + a_y t$ to find the time.

0.500 m/s = 8.87 m/s + (–9.80 m/s^2)t → t = 0.854 s.

EVALUATE: Calculate y using the time from (b) and compare it with the given value of 4.00 m.

$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ = 0 + (8.87 m/s)(0.854 s) + $\frac{1}{2}$(–9.80 m/s^2)(0.854 s)^2 = 4.00 m, which agrees with the given value.

VP2.7.4. **IDENTIFY:** The tennis ball is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: When the ball is at its highest point, its vertical velocity is zero. The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ applies.

EXECUTE: (a) At the highest point, v_y = 0. Use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find v_{0y}.

0 = v_{0y}^2 + 2(–g)(H) → $v_{0y} = \sqrt{2gH}$

(b) We now know H, v_{0y} and want v_y. The same equation gives

$v_y^2 = v_{0y}^2 + 2(-g)(H/2)$ = $2gH - 2gH/2 = 2gH/2$

$$v_y = \sqrt{\frac{2gH}{2}} = \frac{\sqrt{2gH}}{\sqrt{2}} = \frac{v_0}{\sqrt{2}}.$$

(c) We want $y - y_0$, we know v_0, a, and v. The same equation gives

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$\left(\frac{v_0}{2}\right)^2 = v_0^2 + 2(-g)(y - y_0)$$

$$y - y_0 = \frac{3v_0^2}{8g} = \frac{3(2gH)}{8g} = \frac{3H}{4}.$$

EVALUATE: When the ball is half way to the top, $v \neq v_0/2$ because the motion equations involve the *squares* of quanties such as v^2 and t^2.

VP2.8.1. **IDENTIFY:** The rock is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: The equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ applies.

EXECUTE: (a) With the origin at the hand and the y-axis positive upward, $y = 4.00$ m and $y_0 = 0$. We want the time at which this occurs. The equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives

4.00 m = (12.0 m/s)t + $\frac{1}{2}$(–9.80 m/s^2)t^2. Solving this quadratic equation for t gives two answers: t = 0.398 s and t = 2.05 s.

(b) Use the same procedure as in (a) except that y = –4.00 m. This gives

–4.00 m = (12.0 m/s)t + $\frac{1}{2}$(–9.80 m/s^2)t^2. This quadratic equation has two solutions, t = 2.75 s and t = –0.297 s. The negative answer is not physical, so t = 2.75 s.

EVALUATE: The ball is at 4.00 m above your hand twice, when it is going up and when it is going down, so we get two answers. It is at 4.00 m below the hand only once, when it is going down, so we have just one answer.

VP2.8.2. **IDENTIFY:** The ball is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertical with the origin at the hand, the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. We know that a_y = 9.80 m/s^2 downward and v_y is initially 9.00 m/s downward. Since all the quantities are downward, it is convenient to call the +y-axis downward.

EXECUTE: First find v_y when y = 5.00 m. Then use this result to find the time for the ball to reach this height.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ = (9.00 m/s)^2 + 2(9.80 m/s^2)(5.00 m)

v_y = 13.38 m/s

Now use $v_y = v_{0y} + a_y t$ to find the time t.

13.38 m/s = 9.00 m/s + (9.80 m/s^2)t → t = 0.447 s.

EVALUATE: Check using the equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ when t = 0.447 s.

y = (9.00 m/s)(0.447 s) + $\frac{1}{2}$(9.80 m/s^2)(0.447 s)^2 = 5.00 m, which agrees with the given value.

VP2.8.3. **IDENTIFY:** The apple is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertically upward with the origin at the hand, the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. We know that $a_y = 9.80\ \text{m/s}^2$ downward and v_y is initially 5.50 m/s upward.

EXECUTE: (a) Using $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives

$1.30\ \text{m} = (5.50\ \text{m/s})t + \frac{1}{2}(-9.80\ \text{m/s}^2)(0.447\ \text{s})^2$

Solving using the quadratic formula gives $t = 0.338$ s and $t = 0.784$ s. The apple passes through this point twice: going up at 0.338 s and going down at 0784 s.

(b) Use the same approach as in (a).

$1.80\ \text{m} = (5.50\ \text{m/s})t + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$

This equation has no real solutions, so the apple *never* reaches a height of 1.80 m.

EVALUATE: The highest point the apple reaches is when $v_y = 0$. Use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find the maximum height.

$0 = (5.50\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(y - y_0)$

$y - y_0 = 1.54$ m, which is *less than* 1.80 m. This is why we had no solutions to the quadratic equation in part (b).

VP2.8.4. **IDENTIFY:** The orange is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertically upward with the origin at the hand, the formula $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ applies. We know that $a_y = g$ downward and v_y is initially v_0 upward.

EXECUTE: We want the time when $y = \frac{v_0^2}{2g}$, so we use $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$.

$\frac{v_0^2}{2g} = v_0 t - \frac{1}{2}gt^2$. Solving this quadratic equation gives $t = v_0/g$. There is only one solution, so the orange reaches this height only once.

(b) Use the same approach as in part (a). $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives

$\frac{3v_0^2}{8g} = v_0 t - \frac{1}{2}gt^2$. The quadratic formula gives two solutions: $t = v_0/2g$ and $t = 3v_0/2g$.

The orange is going up at the smaller solution and going down at the larger solution.

EVALUATE: We got only one solution for part (a) because $y = \frac{v_0^2}{2g}$ is the highest point the orange reaches, and that occurs only once because the orange stops there. In (b) the height $\frac{3v_0^2}{8g}$ is less than the maximum height, so the orange reaches this height twice, once going up and once going down.

2.1. **IDENTIFY:** $\Delta x = v_{\text{av-}x}\Delta t$

SET UP: We know the average velocity is 6.25 m/s.

EXECUTE: $\Delta x = v_{\text{av-}x}\Delta t = 25.0$ m

EVALUATE: In round numbers, 6 m/s × 4 s = 24 m ≈ 25 m, so the answer is reasonable.

2.2. **IDENTIFY:** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$

SET UP: $13.5 \text{ days} = 1.166\times10^6 \text{ s}$. At the release point, $x = +5.150\times10^6 \text{ m}$.

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{x_2 - x_1}{\Delta t} = \dfrac{-5.150\times10^6 \text{ m}}{1.166\times10^6 \text{ s}} = -4.42 \text{ m/s}$.

(b) For the round trip, $x_2 = x_1$ and $\Delta x = 0$. The average velocity is zero.

EVALUATE: The average velocity for the trip from the nest to the release point is positive.

2.3. **IDENTIFY:** Target variable is the time Δt it takes to make the trip in heavy traffic. Use Eq. (2.2) that relates the average velocity to the displacement and average time.

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so $\Delta x = v_{\text{av-}x}\Delta t$ and $\Delta t = \dfrac{\Delta x}{v_{\text{av-}x}}$.

EXECUTE: Use the information given for normal driving conditions to calculate the distance between the two cities, where the time is 1 h and 50 min, which is 110 min:

$$\Delta x = v_{\text{av-}x}\Delta t = (105 \text{ km/h})(1 \text{ h}/60 \text{ min})(110 \text{ min}) = 192.5 \text{ km}.$$

Now use $v_{\text{av-}x}$ for heavy traffic to calculate Δt; Δx is the same as before:

$$\Delta t = \frac{\Delta x}{v_{\text{av-}x}} = \frac{192.5 \text{ km}}{70 \text{ km/h}} = 2.75 \text{ h} = 2 \text{ h and } 45 \text{ min}.$$

The additional time is (2 h and 45 min) – (1 h and 50 min) = (1 h and 105 min) – (1 h and 50 min) = 55 min.

EVALUATE: At the normal speed of 105 km/s the trip takes 110 min, but at the reduced speed of 70 km/h it takes 165 min. So decreasing your average speed by about 30% adds 55 min to the time, which is 50% of 110 min. Thus a 30% reduction in speed leads to a 50% increase in travel time. This result (perhaps surprising) occurs because the time interval is inversely proportional to the average speed, not directly proportional to it.

2.4. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use the average speed for each segment to find the time traveled in that segment. The average speed is the distance traveled divided by the time.

SET UP: The post is 80 m west of the pillar. The total distance traveled is $200 \text{ m} + 280 \text{ m} = 480 \text{ m}$.

EXECUTE: **(a)** The eastward run takes time $\dfrac{200 \text{ m}}{5.0 \text{ m/s}} = 40.0 \text{ s}$ and the westward run takes $\dfrac{280 \text{ m}}{4.0 \text{ m/s}} = 70.0 \text{ s}$. The average speed for the entire trip is $\dfrac{480 \text{ m}}{110.0 \text{ s}} = 4.4 \text{ m/s}$.

(b) $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{-80 \text{ m}}{110.0 \text{ s}} = -0.73 \text{ m/s}$. The average velocity is directed westward.

EVALUATE: The displacement is much less than the distance traveled, and the magnitude of the average velocity is much less than the average speed. The average speed for the entire trip has a value that lies between the average speed for the two segments.

2.5. **IDENTIFY:** Given two displacements, we want the average velocity and the average speed.

SET UP: The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ and the average speed is just the total distance walked divided by the total time to walk this distance.

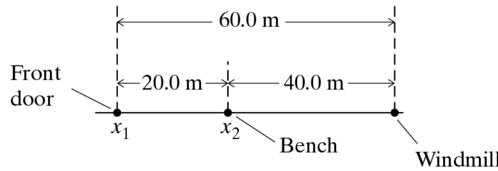


Figure 2.5

EXECUTE: **(a)** Let $+x$ be eastward with the origin at the front door. The trip begins at the front door and ends at the bench as shown in Fig. 2.5. Therefore $x_1 = 0.00$ m and $x_2 = 20.0$ m.
$\Delta x = x_2 - x_1 = 20.0\text{ m} - 0.00\text{ m} = 20.0\text{ m}$. The total time is $\Delta t = 28.0\text{ s} + 36.0\text{ s} = 64.0\text{ s}$. So
$v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{20.0\text{ m}}{64.0\text{ s}} = 0.313\text{ m/s}.$

(b) Average speed $= \dfrac{60.0\text{ m} + 40.0\text{ m}}{64.0\text{ s}} = 1.56\text{ m/s}$.

EVALUATE: The average speed is much greater than the average velocity because the total distance walked is much greater than the magnitude of the displacement vector.

2.6. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use $x(t)$ to find x for each t.

SET UP: $x(0) = 0$, $x(2.00\text{ s}) = 5.60\text{ m}$, and $x(4.00\text{ s}) = 20.8\text{ m}$

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{5.60\text{ m} - 0}{2.00\text{ s}} = +2.80\text{ m/s}$

(b) $v_{\text{av-}x} = \dfrac{20.8\text{ m} - 0}{4.00\text{ s}} = +5.20\text{ m/s}$

(c) $v_{\text{av-}x} = \dfrac{20.8\text{ m} - 5.60\text{ m}}{2.00\text{ s}} = +7.60\text{ m/s}$

EVALUATE: The average velocity depends on the time interval being considered.

2.7. **(a) IDENTIFY:** Calculate the average velocity using $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so use $x(t)$ to find the displacement Δx for this time interval.

EXECUTE: $t = 0$: $x = 0$

$t = 10.0\text{ s}$: $x = (2.40\text{ m/s}^2)(10.0\text{ s})^2 - (0.120\text{ m/s}^3)(10.0\text{ s})^3 = 240\text{ m} - 120\text{ m} = 120\text{ m}.$

Then $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{120\text{ m}}{10.0\text{ s}} = 12.0\text{ m/s}.$

(b) IDENTIFY: Use $v_x = \dfrac{dx}{dt}$ to calculate $v_x(t)$ and evaluate this expression at each specified t.

SET UP: $v_x = \dfrac{dx}{dt} = 2bt - 3ct^2.$

EXECUTE: (i) $t = 0$: $v_x = 0$

(ii) $t = 5.0\text{ s}$: $v_x = 2(2.40\text{ m/s}^2)(5.0\text{ s}) - 3(0.120\text{ m/s}^3)(5.0\text{ s})^2 = 24.0\text{ m/s} - 9.0\text{ m/s} = 15.0\text{ m/s}.$

(iii) $t = 10.0\text{ s}$: $v_x = 2(2.40\text{ m/s}^2)(10.0\text{ s}) - 3(0.120\text{ m/s}^3)(10.0\text{ s})^2 = 48.0\text{ m/s} - 36.0\text{ m/s} = 12.0\text{ m/s}.$

(c) IDENTIFY: Find the value of t when $v_x(t)$ from part (b) is zero.

SET UP: $v_x = 2bt - 3ct^2$

$v_x = 0$ at $t = 0$.

$v_x = 0$ next when $2bt - 3ct^2 = 0$

EXECUTE: $2b = 3ct$ so $t = \dfrac{2b}{3c} = \dfrac{2(2.40\text{ m/s}^2)}{3(0.120\text{ m/s}^3)} = 13.3\text{ s}$

EVALUATE: $v_x(t)$ for this motion says the car starts from rest, speeds up, and then slows down again.

2.8. **IDENTIFY:** We know the position $x(t)$ of the bird as a function of time and want to find its instantaneous velocity at a particular time.

SET UP: The instantaneous velocity is $v_x(t) = \dfrac{dx}{dt} = \dfrac{d\left[28.0\text{ m} + (12.4\text{ m/s})t - (0.0450\text{ m/s}^3)t^3\right]}{dt}$.

EXECUTE: $v_x(t) = \dfrac{dx}{dt} = 12.4\text{ m/s} - (0.135\text{ m/s}^3)t^2$. Evaluating this at $t = 8.0\text{ s}$ gives $v_x = 3.76\text{ m/s}$.

EVALUATE: The acceleration is not constant in this case.

2.9. **IDENTIFY:** The average velocity is given by $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. We can find the displacement Δt for each constant velocity time interval. The average speed is the distance traveled divided by the time.

SET UP: For $t = 0$ to $t = 2.0\text{ s}$, $v_x = 2.0\text{ m/s}$. For $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$, $v_x = 3.0\text{ m/s}$. In part (b), $v_x = -3.0\text{ m/s}$ for $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$. When the velocity is constant, $\Delta x = v_x \Delta t$.

EXECUTE: **(a)** For $t = 0$ to $t = 2.0\text{ s}$, $\Delta x = (2.0\text{ m/s})(2.0\text{ s}) = 4.0\text{ m}$. For $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$, $\Delta x = (3.0\text{ m/s})(1.0\text{ s}) = 3.0\text{ m}$. For the first 3.0 s, $\Delta x = 4.0\text{ m} + 3.0\text{ m} = 7.0\text{ m}$. The distance traveled is also 7.0 m. The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{7.0\text{ m}}{3.0\text{ s}} = 2.33\text{ m/s}$. The average speed is also 2.33 m/s.

(b) For $t = 2.0\text{ s to }3.0\text{ s}$, $\Delta x = (-3.0\text{ m/s})(1.0\text{ s}) = -3.0\text{ m}$. For the first 3.0 s, $\Delta x = 4.0\text{ m} + (-3.0\text{ m}) = +1.0\text{ m}$. The ball travels 4.0 m in the $+x$-direction and then 3.0 m in the $-x$-direction, so the distance traveled is still 7.0 m. $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{1.0\text{ m}}{3.0\text{ s}} = 0.33\text{ m/s}$. The average speed is $\dfrac{7.00\text{ m}}{3.00\text{ s}} = 2.33\text{ m/s}$.

EVALUATE: When the motion is always in the same direction, the displacement and the distance traveled are equal and the average velocity has the same magnitude as the average speed. When the motion changes direction during the time interval, those quantities are different.

2.10. **IDENTIFY** and **SET UP:** The instantaneous velocity is the slope of the tangent to the x versus t graph.

EXECUTE: **(a)** The velocity is zero where the graph is horizontal; point IV.

(b) The velocity is constant and positive where the graph is a straight line with positive slope; point I.

(c) The velocity is constant and negative where the graph is a straight line with negative slope; point V.

(d) The slope is positive and increasing at point II.

(e) The slope is positive and decreasing at point III.

EVALUATE: The sign of the velocity indicates its direction.

2.11. **IDENTIFY:** Find the instantaneous velocity of a car using a graph of its position as a function of time.

SET UP: The instantaneous velocity at any point is the slope of the x versus t graph at that point. Estimate the slope from the graph.

EXECUTE: A: $v_x = 6.7\text{ m/s}$; B: $v_x = 6.7\text{ m/s}$; C: $v_x = 0$; D: $v_x = -40.0\text{ m/s}$; E: $v_x = -40.0\text{ m/s}$; F: $v_x = -40.0\text{ m/s}$; G: $v_x = 0$.

EVALUATE: The sign of v_x shows the direction the car is moving. v_x is constant when x versus t is a straight line.

2.12. **IDENTIFY:** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. $a_x(t)$ is the slope of the v_x versus t graph.

SET UP: $60\text{ km/h} = 16.7\text{ m/s}$

EXECUTE: **(a)** (i) $a_{\text{av-}x} = \dfrac{16.7\text{ m/s} - 0}{10\text{ s}} = 1.7\text{ m/s}^2$. (ii) $a_{\text{av-}x} = \dfrac{0 - 16.7\text{ m/s}}{10\text{ s}} = -1.7\text{ m/s}^2$.

(iii) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$. (iv) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$.

(b) At $t = 20$ s, v_x is constant and $a_x = 0$. At $t = 35$ s, the graph of v_x versus t is a straight line and $a_x = a_{\text{av-}x} = -1.7\text{ m/s}^2$.

EVALUATE: When $a_{\text{av-}x}$ and v_x have the same sign the speed is increasing. When they have opposite signs, the speed is decreasing.

2.13. **IDENTIFY** and **SET UP:** Use $v_x = \dfrac{dx}{dt}$ and $a_x = \dfrac{dv_x}{dt}$ to calculate $v_x(t)$ and $a_x(t)$.

EXECUTE: $v_x = \dfrac{dx}{dt} = 2.00\text{ cm/s} - (0.125\text{ cm/s}^2)t$

$a_x = \dfrac{dv_x}{dt} = -0.125\text{ cm/s}^2$

(a) At $t = 0$, $x = 50.0$ cm, $v_x = 2.00$ cm/s, $a_x = -0.125\text{ cm/s}^2$.

(b) Set $v_x = 0$ and solve for t: $t = 16.0$ s.

(c) Set $x = 50.0$ cm and solve for t. This gives $t = 0$ and $t = 32.0$ s. The turtle returns to the starting point after 32.0 s.

(d) The turtle is 10.0 cm from starting point when $x = 60.0$ cm or $x = 40.0$ cm.
Set $x = 60.0$ cm and solve for t: $t = 6.20$ s and $t = 25.8$ s.
At $t = 6.20$ s, $v_x = +1.23$ cm/s.
At $t = 25.8$ s, $v_x = -1.23$ cm/s.
Set $x = 40.0$ cm and solve for t: $t = 36.4$ s (other root to the quadratic equation is negative and hence nonphysical).
At $t = 36.4$ s, $v_x = -2.55$ cm/s.

(e) The graphs are sketched in Figure 2.13.

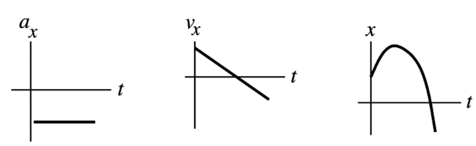


Figure 2.13

EVALUATE: The acceleration is constant and negative. v_x is linear in time. It is initially positive, decreases to zero, and then becomes negative with increasing magnitude. The turtle initially moves farther away from the origin but then stops and moves in the $-x$-direction.

2.14. **IDENTIFY:** We know the velocity $v(t)$ of the car as a function of time and want to find its acceleration at the instant that its velocity is 12.0 m/s.

SET UP: We know that $v_x(t) = (0.860\text{ m/s}^3)t^2$ and that $a_x(t) = \dfrac{dv_x}{dt} = \dfrac{d\left[(0.860\text{ m/s}^3)t^2\right]}{dt}$.

EXECUTE: $a_x(t) = \dfrac{dv_x}{dt} = (1.72\text{ m/s}^3)t$. When $v_x = 12.0$ m/s, $(0.860\text{ m/s}^3)t^2 = 12.0$ m/s, which gives $t = 3.735$ s. At this time, $a_x = 6.42\text{ m/s}^2$.

EVALUATE: The acceleration of this car is not constant.

2.15. **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. Use $v_x(t)$ to find v_x at each t. The instantaneous acceleration is $a_x = \dfrac{dv_x}{dt}$.

SET UP: $v_x(0) = 3.00 \text{ m/s}$ and $v_x(5.00 \text{ s}) = 5.50 \text{ m/s}$.

EXECUTE: **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{5.50 \text{ m/s} - 3.00 \text{ m/s}}{5.00 \text{ s}} = 0.500 \text{ m/s}^2$

(b) $a_x = \dfrac{dv_x}{dt} = (0.100 \text{ m/s}^3)(2t) = (0.200 \text{ m/s}^3)t$. At $t = 0$, $a_x = 0$. At $t = 5.00 \text{ s}$, $a_x = 1.00 \text{ m/s}^2$.

(c) Graphs of $v_x(t)$ and $a_x(t)$ are given in Figure 2.15.

EVALUATE: $a_x(t)$ is the slope of $v_x(t)$ and increases as t increases. The average acceleration for $t = 0$ to $t = 5.00 \text{ s}$ equals the instantaneous acceleration at the midpoint of the time interval, $t = 2.50 \text{ s}$, since $a_x(t)$ is a linear function of t.

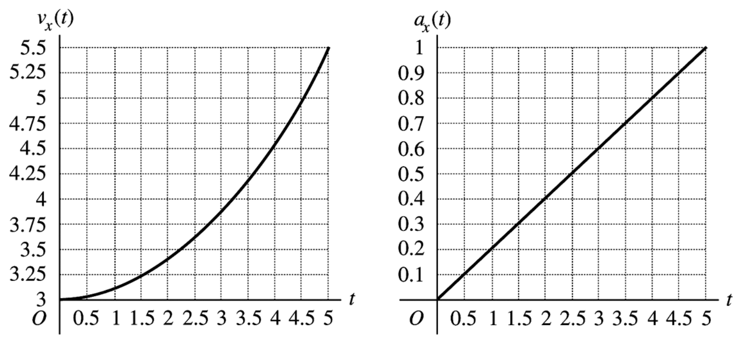


Figure 2.15

2.16. **IDENTIFY:** Use $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$, with $\Delta t = 10 \text{ s}$ in all cases.

SET UP: v_x is negative if the motion is to the left.

EXECUTE: **(a)** $[(5.0 \text{ m/s}) - (15.0 \text{ m/s})]/(10 \text{ s}) = -1.0 \text{ m/s}^2$

(b) $[(-15.0 \text{ m/s}) - (-5.0 \text{ m/s})]/(10 \text{ s}) = -1.0 \text{ m/s}^2$

(c) $[(-15.0 \text{ m/s}) - (+15.0 \text{ m/s})]/(10 \text{ s}) = -3.0 \text{ m/s}^2$

EVALUATE: In all cases, the negative acceleration indicates an acceleration to the left.

2.17. **IDENTIFY:** $v_x(t) = \dfrac{dx}{dt}$ and $a_x(t) = \dfrac{dv_x}{dt}$

SET UP: $\dfrac{d}{dt}(t^n) = nt^{n-1}$ for $n \geq 1$.

EXECUTE: **(a)** $v_x(t) = (9.60 \text{ m/s}^2)t - (0.600 \text{ m/s}^6)t^5$ and $a_x(t) = 9.60 \text{ m/s}^2 - (3.00 \text{ m/s}^6)t^4$. Setting $v_x = 0$ gives $t = 0$ and $t = 2.00 \text{ s}$. At $t = 0$, $x = 2.17 \text{ m}$ and $a_x = 9.60 \text{ m/s}^2$. At $t = 2.00 \text{ s}$, $x = 15.0 \text{ m}$ and $a_x = -38.4 \text{ m/s}^2$.

(b) The graphs are given in Figure 2.17.

EVALUATE: For the entire time interval from $t = 0$ to $t = 2.00 \text{ s}$, the velocity v_x is positive and x increases. While a_x is also positive the speed increases and while a_x is negative the speed decreases.

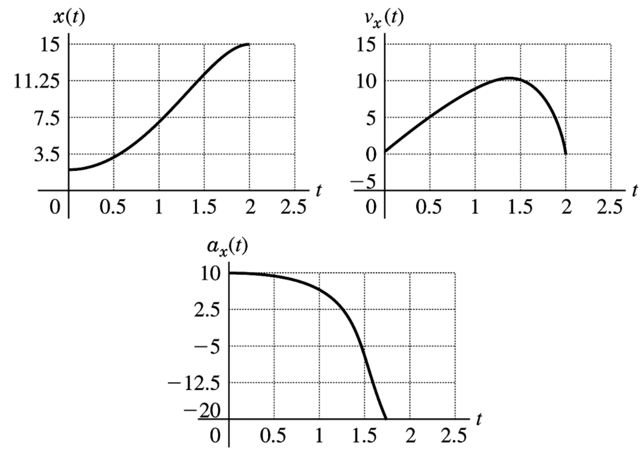


Figure 2.17

2.18. **IDENTIFY:** We have motion with constant acceleration, so the constant-acceleration equations apply. We want to determine the acceleration of a car on the entrance ramp of a highway.

SET UP: Estimate: 300 ft on the entrance ramp. We know the initial and final velocities and the distance traveled, and we want to find the acceleration. So $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ applies.

EXECUTE: Convert units: $30\text{ mph}\left(\dfrac{1.466\text{ ft/s}}{1\text{ mph}}\right) = 44\text{ ft/s}$; $70\text{ mph}\left(\dfrac{1.466\text{ ft/s}}{1\text{ mph}}\right) = 103\text{ ft/s}.$

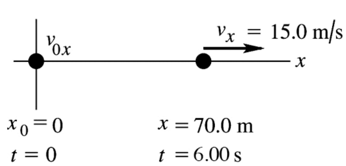
Now use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$: $(103\text{ ft/s})^2 = (44\text{ ft/s})^2 + 2a_x(300\text{ ft})$ $\rightarrow$ $a_x = 14\text{ ft/s}^2$.

In SI units, $a_x = 14\text{ ft/s}^2\left(\dfrac{0.3048\text{ m/s}^2}{1\text{ ft/s}^2}\right) = 4.4\text{ m/s}^2$.

EVALUATE: Compare with acceleration due to gravity: $a/g = (14\text{ ft/s}^2)/(32\text{ ft/s}^2) = 0.45$, so this acceleration is around 45% of g. This seems rather large for an ordinary car. Calculate the time to reach the 70 mph speed, using $v_x = v_{0x} + a_x t$: $103\text{ ft/s} = 44\text{ ft/s} + (14\text{ ft/s}^2)t$ $\rightarrow$ $t = 4.2$ s. This is too small to be reasonable for most cars. My estimate of the length of the on-ramp must be too small.

2.19. **IDENTIFY:** Use the constant acceleration equations to find v_{0x} and a_x.

(a) SET UP: The situation is sketched in Figure 2.19.



$x - x_0 = 70.0$ m
$t = 6.00$ s
$v_x = 15.0$ m/s
$v_{0x} = ?$

Figure 2.19

EXECUTE: Use $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$, so $v_{0x} = \dfrac{2(x - x_0)}{t} - v_x = \dfrac{2(70.0\text{ m})}{6.00\text{ s}} - 15.0\text{ m/s} = 8.33\text{ m/s}.$

(b) Use $v_x = v_{0x} + a_x t$, so $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{15.0\text{ m/s} - 5.0\text{ m/s}}{6.00\text{ s}} = 1.11\text{ m/s}^2$.

EVALUATE: The average velocity is $(70.0\text{ m})/(6.00\text{ s}) = 11.7$ m/s. The final velocity is larger than this, so the antelope must be speeding up during the time interval; $v_{0x} < v_x$ and $a_x > 0$.

2.20. **IDENTIFY:** For constant acceleration, the standard kinematics equations apply.

SET UP: Assume the ball moves in the $+x$ direction.

EXECUTE: **(a)** $v_x = 73.14\text{ m/s}$, $v_{0x} = 0$ and $t = 30.0\text{ ms}$. $v_x = v_{0x} + a_x t$ gives

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{73.14\text{ m/s} - 0}{30.0\times10^{-3}\text{ s}} = 2440\text{ m/s}^2.$$

(b) $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = \left(\frac{0 + 73.14\text{ m/s}}{2}\right)(30.0\times10^{-3}\text{ s}) = 1.10\text{ m}$.

EVALUATE: We could also use $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to calculate $x - x_0$:

$x - x_0 = \frac{1}{2}(2440\text{ m/s}^2)(30.0\times10^{-3}\text{ s})^2 = 1.10\text{ m}$, which agrees with our previous result. The acceleration of the ball is very large.

2.21. **IDENTIFY:** For constant acceleration, the standard kinematics equations apply.

SET UP: Assume the ball starts from rest and moves in the $+x$-direction.

EXECUTE: **(a)** $x - x_0 = 1.50\text{ m}$, $v_x = 45.0\text{ m/s}$ and $v_{0x} = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(45.0\text{ m/s})^2}{2(1.50\text{ m})} = 675\text{ m/s}^2.$$

(b) $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ gives $t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(1.50\text{ m})}{45.0\text{ m/s}} = 0.0667\text{ s}$

EVALUATE: We could also use $v_x = v_{0x} + a_x t$ to find $t = \frac{v_x}{a_x} = \frac{45.0\text{ m/s}}{675\text{ m/s}^2} = 0.0667\text{ s}$ which agrees with our previous result. The acceleration of the ball is very large.

2.22. **IDENTIFY:** A car is slowing down with uniform acceleration, so the constant-acceleration equations apply. We want to determine the acceleration of the car as it slows down.

SET UP: Estimate: It takes 5.0 s to reduce the speed from 70 mph to 30 mph. $v_x = v_{0x} + a_x t$,

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, and $a_{x\text{-av}} = \frac{\Delta v_x}{\Delta t}$ apply. Call the $+x$-axis the direction in which the car is moving.

EXECUTE: **(a)** Δv_x = 30 mph – 70 mph = –40 mph = –59 ft/s, so $a_{x\text{-av}} = \frac{-59\text{ ft/s}}{5.0\text{ s}} = -12\text{ ft/s}^2$. The magnitude is 12 ft/s^2 and its direction is opposite to the velocity of the car.

(b) The initial velocity is 70 mph = 103 ft/s and $v_x = 0$ when the car stops, so $v_x = v_{0x} + a_x t$ gives $0 = 103\text{ ft/s} + (-12\text{ ft/s}^2)t \rightarrow t = 8.6\text{ s}$. This is the time from first hitting the brakes. The time to stop from 30 mph is 8.6 s – 5.0 s = 3.6 s.

(c) Use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ since we know everything in it except the distance traveled.

$0 = (103\text{ ft/s})^2 + 2(-12\text{ ft/s}^2)(x - x_0) \quad \rightarrow \quad x - x_0 = 440\text{ ft}$.

EVALUATE: As a check, use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to calculate the distance in (c). This gives

$x = 0 + (103\text{ ft/s})(8.6\text{ s}) + \frac{1}{2}(-12\text{ ft/s}^2)(8.6\text{ s})^2 = 440\text{ ft}$, which agrees with our answer. Compare the acceleration to g: $a_x g = (12\text{ ft/s}^2)/(32\text{ ft/s}^2) = 0.38$, so a_x is about 38% of g. This is fairly large, but if you really slam on your brakes, it might be reasonable.

2.23. **IDENTIFY:** Assume that the acceleration is constant and apply the constant acceleration kinematic equations. Set $|a_x|$ equal to its maximum allowed value.

SET UP: Let $+x$ be the direction of the initial velocity of the car. $a_x = -250\text{ m/s}^2$.

$105\text{ km/h} = 29.17\text{ m/s}$.

EXECUTE: $v_{0x} = 29.17$ m/s. $v_x = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (29.17 \text{ m/s})^2}{2(-250 \text{ m/s}^2)} = 1.70 \text{ m.}$$

EVALUATE: The car frame stops over a shorter distance and has a larger magnitude of acceleration. Part of your 1.70 m stopping distance is the stopping distance of the car and part is how far you move relative to the car while stopping.

2.24. **IDENTIFY:** Apply constant acceleration equations to the motion of the car.
SET UP: Let $+x$ be the direction the car is moving.

EXECUTE: **(a)** From $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, with $v_{0x} = 0$, $a_x = \dfrac{v_x^2}{2(x - x_0)} = \dfrac{(20 \text{ m/s})^2}{2(120 \text{ m})} = 1.67 \text{ m/s}^2$.

(b) Using $x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$, we have $t = 2(x - x_0)/v_x = 2(120 \text{ m})/(20 \text{ m/s}) = 12$ s.

(c) $(12 \text{ s})(20 \text{ m/s}) = 240$ m.

EVALUATE: The average velocity of the car is half the constant speed of the traffic, so the traffic travels twice as far.

2.25. **IDENTIFY:** If a person comes to a stop in 36 ms while slowing down with an acceleration of 60*g*, how far does he travel during this time?
SET UP: Let $+x$ be the direction the person travels. $v_x = 0$ (he stops), a_x is negative since it is opposite to the direction of the motion, and $t = 36 \text{ ms} = 3.6 \times 10^{-2}$ s. The equations $v_x = v_{0x} + a_x t$ and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ both apply since the acceleration is constant.

EXECUTE: Solving $v_x = v_{0x} + a_x t$ for v_{0x} gives $v_{0x} = -a_x t$. Then $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives $x = -\frac{1}{2}a_x t^2 = -\frac{1}{2}(-588 \text{ m/s}^2)(3.6 \times 10^{-2} \text{ s})^2 = 38$ cm.

EVALUATE: Notice that we were not given the initial speed, but we could find it:

$$v_{0x} = -a_x t = -(-588 \text{ m/s}^2)(36 \times 10^{-3} \text{ s}) = 21 \text{ m/s} = 47 \text{ mph.}$$

2.26. **IDENTIFY:** The acceleration a_x is the slope of the graph of v_x versus *t*.

SET UP: The signs of v_x and of a_x indicate their directions.

EXECUTE: **(a)** Reading from the graph, at $t = 4.0$ s, $v_x = 2.7$ cm/s, to the right and at $t = 7.0$ s, $v_x = 1.3$ cm/s, to the left.

(b) v_x versus *t* is a straight line with slope $-\dfrac{8.0 \text{ cm/s}}{6.0 \text{ s}} = -1.3 \text{ cm/s}^2$. The acceleration is constant and equal to 1.3 cm/s^2, to the left. It has this value at all times.

(c) Since the acceleration is constant, $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$. Call $x_0 = 0$ the cat's position when $t = 0$.

<u>During the first 4.5 s</u>: $x = (8.0 \text{ cm/s})(4.5 \text{ s}) + \frac{1}{2}(-1.3 \text{ cm/s}^2)(4.5 \text{ s})^2 = 22.8$ cm which rounds to 23 cm.

This is the change in the cat's position, but it is also the distance the cat walks.
<u>During the first 7.5 s</u>: After 6.0 s, v_x becomes negative so the cat is walking backward. However the *distance* it is moving does *not* become negative. The position of the car at $t = 6.0$ s is $x = (8.0 \text{ cm/s})(6.0 \text{ s}) + \frac{1}{2}(-1.3 \text{ cm/s}^2)(6.0 \text{ s})^2 = 24.6$ cm. At the end of 7.5 s, the position is

$x = (8.0 \text{ cm/s})(7.5 \text{ s}) + \frac{1}{2}(-1.3 \text{ cm/s}^2)(7.5 \text{ s})^2 = 23.4$ cm. Therefore from $t = 6.0$ s to $t = 7.5$ s, the cat has walked back a *distance* of 24.6 cm – 23.4 cm = 1.2 cm. During the first 7.5 s, the cat has walked a total distance of 24.6 cm + 1.2 cm = 25.8 cm which rounds to 26 cm.
(d) The graphs of a_x and *x* versus *t* are given in Figure 2.26.

EVALUATE: In part (c) we could have instead used $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t.$

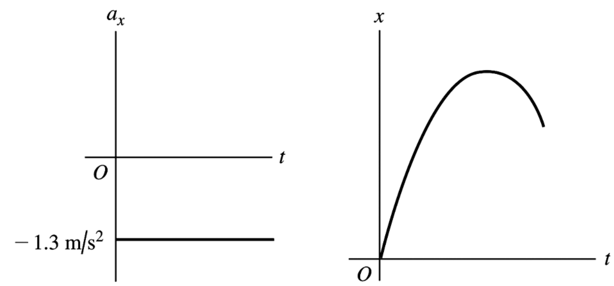


Figure 2.26

2.27. **IDENTIFY:** We know the initial and final velocities of the object, and the distance over which the velocity change occurs. From this we want to find the magnitude and duration of the acceleration of the object.

SET UP: The constant-acceleration kinematics formulas apply. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0),$ where $v_{0x} = 0,$ $v_x = 5.0\times10^3$ m/s, and $x - x_0 = 4.0$ m.

EXECUTE: **(a)** $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(5.0\times10^3\ \text{m/s})^2}{2(4.0\ \text{m})} = 3.1\times10^6\ \text{m/s}^2 = 3.2\times10^5\ g.$$

(b) $v_x = v_{0x} + a_x t$ gives $t = \frac{v_x - v_{0x}}{a_x} = \frac{5.0\times10^3\ \text{m/s}}{3.1\times10^6\ \text{m/s}^2} = 1.6\ \text{ms}.$

EVALUATE: **(c)** The calculated a is less than 450,000 g so the acceleration required doesn't rule out this hypothesis.

2.28. **IDENTIFY:** $v_x(t)$ is the slope of the x versus t graph. Car B moves with constant speed and zero acceleration. Car A moves with positive acceleration; assume the acceleration is constant.

SET UP: For car B, v_x is positive and $a_x = 0.$ For car A, a_x is positive and v_x increases with t.

EXECUTE: **(a)** The motion diagrams for the cars are given in Figure 2.28a.

(b) The two cars have the same position at times when their x-t graphs cross. The figure in the problem shows this occurs at approximately $t = 1$ s and $t = 3$ s.

(c) The graphs of v_x versus t for each car are sketched in Figure 2.28b.

(d) The cars have the same velocity when their x-t graphs have the same slope. This occurs at approximately $t = 2$ s.

(e) Car A passes car B when x_A moves above x_B in the x-t graph. This happens at $t = 3$ s.

(f) Car B passes car A when x_B moves above x_A in the x-t graph. This happens at $t = 1$ s.

EVALUATE: When $a_x = 0,$ the graph of v_x versus t is a horizontal line. When a_x is positive, the graph of v_x versus t is a straight line with positive slope.

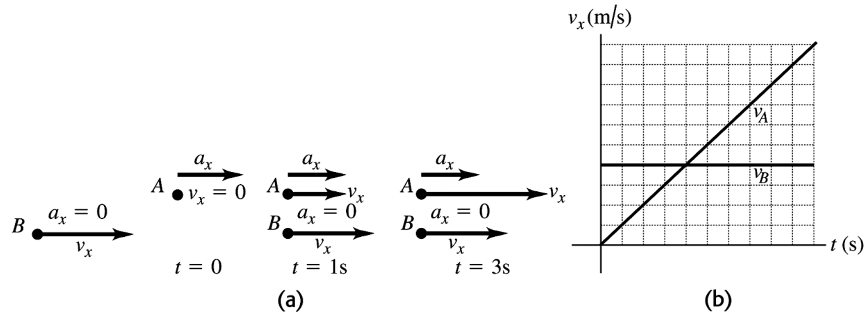


Figure 2.28

2.29. **(a) IDENTIFY** and **SET UP:** The acceleration a_x at time t is the slope of the tangent to the v_x versus t curve at time t.

EXECUTE: At $t = 3$ s, the v_x versus t curve is a horizontal straight line, with zero slope. Thus $a_x = 0$.

At $t = 7$ s, the v_x versus t curve is a straight-line segment with slope $\dfrac{45\text{ m/s} - 20\text{ m/s}}{9\text{ s} - 5\text{ s}} = 6.3\text{ m/s}^2$.

Thus $a_x = 6.3\text{ m/s}^2$.

At $t = 11$ s the curve is again a straight-line segment, now with slope $\dfrac{-0 - 45\text{ m/s}}{13\text{ s} - 9\text{ s}} = -11.2\text{ m/s}^2$.

Thus $a_x = -11.2\text{ m/s}^2$.

EVALUATE: $a_x = 0$ when v_x is constant, $a_x > 0$ when v_x is positive and the speed is increasing, and $a_x < 0$ when v_x is positive and the speed is decreasing.

(b) IDENTIFY: Calculate the displacement during the specified time interval.

SET UP: We can use the constant acceleration equations only for time intervals during which the acceleration is constant. If necessary, break the motion up into constant acceleration segments and apply the constant acceleration equations for each segment. For the time interval $t = 0$ to $t = 5$ s the acceleration is constant and equal to zero. For the time interval $t = 5$ s to $t = 9$ s the acceleration is constant and equal to 6.25 m/s^2. For the interval $t = 9$ s to $t = 13$ s the acceleration is constant and equal to -11.2 m/s^2.

EXECUTE: During the first 5 seconds the acceleration is constant, so the constant acceleration kinematic formulas can be used.

$v_{0x} = 20\text{ m/s}$ $\quad a_x = 0$ $\quad t = 5\text{ s}$ $\quad x - x_0 = ?$

$x - x_0 = v_{0x}t$ ($a_x = 0$ so no $\frac{1}{2}a_x t^2$ term)

$x - x_0 = (20\text{ m/s})(5\text{ s}) = 100\text{ m}$; this is the distance the officer travels in the first 5 seconds.

During the interval $t = 5$ s to 9 s the acceleration is again constant. The constant acceleration formulas can be applied to this 4-second interval. It is convenient to restart our clock so the interval starts at time $t = 0$ and ends at time $t = 4$ s. (Note that the acceleration is *not* constant over the entire $t = 0$ to $t = 9$ s interval.)

$v_{0x} = 20\text{ m/s}$ $\quad a_x = 6.25\text{ m/s}^2$ $\quad t = 4\text{ s}$ $\quad x_0 = 100\text{ m}$ $\quad x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$x - x_0 = (20\text{ m/s})(4\text{ s}) + \frac{1}{2}(6.25\text{ m/s}^2)(4\text{ s})^2 = 80\text{ m} + 50\text{ m} = 130\text{ m}.$

Thus $x - x_0 + 130\text{ m} = 100\text{ m} + 130\text{ m} = 230\text{ m}$.

At $t = 9$ s the officer is at $x = 230$ m, so she has traveled 230 m in the first 9 seconds.

During the interval $t = 9\text{ s}$ to $t = 13\text{ s}$ the acceleration is again constant. The constant acceleration formulas can be applied for this 4-second interval but *not* for the whole $t = 0$ to $t = 13\text{ s}$ interval. To use the equations restart our clock so this interval begins at time $t = 0$ and ends at time $t = 4\text{ s}$.

$v_{0x} = 45\text{ m/s}$ (at the start of this time interval)

$a_x = -11.2\text{ m/s}^2$ $t = 4\text{ s}$ $x_0 = 230\text{ m}$ $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$x - x_0 = (45\text{ m/s})(4\text{ s}) + \frac{1}{2}(-11.2\text{ m/s}^2)(4\text{ s})^2 = 180\text{ m} - 89.6\text{ m} = 90.4\text{ m}.$

Thus $x = x_0 + 90.4\text{ m} = 230\text{ m} + 90.4\text{ m} = 320\text{ m}.$

At $t = 13\text{ s}$ the officer is at $x = 320\text{ m},$ so she has traveled 320 m in the first 13 seconds.

EVALUATE: The velocity v_x is always positive so the displacement is always positive and displacement and distance traveled are the same. The average velocity for time interval Δt is $v_{\text{av-}x} = \Delta x/\Delta t.$ For $t = 0$ to 5 s, $v_{\text{av-}x} = 20\text{ m/s}.$ For $t = 0$ to 9 s, $v_{\text{av-}x} = 26\text{ m/s}.$ For $t = 0$ to 13 s, $v_{\text{av-}x} = 25\text{ m/s}.$ These results are consistent with the figure in the textbook.

2.30. **IDENTIFY:** For constant acceleration, the kinematics formulas apply. We can use the total displacement and final velocity to calculate the acceleration and then use the acceleration and shorter distance to find the speed.

SET UP: Take $+x$ to be down the incline, so the motion is in the $+x$ direction. The formula $v_x^2 = v_{0x}^2 + 2a(x - x_0)$ applies.

EXECUTE: First look at the motion over 6.80 m. We use the following numbers: $v_{0x} = 0$, $x - x_0 = 6.80$ m, and $v_x = 3.80$ /s. Solving the above equation for a_x gives $a_x = 1.062\text{ m/s}^2$. Now look at the motion over the 3.40 m using $v_{0x} = 0$, $a_x = 1.062\text{ m/s}^2$ and $x - x_0 = 3.40$ m. Solving the same equation, but this time for v_x, gives $v_x = 2.69$ m/s.

EVALUATE: Even though the block has traveled half way down the incline, its speed is not half of its speed at the bottom.

2.31. **IDENTIFY:** Apply the constant acceleration equations to the motion of the flea. After the flea leaves the ground, $a_y = g,$ downward. Take the origin at the ground and the positive direction to be upward.

(a) SET UP: At the maximum height $v_y = 0.$

$v_y = 0$ $y - y_0 = 0.440\text{ m}$ $a_y = -9.80\text{ m/s}^2$ $v_{0y} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\text{ m/s}^2)(0.440\text{ m})} = 2.94\text{ m/s}$

(b) SET UP: When the flea has returned to the ground $y - y_0 = 0.$

$y - y_0 = 0$ $v_{0y} = +2.94\text{ m/s}$ $a_y = -9.80\text{ m/s}^2$ $t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(2.94\text{ m/s})}{-9.80\text{ m/s}^2} = 0.600\text{ s}.$

EVALUATE: We can use $v_y = v_{0y} + a_y t$ to show that with $v_{0y} = 2.94\text{ m/s},$ $v_y = 0$ after 0.300 s.

2.32. **IDENTIFY:** The rock has a constant downward acceleration of 9.80 m/s^2. We know its initial velocity and position and its final position.

SET UP: We can use the kinematics formulas for constant acceleration.

EXECUTE: **(a)** $y - y_0 = -30\text{ m}$, $v_{0y} = 22.0\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$. The kinematics formulas give $v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)} = -\sqrt{(22.0\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-30\text{ m})} = -32.74\text{ m/s}$, so the speed is 32.7 m/s.

(b) $v_y = v_{0y} + a_y t$ and $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-32.74\text{ m/s} - 22.0\text{ m/s}}{-9.80\text{ m/s}^2} = 5.59\text{ s}$.

EVALUATE: The vertical velocity in part (a) is negative because the rock is moving downward, but the speed is always positive. The 5.59 s is the total time in the air.

2.33. **IDENTIFY:** The pin has a constant downward acceleration of 9.80 m/s^2 and returns to its initial position.
SET UP: We can use the kinematics formulas for constant acceleration.

EXECUTE: The kinematics formulas give $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. We know that $y - y_0 = 0$, so

$t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(8.20\text{ m/s})}{-9.80\text{ m/s}^2} = +1.67\text{ s}$.

EVALUATE: It takes the pin half this time to reach its highest point and the remainder of the time to return.

2.34. **IDENTIFY:** The putty has a constant downward acceleration of 9.80 m/s^2. We know the initial velocity of the putty and the distance it travels.
SET UP: We can use the kinematics formulas for constant acceleration.
EXECUTE: **(a)** $v_{0y} = 9.50$ m/s and $y - y_0 = 3.60$ m, which gives
$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(9.50\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(3.60\text{ m})} = 4.44\text{ m/s}$

(b) $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{4.44\text{ m/s} - 9.50\text{ m/s}}{-9.8\text{ m/s}^2} = 0.517\text{ s}$

EVALUATE: The putty is stopped by the ceiling, not by gravity.

2.35. **IDENTIFY:** A ball on Mars that is hit directly upward returns to the same level in 8.5 s with a constant downward acceleration of $0.379g$. How high did it go and how fast was it initially traveling upward?
SET UP: Take $+y$ upward. $v_y = 0$ at the maximum height. $a_y = -0.379g = -3.71\text{ m/s}^2$. The constant-acceleration formulas $v_y = v_{0y} + a_y t$ and $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ both apply.

EXECUTE: Consider the motion from the maximum height back to the initial level. For this motion $v_{0y} = 0$ and $t = 4.25$ s. $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.71\text{ m/s}^2)(4.25\text{ s})^2 = -33.5\text{ m}$. The ball went 33.5 m above its original position.
(b) Consider the motion from just after it was hit to the maximum height. For this motion $v_y = 0$ and $t = 4.25$ s. $v_y = v_{0y} + a_y t$ gives $v_{0y} = -a_y t = -(-3.71\text{ m/s}^2)(4.25\text{ s}) = 15.8\text{ m/s}$.
(c) The graphs are sketched in Figure 2.35.

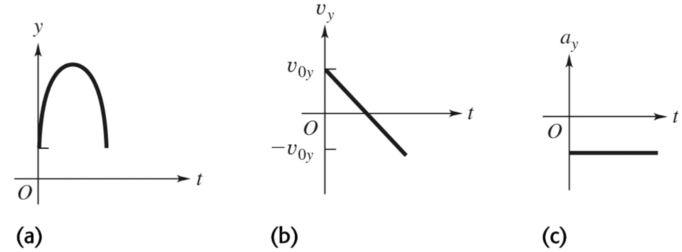


Figure 2.35

EVALUATE: The answers can be checked several ways. For example, $v_y = 0,\ v_{0y} = 15.8\ \text{m/s},$ and

$a_y = -3.71\ \text{m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (15.8\ \text{m/s})^2}{2(-3.71\ \text{m/s}^2)} = 33.6\ \text{m},$

which agrees with the height calculated in (a).

2.36. **IDENTIFY:** A baseball is thrown upward, so its acceleration is downward and uniform. Therefore the constant-acceleration equations apply.

SET UP: Estimate: It is not easy to throw a ball straight up, so estimate 25 ft for the maximum height. Its acceleration is g = 32.2 ft/s^2 downward, and the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. Call the +y-axis upward, with the origin at the point where the ball leaves the hand; this makes $y_0 = 0$ and $a_y = -32.2$ ft/s^2. At its maximum height, the ball stops moving, so $v_y = 0$.

EXECUTE: **(a)** First find the initial speed of the ball using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $v_y = 0$ at the maximum height. $0 = v_{0y}^2 + 2(-32.2\ \text{ft/s}^2)(25\ \text{ft}) \quad \to \quad v_{0y} = 40.1\ \text{ft/s}$. Now use

$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ to get the time t to reach the maximum height.'

$0 = 0 + (40.1\ \text{ft/s})t + \frac{1}{2}(-32.2\ \text{ft/s}^2)t^2 \quad \to \quad t = 2.5\ \text{s}.$

(b) Estimate: The ball moves about 2.5 ft while it is being thrown. We want the average accelerating during this time. The ball starts from rest and reaches a velocity of 40.1 ft/s while traveling 2.5 ft upward. So we use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$.

$(40.1\ \text{ft/s})^2 = 0 + 2a_y(2.5\ \text{ft}) \quad \to \quad a_y = 320\ \text{ft/s}^2 \approx 10g.$

EVALUATE: The result in (b) seems rather large, so maybe the 2.5-ft estimate was too short, or maybe the maximum height of 25 ft was too large.

2.37. **IDENTIFY:** A rock is thrown upward, so its acceleration is downward and uniform. Therefore the constant-acceleration equations apply. We want to know the rock's velocity at times 1.0 s and 3.0 s after it is thrown.

SET UP: The formula $v_y = v_{0y} + a_y t$ applies. Call the +y-axis upward, with the origin at the point where the rock leaves the hand; this makes $y_0 = 0$, $v_{0y} = 24.0$ m/s, and $a_y = -9.80$ m/s^2.

EXECUTE: **(a)** At 1.0 s: $v_y = v_{0y} + a_y t = 24.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(1.0\ \text{s}) = +14.2\ \text{m/s}$. The acceleration is downward. The velocity is upward but the speed is decreasing because the acceleration is downward.

(b) At 3.0 s: $v_y = v_{0y} + a_y t = 24.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(3.0\ \text{s}) = -5.40\ \text{m/s}$. The acceleration is downward. The velocity is downward and the speed is increasing because the acceleration is also downward. The rock has passed its highest point and is now coming down.

EVALUATE: If only gravity acts on an object, its acceleration is always downward with a magnitude of 9.80 m/s^2.

2.38. **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the brick.

SET UP: Let $+y$ be downward. $a_y = 9.80\ \text{m/s}^2$

EXECUTE: **(a)**

$v_{0y} = 0,\ t = 1.90\ \text{s},\ a_y = 9.80\ \text{m/s}^2.\ y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\ \text{m/s}^2)(1.90\ \text{s})^2 = 17.7\ \text{m}.$

The building is 17.7 m tall.

(b) $v_y = v_{0y} + a_y t = 0 + (9.80\ \text{m/s}^2)(1.90\ \text{s}) = 18.6\ \text{m/s}$

(c) The graphs of a_y, v_y and y versus t are given in Figure 2.38. Take $y = 0$ at the ground.

EVALUATE: We could use either $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ or $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to check our results.

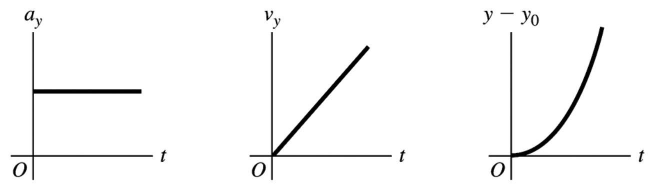


Figure 2.38

2.39. **IDENTIFY:** Apply constant acceleration equations to the motion of the meterstick. The time the meterstick falls is your reaction time.

SET UP: Let $+y$ be downward. The meter stick has $v_{0y} = 0$ and $a_y = 9.80\ \text{m/s}^2$. Let d be the distance the meterstick falls.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $d = (4.90\ \text{m/s}^2)t^2$ and $t = \sqrt{\frac{d}{4.90\ \text{m/s}^2}}$.

(b) $t = \sqrt{\frac{0.176\ \text{m}}{4.90\ \text{m/s}^2}} = 0.190\ \text{s}$

EVALUATE: The reaction time is proportional to the square of the distance the stick falls.

2.40. **IDENTIFY:** Apply constant acceleration equations to the motion of the lander.

SET UP: Let $+y$ be downward. Since the lander is in free-fall, $a_y = +1.6\ \text{m/s}^2$.

EXECUTE: $v_{0y} = 0.8\ \text{m/s}$, $y - y_0 = 5.0\ \text{m}$, $a_y = +1.6\ \text{m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(0.8\ \text{m/s})^2 + 2(1.6\ \text{m/s}^2)(5.0\ \text{m})} = 4.1\ \text{m/s}.$

EVALUATE: The same descent on earth would result in a final speed of 9.9 m/s, since the acceleration due to gravity on earth is much larger than on the moon.

2.41. **IDENTIFY:** When the only force is gravity the acceleration is $9.80\ \text{m/s}^2$, downward. There are two intervals of constant acceleration and the constant acceleration equations apply during each of these intervals.

SET UP: Let $+y$ be upward. Let $y = 0$ at the launch pad. The final velocity for the first phase of the motion is the initial velocity for the free-fall phase.

EXECUTE: **(a)** Find the velocity when the engines cut off. $y - y_0 = 525\ \text{m}$, $a_y = 2.25\ \text{m/s}^2$, $v_{0y} = 0$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(2.25\ \text{m/s}^2)(525\ \text{m})} = 48.6\ \text{m/s}$.

Now consider the motion from engine cut-off to maximum height: $y_0 = 525\ \text{m}$, $v_{0y} = +48.6\ \text{m/s}$, $v_y = 0$ (at the maximum height), $a_y = -9.80\ \text{m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (48.6\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 121\ \text{m}$ and $y = 121\ \text{m} + 525\ \text{m} = 646\ \text{m}$.

(b) Consider the motion from engine failure until just before the rocket strikes the ground: $y - y_0 = -525\ \text{m}$, $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +48.6\ \text{m/s}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = -\sqrt{(48.6\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-525\ \text{m})} = -112\ \text{m/s}$. Then $v_y = v_{0y} + a_y t$ gives

$t = \frac{v_y - v_{0y}}{a_y} = \frac{-112\ \text{m/s} - 48.6\ \text{m/s}}{-9.80\ \text{m/s}^2} = 16.4\ \text{s}.$

(c) Find the time from blast-off until engine failure: $y-y_0=525\text{ m}$, $v_{0y}=0$, $a_y=+2.25\text{ m/s}^2$.

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(525\text{ m})}{2.25\text{ m/s}^2}}=21.6\text{ s}$. The rocket strikes the launch pad $21.6\text{ s}+16.4\text{ s}=38.0\text{ s}$ after blast-off. The acceleration a_y is $+2.25\text{ m/s}^2$ from $t=0$ to $t=21.6\text{ s}$. It is -9.80 m/s^2 from $t=21.6\text{ s}$ to 38.0 s. $v_y=v_{0y}+a_yt$ applies during each constant acceleration segment, so the graph of v_y versus t is a straight line with positive slope of 2.25 m/s^2 during the blast-off phase and with negative slope of -9.80 m/s^2 after engine failure. During each phase $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$. The sign of a_y determines the curvature of $y(t)$. At $t=38.0\text{ s}$ the rocket has returned to $y=0$. The graphs are sketched in Figure 2.41.

EVALUATE: In part (b) we could have found the time from $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$, finding v_y first allows us to avoid solving for t from a quadratic equation.

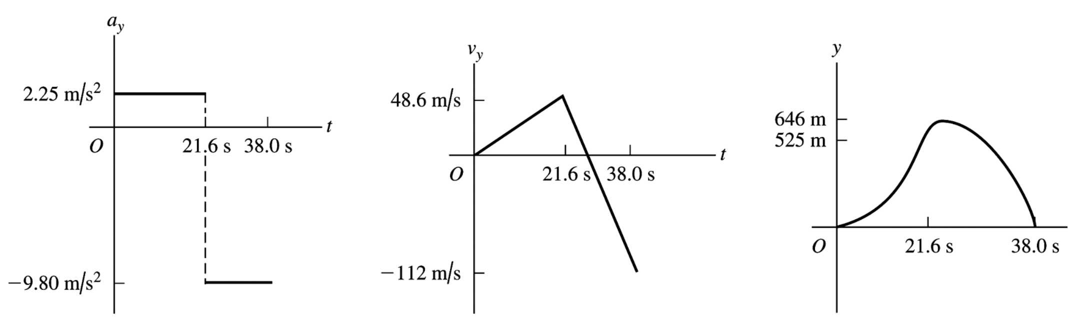


Figure 2.41

2.42. IDENTIFY: Apply constant acceleration equations to the vertical motion of the sandbag.

SET UP: Take $+y$ upward. $a_y=-9.80\text{ m/s}^2$. The initial velocity of the sandbag equals the velocity of the balloon, so $v_{0y}=+5.00\text{ m/s}$. When the balloon reaches the ground, $y-y_0=-40.0\text{ m}$. At its maximum height the sandbag has $v_y=0$.

EXECUTE: (a)

$t=0.250\text{ s}$: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2=(5.00\text{ m/s})(0.250\text{ s})+\frac{1}{2}(-9.80\text{ m/s}^2)(0.250\text{ s})^2=0.94\text{ m}$. The sandbag is 40.9 m above the ground. $v_y=v_{0y}+a_yt=+5.00\text{ m/s}+(-9.80\text{ m/s}^2)(0.250\text{ s})=2.55\text{ m/s}$.

$t=1.00\text{ s}$: $y-y_0=(5.00\text{ m/s})(1.00\text{ s})+\frac{1}{2}(-9.80\text{ m/s}^2)(1.00\text{ s})^2=0.10\text{ m}$. The sandbag is 40.1 m above the ground. $v_y=v_{0y}+a_yt=+5.00\text{ m/s}+(-9.80\text{ m/s}^2)(1.00\text{ s})=-4.80\text{ m/s}$.

(b) $y-y_0=-40.0\text{ m}$, $v_{0y}=5.00\text{ m/s}$, $a_y=-9.80\text{ m/s}^2$. $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $-40.0\text{ m}=(5.00\text{ m/s})t-(4.90\text{ m/s}^2)t^2$. $(4.90\text{ m/s}^2)t^2-(5.00\text{ m/s})t-40.0\text{ m}=0$ and

$t=\dfrac{1}{9.80}\left(5.00\pm\sqrt{(-5.00)^2-4(4.90)(-40.0)}\right)\text{ s}=(0.51\pm 2.90)\text{ s}$. t must be positive, so $t=3.41\text{ s}$.

(c) $v_y=v_{0y}+a_yt=+5.00\text{ m/s}+(-9.80\text{ m/s}^2)(3.41\text{ s})=-28.4\text{ m/s}$

(d) $v_{0y}=5.00\text{ m/s}$, $a_y=-9.80\text{ m/s}^2$, $v_y=0$. $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives

$y-y_0=\dfrac{v_y^2-v_{0y}^2}{2a_y}=\dfrac{0-(5.00\text{ m/s})^2}{2(-9.80\text{ m/s}^2)}=1.28\text{ m}$. The maximum height is 41.3 m above the ground.

(e) The graphs of a_y, v_y, and y versus t are given in Figure 2.42. Take $y = 0$ at the ground.

EVALUATE: The sandbag initially travels upward with decreasing velocity and then moves downward with increasing speed.

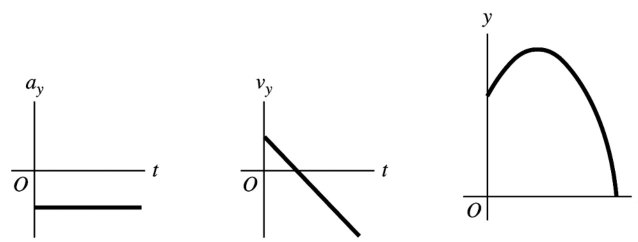


Figure 2.42

2.43. **IDENTIFY:** You throw a rock thrown upward, so its acceleration is downward and uniform. Therefore the constant-acceleration equations apply.

SET UP: We want to know how high the rock went if it returned to your hand 3.60 s after you threw it upward. With constant downward acceleration, the time it takes the rock to reach its highest point is the same as the time to fall back to your hand. Therefore it took 1.80 s to reach that highest point, and the rock's velocity was zero at the point. We can use $v_y = v_{0y} + a_y t$ to find the initial speed and then use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find how high it went.

EXECUTE: First find v_{0y} using $v_y = v_{0y} + a_y t$ with $v_y = 0$ at the highest point.

$0 = v_{0y} - gt$, so $v_{0y} = (9.80\text{ m/s}^2)(1.80\text{ s}) = 17.64\text{ m/s}$. Now use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find the maximum height the rock reached. $0 = (17.64\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(y - y_0) \rightarrow y - y_0 = 15.9\text{ m}$.

EVALUATE: Check by using $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ to calculate y at the maximum height.

$y = 0 + (17.64\text{ m/s})(1.80\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(1.80\text{ s})^2 = 15.9\text{ m}$, which agrees with our result.

2.44. **IDENTIFY:** Since air resistance is ignored, the egg is in free-fall and has a constant downward acceleration of magnitude 9.80 m/s^2. Apply the constant acceleration equations to the motion of the egg.

SET UP: Take $+y$ to be upward. At the maximum height, $v_y = 0$.

EXECUTE: **(a)** $y - y_0 = -30.0\text{ m}$, $t = 5.00\text{ s}$, $a_y = -9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$v_{0y} = \frac{y - y_0}{t} - \tfrac{1}{2}a_y t = \frac{-30.0\text{ m}}{5.00\text{ s}} - \tfrac{1}{2}(-9.80\text{ m/s}^2)(5.00\text{ s}) = +18.5\text{ m/s}.$$

(b) $v_{0y} = +18.5\text{ m/s}$, $v_y = 0$ (at the maximum height), $a_y = -9.80\text{ m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (18.5\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 17.5\text{ m}.$$

(c) At the maximum height $v_y = 0$.

(d) The acceleration is constant and equal to 9.80 m/s^2, downward, at all points in the motion, including at the maximum height.

(e) The graphs are sketched in Figure 2.44.

EVALUATE: The time for the egg to reach its maximum height is $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-18.5\text{ m/s}}{-9.8\text{ m/s}^2} = 1.89\text{ s}.$

The egg has returned to the level of the cornice after 3.78 s and after 5.00 s it has traveled downward from the cornice for 1.22 s.

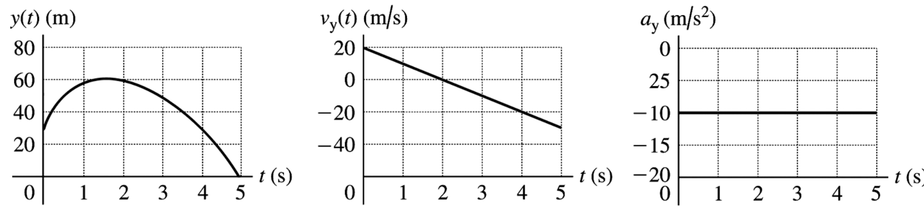


Figure 2.44

2.45. **IDENTIFY:** We can avoid solving for the common height by considering the relation between height, time of fall, and acceleration due to gravity, and setting up a ratio involving time of fall and acceleration due to gravity.

SET UP: Let g_{En} be the acceleration due to gravity on Enceladus and let g be this quantity on earth. Let h be the common height from which the object is dropped. Let $+y$ be downward, so $y - y_0 = h.\ v_{0y} = 0$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = \frac{1}{2}g t_{\text{E}}^2$ and $h = \frac{1}{2}g_{\text{En}} t_{\text{En}}^2$. Combining these two equations gives $g t_{\text{E}}^2 = g_{\text{En}} t_{\text{En}}^2$ and $g_{\text{En}} = g\left(\dfrac{t_{\text{E}}}{t_{\text{En}}}\right)^2 = (9.80\text{ m/s}^2)\left(\dfrac{1.75\text{ s}}{18.6\text{ s}}\right)^2 = 0.0868\text{ m/s}^2.$

EVALUATE: The acceleration due to gravity is inversely proportional to the square of the time of fall.

2.46. **IDENTIFY:** Since air resistance is ignored, the boulder is in free-fall and has a constant downward acceleration of magnitude 9.80 m/s^2. Apply the constant acceleration equations to the motion of the boulder.

SET UP: Take $+y$ to be upward.

EXECUTE: **(a)** $v_{0y} = +40.0\text{ m/s},\ v_y = +20.0\text{ m/s},\ a_y = -9.80\text{ m/s}^2.\ v_y = v_{0y} + a_y t$ gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{20.0\text{ m/s} - 40.0\text{ m/s}}{-9.80\text{ m/s}^2} = +2.04\text{ s}.$$

(b) $v_y = -20.0\text{ m/s}.\ t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-20.0\text{ m/s} - 40.0\text{ m/s}}{-9.80\text{ m/s}^2} = +6.12\text{ s}.$

(c) $y - y_0 = 0,\ v_{0y} = +40.0\text{ m/s},\ a_y = -9.80\text{ m/s}^2.\ y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 0$ and

$$t = -\frac{2v_{0y}}{a_y} = -\frac{2(40.0\text{ m/s})}{-9.80\text{ m/s}^2} = +8.16\text{ s}.$$

(d) $v_y = 0,\ v_{0y} = +40.0\text{ m/s},\ a_y = -9.80\text{ m/s}^2.\ v_y = v_{0y} + a_y t$ gives $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{0 - 40.0\text{ m/s}}{-9.80\text{ m/s}^2} = 4.08\text{ s}.$

(e) The acceleration is 9.80 m/s^2, downward, at all points in the motion.

(f) The graphs are sketched in Figure 2.46.

EVALUATE: $v_y = 0$ at the maximum height. The time to reach the maximum height is half the total time in the air, so the answer in part (d) is half the answer in part (c). Also note that $2.04\text{ s} < 4.08\text{ s} < 6.12\text{ s}$. The boulder is going upward until it reaches its maximum height and after the maximum height it is traveling downward.

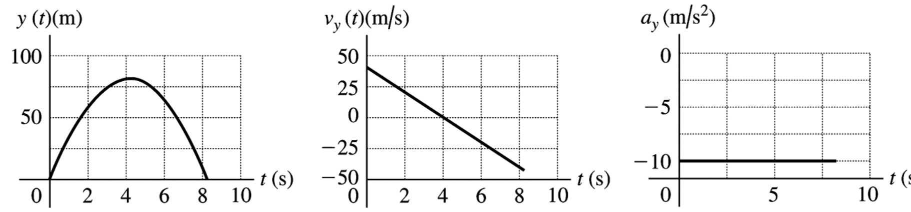


Figure 2.46

2.47. **IDENTIFY:** The rock has a constant downward acceleration of 9.80 m/s^2. The constant-acceleration kinematics formulas apply.

SET UP: The formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply. Call $+y$ upward. First find the initial velocity and then the final speed.

EXECUTE: **(a)** 6.00 s after it is thrown, the rock is back at its original height, so $y = y_0$ at that instant. Using $a_y = -9.80\text{ m/s}^2$ and $t = 6.00$ s, the equation $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$ gives $v_{0y} = 29.4$ m/s. When the rock reaches the water, $y - y_0 = -28.0$ m. The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = -37.6$ m/s, so its speed is 37.6 m/s.

EVALUATE: The final speed is greater than the initial speed because the rock accelerated on its way down below the bridge.

2.48. **IDENTIFY:** We want to interpret a graph of v_x versus t.

SET UP: The area under a graph of v_x versus t is equal to the change in position $x_2 - x_1$. The average velocity is $v_{x\text{-av}} = \dfrac{\Delta x}{\Delta t}$.

EXECUTE: **(a)** The area of a triangle is $\frac{1}{2}bh$, which gives $A = \frac{1}{2}(6.0\text{ s})(8.0\text{ m/s}) = 24$ m.

(b) The area under a graph of v_x versus t is equal to the change in position $x_2 - x_1$, which in this case is the distance traveled, which is 24 m in 6.0 s. Thus $v_{x\text{-av}} = \dfrac{\Delta x}{\Delta t} = \dfrac{24\text{ m}}{6.0\text{ s}} = 4.0$ m/s.

(c) $\Delta x = v_{x\text{-av}}\,\Delta t = (4.0\text{ m/s})(6.0\text{ s}) = 24$ m, which is the same as the area in (a).

EVALUATE: For constant acceleration $v_{x\text{-av}} = \dfrac{v_{x\text{-}0} + v_x}{2} = \dfrac{8.0\text{ m/s} + 0}{2} = 4.0$ m/s, which agrees with our answer in (b).

2.49. **IDENTIFY:** The acceleration is not constant, but we know how it varies with time. We can use the definitions of instantaneous velocity and position to find the rocket's position and speed.

SET UP: The basic definitions of velocity and position are $v_y(t) = v_{0y} + \int_0^t a_y dt$ and $y - y_0 = \int_0^t v_y dt$.

EXECUTE: **(a)** $v_y(t) = \int_0^t a_y dt = \int_0^t (2.80\text{ m/s}^3)t\,dt = (1.40\text{ m/s}^3)t^2$

$y - y_0 = \int_0^t v_y dt = \int_0^t (1.40\text{ m/s}^3)t^2 dt = (0.4667\text{ m/s}^3)t^3$. For $t = 10.0$ s, $y - y_0 = 467$ m.

(b) $y - y_0 = 325$ m so $(0.4667\text{ m/s}^3)t^3 = 325$ m and $t = 8.864$ s. At this time $v_y = (1.40\text{ m/s}^3)(8.864\text{ s})^2 = 110$ m/s.

EVALUATE: The time in part (b) is less than 10.0 s, so the given formulas are valid.

2.50. **IDENTIFY:** The acceleration is not constant, so we must use calculus instead of the standard kinematics formulas.

SET UP: The general calculus formulas are $v_x = v_{0x} + \int_0^t a_x dt$ and $x = x_0 + \int_0^t v_x dt.$ First integrate a_x to find $v(t)$, and then integrate that to find $x(t)$.

EXECUTE: Find $v(t)$: $v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t -(0.0320 \text{ m/s}^3)(15.0 \text{ s} - t)dt.$ Carrying out the integral and putting in the numbers gives $v_x(t) = 8.00 \text{ m/s} - (0.0320 \text{ m/s}^3)[(15.0 \text{ s})t - t^2/2]$. Now use this result to find $x(t)$.

$x = x_0 + \int_0^t v_x dt = x_0 + \int_0^t \left[8.00 \text{ m/s} - (0.0320 \text{ m/s}^3)\left((15.0 \text{ s})t - \frac{t^2}{2}\right)\right]dt,$ which gives $x = x_0 + (8.00 \text{ m/s})t - (0.0320 \text{ m/s}^3)[(7.50 \text{ s})t^2 - t^3/6)]$. Using $x_0 = -14.0$ m and $t = 10.0$ s, we get $x = 47.3$ m.

EVALUATE: The standard kinematics formulas apply only when the acceleration is constant.

2.51. **(a) IDENTIFY:** Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$.

SET UP: $v_x = v_{0x} + \int_0^t a_x \, dt$, $a_x = At - Bt^2$ with $A = 1.50 \text{ m/s}^3$ and $B = 0.120 \text{ m/s}^4$.

EXECUTE: $v_x = v_{0x} + \int_0^t (At - Bt^2)\, dt = v_{0x} + \frac{1}{2}At^2 - \frac{1}{3}Bt^3$

At rest at $t = 0$ says that $v_{0x} = 0,$ so

$v_x = \frac{1}{2}At^2 - \frac{1}{3}Bt^3 = \frac{1}{2}(1.50 \text{ m/s}^3)t^2 - \frac{1}{3}(0.120 \text{ m/s}^4)t^3$

$v_x = (0.75 \text{ m/s}^3)t^2 - (0.040 \text{ m/s}^4)t^3$

SET UP: $x - x_0 + \int_0^t v_x \, dt$

EXECUTE: $x = x_0 + \int_0^t (\frac{1}{2}At^2 - \frac{1}{3}Bt^3)\, dt = x_0 + \frac{1}{6}At^3 - \frac{1}{12}Bt^4$

At the origin at $t = 0$ says that $x_0 = 0,$ so

$x = \frac{1}{6}At^3 - \frac{1}{12}Bt^4 = \frac{1}{6}(1.50 \text{ m/s}^3)t^3 - \frac{1}{12}(0.120 \text{ m/s}^4)t^4$

$x = (0.25 \text{ m/s}^3)t^3 - (0.010 \text{ m/s}^4)t^4$

EVALUATE: We can check our results by using them to verify that $v_x(t) = \frac{dx}{dt}$ and $a_x(t) = \frac{dv_x}{dt}.$

(b) IDENTIFY and SET UP: At time t, when v_x is a maximum, $\frac{dv_x}{dt} = 0.$ (Since $a_x = \frac{dv_x}{dt},$ the maximum velocity is when $a_x = 0.$ For earlier times a_x is positive so v_x is still increasing. For later times a_x is negative and v_x is decreasing.)

EXECUTE: $a_x = \frac{dv_x}{dt} = 0$ so $At - Bt^2 = 0$

One root is $t = 0,$ but at this time $v_x = 0$ and not a maximum.

The other root is $t = \frac{A}{B} = \frac{1.50 \text{ m/s}^3}{0.120 \text{ m/s}^4} = 12.5 \text{ s}$

At this time $v_x = (0.75 \text{ m/s}^3)t^2 - (0.040 \text{ m/s}^4)t^3$ gives

$v_x = (0.75 \text{ m/s}^3)(12.5 \text{ s})^2 - (0.040 \text{ m/s}^4)(12.5 \text{ s})^3 = 117.2 \text{ m/s} - 78.1 \text{ m/s} = 39.1 \text{ m/s}.$

EVALUATE: For $t < 12.5$ s, $a_x > 0$ and v_x is increasing. For $t > 12.5$ s, $a_x < 0$ and v_x is decreasing.

2.52. **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use $v_x = v_{0x} + \int_0^t a_x dt$ and $x = x_0 + \int_0^t v_x dt$. Use the values of v_x and of x at $t = 1.0$ s to evaluate v_{0x} and x_0.

SET UP: $\int t^n dt = \frac{1}{n+1} t^{n+1}$, for $n \geq 0$.

EXECUTE: **(a)** $v_x = v_{0x} + \int_0^t \alpha t dt = v_{0x} + \frac{1}{2}\alpha t^2 = v_{0x} + (0.60 \text{ m/s}^3)t^2$. $v_x = 5.0$ m/s when $t = 1.0$ s gives $v_{0x} = 4.4$ m/s. Then, at $t = 2.0$ s, $v_x = 4.4 \text{ m/s} + (0.60 \text{ m/s}^3)(2.0 \text{ s})^2 = 6.8$ m/s.

(b) $x = x_0 + \int_0^t (v_{0x} + \frac{1}{2}\alpha t^2)dt = x_0 + v_{0x}t + \frac{1}{6}\alpha t^3$. $x = 6.0$ m at $t = 1.0$ s gives $x_0 = 1.4$ m. Then, at $t = 2.0$ s, $x = 1.4 \text{ m} + (4.4 \text{ m/s})(2.0 \text{ s}) + \frac{1}{6}(1.2 \text{ m/s}^3)(2.0 \text{ s})^3 = 11.8$ m.

(c) $x(t) = 1.4 \text{ m} + (4.4 \text{ m/s})t + (0.20 \text{ m/s}^3)t^3$. $v_x(t) = 4.4 \text{ m/s} + (0.60 \text{ m/s}^3)t^2$. $a_x(t) = (1.20 \text{ m/s}^3)t$. The graphs are sketched in Figure 2.52.

EVALUATE: We can verify that $a_x = \frac{dv_x}{dt}$ and $v_x = \frac{dx}{dt}$.

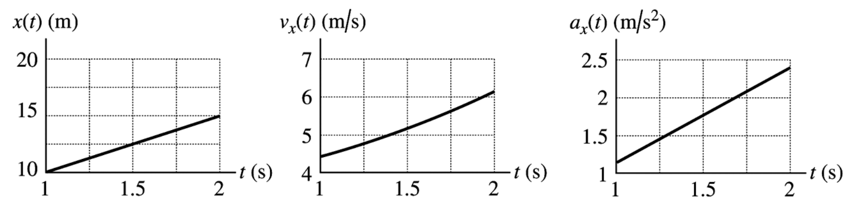


Figure 2.52

2.53. **IDENTIFY:** The sprinter's acceleration is constant for the first 2.0 s but zero after that, so it is not constant over the entire race. We need to break up the race into segments.

SET UP: When the acceleration is constant, the formula $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ applies. The average velocity is $v_{\text{av-}x} = \frac{\Delta x}{\Delta t}$.

EXECUTE: **(a)** $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = \left(\frac{0 + 10.0 \text{ m/s}}{2}\right)(2.0 \text{ s}) = 10.0$ m.

(b) (i) 40.0 m at 10.0 m/s so time at constant speed is 4.0 s. The total time is 6.0 s, so $v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{50.0 \text{ m}}{6.0 \text{ s}} = 8.33$ m/s.

(ii) He runs 90.0 m at 10.0 m/s so the time at constant speed is 9.0 s. The total time is 11.0 s, so $v_{\text{av-}x} = \frac{100 \text{ m}}{11.0 \text{ s}} = 9.09$ m/s.

(iii) He runs 190 m at 10.0 m/s so time at constant speed is 19.0 s. His total time is 21.0 s, so $v_{\text{av-}x} = \frac{200 \text{ m}}{21.0 \text{ s}} = 9.52$ m/s.

EVALUATE: His average velocity keeps increasing because he is running more and more of the race at his top speed.

2.54. IDENTIFY: We know the vertical position of the lander as a function of time and want to use this to find its velocity initially and just before it hits the lunar surface.

SET UP: By definition, $v_y(t) = \dfrac{dy}{dt}$, so we can find v_y as a function of time and then evaluate it for the desired cases.

EXECUTE: **(a)** $v_y(t) = \dfrac{dy}{dt} = -c + 2dt$. At $t = 0$, $v_y(t) = -c = -60.0$ m/s. The initial velocity is 60.0 m/s downward.

(b) $y(t) = 0$ says $b - ct + dt^2 = 0$. The quadratic formula says $t = 28.57\text{ s} \pm 7.38\text{ s}$. It reaches the surface at $t = 21.19$ s. At this time, $v_y = -60.0\text{ m/s} + 2(1.05\text{ m/s}^2)(21.19\text{ s}) = -15.5\text{ m/s}$.

EVALUATE: The given formula for $y(t)$ is of the form $y = y_0 + v_{0y}t + \frac{1}{2}at^2$. For part (a), $v_{0y} = -c = -60$ m/s.

2.55. IDENTIFY: In time t_S the S-waves travel a distance $d = v_S t_S$ and in time t_P the P-waves travel a distance $d = v_P t_P$.

SET UP: $t_S = t_P + 33\text{ s}$

EXECUTE: $\dfrac{d}{v_S} = \dfrac{d}{v_P} + 33\text{ s}$. $d\left(\dfrac{1}{3.5\text{ km/s}} - \dfrac{1}{6.5\text{ km/s}}\right) = 33\text{ s}$ and $d = 250$ km.

EVALUATE: The times of travel for each wave are $t_S = 71\text{ s}$ and $t_P = 38\text{ s}$.

2.56. IDENTIFY: A rock is thrown upward from the edge of a roof and eventually lands on the ground a distance H below the edge. Its acceleration is g downward, so the constant-acceleration equations apply.

SET UP: The time to reach the maximum height is T_{max} and the time after throwing to reach the ground is $3T_{max}$. The equations $v_y = v_{0y} + a_y t$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. Call the y-axis upward with $y = 0$ at the edge of the roof. At the highest point, $v_y = 0$ and at the ground $y = -H$. We want the initial speed V_0 in terms of H. Making a sketch helps to organize the information, as in Fig. 2.56.

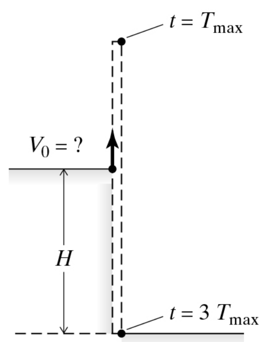


Figure 2.56

EXECUTE: At the highest point: $v_y = v_{0y} + a_y t$ gives $0 = V_0 - gT_{max}$, so $V_0 = gT_{max}$.

At the ground: After the rock was thrown, it took time T_{max} to reach the highest point and time $3T_{max}$ to reach the ground. Therefore the time to fall from the highest point to the ground was $2T_{max}$. Apply $v_y = v_{0y} + a_y t$ to the interval from the highest point to the ground to find the speed v_y when the rock reaches the ground. $v_y = 0 + (-g)(2T_{max}) = -2gT_{max}$. Now look at the entire motion from the instant the rock is thrown up until the instant it reaches the ground a distance H below the edge of the roof. We have already found that $V_0 = gT_{max}$ and $v_y = -2gT_{max}$ at ground level. Therefore $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

gives $(-2gT_{max})^2 = V_0^2 + 2(-g)(-H)$. Using $T_{max} = V_0/g$ gives $\left[-2g\left(\frac{V_0}{g}\right)\right]^2 = V_0^2 + 2gH \quad \rightarrow$

$$V_0 = \sqrt{\frac{2gH}{3}}.$$

EVALUATE: It might seem like a good idea to check the special case if $H = 0$ (the edge of the roof is at ground level). Our result seems to give $V_0 = 0$ in that case. But if the roof were at ground level, the time for the entire up-and-down trip would take $2T_{max}$, not $3T_{max}$, so our result would not apply. Checking units in the final answer would be a good idea.

2.57. **IDENTIFY:** The average velocity is $v_{av\text{-}x} = \frac{\Delta x}{\Delta t}$.

SET UP: Let $+x$ be upward.

EXECUTE: **(a)** $v_{av\text{-}x} = \frac{1000\text{ m} - 63\text{ m}}{4.75\text{ s}} = 197\text{ m/s}$

(b) $v_{av\text{-}x} = \frac{1000\text{ m} - 0}{5.90\text{ s}} = 169\text{ m/s}$

EVALUATE: For the first 1.15 s of the flight, $v_{av\text{-}x} = \frac{63\text{ m} - 0}{1.15\text{ s}} = 54.8\text{ m/s}$. When the velocity isn't constant the average velocity depends on the time interval chosen. In this motion the velocity is increasing.

2.58. **IDENTIFY:** The block has a constant westward acceleration, so we can use the constant-acceleration equations.

SET UP: The equations $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$, $v_x = v_{0x} + a_xt$, $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, and $v_{av\text{-}x} = \frac{\Delta x}{\Delta t}$ apply.

EXECUTE: **(a)** The target variable is the time for the block to return to $x = 0$. Using $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$ gives $0 = (12.0\text{ m/s})t + \frac{1}{2}(-2.00\text{ m/s}^2)t^2$. So $t = 12.0$ s.

(b) The block instantaneously stops when it reaches it maximum distance east, so $v_x = v_{0x} + a_xt$ gives 0 = 12.0 m/s + (–2.00 m/s)t, which gives $t = 6.00$ s. Using $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ we have $0 = (12.0\text{ m/s})^2 + 2(-2.00\text{ m/s}^2)\Delta x$, which gives $\Delta x = 36.0$ m.

EVALUATE: To check, we can use $v_{av\text{-}x} = \frac{\Delta x}{\Delta t}$ to find Δx. This gives $\Delta x = v_{av}\Delta t = \left(\frac{0 + 12.0\text{ m/s}}{2}\right)(6.00\text{ s})$ = 36.0 m, which agrees with our result in (b).

2.59. **IDENTIFY:** A block is moving with constant acceleration on an incline, so the constant-acceleration equations apply.

SET UP: All quantities are *down* the surface of the incline, so choose the x-axis along this surface and pointing downward. We first find the acceleration of the block and then use that to find its speed after sliding 16.0 m starting from rest and how long it takes to slide that distance.

EXECUTE: **(a)** First use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find a_x. $(3.00\text{ m/s})^2 = 0 + 2a_x(8.00\text{ m})$. $a_x = 0.5625\text{ m/s}^2$. Now use the same equation to find the speed when the block has moved 16.0 m.

$v_x^2 = 0 + 2(0.5625\text{ m/s}^2)(16.0\text{ m}) \rightarrow v_x = 4.24$ m/s.

(b) Use $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$ to find the time to travel 16.0 m.

$16.0\text{ m} = 0 + 0 + \frac{1}{2}(0.5625\text{ m/s}^2)t^2 \quad \rightarrow \quad t = 7.54$ s.

EVALUATE: From $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, with $v_{0x} = 0$, we see that $v_x^2 \propto (x - x_0)$, so $v_x \propto \sqrt{x - x_0}$. So if $(x - x_0)$ increases by a factor of 2, v_x should increase by a factor of $\sqrt{2}$. At the end of 8.00 m, the block was moving at 3.00 m/s, so at the end of 16.0 m, it should be moving at (3.00 m/s) $\sqrt{2}$ = 4.24 m/s, which agrees with our result in (a).

2.60. **IDENTIFY:** Use constant acceleration equations to find $x - x_0$ for each segment of the motion.

SET UP: Let $+x$ be the direction the train is traveling.

EXECUTE: $t = 0$ to 14.0 s: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(1.60\text{ m/s}^2)(14.0\text{ s})^2 = 157\text{ m}$.

At $t = 14.0\text{ s}$, the speed is $v_x = v_{0x} + a_x t = (1.60\text{ m/s}^2)(14.0\text{ s}) = 22.4\text{ m/s}$. In the next 70.0 s, $a_x = 0$ and $x - x_0 = v_{0x}t = (22.4\text{ m/s})(70.0\text{ s}) = 1568\text{ m}$.

For the interval during which the train is slowing down, $v_{0x} = 22.4\text{ m/s}$, $a_x = -3.50\text{ m/s}^2$ and $v_x = 0$.

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = \dfrac{0 - (22.4\text{ m/s})^2}{2(-3.50\text{ m/s}^2)} = 72\text{ m}$.

The total distance traveled is $157\text{ m} + 1568\text{ m} + 72\text{ m} = 1800\text{ m}$.

EVALUATE: The acceleration is not constant for the entire motion, but it does consist of constant acceleration segments, and we can use constant acceleration equations for each segment.

2.61. **IDENTIFY:** When the graph of v_x versus t is a straight line the acceleration is constant, so this motion consists of two constant acceleration segments and the constant acceleration equations can be used for each segment. Since v_x is always positive the motion is always in the $+x$ direction and the total distance moved equals the magnitude of the displacement. The acceleration a_x is the slope of the v_x versus t graph.

SET UP: For the $t = 0$ to $t = 10.0\text{ s}$ segment, $v_{0x} = 4.00\text{ m/s}$ and $v_x = 12.0\text{ m/s}$. For the $t = 10.0\text{ s}$ to 12.0 s segment, $v_{0x} = 12.0\text{ m/s}$ and $v_x = 0$.

EXECUTE: **(a)** For $t = 0$ to $t = 10.0\text{ s}$, $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{4.00\text{ m/s} + 12.0\text{ m/s}}{2}\right)(10.0\text{ s}) = 80.0\text{ m}$.

For $t = 10.0\text{ s}$ to $t = 12.0\text{ s}$, $x - x_0 = \left(\dfrac{12.0\text{ m/s} + 0}{2}\right)(2.00\text{ s}) = 12.0\text{ m}$. The total distance traveled is 92.0 m.

(b) $x - x_0 = 80.0\text{ m} + 12.0\text{ m} = 92.0\text{ m}$

(c) For $t = 0$ to 10.0 s, $a_x = \dfrac{12.0\text{ m/s} - 4.00\text{ m/s}}{10.0\text{ s}} = 0.800\text{ m/s}^2$. For $t = 10.0\text{ s}$ to 12.0 s,

$a_x = \dfrac{0 - 12.0\text{ m/s}}{2.00\text{ s}} = -6.00\text{ m/s}^2$. The graph of a_x versus t is given in Figure 2.61.

EVALUATE: When v_x and a_x are both positive, the speed increases. When v_x is positive and a_x is negative, the speed decreases.

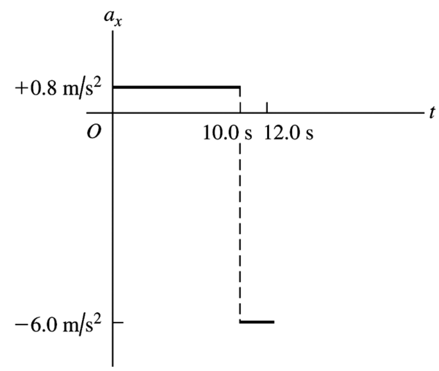


Figure 2.61

2.62. **IDENTIFY:** Apply $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to the motion of each train. A collision means the front of the passenger train is at the same location as the caboose of the freight train at some common time.

SET UP: Let P be the passenger train and F be the freight train. For the front of the passenger train $x_0 = 0$ and for the caboose of the freight train $x_0 = 200$ m. For the freight train $v_F = 15.0$ m/s and $a_F = 0$. For the passenger train $v_P = 25.0$ m/s and $a_P = -0.100\ \text{m/s}^2$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ for each object gives $x_P = v_P t + \frac{1}{2}a_P t^2$ and $x_F = 200\ \text{m} + v_F t$. Setting $x_P = x_F$ gives $v_P t + \frac{1}{2}a_P t^2 = 200\ \text{m} + v_F t$. $(0.0500\ \text{m/s}^2)t^2 - (10.0\ \text{m/s})t + 200\ \text{m} = 0$. The quadratic formula gives $t = \dfrac{1}{0.100}\left(+10.0 \pm \sqrt{(10.0)^2 - 4(0.0500)(200)}\right)\ \text{s} = (100 \pm 77.5)\ \text{s}$. The collision occurs at $t = 100\ \text{s} - 77.5\ \text{s} = 22.5\ \text{s}$. The equations that specify a collision have a physical solution (real, positive t), so a collision does occur.

(b) $x_P = (25.0\ \text{m/s})(22.5\ \text{s}) + \frac{1}{2}(-0.100\ \text{m/s}^2)(22.5\ \text{s})^2 = 537\ \text{m}$. The passenger train moves 537 m before the collision. The freight train moves $(15.0\ \text{m/s})(22.5\ \text{s}) = 337\ \text{m}$.

(c) The graphs of x_F and x_P versus t are sketched in Figure 2.62.

EVALUATE: The second root for the equation for t, $t = 177.5$ s is the time the trains would meet again if they were on parallel tracks and continued their motion after the first meeting.

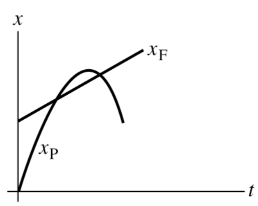


Figure 2.62

2.63. **IDENTIFY** and **SET UP:** Apply constant acceleration kinematics equations.
Find the velocity at the start of the second 5.0 s; this is the velocity at the end of the first 5.0 s. Then find $x - x_0$ for the first 5.0 s.

EXECUTE: For the first 5.0 s of the motion, $v_{0x} = 0$, $t = 5.0$ s.

$v_x = v_{0x} + a_x t$ gives $v_x = a_x(5.0\ \text{s})$.

This is the initial speed for the second 5.0 s of the motion. For the second 5.0 s:

$v_{0x} = a_x(5.0\text{ s}),\ t = 5.0\text{ s},\ x - x_0 = 200\text{ m}.$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $200\text{ m} = (25\text{ s}^2)a_x + (12.5\text{ s}^2)a_x$ so $a_x = 5.333\text{ m/s}^2$.

Use this a_x and consider the first 5.0 s of the motion:

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(5.333\text{ m/s}^2)(5.0\text{ s})^2 = 67\text{ m}.$

EVALUATE: The ball is speeding up so it travels farther in the second 5.0 s interval than in the first.

2.64. IDENTIFY: The rock has a constant acceleration, so we can apply the constant-acceleration equations.

SET UP: We use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ and $v_x = v_{0x} + a_x t$. The force is applied starting at $t = 0$ and maintains a constant acceleration of 4.00 m/s^2 in the $-x$ direction. Our target variables are the three times when the rock is 24.0 m from where it was when the force began to be applied. We also want its velocity at those instants.

EXECUTE: When the rock is 24.0 m on the $+x$ side of the origin, $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives

$24.0\text{ m} = (16.0\text{ m/s})t + \frac{1}{2}(-4.00\text{ m/s}^2)t^2$. Using the quadratic formula we get $t = 2.00$ s and $t = 6.00$ s.

At these times $v_x = v_{0x} + a_x t$ gives $v_x = 16.0\text{ m/s} + (-4.00\text{ m/s}^2)(2.00\text{ s}) = 8.00\text{ m/s}$ and $v_x = 16.0\text{ m/s} + (-4.00\text{ m/s}^2)(6.00\text{ s}) = -8.00\text{ m/s}$. But the rock is also 24.0 m from the origin when it is 24.0 m on the $-x$ side. In this case we get $-24.0\text{ m} = (16.0\text{ m/s})t + \frac{1}{2}(-4.00\text{ m/s}^2)t^2$. This quadratic equation has two roots, one of which is negative. We discard that root, leaving only $t = 9.29$ s. At this time, the velocity is $v_x = 16.0\text{ m/s} + (-4.00\text{ m/s}^2)(9.29\text{ s}) = -21.2\text{ m/s}$.

EVALUATE: To find the third time when the rock was 24.0 m from the origin, we had to think about the rock's behavior. It was not sufficient just to plug into an equation and get all three answers.

2.65. IDENTIFY: Apply constant acceleration equations to each object.
Take the origin of coordinates to be at the initial position of the truck, as shown in Figure 2.65a.
Let d be the distance that the car initially is behind the truck, so $x_0(\text{car}) = -d$ and $x_0(\text{truck}) = 0$. Let T be the time it takes the car to catch the truck. Thus at time T the truck has undergone a displacement $x - x_0 = 60.0\text{ m}$, so is at $x = x_0 + 60.0\text{ m} = 60.0\text{ m}$. The car has caught the truck so at time T is also at $x = 60.0\text{ m}$.

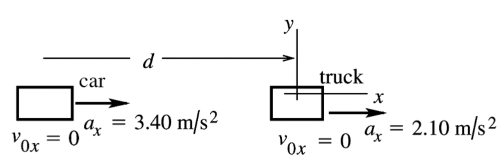


Figure 2.65a

(a) SET UP: Use the motion of the truck to calculate T:

$x - x_0 = 60.0\text{ m},\ v_{0x} = 0$ (starts from rest), $a_x = 2.10\text{ m/s}^2,\ t = T$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

Since $v_{0x} = 0$, this gives $t = \sqrt{\dfrac{2(x - x_0)}{a_x}}$

EXECUTE: $T = \sqrt{\dfrac{2(60.0\text{ m})}{2.10\text{ m/s}^2}} = 7.56\text{ s}$

(b) **SET UP:** Use the motion of the car to calculate d:

$x - x_0 = 60.0\ \text{m} + d,\ v_{0x} = 0,\ a_x = 3.40\ \text{m/s}^2,\ t = 7.56\ \text{s}$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

EXECUTE: $d + 60.0\ \text{m} = \frac{1}{2}(3.40\ \text{m/s}^2)(7.56\ \text{s})^2$

$d = 97.16\ \text{m} - 60.0\ \text{m} = 37.2\ \text{m}$.

(c) car: $v_x = v_{0x} + a_x t = 0 + (3.40\ \text{m/s}^2)(7.56\ \text{s}) = 25.7\ \text{m/s}$

truck: $v_x = v_{0x} + a_x t = 0 + (2.10\ \text{m/s}^2)(7.56\ \text{s}) = 15.9\ \text{m/s}$

(d) The graph is sketched in Figure 2.65b.

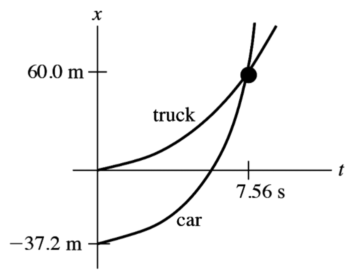


Figure 2.65b

EVALUATE: In part (c) we found that the auto was traveling faster than the truck when they came abreast. The graph in part (d) agrees with this: at the intersection of the two curves the slope of the *x-t* curve for the auto is greater than that of the truck. The auto must have an average velocity greater than that of the truck since it must travel farther in the same time interval.

2.66. **IDENTIFY:** The bus has a constant velocity but you have a constant acceleration, starting from rest.
SET UP: When you catch the bus, you and the bus have been traveling for the same time, but you have traveled an extra 12.0 m during that time interval. The constant-acceleration kinematics formula $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ applies.

EXECUTE: Call d the distance the bus travels after you start running and t the time until you catch the bus. For the bus we have $d = (5.00\ \text{m/s})t$, and for you we have $d + 12.0\ \text{m} = (1/2)(0.960\ \text{m/s}^2)t^2$. Solving these two equations simultaneously, and using the positive root, gives $t = 12.43$ s and $d = 62.14$ m. The distance you must run is 12.0 m + 62.14 m = 74.1 m. Your final speed just as you reach the bus is $v_x = (0.960\ \text{m/s}^2)(12.43\text{s}) = 11.9$ m/s. This might be possible for a college runner for a brief time, but it would be highly demanding!
EVALUATE: Note that when you catch the bus, you are moving much faster than it is.

2.67. **IDENTIFY:** The runner has constant acceleration in each segment of the dash, but it is not the same acceleration in the two segments. Therefore we must solve this problem in two segments.
SET UP: In the first segment, her acceleration is a constant a_x for 3.0 s, and in the next 9.0 s her speed remains constant. The total distance she runs is 100 m. A sketch as in Fig. 2.67 helps to organize the information. The constant-acceleration equations apply to each segment. We want a_x.

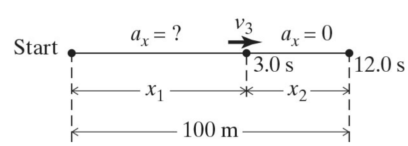


Figure 2.67

EXECUTE: Calling v_3 the velocity at the end of the first 3.0 s and using the notation shown in the figure, we look at one segment at a time. We realize that $v_0 = 0$ and v_3 is the constant speed during the second segment because she is not accelerating during that segment.

First segment: $v_x = v_{0x} + a_x t$ gives $v_3 = (3.0\text{ s})a_x$

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives $x_1 = \frac{1}{2}a_x(3.0\text{ s})^2 = (4.5\text{ s}^2)\,a_x$.

Second segment: She has no acceleration, so $x_2 = v_3 t_2 = [(3.0\text{ s})a_x](9.0\text{ s}) = (27\text{ s}^2)a_x$.

The dash is 100 m long, so $x_1 + x_2 = 100$ m, so $(4.5\text{ s}^2)a_x + (27\text{ s}^2)a_x = 100\text{ m} \rightarrow a_x = 3.2\text{ m/s}^2$.

EVALUATE: We cannot solve this problem in a single step by averaging the accelerations because the accelerations (a_x during the first 3.0 s and zero during the last 9.0 s) do not last for the same time.

2.68. IDENTIFY: The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use $a_x(t) = \dfrac{dv_x}{dt}$ and $x = x_0 + \int_0^t v_x(t)dt$.

SET UP: $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$ for $n \ge 0$.

EXECUTE: **(a)** $x(t) = x_0 + \int_0^t [\alpha - \beta t^2]dt = x_0 + \alpha t - \frac{1}{3}\beta t^3$. $x = 0$ at $t = 0$ gives $x_0 = 0$ and

$x(t) = \alpha t - \frac{1}{3}\beta t^3 = (4.00\text{ m/s})t - (0.667\text{ m/s}^3)t^3$. $a_x(t) = \dfrac{dv_x}{dt} = -2\beta t = -(4.00\text{ m/s}^3)t$.

(b) The maximum positive x is when $v_x = 0$ and $a_x < 0$. $v_x = 0$ gives $\alpha - \beta t^2 = 0$ and

$t = \sqrt{\dfrac{\alpha}{\beta}} = \sqrt{\dfrac{4.00\text{ m/s}}{2.00\text{ m/s}^3}} = 1.41\text{ s}$. At this t, a_x is negative. For $t = 1.41$ s,

$x = (4.00\text{ m/s})(1.41\text{ s}) - (0.667\text{ m/s}^3)(1.41\text{ s})^3 = 3.77\text{ m}$.

EVALUATE: After $t = 1.41$ s the object starts to move in the $-x$ direction and goes to $x = -\infty$ as $t \to \infty$.

2.69. IDENTIFY: In this problem the acceleration is not constant, so the constant-acceleration equations do not apply. We need to go to the basic definitions of velocity and acceleration and use calculus.

SET UP: We know the object starts at $x = 0$ and its velocity is given by $v_x = \alpha t - \beta t^3$. We want to find out when it returns to the origin and what are its velocity and acceleration at that instant. We need to use the definitions $v_x = dx/dt$ and $a_x = dv_x/dt$.

EXECUTE: **(a)** To find when the object returns to the origin we need to find $x(t)$ and use it to find t when $x = 0$. Using $v_x = dx/dt$ gives $dx = v_x dt$. Using $v_x = \alpha t - \beta t^3$, we integrate to find $x(t)$.

$x - x_0 = \int v_x dt = \int\left(\alpha t - \beta t^3\right)dt = \dfrac{\alpha t^2}{2} - \dfrac{\beta t^4}{4}$. The object starts from the origin, so $x_0 = 0$ and when $t = 0$, $x = 0$. This gives $\dfrac{\alpha t^2}{2} - \dfrac{\beta t^4}{4} = 0$. Solving for t gives $t = \sqrt{\dfrac{2\alpha}{\beta}} = \sqrt{\dfrac{2(8.0\text{ m/s}^2)}{4.0\text{ m/s}^4}} = 2.0$ s.

(b) When $t = 2.0$ s, $v_x = \alpha t - \beta t^3 = (8.0\text{ m/s}^2)(2.0\text{ s}) - (4.0\text{ m/s}^4)(2.0\text{ s})^3 = -16$ m/s. The minus sign tells us the object is moving in the $-x$ direction. To find the acceleration, we $a_x = dv_x/dt$.

$a_x = \dfrac{d\left(\alpha t - \beta t^3\right)}{dt} = \alpha - 3\beta t^2 = 8.0\text{ m/s}^2 - 3(4.0\text{ m/s}^4)(2.0\text{ s})^2 = -40\text{ m/s}^2$, in the $-x$ direction.

EVALUATE: The standard constant-acceleration formulas do not apply when the acceleration is a function of time.

2.70. **IDENTIFY:** Find the distance the professor walks during the time t it takes the egg to fall to the height of his head.

SET UP: Let $+y$ be downward. The egg has $v_{0y}=0$ and $a_y=9.80\text{ m/s}^2$. At the height of the professor's head, the egg has $y-y_0=44.2\text{ m}$.

EXECUTE: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(44.2\text{ m})}{9.80\text{ m/s}^2}}=3.00\text{ s}$. The professor walks a distance $x-x_0=v_{0x}t=(1.20\text{ m/s})(3.00\text{ s})=3.60\text{ m}$. Release the egg when your professor is 3.60 m from the point directly below you.

EVALUATE: Just before the egg lands its speed is $(9.80\text{ m/s}^2)(3.00\text{ s})=29.4\text{ m/s}$. It is traveling much faster than the professor.

2.71. **(a) IDENTIFY** and **SET UP:** Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$. We know $a_x(t)=\alpha+\beta t$, with $\alpha=-2.00\text{ m/s}^2$ and $\beta=3.00\text{ m/s}^3$.

EXECUTE: $v_x=v_{0x}+\int_0^t a_x\,dt=v_{0x}+\int_0^t(\alpha+\beta t)\,dt=v_{0x}+\alpha t+\frac{1}{2}\beta t^2$

$x=x_0+\int_0^t v_x\,dt=x_0+\int_0^t(v_{0x}+\alpha t+\frac{1}{2}\beta t^2)\,dt=x_0+v_{0x}t+\frac{1}{2}\alpha t^2+\frac{1}{6}\beta t^3$

At $t=0$, $x=x_0$.

To have $x=x_0$ at $t_1=4.00\text{ s}$ requires that $v_{0x}t_1+\frac{1}{2}\alpha t_1^2+\frac{1}{6}\beta t_1^3=0$.

Thus $v_{0x}=-\frac{1}{6}\beta t_1^2-\frac{1}{2}\alpha t_1=-\frac{1}{6}(3.00\text{ m/s}^3)(4.00\text{ s})^2-\frac{1}{2}(-2.00\text{ m/s}^2)(4.00\text{ s})=-4.00\text{ m/s}$.

(b) With v_{0x} as calculated in part (a) and $t=4.00\text{ s}$,

$v_x=v_{0x}+\alpha t+\frac{1}{2}\beta t^2=-4.00\text{ m/s}+(-2.00\text{ m/s}^2)(4.00\text{ s})+\frac{1}{2}(3.00\text{ m/s}^3)(4.00\text{ s})^2=+12.0\text{ m/s}$.

EVALUATE: $a_x=0$ at $t=0.67\text{ s}$. For $t>0.67\text{ s}$, $a_x>0$. At $t=0$, the particle is moving in the $-x$-direction and is speeding up. After $t=0.67\text{ s}$, when the acceleration is positive, the object slows down and then starts to move in the $+x$-direction with increasing speed.

2.72. **IDENTIFY:** After it is thrown upward, the rock has a constant downward acceleration g, so the constant-acceleration equations apply. It lands on the ground, a distance H below the point where it started.

SET UP: For constant acceleration, $v_{\text{av-}y}=\dfrac{v_1+v_2}{2}$. We know $v_1=v_0$ and can use $v_y^2=v_{0y}^2+2a_y(y-y_0)$ to find v_2.

EXECUTE: **(a)** Use $v_{\text{av-}y}=\dfrac{\Delta y}{\Delta t}$. The rock starts at $y=0$ and ends up at $y=-H$, so $\Delta y=y_2-y_1=-H-0=-H$. We need the total time from the instant the rock is thrown up until it hits the ground. Break the motion into two parts: the upward motion to the highest point and the fall from the highest point to the ground.

<u>Upward motion</u>: Using $v_y=v_{0y}+a_yt$ gives $t_{\text{up}}=v_0/g$. Using $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives the maximum height: $0=v_0^2-2gy_{\text{top}}\ \rightarrow\ y_{\text{top}}=v_0^2/2g$.

<u>Downward motion</u>: The downward motion starts from rest, and the rock falls a distance $y_{\text{top}}+H$, so $y=y_0+v_{0y}t+\dfrac{1}{2}a_yt^2$ gives $\dfrac{v_0^2}{2g}+H=\dfrac{1}{2}gt_{\text{fall}}^2\ \rightarrow\ t_{\text{fall}}=\sqrt{\dfrac{v_0^2}{g^2}+\dfrac{2H}{g}}$.

The total time for the entire motion is $t_{\text{tot}}=t_{\text{up}}+t_{\text{fall}}=\dfrac{v_0}{g}+\sqrt{\dfrac{v_0^2}{g^2}+\dfrac{2H}{g}}$. Now use $v_{\text{av-}y}=\dfrac{\Delta y}{\Delta t}$.

$v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t} = \dfrac{-H}{\dfrac{v_0}{g}+\sqrt{\dfrac{v_0^2}{g^2}+\dfrac{2H}{g}}} = \dfrac{-H}{v_0+\sqrt{v_0^2+2gH}}$. Rationalizing the denominator and simplifying gives

$v_{\text{av-}y} = \dfrac{v_0-\sqrt{v_0^2+2gH}}{2}$.

(b) When $H = 0$, $v_{\text{av-}y} = \dfrac{v_0-\sqrt{v_0^2+2gH}}{2} = \dfrac{v_0-\sqrt{v_0^2}}{2} = 0$. This result is reasonable because in this case $v_2 = -v_0$, so $v_{\text{av-}y} = \dfrac{v_1+v_2}{2} = \dfrac{v_0-v_0}{2} = 0$.

EVALUATE: (c) Using $v_2^2 = v_0^2 - 2g(-H)$ gives $v_2 = -\sqrt{v_0^2+2gH}$. We used the negative root because the rock is moving downward. Now use $v_{\text{av-}y} = \dfrac{v_1+v_2}{2}$, which gives $v_{\text{av-}y} = \dfrac{v_0-\sqrt{v_0^2+2gH}}{2}$. We get the same result as in part (a). If $H = 0$, $y_1 = y_2$, so $\Delta y = 0$, so $v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t} = 0$.

2.73. IDENTIFY: The watermelon is in freefall so it has a constant downward acceleration of g. The constant-acceleration equations apply to its motion.

SET UP: The melon starts from rest at the top of a building. You observe that 1.50 s after it is 30.0 m above the ground, it hits the ground, and you want to find the height of the building. It is very helpful to make a sketch to organize the information, as in Fig. 2.73. In the figure, we see that the height h of the building is $h = y_1 + 30.0$ m, and it takes 1.50 s to travel those last 30.0 m of the fall. We know that $v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t}$, and for constant acceleration it is also true that $v_{\text{av-}y} = \dfrac{v_1+v_2}{2}$. All quantities are downward, so choose the +y-axis downward.

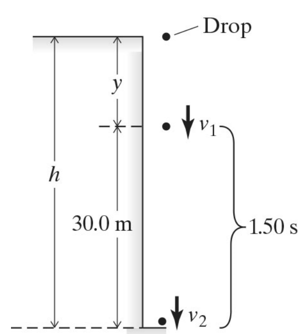


Figure 2.73

EXECUTE: We need to find y_1, but first we need to get v_1. Using $v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t}$ during the last 1.50 s of the fall, we have $v_{\text{av-}y} = \dfrac{30.0\text{ m}}{1.50\text{ s}} = 20.0$ m/s. It is also true that $v_{\text{av-}y} = \dfrac{v_1+v_2}{2}$, so $\dfrac{v_1+v_2}{2} = 20.0$ m/s, which gives $v_2 = 40.0\text{ m/s} - v_1$. We can also use $v_y = v_{0y} + a_y t$ to get v_2 in terms of v_1.

$v_2 = v_1 + gt = v_1 + (9.80\text{ m/s}^2)(1.50\text{ s}) = v_1 + 14.7$ m/s. Equating our two expressions for v_2 gives $v_1 + 14.7\text{ m/s} = 40.0\text{ m/s} - v_1$. Solving for v_1 gives $v_1 = 12.65$ m/s. Now use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find y_1. $(12.65\text{ m/s})^2 = 0 + 2(9.80\text{ m/s}^2)y_1 \quad\rightarrow\quad y_1 = 8.16$ m. The total height h of the building is $h = y_1 + 30.0\text{ m} = 8.16\text{ m} + 30.0\text{ m} = 38.2$ m.

EVALUATE: Use our results to find the time to fall the last 30.0 m; it should be 1.50 s if our answer is correct.

Time to fall 8.16 m from rest: $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$: 8.16 m = 0 + $\frac{1}{2}$(9.80 m/s²)t^2 $\rightarrow$ $t = 1.29$ s.

Time to fall 38.2 m from rest: 38.2 m = $\frac{1}{2}$(9.80 m/s²)t^2 $\rightarrow$ $t = 2.79$ s.

Time to fall the last 30.0 m: 2.79 s – 1.29 s = 1.50 s, which agrees with the observed time.

2.74. **IDENTIFY:** The flowerpot is in free-fall. Apply the constant acceleration equations. Use the motion past the window to find the speed of the flowerpot as it reaches the top of the window. Then consider the motion from the windowsill to the top of the window.

SET UP: Let $+y$ be downward. Throughout the motion $a_y = +9.80\text{ m/s}^2$. The constant-acceleration kinematics formulas all apply.

EXECUTE: Motion past the window:

$y - y_0 = 1.90$ m, $t = 0.380$ s, $a_y = +9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$v_{0y} = \dfrac{y - y_0}{t} - \frac{1}{2}a_y t = \dfrac{1.90\text{ m}}{0.380\text{ s}} - \frac{1}{2}(9.80\text{ m/s}^2)(0.380\text{ s}) = 3.138\text{ m/s}$. This is the velocity of the flowerpot when it is at the top of the window.

Motion from the windowsill to the top of the window: $v_{0y} = 0$, $v_y = 2.466$ m/s, $a_y = +9.80\text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{(3.138\text{ m/s})^2 - 0}{2(9.80\text{ m/s}^2)} = 0.502\text{ m}$. The top of the window is 0.502 m below the windowsill.

EVALUATE: It takes the flowerpot $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{3.138\text{ m/s}}{9.80\text{ m/s}^2} = 0.320\text{ s}$ to fall from the sill to the top of the window. Our result says that from the windowsill the pot falls $0.502\text{ m} + 1.90\text{ m} = 2.4\text{ m}$ in $0.320\text{ s} + 0.380\text{ s} = 0.700\text{ s}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\text{ m/s}^2)(0.700\text{ s})^2 = 2.4\text{ m}$, which checks.

2.75. **(a) IDENTIFY:** Consider the motion from when he applies the acceleration to when the shot leaves his hand.

SET UP: Take positive y to be upward. $v_{0y} = 0$, $v_y = ?$, $a_y = 35.0\text{ m/s}^2$, $y - y_0 = 0.640$ m,

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(35.0\text{ m/s}^2)(0.640\text{ m})} = 6.69\text{ m/s}$

(b) IDENTIFY: Consider the motion of the shot from the point where he releases it to its maximum height, where $v = 0$. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20$ m, $y = ?$, $a_y = -9.80\text{ m/s}^2$ (free fall), $v_{0y} = 6.69$ m/s (from part (a), $v_y = 0$ at maximum height), $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (6.69\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 2.29\text{ m}$, $y = 2.20\text{ m} + 2.29\text{ m} = 4.49\text{ m}$.

(c) IDENTIFY: Consider the motion of the shot from the point where he releases it to when it returns to the height of his head. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20$ m, $y = 1.83$ m, $a_y = -9.80\text{ m/s}^2$, $v_{0y} = +6.69$ m/s, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $1.83\text{ m} - 2.20\text{ m} = (6.69\text{ m/s})t + \frac{1}{2}(-9.80\text{ m/s}^2)t^2 = (6.69\text{ m/s})t - (4.90\text{ m/s}^2)t^2$,

$4.90t^2 - 6.69t - 0.37 = 0$, with t in seconds. Use the quadratic formula to solve for t:

$t = \dfrac{1}{9.80}\left(6.69 \pm \sqrt{(6.69)^2 - 4(4.90)(-0.37)}\right) = 0.6830 \pm 0.7362$. Since t must be positive, $t = 0.6830\ \text{s} + 0.7362\ \text{s} = 1.42\ \text{s}$.

EVALUATE: Calculate the time to the maximum height: $v_y = v_{0y} + a_y t$, so $t = (v_y - v_{0y})/a_y = -(6.69\ \text{m/s})/(-9.80\ \text{m/s}^2) = 0.68\ \text{s}$. It also takes 0.68 s to return to 2.2 m above the ground, for a total time of 1.36 s. His head is a little lower than 2.20 m, so it is reasonable for the shot to reach the level of his head a little later than 1.36 s after being thrown; the answer of 1.42 s in part (c) makes sense.

2.76. **IDENTIFY:** The motion of the rocket can be broken into 3 stages, each of which has constant acceleration, so in each stage we can use the standard kinematics formulas for constant acceleration. But the acceleration is not the same throughout all 3 stages.

SET UP: The formulas $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$, $y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2$, and $v_y = v_{0y} + a_y t$ apply.

EXECUTE: **(a)** Let $+y$ be upward. At $t = 25.0\ \text{s}$, $y - y_0 = 1094\ \text{m}$ and $v_y = 87.5\ \text{m/s}$. During the next 10.0 s the rocket travels upward an additional distance

$y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t = \left(\dfrac{87.5\ \text{m/s} + 132.5\ \text{m/s}}{2}\right)(10.0\ \text{s}) = 1100\ \text{m}$. The height above the launch pad when the second stage quits therefore is $1094\ \text{m} + 1100\ \text{m} = 2194\ \text{m}$. For the free-fall motion after the second stage quits: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (132.5\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 896\ \text{m}$. The maximum height above the launch pad that the rocket reaches is $2194\ \text{m} + 896\ \text{m} = 3090\ \text{m}$.

(b) $y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2$ gives $-2194\ \text{m} = (132.5\ \text{m/s})t - (4.9\ \text{m/s}^2)t^2$. From the quadratic formula the positive root is $t = 38.6\ \text{s}$.

(c) $v_y = v_{0y} + a_y t = 132.5\ \text{m/s} + (-9.8\ \text{m/s}^2)(38.6\ \text{s}) = -246\ \text{m/s}$. The rocket's speed will be 246 m/s just before it hits the ground.

EVALUATE: We cannot solve this problem in a single step because the acceleration, while constant in each stage, is not constant over the entire motion. The standard kinematics equations apply to each stage but not to the motion as a whole.

2.77. **IDENTIFY:** Two stones are thrown up with different speeds. (a) Knowing how soon the faster one returns to the ground, how long it will take the slow one to return? (b) Knowing how high the slower stone went, how high did the faster stone go?

SET UP: Use subscripts f and s to refer to the faster and slower stones, respectively. Take $+y$ to be upward and $y_0 = 0$ for both stones. $v_{0\text{f}} = 3v_{0\text{s}}$. When a stone reaches the ground, $y = 0$. The constant-acceleration formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply.

EXECUTE: **(a)** $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $a_y = -\dfrac{2v_{0y}}{t}$. Since both stones have the same a_y, $\dfrac{v_{0\text{f}}}{t_\text{f}} = \dfrac{v_{0\text{s}}}{t_\text{s}}$ and $t_\text{s} = t_\text{f}\left(\dfrac{v_{0\text{s}}}{v_{0\text{f}}}\right) = \left(\frac{1}{3}\right)(10\ \text{s}) = 3.3\ \text{s}$.

(b) Since $v_y = 0$ at the maximum height, then $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = -\dfrac{v_{0y}^2}{2y}$. Since both have the same a_y, $\dfrac{v_{0\text{f}}^2}{y_\text{f}} = \dfrac{v_{0\text{s}}^2}{y_\text{s}}$ and $y_\text{f} = y_\text{s}\left(\dfrac{v_{0\text{f}}}{v_{0\text{s}}}\right)^2 = 9H$.

EVALUATE: The faster stone reaches a greater height so it travels a greater distance than the slower stone and takes more time to return to the ground.

2.78. **IDENTIFY:** The rocket accelerates uniformly upward at 16.0 m/s^2 with the engines on. After the engines are off, it moves upward but accelerates downward at 9.80 m/s^2.

SET UP: The formulas $y - y_0 = v_{0_y} t = \frac{1}{2} a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply to both parts of the motion since the accelerations are both constant, but the accelerations are different in both cases. Let $+y$ be upward.

EXECUTE: With the engines on, $v_{0y} = 0$, $a_y = 16.0$ m/s^2 upward, and $t = T$ at the instant the engines just shut off. Using these quantities, we get

$y - y_0 = v_{0_y} t + \frac{1}{2} a_y t^2$ = (8.00 m/s^2)T^2 and $v_y = v_{0y} + a_y t =$ (16.0 m/s^2)T.

With the engines off (free fall), the formula $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for the highest point gives

$y - y_0$ = (13.06 m/s^2)T^2, using v_{0y} = (16.0 m/s^2)T, $v_y = 0$, and $a_y = -9.80$ m/s^2.

The total height reached is 960 m, so (distance in free-fall) + (distance with engines on) = 960 m.

Therefore (13.06 m/s^2) T^2 + (8.00 m/s^2) T^2 = 960 m, which gives T = 6.75 s.

EVALUATE: It we put in 6.75 s for T, we see that the rocket travels considerably farther during free fall than with the engines on.

2.79. **IDENTIFY:** The helicopter has two segments of motion with constant acceleration: upward acceleration for 10.0 s and then free-fall until it returns to the ground. Powers has three segments of motion with constant acceleration: upward acceleration for 10.0 s, free-fall for 7.0 s and then downward acceleration of 2.0 m/s^2.

SET UP: Let $+y$ be upward. Let $y = 0$ at the ground.

EXECUTE: **(a)** When the engine shuts off both objects have upward velocity $v_y = v_{0y} + a_y t =$ $(5.0 \text{ m/s}^2)(10.0 \text{ s}) = 50.0 \text{ m/s}$ and are at $y = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(5.0 \text{ m/s}^2)(10.0 \text{ s})^2 = 250 \text{ m}$.

For the helicopter, $v_y = 0$ (at the maximum height), $v_{0y} = +50.0 \text{ m/s}$, $y_0 = 250 \text{ m}$, and $a_y = -9.80 \text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y = \dfrac{v_y^2 - v_{0y}^2}{2a_y} + y_0 = \dfrac{0 - (50.0 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} + 250 \text{ m} = 378 \text{ m}$, which rounds to 380 m.

(b) The time for the helicopter to crash from the height of 250 m where the engines shut off can be found using $v_{0y} = +50.0 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, and $y - y_0 = -250 \text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-250 \text{ m} = (50.0 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$. $(4.90 \text{ m/s}^2)t^2 - (50.0 \text{ m/s})t - 250 \text{ m} = 0$. The quadratic formula gives $t = \dfrac{1}{9.80}\left(50.0 \pm \sqrt{(50.0)^2 + 4(4.90)(250)}\right)$ s. Only the positive solution is physical, so $t = 13.9$ s.

Powers therefore has free-fall for 7.0 s and then downward acceleration of 2.0 m/s^2 for $13.9 \text{ s} - 7.0 \text{ s} = 6.9 \text{ s}$. After 7.0 s of free-fall he is at $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 250 \text{ m} + (50.0 \text{ m/s})(7.0 \text{ s}) +$ $\frac{1}{2}(-9.80 \text{ m/s}^2)(7.0 \text{ s})^2 = 360 \text{ m}$ and has velocity $v_x = v_{0x} + a_x t = 50.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(7.0 \text{ s}) =$ -18.6 m/s. After the next 6.9 s he is at $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 360 \text{ m} + (-18.6 \text{ m/s})(6.9 \text{ s}) +$ $\frac{1}{2}(-2.00 \text{ m/s}^2)(6.9 \text{ s})^2 = 184 \text{ m}$. Powers is 184 m above the ground when the helicopter crashes.

EVALUATE: When Powers steps out of the helicopter he retains the initial velocity he had in the helicopter but his acceleration changes abruptly from 5.0 m/s^2 upward to 9.80 m/s^2 downward. Without the jet pack he would have crashed into the ground at the same time as the helicopter. The jet pack slows his descent so he is above the ground when the helicopter crashes.

2.80. **IDENTIFY:** Apply constant acceleration equations to the motion of the rock. Sound travels at constant speed.

SET UP: Let t_{f} be the time for the rock to fall to the ground and let t_{s} be the time it takes the sound to travel from the impact point back to you. $t_{\text{f}} + t_{\text{s}} = 8.00\text{ s}$. Both the rock and sound travel a distance h that is equal to the height of the cliff. Take $+y$ downward for the motion of the rock. The rock has $v_{0y} = 0$ and $a_y = g = 9.80\text{ m/s}^2$.

EXECUTE: **(a)** For the falling rock, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = \frac{1}{2}gt_{\text{f}}^2$. For the sound, $h = v_s t_s$. Equating these two equations for h and using the fact that $t_{\text{f}} + t_{\text{s}} = 8.00\text{ s}$, we get $\frac{1}{2}gt_{\text{f}}^2 = v_s t_s = v_s(8.00\text{ s} - t_f)$. Using $v_s = 330$ m/s and $g = 9.80\text{ m/s}^2$, we get a quadratic equation. Solving it using the quadratic formula and using the positive square root, we get $t_f = 7.225$ s. Therefore $h = \frac{1}{2}gt_{\text{f}}^2 = (1/2)(9.80\text{ m/s}^2)(7.225\text{ s})^2 = 256$ m.

(b) Ignoring sound you would calculate $d = \frac{1}{2}(9.80\text{ m/s}^2)(8.00\text{ s})^2 = 314\text{ m}$, which is greater than the actual distance. So you would have overestimated the height of the cliff. It actually takes the rock less time than 8.00 s to fall to the ground.

EVALUATE: Once we know h we can calculate that $t_{\text{f}} = 7.225\text{ s}$ and $t_{\text{s}} = 0.775\text{ s}$. The time for the sound of impact to travel back to you is 6% of the total time and should not be neglected for best precision.

2.81. **(a) IDENTIFY:** We have nonconstant acceleration, so we must use calculus instead of the standard kinematics formulas.

SET UP: We know the acceleration as a function of time is $a_x(t) = -Ct$, so we can integrate to find the velocity and then the x-coordinate of the object. We know that $v_x(t) = v_{0x} + \int_0^t a_x dt$ and $x(t) = x_0 + \int_0^t v_x(t)dt$.

EXECUTE: **(a)** We have information about the velocity, so we need to find that by integrating the acceleration. $v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t -Ctdt = v_{0x} - \frac{1}{2}Ct^2$. Using the facts that the initial velocity is 20.0 m/s and $v_x = 0$ when $t = 8.00$ s, we have $0 = 20.0\text{ m/s} - C(8.00\text{ s})^2/2$, which gives $C = 0.625\text{ m/s}^3$.

(b) We need the change in position during the first 8.00 s. Using $x(t) = x_0 + \int_0^t v_x(t)dt$ gives

$$x - x_0 = \int_0^t \left(-\tfrac{1}{2}Ct^2 + (20.0\text{ m/s})\right)dt = -Ct^3/6 + (20.0\text{ m/s})t$$

Putting in $C = 0.625\text{ m/s}^3$ and $t = 8.00$ s gives an answer of 107 m.

EVALUATE: The standard kinematics formulas are of no use in this problem since the acceleration varies with time.

2.82. **IDENTIFY:** Both objects are in free-fall and move with constant acceleration 9.80 m/s^2, downward. The two balls collide when they are at the same height at the same time.

SET UP: Let $+y$ be upward, so $a_y = -9.80\text{ m/s}^2$ for each ball. Let $y = 0$ at the ground. Let ball A be the one thrown straight up and ball B be the one dropped from rest at height H. $y_{0A} = 0$, $y_{0B} = H$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applied to each ball gives $y_A = v_0 t - \frac{1}{2}gt^2$ and $y_B = H - \frac{1}{2}gt^2$. $y_A = y_B$ gives $v_0 t - \frac{1}{2}gt^2 = H - \frac{1}{2}gt^2$ and $t = \dfrac{H}{v_0}$.

(b) For ball A at its highest point, $v_{yA} = 0$ and $v_y = v_{0y} + a_y t$ gives $t = \frac{v_0}{g}$. Setting this equal to the time in part (a) gives $\frac{H}{v_0} = \frac{v_0}{g}$ and $H = \frac{v_0^2}{g}$.

EVALUATE: In part (a), using $t = \frac{H}{v_0}$ in the expressions for y_A and y_B gives $y_A = y_B = H\left(1 - \frac{gH}{2v_0^2}\right)$.

H must be less than $\frac{2v_0^2}{g}$ in order for the balls to collide before ball A returns to the ground. This is because it takes ball A time $t = \frac{2v_0}{g}$ to return to the ground and ball B falls a distance $\frac{1}{2}gt^2 = \frac{2v_0^2}{g}$ during this time. When $H = \frac{2v_0^2}{g}$ the two balls collide just as ball A reaches the ground and for H greater than this ball A reaches the ground before they collide.

2.83. **IDENTIFY** and **SET UP:** Use $v_x = dx/dt$ and $a_x = dv_x/dt$ to calculate $v_x(t)$ and $a_x(t)$ for each car. Use these equations to answer the questions about the motion.

EXECUTE: $x_A = \alpha t + \beta t^2$, $v_{Ax} = \frac{dx_A}{dt} = \alpha + 2\beta t$, $a_{Ax} = \frac{dv_{Ax}}{dt} = 2\beta$

$x_B = \gamma t^2 - \delta t^3$, $v_{Bx} = \frac{dx_B}{dt} = 2\gamma t - 3\delta t^2$, $a_{Bx} = \frac{dv_{Bx}}{dt} = 2\gamma - 6\delta t$

(a) IDENTIFY and **SET UP:** The car that initially moves ahead is the one that has the larger v_{0x}.

EXECUTE: At $t = 0$, $v_{Ax} = \alpha$ and $v_{Bx} = 0$. So initially car A moves ahead.

(b) IDENTIFY and **SET UP:** Cars at the same point implies $x_A = x_B$.

$\alpha t + \beta t^2 = \gamma t^2 - \delta t^3$

EXECUTE: One solution is $t = 0$, which says that they start from the same point. To find the other solutions, divide by t: $\alpha + \beta t = \gamma t - \delta t^2$

$\delta t^2 + (\beta - \gamma)t + \alpha = 0$

$t = \frac{1}{2\delta}\left(-(\beta - \gamma) \pm \sqrt{(\beta - \gamma)^2 - 4\delta\alpha}\right) = \frac{1}{0.40}\left(+1.60 \pm \sqrt{(1.60)^2 - 4(0.20)(2.60)}\right) = 4.00 \text{ s} \pm 1.73 \text{ s}$

So $x_A = x_B$ for $t = 0$, $t = 2.27$ s and $t = 5.73$ s.

EVALUATE: Car A has constant, positive a_x. Its v_x is positive and increasing. Car B has $v_{0x} = 0$ and a_x that is initially positive but then becomes negative. Car B initially moves in the $+x$-direction but then slows down and finally reverses direction. At $t = 2.27$ s car B has overtaken car A and then passes it. At $t = 5.73$ s, car B is moving in the $-x$-direction as it passes car A again.

(c) IDENTIFY: The distance from A to B is $x_B - x_A$. The rate of change of this distance is $\frac{d(x_B - x_A)}{dt}$.

If this distance is not changing, $\frac{d(x_B - x_A)}{dt} = 0$. But this says $v_{Bx} - v_{Ax} = 0$. (The distance between A and B is neither decreasing nor increasing at the instant when they have the same velocity.)

SET UP: $v_{Ax} = v_{Bx}$ requires $\alpha + 2\beta t = 2\gamma t - 3\delta t^2$

EXECUTE: $3\delta t^2 + 2(\beta - \gamma)t + \alpha = 0$

$t = \frac{1}{6\delta}\left(-2(\beta - \gamma) \pm \sqrt{4(\beta - \gamma)^2 - 12\delta\alpha}\right) = \frac{1}{1.20}\left(3.20 \pm \sqrt{4(-1.60)^2 - 12(0.20)(2.60)}\right)$

$t = 2.667 \text{ s} \pm 1.667\ s$, so $v_{Ax} = v_{Bx}$ for $t = 1.00$ s and $t = 4.33$ s.

EVALUATE: At $t = 1.00$ s, $v_{Ax} = v_{Bx} = 5.00$ m/s. At $t = 4.33$ s, $v_{Ax} = v_{Bx} = 13.0$ m/s. Now car *B* is slowing down while *A* continues to speed up, so their velocities aren't ever equal again.
(d) IDENTIFY and **SET UP:** $a_{Ax} = a_{Bx}$ requires $2\beta = 2\gamma - 6\delta t$

EXECUTE: $t = \dfrac{\gamma - \beta}{3\delta} = \dfrac{2.80 \text{ m/s}^2 - 1.20 \text{ m/s}^2}{3(0.20 \text{ m/s}^3)} = 2.67 \text{ s}.$

EVALUATE: At $t = 0$, $a_{Bx} > a_{Ax}$, but a_{Bx} is decreasing while a_{Ax} is constant. They are equal at $t = 2.67$ s but for all times after that $a_{Bx} < a_{Ax}$.

2.84. IDENTIFY: Interpret the data on a graph to draw conclusions about the motion of a glider having constant acceleration down a frictionless air track, starting from rest at the top.
SET UP: The constant-acceleration kinematics formulas apply. Take the +*x*-axis along the surface of the track pointing downward.

EXECUTE: **(a)** For constant acceleration starting from rest, we have $x = \dfrac{1}{2}a_x t^2$. Therefore a plot of *x* versus t^2 should be a straight line, and the slope of that line should be $a_x/2$.
(b) To construct the graph of *x* versus t^2, we can use readings from the graph given in the text to construct a table of values for *x* and t^2, or we could use graphing software if available. The result is a graph similar to the one shown in Figure 2.84, which was obtained using software. A graph done by hand could vary slightly from this one, depending on how one reads the values on the graph in the text. The graph shown is clearly a straight line having slope 3.77 m/s^2 and *x*-intercept 0.0092 m. Using the slope *y*-intercept form of the equation of a straight line, the equation of this line is $x = 3.77t^2 + 0.0092$, where *x* is in meters and *t* in seconds.

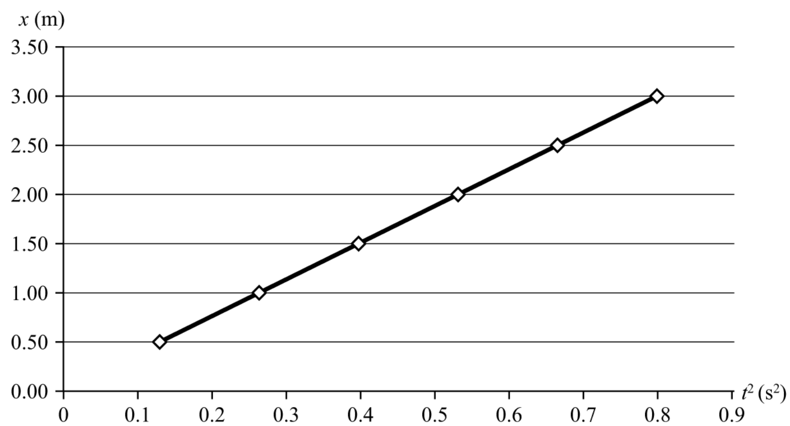


Figure 2.84

(c) The slope of the straight line in the graph is $a_x/2$, so $a_x = 2(3.77 \text{ m/s}^2) = 7.55 \text{ m/s}^2$.
(d) We know the distance traveled is 1.35 m, the acceleration is 7.55 m/s^2, and the initial velocity is zero, so we use the equation $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ and solve for v_x, giving $v_x = 4.51$ m/s.

EVALUATE: For constant acceleration in part (d), the average velocity is (4.51 m/s)/2 = 2.25 m/s. With this average velocity, the time for the glider to travel 1.35 m is x/v_{av} = (1.35 m)/(2.25 m) = 0.6 s, which is approximately the value of *t* read from the graph in the text for *x* = 1.35 m.

2.85. **IDENTIFY:** A ball is dropped from rest and falls from various heights with constant acceleration. Interpret a graph of the square of its velocity just as it reaches the floor as a function of its release height.

SET UP: Let $+y$ be downward since all motion is downward. The constant-acceleration kinematics formulas apply for the ball.

EXECUTE: **(a)** The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ applies to the falling ball. Solving for $y - y_0$ and using $v_{0y} = 0$ and $a_y = g$, we get $y - y_0 = \dfrac{v_y^2}{2g}$. A graph of $y - y_0$ versus v_y^2 will be a straight line with slope $1/2g = 1/(19.6\ \text{m/s}^2) = 0.0510\ \text{s}^2/\text{m}$.

(b) With air resistance the acceleration is less than 9.80 m/s^2, so the final speed will be smaller.

(c) The graph will not be a straight line because the acceleration will vary with the speed of the ball. For a given release height, v_y with air resistance is less than without it. Alternatively, with air resistance the ball will have to fall a greater distance to achieve a given velocity than without air resistance. The graph is sketched in Figure 2.85.

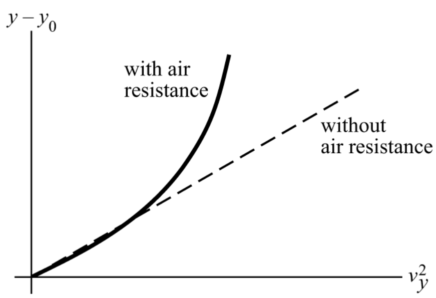


Figure 2.85

EVALUATE: Graphing $y - y_0$ versus v_y^2 for a set of data will tell us if the acceleration is constant. If the graph is a straight line, the acceleration is constant; if not, the acceleration is not constant.

2.86. **IDENTIFY:** Use data of acceleration and time for a model car to find information about its velocity and position.

SET UP: From the table of data in the text, we can see that the acceleration is not constant, so the constant-acceleration kinematics formlas do not apply. Therefore we must use calculus. The equations $v_x(t) = v_{0x} + \int_0^t a_x dt$ and $x(t) = x_0 + \int_0^t v_x dt$ apply.

EXECUTE: **(a)** Figure 2.86a shows the graph of a_x versus t. From the graph, we find that the slope of the line is –0.5131 m/s^3 and the a-intercept is 6.026 m/s^2. Using the slope y-intercept equation of a straight line, the equation is $a(t) = -0.513\ \text{m/s}^3\ t + 6.026\ \text{m/s}^2$, where t is in seconds and a is in m/s^2.

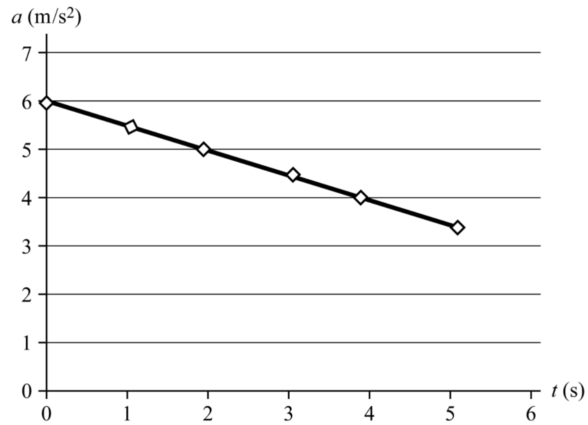


Figure 2.86a

(b) Integrate the acceleration to find the velocity, with the initial velocity equal to zero.

$v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t \left(6.026 \text{ m/s}^2 - 0.513 \text{ m/s}^3\ t\right) dt\ = 6.026 \text{ m/s}^2\ t - 0.2565 \text{ m/s}^3\ t^2.$

Figure 2.86b shows a sketch of the graph of v_x versus t.

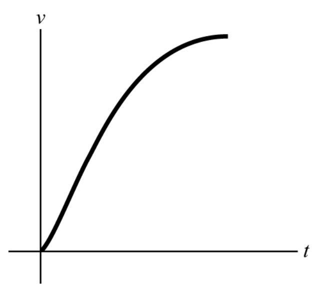


Figure 2.86b

(c) Putting $t = 5.00$ s into the equation we found in (b) gives $v_x = 23.7$ m/s.
(d) Integrate the velocity to find the change in position of the car.

$x - x_0 = \int_0^t v_x dt = \int_0^t \left[\left(6.026 \text{ m/s}^2\right)t - \left(0.2565 \text{ m/s}^3\right)t^2\right]dt\ = 3.013 \text{ m/s}^2\ t^2 - 0.0855 \text{ m/s}^3\ t^3$

At $t = 5.00$ s, this gives $x - x_0 = 64.6$ m.
EVALUATE: Since the acceleration is not constant, the standard kinematics formulas do not apply, so we must go back to basic definitions involving calculus.

2.87. **IDENTIFY:** Apply $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ to the motion from the maximum height, where $v_{0y} = 0$. The time spent above $y_{\text{max}}/2$ on the way down equals the time spent above $y_{\text{max}}/2$ on the way up.

SET UP: Let $+y$ be downward. $a_y = g$. $y - y_0 = y_{\text{max}}/2$ when he is a distance $y_{\text{max}}/2$ above the floor.

EXECUTE: The time from the maximum height to $y_{\text{max}}/2$ above the floor is given by $y_{\text{max}}/2 = \frac{1}{2}gt_1^2$. The time from the maximum height to the floor is given by $y_{\text{max}} = \frac{1}{2}gt_{\text{tot}}^2$ and the time from a height of $y_{\text{max}}/2$ to the floor is $t_2 = t_{\text{tot}} - t_1$.

$$\frac{2t_1}{t_2} = \frac{2\sqrt{y_{\text{max}}/2}}{\sqrt{y_{\text{max}}} - \sqrt{y_{\text{max}}/2}} = \frac{2}{\sqrt{2}-1} = 4.8.$$

EVALUATE: The person spends over twice as long above $y_{max}/2$ as below $y_{max}/2$. His average speed is less above $y_{max}/2$ than it is when he is below this height.

2.88. **IDENTIFY:** Apply constant acceleration equations to the motion of the two objects, the student and the bus.

SET UP: For convenience, let the student's (constant) speed be v_0 and the bus's initial position be x_0. Note that these quantities are for separate objects, the student and the bus. The initial position of the student is taken to be zero, and the initial velocity of the bus is taken to be zero. The positions of the student x_1 and the bus x_2 as functions of time are then $x_1 = v_0 t$ and $x_2 = x_0 + (1/2)at^2$.

EXECUTE: **(a)** Setting $x_1 = x_2$ and solving for the times *t* gives $t = \frac{1}{a}\left(v_0 \pm \sqrt{v_0^2 - 2ax_0}\right)$.

$$t = \frac{1}{0.170\ \text{m/s}^2}\left(5.0\ \text{m/s} \pm \sqrt{(5.0\ \text{m/s})^2 - 2(0.170\ \text{m/s}^2)(40.0\ \text{m})}\right) = 9.55\ \text{s and } 49.3\ \text{s}.$$

The student will be likely to hop on the bus the first time she passes it (see part (d) for a discussion of the later time). During this time, the student has run a distance $v_0 t = (5\ \text{m/s})(9.55\ \text{s}) = 47.8\ \text{m}$.

(b) The speed of the bus is $(0.170\ \text{m/s}^2)(9.55\ \text{s}) = 1.62\ \text{m/s}$.

(c) The results can be verified by noting that the *x* lines for the student and the bus intersect at two points, as shown in Figure 2.88a.

(d) At the later time, the student has passed the bus, maintaining her constant speed, but the accelerating bus then catches up to her. At this later time the bus's velocity is $(0.170\ \text{m/s}^2)(49.3\ \text{s}) = 8.38\ \text{m/s}$.

(e) No; $v_0^2 < 2ax_0$, and the roots of the quadratic are imaginary. When the student runs at $3.5\ \text{m/s}$, Figure 2.88b shows that the two lines do *not* intersect.

(f) For the student to catch the bus, $v_0^2 > 2ax_0$. And so the minimum speed is $\sqrt{2(0.170\ \text{m/s}^2)(40\ \text{m/s})} = 3.688\ \text{m/s}$. She would be running for a time $\frac{3.69\ \text{m/s}}{0.170\ \text{m/s}^2} = 21.7\ \text{s}$, and covers a distance $(3.688\ \text{m/s})(21.7\ \text{s}) = 80.0\ \text{m}$. However, when the student runs at $3.688\ \text{m/s}$, the lines intersect at *one* point, at $x = 80\ \text{m}$, as shown in Figure 2.88c.

EVALUATE: The graph in part (c) shows that the student is traveling faster than the bus the first time they meet but at the second time they meet the bus is traveling faster.

$t_2 = t_{tot} - t_1$

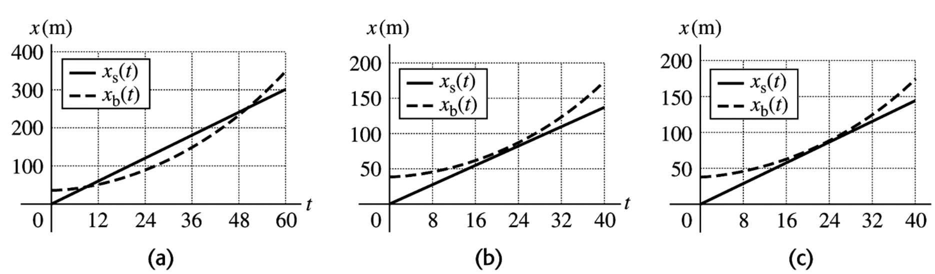


Figure 2.88

2.89. **IDENTIFY:** Apply constant acceleration equations to both objects.

SET UP: Let $+y$ be upward, so each ball has $a_y = -g$. For the purpose of doing all four parts with the least repetition of algebra, quantities will be denoted symbolically. That is, let $y_1 = h + v_0 t - \frac{1}{2}gt^2$, $y_2 = h - \frac{1}{2}g(t - t_0)^2$. In this case, $t_0 = 1.00$ s.

EXECUTE: **(a)** Setting $y_1 = y_2 = 0$, expanding the binomial $(t - t_0)^2$ and eliminating the common term $\frac{1}{2}gt^2$ yields $v_0 t = gt_0 t - \frac{1}{2}gt_0^2$. Solving for t: $t = \frac{\frac{1}{2}gt_0^2}{gt_0 - v_0} = \frac{t_0}{2}\left(\frac{1}{1 - v_0/(gt_0)}\right)$.

Substitution of this into the expression for y_1 and setting $y_1 = 0$ and solving for h as a function of v_0 yields, after some algebra, $h = \frac{1}{2}gt_0^2 \frac{\left(\frac{1}{2}gt_0 - v_0\right)^2}{(gt_0 - v_0)^2}$. Using the given value $t_0 = 1.00$ s and $g = 9.80\ \text{m/s}^2$,

$$h = 20.0\ \text{m} = (4.9\ \text{m})\left(\frac{4.9\ \text{m/s} - v_0}{9.8\ \text{m/s} - v_0}\right)^2.$$

This has two solutions, one of which is unphysical (the first ball is still going up when the second is released; see part (c)). The physical solution involves taking the negative square root before solving for v_0, and yields 8.2 m/s. The graph of y versus t for each ball is given in Figure 2.89.

(b) The above expression gives for (i) 0.411 m and for (ii) 1.15 km.

(c) As v_0 approaches 9.8 m/s, the height h becomes infinite, corresponding to a relative velocity at the time the second ball is thrown that approaches zero. If $v_0 > 9.8$ m/s, the first ball can never catch the second ball.

(d) As v_0 approaches 4.9 m/s, the height approaches zero. This corresponds to the first ball being closer and closer (on its way down) to the top of the roof when the second ball is released. If $v_0 < 4.9$ m/s, the first ball will already have passed the roof on the way down before the second ball is released, and the second ball can never catch up.

EVALUATE: Note that the values of v_0 in parts (a) and (b) are all greater than v_{min} and less than v_{max}.

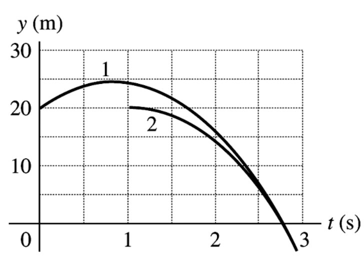


Figure 2.89

2.90. **IDENTIFY:** We know the change in velocity and the time for that change. We can use these quantities to find the average acceleration.

SET UP: The average acceleration is the change in velocity divided by the time for that change.

EXECUTE: $a_{\text{av}} = (v - v_0)/t = (0.80\ \text{m/s} - 0)/(250 \times 10^{-3}\ \text{s}) = 32\ \text{m/s}^2$, which is choice (c).

EVALUATE: This is about 1/3 the acceleration due to gravity, which is a reasonable acceleration for an organ.

2.91. **IDENTIFY:** The original area is divided into two equal areas. We want the diameter of these two areas, assuming the original and final areas are circular.
SET UP: The area A of a circle or radius r is $A = \pi r^2$ and the diameter d is $d = 2r$. $A_i = 2A_f$, and $r = d/2$, where A_f is the area of each of the two arteries.
EXECUTE: Call d the diameter of each artery. $A_i = \pi(d_a/2)^2 = 2[\pi(d/2)^2]$, which gives s $d = d_a/\sqrt{2}$, which is choice (b).
EVALUATE: The area of each artery is half the area of the aorta, but the diameters of the arteries are not half the diameter of the aorta.

2.92. **IDENTIFY:** We must interpret a graph of blood velocity during a heartbeat as a function of time.
SET UP: The instantaneous acceleration of a blood molecule is the slope of the velocity-versus-time graph.
EXECUTE: The magnitude of the acceleration is greatest when the slope of the v-t graph is steepest. That occurs at the upward sloping part of the graph, around $t = 0.10$ s, which makes choice (d) the correct one.
EVALUATE: The slope of the given graph is positive during the first 0.25 s and negative after that. Yet the velocity is positive throughout. Therefore the blood is always flowing forward, but it is increasing in speed during the first 0.25 s and slowing down after that.

3

MOTION IN TWO OR THREE DIMENSIONS

VP3.9.1. **IDENTIFY:** This is a projectile problem with only vertical acceleration.

SET UP: The formulas $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$, $v_x = v_0 \cos\alpha_0$, $v_y = v_0 \sin\alpha_0 - gt$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply.

EXECUTE: We know the launch speed and launch angle of the projectile, and want to know its maximum height, maximum range, and the time it is in the air.
(a) At the maximum height, $v_y = 0$. We use $v_y = v_0 \sin\alpha_0 - gt$ to find the time.

$0 = (25.0 \text{ m/s})(\sin 36.9°) - (9.80 \text{ m/s}^2)t \quad \rightarrow \quad t = 1.53$ s. Now use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$.

$0 = [(25.0 \text{ m/s})(\sin 36.9°)]^2 - 2(9.80 \text{ m/s}^2)(y - y_0) \quad \rightarrow y - y_0 = 11.5$ m.

(b) When the projectile returns to ground level, $v_y = -v_0 \sin\alpha_0$. So

$v_y = v_0 \sin\alpha_0 - gt = -v_0 \sin\alpha_0$

$2v_0 \sin\alpha_0 = gt$

$2(25.0 \text{ m/s})(\sin 36.9°) = (9.80 \text{ m/s}^2)t \quad \rightarrow \quad t = 3.06$ s.

For the horizontal motion, we have

$x = (v_0 \cos\alpha_0)t = (25.0 \text{ m/s})(\cos 36.9°)(3.06 \text{ s}) = 61.2$ m.

EVALUATE: (a) Calculate y using $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$.

$y = (25.0 \text{ m/s})(\sin 36.9°)(1.53 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.53 \text{ s})^2 = 11.5$ m, which agrees with our result. (b) The acceleration is constant, so the time for the upward motion is equal to the time for the downward motion. Thus $t_{\text{tot}} = 1.53 \text{ s} + 1.53 \text{ s} = 3.06$ s, which agrees with our result.

VP3.9.2. **IDENTIFY:** This is a projectile problem with only vertical acceleration.

SET UP: The formulas $v_x = v_0 \cos\alpha_0$, $v_y = v_0 \sin\alpha_0 - gt$, $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. At the highest point in the baseball's trajectory, its vertical velocity is zero, but its horizontal velocity is the same as when it left the ground.

EXECUTE: (a) At the highest point, $v_y = 0$, so $v_y = v_0 \sin\alpha_0 - gt$ gives

$0 = v_0 \sin(30.0°) - (9.80 \text{ m/s}^2)(1.05 \text{ s})^2 \quad \rightarrow \quad v_0 = 20.6$ m/s.

(b) Using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$0 = [(20.6 \text{ m/s})(\sin 30.0°)]^2 - 2(9.80 \text{ m/s}^2)(y - y_0) \quad \rightarrow \quad y - y_0 = 5.40$ m

EVALUATE: We can check by finding y when $t = 1.05$ s, using $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$.

$y = (20.6 \text{ m/s})(\sin 30.0°)(1.05 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.05 \text{ s})^2 = 5.40$, which agrees with our result.

VP3.9.3. **IDENTIFY:** This is a projectile problem with only vertical acceleration.

SET UP: The formulas $v_x = v_0 \cos\alpha_0$, $v_y = v_0 \sin\alpha_0 - gt$, $x = (v_0 \cos\alpha_0)t$, and $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$ apply. Call the origin the launch point with the $+y$-axis vertically upward.

EXECUTE: (a) When the walnut reaches the ground, $y = -20.0$ m. Use $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$.

$-20.0 \text{ m} = (15.0 \text{ m/s})(\sin 50.0°)t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$. Using the quadratic formula gives two roots, only one of which is positive: $t = 3.51$ s.

(b) There is no horizontal acceleration, so we use $x = (v_0 \cos\alpha_0)t$ to find the distance x.

$x = (v_0 \cos\alpha_0)t = (15.0 \text{ m/s})(\cos 50.0°)(3.51 \text{ s}) = 33.8$ m.

(c) There is no horizontal acceleration, so $v_x = v_0 \cos\alpha_0 = (15.0 \text{ m/s})(\cos 50.0°) = 9.64$ m/s. The vertical velocity is

$v_y = v_0 \sin\alpha_0 - gt = (15.0 \text{ m/s}) \sin 50.0° - (9.80 \text{ m/s}^2)(3.51 \text{ s}) = -22.9$ m/s. The minus sign tells us the walnut is moving downward.

EVALUATE: We cannot use the projectile range formula because the landing point is not on the same level as the launch point.

VP3.9.4. **IDENTIFY:** This is a projectile problem with only vertical acceleration.

SET UP: The formulas $v_x = v_0 \cos\alpha_0$, $v_y = v_0 \sin\alpha_0 - gt$, $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$, and $x = (v_0 \cos\alpha_0)t$ apply. Call the origin the launch point with the $+y$-axis vertically upward.

EXECUTE: (a) The horizontal and vertical distances are equal, so $y = -x$, so

$-(v_0 \cos\alpha_0)t = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$. Since $\alpha_0 = 0$, we have $v_0t = \frac{1}{2}gt^2$, which gives $t = 2v_0/g$. At this time $x = v_0t = v_0(2v_0/g) = 2v_0^2/g$. Since $-x = y$, $y = -2v_0^2/g$.

(b) When the potato is moving at 45° below the horizontal, $v_y = -v_x$. $v_x = v_0$ and $v_y = -v_0 = -gt$. Therefore $t = v_0/g$. At this time, $x = v_0t = v_0(v_0/g) = v_0^2/g$, and $y = -\frac{1}{2}gt^2 = -\frac{1}{2}g(v_0/g)^2 = -v_0^2/2g$.

EVALUATE: As a check, solve for y using the time found in part (a).

$y = -\frac{1}{2}gt^2 = -\frac{1}{2}g(2v_0/g)^2 = -2v_0^2/g$, just as we found. Notice in part (b) that $x \neq y$ when the potato is traveling at 45° with the horizontal, but the magnitudes of the velocity are equal. We cannot use the projectile range formula because the landing point is not on the same level as the launch point.

VP3.12.1. **IDENTIFY:** This problem involves circular motion at constant speed. The acceleration of the cyclist is toward the center of the circle.

SET UP: The radial acceleration of the cyclist is $a_{\text{rad}} = \frac{v^2}{R}$ toward the center of the circular track. We know the speed and acceleration and want the radius of the circle.

EXECUTE: Solve $a_{\text{rad}} = \frac{v^2}{R}$ for R, giving $R = v^2/a_{\text{rad}} = (10.0 \text{ m/s})^2/(5.00 \text{ m/s}^2) = 20.0$ m.

EVALUATE: The speed of the cyclist is constant, but not the velocity since it is tangent to the circular path and is always changing direction.

VP3.12.2. **IDENTIFY:** This problem involves circular motion at constant speed. The acceleration of the car is toward the center of the circle.

SET UP: The radial acceleration of the car is $a_{\text{rad}} = \dfrac{v^2}{R}$ toward the center of the circular track. In terms of the period of motion, the acceleration is $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$. We know the speed of the car and the radius of the circle and want to find the period of the motion and centripetal acceleration of the car.

EXECUTE: (a) Equate the two expressions for a_{rad}, giving $\dfrac{4\pi^2 R}{T^2} = \dfrac{v^2}{R}$. Now solve for T.

$T = \sqrt{\dfrac{4\pi^2 R^2}{v^2}} = 2\pi R/v = 2\pi(265\text{ m})/(40.0\text{ m/s}) = 41.6\text{ s}$.

(b) $a_{\text{rad}} = \dfrac{v^2}{R} = (40.0\text{ m/s})^2/(265\text{ m}) = 6.04\text{ m/s}^2$.

EVALUATE: We can find the period by realizing that vT is the circumference of the track, which is $2\pi R$. Therefore $vT = 2\pi R$, so $T = 2\pi R/v = 2\pi(265\text{ m})/(40.0\text{ m/s}) = 41.6\text{ s}$, which is agrees with our answer in (a). The acceleration of the car (and the driver inside) is quite large, over 60% of g.

VP3.12.3. **IDENTIFY:** This problem involves circular motion at constant speed. The radial acceleration is toward the center of the circle.

SET UP: The equation $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ applies and the speed is $v = 2\pi R/T$. The period is the same for all points on the wheel, but the speed (and hence acceleration) is not.

EXECUTE: (a) $v_{10} = 2\pi R/T = 2\pi(10.0\text{ cm})/(0.670\text{ s}) = 93.8\text{ cm/s} = 0.938\text{ m/s}$. Since R is twice as great at 20.0 cm, $v_{20} = 2v_{10} = 2(0.938\text{ m/s}) = 1.88\text{ m/s}$.

(b) $a_{10} = 4\pi^2 R/T^2 = 4\pi^2(0.100\text{ m})/(0.670\text{ s})^2 = 8.79\text{ m/s}^2$. As in (a), we see that $a_{20} = 2a_{10} = 2(8.79\text{ m/s}^2) = 17.6\text{ m/s}^2$.

(c) Both the speed and radial acceleration increase as R increases.

EVALUATE: As R increases, the points farther from the center must travel a greater distance than points closer to the center. So the speed and acceleration are greater for those distant points.

VP3.12.4. **IDENTIFY:** This problem involves circular motion at constant speed. The radial acceleration of a planet is toward the center of the circle, which is essentially the sun.

SET UP: The equation $a_{\text{rad}} = \dfrac{v^2}{R}$ applies and the speed of a planet is $v = 2\pi R/T$.

EXECUTE: (a) Apply $v = 2\pi R/T$ to each planet using the data given.

$v_V = 2\pi(1.08\times 10^{11}\text{ m})/[(225)(86{,}500)\text{s}] = 3.49\times 10^4\text{ m/s}$. Similar calculations for the earth and Mars gives $v_E = 2.99.\times.10^4\text{ m/s}$ and $v_M = 2.41\times 10^4\text{ m/s}$.

(b) Use $a_{\text{rad}} = \dfrac{v^2}{R}$ with the speeds found in part (a).

$a_V = (3.49\times 10^4\text{ m/s})^2/(1.08\times 10^{11}\text{ m}) = 1.13\times 10^{-2}\text{ m/s}^2$. Likewise we get $a_E = 5.95\times 10^{-3}\text{ m/s}^2$ and $a_M = 2.55\times 10^{-3}\text{ m/s}^2$.

(c) As the size of the orbit increases, both the orbital speed and the radial acceleration decrease.

EVALUATE: Since R increases and v decreases with distance from the sun, it must follow that the radial acceleration also decreases with distance. This is reasonable because the gravitational pull of the sun (to be studied in a later chapter) is weaker for distant planets than for closer ones.

VP3.12.5. **IDENTIFY:** This problem involves circular motion at constant speed.

SET UP: The equation $a_{\text{rad}} = \dfrac{v^2}{R}$ applies and the speed of an object moving in a circle is $v = 2\pi R/T$.

EXECUTE: (a) $v = 2\pi R/T$, so $T = 2\pi R/v$. Take the ratio of the two periods, giving $\dfrac{T_A}{T_B} = \dfrac{2\pi R_A / v_A}{2\pi R_B / v_B} = \dfrac{R_A}{R_B}\dfrac{v_B}{v_A}$. Putting in the given values for the speeds and radii gives $\dfrac{T_A}{T_B} = \dfrac{R(2v)}{(R/2)v} = 4.$

(b) Use $a_{\text{rad}} = \dfrac{v^2}{R}$, take the ratio of the accelerations, and use the given speeds and radii.

$$\frac{a_A}{a_B} = \frac{v_A^2 / R_A}{v_B^2 / R_B} = \left(\frac{v_A}{v_B}\right)^2 \frac{R_B}{R_A} = \left(\frac{v}{2v}\right)^2 \frac{R/2}{R} = \frac{1}{8}.$$

EVALUATE: Since $a_{\text{rad}} = \dfrac{v^2}{R}$, doubling the speed will increase a_{rad} by a factor of 4. Then halving the radius will increase a_{rad} by another factor of 2, giving a total factor of 8. This tells us that the inner object will have an acceleration 8 times that of the outer object, which is what we found in part (b).

VP3.15.1. **IDENTIFY:** This problem is about relative velocities.

SET UP: If object P is moving relative to object B and B is moving relative to A, then the velocity of P relative to A is given by $\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$. Let subscript P denote the police car, S the SUV, and E the earth.

EXECUTE: (a) We have one-dimensional motion, so the relative velocities in the x-direction are given by $v_{P/S} = v_{P/E} + v_{E/S}$. Using the given values gives

$v_{P/S} = 35.0\ \text{m/s} + 18.0\ \text{m/s} = 53.0\ \text{m/s}$.

(b) In this case, we want $v_{S/P}$, so

$v_{S/P} = v_{S/E} + v_{E/P} = -18.0\ \text{m/s} + (-35.0\ \text{m/s}) = -53.0\ \text{m/s}$.

EVALUATE: A rider in the SUV sees the police car going north at 53.0 m/s, but a rider in the police car sees the SUV going south at 53.0 m/s. Since they are going in opposite directions, their speeds relative to the earth add.

VP3.15.2. **IDENTIFY:** This problem is about relative velocities.

SET UP: If object P is moving relative to object B and B is moving relative to A, then the velocity of P relative to A is given by $\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$. Let subscript A denote the car A, B car B, and E the earth. The cars have only east-west velocities, so we look at velocity components along those directions.

EXECUTE: (a) We want the velocity of car A relative to car B.

$v_{A/B} = v_{A/E} + v_{E/B} = 45.0\ \text{m/s} + 45.0\ \text{m/sk} = 90.0\ \text{m/s}$, eastward.

(b) $v_{B/A} = v_{B/E} + v_{E/A} = -45.0\ \text{m/s} + (-45.0\ \text{m/s}) = -90.0\ \text{m/s}$, so the magnitude is 90.0 m/s and the direction is westward.

(c) At the points in this problem, the cars have only east-west velocities, so they have no relative velocity component along the line connecting them. Since both cars are traveling at the same speed in the same clockwise sense in a circle, they always remain the same distance apart. So they are neither approaching nor moving away from each other.

EVALUATE: In both cases the cars have east-west velocities in opposite directions, so their velocities relative to the earth add. When A sees B moving eastward at 90.0 m/s, B sees A moving westward at 90.0 m/s.

VP3.15.3. **IDENTIFY:** This problem is about relative velocities.

SET UP: If object P is moving relative to object B and B is moving relative to A, then the velocity of P relative to A is given by $\vec{\boldsymbol{v}}_{P/A} = \vec{\boldsymbol{v}}_{P/B} + \vec{\boldsymbol{v}}_{B/A}$. Let subscript T denote the truck, S the SUV, and E the earth.

EXECUTE: For this case, we have $\vec{\boldsymbol{v}}_{T/S} = \vec{\boldsymbol{v}}_{T/E} + \vec{\boldsymbol{v}}_{E/S}$. We want $\vec{\boldsymbol{v}}_{T/S}$. We know that $\vec{\boldsymbol{v}}_{T/E}$ = 16.0 m/s eastbound and $\vec{\boldsymbol{v}}_{S/E}$ = 20.0 m/s southbound. Therefore $\vec{\boldsymbol{v}}_{E/S}$ = 20.0 m/s northbound. Figure VP3.15.3 illustrates the velocity vectors.

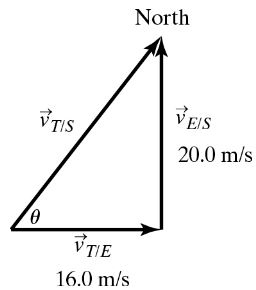


Figure VP3.15.3

Applying $A = \sqrt{A_x^2 + A_y^2}$ for the magnitude of a vector, we have

$v_{T/S} = \sqrt{(16.0 \text{ m/s})^2 + (20.0 \text{ m/s})^2}$ = 25.6 m/s. From the figure, we see that

θ = arctan[(20.0 m/s)/(16.0 m/s)] = 51.3° north of east.

(b) Using $\vec{\boldsymbol{v}}_{S/T} = -\vec{\boldsymbol{v}}_{T/S}$, the speed is 25.6 m/s, and the direction is 51.3° south of west.

EVALUATE: Check (b) by applying the relative velocity formula.

$\vec{\boldsymbol{v}}_{S/T} = \vec{\boldsymbol{v}}_{S/E} + \vec{\boldsymbol{v}}_{E/T} = -20.0 \text{ m/s } \hat{\boldsymbol{i}} + (-16.0 \text{ m/s}) \hat{\boldsymbol{j}} = -(16.0 \text{ m/s } \hat{\boldsymbol{i}} + 20.0 \text{ m/s } \hat{\boldsymbol{j}}) = -\vec{\boldsymbol{v}}_{T/S}$.

VP3.15.4. **IDENTIFY:** This problem is about relative velocities.

SET UP: If object P is moving relative to object B and B is moving relative to A, then the velocity of P relative to A is given by $\vec{\boldsymbol{v}}_{P/A} = \vec{\boldsymbol{v}}_{P/B} + \vec{\boldsymbol{v}}_{B/A}$. Let subscript J denote the jet, A the air, and E the earth.

EXECUTE: The jet's velocity relative to the earth is $\vec{\boldsymbol{v}}_{J/E} = \vec{\boldsymbol{v}}_{J/A} + \vec{\boldsymbol{v}}_{A/E}$. Figure VP3.15.4 illustrates these vectors. We want to find the magnitude of $\vec{\boldsymbol{v}}_{J/A}$ (the airspeed) and its direction (θ in the figure). Therefore we first find the components of $\vec{\boldsymbol{v}}_{J/A}$.

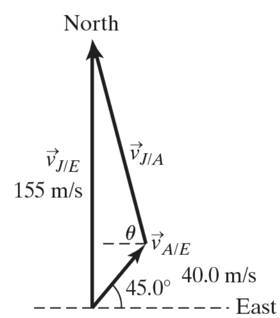


Figure VP3.15.4

Since $\vec{\boldsymbol{v}}_{J/E}$ is due north, it has no east-west component. From the figure, we can therefore see that the east-west components of $\vec{\boldsymbol{v}}_{A/E}$ and $\vec{\boldsymbol{v}}_{J/A}$ must have opposite sign and equal magnitudes.

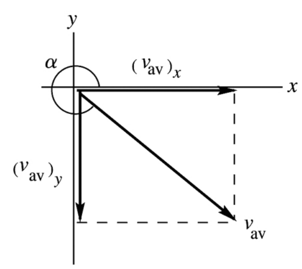
$\vec{v}_{J/A}$ (east component) = $-\vec{v}_{A/E}$ (west component) = –(40.0 m/s) cos 45.0° = –28.28 m/s. From the figure, we also see that
$\vec{v}_{J/A}$ (north component) + $\vec{v}_{A/E}$ (north component) = $\vec{v}_{J/E}$ (north component)
$\vec{v}_{J/A}$ (north component) + (40.0 m/s) cos 45.0° = 155 m/s
$\vec{v}_{J/A}$ (north component) = 126.7 m/s. Now find the magnitude of $\vec{v}_{J/A}$ using its components.
$v_{J/A} = \sqrt{(-28.28\text{ m/s})^2 + (126.7\text{ m/s})^2} = 130$ m/s. From the figure we see that
θ = arctan[(126.7 m/s)/(28.28 m/s)] = 77.4°. Therefore the airspeed of the jet is 130 m/s and the pilot must point it at 77.4° north of west.
EVALUATE: The pilot points the jet in a direction to offset the wind so the plane is flying directly north at 155 m/s as observed by someone standing on the ground.

3.1. **IDENTIFY** and **SET UP:** Use $\vec{v}_{\text{av}} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form.

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5.3\text{ m} - 1.1\text{ m}}{3.0\text{ s} - 0} = 1.4\text{ m/s}$

$$v_{\text{av-}y} = \frac{\Delta y}{\Delta t} = \frac{y_2 - y_1}{t_2 - t_1} = \frac{-0.5\text{ m} - 3.4\text{ m}}{3.0\text{ s} - 0} = -1.3\text{ m/s}$$

(b)



$$\tan\alpha = \frac{(v_{\text{av}})_y}{(v_{\text{av}})_x} = \frac{-1.3\text{ m/s}}{1.4\text{ m/s}} = -0.9286$$

$$\alpha = 360° - 42.9° = 317°$$

$$v_{\text{av}} = \sqrt{(v_{\text{av}})_x^2 + (v_{\text{av}})_y^2}$$

$$v_{\text{av}} = \sqrt{(1.4\text{ m/s})^2 + (-1.3\text{ m/s})^2} = 1.9\text{ m/s}$$

Figure 3.1

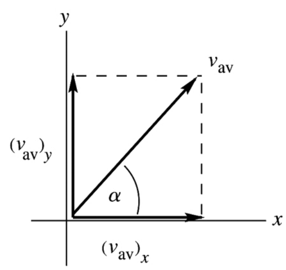
EVALUATE: Our calculation gives that $\vec{v}_{\text{av}}$ is in the 4th quadrant. This corresponds to increasing x and decreasing y.

3.2. **IDENTIFY:** Use $\vec{v}_{\text{av}} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form. The distance from the origin is the magnitude of $\vec{r}$.
SET UP: At time t_1, $x_1 = y_1 = 0$.
EXECUTE: **(a)** $x = (v_{\text{av-}x})\Delta t = (-3.8\text{ m/s})(12.0\text{ s}) = -45.6\text{ m}$ and $y = (v_{\text{av-}y})\Delta t = (4.9\text{ m/s})(12.0\text{ s}) = 58.8\text{ m}$.
(b) $r = \sqrt{x^2 + y^2} = \sqrt{(-45.6\text{ m})^2 + (58.8\text{ m})^2} = 74.4\text{ m}$.
EVALUATE: $\Delta\vec{r}$ is in the direction of $\vec{v}_{\text{av}}$. Therefore, Δx is negative since $v_{\text{av-}x}$ is negative and Δy is positive since $v_{\text{av-}y}$ is positive.

3.3. **(a) IDENTIFY** and **SET UP:** From $\vec{r}$ we can calculate x and y for any t.
Then use $\vec{v}_{\text{av}} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form.
EXECUTE: $\vec{r} = [4.0\text{ cm} + (2.5\text{ cm/s}^2)t^2]\hat{i} + (5.0\text{ cm/s})t\hat{j}$
At $t = 0$, $\vec{r} = (4.0\text{ cm})\,\hat{i}$.
At $t = 2.0$ s, $\vec{r} = (14.0\text{ cm})\,\hat{i} + (10.0\text{ cm})\,\hat{j}$.

$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$$

$$v_{\text{av-}y} = \frac{\Delta y}{\Delta t} = \frac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$$



$$v_{\text{av}} = \sqrt{(v_{\text{av}})_x^2 + (v_{\text{av}})_y^2} = 7.1\text{ cm/s}$$

$$\tan\alpha = \frac{(v_{\text{av}})_y}{(v_{\text{av}})_x} = 1.00$$

$$\theta = 45°.$$

Figure 3.3a

EVALUATE: Both x and y increase, so $\vec{v}_{\text{av}}$ is in the 1st quadrant.

(b) IDENTIFY and SET UP: Calculate $\vec{r}$ by taking the time derivative of $\vec{r}(t)$.

EXECUTE: $\vec{v} = \dfrac{d\vec{r}}{dt} = ([5.0\text{ cm/s}^2]t)\hat{i} + (5.0\text{ cm/s})\hat{j}$

$\underline{t=0}$: $v_x = 0$, $v_y = 5.0$ cm/s; $v = 5.0$ cm/s and $\theta = 90°$

$\underline{t = 1.0\text{ s}}$: $v_x = 5.0$ cm/s, $v_y = 5.0$ cm/s; $v = 7.1$ cm/s and $\theta = 45°$

$\underline{t = 2.0\text{ s}}$: $v_x = 10.0$ cm/s, $v_y = 5.0$ cm/s; $v = 11$ cm/s and $\theta = 27°$

(c) The trajectory is a graph of y versus x.

$x = 4.0\text{ cm} + (2.5\text{ cm/s}^2)t^2$, $y = (5.0\text{ cm/s})t$

For values of t between 0 and 2.0 s, calculate x and y and plot y versus x.

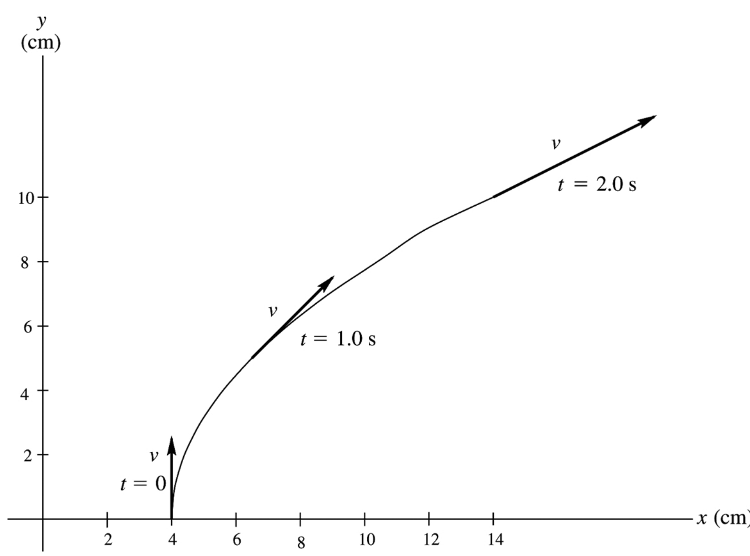


Figure 3.3b

EVALUATE: The sketch shows that the instantaneous velocity at any t is tangent to the trajectory.

3.4. **IDENTIFY:** Given the position vector of a squirrel, find its velocity components in general, and at a specific time find its velocity components and the magnitude and direction of its position vector and velocity.

SET UP: $v_x = dx/dt$ and $v_y = dy/dt$; the magnitude of a vector is $A = \sqrt{(A_x^2 + A_y^2)}$.

EXECUTE: **(a)** Taking the derivatives gives $v_x(t) = 0.280 \text{ m/s} + (0.0720 \text{ m/s}^2)t$ and $v_y(t) = (0.0570 \text{ m/s}^3)t^2$.

(b) Evaluating the position vector at $t = 5.00$ s gives $x = 2.30$ m and $y = 2.375$ m, which gives $r = 3.31$ m.

(c) At $t = 5.00$ s, $v_x = +0.64$ m/s, $v_y = 1.425$ m/s, which gives $v = 1.56$ m/s and $\tan\theta = \dfrac{1.425}{0.64}$ so the direction is $\theta = 65.8^\circ$ (counterclockwise from +x-axis)

EVALUATE: The acceleration is not constant, so we cannot use the standard kinematics formulas.

3.5. **IDENTIFY** and **SET UP:** Use Eq. $\vec{a}_{\text{av}} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ in component form to calculate $a_{\text{av-}x}$ and $a_{\text{av-}y}$.

EXECUTE: **(a)** The velocity vectors at $t_1 = 0$ and $t_2 = 30.0$ s are shown in Figure 3.5a.

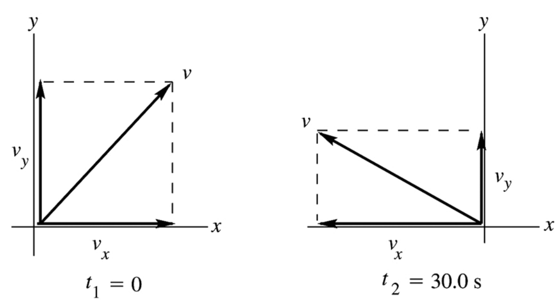
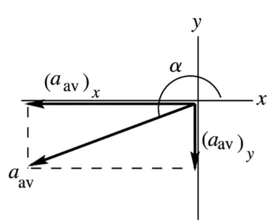


Figure 3.5a

(b) $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{v_{2x} - v_{1x}}{t_2 - t_1} = \dfrac{-170 \text{ m/s} - 90 \text{ m/s}}{30.0 \text{ s}} = -8.67 \text{ m/s}^2$

$a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t} = \dfrac{v_{2y} - v_{1y}}{t_2 - t_1} = \dfrac{40 \text{ m/s} - 110 \text{ m/s}}{30.0 \text{ s}} = -2.33 \text{ m/s}^2$

(c)



$a = \sqrt{(a_{\text{av-}x})^2 + (a_{\text{av-}y})^2} = 8.98 \text{ m/s}^2$

$\tan\alpha = \dfrac{a_{\text{av-}y}}{a_{\text{av-}x}} = \dfrac{-2.33 \text{ m/s}^2}{-8.67 \text{ m/s}^2} = 0.269$

$\alpha = 15^\circ + 180^\circ = 195^\circ$

Figure 3.5b

EVALUATE: The changes in v_x and v_y are both in the negative x or y direction, so both components of $\vec{a}_{\text{av}}$ are in the 3rd quadrant.

3.6. **IDENTIFY:** Use $\vec{a}_{\text{av}} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ in component form.

SET UP: $a_x = (0.45\text{ m/s}^2)\cos 31.0° = 0.39\text{ m/s}^2$, $a_y = (0.45\text{ m/s}^2)\sin 31.0° = 0.23\text{ m/s}^2$

EXECUTE: **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$ and $v_x = 2.6\text{ m/s} + (0.39\text{ m/s}^2)(10.0\text{ s}) = 6.5\text{ m/s}$. $a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t}$ and $v_y = -1.8\text{ m/s} + (0.23\text{ m/s}^2)(10.0\text{ s}) = 0.52\text{ m/s}$.

(b) $v = \sqrt{(6.5\text{ m/s})^2 + (0.52\text{ m/s})^2} = 6.52\text{ m/s}$, at an angle of $\arctan\left(\dfrac{0.52}{6.5}\right) = 4.6°$ counterclockwise from the $+x$-axis.

(c) The velocity vectors $\vec{v}_1$ and $\vec{v}_2$ are sketched in Figure 3.6. The two velocity vectors differ in magnitude and direction.

EVALUATE: $\vec{v}_1$ is at an angle of $35°$ below the $+x$-axis and has magnitude $v_1 = 3.2\text{ m/s}$, so $v_2 > v_1$ and the direction of $\vec{v}_2$ is rotated counterclockwise from the direction of $\vec{v}_1$.

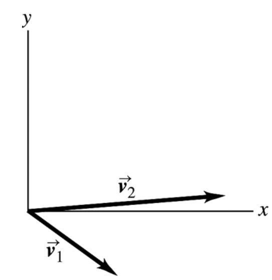


Figure 3.6

3.7. **IDENTIFY** and **SET UP:** Use $\vec{v} = \dfrac{d\vec{r}}{dt}$ and $\vec{a} = \dfrac{d\vec{v}}{dt}$ to find v_x, v_y, a_x, and a_y as functions of time. The magnitude and direction of $\vec{r}$ and $\vec{a}$ can be found once we know their components.

EXECUTE: **(a)** Calculate x and y for t values in the range 0 to 2.0 s and plot y versus x. The results are given in Figure 3.7a.

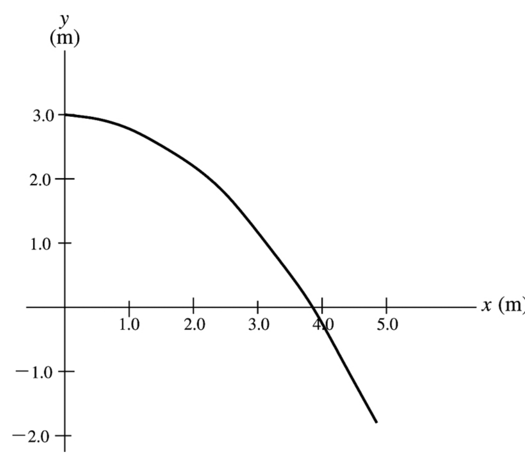


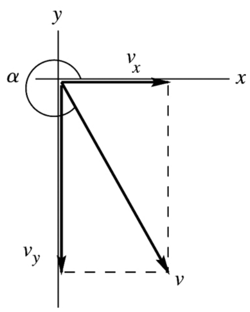
Figure 3.7a

(b) $v_x = \dfrac{dx}{dt} = \alpha \quad v_y = \dfrac{dy}{dt} = -2\beta t$

$a_x = \dfrac{dv_x}{dt} = 0 \quad a_y = \dfrac{dv_y}{dt} = -2\beta$

Thus $\vec{v} = \alpha\hat{i} - 2\beta t\hat{j}$, $\vec{a} = -2\beta\hat{j}$

(c) Velocity: At $t = 2.0$ s, $v_x = 2.4$ m/s, $v_y = -2(1.2\ \text{m/s}^2)(2.0\ \text{s}) = -4.8$ m/s



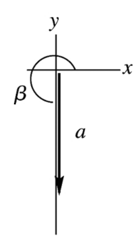
$$v = \sqrt{v_x^2 + v_y^2} = 5.4\ \text{m/s}$$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-4.8\ \text{m/s}}{2.4\ \text{m/s}} = -2.00$$

$$\alpha = -63.4° + 360° = 297°$$

Figure 3.7b

Acceleration: At $t = 2.0$ s, $a_x = 0$, $a_y = -2\,(1.2\ \text{m/s}^2) = -2.4\ \text{m/s}^2$



$$a = \sqrt{a_x^2 + a_y^2} = 2.4\ \text{m/s}^2$$

$$\tan\beta = \frac{a_y}{a_x} = \frac{-2.4\ \text{m/s}^2}{0} = -\infty$$

$$\beta = 270°$$

Figure 3.7c

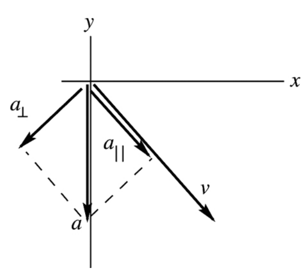


Figure 3.7d

EVALUATE: **(d)** $\vec{a}$ has a component $a_{\parallel}$ in the same direction as $\vec{v}$, so we know that v is increasing (the bird is speeding up). $\vec{a}$ also has a component $a_\perp$ perpendicular to $\vec{v}$, so that the direction of $\vec{v}$ is changing; the bird is turning toward the $-y$-direction (toward the right)

$\vec{v}$ is always tangent to the path; $\vec{v}$ at $t = 2.0$ s shown in part (c) is tangent to the path at this t, conforming to this general rule. $\vec{a}$ is constant and in the $-y$-direction; the direction of $\vec{v}$ is turning toward the $-y$-direction.

3.8. **IDENTIFY:** Use the velocity components of a car (given as a function of time) to find the acceleration of the car as a function of time and to find the magnitude and direction of the car's velocity and acceleration at a specific time.

SET UP: $a_x = dv_x/dt$ and $a_y = dv_y/dt$; the magnitude of a vector is $A = \sqrt{(A_x^2 + A_y^2)}$.

EXECUTE: **(a)** Taking the derivatives gives $a_x(t) = (-0.0360\ \text{m/s}^3)t$ and $a_y(t) = 0.550\ \text{m/s}^2$.

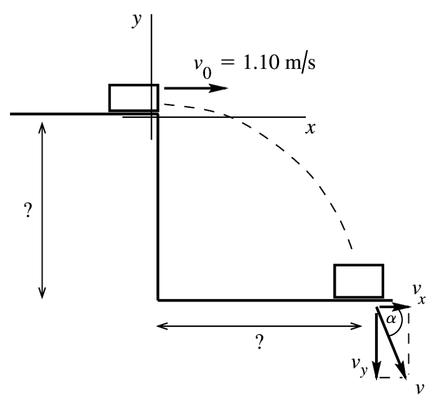
(b) Evaluating the velocity components at $t = 8.00$ s gives $v_x = 3.848$ m/s and $v_y = 6.40$ m/s, which gives $v = 7.47$ m/s. The direction is $\tan\theta = \dfrac{6.40}{3.848}$ so $\theta = 59.0°$ (counterclockwise from +x-axis).

(c) Evaluating the acceleration components at $t = 8.00$ s gives $a_x = 20.288$ m/s^2 and $a_y = 0.550$ m/s^2, which gives $a = 0.621$ m/s^2. The angle with the +y axis is given by $\tan\theta = \dfrac{0.288}{0.550}$, so $\theta = 27.6°$. The direction is therefore 118° counterclockwise from +x-axis.

EVALUATE: The acceleration is not constant, so we cannot use the standard kinematics formulas.

3.9. IDENTIFY: The book moves in projectile motion once it leaves the tabletop. Its initial velocity is horizontal.

SET UP: Take the positive y-direction to be upward. Take the origin of coordinates at the initial position of the book, at the point where it leaves the table top.



x-component:
$a_x = 0$, $v_{0x} = 1.10$ m/s,
$t = 0.480$ s
y-component:
$a_y = -9.80$ m/s^2, $v_{0y} = 0$,
$t = 0.480$ s

Figure 3.9a

Use constant acceleration equations for the x and y components of the motion, with $a_x = 0$ and $a_y = -g$.

EXECUTE: **(a)** $y - y_0 = ?$

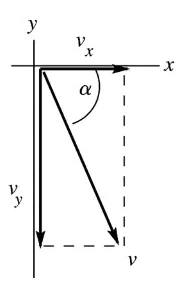
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.480 \text{ s})^2 = -1.129$ m. The tabletop is therefore 1.13 m above the floor.

(b) $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.10 \text{ m/s})(0.480 \text{ s}) + 0 = 0.528$ m.

(c) $v_x = v_{0x} + a_x t = 1.10$ m/s (The x-component of the velocity is constant, since $a_x = 0$.)

$v_y = v_{0y} + a_y t = 0 + (-9.80 \text{ m/s}^2)(0.480 \text{ s}) = -4.704$ m/s



$v = \sqrt{v_x^2 + v_y^2} = 4.83$ m/s

$\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{-4.704 \text{ m/s}}{1.10 \text{ m/s}} = -4.2764$

$\alpha = -76.8°$

Direction of $\vec{v}$ is 76.8° below the horizontal

Figure 3.9b

(d) The graphs are given in Figure 3.9c.

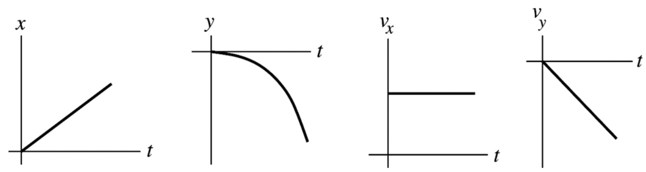


Figure 3.9c

EVALUATE: In the *x*-direction, $a_x = 0$ and v_x is constant. In the *y*-direction, $a_y = -9.80\ \text{m/s}^2$ and v_y is downward and increasing in magnitude since a_y and v_y are in the same directions. The *x* and *y* motions occur independently, connected only by the time. The time it takes the book to fall 1.13 m is the time it travels horizontally.

3.10. **IDENTIFY:** The person moves in projectile motion. She must travel 1.75 m horizontally during the time she falls 9.00 m vertically.

SET UP: Take $+y$ downward. $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

EXECUTE: Time to fall 9.00 m: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(9.00\ \text{m})}{9.80\ \text{m/s}^2}} = 1.36\ \text{s}$.

Speed needed to travel 1.75 m horizontally during this time: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$$v_0 = v_{0x} = \frac{x - x_0}{t} = \frac{1.75\ \text{m}}{1.36\ \text{s}} = 1.29\ \text{m/s}.$$

EVALUATE: If she increases her initial speed she still takes 1.36 s to reach the level of the ledge, but has traveled horizontally farther than 1.75 m.

3.11. **IDENTIFY:** Each object moves in projectile motion.

SET UP: Take $+y$ to be downward. For each cricket, $a_x = 0$ and $a_y = +9.80\ \text{m/s}^2$. For Chirpy, $v_{0x} = v_{0y} = 0$. For Milada, $v_{0x} = 0.950\ \text{m/s}$, $v_{0y} = 0$.

EXECUTE: Milada's horizontal component of velocity has no effect on her vertical motion. She also reaches the ground in 2.70 s. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (0.950\ \text{m/s})(2.70\ \text{s}) = 2.57\ \text{m}$.

EVALUATE: The *x* and *y* components of motion are totally separate and are connected only by the fact that the time is the same for both.

3.12. **IDENTIFY:** The football moves in projectile motion.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. At the highest point in the trajectory, $v_y = 0$.

EXECUTE: **(a)** $v_y = v_{0y} + a_y t$. The time t is $\frac{v_{0y}}{g} = \frac{12.0\ \text{m / s}}{9.80\ \text{m/s}^2} = 1.224\ \text{s}$, which we round to 1.22 s.

(b) Different constant acceleration equations give different expressions but the same numerical result:

$$\tfrac{1}{2}gt^2 = \tfrac{1}{2}v_{y0}t = \frac{v_{0y}^2}{2g} = 7.35\ \text{m}.$$

(c) Regardless of how the algebra is done, the time will be twice that found in part (a), which is 2(1.224 s) = 2.45 s.

(d) $a_x = 0$, so $x - x_0 = v_{0x}t = (20.0\ \text{m/s})(2.45\ \text{s}) = 49.0\ \text{m}$.

(e) The graphs are sketched in Figure 3.12.

EVALUATE: When the football returns to its original level, $v_x = 20.0\ \text{m/s}$ and $v_y = -12.0\ \text{m/s}$.

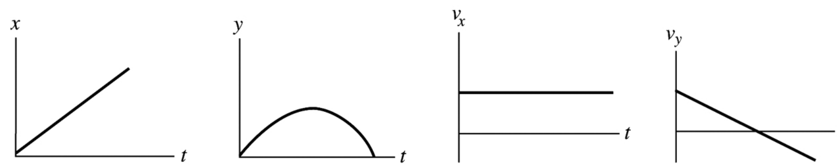


Figure 3.12

3.13. **IDENTIFY:** The car moves in projectile motion. The car travels $21.3\text{ m} - 1.80\text{ m} = 19.5\text{ m}$ downward during the time it travels 48.0 m horizontally.

SET UP: Take $+y$ to be downward. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

EXECUTE: **(a)** Use the vertical motion to find the time in the air:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(19.5\text{ m})}{9.80\text{ m/s}^2}} = 1.995\text{ s}$

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{48.0\text{ m}}{1.995\text{ s}} = 24.1\text{ m/s}$.

(b) $v_x = 24.06\text{ m/s}$ since $a_x = 0$. $v_y = v_{0y} + a_y t = -19.55\text{ m/s}$. $v = \sqrt{v_x^2 + v_y^2} = 31.0\text{ m/s}$.

EVALUATE: Note that the speed is considerably less than the algebraic sum of the x- and y-components of the velocity.

3.14. **IDENTIFY:** Knowing the maximum reached by the froghopper and its angle of takeoff, we want to find its takeoff speed and the horizontal distance it travels while in the air.

SET UP: Use coordinates with the origin at the ground and $+y$ upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. At the maximum height $v_y = 0$. The constant-acceleration formulas $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ apply.

EXECUTE: **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\text{ m/s}^2)(0.587\text{ m})} = 3.39\text{ m/s}$. $v_{0y} = v_0 \sin\theta_0$ so

$v_0 = \dfrac{v_{0y}}{\sin\theta_0} = \dfrac{3.39\text{ m/s}}{\sin 58.0°} = 4.00\text{ m/s}$.

(b) Use the vertical motion to find the time in the air. When the froghopper has returned to the ground, $y - y_0 = 0$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(3.39\text{ m/s})}{-9.80\text{ m/s}^2} = 0.692\text{ s}$.

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (v_0 \cos\theta_0)t = (4.00\text{ m/s})(\cos 58.0°)(0.692\text{ s}) = 1.47\text{ m}$.

EVALUATE: $v_y = 0$ when $t = -\dfrac{v_{0y}}{a_y} = -\dfrac{3.39\text{ m/s}}{-9.80\text{ m/s}^2} = 0.346\text{ s}$. The total time in the air is twice this.

3.15. **IDENTIFY:** The ball moves with projectile motion with an initial velocity that is horizontal and has magnitude v_0. The height h of the table and v_0 are the same; the acceleration due to gravity changes from $g_E = 9.80\text{ m/s}^2$ on earth to g_X on planet X.

SET UP: Let $+x$ be horizontal and in the direction of the initial velocity of the marble and let $+y$ be upward. $v_{0x} = v_0$, $v_{0y} = 0$, $a_x = 0$, $a_y = -g$, where g is either g_E or g_X.

EXECUTE: Use the vertical motion to find the time in the air: $y - y_0 = -h$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2h}{g}}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = v_{0x}t = v_0\sqrt{\dfrac{2h}{g}}$. $x - x_0 = D$ on earth and $2.76D$ on

Planet X. $(x-x_0)\sqrt{g}=v_0\sqrt{2h}$, which is constant, so $D\sqrt{g_E}=2.76D\sqrt{g_X}$.

$g_X=\dfrac{g_E}{(2.76)^2}=0.131g_E=1.28\text{ m/s}^2$.

EVALUATE: On Planet X the acceleration due to gravity is less, it takes the ball longer to reach the floor and it travels farther horizontally.

3.16. **IDENTIFY:** The shell moves in projectile motion.

SET UP: Let $+x$ be horizontal, along the direction of the shell's motion, and let $+y$ be upward. $a_x=0$, $a_y=-9.80\text{ m/s}^2$.

EXECUTE: **(a)** $v_{0x}=v_0\cos\alpha_0=(40.0\text{ m/s})\cos 60.0°=20.0\text{ m/s}$,
$v_{0y}=v_0\sin\alpha_0=(40.0\text{ m/s})\sin 60.0°=34.6\text{ m/s}$.

(b) At the maximum height $v_y=0$. $v_y=v_{0y}+a_yt$ gives $t=\dfrac{v_y-v_{0y}}{a_y}=\dfrac{0-34.6\text{ m/s}}{-9.80\text{ m/s}^2}=3.53\text{ s}$.

(c) $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives $y-y_0=\dfrac{v_y^2-v_{0y}^2}{2a_y}=\dfrac{0-(34.6\text{ m/s})^2}{2(-9.80\text{ m/s}^2)}=61.2\text{ m}$.

(d) The total time in the air is twice the time to the maximum height, so
$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2=(20.0\text{ m/s})(2)(3.53\text{ s})=141\text{ m}$.

(e) At the maximum height, $v_x=v_{0x}=20.0\text{ m/s}$ and $v_y=0$. At all points in the motion, $a_x=0$ and $a_y=-9.80\text{ m/s}^2$.

EVALUATE: The equation for the horizontal range R derived in the text is $R=\dfrac{v_0^2\sin 2\alpha_0}{g}$. This gives $R=\dfrac{(40.0\text{ m/s})^2\sin(120.0°)}{9.80\text{ m/s}^2}=141\text{ m}$, which agrees with our result in part (d).

3.17. **IDENTIFY:** The baseball moves in projectile motion. In part (c) first calculate the components of the velocity at this point and then get the resultant velocity from its components.

SET UP: First find the x- and y-components of the initial velocity. Use coordinates where the $+y$-direction is upward, the $+x$-direction is to the right and the origin is at the point where the baseball leaves the bat.

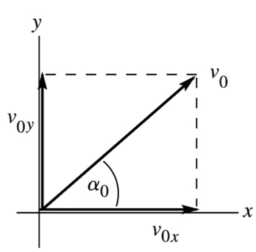


Figure 3.17a

$v_{0x}=v_0\cos\alpha_0=(30.0\text{ m/s})\cos 36.9°=24.0\text{ m/s}$

$v_{0y}=v_0\sin\alpha_0=(30.0\text{ m/s})\sin 36.9°=18.0\text{ m/s}$

Use constant acceleration equations for the x and y motions, with $a_x=0$ and $a_y=-g$.

EXECUTE: **(a)** y-component (vertical motion):

$y-y_0=+10.0\text{ m}$, $v_{0y}=18.0\text{ m/s}$, $a_y=-9.80\text{ m/s}^2$, $t=?$

$y-y_0=v_{0y}+\frac{1}{2}a_yt^2$

$10.0\text{ m}=(18.0\text{ m/s})t-(4.90\text{ m/s}^2)t^2$

$(4.90\ \text{m/s}^2)t^2 - (18.0\ \text{m/s})t + 10.0\ \text{m} = 0$

Apply the quadratic formula: $t = \frac{1}{9.80}\left[18.0 \pm \sqrt{(-18.0)^2 - 4\,(4.90)(10.0)}\right]\ \text{s} = (1.837 \pm 1.154)\ \text{s}$

The ball is at a height of 10.0 above the point where it left the bat at $t_1 = 0.683\ \text{s}$ and at $t_2 = 2.99\ \text{s}$. At the earlier time the ball passes through a height of 10.0 m as its way up and at the later time it passes through 10.0 m on its way down.

(b) $v_x = v_{0x} = +24.0\ \text{m/s}$, at all times since $a_x = 0$.

$v_y = v_{0y} + a_y t$

$\underline{t_1 = 0.683\ \text{s}}$: $v_y = +18.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(0.683\ \text{s}) = +11.3\ \text{m/s}$. ($v_y$ is positive means that the ball is traveling upward at this point.)

$\underline{t_2 = 2.99\ \text{s}}$: $v_y = +18.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(2.99\ \text{s}) = -11.3\ \text{m/s}$. ($v_y$ is negative means that the ball is traveling downward at this point.)

(c) $v_x = v_{0x} = 24.0\ \text{m/s}$

Solve for v_y:

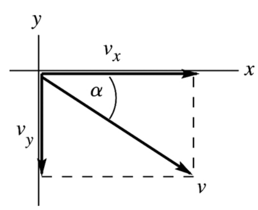
$v_y = ?$, $y - y_0 = 0$ (when ball returns to height where motion started),

$a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +18.0\ \text{m/s}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -v_{0y} = -18.0\ \text{m/s}$ (negative, since the baseball must be traveling downward at this point)

Now solve for the magnitude and direction of $\vec{v}$.



$v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{(24.0\ \text{m/s})^2 + (-18.0\ \text{m/s})^2} = 30.0\ \text{m/s}$

$\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{-18.0\ \text{m/s}}{24.0\ \text{m/s}}$

$\alpha = -36.9°$, $36.9°$ below the horizontal

Figure 3.17b

The velocity of the ball when it returns to the level where it left the bat has magnitude 30.0 m/s and is directed at an angle of $36.9°$ below the horizontal.

EVALUATE: The discussion in parts (a) and (b) explains the significance of two values of t for which $y - y_0 = +10.0\ \text{m}$. When the ball returns to its initial height, our results give that its speed is the same as its initial speed and the angle of its velocity below the horizontal is equal to the angle of its initial velocity above the horizontal; both of these are general results.

3.18. **IDENTIFY:** The shot moves in projectile motion.

SET UP: Let $+y$ be upward.

EXECUTE: **(a)** If air resistance is to be ignored, the components of acceleration are 0 horizontally and $-g = -9.80\ \text{m/s}^2$ vertically downward.

(b) The x-component of velocity is constant at $v_x = (12.0\ \text{m/s})\cos 51.0° = 7.55\ \text{m/s}$. The y-component is $v_{0y} = (12.0\ \text{m/s})\sin 51.0° = 9.32\ \text{m/s}$ at release and

$v_y = v_{0y} - gt = (9.32\ \text{m/s}) - (9.80\ \text{m/s})(2.08\ \text{s}) = -11.06\ \text{m/s}$ when the shot hits.

(c) $x - x_0 = v_{0x}t = (7.55\ \text{m/s})(2.08\ \text{s}) = 15.7\ \text{m}$.

(d) The initial and final heights are not the same.
(e) With $y = 0$ and v_{0y} as found above, the equation for $y - y_0$ as a function of time gives $y_0 = 1.81\text{m}$.
(f) The graphs are sketched in Figure 3.18.
EVALUATE: When the shot returns to its initial height, $v_y = -9.32$ m/s. The shot continues to accelerate downward as it travels downward 1.81 m to the ground and the magnitude of v_y at the ground is larger than 9.32 m/s.

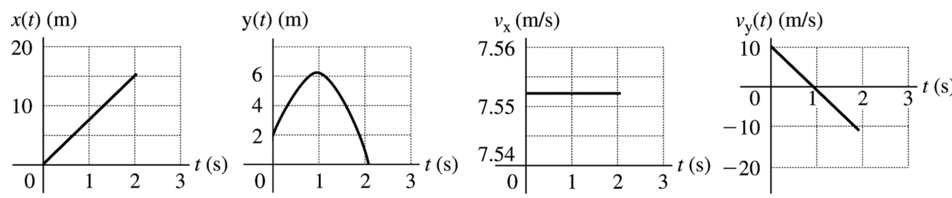
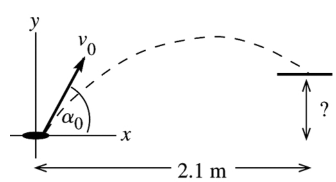


Figure 3.18

3.19. IDENTIFY: Take the origin of coordinates at the point where the quarter leaves your hand and take positive *y* to be upward. The quarter moves in projectile motion, with $a_x = 0$, and $a_y = -g$. It travels vertically for the time it takes it to travel horizontally 2.1 m.



$v_{0x} = v_0 \cos\alpha_0 = (6.4 \text{ m/s}) \cos 60°$
$v_{0x} = 3.20$ m/s
$v_{0y} = v_0 \sin\alpha_0 = (6.4 \text{ m/s}) \sin 60°$
$v_{0y} = 5.54$ m/s

Figure 3.19

(a) SET UP: Use the horizontal (*x*-component) of motion to solve for *t*, the time the quarter travels through the air:
$t = ?,\ x - x_0 = 2.1 \text{ m},\ v_{0x} = 3.2 \text{ m/s},\ a_x = 0$
$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = v_{0x}t,$ since $a_x = 0$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{2.1 \text{ m}}{3.2 \text{ m/s}} = 0.656 \text{ s}$

SET UP: Now find the vertical displacement of the quarter after this time:
$y - y_0 = ?,\ a_y = -9.80 \text{ m/s}^2,\ v_{0y} = +5.54 \text{ m/s},\ t = 0.656 \text{ s}$
$y - y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $y - y_0 = (5.54 \text{ m/s})(0.656 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.656 \text{ s})^2 = 3.63 \text{ m} - 2.11 \text{ m} = 1.5 \text{ m}.$

(b) SET UP: $v_y = ?,\ t = 0.656 \text{ s},\ a_y = -9.80 \text{ m/s}^2,\ v_{0y} = +5.54 \text{ m/s} \quad v_y = v_{0y} + a_y t$

EXECUTE: $v_y = 5.54 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.656 \text{ s}) = -0.89 \text{ m/s}.$

EVALUATE: The minus sign for v_y indicates that the *y*-component of $\vec{v}$ is downward. At this point the quarter has passed through the highest point in its path and is on its way down. The horizontal range if it returned to its original height (it doesn't!) would be 3.6 m. It reaches its maximum height after traveling horizontally 1.8 m, so at $x - x_0 = 2.1$ m it is on its way down.

3.20. IDENTIFY: Consider the horizontal and vertical components of the projectile motion. The water travels 45.0 m horizontally in 3.00 s.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$. $v_{0x} = v_0 \cos\theta_0$, $v_{0y} = v_0 \sin\theta_0$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = v_0(\cos\theta_0)t$ and $\cos\theta_0 = \dfrac{45.0\ \text{m}}{(25.0\ \text{m/s})(3.00\ \text{s})} = 0.600$; $\theta_0 = 53.1°$

(b) At the highest point $v_x = v_{0x} = (25.0\ \text{m/s})\cos 53.1° = 15.0\ \text{m/s}$, $v_y = 0$ and $v = \sqrt{v_x^2 + v_y^2} = 15.0\ \text{m/s}$. At all points in the motion, $a = 9.80\ \text{m/s}^2$ downward.

(c) Find $y - y_0$ when $t = 3.00\text{s}$:

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 = (25.0\ \text{m/s})(\sin 53.1°)(3.00\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(3.00\ \text{s})^2 = 15.9\ \text{m}$$

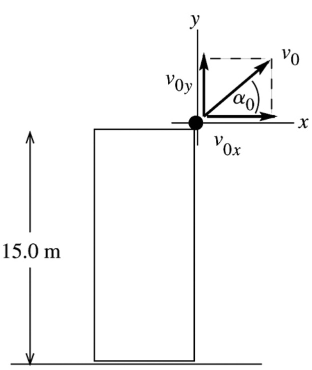
$v_x = v_{0x} = 15.0\ \text{m/s}$, $v_y = v_{0y} + a_y t = (25.0\ \text{m/s})(\sin 53.1°) - (9.80\text{m/s}^2)(3.00\ \text{s}) = -9.41\ \text{m/s}$, and

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.0\ \text{m/s})^2 + (-9.41\ \text{m/s})^2} = 17.7\ \text{m/s}$$

EVALUATE: The acceleration is the same at all points of the motion. It takes the water $t = -\dfrac{v_{0y}}{a_y} = -\dfrac{20.0\ \text{m/s}}{-9.80\ \text{m/s}^2} = 2.04\ \text{s}$ to reach its maximum height. When the water reaches the building it has passed its maximum height and its vertical component of velocity is downward.

3.21. IDENTIFY: Take the origin of coordinates at the roof and let the $+y$-direction be upward. The rock moves in projectile motion, with $a_x = 0$ and $a_y = -g$. Apply constant acceleration equations for the x and y components of the motion.

SET UP:



$v_{0x} = v_0 \cos\alpha_0 = 25.2\ \text{m/s}$

$v_{0y} = v_0 \sin\alpha_0 = 16.3\ \text{m/s}$

Figure 3.21a

(a) At the maximum height $v_y = 0$.

$a_y = -9.80\ \text{m/s}^2$, . $v_y = 0$, . $v_{0y} = +16.3\ \text{m/s}$, $y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y\,(y - y_0)$

EXECUTE: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (16.3\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = +13.6\ \text{m}$

(b) SET UP: Find the velocity by solving for its x and y components.

$v_x = v_{0x} = 25.2\ \text{m/s}$ (since $a_x = 0$)

$v_y = ?,\ a_y = -9.80\ \text{m/s}^2,\ y - y_0 = -15.0\ \text{m}$ (negative because at the ground the rock is below its initial position), $v_{0y} = 16.3\ \text{m/s}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)}$ (v_y is negative because at the ground the rock is traveling downward.)

EXECUTE: $v_y = -\sqrt{(16.3\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-15.0\ \text{m})} = -23.7\ \text{m/s}$

Then $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(25.2\ \text{m/s})^2 + (-23.7\ \text{m/s})^2} = 34.6\ \text{m/s}$.

(c) SET UP: Use the vertical motion (y-component) to find the time the rock is in the air:
$t = ?,\ v_y = -23.7\ \text{m/s}$ (from part (b)), $a_y = -9.80\ \text{m/s}^2,\ v_{0y} = +16.3\ \text{m/s}$

EXECUTE: $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-23.7\ \text{m/s} - 16.3\ \text{m/s}}{-9.80\ \text{m/s}^2} = +4.08\ \text{s}$

SET UP: Can use this t to calculate the horizontal range:
$t = 4.08\ \text{s},\ v_{0x} = 25.2\ \text{m/s},\ a_x = 0,\ x - x_0 = ?$

EXECUTE: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (25.2\ \text{m/s})(4.08\ \text{s}) + 0 = 103\ \text{m}$

(d) Graphs of x versus t, y versus t, v_x versus t and v_y versus t:

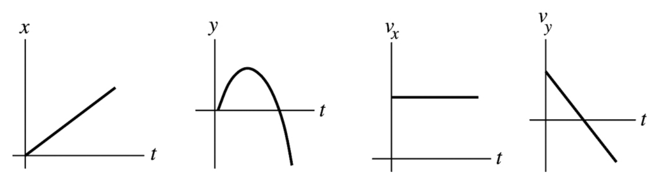


Figure 3.21b

EVALUATE: The time it takes the rock to travel vertically to the ground is the time it has to travel horizontally. With $v_{0y} = +16.3\ \text{m/s}$ the time it takes the rock to return to the level of the roof $(y = 0)$ is $t = 2v_{0y}/g = 3.33\ \text{s}$. The time in the air is greater than this because the rock travels an additional 15.0 m to the ground.

3.22. IDENTIFY: This is a problem in projectile motion. The acceleration is g downward with constant horizontal velocity. The constant-acceleration equations apply.

SET UP: We use $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$ and $v_y = v_0 \sin\alpha_0 - gt$ for the vertical motion and $v_x = v_0 \cos\alpha_0$ for the horizontal motion. We want the time when $y = 5.00$ m.

EXECUTE: Use $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$ to find the times.

$5.00\ \text{m} = (15.0\ \text{m/s})(\sin 53.0°)t - \frac{1}{2}(9.80\ \text{m/s}^2)t^2$

Using the quadratic formula to solve this equation gives $t = 0.534$ s and $t = 1.91$ s.
Now look at the horizontal velocity.
$v_x = v_0 \cos\alpha_0 = (15.0\ \text{m/s}) \cos 53.0° = 9.03\ \text{m/s}$ at both times.
At $t = 0.534$ s: $v_y = v_0 \sin\alpha_0 - gt = (15.0\ \text{m/s}) \sin 53.0° - (9.80\ \text{m/s}^2)(0.534\ \text{s}) = 6.75\ \text{m/s}$ upward.
At $t = 1.92$ s: $v_y = (15.0\ \text{m/s}) \sin 53.0° - (9.80\ \text{m/s}^2)(1.92\ \text{s}) = -6.75\ \text{m/s}$ downward.

EVALUATE: To check, use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find v_y when y = 5.00 m, which gives $v_y^2 =$ [(15.0 m/s)(sin 53.0°)]^2 – 2(9.80 m/s^2)(5.00 m) $\rightarrow$ v_y = ±6.75 m/s, which agrees with our results. We get two answers because the rock is going upward at t = 0.534 s and downward at t = 1.91 s.

3.23. **IDENTIFY:** This is a problem in projectile motion. The acceleration is g downward with constant horizontal velocity. The constant-acceleration equations apply.

SET UP: Estimate: 75 ft which is about 25 m. The range of a projectile is $R = \dfrac{v_0^2}{g}\sin 2\alpha_0$. For the maximum range, $2\alpha_0$ = 90°, so α_0 = 45°, so we assume that we throw it at 45° above the horizontal.

EXECUTE: **(a)** Solve the range formula for v_0: $v_0 = \sqrt{Rg} = \sqrt{(25\text{ m})(9.80\text{ m/s}^2)}$ = 15.65 m/s, which rounds to 16 m/s.

(b) At the maximum height, v_y = 0. Using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $y - y_0 = h$ gives 0 = [(15.65 m/s)(sin 45°)]^2 – 2(9.80 m/s^2)h $\rightarrow$ h = 6.5 m.

(c) Apply the range formula on Mars: $R = \dfrac{(15.65\text{ m/s})^2}{3.7\text{ m/s}^2}\sin 90°$ = 66 m.

EVALUATE: The range is inversely proportional to g, so $R_M/R_E = g_E/g_M$ = 9.8/3.7 = 2.65. So the range of Mars would be 2.65 times the range on earth, which is (2.65)(25 m) = 66 m, in agreement with our result.

3.24. **IDENTIFY:** The merry-go-round is rotating at a constant rate.

SET UP: Estimate: T = 5.0 s for one rotation. Convert units: R = 6.0 ft = 1.829 m. The radial acceleration is $a_{rad} = \dfrac{v^2}{R}$ and $v = 2\pi R/T$.

EXECUTE: **(a)** $v = 2\pi R/T = 2\pi(1.829\text{ m})/(5.0\text{ s})$ = 2.3 m/s.

(b) $a_{rad} = \dfrac{v^2}{R}$ = (2.3 m/s)^2/(1.829 m) = 2.9 m/s^2.

(c) If $T \rightarrow T/2$, $v \rightarrow 2v$ so $v^2 \rightarrow 4v^2$. Since $a_{rad} \propto v^2$, $a_{rad} \rightarrow 4a_{rad}$ = 4(2.9 m/s^2) = 12 m/s^2.

EVALUATE: The acceleration in part (b) is 2.9/9.8 g = 0.30g, which seems a bit large for a merry-go-round for young children. Perhaps the estimate of 5.0 s is too small. If we revise the estimate to 10 s, v will become 1.15 m/s and a_{rad} will become 0.72 m/s^2, which is 0.72/9.8 g = 0.074g. This is about 7.4% of g, which seems a bit more reasonable.

3.25. **IDENTIFY:** This problem deals with circular motion.

SET UP: Apply the equation $a_{rad} = 4\pi^2 R/T^2$, where T = 24 h.

EXECUTE: **(a)** $a_{rad} = \dfrac{4\pi^2(6.38\times10^6\text{ m})}{[(24\text{ h})(3600\text{ s/h})]^2} = 0.034\text{ m/s}^2 = 3.4\times10^{-3}g.$

(b) Solving the equation $a_{rad} = 4\pi^2 R/T^2$ for the period T with $a_{rad} = g$,

$$T = \sqrt{\frac{4\pi^2(6.38\times10^6\text{m})}{9.80\text{ m/s}^2}} = 5070\text{ s} = 1.4\text{ h}.$$

EVALUATE: a_{rad} is proportional to $1/T^2$, so to increase a_{rad} by a factor of $\dfrac{1}{3.4\times10^{-3}} = 294$ requires that T be multiplied by a factor of $\dfrac{1}{\sqrt{294}}$. $\dfrac{24\text{ h}}{\sqrt{294}} = 1.4\text{ h}.$

3.26. **IDENTIFY:** We want to find the acceleration of the inner ear of a dancer, knowing the rate at which she spins.

SET UP: $R = 0.070$ m. For 3.0 rev/s, the period T (time for one revolution) is $T = \dfrac{1.0 \text{ s}}{3.0 \text{ rev}} = 0.333$ s.

The speed is $v = d/T = (2\pi R)/T$, and $a_{\text{rad}} = v^2/R$.

EXECUTE: $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(2\pi R/T)^2}{R} = \dfrac{4\pi^2 R}{T^2} = \dfrac{4\pi^2(0.070 \text{ m})}{(0.333 \text{ s})^2} = 24.92 \text{ m/s}^2$, which rounds to 25 m/s^2 with two significant figures. As a fraction of g, this acceleration is $(24.92 \text{ m/s}^2)/(9.80 \text{ m/s}^2) = 2.54$, which rounds to 2.5 to two significant figures.

EVALUATE: The acceleration is large and the force on the fluid must be 2.5 times its weight.

3.27. **IDENTIFY:** The ball has no vertical acceleration, but it has a horizontal acceleration toward the center of the circle. This acceleration is the radial acceleration.

SET UP: $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$, $R = L \sin\theta$.

EXECUTE: Use $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ with $R = L \sin\theta$. $a_{\text{rad}} = \dfrac{4\pi^2 L \sin\theta}{T^2}$. Using $L = 0.800$ m, $\theta = 37.0°$, and $T = 0.600$ s, this gives $a_{\text{rad}} = 52.8 \text{ m/s}^2$.

EVALUATE: Compare this acceleration to g: $a_{\text{rad}}/g = 52.8/9.80 = 5.39$, so $a_{\text{rad}} = 5.39g$, which is fairly large.

3.28. **IDENTIFY:** Each blade tip moves in a circle of radius $R = 3.40$ m and therefore has radial acceleration $a_{\text{rad}} = v^2/R$.

SET UP: 550 rev/min = 9.17 rev/s, corresponding to a period of $T = \dfrac{1}{9.17 \text{ rev/s}} = 0.109$ s.

EXECUTE: **(a)** $v = \dfrac{2\pi R}{T} = 196$ m/s.

(b) $a_{\text{rad}} = \dfrac{v^2}{R} = 1.13\times10^4 \text{ m/s}^2 = 1.15\times10^3 g$.

EVALUATE: $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ gives the same results for a_{rad} as in part (b).

3.29. **IDENTIFY:** For the curved lowest part of the dive, the pilot's motion is approximately circular. We know the pilot's acceleration and the radius of curvature, and from this we want to find the pilot's speed.

SET UP: $a_{\text{rad}} = 5.5g = 53.9 \text{ m/s}^2$. 1 mph = 0.4470 m/s. $a_{\text{rad}} = \dfrac{v^2}{R}$.

EXECUTE: $a_{\text{rad}} = \dfrac{v^2}{R}$, so $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(280 \text{ m})(53.9 \text{ m/s}^2)} = 122.8 \text{ m/s} = 274.8$ mph. Rounding these answers to 2 significant figures (because of $5.5g$), gives $v = 120$ m/s = 270 mph.

EVALUATE: This speed is reasonable for the type of plane flown by a test pilot.

3.30. **IDENTIFY:** The object has constant horizontal speed (but *not* constant velocity), so its acceleration points toward the center of the circle. This is the radial acceleration.

SET UP: We know v and T and we want a_{rad} in terms of v and T. $a_{\text{rad}} = \dfrac{v^2}{R}$ and $v = 2\pi R/T$.

EXECUTE: We want to express a_{rad} in terms of v and T. From $v = 2\pi R/T$ we get $R = vT/2\pi$, so

$$a_{\text{rad}} = \frac{v^2}{(vT/2\pi)} = 2\pi v/T.$$

(a) Using $a_{\text{rad}} = 2\pi v/T$, we see that if v doubles, T must also double to keep a_{rad} the same.

(b) First express a_{rad} in terms of R and T as $a_{rad} = \dfrac{4\pi^2 R}{T^2}$. If R doubles, the T^2 must also double to keep a_{rad} the same. Therefore T must increase by a factor of $\sqrt{2}$.

EVALUATE: Using $a_{rad} = 2\pi v/T$ would not be helpful in part (b) because the equation does not contain R.

3.31. **IDENTIFY:** Uniform circular motion.

SET UP: Since the magnitude of $\vec{v}$ is constant, $v_{tan} = \dfrac{d|\vec{v}|}{dt} = 0$ and the resultant acceleration is equal to the radial component. At each point in the motion the radial component of the acceleration is directed in toward the center of the circular path and its magnitude is given by v^2/R.

EXECUTE: **(a)** $a_{rad} = \dfrac{v^2}{R} = \dfrac{(6.00\text{ m/s})^2}{14.0\text{ m}} = 2.57\text{ m/s}^2$, upward.

(b) The radial acceleration has the same magnitude as in part (a), but now the direction toward the center of the circle is downward. The acceleration at this point in the motion is 2.57 m/s^2, downward.

(c) SET UP: The time to make one rotation is the period T, and the speed v is the distance for one revolution divided by T.

EXECUTE: $v = \dfrac{2\pi R}{T}$ so $T = \dfrac{2\pi R}{v} = \dfrac{2\pi\,(14.0\text{ m})}{6.00\text{ m/s}} = 14.7\text{ s}$.

EVALUATE: The radial acceleration is constant in magnitude since v is constant and is at every point in the motion directed toward the center of the circular path. The acceleration is perpendicular to $\vec{v}$ and is nonzero because the direction of $\vec{v}$ changes.

3.32. **IDENTIFY:** The roller coaster car is going in a circle. Its radial acceleration is always toward the center of the circle.

SET UP: $a_{rad} = \dfrac{v^2}{R}$

EXECUTE: **(a)** The radial acceleration is toward the center of the circle, which is directly downward when the car is at the top.

(b) By the same reasoning as in (a), the direction is vertically upward when the car is at the bottom.

(c) Use $a_{rad} = \dfrac{v^2}{R}$ and take the ratio of the accelerations: $\dfrac{a_{rad,bottom}}{a_{rad,top}} = \dfrac{v_2^2/R}{v_1^2/R} = (v_2/v_1)^2$.

EVALUATE: Our result shows that the acceleration is greater at the bottom since $v_2 > v_1$.

3.33. **IDENTIFY:** Each part of his body moves in uniform circular motion, with $a_{rad} = \dfrac{v^2}{R}$. The speed in rev/s is $1/T$, where T is the period in seconds (time for 1 revolution). The speed v increases with R along the length of his body but all of him rotates with the same period T.

SET UP: For his head $R = 8.84$ m and for his feet $R = 6.84$ m.

EXECUTE: **(a)** $v = \sqrt{Ra_{rad}} = \sqrt{(8.84\text{ m})(12.5)(9.80\text{ m/s}^2)} = 32.9\text{ m/s}$

(b) Use $a_{rad} = \dfrac{4\pi^2 R}{T^2}$. Since his head has $a_{rad} = 12.5g$ and $R = 8.84$ m,

$T = 2\pi\sqrt{\dfrac{R}{a_{rad}}} = 2\pi\sqrt{\dfrac{8.84\text{m}}{12.5(9.80\text{m/s}^2)}} = 1.688\text{s}$. Then his feet have $a_{rad} = \dfrac{R}{T^2} = \dfrac{4\pi^2(6.84\text{m})}{(1.688\text{s})^2} = 94.8\text{m/s}^2 =$ 9.67 g. The difference between the acceleration of his head and his feet is $12.5g - 9.67g = 2.83g = 27.7\text{ m/s}^2$.

(c) $\dfrac{1}{T} = \dfrac{1}{1.69\text{ s}} = 0.592\text{ rev/s} = 35.5\text{ rpm}$

EVALUATE: His feet have speed $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(6.84\text{ m})(94.8\text{ m/s}^2)} = 25.5\text{ m/s}.$

3.34. **IDENTIFY:** Each planet moves in a circular orbit and therefore has acceleration $a_{\text{rad}} = v^2/R.$

SET UP: The radius of the earth's orbit is $r = 1.50 \times 10^{11}\text{ m}$ and its orbital period is $T = 365\text{ days} = 3.16 \times 10^7\text{ s}.$ For Mercury, $r = 5.79 \times 10^{10}\text{ m}$ and $T = 88.0\text{ days} = 7.60 \times 10^6\text{ s}.$

EXECUTE: **(a)** $v = \dfrac{2\pi r}{T} = 2.98 \times 10^4\text{ m/s}$

(b) $a_{\text{rad}} = \dfrac{v^2}{r} = 5.91 \times 10^{-3}\text{ m/s}^2.$

(c) $v = 4.79 \times 10^4\text{ m/s},$ and $a_{\text{rad}} = 3.96 \times 10^{-2}\text{ m/s}^2.$

EVALUATE: Mercury has a larger orbital velocity and a larger radial acceleration than earth.

3.35. **IDENTIFY:** Relative velocity problem. The time to walk the length of the moving sidewalk is the length divided by the velocity of the woman relative to the ground.

SET UP: Let W stand for the woman, G for the ground and S for the sidewalk. Take the positive direction to be the direction in which the sidewalk is moving.

The velocities are $v_{\text{W/G}}$ (woman relative to the ground), $v_{\text{W/S}}$ (woman relative to the sidewalk), and $v_{\text{S/G}}$ (sidewalk relative to the ground).

The equation for relative velocity becomes $v_{\text{W/G}} = v_{\text{W/S}} + v_{\text{S/G}}.$

The time to reach the other end is given by $t = \dfrac{\text{distance traveled relative to ground}}{v_{\text{W/G}}}$

EXECUTE: **(a)** $v_{\text{S/G}} = 1.0\text{ m/s}$

$v_{\text{W/S}} = +1.5\text{ m/s}$

$v_{\text{W/G}} = v_{\text{W/S}} + v_{\text{S/G}} = 1.5\text{ m/s} + 1.0\text{ m/s} = 2.5\text{ m/s}.$

$t = \dfrac{35.0\text{ m}}{v_{\text{W/G}}} = \dfrac{35.0\text{ m}}{2.5\text{ m/s}} = 14\text{ s}.$

(b) $v_{\text{S/G}} = 1.0\text{ m/s}$

$v_{\text{W/S}} = -1.5\text{ m/s}$

$v_{\text{W/G}} = v_{\text{W/S}} + v_{\text{S/G}} = -1.5\text{ m/s} + 1.0\text{ m/s} = -0.5\text{ m/s}.$ (Since $v_{\text{W/G}}$ now is negative, she must get on the moving sidewalk at the opposite end from in part (a).)

$t = \dfrac{-35.0\text{ m}}{v_{\text{W/G}}} = \dfrac{-35.0\text{ m}}{-0.5\text{ m/s}} = 70\text{ s}.$

EVALUATE: Her speed relative to the ground is much greater in part (a) when she walks with the motion of the sidewalk.

3.36. **IDENTIFY:** The relative velocities are $\vec{v}_{\text{S/F}}$, the velocity of the scooter relative to the flatcar, $\vec{v}_{\text{S/G}}$, the scooter relative to the ground and $\vec{v}_{\text{F/G}}$, the flatcar relative to the ground. $\vec{v}_{\text{S/G}} = \vec{v}_{\text{S/F}} + \vec{v}_{\text{F/G}}$. Carry out the vector addition by drawing a vector addition diagram.

SET UP: $\vec{v}_{\text{S/F}} = \vec{v}_{\text{S/G}} - \vec{v}_{\text{F/G}}$. $\vec{v}_{\text{F/G}}$ is to the right, so $-\vec{v}_{\text{F/G}}$ is to the left.

EXECUTE: In each case the vector addition diagram gives

(a) 5.0 m/s to the right

(b) 16.0 m/s to the left

(c) 13.0 m/s to the left.

EVALUATE: The scooter has the largest speed relative to the ground when it is moving to the right relative to the flatcar, since in that case the two velocities $\vec{v}_{\text{S/F}}$ and $\vec{v}_{\text{F/G}}$ are in the same direction and their magnitudes add.

3.37. **IDENTIFY:** Apply the relative velocity relation.
SET UP: The relative velocities are $\vec{v}_{\text{C/E}}$, the canoe relative to the earth, $\vec{v}_{\text{R/E}}$, the velocity of the river relative to the earth and $\vec{v}_{\text{C/R}}$, the velocity of the canoe relative to the river.
EXECUTE: $\vec{v}_{\text{C/E}} = \vec{v}_{\text{C/R}} + \vec{v}_{\text{R/E}}$ and therefore $\vec{v}_{\text{C/R}} = \vec{v}_{\text{C/E}} - \vec{v}_{\text{R/E}}$. The velocity components of $\vec{v}_{\text{C/R}}$ are $-0.50\text{ m/s} + (0.40\text{ m/s})/\sqrt{2}$, east and $(0.40\text{ m/s})/\sqrt{2}$, south, for a velocity relative to the river of 0.36 m/s, at $52.5°$ south of west.
EVALUATE: The velocity of the canoe relative to the river has a smaller magnitude than the velocity of the canoe relative to the earth.

3.38. **IDENTIFY:** Calculate the rower's speed relative to the shore for each segment of the round trip.
SET UP: The boat's speed relative to the shore is 6.8 km/h downstream and 1.2 km/h upstream.
EXECUTE: The walker moves a total distance of 3.0 km at a speed of 4.0 km/h, and takes a time of three fourths of an hour (45.0 min).
The total time the rower takes is $\dfrac{1.5\text{ km}}{6.8\text{ km/h}} + \dfrac{1.5\text{ km}}{1.2\text{ km/h}} = 1.47\text{ h} = 88.2\text{ min}.$
EVALUATE: It takes the rower longer, even though for half the distance his speed is greater than 4.0 km/h. The rower spends more time at the slower speed.

3.39. **IDENTIFY:** The resultant velocity, relative to the ground, is directly southward. This velocity is the sum of the velocity of the bird relative to the air and the velocity of the air relative to the ground.
SET UP: $v_{\text{B/A}} = 100\text{ km/h}$. $\vec{v}_{\text{A/G}} = 40\text{ km/h}$, east. $\vec{v}_{\text{B/G}} = \vec{v}_{\text{B/A}} + \vec{v}_{\text{A/G}}$.
EXECUTE: We want $\vec{v}_{\text{B/G}}$ to be due south. The relative velocity addition diagram is shown in Figure 3.39.

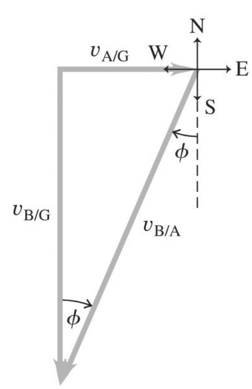


Figure 3.39

(a) $\sin\phi = \dfrac{v_{\text{A/G}}}{v_{\text{B/A}}} = \dfrac{40\text{ km/h}}{100\text{ km/h}}$, $\phi = 24°$, west of south.

(b) $v_{\text{B/G}} = \sqrt{v_{\text{B/A}}^2 - v_{\text{A/G}}^2} = 91.7\text{ km/h}$. $t = \dfrac{d}{v_{\text{B/G}}} = \dfrac{500\text{ km}}{91.7\text{ km/h}} = 5.5\text{ h}.$

EVALUATE: The speed of the bird relative to the ground is less than its speed relative to the air. Part of its velocity relative to the air is directed to oppose the effect of the wind.

3.40. **IDENTIFY:** Relative velocity problem in two dimensions.

(a) SET UP: $\vec{v}_{P/A}$ is the velocity of the plane relative to the air. The problem states that $\vec{v}_{P/A}$ has magnitude 35 m/s and direction south.

$\vec{v}_{A/E}$ is the velocity of the air relative to the earth. The problem states that $\vec{v}_{A/E}$ is to the southwest ($45°$ S of W) and has magnitude 10 m/s.

The relative velocity equation is $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$.

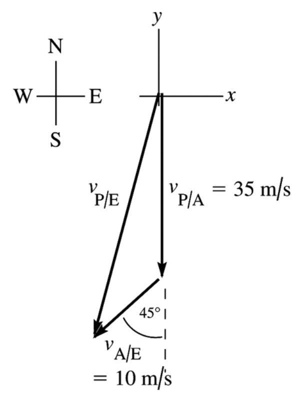


Figure 3.40a

EXECUTE: **(b)** $(v_{P/A})_x = 0,\ (v_{P/A})_y = -35$ m/s

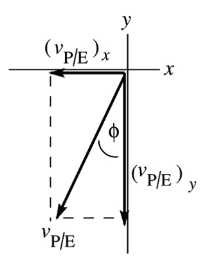
$(v_{A/E})_x = -(10 \text{ m/s})\cos 45° = -7.07$ m/s,

$(v_{A/E})_y = -(10 \text{ m/s})\sin 45° = -7.07$ m/s

$(v_{P/E})_x = (v_{P/A})_x + (v_{A/E})_x = 0 - 7.07 \text{ m/s} = -7.1$ m/s

$(v_{P/E})_y = (v_{P/A})_y + (v_{A/E})_y = -35 \text{ m/s} - 7.07 \text{ m/s} = -42$ m/s

(c)



$$v_{P/E} = \sqrt{(v_{P/E})_x^2 + (v_{P/E})_y^2}$$

$$v_{P/E} = \sqrt{(-7.1 \text{ m/s})^2 + (-42 \text{ m/s})^2} = 43 \text{ m/s}$$

$$\tan\phi = \frac{(v_{P/E})_x}{(v_{P/E})_y} = \frac{-7.1}{-42} = 0.169$$

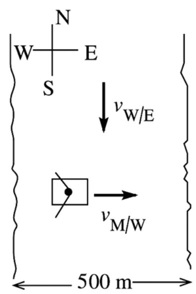
$\phi = 9.6°$; ($9.6°$ west of south)

Figure 3.40b

EVALUATE: The relative velocity addition diagram does not form a right triangle so the vector addition must be done using components. The wind adds both southward and westward components to the velocity of the plane relative to the ground.

3.41. **IDENTIFY:** Relative velocity problem in two dimensions. His motion relative to the earth (time displacement) depends on his velocity relative to the earth so we must solve for this velocity.

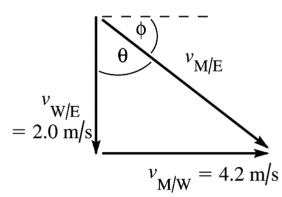
(a) SET UP: View the motion from above.



The velocity vectors in the problem are:
$\vec{v}_{M/E}$, the velocity of the man relative to the earth
$\vec{v}_{W/E}$, the velocity of the water relative to the earth
$\vec{v}_{M/W}$, the velocity of the man relative to the water
The rule for adding these velocities is
$\vec{v}_{M/E} = \vec{v}_{M/W} + \vec{v}_{W/E}$

Figure 3.41a

The problem tells us that $\vec{v}_{W/E}$ has magnitude 2.0 m/s and direction due south. It also tells us that $\vec{v}_{M/W}$ has magnitude 4.2 m/s and direction due east. The vector addition diagram is then as shown in Figure 3.41b.



This diagram shows the vector addition
$\vec{v}_{M/E} = \vec{v}_{M/W} + \vec{v}_{W/E}$
and also has $\vec{v}_{M/W}$ and $\vec{v}_{W/E}$ in their specified directions. Note that the vector diagram forms a right triangle.

Figure 3.41b

The Pythagorean theorem applied to the vector addition diagram gives $v_{M/E}^2 = v_{M/W}^2 + v_{W/E}^2$.

EXECUTE: $v_{M/E} = \sqrt{v_{M/W}^2 + v_{W/E}^2} = \sqrt{(4.2\text{ m/s})^2 + (2.0\text{ m/s})^2} = 4.7\text{ m/s}$; $\tan\theta = \dfrac{v_{M/W}}{v_{W/E}} = \dfrac{4.2\text{ m/s}}{2.0\text{ m/s}} = 2.10$; $\theta = 65°$; or $\phi = 90° - \theta = 25°$. The velocity of the man relative to the earth has magnitude 4.7 m/s and direction 25° S of E.

(b) This requires careful thought. To cross the river the man must travel 500 m due east relative to the earth. The man's velocity relative to the earth is $\vec{v}_{M/E}$. But, from the vector addition diagram the eastward component of $v_{M/E}$ equals $v_{M/W} = 4.2$ m/s.

Thus $t = \dfrac{x - x_0}{v_x} = \dfrac{500\text{ m}}{4.2\text{ m/s}} = 119\text{ s}$, which we round to 120 s.

(c) The southward component of $\vec{v}_{M/E}$ equals $v_{W/E} = 2.0$ m/s. Therefore, in the 120 s it takes him to cross the river, the distance south the man travels relative to the earth is

$$y - y_0 = v_y t = (2.0\text{ m/s})(119\text{ s}) = 240\text{ m}.$$

EVALUATE: If there were no current he would cross in the same time, $(500\text{ m})/(4.2\text{ m/s}) = 120\text{ s}$. The current carries him downstream but doesn't affect his motion in the perpendicular direction, from bank to bank.

3.42. **IDENTIFY:** Use the relation that relates the relative velocities.

SET UP: The relative velocities are the water relative to the earth, $\vec{v}_{W/E}$, the boat relative to the water, $\vec{v}_{B/W}$, and the boat relative to the earth, $\vec{v}_{B/E}$. $\vec{v}_{B/E}$ is due east, $\vec{v}_{W/E}$ is due south and has magnitude 2.0 m/s. $v_{B/W} = 4.2$ m/s. $\vec{v}_{B/E} = \vec{v}_{B/W} + \vec{v}_{W/E}$. The velocity addition diagram is given in Figure 3.42.

EXECUTE: **(a)** Find the direction of $\vec{v}_{B/W}$. $\sin\theta = \dfrac{v_{W/E}}{v_{B/W}} = \dfrac{2.0\text{ m/s}}{4.2\text{ m/s}}$. $\theta = 28.4°$, north of east.

(b) $v_{B/E} = \sqrt{v_{B/W}^2 - v_{W/E}^2} = \sqrt{(4.2\text{ m/s})^2 - (2.0\text{ m/s})^2} = 3.7\text{ m/s}$

(c) $t = \dfrac{800\text{ m}}{v_{B/E}} = \dfrac{800\text{ m}}{3.7\text{ m/s}} = 216\text{ s}.$

EVALUATE: It takes longer to cross the river in this problem than it did in Problem 3.41. In the direction straight across the river (east) the component of his velocity relative to the earth is less than 4.2 m/s.

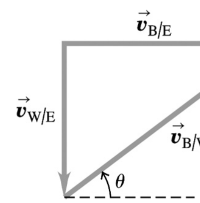


Figure 3.42

3.43. **IDENTIFY:** Use the relation that relates the relative velocities.

SET UP: The relative velocities are the velocity of the plane relative to the ground, $\vec{v}_{P/G}$, the velocity of the plane relative to the air, $\vec{v}_{P/A}$, and the velocity of the air relative to the ground, $\vec{v}_{A/G}$. $\vec{v}_{P/G}$ must be due west and $\vec{v}_{A/G}$ must be south. $v_{A/G} = 80\text{ km/h}$ and $v_{P/A} = 320\text{ km/h}$. $\vec{v}_{P/G} = \vec{v}_{P/A} + \vec{v}_{A/G}$. The relative velocity addition diagram is given in Figure 3.43.

EXECUTE: **(a)** $\sin\theta = \dfrac{v_{A/G}}{v_{P/A}} = \dfrac{80\text{ km/h}}{320\text{ km/h}}$ and $\theta = 14°$, north of west.

(b) $v_{P/G} = \sqrt{v_{P/A}^2 - v_{A/G}^2} = \sqrt{(320\text{ km/h})^2 - (80.0\text{ km/h})^2} = 310\text{ km/h}.$

EVALUATE: To travel due west the velocity of the plane relative to the air must have a westward component and also a component that is northward, opposite to the wind direction.

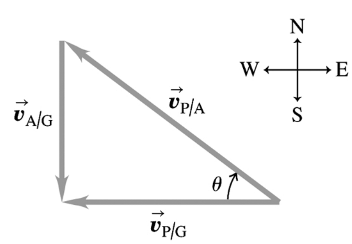


Figure 3.43

3.44. **IDENTIFY:** The dog runs away from a tree with a non-constant acceleration, so the constant-acceleration equations do not apply. We need to go to the basic definitions of *a* and *v* in terms of calculus.

SET UP: The dog starts from rest at the tree when $t = 0$, so $x_0 = 0$ and $v_0 = 0$. The dog's acceleration is given by $\vec{a} = (0.400\ \text{m/s}^2)\hat{i} - (0.180\ \text{m/s}^3)t\,\hat{j}$, so we see that $a_x = 0.400\ \text{m/s}^2$ and $a_y = -0.180\ \text{m/s}^3\ t$. The basic definitions are $a_x = dv_x/dt$ and $v_x = dx/dt$, and likewise for the y-components. We want to know how far the dog is from the tree at the end of 8.00 s of running. By integration of the acceleration we can find the x and y components of the dog's position vector at 8.00 s. From these components we can find the magnitude of the dog's position vector.

EXECUTE: First find the components of the dog's velocity using $a_x = dv_x/dt$ and $a_y = dv_y/dt$.

x-coordinate: We can save some time because $a_x = 0.400\ \text{m/s}^2$ is a constant. So we can use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, which gives $x = \frac{1}{2}a_x t^2 = \frac{1}{2}(0.400\ \text{m/s}^2)(8.00\ \text{s})^2 = 12.80\ \text{m}$.

y-coordinate: First find v_y using $a_y = dv_y/dt$. This gives $v_y = \int a_y dt = \int kt\,dt = \frac{kt^2}{2}$, where we let $k = -0.180\ \text{m/s}^3$ for convenience. We have used $v_y = 0$ when $t = 0$. Now use $v_y = dy/dt$ to find $y(t)$.

$y = \int v_y dt = \int \frac{kt^2}{2} dt = \frac{kt^3}{6}$, where we have used $y = 0$ when $t = 0$. At 8.00 s, we have

$$y = \frac{(-0.180\ \text{m/s}^3)(8.00\ \text{s})^3}{6} = -15.36\ \text{m}.$$

Now use $A = \sqrt{A_x^2 + A_y^2}$ to find the dog's distance d from the tree. $d = \sqrt{(12.80\ \text{m})^2 + (-15.36\ \text{m})^2} = 20.0\ \text{m}$.

EVALUATE: The use of components makes the solution to this problem quite straightforward.

3.45. **IDENTIFY:** $\vec{v} = d\vec{r}/dt$. This vector will make a $45°$ angle with both axes when its x- and y-components are equal.

SET UP: $\dfrac{d(t^n)}{dt} = nt^{n-1}$.

EXECUTE: $\vec{v} = 2bt\hat{i} + 3ct^2\hat{j}$. $v_x = v_y$ gives $t = 2b/3c$.

EVALUATE: Both components of $\vec{v}$ change with t.

3.46. **IDENTIFY:** The acceleration is not constant but is known as a function of time.

SET UP: Integrate the acceleration to get the velocity and the velocity to get the position. At the maximum height $v_y = 0$.

EXECUTE: **(a)** $v_x = v_{0x} + \frac{\alpha}{3}t^3$, $v_y = v_{0y} + \beta t - \frac{\gamma}{2}t^2$, and $x = v_{0x}t + \frac{\alpha}{12}t^4$, $y = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3$.

(b) Setting $v_y = 0$ yields a quadratic in t, $0 = v_{0y} + \beta t - \frac{\gamma}{2}t^2$. Using the numerical values given in the problem, this equation has as the positive solution $t = \frac{1}{\gamma}\left[\beta + \sqrt{\beta^2 + 2v_{0y}\gamma}\right] = 13.59\ \text{s}$. Using this time in the expression for $y(t)$ gives a maximum height of 341 m.

(c) $y = 0$ gives $0 = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3$ and $\frac{\gamma}{6}t^2 - \frac{\beta}{2}t - v_{0y} = 0$. Using the numbers given in the problem, the positive solution is $t = 20.73$ s. For this t, $x = 3.85 \times 10^4$ m.

EVALUATE: We cannot use the constant-acceleration kinematics formulas, but calculus provides the solution.

3.47. **IDENTIFY:** Once the rocket leaves the incline it moves in projectile motion. The acceleration along the incline determines the initial velocity and initial position for the projectile motion.

SET UP: For motion along the incline let $+x$ be directed up the incline. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = \sqrt{2(1.90\text{ m/s}^2)(200\text{ m})} = 27.57\text{ m/s}$. When the projectile motion begins the rocket has $v_0 = 27.57\text{ m/s}$ at $35.0°$ above the horizontal and is at a vertical height of $(200.0\text{ m})\sin 35.0° = 114.7\text{ m}$. For the projectile motion let $+x$ be horizontal to the right and let $+y$ be upward. Let $y = 0$ at the ground. Then $y_0 = 114.7\text{ m}$, $v_{0x} = v_0\cos 35.0° = 22.57\text{ m/s}$, $v_{0y} = v_0 \sin 35.0° = 15.81\text{ m/s}$, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. Let $x = 0$ at point *A*, so $x_0 = (200.0\text{ m})\cos 35.0° = 163.8\text{ m}$.

EXECUTE: **(a)** At the maximum height $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (15.81\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 12.77\text{ m}$ and $y = 114.7\text{ m} + 12.77\text{ m} = 128\text{ m}$. The maximum height above ground is 128 m.

(b) The time in the air can be calculated from the vertical component of the projectile motion: $y - y_0 = -114.7\text{ m}$, $v_{0y} = 15.81\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $(4.90\text{ m/s}^2)t^2 - (15.81\text{ m/s})t - 114.7\text{ m}$. The quadratic formula gives $t = 6.713\text{ s}$ for the positive root. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (22.57\text{ m/s})(6.713\text{ s}) = 151.6\text{ m}$ and $x = 163.8\text{ m} + 151.6\text{ m} = 315\text{ m}$. The horizontal range of the rocket is 315 m.

EVALUATE: The expressions for *h* and *R* derived in the range formula do not apply here. They are only for a projectile fired on level ground.

3.48. **IDENTIFY:** Use the position vector of a dragonfly to determine information about its velocity vector and acceleration vector.

SET UP: Use the definitions $v_x = dx/dt$, $v_y = dy/dt$, $a_x = dv_x/dt$, and $a_y = dv_y/dt$.

EXECUTE: **(a)** Taking derivatives of the position vector gives the components of the velocity vector: $v_x(t) = (0.180\text{ m/s}^2)t$, $v_y(t) = (-0.0450\text{ m/s}^3)t^2$. Use these components and the given direction:

$\tan 30.0° = \dfrac{(0.0450\text{ m/s}^3)t^2}{(0.180\text{ m/s}^2)t}$, which gives $t = 2.31\text{ s}$.

(b) Taking derivatives of the velocity components gives the acceleration components: $a_x = 0.180\text{ m/s}^2$, $a_y(t) = -(0.0900\text{ m/s}^3)t$. At $t = 2.31\text{ s}$, $a_x = 0.180\text{ m/s}^2$ and $a_y = -0.208\text{ m/s}^2$, giving $a = 0.275\text{ m/s}^2$. The direction is $\tan\theta = \dfrac{0.208}{0.180}$, so $\theta = 49.1°$ clockwise from +*x*-axis.

EVALUATE: The acceleration is not constant, so we cannot use the standard kinematics formulas.

3.49. **IDENTIFY:** The cannister moves in projectile motion. Its initial velocity is horizontal. Apply constant acceleration equations for the *x* and *y* components of motion.

SET UP:

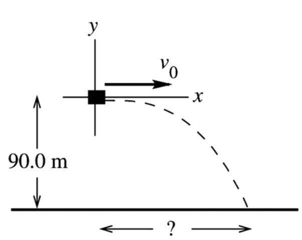


Figure 3.49

Take the origin of coordinates at the point where the cannister is released. Take $+y$ to be upward. The initial velocity of the cannister is the velocity of the plane, 64.0 m/s in the $+x$-direction.

Use the vertical motion to find the time of fall: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ where $t=?$, $v_{0y}=0$, $a_y=-9.80\ \text{m/s}^2$, $y-y_0=-90.0\ \text{m}$ (When the cannister reaches the ground it is 90.0 m *below* the origin.)

EXECUTE: Since $v_{0y}=0$, $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(-90.0\ \text{m})}{-9.80\ \text{m/s}^2}}=4.286\ \text{s}.$

SET UP: Then use the horizontal component of the motion to calculate how far the cannister falls in this time: $x-x_0=?$, a_x-0, $v_{0x}=64.0\ \text{m/s}$.

EXECUTE: $x-x_0=v_0t+\frac{1}{2}at^2=(64.0\ \text{m/s})(4.286\ \text{s})+0=274\ \text{m}.$

EVALUATE: The time it takes the cannister to fall 90.0 m, starting from rest, is the time it travels horizontally at constant speed.

3.50. **IDENTIFY:** $\vec{r}=\vec{r}_0+\int_0^t \vec{v}(t)dt$ and $\boldsymbol{a}=\dfrac{d\vec{v}}{dt}$.

SET UP: At $t=0$, $x_0=0$ and $y_0=0$.

EXECUTE: **(a)** Integrating, $\vec{r}=\left(\alpha t-\dfrac{\beta}{3}t^3\right)\hat{\boldsymbol{i}}+\left(\dfrac{\gamma}{2}t^2\right)\hat{\boldsymbol{j}}$. Differentiating, $\vec{a}=(-2\beta t)\hat{\boldsymbol{i}}+\gamma\hat{\boldsymbol{j}}$.

(b) The positive time at which $x=0$ is given by $t^2=3\alpha/\beta$. At this time, the y-coordinate is

$$y=\frac{\gamma}{2}t^2=\frac{3\alpha\gamma}{2\beta}=\frac{3(2.4\ \text{m/s})(4.0\ \text{m/s}^2)}{2(1.6\ \text{m/s}^3)}=9.0\ \text{m}.$$

EVALUATE: The acceleration is not constant.

3.51. **IDENTIFY:** The person moves in projectile motion. Her vertical motion determines her time in the air.

SET UP: Take $+y$ upward. $v_{0x}=15.0\ \text{m/s}$, $v_{0y}=+10.0\ \text{m/s}$, $a_x=0$, $a_y=-9.80\ \text{m/s}^2$.

EXECUTE: **(a)** Use the vertical motion to find the time in the air: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ with $y-y_0=-30.0\ \text{m}$ gives $-30.0\ \text{m}=(10.0\ \text{m/s})t-(4.90\ \text{m/s}^2)t^2$. The quadratic formula gives $t=\dfrac{1}{2(4.9)}\left(+10.0\pm\sqrt{(-10.0)^2-4(4.9)(-30)}\right)$ s. The positive solution is $t=3.70$ s. During this time she travels a horizontal distance $x-x_0=v_{0x}t+\frac{1}{2}a_xt^2=(15.0\ \text{m/s})(3.70\ \text{s})=55.5\ \text{m}$. She will land 55.5 m south of the point where she drops from the helicopter and this is where the mats should have been placed.

(b) The *x-t*, *y-t*, v_x *-t* and v_y *-t* graphs are sketched in Figure 3.51.

EVALUATE: If she had dropped from rest at a height of 30.0 m it would have taken her $t=\sqrt{\dfrac{2(30.0\ \text{m})}{9.80\ \text{m/s}^2}}=2.47\ \text{s}$. She is in the air longer than this because she has an initial vertical component of velocity that is upward.

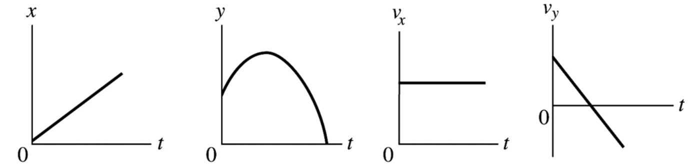


Figure 3.51

3.52. **IDENTIFY:** The shell moves as a projectile. To just clear the top of the cliff, the shell must have $y - y_0 = 25.0$ m when it has $x - x_0 = 60.0$ m.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. $v_{0x} = v_0 \cos 43°$, $v_{0y} = v_0 \sin 43°$.

EXECUTE: **(a)** horizontal motion: $x - x_0 = v_{0x}t$ so $t = \dfrac{60.0 \text{ m}}{(v_0 \cos 43°)}$.

vertical motion: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $25.0 \text{ m} = (v_0 \sin 43.0°)\, t + \frac{1}{2}(-9.80 \text{ m/s}^2)\, t^2$.

Solving these two simultaneous equations for v_0 and t gives $v_0 = 32.6$ m/s and $t = 2.51$ s.

(b) v_y when shell reaches cliff:

$$v_y = v_{0y} + a_y t = (32.6 \text{ m/s}) \sin 43.0° - (9.80 \text{ m/s}^2)(2.51 \text{ s}) = -2.4 \text{ m/s}$$

The shell is traveling downward when it reaches the cliff, so it lands right at the edge of the cliff.

EVALUATE: The shell reaches its maximum height at $t = -\dfrac{v_{0y}}{a_y} = 2.27$ s, which confirms that at $t = 2.51$ s it has passed its maximum height and is on its way down when it strikes the edge of the cliff.

3.53. **IDENTIFY:** Find the horizontal distance a rocket moves if it has a non-constant horizontal acceleration but a constant vertical acceleration of g downward.

SET UP: The vertical motion is g downward, so we can use the constant acceleration formulas for that component of the motion. We must use integration for the horizontal motion because the acceleration is not constant. Solving for t in the kinematics formula for y gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}}$. In the horizontal direction we must use $v_x(t) = v_{0x} + \int_0^t a_x(t')dt'$ and $x - x_0 = \int_0^t v_x(t')dt'$.

EXECUTE: Use vertical motion to find t. $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(30.0 \text{ m})}{9.80 \text{ m/s}^2}} = 2.474$ s.

In the horizontal direction we have

$v_x(t) = v_{0x} + \int_0^t a_x(t')dt' = v_{0x} + (0.800 \text{ m/s}^3)t^2 = 12.0 \text{ m/s} + (0.800 \text{ m/s}^3)t^2$. Integrating $v_x(t)$ gives $x - x_0 = (12.0 \text{ m/s})t + (0.2667 \text{ m/s}^3)t^3$. At $t = 2.474$ s, $x - x_0 = 29.69 \text{ m} + 4.04 \text{ m} = 33.7$ m.

EVALUATE: The vertical part of the motion is familiar projectile motion, but the horizontal part is not.

3.54. **IDENTIFY:** The equipment moves in projectile motion. The distance D is the horizontal range of the equipment plus the distance the ship moves while the equipment is in the air.

SET UP: For the motion of the equipment take $+x$ to be to the right and $+y$ to be upward. Then $a_x = 0$, $a_y = -9.80 \text{ m/s}^2$, $v_{0x} = v_0 \cos\alpha_0 = 7.50$ m/s and $v_{0y} = v_0 \sin\alpha_0 = 13.0$ m/s. When the equipment lands in the front of the ship, $y - y_0 = -8.75$ m.

EXECUTE: Use the vertical motion of the equipment to find its time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \dfrac{1}{9.80}\left(13.0 \pm \sqrt{(-13.0)^2 + 4(4.90)(8.75)}\right)$ s. The positive root is $t = 3.21$ s. The horizontal range of the equipment is $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (7.50 \text{ m/s})(3.21 \text{ s}) = 24.1$ m. In 3.21 s the ship moves a horizontal distance $(0.450 \text{ m/s})(3.21 \text{ s}) = 1.44$ m, so $D = 24.1 \text{ m} + 1.44 \text{ m} = 25.5$ m.

EVALUATE: The range equation $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$ cannot be used here because the starting and ending points of the projectile motion are at different heights.

3.55. **IDENTIFY:** Two-dimensional projectile motion.

SET UP: Let $+y$ be upward. $a_x = 0,\ a_y = -9.80\ \text{m/s}^2$. With $x_0 = y_0 = 0,$ algebraic manipulation of the equations for the horizontal and vertical motion shows that x and y are related by

$$y = (\tan\theta_0)x - \frac{g}{2v_0^2\cos^2\theta_0}x^2.$$

$\theta_0 = 60.0°.\ \ y = 8.00\ \text{m}$ when $x = 18.0\ \text{m}.$

EXECUTE: **(a)** Solving for v_0 gives $v_0 = \sqrt{\dfrac{gx^2}{2(\cos^2\theta_0)(x\tan\theta_0 - y)}} = 16.6\ \text{m/s}.$

(b) We find the horizontal and vertical velocity components:

$v_x = v_{0x} = v_0\cos\theta_0 = 8.3\ \text{m/s}.$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = -\sqrt{(v_0\sin\theta_0)^2 + 2a_y(y - y_0)} = -\sqrt{(14.4\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(8.00\ \text{m})} = -7.1\ \text{m/s}$

$v = \sqrt{v_x^2 + v_y^2} = 10.9\ \text{m/s}.\ \ \tan\theta = \dfrac{|v_y|}{|v_x|} = \dfrac{7.1}{8.3}$ and $\theta = 40.5°,$ below the horizontal.

EVALUATE: We can check our calculated v_0.

$$t = \frac{x - x_0}{v_{0x}} = \frac{18.0\ \text{m}}{8.3\ \text{m/s}} = 2.17\ \text{s}.$$

Then $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = (14.4\ \text{m/s})(2.17\ \text{s}) - (4.9\ \text{m/s}^2)(2.17\ \text{s})^2 = 8\ \text{m},$ which checks.

3.56. **IDENTIFY:** While the hay falls 150 m with an initial upward velocity and with a downward acceleration of g, it must travel a horizontal distance (the target variable) with constant horizontal velocity.

SET UP: Use coordinates with $+y$ upward and $+x$ horizontal. The bale has initial velocity components $v_{0x} = v_0\cos\alpha_0 = (75\ \text{m/s})\cos 55° = 43.0\ \text{m/s}$ and $v_{0y} = v_0\sin\alpha_0 = (75\ \text{m/s})\sin 55° = 61.4\ \text{m/s}.\ \ y_0 = 150\ \text{m}$ and $y = 0.$ The equation $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ applies to the vertical motion and a similar equation to the horizontal motion.

EXECUTE: Use the vertical motion to find t: $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $-150\ \text{m} = (61.4\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2.$ The quadratic formula gives $t = 6.27 \pm 8.36\ \text{s}.$ The physical value is the positive one, and $t = 14.6\ \text{s}.$ Then $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2 = (43.0\ \text{m/s})(14.6\ \text{s}) = 630\ \text{m}.$

EVALUATE: If the airplane maintains constant velocity after it releases the bales, it will also travel horizontally 630 m during the time it takes the bales to fall to the ground, so the airplane will be directly over the impact spot when the bales land.

3.57. **IDENTIFY:** From the figure in the text, we can read off the maximum height and maximum horizontal distance reached by the grasshopper. Knowing its acceleration is g downward, we can find its initial speed and the height of the cliff (the target variables).

SET UP: Use coordinates with the origin at the ground and $+y$ upward. $a_x = 0,\ a_y = -9.80\ \text{m/s}^2.$ The constant-acceleration kinematics formulas $v_y^2 = v_{0y}^2 + 2a_y\,(y - y_0)$ and $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ apply.

EXECUTE: **(a)** $v_y = 0$ when $y - y_0 = 0.0674\ \text{m}.\ \ v_y^2 = v_{0y}^2 + 2a_y\,(y - y_0)$ gives

$v_{0y} = \sqrt{-2a_y\,(y - y_0)} = \sqrt{-2\,(-9.80\ \text{m/s}^2)(0.0674\ \text{m})} = 1.15\ \text{m/s}.\ \ v_{0y} = v_0\sin\alpha_0$ so

$v_0 = \dfrac{v_{0y}}{\sin\alpha_0} = \dfrac{1.15\ \text{m/s}}{\sin 50.0°} = 1.50\ \text{m/s}.$

(b) Use the horizontal motion to find the time in the air. The grasshopper travels horizontally $x - x_0 = 1.06$ m. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{x - x_0}{v_0 \cos 50.0°} = 1.10$ s. Find the vertical displacement of the grasshopper at $t = 1.10$ s:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.15 \text{ m/s})(1.10 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.10 \text{ s})^2 = -4.66$ m. The height of the cliff is 4.66 m.

EVALUATE: The grasshopper's maximum height (6.74 cm) is physically reasonable, so its takeoff speed of 1.50 m/s must also be reasonable. Note that the equation $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$ does *not* apply here since the launch point is not at the same level as the landing point.

3.58. **IDENTIFY:** The water moves in projectile motion.

SET UP: Let $x_0 = y_0 = 0$ and take $+y$ to be positive. $a_x = 0$, $a_y = -g$.

EXECUTE: The equations of motions are $y = (v_0 \sin \alpha)t - \frac{1}{2}gt^2$ and $x = (v_0 \cos \alpha)t$. When the water goes in the tank for the *minimum* velocity, $y = 2D$ and $x = 6D$. When the water goes in the tank for the *maximum* velocity, $y = 2D$ and $x = 7D$. In both cases, $\sin \alpha = \cos \alpha = \sqrt{2}/2$.

To reach the *minimum* distance: $6D = \dfrac{\sqrt{2}}{2}v_0 t$, and $2D = \dfrac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for t gives $t = \dfrac{6D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 6D - \frac{1}{2}g\left(\dfrac{6D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = 3\sqrt{gD}$.

To reach the *maximum* distance: $7D = \dfrac{\sqrt{2}}{2}v_0 t$, and $2D = \dfrac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for t gives $t = \dfrac{7D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 7D - \frac{1}{2}g\left(\dfrac{7D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = \sqrt{49gD/5} = 3.13\sqrt{gD}$, which, as expected, is larger than the previous result.

EVALUATE: A launch speed of $v_0 = \sqrt{6}\sqrt{gD} = 2.45\sqrt{gD}$ is required for a horizontal range of $6D$. The minimum speed required is greater than this, because the water must be at a height of at least $2D$ when it reaches the front of the tank.

3.59. **IDENTIFY:** This is a projectile motion problem. The vertical acceleration is g downward and the horizontal acceleration is zero. The constant-acceleration equations apply.

SET UP: Apply the constant-acceleration formulas.

EXECUTE: **(a)** The object has only vertical motion, so $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v = \sqrt{v_0^2 + 2gH}$.

(b) The procedure is exactly the as in part (a) except that $v_{0y} = -v_0$, so the result is the same.

(c) Use the same approach as in part (a). $v_x = v_0 \cos\alpha_0$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y^2 = \left(v_0 \sin\alpha_0\right)^2 + 2(-g)(-H)$, so $v_y = \sqrt{v_0^2 \sin^2\alpha_0 + 2gH}$. The speed is the magnitude of the velocity vector, so $v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(v_0^2 \cos^2\alpha_0\right) + \left(v_0^2 \sin^2\alpha_0 + 2gH\right)} = \sqrt{v_0^2 + 2gH}$.

EVALUATE: **(d)** From part (c), we see that v does not depend on α_0, so v stays the same as α_0 is changed.

3.60. **IDENTIFY:** To clear the bar the ball must have a height of 10.0 ft when it has a horizontal displacement of 36.0 ft. The ball moves as a projectile. When v_0 is very large, the ball reaches the goal posts in a very short time and the acceleration due to gravity causes negligible downward displacement.

SET UP: $36.0 \text{ ft} = 10.97 \text{ m}$; $10.0 \text{ ft} = 3.048 \text{ m}$. Let $+x$ be to the right and $+y$ be upward, so $a_x = 0$, $a_y = -g$, $v_{0x} = v_0 \cos\alpha_0$ and $v_{0y} = v_0 \sin\alpha_0$.

EXECUTE: **(a)** The ball cannot be aimed lower than directly at the bar. $\tan\alpha_0 = \dfrac{10.0 \text{ ft}}{36.0 \text{ ft}}$ and $\alpha_0 = 15.5°$.

(b) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{x - x_0}{v_0 \cos\alpha_0}$. Then $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$y - y_0 = (v_0 \sin\alpha_0)\left(\frac{x - x_0}{v_0 \cos\alpha_0}\right) - \frac{1}{2}g\frac{(x - x_0)^2}{v_0^2 \cos^2\alpha_0} = (x - x_0)\tan\alpha_0 - \frac{1}{2}g\frac{(x - x_0)^2}{v_0^2 \cos^2\alpha_0}.$$

$$v_0 = \frac{(x - x_0)}{\cos\alpha_0}\sqrt{\frac{g}{2[(x - x_0)\tan\alpha_0 - (y - y_0)]}} = \frac{10.97 \text{ m}}{\cos 45.0°}\sqrt{\frac{9.80 \text{ m/s}^2}{2[10.97 \text{ m} - 3.048 \text{ m}]}} = 12.2 \text{ m/s} = 43.9 \text{ km/h}.$$

EVALUATE: With the v_0 and 45° launch angle in part (b), the horizontal range of the ball is $R = \dfrac{v_0^2 \sin 2\alpha_0}{g} = 15.2 \text{ m} = 49.9 \text{ ft}$. The ball reaches the highest point in its trajectory when $x - x_0 = R/2$, which is 25 ft, so when it reaches the goal posts it is on its way down.

3.61. **IDENTIFY:** The snowball moves in projectile motion. In part (a) the vertical motion determines the time in the air. In part (c), find the height of the snowball above the ground after it has traveled horizontally 4.0 m.

SET UP: Let $+y$ be downward. $a_x = 0$, $a_y = +9.80 \text{ m/s}^2$. $v_{0x} = v_0 \cos\theta_0 = 5.36 \text{ m/s}$, $v_{0y} = v_0 \sin\theta_0 = 4.50 \text{ m/s}$.

EXECUTE: **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 14.0 \text{ m}$ gives $14.0 \text{ m} = (4.50 \text{ m/s})\,t + (4.9 \text{ m/s}^2)\,t^2$. The quadratic formula gives $t = \dfrac{1}{2(4.9)}\left(-4.50 \pm \sqrt{(4.50)^2 - 4\,(4.9)(-14.0)}\right)$ s. The positive root is $t = 1.29$ s. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (5.36 \text{ m/s})(1.29 \text{ s}) = 6.91 \text{ m}$.

(b) The x-t, y-t, v_x-t and v_y-t graphs are sketched in Figure 3.61.

(c) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{4.0 \text{ m}}{5.36 \text{ m/s}} = 0.746 \text{ s}$. In this time the snowball travels downward a distance $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 6.08 \text{ m}$ and is therefore $14.0 \text{ m} - 6.08 \text{ m} = 7.9 \text{ m}$ above the ground. The snowball passes well above the man and doesn't hit him.

EVALUATE: If the snowball had been released from rest at a height of 14.0 m it would have reached the ground in $t = \sqrt{\dfrac{2(14.0 \text{ m})}{9.80 \text{ m/s}^2}} = 1.69 \text{ s}$. The snowball reaches the ground in a shorter time than this because of its initial downward component of velocity.

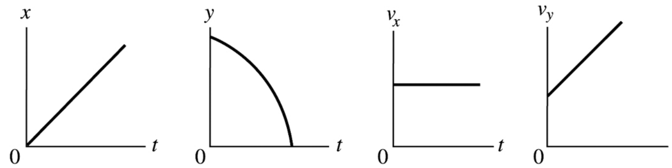


Figure 3.61

3.62. **IDENTIFY:** The ball moves in projectile motion.

SET UP: The woman and ball travel for the same time and must travel the same horizontal distance, so for the ball $v_{0x} = 6.00$ m/s.

EXECUTE: **(a)** $v_{0x} = v_0 \cos\theta_0$. $\cos\theta_0 = \dfrac{v_{0x}}{v_0} = \dfrac{6.00\text{ m/s}}{20.0\text{ m/s}}$ and $\theta_0 = 72.5°$. The ball is in the air for 5.55s and she runs a distance of $(6.00\text{ m/s})(5.55\text{ s}) = 33.3$ m.

(b) Relative to the ground the ball moves in a parabola. The ball and the runner have the same horizontal component of velocity, so relative to the runner the ball has only vertical motion. The trajectories as seen by each observer are sketched in Figure 3.62.

EVALUATE: The ball could be thrown with a different speed, so long as the angle at which it was thrown was adjusted to keep $v_{0x} = 6.00$ m/s.

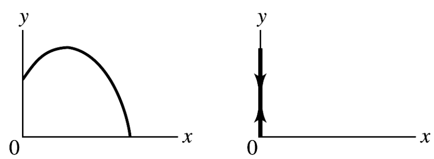
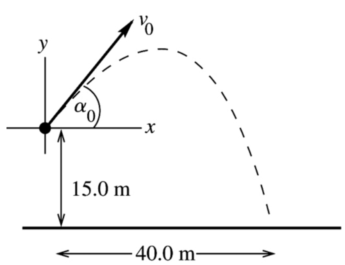


Figure 3.62

3.63. **(a) IDENTIFY:** Projectile motion.



Take the origin of coordinates at the top of the ramp and take $+y$ to be upward.

The problem specifies that the object is displaced 40.0 m to the right when it is 15.0 m below the origin.

Figure 3.63

We don't know *t*, the time in the air, and we don't know v_0. Write down the equations for the horizontal and vertical displacements. Combine these two equations to eliminate one unknown.

SET UP: <u>*y*-component</u>:

$y - y_0 = -15.0$ m, $a_y = -9.80$ m/s^2, $v_{0y} = v_0 \sin 53.0°$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $-15.0\text{ m} = (v_0 \sin 53.0°)\, t - (4.90\text{ m/s}^2)\, t^2$

SET UP: <u>*x*-component</u>:

$x - x_0 = 40.0 \text{ m}, \;\; a_x = 0, \;\; v_{0x} = v_0 \cos 53.0°$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

EXECUTE: $40.0 \text{ m} = (v_0 t)\cos 53.0°$

The second equation says $v_0 t = \dfrac{40.0 \text{ m}}{\cos 53.0°} = 66.47 \text{ m}.$

Use this to replace $v_0 t$ in the first equation:

$-15.0 \text{ m} = (66.47 \text{ m}) \sin 53° - (4.90 \text{ m/s}^2)\, t^2$

$$t = \sqrt{\frac{(66.47 \text{ m})\sin 53° + 15.0 \text{ m}}{4.90 \text{ m/s}^2}} = \sqrt{\frac{68.08 \text{ m}}{4.90 \text{ m/s}^2}} = 3.727 \text{ s}.$$

Now that we have t we can use the x-component equation to solve for v_0:

$$v_0 = \frac{40.0 \text{ m}}{t \cos 53.0°} = \frac{40.0 \text{ m}}{(3.727 \text{ s}) \cos 53.0°} = 17.8 \text{ m/s}.$$

EVALUATE: Using these values of v_0 and t in the $y = y_0 = v_{0y} + \frac{1}{2}a_y t^2$ equation verifies that $y - y_0 = -15.0 \text{ m}.$

(b) IDENTIFY: $v_0 = (17.8 \text{ m/s})/2 = 8.9 \text{ m/s}$

This is less than the speed required to make it to the other side, so he lands in the river.
Use the vertical motion to find the time it takes him to reach the water:

SET UP: $y - y_0 = -100 \text{ m}; \;\; v_{0y} = +v_0 \sin 53.0° = 7.11 \text{ m/s}; \;\; a_y = -9.80 \text{ m/s}^2$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-100 = 7.11t - 4.90t^2$

EXECUTE: $4.90t^2 - 7.11t - 100 = 0$ and $t = \frac{1}{9.80}\left(7.11 \pm \sqrt{(7.11)^2 - 4\,(4.90)(-100)}\right)$

$t = 0.726 \text{ s} \pm 4.57 \text{ s}$ so $t = 5.30 \text{ s}.$

The horizontal distance he travels in this time is
$x - x_0 = v_{0x}t = (v_0 \cos 53.0°)\, t = (5.36 \text{ m/s})(5.30 \text{ s}) = 28.4 \text{ m}.$

He lands in the river a horizontal distance of 28.4 m from his launch point.

EVALUATE: He has half the minimum speed and makes it only about halfway across.

3.64. IDENTIFY: The bagels move in projectile motion. Find Henrietta's location when the bagels reach the ground, and require the bagels to have this horizontal range.

SET UP: Let $+y$ be downward and let $x_0 = y_0 = 0.$ $a_x = 0,$ $a_y = +g.$ When the bagels reach the ground, $y = 38.0 \text{ m}.$

EXECUTE: **(a)** When she catches the bagels, Henrietta has been jogging for 9.00 s plus the time for the bagels to fall 38.0 m from rest. Get the time to fall: $y = \dfrac{1}{2}gt^2,$ $38.0 \text{ m} = \dfrac{1}{2}(9.80 \text{ m/s}^2)\, t^2$ and $t = 2.78 \text{ s}.$

So, she has been jogging for $9.00 \text{ s} + 2.78 \text{ s} = 11.78 \text{ s}.$ During this time she has gone $x = vt = (3.05 \text{ m/s})(11.78 \text{ s}) = 35.9 \text{ m}.$ Bruce must throw the bagels so they travel 35.9 m horizontally in 2.78 s. This gives $x = vt.$ $35.9 \text{ m} = v\,(2.78 \text{ s})$ and $v = 12.9 \text{ m/s}.$

(b) 35.9 m from the building.

EVALUATE: If $v > 12.9 \text{ m/s}$ the bagels land in front of her and if $v < 12.9 \text{ m/s}$ they land behind her. There is a range of velocities greater than 12.9 m/s for which she would catch the bagels in the air, at some height above the sidewalk.

3.65. IDENTIFY: The boulder moves in projectile motion.

SET UP: Take $+y$ downward. $v_{0x} = v_0,$ $a_x = 0,$ $a_x = 0,$ $a_y = +9.80 \text{ m/s}^2.$

EXECUTE: **(a)** Use the vertical motion to find the time for the boulder to reach the level of the lake: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ with $y-y_0=+20\text{ m}$ gives $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(20\text{ m})}{9.80\text{ m/s}^2}}=2.02\text{ s}$. The rock must travel horizontally 100 m during this time. $x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$ gives

$$v_0=v_{0x}=\frac{x-x_0}{t}=\frac{100\text{ m}}{2.02\text{ s}}=49.5\text{ m/s}$$

(b) In going from the edge of the cliff to the plain, the boulder travels downward a distance of $y-y_0=45\text{ m}$. $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(45\text{ m})}{9.80\text{ m/s}^2}}=3.03\text{ s}$ and $x-x_0=v_{0x}t=(49.5\text{ m/s})(3.03\text{ s})=150\text{ m}$.

The rock lands $150\text{ m}-100\text{ m}=50\text{ m}$ beyond the foot of the dam.

EVALUATE: The boulder passes over the dam 2.02 s after it leaves the cliff and then travels an additional 1.01 s before landing on the plain. If the boulder has an initial speed that is less than 49 m/s, then it lands in the lake.

3.66. **IDENTIFY:** The water follows a parabolic trajectory since it is affected only by gravity, so we apply the principles of projectile motion to it.

SET UP: Use coordinates with $+y$ upward. Once the water leaves the cannon it is in free-fall and has $a_x=0$ and $a_y=-9.80\text{ m/s}^2$. The water has $v_{0x}=v_0\cos\theta_0=15.0\text{ m/s}$ and $v_{0y}=v_0\sin\theta_0=20.0\text{ m/s}$.

EXECUTE: Use the vertical motion to find t that gives $y-y_0=10.0\text{ m}$: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $10.0\text{ m}=(20.0\text{ m/s})t-(4.90\text{ m/s}^2)t^2$.

The quadratic formula gives $t=2.04\pm1.45\text{ s}$, and $t=0.59\text{ s}$ or $t=3.49\text{ s}$. Both answers are physical.

For $t=0.59$ s: $x-x_0=v_{0x}t=(15.0\text{ m/s})(0.59\text{ s})=8.8\text{ m}$.

For $t=3.49$ s: $x-x_0=v_{0x}t=(15.0\text{ m/s})(3.49\text{ s})=52.4\text{ m}$.

When the cannon is 8.8 m from the building, the water hits this spot on the wall on its way up to its maximum height. When is it 52.4 m from the building it hits this spot after it has passed through its maximum height.

EVALUATE: The fact that we have two possible answers means that the firefighters have some choice on where to stand. If the fire is extremely fierce, they would no doubt prefer to stand at the more distant location.

3.67. **IDENTIFY:** This is a projectile motion problem. The vertical acceleration is g downward and the horizontal acceleration is zero. The constant-acceleration equations apply.

SET UP: Apply the constant-acceleration formulas. We know that the ball travels 50.0 m horizontally and has a speed of 8.0 m/s at its maximum height. We want to know how long the ball is in the air.

EXECUTE: The horizontal velocity is constant, so $v_x = 8.0$ m/s. The ball moves 50.0 m at this velocity, so $x = v_xt$ gives 50.0 m = (8.0 m/s)t → $t = 6.3$ s.

EVALUATE: The ball's vertical velocity keeps changing, but its horizontal velocity remains constant. At its highest point, the ball does *not stop*. Only its vertical velocity becomes zero there.

3.68. **IDENTIFY:** This is a projectile motion problem. The vertical acceleration is g downward and the horizontal acceleration is zero. The constant-acceleration equations apply.

SET UP: The box leaves the top of the ramp horizontally with speed v_0 and is in freefall until it hits the ramp. Fig. 3.68 illustrates the motion. When it hits the ramp, the box has been moving horizontally at speed v_0 for the same time that it has been falling vertically from rest. We want to know how far the box falls before hitting the ramp.

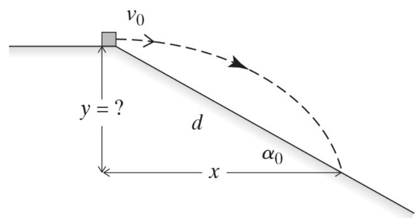


Figure 3.68

EXECUTE: When the box hits the ramp, the constant-acceleration equations give us $x = v_0 t$ and $y = \frac{1}{2}gt^2$. From the figure, we can see that $x = d\cos\alpha_0$ and $y = d\sin\alpha_0$. Therefore $v_0 t = d\cos\alpha_0$ and $\frac{1}{2}gt^2 = d\sin\alpha_0$. Combining the last two equations gives $t = (d\cos\alpha_0)/v_0$ so $\frac{1}{2}g\left(\frac{d\cos\alpha_0}{v_0}\right)^2 = d\sin\alpha_0$.

Solving for d gives $d = \frac{2v_0^2\sin\alpha_0}{g\cos^2\alpha_0}$. We want to find y, so we use the fact that $y = d\sin\alpha_0$ and use the value of d we just found, which gives

$$y = \frac{2v_0^2\sin\alpha_0}{g\cos^2\alpha_0}\cdot\sin\alpha_0 = \frac{2v_0^2\tan^2\alpha_0}{g}.$$

EVALUATE: Check some special cases. If $\alpha_0 \to 0$, the ramp is nearly vertical, so y gets very large. That is reasonable because the box would fall a very long distance before it hit the ramp. If $\alpha_0 \to 0°$, our result gives $y = 0$. This is reasonable because the box would already be on the ramp when it slid off the loading dock, so it would not have to fall any distance to reach the ramp.

3.69. **IDENTIFY:** The rock is in free fall once it is in the air, so it has only a downward acceleration of 9.80 m/s^2, and we apply the principles of two-dimensional projectile motion to it. The constant-acceleration kinematics formulas apply.

SET UP: The vertical displacement must be $\Delta y = y - y_0 = 5.00$ m – 1.60 m = 3.40 m at the instant that the horizontal displacement $\Delta x = x - x_0 = 14.0$ m, and $a_y = -9.80$ m/s^2 with $+y$ upward.

EXECUTE: **(a)** There is no horizontal acceleration, so 14.0 m = $v_0\cos(56.0°)t$, which gives $t = \frac{14.0\text{ m}}{v_0\cos 56.0°}$. Putting this quantity, along with the numerical quantities, into the equation $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ and solving for v_0 we get $v_0 = 13.3$ m/s.

(b) The initial horizontal velocity of the rock is (13.3 m/s)(cos 56.0°), and when it lands on the ground, $y - y_0 = -1.60$ m. Putting these quantities into the equation $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ leads to a quadratic equation in t. Using the positive square root, we get $t = 2.388$ s when the rock lands. The horizontal position at that instant is $x - x_0 =$ (13.3 m/s)(cos 56.0°)(2.388 s) = 17.8 m from the launch point. So the distance beyond the fence is 17.8 m – 14.0 m = 3.8 m.

EVALUATE: We cannot use the range formula to find the distance in (b) because the rock's motion does not start and end at the same height.

3.70. **IDENTIFY:** This is a projectile motion problem. The vertical acceleration is g downward and the horizontal acceleration is zero. The constant-acceleration equations apply.

SET UP: The horizontal range is $R = \frac{v_0^2}{g}\sin 2\alpha_0$.

EXECUTE: **(a)** We want to know how far the object has moved horizontally when it reaches its maximum height. At the maximum height, $v_y = 0$, so $v_y = v_0\sin\alpha_0 - gt$ gives

$t = \dfrac{v_0 \sin\alpha_0}{g}$ = (16.0 m/s)(sin 60.0°)/(9.80 m/s^2) = 1.414 s. This is the time to reach its maximum height.

The horizontal motion during that time gives
$x = (v_0 \cos\alpha_0)t$ = (16.0 m/s)(cos 60.0°)(1.414 s) = 11.3 m.

The horizontal range is $R = \dfrac{v_0^2}{g}\sin 2\alpha_0$ = (16.0 m/s)^2(sin 120°)/(9.80 m/s^2) = 22.6 m. So we see that at the highest point, the object's horizontal displacement is one-half of its horizontal range.

(b) In this case, $x = 0.800R$ = (0.800)(22.6 m) = 18.1 m. We want to know the vertical position of the object when it has traveled this far horizontally. To find the time to reach 18.1 m horizontally, apply $x = (v_0 \cos\alpha_0)t$ and solve for t, giving t = (18.1 m)/[(16.0 m/s)(cos 60.0°)] = 2.262 s. Now find the value of y at this time using $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$. This gives

y = (16.0 m/s)(sin 60.0°)(2.262 s) – $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$ = 6.27 m.

In part (a) we saw that the object reaches its maximum height in 1.414 s. Using $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$, we get

y_{max} = (16.0 m/s)(sin 60.0°)(1.414 s) – $\frac{1}{2}$(9.80 m/s^2)(1.414 s)^2 = 9.80 m.

Comparing the height at 2.262 s to its maximum height gives h/h_{max} = (6.27 m)/(9.80 m) = 0.640, so it is at 64.0% of its maximum height.

(c) When $x - x_0 = \alpha R$, we want to find $\dfrac{y - y_0}{h_{max}}$. We use the same general approach as in part (b). First find h_{max} using the fact that $v_y = 0$ at that point. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $0 = (v_0 \sin\alpha_0)^2 - 2gh_{max}$, so $h_{max} = \dfrac{v_0^2 \sin^2\alpha_0}{2g}$. Now use the horizontal motion to find t when $x - x_0 = \alpha R$, giving $v_0 \cos\alpha_0\, t$ = αR, so t = $t = \dfrac{\alpha R}{v_0 \cos\alpha_0}$. Using the range formula $R = \dfrac{2v_0^2 \sin\alpha_0 \cos\alpha_0}{g}$, the time becomes

$t = \dfrac{\alpha}{v_0 \cos\alpha_0}\left(\dfrac{2v_0^2 \sin\alpha_0 \cos\alpha_0}{g}\right) = \dfrac{2v_0 \alpha \sin\alpha_0}{g}$. Now get $y - y_0$ for this time. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 =$ $v_0 \sin\alpha_0\left(\dfrac{2v_0\alpha \sin\alpha_0}{g}\right) - \dfrac{1}{2}g\left(\dfrac{2v_0 \sin\alpha_0}{g}\right)^2$. Simplifying gives $y - y_0 = \dfrac{2(v_0 \sin\alpha_0)^2}{g}\alpha(1-\alpha)$. Now find the ratio $\dfrac{y - y_0}{h_{max}}$ using our results above. $\dfrac{y - y_0}{h_{max}} = \dfrac{\dfrac{2(v_0 \sin\alpha_0)^2}{g}\alpha(1-\alpha)}{\dfrac{(v_0 \sin\alpha_0)^2}{2g}} = 4\alpha(\alpha - 1)$.

EVALUATE: Check our result in some special cases.

α = ½: $\dfrac{y - y_0}{h_{max}}$ = 4(½)(1 – ½) = 1. This is correct because x is half-way to R, so the object is at its highest point, which makes $y - y_0 = h_{max}$.

α = 0: $\dfrac{y - y_0}{h_{max}}$ = 0. This is reasonable because the object is at its take-off point at ground level.

α = 1: $\dfrac{y - y_0}{h_{max}}$ = 0. This is reasonable because the object is at $x = R$, so it has returned to the ground.

$\alpha = 0.800$: $\dfrac{y - y_0}{h_{\text{max}}} = 4(0.800)(1 - 0.800) = 0.640$. This agrees with our result in part (b).

3.71. **IDENTIFY:** Relative velocity problem. The plane's motion relative to the earth is determined by its velocity relative to the earth.

SET UP: Select a coordinate system where $+y$ is north and $+x$ is east.

The velocity vectors in the problem are:

$\vec{v}_{\text{P/E}}$, the velocity of the plane relative to the earth.

$\vec{v}_{\text{P/A}}$, the velocity of the plane relative to the air (the magnitude $v_{\text{P/A}}$ is the airspeed of the plane and the direction of $\vec{v}_{\text{P/A}}$ is the compass course set by the pilot).

$\vec{v}_{\text{A/E}}$, the velocity of the air relative to the earth (the wind velocity).

The rule for combining relative velocities gives $\vec{v}_{\text{P/E}} = \vec{v}_{\text{P/A}} + \vec{v}_{\text{A/E}}$.

(a) We are given the following information about the relative velocities:

$\vec{v}_{\text{P/A}}$ has magnitude 220 km/h and its direction is west. In our coordinates it has components $(v_{\text{P/A}})_x = -220$ km/h and $(v_{\text{P/A}})_y = 0$.

From the displacement of the plane relative to the earth after 0.500 h, we find that $\vec{v}_{\text{P/E}}$ has components in our coordinate system of

$$(v_{\text{P/E}})_x = -\frac{120\text{ km}}{0.500\text{ h}} = -240\text{ km/h (west)}$$

$$(v_{\text{P/E}})_y = -\frac{20\text{ km}}{0.500\text{ h}} = -40\text{ km/h (south)}$$

With this information the diagram corresponding to the velocity addition equation is shown in Figure 3.71a.

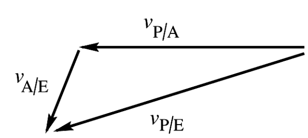


Figure 3.71a

We are asked to find $\vec{v}_{\text{A/E}}$, so solve for this vector:

$\vec{v}_{\text{P/E}} = \vec{v}_{\text{P/A}} + \vec{v}_{\text{A/E}}$ gives $\vec{v}_{\text{A/E}} = \vec{v}_{\text{P/E}} - \vec{v}_{\text{P/A}}$.

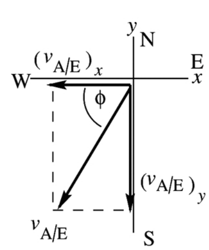
EXECUTE: The x-component of this equation gives

$(v_{\text{A/E}})_x = (v_{\text{P/E}})_x - (v_{\text{P/A}})_x = -240\text{ km/h} - (-220\text{ km/h}) = -20$ km/h.

The y-component of this equation gives

$(v_{\text{A/E}})_y = (v_{\text{P/E}})_y - (v_{\text{P/A}})_y = -40$ km/h.

Now that we have the components of $\vec{v}_{\text{A/E}}$ we can find its magnitude and direction.



$$v_{\text{A/E}} = \sqrt{(v_{\text{A/E}})_x^2 + (v_{\text{A/E}})_y^2}$$

$$v_{\text{A/E}} = \sqrt{(-20\text{ km/h})^2 + (-40\text{ km/h})^2} = 44.7\text{ km/h}$$

$$\tan\phi = \frac{40\text{ km/h}}{20\text{ km/h}} = 2.00;\quad \phi = 63.4^\circ$$

The direction of the wind velocity is 63.4° S of W, or 26.6° W of S.

Figure 3.71b

EVALUATE: The plane heads west. It goes farther west than it would without wind and also travels south, so the wind velocity has components west and south.

(b) SET UP: The rule for combining the relative velocities is still $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$, but some of these velocities have different values than in part (a).

$\vec{v}_{P/A}$ has magnitude 220 km/h but its direction is to be found.

$\vec{v}_{A/E}$ has magnitude 40 km/h and its direction is due south.

The direction of $\vec{v}_{P/E}$ is west; its magnitude is not given.

The vector diagram for $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ and the specified directions for the vectors is shown in Figure 3.71c.

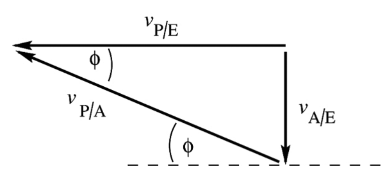


Figure 3.71c

The vector addition diagram forms a right triangle.

EXECUTE: $\sin\phi = \dfrac{v_{A/E}}{v_{P/A}} = \dfrac{40 \text{ km/h}}{220 \text{ km/h}} = 0.1818;\ \ \phi = 10.5°.$

The pilot should set her course $10.5°$ north of west.

EVALUATE: The velocity of the plane relative to the air must have a northward component to counteract the wind and a westward component in order to travel west.

3.72. **IDENTIFY:** Use the relation that relates the relative velocities.

SET UP: The relative velocities are the raindrop relative to the earth, $\vec{v}_{R/E}$, the raindrop relative to the train, $\vec{v}_{R/T}$, and the train relative to the earth, $\vec{v}_{T/E}$. $\vec{v}_{R/E} = \vec{v}_{R/T} + \vec{v}_{T/E}$. $\vec{v}_{T/E}$ is due east and has magnitude 12.0 m/s. $\vec{v}_{R/T}$ is $30.0°$ west of vertical. $\vec{v}_{R/E}$ is vertical. The relative velocity addition diagram is given in Figure 3.72.

EXECUTE: **(a)** $\vec{v}_{R/E}$ is vertical and has zero horizontal component. The horizontal component of $\vec{v}_{R/T}$ is $-\vec{v}_{T/E}$, so is 12.0 m/s westward.

(b) $v_{R/E} = \dfrac{v_{T/E}}{\tan 30.0°} = \dfrac{12.0 \text{ m/s}}{\tan 30.0°} = 20.8 \text{ m/s}.$ $v_{R/T} = \dfrac{v_{T/E}}{\sin 30.0°} = \dfrac{12.0 \text{ m/s}}{\sin 30.0°} = 24.0 \text{ m/s}.$

EVALUATE: The speed of the raindrop relative to the train is greater than its speed relative to the earth, because of the motion of the train.

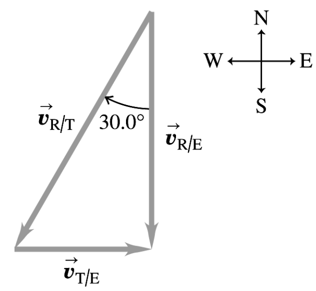


Figure 3.72

3.73. **IDENTIFY:** This is a relative velocity problem.

SET UP: The three relative velocities are:

$\vec{v}_{J/G}$: Juan relative to the ground. This velocity is due north and has magnitude $v_{J/G} = 8.00$ m/s.

$\vec{v}_{B/G}$: the ball relative to the ground. This vector is $37.0°$ east of north and has magnitude $v_{B/G} = 12.00$ m/s.

$\vec{v}_{B/J}$: the ball relative to Juan. We are asked to find the magnitude and direction of this vector.

The relative velocity addition equation is $\vec{v}_{B/G} = \vec{v}_{B/J} + \vec{v}_{J/G}$, so $\vec{v}_{B/J} = \vec{v}_{B/G} - \vec{v}_{J/G}$.

The relative velocity addition diagram does not form a right triangle so we must do the vector addition using components.

Take $+y$ to be north and $+x$ to be east.

EXECUTE: $v_{B/Jx} = +v_{B/G}\sin 37.0° = 7.222$ m/s

$v_{B/Jy} = +v_{B/G}\cos 37.0° - v_{J/G} = 1.584$ m/s

These two components give $v_{B/J} = 7.39$ m/s at $12.4°$ north of east.

EVALUATE: Since Juan is running due north, the ball's eastward component of velocity relative to him is the same as its eastward component relative to the earth. The northward component of velocity for Juan and the ball are in the same direction, so the component for the ball relative to Juan is the difference in their components of velocity relative to the ground.

3.74. **IDENTIFY:** This problem involves relative velocities and vector addition.

SET UP: The catcher is standing still, so the velocity of the ball relative to the catcher is the same as its velocity relative to the ground. We use $\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$. Fig. 3.74 shows the vector sum, where B refers to the baseball, S refers to the shortstop, and G is the ground.

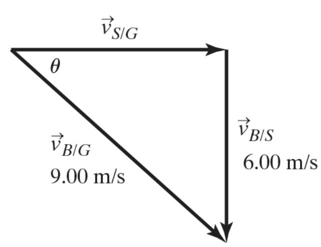


Figure 3.74

EXECUTE: From the figure, we see that $\sin\theta = \dfrac{6.00\text{ m/s}}{9.00\text{ m/s}} = 0.6667 \quad \rightarrow \quad \theta = 41.81°$. We also see that $v_{S/G}$ = (9.00 m/s) cos θ = (9.00 m/s) cos 41.81° = 6.71 m/s.

EVALUATE: As seen by the catcher, the ball is moving eastward at 6.71 m/s and southward at 6.00 m/s.

3.75. IDENTIFY: We need to use relative velocities.

SET UP: If B is moving relative to M and M is moving relative to E, the velocity of B relative to E is $\vec{v}_{B/E} = \vec{v}_{B/M} + \vec{v}_{M/E}$.

EXECUTE: Let $+x$ be east and $+y$ be north. We have $v_{B/M,x} = 2.50$ m/s, $v_{B/M,y} = -4.33$ m/s, $v_{M/E,x} = 0$, and $v_{M/E,y} = 6.00$ m/s. Therefore $v_{B/E,x} = v_{B/M,x} + v_{M/E,x} = 2.50$ m/s and $v_{B/E,y} = v_{B/M,y} + v_{M/E,y} = -4.33 \text{ m/s} + 6.00 \text{ m/s} = +1.67$ m/s. The magnitude is $v_{B/E} = \sqrt{(2.50 \text{ m/s})^2 + (1.67 \text{ m/s})^2} = 3.01$ m/s, and the direction is $\tan\theta = \dfrac{1.67}{2.50}$, which gives $\theta = 33.7^\circ$ north of east.

EVALUATE: Since Mia is moving, the velocity of the ball relative to her is different from its velocity relative to the ground or relative to Alice.

3.76. IDENTIFY: You have a graph showing the horizontal range of the rock as a function of the angle at which it was launched and want to find its initial velocity. Because air resistance is negligible, the rock is in free fall. The range formula applies since the rock rock was launced from the ground and lands at the ground.

SET UP: **(a)** The range formula is $R = \dfrac{v_0^2 \sin(2\theta)}{g}$, so a plot of R versus $\sin(2\theta_0)$ will give a straight line having slope equal to v_0^2/g. We can use that data in the graph in the problem to construct our graph by hand, or we can use graphing software. The resulting graph is shown in Figure 3.76.

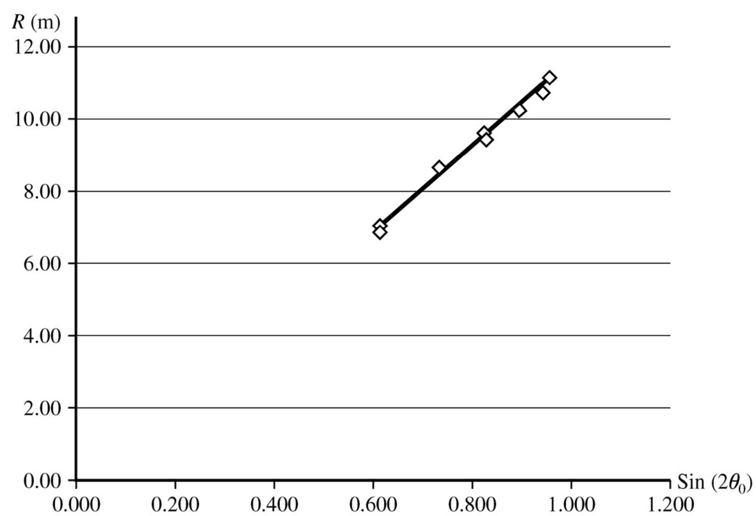


Figure 3.76

(b) The slope of the graph is 10.95 m, so 10.95 m = v_0^2/g. Solving for v_0 we get v_0 = 10.4 m/s.

(c) Solving the formula $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for $y - y_0$ with $v_y = 0$ at the highest point, we get $y - y_0$ = 1.99 m.

EVALUATE: This approach to finding the launch speed v_0 requires only simple measurements: the range and the launch angle. It would be difficult and would require special equipment to measure v_0 directly.

3.77. **IDENTIFY:** The table gives data showing the horizontal range of the potato for various launch heights. You want to use this information to determine the launch speed of the potato, assuming negligible air resistance.

SET UP: The potatoes are launched horizontally, so $v_{0y} = 0$, and they are in free fall, so $a_y = 9.80\text{ m/s}^2$ downward and $a_x = 0$. The time a potato is in the air is just the time it takes for it to fall vertically from the launch point to the ground, a distance h.

EXECUTE: **(a)** For the vertical motion of a potato, we have $h = \frac{1}{2} gt^2$, so $t = \sqrt{2h/g}$. The horizontal range R is given by $R = v_0 t = v_0\sqrt{2h/g}$. Squaring gives $R^2 = \left(\dfrac{2v_0^2}{g}\right)h$. Graphing R^2 versus h will give a straight line with slope $2v_0^2/g$. We can graph the data from the table in the text by hand, or we could use graphing software. The result is shown in Figure 3.77.

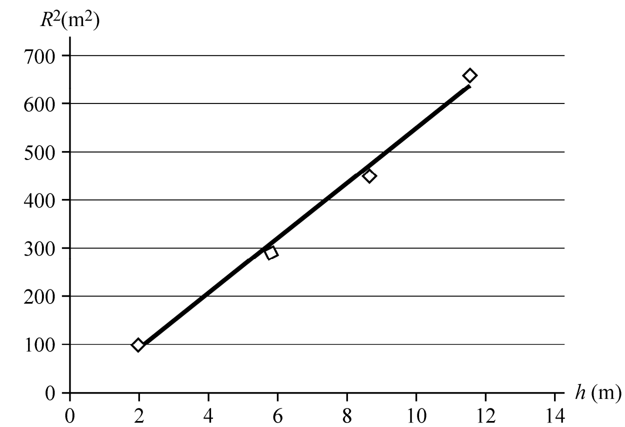


Figure 3.77

(b) The slope of the graph is 55.2 m, so $v_0 = \sqrt{\dfrac{(9.80\text{ m/s}^2)(55.2\text{ m})}{2}} = 16.4\text{ m/s}.$

(c) In this case, the potatoes are launched and land at ground level, so we can use the range formula with $\theta = 30.0°$ and $v_0 = 16.4$ m/s. The result is $R = \dfrac{v_0^2 \sin(2\theta)}{g} = 23.8\text{ m}.$

EVALUATE: This approach to finding the launch speed v_0 requires only simple measurements: the range and the launch height. It would be difficult and would require special equipment to measure v_0 directly.

3.78. **IDENTIFY:** This is a vector addition problem. The boat moves relative to the water and the water moves relative to the earth. We know the speed of the boat relative to the water and the times for the boat to go directly across the river, and from these things we want to find out how fast the water is moving and the width of the river.

SET UP: For both trips of the boat, $\vec{v}_{B/E} = \vec{v}_{B/W} + \vec{v}_{W/E}$, where the subscripts refer to the boat, earth, and water. The speed of the boat relative to the earth is $v_{B/E} = d/t$, where d is the width of the river and t is the time to cross the river, which is different in the two crossings.

EXECUTE: Figure 3.78 shows a vector sum for the first trip and for the return trip.

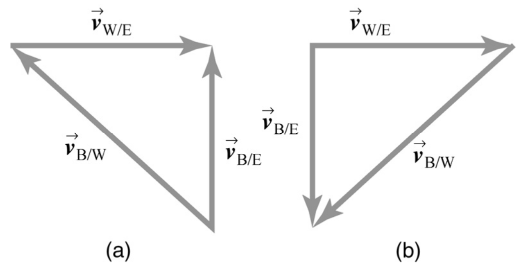


Figure 3.78a-b

(a) For both trips, the vectors in Figures 3.78 a & b form right triangles, so we can apply the Pythagorean theorem. $v_{B/E}^2 = v_{B/W}^2 - v_{W/E}^2$ and $v_{B/E} = d/t$. For the first trip, $v_{B/W} = 6.00$ m/s and $t = 20.1$ s, giving
$d^2/(20.1\text{ s})^2 = (60.00\text{ m/s}^2) - (v_{W/E})^2$. For the return trip, $v_{B/W} = 9.0$ m/s and $t = 11.2$ s, which gives $d^2/(11.2\text{ s})^2 = (9.00\text{ m/s}^2) - (v_{W/E})^2$. Solving these two equations together gives $d = 90.48$ m, which rounds to 90.5 m (the width of the river) and $v_{W/E} = 3.967$ m/s which rounds to 3.97 m/s (the speed of the current).
(b) The shortest time is when the boat heads perpendicular to the current, which is due north. Figure 3.78c illustrates this situation. The time to cross is $t = d/v_{B/W} = (90.48\text{ m})/(6.00\text{ m/s}) = 15.1$ s. The distance x east (down river) that you travel is $x = v_{W/E}t = (3.967\text{ m/s})(15.1\text{ s}) = 59.9$ m east of your starting point.

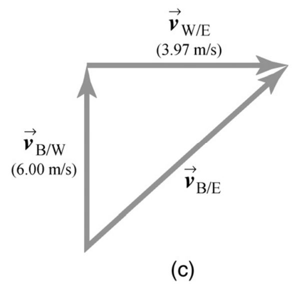


Figure 3.78c

EVALUATE: In part (a), the boat must have a velocity component up river to cancel out the current velocity. In part (b), velocity of the current has no effect on the crossing time, but it does affect the landing position of the boat.

3.79. **IDENTIFY:** Write an expression for the square of the distance (D^2) from the origin to the particle, expressed as a function of time. Then take the derivative of D^2 with respect to t, and solve for the value of t when this derivative is zero. If the discriminant is zero or negative, the distance D will never decrease.
SET UP: $D^2 = x^2 + y^2$, with $x(t)$ and $y(t)$ given by Eqs. (3.19) and (3.20).
EXECUTE: Following this process, $\sin^{-1}\sqrt{8/9} = 70.5°$.
EVALUATE: We know that if the object is thrown straight up it moves away from P and then returns, so we are not surprised that the projectile angle must be less than some maximum value for the distance to always increase with time.

3.80. **IDENTIFY:** Apply the relative velocity relation.

SET UP: Let $v_{\text{C/W}}$ be the speed of the canoe relative to water and $v_{\text{W/G}}$ be the speed of the water relative to the ground.

EXECUTE: **(a)** Taking all units to be in km and h, we have three equations. We know that heading upstream $v_{\text{C/W}} - v_{\text{W/G}} = 2.$ We know that heading downstream for a time t, $(v_{\text{C/W}} + v_{\text{W/G}})t = 5.$ We also know that for the bottle $v_{\text{W/G}}(t+1) = 3.$ Solving these three equations for $v_{\text{W/G}} = x$, $v_{\text{C/W}} = 2 + x$, therefore $(2 + x + x)t = 5$ or $(2 + 2x)t = 5.$ Also $t = 3/x - 1,$ so $(2+2x)\left(\dfrac{3}{x} - 1\right) = 5$ or $2x^2 + x - 6 = 0.$

The positive solution is $x = v_{\text{W/G}} = 1.5$ km/h.

(b) $v_{\text{C/W}} = 2\text{ km/h} + v_{\text{W/G}} = 3.5\text{ km/h}.$

EVALUATE: When they head upstream, their speed relative to the ground is $3.5\text{ km/h} - 1.5\text{ km/h} = 2.0\text{ km/h}.$ When they head downstream, their speed relative to the ground is $3.5\text{ km/h} + 1.5\text{ km/h} = 5.0\text{ km/h}.$ The bottle is moving downstream at 1.5 km/s relative to the earth, so they are able to overtake it.

3.81. **IDENTIFY:** The rocket has two periods of constant acceleration motion.

SET UP: Let $+y$ be upward. During the free-fall phase, $a_x = 0$ and $a_y = -g.$ After the engines turn on, $a_x = (3.00g)\cos 30.0°$ and $a_y = (3.00g)\sin 30.0°.$ Let *t* be the total time since the rocket was dropped and let *T* be the time the rocket falls before the engine starts.

EXECUTE: (i) The diagram is given in Figure 3.81 a.

(ii) The *x*-position of the plane is $(236\text{ m/s})t$ and the *x*-position of the rocket is $(236\text{ m/s})t + (1/2)(3.00)(9.80\text{ m/s}^2)\cos 30°(t-T)^2.$ The graphs of these two equations are sketched in Figure 3.81 b.

(iii) If we take $y = 0$ to be the altitude of the airliner, then $y(t) = -1/2gT^2 - gT(t-T) + 1/2(3.00)(9.80\text{ m/s}^2)(\sin 30°)(t-T)^2$ for the rocket. The airliner has constant *y*. The graphs are sketched in Figure 3.81b.

In each of the Figures 3.81a–c, the rocket is dropped at $t = 0$ and the time *T* when the motor is turned on is indicated.

By setting $y = 0$ for the rocket, we can solve for *t* in terms of *T*:

$0 = -(4.90\text{ m/s}^2)T^2 - (9.80\text{ m/s}^2)T(t-T) + (7.35\text{ m/s}^2)(t-T)^2.$ Using the quadratic formula for the variable $x = t - T$ we find $x = t - T = \dfrac{(9.80\text{ m/s}^2)T + \sqrt{(9.80\text{ m/s}^2T)^2 + (4)(7.35\text{ m/s}^2)(4.9)T^2}}{2(7.35\text{ m/s}^2)}$, or $t = 2.72\,T.$ Now, using the condition that $x_{\text{rocket}} - x_{\text{plane}} = 1000\text{ m},$ we find

$(236\text{ m/s})t + (12.7\text{ m/s}^2)(t-T)^2 - (236\text{ m/s})t = 1000\text{ m},$ or $(1.72T)^2 = 78.6\text{ s}^2.$ Therefore $T = 5.15\text{ s}.$

EVALUATE: During the free-fall phase the rocket and airliner have the same *x* coordinate but the rocket moves downward from the airliner. After the engines fire, the rocket starts to move upward and its horizontal component of velocity starts to exceed that of the airliner.

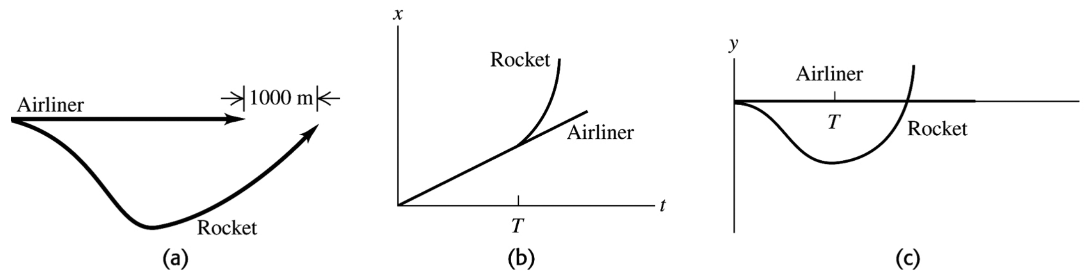


Figure 3. 81

3.82. **IDENTIFY:** We know the speed of the seeds and the distance they travel.
SET UP: We can treat the speed as constant over a very short distance, so $v = d/t$. The minimum frame rate is determined by the maximum speed of the seeds, so we use $v = 4.6$ m/s.
EXECUTE: Solving for t gives $t = d/v = (0.20 \times 10^{-3}\text{ s})/(4.6\text{ m/s}) = 4.3 \times 10^{-5}$ s per frame.
The frame rate is $1/(4.3 \times 10^{-5}$ s per frame) = 23,000 frames/second. Choice (c) 25,000 frames per second is closest to this result, so choice (c) is the best one.
EVALUATE: This experiment would clearly require high-speed photography.

3.83. **IDENTIFY:** A seed launched at 90° goes straight up. Since we are ignoring air resistance, its acceleration is 9.80 m/s^2 downward.
SET UP: For the highest possible speed $v_{0y} = 4.6$ m/s, and $v_y = 0$ at the highest point.
EXECUTE: $v_y = v_{0y} - gt$ gives $t = v_{0y}/g = (4.6\text{ m/s})/(9.80\text{ m/s}^2) = 0.47$ s, which is choice (b).
EVALUATE: Seeds are rather light and 4.6 m/s is fairly fast, so it might not be such a good idea to ignore air resistance. But doing so is acceptable to get a first approximation to the time.

3.84. **IDENTIFY:** A seed launched at 0° starts out traveling horizontally from a height of 20 cm above the ground. Since we are ignoring air resistance, its acceleration is 9.80 m/s^2 downward.
SET UP: Its horizontal distance is determined by the time it takes the seed to fall 20 cm, starting from rest vertically.
EXECUTE: The time to fall 20 cm is $0.20\text{ m} = \frac{1}{2}gt^2$, which gives $t = 0.202$ s. The horizontal distance traveled during this time is $x = (4.6\text{ m/s})(0.202\text{ s}) = 0.93\text{ m} = 93$ cm, which is choice (b).
EVALUATE: In reality the seed would travel a bit less distance due to air resistance.

3.85. **IDENTIFY:** About 2/3 of the seeds are launched between 6° and 56° above the horizontal, and the average for all the seeds is 31°. So clearly most of the seeds are launched above the horizontal.
SET UP and EXECUTE: For choice (a) to be correct, the seeds would need to cluster around 90°, which they do not. For choice (b), most seeds would need to launch below the horizontal, which is not the case. For choice (c), the launch angle should be around +45°. Since 31° is not far from 45°, this is the best choice. For choice (d), the seeds should go straight downward. This would require a launch angle of –90°, which is not the case.
EVALUATE: Evolutionarily it would be an advantage for the seeds to get as far from the parent plant as possible so the young plants would not compete with the parent for water and soil nutrients, so 45° is a biologically plausible result. Natural selection would tend to favor plants that launched their seeds at this angle over those that did not.

4

NEWTON'S LAWS OF MOTION

VP4.1.1. **IDENTIFY:** This is a problem about vector addition. We know the magnitude and direction of three forces and want to find the magnitude and direction of their resultant force.

SET UP: The components of a vector of magnitude A that make an angle θ with the $+x$-axis are $A_x = A \cos\theta$ and $A_y = A \sin\theta$. The magnitude and direction are $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \arctan\dfrac{A_y}{A_x}$. The components of the resultant are $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$.

EXECUTE: For the three given vectors, the components of the resultant are

R_x = 40.0 N + 0 N + (60.0 N) cos 36.9° = 88.0 N

R_y = 0 N + (–80.0 N) + (60.0 N) sin 36.9° = –44.0 N

$R = \sqrt{(88.0\text{ N})^2 + (-44.0\text{ N})^2} = 98.4\text{ N}$.

θ = arctan [(–44.0 N)/(88.0 N)] = –26.6°. The minus sign tells us that θ is clockwise from the $+x$-axis.

EVALUATE: Since R_x is positive and R_y is negative, the resultant should point into the fourth quadrant, which agrees with our result.

VP4.1.2. **IDENTIFY:** This is a problem about vector addition. We know the magnitude and direction of three forces and want to find the magnitude and direction of their resultant force.

SET UP: The components of a vector of magnitude A that make an angle θ with the $+x$-axis are $A_x = A \cos\theta$ and $A_y = A \sin\theta$. The magnitude and direction are $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \arctan\dfrac{A_y}{A_x}$. The components of the resultant are $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$.

EXECUTE: The components of the resultant are

R_x = 45.0 N + 0 N + (235 N) cos 143.1° = –143 N

R_y = 0 N + 105 N + 235 N sin 143.1° = 246 N

$R = \sqrt{(-143\text{ N})^2 + (246\text{ N})^2} = 285\text{ N}$.

θ = arctan[(246 N)/(–143 N)] = –60.0°, so θ = 120° counterclockwise from the $+x$-axis.

EVALUATE: The resultant has a negative x-component and a positive y-component, so it should point into the second quadrant, which is what our result shows.

VP4.1.3. **IDENTIFY:** This is a problem about vector addition. We know the magnitude and direction of three forces and want to find the magnitude and direction of their resultant force.

SET UP: The components of a vector of magnitude A that make an angle θ with the $+x$-axis are $A_x = A \cos\theta$ and $A_y = A \sin\theta$. The magnitude and direction are $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \arctan\dfrac{A_y}{A_x}$. The components of the resultant are $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$.

EXECUTE: The components of the resultant are

R_x = 0 N + (60.0 N) cos 70.0° + (15.0 N) cos 160.0° = 6.4 N

R_y = –60.0 N + (60.0 N) sin 70.0° + (15.0 N) sin 160.0° = 1.5 N

$R = \sqrt{(6.4\text{ N})^2+(1.5\text{ N})^2} = 6.6\text{ N}$

θ = arctan[(1.5 N)/(64 N)] = 13° counterclockwise from the +x-axis.

EVALUATE: Since all the components of the resultant are positive, it should point into the first quadrant, which is what we found.

VP4.1.4. **IDENTIFY:** We know the resultant of three vectors, and we know two of them. We want to find the magnitude and direction of the unknown third vector.

SET UP: The components of a vector of magnitude A that make an angle θ with the +x-axis are $A_x = A \cos\theta$ and $A_y = A \sin\theta$. The magnitude and direction are $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \arctan\frac{A_y}{A_x}$. The components of the resultant are $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$.

EXECUTE: Let T refer to Ernesto's force, K for Kamala's force, and T for Tsuroku's unknown force. The components of the resultant force are

R_x = 35.0 N + 0 N + T_x = (24.0 N) cos 210° $\rightarrow$ T_x = –55.8 N

R_y = 0 N + 50.0 N + T_y = (24.0 N) sin 210° $\rightarrow$ T_y = –62.0 N

$T = \sqrt{(-55.8\text{ N})^2+(-62.0\text{ N})^2} = 83.4\text{ N}$

θ = arctan[(–62.0 N)/(–55.8 N)] = 48.0°

Since both components of Tsuroku's force are negative, its direction is 48.0° below the –x-axis, which is 48.0° south of west.

EVALUATE: A graphical sum will confirm this result.

VP4.4.1. **IDENTIFY:** Apply Newton's second law to the box.

SET UP: Take the x-axis along the floor. Use $\sum F_x = ma_x$ to find a_x. The only horizontal force acting on the box is the force due to the worker.

EXECUTE: $\sum F_x = ma_x$ gives 25 N = (55 kg) a_x $\rightarrow$ a_x = 0.45 m/s^2. This acceleration is in the direction of the worker's force.

EVALUATE: Other forces act on the box, such as gravity downward and the upward push of the floor. But these do not affect the horizontal acceleration since they have no horizontal components.

VP4.4.2. **IDENTIFY:** Apply Newton's second law to the cheese.

SET UP: Take the x-axis along the surface of the table. Use $\sum F_x = ma_x$ to find a_x. The only horizontal force acting on the box is the 0.50-N force.

EXECUTE: **(a)** The forces are gravity acting vertically downward, the normal force due to the tabletop acting vertically upward, and the 0.50-N force due to your hand acting horizontally.

(b) $\sum F_x = ma_x$ gives 0.50 N = (2.0 kg) a_x $\rightarrow$ a_x = 0.25 m/s^2.

EVALUATE: The vertical forces do not affect the horizontal motion, so only the 0.50-N force causes the acceleration.

VP4.4.3. **IDENTIFY:** Apply Newton's second law to the puck.

SET UP: Take the x-axis along the direction of the horizontal hit on the puck. We know the acceleration of the puck, so use $\sum F_x = ma_x$ to find the force of the hit. The only horizontal force acting on the puck is the hit. For the vertical forces, use $\sum F_y = ma_y$.

EXECUTE: (a) With the x-axis horizontal, $\sum F_x = ma_x$ gives

F_x = (0.16 kg)(75 m/s^2) = 12 N.

(b) The vertical forces are gravity (the weight w) and the normal force n due to the ice. Using $\sum F_y = ma_y$, we have $n - w = 0$ since $a_y = 0$. So the normal force must be equal to the weight of the puck.

EVALUATE: The vertical forces do not affect the horizontal motion.

VP4.4.4. **IDENTIFY:** We apply Newton's second law to the plate. We know its horizontal acceleration and mass and one of the horizontal forces acting on it, so we can find the friction force, which is horizontal.

SET UP: Apply $\sum F_x = ma_x$ with the x-axis horizontal. The two horizontal forces are friction f and the push P.

EXECUTE: (a) $\sum F_x = ma_x$ = (0.800 kg)(12.0 m/s^2) = 9.60 N. This is the net force.

(b) $F_{\text{net}} = P - f$, so 9.60 N = 14.0 N – f, so f = 4.4 N. The direction is opposite to the push.

EVALUATE: Friction is less than the push, which it should be since the plate accelerates in the direction of the push.

VP4.5.1. **IDENTIFY:** The forces on the child are constant, so her acceleration is constant. Thus we can use the constant-acceleration motion equations to find her acceleration. Then apply Newton's second law to find the friction force.

SET UP: The formulas $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ and $\sum F_x = ma_x$ both apply.

EXECUTE: First find the girl's acceleration using $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$. Putting in the known numbers gives 0^2 = (3.00 m/s)^2 + $2a_x$(2.25 m), giving a_x = –2.00 m/s^2. The minus sign means that a_x is opposite to v_x. Now use $\sum F_x = ma_x$ to find the friction force. Since friction is the only horizontal force acting on her, it must be in the same direction as her acceleration. This gives $f = ma_x$ = (20.0 kg)(–2.00 m/s^2) = –40.0 N. The magnitude is 40.0 N and the direction is the same as the acceleration, which is opposite to the velocity.

EVALUATE: The other forces (gravity and the normal force due to the ice) are vertical, so they do not affect the horizontal motion.

VP4.5.2. **IDENTIFY:** This problem involves Newton's second law and motion with uniform acceleration. Thus we can use the constant-acceleration motion equations.

SET UP: First use $\sum F_x = ma_x$ to find the plane's acceleration. Then use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find how far it travels while stopping.

EXECUTE: Using $\sum F_x = ma_x$ gives 2.90 N = (1.70×10^5 kg) a_x → a_x = 1.706 m/s^2. Now use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find $x - x_0$. Call the +x-direction to be that of the velocity, so a_x will be negative. Thus

0^2 = (75.0 m/s)^2 + 2(–1.706 m/s^2)($x - x_0$) → $x - x_0$ = 1650 m = 1.65 km.

EVALUATE: This distance is about a mile, which is not so unreasonable for stopping a large plane landing at a fairly high speed.

VP4.5.3. **IDENTIFY:** This problem involves Newton's second law and motion with uniform acceleration. Thus we can use the constant-acceleration motion equations. We know the truck's mass, its initial speed, and the distance it travels while stopping. We want to find how long it takes to stop, its acceleration while stopping, and the braking force while stopping.

SET UP: The braking force is opposite to the truck's velocity but is in the same direction as the truck's acceleration. The equations $v_x = v_{0x} + a_x t$ and $\sum F_x = ma_x$ apply. In addition, the average velocity is $v_{av-x} = \dfrac{\Delta x}{\Delta t}$, and for uniform acceleration, it is also true that $v_{av} = \dfrac{v_0 + v}{2}$.

EXECUTE: **(a)** Combining the two equations for $v_{\text{av-}x}$ gives

$$\Delta t = \frac{\Delta x}{v_{av-x}} = \frac{\Delta x}{\left(\frac{v_0 + v}{2}\right)} = \frac{48.0 \text{ m}}{\frac{25.0 \text{ m/s} + 0}{2}} = 3.84 \text{ s.}$$

(b) Using $v_x = v_{0x} + a_x t$ gives $0 = 25.0$ m/s + a_x (3.84 s)

$a_x = -6.51$ m/s^2. The minus sign means that a_x is opposite to v_x. The magnitude is 6.51 m/s^2.

(c) $\sum F_x = ma_x$ = (2400 kg)(6.51 m/s^2) = 1.56×10^4 N.

EVALUATE: This may seem like a large force, but it is the only force stopping a massive object with a large acceleration.

VP4.5.4. **IDENTIFY:** This problem involves Newton's second law and motion with uniform acceleration. Thus we can use the constant-acceleration motion equations.

SET UP: The equations $\sum F_x = ma_x$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ apply.

EXECUTE: **(a)** Gravity acts downward and the normal force due to the road acts upward. The horizontal forces are the push and the friction force from the road. The friction force is opposite to the push.

(b) Use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find the acceleration of the car, giving

(1.40 m/s)^2 = 0 + 2a_x(5.00 m) $\rightarrow$ $a_x = 0.1960$ m/s^2. Now apply $\sum F_x = ma_x$.

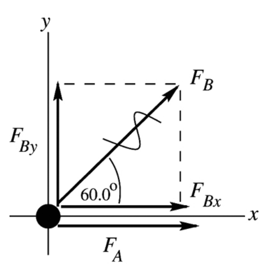
$P - f = ma_x$ $\rightarrow$ 8.00×10^2 N $- f = (1.15 \times 10^3$ kg)(0.1960 m/s^2) $\rightarrow$ $f = 575$ N.

EVALUATE: The push is 800 N and the opposing friction force is 575 N, so the car accelerates in the direction of the push.

4.1. **IDENTIFY:** Vector addition.

SET UP: Use a coordinate system where the $+x$-axis is in the direction of $\vec{F}_A$, the force applied by dog A. The forces are sketched in Figure 4.1.

EXECUTE:



$F_{Ax} = +270$ N, $F_{Ay} = 0$

$F_{Bx} = F_B \cos 60.0° = (300 \text{ N}) \cos 60.0° = +150$ N

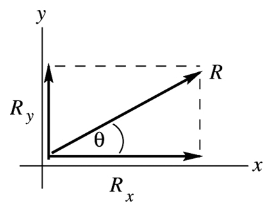
$F_{By} = F_B \sin 60.0° = (300 \text{ N}) \sin 60.0° = +260$ N

Figure 4.1a

$\vec{R} = \vec{F}_A + \vec{F}_B$

$R_x = F_{Ax} + F_{Bx} = +270 \text{ N} + 150 \text{ N} = +420$ N

$R_y = F_{Ay} + F_{By} = 0 + 260 \text{ N} = +260$ N



$$R = \sqrt{R_x^2 + R_y^2}$$
$$R = \sqrt{(420\text{ N})^2 + (260\text{ N})^2} = 494\text{ N}$$
$$\tan\theta = \frac{R_y}{R_x} = 0.619$$
$$\theta = 31.8°$$

Figure 4.1b

EVALUATE: The forces must be added as vectors. The magnitude of the resultant force is less than the sum of the magnitudes of the two forces and depends on the angle between the two forces.

4.2. **IDENTIFY:** We know the magnitudes and directions of three vectors and want to use them to find their components, and then to use the components to find the magnitude and direction of the resultant vector.
SET UP: Let $F_1 = 985$ N, $F_2 = 788$ N, and $F_3 = 411$ N. The angles θ that each force makes with the $+x$ axis are $\theta_1 = 31°$, $\theta_2 = 122°$, and $\theta_3 = 233°$. The components of a force vector are $F_x = F\cos\theta$ and $F_y = F\sin\theta$, and $R = \sqrt{R_x^2 + R_y^2}$ and $\tan\theta = \dfrac{R_y}{R_x}$.

EXECUTE: **(a)** $F_{1x} = F_1\cos\theta_1 = 844$ N, $F_{1y} = F_1\sin\theta_1 = 507$ N, $F_{2x} = F_2\cos\theta_2 = -418$ N, $F_{2y} = F_2\sin\theta_2 = 668$ N, $F_{3x} = F_3\cos\theta_3 = -247$ N, and $F_{3y} = F_3\sin\theta_3 = -328$ N.

(b) $R_x = F_{1x} + F_{2x} + F_{3x} = 179$ N; $R_y = F_{1y} + F_{2y} + F_{3y} = 847$ N. $R = \sqrt{R_x^2 + R_y^2} = 886$ N; $\tan\theta = \dfrac{R_y}{R_x}$ so $\theta = 78.1°$. $\vec{R}$ and its components are shown in Figure 4.2.

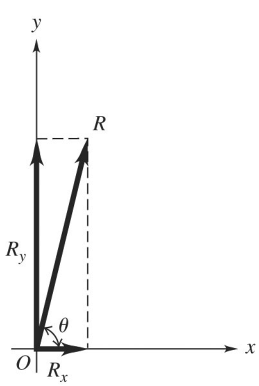


Figure 4.2

EVALUATE: A graphical sketch of the vector sum should agree with the results found in (b). Adding the forces as vectors gives a very different result from adding their magnitudes.

4.3. **IDENTIFY:** We know the resultant of two vectors of equal magnitude and want to find their magnitudes. They make the same angle with the vertical.

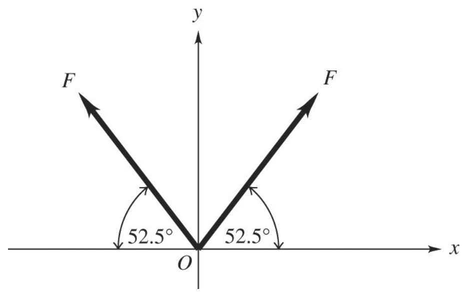


Figure 4.3

SET UP: Take $+y$ to be upward, so $\sum F_y = 5.00$ N. The strap on each side of the jaw exerts a force F directed at an angle of 52.5° above the horizontal, as shown in Figure 4.3.
EXECUTE: $\sum F_y = 2F \sin 52.5° = 5.00$ N, so $F = 3.15$ N.

EVALUATE: The resultant force has magnitude 5.00 N which is *not* the same as the sum of the magnitudes of the two vectors, which would be 6.30 N.

4.4. **IDENTIFY:** $F_x = F\cos\theta$, $F_y = F\sin\theta$.

SET UP: Let $+x$ be parallel to the ramp and directed up the ramp. Let $+y$ be perpendicular to the ramp and directed away from it. Then $\theta = 30.0°$.

EXECUTE: **(a)** $F = \dfrac{F_x}{\cos\theta} = \dfrac{90.0 \text{ N}}{\cos 30°} = 104$ N.

(b) $F_y = F\sin\theta = F_x \tan\theta = (90 \text{ N})(\tan 30°) = 52.0$ N.

EVALUATE: We can verify that $F_x^2 + F_y^2 = F^2$. The signs of F_x and F_y show their direction.

4.5. **IDENTIFY:** Add the two forces using components.
SET UP: $F_x = F\cos\theta$, $F_y = F\sin\theta$, where θ is the angle $\vec{F}$ makes with the $+x$ axis.

EXECUTE: **(a)** $F_{1x} + F_{2x} = (9.00 \text{ N})\cos 120° + (6.00 \text{ N})\cos(233.1°) = -8.10$ N

$F_{1y} + F_{2y} = (9.00 \text{ N})\sin 120° + (6.00 \text{ N})\sin(233.1°) = +3.00$ N.

(b) $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(8.10 \text{ N})^2 + (3.00 \text{ N})^2} = 8.64$ N.

EVALUATE: Since $F_x < 0$ and $F_y > 0$, $\vec{F}$ is in the second quadrant.

4.6. **IDENTIFY:** Use constant acceleration equations to calculate a_x and t. Then use $\sum \vec{F} = m\vec{a}$ to calculate the net force.
SET UP: Let $+x$ be in the direction of motion of the electron.

EXECUTE: **(a)** $v_{0x} = 0$, $(x - x_0) = 1.80\times10^{-2}$ m, $v_x = 3.00\times10^6$ m/s. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(3.00\times10^6 \text{ m/s})^2 - 0}{2(1.80\times10^{-2} \text{ m})} = 2.50\times10^{14} \text{ m/s}^2$$

(b) $v_x = v_{0x} + a_x t$ gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{3.00\times10^6 \text{ m/s} - 0}{2.50\times10^{14} \text{ m/s}^2} = 1.2\times10^{-8}$ s

(c) $\sum F_x = ma_x = (9.11\times10^{-31} \text{ kg})(2.50\times10^{14} \text{ m/s}^2) = 2.28\times10^{-16}$ N.

EVALUATE: The acceleration is in the direction of motion since the speed is increasing, and the net force is in the direction of the acceleration.

4.7. **IDENTIFY:** Friction is the only horizontal force acting on the skater, so it must be the one causing the acceleration. Newton's second law applies.

SET UP: Take $+x$ to be the direction in which the skater is moving initially. The final velocity is $v_x = 0$, since the skater comes to rest. First use the kinematics formula $v_x = v_{0x} + a_x t$ to find the acceleration, then apply $\sum \vec{F} = m\vec{a}$ to the skater.

EXECUTE: $v_x = v_{0x} + a_x t$ so $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{0 - 2.40\text{ m/s}}{3.52\text{ s}} = -0.682\text{ m/s}^2$. The only horizontal force on the skater is the friction force, so $f_x = ma_x = (68.5\text{ kg})(-0.682\text{ m/s}^2) = -46.7\text{ N}$. The force is 46.7 N, directed opposite to the motion of the skater.

EVALUATE: Although other forces are acting on the skater (gravity and the upward force of the ice), they are vertical and therefore do not affect the horizontal motion.

4.8. **IDENTIFY:** The elevator and everything in it are accelerating upward, so we apply Newton's second law in the vertical direction.

SET UP: Your mass is $m = w/g = 63.8\text{ kg}$. Both you and the package have the same acceleration as the elevator. Take $+y$ to be upward, in the direction of the acceleration of the elevator, and apply $\sum F_y = ma_y$.

EXECUTE: **(a)** Your free-body diagram is shown in Figure 4.8a, where *n* is the scale reading. $\sum F_y = ma_y$ gives $n - w = ma$. Solving for *n* gives

$n = w + ma = 625\text{ N} + (63.8\text{ kg})(2.50\text{ m/s}^2) = 784\text{ N}$.

(b) The free-body diagram for the package is given in Figure 4.8b. $\sum F_y = ma_y$ gives $T - w = ma$, so $T = w + ma = (3.85\text{ kg})(9.80\text{ m/s}^2 + 2.50\text{ m/s}^2) = 47.4\text{ N}$.

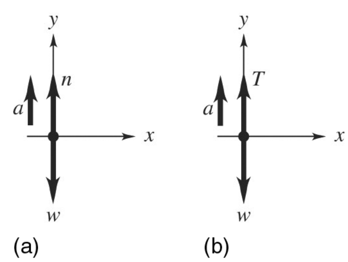


Figure 4.8

EVALUATE: The objects accelerate upward so for each of them the upward force is greater than the downward force.

4.9. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box.

SET UP: Let $+x$ be the direction of the force and acceleration. $\sum F_x = 48.0\text{ N}$.

EXECUTE: $\sum F_x = ma_x$ gives $m = \dfrac{\sum F_x}{a_x} = \dfrac{48.0\text{ N}}{2.20\text{ m/s}^2} = 21.8\text{ kg}$.

EVALUATE: The vertical forces sum to zero and there is no motion in that direction.

4.10. **IDENTIFY:** Use the information about the motion to find the acceleration and then use $\sum F_x = ma_x$ to calculate *m*.

SET UP: Let $+x$ be the direction of the force. $\sum F_x = 80.0\text{ N}$.

EXECUTE: **(a)** $x - x_0 = 11.0\text{ m}$, $t = 5.00\text{ s}$, $v_{0x} = 0$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(11.0\text{ m})}{(5.00\text{ s})^2} = 0.880\text{ m/s}^2$. $m = \dfrac{\sum F_x}{a_x} = \dfrac{80.0\text{ N}}{0.880\text{ m/s}^2} = 90.9\text{ kg}$.

(b) $a_x = 0$ and v_x is constant. After the first 5.0 s, $v_x = v_{0x} + a_x t =$ (0.880 m/s^2) (5.00 s) = 4.40 m/s.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 =$ (4.40 m/s)(5.00 s) = 22.0 m.

EVALUATE: The mass determines the amount of acceleration produced by a given force. The block moves farther in the second 5.00 s than in the first 5.00 s.

4.11. **IDENTIFY** and **SET UP:** Use Newton's second law in component form to calculate the acceleration produced by the force. Use constant acceleration equations to calculate the effect of the acceleration on the motion.

EXECUTE: **(a)** During this time interval the acceleration is constant and equal to

$$a_x = \frac{F_x}{m} = \frac{0.250\ \text{N}}{0.160\ \text{kg}} = 1.562\ \text{m/s}^2$$

We can use the constant acceleration kinematic equations from Chapter 2.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(1.562\ \text{m/s}^2)(2.00\ \text{s})^2 = 3.12\ \text{m}$, so the puck is at $x = 3.12$ m.

$v_x = v_{0x} + a_x t = 0 + (1.562\ \text{m/s}^2)(2.00\ \text{s}) = 3.12\ \text{m/s}.$

(b) In the time interval from $t = 2.00$ s to 5.00 s the force has been removed so the acceleration is zero. The speed stays constant at $v_x = 3.12$ m/s. The distance the puck travels is

$x - x_0 = v_{0x}t = (3.12\ \text{m/s})(5.00\ \text{s} - 2.00\ \text{s}) = 9.36\ \text{m}.$ At the end of the interval it is at

$x = x_0 + 9.36\ \text{m} = 12.5\ \text{m}.$

In the time interval from $t = 5.00$ s to 7.00 s the acceleration is again $a_x = 1.562\ \text{m/s}^2$. At the start of this interval $v_{0x} = 3.12$ m/s and $x_0 = 12.5$ m.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (3.12\ \text{m/s})(2.00\ \text{s}) + \frac{1}{2}(1.562\ \text{m/s}^2)(2.00\ \text{s})^2.$

$x - x_0 = 6.24\ \text{m} + 3.12\ \text{m} = 9.36\ \text{m}.$

Therefore, at $t = 7.00$ s the puck is at $x = x_0 + 9.36\ \text{m} = 12.5\ \text{m} + 9.36\ \text{m} = 21.9\ \text{m}.$

$v_x = v_{0x} = a_x t = 3.12\ \text{m/s} + (1.562\ \text{m/s}^2)(2.00\ \text{s}) = 6.24\ \text{m/s}.$

EVALUATE: The acceleration says the puck gains 1.56 m/s of velocity for every second the force acts. The force acts a total of 4.00 s so the final velocity is (1.56 m/s)(4.0 s) = 6.24 m/s.

4.12. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$. Then use a constant acceleration equation to relate the kinematic quantities.

SET UP: Let $+x$ be in the direction of the force.

EXECUTE: **(a)** $a_x = F_x / m$ = (14.0 N)/(32.5 kg) = 0.4308 m/s^2, which rounds to 0.431 m/s^2 for the final answer.

(b) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$. With $v_{0x} = 0$, $x = \frac{1}{2}a_x t^2 = \frac{1}{2}(0.4308\ \text{m/s}^2)(10.0\ \text{s})^2 = 21.5\ \text{m}.$

(c) $v_x = v_{0x} + a_x t$. With $v_{0x} = 0$, $v_x = a_x t$ = (0.4308 m/s^2)(10.0 s) = 4.31 m/s.

EVALUATE: The acceleration connects the motion to the forces.

4.13. **IDENTIFY:** The force and acceleration are related by Newton's second law.

SET UP: $\sum F_x = ma_x$, where $\sum F_x$ is the net force. $m = 4.50$ kg.

EXECUTE: **(a)** The maximum net force occurs when the acceleration has its maximum value.

$\sum F_x = ma_x = (4.50\ \text{kg})(10.0\ \text{m/s}^2) = 45.0\ \text{N}.$ This maximum force occurs between 2.0 s and 4.0 s.

(b) The net force is constant when the acceleration is constant. This is between 2.0 s and 4.0 s.

(c) The net force is zero when the acceleration is zero. This is the case at $t = 0$ and $t = 6.0$ s.

EVALUATE: A graph of $\sum F_x$ versus t would have the same shape as the graph of a_x versus t.

4.14. **IDENTIFY:** The force and acceleration are related by Newton's second law. $a_x = \dfrac{dv_x}{dt}$, so a_x is the slope of the graph of v_x versus t.

SET UP: The graph of v_x versus t consists of straight-line segments. For $t = 0$ to $t = 2.00$ s, $a_x = 4.00\ \text{m/s}^2$. For $t = 2.00$ s to 6.00 s, $a_x = 0$. For $t = 6.00$ s to 10.0 s, $a_x = 1.00\ \text{m/s}^2$. $\sum F_x = ma_x$, with $m = 2.75$ kg. $\sum F_x$ is the net force.

EXECUTE: **(a)** The maximum net force occurs when the acceleration has its maximum value. $\sum F_x = ma_x = (2.75\ \text{kg})(4.00\ \text{m/s}^2) = 11.0$ N. This maximum occurs in the interval $t = 0$ to $t = 2.00$ s.

(b) The net force is zero when the acceleration is zero. This is between 2.00 s and 6.00 s.

(c) Between 6.00 s and 10.0 s, $a_x = 1.00\ \text{m/s}^2$, so $\sum F_x = (2.75\ \text{kg})(1.00\ \text{m/s}^2) = 2.75$ N.

EVALUATE: The net force is largest when the velocity is changing most rapidly.

4.15. **IDENTIFY:** The net force and the acceleration are related by Newton's second law. When the rocket is near the surface of the earth the forces on it are the upward force $\vec{F}$ exerted on it because of the burning fuel and the downward force $\vec{F}_{\text{grav}}$ of gravity. $F_{\text{grav}} = mg$.

SET UP: Let $+y$ be upward. The weight of the rocket is $F_{\text{grav}} = (8.00\ \text{kg})(9.80\ \text{m/s}^2) = 78.4$ N.

EXECUTE: **(a)** At $t = 0$, $F = A = 100.0$ N. At $t = 2.00$ s, $F = A + (4.00\ \text{s}^2)B = 150.0$ N and $B = \dfrac{150.0\ \text{N} - 100.0\ \text{N}}{4.00\ \text{s}^2} = 12.5\ \text{N/s}^2$.

(b) (i) At $t = 0$, $F = A = 100.0$ N. The net force is $\sum F_y = F - F_{\text{grav}} = 100.0\ \text{N} - 78.4\ \text{N} = 21.6$ N. $a_y = \dfrac{\sum F_y}{m} = \dfrac{21.6\ \text{N}}{8.00\ \text{kg}} = 2.70\ \text{m/s}^2$. (ii) At $t = 3.00$ s, $F = A + B(3.00\ \text{s})^2 = 212.5$ N. $\sum F_y = 212.5\ \text{N} - 78.4\ \text{N} = 134.1$ N. $a_y = \dfrac{\sum F_y}{m} = \dfrac{134.1\ \text{N}}{8.00\ \text{kg}} = 16.8\ \text{m/s}^2$.

(c) Now $F_{\text{grav}} = 0$ and $\sum F_y = F = 212.5$ N. $a_y = \dfrac{212.5\ \text{N}}{8.00\ \text{kg}} = 26.6\ \text{m/s}^2$.

EVALUATE: The acceleration increases as F increases.

4.16. **IDENTIFY:** Weight and mass are related by $w = mg$. The mass is constant but g and w depend on location.

SET UP: On earth, $g = 9.80\ \text{m/s}^2$.

EXECUTE: **(a)** $\dfrac{w}{g} = m$, which is constant, so $\dfrac{w_{\text{E}}}{g_{\text{E}}} = \dfrac{w_{\text{A}}}{g_{\text{A}}}$. $w_{\text{E}} = 17.5$ N, $g_{\text{E}} = 9.80\ \text{m/s}^2$, and $w_{\text{M}} = 3.24$ N. $g_{\text{M}} = \left(\dfrac{w_{\text{A}}}{w_{\text{E}}}\right) g_{\text{E}} = \left(\dfrac{3.24\ \text{N}}{17.5\ \text{N}}\right)(9.80\ \text{m/s}^2) = 1.81\ \text{m/s}^2$.

(b) $m = \dfrac{w_{\text{E}}}{g_{\text{E}}} = \dfrac{17.5\ \text{N}}{9.80\ \text{m/s}^2} = 1.79$ kg.

EVALUATE: The weight at a location and the acceleration due to gravity at that location are directly proportional.

4.17. **IDENTIFY** and **SET UP:** $F = ma$. We must use $w = mg$ to find the mass of the boulder.

EXECUTE: $m = \dfrac{w}{g} = \dfrac{2400\ \text{N}}{9.80\ \text{m/s}^2} = 244.9$ kg

Then $F = ma = (244.9\text{ kg})(12.0\text{ m/s}^2) = 2940\text{ N}$.

EVALUATE: We must use mass in Newton's second law. Mass and weight are proportional.

4.18. **IDENTIFY:** Find weight from mass and vice versa.

SET UP: Equivalencies we'll need are: $1\,\mu\text{g} = 10^{-6}\text{ g} = 10^{-9}\text{ kg}$, $1\text{ mg} = 10^{-3}\text{ g} = 10^{-6}\text{ kg}$, $1\text{ N} = 0.2248\text{ lb}$, and $g = 9.80\text{ m/s}^2 = 32.2\text{ ft/s}^2$.

EXECUTE: **(a)** $m = 210\,\mu\text{g} = 2.10\times10^{-7}\text{ kg}$. $w = mg = (2.10\times10^{-7}\text{ kg})(9.80\text{ m/s}^2) = 2.06\times10^{-6}\text{ N}$.

(b) $m = 12.3\text{ mg} = 1.23\times10^{-5}\text{ kg}$. $w = mg = (1.23\times10^{-5}\text{ kg})(9.80\text{ m/s}^2) = 1.21\times10^{-4}\text{ N}$.

(c) $(45\text{ N})\left(\dfrac{0.2248\text{ lb}}{1\text{ N}}\right) = 10.1\text{ lb}$. $m = \dfrac{w}{g} = \dfrac{45\text{ N}}{9.80\text{ m/s}^2} = 4.6\text{ kg}$.

EVALUATE: We are not converting mass to weight (or vice versa) since they are different types of quantities. We are finding what a given mass will weigh and how much mass a given weight contains.

4.19. **IDENTIFY** and **SET UP:** $w = mg$. The mass of the watermelon is constant, independent of its location. Its weight differs on earth and Jupiter's moon. Use the information about the watermelon's weight on earth to calculate its mass:

EXECUTE: **(a)** $w = mg$ gives that $m = \dfrac{w}{g} = \dfrac{44.0\text{ N}}{9.80\text{ m/s}^2} = 4.49\text{ kg}$.

(b) On Jupiter's moon, $m = 4.49\text{ kg}$, the same as on earth. Thus the weight on Jupiter's moon is $w = mg = (4.49\text{ kg})(1.81\text{ m/s}^2) = 8.13\text{ N}$.

EVALUATE: The weight of the watermelon is less on Io, since g is smaller there.

4.20. **IDENTIFY and SET UP:** This problem requires some estimation and a web search.

EXECUTE: Web search: The mass of a Sumo wrestler is approximately 148 kg, which is about 326 lb. Estimate: The average student in class weighs about 165 lb which is about 75 kg.

EVALUATE: These are average values.

4.21. **IDENTIFY:** Apply $\sum F_x = ma_x$ to find the resultant horizontal force.

SET UP: Let the acceleration be in the $+x$ direction.

EXECUTE: $\sum F_x = ma_x = (55\text{ kg})(15\text{ m/s}^2) = 825\text{ N}$. The force is exerted by the blocks. The blocks push on the sprinter because the sprinter pushes on the blocks.

EVALUATE: The force the blocks exert on the sprinter has the same magnitude as the force the sprinter exerts on the blocks. The harder the sprinter pushes, the greater the force on her.

4.22. **IDENTIFY:** Newton's third law applies.

SET UP: The car exerts a force on the truck and the truck exerts a force on the car.

EXECUTE: The force and the reaction force are always exactly the same in magnitude, so the force that the truck exerts on the car is 1600 N, by Newton's third law.

EVALUATE: Even though the truck is much larger and more massive than the car, it cannot exert a larger force on the car than the car exerts on it.

4.23. **IDENTIFY:** The system is accelerating so we use Newton's second law.

SET UP: The acceleration of the entire system is due to the 250-N force, but the acceleration of box B is due to the force that box A exerts on it. $\sum F = ma$ applies to the two-box system and to each box individually.

EXECUTE: For the two-box system: $a_x = \dfrac{250\text{ N}}{25.0\text{ kg}} = 10.0\text{ m/s}^2$. Then for box B, where F_A is the force exerted on B by A, $F_A = m_B a = (5.0\text{ kg})(10.0\text{ m/s}^2) = 50\text{ N}$.

EVALUATE: The force on B is less than the force on A.

4.24. IDENTIFY: The reaction forces in Newton's third law are always between a pair of objects. In Newton's second law all the forces act on a single object.
SET UP: Let $+y$ be downward. $m = w/g$.

EXECUTE: The reaction to the upward normal force on the passenger is the downward normal force, also of magnitude 620 N, that the passenger exerts on the floor. The reaction to the passenger's weight is the
gravitational force that the passenger exerts on the earth, upward and also of magnitude 650 N.
$\dfrac{\sum F_y}{m} = a_y$ gives $a_y = \dfrac{650\text{ N} - 620\text{ N}}{(650\text{ N})/(9.80\text{ m/s}^2)} = 0.452\text{ m/s}^2$. The passenger's acceleration is 0.452 m/s^2, downward.
EVALUATE: There is a net downward force on the passenger, and the passenger has a downward acceleration.

4.25. IDENTIFY: Apply Newton's second law to the earth.
SET UP: The force of gravity that the earth exerts on her is her weight,
$w = mg = (45\text{ kg})(9.8\text{ m/s}^2) = 441\text{ N}$. By Newton's third law, she exerts an equal and opposite force on the earth.
Apply $\sum\vec{F} = m\vec{a}$ to the earth, with $\left|\sum\vec{F}\right| = w = 441\text{ N}$, but must use the mass of the earth for m.

EXECUTE: $a = \dfrac{w}{m} = \dfrac{441\text{ N}}{6.0\times10^{24}\text{ kg}} = 7.4\times10^{-23}\text{ m/s}^2$.

EVALUATE: This is *much* smaller than her acceleration of 9.8 m/s^2. The force she exerts on the earth equals in magnitude the force the earth exerts on her, but the acceleration the force produces depends on the mass of the object and her mass is *much* less than the mass of the earth.

4.26. IDENTIFY: The surface of block B can exert both a friction force and a normal force on block A. The friction force is directed so as to oppose relative motion between blocks B and A. Gravity exerts a downward force w on block A.
SET UP: The pull is a force on B not on A.
EXECUTE: **(a)** If the table is frictionless there is a net horizontal force on the combined object of the two blocks, and block B accelerates in the direction of the pull. The friction force that B exerts on A is to the right, to try to prevent A from slipping relative to B as B accelerates to the right. The free-body diagram is sketched in Figure 4.26a (next page). f is the friction force that B exerts on A and n is the normal force that B exerts on A.
(b) The pull and the friction force exerted on B by the table cancel and the net force on the system of two blocks is zero. The blocks move with the same constant speed and B exerts no friction force on A. The free-body diagram is sketched in Figure 4.26b (next page).
EVALUATE: If in part (b) the pull force is decreased, block B will slow down, with an acceleration directed to the left. In this case the friction force on A would be to the left, to prevent relative motion between the two blocks by giving A an acceleration equal to that of B.

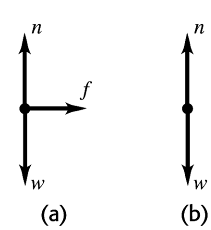


Figure 4.26

4.27. **IDENTIFY:** Identify the forces on each object.

SET UP: In each case the forces are the noncontact force of gravity (the weight) and the forces applied by objects that are in contact with each crate. Each crate touches the floor and the other crate, and some object applies $\vec{F}$ to crate A.

EXECUTE: **(a)** The free-body diagrams for each crate are given in Figure 4.27.

F_{AB} (the force on m_A due to m_B) and F_{BA} (the force on m_B due to m_A) form an action-reaction pair.

(b) Since there is no horizontal force opposing F, any value of F, no matter how small, will cause the crates to accelerate to the right. The weight of the two crates acts at a right angle to the horizontal, and is in any case balanced by the upward force of the surface on them.

EVALUATE: Crate B is accelerated by F_{BA} and crate A is accelerated by the net force $F - F_{AB}$. The greater the total weight of the two crates, the greater their total mass and the smaller will be their acceleration.

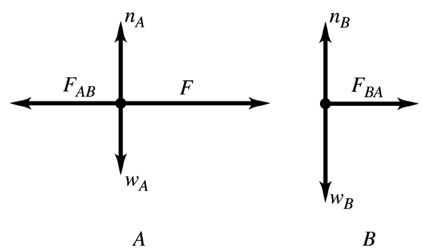


Figure 4.27

4.28. **IDENTIFY:** Use a constant acceleration equation to find the stopping time and acceleration. Then use $\sum\vec{F} = m\vec{a}$ to calculate the force.

SET UP: Let $+x$ be in the direction the bullet is traveling. $\vec{F}$ is the force the wood exerts on the bullet.

EXECUTE: **(a)** $v_{0x} = 350$ m/s, $v_x = 0$ and $(x - x_0) = 0.130$ m. $(x - x_0) = \left(\dfrac{v_{0x} + v_x}{2}\right)t$ gives

$$t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(0.130\text{ m})}{350\text{ m/s}} = 7.43\times10^{-4}\text{ s}.$$

(b) $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \dfrac{0 - (350\text{ m/s})^2}{2(0.130\text{ m})} = -4.71\times10^5\text{ m/s}^2$

$\sum F_x = ma_x$ gives $-F = ma_x$ and $F = -ma_x = -(1.80\times10^{-3}\text{ kg})(-4.71\times10^5\text{ m/s}^2) = 848$ N.

EVALUATE: The acceleration and net force are opposite to the direction of motion of the bullet.

4.29. **IDENTIFY:** Since the observer in the train sees the ball hang motionless, the ball must have the same acceleration as the train car. By Newton's second law, there must be a net force on the ball in the same direction as its acceleration.

SET UP: The forces on the ball are gravity, which is w, downward, and the tension $\vec{T}$ in the string, which is directed along the string.

EXECUTE: **(a)** The acceleration of the train is zero, so the acceleration of the ball is zero. There is no net horizontal force on the ball and the string must hang vertically. The free-body diagram is sketched in Figure 4.29a.

(b) The train has a constant acceleration directed east so the ball must have a constant eastward acceleration. There must be a net horizontal force on the ball, directed to the east. This net force must come from an eastward component of $\vec{T}$ and the ball hangs with the string displaced west of vertical. The free-body diagram is sketched in Figure 4.29b.

EVALUATE: When the motion of an object is described in an inertial frame, there must be a net force in the direction of the acceleration.

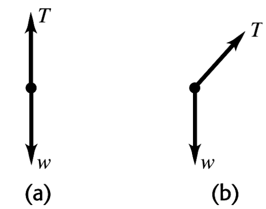


Figure 4.29

4.30. **IDENTIFY:** Identify the forces on the chair. The floor exerts a normal force and a friction force.
SET UP: Let $+y$ be upward and let $+x$ be in the direction of the motion of the chair.

EXECUTE: **(a)** The free-body diagram for the chair is given in Figure 4.30.
(b) For the chair, $a_y = 0$ so $\sum F_y = ma_y$ gives $n - mg - F\sin 37° = 0$ and $n = 142$ N.

EVALUATE: n is larger than the weight because $\vec{F}$ has a downward component.

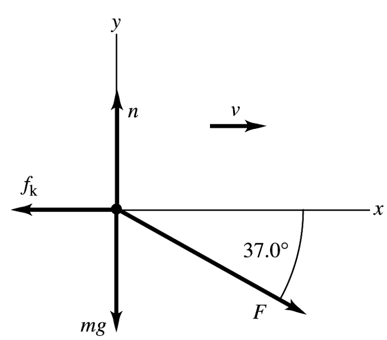


Figure 4.30

4.31. **IDENTIFY:** This problem requires some estimation and a web search.
SET UP: We want to find the force the pitcher exerts on a baseball while pitching a fastball. Estimate: The distance a pitcher moves the ball during a pitch is about twice an arm length, which is about 3.0 ft. Web search: A major league baseball weighs between 5.00 oz and 5.25 oz, so use an average of 5.125 oz, which is 0.320 lb with a mass of 1.0×10^{-2} slugs. The average speed of a pitched fastball is 92 mph which is 135 ft/s.
Assumptions: The ball moves in a straight horizontal line with constant acceleration during the pitch. We use the information above to calculate the acceleration of the ball during the pitch. Then apply Newton's second law to find the average force on the ball. $\sum F_x = ma_x$ applies to the ball and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ for constant acceleration.

EXECUTE: Use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find the acceleration: $(135 \text{ ft/s})^2 = 0 + 2a_x(3.0 \text{ ft})$, which gives $a_x = 3.0 \times 10^3 \text{ ft/s}^2$. Now apply $\sum F_x = ma_x$ to the ball. The only force accelerating it is the push P of the pitcher, so $P = ma = (1.0 \times 10^{-2} \text{ slugs})(3.0 \times 10^3 \text{ ft/s}^2) = 30$ lb. This push is about 130 N.
EVALUATE: A 30-lb push may not seem like much, but the ball has a rather small mass, so this push can produce a large acceleration. In this case, $a_x/g = (3000 \text{ ft/s}^2)/(32.2 \text{ ft/s}^2) = 93$, so a_x is nearly $100g$!

4.32. **IDENTIFY:** Use the motion of the ball to calculate g, the acceleration of gravity on the planet. Then $w = mg$.

SET UP: Let $+y$ be downward and take $y_0 = 0$. $v_{0y} = 0$ since the ball is released from rest.

EXECUTE: Get g on X: $y = \frac{1}{2}gt^2$ gives $10.0\text{ m} = \frac{1}{2}g(3.40\text{ s})^2$. $g = 1.73\text{ m/s}^2$ and then

$w_X = mg_X = (0.100\text{ kg})(1.73\text{ m/s}^2) = 0.173\text{ N}$.

EVALUATE: g on Planet X is smaller than on earth and the object weighs less than it would on earth.

4.33. **IDENTIFY:** Apply Newton's second law to the bucket and constant-acceleration kinematics.

SET UP: The minimum time to raise the bucket will be when the tension in the cord is a maximum since this will produce the greatest acceleration of the bucket.

EXECUTE: Apply Newton's second law to the bucket: $T - mg = ma$. For the maximum acceleration, the tension is greatest, so $a = \frac{T - mg}{m} = \frac{75.0\text{ N} - (5.60\text{ kg})(9.8\text{ m/s}^2)}{5.60\text{ kg}} = 3.593\text{ m/s}^2$.

The kinematics equation for $y(t)$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(12.0\text{ m})}{3.593\text{ m/s}^2}} = 2.58\text{ s}$.

EVALUATE: A shorter time would require a greater acceleration and hence a stronger pull, which would break the cord.

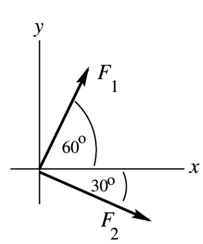
4.34. **IDENTIFY:** The pull accelerates both blocks, so we apply Newton's second law to each one.

SET UP: Apply $\sum F_x = ma_x$ to each block. Using B we can find the friction force on each block, which is also the friction force on A, by Newton's third law. Then using A we can find its acceleration. The target variable is the acceleration of block A.

EXECUTE: Block B: The friction force on B due to A opposes the pull. So $\sum F_x = ma_x$ gives $P - f = m_B a_B$, so $f = P - m_B a_B = 12.0\text{ N} - (6.00\text{ kg})(1.80\text{ m/s}^2) = 1.20\text{ N}$. Now apply $\sum F_x = ma_x$ to block A. Only friction accelerates this block forward, and it must be 1.20 N. Thus $f = m_A a_A$, which becomes $1.20\text{ N} = (2.00\text{ kg})a_A$, so $a_A = 0.600\text{ m/s}^2$.

EVALUATE: Block B accelerates forward at 1.80 m/s^2 but block A accelerates forward at only 0.600 m/s^2. Thus A is not keeping up with B, which means it is sliding *backward* as observed by a person moving with B.

4.35. **IDENTIFY:** If the box moves in the $+x$-direction it must have $a_y = 0$, so $\sum F_y = 0$.



The smallest force the child can exert and still produce such motion is a force that makes the y-components of all three forces sum to zero, but that doesn't have any x-component.

Figure 4.35

SET UP: $\vec{F}_1$ and $\vec{F}_2$ are sketched in Figure 4.35. Let $\vec{F}_3$ be the force exerted by the child.

$\sum F_y = ma_y$ implies $F_{1y} + F_{2y} + F_{3y} = 0$, so $F_{3y} = -(F_{1y} + F_{2y})$.

EXECUTE: $F_{1y} = +F_1 \sin 60° = (100\text{ N})\sin 60° = 86.6\text{ N}$

$F_{2y} = +F_2 \sin(-30°) = -F_2 \sin 30° = -(140\text{ N})\sin 30° = -70.0\text{ N}$

Then $F_{3y} = -(F_{1y} + F_{2y}) = -(86.6\text{ N} - 70.0\text{ N}) = -16.6\text{ N}$; $F_{3x} = 0$

The smallest force the child can exert has magnitude 17 N and is directed at $90°$ clockwise from the $+x$-axis shown in the figure.

(b) IDENTIFY and SET UP: Apply $\sum F_x = ma_x$. We know the forces and a_x so can solve for m. The force exerted by the child is in the $-y$-direction and has no x-component.

EXECUTE: $F_{1x} = F_1 \cos 60° = 50\ \text{N}$

$F_{2x} = F_2 \cos 30° = 121.2\ \text{N}$

$\sum F_x = F_{1x} + F_{2x} = 50\ \text{N} + 121.2\ \text{N} = 171.2\ \text{N}$

$$m = \frac{\sum F_x}{a_x} = \frac{171.2\ \text{N}}{2.00\ \text{m/s}^2} = 85.6\ \text{kg}$$

Then $w = mg = 840\ \text{N}$.

EVALUATE: In part (b) we don't need to consider the y-component of Newton's second law. $a_y = 0$ so the mass doesn't appear in the $\sum F_y = ma_y$ equation.

4.36. IDENTIFY: Use constant acceleration equations to calculate the acceleration a_x that would be required. Then use $\sum F_x = ma_x$ to find the necessary force.

SET UP: Let $+x$ be the direction of the initial motion of the auto.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $v_x = 0$ gives $a_x = -\dfrac{v_{0x}^2}{2(x - x_0)}$. The force F is directed opposite

to

the motion and $a_x = -\dfrac{F}{m}$. Equating these two expressions for a_x gives

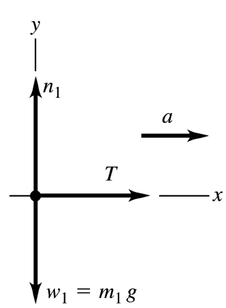
$$F = m\frac{v_{0x}^2}{2(x - x_0)} = (850\ \text{kg})\frac{(12.5\ \text{m/s})^2}{2(1.8\times10^{-2}\ \text{m})} = 3.7\times10^6\ \text{N}.$$

EVALUATE: A very large force is required to stop such a massive object in such a short distance.

4.37. IDENTIFY: Use Newton's second law to relate the acceleration and forces for each crate.

(a) SET UP: Since the crates are connected by a rope, they both have the same acceleration, $2.50\ \text{m/s}^2$.

(b) The forces on the 4.00 kg crate are shown in Figure 4.37a.



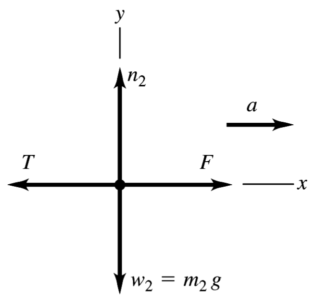
EXECUTE:

$\sum F_x = ma_x$

$T = m_1 a = (4.00\ \text{kg})(2.50\ \text{m/s}^2) = 10.0\ \text{N}.$

Figure 4.37a

(c) SET UP: Forces on the 6.00 kg crate are shown in Figure 4.37b.



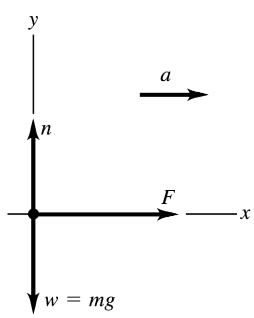
The crate The crate accelerates to the right, so the net force is to the right. F must be larger than T.

Figure 4.37b

(d) EXECUTE: $\sum F_x = ma_x$ gives $F - T = m_2 a$

$F = T + m_2 a = 10.0\text{ N} + (6.00\text{ kg})(2.50\text{ m/s}^2) = 10.0\text{ N} + 15.0\text{ N} = 25.0\text{ N}$

EVALUATE: We can also consider the two crates and the rope connecting them as a single object of mass $m = m_1 + m_2 = 10.0$ kg. The free-body diagram is sketched in Figure 4.37c.



$\sum F_x = ma_x$

$F = ma = (10.0\text{ kg})(2.50\text{ m/s}^2) = 25.0\text{ N}$

This agrees with our answer in part (d).

Figure 4.37c

4.38. **IDENTIFY:** Use kinematics to find the acceleration and then apply Newton's second law.
SET UP: The 60.0-N force accelerates both blocks, but only the tension in the rope accelerates block B. The force F is constant, so the acceleration is constant, which means that the standard kinematics formulas apply. There is no friction.
EXECUTE: **(a)** First use kinematics to find the acceleration of the system. Using $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ with $x - x_0 = 18.0$ m, $v_{0x} = 0$, and $t = 5.00$ s, we get $a_x = 1.44\text{ m/s}^2$. Now apply Newton's second law to the horizontal motion of block A, which gives $F - T = m_A a$. $T = 60.0\text{ N} - (15.0\text{ kg})(1.44\text{ m/s}^2) = 38.4$ N.
(b) Apply Newton's second law to block B, giving $T = m_B a$. $m_B = T/a = (38.4\text{ N})/(1.44\text{ m/s}^2) = 26.7$ kg.
EVALUATE: As an alternative approach, consider the two blocks as a single system, which makes the tension an internal force. Newton's second law gives $F = (m_A + m_B)a$. Putting in numbers gives $60.0\text{ N} = (15.0\text{ kg} + m_B)(1.44\text{ m/s}^2)$, and solving for m_B gives 26.7 kg. Now apply Newton's second law to either block A or block B and find the tension.

4.39. **IDENTIFY** and **SET UP:** Take derivatives of $x(t)$ to find v_x and a_x. Use Newton's second law to relate the acceleration to the net force on the object.
EXECUTE:
(a) $x = (9.0\times10^3\text{ m/s}^2)t^2 - (8.0\times10^4\text{ m/s}^3)t^3$

$x = 0$ at $t = 0$

When $t = 0.025$ s, $x = (9.0\times10^3\text{ m/s}^2)(0.025\text{ s})^2 - (8.0\times10^4\text{ m/s}^3)(0.025\text{ s})^3 = 4.4$ m.

The length of the barrel must be 4.4 m.

(b) $v_x = \dfrac{dx}{dt} = (18.0\times10^3 \text{ m/s}^2)t - (24.0\times10^4 \text{ m/s}^3)t^2$

At $t = 0,\ v_x = 0$ (object starts from rest).

At $t = 0.025$ s, when the object reaches the end of the barrel,

$v_x = (18.0\times10^3 \text{ m/s}^2)(0.025 \text{ s}) - (24.0\times10^4 \text{ m/s}^3)(0.025 \text{ s})^2 = 300 \text{ m/s}$

(c) $\sum F_x = ma_x$, so must find a_x.

$a_x = \dfrac{dv_x}{dt} = 18.0\times10^3 \text{ m/s}^2 - (48.0\times10^4 \text{ m/s}^3)t$

(i) At $t = 0,\ a_x = 18.0\times10^3 \text{ m/s}^2$ and $\sum F_x = (1.50 \text{ kg})(18.0\times10^3 \text{ m/s}^2) = 2.7\times10^4$ N.

(ii) At $t = 0.025$ s, $a_x = 18\times10^3 \text{ m/s}^2 - (48.0\times10^4 \text{ m/s}^3)(0.025 \text{ s}) = 6.0\times10^3 \text{ m/s}^2$ and

$\sum F_x = (1.50 \text{ kg})(6.0\times10^3 \text{ m/s}^2) = 9.0\times10^3$ N.

EVALUATE: The acceleration and net force decrease as the object moves along the barrel.

4.40. **IDENTIFY:** This problem involves Newton's second law. The rocket's motion occurs in two stages: during the first stage, its engines produce a constant upward force, and during the second stage the engines turn off. Gravity acts during both stages.

SET UP: Apply $\sum F_y = ma_y$ during both stages. During the first stage, the engine force F acts upward and gravity mg acts downward. During the second stage, only gravity acts on the rocket. The constant-acceleration formulas apply during both stages, but with different acceleration in each stage. Fig. 4.40 shows the information. The rocket stops moving at its highest point.

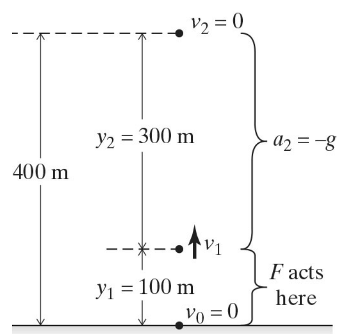


Figure 4.40

EXECUTE: Look at the stages one at a time.

First stage: Apply $\sum F_y = ma_y$. $T - mg = ma_1$ gives $a_1 = (F - mg)/m$. Now apply

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to relate v_1 to F, using the a_1 we just found. This gives $v_1^2 = 0 + 2\left(\dfrac{F - mg}{m}\right)y_1$.

Second stage: Apply $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. The v_{0y} for the second stage is v_1 from the first stage. Using the result for v_1^2 from the first stage, this equation gives $0 = 2\left(\dfrac{F - mg}{m}\right)y_1 - 2gy_2$.

Solving for F gives $F = \dfrac{mg}{y_1}(y_1 + y_2) = \dfrac{(400 \text{ kg})(9.80 \text{ m/s})}{100 \text{ m}}(400 \text{ m}) = 1.57\times10^4$ N.

EVALUATE: We cannot do this problem in a single step because the acceleration is different in the two stages. We cannot treat this as a single process using the average acceleration because the accelerations last for different times.

4.41. **IDENTIFY:** You observe that your weight is different from your normal weight in an elevator, so you must have acceleration. Apply $\sum \vec{F} = m\vec{a}$ to your body inside the elevator.

SET UP: The quantity $w = 683$ N is the force of gravity exerted on you, independent of your motion. Your mass is $m = w/g = 69.7$ kg. Use coordinates with $+y$ upward. Your free-body diagram is shown in Figure 4.41, where n is the scale reading, which is the force the scale exerts on you. You and the elevator have the same acceleration.

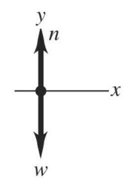


Figure 4.41

EXECUTE: $\sum F_y = ma_y$ gives $n - w = ma_y$ so $a_y = \dfrac{n-w}{m}$.

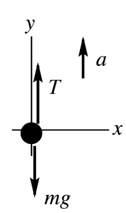
(a) $n = 725$ N, so $a_y = \dfrac{725\text{ N} - 683\text{ N}}{69.7\text{ kg}} = 0.603\text{ m/s}^2$. a_y is positive so the acceleration is upward.

(b) $n = 595$ N, so $a_y = \dfrac{595\text{ N} - 683\text{ N}}{69.7\text{ kg}} = -1.26\text{ m/s}^2$. a_y is negative so the acceleration is downward.

EVALUATE: If you appear to weigh less than your normal weight, you must be accelerating downward, but not necessarily *moving* downward. Likewise if you appear to weigh more than your normal weight, you must be acceleration upward, but you could be *moving* downward.

4.42. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the elevator to relate the forces on it to the acceleration.

(a) SET UP: The free-body diagram for the elevator is sketched in Figure 4.42.



The net force is $T - mg$ (upward).

Figure 4.42

Take the +y-direction to be upward since that is the direction of the acceleration. The maximum upward acceleration is obtained from the maximum possible tension in the cables.

EXECUTE: $\sum F_y = ma_y$ gives $T - mg = ma$

$$a = \frac{T - mg}{m} = \frac{28{,}000\text{ N} - (2200\text{ kg})(9.80\text{ m/s}^2)}{2200\text{ kg}} = 2.93\text{ m/s}^2.$$

(b) What changes is the weight mg of the elevator.

$$a = \frac{T - mg}{m} = \frac{28{,}000\text{ N} - (2200\text{ kg})(1.62\text{ m/s}^2)}{2200\text{ kg}} = 11.1\text{ m/s}^2.$$

EVALUATE: The cables can give the elevator a greater acceleration on the moon since the downward force of gravity is less there and the same T then gives a greater net force.

4.43. **IDENTIFY:** The ball changes velocity, so it has acceleration. Therefore Newton's second law applies to it.

SET UP: Apply $\sum F_x = ma_x$ to the ball. Assume that the acceleration is constant, so we can use the constant-acceleration equation $v_x = v_{0x} + a_x t$. Call the x-axis horizontal with $+x$ in the direction of the ball's original velocity.

EXECUTE: First use $v_x = v_{0x} + a_x t$ to find the acceleration.

$-50.0\text{ m/s} = 40.0\text{ m/s} + a_x(8.00\times10^{-3}\text{ s}) \quad\rightarrow\quad a_x = -1.125\times10^{4}\text{ m/s}^2$.

Now apply $\sum F_x = ma_x$ to the ball. Only the bat exerts a horizontal force on the ball.

$F_{\text{bat}} = ma_x = (0.145\text{ kg})(-1.125\times10^{4}\text{ m/s}^2) = -1630\text{ N}$. The minus sign tells us that the force and the acceleration are directed opposite to the original velocity of the ball, which is away from the batter.

EVALUATE: Compare the acceleration to g: $a/g = (1.125\times10^{4}\text{ m/s}^2)/(9.80\text{ m/s}^2) = 1150$, so the acceleration is $1150g$ – a huge acceleration but for only a very brief time.

4.44. **IDENTIFY:** The object has acceleration, so Newton's second law applies to it. We need to find the acceleration using the equation for its position as a function of time.

SET UP: First find the acceleration using $v_x = dx/dt$ and $a_x = dv_x/dt$. Then apply $\sum F_x = ma_x$. We know that $x(t) = \alpha t^2 - 2\beta t$.

EXECUTE: $v_x = d(\alpha t^2 - 2\beta t)/dt = 2\alpha t - 2\beta$, $a_x = d($a $2\alpha t - 2\beta)/dt = 2\alpha$. Now apply $\sum F_x = ma_x$, giving $F_{\text{net}} = m(2\alpha) = 2\alpha m$.

EVALUATE: Check units: αt^2 must have SI units of m, so α has units of m/s^2. Thus the units of $2\alpha m$ are $(\text{m/s}^2)(\text{kg}) = \text{kg}\cdot\text{m/s}^2 = \text{N}$, so our answer has the proper units.

4.45. **IDENTIFY:** The system is accelerating, so we apply Newton's second law to each box and can use the constant acceleration kinematics for formulas to find the acceleration.

SET UP: First use the constant acceleration kinematics for formulas to find the acceleration of the system. Then apply $\sum F = ma$ to each box.

EXECUTE: **(a)** The kinematics formula $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$a_y = \dfrac{2(y-y_0)}{t^2} = \dfrac{2(12.0\text{ m})}{(4.0\text{ s})^2} = 1.5\text{ m/s}^2$. For box B, $mg - T = ma$ and

$m = \dfrac{T}{g-a} = \dfrac{36.0\text{ N}}{9.8\text{ m/s}^2 - 1.5\text{ m/s}^2} = 4.34\text{ kg}$.

(b) For box A, $T + mg - F = ma$ and $m = \dfrac{F-T}{g-a} = \dfrac{80.0\text{ N} - 36.0\text{ N}}{9.8\text{ m/s}^2 - 1.5\text{ m/s}^2} = 5.30\text{ kg}$.

EVALUATE: The boxes have the same acceleration but experience different forces because they have different masses.

4.46. **IDENTIFY:** Note that in this problem the mass of the rope is given, and that it is not negligible compared to the other masses. Apply $\sum\vec{F} = m\vec{a}$ to each object to relate the forces to the acceleration.

(a) SET UP: The free-body diagrams for each block and for the rope are given in Figure 4.46a.

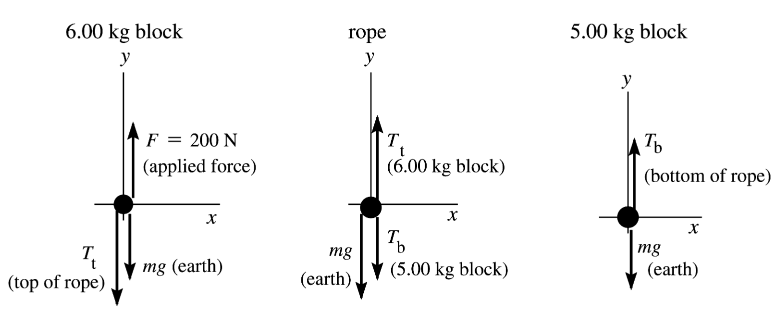
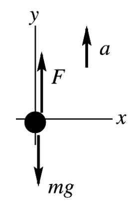


Figure 4.46a

T_{t} is the tension at the top of the rope and T_{b} is the tension at the bottom of the rope.

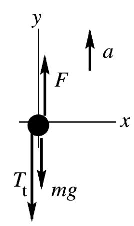
EXECUTE: (b) Treat the rope and the two blocks together as a single object, with mass $m = 6.00\text{ kg} + 4.00\text{ kg} + 5.00\text{ kg} = 15.0\text{ kg}$. Take $+y$ upward, since the acceleration is upward. The free-body diagram is given in Figure 4.46b.



$$\sum F_y = ma_y$$
$$F - mg = ma$$
$$a = \frac{F - mg}{m}$$
$$a = \frac{200\text{ N} - (15.0\text{ kg})(9.80\text{ m/s}^2)}{15.0\text{ kg}} = 3.53\text{ m/s}^2$$

Figure 4.46b

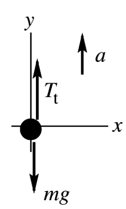
(c) Consider the forces on the top block $(m = 6.00\text{ kg})$, since the tension at the top of the rope (T_{t}) will be one of these forces.



$$\sum F_y = ma_y$$
$$F - mg - T_{\text{t}} = ma$$
$$T_{\text{t}} = F - m(g + a)$$
$$T_{\text{t}} = 200\text{ N} - (6.00\text{ kg})(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 120\text{ N}.$$

Figure 4.46c

Alternatively, you can consider the forces on the combined object rope plus bottom block $(m = 9.00\text{ kg})$:

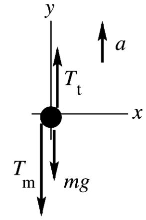


$$\sum F_y = ma_y$$
$$T_{\text{t}} - mg = ma$$
$$T_{\text{t}} = m(g + a) = 9.00\text{ kg}(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 120$$

which checks

Figure 4.46d

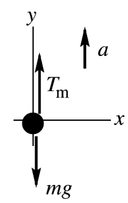
(d) One way to do this is to consider the forces on the top half of the rope $(m = 2.00\text{ kg})$. Let T_m be the tension at the midpoint of the rope.



$$\sum F_y = ma_y$$
$$T_{\text{t}} - T_{\text{m}} - mg = ma$$
$$T_{\text{m}} = T_{\text{t}} - m(g + a) = 120\text{ N} - 2.00\text{ kg}(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2)$$

Figure 4.46e

To check this answer we can alternatively consider the forces on the bottom half of the rope plus the lower block taken together as a combined object $(m = 2.00\text{ kg} + 5.00\text{ kg} = 7.00\text{ kg})$:



$\sum F_y = ma_y$

$T_m - mg = ma$

$T_m = m(g + a) = 7.00\text{ kg}(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 93.$

which checks

Figure 4.46f

Evaluate: The tension in the rope is not constant but increases from the bottom of the rope to the top. The tension at the top of the rope must accelerate the rope as well the 5.00-kg block. The tension at the top of the rope is less than F; there must be a net upward force on the 6.00-kg block.

4.47. **Identify:** The rocket engines reduce the speed of the rocket, so Newton's second law applies to the rocket. Two vertical forces act on the rocket: gravity and the force F of the engines. The constant-acceleration equations apply because both F and gravity are constant, which makes the acceleration constant.

Set Up: The rocket needs to reduce its speed from 30.0 m/s to zero while traveling 80.0 m. We can find the acceleration from this information using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. Then we can apply $\sum F_y = ma_y$ to find the force F. Choose the $+y$-axis upward with the origin at the ground.

Execute: We know that $v_y = 0$ as the rocket reaches the ground. Using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ we get $0 = (-30.0\text{ m/s})^2 + 2a_y(0 - 80.0\text{ m})$ $\rightarrow$ $a_y = 5.625\text{ m/s}^2$. Now apply $\sum F_y = ma_y$.

$F - mg = ma_y$ $\rightarrow$ $F - (20.0\text{ kg})(9.80\text{ m/s}^2) = (20.0\text{ kg})(5.625\text{ m/s}^2)$ $\rightarrow$ $F = 309\text{ N}$.

Evaluate: The weight of this rocket is $w = mg = (20.0\text{ kg})(9.80\text{ m/s}^2) = 196\text{ N}$, so we see that $F > w$. This is reasonable because the engine force must oppose gravity to reduce the rocket's speed. Notice that the rocket is moving downward but its acceleration is upward. This is reasonable because the rocket is slowing down, so its acceleration must be opposite to its velocity.

4.48. **Identify:** On the planet Newtonia, you make measurements on a tool by pushing on it and by dropping it. You want to use those results to find the weight of the object on that planet and on earth.

Set Up: Using $w = mg$, you could find the weight if you could calculate the mass of the tool and the acceleration due to gravity on Newtonia. Newton's laws of motion are applicable on Newtonia, as is your knowledge of falling objects. Let m be the mass of the tool. There is no appreciable friction. Use coordinates where $+x$ is horizontal, in the direction of the 12.0 N force, and let $+y$ be downward.

Execute: First find the mass m: $x - x_0 = 16.0\text{ m}$, $t = 2.00\text{ s}$, $v_{0x} = 0$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(16.0\text{ m})}{(2.00\text{ s})^2} = 8.00\text{ m/s}^2$. Now apply Newton's second law to the tool. $\sum F_x = ma_x$

gives $F = ma_x$ and $m = \dfrac{F}{a_x} = \dfrac{12.0\text{ N}}{8.00\text{ m/s}^2} = 1.50\text{ kg}$. Find g_N, the acceleration due to gravity on

Newtonia. $y - y_0 = 10.0\text{ m}$, $v_{0y} = 0$, $t = 2.58\text{ s}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$a_y = \dfrac{2(y - y_0)}{t^2} = \dfrac{2(10.0\text{ m})}{(2.58\text{ s})^2} = 3.00\text{ m/s}^2$; $g_N = 3.00\text{ m/s}^2$. The weight on Newtonia is

$w_N = mg_N = (1.50\text{ kg})(3.00\text{ m/s}^2) = 4.50\text{ N}$. The weight on earth is

$w_E = mg_E = (1.50\text{ kg})(9.80\text{ m/s}^2) = 14.7\text{ N}$.

EVALUATE: The tool weighs about 1/3 on Newtonia of what it weighs on earth since the acceleration due to gravity on Newtonia is about 1/3 what it is on earth.

4.49. IDENTIFY: The rocket accelerates due to a variable force, so we apply Newton's second law. But the acceleration will not be constant because the force is not constant.

SET UP: We can use $a_x = F_x/m$ to find the acceleration, but must integrate to find the velocity and then the distance the rocket travels.

EXECUTE: Using $a_x = F_x/m$ gives $a_x(t) = \dfrac{(16.8\ \text{N/s})t}{45.0\ \text{kg}} = (0.3733\ \text{m/s}^3)t$. Now integrate the acceleration to get the velocity, and then integrate the velocity to get the distance moved. $v_x(t) = v_{0x} + \int_0^t a_x(t')dt' = (0.1867\ \text{m/s}^3)t^2$ and $x - x_0 = \int_0^t v_x(t')dt' = (0.06222\ \text{m/s}^3)t^3$. At $t = 5.00$ s, $x - x_0 = 7.78$ m.

EVALUATE: The distance moved during the next 5.0 s would be considerably greater because the acceleration is increasing with time.

4.50. IDENTIFY: A constant force is applied to an object, which gives it a constant acceleration. We apply Newton's second law and can use the constant-acceleration equations.

SET UP: The object starts from rest reaches a speed v_1 due to force F_1 which gives it an acceleration $a_1 = F_1/m$. $\sum F_x = ma_x$ applies to the object.

EXECUTE: **(a)** With F_1 acting, the object travels a distance d. Now with $F_2 = 2F_1$ acting, the object still travels a distance d but reaches speed v_2. We want to find v_2 in terms of v_1. In this case, twice the force acts over the same distance d. We know that $a_1 = F_1/m$ and $a_2 = F_2/m = 2F_1/m = 2a_1$. Applying $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to both situations gives $v_1^2 = 2a_1 d$ and $v_2^2 = 2a_2 d$. Since $a_2 = 2a_1$, the last equation gives $v_2^2 = 2(2a_1)d = 2(2a_1 d)$, which tells us that $v_2^2 = 2v_1^2$, so $v_2 = \sqrt{2}v_1$.

(b) In this case, twice the force acts for the same amount of time T. As we saw above, $a_2 = 2a_1$. Applying $v_x = v_{0x} + a_x t$ to both situations gives $v_1 = a_1 T$ and $v_2 = a_2 T = (2a_1)T = 2(a_1 T) = 2v_2$, so we find that $v_2 = 2v_1$.

EVALUATE: We have seen in (a) that doubling the force over the same distance increases the speed by a factor of $\sqrt{2}$, but in (b) we saw that doubling the force for the same amount of time increases the speed by a factor of 2.

4.51. IDENTIFY: Kinematics will give us the average acceleration of each car, and Newton's second law will give us the average force that is accelerating each car.

SET UP: The cars start from rest and all reach a final velocity of 60 mph (26.8 m/s). We first use kinematics to find the average acceleration of each car, and then use Newton's second law to find the average force on each car.

EXECUTE: **(a)** We know the initial and final velocities of each car and the time during which this change in velocity occurs. The definition of average acceleration gives $a_{\text{av}} = \dfrac{\Delta v}{\Delta t}$. Then $F = ma$ gives the force on each car. For the Alpha Romeo, the calculations are $a_{\text{av}} = (26.8\ \text{m/s})/(4.4\ \text{s}) = 6.09\ \text{m/s}^2$. The force is $F = ma = (895\ \text{kg})(6.09\ \text{m/s}^2) = 5.451 \times 10^3\ \text{N} = 5.451$ kN, which we should round to 5.5 kN for 2 significant figures. Repeating this calculation for the other cars and rounding the force to 2 significant figures gives:

Alpha Romeo: $a = 6.09\ \text{m/s}^2$, $F = 5.5$ kN
Honda Civic: $a = 4.19\ \text{m/s}^2$, $F = 5.5$ kN
Ferrari: $a = 6.88\ \text{m/s}^2$, $F = 9.9$ kN
Ford Focus: $a = 4.97\ \text{m/s}^2$, $F = 7.3$ kN
Volvo: $a = 3.72\ \text{m/s}^2$, $F = 6.1$ kN

The smallest net force is on the Alpha Romeo and Honda Civic, to two-figure accuracy. The largest net force is on the Ferrari.
(b) The largest force would occur for the largest acceleration, which would be in the Ferrari. The smallest force would occur for the smallest acceleration, which would be in the Volvo.
(c) We use the same approach as in part (a), but now the final velocity is 100 mph (44.7 m/s).
a_{av} = (44.7 m/s)/(8.6 s) = 5.20 m/s^2, and $F = ma$ = (1435 kg)(5.20 m/s^2) = 7.5 kN. The average force is considerably smaller in this case. This is because air resistance increases with speed.
(d) As the speed increases, so does the air resistance. Eventually the air resistance will be equal to the force from the roadway, so the new force will be zero and the acceleration will also be zero, so the speed will remain constant.
EVALUATE: The actual forces and accelerations involved with auto dynamics can be quite complicated because the forces (and hence the accelerations) are not constant but depend on the speed of the car.

4.52. **IDENTIFY:** Calculate $\vec{a}$ from $\vec{a} = d^2\vec{r}/dt^2$. Then $\vec{F}_{\text{net}} = m\vec{a}$.
SET UP: $w = mg$
EXECUTE: Differentiating twice, the acceleration of the helicopter as a function of time is
$\vec{a} = (0.120\text{ m/s}^3)t\hat{\imath} - (0.12\text{ m/s}^2)\hat{k}$ and at $t = 5.0$ s, the acceleration is $\vec{a} = (0.60\text{ m/s}^2)\hat{\imath} - (0.12\text{ m/s}^2)\hat{k}$.
The force is then

$$\vec{F} = m\vec{a} = \frac{w}{g}\vec{a} = \frac{(2.75\times 10^5\text{ N})}{(9.80\text{ m/s}^2)}\left[(0.60\text{ m/s}^2)\hat{\imath} - (0.12\text{ m/s}^2)\hat{k}\right] = (1.7\times 10^4\text{ N})\hat{\imath} - (3.4\times 10^3\text{ N})\hat{k}$$

EVALUATE: The force and acceleration are in the same direction. They are both time dependent.

4.53. **IDENTIFY:** A block is accelerated upward by a force of magnitude F. For various forces, we know the time for the block to move upward a distance of 8.00 m starting from rest. Since the upward force is constant, so is the acceleration. Newton's second law applies to the accelerating block.
SET UP: The acceleration is constant, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applies, and $\sum F_y = ma_y$ also applies to the block.
EXECUTE: **(a)** Using the above formula with $v_{0y} = 0$ and $y - y_0 = 8.00$ m, we get $a_y = (16.0\text{ m})/t^2$. We use this formula to calculate the acceleration for each value of the force F. For example, when F= 250 N, we have a = (16.0 m)/(3.3 s)^2 = 1.47 m/s^2. We make similar calculations for all six values of F and then graph F versus a. We can do this graph by hand or using graphing software. The result is shown in Figure 4.53.

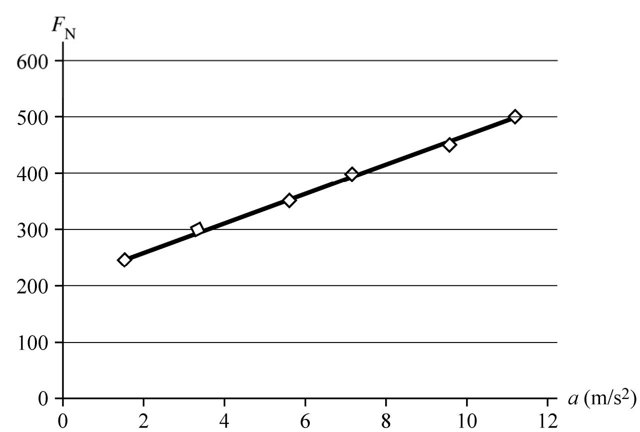


Figure 4.53

(b) Applying Newton's second law to the block gives $F - mg = ma$, so $F = mg + ma$. The equation of our best-fit graph in part (a) is $F = (25.58\text{ kg})a + 213.0\text{ N}$. The slope of the graph is the mass m, so the mass of the block is $m = 26$ kg. The y intercept is mg, so $mg = 213$ N, which gives $g = (213\text{ N})/(25.58\text{ kg}) = 8.3$ m/s^2 on the distant planet.

EVALUATE: The acceleration due to gravity on this planet is not too different from what it is on earth.

4.54. IDENTIFY: The box comes to a stop, so it must have acceleration, so Newton's second law applies. For constant acceleration, the standard kinematics formulas apply.

SET UP: For constant acceleration, $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and $v_x = v_{0x} + a_x t$ apply. For any motion, $\vec{F}_{\text{net}} = m\vec{a}$.

EXECUTE: **(a)** If the box comes to rest with constant acceleration, its final velocity is zero so $v_{0x} = -a_x t$. And if during this time it travels a distance $x - x_0 = d$, the distance formula above can be put into the form
$d = (-a_x t) + \frac{1}{2} a_x t^2 = -\frac{1}{2} a_x t^2$. This gives $a_x = -2d/t^2$. For the first push on the box, this gives $a_x = -2(8.22\text{ m})/(2.8\text{ s})^2 = -2.1\text{ m/s}^2$. If the acceleration is constant, the distance the box should travel after the second push is $d = -\frac{1}{2} a_x t^2 = -(\frac{1}{2})(-2.1\text{ m/s}^2)(2.0\text{ s})^2 = 4.2$ m, which is in fact the distance the box did travel. Therefore the acceleration was constant.

(b) The total mass m_T of the box is the initial mass (8.00 kg) plus the added mass. Since $v_x = 0$ and $a_x = 2d/t^2$ as shown in part (a), the magnitude of the initial speed v_{0x} is $v_{0x} = a_x t = (2d/t^2)t = 2d/t$. For no added mass, this calculation gives $v_{0x} = 2(8.22\text{ m})/(2.8\text{ s}) = 5.87$ m/s. Similar calculations with added mass give

$m_T = 8.00$ kg, $v_{0x} = 5.87\text{ m/s} \approx 5.9$ m/s
$m_T = 11.00$ kg, $v_{0x} = 6.72\text{ m/s} \approx 6.7$ m/s
$m_T = 15.00$ kg, $v_{0x} = 6.30\text{ m/s} \approx 6.3$ m/s
$m_T = 20.00$ kg, $v_{0x} = 5.46\text{ m/s} \approx 5.5$ m/s

where all answers have been rounded to 2 significant figures. It is obvious that the initial speed was *not* the same in each case. The ratio of maximum speed to minimum speed is
$v_{0,\max}/v_{0,\min} = (6.72\text{ m/s})/(5.46\text{ m/s}) = 1.2$

(c) We calculate the magnitude of the force f using $f = ma$, getting a using $a = -2d/t^2$, as we showed in part (a). In each case the acceleration is 2.1 m/s^2. So for example, when $m = 11.00$ kg, the force is $f = (11.00\text{ kg})(2.1\text{ m/s}^2) = 23$ N. Similar calculations produce a set of values for f and m. These can be graphed by hand or using graphing software. The resulting graph is shown in Figure 4.54. The slope of this straight-line graph is 2.1 m/s^2 and it passes through the origin, so the slope-y intercept equation of the line is $f = (2.1\text{ m/s}^2)m$.

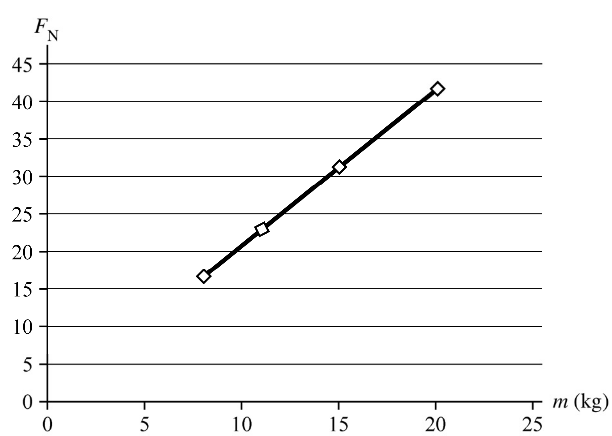


Figure 4.54

Evaluate: The results of the graph certainly agree with Newton's second law. A graph of F versus m should have slope equal to the acceleration a. This is in fact just what we get, since the acceleration is 2.1 m/s^2 which is the same as the slope of the graph.

4.55. **Identify:** The force on the block is a function of time, so the acceleration will also be a function of time. Therefore we cannot use the constant-acceleration formulas, but instead must use the basic velocity and acceleration definitions. We also need to apply Newton's second law.

Set Up: Using $\sum F_x = ma_x$ we can determine the acceleration and use it to find the velocity and position of the block. We use $a_x = dv_x/dt$ and $v_x = dx/dt$. We know that $F(t) = \beta - \alpha t$ and that the object starts from rest.

Execute: **(a)** We want the largest positive value of x. Using $\sum F_x = ma_x$ we can find $a_x(t)$, and from that we can find $v_x(t)$ and then find $x(t)$. First find a_x: $a_x(t) = F_x/m = (\beta - \alpha t)/m$.

Now use $a_x = dv_x/dt$ to find $v_x(t)$. $v_x = \int a_x dt = \int \dfrac{\beta - \alpha t}{m} dt = \dfrac{\beta t - \dfrac{\alpha t^2}{2}}{m}$, where we have used $v_x = 0$ when $t = 0$. Now use $v_x = dx/dt$ to find $x(t)$. $x(t) = \int v_x dt = \int \dfrac{\beta t - \dfrac{\alpha t^2}{2}}{m} dt = \dfrac{\dfrac{\beta t^2}{2} - \dfrac{\alpha t^3}{6}}{m}$, where we have used $x = 0$ when $t = 0$. The largest value of x will occur when $v_x = 0$ because the block will go no further.

Equating v_x to zero gives $v_x = \dfrac{\beta t - \dfrac{\alpha t^2}{2}}{m} = 0$. Solving for t and calling it $t_{\max}$ gives $t_{\max} = 2\beta/\alpha$.

Now use $t_{\max}$ in our equation for $x(t)$ to find the largest value of x. This gives

$$x(t_{\max}) = \frac{\dfrac{\beta(2\beta/\alpha)^2}{2} - \dfrac{\alpha(2\beta/\alpha)^3}{6}}{m} = \frac{2\beta^3}{3m\alpha^2}.$$

The force at $t_{\max}$ is $F_x = \beta - \alpha t_{\max} = \beta - \alpha\left(\dfrac{2\beta}{\alpha}\right) = -\beta$.

Putting in the values for α and β gives the following results:

$t_{\max} = 2\beta/\alpha = 2(4.00\text{ N})/(6.00\text{ N/s}) = 1.33$ s.

$x_{\max} = \dfrac{2\beta^3}{3m\alpha^2} = \dfrac{2(4.00\text{ N})^3}{3(2.00\text{ kg})(6.00\text{ N/s})^2} = 0.593$ m.

$F_{\max} = -\beta = -4.00$ N, but we only want the magnitude, so $F_{\max} = 4.00$ N.

(b) When $x = 0$, the block has returned to where it started. Using our equation for $x(t)$, we get

$x(t) = \dfrac{\dfrac{\beta t^2}{2} - \dfrac{\alpha t^3}{6}}{m} = 0$, which gives $t = 3\beta/\alpha = 3(4.00\text{ N})/(6.00\text{ N/s}) = 2.00$ s.

Using our equation for $v_x(t)$ and evaluating it at $t = 2.00$ s gives

$v_x = \dfrac{\beta t - \dfrac{\alpha t^2}{2}}{m} = \dfrac{(4.00\text{ N})(2.00\text{ s}) - \dfrac{(6.00\text{ N/s})(2.00\text{ s})^2}{2}}{(2.00\text{ kg})} = -2.00$ m/s. Its speed is 2.00 m/s.

Evaluate: The constant-acceleration equations would be of no help with this type of time-dependent force.

4.56. **Identify:** $x = \int_0^t v_x dt$ and $v_x = \int_0^t a_x dt$, and similar equations apply to the y-component.

Set Up: In this situation, the x-component of force depends explicitly on the y-component of position. As the y-component of force is given as an explicit function of time, v_y and y can be found as functions of time and used in the expression for $a_x(t)$.

EXECUTE: $a_y = (k_3/m)t$, so $v_y = (k_3/2m)t^2$ and $y = (k_3/6m)t^3$, where the initial conditions $v_{0y} = 0, y_0 = 0$ have been used. Then, the expressions for a_x, v_x, and x are obtained as functions of time:

$$a_x = \frac{k_1}{m} + \frac{k_2 k_3}{6m^2}t^3, \quad v_x = \frac{k_1}{m}t + \frac{k_2 k_3}{24m^2}t^4 \text{ and } x = \frac{k_1}{2m}t^2 + \frac{k_2 k_3}{120m^2}t^5.$$

In vector form, $\vec{r} = \left(\frac{k_1}{2m}t^2 + \frac{k_2 k_3}{120m^2}t^5\right)\hat{\boldsymbol{i}} + \left(\frac{k_3}{6m}t^3\right)\hat{\boldsymbol{j}}$ and $\vec{v} = \left(\frac{k_1}{m}t + \frac{k_2 k_3}{24m^2}t^4\right)\hat{\boldsymbol{i}} + \left(\frac{k_3}{2m}t^2\right)\hat{\boldsymbol{j}}$.

EVALUATE: a_x depends on time because it depends on y, and y is a function of time.

4.57. **IDENTIFY:** Newton's second law applies to the dancer's head.

SET UP: We use $a_{\text{av}} = \frac{\Delta v}{\Delta t}$ and $\vec{F}_{\text{net}} = m\vec{a}$.

EXECUTE: First find the average acceleration: $a_{\text{av}} = (4.0 \text{ m/s})/(0.20 \text{ s}) = 20 \text{ m/s}^2$. Now apply Newton's second law to the dancer's head. Two vertical forces act on the head: $F_{\text{neck}} - mg = ma$, so $F_{\text{neck}} = m(g + a)$, which gives $F_{\text{neck}} = (0.094)(65 \text{ kg})(9.80 \text{ m/s}^2 + 20 \text{ m/s}^2) = 180 \text{ N}$, which is choice (d).

EVALUATE: The neck force is not simply ma because the neck must balance her head against gravity, even if the head were not accelerating. That error would lead one to incorrectly select choice (c).

4.58. **IDENTIFY:** Newton's third law of motion applies.

SET UP: The force the neck exerts on her head is the same as the force the head exerts on the neck.

EXECUTE: Choice (a) is correct.

EVALUATE: These two forces form an action-reaction pair.

4.59. **IDENTIFY:** The dancer is in the air and holding a pose, so she is in free fall.

SET UP: The dancer, including all parts of her body, are in free fall, so they all have the same downward acceleration of 9.80 m/s^2.

EXECUTE: Since her head and her neck have the same downward acceleration, and that is produced by gravity, her neck does not exert any force on her head, so choice (a) 0 N is correct.

EVALUATE: During falling motion such as this, a person (including her head) is often described as being "weightless."

4.60. **IDENTIFY:** The graph shows the vertical force that a force plate exerts on her body.

SET UP and **EXECUTE:** When the dancer is not moving, the force that the force plate exerts on her will be her weight, which appears to be about 650 N. Between 0.0 s and 0.4 s, the force on her is less than her weight and is decreasing, so she must be accelerating downward. At 0.4 s, the graph reaches a relative minimum of around 300 N and then begins to increase after that. Only choice (a) is consistent with this part of the graph.

EVALUATE: At the high points in the graph, the force on her is over twice her weight.

5

APPLYING NEWTON'S LAWS

VP5.5.1. **IDENTIFY:** The cart and bucket move with constant speed, so their acceleration is zero, which means that the forces on each of them must balance.

SET UP: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the cart and the bucket. For the cart, take the +x-axis parallel to the surface of the incline pointing upward. For the bucket, take the +y-axis vertically upward.

EXECUTE: **(a)** Isolate the bucket and apply $\sum F_y = 0$. $T - w = 0 \quad \rightarrow \quad T = w = 255$ N.

Now apply $\sum F_x = 0$ to the cart. $T - w \sin 36.9° = 0$. $255 \text{ N} - w \sin 36.9° = 0$. $w = 425$ N.

(b) From above, $T = 255$ N.

EVALUATE: The bucket can balance a much heavier cart because it only needs to balance the component of the cart's weight that is parallel to the surface of the incline.

VP5.5.2. **IDENTIFY:** The cart and bucket are at rest, which means that the forces on each of them must balance.

SET UP: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the cart and the bucket. For the cart, take the +x-axis parallel to the surface of the incline pointing upward. For the bucket, take the +y-axis vertically upward.

EXECUTE: **(a)** Apply $\sum F_x = 0$ to the cart, giving $T - w_C \sin 25.0° = 0$.

$155 \text{ N} - w_C \sin 25.0°$, so $w_C = 367$ N.

(b) Isolate the bucket and apply $\sum F_y = 0$. $T - w_B = 0$, so $w_B = T = 367$ N.

The total weight is 367 N + 155 N = 522 N.

EVALUATE: A light bucket can balance a heavy cart because it must balance only the weight component of the cart that is parallel to the surface of the incline.

VP5.5.3. **IDENTIFY:** The cart and bucket move with constant speed, so their acceleration is zero, which means that the forces on each of them must balance.

SET UP: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the cart and the bucket. For the cart, take the +x-axis parallel to the surface of the incline pointing upward. For the bucket, take the +y-axis vertically upward. We want to find the angle θ of the slope. Call T the tension in the cable.

EXECUTE: **(a)** Applying $\sum F_y = 0$ to the bucket gives $T = w_B$. Applying $\sum F_x = 0$ to the cart gives $T = w_C \sin \theta$. Equating the two expressions for T gives $w_B = w_C \sin \theta$, which tells us

$\sin\theta = \dfrac{w_B}{w_C} = \dfrac{m_B g}{m_C g} = \dfrac{65.0 \text{ kg}}{175 \text{ kg}} = 0.371$, so $\theta = 21.8°$.

(b) From part (a), $T = w_B = (65.0 \text{ kg})(9.80 \text{ m/s}^2) = 637$ N.

EVALUATE: It must also be true that $T = w_C \sin\theta$. $T = (175 \text{ kg})(9.80 \text{ m/s}^2) \sin 21.8° = 637$ N, which agrees with our answer in (b).

VP5.5.4. **IDENTIFY:** The cart and bucket remain at rest, so the forces on each of them must balance.

SET UP: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the cart and the bucket. For the cart, take the +*x*-axis parallel to the surface of the incline pointing upward. For the bucket, take the +*y*-axis vertically upward. We want to find the angle θ of the slope. Call *T* the tension in the cable and *f* the friction force. If there were no friction, the cart would slide *up* the incline because $w > w \sin\theta$. Since friction opposes motion, it must act *down* the incline.

EXECUTE: Applying $\sum F_y = 0$ to the bucket gives $T = w_B = w$. Applying $\sum F_x = 0$ to the cart gives $T - w_C \sin\theta - f = 0$, which becomes $T - w_C \sin\theta - f = 0$. Combining the results gives $f = w(1 - \sin\theta)$. Since $\sin\theta < 1$, we know that $f < w$.

EVALUATE: The weight of the bucket must now balance *two* forces: the weight of the cart acting down the incline and the friction force down the incline.

VP5.15.1. **IDENTIFY:** The crate moves at constant velocity, so the forces on it must balance.

SET UP: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the crate. Take the *x*-axis horizontal and the *y*-axis vertical. Make a free-body diagram as in Fig. VP5.15.1.

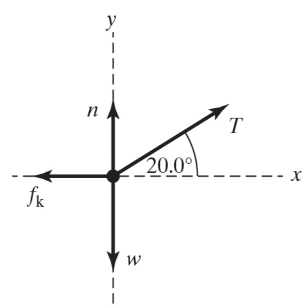


Figure VP5.15.1

EXECUTE: **(a)** Using the notation in Fig. VP5.15.1, $\sum F_x = 0$ gives $T \cos 20.0° - f_k = 0$ and $\sum F_y = 0$ gives $T \sin 20.0° + n - w = 0$, so $n = w - T \sin 20.0°$. For sliding friction $f_k = \mu_k n$. Combining these results gives $T \cos 20.0° - \mu_k (w - T \sin 20.0°) = 0$. Putting in the numbers:

$T \cos 20.0° - (0.250)(325\text{ N}) + (0.250)T \sin 20.0° = 0 \quad \rightarrow \quad T = 79.3\text{ N}$.

(b) $n = w - T \sin 20.0° = 325\text{ N} - (79.3\text{ N}) \sin 20.0° = 298\text{ N}$.

EVALUATE: We find that $n < w$. This is reasonable because the upward component of the tension balances part of the weight of the crate.

VP5.15.2. **IDENTIFY:** The crate moves at constant velocity, so the forces on it must balance.

SET UP: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the crate. Take the *x*-axis horizontal and the *y*-axis vertical. Make a free-body diagram as in Fig. 5.15.2.

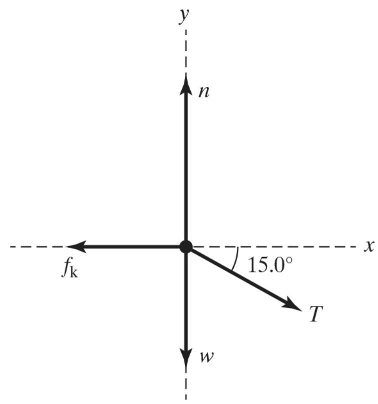


Figure VP5.15.2

Execute: The procedure is exactly the same as for VP5.15.1 *except* that the tension is now directed at 15.0° *below* the horizontal.

(a) $\sum F_x = 0$ gives $n - w - T\sin 15.0° = 0$, so $n = w + T\sin 15.0°$.

$\sum F_y = 0$ gives $T\cos 15.0° - f_k = T\cos 15.0° - \mu_k n = 0$.

Combining these equations and solving for T gives $T = 90.2$ N.

(b) $n = w + T\sin 15.0° = 325\text{ N} + (90.2\text{ N})\sin 15.0° = 348$ N.

Evaluate: The normal force must balance the downward component of the tension in addition to the weight of the crate, so it is greater than the weight.

VP5.15.3. **Identify:** The sled is accelerated horizontally, so Newton's second law applies to it.

Set Up: $\sum F_x = ma_x$ applies to the horizontal motion and $\sum F_y = 0$ applies to the vertical motion. Use $\sum F_y = 0$ to find the normal force n. The kinetic friction force is $f_k = \mu_k n$. Call P the magnitude of the pull and w the weight of the sled. Take the $+x$-axis to be horizontal in the direction of the horizontal component of the pull.

Execute: **(a)** $\sum F_y = 0$: $T\sin 12.0° + n - w = 0$.

$n = w - T\sin 12.0° = 475\text{ N} - (125\text{ N})\sin 12.0° = 449$ N.

(b) $\sum F_x = ma_x$: $P\cos 12.0° - f_k = ma_x = P\cos 12.0° - \mu_k n$

$(125\text{ N})\cos 12.0° - (0.200)(449\text{ N} = [(475\text{ N})/(9.80\text{ m/s}^2)]\, a_x \rightarrow a_x = 0.670\text{ m/s}^2$. We have chosen the $+x$-axis in the same direction as the horizontal component of the pull. Since a_x is positive, the acceleration is in the same direction as that pull, so the sled is *speeding up*.

Evaluate: If the sled were slowing down, that would mean that friction was greater than the horizontal component of the pull. In that case, the sled could never have started moving in the first place, so our answer is reasonable.

VP5.15.4. **Identify:** Before it slides, the forces on the box must balance.

Set Up: For the minimum pull, the box is just ready to slide, so static friction is at its maximum, which is $f_s = \mu_s n$. Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the box just as it is ready to slide.

Execute: $\sum F_x = 0$: $T_{min}\cos\theta - f_s = 0 \rightarrow T_{min}\cos\theta - \mu_s n = 0$.

$\sum F_y = 0$: $T_{min}\sin\theta + n - mg = 0 \rightarrow n = mg - T_{min}\sin\theta$.

Combine these results and solve for μ_s.

$$T_{min}\cos\theta - \mu_s(mg - T_{min}\sin\theta) = 0 \rightarrow \mu_s = \frac{T_{min}\cos\theta}{mg - T_{min}\sin\theta}.$$

Evaluate: If the box is *not* just ready to slide, our analysis is not valid. The forces still balance, but $f_s \neq \mu_s n$ in that case.

VP5.22.1. **IDENTIFY:** The pendulum bob is moving in a horizontal circle at constant speed. Therefore it has horizontal acceleration toward the center of the circle, but it has no vertical acceleration. The vertical forces on it must balance, but we need to use Newton's second law for the horizontal motion.

SET UP: Vertically $\sum F_y = 0$ and horizontally $\sum F_x = ma_x$, where a_x is the radial acceleration $a_{rad} = v^2/R$. Therefore horizontally we use $\sum F = m\frac{v^2}{R}$. The speed is $v = 2\pi R/t$, where t is the time for one cycle (do not use T for the period to avoid confusion with the tension T). Make a free-body diagram like Fig. 5.32b in the text.

EXECUTE: **(a)** $R = L \sin\beta = (0.800\text{ m}) \sin 20.0° = 0.274\text{ m}$.

(b) $\sum F = m\frac{v^2}{R}$ gives $T\sin\beta = m\frac{v^2}{R}$. Using $v = 2\pi R/t$, this becomes $T\sin\beta = \frac{m}{R}\left(\frac{2\pi R}{t}\right)^2$. Solving for t we get $t = \sqrt{\frac{4\pi^2 Rm}{T\sin\beta}}$, so we need to find T.

(c) $\sum F_y = 0$ gives $T\cos\beta = W = mg \rightarrow T = (0.250\text{ kg})(9.80\text{ m/s}^2)/(\cos 20.0°) = 2.61\text{ N}$. Now return to part (b) to find the time $t = \sqrt{\frac{4\pi^2 Rm}{T\sin\beta}}$. Putting in the numbers gives

Ita $t = 2\pi\sqrt{\frac{(0.274\text{ m})(0.250\text{ kg})}{(2.61\text{ N})(\sin 20.0°)}} = 1.74\text{ s}$.

EVALUATE: Even though the bob has constant speed, the horizontal forces do not balance because its *velocity* is changing direction, so it has acceleration.

VP5.22.2. **IDENTIFY:** The cyclist is moving in a horizontal circle at constant speed. Therefore she has horizontal acceleration toward the center of the circle, but no vertical acceleration. The vertical forces on her must balance, but we need to use Newton's second law for her horizontal motion.

SET UP: Vertically $\sum F_y = 0$ and horizontally $\sum F_x = ma_x$, where a_x is the radial acceleration $a_{rad} = v^2/R$. Therefore horizontally we use $\sum F = m\frac{v^2}{R}$. Make a free-body diagram as shown in Fig. VP5.22.2.

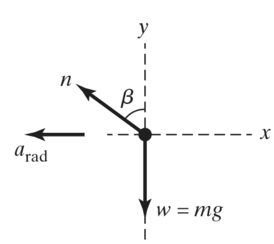


Figure VP5.22.2

EXECUTE: **(a)** $\sum F_x = ma_x = n\sin\beta = mv^2/R$

$\sum F_y = 0 = n\cos\beta - mg$.

Combining these two equations gives $\tan\beta = v^2/Rg$, which gives

$R = v^2/(g\tan\beta) = (12.5\text{ m/s})^2/[(9.80\text{ m/s}^2)(\tan 40.0°)] = 19.0\text{ m}$.

(b) $a_{rad} = v^2/R = (12.5\text{ m/s})^2/(19.0\text{ m}) = 8.22\text{ m/s}^2$.

(c) Using $n\cos\beta - mg = 0$, we get $n = mg/\cos\beta = (64.0\text{ kg})(9.80\text{ m/s}^2)/(\cos 40.0°) = 819\text{ N}$.

EVALUATE: We found that $n > w$. This is reasonable since only the vertical component of n balances her weight, which means that n has to be greater then w.

VP5.22.3. **IDENTIFY:** The plane is moving in a horizontal circle at constant speed. Therefore it has horizontal acceleration toward the center of the circle, but no vertical acceleration. The vertical forces on it must balance, but we need to use Newton's second law for its horizontal motion.

SET UP: Vertically $\sum F_y = 0$ and horizontally $\sum F_x = ma_x$, where a_x is the radial acceleration $a_{rad} = v^2/R$. Therefore horizontally we use $\sum F = m\dfrac{v^2}{R}$. Make a free-body diagram like Fig. 5.35 in the text.

EXECUTE: **(a)** $\sum F_x = ma_x = n \sin\beta = mv^2/R$

$\sum F_y = 0 = n\cos\beta - mg$.

Combining these two equations gives $\tan\beta = v^2/Rg = (80.0\text{ m/s})^2/[(175\text{ m})(9.80\text{ m/s}^2)]$, which gives $\beta = 75.0°$.

(b) The pilot's apparent weight will be the force n due to the seat. Using $\sum F_y = 0$ gives $w = n\cos\beta$, so $n = w/\cos\beta = (80.0\text{ kg})(9.80\text{ m/s}^2)/(\cos 75.0°) = 3.03\times10^3$ N.

$w_{apparent}/w_{actual} = (3.03\times10^3\text{ N})/[(80.0\text{ kg})(9.80\text{ m/s}^2)] = 3.86$, which means that is apparent weight is 3.86 times great than his actual weight.

EVALUATE: Notice that $\tan\beta \propto v^2$, so a large speed means a large bank angle. We also saw that $n = w/\cos\beta$, so as v gets larger and larger, β gets closer and closer to 90°, and $\cos\beta$ gets closer and closer to zero. Therefore n gets larger and larger. This can be dangerous for pilots in high speed turns. The effects from such turns can cause a pilot to black out if the speed is great enough.

VP5.22.4. **IDENTIFY:** The driver is moving in a horizontal circle at constant speed. Therefore she has horizontal acceleration toward the center of the circle, but no vertical acceleration. The vertical forces on her must balance, but we need to use Newton's second law for her horizontal motion.

SET UP: Vertically $\sum F_y = 0$ and horizontally $\sum F_x = ma_x$, where a_x is the radial acceleration $a_{rad} = v^2/R$. Therefore horizontally we use $\sum F = m\dfrac{v^2}{R}$. Make a free-body diagram like Fig. 5.34b in the textbook. Her apparent weight x times her actual weight, which means that the normal force n on her due to the seat is $n = xmg$.

EXECUTE: **(a)** Using $\sum F = m\dfrac{v^2}{R}$, we see that the net force on her is $F_{net} = ma_x = mv^2/R$. From the free-body diagram, we see that $F_{rad} = n\sin\beta$. So $F_{net} = F_{rad} = n\sin\beta = xmg\sin\beta$. Use $\sum F_y = 0$ to find β: $n\cos\beta - mg = 0 \quad\rightarrow\quad \cos\beta = mg/n = mg/xmg = 1/x$. This means that $\sin\beta = \dfrac{\sqrt{x^2-1}}{x}$.

We showed that $F_{net} = xmg\sin\beta$, so $F_{net} = xmg\dfrac{\sqrt{x^2-1}}{x} = mg\sqrt{x^2-1}$.

(b) $F_{rad} = F_{net} = mv^2/R$. Use the result from (a) for F_{net}.

$mg\sqrt{x^2-1} = mv^2/R \quad\rightarrow\quad R = \dfrac{v^2}{g\sqrt{x^2-1}}$.

EVALUATE: Our result in (b) says that as x gets larger and larger, R gets smaller and smaller. This is reasonable, since the larger x is, the greater the apparent weight of the driver. So for a given speed, a sharper turn will produce a greater apparent weight than a wide turn.

5.1. **IDENTIFY:** $a = 0$ for each object. Apply $\Sigma F_y = ma_y$ to each weight and to the pulley.

SET UP: Take $+y$ upward. The pulley has negligible mass. Let T_r be the tension in the rope and let T_c be the tension in the chain.

EXECUTE: **(a)** The free-body diagram for each weight is the same and is given in Figure 5.1a. $\Sigma F_y = ma_y$ gives $T_r = w = 25.0$ N.

(b) The free-body diagram for the pulley is given in Figure 5.1b. $T_c = 2T_r = 50.0$ N.

EVALUATE: The tension is the same at all points along the rope.

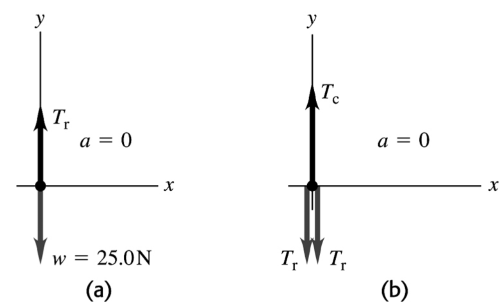


Figure 5.1

5.2. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each weight.

SET UP: Two forces act on each mass: w down and $T(= w)$ up.

EXECUTE: In all cases, each string is supporting a weight w against gravity, and the tension in each string is w.

EVALUATE: The tension is the same in all three cases.

5.3. **IDENTIFY:** Both objects are at rest and $a = 0$. Apply Newton's first law to the appropriate object. The maximum tension T_{max} is at the top of the chain and the minimum tension is at the bottom of the chain.

SET UP: Let $+y$ be upward. For the maximum tension take the object to be the chain plus the ball. For the minimum tension take the object to be the ball. For the tension T three-fourths of the way up from the bottom of the chain, take the chain below this point plus the ball to be the object. The free-body diagrams in each of these three cases are sketched in Figure 5.3. $m_{\text{b+c}} = 75.0\text{ kg} + 26.0\text{ kg} = 101.0\text{ kg}$. $m_{\text{b}} = 75.0$ kg. m is the mass of three-fourths of the chain: $m = \frac{3}{4}(26.0\text{ kg}) = 19.5\text{ kg}$.

EXECUTE: **(a)** From Figure 5.3a, $\Sigma F_y = 0$ gives $T_{\text{max}} - m_{\text{b+c}}g = 0$ and $T_{\text{max}} = (101.0\text{ kg})(9.80\text{ m/s}^2) = 990$ N. From Figure 5.3b, $\Sigma F_y = 0$ gives $T_{\text{min}} - m_{\text{b}}g = 0$ and $T_{\text{min}} = (75.0\text{ kg})(9.80\text{ m/s}^2) = 735$ N.

(b) From Figure 5.3c, $\Sigma F_y = 0$ gives $T - (m + m_{\text{b}})g = 0$ and $T = (19.5\text{ kg} + 75.0\text{ kg})(9.80\text{ m/s}^2) = 926$ N.

EVALUATE: The tension in the chain increases linearly from the bottom to the top of the chain.

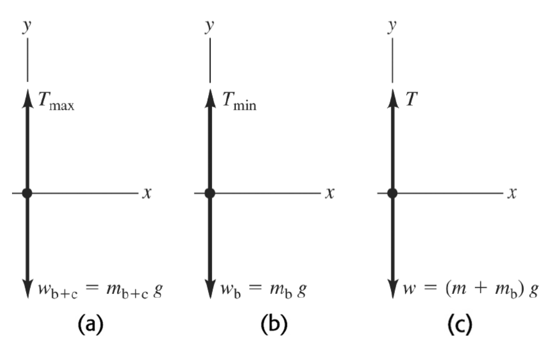
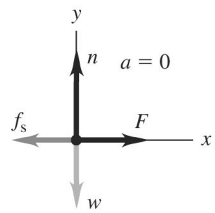


Figure 5.3

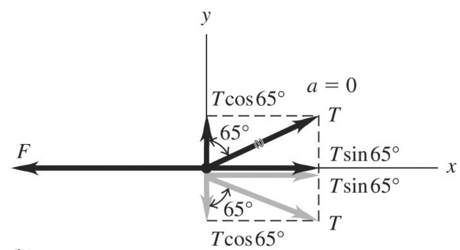
5.4. **IDENTIFY:** For the maximum tension, the patient is just ready to slide so static friction is at its maximum and the forces on him add to zero.

SET UP: **(a)** The free-body diagram for the person is given in Figure 5.4a. F is magnitude of the traction force along the spinal column and $w = mg$ is the person's weight. At maximum static friction, $f_s = \mu_s n$.

(b) The free-body diagram for the collar where the cables are attached is given in Figure 5.4b. The tension in each cable has been resolved into its x- and y-components.



(a)



(b)

Figure 5.4

EXECUTE: **(a)** $n = w$ and $F = f_s = \mu_s n = 0.75w = 0.75(9.80\text{ m/s}^2)(78.5\text{ kg}) = 577\text{ N}$.

(b) $2T\sin 65° - F = 0$ so $T = \dfrac{F}{2\sin 65°} = \dfrac{0.75w}{2\sin 65°} = 0.41w = (0.41)(9.80\text{ m/s}^2)(78.5\text{ kg}) = 315\text{ N}$.

EVALUATE: The two tensions add up to 630 N, which is more than the traction force, because the cables do not pull directly along the spinal column.

5.5. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the frame.

SET UP: Let w be the weight of the frame. Since the two wires make the same angle with the vertical, the tension is the same in each wire. $T = 0.75w$.

EXECUTE: The vertical component of the force due to the tension in each wire must be half of the weight, and this in turn is the tension multiplied by the cosine of the angle each wire makes with the vertical. $\dfrac{w}{2} = \dfrac{3w}{4}\cos\theta$ and $\theta = \arccos\frac{2}{3} = 48°$.

EVALUATE: If $\theta = 0°$, $T = w/2$ and $T \to \infty$ as $\theta \to 90°$. Therefore, there must be an angle where $T = 3w/4$.

5.6. **IDENTIFY:** Apply Newton's first law to the wrecking ball. Each cable exerts a force on the ball, directed along the cable.

SET UP: The force diagram for the wrecking ball is sketched in Figure 5.6.

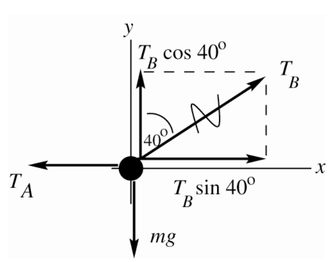


Figure 5.6

EXECUTE: **(a)** $\Sigma F_y = ma_y$

$T_B\cos 40° - mg = 0$

$$T_B = \frac{mg}{\cos 40°} = \frac{(3620\text{ kg})(9.80\text{ m/s}^2)}{\cos 40°} = 4.63\times 10^4\text{ N} = 46.3\text{ kN}$$

(b) $\Sigma F_x = ma_x$

$T_B \sin 40° - T_A = 0$

$T_A = T_B \sin 40° = 2.98 \times 10^4 \text{ N} = 29.8 \text{ kN}$

EVALUATE: If the angle 40° is replaced by 0° (cable B is vertical), then $T_B = mg$ and $T_A = 0$.

5.7. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the object and to the knot where the cords are joined.

SET UP: Let $+y$ be upward and $+x$ be to the right.

EXECUTE: **(a)** $T_C = w$, $T_A \sin 30° + T_B \sin 45° = T_C = w$, and $T_A \cos 30° - T_B \cos 45° = 0$. Since $\sin 45° = \cos 45°$, adding the last two equations gives $T_A(\cos 30° + \sin 30°) = w$, and so

$T_A = \dfrac{w}{1.366} = 0.732w$. Then, $T_B = T_A \dfrac{\cos 30°}{\cos 45°} = 0.897w$.

(b) Similar to part (a), $T_C = w$, $-T_A \cos 60° + T_B \sin 45° = w$, and $T_A \sin 60° - T_B \cos 45° = 0$.

Adding these two equations, $T_A = \dfrac{w}{(\sin 60° - \cos 60°)} = 2.73w$, and $T_B = T_A \dfrac{\sin 60°}{\cos 45°} = 3.35w$.

EVALUATE: In part (a), $T_A + T_B > w$ since only the vertical components of T_A and T_B hold the object against gravity. In part (b), since T_A has a downward component T_B is greater than w.

5.8. **IDENTIFY:** Apply Newton's first law to the hanging weight and to each knot. The tension force at each end of a string is the same.

(a) Let the tensions in the three strings be T, T', and T'', as shown in Figure 5.8a.

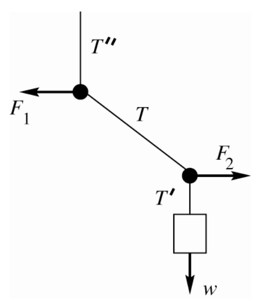
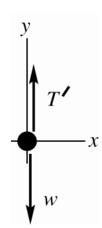


Figure 5.8a

SET UP: The free-body diagram for the block is given in Figure 5.8b.



EXECUTE:

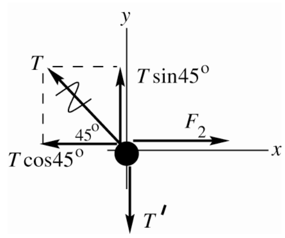
$\Sigma F_y = 0$

$T' - w = 0$

$T' = w = 60.0 \text{ N}$

Figure 5.8b

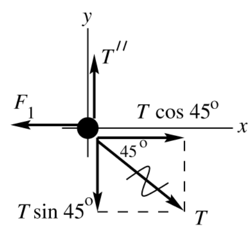
SET UP: The free-body diagram for the lower knot is given in Figure 5.8c.



EXECUTE:
$\Sigma F_y = 0$
$T\sin 45° - T' = 0$
$T = \dfrac{T'}{\sin 45°} = \dfrac{60.0\text{ N}}{\sin 45°} = 84.9\text{ N}$

Figure 5.8c

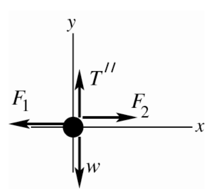
(b) Apply $\Sigma F_x = 0$ to the force diagram for the lower knot:
$\Sigma F_x = 0$
$F_2 = T\cos 45° = (84.9\text{ N})\cos 45° = 60.0\text{ N}$
SET UP: The free-body diagram for the upper knot is given in Figure 5.8d.



EXECUTE:
$\Sigma F_x = 0$
$T\cos 45° - F_1 = 0$
$F_1 = (84.9\text{ N})\cos 45°$
$F_1 = 60.0\text{ N}$

Figure 5.8d

Note that $F_1 = F_2$.
EVALUATE: Applying $\Sigma F_y = 0$ to the upper knot gives $T'' = T\sin 45° = 60.0\text{ N} = w$. If we treat the whole system as a single object, the force diagram is given in Figure 5.8e.



$\Sigma F_x = 0$ **gives** $F_2 = F_1$, **which checks**
$\Sigma F_y = 0$ **gives** $T'' = w$, **which checks**

Figure 5.8e

5.9. **IDENTIFY:** Since the velocity is constant, apply Newton's first law to the piano. The push applied by the man must oppose the component of gravity down the incline.
SET UP: The free-body diagrams for the two cases are shown in Figure 5.9. $\vec{F}$ is the force applied by the man. Use the coordinates shown in the figure.
EXECUTE: **(a)** $\Sigma F_x = 0$ gives $F - w\sin 19.0° = 0$ and $F = (180\text{ kg})(9.80\text{ m/s}^2)\sin 19.0° = 574\text{ N}$.
(b) $\Sigma F_y = 0$ gives $n\cos 19.0° - w = 0$ and $n = \dfrac{w}{\cos 19.0°}$. $\Sigma F_x = 0$ gives $F - n\sin 19.0° = 0$ and $F = \left(\dfrac{w}{\cos 19.0°}\right)\sin 19.0° = w\tan 19.0° = 607\text{ N}$.

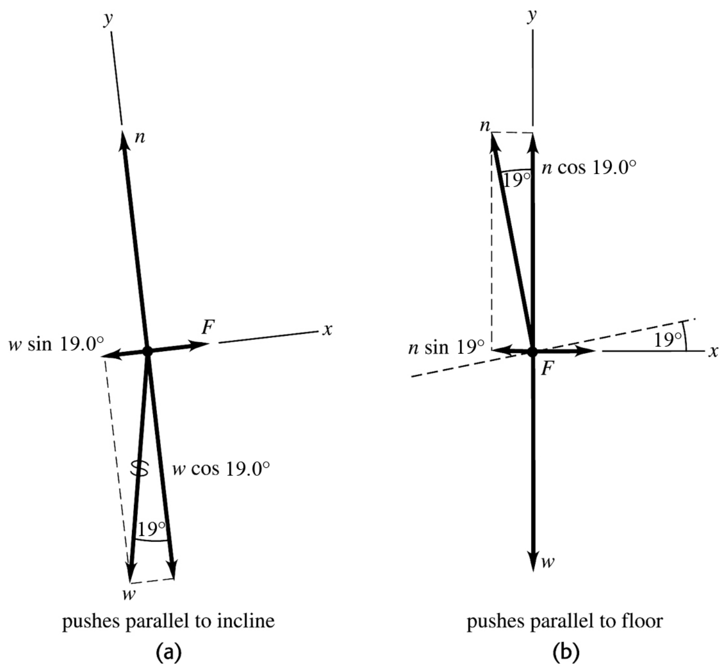


Figure 5.9

EVALUATE: When pushing parallel to the floor only part of the push is up the ramp to balance the weight of the piano, so you need a larger push in this case than if you push parallel to the ramp.

5.10. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object of elevator plus student $(m_{\text{tot}} = 850 \text{ kg})$ and also to the student $(w = 550 \text{ N})$. The elevator and the student have the same acceleration.

SET UP: Let $+y$ be upward. The free-body diagrams for the composite object and for the student are given in Figure 5.10. T is the tension in the cable and n is the scale reading, the normal force the scale exerts on the student. The mass of the student is $m = w/g = 56.1$ kg.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ applied to the student gives $n - mg = ma_y$.

$a_y = \dfrac{n - mg}{m} = \dfrac{450 \text{ N} - 550 \text{ N}}{56.1 \text{ kg}} = -1.78 \text{ m/s}^2$. The elevator has a downward acceleration of 1.78 m/s^2.

(b) $a_y = \dfrac{670 \text{ N} - 550 \text{ N}}{56.1 \text{ kg}} = 2.14 \text{ m/s}^2$.

(c) $n = 0$ means $a_y = -g$. The student should worry; the elevator is in free fall.

(d) $\Sigma F_y = ma_y$ applied to the composite object gives $T - m_{\text{tot}}g = m_{\text{tot}}a_y$. $T = m_{\text{tot}}(a_y + g)$. In part (a), $T = (850 \text{ kg})(-1.78 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 6820$ N. In part (c), $a_y = -g$ and $T = 0$.

EVALUATE: In part (b), $T = (850 \text{ kg})(2.14 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 10{,}150$ N. The weight of the composite object is 8330 N. When the acceleration is upward the tension is greater than the weight and when the acceleration is downward the tension is less than the weight.

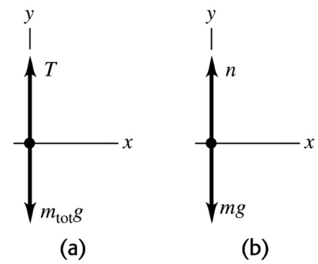


Figure 5.10

5.11. **IDENTIFY:** We apply Newton's second law to the rocket and the astronaut in the rocket. A constant force means we have constant acceleration, so we can use the standard kinematics equations.

SET UP: The free-body diagrams for the rocket (weight w_r) and astronaut (weight w) are given in Figure 5.11. F_T is the thrust and n is the normal force the rocket exerts on the astronaut. The speed of sound is 331 m/s. We use $\Sigma F_y = ma_y$ and $v = v_0 + at$.

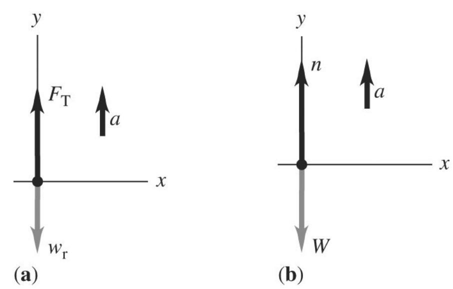


Figure 5.11

EXECUTE: **(a)** Apply $\Sigma F_y = ma_y$ to the rocket: $F_T - w_r = ma$. $a = 4g$ and $w_r = mg$, so $F = m\,(5g) = (2.25\times 10^6\text{ kg})\,(5)\,(9.80\text{ m/s}^2) = 1.10\times 10^8\text{ N}$.

(b) Apply $\Sigma F_y = ma_y$ to the astronaut: $n - w = ma$. $a = 4g$ and $m = \dfrac{w}{g}$, so $n = w + \left(\dfrac{w}{g}\right)(4g) = 5w$.

(c) $v_0 = 0$, $v = 331$ m/s and $a = 4g = 39.2\text{ m/s}^2$. $v = v_0 + at$ gives $t = \dfrac{v - v_0}{a} = \dfrac{331\text{ m/s}}{39.2\text{ m/s}^2} = 8.4\text{ s}$.

EVALUATE: The 8.4 s is probably an unrealistically short time to reach the speed of sound because you would not want your astronauts at the brink of blackout during a launch.

5.12. **IDENTIFY:** Apply Newton's second law to the rocket plus its contents and to the power supply. Both the rocket and the power supply have the same acceleration.

SET UP: The free-body diagrams for the rocket and for the power supply are given in Figure 5.12. Since the highest altitude of the rocket is 120 m, it is near to the surface of the earth and there is a downward gravity force on each object. Let $+y$ be upward, since that is the direction of the acceleration. The power supply has mass $m_{ps} = (15.5\text{ N})/(9.80\text{ m/s}^2) = 1.58\text{ kg}$.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ applied to the rocket gives $F - m_r g = m_r a$.

$$a = \frac{F - m_r g}{m_r} = \frac{1720\text{ N} - (125\text{ kg})(9.80\text{ m/s}^2)}{125\text{ kg}} = 3.96\text{ m/s}^2.$$

(b) $\Sigma F_y = ma_y$ applied to the power supply gives $n - m_{ps}g = m_{ps}a$.

$n = m_{ps}(g + a) = (1.58\text{ kg})(9.80\text{ m/s}^2 + 3.96\text{ m/s}^2) = 21.7\text{ N}$.

EVALUATE: The acceleration is constant while the thrust is constant, and the normal force is constant while the acceleration is constant. The altitude of 120 m is not used in the calculation.

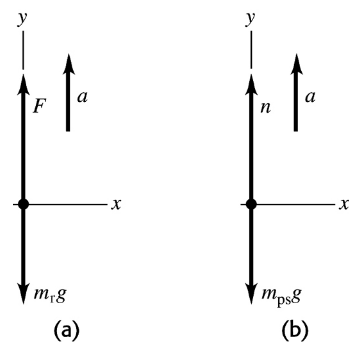


Figure 5.12

5.13. **IDENTIFY:** Use the kinematic information to find the acceleration of the capsule and the stopping time. Use Newton's second law to find the force *F* that the ground exerted on the capsule during the crash.

SET UP: Let $+y$ be upward. $311 \text{ km/h} = 86.4 \text{ m/s}$. The free-body diagram for the capsule is given in Figure 5.13.

EXECUTE: $y - y_0 = -0.810 \text{ m}$, $v_{0y} = -86.4 \text{ m/s}$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{0 - (-86.4 \text{ m/s})^2}{2(-0.810) \text{ m}} = 4610 \text{ m/s}^2 = 470g.$$

(b) $\Sigma F_y = ma_y$ applied to the capsule gives $F - mg = ma$ and

$$F = m(g + a) = (210 \text{ kg})(9.80 \text{ m/s}^2 + 4610 \text{ m/s}^2) = 9.70 \times 10^5 \text{ N} = 471w.$$

(c) $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $t = \dfrac{2(y - y_0)}{v_{0y} + v_y} = \dfrac{2(-0.810 \text{ m})}{-86.4 \text{ m/s} + 0} = 0.0187 \text{ s}$

EVALUATE: The upward force exerted by the ground is much larger than the weight of the capsule and stops the capsule in a short amount of time. After the capsule has come to rest, the ground still exerts a force *mg* on the capsule, but the large 9.70×10^5 N force is exerted only for 0.0187 s.

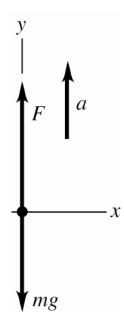


Figure 5.13

5.14. **IDENTIFY:** Apply Newton's second law to the three sleds taken together as a composite object and to each individual sled. All three sleds have the same horizontal acceleration *a*.

SET UP: The free-body diagram for the three sleds taken as a composite object is given in Figure 5.14a and for each individual sled in Figures 5.14b–d. Let $+x$ be to the right, in the direction of the acceleration. $m_{tot} = 60.0$ kg.

Execute: **(a)** $\Sigma F_x = ma_x$ for the three sleds as a composite object gives $P = m_{\text{tot}}a$ and
$a = \dfrac{P}{m_{\text{tot}}} = \dfrac{190 \text{ N}}{60.0 \text{ kg}} = 3.17 \text{ m/s}^2$.

(b) $\Sigma F_x = ma_x$ applied to the 10.0 kg sled gives $P - T_A = m_{10}a$ and
$T_A = P - m_{10}a = 190 \text{ N} - (10.0 \text{ kg})(3.17 \text{ m/s}^2) = 158 \text{ N}$. $\Sigma F_x = ma_x$ applied to the 30.0 kg sled gives
$T_B = m_{30}a = (30.0 \text{ kg})(3.17 \text{ m/s}^2) = 95.1 \text{ N}$.

Evaluate: If we apply $\Sigma F_x = ma_x$ to the 20.0 kg sled and calculate a from T_A and T_B found in part (b), we get $T_A - T_B = m_{20}a$. $a = \dfrac{T_A - T_B}{m_{20}} = \dfrac{158 \text{ N} - 95.1 \text{ N}}{20.0 \text{ kg}} = 3.15 \text{ m/s}^2$, which agrees closely with the value we calculated in part (a), the difference being due to rounding.

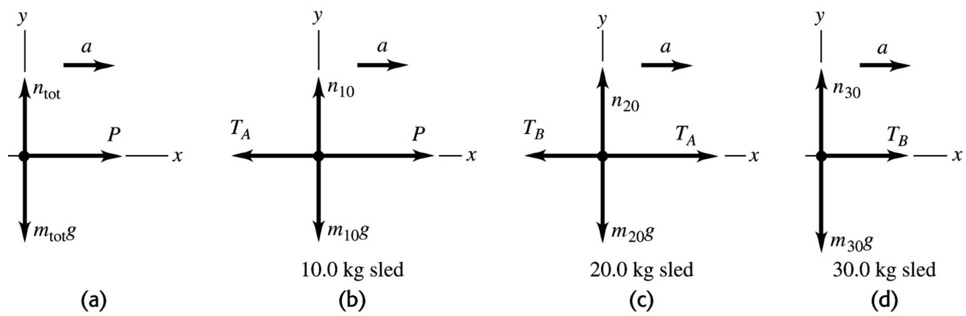


Figure 5.14

5.15. **Identify:** Apply $\Sigma \vec{F} = m\vec{a}$ to the load of bricks and to the counterweight. The tension is the same at each end of the rope. The rope pulls up with the same force (T) on the bricks and on the counterweight. The counterweight accelerates downward and the bricks accelerate upward; these accelerations have the same magnitude.

(a) Set Up: The free-body diagrams for the bricks and counterweight are given in Figure 5.15.

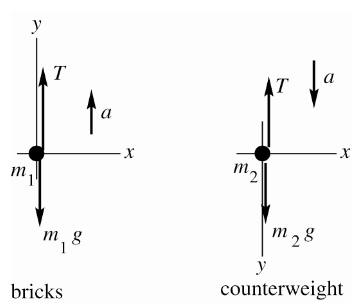


Figure 5.15

(b) Execute: Apply $\Sigma F_y = ma_y$ to each object. The acceleration magnitude is the same for the two objects. For the bricks take $+y$ to be upward since $\vec{a}$ for the bricks is upward. For the counterweight take $+y$ to be downward since $\vec{a}$ is downward.

<u>bricks</u>: $\Sigma F_y = ma_y$

$T - m_1 g = m_1 a$

<u>counterweight</u>: $\Sigma F_y = ma_y$

$m_2 g - T = m_2 a$

Add these two equations to eliminate T:

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g = \left(\frac{28.0\text{ kg} - 15.0\text{ kg}}{15.0\text{ kg} + 28.0\text{ kg}}\right)(9.80\text{ m/s}^2) = 2.96\text{ m/s}^2$$

(c) $T - m_1 g = m_1 a$ gives $T = m_1(a + g) = (15.0\text{ kg})(2.96\text{ m/s}^2 + 9.80\text{ m/s}^2) = 191\text{ N}$

As a check, calculate T using the other equation.

$m_2 g - T = m_2 a$ gives $T = m_2(g - a) = 28.0\text{ kg}(9.80\text{ m/s}^2 - 2.96\text{ m/s}^2) = 191\text{ N}$, which checks.

EVALUATE: The tension is 1.30 times the weight of the bricks; this causes the bricks to accelerate upward. The tension is 0.696 times the weight of the counterweight; this causes the counterweight to accelerate downward. If $m_1 = m_2$, $a = 0$ and $T = m_1 g = m_2 g$. In this special case the objects don't move. If $m_1 = 0$, $a = g$ and $T = 0$; in this special case the counterweight is in free fall. Our general result is correct in these two special cases.

5.16. IDENTIFY: In part (a) use the kinematic information and the constant acceleration equations to calculate the acceleration of the ice. Then apply $\Sigma\vec{F} = m\vec{a}$. In part (b) use $\Sigma\vec{F} = m\vec{a}$ to find the acceleration and use this in the constant acceleration equations to find the final speed.

SET UP: Figure 5.16 gives the free-body diagrams for the ice both with and without friction. Let $+x$ be directed down the ramp, so $+y$ is perpendicular to the ramp surface. Let ϕ be the angle between the ramp and the horizontal. The gravity force has been replaced by its x- and y-components.

EXECUTE: **(a)** $x - x_0 = 1.50\text{ m}$, $v_{0x} = 0$. $v_x = 2.50\text{ m/s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \dfrac{(2.50\text{ m/s})^2 - 0}{2(1.50\text{ m})} = 2.08\text{ m/s}^2$. $\Sigma F_x = ma_x$ gives $mg\sin\phi = ma$ and

$\sin\phi = \dfrac{a}{g} = \dfrac{2.08\text{ m/s}^2}{9.80\text{ m/s}^2}$. $\phi = 12.3°$.

(b) $\Sigma F_x = ma_x$ gives $mg\sin\phi - f = ma$ and

$a = \dfrac{mg\sin\phi - f}{m} = \dfrac{(8.00\text{ kg})(9.80\text{ m/s}^2)\sin 12.3° - 10.0\text{ N}}{8.00\text{ kg}} = 0.838\text{ m/s}^2$.

Then $x - x_0 = 1.50\text{ m}$, $v_{0x} = 0$. $a_x = 0.838\text{ m/s}^2$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$v_x = \sqrt{2a_x(x - x_0)} = \sqrt{2(0.838\text{ m/s}^2)(1.50\text{ m})} = 1.59\text{ m/s}$$

EVALUATE: With friction present the speed at the bottom of the ramp is less.

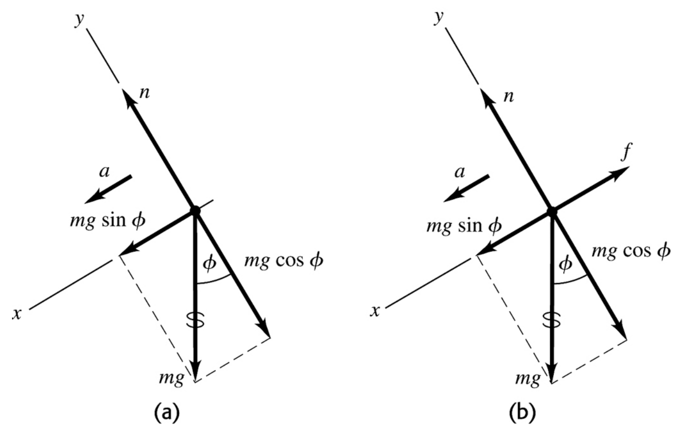


Figure 5.16

5.17. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. Each block has the same magnitude of acceleration a.

SET UP: Assume the pulley is to the right of the 4.00 kg block. There is no friction force on the 4.00 kg block; the only force on it is the tension in the rope. The 4.00 kg block therefore accelerates to the right and the suspended block accelerates downward. Let $+x$ be to the right for the 4.00 kg block, so for it $a_x = a$, and let $+y$ be downward for the suspended block, so for it $a_y = a$.

EXECUTE: **(a)** The free-body diagrams for each block are given in Figures 5.17a and b.

(b) $\Sigma F_x = ma_x$ applied to the 4.00 kg block gives $T = (4.00\text{ kg})a$ and

$$a = \frac{T}{4.00\text{ kg}} = \frac{15.0\text{ N}}{4.00\text{ kg}} = 3.75\text{ m/s}^2.$$

(c) $\Sigma F_y = ma_y$ applied to the suspended block gives $mg - T = ma$ and

$$m = \frac{T}{g-a} = \frac{15.0\text{ N}}{9.80\text{ m/s}^2 - 3.75\text{ m/s}^2} = 2.48\text{ kg}.$$

(d) The weight of the hanging block is $mg = (2.48\text{ kg})(9.80\text{ m/s}^2) = 24.3\text{ N}$. This is greater than the tension in the rope; $T = 0.617mg$.

EVALUATE: Since the hanging block accelerates downward, the net force on this block must be downward and the weight of the hanging block must be greater than the tension in the rope. Note that the blocks accelerate no matter how small m is. It is not necessary to have $m > 4.00\text{ kg}$, and in fact in this problem m is less than 4.00 kg.

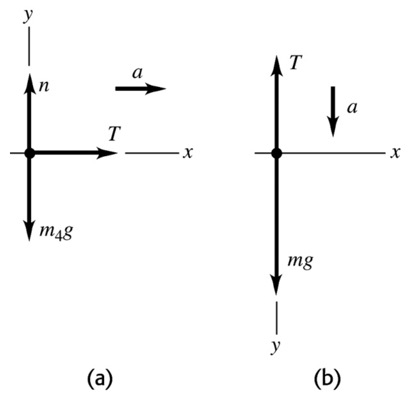


Figure 5.17

5.18. **IDENTIFY:** **(a)** Consider both gliders together as a single object, apply $\Sigma\vec{F} = m\vec{a}$, and solve for a. Use a in a constant acceleration equation to find the required runway length.

(b) Apply $\Sigma\vec{F} = m\vec{a}$ to the second glider and solve for the tension T_g in the towrope that connects the two gliders.

SET UP: In part (a), set the tension T_t in the towrope between the plane and the first glider equal to its maximum value, $T_t = 12{,}000$ N.

EXECUTE: **(a)** The free-body diagram for both gliders as a single object of mass $2m = 1400$ kg is given in Figure 5.18a. $\Sigma F_x = ma_x$ gives $T_t - 2f = (2m)a$ and $a = \dfrac{T_t - 2f}{2m} = \dfrac{12{,}000\text{ N} - 5000\text{ N}}{1400\text{ kg}} = 5.00\text{ m/s}^2$. Then $a_x = 5.00\text{ m/s}^2$, $v_{0x} = 0$ and $v_x = 40$ m/s in $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $(x - x_0) = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = 160$ m.

(b) The free-body diagram for the second glider is given in Figure 5.18b.
$\Sigma F_x = ma_x$ gives $T_g - f = ma$ and $T_g = f + ma = 2500\text{ N} + (700\text{ kg})(5.00\text{ m/s}^2) = 6000$ N.

EVALUATE: We can verify that $\Sigma F_x = ma_x$ is also satisfied for the first glider.

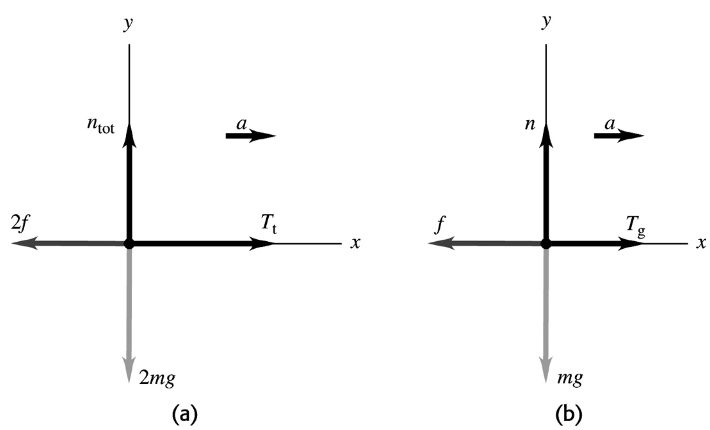


Figure 5.18

5.19. IDENTIFY: While the person is in contact with the ground, he is accelerating upward and experiences two forces: gravity downward and the upward force of the ground. Once he is in the air, only gravity acts on him so he accelerates downward. Newton's second law applies during the jump (and at all other times).

SET UP: Take $+y$ to be upward. After he leaves the ground the person travels upward 60 cm and his acceleration is $g = 9.80\text{ m/s}^2$, downward. His weight is w so his mass is w/g. $\Sigma F_y = ma_y$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply to the jumper.

EXECUTE: **(a)** $v_y = 0$ (at the maximum height), $y - y_0 = 0.60\text{ m}$, $a_y = -9.80\text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\text{ m/s}^2)(0.60\text{ m})} = 3.4\text{ m/s}$.

(b) The free-body diagram for the person while he is pushing up against the ground is given in Figure 5.19.

(c) For the jump, $v_{0y} = 0$, $v_y = 3.4\text{ m/s}$ (from part (a)), and $y - y_0 = 0.50\text{ m}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{(3.4\text{ m/s})^2 - 0}{2(0.50\text{ m})} = 11.6\text{ m/s}^2$. $\Sigma F_y = ma_y$ gives $n - w = ma$.

$$n = w + ma = w\left(1 + \frac{a}{g}\right) = 2.2w.$$

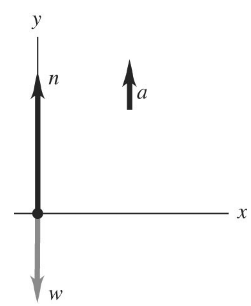


Figure 5.19

EVALUATE: To accelerate the person upward during the jump, the upward force from the ground must exceed the downward pull of gravity. The ground pushes up on him because he pushes down on the ground.

5.20. IDENTIFY: Acceleration and velocity are related by $a_y = \dfrac{dv_y}{dt}$. Apply $\Sigma\vec{F} = m\vec{a}$ to the rocket.

SET UP: Let $+y$ be upward. The free-body diagram for the rocket is sketched in Figure 5.20. $\vec{F}$ is the thrust force.

EXECUTE: **(a)** $v_y = At + Bt^2$. $a_y = A + 2Bt$. At $t = 0$, $a_y = 1.50\text{ m/s}^2$ so $A = 1.50\text{ m/s}^2$. Then $v_y = 2.00\text{ m/s}$ at $t = 1.00\text{ s}$ gives $2.00\text{ m/s} = (1.50\text{ m/s}^2)(1.00\text{ s}) + B(1.00\text{ s})^2$ and $B = 0.50\text{ m/s}^3$.

(b) At $t = 4.00\text{ s}$, $a_y = 1.50\text{ m/s}^2 + 2(0.50\text{ m/s}^3)(4.00\text{ s}) = 5.50\text{ m/s}^2$.

(c) $\Sigma F_y = ma_y$ applied to the rocket gives $T - mg = ma$ and

$T = m(a + g) = (2540\text{ kg})(9.80\text{ m/s}^2 + 5.50\text{ m/s}^2) = 3.89 \times 10^4\text{ N}$. $T = 1.56w$.

(d) When $a = 1.50\text{ m/s}^2$, $T = (2540\text{ kg})(9.80\text{ m/s}^2 + 1.50\text{ m/s}^2) = 2.87 \times 10^4\text{ N}$.

EVALUATE: During the time interval when $v(t) = At + Bt^2$ applies the magnitude of the acceleration is increasing, and the thrust is increasing.

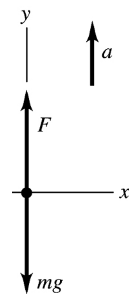


Figure 5.20

5.21. **IDENTIFY:** We know the external forces on the box and want to find the distance it moves and its speed. The force is not constant, so the acceleration will not be constant, so we cannot use the standard constant-acceleration kinematics formulas. But Newton's second law will apply.

SET UP: First use Newton's second law to find the acceleration as a function of time: $a_x(t) = \dfrac{F_x}{m}$. Then integrate the acceleration to find the velocity as a function of time, and next integrate the velocity to find the position as a function of time.

EXECUTE: Let $+x$ be to the right. $a_x(t) = \dfrac{F_x}{m} = \dfrac{(-6.00\ \text{N/s}^2)t^2}{2.00\ \text{kg}} = -(3.00\ \text{m/s}^4)t^2$. Integrate the acceleration to find the velocity as a function of time: $v_x(t) = -(1.00\ \text{m/s}^4)t^3 + 9.00\ \text{m/s}$. Next integrate the velocity to find the position as a function of time: $x(t) = -(0.250\ \text{m/s}^4)t^4 + (9.00\ \text{m/s})t$. Now use the given values of time.

(a) $v_x = 0$ when $(1.00\ \text{m/s}^4)t^3 = 9.00\ \text{m/s}$. This gives $t = 2.08\ \text{s}$. At $t = 2.08\ \text{s}$,
$x = (9.00\ \text{m/s})(2.08\ \text{s}) - (0.250\ \text{m/s}^4)(2.08\ \text{s})^4 = 18.72\ \text{m} - 4.68\ \text{m} = 14.0\ \text{m}$.

(b) At $t = 3.00\ \text{s}$, $v_x(t) = -(1.00\ \text{m/s}^4)(3.00\ \text{s})^3 + 9.00\ \text{m/s} = -18.0\ \text{m/s}$, so the speed is $18.0\ \text{m/s}$.

EVALUATE: The box starts out moving to the right. But because the acceleration is to the left, it reverses direction and v_x is negative in part (b).

5.22. **IDENTIFY:** We know the position of the crate as a function of time, so we can differentiate to find its acceleration. Then we can apply Newton's second law to find the upward force.

SET UP: $v_y(t) = dy/dt$, $a_y(t) = dv_y/dt$, and $\Sigma F_y = ma_y$.

EXECUTE: Let $+y$ be upward. $dy/dt = v_y(t) = 2.80\ \text{m/s} + (1.83\ \text{m/s}^3)t^2$ and $dv_y/dt = a_y(t) = (3.66\ \text{m/s}^3)t$. At $t = 4.00\ \text{s}$, $a_y = 14.64\ \text{m/s}^2$. Newton's second law in the y direction gives $F - mg = ma$. Solving for F gives $F = 49\ \text{N} + (5.00\ \text{kg})(14.64\ \text{m/s}^2) = 122\ \text{N}$.

EVALUATE: The force is greater than the weight since it is accelerating the crate upward.

5.23. **IDENTIFY:** At the maximum tilt angle, the patient is just ready to slide down, so static friction is at its maximum and the forces on the patient balance.

SET UP: Take $+x$ to be down the incline. At the maximum angle $f_s = \mu_s n$ and $\Sigma F_x = ma_x = 0$.

EXECUTE: The free-body diagram for the patient is given in Figure 5.23. $\Sigma F_y = ma_y$ gives $n = mg\cos\theta$. $\Sigma F_x = 0$ gives $mg\sin\theta - \mu_s n = 0$. $mg\sin\theta - \mu_s mg\cos\theta = 0$. $\tan\theta = \mu_s$ so $\theta = 50°$.

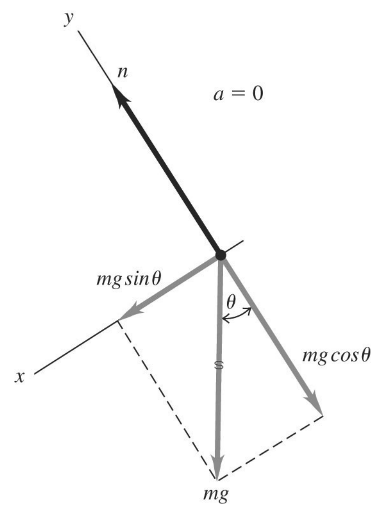


Figure 5.23

EVALUATE: A larger angle of tilt would cause more blood to flow to the brain, but it would also cause the patient to slide down the bed.

5.24. **IDENTIFY:** $f_s \le \mu_s n$ and $f_k = \mu_k n.$ The normal force n is determined by applying $\Sigma\vec{F} = m\vec{a}$ to the block. Normally, $\mu_k \le \mu_s$. f_s is only as large as it needs to be to prevent relative motion between the two surfaces.

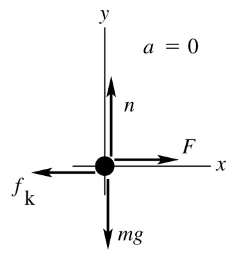
SET UP: Since the table is horizontal, with only the block present $n = 135$ N. With the brick on the block,
$n = 270$ N.

EXECUTE: **(a)** The friction is static for $P = 0$ to $P = 75.0$ N. The friction is kinetic for $P > 75.0$ N.
(b) The maximum value of f_s is $\mu_s n.$ From the graph the maximum f_s is $f_s = 75.0$ N, so
$\mu_s = \dfrac{\max f_s}{n} = \dfrac{75.0\text{ N}}{135\text{ N}} = 0.556.$ $f_k = \mu_k n.$ From the graph, $f_k = 50.0$ N and $\mu_k = \dfrac{f_k}{n} = \dfrac{50.0\text{ N}}{135\text{ N}} = 0.370.$
(c) When the block is moving the friction is kinetic and has the constant value $f_k = \mu_k n,$ independent of P. This is why the graph is horizontal for $P > 75.0$ N. When the block is at rest, $f_s = P$ since this prevents relative motion. This is why the graph for $P < 75.0$ N has slope $+1$.
(d) max f_s and f_k would double. The values of f on the vertical axis would double but the shape of the graph would be unchanged.

EVALUATE: The coefficients of friction are independent of the normal force.

5.25. **(a) IDENTIFY:** Constant speed implies $a = 0$. Apply Newton's first law to the box. The friction force is directed opposite to the motion of the box.

SET UP: Consider the free-body diagram for the box, given in Figure 5.25a. Let $\vec{F}$ be the horizontal force applied by the worker. The friction is kinetic friction since the box is sliding along the surface.



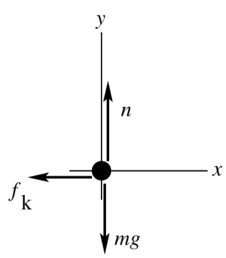
EXECUTE:
$\Sigma F_y = ma_y$
$n - mg = 0$
$n = mg$
so $f_k = \mu_k n = \mu_k mg$

Figure 5.25a

$\Sigma F_x = ma_x$
$F - f_k = 0$
$F = f_k = \mu_k mg = (0.20)(16.8\text{ kg})(9.80\text{ m/s}^2) = 33\text{ N}$

(b) IDENTIFY: Now the only horizontal force on the box is the kinetic friction force. Apply Newton's second law to the box to calculate its acceleration. Once we have the acceleration, we can find the distance using a constant acceleration equation. The friction force is $f_k = \mu_k mg$, just as in part (a).

SET UP: The free-body diagram is sketched in Figure 5.25b.



EXECUTE:
$\Sigma F_x = ma_x$
$-f_k = ma_x$
$-\mu_k mg = ma_x$
$a_x = -\mu_k g = -(0.20)(9.80\text{ m/s}^2) = -1.96\text{ m/s}^2$

Figure 5.25b

Use the constant acceleration equations to find the distance the box travels:
$v_x = 0,\ v_{0x} = 3.50\text{ m/s},\ a_x = -1.96\text{ m/s}^2,\ x - x_0 = ?$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$
$$x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (3.50\text{ m/s})^2}{2(-1.96\text{ m/s}^2)} = 3.1\text{ m}$$

EVALUATE: The normal force is the component of force exerted by a surface perpendicular to the surface. Its magnitude is determined by $\Sigma\vec{F} = m\vec{a}$. In this case *n* and *mg* are the only vertical forces and $a_y = 0$, so $n = mg$. Also note that f_k and *n* are proportional in magnitude but perpendicular in direction.

5.26. IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the box.

SET UP: Since the only vertical forces are *n* and *w*, the normal force on the box equals its weight. Static friction is as large as it needs to be to prevent relative motion between the box and the surface, up to its maximum possible value of $f_s^{\max} = \mu_s n$. If the box is sliding then the friction force is $f_k = \mu_k n$.

EXECUTE: **(a)** If there is no applied force, no friction force is needed to keep the box at rest.

(b) $f_s^{\max} = \mu_s n = (0.40)(40.0\text{ N}) = 16.0\text{ N}$. If a horizontal force of 6.0 N is applied to the box, then $f_s = 6.0\text{ N}$ in the opposite direction.

(c) The monkey must apply a force equal to f_s^{max}, 16.0 N.

(d) Once the box has started moving, a force equal to $f_k = \mu_k n = 8.0\text{ N}$ is required to keep it moving at constant velocity.

(e) $f_k = 8.0\text{ N}$. $a = (18.0\text{ N} - 8.0\text{ N})/(40.0\text{ N}/9.80\text{ m/s}^2) = 2.45\text{ m/s}^2$

EVALUATE: $\mu_k < \mu_s$ and less force must be applied to the box to maintain its motion than to start it moving.

5.27. IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the crate. $f_s \le \mu_s n$ and $f_k = \mu_k n$.

SET UP: Let $+y$ be upward and let $+x$ be in the direction of the push. Since the floor is horizontal and the push is horizontal, the normal force equals the weight of the crate: $n = mg = 441\text{ N}$. The force it takes to start the crate moving equals max f_s and the force required to keep it moving equals f_k.

EXECUTE: (a) max $f_s = 313\text{ N}$, so $\mu_s = \dfrac{313\text{ N}}{441\text{ N}} = 0.710$. $f_k = 208\text{ N}$, so $\mu_k = \dfrac{208\text{ N}}{441\text{ N}} = 0.472$.

(b) The friction is kinetic. $\Sigma F_x = ma_x$ gives $F - f_k = ma$ and

$F = f_k + ma = 208\text{ N} + (45.0\text{ kg})(1.10\text{ m/s}^2) = 258\text{ N}$.

(c) (i) The normal force now is $mg = 72.9\text{ N}$. To cause it to move,

$F = \max f_s = \mu_s n = (0.710)(72.9\text{ N}) = 51.8\text{ N}$.

(ii) $F = f_k + ma$ and $a = \dfrac{F - f_k}{m} = \dfrac{258\text{ N} - (0.472)(72.9\text{ N})}{45.0\text{ kg}} = 4.97\text{ m/s}^2$

EVALUATE: The kinetic friction force is independent of the speed of the object. On the moon, the mass of the crate is the same as on earth, but the weight and normal force are less.

5.28. IDENTIFY: On the level floor and on the ramp the box moves with constant velocity, so its acceleration is zero. Therefore the forces on it must balance. In addition to your push, kinetic friction and gravity act on the box.

SET UP: Estimate: Heaviest box is about 150 lb with sustained effort. $\sum F_x = 0$, $\sum F_y = 0$, and the kinetic friction force is $f_k = \mu_k n$.

EXECUTE: To push a 150-lb box on a horizontal surface, the force needed would be equal to the kinetic friction force, which is $f_k = \mu_k n = (0.50)(150\text{ lb}) = 75\text{ lb}$. We now exert this push on a box on a ramp and want to know the weight of the heaviest box we can push up the ramp at constant speed. μ_k is the same on the ramp as it is on the level floor. Fig. 5.28 shows a free-body diagram of the box on the ramp. Take the *x*-axis parallel to the ramp surface pointing up the ramp, and call β the angle the ramp makes above the horizontal .

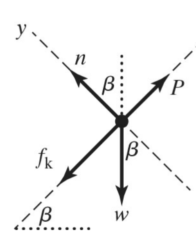


Figure 5.28

Using the notation in Fig. 5.28, $\sum F_y = 0$ gives $n - w\cos\beta = 0$, so $n = w\cos\beta$.

$\sum F_x = 0$ gives $P - f_k - w\sin\beta = 0$. We also have $f_k = \mu_k n$, so the last equation becomes

$P - \mu_k w \cos\beta - w \sin\beta = 0$. Solve for w: $w = \dfrac{P}{\mu_k \cos\beta + \sin\beta} = \dfrac{75 \text{ lb}}{(0.50)\cos 60° + \sin 60°}$, so $w = 67$ lb.

EVALUATE: This weight is less than you can push on a level surface because you need to balance the weight component down the ramp in addition to the friction force.

5.29. **IDENTIFY:** The children are accelerated as they move down the slide, so Newton's second law applies. The acceleration is constant, so the constant-acceleration formulas apply.

SET UP: Estimations: The height of the slide is about 6 ft or 2.0 m, and it rises at an angle of 30° above the horizontal. The forces on the child are gravity, the normal force due to the slide, and friction. Apply $\sum F_x = ma_x$ with the x-axis is along the surface of the slide pointing downward, and $\sum F_y = 0$ with the y-axis perpendicular to the slide surface. Fig. 5.29 shows free-body diagrams of the child with and without friction.

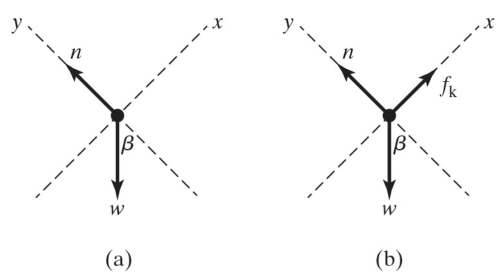


Figure 5.29

EXECUTE: **(a)** First find the acceleration and then use it to find the velocity at the bottom of the slide. Use $\sum F_x = ma_x$ with the notation in Fig. 5.29a. $mg \sin\beta = ma_x$, so $a_x = g \sin\beta = g \sin 30° = g/2$. Now use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find v_x at the bottom. The distance $x - x_0 = (2.0 \text{ m})/(\sin\beta) = (2.0 \text{ m})/(\sin 30°) = 4.0$ m. This gives $v_x^2 = 0 + 2(g/2)(4.0 \text{ m})$, so $v_x = 6.3$ m/s.

(b) Now friction is acting up the slide, as shown in Fig. 5.29b, and we want to find μ_k. In this case, v_x is half of what it was without friction, so $v_x = (0.50)(6.3 \text{ m/s}) = 3.13$ m/s. Now use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find the acceleration. $(3.13 \text{ m/s})^2 = 2a_x(4.0 \text{ m}) \quad \rightarrow \quad a_x = 1.225 \text{ m/s}^2$. Using $\sum F_x = ma_x$ gives $mg \sin\beta - f_k = mg \sin\beta - \mu_k n = ma_x$. Now use $\sum F_y = 0$ to find the normal force. $n - mg \cos\beta = 0$, so $n = mg \cos\beta$. Combining these two results gives

$mg \sin\beta - \mu_k mg \cos\beta = ma_x$. Solving for μ_k gives $\mu_k = \dfrac{g \sin\beta - a_x}{g \cos\beta}$. Using $a_x = 1.225 \text{ m/s}^2$ and $\beta = 30°$, we get $\mu_k = 0.43$.

(c) We now have static friction. For μ_s to have its minimum value, the child must be just ready to slide, so static friction is at its maximum value, which is $f_s = \mu_s n$. As before, $n = mg \cos\beta$. Now we have $\sum F_x = 0$, giving $\mu_s mg \cos\beta - mg \sin\beta = 0$, so $\mu_s = \tan\beta = \tan 30° = 0.58$.

EVALUATE: Table 5.1 shows that 0.58 is a reasonable value for μ_s, and it is larger than the value for μ_k we used in part (b).

5.30. **IDENTIFY:** Newton's second law applies to the rocks on the hill. When they are moving, kinetic friction acts on them, but when they are at rest, static friction acts.

SET UP: Use coordinates with axes parallel and perpendicular to the incline, with $+x$ in the direction of the acceleration. $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y = 0$.

EXECUTE: With the rock sliding up the hill, the friction force is down the hill. The free-body diagram is given in Figure 5.30a.

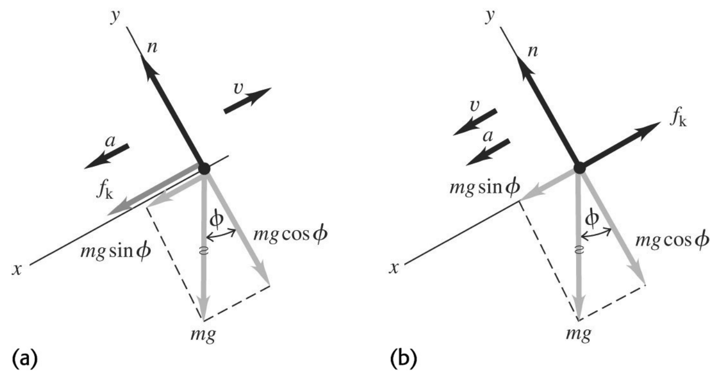


Figure 5.30

$\Sigma F_y = ma_y = 0$ gives $n = mg\cos\phi$ and $f_k = \mu_k n = \mu_k mg\cos\phi$. $\Sigma F_x = ma_x$ gives $mg\sin\phi + \mu_k mg\cos\phi = ma$.

$a = g(\sin\phi + \mu_k\cos\phi) = (9.80\text{ m/s}^2)[\sin 36° + (0.45)\cos 36°]$. $a = 9.33\text{ m/s}^2$, down the incline.

(b) The component of gravity down the incline is $mg\sin\phi = 0.588mg$. The maximum possible static friction force is $f_s = \mu_s n = \mu_s mg\cos\phi = 0.526mg$. f_s can't be as large as $mg\sin\phi$ and the rock slides back down. As the rock slides down, f_k is up the incline. The free-body diagram is given in Figure 5.30b. $\Sigma F_y = ma_y = 0$ gives $n = mg\cos\phi$ and $f_k = \mu_k n = \mu_k mg\cos\phi$. $\Sigma F_x = ma_x$ gives $mg\sin\phi - \mu_k mg\cos\phi = ma$, so $a = g(\sin\phi - \mu_k\cos\phi) = 2.19\text{ m/s}^2$, down the incline.

EVALUATE: The acceleration down the incline in (a) is greater than that in (b) because in (a) the static friction and gravity are both acting down the incline, whereas in (b) friction is up the incline, opposing gravity which still acts down the incline.

5.31. **IDENTIFY:** A 10.0-kg box is pushed on a ramp, causing it to accelerate. Newton's second law applies.

SET UP: Choose the x-axis along the surface of the ramp and the y-axis perpendicular to the surface. The only acceleration of the box is in the x-direction, so $\Sigma F_x = ma_x$ and $\Sigma F_y = 0$. The external forces acting on the box are the push P along the surface of the ramp, friction f_k, gravity mg, and the normal force n. The ramp rises at 55.0° above the horizontal, and $f_k = \mu_k n$. The friction force opposes the sliding, so it is directed up the ramp in part (a) and down the ramp in part (b).

EXECUTE: **(a)** Applying $\Sigma F_y = 0$ gives $n = mg\cos(55.0°)$, so the force of kinetic friction is $f_k = \mu_k n =$ (0.300)(10.0 kg)(9.80 m/s^2)(cos 55.0°) = 16.86 N. Call the +x-direction down the ramp since that is the direction of the acceleration of the box. Applying $\Sigma F_x = ma_x$ gives $P + mg\sin(55.0°) - f_k = ma$. Putting in the numbers gives (10.0 kg)a = 120 N + (98.0 N)(sin 55.0°) – 16.86 N; a = 18.3 m/s^2.

(b) Now P is up the up the ramp and f_k is down the ramp, but the other force components are unchanged, so f_k = 16.86 N as before. We now choose +x to be up the ramp, so $\Sigma F_x = ma_x$ gives $P - mg\sin(55.0°) - f_k = ma$. Putting in the same numbers as before gives a = 2.29 m/s^2.

EVALUATE: Pushing up the ramp produces a much smaller acceleration than pushing down the ramp because gravity helps the downward push but opposes the upward push.

5.32. **IDENTIFY:** For the shortest time, the acceleration is a maximum, so the toolbox is just ready to slide relative to the bed of the truck. The box is at rest relative to the truck, but it is accelerating relative to the ground because the truck is accelerating. Therefore Newton's second law will be useful.

SET UP: If the truck accelerates to the right the static friction force on the box is to the right, to try to prevent the box from sliding relative to the truck. The free-body diagram for the box is given in Figure 5.32. The maximum acceleration of the box occurs when f_s has its maximum value, so $f_s = \mu_s n$. If the box doesn't slide, its acceleration equals the acceleration of the truck. The constant-acceleration equation $v_x = v_{0x} + a_x t$ applies.

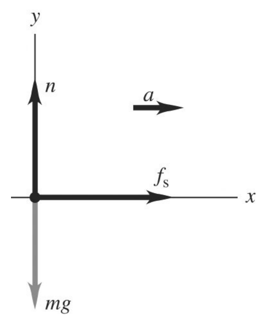


Figure 5.32

EXECUTE: $n = mg$. $\Sigma F_x = ma_x$ gives $f_s = ma$ so $\mu_s mg = ma$ and $a = \mu_s g = 6.37\ \text{m/s}^2$. $v_{0x} = 0$, $v_x = 30.0$ m/s. $v_x = v_{0x} + a_x t$ gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{30.0\ \text{m/s} - 0}{6.37\ \text{m/s}^2} = 4.71$ s.

EVALUATE: If the truck has a smaller acceleration it is still true that $f_s = ma$, but now $f_s < \mu_s n$.

5.33. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the composite object consisting of the two boxes and to the top box. The friction the ramp exerts on the lower box is kinetic friction. The upper box doesn't slip relative to the lower box, so the friction between the two boxes is static. Since the speed is constant the acceleration is zero.

SET UP: Let $+x$ be up the incline. The free-body diagrams for the composite object and for the upper box are given in Figure 5.33. The slope angle ϕ of the ramp is given by $\tan\phi = \dfrac{2.50\ \text{m}}{4.75\ \text{m}}$, so $\phi = 27.76°$. Since the boxes move down the ramp, the kinetic friction force exerted on the lower box by the ramp is directed up the incline. To prevent slipping relative to the lower box the static friction force on the upper box is directed up the incline. $m_{\text{tot}} = 32.0\ \text{kg} + 48.0\ \text{kg} = 80.0\ \text{kg}$.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ applied to the composite object gives $n_{\text{tot}} = m_{\text{tot}} g\cos\phi$ and $f_k = \mu_k m_{\text{tot}} g\cos\phi$. $\Sigma F_x = ma_x$ gives $f_k + T - m_{\text{tot}} g\sin\phi = 0$ and

$T = (\sin\phi - \mu_k\cos\phi)m_{\text{tot}} g = (\sin 27.76° - [0.444]\cos 27.76°)(80.0\ \text{kg})(9.80\ \text{m/s}^2) = 57.1\ \text{N}$.

The person must apply a force of 57.1 N, directed up the ramp.

(b) $\Sigma F_x = ma_x$ applied to the upper box gives $f_s = mg\sin\phi = (32.0\ \text{kg})(9.80\ \text{m/s}^2)\sin 27.76° = 146\ \text{N}$, directed up the ramp.

EVALUATE: For each object the net force is zero.

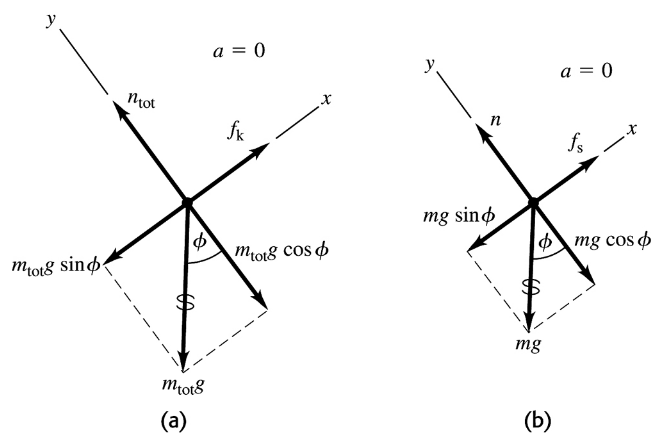


Figure 5.33

5.34. **IDENTIFY:** Constant speed means zero acceleration for each block. If the block is moving, the friction force the tabletop exerts on it is kinetic friction. Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

SET UP: The free-body diagrams and choice of coordinates for each block are given by Figure 5.34. $m_A = 4.59$ kg and $m_B = 2.55$ kg.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ with $a_y = 0$ applied to block *B* gives $m_B g - T = 0$ and $T = 25.0$ N. $\Sigma F_x = ma_x$ with $a_x = 0$ applied to block *A* gives $T - f_k = 0$ and $f_k = 25.0$ N. $n_A = m_A g = 45.0$ N and $\mu_k = \dfrac{f_k}{n_A} = \dfrac{25.0\text{ N}}{45.0\text{ N}} = 0.556.$

(b) Now let *A* be block *A* plus the cat, so $m_A = 9.18$ kg. $n_A = 90.0$ N and $f_k = \mu_k n = (0.556)(90.0\text{ N}) = 50.0$ N. $\sum F_x = ma_x$ for *A* gives $T - f_k = m_A a_x$. $\sum F_y = ma_y$ for block *B* gives $m_B g - T = m_B a_y$. a_x for *A* equals a_y for *B*, so adding the two equations gives $m_B g - f_k = (m_A + m_B)a_y$ and $a_y = \dfrac{m_B g - f_k}{m_A + m_B} = \dfrac{25.0\text{ N} - 50.0\text{ N}}{9.18\text{ kg} + 2.55\text{ kg}} = -2.13\text{ m/s}^2$. The acceleration is upward and block *B* slows down.

EVALUATE: The equation $m_B g - f_k = (m_A + m_B)a_y$ has a simple interpretation. If both blocks are considered together then there are two external forces: $m_B g$ that acts to move the system one way and f_k that acts oppositely. The net force of $m_B g - f_k$ must accelerate a total mass of $m_A + m_B$.

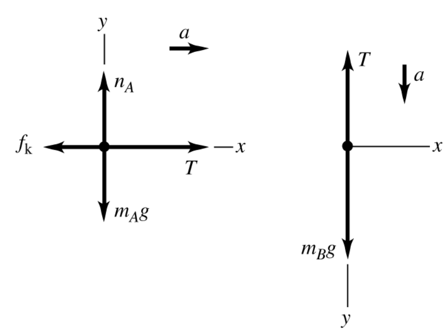


Figure 5.34

5.35. **IDENTIFY:** Use $\Sigma\vec{F} = m\vec{a}$ to find the acceleration that can be given to the car by the kinetic friction force. Then use a constant acceleration equation.

SET UP: Take $+x$ in the direction the car is moving.

EXECUTE: **(a)** The free-body diagram for the car is shown in Figure 5.35. $\Sigma F_y = ma_y$ gives $n = mg$. $\Sigma F_x = ma_x$ gives $-\mu_k n = ma_x$. $-\mu_k mg = ma_x$ and $a_x = -\mu_k g$. Then $v_x = 0$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $(x - x_0) = -\dfrac{v_{0x}^2}{2a_x} = +\dfrac{v_{0x}^2}{2\mu_k g} = \dfrac{(28.7\text{ m/s})^2}{2(0.80)(9.80\text{ m/s}^2)} = 52.5\text{ m}$.

(b) $v_{0x} = \sqrt{2\mu_k g(x - x_0)} = \sqrt{2(0.25)(9.80\text{ m/s}^2)52.5\text{ m}} = 16.0\text{ m/s}$

EVALUATE: For constant stopping distance $\dfrac{v_{0x}^2}{\mu_k}$ is constant and v_{0x} is proportional to $\sqrt{\mu_k}$. The answer to part (b) can be calculated as $(28.7\text{ m/s})\sqrt{0.25/0.80} = 16.0\text{ m/s}$.

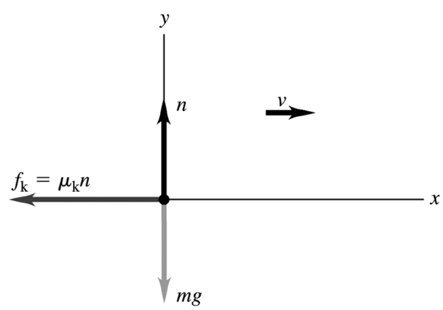


Figure 5.35

5.36. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the box. When the box is ready to slip the static friction force has its maximum possible value, $f_s = \mu_s n$.

SET UP: Use coordinates parallel and perpendicular to the ramp.

EXECUTE: **(a)** The normal force will be $w\cos\alpha$ and the component of the gravitational force along the ramp is $w\sin\alpha$. The box begins to slip when $w\sin\alpha > \mu_s w\cos\alpha$, or $\tan\alpha > \mu_s = 0.35$, so slipping occurs at $\alpha = \arctan(0.35) = 19.3°$.

(b) When moving, the friction force along the ramp is $\mu_k w\cos\alpha$, the component of the gravitational force along the ramp is $w\sin\alpha$, so the acceleration is

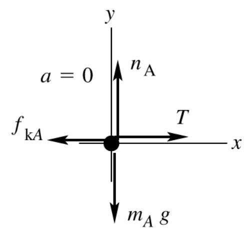
$$(w\sin\alpha - w\mu_k\cos\alpha)/m = g(\sin\alpha - \mu_k\cos\alpha) = 0.92\text{ m/s}^2.$$

(c) Since $v_{0x} = 0$, $2ax = v^2$, so $v = (2ax)^{1/2}$, or $v = [(2)(0.92\text{ m/s}^2)(5\text{ m})]^{1/2} = 3\text{ m/s}$.

EVALUATE: When the box starts to move, friction changes from static to kinetic and the friction force becomes smaller.

5.37. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each crate. The rope exerts force T to the right on crate A and force T to the left on crate B. The target variables are the forces T and F. Constant v implies $a = 0$.

SET UP: The free-body diagram for A is sketched in Figure 5.37a.

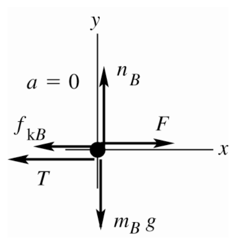


EXECUTE:
$\Sigma F_y = ma_y$
$n_A - m_A g = 0$
$n_A = m_A g$
$f_{kA} = \mu_k n_A = \mu_k m_A g$

Figure 5.37a

$\Sigma F_x = ma_x$
$T - f_{kA} = 0$
$T = \mu_k m_A g$

SET UP: The free-body diagram for *B* is sketched in Figure 5.37b.



EXECUTE:
$\Sigma F_y = ma_y$
$n_B - m_B g = 0$
$n_B = m_B g$
$f_{kB} = \mu_k n_B = \mu_k m_B g$

Figure 5.37b

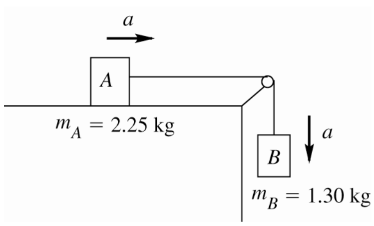
$\Sigma F_x = ma_x$
$F - T - f_{kB} = 0$
$F = T + \mu_k m_B g$
Use the first equation to replace *T* in the second:
$F = \mu_k m_A g + \mu_k m_B g.$
(a) $F = \mu_k (m_A + m_B) g$
(b) $T = \mu_k m_A g$
EVALUATE: We can also consider both crates together as a single object of mass $(m_A + m_B)$. $\Sigma F_x = ma_x$ for this combined object gives $F = f_k = \mu_k (m_A + m_B) g,$ in agreement with our answer in part (a).

5.38. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the box.
SET UP: Let $+y$ be upward and $+x$ be horizontal, in the direction of the acceleration. Constant speed means $a = 0$.
EXECUTE: **(a)** There is no net force in the vertical direction, so $n + F\sin\theta - w = 0,$ or $n = w - F\sin\theta = mg - F\sin\theta.$ The friction force is $f_k = \mu_k n = \mu_k (mg - F\sin\theta).$ The net horizontal force is $F\cos\theta - f_k = F\cos\theta - \mu_k (mg - F\sin\theta),$ and so at constant speed,

$$F = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

(b) Using the given values, $F = \dfrac{(0.35)(90\text{ kg})(9.80\text{ m/s}^2)}{(\cos 25° + (0.35)\sin 25°)} = 290\text{ N}.$
EVALUATE: If $\theta = 0°$, $F = \mu_k mg.$

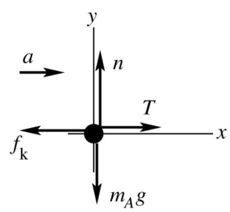
5.39. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. The target variables are the tension T in the cord and the acceleration a of the blocks. Then a can be used in a constant acceleration equation to find the speed of each block. The magnitude of the acceleration is the same for both blocks.
SET UP: The system is sketched in Figure 5.39a.



For each block take a positive coordinate direction to be the direction of the block's acceleration.

Figure 5.39a

Block on the table: The free-body is sketched in Figure 5.39b (next page).

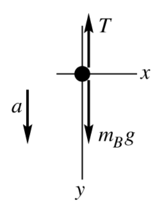


EXECUTE:
$\Sigma F_y = ma_y$
$n - m_A g = 0$
$n = m_A g$
$f_k = \mu_k n = \mu_k m_A g$

Figure 5.39b

$\Sigma F_x = ma_x$
$T - f_k = m_A a$
$T - \mu_k m_A g = m_A a$

SET UP: Hanging block: The free-body is sketched in Figure 5.39c.



EXECUTE:
$\Sigma F_y = ma_y$
$m_B g - T = m_B a$
$T = m_B g - m_B a$

Figure 5.39c

(a) Use the second equation in the first

$$m_B g - m_B a - \mu_k m_A g = m_A a$$

$$(m_A + m_B)a = (m_B - \mu_k m_A)g$$

$$a = \frac{(m_B - \mu_k m_A)g}{m_A + m_B} = \frac{(1.30\text{ kg} - (0.45)(2.25\text{ kg}))(9.80\text{ m/s}^2)}{2.25\text{ kg} + 1.30\text{ kg}} = 0.7937\text{ m/s}^2$$

SET UP: Now use the constant acceleration equations to find the final speed. Note that the blocks have the same speeds. $x - x_0 = 0.0300$ m, $a_x = 0.7937\text{ m/s}^2$, $v_{0x} = 0$, $v_x = ?$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

EXECUTE: $v_x = \sqrt{2a_x(x - x_0)} = \sqrt{2(0.7937\ \text{m/s}^2)(0.0300\ \text{m})} = 0.218\ \text{m/s} = 21.8\ \text{cm/s}.$

(b) $T = m_B g - m_B a = m_B(g - a) = 1.30\ \text{kg}(9.80\ \text{m/s}^2 - 0.7937\ \text{m/s}^2) = 11.7\ \text{N}$

Or, to check, $T - \mu_k m_A g = m_A a$.

$T = m_A(a + \mu_k g) = 2.25\ \text{kg}(0.7937\ \text{m/s}^2 + (0.45)(9.80\ \text{m/s}^2)) = 11.7\ \text{N},$ which checks.

EVALUATE: The force T exerted by the cord has the same value for each block. $T < m_B g$ since the hanging block accelerates downward. Also, $f_k = \mu_k m_A g = 9.92\ \text{N}$. $T > f_k$ and the block on the table accelerates in the direction of T.

5.40. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the ball. At the terminal speed, $f = mg$.

SET UP: The fluid resistance is directed opposite to the velocity of the object. At half the terminal speed, the magnitude of the frictional force is one-fourth the weight.

EXECUTE: **(a)** If the ball is moving up, the frictional force is down, so the magnitude of the net force is $(5/4)w$ and the acceleration is $(5/4)g$, down.

(b) While moving down, the frictional force is up, and the magnitude of the net force is $(3/4)w$ and the acceleration is $(3/4)g$, down.

EVALUATE: The frictional force is less than mg in each case and in each case the net force is downward and the acceleration is downward.

5.41. **(a) IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the crate. Constant v implies $a = 0$. Crate moving says that the friction is kinetic friction. The target variable is the magnitude of the force applied by the woman.

SET UP: The free-body diagram for the crate is sketched in Figure 5.41.

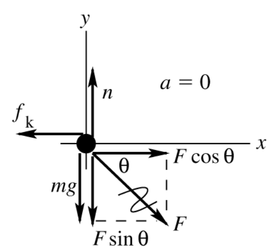


Figure 5.41

EXECUTE:

$\Sigma F_y = ma_y$

$n - mg - F\sin\theta = 0$

$n = mg + F\sin\theta$

$f_k = \mu_k n = \mu_k mg + \mu_k F\sin\theta$

$\Sigma F_x = ma_x$

$F\cos\theta - f_k = 0$

$F\cos\theta - \mu_k mg - \mu_k F\sin\theta = 0$

$F(\cos\theta - \mu_k \sin\theta) = \mu_k mg$

$$F = \frac{\mu_k mg}{\cos\theta - \mu_k \sin\theta}$$

(b) IDENTIFY and **SET UP:** "Start the crate moving" means the same force diagram as in part (a), except that μ_k is replaced by μ_s. Thus $F = \dfrac{\mu_s mg}{\cos\theta - \mu_s \sin\theta}$.

EXECUTE: $F \to \infty$ if $\cos\theta - \mu_s \sin\theta = 0$. This gives $\mu_s = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$.

EVALUATE: $\vec{F}$ has a downward component so $n > mg$. If $\theta = 0$ (woman pushes horizontally), $n = mg$ and $F = f_k = \mu_k mg$.

5.42. **IDENTIFY:** You are accelerated toward the center of the circle, so Newton's second law applies. Static friction is the force preventing you from sliding off the disk.

SET UP: $\sum F = m\dfrac{v^2}{R}$ applies to circular motion. Static friction is at its maximum in this case, so we can use $f_S = \mu_s n$, with $n = mg$. Our target variable is the time for one revolution.

EXECUTE: **(a)** The speed is $v = 2\pi R/T$, so $\sum F = m\dfrac{v^2}{R}$ becomes $f_S = m(2\pi R/T)^2/R$, which becomes $\mu_s mg = m(4\pi^2 R/T^2)$. Solving for T gives $T = \sqrt{\dfrac{4\pi^2 R}{g\mu_s}} = \sqrt{\dfrac{4\pi^2(3.00\text{ m})}{(9.80\text{ m/s}^2)(0.400)}} = 5.50\text{ s}$.

(b) Since $T = \sqrt{\dfrac{4\pi^2 R}{g\mu_s}}$, the period does not depend on the mass (or weight) of the person, so the answer is the same: $T = 5.50$ s.

EVALUATE: In part (a) we could use the formula $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ which leads to the same result.

5.43. **IDENTIFY:** Since the stone travels in a circular path, its acceleration is $a_{\text{rad}} = v^2/R$, directed toward the center of the circle. The only horizontal force on the stone is the tension of the string. Set the tension in the string equal to its maximum value.

SET UP: $\sum F_x = ma_x$ gives $T = m\dfrac{v^2}{R}$.

EXECUTE: **(a)** The free-body diagram for the stone is given in Figure 5.43 (next page). In the diagram the stone is at a point to the right of the center of the path.

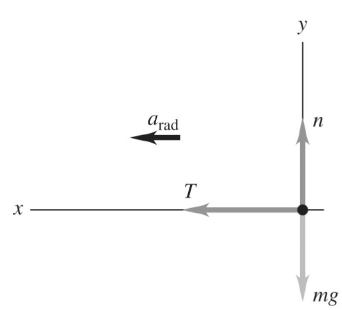


Figure 5.43

(b) Solving for v gives $v = \sqrt{\dfrac{TR}{m}} = \sqrt{\dfrac{(60.0\text{ N})(0.90\text{ m})}{0.80\text{ kg}}} = 8.2\text{ m/s}$.

EVALUATE: The tension is directed toward the center of the circular path of the stone. Gravity plays no role in this case because it is a vertical force and the acceleration is horizontal.

5.44. **IDENTIFY:** The wrist exerts a force on the hand causing the hand to move in a horizontal circle. Newton's second law applies to the hand.

SET UP: Each hand travels in a circle of radius 0.750 m and has mass (0.0125)(52 kg) = 0.65 kg and weight 6.4 N. The period for each hand is $T = (1.0\text{ s})/(2.0) = 0.50$ s. Let $+x$ be toward the center of the circular path. The speed of the hand is $v = 2\pi R/T$, the radial acceleration is $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{4\pi^2 R}{T^2}$, and $\sum F_x = ma_x = ma_{\text{rad}}$.

EXECUTE: **(a)** The free-body diagram for one hand is given in Figure 5.44. $\vec{F}$ is the force exerted on the hand by the wrist. This force has both horizontal and vertical components.

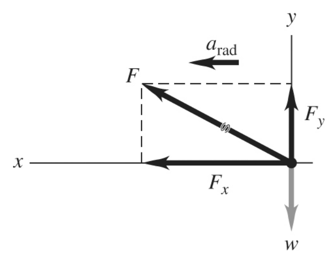


Figure 5.44

(b) $a_{rad} = \dfrac{4\pi^2 R}{T^2} = \dfrac{4\pi^2(0.750\text{ m})}{(0.50\text{ s})^2} = 118\text{ m/s}^2$, so $F_x = ma_{rad} = (0.65\text{ kg})(118\text{ m/s}^2) = 77\text{ N}$.

(c) $\dfrac{F}{w} = \dfrac{77\text{ N}}{6.4\text{ N}} = 12$, so the horizontal force from the wrist is 12 times the weight of the hand.

EVALUATE: The wrist must also exert a vertical force on the hand equal to the weight of the hand.

5.45. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the car. It has acceleration $\vec{a}_{rad}$, directed toward the center of the circular path.

SET UP: The analysis is the same as in Example 5.23.

EXECUTE: **(a)** $F_A = m\left(g + \dfrac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 + \dfrac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = 61.8\text{ N}.$

(b) $F_B = m\left(g - \dfrac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 - \dfrac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = -30.4\text{ N}$, where the minus sign indicates that the track pushes down on the car. The magnitude of this force is 30.4 N.

EVALUATE: $|F_A| > |F_B|$. $|F_A| - 2mg = |F_B|$.

5.46. **IDENTIFY:** The acceleration of the car at the top and bottom is toward the center of the circle, and Newton's second law applies to it.

SET UP: Two forces are acting on the car, gravity and the normal force. At point B (the top), both forces are toward the center of the circle, so Newton's second law gives $mg + n_B = ma$. At point A (the bottom), gravity is downward but the normal force is upward, so $n_A - mg = ma$.

EXECUTE: Solving the equation at B for the acceleration gives
$a = \dfrac{mg + n_B}{m} = \dfrac{(0.800\text{ kg})(9.8\text{ m/s}^2) + 6.00\text{ N}}{0.800\text{ kg}} = 17.3\text{ m/s}^2$. Solving the equation at A for the normal force gives $n_A = m(g + a) = (0.800\text{ kg})(9.8\text{ m/s}^2 + 17.3\text{ m/s}^2) = 21.7\text{ N}$.

EVALUATE: The normal force at the bottom is greater than at the top because it must balance the weight in addition to accelerating the car toward the center of its track.

5.47. **IDENTIFY:** A model car travels in a circle so it has radial acceleration, and Newton's second law applies to it.

SET UP: We use $\Sigma\vec{F} = m\vec{a}$, where the acceleration is $a_{rad} = \dfrac{v^2}{R}$ and the time T for one revolution is $T = 2\pi R/v$.

EXECUTE: At the bottom of the track, taking $+y$ upward, $\Sigma\vec{F} = m\vec{a}$ gives $n - mg = ma$, where n is the normal force. This gives $2.50mg - mg = ma$, so $a = 1.50g$. The acceleration is $a_{\text{rad}} = \dfrac{v^2}{R}$, so

$v = \sqrt{aR} = \sqrt{(1.50)(9.80\ \text{m/s}^2)(5.00\ \text{m})} = 8.573$ m/s, so $T = 2\pi R/v = 2\pi(5.00\ \text{m})/(8.573\ \text{m}) = 3.66$ s.

EVALUATE: We never need the mass of the car because we know the acceleration as a fraction of g and the force as a fraction of mg.

5.48. **IDENTIFY:** Since the car travels in an arc of a circle, it has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the arc. The only horizontal force on the car is the static friction force exerted by the roadway. To calculate the minimum coefficient of friction that is required, set the static friction force equal to its maximum value, $f_s = \mu_s n$. Friction is static friction because the car is not sliding in the radial direction.

SET UP: The free-body diagram for the car is given in Figure 5.48 (next page). The diagram assumes the center of the curve is to the left of the car.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ gives $n = mg$. $\Sigma F_x = ma_x$ gives $\mu_s n = m\dfrac{v^2}{R}$. $\mu_s mg = m\dfrac{v^2}{R}$ and

$$\mu_s = \frac{v^2}{gR} = \frac{(25.0\ \text{m/s})^2}{(9.80\ \text{m/s}^2)(170\ \text{m})} = 0.375$$

(b) $\dfrac{v^2}{\mu_s} = Rg = \text{constant}$, so $\dfrac{v_1^2}{\mu_{s1}} = \dfrac{v_2^2}{\mu_{s2}}$. $v_2 = v_1\sqrt{\dfrac{\mu_{s2}}{\mu_{s1}}} = (25.0\ \text{m/s})\sqrt{\dfrac{\mu_{s1}/3}{\mu_{s1}}} = 14.4$ m/s.

EVALUATE: A smaller coefficient of friction means a smaller maximum friction force, a smaller possible acceleration and therefore a smaller speed.

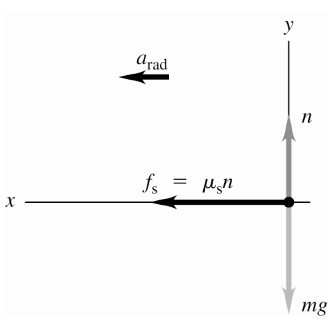


Figure 5.48

5.49. **IDENTIFY:** Apply Newton's second law to the car in circular motion, assume friction is negligible.

SET UP: The acceleration of the car is $a_{\text{rad}} = v^2/R$. As shown in the text, the banking angle β is given by $\tan\beta = \dfrac{v^2}{gR}$. Also, $n = mg/\cos\beta$. $65.0\ \text{mi/h} = 29.1\ \text{m/s}$.

EXECUTE: **(a)** $\tan\beta = \dfrac{(29.1\ \text{m/s})^2}{(9.80\ \text{m/s}^2)(225\ \text{m})}$ and $\beta = 21.0°$. The expression for $\tan\beta$ does not involve the mass of the vehicle, so the truck and car should travel at the same speed.

(b) For the car, $n_{\text{car}} = \dfrac{(1125\ \text{kg})(9.80\ \text{m/s}^2)}{\cos 21.0°} = 1.18\times10^4$ N and $n_{\text{truck}} = 2n_{\text{car}} = 2.36\times10^4$ N, since $m_{\text{truck}} = 2m_{\text{car}}$.

EVALUATE: The vertical component of the normal force must equal the weight of the vehicle, so the normal force is proportional to m.

5.50. **IDENTIFY:** The acceleration of the person is $a_{\text{rad}} = v^2/R$, directed horizontally to the left in the figure in the problem. The time for one revolution is the period $T = \dfrac{2\pi R}{v}$. Apply $\Sigma\vec{F} = m\vec{a}$ to the person.

SET UP: The person moves in a circle of radius $R = 3.00\text{ m} + (5.00\text{ m})\sin 30.0° = 5.50\text{ m}$. The free-body diagram is given in Figure 5.50. $\vec{F}$ is the force applied to the seat by the rod.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ gives $F\cos 30.0° = mg$ and $F = \dfrac{mg}{\cos 30.0°}$. $\Sigma F_x = ma_x$ gives $F\sin 30.0° = m\dfrac{v^2}{R}$. Combining these two equations gives

$v = \sqrt{Rg\tan\theta} = \sqrt{(5.50\text{ m})(9.80\text{ m/s}^2)\tan 30.0°} = 5.58\text{ m/s}$. Then the period is

$$T = \frac{2\pi R}{v} = \frac{2\pi(5.50\text{ m})}{5.58\text{ m/s}} = 6.19\text{ s}.$$

(b) The net force is proportional to m so in $\Sigma\vec{F} = m\vec{a}$ the mass divides out and the angle for a given rate of rotation is independent of the mass of the passengers.

EVALUATE: The person moves in a horizontal circle so the acceleration is horizontal. The net inward force required for circular motion is produced by a component of the force exerted on the seat by the rod.

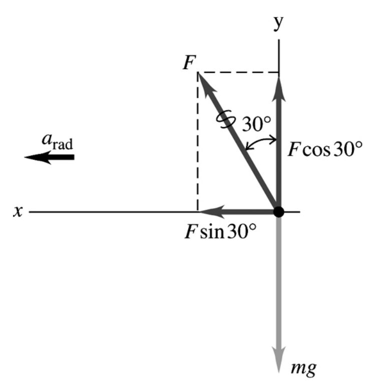


Figure 5.50

5.51. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object of the person plus seat. This object moves in a horizontal circle and has acceleration a_{rad}, directed toward the center of the circle.

SET UP: The free-body diagram for the composite object is given in Figure 5.51. Let $+x$ be to the right, in the direction of $\vec{a}_{\text{rad}}$. Let $+y$ be upward. The radius of the circular path is $R = 7.50\text{ m}$. The total mass is $(255\text{ N} + 825\text{ N})/(9.80\text{ m/s}^2) = 110.2\text{ kg}$. Since the rotation rate is

$28.0\text{ rev/min} = 0.4667\text{ rev/s}$, the period T is $\dfrac{1}{0.4667\text{ rev/s}} = 2.143\text{ s}$.

EXECUTE: $\Sigma F_y = ma_y$ gives $T_A\cos 40.0° - mg = 0$ and $T_A = \dfrac{mg}{\cos 40.0°} = \dfrac{255\text{ N} + 825\text{ N}}{\cos 40.0°} = 1410\text{ N}$.

$\Sigma F_x = ma_x$ gives $T_A\sin 40.0° + T_B = ma_{\text{rad}}$ and

$$T_B = m\frac{4\pi^2 R}{T^2} - T_A\sin 40.0° = (110.2\text{ kg})\frac{4\pi^2(7.50\text{ m})}{(2.143\text{ s})^2} - (1410\text{ N})\sin 40.0° = 6200\text{ N}$$

The tension in the horizontal cable is 6200 N and the tension in the other cable is 1410 N.

EVALUATE: The weight of the composite object is 1080 N. The tension in cable A is larger than this since its vertical component must equal the weight. The tension in cable B is less than ma_{rad} because part of the required inward force comes from a component of the tension in cable A.

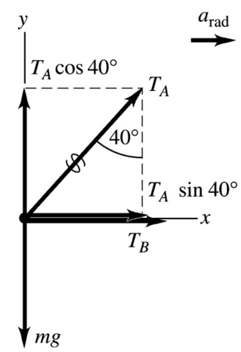


Figure 5.51

5.52. IDENTIFY: Newton's second law applies to the steel ball. Gravity and the tension in the rope are the forces acting on it.

SET UP: $\sum F = m\dfrac{v^2}{R}$

EXECUTE: **(a)** At the lowest point, the tension T is upward and the weight $w = mg$ is downward. $\sum F = m\dfrac{v^2}{R}$ gives $T - mg = ma$, which becomes $3mg - mg = \dfrac{mv^2}{R}$. Solving for v gives $v = \sqrt{2Rg} = \sqrt{2(15.0\text{ m})(9.80\text{ m/s}^2)} = 17.1$ m/s.

(b) If $T = mg$, the vertical forces balance, so $\sum F = m\dfrac{v^2}{R} = 0$, which tells us that the speed is zero.

EVALUATE: If $T = mg$, the ball is just hanging by the rope.

5.53. IDENTIFY: The acceleration due to circular motion is $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$.

SET UP: $R = 400$ m. $1/T$ is the number of revolutions per second.

EXECUTE: **(a)** Setting $a_{\text{rad}} = g$ and solving for the period T gives

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{400\text{ m}}{9.80\text{ m/s}^2}} = 40.1\text{ s},$$

so the number of revolutions per minute is $(60\text{ s/min})/(40.1\text{ s}) = 1.5$ rev/min.

(b) The lower acceleration corresponds to a longer period, and hence a lower rotation rate, by a factor of the square root of the ratio of the accelerations, $T' = (1.5\text{ rev/min}) \times \sqrt{3.70/9.8} = 0.92$ rev/min.

EVALUATE: In part (a) the tangential speed of a point at the rim is given by $a_{\text{rad}} = \dfrac{v^2}{R}$, so $v = \sqrt{Ra_{\text{rad}}} = \sqrt{Rg} = 62.6$ m/s; the space station is rotating rapidly.

5.54. IDENTIFY: $T = \dfrac{2\pi R}{v}$. The apparent weight of a person is the normal force exerted on him by the seat he is sitting on. His acceleration is $a_{\text{rad}} = v^2/R$, directed toward the center of the circle.

SET UP: The period is $T = 60.0$ s. The passenger has mass $m = w/g = 90.0$ kg.

EXECUTE: **(a)** $v = \dfrac{2\pi R}{T} = \dfrac{2\pi(50.0 \text{ m})}{60.0 \text{ s}} = 5.24 \text{ m/s}.$ Note that $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(5.24 \text{ m/s})^2}{50.0 \text{ m}} = 0.549 \text{ m/s}^2.$

(b) The free-body diagram for the person at the top of his path is given in Figure 5.54a. The acceleration is downward, so take $+y$ downward. $\Sigma F_y = ma_y$ gives $mg - n = ma_{\text{rad}}$.

$n = m(g - a_{\text{rad}}) = (90.0 \text{ kg})(9.80 \text{ m/s}^2 - 0.549 \text{ m/s}^2) = 833 \text{ N}.$

The free-body diagram for the person at the bottom of his path is given in Figure 5.54b. The acceleration is upward, so take $+y$ upward. $\Sigma F_y = ma_y$ gives $n - mg = ma_{\text{rad}}$ and

$n = m(g + a_{\text{rad}}) = 931 \text{ N}.$

(c) Apparent weight $= 0$ means $n = 0$ and $mg = ma_{\text{rad}}$. $g = \dfrac{v^2}{R}$ and $v = \sqrt{gR} = 22.1 \text{ m/s}.$ The time for one revolution would be $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(50.0 \text{ m})}{22.1 \text{ m/s}} = 14.2 \text{ s}.$ Note that $a_{\text{rad}} = g$.

(d) $n = m(g + a_{\text{rad}}) = 2mg = 2(882 \text{ N}) = 1760 \text{ N},$ twice his true weight.

EVALUATE: At the top of his path his apparent weight is less than his true weight and at the bottom of his path his apparent weight is greater than his true weight.

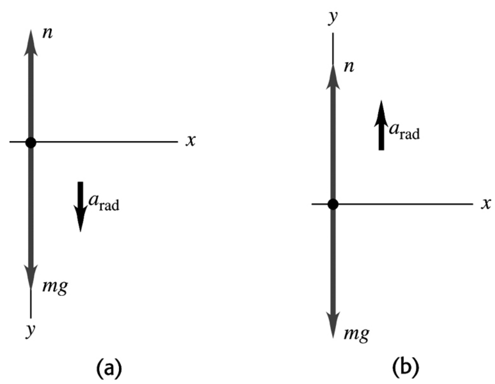


Figure 5.54

5.55. **IDENTIFY:** Newton's second law applies to the rock moving in a vertical circle of radius L. Gravity and the tension in the string are the forces acting on it.

SET UP: $\sum F = m\dfrac{v^2}{R}$, where $R = L$ in this case.

EXECUTE: **(a)** Apply $\sum F = m\dfrac{v^2}{R}$ at the top of the circle. Both gravity and the tension act downward, so $T + mg = mv^2/L$. The smallest that T can be is zero, in which case $mg = mv^2/L$, so $v = \sqrt{Lg}$.

(b) In this case, $v = 2\sqrt{Lg}$. $\sum F = m\dfrac{v^2}{R}$ gives $T - mg = \dfrac{mv^2}{L} = \dfrac{m\left(2\sqrt{Lg}\right)^2}{L} = 4mg$, so $T = 5mg$.

EVALUATE: Note in (a) that the rock does *not stop* at the top of the circle. If it did, it would just fall down and not complete the rest of the circle.

5.56. **IDENTIFY:** $a_{\text{rad}} = v^2/R$, directed toward the center of the circular path. At the bottom of the dive, $\vec{a}_{\text{rad}}$ is upward. The apparent weight of the pilot is the normal force exerted on her by the seat on which she is sitting.

SET UP: The free-body diagram for the pilot is given in Figure 5.56.

EXECUTE: **(a)** $a_{\text{rad}} = \dfrac{v^2}{R}$ gives $R = \dfrac{v^2}{a_{\text{rad}}} = \dfrac{(95.0\text{ m/s})^2}{4.00(9.80\text{ m/s}^2)} = 230\text{ m}.$

(b) $\Sigma F_y = ma_y$ gives $n - mg = ma_{\text{rad}}$.

$n = m(g + a_{\text{rad}}) = m(g + 4.00g) = 5.00mg = (5.00)(50.0\text{ kg})(9.80\text{ m/s}^2) = 2450\text{ N}$

EVALUATE: Her apparent weight is five times her true weight, the force of gravity the earth exerts on her.

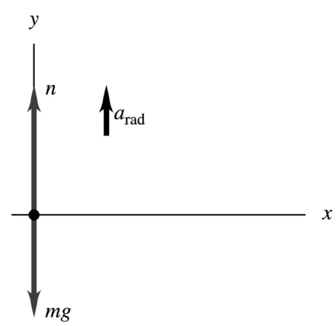
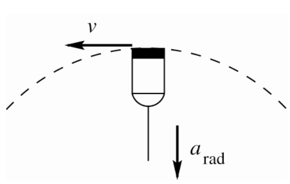


Figure 5.56

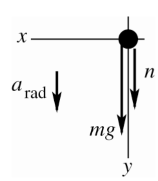
5.57. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the water. The water moves in a vertical circle. The target variable is the speed v; we will calculate a_{rad} and then get v from $a_{\text{rad}} = v^2/R$.

SET UP: Consider the free-body diagram for the water when the pail is at the top of its circular path, as shown in Figures 5.57a and b.



The radial acceleration is in toward the center of the circle so at this point is downward. *n* is the downward normal force exerted on the water by the bottom of the pail.

Figure 5.57a



EXECUTE:

$\Sigma F_y = ma_y$

$n + mg = m\dfrac{v^2}{R}$

Figure 5.57b

At the minimum speed the water is just ready to lose contact with the bottom of the pail, so at this speed, $n \to 0$. (Note that the force n cannot be upward.)

With $n \to 0$ the equation becomes $mg = m\dfrac{v^2}{R}$. $v = \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(0.600\text{ m})} = 2.42\text{ m/s}.$

EVALUATE: At the minimum speed $a_{\text{rad}} = g$. If v is less than this minimum speed, gravity pulls the water (and bucket) out of the circular path.

5.58. **IDENTIFY:** The ball has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the circular path. When the ball is at the bottom of the swing, its acceleration is upward.

SET UP: Take $+y$ upward, in the direction of the acceleration. The bowling ball has mass $m = w/g = 7.27\text{ kg}$.

EXECUTE: **(a)** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(4.20\text{ m/s})^2}{3.80\text{ m}} = 4.64\text{ m/s},$ upward.

(b) The free-body diagram is given in Figure 5.58. $\Sigma F_y = ma_y$ gives $T - mg = ma_{\text{rad}}$.

$T = m(g + a_{\text{rad}}) = (7.27\text{ kg})(9.80\text{ m/s}^2 + 4.64\text{ m/s}^2) = 105\text{ N}$

EVALUATE: The acceleration is upward, so the net force is upward and the tension is greater than the weight.

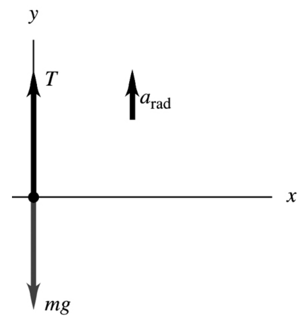


Figure 5.58

5.59. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the knot.

SET UP: $a = 0$. Use coordinates with axes that are horizontal and vertical.

EXECUTE: **(a)** The free-body diagram for the knot is sketched in Figure 5.59.

T_1 is more vertical so supports more of the weight and is larger. You can also see this from $\Sigma F_x = ma_x$:

$T_2\cos 40° - T_1\cos 60° = 0.$ $T_2\cos 40° - T_1\cos 60° = 0.$

(b) T_1 is larger so set $T_1 = 5000\text{ N}$. Then $T_2 = T_1/1.532 = 3263.5\text{ N}$. $\Sigma F_y = ma_y$ gives

$T_1\sin 60° + T_2\sin 40° = w$ and $w = 6400\text{ N}$.

EVALUATE: The sum of the vertical components of the two tensions equals the weight of the suspended object. The sum of the tensions is greater than the weight.

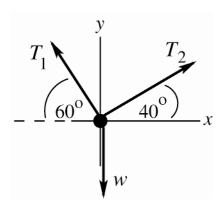
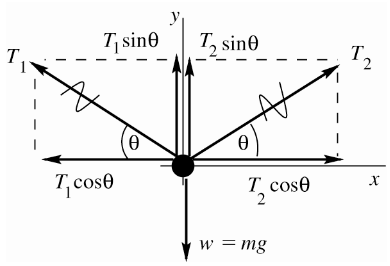


Figure 5.59

5.60. **IDENTIFY:** Apply Newton's first law to the person. Each half of the rope exerts a force on him, directed along the rope and equal to the tension T in the rope.

SET UP: **(a)** The force diagram for the person is given in Figure 5.60.



T_1 and T_2 are the tensions in each half of the rope.

Figure 5.60

EXECUTE: $\Sigma F_x = 0$

$T_2 \cos\theta - T_1 \cos\theta = 0$

This says that $T_1 = T_2 = T$ (The tension is the same on both sides of the person.)

$\Sigma F_y = 0$

$T_1 \sin\theta + T_2 \sin\theta - mg = 0$

But $T_1 = T_2 = T$, so $2T\sin\theta = mg$

$$T = \frac{mg}{2\sin\theta} = \frac{(90.0\text{ kg})(9.80\text{ m/s}^2)}{2\sin 10.0°} = 2540\text{ N}$$

(b) The relation $2T\sin\theta = mg$ still applies but now we are given that $T = 2.50\times10^4$ N (the breaking strength) and are asked to find θ.

$$\sin\theta = \frac{mg}{2T} = \frac{(90.0\text{ kg})(9.80\text{ m/s}^2)}{2(2.50\times10^4\text{ N})} = 0.01764,\quad \theta = 1.01°.$$

EVALUATE: $T = mg/(2\sin\theta)$ says that $T = mg/2$ when $\theta = 90°$ (rope is vertical).

$T \to \infty$ when $\theta \to 0$ since the upward component of the tension becomes a smaller fraction of the tension.

5.61. **IDENTIFY:** The engine is hanging at rest, so its acceleration is zero which means that the forces on it must balance. We balance horizontal components and vertical components.

SET UP: In addition to the tensions in the four cables shown in the text, gravity also acts on the engine. Call $+x$ horizontally to the right and $+y$ vertically upward, and call θ the angle that cable C makes with cable D. The mass of the engine is 409 kg and the tension T_A in cable A is 722 N.

EXECUTE: The tension in cable D is the only force balancing gravity on the engine, so $T_D = mg$. In the x-direction, we have $T_A = T_C \sin\theta$, which gives $T_C = T_A/\sin\theta$ = (722 N)/(sin 37.1°) = 1197 N. In the y-direction, we have $T_B - T_D - T_C \cos\theta = 0$, which gives T_B = (409 kg)(9.80 m/s^2) + (1197 N)cos(37.1°) = 4963 N. Rounding to 3 significant figures gives T_B = 4960 N and T_C = 1200 N.

EVALUATE: The tension in cable B is greater than the weight of the engine because cable C has a downward component that B must also balance.

5.62. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each object. Constant speed means $a = 0$.

SET UP: The free-body diagrams are sketched in Figure 5.62. T_1 is the tension in the lower chain, T_2 is the tension in the upper chain and $T = F$ is the tension in the rope.

EXECUTE: The tension in the lower chain balances the weight and so is equal to w. The lower pulley must have no net force on it, so twice the tension in the rope must be equal to w and the tension in the rope, which equals F, is $w/2$. Then, the downward force on the upper pulley due to the rope is also w, and so the upper chain exerts a force w on the upper pulley, and the tension in the upper chain is also w.

EVALUATE: The pulley combination allows the worker to lift a weight w by applying a force of only $w/2$.

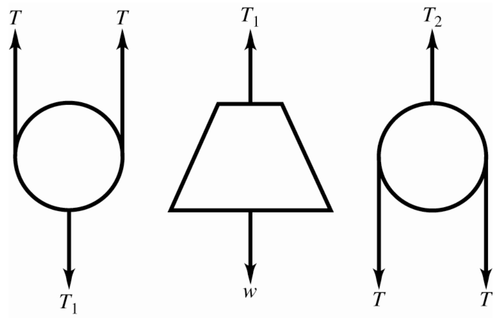


Figure 5.62

5.63. **IDENTIFY:** Newton's second law applies to the block. Gravity, kinetic friction, and the normal force due to the board act upon it.

SET UP: Apply $\sum F_x = ma_x$ and $\sum F_y = 0$ to the block. Choose the +*x*-axis along the surface of the board pointing downward. At the maximum angle α_0 just before slipping, tan α_0 = μ_s = 0.600, so α_0 = 30.96°. Fig. 5.63 shows a free-body diagram of the block after it has slipped.

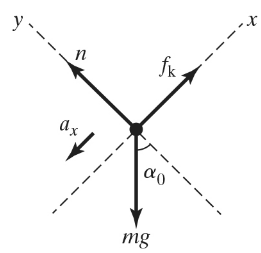


Figure 5.63

EXECUTE: First find the acceleration using Newton's laws. $\sum F_y = 0$ gives $n = mg\cos\alpha_0$ $\sum F_x = ma_x$ gives

$mg\sin\alpha_0 - f_k = ma_x$

$mg\sin\alpha_0 - \mu_k mg\cos\alpha_0 = ma_x$

$a_x = g(\sin\alpha_0 - \mu_k\cos\alpha_0) = (9.80\text{ m/s}^2)[\sin 30.96° - (0.400)\cos 30.96°] = 1.681\text{ m/s}^2$. Now use

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find the speed at the bottom of the board.

$v_x^2 = 0 + 2(1.681\text{ m/s}^2)(3.00\text{ m}) \quad \rightarrow \quad v_x = 3.18\text{ m/s}$.

EVALUATE: We chose the +*x*-axis to be downward because that is the direction of the acceleration. In most cases, it is easiest to make that choice if the direction of the acceleration is known.

5.64. **IDENTIFY:** Apply Newton's first law to the ball. Treat the ball as a particle.

SET UP: The forces on the ball are gravity, the tension in the wire and the normal force exerted by the surface. The normal force is perpendicular to the surface of the ramp. Use *x*- and *y*-axes that are horizontal and vertical.

EXECUTE: **(a)** The free-body diagram for the ball is given in Figure 5.64 (next page). The normal force has been replaced by its *x* and *y* components.

(b) $\Sigma F_y = 0$ gives $n\cos 35.0° - w = 0$ and $n = \dfrac{mg}{\cos 35.0°} = 1.22mg$.

(c) $\Sigma F_x = 0$ gives $T - n\sin 35.0° = 0$ and $T = (1.22mg)\sin 35.0° = 0.700mg$.

EVALUATE: Note that the normal force is greater than the weight, and increases without limit as the angle of the ramp increases toward $90°$. The tension in the wire is $w\tan\phi$, where ϕ is the angle of the ramp and T also increases without limit as $\phi \to 90°$.

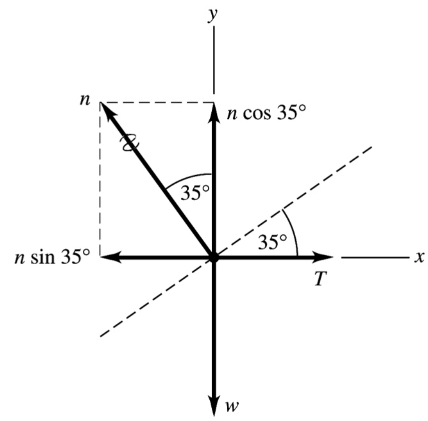


Figure 5.64

5.65. **IDENTIFY:** Newton's second law applies to the accelerating box. The forces acting on it are the force F, gravity, the normal force due to the surface, and kinetic friction.

SET UP: Apply $\sum F_y = 0$ and $\sum F_x = ma_x$. Fig. 5.65 shows a free-body diagram of the box. Choose the $+x$-axis in the direction of the acceleration.

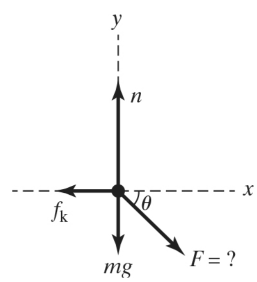


Figure 5.65

EXECUTE: **(a)** We want to find the magnitude of the force F. First use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find a_x.

$(6.00\text{ m/s})^2 = 0 + 2a_x(8.00\text{ m}) \quad \to \quad a_x = 2.250\text{ m/s}^2$.

Now apply $\sum F_x = ma_x$: $F\cos\theta - f_k = ma_x$

We also have $f_k = \mu_k n$

Apply $\sum F_y = 0$: $n - mg - F\sin\theta = 0$

Combine these three results and solve for F: $F = \dfrac{m(a_x + \mu_k g)}{\cos\theta - \mu_k \sin\theta}$. Using $m = 12.0$ kg, $a_x = 2.250\text{ m/s}^2$, $\mu_k = 0.300$, and $\theta = 37.0°$, we get $F = 101$ N.

(b) If $f_k = 0$, we have $F\cos\theta = ma_x \quad \to \quad F\cos 37.0° = (12.0\text{ kg})(2.250\text{ m/s}^2) \quad \to \quad F = 33.8$ N.

(c) If F is horizontal, $\theta = 0°$, $n = mg$, and $f_k = \mu_k mg$, so $F = m(a_x + \mu_k g) = 62.3$ N.

EVALUATE: We see that F is least when there is no friction and greatest when it has a downward component. These results are reasonable since a downward component increases the normal force which increases friction. And the fact that it has a downward component means that there is less horizontal component to cause acceleration.

5.66. **IDENTIFY:** In each rough patch, the kinetic friction (and hence the acceleration) is constant, but the constants are different in the two patches. Newton's second law applies, as well as the constant-acceleration kinematics formulas in each patch.

SET UP: Choose the $+y$-axis upward and the $+x$-axis in the direction of the velocity.

EXECUTE: **(a)** Find the velocity and time when the box is at $x = 2.00$ m. Newton's second law tells us that $n = mg$ and $-f_k = ma_x$ which gives $-\mu_k mg = ma_x$; $a_x = -\mu_k g = -(0.200)(9.80\text{ m/s}^2) = -1.96\text{ m/s}^2$. Now use the kinematics equations involving v_x. Using $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ we get

$v_x = \sqrt{(4.00\text{ m/s})^2 + 2(-1.96\text{ m/s}^2)(2.00\text{ m})} = 2.857$ m/s. Now solve the equation $v_x = v_{0x} + a_x t$ for t to get $t = (2.857\text{ m/s} - 4.00\text{ m/s})/(-1.96\text{ m/s}^2) = 0.5834$ s.

Now look at the motion in the section for which $\mu_k = 0.400$: $a_x = -(0.400)(9.80\text{ m/s}^2) = -3.92\text{ m/s}^2$, $v_x = 0$, $v_{0x} =$ 2.857 m/s. Solving $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ for $x - x_0$ gives $x - x_0 = -(2.857\text{ m/s})^2/[2(-3.92\text{ m/s}^2)] = 1.041$ m.

The box is at the point $x = 2.00\text{ m} + 1.041\text{ m} = 3.04$ m.

Solving $v_x = v_{0x} + a_x t$ for t gives $t = (-2.857\text{ m/s})/(-3.92\text{ m/s}^2) = 0.7288$ s. The total time is $0.5834\text{ s} + 0.7288\text{ s} = 1.31$ s.

EVALUATE: We cannot do this problem in a single process because the acceleration, although constant in each patch, is different in the two patches.

5.67. **IDENTIFY:** Kinematics will give us the acceleration of the person, and Newton's second law will give us the force (the target variable) that his arms exert on the rest of his body.

SET UP: Let the person's weight be W, so $W = 680$ N. Assume constant acceleration during the speeding up motion and assume that the body moves upward 15 cm in 0.50 s while speeding up. The constant-acceleration kinematics formula $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ and $\Sigma F_y = ma_y$ apply. The free-body diagram for the person is given in Figure 5.67. F is the force exerted on him by his arms.

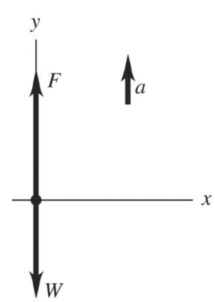


Figure 5.67

EXECUTE: $v_{0y} = 0$, $y - y_0 = 0.15$ m, $t = 0.50$ s. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$a_y = \frac{2(y - y_0)}{t^2} = \frac{2(0.15\text{ m})}{(0.50\text{ s})^2} = 1.2\text{ m/s}^2.\quad \Sigma F_y = ma_y \text{ gives } F - W = ma.\quad m = \frac{W}{g}, \text{ so}$$

$$F = W\left(1 + \frac{a}{g}\right) = 1.12W = 762\text{ N}.$$

EVALUATE: The force is greater than his weight, which it must be if he is to accelerate upward.

5.68. **IDENTIFY:** The force is time-dependent, so the acceleration is not constant. Therefore we must use calculus instead of the standard kinematics formulas. Newton's second law applies.

SET UP: The acceleration is the time derivative of the velocity and $\Sigma F_y = ma_y$.

EXECUTE: Differentiating the velocity gives $a_y = dv_y/dt = 2.00\text{ m/s}^2 + (1.20\text{ m/s}^3)t$. Find the time when $v_y = 9.00$ m/s: $9.00\text{ m/s} = (2.00\text{ m/s}^2)t + (0.600\text{ m/s}^3)t^2$. Solving this quadratic for t and taking the positive value gives $t = 2.549$ s. At this time the acceleration is $a = 2.00\text{ m/s}^2 + (1.20\text{ m/s}^3)(2.549\text{ s}) = 5.059\text{ m/s}^2$.

Now apply Newton's second law to the box, calling T the tension in the rope: $T - mg = ma$, which gives $T = m(g + a) = (2.00\text{ kg})(9.80\text{ m/s}^2 + 5.059\text{ m/s}^2) = 29.7$ N.

EVALUATE: The tension is greater than the weight of the box, which it must be to accelerate the box upward. As time goes on, the acceleration, and hence the tension, would increase.

5.69. **IDENTIFY:** We know the forces on the box and want to find information about its position and velocity. Newton's second law will give us the box's acceleration.

SET UP: $a_y(t) = \dfrac{\Sigma F_y}{m}$. We can integrate the acceleration to find the velocity and the velocity to find the position. At an altitude of several hundred meters, the acceleration due to gravity is essentially the same as it is at the earth's surface.

EXECUTE: Let $+y$ be upward. Newton's second law gives $T - mg = ma_y$, so $a_y(t) = (12.0\text{ m/s}^3)t - 9.8\text{ m/s}^2$. Integrating the acceleration gives $v_y(t) = (6.00\text{ m/s}^3)t^2 - (9.8\text{ m/s}^2)t$.

(a) (i) At $t = 1.00$ s, $v_y = -3.80$ m/s. (ii) At $t = 3.00$ s, $v_y = 24.6$ m/s.

(b) Integrating the velocity gives $y - y_0 = (2.00\text{ m/s}^3)t^3 - (4.9\text{ m/s}^2)t^2$. $v_y = 0$ at $t = 1.63$ s. At $t = 1.63$ s, $y - y_0 = 8.71\text{ m} - 13.07\text{ m} = -4.36$ m.

(c) Setting $y - y_0 = 0$ and solving for t gives $t = 2.45$ s.

EVALUATE: The box accelerates and initially moves downward until the tension exceeds the weight of the box. Once the tension exceeds the weight, the box will begin to accelerate upward and will eventually move upward, as we saw in part (b).

5.70. **IDENTIFY:** We can use the standard kinematics formulas because the force (and hence the acceleration) is constant, and we can use Newton's second law to find the force needed to cause that acceleration. Kinetic friction, not static friction, is acting.

SET UP: From kinematics, we have $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and $\Sigma F_x = ma_x$ applies. Forces perpendicular to the ramp balance. The force of kinetic friction is $f_k = \mu_k mg\cos\theta$.

EXECUTE: Call $+x$ upward along the surface of the ramp. Kinematics gives $a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(8.00\text{ m})}{(6.00\text{ s})^2} = 0.4444\text{ m/s}^2$. $\Sigma F_x = ma_x$ gives $F - mg\sin\theta - \mu_k mg\cos\theta = ma_x$. Solving for F and putting in the numbers for this problem gives
$F = m(a_x + g\sin\theta + \mu_k mg\cos\theta) = (5.00\text{ kg})(0.4444\text{ m/s}^2 + 4.9\text{ m/s}^2 + 3.395\text{ m/s}^2) = 43.7$ N.

EVALUTE: As long as the box is moving, only kinetic friction, not static friction, acts on it. The force is less than the weight of the box because only part of the box's weight acts down the ramp. We should also investigate if the force is great enough to start the box moving in the first place. In that case, static friction would have it maximum value, so $f_s = \mu_s n$. The force F in this would be $F = \mu_s mg\cos(30°) + mg\sin(30°) = mg(\mu_s\cos30° + \sin30°) = (5.00\text{ kg})(9.80\text{ m/s}^2)[(0.43)(\cos30°) + \sin30°] = 42.7$ N. Since the force we found is 43.7 N, it is great enough to overcome static friction and cause the box to move.

5.71. **IDENTIFY:** Newton's second law applies to the accelerating crate. The forces acting on it are the vertical force $\vec{F}$, gravity, the normal force due to the surface, and kinetic friction.

SET UP: Apply $\sum F_x = ma_x$. Fig. 5.71 shows a free-body diagram of the crate. Choose the +*x*-axis in the direction of the acceleration, which is down the surface of the ramp. Call α the angle the ramp makes above the horizontal. We want to find the magnitude of $\vec{F}$ that will give the crate an acceleration of 9.8 m/s^2 down the ramp.

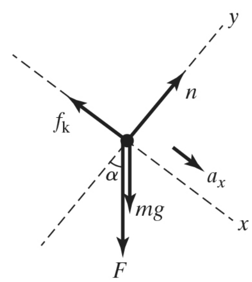


Figure 5.71

EXECUTE: First apply $\sum F_x = ma_x$ without the force $\vec{F}$ and then apply it with $\vec{F}$. Without $\vec{F}$ the normal force is $n = mg\cos\alpha$ and with $\vec{F}$ it is $n = (mg+F)\cos\alpha$.

Without the force: $mg\sin\alpha - \mu_k mg\cos\alpha = ma_1$.

With the force: $(mg+F)\sin\alpha - \mu_k(mg+F)\cos\alpha = ma_2$.

We know that $a_2/a_1 = (9.80\ \text{m/s}^2)/(4.9\ \text{m/s}^2) = 2$, so take the ratio of the equations, giving
$\dfrac{a_2}{a_1} = \dfrac{(mg+F)\sin\alpha - \mu_k(mg+F)\cos\alpha}{mg\sin\alpha - \mu_k mg\cos\alpha} = \dfrac{(mg+F)(\sin\alpha - \mu_k\cos\alpha)}{mg(\sin\alpha - \mu_k\cos\alpha)} = \dfrac{mg+F}{mg} = 2$. Solving for F gives $F = mg = (25.0\ \text{kg})(9.80\ \text{m/s}^2) = 245$ N. The force F is equal to the weight of the crate.

EVALUATE: The addition of the force F doubles the acceleration of the crate. Since F and mg are both downward, we need twice the force to double the acceleration, which tells us that F must be equal to the weight. So our result is reasonable. The addition of $F = mg$ effectively doubles the weight without changing the mass, so it doubles the acceleration.

5.72. **IDENTIFY:** The forces acting on the crate are the force F, gravity, the normal force due to the floor, and static friction. The crate is at rest, so the forces on it must balance. The crate is just ready to slide, so static friction force is a maximum.

SET UP: Apply $\sum F_x = 0$ and $\sum F_y = 0$. Fig. 5.72 shows a free-body diagram of the box. Choose the +*x*-axis horizontally to the right. At its maximum, $f_s = \mu_s n$. We want to find the mass m of the crate.

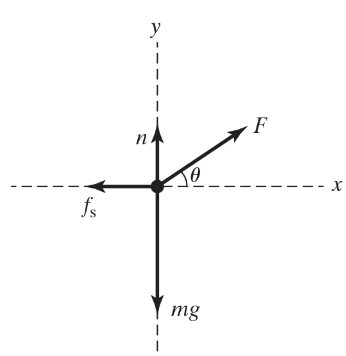


Figure 5.72

EXECUTE: $\sum F_x = 0$: $F\cos\theta - f_s = F\cos\theta - \mu_s n = 0$

$\sum F_y = 0$: $F\sin\theta + n - mg = 0$

Combine these two equations and solve for m: $m = \dfrac{F(\cos\theta + \mu_s \sin\theta)}{\mu_s g}$. Using $F = 380$ N, $\theta = 30.0°$, and $\mu_s = 0.400$ gives $m = 103$ kg.

EVALUATE: If the crate were not just ready to slide, we could *not* use $f_s = \mu_s n$.

5.73. **IDENTIFY:** Newton's second law applies to the box.

SET UP: $f_k = \mu_k n$, $\Sigma F_x = ma_x$, and $\Sigma F_y = ma_y$ apply to the box. Take the $+x$-axis down the surface of the ramp and the $+y$-axis perpendicular to the surface upward.

EXECUTE: $\Sigma F_y = ma_y$ gives $n + F\sin(33.0°) - mg\cos(33.0°) = 0$, which gives $n = 51.59$ N. The friction force is $f_k = \mu_k n = (0.300)(51.59\text{ N}) = 15.48$ N. Parallel to the surface we have $\Sigma F_x = ma_x$ which gives $F\cos(33.0°) + mg\sin(33.0°) - f_k = ma$, which gives $a = 6.129\text{ m/s}^2$. Finally the velocity formula gives us $v_x = v_{0x} + a_x t = 0 + (6.129\text{ m/s}^2)(2.00\text{ s}) = 12.3$ m/s.

EVALUATE: Even though F is horizontal and mg is vertical, it is best to choose the axes as we have done, rather than horizontal-vertical, because the acceleration is then in the x-direction. Taking x and y to be horizontal-vertical would give the acceleration x- and y-components, which would complicate the solution.

5.74. **IDENTIFY:** This is a system having constant acceleration, so we can use the standard kinematics formulas as well as Newton's second law to find the unknown mass m_2.

SET UP: Newton's second law applies to each block. The standard kinematics formulas can be used to find the acceleration because the acceleration is constant. The normal force on m_1 is $m_1 g\cos\alpha$, so the force of friction on it is $f_k = \mu_k m_1 g\cos\alpha$.

EXECUTE: Standard kinematics gives the acceleration of the system to be

$a_y = \dfrac{2(y-y_0)}{t^2} = \dfrac{2(12.0\text{ m})}{(3.00\text{ s})^2} = 2.667\text{ m/s}^2$. For m_1, $n = m_1 g\cos\alpha = 117.7$ N, so the friction force on m_1 is $f_k = (0.40)(117.7\text{ N}) = 47.08$ N. Applying Newton's second law to m_1 gives $T - f_k - m_1 g\sin\alpha = m_1 a$, where T is the tension in the cord. Solving for T gives $T = f_k + m_1 g\sin\alpha + m_1 a = 47.08\text{ N} + 156.7\text{ N} + 53.34\text{ N} = 257.1$ N. Newton's second law for m_2 gives $m_2 g - T = m_2 a$, so $m_2 = \dfrac{T}{g-a} = \dfrac{257.1\text{ N}}{9.8\text{ m/s}^2 - 2.667\text{ m/s}^2} = 36.0$ kg.

EVALUATE: We could treat these blocks as a two-block system. Newton's second law would then give $m_2 g - m_1 g\sin\alpha - \mu_k m_1 g\cos\alpha = (m_1 + m_2)a$, which gives the same result as above.

5.75. **IDENTIFY:** Newton's second law applies, as do the constant-acceleration kinematics equations.

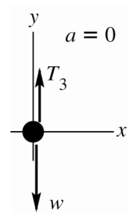
SET UP: Call the $+x$-axis horizontal and to the right and the $+y$-axis vertically upward. $\Sigma F_y = ma_y$ and $\Sigma F_x = ma_x$ both apply to the book.

EXECUTE: The book has no horizontal motion, so $\Sigma F_x = ma_x = 0$, which gives us the normal force n: $n = F\cos(60.0°)$. The kinetic friction force is $f_k = \mu_k n = (0.300)(96.0\text{ N})(\cos 60.0°) = 14.4$ N. In the vertical direction, we have $\Sigma F_y = ma_y$, which gives $F\sin(60.0°) - mg - f_k = ma$. Solving for a gives us $a = [(96.0\text{ N})(\sin 60.0°) - 49.0\text{ N} - 14.4\text{ N}]/(5.00\text{ kg}) = 3.948\text{ m/s}^2$ upward. Now the velocity formula $v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$ gives $v_y = \sqrt{2(3.948\text{ m/s}^2)(0.400\text{ m})} = 1.78$ m/s.

EVALUATE: Only the upward component of the force F makes the book accelerate upward, while the horizontal component of T is the magnitude of the normal force.

5.76. **Identify:** The system is in equilibrium. Apply Newton's first law to block *A*, to the hanging weight and to the knot where the cords meet. Target variables are the two forces.

(a) Set Up: The free-body diagram for the hanging block is given in Figure 5.76a.



Execute:

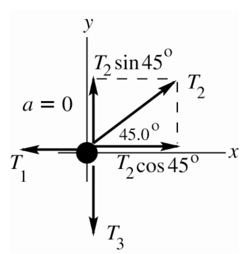
$\Sigma F_y = ma_y$

$T_3 - w = 0$

$T_3 = 12.0\text{ N}$

Figure 5.76a

Set Up: The free-body diagram for the knot is given in Figure 5.76b.



Execute:

$\Sigma F_y = ma_y$

$T_2 \sin 45.0° - T_3 = 0$

$T_2 = \dfrac{T_3}{\sin 45.0°} = \dfrac{12.0\text{ N}}{\sin 45.0°}$

$T_2 = 17.0\text{ N}$

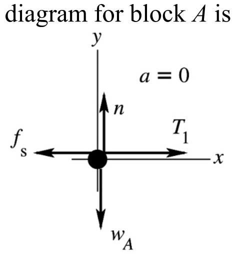
Figure 5.76b

$\Sigma F_x = ma_x$

$T_2 \cos 45.0° - T_1 = 0$

$T_1 = T_2 \cos 45.0° = 12.0\text{ N}$

Set Up: The free-body diagram for block *A* is given in Figure 5.76c.



Execute:

$\Sigma F_x = ma_x$

$T_1 - f_s = 0$

$f_s = T_1 = 12.0\text{ N}$

Figure 5.76c

Evaluate: Also can apply $\Sigma F_y = ma_y$ to this block:

$$n - w_A = 0$$

$$n = w_A = 60.0\text{ N}$$

Then $\mu_s n = (0.25)(60.0\text{ N}) = 15.0\text{ N};$ this is the maximum possible value for the static friction force. We see that $f_s < \mu_s n;$ for this value of *w* the static friction force can hold the blocks in place.

(b) Set Up: We have all the same free-body diagrams and force equations as in part (a) but now the static friction force has its largest possible value, $f_s = \mu_s n = 15.0\text{ N}.$ Then $T_1 = f_s = 15.0\text{ N}.$

Execute: From the equations for the forces on the knot

$T_2 \cos 45.0° - T_1 = 0$ implies $T_2 = T_1/\cos 45.0° = \dfrac{15.0\text{ N}}{\cos 45.0°} = 21.2\text{ N}$

$T_2 \sin 45.0° - T_3 = 0$ implies $T_3 = T_2 \sin 45.0° = (21.2 \text{ N}) \sin 45.0° = 15.0$ N

And finally $T_3 - w = 0$ implies $w = T_3 = 15.0$ N.

EVALUATE: Compared to part (a), the friction is larger in part (b) by a factor of $(15.0/12.0)$ and w is larger by this same ratio.

5.77. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

SET UP: Constant speed means $a = 0$. When the blocks are moving, the friction force is f_k and when they are at rest, the friction force is f_s.

EXECUTE: **(a)** The tension in the cord must be $m_2 g$ in order that the hanging block move at constant speed. This tension must overcome friction and the component of the gravitational force along the incline, so $m_2 g = (m_1 g \sin\alpha + \mu_k m_1 g \cos\alpha)$ and $m_2 = m_1(\sin\alpha + \mu_k \cos\alpha)$.

(b) In this case, the friction force acts in the same direction as the tension on the block of mass m_1, so $m_2 g = (m_1 g \sin\alpha - \mu_k m_1 g \cos\alpha)$, or $m_2 = m_1(\sin\alpha - \mu_k \cos\alpha)$.

(c) Similar to the analysis of parts (a) and (b), the largest m_2 could be is $m_1(\sin\alpha + \mu_s \cos\alpha)$ and the smallest m_2 could be is $m_1(\sin\alpha - \mu_s \cos\alpha)$.

EVALUATE: In parts (a) and (b) the friction force changes direction when the direction of the motion of m_1 changes. In part (c), for the largest m_2 the static friction force on m_1 is directed down the incline and for the smallest m_2 the static friction force on m_1 is directed up the incline.

5.78. **IDENTIFY:** The net force at any time is $F_{\text{net}} = ma$.

SET UP: At $t = 0$, $a = 62g$. The maximum acceleration is 140g at $t = 1.2$ ms.

EXECUTE: **(a)** $F_{\text{net}} = ma = 62mg = 62(210\times10^{-9} \text{ kg})(9.80 \text{ m/s}^2) = 1.3\times10^{-4}$ N. This force is 62 times the flea's weight.

(b) $F_{\text{net}} = 140mg = 2.9\times10^{-4}$ N, at $t = 1.2$ ms.

(c) Since the initial speed is zero, the maximum speed is the area under the $a_x - t$ graph. This gives 1.2 m/s.

EVALUATE: a is much larger than g and the net external force is much larger than the flea's weight.

5.79. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. Use Newton's third law to relate forces on A and on B.

SET UP: Constant speed means $a = 0$.

EXECUTE: **(a)** Treat A and B as a single object of weight $w = w_A + w_B = 1.20$ N + 3.60 N = 4.80 N. The free-body diagram for this combined object is given in Figure 5.79a. $\Sigma F_y = ma_y$ gives $n = w = 4.80$ N. $f_k = \mu_k n = (0.300)(4.80 \text{ N}) = 1.44$ N. $\Sigma F_x = ma_x$ gives $F = f_k = 1.44$ N.

(b) The free-body force diagrams for blocks A and B are given in Figure 5.79b. n and f_k are the normal and friction forces applied to block B by the tabletop and are the same as in part (a). f_{kB} is the friction force that A applies to B. It is to the right because the force from A opposes the motion of B. n_B is the downward force that A exerts on B. f_{kA} is the friction force that B applies to A. It is to the left because block B wants A to move with it. n_A is the normal force that block B exerts on A. By Newton's third law, $f_{kB} = f_{kA}$ and these forces are in opposite directions. Also, $n_A = n_B$ and these forces are in opposite directions.

$\Sigma F_y = ma_y$ for block A gives $n_A = w_A = 1.20$ N, so $n_B = 1.20$ N.

$f_{kA} = \mu_k n_A = (0.300)(1.20 \text{ N}) = 0.360$ N, and $f_{kB} = 0.360$ N.

$\Sigma F_x = ma_x$ for block A gives $T = f_{kA} = 0.360$ N.

$\Sigma F_x = ma_x$ for block B gives $F = f_{kB} + f_k = 0.360\text{ N} + 1.44\text{ N} = 1.80$ N.

EVALUATE: In part (a) block A is at rest with respect to B and it has zero acceleration. There is no horizontal force on A besides friction, and the friction force on A is zero. A larger force F is needed in part (b), because of the friction force between the two blocks.

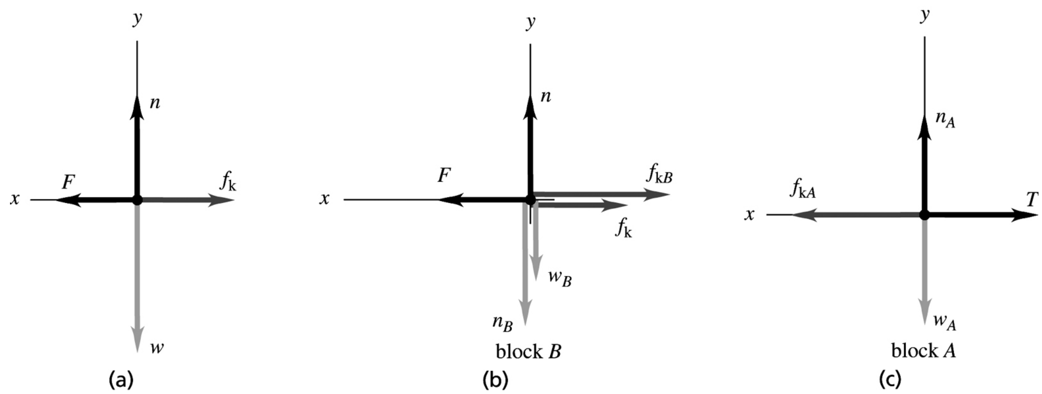


Figure 5.79

5.80. IDENTIFY: Apply $\Sigma \vec{F} = m\vec{a}$ to the passenger to find the maximum allowed acceleration. Then use a constant acceleration equation to find the maximum speed.

SET UP: The free-body diagram for the passenger is given in Figure 5.80.

EXECUTE: $\Sigma F_y = ma_y$ gives $n - mg = ma$. $n = 1.6mg$, so $a = 0.60g = 5.88\text{ m/s}^2$.

$y - y_0 = 3.0$ m, $a_y = 5.88\text{ m/s}^2$, $v_{0y} = 0$ so $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = 5.9$ m/s.

EVALUATE: A larger final speed would require a larger value of a_y, which would mean a larger normal force on the person.

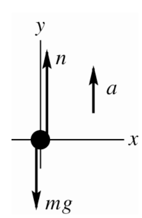


Figure 5.80

5.81. IDENTIFY: $a = dv/dt$. Apply $\Sigma \vec{F} = m\vec{a}$ to yourself.

SET UP: The reading of the scale is equal to the normal force the scale applies to you.

EXECUTE: The elevator's acceleration is $a = \dfrac{dv(t)}{dt} = 3.0\text{ m/s}^2 + 2(0.20\text{ m/s}^3)t = 3.0\text{ m/s}^2 + (0.40\text{ m/s}^3)t$.

At $t = 4.0$ s, $a = 3.0\text{ m/s}^2 + (0.40\text{ m/s}^3)(4.0\text{ s}) = 4.6\text{ m/s}^2$. From Newton's second law, the net force on you is $F_{\text{net}} = F_{\text{scale}} - w = ma$ and $F_{\text{scale}} = w + ma = (64\text{ kg})(9.8\text{ m/s}^2) + (64\text{ kg})(4.6\text{ m/s}^2) = 920$ N.

EVALUATE: a increases with time, so the scale reading is increasing.

5.82. **IDENTIFY:** The blocks are moving together with the same acceleration, so Newton's second law applies to each of them.

SET UP: For block A take the $+x$-axis horizontally to the right, and for B take the $+y$-axis vertically downward. Our choice for B has two advantages: it is in the direction of the acceleration of B and a positive acceleration of A gives a positive acceleration of B. Both blocks have the same acceleration, which we shall simply call a. $\sum F = ma$ applies to both blocks. We want to find the tension T in the rope and the coefficient of kinetic friction μ_k between A and the tabletop.

EXECUTE: **(a)** We know the speed of A and the distance it moved, so use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find its horizontal acceleration. $(3.30 \text{ m/s})^2 = 0 + 2a(2.00 \text{ m})$, so $a = 2.7225 \text{ m/s}^2$. Now look at B to find the tension. $\sum F_y = ma_y$ gives $w_B - T = m_B a = (w_B/g)a$. Solving for T gives $T = w_B\left(1 - \dfrac{a}{g}\right) =$

$$(25.0 \text{ N})\left(1 - \frac{2.7225 \text{ m/s}^2}{9.80 \text{ m/s}^2}\right) = 18.1 \text{ N}.$$

Now look at block A. Balancing vertical forces tells us that $n = w_A$. The friction force is $f_k = \mu_k n = \mu_k w_A$. $\sum F_x = ma_x$ gives $T - f_k = m_A a \quad \rightarrow \quad T - \mu_k w_A = (w_A/g)a$. Solving for μ_k gives $\mu_k = \dfrac{T - (w_A/g)a}{w_A}$. Using $w_A = 45.0$ N, $a = 2.7225 \text{ m/s}^2$, and $T = 18.1$ N gives $\mu_k = 0.123$.

EVALUATE: According to Table 5.1 in the text, 0.123 is not an unreasonable coefficient of kinetic friction.

5.83. **IDENTIFY:** The blocks move together with the same acceleration. Newton's second law applies to each of them. The forces acting on each block are gravity downward the upward tension in the rope.

SET UP: $\sum F_y = ma_y$ applies to each block. The heavier block accelerates downward while the lighter one accelerates upward, both with acceleration a. Call the $+y$-axis downward for the heavier block and upward for the lighter block. We want to find the mass of each block.

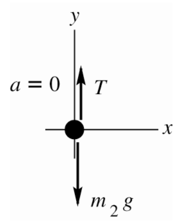
EXECUTE: Heavier block: First find the acceleration using $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, which gives us $y = \frac{1}{2}at^2$, so $5.00 \text{ m} = \frac{1}{2}a(2.00 \text{ s})^2 \quad \rightarrow \quad a = 2.50 \text{ m/s}^2$. Now apply $\sum F_y = ma_y$ to this block, which gives $m_2 g - T = m_2 a$. Now olve for m_2: $m_2 = \dfrac{T}{g - a} = \dfrac{16.0 \text{ N}}{9.80 \text{ m/s}^2 - 2.50 \text{ m/s}^2} = 2.19$ kg.

Lighter block: $\sum F_y = ma_y$ gives $T - m_1 g = m_1 a$, so $m_1 = \dfrac{T}{a + g} = 1.30$ kg.

EVALUATE: As a check, consider the two blocks as a singe system The only external force causing the acceleration is $m_2 g - m_1 g$, so $\sum F_y = ma_y$ gives $m_2 g - m_1 g = (m_1 + m_2)a$. Solving for a using our results for the two masses gives $a = 2.50 \text{ m/s}^2$, which agrees with our result.

5.84. **IDENTIFY:** Apply Newton's first law to the rope. Let m_1 be the mass of that part of the rope that is on the table, and let m_2 be the mass of that part of the rope that is hanging over the edge. ($m_1 + m_2 = m$, the total mass of the rope). Since the mass of the rope is not being neglected, the tension in the rope varies along the length of the rope. Let T be the tension in the rope at that point that is at the edge of the table.

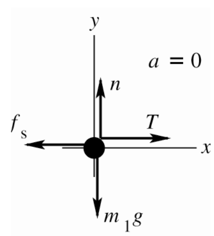
SET UP: The free-body diagram for the hanging section of the rope is given in Figure 5.84a.



EXECUTE:
$\Sigma F_y = ma_y$
$T - m_2 g = 0$
$T = m_2 g$

Figure 5.84a

SET UP: The free-body diagram for that part of the rope that is on the table is given in Figure 5.84b.



EXECUTE:
$\Sigma F_y = ma_y$
$n - m_1 g = 0$
$n = m_1 g$

Figure 5.84b

When the maximum amount of rope hangs over the edge the static friction has its maximum value:
$f_s = \mu_s n = \mu_s m_1 g$
a $\Sigma F_x = ma_x$
$T - f_s = 0$
$T = \mu_s m_1 g$
Use the first equation to replace T:
$m_2 g = \mu_s m_1 g$
$m_2 = \mu_s m_1$
The fraction that hangs over is $\dfrac{m_2}{m} = \dfrac{\mu_s m_1}{m_1 + \mu_s m_1} = \dfrac{\mu_s}{1+\mu_s}.$

EVALUATE: As $\mu_s \to 0,$ the fraction goes to zero and as $\mu_s \to \infty,$ the fraction goes to unity.

5.85. IDENTIFY: Apply $\Sigma \vec{F} = m\vec{a}$ to the point where the three wires join and also to one of the balls. By symmetry the tension in each of the 35.0 cm wires is the same.
SET UP: The geometry of the situation is sketched in Figure 5.85a. The angle ϕ that each wire makes with the vertical is given by $\sin\phi = \dfrac{12.5 \text{ cm}}{47.5 \text{ cm}}$ and $\phi = 15.26°.$ Let T_A be the tension in the vertical wire and let T_B be the tension in each of the other two wires. Neglect the weight of the wires. The free-body diagram for the left-hand ball is given in Figure 5.85b and for the point where the wires join in Figure 5.85c. n is the force one ball exerts on the other.
EXECUTE: (a) $\Sigma F_y = ma_y$ applied to the ball gives $T_B \cos\phi - mg = 0.$

$T_B = \dfrac{mg}{\cos\phi} = \dfrac{(15.0 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 15.26°} = 152 \text{ N}.$ Then $\Sigma F_y = ma_y$ applied in Figure 5.85c gives

$T_A - 2T_B \cos\phi = 0$ and $T_A = 2(152 \text{ N})\cos\phi = 249 \text{ N}.$
(b) $\Sigma F_x = ma_x$ applied to the ball gives $n - T_B \sin\phi = 0$ and $n = (152 \text{ N})\sin 15.26° = 40.0 \text{ N}.$
EVALUATE: T_A equals the total weight of the two balls.

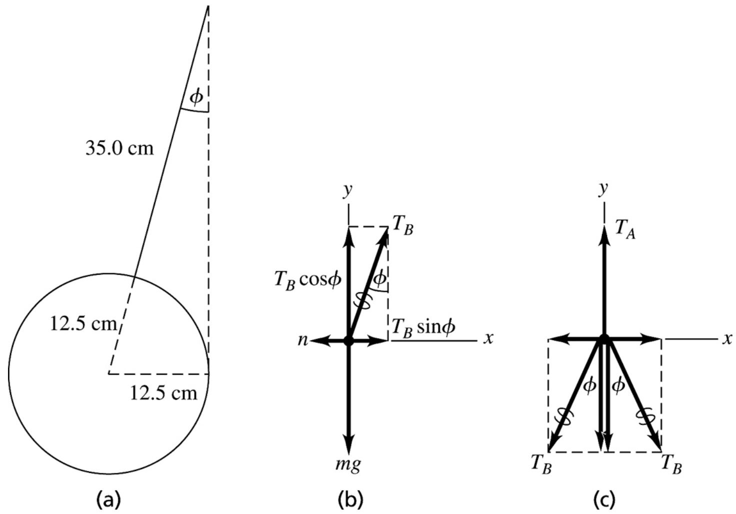


Figure 5.85

5.86. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the car to calculate its acceleration. Then use a constant acceleration equation to find the initial speed.

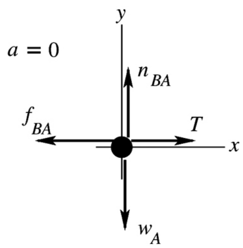
SET UP: Let $+x$ be in the direction of the car's initial velocity. The friction force f_k is then in the $-x$-direction. $192\text{ ft} = 58.52\text{ m}$.

EXECUTE: $n = mg$ and $f_k = \mu_k mg$. $\Sigma F_x = ma_x$ gives $-\mu_k mg = ma_x$ and $a_x = -\mu_k g = -(0.750)(9.80\text{ m/s}^2) = -7.35\text{ m/s}^2$. $v_x = 0$ (stops), $x - x_0 = 58.52\text{ m}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_{0x} = \sqrt{-2a_x(x - x_0)} = \sqrt{-2(-7.35\text{ m/s}^2)(58.52\text{ m})} = 29.3\text{ m/s} = 65.5\text{ mi/h}$. He was guilty.

EVALUATE: $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = -\dfrac{v_{0x}^2}{2a_x}$. If his initial speed had been 45 mi/h he would have stopped in $\left(\dfrac{45\text{ mi/h}}{65.5\text{ mi/h}}\right)^2 (192\text{ ft}) = 91\text{ ft}$.

5.87. **IDENTIFY:** Apply $-\left(\dfrac{M+m}{M}\right)\tan\alpha$ to each block. Forces between the blocks are related by Newton's third law. The target variable is the force F. Block B is pulled to the left at constant speed, so block A moves to the right at constant speed and $a = 0$ for each block.

SET UP: The free-body diagram for block A is given in Figure 5.87a. n_{BA} is the normal force that B exerts on A. $f_{BA} = \mu_k n_{BA}$ is the kinetic friction force that B exerts on A. Block A moves to the right relative to B, and f_{BA} opposes this motion, so f_{BA} is to the left. Note also that F acts just on B, not on A.



EXECUTE:
$\Sigma F_y = ma_y$
$n_{BA} - w_A = 0$
$n_{BA} = 1.90\text{ N}$
$f_{BA} = \mu_k n_{BA} = (0.30)(1.90\text{ N}) = 0.57\text{ N}$

Figure 5.87a

$\Sigma F_x = ma_x$. $T - f_{BA} = 0$. $T = f_{BA} = 0.57\text{ N}$.

SET UP: The free-body diagram for block *B* is given in Figure 5.87b.

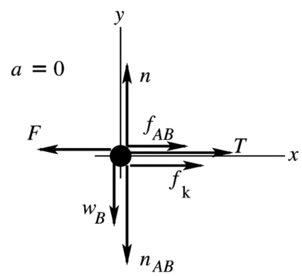


Figure 5.87b

EXECUTE: n_{AB} is the normal force that block *A* exerts on block *B*. By Newton's third law n_{AB} and n_{BA} are equal in magnitude and opposite in direction, so $n_{AB} = 1.90\text{ N}$. f_{AB} is the kinetic friction force that *A* exerts on *B*. Block *B* moves to the left relative to *A* and f_{AB} opposes this motion, so f_{AB} is to the right. $f_{AB} = \mu_k n_{AB} = (0.30)(1.90\text{ N}) = 0.57\text{ N}$. *n* and f_k are the normal and friction force exerted by the floor on block *B*; $f_k = \mu_k n$. Note that block *B* moves to the left relative to the floor and f_k opposes this motion, so f_k is to the right.

$\Sigma F_y = ma_y$: $n - w_B - n_{AB} = 0$. $n = w_B + n_{AB} = 4.20\text{ N} + 1.90\text{ N} = 6.10\text{ N}$. Then
$f_k = \mu_k n = (0.30)(6.10\text{ N}) = 1.83\text{ N}$. $\Sigma F_x = ma_x$: $f_{AB} + T + f_k - F = 0$.
$F = T + f_{AB} + f_k = 0.57\text{ N} + 0.57\text{ N} + 1.83\text{ N} = 3.0\text{ N}$.

EVALUATE: Note that f_{AB} and f_{BA} are a third law action-reaction pair, so they must be equal in magnitude and opposite in direction and this is indeed what our calculation gives.

5.88. **IDENTIFY:** Both blocks accelerate together. The force *P* accelerates the two-block system, but only static friction accelerates block *B*. Newton's second law applies to each block as well as the entire system.

SET UP: When *P* is the largest, block *A* is just ready to slide over block *B* so static friction is at its maximum, which is $f_s = \mu_s n$. Apply $\sum F_x = ma_x$. We want the largest value of *P* for which the blocks move together.

EXECUTE: Isolate *A*: $n = m_A g$ and $f_s = \mu_s m_A g$. $\sum F_x = ma_x$ gives $P - \mu_s m_A g = m_A a_x$. We need a_x. Treat the two blocks as a single system: $\sum F_x = ma_x$ gives $P = (m_A + m_B)a_x$. Now solve for a_x to get $a_x = P/(m_A + m_B)$.

Now use this value of a_x in the equation for block A, giving $P - \mu_s m_A g = m_A\left(\dfrac{P}{m_A + m_B}\right)$. Solving for P and using $m_A = 2.00$ kg, $m_B = 5.00$ kg, and $\mu_s = 0.400$ gives $P = 11.0$ N.

EVALUATE: If P were to exceed 11.0 N, slipping would occur, the friction force would be kinetic friction, and the blocks would not have the same acceleration.

5.89. **IDENTIFY:** Apply $-\left(\dfrac{M+m}{M}\right)\tan\alpha$ to each block. Parts (a) and (b) will be done together.

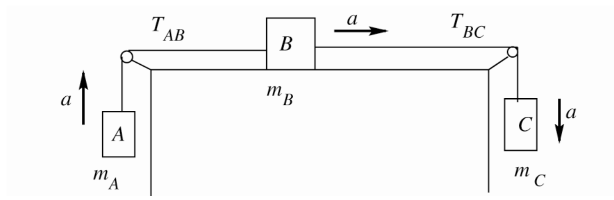
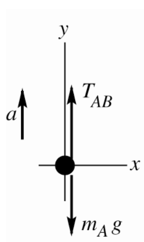


Figure 5.89a

Note that each block has the same magnitude of acceleration, but in different directions. For each block let the direction of $\vec{a}$ be a positive coordinate direction.

SET UP: The free-body diagram for block A is given in Figure 5.89b.



EXECUTE:

$\Sigma F_y = ma_y$

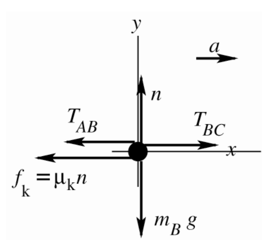
$T_{AB} - m_A g = m_A a$

$T_{AB} = m_A(a + g)$

$T_{AB} = 4.00\text{ kg}(2.00\text{ m/s}^2 + 9.80\text{ m/s}^2) = 47.2\text{ N}$

Figure 5.89b

SET UP: The free-body diagram for block B is given in Figure 5.89c.



EXECUTE:

$\Sigma F_y = ma_y$

$n - m_B g = 0$

$n = m_B g$

Figure 5.89c

$f_k = \mu_k n = \mu_k m_B g = (0.25)(12.0\text{ kg})(9.80\text{ m/s}^2) = 29.4\text{ N}$

$\Sigma F_x = ma_x$

$T_{BC} - T_{AB} - f_k = m_B a$

$T_{BC} = T_{AB} + f_k + m_B a = 47.2\text{ N} + 29.4\text{ N} + (12.0\text{ kg})(2.00\text{ m/s}^2)$

$T_{BC} = 100.6\text{ N}$

SET UP: The free-body diagram for block C is sketched in Figure 5.89d (next page).

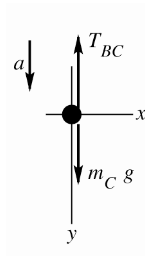


Figure 5.89d

EXECUTE:

$$\Sigma F_y = ma_y$$

$$m_C g - T_{BC} = m_C a$$

$$m_C(g-a) = T_{BC}$$

$$m_C = \frac{T_{BC}}{g-a} = \frac{100.6\ \text{N}}{9.80\ \text{m}/s^2 - 2.00\ \text{m/s}^2} = 12.9\ \text{kg}$$

EVALUATE: If all three blocks are considered together as a single object and $-\left(\frac{M+m}{M}\right)\tan\alpha$ is applied to this combined object, $m_C g - m_A g - \mu_k m_B g = (m_A + m_B + m_C)a$. Using the values for μ_k, m_A and m_B given in the problem and the mass m_C we calculated, this equation gives $a = 2.00\ \text{m/s}^2$, which checks.

5.90. **IDENTIFY:** Apply $-\left(\frac{M+m}{M}\right)\tan\alpha$ to each block. They have the same magnitude of acceleration, a.

SET UP: Consider positive accelerations to be to the right (up and to the right for the left-hand block, down and to the right for the right-hand block).

EXECUTE: **(a)** The forces along the inclines and the accelerations are related by $T - (100\ \text{kg})g\sin 30.0° = (100\ \text{kg})a$ and $(50\ \text{kg})g\sin 53.1° - T = (50\ \text{kg})a$, where T is the tension in the cord and a the mutual magnitude of acceleration. Adding these relations, $(50\ \text{kg}\sin 53.1° - 100\ \text{kg}\sin 30.0°)g = (50\ \text{kg} + 100\ \text{kg})a$, or $a = -0.067g$. Since a comes out negative, the blocks will slide to the left; the 100-kg block will slide down. Of course, if coordinates had been chosen so that positive accelerations were to the left, a would be $+0.067g$.

(b) $a = 0.067(9.80\ \text{m/s}^2) = 0.658\ \text{m/s}^2$.

(c) Substituting the value of a (including the proper sign, depending on choice of coordinates) into either of the above relations involving T yields 424 N.

EVALUATE: For part (a) we could have compared $mg\sin\theta$ for each block to determine which direction the system would move.

5.91. **IDENTIFY:** Let the tensions in the ropes be T_1 and T_2.

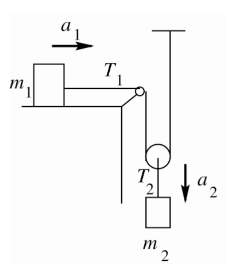
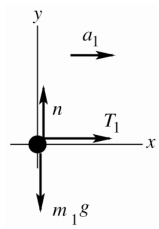


Figure 5.91a

Consider the forces on each block. In each case take a positive coordinate direction in the direction of the acceleration of that block.

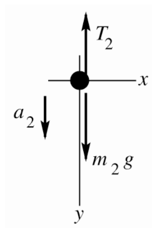
SET UP: The free-body diagram for m_1 is given in Figure 5.91b.



EXECUTE:
$\Sigma F_x = ma_x$
$T_1 = m_1a_1$

Figure 5.91b

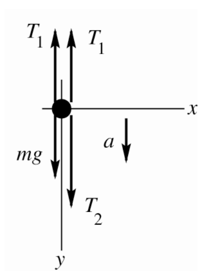
SET UP: The free-body diagram for m_2 is given in Figure 5.91c.



EXECUTE:
$\Sigma F_y = ma_y$
$m_2g - T_2 = m_2a_2$

Figure 5.91c

This gives us two equations, but there are four unknowns (T_1, T_2, a_1 and a_2) so two more equations are required.
SET UP: The free-body diagram for the moveable pulley (mass *m*) is given in Figure 5.91d.



EXECUTE:
$\Sigma F_y = ma_y$
$mg + T_2 - 2T_1 = ma$

Figure 5.91d

But our pulleys have negligible mass, so $mg = ma = 0$ and $T_2 = 2T_1$. Combine these three equations to eliminate T_1 and T_2: $m_2g - T_2 = m_2a_2$ gives $m_2g - 2T_1 = m_2a_2$. And then with $T_1 = m_1a_1$ we have $m_2g - 2m_1a_1 = m_2a_2$.
SET UP: There are still two unknowns, a_1 and a_2. But the accelerations a_1 and a_2 are related. In any time interval, if m_1 moves to the right a distance *d*, then in the same time m_2 moves downward a distance $d/2$. One of the constant acceleration kinematic equations says $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$, so if m_2 moves half the distance it must have half the acceleration of m_1: $a_2 = a_1/2$, or $a_1 = 2a_2$.
EXECUTE: This is the additional equation we need. Use it in the previous equation and get
$m_2g - 2m_1(2a_2) = m_2a_2$.
$a_2(4m_1 + m_2) = m_2g$

$$a_2 = \frac{m_2 g}{4m_1 + m_2} \text{ and } a_1 = 2a_2 = \frac{2m_2 g}{4m_1 + m_2}.$$

EVALUATE: If $m_2 \to 0$ or $m_1 \to \infty$, $a_1 = a_2 = 0$. If $m_2 \gg m_1$, $a_2 = g$ and $a_1 = 2g$.

5.92. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to block B, to block A and B as a composite object, and to block C. If A and B slide together all three blocks have the same magnitude of acceleration.

SET UP: If A and B don't slip, the friction between them is static. The free-body diagrams for block B, for blocks A and B, and for C are given in Figure 5.92. Block C accelerates downward and A and B accelerate to the right. In each case take a positive coordinate direction to be in the direction of the acceleration. Since block A moves to the right, the friction force f_s on block B is to the right, to prevent relative motion between the two blocks. When C has its largest mass, f_s has its largest value: $f_s = \mu_s n$.

EXECUTE: $\Sigma F_x = ma_x$ applied to the block B gives $f_s = m_B a$. $n = m_B g$ and $f_s = \mu_s m_B g$. $\mu_s m_B g = m_B a$ and $a = \mu_s g$. $\Sigma F_x = ma_x$ applied to blocks $A + B$ gives $T = m_{AB} a = m_{AB} \mu_s g$. $\Sigma F_y = ma_y$ applied to block C gives $m_C g - T = m_C a$. $m_C g - m_{AB} \mu_s g = m_C \mu_s g$.

$$m_C = \frac{m_{AB}\mu_s}{1-\mu_s} = (5.00 \text{ kg} + 8.00 \text{ kg})\left(\frac{0.750}{1-0.750}\right) = 39.0 \text{ kg}.$$

EVALUATE: With no friction from the tabletop, the system accelerates no matter how small the mass of C is. If m_C is less than 39.0 kg, the friction force that A exerts on B is less than $\mu_s n$. If m_C is greater than 39.0 kg, blocks C and A have a larger acceleration than friction can give to block B, and A accelerates out from under B.

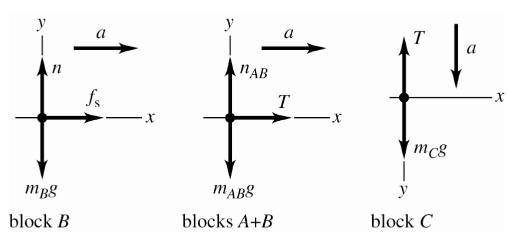


Figure 5.92

5.93. **IDENTIFY:** Both blocks accelerate together. The force F accelerates the two-block system, and the blocks exert kinetic friction forces on each other. Newton's second law applies to each block as well as the entire system.

SET UP: We apply $\sum F_x = ma_x$ and $\sum F_y = 0$ to each block. Block A accelerates to the right and B accelerates to the left. Call the direction of acceleration the $+x$-direction in each case, and call each acceleration a. Fig. 5.93 shows free-body diagrams of each block. It is important to realize two things immediately: the normal force that A exerts on B is equal and opposite to the normal force that B exerts on A, and the blocks exert equal but opposite friction forces on each other. Both of these points are due to Newton's third law (action-reaction). We want to find the tension in the cord and the coefficient of kinetic friction between A and B.

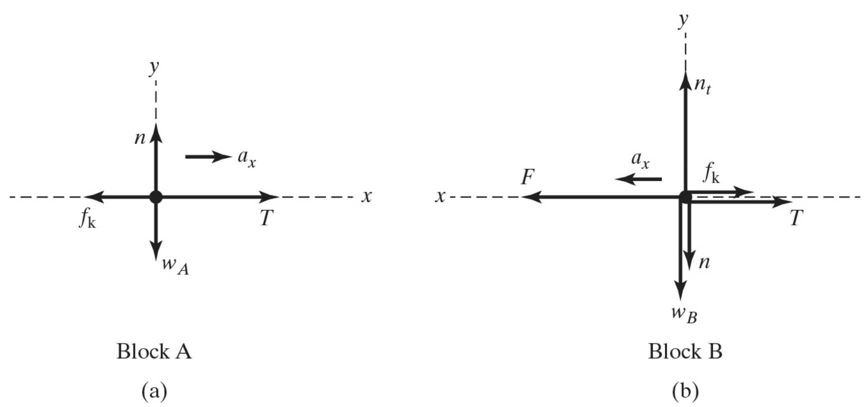


Figure 5.93

EXECUTE: **(a) & (b)** Isolate block A: Call n the normal force that the blocks exert on each other and f_k the kinetic friction force they exert on each other. From Fig. 5.93a, $\sum F_y = 0$ gives $n = w_A$ and $\sum F_x = ma_x$ gives $T - f_k = m_A a$, which becomes $T - \mu_k w_A = m_A a$. (Eq. 1)

Isolate block B: Call n_t the normal force due to the table. From Fig. 5.93b wee see that $\sum F_x = ma_x$ gives $T + f_k - F = m_B a$, which becomes $F - T - \mu_k w_A = m_B a$. (Eq. 2)

Combining Eq. 1 and Eq. 2 and solve for μ_k gives $F - 2\mu_k w_A = (m_A + m_B)a$. Solving for μ_k gives $\mu_k = \dfrac{F - (m_A + m_B)a}{2m_A g}$. Using the given masses, force, and acceleration gives $\mu_k = 0.242$.

Now use Eq. 1 (or Eq. 2) to find T: $T = m_A a + \mu_k m_A g = 7.75$ N.

EVALUATE: The tension is less than the force F, which is reasonable because B could never accelerate if T were greater than F. Also, from Table 5.1 we see that 0.242 is a reasonable coefficient of kinetic friction.

5.94. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the box.

SET UP: The box has an upward acceleration of $a = 1.90 \text{ m/s}^2$.

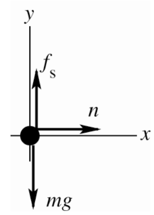
EXECUTE: The floor exerts an upward force n on the box, obtained from $n - mg = ma$, or $n = m(a + g)$. The friction force that needs to be balanced is

$$\mu_k n = \mu_k m(a + g) = (0.32)(36.0 \text{ kg})(1.90 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 135 \text{ N}.$$

EVALUATE: If the elevator were not accelerating the normal force would be $n = mg$ and the friction force that would have to be overcome would be 113 N. The upward acceleration increases the normal force and that increases the friction force.

5.95. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block. The cart and the block have the same acceleration. The normal force exerted by the cart on the block is perpendicular to the front of the cart, so is horizontal and to the right. The friction force on the block is directed so as to hold the block up against the downward pull of gravity. We want to calculate the minimum a required, so take static friction to have its maximum value, $f_s = \mu_s n$.

SET UP: The free-body diagram for the block is given in Figure 5.95.



EXECUTE:
$\Sigma F_x = ma_x$
$n = ma$
$f_s = \mu_s n = \mu_s ma$

Figure 5.95

$\Sigma F_y = ma_y$: $f_s - mg = 0$
$\mu_s ma = mg$, so $a = g/\mu_s$.

EVALUATE: An observer on the cart sees the block pinned there, with no reason for a horizontal force on it because the block is at rest relative to the cart. Therefore, such an observer concludes that $n = 0$ and thus $f_s = 0$, and he doesn't understand what holds the block up against the downward force of gravity. The reason for this difficulty is that $\Sigma\vec{F} = m\vec{a}$ does not apply in a coordinate frame attached to the cart. This reference frame is accelerated, and hence not inertial. The smaller μ_s is, the larger *a* must be to keep the block pinned against the front of the cart.

5.96. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block.
SET UP: Use coordinates where $+x$ is directed down the incline.
EXECUTE: **(a)** Since the larger block (the trailing block) has the larger coefficient of friction, it will need to be pulled down the plane; i.e., the larger block will not move faster than the smaller block, and the blocks will have the same acceleration. For the smaller block, $(4.00\text{ kg})g(\sin 30° - (0.25)\cos 30°) - T = (4.00\text{ kg})a$, or $11.11\text{ N} - T = (4.00\text{ kg})a$, and similarly for the larger, $15.44\text{ N} + T = (8.00\text{ kg})a$. Adding these two relations, $26.55\text{ N} = (12.00\text{ kg})a$, $a = 2.21\text{ m/s}^2$.
(b) Substitution into either of the above relations gives $T = 2.27\text{ N}$.
(c) The string will be slack. The 4.00-kg block will have $a = 2.78\text{ m/s}^2$ and the 8.00-kg block will have $a = 1.93\text{ m/s}^2$, until the 4.00-kg block overtakes the 8.00-kg block and collides with it.
EVALUATE: If the string is cut the acceleration of each block will be independent of the mass of that block and will depend only on the slope angle and the coefficient of kinetic friction. The 8.00-kg block would have a smaller acceleration even though it has a larger mass, since it has a larger μ_k.

5.97. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block and to the plank.
SET UP: Both objects have $a = 0$.
EXECUTE: Let n_B be the normal force between the plank and the block and n_A be the normal force between the block and the incline. Then, $n_B = w\cos\theta$ and $n_A = n_B + 3w\cos\theta = 4w\cos\theta$. The net frictional force on the block is $\mu_k(n_A + n_B) = \mu_k 5w\cos\theta$. To move at constant speed, this must balance the component of the block's weight along the incline, so $3w\sin\theta = \mu_k 5w\cos\theta$, and $\mu_k = \frac{3}{5}\tan\theta = \frac{3}{5}\tan 37° = 0.452$.
EVALUATE: In the absence of the plank the block slides down at constant speed when the slope angle and coefficient of friction are related by $\tan\theta = \mu_k$. For $\theta = 36.9°$, $\mu_k = 0.75$. A smaller μ_k is needed when the plank is present because the plank provides an additional friction force.

5.98. **IDENTIFY:** Apply Newton's second law to Jack in the Ferris wheel.
SET UP: $\Sigma\vec{F} = m\vec{a}$ and Jack's acceleration is $a_{rad} = v^2/R$, and $v = 2\pi R/T$. At the highest point, the normal force that the chair exerts on Jack is ¼ of his weight, or $0.25mg$. Take $+y$ downward.

EXECUTE: $\Sigma F_y = ma_y$ gives $mg - n = mv^2/R$. $mg - 0.25mg = mv^2/R$, so $v^2/R = 0.75g$. Using $T = 2\pi R/T$, we get $v^2/R = 4\pi^2 R/T^2$. Therefore $4\pi^2 R/T^2 = 0.750g$. $T = 1/(0.100 \text{ rev/s}) = 10.0$ s/rev, so $R = (0.750g)T^2/(4\pi^2) = (0.750)(9.80 \text{ m/s}^2)[(10.0 \text{ s})/(2\pi)]^2 = 18.6$ m.

EVALUATE: This Ferris wheel would be about 120 ft in diameter, which is certainly large but not impossible.

5.99. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the person. The person moves in a horizontal circle so his acceleration is $a_{\text{rad}} = v^2/R$, directed toward the center of the circle. The target variable is the coefficient of static friction between the person and the surface of the cylinder.

$$v = (0.60 \text{ rev/s})\left(\frac{2\pi R}{1 \text{ rev}}\right) = (0.60 \text{ rev/s})\left(\frac{2\pi(2.5 \text{ m})}{1 \text{ rev}}\right) = 9.425 \text{ m/s}$$

(a) SET UP: The problem situation is sketched in Figure 5.99a.

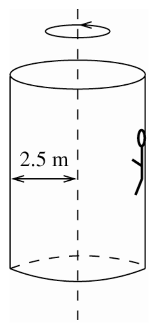
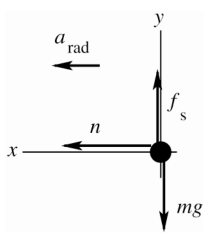


Figure 5.99a



The free-body diagram for the person is sketched in Figure 5.99b.
The person is held up against gravity by the static friction force exerted on him by the wall. The acceleration of the person is a_{rad}, directed in toward the axis of rotation.

Figure 5.99b

(b) EXECUTE: To calculate the minimum μ_s required, take f_s to have its maximum value, $f_s = \mu_s n$.

$\Sigma F_y = ma_y$: $f_s - mg = 0$

$\mu_s n = mg$

$\Sigma F_x = ma_x$: $n = mv^2/R$

Combine these two equations to eliminate n: $\mu_s mv^2/R = mg$

$$\mu_s = \frac{Rg}{v^2} = \frac{(2.5 \text{ m})(9.80 \text{ m/s}^2)}{(9.425 \text{ m/s})^2} = 0.28$$

(c) EVALUATE: No, the mass of the person divided out of the equation for μ_s. Also, the smaller μ_s is, the larger v must be to keep the person from sliding down. For smaller μ_s the cylinder must rotate faster to make n large enough.

5.100. IDENTIFY: The ice is traveling in a circular arc, so Newton's second law applies to it. The radial acceleration is toward the center of the circle, which is downward at the top of the arc.

SET UP: $\sum F = m\dfrac{v^2}{R}$.

EXECUTE: At the top of the arc $\sum F = m\dfrac{v^2}{R}$ gives $mg - n = mv^2/R$. Using $n = mg/2$ gives

$mg - mg/2 = mv^2/R \quad \rightarrow \quad v = \sqrt{\dfrac{Rg}{2}}$.

EVALUATE: The smallest the normal force could be is zero, in which case $v = \sqrt{Rg}$, which is greater than our result. So our answer is reasonable.

5.101. IDENTIFY: The race car is accelerated toward the center of the circle of the curve. If the car goes at the proper speed for the banking angle, there will be no friction force on it, and will not tend to slide either up or down the road. In this case, it is going faster than the proper speed, so it will tend to slide up the road, so the friction force will be down the road. Newton's second law applies to the car.
SET UP: At the maximum speed so the car will not slide up the road, static friction from the road on the tires is at its maximum value, so $f_s = \mu_s n$ and its direction is down the road. Fig. 5.101 shows a free-body diagram of the car. Call the $+x$-axis horizontal pointing toward the center of the circular curve, and the y-axis perpendicular to the road surface. This is one of the few times that it is better *not* to take the x-axis parallel to the surface of the incline. The reason is that the acceleration is horizontal, not parallel to the surface. Apply $\sum F_x = ma_x$ and $\sum F_y = 0$ letting β be the banking angle.

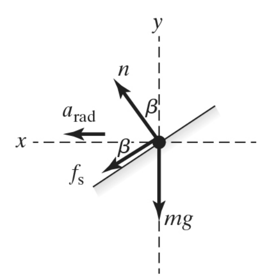


Figure 5.101

EXECUTE: **(a)** Start with $\sum F_x = ma_{rad}$. From the free-body diagram, we see that
$\sum F_x = n \sin\beta + f_s \cos\beta = n \sin\beta + \mu_s n \cos\beta$, so
$ma_{rad} = n(\sin\beta + \mu_s \cos\beta)$ (Eq. 1)
$\sum F_y = 0 = n \cos\beta - f_s \sin\beta - mg = n \cos\beta - \mu_s n \sin\beta - mg = 0$, which gives
$mg = n(\cos\beta - \mu_s \sin\beta)$ (Eq. 2)

Solving for n gives $n = \dfrac{mg}{\cos\beta - \mu_s \sin\beta} = \dfrac{(1200\text{ kg})(9.80\text{ m/s}^2)}{\cos 18.0° - (0.400)\sin 18.0°} = 1.42\times 10^4$ N.

(b) Divide Eq. 1 by Eq. 2, giving $a_{rad} = \left(\dfrac{\sin\beta + \mu_s \cos\beta}{\cos\beta - \mu_s \sin\beta}\right)g$. Using $\beta = 18.0°$ and $\mu_s = 0.400$ gives $a_{rad} = 8.17\text{ m/s}^2$.

(c) $a_{rad} = v^2/R$, so $v = \sqrt{Ra_{rad}} = \sqrt{(90.0\text{ m})(8.17\text{ m/s}^2)} = 27.1$ m/s.

EVALUATE: Check our result in (b) for a very smooth road in which $\mu_s = 0$. This gives $a_{rad} = g\tan\beta$, so $\frac{v^2}{R} = g\tan\beta$, or $\tan\beta = \frac{v^2}{Rg}$. This result give the familiar angle for a properly banked road for speed v. If there were no friction, a car would have to go at the proper speed to avoid slipping. So our result in this special case checks out. The proper speed for $\beta = 18.0°$ should be $v = \sqrt{Rg\tan\beta} = \sqrt{(90.0\text{ m})(9.80\text{ m/s}^2)\tan 18.0°} = 16.9$ m/s, so the speed we just found is obviously much greater than the proper speed.

5.102. **IDENTIFY:** The race car is accelerated toward the center of the circle of the curve. If the car goes at the proper speed for the banking angle, there will be no friction force on it, and will not tend to slide either up or down the road. In this case, it is going slower than the proper speed, so it will tend to slide down the road, so the friction force will be up the road. Newton's second law applies to the car.

SET UP: At the maximum speed so the car will not slide down the road, static friction from the road on the tires is at its maximum value, so $f_S = \mu_s n$ and its direction is up the road. Figure 5.102 shows a free-body diagram of the car. Call the $+x$-axis horizontal pointing toward the center of the circular curve, and the y-axis perpendicular to the road surface. This is one of the few times that it is better *not* to take the x-axis parallel to the surface of the incline. The reason is that the acceleration is horizontal, not parallel to the surface. Apply $\sum F_x = ma_x$ and $\sum F_y = 0$ letting β be the banking angle.

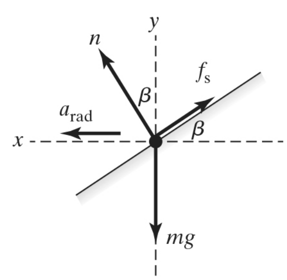


Figure 5.102

EXECUTE: **(a)** The procedure is the same as for problem 5.101 except that f_S is *up* the road surface instead of down. Using the same steps as in 5.101 leads to

$$\frac{mv^2}{R} = n(\sin\beta - \mu_s\cos\beta) \quad \text{(Eq. 1)}$$

$$n(\cos\beta + \mu_s\sin\beta) = mg \quad \text{(Eq. 2)}$$

Solving for n gives $n = \frac{mg}{\cos\beta + \mu_s\sin\beta} = \frac{(900\text{ kg})(9.80\text{ m/s}^2)}{\cos 18.0° + (0.300)\sin 18.0°} = 8450$ N.

(b) Combining Eq. 1 and Eq. 2 we get $v^2 = \left(\frac{\sin\beta - \mu_s\cos\beta}{\cos\beta + \mu_s\sin\beta}\right)Rg$. Using $\mu_s = 0.300$, $\beta = 18.0°$, and $R = 120.0$ m gives $v = 5.17$ m/s.

EVALUATE: The proper speed for a banking angle of 18.0° is given by $\tan\beta = \frac{v^2}{Rg}$, so the proper speed for this angle is $v = \sqrt{Rg\tan\beta} = \sqrt{(120.0\text{ m})(9.80\text{ m/s}^2)\tan 18.0°} = 19.5$ m/s, so the speed we just found is much slower.

5.103. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

SET UP: For block B use coordinates parallel and perpendicular to the incline. Since they are connected by ropes, blocks A and B also move with constant speed.

EXECUTE: **(a)** The free-body diagrams are sketched in Figure 5.103.

(b) The blocks move with constant speed, so there is no net force on block A; the tension in the rope connecting A and B must be equal to the frictional force on block A, $T_1 = (0.35)(25.0\text{ N}) = 8.8\text{ N}$.

(c) The weight of block C will be the tension in the rope connecting B and C; this is found by considering the forces on block B. The components of force along the ramp are the tension in the first rope (8.8 N, from part (b)), the component of the weight along the ramp, the friction on block B and the tension in the second rope. Thus, the weight of block C is

$$w_C = 8.8\text{ N} + w_B(\sin 36.9° + \mu_k \cos 36.9°) = 8.8\text{ N} + (25.0\text{ N})(\sin 36.9° + (0.35)\cos 36.9°) = 30.8\text{ N}$$

The intermediate calculation of the first tension may be avoided to obtain the answer in terms of the common weight w of blocks A and B, $w_C = w(\mu_k + (\sin\theta + \mu_k \cos\theta))$, giving the same result.

(d) Applying Newton's second law to the remaining masses (B and C) gives:

$$a = g(w_C - \mu_k w_B \cos\theta - w_B \sin\theta)/(w_B + w_C) = 1.54\text{ m/s}^2.$$

EVALUATE: Before the rope between A and B is cut the net external force on the system is zero. When the rope is cut the friction force on A is removed from the system and there is a net force on the system of blocks B and C.

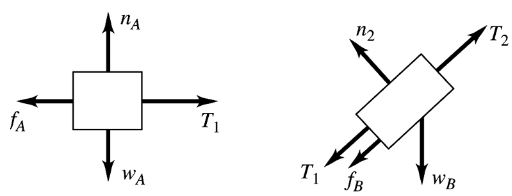


Figure 5.103

5.104. **IDENTIFY:** The block has acceleration $a_{\text{rad}} = v^2/r$, directed to the left in the figure in the problem. Apply $\Sigma\vec{F} = m\vec{a}$ to the block.

SET UP: The block moves in a horizontal circle of radius $r = \sqrt{(1.25\text{ m})^2 - (1.00\text{ m})^2} = 0.75\text{ m}$. Each string makes an angle θ with the vertical. $\cos\theta = \dfrac{1.00\text{ m}}{1.25\text{ m}}$, so $\theta = 36.9°$. The free-body diagram for the block is given in Figure 5.104. Let $+x$ be to the left and let $+y$ be upward.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ gives $T_u \cos\theta - T_l \cos\theta - mg = 0$.

$$T_l = T_u - \frac{mg}{\cos\theta} = 80.0\text{ N} - \frac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 31.0\text{ N}.$$

(b) $\Sigma F_x = ma_x$ gives $(T_u + T_l)\sin\theta = m\dfrac{v^2}{r}$.

$v = \sqrt{\dfrac{r(T_u + T_l)\sin\theta}{m}} = \sqrt{\dfrac{(0.75\text{ m})(80.0\text{ N} + 31.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 3.53\text{ m/s}.$ The number of revolutions per second is $\dfrac{v}{2\pi r} = \dfrac{3.53\text{ m/s}}{2\pi(0.75\text{ m})} = 0.749\text{ rev/s} = 44.9\text{ rev/min}.$

(c) If $T_l \to 0$, $T_u \cos\theta = mg$ and $T_u = \dfrac{mg}{\cos\theta} = \dfrac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 49.0\text{ N}$. $T_u \sin\theta = m\dfrac{v^2}{r}$.

$v = \sqrt{\dfrac{rT_u \sin\theta}{m}} = \sqrt{\dfrac{(0.75\text{ m})(49.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 2.35\text{ m/s}$. The number of revolutions per minute is

$(44.9\text{ rev/min})\left(\dfrac{2.35\text{ m/s}}{3.53\text{ m/s}}\right) = 29.9\text{ rev/min}$.

EVALUATE: The tension in the upper string must be greater than the tension in the lower string so that together they produce an upward component of force that balances the weight of the block.

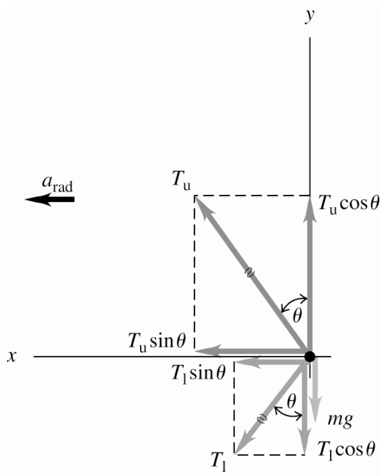


Figure 5.104

5.105. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$, with $f = kv$.

SET UP: Follow the analysis that leads to the equation $v_y = v_t[1 - e^{-(k/m)t}]$, except now the initial speed is $v_{0y} = 3mg/k = 3v_t$ rather than zero.

EXECUTE: The separated equation of motion has a lower limit of $3v_t$ instead of zero; specifically,

$$\int_{3v_t}^{v} \frac{dv}{v - v_t} = \ln\frac{v_t - v}{-2v_t} = \ln\left(\frac{v}{2v_t} - \frac{1}{2}\right) = -\frac{k}{m}t, \text{ or } v = 2v_t\left[\frac{1}{2} + e^{-(k/m)t}\right]$$

where $v_t = mg/k$.

EVALUATE: As $t \to \infty$ the speed approaches v_t. The speed is always greater than v_t and this limit is approached from above.

5.106. **IDENTIFY:** The box on the ramp slows down on the way up and speeds up on the way down. Newton's second law applies to both the upward and downward motion, but the acceleration will not be the same for both parts of the motion.

SET UP: On the way up, kinetic friction acts down the ramp, but on the way down it acts up the ramp. Yet gravity acts down the ramp in both cases. Therefore the acceleration will not be the same in both on the upward motion as on the downward motion. We must break this problem up into two segments: motion up the ramp and motion down the ramp. Fig. 5.106(a) shows a free-body diagram for the upward part of the motion. The free-body diagram for the downward segment is shown in Fig. 5.106(b). It is the same as for the upward motion except that friction acts up the ramp. We appy $\sum F_x = ma_x$ in both segments and use the constant-acceleration equations as needed. In both segments, the acceleration is down the ramp, so we call that the $+x$-axis in both parts. We use $\sum F_y = 0$ and $f_k = \mu_k n$ in both segments. At its highest point, the speed of the box is zero. We want to find μ_k in terms of α.

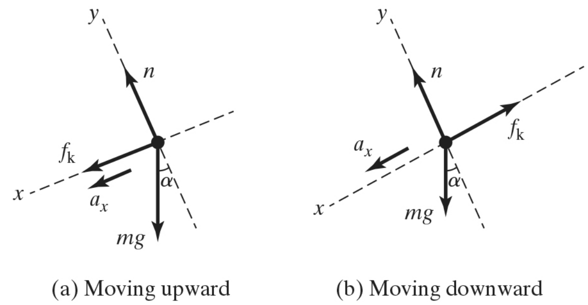


Figure 5.106

EXECUTE: First find the acceleration of the box on both segments of the motion. Call a_{up} the magnitude of the acceleration on the upward segment and a_{down} the magnitude of the acceleration on the downward segment.

Upward segment: Use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $v = 0$ at the highest point. This gives $0 = v_0^2 + 2a_{up}(-d)$. Note that we used $-d$ because down the ramp is positive and the displacement d is up the ramp. This gives $a_{up} = v_0^2 / 2d$.

Downward segment: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $\left(\dfrac{v_0}{2}\right)^2 = 0 + 2a_{down}d$. The displacement is down the ramp so we use $+d$ this time, so we get $a_{down} = v_0^2 / 8d$.

Now apply $\sum F_x = ma_x$ and $\sum F_y = 0$ for both segments. Referring to Fig. 5.106 shows us the components.

Upward segment: $\sum F_y = 0$ gives $n = mg\cos\alpha$

$\sum F_x = ma_x$ gives $mg\sin\alpha + f_k = ma_{up}$ $\rightarrow$ $mg\sin\alpha + \mu_k mg\cos\alpha = ma_{up} = mv_0^2 / 2d$. This simplifies to $g(\sin\alpha + \mu_k\cos\alpha) = \dfrac{v_0^2}{2d}$. (Eq. 1)

Downward segment: The normal force is the same as before, as is the magnitude of the friction force. But now the friction force acts *up* the ramp and $a_{down} = v_0^2 / 8d$. $\sum F_x = ma_x$ gives

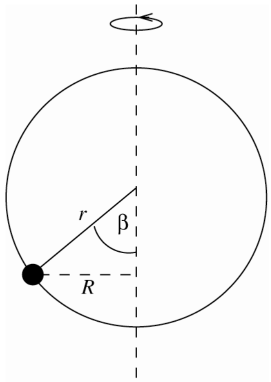
$g(\sin\alpha - \mu_k\cos\alpha) = \dfrac{v_0^2}{8d}$. (Eq. 2)

Combining Eq. 1 and Eq. 2 gives $\dfrac{\sin\alpha - \mu_k\cos\alpha}{\sin\alpha + \mu_k\cos\alpha} = \dfrac{1}{4}$, from which we get $\mu_k = \dfrac{3}{5}\tan\alpha.$

EVALUATE: We found the $a_{up} > a_{down}$. This is reasonable because on the upward segment both gravity and friction oppose the motion, whereas on the downward segment, friction still opposes the motion but gravity does not.

5.107. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the circular motion of the bead. Also use $a_{rad} = 4\pi^2R/T^2$ to relate a_{rad} to the period of rotation T.

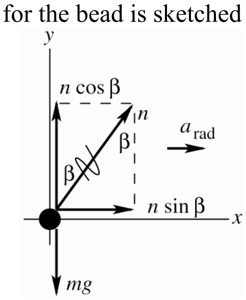
SET UP: The bead and hoop are sketched in Figure 5.107a.



The bead moves in a circle of radius $R = r\sin\beta$. The normal force exerted on the bead by the hoop is radially inward.

Figure 5.107a

The free-body diagram for the bead is sketched in Figure 5.107b.



EXECUTE:

$\Sigma F_y = ma_y$

$n\cos\beta - mg = 0$

$n = mg/\cos\beta$

$\Sigma F_x = ma_x$

$n\sin\beta = ma_{\text{rad}}$

Figure 5.107b

Combine these two equations to eliminate n:

$$\left(\frac{mg}{\cos\beta}\right)\sin\beta = ma_{\text{rad}}$$

$$\frac{\sin\beta}{\cos\beta} = \frac{a_{\text{rad}}}{g}$$

$a_{\text{rad}} = v^2/R$ and $v = 2\pi R/T$, so $a_{\text{rad}} = 4\pi^2 R/T^2$, where T is the time for one revolution.

$R = r\sin\beta$, so $a_{\text{rad}} = \dfrac{4\pi^2 r\sin\beta}{T^2}$

Use this in the above equation: $\dfrac{\sin\beta}{\cos\beta} = \dfrac{4\pi^2 r\sin\beta}{T^2 g}$

This equation is satisfied by $\sin\beta = 0$, so $\beta = 0$, or by $\dfrac{1}{\cos\beta} = \dfrac{4\pi^2 r}{T^2 g}$, which gives $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$.

(a) 4.00 rev/s implies $T = (1/4.00)\text{ s} = 0.250\text{ s}$

Then $\cos\beta = \dfrac{(0.250\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2(0.100\text{ m})}$ and $\beta = 81.1°$.

(b) This would mean $\beta = 90°$. But $\cos 90° = 0$, so this requires $T \to 0$. So β approaches $90°$ as the hoop rotates very fast, but $\beta = 90°$ is not possible.

(c) 1.00 rev/s implies $T = 1.00\text{ s}$

The $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$ equation then says $\cos\beta = \dfrac{(1.00\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2(0.100\text{ m})} = 2.48,$ which is not possible. The only way to have the $\Sigma\vec{F} = m\vec{a}$ equations satisfied is for $\sin\beta = 0.$ This means $\beta = 0;$ the bead sits at the bottom of the hoop.

EVALUATE: $\beta \rightarrow 90°$ as $T \rightarrow 0$ (hoop moves faster). The largest value T can have is given by $T^2 g/(4\pi^2 r) = 1$ so $T = 2\pi\sqrt{r/g} = 0.635$ s. This corresponds to a rotation rate of $(1/0.635)$ rev/s $= 1.58$ rev/s. For a rotation rate less than 1.58 rev/s, $\beta = 0$ is the only solution and the bead sits at the bottom of the hoop. Part (c) is an example of this.

5.108. IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the combined object of motorcycle plus rider.

SET UP: The object has acceleration $a_{\text{rad}} = v^2/r,$ directed toward the center of the circular path.

EXECUTE: **(a)** For the tires not to lose contact, there must be a downward force on the tires. Thus, the (downward) acceleration at the top of the sphere must exceed mg, so $m\dfrac{v^2}{R} > mg,$ and

$v > \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(13.0\text{ m})} = 11.3\text{ m/s}.$

(b) The (upward) acceleration will then be $4g$, so the upward normal force must be $5mg = 5(110\text{ kg})(9.80\text{ m/s}^2) = 5390$ N.

EVALUATE: At any nonzero speed the normal force at the bottom of the path exceeds the weight of the object.

5.109. IDENTIFY: The block begins to move when static friction has reached its maximum value. After that, kinetic friction acts and the block accelerates, obeying Newton's second law.

SET UP: $\Sigma F_x = ma_x$ and $f_{s,max} = \mu_s n$, where n is the normal force (the weight of the block in this case).

EXECUTE: **(a) & (b)** $\Sigma F_x = ma_x$ gives $T - \mu_k mg = ma$. The graph with the problem shows the acceleration a of the block versus the tension T in the cord. So we solve the equation from Newton's second law for a versus T, giving $a = (1/m)T - \mu_k g$. Therefore the slope of the graph will be $1/m$ and the intercept with the vertical axis will be $-\mu_k g$. Using the information given in the problem for the best-fit equation, we have $1/m = 0.182\text{ kg}^{-1}$, so $m = 5.4945$ kg and $-\mu_k g = -2.842\text{ m/s}^2,$ so $\mu_k = 0.290$.

When the block is just ready to slip, we have $f_{s,max} = \mu_s n$, which gives $\mu_s = (20.0\text{ N})/[(5.4945\text{ kg})(9.80\text{ m/s}^2)] = 0.371$.

(c) On the Moon, g is less than on earth, but the mass m of the block would be the same as would μ_k. Therefore the slope $(1/m)$ would be the same, but the intercept $(-\mu_k g)$ would be less negative.

EVALUATE: Both coefficients of friction are reasonable for ordinary materials, so our results are believable.

5.110. IDENTIFY: Near the top of the hill the car is traveling in a circular arc, so it has radial acceleration and Newton's second law applies. We have measurements for the force the car exerts on the road at various speeds.

SET UP: The acceleration of the car is $a_{\text{rad}} = v^2/R$ and $\Sigma F_y = ma_y$ applies to the car. Let the $+y$-axis be downward, since that is the direction of the acceleration of the car.

EXECUTE: **(a)** Apply $\Sigma F_y = ma_y$ to the car at the top of the hill: $mg - n = mv^2/R$, where n is the force the road exerts on the car (which is the same as the force the car exerts on the road). Solving for n gives $n = mg - (m/R)v^2$. So if we plot n versus v^2, we should get a straight line having slope equal to $-m/R$ and intercept with the vertical axis at mg. We could make a table of v^2 and n using the given numbers given with the problem, or we could use graphing software. The resulting graph is shown in Figure 5.110.

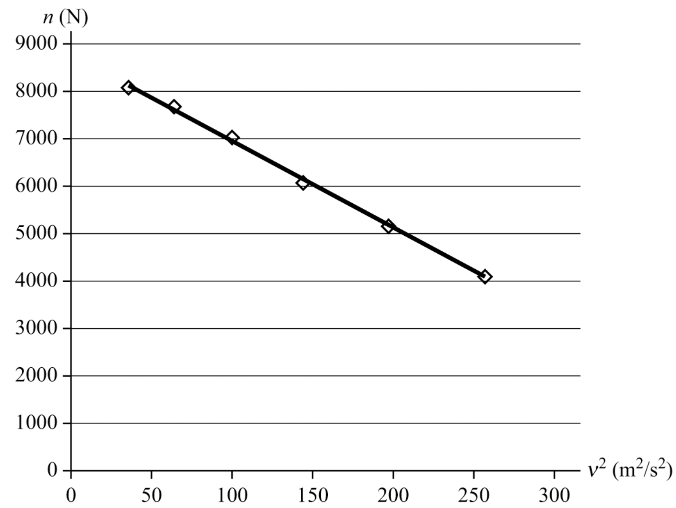


Figure 5.110

(b) The best-fit equation for the graph in Figure 5.110 is $n = [-18.12\text{ N/(m/s)}^2]v^2 + 8794\text{ N}$. Therefore $mg = 8794$ N, which gives $m = (8794\text{ N})/(9.80\text{ m/s}^2) = 897$ kg.
The slope is equal to $-m/R$, so $R = -m/\text{slope} = -(897\text{ kg})/[-18.12\text{ N/(m/s)}^2] = 49.5$ m.

(c) At the maximum speed, $n = 0$. Using $mg - n = mv^2/R$, this gives $v = \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(49.5\text{ m})}$ = 22.0 m/s.

EVALUATE: We can double check (c) using our graph. Putting $n = 0$ into the best-fit equation, we get $v = \sqrt{(8794\text{ N})(18.14\text{ N}\cdot\text{s}^2/\text{m}^2)}$ = 22.0 m/s, which checks. Also 22 m/s is about 49 mph, which is not an unreasonabled speed on a hill.

5.111. **IDENTIFY:** A cable pulling parallel to the surface of a ramp accelerates 2170-kg metal blocks up a ramp that rises at 40.0° above the horizontal. Newton's second law applies to the blocks, and the constant-acceleration kinematics formulas can be used.

SET UP: Call the $+x$-axis parallel to the ramp surface pointing upward because that is the direction of the acceleration of the blocks, and let the y-axis be perpendicular to the surface. There is no acceleration in the y-direction. $\Sigma F_x = ma_x$, $f_k = \mu_k n$, and $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$.

EXECUTE: **(a)** First use $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to find the acceleration of a block. Since $v_{0x} = 0$, we have $a_x = 2(x - x_0)/t^2 = 2(8.00\text{ m})/(4.20\text{ s})^2 = 0.9070\text{ m/s}^2$. The forces in the y-direction balance, so $n = mg\cos(40.0°)$, so $f_k = (0.350)(2170\text{ kg})(9.80\text{ m/s}^2)\cos(40.0°) = 5207$ N. Using $\Sigma F_x = ma_x$, we have $T - mg\sin(40.0°) - f_k = ma$. Solving for T gives $T = (2170\text{ kg})(9.80\text{ m/s}^2)\sin(40.0°) + 5207\text{ N} + (2170\text{ kg})(0.9070\text{ m/s}^2) = 2.13\times 10^4\text{ N} = 21.3$ kN.
From the table shown with the problem, this tension is greater than the safe load of a ½ inch diameter cable (which is 19.0 kN), so we need to use a 5/8-inch cable.

(b) We assume that the safe load (SL) is proportional to the cross-sectional area of the cable, which means that SL $\propto \pi(D/2)^2 \propto (\pi/4)D^2$, where D is the diameter of the cable. Therefore a graph of SL versus D^2 should give a straight line. We could use the data given in the table with the problem to make the graph by hand, or we could use graphing software. The resulting graph is shown in Figure 5.111 (next page). The best-fit line has a slope of 74.09 kN/in.^{2} and a y-intercept of 0.499 kN. For a cable of diameter $D = 9/16$ in., this equation gives SL = (74.09 kN/in.^{2})(9/16 in.)^2 + 0.499 kN = 23.9 kN.

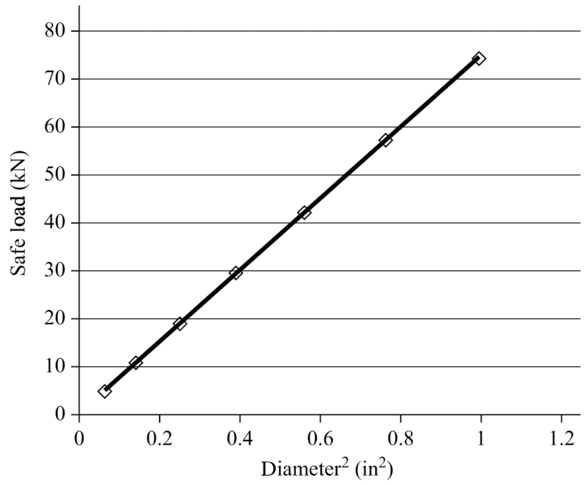


Figure 5.111

(c) The acceleration is now zero, so the forces along the surface balance, giving $T + f_s = mg\sin(40.0°)$. Using the numbers we get $T = 3.57$ kN.
(d) The tension at the top of the cable must accelerate the block and the cable below it, so the tension at the top would be larger. For a 5/8-inch cable, the mass per meter is 0.98 kg/m, so the 9.00-m long cable would have a mass of (0.98 kg/m)(9.00 m) = 8.8 kg. This is only 0.4% of the mass of the block, so neglecting the cable weight has little effect on accuracy.
EVALUATE: It is reasonable that the safe load of a cable is proportional to its cross-sectional area. If we think of the cable as consisting of many tiny strings each pulling, doubling the area would double the number of strings.

5.112. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block and to the wedge.
SET UP: For both parts, take the x-direction to be horizontal and positive to the right, and the y-direction to be vertical and positive upward. The normal force between the block and the wedge is n; the normal force between the wedge and the horizontal surface will not enter, as the wedge is presumed to have zero vertical acceleration. The horizontal acceleration of the wedge is A, and the components of acceleration of the block are a_x and a_y.
EXECUTE: **(a)** The equations of motion are then $MA = -n\sin\alpha$, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$. Note that the normal force gives the wedge a negative acceleration; the wedge is expected to move to the left. These are three equations in four unknowns, A, a_x, a_y and n. Solution is possible with the imposition of the relation between A, a_x and a_y. An observer on the wedge is not in an inertial frame, and should not apply Newton's laws, but the kinematic relation between the components of acceleration are not so restricted. To such an observer, the vertical acceleration of the block is a_y, but the horizontal acceleration of the block is $a_x - A$. To this observer, the block descends at an angle α, so the relation needed is $\dfrac{a_y}{a_x - A} = -\tan\alpha$. At this point, algebra is unavoidable. A possible approach is to eliminate a_x by noting that $a_x = -\dfrac{M}{m}A$, using this in the kinematic constraint to eliminate a_y and then eliminating n. The results are:

$$A = \frac{-gm}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_x = \frac{gM}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_y = \frac{-g(M+m)\tan\alpha}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

(b) When $M \gg m$, $A \to 0$, as expected (the large block won't move). Also,

$a_x \to \dfrac{g}{\tan\alpha + (1/\tan\alpha)} = g\dfrac{\tan\alpha}{\tan^2\alpha + 1} = g\sin\alpha\cos\alpha$ which is the acceleration of the block ($g\sin\alpha$ in this case), with the factor of $\cos\alpha$ giving the horizontal component. Similarly, $a_y \to -g\sin^2\alpha$.

(c) The trajectory is a straight line with slope $-\left(\dfrac{M+m}{M}\right)\tan\alpha$.

EVALUATE: If $m \gg M$, our general results give $a_x = 0$ and $a_y = -g$. The massive block accelerates straight downward, as if it were in free fall.

5.113. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block and to the wedge.

SET UP: From Problem 5.112, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$ for the block. $a_y = 0$ gives $a_x = g\tan\alpha$.

EXECUTE: If the block is not to move vertically, both the block and the wedge have this horizontal acceleration and the applied force must be $F = (M+m)a = (M+m)g\tan\alpha$.

EVALUATE: $F \to 0$ as $\alpha \to 0$ and $F \to \infty$ as $\alpha \to 90°$.

5.114. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each of the three masses and to the pulley B.

SET UP: Take all accelerations to be positive downward. The equations of motion are straightforward, but the kinematic relations between the accelerations, and the resultant algebra, are not immediately obvious. If the acceleration of pulley B is a_B, then $a_B = -a_3$, and a_B is the average of the accelerations of masses 1 and 2, or $a_1 + a_2 = 2a_B = -2a_3$.

EXECUTE: **(a)** There can be no net force on the massless pulley B, so $T_C = 2T_A$. The five equations to be solved are then $m_1g - T_A = m_1a_1$, $m_2g - T_A = m_2a_2$, $m_3g - T_C = m_3a_3$, $a_1 + a_2 + 2a_3 = 0$ and $2T_A - T_C = 0$. These are five equations in five unknowns, and may be solved by standard means.

The accelerations a_1 and a_2 may be eliminated using $2a_3 = -(a_1 + a_2) = -[2g - T_A((1/m_1) + (1/m_2))]$.

The tension T_A may be eliminated by using $T_A = (1/2)T_C = (1/2)m_3(g - a_3)$.

Combining and solving for a_3 gives $a_3 = g\dfrac{-4m_1m_2 + m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(b) The acceleration of the pulley B has the same magnitude as a_3 and is in the opposite direction.

(c) $a_1 = g - \dfrac{T_A}{m_1} = g - \dfrac{T_C}{2m_1} = g - \dfrac{m_3}{2m_1}(g - a_3)$. Substituting the above expression for a_3 gives

$a_1 = g\dfrac{4m_1m_2 - 3m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(d) A similar analysis (or, interchanging the labels 1 and 2) gives $a_2 = g\dfrac{4m_1m_2 - 3m_1m_3 + m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(e), **(f)** Once the accelerations are known, the tensions may be found by substitution into the appropriate equation of motion, giving $T_A = g\dfrac{4m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$, $T_C = g\dfrac{8m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(g) If $m_1 = m_2 = m$ and $m_3 = 2m,$ all of the accelerations are zero, $T_C = 2mg$ and $T_A = mg.$ All masses and pulleys are in equilibrium, and the tensions are equal to the weights they support, which is what is expected.

EVALUATE: It is useful to consider special cases. For example, when $m_1 = m_2 \gg m_3$ our general result gives $a_1 = a_2 = +g$ and $a_3 = g.$

5.115. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the ball at each position.

SET UP: When the ball is at rest, $a = 0.$ When the ball is swinging in an arc it has acceleration component $a_{\text{rad}} = \dfrac{v^2}{R},$ directed inward.

EXECUTE: Before the horizontal string is cut, the ball is in equilibrium, and the vertical component of the tension force must balance the weight, so $T_A \cos\beta = w$ or $T_A = w/\cos\beta.$ At point *B*, the ball is not in equilibrium; its speed is instantaneously 0, so there is no radial acceleration, and the tension force must balance the radial component of the weight, so $T_B = w\cos\beta$ and the ratio $(T_B/T_A) = \cos^2\beta.$

EVALUATE: At point *B* the net force on the ball is not zero; the ball has a tangential acceleration.

5.116. **IDENTIFY:** The forces must balance for the person not to slip.

SET UP and EXECUTE: As was done in earlier problems, balancing forces parallel to and perpendicular to the surface of the rock leads to the equation $\mu_s = \tan\theta = 1.2$, so $\theta = 50°$, which is choice (b).

EVALUATE: The condition $\mu_s = \tan\theta$ applies only when the person is just ready to slip, which would be the case at the maximum angle.

5.117. **IDENTIFY:** Friction changes from static friction to kinetic friction.

SET UP and EXECUTE: When she slipped, static friction must have been at its maximum value, and that was enough to support her weight just before she slipped. But the kinetic friction will be less than the maximum static friction, so the kinetic friction force will not be enough to balance her weight down the incline. Therefore she will slide down the surface and continue to accelerate downward, making (b) the correct choice.

EVALUATE: Shoes with a greater coefficient of static friction would enable her to walk more safely.

5.118. **IDENTIFY:** The person pushes off horizontally and acclerates herself, so Newton's second law applies.

SET UP and EXECUTE: She runs horizontally, so her vertical acceleration is zero, which makes the normal force *n* due to the ground equal to her weight *mg*. In the horizontal direction, static friction accelerates her forward, and it must be its maximum value to achieve her maximum acceleration. Therefore $f_s = ma = \mu_s n = \mu_s mg$, which gives $a = \mu_s g = 1.2g$, making (d) the correct choice.

EVALUATE: Shoes with more friction would allow her to accelerate even faster.

6

WORK AND KINETIC ENERGY

VP6.2.1. **IDENTIFY:** This problem requires the calculation of work knowing the components of the force and displacement vectors.

SET UP: $W = \vec{F} \cdot \vec{S}$, which in terms of components is $W = F_x s_x + F_y s_y$.

EXECUTE: **(a)** $W = F_x s_x + F_y s_y = (126 \text{ N})(3.00 \text{ m}) + (168 \text{ N})(-6.50 \text{ m}) = -714 \text{ J}$

(b) The football player's force is equal and opposite to that of the opponent, but his displacement is the same, so the work he does is the negative of the work done by his opponent. $W = -(-714 \text{ J}) = +714 \text{ J}$.

EVALUATE: As we see in this problem, negative work definitely has physical meaning.

VP6.2.2. **IDENTIFY:** This problem requires a calculation using work. We know the work, force, and displacement and want to find the angle between the force and displacement vectors.

SET UP: To find the angle, we write work in the form $W = \vec{F} \cdot \vec{S} = Fs \cos\phi$.

EXECUTE: $W = Fs \cos\phi \quad \rightarrow \quad 1.47 \times 10^3 \text{ J} = (215 \text{ N})(8.40 \text{ m}) \cos\phi \quad \rightarrow \quad \phi = 35.5°$.

EVALUATE: The angle ϕ is less then 90°, so the work should be positive, which it is.

VP6.2.3. **IDENTIFY:** This problem requires the calculation of work for three forces knowing the magnitudes of the forces and displacements and the angles between them. Work is a scalar quantity, so the total work is the algebraic sum of the individual works.

SET UP: Use $W = Fs \cos\phi$ in each case. Then find the sum of the works.

EXECUTE: **(a)** The friction force is along the surface of the ramp but in the opposite direction from the displacement, so $\phi = 180°$. Thus

$W_f = f_k s \cos\phi° = (30.0 \text{ N})(3.00 \text{ m}) \cos 180° = -90.0 \text{ J}$.

(b) Gravity acts downward and the displacement is along the ramp surface, so $\phi = 62.0°$. Thus $W_g = mgs \cos 62.0° = (15.0 \text{ kg})(9.80 \text{ m/s}^2)(3.00 \text{ m}) \cos 62.0° = 207 \text{ J}$.

(c) The normal force is perpendicular to the displacment, so $\phi = 90°$ and $\cos\phi = 0$, so the normal force does no work.

(d) $W_{tot} = W_f + W_g + W_n = -90.0 \text{ J} + 207 \text{ J} + 0 = 117 \text{ J}$.

EVALUATE: The work is positive because the gravitational force along the ramp is greater than the friction, so the net force does positive work and the displacement is down the ramp.

VP6.2.4. **IDENTIFY:** We want to calculate work using the components of the force and displacement.

SET UP: Since we know the components, we use $W = F_x s_x + F_y s_y$.

EXECUTE: **(a)** $W_1 = F_0(2d) = 2F_0 d$, $W_2 = (-3F_0)d = -3F_0 d$, $W_3 = (-4F_0)(2d) + Gd = -8F_0 d + Gd$.

(b) $W_{tot} = 2F_0 d + (-3F_0 d) + (-8F_0 d + Gd) = 0 \quad \rightarrow \quad G = 9F_0$.

EVALUATE: The equation $W = Fs \cos\phi$ is still correct, but it would not allow us to easily calculate the work for these forces.

VP6.4.1. **IDENTIFY:** This problem requires the work-energy theorem. As the cylinder falls, work is done on it by gravity and friction. This work changes its kinetic energy.

SET UP: $W_{\text{tot}} = \Delta K = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$. Figure VP6.4.1 illustrates the arrangement.

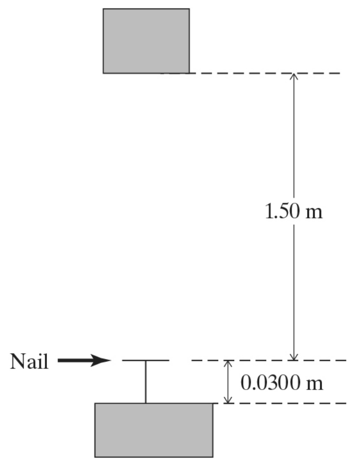


Figure VP6.4.1

EXECUTE: **(a)** Call point 1 where the cylinder is first released, and point 2 where it is just about to hit the nail. $W_g = mgs$ and $W_f = -fs$, so $W_{\text{tot}} = mgs - fs$. $K_1 = 0$ and $K_2 = \frac{1}{2}mv^2$. The work-energy theorem gives $mgs - fs = \frac{1}{2}mv^2$. Using $s = 1.50$ m, $f = 16.0$ N, $m = 20.0$ kg gives $v = 5.20$ m/s.

(b) Now we want to know about the force the cylinder exerts on the nail while hammering it in. Therefore we choose point 1 to be the instant just as the cylinder strikes the nail (that was point 2 in part (a)) and point 2 to be the instant when the nail has first stopped. As the cylinder pushes in the nail, it exerts a force F_N on the nail while pushing 0.0300 m into the block of wood. Likewise the other forces act for the same distance.

$W_{\text{tot}} = W_g + W_f + W_N = mgs - fs - F_N s$, $K_1 = \frac{1}{2}mv^2$ (using $v = 5.20$ m/s), $K_2 = 0$. The work-energy theorem gives $mgs - fs - f_N s = 0 - \frac{1}{2}mv^2$. Putting in the numbers, with $s = 0.0300$ m, gives $F_N = 9180$ N.

EVALUATE: The force in (a) is around a thousand pounds. From experience, we know that it takes a large force to hammer in a nail.

VP6.4.2. **IDENTIFY:** This problem requires the work-energy theorem. Gravity and the tension in the rope do work on the crate.

SET UP: $W_{\text{tot}} = \Delta K = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$. Call point 1 when you just increase the tension to 175 N and point 2 just after you have lifted it the additional 1.25 m.

EXECUTE: **(a)** $W_T = Ts = (175\text{ N})(1.25\text{ m}) = 219$ J.

(b) $W_g = mgs\cos\phi = (14.5\text{ kg})(9.80\text{ m/s}^2)(1.25\text{ m})\cos 180° = -178$ J

(c) $F_{\text{net}} = T - mg$, so $W_{\text{tot}} = F_{\text{net}}s = (T - mg)s = W_T + W_g = 219\text{ J} - 178\text{ J} = 41$ J.

(d) $W_{tot} = K_2 - K_1 = K_2 - \frac{1}{2}mv_1^2$. Solving for K_2 gives $K_2 = W_{tot} + \frac{1}{2}mv_1^2$, so we get

$K_2 = 41\text{ J} + \frac{1}{2}(14.5\text{ kg})(0.500\text{ m/s})^2 = 43\text{ J} = \frac{1}{2}mv_2^2 = \frac{1}{2}(14.5\text{ kg})\,v_2^2 \rightarrow v_2 = 2.4\text{ m/s}$.

EVALUATE: The tension does positive work and therefore increases the crate's kinetic energy.

VP6.4.3. **IDENTIFY:** This problem requires the work-energy theorem. Gravity and the thrust of the engine do work on the helicopter.

SET UP: $W_{tot} = \Delta K = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$. Call point 1 when the pilot just increases the thrust and point 2 just as the speed has decreased to 0.450 m/s.

EXECUTE: **(a)** $K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(1400\text{ kg})(3.00\text{ m/s})^2 = 6300\text{ J}$.

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(1400\text{ kg})(0.450\text{ m/s})^2 = 142\text{ J}$.

(b) $W_{tot} = K_2 - K_1 = 142\text{ J} - 6300\text{ J} = -6160\text{ J}$

$W_g = mg\Delta y = (1400\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 27{,}400\text{ J}$

$W_{tot} = W_{thrust} + W_g \rightarrow -6160\text{ J} = W_{thrust} + 27{,}400\text{ J} \rightarrow W_{thrust} = -3.36\times10^4\text{ J}$.

(c) $W_{thrust} = F_{thrust}s\cos\phi \rightarrow -3.36\times10^4\text{ J} = F_{thrust}(2.00\text{ m})\cos 180° \rightarrow F_{thrust} = 1.68\times10^4\text{ N}$.

EVALUATE: The magnitude of the thrust force is around ten times as great as the force of gravity, so the net work is negative in order to slow down the helicopter.

VP6.4.4. **IDENTIFY:** This problem requires the work-energy theorem.

SET UP: $W_{tot} = \Delta K = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$. Call point 1 the point where the block is released and point 2 the bottom of the ramp. Let d be the distance it moves along the surface of the ramp. We also use $\sum F_y = 0$.

EXECUTE: **(a)** See Fig. VP6.4.4. $W_g = mgd\cos\phi = mgd\sin\theta$.

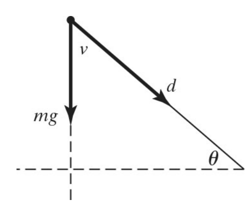


Figure VP6.4.4

(b) $W_f = f_k d\cos 180° = -f_k d = -\mu_k n$. Using $\sum F_y = 0$, where the y-axis is perpendicular to the face of the ramp, gives $n = mg\cos\theta$, so $W_f = -\mu_k mgd\cos\theta$.

(c) $W_{tot} = K_2 - K_1 = W_g + W_f + W_n$. $W_n = 0$ and $K_1 = 0$, so we get

$mgd\sin\theta - \mu_k mgd\cos\theta = \frac{1}{2}mv_2^2 \rightarrow v_2 = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$.

EVALUATE: To check our result, let $\mu_k = 0$ for a frictionless surface. In that case, our result reduces to $v = \sqrt{2gd\sin\theta} = \sqrt{2gh}$. From our study of free-fall, we know that an object falling a distance h from rest has speed $v = \sqrt{2gh}$, which agrees with our result in this special case.

VP6.8.1. **IDENTIFY:** This problem requires the work-energy theorem and involves the work done by a spring.

SET UP: $W_{tot} = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$, and the work done to compress a spring is $W = \frac{1}{2}kx^2$. Call point 1 the point just before the glider hits the spring and point 2 to be at maximum compression of the spring.

EXECUTE: **(a)** $W_{tot} = K_2 - K_1$. The final kinetic energy is zero and the work done by the spring on the glider is $-\frac{1}{2}kd^2$, so we get $-\frac{1}{2}kd^2 = -\frac{1}{2}mv_1^2$, which gives

$(30.0\text{ N/m})d^2 = (0.150\text{ kg})(1.25\text{ m/s})^2 \quad \rightarrow \quad d = 0.0884\text{ m} = 8.84\text{ cm}$.

(b) Both friction and the spring oppose the glider's motion and therefore do negative work on the glider. The work done by the spring is still $-\frac{1}{2}kd^2$, but d is now different. The work done by kinetic friction is $W_f = -f_k d = -\mu_k mgd$. So the work-energy theorem gives

$-\frac{1}{2}kd^2 - \mu_k mgd = 0 - \frac{1}{2}mv_1^2$. Putting in the numbers and simplifying gives

$(30.0\text{ N/m})d^2 + 2(0.320)(0.150\text{ kg})(9.80\text{ m/s}^2)d - (0.150\text{ kg})(1.25\text{ m/s})^2 = 0$. The quadratic formula gives two solutions (one positive and one negative), but we need the positive root, which is $d = 0.0741$ m = 7.41 cm.

EVALUATE: With no friction, the glider traveled 8.84 cm, but with friction it traveled only 7.41 cm. This is reasonable since both friction and the spring slowed down the glider in the case with friction.

VP6.8.2. **IDENTIFY:** This problem requires the work-energy theorem and involves the work done by a spring.

SET UP: $W_{tot} = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$, and the work done to compress a spring is $W = \frac{1}{2}kx^2$. Call point 1 the point just before the glider hits the spring and point 2 to be at maximum compression of the spring.

EXECUTE: $W_{tot} = K_2 - K_1$. The final kinetic energy is zero and the work done by the spring on the glider is $-\frac{1}{2}kd^2$. The work done by kinetic friction is $W_f = -f_k d = -\mu_k mgd$. So the work-energy theorem gives so the work-energy theorem gives $-\frac{1}{2}kd^2 - \mu_k mgd = 0 - \frac{1}{2}mv_1^2$. Solving for μ_k gives

$\mu_k = \dfrac{mv_1^2 - kd^2}{2mgd}$.

EVALUATE: Checking units shows that $\dfrac{mv_1^2 - kd^2}{2mgd}$ is dimensionless, which is correct for a coefficient of friction.

VP6.8.3. **IDENTIFY:** This problem involves the work done on the bob of a swinging pendulum.

SET UP: $W = Fs\cos\phi$. Let L be the length of the string and θ the angle it makes with the vertical.

EXECUTE: **(a)** $W_g = mg\,\Delta y = mgL(1 - \cos\theta)$

$W_g = (0.500\text{ kg})(9.80\text{ m/s}^2)(0.750\text{ m})(1 - \cos 35.0°) = 0.665$ J.

(b) $W_g = mg\,\Delta y$ where Δy is negative, so $W_g = -0.665$ J.

(c) $W_{tot} = W_{AB} + W_{BC} = 0.665\text{ J} - 0.665\text{ J} = 0$.

EVALUATE: The total work is zero because gravity does positive work as the bob is moving downward and negative work as it is moving upward.

VP6.8.4. **IDENTIFY:** This problem requires application of the work-energy theorem to a pendulum.

SET UP: $W_{tot} = K_2 - K_1$, where $K = \frac{1}{2}mv^2$, and work is $W = Fs\cos\phi$. Call point 1 the highest point in its swing point 2 the lowest point.

EXECUTE: The tension in the silk strand does zero work because the tension is perpendicular to the path of the spider, so only gravity does work. $K_1 = 0$ and $K_2 = \frac{1}{2}mv^2$. Therefore

$$mgL(1 - \cos\theta) = \frac{1}{2}mv^2 \qquad \rightarrow \qquad v = \sqrt{2gL(1-\cos\theta)}.$$

EVALUATE: Using our result, we see that as θ approaches 90°, $\cos\theta$ approaches zero, so v gets larger and larger. This is reasonable because the spider starts swinging from a higher and higher elevation.

6.1. **IDENTIFY** and **SET UP:** For parts (a) through (d), identify the appropriate value of ϕ and use the relation $W = F_P s = (F\cos\phi)s$. In part (e), apply the relation $W_{\text{net}} = W_{\text{student}} + W_{\text{grav}} + W_n + W_f$.

EXECUTE: **(a)** Since you are applying a horizontal force, $\phi = 0°$. Thus,

$W_{\text{student}} = (2.40\text{ N})(\cos 0°)(1.50\text{ m}) = 3.60\text{ J}.$

(b) The friction force acts in the horizontal direction, opposite to the motion, so $\phi = 180°$.

$W_f = (F_f\cos\phi)s = (0.600\text{ N})(\cos 180°)(1.50\text{ m}) = -0.900\text{ J}.$

(c) Since the normal force acts upward and perpendicular to the tabletop, $\phi = 90°$.

$W_n = (n\cos\phi)s = (ns)(\cos 90°) = 0.0\text{ J}.$

(d) Since gravity acts downward and perpendicular to the tabletop, $\phi = 270°$.

$W_{\text{grav}} = (mg\cos\phi)s = (mgs)(\cos 270°) = 0.0\text{ J}.$

(e) $W_{\text{net}} = W_{\text{student}} + W_{\text{grav}} + W_n + W_f = 3.60\text{ J} + 0.0\text{ J} + 0.0\text{ J} - 0.900\text{ J} = 2.70\text{ J}.$

EVALUATE: Whenever a force acts perpendicular to the direction of motion, its contribution to the net work is zero.

6.2. **IDENTIFY:** In each case the forces are constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

SET UP: In part **(a)**, when the cable pulls horizontally $\phi = 0°$ and when it pulls at 35.0° above the horizontal $\phi = 35.0°$. In part **(b)**, if the cable pulls horizontally $\phi = 180°$. If the cable pulls on the car at 35.0° above the horizontal it pulls on the truck at 35.0° below the horizontal and ϕ 145.0°. For the gravity force $\phi = 90°$, since the force is vertical and the displacement is horizontal.

EXECUTE: **(a)** When the cable is horizontal, $W = (1350\text{ N})(5.00\times 10^3\text{ m})\cos 0° = 6.75\times 10^6\text{ J}$. When the cable is 35.0° above the horizontal, $W = (1350\text{ N})(5.00\times 10^3\text{ m})\cos 35.0° = 5.53\times 10^6\text{ J}$.

(b) $\cos 180° = -\cos 0°$ and $\cos 145.0° = -\cos 35.0°$, so the answers are -6.75×10^6 J and -5.53×10^6 J.

(c) Since $\cos\phi = \cos 90° = 0$, $W = 0$ in both cases.

EVALUATE: If the car and truck are taken together as the system, the tension in the cable does no net work.

6.3. **IDENTIFY:** Each force can be used in the relation $W = F_{\parallel}s = (F\cos\phi)s$ for parts (b) through (d). For part (e), apply the net work relation as $W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f$.

SET UP: In order to move the crate at constant velocity, the worker must apply a force that equals the force of friction, $F_{\text{worker}} = f_k = \mu_k n$.

EXECUTE: **(a)** The magnitude of the force the worker must apply is:

$$F_{\text{worker}} = f_k = \mu_k n = \mu_k mg = (0.25)(30.0\text{ kg})(9.80\text{ m/s}^2) = 74\text{ N}$$

(b) Since the force applied by the worker is horizontal and in the direction of the displacement, $\phi = 0°$ and the work is:

$$W_{\text{worker}} = (F_{\text{worker}} \cos\phi)s = [(74 \text{ N})(\cos 0°)](4.5 \text{ m}) = +333 \text{ J}$$

(c) Friction acts in the direction opposite of motion, thus $\phi = 180°$ and the work of friction is:

$$W_f = (f_k \cos\phi)s = [(74 \text{ N})(\cos 180°)](4.5 \text{ m}) = -333 \text{ J}$$

(d) Both gravity and the normal force act perpendicular to the direction of displacement. Thus, neither force does any work on the crate and $W_{\text{grav}} = W_n = 0.0$ J.

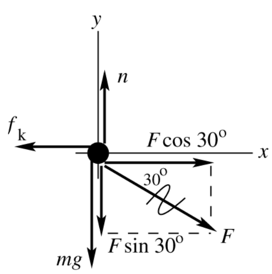
(e) Substituting into the net work relation, the net work done on the crate is:

$$W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f = +333 \text{ J} + 0.0 \text{ J} + 0.0 \text{ J} - 333 \text{ J} = 0.0 \text{ J}$$

EVALUATE: The net work done on the crate is zero because the two contributing forces, F_{worker} and F_f, are equal in magnitude and opposite in direction.

6.4. IDENTIFY: The forces are constant so we can use $W = Fs\cos\phi$ to calculate the work. Constant speed implies $a = 0$. We must use $\Sigma\vec{F} = m\vec{a}$ applied to the crate to find the forces acting on it.

(a) SET UP: The free-body diagram for the crate is given in Figure 6.4.



EXECUTE: $\Sigma F_y = ma_y$

$n - mg - F\sin 30° = 0$

$n = mg + F\sin 30°$

$f_k = \mu_k n = \mu_k mg + F\mu_k \sin 30°$

Figure 6.4

$\Sigma F_x = ma_x$

$F\cos 30° - f_k = 0$

$F\cos 30° - \mu_k mg - \mu_k \sin 30° F = 0$

$$F = \frac{\mu_k mg}{\cos 30° - \mu_k \sin 30°} = \frac{0.25(30.0 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 30° - (0.25)\sin 30°} = 99.2 \text{ N}$$

(b) $W_F = (F\cos\phi)s = (99.2 \text{ N})(\cos 30°)(4.5 \text{ m}) = 387$ J

($F\cos 30°$ is the horizontal component of $\vec{F}$; the work done by $\vec{F}$ is the displacement times the component of $\vec{F}$ in the direction of the displacement.)

(c) We have an expression for f_k from part (a):

$$f_k = \mu_k(mg + F\sin 30°) = (0.250)[(30.0 \text{ kg})(9.80 \text{ m/s}^2) + (99.2 \text{ N})(\sin 30°)] = 85.9 \text{ N}$$

$\phi = 180°$ since f_k is opposite to the displacement. Thus

$W_f = (f_k \cos\phi)s = (85.9 \text{ N})(\cos 180°)(4.5 \text{ m}) = -387$ J.

(d) The normal force is perpendicular to the displacement so $\phi = 90°$ and $W_n = 0$. The gravity force (the weight) is perpendicular to the displacement so $\phi = 90°$ and $W_w = 0$.

(e) $W_{\text{tot}} = W_F + W_f + W_n + W_w = +387 \text{ J} + (-387 \text{ J}) = 0$

EVALUATE: Forces with a component in the direction of the displacement do positive work, forces opposite to the displacement do negative work, and forces perpendicular to the displacement do zero work. The total work, obtained as the sum of the work done by each force, equals the work done by the net force. In this problem, $F_{\text{net}} = 0$ since $a = 0$ and $W_{\text{tot}} = 0,$ which agrees with the sum calculated in part (e).

6.5. **IDENTIFY:** The gravity force is constant and the displacement is along a straight line, so $W = Fs\cos\phi.$

SET UP: The displacement is upward along the ladder and the gravity force is downward, so $\phi = 180.0° - 30.0° = 150.0°.$ $w = mg = 735$ N.

EXECUTE: **(a)** $W = (735\text{ N})(2.75\text{ m})\cos150.0° = -1750$ J.

(b) No, the gravity force is independent of the motion of the painter.

EVALUATE: Gravity is downward and the vertical component of the displacement is upward, so the gravity force does negative work.

6.6. **IDENTIFY** and **SET UP:** $W_F = (F\cos\phi)s,$ since the forces are constant. We can calculate the total work by adding the work done by each force. The forces are sketched in Figure 6.6.

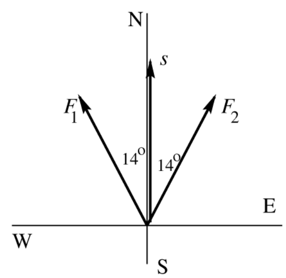


Figure 6.6

EXECUTE: $W_1 = F_1 s\cos\phi_1$

$W_1 = (1.80\times10^6\text{ N})(0.75\times10^3\text{ m})\cos14°$

$W_1 = 1.31\times10^9$ J

$W_2 = F_2 s\cos\phi_2 = W_1$

$W_{\text{tot}} = W_1 + W_2 = 2(1.31\times10^9\text{ J}) = 2.62\times10^9$ J

EVALUATE: Only the component $F\cos\phi$ of force in the direction of the displacement does work. These components are in the direction of $\vec{s}$ so the forces do positive work.

6.7. **IDENTIFY:** All forces are constant and each block moves in a straight line, so $W = Fs\cos\phi.$ The only direction the system can move at constant speed is for the 12.0 N block to descend and the 20.0 N block to move to the right.

SET UP: Since the 12.0 N block moves at constant speed, $a = 0$ for it and the tension T in the string is $T = 12.0$ N. Since the 20.0 N block moves to the right at constant speed, the friction force f_k on it is to the left and $f_k = T = 12.0$ N.

EXECUTE: **(a)** (i) $\phi = 0°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos0° = 9.00$ J. (ii) $\phi = 180°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos180° = -9.00$ J.

(b) (i) $\phi = 90°$ and $W = 0.$ (ii) $\phi = 0°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos0° = 9.00$ J. (iii) $\phi = 180°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos180° = -9.00$ J. (iv) $\phi = 90°$ and $W = 0.$

(c) $W_{\text{tot}} = 0$ for each block.

EVALUATE: For each block there are two forces that do work, and for each block the two forces do work of equal magnitude and opposite sign. When the force and displacement are in opposite directions, the work done is negative.

6.8. **IDENTIFY:** Apply $W = Fs\cos\phi$.

SET UP: $\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{i}} = \hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{j}} = 1$ and $\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{j}} = \hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{i}} = 0$

EXECUTE: The work you do is $\vec{F}\cdot\vec{s}=\left[(30\text{ N})\hat{\boldsymbol{i}}-(40\text{ N})\hat{\boldsymbol{j}}\right]\cdot\left[(-9.0\text{ m})\hat{\boldsymbol{i}}-(3.0\text{ m})\hat{\boldsymbol{j}}\right]$

$\vec{F}\cdot\vec{s}=(30\text{ N})(-9.0\text{ m})+(-40\text{ N})(-3.0\text{ m})=-270\text{ N}\cdot\text{m}+120\text{ N}\cdot\text{m}=-150\text{ J}.$

EVALUATE: The x-component of $\vec{F}$ does negative work and the y-component of $\vec{F}$ does positive work. The total work done by $\vec{F}$ is the sum of the work done by each of its components.

6.9. IDENTIFY: We want to compare the work done by friction over two different paths.
SET UP: Work is $W = Fs\cos\phi$. Call A the starting point and C the ending point for the first trip, and call B the end of the first segment of the second trip. On the first trip, $W_{AC} = -4.8$ J. We want to find the work done on the second trip, which is $W_{ABC} = W_{AB} + W_{BC}$. Call f_k the friction force, which is the same on all parts of the trips.
EXECUTE: $W_{AC} = -f_k s_{AC} = -f_k(0.500\text{ m})$. Likewise $W_{ABC} = W_{AB} + W_{BC}$ which gives $W_{ABC} = -f_k(0.300\text{ m}) - f_k(0.400\text{ m}) = -f_k(0.700\text{ m})$. Comparing the work in the two paths gives $\dfrac{W_{ABC}}{W_{AC}}=\dfrac{(-0.700\text{ m})f_k}{(-0.500\text{ m})f_k}=1.40$. So $W_{ABC} = 1.40W_{AC} = (1.40)(-4.8\text{ J}) = -6.7$ J.
EVALUATE: We found that friction does more work during the ABC path than during the AC path. This is reasonable because the force is the same but the distance is greater during the ABC path. Notice that the work done by friction between two points depends on the *path* taken between those points.

6.10. IDENTIFY and SET UP: Use $W = F_p s = (F\cos\phi)s$ to calculate the work done in each of parts (a) through (c). In part (d), the net work consists of the contributions due to all three forces, or $w_{\text{net}} = w_{\text{grav}} + w_{\text{n}} + w_{\text{f}}$.

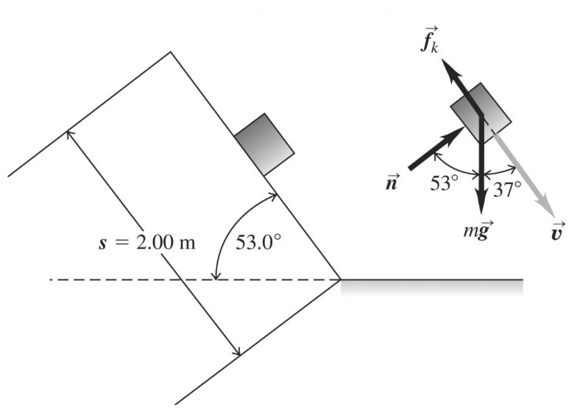


Figure 6.10

EXECUTE: **(a)** As the package slides, work is done by the frictional force which acts at $\phi = 180°$ to the displacement. The normal force is $mg\cos 53.0°$. Thus for $\mu_k = 0.40$,

$W_f = F_p s = (f_k\cos\phi)s = (\mu_k n\cos\phi)s = [\mu_k(mg\cos 53.0°)](\cos 180°)s.$

$W_f = (0.40)[(12.0\text{ kg})(9.80\text{ m/s}^2)(\cos 53.0°)](\cos 180°)(2.00\text{ m}) = -57\text{ J}.$

(b) Work is done by the component of the gravitational force parallel to the displacement. $\phi = 90° - 53° = 37°$ and the work of gravity is

$W_{\text{grav}} = (mg\cos\phi)s = [(12.0\text{ kg})(9.80\text{ m/s}^2)(\cos 37.0°)](2.00\text{ m}) = +188\text{ J}.$

(c) $W_n = 0$ since the normal force is perpendicular to the displacement.

(d) The net work done on the package is $W_{\text{net}} = W_{\text{grav}} + W_n + W_f = 188\text{ J} + 0.0\text{ J} - 57\text{ J} = 131\text{ J}.$

EVALUATE: The net work is positive because gravity does more positive work than the magnitude of the negative work done by friction.

6.11. IDENTIFY: We need to calculate work and use the work-energy theorem.

SET UP: $W = Fs\cos\phi$, $W_{tot} = K_2 - K_1$, $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** Using $W = Fs\cos\phi$, we see that to double the work for the same force and displacement, the angle $\cos\phi$ must double. Thus $\cos\phi = 2\cos 60° = 1.00$, so $\phi = 0°$. We should pull horizontally.

(b) The block starts from rest so $K_1 = 0$. Using the work-energy theorem tells us that in this case $W_1 = \frac{1}{2}mv^2$. To double v, the work must be $W_2 = \frac{1}{2}m(2v)^2 = 4\left(\frac{1}{2}mv^2\right) = 4K_1 = 4W_1$. Since only the force F varies, F must increase by a factor of 4 to double the speed.

EVALUATE: To double the speed does not take twice as much work; it takes 4 times as much.

6.12. IDENTIFY: Since the speed is constant, the acceleration and the net force on the monitor are zero.

SET UP: Use the fact that the net force on the monitor is zero to develop expressions for the friction force, f_k, and the normal force, n. Then use $W = F_P s = (F\cos\phi)s$ to calculate W.

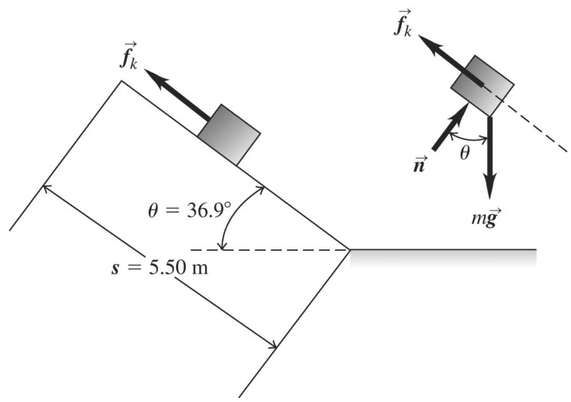


Figure 6.12

EXECUTE: **(a)** Summing forces along the incline, $\Sigma F = ma = 0 = f_k - mg\sin\theta$, giving $f_k = mg\sin\theta$, directed up the incline. Substituting gives $W_f = (f_k\cos\phi)s = [(mg\sin\theta)\cos\phi]s$.

$W_f = [(10.0\text{ kg})(9.80\text{ m/s}^2)(\sin 36.9°)](\cos 0°)(5.50\text{ m}) = +324\text{ J}.$

(b) The gravity force is downward and the displacement is directed up the incline so $\phi = 126.9°$.

$W_{grav} = (10.0\text{ kg})(9.80\text{ m/s}^2)(\cos 126.9°)(5.50\text{ m}) = -324\text{ J}.$

(c) The normal force, n, is perpendicular to the displacement and thus does zero work.

EVALUATE: Friction does positive work and gravity does negative work. The net work done is zero.

6.13. IDENTIFY: We want the work done by a known force acting through a known displacement.

SET UP: $W = Fs\cos\phi$

EXECUTE: $W = (48.0\text{ N})(12.0\text{ m})\cos(173°) = -572\text{ J}.$

EVALUATE: The force has a component opposite to the displacement, so it does negative work.

6.14. IDENTIFY: We want to find the work done by a known force acting through a known displacement.

SET UP: $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$. We know the components of $\vec{F}$ but need to find the components of the displacement $\vec{s}$.

EXECUTE: Using the magnitude and direction of $\vec{s}$, its components are $x = (48.0\text{ m})\cos 240.0° = -24.0\text{ m}$ and $y = (48.0\text{ m})\sin 240.0° = -41.57\text{ m}$. Therefore,

$\vec{s} = (-24.0\text{ m})\hat{\boldsymbol{i}} + (-41.57\text{ m})\hat{\boldsymbol{j}}$. The definition of work gives

$W = \vec{F}\cdot\vec{s} = (-68.0\text{ N})(-24.0\text{ m}) + (36.0\text{ N})(-41.57\text{ m}) = +1632\text{ J} - 1497\text{ J} = +135\text{ J}.$

EVALUATE: The mass of the car is not needed since it is the given force that is doing the work.

6.15. **IDENTIFY:** We want the work done by the force, and we know the force and the displacement in terms of their components.

SET UP: We can use either $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$ or $W = Fs\cos\phi$, depending on what we know.

EXECUTE: (a) We know the magnitudes of the two given vectors and the angle between them, so $W = Fs\cos\phi$ = (30.0 N)(5.00 m)(cos37°) = 120 J.

(b) As in (a), we have $W = Fs\cos\phi$ = (30.0 N)(6.00 m)(cos127°) = –108 J.

(c) We know the components of both vectors, so we use $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$.

$W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$ = (30.0 N)(cos37°)(–2.00 m) + (30.00 N)(sin37°)(4.00 m) = 24.3 J.

EVALUATE: We could check parts (a) and (b) using the method from part (c).

6.16. **IDENTIFY:** The book changes its speed and hence its kinetic energy, so work must have been done on it.

SET UP: Use the work-kinetic energy theorem $W_{\text{net}} = K_{\text{f}} - K_{\text{i}}$, with $K = \frac{1}{2}mv^2$. In part (a) use K_{i} and K_{f} to calculate W. In parts (b) and (c) use K_{i} and W to calculate K_{f}.

EXECUTE: **(a)** Substituting the notation $\text{i} = A$ and $\text{f} = B$,

$W_{\text{net}} = K_B - K_A = \frac{1}{2}(1.50\text{ kg})[(1.25\text{ m/s})^2 - (3.21\text{ m/s})^2] = -6.56\text{ J}.$

(b) Noting $\text{i} = B$ and $\text{f} = C$,

$K_C = K_B + W_{\text{net}} = \frac{1}{2}(1.50\text{ kg})(1.25\text{ m/s})^2 - 0.750\text{ J} = +0.422\text{ J}.$ $K_C = \frac{1}{2}mv_C^2$ so

$v_C = \sqrt{2K_C/m} = 0.750\text{ m/s}.$

(c) Similarly, $K_C = \frac{1}{2}(1.50\text{ kg})(1.25\text{ m/s})^2 + 0.750\text{ J} = 1.922\text{ J}$ and $v_C = 1.60\text{ m/s}.$

EVALUATE: Negative W_{net} corresponds to a decrease in kinetic energy (slowing down) and positive W_{net} corresponds to an increase in kinetic energy (speeding up).

6.17. **IDENTIFY:** Find the kinetic energy of the cheetah knowing its mass and speed.

SET UP: Use $K = \frac{1}{2}mv^2$ to relate v and K.

EXECUTE: **(a)** $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(70\text{ kg})(32\text{ m/s})^2 = 3.6\times10^4\text{ J}.$

(b) K is proportional to v^2, so K increases by a factor of 4 when v doubles.

EVALUATE: A running person, even with a mass of 70 kg, would have only 1/100 of the cheetah's kinetic energy since a person's top speed is only about 1/10 that of the cheetah.

6.18. **IDENTIFY:** Apply the work-energy theorem to the ball.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $K = \dfrac{1}{2}mv^2$.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1 = \dfrac{1}{2}mv_2^2 - 0 = \dfrac{1}{2}(0.145\text{ kg})(30.0\text{ m/s})^2 = 65.3\text{ J}.$

(b) $W_{\text{tot}} = \dfrac{1}{2}mv_2^2 - \dfrac{1}{2}mv_1^2 = \dfrac{1}{2}(0.145\text{ kg})(-30.0\text{ m/s})^2 - \dfrac{1}{2}(0.145\text{ kg})(20.0\text{ m/s})^2 = 36.3\text{ J}.$

(c) The work was greater in part (a) than in part (b) because the change in kinetic energy was greater.

EVALUATE: In part (a) the bat does all positive work on the ball because the force and displacement are in the same direction. In part (b) the bat does negative work to stop the ball and then positive work to increase its speed in the opposite direction. So the *total* work the bat does is less than in part (a).

6.19. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Since the meteor comes to rest the energy it delivers to the ground equals its original kinetic energy.

SET UP: $v = 12 \text{ km/s} = 1.2\times10^4 \text{ m/s}$. A 1.0 megaton bomb releases 4.184×10^{15} J of energy.

EXECUTE: **(a)** $K = \frac{1}{2}(1.4\times10^8 \text{ kg})(1.2\times10^4 \text{ m/s})^2 = 1.0\times10^{16} \text{ J}$.

(b) $\dfrac{1.0\times10^{16} \text{ J}}{4.184\times10^{15} \text{ J}} = 2.4$. The energy is equivalent to 2.4 one-megaton bombs.

EVALUATE: Part of the energy transferred to the ground lifts soil and rocks into the air and creates a large crater.

6.20. **IDENTIFY:** Only gravity does work on the watermelon, so $W_{\text{tot}} = W_{\text{grav}}$. $W_{\text{tot}} = \Delta K$ and $K = \frac{1}{2}mv^2$.

SET UP: Since the watermelon is dropped from rest, $K_1 = 0$.

EXECUTE: **(a)** $W_{\text{grav}} = mgs = (4.80 \text{ kg})(9.80 \text{ m/s}^2)(18.0 \text{ m}) = 847 \text{ J}$.

(b) **(i)** $W_{\text{tot}} = K_2 - K_1$ so $K_2 = 847$ J. **(ii)** $v = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(847 \text{ J})}{4.80 \text{ kg}}} = 18.8 \text{ m/s}$.

(c) The work done by gravity would be the same. Air resistance would do negative work and W_{tot} would be less than W_{grav}. The answer in (a) would be unchanged and both answers in (b) would decrease.

EVALUATE: The gravity force is downward and the displacement is downward, so gravity does positive work.

6.21. **IDENTIFY:** We need to calculate work.

SET UP: $W = Fs\cos\phi$. Since d and ϕ are the same in both cases, but F is different.

EXECUTE: If the box travels a distance d in time t starting from rest, $x = x_0 + v_{0x}t + \dfrac{1}{2}a_x t^2$ tells us that $d = \dfrac{1}{2}a_1 t^2$. If it travels the same distance in half the time, we have $d = \dfrac{1}{2}a_2\left(\dfrac{t}{2}\right)^2 = \dfrac{1}{2}a_2\dfrac{t^2}{4} = \dfrac{1}{2}\left(\dfrac{a_2}{4}\right)t^2$.

Comparing these two equations for d shows that $a_2/4 = a_1$ so $a_2 = 4a_1$. $\sum F_x = ma_x$ tells us that the force F is 4 times as great, and so the work $W = Fs\cos\phi$ done by the force is 4 times as great. Therefore the work is $4W_1$.

EVALUATE: We must be careful of using ratios when quantities are squared.

6.22. **IDENTIFY:** $W_{\text{tot}} = K_2 - K_1$. In each case calculate W_{tot} from what we know about the force and the displacement.

SET UP: The gravity force is mg, downward. The friction force is $f_k = \mu_k n = \mu_k mg$ and is directed opposite to the displacement. The mass of the object isn't given, so we expect that it will divide out in the calculation.

EXECUTE: **(a)** $K_1 = \frac{1}{2}mv_1^2$. $K_2 = 0$. $W_{\text{tot}} = W_f = -\mu_k mgs$. $-\mu_k mgs = -\frac{1}{2}mv_1^2$.

$$s = \frac{v_1^2}{2\mu_k g} = \frac{(5.00 \text{ m/s})^2}{2(0.220)(9.80 \text{ m/s}^2)} = 5.80 \text{ m}.$$

(b) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = \frac{1}{2}mv_2^2$. $W_{\text{tot}} = W_f = -\mu_k mgs$. $K_2 = W_{\text{tot}} + K_1$. $\frac{1}{2}mv_2^2 = -\mu_k mgs + \frac{1}{2}mv_1^2$.

$$v_2 = \sqrt{v_1^2 - 2\mu_k gs} = \sqrt{(5.00 \text{ m/s})^2 - 2(0.220)(9.80 \text{ m/s}^2)(2.90 \text{ m})} = 3.53 \text{ m/s}.$$

(c) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = 0$. $W_{\text{grav}} = -mgy_2$, where y_2 is the vertical height. $-mgy_2 = -\frac{1}{2}mv_1^2$ and

$$y_2 = \frac{v_1^2}{2g} = \frac{(12.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 7.35 \text{ m}.$$

EVALUATE: In parts (a) and (b), friction does negative work and the kinetic energy is reduced. In part (c), gravity does negative work and the speed decreases. The vertical height in part (c) is independent of the slope angle of the hill.

6.23. IDENTIFY and **SET UP:** Apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the box. Let point 1 be at the bottom of the incline and let point 2 be at the skier. Work is done by gravity and by friction. Solve for K_1 and from that obtain the required initial speed.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2, \;\; K_2 = 0$

Work is done by gravity and friction, so $W_{\text{tot}} = W_{mg} + W_f$.

$W_{mg} = -mg(y_2 - y_1) = -mgh$

$W_f = -fs$. The normal force is $n = mg\cos\alpha$ and $s = h/\sin\alpha$, where s is the distance the box travels along the incline.

$W_f = -(\mu_k mg\cos\alpha)(h/\sin\alpha) = -\mu_k mgh/\tan\alpha$

Substituting these expressions into the work-energy theorem gives $-mgh - \mu_k mgh/\tan\alpha = -\frac{1}{2}mv_0^2$.

Solving for v_0 then gives $v_0 = \sqrt{2gh(1+\mu_k/\tan\alpha)}$.

EVALUATE: The result is independent of the mass of the box. As $\alpha \to 90°$, $h = s$ and $v_0 = \sqrt{2gh}$, the same as throwing the box straight up into the air. For $\alpha = 90°$ the normal force is zero so there is no friction.

6.24. IDENTIFY: From the work-energy relation, $W = W_{\text{grav}} = \Delta K_{\text{rock}}$.

SET UP: As the rock rises, the gravitational force, $F = mg$, does work on the rock. Since this force acts in the direction opposite to the motion and displacement, s, the work is negative. Let h be the vertical distance the rock travels.

EXECUTE: **(a)** Applying $W_{\text{grav}} = K_2 - K_1$ we obtain $-mgh = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$. Dividing by m and solving for v_1, $v_1 = \sqrt{v_2^2 + 2gh}$. Substituting $h = 15.0$ m and $v_2 = 25.0$ m/s,

$$v_1 = \sqrt{(25.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(15.0 \text{ m})} = 30.3 \text{ m/s}$$

(b) Solve the same work-energy relation for h. At the maximum height $v_2 = 0$.

$$-mgh = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \text{ and } h = \frac{v_1^2 - v_2^2}{2g} = \frac{(30.3 \text{ m/s})^2 - (0.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 46.8 \text{ m}.$$

EVALUATE: Note that the weight of the rock was never used in the calculations because both gravitational potential and kinetic energy are proportional to mass, m. Thus any object, that attains 25.0 m/s at a height of 15.0 m, must have an initial velocity of 30.3 m/s. As the rock moves upward gravity does negative work and this reduces the kinetic energy of the rock.

6.25. IDENTIFY: Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = K_2 - K_1$.

SET UP: $\phi = 0°$

EXECUTE: Use $W = Fs\cos\phi$, $W_{\text{tot}} = K_2 - K_1$, and $K = \dfrac{1}{2}mv^2$ and solve for F, giving

$$F = \frac{\Delta K}{s} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{s} = \frac{\frac{1}{2}(12.0\text{ kg})\left[(6.00\text{ m/s})^2 - (4.00\text{ m/s})^2\right]}{(2.50\text{ m})} = 48.0 \text{ N}$$

EVALUATE: The force is in the direction of the displacement, so the force does positive work and the kinetic energy of the object increases.

6.26. **IDENTIFY** and **SET UP:** Use the work-energy theorem to calculate the work done by the foot on the ball. Then use $W = Fs\cos\phi$ to find the distance over which this force acts.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.420 \text{ kg})(2.00 \text{ m/s})^2 = 0.84 \text{ J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.420 \text{ kg})(6.00 \text{ m/s})^2 = 7.56 \text{ J}$

$W_{\text{tot}} = K_2 - K_1 = 7.56 \text{ J} - 0.84 \text{ J} = 6.72 \text{ J}$

The 40.0 N force is the only force doing work on the ball, so it must do 6.72 J of work. $W_F = (F\cos\phi)s$ gives that $s = \dfrac{W}{F\cos\phi} = \dfrac{6.72 \text{ J}}{(40.0 \text{ N})(\cos 0)} = 0.168 \text{ m}.$

EVALUATE: The force is in the direction of the motion so positive work is done and this is consistent with an increase in kinetic energy.

6.27. **IDENTIFY:** Apply $W_{\text{tot}} = \Delta K.$

SET UP: $v_1 = 0,\ v_2 = v.\ f_k = \mu_k mg$ and f_k does negative work. The force $F = 36.0 \text{ N}$ is in the direction of the motion and does positive work.

EXECUTE: **(a)** If there is no work done by friction, the final kinetic energy is the work done by the applied force, and solving for the speed,

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2(36.0 \text{ N})(1.20 \text{ m})}{(4.30 \text{ kg})}} = 4.48 \text{ m/s}.$$

(b) The net work is $Fs - f_k s = (F - \mu_k mg)s,$ so

$$v = \sqrt{\frac{2(F - \mu_k mg)s}{m}} = \sqrt{\frac{2(36.0 \text{ N} - (0.30)(4.30 \text{ kg})(9.80 \text{ m/s}^2)(1.20 \text{ m})}{(4.30 \text{ kg})}} = 3.61 \text{ m/s}$$

EVALUATE: The total work done is larger in the absence of friction and the final speed is larger in that case.

6.28. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1.$

SET UP: $K_1 = 0.$ The normal force does no work. The work W done by gravity is $W = mgh,$ where $h = L\sin\theta$ is the vertical distance the block has dropped when it has traveled a distance L down the incline and θ is the angle the plane makes with the horizontal.

EXECUTE: The work-energy theorem gives $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2W}{m}} = \sqrt{2gh} = \sqrt{2gL\sin\theta}.$ Using the given numbers, $v = \sqrt{2(9.80 \text{ m/s}^2)(1.35 \text{ m})\sin 36.9°} = 3.99 \text{ m/s}.$

6.29. **IDENTIFY:** We use the work-energy theorem.

SET UP: $W_{\text{tot}} = K_2 - K_1,\ K = \dfrac{1}{2}mv^2.$

EXECUTE: **(a)** $K_A = \dfrac{1}{2}m_A v_A^2 = 27 \text{ J}$ and $K_B = \dfrac{1}{2}\left(\dfrac{m_A}{4}\right)v_B^2 = 27 \text{ J}.$ Equate both expressions, which gives $\dfrac{1}{2}m_A v_A^2 = = \dfrac{1}{2}\left(\dfrac{m_A}{4}\right)v_B^2.$ Solving for v_B we have $v_B = 2v_A.$ So B is moving faster and its speed is twice that of A.

(b) Apply the work-energy theorem to A. –18 J = K_2 – 27 J, so K_2 = 9 J. Take the ratio of its initial and final kinetic energy, giving $\dfrac{K_2}{K_1} = \dfrac{\frac{1}{2}mv_2^2}{\frac{1}{2}mv_1^2} = \dfrac{9 \text{ J}}{27 \text{ J}} = \dfrac{1}{3} = \dfrac{v_2^2}{v_1^2}.$ Thus $v_2 = \dfrac{v_1}{\sqrt{3}}.$ We will get the same result for B because both objects have the same initial kinetic energy (27 J) and the same amount of work (–18 J) is done on them.

EVALUATE: The result to part (a) is reasonable since a lighter object must move faster to have the same kinetic energy as a heavier object. Part (b) is reasonable because the kinetic energy decreases since the work done on the objects is negative.

6.30. IDENTIFY: We know (or can calculate) the change in the kinetic energy of the crate and want to find the work needed to cause this change, so the work-energy theorem applies.

SET UP: $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2.$

EXECUTE: $W_{\text{tot}} = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}(30.0\text{ kg})(5.62\text{ m/s})^2 - \frac{1}{2}(30.0\text{ kg})(3.90\text{ m/s})^2.$

$W_{\text{tot}} = 473.8\text{ J} - 228.2\text{ J} = 246\text{ J}.$

EVALUATE: Kinetic energy is a scalar and does not depend on direction, so only the initial and final speeds are relevant.

6.31. IDENTIFY: $W_{\text{tot}} = K_2 - K_1$. Only friction does work.

SET UP: $W_{\text{tot}} = W_{f_{\text{k}}} = -\mu_{\text{k}}mgs.$ $K_2 = 0$ (car stops). $K_1 = \frac{1}{2}mv_0^2.$

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$ gives $-\mu_{\text{k}}mgs = -\frac{1}{2}mv_0^2.$ $s = \dfrac{v_0^2}{2\mu_{\text{k}}g}.$

(b) (i) $\mu_{\text{k}b} = 2\mu_{\text{k}a}.$ $s\mu_{\text{k}} = \dfrac{v_0^2}{2g} = \text{constant}$ so $s_a\mu_{\text{k}a} = s_b\mu_{\text{k}b}.$ $s_b = \left(\dfrac{\mu_{\text{k}a}}{\mu_{\text{k}b}}\right)s_a = s_a/2.$ The minimum stopping distance would be halved. (ii) $v_{0b} = 2v_{0a}.$ $\dfrac{s}{v_0^2} = \dfrac{1}{2\mu_{\text{k}}g} = \text{constant},$ so $\dfrac{s_a}{v_{0a}^2} = \dfrac{s_b}{v_{0b}^2}.$

$s_b = s_a\left(\dfrac{v_{0b}}{v_{0a}}\right)^2 = 4s_a.$ The stopping distance would become 4 times as great. (iii) $v_{0b} = 2v_{0a},$

$\mu_{\text{k}b} = 2\mu_{\text{k}a}.$ $\dfrac{s\mu_{\text{k}}}{v_0^2} = \dfrac{1}{2g} = \text{constant},$ so $\dfrac{s_a\mu_{\text{k}a}}{v_{0a}^2} = \dfrac{s_b\mu_{\text{k}b}}{v_{0b}^2}.$ $s_b = s_a\left(\dfrac{\mu_{\text{k}a}}{\mu_{\text{k}b}}\right)\left(\dfrac{v_{0b}}{v_{0a}}\right)^2 = s_a\left(\dfrac{1}{2}\right)(2)^2 = 2s_a.$ The stopping distance would double.

EVALUATE: The stopping distance is directly proportional to the square of the initial speed and indirectly proportional to the coefficient of kinetic friction.

6.32. IDENTIFY: The work that must be done to move the end of a spring from x_1 to x_2 is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2.$ The force required to hold the end of the spring at displacement x is $F_x = kx.$

SET UP: When the spring is at its unstretched length, $x = 0.$ When the spring is stretched, $x > 0,$ and when the spring is compressed, $x < 0.$

EXECUTE: **(a)** $x_1 = 0$ and $W = \frac{1}{2}kx_2^2.$ $k = \dfrac{2W}{x_2^2} = \dfrac{2(12.0\text{ J})}{(0.0300\text{ m})^2} = 2.67\times10^4\text{ N/m}.$

(b) $F_x = kx = (2.67\times10^4\text{ N/m})(0.0300\text{ m}) = 801\text{ N}.$

(c) $x_1 = 0,$ $x_2 = -0.0400\text{ m}.$ $W = \frac{1}{2}(2.67\times10^4\text{ N/m})(-0.0400\text{ m})^2 = 21.4\text{ J}.$

$F_x = kx = (2.67\times10^4\text{ N/m})(0.0400\text{ m}) = 1070\text{ N}.$

EVALUATE: When a spring, initially unstretched, is either compressed or stretched, positive work is done by the force that moves the end of the spring.

6.33. IDENTIFY: The springs obey Hooke's law and balance the downward force of gravity.

SET UP: Use coordinates with $+y$ upward. Label the masses 1, 2, and 3, with 1 the top mass and 3 the bottom mass, and call the amounts the springs are stretched $x_1,$ $x_2,$ and $x_3.$ Each spring force is kx.

EXECUTE: **(a)** The three free-body diagrams are shown in Figure 6.33.

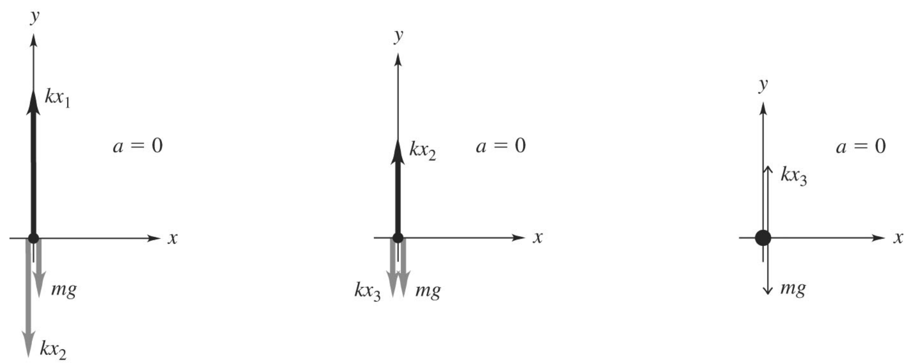


Figure 6.33

(b) Balancing forces on each of the masses and using $F = kx$ gives $kx_3 = mg$ so

$x_3 = \dfrac{mg}{k} = \dfrac{(8.50\text{ kg})(9.80\text{ m/s}^2)}{7.80\times 10^3\text{ N/m}} = 1.068\text{ cm}$. $kx_2 = mg + kx_3 = 2mg$ so $x_2 = 2\left(\dfrac{mg}{k}\right) = 2.136\text{ cm}$.

$kx_1 = mg + kx_2 = 3mg$ so $x_3 = 3\left(\dfrac{mg}{k}\right) = 3.204\text{ cm}$. Adding the original lengths to the distance stretched, the lengths of the springs, starting from the bottom one, are 13.1 cm, 14.1 cm, and 15.2 cm.

N**EVALUATE:** The top spring stretches most because it supports the most weight, while the bottom spring stretches least because it supports the least weight.

6.34. **IDENTIFY:** The magnitude of the work can be found by finding the area under the graph.
SET UP: The area under each triangle is 1/2 base$\times$height. $F_x > 0$, so the work done is positive when x increases during the displacement.
EXECUTE: **(a)** $1/2\ (8\text{ m})(10\text{ N}) = 40\text{ J}$.
(b) $1/2\ (4\text{ m})(10\text{ N}) = 20\text{ J}$.
(c) $1/2\ (12\text{ m})(10\text{ N}) = 60\text{ J}$.
EVALUATE: The sum of the answers to parts (a) and (b) equals the answer to part (c).

6.35. **IDENTIFY:** Use the work-energy theorem and the results of Problem 6.36.
SET UP: For $x = 0$ to $x = 8.0\text{ m}$, $W_{\text{tot}} = 40\text{ J}$. For $x = 0$ to $x = 12.0\text{ m}$, $W_{\text{tot}} = 60\text{ J}$.

EXECUTE: **(a)** $v = \sqrt{\dfrac{(2)(40\text{ J})}{10\text{ kg}}} = 2.83\text{ m/s}$

(b) $v = \sqrt{\dfrac{(2)(60\text{ J})}{10\text{ kg}}} = 3.46\text{ m/s}$.

EVALUATE: $\vec{F}$ is always in the $+x$-direction. For this motion $\vec{F}$ does positive work and the speed continually increases during the motion.

6.36. **IDENTIFY:** The spring obeys Hooke's law.
SET UP: Solve $F = kx$ for x to determine the length of stretch and use $W = +\frac{1}{2}kx^2$ to assess the corresponding work.

EXECUTE: $x = \dfrac{F}{k} = \dfrac{15.0\text{ N}}{300.0\text{ N/m}} = 0.0500\text{ m}$. The new length will be $0.240\text{ m} + 0.0500\text{ m} = 0.290\text{ m}$.

The corresponding work done is $W = \dfrac{1}{2}(300.0\text{ N/m})(0.0500\text{ m})^2 = 0.375\text{ J}$.

EVALUATE: In $F = kx$, F is always the force applied to one end of the spring, thus we did not need to double the 15.0 N force. Consider a free-body diagram of a spring at rest; forces of equal magnitude and opposite direction are always applied to both ends of every section of the spring examined.

6.37. IDENTIFY: Apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the box.

SET UP: Let point 1 be just before the box reaches the end of the spring and let point 2 be where the spring has maximum compression and the box has momentarily come to rest.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2,\ \ K_2 = 0$

Work is done by the spring force. $W_{\text{tot}} = -\frac{1}{2}kx_2^2$, where x_2 is the amount the spring is compressed.

$-\frac{1}{2}kx_2^2 = -\frac{1}{2}mv_0^2$ and $x_2 = v_0\sqrt{m/k} = (3.0\text{ m/s})\sqrt{(6.0\text{ kg})/(7500\text{ N/m})} = 8.5\text{ cm}$

EVALUATE: The compression of the spring increases when either v_0 or m increases and decreases when k increases (stiffer spring).

6.38. IDENTIFY: The force applied to the springs is $F_x = kx$. The work done on a spring to move its end from x_1 to x_2 is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. Use the information that is given to calculate k.

SET UP: When the springs are compressed 0.200 m from their uncompressed length, $x_1 = 0$ and $x_2 = -0.200\text{ m}$. When the platform is moved 0.200 m farther, x_2 becomes -0.400 m.

EXECUTE: **(a)** $k = \dfrac{2W}{x_2^2 - x_1^2} = \dfrac{2(80.0\text{ J})}{(0.200\text{ m})^2 - 0} = 4000\text{ N/m}$. $F_x = kx = (4000\text{ N/m})(-0.200\text{ m}) = -800\text{ N}$.

The magnitude of force that is required is 800 N.

(b) To compress the springs from $x_1 = 0$ to $x_2 = -0.400\text{ m}$, the work required is

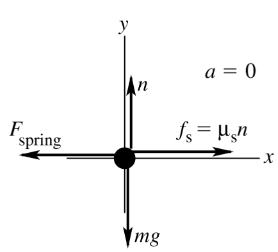
$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = \frac{1}{2}(4000\text{ N/m})(-0.400\text{ m})^2 = 320\text{ J}$. The additional work required is

$320\text{ J} - 80\text{ J} = 240\text{ J}$. For $x = -0.400\text{ m}$, $F_x = kx = -1600\text{ N}$. The magnitude of force required is 1600 N.

EVALUATE: More work is required to move the end of the spring from $x = -0.200\text{ m}$ to $x = -0.400\text{ m}$ than to move it from $x = 0$ to $x = -0.200\text{ m}$, even though the displacement of the platform is the same in each case. The magnitude of the force increases as the compression of the spring increases.

6.39. IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to calculate the μ_s required for the static friction force to equal the spring force.

SET UP: **(a)** The free-body diagram for the glider is given in Figure 6.39.



EXECUTE: $\Sigma F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_s = \mu_s mg$

Figure 6.39

$\Sigma F_x = ma_x$

$f_s - F_{\text{spring}} = 0$

$\mu_s mg - kd = 0$

$$\mu_s = \frac{kd}{mg} = \frac{(20.0\text{ N/m})(0.086\text{ m})}{(0.100\text{ kg})(9.80\text{ m/s}^2)} = 1.76$$

(b) IDENTIFY and **SET UP:** Apply $\Sigma\vec{F} = m\vec{a}$ to find the maximum amount the spring can be compressed and still have the spring force balanced by friction. Then use $W_{\text{tot}} = K_2 - K_1$ to find the initial speed that results in this compression of the spring when the glider stops.

EXECUTE: $\mu_s mg = kd$

$$d = \frac{\mu_s mg}{k} = \frac{(0.60)(0.100\text{ kg})(9.80\text{ m/s}^2)}{20.0\text{ N/m}} = 0.0294\text{ m}$$

Now apply the work-energy theorem to the motion of the glider:

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2$, $K_2 = 0$ (instantaneously stops)

$W_{\text{tot}} = W_{\text{spring}} + W_{\text{fric}} = -\frac{1}{2}kd^2 - \mu_k mgd$ (as in Example 6.7)

$W_{\text{tot}} = -\frac{1}{2}(20.0\text{ N/m})(0.0294\text{ m})^2 - 0.47(0.100\text{ kg})(9.80\text{ m/s}^2)(0.0294\text{ m}) = -0.02218\text{ J}$

Then $W_{\text{tot}} = K_2 - K_1$ gives $-0.02218\text{ J} = -\frac{1}{2}mv_1^2$.

$$v_1 = \sqrt{\frac{2(0.02218\text{ J})}{0.100\text{ kg}}} = 0.67\text{ m/s}.$$

EVALUATE: In Example 6.7 an initial speed of 1.50 m/s compresses the spring 0.086 m and in part (a) of this problem we found that the glider doesn't stay at rest. In part (b) we found that a smaller displacement of 0.0294 m when the glider stops is required if it is to stay at rest. And we calculate a smaller initial speed (0.67 m/s) to produce this smaller displacement.

6.40. IDENTIFY: For the spring, $W = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $x_1 = -0.025$ m and $x_2 = 0$.

EXECUTE: **(a)** $W = \frac{1}{2}kx_1^2 = \frac{1}{2}(200\text{ N/m})(-0.025\text{ m})^2 = 0.0625\text{ J}$, which rounds to 0.063 J.

(b) The work-energy theorem gives $v_2 = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(0.0625\text{ J})}{(4.0\text{ kg})}} = 0.18\text{ m/s}$.

EVALUATE: The block moves in the direction of the spring force, the spring does positive work and the kinetic energy of the block increases.

6.41. IDENTIFY and **SET UP:** The magnitude of the work done by F_x equals the area under the F_x versus x curve. The work is positive when F_x and the displacement are in the same direction; it is negative when they are in opposite directions.

EXECUTE: **(a)** F_x is positive and the displacement Δx is positive, so $W > 0$.

$W = \frac{1}{2}(2.0\text{ N})(2.0\text{ m}) + (2.0\text{ N})(1.0\text{ m}) = +4.0\text{ J}$

(b) During this displacement $F_x = 0$, so $W = 0$.

(c) F_x is negative, Δx is positive, so $W < 0$. $W = -\frac{1}{2}(1.0\text{ N})(2.0\text{ m}) = -1.0\text{ J}$

(d) The work is the sum of the answers to parts (a), (b), and (c), so $W = 4.0\text{ J} + 0 - 1.0\text{ J} = +3.0\text{ J}$.

(e) The work done for $x = 7.0$ m to $x = 3.0$ m is $+1.0$ J. This work is positive since the displacement and the force are both in the $-x$-direction. The magnitude of the work done for $x = 3.0$ m to $x = 2.0$ m is 2.0 J, the area under F_x versus x. This work is negative since the displacement is in the $-x$-direction and the force is in the $+x$-direction. Thus $W = +1.0\text{ J} - 2.0\text{ J} = -1.0\text{ J}$.

EVALUATE: The work done when the car moves from $x = 2.0$ m to $x = 0$ is $-\frac{1}{2}(2.0\text{ N})(2.0\text{ m}) = -2.0$ J. Adding this to the work for $x = 7.0$ m to $x = 2.0$ m gives a total of $W = -3.0$ J for $x = 7.0$ m to $x = 0$. The work for $x = 7.0$ m to $x = 0$ is the negative of the work for $x = 0$ to $x = 7.0$ m.

6.42. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $K_1 = 0$. From Exercise 6.41, the work for $x = 0$ to $x = 3.0$ m is 4.0 J. W for $x = 0$ to $x = 4.0$ m is also 4.0 J. For $x = 0$ to $x = 7.0$ m, $W = 3.0$ J.

EXECUTE: **(a)** $K = 4.0$ J, so $v = \sqrt{2K/m} = \sqrt{2(4.0\text{ J})/(2.0\text{ kg})} = 2.00$ m/s.

(b) No work is done between $x = 3.0$ m and $x = 4.0$ m, so the speed is the same, 2.00 m/s.

(c) $K = 3.0$ J, so $v = \sqrt{2K/m} = \sqrt{2(3.0\text{ J})/(2.0\text{ kg})} = 1.73$ m/s.

EVALUATE: In each case the work done by F is positive and the car gains kinetic energy.

6.43. **IDENTIFY** and **SET UP:** Apply the work-energy theorem. Let point 1 be where the sled is released and point 2 be at $x = 0$ for part (a) and at $x = -0.200$ m for part (b). Use $W = \dfrac{1}{2}kx^2$ for the work done by the spring and calculate K_2. Then $K_2 = \frac{1}{2}mv_2^2$ gives v_2.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$ so $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$ (released with no initial velocity), $K_2 = \frac{1}{2}mv_2^2$

The only force doing work is the spring force. $W = \dfrac{1}{2}kx^2$ gives the work done *on* the spring to move its end from x_1 to x_2. The force the spring exerts on an object attached to it is $F = -kx$, so the work the spring does is

$W_{\text{spr}} = -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right) = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Here $x_1 = -0.375$ m and $x_2 = 0$. Thus

$W_{\text{spr}} = \frac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - 0 = 281$ J.

$K_2 = K_1 + W_{\text{tot}} = 0 + 281\text{ J} = 281$ J.

Then $K_2 = \frac{1}{2}mv_2^2$ implies $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(281\text{ J})}{70.0\text{ kg}}} = 2.83$ m/s.

(b) $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$

$W_{\text{tot}} = W_{\text{spr}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Now $x_2 = -0.200$ m, so

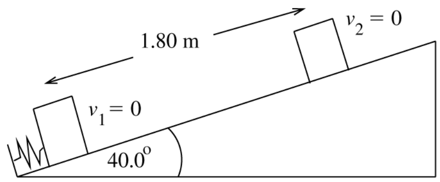
$W_{\text{spr}} = \frac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - \frac{1}{2}(4000\text{ N/m})(-0.200\text{ m})^2 = 281\text{ J} - 80\text{ J} = 201$ J

Thus $K_2 = 0 + 201\text{ J} = 201$ J and $K_2 = \frac{1}{2}mv_2^2$ gives $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(201\text{ J})}{70.0\text{ kg}}} = 2.40$ m/s.

EVALUATE: The spring does positive work and the sled gains speed as it returns to $x = 0$. More work is done during the larger displacement in part (a), so the speed there is larger than in part (b).

6.44. **IDENTIFY** and **SET UP:** Apply the work-energy theorem to the glider. Work is done by the spring and by gravity. Take point 1 to be where the glider is released. In part (a) point 2 is where the glider has traveled 1.80 m and $K_2 = 0$. There are two points shown in Figure 6.44a. In part (b) point 2 is where the glider has traveled 0.80 m.

EXECUTE: (a) $W_{\text{tot}} = K_2 - K_1 = 0$. Solve for x_1, the amount the spring is initially compressed.



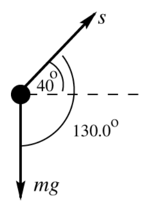
$W_{tot} = W_{spr} + W_w = 0$

So $W_{spr} = -W_w$

(The spring does positive work on the glider since the spring force is directed up the incline, the same as the direction of the displacement.)

Figure 6.44a

The directions of the displacement and of the gravity force are shown in Figure 6.44b.



$W_w = (w\cos\phi)s = (mg\cos 130.0°)s$

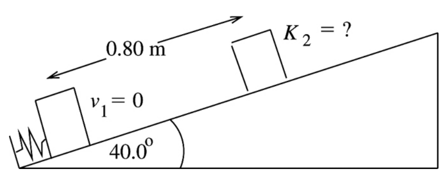
$W_w = (0.0900\text{ kg})(9.80\text{ m/s}^2)(\cos 130.0°)(1.80\text{ m}) = -1.020\text{ J}$ (The component of w parallel to the incline is directed down the incline, opposite to the displacement, so gravity does negative work.)

Figure 6.44b

$W_{spr} = -W_w = +1.020\text{ J}$

$W_{spr} = \frac{1}{2}kx_1^2$ so $x_1 = \sqrt{\dfrac{2W_{spr}}{k}} = \sqrt{\dfrac{2(1.020\text{ J})}{640\text{ N/m}}} = 0.0565\text{ m}$

(b) The spring was compressed only 0.0565 m so at this point in the motion the glider is no longer in contact with the spring. Points 1 and 2 are shown in Figure 6.44c.



$W_{tot} = K_2 - K_1$

$K_2 = K_1 + W_{tot}$

$K_1 = 0$

Figure 6.44c

$W_{tot} = W_{spr} + W_w$

From part (a), $W_{spr} = 1.020\text{ J}$ and

$W_w = (mg\cos 130.0°)s = (0.0900\text{ kg})(9.80\text{ m/s}^2)(\cos 130.0°)(0.80\text{ m}) = -0.454\text{ J}$

Then $K_2 = W_{spr} + W_w = +1.020\text{ J} - 0.454\text{ J} = +0.57\text{ J}.$

EVALUATE: The kinetic energy in part (b) is positive, as it must be. In part (a), $x_2 = 0$ since the spring force is no longer applied past this point. In computing the work done by gravity we use the full 0.80 m the glider moves.

6.45. IDENTIFY: The force does work on the box, which gives it kinetic energy, so the work-energy theorem applies. The force is variable so we must integrate to calculate the work it does on the box.

SET UP: $W_{tot} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ and $W_{tot} = \int_{x_1}^{x_2} F(x)dx.$

EXECUTE: $W_{tot} = \int_{x_1}^{x_2} F(x)dx = \int_0^{14.0\text{ m}} [18.0\text{ N} - (0.530\text{ N/m})x]dx$

$W_{\text{tot}} = (18.0\text{ N})(14.0\text{ m}) - (0.265\text{ N/m})(14.0\text{ m})^2 = 252.0\text{ J} - 51.94\text{ J} = 200.1\text{ J}$. The initial kinetic energy is zero, so $W_{\text{tot}} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2$. Solving for v_f gives $v_f = \sqrt{\frac{2W_{\text{tot}}}{m}} = \sqrt{\frac{2(200.1\text{ J})}{6.00\text{ kg}}} = 8.17\text{ m/s}$.

EVALUATE: We could not readily do this problem by integrating the acceleration over time because we know the force as a function of *x*, not of *t*. The work-energy theorem provides a much simpler method.

6.46. **IDENTIFY:** The force acts through a distance over time, so it does work on the crate and hence supplies power to it. The force exerted by the worker is variable but the acceleration of the cart is constant.

SET UP: Use $P = Fv$ to find the power, and we can use $v = v_0 + at$ to find the instantaneous velocity.

EXECUTE: First find the instantaneous force and velocity: $F = (5.40\text{ N/s})(5.00\text{ s}) = 27.0\text{ N}$ and $v = v_0 + at = (2.80\text{ m/s}^2)(5.00\text{ s}) = 14.0\text{ m/s}$. Now find the power: $P = (27.0\text{ N})(14.0\text{ m/s}) = 378\text{ W}$.

EVALUATE: The instantaneous power will increase as the worker pushes harder and harder.

6.47. **IDENTIFY:** Apply the relation between energy and power.

SET UP: Use $P = \frac{W}{\Delta t}$ to solve for *W*, the energy the bulb uses. Then set this value equal to $\frac{1}{2}mv^2$ and solve for the speed.

EXECUTE: $W = P\Delta t = (100\text{ W})(3600\text{ s}) = 3.6\times10^5\text{ J}$

$K = 3.6\times10^5\text{ J}$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.6\times10^5\text{ J})}{70\text{ kg}}} = 100\text{ m/s}$

EVALUATE: Olympic runners achieve speeds up to approximately 10 m/s, or roughly one-tenth the result calculated.

6.48. **IDENTIFY:** Knowing the rate at which energy is consumed, we want to find out the total energy used.

SET UP: Find the elapsed time Δt in each case by dividing the distance by the speed, $\Delta t = d / v$. Then calculate the energy as $W = P\Delta t$.

EXECUTE: Running: $\Delta t = (5.0\text{ km})/(10\text{ km/h}) = 0.50\text{ h} = 1.8\times10^3\text{ s}$. The energy used is

$W = (700\text{ W})(1.8\times10^3\text{ s}) = 1.3\times10^6\text{ J}$.

Walking: $\Delta t = \frac{5.0\text{ km}}{3.0\text{ km/h}}\left(\frac{3600\text{ s}}{\text{h}}\right) = 6.0\times10^3\text{ s}$. The energy used is

$W = (290\text{ W})(6.0\times10^3\text{ s}) = 1.7\times10^6\text{ J}$.

EVALUATE: The less intense exercise lasts longer and therefore burns up more energy than the intense exercise.

6.49. **IDENTIFY:** We want to calculate power.

SET UP: Estimate: 4 bags/min, so 20 bags in 5.0 min. $P_{\text{av}} = W/t$.

EXECUTE: $W = wh = (30\text{ lb})(4.0\text{ ft}) = 120\text{ ft}\cdot\text{lb}$ per bag. The total work is $(120\text{ ft}\cdot\text{lb})(20\text{ bags}) = 2400\text{ ft}\cdot\text{lb}$. The time is 5.0 min =300 s. So the power is

$P_{\text{av}} = \frac{W}{t} = \frac{2400\text{ ft}\cdot\text{lb}}{300\text{ s}} = 8.0\text{ ft}\cdot\text{lb/s}$. Convert to horsepower: $(8.0\text{ ft}\cdot\text{lb/s})\left(\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lb/s}}\right) = 1.5\times10^{-2}\text{ hp}$.

Convert to watts: $\left(1.5\times10^{-2}\text{ hp}\right)\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 11\text{ W}$.

EVALUATE: This college student puts a lot less power than the marathon runner in Example 6.10 in the text!

6.50. **IDENTIFY:** The thermal energy is produced as a result of the force of friction, $F = \mu_k mg$. The average thermal power is thus the average rate of work done by friction or $P = F_{\|}v_{av}$.

SET UP: $v_{av} = \dfrac{v_2 + v_1}{2} = \left(\dfrac{8.00\text{ m/s} + 0}{2}\right) = 4.00\text{ m/s}$

EXECUTE: $P = Fv_{av} = [(0.200)(20.0\text{ kg})(9.80\text{ m/s}^2)](4.00\text{ m/s}) = 157\text{ W}$

EVALUATE: The power could also be determined as the rate of change of kinetic energy, $\Delta K / t$, where the time is calculated from $v_f = v_i + at$ and a is calculated from a force balance, $\Sigma F = ma = \mu_k mg$.

6.51. **IDENTIFY:** We need to use power.
SET UP: Estimates: student weight is 150 lb which is about 667 N. $P_{av} = W/t$.
EXECUTE: Convert units: 50 ft = 15.2 m. The work is $W = wy = (667\text{ N})(15.2\text{ m}) = 10{,}000\text{ J}$. Solve $P_{av} = W/t$ for t: $t = W/P_{av} = (10{,}000\text{ J})/(500\text{ W}) = 20\text{ s}$.
EVALUATE: 20 s seems like a reasonable time to climb three flights of stairs.

6.52. **IDENTIFY** and **SET UP:** Calculate the power used to make the plane climb against gravity. Consider the vertical motion since gravity is vertical.
EXECUTE: The rate at which work is being done against gravity is

$P = Fv = mgv = (700\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m/s}) = 17.15\text{ kW}$.

This is the part of the engine power that is being used to make the airplane climb. The fraction this is of the total is $17.15\text{ kW}/75\text{ kW} = 0.23$.
EVALUATE: The power we calculate for making the airplane climb is considerably less than the power output of the engine.

6.53. **IDENTIFY:** $P_{av} = \dfrac{\Delta W}{\Delta t}$. The work you do in lifting mass m a height h is mgh.

SET UP: $1\text{ hp} = 746\text{ W}$

EXECUTE: **(a)** The number per minute would be the average power divided by the work (mgh) required to lift one box, $\dfrac{(0.50\text{ hp})(746\text{ W/hp})}{(30\text{ kg})(9.80\text{ m/s}^2)(0.90\text{ m})} = 1.41/\text{s}$, or $84.6/\text{min}$.

(b) Similarly, $\dfrac{(100\text{ W})}{(30\text{ kg})(9.80\text{ m/s}^2)(0.90\text{ m})} = 0.378/\text{s}$, or $22.7/\text{min}$.

EVALUATE: A 30-kg crate weighs about 66 lbs. It is not possible for a person to perform work at this rate.

6.54. **IDENTIFY** and **SET UP:** Use $P_{av} = \dfrac{\Delta W}{\Delta t}$ to relate the power provided and the amount of work done against gravity in 16.0 s. The work done against gravity depends on the total weight which depends on the number of passengers.
EXECUTE: Find the total mass that can be lifted:

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{mgh}{t}, \text{ so } m = \frac{P_{av}t}{gh}$$

$$P_{av} = (40\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 2.984\times 10^4\text{ W}$$

$$m = \frac{P_{av}t}{gh} = \frac{(2.984\times 10^4\text{ W})(16.0\text{ s})}{(9.80\text{ m/s}^2)(20.0\text{ m})} = 2.436\times 10^3\text{ kg}$$

This is the total mass of elevator plus passengers. The mass of the passengers is

$2.436\times10^3\ \text{kg} - 600\ \text{kg} = 1.836\times10^3\ \text{kg}.$ The number of passengers is $\dfrac{1.836\times10^3\ \text{kg}}{65.0\ \text{kg}} = 28.2.$ 28

passengers can ride.

EVALUATE: Typical elevator capacities are about half this, in order to have a margin of safety.

6.55. **IDENTIFY:** To lift the skiers, the rope must do positive work to counteract the negative work developed by the component of the gravitational force acting on the total number of skiers, $F_{\text{rope}} = Nmg\sin\alpha.$

SET UP: $P = F_{\|}v = F_{\text{rope}}v$

EXECUTE: $P_{\text{rope}} = F_{\text{rope}}v = [+Nmg(\cos\phi)]v.$

$$P_{\text{rope}} = [(50\ \text{riders})(70.0\ \text{kg})(9.80\ \text{m/s}^2)(\cos 75.0)]\left[(12.0\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.60\ \text{km/h}}\right)\right].$$

$P_{\text{rope}} = 2.96\times10^4\ \text{W} = 29.6\ \text{kW}.$

EVALUATE: Some additional power would be needed to give the riders kinetic energy as they are accelerated from rest.

6.56. **IDENTIFY:** We want to find the power supplied by a known force acting on a crate at a known velocity.

SET UP: We know the vector components, so we use $P = \vec{F}\cdot\vec{v} = F_xv_x + F_yv_y$

EXECUTE: $P = F_xv_x + F_yv_y = (-8.00\ \text{N})(3.20\ \text{m/s}) + (3.00\ \text{N})(2.20\ \text{m/s}) = -19.0\ \text{W}.$

EVALUATE: The power is negative because the x-component of the force is opposite to the x-component of the velocity and hence opposes the motion of the crate.

6.57. **IDENTIFY:** Relate power, work, and time.

SET UP: Work done in each stroke is $W = Fs$ and $P_{\text{av}} = W/t.$

EXECUTE: 100 strokes per second means $P_{\text{av}} = 100Fs/t$ with $t = 1.00$ s, $F = 2mg$ and $s = 0.010$ m.

$P_{\text{av}} = 0.20$ W.

EVALUATE: For a 70-kg person to apply a force of twice his weight through a distance of 0.50 m for 100 times per second, the average power output would be 7.0×10^4 W. This power output is very far beyond the capability of a person.

6.58. **IDENTIFY:** The force has only an x-component and the motion is along the x-direction, so $W = \int_{x_1}^{x_2} F_x dx.$

SET UP: $x_1 = 0$ and $x_2 = 6.9$ m.

EXECUTE: The work you do with your changing force is

$$W = \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2}(-20.0\ \text{N})dx - \int_{x_1}^{x_2}(3.0\ \text{N/m})x dx = (-20.0\ \text{N})x\,|_{x_1}^{x_2} - (3.0\ \text{N/m})(x^2/2)\,|_{x_1}^{x_2}$$

$W = -138\ \text{N}\cdot\text{m} - 71.4\ \text{N}\cdot\text{m} = -209\ \text{J}.$

EVALUATE: The work is negative because the cow continues to move forward (in the $+x$-direction) as you vainly attempt to push her backward.

6.59. **IDENTIFY** and **SET UP:** Since the forces are constant, $W_F = (F\cos\phi)s$ can be used to calculate the work done by each force. The forces on the suitcase are shown in Figure 6.59a.

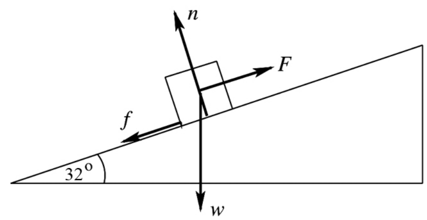


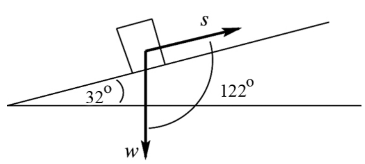
Figure 6.59a

In part (f), the work-energy theorem is used to relate the total work to the initial and final kinetic energy.

EXECUTE: **(a)** $W_F = (F\cos\phi)s$

Both $\vec{F}$ and $\vec{s}$ are parallel to the incline and in the same direction, so $\phi = 90°$ and $W_F = Fs = (160\text{ N})(3.80\text{ m}) = 608\text{ J}.$

(b) The directions of the displacement and of the gravity force are shown in Figure 6.59b.



$W_w = (w\cos\phi)s$

$\phi = 122°,$ so

$W_w = (196\text{ N})(\cos 122°)(3.80\text{ m})$

$W_w = -395\text{ J}$

Figure 6.59b

Alternatively, the component of w parallel to the incline is $w\sin 32°$. This component is down the incline so its angle with $\vec{s}$ is $\phi = 180°$. $W_{w\sin 32°} = (196\text{ N}\sin 32°)(\cos 180°)(3.80\text{ m}) = -395\text{ J}$. The other component of w, $w\cos 32°$, is perpendicular to $\vec{s}$ and hence does no work. Thus $W_w = W_{w\sin 25°} = -315\text{ J},$ which agrees with the above.

(c) The normal force is perpendicular to the displacement $(\phi = 90°)$, so $W_n = 0$.

(d) $n = w\cos 32°$ so $f_k = \mu_k n = \mu_k w\cos 32° = (0.30)(196\text{ N})\cos 32° = 49.87\text{ N}$

$W_f = (f_k\cos\phi)x = (49.87\text{ N})(\cos 180°)(3.80\text{ m}) = -189\text{ J}.$

(e) $W_{\text{tot}} = W_F + W_w + W_n + W_f = +608\text{ J} - 395\text{ J} + 0 - 189\text{ J} = 24\text{ J}.$

(f) $W_{\text{tot}} = K_2 - K_1,\ K_1 = 0,$ so $K_2 = W_{\text{tot}}$

$\tfrac{1}{2}mv_2^2 = W_{\text{tot}}$ so $v_2 = \sqrt{\dfrac{2W_{\text{tot}}}{m}} = \sqrt{\dfrac{2(24\text{ J})}{20.0\text{ kg}}} = 1.5\text{ m/s}.$

EVALUATE: The total work done is positive and the kinetic energy of the suitcase increases as it moves up the incline.

6.60. **IDENTIFY:** We need to use projectile motion and the work-energy theorem.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $K = \dfrac{1}{2}mv^2$. First use projectile motion to find the initial velocity, and then use the work-energy theorem to find the work.

EXECUTE: **(a)** When the can returns to the ground, $y = 0$, so $y = (v_0\sin\alpha_0)t - \dfrac{1}{2}gt^2$ gives us $0 = v_0\sin\alpha_0 T - \dfrac{1}{2}gT^2$, so $T = \dfrac{2v_0\sin\alpha_0}{g}$, which is the time in the air, and $v_0 = \dfrac{gT}{2\sin\alpha_0}$. Now use $W_{\text{tot}} = K_2 - K_1$ with $K_1 = 0$ to find the work. $W = \dfrac{1}{2}Mv_0^2 = \dfrac{1}{2}M\left(\dfrac{gT}{2\sin\alpha_0}\right)^2 = \dfrac{M}{8}\left(\dfrac{gT}{\sin}\alpha_0\right)^2.$

(b) If it is in the air twice as long, T is doubled. Using our result from part (a), we see that T is *squared*, so $(2T)^2$ would become $4T$. Therefore the work would become $W = \frac{M}{8}\left(\frac{g2T}{\sin}\alpha_0\right)^2 = \frac{M}{2}\left(\frac{gT}{\sin}\alpha_0\right)^2$, which is 4 times as much as in part (a).

EVALUATE: Another approach is to use $T = \frac{2v_0 \sin\alpha_0}{g}$ to see that if T is doubled, so is v_0. Therefore the kinetic energy $\frac{1}{2}mv^2$ is 4 times as large, so the work must have 4 times as much.

6.61. IDENTIFY: This problem requires use of the work-energy theorem. Both friction and gravity do work on the block as it slides down the ramp, but the normal force does no work.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $W = Fs\cos\phi$, $K = \frac{1}{2}mv^2$, $K_1 = 0$ when the block is released. Fig. 6.61 shows the information in the problem.

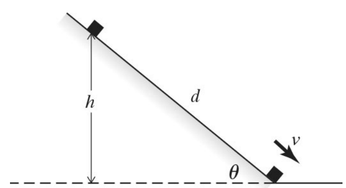


Figure 6.61

EXECUTE: **(a)** We want the work done by friction. $W_{\text{tot}} = K_2 - K_1$, where $K_1 = 0$, $K_2 = \frac{1}{2}mv^2$, $W_{\text{tot}} = W_f + W_g$. So $W_f = K_2 - W_g = \frac{1}{2}mv^2 - mgh$.

$W_f = \frac{1}{2}(5.00\text{ kg})(5.00\text{ m/s})^2 - (5.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = -35.5\text{ J}$.

(b) Now the ramp is lowered to 50.0° and the block is released from higher up but still 2.00 m above the bottom. Using $W = Fs\cos\phi$, we have $W_f = -f_k d = -\mu_k nd$. Using $\sum F_y = 0$ perpendicular to the surface of the ramp tells us that $n = mg\cos\theta$, and Fig. 6.61 shows that $d = \frac{h}{\sin\theta}$, so

$W_f = -\mu_k mg\cos\theta\left(\frac{h}{\sin\theta}\right) = -\frac{\mu_k mgh}{\tan\theta}$. From this result we see that as θ decreases, $\tan\theta$ decreases, so $1/\tan\theta$ increases. Therefore the magnitude of the work done by friction *increases* as θ decreases providing that h remains the same.

(c) Take the ratio of the works to find $W_{50°}$. $\frac{W_{50°}}{W_{60°}} = \frac{-\frac{\mu_k mgh}{\tan 50.0°}}{-\frac{\mu_k mgh}{\tan 60.0°}} = \frac{\tan 60.0°}{\tan 50.0°} = 1.45$, which gives $W_{50°} = (1.45)W_{60°} = (1.45)(-35.5\text{ J}) = -51.6\text{ J}$.

EVALUATE: As the slope angle θ decreases, the normal force increases so friction increases, and the distance d that the block slides also increases. So the magnitude of the work $W_f = f_k d$ increases, which agrees with our result.

6.62. **IDENTIFY:** We want to calculate the work that friction does on a block as it slides down an incline.

SET UP: $W = Fs\cos\phi$

EXECUTE: In this case, F is the friction force f, s is the distance that the block slides along the surface of the incline, and ϕ is the angle between the friction force and the displacement of the block, which is 180°. $W_f = fs \cos 180° = -fs$. In terms of h, $s = h/\sin\alpha$, and $f = \mu_k n$. $\sum F_y = 0$ perpendicular to the surface of the incline tells us that $n = mg\cos\alpha$, so $f = \mu_k mg\cos\alpha$. Combining these relations gives $W_f = -\mu_k mg\cos\alpha\left(\dfrac{h}{\sin\alpha}\right) = -\dfrac{\mu_k mgh}{\tan\alpha}$.

EVALUATE: As α is decreased, $\tan\alpha$ decreases, so W_f increases in magnitude.

6.63. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block to find the tension in the string. Each force is constant and $W = Fs\cos\phi$.

SET UP: The free-body diagram for each block is shown in Figure 6.63. $m_A = \dfrac{20.0\text{ N}}{g} = 2.04\text{ kg}$ and $m_B = \dfrac{12.0\text{ N}}{g} = 1.22\text{ kg}$.

EXECUTE: $T - f_k = m_A a$. $w_B - T = m_B a$. $w_B - f_k = (m_A + m_B)a$.

(a) $f_k = 0$. $a = \left(\dfrac{w_B}{m_A + m_B}\right)$ and $T = w_B\left(\dfrac{m_A}{m_A + m_B}\right) = w_B\left(\dfrac{w_A}{w_A + w_B}\right) = 7.50\text{ N}$.

20.0 N block: $W_{tot} = Ts = (7.50\text{ N})(0.750\text{ m}) = 5.62\text{ J}$.

12.0 N block: $W_{tot} = (w_B - T)s = (12.0\text{ N} - 7.50\text{ N})(0.750\text{ m}) = 3.38\text{ J}$.

(b) $f_k = \mu_k w_A = 6.50\text{ N}$. $a = \dfrac{w_B - \mu_k w_A}{m_A + m_B}$.

$$T = f_k + (w_B - \mu_k w_A)\left(\frac{m_A}{m_A + m_B}\right) = \mu_k w_A + (w_B - \mu_k w_A)\left(\frac{w_A}{w_A + w_B}\right).$$

$T = 6.50\text{ N} + (5.50\text{ N})(0.625) = 9.94\text{ N}$.

20.0 N block: $W_{tot} = (T - f_k)s = (9.94\text{ N} - 6.50\text{ N})(0.750\text{ m}) = 2.58\text{ J}$.

12.0 N block: $W_{tot} = (w_B - T)s = (12.0\text{ N} - 9.94\text{ N})(0.750\text{ m}) = 1.54\text{ J}$.

EVALUATE: Since the two blocks move with equal speeds, for each block $W_{tot} = K_2 - K_1$ is proportional to the mass (or weight) of that block. With friction the gain in kinetic energy is less, so the total work on each block is less.

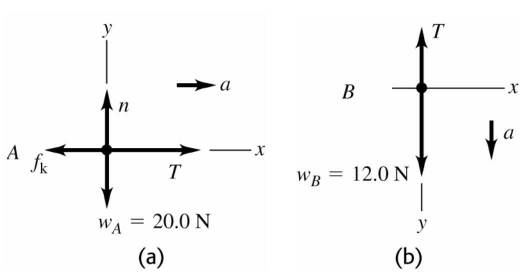


Figure 6.63

6.64. **IDENTIFY:** $W = Fs\cos\phi$ and $W_{tot} = K_2 - K_1$.

SET UP: $f_k = \mu_k n$. The normal force is $n = mg\cos\theta$, with $\theta = 24.0°$. The component of the weight parallel to the incline is $mg\sin\theta$.

EXECUTE: **(a)** $\phi = 180°$ and

$W_f = -f_k s = -(\mu_k mg\cos\theta)s = -(0.31)(5.00\text{ kg})(9.80\text{ m/s}^2)(\cos 24.0°)(2.80\text{ m}) = -38.9\text{ J}.$

(b) $(5.00\text{ kg})(9.80\text{ m/s}^2)(\sin 24.0°)(2.80\text{ m}) = 55.8\text{ J}.$

(c) The normal force does no work.

(d) $W_{\text{tot}} = 55.8\text{ J} - 38.9\text{ J} = +16.9\text{ J}.$

(e) $K_2 = K_1 + W_{\text{tot}} = (1/2)(5.00\text{ kg})(2.20\text{ m/s})^2 + 16.9\text{ J} = 29.0\text{ J},$ and so

$v_2 = \sqrt{2(29.0\text{ J})/(5.00\text{ kg})} = 3.41\text{ m/s}.$

EVALUATE: Friction does negative work and gravity does positive work. The net work is positive and the kinetic energy of the object increases.

6.65. **IDENTIFY:** The initial kinetic energy of the head is absorbed by the neck bones during a sudden stop. Newton's second law applies to the passengers as well as to their heads.

SET UP: In part (a), the initial kinetic energy of the head is absorbed by the neck bones, so $\frac{1}{2}mv_{\text{max}}^2 = 8.0\text{ J}.$ For part (b), assume constant acceleration and use $v_\text{f} = v_\text{i} + at$ with $v_\text{i} = 0,$ to calculate a; then apply $F_{\text{net}} = ma$ to find the net accelerating force.

Solve: (a) $v_{\text{max}} = \sqrt{\dfrac{2(8.0\text{ J})}{5.0\text{ kg}}} = 1.8\text{ m/s} = 4.0\text{ mph}.$

(b) $a = \dfrac{v_\text{f} - v_\text{i}}{t} = \dfrac{1.8\text{ m/s} - 0}{10.0\times 10^{-3}\text{ s}} = 180\text{ m/s}^2 \approx 18g,$ and $F_{\text{net}} = ma = (5.0\text{ kg})(180\text{ m/s}^2) = 900\text{ N}.$

EVALUATE: The acceleration is very large, but if it lasts for only 10 ms it does not do much damage.

6.66. **IDENTIFY:** The force does work on the object, which changes its kinetic energy, so the work-energy theorem applies. The force is variable so we must integrate to calculate the work it does on the object.

SET UP: $W_{\text{tot}} = \Delta K = K_\text{f} - K_\text{i} = \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2$ and $W_{\text{tot}} = \int_{x_1}^{x_2} F(x)dx.$

EXECUTE: $W_{\text{tot}} = \int_{x_1}^{x_2} F(x)dx = \int_0^{5.00\text{ m}} [-12.0\text{ N} + (0.300\text{ N/m}^2)x^2]dx.$

$W_{\text{tot}} = -(12.0\text{ N})(5.00\text{ m}) + (0.100\text{ N/m}^2)(5.00\text{ m})^3 = -60.0\text{ J} + 12.5\text{ J} = -47.5\text{ J}.$

$W_{\text{tot}} = \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2 = -47.5\text{ J},$ so the final velocity is

$v_\text{f} = \sqrt{v_\text{i}^2 - \dfrac{2(47.5\text{ J})}{m}} = \sqrt{(6.00\text{ m/s})^2 - \dfrac{2(47.5\text{ J})}{5.00\text{ kg}}} = 4.12\text{ m/s}.$

EVALUATE: We could not readily do this problem by integrating the acceleration over time because we know the force as a function of x, not of t. The work-energy theorem provides a much simpler method.

6.67. **IDENTIFY:** Calculate the work done by friction and apply $W_{\text{tot}} = K_2 - K_1.$ Since the friction force is not constant, use $W = \int F_x dx$ to calculate the work.

SET UP: Let x be the distance past P. Since μ_k increases linearly with x, $\mu_k = 0.100 + Ax.$ When $x = 12.5\text{ m},$ $\mu_k = 0.600,$ so $A = 0.500/(12.5\text{ m}) = 0.0400/\text{m}.$

EXECUTE: **(a)** $W_{\text{tot}} = \Delta K = K_2 - K_1$ gives $-\int \mu_k mg dx = 0 - \dfrac{1}{2}mv_1^2.$ Using the above expression for μ_k, $g\int_0^{x_2}(0.100 + Ax)dx = \dfrac{1}{2}v_1^2$ and $g\left[(0.100)x_2 + A\dfrac{x_2^2}{2}\right] = \dfrac{1}{2}v_1^2.$

$(9.80\text{ m/s}^2)\left[(0.100)x_2 + (0.0400/\text{m})\dfrac{x_2^2}{2}\right] = \dfrac{1}{2}(4.50\text{ m/s})^2.$ Solving for x_2 gives $x_2 = 5.11\text{ m}.$

(b) $\mu_k = 0.100 + (0.0400/\text{m})(5.11\text{ m}) = 0.304$

(c) $W_{tot} = K_2 - K_1$ gives $-\mu_k mgx_2 = 0 - \frac{1}{2}mv_1^2$. $x_2 = \frac{v_1^2}{2\mu_k g} = \frac{(4.50\text{ m/s})^2}{2(0.100)(9.80\text{ m/s}^2)} = 10.3\text{ m}$.

EVALUATE: The box goes farther when the friction coefficient doesn't increase.

6.68. **IDENTIFY:** Use $W = \int F_x dx$ to calculate W.

SET UP: $x_1 = 0$. In part (a), $x_2 = 0.050\text{ m}$. In part (b), $x_2 = -0.050\text{ m}$.

EXECUTE: **(a)** $W = \int_0^{x_2} F_x dx = \int_0^{x_2} (kx - bx^2 + cx^3)\, dx = \frac{k}{2}x_2^2 - \frac{b}{3}x_2^3 + \frac{c}{4}x_2^4$.

$W = (50.0\text{ N/m})\, x_2^2 - (233\text{ N/m}^2)\, x_2^3 + (3000\text{ N/m}^3)\, x_2^4$. When $x_2 = 0.050\text{ m}$, $W = 0.12\text{ J}$.

(b) When $x_2 = -0.050\text{ m}$, $W = 0.17\text{ J}$.

(c) It's easier to stretch the spring; the quadratic $-bx^2$ term is always in the –x-direction, and so the needed force, and hence the needed work, will be less when $x_2 > 0$.

EVALUATE: When $x = 0.050\text{ m}$, $F_x = 4.75\text{ N}$. When $x = -0.050\text{ m}$, $F_x = -8.25\text{ N}$.

6.69. **IDENTIFY:** Use the work-energy theorem.

SET UP: $W_{tot} = K_2 - K_1$, $K = \frac{1}{2}mv^2$. When a force does work W_D on the box, its speed is V starting from rest, so $W_D = \frac{1}{2}mV^2$.

EXECUTE: **(a)** We want to find the speed v of the box when half the total work has been done on it. Using $W_{tot} = K_2 - K_1$, we have $\frac{1}{2}W_D = \frac{1}{2}mv^2$. But as we saw above, $W_D = \frac{1}{2}mV^2$. So $\frac{1}{2}\left(\frac{1}{2}mV^2\right) = \frac{1}{2}mv^2$ which gives $v = \frac{V}{\sqrt{2}}$. Since $v \approx 0.707V$, $v > V/2$.

(b) Now we want to find how much work has been done on the box to reach half of its maximum speed, which is $\frac{1}{2}V$. The work-energy theorem gives $W = \frac{1}{2}m\left(\frac{V}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}mV^2\right)$. From above, we know that $W_D = \frac{1}{2}mV^2$, so $W = \frac{1}{4}W_D$, which is *less than* $\frac{1}{2}W_D$.

EVALUATE: It takes $\frac{1}{4}$ of the total work to get the box to half its maximum velocity, and $\frac{3}{4}$ of the total work to get the box from $V/2$ to V.

6.70. **IDENTIFY:** We are dealing with Hooke's law and the work to compress a spring.

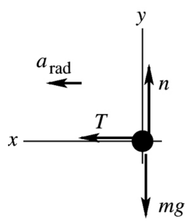
SET UP: Hooke's law: The force to stretch a spring a distance x is $F = kx$. The work to compress a spring a distance x from its equilibrium position is $W = \frac{1}{2}kx^2$.

EXECUTE: First find the force constant k. With you alone on the spring at maximum compression, $kx_1 = mg$, so $k = mg/x_1 = (530\text{ N})/(0.0180\text{ m}) = 2.94\times 10^4\text{ N/m}$. The spring compression with you and the dog is given by $kx_2 = w_{you} + w_{dog}$, so $x_2 = (710\text{ N})/(2.94\times 10^4\text{ N/m}) = 2.411\times 10^{-2}\text{ m} = 0.02411\text{ m}$. The work to compress the spring from x_1 to x_2 is $W_{1\to 2} = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. Using $x_1 = 0.0180\text{ m}$, $x_2 = 0.02411\text{ m}$, and $k = 2.94\times 10^4\text{ N/m}$, we get $W_{1\to 2} = 3.79\text{ J}$.

EVALUATE: The work to bring you to rest without your dog is $\frac{1}{2}kx_1^2 = \frac{1}{2}(2.944\times10^4 \text{ N/m})(0.0180 \text{ m})^2$ $= 4.77$ J. This is greater than the work to compress the spring with you and your dog because the amount of compression was less. You alone compressed the spring by 1.80 cm, but you with your dog compressed it only an additional 0.61 cm.

6.71. IDENTIFY and **SET UP:** Use $\Sigma\vec{F} = m\vec{a}$ to find the tension force T. The block moves in uniform circular motion and $\vec{a} = \vec{a}_{\text{rad}}$.

(a) The free-body diagram for the block is given in Figure 6.71.



EXECUTE: $\Sigma F_x = ma_x$

$$T = m\frac{v^2}{R}$$

$$T = (0.0600 \text{ kg})\frac{(0.70 \text{ m/s})^2}{0.40 \text{ m}} = 0.074 \text{ N}.$$

Figure 6.71

(b) $T = m\frac{v^2}{R} = (0.0600 \text{ kg})\frac{(2.80 \text{ m/s})^2}{0.10 \text{ m}} = 4.7 \text{ N}.$

(c) SET UP: The tension changes as the distance of the block from the hole changes. We could use $W = \int_{x_1}^{x_2} F_x\, dx$ to calculate the work. But a much simpler approach is to use $W_{\text{tot}} = K_2 - K_1$.

EXECUTE: The only force doing work on the block is the tension in the cord, so $W_{\text{tot}} = W_T$.

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0600 \text{ kg})(0.70 \text{ m/s})^2 = 0.01470 \text{ J}$, $K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.0600 \text{ kg})(2.80 \text{ m/s})^2 = 0.2352 \text{ J}$, so $W_{\text{tot}} = K_2 - K_1 = 0.2352 \text{ J} - 0.01470 \text{ J} = 0.22 \text{ J}$. This is the amount of work done by the person who pulled the cord.

EVALUATE: The block moves inward, in the direction of the tension, so T does positive work and the kinetic energy increases.

6.72. IDENTIFY: Use $W = \int F_x dx$ to find the work done by F. Then apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $\int \frac{dx}{x^2} = -\frac{1}{x}$.

EXECUTE: $W = \int_{x_1}^{x_2} \frac{\alpha}{x^2}\, dx = \alpha\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$.

$W = (2.12\times10^{-26} \text{ N}\cdot\text{m}^2)\left[(0.200 \text{ m}^{-1}) - (1.25\times10^9 \text{ m}^{-1})\right] = -2.65\times10^{-17} \text{ J}.$

Note that x_1 is so large compared to x_2 that the term $1/x_1$ is negligible. Then, using the work-energy theorem and solving for v_2,

$$v_2 = \sqrt{v_1^2 + \frac{2W}{m}} = \sqrt{(3.00\times10^5 \text{ m/s})^2 + \frac{2(-2.65\times10^{-17} \text{ J})}{(1.67\times10^{-27} \text{ kg})}} = 2.41\times10^5 \text{ m/s}.$$

(b) With $K_2 = 0, W = -K_1$. Using $W = -\frac{\alpha}{x_2}$,

$$x_2 = \frac{\alpha}{K_1} = \frac{2\alpha}{mv_1^2} = \frac{2(2.12\times10^{-26} \text{ N}\cdot\text{m}^2)}{(1.67\times10^{-27} \text{ kg})(3.00\times10^5 \text{ m/s})^2} = 2.82\times10^{-10} \text{ m}.$$

(c) The repulsive force has done no net work, so the kinetic energy and hence the speed of the proton have their original values, and the speed is 3.00×10^5 m/s.

EVALUATE: As the proton moves toward the uranium nucleus the repulsive force does negative work and the kinetic energy of the proton decreases. As the proton moves away from the uranium nucleus the repulsive force does positive work and the kinetic energy of the proton increases.

6.73. IDENTIFY: The negative work done by the spring equals the change in kinetic energy of the car.

SET UP: The work done by a spring when it is compressed a distance x from equilibrium is $-\frac{1}{2}kx^2$. $K_2 = 0$.

EXECUTE: $-\frac{1}{2}kx^2 = K_2 - K_1$ gives $\frac{1}{2}kx^2 = \frac{1}{2}mv_1^2$ and

$k = (mv_1^2)/x^2 = [(1200\text{ kg})(0.65\text{ m/s})^2]/(0.090\text{ m})^2 = 6.3\times 10^4$ N/m.

EVALUATE: When the spring is compressed, the spring force is directed opposite to the displacement of the object and the work done by the spring is negative.

6.74. IDENTIFY and SET UP: Use the work-energy theorem $W_{\text{tot}} = K_2 - K_1$. You do positive work and gravity does negative work. Let point 1 be at the base of the bridge and point 2 be at the top of the bridge.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2 = 1000\text{ J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(80.0\text{ kg})(1.50\text{ m/s})^2 = 90\text{ J}$

$W_{\text{tot}} = 90\text{ J} - 1000\text{ J} = -910\text{ J}$

(b) Neglecting friction, work is done by you (with the force you apply to the pedals) and by gravity: $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$. The gravity force is $w = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$, downward. The displacement is 5.20 m, upward. Thus $\phi = 180°$ and

$$W_{\text{gravity}} = (F\cos\phi)s = (784\text{ N})(5.20\text{ m})\cos 180° = -4077\text{ J}$$

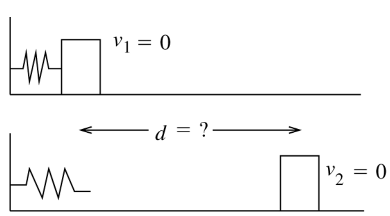
Then $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$ gives

$$W_{\text{you}} = W_{\text{tot}} - W_{\text{gravity}} = -910\text{ J} - (-4077\text{ J}) = +3170\text{ J}$$

EVALUATE: The total work done is negative and you lose kinetic energy.

6.75. IDENTIFY and SET UP: Use the work-energy theorem $W_{\text{tot}} = K_2 - K_1$. Work is done by the spring and by gravity. Let point 1 be where the textbook is released and point 2 be where it stops sliding. $x_2 = 0$ since at point 2 the spring is neither stretched nor compressed. The situation is sketched in Figure 6.75.

EXECUTE:



$W_{\text{tot}} = K_2 - K_1$

$K_1 = 0,\ \ K_2 = 0$

$W_{\text{tot}} = W_{\text{fric}} + W_{\text{spr}}$

Figure 6.75

$W_{\text{spr}} = \frac{1}{2}kx_1^2$, where $x_1 = 0.250$ m (the spring force is in direction of motion of block so it does positive work).

$W_{\text{fric}} = -\mu_k mgd$

Then $W_{\text{tot}} = K_2 - K_1$ gives $\frac{1}{2}kx_1^2 - \mu_k mgd = 0$

$$d = \frac{kx_1^2}{2\mu_k mg} = \frac{(250\text{ N/m})(0.250\text{ m})^2}{2(0.30)(2.50\text{ kg})(9.80\text{ m/s}^2)} = 1.1\text{ m},$$ measured from the point where the block was released.

EVALUATE: The positive work done by the spring equals the magnitude of the negative work done by friction. The total work done during the motion between points 1 and 2 is zero, and the textbook starts and ends with zero kinetic energy.

6.76. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: Let x_0 be the initial distance the spring is compressed. The work done by the spring is $\frac{1}{2}kx_0^2 - \frac{1}{2}kx^2$, where x is the final distance the spring is compressed.

EXECUTE: **(a)** Equating the work done by the spring to the gain in kinetic energy, $\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$, so

$$v = \sqrt{\frac{k}{m}}x_0 = \sqrt{\frac{400\text{ N/m}}{0.0300\text{ kg}}}(0.060\text{ m}) = 6.93\text{ m/s}.$$

(b) W_{tot} must now include friction, so $\frac{1}{2}mv^2 = W_{\text{tot}} = \frac{1}{2}kx_0^2 - fx_0$, where f is the magnitude of the friction force. Then,

$$v = \sqrt{\frac{k}{m}x_0^2 - \frac{2f}{m}x_0} = \sqrt{\frac{400\text{ N/m}}{0.0300\text{ kg}}(0.060\text{ m})^2 - \frac{2(6.00\text{ N})}{(0.0300\text{ kg})}(0.060\text{ m})} = 4.90\text{ m/s}.$$

(c) The greatest speed occurs when the acceleration (and the net force) are zero. Let x be the amount the spring is still compressed, so the distance the ball has moved is $x_0 - x$. $kx = f$, $x = \frac{f}{k} = \frac{6.00\text{ N}}{400\text{ N/m}} =$ 0.0150 m.

The ball is 0.0150 m from the end of the barrel, or 0.0450 m from its initial position.

To find the speed, the net work is $W_{\text{tot}} = \frac{1}{2}k(x_0^2 - x^2) - f(x_0 - x)$, so the maximum speed is

$$v_{\max} = \sqrt{\frac{k}{m}(x_0^2 - x^2) - \frac{2f}{m}(x_0 - x)}.$$

$$v_{\max} = \sqrt{\frac{400\text{ N/m}}{(0.0300\text{ kg})}\left[(0.060\text{ m})^2 - (0.0150\text{ m})^2\right] - \frac{2(6.00\text{ N})}{(0.0300\text{ kg})}(0.060\text{ m} - 0.0150\text{ m})} = 5.20\text{ m/s}$$

EVALUATE: The maximum speed with friction present (part (c)) is larger than the result of part (b) but smaller than the result of part (a).

6.77. **IDENTIFY:** A constant horizontal force pushes a block against a spring on a rough floor. The work-energy theorem and Newton's second law both apply.

SET UP: In part (a), we apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the block. $f_k = \mu_k n$ and $W_{\text{spring}} = -\frac{1}{2} kx^2$. In part (b), we apply Newton's second law to the block.

EXECUTE: **(a)** $W_F + W_{\text{spring}} + W_f = K_2 - K_1$. $Fx - \frac{1}{2} kx^2 - \mu_k mgx = \frac{1}{2} mv^2 - 0$. Putting in the numbers from the problem gives (82.0 N)(0.800 m) – (130.0 N/m)(0.800 m)^2/2 – (0.400)(4.00 kg)(9.80 m/s^2)(0.800 m) = (4.00 kg)v^2/2, v = 2.39 m/s.

(b) Looking at quantities parallel to the floor, with the positive direction toward the wall, Newton's second law gives $F - f_k - F_{\text{spring}} = ma$.

$F - \mu_k mg - kx = ma$: 82.0 N – (0.400)(4.00 kg)(9.80 m/s^2) – (130.0 N/m)(0.800 m) = (4.00 kg)a

a = –9.42 m/s^2. The minus sign means that the acceleration is away from the wall.

EVALUATE: The force you apply is toward the wall but the block is accelerating away from the wall.

6.78. **IDENTIFY:** A constant horizontal force pushes a frictionless block of ice against a spring on the floor. The work-energy theorem and Newton's second law both apply.

SET UP: In part (a), we apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the ice. $W_{\text{spring}} = -\frac{1}{2}kx^2$. In part (b), we apply Newton's second law to the ice.

EXECUTE: **(a)** $W_F + W_{\text{spring}} = K_2 - K_1$. $Fx - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$. Putting in the numbers from the problem gives $(54.0\text{ N})(0.400\text{ m}) - (76.0\text{ N/m})(0.400\text{ m})^2/2 = (2.00\text{ kg})v^2/2$, $v = 3.94$ m/s.

(b) Looking at quantities parallel to the floor, with the positive direction away from the post, Newton's second law gives $F - F_{\text{spring}} = ma$, so $F - kx = ma$.

54.0 N – (76.0 N/m)(0.400 m) = (2.00 kg)a, which gives $a = 11.8\text{ m/s}^2$. The acceleration is positive, so the block is accelerating away from the post.

EVALUATE: The given force must be greater than the spring force since the ice is accelerating away from the post.

6.79. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the blocks.

SET UP: If X is the distance the spring is compressed, the work done by the spring is $-\frac{1}{2}kX^2$. At maximum compression, the spring (and hence the block) is not moving, so the block has no kinetic energy.

EXECUTE: **(a)** The work done *by* the block is equal to its initial kinetic energy, and the maximum compression is found from $\frac{1}{2}kX^2 = \frac{1}{2}mv_0^2$ and $X = \sqrt{\frac{m}{k}}v_0 = \sqrt{\frac{5.00\text{ kg}}{500\text{ N/m}}}(6.00\text{ m/s}) = 0.600\text{ m}.$

(b) Solving for v_0 in terms of a known X, $v_0 = \sqrt{\frac{k}{m}}X = \sqrt{\frac{500\text{ N/m}}{5.00\text{ kg}}}(0.150\text{ m}) = 1.50\text{ m/s}.$

EVALUATE: The negative work done by the spring removes the kinetic energy of the block.

6.80. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$. $W = Fs\cos\phi$.

SET UP: The students do positive work, and the force that they exert makes an angle of $30.0°$ with the direction of motion. Gravity does negative work, and is at an angle of $120.0°$ with the chair's motion.

EXECUTE: The total work done is $W_{\text{tot}} =$
$\left[(600\text{ N})\cos 30.0° + (85.0\text{ kg})(9.80\text{ m/s}^2)\cos 120.0°\right](2.50\text{ m}) = 257.8\text{ J}$, and so the speed at the top of the ramp is $v_2 = \sqrt{v_1^2 + \frac{2W_{\text{tot}}}{m}} = \sqrt{(2.00\text{ m/s})^2 + \frac{2(257.8\text{ J})}{(85.0\text{ kg})}} = 3.17\text{ m/s}.$

EVALUATE: The component of gravity down the incline is $mg\sin 30° = 417\text{ N}$ and the component of the push up the incline is $(600\text{ N})\cos 30° = 520\text{ N}$. The force component up the incline is greater than the force component down the incline; the net work done is positive and the speed increases.

6.81. **IDENTIFY** and **SET UP:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system consisting of both blocks. Since they are connected by the cord, both blocks have the same speed at every point in the motion. Also, when the 6.00-kg block has moved downward 1.50 m, the 8.00-kg block has moved 1.50 m to the right. The target variable, μ_k, will be a factor in the work done by friction. The forces on each block are shown in Figure 6.81.

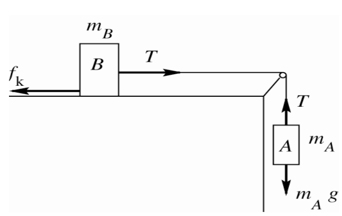


Figure 6.81

EXECUTE: $K_1 = \frac{1}{2}m_A v_1^2 + \frac{1}{2}m_B v_1^2 = \frac{1}{2}(m_A + m_B)v_1^2$

$K_2 = 0$

The tension T in the rope does positive work on block B and the same magnitude of negative work on block A, so T does no net work on the system. Gravity does work $W_{mg} = m_A gd$ on block A, where $d = 2.00$ m. (Block B moves horizontally, so no work is done on it by gravity.) Friction does work $W_{\text{fric}} = -\mu_k m_B gd$ on block B. Thus $W_{\text{tot}} = W_{mg} + W_{\text{fric}} = m_A gd - \mu_k m_B gd$. Then $W_{\text{tot}} = K_2 - K_1$ gives $m_A gd - \mu_k m_B gd = -\frac{1}{2}(m_A + m_B)v_1^2$ and

$$\mu_k = \frac{m_A}{m_B} + \frac{\frac{1}{2}(m_A + m_B)v_1^2}{m_B gd} = \frac{6.00\text{ kg}}{8.00\text{ kg}} + \frac{(6.00\text{ kg} + 8.00\text{ kg})(0.900\text{ m/s})^2}{2(8.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})} = 0.786$$

EVALUATE: The weight of block A does positive work and the friction force on block B does negative work, so the net work is positive and the kinetic energy of the blocks increases as block A descends. Note that K_1 includes the kinetic energy of both blocks. We could have applied the work-energy theorem to block A alone, but then W_{tot} includes the work done on block A by the tension force.

6.82. IDENTIFY: Apply $W_{\text{tot}} = K_2 - K_1$ to the system of the two blocks. The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table).

SET UP: Let h be the distance the 6.00 kg block descends. The work done by gravity is (6.00 kg)gh and the work done by friction is $-\mu_k(8.00\text{ kg})gh$.

EXECUTE: $W_{\text{tot}} = (6.00\text{ kg} - (0.25)(8.00\text{ kg}))(9.80\text{ m/s}^2)(1.50\text{ m}) = 58.8\text{ J}$. This work increases the kinetic energy of both blocks: $W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2$, so $v = \sqrt{\dfrac{2(58.8\text{ J})}{(14.00\text{ kg})}} = 2.90\text{ m/s}$.

EVALUATE: Since the two blocks are connected by the rope, they move the same distance h and have the same speed v.

6.83. IDENTIFY: Apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the skater.

SET UP: Let point 1 be just before she reaches the rough patch and let point 2 be where she exits from the patch. Work is done by friction. We don't know the skater's mass so can't calculate either friction or the initial kinetic energy. Leave her mass m as a variable and expect that it will divide out of the final equation.

EXECUTE: $f_k = 0.25mg$ so $W_f = W_{\text{tot}} = -(0.25mg)s$, where s is the length of the rough patch.

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2$, $K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m(0.55v_0)^2 = 0.3025\left(\frac{1}{2}mv_0^2\right)$

The work-energy relation gives $-(0.25mg)s = (0.3025 - 1)\frac{1}{2}mv_0^2$.

The mass divides out, and solving gives $s = 1.3$ m.

EVALUATE: Friction does negative work and this reduces her kinetic energy.

6.84. IDENTIFY and **SET UP:** $W = Pt$

EXECUTE: **(a)** The hummingbird produces energy at a rate of 0.7 J/s to 1.75 J/s. At 10 beats/s, the bird must expend between 0.07 J/beat and 0.175 J/beat.

(b) The steady output of the athlete is (500 W)/(70 kg) = 7 W/kg, which is below the 10 W/kg necessary to stay aloft. Though the athlete can expend 1400 W/70 kg = 20 W/kg for short periods of time, no human-powered aircraft could stay aloft for very long.

EVALUATE: Movies of early attempts at human-powered flight bear out our results.

6.85. IDENTIFY: To lift a mass m a height h requires work $W = mgh$. To accelerate mass m from rest to speed v requires $W = K_2 - K_1 = \frac{1}{2}mv^2$. $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$.

SET UP: $t = 60$ s

EXECUTE: **(a)** $(800\text{ kg})(9.80\text{ m/s}^2)(14.0\text{ m}) = 1.10\times 10^5\text{ J}.$

(b) $(1/2)(800\text{ kg})(18.0\text{ m/s}^2) = 1.30\times 10^5\text{ J}.$

(c) $\dfrac{1.10\times 10^5\text{ J} + 1.30\times 10^5\text{ J}}{60\text{ s}} = 3.99\text{ kW}.$

EVALUATE: Approximately the same amount of work is required to lift the water against gravity as to accelerate it to its final speed.

6.86. **IDENTIFY:** This problem requires the work-energy theorem. Gravity and the force $\vec{F}$ do work on the steel ball, the tension in the rope does none since it is perpendicular to the displacement.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $K = \frac{1}{2}mv^2$, $W = Fs\cos\phi$. The work done by gravity is $-mgL(1-\cos\alpha)$ and $K_1 = 0$. The work done by $\vec{F}$ is Fs, where s is the arc length subtended by α. So $s = L\alpha$, but α must be in radians. Converting gives $\alpha = 37.0° = 0.6458$ rad.

EXECUTE: Applying $W_{\text{tot}} = K_2 - K_1$ we have $W_F + W_g = \frac{1}{2}mv^2$, which gives

$FL\alpha(\text{rad}) - mgL(1-\cos\alpha) = \frac{1}{2}mv^2.$ Putting in the numbers gives us

$(0.760\text{ N})(0.600\text{ m})(0.6458\text{ rad}) - (0.200\text{ kg})(9.80\text{ m/s}^2)(0.600\text{ m})(1 - \cos 37.0°) = \frac{1}{2}(0.200\text{ kg})v^2$

which gives $v = 0.759$ m/s.

EVALUATE: Note that we used $\alpha = 0.6458$ rad for the arc length but $\alpha = 37.0°$ for $\cos\alpha$. We could have used $\alpha = 0.6458$ rad in $\cos\alpha$, but we would have had to put our calculator in the radian mode for angles, and most people would forget to do this!

6.87. **IDENTIFY:** This problem requires the work-energy theorem. Gravity does work on the system, the tension in the rope does work on both of the blocks, and friction does work on the 8.00-kg block.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $K = \frac{1}{2}mv^2$, $W = Fs\cos\phi$, and $K_1 = 0$. The blocks move together and therefore have the same speed.

EXECUTE: **(a)** $W_g = mgs = (6.00\text{ kg})(9.80\text{ m/s}^2)(0.800\text{ m}) = 47.0$ J.

$W_T = -Ts = -(37.0\text{ N})(0.800\text{ m}) = -29.6$ J.

$W_{\text{tot}} = K_2 - K_1$, where $W_{\text{tot}} = 47.0\text{ J} - 29.6\text{ J} = 17.4$ J. So $17.4\text{ J} = \frac{1}{2}mv^2 = \frac{1}{2}(6.00\text{ kg})v^2$, which gives $v = 2.41$ m/s.

(b) $W_{\text{tot}} = \frac{1}{2}mv^2 = \frac{1}{2}(8.00\text{ kg})(2.41\text{ m/s})^2 = 23.2$ J.

$W_T = Ts = (37.0\text{ N})(0.800\text{ m}) = 29.6$ J.

$W_{\text{tot}} = W_T + W_f \rightarrow \quad 23.2\text{ J} = 29.6\text{ J} + W_f \quad \rightarrow \quad W_f = -6.4$ J.

(c) $W_{\text{tot}} = \frac{1}{2}mv^2 = \frac{1}{2}(14.0\text{ kg})(2.41\text{ m/s})^2 = 40.6$ J.

$W_g = 47.0$ J (from part (a)).

$W_f = -6.4$ J (from part (b)).

$W_{\text{tot}} = 0$ since the tension is internal. Another way to see this is that the tension does positive work on the 8.00-kg block and an equal amount of negative work on the 6.00-kg block, so its total work is zero.

EVALUATE: In part (c), the total work on the system is 47.0 J – 6.4 J = 40.6 J, which agrees from our result using the work-energy theorem.

6.88. **IDENTIFY:** $W = \int_{x_1}^{x_2} F_x dx$, and F_x depends on both x and y.

SET UP: In each case, use the value of y that applies to the specified path. $\int x dx = \frac{1}{2}x^2$. $\int x^2 dx = \frac{1}{3}x^3$.

EXECUTE: **(a)** Along this path, y is constant, with the value $y = 3.00$ m.

$$W = \alpha y \int_{x_1}^{x_2} x dx = (2.50\ \text{N/m}^2)(3.00\ \text{m})\frac{(2.00\ \text{m})^2}{2} = 15.0\ \text{J},\ \text{since}\ x_1 = 0\ \text{and}\ x_2 = 2.00\ \text{m}.$$

(b) Since the force has no y-component, no work is done moving in the y-direction.

(c) Along this path, y varies with position along the path, given by $y = 1.5x$, so $F_x = \alpha(1.5x)x = 1.5\alpha x^2$, and

$$W = \int_{x_1}^{x_2} F dx = 1.5\alpha \int_{x_1}^{x_2} x^2 dx = 1.5(2.50\ \text{N/m}^2)\frac{(2.00\ \text{m})^3}{3} = 10.0\ \text{J}.$$

EVALUATE: The force depends on the position of the object along its path.

6.89. **IDENTIFY** and **SET UP:** For part (a) calculate m from the volume of blood pumped by the heart in one day. For part (b) use W calculated in part (a) in $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$.

EXECUTE: (a) The work to life the blood is $W = mgh$. We need the mass of blood lifted; we are given the volume $V = (7500\ \text{L})\left(\dfrac{1\times10^{-3}\ \text{m}^3}{1\ \text{L}}\right) = 7.50\ \text{m}^3$.

$$m = \text{density}\times\text{volume} = (1.05\times10^3\ \text{kg/m}^3)(7.50\ \text{m}^3) = 7.875\times10^3\ \text{kg}$$

Then $W = mgh = (7.875\times10^3\ \text{kg})(9.80\ \text{m/s}^2)(1.63\ \text{m}) = 1.26\times10^5\ \text{J}$.

(b) $P_{\text{av}} = \dfrac{\Delta W}{\Delta t} = \dfrac{1.26\times10^5\ \text{J}}{(24\ \text{h})(3600\ \text{s/h})} = 1.46\ \text{W}.$

EVALUATE: Compared to light bulbs or common electrical devices, the power output of the heart is rather small.

6.90. **IDENTIFY:** We know information about the force exerted by a stretched rubber band and want to know if it obeys Hooke's law.

SET UP: Hooke's law is $F = kx$. The graph fits the equation $F = 33.55x^{0.4871}$, with F in newtons and x in meters.

EXECUTE: **(a)** For Hooke's law, a graph of F versus x is a straight line through the origin. This graph is not a straight line, so the rubber band does not obey Hooke's law.

(b) $k_{\text{eff}} = \dfrac{dF}{dx} = \dfrac{d}{dx}(33.55x^{0.4871}) = 16.34x^{-0.5129}$. Because of the negative exponent for x, as x increases, k_{eff} decreases.

(c) The definition of work gives $W = \int_a^b F_x dx = \int_0^{0.0400\ \text{m}} 0.3355x^{0.4871} dx = (33.55/1.4871)\ 0.0400^{1.4871}$

$W = 0.188$ J. From 0.0400 m to 0.0800 m, we follow the same procedure but with different limits of integration. The result is $W = (33.55/1.4871)\ (0.0800^{1.4871} - 0.0400^{1.4871}) = 0.339$ J.

(d) $W = K_2 - K_1 = ½\ mv^2 - 0$, which gives 0.339 J = (0.300 kg)$v^2/2$, $v = 1.50$ m/s.

EVALUATE: The rubber band does not obey Hooke's law, but it does obey the work-energy theorem.

6.91. **IDENTIFY:** We know a spring obeys Hooke's law, and we want to use observations of the motion of a block attached to this spring to determine its force constant and the coefficient of friction between the block and the surface on which it is sliding. The work-energy theorem applies.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $W_{\text{spring}} = ½\ kx^2$.

EXECUTE: **(a)** The spring force is initially greater than friction, so the block accelerates forward. But eventually the spring force decreases enough so that it is less than the force of friction, and the block then slows down (decelerates).

(b) The spring is initially compressed a distance x_0, and after the block has moved a distance d, the spring is compressed a distance $x = x_0 - d$. Therefore the work done by the spring is

$W_{\text{spring}} = \frac{1}{2}kx_0^2 - \frac{1}{2}k(x_0 - d)^2$. The work done by friction is $W_f = -\mu_k mgd$.

The work-energy theorem gives $W_{\text{spring}} + W_f = K_2 - K_1 = \frac{1}{2}mv^2$. Using our previous results, we get $\frac{1}{2}kx_0^2 - \frac{1}{2}k(x_0 - d)^2 - \mu_k mgd = \frac{1}{2}mv^2$. Solving for v^2 gives $v^2 = -\frac{k}{m}d^2 + 2d\left(\frac{k}{m}x_0 - \mu_k g\right)$, where $x_0 =$ 0.400 m.

(c) Figure 6.91 shows the resulting graph of v^2 versus d. Using a graphing program and a quadratic fit gives $v^2 = -39.96d^2 + 16.31d$. The maximum speed occurs when $dv^2/dd = 0$, which gives $(-39.96)(2d) + 16.31 = 0$, so $d = 0.204$ m. For this value of d, we have $v^2 = (-39.96)(0.204\text{ m})^2 + (16.31)(0.204\text{ m})$, giving $v = 1.29$ m/s.

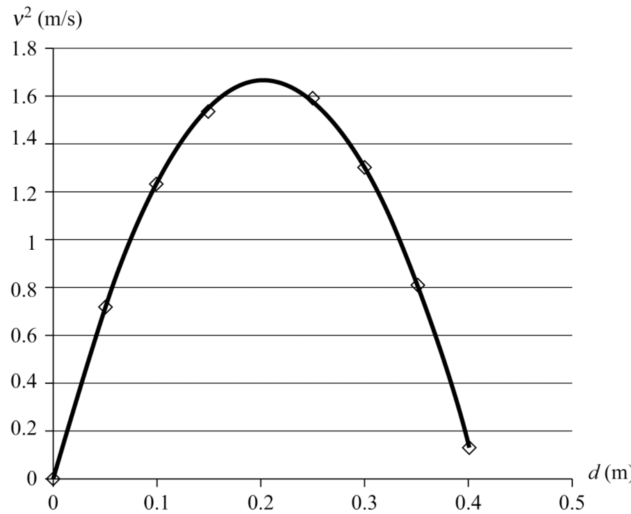


Figure 6.91

(d) From our work in (b) and (c), we know that $-k/m$ is the coefficient of d^2, so $-k/m = -39.96$, which gives $k = (39.96)(0.300\text{ kg}) = 12.0$ N/m. We also know that $2(kx_0/m - \mu_k g)$ is the coefficient of d. Solving for μ_k and putting in the numbers gives $\mu_k = 0.800$.

EVALUATE: The graphing program makes analysis of complicated behavior relatively easy.

6.92. **IDENTIFY:** The power output of the runners is the work they do in running from the basement to the top floor divided by the time it takes to make this run.

SET UP: $P = W/t$ and $W = mgh$.

EXECUTE: **(a)** For each runner, $P = mgh/t$. We must read the time of each runner from the figure shown with the problem. For example, for Tatiana we have $P = (50.2\text{ kg})(9.80\text{ m/s}^2)(16.0\text{ m})/32\text{ s} =$ 246.0 W, which we must round to 2 significant figures because we cannot read the times any more accurate than that using the figure in the text. Carrying out these calculations for all the runners, we get the following results.

Tatiana: 250 W, Bill: 210 W, Ricardo: 290 W, Melanie: 170 W. Ricardo had the greatest power output, and Melanie had the least.

(b) Solving $P = mgh/t$ for t gives $t = mgh/P = (62.3\text{ kg})(9.80\text{ m/s}^2)(16.0\text{ m})/(746\text{ W}) = 13.1\text{ s}$, where we have used the fact that 1 hp = 746 W.

EVALUATE: Even though Tatiana had the shortest time, her power output was less than Ricardo's because she weighs less than he does.

6.93. **IDENTIFY:** In part (a) follow the steps outlined in the problem. For parts (b), (c), and (d) apply the work-energy theorem.

SET UP: $\int x^2 dx = \frac{1}{3}x^3$

EXECUTE: **(a)** Denote the position of a piece of the spring by l; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to l, denoted u, is $u(l) = v(l/L)$ (when the spring is moving, l will be a function of time, and so u is an implicit function of time). The mass of a piece of length dl is $dm = (M/L)dl$, and so $dK = \frac{1}{2}(dm)u^2 = \frac{1}{2}\frac{Mv^2}{L^3}l^2 dl$, and

$$K = \int dK = \frac{Mv^2}{2L^3}\int_0^L l^2 dl = \frac{Mv^2}{6}.$$

(b) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = \sqrt{(k/m)x} = \sqrt{(3200\text{ N/m})/(0.053\text{ kg})}\,(2.50\times 10^{-2}\text{ m}) = 6.1\text{ m/s}$.

(c) With the mass of the spring included, the work that the spring does goes into the kinetic energies of both the ball and the spring, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{6}Mv^2$. Solving for v,

$$v = \sqrt{\frac{k}{m + M/3}}\,x = \sqrt{\frac{(3200\text{ N/m})}{(0.053\text{ kg}) + (0.243\text{ kg})/3}}(2.50\times 10^{-2}\text{ m}) = 3.9\text{ m/s}.$$

(d) Algebraically, $\frac{1}{2}mv^2 = \frac{(1/2)kx^2}{(1 + M/3m)} = 0.40\text{ J}$ and $\frac{1}{6}Mv^2 = \frac{(1/2)kx^2}{(1 + 3m/M)} = 0.60\text{ J}$.

EVALUATE: For this ball and spring, $\frac{K_{\text{ball}}}{K_{\text{spring}}} = \frac{3m}{M} = 3\left(\frac{0.053\text{ kg}}{0.243\text{ kg}}\right) = 0.65$. The percentage of the final kinetic energy that ends up with each object depends on the ratio of the masses of the two objects. As expected, when the mass of the spring is a small fraction of the mass of the ball, the fraction of the kinetic energy that ends up in the spring is small.

6.94. **IDENTIFY:** In both cases, a given amount of fuel represents a given amount of work W_0 that the engine does in moving the plane forward against the resisting force. Write W_0 in terms of the range R and speed v and in terms of the time of flight T and v.

SET UP: In both cases assume v is constant, so $W_0 = RF$ and $R = vT$.

EXECUTE: In terms of the range R and the constant speed v, $W_0 = RF = R\left(\alpha v^2 + \frac{\beta}{v^2}\right)$.

In terms of the time of flight $T, R = vt$, so $W_0 = vTF = T\left(\alpha v^3 + \frac{\beta}{v}\right)$.

(a) Rather than solve for R as a function of v, differentiate the first of these relations with respect to v, setting $\frac{dW_0}{dv} = 0$ to obtain $\frac{dR}{dv}F + R\frac{dF}{dv} = 0$. For the maximum range, $\frac{dR}{dv} = 0$, so $\frac{dF}{dv} = 0$. Performing the differentiation, $\frac{dF}{dv} = 2\alpha v - 2\beta/v^3 = 0$, which is solved for

$$v = \left(\frac{\beta}{\alpha}\right)^{1/4} = \left(\frac{3.5\times 10^5\text{ N}\cdot\text{m}^2/\text{s}^2}{0.30\text{ N}\cdot\text{s}^2/\text{m}^2}\right)^{1/4} = 32.9\text{ m/s} = 118\text{ km/h}.$$

(b) Similarly, the maximum time is found by setting $\frac{d}{dv}(Fv)=0;$ performing the differentiation,

$3\alpha v^2-\beta/v^2=0.$ $v=\left(\frac{\beta}{3\alpha}\right)^{1/4}=\left(\frac{3.5\times10^5\ \text{N}\cdot\text{m}^2/\text{s}^2}{3(0.30\ \text{N}\cdot\text{s}^2/\text{m}^2)}\right)^{1/4}=25\ \text{m/s}=90\ \text{km/h}.$

EVALUATE: When $v=(\beta/\alpha)^{1/4}$, F_{air} has its minimum value $F_{\text{air}}=2\sqrt{\alpha\beta}$. For this v, $R_1=(0.50)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_1=(0.50)\alpha^{-1/4}\beta^{-3/4}$. When $v=(\beta/3\alpha)^{1/4}$, $F_{\text{air}}=2.3\sqrt{\alpha\beta}$. For this v, $R_2=(0.43)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_2=(0.57)\alpha^{-1/4}\beta^{-3/4}$. $R_1>R_2$ and $T_2>T_1$, as they should be.

6.95. **IDENTIFY:** Using 300 W of metabolic power, the person travels 3 times as fast when biking than when walking.
SET UP: $P = W/t$, so $W = Pt$.
EXECUTE: When biking, the person travels 3 times as fast as when walking, so the bike trip takes 1/3 the time. Since $W = Pt$ and the power is the same, the energy when biking will be 1/3 of the energy when walking, which makes choice (a) the correct one.
EVALUATE: Walking is obviously a better way to burn calories than biking.

6.96. **IDENTIFY:** When walking on a grade, metabolic power is required for walking horizontally as well as the vertical climb.
SET UP: $P = W/t$, $W = mgh$.
EXECUTE: $P_{\text{tot}} = P_{\text{horiz}} + P_{\text{vert}} = P_{\text{horiz}} + mgh/t = P_{\text{horiz}} + mg(v_{\text{vert}})$. The slope is a 5% grade, so $v_{\text{vert}} = 0.05v_{\text{horiz}}$. Therefore $P_{\text{tot}} = 300\ \text{W} + (70\ \text{kg})(9.80\ \text{m/s}^2)(0.05)(1.4\ \text{m/s}) = 348\ \text{W} \approx 350\ \text{W}$, which makes choice (c) correct.
EVALUATE: Even a small grade of only 5% makes a difference of about 17% in power output.

6.97. **IDENTIFY:** Using 300 W of metabolic power, the person travels 3 times as fast when biking than when walking.
SET UP: $K = \frac{1}{2} mv^2$.
EXECUTE: The speed when biking is 3 times the speed when walking. Since the kinetic energy is proportional to the square of the speed, the kinetic energy will be $3^2 = 9$ times as great when biking, making choice (d) correct.
EVALUATE: Even a small increase in speed gives a considerable increase in kinetic energy due to the factor of v^2 in the kinetic energy.

7

POTENTIAL ENERGY AND ENERGY CONSERVATION

VP7.2.1. **IDENTIFY:** We use energy conservation. The ball has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ Call point 1 the place where the ball leaves your hand and point 2 the height where it has the desired speed. In this case, $W_{\text{other}} = 0$.

EXECUTE: **(a)** In this case, $v_2 = v_1/2$ and $U_1 = 0$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ gives

$0 + \frac{1}{2}mv_1^2 + 0 = mgy_2 + \frac{1}{2}mv_2^2 = mgy_2 + \frac{1}{2}m\left(\frac{v_1}{2}\right)^2$. Solve for y_2 gives $y_2 = \frac{3v_1^2}{8g} = \frac{3(12.0\text{ m/s})^2}{8(9.80\text{ m/s}^2)} = 5.51$ m.

(b) In this case, $K_2 = K_1/2$ and $U_1 = 0$, so $0 + K_1 + 0 = mgy_2 + K_1/2$. Solving for y_2 gives

$y_2 = \frac{K_1}{2mg} = \frac{\frac{1}{2}mv_1^2}{2mg} = \frac{v_1^2}{4g} = \frac{(12.0\text{ m/s})^2}{4(9.80\text{ m/s}^2)} = 3.67$ m.

EVALUATE: Notice that the ball does *not* have half of its initial kinetic energy when it is half-way to the top. Likewise when it has half of its initial kinetic energy, it is *not* half-way to the top.

VP7.2.2. **IDENTIFY:** We use energy conservation. The rock has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ .Call point 1 the place where the rock leaves your hand and point 2 the height where it has the desired height. In this case, $W_{\text{other}} = 0$ and $U_1 =$ 0.

EXECUTE: First find the initial speed in terms of h. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ gives

$K_1 = U_2 + K_2 \quad \rightarrow \quad \frac{1}{2}mv_1^2 = mgh \quad \rightarrow \quad v_1^2 = 2gh$.

(a) In this case, $y_2 = h/4$ and we use , so $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ gives

$\frac{1}{2}mv_1^2 = mg(h/4) + \frac{1}{2}mv_2^2 \quad \rightarrow \quad 2(2gh) - gh = 2v_2^2 \quad \rightarrow \quad v_2 = \sqrt{\frac{3gh}{2}}$.

(b) Follow the same procedure as in (a) *except* that $y_2 = 3h/4$. Energy conservation gives

$\frac{1}{2}mv_1^2 = mg(3h/4) + \frac{1}{2}mv_2^2$. Using $v_1^2 = 2gh$ gives $v_2 = \sqrt{\frac{gh}{2}}$.

EVALUATE: Our result says that the speed when $y = 3h/4$ is less than when $y_2 = h/4$, which is reasonable because the rock is slowing down as it rises.

VP7.2.3. **IDENTIFY:** We use energy conservation. The ball has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Call point 1 the place where the ball leaves your hand and point 2 the height where it reaches its maximum height, so $U_1 = 0$ and $K_2 = 0$. Call E the total mechanical energy of the ball, and use $W = Fs\cos\phi$.

EXECUTE: **(a)** $E_1 = K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0570 \text{ kg})(15.0 \text{ m/s})^2 = 6.4125 \text{ J}$.

$E_2 = U_2 = mgy_2 = (0.0570 \text{ kg})(9.80 \text{ m/s}^2)(8.00 \text{ m}) = 4.4688 \text{ J}$.

$\Delta E = E_2 - E_1 = 4.4688 \text{ J} - 6.4125 \text{ J} = -1.94 \text{ J}$, so the total mechanical energy has *decreased* by 1.94 J.

(b) The loss of mechanical energy is equal to the work done by the friction of air resistance.

$Fs\cos\phi = \Delta E \quad \to \quad F(8.00 \text{ m})\cos 180° = -1.94 \text{ J} \quad \to \quad F = 0.243 \text{ N}$.

EVALUATE: From our results, we have $E_2/E_1 = (4.4688 \text{ J})/(6.4125 \text{ J}) = 0.700$. We can also calculate this ratio as $E_2/E_1 = \frac{U_2}{U_1} = \frac{mgh}{\frac{1}{2}mv_1^2} = \frac{2gh}{v_1^2} = \frac{2(9.80 \text{ m/s}^2)}{(15.0 \text{ m/s})^2} = 0.700$, so $E_2 = 0.700E_1$. The mechanical energy has decreased by 30%.

VP7.2.4. **IDENTIFY:** We use energy conservation. The ball has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Call point 1 the place where you catch the ball and point 2 the place where the ball stops, so $U_1 = 0$, $y_2 = -0.150$ m, and $K_2 = 0$. Use $W = Fs\cos\phi$. W_{other} is the work done by your hands in stopping the ball.

EXECUTE: **(a)** Using $U_1 = 0$, $K_2 = 0$, $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ becomes

$\frac{1}{2}mv_1^2 + W_{\text{hands}} = mgy_2$

$W_{\text{hands}} = m\left(gy_2 - \frac{v_1^2}{2}\right) = (0.270 \text{ kg})\left[(9.80 \text{ m/s}^2)(-0.150 \text{ m}) - \frac{(7.50 \text{ m/s})^2}{2}\right] = -7.99 \text{ J}$.

(b) $W = Fs\cos\phi \quad \to \quad -7.99 \text{ J} = F(0.150 \text{ m})\cos 180° \quad \to \quad F = 53.3 \text{ N}$.

EVALUATE: The work done by gravity is $W_g = mgy = (0.270 \text{ kg})(9.80 \text{ m/s}^2)(0.150 \text{ m}) = 39.7 \text{ J}$, which is much more than the magnitude of the work your hands do. This is reasonable because your hands also reduce (to zero) the ball's initial kinetic energy.

VP7.5.1. **IDENTIFY** We use energy conservation. The butter has kinetic energy and gravitational potential energy. Use Newton's second law for circular motion.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Call point 1 the place at the rim of the bowl and point 2 the bottom of the bowl, so $K_1 = 0$, $y_2 = 0$, and $W_{\text{other}} = 0$. Use $\sum F_y = ma_y$ at the bottom of the bowl, where $a_y = \frac{v^2}{R}$. At the top of the bowl, $y_1 = R$ (the radius of the bowl).

EXECUTE: **(a)** Using $K_1 = 0$, $U_2 = 0$, and $W_{\text{other}} = 0$, $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ becomes

$mgR = \frac{1}{2}mv_2^2 \quad \to \quad v_2 = \sqrt{2Rg} = \sqrt{2(0.150 \text{ m})(9.80 \text{ m/s}^2)} = 1.71 \text{ m/s}$.

(b) At the bottom of the bowl, apply Newton's second law for circular motion.

$\sum F_y = ma_y = \frac{v^2}{R} \quad \to \quad F_{\text{bowl}} - mg = mv^2/R \quad \to \quad F_{\text{bowl}} = m(g + v^2/R)$

For the numbers here we get

$F_{\text{bowl}} = (5.00 \times 10^{-3} \text{ kg})[9.80 \text{ m/s}^2 + (1.71 \text{ m/s})^2/(0.150 \text{ m})] = 0.147 \text{ N}$.

$w = mg = (5.00 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) = 0.0490 \text{ N}$.

F_{bowl}/w = (0.147 N)/(0.0490 N) = 3.00, so the force due to the bowl is 3 times the weight of the butter.
EVALUATE: The ratio F_{bowl}/w is $m(g + v^2/R)/mg = g + v^2/R$, which is independent of the butter's mass. Therefore *any* object sliding down this bowl under the same conditions would experience a force from the bowl equal to 3 times the weight of the object.

VP7.5.2. **IDENTIFY:** We use energy conservation. The snowboarder has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{other} = U_2 + K_2$. Call point 1 the bottom of the ditch and point 2 the highest point she reaches, so $K_2 = 0$, $U_1 = 0$.
EXECUTE: **(a)** If there is no friction, $W_{other} = 0$, so $U_1 + K_1 + W_{other} = U_2 + K_2$ becomes $K_1 = U_2$ → $\frac{1}{2}mv^2 = mgh$ → $h = v^2/2g$ = (9.30 m/s²)/[2(9.80 m/s²)] = 4.41 m. This answer does not depend on the shape of the ditch since there is no friction.
(b) The work done by friction is equal to the loss of mechanical energy, so $W_f = U_2 - K_1$.
$W_f = mgh - \frac{1}{2}mv^2 = m(gh - v^2/2)$ = (40.0 kg)[(9.80 m/s²)(3.50 m) – (9.30 m/s)²/2] = –358 J.

EVALUATE: The work done by friction is negative because the friction force is opposite to the displacement of the snowboard, which agrees with our result.

VP7.5.3. **IDENTIFY:** We apply energy conservation and Newton's second law to a swinging pendulum. The small sphere (the bob) has kinetic energy and potential energy. Use Newton's second law for circular motion.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{other} = U_2 + K_2$. Call point 1 the high point of the swing and point 2 its low point, so $K_1 = 0$, $U_2 = 0$, and $W_{other} = 0$. Use $\sum F_y = ma_y$ at the bottom of the swing, where $a_y = \frac{v^2}{R}$. At the top of the swing, $h = L(1 - \cos\theta)$, where L is the length of the string and θ is the largest angle it makes with the vertical.
EXECUTE: **(a)** For conditions here, we find that $U_1 = K_2 = mgh = mgL(1 - \cos\theta)$, so
K_2 = (0.250 kg)(9.80 m/s²)(1.20 m)(1 – cos 34.0°) = 0.503 J.
(b) Apply $\sum F_y = ma_y = \frac{v^2}{R}$ at the bottom of the swing: $T - mg = \frac{mv^2}{L} = \frac{2}{L}\left(\frac{1}{2}mv^2\right) = \frac{2K}{L}$.
$T = mg + 2K/L$ = (0.250 kg)(9.80 m/s²) + 2(0.503 J)/(1.20 m) = 3.29 m.
EVALUATE: As a check, find v^2 from the known kinetic energy, giving v^2 = 4.021 m²/s². Then use $T - mg = mv^2/L$ to find T. The result is the same as we found in (b).

VP7.5.4. **IDENTIFY:** We use energy conservation. The car has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{other} = U_2 + K_2$. Call A point 1 and B point 2, so U_1 = 0. In this case, $K_A = K_i$, $K_B = K_i/4$, $U_B = K_i/2$, and $y_B = 2R$.

EXECUTE: **(a)** $\Delta U_{AB} = mg(2R) = \frac{1}{2}K_i = \frac{1}{2}\left(\frac{1}{2}mv_A^2\right)$ → $v_A = \sqrt{8gR}$.

(b) Energy conservation gives $K_A + W_{other} = K_B + (U_B - U_B)$. Since W_{other} is due to friction, this becomes $K_i + W_f = \frac{1}{4}K_i + \frac{1}{2}K_i = \frac{3}{4}K_i$. Solving for W_f gives
$W_f = -\frac{1}{4}K_i = -\frac{1}{4}\left(\frac{1}{2}mv_A^2\right) = -\frac{1}{8}m(8gR) = -mgR$.

(c) Using $W_f = fs\cos\phi$ and the result from (b) gives $-mgR = f(\pi R)\cos 180° = -\pi Rf$

$f = mg/\pi$.

EVALUATE: From (c) we see that the heavier the roller coaster, the greater the friction force, which is reasonable since friction depends on the normal force at the surface of contact.

VP7.9.1. **IDENTIFY:** We use energy conservation. The system has kinetic energy and elastic potential energy in the spring.

SET UP: $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$. The mechanical energy is the kinetic energy plus the potential energy: $E = K + U$.

EXECUTE: **(a)** $E = K + U = \frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2$

$E = \frac{1}{2}(0.240\text{ kg})(0.400\text{ m/s})^2 + \frac{1}{2}(6.00\text{ N/m})(0.100\text{ m})^2 = 0.0492\text{ J}.$

(b) There is no friction, so E is constant. So when the glider stops, $U = 0.0492$ J. Therefore

$\frac{1}{2}kx_1^2 = 0.0492\text{ J} \quad\rightarrow\quad \frac{1}{2}(6.00\text{ N/m})\,x^2 = 0.0492\text{ J} \quad\rightarrow\quad x = 0.128\text{ m}.$

EVALUATE: During most of the motion, the glider has kinetic energy and potential energy, but the sum of the two is always equal to 0.0492 J.

VP7.9.2. **IDENTIFY:** We use energy conservation. The system has kinetic energy and elastic potential energy in the spring and there is friction.

SET UP: $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. $W = Fs\cos\phi$. Call position 1 when the spring is stretched by 0.100 m and position 2 when the glider has instantaneously stopped, so $K_2 = 0$.

EXECUTE: **(a)** $U_1 + K_1 + W_f = U_2 + K_2$ gives $\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 + W_f = \frac{1}{2}kx_2^2$. Using $k = 6.00$ N/m, $m =$ 0.240 kg, $x_1 = 0.100$ m, $x_2 = 0.112$ m, and $v_1 = 0.400$ m/s gives $W_f = -0.0116$ J.

(b) $W_f = fs\cos\phi = \mu_k n(x_2 - x_1)\cos 180° = -\mu_k mg(x_2 - x_1)$. Use the result from (a) for W_f.

$-\mu_k(0.240\text{ kg})(9.80\text{ m/s}^2)(0.112\text{ m} - 0.100\text{ m}) = -0.0116\text{ J} \quad\rightarrow\quad \mu_k = 0.410.$

EVALUATE: From Table 5.1 we see that our result is very reasonable for metal-on-metal coefficients of kinetic friction. For example, $\mu_k = 0.47$ for aluminum on steel.

VP7.9.3. **IDENTIFY:** We use energy conservation. The system has kinetic energy, gravitational potential energy, and elastic potential energy, but there is no friction.

SET UP: $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, and $W_{\text{other}} = 0$.

EXECUTE: **(a)** Take point 1 to be the the instant the compressed spring is released and point 2 to be just when the block loses contact with the spring, so $K_1 = 0$ and $U_2 = 0$. This gives

$U_{\text{spring}} + U_g = K_2 \quad\rightarrow\quad \frac{1}{2}kd^2 - mgd = \frac{1}{2}mv_2^2 \quad\rightarrow\quad v_2 = \sqrt{\frac{kd^2}{m} - 2gd}.$

(b) After the block clears the spring, we reapply $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Now call point 1 to be the instant it has left the spring and point 2 to be its maximum height. The v_1 in this part is the v_2 we found in part (a). Let maximum height be h.

$\frac{1}{2}mv_1^2 = mgh \quad \rightarrow \qquad h = \frac{v_1^2}{2g}$ The *total* vertical distance the block travels is $D = d + h$, so

$D = d + \frac{v_1^2}{2g}$. Using the value of v_2 (which is v_1 for this part) gives

$$D = d + \frac{\frac{kd^2}{m} - 2gd}{2g} = d + \frac{kd^2}{2mg} - d = \frac{kd^2}{2mg}.$$

EVALUATE: Check units to be sure that both answers have the proper dimensions of length.

VP7.9.4. **IDENTIFY:** We use energy conservation. The system has kinetic energy, gravitational potential energy, and elastic potential energy, but there is no friction.

SET UP: $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, and $W_{\text{other}} = W_{\text{f}}$. Call point 1 to be at the place where the cylinder is released with the spring relaxed, and point 2 to be at the maximum elongation of the spring, so $K_1 = 0$, $U_1 = 0$, and $K_2 = 0$. Call x the maximum elongation of the spring. The friction force and gravity both act through a distance x, so $W_{\text{f}} = -fx$ and $U_{2\text{-grav}} = -mgx$. This energy is negative because the cylinder is *below* point 1.

EXECUTE: $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ gives $-fx = \frac{1}{2}kx^2 - mgx$. Solving for x gives $x = 2(mg - f)/k$.

EVALUATE: Check units to be sure that the answer has the proper dimensions of length.

7.1. **IDENTIFY:** $U_{\text{grav}} = mgy$ so $\Delta U_{\text{grav}} = mg(y_2 - y_1)$

SET UP: $+y$ is upward.

EXECUTE: **(a)** $\Delta U = (75 \text{ kg})(9.80 \text{ m/s}^2)(2400 \text{ m} - 1500 \text{ m}) = +6.6\times10^5 \text{ J}$

(b) $\Delta U = (75 \text{ kg})(9.80 \text{ m/s}^2)(1350 \text{ m} - 2400 \text{ m}) = -7.7\times10^5 \text{ J}$

EVALUATE: U_{grav} increases when the altitude of the object increases.

7.2. **IDENTIFY:** The change in height of a jumper causes a change in their potential energy.
SET UP: Use $\Delta U_{\text{grav}} = mg(y_2 - y_1)$.

EXECUTE: $\Delta U_{\text{grav}} = (72 \text{ kg})(9.80 \text{ m/s}^2)(0.60 \text{ m}) = 420 \text{ J}$.

EVALUATE: This gravitational potential energy comes from elastic potential energy stored in the jumper's tensed muscles.

7.3. **IDENTIFY:** Use the free-body diagram for the bag and Newton's first law to find the force the worker applies. Since the bag starts and ends at rest, $K_2 - K_1 = 0$ and $W_{\text{tot}} = 0$.

SET UP: A sketch showing the initial and final positions of the bag is given in Figure 7.3a. $\sin\phi = \frac{2.0 \text{ m}}{3.5 \text{ m}}$ and $\phi = 34.85°$. The free-body diagram is given in Figure 7.3b. $\vec{F}$ is the horizontal force applied by the worker. In the calculation of U_{grav} take $+y$ upward and $y = 0$ at the initial position of the bag.

EXECUTE: **(a)** $\Sigma F_y = 0$ gives $T\cos\phi = mg$ and $\Sigma F_x = 0$ gives $F = T\sin\phi$. Combining these equations to eliminate T gives $F = mg\tan\phi = (90.0 \text{ kg})(9.80 \text{ m/s}^2)\tan 34.85° = 610 \text{ N}$.

(b) (i) The tension in the rope is radial and the displacement is tangential so there is no component of T in the direction of the displacement during the motion and the tension in the rope does no work.
(ii) $W_{\text{tot}} = 0$ so

$$W_{\text{worker}} = -W_{\text{grav}} = U_{\text{grav,2}} - U_{\text{grav,1}} = mg(y_2 - y_1) = (90.0 \text{ kg})(9.80 \text{ m/s}^2)(0.6277 \text{ m}) = 550 \text{ J}.$$

EVALUATE: The force applied by the worker varies during the motion of the bag and it would be difficult to calculate W_{worker} directly.

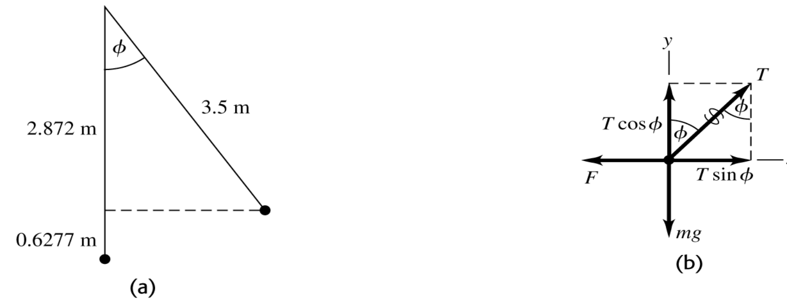


Figure 7.3

7.4. IDENTIFY: The energy from the food goes into the increased gravitational potential energy of the hiker. We must convert food calories to joules.

SET UP: The change in gravitational potential energy is $\Delta U_{\text{grav}} = mg\,(y_f - y_i)$, while the increase in kinetic energy is negligible. Set the food energy, expressed in joules, equal to the mechanical energy developed.

EXECUTE: **(a)** The food energy equals $mg(y_2 - y_1)$, so

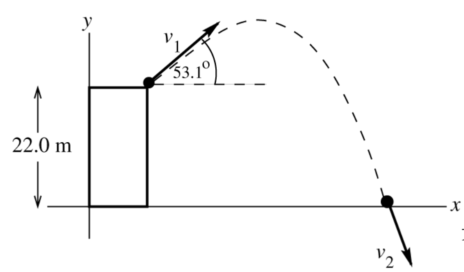
$$y_2 - y_1 = \frac{(140\text{ food calories})(4186\text{ J/1 food calorie})}{(65\text{ kg})(9.80\text{ m/s}^2)} = 920\text{ m}.$$

(b) The mechanical energy would be 20% of the results of part (a), so $\Delta y = (0.20)(920\text{ m}) = 180\text{ m}$.

EVALUATE: Since only 20% of the food calories go into mechanical energy, the hiker needs much less of a climb to turn off the calories in the bar.

7.5. IDENTIFY and SET UP: Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 are shown in Figure 7.5.

(a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Solve for K_2 and then use $K_2 = \frac{1}{2}mv_2^2$ to obtain v_2.



$W_{\text{other}} = 0$ (The only force on the ball while it is in the air is gravity.)

$K_1 = \frac{1}{2}mv_1^2$; $K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1$, $y_1 = 22.0\text{ m}$

$U_2 = mgy_2 = 0$, since $y_2 = 0$ for our choice of coordinates.

Figure 7.5

EXECUTE: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(12.0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(22.0\text{ m})} = 24.0\text{ m/s}$$

EVALUATE: The projection angle of 53.1° doesn't enter into the calculation. The kinetic energy depends only on the magnitude of the velocity; it is independent of the direction of the velocity.

(b) Nothing changes in the calculation. The expression derived in part (a) for v_2 is independent of the angle, so $v_2 = 24.0\text{ m/s}$, the same as in part (a).

(c) The ball travels a shorter distance in part (b), so in that case air resistance will have less effect.

7.6. IDENTIFY: The normal force does no work, so only gravity does work and $K_1 + U_1 = K_2 + U_2$ applies.

SET UP: $K_1 = 0$. The crate's initial point is at a vertical height of $d\sin\alpha$ above the bottom of the ramp.

EXECUTE: **(a)** $y_2 = 0$, $y_1 = d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $U_{\text{grav},1} = K_2$. $mgd\sin\alpha = \frac{1}{2}mv_2^2$ and $v_2 = \sqrt{2gd\sin\alpha}$.

(b) $y_1 = 0$, $y_2 = -d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $0 = K_2 + U_{\text{grav},2}$. $0 = \frac{1}{2}mv_2^2 + (-mgd\sin\alpha)$ and $v_2 = \sqrt{2gd\sin\alpha}$, the same as in part (a).

(c) The normal force is perpendicular to the displacement and does no work.

EVALUATE: When we use $U_{\text{grav}} = mgy$ we can take any point as $y = 0$ but we must take $+y$ to be upward.

7.7. IDENTIFY: The take-off kinetic energy of the flea goes into gravitational potential energy.

SET UP: Use $K_1 + U_1 = K_2 + U_2$. Let $y_1 = 0$ and $y_2 = h$ and note that $U_1 = 0$ while $K_2 = 0$ at the maximum height. Consequently, conservation of energy becomes $mgh = \frac{1}{2}mv_1^2$.

EXECUTE: **(a)** $v_1 = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(0.20\text{ m})} = 2.0\text{ m/s}$.

(b) $K_1 = mgh = (0.50\times10^{-6}\text{ kg})(9.80\text{ m/s}^2)(0.20\text{ m}) = 9.8\times10^{-7}\text{ J}$. The kinetic energy per kilogram is

$$\frac{K_1}{m} = \frac{9.8\times10^{-7}\text{ J}}{0.50\times10^{-6}\text{ kg}} = 2.0\text{ J/kg}.$$

(c) The human can jump to a height of $h_{\text{h}} = h_{\text{f}}\left(\frac{l_{\text{h}}}{l_{\text{f}}}\right) = (0.20\text{ m})\left(\frac{2.0\text{ m}}{2.0\times10^{-3}\text{ m}}\right) = 200\text{ m}$. To attain this height, he would require a takeoff speed of: $v_1 = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(200\text{ m})} = 63\text{ m/s}$.

(d) The human's kinetic energy per kilogram is $\frac{K_1}{m} = gh = (9.80\text{ m/s}^2)(0.60\text{ m}) = 5.9\text{ J/kg}$.

(e) EVALUATE: The flea stores the energy in its tensed legs.

7.8. IDENTIFY: This problem involves kinetic energy and gravitational potential energy.

SET UP: Estimates: maximum speed is 2.5 m/s, mass is 70 kg. $U_g = mgh$, $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** $K = \frac{1}{2}mv^2 = \frac{1}{2}(70\text{ kg})(2.5\text{ m/s})^2 = 220\text{ J}$.

(b) $U_g = mgh$, so $h = U_g/mg = (220\text{ J})/[(70\text{ kg})(9.8\text{ m/s}^2)] = 0.32\text{ m}$.

EVALUATE: These are reasonable values since we put in reasonable estimates.

7.9. IDENTIFY: $W_{\text{tot}} = K_B - K_A$. The forces on the rock are gravity, the normal force and friction.

SET UP: Let $y = 0$ at point B and let $+y$ be upward. $y_A = R = 0.50\text{ m}$. The work done by friction is negative; $W_f = -0.22\text{ J}$. $K_A = 0$. The free-body diagram for the rock at point B is given in Figure 7.9. The acceleration of the rock at this point is $a_{\text{rad}} = v^2/R$, upward.

EXECUTE: **(a)** (i) The normal force is perpendicular to the displacement and does zero work.

(ii) $W_{\text{grav}} = U_{\text{grav},A} - U_{\text{grav},B} = mgy_A = (0.20\text{ kg})(9.80\text{ m/s}^2)(0.50\text{ m}) = 0.98\text{ J}$.

(b) $W_{\text{tot}} = W_n + W_f + W_{\text{grav}} = 0 + (-0.22\text{ J}) + 0.98\text{ J} = 0.76\text{ J}$. $W_{\text{tot}} = K_B - K_A$ gives $\frac{1}{2}mv_B^2 = W_{\text{tot}}$.

$$v_B = \sqrt{\frac{2W_{\text{tot}}}{m}} = \sqrt{\frac{2(0.76\text{ J})}{0.20\text{ kg}}} = 2.8\text{ m/s}.$$

(c) Gravity is constant and equal to mg. n is not constant; it is zero at A and not zero at B. Therefore, $f_{\text{k}} = \mu_{\text{k}}n$ is also not constant.

(d) $\Sigma F_y = ma_y$ applied to Figure 7.9 gives $n - mg = ma_{\text{rad}}$.

$$n = m\left(g + \frac{v^2}{R}\right) = (0.20\text{ kg})\left(9.80\text{ m/s}^2 + \frac{[2.8\text{ m/s}]^2}{0.50\text{ m}}\right) = 5.1\text{ N}.$$

EVALUATE: In the absence of friction, the speed of the rock at point *B* would be $\sqrt{2gR} = 3.1$ m/s. As the rock slides through point *B*, the normal force is greater than the weight $mg = 2.0$ N of the rock.

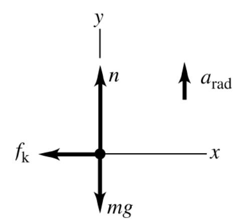


Figure 7.9

7.10. IDENTIFY: The child's energy is transformed from gravitational potential energy to kinetic energy as she swings downward.

SET UP: Let $y_2 = 0$. For part (a), $U_1 = mgy_1$. For part (b) use $K_2 + U_2 = K_1 + U_1$ with $U_2 = K_1 = 0$ and $K_2 = \frac{1}{2}mv_2{}^2$; the result is $\frac{1}{2}mv_2{}^2 = mgy_1$.

EXECUTE: **(a)** Figure 7.10 shows that the difference in potential energy at the top of the swing is proportional to the height difference, $y_1 = (2.20\text{ m})(1 - \cos 42°) = 0.56$ m. The difference in potential energy is thus $U_1 = mgy_1 = (25\text{ kg})(9.80\text{ m/s}^2)(0.56\text{ m}) = 140$ J.

(b) $v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(0.56\text{ m})} = 3.3$ m/s.

EVALUATE: (c) The tension is radial while the displacement is tangent to the circular path; thus there is no component of the tension along the direction of the displacement and the tension in the ropes does no work on the child.

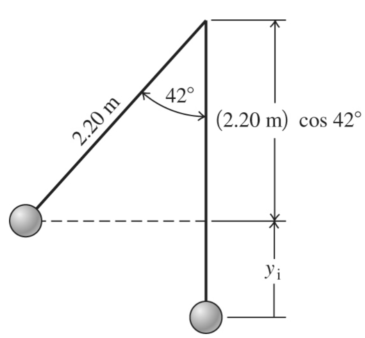


Figure 7.10

7.11. IDENTIFY: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the car.

SET UP: Take $y = 0$ at point *A*. Let point 1 be *A* and point 2 be *B*.

EXECUTE: $U_1 = 0$, $U_2 = mg(2R) = 28{,}224$ J, $W_{\text{other}} = W_f$

$K_1 = \frac{1}{2}mv_1^2 = 37{,}500$ J, $K_2 = \frac{1}{2}mv_2^2 = 3840$ J

The work-energy relation then gives $W_f = K_2 + U_2 - K_1 = -5400$ J.

EVALUATE: Friction does negative work. The final mechanical energy $(K_2 + U_2 = 32{,}064\ \text{J})$ is less than the initial mechanical energy $(K_1 + U_1 = 37{,}500\ \text{J})$ because of the energy removed by friction work.

7.12. IDENTIFY: Only gravity does work, so apply $K_1 + U_1 = K_2 + U_2$.

SET UP: $v_1 = 0$, so $\frac{1}{2}mv_2^2 = mg(y_1 - y_2)$.

EXECUTE: Tarzan is lower than his original height by a distance $y_1 - y_2 = l(\cos 30° - \cos 45°)$ so his speed is $v = \sqrt{2gl(\cos 30° - \cos 45°)} = 7.9$ m/s, a bit quick for conversation.

EVALUATE: The result is independent of Tarzan's mass.

7.13. IDENTIFY: This problem involves kinetic energy, gravitational potential energy, and energy conservation.

SET UP: $U_g = mgh$, $K = \dfrac{1}{2}mv^2$, $U_1 + K_1 = U_2 + K_2$.

EXECUTE: (a) $\Delta U_6 = m_6 g \Delta y$ = (6.00 kg)(9.80 /s^2)(0.200 m) = 11.8 J

$\Delta U_8 = m_8 g \Delta y$ = (8.00 kg)(9.80 m/s^2)(–0.200 m) = –15.7 J.

(b) $W_6 = T\Delta y$ = (0.200 m)T and $W_8 = T\Delta y$ = (–0.200 m)T.

(c) From part (b), we have W_T = (0.200 m)T + (–0.200 m)T = 0.

$\Delta U_g = \Delta U_6 + \Delta U_8$ = 11.8 J – 15.7 J = –3.9 J, and K_1 = 0.

$U_1 + K_1 = U_2 + K_2$ gives $K_2 = U_1 - U_2 = -(U_2 - U_1) = -\Delta U$ = –(–3.9 J) = 3.9 J.

$$K_2 = \frac{1}{2}mv_2^2 = -\Delta U \quad \rightarrow \quad v_2 = \sqrt{\frac{-2\Delta U}{m}} = \sqrt{\frac{-2(-3.9\ \text{J})}{14\ \text{kg}}} = 0.75\ \text{m/s}.$$

EVALUATE: To check, we could find v_2 using Newton's second law. Treating the two masses as a single system gives $m_8 g - m_6 g = (m_8 + m_6)a$. Kinematics gives $a = \dfrac{v_2^2}{2\Delta y}$. Combining these equations and putting in the numbers gives v_2 = 0.75 m/s.

7.14. IDENTIFY: Use the information given in the problem with $F = kx$ to find k. Then $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: x is the amount the spring is stretched. When the weight is hung from the spring, $F = mg$.

EXECUTE: $k = \dfrac{F}{x} = \dfrac{mg}{x} = \dfrac{(3.15\ \text{kg})(9.80\ \text{m/s}^2)}{0.1340\ \text{m} - 0.1200\ \text{m}} = 2205\ \text{N/m}.$

$x = \pm\sqrt{\dfrac{2U_{\text{el}}}{k}} = \pm\sqrt{\dfrac{2(10.0\ \text{J})}{2205\ \text{N/m}}} = \pm 0.0952\ \text{m} = \pm 9.52\ \text{cm}.$ The spring could be either stretched 9.52 cm or compressed 9.52 cm. If it were stretched, the total length of the spring would be $12.00\ \text{cm} + 9.52\ \text{cm} = 21.52\ \text{cm}.$ If it were compressed, the total length of the spring would be $12.00\ \text{cm} - 9.52\ \text{cm} = 2.48\ \text{cm}.$

EVALUATE: To stretch or compress the spring 9.52 cm requires a force $F = kx = 210$ N.

7.15. IDENTIFY: Apply $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: $kx = F$, so $U_{\text{el}} = \frac{1}{2}Fx$, where F is the magnitude of force required to stretch or compress the spring a distance x.

EXECUTE: **(a)** $(1/2)(520\ \text{N})(0.200\ \text{m}) = 52.0\ \text{J}.$

(b) The potential energy is proportional to the square of the compression or extension;

$(52.0\ \text{J})\,(0.050\ \text{m}/0.200\ \text{m})^2 = 3.25\ \text{J}.$

EVALUATE: We could have calculated $k = \dfrac{F}{x} = \dfrac{520\text{ N}}{0.200\text{ m}} = 2600\text{ N/m}$ and then used $U_{el} = \frac{1}{2}kx^2$ directly.

7.16. IDENTIFY: We treat the tendon like a spring and apply Hooke's law to it. Knowing the force stretching the tendon and how much it stretched, we can find its force constant.

SET UP: Use $F_{\text{on tendon}} = kx$. In part (a), $F_{\text{on tendon}}$ equals mg, the weight of the object suspended from it. In part (b), also apply $U_{el} = \frac{1}{2}kx^2$ to calculate the stored energy.

EXECUTE: **(a)** $k = \dfrac{F_{\text{on tendon}}}{x} = \dfrac{(0.250\text{ kg})(9.80\text{ m/s}^2)}{0.0123\text{ m}} = 199\text{ N/m}.$

(b) $x = \dfrac{F_{\text{on tendon}}}{k} = \dfrac{138\text{ N}}{199\text{ N/m}} = 0.693\text{m} = 69.3\text{ cm};\ U_{el} = \frac{1}{2}(199\text{ N/m})(0.693\text{ m})^2 = 47.8\text{ J}.$

EVALUATE: The 250 g object has a weight of 2.45 N. The 138 N force is much larger than this and stretches the tendon a much greater distance.

7.17. IDENTIFY: Apply $U_{el} = \frac{1}{2}kx^2$.

SET UP: $U_0 = \frac{1}{2}kx_0^2$. x is the distance the spring is stretched or compressed.

EXECUTE: **(a)** (i) $x = 2x_0$ gives $U_{el} = \frac{1}{2}k(2x_0)^2 = 4(\frac{1}{2}kx_0^2) = 4U_0$. (ii) $x = x_0/2$ gives $U_{el} = \frac{1}{2}k(x_0/2)^2 = \frac{1}{4}(\frac{1}{2}kx_0^2) = U_0/4$.

(b) (i) $U = 2U_0$ gives $\dfrac{1}{2}kx^2 = 2(\dfrac{1}{2}kx_0^2)$ and $x = x_0\sqrt{2}$. (ii) $U = U_0/2$ gives $\frac{1}{2}kx^2 = \frac{1}{2}(\frac{1}{2}kx_0^2)$ and $x = x_0/\sqrt{2}$.

EVALUATE: U is proportional to x^2 and x is proportional to $\sqrt{U}$.

7.18. IDENTIFY: This problem involves energy conservation, elastic potential energy, and kinetic energy.

SET UP: $K = \dfrac{1}{2}mv^2$ and $U_{\text{spring}} = \dfrac{1}{2}kx^2$. The elastic potential energy in the spring is transferred to the block as kinetic energy.

EXECUTE: $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kx^2 \quad \rightarrow \quad v = x\sqrt{k/m} = d\sqrt{k/m}.$

EVALUATE: Our result says that for a given mass and compression distance, the larger that k is the greater that v will be. A large k means a stiff spring, so our result is reasonable.

7.19. IDENTIFY and SET UP: Use energy methods. There are changes in both elastic and gravitational potential energy; elastic; $U = \frac{1}{2}kx^2$, gravitational: $kx - mg = ma$,

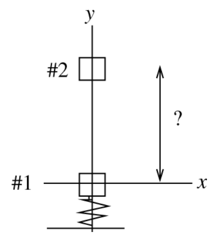
EXECUTE: **(a)** $U_{el} = \frac{1}{2}kx^2$ so $x = \sqrt{\dfrac{2U_{el}}{k}} = \sqrt{\dfrac{2(1.20\text{ J})}{800\text{ N/m}}} = 0.0548\text{ m} = 5.48\text{ cm}.$

(b) The work done by gravity is equal to the gain in elastic potential energy: $W_{grav} = U_{el}$.
$mgx = ½ kx^2$, so $x = 2mg/k = 2(1.60\text{ kg})(9.80\text{ m/s}^2)/(800\text{ N/m}) = 0.0392\text{ m} = 3.92\text{ cm}$.

EVALUATE: When the spring is compressed 3.92 cm, it exerts an upward force of 31.4 N on the book, which is greater than the weight of the book (15.6 N). The book will be accelerated upward from this position.

7.20. IDENTIFY: Use energy methods. There are changes in both elastic and gravitational potential energy.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 in the motion are sketched in Figure 7.20.



The spring force and gravity are the only forces doing work on the cheese, so $W_{\text{other}} = 0$ and $U = U_{\text{grav}} + U_{\text{el}}$.

Figure 7.20

EXECUTE: Cheese released from rest implies $K_1 = 0$.
At the maximum height $v_2 = 0$ so $K_2 = 0$. $U_1 = U_{1,\text{el}} + U_{1,\text{grav}}$
$y_1 = 0$ implies $U_{1,\text{grav}} = 0$
$U_{1,\text{el}} = \frac{1}{2}kx_1^2 = \frac{1}{2}(1800\text{ N/m})(0.15\text{ m})^2 = 20.25\text{ J}$
(Here x_1 refers to the amount the spring is stretched or compressed when the cheese is at position 1; it is *not* the *x*-coordinate of the cheese in the coordinate system shown in the sketch.)
$U_2 = U_{2,\text{el}} + U_{2,\text{grav}}$ $U_{2,\text{grav}} = mgy_2$, where y_2 is the height we are solving for. $U_{2,\text{el}} = 0$ since now the spring is no longer compressed. Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $U_{1,\text{el}} = U_{2,\text{grav}}$

$$y_2 = \frac{20.25\text{ J}}{mg} = \frac{20.25\text{ J}}{(1.20\text{ kg})(9.80\text{ m/s}^2)} = 1.72\text{ m}$$

EVALUATE: The description in terms of energy is very simple; the elastic potential energy originally stored in the spring is converted into gravitational potential energy of the system.

7.21. IDENTIFY: The energy of the book-spring system is conserved. There are changes in both elastic and gravitational potential energy.
SET UP: $U_{\text{el}} = \frac{1}{2}kx^2$, $U_{\text{grav}} = mgy$, $W_{\text{other}} = 0$.

EXECUTE: **(a)** $U = \frac{1}{2}kx^2$ so $x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(3.20\text{ J})}{1600\text{ N/m}}} = 0.0632\text{ m} = 6.32\text{ cm}$

(b) Points 1 and 2 in the motion are sketched in Figure 7.21. We have $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $W_{\text{other}} = 0$ (only work is that done by gravity and spring force), $K_1 = 0$, $K_2 = 0$, and $y = 0$ at final position of book. Using $U_1 = mg(h+d)$ and $U_2 = \frac{1}{2}kd^2$ we obtain $0 + mg(h+d) + 0 = \frac{1}{2}kd^2$. The original gravitational potential energy of the system is converted into potential energy of the compressed spring. Finally, we use the quadratic formula to solve for *d*: $\frac{1}{2}kd^2 - mgd - mgh = 0$, which gives

$d = \frac{1}{k}\left(mg \pm \sqrt{(mg)^2 + 4\left(\frac{1}{2}k\right)(mgh)}\right)$. In our analysis we have assumed that *d* is positive, so we get

$$d = \frac{(1.20\text{ kg})(9.80\text{ m/s}^2) + \sqrt{\left[(1.20\text{ kg})(9.80\text{ m/s}^2)\right]^2 + 2(1600\text{ N/m})(1.20\text{ kg})(9.80\text{ m/s}^2)(0.80\text{ m})}}{1600\text{ N/m}},$$

which gives *d* = 0.12 m = 12 cm.
EVALUATE: It was important to recognize that the total displacement was *h* + *d*; gravity continues to do work as the book moves against the spring. Also note that with the spring compressed 0.12 m it exerts an upward force (192 N) greater than the weight of the book (11.8 N). The book will be accelerated upward from this position.

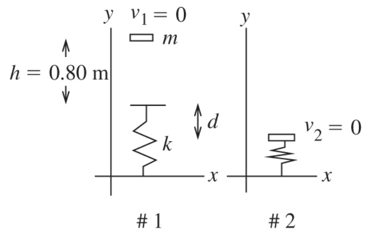


Figure 7.21

7.22. **(a) IDENTIFY** and **SET UP:** Use energy methods. Both elastic and gravitational potential energy changes. Work is done by friction.

Choose point 1 and let that be the origin, so $y_1 = 0$. Let point 2 be 1.00 m below point 1, so $y_2 = -1.00$ m.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(2000 \text{ kg})(4.0 \text{ m/s})^2 = 16{,}000 \text{ J},\ U_1 = 0$

$W_{\text{other}} = -f|y_2| = -(17{,}000 \text{ N})(1.00 \text{ m}) = -17{,}000 \text{ J}$

$K_2 = \frac{1}{2}mv_2^2$

$U_2 = U_{2,\text{grav}} + U_{2,\text{el}} = mgy_2 + \frac{1}{2}ky_2^2$

$U_2 = (2000 \text{ kg})(9.80 \text{ m/s}^2)(-1.00 \text{ m}) + \frac{1}{2}(1.06\times10^4 \text{ N/m})(1.00 \text{ m})^2$

$U_2 = -19{,}600 \text{ J} + 5300 \text{ J} = -14{,}300 \text{ J}$

Thus $16{,}000 \text{ J} - 17{,}000 \text{ J} = \frac{1}{2}mv_2^2 - 14{,}300 \text{ J}$

$\frac{1}{2}mv_2^2 = 13{,}300 \text{ J}$

$$v_2 = \sqrt{\frac{2(13{,}300 \text{ J})}{2000 \text{ kg}}} = 3.65 \text{ m/s}$$

EVALUATE: The elevator stops after descending 3.00 m. After descending 1.00 m it is still moving but has slowed down.

(b) IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the elevator. We know the forces and can solve for $\vec{a}$.

SET UP: The free-body diagram for the elevator is given in Figure 7.22.

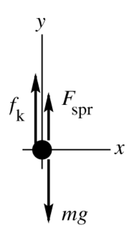


Figure 7.22

EXECUTE: $F_{\text{spr}} = kd$, where d is the distance the spring is compressed

$\Sigma F_y = ma_y$

$f_k + F_{\text{spr}} - mg = ma$

$f_k + kd - mg = ma$

$$a = \frac{f_k + kd - mg}{m} = \frac{17{,}000 \text{ N} + (1.06\times10^4 \text{ N/m})(1.00 \text{ m}) - (2000 \text{ kg})(9.80 \text{ m/s}^2)}{2000 \text{ kg}} = 4.00 \text{ m/s}^2$$

We calculate that a is positive, so the acceleration is upward.

EVALUATE: The velocity is downward and the acceleration is upward, so the elevator is slowing down at this point.

7.23. **IDENTIFY:** Only the spring does work and $K_1 + U_1 = K_2 + U_2$ applies. $a = \frac{F}{m} = \frac{-kx}{m}$, where F is the force the spring exerts on the mass.

SET UP: Let point 1 be the initial position of the mass against the compressed spring, so $K_1 = 0$ and $U_1 = 11.5$ J. Let point 2 be where the mass leaves the spring, so $U_{\text{el},2} = 0$.

EXECUTE: **(a)** $K_1 + U_{\text{el},1} = K_2 + U_{\text{el},2}$ gives $U_{\text{el},1} = K_2$. $\frac{1}{2}mv_2^2 = U_{\text{el},1}$ and

$$v_2 = \sqrt{\frac{2U_{\text{el},1}}{m}} = \sqrt{\frac{2(11.5 \text{ J})}{2.50 \text{ kg}}} = 3.03 \text{ m/s}.$$

K is largest when U_{el} is least and this is when the mass leaves the spring. The mass achieves its maximum speed of 3.03 m/s as it leaves the spring and then slides along the surface with constant speed.

(b) The acceleration is greatest when the force on the mass is the greatest, and this is when the spring has its maximum compression. $U_{\text{el}} = \frac{1}{2}kx^2$ so $x = -\sqrt{\frac{2U_{\text{el}}}{k}} = -\sqrt{\frac{2(11.5 \text{ J})}{2500 \text{ N/m}}} = -0.0959$ m. The minus sign indicates compression. $F = -kx = ma_x$ and $a_x = -\frac{kx}{m} = -\frac{(2500 \text{ N/m})(-0.0959 \text{ m})}{2.50 \text{ kg}} = 95.9 \text{ m/s}^2$.

EVALUATE: If the end of the spring is displaced to the left when the spring is compressed, then a_x in part (b) is to the right, and vice versa.

7.24. **IDENTIFY:** The spring force is conservative but the force of friction is nonconservative. Energy is conserved during the process. Initially all the energy is stored in the spring, but part of this goes to kinetic energy, part remains as elastic potential energy, and the rest does work against friction.

SET UP: Energy conservation: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, the elastic energy in the spring is $U = \frac{1}{2}kx^2$, and the work done by friction is $W_f = -f_{\text{k}}s = -\mu_{\text{k}}mgs$.

EXECUTE: The initial and final elastic potential energies are

$U_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(840 \text{ N/m})(0.0300 \text{ m})^2 = 0.378$ J and $U_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(840 \text{ N/m})(0.0100 \text{ m})^2 = 0.0420$ J.

The initial and final kinetic energies are $K_1 = 0$ and $K_2 = \frac{1}{2}mv_2^2$. The work done by friction is

$W_{\text{other}} = W_{f_{\text{k}}} = -f_{\text{k}}s = -\mu_{\text{k}}mgs = -(0.40)(2.50 \text{ kg})(9.8 \text{ m/s}^2)(0.0200 \text{ m}) = -0.196$ J. Energy conservation gives $K_2 = \frac{1}{2}mv_2^2 = K_1 + U_1 + W_{\text{other}} - U_2 = 0.378 \text{ J} + (-0.196 \text{ J}) - 0.0420 \text{ J} = 0.140$ J. Solving for v_2 gives $v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.140 \text{ J})}{2.50 \text{ kg}}} = 0.335$ m/s.

EVALUATE: Mechanical energy is not conserved due to friction.

7.25. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ and $F = ma$.

SET UP: $W_{\text{other}} = 0$. There is no change in U_{grav}. $K_1 = 0$, $U_2 = 0$.

EXECUTE: $\frac{1}{2}kx^2 = \frac{1}{2}mv_x^2$. The relations for m, v_x, k and x are $kx^2 = mv_x^2$ and $kx = 5mg$.

Dividing the first equation by the second gives $x = \frac{v_x^2}{5g}$, and substituting this into the second gives

$$k = 25\frac{mg^2}{v_x^2}.$$

(a) $k = 25\frac{(1160 \text{ kg})(9.80 \text{ m/s}^2)^2}{(2.50 \text{ m/s})^2} = 4.46 \times 10^5$ N/m

(b) $x = \frac{(2.50 \text{ m/s})^2}{5(9.80 \text{ m/s}^2)} = 0.128$ m

EVALUATE: Our results for k and x do give the required values for a_x and v_x:

$$a_x = \frac{kx}{m} = \frac{(4.46\times10^5\ \text{N/m})(0.128\ \text{m})}{1160\ \text{kg}} = 49.2\ \text{m/s}^2 = 5.0g \ \text{ and } \ v_x = x\sqrt{\frac{k}{m}} = 2.5\ \text{m/s}.$$

7.26. IDENTIFY: This problem involves the work to stretch a spring and the energy stored in the spring.

SET UP: $U_{\text{spring}} = \frac{1}{2}kx^2$ and $F = kx$ (Hooke's law).

EXECUTE: **(a)** At a 2.00-cm stretch: $F = kx$ becomes $5.00\ \text{N} = k(2.00\ \text{cm})$. At the additional stretch, the spring is now stretched 6.00 cm, so $F = k(6.00\ \text{cm})$. This gives $\frac{F}{5.00\ \text{N}} = \frac{k(6.00\ \text{cm})}{k(2.00\ \text{cm})}$, which gives $F = 15.0$ N.

(b) At 2.00-cm stretch, $U_2 = \frac{1}{2}kx^2 = \frac{1}{2}k(2.00\ \text{cm})^2$ and $U_6 = \frac{1}{2}k(6.00\ \text{cm})^2$. Therefore

$$\frac{U_6}{U_2} = \frac{\frac{1}{2}k(6.00\ \text{cm})^2}{\frac{1}{2}k(2.00\ \text{cm})^2} = 9.00.$$

EVALUATE: The force increases by a factor of 3 but the stored energy increases by a factor of 9 because the energy is proportional to the *square* of x while the force is only proportional to x. We could have solved this problem by first finding k and then using the force and energy formulas. However by taking ratios we never needed to know k and we did not have to convert any units.

7.27. IDENTIFY: Since the force is constant, use $W = Fs\cos\phi$.

SET UP: For both displacements, the direction of the friction force is opposite to the displacement and $\phi = 180°$.

EXECUTE: **(a)** When the book moves to the left, the friction force is to the right, and the work is $-(1.8\ \text{N})(3.0\ \text{m}) = -5.4\ \text{J}$.

(b) The friction force is now to the left, and the work is again -5.4 J.

(c) The total work is sum of the work in both directions, which is –10.8 J.

(d) The net work done by friction for the round trip is not zero, so friction is not a conservative force.

EVALUATE: The direction of the friction force depends on the motion of the object. For the gravity force, which is conservative, the force does not depend on the motion of the object.

7.28. IDENTIFY and SET UP: The force is not constant so we must integrate to calculate the work. $W = \int_1^2 \vec{F}\cdot d\vec{l}$, $\vec{F} = -\alpha x^2\hat{i}$.

EXECUTE: **(a)** $d\vec{l} = dy\hat{j}$ (x is constant; the displacement is in the $+y$-direction)

$\vec{F}\cdot d\vec{l} = 0$ (since $\hat{i}\cdot\hat{j} = 0$) and thus $W = 0$.

(b) $d\vec{l} = dx\hat{i}$

$\vec{F}\cdot d\vec{l} = (-\alpha x^2\hat{i})\cdot(dx\hat{i}) = -\alpha x^2\,dx$

$$W = \int_{x_1}^{x_2}(-\alpha x^2)\,dx = -\tfrac{1}{3}\alpha x^3\Big|_{x_1}^{x_2} = -\tfrac{1}{3}\alpha\,(x_2^3 - x_1^3) = -\frac{12\ \text{N/m}^2}{3}\left[(0.300\ \text{m})^3 - (0.10\ \text{m})^3\right] = -0.10\ \text{J}$$

(c) $d\vec{l} = dx\hat{i}$ as in part (b), but now $x_1 = 0.30$ m and $x_2 = 0.10$ m, so $W = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = +0.10$ J.

(d) EVALUATE: The total work for the displacement along the x-axis from 0.10 m to 0.30 m and then back to 0.10 m is the sum of the results of parts (b) and (c), which is zero. The total work is zero when the starting and ending points are the same, so the force is conservative.

EXECUTE: $W_{x_1 \to x_2} = -\frac{1}{3}\alpha(x_2^3 - x_1^3) = \frac{1}{3}\alpha x_1^3 - \frac{1}{3}\alpha x_2^3$

The definition of the potential energy function is $W_{x_1 \to x_2} = U_1 - U_2$. Comparison of the two expressions for *W* gives $U = \frac{1}{3}\alpha x^3$. This does correspond to $U = 0$ when $x = 0$.

EVALUATE: In part (a) the work done is zero because the force and displacement are perpendicular. In part (b) the force is directed opposite to the displacement and the work done is negative. In part (c) the force and displacement are in the same direction and the work done is positive.

7.29. **IDENTIFY:** Some of the mechanical energy of the skier is converted to internal energy by the nonconservative force of friction on the rough patch. Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

SET UP: For part (a) use $E_{\text{mech, 2}} = E_{\text{mech, 1}} - f_k s$ where $f_k = \mu_k mg$. Let $y_2 = 0$ at the bottom of the hill; then $y_1 = 2.50\text{ m}$ along the rough patch. The energy equation is $\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + mgy_1 - \mu_k mgs$. Solving for her final speed gives $v_2 = \sqrt{v_1^2 + 2gy_1 - 2\mu_k gs}$. For part (b), the internal energy is calculated as the negative of the work done by friction: $-W_f = +f_k s = +\mu_k mgs$.

EXECUTE: **(a)** $v_2 = \sqrt{(6.50\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(2.50\text{ m}) - 2(0.300)(9.80\text{ m/s}^2)(4.20\text{ m})} = 8.16\text{ m/s}$.

(b) Internal energy $= \mu_k mgs = (0.300)(62.0\text{ kg})(9.80\text{ m/s}^2)(4.20\text{ m}) = 766\text{ J}$.

EVALUATE: Without friction the skier would be moving faster at the bottom of the hill than at the top, but in this case she is moving *slower* because friction converted some of her initial kinetic energy into internal energy.

7.30. **IDENTIFY:** Some of the initial gravitational potential energy is converted to kinetic energy, but some of it is lost due to work by the nonconservative friction force.

SET UP: The energy of the box at the edge of the roof is given by: $E_{\text{mech, f}} = E_{\text{mech, i}} - f_k s$. Setting $y_f = 0$ at this point, $y_i = (4.25\text{ m})\sin 36° = 2.50\text{ m}$. Furthermore, by substituting $K_i = 0$ and $K_f = \frac{1}{2}mv_f^2$ into the conservation equation, $\frac{1}{2}mv_f^2 = mgy_i - f_k s$ or $v_f = \sqrt{2gy_i - 2f_k sg/w} = \sqrt{2g(y_i - f_k s/w)}$.

EXECUTE: $v_f = \sqrt{2(9.80\text{ m/s}^2)[(2.50\text{ m}) - (22.0\text{ N})(4.25\text{ m})/(85.0\text{ N})]} = 5.24\text{ m/s}$.

EVALUATE: Friction does negative work and removes mechanical energy from the system. In the absence of friction the final speed of the toolbox would be 7.00 m/s.

7.31. **IDENTIFY:** We know the potential energy function and want to find the force causing this energy.

SET UP: $F_x = -\dfrac{dU}{dx}$. The sign of F_x indicates its direction.

EXECUTE: $F_x = -\dfrac{dU}{dx} = -4\alpha x^3 = -4(0.630\text{ J/m}^4)x^3$.

$F_x(-0.800\text{ m}) = -4(0.630\text{ J/m}^4)(-0.80\text{ m})^3 = 1.29\text{ N}$. The force is in the $+x$-direction.

EVALUATE: $F_x > 0$ when $x < 0$ and $F_x < 0$ when $x > 0$, so the force is always directed toward the origin.

7.32. **IDENTIFY** and **SET UP:** Use $F_x = -\dfrac{dU}{dx}$ to calculate the force from $U(x)$. Use coordinates where the origin is at one atom. The other atom then has coordinate *x*.

EXECUTE:

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(-\frac{C_6}{x^6}\right) = +C_6\frac{d}{dx}\left(\frac{1}{x^6}\right) = -\frac{6C_6}{x^7}$$

The minus sign means that F_x is directed in the $-x$-direction, toward the origin. The force has magnitude $6C_6/x^7$ and is attractive.

EVALUATE: U depends only on x so $\vec{F}$ is along the x-axis; it has no y- or z-components.

7.33. **IDENTIFY:** From the potential energy function of the block, we can find the force on it, and from the force we can use Newton's second law to find its acceleration.

SET UP: The force components are $F_x = -\dfrac{\partial U}{\partial x}$ and $F_y = -\dfrac{\partial U}{\partial y}$. The acceleration components are $a_x = F_x/m$ and $a_y = F_y/m$. The magnitude of the acceleration is $a = \sqrt{a_x^2 + a_y^2}$ and we can find its angle with the $+x$ axis using $\tan\theta = a_y/a_x$.

EXECUTE: $F_x = -\dfrac{\partial U}{\partial x} = -(11.6\text{ J/m}^2)x$ and $F_y = -\dfrac{\partial U}{\partial y} = (10.8\text{ J/m}^3)y^2$. At the point ($x = 0.300$ m, $y = 0.600$ m), $F_x = -(11.6\text{ J/m}^2)(0.300\text{ m}) = -3.48$ N and $F_y = (10.8\text{ J/m}^3)(0.600\text{ m})^2 = 3.89$ N. Therefore $a_x = \dfrac{F_x}{m} = -87.0\text{ m/s}^2$ and $a_y = \dfrac{F_y}{m} = 97.2\text{ m/s}^2$, giving $a = \sqrt{a_x^2 + a_y^2} = 130\text{ m/s}^2$ and $\tan\theta = \dfrac{97.2}{87.0}$, so $\theta = 48.2°$. The direction is $132°$ counterclockwise from the $+x$-axis.

EVALUATE: The force is not constant, so the acceleration will not be the same at other points.

7.34. **IDENTIFY:** Apply $\vec{F} = -\dfrac{\partial U}{\partial x}\hat{\boldsymbol{i}} - \dfrac{\partial U}{\partial y}\hat{\boldsymbol{j}}$.

SET UP: $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right) = -\dfrac{2}{x^3}$ and $\dfrac{d}{dy}\left(\dfrac{1}{y^2}\right) = -\dfrac{2}{y^3}$.

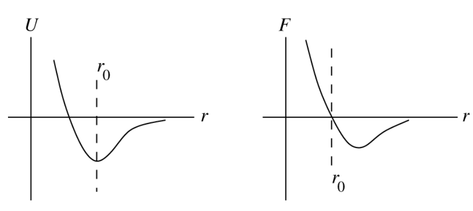
EXECUTE: $\vec{F} = -\dfrac{\partial U}{\partial x}\hat{\boldsymbol{i}} - \dfrac{\partial U}{\partial y}\hat{\boldsymbol{j}}$ since U has no z-dependence. $\dfrac{\partial U}{\partial x} = \dfrac{-2\alpha}{x^3}$ and $\dfrac{\partial U}{\partial y} = \dfrac{-2\alpha}{y^3}$, so

$$\vec{F} = -\alpha\left(\frac{-2}{x^3}\hat{\boldsymbol{i}} + \frac{-2}{y^3}\hat{\boldsymbol{j}}\right) = 2\alpha\left(\frac{\vec{\boldsymbol{i}}}{x^3} + \frac{\vec{\boldsymbol{j}}}{y^3}\right).$$

EVALUATE: F_x and x have the same sign and F_y and y have the same sign. When $x > 0$, F_x is in the $+x$-direction, and so forth.

7.35. **IDENTIFY** and **SET UP:** Use $F = -dU/dr$ to calculate the force from U. At equilibrium $F = 0$.

(a) EXECUTE: The graphs are sketched in Figure 7.35.



$$U = \frac{a}{r^{12}} - \frac{b}{r^6}$$

$$F = -\frac{dU}{dr} = +\frac{12a}{r^{13}} - \frac{6b}{r^7}$$

Figure 7.35

(b) At equilibrium $F = 0$, so $\dfrac{dU}{dr} = 0$

$$F = 0 \text{ implies } \frac{+12a}{r^{13}} - \frac{6b}{r^7} = 0$$

$6br^6 = 12a$; solution is the equilibrium distance $r_0 = (2a/b)^{1/6}$

U is a minimum at this r; the equilibrium is stable.

(c) At $r = (2a/b)^{1/6}$, $U = a/r^{12} - b/r^6 = a(b/2a)^2 - b(b/2a) = -b^2/4a$.

At $r \to \infty$, $U = 0$. The energy that must be added is $-\Delta U = b^2/4a$.

(d) $r_0 = (2a/b)^{1/6} = 1.13\times10^{-10}$ m gives that

$2a/b = 2.082\times10^{-60}\ \text{m}^6$ and $b/4a = 2.402\times10^{59}\ \text{m}^{-6}$

$b^2/4a = b(b/4a) = 1.54\times10^{-18}$ J

$b(2.402\times10^{59}\ \text{m}^{-6}) = 1.54\times10^{-18}$ J and $b = 6.41\times10^{-78}\ \text{J}\cdot\text{m}^6$.

Then $2a/b = 2.082\times10^{-60}\ \text{m}^6$ gives $a = (b/2)(2.082\times10^{-60}\ \text{m}^6) =$

$\frac{1}{2}(6.41\times10^{-78}\ \text{J}\cdot\text{m}^6)\ (2.082\times10^{-60}\ \text{m}^6)\ = 6.67\times10^{-138}\ \text{J}\cdot\text{m}^{12}$

EVALUATE: As the graphs in part (a) show, $F(r)$ is the slope of $U(r)$ at each r. $U(r)$ has a minimum where $F = 0$.

7.36. **IDENTIFY:** Apply $F_x = -\dfrac{dU}{dx}$.

SET UP: $\dfrac{dU}{dx}$ is the slope of the U versus x graph.

EXECUTE: **(a)** Considering only forces in the x-direction, $F_x = -\dfrac{dU}{dx}$ and so the force is zero when the slope of the U *vs* x graph is zero, at points b and d.

(b) Point b is at a potential minimum; to move it away from b would require an input of energy, so this point is stable.

(c) Moving away from point d involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so d is an unstable point.

EVALUATE: At point b, F_x is negative when the marble is displaced slightly to the right and F_x is positive when the marble is displaced slightly to the left, the force is a restoring force, and the equilibrium is stable. At point d, a small displacement in either direction produces a force directed away from d and the equilibrium is unstable.

7.37. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the bag and to the box. Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the system of the box and bucket after the bag is removed.

SET UP: Let $y = 0$ at the final height of the bucket, so $y_1 = 2.00$ m and $y_2 = 0$. $K_1 = 0$. The box and the bucket move with the same speed v, so $K_2 = \frac{1}{2}(m_{\text{box}} + m_{\text{bucket}})v^2$. $W_{\text{other}} = -f_k d$, with $d = 2.00$ m and $f_k = \mu_k m_{\text{box}} g$. Before the bag is removed, the maximum possible friction force the roof can exert on the box is $(0.700)(80.0\ \text{kg} + 50.0\ \text{kg})(9.80\ \text{m/s}^2) = 892$ N. This is larger than the weight of the bucket (637 N), so before the bag is removed the system is at rest.

EXECUTE: **(a)** The friction force on the bag of gravel is zero, since there is no other horizontal force on the bag for friction to oppose. The static friction force on the box equals the weight of the bucket, 637 N.

(b) Applying $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $m_{\text{bucket}} g y_1 - f_k d = \frac{1}{2} m_{\text{tot}} v^2$, with $m_{\text{tot}} = 145.0$ kg.

$$v = \sqrt{\frac{2}{m_{\text{tot}}}(m_{\text{bucket}} g y_1 - \mu_k m_{\text{box}} g d)}.$$

$$v = \sqrt{\frac{2}{145.0\ \text{kg}}\left[(65.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m}) - (0.400)(80.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})\right]} = 2.99\ \text{m/s}.$$

EVALUATE: If we apply $\Sigma\vec{F} = m\vec{a}$ to the box and to the bucket we can calculate their common acceleration a. Then a constant acceleration equation applied to either object gives $v = 2.99$ m/s, in agreement with our result obtained using energy methods.

7.38. IDENTIFY: This problem involves projectile motion and energy conservation.

SET UP: Estimate: horizontal range is about 75 ft, which is about 25 m. The range of a projectile is $R = \dfrac{v_0^2}{g}\sin 2\alpha_0$, energy conservation says $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, $K = \dfrac{1}{2}mv^2$, $U_g = mgy$.

EXECUTE: **(a)** Use range formula to find v_0. At 45° $R = \dfrac{v_0^2}{g}\sin 2\alpha_0$ gives $v_0 = \sqrt{Rg} = \sqrt{(25\text{ m})(9.80\text{ m/s}^2)}$ = 15.65 m/s. $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(0.145\text{ kg})(15.65\text{ m/s})^2 = 18$ J.

(b) At the highest point, $v = v_x = v_0\cos\alpha_0$ = (15.65 m/s) cos 45° = 11.1 m/s, so $\dfrac{1}{2}(0.145\text{ kg})(11.1\text{ m/s})^2 = 8.88$ J. At the maximum height, $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $0 = (v_0\sin\alpha_0)^2 - 2gh$. This gives $h = \dfrac{(v_0\sin\alpha_0)^2}{2g}$ = 6.248 m. The gravitational potential energy is $U_g = mgh$ = (0.145 kg)(9.80 m/s^2)(6.248 m) = 8.88 J. As we have just found, at the maximum height $U_g = K$ = 8.88 J. So with α_0 = 45°, half of the mechanical energy is kinetic energy and half is potential energy.

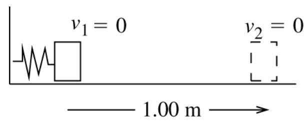
(c) At the highest point $K = \dfrac{1}{2}mv_x^2 = \dfrac{1}{2}m(v_0\cos\alpha_0)^2 = \dfrac{1}{2}mv_0^2\cos^2\alpha_0$. In part (b) we saw that the maximum height is $h = \dfrac{(v_0\sin\alpha_0)^2}{2g}$, so $U_g = mgh = mg\dfrac{(v_0\sin\alpha_0)^2}{2g} = \dfrac{1}{2}mv_0^2\sin^2\alpha_0$. The total energy is $E = K + U_g = \dfrac{1}{2}mv_0^2\cos^2\alpha_0 + \dfrac{1}{2}mv_0^2\sin^2\alpha_0 = \dfrac{1}{2}mv_0^2$. The fraction that is kinetic energy is $\dfrac{K}{E} = \dfrac{\frac{1}{2}mv_0^2\cos^2\alpha_0}{\frac{1}{2}mv_0^2} = \cos^2\alpha_0 = \cos^2 60° = \dfrac{1}{4}$. The fraction of the total energy that is potential energy is $\dfrac{U_g}{E} = \dfrac{\frac{1}{2}mv_0^2\sin^2\alpha_0}{\frac{1}{2}mv_0^2} = \sin^2\alpha_0 = \sin^2 60° = \dfrac{3}{4}$. So ¼ of the energy is kinetic energy and ¾ of the energy is potential energy.

EVALUATE: **(d)** If α_0 = 0: The ball is thrown horizontally. In this case, $K/E = \cos^2 0° = 1$ and $U_g/E = \sin^2 0° = 0$. All of the energy is kinetic and none is potential. This is reasonable because the ball never leaves the ground. If α_0 = 90°: The ball is thrown vertically upward. In this case, $K/E = \cos^2 90° = 0$ and $U_g/E = \sin^2 90° = 1$. All of the energy is potential and none is kinetic. This is reasonable because at its highest point the ball has stopped moving vertically and has no horizontal velocity, so its kinetic energy is zero.

7.39. IDENTIFY: Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. The target variable μ_k will be a factor in the work done by friction.

SET UP: Let point 1 be where the block is released and let point 2 be where the block stops, as shown in Figure 7.39.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$



Work is done on the block by the spring and by friction, so $W_{\text{other}} = W_f$ and $U = U_{\text{el}}$.

Figure 7.39

EXECUTE: $K_1 = K_2 = 0$

$U_1 = U_{1,\text{el}} = \frac{1}{2}kx_1^2 = \frac{1}{2}(100\ \text{N/m})(0.200\ \text{m})^2 = 2.00\ \text{J}$

$U_2 = U_{2,\text{el}} = 0,$ since after the block leaves the spring has given up all its stored energy

$W_{\text{other}} = W_f = (f_k \cos\phi)s = \mu_k mg(\cos\phi)s = -\mu_k mgs,$ since $\phi = 180°$ (The friction force is directed opposite to the displacement and does negative work.)

Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$U_{1,\text{el}} + W_f = 0$

$\mu_k mgs = U_{1,\text{el}}$

$$\mu_k = \frac{U_{1,\text{el}}}{mgs} = \frac{2.00\ \text{J}}{(0.50\ \text{kg})(9.80\ \text{m/s}^2)(1.00\ \text{m})} = 0.41.$$

EVALUATE: $U_{1,\text{el}} + W_f = 0$ says that the potential energy originally stored in the spring is taken out of the system by the negative work done by friction.

7.40. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2.$

SET UP: Only the spring force and gravity do work, so $W_{\text{other}} = 0.$ Let $y = 0$ at the horizontal surface.

EXECUTE: **(a)** Equating the potential energy stored in the spring to the block's kinetic energy,

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2,$ or $v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{400\ \text{N/m}}{2.00\ \text{kg}}}(0.220\ \text{m}) = 3.11\ \text{m/s}.$

(b) Using energy methods directly, the initial potential energy of the spring equals the final gravitational potential energy, $\frac{1}{2}kx^2 = mgL\sin\theta,$ or $L = \frac{\frac{1}{2}kx^2}{mg\sin\theta} = \frac{\frac{1}{2}(400\ \text{N/m})(0.220\ \text{m})^2}{(2.00\ \text{kg})(9.80\ \text{m/s}^2)\sin 37.0°} = 0.821\ \text{m}.$

EVALUATE: The total energy of the system is constant. Initially it is all elastic potential energy stored in the spring, then it is all kinetic energy and finally it is all gravitational potential energy.

7.41. **IDENTIFY:** The mechanical energy of the roller coaster is conserved since there is no friction with the track. We must also apply Newton's second law for the circular motion.

SET UP: For part (a), apply conservation of energy to the motion from point *A* to point *B*: $K_B + U_{\text{grav},B} = K_A + U_{\text{grav},A}$ with $K_A = 0.$ Defining $y_B = 0$ and $y_A = 13.0\ \text{m},$ conservation of energy becomes $\frac{1}{2}mv_B^2 = mgy_A$ or $v_B = \sqrt{2gy_A}.$ In part (b), the free-body diagram for the roller coaster car at point *B* is shown in Figure 7.41. $\Sigma F_y = ma_y$ gives $mg + n = ma_{\text{rad}},$ where $a_{\text{rad}} = v^2/r.$ Solving for the normal force gives $n = m\left(\frac{v^2}{r} - g\right).$

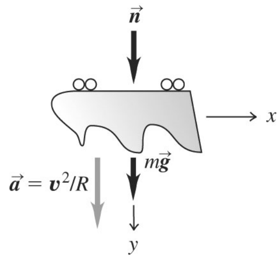


Figure 7.41

EXECUTE: **(a)** $v_B = \sqrt{2(9.80\ \text{m/s}^2)(13.0\ \text{m})} = 16.0\ \text{m/s}.$

(b) $n = (350\ \text{kg})\left[\dfrac{(16.0\ \text{m/s})^2}{6.0\ \text{m}} - 9.80\ \text{m/s}^2\right] = 1.15\times 10^4\ \text{N}.$

EVALUATE: The normal force n is the force that the tracks exert on the roller coaster car. The car exerts a force of equal magnitude and opposite direction on the tracks.

7.42. **IDENTIFY:** In this problem, we need to use Newton's second law and energy conservation.

SET UP: $\sum F = m\dfrac{v^2}{R}$ and $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, where W_{other} is the work done by friction. At the bottom of the bowl, the normal force on the rock is equal to twice its weight, so $n = 2mg$.

EXECUTE: First use $\sum F = m\dfrac{v^2}{R}$ to find the speed at B. $n - mg = m\dfrac{v_B^2}{R} \rightarrow 2mg - mg = m\dfrac{v_B^2}{R}$, which gives $v_B^2 = Rg$. Now use energy conservation. $U_A + K_A + W_\text{f} = U_B + K_B$ gives us $W_\text{f} = -(U_A - U_B) + K_B = -mgR + \dfrac{1}{2}mRg = -\dfrac{mgR}{2}.$

EVALUATE: If there were no friction, energy conservation gives $mgR = \dfrac{1}{2}mv_B^2$, so $v_B^2 = 2Rg$. Newton's second law gives $n - mg = m\dfrac{v_B^2}{R} = m\left(\dfrac{2Rg}{R}\right)$, so $n = 3mg$. With friction the normal force was $2mg$, which is reasonable because friction slowed down the rock.

7.43. **(a) IDENTIFY:** Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to find the kinetic energy of the wood as it enters the rough bottom.

SET UP: Let point 1 be where the piece of wood is released and point 2 be just before it enters the rough bottom. Let $y = 0$ be at point 2.

EXECUTE: $U_1 = K_2$ gives $K_2 = mgy_1 = 78.4\ \text{J}$.

IDENTIFY: Now apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion along the rough bottom.

SET UP: Let point 1 be where it enters the rough bottom and point 2 be where it stops.
$K_1 + U_1 + W_{\text{other}} = K_2 + U_2.$

EXECUTE: $W_{\text{other}} = W_f = -\mu_\text{k} mgs,\ \ K_2 = U_1 = U_2 = 0;\ \ K_1 = 78.4\ \text{J}$

$78.4\ \text{J} - \mu_\text{k} mgs = 0;$ solving for s gives $s = 20.0\ \text{m}$.

The wood stops after traveling 20.0 m along the rough bottom.

(b) Friction does $-78.4\ \text{J}$ of work.

EVALUATE: The piece of wood stops before it makes one trip across the rough bottom. The final mechanical energy is zero. The negative friction work takes away all the mechanical energy initially in the system.

7.44. IDENTIFY: In this problem, we need to use Newton's second law and energy conservation.

SET UP: $\sum F = m\dfrac{v^2}{R}$ and $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, where $W_{\text{other}} = 0$. In order that the block will not fall off the track at the top, it must be moving; otherwise it would simply drop down. First find the minimum speed at the top and then use that result to find the speed at the bottom.

EXECUTE: At the top n and mg are both downward, so $\sum F = m\dfrac{v^2}{R}$ gives $n + mg = m\dfrac{v^2}{R}$. The smallest that n can be is zero, so the minimum speed at the top is $v_{\text{min}}^2 = Rg$. Now use energy conservation to find the speed v_B at the bottom. $U_T + K_T = U_B + K_B a$ gives $\dfrac{1}{2}mv_B^2 = \dfrac{1}{2}mv_T^2 + 2mgR$, which gives $v_B = \sqrt{v_T^2 + 4Rg} = \sqrt{Rg + 4Rg} = \sqrt{5Rg}$.

EVALUATE: The larger R, the greater the speed at the bottom.

7.45. IDENTIFY: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the stone.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Let point 1 be point A and point 2 be point B. Take $y = 0$ at B.

EXECUTE: $mgy_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$, with $h = 20.0\text{ m}$ and $v_1 = 10.0\text{ m/s}$, so $v_2 = \sqrt{v_1^2 + 2gh} = 22.2\text{ m/s}$.

EVALUATE: The loss of gravitational potential energy equals the gain of kinetic energy.

(b) IDENTIFY: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the stone from point B to where it comes to rest against the spring.

SET UP: Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, with point 1 at B and point 2 where the spring has its maximum compression x.

EXECUTE: $U_1 = U_2 = K_2 = 0$; $K_1 = \frac{1}{2}mv_1^2$ with $v_1 = 22.2\text{ m/s}$. $W_{\text{other}} = W_f + W_{\text{el}} = -\mu_k mgs - \frac{1}{2}kx^2$, with $s = 100\text{ m} + x$. The work-energy relation gives $K_1 + W_{\text{other}} = 0$. $\frac{1}{2}mv_1^2 - \mu_k mgs - \frac{1}{2}kx^2 = 0$.

Putting in the numerical values gives $x^2 + 29.4x - 750 = 0$. The positive root to this equation is $x = 16.4\text{ m}$.

EVALUATE: Part of the initial mechanical (kinetic) energy is removed by friction work and the rest goes into the potential energy stored in the spring.

(c) IDENTIFY and **SET UP:** Consider the forces.

EXECUTE: When the spring is compressed $x = 16.4\text{ m}$ the force it exerts on the stone is $F_{\text{el}} = kx = 32.8\text{ N}$. The maximum possible static friction force is

$\max f_s = \mu_s mg = (0.80)(15.0\text{ kg})(9.80\text{ m/s}^2) = 118\text{ N}$.

EVALUATE: The spring force is less than the maximum possible static friction force so the stone remains at rest.

7.46. IDENTIFY: Once the block leaves the top of the hill it moves in projectile motion. Use $K_1 + U_1 = K_2 + U_2$ to relate the speed v_{B} at the bottom of the hill to the speed v_{Top} at the top and the 70 m height of the hill.

SET UP: For the projectile motion, take $+y$ to be downward. $a_x = 0$, $a_y = g$. $v_{0x} = v_{\text{Top}}$, $v_{0y} = 0$. For the motion up the hill only gravity does work. Take $y = 0$ at the base of the hill.

EXECUTE: First get speed at the top of the hill for the block to clear the pit. $y = \dfrac{1}{2}gt^2$.

$20\text{ m} = \dfrac{1}{2}(9.8\text{ m/s}^2)t^2$. $t = 2.0\text{ s}$. Then $v_{\text{Top}}t = 40\text{ m}$ gives $v_{\text{Top}} = \dfrac{40\text{ m}}{2.0\text{ s}} = 20\text{ m/s}$.

Energy conservation applied to the motion up the hill: $K_{\text{Bottom}} = U_{\text{Top}} + K_{\text{Top}}$ gives

$\frac{1}{2}mv_B^2 = mgh + \frac{1}{2}mv_{Top}^2$. $v_B = \sqrt{v_{Top}^2 + 2gh} = \sqrt{(20\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(70\text{ m})} = 42\text{ m/s}$.

EVALUATE: The result does not depend on the mass of the block.

7.47. **IDENTIFY:** We must use energy conservation. This system has elastic potential energy and gravitational potential energy, and friction does work on it.

SET UP: $U_1 + K_1 + W_{other} = U_2 + K_2$, $U_g = mgy$, $U_{spring} = \frac{1}{2}kx^2$. For the minimum elastic potential energy, the box just stops at the top of the incline. So $K_1 = K_2$ in this case. We want to find the energy stored in the spring.

EXECUTE: $U_1 + K_1 + W_{other} = U_2 + K_2$ gives $U_{1,spring} + W_f = U_{2,g}$, so $U_{1,spring} = U_{2,g} - W_f$. Therefore $U_{1,spring} = mgh - (-f_k)s = mgh + \mu_k ns = mgh + \mu_k mgs\cos\theta = mg(h + \mu_k s\cos\theta)$. Using m = 0.600 kg, s = 2.00 m, θ = 37.0°, h = (2.00 m) sin 37.0°, and μ_k = 0.400 gives $U_{1,spring}$ = 10.8 J.

EVALUATE: The elastic potential energy in the spring must be greater than the gravitational potential energy because friction is doing negative work on the box. $U_g = mgh$ = (0.600 kg)(9.80 m/s^2)(2.00 m)(sin 37.0°) = 7.08 J, which agrees with our expectation.

7.48. **IDENTIFY:** To be at equilibrium at the bottom, with the spring compressed a distance x_0, the spring force must balance the component of the weight down the ramp plus the largest value of the static friction, or $kx_0 = w\sin\theta + f$. Apply energy conservation to the motion down the ramp.

SET UP: $K_2 = 0$, $K_1 = \frac{1}{2}mv^2$, where v is the speed at the top of the ramp. Let $U_2 = 0$, so $U_1 = wL\sin\theta$, where L is the total length traveled down the ramp.

EXECUTE: Energy conservation gives $\frac{1}{2}kx_0^2 = (w\sin\theta - f)L + \frac{1}{2}mv^2$. With the given parameters, $\frac{1}{2}kx_0^2 = 421\text{ J}$ and $kx_0 = 1.066\times10^3\text{ N}$. Solving for k gives $k = 1350\text{ N/m}$.

EVALUATE: $x_0 = 0.790\text{ m}$. $w\sin\theta = 551\text{ N}$. The decrease in gravitational potential energy is larger than the amount of mechanical energy removed by the negative work done by friction. $\frac{1}{2}mv^2 = 243\text{ J}$. The energy stored in the spring is larger than the initial kinetic energy of the crate at the top of the ramp.

7.49. **IDENTIFY:** Use $K_1 + U_1 + W_{other} = K_2 + U_2$. Solve for K_2 and then for v_2.

SET UP: Let point 1 be at his initial position against the compressed spring and let point 2 be at the end of the barrel, as shown in Figure 7.49. Use $F = kx$ to find the amount the spring is initially compressed by the 4400 N force.

$K_1 + U_1 + W_{other} = K_2 + U_2$

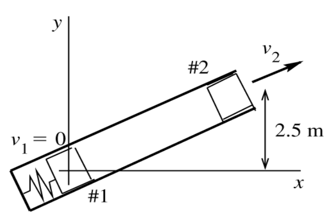


Figure 7.49

Take $y = 0$ at his initial position.

EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$

$W_{other} = W_{fric} = -fs$

$W_{other} = -(40\text{ N})(4.0\text{ m}) = -160\text{ J}$

$U_{1,grav} = 0$, $U_{1,el} = \frac{1}{2}kd^2$, where d is the distance the spring is initially compressed.

$F = kd$ so $d = \frac{F}{k} = \frac{4400\text{ N}}{1100\text{ N/m}} = 4.00\text{ m}$

and $U_{1,\text{el}} = \frac{1}{2}(1100\text{ N/m})(4.00\text{ m})^2 = 8800\text{ J}$

$U_{2,\text{grav}} = mgy_2 = (60\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m}) = 1470\text{ J}, \;\; U_{2,\text{el}} = 0$

Then $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$8800\text{ J} - 160\text{ J} = \frac{1}{2}mv_2^2 + 1470\text{ J}$

$\frac{1}{2}mv_2^2 = 7170\text{ J}$ and $v_2 = \sqrt{\dfrac{2(7170\text{ J})}{60\text{ kg}}} = 15.5\text{ m/s}.$

Evaluate: Some of the potential energy stored in the compressed spring is taken away by the work done by friction. The rest goes partly into gravitational potential energy and partly into kinetic energy.

7.50. **Identify:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the rocket from the starting point to the base of the ramp. W_{other} is the work done by the thrust and by friction.

Set Up: Let point 1 be at the starting point and let point 2 be at the base of the ramp. $v_1 = 0$, $v_2 = 50.0\text{ m/s}$. Let $y = 0$ at the base and take $+y$ upward. Then $y_2 = 0$ and $y_1 = d\sin 53°$, where d is the distance along the ramp from the base to the starting point. Friction does negative work.

Execute: $K_1 = 0$, $U_2 = 0$. $U_1 + W_{\text{other}} = K_2$. $W_{\text{other}} = (2000\text{ N})d - (500\text{ N})d = (1500\text{ N})d$.

$mgd\sin 53° + (1500\text{ N})d = \frac{1}{2}mv_2^2$.

$$d = \frac{mv_2^2}{2[mg\sin 53° + 1500\text{ N}]} = \frac{(1500\text{ kg})(50.0\text{ m/s})^2}{2[(1500\text{ kg})(9.80\text{ m/s}^2)\sin 53° + 1500\text{ N}]} = 142\text{ m}.$$

Evaluate: The initial height is $y_1 = (142\text{ m})\sin 53° = 113\text{ m}$. An object free-falling from this distance attains a speed $v = \sqrt{2gy_1} = 47.1\text{ m/s}$. The rocket attains a greater speed than this because the forward thrust is greater than the friction force.

7.51. **Identify:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the system consisting of the two buckets. If we ignore the inertia of the pulley we ignore the kinetic energy it has.

Set Up: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 in the motion are sketched in Figure 7.51.

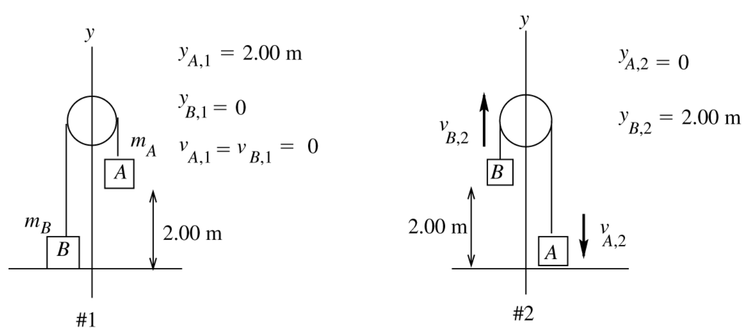


Figure 7.51

The tension force does positive work on the 4.0 kg bucket and an equal amount of negative work on the 12.0 kg bucket, so the net work done by the tension is zero.

Work is done on the system only by gravity, so $W_{\text{other}} = 0$ and $U = U_{\text{grav}}$.

Execute: $K_1 = 0$, $K_2 = \frac{1}{2}m_A v_{A,2}^2 + \frac{1}{2}m_B v_{B,2}^2$. But since the two buckets are connected by a rope they move together and have the same speed: $v_{A,2} = v_{B,2} = v_2$. Thus $K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = (8.00\text{ kg})v_2^2$.

$U_1 = m_A g y_{A,1} = (12.0 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m}) = 235.2 \text{ J}.$

$U_2 = m_B g y_{B,2} = (4.0 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m}) = 78.4 \text{ J}.$

Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $U_1 = K_2 + U_2$.

$235.2 \text{ J} = (8.00 \text{ kg})v_2^2 + 78.4 \text{ J}.$ $v_2 = \sqrt{\dfrac{235.2 \text{ J} - 78.4 \text{ J}}{8.00 \text{ kg}}} = 4.4 \text{ m/s}$

EVALUATE: The gravitational potential energy decreases and the kinetic energy increases by the same amount. We could apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to one bucket, but then we would have to include in W_{other} the work done on the bucket by the tension T.

7.52. **IDENTIFY:** We use energy conservation.

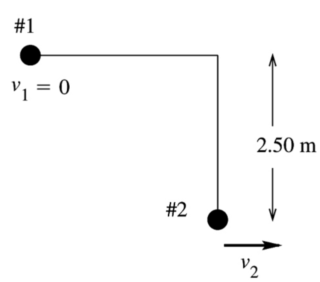
SET UP: $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, $U_g = mgy$, $K = \frac{1}{2}mv^2$. Call point 1 the instant the block is released against the spring and point 2 when its speed is 4.00 m/s. We want to find the magnitude of the friction force f.

EXECUTE: $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ becomes $U_{\text{spr}} + W_f = K_2 + U_g$. This gives

$U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, so $f = \dfrac{U_{\text{spr}} - m\left(\dfrac{v_2^2}{2} + gh\right)}{s}$. Using $m = 0.200$ kg, $v_2 = 4.00$ m/s, $s = 3.00$ m, $h = s \sin 53.0° = 2.40$ m, and $U_{\text{spr}} = 8.00$ J, this gives $U_{\text{spr}} = 0.568$ N.

EVALUATE: The coefficient of kinetic friction for this value of kinetic friction would be $\mu_k = f/n = f/(mg \cos\theta) = (0.568 \text{ N})/[(0.200 \text{ kg})(9.80 \text{ m/s}^2)(\cos 53.0°)] = 0.482$, which is a reasonable value according to Table 5.1.

7.53. **(a) IDENTIFY and SET UP:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the potato. Let point 1 be where the potato is released and point 2 be at the lowest point in its motion, as shown in Figure 7.53a.



$y_1 = 2.50$ m

$y_2 = 0$

The tension in the string is at all points in the motion perpendicular to the displacement, so $W_r = 0$

The only force that does work on the potato is gravity, so $W_{\text{other}} = 0$.

Figure 7.53a

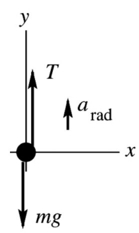
EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$, $U_1 = mgy_1$, $U_2 = 0$. Thus $U_1 = K_2$. $mgy_1 = \frac{1}{2}mv_2^2$, which gives

$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80 \text{ m/s}^2)(2.50 \text{ m})} = 7.00 \text{ m/s}.$

EVALUATE: The speed v_2 is the same as if the potato fell through 2.50 m.

(b) IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the potato. The potato moves in an arc of a circle so its acceleration is $\vec{a}_{\text{rad}}$, where $a_{\text{rad}} = v^2/R$ and is directed toward the center of the circle. Solve for one of the forces, the tension T in the string.

SET UP: The free-body diagram for the potato as it swings through its lowest point is given in Figure 7.53b.



The acceleration $\vec{a}_{rad}$ is directed in toward the center of the circular path, so at this point it is upward.

Figure 7.53b

EXECUTE: $\Sigma F_y = ma_y$ gives $T - mg = ma_{rad}$. Solving for T gives $T = m(g + a_{rad}) = m\left(g + \frac{v_2^2}{R}\right)$, where the radius R for the circular motion is the length L of the string. It is instructive to use the algebraic expression for v_2 from part (a) rather than just putting in the numerical value: $v_2 = \sqrt{2gy_1} = \sqrt{2gL}$, so $v_2^2 = 2gL$. Then $T = m\left(g + \frac{v_2^2}{L}\right) = m\left(g + \frac{2gL}{L}\right) = 3mg$. The tension at this point is three times the weight of the potato, so $T = 3mg = 3(0.300\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$.

EVALUATE: The tension is greater than the weight; the acceleration is upward so the net force must be upward.

7.54. **IDENTIFY:** Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to each stage of the motion.

SET UP: Let $y = 0$ at the bottom of the slope. In part (a), W_{other} is the work done by friction. In part (b), W_{other} is the work done by friction and the air resistance force. In part (c), W_{other} is the work done by the force exerted by the snowdrift.

EXECUTE: **(a)** The skier's kinetic energy at the bottom can be found from the potential energy at the top minus the work done by friction, $K_1 = mgh - W_f = (60.0\text{ kg})(9.8\text{ N/kg})(65.0\text{ m}) - 10{,}500\text{ J}$, or

$K_1 = 38{,}200\text{ J} - 10{,}500\text{ J} = 27{,}720\text{ J}$. Then $v_1 = \sqrt{\frac{2K_1}{m}} = \sqrt{\frac{2(27{,}720\text{ J})}{60\text{ kg}}} = 30.4\text{ m/s}$.

(b) $K_2 = K_1 - (W_f + W_{air}) = 27{,}720\text{ J} - (\mu_k mgd + f_{air}d)$.

$K_2 = 27{,}720\text{ J} - [(0.2)(588\text{ N})(82\text{ m}) + (160\text{ N})(82\text{ m})]$ or $K_2 = 27{,}720\text{ J} - 22{,}763\text{ J} = 4957\text{ J}$. Then,

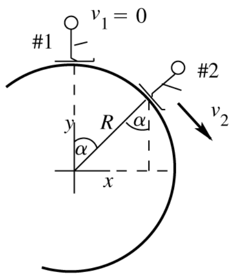
$v_2 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4957\text{ J})}{60\text{ kg}}} = 12.9\text{ m/s}$.

(c) Use the work-energy theorem to find the force. $W = \Delta K$, $F = K/d = (4957\text{ J})/(2.5\text{ m}) = 2000\text{ N}$.

EVALUATE: In each case, W_{other} is negative and removes mechanical energy from the system.

7.55. **IDENTIFY** and **SET UP:** First apply $\Sigma\vec{F} = m\vec{a}$ to the skier.

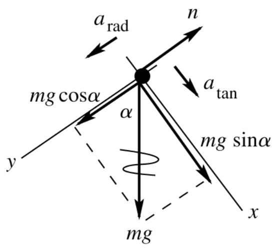
Find the angle α where the normal force becomes zero, in terms of the speed v_2 at this point. Then apply the work-energy theorem to the motion of the skier to obtain another equation that relates v_2 and α. Solve these two equations for α.



Let point 2 be where the skier loses contact with the snowball, as sketched in Figure 7.55a
Loses contact implies $n \to 0$.
$y_1 = R,\ \ y_2 = R\cos\alpha$

Figure 7.55a

First, analyze the forces on the skier when she is at point 2. The free-body diagram is given in Figure 7.55b. For this use coordinates that are in the tangential and radial directions. The skier moves in an arc of a circle, so her acceleration is $a_{\text{rad}} = v^2/R,$ directed in toward the center of the snowball.



EXECUTE: $\Sigma F_y = ma_y$

$mg\cos\alpha - n = mv_2^2/R$

But $n = 0$ so $mg\cos\alpha = mv_2^2/R$

$v_2^2 = Rg\cos\alpha$

Figure 7.55b

Now use conservation of energy to get another equation relating v_2 to α:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the skier is gravity, so $W_{\text{other}} = 0.$

$K_1 = 0,\ \ K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1 = mgR,\ \ U_2 = mgy_2 = mgR\cos\alpha$

Then $mgR = \frac{1}{2}mv_2^2 + mgR\cos\alpha$

$v_2^2 = 2gR(1-\cos\alpha)$

Combine this with the $\Sigma F_y = ma_y$ equation:

$Rg\cos\alpha = 2gR(1-\cos\alpha)$

$\cos\alpha = 2 - 2\cos\alpha$

$3\cos\alpha = 2$ so $\cos\alpha = 2/3$ and $\alpha = 48.2°$

EVALUATE: She speeds up and her a_{rad} increases as she loses gravitational potential energy. She loses contact when she is going so fast that the radially inward component of her weight isn't large enough to keep her in the circular path. Note that α where she loses contact does not depend on her mass or on the radius of the snowball.

7.56. **IDENTIFY:** We use energy conservation.

SET UP: $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ where $W_{\text{other}} = 0$, $U_{\text{spr}} = \dfrac{1}{2}kx^2$, $K = \dfrac{1}{2}mv^2$. Call point 1 when the block's speed is 3.00 m/s and point 2 at maximum compression of the spring. This makes v_1 = 3.00 m/s, x_1 = 0.160 m, and $K_2 = 0$.

EXECUTE: **(a)** $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ becomes $\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_{\max}^2$. Now solve for $v_{\max}$:

$v_{\max} = \sqrt{\dfrac{kx_1^2 + mv_1^2}{m}}$. Using $x_1 = 0.160$ m, $v_1 = 3.00$ m/s, and $k = 200$ N/m gives $v_{\max} = 4.67$ m/s.

(b) The maximum compression occurs when the kinetic energy is zero, so $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx_{\max}^2$,

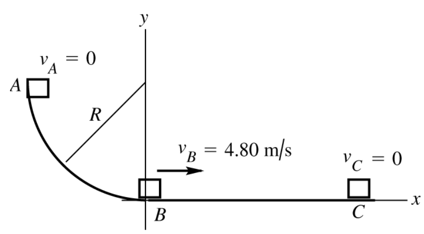
which gives $x_{\max} = v_{\max}\sqrt{m/k} = (4.67\text{ m/s})\sqrt{\dfrac{0.400\text{ kg}}{200\text{ N/m}}} = 0.209$ m.

(c) At maximum compression $v = 0$ and a is a maximum. So $\sum F_x = ma_x$ gives $kx_{\max} = ma$, so

$a = \dfrac{kx_{\max}}{m} = \dfrac{(200\text{ N/m})(0.209\text{ m})}{0.400\text{ kg}} = 104\text{ m/s}^2$.

EVALUATE: At maximum compression, the spring force is a maximum because x is a maximum, so the acceleration is a maximum. But at this instant the block has stopped, so $v = 0$ at maximum acceleration.

7.57. **IDENTIFY** and **SET UP:**



$y_A = R$

$y_B = y_C = 0$

Figure 7.57

(a) Apply conservation of energy to the motion from B to C:

$K_B + U_B + W_{\text{other}} = K_C + U_C$. The motion is described in Figure 7.57.

EXECUTE: The only force that does work on the package during this part of the motion is friction, so $W_{\text{other}} = W_f = f_k(\cos\phi)s = \mu_k\, mg(\cos 180°)s = \mu_k\, mgs$

$K_B = \frac{1}{2}mv_B^2,\quad K_C = 0$

$U_B = 0,\quad U_C = 0$

Thus $K_B + W_f = 0$

$\frac{1}{2}mv_B^2 - \mu_k mgs = 0$

$\mu_k = \dfrac{v_B^2}{2gs} = \dfrac{(4.80\text{ m/s})^2}{2(9.80\text{ m/s}^2)(3.00\text{ m})} = 0.392.$

EVALUATE: The negative friction work takes away all the kinetic energy.

(b) IDENTIFY and **SET UP:** Apply conservation of energy to the motion from A to B:

$$K_A + U_A + W_{\text{other}} = K_B + U_B$$

EXECUTE: Work is done by gravity and by friction, so $W_{\text{other}} = W_f$.

$K_A = 0,\quad K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200\text{ kg})(4.80\text{ m/s})^2 = 2.304\text{ J}$

$U_A = mgy_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(1.60\text{ m}) = 3.136\text{ J},\quad U_B = 0$

Thus $U_A + W_f = K_B$

$W_f = K_B - U_A = 2.304\text{ J} - 3.136\text{ J} = \text{-}0.83\text{ J}$

EVALUATE: W_f is negative as expected; the friction force does negative work since it is directed opposite to the displacement.

7.58. IDENTIFY: Apply $K_1+U_1+W_{\text{other}}=K_2+U_2$ to the initial and final positions of the truck.

SET UP: Let $y=0$ at the lowest point of the path of the truck. W_{other} is the work done by friction. $f_{\text{r}}=\mu_{\text{r}}n=\mu_{\text{r}}mg\cos\beta.$

EXECUTE: Denote the distance the truck moves up the ramp by x. $K_1=\frac{1}{2}mv_0^2$, $U_1=mgL\sin\alpha$, $K_2=0$, $U_2=mgx\sin\beta$ and $W_{\text{other}}=-\mu_{\text{r}}mgx\cos\beta$. From $W_{\text{other}}=(K_2+U_2)-(K_1+U_1)$, and solving for x, we get $x=\dfrac{K_1+mgL\sin\alpha}{mg(\sin\beta+\mu_{\text{r}}\cos\beta)}=\dfrac{(v_0^2/2g)+L\sin\alpha}{\sin\beta+\mu_{\text{r}}\cos\beta}$.

EVALUATE: x increases when v_0 increases and decreases when μ_{r} increases.

7.59. (a) IDENTIFY: We are given that $F_x=-\alpha x-\beta x^2$, $\alpha=60.0\text{ N/m}$ and $\beta=18.0\text{ N/m}^2$. Use $W_{F_x}=\int_{x_1}^{x_2}F_x(x)\,dx$ to calculate W and then use $W=-\Delta U$ to identify the potential energy function $U(x)$.

SET UP: $W_{F_x}=U_1-U_2=\int_{x_1}^{x_2}F_x(x)\,dx$

Let $x_1=0$ and $U_1=0$. Let x_2 be some arbitrary point x, so $U_2=U(x)$.

EXECUTE: $U(x)=-\int_0^x F_x(x)\,dx=-\int_0^x(-\alpha x-\beta x^2)\,dx=\int_0^x(\alpha x+\beta x^2)\,dx=\frac{1}{2}\alpha x^2+\frac{1}{3}\beta x^3.$

EVALUATE: If $\beta=0$, the spring does obey Hooke's law, with $k=\alpha$, and our result reduces to $\frac{1}{2}kx^2$.

(b) IDENTIFY: Apply $K_1+U_1+W_{\text{other}}=K_2+U_2$ to the motion of the object.

SET UP: The system at points 1 and 2 is sketched in Figure 7.59.

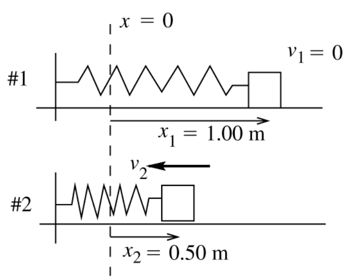


Figure 7.59

$K_1+U_1+W_{\text{other}}=K_2+U_2$

The only force that does work on the object is the spring force, so $W_{\text{other}}=0$.

EXECUTE: $K_1=0$, $K_2=\frac{1}{2}mv_2^2$

$U_1=U(x_1)=\frac{1}{2}\alpha x_1^2+\frac{1}{3}\beta x_1^3=\frac{1}{2}(60.0\text{ N/m})(1.00\text{ m})^2+\frac{1}{3}(18.0\text{ N/m}^2)(1.00\text{ m})^3=36.0\text{ J}$

$U_2=U(x_2)=\frac{1}{2}\alpha x_2^2+\frac{1}{3}\beta x_2^3=\frac{1}{2}(60.0\text{ N/m})(0.500\text{ m})^2+\frac{1}{3}(18.0\text{ N/m}^2)(0.500\text{ m})^3=8.25\text{ J}$

Thus $36.0\text{ J}=\frac{1}{2}mv_2^2+8.25\text{ J}$, which gives $v_2=\sqrt{\dfrac{2(36.0\text{ J}-8.25\text{ J})}{0.900\text{ kg}}}=7.85\text{ m/s}.$

EVALUATE: The elastic potential energy stored in the spring decreases and the kinetic energy of the object increases.

7.60. **IDENTIFY:** Mechanical energy is conserved on the hill, which gives us the speed of the sled at the top. After it leaves the cliff, we must use projectile motion.

SET UP: Use conservation of energy to find the speed of the sled at the edge of the cliff. Let $y_i = 0$ so $y_f = h = 11.0$ m. $K_f + U_f = K_i + U_i$ gives $\frac{1}{2}mv_f^2 + mgh = \frac{1}{2}mv_i^2$ or $v_f = \sqrt{v_i^2 - 2gh}$. Then analyze the projectile motion of the sled: use the vertical component of motion to find the time *t* that the sled is in the air; then use the horizontal component of the motion with $a_x = 0$ to find the horizontal displacement.

EXECUTE: $v_f = \sqrt{(22.5\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(11.0\text{ m})} = 17.1$ m/s. $y_f = v_{i,y}t + \frac{1}{2}a_y t^2$ gives

$t = \sqrt{\dfrac{2y_f}{a_y}} = \sqrt{\dfrac{2(-11.0\text{ m})}{-9.80\text{ m/s}^2}} = 1.50$ s. $x_f = v_{i,x}t + \frac{1}{2}a_x t^2$ gives $x_f = v_{i,x}t = (17.1\text{ m/s})(1.50\text{ s}) = 25.6$ m.

EVALUATE: Conservation of energy can be used to find the speed of the sled at any point of the motion but does not specify how far the sled travels while it is in the air.

7.61. **IDENTIFY:** We have a conservative force, so we can relate the force and the potential energy function. Energy conservation applies.

SET UP: $F_x = -dU/dx$, U goes to 0 as x goes to infinity, and $F(x) = \dfrac{\alpha}{(x+x_0)^2}$.

EXECUTE: **(a)** Using $dU = -F_x dx$, we get $U_x - U_\infty = -\displaystyle\int_\infty^x \frac{\alpha}{(x+x_0)^2}dx = \frac{\alpha}{x+x_0}$.

(b) Energy conservation tells us that $U_1 = K_2 + U_2$. Therefore $\dfrac{\alpha}{x_1+x_0} = \dfrac{1}{2}mv_x^2 + \dfrac{\alpha}{x_2+x_0}$. Putting in $m =$ 0.500 kg, $\alpha = 0.800\text{ N}\cdot\text{m}$, $x_0 = 0.200$ m, $x_1 = 0$, and $x_2 = 0.400$ m, solving for v gives $v = 3.27$ m/s.

EVALUATE: The potential energy is not infinite even though the integral in (a) is taken over an infinite distance because the force rapidly gets smaller with increasing distance x.

7.62. **IDENTIFY:** We need to use energy conservation and apply Newton's second law.

SET UP: $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ where $W_{\text{other}} = 0$, $U_g = mgy$, and $K = \dfrac{1}{2}mv^2$. $\sum F_{\text{rad}} = ma_{\text{rad}}$, where $a_{\text{rad}} = \dfrac{v^2}{R}$. The ball has momentarily stopped swinging when $\theta = 37.0°$, but it is swinging when $\theta = 25.0°$. Since the ball is swinging in a circular arc, it has radial acceleration toward the center of the circle. The forces causing that acceleration are the tension and the component of gravity toward (or away from) the center of the circle. Fig. 7.62 shows a free-body diagram of the ball. We want to find the tension in the rope at two different angles.

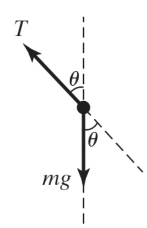


Figure 7.62

EXECUTE: **(a)** At the maximum angle of swing, the ball has stopped, so its radial acceleration (v^2/R) is zero at that instant. This means that the net radial force must be zero. From Fig. 7.62 we can see the force components. Thus $T - mg\cos\theta = 0$, and solving for T gives

$T = mg\cos\theta = (0.200 \text{ kg})(9.80 \text{ m/s}^2)(\cos 37.0°) = 1.57$ N.

(b) The ball is now moving, so it has radial acceleration. Therefore $\sum F_{\text{rad}} = ma_{\text{rad}}$ gives $T - mg\cos\theta = m\dfrac{v^2}{R}$. Solving for T and using $R = L$, we get $T = mg\cos\theta + m\dfrac{v^2}{L}$. To find T, we need to find v^2 when $\theta = 25.0°$. We use $U_1 + K_1 = U_2 + K_2$. Call point 1 when $\theta = 37.0°$ and point 2 when $\theta = 25.0°$. In that case we have $U_1 = K_2 + U_2$, which gives $K_2 = -(U_2 - U_1) = -mg(y_2 - y_1) = -mgL(\cos\theta_1 - \cos\theta_2)$. In terms of v_2, this is $\frac{1}{2}mv_2^2 = -mgL(\cos\theta_1 - \cos\theta_2)$, which simplifies to $v_2^2 = -2gL(\cos\theta_1 - \cos\theta_2)$, so $v_2^2 = -2g(1.40 \text{ m})(\cos 37.0° - \cos 25.0°) = 2.95 \text{ m}^2/\text{s}^2$. Now return to the tension. From earlier work, we have $T = mg\cos\theta_2 + m\dfrac{v^2}{L} = m\left(g\cos\theta_2 + \dfrac{v^2}{L}\right)$. Using $v_2^2 = 2.95 \text{ m}^2/\text{s}^2$, $m = 0.200$ kg, $\theta = 25.0°$, and $L = 1.40$ m, we find that $T = 2.20$ N.

EVALUATE: We have found that the tension T is greater at 25° than at 37° because it helps to accelerate the ball toward the center at 25°.

7.63. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the block.

SET UP: Let $y = 0$ at the floor. Let point 1 be the initial position of the block against the compressed spring and let point 2 be just before the block strikes the floor.

EXECUTE: With $U_2 = 0, K_1 = 0,$ $K_2 = U_1$. $\frac{1}{2}mv_2^2 = \frac{1}{2}kx^2 + mgh$. Solving for v_2,

$$v_2 = \sqrt{\frac{kx^2}{m} + 2gh} = \sqrt{\frac{(1900 \text{ N/m})(0.045 \text{ m})^2}{(0.150 \text{ kg})} + 2(9.80 \text{ m/s}^2)(1.20 \text{ m})} = 7.01 \text{ m/s}.$$

EVALUATE: The potential energy stored in the spring and the initial gravitational potential energy all go into the final kinetic energy of the block.

7.64. **IDENTIFY:** At equilibrium the upward spring force equals the weight mg of the object. Apply conservation of energy to the motion of the fish.

SET UP: The distance that the mass descends equals the distance the spring is stretched. $K_1 = K_2 = 0$, so $U_1(\text{gravitational}) = U_2(\text{spring})$

EXECUTE: Following the hint, the force constant k is found from $mg = kd$, or $k = mg/d$. When the fish falls from rest, its gravitational potential energy decreases by mgy; this becomes the potential energy of the spring, which is $\frac{1}{2}ky^2 = \frac{1}{2}(mg/d)y^2$. Equating these, $\frac{1}{2}\frac{mg}{d}y^2 = mgy$, or $y = 2d$.

EVALUATE: At its lowest point the fish is not in equilibrium. The upward spring force at this point is $ky = 2kd$, and this is equal to twice the weight. At this point the net force is mg, upward, and the fish has an upward acceleration equal to g.

7.65. **IDENTIFY:** The spring does positive work on the box but friction does negative work.

SET UP: $U_{\text{el}} = \frac{1}{2}kx^2$ and $W_{\text{other}} = W_f = -\mu_k mgx$.

EXECUTE: **(a)** $U_{\text{el}} + W_{\text{other}} = K$ gives $\frac{1}{2}kx^2 + (-\mu_k mgx) = \frac{1}{2}mv^2$. Using the numbers for the problem, $k = 45.0$ N/m, $x = 0.280$ m, $\mu_k = 0.300$, and $m = 1.60$ kg, solving for v gives $v = 0.747$ m/s.

(b) Call x the distance the spring is compressed when the speed of the box is a maximum and x_0 the initial compression distance of the spring. Using an approach similar to that in part (a) gives $\frac{1}{2}kx_0^2 - \mu_k mg(x_0 - x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$. Rearranging gives $mv^2 = kx_0^2 - kx^2 - 2\mu_k mg(x_0 - x)$. For the maximum speed, $d(v^2)/dx = 0$, which gives $-2kx + 2\mu_k mg = 0$. Solving for $x_{\max}$, the compression distance at maximum speed, gives $x_{\max} = \mu_k mg/k$. Now substitute this result into the expression above for mv^2, put in the numbers, and solve for v, giving $v = 0.931$ m/s.

EVALUATE: Another way to find the result in (b) is to realize that the spring force decreases as x decreases, but the friction force remains constant. Eventually these two forces will be equal in magnitude. After that the friction force will be greater than the spring force, and friction will begin to slow down the box. So the maximum box speed occurs when the spring force is equal to the friction force. At that instant, $kx = f_k$, which gives $x = 0.105$ m. Then energy conservation can be used to find v with this value of x.

7.66. **IDENTIFY:** The spring obeys Hooke's law. Gravity and the spring provide the vertical forces on the brick. The mechanical energy of the system is conserved.

SET UP: Use $K_f + U_f = K_i + U_i$. In part (a), setting $y_f = 0$, we have $y_i = x$, the amount the spring will stretch. Also, since $K_i = K_f = 0$, $\frac{1}{2}kx^2 = mgx$. In part (b), $y_i = h + x$, where $h = 1.0$ m.

EXECUTE: **(a)** $x = \dfrac{2mg}{k} = \dfrac{2(3.0\text{ kg})(9.80\text{ m/s}^2)}{1500\text{ N/m}} = 0.039\text{ m} = 3.9\text{ cm}.$

(b) $\frac{1}{2}kx^2 = mg(h + x)$, $kx^2 - 2mgx - 2mgh = 0$ and $x = \dfrac{mg}{k}\left(1 \pm \sqrt{1 + \dfrac{2hk}{mg}}\right)$. Since x must be positive,

we have $x = \dfrac{mg}{k}\left(1+\sqrt{1+\dfrac{2hk}{mg}}\right) = \dfrac{(3.0\text{ kg})(9.80\text{ m/s}^2)}{1500\text{ N/m}}\left(1+\sqrt{1+\dfrac{2(1.0\text{ m})(1500\text{ N/m})}{3.0\text{ kg}(9.80\text{ m/s}^2)}}\right) = 0.22\text{ m} = 22\text{ cm}.$

EVALUATE: In part (b) there is additional initial energy (from gravity), so the spring is stretched more.

7.67. **IDENTIFY:** Only conservative forces (gravity and the spring) act on the fish, so its mechanical energy is conserved.

SET UP: Energy conservation tells us $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $W_{\text{other}} = 0$. $U_g = mgy$, $K = \frac{1}{2}mv^2$, and $U_{spring} = \frac{1}{2}ky^2$.

EXECUTE: **(a)** $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Let y be the distance the fish has descended, so $y = 0.0500$ m. $K_1 = 0$, $W_{\text{other}} = 0$, $U_1 = mgy$, $K_2 = \dfrac{1}{2}mv_2^2$, and $U_2 = \dfrac{1}{2}ky^2$. Solving for K_2 gives

$K_2 = U_1 - U_2 = mgy - \dfrac{1}{2}ky^2 = (3.00\text{ kg})(9.8\text{ m/s}^2)(0.0500\text{ m}) - \dfrac{1}{2}(900\text{ N/m})(0.0500\text{ m})^2$

$K_2 = 1.47\text{ J} - 1.125\text{ J} = 0.345\text{ J}$. Solving for v_2 gives $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(0.345\text{ J})}{3.00\text{ kg}}} = 0.480\text{ m/s}.$

(b) The maximum speed is when K_2 is maximum, which is when $dK_2/dy = 0$. Using $K_2 = mgy - \dfrac{1}{2}ky^2$ gives $\dfrac{dK_2}{dy} = mg - ky = 0$. Solving for y gives

$y = \dfrac{mg}{k} = \dfrac{(3.00\text{ kg})(9.8\text{ m/s}^2)}{900\text{ N/m}} = 0.03267\text{ m}$. At this y,

$K_2 = (3.00\text{ kg})(9.8\text{ m/s}^2)(0.03267\text{ m}) - \dfrac{1}{2}(900\text{ N/m})(0.03267\text{ m})^2.$

$K_2 = 0.9604\text{ J} - 0.4803\text{ J} = 0.4801\text{ J}$, so $v_2 = \sqrt{\dfrac{2K_2}{m}} = 0.566\text{ m/s}.$

EVALUATE: The speed in part (b) is greater than the speed in part (a), as it should be since it is the maximum speed.

7.68. **IDENTIFY:** We need to use projectile motion and energy conservation.

SET UP: We work "backwards" in a sense. First use projectile motion to find the horizontal velocity of the wood at the base of the slide. Then use energy conservation to find how much work friction did as the wood slid down the slide. Use $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, where W_{other} is the work done by friction on the slide, which is what we want to find. The sketch in Fig. 7.68 organizes the quantities in the problem.

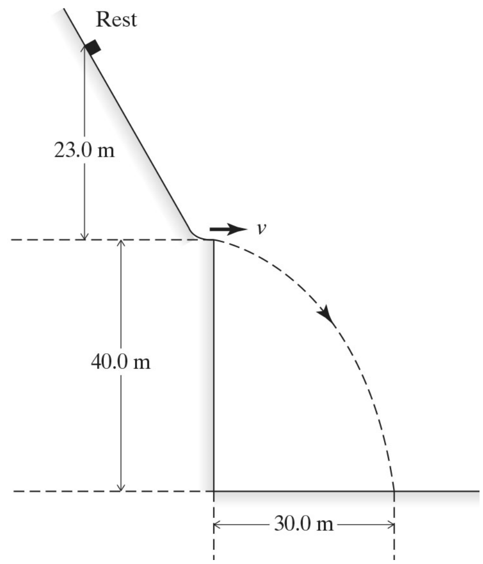


Figure 7.68

EXECUTE: Once the wood leaves the slide, it falls 40.0 m from rest while traveling 30.0 m horizontally at constant horizontal velocity. Using $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $y = \frac{1}{2}gt^2$, so

$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{80.0\text{ m}}{9.80\text{ m/s}^2}} = 2.857$ s. The constant speed v needed to travel 30.0 m horizontally in 2.857 s is $v = x/t = (30.0\text{ m})/(2.857\text{ s}) = 10.5$ m/s. This is the speed at the bottom of the slide. Now use energy conservation, calling point 1 at the top of the slide and point 2 the bottom of the slide. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ becomes $mgh + W_{\text{f}} = \frac{1}{2}mv^2$. Solve for W_{f}: $W_{\text{f}} = m\left(\frac{v^2}{2} - gh\right)$. Using $m =$ 3.00 kg, $v = 10.5$ m/s, and $h = 23.0$ m gives $W_{\text{f}} = -511$ J.

EVALUATE: The work must be negative because friction works *against* the block's motion and therefore takes away mechanical energy.

7.69. **(a) IDENTIFY and SET UP:** Apply $K_A + U_A + W_{\text{other}} = K_B + U_B$ to the motion from A to B.

EXECUTE: $K_A = 0,\ K_B = \frac{1}{2}mv_B^2,\ U_A = 0,\ U_B = U_{\text{el},B} = \frac{1}{2}kx_B^2,$ where $x_B = 0.25$ m, and $W_{\text{other}} = W_F = Fx_B$. Thus $Fx_B = \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2$. (The work done by F goes partly to the potential energy of the stretched spring and partly to the kinetic energy of the block.)

$Fx_B = (20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ and $\frac{1}{2}kx_B^2 = \frac{1}{2}(40.0\text{ N/m})(0.25\text{ m})^2 = 1.25\text{ J}$

Thus $5.0\text{ J} = \frac{1}{2}mv_B^2 + 1.25\text{ J}$ and $v_B = \sqrt{\frac{2(3.75\text{ J})}{0.500\text{ kg}}} = 3.87\text{ m/s}.$

(b) IDENTIFY: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the block. Let point C be where the block is closest to the wall. When the block is at point C the spring is compressed an amount $|x_C|$, so the block is $0.60\text{ m} - |x_C|$ from the wall, and the distance between B and C is $x_B + |x_C|$.

SET UP: The motion from A to B to C is described in Figure 7.69.

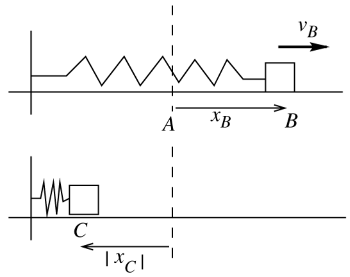


Figure 7.69

$K_B + U_B + W_{\text{other}} = K_C + U_C$

EXECUTE: $W_{\text{other}} = 0$

$K_B = \frac{1}{2}mv_B^2 = 5.0\ \text{J} - 1.25\ \text{J} = 3.75\ \text{J}$ (from part (a))

$U_B = \frac{1}{2}kx_B^2 = 1.25\ \text{J}$

$K_C = 0$ (instantaneously at rest at point closest to wall)

$U_C = \frac{1}{2}k|x_C|^2$

Thus $3.75\ \text{J} + 1.25\ \text{J} = \frac{1}{2}k|x_C|^2$, giving $|x_c| = \sqrt{\dfrac{2(5.0\ \text{J})}{40.0\ \text{N/m}}} = 0.50\ \text{m}$. The distance of the block from the wall is $0.60\ \text{m} - 0.50\ \text{m} = 0.10\ \text{m}$.

EVALUATE: The work $(20.0\ \text{N})(0.25\ \text{m}) = 5.0\ \text{J}$ done by F puts 5.0 J of mechanical energy into the system. No mechanical energy is taken away by friction, so the total energy at points B and C is 5.0 J.

7.70. **IDENTIFY:** Applying Newton's second law, we can use the known normal forces to find the speeds of the block at the top and bottom of the circle. We can then use energy conservation to find the work done by friction, which is the target variable.

SET UP: For circular motion $\Sigma F = m\dfrac{v^2}{R}$. Energy conservation tells us that $K_A + U_A + W_{\text{other}} = K_B + U_B$, where W_{other} is the work done by friction. $U_{\text{g}} = mgy$ and $K = \frac{1}{2}mv^2$.

EXECUTE: Use the given values for the normal force to find the block's speed at points A and B. At point A, Newton's second law gives $n_A - mg = m\dfrac{v_A^2}{R}$. So

$v_A = \sqrt{\dfrac{R}{m}(n_A - mg)} = \sqrt{\dfrac{0.500\ \text{m}}{0.0400\ \text{kg}}(3.95\ \text{N} - 0.392\ \text{N})} = 6.669\ \text{m/s}$. Similarly at point B,

$n_B + mg = m\dfrac{v_B^2}{R}$. Solving for v_{B} gives

$v_B = \sqrt{\dfrac{R}{m}(n_B + mg)} = \sqrt{\dfrac{0.500\ \text{m}}{0.0400\ \text{kg}}(0.680\ \text{N} + 0.392\ \text{N})} = 3.660\ \text{m/s}$. Now apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to find the work done by friction. $K_A + U_A + W_{\text{other}} = K_B + U_B$.

$W_{\text{other}} = K_B + U_B - K_A$.

$W_{\text{other}} = \dfrac{1}{2}(0.040\ \text{kg})(3.66\ \text{m/s})^2 + (0.04\ \text{kg})(9.8\ \text{m/s}^2)(1.0\ \text{m}) - \dfrac{1}{2}(0.04\ \text{kg})(6.669\ \text{m/s})^2$.

$W_{\text{other}} = 0.2679\ \text{J} + 0.392\ \text{J} - 0.8895\ \text{J} = -0.230\ \text{J}$.

EVALUATE: The work done by friction is negative, as it should be. This work is equal to the loss of mechanical energy between the top and bottom of the circle.

7.71. **IDENTIFY:** We can apply Newton's second law to the block. The only forces acting on the block are gravity downward and the normal force from the track pointing toward the center of the circle. The mechanical energy of the block is conserved since only gravity does work on it. The normal force does no work since it is perpendicular to the displacement of the block. The target variable is the normal force at the top of the track.

SET UP: For circular motion $\Sigma F = m\dfrac{v^2}{R}$. Energy conservation tells us that $K_A + U_A + W_{\text{other}} = K_B + U_B$, where $W_{\text{other}} = 0$. $U_g = mgy$ and $K = \frac{1}{2}mv^2$.

EXECUTE: Let point A be at the bottom of the path and point B be at the top of the path. At the bottom of the path, $n_A - mg = m\dfrac{v^2}{R}$ (from Newton's second law).

$v_A = \sqrt{\dfrac{R}{m}(n_A - mg)} = \sqrt{\dfrac{0.800\text{ m}}{0.0500\text{ kg}}(3.40\text{ N} - 0.49\text{ N})} = 6.82\text{ m/s}$. Use energy conservation to find the speed at point B. $K_A + U_A + W_{\text{other}} = K_B + U_B$, giving $\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mg(2R)$. Solving for v_B gives $v_B = \sqrt{v_A^2 - 4Rg} = \sqrt{(6.82\text{ m/s})^2 - 4(0.800\text{ m})(9.8\text{ m/s}^2)} = 3.89\text{ m/s}$. Then at point B, Newton's second law gives $n_B + mg = m\dfrac{v_B^2}{R}$. Solving for n_B gives $n_B = m\dfrac{v_B^2}{R} - mg =$

$$(0.0500\text{ kg})\left(\frac{(3.89\text{ m/s})^2}{0.800\text{ m}} - 9.8\text{ m/s}^2\right) = 0.456\text{ N}.$$

EVALUATE: The normal force at the top is considerably less than it is at the bottom for two reasons: the block is moving slower at the top and the downward force of gravity at the top aids the normal force in keeping the block moving in a circle.

7.72. IDENTIFY: Only gravity does work, so apply $K_1 + U_1 = K_2 + U_2$. Use $\Sigma\vec{F} = m\vec{a}$ to calculate the tension.

SET UP: Let $y = 0$ at the bottom of the arc. Let point 1 be when the string makes a $45°$ angle with the vertical and point 2 be where the string is vertical. The rock moves in an arc of a circle, so it has radial acceleration $a_{\text{rad}} = v^2/r$.

EXECUTE: **(a)** At the top of the swing, when the kinetic energy is zero, the potential energy (with respect to the bottom of the circular arc) is $mgl(1 - \cos\theta)$, where l is the length of the string and θ is the angle the string makes with the vertical. At the bottom of the swing, this potential energy has become kinetic energy, so $mgl(1 - \cos\theta) = \frac{1}{2}mv^2$, which gives

$v = \sqrt{2gl(1 - \cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)(0.80\text{ m})(1 - \cos 45°)} = 2.1\text{ m/s}$.

(b) At $45°$ from the vertical, the speed is zero, and there is no radial acceleration; the tension is equal to the radial component of the weight, or $mg\cos\theta = (0.12\text{ kg})(9.80\text{ m/s}^2)\cos 45° = 0.83\text{ N}$.

(c) At the bottom of the circle, the tension is the sum of the weight and the mass times the radial acceleration, $mg + mv_2^2/l = mg(1 + 2(1 - \cos 45°)) = 1.9\text{ N}$.

EVALUATE: When the string passes through the vertical, the tension is greater than the weight because the acceleration is upward.

7.73. IDENTIFY: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the block.

SET UP: The motion from A to B is described in Figure 7.73.

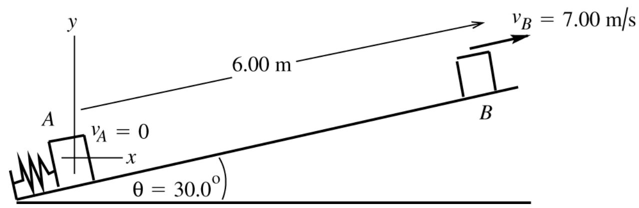


Figure 7.73

The normal force is $n = mg\cos\theta$, so $f_k = \mu_k n = \mu_k mg\cos\theta$. $y_A = 0$; $y_B = (6.00\text{ m})\sin 30.0° = 3.00$ m.
$K_A + U_A + W_{\text{other}} = K_B + U_B$

Execute: Work is done by gravity, by the spring force, and by friction, so $W_{\text{other}} = W_f$ and
$U = U_{\text{el}} + U_{\text{grav}}$

$K_A = 0$, $K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(1.50\text{ kg})(7.00\text{ m/s})^2 = 36.75$ J

$U_A = U_{\text{el},A} + U_{\text{grav},A} = U_{\text{el},A}$, since $U_{\text{grav},A} = 0$

$U_B = U_{\text{el},B} + U_{\text{grav},B} = 0 + mgy_B = (1.50\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m}) = 44.1$ J

$W_{\text{other}} = W_f = (f_k\cos\phi)s = \mu_k mg\cos\theta(\cos 180°)s = -\mu_k mg\cos\theta s$

$W_{\text{other}} = -(0.50)(1.50\text{ kg})(9.80\text{ m/s}^2)(\cos 30.0°)(6.00\text{ m}) = -38.19$ J

Thus $U_{\text{el},A} - 38.19\text{ J} = 36.75\text{ J} + 44.10\text{ J}$, giving $U_{\text{el},A} = 38.19\text{ J} + 36.75\text{ J} + 44.10\text{ J} = 119$ J.

Evaluate: U_{el} must always be positive. Part of the energy initially stored in the spring was taken away by friction work; the rest went partly into kinetic energy and partly into an increase in gravitational potential energy.

7.74. **Identify:** We know the potential energy function for a conservative force. Mechanical energy is conserved.

Set Up: $F_x = -dU/dx$ and $U(x) = -\alpha x^2 + \beta x^3$.

Execute: **(a)** $U_1 + K_1 = U_2 + K_2$ gives $0 + 0 = U_2 + K_2$, so $K_2 = -U_2 = -(-\alpha x_2^2 + \beta x_2^3) = \frac{1}{2}mv^2$. Using $m = 0.0900$ kg, $x = 4.00$ m, $\alpha = 2.00$ J/m^2, and $\beta = 0.300$ J/m^3, solving for v gives $v = 16.9$ m/s.

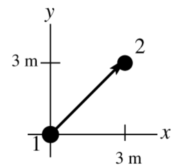
(b) $F_x = -dU/dx = -(-2\alpha x + 3\beta x^2)$. In addition, $F_x = ma_x$, so $a_x = F_x/m$. Using the numbers from (a), gives $a = 17.8$ m/s^2.

(c) The maximum x will occur when $U = 0$ since the total energy is zero. Therefore $-\alpha x^2 + \beta x^3 = 0$, so $x_{\max} = \alpha/\beta = (2.00\text{ J/m}^2)/(0.300\text{ J/m}^3) = 6.67$ m.

Evaluate: The object is released from rest but at a small (but not zero) x. Therefore F_x is small but not zero initially, so it will start the object moving.

7.75. **Identify:** We are given that $\vec{F} = -\alpha xy^2\hat{j}$, $\alpha = 2.50\text{ N/m}^3$. $\vec{F}$ is not constant so use $W = \int_1^2 \vec{F}\cdot d\vec{l}$ to calculate the work. $\vec{F}$ must be evaluated along the path.

(a) SET UP: The path is sketched in Figure 7.75a.



$d\vec{l} = dx\hat{\imath} + dy\hat{\jmath}$

$\vec{F} \cdot d\vec{l} = -\alpha xy^2\, dy$

On the path, $x = y$ so $\vec{F} \cdot d\vec{l} = -\alpha y^3\, dy$

Figure 7.75a

EXECUTE: $W = \int_1^2 \vec{F} \cdot d\vec{l} = \int_{y_1}^{y_2} (-\alpha y^3)\, dy = -(\alpha/4)(y_2^4 - y_1^4)$

$y_1 = 0,\ \ y_2 = 3.00\text{ m},$ so $W = -\frac{1}{4}(2.50\text{ N/m}^3)(3.00\text{ m})^4 = -50.6\text{ J}$

(b) SET UP: The path is sketched in Figure 7.75b.

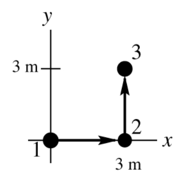


Figure 7.75b

For the displacement from point 1 to point 2, $d\vec{l} = dx\hat{\imath}$, so $\vec{F} \cdot d\vec{l} = 0$ and $W = 0$. (The force is perpendicular to the displacement at each point along the path, so $W = 0$.)

For the displacement from point 2 to point 3, $d\vec{l} = dy\hat{\jmath}$, so $\vec{F} \cdot d\vec{l} = -\alpha xy^2\, dy$. On this path, $x = 3.00$ m, so

$$\vec{F} \cdot d\vec{l} = -(2.50\text{ N/m}^3)(3.00\text{ m})y^2\, dy = -(7.50\text{ N/m}^2)y^2\, dy.$$

EXECUTE: $W = \int_2^3 \vec{F} \cdot d\vec{l} = -(7.50\text{ N/m}^2)\int_{y_2}^{y_3} y^2\, dy = -(7.50\text{ N/m}^2)\frac{1}{3}(y_3^3 - y_2^3)$

$W = -(7.50\text{ N/m}^2)\left(\frac{1}{3}\right)(3.00\text{ m})^3 = -67.5\text{ J}.$

(c) EVALUATE: For these two paths between the same starting and ending points the work is different, so the force is nonconservative.

7.76. IDENTIFY: Use $F_x = -\dfrac{dU}{dx}$ to relate F_x and $U(x)$. The equilibrium is stable where $U(x)$ is a local minimum and the equilibrium is unstable where $U(x)$ is a local maximum.

SET UP: dU/dx is the slope of the graph of U versus x. $K = E - U$, so K is a maximum when U is a minimum. The maximum x is where $E = U$.

EXECUTE: **(a)** The slope of the U *vs*. x curve is negative at point A, so F_x is positive because $F_x = -dU/dx$.

(b) The slope of the curve at point B is positive, so the force is negative.

(c) The kinetic energy is a maximum when the potential energy is a minimum, and that figures to be at around 0.75 m.

(d) The curve at point C looks pretty close to flat, so the force is zero.

(e) The object had zero kinetic energy at point A, and in order to reach a point with more potential energy than $U(A)$, the kinetic energy would need to be negative. Kinetic energy is never negative, so the object can never be at any point where the potential energy is larger than $U(A)$. On the graph, that looks to be at about 2.2 m.

(f) The point of minimum potential (found in part (c)) is a stable point, as is the relative minimum near 1.9 m.

(g) The only potential maximum, and hence the only point of unstable equilibrium, is at point C.

EVALUATE: If E is less than U at point C, the particle is trapped in one or the other of the potential "wells" and cannot move from one allowed region of x to the other.

7.77. IDENTIFY: The mechanical energy of the system is conserved, and Newton's second law applies. As the pendulum swings, gravitational potential energy gets transformed to kinetic energy.

SET UP: For circular motion, $F = mv^2/r$. $U_{\text{grav}} = mgh$.

EXECUTE: **(a)** Conservation of mechanical energy gives $mgh = ½\, mv^2 + mgh_0$, where $h_0 = 0.800$ m. Applying Newton's second law at the bottom of the swing gives $T = mv^2/L + mg$. Combining these two equations and solving for T as a function of h gives $T = (2mg/L)h + mg(1 - 2h_0/L)$. In a graph of T versus h, the slope is $2mg/L$. Graphing the data given in the problem, we get the graph shown in Figure 7.77. Using the best-fit equation, we get $T = (9.293\text{ N/m})h + 257.3$ N. Therefore $2mg/L = 9.293$ N/m. Using $mg = 265$ N and solving for L, we get $L = 2(265\text{ N})/(9.293\text{ N/m}) = 57.0$ m.

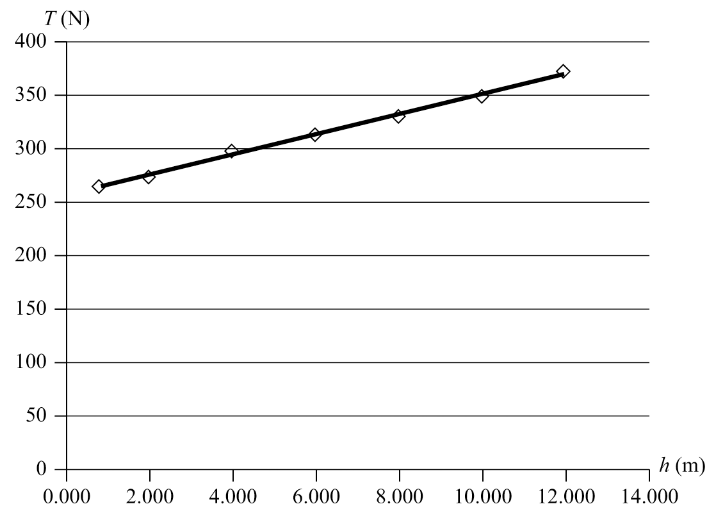


Figure 7.77

(b) $T_{\max} = 822$ N, so $T = T_{\max}/2 = 411$ N. We use the equation for the graph with $T = 411$ N and solve for h. $411\text{ N} = (9.293\text{ N/m})h + 257.3$ N, which gives $h = 16.5$ m.

(c) The pendulum is losing energy because negative work is being done on it by friction with the air and at the point of contact where it swings.

EVALUATE: The length of this pendulum may seem extremely large, but it is not unreasonable for a museum exhibit, which can cover a height of several floor levels.

7.78. IDENTIFY: Friction does negative work, and we can use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

SET UP: $U_1 + W_{\text{other}} = K_2$

EXECUTE: **(a)** Using $K_2 = U_1 + W_{\text{other}}$ gives $\frac{1}{2}mv^2 = mgh - (\mu_k mg\cos\theta)s$ and geometry gives $s = \frac{h}{\sin\theta}$. Combining these equations and solving for h gives $h = \frac{v^2}{2g\left(1 - \frac{\mu_k}{\tan\theta}\right)}$. For each material, θ = 52.0° and $v = 4.00$ m/s. Using the coefficients of sliding friction from the table in the problem, this formula gives the following results for h. (i) 0.92 m (ii) 1.1 m (iii) 2.4 m.

(b) The mass divides out, so h is unchanged and remains at 1.1 m.

(c) In the formula for h in part (a), we solve for v^2 giving $v^2 = 2gh\left(1 - \frac{\mu_k}{\tan\theta}\right)$. As θ increases (but h remains the same), $\tan\theta$ increases, so the quantity in parentheses increases since $\tan\theta$ is in the denominator. Therefore v increases.

EVALUATE: The answer in (c) makes physical sense because with h constant, a larger value for θ means that the normal force decreases so the magnitude of the friction force also decreases, and therefore friction is less able to oppose the motion of the block as it slides down the slope.

7.79. **IDENTIFY:** For a conservative force, mechanical energy is conserved and we can relate the force to its potential energy function.

SET UP: $F_x = -dU/dx$.

EXECUTE: **(a)** $U + K = E =$ constant. If two points have the same kinetic energy, they must have the same potential energy since the sum of U and K is constant. Since the kinetic energy curve symmetric, the potential energy curve must also be symmetric.

(b) At $x = 0$ we can see from the graph with the problem that $E = K + 0 = 0.14$ J. Since E is constant, if $K = 0$ at $x = -1.5$ m, then U must be equal to 0.14 J at that point.

(c) $U(x) = E - K(x) = 0.14\text{ J} - K(x)$, so the graph of $U(x)$ is like the sketch in Figure 7.79.

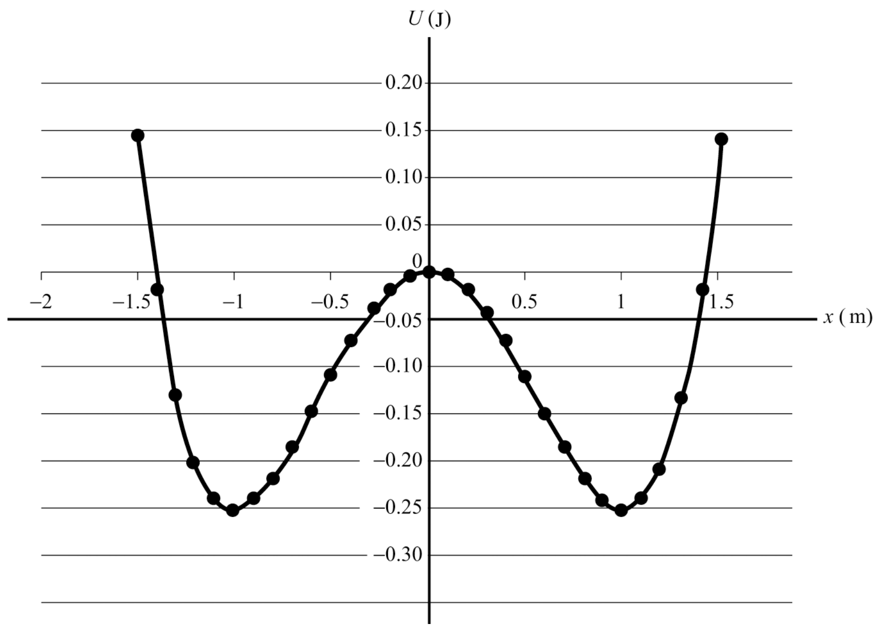


Figure 1.79

(d) Since $F_x = -dU/dx$, $F(x) = 0$ at $x = 0$, +1.0 m, and –1.0 m.

(e) $F(x)$ is positive when the slope of the $U(x)$ curve is negative, and $F(x)$ is negative when the slope of the $U(x)$ curve is positive. Therefore $F(x)$ is positive between $x = -1.5$ m and $x = -1.0$ m and between $x = 0$ and $x = 1.0$ m. $F(x)$ is negative between $x = -1.0$ m and 0 and between $x = 1.0$ m and $x = 1.5$ m.

(f) When released from $x = -1.30$ m, the sphere will move to the right until it reaches $x = -0.55$ m, at which point it has 0.12 J of potential energy, the same as at is original point of release.

EVALUATE: Even though we do not have the equation of the kinetic energy function, we can still learn much about the behavior of the system by studying its graph.

7.80. IDENTIFY: $K = E - U$ determines $v(x)$.

SET UP: v is a maximum when U is a minimum and v is a minimum when U is a maximum. $F_x = -dU/dx$. The extreme values of x are where $E = U(x)$.

EXECUTE: (a) Eliminating β in favor of α and $x_0 (\beta = \alpha/x_0)$,

$$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x} = \frac{\alpha}{x_0^2}\frac{x_0^2}{x^2} - \frac{\alpha}{x_0 x} = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right)\right].$$

$U(x_0) = \left(\frac{\alpha}{x_0^2}\right)(1-1) = 0.$ $U(x)$ is positive for $x < x_0$ and negative for $x > x_0$ (α and β must be taken as positive). The graph of $U(x)$ is sketched in Figure 7.80a.

(b) $v(x) = \sqrt{-\frac{2}{m}U} = \sqrt{\left(\frac{2\alpha}{mx_0^2}\right)\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2\right)}$. The proton moves in the positive x-direction, speeding up until it reaches a maximum speed (see part (c)), and then slows down, although it never stops. The minus sign in the square root in the expression for $v(x)$ indicates that the particle will be found only in the region where $U < 0$, that is, $x > x_0$. The graph of $v(x)$ is sketched in Figure 7.80b.

(c) The maximum speed corresponds to the maximum kinetic energy, and hence the minimum potential energy. This minimum occurs when $\frac{dU}{dx} = 0$, or $\frac{dU}{dx} = \frac{\alpha}{x_0}\left[-2\left(\frac{x_0}{x}\right)^3 + \left(\frac{x_0}{x}\right)^2\right] = 0,$

which has the solution $x = 2x_0$. $U(2x_0) = -\frac{\alpha}{4x_0^2}$, so $v = \sqrt{\frac{\alpha}{2mx_0^2}}$.

(d) The maximum speed occurs at a point where $\frac{dU}{dx} = 0$, and since $F_x = -\frac{dU}{dx}$, the force at this point is zero.

(e) $x_1 = 3x_0$, and $U(3x_0) = -\frac{2\alpha}{9x_0^2}$.

$$v(x) = \sqrt{\frac{2}{m}(U(x_1) - U(x))} = \sqrt{\frac{2}{m}\left[\left(\frac{-2}{9}\frac{\alpha}{x_0^2}\right) - \frac{\alpha}{x_0^2}\left(\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right)\right]} = \sqrt{\frac{2\alpha}{mx_0^2}\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2 - \frac{2}{9}\right)}.$$

The particle is confined to the region where $U(x) < U(x_1)$. The maximum speed still occurs at $x = 2x_0$, but now the particle will oscillate between x_1 and some minimum value (see part (f)).

(f) Note that $U(x) - U(x_1)$ can be written as

$$\frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right) + \left(\frac{2}{9}\right)\right] = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right) - \frac{1}{3}\right]\left[\left(\frac{x_0}{x}\right) - \frac{2}{3}\right],$$

which is zero (and hence the kinetic energy is zero) at $x = 3x_0 = x_1$ and $x = \frac{3}{2}x_0$. Thus, when the particle is released from x_0, it goes on to infinity, and doesn't reach any maximum distance. When released from x_1, it oscillates between $\frac{3}{2}x_0$ and $3x_0$.

EVALUATE: In each case the proton is released from rest and $E = U(x_i)$, where x_i is the point where it is released. When $x_i = x_0$ the total energy is zero. When $x_i = x_1$ the total energy is negative. $U(x) \to 0$ as $x \to \infty$, so for this case the proton can't reach $x \to \infty$ and the maximum x it can have is limited.

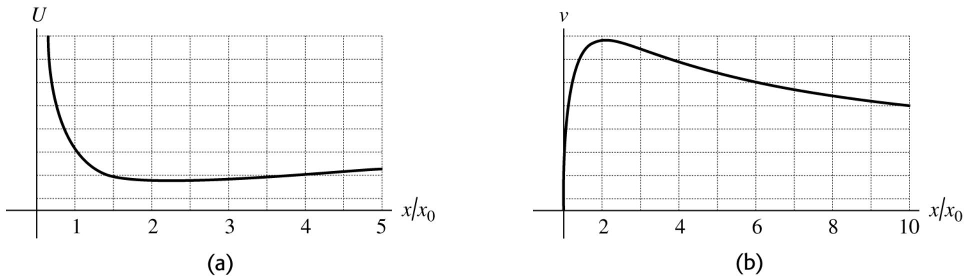


Figure 7.80

7.81. **IDENTIFY:** We model the DNA molecule as an ideal spring.
SET UP: Hooke's law is $F = kx$.
EXECUTE: Since F is proportional to x, if a 3.0-pN force causes a 0.10-nm deflection, a 6.0-pN force, which is twice as great, should use twice as much deflection, or 0.2 nm. This makes choice (c) correct.
EVALUATE: A simple model can give rough but often meaningful insight into the behavior of a complicated system.

7.82. **IDENTIFY** and **SET UP:** If a system obeys Hooke's law, a graph of force versus displacement will be a straight line through the origin having positive slope equal to the force constant.
EXECUTE: The graph is a straight line. Reading its slope from the graph gives (2.0 pN)/(20 nm) = 0.1 pN/nm, which makes choice (b) correct.
EVALUATE: The molecule would obey Hooke's law only over a restricted range of displacements.

7.83. **IDENTIFY** and **SET UP:** The energy is the area under the force-displacement curve.
EXECUTE: Using the area under the triangular section from 0 to 50 nm, we have
$A = \frac{1}{2}(5.0\text{ pN})(50\text{ nm}) = 1.25\times10^{-19}\text{ J} \approx 1.2\times10^{-19}\text{ J}$, which makes choice (b) correct.
EVALUATE: This amount of energy is quite small, but recall that this is the energy of a microscopic molecule.

7.84. **IDENTIFY** and **SET UP:** $P = Fv$ and at constant speed $x = vt$. The DNA follows Hooke's law, so $F = kx$.
EXECUTE: $P = Fv = kxv = k(vt)v = kv^2t$. Since k and v are constant, the power is proportional to the time, so the graph of power versus time should be a straight line through the origin, which fits choice (a).
EVALUATE: The power increases with time because the force increases with x and x increases with t.

8

MOMENTUM, IMPULSE, AND COLLISIONS

VP8.6.1. **IDENTIFY:** This problem involves a one-dimensional collision, so we use momentum conservation. The total momentum before the collision must equal the total momentum after the collision.

SET UP: $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$. Call the x-axis positive horizontally to the right.

EXECUTE: **(a)** After the gliders move free of the spring, their total momentum is conserved. Before the gliders were released, they were at rest so their total momentum was zero. This means that their total momentum will be zero later also. So $P_{1x} = P_{2x}$.

$m_A v_{Ax} + m_B v_{Bx} = 0$

$(0.125 \text{ kg})(0.600 \text{ m/s}) + (0.375 \text{ kg})v_{Bx} \quad \rightarrow \quad v_{Bx} = -0.200 \text{ m/s}$, to the left.

(b) Figure VP8.6.1 shows before and after sketches of the collision. Use momentum conservation.

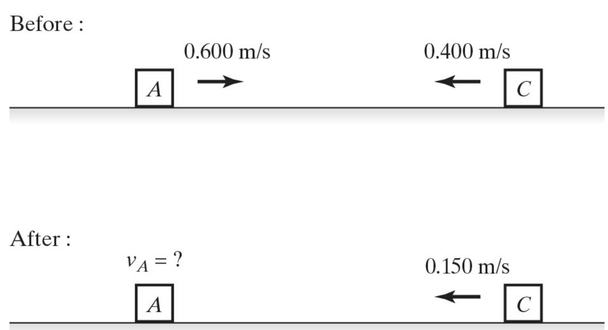


Figure VP8.6.1

$$m_A v_{A1x} + m_C v_{C1x} = m_A v_{A2x} + m_C v_{C2x}$$

$(0.125 \text{ kg})(0.600 \text{ m/s}) + (0.750 \text{ kg})(-0.400 \text{ m/s}) = (0.125 \text{ kg})\, v_{A2x} + (0.750 \text{ kg})(-0.150 \text{ m/s})$

$v_{A2x} = -0.900$ m/s, to the left.

EVALUATE: Since momentum is a vector, we need to pay close attention to the *signs* of its components.

VP8.6.2. **IDENTIFY:** This problem involves a one-dimensional collision, so we use momentum conservation. The total momentum before the collision must equal the total momentum after the collision.

SET UP: $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$. Call the x-axis positive horizontally to the right. Let B stand for Buffy and M for Madeleine. Fig. VP8.6.2 shows before and after sketches.

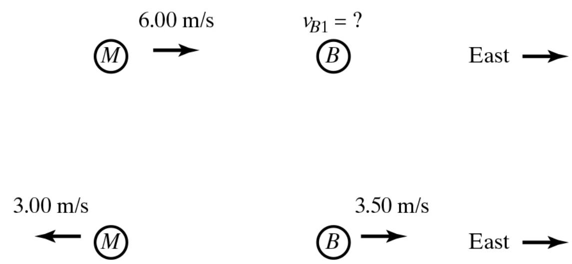


Figure VP8.6.2

EXECUTE: **(a)** $P_{1x} = P_{2x} \quad \rightarrow \quad m_M v_{M1x} + m_B v_{B1x} = m_M v_{M2x} + m_B v_{B2x}$

(65.0 kg)(6.00 m/s) + (55.0 kg) v_{B1x} = (65.0 kg)(–3.00 m/s) + (55.0 kg)(3.50 m/s)

v_{B1x} = –7.14 m/s. The minus sign means it is to the west.

(b) $\Delta v_{Mx} = v_{M2x} - v_{M1x}$ = –3.00 m/s – 6.00 m/s = –9.00 m/s, to the west.

$\Delta v_{Bx} = v_{B2x} - v_{B1x}$ = 3.50 m/s – (–7.14 m/s) = 10.6 m/s, to the east. Buffy has the greater magnitude velocity change.

EVALUATE: It is reasonable that the magnitude of light-weight Buffy's velocity change is greater than that of the heavier Madeleine. Both experience the same magnitude change in *momentum*, but the smaller-mass Buffy needs a larger velocity change than Madeleine so their momentum changes can be equal in magnitude.

VP8.6.3. **IDENTIFY:** This problem involves a two-dimensional collision, so we use momentum conservation. The total momentum before the collision must equal the total momentum after the collision.

SET UP: $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$ and likewise for y-axis. Call A the 2.40-kg stone and B the 4.00-kg stone. Fig. VP8.6.3 shows before and after sketches.

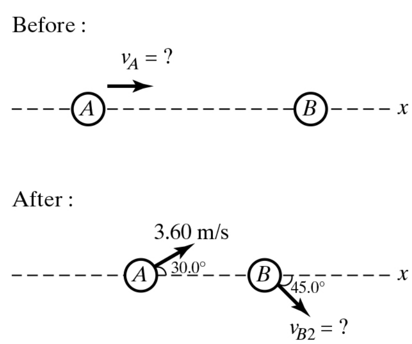


Figure VP8.6.3

EXECUTE: **(a)** $m_A v_{A1y}$ = (2.40 kg)(3.60 m/s) sin 30.0° = 4.32 kg·m/s. The initial y-component of the momentum is zero, so the final y-component must be zero. So p_{B2y} = –4.32 kg·m/s.

(b) $p_{B2y} = m_B v_{B2y} \sin 45.0° \quad \rightarrow \quad$ –4.32 kg·m/s = (4.00 kg) v_{B2y} sin 45.0°

v_{B2y} = –1.53 m/s, so its speed is 1.53 m/s.

(c) $P_x = p_{Ax} + p_{Bx}$

P_x = (2.40 kg)(3.60 m/s) cos 30.0° + (4.00 kg)(1.53 m/s) cos 45.0° = 11.8 kg·m/s.

(d) Initially A has all the x-momentum. Since the x-component of the momentum is conserved (2.40 kg)v_{Ax}= P_x = 11.8 kg·m/s $\quad \rightarrow \quad v_{Ax}$ = 4.92 m/s.

EVALUATE: In a two-dimensional collision, the x-components of the momentum must *always* be treated separately from the y-components.

VP8.6.4. **IDENTIFY:** This problem involves a two-dimensional collision, so we use momentum conservation. The total momentum before the collision must equal the total momentum after the collision.

SET UP: $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$ and likewise for y-axis. Call P the hockey puck and S the stone. Figure VP8.6.4 shows before and after sketches.

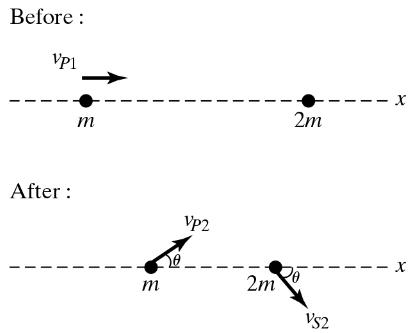


Figure VP8.6.4

EXECUTE: **(a)** The puck and stone must have equal-magnitude y-components of their momentum. $mv_{P2}\sin\theta = (2m)v_{S2} \quad \sin\theta \quad \rightarrow \quad v_{S2}/v_{P2} = ½$.

(b) The x-components of the momentum give $mv_{P1} = mv_{P2}\cos\theta + (2m)v_{S2}\cos\theta$. From part (a) we have $v_{P2} = 2v_{S2}$. Combining these two equations gives $v_{S2} = \dfrac{v_{P1}}{4\cos\theta}$ and $v_{P2} = \dfrac{v_{P1}}{2\cos\theta}$.

EVALUATE: In a two-dimensional collision, the x-components of the momentum must *always* be treated separately from the y-components.

VP8.9.1. **IDENTIFY:** This problem involves a one-dimensional collision in which the colliding objects stick together. This makes it a completely *inelastic* collision. Momentum is conserved.

SET UP: $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$. $K = \dfrac{1}{2}mv^2$.

EXECUTE: **(a)** $K_1 = \dfrac{1}{2}mv_1^2 = \dfrac{1}{2}(1.00\text{ kg})\, v_1^2 = 32.0\text{ J} \quad \rightarrow \quad v_1 = 8.00\text{ m/s}$.

Momentum conservation gives $m_1v_1 = (m_1 + m_2)v_2 \quad \rightarrow \quad (1.00\text{ kg})(8.00\text{ m/s}) = (5.00\text{ kg})v_2$, so $v_2 = 1.60$ m/s.

The lost kinetic energy is

$K_1 - K_2 = K_1 - \dfrac{1}{2}(m_1 + m_2)\, v_2^2 = 32.0\text{ J} - \dfrac{1}{2}(5.00\text{ kg})(1.60\text{ m/s})^2 = 25.6\text{ J}$.

(b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(4.00\text{ kg})v^2 = 32\text{ J} \quad \rightarrow \quad v = 4.00\text{ m/s}$.

Momentum conservation gives $m_1v_1 = (m_1 + m_2)v_2 \quad \rightarrow \quad (4.00\text{ kg})(4.00\text{ m/s}) = (5.00\text{ kg})v_2$

$v_2 = 3.20$ m/s. The loss of kinetic energy is $K_1 - K_2 = K_1 - \dfrac{1}{2}(m_1 + m_2)v^2$

$K_1 - K_2 = 32.0\text{ J} - \dfrac{1}{2}(5.00\text{ kg})(3.20\text{ m/s})^2 = 6.4\text{ J}$.

(c) More kinetic energy is lost if a light object collides with a stationary heavy object, which is case (i).

EVALUATE: Careful! Even though momentum is always conserved during a collision, kinetic energy may or may not be conserved. The amount of energy lost depends on the nature of the collision and the relative masses of the colliding objects.

VP8.9.2. **IDENTIFY:** During the collision, momentum is conserved. After the collision, energy is conserved. We must break this problem up into two parts.

SET UP: During the collision, we use $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$ and momentum conservation. After the collision we use $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, where $W_{\text{tot}} = 0$. Take the x-axis to be horizontal in the direction in which the cheese is originally moving. Figure VP8.9.2 shows the given information.

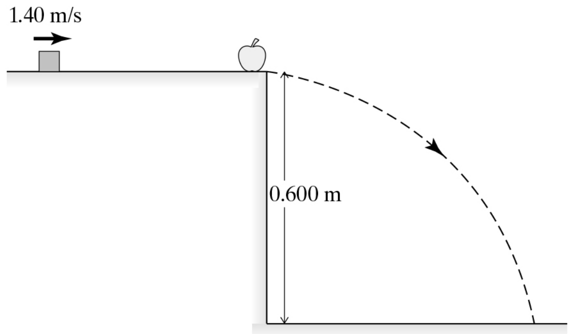


Figure VP8.9.2

EXECUTE: (a) During the collision: $p_{\text{cheese}} = p_{\text{apple + cheese}}$, so $m_c v_c = (m_c + m_a)v$
(0.500 kg)(1.40 m/s) = (0.700 kg)v $\rightarrow$ v = 1.00 m/s. Only momentum is conserved because no net external forces to the apple-cheese system due to the collision. The kinetic energy is *not* conserved because the objects stick together.

(b) Energy conservation after the collision gives $K_1 + U_1 = K_2 + U_2$. Call $y = 0$ at the floor level.
$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv_2^2$, which gives $v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(1.00 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(0.600 \text{ m})} = 3.57$ m/s. Only the total mechanical energy is conserved during the fall. Gravity is an external force acting on the falling object, so its momentum is *not* conserved.

EVALUATE: We cannot do this type of problem in a single step. Some of the initial mechanical energy of the cheese is lost during the collision, and we have no way of knowing how much without examining the collision.

VP8.9.3. **IDENTIFY:** This is a collision, but we do not know if it's elastic or inelastic from the given information. Momentum is conserved during the collision regardless of whether it is elastic or inelastic.

SET UP: During the collision, we use $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$ and momentum conservation. Let C refer to the coffee can and M refer to the macaroni.

EXECUTE: **(a)** The momentum before the collision is equal to the momentum after the collision. Using $m_C v_{C1} = m_C v_{C2} + m_M v_{M2}$ gives (2.40 kg)(1.50 m/s) = (2.40 kg)(0.825 m/s) + (1.20 kg)v_{M2}, so v_{M2} = 1.35 m/s in the $+x$ direction.

(b) $K_{C1} = \frac{1}{2}m_C v_{C1}^2 = \frac{1}{2}(2.40 \text{ kg})(1.50 \text{ m/s})^2 = 2.70$ J.

$K_{C2} = \frac{1}{2}m_C v_{C2}^2 = \frac{1}{2}(2.40 \text{ kg})(0.825 \text{ m/s})^2 = 0.817$ J.

$K_{M2} = \frac{1}{2}m_M v_{M2}^2 = \frac{1}{2}(1.20 \text{ kg})(1.35 \text{ m/s})^2 = 1.09$ J.

(c) $K_1 = K_{C1} = 2.70$ J
$K_2 = K_{C2} + K_{M2} = 0.817 \text{ J} + 1.09 \text{ J} = 1.91$ J
$K_2 < K_1$, so the collision is *inelastic*.

EVALUATE: This collision is inelastic, but it is not *perfectly* elastic because the objects do not stick together.

VP8.9.4. **IDENTIFY:** This is a two-dimensional collision in which the objects stick together, so it is perfectly inelastic. Momentum is conserved during the collision regardless of whether it is elastic or inelastic. We treat the x-components of the momentum separately from the y-components.

SET UP: During the collision, we use $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$ and likewise for P_y. Call the x-axis positive toward the east and y-axis positive toward the north. Let θ be the angle with the $+x$-axis at which the blocks move together after the collision, and let V be their common speed. Figure VP8.9.4 shows before and after sketches.

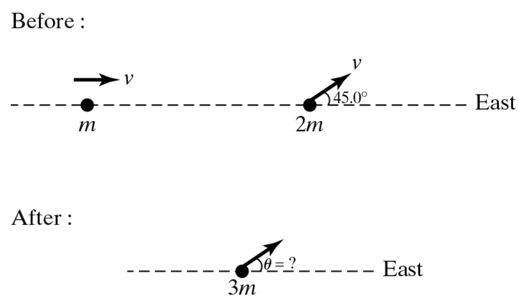


Figure VP8.9.4

EXECUTE: Before the collision the x-components of the momentum are mv and $(2m)v\cos 45.0°$ and the y-component is $(2m)v \sin 45.0°$. After the collision the x-component is $(3m)V\cos\theta$ and the y-component is $(3m)V\sin\theta$. These must also be the components after the collision because momentum is conserved.

Easterly components: $mv + 2mv \cos 45.0° = 3mV\cos\theta \quad \rightarrow \quad v(1 + 2\cos 45.0°) = 3V\cos\theta$

Northerly components: $2mv \sin 45.0° = 3mV\sin\theta \quad \rightarrow \quad 2v \sin 45.0° = 3V\sin\theta$

Dividing these two equations gives $\dfrac{3V\sin\theta}{3V\cos\theta} = \dfrac{2v\sin 45.0°}{v(1+2\cos 45.0°)}$ which simplifies to

$\tan\theta = \dfrac{2\sin 45.0°}{1+2\cos 45.0°} \quad \rightarrow \quad \theta = 30.4°$ north of east.

EVALUATE: The momentum is conserved because the collision generates no external forces on the system of colliding objects. Kinetic energy is not conserved since the objects stick together.

VP8.14.1. **IDENTIFY:** We want to find the location of the center of mass of a system of particles.

SET UP: The x-coordinate of the center of mass is $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ and likewise for the y-coordinate.

EXECUTE: **(a)** $x_{\text{cm}} = \dfrac{(0.500\text{ kg})(0) + (1.25\text{ kg})(0.150\text{ m}) + (0.750\text{ kg})(0.200\text{ m})}{2.50\text{ kg}} = 0.135\text{ m}.$

$y_{\text{cm}} = \dfrac{(0.500\text{ kg})(0) + (1.25\text{ kg})(0.200\text{ m}) + (0.750\text{ kg})(-0.800\text{ m})}{2.50\text{ kg}} = -0.140\text{ m}.$

(b) Calling d the distance, we can calculate the distance between the center of mass and a given particle. For the 0.500-kg particle, we have

$d_{0.500} = \sqrt{(x_{\text{cm}} - x_{0.5})^2 + (y_{\text{cm}} - y_{0.5})^2} = \sqrt{(0.135\text{ m} - 0)^2 + (-0.140\text{ m} - 0)^2} = 0.1945$ m. Likewise for the 1.25-kg particle we find $d_{1.25} = 0.3403$ m. The result is that the center of mass is closest to the 0.500-kg particle.

EVALUATE: A plot on graph paper indicates that the center of mass is closest to the 0.500-kg particle, as we just calculated.

VP8.14.2. **IDENTIFY:** We know the location of the center of mass of a system of particles and the location of two of them. We want to find the location of the third particle. The particles all lie on the x-axis.

SET UP: The x-coordinate of the center of mass is $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$.

EXECUTE: Putting the known x-coordinates into the center of mass formula gives

$$-0.200\text{ m} = \frac{(3.00\text{ kg})(0) + (2.00\text{ kg})(1.50\text{ m}) + (1.20\text{ kg})x}{6.20\text{ kg}} \quad \to \quad x = -3.53\text{ m}.$$

EVALUATE: Since x_{cm} is negative, the 1.20-kg object must lie on the $-x$ side of the origin and farther from the origin than the center of mass, which it does. So our result is reasonable.

VP8.14.3. **IDENTIFY:** The force of the spring is internal to the two-glider system, so the center of mass of that system does not move.

SET UP: Use $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$. Take the origin as the original position of the center of mass of the gliders, which makes $x_{\text{cm}} = 0$.

EXECUTE: $0 = \dfrac{(0.125\text{ kg})(-0.960\text{ m}) + (0.500\text{ kg})x}{0.625\text{ kg}} \quad \to \quad x = 0.240\text{ m}.$

EVALUATE: Since B is more massive than A,it should have moved a shorter distance since both of them felt the same force from the spring. This agrees with our result.

VP8.14.4. **IDENTIFY:** The objects exert forces on each other, but no external forces act on the system of three objects. Therefore the center of mass of this system does not move.

SET UP: The x-coordinate of the center of mass is $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$ and likewise for the y-coordinate. Using the initial conditions, first find the location of the center of mass of the system. Then find the center of mass (which is still the same) after the objects have moved and use it to find the position x of the third object.

EXECUTE: Initially: $x_{\text{cm}} = \dfrac{m(-L) + m(0) + m(L)}{3m} = 0$ and $y_{\text{cm}} = \dfrac{m(0) + m(L) + m(0)}{3m} = \dfrac{L}{3}$. So the center of mass is at $(0, L/3)$ and does not change location.

For the new arrangement: $x_{\text{cm}} = \dfrac{m(-L/3) + m(L/2) + mx}{3m} = 0 \quad \to \quad x = -\dfrac{L}{6}$.

$y_{\text{cm}} = \dfrac{m(L/4) + m(-L) + my}{3m} = \dfrac{L}{3} \quad \to \quad y = \dfrac{7L}{4}$.

EVALUATE: As a check, use the coordinates of the third mass $(-L/6, 7L/4)$ to calculate the location of the center of mass of the new arrangement. The result should come out $(0, L/3)$.

8.1. **IDENTIFY** and **SET UP:** $p = mv$. $K = \frac{1}{2}mv^2$.

EXECUTE: (a) $p = (10{,}000\text{ kg})(12.0\text{ m/s}) = 1.20 \times 10^5\text{ kg} \cdot \text{m/s}$

(b) (i) $v = \dfrac{p}{m} = \dfrac{1.20 \times 10^5\text{ kg} \cdot \text{m/s}}{2000\text{ kg}} = 60.0\text{ m/s}$. (ii) $\frac{1}{2}m_{\text{T}}v_{\text{T}}^2 = \frac{1}{2}m_{\text{SUV}}v_{\text{SUV}}^2$, so

$$v_{\text{SUV}} = \sqrt{\frac{m_{\text{T}}}{m_{\text{SUV}}}}v_{\text{T}} = \sqrt{\frac{10{,}000\text{ kg}}{2000\text{ kg}}}(12.0\text{ m/s}) = 26.8\text{ m/s}$$

EVALUATE: The SUV must have less speed to have the same kinetic energy as the truck than to have the same momentum as the truck.

8.2. IDENTIFY: Each momentum component is the mass times the corresponding velocity component.
SET UP: Let $+x$ be along the horizontal motion of the shotput. Let $+y$ be vertically upward.
$v_x = v\cos\theta,\ \ v_y = v\sin\theta.$

EXECUTE: The horizontal component of the initial momentum is

$$p_x = mv_x = mv\cos\theta = (7.30\text{ kg})(15.0\text{ m/s})\cos 40.0° = 83.9\text{ kg}\cdot\text{m/s}.$$

The vertical component of the initial momentum is

$$p_y = mv_y = mv\sin\theta = (7.30\text{ kg})(15.0\text{ m/s})\sin 40.0° = 70.4\text{ kg}\cdot\text{m/s}.$$

EVALUATE: The initial momentum is directed at $40.0°$ above the horizontal.

8.3. IDENTIFY and SET UP: We use $p = mv$ and add the respective components.
EXECUTE: (a) $P_x = p_{Ax} + p_{Cx} = 0 + (10.0\text{ kg})(-3.0\text{ m/s}) = -30\text{ kg}\cdot\text{m/s}$

$P_y = p_{Ay} + p_{Cy} = (5.0\text{ kg})(-11.0\text{ m/s}) + 0 = -55\text{ kg}\cdot\text{m/s}$

(b) $P_x = p_{Bx} + p_{Cx} = (6.0\text{ kg})(10.0\text{ m/s}\cos 60°) + (10.0\text{ kg})(-3.0\text{ m/s}) = 0$

$P_y = p_{By} + p_{Cy} = (6.0\text{ kg})(10.0\text{ m/s}\sin 60°) + 0 = 52\text{ kg}\cdot\text{m/s}$

(c) $P_x = p_{Ax} + p_{Bx} + p_{Cx} = 0 + (6.0\text{ kg})(10.0\text{ m/s}\cos 60°) + (10.0\text{ kg})(-3.0\text{ m/s}) = 0$

$P_y = p_{Ay} + p_{By} + p_{Cy} = (5.0\text{ kg})(-11.0\text{ m/s}) + (6.0\text{ kg})(10.0\text{ m/s}\sin 60°) + 0 = -3.0\text{ kg}\cdot\text{m/s}$

EVALUATE: A has no x-component of momentum so P_x is the same in (b) and (c). C has no y-component of momentum so P_y in (c) is the sum of P_y in (a) and (b).

8.4. IDENTIFY: For each object $\vec{p} = m\vec{v}$ and the net momentum of the system is $\vec{P} = \vec{p}_A + \vec{p}_B$. The momentum vectors are added by adding components. The magnitude and direction of the net momentum is calculated from its x- and y-components.
SET UP: Let object A be the pickup and object B be the sedan. $v_{Ax} = -14.0\text{ m/s},\ v_{Ay} = 0.\ v_{Bx} = 0,$ $v_{By} = +23.0\text{ m/s}.$

EXECUTE: (a) $P_x = p_{Ax} + p_{Bx} = m_A v_{Ax} + m_B v_{Bx} = (2500\text{ kg})(-14.0\text{ m/s}) + 0 = -3.50\times 10^4\text{ kg}\cdot\text{m/s}$

$$P_y = p_{Ay} + p_{By} = m_A v_{Ay} + m_B v_{By} = (1500\text{ kg})(+23.0\text{ m/s}) = +3.45\times 10^4\text{ kg}\cdot\text{m/s}$$

(b) $P = \sqrt{P_x^2 + P_y^2} = 4.91\times 10^4\text{ kg}\cdot\text{m/s}$. From Figure 8.4, $\tan\theta = \left|\dfrac{P_x}{P_y}\right| = \dfrac{3.50\times 10^4\text{ kg}\cdot\text{m/s}}{3.45\times 10^4\text{ kg}\cdot\text{m/s}}$ and

$\theta = 45.4°$. The net momentum has magnitude $4.91\times 10^4\text{ kg}\cdot\text{m/s}$ and is directed at $45.4°$ west of north.

EVALUATE: The momenta of the two objects must be added as vectors. The momentum of one object is west and the other is north. The momenta of the two objects are nearly equal in magnitude, so the net momentum is directed approximately midway between west and north.

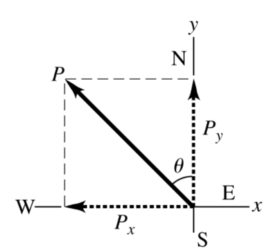


Figure 8.4

8.5. **IDENTIFY:** For each object, $\vec{p} = m\vec{v}$ and $K = \frac{1}{2}mv^2$. The total momentum is the vector sum of the momenta of each object. The total kinetic energy is the scalar sum of the kinetic energies of each object.
SET UP: Let object A be the 110 kg lineman and object B the 125 kg lineman. Let $+x$ be to the right, so $v_{Ax} = +2.75$ m/s and $v_{Bx} = -2.60$ m/s.

EXECUTE: **(a)** $P_x = m_A v_{Ax} + m_B v_{Bx} = (110\text{ kg})(2.75\text{ m/s}) + (125\text{ kg})(-2.60\text{ m/s}) = -22.5\text{ kg}\cdot\text{m/s}$. The net momentum has magnitude $22.5\text{ kg}\cdot\text{m/s}$ and is directed to the left.

(b) $K = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(110\text{ kg})(2.75\text{ m/s})^2 + \frac{1}{2}(125\text{ kg})(2.60\text{ m/s})^2 = 838\text{ J}$

EVALUATE: The kinetic energy of an object is a scalar and is never negative. It depends only on the magnitude of the velocity of the object, not on its direction. The momentum of an object is a vector and has both magnitude and direction. When two objects are in motion, their total kinetic energy is greater than the kinetic energy of either one. But if they are moving in opposite directions, the net momentum of the system has a smaller magnitude than the magnitude of the momentum of either object.

8.6. **IDENTIFY:** We know the contact time of the ball with the racket, the change in velocity of the ball, and the mass of the ball. From this information we can use the fact that the impulse is equal to the change in momentum to find the force exerted on the ball by the racket.
SET UP: $J_x = \Delta p_x$ and $J_x = F_x \Delta t$. In part (a), take the $+x$-direction to be along the final direction of motion of the ball. The initial speed of the ball is zero. In part (b), take the $+x$-direction to be in the direction the ball is traveling before it is hit by the opponent's racket.

EXECUTE: **(a)** $J_x = mv_{2x} - mv_{1x} = (57\times10^{-3}\text{ kg})(73\text{ m/s} - 0) = 4.16\text{ kg}\cdot\text{m/s}$. Using $J_x = F_x\Delta t$ gives

$$F_x = \frac{J_x}{\Delta t} = \frac{4.16\text{ kg}\cdot\text{m/s}}{30.0\times10^{-3}\text{ s}} = 140\text{ N}.$$

(b) $J_x = mv_{2x} - mv_{1x} = (57\times10^{-3}\text{ kg})(-55\text{ m/s} - 73\text{ m/s}) = -7.30\text{ kg}\cdot\text{m/s}$.

$$F_x = \frac{J_x}{\Delta t} = \frac{-7.30\text{ kg}\cdot\text{m/s}}{30.0\times10^{-3}\text{ s}} = -240\text{ N}.$$

EVALUATE: The signs of J_x and F_x show their direction. $140\text{ N} = 31\text{ lb}$. This very attainable force has a large effect on the light ball. 140 N is 250 times the weight of the ball.

8.7. **IDENTIFY:** The average force on an object and the object's change in momentum are related by $(F_{\text{av}})_x = \dfrac{J_x}{\Delta t}$. The weight of the ball is $w = mg$.

SET UP: Let $+x$ be in the direction of the final velocity of the ball, so $v_{1x} = 0$ and $v_{2x} = 25.0$ m/s.

EXECUTE: $(F_{\text{av}})_x(t_2 - t_1) = mv_{2x} - mv_{1x}$ gives $(F_{\text{av}})_x = \dfrac{mv_{2x} - mv_{1x}}{t_2 - t_1} = \dfrac{(0.0450\text{ kg})(25.0\text{ m/s})}{2.00\times10^{-3}\text{ s}} = 562\text{ N}$.

$w = (0.0450\text{ kg})(9.80\text{ m/s}^2) = 0.441\text{ N}$. The force exerted by the club is much greater than the weight of the ball, so the effect of the weight of the ball during the time of contact is not significant.
EVALUATE: Forces exerted during collisions typically are very large but act for a short time.

8.8. **IDENTIFY:** The change in momentum, the impulse, and the average force are related by $J_x = \Delta p_x$ and $(F_{\text{av}})_x = \dfrac{J_x}{\Delta t}$.

SET UP: Let the direction in which the batted ball is traveling be the $+x$-direction, so $v_{1x} = -45.0$ m/s and $v_{2x} = 55.0$ m/s.

EXECUTE: **(a)** $\Delta p_x = p_{2x} - p_{1x} = m(v_{2x} - v_{1x}) = (0.145\text{ kg})[55.0\text{ m/s} - (-45.0\text{ m/s})] = 14.5\text{ kg}\cdot\text{m/s}$.
$J_x = \Delta p_x$, so $J_x = 14.5\text{ kg}\cdot\text{m/s}$. Both the change in momentum and the impulse have magnitude $14.5\text{ kg}\cdot\text{m/s}$.

(b) $(F_{\text{av}})_x = \dfrac{J_x}{\Delta t} = \dfrac{14.5\ \text{kg}\cdot\text{m/s}}{2.00\times10^{-3}\ \text{s}} = 7250\ \text{N}.$

EVALUATE: The force is in the direction of the momentum change.

8.9. **IDENTIFY:** Use $J_x = p_{2x} - p_{1x}$. We know the initial momentum and the impulse so can solve for the final momentum and then the final velocity.

SET UP: Take the x-axis to be toward the right, so $v_{1x} = +3.00$ m/s. Use $J_x = F_x\Delta t$ to calculate the impulse, since the force is constant.

EXECUTE: **(a)** $J_x = p_{2x} - p_{1x}$

$$J_x = F_x(t_2 - t_1) = (+25.0\ \text{N})(0.050\ \text{s}) = +1.25\ \text{kg}\cdot\text{m/s}$$

Thus $p_{2x} = J_x + p_{1x} = +1.25\ \text{kg}\cdot\text{m/s} + (0.160\ \text{kg})(+3.00\ \text{m/s}) = +1.73\ \text{kg}\cdot\text{m/s}$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{1.73\ \text{kg}\cdot\text{m/s}}{0.160\ \text{kg}} = +10.8\ \text{m/s (to the right)}$$

(b) $J_x = F_x(t_2 - t_1) = (-12.0\ \text{N})(0.050\ \text{s}) = -0.600\ \text{kg}\cdot\text{m/s}$ (negative since force is to left)

$$p_{2x} = J_x + p_{1x} = -0.600\ \text{kg}\cdot\text{m/s} + (0.160\ \text{kg})(+3.00\ \text{m/s}) = -0.120\ \text{kg}\cdot\text{m/s}$$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{-0.120\ \text{kg}\cdot\text{m/s}}{0.160\ \text{kg}} = -0.75\ \text{m/s (to the left)}$$

EVALUATE: In part (a) the impulse and initial momentum are in the same direction and v_x increases. In part (b) the impulse and initial momentum are in opposite directions and the velocity decreases.

8.10. **IDENTIFY:** Apply $J_x = \Delta p_x = mv_{2x} - mv_{1x}$ and $J_y = \Delta p_y = mv_{2y} - mv_{1y}$ to relate the change in momentum to the components of the average force on it.

SET UP: Let $+x$ be to the right and $+y$ be upward.

EXECUTE: $J_x = \Delta p_x = mv_{2x} - mv_{1x} = (0.145\ \text{kg})[-(52.0\ \text{m/s})\cos 30° - 40.0\ \text{m/s}] = -12.33\ \text{kg}\cdot\text{m/s}.$

$J_y = \Delta p_y = mv_{2y} - mv_{1y} = (0.145\ \text{kg})[(52.0\ \text{m/s})\sin 30° - 0] = 3.770\ \text{kg}\cdot\text{m/s}.$

The horizontal component is $12.33\ \text{kg}\cdot\text{m/s}$, to the left and the vertical component is $3.770\ \text{kg}\cdot\text{m/s}$, upward.

$F_{\text{av-}x} = \dfrac{J_x}{\Delta t} = \dfrac{-12.33\ \text{kg}\cdot\text{m/s}}{1.75\times10^{-3}\ \text{s}} = -7050\ \text{N}.$ $F_{\text{av-}y} = \dfrac{J_y}{\Delta t} = \dfrac{3.770\ \text{kg}\cdot\text{m/s}}{1.75\times10^{-3}\ \text{s}} = 2150\ \text{N}.$

The horizontal component is 7050 N, to the left, and the vertical component is 2150 N, upward.

EVALUATE: The ball gains momentum to the left and upward and the force components are in these directions.

8.11. **IDENTIFY:** The force is not constant so $\vec{J} = \int_{t_1}^{t_2} \vec{F}dt$. The impulse is related to the change in velocity by $J_x = m(v_{2x} - v_{1x})$.

SET UP: Only the x-component of the force is nonzero, so $J_x = \int_{t_1}^{t_2} F_x dt$ is the only nonzero component of $\vec{J}$. $J_x = m(v_{2x} - v_{1x})$. $t_1 = 2.00$ s, $t_2 = 3.50$ s.

EXECUTE: **(a)** $A = \dfrac{F_x}{t^2} = \dfrac{781.25\ \text{N}}{(1.25\ \text{s})^2} = 500\ \text{N/s}^2.$

(b) $J_x = \int_{t_1}^{t_2} At^2 dt = \frac{1}{3}A(t_2^3 - t_1^3) = \frac{1}{3}(500\ \text{N/s}^2)([3.50\ \text{s}]^3 - [2.00\ \text{s}]^3) = 5.81\times10^3\ \text{N}\cdot\text{s}.$

(c) $\Delta v_x = v_{2x} - v_{1x} = \dfrac{J_x}{m} = \dfrac{5.81\times10^3\ \text{N}\cdot\text{s}}{2150\ \text{kg}} = 2.70\ \text{m/s}.$ The x-component of the velocity of the rocket increases by 2.70 m/s.

EVALUATE: The change in velocity is in the same direction as the impulse, which in turn is in the direction of the net force. In this problem the net force equals the force applied by the engine, since that is the only force on the rocket.

8.12. **IDENTIFY:** This problem requires the graphical interpretation of impulse and momentum.

SET UP: The impulse is the area under the curve on a graph of force versus time. Impulse is equal to the change in momentum: $J_x = p_{2x} - p_{1x}$, where $p_x = mv_x$. Fig. 8.12 shows a graph of F versus t.

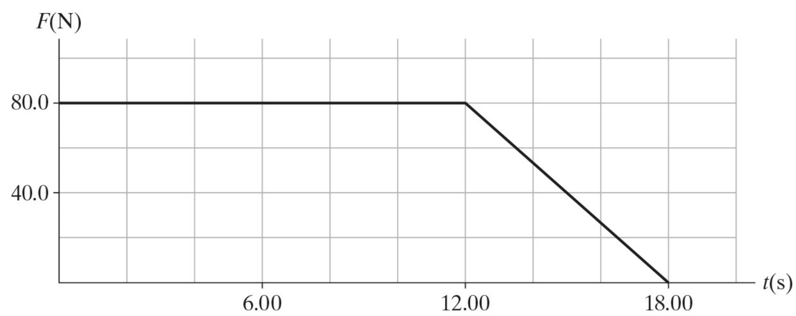


Figure 8.12

EXECUTE: To compute the area under the F-t graph, break it into two figures, a rectangle and a triangle. Using the numbers on the graph, we have $J_x = (80.0\text{ N})(12.0\text{ s}) + \frac{1}{2}(80.0\text{ N})(6.0\text{ s}) = 1200$ $\text{N}\cdot\text{s}$. $J_x = p_{2x} - p_{1x} = mv - 0 \quad \rightarrow \quad (80.0\text{ kg})v = 1200\ \text{N}\cdot\text{s} \quad \rightarrow \quad v = 15.0\text{ m/s}$.

EVALUATE: The longer a force acts, the more it changes the momentum of an object.

8.13. **IDENTIFY:** The force is constant during the 1.0 ms interval that it acts, so $\vec{J} = \vec{F}\Delta t$.
$\vec{J} = \vec{p}_2 - \vec{p}_1 = m(\vec{v}_2 - \vec{v}_1)$.

SET UP: Let $+x$ be to the right, so $v_{1x} = +5.00\text{ m/s}$. Only the x-component of $\vec{J}$ is nonzero, and $J_x = m(v_{2x} - v_{1x})$.

EXECUTE: **(a)** The magnitude of the impulse is $J = F\Delta t = (2.50\times 10^3\text{ N})(1.00\times 10^{-3}\text{ s}) = 2.50\text{ N}\cdot\text{s}$. The direction of the impulse is the direction of the force.

(b) (i) $v_{2x} = \dfrac{J_x}{m} + v_{1x}$. $J_x = +2.50\text{ N}\cdot\text{s}$. $v_{2x} = \dfrac{+2.50\text{ N}\cdot\text{s}}{2.00\text{ kg}} + 5.00\text{ m/s} = 6.25\text{ m/s}$. The stone's velocity has magnitude 6.25 m/s and is directed to the right. (ii) Now $J_x = -2.50\text{ N}\cdot\text{s}$ and

$$v_{2x} = \frac{-2.50\text{ N}\cdot\text{s}}{2.00\text{ kg}} + 5.00\text{ m/s} = 3.75\text{ m/s}.$$ The stone's velocity has magnitude 3.75 m/s and is directed to the right.

EVALUATE: When the force and initial velocity are in the same direction the speed increases, and when they are in opposite directions the speed decreases.

8.14. **IDENTIFY:** We know the force acting on a box as a function of time and its initial momentum and want to find its momentum at a later time. The target variable is the final momentum.

SET UP: Use $\int_{t_1}^{t_2} \vec{F}(t)dt = \vec{p}_2 - \vec{p}_1$ to find $\vec{p}_2$ since we know $\vec{p}_1$ and $\vec{F}(t)$.

EXECUTE: $\vec{p}_1 = (-3.00 \text{ kg}\cdot\text{m/s})\hat{i} + (4.00 \text{ kg}\cdot\text{m/s})\hat{j}$ at $t_1 = 0$, and $t_2 = 2.00$ s. Work with the components of the force and momentum. $\int_{t_1}^{t_2} F_x(t)dt = (0.280 \text{ N/s})\int_{t_1}^{t_2} t\,dt = (0.140 \text{ N/s})t_2^2 = 0.560 \text{ N}\cdot\text{s}$

$p_{2x} = p_{1x} + 0.560 \text{ N}\cdot\text{s} = -3.00 \text{ kg}\cdot\text{m/s} + 0.560 \text{ N}\cdot\text{s} = -2.44 \text{ kg}\cdot\text{m/s}.$

$\int_{t_1}^{t_2} F_y(t)dt = (-0.450 \text{ N/s}^2)\int_{t_1}^{t_2} t^2 dt = (-0.150 \text{ N/s}^2)t_2^3 = -1.20 \text{ N}\cdot\text{s}.$

$p_{2y} = p_{1y} + (-1.20 \text{ N}\cdot\text{s}) = 4.00 \text{ kg}\cdot\text{m/s} + (-1.20 \text{ N}\cdot\text{s}) = +2.80 \text{ kg}\cdot\text{m/s}.$ So

$\vec{p}_2 = (-2.44 \text{ kg}\cdot\text{m/s})\hat{i} + (2.80 \text{ kg}\cdot\text{m/s})\hat{j}$

EVALUATE: Since the given force has *x*- and *y*-components, it changes both components of the box's momentum.

8.15. **IDENTIFY:** This problem requires the use of momentum, impulse, and kinetic energy.

SET UP: $p_x = mv_x$, $J_x = p_{2x} - p_{1x}$, $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** $p_x = mv_x = (40.0 \text{ kg})(20.0 \text{ m/s}) = 800 \text{ kg}\cdot\text{m/s}$.

(b) $K = \frac{1}{2}mv^2 = \frac{1}{2}(40.0 \text{ kg})(20.0 \text{ m/s})^2 = 8.00\times10^3$ J.

(c) $J_x = p_{2x} - p_{1x} = 0 - p_{1x}$, so $Ft = -p_1$ gives $F(5.00 \text{ s}) = -800 \text{ kg}\cdot\text{m/s}$. $F = -160$ N. The minus sign tells us that the force is opposite to her velocity.

EVALUATE: Unlike kinetic energy, momentum has components that can be negative as well as positive.

8.16. **IDENTIFY:** Apply conservation of momentum to the system of the astronaut and tool.

SET UP: Let *A* be the astronaut and *B* be the tool. Let $+x$ be the direction in which she throws the tool, so $v_{B2x} = +3.20$ m/s. Assume she is initially at rest, so $v_{A1x} = v_{B1x} = 0$. Solve for v_{A2x}.

EXECUTE: $P_{1x} = P_{2x}$. $P_{1x} = m_A v_{A1x} + m_B v_{B1x} = 0$. $P_{2x} = m_A v_{A2x} + m_B v_{B2x} = 0$ and

$v_{A2x} = -\frac{m_B v_{A2x}}{m_A} = -\frac{(2.25 \text{ kg})(3.20 \text{ m/s})}{68.5 \text{ kg}} = -0.105$ m/s. Her speed is 0.105 m/s and she moves opposite to the direction in which she throws the tool.

EVALUATE: Her mass is much larger than that of the tool, so to have the same magnitude of momentum as the tool her speed is much less.

8.17. **IDENTIFY:** Since the rifle is loosely held there is no net external force on the system consisting of the rifle, bullet, and propellant gases and the momentum of this system is conserved. Before the rifle is fired everything in the system is at rest and the initial momentum of the system is zero.

SET UP: Let $+x$ be in the direction of the bullet's motion. The bullet has speed $601 \text{ m/s} - 1.85 \text{ m/s} = 599 \text{ m/s}$ relative to the earth. $P_{2x} = p_{rx} + p_{bx} + p_{gx}$, the momenta of the rifle, bullet, and gases. $v_{rx} = -1.85$ m/s and $v_{bx} = +599$ m/s.

EXECUTE: $P_{2x} = P_{1x} = 0$. $p_{rx} + p_{bx} + p_{gx} = 0$.

$p_{gx} = -p_{rx} - p_{bx} = -(2.80 \text{ kg})(-1.85 \text{ m/s}) - (0.00720 \text{ kg})(599 \text{ m/s})$ and

$p_{gx} = +5.18 \text{ kg}\cdot\text{m/s} - 4.31 \text{ kg}\cdot\text{m/s} = 0.87 \text{ kg}\cdot\text{m/s}.$ The propellant gases have momentum $0.87 \text{ kg}\cdot\text{m/s}$, in the same direction as the bullet is traveling.

EVALUATE: The magnitude of the momentum of the recoiling rifle equals the magnitude of the momentum of the bullet plus that of the gases as both exit the muzzle.

8.18. **IDENTIFY:** The total momentum of the two skaters is conserved, but not their kinetic energy.

SET UP: There is no horizontal external force so, $P_{i,x} = P_{f,x}$, $p = mv$, $K = ½\, mv^2$.

EXECUTE: **(a)** $P_{i,x} = P_{f,x}$. The skaters are initially at rest so $P_{i,x} = 0$. $0 = m_A(v_{A,f})_x + m_B(v_{B,f})_x$

$(v_{A,f})_x = -\dfrac{m_B(v_{B,f})_x}{m_A} = -\dfrac{(74.0\text{ kg})(1.50\text{ m/s})}{63.8\text{ kg}} = -1.74\text{ m/s}$. The lighter skater travels to the left at 1.74 m/s.

(b) $K_i = 0$. $K_f = \frac{1}{2}m_A v_{A,f}{}^2 + \frac{1}{2}m_B v_{B,f}{}^2 = \frac{1}{2}(63.8\text{ kg})(1.74\text{ m/s})^2 + \frac{1}{2}(74.0\text{ kg})(1.50\text{ m/s})^2 = 180\text{ J}$.

EVALUATE: The kinetic energy of the system was produced by the work the two skaters do on each other.

8.19. **IDENTIFY:** Since drag effects are neglected, there is no net external force on the system of squid plus expelled water, and the total momentum of the system is conserved. Since the squid is initially at rest, with the water in its cavity, the initial momentum of the system is zero. For each object, $K = \frac{1}{2}mv^2$.

SET UP: Let A be the squid and B be the water it expels, so $m_A = 6.50\text{ kg} - 1.75\text{ kg} = 4.75\text{ kg}$. Let $+x$ be the direction in which the water is expelled. $v_{A2x} = -2.50\text{ m/s}$. Solve for v_{B2x}.

EXECUTE: **(a)** $P_{1x} = 0$. $P_{2x} = P_{1x}$, so $0 = m_A v_{A2x} + m_B v_{B2x}$.

$v_{B2x} = -\dfrac{m_A v_{A2x}}{m_B} = -\dfrac{(4.75\text{ kg})(-2.50\text{ m/s})}{1.75\text{ kg}} = +6.79\text{ m/s}$.

(b) $K_2 = K_{A2} + K_{B2} = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(4.75\text{ kg})(2.50\text{ m/s})^2 + \frac{1}{2}(1.75\text{ kg})(6.79\text{ m/s})^2 = 55.2\text{ J}$. The initial kinetic energy is zero, so the kinetic energy produced is $K_2 = 55.2\text{ J}$.

EVALUATE: The two objects end up with momenta that are equal in magnitude and opposite in direction, so the total momentum of the system remains zero. The kinetic energy is created by the work done by the squid as it expels the water.

8.20. **IDENTIFY:** Apply conservation of momentum to the system of you and the ball. In part (a) both objects have the same final velocity.

SET UP: Let $+x$ be in the direction the ball is traveling initially. $m_A = 0.600\text{ kg}$ (ball). $m_B = 70.0\text{ kg}$ (you).

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $(0.600\text{ kg})(10.0\text{ m/s}) = (0.600\text{ kg} + 70.0\text{ kg})v_2$ so $v_2 = 0.0850\text{ m/s}$.

(b) $P_{1x} = P_{2x}$ gives $(0.600\text{ kg})(10.0\text{ m/s}) = (0.600\text{ kg})(-8.00\text{ m/s}) + (70.0\text{ kg})v_{B2}$ so $v_{B2} = 0.154\text{ m/s}$.

EVALUATE: When the ball bounces off it has a greater change in momentum and you acquire a greater final speed.

8.21. **IDENTIFY:** Apply conservation of momentum to the system of the two pucks.

SET UP: Let $+x$ be to the right.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ says $(0.250\text{ kg})v_{A1} = (0.250\text{ kg})(-0.120\text{ m/s}) + (0.350\text{ kg})(0.650\text{ m/s})$ and $v_{A1} = 0.790\text{ m/s}$.

(b) $K_1 = \frac{1}{2}(0.250\text{ kg})(0.790\text{ m/s})^2 = 0.0780\text{ J}$.

$K_2 = \frac{1}{2}(0.250\text{ kg})(0.120\text{ m/s})^2 + \frac{1}{2}(0.350\text{ kg})(0.650\text{ m/s})^2 = 0.0757\text{ J}$ and $\Delta K = K_2 - K_1 = -0.0023\text{ J}$.

EVALUATE: The total momentum of the system is conserved but the total kinetic energy decreases.

8.22. **IDENTIFY:** Since road friction is neglected, there is no net external force on the system of the two cars and the total momentum of the system is conserved. For each object, $K = \frac{1}{2}mv^2$.

SET UP: Let A be the 1750 kg car and B be the 1450 kg car. Let $+x$ be to the right, so $v_{A1x} = +1.50\text{ m/s}$, $v_{B1x} = -1.10\text{ m/s}$, and $v_{A2x} = +0.250\text{ m/s}$. Solve for v_{B2x}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$. $v_{B2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B}$.

$$v_{B2x} = \frac{(1750 \text{ kg})(1.50 \text{ m/s}) + (1450 \text{ kg})(-1.10 \text{ m/s}) - (1750 \text{ kg})(0.250 \text{ m/s})}{1450 \text{ kg}} = 0.409 \text{ m/s}.$$

After the collision the lighter car is moving to the right with a speed of 0.409 m/s.

(b) $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(1750 \text{ kg})(1.50 \text{ m/s})^2 + \frac{1}{2}(1450 \text{ kg})(1.10 \text{ m/s})^2 = 2846 \text{ J}.$

$$K_2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(1750 \text{ kg})(0.250 \text{ m/s})^2 + \frac{1}{2}(1450 \text{ kg})(0.409 \text{ m/s})^2 = 176 \text{ J}.$$

The change in kinetic energy is $\Delta K = K_2 - K_1 = 176 \text{ J} - 2846 \text{ J} = -2670 \text{ J}.$

EVALUATE: The total momentum of the system is constant because there is no net external force during the collision. The kinetic energy of the system decreases because of negative work done by the forces the cars exert on each other during the collision.

8.23. IDENTIFY: The momentum and the mechanical energy of the system are both conserved. The mechanical energy consists of the kinetic energy of the masses and the elastic potential energy of the spring. The potential energy stored in the spring is transformed into the kinetic energy of the two masses.

SET UP: Let the system be the two masses and the spring. The system is sketched in Figure 8.23, in its initial and final situations. Use coordinates where $+x$ is to the right. Call the masses A and B.

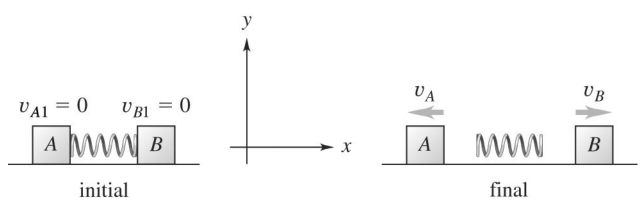


Figure 8.23

EXECUTE: $P_{1x} = P_{2x}$ so $0 = (0.900 \text{ kg})(-v_A) + (0.900 \text{ kg})(v_B)$ and, since the masses are equal, $v_A = v_B$. Energy conservation says the potential energy originally stored in the spring is all converted into kinetic energy of the masses, so $\frac{1}{2}kx_1^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$. Since $v_A = v_B$, this equation gives

$$v_A = x_1\sqrt{\frac{k}{2m}} = (0.200 \text{ m})\sqrt{\frac{175 \text{ N/m}}{2(0.900 \text{ kg})}} = 1.97 \text{ m/s}.$$

EVALUATE: If the objects have different masses they will end up with different speeds. The lighter one will have the greater speed, since they end up with equal magnitudes of momentum.

8.24. IDENTIFY: In part (a) no horizontal force implies P_x is constant. In part (b) use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to find the potential energy initially in the spring.

SET UP: Initially both blocks are at rest.

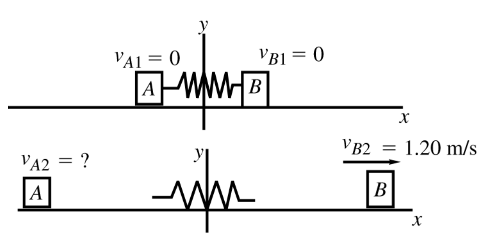


Figure 8.24

EXECUTE: **(a)** $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$

$$0 = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{A2x} = -\left(\frac{m_B}{m_A}\right) v_{B2x} = -\left(\frac{3.00\text{ kg}}{1.00\text{ kg}}\right)(+1.20\text{ m/s}) = -3.60\text{ m/s}$$

Block A has a final speed of 3.60 m/s, and moves off in the opposite direction to B.

(b) Use energy conservation: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

Only the spring force does work so $W_{\text{other}} = 0$ and $U = U_{\text{el}}$.

$K_1 = 0$ (the blocks initially are at rest)

$U_2 = 0$ (no potential energy is left in the spring)

$$K_2 = \tfrac{1}{2} m_A v_{A2}^2 + \tfrac{1}{2} m_B v_{B2}^2 = \tfrac{1}{2}(1.00\text{ kg})(3.60\text{ m/s})^2 + \tfrac{1}{2}(3.00\text{ kg})(1.20\text{ m/s})^2 = 8.64\text{ J}$$

$U_1 = U_{1,\text{el}}$ the potential energy stored in the compressed spring.

Thus $U_{1,\text{el}} = K_2 = 8.64$ J.

EVALUATE: The blocks have equal and opposite momenta as they move apart, since the total momentum is zero. The kinetic energy of each block is positive and doesn't depend on the direction of the block's velocity, just on its magnitude.

8.25. **IDENTIFY:** Since friction at the pond surface is neglected, there is no net external horizontal force, and the horizontal component of the momentum of the system of hunter plus bullet is conserved. Both objects are initially at rest, so the initial momentum of the system is zero. Gravity and the normal force exerted by the ice together produce a net vertical force while the rifle is firing, so the vertical component of momentum is not conserved.

SET UP: Let object A be the hunter and object B be the bullet. Let $+x$ be the direction of the horizontal component of velocity of the bullet. Solve for v_{A2x}.

EXECUTE: **(a)** $v_{B2x} = +965$ m/s. $P_{1x} = P_{2x} = 0$. $0 = m_A v_{A2x} + m_B v_{B2x}$ and

$$v_{A2x} = -\frac{m_B}{m_A} v_{B2x} = -\left(\frac{4.20\times10^{-3}\text{ kg}}{72.5\text{ kg}}\right)(965\text{ m/s}) = -0.0559\text{ m/s}.$$

(b) $v_{B2x} = v_{B2}\cos\theta = (965\text{ m/s})\cos 56.0° = 540$ m/s. $v_{A2x} = -\left(\dfrac{4.20\times10^{-3}\text{ kg}}{72.5\text{ kg}}\right)(540\text{ m/s}) = -0.0313$ m/s.

EVALUATE: The mass of the bullet is much less than the mass of the hunter, so the final mass of the hunter plus gun is still 72.5 kg, to three significant figures. Since the hunter has much larger mass, his final speed is much less than the speed of the bullet.

8.26. **IDENTIFY:** Assume the nucleus is initially at rest. $K = \tfrac{1}{2}mv^2$.

SET UP: Let $+x$ be to the right. $v_{A2x} = -v_A$ and $v_{B2x} = +v_B$.

EXECUTE: **(a)** $P_{2x} = P_{1x} = 0$ gives $m_A v_{A2x} + m_B v_{B2x} = 0$. $v_B = \left(\dfrac{m_A}{m_B}\right) v_A$.

(b) $\dfrac{K_A}{K_B} = \dfrac{\tfrac{1}{2} m_A v_A^2}{\tfrac{1}{2} m_B v_B^2} = \dfrac{m_A v_A^2}{m_B (m_A v_A / m_B)^2} = \dfrac{m_B}{m_A}$.

EVALUATE: The lighter fragment has the greater kinetic energy.

8.27. **IDENTIFY:** Each horizontal component of momentum is conserved. $K = \tfrac{1}{2}mv^2$.

SET UP: Let $+x$ be the direction of Rebecca's initial velocity and let the $+y$ axis make an angle of $36.9°$ with respect to the direction of her final velocity. $v_{\text{D}1x} = v_{\text{D}1y} = 0$. $v_{\text{R}1x} = 13.0$ m/s; $v_{\text{R}1y} = 0$.

$v_{R2x} = (8.00 \text{ m/s})\cos 53.1° = 4.80 \text{ m/s}$; $v_{R2y} = (8.00 \text{ m/s})\sin 53.1° = 6.40 \text{ m/s}$. Solve for v_{D2x} and v_{D2y}.

EXECUTE: (a) $P_{1x} = P_{2x}$ gives $m_R v_{R1x} = m_R v_{R2x} + m_D v_{D2x}$.

$$v_{D2x} = \frac{m_R(v_{R1x} - v_{R2x})}{m_D} = \frac{(45.0 \text{ kg})(13.0 \text{ m/s} - 4.80 \text{ m/s})}{65.0 \text{ kg}} = 5.68 \text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $0 = m_R v_{R2y} + m_D v_{D2y}$. $v_{D2y} = -\frac{m_R}{m_D} v_{R2y} = -\left(\frac{45.0 \text{ kg}}{65.0 \text{ kg}}\right)(6.40 \text{ m/s}) = -4.43 \text{ m/s}$.

The directions of $\vec{v}_{R1}$, $\vec{v}_{R2}$ and $\vec{v}_{D2}$ are sketched in Figure 8.27. $\tan\theta = \left|\frac{v_{D2y}}{v_{D2x}}\right| = \frac{4.43 \text{ m/s}}{5.68 \text{ m/s}}$ and $\theta = 38.0°$. $v_D = \sqrt{v_{D2x}^2 + v_{D2y}^2} = 7.20 \text{ m/s}$.

(b) $K_1 = \frac{1}{2} m_R v_{R1}^2 = \frac{1}{2}(45.0 \text{ kg})(13.0 \text{ m/s})^2 = 3.80 \times 10^3 \text{ J}$.

$K_2 = \frac{1}{2} m_R v_{R2}^2 + \frac{1}{2} m_D v_{D2}^2 = \frac{1}{2}(45.0 \text{ kg})(8.00 \text{ m/s})^2 + \frac{1}{2}(65.0 \text{ kg})(7.20 \text{ m/s})^2 = 3.12 \times 10^3 \text{ J}$.

$\Delta K = K_2 - K_1 = -680 \text{ J}$.

EVALUATE: Each component of momentum is separately conserved. The kinetic energy of the system decreases.

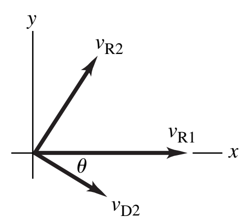


Figure 8.27

8.28. IDENTIFY and **SET UP:** Let the $+x$-direction be horizontal, along the direction the rock is thrown. There is no net horizontal force, so P_x is constant. Let object A be you and object B be the rock.

EXECUTE: $0 = -m_A v_A + m_B v_B \cos 35.0°$ gives $v_A = \frac{m_B v_B \cos 35.0°}{m_A} = 0.421 \text{ m/s}$.

EVALUATE: P_y is not conserved because there is a net external force in the vertical direction; as you throw the rock the normal force exerted on you by the ice is larger than the total weight of the system.

8.29. IDENTIFY: In the absence of a horizontal force, we know that momentum is conserved.
SET UP: $p = mv$. Let $+x$ be the direction you are moving. Before you catch it, the flour sack has no momentum along the x-axis. The total mass of you and your skateboard is 60 kg. You, the skateboard, and the flour sack are all moving with the same velocity, after the catch.
EXECUTE: (a) Since $P_{i,x} = P_{f,x}$, we have $(60 \text{ kg})(4.5 \text{ m/s}) = (62.5 \text{ kg})v_{f,x}$. Solving for the final velocity we obtain $v_{f,x} = 4.3 \text{ m/s}$.

(b) To bring the flour sack up to your speed, you must exert a horizontal force on it. Consequently, it exerts an equal and opposite force on you, which slows you down.
(c) Since you exert a vertical force on the flour sack, your horizontal speed does not change and remains at 4.3 m/s. Since the flour sack is only accelerated in the vertical direction, its horizontal velocity-component remains at 4.3 m/s as well.
EVALUATE: Unless you or the flour sack are deflected by an outside force, you will need to be ready to catch the flour sack as it returns to your arms!

8.30. **IDENTIFY:** The x- and y-components of the momentum of the system of the two asteroids are separately conserved.
SET UP: The before and after diagrams are given in Figure 8.30 and the choice of coordinates is indicated. Each asteroid has mass m.
EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $mv_{A1} = mv_{A2}\cos 30.0° + mv_{B2}\cos 45.0°$.
$40.0\text{ m/s} = 0.866v_{A2} + 0.707v_{B2}$ and $0.707v_{B2} = 40.0\text{ m/s} - 0.866v_{A2}$.
$P_{2y} = P_{2y}$ gives $0 = mv_{A2}\sin 30.0° - mv_{B2}\sin 45.0°$ and $0.500v_{A2} = 0.707v_{B2}$.
Combining these two equations gives $0.500v_{A2} = 40.0\text{ m/s} - 0.866v_{A2}$ and $v_{A2} = 29.3\text{ m/s}$. Then

$$v_{B2} = \left(\frac{0.500}{0.707}\right)(29.3\text{ m/s}) = 20.7\text{ m/s}.$$

(b) $K_1 = \frac{1}{2}mv_{A1}^2$. $K_2 = \frac{1}{2}mv_{A2}^2 + \frac{1}{2}mv_{B2}^2$. $\dfrac{K_2}{K_1} = \dfrac{v_{A2}^2 + v_{B2}^2}{v_{A1}^2} = \dfrac{(29.3\text{ m/s})^2 + (20.7\text{ m/s})^2}{(40.0\text{ m/s})^2} = 0.804.$

$$\frac{\Delta K}{K_1} = \frac{K_2 - K_1}{K_1} = \frac{K_2}{K_1} - 1 = -0.196.$$

19.6% of the original kinetic energy is dissipated during the collision.
EVALUATE: We could use any directions we wish for the x- and y-coordinate directions, but the particular choice we have made is especially convenient.

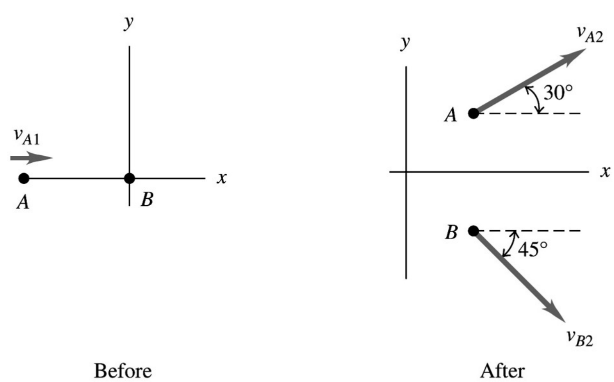


Figure 8.30

8.31. **IDENTIFY:** Momentum is conserved during the collision.
SET UP: $p_x = mv_x$. Call the $+x$-axis pointing northward. We want the speed v of the hockey players after they collide and become intertwined.
EXECUTE: The momentum before the collision is equal to the momentum after the collision.
$m_1v_1 + m_2v_2 = (m_1 + m_2)v \rightarrow$ (70 kg)(5.5 m/s) + (110 kg)(–4.0 m/s) = (180 kg)v $\rightarrow$ $v = -0.31$ m/s. The minus sign tells that they are traveling toward the south.
EVALUATE: Even though the heavier player was traveling slower than the lighter player, his larger mass gave him greater momentum than that of the faster lighter player.

8.32. **IDENTIFY:** There is no net external force on the system of the two skaters and the momentum of the system is conserved.
SET UP: Let object A be the skater with mass 70.0 kg and object B be the skater with mass 65.0 kg. Let $+x$ be to the right, so $v_{A1x} = +4.00\text{ m/s}$ and $v_{B1x} = -2.50\text{ m/s}$. After the collision, the two objects are combined and move with velocity $\vec{v}_2$. Solve for v_{2x}.
EXECUTE: $P_{1x} = P_{2x}$. $m_Av_{A1x} + m_Bv_{B1x} = (m_A + m_B)v_{2x}$.

$v_{2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \dfrac{(70.0\text{ kg})(4.00\text{ m/s}) + (65.0\text{ kg})(-2.50\text{ m/s})}{70.0\text{ kg} + 65.0\text{ kg}} = 0.870\text{ m/s}$. The two skaters move to the right at 0.870 m/s.

EVALUATE: There is a large decrease in kinetic energy.

8.33. **IDENTIFY:** Since drag effects are neglected there is no net external force on the system of two fish and the momentum of the system is conserved. The mechanical energy equals the kinetic energy, which is $K = \frac{1}{2}mv^2$ for each object.

SET UP: Let object A be the 15.0 kg fish and B be the 4.50 kg fish. Let $+x$ be the direction the large fish is moving initially, so $v_{A1x} = 1.10$ m/s and $v_{B1x} = 0$. After the collision the two objects are combined and move with velocity $\vec{v}_2$. Solve for v_{2x}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(15.0\text{ kg})(1.10\text{ m/s}) + 0}{15.0\text{ kg} + 4.50\text{ kg}} = 0.846\text{ m/s}.$$

(b) $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(15.0\text{ kg})(1.10\text{ m/s})^2 = 9.08$ J.

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = \frac{1}{2}(19.5\text{ kg})(0.846\text{ m/s})^2 = 6.98$ J.

$\Delta K = K_2 - K_1 = \mathbf{2}2.10$ J. 2.10 J of mechanical energy is dissipated.

EVALUATE: The total kinetic energy always decreases in a collision where the two objects become combined.

8.34. **IDENTIFY:** This problem requires use of conservation of momentum and conservation of energy. We need to break it into two parts: the collision and the motion after the collision.

SET UP: During the collision momentum is conserved. After the collision energy is conserved. We use $p_x = mv_x$, $K = \dfrac{1}{2}mv^2$, $U_g = mgh$, and $U_1 + K_1 = U_2 + K_2$. We want to find the maximum height the apple will reach after being hit by the dart.

EXECUTE: <u>Collision</u>: Use momentum conservation to find the speed v of the apple-dart system just after the collision. $\dfrac{M}{4}v_0 = \left(M + \dfrac{M}{4}\right)v$, so $v = \dfrac{v_0}{5}$.

<u>After collision</u>: Use energy conservation with the initial speed $v = \dfrac{v_0}{5}$ and the final speed zero.

$U_1 + K_1 = U_2 + K_2$ gives $\dfrac{1}{2}\left(M + \dfrac{M}{4}\right)\left(\dfrac{v_0}{5}\right)^2 = \left(M + \dfrac{M}{4}\right)gh \quad \rightarrow \quad h = \dfrac{v_0^2}{50g}$.

EVALUATE: The momentum is conserved but the kinetic energy is *not* conserved. After the collision the mechanical energy is conserved but the momentum is *not* conserved.

8.35. **IDENTIFY:** This problem involves a collision, so momentum is conserved.

SET UP: We use $p_x = mv_x$ and $K = \dfrac{1}{2}mv^2$. The total momentum P_x before the collision is equal to the total momentum after the collision, where $P_x = p_{1x} + p_{2x}$. Call the $+x$-axis eastward. We want to find the decrease in kinetic energy during the collision. Start by making a before-and-after sketch of the collision, as shown in Fig. 8.35.

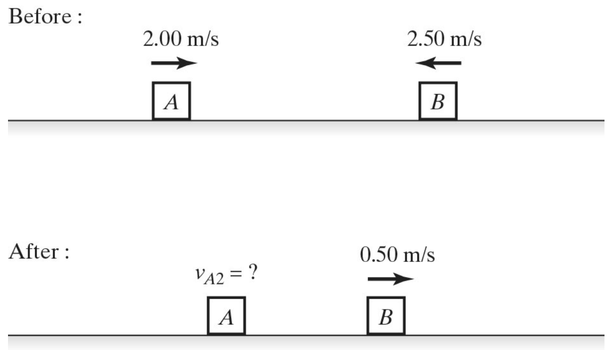


Figure 8.35

EXECUTE: We first need to find the velocity of A after the collision. Using $P_{1x} = P_{2x}$ gives $p_{A1} + p_{B1} = p_{A2} + p_{B2}$, so $m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$. Putting in the numbers gives us (4.00 kg)(2.00 m/s) – (6.00 kg)(2.50 m/s) = (4.00 kg)v_{A2} + (6.00 kg)(0.50 m/s) , so $v_{A2} = -2.50$ m/s.

Now find the decrease in kinetic energy, which is the initial value minus the final value. Using v_{A2} = –2.50 m/s and the quantities given in the problem, we get

$$\text{Decrease} = K_1 - K_2 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 - \left(\frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2\right) = 13.5\text{ J.}$$

EVALUATE: We see that the momentum is conserved during this collision but the kinetic energy is not conserved. The final kinetic energy is 13.5 J less than the initial kinetic energy.

8.36. **IDENTIFY:** The forces the two vehicles exert on each other during the collision are much larger than the horizontal forces exerted by the road, and it is a good approximation to assume momentum conservation.

SET UP: Let $+x$ be eastward. After the collision two vehicles move with a common velocity $\vec{v}_2$.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $m_{\text{SC}}v_{\text{SC}x} + m_{\text{T}}v_{\text{T}x} = (m_{\text{SC}} + m_{\text{T}})v_{2x}$.

$$v_{2x} = \frac{m_{\text{SC}}v_{\text{SC}x} + m_{\text{T}}v_{\text{T}x}}{m_{\text{SC}} + m_{\text{T}}} = \frac{(1050\text{ kg})(-15.0\text{ m/s}) + (6320\text{ kg})(+10.0\text{ m/s})}{1050\text{ kg} + 6320\text{ kg}} = 6.44\text{ m/s.}$$

The final velocity is 6.44 m/s, eastward.

(b) $P_{1x} = P_{2x} = 0$ gives $m_{\text{SC}}v_{\text{SC}x} + m_{\text{T}}v_{\text{T}x} = 0$. $v_{\text{T}x} = -\left(\frac{m_{\text{SC}}}{m_{\text{T}}}\right)v_{\text{SC}x} = -\left(\frac{1050\text{ kg}}{6320\text{ kg}}\right)(-15.0\text{ m/s}) = 2.50\text{ m/s.}$

The truck would need to have initial speed 2.50 m/s.

(c) part (a):

$\Delta K = \frac{1}{2}(7370\text{ kg})(6.44\text{ m/s})^2 - \frac{1}{2}(1050\text{ kg})(15.0\text{ m/s})^2 - \frac{1}{2}(6320\text{ kg})(10.0\text{ m/s})^2 = -2.81\times10^5\text{ J}$

part (b): $\Delta K = 0 - \frac{1}{2}(1050\text{ kg})(15.0\text{ m/s})^2 - \frac{1}{2}(6320\text{ kg})(2.50\text{ m/s})^2 = -1.38\times10^5\text{ J}$. The change in kinetic energy has the greater magnitude in part (a).

EVALUATE: In part (a) the eastward momentum of the truck has a greater magnitude than the westward momentum of the car and the wreckage moves eastward after the collision. In part (b) the two vehicles have equal magnitudes of momentum, the total momentum of the system is zero and the wreckage is at rest after the collision.

8.37. **IDENTIFY:** The forces the two players exert on each other during the collision are much larger than the horizontal forces exerted by the slippery ground and it is a good approximation to assume momentum conservation. Each component of momentum is separately conserved.

SET UP: Let $+x$ be east and $+y$ be north. After the collision the two players have velocity $\vec{v}_2$. Let the linebacker be object *A* and the halfback be object *B*, so $v_{A1x} = 0$, $v_{A1y} = 8.8$ m/s, $v_{B1x} = 7.2$ m/s and $v_{B1y} = 0$. Solve for v_{2x} and v_{2y}.

EXECUTE: $P_{1x} = P_{2x}$ gives $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(85 \text{ kg})(7.2 \text{ m/s})}{110 \text{ kg} + 85 \text{ kg}} = 3.14 \text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{A1y} + m_B v_{B1y} = (m_A + m_B)v_{2y}$.

$$v_{2y} = \frac{m_A v_{A1y} + m_B v_{B1y}}{m_A + m_B} = \frac{(110 \text{ kg})(8.8 \text{ m/s})}{110 \text{ kg} + 85 \text{ kg}} = 4.96 \text{ m/s}.$$

$v = \sqrt{v_{2x}^2 + v_{2y}^2} = 5.9$ m/s.

$$\tan\theta = \frac{v_{2y}}{v_{2x}} = \frac{4.96 \text{ m/s}}{3.14 \text{ m/s}} \text{ and } \theta = 58°.$$

The players move with a speed of 5.9 m/s and in a direction 58° north of east.

EVALUATE: Each component of momentum is separately conserved.

8.38. **IDENTIFY:** The momentum is conserved during the collision. Since the motions involved are in two dimensions, we must consider the components separately.

SET UP: Use coordinates where $+x$ is east and $+y$ is south. The system of two cars before and after the collision is sketched in Figure 8.38. Neglect friction from the road during the collision. The enmeshed cars have a total mass of $2000 \text{ kg} + 1500 \text{ kg} = 3500 \text{ kg}$. Momentum conservation tells us that $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

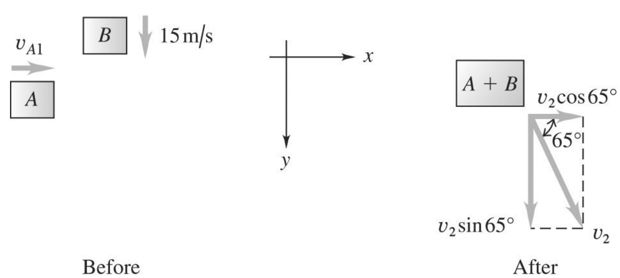


Figure 8.38

EXECUTE: There are no external horizontal forces during the collision, so $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

(a) $P_{1x} = P_{2x}$ gives $(1500 \text{ kg})(15 \text{ m/s}) = (3500 \text{ kg})v_2 \sin 65°$ and $v_2 = 7.1$ m/s.

(b) $P_{1y} = P_{2y}$ gives $(2000 \text{ kg})v_{A1} = (3500 \text{ kg})v_2 \cos 65°$. And then using $v_2 = 7.1$ m/s, we have $v_{A1} = 5.2$ m/s.

EVALUATE: Momentum is a vector so we must treat each component separately.

8.39. **IDENTIFY:** The collision generates only internal forces to the Jack-Jill system, so momentum is conserved.

SET UP: Call the *x*-axis Jack's initial direction (eastward), and the *y*-axis perpendicular to that (northward). The initial *y*-component of the momentum is zero. Call *v* Jill's speed just after the collision and call θ the angle her velocity makes with the $+x$-axis.

EXECUTE: In the *x*-direction: (55.0 kg)(8.00 m/s) = (55.0 kg)(5.00 m/s)(cos34.0°) + (48.0 kg)$v \cos\theta$.
In the *y*-direction: (55.0 kg)(5.00 m/s)(sin34.0°) = (48.0 kg)$v \sin\theta$.
Separating $v \sin\theta$ and $v \cos\theta$ and dividing gives

$\tan\theta$ = (5.00 m/s)(sin34.0°)/[8.00 m/s – (5.00 m/s)(cos34.0°)] = 0.72532, so θ = 36.0° south of east. Using the y-direction momentum equation gives
v = (55.0 kg)(5.00 m/s)(sin34.0°)/[(48.0 kg)(sin36.0°) = 5.46 m/s.
EVALUATE: Jill has a bit less mass than Jack, so the angle her momentum makes with the $+x$-axis (36.0°) has to be a bit larger than Jack's (34.0°) for their y-component momenta to be equal in magnitude.

8.40. **IDENTIFY:** The collision forces are large so gravity can be neglected during the collision. Therefore, the horizontal and vertical components of the momentum of the system of the two birds are conserved.
SET UP: The system before and after the collision is sketched in Figure 8.40. Use the coordinates shown.

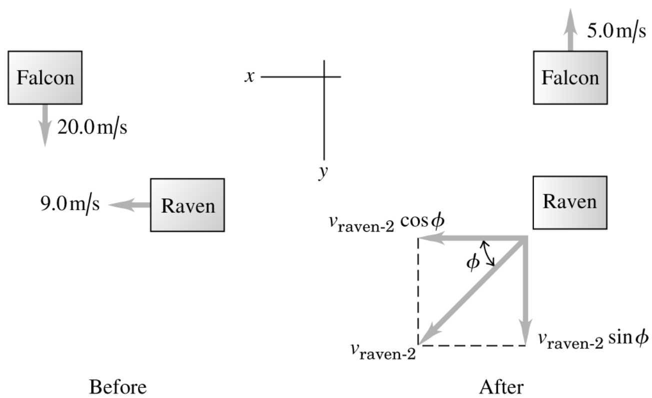


Figure 8.40

EXECUTE: **(a)** There is no external force on the system so $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.
$P_{1x} = P_{2x}$ gives $(1.5\text{ kg})(9.0\text{ m/s}) = (1.5\text{ kg})v_{\text{raven-2}}\cos\phi$ and $v_{\text{raven-2}}\cos\phi = 9.0\text{ m/s}$.
$P_{1y} = P_{2y}$ gives $(0.600\text{ kg})(20.0\text{ m/s}) = (0.600\text{ kg})(-5.0\text{ m/s}) + (1.5\text{ kg})v_{\text{raven-2}}\sin\phi$ and $v_{\text{raven-2}}\sin\phi = 10.0\text{ m/s}$.

Combining these two equations gives $\tan\phi = \dfrac{10.0\text{ m/s}}{9.0\text{ m/s}}$ and $\phi = 48°$.

(b) $v_{\text{raven-2}} = 13.5$ m/s
EVALUATE: Due to its large initial speed the lighter falcon was able to produce a large change in the raven's direction of motion.

8.41. **IDENTIFY:** Since friction forces from the road are ignored, the x- and y-components of momentum are conserved.
SET UP: Let object A be the subcompact and object B be the truck. After the collision the two objects move together with velocity $\vec{v}_2$. Use the x- and y-coordinates given in the problem. $v_{A1y} = v_{B1x} = 0$.
$v_{2x} = (16.0\text{ m/s})\sin 24.0° = 6.5\text{ m/s}$; $v_{2y} = (16.0\text{ m/s})\cos 24.0° = 14.6\text{ m/s}$.
EXECUTE: $P_{1x} = P_{2x}$ gives $m_A v_{A1x} = (m_A + m_B)v_{2x}$.

$$v_{A1x} = \left(\frac{m_A + m_B}{m_A}\right)v_{2x} = \left(\frac{950\text{ kg} + 1900\text{ kg}}{950\text{ kg}}\right)(6.5\text{ m/s}) = 19.5\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_B v_{B1y} = (m_A + m_B)v_{2y}$.

$$v_{B1y} = \left(\frac{m_A + m_B}{m_B}\right)v_{2y} = \left(\frac{950\text{ kg} + 1900\text{ kg}}{1900\text{ kg}}\right)(14.6\text{ m/s}) = 21.9\text{ m/s}.$$

Before the collision the subcompact car has speed 19.5 m/s and the truck has speed 21.9 m/s.

EVALUATE: Each component of momentum is independently conserved.

8.42. **IDENTIFY:** Apply conservation of momentum to the collision. Apply conservation of energy to the motion of the block after the collision.

SET UP: Conservation of momentum applied to the collision between the bullet and the block: Let object A be the bullet and object B be the block. Let v_A be the speed of the bullet before the collision and let V be the speed of the block with the bullet inside just after the collision.

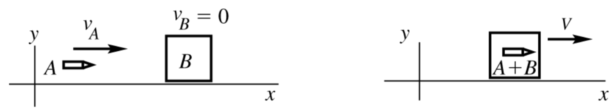


Figure 8.42a

P_x is constant gives $m_A v_A = (m_A + m_B)V$.

Conservation of energy applied to the motion of the block after the collision:

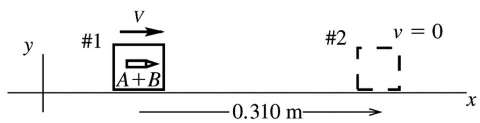


Figure 8.42b

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

EXECUTE: Work is done by friction so $W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$

$U_1 = U_2 = 0$ (no work done by gravity)

$K_1 = \frac{1}{2}mV^2$; $+y$ (block has come to rest)

Thus $\frac{1}{2}mV^2 - \mu_k mgs = 0$

$V = \sqrt{2\mu_k gs} = \sqrt{2(0.20)(9.80\text{ m/s}^2)(0.310\text{ m})} = 1.1\text{ m/s}$

Use this result in the conservation of momentum equation

$v_A = \left(\dfrac{m_A + m_B}{m_A}\right)V = \left(\dfrac{5.00\times10^{-3}\text{ kg} + 1.20\text{ kg}}{5.00\times10^{-3}\text{ kg}}\right)(1.1\text{ m/s}) = 266\text{ m/s}$, which rounds to 270 m/s.

EVALUATE: When we apply conservation of momentum to the collision we are ignoring the impulse of the friction force exerted by the surface during the collision. This is reasonable since this force is much smaller than the forces the bullet and block exert on each other during the collision. This force does work as the block moves after the collision, and takes away all the kinetic energy.

8.43. **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion after the collision. After the collision the kinetic energy of the combined object is converted to gravitational potential energy.

SET UP: Immediately after the collision the combined object has speed V. Let h be the vertical height through which the pendulum rises.

EXECUTE: **(a)** Conservation of momentum applied to the collision gives

$(12.0\times10^{-3}\text{ kg})(380\text{ m/s}) = (6.00\text{ kg} + 12.0\times10^{-3}\text{ kg})V$ and $V = 0.758\text{ m/s}$.

Conservation of energy applied to the motion after the collision gives $\frac{1}{2}m_{\text{tot}}V^2 = m_{\text{tot}}gh$ and

$$h = \frac{V^2}{2g} = \frac{(0.758\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 0.0293\text{ m} = 2.93\text{ cm}.$$

(b) $K = \frac{1}{2}m_b v_b^2 = \frac{1}{2}(12.0\times10^{-3}\text{ kg})(380\text{ m/s})^2 = 866\text{ J}.$

(c) $K = \frac{1}{2}m_{tot}V^2 = \frac{1}{2}(6.00\text{ kg} + 12.0\times10^{-3}\text{ kg})(0.758\text{ m/s})^2 = 1.73\text{ J}.$

EVALUATE: Most of the initial kinetic energy of the bullet is dissipated in the collision.

8.44. **IDENTIFY:** During the collision, momentum is conserved. After the collision, mechanical energy is conserved.

SET UP: The collision occurs over a short time interval and the block moves very little during the collision, so the spring force during the collision can be neglected. Use coordinates where $+x$ is to the right. During the collision, momentum conservation gives $P_{1x} = P_{2x}$. After the collision, $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$.

EXECUTE: *Collision*: There is no external horizontal force during the collision and $P_{1x} = P_{2x}$, so $(3.00\text{ kg})(8.00\text{ m/s}) = (15.0\text{ kg})v_{\text{block, 2}} - (3.00\text{ kg})(2.00\text{ m/s})$ and $v_{\text{block, 2}} = 2.00\text{ m/s}$.

Motion after the collision: When the spring has been compressed the maximum amount, all the initial kinetic energy of the block has been converted into potential energy $\frac{1}{2}kx^2$ that is stored in the compressed spring. Conservation of energy gives $\frac{1}{2}(15.0\text{ kg})(2.00\text{ m/s})^2 = \frac{1}{2}(500.0\text{ kg})x^2$, so $x = 0.346\text{ m}$.

EVALUATE: We cannot say that the momentum was converted to potential energy, because momentum and energy are different types of quantities.

8.45. **IDENTIFY:** The missile gives momentum to the ornament causing it to swing in a circular arc and thereby be accelerated toward the center of the circle.

SET UP: After the collision the ornament moves in an arc of a circle and has acceleration $a_{rad} = \frac{v^2}{r}$.

During the collision, momentum is conserved, so $P_{1x} = P_{2x}$. The free-body diagram for the ornament plus missile is given in Figure 8.45. Take $+y$ to be upward, since that is the direction of the acceleration. Take the $+x$-direction to be the initial direction of motion of the missile.

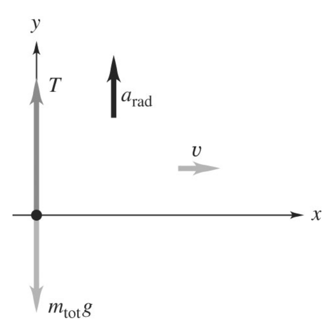


Figure 8.45

EXECUTE: Apply conservation of momentum to the collision. Using $P_{1x} = P_{2x}$, we get $(0.200\text{ kg})(12.0\text{ m/s}) = (1.00\text{ kg})V$, which gives $V = 2.40\text{ m/s}$, the speed of the ornament immediately after the collision. Then $\Sigma F_y = ma_y$ gives $T - m_{tot}g = m_{tot}\frac{v^2}{r}$. Solving for T gives

$$T = m_{tot}\left(g + \frac{v^2}{r}\right) = (1.00\text{ kg})\left(9.80\text{ m/s}^2 + \frac{(2.40\text{ m/s})^2}{1.50\text{ m}}\right) = 13.6\text{ N}.$$

Evaluate: We cannot use energy conservation during the collision because it is an inelastic collision (the objects stick together).

8.46. **Identify:** No net external horizontal force so P_x is conserved. Elastic collision so $K_1 = K_2$ and can use $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$.

Set Up:

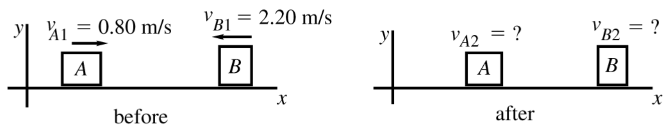


Figure 8.46

Execute: From conservation of x-component of momentum:

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$m_A v_{A1} - m_B v_{B1} = m_A v_{A2x} + m_B v_{B2x}$$

$$(0.150 \text{ kg})(0.80 \text{ m/s}) - (0.300 \text{ kg})(2.20 \text{ m/s}) = (0.150 \text{ kg})v_{A2x} + (0.300 \text{ kg})v_{B2x}$$

$$-3.60 \text{ m/s} = v_{\text{A}2x} + 2v_{B2x}$$

From the relative velocity equation for an elastic collision Eq. 8.27:

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x}) = -(-2.20 \text{ m/s} - 0.80 \text{ m/s}) = +3.00 \text{ m/s}$$

$$3.00 \text{ m/s} = -v_{\text{A}2x} + v_{B2x}$$

Adding the two equations gives $-0.60 \text{ m/s} = 3v_{B2x}$ and $v_{B2x} = -0.20$ m/s. Then $v_{A2x} = v_{B2x} - 3.00 \text{ m/s} = -3.20$ m/s.

The 0.150 kg glider (A) is moving to the left at 3.20 m/s and the 0.300 kg glider (B) is moving to the left at 0.20 m/s.

Evaluate: We can use our v_{A2x} and v_{B2x} to show that P_x is constant and $K_1 = K_2$.

8.47. **Identify:** When the spring is compressed the maximum amount the two blocks aren't moving relative to each other and have the same velocity $\vec{V}$ relative to the surface. Apply conservation of momentum to find V and conservation of energy to find the energy stored in the spring. Since the collision is elastic, $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x}$ and $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_{A1x}$ give the final velocity of each block after the collision.

Set Up: Let $+x$ be the direction of the initial motion of A.

Execute: **(a)** Momentum conservation gives $(2.00 \text{ kg})(2.00 \text{ m/s}) = (8.00 \text{ kg})V$ so $V = 0.500$ m/s. Both blocks are moving at 0.500 m/s, in the direction of the initial motion of block A. Conservation of energy says the initial kinetic energy of A equals the total kinetic energy at maximum compression plus the potential energy U_b stored in the bumpers: $\frac{1}{2}(2.00 \text{ kg})(2.00 \text{ m/s})^2 = U_\text{b} + \frac{1}{2}(8.00 \text{ kg})(0.500 \text{ m/s})^2$ so $U_\text{b} = 3.00$ J.

(b) $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \left(\dfrac{2.00 \text{ kg} - 6.0 \text{ kg}}{8.00 \text{ kg}}\right)(2.00 \text{ m/s}) = -1.00$ m/s. Block A is moving in the $-x$-direction at 1.00 m/s.

$v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right) v_{A1x} = \dfrac{2(2.00\text{ kg})}{8.00\text{ kg}}(2.00\text{ m/s}) = +1.00\text{ m/s}$. Block B is moving in the $+x$-direction at 1.00 m/s.

EVALUATE: When the spring is compressed the maximum amount, the system must still be moving in order to conserve momentum.

8.48. **IDENTIFY:** Since the collision is elastic, both momentum conservation and equation $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ apply.

SET UP: Let object A be the 30.0 g marble and let object B be the 10.0 g marble. Let $+x$ be to the right.

EXECUTE: **(a)** Conservation of momentum gives
$(0.0300\text{ kg})(0.200\text{ m/s}) + (0.0100\text{ kg})(-0.400\text{ m/s}) = (0.0300\text{ kg})v_{A2x} + (0.0100\text{ kg})v_{B2x}$.
$3v_{A2x} + v_{B2x} = 0.200\text{ m/s}$. $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ says
$v_{B2x} - v_{A2x} = -(-0.400\text{ m/s} - 0.200\text{ m/s}) = +0.600\text{ m/s}$. Solving this pair of equations gives
$v_{A2x} = -0.100\text{ m/s}$ and $v_{B2x} = +0.500\text{ m/s}$. The 30.0 g marble is moving to the left at 0.100 m/s and the 10.0 g marble is moving to the right at 0.500 m/s.

(b) For marble A, $\Delta P_{Ax} = m_A v_{A2x} - m_A v_{A1x} = (0.0300\text{ kg})(-0.100\text{ m/s} - 0.200\text{ m/s}) = -0.00900\text{ kg}\cdot\text{m/s}$.
For marble B, $\Delta P_{Bx} = m_B v_{B2x} - m_B v_{B1x} = (0.0100\text{ kg})(0.500\text{ m/s} - [-0.400\text{ m/s}]) = +0.00900\text{ kg}\cdot\text{m/s}$.
The changes in momentum have the same magnitude and opposite sign.

(c) For marble A, $\Delta K_A = \frac{1}{2}m_A v_{A2}^2 - \frac{1}{2}m_A v_{A1}^2 = \frac{1}{2}(0.0300\text{ kg})([0.100\text{ m/s}]^2 - [0.200\text{ m/s}]^2) = -4.5\times10^{-4}\text{ J}$.
For marble B, $\Delta K_B = \frac{1}{2}m_B v_{B2}^2 - \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(0.0100\text{ kg})([0.500\text{ m/s}]^2 - [0.400\text{ m/s}]^2) = +4.5\times10^{-4}\text{ J}$.
The changes in kinetic energy have the same magnitude and opposite sign.

EVALUATE: The results of parts (b) and (c) show that momentum and kinetic energy are conserved in the collision.

8.49. **IDENTIFY:** Equation $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right) v_{A1x}$ applyies, with object A being the neutron.

SET UP: Let $+x$ be the direction of the initial momentum of the neutron. The mass of a neutron is $m_\text{n} = 1.0\text{ u}$.

EXECUTE: **(a)** $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right) v_{A1x} = \dfrac{1.0\text{ u} - 2.0\text{ u}}{1.0\text{ u} + 2.0\text{ u}} v_{A1x} = -v_{A1x}/3.0$. The speed of the neutron after the collision is one-third its initial speed.

(b) $K_2 = \frac{1}{2}m_\text{n}v_\text{n}^2 = \frac{1}{2}m_\text{n}(v_{A1}/3.0)^2 = \dfrac{1}{9.0}K_1$.

(c) After n collisions, $v_{A2} = \left(\dfrac{1}{3.0}\right)^n v_{A1}$. $\left(\dfrac{1}{3.0}\right)^n = \dfrac{1}{59{,}000}$, so $3.0^n = 59{,}000$. $n\log 3.0 = \log 59{,}000$ and $n = 10$.

EVALUATE: Since the collision is elastic, in each collision the kinetic energy lost by the neutron equals the kinetic energy gained by the deuteron.

8.50. **IDENTIFY:** Elastic collision. Solve for mass and speed of target nucleus.

SET UP: **(a)** Let A be the proton and B be the target nucleus. The collision is elastic, all velocities lie along a line, and B is at rest before the collision. Hence the results of equations $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right) v_{A1x}$ and $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right) v_{A1x}$ apply.

EXECUTE: $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right) v_{A1x}$: $m_B(v_x + v_{Ax}) = m_A(v_x - v_{Ax})$, where v_x is the velocity component of *A* before the collision and v_{Ax} is the velocity component of *A* after the collision. Here, $v_x = 1.50\times10^7$ m/s (take direction of incident beam to be positive) and $v_{Ax} = -1.20\times10^7$ m/s (negative since traveling in direction opposite to incident beam).

$$m_B = m_A\left(\frac{v_x - v_{Ax}}{v_x + v_{Ax}}\right) = m\left(\frac{1.50\times10^7 \text{ m/s} + 1.20\times10^7 \text{ m/s}}{1.50\times10^7 \text{ m/s} - 1.20\times10^7 \text{ m/s}}\right) = m\left(\frac{2.70}{0.30}\right) = 9.00m.$$

(b) $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right) v_{A1x}$: $v_{Bx} = \left(\dfrac{2m_A}{m_A + m_B}\right) v_x = \left(\dfrac{2m}{m + 9.00m}\right)(1.50\times10^7 \text{ m/s}) = 3.00\times10^6 \text{ m/s}.$

EVALUATE: Can use our calculated v_{Bx} and m_B to show that P_x is constant and that $K_1 = K_2$.

8.51. **IDENTIFY:** In any collision, momentum is conserved. But in this one, kinetic energy is conserved because it is an *elastic* collision.

SET UP: We use $p_x = mv_x$ and $K = \dfrac{1}{2}mv^2$. The total momentum is $P_x = p_{1x} + p_{2x}$. Call the +*x*-axis the original direction of object *A*. Start by making a before-and-after sketch of the collision, as shown in Fig. 8.51. Call v_0 the original speed of *A*. We want to find v_A after the collision.

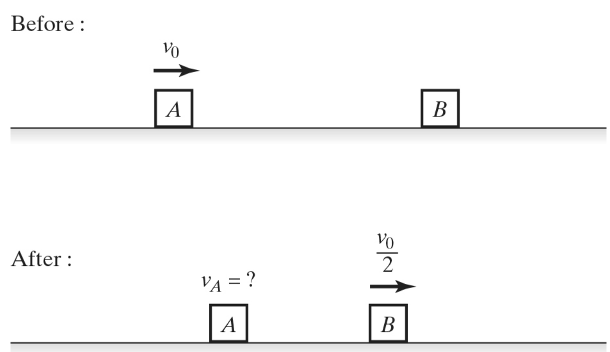


Figure 8.51

EXECUTE: **(a)** Which has greater mass, *A* or *B*? Apply momentum conservation and energy conservation to the collision. We know that $v_B = v_0/2$ after the collision. See the figure for the quantities used.

Momentum conservation: $m_A v_0 = m_A v_A + m_B \dfrac{v_0}{2}$ (Eq. 1)

Energy conservation: $\dfrac{1}{2}m_A v_0^2 = \dfrac{1}{2}m_A v_A^2 + \dfrac{1}{2}m_B\left(\dfrac{v_0}{2}\right)^2$ (Eq. 2)

Defining $R = m_B/m_A$, Eq. 1 becomes $v_A = v_0\left(1 - \dfrac{R}{2}\right)$. Using this result and simplifying, Eq. 2 becomes $1 = \left(1 - \dfrac{R}{2}\right)^2 + \dfrac{R}{4}$. Squaring and solving for *R* gives *R* =3.

(b) Therefore $m_B/m_A = 3$, so *B* has 3 times the mass of *A*.

(c) From our result in part (a), $v_A = v_0\left(1 - \dfrac{R}{2}\right) = v_0\left(1 - \dfrac{3}{2}\right) = -\dfrac{v_0}{2} = -\dfrac{6.0 \text{ m/s}}{2} = -3.0 \text{ m/s}$. The minus sign tells us that *A* is moving opposite to its original direction.

EVALUATE: Use our results to calculate the kinetic energy before and after the collision. $K_1 = \frac{1}{2}m_A v_0^2$ and $K_2 = \frac{1}{2}m_A\left(\frac{v_0}{2}\right)^2 + \frac{1}{2}(3m_A)\left(\frac{v_0}{2}\right)^2 = \frac{1}{2}m_A v_0^2 = K_1$. This agrees with the fact that it is an elastic collision.

8.52. IDENTIFY: Calculate x_{cm}.

SET UP: Apply $x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ with the sun as mass 1 and Jupiter as mass 2. Take the origin at the sun and let Jupiter lie on the positive x-axis.

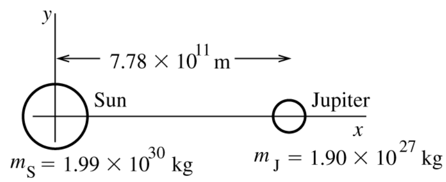


Figure 8.52

$$x_{\text{cm}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

EXECUTE: $x_1 = 0$ and $x_2 = 7.78\times10^{11}$ m

$$x_{\text{cm}} = \frac{(1.90\times10^{27}\text{ kg})(7.78\times10^{11}\text{ m})}{1.99\times10^{30}\text{ kg} + 1.90\times10^{27}\text{ kg}} = 7.42\times10^{8}\text{ m}$$

The center of mass is 7.42×10^8 m from the center of the sun and is on the line connecting the centers of the sun and Jupiter. The sun's radius is 6.96×10^8 m so the center of mass lies just outside the sun.

EVALUATE: The mass of the sun is much greater than the mass of Jupiter, so the center of mass is much closer to the sun. For each object we have considered all the mass as being at the center of mass (geometrical center) of the object.

8.53. IDENTIFY: The location of the center of mass is given by $x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$. The mass can be expressed in terms of the diameter. Each object can be replaced by a point mass at its center.

SET UP: Use coordinates with the origin at the center of Pluto and the $+x$-direction toward Charon, so $x_{\text{P}} = 0$, $x_{\text{C}} = 19{,}700$ km. $m = \rho V = \rho\frac{4}{3}\pi r^3 = \frac{1}{6}\rho\pi d^3$.

EXECUTE: $x_{\text{cm}} = \frac{m_{\text{P}}x_{\text{P}} + m_{\text{C}}x_{\text{C}}}{m_{\text{P}} + m_{\text{C}}} = \left(\frac{m_{\text{C}}}{m_{\text{P}} + m_{\text{C}}}\right)x_{\text{C}} = \left(\frac{\frac{1}{6}\rho\pi d_{\text{C}}^3}{\frac{1}{6}\rho\pi d_{\text{P}}^3 + \frac{1}{6}\rho\pi d_{\text{C}}^3}\right)x_{\text{C}} = \left(\frac{d_{\text{C}}^3}{d_{\text{P}}^3 + d_{\text{C}}^3}\right)x_{\text{C}}$.

$$x_{\text{cm}} = \left(\frac{[1250\text{ km}]^3}{[2370\text{ km}]^3 + [1250\text{ km}]^3}\right)(19{,}700\text{ km}) = 2.52\times10^3\text{ km}.$$

The center of mass of the system is 2.52×10^3 km from the center of Pluto.

EVALUATE: The center of mass is closer to Pluto because Pluto has more mass than Charon.

8.54. IDENTIFY: Apply $x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$, $v_{\text{cm},x} = \frac{m_Av_{A,x} + m_Bv_{B,x}}{m_A + m_B}$, and $P_x = Mv_{\text{cm}-x}$. There is only one component of position and velocity.

SET UP: $m_A = 1200\text{ kg}$, $m_B = 1800\text{ kg}$. $M = m_A + m_B = 3000\text{ kg}$. Let $+x$ be to the right and let the origin be at the center of mass of the station wagon.

EXECUTE: **(a)** $x_{\text{cm}} = \dfrac{m_A x_A + m_B x_B}{m_A + m_B} = \dfrac{0 + (1800\text{ kg})(40.0\text{ m})}{1200\text{ kg} + 1800\text{ kg}} = 24.0\text{ m}.$

The center of mass is between the two cars, 24.0 m to the right of the station wagon and 16.0 m behind the lead car.

(b) $P_x = m_A v_{A,x} + m_B v_{B,x} = (1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s}) = 5.04 \times 10^4\text{ kg} \cdot \text{m/s}.$

(c) $v_{\text{cm},x} = \dfrac{m_A v_{A,x} + m_B v_{B,x}}{m_A + m_B} = \dfrac{(1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s})}{1200\text{ kg} + 1800\text{ kg}} = 16.8\text{ m/s}.$

(d) $P_x = M v_{\text{cm}-x} = (3000\text{ kg})(16.8\text{ m/s}) = 5.04 \times 10^4\text{ kg} \cdot \text{m/s}$, the same as in part (b).

EVALUATE: The total momentum can be calculated either as the vector sum of the momenta of the individual objects in the system, or as the total mass of the system times the velocity of the center of mass.

8.55. **IDENTIFY:** This problem requires finding the location of the center of mass of a system of two objects.

SET UP: We can treat each object as a point mass located at its center of mass and use the formula $y_{\text{cm}} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$. We want to find the location of the center of mass of the two-mass sytstem.

EXECUTE: We'll use $y_{\text{cm}} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$, but first we find the dimensions of the cube. Its volume is x^3 = 0.0270 m^3, so $x = 0.300\text{ m}$, so its center of mass is 0.150 m above the floor. The center of mass of the sphere is 0.400 m above the top of the cube. The locations of each center of mass are $y_1 = y_{\text{cube}} = 0.150$ m and $y_2 = y_{\text{sphere}} = 0.400\text{ m} + 0.300\text{ m} = 0.700\text{ m}$. Now use the center of mass formula with the origin at the floor. $y_{\text{cm}} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \dfrac{(0.500\text{ kg})(0.150\text{ m}) + (0.800\text{ kg})(0.700\text{ m})}{1.300\text{ kg}} = 0.488\text{ m}$ above the floor.

EVALUATE: The center of mass is closer to the center of the sphere than to the center of the cube since the sphere has more mass.

8.56. **IDENTIFY:** Use $x_{\text{cm}} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$.

SET UP: The target variable is m_1.

EXECUTE: $x_{\text{cm}} = 2.0\text{ m}$, $y_{\text{cm}} = 0$

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1(0) + (0.10\text{ kg})(8.0\text{ m})}{m_1 + (0.10\text{ kg})} = \frac{0.80\text{ kg} \cdot \text{m}}{m_1 + 0.10\text{ kg}}.$$

$$x_{\text{cm}} = 2.0\text{ m gives } 2.0\text{ m} = \frac{0.80\text{ kg} \cdot \text{m}}{m_1 + 0.10\text{ kg}}.$$

$$m_1 + 0.10\text{ kg} = \frac{0.80\text{ kg} \cdot \text{m}}{2.0\text{ m}} = 0.40\text{ kg}.$$

$$m_1 = 0.30\text{ kg}.$$

EVALUATE: The cm is closer to m_1 so its mass is larger then m_2.

(b) IDENTIFY: Use $\vec{P} = M\vec{v}_{\text{cm}}$ to calculate $\vec{P}$.

SET UP: $\vec{v}_{\text{cm}} = (5.0\text{ m/s})\hat{i}$.

$$\vec{P} = M\vec{v}_{\text{cm}} = (0.10\text{ kg} + 0.30\text{ kg})(5.0\text{ m/s})\hat{i} = (2.0\text{ kg} \cdot \text{m/s})\hat{i}.$$

(c) IDENTIFY: Use $\vec{v}_{\text{cm}} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$.

SET UP: $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$. The target variable is $\vec{v}_1$. Particle 2 at rest says $v_2 = 0$.

EXECUTE: $\vec{v}_1 = \left(\dfrac{m_1 + m_2}{m_1}\right)\vec{v}_{cm} = \left(\dfrac{0.30 \text{ kg} + 0.10 \text{ kg}}{0.30 \text{ kg}}\right)(5.00 \text{ m/s})\hat{\boldsymbol{i}} = (6.7 \text{ m/s})\hat{\boldsymbol{i}}$.

EVALUATE: Using the result of part (c) we can calculate $\vec{p}_1$ and $\vec{p}_2$ and show that $\vec{P}$ as calculated in part (b) does equal $\vec{p}_1 + \vec{p}_2$.

8.57. IDENTIFY: There is no net external force on the system of James, Ramon, and the rope; the momentum of the system is conserved, and the velocity of its center of mass is constant. Initially there is no motion, and the velocity of the center of mass remains zero after Ramon has started to move.

SET UP: Let $+x$ be in the direction of Ramon's motion. Ramon has mass $m_R = 60.0$ kg and James has mass $m_J = 90.0$ kg.

EXECUTE: $v_{cm\text{-}x} = \dfrac{m_R v_{Rx} + m_J v_{Jx}}{m_R + m_J} = 0.$

$v_{Jx} = -\left(\dfrac{m_R}{m_J}\right)v_{Rx} = -\left(\dfrac{60.0 \text{ kg}}{90.0 \text{ kg}}\right)(1.10 \text{ m/s}) = -0.733 \text{ m/s}$. James' speed is 0.733 m/s.

EVALUATE: As they move, the two men have momenta that are equal in magnitude and opposite in direction, and the total momentum of the system is zero. Also, Example 8.14 shows that Ramon moves farther than James in the same time interval. This is consistent with Ramon having a greater speed.

8.58. (a) IDENTIFY and SET UP: Apply $y_{cm} = \dfrac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ and solve for m_1 and m_2.

EXECUTE: $y_{cm} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

$m_1 + m_2 = \dfrac{m_1 y_1 + m_2 y_2}{y_{cm}} = \dfrac{m_1(0) + (0.50 \text{ kg})(6.0 \text{ m})}{2.4 \text{ m}} = 1.25 \text{ kg}$ and $m_1 = 0.75$ kg.

EVALUATE: y_{cm} is closer to m_1 since $m_1 > m_2$.

(b) IDENTIFY and SET UP: Apply $\vec{a} = d\vec{v}/dt$ for the cm motion.

EXECUTE: $\vec{a}_{cm} = \dfrac{d\vec{v}_{cm}}{dt} = (1.5 \text{ m/s}^3)t\hat{\boldsymbol{i}}$.

(c) IDENTIFY and SET UP: Apply $\sum\vec{F}_{ext} = M\vec{a}_{cm}$.

EXECUTE: $\sum\vec{F}_{ext} = M\vec{a}_{cm} = (1.25 \text{ kg})(1.5 \text{ m/s}^3)t\hat{\boldsymbol{i}}$.

At $t = 3.0$ s, $\sum\vec{F}_{ext} = (1.25 \text{ kg})(1.5 \text{ m/s}^3)(3.0 \text{ s})\hat{\boldsymbol{i}} = (5.6 \text{ N})\hat{\boldsymbol{i}}$.

EVALUATE: $v_{cm\text{-}x}$ is positive and increasing so $a_{cm\text{-}x}$ is positive and $\vec{F}_{ext}$ is in the $+x$-direction. There is no motion and no force component in the y-direction.

8.59. IDENTIFY: Apply $\sum\vec{F} = \dfrac{d\vec{P}}{dt}$ to the airplane.

SET UP: $\dfrac{d}{dt}(t^n) = nt^{n-1}$. $1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2$

EXECUTE: $\dfrac{d\vec{P}}{dt} = [-(1.50 \text{ kg}\cdot\text{m/s}^3)t]\hat{\boldsymbol{i}} + (0.25 \text{ kg}\cdot\text{m/s}^2)\hat{\boldsymbol{j}}$. $F_x = -(1.50 \text{ N/s})t$, $F_y = 0.25$ N, $F_z = 0$.

EVALUATE: There is no momentum or change in momentum in the z-direction and there is no force component in this direction.

8.60. IDENTIFY: This problem requires finding the location of the center of mass of a system of three objects.

SET UP: We can treat each object as a point mass located at its center of mass and use the formula $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ We want to find the location of the center of mass of the three-mass sytstem. The center of mass of the bottom piece is a distance $L/4$ before the edge of the table, that of the middle piece is above the edge of the table, and that of the upper piece is $L/4$ beyond the edge of the table. We treat each square of wood as a point mass located at its center of mass. Call the origin of the x-axis at the edge of the table.

EXECUTE: Using $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{m(-L/4) + m(0) + m(L/4)}{3m} = 0$, so the center of mass is directly above the edge of the table.

EVALUATE: Our result is reasonable because of the symmetry of the arrangement. We have a mass $L/4$ before the edge, an equal mass mass $L/4$ beyond the edge, and a mass at the edge, so the center of mass should be at the edge. The vertical location of the center of mass would be at the middle of the middle piece.

8.61. IDENTIFY: $a = -\dfrac{v_{ex}}{m}\dfrac{dm}{dt}$. Assume that dm/dt is constant over the 5.0 s interval, since m doesn't change much during that interval. The thrust is $F = -v_{ex}\dfrac{dm}{dt}$.

SET UP: Take m to have the constant value $110\text{ kg} + 70\text{ kg} = 180\text{ kg}$. dm/dt is negative since the mass of the MMU decreases as gas is ejected.

EXECUTE: (a) $\dfrac{dm}{dt} = -\dfrac{m}{v_{ex}}a = -\left(\dfrac{180\text{ kg}}{490\text{ m/s}}\right)(0.029\text{ m/s}^2) = -0.0106\text{ kg/s}$. In 5.0 s the mass that is ejected is $(0.0106\text{ kg/s})(5.0\text{ s}) = 0.053\text{ kg}$.

(b) $F = -v_{ex}\dfrac{dm}{dt} = -(490\text{ m/s})(-0.0106\text{ kg/s}) = 5.19\text{ N}$.

EVALUATE: The mass change in the 5.0 s is a very small fraction of the total mass m, so it is accurate to take m to be constant.

8.62. IDENTIFY: Use $F = -v_{ex}\dfrac{\Delta m}{\Delta t}$, applied to a finite time interval.

SET UP: $v_{ex} = 1600\text{ m/s}$

EXECUTE: (a) $F = -v_{ex}\dfrac{\Delta m}{\Delta t} = -(1600\text{ m/s})\dfrac{-0.0500\text{ kg}}{1.00\text{ s}} = +80.0\text{ N}$.

(b) The absence of atmosphere would not prevent the rocket from operating. The rocket could be steered by ejecting the gas in a direction with a component perpendicular to the rocket's velocity and braked by ejecting it in a direction parallel (as opposed to antiparallel) to the rocket's velocity.

EVALUATE: The thrust depends on the speed of the ejected gas relative to the rocket and on the mass of gas ejected per second.

8.63. IDENTIFY: This problem involves both static and kinetic friction as well as the impulse-momentum theorem and energy conservation.

SET UP: Once we start the crate moving, the friction force on it is kinetic friction. Impulse is $J_x = F_xt$, where F_x is the net force acting. The impulse-momentum theorem is $J_x = p_{2x} - p_{1x}$, where $p_x = mv_x$. Energy conservation says that $U_1 + K_1 + W_{other} = U_2 + K_2$, and $W = Fs\cos\phi$. Call the push P.

EXECUTE: (a) Estimate: The heaviest crate weighs about 100 lb, so the maximum force need to start it moving is $f_{max} = \mu_s n = (0.500)(100\text{ lb}) = 50\text{ lb} \approx 220\text{ N}$. The mass of a 100-lb weight is about 45 kg.

(b) Now the friction force is due to kinetic friction, so we use $f_k = \mu_k n = \mu_k mg$, so the net force is $F_{net} = P - f_k = P - \mu_k mg$. The impulse-momentum theorem gives $F_{net} = P - f_k = P - \mu_k mg$, so $t = \dfrac{mv}{P - \mu_k mg} = \dfrac{(45 \text{ kg})(8.0 \text{ m/s})}{220 \text{ N} - (0.300)(450 \text{ N})} = 4.2 \text{ s}.$

(c) Using $U_1 + K_1 + W_{other} = U_2 + K_2$, we have $U_1 = U_2 = 0$, $K_1 = 0$, and $W_{tot} = W_P + W_f$. Using $W = Fs\cos\phi$ gives $W_{tot} = -\mu_k mgs + Ps$, so energy conservation gives $-\mu_k mgs + Ps = \frac{1}{2}mv^2$. Solving for s gives $s = \dfrac{\frac{1}{2}mv^2}{P - \mu_k mg} = s = \dfrac{\frac{1}{2}(45 \text{ kg})(8.0 \text{ m/s})^2}{220 \text{ N} - (0.300)(450 \text{ N})} = 17 \text{ m}.$

EVALUATE: As a check, use $\sum F_x = ma_x$: $-\mu_k mg + P = ma_x$ gives $a_x = 1.89 \text{ m/s}^2$. Then use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, which gives $x - x_0 = 17$ m, as we just found.

8.64. **IDENTIFY:** Use the heights to find v_{1y} and v_{2y}, the velocity of the ball just before and just after it strikes the slab. Then apply $J_y = F_y\Delta t = \Delta p_y$.

SET UP: Let $+y$ be downward.

EXECUTE: **(a)** $\frac{1}{2}mv^2 = mgh$ so $v = \pm\sqrt{2gh}$.

$v_{1y} = +\sqrt{2(9.80 \text{ m/s}^2)(2.00 \text{ m})} = 6.26 \text{ m/s}.$ $v_{2y} = -\sqrt{2(9.80 \text{ m/s}^2)(1.60 \text{ m})} = -5.60 \text{ m/s}.$

$$J_y = \Delta p_y = m(v_{2y} - v_{1y}) = (40.0\times10^{-3} \text{ kg})(-5.60 \text{ m/s} - 6.26 \text{ m/s}) = -0.474 \text{ kg}\cdot\text{m/s}.$$

The impulse is $0.474 \text{ kg}\cdot\text{m/s}$, upward.

(b) $F_y = \dfrac{J_y}{\Delta t} = \dfrac{-0.474 \text{ kg}\cdot\text{m/s}}{2.00\times10^{-3} \text{ s}} = -237 \text{ N}.$ The average force on the ball is 237 N, upward.

EVALUATE: The upward force, on the ball changes the direction of its momentum.

8.65. **IDENTIFY:** The impulse, force, and change in velocity are related by $J_x = F_x\Delta t$.

SET UP: $m = w/g = 0.0571$ kg. Since the force is constant, $\vec{F} = \vec{F}_{av}$.

EXECUTE: **(a)** $J_x = F_x\Delta t = (-380 \text{ N})(3.00\times10^{-3} \text{ s}) = -1.14 \text{ N}\cdot\text{s}.$

$J_y = F_y\Delta t = (110 \text{ N})(3.00\times10^{-3} \text{ s}) = 0.330 \text{ N}\cdot\text{s}.$

(b) $v_{2x} = \dfrac{J_x}{m} + v_{1x} = \dfrac{-1.14 \text{ N}\cdot\text{s}}{0.0571 \text{ kg}} + 20.0 \text{ m/s} = 0.04 \text{ m/s}.$

$v_{2y} = \dfrac{J_y}{m} + v_{1y} = \dfrac{0.330 \text{ N}\cdot\text{s}}{0.0571 \text{ kg}} + (-4.0 \text{ m/s}) = +1.8 \text{ m/s}.$

EVALUATE: The change in velocity $\Delta\vec{v}$ is in the same direction as the force, so $\Delta\vec{v}$ has a negative x-component and a positive y-component.

8.66. **IDENTIFY:** We use the impulse-momentum theorem.

SET UP: The impulse-momentum theorem is $J_x = p_{2x} - p_{1x}$, where $p_x = mv_x$. For a variable force, we use $J_x = \int F_x dt$. From the information given, $F_x = (3.00 \text{ N/s})t$.

EXECUTE: The impulse is $J_x = \int F_x dt = \int (3.00 \text{ N/s})t dt = (1.50 \text{ N/s})t^2$, where we have used $J_x = 0$ when $t = 0$. Now use $J_x = p_{2x} - p_{1x}$, giving $(1.50 \text{ N/s})t^2 = mv - 0$. Solving for t gives

$t = \sqrt{\dfrac{mv}{1.50 \text{ N/s}}} = \sqrt{\dfrac{(2.00 \text{ kg})(9.00 \text{ m/s})}{1.50 \text{ N/s}}} = 3.46$ s. Therefore $F = (3.00 \text{ N/s})(3.464 \text{ s}) = 10.4$ N.

EVALUATE: We could find the impulse graphically as the area under the F versus t graph. This area is a triangle of base t and height F, where F = (3.00 N/s)t. The area of a triangle is $\frac{1}{2}bh$, so the impulse is $J_x = \frac{1}{2}Ft = \frac{1}{2}(3.00\text{ N/s})t = (1.50\text{ N/s})t^2.$

8.67. **IDENTIFY** and **SET UP:** When the spring is compressed the maximum amount the two blocks aren't moving relative to each other and have the same velocity V relative to the surface. Apply conservation of momentum to find V and conservation of energy to find the energy stored in the spring. Let $+x$ be the direction of the initial motion of A. The collision is elastic.
SET UP: $p = mv$, $K = ½\, mv^2$, $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ for an elastic collision.

EXECUTE: **(a)** The maximum energy stored in the spring is at maximum compression, at which time the blocks have the same velocity. Momentum conservation gives $m_A v_{A1} + m_B v_{B1} = (m_A + m_B)V$. Putting in the numbers we have (2.00 kg)(2.00 m/s) + (10.0 kg)(–0.500 m/s) = (12.0 kg)V, giving V = –0.08333 m/s. The energy U_{spring} stored in the spring is the loss of kinetic of the system. Therefore $U_{\text{spring}} = K_1 - K_2 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{V1}^2 - \frac{1}{2}(m_A + m_B)V^2$. Putting in the same set of numbers as above, and using V = –0.08333 m/s, we get U_{spring} = 5.21 J. At this time, the blocks are both moving to the left, so their velocities are each –0.0833 m/s.
(b) Momentum conservation gives $m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$. Putting in the numbers gives –1 m/s = $2v_{A2} + 10v_{B2}$. Using $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ we get $v_{B2x} - v_{A2x}$ = –(–0.500 m/s – 2.00 m/s) = +2.50 m/s. Solving this equation and the momentum equation simultaneously gives v_{A2x} = 2.17 m/s and v_{B2x} = 0.333 m/s.
EVALUATE: The total kinetic energy before the collision is 5.25 J, and it is the same after, which is consistent with an elastic collision.

8.68. **IDENTIFY:** Use a coordinate system attached to the ground. Take the x-axis to be east (along the tracks) and the y-axis to be north (parallel to the ground and perpendicular to the tracks). Then P_x is conserved and P_y is *not* conserved, due to the sideways force exerted by the tracks, the force that keeps the handcar on the tracks.
(a) SET UP: Let A be the 25.0 kg mass and B be the car (mass 175 kg). After the mass is thrown sideways relative to the car it still has the same eastward component of velocity, 5.00 m/s as it had before it was thrown.

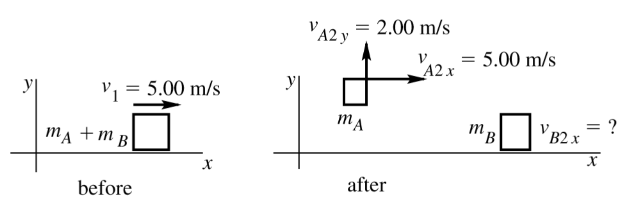


Figure 8.68a

P_x is conserved so $(m_A + m_B)v_1 = m_A v_{A2x} + m_B v_{B2x}$
EXECUTE: $(200\text{ kg})(5.00\text{ m/s}) = (25.0\text{ kg})(5.00\text{ m/s}) + (175\text{ kg})v_{B2x}$.

$$v_{B2x} = \frac{1000\text{ kg}\cdot\text{m/s} - 125\text{ kg}\cdot\text{m/s}}{175\text{ kg}} = 5.00\text{ m/s}.$$

The final velocity of the car is 5.00 m/s, east (unchanged).

EVALUATE: The thrower exerts a force on the mass in the y-direction and by Newton's third law the mass exerts an equal and opposite force in the $-y$-direction on the thrower and car.

(b) SET UP: We are applying $P_x = \text{constant}$ in coordinates attached to the ground, so we need the final velocity of A relative to the ground. Use the relative velocity addition equation. Then use $P_x = \text{constant}$ to find the final velocity of the car.

EXECUTE: $\vec{v}_{A/E} = \vec{v}_{A/B} + \vec{v}_{B/E}$

$v_{B/E} = +5.00\text{ m/s}$

$v_{A/B} = -5.00\text{ m/s}$ (minus since the mass is moving west relative to the car). This gives $v_{A/E} = 0$; the mass is at rest relative to the earth after it is thrown backwards from the car.

As in part (a) $(m_A + m_B)v_1 = m_A v_{A2x} + m_B v_{B2x}$.

Now $v_{A2x} = 0$, so $(m_A + m_B)v_1 = m_B v_{B2x}$.

$$v_{B2x} = \left(\frac{m_A + m_B}{m_B}\right)v_1 = \left(\frac{200\text{ kg}}{175\text{ kg}}\right)(5.00\text{ m/s}) = 5.71\text{ m/s}.$$

The final velocity of the car is 5.71 m/s, east.

EVALUATE: The thrower exerts a force in the $-x$-direction so the mass exerts a force on him in the $+x$-direction, and he and the car speed up.

(c) SET UP: Let A be the 25.0 kg mass and B be the car (mass $m_B = 200\text{ kg}$).

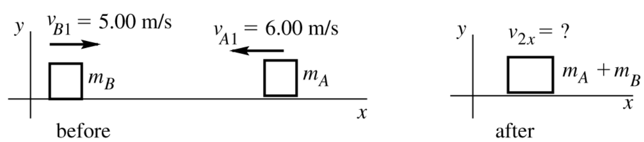


Figure 8.68b

P_x is conserved so $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

EXECUTE: $-m_A v_{A1} + m_B v_{B1} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_B v_{B1} - m_A v_{A1}}{m_A + m_B} = \frac{(200\text{ kg})(5.00\text{ m/s}) - (25.0\text{ kg})(6.00\text{ m/s})}{200\text{ kg} + 25.0\text{ kg}} = 3.78\text{ m/s}.$$

The final velocity of the car is 3.78 m/s, east.

EVALUATE: The mass has negative p_x so reduces the total P_x of the system and the car slows down.

8.69. IDENTIFY: The x- and y-components of the momentum of the system are conserved.

SET UP: After the collision the combined object with mass $m_{\text{tot}} = 0.100\text{ kg}$ moves with velocity $\vec{v}_2$. Solve for v_{Cx} and v_{Cy}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx} = m_{\text{tot}} v_{2x}$.

$$v_{Cx} = -\frac{m_A v_{Ax} + m_B v_{Bx} - m_{\text{tot}} v_{2x}}{m_C}$$

$$v_{Cx} = -\frac{(0.020\text{ kg})(-1.50\text{ m/s}) + (0.030\text{ kg})(-0.50\text{ m/s})\cos 60° - (0.100\text{ kg})(0.50\text{ m/s})}{0.050\text{ kg}}.$$

$$v_{Cx} = 1.75\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{Ay} + m_B v_{By} + m_C v_{Cy} = m_{\text{tot}} v_{2y}$.

$$v_{Cy} = -\frac{m_A v_{Ay} + m_B v_{By} - m_{\text{tot}} v_{2y}}{m_C} = -\frac{(0.030\text{ kg})(-0.50\text{ m/s})\sin 60°}{0.050\text{ kg}} = +0.260\text{ m/s}.$$

(b) $v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = 1.77$ m/s. $\Delta K = K_2 - K_1$.

$\Delta K = \frac{1}{2}(0.100 \text{ kg})(0.50 \text{ m/s})^2 - [\frac{1}{2}(0.020 \text{ kg})(1.50 \text{ m/s})^2 + \frac{1}{2}(0.030 \text{ kg})(0.50 \text{ m/s})^2 + \frac{1}{2}(0.050 \text{ kg})(1.77 \text{ m/s})^2]$
$\Delta K = -0.092$ J.

EVALUATE: Since there is no horizontal external force the vector momentum of the system is conserved. The forces the spheres exert on each other do negative work during the collision and this reduces the kinetic energy of the system.

8.70. **IDENTIFY:** We need to use the impulse-momentum theorem.
SET UP: The impulse-momentum theorem is $J_x = p_{2x} - p_{1x}$, where $p_x = mv_x$. The impulse is equal to the area under a graph of *F* versus *t*.
EXECUTE: **(a)** Figure 8.70 shows the graph of *F* versus *t*.

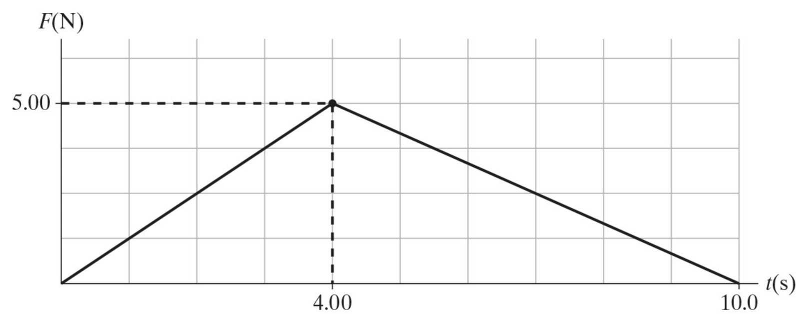


Figure 8.70

(b) Using the area of a triangle, the area under the graph is $\frac{1}{2}(5.00 \text{ N})(10.0 \text{ s}) = 25.0 \text{ N} \cdot \text{s}$. Now use $J_x = p_{2x} - p_{1x}$: $25.0 \text{ N} \cdot \text{s} = (3.00 \text{ kg})v$, so $v = 8.33$ m/s.

EVALUATE: Using the area under the curve to find the J_x is much easier than integrating $\int F_x dt$ because the force changes at 4.00 s, so the integral would have to be done in two parts.

8.71. **IDENTIFY:** Momentum is conserved during the collision, and the wood (with the clay attached) is in free fall as it falls since only gravity acts on it.
SET UP: Apply conservation of momentum to the collision to find the velocity *V* of the combined object just after the collision. After the collision, the wood's downward acceleration is *g* and it has no horizontal acceleration, so we can use the standard kinematics equations: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ and $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$.

EXECUTE: Momentum conservation gives $(0.500 \text{ kg})(24.0 \text{ m/s}) = (8.50 \text{ kg})V$, so $V = 1.412$ m/s. Consider the projectile motion after the collision: $a_y = +9.8 \text{ m/s}^2$, $v_{0y} = 0$, $y - y_0 = +2.20$ m, and *t* is unknown. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(2.20 \text{ m})}{9.8 \text{ m/s}^2}} = 0.6701$ s. The horizontal acceleration is zero so $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.412 \text{ m/s})(0.6701 \text{ s}) = 0.946$ m.

EVALUATE: The momentum is *not* conserved after the collision because an external force (gravity) acts on the system. Mechanical energy is *not* conserved during the collision because the clay and block stick together, making it an inelastic collision.

8.72. **IDENTIFY:** An inelastic collision (the objects stick together) occurs during which momentum is conserved, followed by a swing during which mechanical energy is conserved. The target variable is the initial speed of the bullet.

SET UP: Newton's second law, $\Sigma\vec{F} = m\vec{a}$, will relate the tension in the cord to the speed of the block during the swing. Mechanical energy is conserved after the collision, and momentum is conserved during the collision.

EXECUTE: First find the speed v of the block, at a height of 0.800 m. The mass of the combined object is 0.812 kg. $\cos\theta = \dfrac{0.8\text{ m}}{1.6\text{ m}} = 0.50$ so $\theta = 60.0^\circ$ is the angle the cord makes with the vertical. At this position, Newton's second law gives $T - mg\cos\theta = m\dfrac{v^2}{R}$, where we have taken force components toward the center of the circle. Solving for v gives

$v = \sqrt{\dfrac{R}{m}(T - mg\cos\theta)} = \sqrt{\dfrac{1.6\text{ m}}{0.812\text{ kg}}(4.80\text{ N} - 3.979\text{ N})} = 1.272\text{ m/s}.$ Now apply conservation of energy to find the velocity V of the combined object just after the collision: $\dfrac{1}{2}mV^2 = mgh + \dfrac{1}{2}mv^2$. Solving for V gives $V = \sqrt{2gh + v^2} = \sqrt{2(9.8\text{ m/s}^2)(0.8\text{ m}) + (1.272\text{ m/s})^2} = 4.159\text{ m/s}.$ Now apply conservation of momentum to the collision: $(0.012\text{ kg})v_0 = (0.812\text{ kg})(4.159\text{ m/s}),$ which gives $v_0 = 281\text{ m/s}.$

EVALUATE: We cannot solve this problem in a single step because different conservation laws apply to the collision and the swing.

8.73. **IDENTIFY:** During the collision, momentum is conserved, but after the collision mechanical energy is conserved. We cannot solve this problem in a single step because the collision and the motion after the collision involve different conservation laws.

SET UP: Use coordinates where $+x$ is to the right and $+y$ is upward. Momentum is conserved during the collision, so $P_{1x} = P_{2x}$. Energy is conserved after the collision, so $K_1 = U_2$, where $K = \frac{1}{2}mv^2$ and $U = mgh$.

EXECUTE: *Collision*: There is no external horizontal force during the collision so $P_{1x} = P_{2x}$. This gives $(5.00\text{ kg})(12.0\text{ m/s}) = (10.0\text{ kg})v_2$ and $v_2 = 6.0\text{ m/s}.$

Motion after the collision: Only gravity does work and the initial kinetic energy of the combined chunks is converted entirely to gravitational potential energy when the chunk reaches its maximum height h above the valley floor. Conservation of energy gives $\frac{1}{2}m_{\text{tot}}v^2 = m_{\text{tot}}gh$ and

$$h = \frac{v^2}{2g} = \frac{(6.0\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 1.8\text{ m}.$$

EVALUATE: After the collision the energy of the system is $\frac{1}{2}m_{\text{tot}}v^2 = \frac{1}{2}(10.0\text{ kg})(6.0\text{ m/s})^2 = 180\text{ J}$ when it is all kinetic energy and the energy is $m_{\text{tot}}gh = (10.0\text{ kg})(9.8\text{ m/s}^2)(1.8\text{ m}) = 180\text{ J}$ when it is all gravitational potential energy. Mechanical energy is conserved during the motion after the collision. But before the collision the total energy of the system is $\frac{1}{2}(5.0\text{ kg})(12.0\text{ m/s})^2 = 360\text{ J}$; 50% of the mechanical energy is dissipated during the inelastic collision of the two chunks.

8.74. **IDENTIFY:** Momentum is conserved during the collision. After that we use energy conservation for B.

SET UP: $P_1 = P_2$ during the collision. For B, $K_1 + U_1 = K_2 + U_2$ after the collision.

EXECUTE: For the collision, $P_1 = P_2$: (2.00 kg)(8.00 m/s) = (2.00 kg)(–2.00 m/s) + (4.00 kg)v_B, which gives v_B = 5.00 m/s. Now look at B after the collision and apply $K_1 + U_1 = K_2 + U_2$.

$K_1 + U_1 = K_2 + 0$: $\frac{1}{2} m v_B^2 + mgh = \frac{1}{2} mv^2$

$v^2 = (5.00\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(2.60\text{ m})$, which gives v = 8.72 m/s.

EVALUATE: We cannot do this problem in a single step because we have two different conservation laws involved: momentum during the collision and energy after the collision. The energy is not conserved during the collision, and the momentum of B is not conserved after the collision.

8.75. **IDENTIFY:** We use momentum conservation during the collision of the carts. Half the initial kinetic energy is lost during the collision.

SET UP: Use $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$ and $K = \frac{1}{2}mv^2$. Call the $+x$-axis the original direction that cart A is moving. Fig. 8.75 shows a before and after sketch. The carts have equal masses, the final kinetic energy is one-half the initial kinetic energy, and we want to find the speed of each cart after the collision.

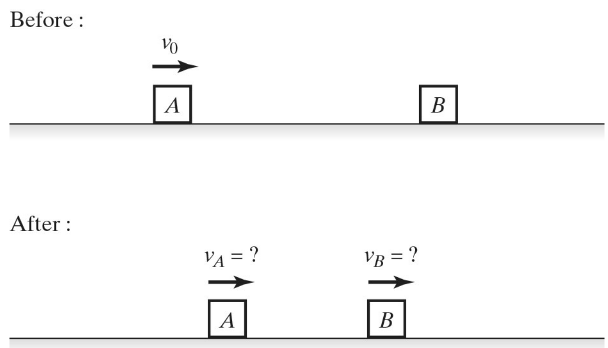


Figure 8.75

EXECUTE: Using the notation in the figure, momentum conservation gives us $mv_0 = mv_A + mv_B$, so $v_0 = v_A + v_B$. (Eq. 1)

Half the kinetic energy is lost, so $K_2 = \frac{1}{2}K_1 = \frac{1}{2}\left(\frac{1}{2}mv_0^2\right) = \frac{1}{4}mv_0^2$, which simplifies to

$v_A^2 + v_B^2 = \frac{1}{2}v_0^2$. (Eq. 2)

Solve Eq. 1 and Eq. 2 together, which gives $v_B = \frac{v_0}{2}$. Now find v_A using Eq. 1:

$v_A = v_0 - v_B = v_0 - \frac{v_0}{2} = \frac{v_0}{2}$. Returning to the notation stated in the problem, we find that each cart has speed $\frac{v_A}{2}$ after the collision.

EVALUATE: To check, we calculate the kinetic energy before and after the collision. $K_1 = \frac{1}{2}mv_0^2$ and

$K_2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 = \frac{1}{2}m\left(\frac{v_0^2}{4} + \frac{v_0^2}{4}\right) = \frac{1}{2}\left(\frac{1}{2}mv_0^2\right) = \frac{1}{2}K_1$. Our result checks.

8.76. **IDENTIFY:** During the inelastic collision, momentum is conserved but not mechanical energy. After the collision, momentum is not conserved and the kinetic energy of the cars is dissipated by nonconservative friction.

SET UP: Treat the collision and motion after the collision as separate events. Apply conservation of momentum to the collision and conservation of energy to the motion after the collision. The friction force on the combined cars is $\mu_k(m_A + m_B)g$.

EXECUTE: *Motion after the collision*: The kinetic energy of the combined cars immediately after the collision is taken away by the negative work done by friction: $\frac{1}{2}(m_A + m_B)V^2 = \mu_k(m_A + m_B)gd$, where $d = 7.15$ m. This gives $V = \sqrt{2\mu_k gd} = 9.54$ m/s.

Collision: Momentum conservation gives $m_A v_A = (m_A + m_B)V$, which gives

$$v_A = \left(\frac{m_A + m_B}{m_A}\right)V = \left(\frac{1500 \text{ kg} + 1900 \text{ kg}}{1500 \text{ kg}}\right)(9.54 \text{ m/s}) = 21.6 \text{ m/s}.$$

(b) $v_A = 21.6$ m/s $= 48$ mph, which is 13 mph greater than the speed limit.

EVALUATE: We cannot solve this problem in a single step because the collision and the motion after the collision involve different principles (momentum conservation and energy conservation).

8.77. IDENTIFY: During the inelastic collision, momentum is conserved (in two dimensions), but after the collision we must use energy principles.

SET UP: The friction force is $\mu_k m_{\text{tot}} g$. Use energy considerations to find the velocity of the combined object immediately after the collision. Apply conservation of momentum to the collision. Use coordinates where $+x$ is west and $+y$ is south. For momentum conservation, we have $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

EXECUTE: *Motion after collision*: The negative work done by friction takes away all the kinetic energy that the combined object has just after the collision. Calling ϕ the angle south of west at which the enmeshed cars slid, we have $\tan\phi = \dfrac{6.43 \text{ m}}{5.39 \text{ m}}$ and $\phi = 50.0°$. The wreckage slides 8.39 m in a direction 50.0° south of west. Energy conservation gives $\frac{1}{2}m_{\text{tot}}V^2 = \mu_k m_{\text{tot}} gd$, so

$V = \sqrt{2\mu_k gd} = \sqrt{2(0.75)(9.80 \text{ m/s}^2)(8.39 \text{ m})} = 11.1$ m/s. The velocity components are

$V_x = V\cos\phi = 7.13$ m/s; $V_y = V\sin\phi = 8.50$ m/s.

Collision: $P_{1x} = P_{2x}$ gives $(2200 \text{ kg})v_{\text{SUV}} = (1500 \text{ kg} + 2200 \text{ kg})V_x$ and $v_{\text{SUV}} = 12$ m/s. $P_{1y} = P_{2y}$ gives $(1500 \text{ kg})v_{\text{sedan}} = (1500 \text{ kg} + 2200 \text{ kg})V_y$ and $v_{\text{sedan}} = 21$ m/s.

EVALUATE: We cannot solve this problem in a single step because the collision and the motion after the collision involve different principles (momentum conservation and energy conservation).

8.78. IDENTIFY: Find k for the spring from the forces when the frame hangs at rest, use constant acceleration equations to find the speed of the putty just before it strikes the frame, apply conservation of momentum to the collision between the putty and the frame, and then apply conservation of energy to the motion of the frame after the collision.

SET UP: Use the free-body diagram in Figure 8.78a for the frame when it hangs at rest on the end of the spring to find the force constant k of the spring. Let s be the amount the spring is stretched.

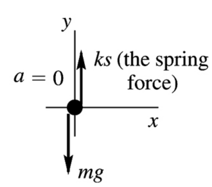


Figure 8.78a

EXECUTE: $\Sigma F_y = ma_y$ gives $-mg + ks = 0$. $k = \dfrac{mg}{s} = \dfrac{(0.150 \text{ kg})(9.80 \text{ m/s}^2)}{0.0400 \text{ m}} = 36.75$ N/m.

SET UP: Next find the speed of the putty when it reaches the frame. The putty falls with acceleration $a = g$, downward (see Figure 8.78b).

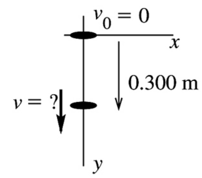


Figure 8.78b

$v_0 = 0,\ \ y - y_0 = 0.300$ m, $a = +9.80\ \text{m/s}^2$, and we want to find v. The constant-acceleration $v^2 = v_0^2 + 2a(y - y_0)$ applies to this motion.

EXECUTE: $v = \sqrt{2a(y - y_0)} = \sqrt{2(9.80\ \text{m/s}^2)(0.300\ \text{m})} = 2.425\ \text{m/s}$.

SET UP: Apply conservation of momentum to the collision between the putty (A) and the frame (B). See Figure 8.78c.

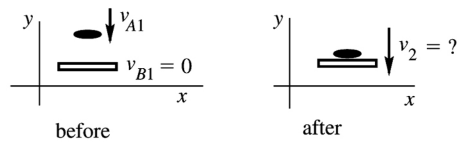


Figure 8.78c

P_y is conserved, so $-m_A v_{A1} = -(m_A + m_B)v_2$.

EXECUTE: $v_2 = \left(\dfrac{m_A}{m_A + m_B}\right)v_{A1} = \left(\dfrac{0.200\ \text{kg}}{0.350\ \text{kg}}\right)(2.425\ \text{m/s}) = 1.386\ \text{m/s}$.

SET UP: Apply conservation of energy to the motion of the frame on the end of the spring after the collision. Let point 1 be just after the putty strikes and point 2 be when the frame has its maximum downward displacement. Let d be the amount the frame moves downward (see Figure 8.78d).

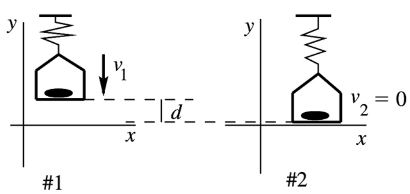


Figure 8.78d

When the frame is at position 1 the spring is stretched a distance $x_1 = 0.0400$ m. When the frame is at position 2 the spring is stretched a distance $x_2 = 0.040\ \text{m} + d$. Use coordinates with the y-direction upward and $y = 0$ at the lowest point reached by the frame, so that $y_1 = d$ and $y_2 = 0$. Work is done on the frame by gravity and by the spring force, so $W_{\text{other}} = 0$, and $U = U_{\text{el}} + U_{\text{gravity}}$.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. $W_{\text{other}} = 0$.

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.350\ \text{kg})(1.386\ \text{m/s})^2 = 0.3362\ \text{J}$.

$U_1 = U_{1,\text{el}} + U_{1,\text{grav}} = \frac{1}{2}kx_1^2 + mgy_1 = \frac{1}{2}(36.75\ \text{N/m})(0.0400\ \text{m})^2 + (0.350\ \text{kg})(9.80\ \text{m/s}^2)d$.

$U_1 = 0.02940\ \text{J} + (3.43\ \text{N})d$. $U_2 = U_{2,\text{el}} + U_{2,\text{grav}} = \frac{1}{2}kx_2^2 + mgy_2 = \frac{1}{2}(36.75\ \text{N/m})(0.0400\ \text{m} + d)^2$.

$U_2 = 0.02940\ \text{J} + (1.47\ \text{N})d + (18.375\ \text{N/m})d^2$. Thus

$0.3362\ \text{J} + 0.02940\ \text{J} + (3.43\ \text{N})d = 0.02940\ \text{J} + (1.47\ \text{N})d + (18.375\ \text{N/m})d^2$.

$(18.375\text{ N/m})d^2 - (1.96\text{ N})d - 0.3362\text{ J} = 0.$ Using the quadratic formula, with the positive solution, we get $d = 0.199$ m.

EVALUATE: The collision is inelastic and mechanical energy is lost. Thus the decrease in gravitational potential energy is *not* equal to the increase in potential energy stored in the spring.

8.79. **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion after the collision.

SET UP: Let $+x$ be to the right. The total mass is $m = m_{\text{bullet}} + m_{\text{block}} = 1.00$ kg. The spring has force constant $k = \dfrac{|F|}{|x|} = \dfrac{0.750\text{ N}}{0.250\times10^{-2}\text{ m}} = 300$ N/m. Let V be the velocity of the block just after impact.

EXECUTE: **(a)** Conservation of energy for the motion after the collision gives $K_1 = U_{\text{el}2}$. $\frac{1}{2}mV^2 = \frac{1}{2}kx^2$ and

$$V = x\sqrt{\frac{k}{m}} = (0.150\text{ m})\sqrt{\frac{300\text{ N/m}}{1.00\text{ kg}}} = 2.60\text{ m/s}.$$

(b) Conservation of momentum applied to the collision gives $m_{\text{bullet}}v_1 = mV$.

$$v_1 = \frac{mV}{m_{\text{bullet}}} = \frac{(1.00\text{ kg})(2.60\text{ m/s})}{8.00\times10^{-3}\text{ kg}} = 325\text{ m/s}.$$

EVALUATE: The initial kinetic energy of the bullet is 422 J. The energy stored in the spring at maximum compression is 3.38 J. Most of the initial mechanical energy of the bullet is dissipated in the collision.

8.80. **IDENTIFY:** The horizontal components of momentum of the system of bullet plus stone are conserved. The collision is elastic if $K_1 = K_2$.

SET UP: Let A be the bullet and B be the stone.

(a)

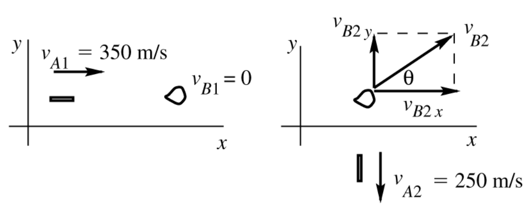


Figure 8.80

EXECUTE: P_x is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$.

$m_A v_{A1} = m_B v_{B2x}$.

$$v_{B2x} = \left(\frac{m_A}{m_B}\right)v_{A1} = \left(\frac{6.00\times10^{-3}\text{ kg}}{0.100\text{ kg}}\right)(350\text{ m/s}) = 21.0\text{ m/s}$$

P_y is conserved so $m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}$.

$0 = -m_A v_{A2} + m_B v_{B2y}$.

$$v_{B2y} = \left(\frac{m_A}{m_B}\right)v_{A2} = \left(\frac{6.00\times10^{-3}\text{ kg}}{0.100\text{ kg}}\right)(250\text{ m/s}) = 15.0\text{ m/s}.$$

$$v_{B2} = \sqrt{v_{B2x}^2 + v_{B2y}^2} = \sqrt{(21.0\text{ m/s})^2 + (15.0\text{ m/s})^2} = 25.8\text{ m/s}.$$

$\tan\theta = \dfrac{v_{B2y}}{v_{B2x}} = \dfrac{15.0\text{ m/s}}{21.0\text{ m/s}} = 0.7143;$ $\theta = 35.5°$ (defined in the sketch).

(b) To answer this question compare K_1 and K_2 for the system:

$$K_1 = \tfrac{1}{2}m_A v_{A1}^2 + \tfrac{1}{2}m_B v_{B1}^2 = \tfrac{1}{2}(6.00\times10^{-3}\text{ kg})(350\text{ m/s})^2 = 368\text{ J}.$$

$$K_2 = \tfrac{1}{2}m_A v_{A2}^2 + \tfrac{1}{2}m_B v_{B2}^2 = \tfrac{1}{2}(6.00\times10^{-3}\text{ kg})(250\text{ m/s})^2 + \tfrac{1}{2}(0.100\text{ kg})(25.8\text{ m/s})^2 = 221\text{ J}.$$

$$\Delta K = K_2 - K_1 = 221\text{ J} - 368\text{ J} = -147\text{ J}.$$

EVALUATE: The kinetic energy of the system decreases by 147 J as a result of the collision; the collision is *not* elastic. Momentum is conserved because $\Sigma \boldsymbol{F}_{\text{ext},x} = 0$ and $\Sigma \boldsymbol{F}_{\text{ext},y} = 0$. But there are internal forces between the bullet and the stone. These forces do negative work that reduces K.

8.81. **IDENTIFY:** Apply conservation of momentum to the collision between the two people. Apply conservation of energy to the motion of the stuntman before the collision and to the entwined people after the collision.

SET UP: For the motion of the stuntman, $y_1 - y_2 = 5.0$ m. Let v_S be the magnitude of his horizontal velocity just before the collision. Let V be the speed of the entwined people just after the collision. Let d be the distance they slide along the floor.

EXECUTE: **(a)** Motion before the collision: $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$ and $\tfrac{1}{2}mv_S^2 = mg(y_1 - y_2)$.

$v_S = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80\text{ m/s}^2)(5.0\text{ m})} = 9.90\text{ m/s}.$

Collision: $m_S v_S = m_{\text{tot}}V$. $V = \dfrac{m_S}{m_{\text{tot}}}v_S = \left(\dfrac{80.0\text{ kg}}{150.0\text{ kg}}\right)(9.90\text{ m/s}) = 5.28\text{ m/s}.$

(b) Motion after the collision: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $\tfrac{1}{2}m_{\text{tot}}V^2 - \mu_k m_{\text{tot}}gd = 0$.

$d = \dfrac{V^2}{2\mu_k g} = \dfrac{(5.28\text{ m/s})^2}{2(0.250)(9.80\text{ m/s}^2)} = 5.7\text{ m}.$

EVALUATE: Mechanical energy is dissipated in the inelastic collision, so the kinetic energy just after the collision is less than the initial potential energy of the stuntman.

8.82. **IDENTIFY:** Apply conservation of energy to the motion before and after the collision and apply conservation of momentum to the collision.

SET UP: Let v be the speed of the mass released at the rim just before it strikes the second mass. Let each object have mass m.

EXECUTE: Conservation of energy says $\tfrac{1}{2}mv^2 = mgR$; $v = \sqrt{2gR}$.

SET UP: This is speed v_1 for the collision. Let v_2 be the speed of the combined object just after the collision.

EXECUTE: Conservation of momentum applied to the collision gives $mv_1 = 2mv_2$ so $v_2 = v_1/2 = \sqrt{gR/2}$.

SET UP: Apply conservation of energy to the motion of the combined object after the collision. Let y_3 be the final height above the bottom of the bowl.

EXECUTE: $\tfrac{1}{2}(2m)v_2^2 = (2m)gy_3$.

$y_3 = \dfrac{v_2^2}{2g} = \dfrac{1}{2g}\left(\dfrac{gR}{2}\right) = R/4.$

EVALUATE: Mechanical energy is lost in the collision, so the final gravitational potential energy is less than the initial gravitational potential energy.

8.83. **IDENTIFY:** This collision is elastic, so kinetic energy and momentum are both conserved.

SET UP: Use $v_A = \dfrac{m_A - m_B}{m_A + m_B} v_0$ (Eq. 8.24) and $v_B = \dfrac{2m_A}{m_A + m_B} v_0$ (Eq. 8.25), as well as $K = \dfrac{1}{2}mv^2$. Call the $+x$-axis the original direction that object A is moving. Where we let $v_{Ai} = v_0$ and we have simplified the notation of Eq. 8.24 and Eq. 8.25 somewhat. We have $m_A = \alpha m_B$.

EXECUTE: **(a)** Using Eq. 8.25 gives the final kinetic energy of B.

$K_{B,f} = \dfrac{1}{2}m_B v_B^2 = \dfrac{1}{2}m_B\left(\dfrac{2m_A v_0}{m_A + m_B}\right)^2 = \dfrac{1}{2}m_B\left(\dfrac{2m_A}{m_A + m_B}\right)^2 v_0^2$. This is equal to the initial kinetic energy of A, which is $K_{A,i} = \dfrac{1}{2}m_A v_0^2 = \dfrac{1}{2}\alpha m_B v_0^2$. Equating the two kinetic energies gives

$\dfrac{1}{2}\alpha m_B v_0^2 = \dfrac{1}{2}m_B\left(\dfrac{2m_A}{m_A + m_B}\right)^2 v_0^2$, which simplifies to $\alpha = \left(\dfrac{2m_A}{m_A + m_B}\right)^2$. Using $m_A = \alpha m_B$ gives

$\alpha = \left(\dfrac{2\alpha m_B}{\alpha m_B + m_B}\right)^2 = \left(\dfrac{2\alpha}{\alpha + 1}\right)^2$. Solving for α gives $\alpha = 1$. This means that the masses are equal.

(b) In this case, after the collision $K_A = K_B$, so $\dfrac{1}{2}m_A v_A^2 = \dfrac{1}{2}m_B v_B^2$. Using $m_A = \alpha m_B$ and simplifying gives $\alpha v_A^2 = v_B^2$. Now use Eq. 24 and Eq. 25 in the last equation, which gives

$\alpha\left(\dfrac{m_A - m_B}{m_A + m_B} v_0\right)^2 = \left(\dfrac{2m_A v_0}{m_A + m_B}\right)^2$. Using $m_A = \alpha m_B$ and simplifying gives $\alpha(\alpha - 1)^2 = 4\alpha^2$. The resulting quadratic equation has solutions $\alpha = 3 + 2\sqrt{2} \approx 5.83$ and $\alpha = 3 - 2\sqrt{2} \approx 0.172$.

EVALUATE: When $\alpha = 1$, $m_A = m_B$. Object A stops and object B moves ahead with the same speed that A had. Object A has lost all of its momentum and kinetic energy and object B has gained it all, so both momentum and kinetic energy are conserved.

8.84. **IDENTIFY:** Apply conservation of energy to the motion before and after the collision. Apply conservation of momentum to the collision.

SET UP: First consider the motion after the collision. The combined object has mass $m_{\text{tot}} = 25.0$ kg. Apply $\Sigma\vec{F} = m\vec{a}$ to the object at the top of the circular loop, where the object has speed v_3. The acceleration is $a_{\text{rad}} = v_3^2/R$, downward.

EXECUTE: $T + mg = m\dfrac{v_3^2}{R}$.

The minimum speed v_3 for the object not to fall out of the circle is given by setting $T = 0$. This gives $v_3 = \sqrt{Rg}$, where $R = 2.80$ m.

SET UP: Next, use conservation of energy with point 2 at the bottom of the loop and point 3 at the top of the loop. Take $y = 0$ at point 2. Only gravity does work, so $K_2 + U_2 = K_3 + U_3$

EXECUTE: $\frac{1}{2}m_{\text{tot}}v_2^2 = \frac{1}{2}m_{\text{tot}}v_3^2 + m_{\text{tot}}g(2R)$.

Use $v_3 = \sqrt{Rg}$ and solve for v_2: $v_2 = \sqrt{5gR} = 11.71$ m/s.

SET UP: Now apply conservation of momentum to the collision between the dart and the sphere. Let v_1 be the speed of the dart before the collision.

EXECUTE: $(5.00\text{ kg})v_1 = (25.0\text{ kg})(11.71\text{ m/s})$, which gives $v_1 = 58.6$ m/s.

EVALUATE: The collision is inelastic and mechanical energy is removed from the system by the negative work done by the forces between the dart and the sphere.

8.85. **IDENTIFY:** Apply conservation of momentum to the collision between the bullet and the block and apply conservation of energy to the motion of the block after the collision.

(a) SET UP: For the collision between the bullet and the block, let object A be the bullet and object B be the block. Apply momentum conservation to find the speed v_{B2} of the block just after the collision (see Figure 8.85a).

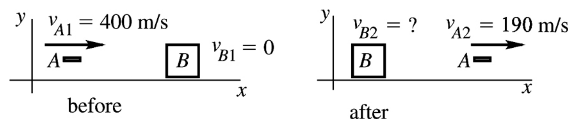


Figure 8.85a

EXECUTE: P_x is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$. $m_A v_{A1} = m_A v_{A2} + m_B v_{B2x}$.

$$v_{B2x} = \frac{m_A(v_{A1} - v_{A2})}{m_B} = \frac{4.00\times10^{-3}\ \text{kg}(400\ \text{m/s} - 190\ \text{m/s})}{0.800\ \text{kg}} = 1.05\ \text{m/s}.$$

SET UP: For the motion of the block after the collision, let point 1 in the motion be just after the collision, where the block has the speed 1.05 m/s calculated above, and let point 2 be where the block has come to rest (see Figure 8.85b).
$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

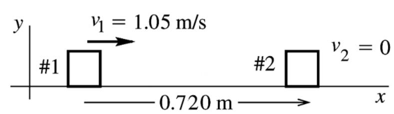


Figure 8.85b

EXECUTE: Work is done on the block by friction, so $W_{\text{other}} = W_f$.

$W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$, where $s = 0.720$ m. $U_1 = 0$, $U_2 = 0$, $K_1 = \frac{1}{2}mv_1^2$, $K_2 = 0$ (the block has come to rest). Thus $\frac{1}{2}mv_1^2 - \mu_k mgs = 0$. Therefore

$$\mu_k = \frac{v_1^2}{2gs} = \frac{(1.05\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(0.720\ \text{m})} = 0.0781.$$

(b) For the bullet, $K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(4.00\times10^{-3}\ \text{kg})(400\ \text{m/s})^2 = 320\ \text{J}$ and

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(4.00\times10^{-3}\ \text{kg})(190\ \text{m/s})^2 = 72.2\ \text{J}$. $\Delta K = K_2 - K_1 = 72.2\ \text{J} - 320\ \text{J} = -248\ \text{J}$. The kinetic energy of the bullet decreases by 248 J.

(c) Immediately after the collision the speed of the block is 1.05 m/s, so its kinetic energy is
$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.800\ \text{kg})(1.05\ \text{m/s})^2 = 0.441\ \text{J}$.

EVALUATE: The collision is highly inelastic. The bullet loses 248 J of kinetic energy but only 0.441 J is gained by the block. But momentum is conserved in the collision. All the momentum lost by the bullet is gained by the block.

8.86. IDENTIFY: Apply conservation of momentum to the collision and conservation of energy to the motion of the block after the collision.

SET UP: Let $+x$ be to the right. Let the bullet be A and the block be B. Let V be the velocity of the block just after the collision.

EXECUTE: *Motion of block after the collision*: $K_1 = U_{\text{grav}2}$. $\frac{1}{2}m_B V^2 = m_B gh$.

$$V = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(0.38\times10^{-2}\ \text{m})} = 0.273\ \text{m/s}.$$

Collision: $v_{B2} = 0.273$ m/s. $P_{1x} = P_{2x}$ gives $m_A v_{A1} = m_A v_{A2} + m_B v_{B2}$.

$$v_{A2} = \frac{m_A v_{A1} - m_B v_{B2}}{m_A} = \frac{(5.00\times10^{-3}\text{ kg})(450\text{ m/s}) - (1.00\text{ kg})(0.273\text{ m/s})}{5.00\times10^{-3}\text{ kg}} = 395\text{ m/s}.$$

EVALUATE: We assume the block moves very little during the time it takes the bullet to pass through it.

8.87. **IDENTIFY:** Apply conservation of energy to the motion of the package before the collision and apply conservation of the horizontal component of momentum to the collision.

(a) SET UP: Apply conservation of energy to the motion of the package from point 1 as it leaves the chute to point 2 just before it lands in the cart. Take $y = 0$ at point 2, so $y_1 = 4.00$ m. Only gravity does work, so

$$K_1 + U_1 = K_2 + U_2.$$

EXECUTE: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$.

$v_2 = \sqrt{v_1^2 + 2gy_1} = 9.35$ m/s.

(b) SET UP: In the collision between the package and the cart, momentum is conserved in the horizontal direction. (But not in the vertical direction, due to the vertical force the floor exerts on the cart.) Take $+x$ to be to the right. Let *A* be the package and *B* be the cart.

EXECUTE: P_x is constant gives $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$v_{B1x} = -5.00$ m/s.

$v_{A1x} = (3.00\text{ m/s})\cos 37.0°$. (The horizontal velocity of the package is constant during its free fall.)

Solving for v_{2x} gives $v_{2x} = -3.29$ m/s. The cart is moving to the left at 3.29 m/s after the package lands in it.

EVALUATE: The cart is slowed by its collision with the package, whose horizontal component of momentum is in the opposite direction to the motion of the cart.

8.88. **IDENTIFY:** Apply conservation of momentum to the system of the neutron and its decay products.

SET UP: Let the proton be moving in the $+x$-direction with speed v_p after the decay. The initial momentum of the neutron is zero, so to conserve momentum the electron must be moving in the $-x$-direction after the decay. Let the speed of the electron be v_e.

EXECUTE: $P_{1x} = P_{2x}$ gives $0 = m_p v_p - m_e v_e$ and $v_e = \left(\frac{m_p}{m_e}\right)v_p$. The total kinetic energy after the decay is $K_{tot} = \frac{1}{2}m_e v_e^2 + \frac{1}{2}m_p v_p^2 = \frac{1}{2}m_e\left(\frac{m_p}{m_e}\right)^2 v_p^2 + \frac{1}{2}m_p v_p^2 = \frac{1}{2}m_p v_p^2\left(1 + \frac{m_p}{m_e}\right)$.

Thus, $\dfrac{K_p}{K_{tot}} = \dfrac{1}{1 + m_p/m_e} = \dfrac{1}{1+1836} = 5.44\times10^{-4} = 0.0544\%$.

EVALUATE: Most of the released energy goes to the electron, since it is much lighter than the proton.

8.89. **IDENTIFY:** We have a collision, so we use momentum conservation. After the collision we have projectile motion. Fig. 8.89 illustrates this process.

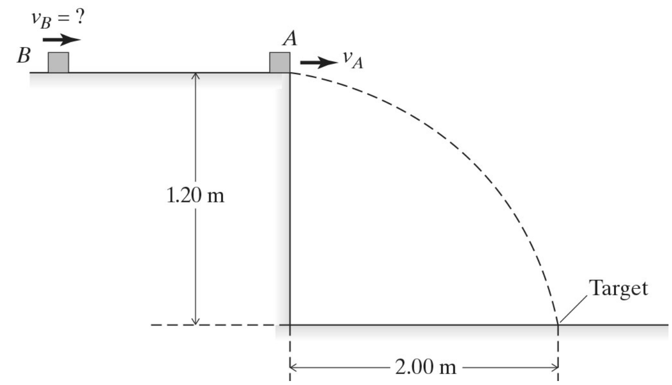


Figure 8.89

SET UP: We use projectile motion to find the speed that A needs at the top of the table to hit the target. We use momentum conservation during the collision to find the speed that B needed to give A the necessary speed. The collision is elastic, so we can use $v_A = \dfrac{m_A - m_B}{m_A + m_B} v_0$ (Eq. 8.24) and $v_B = \dfrac{2m_A}{m_A + m_B} v_0$ (Eq. 8.25). In this problem A and B are interchanged from the equations in the text because B is moving and A is initially at rest, so we must be careful. Eq. 8.24 tells us that if the masses are equal, B stops and A moves forward with the same velocity that B had.

EXECUTE: Look at the projectile motion after the collision. The constant-acceleration equations apply. The block needs to travel 2.00 m at constant horizontal velocity v_A in the same time that it falls 1.20 m starting from rest vertically. The vertical motion gives $y = \dfrac{1}{2}gt^2$, so $t = \sqrt{2y/g} = \sqrt{\dfrac{2(1.20\text{ m})}{9.80\text{ m/s}^2}} = 0.4949$ s. Horizontally $x = v_A t = 2.00$ m, so $v_A = (2.00\text{ m})/(0.4949\text{ s}) = 4.04\text{ m/s} = v_B$.

EVALUATE: If the collision were inelastic, B would have had a velocity after the collision.

8.90. **IDENTIFY:** Since there is no friction, the horizontal component of momentum of the system of Jonathan, Jane, and the sleigh is conserved.

SET UP: Let $+x$ be to the right. $w_A = 800$ N, $w_B = 600$ N and $w_C = 1000$ N.

EXECUTE: $P_{1x} = P_{2x}$ gives $0 = m_A v_{A2x} + m_B v_{B2x} + m_C v_{C2x}$.

$$v_{C2x} = -\frac{m_A v_{A2x} + m_B v_{B2x}}{m_C} = -\frac{w_A v_{A2x} + w_B v_{B2x}}{w_C}.$$

$$v_{C2x} = -\frac{(800\text{ N})[-(5.00\text{ m/s})\cos 30.0°] + (600\text{ N})[+(7.00\text{ m/s})\cos 36.9°]}{1000\text{ N}} = 0.105\text{ m/s}.$$

The sleigh's velocity is 0.105 m/s, to the right.

EVALUATE: The vertical component of the momentum of the system consisting of the two people and the sleigh is not conserved, because of the net force exerted on the sleigh by the ice while they jump.

8.91. **IDENTIFY:** No net external force acts on the Burt-Ernie-log system, so the center of mass of the system does not move.

SET UP: $x_{\text{cm}} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$.

EXECUTE: Use coordinates where the origin is at Burt's end of the log and where $+x$ is toward Ernie, which makes $x_1 = 0$ for Burt initially. The initial coordinate of the center of mass is

$x_{\text{cm},1} = \dfrac{(20.0\text{ kg})(1.5\text{ m}) + (40.0\text{ kg})(3.0\text{ m})}{90.0\text{ kg}}$. Let d be the distance the log moves toward Ernie's original position. The final location of the center of mass is

$x_{\text{cm},2} = \dfrac{(30.0\text{ kg})d + (1.5\text{ kg} + d)(20.0\text{ kg}) + (40.0\text{ kg})d}{90.0\text{ kg}}$. The center of mass does not move, so

$x_{\text{cm},1} = x_{\text{cm},2}$, which gives

$(20.0\text{ kg})(1.5\text{ m}) + (40.0\text{ kg})(3.0\text{ m}) = (30.0\text{ kg})d + (20.0\text{ kg})(1.5\text{ m} + d) + (40.0\text{ kg})d$. Solving for d gives $d = 1.33$ m.

EVALUATE: Burt, Ernie, and the log all move, but the center of mass of the system does not move.

8.92. **IDENTIFY:** There is no net horizontal external force so v_{cm} is constant.

SET UP: Let $+x$ be to the right, with the origin at the initial position of the left-hand end of the canoe. $m_{\text{A}} = 45.0$ kg, $m_B = 60.0$ kg. The center of mass of the canoe is at its center.

EXECUTE: Initially, $v_{\text{cm}} = 0$, so the center of mass doesn't move. Initially, $x_{\text{cm1}} = \dfrac{m_A x_{A1} + m_B x_{B1}}{m_A + m_B}$.

After she walks, $x_{\text{cm2}} = \dfrac{m_A x_{A2} + m_B x_{B2}}{m_A + m_B}$. $x_{\text{cm1}} = x_{\text{cm2}}$ gives $m_A x_{A1} + m_B x_{B1} = m_A x_{A2} + m_B x_{B2}$. She walks to a point 1.00 m from the right-hand end of the canoe, so she is 1.50 m to the right of the center of mass of the canoe and $x_{A2} = x_{B2} + 1.50$ m.

$$(45.0\text{ kg})(1.00\text{ m}) + (60.0\text{ kg})(2.50\text{ m}) = (45.0\text{ kg})(x_{B2} + 1.50\text{ m}) + (60.0\text{ kg})x_{B2}.$$

$(105.0\text{ kg})x_{B2} = 127.5\text{ kg}\cdot\text{m}$ and $x_{B2} = 1.21$ m. $x_{B2} - x_{B1} = 1.21\text{ m} - 2.50\text{ m} = -1.29$ m. The canoe moves 1.29 m to the left.

EVALUATE: When the woman walks to the right, the canoe moves to the left. The woman walks 3.00 m to the right relative to the canoe and the canoe moves 1.29 m to the left, so she moves $3.00\text{ m} - 1.29\text{ m} = 1.71\text{ m}$ to the right relative to the water. Note that this distance is $(60.0\text{ kg}/45.0\text{ kg})(1.29\text{ m})$.

8.93. **IDENTIFY:** This process involves a swing, a collision, and another swing. Energy is conserved during the two swings and momentum is conserved during the collision.

SET UP: We must break this problem into three parts: the first swing, the collision, and the second swing. We cannot solve it in a single step. We want to find h_{max}, the maximum height the combined spheres reach during the second swing after the collision.

EXECUTE: Swing of B: Energy conservation gives $mgH = \frac{1}{2}mv_B^2$, so $v_B = \sqrt{2gH}$.

Collision: Momentum conservation gives $mv_B = (3m)v_{AB}$, so $v_{AB} = \dfrac{v_B}{3} = \dfrac{1}{3}\sqrt{2gH}$.

Swing of $A + B$: Energy conservation gives $\frac{1}{2}(3m)v_{AB}^2 = 3mgh_{\text{max}}$. Using v_{AB}, this becomes

$\dfrac{1}{2}\left(\dfrac{1}{3}\sqrt{2gH}\right)^2 = gh_{\text{max}}$. Solving for h_{max} gives $h_{\text{max}} = \dfrac{H}{9}$.

EVALUATE: It is reasonable that we get $h_{\text{max}} < H$ because mechanical energy is lost during the inelastic collision.

8.94. **IDENTIFY:** The explosion produces only internal forces for the fragments, so the momentum of the two-fragment system is conserved. Therefore the explosion does not affect the motion of the center of mass of this system.

SET UP: The center of mass follows a parabolic path just as a single particle would. Its horizontal range is $R = \dfrac{v_0^2 \sin(2\alpha)}{g}$. The center of mass of a two-particle system is $x_{\text{cm}} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$.

EXECUTE: **(a)** The range formula gives $R = (18.0\text{ m/s})^2(\sin 102°)/(9.80\text{ m/s}^2) = 32.34$ m, which rounds to 32.3 m.

(b) The center of mass is 32.3 m from the firing point and one fragment lands at $x_2 = 26.0$ m. Using the center of mass formula, with the origin at the firing point and calling m the mass of each fragment, we have 32.34 m = $[m(26.0\text{ m}) + mx_2]/(2m)$, which gives $x_2 = 38.68$ m, which rounds to 38.7 m.

EVALUATE: Since the fragments have equal masses, their center of mass should be midway between them. So it should be at (26.0 m + 38.68 m)/2 = 32.3 m, which it is.

8.95. **IDENTIFY:** The collision is inelastic since the blocks stick together. Momentum is conserved during the collision and energy is conserved before and after the collision.

SET UP: Hooke's law: $F = kx$. The elastic energy stored in a spring is $U_{\text{spr}} = \frac{1}{2}kx^2$. Energy conservation gives $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ and momentum is $p_x = mv_x$.

EXECUTE: **(a)** First find the distance the spring is compressed using Hooke's law. $mg = kx$, so $x = \dfrac{mg}{k} = \dfrac{(0.500\text{ kg})(9.80\text{ m/s}^2)}{80.0\text{ N/m}} = 0.06125$ m. The energy stored with this compression is

$$U_{\text{spr}} = \frac{1}{2}kx^2 = \frac{1}{2}(80.0\text{ N})(0.06125\text{ m})^2 = 0.150\text{ J}.$$

(b) After the collision, the two-block system has kinetic energy, which can be transferred to the spring. As the spring compresses, gravity also does work on the masses. Fig. 8.95 shows the system just after the collision.

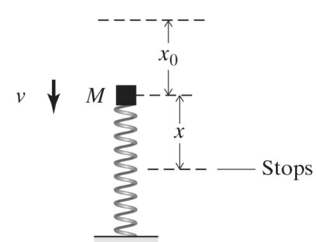


Figure 8.95

First we need to find v, the speed of the two-block system just after the collision. We do this in two steps: conservation of energy as the first block is dropped and reaches the block on the spring, followed by momentum conservation during the collision. Call v_0 the speed of the single block just before it hits the block on the spring. Energy conservation gives $mgh = \frac{1}{2}mv_0^2$, so $v_0 = \sqrt{2gh}$. Momentum conservation during the collision gives $mv_0 = (2m)v$, which gives

$$v = \frac{v_0}{2} = \frac{\sqrt{2gh}}{2} = \frac{\sqrt{2(9.80\text{ m/s}^2)(4.00\text{ m})}}{2} = 19.60\text{ m/s}.$$

Now we use energy conservation after the collision. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ with $W_{\text{other}} = 0$. Choose point 1 to be the instant after the collision and point 2 to be when the spring has its maximum compression. At that point, the blocks stop, so $K_2 = 0$. Call x the maximum distance that the spring will compress after the collision (see Fig. 8.25). At point 1 the blocks have gravitational potential energy, which is $U_{\text{g}} = mgx$. At point 1 the system has two forms of potential, U_{g} and the elastic potential energy that is already stored in the spring. In part (a) we saw that this is 0.150 J. At point 2 the system has only elastic potential energy because the spring is now compressed a *total* distance of $x_0 + x$, where x_0 = 0.06125 m from part (a). Calling M the total mass of the system, $U_1 + K_1 = U_2 + K_2$ becomes $Mgx + U_1 + \frac{1}{2}Mv^2 = \frac{1}{2}k(x_0 + x)^2$. Expanding the square, realizing that $U_1 = \frac{1}{2}kx_0^2$, and collecting terms, this equation becomes

$-kx^2 + (2Mg - 2kx_0)x + Mv^2 = 0.$ Using k = 80.0 N/m, v = 19.60 m/s, M = 1.00 kg, and x_0 = 0.06125 m, the quadratic formula gives two solutions. One of the solutions is negative, so we discard it as nonphysical. The other solution is x = 0.560 m. This is the maximum distance that the system compresses the spring *after* the collision. But the spring was already compressed by 0.06125 m before the collision, so the *total* distance that the spring is compressed at the instant the blocks stop moving is x_{total} = 0.560 m + 0.06125 m. Therefore the maximum elastic energy stored in the spring is

$U_{max} = \frac{1}{2}kx_{max}^2 = \frac{1}{2}k(x + x_0)^2 = \frac{1}{2}(80 \text{ N/m})(0.560 \text{ m} + 0.06125 \text{ m})^2 = 15.4 \text{ J}.$

(c) As shown in part (b), x = 0.560 m.

EVALUATE: We cannot overlook the gravitational potential energy in solving this problem because it could be significant compared to the elastic energy in the spring.

8.96. **IDENTIFY:** Conservation of x- and y-components of momentum applies to the collision. At the highest point of the trajectory the vertical component of the velocity of the projectile is zero.

SET UP: Let $+y$ be upward and $+x$ be horizontal and to the right. Let the two fragments be A and B, each with mass m. For the projectile before the explosion and the fragments after the explosion. $a_x = 0$, $a_y = -9.80 \text{ m/s}^2$.

EXECUTE: **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_y = 0$ gives that the maximum height of the projectile is

$h = -\frac{v_{0y}^2}{2a_y} = -\frac{[(80.0 \text{ m/s})\sin 60.0°]^2}{2(-9.80 \text{ m/s}^2)} = 244.9 \text{ m}.$ Just before the explosion the projectile is moving to the right with horizontal velocity $v_x = v_{0x} = v_0 \cos 60.0° = 40.0 \text{ m/s}.$ After the explosion $v_{Ax} = 0$ since fragment A falls vertically. Conservation of momentum applied to the explosion gives $(2m)(40.0 \text{ m/s}) = mv_{Bx}$ and $v_{Bx} = 80.0 \text{ m/s}.$ Fragment B has zero initial vertical velocity so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives a time of fall of $t = \sqrt{-\frac{2h}{a_y}} = \sqrt{-\frac{2(244.9 \text{ m})}{-9.80 \text{ m/s}^2}} = 7.07 \text{ s}.$ During this time the fragment travels horizontally a distance $(80.0 \text{ m/s})(7.07 \text{ s}) = 566 \text{ m}.$ It also took the projectile 7.07 s to travel from launch to maximum height and during this time it travels a horizontal distance of $([80.0 \text{ m/s}]\cos 60.0°)(7.07 \text{ s}) = 283 \text{ m}.$ The second fragment lands $283 \text{ m} + 566 \text{ m} = 849 \text{ m}$ from the firing point.

(b) For the explosion, $K_1 = \frac{1}{2}(20.0 \text{ kg})(40.0 \text{ m/s})^2 = 1.60 \times 10^4 \text{ J}.$

$K_2 = \frac{1}{2}(10.0 \text{ kg})(80.0 \text{ m/s})^2 = 3.20 \times 10^4 \text{ J}.$ The energy released in the explosion is 1.60×10^4 J.

EVALUATE: The kinetic energy of the projectile just after it is launched is 6.40×10^4 J. We can calculate the speed of each fragment just before it strikes the ground and verify that the total kinetic energy of the fragments just before they strike the ground is $6.40 \times 10^4 \text{ J} + 1.60 \times 10^4 \text{ J} = 8.00 \times 10^4 \text{ J}.$ Fragment A has speed 69.3 m/s just before it strikes the ground, and hence has kinetic energy 2.40×10^4 J. Fragment B has speed $\sqrt{(80.0 \text{ m/s})^2 + (69.3 \text{ m/s})^2} = 105.8 \text{ m/s}$ just before it strikes the ground, and hence has kinetic energy 5.60×10^4 J. Also, the center of mass of the system has the same horizontal range $R = \frac{v_0^2}{g}\sin(2\alpha_0) = 565 \text{ m}$ that the projectile would have had if no explosion had occurred. One fragment lands at $R/2$ so the other, equal mass fragment lands at a distance $3R/2$ from the launch point.

8.97. **IDENTIFY:** The rocket moves in projectile motion before the explosion and its fragments move in projectile motion after the explosion. Apply conservation of energy and conservation of momentum to the explosion.

(a) SET UP: Apply conservation of energy to the explosion. Just before the explosion the rocket is at its maximum height and has zero kinetic energy. Let *A* be the piece with mass 1.40 kg and *B* be the piece with mass 0.28 kg. Let v_A and v_B be the speeds of the two pieces immediately after the collision.

EXECUTE: $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = 860\text{ J}$

SET UP: Since the two fragments reach the ground at the same time, their velocities just after the explosion must be horizontal. The initial momentum of the rocket before the explosion is zero, so after the explosion the pieces must be moving in opposite horizontal directions and have equal magnitude of momentum: $m_A v_A = m_B v_B$.

EXECUTE: Use this to eliminate v_A in the first equation and solve for v_B:

$$\tfrac{1}{2}m_B v_B^2(1 + m_B / m_A) = 860\text{ J} \text{ and } v_B = 71.6\text{ m/s}.$$

Then $v_A = (m_B/m_A)v_B = 14.3\text{ m/s}$.

(b) SET UP: Use the vertical motion from the maximum height to the ground to find the time it takes the pieces to fall to the ground after the explosion. Take $+y$ downward.

$$v_{0y} = 0,\ \ a_y = +9.80\text{ m/s}^2,\ \ y - y_0 = 80.0\text{ m},\ \ t = ?$$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 4.04\text{ s}$.

During this time the horizontal distance each piece moves is $x_A = v_A t = 57.8\text{ m}$ and $x_B = v_B t = 289.1\text{ m}$. They move in opposite directions, so they are $x_A + x_B = 347\text{ m}$ apart when they land.

EVALUATE: Fragment *A* has more mass so it is moving slower right after the collision, and it travels horizontally a smaller distance as it falls to the ground.

8.98. **IDENTIFY:** Apply conservation of momentum to the explosion. At the highest point of its trajectory the shell is moving horizontally. If one fragment received some upward momentum in the explosion, the other fragment would have had to receive a downward component. Since they each hit the ground at the same time, each must have zero vertical velocity immediately after the explosion.

SET UP: Let $+x$ be horizontal, along the initial direction of motion of the projectile and let $+y$ be upward. At its maximum height the projectile has $v_x = v_0\cos 55.0° = 86.0\text{ m/s}$. Let the heavier fragment be *A* and the lighter fragment be *B*. $m_A = 9.00\text{ kg}$ and $m_B = 3.00\text{ kg}$.

EXECUTE: Since fragment *A* returns to the launch point, immediately after the explosion it has $v_{Ax} = -86.0\text{ m/s}$. Conservation of momentum applied to the explosion gives $(12.0\text{ kg})(86.0\text{ m/s}) = (9.00\text{ kg})(-86.0\text{ m/s}) + (3.00\text{ kg})v_{Bx}$ and $v_{Bx} = 602\text{ m/s}$. The horizontal range of the projectile, if no explosion occurred, would be $R = \dfrac{v_0^2}{g}\sin(2\alpha_0) = 2157\text{ m}$. The horizontal distance each fragment travels is proportional to its initial speed and the heavier fragment travels a horizontal distance $R/2 = 1078\text{ m}$ after the explosion, so the lighter fragment travels a horizontal distance $\left(\dfrac{602\text{ m}}{86\text{ m}}\right)(1078\text{ m}) = 7546\text{ m}$ from the point of explosion and $1078\text{ m} + 7546\text{ m} = 8624\text{ m}$ from the launch point. The energy released in the explosion is

$K_2 - K_1 = \frac{1}{2}(9.00\text{ kg})(86.0\text{ m/s})^2 + \frac{1}{2}(3.00\text{ kg})(602\text{ m/s})^2 - \frac{1}{2}(12.0\text{ kg})(86.0\text{ m/s})^2 = 5.33\times 10^5\text{ J}.$

EVALUATE: The center of mass of the system has the same horizontal range $R = 2157$ m as if the explosion didn't occur. This gives $(12.0 \text{ kg})(2157 \text{ m}) = (9.00 \text{ kg})(0) + (3.00 \text{ kg})d$ and $d = 8630$ m, where d is the distance from the launch point to where the lighter fragment lands. This agrees with our calculation.

8.99. **IDENTIFY:** Apply conservation of energy to the motion of the wagon before the collision. After the collision the combined object moves with constant speed on the level ground. In the collision the horizontal component of momentum is conserved.

SET UP: Let the wagon be object A and treat the two people together as object B. Let $+x$ be horizontal and to the right. Let V be the speed of the combined object after the collision.

EXECUTE: **(a)** The speed v_{A1} of the wagon just before the collision is given by conservation of energy applied to the motion of the wagon prior to the collision. $U_1 = K_2$ says

$m_A g([50 \text{ m}][\sin 6.0°]) = \frac{1}{2} m_A v_{A1}^2$. $v_{A1} = 10.12$ m/s. $P_{1x} = P_{2x}$ for the collision says $m_A v_{A1} = (m_A + m_B)V$

and $V = \left(\dfrac{300 \text{ kg}}{300 \text{ kg} + 75.0 \text{ kg} + 60.0 \text{ kg}}\right)(10.12 \text{ m/s}) = 6.98$ m/s. In 5.0 s the wagon travels

$(6.98 \text{ m/s})(5.0 \text{ s}) = 34.9$ m, and the people will have time to jump out of the wagon before it reaches the edge of the cliff.

(b) For the wagon, $K_1 = \frac{1}{2}(300 \text{ kg})(10.12 \text{ m/s})^2 = 1.54 \times 10^4$ J. Assume that the two heroes drop from a small height, so their kinetic energy just before the wagon can be neglected compared to K_1 of the wagon. $K_2 = \frac{1}{2}(435 \text{ kg})(6.98 \text{ m/s})^2 = 1.06 \times 10^4$ J. The kinetic energy of the system decreases by $K_1 - K_2 = 4.8 \times 10^3$ J.

EVALUATE: The wagon slows down when the two heroes drop into it. The mass that is moving horizontally increases, so the speed decreases to maintain the same horizontal momentum. In the collision the vertical momentum is not conserved, because of the net external force due to the ground.

8.100. **IDENTIFY:** Impulse is equal to the area under the curve in a graph of force versus time.

SET UP: $J_x = \Delta p_x = F_x \Delta t$.

EXECUTE: **(a)** Impulse is the area under *F-t* curve

$J_x = [7500 \text{ N} + ½ (7500 \text{ N} + 3500 \text{ N}) + 3500 \text{ N}](1.50 \text{ s}) = 2.475 \times 10^4 \text{ N} \cdot \text{s}$.

(b) The total mass of the car and driver is $(3071 \text{ lb})(4.448 \text{ N/lb})/(9.80 \text{ m/s}^2) = 1394$ kg.

$J_x = \Delta p_x = mv_x - 0$, so $v_x = J_x/m = (2.475 \times 10^4 \text{ N} \cdot \text{s})/(1394 \text{ kg}) = 17.8$ m/s.

(c) The braking force must produce an impulse opposite to the one that accelerated the car, so $J_x = -2.475 \times 10^4 \text{ N} \cdot \text{s}$. Therefore $J_x = F_x \Delta t$ gives $\Delta t = J_x/F_x = (-24{,}750 \text{ N} \cdot \text{s})/(-5200 \text{ N}) = 4.76$ s.

(d) $W_{\text{brake}} = \Delta K = -K = -½ mv^2 = -½ (1394 \text{ kg})(17.76 \text{ m/s})^2 = -2.20 \times 10^5$ J.

(e) $W_{\text{brake}} = -B_x s$, so $s = -W_{\text{brake}}/B_x = -(2.20 \times 10^5 \text{ J})/(-5200 \text{ N}) = 42.3$ m.

EVALUATE: The result in (e) could be checked by using kinematics with an average velocity of (17.8 m/s)/2 for 4.76 s.

8.101. **IDENTIFY:** As the bullet strikes and embeds itself in the block, momentum is conserved. After that, we use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where W_{other} is due to kinetic friction.

SET UP: Momentum conservation during the collision gives $m_b v_b = (m_b + m)V$, where m is the mass of the block and m_b is the mass of the bullet. After the collision, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$\frac{1}{2}MV^2 - \mu_k Mgd = \frac{1}{2}kd^2$, where M is the mass of the block plus the bullet.

EXECUTE: **(a)** From the energy equation above, we can see that the greatest compression of the spring will occur for the greatest V (since $M >> m_b$), and the greatest V will occur for the bullet with the greatest initial momentum. Using the data in the table with the problem, we get the following momenta expressed in units of grain · ft/s.

A: 1.334×10^5 grain · ft/s B: 1.181×10^5 grain · ft/s C: 2.042×10^5 grain · ft/s

D: 1.638×10^5 grain · ft/s E: 1.869×10^5 grain · ft/s

From these results, it is clear that bullet C will produce the maximum compression of the spring and bullet B will produce the least compression.

(b) For bullet C, we use $p_b = m_b v_b = (m_b + m)V$. Converting mass (in grains) and speed to SI units gives $m_b = 0.01555$ kg and $v_b = 259.38$ m/s, we have

(0.01555 kg)(259.38 m/s) = (0.01555 kg + 2.00 kg)V, so $V = 2.001$ m/s.

Now use $\frac{1}{2}MV^2 - \mu_k Mgd = \frac{1}{2}kd^2$ and solve for k, giving

$k = (2.016\text{ kg})[(2.001\text{ m/s})^2 - 2(0.38)(9.80\text{ m/s}^2)(0.25\text{ m})]/(0.25\text{ m})^2 = 69.1$ N/m, which rounds to 69 N/m.

(c) For bullet B, m_b =125 grains = 0.00810 kg and v_b = 945 ft/s = 288.0 m/s. Momentum conservation gives

$V = (0.00810\text{ kg})(288.0\text{ m/s})/(2.00810\text{ kg}) = 1.162$ m/s.

Using $\frac{1}{2}MV^2 - \mu_k Mgd = \frac{1}{2}kd^2$, the above numbers give $33.55d^2 + 7.478d - 1.356 = 0$. The quadratic formula, using the positive square root, gives $d = 0.118$ m, which rounds to 0.12 m.

EVALUATE: This method for measuring muzzle velocity involves a spring displacement of around 12 cm, which should be readily measurable.

8.102. **IDENTIFY** Momentum is conserved during the collision. After the collision, we can use energy methods.

SET UP: $p = mv$, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where W_{other} is due to kinetic friction. We need to use components of momentum. Call $+x$ eastward and $+y$ northward.

EXECUTE: **(a)** Momentum conservation gives

$p_x = [6500\text{ lb})/g]v_D = [(9542\text{ lb})/g]v_w\cos(39°)$

$p_y = [(3042\text{ lb})/g](50\text{ mph}) = [(9542\text{ lb})/g]v_w\sin(39°)$

Solving for v_D gives v_D = 28.9 mph, which rounds to 29 mph.

(b) The above equations also give that the velocity of the wreckage just after impact is 25.3 mph = 37.1 ft/s. Using $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, we have $\frac{1}{2}mv_1^2 - \mu_k mgd = \frac{1}{2}mv_2^2$. Solving for v_2 gives

$v_2 = \sqrt{v_1^2 - 2\mu_k gd}$. Using v_1 = 37.1 ft/s, g = 32.2 ft/s^2 and d = 35 ft, we get v_2 = 19.1 ft/s = 13 mph.

EVALUATE: We were able to minimize unit conversions by working in British units instead of SI units since the data was given in British units.

8.103. **IDENTIFY:** From our analysis of motion with constant acceleration, if $v = at$ and a is constant, then $x - x_0 = v_0 t + \frac{1}{2}at^2$.

SET UP: Take $v_0 = 0$, $x_0 = 0$ and let $+x$ downward.

EXECUTE: **(a)** $\frac{dv}{dt} = a$, $v = at$ and $x = \frac{1}{2}at^2$. Substituting into $xg = x\frac{dv}{dt} + v^2$ gives

$\frac{1}{2}at^2 g = \frac{1}{2}at^2 a + a^2t^2 = \frac{3}{2}a^2t^2$. The nonzero solution is $a = g/3$.

(b) $x = \frac{1}{2}at^2 = \frac{1}{6}gt^2 = \frac{1}{6}(9.80\text{ m/s}^2)(3.00\text{ s})^2 = 14.7$ m.

(c) $m = kx = (2.00\text{ g/m})(14.7\text{ m}) = 29.4$ g.

EVALUATE: The acceleration is less than g because the small water droplets are initially at rest, before they adhere to the falling drop. The small droplets are suspended by buoyant forces that we ignore for the raindrops.

8.104. **IDENTIFY** and **SET UP:** $dm = \rho dV$. $dV = Adx$. Since the thin rod lies along the x-axis, $y_{\text{cm}} = 0$. The mass of the rod is given by $M = \int dm$.

EXECUTE: **(a)** $x_{\text{cm}} = \dfrac{1}{M}\int_0^L x dm = \dfrac{\rho}{M} A\int_0^L x dx = \dfrac{\rho A}{M}\dfrac{L^2}{2}$. The volume of the rod is AL and $M = \rho AL$.

$x_{\text{cm}} = \dfrac{\rho AL^2}{2\rho AL} = \dfrac{L}{2}$. The center of mass of the uniform rod is at its geometrical center, midway between its ends.

(b) $x_{\text{cm}} = \dfrac{1}{M}\int_0^L x dm = \dfrac{1}{M}\int_0^L x\rho A dx = \dfrac{A\alpha}{M}\int_0^L x^2 dx = \dfrac{A\alpha L^3}{3M}$. $M = \int dm = \int_0^L \rho A dx = \alpha A\int_0^L x dx = \dfrac{\alpha AL^2}{2}$.

Therefore, $x_{\text{cm}} = \left(\dfrac{A\alpha L^3}{3}\right)\left(\dfrac{2}{\alpha AL^2}\right) = \dfrac{2L}{3}$.

EVALUATE: When the density increases with x, the center of mass is to the right of the center of the rod.

8.105. **IDENTIFY:** $x_{\text{cm}} = \dfrac{1}{M}\int x dm$ and $y_{\text{cm}} = \dfrac{1}{M}\int y dm$. At the upper surface of the plate, $y^2 + x^2 = a^2$.

SET UP: To find x_{cm}, divide the plate into thin strips parallel to the y-axis, as shown in Figure 8.105a. To find y_{cm}, divide the plate into thin strips parallel to the x-axis as shown in Figure 8.105b. The plate has volume one-half that of a circular disk, so $V = \frac{1}{2}\pi a^2 t$ and $M = \frac{1}{2}\rho\pi a^2 t$.

EXECUTE: In Figure 8.105a each strip has length $y = \sqrt{a^2 - x^2}$. $x_{\text{cm}} = \dfrac{1}{M}\int x dm$, where

$dm = \rho t y dx = \rho t\sqrt{a^2 - x^2}\, dx$. $x_{\text{cm}} = \dfrac{\rho t}{M}\int_{-a}^{a} x\sqrt{a^2 - x^2}\, dx = 0$, since the integrand is an odd function of x.

$x_{\text{cm}} = 0$ because of symmetry. In Figure 8.105b each strip has length $2x = 2\sqrt{a^2 - y^2}$. $y_{\text{cm}} = \dfrac{1}{M}\int y dm$,

where $dm = 2\rho t x dy = 2\rho t\sqrt{a^2 - y^2}\, dy$. $y_{\text{cm}} = \dfrac{2\rho t}{M}\int_{-a}^{a} y\sqrt{a^2 - y^2}\, dy$. The integral can be evaluated using

$u = a^2 - y^2$, $du = -2y dy$. This substitution gives

$$y_{\text{cm}} = \frac{2\rho t}{M}\left(-\frac{1}{2}\right)\int_{a^2}^{0} u^{1/2} du = \frac{2\rho t a^3}{3M} = \left(\frac{2\rho t a^3}{3}\right)\left(\frac{2}{\rho\pi a^2 t}\right) = \frac{4a}{3\pi}.$$

EVALUATE: $\dfrac{4}{3\pi} = 0.424$. y_{cm} is less than $a/2$, as expected, since the plate becomes wider as y decreases.

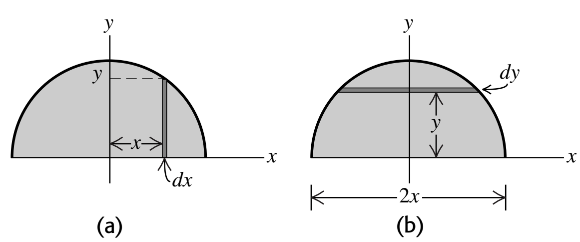


Figure 8.105

8.106. **IDENTIFY** and **SET UP:** $p = mv$.
EXECUTE: $p = mv = (0.30 \times 10^{-3}\text{ kg})(2.5\text{ m/s}) = 7.5 \times 10^{-4}\text{ kg}\cdot\text{m/s}$, which makes choice (a) correct.

EVALUATE: This is a small amount of momentum for a speed of 2.5 m/s, but the water drop is very light.

8.107. **IDENTIFY** and **SET UP:** Momentum is conserved, $p = mv$.
EXECUTE: $(65 \times 10^{-3}\text{ kg})v_{\text{fish}} = 7.5 \times 10^{-4}\text{ kg}\cdot\text{m/s}$, so $v_{\text{fish}} = 0.012$ m/s, which makes choice (b) correct.
EVALUATE: The fish is much ligher than the water drop and thus moves much slower.

8.108. **IDENTIFY** and **SET UP:** $J = F_{\text{av}}t = \Delta p$.
EXECUTE: $F_{\text{av}} = \Delta p/t = (7.5\times10^{-4}\text{ kg}\cdot\text{m/s})/(0.0050\text{ s}) = 0.15\text{ N}$, which is choice (d).

EVALUATE: This is a rather small force, but it acts on a very light-weight water drop, so it can give the water considerable speed.

8.109. **IDENTIFY** and **SET UP:** Momentum is conserved in the collision with the insect. $p = mv$.
EXECUTE: Using $P_1 = P_2$ gives $7.5 \times 10^{-4}\text{ kg}\cdot\text{m/s} = (m_{\text{insect}} + 3.0 \times 10^{-4}\text{ kg})(2.0\text{ m/s})$, which gives $m_{\text{insect}} = 0.075$ g, so choice (b) is correct.
EVALUATE: The insect has considerably less mass than the water drop.

9

ROTATION OF RIGID BODIES

VP9.3.1. **IDENTIFY:** We are dealing with angular motion having constant angular acceleration, so the constant angular acceleration formulas apply.

SET UP: A $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

EXECUTE: **(a)** (3.25 rad/s)^2 = (0.500 rad/s)^2 + 2(2.50 rad/s^2) $(\theta - \theta_0)$ $\rightarrow$ $\theta - \theta_0$ = 2.06 rad.

(b) Convert using π rad = 180°: 2.06 rad $\left(\dfrac{180°}{\pi\text{ rad}}\right)$ = 118°.

(c) Convert using 1 rev = 2π rad: 2.06 rad $\left(\dfrac{1\text{ rev}}{2\pi\text{ rad}}\right)$ = 0.328 rev.

EVALUATE: The conversion factors in (b) and (c) are very useful to remember.

VP9.3.2. **IDENTIFY:** The constant angular acceleration formulas apply.

SET UP: $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ applies.

EXECUTE: **(a)** Use $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ to find α_z .

8.00 rad = (1.25 rad/s)(2.00 s) + $\frac{1}{2}$ α_z (2.00 s)^2 $\rightarrow$ α_z = 2.75 rad/s^2.

(b) Now use $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ again to find $\theta - \theta_0$.

$\theta - \theta_0$ = (1.25 rad/s)(4.00 s) + $\frac{1}{2}$(2.75 rad/s^2)(4.00 s)^2 = 27.0 rad.

EVALUATE: During the first 2.00 s the rotor turned through 8.00 rad and during the first 4.00 s it turned through 27.0 rad. This means that during the *second* 2.00 s it turned through 19.0 rad, which is considerably more than 8.00 rad. The difference shows us that the rotor is speeding up due to angular acceleration, so it turns much farther during the second 2.00 s than during the first 2.00 s.

VP9.3.3. **IDENTIFY:** The constant angular acceleration formulas apply.

SET UP: We need α_z to find the time to slow down from 185 rad/s to 105 rad/s, so we use $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ and $\omega_z = \omega_{0z} + \alpha_z t$.

EXECUTE: First convert 16.0 rev to rad: 16.0 rev = (16.0)(2π) rad.

Now use $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ to find α_z .

(105 rad/s)^2 = (185 rad/s)^2 + 2 α_z (16.0)(2π rad) $\rightarrow$ α_z = –115. 4 rad/s^2.

Now use $\omega_z = \omega_{0z} + \alpha_z t$ to find the time for the wheel to slow down.

105 rad/s = 185 rad/s + (–115.4 rad/s^2)t $\rightarrow$ t = 0.693 s.

EVALUATE: The minus sign for α_z means that it is in the opposite angular direction from ω_z. This is reasonable because the wheel is slowing down. If α_z and ω_z were in the same direction, the wheel would be increasing its angular speed.

VP9.3.4. **IDENTIFY:** The constant angular acceleration formulas apply.

SET UP: We need the time to turn through a given angle θ, so we use $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$.

EXECUTE: The angular displacement is θ, so we use $\theta = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ and solve for *t*. Using the quadratic formula gives $t = \dfrac{-\omega_{0z} + \sqrt{\omega_{0z}^2 + 2\alpha_z\theta}}{\alpha_z}$.

EVALUATE: We used the positive square root because the time *t* should be positive.

VP9.5.1. **IDENTIFY:** We need to relate angular quantities to linear quantities.

SET UP: $a_{\tan} = r\alpha$, $a_{\text{rad}} = \omega^2 r$, and $a = \sqrt{a_{\tan}^2 + a_{\text{rad}}^2}$ all apply.

EXECUTE: **(a)** $a_{\tan} = r\alpha = (0.152\text{ m})(8.00\text{ rad/s}) = 1.22\text{ m/s}^2$.

(b) $a_{\text{rad}} = \omega^2 r = (1.60\text{ rad/s})^2(0.152\text{ m}) = 0.389\text{ m/s}^2$.

(c) $a = \sqrt{a_{\tan}^2 + a_{\text{rad}}^2} = \sqrt{(0.122\text{ m/s}^2)^2 + (0.399\text{ m/s}^2)^2} = 1.28\text{ m/s}^2$.

EVALUATE: Careful! In order to use the formulas $a_{\tan} = r\alpha$ and $a_{\text{rad}} = \omega^2 r$, the angular quantities *must* be in *radian* measure. If they are in revolutions or degrees, you must convert them to radians before using these formulas.

VP9.5.2. **IDENTIFY:** We need to relate angular quantities to linear quantities.

SET UP: $v_{\tan} = r\omega$, $a_{\tan} = r\alpha$, and $a_{\text{rad}} = \omega^2 r$ apply. The magnitude *A* of a vector is $A = \sqrt{A_x^2 + A_y^2}$.

EXECUTE: **(a)** The end of the hammer has two components to its velocity: a horizontal component due to its rotation and the vertical component of 2.00 m/s. The horizontal component is its tangential speed: $v_{\tan} = r\omega = (1.50\text{ m})(6.00\text{ rad/s}) = 9.00\text{ m/s}$. Its speed *v* is

$v = \sqrt{v_{\tan}^2 + v_y^2} = \sqrt{(9.00\text{ m/s})^2 + (2.00\text{ m/s})^2} = 9.22\text{ m/s}$.

(b) $a_{\tan} = 0$ and $a_y = 0$. $a_{\text{rad}} = \omega^2 r = (6.00\text{ rad/s})^2(1.50\text{ m}) = 54.0\text{ m/s}^2$. The direction is toward the center of the circular path of the end of the hammer.

EVALUATE: The end of the hammer has a very large acceleration, so it must take a large force to give it this acceleration.

VP9.5.3. **IDENTIFY:** We want to relate angular quantities to linear quantities.

SET UP: $a_{\tan} = r\alpha$, $a_{\text{rad}} = \omega^2 r$, and $a = \sqrt{a_{\tan}^2 + a_{\text{rad}}^2}$ apply.

EXECUTE: The maximum acceleration of the blade tip is $a_{\max}$. We need to relate this to ω.

$a_{\max} = \sqrt{a_{\tan}^2 + a_{\text{rad}}^2} = \sqrt{\left(\omega^2 r\right)^2 + (r\alpha)^2} = r\sqrt{\omega^4 + \alpha^2}$. Squaring and solving for sa ω gives

$$\omega = \left(\frac{a_{\max}^2}{r^2} - \alpha^2\right)^{1/4}.$$

EVALUATE: Our result says that the smaller $a_{\max}$, the smaller ω can be. This is reasonable because it is the fast spin that causes a large acceleration of the tip.

VP9.5.4. **IDENTIFY:** We want to relate angular quantities to linear quantities.

SET UP: $a_{\tan} = r\alpha$ and $a_{\text{rad}} = \omega^2 r$ apply. The angle between the acceleration vector and the velocity vector is 30°. The velocity vector is tangent to the circular path, so the angle between the acceleration vector and the tangent is also 30°. This tells us that tan 30° = $a_{\text{rad}}/a_{\tan}$. (See Fig. VP9.5.4.)

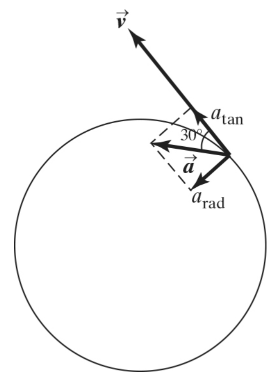


Figure VP9.5.4

EXECUTE: **(a)** $\tan 30° = \dfrac{a_{\text{rad}}}{a_{\tan}} = \dfrac{\omega^2 R}{R\alpha} = \dfrac{\omega^2}{\alpha} \quad \rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{\omega^2}{\alpha} \quad \rightarrow \quad \alpha = \omega^2\sqrt{3}$.

(b) This point is $R/2$ from the center of the circle. Using our work in (a) and using θ for the unknown angle, we see that $\tan\theta = \dfrac{a_{\text{rad}}}{a_{\tan}} = \dfrac{\omega^2 R/2}{(R/2)\alpha} = \dfrac{\omega^2}{\alpha}$. The result is independent of R, so the angle is 30° as before.

EVALUATE: Careful! In our analysis we cannot say that tan 30° = a/v because a and v have different units. This would give tan 30° units of s^{-1}, which is not possible because the $\tan\theta$ is always dimensionless.

VP9.8.1. **IDENTIFY:** We are dealing with the rotation of a solid object having rotational kinetic energy. Work is done on it, so we can apply the work-energy theorem.

SET UP: $K = \dfrac{1}{2}I\omega^2$ for a rotating object, $I = \dfrac{1}{12}M\left(a^2 + b^2\right)$ for a rectangular plate, and the work-energy theorem is $W = K_2 - K_1$.

EXECUTE: **(a)** We want to find the work to change the angular speed.

$W = K_2 = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\left[\dfrac{1}{12}M\left(a^2 + b^2\right)\right]\omega^2$. In this case, $a = b = 0.150$ m, so the equation reduces to $W = \dfrac{1}{12}Ma^2\omega^2 = \dfrac{1}{12}(0.600\text{ kg})(0.150\text{ m})^2(40.0\text{ rad/s})^2 = 1.80$ J.

(b) $W = K_2 - K_1 = \dfrac{1}{12}Ma^2\left(\omega_2^2 - \omega_1^2\right) = \dfrac{1}{12}(0.600\text{ kg})(0.150\text{ m})^2[(80.0\text{ rad/s})^2 - (40.0\text{ rad/s})^2]$.

$W = 5.4$ J.

EVALUATE: Notice that in both cases, we changed the angular speed by 40.0 rad/s, yet the work required to do this was different. This is because the kinetic energy depends on the *square* of ω. Also note that in all of these formulas relating linear and rotational quantities, the angular measure must always be in terms of *radians*.

VP9.8.2. **IDENTIFY:** We are dealing with the rotation of a solid object having rotational kinetic energy. Work is done on it, so we can apply the work-energy theorem.

SET UP: $K = \frac{1}{2}I\omega^2$ for a rotating object, $I = \frac{1}{2}MR^2$ for a solid cylinder, and the work-energy theorem is $W = K_2 - K_1$.

EXECUTE: **(a)** We know the work and want to find the final angular velocity if $\omega_1 = 0$.

$$W = K_2 - K_1 = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{1}{2}MR^2\right]\omega^2 \quad \rightarrow \quad \omega = \sqrt{\frac{4(75.0\text{ J})}{(12.0\text{ kg})(0.250\text{ m})^2}} = 20.0\text{ rad/s}.$$

(b) We follow exactly the same procedure except that $K_1 = \frac{1}{2}I\omega_1^2 \neq 0$. This gives

$W = K_2 - K_1 = \frac{1}{2}I\left(\omega_2^2 - \omega_1^2\right) = \frac{1}{2}\left[\frac{1}{2}MR^2\right]\left(\omega_2^2 - \omega_1^2\right)$. Solving for ω_2 and using the same numbers as in (a) except $\omega_1 = 12.0$ rad/s, we get $\omega_2 = 23.2$ rad/s.

EVALUATE: In both cases, 75.0 J of work was done on the cylinder. When starting from rest, the angular velocity increased by 20.0 rad/s, but when starting from 12.0 rad/s, it increased by only 11.2 rad/s (23.2 rad/s – 12.0 rad/s). The kinetic energy increased by the same amount (75.0 J) in both cases, but the increase in the angular velocity was different.

VP9.8.3. **IDENTIFY:** We are dealing with the rotation of a cylinder that is connected to a piece of cheese. As the cheese falls, it causes the cylinder to turn. Work is done on the system by gravity, so we can apply energy conservation. The kinetic energy of the cheese is due to its linear motion and that of the cylinder is due to its rotation.

SET UP: $K = \frac{1}{2}I\omega^2$ for a rotating object and $K = \frac{1}{2}mv^2$ for a moving object. For a hollow cylinder $I = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$ and energy conservation is $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Measure y from the floor level, so $U_2 = 0$ and $U_1 = mgy_1 = mgh$. There is no friction, so $W_{\text{other}} = 0$, and $K_1 = 0$. The speed v of the cheese is $v = v_{\text{tan}} = r\omega$, so $\omega = v/R_2$. Call m the mass of the cheese and M the mass of the cylinder.

EXECUTE: **(a)** We want to find h. Energy conservation gives $mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$. Putting in the expressions for I and ω and solving for h, we get $h = \dfrac{\frac{M}{4}\left(R_1^2 + R_2^2\right)\left(\frac{v}{R_2}\right)^2 + \frac{1}{2}mv^2}{mg}$. Using $m = 0.500$ kg, $M = 2.00$ kg, $R_1 = 0.100$ m, $R_2 = 0.200$ m, and $v = 4.00$ m/s, we get $h = 2.86$ m.

(b) $\omega = v_{\text{tan}}/R_2 = v/R_2 = (4.00\text{ m/s})/(0.200\text{ m}) = 20.0$ rad/s.

EVALUATE: Checking for the proper units in the equation for h is a good way to spot algebraic errors in the solution.

VP9.8.4. **IDENTIFY:** We are dealing with the rotation of a cylinder that is connected to a textbook. As the book falls, it causes the cylinder to turn. Work is done on the system by gravity and by friction at the shaft of the cylinder, so we can apply energy conservation. The kinetic energy of the book is due to its linear motion and that of the cylinder is due to its rotation.

SET UP: $K = \frac{1}{2}I\omega^2$ for a rotating object and $K = \frac{1}{2}mv^2$ for a moving object. For a solid cylinder $I = \frac{1}{2}MR^2$ and energy conservation is $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, where $K_1 = 0$. Measure y from the floor level, so $U_2 = 0$ and $U_1 = mgh$. The speed v of the textbook is $v = v_{\text{tan}} = R\omega$. Call m the mass of the book and M the mass of the cylinder.

EXECUTE: **(a)** $v = v_{\text{tan}} = R\omega = (0.240\text{ m})(10.0\text{ rad/s}) = 2.40$ m/s.

(b) From $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ we get $mgh + W_{\text{f}} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$. Solving for W_{f} using the expressions for v and I gives $W_{\text{f}} = \frac{1}{2}\left(m + \frac{M}{2}\right)(R\omega)^2 - mgh$. Putting in m = 2.00 kg, M = 1.50 kg, R = 0.240 m, and ω = 10.0 rad/s gives $W_{\text{f}} = -9.72$ J.

EVALUATE: Friction does negative work, which is reasonable because it opposes the turning motion at the axle of the cylinder.

9.1. **IDENTIFY:** $s = r\theta,$ with θ in radians.

SET UP: $\pi \text{ rad} = 180°.$

EXECUTE: **(a)** $\theta = \dfrac{s}{r} = \dfrac{1.50 \text{ m}}{2.50 \text{ m}} = 0.600 \text{ rad} = 34.4°$

(b) $r = \dfrac{s}{\theta} = \dfrac{14.0 \text{ cm}}{(128°)(\pi \text{ rad}/180°)} = 6.27 \text{ cm}$

(c) $s = r\theta = (1.50 \text{ m})(0.700 \text{ rad}) = 1.05 \text{ m}$

EVALUATE: An angle is the ratio of two lengths and is dimensionless. But, when $s = r\theta$ is used, θ must be in radians. Or, if $\theta = s/r$ is used to calculate $\theta,$ the calculation gives θ in radians.

9.2. **IDENTIFY:** $\theta - \theta_0 = \omega t,$ since the angular velocity is constant.

SET UP: $1 \text{ rpm} = (2\pi/60) \text{ rad/s}.$

EXECUTE: **(a)** $\omega = (1900)(2\pi \text{ rad}/60 \text{ s}) = 199 \text{ rad/s}$

(b) $35° = (35°)(\pi/180°) = 0.611 \text{ rad}.$ $t = \dfrac{\theta - \theta_0}{\omega} = \dfrac{0.611 \text{ rad}}{199 \text{ rad/s}} = 3.1\times10^{-3} \text{ s}$

EVALUATE: In $t = \dfrac{\theta - \theta_0}{\omega}$ we must use the same angular measure (radians, degrees or revolutions) for both $\theta - \theta_0$ and $\omega.$

9.3. **IDENTIFY:** $\alpha_z(t) = \dfrac{d\omega_z}{dt}.$ Using $\omega_z = d\theta/dt$ gives $\theta - \theta_0 = \int_{t_1}^{t_2} \omega_z dt.$

SET UP: $\dfrac{d}{dt}t^n = nt^{n-1}$ and $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$

EXECUTE: **(a)** A must have units of rad/s and B must have units of $\text{rad/s}^3.$

(b) $\alpha_z(t) = 2Bt = (3.00 \text{ rad/s}^3)t.$ (i) For $t = 0,$ $\alpha_z = 0.$ (ii) For $t = 5.00$ s, $\alpha_z = 15.0 \text{ rad/s}^2.$

(c) $\theta_2 - \theta_1 = \int_{t_1}^{t_2} (A + Bt^2)dt = A(t_2 - t_1) + \frac{1}{3}B(t_2^3 - t_1^3).$ For $t_1 = 0$ and $t_2 = 2.00$ s,

$\theta_2 - \theta_1 = (2.75 \text{ rad/s})(2.00 \text{ s}) + \frac{1}{3}(1.50 \text{ rad/s}^3)(2.00 \text{ s})^3 = 9.50 \text{ rad}.$

EVALUATE: Both α_z and ω_z are positive and the angular speed is increasing.

9.4. **IDENTIFY:** $\alpha_z = d\omega_z/dt.$ $\alpha_{\text{av-}z} = \dfrac{\Delta\omega_z}{\Delta t}.$

SET UP: $\dfrac{d}{dt}(t^2) = 2t$

EXECUTE: **(a)** $\alpha_z(t) = \dfrac{d\omega_z}{dt} = -2\beta t = (-1.60 \text{ rad/s}^3)t.$

(b) $\alpha_z(3.0 \text{ s}) = (-1.60 \text{ rad/s}^3)(3.0 \text{ s}) = -4.80 \text{ rad/s}^2.$

$$\alpha_{\text{av-}z} = \frac{\omega_z(3.0 \text{ s}) - \omega_z(0)}{3.0 \text{ s}} = \frac{-2.20 \text{ rad/s} - 5.00 \text{ rad/s}}{3.0 \text{ s}} = -2.40 \text{ rad/s}^2,$$

which is half as large (in magnitude) as the acceleration at $t = 3.0$ s.

EVALUATE: $\alpha_z(t)$ increases linearly with time, so $\alpha_{\text{av-}z} = \dfrac{\alpha_z(0) + \alpha_z(3.0\text{ s})}{2}$. $\alpha_z(0) = 0$.

9.5. **IDENTIFY** and **SET UP:** Use $\omega_z = \dfrac{d\theta}{dt}$ to calculate the angular velocity and $\omega_{\text{av-}z} = \dfrac{\Delta\theta}{\Delta t} = \dfrac{\theta_2 - \theta_1}{t_2 - t_1}$ to calculate the average angular velocity for the specified time interval.

EXECUTE: $\theta = \gamma t + \beta t^3$; $\gamma = 0.400$ rad/s, $\beta = 0.0120\text{ rad/s}^3$

(a) $\omega_z = \dfrac{d\theta}{dt} = \gamma + 3\beta t^2$

(b) At $t = 0$, $\omega_z = \gamma = 0.400$ rad/s

(c) At $t = 5.00$ s, $\omega_z = 0.400\text{ rad/s} + 3(0.0120\text{ rad/s}^3)(5.00\text{ s})^2 = 1.30$ rad/s

$$\omega_{\text{av-}z} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

For $t_1 = 0$, $\theta_1 = 0$.

For $t_2 = 5.00$ s, $\theta_2 = (0.400\text{ rad/s})(5.00\text{ s}) + (0.012\text{ rad/s}^3)(5.00\text{ s})^3 = 3.50$ rad

So $\omega_{\text{av-}z} = \dfrac{3.50\text{ rad} - 0}{5.00\text{ s} - 0} = 0.700$ rad/s.

EVALUATE: The average of the instantaneous angular velocities at the beginning and end of the time interval is $\frac{1}{2}(0.400\text{ rad/s} + 1.30\text{ rad/s}) = 0.850$ rad/s. This is larger than $\omega_{\text{av-}z}$, because $\omega_z(t)$ is increasing faster than linearly.

9.6. **IDENTIFY:** $\omega_z(t) = \dfrac{d\theta}{dt}$. $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. $\omega_{\text{av}-z} = \dfrac{\Delta\theta}{\Delta t}$.

SET UP: $\omega_z = (250\text{ rad/s}) - (40.0\text{ rad/s}^2)t - (4.50\text{ rad/s}^3)t^2$. $\alpha_z = -(40.0\text{ rad/s}^2) - (9.00\text{ rad/s}^3)t$.

EXECUTE: **(a)** Setting $\omega_z = 0$ results in a quadratic in *t*. The only positive root is $t = 4.23$ s.

(b) At $t = 4.23$ s, $\alpha_z = -78.1\text{ rad/s}^2$.

(c) At $t = 4.23$ s, $\theta = 586\text{ rad} = 93.3$ rev.

(d) At $t = 0$, $\omega_z = 250$ rad/s.

(e) $\omega_{\text{av-}z} = \dfrac{586\text{ rad}}{4.23\text{ s}} = 138$ rad/s.

EVALUATE: Between $t = 0$ and $t = 4.23$ s, ω_z decreases from 250 rad/s to zero. ω_z is not linear in *t*, so $\omega_{\text{av-}z}$ is not midway between the values of ω_z at the beginning and end of the interval.

9.7. **IDENTIFY:** $\omega_z(t) = \dfrac{d\theta}{dt}$. $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. Use the values of θ and ω_z at $t = 0$ and α_z at 1.50 s to calculate *a*, *b*, and *c*.

SET UP: $\dfrac{d}{dt}t^n = nt^{n-1}$

EXECUTE: **(a)** $\omega_z(t) = b - 3ct^2$. $\alpha_z(t) = -6ct$. At $t = 0$, $\theta = a = \pi/4$ rad and $\omega_z = b = 2.00$ rad/s. At $t = 1.50$ s, $\alpha_z = -6c(1.50\text{ s}) = 1.25\text{ rad/s}^2$ and $c = -0.139\text{ rad/s}^3$.

(b) $\theta = \pi/4$ rad and $\alpha_z = 0$ at $t = 0$.

(c) $\alpha_z = 3.50\text{ rad/s}^2$ at $t = -\dfrac{\alpha_z}{6c} = -\dfrac{3.50\text{ rad/s}^2}{6(-0.139\text{ rad/s}^3)} = 4.20$ s. At $t = 4.20$ s,

$$\theta = \frac{\pi}{4}\text{ rad} + (2.00\text{ rad/s})(4.20\text{ s}) - (-0.139\text{ rad/s}^3)(4.20\text{ s})^3 = 19.5\text{ rad}.$$

$\omega_z = 2.00\ \text{rad/s} - 3(-0.139\ \text{rad/s}^3)(4.20\ \text{s})^2 = 9.36\ \text{rad/s}.$

EVALUATE: θ, ω_z, and α_z all increase as t increases.

9.8. IDENTIFY: $\alpha_z = \dfrac{d\omega_z}{dt}$. $\theta - \theta_0 = \omega_{\text{av-}z}t$. When ω_z is linear in t, $\omega_{\text{av-}z}$ for the time interval t_1 to t_2 is $\omega_{\text{av-}z} = \dfrac{\omega_{z1} + \omega_{z2}}{t_2 - t_1}$.

SET UP: From the information given, $\alpha_z = \dfrac{\Delta\omega}{\Delta t} = \dfrac{4.00\ \text{rad/s} - (-6.00\ \text{rad/s})}{7.00\ \text{s}} = 1.429\ \text{rad/s}^2$.

$\omega_z(t) = -6.00\ \text{rad/s} + (1.429\ \text{rad/s}^2)t.$

EXECUTE: **(a)** The angular acceleration is positive, since the angular velocity increases steadily from a negative value to a positive value.

(b) It takes time $t = -\dfrac{\omega_{0z}}{\alpha_z}$ = –(–6.00 rad/s)/(1.429 rad/s^2) = 4.20 s for the wheel to stop ($\omega_z = 0$). During this time its speed is decreasing. For the next 2.80 s its speed is increasing from 0 rad/s to +4.00 rad/s.

(c) The average angular velocity is $\dfrac{-6.00\ \text{rad/s} + 4.00\ \text{rad/s}}{2} = -1.00\ \text{rad/s}$. $\theta - \theta_0 = \omega_{\text{av-}z}t$ then leads to displacement of –7.00 rad after 7.00 s.

EVALUATE: When α_z and ω_z have the same sign, the angular speed is increasing; this is the case for $t = 4.20$ s to $t = 7.00$ s. When α_z and ω_z have opposite signs, the angular speed is decreasing; this is the case between $t = 0$ and $t = 4.20$ s.

9.9. IDENTIFY: Apply the constant angular acceleration equations.

SET UP: Let the direction the wheel is rotating be positive.

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t = 1.50\ \text{rad/s} + (0.200\ \text{rad/s}^2)(2.50\ \text{s}) = 2.00\ \text{rad/s}.$

(b) $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (1.50\ \text{rad/s})(2.50\ \text{s}) + \frac{1}{2}(0.200\ \text{rad/s}^2)(2.50\ \text{s})^2 = 4.38\ \text{rad}.$

EVALUATE: $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t = \left(\dfrac{1.50\ \text{rad/s} + 2.00\ \text{rad/s}}{2}\right)(2.50\ \text{s}) = 4.38\ \text{rad}$, the same as calculated with another equation in part (b).

9.10. IDENTIFY: Apply the constant angular acceleration equations to the motion of the fan.

(a) SET UP: $\omega_{0z} = (500\ \text{rev/min})(1\ \text{min}/60\ \text{s}) = 8.333\ \text{rev/s}$,

$\omega_z = (200\ \text{rev/min})(1\ \text{min}/60\ \text{s}) = 3.333\ \text{rev/s}$, $t = 4.00$ s, $\alpha_z = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{3.333\ \text{rev/s} - 8.333\ \text{rev/s}}{4.00\ \text{s}} = -1.25\ \text{rev/s}^2$

$\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (8.333\ \text{rev/s})(4.00\ \text{s}) + \frac{1}{2}(-1.25\ \text{rev/s}^2)(4.00\ \text{s})^2 = 23.3\ \text{rev}$

(b) SET UP: $\omega_z = 0$ (comes to rest); $\omega_{0z} = 3.333\ \text{rev/s}$; $\alpha_z = -1.25\ \text{rev/s}^2$; $t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{0 - 3.333\ \text{rev/s}}{-1.25\ \text{rev/s}^2} = 2.67\ \text{s}$

EVALUATE: The angular acceleration is negative because the angular velocity is decreasing. The average angular velocity during the 4.00 s time interval is 350 rev/min and $\theta - \theta_0 = \omega_{\text{av-}z} t$ gives $\theta - \theta_0 = 23.3\ \text{rev},$ which checks.

9.11. **IDENTIFY:** Apply the constant angular acceleration equations to the motion. The target variables are t and $\theta - \theta_0$.

SET UP: **(a)** $\alpha_z = 1.50\ \text{rad/s}^2$; $\omega_{0z} = 0$ (starts from rest); $\omega_z = 36.0\ \text{rad/s}$; $t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{36.0\ \text{rad/s} - 0}{1.50\ \text{rad/s}^2} = 24.0\ \text{s}$

(b) $\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z} t + \frac{1}{2}\alpha_z t^2 = 0 + \frac{1}{2}(1.50\ \text{rad/s}^2)(24.0\ \text{s})^2 = 432\ \text{rad}$

$\theta - \theta_0 = 432\ \text{rad}(1\ \text{rev}/2\pi\ \text{rad}) = 68.8\ \text{rev}$

EVALUATE: We could use $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$ to calculate

$\theta - \theta_0 = \frac{1}{2}(0 + 36.0\ \text{rad/s})(24.0\ \text{s}) = 432\ \text{rad},$ which checks.

9.12. **IDENTIFY:** We use the constant-angular acceleration equations.

SET UP: $\theta - \theta_0 = \omega_{0z} t + \dfrac{1}{2}\alpha_z t^2$. The target variable is the angle the wheel turns through.

EXECUTE: First us $\theta - \theta_0 = \omega_{0z} t + \dfrac{1}{2}\alpha_z t^2$ to find α_z : 8.00 rev = $\dfrac{1}{2}$ α_z $(2.50\ \text{s})^2$, which gives α_z = $2.56\ \text{rev/s}^2$. Now use the same formula to find $\theta - \theta_0$ during the first 7.50 s. This gives $\theta - \theta_0 = \dfrac{1}{2}\alpha_z t^2$ $= \dfrac{1}{2}(2.56\ \text{rev/s}^2)(7.50\ \text{s})^2$ = 72.0 rev. The wheel turned through 72.0 rev during the first 5.00 s and 8.00 rev during the first 2.50 s, so during the second 5.00 s it turned through an angle of 72.0 rev – 8.00 rev = 64.0 rev.

EVALUATE: It turned through a greater angle during the next 5.00 s because the time was longer and because it was turning faster than during the first 2.50 s.

9.13. **IDENTIFY:** Use a constant angular acceleration equation and solve for ω_{0z}.

SET UP: Let the direction of rotation of the flywheel be positive.

EXECUTE: $\theta - \theta_0 = \omega_{0z} t + \frac{1}{2}\alpha_z t^2$ gives

$\omega_{0z} = \dfrac{\theta - \theta_0}{t} - \frac{1}{2}\alpha_z\, t = \dfrac{30.0\ \text{rad}}{4.00\ \text{s}} - \frac{1}{2}(2.25\ \text{rad/s}^2)(4.00\ \text{s}) = 3.00\ \text{rad/s}.$

EVALUATE: At the end of the 4.00 s interval, $\omega_z = \omega_{0z} + \alpha_z t = 12.0\ \text{rad/s}.$

$\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t = \left(\dfrac{3.00\ \text{rad/s} + 12.0\ \text{rad/s}}{2}\right)(4.00\ \text{s}) = 30.0\ \text{rad},$ which checks.

9.14. **IDENTIFY:** Apply the constant angular acceleration equations.

SET UP: Let the direction of the rotation of the blade be positive. $\omega_{0z} = 0.$

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{140\ \text{rad/s} - 0}{6.00\ \text{s}} = 23.3\ \text{rad/s}^2.$

$(\theta - \theta_0) = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t = \left(\dfrac{0 + 140\ \text{rad/s}}{2}\right)(6.00\ \text{s}) = 420\ \text{rad}.$

EVALUATE: We could also use $\theta-\theta_0=\omega_{0z}t+\frac{1}{2}\alpha_z t^2$. This equation gives $\theta-\theta_0=\frac{1}{2}(23.3\text{ rad/s}^2)(6.00\text{ s})^2=419\text{ rad},$ in agreement with the result obtained above.

9.15. **IDENTIFY:** Apply constant angular acceleration equations.
SET UP: Let the direction the flywheel is rotating be positive.
$\theta-\theta_0=200\text{ rev}$, $\omega_{0z}=500\text{ rev/min}=8.333\text{ rev/s}$, $t=30.0\text{ s}$.

EXECUTE: **(a)** $\theta-\theta_0=\left(\dfrac{\omega_{0z}+\omega_z}{2}\right)t$ gives $\omega_z=5.00\text{ rev/s}=300\text{ rpm}$.

(b) Use the information in part (a) to find α_z: $\omega_z=\omega_{0z}+\alpha_z t$ gives $\alpha_z=-0.1111\text{ rev/s}^2$. Then $\omega_z=0$, $\alpha_z=-0.1111\text{ rev/s}^2$, $\omega_{0z}=8.333\text{ rev/s}$ in $\omega_z=\omega_{0z}+\alpha_z t$ gives $t=75.0\text{ s}$ and $\theta-\theta_0=\left(\dfrac{\omega_{0z}+\omega_z}{2}\right)t$ gives $\theta-\theta_0=312\text{ rev}$.

EVALUATE: The mass and diameter of the flywheel are not used in the calculation.

9.16. **IDENTIFY:** Apply the constant angular acceleration equations separately to the time intervals 0 to 2.00 s and 2.00 s until the wheel stops.
(a) SET UP: Consider the motion from $t=0$ to $t=2.00\text{ s}$:
$\theta-\theta_0=?$; $\omega_{0z}=24.0\text{ rad/s}$; $\alpha_z=30.0\text{ rad/s}^2$; $t=2.00\text{ s}$

EXECUTE: $\theta-\theta_0=\omega_{0z}t+\frac{1}{2}\alpha_z t^2=(24.0\text{ rad/s})(2.00\text{ s})+\frac{1}{2}(30.0\text{ rad/s}^2)(2.00\text{ s})^2$
$\theta-\theta_0=48.0\text{ rad}+60.0\text{ rad}=108\text{ rad}$
Total angular displacement from $t=0$ until stops: $108\text{ rad}+432\text{ rad}=540\text{ rad}$

Note: At $t=2.00\text{ s}$, $\omega_z=\omega_{0z}+\alpha_z t=24.0\text{ rad/s}+(30.0\text{ rad/s}^2)(2.00\text{ s})=84.0\text{ rad/s}$; angular speed when breaker trips.
(b) SET UP: Consider the motion from when the circuit breaker trips until the wheel stops. For this calculation let $t=0$ when the breaker trips.

$t=?$; $\theta-\theta_0=432\text{ rad}$; $\omega_z=0$; $\omega_{0z}=84.0\text{ rad/s}$ (from part (a)). Use $\theta-\theta_0=\left(\dfrac{\omega_{0z}+\omega_z}{2}\right)t$.

EXECUTE: $t=\dfrac{2(\theta-\theta_0)}{\omega_{0z}+\omega_z}=\dfrac{2(432\text{ rad})}{84.0\text{ rad/s}+0}=10.3\text{ s}$

The wheel stops 10.3 s after the breaker trips so $2.00\text{ s}+10.3\text{ s}=12.3\text{ s}$ from the beginning.
(c) SET UP: $\alpha_z=?$; consider the same motion as in part (b):
$\omega_z=\omega_{0z}+\alpha_z t$

EXECUTE: $\alpha_z=\dfrac{\omega_z-\omega_{0z}}{t}=\dfrac{0-84.0\text{ rad/s}}{10.3\text{ s}}=-8.16\text{ rad/s}^2$.

EVALUATE: The angular acceleration is positive while the wheel is speeding up and negative while it is slowing down. We could also use $\omega_z^2=\omega_{0z}^2+2\alpha_z(\theta-\theta_0)$ to calculate

$\alpha_z=\dfrac{\omega_z^2-\omega_{0z}^2}{2(\theta-\theta_0)}=\dfrac{0-(84.0\text{ rad/s})^2}{2(432\text{ rad})}=-8.16\text{ rad/s}^2$ for the acceleration after the breaker trips.

9.17. **IDENTIFY:** Apply $\omega_z^2=\omega_{z0}^2+2\alpha_z(\theta-\theta_0)$ to relate ω_z to $\theta-\theta_0$.
SET UP: Establish a proportionality.
EXECUTE: From $\omega_z^2=\omega_{z0}^2+2\alpha_z(\theta-\theta_0)$, with $\omega_{0z}=0$, the number of revolutions is proportional to the square of the initial angular velocity, so tripling the initial angular velocity increases the number of revolutions by 9, to 9.00 rev.

Evaluate: We don't have enough information to calculate α_z; all we need to know is that it is constant.

9.18. **Identify:** The linear distance the elevator travels, its speed and the magnitude of its acceleration are equal to the tangential displacement, speed and acceleration of a point on the rim of the disk. $s = r\theta$, $v = r\omega$ and $a = r\alpha$. In these equations the angular quantities must be in radians.

Set Up: 1 rev $= 2\pi$ rad. 1 rpm $= 0.1047$ rad/s. π rad $= 180°$. For the disk, $r = 1.25$ m.

Execute: **(a)** $v = 0.250$ m/s so $\omega = \dfrac{v}{r} = \dfrac{0.250\text{ m/s}}{1.25\text{ m}} = 0.200\text{ rad/s} = 1.91\text{ rpm}.$

(b) $a = \frac{1}{8}g = 1.225\text{ m/s}^2$. $\alpha = \dfrac{a}{r} = \dfrac{1.225\text{ m/s}^2}{1.25\text{ m}} = 0.980\text{ rad/s}^2.$

(c) $s = 3.25$ m. $\theta = \dfrac{s}{r} = \dfrac{3.25\text{ m}}{1.25\text{ m}} = 2.60\text{ rad} = 149°.$

Evaluate: When we use $s = r\theta$, $v = r\omega$ and $a_{\text{tan}} = r\alpha$ to solve for θ, ω and α, the results are in rad, rad/s, and rad/s^2.

9.19. **Identify:** We relate angular quantities to linear quantities.

Set Up: Estimate: Tub diameter ≈ 50 cm, so $r \approx 25$ cm. $v_{\text{tan}} = r\omega$ and $a_{\text{rad}} = \omega^2 r$.

Execute: **(a)** $a_{\text{rad}} = \omega^2 r = 3g$, so $\omega = \sqrt{\dfrac{3g}{r}} = \sqrt{\dfrac{3(9.80\text{ m/s}^2)}{0.25\text{ m}}} = 10.8$ rad/s, which rounds to 11 rad/s.

Now convert this result to rpm (rev/min). $\omega = \left(10.8\dfrac{\text{rad}}{\text{s}}\right)\left(\dfrac{60\text{ s}}{1\text{ min}}\right)\left(\dfrac{1\text{ rev}}{2\pi\text{ rad}}\right) \approx 100$ rev/min.

(b) $v_{\text{tan}} = r\omega = (0.25\text{ m})(10.8\text{ rad/s}) = 2.7\text{ m/s}.$

Evaluate: When using the formulas relating linear and angular quantities, the angular measures must *always* be in *radian* measure, such as rad/s or rad/s^2.

9.20. **Identify:** Linear and angular velocities are related by $v = r\omega$. Use $\omega_z = \omega_{0z} + \alpha_z t$ to calculate α_z.

Set Up: $\omega = v/r$ gives ω in rad/s.

Execute: **(a)** $\dfrac{1.25\text{ m/s}}{25.0\times10^{-3}\text{ m}} = 50.0\text{ rad/s}$, $\dfrac{1.25\text{ m/s}}{58.0\times10^{-3}\text{ m}} = 21.6\text{ rad/s}.$

(b) $(1.25\text{ m/s})(74.0\text{ min})(60\text{ s/min}) = 5.55\text{ km}.$

(c) $\alpha_z = \dfrac{21.55\text{ rad/s} - 50.0\text{ rad/s}}{(74.0\text{ min})(60\text{ s/min})} = -6.41\times10^{-3}\text{ rad/s}^2.$

Evaluate: The width of the tracks is very small, so the total track length on the disc is huge.

9.21. **Identify:** Use constant acceleration equations to calculate the angular velocity at the end of two revolutions. $v = r\omega$.

Set Up: 2 rev $= 4\pi$ rad. $r = 0.200$ m.

Execute: **(a)** $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$. $\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(3.00\text{ rad/s}^2)(4\pi\text{ rad})} = 8.68\text{ rad/s}.$

$a_{\text{rad}} = r\omega^2 = (0.200\text{ m})(8.68\text{ rad/s})^2 = 15.1\text{ m/s}^2.$

(b) $v = r\omega = (0.200\text{ m})(8.68\text{ rad/s}) = 1.74\text{ m/s}$. $a_{\text{rad}} = \dfrac{v^2}{r} = \dfrac{(1.74\text{ m/s})^2}{0.200\text{ m}} = 15.1\text{ m/s}^2.$

Evaluate: $r\omega^2$ and v^2/r are completely equivalent expressions for a_{rad}.

9.22. **Identify:** $v = r\omega$ and $a_{\text{tan}} = r\alpha$.

Set Up: The linear acceleration of the bucket equals a_{tan} for a point on the rim of the axle.

EXECUTE: **(a)** $v = R\omega$. $2.00\text{ cm/s} = R\left(\dfrac{7.5\text{ rev}}{\text{min}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)$ gives $R = 2.55$ cm.

$D = 2R = 5.09$ cm.

(b) $a_{\tan} = R\alpha$. $\alpha = \dfrac{a_{\tan}}{R} = \dfrac{0.400\text{ m/s}^2}{0.0255\text{ m}} = 15.7\text{ rad/s}^2$.

EVALUATE: In $v = R\omega$ and $a_{\tan} = R\alpha$, ω and α must be in radians.

9.23. **IDENTIFY:** We relate angular speed to linear speed.

SET UP: Estimate: Diameter is about 7 in., so $r \approx 3.5$ in. $v_{\tan} = r\omega$. We want the tangential velocity.

EXECUTE: First convert 2600 rev/min to rad/s and 3.5 in. to m.

$r =$ 3.5 in.$\left(\dfrac{1\text{ m}}{39.37\text{ in.}}\right)$ = 0.0889 m and $\omega = \left(2600\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)$ = 26,000 rad/s. Now use

$v_{\tan} = r\omega$ = (0.0889 m)(26,000 rad/s) = 2300 m/s.

EVALUATE: When using the formula $v_{\tan} = r\omega$, ω *must* be in *radian* measure, such as rad/s or rad/min.

9.24. **IDENTIFY:** Apply constant angular acceleration equations. $v = r\omega$. A point on the rim has both tangential and radial components of acceleration.

SET UP: $a_{\tan} = r\alpha$ and $a_{\text{rad}} = r\omega^2$.

EXECUTE: **(a)** a_{rad}

(Note that since ω_{0z} and a α_z are given in terms of revolutions, it's not necessary to convert to radians).

(b) $\omega_{\text{av-}z}\Delta t = (0.340\text{ rev/s})(0.2\text{ s}) = 0.068$ rev.

(c) Here, the conversion to radians must be made to use $v = r\omega$, and

$$v = r\omega = \left(\frac{0.750\text{ m}}{2}\right)(0.430\text{ rev/s})(2\pi\text{ rad/rev}) = 1.01\text{ m/s}.$$

(d) Combining $a_{\text{rad}} = r\omega^2$ and $a_{\tan} = R\alpha$,

$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2}$.

$a = \sqrt{\left[[(0.430\text{ rev/s})(2\pi\text{ rad/rev})]^2(0.375\text{ m})\right]^2 + \left[(0.900\text{ rev/s}^2)(2\pi\text{ rad/rev})(0.375\text{ m})\right]^2}$.

$a = 3.46\text{ m/s}^2$.

EVALUATE: If the angular acceleration is constant, $a_{\tan}$ is constant but a_{rad} increases as ω increases.

9.25. **IDENTIFY:** Use $a_{\text{rad}} = r\omega^2$ and solve for r.

SET UP: $a_{\text{rad}} = r\omega^2$ so $r = a_{\text{rad}}/\omega^2$, where ω must be in rad/s

EXECUTE: $a_{\text{rad}} = 3000g = 3000(9.80\text{ m/s}^2) = 29{,}400\text{ m/s}^2$

$\omega = (5000\text{ rev/min})\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) = 523.6$ rad/s

Then $r = \dfrac{a_{\text{rad}}}{\omega^2} = \dfrac{29{,}400\text{ m/s}^2}{(523.6\text{ rad/s})^2} = 0.107$ m.

EVALUATE: The diameter is then 0.214 m, which is larger than 0.127 m, so the claim is *not* realistic.

9.26. **IDENTIFY:** $a_{\tan} = r\alpha$, $v = r\omega$ and $a_{\text{rad}} = v^2/r$. $\theta - \theta_0 = \omega_{\text{av-}z}t$.

SET UP: When α_z is constant, $\omega_{\text{av-}z} = \dfrac{\omega_{0z} + \omega_z}{2}$. Let the direction the wheel is rotating be positive.

EXECUTE: **(a)** $\alpha = \dfrac{a_{\tan}}{r} = \dfrac{-10.0\text{ m/s}^2}{0.200\text{ m}} = -50.0\text{ rad/s}^2$

(b) At $t = 3.00$ s, $v = 50.0$ m/s and $\omega = \dfrac{v}{r} = \dfrac{50.0\text{ m/s}}{0.200\text{ m}} = 250\text{ rad/s}$ and at $t = 0$,

$v = 50.0\text{ m/s} + (-10.0\text{ m/s}^2)(0 - 3.00\text{ s}) = 80.0\text{ m/s},\ \ \omega = 400\text{ rad/s}.$

(c) $\omega_{\text{av-}z}t = (325\text{ rad/s})(3.00\text{ s}) = 975\text{ rad} = 155\text{ rev}.$

(d) $v = \sqrt{a_{\text{rad}}r} = \sqrt{(9.80\text{ m/s}^2)(0.200\text{ m})} = 1.40\text{ m/s}$. This speed will be reached at time $\dfrac{50.0\text{ m/s} - 1.40\text{ m/s}}{10.0\text{ m/s}^2} = 4.86\text{ s}$ after $t = 3.00$ s, or at $t = 7.86$ s. (There are many equivalent ways to do this calculation.)

EVALUATE: At $t = 0$, $a_{\text{rad}} = r\omega^2 = 3.20 \times 10^4\text{ m/s}^2$. At $t = 3.00$ s, $a_{\text{rad}} = 1.25 \times 10^4\text{ m/s}^2$. For $a_{\text{rad}} = g$ the wheel must be rotating more slowly than at 3.00 s so it occurs some time after 3.00 s.

9.27. **IDENTIFY:** We relate angular speed to linear speed and use the constant-angular acceleration equations.

SET UP: Use $v_{\tan} = r\omega$ and $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$. We want the angular acceleration.

EXECUTE: First find the initial and final angular velocities. $v_{\tan} = r\omega$ gives $\omega = \dfrac{v_{\tan}}{r}$. Therefore $\omega_1 = \dfrac{3.00\text{ m/s}}{0.300\text{ m}} = 10.0\text{ rad/s}$ and $\omega_2 = 20.0$ rad/s. Now use $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ to find α_z. The wheel turns through 4 rev, which is 8π rad, so $(20.0\text{ rad/s})^2 = (10.0\text{ rad/s})^2 + 2\alpha_z(8\pi\text{ rad})$, so $\alpha_z = 5.97$ rad/s^2.

EVALUATE: Notice that in many cases, the units of radians do not actually emerge, but must be inferred. For example in the calculation $\omega_1 = \dfrac{3.00\text{ m/s}}{0.300\text{ m}}$, the units are 1/s (or s^{-1}), so the radians must be put in. This is not a problem because a radian has no dimensions; it is a pure number since it is defined as the length of an arc of a circle divided by the radius of the circle: θ (in rad) = *s/r*.

9.28. **IDENTIFY:** We relate angular quantities to linear quantities.

SET UP: Use $v_{\tan} = r\omega$ and $a_{\text{rad}} = \omega^2 r$. For constant angular speed, $\omega = \dfrac{\Delta\theta}{\Delta t}$. We want the tangential speed and radial acceleration of a point on the earth's equator.

EXECUTE: First find the angular speed using $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{2\pi\text{ rad}}{24\text{ h}} = \dfrac{2\pi\text{ rad}}{(24)(3600\text{ s})} = 7.27 \times 10^{-5}\text{ rad/s}.$

Therefore $v_{\tan} = r\omega = \dfrac{1}{2}(1.27 \times 10^7\text{ m})(7.27 \times 10^{-5}\text{ rad/s}) = 462\text{ m/s}$, and now use $a_{\text{rad}} = \omega^2 r$ to get $a_{\text{rad}} = (7.27 \times 10^{-5}\text{ rad/s})^2 \dfrac{1}{2}(1.27 \times 10^7\text{ m}) = 0.0336\text{ m/s}^2$.

EVALUATE: The radial acceleration is *much* less than *g*, so we normally are not even aware of it and can ignore it in most calculations.

9.29. **IDENTIFY:** We relate angular quantities to linear quantities and use the constant-angular accleration equations.

SET UP: Use $a_{\tan} = r\alpha$, $a_{\text{rad}} = \omega^2 r$, $a = \sqrt{a_{\tan}^2 + a_{\text{rad}}^2}$, and $\omega_z = \omega_{0z} + \alpha_z t$.

EXECUTE: First find ω at the end of 2.00 s.

$\omega_z = \omega_{0z} + \alpha_z t = 0 + (0.600\text{ rad/s}^2)(2.00\text{ s})^2 = 1.20\text{ rad/s}.$

$a_{tan} = r\alpha = (0.300\text{ m})(0.600\text{ rad/s}^2) = 0.180\text{ m/s}^2$.

$a_{rad} = \omega^2 r = (1.20\text{ rad/s})^2(0.300\text{ m}) = 0.432\text{ m/s}^2$.

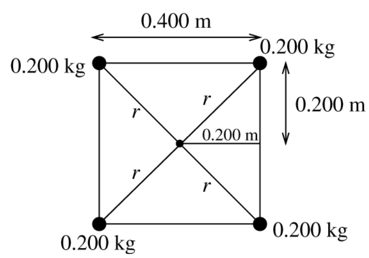
$a = \sqrt{a_{tan}^2 + a_{rad}^2} = \sqrt{(0.180\text{ m/s}^2)^2 + (0.432\text{ m/s}^2)^2} = 0.468\text{ m/s}^2$.

EVALUATE: There is only a tangential acceleration when the object has an angular acceleration, but there is a radial acceleration even if there is no angular acceleration.

9.30. IDENTIFY and **SET UP:** Use $I = \sum m_i r_i^2$. Treat the spheres as point masses and ignore I of the light rods.

EXECUTE: The object is shown in Figure 9.30a.

(a)



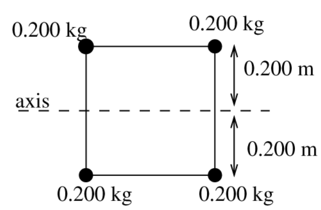
$r = \sqrt{(0.200\text{ m})^2 + (0.200\text{ m})^2} = 0.2828\text{ m}$

$I = \sum m_i r_i^2 = 4(0.200\text{ kg})(0.2828\text{ m})^2$

$I = 0.0640\text{ kg}\cdot\text{m}^2$

Figure 9.30a

(b) The object is shown in Figure 9.30b.



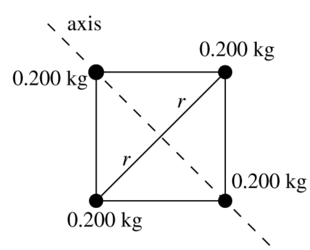
$r = 0.200\text{ m}$

$I = \sum m_i r_i^2 = 4(0.200\text{ kg})(0.200\text{ m})^2$

$I = 0.0320\text{ kg}\cdot\text{m}^2$

Figure 9.30b

(c) The object is shown in Figure 9.30c.



$r = 0.2828\text{ m}$

$I = \sum m_i r_i^2 = 2(0.200\text{ kg})(0.2828\text{ m})^2$

$I = 0.0320\text{ kg}\cdot\text{m}^2$

Figure 9.30c

EVALUATE: In general I depends on the axis and our answer for part (a) is larger than for parts (b) and (c). It just happens that I is the same in parts (b) and (c).

9.31. IDENTIFY: Use Table 9.2. The correct expression to use in each case depends on the shape of the object and the location of the axis.

SET UP: In each case express the mass in kg and the length in m, so the moment of inertia will be in $\text{kg}\cdot\text{m}^2$.

EXECUTE: **(a)** (i) $I=\frac{1}{3}ML^2=\frac{1}{3}(2.50\text{ kg})(0.750\text{ m})^2=0.469\text{ kg}\cdot\text{m}^2.$

(ii) $I=\frac{1}{12}ML^2=\frac{1}{4}(0.469\text{ kg}\cdot\text{m}^2)=0.117\text{ kg}\cdot\text{m}^2.$ (iii) For a very thin rod, all of the mass is at the axis and $I=0.$

(b) (i) $I=\frac{2}{5}MR^2=\frac{2}{5}(3.00\text{ kg})(0.190\text{ m})^2=0.0433\text{ kg}\cdot\text{m}^2.$

(ii) $I=\frac{2}{3}MR^2=\frac{5}{3}(0.0433\text{ kg}\cdot\text{m}^2)=0.0722\text{ kg}\cdot\text{m}^2.$

(c) (i) $I=MR^2=(8.00\text{ kg})(0.0600\text{ m})^2=0.0288\text{ kg}\cdot\text{m}^2.$

(ii) $I=\frac{1}{2}MR^2=\frac{1}{2}(8.00\text{ kg})(0.0600\text{ m})^2=0.0144\text{ kg}\cdot\text{m}^2.$

EVALUATE: I depends on how the mass of the object is distributed relative to the axis.

9.32. **IDENTIFY:** Treat each block as a point mass, so for each block $I=mr^2,$ where r is the distance of the block from the axis. The total I for the object is the sum of the I for each of its pieces.

SET UP: In part (a) two blocks are a distance $L/2$ from the axis and the third block is on the axis. In part (b) two blocks are a distance $L/4$ from the axis and one is a distance $3L/4$ from the axis.

EXECUTE: **(a)** $I=2m(L/2)^2=\frac{1}{2}mL^2.$

(b) $I=2m(L/4\,)^2+m(3L/4)^2=\dfrac{1}{16}mL^2(2+9)=\dfrac{11}{16}mL^2.$

EVALUATE: For the same object I is in general different for different axes.

9.33. **IDENTIFY:** I for the object is the sum of the values of I for each part.

SET UP: For the bar, for an axis perpendicular to the bar, use the appropriate expression from Table 9.2. For a point mass, $I=mr^2,$ where r is the distance of the mass from the axis.

EXECUTE: **(a)** $I=I_{\text{bar}}+I_{\text{balls}}=\dfrac{1}{12}M_{\text{bar}}L^2+2m_{\text{balls}}\left(\dfrac{L}{2}\right)^2.$

$$I=\frac{1}{12}(4.00\text{ kg})(2.00\text{ m})^2+2(0.300\text{ kg})(1.00\text{ m})^2=1.93\text{ kg}\cdot\text{m}^2$$

(b) $I=\dfrac{1}{3}m_{\text{bar}}L^2+m_{\text{ball}}L^2=\dfrac{1}{3}(4.00\text{ kg})(2.00\text{ m})^2+(0.300\text{ kg})(2.00\text{ m})^2=6.53\text{ kg}\cdot\text{m}^2$

(c) $I=0$ because all masses are on the axis.

(d) All the mass is a distance $d=0.500$ m from the axis and

$I=m_{\text{bar}}d^2+2m_{\text{ball}}d^2=M_{\text{Total}}d^2=(4.60\text{ kg})(0.500\text{ m})^2=1.15\text{ kg}\cdot\text{m}^2.$

EVALUATE: I for an object depends on the location and direction of the axis.

9.34. **IDENTIFY:** Moment of inertia of a bar.

SET UP: $I_{\text{end}}=\dfrac{1}{3}ML^2,\ I_{\text{center}}=\dfrac{1}{12}ML^2$

EXECUTE: **(a)** $\dfrac{1}{12}ML^2=(0.400\text{ kg})(0.600\text{ m})^2/12=0.0120\text{ kg}\cdot\text{m}^2.$

(b) Now we want the moment of inertia of two bars about their ends. Each has mass $M/2$ and length $L/2$.

$\dfrac{1}{3}ML^2=\dfrac{1}{3}\left(\dfrac{M}{2}\right)\left(\dfrac{L}{2}\right)^2+\dfrac{1}{3}\left(\dfrac{M}{2}\right)\left(\dfrac{L}{2}\right)^2=\dfrac{1}{12}ML^2=0.0120\text{ kg}\cdot\text{m}^2.$

EVALUATE: Neither the bend nor the 60° angle affects the moment of inertia. In (a) and (b), we can think of the rod as two 0.200-kg rods, each 0.300 m long, with the moment of inertia calculated about one end.

9.35. **IDENTIFY** and **SET UP:** $I = \sum m_i r_i^2$ implies $I = I_{\text{rim}} + I_{\text{spokes}}$

EXECUTE: $I_{\text{rim}} = MR^2 = (1.40 \text{ kg})(0.300 \text{ m})^2 = 0.126 \text{ kg} \cdot \text{m}^2$

Each spoke can be treated as a slender rod with the axis through one end, so

$I_{\text{spokes}} = 8\left(\frac{1}{3}ML^2\right) = \frac{8}{3}(0.280 \text{ kg})(0.300 \text{ m})^2 = 0.0672 \text{ kg} \cdot \text{m}^2$

$I = I_{\text{rim}} + I_{\text{spokes}} = 0.126 \text{ kg} \cdot \text{m}^2 + 0.0672 \text{ kg} \cdot \text{m}^2 = 0.193 \text{ kg} \cdot \text{m}^2.$

EVALUATE: Our result is smaller than $m_{\text{tot}}R^2 = (3.64 \text{ kg})(0.300 \text{ m})^2 = 0.328 \text{ kg} \cdot \text{m}^2$, since the mass of each spoke is distributed between $r = 0$ and $r = R$.

9.36. **IDENTIFY:** This problem requires moment of inertia calculations.

SET UP: The disk has the same mass and radius as the sphere. $I_\text{d} = \frac{1}{2}MR^2$, $I_1 = \frac{2}{5}MR^2$.

EXECUTE: Take the ratio of I_d to I_1. $\frac{I_\text{d}}{I_1} = \frac{\frac{1}{2}MR^2}{\frac{2}{5}MR^2} = \frac{5}{4}$, so $I_\text{d} = 5I_1/4$.

EVALUATE: Even though the disk and sphere have the same mass and radius, their moments of inertia are different because the mass is distributed differently.

9.37. **IDENTIFY:** The flywheel has kinetic energy due to its rotation, but it is slowing down. We need to use rotational kinetic energy and constant-angular acceleration equations.

SET UP: $K = \frac{1}{2}I\omega^2$, $\omega_z = \omega_{0z} + \alpha_z t$. We want to find the time for the flywheel to lose half of its kinetic energy.

EXECUTE: First use kinetic energy to find the initial and final angular velocities. We know that $K_2 = \frac{1}{2}K_1$, so $\frac{1}{2}I\omega_2^2 = \frac{1}{2}\left(\frac{1}{2}I\omega_1^2\right)$, which gives $\omega_2 = \frac{\omega_1}{\sqrt{2}}$. Now use $K = \frac{1}{2}I\omega^2$ to find ω_1. $\frac{1}{2}I\omega_1^2 = K_1$ gives $\omega_1 = \sqrt{\frac{2K_1}{I}} = \sqrt{\frac{2(30.0 \text{ J})}{12.0 \text{ kg} \cdot \text{m}^2}} = 2.236$ rad/s. $\omega_2 = \frac{\omega_1}{\sqrt{2}} = \frac{2.236 \text{ rad/s}}{\sqrt{2}} = 1.581$ rad/s. Now use $\omega_z = \omega_{0z} + \alpha_z t$ to find t: 1.581 rad/s = 2.236 rad/s + $\left((-0.500)(2\pi) \text{ rad/s}\right)t$, which gives $t = 0.208$ s.

EVALUATE: We must use radian measure in the formula $K = \frac{1}{2}I\omega^2$, but we could use any type of angular measure in the equation $\omega_z = \omega_{0z} + \alpha_z t$.

9.38. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to calculate I.

SET UP: $I = \frac{1}{12}ML^2$. 1 rpm = 0.1047 rad/s

EXECUTE: **(a)** $I = \frac{1}{12}(117 \text{ kg})(2.08 \text{ m})^2 = 42.2 \text{ kg} \cdot \text{m}^2$.

$\omega = (2400 \text{ rev/min})\left(\frac{0.1047 \text{ rad/s}}{1 \text{ rev/min}}\right) = 251 \text{ rad/s}.$ $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2 \text{ kg} \cdot \text{m}^2)(251 \text{ rad/s})^2 = 1.33 \times 10^6 \text{ J}.$

(b) $K_1 = \frac{1}{12}M_1L_1^2\omega_1^2$, $K_2 = \frac{1}{12}M_2L_2^2\omega_2^2$. $L_1 = L_2$ and $K_1 = K_2$, so $M_1\omega_1^2 = M_2\omega_2^2$.

$\omega_2 = \omega_1\sqrt{\frac{M_1}{M_2}} = (2400 \text{ rpm})\sqrt{\frac{M_1}{0.750M_1}} = 2770 \text{ rpm}$

EVALUATE: The rotational kinetic energy is proportional to the square of the angular speed and directly proportional to the mass of the object.

9.39. **IDENTIFY:** This problem requires moment of inertia calculations.

SET UP: The two spheres have the same mass, radius, and kinetic energy, but one is solid and the other is a hollow shell. $I_h = \frac{2}{3}MR^2$ and $I_s = \frac{2}{5}MR^2$. $K = \frac{1}{2}I\omega^2$. The solid sphere has angular speed ω_1 and we want to find the angular speed ω_h of the hollow sphere.

EXECUTE: Equate the kinetic energies of the two spheres: $\frac{2}{5}MR^2\omega_1^2 = \frac{2}{3}MR^2\omega_h^2$ which gives $\omega_h = \omega_1\sqrt{3/5}$.

EVALUATE: From our result, we see that the hollow sphere is spinning slower than the solid sphere. This is reasonable because the hollow sphere has a greater moment of inertia than the solid sphere because more of its mass is located farther from the axis of rotation.

9.40. **IDENTIFY:** We can use angular kinematics (for constant angular acceleration) to find the angular velocity of the wheel. Then knowing its kinetic energy, we can find its moment of inertia, which is the target variable.

SET UP: $\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$ and $K = \frac{1}{2}I\omega^2$.

EXECUTE: Converting the angle to radians gives $\theta - \theta_0 = (8.20 \text{ rev})(2\pi \text{ rad/1 rev}) = 51.52 \text{ rad}$.

$\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = \frac{2(\theta - \theta_0)}{t} = \frac{2(51.52 \text{ rad})}{12.0 \text{ s}} = 8.587 \text{ rad/s}$. Solving $K = \frac{1}{2}I\omega^2$ for I gives $I = \frac{2K}{\omega^2} = \frac{2(36.0 \text{ J})}{(8.587 \text{ rad/s})^2} = 0.976 \text{ kg}\cdot\text{m}^2$.

EVALUATE: The angular velocity must be in radians to use the formula $K = \frac{1}{2}I\omega^2$.

9.41. **IDENTIFY:** Knowing the kinetic energy, mass and radius of the sphere, we can find its angular velocity. From this we can find the tangential velocity (the target variable) of a point on the rim.

SET UP: $K = \frac{1}{2}I\omega^2$ and $I = \frac{2}{5}MR^2$ for a solid uniform sphere. The tagential velocity is $v = r\omega$.

EXECUTE: $I = \frac{2}{5}MR^2 = \frac{2}{5}(28.0 \text{ kg})(0.380 \text{ m})^2 = 1.617 \text{ kg}\cdot\text{m}^2$. $K = \frac{1}{2}I\omega^2$ so

$\omega = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(236 \text{ J})}{1.617 \text{ kg}\cdot\text{m}^2}} = 17.085 \text{ rad/s}$.

$v = r\omega = (0.380 \text{ m})(17.085 \text{ rad/s}) = 6.49 \text{ m/s}$.

EVALUATE: This is the speed of a point on the surface of the sphere that is farthest from the axis of rotation (the "equator" of the sphere). Points off the "equator" would have smaller tangential velocity but the same angular velocity.

9.42. **IDENTIFY:** Knowing the angular acceleration of the sphere, we can use angular kinematics (with constant angular acceleration) to find its angular velocity. Then using its mass and radius, we can find its kinetic energy, the target variable.

SET UP: $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, $K = \frac{1}{2}I\omega^2$, and $I = \frac{2}{3}MR^2$ for a uniform hollow spherical shell.

EXECUTE: $I = \frac{2}{3}MR^2 = \frac{2}{3}(8.20 \text{ kg})(0.220 \text{ m})^2 = 0.2646 \text{ kg}\cdot\text{m}^2$. Converting the angle to radians gives $\theta - \theta_0 = (6.00 \text{ rev})(2\pi \text{ rad/1 rev}) = 37.70 \text{ rad}$. The angular velocity is $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, which gives $\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.890 \text{ rad/s}^2)(37.70 \text{ rad})} = 8.192 \text{ rad/s}$.

$K = \frac{1}{2}(0.2646 \text{ kg}\cdot\text{m}^2)(8.192 \text{ rad/s})^2 = 8.88 \text{ J}$.

EVALUATE: The angular velocity must be in radians to use the formula $K = \frac{1}{2}I\omega^2$.

9.43. **IDENTIFY:** We need to use rotational kinetic energy.

SET UP: $K = \frac{1}{2}I\omega^2$. We know that $I_A = 3I_B$ and $\omega_B = 4\omega_A$, and we want to compare the kinetic energies of the wheels.

EXECUTE: **(a)** Using what we know gives $K_A = \frac{1}{2}I_A\omega_A^2$ and $K_B = \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}\left(\frac{I_A}{3}\right)(4\omega_A)^2$. For K_B this becomes $K_B = \frac{1}{2}\left(\frac{I_A}{3}\right)(4\omega_A)^2 = \frac{16}{3}\left(\frac{1}{2}I_A\omega_A^2\right) = \frac{16}{3}K_A$, which tells us the B has more kinetic energy than A.

(b) $\dfrac{K_A}{K_B} = \dfrac{\frac{1}{2}I_A\omega_A^2}{\left(\frac{16}{3}\right)\left(\frac{1}{2}I_A\omega_A^2\right)} = \dfrac{3}{16}.$

EVALUATE: Wheel A has 3 times the moment of inertia of B but only ¼ the angular speed and therefore only 1/16 the *square* of the angular speed, so it has less kinetic energy than B.

9.44. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to relate I to the mass M of the disk.

SET UP: 45.0 rpm = 4.71 rad/s. For a uniform solid disk, $I = \frac{1}{2}MR^2$.

EXECUTE: **(a)** $I = \dfrac{2K}{\omega^2} = \dfrac{2(0.250\text{ J})}{(4.71\text{ rad/s})^2} = 0.0225\text{ kg}\cdot\text{m}^2.$

(b) $I = \frac{1}{2}MR^2$ and $M = \dfrac{2I}{R^2} = \dfrac{2(0.0225\text{ kg}\cdot\text{m}^2)}{(0.300\text{ m})^2} = 0.500\text{ kg}.$

EVALUATE: No matter what the shape is, the rotational kinetic energy is proportional to the mass of the object.

9.45. **IDENTIFY** and **SET UP:** Combine $K = \frac{1}{2}I\omega^2$ and $a_{\text{rad}} = r\omega^2$ to solve for K. Use Table 9.2 to get I.

EXECUTE: $K = \frac{1}{2}I\omega^2$

$a_{\text{rad}} = R\omega^2$, so $\omega = \sqrt{a_{\text{rad}}/R} = \sqrt{(3500\text{ m/s}^2)/1.20\text{ m}} = 54.0\text{ rad/s}$

For a disk, $I = \frac{1}{2}MR^2 = \frac{1}{2}(70.0\text{ kg})(1.20\text{ m})^2 = 50.4\text{ kg}\cdot\text{m}^2$

Thus $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(50.4\text{ kg}\cdot\text{m}^2)(54.0\text{ rad/s})^2 = 7.35\times10^4\text{ J}$

EVALUATE: The limit on a_{rad} limits ω which in turn limits K.

9.46. **IDENTIFY:** In order to meet the specifications, we need to use rotational kinetic energy and constant-angular acceleration equations.
SET UP: The flywheel should have 240 J of kinetic energy after it has turned through 30.0 rev starting from rest. We want to know its moment of inertia. We know that $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ and $K = \frac{1}{2}I\omega^2$.

EXECUTE: First use $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ to find ω^2 and then use $K = \frac{1}{2}I\omega^2$ to find I. Using $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega^2 = 2\alpha_z\Delta\theta$. Now solve $K = \frac{1}{2}I\omega^2$ for I, which gives

$$I = \frac{2K}{\omega^2} = \frac{2K}{2\alpha_z\Delta\theta} = \frac{240\text{ J}}{(0.400\pi\text{ rad/s}^2)(60.0\pi\text{ rad})} = 1.01\text{ kg}\cdot\text{m}^2.$$

EVALUATE: Since $I = \frac{1}{2}MR^2$, we could increase either M or R to increase I, but it would be more effective to increase R since I is proportional to the *square* of R.

9.47. **IDENTIFY:** Apply conservation of energy to the system of stone plus pulley. $v = r\omega$ relates the motion of the stone to the rotation of the pulley.

SET UP: For a uniform solid disk, $I = \frac{1}{2}MR^2$. Let point 1 be when the stone is at its initial position and point 2 be when it has descended the desired distance. Let $+y$ be upward and take $y = 0$ at the initial position of the stone, so $y_1 = 0$ and $y_2 = -h$, where h is the distance the stone descends.

EXECUTE: **(a)** $K_p = \frac{1}{2}I_p\omega^2$. $I_p = \frac{1}{2}M_pR^2 = \frac{1}{2}(2.50 \text{ kg})(0.200 \text{ m})^2 = 0.0500 \text{ kg}\cdot\text{m}^2$.

$\omega = \sqrt{\dfrac{2K_p}{I_p}} = \sqrt{\dfrac{2(4.50 \text{ J})}{0.0500 \text{ kg}\cdot\text{m}^2}} = 13.4 \text{ rad/s}$. The stone has speed

$v = R\omega = (0.200 \text{ m})(13.4 \text{ rad/s}) = 2.68 \text{ m/s}$. The stone has kinetic energy

$K_s = \frac{1}{2}mv^2 = \frac{1}{2}(1.50 \text{ kg})(2.68 \text{ m/s})^2 = 5.39 \text{ J}$. $K_1 + U_1 = K_2 + U_2$ gives $0 = K_2 + U_2$.

$0 = 4.50 \text{ J} + 5.39 \text{ J} + mg(-h)$. $h = \dfrac{9.89 \text{ J}}{(1.50 \text{ kg})(9.80 \text{ m/s}^2)} = 0.673 \text{ m}$.

(b) $K_{tot} = K_p + K_s = 9.89 \text{ J}$. $I = 2MR^2$,

EVALUATE: The gravitational potential energy of the pulley doesn't change as it rotates. The tension in the wire does positive work on the pulley and negative work of the same magnitude on the stone, so no net work on the system.

9.48. **IDENTIFY:** $K_p = \frac{1}{2}I\omega^2$ for the pulley and $K_b = \frac{1}{2}mv^2$ for the bucket. The speed of the bucket and the rotational speed of the pulley are related by $v = R\omega$.

SET UP: $K_p = \frac{1}{2}K_b$

EXECUTE: $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mv^2) = \frac{1}{4}mR^2\omega^2$. $I = \frac{1}{2}mR^2$.

EVALUATE: The result is independent of the rotational speed of the pulley and the linear speed of the mass.

9.49. **IDENTIFY:** With constant acceleration, we can use kinematics to find the speed of the falling object. Then we can apply the work-energy expression to the entire system and find the moment of inertia of the wheel. Finally, using its radius we can find its mass, the target variable.

SET UP: With constant acceleration, $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$. The angular velocity of the wheel is related to the linear velocity of the falling mass by $\omega_z = \dfrac{v_y}{R}$. The work-energy theorem is $K_1 + U_1 + W_{other} = K_2 + U_2$, and the moment of inertia of a uniform disk is $I = \frac{1}{2}MR^2$.

EXECUTE: Find v_y, the velocity of the block after it has descended 3.00 m. $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = \dfrac{2(y - y_0)}{t} = \dfrac{2(3.00 \text{ m})}{2.00 \text{ s}} = 3.00 \text{ m/s}$. For the wheel, $\omega_z = \dfrac{v_y}{R} = \dfrac{3.00 \text{ m/s}}{0.280 \text{ m}} = 10.71 \text{ rad/s}$. Apply the work-energy expression: $K_1 + U_1 + W_{other} = K_2 + U_2$, giving $mg(3.00 \text{ m}) = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$. Solving for I gives $I = \dfrac{2}{\omega^2}\left[mg(3.00 \text{ m}) - \dfrac{1}{2}mv^2\right]$.

$I = \dfrac{2}{(10.71\ \text{rad/s})^2}\left[(4.20\ \text{kg})(9.8\ \text{m/s}^2)(3.00\ \text{m}) - \dfrac{1}{2}(4.20\ \text{kg})(3.00\ \text{m/s})^2\right]$. $I = 1.824\ \text{kg}\cdot\text{m}^2$. For a solid disk, $I = \frac{1}{2}MR^2$ gives $M = \dfrac{2I}{R^2} = \dfrac{2(1.824\ \text{kg}\cdot\text{m}^2)}{(0.280\ \text{m})^2} = 46.5\ \text{kg}$.

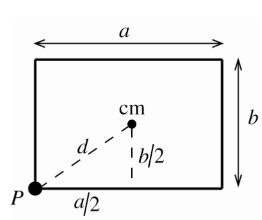
EVALUATE: The gravitational potential of the falling object is converted into the kinetic energy of that object and the rotational kinetic energy of the wheel.

9.50. **IDENTIFY:** Apply the parallel-axis theorem.
SET UP: The center of mass of the hoop is at its geometrical center.
EXECUTE: In the parallel-axis theorem, $I_{\text{cm}} = MR^2$ and $d = R^2$, so $I_P = 2MR^2$.
EVALUATE: I is larger for an axis at the edge than for an axis at the center. Some mass is closer than distance R from the axis but some is also farther away. Since I for each piece of the hoop is proportional to the square of the distance from the axis, the increase in distance has a larger effect.

9.51. **IDENTIFY:** Use the parallel-axis theorem to relate I for the wood sphere about the desired axis to I for an axis along a diameter.
SET UP: For a thin-walled hollow sphere, axis along a diameter, $I = \frac{2}{3}MR^2$.
For a solid sphere with mass M and radius R, $I_{\text{cm}} = \frac{2}{5}MR^2$, for an axis along a diameter.
EXECUTE: Find d such that $I_P = I_{\text{cm}} + Md^2$ with $I_P = \frac{2}{3}MR^2$:
$\frac{2}{3}MR^2 = \frac{2}{5}MR^2 + Md^2$
The factors of M divide out and the equation becomes $(\frac{2}{3} - \frac{2}{5})R^2 = d^2$
$d = \sqrt{(10-6)/15}R = 2R/\sqrt{15} = 0.516R$.
The axis is parallel to a diameter and is $0.516R$ from the center.
EVALUATE: $I_{\text{cm}}(\text{lead}) > I_{\text{cm}}(\text{wood})$ even though M and R are the same since for a hollow sphere all the mass is a distance R from the axis. The parallel-axis theorem says $I_P > I_{\text{cm}}$, so there must be a d where $I_P(\text{wood}) = I_{\text{cm}}(\text{lead})$.

9.52. **IDENTIFY:** Consider the plate as made of slender rods placed side-by-side.
SET UP: The expression in Table 9.2 gives I for a rod and an axis through the center of the rod.
EXECUTE: **(a)** I is the same as for a rod with length a: $I = \frac{1}{12}Ma^2$.
(b) I is the same as for a rod with length b: $I = \frac{1}{12}Mb^2$.
EVALUATE: I is smaller when the axis is through the center of the plate than when it is along one edge.

9.53. **IDENTIFY** and **SET UP:** Use the parallel-axis theorem. The cm of the sheet is at its geometrical center. The object is sketched in Figure 9.53.
EXECUTE: $I_P = I_{\text{cm}} + Md^2$.



From Table 9.2,
$I_{\text{cm}} = \frac{1}{12}M(a^2 + b^2)$.
The distance d of P from the cm is
$d = \sqrt{(a/2)^2 + (b/2)^2}$.

Figure 9.53

Thus $I_P = I_{\text{cm}} + Md^2 = \frac{1}{12}M(a^2+b^2) + M(\frac{1}{4}a^2 + \frac{1}{4}b^2) = (\frac{1}{12}+\frac{1}{4})M(a^2+b^2) = \frac{1}{3}M(a^2+b^2)$

EVALUATE: $I_P = 4I_{\text{cm}}$. For an axis through P mass is farther from the axis.

9.54. **IDENTIFY:** Use the equations in Table 9.2. I for the rod is the sum of I for each segment. The parallel-axis theorem says $I_{\text{p}} = I_{\text{cm}} + Md^2$.

SET UP: The bent rod and axes a and b are shown in Figure 9.54. Each segment has length $L/2$ and mass $M/2$.

EXECUTE: **(a)** For each segment the moment of inertia is for a rod with mass $M/2$, length $L/2$ and the axis through one end. For one segment, $I_{\text{s}} = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{24}ML^2$. For the rod, $I_{\text{a}} = 2I_{\text{s}} = \frac{1}{12}ML^2$.

(b) The center of mass of each segment is at the center of the segment, a distance of $L/4$ from each end. For each segment, $I_{\text{cm}} = \frac{1}{12}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{96}ML^2$. Axis b is a distance $L/4$ from the cm of each segment, so for each segment the parallel axis theorem gives I for axis b to be

$I_{\text{s}} = \frac{1}{96}ML^2 + \frac{M}{2}\left(\frac{L}{4}\right)^2 = \frac{1}{24}ML^2$ and $I_{\text{b}} = 2I_{\text{s}} = \frac{1}{12}ML^2$.

EVALUATE: I for these two axes are the same.

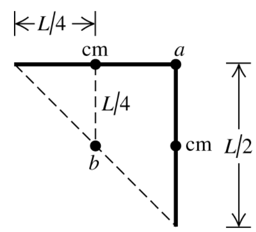


Figure 9.54

9.55. **IDENTIFY:** Apply $I = \int r^2\, dm$.

SET UP: $dm = \rho dV = \rho(2\pi rL\; dr)$, where L is the thickness of the disk. $M = \pi L\rho R^2$.

EXECUTE: The analysis is identical to that of Example 9.10, with the lower limit in the integral being zero and the upper limit being R. The result is $I = \frac{1}{2}MR^2$.

9.56. **IDENTIFY:** Use $I = \int r^2\, dm$.

SET UP:

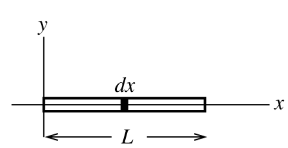


Figure 9.56

Take the x-axis to lie along the rod, with the origin at the left end. Consider a thin slice at coordinate x and width dx, as shown in Figure 9.56. The mass per unit length for this rod is M/L, so the mass of this slice is $dm = (M/L)\, dx$.

EXECUTE: $I = \int_0^L x^2(M/L)\,dx = (M/L)\int_0^L x^2\,dx = (M/L)(L^3/3) = \frac{1}{3}ML^2$

EVALUATE: This result agrees with the table in the text.

9.57. **IDENTIFY:** Apply $I = \int r^2\,dm$ and $M = \int dm$.

SET UP: For this case, $dm = \gamma x\,dx$.

EXECUTE: **(a)** $M = \int dm = \int_0^L \gamma x\,dx = \gamma \frac{x^2}{2}\Big|_0^L = \frac{\gamma L^2}{2}$

(b) $I = \int_0^L x^2(\gamma x)dx = \gamma \frac{x^4}{4}\Big|_0^L = \frac{\gamma L^4}{4} = \frac{M}{2}L^2$. This is larger than the moment of inertia of a uniform rod of the same mass and length, since the mass density is greater farther away from the axis than nearer the axis.

(c) $I = \int_0^L (L-x)^2 \gamma x dx = \gamma \int_0^L (L^2 x - 2Lx^2 + x^3)dx = \gamma\left(L^2\frac{x^2}{2} - 2L\frac{x^3}{3} + \frac{x^4}{4}\right)\Bigg|_0^L = \gamma\frac{L^4}{12} = \frac{M}{6}L^2$.

This is a third of the result of part (b), reflecting the fact that more of the mass is concentrated at the right end.

EVALUATE: For a uniform rod with an axis at one end, $I = \frac{1}{3}ML^2$. The result in (b) is larger than this and the result in (c) is smaller than this.

9.58. **IDENTIFY:** Using the equation for the angle as a function of time, we can find the angular acceleration of the disk at a given time and use this to find the linear acceleration of a point on the rim (the target variable).

SET UP: We can use the definitions of the angular velocity and the angular acceleration: $\omega_z(t) = \frac{d\theta}{dt}$ and $\alpha_z(t) = \frac{d\omega_z}{dt}$. The acceleration components are $a_{\text{rad}} = R\omega^2$ and $a_{\text{tan}} = R\alpha$, and the magnitude of the acceleration is $a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2}$.

EXECUTE: $\omega_z(t) = \frac{d\theta}{dt} = 1.10\text{ rad/s} + (12.6\text{ rad/s}^2)t$. $\alpha_z(t) = \frac{d\omega_z}{dt} = 12.6\text{ rad/s}^2$ (constant).

$\theta = 0.100\text{ rev} = 0.6283\text{ rad}$ gives $6.30t^2 + 1.10t - 0.6283 = 0$, so $t = 0.2403\text{ s}$, using the positive root. At this t, $\omega_z(t) = 4.1278\text{ rad/s}$ and $\alpha_z(t) = 12.6\text{ rad/s}^2$. For a point on the rim, $a_{\text{rad}} = R\omega^2 = 6.815\text{ m/s}^2$ and $a_{\text{tan}} = R\alpha = 5.04\text{ m/s}^2$, so $a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = 8.48\text{ m/s}^2$.

EVALUATE: Since the angular acceleration is constant, we could use the constant acceleration formulas as a check. For example, the coefficient of t^2 is $\frac{1}{2}\alpha_z = 6.30\text{ rad/s}^2$ gives $\alpha_z = 12.6\text{ rad/s}^2$.

9.59. **IDENTIFY:** The target variable is the horizontal distance the piece travels before hitting the floor. Using the angular acceleration of the blade, we can find its angular velocity when the piece breaks off. This will give us the linear horizontal speed of the piece. It is then in free fall, so we can use the linear kinematics equations.

SET UP: $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ for the blade, and $v = r\omega$ is the horizontal velocity of the piece. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ for the falling piece.

EXECUTE: Find the initial horizontal velocity of the piece just after it breaks off.
$\theta-\theta_0=(155\text{ rev})(2\pi\text{ rad/1 rev})=973.9\text{ rad}.$

$\alpha_z=(2.00\text{ rev/s}^2)(2\pi\text{ rad/1 rev})=12.566\text{ rad/s}^2.$ $\omega_z^2=\omega_{0z}^2+2\alpha_z(\theta-\theta_0).$

$\omega_z=\sqrt{2\alpha_z(\theta-\theta_0)}=\sqrt{2(12.566\text{ rad/s}^2)(973.9\text{ rad})}=156.45\text{ rad/s}.$ The horizontal velocity of the piece is $v=r\omega=(0.120\text{ m})(156.45\text{ rad/s})=18.774\text{ m/s}.$ Now consider the projectile motion of the piece. Take $+y$ downward and use the vertical motion to find t. Solving $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ for t gives

$t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(0.820\text{ m})}{9.8\text{ m/s}^2}}=0.4091\text{ s}.$ Then

$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2=(18.774\text{ m/s})(0.4091\text{ s})=7.68\text{ m}.$

EVALUATE: Once the piece is free of the blade, the only force acting on it is gravity so its acceleration is g downward.

9.60. **IDENTIFY** and **SET UP:** Use $\omega_z=\dfrac{d\theta}{dt}$ and $\alpha_z=\dfrac{d\omega_z}{dt}$. As long as $\alpha_z>0$, ω_z increases. At the t when $\alpha_z=0$, ω_z is at its maximum positive value and then starts to decrease when α_z becomes negative.

$\theta(t)=\gamma t^2-\beta t^3;$ $\gamma=3.20\text{ rad/s}^2,$ $\beta=0.500\text{ rad/s}^3$

EXECUTE: **(a)** $\omega_z(t)=\dfrac{d\theta}{dt}=\dfrac{d(\gamma t^2-\beta t^3)}{dt}=2\gamma t-3\beta t^2$

(b) $\alpha_z(t)=\dfrac{d\omega_z}{dt}=\dfrac{d(2\gamma t-3\beta t^2)}{dt}=2\gamma-6\beta t$

(c) The maximum angular velocity occurs when $\alpha_z=0.$

$2\gamma-6\beta t=0$ implies $t=\dfrac{2\gamma}{6\beta}=\dfrac{\gamma}{3\beta}=\dfrac{3.20\text{ rad/s}^2}{3(0.500\text{ rad/s}^3)}=2.133\text{ s}$

At this t, $\omega_z=2\gamma t-3\beta t^2=2(3.20\text{ rad/s}^2)(2.133\text{ s})-3(0.500\text{ rad/s}^3)(2.133\text{ s})^2=6.83\text{ rad/s}$

The maximum positive angular velocity is 6.83 rad/s and it occurs at 2.13 s.

EVALUATE: For large t both ω_z and α_z are negative and ω_z increases in magnitude. In fact, $\omega_z\to-\infty$ at $t\to\infty$. So the answer in (c) is not the largest angular speed, just the largest positive angular velocity.

9.61. **IDENTIFY:** The angular acceleration α of the disk is related to the linear acceleration a of the ball by $a=R\alpha$. Since the acceleration is not constant, use $\omega_z-\omega_{0z}=\int_0^t\alpha_z dt$ and $\theta-\theta_0=\int_0^t\omega_z dt$ to relate θ, ω_z, α_z, and t for the disk. $\omega_{0z}=0.$

SET UP: $\int t^n dt=\dfrac{1}{n+1}t^{n+1}.$ In $a=R\alpha$, α is in rad/s^2.

EXECUTE: **(a)** $A=\dfrac{a}{t}=\dfrac{1.80\text{ m/s}^2}{3.00\text{ s}}=0.600\text{ m/s}^3$

(b) $\alpha=\dfrac{a}{R}=\dfrac{(0.600\text{ m/s}^3)t}{0.250\text{ m}}=(2.40\text{ rad/s}^3)t$

(c) $\omega_z=\int_0^t(2.40\text{ rad/s}^3)tdt=(1.20\text{ rad/s}^3)t^2.$ $\omega_z=15.0\text{ rad/s}$ for $t=\sqrt{\dfrac{15.0\text{ rad/s}}{1.20\text{ rad/s}^3}}=3.54\text{ s}.$

(d) $\theta-\theta_0=\int_0^t\omega_z dt=\int_0^t(1.20\text{ rad/s}^3)t^2dt=(0.400\text{ rad/s}^3)t^3.$ For $t=3.54$ s, $\theta-\theta_0=17.7$ rad.

Evaluate: If the disk had turned at a constant angular velocity of 15.0 rad/s for 3.54 s it would have turned through an angle of 53.1 rad in 3.54 s. It actually turns through less than half this because the angular velocity is increasing in time and is less than 15.0 rad/s at all but the end of the interval.

9.62. Identify: The flywheel gains rotational kinetic energy as it spins. This kinetic energy depends on the flywheel's rate of spin but also on its moment of inertia. The angular acceleration is constant.

Set Up: $K = \frac{1}{2}I\omega^2,\ I = \frac{1}{2}mR^2,\ \ \omega = \omega_0 + \alpha t,\ \ m = \rho V = \rho\pi R^2 h.$

Execute: $K = \frac{1}{2}I\omega^2\ =\ \frac{1}{2}(\frac{1}{2}mR^2)(\omega_0 + \alpha t)^2\ =\ \frac{1}{4}\left[(\rho\pi R^2 h)R^2\right](0 + \alpha t)^2$. Solving for h gives

$h = \dfrac{4K}{\rho\pi R^4(\alpha t)^2}$ = 4(800 J)/[π(8600 kg/m^3)(0.250 m)^4(3.00 rad/s^2)^2(8.00 s)^2] = 0.0526 m = 5.26 cm.

Evaluate: If we could turn the disk into a thin-walled cylinder of the same mass and radius, the moment of inertia would be twice as great, so we could store twice as much energy as for the given disk.

9.63. Identify: We use energy conservation for this problem. The accelerations are constant, so we can use the constant-acceleration equations.

Set Up: We can use $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, $K_{\text{rot}} = \dfrac{1}{2}I\omega^2$, $K_{\text{tr}} = \dfrac{1}{2}mv^2$, $U_{\text{g}} = mgy$, $\omega_z = \omega_{0z} + \alpha_z t$, $I = \dfrac{1}{2}MR^2$, and $\theta - \theta_0 = \omega_{0z}t + \dfrac{1}{2}\alpha_z t^2$. The system starts from rest, and the wheel turns through 8.00 rev in the first 5.00 s. We want to find the mass of the block.

Execute: First use $\theta - \theta_0 = \omega_{0z}t + \dfrac{1}{2}\alpha_z t^2$ to find α. 16π rad = 0 + $\dfrac{1}{2}\alpha(5.00\text{ s})^2$, which gives α = 4.021 rad/s^2. Now use $\omega_z = \omega_{0z} + \alpha_z t$ to find the angular speed of the wheel at the end of 5.00 s. $\omega = 0 + (4.021\text{ rad/s}^2)(5.00\text{ s})$ = 20.11 rad/s. Now use energy conservation for the system, $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. $W_{\text{other}} = 0$ and $K_1 = 0$. For gravitational potential energy, call $y = 0$ the level of the block at the end of the first 5.00 s. This makes $U_2 = 0$ and $U_1 = mgy$, where y is the length of rope that has come off the rim of the wheel as it turns through 8.00 rev (which is 16.00π rad). This length is R(16.00π rad) = (0.800 m)(16.00π rad) = 40.21 m; therefore y = 40.21 m. Putting this into the energy equation gives $m_B gy = \dfrac{1}{2}m_B v^2 + \dfrac{1}{2}I\omega^2$. Using $I = \dfrac{1}{2}MR^2$ and $v = R\omega$, the energy equation becomes

$m_B gy = \dfrac{1}{2}m_B(R\omega)^2 + \dfrac{1}{2}\left(\dfrac{1}{2}m_{\text{wheel}}R^2\right)\omega^2$. Solving for m_B gives $m_B = \dfrac{\frac{1}{4}m_{\text{wheel}}R^2\omega^2}{gy - \frac{1}{2}R^2\omega^2}$. Putting in m_{wheel} = 5.00 kg, R = 0.800 m, ω = 20.11 rad/s, and y = 40.21 m, we get m_B = 1.22 kg.

Evaluate: As the block falls, its gravitational potential energy is converted to kinetic energy, but part of that kinetic energy goes to the wheel and part goes to the block.

9.64. Identify: Apply conservation of energy to the system of drum plus falling mass, and compare the results for earth and for Mars.

Set Up: $K_{\text{drum}} = \frac{1}{2}I\omega^2.\ \ K_{\text{mass}} = \frac{1}{2}mv^2.\ \ v = R\omega$ so if K_{drum} is the same, ω is the same and v is the same on both planets. Therefore, K_{mass} is the same. Let $y = 0$ at the initial height of the mass and take $+y$ upward. Configuration 1 is when the mass is at its initial position and 2 is when the mass has descended 5.00 m, so $y_1 = 0$ and $y_2 = -h$, where h is the height the mass descends.

Execute: (a) $K_1 + U_1 = K_2 + U_2$ gives $0 = K_{\text{drum}} + K_{\text{mass}} - mgh.$ $K_{\text{drum}} + K_{\text{mass}}$ are the same on both planets, so $mg_{\text{E}}h_{\text{E}} = mg_{\text{M}}h_{\text{M}}.$ $h_{\text{M}} = h_{\text{E}}\left(\dfrac{g_{\text{E}}}{g_{\text{M}}}\right) = (5.00\text{ m})\left(\dfrac{9.80\text{ m/s}^2}{3.71\text{ m/s}^2}\right) = 13.2\text{ m}.$

(b) $mg_{\rm M}h_{\rm M} = K_{\rm drum} + K_{\rm mass}$. $\tfrac{1}{2}mv^2 = mg_{\rm M}h_{\rm M} - K_{\rm drum}$ and

$$v = \sqrt{2g_{\rm M}h_{\rm M} - \frac{2K_{\rm drum}}{m}} = \sqrt{2(3.71\text{ m/s}^2)(13.2\text{ m}) - \frac{2(250.0\text{ J})}{15.0\text{ kg}}} = 8.04\text{ m/s}$$

EVALUATE: We did the calculations without knowing the moment of inertia I of the drum, or the mass and radius of the drum.

9.65. **IDENTIFY:** We use energy conservation. The kinetic energy is shared between the two blocks and the pulley.

SET UP: We can use $U_1 + K_1 + W_{\rm other} = U_2 + K_2$, $K_{\rm rot} = \dfrac{1}{2}I\omega^2$, $K_{\rm tr} = \dfrac{1}{2}mv^2$, and $U_{\rm g} = mgy$. The system starts from rest, and the pulley is turning at 8.00 rad/s after block B has descended 1.20 m. We want to find the mass of block A.

EXECUTE: First use $v_{\rm tan} = r\omega$ to find the speed of the block when ω = 8.00 rad/s.
$v_B = v_{\rm tan} = R\omega = (0.200\text{ m})(8.00\text{ rad/s}) = 1.60\text{ m/s}$. Now use energy conservation,
$U_1 + K_1 + W_{\rm other} = U_2 + K_2$. $K_1 = 0$ and $W_{\rm other} = 0$. $K_2 = K_A + K_B + K_{\rm pulley}$. Choose $y = 0$ at 1.20 m below the starting point of B, which makes $U_2 = 0$ and $U_1 = mgy$, where $y = 1.20$ m. We also know that $v_A = v_B = v = 1.60$ m/s. Therefore we have $m_Bgy = \dfrac{1}{2}m_Av^2 + \dfrac{1}{2}m_Bv^2 + \dfrac{1}{2}I\omega^2$. Solving for m_A gives

$m_A = \dfrac{2m_Bgy - m_Bv^2 - I\omega^2}{v^2}$. Using $m_B = 5.00$ kg, $y = 1.20$ m, $I = 1.30\ \text{kg}\cdot\text{m}^2$, $v = 1.60$ m/s, and ω = 8.00 rad/s, we get $m_A = 8.44$ kg.

EVALUATE: As block B falls, its gravitational potential energy gets converted to kinetic energy which is shared between itself, block A, and the pulley. This is a case where the pulley is *not* massless!

9.66. **IDENTIFY:** The speed of all points on the belt is the same, so $r_1\omega_1 = r_2\omega_2$ applies to the two pulleys.

SET UP: The second pulley, with half the diameter of the first, must have twice the angular velocity, and this is the angular velocity of the saw blade. $a_{\rm tan} = R\alpha$

EXECUTE: **(a)** $v_2 = (2(3450\text{ rev/min}))\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)\left(\dfrac{0.208\text{ m}}{2}\right) = 75.1\text{ m/s}.$

(b) $a_{\rm rad} = \omega^2 r = \left(2(3450\text{ rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)\right)^2\left(\dfrac{0.208\text{ m}}{2}\right) = 5.43\times10^4\text{ m/s}^2,$

so the force holding sawdust on the blade would have to be about 5500 times as strong as gravity.

EVALUATE: In $v = r\omega$ and $a_{\rm rad} = r\omega^2$, ω must be in rad/s.

9.67. **IDENTIFY:** Apply $v = r\omega$.

SET UP: Points on the chain all move at the same speed, so $r_{\rm r}\omega_{\rm r} = r_{\rm f}\omega_{\rm f}$.

EXECUTE: The angular velocity of the rear wheel is $\omega_{\rm r} = \dfrac{v_{\rm r}}{r_{\rm r}} = \dfrac{5.00\text{ m/s}}{0.330\text{ m}} = 15.15\text{ rad/s}.$

The angular velocity of the front wheel is $\omega_{\rm f} = 0.600\text{ rev/s} = 3.77\text{ rad/s}$. $r_{\rm r} = r_{\rm f}(\omega_{\rm f}/\omega_{\rm r}) = 2.99\text{ cm}$.

EVALUATE: The rear sprocket and wheel have the same angular velocity and the front sprocket and wheel have the same angular velocity. α is the same for both, so the rear sprocket has a smaller radius since it has a larger angular velocity. The speed of a point on the chain is
$v = r_{\rm r}\omega_{\rm r} = (2.99\times10^{-2}\text{ m})(15.15\text{ rad/s}) = 0.453\text{ m/s}$. The linear speed of the bicycle is 5.00 m/s.

9.68. **IDENTIFY:** Use the constant angular acceleration equations, applied to the first revolution and to the first two revolutions.

SET UP: Let the direction the disk is rotating be positive. $\text{kg}\cdot\text{m}^2$. Let t be the time for the first revolution. The time for the first two revolutions is $t + 0.0865\text{ s}$.

EXECUTE: **(a)** $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ applied to the first revolution and then to the first two revolutions gives $2\pi\text{ rad} = \frac{1}{2}\alpha_z t^2$ and $4\pi\text{ rad} = \frac{1}{2}\alpha_z (t + 0.0865\text{ s})^2$. Eliminating α_z between these equations gives $4\pi\text{ rad} = \dfrac{2\pi\text{ rad}}{t^2}(t + 0.0865\text{ s})^2$. $2t^2 = (t + 0.0865\text{ s})^2$. $\sqrt{2}t = \pm(t + 0.0865\text{ s})$. The positive root is $t = \dfrac{0.0865\text{ s}}{\sqrt{2} - 1} = 0.209\text{ s}$.

(b) $2\pi\text{ rad} = \frac{1}{2}\alpha_z t^2$ and $t = 0.209\text{ s}$ gives $\alpha_z = 288\text{ rad/s}^2$

EVALUATE: At the start of the second revolution, $\omega_{0z} = (288\text{ rad/s}^2)(0.209\text{ s}) = 60.19\text{ rad/s}$. The distance the disk rotates in the next 0.0865 s is $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (60.19\text{ rad/s})(0.0865\text{ s}) + \frac{1}{2}(288\text{ rad/s}^2)(0.0865\text{ s})^2 = 6.28\text{ rad}$, which is two revolutions.

9.69. **IDENTIFY:** This problem involves energy conservation and Newton's second law.

SET UP: Only conservative forces act on the system, so $U_1 + K_1 = U_2 + K_2$. The kinetic energy is due to the translation of the block and the rotation of the wheel, and $K_1 = 0$. Call point 2 the location of the block after it has fallen for 2.00 s. This makes $U_2 = 0$ and $U_1 = mgy$. We also use $v_y = v_{0y} + a_y t$, $y = y_0 + v_{0y}t + \dfrac{1}{2}a_y t^2$, and $\sum F_y = ma_y$. We want to find the kinetic energy of the wheel 2.00 s after motion has begun.

EXECUTE: $U_1 + K_1 = U_2 + K_2$ gives $mgy = \dfrac{1}{2}mv^2 + K_{\text{wheel}}$. We need to find v, so apply $K = \frac{1}{2}I\omega^2 + \frac{1}{2}m(\omega R)^2 = \frac{1}{2}(I + mR^2)\omega^2$. to the block, taking downward positive because that is the direction of its acceleration. This gives $mg - T = ma$ so Using $I = \frac{1}{2}mR^2$ and solving for ω, = $9.80\text{ m/s}^2 - \dfrac{9.00\text{ N}}{1.50\text{ kg}}$ = 3.80 m/s². Now use $v_y = v_{0y} + a_y t$ to find v, which gives

$v = 0 + (3.80\text{ m/s}^2)(2.00\text{ s})$ = 7.60 m/s. Find y using $y = y_0 + v_{0y}t + \dfrac{1}{2}a_y t^2$, from which we get

$y = \dfrac{1}{2}at^2 = \dfrac{1}{2}(3.80\text{ m/s}^2)(2.00\text{ s})^2$ = 7.60 m. Now return to energy conservation, where we found that $mgy = \dfrac{1}{2}mv^2 + K_{\text{wheel}}$. Solve for K_{wheel} to get

$$K_{\text{wheel}} = m\left(gy - \frac{v^2}{2}\right) = (1.50\text{ kg})\left[(9.80\text{ m/s}^2)(7.60\text{ m}) - \frac{(7.60\text{ m/s})^2}{2}\right] = 68.4\text{ J}.$$

EVALUATE: The kinetic energy of the block is $K_{\text{block}} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(1.50\text{ kg})(7.60\text{ m/s})^2$ = 43.3 J, which is around 2/3 the kinetic energy of the wheel. So the wheel has a large effect on the motion.

9.70. **IDENTIFY:** The moment of inertia of the section that is removed must be one-half the moment of inertia of the original disk.

SET UP: For a solid disk, $I = \frac{1}{2}mR^2$. Call m the mass of the removed piece and R its radius.

$I_m = \dfrac{1}{2}I_{M_0}$.

EXECUTE: $I_m = \frac{1}{2} I_{M_0}$ gives $\frac{1}{2}mR^2 = \frac{1}{2}(\frac{1}{2}M_0R_0^2)$. We need to find m. Since the disk is uniform, the mass of a given segment will be proportional to the area of that segment. In this case, the segment is the piece cut out of the center. So $\dfrac{m}{M_0} = \dfrac{A_R}{A_{R_0}} = \dfrac{\pi R^2}{\pi R_0^2} = \dfrac{R^2}{R_0^2}$, which gives $m = M_0\left(\dfrac{R^2}{R_0^2}\right)$. Combining the two results gives $\frac{1}{2}M_0\left(\dfrac{R^2}{R_0^2}\right)R^2 = \frac{1}{2}(\frac{1}{2}M_0R_0^2)$, from which we get $R = \dfrac{R_0}{2^{1/4}} = 0.841R_0$.

EVALUATE: Notice that the piece that is removed does not have one-half the mass of the original disk, nor is its radius one-half the original radius.

9.71. **IDENTIFY:** The falling wood accelerates downward as the wheel undergoes angular acceleration. Newton's second law applies to the wood and the wheel, and the linear kinematics formulas apply to the wood because it has constant acceleration.

SET UP: $\Sigma\vec{F} = m\vec{a}$, $\tau = I\alpha$, $a_{\text{tan}} = R\alpha$, $y - y_0 = v_{0y}t + \dfrac{1}{2}a_yt^2$.

EXECUTE: First use $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ to find the downward acceleration of the wood. With $v_0 = 0$, we have $a_y = 2(y - y_0)/t^2 = 2(12.0\text{ m})/(4.00\text{ s})^2 = 1.50\text{ m/s}^2$. Now apply Newton's second to the wood to find the tension in the rope. $\Sigma\vec{F} = m\vec{a}$ gives $mg - T = ma$, $T = m(g - a)$, which gives
$T = (8.20\text{ kg})(9.80\text{ m/s}^2 - 1.50\text{ m/s}^2) = 68.06\text{ N}$. Now use $a_{\text{tan}} = R\alpha$ and apply Newton's second law (in its rotational form) to the wheel. $\tau = I\alpha$ gives $TR = I\alpha$, $I = TR/\alpha = TR/(a/R) = TR^2/a$

$I = (68.06\text{ N})(0.320\text{ m})^2/(1.50\text{ m/s}^2) = 4.65\ \text{kg}\cdot\text{m}^2$.

EVALUATE: The tension in the rope affects the acceleration of the wood and causes the angular acceleration of the wheel.

9.72. **IDENTIFY:** Using energy considerations, the system gains as kinetic energy the lost potential energy, mgR.

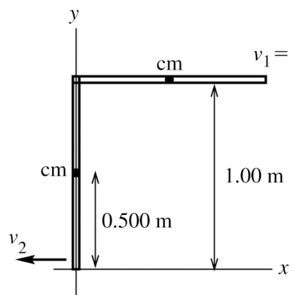
SET UP: The kinetic energy is $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$, with $I = \frac{1}{2}mR^2$ for the disk. $v = R\omega$.

EXECUTE: $K = \frac{1}{2}I\omega^2 + \frac{1}{2}m(\omega R)^2 = \frac{1}{2}(I + mR^2)\omega^2$. Using $I = \frac{1}{2}mR^2$ and solving for ω, $\omega^2 = \dfrac{4}{3}\dfrac{g}{R}$ and $\omega = \sqrt{\dfrac{4}{3}\dfrac{g}{R}}$.

EVALUATE: The small object has speed $v = \sqrt{\dfrac{2}{3}}\sqrt{2gR}$. If it was not attached to the disk and was dropped from a height h, it would attain a speed $\sqrt{2gR}$. Being attached to the disk reduces its final speed by a factor of $\sqrt{\dfrac{2}{3}}$.

9.73. **IDENTIFY:** Use conservation of energy. The stick rotates about a fixed axis so $K = \frac{1}{2}I\omega^2$. Once we have ω use $v = r\omega$ to calculate v for the end of the stick.

SET UP: The object is sketched in Figure 9.73.



Take the origin of coordinates at the lowest point reached by the stick and take the positive y-direction to be upward.

Figure 9.73

EXECUTE: **(a)** Use $U = Mgy_{\text{cm}}$. $\Delta U = U_2 - U_1 = Mg(y_{\text{cm}2} - y_{\text{cm}1})$. The center of mass of the meter stick is at its geometrical center, so $y_{\text{cm}1} = 1.00\text{ m}$ and $y_{\text{cm}2} = 0.50\text{ m}$. Then

$\Delta U = (0.180\text{ kg})(9.80\text{ m/s}^2)(0.50\text{ m} - 1.00\text{ m}) = -0.882\text{ J}$.

(b) Use conservation of energy: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Gravity is the only force that does work on the meter stick, so $W_{\text{other}} = 0$. $K_1 = 0$. Thus $K_2 = U_1 - U_2 = -\Delta U$, where ΔU was calculated in part (a). $K_2 = \frac{1}{2}I\omega_2^2$ so $\frac{1}{2}I\omega_2^2 = -\Delta U$ and $\omega_2 = \sqrt{2(-\Delta U)/I}$. For stick pivoted about one end, $I = \frac{1}{3}ML^2$ where $L = 1.00\text{ m}$, so $\omega_2 = \sqrt{\dfrac{6(-\Delta U)}{ML^2}} = \sqrt{\dfrac{6(0.882\text{ J})}{(0.180\text{ kg})(1.00\text{ m})^2}} = 5.42\text{ rad/s}$.

(c) $v = r\omega = (1.00\text{ m})(5.42\text{ rad/s}) = 5.42\text{ m/s}$.

(d) For a particle in free fall, with $+y$ upward, $v_{0y} = 0$; $y - y_0 = -1.00\text{ m}$; $a_y = -9.80\text{ m/s}^2$; and $v_y = ?$ Solving the equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for v_y gives

$v_y = -\sqrt{2a_y(y - y_0)} = -\sqrt{2(-9.80\text{ m/s}^2)(-1.00\text{ m})} = -4.43\text{ m/s}$.

EVALUATE: The magnitude of the answer in part (c) is larger. $U_{1,\text{grav}}$ is the same for the stick as for a particle falling from a height of 1.00 m. For the stick $K = \frac{1}{2}I\omega_2^2 = \frac{1}{2}(\frac{1}{3}ML^2)(v/L)^2 = \frac{1}{6}Mv^2$. For the stick and for the particle, K_2 is the same but the same K gives a larger v for the end of the stick than for the particle. The reason is that all the other points along the stick are moving slower than the end opposite the axis.

9.74. **IDENTIFY:** The student accelerates downward and causes the wheel to turn. Newton's second law applies to the student and to the wheel. The acceleration is constant so the kinematics formulas apply.

SET UP: $\Sigma\tau = I\alpha$, $\Sigma\vec{F} = m\vec{a}$, $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$, $v_y = v_{0y} + a_yt$.

EXECUTE: Apply $\Sigma\tau = I\alpha$ to the wheel: $TR = I\alpha = I(\alpha/R)$, so $T = I\alpha/R^2$.

Apply $\Sigma\vec{F} = m\vec{a}$ to the student: $mg - T = ma$, so $T = m(g - a)$.

Equating these two expressions for T and solving for the acceleration gives $a = \dfrac{mg}{m + I/R^2}$. Now apply kinematics for $y - y_0$ to the student, using $v_{0y} = 0$, and solve for t.

$t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(y - y_0)(m + I/R^2)}{mg}}$. Putting in $y - y_0 = 12.0$ m, $m = 43.0$ kg, $I = 9.60\text{ kg}\cdot\text{m}^2$, and $R = 0.300$ m, we get $t = 2.92$ s.

Now use $v_y = v_{0y} + a_y t$ to get v_y, where $a = \dfrac{mg}{m + I/R^2}$. Putting in the numbers listed above, the result is $v_y = 8.22$ m/s.

EVALUATE: If the wheel were massless, her speed would simply be $v = \sqrt{2gy} = 15.3$ m/s, so the effect of the massive wheel reduces her speed by nearly half.

9.75. IDENTIFY: Mechanical energy is conserved since there is no friction.

SET UP: $K_1 + U_1 = K_2 + U_2$, $K = \frac{1}{2}I\omega^2$ (for rotational motion), $K = \frac{1}{2}mv^2$ (for linear motion), $I = \dfrac{1}{12}ML^2$ for a slender rod.

EXECUTE: Take the initial position with the rod horizontal, and the final position with the rod vertical. The heavier sphere will be at the bottom and the lighter one at the top. Call the gravitational potential energy zero with the rod horizontal, which makes the initial potential energy zero. The initial kinetic energy is also zero. Applying $K_1 + U_1 = K_2 + U_2$ and calling A and B the spheres gives $0 = K_A + K_B + K_{rod} + U_A + U_B + U_{rod}$. $U_{rod} = 0$ in the final position since its center of mass has not moved. Therefore $0 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}I\omega^2 + m_A g\dfrac{L}{2} - m_B g\dfrac{L}{2}$. We also know that $v_A = v_B = (L/2)\,\omega$. Calling v the speed of the spheres, we get

$0 = \frac{1}{2}m_A v^2 + \frac{1}{2}m_B v^2 + \frac{1}{2}(\frac{1}{12})(ML^2)(2v/L)^2 + m_A g\frac{L}{2} - m_B g\frac{L}{2}$ Putting in $m_A = 0.0200$ kg, $m_B = 0.0500$ kg, $M = 0.120$ kg, and $L = 800$ m, we get $v = 1.46$ m/s.

EVALUATE: As the rod turns, the heavier sphere loses potential energy but the lighter one gains potential energy.

9.76. IDENTIFY: Apply conservation of energy to the system of cylinder and rope.

SET UP: Taking the zero of gravitational potential energy to be at the axle, the initial potential energy is zero (the rope is wrapped in a circle with center on the axle).When the rope has unwound, its center of mass is a distance πR below the axle, since the length of the rope is $2\pi R$ and half this distance is the position of the center of the mass. Initially, every part of the rope is moving with speed $\omega_0 R$, and when the rope has unwound, and the cylinder has angular speed ω, the speed of the rope is ωR (the upper end of the rope has the same tangential speed at the edge of the cylinder). $I = (1/2)MR^2$ for a uniform cylinder.

EXECUTE: $K_1 = K_2 + U_2$. $\left(\dfrac{M}{4} + \dfrac{m}{2}\right)R^2\omega_0^2 = \left(\dfrac{M}{4} + \dfrac{m}{2}\right)R^2\omega^2 - mg\pi R$. Solving for ω gives

$\omega = \sqrt{\omega_0^2 + \dfrac{(4\pi mg/R)}{(M + 2m)}}$, and the speed of any part of the rope is $v = \omega R$.

EVALUATE: When $m \to 0$, $\omega \to \omega_0$, When $m >> M$, $\omega = \sqrt{\omega_0^2 + \dfrac{2\pi g}{R}}$ and $v = \sqrt{v_0^2 + 2\pi gR}$. This is the final speed when an object with initial speed v_0 descends a distance πR.

9.77. IDENTIFY: Apply conservation of energy to the system consisting of blocks A and B and the pulley.

SET UP: The system at points 1 and 2 of its motion is sketched in Figure 9.77.

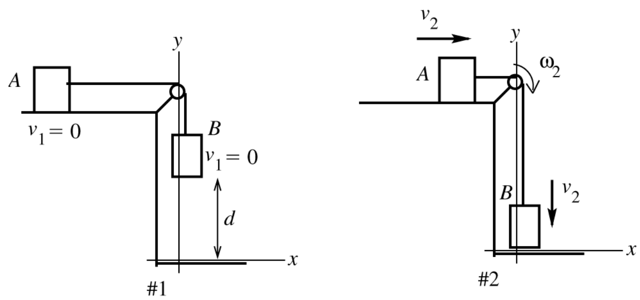


Figure 9.77

Use the work-energy relation $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Use coordinates where $+y$ is upward and where the origin is at the position of block *B* after it has descended. The tension in the rope does positive work on block *A* and negative work of the same magnitude on block *B*, so the net work done by the tension in the rope is zero. Both blocks have the same speed.

EXECUTE: Gravity does work on block *B* and kinetic friction does work on block *A*. Therefore $W_{\text{other}} = W_f = -\mu_k m_A g d.$

$K_1 = 0$ (system is released from rest)

$U_1 = m_B g y_{B1} = m_B g d;\; U_2 = m_B g y_{B2} = 0$

$K_2 = \frac{1}{2} m_A v_2^2 + \frac{1}{2} m_B v_2^2 + \frac{1}{2} I \omega_2^2.$

But $v(\text{blocks}) = R\omega(\text{pulley})$, so $\omega_2 = v_2/R$ and

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 + \frac{1}{2} I (v_2/R)^2 = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

Putting all this into the work-energy relation gives

$m_B g d - \mu_k m_A g d = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

$(m_A + m_B + I/R^2)v_2^2 = 2gd(m_B - \mu_k m_A)$

$$v_2 = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$$

EVALUATE: If $m_B >> m_A$ and s I/R^2, then $v_2 = \sqrt{2gd}$; block *B* falls freely. If *I* is very large, v_2 is very small. Must have $m_B > \mu_k m_A$ for motion, so the weight of *B* will be larger than the friction force on *A*. I/R^2 has units of mass and is in a sense the "effective mass" of the pulley.

9.78. **IDENTIFY:** Apply conservation of energy to the system of two blocks and the pulley.

SET UP: Let the potential energy of each block be zero at its initial position. The kinetic energy of the system is the sum of the kinetic energies of each object. $v = R\omega$, where *v* is the common speed of the blocks and ω is the angular velocity of the pulley.

EXECUTE: The amount of gravitational potential energy which has become kinetic energy is $K = (4.00\text{ kg} - 2.00\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 98.0\text{ J}$. In terms of the common speed *v* of the blocks, the kinetic energy of the system is $K = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2} I \left(\frac{v}{R}\right)^2.$

$K = v^2 \frac{1}{2}\left(4.00\text{ kg} + 2.00\text{ kg} + \frac{(0.380\text{ kg}\cdot\text{m}^2)}{(0.160\text{ m})^2}\right) = v^2(10.422\text{ kg})$. Solving for *v* gives

$$v = \sqrt{\frac{98.0\text{ J}}{10.422\text{ kg}}} = 3.07\text{ m/s}.$$

EVALUATE: If the pulley is massless, $98.0\text{ J} = \frac{1}{2}(4.00\text{ kg} + 2.00\text{ kg})v^2$ and $v = 5.72\text{ m/s}$. The moment of inertia of the pulley reduces the final speed of the blocks.

9.79. **IDENTIFY:** $I = I_1 + I_2$. Apply conservation of energy to the system. The calculation is similar to Example 9.8.

SET UP: $\omega = \dfrac{v}{R_1}$ for part (b) and $\omega = \dfrac{v}{R_2}$ for part (c).

EXECUTE: **(a)** $I = \frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2 = \frac{1}{2}((0.80\text{ kg})(2.50\times10^{-2}\text{ m})^2 + (1.60\text{ kg})(5.00\times10^{-2}\text{ m})^2)$

$I = 2.25\times10^{-3}\text{ kg}\cdot\text{m}^2$.

(b) The method of Example 9.8 yields $v = \sqrt{\dfrac{2gh}{1+(I/mR_1^2)}}$.

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(2.00\text{ m})}{(1+((2.25\times10^{-3}\text{ kg}\cdot\text{m}^2)/(1.50\text{ kg})(0.025\text{ m})^2))}} = 3.40\text{ m/s}.$$

(c) The same calculation, with R_2 instead of R_1 gives $v = 4.95\text{ m/s}$.

EVALUATE: The final speed of the block is greater when the string is wrapped around the larger disk. $v = R\omega$, so when $R = R_2$ the factor that relates v to ω is larger. For $R = R_2$ a larger fraction of the total kinetic energy resides with the block. The total kinetic energy is the same in both cases (equal to mgh), so when $R = R_2$ the kinetic energy and speed of the block are greater.

9.80. **IDENTIFY:** The potential energy of the falling block is transformed into kinetic energy of the block and kinetic energy of the turning wheel, but some of it is lost to the work by friction. Energy conservation applies, with the target variable being the angular velocity of the wheel when the block has fallen a given distance.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $K = \frac{1}{2}mv^2$, $U = mgh$, and W_{other} is the work done by friction.

EXECUTE: Energy conservation gives $mgh + (-9.00\text{ J}) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $v = R\omega$, so $\frac{1}{2}mv^2 = \frac{1}{2}mR^2\omega^2$ and $mgh + (-9.00\text{ J}) = \frac{1}{2}(mR^2 + I)\omega^2$. Solving for ω gives

$$\omega = \sqrt{\frac{2[mgh + (-9.00\text{ J})]}{mR^2 + I}} = \sqrt{\frac{2[(0.340\text{ kg})(9.8\text{ m/s}^2)(3.00\text{ m}) - 9.00\text{ J}]}{(0.340\text{ kg})(0.180\text{ m})^2 + 0.480\text{ kg}\cdot\text{m}^2}} = 2.01\text{ rad/s}.$$

EVALUATE: Friction does negative work because it opposes the turning of the wheel.

9.81. **IDENTIFY:** Apply conservation of energy to relate the height of the mass to the kinetic energy of the cylinder.

SET UP: First use $K(\text{cylinder}) = 480\text{ J}$ to find ω for the cylinder and v for the mass.

EXECUTE: $I = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(0.150\text{ m})^2 = 0.1125\text{ kg}\cdot\text{m}^2$. $K = \frac{1}{2}I\omega^2$ so $\omega = \sqrt{2K/I} = 92.38\text{ rad/s}$. $v = R\omega = 13.86\text{ m/s}$.

SET UP: Use conservation of energy $K_1 + U_1 = K_2 + U_2$ to solve for the distance the mass descends. Take $y = 0$ at lowest point of the mass, so $y_2 = 0$ and $y_1 = h$, the distance the mass descends.

EXECUTE: $K_1 = U_2 = 0$ so $U_1 = K_2$. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where $m = 12.0\text{ kg}$. For the cylinder, $I = \frac{1}{2}MR^2$ and $\omega = v/R$, so $\frac{1}{2}I\omega^2 = \frac{1}{4}Mv^2$. Solving $mgh = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2$ for h gives

$$h = \frac{v^2}{2g}\left(1 + \frac{M}{2m}\right) = 13.9\text{ m}.$$

EVALUATE: For the cylinder $K_{cyl} = \frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}MR^2)(v/R)^2 = \frac{1}{4}Mv^2$. $K_{mass} = \frac{1}{2}mv^2$, so $K_{mass} = (2m/M)K_{cyl} = [2(12.0 \text{ kg})/10.0 \text{ kg}](480 \text{ J}) = 1150 \text{ J}$. The mass has 1150 J of kinetic energy when the cylinder has 480 J of kinetic energy and at this point the system has total energy 1630 J since $U_2 = 0$. Initially the total energy of the system is $U_1 = mgy_1 = mgh = 1630$ J, so the total energy is shown to be conserved.

9.82. IDENTIFY: Energy conservation: Loss of U of box equals gain in K of system. Both the cylinder and pulley have kinetic energy of the form $K = \frac{1}{2}I\omega^2$.

$$m_{box}gh = \frac{1}{2}m_{box}v_{box}^2 + \frac{1}{2}I_{pulley}\omega_{pulley}^2 + \frac{1}{2}I_{cylinder}\omega_{cylinder}^2.$$

SET UP: $\omega_{pulley} = \dfrac{v_{box}}{r_{pulley}}$ and $\omega_{cylinder} = \dfrac{v_{box}}{r_{cylinder}}$.

Let B = box, P = pulley, and C = cylinder.

EXECUTE: $m_B gh = \frac{1}{2}m_B v_B^2 + \frac{1}{2}\left(\frac{1}{2}m_P r_P^2\right)\left(\dfrac{v_B}{r_P}\right)^2 + \frac{1}{2}\left(\frac{1}{2}m_C r_C^2\right)\left(\dfrac{v_B}{r_C}\right)^2$. $m_B gh = \dfrac{1}{2}m_B v_B^2 + \dfrac{1}{4}m_P v_B^2 + \dfrac{1}{4}m_C v_B^2$

and $v_B = \sqrt{\dfrac{m_B gh}{\frac{1}{2}m_B + \frac{1}{4}m_P + \frac{1}{4}m_C}} = \sqrt{\dfrac{(3.00 \text{ kg})(9.80 \text{ m/s}^2)(2.50 \text{ m})}{1.50 \text{ kg} + \frac{1}{4}(7.00 \text{ kg})}} = 4.76 \text{ m / s}.$

EVALUATE: If the box was disconnected from the rope and dropped from rest, after falling 2.50 m its speed would be $v = \sqrt{2g(2.50 \text{ m})} = 7.00$ m/s. Since in the problem some of the energy of the system goes into kinetic energy of the cylinder and of the pulley, the final speed of the box is less than this.

9.83. IDENTIFY: We know (or can calculate) the masses and geometric measurements of the various parts of the body. We can model them as familiar objects, such as uniform spheres, rods, and cylinders, and calculate their moments of inertia and kinetic energies.

SET UP: My total mass is $m = 90$ kg. I model my head as a uniform sphere of radius 8 cm. I model my trunk and legs as a uniform solid cylinder of radius 12 cm. I model my arms as slender rods of length 60 cm. $\omega = 72$ rev/min = 7.5 rad/s. For a solid uniform sphere, $I = 2/5\ MR^2$, for a solid cylinder, $I = \frac{1}{2}MR^2$, and for a rod rotated about one end $I = 1/3\ ML^2$.

EXECUTE: **(a)** Using the formulas indicated above, we have $I_{tot} = I_{head} + I_{trunk+legs} + I_{arms}$, which gives $I_{tot} = \frac{2}{5}(0.070m)(0.080 \text{ m})^2 + \frac{1}{2}(0.80m)(0.12 \text{ m})^2 + 2\left(\frac{1}{3}\right)(0.13m)(0.60 \text{ m})^2 = 3.3 \text{ kg}\cdot\text{m}^2$ where we have used $m = 90$ kg.

(b) $K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(3.3 \text{ kg}\cdot\text{m}^2)(7.5 \text{ rad/s})^2 = 93$ J.

EVALUATE: According to these estimates about 85% of the total I is due to the outstretched arms. If the initial translational kinetic energy $\frac{1}{2}mv^2$ of the skater is converted to this rotational kinetic energy as he goes into a spin, his initial speed must be 1.4 m/s.

9.84. IDENTIFY: Apply the parallel-axis theorem to each side of the square.

SET UP: Each side has length a and mass $M/4$, and the moment of inertia of each side about an axis perpendicular to the side and through its center is $\frac{1}{12}\left(\frac{1}{4}Ma^2\right) = \frac{1}{48}Ma^2$.

EXECUTE: The moment of inertia of each side about the axis through the center of the square is, from the perpendicular axis theorem, $\dfrac{Ma^2}{48} + \dfrac{M}{4}\left(\dfrac{a}{2}\right)^2 = \dfrac{Ma^2}{12}$. The total moment of inertia is the sum of the contributions from the four sides, or $4\times\dfrac{Ma^2}{12} = \dfrac{Ma^2}{3}$.

EVALUATE: If all the mass of a side were at its center, a distance $a/2$ from the axis, we would have $I = 4\left(\frac{M}{4}\right)\left(\frac{a}{2}\right)^2 = \frac{1}{4}Ma^2$. If all the mass was divided equally among the four corners of the square, a distance $a/\sqrt{2}$ from the axis, we would have $I = 4\left(\frac{M}{4}\right)\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{1}{2}Ma^2$. The actual I is between these two values.

9.85. IDENTIFY: The density depends on the distance from the center of the sphere, so it is a function of r. We need to integrate to find the mass and the moment of inertia.

SET UP: $M = \int dm = \int \rho dV$ and $I = \int dI$.

EXECUTE: **(a)** Divide the sphere into thin spherical shells of radius r and thickness dr. The volume of each shell is $dV = 4\pi r^2 dr$. $\rho(r) = a - br$, with $a = 3.00\times10^3\ \text{kg/m}^3$ and $b = 9.00\times10^3\ \text{kg/m}^4$.

Integrating gives $M = \int dm = \int \rho dV = \int_0^R (a - br)4\pi r^2 dr = \frac{4}{3}\pi R^3\left(a - \frac{3}{4}bR\right)$.

$$M = \frac{4}{3}\pi(0.200\ \text{m})^3\left(3.00\times10^3\ \text{kg/m}^3 - \frac{3}{4}(9.00\times10^3\ \text{kg/m}^4)(0.200\ \text{m})\right) = 55.3\ \text{kg}.$$

(b) The moment of inertia of each thin spherical shell is

$$dI = \frac{2}{3}r^2 dm = \frac{2}{3}r^2\rho dV = \frac{2}{3}r^2(a - br)4\pi r^2 dr = \frac{8\pi}{3}r^4(a - br)dr.$$

$$I = \int_0^R dI = \frac{8\pi}{3}\int_0^R r^4(a - br)dr = \frac{8\pi}{15}R^5\left(a - \frac{5b}{6}R\right).$$

$$I = \frac{8\pi}{15}(0.200\ \text{m})^5\left(3.00\times10^3\ \text{kg/m}^3 - \frac{5}{6}(9.00\times10^3\ \text{kg/m}^4)(0.200\ \text{m})\right) = 0.804\ \text{kg}\cdot\text{m}^2.$$

EVALUATE: We cannot use the formulas $M = \rho V$ and $I = \frac{1}{2}MR^2$ because this sphere is not uniform throughout. Its density increases toward the surface. For a uniform sphere with density $3.00\times10^3\ \text{kg/m}^3$, the mass is $\frac{4}{3}\pi R^3\rho = 100.5\ \text{kg}$. The mass of the sphere in this problem is less than this. For a uniform sphere with mass 55.3 kg and $R = 0.200$ m, $I = \frac{2}{5}MR^2 = 0.885\ \text{kg}\cdot\text{m}^2$. The moment of inertia for the sphere in this problem is less than this, since the density decreases with distance from the center of the sphere.

9.86. IDENTIFY: Write K in terms of the period T and take derivatives of both sides of this equation to relate dK/dt to dT/dt.

SET UP: $\omega = \frac{2\pi}{T}$ and $K = \frac{1}{2}I\omega^2$. The speed of light is $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $K = \frac{2\pi^2 I}{T^2}$. $\frac{dK}{dt} = -\frac{4\pi^2 I}{T^3}\frac{dT}{dt}$. The rate of energy loss is $\frac{4\pi^2 I}{T^3}\frac{dT}{dt}$. Solving for the moment of inertia I in terms of the power P,

$$I = \frac{PT^3}{4\pi^2}\frac{1}{dT/dt} = \frac{(5\times10^{31}\ \text{W})(0.0331\ \text{s})^3}{4\pi^2}\frac{1\ \text{s}}{4.22\times10^{-13}\ \text{s}} = 1.09\times10^{38}\ \text{kg}\cdot\text{m}^2$$

(b) $R = \sqrt{\frac{5I}{2M}} = \sqrt{\frac{5(1.08\times10^{38}\ \text{kg}\cdot\text{m}^2)}{2(1.4)(1.99\times10^{30}\ \text{kg})}} = 9.9\times10^3\ \text{m}$, about 10 km.

(c) $v = \dfrac{2\pi R}{T} = \dfrac{2\pi(9.9\times10^3\,\text{m})}{(0.0331\text{ s})} = 1.9\times10^6\text{ m/s} = 6.3\times10^{-3}c.$

(d) $\rho = \dfrac{M}{V} = \dfrac{M}{(4\pi/3)R^3} = 6.9\times10^{17}\,\text{kg}/\text{m}^3$, which is much higher than the density of ordinary rock by 14 orders of magnitude, and is comparable to nuclear mass densities.

EVALUATE: I is huge because M is huge. A small rate of change in the period corresponds to a large release of energy.

9.87. **IDENTIFY:** The graph with the problem in the text shows that the angular acceleration increases linearly with time and is therefore not constant.

SET UP: $\omega_z = d\theta/dt$, $\alpha_z = d\omega_z/dt$.

EXECUTE: **(a)** Since the angular acceleration is not constant, Eq. (9.11) cannot be used, so we must use $\alpha_z = d\omega_z/dt$ and $\omega_z = d\theta/dt$ and integrate to find the angle. The graph passes through the origin and has a constant positive slope of 6/5 rad/s^3, so the equation for α_z is α_z = (1.2 rad/s^3)t. Using $\alpha_z = d\omega_z/dt$ gives $\omega_z = \omega_{0z} + \int_0^t \alpha_z dt = 0 + \int_0^t (1.2\text{ rad/s}^3)t dt = (0.60\text{ rad/s}^3)t^2$. Now we must use $\omega_z = d\theta/dt$ and integrate again to get the angle.

$\theta_2 - \theta_1 = \int_0^t \omega_z dt = \int_0^t (0.60\text{ rad/s}^3)t^2 dt = (0.20\text{ rad/s}^3)t^3$ = (0.20 rad/s^3)(5.0 s)^3 = 25 rad.

(b) The result of our first integration gives ω_z = (0.60 rad/s^3)(5.0 s)^2 = 15 rad/s.

(c) The result of our second integration gives 4π rad = (0.20 rad/s^3)t^3, so t = 3.98 s. Therefore ω_z = (0.60 rad/s^3)(3.98 s)^2 = 9.48 rad/s.

EVALUATE: When the constant-acceleration angular kinematics formulas do not apply, we must go back to basic definitions.

9.88. **IDENTIFY** and **SET UP:** The graph of a^2 versus $(\theta-\theta_0)^2$ is shown in Figure 9.88. It is a straight line with a positive slope. The angular acceleration is constant.

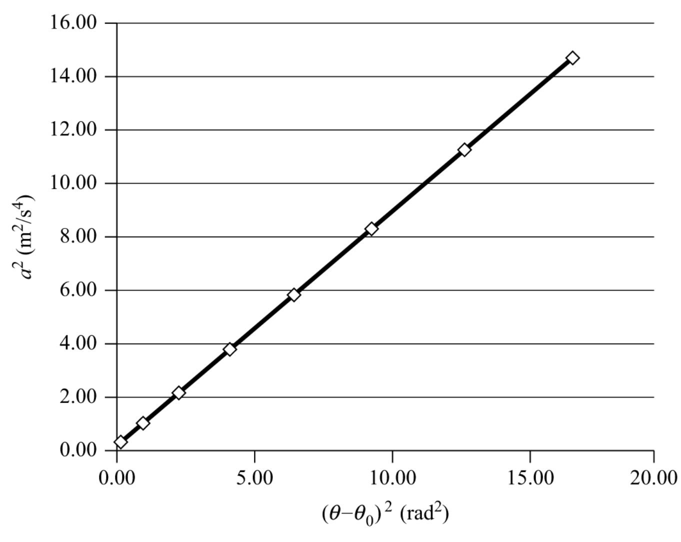


Figure 9.88

EXECUTE: **(a)** From graphing software, the slope is 0.921 m^2/s^4 and the y-intercept is 0.233 m^2/s^4.

(b) The resultant acceleration is $a^2 = a_{\tan}^2 + a_{\text{rad}}^2$. $a_{\tan} = r\alpha_z$ and $a_{\text{rad}} = r\omega_z^2$, where $\omega_z^2 = \omega_{z0}^2 + 2\alpha_z(\theta - \theta_0) = 0 + 2\alpha_z(\theta - \theta_0)$. Therefore the resultant acceleration is

$a^2 = (r\,\alpha_z)^2 + [2r\,\alpha_z(\theta - \theta_0)]^2$

$a^2 = 4r^2\alpha_z^2(\theta - \theta_0)^2 + (r\alpha_z)^2$.

From this result, we see that the slope of the graph is $4r^2\alpha_z^2$, so $4r^2\alpha_z^2 = 0.921\ \text{m}^2/\text{s}^4$. Solving for α_z gives $\alpha_z = \sqrt{\dfrac{0.921\ \text{m}^2/\text{s}^4}{4(0.800\ \text{m})^2}} = 0.600\ \text{rad/s}^2$.

(c) Using $\omega_z^2 = \omega_{z0}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega_z^2 = 0 + 2(0.600\ \text{rad/s}^2)(3\pi/4\ \text{rad})$, $\omega_z = 1.6815$ rad/s. The speed is $v = r\,\omega_z = (0.800\ \text{m})(1.6815\ \text{rad/s}) = 1.35$ m/s.

(d) Call ϕ the angle between the linear velocity and the resultant acceleration. The resultant velocity is tangent to the circle, so $\tan\phi = \dfrac{a_{\text{rad}}}{a_{\tan}} = \dfrac{r\omega_z^2}{r\alpha_z} = \dfrac{\omega_z^2}{\alpha_z}$. It is also the case that $\omega_z^2 = 2\alpha_z\Delta\theta$, so

$\tan\phi = \dfrac{2\alpha_z\Delta\theta}{\alpha_z} = 2\Delta\theta = 2(\pi/2) = \pi$. Thus $\phi = \arctan \pi = 72.3°$.

EVALUATE: According to the work in parts (a) and (b), the *y*-intercept of the graph is $(r\alpha_z)^2$ and is equal to 0.233 m^2/s^4. Solving for α_z gives $\alpha_z = \sqrt{\dfrac{0.233\ \text{m}^2/\text{s}^4}{(0.800\ \text{m})^2}} = 0.60\ \text{rad/s}^2$, as we found in part (b).

9.89. IDENTIFY and SET UP: The equation of the graph in the text is $d = (165\ \text{cm/s}^2)t^2$. For constant acceleration, the second time derivative of the position (*d* in this case) is a constant.

EXECUTE: (a) $\dfrac{d(d)}{dt} = (330\ \text{cm/s}^2)t$ and $\dfrac{d^2(d)}{dt^2} = 330\ \text{cm/s}^2$, which is a constant. Therefore the acceleration of the metal block is a constant $330\ \text{cm/s}^2 = 3.30\ \text{m/s}^2$.

(b) $v = \dfrac{d(d)}{dt} = (330\ \text{cm/s}^2)t$. When $d = 1.50$ m = 150 cm, we have 150 cm = $(165\ \text{cm/s}^2)t^2$, which gives $t = 0.9535$ s. Thus $v = 330\ \text{cm/s}^2)(0.9535\ \text{s}) = 315\ \text{cm/s} = 3.15$ m/s.

(c) Energy conservation $K_1 + U_1 = K_2 + U_2$ gives $mgd = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}mv^2$. Using $\omega = v/r$, solving for *I* and putting in the numbers $m = 5.60$ kg, $d = 1.50$ m, $r = 0.178$ m, $v = 3.15$ m/s, we get $I = 0.348$ $\text{kg}\cdot\text{m}^2$.

(d) Newton's second law gives $mg - T = ma$, $T = m(g - a) = (5.60\ \text{kg})(9.80\ \text{m/s}^2 - 3.30\ \text{m/s}^2) = 36.4$ N.

EVALUATE: When dealing with non-uniform objects, such as this flywheel, we cannot use the standard moment of inertia formulas and must resort to other ways.

9.90. IDENTIFY: Apply $I = \int r^2\, dm$.

SET UP: Let *z* be the coordinate along the vertical axis. $r(z) = \dfrac{zR}{h}$. $dm = \pi\rho\dfrac{R^2z^2}{h^2}$ and

$$dI = \frac{\pi\rho}{2}\frac{R^4}{h^4}z^4\, dz.$$

EXECUTE: $I = \int dI = \dfrac{\pi\rho}{2}\dfrac{R^4}{h^4}\int_0^h z^4\, dz = \dfrac{\pi\rho}{10}\dfrac{R^4}{h^4}\left[z^5\right]_0^h = \dfrac{1}{10}\pi\rho R^4 h$. The volume of a right circular cone is $V = \tfrac{1}{3}\pi R^2 h$, the mass is $\tfrac{1}{3}\pi\rho R^2 h$ and so $I = \dfrac{3}{10}\left(\dfrac{\pi\rho R^2 h}{3}\right)R^2 = \dfrac{3}{10}MR^2$.

EVALUATE: For a uniform cylinder of radius R and for an axis through its center, $I = \frac{1}{2}MR^2$. I for the cone is less, as expected, since the cone is constructed from a series of parallel discs whose radii decrease from R to zero along the vertical axis of the cone.

9.91. **IDENTIFY:** Follow the steps outlined in the problem.

SET UP: $\omega_z = d\theta/dt$. $\alpha_z = d^2\omega_z/dt^2$.

EXECUTE: **(a)** $ds = r\,d\theta = r_0 d\theta + \beta\theta\, d\theta$ so $s(\theta) = r_0\theta + \frac{\beta}{2}\theta^2$. θ must be in radians.

(b) Setting $s = vt = r_0\theta + \frac{\beta}{2}\theta^2$ gives a quadratic in θ. The positive solution is

$$\theta(t) = \frac{1}{\beta}\left[\sqrt{r_0{}^2 + 2\beta vt} - r_0\right].$$

(The negative solution would be going backwards, to values of r smaller than r_0.)

(c) Differentiating, $\omega_z(t) = \dfrac{d\theta}{dt} = \dfrac{v}{\sqrt{r_0^2 + 2\beta vt}}$, $\alpha_z = \dfrac{d\omega_z}{dt} = -\dfrac{\beta v^2}{(r_0^2 + 2\beta vt)^{3/2}}$. The angular acceleration α_z is not constant.

(d) $r_0 = 25.0$ mm. θ must be measured in radians, so $\beta = (1.55\,\mu\text{m/rev})(1\text{ rev}/2\pi\text{ rad}) = 0.247\,\mu\text{m/rad}$. Using $\theta(t)$ from part (b), the total angle turned in $74.0\text{ min} = 4440\text{ s}$ is

$$\theta = \frac{1}{2.47\times10^{-7}\text{ m/rad}}\left(\sqrt{2(2.47\times10^{-7}\text{ m/rad})(1.25\text{ m/s})(4440\text{ s}) + (25.0\times10^{-3}\text{ m})^2} - 25.0\times10^{-3}\text{ m}\right)$$

$\theta = 1.337\times10^5$ rad, which is 2.13×10^4 rev.

(e) The graphs are sketched in Figure 9.91.

EVALUATE: ω_z must decrease as r increases, to keep $v = r\omega$ constant. For ω_z to decrease in time, α_z must be negative.

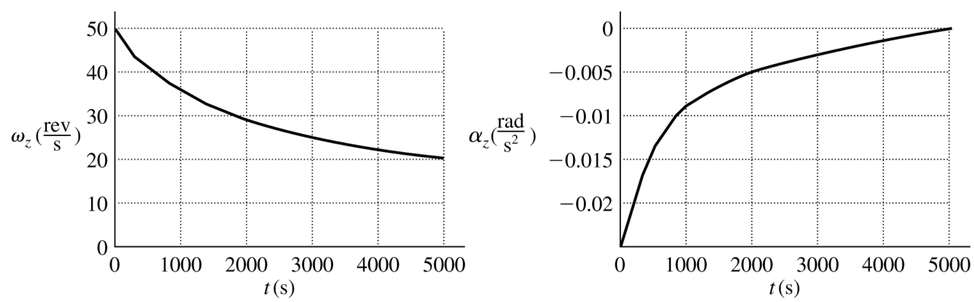


Figure 9.91

9.92. **IDENTIFY** and **SET UP:** For constant angular speed $\theta = \omega t$.

EXECUTE: **(a)** $\theta = \omega t$ = (14 rev/s)(2π rad/rev)(1/120 s) = 42°, which is choice (d).

EVALUATE: This is quite a large rotation in just one frame.

9.93. **IDENTIFY** and **SET UP:** The average angular acceleration is $\alpha_{\text{av}} = \dfrac{\omega - \omega_0}{t}$.

EXECUTE: **(a)** $\alpha_{\text{av}} = \dfrac{\omega - \omega_0}{t}$ = [8 rev/s – (–14 rev/s)]/(10 s) = (2.2 rev/s)(2π rad/rev) = 44π/10 rad/s^2 which is choice (d).

EVALUATE: This is nearly 14 rad/s^2.

9.94. **IDENTIFY and SET UP:** The rotational kinetic energy is $K = \frac{1}{2}I\omega^2$ and the kinetic energy due to running is $K = \frac{1}{2}mv^2$.

EXECUTE: Equating the two kinetic energies gives $\frac{1}{2}mv^2 = \frac{1}{2}I\omega^2$. Using $I = \frac{1}{2}mR^2$, we have $\frac{1}{2}(\frac{1}{2}mr^2)\omega^2 = \frac{1}{2}mv^2$, which gives $v = \dfrac{r\omega}{\sqrt{2}} = \dfrac{(0.05\text{ m})(14\text{ rev/s})(2\pi\text{ rad/rev})}{\sqrt{2}} = 3.11$ m/s, choice (c).

EVALUATE: This is about 3 times as fast as a human walks.

9.95. **IDENTIFY and SET UP:** $I = \frac{1}{2}mR^2$.

EXECUTE: **(a)** $I = \frac{1}{2}mR^2$, so if we double the radius but keep the mass fixed, the moment of inertia increases by a factor of 4, which is choice (d).

EVALUATE: The difference in length of the two eels plays no part in their moment of inertia if their mass is the same in both cases.

10

DYNAMICS OF ROTATIONAL MOTION

VP10.3.1. **IDENTIFY:** The force of the cable produces a torque on the cylinder, giving it an angular acceleration. We apply the rotational analog of Newton's second law.

SET UP: $\sum\tau_z = I\alpha_z$, $I = \frac{1}{2}MR^2$ for a solid cylinder, $a_{\tan} = r\alpha_z$ apply in this case.

EXECUTE: **(a)** $a_{\tan} = r\alpha_z$ gives $\alpha_z = a_{\tan}/R = (0.60\ \text{m/s}^2)/(0.060\ \text{m}) = 10\ \text{rad/s}^2$.

(b) Apply $\sum\tau_z = I\alpha_z$ to the cylinder. There is only one torque, so $\tau_z = I\alpha_z = \frac{1}{2}MR^2\alpha_z$.

$\tau_z = \frac{1}{2}(50\ \text{kg})(0.060\ \text{m})^2(10\ \text{rad/s}^2) = 0.90\ \text{N}\cdot\text{m}$.

(c) $\tau_z = FR \quad \rightarrow \quad F = \tau_z/R = (0.90\ \text{N}\cdot\text{m})/(0.060\ \text{m}) = 15\ \text{N}$.

EVALUATE: When using $\sum\tau_z = I\alpha_z$, α_z *must* be in radian measure.

VP10.3.2. **IDENTIFY:** Gravity pulls the block, causing tension in the cable. This tension produces a torque on the cylinder, giving it an angular acceleration. We apply the rotational analog of Newton's second law to the wheel and the linear form to the falling block.

SET UP: $\sum\tau_z = I\alpha_z$, $I = MR^2$ for a hollow cylinder, $a_{\tan} = r\alpha_z$, $\sum F_y = ma_y$ apply in this case. Call m the mass of the block and M the mass of the wheel.

EXECUTE: **(a)** Apply $\sum F_y = ma_y$ to the block. Since the block accelerates downward, it is best to call the y-axis positive downward. This gives $mg - T = ma_y$ (Eq. 1)

Now apply $\sum\tau_z = I\alpha_z$ to the wheel. $TR = MR^2\alpha_z$. Using $a_y = a_{\tan} = R\alpha_z$, the second equation becomes $T = MR(a_y/R) = Ma_y$ (Eq. 2)

Combining Eq. 1 and Eq. 2 gives $mg - Ma_y = ma_y$. Solving for a_y gives

$a_y = \dfrac{mg}{m+M} = \dfrac{g}{1+M/m}$.

(b) Eq. 2 gives $T = Ma_y = \dfrac{Mg}{1+M/m}$, which can also be written as $T = \dfrac{mg}{1+m/M}$.

EVALUATE: Careful! The torque on the wheel is *not* equal to mgR! It is the *tension* that turns the wheel, and $T \neq mg$. Look at our answers in some limiting cases. If $m >> M$, the ratio M/m is very small, so the acceleration approaches g. This is reasonable because the wheel has very little effect on the block, so the block is essentially in freefall. The tension approaches zero because M approaches zero. This is reasonable because the block is essentially in freefall. If $M >> m$, the acceleration approaches zero since the small m produces very little acceleration of the much heavier wheel. The tension approaches mg because $m/M << 1$, so the block is essentially just hanging with almost no acceleration.

VP10.3.3. **IDENTIFY:** Gravity pulls the block, causing tension in the cable. The torque on the cylinder is due to the tension in the cable. We apply the rotational analog of Newton's second law to the cylinder and the linear form to the block. We do not know the moment of inertia of the drum.

SET UP: $\sum\tau_z = I\alpha_z$, $a_{\tan} = r\alpha_z$, $\sum F_y = ma_y$ apply in this case. Call m the mass of the block.

EXECUTE: **(a)** Apply $\sum F_y = ma_y$ to the block. Since the block accelerates downward, it is best to call the y-axis positive downward. This gives $mg - T = ma$. Solving for T gives

$T = m(g - a)$ (Eq. 1)

(b) Applying $\sum\tau_z = I\alpha_z$ to the drum gives $TR = I\alpha_z$ (Eq. 2)

We also have $a_y = a_{\tan} = R\alpha_z$, which gives $\alpha_z = a_y/R$ (Eq. 3)

Combining Equations 1, 2, and 3 and solving for I gives $I = mR^2(g/a - 1)$.

EVALUATE: Check our result for $a \to g$, which gives $I \to 0$. This is reasonable because the block is then essentially in freefall, meaning that the drum had almost no effect on it. This would be the case of the drum's moment of inertia was very very small. You could also interpret our result for $a > g$ to understand its meaning in that case.

VP10.3.4. **IDENTIFY:** Gravity pulls the block, causing tension in the cable. The torque on the cylinder is due to the tension in the cable and the torque produced by the motor. We apply the rotational analog of Newton's second law to the cylinder and the linear form to the block.

SET UP: $\sum\tau_z = I\alpha_z$, $a_{\tan} = r\alpha_z$, $\sum F_y = ma_y$ apply in this case. Call m the mass of the block and M the mass of the cylinder. $I = \frac{1}{2}MR^2$ for a solid uniform cylinder.

EXECUTE: **(a)** Apply $\sum F_y = ma_y$ to the block. Since the block accelerates upward, it is best to call the y-axis positive upward. This gives $T - mg = ma$. Solving for T gives $T = m(g + a)$.

(b) $\tau_z = TR = mR(g + a)$.

(c) Apply $\sum\tau_z = I\alpha_z$ to the cylinder. The torque due to the motor must be opposite in direction to that of the tension and it must have a greater magnitude.

$\tau_{\text{motor}} + \tau_{\text{tension}} = I\alpha_z \quad \to \quad \tau_{\text{motor}} - TR = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$. Using the tension from part (a) and solving for τ_{motor}, we get $\tau_{\text{motor}} = mR(g + a) +$ fc MRa.

EVALUATE: Check if our result is reasonable. If either m or M are large, the torque must be large to produce the given acceleration.

VP10.7.1. **IDENTIFY:** The cylinder is rotating and translating at the same time. The rotational analog of Newton's second law applies to its rotational motion, and the linear form applies to its center of mass motion.

SET UP: $\sum F_y = ma_y$ applies to the vertical motion and $\sum\tau_z = I\alpha_z$ applies to the rotational motion about the center of mass. For a solid hollow cylinder, $I = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$ and $a = r\alpha$.

EXECUTE: **(a)** We want the acceleration, so we apply $\sum F_y = ma_y$: $Mg - T = Ma_y$ (Eq. 1)

Now apply $\sum\tau_z = I\alpha_z$. For this cylinder, $R_1 = R$ and $R_2 = R/2$, so $I = 5MR^2/4$. Using this result and $\alpha = a_y / R$, we have $TR = \frac{1}{2}\left(\frac{5R^2}{4}\right)\left(\frac{a_y}{R}\right)M$, so $T = \frac{5}{8}Ma_y$ (Eq. 2)

Combining Eq. 1 and Eq. 2 and solving for a_y gives $a_y = \frac{8}{13}g$.

(b) From Eq. 2, we have $T = \frac{5}{8}Ma_y = \frac{5}{8}M\left(\frac{8}{13}g\right) = \frac{5}{13}Mg$.

EVALUATE: Note the difference between the answer to this problem and Ex. 10.6. In both cases, the yo-yo has the same mass but a different distribution of that mass. The acceleration is less in this problem than in Example 10.6 because the moment of inertia is greater than in the example.

VP10.7.2. **IDENTIFY:** As the solid sphere rolls down a ramp its linear velocity and its angular velocity both increase. Newton's second law in both its linear and rotational forms applies.
SET UP: $\sum F_x = ma_x$ applies to the linear motion and $\sum \tau_z = I\alpha_z$ applies to the rotational motion about the center of mass. For a solid sphere $I = \frac{2}{3}MR^2$, and $a_x = R\alpha$ because the sphere does not slip. Take the x-axis along the surface of the ramp, pointing downward.
EXECUTE: Apply $\sum F_x = ma_x$. The friction force is up the ramp, so we get
$Mg \sin\beta - f = Ma_x$ (Eq. 1)
Apply $\sum \tau_z = I\alpha_z$, which gives $fR = \frac{2}{3}MR^2\ \alpha_z = \frac{2}{3}MR^2\ (a_x/R)$, which simplifies to
$f = \frac{2}{3}Ma_x$ (Eq. 2)
Combining Eq. 1 and Eq. 2 and solving for a_x gives $a_x = \frac{3}{5}g\sin\beta$.
(b) From Eq. 2 and the result from (a), we have $f = \frac{2}{3}Ma_x = \frac{2}{3}M\left(\frac{3}{5}g\sin\beta\right) = \frac{2}{5}Mg\sin\beta$.
(c) $\tau_z = fR = \frac{2}{5}MgR\sin\beta$.
EVALUATE: If there were no friction and the ball just slid down the incline, its acceleration would be $g\sin\beta$. But with rolling, friction is *up* the ramp, so it opposes the component of gravity down the ramp. Therefore the acceleration is *less* than $g\sin\beta$.

VP10.7.3. **IDENTIFY:** The yo-yo is rotating and translating at the same time. The rotational analog of Newton's second law applies to its rotational motion, and the linear form applies to its center of mass motion.
SET UP: $\sum F_y = ma_y$ applies to the vertical motion and $\sum \tau_z = I\alpha_z$ applies to the rotational motion about the center of mass. For a solid cylinder, $I = \frac{1}{2}MR^2$. We know that the tension in the string is 2/3 the weight of the yo-yo, and we want to find the acceleration of its center of mass and the angular acceleration about its center of mass. The only torque about the center of mass is due to the tension in the string.
EXECUTE: **(a)** We want the acceleration of the center of mass, so we apply $\sum F_y = ma_y$.
$Mg - T = Ma_y \rightarrow Mg - 2Mg/3 = Ma_y \rightarrow a_y = -g/3$. The magnitude is $g/3$ and the direction is downward.
(b) We want the angular acceleration about the center of mass, so we apply $\sum \tau_z = I\alpha_z$.
$TR = \frac{1}{2}MR^2\alpha_{\text{cm}} \rightarrow \alpha_{\text{cm}} = \frac{4g}{3R}$.
EVALUATE: Notice that a_{cm} is *not* equal to $R\alpha_{\text{cm}}$. The cylinder is turning and moving, but it is not rolling.

VP10.7.4. **IDENTIFY:** The cylinder is rotating and translating at the same time and is rolling without slipping. The rotational analog of Newton's second law applies to its rotational motion, and the linear form applies to its center of mass motion.
SET UP: $\sum F_x = ma_x$ applies to the center of mass motion and $\sum \tau_z = I\alpha_z$ applies to the rotational motion about the center of mass. For a solid cylinder, $I = \frac{1}{2}MR^2$, and for rolling without slipping $a_{\text{cm}} =$

$r\alpha_{cm}$. We want to find the friction force on the cylinder and the maximum angle of the ramp for which slipping will not occur. Call the *x*-axis along the surface of the ramp pointing downward.

EXECUTE: **(a)** We want the acceleration of the center of mass, so we apply $\sum F_x = ma_x$. The friction force *f* acts up the ramp to prevent sliding, so $Mg \sin\beta - f = Ma_x$ (Eq. 1)

Applying $\sum \tau_z = I\alpha_z$ about the center of mass of the cylinder gives $fR = \frac{1}{2}MR^2\alpha_{cm}$. For rolling we also have $a_{cm} = r\alpha_{cm}$, so this becomes $fR = \frac{1}{2}MR^2(a_{cm}/R)$, which gives $f = \frac{1}{2}Ma_{cm}$ (Eq. 2)

Putting Eq. 2 into Eq. 1 gives $f = \frac{Mg}{3}\sin\beta$.

(b) When the ramp is at the maximum angle β, the cylinder is just ready to slip, so f_s is at its maximum value of $\mu_s n$. Applying $\sum F_y = 0$ to the cylinder gives $n = Mg\cos\beta$. Combining this result with the answer to (a) gives $\frac{Mg}{3}\sin\beta = \mu_s Mg\cos\beta \rightarrow \tan\beta = 3\mu_s$, so

$\beta = \arctan(3\mu_s)$.

EVALUATE: Check a special case: If $\mu_s = 0$ (perfectly smooth ramp), then $\beta = 0$, which means that any elevation at all will cause slipping. This is a reasonable result for a perfectly smooth ramp.

VP10.12.1. **IDENTIFY:** The two-disk system does not experience any external torque, so its angular momentum is conserved.

SET UP: Angular momentum is $L = I\omega$, $I = \frac{1}{2}MR^2$ for a solid disk, and rotational kinetic energy is $K = \frac{1}{2}I\omega^2$.

EXECUTE: **(a)** Conservation of angular momentum tells us that $L_1 = L_2$, so $I_A\omega_A + I_B\omega_B = I_{A+B}\omega_2$. We know that $I_B = I_A/4$ and $\omega_B = \omega_A/2$, so this gives $I_A\omega_A + \frac{I_A}{4}\cdot\frac{\omega_A}{2} = \left(I_A + \frac{I_A}{4}\right)\omega_2$. Solving for ω_2 gives $\omega_2 = \frac{9}{10}\omega_A$.

(b) We want K_2/K_1. $K_1 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}\left(\frac{I_A}{4}\right)\left(\frac{\omega_A}{2}\right)^2 = \frac{17}{32}I_A\omega_A^2$.

Using the result from (a) gives $K_2 = \frac{1}{2}I_{A+B}\omega^2 = \frac{1}{2}\left(I_A + \frac{I_A}{4}\right)\left(\frac{9}{10}\omega_A\right)^2 = \frac{405}{800}I_A\omega_A^2$.

The fraction of the initial rotational kinetic energy that remains is $\frac{K_2}{K_1} = \frac{\frac{405}{800}I_A\omega_A^2}{\frac{17}{32}I_A\omega_A^2} = \frac{81}{85} = 0.953$.

EVALUATE: The disks stick together so kinetic energy is lost, just as when objects collide in an inelastic collision. In this case, 95.3% of the kinetic energy remains, so only 4.7% is lost.

VP10.12.2. **IDENTIFY:** We follow exactly the same procedure as in VP10.12.1 *except* that the initial angular velocities are in *opposite* directions.

SET UP: The set up is the same as in VP10.12.1 except that $\omega_B = -\omega_A/2$

EXECUTE: **(a)** Conservation of angular momentum tells us that $L_1 = L_2$, so $I_A\omega_A + I_B\omega_B = I_{A+B}\omega_2$. We know that $I_B = I_A/4$ and $\omega_B = -\omega_A/2$, so this gives $I_A\omega_A - \frac{I_A}{4}\cdot\frac{\omega_A}{2} = \left(I_A + \frac{I_A}{4}\right)\omega_2$. Solving for ω_2 gives $\omega_2 = \frac{7}{10}\omega_A$.

(b) We want K_2/K_1. $K_1 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}\left(\frac{I_A}{4}\right)\left(\frac{\omega_A}{2}\right)^2 = \frac{17}{32}I_A\omega_A^2$.

Using the result from (a) gives $K_2 = \frac{1}{2}I_{A+B}\omega^2 = \frac{1}{2}\left(I_A + \frac{I_A}{4}\right)\left(\frac{7}{10}\omega_A\right)^2 = \frac{49}{160}I_A\omega_A^2$.

The fraction of the initial rotational kinetic energy that remains is $\dfrac{K_2}{K_1} = \dfrac{\frac{49}{160}I_A\omega_A^2}{\frac{17}{32}I_A\omega_A^2} = \dfrac{49}{85} = 0.576.$

EVALUATE: The disks stick together so kinetic energy is lost, just as when objects collide in an inelastic collision. In this case, only 57.6% of the kinetic energy remains, so 42.4% is lost.

VP10.12.3. **IDENTIFY:** During the collision, the hinge exerts a force on the door. But if we look at the angular momentum about the hinge, the hinge exerts no torque, so the angular momentum of the bullet-door system is conserved about the hinge.

SET UP: About the hinge $L_{\text{bullet}} = mvd$, $L_{\text{door}} = I_{\text{door}}\,\omega$, and $I_{\text{door}} = \frac{1}{3}Md^2$. $K = \frac{1}{2}I\omega^2$ for rotation and $K = \frac{1}{2}mv^2$ for linear motion. Figure VP10.12.3 shows before and after sketches.

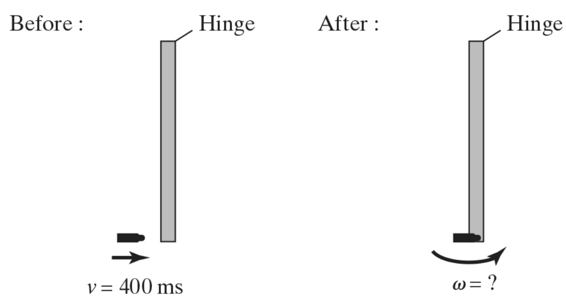


Figure VP10.12.3

EXECUTE: **(a)** We want the angular velocity ω just after the bullet hits the door, so use conservation of angular momentum about the hinge. $L_{\text{bullet}} + L_{\text{door}} = L_{\text{door + bullet}}$.

$mvd = (I_{\text{door}} + I_{\text{bullet}})\,\omega = \left(\frac{1}{3}Md^2 + md^2\right)\omega$

$$\omega = \frac{mv}{d\left(\frac{M}{3}+m\right)} = \frac{(0.0100\text{ kg})(400\text{ m/s})}{(1.00\text{ m})\left(\frac{15\text{ kg}}{3} + 0.0100\text{ kg}\right)} = 0.80\text{ rad/s.}$$

(b) $K_1 = K_{\text{bullet}} = \frac{1}{2}mv^2 = \frac{1}{2}(0.0100\text{ kg})(400\text{ m/s})^2 = 800\text{ J}$

$$K_2 = K_{\text{bullet}} + K_{\text{door}} = \frac{1}{2}\left(md^2\omega^2 + \frac{1}{3}Md^2\omega^2\right) = \left(m + \frac{M}{3}\right)\frac{(d\omega)^2}{2}$$

$$K_2 = \left(0.010\text{ kg} + \frac{15\text{ kg}}{3}\right)\frac{[(1.00\text{ m})(0.80\text{ rad/s})]^2}{2} = 1.6\text{ J}$$

$K_2/K_1 = (1.6\text{ J})/(800\text{ J}) = 0.0020 = 1/500$.

EVALUATE: This is a *very* inelastic collision. Only 0.20% of the original kinetic energy remained, so 99.8% was lost.

VP10.12.4. IDENTIFY: During the collision, the angular momentum of the clay-sphere system is conserved about the vertical shaft.

SET UP: About the shaft $L_{\text{clay}} = MvR$, $L_{\text{sphere}} = I_{\text{sphere}}\,\omega$, and $I_{\text{sphere}} = \frac{2}{3}MR^2$. $K = \frac{1}{2}I\omega^2$ for rotation and $K = \frac{1}{2}mv^2$ for linear motion.

EXECUTE: **(a)** Before: $L = MvR$. After: $L = M(v/2)R = \frac{1}{2}MvR$.

(b) The clay clay lost half of its angular momentum, so the sphere must have gained that amount by conservation of angular momentum. Therefore $L_{\text{sphere}} = \frac{1}{2}MvR = I_{\text{sphere}}\,\omega$. This gives $\frac{2}{3}MR^2\,\omega = \frac{1}{2}MvR$, so $\omega = \frac{3v}{4R}$.

(c) $K_1 = K_{\text{clay}} = \frac{1}{2}Mv^2$

$$K_2 = K_{\text{clay}} + K_{\text{sphere}} = \frac{1}{2}M(v/2)^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}M\left(\frac{v}{2}\right)^2 + \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{3v}{4R}\right)^2 = \frac{5}{16}Mv^2$$

$$\frac{K_2}{K_1} = \frac{\frac{5}{16}Mv^2}{\frac{1}{2}Mv^2} = \frac{5}{8}.$$

EVALUATE: The system lost 3/8 of its kinetic energy during this collision, so it was *not* an elastic collision.

10.1. IDENTIFY: Use $\tau = Fl$ to calculate the magnitude of the torque and use the right-hand rule illustrated in Section 10.1 in the textbook to calculate the torque direction.

(a) SET UP: Consider Figure 10.1a.

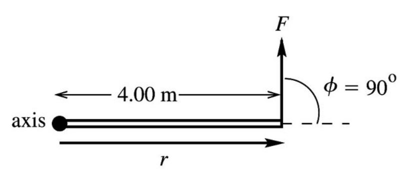


Figure 10.1a

EXECUTE: $\tau = Fl$

$l = r\sin\phi = (4.00\text{ m})\sin 90°$

$l = 4.00\text{ m}$

$\tau = (10.0\text{ N})(4.00\text{ m}) = 40.0\text{ N}\cdot\text{m}$

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(b) SET UP: Consider Figure 10.1b.

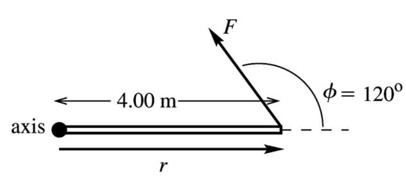


Figure 10.1b

EXECUTE: $\tau = Fl$

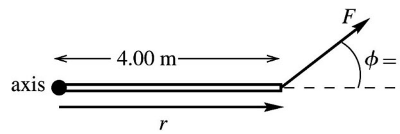
$l = r\sin\phi = (4.00\text{ m})\sin 120°$

$l = 3.464\text{ m}$

$\tau = (10.0\text{ N})(3.464\text{ m}) = 34.6\text{ N}\cdot\text{m}$

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(c) SET UP: Consider Figure 10.1c.



EXECUTE: $\tau = Fl$

$l = r\sin\phi = (4.00\text{ m})\sin 30°$

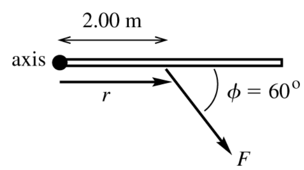
$l = 2.00\text{ m}$

$\tau = (10.0\text{ N})(2.00\text{ m}) = 20.0\text{ N}\cdot\text{m}$

Figure 10.1c

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(d) SET UP: Consider Figure 10.1d.



EXECUTE: $\tau = Fl$

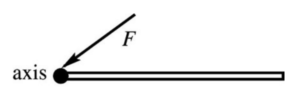
$l = r\sin\phi = (2.00\text{ m})\sin 60° = 1.732\text{ m}$

$\tau = (10.0\text{ N})(1.732\text{ m}) = 17.3\text{ N}\cdot\text{m}$

Figure 10.1d

This force tends to produce a clockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed into the plane of the figure.

(e) SET UP: Consider Figure 10.1e.

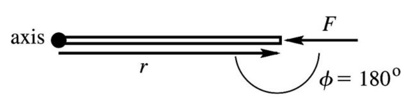


EXECUTE: $\tau = Fl$

$r = 0$ so $l = 0$ and $\tau = 0$

Figure 10.1e

(f) SET UP: Consider Figure 10.1f.



EXECUTE: $\tau = Fl$

$l = r\sin\phi$, $\phi = 180°$,

so $l = 0$ and $\tau = 0$

Figure 10.1f

EVALUATE: The torque is zero in parts (e) and (f) because the moment arm is zero; the line of action of the force passes through the axis.

10.2. **IDENTIFY:** $\tau = Fl$ with $l = r\sin\phi$. Add the two torques to calculate the net torque.

SET UP: Let counterclockwise torques be positive.

EXECUTE: $\tau_1 = -F_1 l_1 = -(8.00\text{ N})(5.00\text{ m}) = -40.0\text{ N}\cdot\text{m}$.

$\tau_2 = +F_2 l_2 = (12.0\text{ N})(2.00\text{ m})\sin 30.0° = +12.0\text{ N}\cdot\text{m}$. $\sum\tau = \tau_1 + \tau_2 = -28.0\text{ N}\cdot\text{m}$. The net torque is $28.0\text{ N}\cdot\text{m}$, clockwise.

EVALUATE: Even though $F_1 < F_2$, the magnitude of τ_1 is greater than the magnitude of τ_2, because F_1 has a larger moment arm.

10.3. **IDENTIFY** and **SET UP:** Use $\tau = Fl$ to calculate the magnitude of each torque and use the right-hand rule (Figure 10.4 in the textbook) to determine the direction. Consider Figure 10.3.

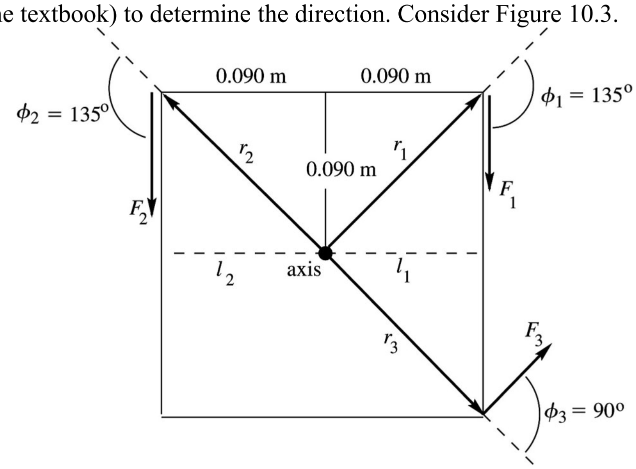


Figure 10.3

Let counterclockwise be the positive sense of rotation.

EXECUTE: $r_1 = r_2 = r_3 = \sqrt{(0.090\text{ m})^2 + (0.090\text{ m})^2} = 0.1273\text{ m}$

$\tau_1 = -F_1 l_1$

$l_1 = r_1 \sin\phi_1 = (0.1273\text{ m})\sin 135° = 0.0900\text{ m}$

$\tau_1 = -(18.0\text{ N})(0.0900\text{ m}) = -1.62\text{ N}\cdot\text{m}$

$\vec{\tau}_1$ is directed into paper

$\tau_2 = +F_2 l_2$

$l_2 = r_2 \sin\phi_2 = (0.1273\text{ m})\sin 135° = 0.0900\text{ m}$

$\tau_2 = +(26.0\text{ N})(0.0900\text{ m}) = +2.34\text{ N}\cdot\text{m}$

$\vec{\tau}_2$ is directed out of paper

$\tau_3 = +F_3 l_3$

$l_3 = r_3 \sin\phi_3 = (0.1273\text{ m})\sin 90° = 0.1273\text{ m}$

$\tau_3 = +(14.0\text{ N})(0.1273\text{ m}) = +1.78\text{ N}\cdot\text{m}$

$\vec{\tau}_3$ is directed out of paper

$\sum\tau = \tau_1 + \tau_2 + \tau_3 = -1.62\text{ N}\cdot\text{m} + 2.34\text{ N}\cdot\text{m} + 1.78\text{ N}\cdot\text{m} = 2.50\text{ N}\cdot\text{m}$

EVALUATE: The net torque is positive, which means it tends to produce a counterclockwise rotation; the vector torque is directed out of the plane of the paper. In summing the torques it is important to include + or − signs to show direction.

10.4. **IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ to calculate the magnitude of each torque and use the right-hand rule to determine the direction of each torque. Add the torques to find the net torque.

SET UP: Let counterclockwise torques be positive. For the 11.9 N force (F_1), $r = 0$. For the 14.6 N force (F_2), $r = 0.350\text{ m}$ and $\phi = 40.0°$. For the 8.50 N force (F_3), $r = 0.350\text{ m}$ and $\phi = 90.0°$.

EXECUTE: $\tau_1 = 0$. $\tau_2 = -(14.6\text{ N})(0.350\text{ m})\sin 40.0° = -3.285\text{ N}\cdot\text{m}$.

$\tau_3 = +(8.50\text{ N})(0.350\text{ m})\sin 90.0° = +2.975\text{ N}\cdot\text{m}$. $\sum\tau = -3.285\text{ N}\cdot\text{m} + 2.975\text{ N}\cdot\text{m} = -0.31\text{ N}\cdot\text{m}$. The net torque is $0.31\text{ N}\cdot\text{m}$ and is clockwise.

EVALUATE: If we treat the torques as vectors, $\vec{\tau}_2$ is into the page and $\vec{\tau}_3$ is out of the page.

10.5. **IDENTIFY** and **SET UP:** Calculate the torque using Eq. (10.3) and also determine the direction of the torque using the right-hand rule.

(a) $\vec{r} = (-0.450\text{ m})\hat{i} + (0.150\text{ m})\hat{j}$; $\vec{F} = (-5.00\text{ N})\hat{i} + (4.00\text{ N})\hat{j}$. The sketch is given in Figure 10.5.

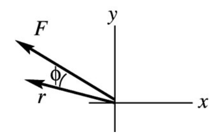


Figure 10.5

EXECUTE: **(b)** When the fingers of your right hand curl from the direction of $\vec{r}$ into the direction of $\vec{F}$ (through the smaller of the two angles, angle ϕ) your thumb points into the page (the direction of $\vec{\tau}$, the $-z$-direction).

(c) $\vec{\tau} = \vec{r} \times \vec{F} = [(-0.450\text{ m})\hat{i} + (0.150\text{ m})\hat{j}] \times [(-5.00\text{ N})\hat{i} + (4.00\text{ N})\hat{j}]$

$\vec{\tau} = +(2.25\text{ N}\cdot\text{m})\hat{i}\times\hat{i} - (1.80\text{ N}\cdot\text{m})\hat{i}\times\hat{j} - (0.750\text{ N}\cdot\text{m})\hat{j}\times\hat{i} + (0.600\text{ N}\cdot\text{m})\hat{j}\times\hat{j}$

$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = 0$

$\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{i} = -\hat{k}$

Thus $\vec{\tau} = -(1.80\text{ N}\cdot\text{m})\hat{k} - (0.750\text{ N}\cdot\text{m})(-\hat{k}) = (-1.05\text{ N}\cdot\text{m})\hat{k}$.

EVALUATE: The calculation gives that $\vec{\tau}$ is in the $-z$-direction. This agrees with what we got from the right-hand rule.

10.6. **IDENTIFY:** Knowing the force on a bar and the point where it acts, we want to find the position vector for the point where the force acts and the torque the force exerts on the bar.

SET UP: The position vector is $\vec{r} = x\hat{i} + y\hat{j}$ and the torque is $\vec{\tau} = \vec{r} \times \vec{F}$.

EXECUTE: **(a)** Using $x = 3.00\text{ m}$ and $y = 4.00\text{ m}$, we have $\vec{r} = (3.00\text{ m})\hat{i} + (4.00\text{ m})\hat{j}$.

(b) $\vec{\tau} = \vec{r} \times \vec{F} = [(3.00\text{ m})\hat{i} + (4.00\text{ m})\hat{j}] \times [(7.00\text{ N})\hat{i} + (-3.00\text{ N})\hat{j}]$.

$\vec{\tau} = (-9.00\text{ N}\cdot\text{m})\hat{k} + (-28.0\text{ N}\cdot\text{m})(-\hat{k}) = (-37.0\text{ N}\cdot\text{m})\hat{k}$. The torque has magnitude $37.0\text{ N}\cdot\text{m}$ and is in the $-z$-direction.

EVALUATE: Applying the right-hand rule for the vector product to $\vec{r} \times \vec{F}$ shows that the torque must be in the $-z$-direction because it is perpendicular to both $\vec{r}$ and $\vec{F}$, which are both in the x-y plane.

10.7. **IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ for the magnitude of the torque and the right-hand rule for the direction.

SET UP: In part (a), $r = 0.250\text{ m}$ and $\phi = 37°$.

EXECUTE: **(a)** $\tau = (17.0\text{ N})(0.250\text{ m})\sin 37° = 2.56\text{ N}\cdot\text{m}$. The torque is counterclockwise.

(b) The torque is maximum when $\phi = 90°$ and the force is perpendicular to the wrench. This maximum torque is $(17.0\text{ N})(0.250\text{ m}) = 4.25\text{ N}\cdot\text{m}$.

EVALUATE: If the force is directed along the handle then the torque is zero. The torque increases as the angle between the force and the handle increases.

10.8. **IDENTIFY:** The constant force produces a torque which gives a constant angular acceleration to the disk and a linear acceleration to points on the disk.

SET UP: $\sum\tau_z = I\alpha_z$ applies to the disk, $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ because the angular acceleration is constant. The acceleration components of the rim are $a_{\text{tan}} = r\alpha$ and $a_{\text{rad}} = r\omega^2$, and the magnitude of the acceleration is $a = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2}$.

EXECUTE: **(a)** $\sum \tau_z = I\alpha_z$ gives $Fr = I\alpha_z$. For a uniform disk,

$I = \frac{1}{2}MR^2 = \frac{1}{2}(40.0\ \text{kg})(0.200\ \text{m})^2 = 0.800\ \text{kg}\cdot\text{m}^2.$ $\alpha_z = \dfrac{Fr}{I} = \dfrac{(30.0\ \text{N})(0.200\ \text{m})}{0.800\ \text{kg}\cdot\text{m}^2} = 7.50\ \text{rad/s}^2.$

$\theta - \theta_0 = 0.200\ \text{rev} = 1.257\ \text{rad}.$ $\omega_{0z} = 0,$ so $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives

$\omega_z = \sqrt{2(7.50\ \text{rad/s}^2)(1.257\ \text{rad})} = 4.342\ \text{rad/s}.$ $v = r\omega = (0.200\ \text{m})(4.342\ \text{rad/s}) = 0.868\ \text{m/s}.$

(b) $a_{\text{tan}} = r\alpha = (0.200\ \text{m})(7.50\ \text{rad/s}^2) = 1.50\ \text{m/s}^2.$ $a_{\text{rad}} = r\omega^2 = (0.200\ \text{m})(4.342\ \text{rad/s})^2 = 3.771\ \text{m/s}^2.$

$a = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2} = 4.06\ \text{m/s}^2.$

EVALUATE: The net acceleration is neither toward the center nor tangent to the disk.

10.9. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$.

SET UP: $\omega_{0z} = 0.$ $\omega_z = (400\ \text{rev/min})\left(\dfrac{2\pi\ \text{rad/rev}}{60\ \text{s/min}}\right) = 41.9\ \text{rad/s}$

EXECUTE: $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t} = (1.60\ \text{kg}\cdot\text{m}^2)\dfrac{41.9\ \text{rad/s}}{8.00\ \text{s}} = 8.38\ \text{N}\cdot\text{m}.$

EVALUATE: In $\tau_z = I\alpha_z$, α_z must be in rad/s^2.

10.10. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the wheel. The acceleration a of a point on the cord and the angular acceleration α of the wheel are related by $a = R\alpha$.

SET UP: Let the direction of rotation of the wheel be positive. The wheel has the shape of a disk and $I = \frac{1}{2}MR^2$. The free-body diagram for the wheel is sketched in Figure 10.10a for a horizontal pull and in Figure 10.10b for a vertical pull. P is the pull on the cord and F is the force exerted on the wheel by the axle.

EXECUTE: **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{(40.0\ \text{N})(0.250\ \text{m})}{\frac{1}{2}(9.20\ \text{kg})(0.250\ \text{m})^2} = 34.8\ \text{rad/s}^2.$

$a = R\alpha = (0.250\ \text{m})(34.8\ \text{rad/s}^2) = 8.70\ \text{m/s}^2.$

(b) $F_x = -P,$ $F_y = Mg.$ $F = \sqrt{P^2 + (Mg)^2} = \sqrt{(40.0\ \text{N})^2 + ([9.20\ \text{kg}][9.80\ \text{m/s}^2])^2} = 98.6\ \text{N}.$

$\tan\phi = \dfrac{|F_y|}{|F_x|} = \dfrac{Mg}{P} = \dfrac{(9.20\ \text{kg})(9.80\ \text{m/s}^2)}{40.0\ \text{N}}$ and $\phi = 66.1°.$ The force exerted by the axle has magnitude 98.6 N and is directed at $66.1°$ above the horizontal, away from the direction of the pull on the cord.

(c) The pull exerts the same torque as in part (a), so the answers to part (a) don't change. In part (b), $F + P = Mg$ and $F = Mg - P = (9.20\ \text{kg})(9.80\ \text{m/s}^2) - 40.0\ \text{N} = 50.2\ \text{N}.$ The force exerted by the axle has magnitude 50.2 N and is upward.

EVALUATE: The weight of the wheel and the force exerted by the axle produce no torque because they act at the axle.

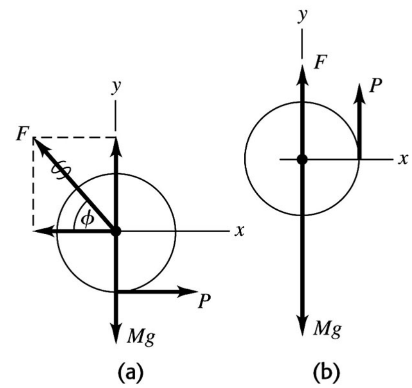


Figure 10.10

10.11. **IDENTIFY:** Use $\sum \tau_z = I\alpha_z$ to calculate α. Use a constant angular acceleration kinematic equation to relate α_z, ω_z, and t.

SET UP: For a solid uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$. Let the direction the sphere is spinning be the positive sense of rotation. The moment arm for the friction force is $l = 0.0150\text{ m}$ and the torque due to this force is negative.

EXECUTE: **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{-(0.0200\text{ N})(0.0150\text{ m})}{\frac{2}{5}(0.225\text{ kg})(0.0150\text{ m})^2} = -14.8\text{ rad/s}^2$

(b) $\omega_z - \omega_{0z} = -22.5\text{ rad/s}$. $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{-22.5\text{ rad/s}}{-14.8\text{ rad/s}^2} = 1.52\text{ s}$.

EVALUATE: The fact that α_z is negative means its direction is opposite to the direction of spin. The negative α_z causes ω_z to decrease.

10.12. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the stone and $\sum \tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find a for the stone.

SET UP: For the motion of the stone take $+y$ to be downward. The pulley has $I = \frac{1}{2}MR^2$. $a = R\alpha$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $12.6\text{ m} = \frac{1}{2}a_y(3.00\text{ s})^2$ and $a_y = 2.80\text{ m/s}^2$.

Then $\sum F_y = ma_y$ applied to the stone gives $mg - T = ma$.

$\sum \tau_z = I\alpha_z$ applied to the pulley gives $TR = \frac{1}{2}MR^2\alpha = \frac{1}{2}MR^2(a/R)$. $T = \frac{1}{2}Ma$.

Combining these two equations to eliminate T gives

$$m = \frac{M}{2}\left(\frac{a}{g-a}\right) = \left(\frac{10.0\text{ kg}}{2}\right)\left(\frac{2.80\text{ m/s}^2}{9.80\text{ m/s}^2 - 2.80\text{ m/s}^2}\right) = 2.00\text{ kg}.$$

(b) $T = \frac{1}{2}Ma = \frac{1}{2}(10.0\text{ kg})(2.80\text{ m/s}^2) = 14.0\text{ N}$

EVALUATE: The tension in the wire is less than the weight $mg = 19.6\text{ N}$ of the stone, because the stone has a downward acceleration.

10.13. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each book and apply $\sum \tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find the common acceleration of the books.

SET UP: $m_1 = 2.00\text{ kg}$, $m_2 = 3.00\text{ kg}$. Let T_1 be the tension in the part of the cord attached to m_1 and T_2 be the tension in the part of the cord attached to m_2. Let the $+x$-direction be in the direction of the acceleration of each book. $a = R\alpha$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(1.20 \text{ m})}{(0.800 \text{ s})^2} = 3.75 \text{ m/s}^2$. $a_1 = 3.75 \text{ m/s}^2$ so $T_1 = m_1 a_1 = 7.50 \text{ N}$ and $T_2 = m_2(g - a_1) = 18.2 \text{ N}$.

(b) The torque on the pulley is $(T_2 - T_1)R = 0.803 \text{ N} \cdot \text{m}$, and the angular acceleration is $\alpha = a_1/R = 50 \text{ rad/s}^2$, so $I = \tau/\alpha = 0.016 \text{ kg} \cdot \text{m}^2$.

EVALUATE: The tensions in the two parts of the cord must be different, so there will be a net torque on the pulley.

10.14. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the bucket, with $+y$ downward. Apply $\sum \tau_z = I\alpha_z$ to the cylinder, with the direction the cylinder rotates positive.

SET UP: The free-body diagram for the bucket is given in Figure 10.14a and the free-body diagram for the cylinder is given in Figure 10.14b. $I = \frac{1}{2}MR^2$. $a(\text{bucket}) = R\alpha(\text{cylinder})$

EXECUTE: **(a)** For the bucket, $mg - T = ma$. For the cylinder, $\sum \tau_z = I\alpha_z$ gives $TR = \frac{1}{2}MR^2\alpha$. $\alpha = a/R$ then gives $T = \frac{1}{2}Ma$. Combining these two equations gives $mg - \frac{1}{2}Ma = ma$ and

$$a = \frac{mg}{m + M/2} = \left(\frac{15.0 \text{ kg}}{15.0 \text{ kg} + 6.0 \text{ kg}}\right)(9.80 \text{ m/s}^2) = 7.00 \text{ m/s}^2.$$

$T = m(g - a) = (15.0 \text{ kg})(9.80 \text{ m/s}^2 - 7.00 \text{ m/s}^2) = 42.0 \text{ N}$.

(b) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(7.00 \text{ m/s}^2)(10.0 \text{ m})} = 11.8 \text{ m/s}$.

(c) $a_y = 7.00 \text{ m/s}^2$, $v_{0y} = 0$, $y - y_0 = 10.0 \text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}\alpha_y t^2$ gives

$$t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(10.0 \text{ m})}{7.00 \text{ m/s}^2}} = 1.69 \text{ s}$$

(d) $\sum F_y = ma_y$ applied to the cylinder gives $n - T - Mg = 0$ and

$n = T + mg = 42.0 \text{ N} + (12.0 \text{ kg})(9.80 \text{ m/s}^2) = 160 \text{ N}$.

EVALUATE: The tension in the rope is less than the weight of the bucket, because the bucket has a downward acceleration. If the rope were cut, so the bucket would be in free fall, the bucket would strike the water in $t = \sqrt{\dfrac{2(10.0 \text{ m})}{9.80 \text{ m/s}^2}} = 1.43 \text{ s}$ and would have a final speed of 14.0 m/s. The presence of the cylinder slows the fall of the bucket.

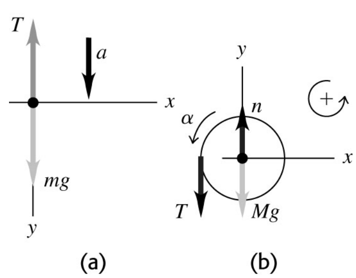


Figure 10.14

10.15. **IDENTIFY:** The constant force produces a torque which gives a constant angular acceleration to the wheel.

SET UP: $\omega_z = \omega_{0z} + \alpha_z t$ because the angular acceleration is constant, and $\sum \tau_z = I\alpha_z$ applies to the wheel.

Execute: $\omega_{0z} = 0$ and $\omega_z = 12.0 \text{ rev/s} = 75.40 \text{ rad/s}$. $\omega_z = \omega_{0z} + \alpha_z t$, so

$\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{75.40 \text{ rad/s}}{2.00 \text{ s}} = 37.70 \text{ rad/s}^2$. $\sum \tau_z = I\alpha_z$ gives

$I = \dfrac{Fr}{\alpha_z} = \dfrac{(80.0 \text{ N})(0.120 \text{ m})}{37.70 \text{ rad/s}^2} = 0.255 \text{ kg} \cdot \text{m}^2$.

Evaluate: The units of the answer are the proper ones for moment of inertia.

10.16. Identify: Apply $\sum \vec{F} = m\vec{a}$ to each box and $\sum \tau_z = I\alpha_z$ to the pulley. The magnitude a of the acceleration of each box is related to the magnitude of the angular acceleration α of the pulley by $a = R\alpha$.

Set Up: The free-body diagrams for each object are shown in Figure 10.16. For the pulley, $R = 0.250$ m and $I = \frac{1}{2}MR^2$. T_1 and T_2 are the tensions in the wire on either side of the pulley. $m_1 = 12.0$ kg, $m_2 = 5.00$ kg and $M = 2.00$ kg. $\vec{F}$ is the force that the axle exerts on the pulley. For the pulley, let clockwise rotation be positive.

Execute: (a) $\sum F_x = ma_x$ for the 12.0 kg box gives $T_1 = m_1 a$. $\sum F_y = ma_y$ for the 5.00 kg weight gives $m_2 g - T_2 = m_2 a$. $\sum \tau_z = I\alpha_z$ for the pulley gives $(T_2 - T_1)R = (\frac{1}{2}MR^2)\alpha$. $a = R\alpha$ and $T_2 - T_1 = \frac{1}{2}Ma$. Adding these three equations gives $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ and

$a = \left(\dfrac{m_2}{m_1 + m_2 + \frac{1}{2}M}\right) g = \left(\dfrac{5.00 \text{ kg}}{12.0 \text{ kg} + 5.00 \text{ kg} + 1.00 \text{ kg}}\right)(9.80 \text{ m/s}^2) = 2.72 \text{ m/s}^2$. Then

$T_1 = m_1 a = (12.0 \text{ kg})(2.72 \text{ m/s}^2) = 32.6$ N. $m_2 g - T_2 = m_2 a$ gives

$T_2 = m_2(g - a) = (5.00 \text{ kg})(9.80 \text{ m/s}^2 - 2.72 \text{ m/s}^2) = 35.4$ N. The tension to the left of the pulley is 32.6 N and below the pulley it is 35.4 N.

(b) $a = 2.72 \text{ m/s}^2$

(c) For the pulley, $\sum F_x = ma_x$ gives $F_x = T_1 = 32.6$ N and $\sum F_y = ma_y$ gives

$F_y = Mg + T_2 = (2.00 \text{ kg})(9.80 \text{ m/s}^2) + 35.4 \text{ N} = 55.0$ N.

Evaluate: The equation $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ says that the external force $m_2 g$ must accelerate all three objects.

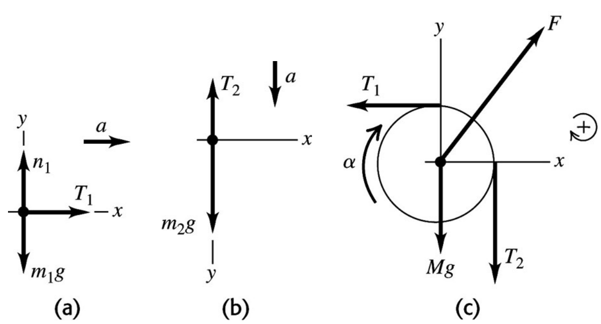


Figure 10.16

10.17. Identify: The rotational form of Newton's second law applies to the cylinder. Interpretation of graphical data is necessary.

Set Up: Since $\theta - \theta_0$ is proportional to t^2, the equation $\theta - \theta_0 = \omega_{0z} t + \dfrac{1}{2}\alpha_z t^2$ applies to the rotational motion. Since the rotation starts from rest, $\omega_{0z} = 0$, so we have $\theta - \theta_0 = \dfrac{1}{2}\alpha_z t^2$. Therefore a graph of

$\theta-\theta_0$ versus t^2 should be a straight line having slope $\frac{1}{2}\alpha_z$. Our target variable is the moment of inertia of the cylinder. Once we know α_z, we can apply $\sum\tau_z = I\alpha_z$ to find I.

EXECUTE: Use the slope of the graph to find α_z. As we discussed above, slope = $\frac{1}{2}\alpha_z$, so α_z = 2(slope) = 2(16.0 rad/s^2) = 32.0 rad/s^2. Now use $\sum\tau_z = I\alpha_z$ to find I. $FR = I\alpha_z$, so

$I = \dfrac{FR}{\alpha_z} = \dfrac{(3.00\text{ N})(0.140\text{ m})}{32.0\text{ rad/s}^2} = 0.0131\ \text{kg}\cdot\text{m}^2$.

EVALUATE: Using the slope to find α_z is more accurate than using individual data points because individual measurements vary, but finding the slope essentially "averages out" the data points.

10.18. IDENTIFY: The spheres have kinetic energy due to the motion of their center of mass and the rotation about the center of mass.

SET UP: $K_{\text{total}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2$, $I = \frac{2}{5}MR^2$ for a solid sphere, and $v_{\text{cm}} = r\omega$ when there is no slipping. The work done on each sphere is equal to its loss of kinetic energy. The work is the target variable.

EXECUTE: $W_A = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2$.

$W_A = \frac{7}{10}mv_{\text{cm}}^2 = \frac{7}{10}(5.00\text{ kg})(4.00\text{ m/s})^2 = 56.0$ J. A similar calculation for sphere B, except using $I = \frac{2}{3}MR^2$, gives $W_B = \frac{5}{6}mv_{\text{cm}}^2 = \frac{5}{6}(5.00\text{ kg})(4.00\text{ m/s})^2 = 66.7$ J.

EVALUATE: Sphere B requires more work because it has a larger moment of inertia than A. Even though both spheres have the same size, mass, and linear speed, B's mass is farther from the rotation axis so its moment of inertia is greater than that of A.

10.19. IDENTIFY: Since there is rolling without slipping, $v_{\text{cm}} = R\omega$. The kinetic energy is given by $K_{\text{tot}} = K_{\text{cm}} + K_{\text{rot}}$ where $K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}^2$ and $K_{\text{rot}} = \frac{1}{2}I_{\text{cm}}\omega^2$. The velocity of any point on the rim of the hoop is the vector sum of the tangential velocity of the rim and the velocity of the center of mass of the hoop.

SET UP: $\omega = 2.60$ rad/s and $R = 0.600$ m. For a hoop rotating about an axis at its center, $I = MR^2$.

EXECUTE: **(a)** $v_{\text{cm}} = R\omega = (0.600\text{ m})(2.60\text{ rad/s}) = 1.56$ m/s.

(b) $K = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}(MR^2)(v_{\text{cm}}/R^2) = Mv_{\text{cm}}^2 = (2.20\text{ kg})(1.56\text{ m/s})^2 = 5.35$ J

(c) (i) $v = 2v_{\text{cm}} = 3.12$ m/s. $\vec{v}$ is to the right. (ii) $v = 0$

(iii) $v = \sqrt{v_{\text{cm}}^2 + v_{\tan}^2} = \sqrt{v_{\text{cm}}^2 + (R\omega)^2} = \sqrt{2}v_{\text{cm}} = 2.21$ m/s. $\vec{v}$ at this point is at $45°$ below the horizontal.

(d) To someone moving to the right at $v = v_{\text{cm}}$, the hoop appears to rotate about a stationary axis at its center. (i) $v = R\omega = 1.56$ m/s, to the right. (ii) $v = 1.56$ m/s, to the left. (iii) $v = 1.56$ m/s, downward.

EVALUATE: For the special case of a hoop, the total kinetic energy is equally divided between the motion of the center of mass and the rotation about the axis through the center of mass. In the rest frame of the ground, different points on the hoop have different speed.

10.20. IDENTIFY: Newton's second law applies to the sphere rolling down the incline.

SET UP: From Example 10.7, we have $f_s = \frac{2}{7}Mg\sin\beta$. For maximum static friction, $f_s = \mu_s n$. The target variable is μ_s.

EXECUTE: (a) Balancing forces perpendicular to the surface of the incline gives $n = Mg\cos\beta$. Using this in the result from Example 10.7 gives $\mu_s Mg\cos\beta = \frac{2}{7}Mg\sin\beta$. Solving for μ_s we get $\mu_s = \frac{2}{7}\tan\beta$.

(b) As $\beta \to 0°$, $\mu_s \to 0$. As $\beta \to 90°$, $\mu_s \to \infty$.

EVALUATE: The results in part (a) are reasonable. As $\beta \to 0°$, the ball will roll on a smooth horizontal surface. If it is already moving and rotating, it will continue to do so. If it is not already rotating, it will not start to do so. As $\beta \to 90°$, $\mu_s \to \infty$. As $\beta \to 0°$, the normal force gets smaller and smaller, so we would need a larger and larger value of μ_s to prevent slipping.

10.21. IDENTIFY: Apply $K = K_{cm} + K_{rot}$.

SET UP: For an object that is rolling without slipping, $v_{cm} = R\omega$.

EXECUTE: The fraction of the total kinetic energy that is rotational is

$$\frac{(1/2)I_{cm}\omega^2}{(1/2)Mv_{cm}^2 + (1/2)I_{cm}\omega^2} = \frac{1}{1 + (M/I_{cm})v_{cm}^2/\omega^2} = \frac{1}{1 + (MR^2/I_{cm})}$$

(a) $I_{cm} = (1/2)MR^2$, so the above ratio is 1/3.

(b) $I_{cm} = (2/5)MR^2$ so the above ratio is 2/7.

(c) $I_{cm} = (2/3)MR^2$ so the ratio is 2/5.

(d) $I_{cm} = (5/8)MR^2$ so the ratio is 5/13.

EVALUATE: The moment of inertia of each object takes the form $I = \beta MR^2$. The ratio of rotational kinetic energy to total kinetic energy can be written as $\frac{1}{1+1/\beta} = \frac{\beta}{1+\beta}$. The ratio increases as β increases.

10.22. IDENTIFY: Only gravity does work, so $W_{other} = 0$ and conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $K_2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$.

SET UP: Let $y_2 = 0$, so $U_2 = 0$ and $y_1 = 0.750$ m. The hoop is released from rest so $K_1 = 0$. $v_{cm} = R\omega$. For a hoop with an axis at its center, $I_{cm} = MR^2$.

EXECUTE: (a) Conservation of energy gives $U_1 = K_2$. $K_2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2$, so $MR^2\omega^2 = Mgy_1$. $\omega = \frac{\sqrt{gy_1}}{R} = \frac{\sqrt{(9.80\text{ m/s}^2)(0.750\text{ m})}}{0.0800\text{ m}} = 33.9\text{ rad/s}$.

(b) $v = R\omega = (0.0800\text{ m})(33.9\text{ rad/s}) = 2.71\text{ m/s}$

EVALUATE: An object released from rest and falling in free fall for 0.750 m attains a speed of $\sqrt{2g(0.750\text{ m})} = 3.83\text{ m/s}$. The final speed of the hoop is less than this because some of its energy is in kinetic energy of rotation. Or, equivalently, the upward tension causes the magnitude of the net force of the hoop to be less than its weight.

10.23. IDENTIFY: Apply $\sum \vec{F}_{ext} = m\vec{a}_{cm}$ and $\sum \tau_z = I_{cm}\alpha_z$ to the motion of the ball.

(a) SET UP: The free-body diagram is given in Figure 10.23a.

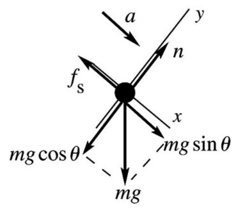


Figure 10.23a

EXECUTE: $\sum F_y = ma_y$

$n = mg\cos\theta$ and $f_s = \mu_s mg\cos\theta$

$\sum F_x = ma_x$

$mg\sin\theta - \mu_s mg\cos\theta = ma$

$g(\sin\theta - \mu_s\cos\theta) = a$ (Eq. 1)

SET UP: Consider Figure 10.23b.

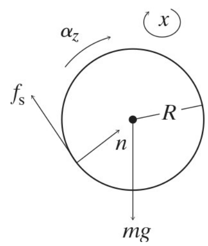


Figure 10.23b

The normal force n is directed through the center of the ball and mg acts at the center of the ball, so neither of them produces a torque about the center.

EXECUTE: $\sum\tau = \tau_f = \mu_s mg(\cos\theta)R$; $I = \frac{2}{5}mR^2$

$\sum\tau_z = I_{cm}\alpha_z$ gives $\mu_s\, mg(\cos\theta)R = \frac{2}{5}mR^2\alpha$

No slipping means $\alpha = a/R$, so $\mu_s g\cos\theta = \frac{2}{5}a$ (Eq. 2)

We have two equations in the two unknowns a and μ_s. Solving gives $a = \frac{5}{7}g\sin\theta$ and

$\mu_s = \frac{2}{7}\tan\theta = \frac{2}{7}\tan 65.0° = 0.613.$

(b) Repeat the calculation of part (a), but now $I = \frac{2}{3}mR^2$. $a = \frac{3}{5}g\sin\theta$ and

$\mu_s = \frac{2}{5}\tan\theta = \frac{2}{5}\tan 65.0° = 0.858$

The value of μ_s calculated in part (a) is not large enough to prevent slipping for the hollow ball.

(c) EVALUATE: There is no slipping at the point of contact. More friction is required for a hollow ball since for a given m and R it has a larger I and more torque is needed to provide the same α. Note that the required μ_s is independent of the mass or radius of the ball and only depends on how that mass is distributed.

10.24. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to the translational motion of the center of mass and $\sum\tau_z = I\alpha_z$ to the rotation about the center of mass.

SET UP: Let $+x$ be down the incline and let the shell be turning in the positive direction. The free-body diagram for the shell is given in Figure 10.24. From Table 9.2, $I_{cm} = \frac{2}{3}mR^2$.

EXECUTE: **(a)** $\sum F_x = ma_x$ gives $mg\sin\beta - f = ma_{cm}$. $\sum\tau_z = I\alpha_z$ gives $fR = \left(\frac{2}{3}mR^2\right)\alpha$. With $\alpha = a_{cm}/R$ this becomes $f = \frac{2}{3}ma_{cm}$. Combining the equations gives $mg\sin\beta - \frac{2}{3}ma_{cm} = ma_{cm}$ and

$a_{cm} = \frac{3g\sin\beta}{5} = \frac{3(9.80\text{ m/s}^2)(\sin 38.0°)}{5} = 3.62\text{ m/s}^2$. $f = \frac{2}{3}ma_{cm} = \frac{2}{3}(2.00\text{ kg})(3.62\text{ m/s}^2) = 4.83\text{ N}$.

The friction is static since there is no slipping at the point of contact. $n = mg\cos\beta = 15.45\text{ N}$.

$\mu_s = \frac{f}{n} = \frac{4.83\text{ N}}{15.45\text{ N}} = 0.313.$

(b) The acceleration is independent of m and doesn't change. The friction force is proportional to m so will double; $f = 9.66$ N. The normal force will also double, so the minimum μ_s required for no slipping wouldn't change.

EVALUATE: If there is no friction and the object slides without rolling, the acceleration is $g\sin\beta$. Friction and rolling without slipping reduce a to 0.60 times this value.

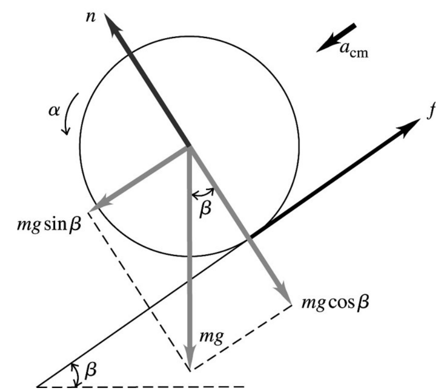
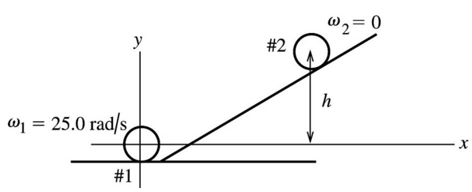


Figure 10.24

10.25. IDENTIFY: Apply conservation of energy to the motion of the wheel.

SET UP: The wheel at points 1 and 2 of its motion is shown in Figure 10.25.



Take $y = 0$ at the center of the wheel when it is at the bottom of the hill.

Figure 10.25

The wheel has both translational and rotational motion so its kinetic energy is $K = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv_{cm}^2$.

EXECUTE: $K_1 + U_1 + W_{other} = K_2 + U_2$

$W_{other} = W_{fric} = -2600$ J (the friction work is negative)

$K_1 = \frac{1}{2}I\omega_1^2 + \frac{1}{2}Mv_1^2$; $v = R\omega$ and $I = 0.800MR^2$ so

$K_1 = \frac{1}{2}(0.800)MR^2\omega_1^2 + \frac{1}{2}MR^2\omega_1^2 = 0.900MR^2\omega_1^2$

$K_2 = 0$, $U_1 = 0$, $U_2 = Mgh$

Thus $0.900MR^2\omega_1^2 + W_{fric} = Mgh$

$M = w/g = 392\text{ N}/(9.80\text{ m/s}^2) = 40.0$ kg

$$h = \frac{0.900MR^2\omega_1^2 + W_{fric}}{Mg}$$

$$h = \frac{(0.900)(40.0\text{ kg})(0.600\text{ m})^2(25.0\text{ rad/s})^2 - 2600\text{ J}}{(40.0\text{ kg})(9.80\text{ m/s}^2)} = 14.0\text{ m}.$$

EVALUATE: Friction does negative work and reduces h.

10.26. **IDENTIFY:** Apply conservation of energy to the motion of the marble.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, with $I = \frac{2}{5}MR^2$. $v_{cm} = R\omega$ for no slipping.

Let $y = 0$ at the bottom of the bowl. The marble at its initial and final locations is sketched in Figure 10.26.

EXECUTE: **(a)** Motion from the release point to the bottom of the bowl: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$ and $v = \sqrt{\frac{10}{7}gh}$.

Motion along the smooth side: The rotational kinetic energy does not change, since there is no friction torque on the marble, $\frac{1}{2}mv^2 + K_{rot} = mgh' + K_{rot}$. $h' = \frac{v^2}{2g} = \frac{\frac{10}{7}gh}{2g} = \frac{5}{7}h$

(b) $mgh = mgh'$ so $h' = h$.

EVALUATE: **(c)** With friction on both halves, all the initial potential energy gets converted back to potential energy. Without friction on the right half some of the energy is still in rotational kinetic energy when the marble is at its maximum height.

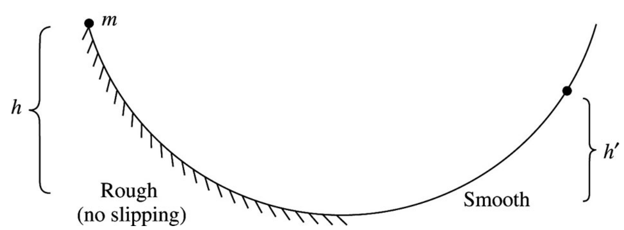


Figure 10.26

10.27. **IDENTIFY:** We want to investigate the kinetic energy of a bowling ball as it rolls down the bowling lane.

SET UP: $K_{total} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$, $I = \frac{2}{5}MR^2$ for a solid sphere, and $v_{cm} = r\omega$ when there is no slipping. Estimate: It takes 5.0 s to travel the 60 ft.

EXECUTE: **(a)** The target variable is the rotation rate of the ball. v_{cm} = (60 ft)/(5.0 s) = 12 ft/s. With no slipping $v_{cm} = r\omega$, so $\omega = \frac{v_{cm}}{R} = \frac{12 \text{ ft/s}}{\left(\frac{8.5}{2}\right)\left(\frac{1}{12}\right)\text{ft}}$ = 34 rad/s = 5.4 rev/s.

(b) We want to find out what fraction of the ball's kinetic energy is rotational.

$K_{tot} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{cm}}{R}\right)^2$, which gives

$K_{tot} = \frac{7}{10}mv_{cm}^2 = \frac{7}{10}\left(\frac{12 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(12 \text{ ft/s})^2 = 37.6 \text{ ft}\cdot\text{lb}$. The fraction that is rotational is

$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{\frac{7}{10}mv_{cm}^2} = \frac{\frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{cm}}{R}\right)^2}{\frac{7}{10}mv_{cm}^2} = \frac{2}{7}$. Therefore 2/7 of the kinetic energy is rotational.

EVALUATE: The distribution of the kinetic between rotational and translational forms depends only on the *geometry* of the ball, not on its mass or size. If the ball were hollow, its moment of inertia would be different which would give a different result for the fraction we just found.

10.28. **IDENTIFY:** We want to compare the kinetic energy of two rolling balls.

SET UP: $K_{\text{tot}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2$, $I = \frac{2}{5}MR^2$ for a solid sphere, and $v_{\text{cm}} = r\omega$ when there is no slipping. We know the kinetic energy of ball 2 is 27.0 J, and our target variable is the kinetic energy of ball 1.

EXECUTE: $K_{\text{tot}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2$, which gives $K_1 = \frac{7}{10}m_1v_1^2$. We get a similar result for ball 2: $K_2 = \frac{7}{10}m_2v_2^2$. Taking the ratio gives

$$\frac{K_1}{K_2} = \frac{\frac{7}{10}m_1v_1^2}{\frac{7}{10}m_2v_2^2} = \left(\frac{m_1}{m_2}\right)\left(\frac{v_1}{v_2}\right)^2 = \left(\frac{\frac{1}{2}m_2}{m_2}\right)\left(\frac{\frac{1}{3}v_2}{v_2}\right)^2 = \frac{1}{18}.$$ So $K_1 = K_2/18 = (27.0\text{ J})/18 = 1.50$ J.

EVALUATE: The result does *not* depend on the relative size of the balls since R cancels out.

10.29. **IDENTIFY:** As the cylinder falls, its potential energy is transformed into both translational and rotational kinetic energy. Its mechanical energy is conserved.

SET UP: The hollow cylinder has $I = \frac{1}{2}m(R_a^2 + R_b^2)$, where $R_a = 0.200$ m and $R_b = 0.350$ m. Use coordinates where $+y$ is upward and $y = 0$ at the initial position of the cylinder. Then $y_1 = 0$ and $y_2 = -d$, where d is the distance it has fallen. $v_{\text{cm}} = R\omega$. $K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}^2$ and $\omega_{0z} = 10.47$ rad/s,

EXECUTE: **(a)** Conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$, $U_1 = 0$. $0 = U_2 + K_2$ and $0 = -mgd + \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$. $\frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{1}{2}m(R_a^2 + R_b^2)\right](v_{\text{cm}}/R_b)^2 = \frac{1}{4}m\left[1 + (R_a/R_b)^2\right]v_{\text{cm}}^2$, so

$\frac{1}{2}\left\{1 + \frac{1}{2}\left[1 + (R_a/R_b)^2\right]\right\}v_{\text{cm}}^2 = gd$ and

$$d = \frac{\left\{1 + \frac{1}{2}\left[1 + (R_a/R_b)^2\right]\right\}v_{\text{cm}}^2}{2g} = \frac{(1 + 0.663)(6.66\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 3.76\text{ m}.$$

(b) $K_2 = \frac{1}{2}mv_{\text{cm}}^2$ since there is no rotation. So $mgd = \frac{1}{2}mv_{\text{cm}}^2$ which gives

$v_{\text{cm}} = \sqrt{2gd} = \sqrt{2(9.80\text{ m/s}^2)(3.76\text{ m})} = 8.58$ m/s.

(c) In part (a) the cylinder has rotational as well as translational kinetic energy and therefore less translational speed at a given kinetic energy. The kinetic energy comes from a decrease in gravitational potential energy and that is the same, so in (a) the translational speed is less.

EVALUATE: If part (a) were repeated for a solid cylinder, $R_a = 0$ and $d = 3.39$ m. For a thin-walled hollow cylinder, $R_a = R_b$ and $d = 4.52$ cm. Note that all of these answers are independent of the mass m of the cylinder.

10.30. **IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$ and $\sum\vec{F} = m\vec{a}$ to the motion of the bowling ball.

SET UP: $a_{\text{cm}} = R\alpha$. $f_s = \mu_s n$. Let $+x$ be directed down the incline.

EXECUTE: **(a)** The free-body diagram is sketched in Figure 10.30.

The angular speed of the ball must decrease, and so the torque is provided by a friction force that acts up the hill.

(b) The friction force results in an angular acceleration, given by $I\alpha = fR$. $\sum\vec{F} = m\vec{a}$ applied to the motion of the center of mass gives $mg\sin\beta - f = ma_{\text{cm}}$, and the acceleration and angular acceleration are related by $a_{\text{cm}} = R\alpha$.

Combining, $mg\sin\beta = ma_{\text{cm}}\left(1+\dfrac{I}{mR^2}\right) = ma_{\text{cm}}(7/5).$ $a_{\text{cm}} = (5/7)g\sin\beta.$

(c) From either of the above relations between f and a_{cm}, $f = \dfrac{2}{5}ma_{\text{cm}} = \dfrac{2}{7}mg\sin\beta \le \mu_s n = \mu_s mg\cos\beta.$

$\mu_s \ge (2/7)\tan\beta.$

EVALUATE: If $\mu_s = 0$, $a_{\text{cm}} = mg\sin\beta.$ a_{cm} is less when friction is present. The ball rolls farther uphill when friction is present, because the friction removes the rotational kinetic energy and converts it to gravitational potential energy. In the absence of friction the ball retains the rotational kinetic energy that is has initially.

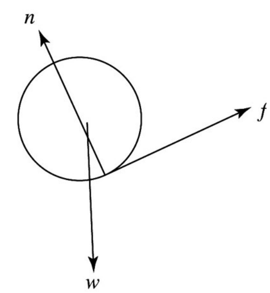


Figure 10.30

10.31. **IDENTIFY:** As the ball rolls up the hill, its kinetic energy (translational and rotational) is transformed into gravitational potential energy. Since there is no slipping, its mechanical energy is conserved.

SET UP: The ball has moment of inertia $I_{\text{cm}} = \frac{2}{3}mR^2$. Rolling without slipping means $v_{\text{cm}} = R\omega$. Use coordinates where $+y$ is upward and $y = 0$ at the bottom of the hill, so $y_1 = 0$ and $y_2 = h = 5.00$ m. The ball's kinetic energy is $K = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$ and its potential energy is $U = mgh.$

EXECUTE: **(a)** Conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $U_1 = 0$, $K_2 = 0$ (the ball stops).

Therefore $K_1 = U_2$ and $\frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2 = mgh.$ $\frac{1}{2}I_{\text{cm}}\omega^2 = \frac{1}{2}(\frac{2}{3}mR^2)\left(\dfrac{v_{\text{cm}}}{R}\right)^2 = \frac{1}{3}mv_{\text{cm}}^2$, so

$\frac{5}{6}mv_{\text{cm}}^2 = mgh.$ Therefore $v_{\text{cm}} = \sqrt{\dfrac{6gh}{5}} = \sqrt{\dfrac{6(9.80\text{ m/s}^2)(5.00\text{ m})}{5}} = 7.67$ m/s and

$\omega = \dfrac{v_{\text{cm}}}{R} = \dfrac{7.67\text{ m/s}}{0.113\text{ m}} = 67.9$ rad/s.

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{3}mv_{\text{cm}}^2 = \frac{1}{3}(0.426\text{ kg})(7.67\text{ m/s})^2 = 8.35$ J.

EVALUATE: Its translational kinetic energy at the base of the hill is $\frac{1}{2}mv_{\text{cm}}^2 = \frac{3}{2}K_{\text{rot}} = 12.52$ J. Its total kinetic energy is 20.9 J, which equals its final potential energy:

$mgh = (0.426\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 20.9$ J.

10.32. **IDENTIFY:** Apply $P = \tau\omega$ and $W = \tau\Delta\theta.$

SET UP: P must be in watts, $\Delta\theta$ must be in radians, and ω must be in rad/s. 1 rev $= 2\pi$ rad. 1 hp $= 746$ W. π rad/s $= 30$ rev/min.

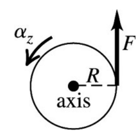
EXECUTE: **(a)** $\tau = \dfrac{P}{\omega} = \dfrac{(175\text{ hp})(746\text{ W/hp})}{(2400\text{ rev/min})\left(\dfrac{\pi\ \text{rad/s}}{30\text{ rev/min}}\right)} = 519\text{ N}\cdot\text{m}.$

(b) $W = \tau\Delta\theta = (519\ \text{N}\cdot\text{m})(2\pi\ \text{rad}) = 3260\ \text{J}$

EVALUATE: $\omega = 40$ rev/s, so the time for one revolution is 0.025 s. $P = 1.306\times 10^5$ W, so in one revolution, $W = Pt = 3260$ J, which agrees with our result.

10.33. **(a) IDENTIFY:** Use $\sum\tau_z = I\alpha_z$ to find α_z and then use a constant angular acceleration equation to find ω_z.

SET UP: The free-body diagram is given in Figure 10.33.



EXECUTE: Apply $\sum\tau_z = I\alpha_z$ to find the angular acceleration:

$FR = I\alpha_z$

$$\alpha_z = \frac{FR}{I} = \frac{(18.0\ \text{N})(2.40\ \text{m})}{2100\ \text{kg}\cdot\text{m}^2} = 0.02057\ \text{rad/s}^2$$

Figure 10.33

SET UP: Use the constant α_z kinematic equations to find ω_z.

$\omega_z = ?$; ω_{0z} (initially at rest); $\alpha_z = 0.02057\ \text{rad/s}^2$; $t = 15.0$ s

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t = 0 + (0.02057\ \text{rad/s}^2)(15.0\ \text{s}) = 0.309\ \text{rad/s}$

(b) IDENTIFY and **SET UP:** Calculate the work from $W = \tau_z\Delta\theta$, using a constant angular acceleration equation to calculate $\theta - \theta_0$, or use the work-energy theorem. We will do it both ways.

EXECUTE: (1) $W = \tau_z\Delta\theta$

$\Delta\theta = \theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = 0 + \frac{1}{2}(0.02057\ \text{rad/s}^2)(15.0\ \text{s})^2 = 2.314\ \text{rad}$

$\tau_z = FR = (18.0\ \text{N})(2.40\ \text{m}) = 43.2\ \text{N}\cdot\text{m}$

Then $W = \tau_z\Delta\theta = (43.2\ \text{N}\cdot\text{m})(2.314\ \text{rad}) = 100\ \text{J}$.

or

(2) $W_{\text{tot}} = K_2 - K_1$

$W_{\text{tot}} = W$, the work done by the child

$K_1 = 0$; $K_2 = \frac{1}{2}I\omega^2 = \frac{1}{2}(2100\ \text{kg}\cdot\text{m}^2)(0.309\ \text{rad/s})^2 = 100\ \text{J}$

Thus $W = 100$ J, the same as before.

EVALUATE: Either method yields the same result for W.

(c) IDENTIFY and **SET UP:** Use $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$ to calculate P_{av}.

EXECUTE: $P_{\text{av}} = \dfrac{\Delta W}{\Delta t} = \dfrac{100\ \text{J}}{15.0\ \text{s}} = 6.67\ \text{W}$.

EVALUATE: Work is in joules, power is in watts.

10.34. **IDENTIFY:** The power output of the motor is related to the torque it produces and to its angular velocity by $P = \tau_z\omega_z$, where ω_z must be in rad/s.

SET UP: The work output of the motor in 60.0 s is $\frac{2}{3}(9.00\ \text{kJ}) = 6.00\ \text{kJ}$, so $P = \dfrac{6.00\ \text{kJ}}{60.0\ \text{s}} = 100\ \text{W}$.

$\omega_z = 2500\ \text{rev/min} = 262\ \text{rad/s}$.

EXECUTE: $\tau_z = \dfrac{P}{\omega_z} = \dfrac{100\ \text{W}}{262\ \text{rad/s}} = 0.382\ \text{N}\cdot\text{m}$.

EVALUATE: For a constant power output, the torque developed decreases when the rotation speed of the motor increases.

10.35. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ and constant angular acceleration equations to the motion of the wheel.

SET UP: 1 rev $= 2\pi$ rad. π rad/s $= 30$ rev/min.

EXECUTE: **(a)** $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t}$.

$$\tau_z = \frac{\left[(1/2)(2.80\text{ kg})(0.100\text{ m})^2\right](1200\text{ rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)}{2.5\text{ s}} = 0.704\text{ N}\cdot\text{m}.$$

(b) $\omega_{\text{av}}\Delta t = \dfrac{(600\text{ rev/min})(2.5\text{ s})}{60\text{ s/min}} = 25.0\text{ rev} = 157\text{ rad}.$

(c) $W = \tau\Delta\theta = (0.704\text{ N}\cdot\text{m})(157\text{ rad}) = 111\text{ J}.$

(d) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[(1/2)(2.80\text{ kg})(0.100\text{ m})^2\right]\left[(1200\text{ rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)\right]^2 = 111\text{ J}.$

the same as in part (c).

EVALUATE: The agreement between the results of parts (c) and (d) illustrates the work-energy theorem.

10.36. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the motion of the propeller and then use constant acceleration equations to analyze the motion. $W = \tau\Delta\theta$.

SET UP: $I = \frac{1}{12}mL^2 = \frac{1}{12}(117\text{ kg})(2.08\text{ m})^2 = 42.2\text{ kg}\cdot\text{m}^2.$

EXECUTE: **(a)** $\alpha = \dfrac{\tau}{I} = \dfrac{1950\text{ N}\cdot\text{m}}{42.2\text{ kg}\cdot\text{m}^2} = 46.2\text{ rad/s}^2.$

(b) $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega = \sqrt{2\alpha\theta} = \sqrt{2(46.2\text{ rad/s}^2)(5.0\text{ rev})(2\pi\text{ rad/rev})} = 53.9\text{ rad/s}.$

(c) $W = \tau\theta = (1950\text{ N}\cdot\text{m})(5.00\text{ rev})(2\pi\text{ rad/rev}) = 6.13\times10^4\text{ J}.$

(d) $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{53.9\text{ rad/s}}{46.2\text{ rad/s}^2} = 1.17\text{ s}.$ $P_{\text{av}} = \dfrac{W}{\Delta t} = \dfrac{6.13\times10^4\text{ J}}{1.17\text{ s}} = 52.5\text{ kW}.$

(e) $P = \tau\omega = (1950\text{ N}\cdot\text{m})(53.9\text{ rad/s}) = 105\text{ kW}.$

EVALUATE: $P = \tau\omega$. τ is constant and ω is linear in t, so P_{av} is half the instantaneous power at the end of the 5.00 revolutions. We could also calculate W from $W = \Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2\text{ kg}\cdot\text{m}^2)(53.9\text{ rad/s})^2 = 6.13\times10^4\text{ J}.$

10.37. **(a) IDENTIFY:** Use $L = mvr\sin\phi$.

SET UP: Consider Figure 10.37.

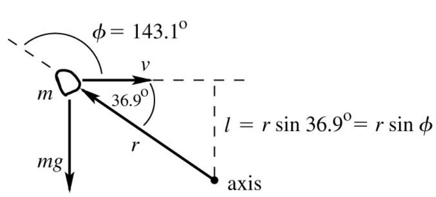


Figure 10.37

EXECUTE: $L = mvr\sin\phi =$ $(2.00\text{ kg})(12.0\text{ m/s})(8.00\text{ m})\sin 143.1°$

$L = 115\text{ kg}\cdot\text{m}^2/\text{s}$

To find the direction of $\vec{L}$ apply the right-hand rule by turning $\vec{r}$ into the direction of $\vec{v}$ by pushing on it with the fingers of your right hand. Your thumb points into the page, in the direction of $\vec{L}$.

(b) IDENTIFY and **SET UP:** By $\vec{\tau} = \dfrac{d\vec{L}}{dt}$ the rate of change of the angular momentum of the rock equals the torque of the net force acting on it.

EXECUTE: $\tau = mg(8.00\text{ m})\cos 36.9° = 125\text{ kg}\cdot\text{m}^2/\text{s}^2$

To find the direction of $\vec{\boldsymbol{\tau}}$ and hence of $d\vec{\boldsymbol{L}}/dt$, apply the right-hand rule by turning $\vec{\boldsymbol{r}}$ into the direction of the gravity force by pushing on it with the fingers of your right hand. Your thumb points out of the page, in the direction of $d\vec{\boldsymbol{L}}/dt$.

EVALUATE: $\vec{\boldsymbol{L}}$ and $d\vec{\boldsymbol{L}}/dt$ are in opposite directions, so L is decreasing. The gravity force is accelerating the rock downward, toward the axis. Its horizontal velocity is constant but the distance l is decreasing and hence L is decreasing.

10.38. **IDENTIFY:** $L = I\omega$ and $I = I_{\text{disk}} + I_{\text{woman}}$.

SET UP: $\omega = 0.80\text{ rev/s} = 5.026\text{ rad/s}$. $I_{\text{disk}} = \frac{1}{2}m_{\text{disk}}R^2$ and $I_{\text{woman}} = m_{\text{woman}}R^2$.

EXECUTE: $I = (55\text{ kg} + 50.0\text{ kg})(4.0\text{ m})^2 = 1680\text{ kg}\cdot\text{m}^2$.

$L = (1680\text{ kg}\cdot\text{m}^2)(5.026\text{ rad/s}) = 8.4\times10^3\text{ kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: The disk and the woman have similar values of I, even though the disk has twice the mass.

10.39. **IDENTIFY** and **SET UP:** Use $L = I\omega$.

EXECUTE: The second hand makes 1 revolution in 1 minute, so
$\omega = (1.00\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 0.1047\text{ rad/s}$.

For a slender rod, with the axis about one end,
$I = \frac{1}{3}ML^2 = \frac{1}{3}(6.00\times10^{-3}\text{ kg})(0.150\text{ m})^2 = 4.50\times10^{-5}\text{ kg}\cdot\text{m}^2$.

Then $L = I\omega = (4.50\times10^{-5}\text{ kg}\cdot\text{m}^2)(0.1047\text{ rad/s}) = 4.71\times10^{-6}\text{ kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: $\vec{\boldsymbol{L}}$ is clockwise.

10.40. **IDENTIFY:** $L_z = I\omega_z$

SET UP: For a particle of mass m moving in a circular path at a distance r from the axis, $I = mr^2$ and $v = r\omega$. For a uniform sphere of mass M and radius R and an axis through its center, $I = \frac{2}{5}MR^2$. The earth has mass $m_{\text{E}} = 5.97\times10^{24}\text{ kg}$, radius $R_{\text{E}} = 6.37\times10^6\text{ m}$ and orbit radius $r = 1.50\times10^{11}\text{ m}$. The earth completes one rotation on its axis in $24\text{ h} = 86{,}400\text{ s}$ and one orbit in $1\text{ y} = 3.156\times10^7\text{ s}$.

EXECUTE: **(a)**

$$L_z = I\omega_z = mr^2\omega_z = (5.97\times10^{24}\text{ kg})(1.50\times10^{11}\text{ m})^2\left(\frac{2\pi\text{ rad}}{3.156\times10^7\text{ s}}\right) = 2.67\times10^{40}\text{ kg}\cdot\text{m}^2/\text{s}.$$

The radius of the earth is much less than its orbit radius, so it is very reasonable to model it as a particle for this calculation.

(b) $L_z = I\omega_z = (\frac{2}{5}MR^2)\omega = \frac{2}{5}(5.97\times10^{24}\text{ kg})(6.38\times10^6\text{ m})^2\left(\frac{2\pi\text{ rad}}{86{,}400\text{ s}}\right) = 7.07\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}$

EVALUATE: The angular momentum associated with each of these motions is very large.

10.41. **IDENTIFY:** $\omega_z = d\theta/dt$. $L_z = I\omega_z$ and $\tau_z = dL_z/dt$.

SET UP: For a hollow, thin-walled sphere rolling about an axis through its center, $I = \frac{2}{3}MR^2$. $R = 0.240\text{ m}$.

EXECUTE: **(a)** $A = 1.50\text{ rad/s}^2$ and $B = 1.10\text{ rad/s}^4$, so that $\theta(t)$ will have units of radians.

(b) (i) $\omega_z = \dfrac{d\theta}{dt} = 2At + 4Bt^3$. At $t = 3.00\text{ s}$,

$\omega_z = 2(1.50\text{ rad/s}^2)(3.00\text{ s}) + 4(1.10\text{ rad/s}^4)(3.00\text{ s})^3 = 128\text{ rad/s}$.

$L_z = (\frac{2}{3}MR^2)\omega_z = \frac{2}{3}(12.0\text{ kg})(0.240\text{ m})^2(128\text{ rad/s}) = 59.0\text{ kg}\cdot\text{m}^2/\text{s}$.

(ii) $\tau_z = \dfrac{dL_z}{dt} = I\dfrac{d\omega_z}{dt} = I(2A + 12Bt^2)$ and

$\tau_z = \frac{2}{3}(12.0\text{ kg})(0.240\text{ m})^2\left[2(1.50\text{ rad/s}^2) + 12(1.10\text{ rad/s}^4)(3.00\text{ s})^2\right] = 56.1\text{ N}\cdot\text{m}.$

EVALUATE: The angular speed of rotation is increasing. This increase is due to an acceleration α_z that is produced by the torque on the sphere. When I is constant, as it is here, $\tau_z = dL_z/dt = Id\omega_z/dt = I\alpha_z$.

10.42. IDENTIFY and SET UP: $\vec{L}$ is conserved if there is no net external torque.
Use conservation of angular momentum to find ω at the new radius and use $K = \frac{1}{2}I\omega^2$ to find the change in kinetic energy, which is equal to the work done on the block.

EXECUTE: **(a)** Yes, angular momentum is conserved. The moment arm for the tension in the cord is zero so this force exerts no torque and there is no net torque on the block.

(b) $L_1 = L_2$ so $I_1\omega_1 = I_2\omega_2$. Block treated as a point mass, so $I = mr^2$, where r is the distance of the block from the hole.

$mr_1^2\omega_1 = mr_2^2\omega_2$

$$\omega_2 = \left(\frac{r_1}{r_2}\right)^2\omega_1 = \left(\frac{0.300\text{ m}}{0.150\text{ m}}\right)^2(2.85\text{ rad/s}) = 11.4\text{ rad/s}$$

(c) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}mr_1^2\omega_1^2 = \frac{1}{2}mv_1^2$

$v_1 = r_1\omega_1 = (0.300\text{ m})(2.85\text{ rad/s}) = 0.855\text{ m/s}$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0250\text{ kg})(0.855\text{ m/s})^2 = 0.00914\text{ J}$

$K_2 = \frac{1}{2}mv_2^2$

$v_2 = r_2\omega_2 = (0.150\text{ m})(11.4\text{ rad/s}) = 1.71\text{ m/s}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.0250\text{ kg})(1.71\text{ m/s}^2)^2 = 0.03655\text{ J}$

$\Delta K = K_2 - K_1 = 0.03655\text{ J} - 0.00914\text{ J} = 0.0274\text{ J} = 27.4\text{ mJ}.$

(d) $W_{\text{tot}} = \Delta K$

But $W_{\text{tot}} = W$, the work done by the tension in the cord, so $W = 0.0274$ J.

EVALUATE: Smaller r means smaller I. $L = I\omega$ is constant so ω increases and K increases. The work done by the tension is positive since it is directed inward and the block moves inward, toward the hole.

10.43. IDENTIFY: Apply conservation of angular momentum.

SET UP: For a uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$.

EXECUTE: The moment of inertia is proportional to the square of the radius, and so the angular velocity will be proportional to the inverse of the square of the radius, and the final angular velocity is

$$\omega_2 = \omega_1\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2\pi\text{ rad}}{(30\text{ d})(86{,}400\text{ s/d})}\right)\left(\frac{7.0\times10^5\text{ km}}{16\text{ km}}\right)^2 = 4.6\times10^3\text{ rad/s}.$$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$. L is constant and ω increases by a large factor, so there is a large increase in the rotational kinetic energy of the star. This energy comes from potential energy associated with the gravity force within the star.

10.44. IDENTIFY and SET UP: Apply conservation of angular momentum to the diver.

SET UP: The number of revolutions she makes in a certain time is proportional to her angular velocity. The ratio of her untucked to tucked angular velocity is $(3.6\text{ kg}\cdot\text{m}^2)/(18\text{ kg}\cdot\text{m}^2)$.

EXECUTE: If she had not tucked, she would have made $(2\text{ rev})(3.6\text{ kg}\cdot\text{m}^2)/(18\text{ kg}\cdot\text{m}^2) = 0.40\text{ rev}$ in the last 1.0 s, so she would have made $(0.40\text{ rev})(1.5/1.0) = 0.60\text{ rev}$ in the total 1.5 s.

EVALUATE: Untucked she rotates slower and completes fewer revolutions.

10.45. **IDENTIFY:** Apply conservation of angular momentum to the motion of the skater.

SET UP: For a thin-walled hollow cylinder $I = mR^2$. For a slender rod rotating about an axis through its center, $I = \frac{1}{12}ml^2$.

EXECUTE: $L_\text{i} = L_\text{f}$ so $I_\text{i}\omega_\text{i} = I_\text{f}\omega_\text{f}$.

$I_\text{i} = 0.40\text{ kg}\cdot\text{m}^2 + \frac{1}{12}(8.0\text{ kg})(1.8\text{ m})^2 = 2.56\text{ kg}\cdot\text{m}^2.$

$I_\text{f} = 0.40\text{ kg}\cdot\text{m}^2 + (8.0\text{ kg})(0.25\text{ m})^2 = 0.90\text{ kg}\cdot\text{m}^2.$

$$\omega_\text{f} = \left(\frac{I_\text{i}}{I_\text{f}}\right)\omega_\text{i} = \left(\frac{2.56\text{ kg}\cdot\text{m}^2}{0.90\text{ kg}\cdot\text{m}^2}\right)(0.40\text{ rev/s}) = 1.14\text{ rev/s}.$$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega.$ ω increases and L is constant, so K increases. The increase in kinetic energy comes from the work done by the skater when he pulls in his hands.

10.46. **IDENTIFY:** Apply conservation of angular momentum to the collision.

SET UP: Let the width of the door be l. The initial angular momentum of the mud is $mv(l/2)$, since it strikes the door at its center. For the axis at the hinge, $I_\text{door} = \frac{1}{3}Ml^2$ and $I_\text{mud} = m(l/2)^2$.

EXECUTE: $\omega = \dfrac{L}{I} = \dfrac{mv(l/2)}{(1/3)Ml^2 + m(l/2)^2}.$

$$\omega = \frac{(0.500\text{ kg})(12.0\text{ m/s})(0.500\text{ m})}{(1/3)(40.0\text{ kg})(1.00\text{ m})^2 + (0.500\text{ kg})(0.500\text{ m})^2} = 0.223\text{ rad/s}.$$

Ignoring the mass of the mud in the denominator of the above expression gives $\omega = 0.225\text{ rad/s}$, so the mass of the mud in the moment of inertia does affect the third significant figure.

EVALUATE: Angular momentum is conserved but there is a large decrease in the kinetic energy of the system.

10.47. **IDENTIFY** and **SET UP:** There is no net external torque about the rotation axis so the angular momentum $L = I\omega$ is conserved.

EXECUTE: **(a)** $L_1 = L_2$ gives $I_1\omega_1 = I_2\omega_2$, so $\omega_2 = (I_1/I_2)\omega_1$

$I_1 = I_\text{tt} = \frac{1}{2}MR^2 = \frac{1}{2}(120\text{ kg})(2.00\text{ m})^2 = 240\text{ kg}\cdot\text{m}^2$

$I_2 = I_\text{tt} + I_\text{p} = 240\text{ kg}\cdot\text{m}^2 + mR^2 = 240\text{ kg}\cdot\text{m}^2 + (70\text{ kg})(2.00\text{ m})^2 = 520\text{ kg}\cdot\text{m}^2$

$\omega_2 = (I_1/I_2)\omega_1 = (240\text{ kg}\cdot\text{m}^2/520\text{ kg}\cdot\text{m}^2)(3.00\text{ rad/s}) = 1.38\text{ rad/s}$

(b) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(240\text{ kg}\cdot\text{m}^2)(3.00\text{ rad/s})^2 = 1080\text{ J}$

$K_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(520\text{ kg}\cdot\text{m}^2)(1.38\text{ rad/s})^2 = 495\text{ J}$

EVALUATE: The kinetic energy decreases because of the negative work done on the turntable and the parachutist by the friction force between these two objects.
The angular speed decreases because I increases when the parachutist is added to the system.

10.48. **IDENTIFY:** Apply conservation of angular momentum to the system of earth plus asteroid.

SET UP: Take the axis to be the earth's rotation axis. The asteroid may be treated as a point mass and it has zero angular momentum before the collision, since it is headed toward the center of the earth. For the earth, $L_z = I\omega_z$ and $I = \frac{2}{5}MR^2$, where M is the mass of the earth and R is its radius. The length of a day is $T = \dfrac{2\pi\text{ rad}}{\omega}$, where ω is the earth's angular rotation rate.

EXECUTE: Conservation of angular momentum applied to the collision between the earth and asteroid gives $\frac{2}{5}MR^2\omega_1 = (mR^2 + \frac{2}{5}MR^2)\omega_2$ and $m = \frac{2}{5}M\left(\frac{\omega_1 - \omega_2}{\omega_2}\right)$. $T_2 = 1.250T_1$ gives $\frac{1}{\omega_2} = \frac{1.250}{\omega_1}$ and

$\omega_1 = 1.250\omega_2$. $\frac{\omega_1 - \omega_2}{\omega_2} = 0.250$. $m = \frac{2}{5}(0.250)M = 0.100M$.

EVALUATE: If the asteroid hit the surface of the earth tangentially it could have some angular momentum with respect to the earth's rotation axis, and could either speed up or slow down the earth's rotation rate.

10.49. **(a) IDENTIFY** and **SET UP:** Apply conservation of angular momentum $\vec{L}$, with the axis at the nail. Let object A be the bug and object B be the bar. Initially, all objects are at rest and $L_1 = 0$. Just after the bug jumps, it has angular momentum in one direction of rotation and the bar is rotating with angular velocity ω_B in the opposite direction.

EXECUTE: $L_2 = m_A v_A r - I_B\omega_B$ where $r = 1.00\text{ m}$ and $I_B = \frac{1}{3}m_B r^2$

$L_1 = L_2$ gives $m_A v_A r = \frac{1}{3}m_B r^2\omega_B$

$$\omega_B = \frac{3m_A v_A}{m_B r} = 0.120\text{ rad/s}$$

(b) $K_1 = 0$;

$K_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}(0.0100\text{ kg})(0.200\text{ m/s})^2 + \frac{1}{2}(\frac{1}{3}(0.0500\text{ kg})(1.00\text{ m})^2)(0.120\text{ rad/s})^2 = 3.2\times10^{-4}\text{ J}.$

(c) The increase in kinetic energy comes from work done by the bug when it pushes against the bar in order to jump.

EVALUATE: There is no external torque applied to the system and the total angular momentum of the system is constant. There are internal forces, forces the bug and bar exert on each other. The forces exert torques and change the angular momentum of the bug and the bar, but these changes are equal in magnitude and opposite in direction. These internal forces do positive work on the two objects and the kinetic energy of each object and of the system increases.

10.50. **IDENTIFY:** As the bug moves outward, it increases the moment of inertia of the rod-bug system. The angular momentum of this system is conserved because no unbalanced external torques act on it.

SET UP: The moment of inertia of the rod is $I = \frac{1}{3}ML^2$, and conservation of angular momentum gives $I_1\omega_1 = I_2\omega_2$.

EXECUTE: **(a)** $I = \frac{1}{3}ML^2$ gives $M = \frac{3I}{L^2} = \frac{3(3.00\times10^{-3}\text{ kg}\cdot\text{m}^2)}{(0.500\text{ m})^2} = 0.0360\text{ kg}.$

(b) $L_1 = L_2$, so $I_1\omega_1 = I_2\omega_2$. $\omega_2 = \frac{v}{r} = \frac{0.160\text{ m/s}}{0.500\text{ m}} = 0.320\text{ rad/s}$, so

$(3.00\times10^{-3}\text{ kg}\cdot\text{m}^2)(0.400\text{ rad/s}) = (3.00\times10^{-3}\text{ kg}\cdot\text{m}^2 + m_{\text{bug}}(0.500\text{ m})^2)(0.320\text{ rad/s}).$

$$m_{\text{bug}} = \frac{(3.00\times10^{-3}\text{ kg}\cdot\text{m}^2)(0.400\text{ rad/s} - 0.320\text{ rad/s})}{(0.320\text{ rad/s})(0.500\text{ m})^2} = 3.00\times10^{-3}\text{ kg}.$$

EVALUATE: This is a 3.00 mg bug, which is not unreasonable.

10.51. **IDENTIFY:** Energy is conserved as the cylinder rolls down the incline without slipping.

SET UP: The total energy of the cylinder is $K_{\text{tot}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2$, $I_{\text{cyl}} = \frac{1}{2}MR^2$. As the cylinder rolls down the incline, gravitational potential energy is transformed into kinetic energy, so we use $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Our target variable is the acceleration due to gravity on the planet.

EXECUTE: Energy conservation gives $mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$, which, for a solid cylinder, gives $mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v_{cm}}{R}\right)^2$. Solving for v_{cm}^2 gives $v_{cm}^2 = \left(\frac{4}{3}g\right)h.$ The graph of v_{cm}^2 versus h should be a straight line having slope equal to $4g/3$. Thus $g = \frac{3}{4}(\text{slope}) = \frac{3}{4}(6.42\text{ m/s}^2) = 4.82\text{ m/s}^2$.

EVALUATE: On this planet, g is about half of what it is on the earth.

10.52. **IDENTIFY:** If we take the raven and the gate as a system, the torque about the pivot is zero, so the angular momentum of the system about the pivot is conserved.

SET UP: The system before and after the collision is sketched in Figure 10.52. The gate has $I = \frac{1}{3}ML^2$. Take counterclockwise torques to be positive.

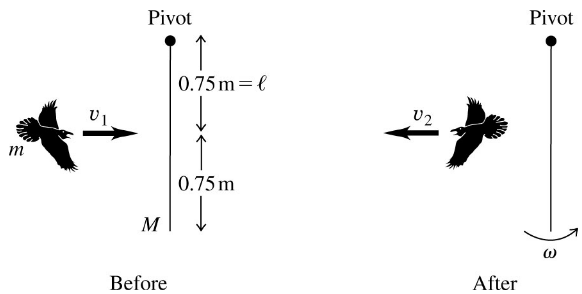


Figure 10.52

EXECUTE: **(a)** The gravity forces exert no torque at the moment of collision and angular momentum is conserved. $L_1 = L_2$. $mv_1 l = -mv_2 l + I_{gate}\omega$ with $l = L/2$.

$$\omega = \frac{m(v_1 + v_2)l}{\frac{1}{3}ML^2} = \frac{3m(v_1 + v_2)}{2ML} = \frac{3(1.1\text{ kg})(5.0\text{ m/s} + 2.0\text{ m/s})}{2(4.5\text{ kg})(1.5\text{ m})} = 1.71\text{ rad/s}.$$

(b) Linear momentum is not conserved; there is an external force exerted by the pivot. But the force on the pivot has zero torque. There is no external torque and angular momentum is conserved.

EVALUATE: $K_1 = \frac{1}{2}(1.1\text{ kg})(5.0\text{ m/s})^2 = 13.8\text{ J}$.

$K_2 = \frac{1}{2}(1.1\text{ kg})(2.0\text{ m/s})^2 + \frac{1}{2}(\frac{1}{3}[4.5\text{ kg}][1.5\text{ m/s}]^2)(1.71\text{ rad/s})^2 = 7.1\text{ J}$. This is an inelastic collision and $K_2 < K_1$.

10.53. **IDENTIFY:** When the teenager throws the rock, it causes the wooden disk to spin. Because no external torques act on the system due to the throw, the angular momentum of the system is conserved.

SET UP: $L = I\omega$ for a rotating object, and $L = mvr$ for a small object. $I_{disk} = \frac{1}{2}MR^2$. The target variable is the angular speed of the disk just after the rock is thrown.

EXECUTE: The initial angular momentum is zero since nothing is turning. Conservation of angular momentum tells us that $L_1 = L_2$, so $0 = L_{teen} + L_{disk} + L_{rock}$. The rock's motion is opposite to that of the teen and disk. This gives us $0 = mR^2\omega + \frac{1}{2}MR^2\omega - m_{rock}vR$. Solving for ω gives $\omega = \frac{m_{rock}v}{R\left(m + \frac{M}{2}\right)}$.

EVALUATE: Check some special cases. If $m_{rock} \to 0$, then $\omega \to 0$, which means that throwing the very light rock had no effect on the disk. If m or M are very large, $\omega \to 0$, which means that the teen or the disk were too massive to be moved by the light rock. If the teen and the rock are both much more massive than the disk, then $\omega \rightarrow v/R$, which means that the teen and rock have the same speed but in opposite directions.

10.54. **IDENTIFY:** The bullet collides with (and embeds itself in) the wooden disk, causing the disk and bullet to rotate. The angular momentum of the system is conserved because the collision did not cause any external torques on it.

SET UP: $L = I\omega$ for a rotating object, and $L = mvr$ for a small object. $I_{\text{disk}} = \frac{1}{2}MR^2$. The target variable is the speed of the bullet just before it hit the disk.

EXECUTE: Angular momentum conservation tells us that $L_{\text{bullet}} = L_{\text{bullet+disk}}$. Calling m the bullet mass and M the disk mass gives $mvR = (I_b + I_d)\omega = \left(mR^2 + \frac{1}{2}MR^2\right)\omega$. Solving for v gives

$$v = R\omega\left(1+\frac{M}{2m}\right) = (0.600\text{ m})(4.00\text{ rad/s})\left(1+\frac{1.60\text{ kg}}{0.0400\text{ kg}}\right) = 98.4\text{ m/s}.$$

EVALUATE: Our result tells us that if m is small, v will need to be large to give the disk angular speed. If m is large, v can be small to give the disk angular speed.

10.55. **IDENTIFY:** An external torque will cause precession of the telescope.

SET UP: $I = MR^2$, with $R = 2.5\times10^{-2}$ m. 1.0×10^{-6} degree $= 1.745\times10^{-8}$ rad.

$\omega = 19{,}200$ rpm $= 2.01\times10^3$ rad/s. $t = 5.0$ h $= 1.8\times10^4$ s.

EXECUTE: $\Omega = \dfrac{\Delta\phi}{\Delta t} = \dfrac{1.745\times10^{-8}\text{ rad}}{1.8\times10^4\text{ s}} = 9.694\times10^{-13}$ rad/s. $\Omega = \dfrac{\tau}{I\omega}$ so $\tau = \Omega I\omega = \Omega MR^2\omega$. Putting in the numbers gives

$\tau = (9.694\times10^{-13}\text{ rad/s})(2.0\text{ kg})(2.5\times10^{-2}\text{ m})^2(2.01\times10^3\text{ rad/s}) = 2.4\times10^{-12}\text{ N}\cdot\text{m}.$

EVALUATE: The external torque must be very small for this degree of stability.

10.56. **IDENTIFY:** The precession angular speed is related to the acceleration due to gravity by $\Omega = \dfrac{mgr}{I\omega}$, with $w = mg$.

SET UP: $\Omega_E = 0.50$ rad/s, $g_E = g$ and $g_M = 0.165g$. For the gyroscope, m, r, I, and ω are the same on the moon as on the earth.

EXECUTE: $\Omega = \dfrac{mgr}{I\omega}$. $\dfrac{\Omega}{g} = \dfrac{mr}{I\omega} =$ constant, so $\dfrac{\Omega_E}{g_E} = \dfrac{\Omega_M}{g_M}$.

$$\Omega_M = \Omega_E\left(\frac{g_M}{g_E}\right) = 0.165\Omega_E = (0.165)(0.50\text{ rad/s}) = 0.0825\text{ rad/s}.$$

EVALUATE: In the limit that $g \rightarrow 0$ the precession rate $\rightarrow 0$.

10.57. **IDENTIFY:** As you pedal your bike, you turn a large sprocket which then turns a smaller sprocket on the wheel, and this causes the wheel to turn. Fig. 10.57 illustrates this arrangement.

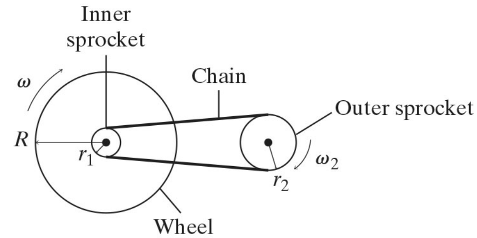


Figure 10.57

SET UP: Estimates: Bicycle wheel is 30 in. in diameter, large sprocket is 8.0 in. in diameter, and small sprocket is 4.0 in. in diameter. This means that in Fig. 10.57, $R = 15$ in., $r_1 = 2.0$ in., and $r_2 = 4.0$ in. We assume that the bike's wheel does not slip on the pavement, so $v_{cm} = R\omega$. The target variable is the angular speed of the large sprocket while the bike is traveling at 30 mph.

EXECUTE: <u>Wheel</u>: $v_{cm} = R\omega = 30$ mph $= 44$ ft/s, so $\omega = (44\text{ ft/s})/(15/12\text{ ft}) = 35.2$ rad/s.

<u>Inner sprocket</u>: It turns at the same angular speed as the wheel, so its speed is $v_1 = r_1\omega$.

<u>Outer sprocket</u>: It is connected by a chain to the inner sprocket, so its tangential speed is the same as that of the inner sprocket, which means that $v_1 = v_2$ and $v_2 = r_2\omega_2$. Equating these two speeds gives $r_1\omega = r_2\omega_2$, so $\omega_2 = \frac{r_1}{r_2}\omega = \frac{2.0\text{ in.}}{4.0\text{ in.}}(35.2\text{ rad/s}) = 17.6$ rad/s. Converting to rpm gives

$$\omega_2 = \left(17.6\frac{\text{rad}}{\text{s}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 170\text{ rev/min}.$$

EVALUATE: By contrast, the bike's wheel turns at $\omega = 35$ rad/s $= 330$ rpm.

10.58. **IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$ and constant acceleration equations to the motion of the grindstone.

SET UP: Let the direction of rotation of the grindstone be positive. The friction force is $f = \mu_k n$ and produces torque fR. $\omega = (120\text{rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 4\pi$ rad/s. $I = \frac{1}{2}MR^2 = 1.69\text{ kg}\cdot\text{m}^2$.

EXECUTE: **(a)** The net torque must be

$$\tau = I\alpha = I\frac{\omega_z - \omega_{0z}}{t} = (1.69\text{ kg}\cdot\text{m}^2)\frac{4\pi\text{ rad/s}}{9.00\text{ s}} = 2.36\text{ N}\cdot\text{m}.$$

This torque must be the sum of the applied force FR and the opposing frictional torques τ_f at the axle and $fR = \mu_k nR$ due to the knife. $F = \frac{1}{R}(\tau + \tau_f + \mu_k nR)$.

$$F = \frac{1}{0.500\text{ m}}\left[(2.36\text{ N}\cdot\text{m}) + (6.50\text{ N}\cdot\text{m}) + (0.60)(160\text{ N})(0.260\text{ m})\right] = 67.6\text{ N}.$$

(b) To maintain a constant angular velocity, the net torque τ is zero, and the force F' is

$$F' = \frac{1}{0.500\text{ m}}(6.50\text{ N}\cdot\text{m} + 24.96\text{ N}\cdot\text{m}) = 62.9\text{ N}.$$

(c) The time t needed to come to a stop is found by taking the magnitudes in $\vec{\tau} = \frac{d\vec{\boldsymbol{L}}}{dt}$, with $\tau = \tau_f$ constant; $t = \frac{L}{\tau_f} = \frac{\omega I}{\tau_f} = \frac{(4\pi\text{ rad/s})(1.69\text{ kg}\cdot\text{m}^2)}{6.50\text{ N}\cdot\text{m}} = 3.27$ s.

EVALUATE: The time for a given change in ω is proportional to α, which is in turn proportional to the net torque, so the time in part (c) can also be found as $t = (9.00\text{ s})\frac{2.36\text{ N}\cdot\text{m}}{6.50\text{ N}\cdot\text{m}}$.

10.59. **IDENTIFY:** Use the kinematic information to solve for the angular acceleration of the grindstone. Assume that the grindstone is rotating counterclockwise and let that be the positive sense of rotation. Then apply $\sum\tau_z = I\alpha_z$ to calculate the friction force and use $f_k = \mu_k n$ to calculate μ_k.

SET UP: $\omega_{0z} = 850\text{ rev/min}(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 89.0\text{ rad/s}$

$t = 7.50$ s; $\omega_z = 0$ (comes to rest); $\alpha_z = ?$

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t$

$$\alpha_z = \frac{0 - 89.0\text{ rad/s}}{7.50\text{ s}} = -11.9\text{ rad/s}^2$$

SET UP: Apply $\sum\tau_z = I\alpha_z$ to the grindstone. The free-body diagram is given in Figure 10.59.

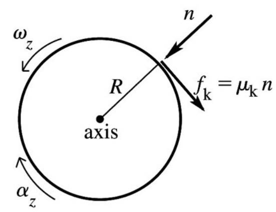


Figure 10.59

The normal force has zero moment arm for rotation about an axis at the center of the grindstone, and therefore zero torque. The only torque on the grindstone is that due to the friction force f_k exerted by the ax; for this force the moment arm is $l = R$ and the torque is negative.

EXECUTE: $\sum\tau_z = -f_k R = -\mu_k nR$

$I = \frac{1}{2}MR^2$ (solid disk, axis through center)

Thus $\sum\tau_z = I\alpha_z$ gives $-\mu_k nR = (\frac{1}{2}MR^2)\alpha_z$

$$\mu_k = -\frac{MR\alpha_z}{2n} = -\frac{(50.0\text{ kg})(0.260\text{ m})(-11.9\text{ rad/s}^2)}{2(160\text{ N})} = 0.483$$

EVALUATE: The friction torque is clockwise and slows down the counterclockwise rotation of the grindstone.

10.60. IDENTIFY: This problem involves moving blocks and a turning pulley, so we need to use Newton's second law in its linear and rotational forms. The constant-acceleration formulas also apply.

SET UP: Apply $\sum F_x = ma_x$ and $\sum F_y = ma_y$ to the blocks and $\sum\tau_z = I\alpha_z$ to the pulley. We also apply $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ to block B. The target variables are the tension in the rope on both sides of the pulley and the moment of inertia of the pulley.

EXECUTE: **(a)** First use $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ to find the acceleration of the blocks, giving $\Delta y = \frac{1}{2}at^2$. $a = 2(1.80\text{ m})/(2.00\text{ s})^2 = 0.900\text{ m/s}^2$. Now apply $\sum F_y = ma_y$ to block B. Choose $+y$ downward since that is the direction of the acceleration. This gives $m_B g - T_B = m_B a$, and solving for T_B gives

$T_B = m_B(g-a) = (6.00\text{ kg})(9.80\text{ m/s}^2 - 0.900\text{ m/s}^2) = 53.4\text{ N}$.

(b) Apply $\sum F_x = ma_x$ to block A: $T_A = m_A a = (2.50\text{ kg})(0.900\text{ m/s}^2) = 2.25\text{ N}$.

(c) Apply $\sum\tau_z = I\alpha_z$ to the pulley. It is important to realize that the tensions on the pulley are *not* the same because the pulley has mass. The acceleration of the blocks is the same as the tangential acceleration of the rope on the pulley, so $\alpha_z = a/R$, where R is the pulley radius. Therefore we get

$T_B R - T_A R = I(a/R)$, so $I = \frac{R^2}{a}(T_B - T_A) = \frac{(0.0800\text{ m})^2}{0.900\text{ m/s}^2}(53.4\text{ N} - 2.25\text{ N}) = 0.364\text{ kg}\cdot\text{m}^2$.

EVALUATE: We found that $T_B > T_A$ which is reasonable since their torques are what give the pulley its angular acceleration causing it to turn in a direction that allows B to go downward.

10.61. IDENTIFY: Use $\sum\tau_z = I\alpha_z$ to find the angular acceleration just after the ball falls off and use conservation of energy to find the angular velocity of the bar as it swings through the vertical position.

SET UP: The axis of rotation is at the axle. For this axis the bar has $I = \frac{1}{12}m_{\text{bar}}L^2$, where $m_{\text{bar}} = 3.80\text{ kg}$ and $L = 0.800\text{ m}$. Energy conservation gives $K_1 + U_1 = K_2 + U_2$. The gravitational potential energy of the bar doesn't change. Let $mg(\sin 36.9° - \mu_k\cos 36.9°) - T = ma$ so $y_2 = -L/2$.

EXECUTE: **(a)** $\tau_z = m_{\text{ball}}g(L/2)$ and $I = I_{\text{ball}} + I_{\text{bar}} = \frac{1}{12}m_{\text{bar}}L^2 + m_{\text{ball}}(L/2)^2$. $\sum\tau_z = I\alpha_z$ gives

$$\alpha_z = \frac{m_{\text{ball}}g(L/2)}{\frac{1}{12}m_{\text{bar}}L^2 + m_{\text{ball}}(L/2)^2} = \frac{2g}{L}\left(\frac{m_{\text{ball}}}{m_{\text{ball}} + m_{\text{bar}}/3}\right) \text{ and}$$

$$\alpha_z = \frac{2(9.80\text{ m/s}^2)}{0.800\text{ m}}\left(\frac{2.50\text{ kg}}{2.50\text{ kg} + [3.80\text{ kg}]/3}\right) = 16.3\text{ rad/s}^2.$$

(b) As the bar rotates, the moment arm for the weight of the ball decreases and the angular acceleration of the bar decreases.

(c) $K_1 + U_1 = K_2 + U_2$. $0 = K_2 + U_2$. $\frac{1}{2}(I_{\text{bar}} + I_{\text{ball}})\omega^2 = -m_{\text{ball}}g(-L/2)$.

$$\omega = \sqrt{\frac{m_{\text{ball}}gL}{m_{\text{ball}}L^2/4 + m_{\text{bar}}L^2/12}} = \sqrt{\frac{g}{L}\left(\frac{4m_{\text{ball}}}{m_{\text{ball}} + m_{\text{bar}}/3}\right)} = \sqrt{\frac{9.80\text{ m/s}^2}{0.800\text{ m}}\left(\frac{4(2.50\text{ kg})}{2.50\text{ kg} + (3.80\text{ kg})/3}\right)}$$

$\omega = 5.70$ rad/s.

EVALUATE: As the bar swings through the vertical, the linear speed of the ball that is still attached to the bar is $v = (0.400\text{ m})(5.70\text{ rad/s}) = 2.28\text{ m/s}$. A point mass in free-fall acquires a speed of 2.80 m/s after falling 0.400 m; the ball on the bar acquires a speed less than this.

10.62. **IDENTIFY:** A solid sphere rolls up a ramp without slipping. Newton's second law in its linear and angular forms applies to the sphere.

SET UP: Apply $\sum\tau_z = I\alpha_z$ and $\sum F_x = ma_x$ to the sphere. Choose the x-axis to be parallel to the surface of the ramp with $+x$ down the ramp since that is the direction of the acceleration. $I_{\text{sphere}} = \frac{2}{5}MR^2$. Our target variables are the linear acceleration of the sphere and the friction force on it.

EXECUTE: **(a)** $\sum F_x = ma_x$: $mg\sin\beta - f = ma_{\text{cm}}$ (Eq. 1)

$\sum\tau_z = I\alpha_z$: $fR = \frac{2}{5}mR^2\alpha$. There is no slipping, so $\alpha = a_{\text{cm}}/R$, which gives $fR = \frac{2}{5}mR^2\left(\frac{a_{\text{cm}}}{R}\right)$.

Solving for f gives $f = \frac{2}{5}ma_{\text{cm}}$ (Eq. 2)

Combining Eq. 1 and Eq. 2 gives $mg\sin\beta - \frac{2}{5}ma_{\text{cm}} = ma_{\text{cm}}$, from which we get $a_{\text{cm}} = \frac{5}{7}g\sin\beta$. This is the same result as in Example 10.7.

(b) Eq. 2 gives $f = \frac{2}{5}ma_{\text{cm}} = \frac{2}{5}m\left(\frac{5}{7}g\sin\beta\right) = \frac{2}{7}mg\sin\beta$. This is the same as required to prevent the sphere to slip while rolling down the ramp.

EVALUATE: Whether rolling up the ramp or down the ramp, friction is up the ramp and gravity has a component down the ramp, so we get the same answers in Example 10.7.

10.63. **IDENTIFY:** Blocks A and B have linear acceleration and therefore obey the linear form of Newton's second law $\sum F_y = ma_y$. The wheel C has angular acceleration, so it obeys the rotational form of Newton's second law $\sum\tau_z = I\alpha_z$.

SET UP: A accelerates downward, B accelerates upward and the wheel turns clockwise. Apply $\sum F_y = ma_y$ to blocks A and B. Let $+y$ be downward for A and $+y$ be upward for B. Apply $\sum\tau_z = I\alpha_z$ to the wheel, with the clockwise sense of rotation positive. Each block has the same magnitude of acceleration, a, and $a = R\alpha$. Call the T_A the tension in the cord between C and A and T_B the tension between C and B.

EXECUTE: For A, $\sum F_y = ma_y$ gives $m_A g - T_A = m_A a$. For B, $\sum F_y = ma_y$ gives $T_B - m_B g = m_B a$. For the wheel, $\sum \tau_z = I\alpha_z$ gives $T_A R - T_B R = I\alpha = I(a/R)w$ and $T_A - T_B = \left(\frac{I}{R^2}\right)a$. Adding these three equations gives $(m_A - m_B)g = \left(m_A + m_B + \frac{I}{R^2}\right)a$. Solving for a, we have

$$a = \left(\frac{m_A - m_B}{m_A + m_B + I/R^2}\right)g = \left(\frac{4.00\text{ kg} - 2.00\text{ kg}}{4.00\text{ kg} + 2.00\text{ kg} + (0.220\text{ kg}\cdot\text{m}^2)/(0.120\text{ m})^2}\right)(9.80\text{ m/s}^2) = 0.921\text{ m/s}^2.$$

$$\alpha = \frac{a}{R} = \frac{0.921\text{ m/s}^2}{0.120\text{ m}} = 7.68\text{ rad/s}^2.$$

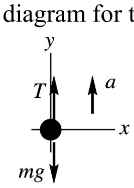
$$T_A = m_A(g - a) = (4.00\text{ kg})(9.80\text{ m/s}^2 - 0.921\text{ m/s}^2) = 35.5\text{ N}.$$

$$T_B = m_B(g + a) = (2.00\text{ kg})(9.80\text{ m/s}^2 + 0.921\text{ m/s}^2) = 21.4\text{ N}.$$

EVALUATE: The tensions must be different in order to produce a torque that accelerates the wheel when the blocks accelerate.

10.64. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the crate and $\sum \tau_z = I\alpha_z$ to the cylinder. The motions are connected by $a(\text{crate}) = R\alpha(\text{cylinder})$.

SET UP: The force diagram for the crate is given in Figure 10.64a.

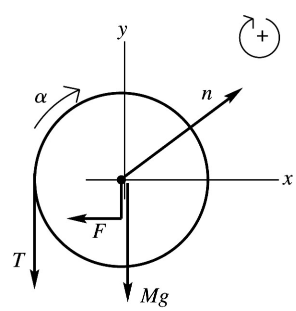


EXECUTE: Applying $\sum F_y = ma_y$ gives $T - mg = ma$. Solving for T gives

$$T = m(g + a) = (50\text{ kg})(9.80\text{ m/s}^2 + 1.40\text{ m/s}^2) = 5$$

Figure 10.64a

SET UP: The force diagram for the cylinder is given in Figure 10.64b.



EXECUTE: $\sum \tau_z = I\alpha_z$ gives $Fl - TR = I\alpha_z$, where $l = 0.12$ m and $R = 0.25$ m. $a = R\alpha$ so $\alpha_z = a/R$. Therefore $Fl = TR + Ia/R$.

Figure 10.64b

$$F = T\left(\frac{R}{l}\right) + \frac{Ia}{Rl} = (560\text{ N})\left(\frac{0.25\text{ m}}{0.12\text{ m}}\right) + \frac{(2.9\text{ kg}\cdot\text{m}^2)(1.40\text{ m/s}^2)}{(0.25\text{ m})(0.12\text{ m})} = 1300\text{ N}.$$

EVALUATE: The tension in the rope is greater than the weight of the crate since the crate accelerates upward. If F were applied to the rim of the cylinder ($l = 0.25$ m), it would have the value $F = 625$ N. This is greater than T because it must accelerate the cylinder as well as the crate. And F is larger than this because it is applied closer to the axis than R so has a smaller moment arm and must be larger to give the same torque.

10.65. IDENTIFY: A hollow sphere and a solid sphere roll up a ramp without slipping starting with the same speed at the base. Energy conservation applies to both of them.

SET UP: We use $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, with $K_2 = 0$, $U_1 = 0$, $U_2 = mgh$, and $W_{\text{other}} = 0$. The total kinetic energy is $K_{\text{tot}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2$, where $I_{\text{solid}} = \frac{2}{5}MR^2$ and $I_{\text{hollow}} = \frac{2}{3}MR^2$. We want to know which sphere reaches the greater height on the ramp.

EXECUTE: Apply energy conservation to each sphere to find h in each case.

<u>Solid sphere</u>: $mgh_{\text{s}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2$, so $h_{\text{s}} = \frac{7}{10}\frac{v_{\text{cm}}^2}{g} = 0.700\frac{v_{\text{cm}}^2}{g}$.

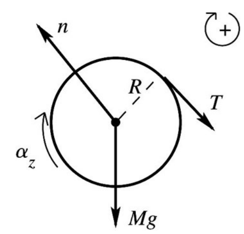
<u>Hollow sphere</u>: $mgh_{\text{H}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2$, so $h_{\text{H}} = \frac{5}{6}\frac{v_{\text{cm}}^2}{g} = 0.833\frac{v_{\text{cm}}^2}{g}$.

The hollow sphere reaches a greater height than the solid sphere.

EVALUATE: Even though the two spheres have the same size, mass, linear speed, and angular speed at the bottom of the ramp, they do not have the same kinetic energy because the hollow sphere has a greater moment of inertia. Therefore the hollow sphere goes higher up the ramp.

10.66. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ to the flywheel and $\sum\vec{F} = m\vec{a}$ to the block. The target variables are the tension in the string and the acceleration of the block.

(a) SET UP: Apply $\sum\tau_z = I\alpha_z$ to the rotation of the flywheel about the axis. The free-body diagram for the flywheel is given in Figure 10.66a.



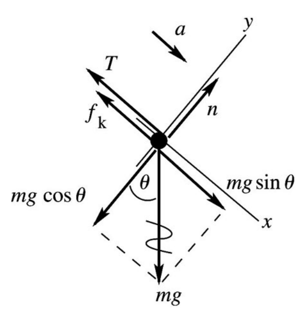
EXECUTE: The forces n and Mg act at the axis so have zero torque.

$\sum\tau_z = TR$

$TR = I\alpha_z$

Figure 10.66a

SET UP: Apply $\sum\vec{F} = m\vec{a}$ to the translational motion of the block. The free-body diagram for the block is given in Figure 10.66b.



EXECUTE: $\Sigma F_y = ma_y$

$n - mg\cos 36.9° = 0$

$n = mg\cos 36.9°$

$f_{\text{k}} = \mu_{\text{k}}n = \mu_{\text{k}}mg\cos 36.9°$

Figure 10.66b

$\sum F_x = ma_x$

$mg\sin 36.9° - T - \mu_{\text{k}}mg\cos 36.9° = ma$

$mg(\sin 36.9° - \mu_{\text{k}}\cos 36.9°) - T = ma$

But we also know that $a_{\text{block}} = R\alpha_{\text{wheel}}$, so $\alpha = a/R$. Using this in the $\sum\tau_z = I\alpha_z$ equation gives $TR = Ia/R$ and $T = (I/R^2)a$. Use this to replace T in the $\sum F_x = ma_x$ equation:

$$mg(\sin 36.9° - \mu_k \cos 36.9°) - (I/R^2)a = ma$$

$$a = \frac{mg(\sin 36.9° - \mu_k \cos 36.9°)}{m + I/R^2}$$

$$a = \frac{(5.00\text{ kg})(9.80\text{ m/s}^2)[\sin 36.9° - (0.25)\cos 36.9°]}{5.00\text{ kg} + 0.500\text{ kg}\cdot\text{m}^2/(0.200\text{ m})^2} = 1.12\text{ m/s}^2.$$

(b) $T = \dfrac{0.500\text{ kg}\cdot\text{m}^2}{(0.200\text{ m})^2}(1.12\text{ m/s}^2) = 14.0\text{ N}$

EVALUATE: If the string is cut the block will slide down the incline with $a = g\sin 36.9° - \mu_k g\cos 36.9° = 3.92\text{ m/s}^2$. The actual acceleration is less than this because $mg\sin 36.9°$ must also accelerate the flywheel. $mg\sin 36.9° - f_k = 19.6$ N. T is less than this; there must be more force on the block directed down the incline than up the incline since the block accelerates down the incline.

10.67. **IDENTIFY:** A force produces a torque on a wheel, giving it an angular acceleration. But the force is not constant, so the angular acceleration is not constant.

SET UP: The force is $F = kt$, where $k = 5.00$ N/s. Our target variable is the magnitude of the force at the instant the wheel has turned through 8.00 rev (which is 16.0π rad). We can apply $\sum\tau_z = I\alpha_z$ to the wheel but we cannot use the constant-acceleration equations. We must return to the basic definitions $\omega_z = d\theta/dt$ and $\alpha_z = d\omega_z/dt$ and integrate.

EXECUTE: First apply $\sum\tau_z = I\alpha_z$ to get α_z, calling R the wheel radius. $FR = ktR = I\alpha_z$, so $\alpha_z = Rkt/I = Bt$, where $B = Rk/I$. Since α_z is a function of time, we must integrate to get the angle turned through $\Delta\theta$. $\omega_z = \int \alpha_z dt = \int Bt\,dt = \dfrac{Bt^2}{2}$, where we have used $\omega_z = 0$ at $t = 0$. Now integrate ω_z to find $\Delta\theta$. $\Delta\theta = \int \omega dt = \int \dfrac{Bt^2}{2} dt = \dfrac{Bt^3}{6}$, so $t = \left(\dfrac{6\Delta\theta}{B}\right)^{1/3}$. The force at this time is

$$F = kt = k\left(\frac{6\Delta\theta}{B}\right)^{1/3} = k\left(\frac{96\pi I}{kR}\right)^{1/3} = (5.00\text{ N/s})\left[\frac{96\pi(2.50\text{ kg}\cdot\text{m}^2)}{(5.00\text{ N/s})(0.0600\text{ m})}\right]^{1/3} = 68.0\text{ N}.$$

EVALUATE: When the acceleration (linear or angular) is not constant, we have little choice but to return to basic definitions and use calculus.

10.68. **IDENTIFY:** Apply both $\sum\vec{F} = m\vec{a}$ and $\sum\tau_z = I\alpha_z$ to the motion of the roller. Rolling without slipping means $a_{\text{cm}} = R\alpha$. Target variables are a_{cm} and f.

SET UP: The free-body diagram for the roller is given in Figure 10.68.

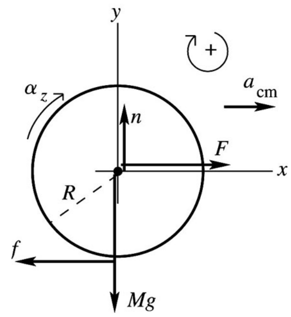


Figure 10.68

EXECUTE: Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the center of mass:

$\sum F_x = ma_x$

$F - f = Ma_{\text{cm}}$

Apply $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass:

$\sum \tau_z = fR$

thin-walled hollow cylinder: $I = MR^2$

Then $\sum \tau_z = I\alpha_z$ implies $fR = MR^2\alpha.$

But $\alpha_{\text{cm}} = R\alpha,$ so $f = Ma_{\text{cm}}.$

Using this in the $\sum F_x = ma_x$ equation gives $F - Ma_{\text{cm}} = Ma_{\text{cm}}.$

$a_{\text{cm}} = F/2M,$ and then $f = Ma_{\text{cm}} = M(F/2M) = F/2.$

EVALUATE: If the surface were frictionless the object would slide without rolling and the acceleration would be $a_{\text{cm}} = F/M$. The acceleration is less when the object rolls.

10.69. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each object and apply $\sum \tau_z = I\alpha_z$ to the pulley.

SET UP: Call the 75.0 N weight *A* and the 125 N weight *B*. Let T_A and T_B be the tensions in the cord to the left and to the right of the pulley. For the pulley, $I = \frac{1}{2}MR^2,$ where $Mg = 80.0\text{ N}$ and $R = 0.300\text{ m}.$ The 125 N weight accelerates downward with acceleration *a*, the 75.0 N weight accelerates upward with acceleration *a* and the pulley rotates clockwise with angular acceleration α, where $a = R\alpha.$

EXECUTE: $\sum \vec{F} = m\vec{a}$ applied to the 75.0 N weight gives $T_A - w_A = m_A a.$ $\sum \vec{F} = m\vec{a}$ applied to the 125.0 N weight gives $w_B - T_B = m_B a.$ $\sum \tau_z = I\alpha_z$ applied to the pulley gives $(T_B - T_A)R = (\frac{1}{2}MR^2)\alpha_z$ and $T_B - T_A = \frac{1}{2}Ma.$ Combining these three equations gives $w_B - w_A = (m_A + m_B + M/2)a$ and

$$a = \left(\frac{w_B - w_A}{w_A + w_B + w_{\text{pulley}}/2}\right)g = \left(\frac{125\text{ N} - 75.0\text{ N}}{75.0\text{ N} + 125\text{ N} + 40.0\text{ N}}\right)g = 0.2083g.$$

$T_A = w_A(1 + a/g) = 1.2083w_A = 90.62\text{ N}.$ $T_B = w_B(1 - a/g) = 0.792w_B = 98.96\text{ N}.$ $\sum \vec{F} = m\vec{a}$ applied to the pulley gives that the force *F* applied by the hook to the pulley is $F = T_A + T_B + w_{\text{pulley}} = 270\text{ N}.$ The force the ceiling applies to the hook is 270 N.

EVALUATE: The force the hook exerts on the pulley is less than the total weight of the system, since the net effect of the motion of the system is a downward acceleration of mass.

10.70. **IDENTIFY:** Dropping the object on the rotating turntable slows down the turntable and speeds up the object, but it does not change the total angular momentum of the object-turntable system.

SET UP: Angular momentum is conserved, so $L_{\text{table}} = L_{\text{table+object}}$, where $L = I\omega$. Our target variable is the moment of inertia *I* of the table.

EXECUTE: Using so $L_{\text{table}} = L_{\text{table+object}}$ gives $I\omega = \left(I + I_{object}\right)\omega_f = \left(I + mR^2\right)\omega_f$. Now solve for $\dfrac{\omega - \omega_f}{\omega_f}$, which gives $\dfrac{\omega - \omega_f}{\omega_f} = \left(\dfrac{R^2}{I}\right)m$. From this we see that a graph of $\dfrac{\omega - \omega_f}{\omega_f}$ versus m should give a straight line having slope equal to R^2/I. Thus $I = \dfrac{R^2}{\text{slope}} = \dfrac{(3.00\text{ m})^2}{0.250\text{ kg}^{-1}} = 36.0\text{ kg}\cdot\text{m}^2$.

EVALUATE: It would be quite difficult to measure directly the moment of a large heavy turntable, especially if it was not uniform throughout. But the measurements described here would be fairly easy to make.

10.71. IDENTIFY: Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ to the motion of the center of mass and apply $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the rotation about the center of mass.

SET UP: $I = 2(\tfrac{1}{2}mR^2) = mR^2$. The moment arm for T is b.

EXECUTE: The tension is related to the acceleration of the yo-yo by $(2m)g - T = (2m)a$, and to the angular acceleration by $Tb = I\alpha = I\dfrac{a}{b}$. Dividing the second equation by b and adding to the first to eliminate T yields $a = g\dfrac{2m}{(2m + I/b^2)} = g\dfrac{2}{2 + (R/b)^2}$, $\alpha = g\dfrac{2}{2b + R^2/b}$. The tension is found by substitution into either of the two equations:

$$T = (2m)(g - a) = (2mg)\left(1 - \frac{2}{2 + (R/b)^2}\right) = 2mg\frac{(R/b)^2}{2 + (R/b)^2} = \frac{2mg}{(2(b/R)^2 + 1)}.$$

EVALUATE: $a \to 0$ when $b \to 0$. As $b \to R$, $a \to 2g/3$.

10.72. IDENTIFY: Apply conservation of energy to the motion of the shell, to find its linear speed v at points A and B. Apply $\sum \vec{F} = m\vec{a}$ to the circular motion of the shell in the circular part of the track to find the normal force exerted by the track at each point. Since $r << R$ the shell can be treated as a point mass moving in a circle of radius R when applying $\sum \vec{F} = m\vec{a}$. But as the shell rolls along the track, it has both translational and rotational kinetic energy.

SET UP: $K_1 + U_1 = K_2 + U_2$. Let 1 be at the starting point and take $y = 0$ to be at the bottom of the track, so $y_1 = h_0$. $K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$. $I = \tfrac{2}{3}mr^2$ and $\omega = v/r$, so $K = \tfrac{5}{6}mv^2$. During the circular motion, $a_{\text{rad}} = v^2/R$.

EXECUTE: **(a)** $\sum \vec{F} = m\vec{a}$ at point A gives $n + mg = m\dfrac{v^2}{R}$. The minimum speed for the shell not to fall off the track is when $n \to 0$ and $v^2 = gR$. Let point 2 be A, so $y_2 = 2R$ and $v_2^2 = gR$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = 2mgR + \tfrac{5}{6}m(gR)$. $h_0 = (2 + \tfrac{5}{6})R = \tfrac{17}{6}R$.

(b) Let point 2 be B, so $y_2 = R$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = mgR + \tfrac{5}{6}mv_2^2$. With $h = \tfrac{17}{6}R$ this gives $v^2 = \tfrac{11}{5}gR$. Then $\sum \vec{F} = m\vec{a}$ at B gives $n = m\dfrac{v^2}{R} = \tfrac{11}{5}mg$.

(c) Now $K = \tfrac{1}{2}mv^2$ instead of $\tfrac{5}{6}mv^2$. The shell would be moving faster at A than with friction and would still make the complete loop.

(d) In part (c): $mgh_0 = mg(2R) + \frac{1}{2}mv^2$. $h_0 = \frac{17}{6}R$ gives $v^2 = \frac{5}{3}gR$. $\sum \vec{F} = m\vec{a}$ at point A gives $mg + n = m\dfrac{v^2}{R}$ and $n = m\left(\dfrac{v^2}{R} - g\right) = \frac{2}{3}mg$. In part (a), $n = 0$, since at this point gravity alone supplies the net downward force that is required for the circular motion.

EVALUATE: The normal force at A is greater when friction is absent because the speed of the shell at A is greater when friction is absent than when there is rolling without slipping.

10.73. **IDENTIFY:** As it rolls down the rough slope, the basketball gains rotational kinetic energy as well as translational kinetic energy. But as it moves up the smooth slope, its rotational kinetic energy does not change since there is no friction.

SET UP: $I_{cm} = \frac{2}{3}mR^2$. When it rolls without slipping, $v_{cm} = R\omega$. When there is no friction the angular speed of rotation is constant. Take $+y$ upward and let $y = 0$ in the valley.

EXECUTE: **(a)** Find the speed v_{cm} in the level valley: $K_1 + U_1 = K_2 + U_2$. $y_1 = H_0$, $y_2 = 0$. $K_1 = 0$, $U_2 = 0$. Therefore, $U_1 = K_2$. $mgH_0 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$. $\frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}(\frac{2}{3}mR^2)\left(\dfrac{v_{cm}}{R}\right)^2 = \frac{1}{3}mv_{cm}^2$, so $mgH_0 = \frac{5}{6}mv_{cm}^2$ and $v_{cm}^2 = \dfrac{6gH_0}{5}$. Find the height H it goes up the other side. Its rotational kinetic energy stays constant as it rolls on the frictionless surface. $\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}I_{cm}\omega^2 + mgH$.

$H = \dfrac{v_{cm}^2}{2g} = \frac{3}{5}H_0$.

(b) Some of the initial potential energy has been converted into rotational kinetic energy so there is less potential energy at the second height H than at the first height H_0.

EVALUATE: Mechanical energy is conserved throughout this motion. But the initial gravitational potential energy on the rough slope is not all transformed into potential energy on the smooth slope because some of that energy remains as rotational kinetic energy at the highest point on the smooth slope.

10.74. **IDENTIFY:** Apply conservation of energy to the motion of the ball as it rolls up the hill. After the ball leaves the edge of the cliff it moves in projectile motion and constant acceleration equations can be used.

(a) SET UP: Use conservation of energy to find the speed v_2 of the ball just before it leaves the top of the cliff. Let point 1 be at the bottom of the hill and point 2 be at the top of the hill. Take $y = 0$ at the bottom of the hill, so $y_1 = 0$ and $y_2 = 28.0$ m.

EXECUTE: $K_1 + U_1 = K_2 + U_2$

$\frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 = mgy_2 + \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2$

Rolling without slipping means $\omega = v/r$ and $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{2}{5}mr^2)(v/r)^2 = \frac{1}{5}mv^2$.

$\frac{7}{10}mv_1^2 = mgy_2 + \frac{7}{10}mv_2^2$

$v_2 = \sqrt{v_1^2 - \frac{10}{7}gy_2} = 15.26$ m/s

SET UP: Consider the projectile motion of the ball, from just after it leaves the top of the cliff until just before it lands. Take $+y$ to be downward. Use the vertical motion to find the time in the air:

$v_{0y} = 0$, $a_y = 9.80$ m/s^2, $y - y_0 = 28.0$ m, $t = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 2.39$ s

During this time the ball travels horizontally

$x - x_0 = v_{0x}t = (15.26 \text{ m/s})(2.39 \text{ s}) = 36.5$ m.

Just before it lands, $v_y = v_{0y} + a_y t = 23.4$ m/s and $v_x = v_{0x} = 15.3$ m/s

$v = \sqrt{v_x^2 + v_y^2} = 28.0$ m/s

(b) EVALUATE: At the bottom of the hill, $\omega = v/r = (25.0 \text{ m/s})/r$. The rotation rate doesn't change while the ball is in the air, after it leaves the top of the cliff, so just before it lands $\omega = (15.3 \text{ m/s})/r$. The total kinetic energy is the same at the bottom of the hill and just before it lands, but just before it lands less of this energy is rotational kinetic energy, so the translational kinetic energy is greater.

10.75. IDENTIFY: Apply conservation of energy to the motion of the boulder.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ and $v = R\omega$ when there is rolling without slipping. $I = \frac{2}{5}mR^2$.

EXECUTE: Break into two parts, the rough and smooth sections.

Rough: $mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$. $v^2 = \frac{10}{7}gh_1$.

Smooth: Rotational kinetic energy does not change. $mgh_2 + \frac{1}{2}mv^2 + K_{\text{rot}} = \frac{1}{2}mv_{\text{Bottom}}^2 + K_{\text{rot}}$.

$$gh_2 + \frac{1}{2}\left(\frac{10}{7}gh_1\right) = \frac{1}{2}v_{\text{Bottom}}^2.\ v_{\text{Bottom}} = \sqrt{\frac{10}{7}gh_1 + 2gh_2} = \sqrt{\frac{10}{7}(9.80 \text{ m/s}^2)(25 \text{ m}) + 2(9.80 \text{ m/s}^2)(25 \text{ m})} = 29.0 \text{ m/s}.$$

EVALUATE: If all the hill was rough enough to cause rolling without slipping,

$v_{\text{Bottom}} = \sqrt{\frac{10}{7}g(50 \text{ m})} = 26.5$ m/s. A smaller fraction of the initial gravitational potential energy goes into translational kinetic energy of the center of mass than if part of the hill is smooth. If the entire hill is smooth and the boulder slides without slipping, $v_{\text{Bottom}} = \sqrt{2g(50 \text{ m})} = 31.3$ m/s. In this case all the initial gravitational potential energy goes into the kinetic energy of the translational motion.

10.76. IDENTIFY: Apply Newton's second law in its linear and rotational form to the cylinder. The cylinder does not slip on the surface of the ramp.

SET UP: $\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}}$, $\Sigma\tau_z = I\alpha_z$, $I = \frac{1}{2}mR^2$, and $a_{\text{cm}} = R\alpha$ for no slipping. Take the x-axis parallel to the surface of the ramp; call up the ramp positive since that is the direction in which the cylinders must accelerate. Take the y-axis perpendicular to the surface. For uniform acceleration $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$.

EXECUTE: (a) The forces balance in the y-direction, so the normal force n is $n = mg\cos\theta$. In the x-direction, $\Sigma F_x = ma_x$ gives

$F - f_s - mg\sin\theta = ma$.

Now apply $\Sigma\tau_z = I\alpha_z$.

$f_s R = (\frac{1}{2}mR^2)(a/R)$, which gives $a = 2f_s/m$. Putting this result into the previous result gives

$F - f_s - mg\sin\theta = m(2f_s/m) = 2f_s$.

Solving for F gives

$F = 3f_s + mg\sin\theta = 3\mu_s n + mg\sin\theta = 3\mu_s mg\cos\theta + mg\sin\theta = mg(3\mu_s\cos\theta + \sin\theta)$

$F = (460 \text{ kg})(9.80 \text{ m/s}^2)[3(0.120)\cos 37° + \sin 37°] = 4010$ N.

(b) From part (a) we have
$a = 2f_s/m = (2\mu_s mg\cos\theta)/m = 2\mu_s g\cos\theta$.
Linear kinematics using $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$$t = \sqrt{\frac{2(x-x_0)}{a}} = \sqrt{\frac{2(x-x_0)}{2\mu_s g\cos\theta}} = \sqrt{\frac{6.00\text{ m}}{(0.120)(9.80\text{ m/s}^2)\cos 37°}} = 2.53\text{ s}.$$

EVALUATE: Just lifting the 460-kg vertically would require a force of $mg = 4510$ N, so we don't do very much better by rolling them up the slope since friction opposes the linear motion.

10.77. IDENTIFY: Apply conservation of energy to the motion of the wheel.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. No slipping means that $\omega = v/R$. Uniform density means $m_r = \lambda 2\pi R$ and $m_s = \lambda R$, where m_r is the mass of the rim and m_s is the mass of each spoke. For the wheel, $I = I_{\text{rim}} + I_{\text{spokes}}$. For each spoke, $I = \frac{1}{3}m_s R^2$.

EXECUTE: **(a)** $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $I = I_{\text{rim}} + I_{\text{spokes}} = m_r R^2 + 6(\frac{1}{3}m_s R^2)$

Also, $m = m_r + m_s = 2\pi R\lambda + 6R\lambda = 2R\lambda(\pi + 3)$. Substituting into the conservation of energy equation gives $2R\lambda(\pi+3)gh = \frac{1}{2}(2R\lambda)(\pi+3)(R\omega)^2 + \frac{1}{2}\left[2\pi R\lambda R^2 + 6(\frac{1}{3}\lambda RR^2)\right]\omega^2$.

$$\omega = \sqrt{\frac{(\pi+3)gh}{R^2(\pi+2)}} = \sqrt{\frac{(\pi+3)(9.80\text{ m/s}^2)(58.0\text{ m})}{(0.210\text{ m})^2(\pi+2)}} = 124\text{ rad/s} \quad \text{and } v = R\omega = 26.0\text{ m/s}$$

(b) Doubling the density would have no effect because it does not appear in the answer. ω is inversely proportional to R so doubling the diameter would double the radius which would reduce ω by half, but $v = R\omega$ would be unchanged.

EVALUATE: Changing the masses of the rim and spokes by different amounts would alter the speed v at the bottom of the hill.

10.78. IDENTIFY: The rings and the rod exert forces on each other, but there is no net force or torque on the system, and so the angular momentum will be constant.

SET UP: For the rod, $I = \frac{1}{12}ML^2$. For each ring, $I = mr^2$, where r is their distance from the axis.

EXECUTE: **(a)** As the rings slide toward the ends, the moment of inertia changes, and the final angular velocity is given by $\omega_2 = \omega_1\frac{I_1}{I_2} = \omega_1\left[\frac{\frac{1}{12}ML^2 + 2mr_1^2}{\frac{1}{12}ML^2 + 2mr_2^2}\right] = \omega_1\left(\frac{5.00\times10^{-4}\text{ kg}\cdot\text{m}^2}{2.00\times10^{-3}\text{ kg}\cdot\text{m}^2}\right) = \frac{\omega_1}{4}$, so

$\omega_2 = 12.0$ rev/min.

(b) The forces and torques that the rings and the rod exert on each other will vanish, but the common angular velocity will be the same, 12.0 rev/min.

EVALUATE: Note that conversion from rev/min to rad/s was not necessary. The angular velocity of the rod decreases as the rings move away from the rotation axis.

10.79. IDENTIFY: Use conservation of energy to relate the speed of the block to the distance it has descended. Then use a constant acceleration equation to relate these quantities to the acceleration.

SET UP: For the cylinder, $I = \frac{1}{2}M(2R)^2$, and for the pulley, $I = \frac{1}{2}MR^2$.

EXECUTE: Doing this problem using kinematics involves four unknowns (six, counting the two angular accelerations), while using energy considerations simplifies the calculations greatly. If the block and the cylinder both have speed v, the pulley has angular velocity v/R and the cylinder has angular velocity $v/2R$, the total kinetic energy is

$$K = \frac{1}{2}\left[Mv^2 + \frac{M(2R)^2}{2}(v/2R)^2 + \frac{MR^2}{2}(v/R)^2 + Mv^2\right] = \frac{3}{2}Mv^2.$$

This kinetic energy must be the work done by gravity; if the hanging mass descends a distance y, $K = Mgy$, or $v^2 = (2/3)gy$. For constant acceleration, $v^2 = 2ay$, and comparison of the two expressions gives $a = g/3$.

EVALUATE: If the pulley were massless and the cylinder slid without rolling, $Mg = 2Ma$ and $a = g/2$. The rotation of the objects reduces the acceleration of the block.

10.80. **IDENTIFY:** Apply conservation of energy to the motion of the first ball before the collision and to the motion of the second ball after the collision. Apply conservation of angular momentum to the collision between the first ball and the bar.

SET UP: The speed of the ball just before it hits the bar is $v = \sqrt{2gy} = 15.34\text{ m/s}$. Use conservation of angular momentum to find the angular velocity ω of the bar just after the collision. Take the axis at the center of the bar.

EXECUTE: $L_1 = mvr = (5.00\text{ kg})(15.34\text{ m/s})(2.00\text{ m}) = 153.4\text{ kg}\cdot\text{m}^2/\text{s}$

Immediately after the collision the bar and both balls are rotating together.

$L_2 = I_{\text{tot}}\omega$

$I_{\text{tot}} = \frac{1}{12}Ml^2 + 2mr^2 = \frac{1}{12}(8.00\text{ kg})(4.00\text{ m})^2 + 2(5.00\text{ kg})(2.00\text{ m})^2 = 50.67\text{ kg}\cdot\text{m}^2$

$L_2 = L_1 = 153.4\text{ kg}\cdot\text{m}^2/\text{s}$

$\omega = L_2/I_{\text{tot}} = 3.027\text{ rad/s}$

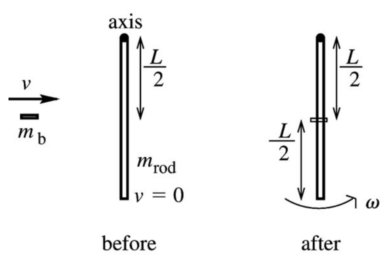
Just after the collision the second ball has linear speed $v = r\omega = (2.00\text{ m})(3.027\text{ rad/s}) = 6.055\text{ m/s}$ and is moving upward. $\frac{1}{2}mv^2 = mgy$ gives $y = 1.87\text{ m}$ for the height the second ball goes.

EVALUATE: Mechanical energy is lost in the inelastic collision and some of the final energy is in the rotation of the bar with the first ball stuck to it. As a result, the second ball does not reach the height from which the first ball was dropped.

10.81. **IDENTIFY:** Apply conservation of angular momentum to the collision. Linear momentum is not conserved because of the force applied to the rod at the axis. But since this external force acts at the axis, it produces no torque and angular momentum is conserved.

SET UP: The system before and after the collision is sketched in Figure 10.81.

EXECUTE: **(a)** $m_b = \frac{1}{4}m_{\text{rod}}$



EXECUTE: $L_1 = m_b vr = \frac{1}{4}m_{\text{rod}}v(L/2)$

$L_1 = \frac{1}{8}m_{\text{rod}}vL$

$L_2 = (I_{\text{rod}} + I_b)\omega$

$I_{\text{rod}} = \frac{1}{3}m_{\text{rod}}L^2$

$I_b = m_b r^2 = \frac{1}{4}m_{\text{rod}}(L/2)^2$

$I_b = \frac{1}{16}m_{\text{rod}}L^2$

Figure 10.81

Thus $L_1 = L_2$ gives $\frac{1}{8}m_{\text{rod}}vL = (\frac{1}{3}m_{\text{rod}}L^2 + \frac{1}{16}m_{\text{rod}}L^2)\omega$

$\frac{1}{8}v = \frac{19}{48}L\omega$

$\omega = \frac{6}{19}v/L$

(b) $K_1 = \frac{1}{2}mv^2 = \frac{1}{8}m_{\text{rod}}v^2$

$K_2 = \frac{1}{2}I\omega^2 = \frac{1}{2}(I_{\text{rod}} + I_{\text{b}})\omega^2 = \frac{1}{2}(\frac{1}{3}m_{\text{rod}}L^2 + \frac{1}{16}m_{\text{rod}}L^2)(6v/19L)^2$

$K_2 = \frac{1}{2}(\frac{19}{48})(\frac{6}{19})^2 m_{\text{rod}}v^2 = \frac{3}{152}m_{\text{rod}}v^2$

Then $\dfrac{K_2}{K_1} = \dfrac{\frac{3}{152}m_{\text{rod}}v^2}{\frac{1}{8}m_{\text{rod}}v^2} = 3/19.$

EVALUATE: The collision is inelastic and $K_2 < K_1$.

10.82. **IDENTIFY:** As you walk toward the center of the turntable, the angular momentum of the system (you plus turntable) is conserved. By getting closer to the center, you are decreasing the moment of inertia of the system. Newton's second law applies to you, and static friction provides the centripetal force on you.

SET UP: $I_0\omega_0 = I_2\omega_2$, $I = mr^2$ for a point mass, $a_{\text{rad}} = r\omega^2$, $f_{\text{s}}^{\max} = \mu_{\text{s}}n$, and $\Sigma\vec{F} = m\vec{a}$.

EXECUTE: At the closest distance, the friction force is

$f_{\text{s}} = \mu_{\text{s}}n = \mu_{\text{s}}mg$

Newton's second law gives

$f_{\text{s}} = ma = mr\omega^2$

Combining these two equations gives

$\mu_{\text{s}}mg = mr\omega^2$

Conservation of angular momentum gives $\omega = \dfrac{I_0}{I}\omega_0 = \left(\dfrac{I_{\text{t}} + mr_0^2}{I_{\text{t}} + mr^2}\right)\omega_0$. Solving the earlier equation for μ_{s} and using the previous result gives $\mu_{\text{s}} = \dfrac{\omega^2 r}{g} = \left(\dfrac{I_{\text{t}} + mr_0^2}{I_{\text{t}} + mr^2}\right)^2 \dfrac{\omega_0^2 r}{g}$. Putting in $m = 70.0$ kg, $r = 3.00$ m, and $I_{\text{t}} = 1200\ \text{kg}\cdot\text{m}^2$, and using $\omega_0 = 2\pi/(8.0\ \text{s})$, we get $\mu_{\text{s}} = 0.780$.

EVALUATE: This coefficient of static friction is physically reasonable.

10.83. **IDENTIFY:** As the disks are connected, their angular momentum is conserved, but some of their initial kinetic energy is converted to thermal energy. The 2400 J of thermal energy is equal to the loss of rotational kinetic energy.

SET UP: $I_1\omega_1 = I_2\omega_2$, $K = \frac{1}{2}I\omega^2$.

EXECUTE: Angular momentum conservation gives $I_A\omega_A = (I_A + I_B)\omega \rightarrow \omega = \dfrac{I_A\omega_A}{I_A + I_B}$. The loss of kinetic energy is $\Delta K = K_1 - K_2 = \frac{1}{2}I_A\omega_0^2 - \frac{1}{2}(I_A + I_B)\omega^2$. Combining these two equations gives $\Delta K = \dfrac{I_A\omega_0^2}{2}\left(1 - \dfrac{I_A}{I_A + I_B}\right)$. The loss of kinetic energy should be no more than 2400 J, so $\dfrac{I_A\omega_0^2}{2}\left(1 - \dfrac{I_A}{I_A + I_B}\right) \le 2400\ \text{J}$. The quantity $\dfrac{I_A\omega_0^2}{2}$ is the kinetic energy of A, K_A. Therefore we can solve the inequality for K_A, giving $K_A \le (2400\ \text{J})\left(\dfrac{I_A + I_B}{I_B}\right)$. Since $I_A = I_B/3$, the maximum kinetic energy of A is 3200 J.

EVALUATE: This situation is the rotational analog to a collision in which one object is initially at rest and they stick together. As in that situation, the momentum (angular in this case) is conserved but the kinetic energy is not.

10.84. **IDENTIFY:** This is a collision in which one object is initially stationary and they stick together. The rod is pivoted at one end, so it can only rotate after it is struck. The puck has angular momentum, some of which is transferred to the rod, but the angular momentum of the puck-rod system is conserved.

SET UP: The initial angular momentum of the puck is mvr, the final angular momentum of the rod is $I\omega$, and $I_{rod} = \frac{1}{3}ML^2$.

EXECUTE: After the collision, $\omega = 2\pi/T$, where $T = 0.736$ s, $r = L$, and $I = I_{rod} + I_{puck}$. Conservation of angular momentum gives $mvr = (\frac{1}{3}ML^2 + mL^2)\omega$. Solving for v gives $v = \dfrac{(\frac{1}{3}ML^2 + mL^2)\left(\frac{2\pi}{T}\right)}{mL}$. Putting in $m = 0.163$ kg, $M = 0.800$ kg, $L = 2.00$ m, $T = 0.736$ s gives $v = 45.0$ m/s.

EVALUATE: This situation is the rotational analog to a collision in which one object is initially at rest and they stick together. As in that situation, the momentum (angular in this case) is conserved but the kinetic energy is not.

10.85. **IDENTIFY:** Apply conservation of angular momentum to the collision between the bird and the bar and apply conservation of energy to the motion of the bar after the collision.

SET UP: For conservation of angular momentum take the axis at the hinge. For this axis the initial angular momentum of the bird is $m_{bird}(0.500\text{ m})v$, where $m_{bird} = 0.500$ kg and $v = 2.25$ m/s. For this axis the moment of inertia is $I = \frac{1}{3}m_{bar}L^2 = \frac{1}{3}(1.50\text{ kg})(0.750\text{ m})^2 = 0.281\text{ kg}\cdot\text{m}^2$. For conservation of energy, the gravitational potential energy of the bar is $U = m_{bar}gy_{cm}$, where y_{cm} is the height of the center of the bar. Take $y_{cm,1} = 0$, so $y_{cm,2} = -0.375$ m.

EXECUTE: **(a)** $L_1 = L_2$ gives $m_{bird}(0.500\text{ m})v = (\frac{1}{3}m_{bar}L^2)\omega$.

$$\omega = \frac{3m_{bird}(0.500\text{ m})v}{m_{bar}L^2} = \frac{3(0.500\text{ kg})(0.500\text{ m})(2.25\text{ m/s})}{(1.50\text{ kg})(0.750\text{ m})^2} = 2.00\text{ rad/s}.$$

(b) $U_1 + K_1 = U_2 + K_2$ applied to the motion of the bar after the collision gives

$$\tfrac{1}{2}I\omega_1^2 = m_{bar}g(-0.375\text{ m}) + \tfrac{1}{2}I\omega_2^2. \quad \omega_2 = \sqrt{\omega_1^2 + \frac{2}{I}m_{bar}g(0.375\text{ m})}.$$

$$\omega_2 = \sqrt{(2.00\text{ rad/s})^2 + \frac{2}{0.281\text{ kg}\cdot\text{m}^2}(1.50\text{ kg})(9.80\text{ m/s}^2)(0.375\text{ m})} = 6.58\text{ rad/s}.$$

EVALUATE: Mechanical energy is not conserved in the collision. The kinetic energy of the bar just after the collision is less than the kinetic energy of the bird just before the collision.

10.86. **IDENTIFY:** Angular momentum is conserved, since the tension in the string is in the radial direction and therefore produces no torque. Apply $\sum\vec{F} = m\vec{a}$ to the block, with $a = a_{rad} = v^2/r$.

SET UP: The block's angular momentum with respect to the hole is $L = mvr$.

EXECUTE: The tension is related to the block's mass and speed, and the radius of the circle, by $T = m\dfrac{v^2}{r}$. $T = mv^2\dfrac{1}{r} = \dfrac{m^2v^2}{m}\dfrac{r^2}{r^3} = \dfrac{(mvr)^2}{mr^3} = \dfrac{L^2}{mr^3}$. The radius at which the string breaks is

$$r^3 = \frac{L^2}{mT_{max}} = \frac{(mv_1r_1)^2}{mT_{max}} = \frac{[(0.130\text{ kg})(4.00\text{ m/s})(0.800\text{ m})]^2}{(0.130\text{ kg})(30.0\text{ N})},$$ from which $r = 0.354$ m.

EVALUATE: Just before the string breaks, the speed of the rock is $(4.00\text{ m/s})\left(\dfrac{0.800\text{ m}}{0.354\text{ m}}\right) = 9.04$ m/s.

We can verify that using $T = mv^2/R$ that $v = 9.04$ m/s and $r = 0.354$ m do give $T = 30.0$ N.

10.87. **IDENTIFY:** Apply conservation of momentum to the system of the runner and turntable.
SET UP: Let the positive sense of rotation be the direction the turntable is rotating initially.
EXECUTE: The initial angular momentum is $I\omega_1 - mRv_1,$ with the minus sign indicating that runner's motion is opposite the motion of the part of the turntable under his feet. The final angular momentum is $\omega_2(I+mR^2),$ so $\omega_2 = \dfrac{I\omega_1 - mRv_1}{I+mR^2}.$

$$\omega_2 = \frac{(80\text{ kg}\cdot\text{m}^2)(0.200\text{ rad/s}) - (55.0\text{ kg})(3.00\text{ m})(2.8\text{ m/s})}{(80\text{ kg}\cdot\text{m}^2) + (55.0\text{ kg})(3.00\text{ m})^2} = -0.776\text{ rad/s}.$$

EVALUATE: The minus sign indicates that the turntable has reversed its direction of motion. This happened because the man had the larger magnitude of angular momentum initially.

10.88. **IDENTIFY:** We use the power and angular velocity to calculate the torque.
SET UP: $P = \tau\omega,$ 1 hp = 746 W.
EXECUTE: **(a)** First make the necessary conversions: $1\text{ ft}\cdot\text{lb} = (0.3048\text{ m})(4.448\text{ N}) = 1.356\text{ N}\cdot\text{m}$
1 rpm = 1 rev/min = (2π rad)/(60 s) = 0.1047 rad/s.
Solve for torque and use the above conversions:
$\tau = P/\omega$ = [(285 hp)/(5300 rpm)]{(746 W/hp)/[(0.1047 rad/s)/rpm]} = 383 N·m = 283 ft·lb.
As we can see, 283 ft·lb is less than the maximum 305 ft·lb.
(b) $P = \tau\omega$ = (305 ft·lb) (3900 rpm)(1.356 N·m / ft·lb) [(0.1047 rad/s)/rpm] = 169 kW = 226 hp.
The power of 226 hp is smaller than the maximum of 285 hp.
(c) Make the following conversions:
$\text{hp} = \tau(\text{ft}\cdot\text{lb})\omega(\text{rpm})\left(\dfrac{1.356\text{ N}\cdot\text{m}}{1\text{ ft}\cdot\text{lb}}\right)\left(\dfrac{0.1047\text{ rad/s}}{1\text{ rpm}}\right)\left(\dfrac{1\text{ hp}}{746\text{ W}}\right) = 1.9031\times10^{-4}\ \tau(\text{ft}\cdot\text{lb})\omega(\text{rpm}),$ so $1/c$ = 1.9031×10^{-4}, which gives c = 5254.
(d) From (c), $P = \tau\omega$ gives 580 hp = τ(6000 rpm)/5254, so τ = 508 ft·lb.
EVALUATE: Torque, power, and angular velocity are often expressed in diverse units, so conversions are frequently necessary.

10.89. **IDENTIFY:** All the objects have the same mass and start from rest at the same height h. They roll without slipping, so their mechanical energy is conserved. Newton's second law, in its linear and rotational forms, applies to each object. Since the objects have different mass distributions, they will take different times to reach the bottom of the ramp.
SET UP: $K_1 + U_1 = K_2 + U_2,\ \ \sum\vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}},\ \ \Sigma\tau = I\alpha,\ \ K_{\text{tot}} = K_{\text{cm}} + K_{\text{rot}},\ \ K_{\text{cm}} = \tfrac{1}{2}Mv_{\text{cm}}^2,$ $K_{\text{rot}} = \tfrac{1}{2}I_{\text{cm}}\omega^2.$
EXECUTE: **(a)** We can express the moment of inertia of a round object as $I = cmR^2$, where c depends on the shape and mass distribution. Energy conservation gives $K_1 + U_1 = K_2 + U_2,$ so
$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}cmR^2\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}cmR^2\left(\dfrac{v}{R}\right)^2 = \tfrac{1}{2}v^2(1+c).$ Solving for v^2 gives $v^2 = \dfrac{2gh}{1+c}.$ This v is the speed at the bottom of the ramp. The object with the greatest speed v will also have the greatest average speed down the ramp and will therefore take the shortest time to reach the bottom. Thus the object with the smallest c will have the greatest v and therefore the shortest time in the bar graph shown with the problem. For a solid cylinder, $I = \tfrac{1}{2}mR^2$ so c = ½, for a hollow cylinder, $I = mR^2$, so c = 1, and likewise we get c = 2/5 for a solid sphere and c = 2/3 for a hollow sphere. The smallest value of c is 2/5 for a solid sphere, so that object must take the shortest time, which makes it

object *A*. The largest value of c is 1 for a hollow cylinder, so that object takes the longest time, which makes it object *D*. The hollow sphere has a larger c than the solid cylinder, so it takes longer than the solid cylinder, so *C* must be the hollow sphere and *B* the solid cylinder. Summarizing these results, we have

A: solid sphere, $c = 2/5$

B: solid cylinder, $c = 1/2$

C: hollow sphere, $c = 2/3$

D: hollow cylinder, $c = 1$

(b) All the objects start from rest at the same initial height and roll without slipping, so they all have the same kinetic energy at the bottom of the ramp.

(c) Using $K_{\text{rot}} = \frac{1}{2}I_{\text{cm}}\omega^2$, we have $K_{\text{rot}} = ½\,(cmR^2)(v/R)^2 = ½\,mcv^2$. Using our result for v^2 from (a) gives

$K_{\text{rot}} = \frac{1}{2}mc\left(\frac{2gh}{1+c}\right) = mgh\left(\frac{1}{1+\frac{1}{c}}\right)$. From this result, we see that the object with the largest c has the largest rotational kinetic energy because the denominator in the parentheses is the smallest. Therefore the hollow cylinder, with $c = 1$, has the largest rotational kinetic energy.

(d) Apply Newton's second law. Perpendicular to the ramp surface, we get $n = mg\cos\theta$ for the normal force. Parallel to the surface, with down the ramp as positive, we get $mg\sin\theta - f_s = ma$. Taking torques about the center of the rolling object gives $fR = I\alpha = (mcR^2)(a/R)$, which gives $f_s = mca$, so $ma = f_s/c$. Putting this into the previous equation gives $mg\sin\theta - f_s = f_s/c$, which can be written as $mg\sin\theta = f_s(1 + 1/c)$. We want the minimum coefficient of friction to prevent slipping, so $f_s = \mu_s n = \mu_s mg\cos\theta$. Putting this into the previous equation gives $mg\sin\theta = (\mu_s mg\cos\theta)(1 + 1/c)$.

Solving for μ_s gives $\mu_s = \dfrac{\tan\theta}{1+\frac{1}{c}}$. We want μ_s such that none of the objects will slip, so we must find the maximum μ_s. That will occur when c has its largest value since that will make the denominator smallest, and that is for the hollow cylinder for which $c = 1$. This gives $\mu_s = (\tan 35.0°)/2 = 0.350$.

EVALUATE: As a check, part (a) could be solved using Newton's second law, as we did in part (d). As a check in part (d), find μ_s for the solid sphere which has the smallest value of c. This gives

$\mu_s = \dfrac{\tan 35.0°}{1+\frac{1}{2/5}} = \dfrac{\tan 35.0°}{3.5} = 0.200$. This is less than the 0.350 we found in (d), so a coefficient of friction of 0.350 is more than enough to prevent slipping of the solid sphere.

10.90. **IDENTIFY:** The work done by the force F is equal to the kinetic energy gained by the flywheel. This work is the area under the curve in a F-versus-d graph.

SET UP: $W = Fd$, $K = \frac{1}{2}I\omega^2$, $v = r\omega$.

EXECUTE: **(a)** The pull is constant, so the linear and angular accelerations are constant. Therefore $v = 2v_{\text{av}} = 2(d/t)$, so $\omega = v/R = 2d/tR$. The work done is equal to the kinetic energy of the flywheel, so

$Fd = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{2d}{tR}\right)^2$. Solving for I gives

$I = Ft^2R^2/2d = (25.0\text{ N})(2.00\text{ s})^2(0.166\text{ m})^2/[2(8.35\text{ m})] = 0.165\ \text{kg}\cdot\text{m}^2$.

(b) The kinetic energy gained is equal to the work done which is equal to the area under the curve on the F-d graph. This gives

$K = (60.0\text{ N})(3.00\text{ m}) + ½\,(60.0\text{ N})(3.00\text{ m}) = 270\text{ J}$.

(c) $K = \frac{1}{2}I\omega^2$ so $\omega = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(270\text{ J})}{0.165\text{ kg}\cdot\text{m}^2}} = 57.2$ rad/s. Converting to rpm gives

(57.2 rad/s)[(60 s)/(1 min)][(1 rev)/(2π rad)] = 546 rpm.

EVALUATE: In this case, we could have deduced the equation for *F* as a function of *d* from the graph and integrated to find the work. But for a more complicated *F*-*d* dependence, that would have been impossible, but we could still estimate the area quite accurately from the graph.

10.91. IDENTIFY: The answer to part (a) can be taken from the solution to Problem 10.86. The work-energy theorem says $W = \Delta K$.

SET UP: Problem 10.86 uses conservation of angular momentum to show that $r_1 v_1 = r_2 v_2$.

EXECUTE: **(a)** $T = m v_1^2 r_1^2 / r^3$.

(b) $\vec{T}$ and $d\vec{r}$ are always antiparallel. $\vec{T}\cdot d\vec{r} = -T dr$.

$$W = -\int_{r_1}^{r_2} T\,dr = m v_1^2 r_1^2 \int_{r_2}^{r_1} \frac{dr}{r^3} = \frac{m v_1^2}{2} r_1^2 \left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right].$$

(c) $v_2 = v_1(r_1/r_2)$, so $\Delta K = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{m v_1^2}{2}\left[(r_1/r_2)^2 - 1\right]$, which is equal to the work found in part (b).

EVALUATE: The work done by *T* is positive, since $\vec{T}$ is toward the hole in the surface and the block moves toward the hole. Positive work means the kinetic energy of the object increases.

10.92. IDENTIFY: Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ and $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the motion of the cylinder. Use constant acceleration equations to relate a_x to the distance the object travels. Use the work-energy theorem to find the work done by friction.

SET UP: The cylinder has $I_{\text{cm}} = \frac{1}{2}MR^2$.

EXECUTE: **(a)** The free-body diagram is sketched in Figure 10.92. The friction force is $f = \mu_k n = \mu_k Mg$, so $a = \mu_k g$. The magnitude of the angular acceleration is $\frac{fR}{I} = \frac{\mu_k MgR}{(1/2)MR^2} = \frac{2\mu_k g}{R}$.

(b) Setting $v = at = \omega R = (\omega_0 - \alpha t)R$ and solving for *t* gives $t = \frac{R\omega_0}{a + R\alpha} = \frac{R\omega_0}{\mu_k g + 2\mu_k g} = \frac{R\omega_0}{3\mu_k g}$,

and $d = \frac{1}{2}at^2 = \frac{1}{2}(\mu_k g)\left(\frac{R\omega_0}{3\mu_k g}\right)^2 = \frac{R^2\omega_0^2}{18\mu_k g}$.

(c) The final kinetic energy is $(3/4)Mv^2 = (3/4)M(at)^2$, so the change in kinetic energy is

$$\Delta K = \frac{3}{4}M\left(\mu_k g \frac{R\omega_0}{3\mu_k g}\right)^2 - \frac{1}{4}MR^2\omega_0^2 = -\frac{1}{6}MR^2\omega_0^2.$$

EVALUATE: The fraction of the initial kinetic energy that is removed by friction work is $\frac{\frac{1}{6}MR\omega_0^2}{\frac{1}{4}MR\omega_0^2} = \frac{2}{3}$.

This fraction is independent of the initial angular speed ω_0.

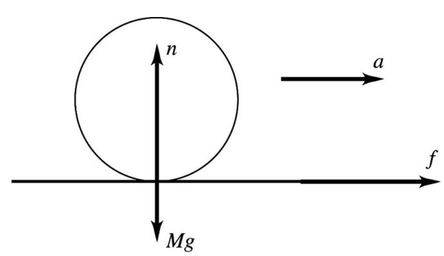


Figure 10.92

10.93. **IDENTIFY:** The vertical forces must sum to zero. Apply $\Omega = \dfrac{\tau_z}{L_z} = \dfrac{wr}{I\omega}$.

SET UP: Denote the upward forces that the hands exert as F_L and F_R. $\tau = (F_L - F_R)r$, where $r = 0.200$ m.

EXECUTE: The conditions that F_L and F_R must satisfy are $F_L + F_R = w$ and $F_L - F_R = \Omega \dfrac{I\omega}{r}$, where the second equation is $\tau = \Omega L$, divided by r. These two equations can be solved for the forces by first adding and then subtracting, yielding $F_L = \frac{1}{2}\left(w + \Omega \dfrac{I\omega}{r}\right)$ and $F_R = \frac{1}{2}\left(w - \Omega \dfrac{I\omega}{r}\right)$. Using the values

$w = mg = (8.00 \text{ kg})(9.80 \text{ m/s}^2) = 78.4 \text{ N}$ and

$\dfrac{I\omega}{r} = \dfrac{(8.00 \text{ kg})(0.325 \text{ m})^2(5.00 \text{ rev/s} \times 2\pi \text{ rad/rev})}{(0.200 \text{ m})} = 132.7 \text{ kg} \cdot \text{m/s}$ gives

$F_L = 39.2 \text{ N} + \Omega(66.4 \text{ N} \cdot \text{s})$, $F_R = 39.2 \text{ N} - \Omega(66.4 \text{ N} \cdot \text{s})$.

(a) $\Omega = 0, F_L = F_R = 39.2$ N.

(b) $\Omega = 0.05 \text{ rev/s} = 0.314 \text{ rad/s}, F_L = 60.0 \text{ N}, F_R = 18.4$ N.

(c) $\Omega = 0.3 \text{ rev/s} = 1.89 \text{ rad/s}, F_L = 165 \text{ N}, F_R = -86.2$ N, with the minus sign indicating a downward force.

(d) $F_R = 0$ gives $\Omega = \dfrac{39.2 \text{ N}}{66.4 \text{ N} \cdot \text{s}} = 0.590$ rad/s, which is 0.0940 rev/s.

EVALUATE: The larger the precession rate Ω, the greater the torque on the wheel and the greater the difference between the forces exerted by the two hands.

10.94. **IDENTIFY:** The rotational form of Newton's second law applies.
SET UP: $\Sigma\tau = I\alpha$ and $\omega_z = \omega_{0z} + \alpha_z t$.
EXECUTE: $\Sigma\tau = I\alpha = \Delta\omega/\Delta t$, where $I = I_{\text{person}} + I_0$. Solving for I_{person} gives $I_{\text{person}} = \tau/\alpha - I_0$.

$I_{\text{person}} = \dfrac{2.5 \text{ N} \times \text{m}}{\left(\dfrac{1.0 \text{ rad/s}}{3.0 \text{ s}}\right)} - 1.5 \text{ kg} \times \text{m}^2 = 6.0 \text{ kg} \times \text{m}^2$, which is choice (b).

EVALUATE: The moment of inertia of the turntable is considerably less than that of the person, which is a good thing. If the moment of inertia of the table were much greater than that of the person, the person's body would have a small effect on the angular acceleration of the table, making it hard to get an accurate measurement.

10.95. **IDENTIFY** and **SET UP:** Moment of inertia depends on the distribution of mass.
EXECUTE: Extending her legs increases the person's moment of inertia to increase. With a constant torque on the turntable, this would decrease her angular acceleration, which is choice (c).
EVALUATE: The person being studied should be told to lie still during the procedure.

10.96. **IDENTIFY** and **SET UP:** The torque is the product of the force times the lever arm, and $\Sigma\tau = I\alpha$.
EXECUTE: Doubling the lever arm with a constant force doubles the torque, which then doubles the angular acceleration, so choice (b) is correct.
EVALUATE: Doubling the diameter of the pulley would also allow the tension to be decreased by a factor of 2 and still keep the same original angular acceleration.

10.97. **IDENTIFY** and **SET UP:** The parallel-axis theorem, $I = I_{\text{cm}} + md^2$, applies to the person.
EXECUTE: The measured moment of inertia would be I, but this would be greater than I_{cm}, so the measured value would be too large, choice (a).
EVALUATE: Care is essential to position the person properly on the turntable.

11

EQUILIBRIUM AND ELASTICITY

VP11.1.1. **IDENTIFY:** We are dealing with center of gravity. If the center of gravity of the mass-plank system moves beyond the right-hand support point, the plank will tip over.

SET UP: $x_{cg} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$. Call the origin the right-hand support point. The center of mass of the system is at that point, so $x_{cm} = 0$. Call m the unknown mass.

EXECUTE: Applying the center of gravity formula $x_{cg} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$ gives

$$0 = \frac{(40.0\text{ kg})(-1.00\text{ m}) + (3.00\text{ m})\,m}{40.0\text{ kg} + m} \quad \rightarrow \quad m = 13.3\text{ kg}.$$

EVALUATE: The added mass of 13.3 kg is much less than the 40.0-kg mass of the plank because this added mass is much farther from the pivot point than the center of mass is.

VP11.1.2. **IDENTIFY:** We are dealing with center of gravity. If the center of gravity of the two-ball system is at the bowling ball, the system will balance.

SET UP: $x_{cg} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$. Call the origin the center of the bowling ball and call d the distance between the centers of the two balls.

EXECUTE: With our choice of origin, $x_{cm} = 0.216$ m, so we have

$$0.216\text{ m} = \frac{(7.26\text{ kg})(0) + (0.145\text{ kg})d}{7.26\text{ kg} + 0.145\text{ kg}} \quad \rightarrow \quad d = 11.0\text{ m}.$$

EVALUATE: To check, we can balance torques about the surface of the bowling ball with the rod horizontal. Call M the mass of the bowling ball and R its radius and m the mass of the baseball. This gives $MgR - mg(d - R) = 0 \quad \rightarrow \quad d = R(M + m)/m = 11.0$ m, which agrees with our result.

VP11.1.3. **IDENTIFY:** For balance, the center of gravity of the system of three objects and the rod must be at the support point.

SET UP: Call the center of the rod the origin and call x the distance of the support point from the center of the rod; this is also the distance of the center of gravity from the center. We treat the bar as a point-mass of m located at its center.

EXECUTE: $x_{cg} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \dfrac{m(-L/2) + 2m(0) + 2m(L/2)}{5m} = \dfrac{L}{10}$. The support point should be a distance $L/10$ to the right of the center of the bar, which is the location of the center of gravity of the system.

EVALUATE: To check, substitute the calculated answer back into the center-of-mass formula but using a different origin and use it to calculate the location of the center of mass. For example, using the left end of the bar as the origin, we have $x_{cg} = \dfrac{2m(L/2)+2m(L)}{5m}$ = 6*L*/10, which is *L*/10 to the right of the center of the bar, as we just found.

VP11.1.4. **IDENTIFY:** We know where the center of gravity of the loaded plane should be, and we want to find out how much mass we can have in the baggage compartment.

SET UP: Use $x_{cg} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$. Let the *x*-axis point horizontally to the right with the nose of the plane on the left end and the origin at the center of gravity of the *loaded* plane. With this choice, $x_{cm} = 0$.

EXECUTE: Let *M* be the maximum mass in the baggage compartment.

$$0 = \frac{-(1170\text{ kg})(2.76\text{ m} - 2.58\text{ m}) - (75.0\text{ kg})(2.76\text{ m} - 2.67\text{ m}) + M(4.30\text{ m} - 2.76\text{ m})}{m_{\text{total}}}$$

M = 141 kg.

EVALUATE: Choosing the origin at the center of gravity of the loaded plane makes the algebra rather simple compared to other choices because we do not have to deal with m_{total} in the denominator. Since m_{total} includes *M*, we would have more work (and more chances for error) with other choices of the origin. It helps to plan ahead!

VP11.4.1. **IDENTIFY:** The truck is in equilibrium, so the torques on it must balance.

SET UP: Apply $\sum \tau_z = 0$. Take torques about the rear wheel. Call *d* the wheelbase and *w* the weight of the truck. See Fig. VP11.4.1.

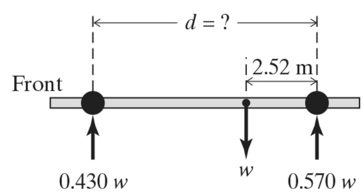


Figure VP11.4.1

EXECUTE: $\sum \tau_z = 0$ gives $(0.430\,w)d = (2.52\text{ m})w \;\rightarrow\; d = 5.86\text{ m}$.

EVALUATE: To check, calculate the center of gravity of the truck. This gives $(0.570\,w)(2.52\text{ m}) - (0.430\,w)(5.86\text{ m} - 2.52\text{ m}) = 0$, as it should since the truck is in equilibrium.

VP11.4.2. **IDENTIFY:** The plane is in equilibrium, so the torques on it must balance.

SET UP: Apply $\sum \tau_z = 0$. Take torques about the nose wheel. Call *F* the force due to the main wheel and *w* the weight of the plane.

EXECUTE: $\sum \tau_z = 0$ gives $-w(2.58\text{ m} - 0.800\text{ m}) + F(3.02\text{ m} - 0.800\text{ m}) = 0$, so $F = 0.802w$. The main wheel supports 80.2% of the weight of the plane, so the nose wheels must support 19.8% of the weight.

EVALUATE: To check, calculate torques about another point.

VP11.4.3. **IDENTIFY:** The sign is in equilibrium, so the forces and torques on the rod must balance.

SET UP: Apply $\sum \tau_z = 0$, $\sum F_x = 0$, and $\sum F_y = 0$.

EXECUTE: **(a)** Apply $\sum \tau_z = 0$ about the right end of the rod, giving

$F_{\text{hinge-}y} L + F_{\text{hinge-}x}(0) + T(0) + w(0) = 0 \;\rightarrow\; F_{\text{hinge-}y} = 0$.

(b) $\sum F_y = 0$ gives $F_{\text{hinge-}y} + T\sin\theta - w = 0 \;\rightarrow\; 0 + T\sin\theta - w = 0 \;\rightarrow\; T = w/\sin\theta$.

(c) $\sum F_x = 0$ gives $F_{\text{hinge-}x} - T\cos\theta = 0$. Using *T* from part (b) gives

$F_{\text{hinge-}x} = \left(\dfrac{w}{\sin\theta}\right)\cos\theta = w/\tan\theta.$

EVALUATE: Our result in (c) says that if θ is small, the hinge exerts a large horizontal force on the rod. This is reasonable because the cable pulls nearly horizontally against the rod which causes it to push very hard against the hinge.

VP11.4.4. **IDENTIFY:** The sign is in equilibrium, so the forces and torques on the rod must balance.

SET UP: Apply $\sum\tau_z = 0$, $\sum F_x = 0$, and $\sum F_y = 0$. The rod now also has weight w.

EXECUTE: **(a)** Apply $\sum\tau_z = 0$ about the right end of the rod, giving

$F_{\text{hinge-}y}L + F_{\text{hinge-}x}(0) + wL/2 + T(0) + w(0) = 0 \quad \rightarrow \quad F_{\text{hinge-}y} = w/2.$

(b) $\sum F_y = 0$ gives $F_{\text{hinge-}y} + T\sin\theta - w/2 - w = 0 \quad \rightarrow \quad T\sin\theta - 3w/2 = 0$

so $T = \dfrac{3w}{2\sin\theta}$.

(c) $\sum F_x = 0$ gives $F_{\text{hinge-}x} - T\cos\theta = 0$. Using T from part (b) gives

$F_{\text{hinge-}x} = \left(\dfrac{3w}{2\sin\theta}\right)\cos\theta = \dfrac{3w}{2\tan\theta}.$

EVALUATE: Our result in (c) says that if θ is small, the hinge exerts a large horizontal force on the rod. This is reasonable because the cable pulls nearly horizontally against the rod, which causes it to push very hard against the hinge. We also find that if the rod weighs as much as the sign, the tension and hinge force each are 1.5 times as great as before.

VP11.7.1. **IDENTIFY:** This problem involves elasticity, tensile stress, tensile stress, and Young's modulus.

SET UP: Young's modulus is $Y = \dfrac{\text{Tensile stress}}{\text{Tensile strain}}$, tensile strain = $\dfrac{\Delta\ell}{\ell_0}$, tensile stress = $\dfrac{F_\perp}{A}$.

EXECUTE: **(a)** Tensile stress = Y (tensile strain). $\dfrac{\Delta\ell}{\ell_0} = \dfrac{5.0\times10^{-3}\ell_0}{\ell_0} = 5.0\times10^{-3}$, so

tensile stress = $(11\times10^{10}\text{ Pa})(5.0\times10^{-3}) = 5.5\times10^{8}$ Pa.

(b) Tensile stress = $\dfrac{F_\perp}{A}$, where $A = \pi r^2$. Therefore $F_\perp$ = (tensile strain)(πr^2) so

$F_\perp = (5.5\times10^{8}\text{ Pa})(\pi)(4.5\times10^{-3}\text{ m})^2 = 3.5\times10^{4}$ N.

EVALUATE: The pressure is much less than atmospheric pressure, but the force is about 7600 lb, which is very large. But a wire 4.5 mm in radius is quite thick compared to ordinary electrical copper wires.

VP11.7.2. **IDENTIFY:** This problem involves elasticity, compressive stress and compressive strain, and Young's modulus.

SET UP: Young's modulus is $Y = \dfrac{\text{Compressive stress}}{\text{Compressive strain}}$, compressive strain = $\dfrac{\Delta\ell}{\ell_0}$, compressive stress = $\dfrac{F_\perp}{A}$. From Table 11.1, $Y = 7.0\times10^{10}$ Pa for aluminum.

EXECUTE: **(a)** We want the compressive strain. $Y = \dfrac{\text{Compressive stress}}{\text{Compressive strain}}$ tells us that compressive strain = $\dfrac{\text{Compressive stress}}{Y} = \dfrac{F_\perp}{AY} = \dfrac{3.2\times10^{4}\text{ N}}{\pi(0.025\text{ m})^2(7.0\times10^{10}\text{ Pa})} = 2.3\times10^{-4}$.

(b) Compressive strain = $\dfrac{\Delta\ell}{\ell_0} \quad \rightarrow \quad \Delta\ell = \ell_0 \times$ compressive strain, which gives

$\Delta\ell = (82\text{ cm})(2.3\times10^{-4}) = 1.9\times10^{-2}\text{ cm} = 0.19$ mm.

EVALUATE: Note that an 82-cm cylinder compresses only 0.19 mm under a force of 32,000 N, which is about 7200 lb. So fractional changes in length are normally quite small.

VP11.7.3. **IDENTIFY:** The increase in pressure compresses the sphere. Using the bulk modulus we can find the change in volume.

SET UP: Bulk modulus is $B = -\dfrac{\Delta p}{\Delta V / V_0}$. To find the *decrease* in volume, we can neglect the minus sign. From Table 11.1, $B = 4.1\times10^{10}$ Pa for lead, and from Table 11.2, $B = 1/k = 2.7\times10^{10}$ Pa for mercury.

EXECUTE: The decrease in volume is $\Delta V = \dfrac{V_0\Delta p}{B}$.

(a) Lead: $\Delta V = \dfrac{V_0\Delta p}{B} = \dfrac{\frac{4}{3}\pi(0.012\text{ m})^3(2.5\times10^7\text{ Pa})}{4.1\times10\text{ Pa}} = 4.4\times10^{-9}\text{ m}^3.$

(b) Mercury: Table 11.2 gives compressibility (k), so we use $B = 1/k$ to find $B = 2.7\times10^{10}$ Pa. Using the same formula as in (a), but with a different B, gives $\Delta V = 6.7\times10^{-9}\text{ m}^3$.

EVALUATE: If we divide the equations for ΔV, we get $\Delta V_M = \Delta V_L(B_L/B_M)$ $\Delta V_M = (4.4\times10^{-9}\text{ m}^3)(4.1/2.7) = 6.7\times10^{-9}\text{ m}^3$, which agrees with our result. Also note that since $B_L \approx 1.5\,B_M$, we would expect ΔV_M to be about 1.5 times as great as ΔV_L (since ΔV is *inversely* proportional to B), which is what we find.

VP11.7.4. **IDENTIFY:** The shear forces distort the cube by a small distance x.

SET UP: The shear modulus is $S = \dfrac{F_{\parallel}h}{Ax}$, so $x = \dfrac{F_{\parallel}h}{AS}$. For brass $S = 3.5\times10^{10}$ Pa from Table 11.1.

EXECUTE: $x = \dfrac{(4.2\times10^4\text{ N})(0.025\text{ m})}{(0.025\text{ m})^2(3.5\times10^{10}\text{ Pa})} = 4.8\times10^{-5}\text{ m}.$

EVALUATE: The length of a side of this cube is 2.5 cm, but the shear displacement is only 4.8×10^{-3} cm, which is 4.8 thousandths of a centimeter. Shear displacements are typically very small.

11.1. **IDENTIFY:** Use $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$ to calculate x_{cm}. The center of gravity of the bar is at its center and it can be treated as a point mass at that point.

SET UP: Use coordinates with the origin at the left end of the bar and the $+x$-axis along the bar. $m_1 = 0.120$ kg, $m_2 = 0.055$ kg, $m_3 = 0.110$ kg.

EXECUTE: $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(0.120\text{ kg})(0.250\text{ m}) + 0 + (0.110\text{ kg})(0.500\text{ m})}{0.120\text{ kg} + 0.055\text{ kg} + 0.110\text{ kg}} = 0.298\text{ m}.$ The fulcrum should be placed 29.8 cm to the right of the left-hand end.

EVALUATE: The mass at the right-hand end is greater than the mass at the left-hand end. So the center of gravity is to the right of the center of the bar.

11.2. **IDENTIFY:** Use $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$ to calculate x_{cm} of the composite object.

SET UP: Use coordinates where the origin is at the original center of gravity of the object and $+x$ is to the right. With the 1.50 kg mass added, $x_{\text{cm}} = -2.20$ cm, $m_1 = 5.00$ kg and $m_2 = 1.50$ kg. $x_1 = 0$.

EXECUTE: $x_{\text{cm}} = \dfrac{m_2x_2}{m_1 + m_2}$. $x_2 = \left(\dfrac{m_1 + m_2}{m_2}\right)x_{\text{cm}} = \left(\dfrac{5.00\text{ kg} + 1.50\text{ kg}}{1.50\text{ kg}}\right)(-2.20\text{ cm}) = -9.53\text{ cm}.$

The additional mass should be attached 9.53 cm to the left of the original center of gravity.

EVALUATE: The new center of gravity is somewhere between the added mass and the original center of gravity.

11.3. IDENTIFY: Treat the rod and clamp as point masses. The center of gravity of the rod is at its midpoint, and we know the location of the center of gravity of the rod-clamp system.

SET UP: $x_{\text{cm}} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$.

EXECUTE: $1.20 \text{ m} = \dfrac{(1.80 \text{ kg})(1.00 \text{ m}) + (2.40 \text{ kg})x_2}{1.80 \text{ kg} + 2.40 \text{ kg}}$.

$$x_2 = \frac{(1.20 \text{ m})(1.80 \text{ kg} + 2.40 \text{ kg}) - (1.80 \text{ kg})(1.00 \text{ m})}{2.40 \text{ kg}} = 1.35 \text{ m}$$

EVALUATE: The clamp is to the right of the center of gravity of the system, so the center of gravity of the system lies between that of the rod and the clamp, which is reasonable.

11.4. IDENTIFY: This problem involves center of gravity and torque.

SET UP: Refer to Fig. 11.9(b) in the textbook. The equation $x_{\text{cg}} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ gives the location of the center of gravity, and torque is equal to the force times the lever arm. In the center of gravity formula, we should use the masses of the objects. However since mass is $m = w/g$, we do not have to divide each weight by g since the g will cancel out because it occurs in each term in the numerator and denominator.

EXECUTE: **(a)** $x_{\text{cg}} = \dfrac{(180 \text{ N})(1.5 \text{ m}) + (800 \text{ N})(1.0 \text{ m})}{180 \text{ N} + 800 \text{ N}}$ = 1.1 m to the right of the foot of the ladder.

(b) $\tau = (980 \text{ N})(1.1 \text{ m}) = 1070 \text{ N} \cdot \text{m}$.

EVALUATE: **(c)** In the text the torque is $\tau = (180 \text{ N})(1.5 \text{ m}) + (800 \text{ N})(1.0 \text{ m}) = 1070 \text{ N} \cdot \text{m}$. Our answer in (b) agrees with the text answer.

11.5. IDENTIFY: We need to calculate the center of gravity of a compound object consisting of two spheres and a steel rod. The center of gravity of each of them is at their midpoint since they are all uniform.

SET UP: Use $x_{\text{cg}} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$ with the origin at the center of the rod. Let the 0.900-kg sphere be on the left end of the rod and the 0380-kg sphere on the right end.

EXECUTE: $x_{\text{cg}} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \dfrac{(0.900 \text{ kg})(-28.0 \text{ m}) + (0.300 \text{ kg})(0 \text{ m}) + (0.380 \text{ kg})(26.0 \text{ cm})}{0.900 \text{ kg} + 0.300 \text{ kg} + 0.380 \text{ kg}}$

$= -9.70$ cm. So the center of gravity is 9.70 cm from the center of the rod toward the 0.900-kg sphere.

EVALUATE: The center of gravity is toward the heavier sphere, which is reasonable.

11.6. IDENTIFY: Apply the first and second conditions for equilibrium to the trap door.

SET UP: For $\sum \tau_z = 0$ take the axis at the hinge. Then the torque due to the applied force must balance the torque due to the weight of the door.

EXECUTE: **(a)** The force is applied at the center of gravity, so the applied force must have the same magnitude as the weight of the door, or 300 N. In this case the hinge exerts no force.

(b) With respect to the hinges, the moment arm of the applied force is twice the distance to the center of mass, so the force has half the magnitude of the weight, or 150 N.

The hinges supply an upward force of $300 \text{ N} - 150 \text{ N} = 150 \text{ N}$.

EVALUATE: Less force must be applied when it is applied farther from the hinges.

11.7. IDENTIFY: Apply $\sum \tau_z = 0$ to the ladder.

SET UP: Take the axis to be at point A. The free-body diagram for the ladder is given in Figure 11.7. The torque due to F must balance the torque due to the weight of the ladder.

EXECUTE: $F(8.0 \text{ m})\sin 40° = (3400 \text{ N})(10.0 \text{ m})$, so $F = 6.6$ kN.

EVALUATE: The force required is greater than the weight of the ladder, because the moment arm for F is less than the moment arm for w.

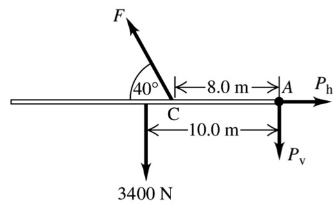


Figure 11.7

11.8. **IDENTIFY:** Apply the first and second conditions of equilibrium to the board.
SET UP: The free-body diagram for the board is given in Figure 11.8. Since the board is uniform its center of gravity is 1.50 m from each end. Apply $\sum F_y = 0$, with $+y$ upward. Apply $\sum \tau_z = 0$ with the axis at the end where the first person applies a force and with counterclockwise torques positive.
EXECUTE: $\sum F_y = 0$ gives $F_1 + F_2 - w = 0$ and $F_2 = w - F_1 = 160\text{ N} - 60\text{ N} = 100\text{ N}$. $\sum \tau_z = 0$ gives $F_2 x - w(1.50\text{ m}) = 0$ and $x = \left(\dfrac{w}{F_2}\right)(1.50\text{ m}) = \left(\dfrac{160\text{ N}}{100\text{ N}}\right)(1.50\text{ m}) = 2.40\text{ m}$. The other person lifts with a force of 100 N at a point 2.40 m from the end where the other person lifts.
EVALUATE: By considering the axis at the center of gravity we can see that a larger force is applied by the person who pushes closer to the center of gravity.

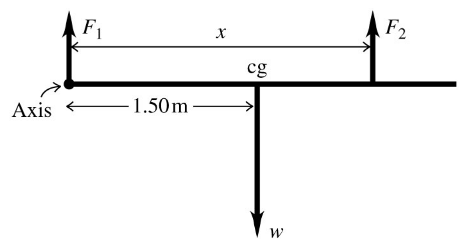


Figure 11.8

11.9. **IDENTIFY:** Apply $\sum F_y = 0$ and $\sum \tau_z = 0$ to the board.
SET UP: Let $+y$ be upward. Let x be the distance of the center of gravity of the motor from the end of the board where the 400 N force is applied.
EXECUTE: **(a)** If the board is taken to be massless, the weight of the motor is the sum of the applied forces, 1000 N. The motor is a distance $\dfrac{(2.00\text{ m})(600\text{ N})}{(1000\text{ N})} = 1.20\text{ m}$ from the end where the 400 N force is applied, and so is 0.800 m from the end where the 600 N force is applied.
(b) The weight of the motor is $400\text{ N} + 600\text{ N} - 200\text{ N} = 800\text{ N}$. Applying $\sum \tau_z = 0$ with the axis at the end of the board where the 400 N acts gives $(600\text{ N})(2.00\text{ m}) = (200\text{ N})(1.00\text{ m}) + (800\text{ N})x$ and $x = 1.25\text{ m}$. The center of gravity of the motor is 0.75 m from the end of the board where the 600 N force is applied.
EVALUATE: The motor is closest to the end of the board where the larger force is applied.

11.10. **IDENTIFY:** Apply the first and second conditions of equilibrium to the shelf.
SET UP: The free-body diagram for the shelf is given in Figure 11.10. Take the axis at the left-hand end of the shelf and let counterclockwise torque be positive. The center of gravity of the uniform shelf is at its center.
EXECUTE: **(a)** $\sum \tau_z = 0$ gives $-w_t(0.200\text{ m}) - w_s(0.300\text{ m}) + T_2(0.400\text{ m}) = 0$.

$$T_2 = \frac{(25.0\text{ N})(0.200\text{ m}) + (50.0\text{ N})(0.300\text{ m})}{0.400\text{ m}} = 50.0\text{ N}$$

$\sum F_y = 0$ gives $T_1 + T_2 - w_t - w_s = 0$ and $T_1 = 25.0$ N. The tension in the left-hand wire is 25.0 N and the tension in the right-hand wire is 50.0 N.

EVALUATE: We can verify that $\sum \tau_z = 0$ is zero for any axis, for example for an axis at the right-hand end of the shelf.

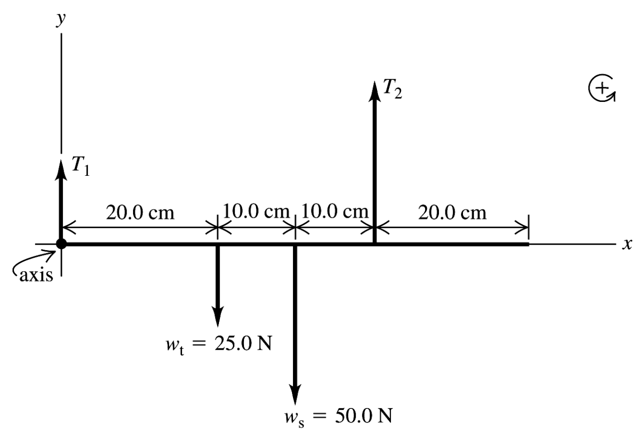


Figure 11.10

11.11. **IDENTIFY:** Apply the conditions for equilibrium to the bar. Set each tension equal to its maximum value.

SET UP: Let cable *A* be at the left-hand end. Take the axis to be at the left-hand end of the bar and *x* be the distance of the weight *w* from this end. The free-body diagram for the bar is given in Figure 11.11.

EXECUTE: **(a)** $\sum F_y = 0$ gives $T_A + T_B - w - w_{\text{bar}} = 0$ and

$w = T_A + T_B - w_{\text{bar}} = 500.0\ \text{N} + 400.0\ \text{N} - 350.0\ \text{N} = 550\ \text{N}.$

(b) $\sum \tau_z = 0$ gives $T_B(1.50\ \text{m}) - wx - w_{\text{bar}}(0.750\ \text{m}) = 0.$

$x = \dfrac{T_B(1.50\ \text{m}) - w_{\text{bar}}(0.750\ \text{m})}{w} = \dfrac{(400.0\ \text{N})(1.50\ \text{m}) - (350\ \text{N})(0.750\ \text{m})}{550\ \text{N}} = 0.614\ \text{m}.$ The weight should be placed 0.614 m from the left-hand end of the bar (cable *A*).

EVALUATE: If the weight is moved to the left, T_A exceeds 500.0 N and if it is moved to the right T_B exceeds 400.0 N.

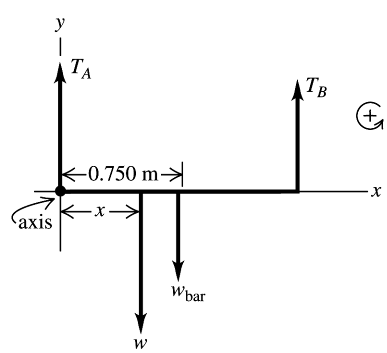


Figure 11.11

11.12. **IDENTIFY:** Apply the first and second conditions for equilibrium to the ladder.

SET UP: Let n_2 be the upward normal force exerted by the ground and let n_1 be the horizontal normal force exerted by the wall. The maximum possible static friction force that can be exerted by the ground is $\mu_s n_2$.

EXECUTE: **(a)** Since the wall is frictionless, the only vertical forces are the weights of the man and the ladder, and the normal force n_2. For the vertical forces to balance,

$n_2 = w_1 + w_m = 160\text{ N} + 740\text{ N} = 900\text{ N}$, and the maximum frictional force is

$\mu_s n_2 = (0.40)(900\text{ N}) = 360\text{ N}$.

(b) Note that the ladder makes contact with the wall at a height of 4.0 m above the ground. Balancing torques about the point of contact with the ground,

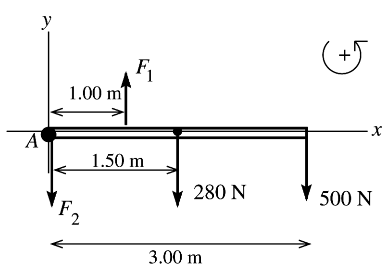
$(4.0\text{ m})n_1 = (1.5\text{ m})(160\text{ N}) + (1.0\text{ m})(3/5)(740\text{ N}) = 684\text{ N}\cdot\text{m}$, so $n_1 = 171.0\text{ N}$. This horizontal force must be balanced by the friction force, which must then be 170 N to two figures.

(c) Setting the friction force, and hence n_1, equal to the maximum of 360 N and solving for the distance x along the ladder, $(4.0\text{ m})(360\text{ N}) = (1.50\text{ m})(160\text{ N}) + x(3/5)(740\text{ N})$, so $x = 2.7\text{ m}$.

EVALUATE: The normal force exerted by the ground doesn't change as the man climbs up the ladder. But the normal force exerted by the wall and the friction force exerted by the ground both increase as he moves up the ladder.

11.13. **IDENTIFY:** The system of the person and diving board is at rest so the two conditions of equilibrium apply.

(a) SET UP: The free-body diagram for the diving board is given in Figure 11.13. Take the origin of coordinates at the left-hand end of the board (point A).



$\vec{F}_1$ is the force applied at the support point and $\vec{F}_2$ is the force at the end that is held down.

Figure 11.13

EXECUTE: $\sum\tau_A = 0$ gives $+F_1(1.0\text{ m}) - (500\text{ N})(3.00\text{ m}) - (280\text{ N})(1.50\text{ m}) = 0$

$$F_1 = \frac{(500\text{ N})(3.00\text{ m}) + (280\text{ N})(1.50\text{ m})}{1.00\text{ m}} = 1920\text{ N}$$

(b) $\sum F_y = ma_y$

$F_1 - F_2 - 280\text{ N} - 500\text{ N} = 0$

$F_2 = F_1 - 280\text{ N} - 500\text{ N} = 1920\text{ N} - 280\text{ N} - 500\text{ N} = 1140\text{ N}$

EVALUATE: We can check our answers by calculating the net torque about some point and checking that $\sum\tau_z = 0$ for that point also. Net torque about the right-hand end of the board:

$(1140\text{ N})(3.00\text{ m}) + (280\text{ N})(1.50\text{ m}) - (1920\text{ N})(2.00\text{ m}) = 3420\text{ N}\cdot\text{m} + 420\text{ N}\cdot\text{m} - 3840\text{ N}\cdot\text{m} = 0$, which checks.

11.14. **IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

SET UP: The boy exerts a downward force on the beam that is equal to his weight.

EXECUTE: **(a)** The graphs are given in Figure 11.14.

(b) $x = 6.25$ m when $F_A = 0$, which is 1.25 m beyond point B.

(c) Take torques about the right end. When the beam is just balanced, $F_A = 0$, so $F_B = 900$ N.

The distance that point B must be from the right end is then $\dfrac{(300\text{ N})(4.50\text{ m})}{(900\text{ N})} = 1.50$ m.

EVALUATE: When the beam is on the verge of tipping it starts to lift off the support A and the normal force F_A exerted by the support goes to zero.

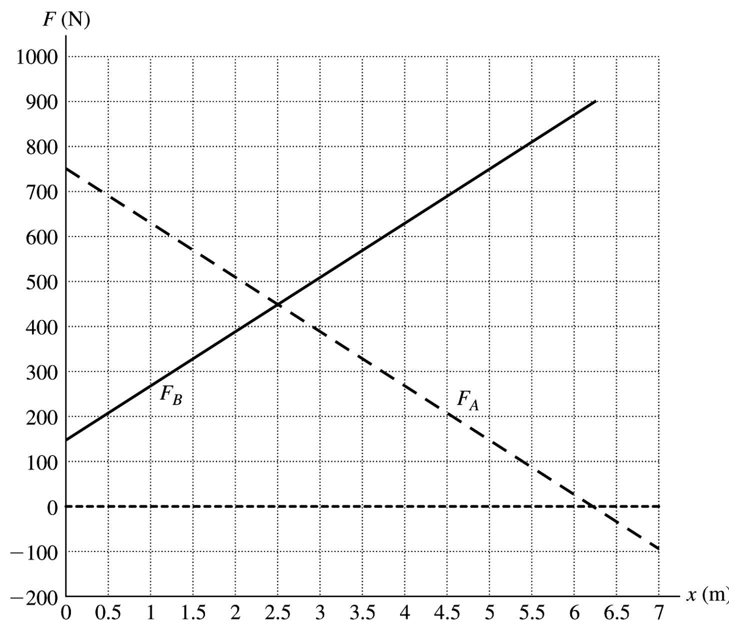


Figure 11.14

11.15. IDENTIFY: Apply the first and second conditions of equilibrium to the strut.

(a) SET UP: The free-body diagram for the strut is given in Figure 11.15a. Take the origin of coordinates at the hinge (point A) and $+y$ upward. Let F_h and F_v be the horizontal and vertical components of the force $\vec{F}$ exerted on the strut by the pivot. The tension in the vertical cable is the weight w of the suspended object. The weight w of the strut can be taken to act at the center of the strut. Let L be the length of the strut.

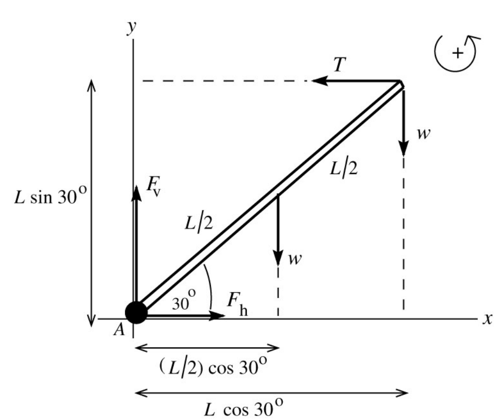


Figure 11.15a

EXECUTE:

$\sum F_y = ma_y$

$F_v - w - w = 0$

$F_v = 2w$

Sum torques about point A. The pivot force has zero moment arm for this axis and so doesn't enter into the torque equation.

$\tau_A = 0$

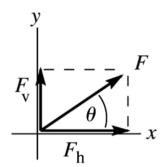
$TL\sin 30.0° - w((L/2)\cos 30.0°) - w(L\cos 30.0°) = 0$

$T\sin 30.0° - (3w/2)\cos 30.0° = 0$

$T = \dfrac{3w\cos 30.0°}{2\sin 30.0°} = 2.60w$

Then $\sum F_x = ma_x$ implies $T - F_\text{h} = 0$ and $F_\text{h} = 2.60w$.

We now have the components of $\vec{F}$ so can find its magnitude and direction (Figure 11.15b).



$F = \sqrt{F_\text{h}^2 + F_\text{v}^2}$

$F = \sqrt{(2.60w)^2 + (2.00w)^2}$

$F = 3.28w$

$\tan\theta = \dfrac{F_\text{v}}{F_\text{h}} = \dfrac{2.00w}{2.60w}$

$\theta = 37.6°$

Figure 11.15b

(b) SET UP: The free-body diagram for the strut is given in Figure 11.15c.

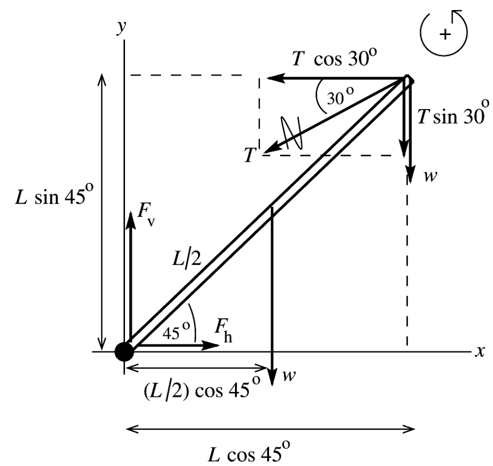


Figure 11.15c

The tension T has been replaced by its x and y components. The torque due to T equals the sum of the torques of its components, and the latter are easier to calculate.

EXECUTE: $\sum\tau_A = 0 + (T\cos 30.0°)(L\sin 45.0°) - (T\sin 30.0°)(L\cos 45.0°) -$

$$w[(L/2)\cos 45.0°] - w(L\cos 45.0°) = 0$$

The length L divides out of the equation. The equation can also be simplified by noting that $\sin 45.0° = \cos 45.0°$.

Then $T(\cos 30.0° - \sin 30.0°) = 3w/2$.

$T = \dfrac{3w}{2(\cos 30.0° - \sin 30.0°)} = 4.10w$

$\sum F_x = ma_x$

$F_h - T\cos 30.0° = 0$

$F_h = T\cos 30.0° = (4.10w)(\cos 30.0°) = 3.55w$

$\sum F_y = ma_y$

$F_v - w - w - T\sin 30.0° = 0$

$F_v = 2w + (4.10w)\sin 30.0° = 4.05w$

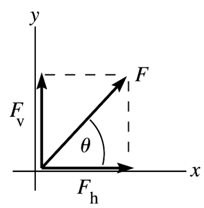


Figure 11.15d

From Figure 11.15d,

$F = \sqrt{F_h^2 + F_v^2}$

$F = \sqrt{(3.55w)^2 + (4.05w)^2} = 5.39w$

$\tan\theta = \dfrac{F_v}{F_h} = \dfrac{4.05w}{3.55w}$

$\theta = 48.8°$

EVALUATE: In each case the force exerted by the pivot does not act along the strut. Consider the net torque about the upper end of the strut. If the pivot force acted along the strut, it would have zero torque about this point. The two forces acting at this point also have zero torque and there would be one nonzero torque, due to the weight of the strut. The net torque about this point would then not be zero, violating the second condition of equilibrium.

11.16. IDENTIFY: Apply the first and second conditions of equilibrium to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.16. H_v and H_h are the vertical and horizontal components of the force exerted on the beam at the wall (by the hinge). Since the beam is uniform, its center of gravity is 2.00 m from each end. The angle θ has $\cos\theta = 0.800$ and $\sin\theta = 0.600$. The tension *T* has been replaced by its *x*- and *y*-components.

EXECUTE: **(a)** H_v, H_h and $T_x = T\cos\theta$ all produce zero torque. $\sum\tau_z = 0$ gives $-w(2.00\text{ m}) - w_{load}(4.00\text{ m}) + T\sin\theta(4.00\text{ m}) = 0$ and

$T = \dfrac{(190\text{ N})(2.00\text{ m}) + (300\text{ N})(4.00\text{ m})}{(4.00\text{ m})(0.600)} = 658.3\text{ N}$, which rounds to 658 N.

(b) $\sum F_x = 0$ gives $H_h - T\cos\theta = 0$ and $H_h = (658.3\text{ N})(0.800) = 527\text{ N}$. $\sum F_y = 0$ gives $H_v - w - w_{load} + T\sin\theta = 0$ and $H_v = w + w_{load} - T\sin\theta = 190\text{ N} + 300\text{ N} - (658\text{ N})(0.600) = 95\text{ N}$.

EVALUATE: For an axis at the right-hand end of the beam, only *w* and H_v produce torque. The torque due to *w* is counterclockwise so the torque due to H_v must be clockwise. To produce a clockwise torque, H_v must be upward, in agreement with our result from $\sum F_y = 0$.

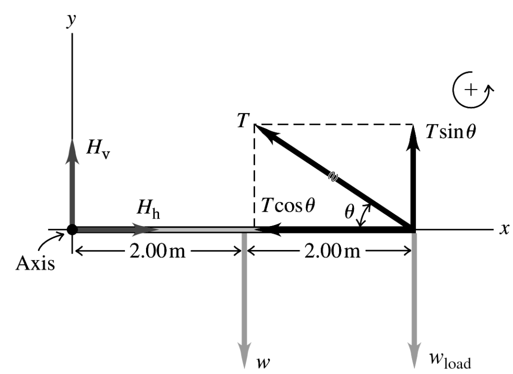


Figure 11.16

11.17. **IDENTIFY:** The boom is at rest, so the forces and torques on it must each balance.
SET UP: $\sum\tau = 0,\ \sum F_x = 0,\ \sum F_y = 0.$ The free-body is shown in Figure 11.17. Call L the length of the boom.

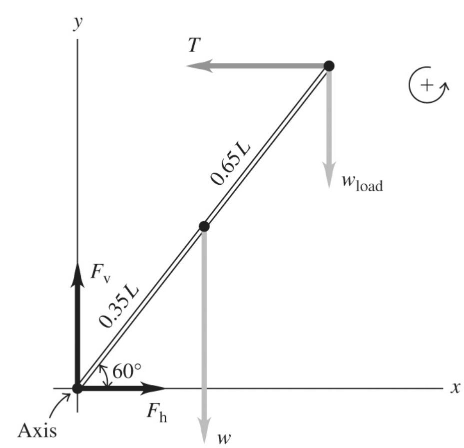


Figure 11.17

EXECUTE: **(a)** $\sum\tau = 0$ gives $T(L\sin 60.0°) - w_{load}(L\cos 60.0°) - w(0.35L\cos 60.0°) = 0$ and

$$T = \frac{w_{load}\cos 60.0° + w(0.35\cos 60.0°)}{\sin 60.0°} = \frac{(5000\text{ N})\cos 60.0° + (2600\text{ N})(0.35\cos 60.0°)}{\sin 60.0°} = 3.41\times 10^3\text{ N}.$$

(b) $\sum F_x = 0$ gives $F_h - T = 0$ and $F_h = 3410$ N.

$\sum F_y = 0$ gives $F_v - w - w_{load} = 0$ and $F_v = 5000\text{ N} + 2600\text{ N} = 7600\text{ N}$

EVALUATE: The bottom of the boom is the best point about which to take torques because only one unknown (the tension) appears in our equation. Using the top (or the center of mass) would give a torque equation with two (or three) unknowns.

11.18. **IDENTIFY:** The wheelbarrow is normally either held at rest or is moving with uniform velocity. Therefore the forces and torques on it must both balance.
SET UP: Estimate: The maximum upward force is 75 lb. Apply $\sum F_y = 0$ and $\sum\tau_z = 0$. Our target variables are the maximum weight of dirt we can life and the force the ground exerts on the wheels with that load in the wheelbarrow. Start with a free-body diagram of the wheelbarrow as shown in Fig. 11.18. Call F the upward force we exert (75 lb), w_{wb} the weight of the wheelbarrow (80.0 N = 18.0 lb), w_d the weight of the load of dirt, and F_{grd} the upward force the ground exerts on the front wheel.

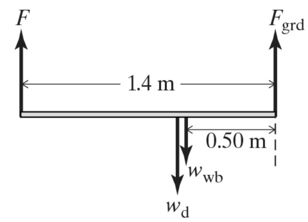


Figure 11.18

EXECUTE: **(a)** Apply $\sum \tau_z = 0$ about the point where the wheel is in contact with the ground. Using the notation in Fig 11.18 gives $-F(1.4\text{ m}) + (w_{wb} + w_d)(0.50\text{ m}) = 0$. Putting in the numbers and solving for w_d gives $w_d = \dfrac{(75\text{ lb})(1.4) - (18.0\text{ lb})(0.50)}{0.50} = 190\text{ lb}$.

(b) Now apply $\sum F_y = 0$: $F + F_{grd} - w_{wb} - w_d = 0$, which gives $F_{grd} =$ 18.0 lb + 190 lb – 75 lb = 130 lb.

EVALUATE: You can lift such a heavy load of dirt with only 75 lb because of your long lever arm compared to that of the dirt.

11.19. **IDENTIFY:** The beam is at rest so the forces and torques on it must each balance.

SET UP: $\sum \tau = 0$, $\sum F_x = 0$, $\sum F_y = 0$. The distance along the beam from the hinge to where the cable is attached is 3.0 m. The angle ϕ that the cable makes with the beam is given by $\sin\phi = \dfrac{4.0\text{ m}}{5.0\text{ m}}$, so $\phi = 53.1°$. The center of gravity of the beam is 4.5 m from the hinge. Use coordinates with $+y$ upward and $+x$ to the right. Take the pivot at the hinge and let counterclockwise torque be positive. Express the hinge force as components H_v and H_h. Assume H_v is downward and that H_h is to the right. If one of these components is actually in the opposite direction we will get a negative value for it. Set the tension in the cable equal to its maximum possible value, $T = 1.00$ kN.

EXECUTE: **(a)** The free-body diagram is shown in Figure 11.19, with $\vec{T}$ resolved into its x- and y-components.

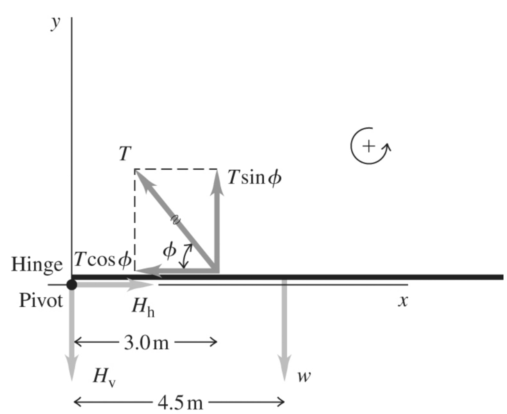


Figure 11.19

(b) $\sum \tau = 0$ gives $(T\sin\phi)(3.0\text{ m}) - w(4.5\text{ m}) = 0$

$$w = \frac{(T\sin\phi)(3.00\text{ m})}{4.50\text{ m}} = \frac{(1000\text{ N})(\sin 53.1°)(3.00\text{ m})}{4.50\text{ m}} = 533\text{ N}$$

(c) $\sum F_x = 0$ gives $H_h - T\cos\phi = 0$ and $H_h = (1.00\text{ kN})(\cos 53.1°) = 600\text{ N}$

$\sum F_y = 0$ gives $T\sin\phi - \text{H}_v - w = 0$ and $H_v = (1.00\text{ kN})(\sin 53.1°) - 533\text{ N} = 267\text{ N}$.

EVALUATE: $T\cos\phi$, H_v and H_h all have zero moment arms for a pivot at the hinge and therefore produce zero torque. If we consider a pivot at the point where the cable is attached we can see that H_v must be downward to produce a torque that opposes the torque due to *w*.

11.20. IDENTIFY: Apply the conditions for equilibrium to the crane.

SET UP: The free-body diagram for the crane is sketched in Figure 11.20. F_h and F_v are the components of the force exerted by the axle. $\vec{T}$ pulls to the left so F_h is to the right. $\vec{T}$ also pulls downward and the two weights are downward, so F_v is upward.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $T[(13\text{ m})\sin 25°] - w_c[(7.0\text{ m})\cos 55°] - w_b[(16.0\text{ m})\cos 55°] = 0$.

$$T = \frac{(11{,}000\text{ N})[(16.0\text{ m})\cos 55°] + (15{,}000\text{ N})[(7.0\text{ m})\cos 55°]}{(13.0\text{ m})\sin 25°} = 2.93\times10^4\text{ N}.$$

(b) $\sum F_x = 0$ gives $F_h - T\cos 30° = 0$ and $F_h = 2.54\times10^4$ N.

$\sum F_y = 0$ gives $F_v - T\sin 30° - w_c - w_b = 0$ and $F_v = 4.06\times10^4$ N.

EVALUATE: $\tan\theta = \dfrac{F_v}{F_h} = \dfrac{4.06\times10^4\text{ N}}{2.54\times10^4\text{ N}}$ and $\theta = 58°$. The force exerted by the axle is not directed along the crane.

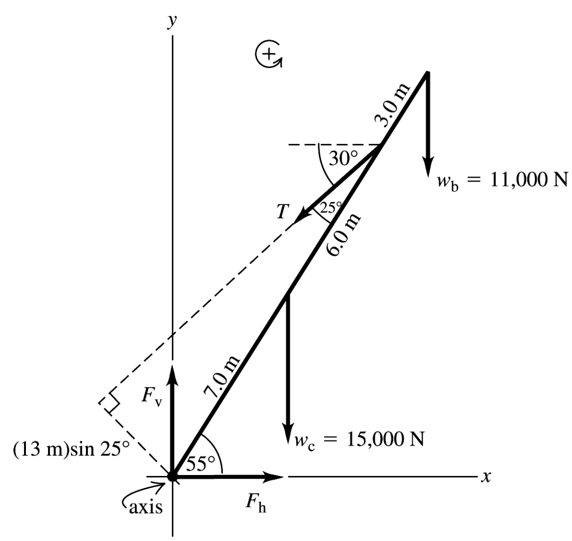


Figure 11.20

11.21. IDENTIFY: Apply the first and second conditions of equilibrium to the rod.

SET UP: The force diagram for the rod is given in Figure 11.21.

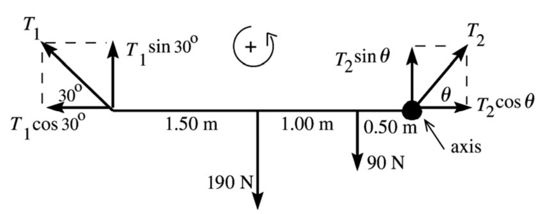


Figure 11.21

EXECUTE: $\sum\tau_z = 0$, axis at right end of rod, counterclockwise torque is positive

$(190\ \text{N})(1.50\ \text{m}) + (90\ \text{N})(0.50\ \text{m}) - (T_1 \sin 30.0°)(3.00\ \text{m}) = 0$

$$T_1 = \frac{285\ \text{N}\cdot\text{m} + 45\ \text{N}\cdot\text{m}}{1.50\ \text{m}} = 220\ \text{N}$$

$\sum F_x = ma_x$

$T_2 \cos\theta - T_1 \cos 30° = 0$ and $T_2 \cos\theta = (220\ \text{N})(\cos 30°) = 190.5\ \text{N}$

$\sum F_y = ma_y$

$T_1 \sin 30° + T_2 \sin\theta - 190\ \text{N} - 90\ \text{N} = 0$

$T_2 \sin\theta = 280\ \text{N} - (220\ \text{N})\sin 30° = 170\ \text{N}$

Then $\dfrac{T_2 \sin\theta}{T_2 \cos\theta} = \dfrac{170\ \text{N}}{190.5\ \text{N}}$ gives $\tan\theta = 0.89239$ and $\theta = 41.7°$

And $T_2 = \dfrac{170\ \text{N}}{\sin 41.7°} = 255\ \text{N}.$

EVALUATE: The monkey is closer to the right rope than to the left one, so the tension is larger in the right rope. The horizontal components of the tensions must be equal in magnitude and opposite in direction. Since $T_2 > T_1$, the rope on the right must be at a greater angle above the horizontal to have the same horizontal component as the tension in the other rope.

11.22. IDENTIFY: Apply the first and second conditions for equilibrium to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.22.

EXECUTE: The cable is given as perpendicular to the beam, so the tension is found by taking torques about the pivot point; $T(3.00\ \text{m}) = (1.40\ \text{kN})(2.00\ \text{m})\cos 25.0° + (5.00\ \text{kN})(4.50\ \text{m})\cos 25.0°$, and $T = 7.64\ \text{kN}$. The vertical component of the force exerted on the beam by the pivot is the net weight minus the upward component of T, $6.00\ \text{kN} - T\cos 25.0° = -0.53\ \text{kN}$. The vertical component is downward. The horizontal force is $T\sin 25.0° = 3.23\ \text{kN}$.

EVALUATE: The vertical component of the tension is nearly the same magnitude as the total weight of the object and the vertical component of the force exerted by the pivot is much less than its horizontal component.

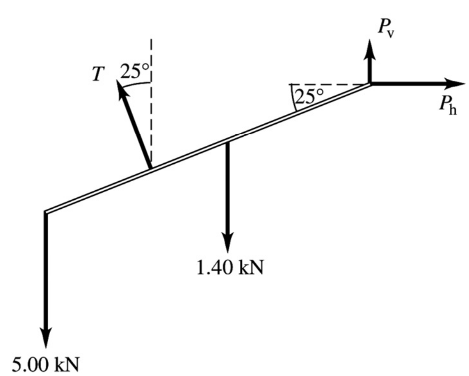
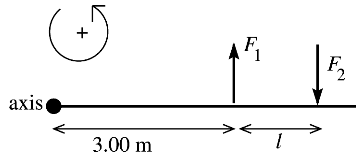


Figure 11.22

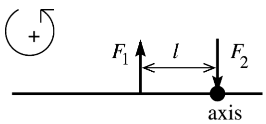
11.23. (a) IDENTIFY and SET UP: Use $\tau = Fl$ to calculate the torque (magnitude and direction) for each force and add the torques as vectors. See Figure 11.23a.



EXECUTE:
$\tau_1 = F_1 l_1 = +(8.00\text{ N})(3.00\text{ m})$
$\tau_1 = +24.0\text{ N}\cdot\text{m}$
$\tau_2 = -F_2 l_2 = -(8.00\text{ N})(l + 3.00\text{ m})$
$\tau_2 = -24.0\text{ N}\cdot\text{m} - (8.00\text{ N})l$

Figure 11.23a

$\sum \tau_z = \tau_1 + \tau_2 = +24.0\text{ N}\cdot\text{m} - 24.0\text{ N}\cdot\text{m} - (8.00\text{ N})l = -(8.00\text{ N})l$
Want l that makes $\sum \tau_z = -6.40\text{ N}\cdot\text{m}$ (net torque must be clockwise)
$-(8.00\text{ N})l = -6.40\text{ N}\cdot\text{m}$
$l = (6.40\text{ N}\cdot\text{m})/8.00\text{ N} = 0.800\text{ m}$
(b) $|\tau_2| > |\tau_1|$ since F_2 has a larger moment arm; the net torque is clockwise.
(c) See Figure 11.23b.



$\tau_1 = -F_1 l_1 = -(8.00\text{ N})l$
$\tau_2 = 0$ since $\vec{F}_2$ is at the axis

Figure 11.23b

$\sum \tau_z = -6.40\text{ N}\cdot\text{m}$ gives $-(8.00\text{ N})l = -6.40\text{ N}\cdot\text{m}$
$l = 0.800\text{ m}$, same as in part (a).
EVALUATE: The force couple gives the same magnitude of torque for the pivot at any point.

11.24. **IDENTIFY:** The person is in equilibrium, so the torques on him must balance. The target variable is the force exerted by the deltoid muscle.
SET UP: The free-body diagram for the arm is given in Figure 11.24. Take the pivot at the shoulder joint and let counterclockwise torques be positive. Use coordinates as shown. Let F be the force exerted by the deltoid muscle. There are also the weight of the arm and forces at the shoulder joint, but none of these forces produce any torque when the arm is in this position. The forces F and T have been replaced by their x- and y-components. $\sum \tau_z = 0$.

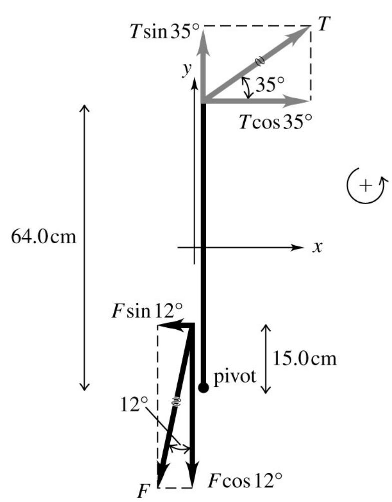


Figure 11.24

EXECUTE: $\sum \tau_z = 0$ gives $(F \sin 12.0°)(15.0 \text{ cm}) - (T \cos 35°)(64.0 \text{ cm}) = 0.$

$$F = \frac{(36.0 \text{ N})(\cos 35°)(64.0 \text{ cm})}{(\sin 12.0°)(15.0 \text{ cm})} = 605 \text{ N}.$$

EVALUATE: The force exerted by the deltoid muscle is much larger than the tension in the cable because the deltoid muscle makes a small angle (only 12.0°) with the humerus.

11.25. **IDENTIFY:** This problem involves the use of torques and graphical interpretation.

SET UP: In order to interpret the graph, apply $\sum \tau_z = 0$ to the rod and solve for the tension T in terms of the angle θ. Take torques about the hinge and call L the length of the rod. The mass of the rod is the target variable.

EXECUTE: $\sum \tau_z = 0$: $TL \sin\theta - mg\frac{L}{2}\cos\theta$, which gives $T = \frac{mg}{2}\frac{\cos\theta}{\sin\theta} = \left(\frac{mg}{2}\right)\cot\theta$. From this we see that a graph of T versus $\cot\theta$ should be a straight line having slope $mg/2$. Using the given slope we have $m = \frac{2(\text{slope})}{g} = \frac{2(30.0 \text{ N})}{9.80 \text{ m/s}^2} = 6.12$ kg.

EVALUATE: A situation like this might occur if you had a very heavy rod that could not easily be removed to weigh it, but could be pivoted about the hinge. The tension could be measured using a strain gauge.

11.26. **IDENTIFY:** Use $Y = \frac{l_0 F_\perp}{A\Delta l}$.

SET UP: $A = 50.0 \text{ cm}^2 = 50.0 \times 10^{-4} \text{ m}^2$.

EXECUTE: Relaxed: $Y = \frac{(0.200 \text{ m})(25.0 \text{ N})}{(50.0 \times 10^{-4} \text{ m}^2)(3.0 \times 10^{-2} \text{ m})} = 3.33 \times 10^4$ Pa.

Maximum tension: $Y = \frac{(0.200 \text{ m})(500 \text{ N})}{(50.0 \times 10^{-4} \text{ m}^2)(3.0 \times 10^{-2} \text{ m})} = 6.67 \times 10^5$ Pa.

EVALUATE: The muscle tissue is much more difficult to stretch when it is under maximum tension.

11.27. **IDENTIFY and SET UP:** Apply $Y = \frac{l_0 F_\perp}{A\Delta l}$ and solve for A and then use $A = \pi r^2$ to get the radius and $d = 2r$ to calculate the diameter.

EXECUTE: $Y = \frac{l_0 F_\perp}{A\Delta l}$ so $A = \frac{l_0 F_\perp}{Y\Delta l}$ (A is the cross-section area of the wire)

For steel, $Y = 2.0 \times 10^{11}$ Pa (Table 11.1)

Thus $A = \frac{(2.00 \text{ m})(700 \text{ N})}{(2.0 \times 10^{11} \text{ Pa})(0.25 \times 10^{-2} \text{ m})} = 2.8 \times 10^{-6} \text{ m}^2$.

$A = \pi r^2$, so $r = \sqrt{A/\pi} = \sqrt{2.8 \times 10^{-6} \text{ m}^2/\pi} = 9.44 \times 10^{-4}$ m

$d = 2r = 1.9 \times 10^{-3} \text{ m} = 1.9$ mm.

EVALUATE: Steel wire of this diameter doesn't stretch much; $\Delta l/l_0 = 0.12\%$.

11.28. **IDENTIFY:** Apply $Y = \frac{l_0 F_\perp}{A\Delta l}$.

SET UP: From Table 11.1, for steel, $Y = 2.0 \times 10^{11}$ Pa and for copper, $Y = 1.1 \times 10^{11}$ Pa. $A = \pi(d^2/4) = 1.77 \times 10^{-4} \text{ m}^2$. $F_\perp = 4000$ N for each rod.

EXECUTE: **(a)** The strain is $\frac{\Delta l}{l_0} = \frac{F}{YA}$. For steel $\frac{\Delta l}{l_0} = \frac{(4000 \text{ N})}{(2.0 \times 10^{11} \text{ Pa})(1.77 \times 10^{-4} \text{ m}^2)} = 1.1 \times 10^{-4}$.

Similarly, the strain for copper is 2.1×10^{-4}.

(b) Steel: $(1.1\times10^{-4})(0.750\text{ m}) = 8.3\times10^{-5}\text{ m}$. Copper: $(2.1\times10^{-4})(0.750\text{ m}) = 1.6\times10^{-4}\text{ m}$.

EVALUATE: Copper has a smaller Y and therefore a greater elongation.

11.29. IDENTIFY: Apply $Y = \dfrac{l_0F_\perp}{A\Delta l}$.

SET UP: $A = 0.50\text{ cm}^2 = 0.50\times10^{-4}\text{ m}^2$

EXECUTE: $Y = \dfrac{(4.00\text{ m})(5000\text{ N})}{(0.50\times10^{-4}\text{ m}^2)(0.20\times10^{-2}\text{ m})} = 2.0\times10^{11}\text{ Pa}$

EVALUATE: Our result is the same as that given for steel in Table 11.1.

11.30. IDENTIFY: Apply $Y = \dfrac{l_0F_\perp}{A\Delta l}$.

SET UP: $A = \pi r^2 = \pi(3.5\times10^{-3}\text{ m})^2 = 3.85\times10^{-5}\text{ m}^2$. The force applied to the end of the rope is the weight of the climber: $F_\perp = (65.0\text{ kg})(9.80\text{ m/s}^2) = 637\text{ N}$.

EXECUTE: $Y = \dfrac{(45.0\text{ m})(637\text{ N})}{(3.85\times10^{-5}\text{ m}^2)(1.10\text{ m})} = 6.77\times10^{8}\text{ Pa}$

EVALUATE: Our result is a lot smaller than the values given in Table 11.1. An object made of rope material is much easier to stretch than if the object were made of metal.

11.31. IDENTIFY: The increased pressure compresses the lead sphere, so we are dealing with bulk stress and strain.

SET UP: The bulk modulus is $B = -\dfrac{\Delta p}{\Delta V / V_0}$, and for lead it is $B = 4.1\times10^{10}$ Pa. The volume compresses by 0.50%, so $\Delta V = -0.0050V_0$. The target variable is the pressure increase that causes this amount of compression.

EXECUTE: Solve $-\dfrac{\Delta p}{\Delta V / V_0}$ for Δp : $\Delta p = -B\left(\dfrac{\Delta V}{V_0}\right) = -\left(4.1\times10^{10}\text{ Pa}\right)\left(\dfrac{-0.0050V_0}{V_0}\right) = 2.05\times10^{8}\text{ Pa} = 2.0\times10^{3}$ atm. The pressure is 2000 atmospheres above atmospheric pressure.

EVALUATE: This is a very large pressure, but it would take a large pressure to compress a lead sphere.

11.32. IDENTIFY: Apply $\text{stress} = \dfrac{F_\perp}{A}$, $\text{strain} = \dfrac{\text{stress}}{Y}$, $Y = \dfrac{l_0F_\perp}{A\Delta l}$.

SET UP: The cross-sectional area of the post is $A = \pi r^2 = \pi(0.125\text{ m})^2 = 0.0491\text{ m}^2$. The force applied to the end of the post is $F_\perp = (8000\text{ kg})(9.80\text{ m/s}^2) = 7.84\times10^4\text{ N}$. The Young's modulus of steel is $Y = 2.0\times10^{11}\text{ Pa}$.

EXECUTE: **(a)** $\text{stress} = \dfrac{F_\perp}{A} = -\dfrac{7.84\times10^4\text{ N}}{0.0491\text{ m}^2} = -1.60\times10^6\text{ Pa}$. The minus sign indicates that the stress is compressive.

(b) $\text{strain} = \dfrac{\text{stress}}{Y} = -\dfrac{1.60\times10^6\text{ Pa}}{2.0\times10^{11}\text{ Pa}} = -8.0\times10^{-6}$. The minus sign indicates that the length decreases.

(c) $\Delta l = l_0(\text{strain}) = (2.50\text{ m})(-8.0\times10^{-6}) = -2.0\times10^{-5}\text{ m}$

EVALUATE: The fractional change in length of the post is very small.

11.33. IDENTIFY: The amount of compression depends on the bulk modulus of the bone.

SET UP: $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$ and $1\text{ atm} = 1.01\times10^5\text{ Pa}$.

EXECUTE: **(a)** $\Delta p = -B\dfrac{\Delta V}{V_0} = -(15\times10^9\ \text{Pa})(-0.0010) = 1.5\times10^7\ \text{Pa} = 150\ \text{atm}.$

(b) The depth for a pressure increase of $1.5\times10^7\ \text{Pa}$ is 1.5 km.

EVALUATE: An extremely large pressure increase is needed for just a 0.10% bone compression, so pressure changes do not appreciably affect the bones. Unprotected dives do not approach a depth of 1.5 km, so bone compression is not a concern for divers.

11.34. **IDENTIFY:** Apply $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}.$

SET UP: $\Delta V = -\dfrac{V_0 \Delta p}{B}.$ Δp is positive when the pressure increases.

EXECUTE: **(a)** The volume would increase slightly.
(b) The volume change would be twice as great.
(c) The volume change is inversely proportional to the bulk modulus for a given pressure change, so the volume change of the lead ingot would be four times that of the gold.

EVALUATE: For lead, $B = 4.1\times10^{10}\ \text{Pa},$ so $\Delta p/B$ is very small and the fractional change in volume is very small.

11.35. **IDENTIFY** and **SET UP:** Use $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$ and $k = 1/B$ to calculate B and k.

EXECUTE: $B = -\dfrac{\Delta p}{\Delta V/V_0} = -\dfrac{(3.6\times10^6\ \text{Pa})(600\ \text{cm}^3)}{(-0.45\ \text{cm}^3)} = +4.8\times10^9\ \text{Pa}$

$k = 1/B = 1/4.8\times10^9\ \text{Pa} = 2.1\times10^{-10}\ \text{Pa}^{-1}$

EVALUATE: k is the same as for glycerine (Table 11.2).

11.36. **IDENTIFY:** Apply $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}.$ Density $= m/V.$

SET UP: At the surface the pressure is $1.0\times10^5\ \text{Pa},$ so $\Delta p = 1.16\times10^8\ \text{Pa}.$ $V_0 = 1.00\ \text{m}^3.$ At the surface $1.00\ \text{m}^3$ of water has mass $1.03\times10^3\ \text{kg}.$

EXECUTE: **(a)** $B = -\dfrac{(\Delta p)V_0}{\Delta V}$ gives $\Delta V = -\dfrac{(\Delta p)V_0}{B} = -\dfrac{(1.16\times10^8\ \text{Pa})(1.00\ \text{m}^3)}{2.2\times10^9\ \text{Pa}} = -0.0527\ \text{m}^3$

(b) At this depth $1.03\times10^3\ \text{kg}$ of seawater has volume $V_0 + \Delta V = 0.9473\ \text{m}^3.$ The density is $\dfrac{1.03\times10^3\ \text{kg}}{0.9473\ \text{m}^3} = 1.09\times10^3\ \text{kg/m}^3.$

EVALUATE: The density is increased because the volume is compressed due to the increased pressure.

11.37. **IDENTIFY:** Apply $S = \dfrac{F_\parallel}{A}\dfrac{h}{x}.$

SET UP: $F_\parallel = 9.0\times10^5\ \text{N}.$ $A = (0.100\ \text{m})(0.500\times10^{-2}\ \text{m}).$ $h = 0.100\ \text{m}.$ From Table 11.1, $S = 7.5\times10^{10}\ \text{Pa}$ for steel.

EXECUTE: **(a)** Shear strain $= \dfrac{F_\parallel}{AS} = \dfrac{(9\times10^5\ \text{N})}{[(0.100\ \text{m})(0.500\times10^{-2}\ \text{m})][7.5\times10^{10}\ \text{Pa}]} = 2.4\times10^{-2}.$

(b) Since shear strain $= x/h,$ $x = (\text{Shear strain})\cdot h = (0.024)(0.100\ \text{m}) = 2.4\times10^{-3}\ \text{m}.$

EVALUATE: This very large force produces a small displacement; $x/h = 2.4\%.$

11.38. **IDENTIFY:** The force components parallel to the face of the cube produce a shear which can deform the cube.

SET UP: $S = \dfrac{F_\parallel}{A\phi}$, where $\phi = x/h$. $F_\parallel$ is the component of the force tangent to the surface, so $F_\parallel = (1375\ \text{N})\cos 8.50° = 1360\ \text{N}$. ϕ must be in radians, $\phi = 1.24° = 0.0216$ rad.

EXECUTE: $S = \dfrac{1360\ \text{N}}{(0.0925\ \text{m})^2(0.0216\ \text{rad})} = 7.36\times 10^6\ \text{Pa}$.

EVALUATE: The shear modulus of this material is much less than the values for metals given in Table 11.1 in the text.

11.39. **IDENTIFY:** The problem involves the stretching of metal wires, so it makes use of tensile stress and strain and Young's modulus.

SET UP: The steel and aluminum wires have the same fractional change in length. We use $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$. We know that $\dfrac{\Delta\ell}{\ell_0}$ is the same for both wires, $r_{\text{al}} = 2r_{\text{st}}$, and $A = \pi r^2$. Table 11.1 gives us the values of Y for both metals. Our target variable is the tension T_{al} in the aluminum wire in terms of the tension T_{st} in the steel wire. Since $\dfrac{\Delta\ell}{\ell_0}$ is the same for both wires, we can equate this expression for both wires and solve for T_{al}.

EXECUTE: <u>Steel</u>: $\dfrac{\Delta\ell}{\ell_0} = \dfrac{T_{\text{st}}}{Y_{\text{st}}\pi r_{\text{st}}^2}$. <u>Aluminum</u>: $\dfrac{T_{\text{al}}}{Y_{\text{al}}\pi r_{\text{al}}^2}$. Equating gives $\dfrac{T_{\text{st}}}{Y_{\text{st}}\pi r_{\text{st}}^2} = \dfrac{T_{\text{al}}}{Y_{\text{al}}\pi r_{\text{al}}^2}$. Using the fact that $r_{\text{al}} = 2r_{\text{st}}$ and $A = \pi r^2$, we have $T_{\text{al}} = \dfrac{Y_{\text{al}}}{Y_{\text{st}}}\left(\dfrac{r_{\text{al}}}{r_{\text{st}}}\right)^2 T_{\text{st}} = \dfrac{7.0\times 10^{10}\ \text{Pa}}{20\times 10^{10}\ \text{Pa}}\left(\dfrac{2r_{\text{st}}}{r_{\text{st}}}\right)^2 T_{\text{st}} = 1.4T_{\text{st}}$.

EVALUATE: Although aluminum has a smaller Young's modulus than does steel, the aluminum wire is twice as thick and hence has 4 times the area as the steel wire, so it can support more tension than the steel for the same fractional stretch.

11.40. **IDENTIFY:** The applied force stretches the wire, so are dealing with tensile stress and strain and Young's modulus.

SET UP: Since we graph $\dfrac{\Delta\ell}{\ell_0}$ versus $F_\perp$, we need to discover a relationship between these quantities so we can interpret the slope of the graph. We start with $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$. Our target variable is Y for the metal of the wire.

EXECUTE: Solve $\dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$ for $\dfrac{\Delta\ell}{\ell_0}$ in terms of $F_\perp$, which gives $\dfrac{\Delta\ell}{\ell_0} = \dfrac{1}{AY}F_\perp$. Therefore a graph of $\dfrac{\Delta\ell}{\ell_0}$ versus $F_\perp$ should be a straight line having slope $\dfrac{1}{AY}$. Thus $Y = \dfrac{1}{A(\text{slope})}$, which gives

$$Y = \frac{1}{(8.00\times 10^{-6}\ \text{m}^2)(8.00\times 10^{-7}\ \text{N}^{-1})} = 1.6\times 10^{11}\ \text{Pa}.$$

EVALUATE: From Table 11.1, we see that this value is about the same as Y for steel, nickel, iron, and copper, so it is a reasonable result.

11.41. **IDENTIFY:** We are dealing with compression due to increased pressure, so we must use bulk stress and strain and the bulk modulus B.

SET UP: Since we are dealing with liquids, we will need to use Table 11.2. It gives the compressibility k, which is $1/B$, so $k = -\frac{\Delta V / V_0}{\Delta p}$. We know that $\Delta V / V_0$ will be negative (for a compression) when Δp is positive, so we can neglect the minus signs. We also know that $\Delta V / V_0$ is the same for both liquids. The target variable is the pressure increase Δp_{a} in the alcohol.

EXECUTE: Equating $\Delta V / V_0$ for both liquids gives $k_{\text{a}}\Delta p_{\text{a}} = k_{\text{g}}\Delta p_{\text{g}} = k_{\text{g}}\Delta p_1$. Solving for Δp_{a} gives

$$\Delta p_{\text{a}} = \frac{k_{\text{g}}}{k_{\text{a}}}\Delta p_1 = \frac{21\times 10^{-11}\ \text{Pa}^{-1}}{110\times 10^{-11}\ \text{Pa}^{-1}}\Delta p_1 = 0.19\Delta p_1.$$

EVALUATE: Since $k_{\text{a}} \approx 5k_{\text{g}}$, alcohol is 5 times more compressible than glycerin, so it takes only about 1/5 the pressure increase to produce the same compression as for glycerin. This is a reasonable result.

11.42. **IDENTIFY:** The breaking stress of the wire is the value of $F_\perp/A$ at which the wire breaks.

SET UP: From Table 11.3, the breaking stress of brass is 4.7×10^8 Pa. The area A of the wire is related to its diameter by $A = \pi d^2/4$.

EXECUTE: $A = \frac{350\ \text{N}}{4.7\times 10^8\ \text{Pa}} = 7.45\times 10^{-7}\ \text{m}^2$, so $d = \sqrt{4A/\pi} = 0.97$ mm.

EVALUATE: The maximum force a wire can withstand without breaking is proportional to the square of its diameter.

11.43. **IDENTIFY** and **SET UP:** Use $\text{stress} = \frac{F_\perp}{A}$.

EXECUTE: $\text{Tensile stress} = \frac{F_\perp}{A} = \frac{F_\perp}{\pi r^2} = \frac{90.8\ \text{N}}{\pi(0.92\times 10^{-3}\ \text{m})^2} = 3.41\times 10^7\ \text{Pa}$

EVALUATE: A modest force produces a very large stress because the cross-sectional area is small.

11.44. **IDENTIFY:** The elastic limit is a value of the stress, $F_\perp/A$. Apply $\sum \vec{F} = m\vec{a}$ to the elevator in order to find the tension in the cable.

SET UP: $\frac{F_\perp}{A} = \frac{1}{3}(2.40\times 10^8\ \text{Pa}) = 0.80\times 10^8\ \text{Pa}$. The free-body diagram for the elevator is given in Figure 11.44. $F_\perp$ is the tension in the cable.

EXECUTE: $F_\perp = A(0.80\times 10^8\ \text{Pa}) = (3.00\times 10^{-4}\ \text{m}^2)(0.80\times 10^8\ \text{Pa}) = 2.40\times 10^4\ \text{N}$. $\sum F_y = ma_y$ applied to the elevator gives $F_\perp - mg = ma$ and $a = \frac{F_\perp}{m} - g = \frac{2.40\times 10^4\ \text{N}}{1200\ \text{kg}} - 9.80\ \text{m/s}^2 = 10.2\ \text{m/s}^2$

EVALUATE: The tension in the cable is about twice the weight of the elevator.

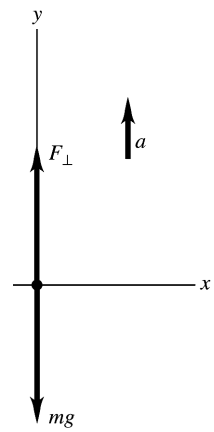


Figure 11.44

11.45. **IDENTIFY:** To pull out the nail, you exert a torque on the handle which in turn produces a torque on the nail.

SET UP: Estimate: $F_2 = 25$ lb in Fig. 11.45 in the text. Use $\sum \tau_z = 0$ about contact point A in the figure. The target variable is the force the hammer exerts on the nail (F_1 in the figure).

EXECUTE: Using the figure in the text, applying $\sum \tau_z = 0$ to the hammer gives $F_2(0.300 \text{ m}) - F_1(\sin 60°)(0.080 \text{ m}) = 0.$ Using $F_2 = 25$ lb and solving for F_1 gives $F_1 = 110$ lb. Note that in the figure $\vec{F}_1$ is the force on the nail, but by Newton's third law, the force on the hammer is equal and opposite to this force.

EVALUATE: The long handle allows the force on the nail to be about 4 times your force. A longer handle would increase this factor even more.

11.46. **IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

SET UP: The free-body diagram for the door is given in Figure 11.46. Let $\vec{H}_1$ and $\vec{H}_2$ be the forces exerted by the upper and lower hinges. Take the origin of coordinates at the bottom hinge (point A) and $+y$ upward.

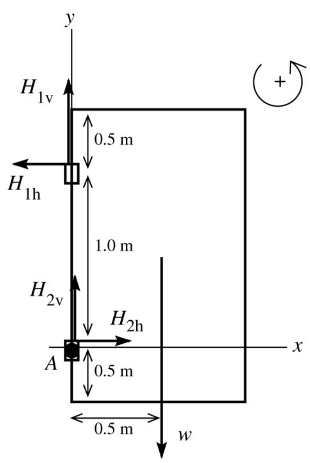


Figure 11.46

EXECUTE:
We are given that
$H_{1v} = H_{2v} = w/2 = 165$ N.
$\sum F_x = ma_x$
$H_{2h} - H_{1h} = 0$
$H_{1h} = H_{2h}$
The horizontal components of the hinge forces are equal in magnitude and opposite in direction.

Sum torques about point A. H_{1v}, H_{2v}, and H_{2h} all have zero moment arm and hence zero torque about an axis at this point. Thus $\sum \tau_A = 0$ gives $H_{1h}(1.00 \text{ m}) - w(0.50 \text{ m}) = 0$

$$H_{1h} = w\left(\frac{0.50\text{ m}}{1.00\text{ m}}\right) = \tfrac{1}{2}(330\text{ N}) = 165\text{ N}.$$

The horizontal component of each hinge force is 165 N.

EVALUATE: The horizontal components of the force exerted by each hinge are the only horizontal forces so must be equal in magnitude and opposite in direction. With an axis at A, the torque due to the horizontal force exerted by the upper hinge must be counterclockwise to oppose the clockwise torque exerted by the weight of the door. So, the horizontal force exerted by the upper hinge must be to the left. You can also verify that the net torque is also zero if the axis is at the upper hinge.

11.47. **IDENTIFY:** The center of gravity of the combined object must be at the fulcrum. Use $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$ to calculate x_{cm}.

SET UP: The center of gravity of the sand is at the middle of the box. Use coordinates with the origin at the fulcrum and $+x$ to the right. Let $m_1 = 25.0$ kg, so $x_1 = 0.500$ m. Let $m_2 = m_{\text{sand}}$, so $x_2 = -0.625$ m. $x_{\text{cm}} = 0$.

EXECUTE: $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2} = 0$ and $m_2 = -m_1\dfrac{x_1}{x_2} = -(25.0\text{ kg})\left(\dfrac{0.500\text{ m}}{-0.625\text{ m}}\right) = 20.0\text{ kg}.$

EVALUATE: The mass of sand required is less than the mass of the plank since the center of the box is farther from the fulcrum than the center of gravity of the plank is.

11.48. **IDENTIFY:** Apply $\sum\tau_z = 0$ to the bridge.

SET UP: Let the axis of rotation be at the left end of the bridge and let counterclockwise torques be positive.

EXECUTE: If Lancelot were at the end of the bridge, the tension in the cable would be (from taking torques about the hinge of the bridge) obtained from

$T(12.0\text{ m}) = (600\text{ kg})(9.80\text{ m/s}^2)(12.0\text{ m}) + (200\text{ kg})(9.80\text{ m/s}^2)(6.0\text{ m})$, so $T = 6860$ N.

This exceeds the maximum tension that the cable can have, so Lancelot is going into the drink. To find the distance x Lancelot can ride, replace the 12.0 m multiplying Lancelot's weight by x and the tension T by $T_{\text{max}} = 5.80\times10^3$ N and solve for x;

$$x = \frac{(5.80\times10^3\text{ N})(12.0\text{ m}) - (200\text{ kg})(9.80\text{ m/s}^2)(6.0\text{ m})}{(600\text{ kg})(9.80\text{ m/s}^2)} = 9.84\text{ m}.$$

EVALUATE: Before Lancelot goes onto the bridge, the tension in the supporting cable is $T = \dfrac{(6.0\text{ m})(200\text{ kg})(9.80\text{ m/s}^2)}{12.0\text{ m}} = 980$ N, well below the breaking strength of the cable. As he moves along the bridge, the increase in tension is proportional to x, the distance he has moved along the bridge.

11.49. **IDENTIFY:** Apply the conditions of equilibrium to the climber. For the minimum coefficient of friction the static friction force has the value $f_s = \mu_s n$.

SET UP: The free-body diagram for the climber is given in Figure 11.49. f_s and n are the vertical and horizontal components of the force exerted by the cliff face on the climber. The moment arm for the force T is $(1.4\text{ m})\cos10°$.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $T(1.4\text{ m})\cos10° - w(1.1\text{ m})\cos35.0° = 0$.

$$T = \frac{(1.1\text{ m})\cos35.0°}{(1.4\text{ m})\cos10°}(82.0\text{ kg})(9.80\text{ m/s}^2) = 525\text{ N}$$

(b) $\sum F_x = 0$ gives $n = T\sin 25.0° = 222\ \text{N}$. $\sum F_y = 0$ gives $f_s + T\cos 25° - w = 0$ and $f_s = (82.0\ \text{kg})(9.80\ \text{m/s}^2) - (525\ \text{N})\cos 25° = 328\ \text{N}$.

(c) $\mu_s = \dfrac{f_s}{n} = \dfrac{328\ \text{N}}{222\ \text{N}} = 1.48$

EVALUATE: To achieve this large value of μ_s the climber must wear special rough-soled shoes.

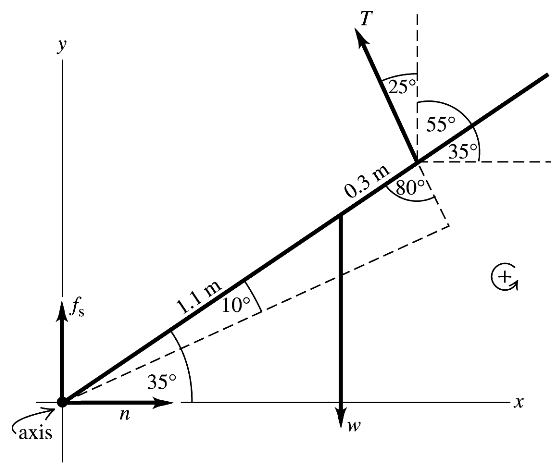


Figure 11.49

11.50. **IDENTIFY:** The beam is at rest, so the forces and torques on it must balance.

SET UP: The weight of the beam acts 4.0 m from each end. Take the pivot at the hinge and let counterclockwise torques be positive. Represent the force exerted by the hinge by its horizontal and vertical components, H_h and H_v. $\sum F_x = 0$, $\sum F_y = 0$ and $\sum \tau_z = 0$.

EXECUTE: **(a)** The free-body diagram for the beam is given in Figure 11.50a.

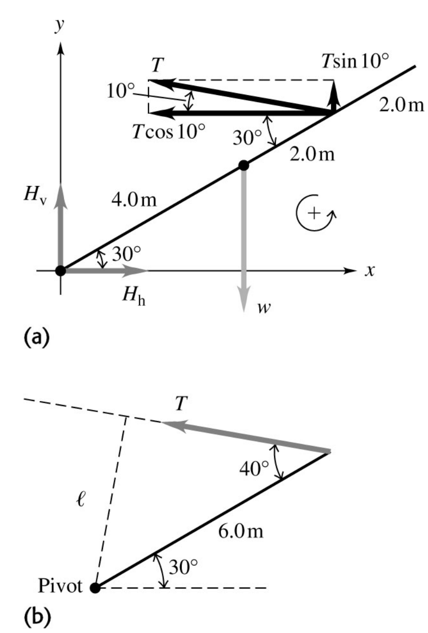


Figure 11.50

(b) The moment arm for T is sketched in Figure 11.50b and is equal to $(6.0\text{ m})\sin 40.0°$. $\sum\tau_z = 0$ gives $T(6.0\text{ m})(\sin 40.0°) - w(4.0\text{ m})(\cos 30.0°) = 0$.

$$T = \frac{(1150\text{ kg})(9.80\text{ m/s}^2)(4.0\text{ m})(\cos 30.0°)}{(6.0\text{ m})(\sin 40.0°)} = 1.01\times 10^4\text{ N}.$$

(c) $\sum F_x = 0$ gives $H_\text{h} - T\cos 10.0° = 0$ and $H_\text{h} = T\cos 10.0° = 9.97\times 10^3$ N.

EVALUATE: The tension is less than the weight of the beam because it has a larger moment arm than the weight force has.

11.51. **IDENTIFY:** In each case, to achieve balance the center of gravity of the system must be at the fulcrum. Use $x_\text{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$ to locate x_cm, with m_i replaced by w_i.

SET UP: Let the origin be at the left-hand end of the rod and take the $+x$-axis to lie along the rod. Let $w_1 = 255$ N (the rod) so $x_1 = 1.00$ m, let $w_2 = 225$ N so $x_2 = 2.00$ m and let $w_3 = W$. In part (a) $x_3 = 0.500$ m and in part (b) $x_3 = 0.750$ m.

EXECUTE: **(a)** $x_\text{cm} = 1.25$ m. $x_\text{cm} = \dfrac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3}$ gives $w_3 = \dfrac{(w_1 + w_2)x_\text{cm} - w_1x_1 - w_2x_2}{x_3 - x_\text{cm}}$ and

$$W = \frac{(480\text{ N})(1.25\text{ m}) - (255\text{ N})(1.00\text{ m}) - (225\text{ N})(2.00\text{ m})}{0.500\text{ m} - 1.25\text{ m}} = 140\text{ N}.$$

(b) Now $w_3 = W = 140$ N and $x_3 = 0.750$ m.

$$x_\text{cm} = \frac{(255\text{ N})(1.00\text{ m}) + (225\text{ N})(2.00\text{ m}) + (140\text{ N})(0.750\text{ m})}{255\text{ N} + 225\text{ N} + 140\text{ N}} = 1.31\text{ m}.$$ W must be moved $1.31\text{ m} - 1.25\text{ m} = 6\text{ cm}$ to the right.

EVALUATE: Moving W to the right means x_cm for the system moves to the right.

11.52. **IDENTIFY:** Apply $\sum\tau_z = 0$ to the hammer.

SET UP: Take the axis of rotation to be at point A.

EXECUTE: The force $\vec{F}_1$ is directed along the length of the nail, and so has a moment arm of $(0.080\text{ m})\sin 60°$. The moment arm of $\vec{F}_2$ is 0.300 m, so

$$F_2 = F_1\frac{(0.0800\text{ m})\sin 60°}{(0.300\text{ m})} = (400\text{ N})(0.231) = 92.4\text{ N}.$$

EVALUATE: The force F_2 that must be applied to the hammer handle is much less than the force that the hammer applies to the nail, because of the large difference in the lengths of the moment arms.

11.53. **IDENTIFY:** Apply the conditions of equilibrium to the horizontal beam. Since the two wires are symmetrically placed on either side of the middle of the sign, their tensions are equal and are each equal to $T_\text{w} = mg/2 = 137$ N.

SET UP: The free-body diagram for the beam is given in Figure 11.53. F_v and F_h are the vertical and horizontal forces exerted by the hinge on the beam. Since the cable is 2.00 m long and the beam is 1.50 m long, $\cos\theta = \dfrac{1.50\text{ m}}{2.00\text{ m}}$ and $\theta = 41.4°$. The tension T_c in the cable has been replaced by its horizontal and vertical components.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $T_\text{c}(\sin 41.4°)(1.50\text{ m}) - w_\text{beam}(0.750\text{ m}) - T_\text{w}(1.50\text{ m}) - T_\text{w}(0.60\text{ m}) = 0$.

$$T_\text{c} = \frac{(16.0\text{ kg})(9.80\text{ m/s}^2)(0.750\text{ m}) + (137\text{ N})(1.50\text{ m} + 0.60\text{ m})}{(1.50\text{ m})(\sin 41.4°)} = 408.6\text{ N},$$ which rounds to 409 N.

(b) $\sum F_y = 0$ gives $F_v + T_c \sin 41.4° - w_{beam} - 2T_w = 0$ and

$F_v = 2T_w + w_{beam} - T_c \sin 41.4° = 2(137 \text{ N}) + (16.0 \text{ kg})(9.80 \text{ m/s}^2) - (408.6 \text{ N})(\sin 41.4°) = 161 \text{ N}$. The hinge must be able to supply a vertical force of 161 N.

EVALUATE: The force from the two wires could be replaced by the weight of the sign acting at a point 0.60 m to the left of the right-hand edge of the sign.

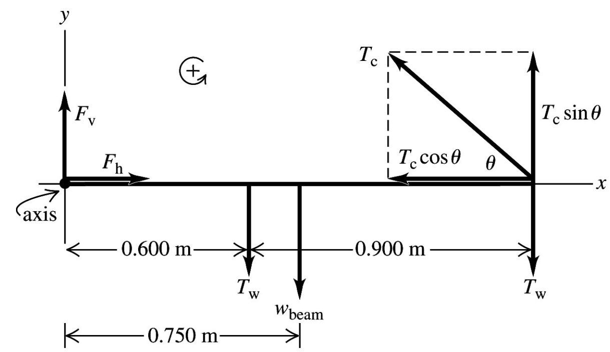


Figure 11.53

11.54. **IDENTIFY:** Apply the first and second conditions of equilibrium to the bar.

SET UP: The free-body diagram for the bar is given in Figure 11.54. n is the normal force exerted on the bar by the surface. There is no friction force at this surface. H_h and H_v are the components of the force exerted on the bar by the hinge. The components of the force of the bar on the hinge will be equal in magnitude and opposite in direction.

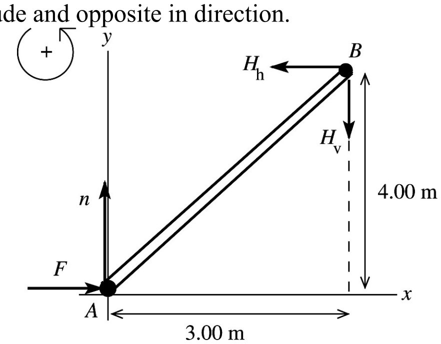


Figure 11.54

EXECUTE:

$\sum F_x = ma_x$

$F = H_h = 220 \text{ N}$

$\sum F_y = ma_y$

$n - H_v = 0$

$H_v = n$, but we don't know either of these forces.

$\sum \tau_B = 0$ gives $F(4.00 \text{ m}) - n(3.00 \text{ m}) = 0$.

$n = (4.00 \text{ m}/3.00 \text{ m})F = \frac{4}{3}(220 \text{ N}) = 293 \text{ N}$ and then $H_v = 293 \text{ N}$.

Force of bar on hinge:
horizontal component 220 N, to right
vertical component 293 N, upward

EVALUATE: $H_h/H_v = 220/293 = 0.75 = 3.00/4.00$, so the force the hinge exerts on the bar is directed along the bar. $\vec{n}$ and $\vec{F}$ have zero torque about point A, so the line of action of the hinge force $\vec{H}$ must pass through this point also if the net torque is to be zero.

11.55. **IDENTIFY:** We want to locate the center of mass of the leg-cast system. We can treat each segment of the leg and cast as a point-mass located at its center of mass.

SET UP: The force diagram for the leg is given in Figure 11.55. The weight of each piece acts at the center of mass of that piece. The mass of the upper leg is $m_{ul} = (0.215)(37 \text{ kg}) = 7.955 \text{ kg}$. The mass of the lower leg is $m_{ll} = (0.140)(37\text{ kg}) = 5.18\text{ kg}$. Use the coordinates shown, with the origin at the hip and the x-axis along the leg, and use $x_{cm} = \dfrac{x_{ul}m_{ul} + x_{ll}m_{ll} + x_{cast}m_{cast}}{m_{ul} + m_{ll} + m_{cast}}$.

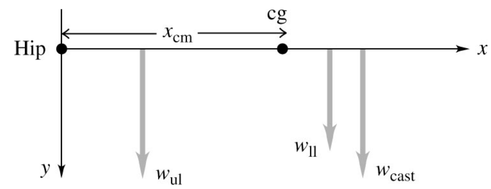


Figure 11.55

EXECUTE: Using $x_{cm} = \dfrac{x_{ul}m_{ul} + x_{ll}m_{ll} + x_{cast}m_{cast}}{m_{ul} + m_{ll} + m_{cast}}$, we have

$$x_{cm} = \frac{(18.0\text{ cm})(7.955\text{ kg}) + (69.0\text{ cm})(5.18\text{ kg}) + (78.0\text{ cm})(5.50\text{ kg})}{7.955\text{ kg} + 5.18\text{ kg} + 5.50\text{ kg}} = 49.9\text{ cm}$$

EVALUATE: The strap is attached to the left of the center of mass of the cast, but it is still supported by the rigid cast since the cast extends beyond its center of mass.

11.56. **IDENTIFY:** Apply the first and second conditions for equilibrium to the bridge.
SET UP: Find torques about the hinge. Use L as the length of the bridge and w_T and w_B for the weights of the truck and the raised section of the bridge. Take $+y$ to be upward and $+x$ to be to the right.
EXECUTE: **(a)** $TL\sin 70° = w_T(\tfrac{3}{4}L)\cos 30° + w_B(\tfrac{1}{2}L)\cos 30°$, so

$$T = \frac{(\tfrac{3}{4}m_T + \tfrac{1}{2}m_B)(9.80\text{ m/s}^2)\cos 30°}{\sin 70°} = 2.84\times 10^5\text{ N}.$$

(b) Horizontal: $T\cos(70° - 30°) = 2.18\times 10^5\text{ N}$ (to the right).

Vertical: $w_T + w_B - T\sin 40° = 2.88\times 10^5\text{ N}$ (upward).

EVALUATE: If ϕ is the angle of the hinge force above the horizontal,

$\tan\phi = \dfrac{2.88\times 10^5\text{ N}}{2.18\times 10^5\text{ N}}$ and $\phi = 52.9°$. The hinge force is not directed along the bridge.

11.57. **IDENTIFY:** The rod is suspended at rest, so the forces and torques on it must balance. Once the wire breaks, it rotates downward about the hinge, so we can use energy conservation.
SET UP: While the rod is at rest, we use $\sum\tau_z = 0$. After the wire breaks, we apply energy conservation $U_1 + K_1 + W_{other} = U_2 + K_2$ with $I = \dfrac{1}{3}ML^2$, $K_1 = 0$, $U_2 = 0$, and $W_{other} = 0$ because the hinge is frictionless. Our target variables are the angle the wire makes with the horizontal and its angular speed after the wire breaks. Begin with a free-body diagram of the rod, as in Fig. 11.57.

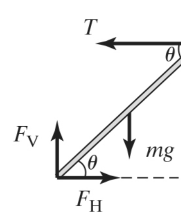


Figure 11.57

EXECUTE: **(a)** $\sum \tau_z = 0$: $TL\sin\theta - mg\dfrac{L}{2}\cos\theta = 0$, which gives $\theta = \arctan\left(\dfrac{mg}{2T}\right)$.

(b) Calling $y = 0$ at the level when the rod is horizontal, $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ gives

$mg\dfrac{L}{2}\sin\theta = \dfrac{1}{2}\left(\dfrac{1}{3}mL^2\right)\omega^2$, from which we get $\omega = \sqrt{\dfrac{3g\sin\theta}{L}}$.

EVALUATE: Check is some special cases: If θ is large, ω is large, which is reasonable because a large θ means that the center of gravity of the rod would be higher than for a small θ. If $\theta = 0$, $\omega = 0$, which is reasonable since the rod started from rest horizontally.

11.58. **IDENTIFY:** The arm is stationary, so the forces and torques must each balance.

SET UP: $\sum\tau = 0$, $\sum F_x = 0$, $\sum F_y = 0$. Let the forearm be at an angle ϕ below the horizontal. Take the pivot at the elbow joint and let counterclockwise torques be positive. Let $+y$ be upward and let $+x$ be to the right. Each forearm has mass $m_{\text{arm}} = \frac{1}{2}(0.0600)(72\text{ kg}) = 2.16\text{ kg}$. The weight held in each hand is $w = mg$, with $m = 7.50\text{ kg}$. $\vec{T}$ is the force the biceps muscle exerts on the forearm. $\vec{E}$ is the force exerted by the elbow and has components E_v and E_h.

EXECUTE: **(a)** The free-body diagram is shown in Figure 11.58.

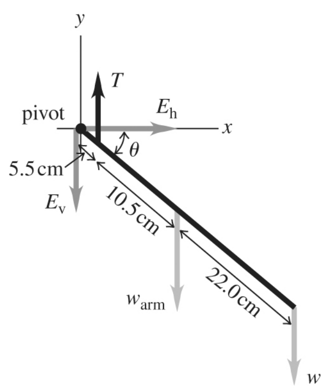


Figure 11.58

(b) $\sum\tau = 0$ gives $T(5.5\text{ cm})(\cos\theta) - w_{\text{arm}}(16.0\text{ cm})(\cos\theta) - w(38.0\text{ cm})(\cos\theta) = 0$

$$T = \frac{16.0w_{\text{arm}} + 38.0w}{5.5} = \frac{16.0(2.16\text{ kg})(9.80\text{ m/s}^2) + 38.0(7.50\text{ kg})(9.80\text{ m/s}^2)}{5.5} = 569\text{ N}$$

(c) $\sum F_x = 0$ gives $E_\text{h} = 0$. $\sum F_y = 0$ gives $T - E_\text{v} - w_{\text{arm}} - w = 0$, so

$$E_\text{v} = T - w_{\text{arm}} - w = 569\text{ N} - (2.16\text{ kg})(9.80\text{ m/s}^2) - (7.50\text{ kg})(9.80\text{ m/s}^2) = 474\text{ N}$$

Since we calculate E_v to be positive, we correctly assumed that it was downward when we drew the free-body diagram.

(d) The weight and the pull of the biceps are both always vertical in this situation, so the factor $\cos\theta$ divides out of the $\sum\tau = 0$ equation in part (b). Therefore the force T stays the same as she raises her arm.

EVALUATE: The biceps force must be much greater than the weight of the forearm and the weight in her hand because it has such a small lever arm compared to those two forces.

11.59. **IDENTIFY:** The rod is held in position, so the forces and torques on it must balance.

SET UP: Start with a free-body diagram of the rod, as in Fig. 11.59. Apply $\sum\tau_z = 0$, $\sum F_x = 0$, and $\sum F_y = 0$. The target variable is the angle β in the figure.

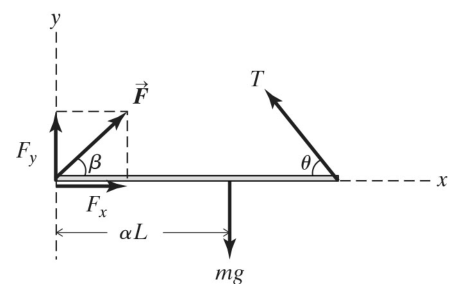


Figure 11.59

EXECUTE: **(a)** If we can find the components F_x and F_y of the hinge force $\vec{F}$, we can use them to find β. Taking torques about the right end of the rod gives $F_y L = mg(L - \alpha L) = mgL(1-\alpha)$, which gives

$F_y = mg(1-\alpha)$ (Eq. 1)

$\sum F_x = 0$: $F_x = T\cos\theta$ (Eq. 2)

$\sum\tau_z = 0$ about the hinge: $TL\sin\theta = mg\alpha L$ (Eq. 3)

Dividing Eq. 3 by Eq. 2 gives $\dfrac{TL\sin\theta}{T\cos\theta} = \dfrac{mg\alpha L}{F_x}$, which gives $F_x = \dfrac{mg\alpha}{\tan\theta}$. Now use this result and Eq. 1 to find β. $\tan\beta = \dfrac{F_y}{F_x} = \dfrac{mg(1-\alpha)}{\dfrac{mg\alpha}{\tan\theta}} = \left(\dfrac{1}{\alpha} - 1\right)\tan\theta$, so $\beta = \arctan\left(\dfrac{1}{\alpha} - 1\right)$.

(b) If $\beta = \theta$ we get $\tan\beta = \left(\dfrac{1}{\alpha} - 1\right)\tan\theta = \tan\theta$, so $\alpha = ½$.

(c) If $\alpha = 1$, we get $\tan\beta = \left(\dfrac{1}{\alpha} - 1\right)\tan\theta = 0$, so $\beta = 0$.

EVALUATE: In part (c), if $\beta = 0$ then $F_y = 0$, so all the weight of m is supported by T_y. Taking torques about the hinge gives $TL\sin\theta - mgL = 0$, so $T\sin\theta = mg$, which agrees with our answer with $\beta = 0$.

11.60. **IDENTIFY:** The rod is held fixed so it is in equilibrium. Therefore the forces and torques on it must balance.

SET UP: Apply $\sum\tau_z = 0$, $\sum F_x = 0$, and $\sum F_y = 0$. The target variable is the friction force f at the wall and the maximum angle θ for which slipping will not occur. Fig. 11.60 shows a free-body diagram of the rod.

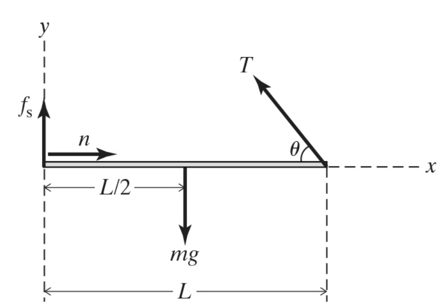


Figure 11.60

EXECUTE: **(a)** Using $\sum\tau_z = 0$ about the right end gives $fL = mg\dfrac{L}{2}$, so $f = \dfrac{mg}{2}$.

(b) $\sum F_x = 0$: $n = T\cos\theta$

$\sum F_y = 0$: $T\sin\theta + f = mg$. Using $f = \dfrac{mg}{2}$ and simplifying gives $T\sin\theta = mg - f = mg - \dfrac{mg}{2} = \dfrac{mg}{2}$,

so $T = \dfrac{mg}{2\sin\theta}$. Using this result, we find n to be $n = T\cos\theta = \left(\dfrac{mg}{2\sin\theta}\right)\cos\theta = \dfrac{mg}{2\tan\theta}$. At the maximum angle θ the rod is just ready to slip, so static friction is at its maximum value of $f_{\max} = \mu_s n$.

Combining this with our results that $f = \dfrac{mg}{2}$ and $n = \dfrac{mg}{2\tan\theta}$, we have $f = \dfrac{mg}{2} = \mu_s n = \dfrac{\mu_s mg}{2\tan\theta}$, which gives $\tan\theta = \mu_s$, so $\theta = \arctan\mu_s$.

EVALUATE: Check in some special cases. As $\theta \to 90°$, $\tan\theta \to \infty$ so $\mu_s \to \infty$. This is reasonable because the normal force $n = \dfrac{mg}{2\tan\theta} \to 0$, so we would need an extremely large μ_s (that is, an extremely rough wall) to hold up the rod. As $\theta \to 0$, $T = \dfrac{mg}{2\sin\theta} \to \infty$. This means that the normal force would get extremely large, so we would need a very small coefficient of friction to hold up the rod. Our results are reasonable.

11.61. **IDENTIFY:** Apply $\sum\tau_z = 0$ to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.61.

EXECUTE: $\sum\tau_z = 0$, axis at hinge, gives $T(6.0\text{ m})(\sin 40°) - (6490\text{ N})(3.75\text{ m})(\cos 30°) = 0$ and $T = 5500\text{ N}$.

EVALUATE: The tension in the cable is less than the weight of the beam. $T\sin 40°$ is the component of T that is perpendicular to the beam.

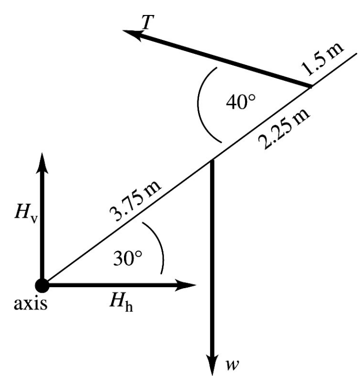


Figure 11.61

11.62. **IDENTIFY:** Apply the first and second conditions of equilibrium to the drawbridge.

SET UP: The free-body diagram for the drawbridge is given in Figure 11.62. H_v and H_h are the components of the force the hinge exerts on the bridge. In part (c), apply $\sum\tau_z = I\alpha$ to the rotating bridge and in part (d) apply energy conservation to the bridge.

EXECUTE: **(a)** $\sum\tau_z = 0$ with the axis at the hinge gives $-w(7.0\text{ m})(\cos 37°) + T(3.5\text{ m})(\sin 37°) = 0$ and $T = 2w\dfrac{\cos 37°}{\sin 37°} = 2\dfrac{(45{,}000\text{ N})}{\tan 37°} = 1.19\times 10^5\text{ N}$.

(b) $\sum F_x = 0$ gives $H_h = T = 1.19\times 10^5\text{ N}$. $\sum F_y = 0$ gives $H_v = w = 4.50\times 10^4\text{ N}$.

$H=\sqrt{H_h^2+H_v^2}=1.27\times10^5$ N. $\tan\theta=\dfrac{H_v}{H_h}$ and $\theta=20.7°$. The hinge force has magnitude 1.27×10^5 N and is directed at 20.7° above the horizontal.

(c) We can treat the bridge as a uniform bar rotating around one end, so $I=1/3\,mL^2$. $\sum\tau_z=I\alpha_z$ gives $mg(L/2)\cos37°=1/3\ mL^2\alpha$. Solving for α gives $\alpha=\dfrac{3g\cos37°}{2L}=\dfrac{3(9.80\text{ m/s}^2)\cos37°}{2(14.0\text{ m})}=0.839\text{ rad/s}^2$.

(d) Energy conservation gives $U_1=K_2$, giving $mgh=1/2\ I\omega^2=(1/2)(1/3\ mL^2)\omega^2$. Trigonometry gives $h=L/2\ \sin37°$. Canceling *m*, the energy conservation equation gives $g(L/2)\sin37°=(1/6)L^2\omega^2$. Solving for ω gives $\omega=\sqrt{\dfrac{3g\sin37°}{L}}=\sqrt{\dfrac{3(9.80\text{ m/s}^2)\sin37°}{14.0\text{ m}}}=1.12$ rad/s.

EVALUATE: The hinge force is not directed along the bridge. If it were, it would have zero torque for an axis at the center of gravity of the bridge and for that axis the tension in the cable would produce a single, unbalanced torque.

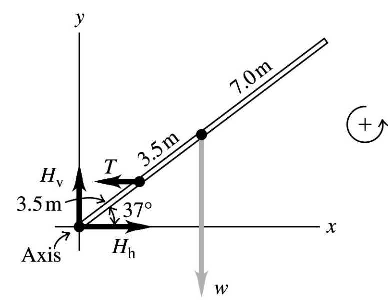


Figure 11.62

11.63. IDENTIFY: The amount the tendon stretches depends on Young's modulus for the tendon material. The foot is in rotational equilibrium, so the torques on it balance.

SET UP: $Y=\dfrac{F_T/A}{\Delta l/l_0}$. The foot is in rotational equilibrium, so $\sum\tau_z=0$.

EXECUTE: (a) The free-body diagram for the foot is given in Figure 11.63. *T* is the tension in the tendon and *A* is the force exerted on the foot by the ankle. $n=(75\text{ kg})g$, the weight of the person.

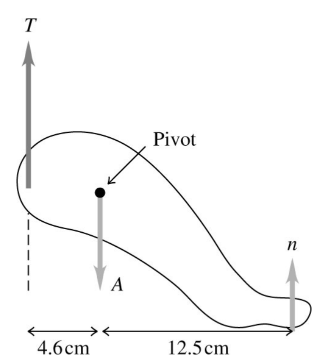


Figure 11.63

(b) Apply $\sum\tau_z=0$, letting counterclockwise torques be positive and with the pivot at the ankle: $T(4.6\text{ cm})-n(12.5\text{ cm})=0$. $T=\left(\dfrac{12.5\text{ cm}}{4.6\text{ cm}}\right)(75\text{ kg})(9.80\text{ m/s}^2)=2000$ N, which is 2.72 times his weight.

(c) The foot pulls downward on the tendon with a force of 2000 N.

$$\Delta l = \left(\frac{F_\text{T}}{YA}\right) l_0 = \frac{2000\text{ N}}{(1470\times 10^6\text{ Pa})(78\times 10^{-6}\text{ m}^2)}(25\text{ cm}) = 4.4\text{ mm}.$$

EVALUATE: The tension is quite large, but the Achilles tendon stretches about 4.4 mm, which is only about 1/6 of an inch, so it must be a strong tendon.

11.64 **IDENTIFY:** Apply $\sum \tau_z = 0$ to the beam.

SET UP: The center of mass of the beam is 1.0 m from the suspension point.

EXECUTE: **(a)** Taking torques about the suspension point,
$w(4.00\text{ m})\sin 30° + (140.0\text{ N})(1.00\text{ m})\sin 30° = (100\text{ N})(2.00\text{ m})\sin 30°.$

The common factor of $\sin 30°$ divides out, from which $w = 15.0$ N.

(b) In this case, a common factor of $\sin 45°$ would be factored out, and the result would be the same.

EVALUATE: All the forces are vertical, so the moments are all horizontal and all contain the factor $\sin\theta$, where θ is the angle the beam makes with the horizontal.

11.65. **IDENTIFY:** The rod is held in place, so the torques on it must balance. The added weight of the object causes the wire to stretch slightly, so we need to use tensile stress and strain.

SET UP: We use $\sum \tau_z = 0$ and $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$. The target variable is the distance the aluminum wire stretches due to the added weight.

EXECUTE: **(a)** A little trigonometry gives cos 30.0° = (1.20 m)/L_wire, so L_wire = 1.39 m.

(b) Fig. 11.65 shows a free-body diagram of the rod with the object attached. The stretching of the wire is due to the *increase* in tension due to the addition of the 90.0-kg object. Therefore in Fig. 11.65 we do not use the weight *mg* of the rod when computing the torque about the hinge.

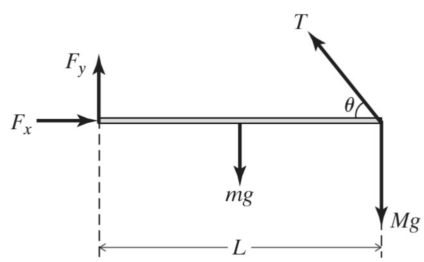


Figure 11.65

$\sum \tau_z = 0:\ TL\sin\theta = MgL \quad\rightarrow\quad T = \dfrac{mg}{\sin\theta} = \dfrac{(90.0\text{ kg})(9.80\text{ m/s}^2)}{\sin 30.0°} = 1764\text{ N}.$ Now use Young's modulus for aluminum (from Table 11.1) to find the increase in the length of the wire. Solving

$Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$ for $\Delta\ell$ gives $\Delta\ell = \dfrac{F_\perp \ell_0}{AY} = \dfrac{F_\perp \ell_0}{\pi r^2 Y} = \dfrac{(1764\text{ N})(1.39\text{ m})}{\pi(0.00250\text{ m})^2(7.0\times 10^{10}\text{ Pa})} = 1.8\times 10^{-3}\text{ m} = 1.8\text{ mm}.$

EVALUATE: Since $\Delta\ell << \ell_0$ we are justified in treating the rod as being horizontal after object is added. Our result is reasonable since most materials stretch very little under ordinary circumstances.

11.66. **IDENTIFY:** Apply $\sum \vec{F} = 0$ to each object, including the point where *D*, *C*, and *B* are joined. Apply $\sum \tau_z = 0$ to the rod.

SET UP: To find T_C and T_D, use a coordinate system with axes parallel to the cords.

EXECUTE: *A* and *B* are straightforward, the tensions being the weights suspended:
$T_A = (0.0360\text{ kg})(9.80\text{ m/s}^2) = 0.353\text{ N}$ and $T_B = (0.0240\text{ kg} + 0.0360\text{ kg})(9.80\text{ m/s}^2) = 0.588\text{ N}.$

Applying $\sum F_x = 0$ and $\sum F_y = 0$ to the point where the cords are joined, $T_C = T_B \cos 36.9° = 0.470$ N and $T_D = T_B \cos 53.1° = 0.353$ N. To find T_E, take torques about the point where string F is attached. $T_E(1.00 \text{ m}) = T_D \sin 36.9°(0.800 \text{ m}) + T_C \sin 53.1°(0.200 \text{ m}) + (0.120 \text{ kg})(9.80 \text{ m/s}^2)(0.500 \text{ m})$ and $T_E = 0.833$ N.

T_F may be found similarly, or from the fact that $T_E + T_F$ must be the total weight of the ornament. $(0.180\text{kg})(9.80\text{m/s}^2) = 1.76$ N, from which $T_F = 0.931$ N.

EVALUATE: The vertical line through the spheres is closer to *F* than to *E*, so we expect $T_F > T_E$, and this is indeed the case.

11.67. IDENTIFY: The torques must balance since the person is not rotating.

SET UP: Figure 11.67a shows the distances and angles. $\theta + \phi = 90°$. $\theta = 56.3°$ and $\phi = 33.7°$. The distances x_1 and x_2 are $x_1 = (90 \text{ cm})\cos\theta = 50.0$ cm and $x_2 = (135 \text{ cm})\cos\phi = 112$ cm. The free-body diagram for the person is given in Figure 11.67b. $w_1 = 277$ N is the weight of his feet and legs, and $w_t = 473$ N is the weight of his trunk. n_f and f_f are the total normal and friction forces exerted on his feet and n_h and f_h are those forces on his hands. The free-body diagram for his legs is given in Figure 11.67c. *F* is the force exerted on his legs by his hip joints. For balance, $\sum \tau_z = 0$.

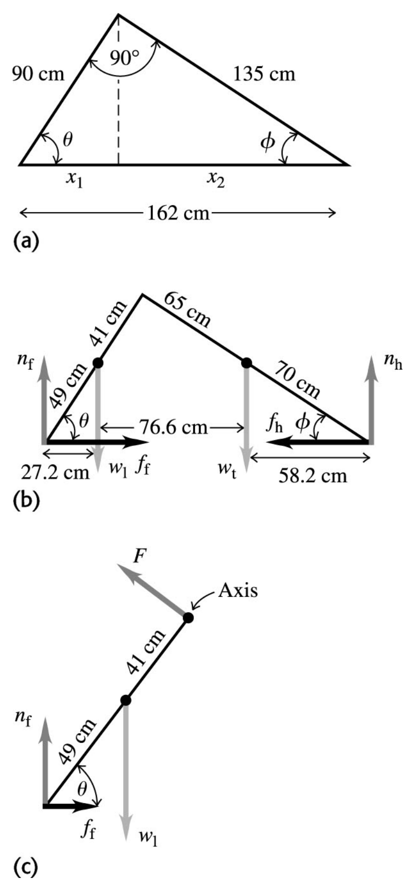


Figure 11.67

EXECUTE: **(a)** Consider the force diagram of Figure 11.67b. $\sum \tau_z = 0$ with the pivot at his feet and counterclockwise torques positive gives $n_h(162 \text{ cm}) - (277 \text{ N})(27.2 \text{ cm}) - (473 \text{ N})(103.8 \text{ cm}) = 0$.

$n_{\text{h}} = 350\text{ N},$ so there is a normal force of 175 N at each hand. $n_{\text{f}} + n_{\text{h}} - w_{\text{l}} - w_{\text{t}} = 0$ so $n_{\text{f}} = w_{\text{l}} + w_{\text{t}} - n_{\text{h}} = 750\text{ N} - 350\text{ N} = 400\text{ N},$ so there is a normal force of 200 N at each foot.

(b) Consider the force diagram of Figure 11.67c. $\sum \tau_z = 0$ with the pivot at his hips and counterclockwise torques positive gives $f_{\text{f}}(74.9\text{ cm}) + w_{\text{l}}(22.8\text{ cm}) - n_{\text{f}}(50.0\text{ cm}) = 0.$

$f_{\text{f}} = \dfrac{(400\text{ N})(50.0\text{ cm}) - (277\text{ N})(22.8\text{ cm})}{74.9\text{ cm}} = 182.7\text{ N}.$ There is a friction force of 91 N at each foot.

$\sum F_x = 0$ in Figure 11.67b gives $f_{\text{h}} = f_{\text{f}},$ so there is a friction force of 91 N at each hand.

EVALUATE: In this position the normal forces at his feet and at his hands don't differ very much.

11.68. IDENTIFY: The ball is going in a circle, so it obeys Newton's second law. Since the brass wire stretches slightly, we must use Young's modulus and stress and strain.

SET UP: We know the fractional change in the wire's length. The target variable is the speed of the ball at the bottom of its circular path. Apply $\sum F = m\dfrac{v^2}{R}$ and then use $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}.$

EXECUTE: At the lowest point, the ball's acceleration is upward. The fractional change in length of the wire is only 2.0×10^{-5}, so we can use ℓ_0 for the radius of the circle. $\sum F = m\dfrac{v^2}{R}$ gives $T - mg = \dfrac{mv^2}{\ell_0},$ so $T = mg + \dfrac{mv^2}{\ell_0}.$ Now use $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell},$ where $F_\perp$ is the tension in the wire. Doing so gives

$Y = \dfrac{mg + \dfrac{mv^2}{\ell_0}}{A\left(\dfrac{\Delta\ell}{\ell_0}\right)}.$ Solving for v gives $v = \sqrt{\dfrac{\ell_0}{m}\left[YA\left(\dfrac{\Delta\ell}{\ell_0}\right) - mg\right]}.$ Using $\dfrac{\Delta\ell}{\ell_0} = 2.0\times10^{-5},$ $\ell_0 = 1.40$ m, A = 6.00 mm^2 = 6.00×10^{-6} m^2, m = 0.0800 kg, and $Y = 9.0\times10^{10}$ Pa for brass (from Table 11.1 in the text), we have v = 13.2 m/s.

EVALUATE: A speed of 13.2 m/s is about 30 mph, yet this speed would only produce a fractional length change of 0.000020. The fractional length change would be even less at the top since the ball is moving slower up there than at the bottom.

11.69. IDENTIFY: Apply the equilibrium conditions to the crate. When the crate is on the verge of tipping it touches the floor only at its lower left-hand corner and the normal force acts at this point. The minimum coefficient of static friction is given by the equation $f_{\text{s}} = \mu_{\text{s}} n.$

SET UP: The free-body diagram for the crate when it is ready to tip is given in Figure 11.69.

EXECUTE: **(a)** $\sum \tau_z = 0$ gives $P(1.50\text{ m})\sin 53.0° - w(1.10\text{ m}) = 0.$

$$P = w\left(\frac{1.10\text{ m}}{[1.50\text{ m}][\sin 53.0°]}\right) = 1.15\times10^3\text{ N}$$

(b) $\sum F_y = 0$ gives $n - w - P\cos 53.0° = 0.$

$$n = w + P\cos 53.0° = 1250\text{ N} + (1.15\times10^3\text{ N})\cos 53° = 1.94\times10^3\text{ N}$$

(c) $\sum F_y = 0$ gives $f_{\text{s}} = P\sin 53.0° = (1.15\times10^3\text{ N})\sin 53.0° = 918\text{ N}.$

(d) $\mu_{\text{s}} = \dfrac{f_{\text{s}}}{n} = \dfrac{918\text{ N}}{1.94\times10^3\text{ N}} = 0.473$

EVALUATE: The normal force is greater than the weight because P has a downward component.

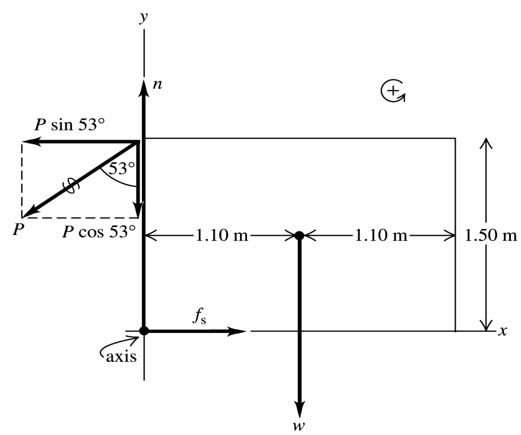


Figure 11.69

11.70. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the meterstick.

SET UP: The wall exerts an upward static friction force *f* and a horizontal normal force *n* on the stick. Denote the length of the stick by *l*. $f = \mu_s n$.

EXECUTE: **(a)** Taking torques about the right end of the stick, the friction force is half the weight of the stick, $f = w/2$. Taking torques about the point where the cord is attached to the wall (the tension in the cord and the friction force exert no torque about this point), and noting that the moment arm of the normal force is $l \tan\theta$, $n \tan\theta = w/2$. Then, $(f/n) = \tan\theta < 0.40$, so $\theta < \arctan(0.40) = 22°$.

(b) Taking torques as in part (a), $fl = w\dfrac{l}{2} + w(l - x)$ and $nl \tan\theta = w\dfrac{l}{2} + wx$. In terms of the coefficient of friction μ_s, $\mu_s > \dfrac{f}{n} = \dfrac{l/2 + (l - x)}{l/2 + x}\tan\theta = \dfrac{3l - 2x}{l + 2x}\tan\theta$. Solving for *x*, $x > \dfrac{l}{2}\dfrac{3\tan\theta - \mu_s}{\mu_s + \tan\theta} = 30.2$ cm.

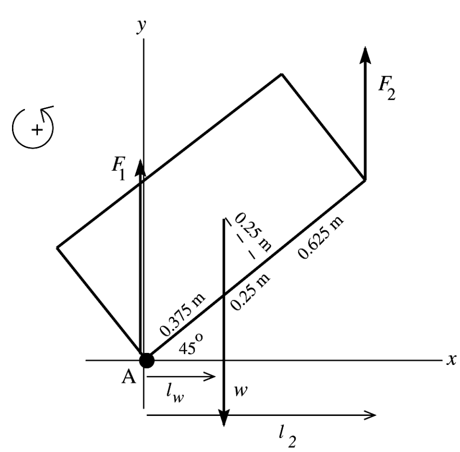
(c) In the above expression, setting $x = 10$ cm and $l = 100$ cm and solving for μ_s gives

$$\mu_s > \frac{(3 - 20/l)\tan\theta}{1 + 20/l} = 0.625.$$

EVALUATE: For $\theta = 15°$ and without the block suspended from the stick, a value of $\mu_s \geq 0.268$ is required to prevent slipping. Hanging the block from the stick increases the value of μ_s that is required.

11.71. **IDENTIFY:** Apply the first and second conditions of equilibrium to the crate.

SET UP: The free-body diagram for the crate is given in Figure 11.71.



$l_w = (0.375 \text{ m})\cos 45°$

$l_2 = (1.25 \text{ m})\cos 45°$

Let $\vec{F}_1$ and $\vec{F}_2$ be the vertical forces exerted by you and your friend. Take the origin at the lower left-hand corner of the crate (point A).

Figure 11.71

EXECUTE: $\sum F_y = ma_y$ gives $F_1 + F_2 - w = 0$

$F_1 + F_2 = w = (200\text{ kg})(9.80\text{ m/s}^2) = 1960\text{ N}$

$\sum \tau_A = 0$ gives $F_2 l_2 - w l_w = 0$

$$F_2 = w\left(\frac{l_w}{l_2}\right) = 1960\text{ N}\left(\frac{0.375\text{ m}\cos 45^\circ}{1.25\text{ m}\cos 45^\circ}\right) = 590\text{ N}$$

Then $F_1 = w - F_2 = 1960\text{ N} - 590\text{ N} = 1370\text{ N}.$

EVALUATE: The person below (you) applies a force of 1370 N. The person above (your friend) applies a force of 590 N. It is better to be the person above. As the sketch shows, the moment arm for $\vec{F}_1$ is less than for $\vec{F}_2$, so must have $F_1 > F_2$ to compensate.

11.72. **IDENTIFY:** The beam is at rest, so the forces and torques on it must all balance.

SET UP: The cables could point inward toward each other or outward away from each other. We shall assume they point away from each other. Call *d* the distance of the center of gravity from the left end, call *w* the weight of the beam, and call *T* the tension in the right-hand cable. $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\sum \tau_z = 0$.

EXECUTE: $\Sigma F_x = 0$ gives (620 N)(sin30.0°) – *T*(sin50.0°) = 0, so *T* = 404.68 N.

$\Sigma F_y = 0$ gives (620 N)(cos30.0°) + (404.68 N)(cos50.0°) – *w* = 0, so *w* = 797 N.

Taking torques about the left end, $\sum \tau_z = 0$ gives (404.68 N)(cos50.0°)(4.00 m) – (797 N)*d* = 0, so *d* = 1.31 m from the left end of the beam, or 2.69 m from the right end.

EVALUATE: The center of gravity is closer to the cable having the greater tension. The answer would be no different if we assumed that the cables pointed inward toward each other.

11.73. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the forearm.

SET UP: The free-body diagram for the forearm is given in Figure 11.10 in the textbook.

EXECUTE: **(a)** $\sum \tau_z = 0$, axis at elbow gives

$$wL - (T\sin\theta)D = 0.\ \sin\theta = \frac{h}{\sqrt{h^2 + D^2}} \text{ so } w = T\frac{hD}{L\sqrt{h^2 + D^2}}.$$

$$w_{\max} = T_{\max}\frac{hD}{L\sqrt{h^2 + D^2}}.$$

(b) $\dfrac{dw_{\max}}{dD} = \dfrac{T_{\max}h}{L\sqrt{h^2 + D^2}}\left(1 - \dfrac{D^2}{h^2 + D^2}\right)$; the derivative is positive.

EVALUATE: **(c)** The result of part (b) shows that $w_{\max}$ increases when *D* increases, since the derivative is positive. $w_{\max}$ is larger for a chimp since *D* is larger.

11.72. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the wheel.

SET UP: Take torques about the upper corner of the curb.

EXECUTE: The force $\vec{F}$ acts at a perpendicular distance $R - h$ and the weight acts at a perpendicular distance $\sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$. Setting the torques equal for the minimum necessary force,

$$F = mg\frac{\sqrt{2Rh - h^2}}{R - h}.$$

(b) The torque due to gravity is the same, but the force $\vec{F}$ acts at a perpendicular distance $2R - h$, so the minimum force is $(mg)\sqrt{2Rh - h^2}/(2R - h)$.

EVALUATE: **(c)** Less force is required when the force is applied at the top of the wheel, since in this case $\vec{F}$ has a larger moment arm.

11.75. IDENTIFY: Apply the first and second conditions of equilibrium to the gate.

SET UP: The free-body diagram for the gate is given in Figure 11.75.

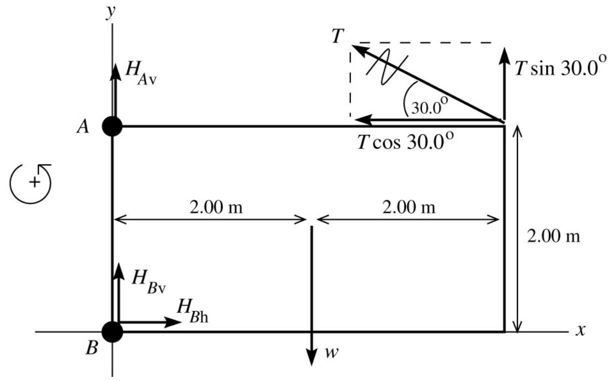


Figure 11.75

Use coordinates with the origin at B. Let $\vec{H}_A$ and $\vec{H}_B$ be the forces exerted by the hinges at A and B. The problem states that $\vec{H}_A$ has no horizontal component. Replace the tension $\vec{T}$ by its horizontal and vertical components.

EXECUTE: (a) $\sum \tau_B = 0$ gives $+(T\sin 30.0^{\circ})(4.00\text{ m}) + (T\cos 30.0^{\circ})(2.00\text{ m}) - w(2.00\text{ m}) = 0$

$T(2\sin 30.0^{\circ} + \cos 30.0^{\circ}) = w$

$$T = \frac{w}{2\sin 30.0^{\circ} + \cos 30.0^{\circ}} = \frac{700\text{ N}}{2\sin 30.0^{\circ} + \cos 30.0^{\circ}} = 375\text{ N}.$$

(b) $\sum F_x = ma_x$ says $H_{Bh} - T\cos 30.0^{\circ} = 0$

$H_{Bh} = T\cos 30.0^{\circ} = (375\text{ N})\cos 30.0^{\circ} = 325\text{ N}.$

(c) $\sum F_y = ma_y$ says $H_{Av} + H_{Bv} + T\sin 30.0^{\circ} - w = 0$

$H_{Av} + H_{Bv} = w - T\sin 30.0^{\circ} = 700\text{ N} - (375\text{ N})\sin 30.0^{\circ} = 512\text{ N}.$

EVALUATE: T has a horizontal component to the left so H_{Bh} must be to the right, as these are the only two horizontal forces. Note that we cannot determine H_{Av} and H_{Bv} separately, only their sum.

11.76. IDENTIFY: Use $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$ to locate the x-coordinate of the center of gravity of the block combinations.

SET UP: The center of mass and the center of gravity are the same point. For two identical blocks, the center of gravity is midway between the center of the two blocks.

EXECUTE: (a) The center of gravity of the top block can be as far out as the edge of the lower block. The center of gravity of this combination is then $3L/4$ to the left of the right edge of the upper block, so the overhang is $3L/4$.

(b) Take the two-block combination from part (a), and place it on top of the third block such that the overhang of $3L/4$ is from the right edge of the third block; that is, the center of gravity of the first two blocks is above the right edge of the third block. The center of mass of the three-block combination, measured from the right end of the bottom block, is $-L/6$ and so the largest possible overhang is $(3L/4) + (L/6) = 11L/12$. Similarly, placing this three-block combination with its center of gravity over the right edge of the fourth block allows an extra overhang of $L/8$, for a total of $25L/24$.

(c) As the result of part (b) shows, with only four blocks, the overhang can be larger than the length of a single block.

EVALUATE: The sequence of maximum overhangs is $\dfrac{18L}{24}, \dfrac{22L}{24}, \dfrac{25L}{24}, \ldots$. The increase of overhang when one more block is added is decreasing.

11.77. **IDENTIFY:** Apply the first and second conditions of equilibrium, first to both marbles considered as a composite object and then to the bottom marble.

(a) SET UP: The forces on each marble are shown in Figure 11.77.

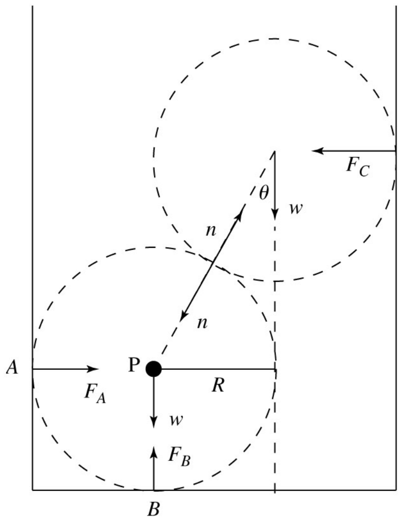


Figure 11.77

EXECUTE:

$F_B = 2w = 1.47$ N

$\sin\theta = R/2R$ so $\theta = 30°$

$\sum\tau_z = 0$, axis at P

$F_C(2R\cos\theta) - wR = 0$

$$F_C = \frac{mg}{2\cos 30°} = 0.424 \text{ N}$$

$F_A = F_C = 0.424$ N

(b) Consider the forces on the bottom marble. The horizontal forces must sum to zero, so $F_A = n\sin\theta$.

$$n = \frac{F_A}{\sin 30°} = 0.848 \text{ N}$$

Could use instead that the vertical forces sum to zero

$F_B - mg - n\cos\theta = 0$

$$n = \frac{F_B - mg}{\cos 30°} = 0.848 \text{ N, which checks.}$$

EVALUATE: If we consider each marble separately, the line of action of every force passes through the center of the marble so there is clearly no torque about that point for each marble. We can use the results we obtained to show that $\sum F_x = 0$ and $\sum F_y = 0$ for the top marble.

11.78. **IDENTIFY:** Apply $\sum\tau_z = 0$ to the right-hand beam.

SET UP: Use the hinge as the axis of rotation and take counterclockwise rotation as positive. If F_{wire} is the tension in each wire and $w = 260$ N is the weight of each beam, $2F_{\text{wire}} - 2w = 0$ and $F_{\text{wire}} = w$. Let L be the length of each beam.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $F_{\text{wire}}L\sin\frac{\theta}{2} - F_c\frac{L}{2}\cos\frac{\theta}{2} - w\frac{L}{2}\sin\frac{\theta}{2} = 0$, where θ is the angle between the beams and F_c is the force exerted by the cross bar. The length drops out, and all other quantities except F_c are known, so $F_c = \dfrac{F_{\text{wire}}\sin(\theta/2) - \frac{1}{2}w\sin(\theta/2)}{\frac{1}{2}\cos(\theta/2)} = (2F_{\text{wire}} - w)\tan\dfrac{\theta}{2}$. Therefore

$$F_c = (260 \text{ N})\tan\frac{53°}{2} = 130 \text{ N}.$$

(b) The crossbar is under compression, as can be seen by imagining the behavior of the two beams if the crossbar were removed. It is the crossbar that holds them apart.

(c) The upward pull of the wire on each beam is balanced by the downward pull of gravity, due to the symmetry of the arrangement. The hinge therefore exerts no vertical force. It must, however, balance the outward push of the crossbar. The hinge exerts a force 130 N horizontally to the left for the right-hand beam and 130 N to the right for the left-hand beam. Again, it's instructive to visualize what the beams would do if the hinge were removed.

EVALUATE: The force exerted on each beam increases as θ increases and exceeds the weight of the beam for $\theta \geq 90°$.

11.79. **IDENTIFY:** Apply the first and second conditions of equilibrium to the bale.

(a) SET UP: Find the angle where the bale starts to tip. When it starts to tip only the lower left-hand corner of the bale makes contact with the conveyor belt. Therefore the line of action of the normal force n passes through the left-hand edge of the bale. Consider $\Sigma\tau_z = 0$ with point A at the lower left-hand corner. Then $\tau_n = 0$ and $\tau_f = 0$, so it must be that $\tau_{mg} = 0$ also. This means that the line of action of the gravity must pass through point A. Thus the free-body diagram must be as shown in Figure 11.79a.

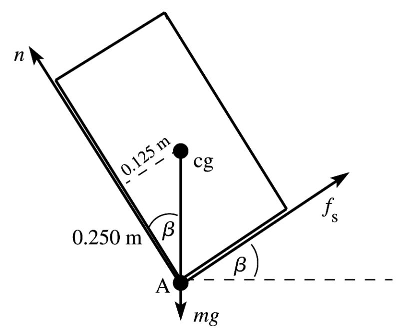


Figure 11.79a

EXECUTE: $\tan\beta = \dfrac{0.125\text{ m}}{0.250\text{ m}}$

$\beta = 27°$, angle where tips

SET UP: At the angle where the bale is ready to slip down the incline f_s has its maximum possible value, $f_s = \mu_s n$. The free-body diagram for the bale, with the origin of coordinates at the cg is given in Figure 11.79b.

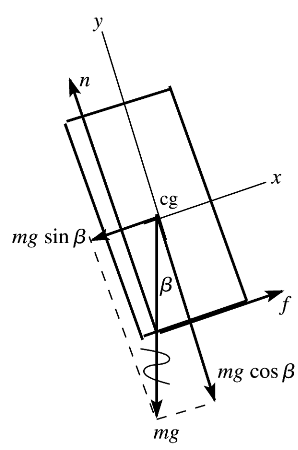


Figure 11.79b

EXECUTE:

$\sum F_y = ma_y$

$n - mg\cos\beta = 0$

$n = mg\cos\beta$

$f_s = \mu_s mg\cos\beta$

(f_s has maximum value when bale ready to slip)

$\sum F_x = ma_x$

$f_s - mg\sin\beta = 0$

$\mu_s mg\cos\beta - mg\sin\beta = 0$

$\tan\beta = \mu_s$

$\mu_s = 0.60$ gives that $\beta = 31°$

$\beta = 27°$ to tip; $\beta = 31°$ to slip, so tips first

(b) The magnitude of the friction force didn't enter into the calculation of the tipping angle; still tips at $\beta = 27°$. For $\mu_s = 0.40$ slips at $\beta = \arctan(0.40) = 22°$.

Now the bale will start to slide down the incline before it tips.

EVALUATE: With a smaller μ_s the slope angle β where the bale slips is smaller.

11.80. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the slab.

SET UP: The free-body diagram is given in Figure 11.80a. $\tan\beta = \dfrac{3.75 \text{ m}}{1.75 \text{ m}}$ so $\beta = 65.0°$.

$20.0° + \beta + \alpha = 90°$ so $\alpha = 5.0°$. The distance from the axis to the center of the block is

$$\sqrt{\left(\frac{3.75 \text{ m}}{2}\right)^2 + \left(\frac{1.75 \text{ m}}{2}\right)^2} = 2.07 \text{ m}.$$

EXECUTE: **(a)** $w(2.07 \text{ m})\sin 5.0° - T(3.75 \text{ m})\sin 52.0° = 0$. $T = 0.061w$. Each worker must exert a force of $0.012w$, where w is the weight of the slab.

(b) As θ increases, the moment arm for w decreases and the moment arm for T increases, so the worker needs to exert less force.

(c) $T \to 0$ when w passes through the support point. This situation is sketched in Figure 11.80b. $\tan\theta = \dfrac{(1.75 \text{ m})/2}{(3.75 \text{ m})/2}$ and $\theta = 25.0°$. If θ exceeds this value the gravity torque causes the slab to tip over.

EVALUATE: The moment arm for T is much greater than the moment arm for w, so the force the workers apply is much less than the weight of the slab.

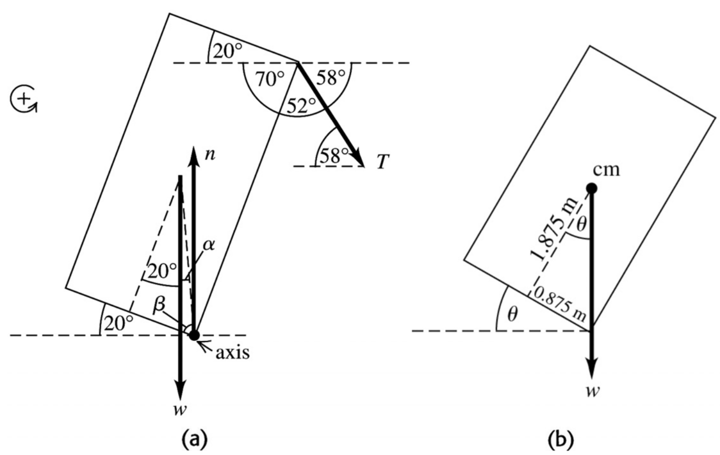


Figure 11.80

11.81. **IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

(a) SET UP: The free-body diagram for the door is given in Figure 11.81.

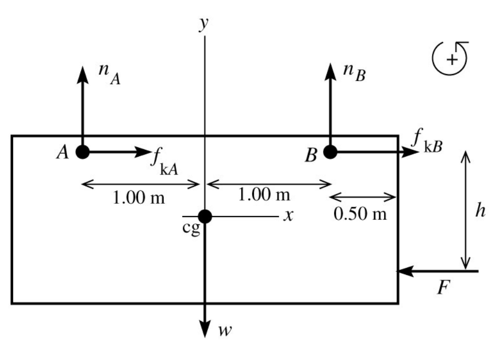


Figure 11.81

Take the origin of coordinates at the center of the door (at the cg). Let n_A, f_{kA}, n_B, and f_{kB} be the normal and friction forces exerted on the door at each wheel.

EXECUTE: $\sum F_y = ma_y$

$n_A + n_B - w = 0$

$n_A + n_B = w = 950\text{ N}$

$\sum F_x = ma_x$

$f_{kA} + f_{kB} - F = 0$

$F = f_{kA} + f_{kB}$

$f_{kA} = \mu_k n_A,\ \ f_{kB} = \mu_k n_B,$ so $F = \mu_k(n_A + n_B) = \mu_k w = (0.52)(950\text{ N}) = 494\text{ N}$

$\sum \tau_B = 0$

$n_B,\ f_{kA},$ and f_{kB} all have zero moment arms and hence zero torque about this point.

Thus $+w(1.00\text{ m}) - n_A(2.00\text{ m}) - F(h) = 0$

$$n_A = \frac{w(1.00\text{ m}) - F(h)}{2.00\text{ m}} = \frac{(950\text{ N})(1.00\text{ m}) - (494\text{ N})(1.60\text{ m})}{2.00\text{ m}} = 80\text{ N}$$

And then $n_B = 950\text{ N} - n_A = 950\text{ N} - 80\text{ N} = 870\text{ N}.$

(b) SET UP: If h is too large the torque of F will cause wheel A to leave the track. When wheel A just starts to lift off the track n_A and f_{kA} both go to zero.

EXECUTE: The equations in part (a) still apply.

$n_A + n_B - w = 0$ gives $n_B = w = 950\text{ N}$

Then $f_{kB} = \mu_k n_B = 0.52(950\text{ N}) = 494\text{ N}$

$F = f_{kA} + f_{kB} = 494\text{ N}$

$+w(1.00\text{ m}) - n_A(2.00\text{ m}) - F(h) = 0$

$$h = \frac{w(1.00\text{ m})}{F} = \frac{(950\text{ N})(1.00\text{ m})}{494\text{ N}} = 1.92\text{ m}$$

EVALUATE: The result in part (b) is larger than the value of h in part (a). Increasing h increases the clockwise torque about B due to F and therefore decreases the clockwise torque that n_A must apply.

11.82. IDENTIFY: Apply Newton's second law to the mass to find the tension in the wire. Then apply $Y = \dfrac{l_0 F_\perp}{A\Delta l}$ to the wire to find the elongation this tensile force produces.

(a) SET UP: Calculate the tension in the wire as the mass passes through the lowest point. The free-body diagram for the mass is given in Figure 11.82a.

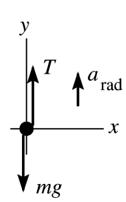


Figure 11.82a

The mass moves in an arc of a circle with radius $R = 0.70\text{ m}.$ It has acceleration $\vec{a}_{\text{rad}}$ directed in toward the center of the circle, so at this point $\vec{a}_{\text{rad}}$ is upward.

EXECUTE: $\sum F_y = ma_y$

$T - mg = mR\omega^2$ so that $T = m(g + R\omega^2).$

But ω must be in rad/s:

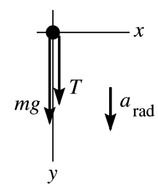
$\omega = (120\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 12.57\text{ rad/s}.$

Then $T = (12.0\text{ kg})\left[9.80\text{ m/s}^2 + (0.70\text{ m})(12.57\text{ rad/s})^2\right] = 1445\text{ N}.$

Now calculate the elongation Δl of the wire that this tensile force produces:

$Y = \dfrac{F_\perp l_0}{A\Delta l}$ so $\Delta l = \dfrac{F_\perp l_0}{YA} = \dfrac{(1445\text{ N})(0.70\text{ m})}{(7.0\times 10^{10}\text{ Pa})(0.014\times 10^{-4}\text{ m}^2)} = 0.0103\text{ m} = 1.0\text{ cm}.$

(b) SET UP: The acceleration $\vec{a}_{\text{rad}}$ is directed in toward the center of the circular path, and at this point in the motion this direction is downward. The free-body diagram is given in Figure 11.82b.



EXECUTE:

$\sum F_y = ma_y$

$mg + T = mR\omega^2$

$T = m(R\omega^2 - g)$

Figure 11.82b

$T = (12.0\text{ kg})\left[(0.70\text{ m})(12.57\text{ rad/s})^2 - 9.80\text{ m/s}^2\right] = 1210\text{ N}.$

$\Delta l = \dfrac{F_\perp l_0}{YA} = \dfrac{(1210\text{ N})(0.70\text{ m})}{(7.0\times 10^{10}\text{ Pa})(0.014\times 10^{-4}\text{ m}^2)} = 8.6\times 10^{-3}\text{ m} = 0.86\text{ cm}.$

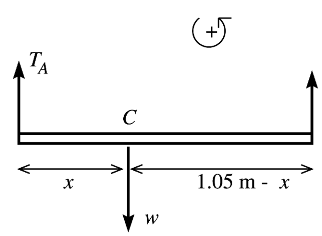
EVALUATE: At the lowest point T and w are in opposite directions and at the highest point they are in the same direction, so T is greater at the lowest point and the elongation is greatest there. The elongation is at most 1.4% of the length.

11.83. IDENTIFY: Use the second condition of equilibrium to relate the tension in the two wires to the distance w is from the left end. Use $\text{stress} = \dfrac{F_\perp}{A}$ and $Y = \dfrac{l_0 F_\perp}{A\Delta l}$ to relate the tension in each wire to its stress and strain.

(a) SET UP: $\text{stress} = F_\perp/A$, so equal stress implies T/A same for each wire.

$T_A/2.00\text{ mm}^2 = T_B/4.00\text{ mm}^2$ so $T_B = 2.00T_A$

The question is where along the rod to hang the weight in order to produce this relation between the tensions in the two wires. Let the weight be suspended at point C, a distance x to the right of wire A. The free-body diagram for the rod is given in Figure 11.83.



EXECUTE:

$\sum \tau_C = 0$

$+T_B(1.05\text{ m} - x) - T_A x = 0$

Figure 11.83

But $T_B = 2.00T_A$ so $2.00T_A(1.05\text{ m} - x) - T_A x = 0$

$2.10\text{ m} - 2.00x = x$ and $x = 2.10\text{ m}/3.00 = 0.70\text{ m}$ (measured from A).

(b) SET UP: $Y = \text{stress/strain}$ gives that $\text{strain} = \text{stress}/Y = F_\perp/AY$.

EXECUTE: Equal strain thus implies

$$\frac{T_A}{(2.00\text{ mm}^2)(1.80\times 10^{11}\text{ Pa})} = \frac{T_B}{(4.00\text{ mm}^2)(1.20\times 10^{11}\text{ Pa})}$$

$$T_B = \left(\frac{4.00}{2.00}\right)\left(\frac{1.20}{1.80}\right)T_A = 1.333T_A.$$

The $\sum \tau_C = 0$ equation still gives $T_B(1.05\text{ m} - x) - T_A x = 0.$

But now $T_B = 1.333T_A$ so $(1.333T_A)(1.05\text{ m} - x) - T_A x = 0.$

$1.40\text{ m} = 2.33x$ and $x = 1.40\text{ m}/2.33 = 0.60\text{ m}$ (measured from A).

EVALUATE: Wire B has twice the diameter so it takes twice the tension to produce the same stress. For equal stress the moment arm for T_B (0.35 m) is half that for T_A (0.70 m), since the torques must be equal. The smaller Y for B partially compensates for the larger area in determining the strain and for equal strain the moment arms are closer to being equal.

11.84. IDENTIFY: Apply $Y = \dfrac{l_0 F_\perp}{A\Delta l}$ and calculate Δl.

SET UP: When the ride is at rest the tension $F_\perp$ in the rod is the weight 1900 N of the car and occupants. When the ride is operating, the tension $F_\perp$ in the rod is obtained by applying $\sum \vec{F} = m\vec{a}$ to a car and its occupants. The free-body diagram is shown in Figure 11.84. The car travels in a circle of radius $r = l\sin\theta$, where l is the length of the rod and θ is the angle the rod makes with the vertical. For steel, $Y = 2.0\times10^{11}$ Pa. $\omega = 12.0\text{ rev/min} = 1.2566\text{ rad/s}$.

EXECUTE: (a) $\Delta l = \dfrac{l_0 F_\perp}{YA} = \dfrac{(15.0\text{ m})(1900\text{ N})}{(2.0\times10^{11}\text{ Pa})(8.00\times10^{-4}\text{ m}^2)} = 1.78\times10^{-4}\text{ m} = 0.18\text{ mm}$

(b) $\sum F_x = ma_x$ gives $F_\perp \sin\theta = mr\omega^2 = ml\sin\theta\omega^2$ and

$$F_\perp = ml\omega^2 = \left(\frac{1900\text{ N}}{9.80\text{ m/s}^2}\right)(15.0\text{ m})(1.2566\text{ rad/s})^2 = 4.592\times10^3\text{ N}.$$

$$\Delta l = \left(\frac{4.592\times10^3\text{ N}}{1900\text{ N}}\right)(0.18\text{ mm}) = 0.44\text{ mm}.$$

EVALUATE: $\sum F_y = ma_y$ gives $F_\perp\cos\theta = mg$ and $\cos\theta = mg/F_\perp$. As ω increases $F_\perp$ increases and $\cos\theta$ becomes small. Smaller $\cos\theta$ means θ increases, so the rods move toward the horizontal as ω increases.

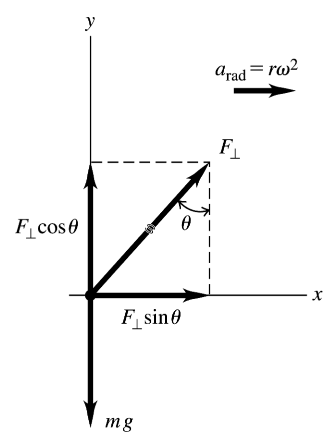


Figure 11.84

11.85. IDENTIFY: Apply $\dfrac{F_\perp}{A} = Y\left(\dfrac{\Delta l}{l_0}\right)$. The height from which he jumps determines his speed at the ground. The acceleration as he stops depends on the force exerted on his legs by the ground.

SET UP: In considering his motion take $+y$ downward. Assume constant acceleration as he is stopped by the floor.

EXECUTE: (a) $F_\perp = YA\left(\dfrac{\Delta l}{l_0}\right) = (3.0\times10^{-4}\text{ m}^2)(14\times10^9\text{ Pa})(0.010) = 4.2\times10^4\text{ N}$

(b) As he is stopped by the ground, the net force on him is $F_{net} = F_\perp - mg$, where $F_\perp$ is the force exerted on him by the ground. From part (a), $F_\perp = 2(4.2\times10^4\ \text{N}) = 8.4\times10^4\ \text{N}$ and $F = 8.4\times10^4\ \text{N} - (70\ \text{kg})(9.80\ \text{m/s}^2) = 8.33\times10^4\ \text{N}$. $F_{net} = ma$ gives $a = 1.19\times10^3\ \text{m/s}^2$. $a_y = -1.19\times10^3\ \text{m/s}^2$ since the acceleration is upward. $v_y = v_{0y} + a_y t$ gives $v_{0y} = -a_y t = (-1.19\times10^3\ \text{m/s}^2)(0.030\ \text{s}) = 35.7\ \text{m/s}$. His speed at the ground therefore is $v = 35.7\ \text{m/s}$. This speed is related to his initial height h above the floor by $\frac{1}{2}mv^2 = mgh$ and

$$h = \frac{v^2}{2g} = \frac{(35.7\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 65\ \text{m}.$$

EVALUATE: Our estimate is based solely on compressive stress; other injuries are likely at a much lower height.

11.86. **IDENTIFY:** The graph gives the change in length of the wire as a function of the weight hanging from it, which is equal to the tension in the wire. Young's modulus Y applies to the stretching of the wire. Energy conservation and Newton's second law apply to the swinging sphere.

SET UP: $Y = \dfrac{l_0 F_\perp}{A\Delta l}$, $K_1 + U_1 = K_2 + U_2$, $\Sigma\vec{F} = m\vec{a}$, $a_{rad} = v^2/R$.

EXECUTE: **(a)** Solve $Y = \dfrac{l_0 F_\perp}{A\Delta l}$ for Δl and realize that $F_\perp = mg$: $\Delta l = \dfrac{gl_0}{AY}m$. Therefore, in the graph of Δl versus m, the slope is equal to gl_0/AY. The equation of the graph is given in the problem as Δl = (0.422 mm/kg)m, so the slope is 0.422 mm/kg, so gl_0/AY = 0.422 mm/kg = 4.22 $\times 10^{-4}$ m/kg. Solving for Y gives $Y = \dfrac{gl_0}{A(4.22\times10^{-4}\ \text{m/kg})}$. Using $A = \pi r^2$ and putting in the given numbers gives

$$Y = \frac{(9.80\ \text{m/s})(22.0\ \text{m})}{\pi(4.30\times10^{-4}\ \text{m})^2(4.22\times10^{-4}\ \text{m/kg})} = 8.80\times10^{11}\ \text{Pa}.$$

(b) Use energy conservation to find the speed of the sphere. $K_1 + U_1 = K_2 + U_2$ gives $mgL(1-\cos\theta) = \dfrac{1}{2}mv^2$. Solving for v using θ = 36.0° and L = 22.0 m gives v = 9.075 m/s.

Now apply Newton's second law to the sphere at the bottom of the swing. $\Sigma\vec{F} = m\vec{a}$ and $a_{rad} = v^2/R$ give
$T - mg = mv^2/L$, so $T = mv^2/L + mg$ = (9.50 kg)(9.075 m/s)^2/(22.0 m) + (9.50 kg)(9.80 m/s^2) = 129 N.
Using the value of Y found in part (a), we have

$$\Delta l = \frac{F_\perp l_0}{AY} = (129\ \text{N})(22.0\ \text{m})/[\pi(4.30\times10^{-4}\ \text{m})^2(8.80\times10^{11}\ \text{Pa})] = 0.00554\ \text{m} = 5.54\ \text{mm}.$$

EVALUATE: For a wire 22 m long, 5.5 mm is a very small stretch, 0.0055/22 = 0.025%.

11.87. **IDENTIFY:** The bar is at rest, so the forces and torques on it must all balance.
SET UP: $\Sigma F_y = 0$, $\Sigma\tau_z = 0$.

EXECUTE: **(a)** The free-body diagram is shown in Figure 11.87a, where F_p is the force due to the knife-edge pivot.

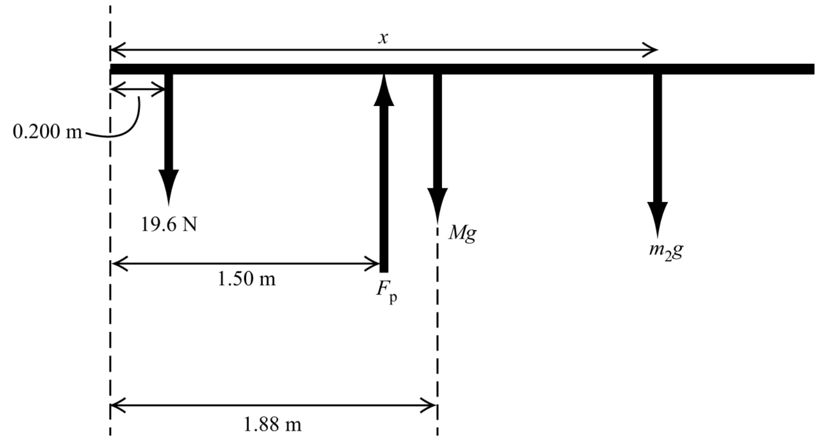


Figure 11.87a

(b) $\sum \tau_z = 0,$ with torques taken about the location of the knife-edge pivot, gives
(2.00 kg)g(1.30 m) – Mg(0.38 m) – $m_2g(x$ – 1.50 m) = 0
Solving for x gives
x = [(2.00 kg)(1.30 m) – M(0.38 m)](1/m_2) + 1.50 m
The graph of this equation (x versus 1/m_2) is a straight line of slope [(2.00 kg)(1.30 m) – M(0.38 m)].
(c) The plot of x versus 1/m_2 is shown in Figure 11.87b. The equation of the best-fit line is
x = (1.9955 $\text{m}\cdot\text{kg}$)/m_2 + 1.504 m. The slope of the best-fit line is 1.9955 $\text{m}\cdot\text{kg}$, so
[(2.00 kg)(1.30 m) – M(0.38 m)] = 1.9955 $\text{m}\cdot\text{kg}$, which gives M = 1.59 kg.

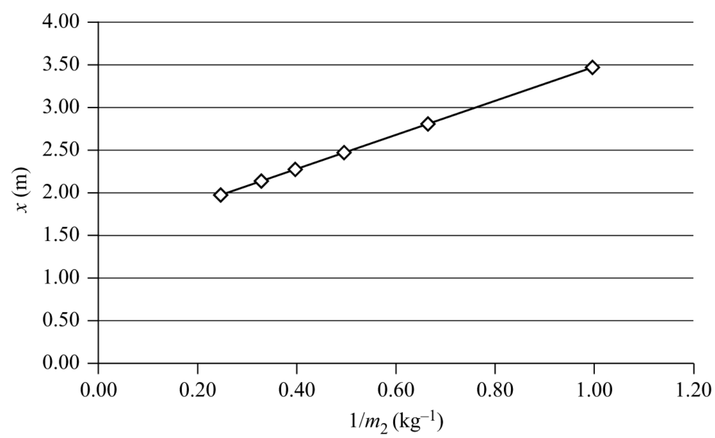


Figure 11.87b

(d) The y-intercept of the best-fit line is 1.50 m. This is plausible. As the graph approaches the y-axis, 1/m_2 approaches zero, which means that m_2 is getting extremely large. In that case, it would be much larger than any other masses involved, so to balance the system, m_2 would have to be at the knife-point pivot, which is at x = 1.50 m.
EVALUATE: The fact that the graph gave a physically plausible result in part (d) suggests that this graphical analysis is reasonable.

11.88. **IDENTIFY:** The bar is at rest, so the forces and torques on it must all balance.
SET UP: $\Sigma F_x = 0,\ \Sigma F_y = 0,\ \ \sum \tau_z = 0.$

EXECUTE: (a) Take torques about the hinge, calling m the mass of the bar and L its length. $\sum \tau_z = 0$ gives $xT\sin\theta = mg\frac{L}{2}$. Solving for T gives $T = \frac{mgL/2}{x\sin\theta}$. Therefore the alternative having the largest value of $x\sin\theta$ will have the smallest tension, and the one with the smallest value of $x\sin\theta$ will have the greatest tension. Calculating $x\sin\theta$ for each alternative gives the following values. A: 1.00 m, B: 1.30 m, C: 0.451 m, D: 0.483 m. Therefore alternative B gives the smallest tension and C produces the largest tension

(b) Calling H the magnitude of the hinge force, $\Sigma F_x = 0$ gives $H_x = T\cos\theta$. Using the value of T from part (a), we get $H_x = \frac{mg\,L/2}{x\sin\theta}.\cos\theta = \frac{mg\,L/2}{x\tan\theta}$. From this result, we can see that H_x is greatest when $x\tan\theta$ is the smallest, and H_x is least when $x\tan\theta$ is greatest. Calculating $x\tan\theta$ for each alternative gives A: 1.15 m, B: 2.60 m, C: 0.565 m, D: 1.87 m. Therefore alternative C gives the greatest H_x and B gives the smallest H_x.

(c) Taking torques about the point where the cable is connected to the bar, $\sum \tau_z = 0$ gives $H_y x = mg(x - L/2)$. Solving for H_y gives $H_y = mg(1 - L/2x)$. Since H_y could be positive or negative, we should calculate all four possibilities. For alternative A, we have $H_y = mg\left(1 - \frac{2.00\text{ m}}{4.00\text{ m}}\right) = 0.500mg$. For B we have $H_y = mg\left(1 - \frac{2.00\text{ m}}{3.00\text{ m}}\right) = 0.333mg$, and likewise we get $H_y = -0.333mg$ for C and $H_y = -1.00mg$ for D. Therefore alternative D gives the largest H_y and B and C both give the smallest value.

(d) Alternative B is clearly optimal because it results in the smallest values for T, H_x, and H_y. It might be a good idea to avoid alternative C because it has the greatest T and H_x.

EVALUATE: As a check, part (c) could be solved by using $\Sigma F_y = 0$.

11.89. IDENTIFY: Apply the equilibrium conditions to the ladder combination and also to each ladder.

SET UP: The geometry of the 3-4-5 right triangle simplifies some of the intermediate algebra. Denote the forces on the ends of the ladders by F_L and F_R (left and right). The contact forces at the ground will be vertical, since the floor is assumed to be frictionless.

EXECUTE: (a) Taking torques about the right end, $F_L(5.00\text{ m}) = (480\text{ N})(3.40\text{ m}) + (360\text{ N})(0.90\text{ m})$, so $F_L = 391\text{ N}$. F_R may be found in a similar manner, or from $F_R = 840\text{ N} - F_L = 449\text{ N}$.

(b) The tension in the rope may be found by finding the torque on each ladder, using the point A as the origin. The lever arm of the rope is 1.50 m. For the left ladder, $T(1.50\text{ m}) = F_L(3.20\text{ m}) - (480\text{ N})(1.60\text{ m})$, so $T = 322.1\text{ N}$ (322 N to three figures). As a check, using the torques on the right ladder, $T(1.50\text{ m}) = F_R(1.80\text{ m}) - (360\text{ N})(0.90\text{ m})$ gives the same result.

(c) The horizontal component of the force at A must be equal to the tension found in part (b). The vertical force must be equal in magnitude to the difference between the weight of each ladder and the force on the bottom of each ladder, $480\text{ N} - 391\text{ N} = 449\text{ N} - 360\text{ N} = 89\text{ N}$. The magnitude of the force at A is then $\sqrt{(322.1\text{ N})^2 + (89\text{ N})^2} = 334\text{ N}$.

(d) The easiest way to do this is to see that the added load will be distributed at the floor in such a way that $F_L' = F_L + (0.36)(800\text{ N}) = 679\text{ N}$, and $F_R' = F_R + (0.64)(800\text{ N}) = 961\text{ N}$. Using these forces in the form for the tension found in part (b) gives

$$T = \frac{F'_L(3.20\text{ m}) - (480\text{ N})(1.60\text{ m})}{(1.50\text{ m})} = \frac{F'_R(1.80\text{ m}) - (360\text{ N})(0.90\text{ m})}{(1.50\text{ m})} = 937\text{ N}.$$

EVALUATE: The presence of the painter increases the tension in the rope, even though his weight is vertical and the tension force is horizontal.

11.90. IDENTIFY: Apply $\sum \tau_z = 0$ to the post, for various choices of the location of the rotation axis.

SET UP: When the post is on the verge of slipping, f_s has its largest possible value, $f_s = \mu_s n.$

EXECUTE: (a) Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is $h/2$ and the lever arm of both the weight and the normal force is $h\tan\theta,$ and so

$$F\frac{h}{2} = (n - w)h\tan\theta.$$

Taking torques about the upper point (where the rope is attached to the post), $fh = F\frac{h}{2}.$ Using $f \le \mu_s n$ and solving for F, $F \le 2w\left(\frac{1}{\mu_s} - \frac{1}{\tan\theta}\right)^{-1} = 2(400\text{ N})\left(\frac{1}{0.30} - \frac{1}{\tan 36.9°}\right)^{-1} = 400\text{ N}.$

(b) The above relations between F, n and f become $F\frac{3}{5}h = (n - w)h\tan\theta, f = \frac{2}{5}F,$ and eliminating f and n and solving for F gives $F \le w\left(\frac{2/5}{\mu_s} - \frac{3/5}{\tan\theta}\right)^{-1}$, and substitution of numerical values gives 750 N to two figures.

(c) If the force is applied a distance y above the ground, the above relations become $Fy = (n - w)h\tan\theta,\ F(h - y) = fh,$ which become, on eliminating n and f, $w \ge F\left[\frac{(1 - y/h)}{\mu_s} - \frac{(y/h)}{\tan\theta}\right].$

As the term in square brackets approaches zero, the necessary force becomes unboundedly large. The limiting value of y is found by setting the term in square brackets equal to zero. Solving for y gives

$$\frac{y}{h} = \frac{\tan\theta}{\mu_s + \tan\theta} = \frac{\tan 36.9°}{0.30 + \tan 36.9°} = 0.71.$$

EVALUATE: For the post to slip, for an axis at the top of the post the torque due to F must balance the torque due to the friction force. As the point of application of F approaches the top of the post, its moment arm for this axis approaches zero.

11.91. IDENTIFY: Apply $Y = \frac{l_0 F_\perp}{A\Delta l}$ to calculate $\Delta l.$

SET UP: For steel, $Y = 2.0\times10^{11}$ Pa.

EXECUTE: (a) From $Y = \frac{l_0 F_\perp}{A\Delta l}$, $\Delta l = \frac{(4.50\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m})}{(20\times10^{10}\text{ Pa})(5.00\times10^{-7}\text{ m}^2)} = 6.62\times10^{-4}$ m, or 0.66 mm to two figures.

(b) $(4.50\text{ kg})(9.80\text{ m/s}^2)(0.0500\times10^{-2}\text{ m}) = 0.022\text{ J}.$

(c) The magnitude F will vary with distance; the average force is $YA(0.0250\text{ cm}/l_0) = 16.7\text{ N},$ and so the work done by the applied force is $(16.7\text{ N})(0.0500\times10^{-2}\text{ m}) = 8.35\times10^{-3}\text{ J}.$

(d) The average force the wire exerts is $(4.50\text{ kg})g + 16.7\text{ N} = 60.8\text{ N}.$ The work done is negative, and equal to $-(60.8\text{ N})(0.0500\times10^{-2}\text{ m}) = -3.04\times10^{-2}\text{ J}.$

(e) The equation $Y = \frac{l_0 F_\perp}{A\Delta l}$ can be put into the form of Hooke's law, with $k = \frac{YA}{l_0}.$ $U_{el} = \frac{1}{2}kx^2,$ so $\Delta U_{el} = \frac{1}{2}k(x_2^2 - x_1^2).$ $x_1 = 6.62\times10^{-4}$ m and $x_2 = 0.500\times10^{-3}\text{ m} + x_1 = 11.62\times10^{-4}$ m. The change in elastic potential energy is

$$\frac{(20\times10^{10}\text{ Pa})(5.00\times10^{-7}\text{ m}^2)}{2(1.50\text{ m})}\left[(11.62\times10^{-4}\text{ m})^2 - (6.62\times10^{-4}\text{ m})^2\right] = 3.04\times10^{-2}\text{ J},$$ the negative of the result of part (d).

EVALUATE: The tensile force in the wire is conservative and obeys the relation $W = -\Delta U$.

11.92. **IDENTIFY** and **SET UP:** The forces and torques on the competitor must balance, so $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\sum \tau_z = 0$.

EXECUTE: Take torques about his feet, giving $(T_1 - T_2)(1.5\text{ m})(\cos 30°) = mg(1.0\text{ m})(\sin 30°)$. Solving for T_2 gives $T_2 = 1160\text{ N} - [(80.0\text{ kg})(9.80\text{ m/s}^2)/(1.5\text{ m})]\tan 30° = 858\text{ N} \approx 860\text{ N}$, which is choice (c).

EVALUATE: We find $T_2 < T_1$ as expected.

11.93. **IDENTIFY** and **SET UP:** The forces and torques on the competitor must balance, so $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\sum \tau_z = 0$.

EXECUTE: As in the previous problem, $T_1 - T_2$ is proportional to $\tan\theta$, so as θ increases, so does $\tan\theta$ and so does $T_1 - T_2$, which makes choice (a) correct.

EVALUATE: The result is physically reasonable. As he leans back, the ropes get lower, which reduces their moment arm, and his weight also gets lower, which increases its moment arm. Therefore to keep balance, the diffrerence in the tensions must be greater than before.

11.94. **IDENTIFY** and **SET UP:** Apply $\tau = Fl$.

EXECUTE: The moment arm for T_1 has increased, so T_1 can be smaller and still produce the same torque needed to balance the torque due to gravity, so choice (c) is correct.

EVALUATE: If the rope is held too high, it will be hard for the competitor to hold it, so there is a limit on how much the holding height can be effectively increased.

11.95. **IDENTIFY** and **SET UP:** The competitor will slip if the static friction force would need to be greater than its maximum possible value. $f_s^{\max} = \mu_s n$.

EXECUTE: From earlier work, we know that $T_1 - T_2 = 1160\text{ N} - 858\text{ N} = 302\text{ N}$. The maximum static friction force is $f_s^{\max} = \mu_s n = (0.50)(80.0\text{ kg})(9.80\text{ m/s}^2) = 392\text{ N}$. He needs only 302 N to balance the tension difference, yet the static friction force could be as great as 392 N, so he is not even ready to slip. Therefore he will not move, choice (d).

EVALUATE: The friction force is 302 N, not 392 N, because he is not just ready to slip.

12

FLUID MECHANICS

VP12.4.1. **IDENTIFY:** We want the pressure at a depth in a fluid.
SET UP: $p = p_0 + \rho gh$ gives the absolute pressure.

EXECUTE: $p = p_0 + \rho gh = 1.22\times 10^5$ Pa + (455 kg/m^3)(9.80 m/s^2)(2.00 m) = 1.31×10^5 Pa.

EVALUATE: This pressure is about 30% above atmospheric pressure.

VP12.4.2. **IDENTIFY:** We are dealing with the gauge pressure at a depth in a fluid.
SET UP: $p = \rho gh$ gives the gauge pressure which is the pressure above atmospheric pressure.

EXECUTE: **(a)** The gauge pressure is due only to the gasoline.
$p = \rho gh$ = (740 kg/m^3)(9.80 m/s^2)(0.150 m) = 1.09×10^3 Pa.

(b) The gauge pressure at the top of the water is 1.09×10^3 Pa, so the pressure at the bottom is
$p = p_0 + \rho gh = 1.09\times 10^3$ Pa + (1000 kg/m^3)(9.80 m/s^2)(0.150 m) = 2.56×10^3 Pa.

EVALUATE: To find the *absolute* pressure in each case we would have to add 1.01×10^5 Pa to our answers. A pressure gauge would read the gauge pressure.

VP12.4.3. **IDENTIFY:** We are dealing with the pressure at a depth in a fluid in a manometer.
SET UP: $p = p_0 + \rho gh$ gives the absolute pressure. Using Fig. 12.8a in the text, and looking at the right-hand column, the pressure in that column at a level y_1 is p. This is also the pressure due to the column from y_1 to the top which is a depth h in the right-hand column. Therefore the difference in heights h is $p - p_{\text{atm}} = \rho gh$.

EXECUTE: Solve for h: $h = \dfrac{p - p_{\text{atm}}}{\rho g} = \dfrac{2.1\times 10^5\ \text{Pa} - 1.01\times 10^5\ \text{Pa}}{(1.36\times 10^4\ \text{kg/m}^3)(9.80\ \text{m/s}^2)}$ = 0.818 m = 81.8 cm.

EVALUATE: According to our results, if the pressure p in the container were greater, h would be greater. If the density of the fluid in the manometer were less, h would also be greater. Both cases are physically reasonable, which suggests our solution is correct.

VP12.4.4. **IDENTIFY:** We are dealing with the pressure at a depth in a fluid in a manometer.
SET UP: $p = \rho gh$ gives the gauge pressure. The gauge pressure at the bottom of the oil is the same as the gauge pressure at the bottom of the water since both tubes are open to the air.
EXECUTE: **(a)** Equating the gauge pressures gives $\rho_{\text{oil}} g h_{\text{oil}} = \rho_{\text{water}} g h_{\text{water}}$.

$h_{\text{water}} = \dfrac{\rho_{\text{oil}}}{\rho_{\text{water}}} h_{\text{oil}} = \dfrac{916\ \text{kg/m}^3}{1000\ \text{kg/m}^3} \cdot 25.0\ \text{cm} = 22.9$ cm.

(b) $p_{\text{gauge}} = \rho gh$ = (1000 kg/m^3)(9.80 m/s^2)(0.150 m) = 1.47×10^3 Pa.

(c) Solve $p_{\text{gauge}} = \rho gh$ for h, giving

$h = p_{\text{gauge}}/\rho g = (1.47\times 10^3\ \text{Pa})/[(916\ \text{kg/m}^3)(9.80\ \text{m/s}^2)]$ = 0.164 m = 16.4 cm.

EVALUATE: From (b) and (c), we see that we would have to be 15.0 cm deep in water to have the same pressure as at 16.4 cm in oil. This is reasonable since the density of oil is less than that of water.

VP12.5.1. **IDENTIFY:** This problem involves, density, buoyancy, and Archimedes's principle.
SET UP: Density is $\rho = m/V$. Archimedes's principle says that the buoyant force B on an immersed object is equal to the weight of the fluid displaced.
EXECUTE: **(a)** $m = \rho V$, so $w = \rho Vg = (1150 \text{ kg/m}^3)(7.50\times10^{-4} \text{ m}^3)(9.80 \text{ m/s}^2) = 8.45$ N.
(b) B equals the weight of the displaced fluid or gas, so $B = \rho Vg$.
(i) $B_{\text{air}} = \rho_{\text{air}} Vg = (1.20 \text{ kg/m}^3)(7.50\times10^{-4} \text{ m}^3)(9.80 \text{ m/s}^2) = 8.82\times10^{-3}$ N. The object would sink since the buoyant force on it is less than its weight.
(ii) $B_{\text{water}} = \rho_{\text{water}} Vg = (1000 \text{ kg/m}^3)(7.50\times10^{-4} \text{ m}^3)(9.80 \text{ m/s}^2) = 7.35$ N. The object would sink since the buoyant force on it is less than its weight.
(iii) $B_{\text{glycerine}} = \rho_{\text{glycerine}} Vg = (1260 \text{ kg/m}^3)(7.50\times10^{-4} \text{ m}^3)(9.80 \text{ m/s}^2) = 9.26$ N. The object would rise since the buoyant force on it is greater than its weight.
EVALUATE: Notice that in each case in (b), the object sinks if its density is greater than the fluid and rises if its density is less than that of the fluid. This is a useful point to keep in mind.

VP12.5.2. **IDENTIFY:** This problem involves density, buoyancy, and Archimedes's principle.
SET UP: Density is $\rho = m/V$. Archimedes's principle says that the buoyant force B on an immersed object is equal to the weight of the fluid displaced. $\sum F_y = 0$ for the sphere.
EXECUTE: **(a)** $B = \rho Vg = (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.20\times10^{-3} \text{ m}^3) = 11.8$ N.
(b) $\sum F_y = 0$ gives $w = T + B = 29.4 \text{ N} + 11.8 \text{ N} = 41.2$ N.
(c) $\rho = m/V = (W/g)/V = (41.2 \text{ N})/[(9.80 \text{ m/s}^2)(1.20\times10^{-3} \text{ m}^3)] = 3.50\times10^3 \text{ kg/m}^3$.
EVALUATE: The sphere is denser than water, so the buoyant force is less than its weight, so there must be a tension in the cable. This agrees with our results.

VP12.5.3. **IDENTIFY:** This problem involves density, buoyancy, and Archimedes's principle.
SET UP: Density is $\rho = m/V$. Archimedes's principle says that the buoyant force B on an immersed object is equal to the weight of the fluid displaced. $\sum F_y = 0$ for the cube. Call V the volume of the cube and ρ_c its density. Let ρ be the density of the fluid.
EXECUTE: Archimedes's principle gives $B = \rho g(V/2)$. $\sum F_y = 0$ for the cube gives

$$T + B - w_{\text{cube}} = 0 \quad \rightarrow \quad T + \rho gV/2 - \rho_c gV = 0 \quad \rightarrow \quad \rho = \frac{2(\rho_c gV - T)}{gV}$$

$$\rho = \frac{2\left[(7.50\times10^3 \text{kg/m}^3)(9.80 \text{ m/s}^2)(5.50\times10^{-3}\text{m}^3) - 375 \text{ N}\right]}{(9.80 \text{ m/s}^2)(5.50\times10^{-3}\text{m}^3)} = 1.1\times10^3 \text{ kg/m}^3.$$

EVALUATE: The density of this fluid is slightly greater than that of water.

VP12.5.4. **IDENTIFY:** This problem involves density, buoyancy, and Archimedes's principle.
SET UP: Density is $\rho = m/V$. Archimedes's principle says that the buoyant force B on an immersed object is equal to the weight of the fluid displaced.
EXECUTE: **(a)** $B = \rho_L Vg$ and $V = Ad$, so $B = \rho_L Adg$.
(b) For floating $B = w_{\text{cyl}} \rightarrow \rho_L Vg = \rho_{\text{cyl}} ALg \rightarrow \rho_{\text{cyl}} = \dfrac{d}{L}\rho_L$.
EVALUATE: If $d = L$, our result gives $\rho_{\text{cyl}} = \rho_L$. In that case the cylinder would be fully submerged. Since the cylinder and liquid have the same densities, the volumes of the cylinder and the displaced liquid would have to be equal for the cylinder to float.

VP12.9.1. **IDENTIFY:** This is a problem in fluid flow involving Bernoulli's equation and the continuity equation.
SET UP: Bernoulli's equation is $p_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$ and the continuity equation is $A_1v_1 = A_2v_2$.

EXECUTE: (a) Use $A_1v_1 = A_2v_2$ to find the speed of flow at ground level.

$A_1v_1 = A_2v_2 \quad \rightarrow \quad \pi r_1^2 v_1 = \pi r_2^2 v_2 \quad \rightarrow \quad v_2 = (r_1/r_2)^2 v_1$

$v_2 = [(1.0\text{ cm})/(0.50\text{ cm})]^2(1.6\text{ m/s}) = 6.4\text{ m/s}.$

(b) Use Bernoulli's equation with point 2 to be floor level. This makes $v_1 = 1.6$ m/s, $v_2 = 6.4$ m/s, $y_1 = 9.0$ m, $y_2 = 0$, $p_1 = 3.0\times 10^5$ Pa, and $\rho = 1000\text{ kg/m}^3$. We want p_2 at ground level. Solving Bernoulli's equation gives $p_2 = 3.7\times 10^5$ Pa.

EVALUATE: The pressure in the pipe is about 3.7 times atmospheric pressure. It would be even greater if the pipe were longer than 9.0 m.

VP12.9.2. IDENTIFY: This problem is about fluid flow. It involves Bernoulli's equation, volume flow rate, and the continuity equation.

SET UP: Bernoulli's equation is $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, the volume flow rate is $dV/dt = Av$, and the continuity equation is $A_1v_1 = A_2v_2$.

EXECUTE: (a) We know the volume flow rate, so use $dV/dt = Av$ to find v_2.

$$v = \frac{dV/dt}{A_2} = \frac{dV/dt}{\pi r_2^2} = \frac{4.4\times 10^{-3}\text{ m}^3/\text{s}}{\pi(0.010\text{ m})^2} = 14\text{ m/s}.$$

(b) Apply Bernoulli's equation. Call point 1 the top of the ethanol and point 2 the level of the out-coming ethanol at the bottom. We want $p_1 - p_{\text{atm}}$ (the *gauge* pressure) and we know that $p_2 = p_{\text{atm}}$, $y_1 = 3.2$ m, $y_2 = 0$, $\rho = 810\text{ kg/m}^3$, $v_2 = 14$ m/s, and $v_1 \approx 0$ (because $A_1 >> A_2$). Solve for $p_1 - p_{\text{atm}}$ gives $p_{\text{gauge}} = p_1 - p_{\text{atm}} = 5.4\times 10^4$ Pa.

EVALUATE: The tank is pressurized because $p_1 > p_{\text{atm}}$.

VP12.9.3. IDENTIFY: This problem deals with a Venturi meter.

SET UP: From Example 12.9, we have $p_1 - p_2 = \frac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$. We want the difference in height h of the liquid levels in the two tubes. We also know from Example 12.9 that $v = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}$. First solve for v_1 and then use that result to solve for h. The volume flow rate is $dV/dt = Av$.

EXECUTE: (a) First use $p_1 - p_2 = \frac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$ to find v_1.

$$810\text{ Pa} = \frac{1}{2}(1000\text{ kg/m}^3)v_1^2\left[\left(\frac{\pi(2.5\text{ cm})^2}{\pi(1.2\text{ cm})^2}\right)^2 - 1\right] \quad \rightarrow \quad v_1 = 0.3014\text{ m/s}.$$

Now use $v_1 = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}$ to solve for h. Use $v_1 = 0.3014$ m/s and the same areas as above. This gives $h = 5.9\times 10^{-4}\text{ m}^3/\text{s}$.

(b) $dV/dt = A_1v_1 = \pi r_1^2 v_1 = \pi(0.025\text{ m})^2(0.3014\text{ m/s}) = 5.9\times 10^{-4}\text{ m}^3/\text{s}.$

EVALUATE: The volume flow rate is the same throughout the pipe, so we calculated it at point 1. But it would also be true at point 2 or any other point in the pipe.

VP12.9.4. IDENTIFY: This problem is about fluid flow. It involves Bernoulli's equation, volume flow rate, and the continuity equation.

SET UP: Bernoulli's equation is $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, the volume flow rate is $dV/dt = Av$, and the continuity equation is $A_1 v_1 = A_2 v_2$. We follow exactly the same procedure as in problem VP12.9.2 *except* that v_1 cannot be neglected.

EXECUTE: The continuity equation gives v_1: $A_1 v_1 = A_2 v_2 \quad \rightarrow \quad v_1 = v_2(A_2/A_1)$. Now use Bernoulli's equation with $p_1 = p_0$, $p_2 = p_{atm}$, $y_1 = h$, $y_2 = 0$, $v_1 = v_2(A_2/A_1)$. This equation becomes $p_0 - p_{atm} + \rho g h = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_2^2\left(\frac{A_2}{A_1}\right)^2$. Solving for v_2 gives $v_2 = \sqrt{\dfrac{2\left(\dfrac{p_0 - p_{atm}}{\rho}\right) + 2gh}{1 - (A_2/A_1)^2}}$.

EVALUATE: To check our result, consider the case of $A_1 >> A_2$. In that case the denominator approaches 1 and $v_2 \rightarrow \sqrt{2\left(\dfrac{p_0 - p_{atm}}{\rho}\right) + 2gh}$. This is the result obtained in Example 2.8 in the text, so our result looks reasonable. Also if $A_1 >> A_2$ and the tank is open at the top, $p_0 = p_{atm}$ and $v_2 \rightarrow \sqrt{2gh}$, which is the as for an object in freefall, as we would expect.

12.1. IDENTIFY: Use $\rho = m/V$ to calculate the mass and then use $w = mg$ to calculate the weight.

SET UP: $\rho = m/V$ so $m = \rho V$ From Table 12.1, $\rho = 7.8\times10^3\ \text{kg/m}^3$.

EXECUTE: For a cylinder of length L and radius R,

$V = (\pi R^2)L = \pi(0.01425\ \text{m})^2(0.858\ \text{m}) = 5.474\times10^{-4}\ \text{m}^3$.

Then $m = \rho V = (7.8\times10^3\ \text{kg/m}^3)(5.474\times10^{-4}\ \text{m}^3) = 4.27\ \text{kg}$, and

$w = mg = (4.27\ \text{kg})(9.80\ \text{m/s}^2) = 41.8\ \text{N}$ (about 9.4 lbs). A cart is not needed.

EVALUATE: The rod is less than 1m long and less than 3 cm in diameter, so a weight of around 10 lbs seems reasonable.

12.2. IDENTIFY: The volume of the remaining object is the volume of a cube minus the volume of a cylinder, and it is this object for which we know the mass. The target variables are the density of the metal of the cube and the original weight of the cube.

SET UP: The volume of a cube with side length L is L^3, the volume of a cylinder of radius r and length L is $\pi r^2 L$, and density is $\rho = m/V$.

EXECUTE: **(a)** The volume of the metal left after the hole is drilled is the volume of the solid cube minus the volume of the cylindrical hole:

$V = L^3 - \pi r^2 L = (5.0\ \text{cm})^3 - \pi(1.0\ \text{cm})^2(5.0\ \text{cm}) = 109\ \text{cm}^3 = 1.09\times10^{-4}\ \text{m}^3$. The cube with the hole has mass $m = \dfrac{w}{g} = \dfrac{6.30\ \text{N}}{9.80\ \text{m/s}^2} = 0.6429\ \text{kg}$ and density $\rho = \dfrac{m}{V} = \dfrac{0.6429\ \text{kg}}{1.09\times10^{-4}\ \text{m}^3} = 5.9\times10^3\ \text{kg/m}^3$.

(b) The solid cube has volume $V = L^3 = 125\ \text{cm}^3 = 1.25\times10^{-4}\ \text{m}^3$ and mass

$m = \rho V = (5.9\times10^3\ \text{kg/m}^3)(1.25\times10^{-4}\ \text{m}^3) = 0.7372\ \text{kg}$. The original weight of the cube was $w = mg = 7.2\ \text{N}$.

EVALUATE: As Table 12.1 shows, the density of this metal is about twice that of aluminum and half that of lead, so it is reasonable.

12.3. IDENTIFY: $\rho = m/V$

SET UP: The density of gold is $19.3\times10^3\ \text{kg/m}^3$.

EXECUTE: $V = (5.0\times10^{-3}\ \text{m})(15.0\times10^{-3}\ \text{m})(30.0\times10^{-3}\ \text{m}) = 2.25\times10^{-6}\ \text{m}^3$.

$\rho = \dfrac{m}{V} = \dfrac{0.0158\ \text{kg}}{2.25\times10^{-6}\ \text{m}^3} = 7.02\times10^3\ \text{kg/m}^3$. The metal is not pure gold.

EVALUATE: The average density is only 36% that of gold, so at most 36% of the mass is gold.

12.4. **IDENTIFY:** Average density is $\rho = m/V$.

SET UP: For a sphere, $V = \frac{4}{3}\pi R^3$. The sun has mass $M_{\text{sun}} = 1.99\times10^{30}\text{ kg}$ and radius $6.96\times10^{8}\text{ m}$.

EXECUTE: **(a)** $\rho = \dfrac{M_{\text{sun}}}{V_{\text{sun}}} = \dfrac{1.99\times10^{30}\text{ kg}}{\frac{4}{3}\pi(6.96\times10^{8}\text{ m})^3} = \dfrac{1.99\times10^{30}\text{ kg}}{1.412\times10^{27}\text{ m}^3} = 1.409\times10^{3}\text{ kg/m}^3$

(b) $\rho = \dfrac{1.99\times10^{30}\text{ kg}}{\frac{4}{3}\pi(2.00\times10^{4}\text{ m})^3} = \dfrac{1.99\times10^{30}\text{ kg}}{3.351\times10^{13}\text{ m}^3} = 5.94\times10^{16}\text{ kg/m}^3$

EVALUATE: For comparison, the average density of the earth is $5.5\times10^{3}\text{ kg/m}^3$. A neutron star is extremely dense.

12.5. **IDENTIFY:** Apply $\rho = m/V$ to relate the densities and volumes for the two spheres.

SET UP: For a sphere, $V = \frac{4}{3}\pi r^3$. For lead, $\rho_1 = 11.3\times10^{3}\text{ kg/m}^3$ and for aluminum, $\rho_{\text{a}} = 2.7\times10^{3}\text{ kg/m}^3$.

EXECUTE: $m = \rho V = \frac{4}{3}\pi r^3\rho$. Same mass means $r_{\text{a}}^3\rho_{\text{a}} = r_1^3\rho_1$. $\dfrac{r_{\text{a}}}{r_1} = \left(\dfrac{\rho_1}{\rho_{\text{a}}}\right)^{1/3} = \left(\dfrac{11.3\times10^{3}}{2.7\times10^{3}}\right)^{1/3} = 1.6.$

EVALUATE: The aluminum sphere is larger, since its density is less.

12.6. **IDENTIFY:** $w = mg$ and $m = \rho V$. Find the volume V of the pipe.

SET UP: For a hollow cylinder with inner radius R_1, outer radius R_2, and length L the volume is $V = \pi(R_2^2 - R_1^2)L$. $R_1 = 1.25\times10^{-2}\text{ m}$ and $R_2 = 1.75\times10^{-2}\text{ m}$.

EXECUTE: $V = \pi[(0.0175\text{ m})^2 - (0.0125\text{ m})^2](1.50\text{ m}) = 7.07\times10^{-4}\text{ m}^3$.

$m = \rho V = (8.9\times10^{3}\text{ kg/m}^3)(7.07\times10^{-4}\text{ m}^3) = 6.29\text{ kg}$. $w = mg = 61.6\text{ N}$.

EVALUATE: The pipe weighs about 14 pounds.

12.7. **IDENTIFY:** The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho g h$.

SET UP: Freshwater has density $1.00\times10^{3}\text{ kg/m}^3$ and seawater has density $1.03\times10^{3}\text{ kg/m}^3$.

EXECUTE: **(a)** $p - p_0 = (1.00\times10^{3}\text{ kg/m}^3)(3.71\text{ m/s}^2)(500\text{ m}) = 1.86\times10^{6}\text{ Pa}$.

(b) $h = \dfrac{p - p_0}{\rho g} = \dfrac{1.86\times10^{6}\text{ Pa}}{(1.03\times10^{3}\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 184\text{ m}$

EVALUATE: The pressure at a given depth is greater on earth because a cylinder of water of that height weighs more on earth than on Mars.

12.8. **IDENTIFY:** The difference in pressure at points with heights y_1 and y_2 is $p - p_0 = \rho g(y_1 - y_2)$. The outward force $F_\perp$ is related to the surface area A by $F_\perp = pA$.

SET UP: For blood, $\rho = 1.06\times10^{3}\text{ kg/m}^3$. $y_1 - y_2 = 1.65\text{ m}$. The surface area of the segment is πDL, where $D = 1.50\times10^{-3}\text{ m}$ and $L = 2.00\times10^{-2}\text{ m}$.

EXECUTE: **(a)** $p_1 - p_2 = (1.06\times10^{3}\text{ kg/m}^3)(9.80\text{ m/s}^2)(1.65\text{ m}) = 1.71\times10^{4}\text{ Pa}$.

(b) The additional force due to this pressure difference is $\Delta F_\perp = (p_1 - p_2)A$.

$A = \pi DL = \pi(1.50\times10^{-3}\text{ m})(2.00\times10^{-2}\text{ m}) = 9.42\times10^{-5}\text{ m}^2$.

$\Delta F_\perp = (1.71\times10^{4}\text{ Pa})(9.42\times10^{-5}\text{ m}^2) = 1.61\text{ N}$.

EVALUATE: The pressure difference is about $\frac{1}{6}$ atm.

12.9. **IDENTIFY:** Apply $p = p_0 + \rho gh.$

SET UP: Gauge pressure is $p - p_{\text{air}}$.

EXECUTE: The pressure difference between the top and bottom of the tube must be at least 5980 Pa in order to force fluid into the vein: $\rho gh = 5980$ Pa and

$$h = \frac{5980\ \text{Pa}}{\rho g} = \frac{5980\ \text{N/m}^2}{(1050\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 0.581\ \text{m}.$$

EVALUATE: The bag of fluid is typically hung from a vertical pole to achieve this height above the patient's arm.

12.10. **IDENTIFY:** $p_0 = p_{\text{surface}} + \rho gh$ where p_{surface} is the pressure at the surface of a liquid and p_0 is the pressure at a depth h below the surface.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** For the oil layer, $p_{\text{surface}} = p_{\text{atm}}$ and p_0 is the pressure at the oil-water interface.

$p_0 - p_{\text{atm}} = p_{\text{gauge}} = \rho gh = (600\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.120\ \text{m}) = 706\ \text{Pa}$

(b) For the water layer, $p_{\text{surface}} = 706\ \text{Pa} + p_{\text{atm}}$.

$p_0 - p_{\text{atm}} = p_{\text{gauge}} = 706\ \text{Pa} + \rho gh = 706\ \text{Pa} + (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.250\ \text{m}) = 3.16\times10^3\ \text{Pa}$

EVALUATE: The gauge pressure at the bottom of the barrel is due to the combined effects of the oil layer and water layer. The pressure at the bottom of the oil layer is the pressure at the top of the water layer.

12.11. **IDENTIFY:** The pressure due to the glycerin balances the pressure due to the unknown liquid, so we are dealing with the pressure at a depth in a liquid.

SET UP: Both sides of the U-shaped tube are open to the air, so it is the gauge pressures that balance. Therefore we use $p_g = \rho gh.$ At the level of the bottom of the column of the unknown, the pressure in the right side is equal to the pressure in the left side. (See Fig. 12.11.) On the right side, we have a 35.0-cm column of the unknown, and on the left side we have a column of glycerin whose top is 12.0 cm below that of the unknown, so its height is 35.0 cm – 12.0 cm = 23.0 cm above the bottom of the bottom of the column of the unknown liquid. The target variable is the density of the unknown liquid.

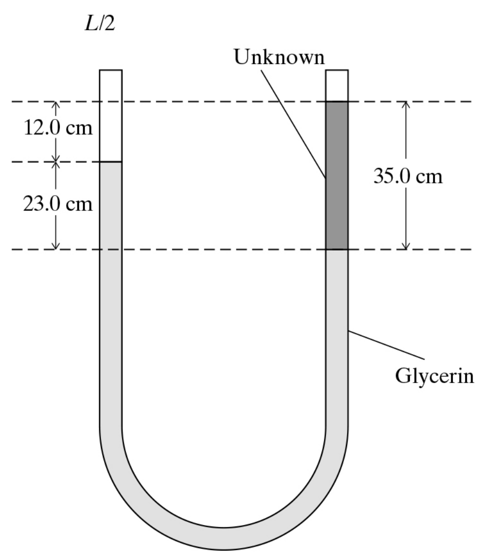


Figure 12.11

EXECUTE: We use $p_g = \rho gh$ to find the gauge pressure at the bottom of each column.

Right-hand column (the unknown): $p = \rho_x g(35.0\ \text{cm})$

Left-hand column (glycerin): $p = \rho_g g(35.0\ \text{cm} - 12.0\ \text{cm}) = \rho_g g(23.0\ \text{cm})$

Equating these pressures and solving for ρ_x gives $\rho_x = \dfrac{23\ \text{cm}}{35\ \text{cm}}\rho_g$. Using ρ_g = 1260 kg/m³ from Table 12.1, we have $\rho_x = \left(\dfrac{23\ \text{cm}}{35\ \text{cm}}\right)(1260\ \text{kg/m}^3) = 828\ \text{kg/m}^3$.

EVALUATE: Our result shows that the unknown is less dense than glycerin. This is reasonable because it takes only a 23-cm column of glycerin to balance a 35-cm column of the unknown.

12.12. **IDENTIFY** and **SET UP:** Use $p_g = \rho gh$ to calculate the gauge pressure at this depth. Use $F = pA$ to calculate the force the inside and outside pressures exert on the window, and combine the forces as vectors to find the net force.

EXECUTE: **(a)** gauge pressure $= p - p_0 = \rho gh$ From Table 12.1 the density of seawater is $1.03 \times 10^3\ \text{kg/m}^3$, so

$$p - p_0 = \rho gh = (1.03 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(250\ \text{m}) = 2.52 \times 10^6\ \text{Pa}.$$

(b) The force on each side of the window is $F = pA$. Inside the pressure is x p_0 and outside in the water the pressure is $p = p_0 + \rho gh$. The forces are shown in Figure 12.12.

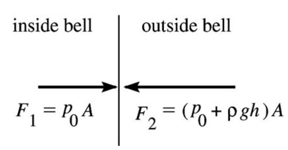


Figure 12.12

The net force is

$$F_2 - F_1 = (p_0 + \rho gh)A - p_0 A = (\rho gh)A$$
$$F_2 - F_1 = (2.52 \times 10^6\ \text{Pa})\pi(0.150\ \text{m})^2$$
$$F_2 - F_1 = 1.78 \times 10^5\ \text{N}$$

EVALUATE: The pressure at this depth is very large, over 20 times normal air pressure, and the net force on the window is huge. Diving bells used at such depths must be constructed to withstand these large forces.

12.13. **IDENTIFY:** The external pressure on the eardrum increases with depth in the ocean. This increased pressure could damage the eardrum.

SET UP: The density of seawater is $1.03 \times 10^3\ \text{kg/m}^3$. The area of the eardrum is $A = \pi r^2$, with $r = 4.1$ mm. The pressure increase with depth is $\Delta p = \rho gh$ and $F = pA$.

EXECUTE: $\Delta F = (\Delta p)A = \rho ghA$. Solving for h gives

$$h = \frac{\Delta F}{\rho gA} = \frac{1.5\ \text{N}}{(1.03 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\pi(4.1 \times 10^{-3}\ \text{m})^2} = 2.8\ \text{m}.$$

EVALUATE: 2.8 m is less than 10 ft, so it is probably a good idea to wear ear plugs if you scuba dive.

12.14. **IDENTIFY** and **SET UP:** Use $p = p_0 + \rho gh$ to calculate the pressure at the specified depths in the open tube. The pressure is the same at all points the same distance from the bottom of the tubes, so the pressure calculated in part (b) is the pressure in the tank. Gauge pressure is the difference between the absolute pressure and air pressure.

EXECUTE: $p_a = 980\ \text{millibar} = 9.80 \times 10^4\ \text{Pa}$

(a) Apply $p = p_0 + \rho gh$ to the right-hand tube. The top of this tube is open to the air so $p_0 = p_a$. The density of the liquid (mercury) is $13.6 \times 10^3\ \text{kg/m}^3$.

Thus $p = 9.80 \times 10^4\ \text{Pa} + (13.6 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0700\ \text{m}) = 1.07 \times 10^5\ \text{Pa}$.

(b) a $p = p_0 + \rho gh = 9.80\times10^4\ \text{Pa} + (13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0400\ \text{m}) = 1.03\times10^5\ \text{Pa}.$

(c) Since $y_2 - y_1 = 4.00\ \text{cm}$ the pressure at the mercury surface in the left-hand end tube equals that calculated in part (b). Thus the absolute pressure of gas in the tank is $1.03\times10^5\ \text{Pa}.$

(d) $p - p_0 = \rho gh = (13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0400\ \text{m}) = 5.33\times10^3\ \text{Pa}.$

EVALUATE: If $p = p_0 + \rho gh$ is evaluated with the density of mercury and $p - p_a = 1\ \text{atm} = 1.01\times10^5\ \text{Pa},$ then $h = 76\ \text{cm}.$ The mercury columns here are much shorter than 76 cm, so the gauge pressures are much less than $1.0\times10^5\ \text{Pa}.$

12.15. **IDENTIFY:** Apply $p = p_0 + \rho gh.$

SET UP: For water, $\rho = 1.00\times10^3\ \text{kg/m}^3.$

EXECUTE: $p - p_{\text{air}} = \rho gh = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(6.1\ \text{m}) = 6.0\times10^4\ \text{Pa}.$

EVALUATE: The pressure difference increases linearly with depth.

12.16. **IDENTIFY:** The gauge pressure of the person must be equal to the pressure due to the column of water in the straw.

SET UP: Apply $p = p_0 + \rho gh.$

EXECUTE: **(a)** The gauge pressure is $p_g = \rho gh = -(1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.1\ \text{m}) = -1.1\times10^4\ \text{Pa}.$

(b) In order for water to go up the straw, the pressure at the top of the straw must be lower than atmospheric pressure. Therefore the gauge pressure, $p - p_{\text{atm}}$, must be negative.

EVALUATE: The actual pressure is not negative, just the difference between the pressure at the top of the straw and atmospheric pressure.

12.17. **IDENTIFY:** $p = p_0 + \rho gh.$ $F = pA.$

SET UP: For seawater, $\rho = 1.03\times10^3\ \text{kg/m}^3.$

EXECUTE: The force F that must be applied is the difference between the upward force of the water and the downward forces of the air and the weight of the hatch. The difference between the pressure inside and out is the gauge pressure, so

$$F = (\rho gh)A - w = (1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(30\ \text{m})(0.75\ \text{m}^2) - 300\ \text{N} = 2.27\times10^5\ \text{N}.$$

EVALUATE: The force due to the gauge pressure of the water is much larger than the weight of the hatch and would be impossible for the crew to apply it just by pushing.

12.18. **IDENTIFY** and **SET UP:** Apply $p = p_0 + \rho gh$ to the water and mercury columns. The pressure at the bottom of the water column is the pressure at the top of the mercury column.

EXECUTE: With just the mercury, the gauge pressure at the bottom of the cylinder is $p - p_0 = \rho_m g h_m.$ With the water to a depth h_w, the gauge pressure at the bottom of the cylinder is $p - p_0 = \rho_m g h_m + \rho_w g h_w.$ If this is to be double the first value, then $\rho_w g h_w = \rho_m g h_m.$

$$h_w = h_m(\rho_m/\rho_w) = (0.0800\ \text{m})(13.6\times10^3/1.00\times10^3) = 1.088\ \text{m}$$

The volume of water is $V = hA = (1.088\ \text{m})(12.0\times10^{-4}\ \text{m}^2) = 1.306\times10^{-3}\ \text{m}^3 = 1310\ \text{cm}^3 = 1.31\ \text{L}.$

EVALUATE: The density of mercury is 13.6 times the density of water and $(13.6)(8\ \text{cm}) = 109\ \text{cm},$ so the pressure increase from the top to the bottom of a 109-cm tall column of water is the same as the pressure increase from top to bottom for an 8-cm tall column of mercury.

12.19. **IDENTIFY:** The gauge pressure at the top of the oil column must produce a force on the disk that is equal to its weight.

SET UP: The area of the bottom of the disk is $A = \pi r^2 = \pi(0.150\ \text{m})^2 = 0.0707\ \text{m}^2.$

EXECUTE: **(a)** $p - p_0 = \dfrac{w}{A} = \dfrac{45.0\text{ N}}{0.0707\text{ m}^2} = 636\text{ Pa}.$

(b) The increase in pressure produces a force on the disk equal to the increase in weight. By Pascal's law the increase in pressure is transmitted to all points in the oil.

(i) $\Delta p = \dfrac{83.0\text{ N}}{0.0707\text{ m}^2} = 1170\text{ Pa}.$ (ii) 1170 Pa

EVALUATE: The absolute pressure at the top of the oil produces an upward force on the disk but this force is partially balanced by the force due to the air pressure at the top of the disk.

12.20. **IDENTIFY:** This problem deals with the absolute pressure at a depth in a fluid.

SET UP: The absolute (or total) pressure is $p = p_0 + \rho g h$. The target variable is the depth at which the absolute pressure is twice and four times the surface pressure. The density of seawater is 1030 kg/m³.

EXECUTE: **(a)** Using $p = p_0 + \rho g h$ gives $2p_0 = p_0 + \rho g h$, so $h = \dfrac{p_0}{\rho g}$, which gives

$$h = \frac{1.0\times10^5\text{ Pa}}{(1030\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 10.2\text{ m}.$$

(b) Follow the same procedure as in part (a), giving $4p_0 = p_0 + \rho g h$, so $h = \dfrac{3p_0}{\rho g}$, which gives

$$h = \frac{3p_0}{\rho g} = \frac{3(1.03\times10^5\text{ Pa})}{(1030\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 30.6\text{ m}.$$

EVALUATE: Notice that by tripling the depth we do not triple the absolute pressure. But we would triple the gauge pressure.

12.21. **IDENTIFY:** $F_2 = \dfrac{A_2}{A_1}F_1.$ F_2 must equal the weight $w = mg$ of the car.

SET UP: $A = \pi D^2/4.$ D_1 is the diameter of the vessel at the piston where F_1 is applied and D_2 is the diameter at the car.

EXECUTE: $mg = \dfrac{\pi D_2^2/4}{\pi D_1^2/4}F_1.$ $\dfrac{D_2}{D_1} = \sqrt{\dfrac{mg}{F_1}} = \sqrt{\dfrac{(1520\text{ kg})(9.80\text{ m/s}^2)}{125\text{ N}}} = 10.9$

EVALUATE: The diameter is smaller where the force is smaller, so the pressure will be the same at both pistons.

12.22. **IDENTIFY:** Apply $\Sigma F_y = ma_y$ to the piston, with $+y$ upward. $F = pA.$

SET UP: $1\text{ atm} = 1.013\times10^5\text{ Pa}.$ The force diagram for the piston is given in Figure 12.22. p is the absolute pressure of the hydraulic fluid.

EXECUTE: $pA - w - p_{\text{atm}}A = 0$ and

$$p - p_{\text{atm}} = p_{\text{gauge}} = \frac{w}{A} = \frac{mg}{\pi r^2} = \frac{(1200\text{ kg})(9.80\text{ m/s}^2)}{\pi(0.15\text{ m})^2} = 1.7\times10^5\text{ Pa} = 1.7\text{ atm}$$

EVALUATE: The larger the diameter of the piston, the smaller the gauge pressure required to lift the car.

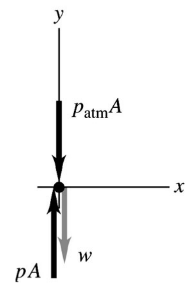


Figure 12.22

12.23. **IDENTIFY:** We are dealing with the pressure at a depth in a fluid.

SET UP: The pressure with the wine is the same as the pressure with mercury, except that the heights of the fluids are different because they have different densities. $p = p_0 + \rho gh$ gives the pressure in each case, with p_0 the same for both cases. The target variable is the height of the wine column.

EXECUTE: $p_0 + \rho_w gh_w = p_0 + \rho_m gh_m$, so $h_w = h_m \dfrac{\rho_m}{\rho_w} = (0.750\ \text{m})\dfrac{13.6\times10^3\ \text{kg/m}^3}{990\ \text{kg/m}^3} = 10.3\ \text{m}.$

EVALUATE: A wine barometer over 10 m high is highly unwieldy compared to one around 1 m high for mercury. Besides, there are better uses for wine!

12.24. **IDENTIFY:** This problem deals with buoyant force on your floating body. Archimedes's principle applies.

SET UP: Estimate: About 5% of the body is above water. Weight is 165 lb ≈ 734 N. The buoyant force B is equal to your weight and is equal to the weight of the seawater displaced by your body. The volume of seawater is 95% of your volume. The weight of a volume of material is $w = \rho gV$ and average density is $\rho_{\text{av}} = m/V$. The target variables are the volume of your body and its average density.

EXECUTE: **(a)** Let subscripts w refer to water and quantities without susbscripts refer to you. When floating, $B = w_{\text{w}} = \rho_{\text{w}} gV_{\text{w}}$ and $V_{\text{w}} = 0.95V$. Using $B = w$ gives $\rho_w g(0.95V) = w$. Solving for V gives

$$V = \frac{w}{0.95\rho_{\text{w}} g} = \frac{734\ \text{N}}{(0.95)(1030\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 7.7\times10^{-2}\ \text{m}^3.$$

(b) $\rho_{\text{av}} = \dfrac{m}{V} = \dfrac{W/g}{V} = \dfrac{(734\ \text{N})/(9.80\ \text{m/s}^2)}{7.7\times10^{-2}\ \text{m}^3} = 970\ \text{kg/m}^3 \approx 95\%\ \rho_{\text{seawater}}$.

EVALUATE: To see if your volume is reasonable, assume you are all pure water and calculate your weight. $w_{\text{w}} = \rho_{\text{w}} gV_{\text{w}} = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.077\ \text{m}^3) = 755$ N, which is very close to your weight. So the volume is reasonable.

12.25. **IDENTIFY:** A buoyant force acts on the athlete, so Archimedes's principle applies. He doesn't sink, so the forces on him must balance.

SET UP: Apply $\sum F_y = 0$ to the athlete. The volume of water he displaces is equal to his volume since he is totally submerged, so $V_{\text{w}} = V_{\text{ath}}$. The buoyant force it exerts is equal to the weight of that volume of water. We want to find the volume and average density of the athlete. $\rho_{\text{av}} = m/V$.

EXECUTE: Using $\sum F_y = 0$ gives $B + 20\ \text{N} - 900\ \text{N} = 0$, so $B = 880$ N. The buoyant force is equal to the weight of the water he displaces, and $V_{\text{w}} = V_{\text{ath}}$, so $B = \rho_{\text{w}} gV_{\text{w}} = \rho_{\text{w}} gV_{\text{ath}}$. Solving for his volume gives $V_{\text{ath}} = \dfrac{B}{\rho_{\text{w}} g} = \dfrac{880\ \text{N}}{(1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 8.98\times10^{-2}\ \text{m}^3$. His average density is $\rho_{\text{av}} = \dfrac{m}{V} = \dfrac{W/g}{V} = \dfrac{(900\ \text{N})/(9.80\ \text{m/s}^2)}{8.98\times10^{-2}\ \text{m}^3} = 1020\ \text{kg/m}^3$.

EVALUATE: The athlete is slightly denser than pure water. This is reasonable because he probably has little fat, which is less dense than muscle.

12.26. **IDENTIFY:** The buoyant force B acts upward on the rock, opposing gravity. Archimedes's principle applies, and the forces must balance.

SET UP: $\rho = m/V$.

EXECUTE: With the rock of mass m in the water: $T + B = mg$. where T is the tension in the string. Call V the volume of the rock and ρ_w the density of water. By Archimedes's principle, $m = \rho_w V$, so we get $T + \rho_w Vg = mg$. Solving for V gives $V = (mg - T)/\rho_w g$. Now look at the rock in the liquid, where ρ is the density of the liquid. For the smallest density liquid, the rock is totally submerged, so the volume of liquid displaced is V. For floating we have $B = mg$, which gives $\rho gV = mg$. Solving for ρ and using the equation for V that we just found, we get

$$\rho = \frac{m}{\frac{mg - T}{\rho_w g}} = \frac{\rho_w mg}{mg - T} = \frac{(1000\ \text{kg/m}^3)(1.80\ \text{kg})(9.80\ \text{m/s}^2)}{(1.80\ \text{kg})(9.80\ \text{m/s}^2) - 12.8\ \text{N}} = 3640\ \text{kg/m}^3.$$

EVALUATE: In the water, the buoyant force was not enough to balance the weight of the rock, so there was a tension of 12.8 N in the string. In the new liquid, the buoyant force is equal to the weight. Therefore the liquid must be denser than water, which in fact it is.

12.27. **IDENTIFY:** By Archimedes's principle, the additional buoyant force will be equal to the additional weight (the man).

SET UP: $V = \dfrac{m}{\rho}$ where $dA = V$ and d is the additional distance the buoy will sink.

EXECUTE: With man on buoy must displace additional 80.0 kg of water.

$$V = \frac{m}{\rho} = \frac{80.0\ \text{kg}}{1030\ \text{kg/m}^3} = 0.07767\ \text{m}^3.\quad dA = V \text{ so } d = \frac{V}{A} = \frac{0.07767\ \text{m}^3}{\pi(0.450\ \text{m})^2} = 0.122\ \text{m}.$$

EVALUATE: We do not need to use the mass of the buoy because it is already floating and hence in balance.

12.28. **IDENTIFY:** Apply Newton's second law to the woman plus slab. The buoyancy force exerted by the water is upward and given by $B = \rho_{\text{water}} V_{\text{displ}} g$, where V_{displ} is the volume of water displaced.

SET UP: The floating object is the slab of ice plus the woman; the buoyant force must support both. The volume of water displaced equals the volume V_{ice} of the ice. The free-body diagram is given in Figure 12.28.

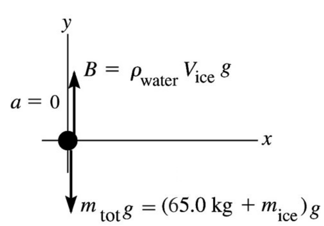


Figure 12.28

EXECUTE: $\Sigma F_y = ma_y$

$B - m_{\text{tot}} g = 0$

$\rho_{\text{water}} V_{\text{ice}} g = (65.0\ \text{kg} + m_{\text{ice}})g$

But $\rho = m/V$ so $m_{\text{ice}} = \rho_{\text{ice}} V_{\text{ice}}$

$$V_{\text{ice}} = \frac{65.0\ \text{kg}}{\rho_{\text{water}} - \rho_{\text{ice}}} = \frac{65.0\ \text{kg}}{1000\ \text{kg/m}^3 - 920\ \text{kg/m}^3} = 0.81\ \text{m}^3.$$

EVALUATE: The mass of ice is $m_{\text{ice}} = \rho_{\text{ice}} V_{\text{ice}} = 750\ \text{kg}$.

12.29. **IDENTIFY:** Apply $\Sigma F_y = ma_y$ to the sample, with $+y$ upward. $B = \rho_{\text{water}} V_{\text{obj}} g$.

SET UP: $w = mg = 17.50\ \text{N}$ and $m = 1.79\ \text{kg}$.

EXECUTE: $T + B - mg = 0$. $a_x = 0$

$$V_{\text{obj}} = \frac{B}{\rho_{\text{water}} g} = \frac{6.30\ \text{N}}{(1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 6.43\times10^{-4}\ \text{m}^3.$$

$$\rho = \frac{m}{V} = \frac{1.79\ \text{kg}}{6.43\times10^{-4}\ \text{m}^3} = 2.78\times10^3\ \text{kg/m}^3.$$

EVALUATE: The density of the sample is greater than that of water and it doesn't float.

12.30. IDENTIFY: The upward buoyant force B exerted by the liquid equals the weight of the fluid displaced by the object. Since the object floats the buoyant force equals its weight.

SET UP: Glycerin has density $\rho_{\text{gly}} = 1.26\times10^3\ \text{kg/m}^3$ and seawater has density $\rho_{\text{sw}} = 1.03\times10^3\ \text{kg/m}^3$. Let V_{obj} be the volume of the apparatus. $g_{\text{E}} = 9.80\ \text{m/s}^2$; $g_{\text{C}} = 5.40\ \text{m/s}^2$. Let V_{sub} be the volume submerged on Caasi.

EXECUTE: On earth $B = \rho_{\text{sw}}(0.250V_{\text{obj}})g_{\text{E}} = mg_{\text{E}}$. $m = (0.250)\rho_{\text{sw}}V_{\text{obj}}$. On Caasi, $B = \rho_{\text{gly}}V_{\text{sub}}g_{\text{C}} = mg_{\text{C}}$. $m = \rho_{\text{gyl}}V_{\text{sub}}$. The two expressions for m must be equal, so

$$(0.250)V_{\text{obj}}\rho_{\text{sw}} = \rho_{\text{gly}}V_{\text{sub}} \text{ and } V_{\text{sub}} = \left(\frac{0.250\rho_{\text{sw}}}{\rho_{\text{gly}}}\right)V_{\text{obj}} = \left(\frac{[0.250][1.03\times10^3\ \text{kg/m}^3]}{1.26\times10^3\ \text{kg/m}^3}\right)V_{\text{obj}} = 0.204V_{\text{obj}}.$$

20.4% of the volume will be submerged on Caasi.

EVALUATE: Less volume is submerged in glycerin since the density of glycerin is greater than the density of seawater. The value of g on each planet cancels out and has no effect on the answer. The value of g changes the weight of the apparatus and the buoyant force by the same factor.

12.31. IDENTIFY: In air and in the liquid, the forces on the rock must balance. Archimedes's principle applies in the liquid.

SET UP: $B = \rho V g$, $\rho = m/V$, call m the mass of the rock, V its volume, and ρ its density; T is the tension in the string and ρ_{L} is the density of the liquid.

EXECUTE: In air: $T = mg = \rho V g$. $V = T/\rho g = (28.0\ \text{N})/[(1200\ \text{kg/m}^3)(9.80\ \text{m/s}^2)] = 0.00238\ \text{m}^3$.

In the liquid: $T + B = mg$, so

$T = mg - B = \rho V g - \rho_{\text{L}} V g = gV(\rho - \rho_{\text{L}}) = (9.80\ \text{m/s}^2)(0.00238\ \text{m}^3)(1200\ \text{kg/m}^3 - 750\ \text{kg/m}^3) = 10.5$ N.

EVALUATE: When the rock is in the liquid, the tension in the string is less than the tension when the rock is in air since the buoyant force helps balance some of the weight of the rock.

12.32. IDENTIFY: $B = \rho_{\text{water}}V_{\text{obj}}g$. The net force on the sphere is zero.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** $B = (1000\ \text{kg/m}^3)(0.650\ \text{m}^3)(9.80\ \text{m/s}^2) = 6.37\times10^3\ \text{N}$

(b) $B = T + mg$ and $m = \dfrac{B - T}{g} = \dfrac{6.37\times10^3\ \text{N} - 1120\ \text{N}}{9.80\ \text{m/s}^2} = 536\ \text{kg}.$

(c) Now $B = \rho_{\text{water}}V_{\text{sub}}g$, where V_{sub} is the volume of the sphere that is submerged. $B = mg$.

$$\rho_{\text{water}}V_{\text{sub}}g = mg \text{ and } V_{\text{sub}} = \frac{m}{\rho_{\text{water}}} = \frac{536\ \text{kg}}{1000\ \text{kg/m}^3} = 0.536\ \text{m}^3. \quad \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{0.536\ \text{m}^3}{0.650\ \text{m}^3} = 0.824 = 82.4\%.$$

EVALUATE: The average density of the sphere is $\rho_{\text{sph}} = \dfrac{m}{V} = \dfrac{536\ \text{kg}}{0.650\ \text{m}^3} = 825\ \text{kg/m}^3$. $\rho_{\text{sph}} < \rho_{\text{water}}$, and that is why it floats with 82.4% of its volume submerged.

12.33. IDENTIFY and SET UP: Use $p = p_0 + \rho g h$ to calculate the gauge pressure at the two depths.

(a) The distances are shown in Figure 12.33a.

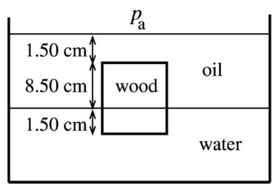


Figure 12.33a

EXECUTE: $p - p_0 = \rho g h$

The upper face is 1.50 cm below the top of the oil, so

$$p - p_0 = (790\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0150$$

$$p - p_0 = 116\ \text{Pa}$$

(b) The pressure at the interface is $p_{\text{interface}} = p_a + \rho_{\text{oil}} g(0.100\ \text{m})$. The lower face of the block is 1.50 cm below the interface, so the pressure there is $p = p_{\text{interface}} + \rho_{\text{water}} g(0.0150\ \text{m})$. Combining these two equations gives

$$p - p_a = \rho_{\text{oil}} g(0.100\ \text{m}) + \rho_{\text{water}} g(0.0150\ \text{m})$$

$$p - p_a = [(790\ \text{kg/m}^3)(0.100\ \text{m}) + (1000\ \text{kg/m}^3)(0.0150\ \text{m})](9.80\ \text{m/s}^2)$$

$$p - p_a = 921\ \text{Pa}$$

(c) IDENTIFY and **SET UP:** Consider the forces on the block. The area of each face of the block is $A = (0.100\ \text{m})^2 = 0.0100\ \text{m}^2$. Let the absolute pressure at the top face be p_t and the pressure at the bottom face be p_b. In $p = \dfrac{F_\perp}{A}$, use these pressures to calculate the force exerted by the fluids at the top and bottom of the block. The free-body diagram for the block is given in Figure 12.33b.

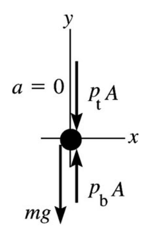


Figure 12.33b

EXECUTE: $\Sigma F_y = m a_y$

$$p_b A - p_t A - mg = 0$$

$$(p_b - p_t)A = mg$$

Note that $(p_b - p_t) = (p_b - p_a) - (p_t - p_a) = 921\ \text{Pa} - 116\ \text{Pa} = 805\ \text{Pa}$; the difference in absolute pressures equals the difference in gauge pressures.

$$m = \frac{(p_b - p_t)A}{g} = \frac{(805\ \text{Pa})(0.0100\ \text{m}^2)}{9.80\ \text{m/s}^2} = 0.821\ \text{kg}.$$

And then $\rho = m/V = 0.821\ \text{kg}/(0.100\ \text{m})^3 = 821\ \text{kg/m}^3$.

EVALUATE: We can calculate the buoyant force as $B = (\rho_{\text{oil}} V_{\text{oil}} + \rho_{\text{water}} V_{\text{water}})g$ where $V_{\text{oil}} = (0.0100\ \text{m}^2)(0.0850\ \text{m}) = 8.50 \times 10^{-4}\ \text{m}^3$ is the volume of oil displaced by the block and $V_{\text{water}} = (0.0100\ \text{m}^2)(0.0150\ \text{m}) = 1.50 \times 10^{-4}\ \text{m}^3$ is the volume of water displaced by the block. This gives $B = (0.821\ \text{kg})g$. The mass of water displaced equals the mass of the block.

12.34. IDENTIFY: The sum of the vertical forces on the ingot is zero. $\rho = m/V$. The buoyant force is $B = \rho_{\text{water}} V_{\text{obj}} g$.

SET UP: The density of aluminum is $2.7 \times 10^3\ \text{kg/m}^3$. The density of water is $1.00 \times 10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** $T = mg = 89\ \text{N}$ so $m = 9.08\ \text{kg}$. $V = \dfrac{m}{\rho} = \dfrac{9.08\ \text{kg}}{2.7 \times 10^3\ \text{kg/m}^3} = 3.36 \times 10^{-3}\ \text{m}^3 = 3.4\ \text{L}$.

(b) When the ingot is totally immersed in the water while suspended, $T + B - mg = 0$.

$B = \rho_{\text{water}} V_{\text{obj}} g = (1.00\times10^3\ \text{kg/m}^3)(3.36\times10^{-3}\ \text{m}^3)(9.80\ \text{m/s}^2) = 32.9\ \text{N}$.

$T = mg - B = 89\ \text{N} - 32.9\ \text{N} = 56\ \text{N}$.

EVALUATE: The buoyant force is equal to the difference between the apparent weight when the object is submerged in the fluid and the actual gravity force on the object.

12.35. **IDENTIFY:** The vertical forces on the rock sum to zero. The buoyant force equals the weight of liquid displaced by the rock. $V = \frac{4}{3}\pi R^3$.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: The rock displaces a volume of water whose weight is $39.2\ \text{N} - 28.4\ \text{N} = 10.8\ \text{N}$. The mass of this much water is thus $10.8\ \text{N}/(9.80\ \text{m/s}^2) = 1.102\ \text{kg}$ and its volume, equal to the rock's volume, is $\dfrac{1.102\ \text{kg}}{1.00\times10^3\ \text{kg/m}^3} = 1.102\times10^{-3}\ \text{m}^3$. The weight of unknown liquid displaced is $39.2\ \text{N} - 21.5\ \text{N} = 17.7\ \text{N}$, and its mass is $(17.7\ \text{N})/(9.80\ \text{m/s}^2) = 1.806\ \text{kg}$. The liquid's density is thus $(1.806\ \text{kg})/(1.102\times10^{-3}\ \text{m}^3) = 1.64\times10^3\ \text{kg/m}^3$.

EVALUATE: The density of the unknown liquid is a little more than 1.5 times the density of water.

12.36. **IDENTIFY:** The block floats in water and then in a second liquid, so we apply Archimedes's principle.

SET UP: In both cases, the buoyant force is equal to the weight of the block and is also equal to the weight of the liquid displaced by the block. Call V the volume of the block and use $w = \rho g V$. The target variable is the density of the second liquid.

EXECUTE: In water: $B = \rho_{\text{w}} g V_{\text{W}} = \rho_{\text{w}} g(0.700V)$

In the second liquid: $B = \rho_{\text{L}} g V_{\text{L}} = \rho_{\text{L}} g(0.800V)$

Equate the buoyant forces and solve for ρ_{L}: $\rho_{\text{L}} = \dfrac{0.700}{0.800}\rho_{\text{w}} = 875\ \text{kg/m}^3$.

EVALUATE: In the second liquid the block has more of its volume submerged than in the water, so the second liquid must be less dense than water. This agrees with our result.

12.37. **IDENTIFY:** The cylinder is partially submerged in water, so we apply Archimedes's principle. The vertical forces on it must balance.

SET UP: The density of this cylinder is 370/1000 = 37% that of water, so it would float with 37% of its volume under the water and 63% above water. But it is partially submerged with 70.0% under water. Therefore the buoyant force B on it must be greater than its weight, and this would force the cylinder upward. The tension in the cable prevents this from happening. The target variable is the tension T in the cable. We apply $\Sigma F_y = 0$ and $w = \rho g V$.

EXECUTE: Start with $\Sigma F_y = 0$: $B - T - w_{\text{c}} = 0$. We see that we first need to find B. Call V the volume of the cylinder and ρ_{c} its density. Therefore $w_{\text{c}} = \rho_{\text{c}} gV$. By Archimedes's principle, the buoyant force B is equal to the weight of the water displaced by the cylinder, and we know that the cylinder has 70% of its volume below the water, which is $0.700V$. Therefore $B = \rho_{\text{w}} g(0.700V)$. Dividing B by w_{c} gives

$\dfrac{B}{w_{\text{c}}} = \dfrac{\rho_{\text{w}} g(0.700V)}{\rho_{\text{c}} gV} = \dfrac{\rho_{\text{w}}}{\rho_{\text{c}}} = \dfrac{1000\ \text{kg/m}^3}{370\ \text{kg/m}^3} = 2.7027$, so $B = 2.7027w_{\text{c}}$. Putting this result into $\Sigma F_y = 0$ gives

$2.7027w_{\text{c}} - T - w_{\text{c}} = 0$. Solving for T gives

$T = 1.7027w_{\text{c}} = (1.7027)(30.0\ \text{kg})(9.80\ \text{m/s}^2) = 501\ \text{N}$.

EVALUATE: If the cable were to break, the net upward force at that instant on the cylinder would be $B - w_{\text{c}} = 2.7027w_{\text{c}} - w_{\text{c}} = 1.70w_{\text{c}}$, so the cylinder would accelerate upward and eventually float with 63% of its volume above water.

12.38. **IDENTIFY:** The volume flow rate is Av.

SET UP: $Av = 0.750\ \text{m}^3/\text{s}$. $A = \pi D^2/4$.

EXECUTE: **(a)** $v\pi D^2/4 = 0.750\ \text{m}^3/\text{s}$. $v = \dfrac{4(0.750\ \text{m}^3/\text{s})}{\pi(4.50\times10^{-2}\ \text{m})^2} = 472\ \text{m/s}$.

(b) vD^2 must be constant, so $v_1D_1^2 = v_2D_2^2$. $v_2 = v_1\left(\dfrac{D_1}{D_2}\right)^2 = (472\ \text{m/s})\left(\dfrac{D_1}{3D_1}\right)^2 = 52.4\ \text{m/s}$.

EVALUATE: The larger the hole, the smaller the speed of the fluid as it exits.

12.39. **IDENTIFY:** Apply the equation of continuity.

SET UP: $A = \pi r^2$, $v_1A_1 = v_2A_2$.

EXECUTE: $v_2 = v_1(A_1/A_2)$.

$A_1 = \pi(0.80\ \text{cm})^2$, $A_2 = 20\pi(0.10\ \text{cm})^2$. $v_2 = (3.0\ \text{m/s})\dfrac{\pi(0.80)^2}{20\pi(0.10)^2} = 9.6\ \text{m/s}$.

EVALUATE: The total area of the shower head openings is less than the cross-sectional area of the pipe, and the speed of the water in the shower head opening is greater than its speed in the pipe.

12.40. **IDENTIFY:** Apply the equation of continuity. The volume flow rate is vA.

SET UP: $1.00\ \text{h} = 3600\ \text{s}$. $v_1A_1 = v_2A_2$.

EXECUTE: **(a)** $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (3.50\ \text{m/s})\left(\dfrac{0.070\ \text{m}^2}{0.105\ \text{m}^2}\right) = 2.3\ \text{m/s}$

(b) $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (3.50\ \text{m/s})\left(\dfrac{0.070\ \text{m}^2}{0.047\ \text{m}^2}\right) = 5.2\ \text{m/s}$

(c) $V = v_1A_1t = (3.50\ \text{m/s})(0.070\ \text{m}^2)(3600\ \text{s}) = 880\ \text{m}^3$.

EVALUATE: The equation of continuity says the volume flow rate is the same at all points in the pipe.

12.41. **IDENTIFY** and **SET UP:** Apply the continuity equation, $v_1A_1 = v_2A_2$. In part (a) the target variable is V. In part (b) solve for A and then from that get the radius of the pipe.

EXECUTE: **(a)** $vA = 1.20\ \text{m}^3/\text{s}$

$$v = \frac{1.20\ \text{m}^3/\text{s}}{A} = \frac{1.20\ \text{m}^3/\text{s}}{\pi r^2} = \frac{1.20\ \text{m}^3/\text{s}}{\pi(0.150\ \text{m})^2} = 17.0\ \text{m/s}$$

(b) $vA = 1.20\ \text{m}^3/\text{s}$

$v\pi r^2 = 1.20\ \text{m}^3/\text{s}$

$$r = \sqrt{\frac{1.20\ \text{m}^3/\text{s}}{v\pi}} = \sqrt{\frac{1.20\ \text{m}^3/\text{s}}{(3.80\ \text{m/s})\pi}} = 0.317\ \text{m}$$

EVALUATE: The speed is greater where the area and radius are smaller.

12.42. **IDENTIFY:** This problem is about fluid flow, so we use Bernoulli's equation and the volume flow rate.

SET UP: Apply $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$ and $dv/dt = Av$. Choose $y = 0$ at the base of the tank, so $y_2 = 0$ and $y_1 = h$. Since the tank is large, assume that $v_2 >> v_1$, so we use $v_1 = 0$. The top of the tank and the end of the pipe are open to the atmosphere, so $p_1 = p_2$. We need to find a relationship between the volume flow rate dV/dt and h in order to interpret the graph. The target variable is g on this planet.

EXECUTE: Bernoulli's equation reduces to $\rho g h = \frac{1}{2}\rho v_2^2$, which gives $v_2 = \sqrt{2gh}$. The continuity equation gives $dV/dt = Av_2$. Using our result for v_2 gives $dV/dt = Av_2 = A\sqrt{2gh}$. Squaring gives $(dV/dt)^2 = 2A^2gh$.

From this result we see that a graph of $(dV/dt)^2$ versus h should be a straight line with slope $2A^2g$. This gives $g = \frac{\text{slope}}{2A^2} = g = \frac{1.94\times10^{-5}\ \text{m}^5/\text{s}^2}{2(9.0\times10^{-4}\ \text{m}^2)} = 9.0\ \text{m/s}^2$.

EVALUATE: This is a reasonable value for g on a solid planet.

12.43. **IDENTIFY:** Water is flowing out of the tank and collecting in a bucket, so we use Bernoulli's equation and the volume flow rate.

SET UP: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, $dV/dt = Av$. Choose $y = 0$ at the base of the tank, so $y_2 = 0$ and $y_1 = h$. The top of the tank and the end of the pipe are open to the atmosphere, so $p_1 = p_2$. The target variable is the time it takes to collect a gallon of water. Call R the radius of the tank and r the radius of the small hole at the bottom.

EXECUTE: **(a)** As 1 gal flows out of the tank the change in volume in the tank is 1 gal = $3.788\times10^{-3}\ \text{m}^3$. The change in volume ΔV of water in the tank due to a height change Δh is $\Delta V = \pi R^2 \Delta h$, so Δh = $\Delta V/(\pi R^2)$ = ($3.788\times10^{-3}\ \text{m}^3$)/[$\pi$(1.50 m)^2] = 5.36×10^{-4} m = 0.536 mm.

(b) Now use $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. Based upon the result in part (a), we can treat the height of water in the tank as constant while a gallon flows out and treat the speed v_1 of the water at the top of the tank as being essentially zero. The top of the tank and the end of the pipe are open to the atmosphere, so $p_1 = p_2$. Take $y = 0$ at the bottom of the tank. We need to find the speed v_2 of the water as it comes out of the small hole at the bottom of the tank. Bernoulli's equation becomes $\rho g h = \frac{1}{2}\rho v_2^2$, which gives $v_2 = \sqrt{2gh}$. Now use the volume flow rate at the bottom. $dV/dt = Av_2 = \pi r^2 \sqrt{2gh}$. This gives $\Delta V = \pi r^2 \sqrt{2gh}\,\Delta t$, so $\Delta t = \frac{\Delta V}{\pi r^2 \sqrt{2gh}}$. Using ΔV = $3.788\times10^{-3}\ \text{m}^3$, r = 0.250 cm = 0.0025 m, and h = 2.00 m gives Δt = 30.8 s.

EVALUATE: It is reasonable to neglect the change in depth of the water in the tank as one gallon flows out because $\Delta h \ll h$: 0.536 mm << 2.00 m.

12.44. **IDENTIFY:** $\rho = m/V$. Apply the equation of continuity and Bernoulli's equation to points 1 and 2.

SET UP: The density of water is 1 kg/L.

EXECUTE: **(a)** $\frac{(220)(0.355\ \text{kg})}{60.0\ \text{s}} = 1.30\ \text{kg/s}$.

(b) The density of the liquid is $\frac{0.355\ \text{kg}}{0.355\times10^{-3}\ \text{m}^3} = 1000\ \text{kg/m}^3$, and so the volume flow rate is $\frac{1.30\ \text{kg/s}}{1000\ \text{kg/m}^3} = 1.30\times10^{-3}\ \text{m}^3/\text{s} = 1.30\ \text{L/s}$. This result may also be obtained from $\frac{(220)(0.355\ \text{L})}{60.0\ \text{s}} = 1.30\ \text{L/s}$.

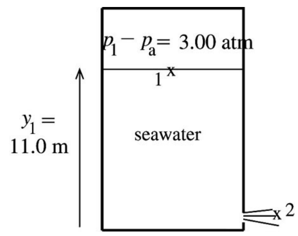
(c) $v_1 = \frac{1.30\times10^{-3}\ \text{m}^3/\text{s}}{2.00\times10^{-4}\ \text{m}^2} = 6.50\ \text{m/s}$. $v_2 = v_1/4 = 1.63\ \text{m/s}$.

(d) $p_1 = p_2 + \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$.

$p_1 = 152\ \text{kPa} + (1000\ \text{kg/m}^3)\left(\tfrac{1}{2}\left[(1.63\ \text{m/s})^2 - (6.50\ \text{m/s})^2\right] + (9.80\ \text{m/s}^2)(-1.35\ \text{m})\right)$. $p_1 = 119\ \text{kPa}$.

EVALUATE: The increase in height and the increase in fluid speed at point 1 both cause the pressure at point 1 to be less than the pressure at point 2.

12.45. **IDENTIFY** and **SET UP:**



Apply Bernoulli's equation with points 1 and 2 chosen as shown in Figure 12.45. Let $y = 0$ at the bottom of the tank so $y_1 = 11.0$ m and $y_2 = 0$. The target variable is v_2.

Figure 12.45

$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$A_1 v_1 = A_2 v_2$, so $v_1 = (A_2/A_1)v_2$. But the cross-sectional area of the tank (A_1) is much larger than the cross-sectional area of the hole (A_2), so $v_1 << v_2$ and the $\frac{1}{2}\rho v_1^2$ term can be neglected.

EXECUTE: This gives $\frac{1}{2}\rho v_2^2 = (p_1 - p_2) + \rho g y_1$.

Use $p_2 = p_a$ and solve for v_2:

$$v_2 = \sqrt{2(p_1 - p_a)/\rho + 2gy_1} = \sqrt{\frac{2(3.039\times 10^5\ \text{Pa})}{1030\ \text{kg/m}^3} + 2(9.80\ \text{m/s}^2)(11.0\ \text{m})}$$

$v_2 = 28.4$ m/s

EVALUATE: If the pressure at the top surface of the water were air pressure, then Toricelli's theorem (Example: 12.8) gives $v_2 = \sqrt{2g(y_1 - y_2)} = 14.7$ m/s. The actual afflux speed is much larger than this due to the excess pressure at the top of the tank.

12.46. **IDENTIFY:** A change in the speed of the blood indicates that there is a difference in the cross-sectional area of the artery. Bernoulli's equation applies to the fluid.

SET UP: Bernoulli's equation is $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. The two points are close together so we can neglect $\rho g(y_1 - y_2)$. $\rho = 1.06\times 10^3$ kg/m^3. The continuity equation is $A_1 v_1 = A_2 v_2$.

EXECUTE: Solve $p_1 - p_2 + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$ for v_2:

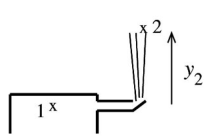
$$v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho} + v_1^2} = \sqrt{\frac{2(1.20\times 10^4\ \text{Pa} - 1.15\times 10^4\ \text{Pa})}{1.06\times 10^3\ \text{kg/m}^3} + (0.300\ \text{m/s})^2} = 1.0\ \text{m/s} = 100\ \text{cm/s}.$$

$v_2 = 1.0$ m/s $= 100$ cm/s.

The continuity equation gives $\frac{A_2}{A_1} = \frac{v_1}{v_2} = \frac{30\ \text{cm/s}}{100\ \text{cm/s}} = 0.30$. $A_2 = 0.30 A_1$, so 70% of the artery is blocked.

EVALUATE: A 70% blockage reduces the blood speed from 100 cm/s to 30 cm/s, which should easily be detectable.

12.47. **IDENTIFY** and **SET UP:**



Apply Bernoulli's equation to points 1 and 2 as shown in Figure 12.47. Point 1 is in the mains and point 2 is at the maximum height reached by the stream, so $v_2 = 0$.

Figure 12.47

Solve for p_1 and then convert this absolute pressure to gauge pressure.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Let $y_1 = 0$, $y_2 = 15.0$ m. The mains have large diameter, so $v_1 \approx 0$.

Thus $p_1 = p_2 + \rho g y_2$.

But $p_2 = p_a$, so $p_1 - p_a = \rho g y_2 = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(15.0\ \text{m}) = 1.47\times10^5$ Pa.

EVALUATE: This is the gauge pressure at the bottom of a column of water 15.0 m high.

12.48. **IDENTIFY:** Toricelli's theorem says the speed of efflux is $v = \sqrt{2gh}$, where h is the distance of the small hole below the surface of the water in the tank. The volume flow rate is vA.

SET UP: $A = \pi D^2/4$, with $D = 6.00\times10^{-3}$ m.

EXECUTE: **(a)** $v = \sqrt{2(9.80\ \text{m/s}^2)(14.0\ \text{m})} = 16.6$ m/s

(b) $vA = (16.6\ \text{m/s})\pi(6.00\times10^{-3}\ \text{m})^2/4 = 4.69\times10^{-4}\ \text{m}^3/\text{s}$. A volume of $4.69\times10^{-4}\ \text{m}^3 = 0.469$ L is discharged each second.

EVALUATE: We have assumed that the diameter of the hole is much less than the diameter of the tank.

12.49. **IDENTIFY:** Apply Bernoulli's equation to the two points.

SET UP: $y_1 = y_2$. $v_1A_1 = v_2A_2$. $A_2 = 2A_1$.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (2.50\ \text{m/s})\left(\dfrac{A_1}{2A_1}\right) = 1.25$ m/s.

$$p_2 = p_1 + \tfrac{1}{2}\rho(v_1^2 - v_2^2) = 1.80\times10^4\ \text{Pa} + \tfrac{1}{2}(1000\ \text{kg/m}^3)[(2.50\ \text{m/s})^2 - (1.25\ \text{m/s})^2] = 2.03\times10^4\ \text{Pa}.$$

EVALUATE: The gauge pressure is higher at the second point because the water speed is less there.

12.50. **IDENTIFY:** Apply Bernoulli's equation to the two points.

SET UP: The continuity equation says $v_1A_1 = v_2A_2$. In Bernoulli's equation, either absolute or gauge pressures can be used at both points. $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$.

EXECUTE: Using $v_2 = \frac{1}{4}v_1$,

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2) = p_1 + \rho\left[\left(\frac{15}{32}\right)v_1^2 + g(y_1 - y_2)\right]$$

$$p_2 = 5.00\times10^4\ \text{Pa} + (1.00\times10^3\ \text{kg/m}^3)\left(\frac{15}{32}(3.00\ \text{m/s})^2 + (9.80\ \text{m/s}^2)(11.0\ \text{m})\right) = 1.62\times10^5\ \text{Pa}.$$

EVALUATE: The decrease in speed and the decrease in height at point 2 both cause the pressure at point 2 to be greater than the pressure at point 1.

12.51. **IDENTIFY** and **SET UP:** Let point 1 be where $r_1 = 4.00$ cm and point 2 be where $r_2 = 2.00$ cm. The volume flow rate vA has the value $7200\ \text{cm}^3/\text{s}$ at all points in the pipe. Apply $v_1A_1 = v_2A_2$ to find the fluid speed at points 1 and 2 and then use Bernoulli's equation for these two points to find p_2.

EXECUTE: $v_1A_1 = v_1\pi r_1^2 = 7200\ \text{cm}^3$, so $v_1 = 1.43$ m/s

$v_2A_2 = v_2\pi r_2^2 = 7200\ \text{cm}^3/\text{s}$, so $v_2 = 5.73$ m/s

$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$y_1 = y_2$ and $p_1 = 2.40\times10^5$ Pa, so $p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 2.25\times10^5$ Pa.

EVALUATE: Where the area decreases the speed increases and the pressure decreases.

12.52. IDENTIFY: Since a pressure difference is needed to keep the fluid flowing, there must be viscosity in the fluid.

SET UP: From Section 12.6, the pressure difference Δp over a length L of cylindrical pipe of radius R is proportional to L/R^4. In this problem, the length L is the same in both cases, so $R^4\,\Delta p$ must be constant. The target variable is the pressure difference.

EXECUTE: Since $R^4\,\Delta p$ is constant, we have $\Delta p_1 R_1^4 = \Delta p_2 R_2^4$.

$$\Delta p_2 = \Delta p_1\left(\frac{R_1}{R_2}\right)^4 = (6.00\times10^4\ \text{Pa})\left(\frac{0.21\ \text{m}}{0.0700\ \text{m}}\right)^4 = 4.86\times10^6\ \text{Pa}.$$

EVALUATE: The pipe is narrower, so the pressure difference must be greater.

12.53. IDENTIFY: Increasing the cross-sectional area of the artery will increase the amount of blood that flows through it per second.

SET UP: The flow rate, $\dfrac{\Delta V}{\Delta t}$, is related to the radius R or diameter D of the artery by Poiseuille's law: $\dfrac{\Delta V}{\Delta t} = \dfrac{\pi R^4}{8\eta}\left(\dfrac{p_1-p_2}{L}\right) = \dfrac{\pi D^4}{128\eta}\left(\dfrac{p_1-p_2}{L}\right)$. Assume the pressure gradient $(p_1-p_2)/L$ in the artery remains the same.

EXECUTE: $(\Delta V/\Delta t)/D^4 = \dfrac{\pi}{128\eta}\left(\dfrac{p_1-p_2}{L}\right) = \text{constant}$, so $(\Delta V/\Delta t)_{\text{old}}/D_{\text{old}}{}^4 = (\Delta V/\Delta t)_{\text{new}}/D_{\text{new}}{}^4$.

$(\Delta V/\Delta t)_{\text{new}} = 2(\Delta V/\Delta t)_{\text{old}}$ and $D_{\text{old}} = D$. This gives $D_{\text{new}} = D_{\text{old}}\left[\dfrac{(\Delta V/\Delta t)_{\text{new}}}{(\Delta V/\Delta t)_{\text{old}}}\right]^{1/4} = 2^{1/4}D = 1.19D.$

EVALUATE: Since the flow rate is proportional to D^4, a 19% increase in D doubles the flow rate.

12.54. IDENTIFY: Apply $p = p_0 + \rho g h$ and $\Delta V = -\dfrac{(\Delta p)V_0}{B}$, where B is the bulk modulus.

SET UP: Seawater has density $\rho = 1.03\times10^3\ \text{kg/m}^3$. The bulk modulus of water is $B = 2.2\times10^9$ Pa. $p_{\text{air}} = 1.01\times10^5$ Pa.

EXECUTE: (a)

$$p_0 = p_{\text{air}} + \rho g h = 1.01\times10^5\ \text{Pa} + (1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(10.92\times10^3\ \text{m}) = 1.10\times10^8\ \text{Pa}$$

(b) At the surface $1.00\ \text{m}^3$ of seawater has mass 1.03×10^3 kg. At a depth of 10.92 km the change in volume is $\Delta V = -\dfrac{(\Delta p)V_0}{B} - \dfrac{(1.10\times10^8\ \text{Pa})(1.00\ \text{m}^3)}{2.2\times10^9\ \text{Pa}} = -0.050\ \text{m}^3$. The volume of this mass of water at this depth therefore is $V = V_0 + \Delta V = 0.950\ \text{m}^3$. $\rho = \dfrac{m}{V} = \dfrac{1.03\times10^3\ \text{kg}}{0.950\ \text{m}^3} = 1.08\times10^3\ \text{kg/m}^3$. The density is 5% larger than at the surface.

EVALUATE: For water B is small and a very large increase in pressure corresponds to a small fractional change in volume.

12.55. IDENTIFY: In part (a), the force is the weight of the water. In part (b), the pressure due to the water at a depth h is $\rho g h$. $F = pA$ and $m = \rho V$.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: (a) The weight of the water is

$$\rho g V = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)((5.00\ \text{m})(4.0\ \text{m})(3.0\ \text{m})) = 5.9\times10^5\ \text{N}.$$

(b) Integration gives the expected result that the force is what it would be if the pressure were uniform and equal to the pressure at the midpoint. If d is the depth of the pool and A is the area of one end of the pool, then $F = \rho g A \dfrac{d}{2} = (1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)((4.0 \text{ m})(3.0 \text{ m}))(1.50 \text{ m}) = 1.76\times10^5$ N.

EVALUATE: The answer to part (a) can be obtained as $F = pA$, where $p = \rho g d$ is the gauge pressure at the bottom of the pool and $A = (5.0 \text{ m})(4.0 \text{ m})$ is the area of the bottom of the pool.

12.56. **IDENTIFY:** A buoyant force acts on the rock when it is suspended in the liquids, so we apply Archimedes's principle. The vertical forces on the rock must balance.

SET UP: Apply $\sum F_y = 0$ and $w = \rho g V$. The target variable is the weight of the rock. In both cases, the rock is totally immersed, so the volume of fluid displaced is equal to the volume V of the rock. We know the densities of the fluids from Table 12.1.

EXECUTE: Call T the tension in the string. $\sum F_y = 0$ gives $T + B = w$, where $B = \rho g V$.

In water: $T_w + \rho_w g V = w$ (Eq. 1)

In ethanol: $T_e + \rho_e g V = w$ (Eq. 2)

Combine Eq. (1) and Eq. (2): $\dfrac{\rho_w g V}{\rho_e g V} = \dfrac{w - T_w}{w - T_e}$. Solving for w gives $w = \dfrac{(\rho_w/\rho_e)T_e - T_w}{(\rho_w/\rho_e) - 1}$. Using ρ_w = 1000 kg/m^3, ρ_e = 810 kg/m^3, T_w = 1.20 N, and T_e = 1.60 N, we get w = 3.3 N.

EVALUATE: The tension in ethanol is greater than the tension in water because ethanol is less than water and therefore produces a smaller buoyant force.

12.57. **IDENTIFY:** Use $p = p_0 + \rho g h$ to find the gauge pressure versus depth, use $p = \dfrac{F_\perp}{A}$ to relate the pressure to the force on a strip of the gate, calculate the torque as force times moment arm, and follow the procedure outlined in the hint to calculate the total torque.

SET UP: The gate is sketched in Figure 12.57a.

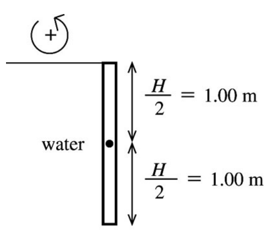


Figure 12.57a

Let τ_u be the torque due to the net force of the water on the upper half of the gate, and τ_l be the torque due to the force on the lower half.

With the indicated sign convention, τ_l is positive and τ_u is negative, so the net torque about the hinge is $\tau = \tau_l - \tau_u$. Let H be the height of the gate.

Upper half of gate:

Calculate the torque due to the force on a narrow strip of height dy located a distance y below the top of the gate, as shown in Figure 12.57b. Then integrate to get the total torque.

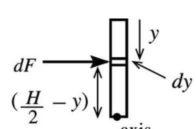


Figure 12.57b

The net force on the strip is $dF = p(y)\,dA$, where $p(y) = \rho g y$ is the pressure at this depth and $dA = W\,dy$ with $W = 4.00$ m.

$dF = \rho g y W\,dy$

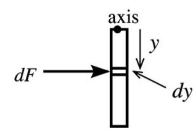
The moment arm is $(H/2-y)$, so $d\tau = \rho gW(H/2-y)y\,dy$.

$\tau_\text{u} = \int_0^{H/2} d\tau = \rho gW \int_0^{H/2}(H/2-y)y\,dy = \rho gW((H/4)y^2 - y^3/3)\Big|_0^{H/2}$

$\tau_\text{u} = \rho gW(H^3/16 - H^3/24) = \rho gW(H^3/48)$

$\tau_\text{u} = (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(4.00\text{ m})(2.00\text{ m})^3/48 = 6.533\times10^3\text{ N}\cdot\text{m}$

<u>Lower half of gate</u>:



Consider the narrow strip shown in Figure 12.57c. The depth of the strip is $(H/2+y)$ so the force dF is $dF = p(y)\,dA = \rho g(H/2+y)W\,dy$.

Figure 12.57c

The moment arm is y, so $d\tau = \rho gW(H/2+y)y\,dy$.

$\tau_1 = \int_0^{H/2} d\tau = \rho gW \int_0^{H/2}(H/2+y)y\,dy = \rho gW((H/4)y^2 + y^3/3)\Big|_0^{H/2}$

$\tau_1 = \rho gW(H^3/16 + H^3/24) = \rho gW(5H^3/48)$

$\tau_1 = (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(4.00\text{ m})5(2.00\text{ m})^3/48 = 3.267\times10^4\text{ N}\cdot\text{m}$

Then $\tau = \tau_1 - \tau_\text{u} = 3.267\times10^4\text{ N}\cdot\text{m} - 6.533\times10^3\text{ N}\cdot\text{m} = 2.61\times10^4\text{ N}\cdot\text{m}$.

EVALUATE: The forces and torques on the upper and lower halves of the gate are in opposite directions so find the net value by subtracting the magnitudes. The torque on the lower half is larger than the torque on the upper half since pressure increases with depth.

12.58. **IDENTIFY:** A dense cube floats in a tank containing water and glycerin, so Archimedes's principle applies and the vertical forces on it must balance.

SET UP: The buoyant force B is equal to the weight of the fluid displaced, and $w = \rho gV$. We want to find out what fraction of the cube's volume is below the surface of the glycerin. The water layer is on top of the glycerin layer, so we should first find out if *any* of the cube gets into the glycerin. Calling m the mass of the cube and V its volume, its density is $\rho_c = m/V = \dfrac{9.20\times10^{-3}\text{ kg}}{(0.0200\text{ m})^3} = 1150\text{ kg/m}^3$. This is greater than the density of water (1000 kg/m^3) but less than the density of glycerin (1260 kg/m^3). Therefore water alone cannot support the cube, so some of it must be in the glycerin. The cube is floating, so $\sum F_y = 0$.

EXECUTE: With V the total volume of the cube and x the volume that is in the glycerin, $V-x$ is the volume in water. By Archimedes's principle, there are *two* buoyant forces on it: one due to the displaced water and the other due to the displaced glycerin. Therefore $\sum F_y = 0$ tells us that $B_w + B_g = mg$. Using $w = \rho gV$, this gives $\rho_w gV_w + \rho_g gV_g = mg$, so $\rho_w g(V-x) + \rho_g gx = mg$. Rearranging gives $x(\rho_g - \rho_w) = m - \rho_w V$. We want the fraction of the cube's volume that is in glycerin, so we want x/V. Solving for this gives $\dfrac{x}{V} = \dfrac{m/V - \rho_w}{\rho_g - \rho_w} = \dfrac{\rho_c - \rho_w}{\rho_g - \rho_w}$. Using the given numbers gives $\dfrac{x}{V} = \dfrac{(1150-1000)\text{ kg/m}^3}{(1260-1000)\text{ kg/m}^3} = 0.577$, so 57.7% of its volume is in glycerin.

EVALUATE: It might seem strange that the water can exert any buoyant force on the cube since the only horizontal surface of the cube that is exposed the water is the *top*, and the force there is *downward*. However, let us look at the buoyant force in terms of the pressure in the fluids. This pressure is

$p = p_0 + \rho gh$. Fig. 12.58 shows the pressure on the upper and lower surfaces of the cube, with p_0 the atmospheric pressure.

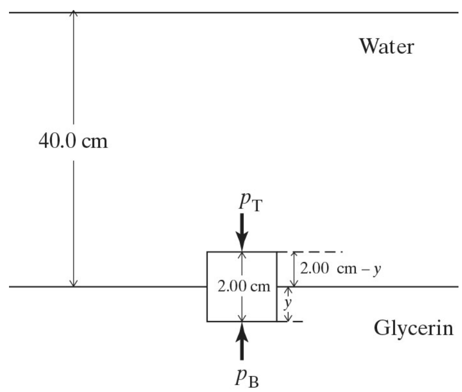


Figure 12.58

The upper surface of the cube is a distance 40 cm – (2 cm – y) = 38 cm + y below the top of the water. The pressure at that depth is the pressure at the top of the cube $p_T = p_0 + \rho_w g(38 \text{ cm} + y)$. The bottom of the cube is a distance y below the surface of the glycerin. The pressure at the surface of the glycerin layer is due to atmospheric pressure and the pressure from 40 cm of water, which is $p_0 + p_w =$ $p_0 + \rho_w g(40 \text{ cm})$. The pressure at a depth y in the glycerin is the pressure at the bottom face of the cube and is $p_B = p_0 + \rho_w g(40 \text{ cm}) + \rho_w gy.$ The force on an area A is pA, so the buoyant force on the cube is $B = p_B A - p_T A = (p_B - p_T)A.$ Using the expressions we just found for p_B and p_T gives $B = \{p_0 + \rho_w g(40 \text{ cm}) + \rho_w gy - [p_0 + \rho_w g(38 \text{ cm} + y)]\}A$. Simplifying gives $B = [\rho_w(2 \text{ cm} - y) + \rho_g y]Ag$. The buoyant force supports the cube, so $B = \rho_c g V_c$, which gives $\rho_c g V_c = [\rho_w(2 \text{ cm} - y) + \rho_g y]Ag.$ In terms of the surface area, $V_c = A(2 \text{ cm})$. Using this we get $\rho_c(2\text{cm}) = \rho_w(2\text{cm} - y) + \rho_g y$. Solving for y gives $y = \dfrac{(\rho_c - \rho_w)(2\text{cm})}{\rho_g - \rho_w}$. The fraction of the volume in the glycerin is $\dfrac{y}{2\text{cm}}$, so we get $\dfrac{y}{2\text{cm}} = \dfrac{\rho_c - \rho_w}{\rho_g - \rho_w}$, which is the same result we got above, so our result checks.

12.59. IDENTIFY: This problem deals with the pressure at a depth in a fluid and requires graphical interpretation.

SET UP: The gauge pressure on the bottom of the block supports the weight of the block and the coins. The graph is a plot of the depth h of the bottom of the block versus the mass m of the coins, so we need to look for a relationship between these quantities. The target variable is the mass M of the block. The gauge pressure is $p = \rho gh$ and the force on an area is $F = pA$.

EXECUTE: The net force on the block is $p_g A$, and that force must be equal to the weight of the box plus the coins. This gives $p_g A = mg + Mg.$ Using $p_g = \rho gh$, this becomes $\rho ghA = mg + Mg$ so $h = \dfrac{1}{\rho A}m + \dfrac{M}{\rho A}$. Therefore a graph of h versus m should have a slope of $\dfrac{1}{\rho A}$ and a y-intercept of $\dfrac{M}{\rho A}$. From the slope we get $A = \dfrac{1}{\rho(\text{slope})}$. The y-intercept gives $y\text{-int} = \dfrac{M}{\rho A}$, so $M = (y\text{-int})\rho A$. Putting the result for A into the equation for M gives $M = (y\text{-int})\rho A = (y\text{-int})\rho\left(\dfrac{1}{\rho(\text{slope})}\right) = \dfrac{y\text{-int}}{\text{slope}} =$

$\dfrac{0.0312\text{ m}}{0.0390\text{ m/kg}} = 0.800\text{ kg}.$

EVALUATE: Careful graphing is important because the answer depends on *both* the slope and y-intercept of the graph.

12.60. **IDENTIFY:** The buoyant force B equals the weight of the air displaced by the balloon.

SET UP: $B = \rho_{\text{air}}Vg$. Let g_{M} be the value of g for Mars. For a sphere $V = \frac{4}{3}\pi R^3$. The surface area of a sphere is given by $A = 4\pi R^2$. The mass of the balloon is $(5.00\times10^{-3}\text{ kg/m}^2)(4\pi R^2)$.

EXECUTE: **(a)** $B = mg_{\text{M}}$. $\rho_{\text{air}}Vg_{\text{M}} = mg_{\text{M}}$. $\rho_{\text{air}}\frac{4}{3}\pi R^3 = (5.00\times10^{-3}\text{ kg/m}^2)(4\pi R^2)$.

$R = \dfrac{3(5.00\times10^{-3}\text{ kg/m}^2)}{\rho_{\text{air}}} = 0.974\text{ m}.$ $m = (5.00\times10^{-3}\text{ kg/m}^2)(4\pi R^2) = 0.0596\text{ kg}.$

(b) $F_{\text{net}} = B - mg = ma$. $B = \rho_{\text{air}}Vg = \rho_{\text{air}}\frac{4}{3}\pi R^3 g = (1.20\text{ kg/m}^3)\left(\dfrac{4\pi}{3}\right)(0.974\text{ m})^3(9.80\text{ m/s}^2) = 45.5\text{ N}.$

$a = \dfrac{B - mg}{m} = \dfrac{45.5\text{ N} - (0.0596\text{ kg})(9.80\text{ m/s}^2)}{0.0596\text{ m}} = 754\text{ m/s}^2,$ upward.

(c) $B = m_{\text{tot}}g$. $\rho_{\text{air}}Vg = (m_{\text{balloon}} + m_{\text{load}})g$. $m_{\text{load}} = \rho_{\text{air}}\frac{4}{3}\pi R^3 - (5.00\times10^{-3}\text{ kg/m}^2)4\pi R^2$.

$m_{\text{load}} = (0.0154\text{ kg/m}^3)\left(\dfrac{4\pi}{3}\right)(5[0.974\text{ m}])^3 - (5.00\times10^{-3}\text{ kg/m}^2)(4\pi)(5[0.974\text{ m}])^2$

$m_{\text{load}} = 7.45\text{ kg} - 1.49\text{ kg} = 5.96\text{ kg}$

EVALUATE: The buoyant force is proportional to R^3 and the mass of the balloon is proportional to R^2, so the load that can be carried increases when the radius of the balloon increases. We calculated the mass of the load. To find the weight of the load we would need to know the value of g for Mars.

12.61. **IDENTIFY:** The buoyant force on an object in a liquid is equal to the weight of the liquid it displaces.

SET UP: $V = \dfrac{m}{\rho}$.

EXECUTE: When it is floating, the ice displaces an amount of glycerin equal to its weight. From Table 12.1, the density of glycerin is 1260 kg/m^3. The volume of this amount of glycerin is $V = \dfrac{m}{\rho} = \dfrac{0.180\text{ kg}}{1260\text{ kg/m}^3} = 1.429\times10^{-4}\text{ m}^3$. The ice cube produces 0.180 kg of water. The volume of this mass of water is $V = \dfrac{m}{\rho} = \dfrac{0.180\text{ kg}}{1000\text{ kg/m}^3} = 1.80\times10^{-4}\text{ m}^3$. The volume of water from the melted ice is greater than the volume of glycerin displaced by the floating cube and the level of liquid in the cylinder rises. The distance the level rises is $\dfrac{1.80\times10^{-4}\text{ m}^3 - 1.429\times10^{-4}\text{ m}^3}{\pi(0.0350\text{ m})^2} = 9.64\times10^{-3}\text{ m} = 0.964\text{ cm}.$

EVALUATE: The melted ice has the same mass as the solid ice, but a different density.

12.62. **IDENTIFY:** The pressure must be the same at the bottom of the tube. Therefore since the liquids have different densities, they must have difference heights.

SET UP: After the barrier is removed the top of the water moves downward a distance x and the top of the oil moves up a distance x, as shown in Figure 12.62. After the heights have changed, the gauge pressure at the bottom of each of the tubes is the same. The gauge pressure p at a depth h is $p - p_{\text{atm}} = \rho gh$.

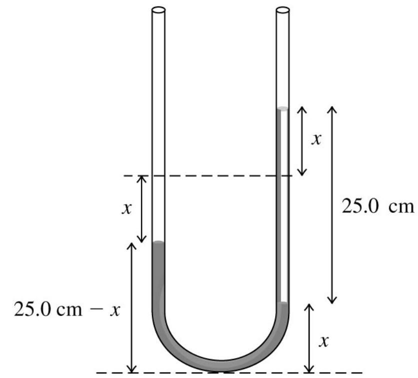


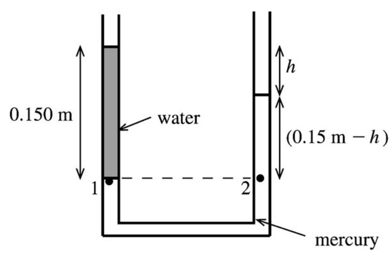
Figure 12.62

EXECUTE: The gauge pressure at the bottom of arm *A* of the tube is $p - p_{atm} = \rho_w g(25.0\text{ cm} - x)$. The gauge pressure at the bottom of arm *B* of the tube is $p - p_{atm} = \rho_{oil} g(25.0\text{ cm}) + \rho_w gx$. The gauge pressures must be equal, so $\rho_w g(25.0\text{ cm} - x) = \rho_{oil} g(25.0\text{ cm}) + \rho_w gx$. Dividing out *g* and using $\rho_{oil} = 0.80\rho_w$, we have $\rho_w(25.0\text{ cm} - x) = 0.80\rho_w(25.0\text{ cm}) + \rho_w x$. ρ_w divides out and leaves $25.0\text{ cm} - x = 20.0\text{ cm} + x$, so $x = 2.5\text{ cm}$. The height of fluid in arm *A* is $25.0\text{ cm} - x = 22.5\text{ cm}$ and the height in arm *B* is $25.0\text{ cm} + x = 27.5\text{ cm}$.

(b) (i) If the densities were the same there would be no reason for a difference in height and the height would be 25.0 cm on each side. (ii) The pressure exerted by the column of oil would be very small and the water would divide equally on both sides. The height in arm *A* would be 12.5 cm and the height in arm *B* would be $25.0\text{ cm} + 12.5\text{ cm} = 37.5\text{ cm}$.

EVALUATE: The less dense fluid rises to a higher height, which is physically reasonable.

12.63. (a) IDENTIFY and SET UP:



Apply $p = p_0 + \rho gh$ to the water in the left-hand arm of the tube. See Figure 12.63.

Figure 12.63

EXECUTE: $p_0 = p_a$, so the gauge pressure at the interface (point 1) is

$p - p_a = \rho gh = (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.150\text{ m}) = 1470\text{ Pa}$.

(b) IDENTIFY and SET UP: The pressure at point 1 equals the pressure at point 2. Apply $p = p_0 + \rho gh$ to the right-hand arm of the tube and solve for *h*.

EXECUTE: $p_1 = p_a + \rho_w g(0.150\text{ m})$ and $p_2 = p_a + \rho_{Hg} g(0.150\text{ m} - h)$

$p_1 = p_2$ implies $\rho_w g(0.150\text{ m}) = \rho_{Hg} g(0.150\text{ m} - h)$

$$0.150\text{ m} - h = \frac{\rho_w(0.150\text{ m})}{\rho_{Hg}} = \frac{(1000\text{ kg/m}^3)(0.150\text{ m})}{13.6\times10^3\text{ kg/m}^3} = 0.011\text{ m}$$

$h = 0.150\text{ m} - 0.011\text{ m} = 0.139\text{ m} = 13.9\text{ cm}$

EVALUATE: The height of mercury above the bottom level of the water is 1.1 cm. This height of mercury produces the same gauge pressure as a height of 15.0 cm of water.

12.64. IDENTIFY: Follow the procedure outlined in the hint. $F = pA$.

SET UP: The circular ring has area $dA = (2\pi R)dy$. The pressure due to the molasses at depth y is ρgy.

EXECUTE: $F = \int_0^h (\rho gy)(2\pi R)dy = \rho g\pi Rh^2$ where R and h are the radius and height of the tank.

Using the given numerical values gives $F = 2.11\times10^8$ N.

EVALUATE: The net outward force is the area of the wall of the tank, $A = 2\pi Rh$, times the average pressure, the pressure $\rho gh/2$ at depth $h/2$.

12.65. IDENTIFY: Archimedes's principle applies.

SET UP: $\rho = m/V$, the buoyant force B is equal to the weight of the liquid displaced. Call m the mass of the block.

EXECUTE: **(a)** The volume of water displaced by the block is 80.0% of the volume of the block. Using $B = mg$: $\rho_w V_w g = mg$ gives $\rho_w (0.800 V_{block}) = m$. Therefore $V_{block} = m/(0.800\,\rho_w)$, so

$V_{block} = (40.0\text{ kg})/[(0.800)(1000\text{ kg/m}^3)] = 0.0500\text{ m}^3$.

(b) With the maximum amount of bricks added on, the block is completely submerged but the bricks are not under water. Therefore $m_{bricks}g + mg = \rho_w V_{block}g$. Solving for m_{bricks} and putting in the numbers gives

$m_{bricks} = \rho_w V_{block} - m = (1000\text{ kg/m}^3)(0.0500\text{ m}^3) - 40.0\text{ kg} = 10.0\text{ kg}$.

EVALUATE: If the bricks were to go under water, the buoyant force would increase because a greater volume of water would be displaced.

12.66. IDENTIFY: The buoyant force on the balloon must equal the total weight of the balloon fabric, the basket and its contents and the gas inside the balloon. $m_{gas} = \rho_{gas}V$. $B = \rho_{air}Vg$.

SET UP: The total weight, exclusive of the gas inside the balloon, is $900\text{ N} + 1700\text{ N} + 3200\text{ N} = 5800\text{ N}$.

EXECUTE: $5800\text{ N} + \rho_{gas}Vg = \rho_{air}Vg$ and $\rho_{gas} = 1.23\text{ kg/m}^3 - \dfrac{(5800\text{ N})}{(9.80\text{ m/s}^2)(2200\text{ m}^3)} = 0.96\text{ kg/m}^3$.

EVALUATE: The volume of a given mass of gas increases when the gas is heated, and the density of the gas therefore decreases.

12.67. IDENTIFY: Apply Newton's second law to the barge plus its contents. Apply Archimedes's principle to express the buoyancy force B in terms of the volume of the barge.

SET UP: The free-body diagram for the barge plus coal is given in Figure 12.67.

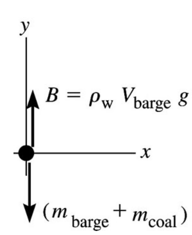


Figure 12.67

EXECUTE: $\sum F_y = ma_y$

$B - (m_{barge} + m_{coal})g = 0$

$\rho_w V_{barge} g = (m_{barge} + m_{coal})g$

$m_{coal} = \rho_w V_{barge} - m_{barge}$

$V_{barge} = (22\text{ m})(12\text{ m})(40\text{ m}) = 1.056\times10^4\text{ m}^3$

The mass of the barge is $m_{barge} = \rho_s V_s$, where s refers to steel.

From Table 12.1, $\rho_s = 7800\ \text{kg/m}^3$. The volume V_s is 0.040 m times the total area of the five pieces of steel that make up the barge

$$V_s = (0.040\ \text{m})[2(22\ \text{m})(12\ \text{m}) + 2(40\ \text{m})(12\ \text{m}) + (22\ \text{m})(40\ \text{m})] = 94.7\ \text{m}^3.$$

Therefore, $m_{\text{barge}} = \rho_s V_s = (7800\ \text{kg/m}^3)(94.7\ \text{m}^3) = 7.39\times10^5\ \text{kg}$.

Then $m_{\text{coal}} = \rho_w V_{\text{barge}} - m_{\text{barge}} = (1000\ \text{kg/m}^3)(1.056\times10^4\ \text{m}^3) - 7.39\times10^5\ \text{kg} = 9.8\times10^6\ \text{kg}$.

The volume of this mass of coal is $V_{\text{coal}} = m_{\text{coal}}/\rho_{\text{coal}} = 9.8\times10^6\ \text{kg}/1500\ \text{kg/m}^3 = 6500\ \text{m}^3$; this is less than V_{barge} so it will fit into the barge.

EVALUATE: The buoyancy force B must support both the weight of the coal and also the weight of the barge. The weight of the coal is about 13 times the weight of the barge. The buoyancy force increases when more of the barge is submerged, so when it holds the maximum mass of coal the barge is fully submerged.

12.68. **IDENTIFY:** For a floating object the buoyant force equals the weight of the object. The buoyant force when the wood sinks is $B = \rho_{\text{water}} V_{\text{tot}} g$, where V_{tot} is the volume of the wood plus the volume of the lead. $\rho = m/V$.

SET UP: The density of lead is $11.3\times10^3\ \text{kg/m}^3$.

EXECUTE: $V_{\text{wood}} = (0.600\ \text{m})(0.250\ \text{m})(0.080\ \text{m}) = 0.0120\ \text{m}^3$.

$m_{\text{wood}} = \rho_{\text{wood}} V_{\text{wood}} = (700\ \text{kg/m}^3)(0.0120\ \text{m}^3) = 8.40\ \text{kg}$.

$B = (m_{\text{wood}} + m_{\text{lead}})g$. Using $B = \rho_{\text{water}} V_{\text{tot}} g$ and $V_{\text{tot}} = V_{\text{wood}} + V_{\text{lead}}$ gives

$\rho_{\text{water}}(V_{\text{wood}} + V_{\text{lead}})g = (m_{\text{wood}} + m_{\text{lead}})g$. $m_{\text{lead}} = \rho_{\text{lead}} V_{\text{lead}}$ then gives

$\rho_{\text{water}} V_{\text{wood}} + \rho_{\text{water}} V_{\text{lead}} = m_{\text{wood}} + \rho_{\text{lead}} V_{\text{lead}}$.

$$V_{\text{lead}} = \frac{\rho_{\text{water}} V_{\text{wood}} - m_{\text{wood}}}{\rho_{\text{lead}} - \rho_{\text{water}}} = \frac{(1000\ \text{kg/m}^3)(0.0120\ \text{m}^3) - 8.40\ \text{kg}}{11.3\times10^3\ \text{kg/m}^3 - 1000\ \text{kg/m}^3} = 3.50\times10^{-4}\ \text{m}^3.$$

$m_{\text{lead}} = \rho_{\text{lead}} V_{\text{lead}} = 3.95\ \text{kg}$.

EVALUATE: The volume of the lead is only 2.9% of the volume of the wood. If the contribution of the volume of the lead to F_B is neglected, the calculation is simplified: $\rho_{\text{water}} V_{\text{wood}} g = (m_{\text{wood}} + m_{\text{lead}})g$ and $m_{\text{lead}} = 3.6\ \text{kg}$. The result of this calculation is in error by about 9%.

12.69. **IDENTIFY:** The water shoots out of the hole at the bottom of the jug and is then in free-fall downward to the floor. We will need to use Bernoulli's equation as well as projectile motion after the milk leaves the jug.

SET UP: Estimates: The milk is 10 in. (25.4 cm = 0.254 m) high in the jug and the table height is 30 in. (0.762 m). We use $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$. The target variable is the horizontal distance the milk travels before hitting the floor. Call h the height of the milk in the jug and H the height of the table.

EXECUTE: Using $p_1 = p_2 = p_{\text{atm}}$, $v_1 = 0$ ($A_{-1} >> A_2$), $y_1 = h$ = 10 in. = 0.54 m, $y_2 = 0$, we get $\rho g h = \frac{1}{2}\rho v_2^2$, which gives $v_2 = \sqrt{2gh}$. Now use projectile motion. Find the time to fall a distance H from rest vertically. $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2H}{g}}$. The horizontal distance traveled in this time is

$$x = v_2 t = \sqrt{2gh}\sqrt{\frac{2H}{g}} = 2\sqrt{hH} = 2\sqrt{(0.254\ \text{m})(0.762\ \text{m})} = 0.88\ \text{m} = 88\ \text{cm} = 35\ \text{in.}$$

EVALUATE: As the jug drains, h will decrease so the distance x will also decrease.

12.70. **IDENTIFY:** The ethanol is flowing through a pipe, so we need to use Bernoulli's equation, the continuity equation, and volume flow rate.

SET UP: Use $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, $A_1 v_1 = A_2 v_2$ (continuity equation) and $dV/dt = Av$. We want to find the pressure at A so that the volume flow rate at B is $0.0800\ \text{m}^3/\text{s}$.

EXECUTE: $p_1 = p$ (unknown), $y_1 = 0$, $y_2 = 3.00$ m, $p_2 = p_{\text{atm}}$, v_1 and v_2 are unknown. Use the volume flow rate to find v_2. Using $dV/dt = A_B v_2$ gives $v_2 = \frac{dV/dt}{A_B} = \frac{0.0800\ \text{m}^3/\text{s}}{0.0200\ \text{m}^2} = 4.00\ \text{m/s}$. Now use the continuity equation to find v_1. Solving $A_1 v_1 = A_2 v_2$ for v_1 gives $v_1 = \frac{A_B}{A_A} v_2$. For the numbers here we have $v_1 = \frac{0.0200\ \text{m}^2}{0.0500\ \text{m}^2}(4.00\ \text{m/s}) = 1.60\ \text{m/s}$. Now return to Bernoulli's equation. We want the gauge pressure at A so we want $p = p_2 - p_{\text{atm}}$. The Bernoulli equation now becomes $p + \frac{1}{2}\rho v_1^2 = \rho g y_2 + \frac{1}{2}\rho v_2^2$. Solving for p gives $p = \rho\left[g y_2 + \frac{1}{2}\left(v_2^2 - v_1^2\right)\right]$. Using (from Table 12.1) $\rho = 810\ \text{kg/m}^3$ for ethanol, $v_1 = 1.60$ m/s, $v_2 = 4.00$ m/s, and $y_2 = 3.00$ m, we get $p = 2.9 \times 10^4$ Pa.

EVALUATE: This is about 30% of atmospheric pressure. The absolute pressure would be about 1.3 atm.

12.71. **IDENTIFY:** After the water leaves the hose the only force on it is gravity. Use conservation of energy to relate the initial speed to the height the water reaches. The volume flow rate is Av.

SET UP: $A = \pi D^2/4$

EXECUTE: **(a)** $\frac{1}{2}mv^2 = mgh$ gives $v = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(28.0\ \text{m})} = 23.4\ \text{m/s}$.

$(\pi D^2/4)v = 0.500\ \text{m/s}^3$. $D = \sqrt{\frac{4(0.500\ \text{m/s}^3)}{\pi v}} = \sqrt{\frac{4(0.500\ \text{m/s}^3)}{\pi(23.4\ \text{m/s})}} = 0.165\ \text{m} = 16.5\ \text{cm}$.

(b) $D^2 v$ is constant so if D is twice as great, then v is decreased by a factor of 4. h is proportional to v^2, so h is decreased by a factor of 16. $h = \frac{28.0\ \text{m}}{16} = 1.75\ \text{m}$.

EVALUATE: The larger the diameter of the nozzle the smaller the speed with which the water leaves the hose and the smaller the maximum height.

12.72. **IDENTIFY:** $B = \rho V_A g$. Apply Newton's second law to the beaker, liquid and block as a combined object and also to the block as a single object.

SET UP: Take $+y$ upward. Let F_D and F_E be the forces corresponding to the scale reading.

EXECUTE: Forces on the combined object: $F_D + F_E - (w_A + w_B + w_C) = 0$. $w_A = F_D + F_E - w_B - w_C$. D and E read mass rather than weight, so write the equation as $m_A = m_D + m_E - m_B - m_C$. $m_D = F_D/g$ is the reading in kg of scale D; a similar statement applies to m_E.

$m_A = 3.50\ \text{kg} + 7.50\ \text{kg} - 1.00\ \text{kg} - 1.80\ \text{kg} = 8.20\ \text{kg}$.

Forces on A: $B + F_D - w_A = 0$. $\rho V_A g + F_D - m_A g = 0$. $\rho V_A + m_D = m_A$.

$$\rho = \frac{m_A - m_D}{V_A} = \frac{8.20\ \text{kg} - 3.50\ \text{kg}}{3.80 \times 10^{-3}\ \text{m}^3} = 1.24 \times 10^3\ \text{kg/m}^3$$

(b) D reads the mass of A: 8.20 kg. E reads the total mass of B and C: 2.80 kg.

EVALUATE: The sum of the readings of the two scales remains the same.

12.73. **IDENTIFY:** As water flows from the tank, the water level changes. This affects the speed with which the water flows out of the tank and the pressure at the bottom of the tank.

SET UP: Bernoulli's equation, $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, and the continuity equation, $A_1 v_1 = A_2 v_2$, both apply.

EXECUTE: **(a)** Let point 1 be at the surface of the water in the tank and let point 2 be in the stream of water that is emerging from the tank. $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. $v_1 = \dfrac{\pi d_2^2}{\pi d_1^2} v_2$, with $d_2 = 0.0200\text{ m}$ and $d_1 = 2.00\text{ m}$. $v_1 << v_2$ so the $\frac{1}{2}\rho v_1^2$ term can be neglected. $v_2 = \sqrt{\dfrac{2p_0}{\rho} + 2gh}$, where $h = y_1 - y_2$ and $p_0 = p_1 - p_2 = 5.00\times10^3\text{ Pa}$. Initially $h = h_0 = 0.800\text{ m}$ and when the tank has drained $h = 0$. At $t = 0$, $v_2 = \sqrt{\dfrac{2(5.00\times10^3\text{ Pa})}{1000\text{ kg/m}^3} + 2(9.8\text{ m/s}^2)(0.800\text{ m})} = \sqrt{10 + 15.68}\text{ m/s} = 5.07\text{ m/s}$.

If the tank is open to the air, $p_0 = 0$ and $v_2 = 3.96\text{ m/s}$. The ratio is 1.28.

(b) $v_1 = -\dfrac{dh}{dt} = \dfrac{A_2}{A_1} v_2 = \left(\dfrac{d_2}{d_1}\right)^2 \sqrt{\dfrac{2p_0}{\rho} + 2gh} = \left(\dfrac{d_2}{d_1}\right)^2 \sqrt{2g}\sqrt{\dfrac{p_0}{g\rho} + h}$. Separating variables gives $\dfrac{dh}{\sqrt{\dfrac{p_0}{g\rho} + h}} = -\left(\dfrac{d_2}{d_1}\right)^2 \sqrt{2g}\, dt$. We now must integrate $\displaystyle\int_{h_0}^{0} \dfrac{dh'}{\sqrt{\dfrac{p_0}{g\rho} + h'}} = -\left(\dfrac{d_2}{d_1}\right)^2 \sqrt{2g} \int_0^t dt'$. To do the left-hand side integral, make the substitution $u = \dfrac{p_0}{g\rho} + h'$, which makes $du = dh'$. The integral is then of the form $\displaystyle\int \dfrac{du}{u^{1/2}}$, which can be readily integrated using $\displaystyle\int u^n\, du = \dfrac{u^{n+1}}{n+1}$. The result is

$2\left(\sqrt{\dfrac{p_0}{g\rho}} - \sqrt{\dfrac{p_0}{g\rho} + h_0}\right) = -\left(\dfrac{d_2}{d_1}\right)^2 \sqrt{2g}\, t$. Solving for t gives $t = \left(\dfrac{d_1}{d_2}\right)^2 \sqrt{\dfrac{2}{g}}\left(\sqrt{\dfrac{p_0}{g\rho} + h_0} - \sqrt{\dfrac{p_0}{g\rho}}\right)$. Since

$\dfrac{p_0}{g\rho} = \dfrac{5.00\times10^3\text{ Pa}}{(9.8\text{ m/s}^2)(1000\text{ kg/m}^3)} = 0.5102\text{ m}$, we get

$t = \left(\dfrac{2.00}{0.0200}\right)^2 \sqrt{\dfrac{2}{9.8\text{ m/s}^2}}\left(\sqrt{0.5102\text{ m} + 0.800\text{ m}} - \sqrt{0.5102}\text{ m}\right) = 1.944\times10^3\text{ s} = 32.4\text{ min}$. When $p_0 = 0$,

$t = \left(\dfrac{2.00}{0.0200}\right)^2 \sqrt{\dfrac{2}{9.8\text{ m/s}^2}}\left(\sqrt{0.800\text{ m}}\right) = 4.04\times10^3\text{ s} = 67.3\text{ min}$. The ratio is 2.08.

EVALUATE: Both ratios are greater than one because a surface pressure greater than atmospheric pressure causes the water to drain with a greater speed and in a shorter time than if the surface were open to the atmosphere with a pressure of one atmosphere.

12.74. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the ball, with $+y$ upward. The buoyant force is given by Archimedes's principle.

SET UP: The ball's volume is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12.0\text{ cm})^3 = 7238\text{ cm}^3$. As it floats, it displaces a weight of water equal to its weight.

EXECUTE: **(a)** By pushing the ball under water, you displace an additional amount of water equal to 76.0% of the ball's volume or $(0.760)(7238\text{ cm}^3) = 5501\text{ cm}^3$. This much water has a mass of

$5501\ g = 5.501\ \text{kg}$ and weighs $(5.501\ \text{kg})(9.80\ \text{m/s}^2) = 53.9\ \text{N}$, which is how hard you'll have to push to submerge the ball.

(b) The upward force on the ball in excess of its own weight was found in part (a): 53.9 N. The ball's mass is equal to the mass of water displaced when the ball is floating:

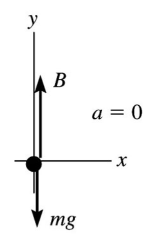
$$(0.240)(7238\ \text{cm}^3)(1.00\ \text{g/cm}^3) = 1737\ \text{g} = 1.737\ \text{kg},$$

and its acceleration upon release is thus $a = \dfrac{F_{\text{net}}}{m} = \dfrac{53.9\ \text{N}}{1.737\ \text{kg}} = 31.0\ \text{m/s}^2$.

EVALUATE: When the ball is totally immersed the upward buoyant force on it is much larger than its weight.

12.75. IDENTIFY: Apply Newton's second law to the block. In part (a), use Archimedes's principle for the buoyancy force. In part (b), use $p = p_0 + \rho g h$ to find the pressure at the lower face of the block and then use $p = \dfrac{F_\perp}{A}$ to calculate the force the fluid exerts.

(a) SET UP: The free-body diagram for the block is given in Figure 12.75a.



EXECUTE: $\sum F_y = ma_y$

$B - mg = 0$

$\rho_{\text{L}} V_{\text{sub}} g = \rho_{\text{B}} V_{\text{obj}} g$

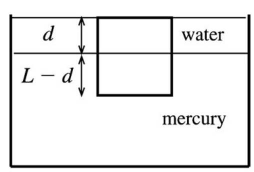
Figure 12.75a

The fraction of the volume that is submerged is $V_{\text{sub}}/V_{\text{obj}} = \rho_{\text{B}}/\rho_{\text{L}}$.

Thus the fraction that is *above* the surface is $V_{\text{above}}/V_{\text{obj}} = 1 - \rho_{\text{B}}/\rho_{\text{L}}$.

EVALUATE: If $\rho_{\text{B}} = \rho_{\text{L}}$ the block is totally submerged as it floats.

(b) SET UP: Let the water layer have depth *d*, as shown in Figure 12.75b.



EXECUTE: $p = p_0 + \rho_{\text{w}} g d + \rho_{\text{L}} g (L - d)$

Applying $\sum F_y = ma_y$ to the block gives

$$(p - p_0)A - mg = 0.$$

Figure 12.75b

$[\rho_{\text{w}} g d + \rho_{\text{L}} g (L - d)]A = \rho_{\text{B}} L A g$

A and *g* divide out and $\rho_{\text{w}} d + \rho_{\text{L}}(L - d) = \rho_{\text{B}} L$

$d(\rho_{\text{w}} - \rho_{\text{L}}) = (\rho_{\text{B}} - \rho_{\text{L}})L$

$$d = \left(\frac{\rho_{\text{L}} - \rho_{\text{B}}}{\rho_{\text{L}} - \rho_{\text{w}}}\right)L$$

(c) $d = \left(\dfrac{13.6\times10^3\ \text{kg/m}^3 - 7.8\times10^3\ \text{kg/m}^3}{13.6\times10^3\ \text{kg/m}^3 - 1000\ \text{kg/m}^3}\right)(0.100\ \text{m}) = 0.0460\ \text{m} = 4.60\ \text{cm}$

EVALUATE: In the expression derived in part (b), if $\rho_B = \rho_L$ the block floats in the liquid totally submerged and no water needs to be added. If $\rho_L \to \rho_w$ the block continues to float with a fraction $1 - \rho_B/\rho_w$ above the water as water is added, and the water never reaches the top of the block $(d \to \infty)$.

12.76. IDENTIFY: For the floating tanker, the buoyant force equals its total weight. The buoyant force is given by Archimedes's principle.

SET UP: When the metal is in the tanker, it displaces its weight of water and after it has been pushed overboard it displaces its volume of water.

EXECUTE: **(a)** The change in height Δy is related to the displaced volume ΔV by $\Delta y = \dfrac{\Delta V}{A}$, where A is the surface area of the water in the lock. ΔV is the volume of water that has the same weight as the metal, so

$$\Delta y = \frac{\Delta V}{A} = \frac{w/(\rho_{\text{water}} g)}{A} = \frac{w}{\rho_{\text{water}} g A} = \frac{(2.50\times10^6 \text{ N})}{(1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)[(60.0 \text{ m})(20.0 \text{ m})]} = 0.213 \text{ m}.$$

(b) In this case, ΔV is the volume of the metal; in the above expression, ρ_{water} is replaced by $\rho_{\text{metal}} = 7.20\rho_{\text{water}}$, which gives , $\Delta y' = \dfrac{\Delta y}{7.2}$, so

$$\Delta y - \Delta y' = \Delta y - \frac{\Delta y}{7.20} = \Delta y\left(1 - \frac{1}{7.20}\right) = (0.213 \text{ m})\left(1 - \frac{1}{7.20}\right) = 0.183 \text{ m};$$ the water level falls this amount.

EVALUATE: The density of the metal is greater than the density of water, so the volume of water that has the same weight as the steel is greater than the volume of water that has the same volume as the steel.

12.77. IDENTIFY: After leaving the tank, the water is in free fall, with $a_x = 0$ and $a_y = +g$.

SET UP: The speed of efflux is $\sqrt{2gh}$.

EXECUTE: **(a)** The time it takes any portion of the water to reach the ground is $t = \sqrt{\dfrac{2(H-h)}{g}}$, in which time the water travels a horizontal distance $R = vt = 2\sqrt{h(H-h)}$.

(b) Note that if $h' = H - h$, $h'(H - h') = (H - h)h$, and so $h' = H - h$ gives the same range. A hole $H - h$ below the water surface is a distance h above the bottom of the tank.

EVALUATE: For the special case of $h = H/2$, $h = h'$ and the two points coincide. For the upper hole the speed of efflux is less but the time in the air during the free fall is greater.

12.78. IDENTIFY: Bernoulli's equation applies to the water.

SET UP: First use Bernoulli's equation, $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$, to find the speed of the water as it enters the hold. Then use $dV/dt = Av$ to find the rate at which water flows into the hole, and then solve for the time for 10.0 L to flow in. The pressure at the top of the water is the same as the pressure of the cabin into which the water flows (atmospheric pressure), and the speed of the water at the surface is zero. 1 m^3 = 1000 L.

EXECUTE: **(a)** Using the above conditions, Bernoulli's equation gives

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p_0 + 0 + 0 = p_0 + \tfrac{1}{2}\rho v^2 + \rho g y$$

Solving for v gives $v = \sqrt{-2gy}$. The rate at which water enters ship is $V/t = Av = A\sqrt{-2gy}$. Thus

$$t = V/Av = \frac{V}{A\sqrt{-2gy}} = \frac{10\times10^{-3} \text{ m}^3}{(1.20\times10^{-4} \text{ m}^2)\sqrt{-2(9.80 \text{ m/s}^2)(-0.900 \text{ m})}} = 19.8 \text{ s}.$$

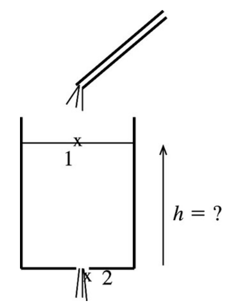
(b) The mass of 10.0 L of water is 10 kg, which will have no appreciable effect on the weight of a ship.
EVALUATE: The speed at which the water enters the hole is the same as if it had just fallen a distance of 0.900 m.

12.79. **IDENTIFY:** As you constrict the hose, you decrease its area, but the equation of continuity applies to the water.
SET UP: $A_1 v_1 = A_2 v_2$. The distance traveled by a projectile that is fired from a height h with an initial horizontal velocity v is $x = vt$ where $t = \sqrt{\dfrac{2h}{g}}$.
EXECUTE: Since h is fixed, t does not change as we constrict the nozzle. Looking at the ratio of distances we obtain $\dfrac{x_1}{x_2} = \dfrac{v_1 t}{v_2 t} = \dfrac{A_2}{A_1} = \dfrac{\pi r_2^2}{\pi r_1^2}$, which gives

$$x_1 = x_2\left(\frac{r_2}{r_1}\right)^2 = (0.950\text{ m})\left(\frac{1.80\text{ cm}}{0.750\text{ cm}}\right)^2 = 5.47\text{ m}.$$

EVALUATE: A smaller constriction results in a higher exit velocity, which results in a greater range, so our result is plausible.

12.80. **IDENTIFY:** Use Bernoulli's equation to find the velocity with which the water flows out the hole.
SET UP: The water level in the vessel will rise until the volume flow rate into the vessel, $2.40\times10^{-4}\text{ m}^3/\text{s}$, equals the volume flow rate out the hole in the bottom.



Let points 1 and 2 be chosen as in Figure 12.80.

Figure 12.80

EXECUTE: Bernoulli's equation: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Volume flow rate out of hole equals volume flow rate from tube gives that $v_2 A_2 = 2.40\times10^{-4}\text{ m}^3/\text{s}$ and

$$v_2 = \frac{2.40\times10^{-4}\text{ m}^3/\text{s}}{1.50\times10^{-4}\text{ m}^2} = 1.60\text{ m/s}$$

$A_1 \gg A_2$ and $v_1 A_1 = v_2 A_2$ says that $\frac{1}{2}\rho v_1^2 \ll \frac{1}{2}\rho v_2^2$; neglect the $\frac{1}{2}\rho v_1^2$ term.

Measure y from the bottom of the bucket, so $y_2 = 0$ and $y_1 = h$.

$p_1 = p_2 = p_a$ (air pressure)

Then $p_a + \rho g h = p_a + \frac{1}{2}\rho v_2^2$ and $h = v_2^2/2g = (1.60\text{ m/s})^2/2(9.80\text{ m/s}^2) = 0.131\text{ m} = 13.1\text{ cm}$

EVALUATE: The greater the flow rate into the bucket, the larger v_2 will be at equilibrium and the higher the water will rise in the bucket.

12.81. **IDENTIFY:** Apply Bernoulli's equation and the equation of continuity.
SET UP: The speed of efflux is $\sqrt{2gh}$, where h is the distance of the hole below the surface of the fluid.

EXECUTE: **(a)** $v_3 A_3 = \sqrt{2g(y_1 - y_3)}A_3 = \sqrt{2(9.80\text{ m/s}^2)(8.00\text{ m})}(0.0160\text{ m}^2) = 0.200\text{ m}^3/\text{s}.$

(b) Since p_3 is atmospheric pressure, the gauge pressure at point 2 is

$$p_2 = \frac{1}{2}\rho(v_3^2 - v_2^2) = \frac{1}{2}\rho v_3^2\left(1 - \left(\frac{A_3}{A_2}\right)^2\right) = \frac{8}{9}\rho g(y_1 - y_3),$$ using the expression for v_3 found above.

Substitution of numerical values gives $p_2 = 6.97\times10^4$ Pa.

EVALUATE: We could also calculate p_2 by applying Bernoulli's equation to points 1 and 2.

12.82 **IDENTIFY:** Apply Bernoulli's equation to the air in the hurricane.
SET UP: For a particle a distance r from the axis, the angular momentum is $L = mvr$.
EXECUTE: **(a)** Using the constancy of angular momentum, the product of the radius and speed is constant, so the speed at the rim is about $(200\text{ km/h})\left(\frac{30}{350}\right) = 17\text{ km/h}$.

(b) The pressure is lower at the eye, by an amount

$$\Delta p = \frac{1}{2}(1.2\text{ kg/m}^3)((200\text{ km/h})^2 - (17\text{ km/h})^2)\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)^2 = 1.8\times10^3\text{ Pa}.$$

(c) $\frac{v^2}{2g} = 160$ m.

(d) The pressure difference at higher altitudes is even greater.
EVALUATE: According to Bernoulli's equation, the pressure decreases when the fluid velocity increases.

12.83. **IDENTIFY:** Apply Bernoulli's equation and the equation of continuity.
SET UP: The speed of efflux at point D is $\sqrt{2gh_1}$.

EXECUTE: Applying the equation of continuity to points at C and D gives that the fluid speed is $\sqrt{8gh_1}$ at C. Applying Bernoulli's equation to points A and C gives that the gauge pressure at C is $\rho gh_1 - 4\rho gh_1 = -3\rho gh_1$, and this is the gauge pressure at the surface of the fluid at E. The height of the fluid in the column is $h_2 = 3h_1$.

EVALUATE: The gauge pressure at C is less than the gauge pressure ρgh_1 at the bottom of tank A because of the speed of the fluid at C.

12.84. **(a) IDENTIFY:** Apply constant acceleration equations to the falling liquid to find its speed as a function of the distance below the outlet. Then apply $v_1 A_1 = v_2 A_2$ to relate the speed to the radius of the stream.
SET UP:

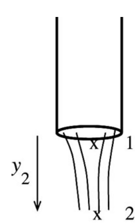


Figure 12.84

Let point 1 be at the end of the pipe and let point 2 be in the stream of liquid at a distance y_2 below the end of the tube, as shown in Figure 12.84.

Consider the free fall of the liquid. Take $+y$ to be downward.

Free fall implies $a_y = g$. v_y is positive, so replace it by the speed v.

EXECUTE: $v_2^2 = v_1^2 + 2a(y - y_0)$ gives $v_2^2 = v_1^2 + 2gy_2$ and $v_2 = \sqrt{v_1^2 + 2gy_2}$.

Equation of continuity says $v_1 A_1 = v_2 A_2$

And since $A = \pi r^2$ this becomes $v_1 \pi r_1^2 = v_2 \pi r_2^2$ and $v_2 = v_1 (r_1/r_2)^2$.

Use this in the above to eliminate v_2: $v_1 (r_1^2/r_2^2) = \sqrt{v_1^2 + 2gy_2}$

$r_2 = r_1 \sqrt{v_1}/(v_1^2 + 2gy_2)^{1/4}$

To correspond to the notation in the problem, let $v_1 = v_0$ and $r_1 = r_0$, since point 1 is where the liquid first leaves the pipe, and let r_2 be r and y_2 be y. The equation we have derived then becomes

$r = r_0 \sqrt{v_0}/(v_0^2 + 2gy)^{1/4}$.

(b) $v_0 = 1.20$ m/s

We want the value of y that gives $r = \frac{1}{2} r_0$, or $r_0 = 2r$.

The result obtained in part (a) says $r^4 (v_0^2 + 2gy) = r_0^4 v_0^2$.

Solving for y gives $y = \dfrac{[(r_0/r)^4 - 1]v_0^2}{2g} = \dfrac{(16-1)(1.20\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 1.10$ m.

EVALUATE: The equation derived in part (a) says that r decreases with distance below the end of the pipe.

12.85. **IDENTIFY** and **SET UP:** We are given the densities of elements in the table and look up their atomic masses in Appendix D.

EXECUTE: For example, for aluminum, the density is 2.7 g/cm^3 and the atomic mass is 26.98 g/mol.

(a) Figure 12.85 shows the graph of density versus atomic mass.

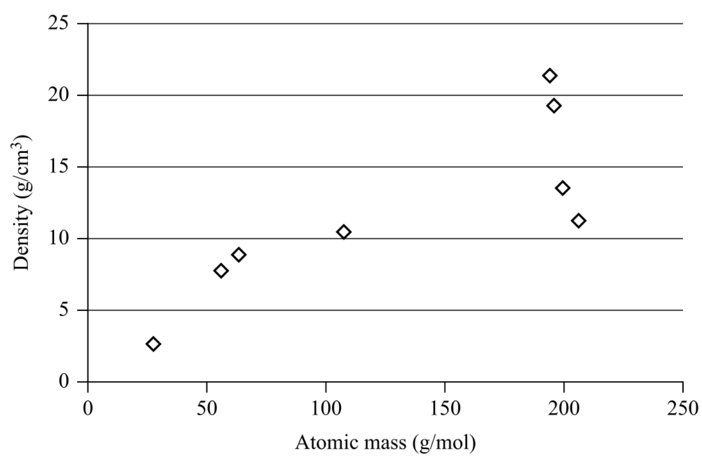


Figure 12.85

(b) From the graph, we see that there is no obvious mathematical relation between the two variables. No straight line or simple curve can be fitted to the data points.

(c) Density depends not only on atomic mass, but also on how tightly atoms are packed together. This packing is determined by the electrical interactions between atoms.

EVALUATE: Not all data can be reduced to straight-line graphs!

12.86. **IDENTIFY:** Archimedes's principle applies. For a floating object, the buoyant force balances the weight.

SET UP: Call m_b the mass of the block and m_n the mass of n quarters, so $m_n = nM$, where $M = 5.670$ g is the mass of a single quarter. $m_b + m_n$. The submerged volume is $(L - h)L^2$, where $L = 8.0$ cm.

EXECUTE: The forces balance, so $(m_b + m_n)g = \rho_{\text{liq}} L^2 (L - h)g$. Solving for h gives

$h = \dfrac{\rho_{\text{liq}}L^3 - m_b}{\rho_{\text{liq}}L^2} - \left(\dfrac{M}{\rho_{\text{liq}}L^2}\right)n$. If we plot h versus n, the slope is $-\dfrac{M}{\rho_{\text{liq}}L^2}$ and the y-intercept is $\dfrac{\rho_{\text{liq}}L^3 - m_b}{\rho_{\text{liq}}L^2}$. For the graph in the problem, we can calculate the slope by reading points from the graph. The top and bottom points seem easiest to read, so they give a slope of (1.2 cm – 3.0 cm)/25 = 0.072 cm. (Due to uncertainties in reading the graph, your answers may differ slightly from the ones here.) Using this slope, we get ρ_{liq} = –(5.670 g)/[(–0.072 cm)(8.0 cm)^2] = 1.24 g/cm^3 , which rounds to 1.2 g/cm^3 = 1200 kg/m^3.

(b) Simplifying the y-intercept gives y-intercept = $L - m_b/[(\rho_{\text{liq}})(L^2)]$. From the graph, the y-intercept is 3.0 cm, so we have 3.0 cm = 8.0 cm – m_b/[(1.24 g/cm^3)(8.0 cm)^2] , which gives m_b = 400 g = 0.40 kg.

EVALUATE: The unknown liquid is 20% denser than water.

12.87. IDENTIFY: Bernoulli's equation applies. We have free-fall projectile motion after the liquid leaves the tank. The pressure at the hole where the liquid exits is atmospheric pressure p_0. The absolute pressure at the top of the liquid is $p_g + p_0$.

SET UP: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$.

EXECUTE: **(a)** The graph of R^2 versus p_g is shown in Figure 12.87.

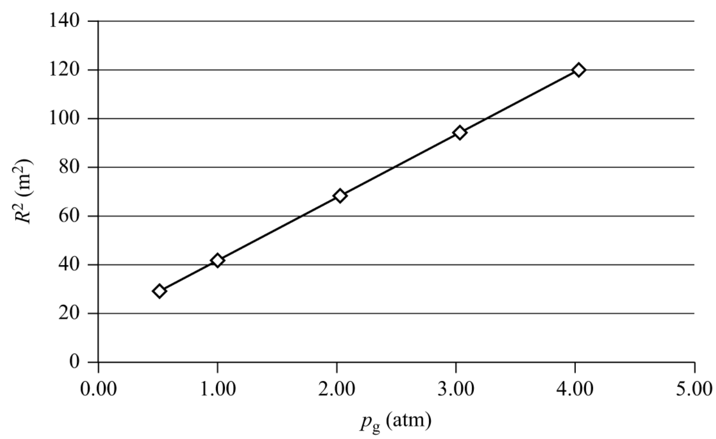


Figure 12.87

Applying Bernoulli's equation between the top and bottom of the liquid in the tank gives $p_0 + p_g + \rho g h = \frac{1}{2}\rho v^2 + p_0$, which simplifies to $p_g + \rho g h = \frac{1}{2}\rho v^2$.

The free-fall motion after leaving the tank gives $vt = R$ and $y = \frac{1}{2}gt^2$, where y = 50.0 cm. Eliminating t between these two equations gives $v^2 = (\rho g/4y)R^2$. Putting this into the result from Bernoulli's equation gives $p_g + \rho g h = (\rho g/4y)R^2$. Solving for R^2 in terms of h gives $R^2 = (4y/\rho g)\ p_g + 4yh$. This is the equation of a straight line of slope $4y/\rho g$, which gives $\rho = 4y/[g(\text{slope})]$ and y-intercept $4yh$. The best-fit equation is R^2 = (25.679 m^2/atm) p_g + 16.385 m^2. The y-intercept gives us h: $4yh$ = y-intercept, so h = (y-intercept)/(4y) = (16.385 m^2)/[4(0.500 m)] = 8.2 m. And the density is ρ = 4y/[g(slope)] = 4(0.500 m)/[(9.80 m/s^2)(25.679 m^2/atm)(1 atm/1.01 × 10^5 Pa)] = 803 kg/m^3.

EVALUATE: The liquid is about 80% as dense as water, and h = 8.2 m which is about 25 ft, so this is a rather large tank.

12.88. IDENTIFY: Apply Bernoulli's equation to the fluid in the siphon.
SET UP: The efflux speed from a small hole a distance h below the surface of fluid in a large open tank is $\sqrt{2gh}$.
EXECUTE: **(a)** The fact that the water first moves upward before leaving the siphon does not change the efflux speed, $\sqrt{2gh}$.
(b) Water will not flow if the absolute (not gauge) pressure would be negative. The hose is open to the atmosphere at the bottom, so the pressure at the top of the siphon is $p_a - \rho g(H+h)$, where the assumption that the cross-sectional area is constant has been used to equate the speed of the liquid at the top and bottom. Setting $p = 0$ and solving for H gives $H = (p_a/\rho g) - h$.
EVALUATE: The analysis shows that $H + h < \dfrac{p_a}{\rho g}$, so there is also a limitation on $H + h$. For water and normal atmospheric pressure, $\dfrac{p_a}{\rho g} = 10.3$ m.

12.89. IDENTIFY and **SET UP:** One atmosphere of pressure is 760 mm Hg. The gauge pressure is $p_g = \rho gh$.
EXECUTE: Since 1 atm is 760 mm Hg, the pressure is (150 mm/750 mm)P_{atm}. Solving for the depth h gives $h = \dfrac{P}{\rho g} = \dfrac{\left(\dfrac{150\text{ mm}}{760\text{ mm}}\right)(1.01\times 10^5\text{ Pa})}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 2.0$ m, which is choice (b).
EVALUATE: This result is reasonable since an elephant can have its chest several meters under water.

12.90. IDENTIFY and **SET UP:** Use the definition of pressure. $\Delta p = F/A$.
EXECUTE: $\Delta p = F/A$ = (24,000 N)/[π(0.60 m)^2] = 21,200 Pa. Converting to mm Hg gives 21,200 Pa [(760 mm Hg)/(1.01×10^5 Pa)] = 160 mm Hg, which is choice (a).
EVALUATE: The force (24,000 N) is large, but the pressure cannot be too large since the area of the diaphragm is quite large.

12.91. IDENTIFY and **SET UP:** The gauge pressure increases with depth, and since the force is proportional to the pressure, so does the force. $p_g = \rho gh$ and $p = F/A$.
EXECUTE: The force is $F = pA = (\rho gh)A$, which tells us that the force increases linearly with distance, as in choice (a).
EVALUATE: The diaphragm experiences greater force as the elephant goes into deeper water.

12.92. IDENTIFY and **SET UP:** The gauge pressure depends on the density of the liquid, $p_g = \rho gh$.
EXECUTE: Since $p_g = \rho gh$, a denser liquid will exert a greater pressure. Since salt water is denser than fresh water, the gauge pressure at a given depth will be greater in salt water than in freshwater. Therefore the maximum depth in salt water would be less than in freshwater, which is choice (b).
EVALUATE: Although the pressure in salt water would be greater than in freshwater, it would not be much greater since the density of seawater (1030 kg/m^3) is only slightly greater than that of freshwater (1000 kg/m^3).

13

GRAVITATION

VP13.5.1. **IDENTIFY:** This problem involves gravitational potential energy.

SET UP: The minimum amount of work is just the magnitude of the spacecraft's initial potential energy at the earth's surface. $U_{grav} = -Gm_1m_2/r$.

EXECUTE: $U_{grav} = -Gm_1m_2/r = -Gm_sm_E/R_E$. The magnitude of this energy is equal to the work, so

$$W = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(478\ \text{kg})\left(5.97\times10^{24}\ \text{kg}\right)}{6.37\times10^6\ \text{m}} = 2.99\times10^{10}\ \text{J}.$$

EVALUATE: Technically this is the energy given to the earth-satellite system, but it essentially all goes to the satellite because the earth is so massive that it does not change its kinetic energy.

VP13.5.2. **IDENTIFY:** This problem involves gravitational potential energy. Energy conservation applies.

SET UP: No air resistance, so $K_1 + U_1 = K_2 + U_2$, $U_{grav} = -Gm_1m_2/r$, $K = \frac{1}{2}mv^2$. Let m be the mass of the debris, R the radius of the earth, and m_E its mass. Call point 1 the original location of the debris and point 2 the surface of the earth.

EXECUTE: $K_1 = 0$, so $-Gmm_E/(3R/2) = -Gmm_E/R + \frac{1}{2}mv^2$, which gives $v = \sqrt{\frac{2Gm_E}{3R}}$. Using $R = 6.37\times10^6$ m and $m_E = 5.97\times10^{24}$ kg gives $v = 6.46\times10^3$ m/s.

EVALUATE: The equation $\sqrt{\frac{2Gm_E}{3R}}$ does not contain the mass of the satellite, so *any* object dropped from the same height would reach the earth's surface with the same speed.

VP13.5.3. **IDENTIFY:** This problem involves gravitational potential energy. Energy conservation applies.

SET UP: No air resistance, so $K_1 + U_1 = K_2 + U_2$, $U_{grav} = -Gm_1m_2/r$, $K = \frac{1}{2}mv^2$. Let m be the mass of the probe, R the radius of Mars, and M its mass. Call point 1 at the surface of Mars and point 2 at the maximum height of the probe. At that point, the probe's distance from the center of Mars is h and its speed is zero. Call v the speed at the surface.

EXECUTE: $K_1 + U_1 = K_2 + U_2$ gives $\frac{1}{2}mv^2 - \frac{GmM}{R} = -\frac{GmM}{h}$. Using $M = 6.42\times10^{23}$ kg, $R = 3.39\times10^6$ m, and $v = 3.00\times10^3$ m/s, we find $h = 5.27\times10^6$ m from the center of Mars. The height H above the surface is $H = h - R = 5.27\times10^6\ \text{m} - 3.39\times10^6\ \text{m} = 1.88\times10^6$ m.

EVALUATE: The answer is independent of the mass of the probe since m cancels from the equations, so *any* probe would reach that height if it started at 3000 m/s.

VP13.5.4. **IDENTIFY:** This problem involves gravitational potential energy. Energy conservation applies.

SET UP: No air resistance, so $K_1 + U_1 = K_2 + U_2$, $U_{grav} = -Gm_1m_2/r$, $K = \frac{1}{2}mv^2$. Let m be the mass of the spacecraft, R the radius of the earth and M its mass. Call point 1 at the surface of the earth and point 2 at the height of the spacecraft when it has reached the desired speed. Call v_1 its initial speed at the surface and v_2 the desired speed when it has reached the desired height r from the earth's center. We want to find v_1.

EXECUTE: Energy conservation gives $\frac{1}{2}mv_1^2 - \frac{GmM}{R} = \frac{1}{2}mv_2^2 - \frac{GmM}{r}$. Using $v_2 = 8.50\times10^3$ m/s, $r = 2.50\times10^8$ m, $M = 5.97\times10^{24}$ kg, and $R = 6.37\times10^6$ m, we get $v_1 = 1.39\times10^4$ m/s = 13.9 km/s.

EVALUATE: Any spacecraft, regardless of mass, would need the same launch speed since the mass m cancels from the equations.

VP13.6.1. **IDENTIFY:** This problem deals with satellite orbits. The force of gravity applies, as well as Newton's second law for circular motion.

SET UP: $F = Gm_1m_2/r^2$. $\sum F = m\frac{v^2}{r}$ for circular motion. Let m be the satellite mass, v its speed, and M the earth's mass. $M = 5.97\times10^{24}$ kg and $v = 4.00\times10^3$ m/s.

EXECUTE: **(a)** $\sum F = m\frac{v^2}{r}$ gives $\frac{GmM}{r^2} = \frac{mv^2}{r} \rightarrow r = \frac{GM}{v^2} = 2.49\times10^7$ m.

(b) The height H above the surface is $H = r - R = 2.49\times10^7\text{ m} - 6.37\times10^6\text{ m} = 1.85\times10^7$ m.

(c) $v = 2\pi r/T$, so $T = 2\pi r/v = 2\pi(2.49\times10^7\text{ m})/(4.00\times10^3\text{ m/s}) = 3.91\times10^4\text{ s} = 10.9$ h.

EVALUATE: The results are independent of the mass of the satellite.

VP13.6.2. **IDENTIFY:** This problem deals with satellite orbits. The force of gravity applies, as well as Newton's second law for circular motion.

SET UP: $F = Gm_1m_2/r^2$. $\sum F = m\frac{v^2}{r}$ for circular motion. Let m be the satellite mass, v its speed, T its orbital period, and M the mass of Mars.

EXECUTE: **(a)** We want the radius r of the orbit. $\sum F = m\frac{v^2}{r}$ gives $\frac{GmM}{r^2} = \frac{m\left(\frac{2\pi r}{T}\right)}{r}$. Solving for r gives $r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$. Using $M = 6.42\times10^{23}$ kg and $T = 24.66\text{ h} = 8.8776\times10^4$ s gives $r = 2.04\times10^7$ m.

(b) Using the $r = 2.04\times10^7$ m and $R = 3.39\times10^6$ m gives $v = 2\pi r/T = 1.45\times10^3$ m/s.

EVALUATE: The orbital radius does not depend on the mass of the satellite, but it *does* depend on the mass of the planet. On the earth the radius would be larger since $r \propto M^{1/3}$.

VP13.6.3. **IDENTIFY:** This problem deals with orbits around the sun and involves gravitational potential energy and kinetic energy. Newton's second law also applies.

SET UP: $U_{grav} = -Gm_1m_2/r$, $K = \frac{1}{2}mv^2$. Let m be the mass of the spacecraft, M the mass of the sun, r_1 the radius of the outer orbit and r_2 that of the inner orbit. Apply $\sum F = m\frac{v^2}{r}$ to the spacecraft.

EXECUTE: **(a)** First find the kinetic energy in both orbits. $\sum F = m\frac{v^2}{r}$ to the spacecraft gives

$\dfrac{GmM}{r^2} = \dfrac{mv^2}{r} = \dfrac{2}{r}\left(\dfrac{1}{2}mv^2\right) = \dfrac{2K}{r}$, so $K = \dfrac{GmM}{2r}$. Therefore change in kinetic energy is

$\Delta K = K_2 - K_1 = \dfrac{GmM}{2}\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right)$. Using m = 1000 kg, $M = 1.99\times10^{30}$ kg, $r_1 = 1.50\times10^{11}$ m, and $r_2 = 1.08\times10^{11}$ m, we get $\Delta K = 1.72\times10^{11}$ J.

(b) $U_2 - U_1 = -\dfrac{GmM}{r_2} - \left(-\dfrac{GmM}{r_1}\right) = GmM\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right) = -3.44\times10^{11}$ J.

(c) $E = K + U$, so $\Delta E = \Delta K + \Delta U = 1.72\times10^{11}\ \text{J} + (-3.44\times10^{11}\ \text{J}) = -1.72\times10^{11}$ J.

EVALUATE: The spacecraft lost twice as much potential energy as it gained in kinetic energy, so it had a net loss equal to the kinetic energy gain.

VP13.6.4 **IDENTIFY:** This problem deals with orbits around the sun and involves gravitational potential energy and kinetic energy. Newton's second law also applies.

SET UP: $U_{\text{grav}} = -Gm_1m_2/r$, $K = \dfrac{1}{2}mv^2$. Let m be the mass of the spacecraft and M the mass of the earth. Apply $\sum F = m\dfrac{v^2}{r}$ to the spacecraft.

EXECUTE: **(a)** The work you would need to do is equal to the change in energy of the satellite. The potential energy does not change because r is the same, so the change in energy is the change in the kinetic energy of the satellite. Apply $\sum F = m\dfrac{v^2}{r}$ for a circular orbit.

$6.37\times10^6\ \text{m} = \dfrac{2}{r}\left(\dfrac{1}{2}mv^2\right) = \dfrac{2K}{r}$, which gives $K = \dfrac{GmM}{2r}$.

$\Delta K = K_2 - K_1 = \dfrac{GmM}{2r} - \dfrac{1}{2}mv_1^2$, where v_1 is the initial speed. Using m = 1500 kg, $M = 5.97\times10^{24}$ kg, v_1 = 7500 m/s, and $r = 3.50\times10^6\ \text{m} + 6.37\times10^6\ \text{m} = 9.87\times10^6$ m, we find that $\Delta E = \Delta K = -1.19\times10^{10}$ J.

(b) The minimum energy you would have to give the satellite for it to escape earth would be its total energy, which is $E_1 = K_1 + U_1 = \dfrac{1}{2}mv_1^2 - \dfrac{GmM}{r}$. Using the numbers from part (a), this energy is $E_1 = -1.83\times10^{10}$ J. The work you would have to would be $+1.83\times10^{10}$ J.

EVALUATE: Orbital parameters such as period, speed, and radius do not depend on the mass of the satellite, but energies do depend on that mass.

VP13.9.1. **IDENTIFY:** We are dealing with a comet in noncircular orbit.

SET UP: The semi-major axis a is $2a = r_p + r_a$, where r_p is the perihelion distance and r_a is the aphelion distance. $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, where m_S is the mass of the sun (or central star for another solar system). $a = ea + r_p$, where e is the eccentricity of the orbit. Refer to Fig. 13.18 in the textbook.

EXECUTE: **(a)** We want a, so we use $2a = = r_p + r_a = 6.00\times10^9\ \text{m} + 3.00\times10^{12}$ m, so $a = 1.50\times10^{12}$ m.

(b) $a = ea + r_p$, so $e = (a - r_{-p})/a = 1 - r_p/a = 1 - (6.00\times10^9\ \text{m})/(1.50\times10^{12}\ \text{m}) = 0.996$.

(c) Use $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ with a from (a) and $m_S = 1.99\times10^{30}$ kg gives $T = 1.00\times10^9$ s = 31.8 y.

EVALUATE: This orbital period is roughly the same as that of Saturn.

VP13.9.2. **IDENTIFY:** We are dealing with an asteroid in a noncircular orbit.

SET UP: The semi-major axis a is $2a = r_p + r_a$, where r_p is the perihelion distance and r_a is the aphelion distance. $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, where m_S is the mass of the sun (or central star for another solar system). $a = ea + r_p$, where e is the eccentricity of the orbit. Refer to Fig. 13.18 in the textbook. $T = 7.85$ y $= 2.4775\times10^8$ s, and $e = 0.250$.

EXECUTE: **(a)** Use $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ to find a. Solve for a: $a = \left[\dfrac{T^2 Gm_S}{4\pi^2}\right]^{1/3}$. Using T from above and $m_S = 1.99\times10^{30}$ kg gives $a = 5.91\times10^{11}$ m.

(b) $a = ea + r_p \rightarrow r_p = a(1-e) = (5.91\times10^{11}\text{ m})(1-0.250) = 4.43\times10^{11}$ m.

(c) $r_a = 2a - r_p = 2(5.91\times10^{11}\text{ m}) - 4.43\times10^{11}\text{ m} = 7.39\times10^{11}$ m.

EVALUATE: This orbit is considerably more eccentric than those of the major planets. Its aphelion distance is about 1.7 times greater than its perihelion distance.

VP13.9.3. **IDENTIFY:** We are dealing with a satellite in a noncircular orbit.

SET UP: The semi-major axis a is $2a = r_p + r_a$, where r_p is the perihelion distance and r_a is the aphelion distance. $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, where m_S is the mass of the sun (or central star for another solar system). $a = ea + r_p$, where e is the eccentricity of the orbit. Refer to Fig. 13.18 in the textbook. The minor planet makes 2 orbits in the time it takes Neptune to make 3 orbits, so the minor planet must have a longer period than Neptune: $T_{mp} = 3/2\, T_N$, and its eccentricity is $e = 0.330$.

EXECUTE: **(a)** We want to find the semi-major axis a. If we take the ratio of the period of Neptune and minor planet, the factors in common will cancel.

$$\frac{T_{mp}}{T_N} = \frac{\dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}}{\dfrac{2\pi r_N^{3/2}}{\sqrt{Gm_S}}} = \left(\frac{a}{r_N}\right)^{3/2} = \frac{3}{2} \quad \rightarrow \quad (a/r_N)^{3/2} = 3/2 \quad \rightarrow \quad a = r_N(3/2)^{2/3}$$

$a = (4.50\times10^{12}\text{ m})(3/2)^{2/3} = 5.90\times10^{12}$ m.

(b) $a = ea + r_p \rightarrow r_p = a(1-e) = (5.90\times10^{12}\text{ m})(1-0.330) = 3.95\times10^{12}$ m.

EVALUATE: At perihelion, this comet is *inside* Neptune's orbit. The eccentricity of its orbit is much greater than that of Neptune's orbit.

VP13.9.4. **IDENTIFY:** We are dealing with a planet in a noncircular orbit around another star.

SET UP: The semi-major axis a is $2a = r_p + r_a$, where r_p is the perihelion distance and r_a is the aphelion distance. $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, where m_S is the mass of the central star. $a = ea + r_p$, where e is the eccentricity of the orbit. In this case, $T = 4.39$ d $= 3.7930\times10^5$ s. Refer to Fig. 13.18 in the textbook.

EXECUTE: **(a)** We want the distance of the planet from its star at perigee (point of closest approach). $a = ea + r_p \rightarrow r_p = a(1-e) = (7.41\times10^9\text{ m})(1-0.173) = 6.13\times10^9$ m.

(b) We want the mass of HATS-43, so we use $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$. Solving for m_S gives $m_S = \dfrac{4\pi^2 a^3}{GT^2}$. Using $a = 7.41\times10^9$ m and $T = 3.793\times10^5$ s, we get $m_S = 1.67\times10^{30}$ kg.

EVALUATE: This planet is considerably closer to its star than Mercury is to our sun. Comparing the mass of this star to that of our sun gives $m_{HATS}/m_{sun} = 1.67/1.99 = 0.839$, so the mass of this star is about 84% the mass of our sun.

13.1. **IDENTIFY** and **SET UP:** Use the law of gravitation, $F_g = \dfrac{Gm_1m_2}{r^2}$, to determine F_g.

EXECUTE: $F_{\text{S on M}} = G\dfrac{m_\text{S} m_\text{M}}{r_\text{SM}^2}$ (S = sun, M = moon); $F_{\text{E on M}} = G\dfrac{m_\text{E} m_\text{M}}{r_\text{EM}^2}$ (E = earth)

$$\frac{F_{\text{S on M}}}{F_{\text{E on M}}} = \left(G\frac{m_\text{S} m_\text{M}}{r_\text{SM}^2}\right)\left(\frac{r_\text{EM}^2}{G m_\text{E} m_\text{M}}\right) = \frac{m_\text{S}}{m_\text{E}}\left(\frac{r_\text{EM}}{r_\text{SM}}\right)^2$$

r_EM, the radius of the moon's orbit around the earth is given in Appendix F as 3.84×10^8 m. The moon is much closer to the earth than it is to the sun, so take the distance r_SM of the moon from the sun to be r_SE, the radius of the earth's orbit around the sun.

$$\frac{F_{\text{S on M}}}{F_{\text{E on M}}} = \left(\frac{1.99\times10^{30}\text{ kg}}{5.98\times10^{24}\text{ kg}}\right)\left(\frac{3.84\times10^8\text{ m}}{1.50\times10^{11}\text{ m}}\right)^2 = 2.18.$$

EVALUATE: The force exerted by the sun is larger than the force exerted by the earth. The moon's motion is a combination of orbiting the sun and orbiting the earth.

13.2. **IDENTIFY:** We want to calculate the gravitational force between two persons.

SET UP: Estimates: Mass of instructor is 75 kg, your mass is 70 kg, distance between the two of you is 3.0 m. Approximation: Treat both of you as point-masses. Use $F_\text{g} = \dfrac{Gm_1m_2}{r^2}$. We want to approximate the gravitational force that your physics instructor exerts on you.

EXECUTE: $F_\text{instr} = Gm_1m_2/r^2 = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(75\text{ kg})(70\text{ kg})/(3.0\text{ m})^2 = 3.9\times10^{-8}$ N.

$F_\text{earth} = mg = (70\text{ kg})(9.80\text{ m/s}^2) = 690$ N.

EVALUATE: $F_\text{instr}/F_\text{earth} = (3.9\times10^{-8}\text{ N})/(690\text{ N}) = 5.7\times10^{-11}$ so F_instr is insignificant compared to the force the earth exerts on you.

13.3. **IDENTIFY:** The gravitational attraction of the astronauts on each other causes them to accelerate toward each other, so Newton's second law of motion applies to their motion.

SET UP: The net force on each astronaut is the gravity force exerted by the other astronaut. Call the astronauts *A* and *B*, where $m_A = 65$ kg and $m_B = 72$ kg. $F_\text{g} = Gm_1m_2/r^2$ and $\Sigma F = ma$.

EXECUTE: **(a)** The free-body diagram for astronaut *A* is given in Figure 13.3(a) and for astronaut *B* in Figure 13.3(b) (next page).

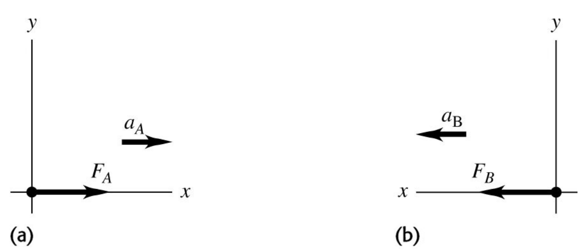


Figure 13.3

$\Sigma F_x = ma_x$ for *A* gives $F_A = m_A a_A$ and $a_A = \dfrac{F_A}{m_A}$. And for *B*, $a_B = \dfrac{F_B}{m_B}$.

$$F_A = F_B = G\frac{m_A m_B}{r^2} = (6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(65\text{ kg})(72\text{ kg})}{(20.0\text{ m})^2} = 7.807\times10^{-10}\text{ N so}$$

$$a_A = \frac{7.807\times10^{-10}\text{ N}}{65\text{ kg}} = 1.2\times10^{-11}\text{ m/s}^2 \text{ and } a_B = \frac{7.807\times10^{-10}\text{ N}}{72\text{ kg}} = 1.1\times10^{-11}\text{ m/s}^2.$$

(b) Using constant-acceleration kinematics, we have $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, which gives $x_A = \frac{1}{2}a_A t^2$ and $x_B = \frac{1}{2}a_B t^2$. $x_A + x_B = 20.0\text{ m}$, so $20.0\text{ m} = \frac{1}{2}(a_A + a_B)t^2$ and

$$t = \sqrt{\frac{2(20.0\text{ m})}{1.2\times10^{-11}\text{ m/s}^2 + 1.1\times10^{-11}\text{ m/s}^2}} = 1.32\times10^6\text{ s} = 15\text{ days.}$$

(c) Their accelerations would increase as they moved closer and the gravitational attraction between them increased.

EVALUATE: Even though the gravitational attraction of the astronauts is much weaker than ordinary forces on earth, if it were the only force acting on the astronauts, it would produce noticeable effects.

13.4. IDENTIFY: Apply $F_g = \dfrac{Gm_1m_2}{r^2}$, generalized to any pair of spherically symmetric objects.

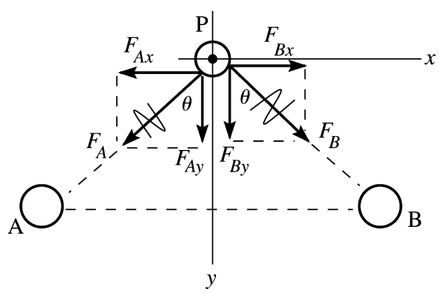
SET UP: The separation of the centers of the spheres is $2R$.

EXECUTE: The magnitude of the gravitational attraction is $GM^2/(2R)^2 = GM^2/4R^2$.

EVALUATE: The formula $F_g = \dfrac{Gm_1m_2}{r^2}$ applies to any pair of spherically symmetric objects; one of the objects doesn't have to be the earth.

13.5. IDENTIFY: Use $F_g = \dfrac{Gm_1m_2}{r^2}$ to find the force exerted by each large sphere. Add these forces as vectors to get the net force and then use Newton's second law to calculate the acceleration.

SET UP: The forces are shown in Figure 13.5.



$\sin\theta = 0.80$
$\cos\theta = 0.60$
Take the origin of coordinate at point P.

Figure 13.5

EXECUTE: $F_A = G\dfrac{m_Am}{r^2} = G\dfrac{(0.26\text{ kg})(0.010\text{ kg})}{(0.100\text{ m})^2} = 1.735\times10^{-11}\text{ N}$

$F_B = G\dfrac{m_Bm}{r^2} = 1.735\times10^{-11}\text{ N}$

$F_{Ax} = -F_A\sin\theta = -(1.735\times10^{-11}\text{ N})(0.80) = -1.39\times10^{-11}\text{ N}$

$F_{Ay} = +F_A\cos\theta = +(1.735\times10^{-11}\text{ N})(0.60) = +1.04\times10^{-11}\text{ N}$

$F_{Bx} = +F_B\sin\theta = +1.39\times10^{-11}\text{ N}$

$F_{By} = +F_B\cos\theta = +1.04\times10^{-11}\text{ N}$

$\Sigma F_x = ma_x$ gives $F_{Ax} + F_{Bx} = ma_x$

$0 = ma_x$ so $a_x = 0$

$\Sigma F_y = ma_y$ gives $F_{Ay} + F_{By} = ma_y$

$2(1.04\times10^{-11}\text{ N}) = (0.010\text{ kg})a_y$

$a_y = 2.1\times10^{-9}\text{ m/s}^2$, directed downward midway between A and B

EVALUATE: For ordinary size objects the gravitational force is very small, so the initial acceleration is very small. By symmetry there is no x-component of net force and the y-component is in the direction of the two large spheres, since they attract the small sphere.

13.6. **IDENTIFY:** The net force on A is the vector sum of the force due to B and the force due to C. In part (a), the two forces are in the same direction, but in (b) they are in opposite directions.
SET UP: Use coordinates where $+x$ is to the right. Each gravitational force is attractive, so is toward the mass exerting it. Treat the masses as uniform spheres, so the gravitational force is the same as for point masses with the same center-to-center distances. The free-body diagrams for (a) and (b) are given in Figures 13.6a and 13.6b. The gravitational force is $F_g = Gm_1m_2/r^2$.

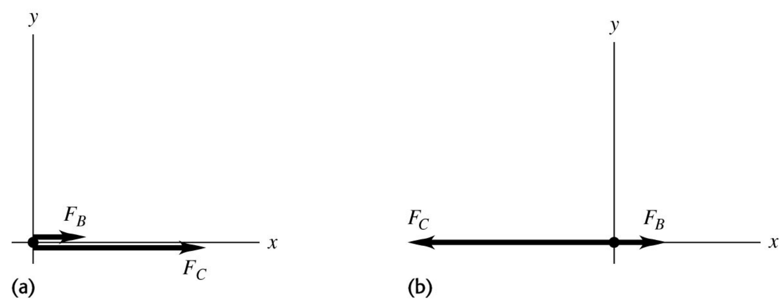


Figure 13.6

EXECUTE: **(a)** Calling F_B the force due to mass B and likewise for C, we have

$$F_B = G\frac{m_A m_B}{r_{AB}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\ \text{kg})^2}{(0.50\ \text{m})^2} = 1.069\times10^{-9}\ \text{N} \text{ and}$$

$$F_C = G\frac{m_A m_C}{r_{AC}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\ \text{kg})^2}{(0.10\ \text{m})^2} = 2.669\times10^{-8}\ \text{N}.$$ The net force is

$$F_{\text{net},x} = F_{Bx} + F_{Cx} = 1.069\times10^{-9}\ \text{N} + 2.669\times10^{-8}\ \text{N} = 2.8\times10^{-8}\ \text{N} \text{ to the right.}$$

(b) Following the same procedure as in (a), we have

$$F_B = G\frac{m_A m_B}{r_{AB}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\ \text{kg})^2}{(0.40\ \text{m})^2} = 1.668\times10^{-9}\ \text{N}$$

$$F_C = G\frac{m_A m_C}{r_{AC}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\ \text{kg})^2}{(0.10\ \text{m})^2} = 2.669\times10^{-8}\ \text{N}$$

$$F_{\text{net},x} = F_{Bx} + F_{Cx} = 1.668\times10^{-9}\ \text{N} - 2.669\times10^{-8}\ \text{N} = -2.5\times10^{-8}\ \text{N}$$

The net force on A is 2.5×10^{-8} N, to the left.

EVALUATE: As with any force, the gravitational force is a vector and must be treated like all other vectors. The formula $F_g = Gm_1m_2/r^2$ only gives the magnitude of this force.

13.7. **IDENTIFY:** The force exerted by the moon is the gravitational force, $F_g = \dfrac{Gm_M m}{r^2}$. The force exerted on the person by the earth is $w = mg$.

SET UP: The mass of the moon is $m_M = 7.35\times10^{22}$ kg. $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** $F_{\text{moon}} = F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(7.35\times10^{22}\ \text{kg})(70\ \text{kg})}{(3.78\times10^8\ \text{m})^2} = 2.4\times10^{-3}\ \text{N}.$

(b) $F_{\text{earth}} = w = (70\ \text{kg})(9.80\ \text{m/s}^2) = 690\ \text{N}.$ $F_{\text{moon}}/F_{\text{earth}} = 3.5\times10^{-6}.$

EVALUATE: The force exerted by the earth is much greater than the force exerted by the moon. The mass of the moon is less than the mass of the earth and the center of the earth is much closer to the person than is the center of the moon.

13.8. IDENTIFY: Use $F_g = Gm_1m_2/r^2$ to find the force each point mass exerts on the particle, find the net force, and use Newton's second law to calculate the acceleration.

SET UP: Each force is attractive. The particle (mass m) is a distance $r_1 = 0.200$ m from $m_1 = 8.00$ kg and therefore a distance $r_2 = 0.300$ m from $m_2 = 12.0$ kg. Let $+x$ be toward the 12.0-kg mass.

EXECUTE: $F_1 = \dfrac{Gm_1m}{r_1^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(8.00\ \text{kg})m}{(0.200\ \text{m})^2} = (1.334\times10^{-8}\ \text{N/kg})m,$ in the $-x$-direction. $F_2 = \dfrac{Gm_2m}{r_2^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(12.0\ \text{kg})m}{(0.300\ \text{m})^2} = (8.893\times10^{-9}\ \text{N/kg})m,$ in the $+x$-direction. The net force is

$F_x = F_{1x} + F_{2x} = (-1.334\times10^{-8}\ \text{N/kg} + 8.893\times10^{-9}\ \text{N/kg})m = (-4.447\times10^{-9}\ \text{N/kg})m.$

$a_x = \dfrac{F_x}{m} = -4.45\times10^{-9}\ \text{m/s}^2$. The acceleration is $4.45\times10^{-9}\ \text{m/s}^2$, toward the 8.00 kg mass.

EVALUATE: The smaller mass exerts the greater force, because the particle is closer to the smaller mass.

13.9. IDENTIFY: Use $F_g = Gm_1m_2/r^2$ to calculate the gravitational force each particle exerts on the third mass. The equilibrium is stable when for a displacement from equilibrium the net force is directed toward the equilibrium position and it is unstable when the net force is directed away from the equilibrium position.

SET UP: For the net force to be zero, the two forces on M must be in opposite directions. This is the case only when M is on the line connecting the two particles and between them. The free-body diagram for M is given in Figure 13.9. $m_1 = 3m$ and $m_2 = m$. If M is a distance x from m_1, it is a distance $1.00\ \text{m} - x$ from m_2.

EXECUTE: **(a)** $F_x = F_{1x} + F_{2x} = -G\dfrac{3mM}{x^2} + G\dfrac{mM}{(1.00\ \text{m} - x)^2} = 0$. Cancelling and simplifying gives $3(1.00\ \text{m} - x)^2 = x^2$. Taking square roots gives $1.00\ \text{m} - x = \pm x/\sqrt{3}$. Since M is between the two particles, x must be less than 1.00 m and $x = \dfrac{1.00\ \text{m}}{1+1/\sqrt{3}} = 0.634$ m. M must be placed at a point that is 0.634 m from the particle of mass $3m$ and 0.366 m from the particle of mass m.

(b) (i) If M is displaced slightly to the right in Figure 13.9, the attractive force from m is larger than the force from $3m$ and the net force is to the right. If M is displaced slightly to the left in Figure 13.9, the attractive force from $3m$ is larger than the force from m and the net force is to the left. In each case the net force is away from equilibrium and the equilibrium is unstable.

(ii) If M is displaced a very small distance along the y-axis in Figure 13.9, the net force is directed opposite to the direction of the displacement and therefore the equilibrium is stable.

EVALUATE: The point where the net force on M is zero is closer to the smaller mass.

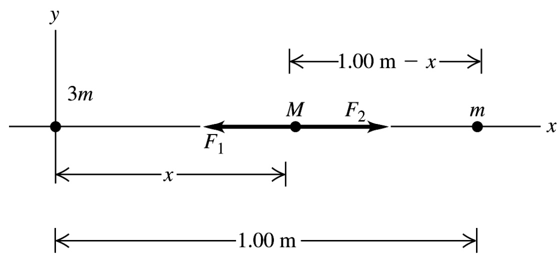


Figure 13.9

13.10. **IDENTIFY:** The force $\vec{F}_1$ exerted by m on M and the force $\vec{F}_2$ exerted by $2m$ on M are each given by $F_g = Gm_1m_2/r^2$ and the net force is the vector sum of these two forces.

SET UP: Each force is attractive. The forces on M in each region are sketched in Figure 13.10a. Let M be at coordinate x on the x-axis.

EXECUTE: **(a)** For the net force to be zero, $\vec{F}_1$ and $\vec{F}_2$ must be in opposite directions and this is the case only for $0 < x < L$. $\vec{F}_1 + \vec{F}_2 = 0$ then requires $F_1 = F_2$. $\dfrac{GmM}{x^2} = \dfrac{G(2m)M}{(L-x)^2}$. $2x^2 = (L-x)^2$ and $L - x = \pm\sqrt{2}x$. x must be less than L, so $x = \dfrac{L}{1+\sqrt{2}} = 0.414L$.

(b) For $x < 0$, $F_x > 0$. $F_x \to 0$ as $x \to -\infty$ and $F_x \to +\infty$ as $x \to 0$. For $x > L$, $F_x < 0$. as $x \to \infty$ and $F_x \to -\infty$ as $x \to L$. For $0 < x < 0.414L$, $F_x < 0$ and F_x increases from $-\infty$ to 0 as x goes from 0 to $0.414L$. For $0.414L < x < L$, $F_x > 0$ and F_x increases from 0 to $+\infty$ as x goes from $0.414L$ to L. The graph of F_x versus x is sketched in Figure 13.10b (next page).

EVALUATE: Any real object is not exactly a point so it is not possible to have both m and M exactly at $x = 0$ or $2m$ and M both exactly at $x = L$. But the magnitude of the gravitational force between two objects approaches infinity as the objects get very close together.

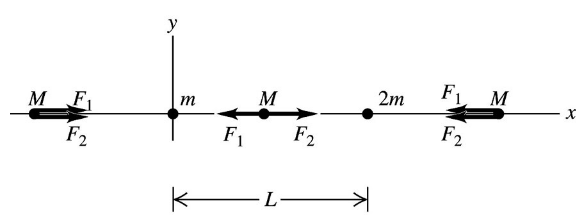


Figure 13.10a

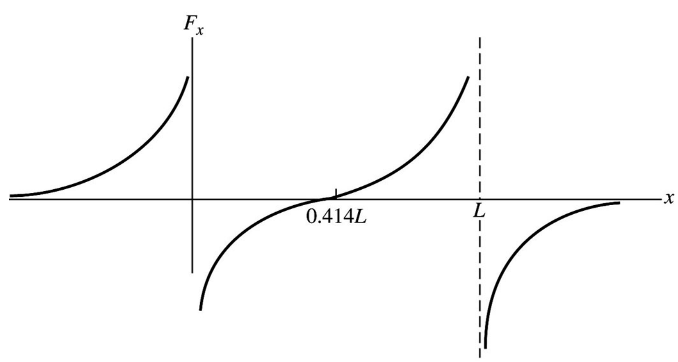


Figure 13.10b

13.11. IDENTIFY: $F_g = G\dfrac{mm_E}{r^2} = mg$, so ssc where r is the distance of the object from the center of the earth.

SET UP: $r = h + R_E$, where h is the distance of the object above the surface of the earth and $R_E = 6.37\times10^6$ m is the radius of the earth.

EXECUTE: To decrease the acceleration due to gravity by one-tenth, the distance from the center of the earth must be increased by a factor of $\sqrt{10}$, and so the distance above the surface of the earth is $(\sqrt{10}-1)R_E = 1.38\times10^7$ m.

EVALUATE: This height is about twice the radius of the earth.

13.12. IDENTIFY: Apply $g = G\dfrac{m_E}{r^2}$ to the earth and to Venus. $w = mg$.

SET UP: $g = \dfrac{Gm_E}{R_E^2} = 9.80\text{ m/s}^2$. $m_V = 0.815m_E$ and $R_V = 0.949R_E$. $w_E = mg_E = 75.0$ N.

EXECUTE: **(a)** $g_V = \dfrac{Gm_V}{R_V^2} = \dfrac{G(0.815m_E)}{(0.949R_E)^2} = 0.905\dfrac{Gm_E}{R_E^2} = 0.905g_E$.

(b) $w_V = mg_V = 0.905mg_E = (0.905)(75.0\text{ N}) = 67.9$ N.

EVALUATE: The mass of the rock is independent of its location but its weight equals the gravitational force on it and that depends on its location.

13.13. (a) IDENTIFY and **SET UP:** Apply $g = G\dfrac{m_E}{r^2}$ to the earth and to Titania. The acceleration due to gravity at the surface of Titania is given by $g_T = Gm_T/R_T^2$, where m_T is its mass and R_T is its radius. For the earth, $g_E = Gm_E/R_E^2$.

EXECUTE: For Titania, $m_T = m_E/1700$ and $R_T = R_E/8$, so

$$g_T = \frac{Gm_T}{R_T^2} = \frac{G(m_E/1700)}{(R_E/8)^2} = \left(\frac{64}{1700}\right)\frac{Gm_E}{R_E^2} = 0.0377g_E.$$

Since $g_E = 9.80\text{ m/s}^2$, $g_T = (0.0377)(9.80\text{ m/s}^2) = 0.37\text{ m/s}^2$.

EVALUATE: g on Titania is much smaller than on earth. The smaller mass reduces g and is a greater effect than the smaller radius, which increases g.

(b) IDENTIFY and **SET UP:** Use density = mass/volume. Assume Titania is a sphere.

EXECUTE: From Section 13.2 we know that the average density of the earth is 5500 kg/m³. For Titania

$$\rho_T = \frac{m_T}{\frac{4}{3}\pi R_T^3} = \frac{m_E/1700}{\frac{4}{3}\pi(R_E/8)^3} = \frac{512}{1700}\rho_E = \frac{512}{1700}(5500\text{ kg/m}^3) = 1700\text{ kg/m}^3.$$

EVALUATE: The average density of Titania is about a factor of 3 smaller than for earth. We can write $a_g = G\dfrac{m_E}{r^2}$ for Titania as $g_T = \frac{4}{3}\pi GR_T\rho_T$. $g_T < g_E$ both because $\rho_T < \rho_E$ and $R_T < R_E$.

13.14. IDENTIFY: Apply $g = G\dfrac{m_E}{r^2}$ to Rhea.

SET UP: $\rho = m/V$. The volume of a sphere is $V = \frac{4}{3}\pi R^3$.

EXECUTE: $M = \dfrac{gR^2}{G} = 2.32\times10^{21}$ kg and $\rho = \dfrac{M}{(4\pi/3)R^3} = 1.24\times10^3\text{ kg/m}^3$.

EVALUATE: The average density of Rhea is a bit less than one-fourth that of the earth.

13.15. IDENTIFY: We are dealing with the acceleration due to gravity on another planet.

SET UP: We want to find g at the surface of a planet. At the surface, $g = GM/R^2$. Since we know the density ρ of the planet, we should put g in terms of ρ using $\rho = m/V$.

EXECUTE: (a) We want g at the surface of this planet. Using $m = \rho V = \rho(4/3\ \pi R^3)$, we have

$g = \dfrac{Gm}{R^2} = \dfrac{G\rho\left(\frac{4}{3}\pi R^3\right)}{R^2} = \dfrac{4}{3}\pi G\rho R.$ Now take the ratio of $g_{\text{planet}}/g_{\text{earth}}$ using the same density for both planets but $R = 1.25R_{\text{earth}}$. We get $\dfrac{g_{\text{p}}}{g_{\text{e}}} = \dfrac{\frac{4}{3}\pi G\rho R_{\text{p}}}{\frac{4}{3}\pi G\rho R_{\text{e}}} = \dfrac{R_{\text{p}}}{R_{\text{e}}} = \dfrac{1.25R_{\text{e}}}{R_{\text{e}}} = 1.25$, from which we get $g_{\text{p}} = 1.25$ $g_{\text{e}} = (1.25)(9.80\ \text{m/s}^2) = 12.3\ \text{m/s}^2$.

(b) We now want to change the density of this planet (a neat trick if you can do it!) so that g at its surface is the same as on earth. Use the result we found in part (a) for g: $g = \frac{4}{3}\pi G\rho R.$ Since $g_{\text{p}} = g_{\text{e}}$, we equate the two expressions for g, giving $\frac{4}{3}\pi G\rho_{\text{p}}R_{\text{p}} = \frac{4}{3}\pi G\rho_{\text{e}}R_{\text{e}}$. Solving for ρ_{p} gives

$\rho_{\text{p}} = \dfrac{R_{\text{e}}}{R_{\text{p}}}\rho_{\text{e}} = \dfrac{R_{\text{e}}}{1.25R_{\text{e}}}\rho_{\text{e}} = 0.800\rho_{\text{e}}.$ The planet should have 80.0% the density of earth.

EVALUATE: The planet is less dense than earth but it has more mass since it is larger, so g at its surface is the same as it is on earth.

13.16. IDENTIFY: The gravity of Io limits the height to which volcanic material will rise. The acceleration due to gravity at the surface of Io depends on its mass and radius.

SET UP: The radius of Io is $R = 1.821\times10^6$ m. Use coordinates where $+y$ is upward. At the maximum height, $v_{0y} = 0$, $a_y = -g_{\text{Io}}$, which is assumed to be constant. Therefore the constant-acceleration kinematics formulas apply. The acceleration due to gravity at Io's surface is given by $g_{\text{Io}} = Gm/R^2$.

EXECUTE: At the surface of Io, $g_{\text{Io}} = \dfrac{Gm}{R^2} = \dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(8.93\times10^{22}\ \text{kg})}{(1.821\times10^6\ \text{m})^2} = 1.797\ \text{m/s}^2.$

For constant acceleration (assumed), the equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ applies, so

$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-1.797\ \text{m/s}^2)(5.00\times10^5\ \text{m})} = 1.3405\times10^3\ \text{m/s}.$ Now solve for $y - y_0$ when $v_{0y} = 1.3405\times10^3$ m/s and $a_y = -9.80\ \text{m/s}^2$. The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{-(1.3405\times10^3\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 9.17\times10^4\ \text{m} = 91.7\ \text{km}.$

EVALUATE: Even though the mass of Io is around 100 times smaller than that of the earth, the acceleration due to gravity at its surface is only about 1/6 of that of the earth because Io's radius is much smaller than earth's radius.

13.17. IDENTIFY: The escape speed, as shown in Example 13.5, is $\sqrt{2GM/R}$.

SET UP: For Mars, $M = 6.42\times10^{23}$ kg and $R = 3.39\times10^6$ m. For Jupiter, $M = 1.90\times10^{27}$ kg and $R = 6.99\times10^7$ m.

EXECUTE: (a) $v = \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42\times10^{23}\ \text{kg})/(3.39\times10^6\ \text{m})} = 5.03\times10^3$ m/s.

(b) $v = \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2(1.90\times10^{27}\ \text{kg})/(6.99\times10^7\ \text{m})} = 6.02\times10^4$ m/s.

(c) Both the kinetic energy and the gravitational potential energy are proportional to the mass of the spacecraft.

EVALUATE: Example 13.5 calculates the escape speed for earth to be 1.12×10^4 m/s. This is larger than our result for Mars and less than our result for Jupiter.

13.18. IDENTIFY: The kinetic energy is $K=\frac{1}{2}mv^2$ and the potential energy is $U=-\dfrac{GMm}{r}$.

SET UP: The mass of the earth is $M_E = 5.97\times10^{24}$ kg.

EXECUTE: **(a)** $K=\frac{1}{2}(629\text{ kg})(3.33\times10^3\text{ m/s})^2 = 3.49\times10^9$ J

(b) $U=-\dfrac{GM_E m}{r} = -\dfrac{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})(629\text{ kg})}{2.87\times10^9\text{ m}} = -8.73\times10^7$ J.

EVALUATE: The total energy $K+U$ is positive.

13.19. IDENTIFY: Mechanical energy is conserved. At the escape speed, the object has no kinetic energy when it is very far away from the planet.

SET UP: Call m the mass of the object, M the mass of the planet, and r its radius. $K_1 + U_1 = K_2 + U_2$, $K = \frac{1}{2}mv^2$, $U = -GmM/r$, $g = GM/r^2$.

EXECUTE: Energy conservation gives $\frac{1}{2}mv^2 - GmM/r = 0 + 0$. $M = rv^2/2G$. Putting this into $g =$ GM/r^2 gives $g = \dfrac{G\left(\dfrac{rv^2}{2G}\right)}{r^2} = \dfrac{v^2}{2r}$. Putting in the numbers gives

$g = (7.65\times10^3\text{ m/s})^2/[2(3.24\times10^6\text{ m})] = 9.03\text{ m/s}^2$.

EVALUATE: This result is not very different from g on earth, so it is physically reasonable for a planet.

13.20. IDENTIFY: This problem involves gravitational potential energy.

SET UP: Estimate: Altitude is h = 30,000 ft (≈10,000 m) above the earth's surface. We want to find the percent change in the gravitational potential energy of the system (you and the earth) when you are at this altitude compared to when you are at the surface. Use $U=-\dfrac{Gm_E m}{r}$, where m is your mass.

EXECUTE: At the surface: $U_s = -\dfrac{Gm_E m}{R}$, and at altitude h: $U_h = -\dfrac{Gm_E m}{R_E+h}$. The fractional change in potential energy is $\dfrac{\Delta U}{U_s} = \dfrac{U_h-U_s}{U_s} = \dfrac{U_h}{U_s}-1$. Using the expressions for U_h and U_s gives

$$\frac{\Delta U}{U_s} = \frac{-\dfrac{Gm_E m}{R_E+h}}{-\dfrac{Gm_E m}{R_E}} - 1 = \frac{R_E}{R_E+h} - 1 = \frac{1}{1+h/R_E} - 1 = \left(1+h/R_E\right)^{-1} - 1.$$ From Appendix B, we have

$(1+x)^n = 1^n + n1^{n-1}x + \dfrac{n(n-1)1^{n-2}}{2!}x^2 + \cdots$. If $x \ll 1$, we can neglect all the x^2 and higher terms, so for $n = -1$, we have $(1+x)^{-1} \approx 1 - x$. In our case, $x = h/R_E \ll 1$, so $\dfrac{1}{1+h/R_E} \approx 1 - \dfrac{h}{R_E}$, which yields

$\dfrac{\Delta U}{U_s} \approx \left(1-\dfrac{h}{R_E}\right) - 1 \approx -\dfrac{h}{R_E}$. This gives $\dfrac{\Delta U}{U_s} \approx -\dfrac{10{,}000\text{ m}}{6.37\times10^6\text{ m}} \approx -1.6\times10^{-3}$. This result gives us only the *magnitude* of the change. Since U is negative, its magnitude decreases with altitude, but it is becoming *less negative*, so it is actually increasing. So the gravitational potential energy is 0.16% greater when you are in the plane.

EVALUATE: Fractional changes in gravitational potential energy only become apparent when $\Delta r/r$ is fairly large.

13.21. IDENTIFY: This problem involves the gravitational force and gravitational potential energy.

SET UP: $F_g = \dfrac{Gm_1m_2}{r^2}$ and $U_g = -\dfrac{Gm_p m}{r}$. We know U_g and want to find the force F_g on you at the surface of the planet when $r = R$.

EXECUTE: Relate F_g and U_g: $F_g = \dfrac{Gmm_p}{R^2} = \left(\dfrac{Gmm_p}{R}\right)\dfrac{1}{R} = -\dfrac{U_g}{R} = -\dfrac{-1.20\times10^9\text{ J}}{5.00\times10^6\text{ m}} = 240\text{ N}.$

EVALUATE: Knowing your mass, we could find g at the surface.

13.22. **IDENTIFY:** The satellite is in orbit, so we need to use Newton's second law, the gravitational force, and gravitational potential energy.

SET UP: Use $\sum F = m\dfrac{v^2}{R}$, $F_g = \dfrac{Gm_1m_2}{r^2}$, and $U_g = -\dfrac{Gm_pm}{r}$. We want to find r_B so that $K_B = 2K_A$.

EXECUTE: $\sum F = m\dfrac{v^2}{R}$ gives $\dfrac{Gmm_p}{r^2} = \dfrac{mv^2}{r} = \dfrac{2}{r}\left(\dfrac{1}{2}mv^2\right) = \dfrac{2K}{r}$, so $K = \dfrac{Gmm_p}{2r}$.

At A: $K_A = \dfrac{Gmm_p}{2r_A}$

At B: $K_B = \dfrac{Gmm_p}{2r_B} = 2K_A = \dfrac{2Gmm_p}{2r_A}$.

Solving the last equation for r_B gives $r_B = r_A/2$.

EVALUATE: It is reasonable that $r_B < r_A$ since when it is closer to the planet, the satellite must move faster to remain in orbit.

13.23. **IDENTIFY:** This problem involves the total energy of the earth-satellite system.

SET UP: Eq. 13.13: $E = -\dfrac{Gm_Em}{2r}$, where E is the total energy of the system. Also use $\sum F = m\dfrac{v^2}{R}$, $F_g = \dfrac{Gm_1m_2}{r^2}$, and $U_g = -\dfrac{Gm_pm}{r}$.

EXECUTE: $\sum F = m\dfrac{v^2}{R} = \dfrac{Gmm_E}{r^2}$ gives $v^2 = \dfrac{Gm_E}{r}$, so $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(\dfrac{Gm_E}{r}\right) = \dfrac{Gmm_E}{2r} =$

2.00×10^6 J. Using Eq. 13.13 gives $E = -\dfrac{Gm_Em}{2r} = -K = -2.00\times10^6$ J. The total energy is $E = K + U$, so $U = E - K = -2.00\times10^6\text{ J} - 2.00\times10^6\text{ J} = -4.00\times10^6$ J.

EVALUATE: Note that $K = -\dfrac{1}{2}U$, which is a general result for orbits.

13.24. **IDENTIFY:** Newton's second law and his law of gravitation both apply to the satellite.

SET UP: $T = \dfrac{2\pi r}{v}$. r and v are also related by applying $\sum\vec{F} = m\vec{a}$ to the motion of the satellite. The satellite has $a_{rad} = v^2/R$, and the only force on the satellite is the gravitational force, $F_g = G\dfrac{m_Em}{r^2}$.

$m_E = 5.97\times10^{24}$ kg.

EXECUTE: **(a)** The free-body diagram of the satellite is shown in Figure 13.24. $F_g = ma_{rad}$ gives $G\dfrac{m_Em}{r^2} = m\dfrac{v^2}{r}$, which simplifies to $G\dfrac{m_E}{r} = v^2$. Solving for r gives

$$r = \frac{Gm_E}{v^2} = \frac{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}{(6200\text{ m/s})^2} = 1.04\times10^7\text{ m}.$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.04\times10^7\text{ m})}{6200\text{ m/s}} = 1.05\times10^4\text{ s} = 176\text{ min} = 2.93\text{ h}.$$

(b) $a_{rad} = \dfrac{v^2}{r} = \dfrac{(6200\text{ m/s})^2}{1.04\times10^7\text{ m}} = 3.70\text{ m/s}^2.$

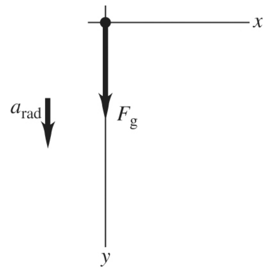
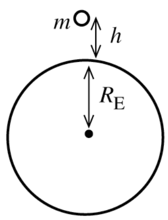


Figure 13.24

EVALUATE: The acceleration of the satellite is 38% of the acceleration due to gravity at the earth's surface.

13.25. **IDENTIFY:** Apply Newton's second law to the motion of the satellite and obtain an equation that relates the orbital speed v to the orbital radius r.

SET UP: The distances are shown in Figure 13.25a.

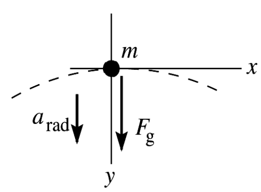


The radius of the orbit is $r = h + R_E$.

$r = 8.90\times10^5 \text{ m} + 6.37\times10^6 \text{ m} = 7.26\times10^6 \text{ m}.$

Figure 13.25a

The free-body diagram for the satellite is given in Figure 13.25b.



(a) EXECUTE: $\Sigma F_y = ma_y$

$F_g = ma_{rad}$

$$G\frac{mm_E}{r^2} = m\frac{v^2}{r}$$

Figure 13.25b

$$v = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{(6.673\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24} \text{ kg})}{7.26\times10^6 \text{ m}}} = 7.408\times10^3 \text{ m/s which rounds to 7410 m/s.}$$

(b) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(7.26\times10^6 \text{ m})}{7.408\times10^3 \text{ m/s}} = 6158 \text{ s} = 1.71 \text{ h}.$

EVALUATE: Note that $r = h + R_E$ is the radius of the orbit, measured from the center of the earth. For this satellite r is greater than for the satellite in Example 13.6, so its orbital speed is less.

13.26. **IDENTIFY:** The time to complete one orbit is the period T, given by $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$. The speed v of the satellite is given by $v = \dfrac{2\pi r}{T}$.

SET UP: If h is the height of the orbit above the earth's surface, the radius of the orbit is $r = h + R_E$. $R_E = 6.37\times10^6$ m and $m_E = 5.97\times10^{24}$ kg.

Execute: **(a)** $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}} = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.37\times10^6\ \text{m})^{3/2}}{\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}} = 5.93\times10^3\ \text{s} = 98.8\ \text{min}$

(b) $v = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.37\times10^6\ \text{m})}{5.93\times10^3\ \text{s}} = 7.50\times10^3\ \text{m/s} = 7.50\ \text{km/s}.$

Evaluate: The satellite in Example 13.6 is at a lower altitude and therefore has a smaller orbit radius than the satellite in this problem. Therefore, the satellite in this problem has a larger period and a smaller orbital speed. But a large percentage change in h corresponds to a small percentage change in r and the values of T and v for the two satellites do not differ very much.

13.27. **Identify:** We know orbital data (speed and orbital radius) for one satellite and want to use it to find the orbital speed of another satellite having a known orbital radius. Newton's second law and the law of universal gravitation apply to both satellites.

Set Up: For circular motion, $F_{\text{net}} = ma = mv^2/r$, which in this case is $G\dfrac{mm_p}{r^2} = m\dfrac{v^2}{r}$.

Execute: Using $G\dfrac{mm_p}{r^2} = m\dfrac{v^2}{r}$, we get $Gm_p = rv^2 = \text{constant}$. $r_1v_1^2 = r_2v_2^2$.

$v_2 = v_1\sqrt{\dfrac{r_1}{r_2}} = (4800\ \text{m/s})\sqrt{\dfrac{7.00\times10^7\ \text{m}}{3.00\times10^7\ \text{m}}} = 7330\ \text{m/s}.$

Evaluate: The more distant satellite moves slower than the closer satellite, which is reasonable since the planet's gravity decreases with distance. The masses of the satellites do not affect their orbits.

13.28. **Identify:** We can calculate the orbital period T from the number of revolutions per day. Then the period and the orbit radius are related by $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$.

Set Up: $m_E = 5.97\times10^{24}$ kg and $R_E = 6.37\times10^6$ m. The height h of the orbit above the surface of the earth is related to the orbit radius r by $r = h + R_E$. $1\ \text{day} = 8.64\times10^4$ s.

Execute: The satellite moves 15.65 revolutions in 8.64×10^4 s, so the time for 1.00 revolution is

$T = \dfrac{8.64\times10^4\ \text{s}}{15.65} = 5.52\times10^3\ \text{s}$. $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives

$r = \left(\dfrac{Gm_ET^2}{4\pi^2}\right)^{1/3} = \left(\dfrac{[6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2][5.97\times10^{24}\ \text{kg}][5.52\times10^3\ \text{s}]^2}{4\pi^2}\right)^{1/3}$. $r = 6.75\times10^6$ m and

$h = r - R_E = 3.8\times10^5\ \text{m} = 380\ \text{km}.$

Evaluate: The period of this satellite is slightly larger than the period for the satellite in Example 13.6 and the altitude of this satellite is therefore somewhat greater.

13.29. **Identify:** Apply $\Sigma\vec{F} = m\vec{a}$ to the motion of the baseball. $v = \dfrac{2\pi r}{T}$.

Set Up: $r_D = 6\times10^3$ m.

Execute: **(a)** $F_g = ma_{\text{rad}}$ gives $G\dfrac{m_Dm}{r_D^2} = m\dfrac{v^2}{r_D}$.

$v = \sqrt{\dfrac{Gm_D}{r_D}} = \sqrt{\dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.5\times10^{15}\ \text{kg})}{6\times10^3\ \text{m}}} = 4.07\ \text{m/s}$ which rounds to 4.1 m/s.

$4.1\ \text{m/s} = 9.1\ \text{mph}$, which is easy to achieve.

(b) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(6\times10^3\ \text{m})}{4.07\ \text{m/s}} = 9263\ \text{s} = 154.4\ \text{min} = 2.6\ \text{h}.$ The game would last a very long time indeed!

EVALUATE: The speed v is relative to the center of Deimos. The baseball would already have some speed before we throw it because of the rotational motion of Deimos.

13.30. IDENTIFY: $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$, where m_{star} is the mass of the star. $v = \dfrac{2\pi r}{T}.$

SET UP: $3.09\ \text{days} = 2.67\times10^5\ \text{s}.$ The orbit radius of Mercury is $5.79\times10^{10}\ \text{m}.$ The mass of our sun is $1.99\times10^{30}\ \text{kg}.$

EXECUTE: **(a)** $T = 2.67\times10^5\ \text{s}.$ $r = (5.79\times10^{10}\ \text{m})/9 = 6.43\times10^9\ \text{m}.$ $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$ gives

$m_{\text{star}} = \dfrac{4\pi^2 r^3}{T^2 G} = \dfrac{4\pi^2(6.43\times10^9\ \text{m})^3}{(2.67\times10^5\ \text{s})^2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 2.21\times10^{30}\ \text{kg}.$ $\dfrac{m_{\text{star}}}{m_{\text{sun}}} = 1.11,$ so $m_{\text{star}} = 1.11 m_{\text{sun}}.$

(b) $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(6.43\times10^9\ \text{m})}{2.67\times10^5\ \text{s}} = 1.51\times10^5\ \text{m/s} = 151\ \text{km/s}$

EVALUATE: The orbital period of Mercury is 88.0 d. The period for this planet is much less primarily because the orbit radius is much less and also because the mass of the star is greater than the mass of our sun.

13.31. IDENTIFY: The orbital speed is given by $v = \sqrt{Gm/r}$, where m is the mass of the star. The orbital period is given by $T = \dfrac{2\pi r}{v}.$

SET UP: The sun has mass $m_S = 1.99\times10^{30}\ \text{kg}.$ The orbit radius of the earth is $1.50\times10^{11}\ \text{m}.$

EXECUTE: **(a)** $v = \sqrt{Gm/r}.$

$v = \sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.85\times1.99\times10^{30}\ \text{kg})/((1.50\times10^{11}\ \text{m})(0.11))} = 8.27\times10^4\ \text{m/s}.$

(b) $2\pi r/v = 1.25\times10^6\ \text{s} = 14.5\ \text{days}$ (about two weeks).

EVALUATE: The orbital period is less than the 88-day orbital period of Mercury; this planet is orbiting very close to its star, compared to the orbital radius of Mercury.

13.32. IDENTIFY: The period of each satellite is given by $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_p}}.$ Set up a ratio involving T and r.

SET UP: $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_p}}$ gives $\dfrac{T}{r^{3/2}} = \dfrac{2\pi}{\sqrt{Gm_p}} = \text{constant},$ so $\dfrac{T_1}{r_1^{3/2}} = \dfrac{T_2}{r_2^{3/2}}.$

EXECUTE: $T_2 = T_1\left(\dfrac{r_2}{r_1}\right)^{3/2} = (6.39\ \text{days})\left(\dfrac{48{,}000\ \text{km}}{19{,}600\ \text{km}}\right)^{3/2} = 24.5\ \text{days}.$ For the other satellite,

$T_2 = (6.39\ \text{days})\left(\dfrac{64{,}000\ \text{km}}{19{,}600\ \text{km}}\right)^{3/2} = 37.7\ \text{days}.$

EVALUATE: T increases when r increases.

13.33. IDENTIFY: Kepler's third law applies.

SET UP: $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, $d_{\min} = a(1 - e)$, $d_{\max} = a(1 + e)$.

EXECUTE: **(a)** Kepler's third law gives

$$T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}} = \frac{2\pi(5.91\times10^{12}\ \text{m})^{3/2}}{\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}} = 7.84\times10^9\ \text{s}\ [(1\ \text{y})/(3.156\times10^7\ \text{s})] = 248\ \text{y}.$$

(b) $d_{min} = a(1 - e) = (5.91\times10^{12}\ \text{m})(1 - 0.249) = 4.44\times10^{12}\ \text{m}$; $d_{max} = a(1 + e) = 7.38\times10^{12}\ \text{m}$.

EVALUATE: $d_{max} = 1.66 d_{min}$, which is *much* greater than for the earth's orbit since the earth moves in a much more circular orbit than Pluto.

13.34. **IDENTIFY:** Knowing the orbital radius and orbital period of a satellite, we can calculate the mass of the object about which it is revolving.

SET UP: The radius of the orbit is $r = 10.5\times10^9$ m and its period is $T = 6.3\ \text{days} = 5.443\times10^5$ s. The mass of the sun is $m_S = 1.99\times10^{30}$ kg. The orbital period is given by $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{HD}}}$.

EXECUTE: Solving $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{HD}}}$ for the mass of the star gives

$$m_{HD} = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2(10.5\times10^9\ \text{m})^3}{(5.443\times10^5\ \text{s})^2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 2.3\times10^{30}\ \text{kg},$$ which is $m_{HD} = 1.2 m_S$.

EVALUATE: The mass of the star is only 20% greater than that of our sun, yet the orbital period of the planet is much shorter than that of the earth, so the planet must be much closer to the star than the earth is.

13.35. **IDENTIFY:** We are dealing with the gravitational force due to spherical shells.

SET UP: Outside a uniform spherical shell, $F_g = \dfrac{Gm_1m_2}{r^2}$, and inside of it $F_g = 0$. Our target variable is the net force F at various distances from the center, where $F = F_A + F_B$.

EXECUTE: **(a)** At $r = 2.00$ m, the point mass is inside both shells, so $F = 0$.

(b) At $r = 5.00$ m, the mass is between the shells, so $F_B = 0$, so $F = F_A$. This gives

$$F = F_A = \frac{Gmm_A}{r^2} = \frac{G(0.0200\ \text{kg})(20.0\ \text{kg})}{(5.00\ \text{m})^2} = 1.07\times10^{-12}\ \text{N}.$$

(c) At $r = 8.00$ m, the mass is outside of both shells. We treat each one as a point mass at its center.

$$F = \frac{G(m_A + m_B)}{r^2} = \frac{G(0.0200\ \text{kg})(60.0\ \text{kg})}{(8.00\ \text{m})^2} = 1.25\times10^{-12}\ \text{N}.$$

EVALUATE: If a shell is not uniform, we cannot treat it as a point mass at its center. And the force on a mass inside of it is not necessarily zero.

13.36. **IDENTIFY:** Section 13.6 states that for a point mass outside a spherical shell the gravitational force is the same as if all the mass of the shell were concentrated at its center. It also states that for a point inside a spherical shell the force is zero.

SET UP: For $r = 5.01$ m the point mass is outside the shell and for $r = 4.99$ m and $r = 2.72$ m the point mass is inside the shell.

EXECUTE: **(a)** (i) $F_g = \dfrac{Gm_1m_2}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1000.0\ \text{kg})(2.00\ \text{kg})}{(5.01\ \text{m})^2} = 5.31\times10^{-9}$ N. (ii) $F_g = 0$. (iii) $F_g = 0$.

(b) For $r < 5.00$ m the force is zero and for $r > 5.00$ m the force is proportional to $1/r^2$. The graph of F_g versus r is sketched in Figure 13.36.

EVALUATE: Inside the shell the gravitational potential energy is constant and the force on a point mass inside the shell is zero.

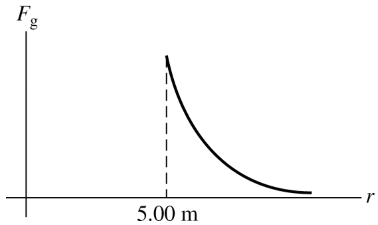


Figure 13.36

13.37. **IDENTIFY:** Section 13.6 states that for a point mass outside a uniform sphere the gravitational force is the same as if all the mass of the sphere were concentrated at its center. It also states that for a point mass a distance r from the center of a uniform sphere, where r is less than the radius of the sphere, the gravitational force on the point mass is the same as though we removed all the mass at points farther than r from the center and concentrated all the remaining mass at the center.

SET UP: The density of the sphere is $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$, where M is the mass of the sphere and R is its radius.

The mass inside a volume of radius $r < R$ is $M_r = \rho V_r = \left(\dfrac{M}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = M\left(\dfrac{r}{R}\right)^3$. $r = 5.01\text{ m}$ is outside the sphere and $r = 2.50\text{ m}$ is inside the sphere. $F_g = \dfrac{Gm_1m_2}{r^2}$.

EXECUTE: **(a)** (i) $F_g = \dfrac{GMm}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1000.0\text{ kg})(2.00\text{ kg})}{(5.01\text{ m})^2} = 5.31\times10^{-9}\text{ N}.$

(ii) $F_g = \dfrac{GM'm}{r^2}$. $M' = M\left(\dfrac{r}{R}\right)^3 = (1000.0\text{ kg})\left(\dfrac{2.50\text{ m}}{5.00\text{ m}}\right)^3 = 125\text{ kg}.$

$F_g = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(125\text{ kg})(2.00\text{ kg})}{(2.50\text{ m})^2} = 2.67\times10^{-9}\text{ N}.$

(b) $F_g = \dfrac{GM(r/R)^3 m}{r^2} = \left(\dfrac{GMm}{R^3}\right)r$ for $r < R$ and $F_g = \dfrac{GMm}{r^2}$ for $r > R$. The graph of F_g versus r is sketched in Figure 13.37.

EVALUATE: At points outside the sphere the force on a point mass is the same as for a shell of the same mass and radius. For $r < R$ the force is different in the two cases of uniform sphere versus hollow shell.

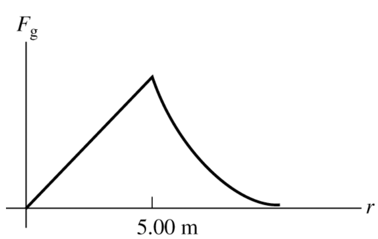


Figure 13.37

13.38. **IDENTIFY:** The gravitational potential energy of a pair of point masses is $U = -G\dfrac{m_1m_2}{r}$. Divide the rod into infinitesimal pieces and integrate to find U.

SET UP: Divide the rod into differential masses dm at position l, measured from the right end of the rod. $dm = dl(M/L)$.

EXECUTE: **(a)** $U = -\dfrac{Gm\,dm}{l+x} = -\dfrac{GmM}{L}\dfrac{dl}{l+x}.$

Integrating, $U = -\dfrac{GmM}{L}\displaystyle\int_0^L \frac{dl}{l+x} = -\frac{GmM}{L}\ln\left(1+\frac{L}{x}\right)$. For $x \gg L$, the natural logarithm is ~(L/x), and $U \to -GmM/x$.

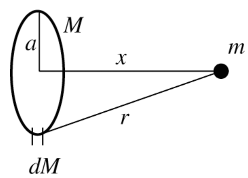
(b) The x-component of the gravitational force on the sphere is

$F_x = -\dfrac{\partial U}{\partial x} = \dfrac{GmM}{L}\dfrac{(-L/x^2)}{1+(L/x)} = -\dfrac{GmM}{x^2+Lx}$, with the minus sign indicating an attractive force. As $x \gg L$, the denominator in the above expression approaches x^2, and $F_x \to -GmM/x^2$, as expected.

EVALUATE: When x is much larger than L the rod can be treated as a point mass, and our results for U and F_x do reduce to the correct expression when $x \gg L$.

13.39. **IDENTIFY:** Find the potential due to a small segment of the ring and integrate over the entire ring to find the total U.

(a) SET UP:



Divide the ring up into small segments dM, as indicated in Figure 13.39.

Figure 13.39

EXECUTE: The gravitational potential energy of dM and m is $dU = -GmdM/r$.

The total gravitational potential energy of the ring and particle is $U = \int dU = -Gm\int dM/r$.

But $r = \sqrt{x^2+a^2}$ is the same for all segments of the ring, so

$U = -\dfrac{Gm}{r}\displaystyle\int dM = -\frac{GmM}{r} = -\frac{GmM}{\sqrt{x^2+a^2}}.$

(b) EVALUATE: When $x \gg a$, $\sqrt{x^2+a^2} \to \sqrt{x^2} = x$ and $U = -GmM/x$. This is the gravitational potential energy of two point masses separated by a distance x. This is the expected result.

(c) IDENTIFY and **SET UP:** Use $F_x = -dU/dx$ with $U(x)$ from part (a) to calculate F_x.

EXECUTE: $F_x = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(-\dfrac{GmM}{\sqrt{x^2+a^2}}\right)$

$F_x = +GmM\dfrac{d}{dx}(x^2+a^2)^{-1/2} = GmM\left(-\dfrac{1}{2}(2x)(x^2+a^2)^{-3/2}\right)$

$F_x = -GmMx/(x^2+a^2)^{3/2}$; the minus sign means the force is attractive.

EVALUATE: **(d)** For $x \gg a$, $(x^2+a^2)^{3/2} \to (x^2)^{3/2} = x^3$

Then $F_x = -GmMx/x^3 = -GmM/x^2$. This is the force between two point masses separated by a distance x and is the expected result.

(e) For $x = 0$, $U = -GMm/a$. Each small segment of the ring is the same distance from the center and the potential is the same as that due to a point charge of mass M located at a distance a.

For $x = 0$, $F_x = 0$. When the particle is at the center of the ring, symmetrically placed segments of the ring exert equal and opposite forces and the total force exerted by the ring is zero.

13.40. **IDENTIFY:** We are investigating the gravitational field $\vec{g}$ for a spherical shell.

SET UP: The field is defined as $\vec{g} = \dfrac{\vec{F}_g}{m}$. We want to find $\vec{g}$ inside and outside a spherical shell.

EXECUTE: **(a)** Inside ($r < R$) the shell $\vec{F}_g = 0$, so $\vec{g} = 0$.

(b) Outside ($r > R$) the shell behaves like a point mass at its center, so $g = \dfrac{GmM / r^2}{m} = \dfrac{GM}{r^2}$.

EVALUATE: The gravitational field obeys an inverse-square law.

13.41. **IDENTIFY** and **SET UP:** At the north pole, $F_g = w_0 = mg_0$, where g_0 is given by $g = G\dfrac{m_E}{r^2}$ applied to Neptune. At the equator, the apparent weight is given by $w = w_0 - mv^2/R$. The orbital speed v is obtained from the rotational period using $v = 2\pi R/T$.

EXECUTE: **(a)** $g_0 = Gm/R^2 = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.02\times10^{26}\ \text{kg})/(2.46\times10^7\ \text{m})^2 = 11.25\ \text{m/s}^2$. This agrees with the value of g given in the problem.

$F = w_0 = mg_0 = (3.00\ \text{kg})(11.25\ \text{m/s}^2) = 33.74\ \text{N}$, which rounds to 33.7 N. This is the true weight of the object.

(b) We have $w = w_0 - mv^2/R$

$T = \dfrac{2\pi r}{v}$ gives $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(2.46\times10^7\ \text{m})}{(16\ \text{h})(3600\ \text{s}/1\ \text{h})} = 2.683\times10^3\ \text{m/s}$

$v^2/R = (2.683\times10^3\ \text{m/s})^2/(2.46\times10^7\ \text{m}) = 0.2927\ \text{m/s}^2$

Then $w = 33.74\ \text{N} - (3.00\ \text{kg})(0.2927\ \text{m/s}^2) = 32.9\ \text{N}$.

EVALUATE: The apparent weight is less than the true weight. This effect is larger on Neptune than on earth.

13.42. **IDENTIFY:** At the North Pole, Sneezy has no circular motion and therefore no acceleration. But at the equator he has acceleration toward the center of the earth due to the earth's rotation.

SET UP: The earth has mass $m_E = 5.97\times10^{24}\ \text{kg}$, radius $R_E = 6.37\times10^6\ \text{m}$ and rotational period $T = 24\ \text{hr} = 8.64\times10^4\ \text{s}$. Use coordinates for which the $+y$ direction is toward the center of the earth. The free-body diagram for Sneezy at the equator is given in Figure 13.42. The radial acceleration due to Sneezy's circular motion at the equator is $a_{rad} = \dfrac{4\pi^2 R}{T^2}$, and Newton's second law applies to Sneezy.

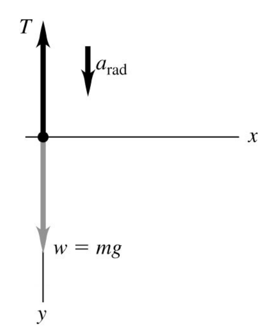


Figure 13.42

EXECUTE: At the north pole Sneezy has $a = 0$ and $T = w = 395.0\ \text{N}$ (the gravitational force exerted by the earth). Sneezy has mass $m = w/g = 40.31\ \text{kg}$. At the equator Sneezy is traveling in a circular path and has radial acceleration $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2} = \dfrac{4\pi^2(6.37\times10^6\ \text{m})}{(8.64\times10^4\ \text{s})^2} = 0.03369\ \text{m/s}^2$. Newton's second law $\Sigma F_y = ma_y$ gives $w - T = ma_{\text{rad}}$. Solving for T gives

$T = w - ma_{\text{rad}} = m(g - a_{\text{rad}}) = (40.31\ \text{kg})(9.80\ \text{m/s}^2 - 0.03369\ \text{m/s}^2) = 394\ \text{N}$.

EVALUATE: At the equator Sneezy has an inward acceleration and the outward tension is less than the true weight, since there is a net inward force.

13.43. **IDENTIFY:** The orbital speed for an object a distance r from an object of mass M is $v = \sqrt{\dfrac{GM}{r}}$. The mass M of a black hole and its Schwarzschild radius R_S are related by $R_S = \dfrac{2GM}{c^2}$.

SET UP: $c = 3.00\times10^8$ m/s. 1 ly $= 9.461\times10^{15}$ m.

EXECUTE:

(a) $M = \dfrac{rv^2}{G} = \dfrac{(7.5\ \text{ly})(9.461\times10^{15}\ \text{m/ly})(200\times10^3\ \text{m/s})^2}{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 4.3\times10^{37}\ \text{kg} = 2.1\times10^7\ \text{M}_\text{S}$.

(b) No, the object has a mass very much greater than 50 solar masses.

(c) $R_S = \dfrac{2GM}{c^2} = \dfrac{2v^2 r}{c^2} = 6.32\times10^{10}$ m, which does fit.

EVALUATE: The Schwarzschild radius of a black hole is approximately the same as the radius of Mercury's orbit around the sun.

13.44. **IDENTIFY:** The clumps orbit the black hole. Their speed, orbit radius and orbital period are related by $v = \dfrac{2\pi r}{T}$. Their orbit radius and period are related to the mass M of the black hole by $T = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$. The radius of the black hole's event horizon is related to the mass of the black hole by $R_S = \dfrac{2GM}{c^2}$.

SET UP: $v = 3.00\times10^7$ m/s. $T = 27\ \text{h} = 9.72\times10^4$ s. $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $r = \dfrac{vT}{2\pi} = \dfrac{(3.00\times10^7\ \text{m/s})(9.72\times10^4\ \text{s})}{2\pi} = 4.64\times10^{11}$ m.

(b) $T = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$ gives $M = \dfrac{4\pi^2 r^3}{GT^2} = \dfrac{4\pi^2(4.64\times10^{11}\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.72\times10^4\ \text{s})^2} = 6.26\times10^{36}$ kg.

$= 3.15\times10^6 M_S$, where M_S is the mass of our sun

(c) $R_S = \dfrac{2GM}{c^2} = \dfrac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.26\times10^{36}\ \text{kg})}{(3.00\times10^8\ \text{m/s})^2} = 9.28\times10^9$ m

EVALUATE: The black hole has a mass that is about 3×10^6 solar masses.

13.45. **IDENTIFY:** Use $F_g = Gm_1m_2/r^2$ to find each gravitational force. Each force is attractive. In part (b) apply conservation of energy.

SET UP: For a pair of masses m_1 and m_2 with separation r, $U = -G\dfrac{m_1m_2}{r}$.

EXECUTE: **(a)** From symmetry, the net gravitational force will be in the direction 45° from the x-axis (bisecting the x- and y-axes), with magnitude

$$F = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.0150\ \text{kg})\left[\frac{(2.0\ \text{kg})}{(2(0.50\ \text{m})^2)} + 2\frac{(1.0\ \text{kg})}{(0.50\ \text{m})^2}\sin 45^\circ\right] = 9.67\times10^{-12}\ \text{N}$$

(b) The initial displacement is so large that the initial potential energy may be taken to be zero. From the work-energy theorem, $\frac{1}{2}mv^2 = Gm\left[\frac{(2.0\text{ kg})}{\sqrt{2}\,(0.50\text{ m})} + 2\frac{(1.0\text{ kg})}{(0.50\text{ m})}\right]$. Cancelling the factor of m, solving for v, and using the numerical values gives $v = 3.02\times10^{-5}$ m/s.

EVALUATE: The result in part (b) is independent of the mass of the particle. It would take the particle a long time to reach point P.

13.46. **IDENTIFY:** Use $g = G\frac{m_E}{r^2}$ to calculate g for Europa. The acceleration of a particle moving in a circular path is $a_{rad} = r\omega^2$.

SET UP: In $a_{rad} = r\omega^2$, ω must be in rad/s. For Europa, $R = 1.560\times10^6$ m.

EXECUTE: $g = \frac{Gm}{R^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(4.80\times10^{22}\text{ kg})}{(1.560\times10^6\text{ m})^2} = 1.316\text{ m/s}^2$. $g = a_{rad}$ gives

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{1.316\text{ m/s}^2}{4.25\text{ m}}} = (0.5565\text{ rad/s})\left(\frac{60\text{ s}}{1\text{ min}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 5.31\text{ rpm}.$$

EVALUATE: The radius of Europa is about one-fourth that of the earth and its mass is about one-hundredth that of earth, so g on Europa is much less than g on earth. The lander would have some spatial extent so different points on it would be different distances from the rotation axis and a_{rad} would have different values. For the ω we calculated, $a_{rad} = g$ at a point that is precisely 4.25 m from the rotation axis.

13.47. **IDENTIFY:** Apply conservation of energy and conservation of linear momentum to the motion of the two spheres.

SET UP: Denote the 50.0-kg sphere by a subscript 1 and the 100-kg sphere by a subscript 2.

EXECUTE: **(a)** Linear momentum is conserved because we are ignoring all other forces, that is, the net external force on the system is zero. Hence, $m_1v_1 = m_2v_2$.

(b) (i) From the work-energy theorem in the form $K_i + U_i = K_f + U_f$, with the initial kinetic energy $K_i = 0$ and $U = -G\frac{m_1m_2}{r}$, $Gm_1m_2\left[\frac{1}{r_f} - \frac{1}{r_i}\right] = \frac{1}{2}(m_1v_1^2 + m_2v_2^2)$. Using the conservation of momentum relation $m_1v_1 = m_2v_2$ to eliminate v_2 in favor of v_1 and simplifying yields $v_1^2 = \frac{2Gm_2^2}{m_1 + m_2}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$, with a similar expression for v_2. Substitution of numerical values gives $v_1 = 1.49\times10^{-5}$ m/s, $v_2 = 7.46\times10^{-6}$ m/s. (ii) The magnitude of the relative velocity is the sum of the speeds, 2.24×10^{-5} m/s.

(c) The distance the centers of the spheres travel $(x_1$ and $x_2)$ is proportional to their acceleration, and $\frac{x_1}{x_2} = \frac{a_1}{a_2} = \frac{m_2}{m_1}$, or $x_1 = 2x_2$. When the spheres finally make contact, their centers will be a distance of $2r$ apart, or $x_1 + x_2 + 2r = 40$ m, or $2x_2 + x_2 + 2r = 40$ m. Thus, $x_2 = 40/3\text{ m} - 2r/3$, and $x_1 = 80/3\text{ m} - 4r/3$. The point of contact of the surfaces is $80/3\text{ m} - r/3 = 26.6$ m from the initial position of the center of the 50.0-kg sphere.

EVALUATE: The result $x_1/x_2 = 2$ can also be obtained from the conservation of momentum result that $\frac{v_1}{v_2} = \frac{m_2}{m_1}$, at every point in the motion.

EVALUATE: The work done by the attractive gravity forces is negative. The work you do is positive.

13.48. IDENTIFY: The gravitational pulls of Titan and Saturn on the *Huygens* probe should be in opposite directions and of equal magnitudes to cancel.

SET UP: The mass of Saturn is $m_S = 5.68\times10^{26}$ kg. When the probe is a distance d from the center of Titan it is a distance $1.22\times10^9\ \text{m} - d$ from the center of Saturn. The magnitude of the gravitational force is given by $F_{grav} = GmM/r^2$.

EXECUTE: Equal gravity forces means the two gravitational pulls on the probe must balance, so $G\dfrac{mm_T}{d^2} = G\dfrac{mm_S}{(1.22\times10^9\ \text{m} - d)^2}$. Simplifying, this becomes $d = \sqrt{\dfrac{m_T}{m_S}}(1.22\times10^9\ \text{m} - d)$. Using the masses from the text and solving for d we get

$$d = \sqrt{\frac{1.35\times10^{23}\ \text{kg}}{5.68\times10^{26}\ \text{kg}}}(1.22\times10^9\ \text{m} - d) = (0.0154)(1.22\times10^9\ \text{m} - d),\ \text{so}\ d = 1.85\times10^7\ \text{m} = 1.85\times10^4\ \text{km}.$$

EVALUATE: For the forces to balance, the probe must be much closer to Titan than to Saturn since Titan's mass is much smaller than that of Saturn.

13.49. IDENTIFY and **SET UP:** **(a)** To stay above the same point on the surface of the earth the orbital period of the satellite must equal the orbital period of the earth:

$T = 1\ \text{d}(24\ \text{h/1 d})(3600\ \text{s/1 h}) = 8.64\times10^4$ s. The equation $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives the relation between the orbit radius and the period.

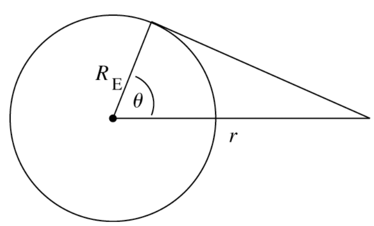
EXECUTE: $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives $T^2 = \dfrac{4\pi^2 r^3}{Gm_E}$. Solving for r gives

$$r = \left(\frac{T^2 Gm_E}{4\pi^2}\right)^{1/3} = \left(\frac{(8.64\times10^4\ \text{s})^2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{4\pi^2}\right)^{1/3} = 4.23\times10^7\ \text{m}.$$

This is the radius of the orbit; it is related to the height h above the earth's surface and the radius R_E of the earth by $r = h + R_E$. Thus $h = r - R_E = 4.23\times10^7\ \text{m} - 6.37\times10^6\ \text{m} = 3.59\times10^7$ m.

EVALUATE: The orbital speed of the geosynchronous satellite is $2\pi r/T = 3080$ m/s. The altitude is much larger and the speed is much less than for the satellite in Example 13.6.

(b) Consider Figure 13.49.



$$\cos\theta = \frac{R_E}{r} = \frac{6.37\times10^6\ \text{m}}{4.23\times10^7\ \text{m}}$$

$$\theta = 81.3°$$

Figure 13.49

A line from the satellite is tangent to a point on the earth that is at an angle of $81.3°$ above the equator. The sketch shows that points at higher latitudes are blocked by the earth from viewing the satellite.

13.50. IDENTIFY: We are dealing with satellites orbiting two different planets.

SET UP: The period is $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_p}}$. We want to compare the period around two different planets. The planets have the same average density ρ, so first express T in terms of ρ.

EXECUTE: $m = \rho V = \rho\left(\frac{4}{3}\pi R^3\right) = \frac{4}{3}\pi\rho R^3$. Now find T. The satellites are just above the surface, so $r = R$. In this case $T = \frac{2\pi R^{3/2}}{\sqrt{Gm_p}} = \frac{2\pi R^{3/2}}{\sqrt{G\left(\frac{4}{3}\pi\rho R^3\right)}} = \frac{2\pi}{\sqrt{4G\pi\rho/3}}$. This result depends only on ρ, which is the same for both planets. Therefore T is the same in both cases, so $T_A = T_B$.

EVALUATE: The planets have the same density, but not the same mass. The smaller planet has less mass than the larger planet, but since it is smaller the satellite orbits closer to it so both satellites have the same period.

13.51. IDENTIFY: From Example 13.5, the escape speed is $v = \sqrt{\frac{2GM}{R}}$. Use $\rho = M/V$ to write this expression in terms of ρ.

SET UP: For a sphere $V = \frac{4}{3}\pi R^3$.

EXECUTE: In terms of the density ρ, the ratio M/R is $(4\pi/3)\rho R^2$, and so the escape speed is

$v = \sqrt{(8\pi/3)(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2500\ \text{kg/m}^3)(150\times10^3\ \text{m})^2} = 177\ \text{m/s}.$

EVALUATE: This is much less than the escape speed for the earth, 11,200 m/s.

13.52. IDENTIFY: Apply $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ to relate the orbital period T and M_P, the planet's mass, and then use $w = \frac{Gm_E m}{r^2}$ applied to the planet to calculate the astronaut's weight.

SET UP: The radius of the orbit of the lander is $5.75\times10^5\ \text{m} + 4.80\times10^6\ \text{m}$.

EXECUTE: From $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$, we get $T^2 = \frac{4\pi^2 r^3}{GM_P}$ and

$$M_P = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(5.75\times10^5\ \text{m} + 4.80\times10^6\ \text{m})^3}{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.8\times10^3\ \text{s})^2} = 2.731\times10^{24}\ \text{kg},$$

or about half the earth's mass. Now we can find the astronaut's weight on the surface from $w = \frac{Gm_E m}{r^2}$. (The landing on the north pole removes any need to account for centripetal acceleration.)

$$w = \frac{GM_P m_a}{r_P^2} = \frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.731\times10^{24}\ \text{kg})(85.6\ \text{kg})}{(4.80\times10^6\ \text{m})^2} = 677\ \text{N}.$$

EVALUATE: At the surface of the earth the weight of the astronaut would be 839 N.

13.53. IDENTIFY: Apply the law of gravitation to the astronaut at the north pole to calculate the mass of the planet. Then apply $\Sigma\vec{F} = m\vec{a}$ to the astronaut, with $a_{rad} = \frac{4\pi^2 R}{T^2}$, toward the center of the planet, to calculate the period T. Apply $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ to the satellite in order to calculate its orbital period.

SET UP: Get radius of X: $\frac{1}{4}(2\pi R) = 18{,}850\ \text{km}$ and $R = 1.20\times10^7\ \text{m}$. Astronaut mass:

$m = \frac{w}{g} = \frac{943\ \text{N}}{9.80\ \text{m/s}^2} = 96.2\ \text{kg}.$

EXECUTE: $\dfrac{GmM_X}{R^2} = w$, where $w = 915.0$ N.

$M_X = \dfrac{mg_x R^2}{Gm} = \dfrac{(915\text{ N})(1.20\times 10^7\text{ m})^2}{(6.67\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(96.2\text{ kg})} = 2.05\times 10^{25}\text{ kg}$

Apply Newton's second law to the astronaut on a scale at the equator of X. $F_{\text{grav}} - F_{\text{scale}} = ma_{\text{rad}}$, so

$F_{\text{grav}} - F_{\text{scale}} = \dfrac{4\pi^2 mR}{T^2}$. $915.0\text{ N} - 850.0\text{ N} = \dfrac{4\pi^2(96.2\text{ kg})(1.20\times 10^7\text{ m})}{T^2}$ and

$T = 2.65\times 10^4\text{ s}\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 7.36\text{ h}.$

(b) For the satellite,

$T = \sqrt{\dfrac{4\pi^2 r^3}{Gm_X}} = \sqrt{\dfrac{4\pi^2(1.20\times 10^7\text{ m} + 2.0\times 10^6\text{ m})^3}{(6.67\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2.05\times 10^{25}\text{ kg})}} = 8.90\times 10^3\text{ s} = 2.47\text{ hours}.$

EVALUATE: The acceleration of gravity at the surface of the planet is $g_X = \dfrac{915.0\text{ N}}{96.2\text{ kg}} = 9.51\text{ m/s}^2$, similar to the value on earth. The radius of the planet is about twice that of earth. The planet rotates more rapidly than earth and the length of a day is about one-third what it is on earth.

13.54. **IDENTIFY:** We are dealing with a planet that is spherically symmetric but not radially uniform. We will need to integrate to find the mass as a function of distance r from the center. To determine the equation for the density we use the slope-intercept form of the equation of a straight line, which is $y = mx + b$.

SET UP: The graph in Fig. 13.9 slopes downward. Its slope is $m = -\dfrac{(13{,}000 - 3000)\text{ kg/m}^3}{R_E} = -\dfrac{10{,}000\text{ kg/m}^3}{R_E}$ and its y-intercept is $b = 13{,}000$ kg/m^3. Using the slope-intercept form of a straight-line equation ($y = mx + b$), the density is $\rho(r) = -\dfrac{10{,}000\text{ kg/m}^3}{R_E}r + 13{,}000\text{ kg/m}^3$, which we simplify as $\rho(r) = mr + b$ for convenience while integrating.

EXECUTE: **(a)** $M(r) = \int \rho(r')dV = \int \rho(r')4\pi r'^2 dr' = \int_0^r (mr' + b)4\pi r'^2 dr' = \pi m r^4 + \dfrac{4\pi b r^3}{3}$. Substitute for m, b and R_E. $M(r) = -(4.932\times 10^{-3}\text{ kg/m}^4)r^4 + (5.4454\times 10^4\text{ kg/m}^3)r^3$ for $r \le R_E$.

(b) For $r \le R_E$ we use the result from (a) for the mass enclosed within a sphere of radius r. To find the force the mass in this shell exerts on a mass m a distance r from the center, we use $F_g = \dfrac{GmM(r)}{r^2} = \dfrac{Gm\left[-(4.932\times 10^{-3}\text{ kg/m}^4)r^4 + (5.4454\times 10^4\text{ kg/m}^3)r^3\right]}{r^2}$, which gives

$F_g = Gm\left[-(4.932\times 10^{-3}\text{ kg/m}^4)r^2 + (5.4454\times 10^4\text{ kg/m}^3)r\right]$. Putting in for G gives

$F_g = m\left[-(3.29\times 10^{-13}\text{ N/kg}\cdot\text{m}^2)r^2 + (3.63\times 10^{-6}\text{ N/kg}\cdot\text{m})r\right]$

For $r \ge R_E$, evaluate the final equation in part (a) for $M(r)$ when $r = R_E = 6.37\times 10^6$ m. This gives 5.95×10^{24} kg. Outside the earth, we can treat it as a point mass at its center, so we use $F_g = \dfrac{GmM}{r^2} = \dfrac{Gm(5.95\times 10^{24}\text{ kg})}{r^2} = \dfrac{m(3.79\times 10^{14}\text{ N}\cdot\text{m}^2/\text{kg})}{r^2}$.

EVALUATE: The mass we calculated in part (a) for the earth (5.95×10^{24} kg) is extremely close to the actual value of 5.97×10^{24} kg , so our model is very good. In addition, in the equation for F_g from part (b), $F_g = m\left[-(3.29\times10^{-13}\ \text{N/kg}\cdot\text{m}^2)r^2+(3.63\times10^{-6}\ \text{N/kg}\cdot\text{m})r\right]$, the quantity in square brackets should give the acceleration due to gravity at the surface of the earth if we use $r = 6.37\times10^6$ m . Doing so gives a value of 9.79 m/s^2, which is extremely close to 9.80 m/s^2.

13.55. IDENTIFY: The free-fall time of the rock will give us the acceleration due to gravity at the surface of the planet. Applying Newton's second law and the law of universal gravitation will give us the mass of the planet since we know its radius.

SET UP: For constant acceleration, $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$. At the surface of the planet, Newton's second law gives $m_{\text{rock}}g=\dfrac{Gm_{\text{rock}}m_{\text{p}}}{R_{\text{p}}^2}$.

EXECUTE: First find $a_y=g$. $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$. $a_y=\dfrac{2(y-y_0)}{t^2}=\dfrac{2(1.90\ \text{m})}{(0.480\ \text{s})^2}=16.49\ \text{m/s}^2=g$.

$g=16.49\ \text{m/s}^2$. $m_{\text{p}}=\dfrac{gR_{\text{p}}^2}{G}=\dfrac{(16.49\ \text{m/s})(8.60\times10^7\ \text{m})^2}{6.674\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}=1.83\times10^{27}\ \text{kg}.$

EVALUATE: The planet's mass is over 100 times that of the earth, which is reasonable since it is larger (in size) than the earth yet has a greater acceleration due to gravity at its surface.

13.56. IDENTIFY: Use the measurements of the motion of the rock to calculate g_{M}, the value of g on Mongo. Then use this to calculate the mass of Mongo. For the ship, $F_g=ma_{\text{rad}}$ and $T=\dfrac{2\pi r}{v}$.

SET UP: Take $+y$ upward. When the stone returns to the ground its velocity is 12.0 m/s, downward. $g_{\text{M}}=G\dfrac{m_{\text{M}}}{R_{\text{M}}^2}$. The radius of Mongo is $R_{\text{M}}=\dfrac{c}{2\pi}=\dfrac{2.00\times10^8\ \text{m}}{2\pi}=3.18\times10^7\ \text{m}$. The ship moves in an orbit of radius $r=3.18\times10^7\ \text{m}+3.00\times10^7\ \text{m}=6.18\times10^7\ \text{m}$.

EXECUTE: **(a)** $v_{0y}=+12.0$ m/s, $v_y=-12.0$ m/s, $a_y=-g_{\text{M}}$ and $t=4.80$ s. $v_y=v_{0y}+a_yt$ gives

$-g_{\text{M}}=\dfrac{v_y-v_{0y}}{t}=\dfrac{-12.0\ \text{m/s}-12.0\ \text{m/s}}{4.80\ \text{s}}$ and $g_{\text{M}}=5.00\ \text{m/s}^2$.

$m_{\text{M}}=\dfrac{g_{\text{M}}R_{\text{M}}^2}{G}=\dfrac{(5.00\ \text{m/s}^2)(3.18\times10^7\ \text{m})^2}{6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}=7.577\times10^{25}\ \text{kg}$ which rounds to 7.58×10^{25} kg.

(b) $F_g=ma_{\text{rad}}$ gives $G\dfrac{m_{\text{M}}m}{r^2}=m\dfrac{v^2}{r}$ and $v^2=\dfrac{Gm_{\text{M}}}{r}$.

$$T=\frac{2\pi r}{v}=2\pi r\sqrt{\frac{r}{Gm_{\text{M}}}}=\frac{2\pi r^{3/2}}{\sqrt{Gm_{\text{M}}}}=\frac{2\pi(6.18\times10^7\ \text{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.577\times10^{25}\ \text{kg})}}$$

$T=4.293\times10^4\ \text{s}=11.9\ \text{h}.$

EVALUATE: $R_{\text{M}}=5.0R_{\text{E}}$ and $m_{\text{M}}=12.7m_{\text{E}}$, so $g_{\text{M}}=\dfrac{12.7}{(5.0)^2}g_{\text{E}}=0.508g_{\text{E}}$, which agrees with the value calculated in part (a).

13.57. IDENTIFY: Use the orbital speed and altitude to find the mass of the planet. Use this mass and the planet's radius to find g at the surface. Use projectile motion to find the horizontal range x, where $x=\dfrac{v_0^2\sin(2\alpha)}{g}$.

SET UP: For an object in a circular orbit, $v = \sqrt{GM/r}$. $g = GM/r^2$. Call r the orbital radius and R the radius of the planet.

EXECUTE: $v = \sqrt{GM/r}$ gives $M = rv^2/G$. Using this to find g gives
$g = GM/R^2 = G(rv^2/G)/R^2 = v^2r/R^2 = (4900 \text{ m/s})^2(4.48\times10^6 \text{ m} + 6.30\times10^5 \text{ m})/(4.48\times10^6 \text{ m})^2 = 6.113$ m/s^2. Now use this acceleration to find the horizontal range.

$x = \dfrac{v_0^2\sin(2\alpha)}{g}$ = (12.6 m/s)^2 sin[2(30.8°)]/(6.113 m/s^2) = 22.8 m.

EVALUATE: On this planet, $g = 0.624g_E$, so the range is about 1.6 times what it would be on earth.

13.58. **IDENTIFY:** The 0.100 kg sphere has gravitational potential energy due to the other two spheres. Its mechanical energy is conserved.

SET UP: From energy conservation, $K_1 + U_1 = K_2 + U_2$, where $K = \frac{1}{2}mv^2$, and $U = -GmM/r$.

EXECUTE: Using $K_1 + U_1 = K_2 + U_2$, we have $K_1 = 0$, $m_A = 5.00$ kg, $m_B = 10.0$ kg and $m = 0.100$ kg.

$$U_1 = -\frac{Gmm_A}{r_{A1}} - \frac{Gmm_B}{r_{B1}} = -(6.674\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.100\ \text{kg})\left(\frac{5.00\ \text{kg}}{0.400\ \text{m}} + \frac{10.0\ \text{kg}}{0.600\ \text{m}}\right)$$

$U_1 = -1.9466\times10^{-10}$ J.

$$U_2 = -\frac{Gmm_A}{r_{A2}} - \frac{Gmm_B}{r_{B2}} = -(6.674\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.100\ \text{kg})\left(\frac{5.00\ \text{kg}}{0.800\ \text{m}} + \frac{10.0\ \text{kg}}{0.200\ \text{m}}\right)$$

$U_2 = -3.7541\times10^{-10}$ J. $K_2 = U_1 - U_2 = -1.9466\times10^{-10}\ \text{J} - (-3.7541\times10^{-10}\ \text{J}) = 1.8075\times10^{-10}$ J.

$$\frac{1}{2}mv^2 = K_2 \text{ and } v = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(1.8075\times10^{-10}\ \text{J})}{0.100\ \text{kg}}} = 6.01\times10^{-5}\ \text{m/s}.$$

EVALUATE: The kinetic energy gained by the sphere is equal to the loss in its potential energy.

13.59. **IDENTIFY:** This problem involves the gravitational flux, as defined in Ex. 13.40.

SET UP: From Ex. 13.40, the gravitational field is $g = GM/r^2$ outside of a spherically symmetric object of mass M. The flux is $\Phi_g = gA$. We want the flux through a spherical surface outside the object.

EXECUTE: $\Phi_g = gA = \left(\dfrac{GM}{r^2}\right)(4\pi r^2) = 4\pi GM$.

EVALUATE: The answer does *not* depend on the radius of the spherical surface since the r^2 factors cancel out.

13.60. **IDENTIFY:** In this problem we must calculate the gravitational force on a point mass M due to a uniform rod, so we need to use the gravitational force formula and integrate.

SET UP: Follow the hint in the problem. The magnitude of the force dF on M due to a tiny mass element dm is $dF_g = \dfrac{GMdm}{r^2}$. We integrate to find the total force. Fig. 13.60 shows the arrangement of masses and the force. Call the y-axis along the rod with the x-axis perpendicular to it at its midpoint. M is on the x-axis. We want to find the force on M.

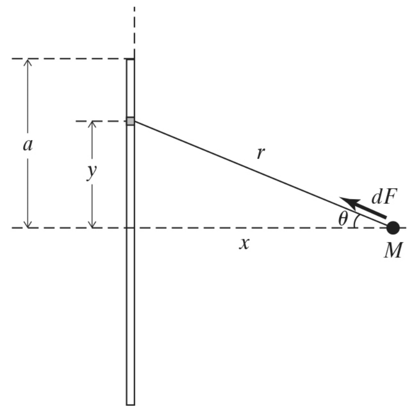


Figure 13.60

EXECUTE: **(a)** The rod is uniform. Therefore for every mass element dm above the x-axis, there is an equal element dm below the x-axis. The mass elements above the x-axis give M an upward pull and those below the x-axis give it a downward pull, and these pulls cancel. The mass elements also give a pull to the left (toward the rod), and these pulls add. Each of these pulls has an x-component $dF_x = dF\ \cos\theta$. Since the y-components cancel, the net force on M is due only to the x-components, so $F = \int dF_x$. We use $dF = \dfrac{GMdm}{r^2}$, where $\cos\theta = x/r$ and $r = \sqrt{x^2 + y^2}$. The mass element dm is the mass of a tiny segment of rod of length dy, so $dm = \rho dy$. Putting this information into the integral gives $F = \int_{-a}^{a} \dfrac{GM}{x^2 + y^2} \dfrac{x}{\sqrt{x^2 + y^2}} \rho dy = GM\rho x \int_{-a}^{a} \dfrac{dy}{\left(x^2 + y^2\right)^{3/2}}$. Using the integral tables in Appendix B gives

$F = GM\rho x \dfrac{1}{x^2} \dfrac{y}{\sqrt{x^2 + y^2}} \Bigg|_{-a}^{+a} = \dfrac{2GM\rho}{x} \dfrac{a}{\sqrt{x^2 + a^2}}$. In the problem statement we are given that r is the perpendicular distance from the center of the rod to M, which is x in our result. So rewriting in terms of r gives $F = \dfrac{2GM\rho a}{r\sqrt{r^2 + a^2}}$. The direction is toward the rod.

(b) If $a >> r$, the factor $\dfrac{a}{\sqrt{r^2 + a^2}} \to 1$, so $F \to \dfrac{2GM\rho}{r}$.

(c) For $a >> r$, we use the result in part (b), so the field is $g = \dfrac{F}{M} = \dfrac{\dfrac{2GM\rho}{r}}{M} = \dfrac{2G\rho}{r}$.

(d) For $a >> r$, we use the result in part (b), so the field is $g = \dfrac{2G\rho}{r}$ and the flux is

$\Phi_g = gA = \left(\dfrac{2G\rho}{r}\right)(2\pi rL) = 4\pi G\rho L$. The mass inside the cylinder is $m_{\text{inside}} = \rho L$, so the flux is

$\Phi_g = 4\pi Gm_{\text{inside}}$.

EVALUATE: In part (b) we see that the force obeys an inverse r law instead of an inverse square law.

13.61. **IDENTIFY** and **SET UP:** Apply conservation of energy. We must use $U = -\dfrac{GMm}{r}$ for the gravitational potential energy since h is not small compared to R_E.

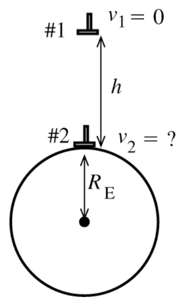


Figure 13.61

As indicated in Figure 13.61, take point 1 to be where the hammer is released and point 2 to be just above the surface of the earth, so $r_1 = R_E + h$ and $r_2 = R_E$.

EXECUTE: $K_1 + U_1 + W_{other} = K_2 + U_2$

Only gravity does work, so $W_{other} = 0$.

$K_1 = 0, \quad K_2 = \frac{1}{2}mv_2^2$

$$U_1 = -G\frac{mm_E}{r_1} = -\frac{Gmm_E}{h + R_E}, \quad U_2 = -G\frac{mm_E}{r_2} = -\frac{Gmm_E}{R_E}$$

Thus, $-G\dfrac{mm_E}{h + R_E} = \dfrac{1}{2}mv_2^2 - G\dfrac{mm_E}{R_E}$

$$v_2^2 = 2Gm_E\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right) = \frac{2Gm_E}{R_E(R_E + h)}(R_E + h - R_E) = \frac{2Gm_E h}{R_E(R_E + h)}$$

$$v_2 = \sqrt{\frac{2Gm_E h}{R_E(R_E + h)}}$$

EVALUATE: If $h \to \infty$, $v_2 \to \sqrt{2Gm_E/R_E}$, which equals the escape speed. In this limit this event is the reverse of an object being projected upward from the surface with the escape speed. If $h \ll R_E$, then $v_2 = \sqrt{2Gm_E h/R_E^2} = \sqrt{2gh}$, the same result if mgh is used for U.

13.62. IDENTIFY: In orbit the total mechanical energy of the satellite is $E = -\dfrac{Gm_E m}{2R_E}$. $U = -G\dfrac{m_E m}{r}$.

$W = E_2 - E_1$.

SET UP: $U \to 0$ as $r \to \infty$.

EXECUTE: **(a)** The energy the satellite has as it sits on the surface of the Earth is $E_1 = \dfrac{-GmM_E}{R_E}$. The energy it has when it is in orbit at a radius $R \approx R_E$ is $E_2 = \dfrac{-GmM_E}{2R_E}$. The work needed to put it in orbit is the difference between these: $W = E_2 - E_1 = \dfrac{GmM_E}{2R_E}$.

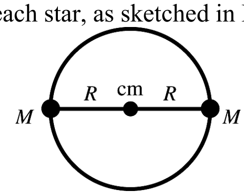
(b) The total energy of the satellite far away from the earth is zero, so the additional work needed is

$$0 - \left(\frac{-GmM_E}{2R_E}\right) = \frac{GmM_E}{2R_E}.$$

EVALUATE: **(c)** The work needed to put the satellite into orbit was the same as the work needed to put the satellite from orbit to the edge of the universe.

13.63. IDENTIFY: Use $F_g = Gm_1m_2/r^2$ to calculate F_g. Apply Newton's second law to circular motion of each star to find the orbital speed and period. Apply the conservation of energy to calculate the energy input (work) required to separate the two stars to infinity.

(a) SET UP: The cm is midway between the two stars since they have equal masses. Let R be the orbit radius for each star, as sketched in Figure 13.63.



The two stars are separated by a distance $2R$, so

$$F_g = GM^2/(2R)^2 = GM^2/4R^2$$

Figure 13.63

(b) EXECUTE: $F_g = ma_{rad}$

$GM^2/4R^2 = M(v^2/R)$ so $v = \sqrt{GM/4R}$

And $T = 2\pi R/v = 2\pi R\sqrt{4R/GM} = 4\pi\sqrt{R^3/GM}$

(c) SET UP: Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the system of the two stars. Separate to infinity implies $K_2 = 0$ and $U_2 = 0$.

EXECUTE: $K_1 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = 2\left(\frac{1}{2}M\right)(GM/4R) = GM^2/4R$

$U_1 = -GM^2/2R$

Thus the energy required is $W_{other} = -(K_1 + U_1) = -(GM^2/4R - GM^2/2R) = GM^2/4R$.

EVALUATE: The closer the stars are and the greater their mass, the larger their orbital speed, the shorter their orbital period and the greater the energy required to separate them.

13.64. IDENTIFY: In the center of mass coordinate system, $r_{cm} = 0$. Apply $\vec{F} = m\vec{a}$ to each star, where F is the gravitational force of one star on the other and $a = a_{rad} = \dfrac{4\pi^2 R}{T^2}$.

SET UP: $v = \dfrac{2\pi R}{T}$ allows R to be calculated from v and T.

EXECUTE: **(a)** The radii R_1 and R_2 are measured with respect to the center of mass, and so $M_1R_1 = M_2R_2$, and $R_1/R_2 = M_2/M_1$.

(b) The forces on each star are equal in magnitude, so the product of the mass and the radial accelerations are equal: $\dfrac{4\pi^2 M_1 R_1}{T_1^2} = \dfrac{4\pi^2 M_2 R_2}{T_2^2}$. From the result of part (a), the numerators of these expressions are equal, and so the denominators are equal, and the periods are the same. To find the period in the symmetric form desired, there are many possible routes. An elegant method, using a bit of hindsight, is to use the above expressions to relate the periods to the force $F_g = \dfrac{GM_1M_2}{(R_1+R_2)^2}$, so that equivalent expressions for the period are $M_2T^2 = \dfrac{4\pi^2 R_1(R_1+R_2)^2}{G}$ and $M_1T^2 = \dfrac{4\pi^2 R_2(R_1+R_2)^2}{G}$.

Adding the expressions gives $(M_1+M_2)T^2 = \dfrac{4\pi^2(R_1+R_2)^3}{G}$ or $T = \dfrac{2\pi(R_1+R_2)^{3/2}}{\sqrt{G(M_1+M_2)}}$.

(c) First we must find the radii of each orbit given the speed and period data. In a circular orbit,

$v = \dfrac{2\pi R}{T}$, or $R = \dfrac{vT}{2\pi}$. Thus $R_\alpha = \dfrac{(36\times10^3\ \text{m/s})(137\ \text{d})(86{,}400\ \text{s/d})}{2\pi} = 6.78\times10^{10}\ \text{m}$ and

$R_\beta = \dfrac{(12\times10^3\ \text{m/s})(137\ \text{d})(86{,}400\ \text{s/d})}{2\pi} = 2.26\times10^{10}\ \text{m}$. Now find the sum of the masses.

$(M_\alpha + M_\beta) = \dfrac{4\pi^2 (R_\alpha + R_\beta)^3}{T^2 G}$. Inserting the values of T and the radii gives

$(M_\alpha + M_\beta) = \dfrac{4\pi^2 (6.78\times10^{10}\ \text{m} + 2.26\times10^{10}\ \text{m})^3}{[(137\ \text{d})(86{,}400\ \text{s/d})]^2 (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 3.12\times10^{30}\ \text{kg}$. Since

$M_\beta = M_\alpha R_\alpha / R_\beta = 3M_\alpha$, $4M_\alpha = 3.12\times10^{30}$ kg, or $M_\alpha = 7.80\times10^{29}$ kg, and $M_\beta = 2.34\times10^{30}$ kg.

(d) Let α refer to the star and β refer to the black hole. Use the relationships derived in parts (a) and (b): $R_\beta = (M_\alpha / M_\beta) R_\alpha = (0.67/3.8) R_\alpha = (0.176) R_\alpha$, $R_\alpha + R_\beta = \sqrt[3]{\dfrac{(M_\alpha + M_\beta) T^2 G}{4\pi^2}}$. For Monocerotis, inserting the values for M and T gives $R_\alpha = 1.9\times10^9$ m, $v_\alpha = 4.4\times10^2$ km/s and for the black hole $R_\beta = 34\times10^8$ m, $v_\beta = 77$ km/s.

EVALUATE: Since T is the same, v is smaller when R is smaller.

13.65. **IDENTIFY** and **SET UP:** Use conservation of energy, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. The gravity force exerted by the sun is the only force that does work on the comet, so $W_{\text{other}} = 0$.

EXECUTE: $K_1 = \frac{1}{2} m v_1^2$, $v_1 = 2.0\times10^4$ m/s

$U_1 = -G m_S m / r_1$, where $r_1 = 2.5\times10^{11}$ m

$K_2 = \frac{1}{2} m v_2^2$

$U_2 = -G m_S m / r_2$, $r_2 = 5.0\times10^{10}$ m

$\frac{1}{2} m v_1^2 - G m_S m / r_1 = \frac{1}{2} m v_2^2 - G m_S m / r_2$

$$v_2^2 = v_1^2 + 2Gm_S\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = v_1^2 + 2Gm_S\left(\frac{r_1 - r_2}{r_1 r_2}\right)$$

$v_2 = 6.8\times10^4$ m/s

EVALUATE: The comet has greater speed when it is closer to the sun.

13.66. **IDENTIFY:** $g = \dfrac{GM}{R^2}$, where M and R are the mass and radius of the planet.

SET UP: Let m_U and R_U be the mass and radius of Uranus and let g_U be the acceleration due to gravity at its poles. The orbit radius of Miranda is $r = h + R_U$, where $h = 1.04\times10^8$ m is the altitude of Miranda above the surface of Uranus.

EXECUTE: **(a)** From the value of g at the poles,

$$m_U = \frac{g_U R_U^2}{G} = \frac{(9.0\ \text{m/s}^2)(2.5360\times10^7\ \text{m})^2}{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 8.674\times10^{25}\ \text{kg}$$ which rounds to 8.7×10^{25} kg.

(b) $G m_U / r^2 = g_U (R_U / r)^2 = 0.35\ \text{m/s}^2$.

(c) $G m_M / R_M^2 = 0.079\ \text{m/s}^2$.

EVALUATE: **(d)** No. Both the object and Miranda are in orbit together around Uranus, due to the gravitational force of Uranus. The object has additional force toward Miranda.

13.67. **(a) IDENTIFY** and **SET UP:** Use $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, applied to the satellites orbiting the earth rather than the sun.

EXECUTE: Find the value of a for the elliptical orbit:

$2a = r_a + r_p = R_E + h_a + R_E + h_p$, where h_a and h_p are the heights at apogee and perigee, respectively.

$a = R_E + (h_a + h_p)/2$

$a = 6.37\times10^6 \text{ m} + (400\times10^3 \text{ m} + 4000\times10^3 \text{ m})/2 = 8.57\times10^6 \text{ m}$

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_E}} = \frac{2\pi(8.57\times10^6 \text{ m})^{3/2}}{\sqrt{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24} \text{ kg})}} = 7.90\times10^3 \text{ s}$$

(b) Conservation of angular momentum gives $r_a v_a = r_p v_p$

$$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{6.37\times10^6 \text{ m} + 4.00\times10^6 \text{ m}}{6.37\times10^6 \text{ m} + 4.00\times10^5 \text{ m}} = 1.53.$$

(c) Conservation of energy applied to apogee and perigee gives $K_a + U_a = K_p + U_p$

$\frac{1}{2}mv_a^2 - Gm_E m/r_a = \frac{1}{2}mv_p^2 - Gm_E m/r_p$

$v_p^2 - v_a^2 = 2Gm_E(1/r_p - 1/r_a) = 2Gm_E(r_a - r_p)/r_a r_p$

But $v_p = 1.532 v_a$, so $1.347 v_a^2 = 2Gm_E(r_a - r_p)/r_a r_p$

$v_a = 5.51\times10^3 \text{ m/s}$, $v_p = 8.43\times10^3 \text{ m/s}$

(d) Need v so that $E = 0$, where $E = K + U$.

at perigee: $\frac{1}{2}mv_p^2 - Gm_E m/r_p = 0$

$$v_p = \sqrt{2Gm_E/r_p} = \sqrt{2(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24} \text{ kg})/(6.77\times10^6 \text{ m})} = 1.085\times10^4 \text{ m/s}$$

This means an increase of $1.085\times10^4 \text{ m/s} - 8.43\times10^3 \text{ m/s} = 2.42\times10^3 \text{ m/s}$.

at apogee:

$$v_a = \sqrt{2Gm_E/r_a} = \sqrt{2(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24} \text{ kg})/(1.037\times10^7 \text{ m})} = 8.763\times10^3 \text{ m/s}$$

This means an increase of $8.763\times10^3 \text{ m/s} - 5.51\times10^3 \text{ m/s} = 3.25\times10^3 \text{ m/s}$.

EVALUATE: Perigee is more efficient. At this point r is smaller so v is larger and the satellite has more kinetic energy and more total energy.

13.68. **IDENTIFY:** The engines do work on the rocket and change its kinetic energy and gravitational potential energy.

SET UP: Call M the mass of the earth and m the mass of the rocket. $U_g = -GMm/r$, $K = \frac{1}{2}mMG/r$ for a circular orbit, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

EXECUTE: **(a)** For a circular orbit, using $K = \frac{1}{2}mMG/r$ gives the difference in kinetic energy: $K_2 - K_1 = \frac{1}{2}mMG(1/r_2 - 1/r_1)$. Using the given numbers $m = 5000$ kg, $M = 5.97\times10^{24}$ kg, $r_2 = 8.80\times10^6$ m, and $r_1 = 7.20\times10^6$ m, we get $K_2 - K_1 = -2.52\times10^{10}$ J. The minus sign tells us that the kinetic energy decreases.

(b) $U_2 - U_1 = -GmM/r_2 - (-GmM/r_1) = GmM(1/r_1 - 1/r_2) = -2(K_2 - K_1) = +5.03\times10^{10}$ J. The plus sign means that the energy increases.

(c) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$W_{\text{other}} = (K_2 - K_1) + (U_2 - U_1) = -2.51\times10^{10} \text{ J} + 5.03\times10^{10} \text{ J} = +2.51\times10^{10}$ J.

EVALUATE: In part (b), the potential energy increases because it becomes less negative. The work is positive because the total energy increases.

13.69. **IDENTIFY** and **SET UP:** Apply conservation of energy, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, and solve for W_{other}. Only $r = h + R_E$ is given, so use $v = \sqrt{GM/r}$ to relate r and v.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$U_1 = -Gm_M m/r_1$, where m_M is the mass of Mars and $r_1 = R_M + h$, where R_M is the radius of Mars and $h = 2000\times10^3$ m.

$$U_1 = -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\ \text{kg})(5000\ \text{kg})}{3.39\times10^6\ \text{m} + 2000\times10^3\ \text{m}} = -3.97230\times10^{10}\ \text{J}$$

$U_2 = -Gm_M m/r_2$, where r_2 is the new orbit radius.

$$U_2 = -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\ \text{kg})(5000\ \text{kg})}{3.39\times10^6\ \text{m} + 4000\times10^3\ \text{m}} = -2.89725\times10^{10}\ \text{J}$$

For a circular orbit $v = \sqrt{Gm_M/r}$, with the mass of Mars rather than the mass of the earth.

Using this gives $K = \frac{1}{2}mv^2 = \frac{1}{2}m(Gm_M/r) = \frac{1}{2}Gm_M m/r$, so $K = -\frac{1}{2}U$.

$K_1 = -\frac{1}{2}U_1 = +1.98615\times10^{10}$ J and $K_2 = -\frac{1}{2}U_2 = +1.44863\times10^{10}$ J

Then $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$W_{\text{other}} = (K_2 - K_1) + (U_2 - U)$

$W_{\text{other}} = (1.44863\times10^{10}\ \text{J} - 1.98615\times10^{10}\ \text{J}) + (+3.97230\times10^{10}\ \text{J} - 2.89725\times10^{10}\ \text{J})$

$W_{\text{other}} = 5.38\times10^9$ J.

EVALUATE: When the orbit radius increases the kinetic energy decreases and the gravitational potential energy increases. $K = -U/2$ so $E = K + U = -U/2$ and the total energy also increases (becomes less negative). Positive work must be done to increase the total energy of the satellite.

13.70. **IDENTIFY:** The engines do work on the rocket and change its kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mMG/r = -\frac{1}{2}U_g$ for a circular orbit; $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ and $U_g = -2K$. Combining these two equations gives $K_1 - 2K_1 + W_{\text{other}} = K_2 - 2K_2$, so $K_2 = K_1 - W_{\text{other}}$. This gives $\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - W_{\text{other}}$. Solving for v_2 gives $v_2 = \sqrt{v_1^2 - \frac{2W_{\text{other}}}{m}} = \sqrt{(9640\ \text{m/s})^2 - \frac{2(-7.50\times10^9\ \text{J})}{848\ \text{kg}}} = 10{,}500$ m/s, which is greater than v_1, so the speed increases.

EVALUATE: The work is negative, yet the speed increases. The potential energy decreases (becomes more negative), so the total energy decreases. Due to the friction, the satellite will go to a lower orbit (closer to the earth), so it must have a greater speed to remain in orbit.

13.71. **IDENTIFY:** Integrate $dm = \rho dV$ to find the mass of the planet. Outside the planet, the planet behaves like a point mass, so at the surface $g = GM/R^2$.

SET UP: A thin spherical shell with thickness dr has volume $dV = 4\pi r^2 dr$. The earth has radius $R_E = 6.37\times10^6$ m.

EXECUTE: Get $M: M = \int dm = \int \rho dV = \int \rho 4\pi r^2 dr$. The density is $\rho = \rho_0 - br$, where $\rho_0 = 15.0\times10^3\ \text{kg/m}^3$ at the center and at the surface, $\rho_S = 2.0\times10^3\ \text{kg/m}^3$, so $b = \frac{\rho_0 - \rho_s}{R}$.

$$M = \int_0^R (\rho_0 - br)\,4\pi r^2 dr = \frac{4\pi}{3}\rho_0 R^3 - \pi b R^4 = \frac{4}{3}\pi R^3 \rho_0 - \pi R^4\left(\frac{\rho_0 - \rho_s}{R}\right) = \pi R^3\left(\frac{1}{3}\rho_0 + \rho_s\right)$$ and

$M = 5.71\times 10^{24}$ kg. Then $g = \dfrac{GM}{R^2} = \dfrac{G\pi R^3(\frac{1}{3}\rho_0 + \rho_s)}{R^2} = \pi R G\left(\dfrac{1}{3}\rho_0 + \rho_s\right)$.

$$g = \pi(6.37\times 10^6\,\text{m})(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{15.0\times 10^3\ \text{kg/m}^3}{3} + 2.0\times 10^3\ \text{kg/m}^3\right).$$

$g = 9.34\ \text{m/s}^2$.

EVALUATE: The average density of the planet is

$\rho_{\text{av}} = \dfrac{M}{V} = \dfrac{M}{\frac{4}{3}\pi R^3} = \dfrac{3(5.71\times 10^{24}\ \text{kg})}{4\pi(6.37\times 10^6\ \text{m})^3} = 5.27\times 10^3\ \text{kg/m}^3$. Note that this is not $(\rho_0 + \rho_s)/2$.

13.72. IDENTIFY and **SET UP:** Use $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ to calculate a.

$T = 30{,}000\ \text{y}(3.156\times 10^7\ \text{s/1 y}) = 9.468\times 10^{11}$ s

EXECUTE: $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, which gives $T^2 = \dfrac{4\pi^2 a^3}{Gm_S}$, so $a = \left(\dfrac{Gm_S T^2}{4\pi^2}\right)^{1/3} = 1.4\times 10^{14}$ m.

EVALUATE: The average orbit radius of Pluto is 5.9×10^{12} m (Appendix F); the semi-major axis for this comet is larger by a factor of 24. Converting to meters gives 4.3 light years = $4.3(9.461\times 10^{15}\ \text{m})$ = 4.1×10^{16} m. The distance of Alpha Centauri is larger by a factor of 300. The orbit of the comet extends well past Pluto but is well within the distance to Alpha Centauri.

13.73. IDENTIFY: The direct calculation of the force that the sphere exerts on the ring is slightly more involved than the calculation of the force that the ring exerts on the sphere. These forces are equal in magnitude but opposite in direction, so it will suffice to do the latter calculation. By symmetry, the force on the sphere will be along the axis of the ring in Figure E13.35 in the textbook, toward the ring.

SET UP: Divide the ring into infinitesimal elements with mass dM.

EXECUTE: Each mass element dM of the ring exerts a force of magnitude $\dfrac{(Gm)dM}{a^2 + x^2}$ on the sphere,and the x-component of this force is $\dfrac{GmdM}{a^2 + x^2}\dfrac{x}{\sqrt{a^2 + x^2}} = \dfrac{GmdMx}{(a^2 + x^2)^{3/2}}$.

Therefore, the force on the sphere is $GmMx/(a^2 + x^2)^{3/2}$, in the $-x$-direction. The sphere attracts the ring with a force of the same magnitude.

EVALUATE: As $x \gg a$ the denominator approaches x^3 and $F \to \dfrac{GMm}{x^2}$, as expected.

13.74. IDENTIFY and **SET UP:** Use $F_g = Gm_1 m_2/r^2$ to calculate the force between the point mass and a small segment of the semicircle.

EXECUTE: The radius of the semicircle is $R = L/\pi$.

Divide the semicircle up into small segments of length $R\,d\theta$, as shown in Figure 13.74.

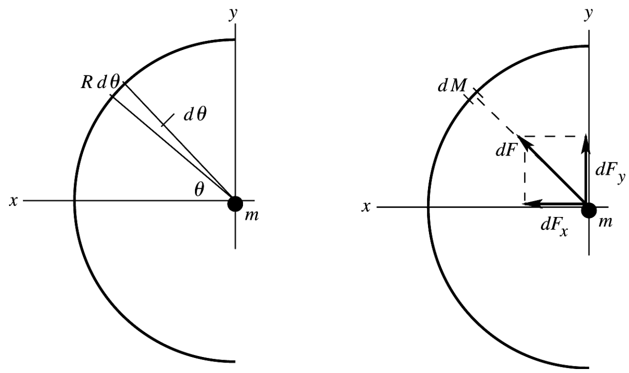


Figure 13.74

$dM = (M/L)R\,d\theta = (M/\pi)\,d\theta$

$d\vec{F}$ is the gravity force on m exerted by dM.

$\int dF_y = 0$; the y-components from the upper half of the semicircle cancel the y-components from the lower half.

The x-components are all in the $+x$-direction and all add.

$$dF = G\frac{mdM}{R^2}$$

$$dF_x = G\frac{mdM}{R^2}\cos\theta = \frac{Gm\pi M}{L^2}\cos\theta\,d\theta$$

$$F_x = \int_{-\pi/2}^{\pi/2} dF_x = \frac{Gm\pi M}{L^2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{Gm\pi M}{L^2}(2)a$$

$$F = \frac{2\pi GmM}{L^2}$$

EVALUATE: If the semicircle were replaced by a point mass M at $x = R$, the gravity force would be $GmM/R^2 = \pi^2 GmM/L^2$. This is $\pi/2$ times larger than the force exerted by the semicirclar wire. For the semicircle it is the x-components that add, and the sum is less than if the force magnitudes were added.

13.75. **IDENTIFY:** Compare F_E to Hooke's law.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg and radius $R_E = 6.37\times10^6$ m.

EXECUTE: **(a)** For $F_x = -kx$, $U = \frac{1}{2}kx^2$. The force here is in the same form, so by analogy $U(r) = \frac{Gm_E m}{2R_E^3}r^2$. This is also given by the integral of F_g from 0 to r with respect to distance.

(b) From part (a), the initial gravitational potential energy is $\frac{Gm_E m}{2R_E}$. Equating initial potential energy and final kinetic energy (initial kinetic energy and final potential energy are both zero) gives $v^2 = \frac{Gm_E}{R_E}$, so $v = 7.91\times10^3$ m/s.

EVALUATE: When $r = 0$, $U(r) = 0$, as specified in the problem.

13.76. **IDENTIFY:** Kepler's third law applies to the planets.

SET UP: $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$.

EXECUTE: Squaring $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ gives $T^2 = \left(\dfrac{4\pi^2}{Gm_S}\right)a^3$. If we graph T^2 versus a^3, the equation is of the slope-y-intercept form of a straight line, $y = mx + b$. In this case, the slope is $(4\pi^2/Gm_S)$, and the y-intercept is zero. Take logs of both sides of the equation in Kepler's third law, giving $\log T = \log\left[\left(\dfrac{2\pi}{\sqrt{Gm_S}}\right)a^{3/2}\right]$, which can be written as $\log T = \dfrac{3}{2}\log a + \log\left(\dfrac{2\pi}{\sqrt{Gm_s}}\right)$. Therefore graphing $\log T$ versus $\log a$ should yield a straight line. Figure 13.76 shows this graph.

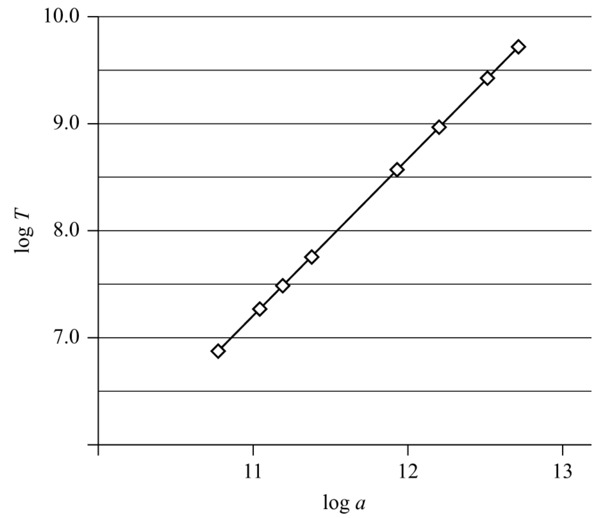


Figure 13.76

(b) In the slope-y-intercept form $y = mx + b$, the slope is 3/2. The best-fit equation of the graph in the figure is $y = 1.4986x - 9.2476$. Since 1.4986 rounds to 1.50, which is equal to 3/2, our graph has the expected slope.

(c) The y-intercept is $\log\left(\dfrac{2\pi}{\sqrt{Gm_S}}\right) = -9.2476$. Solving for m_S, we get $\dfrac{2\pi}{\sqrt{Gm_S}} = 10^{-9.2476}$. Squaring and solving for m_S gives $m_S = 4\pi^2[10^{2(9.2476)}]/G = 1.85\times10^{30}$ kg. From Appendix F, $m_S = 1.99\times10^{30}$ kg. Our result agrees to within about 7% with the value in Appendix F.

(d) Solving Kepler's third law for a gives $a = \left(\dfrac{T\sqrt{Gm_s}}{2\pi}\right)^{2/3}$. Expressing T in seconds, we get $T = 1325.4$ d (24 h/d)(3600 s/h) $= 1.145\times10^8$ s. Putting in $G = 6.674\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ and $m_S = 1.99\times10^{30}$ kg, we get $a = 3.53\times10^{11}$ m $= 353\times10^6$ km. From the table with the problem, we see that Vesta's orbit lies between that of Mars and Jupiter.

EVALUATE: The asteroid belt lies between Mars and Jupiter, which is why Vesta is usually considered an asteroid.

13.77. IDENTIFY and **SET UP:** At the surface of a planet, $g = \dfrac{GM}{R^2}$, and average density is $\rho = m/V$, where $V = 4/3\ \pi R^3$ for a sphere.

EXECUTE: We have expressions for g and M: $g = \dfrac{GM}{R^2}$ and $M = \rho V = \rho\left(\dfrac{4}{3}\pi R^3\right)$. Combining them

we get $g = \dfrac{G\rho\left(\dfrac{4}{3}\pi R^3\right)}{R^2} = \dfrac{4\pi G\rho R}{3}$. Using $R = D/2$ gives $g = \dfrac{2\pi G\rho D}{3}$.

(a) A graph of g versus D is shown in Figure 13.77. As this graph shows, the densities vary considerably and show no apparent pattern.

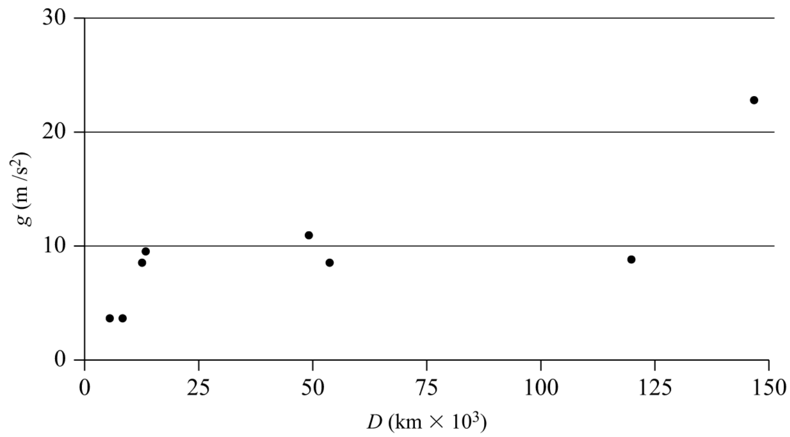


Figure 13.77

(b) Using the equation we just derived, $g = \dfrac{2\pi G\rho D}{3}$, we solve for ρ and use the values from the table given in the problem. For example, for Mercury we have

$\rho = \dfrac{3g}{2\pi DG} = \dfrac{3(3.7\ \text{m/s}^2)}{2\pi(4.879\times10^6\ \text{m})(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 5400\ \text{kg/m}^3$. Continuing the calculations and putting the results in order of decreasing density, we get the following results.

Earth: 5500 kg/m^3
Mercury: 5400 kg/m^3
Venus: 5300 kg/m^3
Mars: 3900 kg/m^3
Neptune: 1600 kg/m^3
Uranus: 1200 kg/m^3
Jupiter: 1200 kg/m^3
Saturn: 534 kg/m^3

(c) For several reasons, it is reasonable that the other planets would be denser toward their centers. Gravity is stronger at close distances, so it would compress matter near the center. In addition, during the formation of planets, heavy elements would tend to sink toward the center and displace light elements, much as a rock sinks in water. This variation in density would have no effect on our analysis however, since the planets are still spherically symmetric.

(d) $g = \dfrac{2\pi G\rho D}{3} = \dfrac{2\pi(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.20536\times10^8\ \text{m})(5500\ \text{kg/m}^3)}{3} = 93\ \text{m/s}^2$.

EVALUATE: Saturn is less dense than water, so it would float if we could throw it into our ocean (which of course is impossible since it is much larger than the earth). This low density is the reason that g at its "surface" is less than g at the earth's surface, even though the mass of Saturn is much greater than that of the earth. Also note in our results in (b) that the inner four planets are much denser than the outer four (the gas giants), with the earth being the densest of all.

13.78. **IDENTIFY** and **SET UP:** $L = MvR$, $v = 2\pi R/T$.

EXECUTE: (a) Combining the two equations gives $L = mvR = m(2\pi R/T)R = 2\pi mR^2/T$.

(b) We use our formula with the quantities in Appendix F. For example, for Mercury we have $L = 2\pi(3.30\times10^{23}\text{ kg})(5.79\times10^{10}\text{ m})^2/[(88.0\text{ d})(24\text{ h/d})(3600\text{ s/h})] = 9.14\times10^{38}\text{ kg}\cdot\text{m}^2/\text{s}$. Similar calculations give the following results.

Mercury: 9.14×10^{38} kg · m^2/s
Venus: 1.84×10^{40} kg · m^2/s
Earth: 2.67×10^{40} kg · m^2/s
Mars: 3.53×10^{39} kg · m^2/s
Jupiter: 1.93×10^{43} kg · m^2/s
Saturn: 7.86×10^{42} kg · m^2/s
Uranus: 1.73×10^{42} kg · m^2/s
Neptune: 2.50×10^{42} kg · m^2/s

The total angular momentum is the sum of all of these since the planets all move in the same direction around the sun. $L_{\text{tot}} = 3.13\times10^{43}\text{ kg}\cdot\text{m}^2/\text{s}$.

(c) Treating the sun as a uniform solid sphere, we have

$$L = I\omega = \left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right) = \frac{4\pi MR^2}{5T} = \frac{4\pi(1.99\times10^{30}\text{ kg})(6.96\times10^8\text{ m})^2}{5(24.6\text{ d})(24\text{ h/d})(3600\text{ s/h})} = 1.14\times10^{42}\text{ kg}\cdot\text{m}^2/\text{s}.$$

(d) $L_S/L_P = (1.14\times10^{42}\text{ kg}\cdot\text{m}^2/\text{s})/(3.14\times10^{43}\text{ kg}\cdot\text{m}^2/\text{s}) = 0.0363$. The angular momentum of the sun is only 3.63% of the angular momentum of the planets. From Appendix F, the total mass of the planets is $m_P = 2.669\times10^{27}$ kg, so $m_S/m_P = (1.99\times10^{30}\text{ kg})/(2.669\times10^{27}\text{ kg}) = 746$, so the sun is 746 times as massive as the planets combined.

EVALUATE: The sun contains nearly all the mass in the solar system, yet it has only 3.6% of the angular momentum. There appears to be no progressive pattern in the angular momentum of the planets as we go from the inner plants to the outer ones.

13.79. **IDENTIFY:** Apply $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ to the transfer orbit.

SET UP: The orbit radius for earth is $r_E = 1.50\times10^{11}$ m and for Mars it is $r_M = 2.28\times10^{11}$ m. From Figure 13.18 in the textbook, $a = \frac{1}{2}(r_E + r_M)$.

EXECUTE: **(a)** To get from the circular orbit of the earth to the transfer orbit, the spacecraft's energy must increase, and the rockets are fired in the direction opposite that of the motion, that is, in the direction that increases the speed. Once at the orbit of Mars, the energy needs to be increased again, and so the rockets need to be fired in the direction opposite that of the motion. From Figure 13.18 in the textbook, the semimajor axis of the transfer orbit is the arithmetic average of the orbit radii of the earth and Mars, and so from $E = -Gm_Sm/2r$, the energy of the spacecraft while in the transfer orbit is intermediate between the energies of the circular orbits. Returning from Mars to the earth, the procedure is reversed, and the rockets are fired against the direction of motion.

(b) The time will be half the period as given in $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, with the semimajor axis equal to

$a = \frac{1}{2}(r_E + r_M) = 1.89\times10^{11}$ ma so

$$t = \frac{T}{2} = \frac{\pi\,(1.89\times10^{11}\text{ m})^{3/2}}{\sqrt{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}} = 2.24\times10^7\text{ s} = 259\text{ days},$$ which is more than $8\frac{1}{2}$ months.

(c) During this time, Mars will pass through an angle of $(360°)\dfrac{(2.24\times10^{7}\text{ s})}{(687\text{ d})(86{,}400\text{ s/d})}=135.9°,$ and the spacecraft passes through an angle of $180°,$ so the angle between the earth-sun line and the Mars-sun line must be $44.1°.$

EVALUATE: The period T for the transfer orbit is 526 days, the average of the orbital periods for earth and Mars.

13.80. IDENTIFY: Apply $\Sigma\vec{F}=m\vec{a}$ to each ear.

SET UP: Denote the orbit radius as r and the distance from this radius to either ear as $\delta.$ Each ear, of mass $m,$ can be modeled as subject to two forces, the gravitational force from the black hole and the tension force (actually the force from the body tissues), denoted by $F.$

EXECUTE: The force equation for either ear is $\dfrac{GMm}{(r+\delta)^2}-F=m\omega^2(r+\delta),$ where δ can be of either sign. Replace the product $m\omega^2$ with the value for $\delta=0,$ $m\omega^2=GMm/r^3,$ and solve for F:

$$F=(GMm)\left[\frac{r+\delta}{r^3}-\frac{1}{(r+\delta)^2}\right]=\frac{GMm}{r^3}\left[r+\delta-r(1+(\delta/r)^{-2}\right].$$

Using the binomial theorem to expand the term in square brackets in powers of $\delta/r,$

$$F\approx\frac{GMm}{r^3}[r+\delta-r(1-2(\delta/r))]=\frac{GMm}{r^3}(3\delta)=2.1\text{ kN}.$$

This tension is much larger than that which could be sustained by human tissue, and the astronaut is in trouble.

(b) The center of gravity is not the center of mass. The gravity force on the two ears is not the same.

EVALUATE: The tension between her ears is proportional to their separation.

13.81. IDENTIFY: As suggested in the problem, divide the disk into rings of radius r and thickness dr.

SET UP: Each ring has an area $dA=2\pi r\,dr$ and mass $dM=\dfrac{M}{\pi a^2}dA=\dfrac{2M}{a^2}r\,dr.$

EXECUTE: The magnitude of the force that this small ring exerts on the mass m is then $(Gm\,dM)(x/(r^2+x^2)^{3/2}).$ The contribution dF to the force is $dF=\dfrac{2GMmx}{a^2}\dfrac{rdr}{(x^2+r^2)^{3/2}}.$

The total force F is then the integral over the range of r;

$$F=\int dF=\frac{2GMmx}{a^2}\int_0^a\frac{r}{(x^2+r^2)^{3/2}}dr.$$

The integral (either by looking in a table or making the substitution $u=r^2+a^2$) is

$$\int_0^a\frac{r}{(x^2+r^2)^{3/2}}dr=\left[\frac{1}{x}-\frac{1}{\sqrt{a^2+x^2}}\right]=\frac{1}{x}\left[1-\frac{x}{\sqrt{a^2+x^2}}\right].$$

Substitution yields the result $F=\dfrac{2GMm}{a^2}\left[1-\dfrac{x}{\sqrt{a^2+x^2}}\right].$ The force on m is directed toward the center of the ring. The second term in brackets can be written as

$$\frac{1}{\sqrt{1+(a/x)^2}}=(1+(a/x)^2)^{-1/2}\approx1-\frac{1}{2}\left(\frac{a}{x}\right)^2$$

if $x\gg a,$ where the binomial expansion has been used. Substitution of this into the above form gives $F\approx\dfrac{GMm}{x^2},$ as it should.

EVALUATE: As $x\to0,$ the force approaches a constant.

13.82. **IDENTIFY** and **SET UP:** Use $\rho = m/V$ to calculate the density of the planet and then use the table given in the problem to estimate its composition.
EXECUTE: Using $\rho = m/V$ gives

$$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{7.9m_{\mathrm{E}}}{\frac{4}{3}\pi(2.3R_{\mathrm{E}})^3} = \frac{7.9}{(2.3)^3}\frac{m_{\mathrm{E}}}{\frac{4}{3}\pi(R_{\mathrm{E}})^3} = 0.65\rho_{\mathrm{E}}.$$

From the table, this density is in the range 0.4–0.9 times the density of the earth, so the planet probably has an iron core with a rock mantle and some lighter elements, which is choice (c).
EVALUATE: A method such as this gives only an estimation of the composition of a planet.

13.83. **IDENTIFY** and **SET UP:** Use $g = GM/R^2$.
EXECUTE: $g = GM/R^2 = G(7.9m_{\mathrm{E}})/(2.3R_{\mathrm{E}})^2 = [(7.9)/(2.3)^2](Gm_{\mathrm{E}}/R_{\mathrm{E}}^2) = 1.5g_{\mathrm{E}}$, which is choice (c).
EVALUATE: Even though this planet has 7.9 times the mass of the earth, g at its surface is only $1.5g_{\mathrm{E}}$ because the planet is 2.3 times the radius of the earth, which makes the surface farther away from its center than is the case with the earth.

13.84. **IDENTIFY** and **SET UP:** Apply Newton's second law and the law of universal gravitation to the planet, calling m the mass of the planet, M the mass of the star, r the orbital radius, and T the time for one orbit. $\Sigma F = ma$, $F_{\mathrm{g}} = Gm_1m_2/r^2$, $a_{\mathrm{rad}} = mv^2/r$, $v = 2\pi r/T$.
EXECUTE: $\frac{GmM}{r^2} = \frac{mv^2}{r} = \frac{m(2\pi r/T)^2}{r} = \frac{4\pi^2 mr}{T^2}$. Now solve for r, which gives

$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{G(0.70M_{\mathrm{sun}})\left(\frac{9.5}{365}T_{\mathrm{earth}}\right)^2}{4\pi^2} = (0.70)\left(\frac{9.5}{365}\right)^2\left(\frac{GM_{\mathrm{sun}}T_{\mathrm{earth}}^2}{4\pi^2}\right) = 0.000474r_{\mathrm{earth}}^3$$

$r = (0.000474)^{1/3}r_{\mathrm{earth}} = 0.078r_{\mathrm{earth}}$, which is choice (b).
EVALUATE: The planet takes only 9.5 days for one orbit, yet the star has 70% the mass of our sun, so the planet must be very close to the star compared to the earth. And this is, in fact, what we have found, since r for this planet iss 7.8% the distance of the earth from the sun.

14

PERIODIC MOTION

VP14.3.1. **IDENTIFY:** The glider undergoes SHM on the spring.

SET UP: For SHM, $T = 1/f$, $\omega = 2\pi f$, $f = \dfrac{1}{2\pi}\sqrt{k/m}$, and $F = kx$ for an ideal spring.

EXECUTE: **(a)** $\omega = 2\pi f = 2\pi(4.15\text{ Hz}) = 26.1\text{ rad/s}$. $T = 1/f = 1/(4.15\text{ Hz}) = 0.241\text{ s}$.

(b) Use $f = \dfrac{1}{2\pi}\sqrt{k/m}$ to solve for k: $k = 4\pi^2 f^2 m = 4\pi^2(4.15\text{ Hz})^2(0.400\text{ kg}) = 272\text{ N/m}$.

(c) $F = kx = (272\text{ N/m})(0.0200\text{ m}) = 5.44\text{ N}$.

EVALUATE: The force in part (c) is around a pound.

VP14.3.2. **IDENTIFY:** The puck is executing SHM on the spring.

SET UP: For SHM $T = 1/f$, $T = 2\pi\sqrt{m/k}$, and $a_{max} = A\omega^2$. Our target variables are the mass of the puck and the amplitude of the oscillations.

EXECUTE: **(a)** Solve $T = 2\pi\sqrt{m/k}$ for m: $m = \dfrac{T^2 k}{4\pi^2} = \dfrac{(1.20\text{ s})^2(4.50\text{ N/m})}{4\pi^2} = 0.164\text{ kg}$.

(b) Solve $a_{max} = A\omega^2$ for A: $A = \dfrac{a_{max}}{\omega^2} = \dfrac{a_{max}}{(2\pi/T)^2} = \dfrac{1.20\text{ m/s}^2}{[2\pi/(1.20\text{ s})]^2} = 0.0438\text{ m} = 4.38\text{ cm}$.

EVALUATE: The maximum acceleration occurs at the instants that the puck has stopped moving, which are at the extremes of its motion when $x = \pm A$.

VP14.3.3. **IDENTIFY:** The piston is moving in SHM.

SET UP: For SHM, $x(t) = A\cos(\omega t + \phi)$ and $v_{max} = A\omega$. We know that the frequency is 50.0 Hz and at a certain instant $x = 0.0300$ m and $v = 12.5$ m/s; but we don't know if either x or v are positive or negative. The target variables are the amplitude of the motion and the maximum speed of the piston.

EXECUTE: **(a)** With $x = A\cos(\omega t + \phi)$, we have $v = dx/dt = -A\omega\sin(\omega t + \phi)$. Since we don't know the sign of x or v at the instant in question, so we can neglect the minus sign for v. Taking the ration of v/x gives $\dfrac{v}{x} = \dfrac{A\omega\sin(\omega t + \phi)}{A\cos(\omega t + \phi)} = \omega\tan(\omega t + \phi)$. We know this ratio at the instant in question, so we can use this result to find $(\omega t + \phi)$. Knowing this, we can use the known value of x to find the amplitude A.

Using the known values, first find $(\omega t + \phi)$. Solving $\dfrac{v}{x} = \omega\tan(\omega t + \phi)$ for $(\omega t + \phi)$ gives

$(\omega t + \phi) = \arctan\left(\dfrac{v}{x\omega}\right) = \arctan\left(\dfrac{v}{2\pi f x}\right)$. Using the known values we have $(\omega t + \phi)$ =

$\arctan\left(\dfrac{12.5\text{ m/s}}{2\pi(50.0\text{ Hz})(0.0300\text{ m})}\right) = 52.984°$. Now use $x = A\cos(\omega t + \phi)$ to find A, giving 0.0300 m = A cos(52.984°), so $A = 0.0498$ m.

(b) $v_{max} = A\omega = 2\pi fA = 2\pi(50.0\text{ Hz})(0.0498\text{ m}) = 15.7\text{ m/s}$.

EVALUATE: From the information give, we don't know the exact position or velocity of the piston. It could be to the right or left of the origin moving either right or left. But none of this affects the amplitude or maximum speed.

VP14.3.4. IDENTIFY: The cat and platform oscillate together in SHM.

SET UP: We use $f = \frac{1}{2\pi}\sqrt{k/m}$ and $a_{max} = \omega^2 A$ with $\omega = 2\pi f$. The target variables are the frequency of vibration and the amplitude of the motion so that the acceleration that will not disturb the sleeping cat.

EXECUTE: **(a)** $f = \frac{1}{2\pi}\sqrt{k/m} = .\frac{1}{2\pi}\sqrt{\frac{185\text{ N/m}}{5.00\text{ kg}}} == 0.968\text{ Hz}.$.

(b) Use $a_{max} = \omega^2 A = (2\pi f)^2 A$ to solve for A: $A = \frac{a_{max}}{(2\pi f)^2} = \frac{1.52\text{ m/s}^2}{[2\pi(0.968\text{ Hz})]^2} = 0.0411\text{ m}$.

EVALUATE: This motion makes about one vibration per second with an amplitude of about 4 cm, so it is not particularly fast.

VP14.4.1. IDENTIFY: This problem deals with the energy of a glider attached to a spring and oscillating with SHM.

SET UP: We use $v_{max} = A\omega$, $\omega = \sqrt{k/m}$, $K = \frac{1}{2}mv^2$, and $U = \frac{1}{2}kx^2$. The target variables are the amplitude A of the motion, the total mechanical energy E of the system, and the potential energy and kinetic energy at a certain point.

EXECUTE: **(a)** Use $v_{max} = A\omega$ and $\omega = \sqrt{k/m}$ to solve for A. We get $A = \frac{v_{max}}{\omega}$, which gives

$$A = v_{max}\sqrt{\frac{m}{k}} = (0.350\text{ m/s})\sqrt{\frac{0.150\text{ kg}}{8.00\text{ N/m}}} = 0.0479\text{ m}.$$

(b) $E = K_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.150\text{ kg})(0.350\text{ m/s})^2 = 9.19\times 10^{-3}\text{ J}$.

(c) $U = \frac{1}{2}kx^2 = \frac{1}{2}(8.00\text{ N/m})(0.0300\text{ m})^2 = 3.60\times 10^{-3}\text{ J}$.

$K = E - U = 9.19\times 10^{-3}\text{ J} - 3.60\times 10^{-3}\text{ J} = 5.59\times 10^{-3}\text{ J}$.

EVALUATE: Another way to find the amplitude is to realize that $K_{max} = U_{max}$. Therefore $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$, which gives $A = \sqrt{\frac{mv_{max}^2}{k}} = \sqrt{\frac{(0.150\text{ kg})(0.350\text{ m/s})^2}{8.00\text{ N/m}}} = 0.479$ m, which agrees with your result.

VP14.4.2. IDENTIFY: The block oscillates in SHM on the spring.

SET UP: The total mechanical energy is $E = K + U$, where $K = \frac{1}{2}mv^2$. We also know that $\omega = \sqrt{k/m}$ and $v_{max} = A\omega$. Our target variables are the force constant of the spring and the speed of the block when the potential energy equals one-half the total mechanical energy.

EXECUTE: **(a)** When $x = 0$, $U = 0$ so $K = K_{max}$. Therefore $v = v_{max} = A\omega$. This tells us that $E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}m(A\omega)^2$. Using $\omega = \sqrt{k/m}$, this becomes $E = \frac{1}{2}m\left(A\sqrt{\frac{k}{m}}\right)^2$, which gives

$$k = \frac{2E}{A^2} = \frac{2(6.00\times 10^{-2}\text{ J})}{(0.0440\text{ m})^2} = 62.0\text{ N/m}.$$

(b) If $U = E/2$, then K must also equal $E/2$, so $\frac{1}{2}mv^2 = \frac{1}{2}E$. This gives $v = \sqrt{\frac{E}{m}} = \sqrt{\frac{6.00\times10^{-2}\text{ J}}{0.300\text{ kg}}} =$ 0.447 m/s.

EVALUATE: Note that we *cannot* say that $v = v_{max}/2$ when $K = K_{max}/2$ because K depends on the *square* of v.

VP14.4.3. **IDENTIFY:** The glider oscillates in SHM on the spring.

SET UP: The total mechanical energy is $E = K + U$, where $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$. We know the total mechanical energy E of the system, the amplitude of the oscillations, and the maximum speed of the glider. We also know that $a_{max} = \omega^2 A$ and $\omega = \sqrt{k/m}$, $x(t) = A\cos(\omega t + \phi)$, and $a(t) = -\omega^2 A\cos(\omega t + \phi)$. Our target variables are the force constant of the spring, the mass m of the glider, maximum acceleration of the glider, and its acceleration when the potential energy is 3.00×10^{-3} J.

EXECUTE: **(a)** When $x = 0$, $U = 0$ so $K = K_{max}$ and $E = K_{max}$. So $\frac{1}{2}mv_{max}^2 = E$, which gives

$$m = \frac{2E}{v_{max}^2} = \frac{2(4.00\times10^{-3}\text{ J})}{(0.125\text{ m/s})^2} = 0.512\text{ kg.}$$

When $x = A$, $K = 0$ so $U = U_{max}$, so $E = U_{max} = \frac{1}{2}kA^2$ which gives $k = \frac{2E}{A^2} = \frac{2(4.00\times10^{-3}\text{ J})}{(0.0300\text{ m})^2} = 8.89$ N/m.

(b) $a_{max} = \omega^2 A = \frac{k}{m}A = \left(\frac{8.89\text{ N/m}}{0.512\text{ kg}}\right)(0.0300\text{ m}) = 0.521\text{ m/s}^2$.

(c) We can use if we can find the value of $\cos(\omega t + \phi)$. We know that $U = 3.00\times10^{-3}\text{ J} = \frac{1}{2}kx^2$ and that $x(t) = A\cos(\omega t + \phi)$. Therefore $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$, which gives $\cos(\omega t + \phi) = \sqrt{\frac{2U}{kA^2}}$. Now find the acceleration for this value of $\cos(\omega t + \phi)$. We can drop the minus sign since we only want the magnitude of the acceleration. $a(t) = \omega^2 A\cos(\omega t + \phi) = \omega^2 A\sqrt{\frac{2U}{kA^2}} = \frac{k}{m}\sqrt{\frac{2U}{k}}$. Using $k = 8.89$ N/m, $m = 0.512$ kg, and $U = 3.00\times10^{-3}$ J, we have $a = 0.451\text{ m/s}^2$.

EVALUATE: In part (c) the acceleration is less than the maximum of 0.521 m/s^2 from part (b), so our result is reasonable.

VP14.4.4. **IDENTIFY:** An object is in SHM, and the energy of the system is conserved.

SET UP: The total energy is $E = K + U$ and is constant, where $U = \frac{1}{2}kx^2$. We want to know the values of x when the kinetic energy is equal to 1/3 of the total mechanical energy and to 1/5 of the total mechanical energy.

EXECUTE: **(a)** If $K = 1/3\ E$, then $U = 2/3\ E$, and $E = \frac{1}{2}kA^2$. This gives $\frac{1}{2}kx^2 = \frac{2}{3}\left(\frac{1}{2}kA^2\right)$, so

$$x = \pm A\sqrt{\frac{2}{3}}.$$

(b) Proceed as in part (a). If $K = 4/5\,E$, then $U = 1/5\,E$, which gives $\frac{1}{2}kx^2 = \frac{1}{5}\left(\frac{1}{2}kA^2\right)$, so $x = \pm\frac{A}{\sqrt{5}}$.

EVALUATE: We get square roots in our answers because U depends on the *square* of x and K depends on the *square* of v.

VP14.9.1. **IDENTIFY:** We are dealing with the small oscillations of a simple pendulum.

SET UP: $T = 1/f$ and $T = 2\pi\sqrt{L/g}$ for small oscillations. We want to find g on the alien planet, but first we need the period T.

EXECUTE: **(a)** $T = 1/f = 1/(0.609\text{ Hz}) = 1.64\text{ s}$.

(b) Solve $T = 2\pi\sqrt{L/g}$ for g, giving $g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(0.500\text{ m})}{(1.64\text{ s})^2} = 7.32\text{ m/s}^2$.

EVALUATE: This would be a very simple way to determine g since measurements of L and T are quite easy to make.

VP14.9.2. **IDENTIFY:** We are comparing the oscillation frequencies of a simple pendulum and an object attached to a spring.

SET UP: For the glider $\omega = \sqrt{k/m}$, and for a simple pendulum $\omega = \sqrt{g/L}$. The target variable is the length L of the pendulum so that it oscillates with the same frequency as the object attached to the spring.

EXECUTE: If the oscillation frequencies are the same, the angular frequencies must also be the same. So $\sqrt{k/m} = \sqrt{g/L}$, which gives $L = \frac{mg}{k} = \frac{(0.350\text{ kg})(9.80\text{ m/s}^2)}{8.75\text{ N/m}} = 0.392\text{ m}$.

EVALUATE: Our result is only accurate if the pendulum makes small oscillations.

VP14.9.3. **IDENTIFY:** The bicycle tire oscillates, but it is a *physical* pendulum, not a simple pendulum.

SET UP: The angular frequency is $\omega = \sqrt{\frac{Mgd}{I}}$ and $\omega = 2\pi f$.

EXECUTE: Using the given moment of inertia, we have $2\pi f = \sqrt{\frac{Mgd}{I}} = \sqrt{\frac{MgR}{2MR^2}} = \sqrt{\frac{g}{2R}}$. The oscillation frequency is $f = \frac{1}{2\pi}\sqrt{\frac{g}{2R}}$.

EVALUATE: If R is large, f is small, meaning that the wheel oscillates with a long period. This is analogous to a long simple pendulum, which oscillates slowly. If g were large, the frequency would be large since gravity could pull it back quickly from its extremes. Both cases suggest that our result is physically plausible.

VP14.9.4. **IDENTIFY:** The rod swings back and forth, but it is not a *simple* pendulum because its mass is spread out. Instead it is a *physical* pendulum.

SET UP: The period of swing is $T = 2\pi\sqrt{\frac{I}{mgd}}$. We want to find the moment of inertia of the rod.

EXECUTE: Use $T = 2\pi\sqrt{\frac{I}{mgd}}$ and solve for I, giving $I = mgd\left(\frac{T}{2\pi}\right)^2$. Therefore we get

$$I = (0.600\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m})\left(\frac{1.59\text{ s}}{2\pi}\right)^2 = 0.188\text{ kg}\cdot\text{m}^2.$$

EVALUATE: If this rod were uniform, its moment of inertia about the pivot would be $I = \frac{1}{3}ML^2 = \frac{1}{3}(0.600\text{ kg})(0.900\text{ m})^2 = 0.162\text{ kg}\cdot\text{m}^2$, which is *less than* we found. This is reasonable because the center of mass of this pendulum is *below* the midpoint of the rod, so it will have a larger moment of inertia about the pivot point than if it were uniform.

14.1. **IDENTIFY:** We want to relate the characteristics of various waves, such as the period, frequency, and angular frequency.
SET UP: The frequency f in Hz is the number of cycles per second. The angular frequency ω is $\omega = 2\pi f$ and has units of radians per second. The period T is the time for one cycle of the wave and has units of seconds. The period and frequency are related by $T = \dfrac{1}{f}$.

EXECUTE: **(a)** $T = \dfrac{1}{f} = \dfrac{1}{466 \text{ Hz}} = 2.15\times10^{-3}$ s.

$\omega = 2\pi f = 2\pi(466 \text{ Hz}) = 2.93\times10^{3}$ rad/s.

(b) $f = \dfrac{1}{T} = \dfrac{1}{50.0\times10^{-6} \text{ s}} = 2.00\times10^{4}$ Hz. $\omega = 2\pi f = 1.26\times10^{5}$ rad/s.

(c) $f = \dfrac{\omega}{2\pi}$ so f ranges from $\dfrac{2.7\times10^{15} \text{ rad/s}}{2\pi \text{ rad}} = 4.3\times10^{14}$ Hz to $\dfrac{4.7\times10^{15} \text{ rad/s}}{2\pi \text{ rad}} = 7.5\times10^{14}$ Hz. $T = \dfrac{1}{f}$ so T ranges from $\dfrac{1}{7.5\times10^{14} \text{ Hz}} = 1.3\times10^{-15}$ s to $\dfrac{1}{4.3\times10^{14} \text{ Hz}} = 2.3\times10^{-15}$ s.

(d) $T = \dfrac{1}{f} = \dfrac{1}{5.0\times10^{6} \text{ Hz}} = 2.0\times10^{-7}$ s and $\omega = 2\pi f = 2\pi(5.0\times10^{6} \text{ Hz}) = 3.1\times10^{7}$ rad/s.

EVALUATE: Visible light has much higher frequency than either sounds we can hear or ultrasound. Ultrasound is sound with frequencies higher than what the ear can hear. Large f corresponds to small T.

14.2. **IDENTIFY** and **SET UP:** The amplitude is the maximum displacement from equilibrium. In one period the object goes from $x = +A$ to $x = -A$ and returns.
EXECUTE: **(a)** $A = 0.120$ m.
(b) $0.800 \text{ s} = T/2$ so the period is 1.60 s.
(c) $f = \dfrac{1}{T} = 0.625$ Hz.

EVALUATE: Whenever the object is released from rest, its initial displacement equals the amplitude of its SHM.

14.3. **IDENTIFY:** The period is the time for one vibration and $\omega = \dfrac{2\pi}{T}$.

SET UP: The units of angular frequency are rad/s.
EXECUTE: The period is $\dfrac{0.50 \text{ s}}{440} = 1.14\times10^{-3}$ s and the angular frequency is $\omega = \dfrac{2\pi}{T} = 5.53\times10^{3}$ rad/s.
EVALUATE: There are 880 vibrations in 1.0 s, so $f = 880$ Hz. This is equal to $1/T$.

14.4. **IDENTIFY:** The period is the time for one cycle and the amplitude is the maximum displacement from equilibrium. Both these values can be read from the graph.
SET UP: The maximum x is 10.0 cm. The time for one cycle is 16.0 s.

EXECUTE: **(a)** $T = 16.0$ s so $f = \dfrac{1}{T} = 0.0625$ Hz.

(b) $A = 10.0$ cm.
(c) $T = 16.0$ s

(d) $\omega = 2\pi f = 0.393$ rad/s

EVALUATE: After one cycle the motion repeats.

14.5. **IDENTIFY:** This displacement is $\frac{1}{4}$ of a period.

SET UP: $T = 1/f = 0.250$ s.

EXECUTE: $t = 0.0625$ s

EVALUATE: The time is the same for $x = A$ to $x = 0$, for $x = 0$ to $x = -A$, for $x = -A$ to $x = 0$ and for $x = 0$ to $x = A$.

14.6. **IDENTIFY:** The swing moves back and forth and can be approximated as a simple pendulum.

SET UP: Estimate: The length is about 6 ft ≈ 2.0 m. The time between pushes is the period T of swing, where $T = 2\pi\sqrt{L/g}$. We want to find the period.

EXECUTE: $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{\dfrac{2.0\text{ m}}{9.80\text{ m/s}^2}} = 2.8$ s.

EVALUATE: A period of around 3 s seems reasonable for a small playground swing.

14.7. **IDENTIFY** and **SET UP:** The period is the time for one cycle. A is the maximum value of x.

EXECUTE: **(a)** From the figure with the problem, $T = 0.800$ s.

(b) $f = \dfrac{1}{T} = 1.25$ Hz.

(c) $\omega = 2\pi f = 7.85$ rad/s.

(d) From the figure with the problem, $A = 3.0$ cm.

(e) $T = 2\pi\sqrt{\dfrac{m}{k}}$, so $k = m\left(\dfrac{2\pi}{T}\right)^2 = (2.40\text{ kg})\left(\dfrac{2\pi}{0.800\text{ s}}\right)^2 = 148$ N/m.

EVALUATE: The amplitude shown on the graph does not change with time, so there must be little or no friction in this system.

14.8. **IDENTIFY:** Apply $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$.

SET UP: The period will be twice the interval between the times at which the glider is at the equilibrium position.

EXECUTE: $k = \omega^2 m = \left(\dfrac{2\pi}{T}\right)^2 m = \left(\dfrac{2\pi}{2(2.60\text{ s})}\right)^2 (0.200\text{ kg}) = 0.292\text{ N/m}$.

EVALUATE: $1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2$, so $1\text{ N/m} = 1\text{ kg/s}^2$.

14.9. **IDENTIFY** and **SET UP:** Use $T = 1/f$ to calculate T, $\omega = 2\pi f$ to calculate ω, and $\omega = \sqrt{k/m}$ for m.

EXECUTE: **(a)** $T = 1/f = 1/6.00\text{ Hz} = 0.167$ s.

(b) $\omega = 2\pi f = 2\pi(6.00\text{ Hz}) = 37.7$ rad/s.

(c) $\omega = \sqrt{k/m}$ implies $m = k/\omega^2 = (120\text{ N/m})/(37.7\text{ rad/s})^2 = 0.0844$ kg.

EVALUATE: We can verify that k/ω^2 has units of mass.

14.10. **IDENTIFY:** The mass and frequency are related by $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

SET UP: $f\sqrt{m} = \dfrac{\sqrt{k}}{2\pi} =$ constant, so $f_1\sqrt{m_1} = f_2\sqrt{m_2}$.

EXECUTE: **(a)** $m_1 = 0.750\text{ kg}$, $f_1 = 1.75\text{ Hz}$ and $m_2 = 0.750\text{ kg} + 0.220\text{ kg} = 0.970\text{ kg}$.

$$f_2 = f_1\sqrt{\frac{m_1}{m_2}} = (1.75\text{ Hz})\sqrt{\frac{0.750\text{ kg}}{0.970\text{ kg}}} = 1.54\text{ Hz}.$$

(b) $m_2 = 0.750\text{ kg} - 0.220\text{ kg} = 0.530\text{ kg}$. $f_2 = (1.75\text{ Hz})\sqrt{\dfrac{0.750\text{ kg}}{0.530\text{ kg}}} = 2.08\text{ Hz}$.

EVALUATE: When the mass increases the frequency decreases, and when the mass decreases the frequency increases.

14.11. **IDENTIFY:** For SHM the motion is sinusoidal.
SET UP: $x(t) = A\cos(\omega t)$.

EXECUTE: $x(t) = A\cos(\omega t)$, where $A = 0.320\text{ m}$ and $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.900\text{ s}} = 6.981\text{ rad/s}$.

(a) $x = 0.320\text{ m}$ at $t_1 = 0$. Let t_2 be the instant when $x = 0.160\text{ m}$. Then we have

$0.160\text{ m} = (0.320\text{ m})\cos(\omega t_2)$. $\cos(\omega t_2) = 0.500$. $\omega t_2 = 1.047\text{ rad}$. $t_2 = \dfrac{1.047\text{ rad}}{6.981\text{ rad/s}} = 0.150\text{ s}$. It takes

$t_2 - t_1 = 0.150\text{ s}$.

(b) Let t_3 be when $x = 0$. Then we have $\cos(\omega t_3) = 0$ and $\omega t_3 = 1.571\text{ rad}$. $t_3 = \dfrac{1.571\text{ rad}}{6.981\text{ rad/s}} = 0.225\text{ s}$.

It takes $t_3 - t_2 = 0.225\text{ s} - 0.150\text{ s} = 0.0750\text{ s}$.

EVALUATE: Note that it takes twice as long to go from $x = 0.320\text{ m}$ to $x = 0.160\text{ m}$ than to go from $x = 0.160\text{ m}$ to $x = 0$, even though the two distances are the same, because the speeds are different over the two distances.

14.12. **IDENTIFY:** For SHM the restoring force is directly proportional to the displacement and the system obeys Newton's second law.

SET UP: $F_x = ma_x$ and $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

EXECUTE: $F_x = ma_x$ gives $a_x = -\dfrac{kx}{m}$, so $\dfrac{k}{m} = -\dfrac{a_x}{x} = -\dfrac{-5.30\text{ m/s}^2}{0.280\text{ m}} = 18.93\text{ s}^{-2}$.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{18.93\text{ s}^{-2}} = 0.692\text{ Hz}$$

EVALUATE: The period is around 1.5 s, so this is a rather slow vibration.

14.13. **IDENTIFY:** Use $A = \sqrt{x_0^2 + \dfrac{v_{0x}^2}{\omega^2}}$ to calculate A. The initial position and velocity of the block determine ϕ. $x(t)$ is given by $x = A\cos(\omega t + \phi)$.

SET UP: $\cos\theta$ is zero when $\theta = \pm\pi/2$ and $\sin(\pi/2) = 1$.

EXECUTE: **(a)** From $A = \sqrt{x_0^2 + \dfrac{v_{0x}^2}{\omega^2}}$, $A = \left|\dfrac{v_0}{\omega}\right| = \left|\dfrac{v_0}{\sqrt{k/m}}\right| = 0.98\text{ m}$.

(b) Since $x(0) = 0$, $x = A\cos(\omega t + \phi)$ requires $\phi = \pm\frac{\pi}{2}$. Since the block is initially moving to the left, $v_{0x} < 0$ and $v_{0x} = -\omega A\sin\phi$ requires that $\sin\phi > 0$, so $\phi = +\frac{\pi}{2}$.

(c) $\cos(\omega t + \pi/2) = -\sin\omega t$, so $x = (-0.98\text{ m})\sin[(12.2\text{ rad/s})t]$.

EVALUATE: The $x(t)$ result in part (c) does give $x = 0$ at $t = 0$ and $x < 0$ for t slightly greater than zero.

14.14. **IDENTIFY** and **SET UP:** We are given k, m, x_0, and v_0. Use $A = \sqrt{x_0^2 + v_{0x}^2/\omega^2} = \sqrt{x_0^2 + mv_{0x}^2/k}$, $\phi = \arctan(-v_{0x}/\omega x_0)$, and $x = A\cos(\omega t + \phi)$.

EXECUTE: **(a)** $A = \sqrt{x_0^2 + v_{0x}^2/\omega^2} = \sqrt{x_0^2 + mv_{0x}^2/k}$:

$A = \sqrt{(0.200\text{ m})^2 + (2.00\text{ kg})(-4.00\text{ m/s})^2/(300\text{ N/m})} = 0.383\text{ m}$

(b) $\phi = \arctan(-v_{0x}/\omega x_0)$:

$\omega = \sqrt{k/m} = \sqrt{(300\text{ N/m})/2.00\text{ kg}} = 12.25\text{ rad/s}$

$$\phi = \arctan\left(-\frac{(-4.00\text{ m/s})}{(12.25\text{ rad/s})(0.200\text{ m})}\right) = \arctan(+1.633) = 58.5^\circ\text{ (or 1.02 rad)}$$

(c) $x = A\cos(\omega t + \phi)$ gives $x = (0.383\text{ m})\cos([12.2\text{rad/s}]t + 1.02\text{ rad})$

EVALUATE: At $t = 0$ the block is displaced 0.200 m from equilibrium but is moving, so $A > 0.200$ m. Since $v_x = -\omega A\cos(\omega t + \phi)$, a phase angle ϕ in the range $0 < \phi < 90^\circ$ gives $v_{0x} < 0$.

14.15. **IDENTIFY:** The block oscillates in SHM.

SET UP: For the given initial conditions $x(t) = A\cos\omega t$ where $\omega = \sqrt{k/m} = 2\pi/T$ and $T = 2\pi\sqrt{m/k}$.

EXECUTE: **(a)** We want to find the time when $x(t) = A/2$ for the first time.
$x(t) = A\cos\omega t = A/2$, so $\omega t = 60^\circ = \pi/3$ rad, which gives $t = \pi/3\omega$. Since $\omega = 2\pi/T$, we have

$$t = \frac{\pi}{3\left(\frac{2\pi}{T}\right)} = T/6.$$

(b) We want to find the time when the block first $v_{max}/2$. The velocity is $v = dx/dt$, which gives $\frac{d(A\cos\omega t)}{dt} = -A\omega\sin\omega t = -v_{max}\sin\omega t$. We can drop the minus sign because we are interested only in the speed. Therefore $v = \frac{1}{2}v_{max} = v_{max}\sin\omega t$, so $\sin\omega t = \frac{1}{2}$, which means that $\omega t = 30^\circ = \pi/6$. Using $\omega = 2\pi/T$, we get $t = \frac{\pi/6}{\omega} = \frac{\pi}{6\left(\frac{2\pi}{T}\right)} = T/12$.

(c) The answer is no, the block does not reach $v_{max}/2$ when $x = A/2$.

EVALUATE: We can check our result using energy conservation. The total mechanical energy E is $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$. We know that $v_{max} = A\omega = A\sqrt{k/m}$. Use energy to find x when $v = v_{max}/2$, which gives $\frac{1}{2}m\left(\frac{v_{max}}{2}\right)^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$. Using $v_{max} = A\omega = A\sqrt{k/m}$, this becomes $\frac{1}{2}m\left(\frac{A\sqrt{k/m}}{2}\right)^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$. Squaring and solving for x gives $x = \frac{\sqrt{3}}{2}A$, which is *not* equal to $A/2$. This agrees with our result in part (c).

14.16. **IDENTIFY:** The motion is SHM, and in each case the motion described is one-half of a complete cycle.

SET UP: For SHM, $x = A\cos(\omega t)$ and $\omega = \frac{2\pi}{T}$.

EXECUTE: **(a)** The time is half a period. The period is independent of the amplitude, so it still takes 2.70 s.

(b) $x = 0.090$ m at time t_1. $T = 5.40$ s and $\omega = \dfrac{2\pi}{T} = 1.164$ rad/s. $x_1 = A\cos(\omega t_1)$. $\cos(\omega t_1) = 0.500$. $\omega t_1 = 1.047$ rad and $t_1 = 0.8997$ s. $x = -0.090$ m at time t_2. $\cos(\omega t_2) = -0.500$ m. $\omega t_2 = 2.094$ rad and $t_2 = 1.800$ s. The elapsed time is $t_2 - t_1 = 1.800\text{ s} - 0.8997\text{ s} = 0.900$ s.

EVALUATE: It takes less time to travel from ±0.090 m in (b) than it originally did because the block has larger speed at ± 0.090 m with the increased amplitude.

14.17. **IDENTIFY:** Apply $T = 2\pi\sqrt{\dfrac{m}{k}}$. Use the information about the empty chair to calculate k.

SET UP: When $m = 42.5$ kg, $T = 1.30$ s.

EXECUTE: Empty chair: $T = 2\pi\sqrt{\dfrac{m}{k}}$. gives $k = \dfrac{4\pi^2 m}{T^2} = \dfrac{4\pi^2(42.5\text{ kg})}{(1.30\text{ s})^2} = 993$ N/m.

With person in chair: $T = 2\pi\sqrt{\dfrac{m}{k}}$ gives $m = \dfrac{T^2 k}{4\pi^2} = \dfrac{(2.54\text{ s})^2(993\text{ N/m})}{4\pi^2} = 162$ kg and $m_{\text{person}} = 162\text{ kg} - 42.5\text{ kg} = 120$ kg.

EVALUATE: For the same spring, when the mass increases, the period increases.

14.18. **IDENTIFY** and **SET UP:** Use $T = 2\pi\sqrt{\dfrac{m}{k}}$ for T and $a_x = -\dfrac{k}{m}x$ to relate a_x and k.

EXECUTE: $T = 2\pi\sqrt{\dfrac{m}{k}}$, $m = 0.400$ kg

Use $a_x = -1.80\text{ m/s}^2$ to calculate k: $-kx = ma_x$ gives

$$k = -\frac{ma_x}{x} = -\frac{(0.400\text{ kg})(-1.80\text{ m/s}^2)}{0.300\text{ m}} = +2.40\text{ N/m, so } T = 2\pi\sqrt{\frac{m}{k}} = 2.57\text{ s.}$$

EVALUATE: a_x is negative when x is positive. ma_x/x has units of N/m and $\sqrt{m/k}$ has units of seconds.

14.19. **IDENTIFY:** $T = 2\pi\sqrt{\dfrac{m}{k}}$. $a_x = -\dfrac{k}{m}x$ so $a_{\max} = \dfrac{k}{m}A$. $F = -kx$.

SET UP: a_x is proportional to x so a_x goes through one cycle when the displacement goes through one cycle. From the graph, one cycle of a_x extends from $t = 0.10$ s to $t = 0.30$ s, so the period is $T = 0.20$ s. $k = 2.50\text{ N/cm} = 250$ N/m. From the graph the maximum acceleration is 12.0 m/s^2.

EXECUTE: **(a)** $T = 2\pi\sqrt{\dfrac{m}{k}}$ gives $m = k\left(\dfrac{T}{2\pi}\right)^2 = (250\text{ N/m})\left(\dfrac{0.20\text{ s}}{2\pi}\right)^2 = 0.253$ kg

(b) $A = \dfrac{ma_{\max}}{k} = \dfrac{(0.253\text{ kg})(12.0\text{ m/s}^2)}{250\text{ N/m}} = 0.0121\text{ m} = 1.21$ cm

(c) $F_{\max} = kA = (250\text{ N/m})(0.0121\text{ m}) = 3.03$ N.

EVALUATE: We can also calculate the maximum force from the maximum acceleration: $F_{\max} = ma_{\max} = (0.253\text{ kg})(12.0\text{ m/s}^2) = 3.04$ N, which agrees with our previous results.

14.20. **IDENTIFY:** The general expression for $v_x(t)$ is $v_x(t) = -\omega A\sin(\omega t + \phi)$. We can determine ω and A by comparing the equation in the problem to the general form.

SET UP: $\omega = 4.71$ rad/s. $\omega A = 3.60\text{ cm/s} = 0.0360$ m/s.

EXECUTE: **(a)** $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi\text{ rad}}{4.71\text{ rad/s}} = 1.33$ s

(b) $A = \dfrac{0.0360\text{ m/s}}{\omega} = \dfrac{0.0360\text{ m/s}}{4.71\text{ rad/s}} = 7.64\times10^{-3}\text{ m} = 7.64\text{ mm}$

(c) $a_{\max} = \omega^2 A = (4.71\text{ rad/s})^2(7.64\times10^{-3}\text{ m}) = 0.169\text{ m/s}^2$

(d) $\omega = \sqrt{\dfrac{k}{m}}$ so $k = m\omega^2 = (0.500\text{ kg})(4.71\text{ rad/s})^2 = 11.1\text{ N/m}.$

EVALUATE: The overall negative sign in the expression for $v_x(t)$ and the factor of $-\pi/2$ both are related to the phase factor ϕ in the general expression.

14.21. **IDENTIFY:** Compare the specific $x(t)$ given in the problem to the general form $x = A\cos(\omega t + \phi)$.

SET UP: $A = 7.40\text{ cm}$, $\omega = 4.16\text{ rad/s}$, and $\phi = -2.42\text{ rad}$.

EXECUTE: **(a)** $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4.16\text{ rad/s}} = 1.51\text{ s}.$

(b) $\omega = \sqrt{\dfrac{k}{m}}$ so $k = m\omega^2 = (1.50\text{ kg})(4.16\text{ rad/s})^2 = 26.0\text{ N/m}$

(c) $v_{\max} = \omega A = (4.16\text{ rad/s})(7.40\text{ cm}) = 30.8\text{ cm/s}$

(d) $F_x = -kx$ so $F_{\max} = kA = (26.0\text{ N/m})(0.0740\text{ m}) = 1.92\text{ N}.$

(e) $x(t)$ evaluated at $t = 1.00\text{ s}$ gives $x = -0.0125\text{ m}$. $v_x = -\omega A\sin(\omega t + \phi) = 30.4\text{ cm/s}.$

$a_x = -kx/m = -\omega^2 x = +0.216\text{ m/s}^2.$

(f) $F_x = -kx = -(26.0\text{ N/m})(-0.0125\text{ m}) = +0.325\text{ N}$

EVALUATE: The maximum speed occurs when $x = 0$ and the maximum force is when $x = \pm A$.

14.22. **IDENTIFY:** The frequency of vibration of a spring depends on the mass attached to the spring. Differences in frequency are due to differences in mass, so by measuring the frequencies we can determine the mass of the virus, which is the target variable.

SET UP: The frequency of vibration is $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}.$

EXECUTE: **(a)** The frequency without the virus is $f_s = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_s}}$, and the frequency with the virus is

$f_{s+v} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_s + m_v}}$. $\dfrac{f_{s+v}}{f_s} = \left(\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_s + m_v}}\right)\left(2\pi\sqrt{\dfrac{m_s}{k}}\right) = \sqrt{\dfrac{m_s}{m_s + m_v}} = \dfrac{1}{\sqrt{1 + m_v/m_s}}.$

(b) $\left(\dfrac{f_{s+v}}{f_s}\right)^2 = \dfrac{1}{1 + m_v/m_s}$. Solving for m_v gives

$m_v = m_s\left(\left[\dfrac{f_s}{f_{s+v}}\right]^2 - 1\right) = (2.10\times10^{-16}\text{ g})\left(\left[\dfrac{2.00\times10^{15}\text{ Hz}}{2.87\times10^{14}\text{ Hz}}\right]^2 - 1\right) = 9.99\times10^{-15}\text{ g}$, or

$m_v = 9.99$ femtograms.

EVALUATE: When the mass increases, the frequency of oscillation increases.

14.23. **IDENTIFY:** The *jerk* is defined as *da/dt*. We want to investigate the jerk for an object in SHM.

SET UP: $j_x = da_x/dt$, $v_x = -A\omega\sin\omega t$, and $a_x = dv_x/dt$.

EXECUTE: **(a)** First find a_x: $a_x = dv_x/dt = d(-A\omega\sin\omega t)/dt = -\omega^2 A\cos\omega t$. Now find j_x:

$j_x = da_x/dt = d(-\omega^2 A\cos\omega t)/dt = \omega^3 A\sin\omega t.$

(b) The jerk has its largest positive value when $\sin\omega t$ is 1, so $\omega t = \pi/2$. Since $v_x = -A\omega\sin\omega t$, integration tells us that $x(t) = A\cos\omega t$. So when j_x is a positive maximum, $x = A\cos(\pi/2) = 0$.

(c) The jerk is most negative when $\sin\omega t$ is -1, so $\omega t = 3\pi/2$. So when j_x is most negative, $x = A\cos(3\pi/2) = 0$.

(d) The jerk is zero when $\omega t = 0, \pi, 2\pi, \ldots$. At these values, $\cos\omega t$ is either +1 or –1, so $x = \pm A$.
(e) We know that $v_x = -Kj_x$ where $K = +0.040\ \text{s}^2$ and that $v_x = -A\omega\sin\omega t$. Using our result for j_x from part (a), we get $v_x = -K\omega^3 A\sin\omega t$. Equating these two expressions for v_x gives $K\omega^3 A = \omega A$, so $\omega = 1/\sqrt{K} = 1/\sqrt{0.040\ \text{s}^2} = 5.0$ rad/s. The period is $T = 2\pi/\omega,$ which gives $T = 2\pi/(5.0\ \text{rad/s}) = 1.3\ \text{s}.$

EVALUATE: The jerk also has simple harmonic behavior.

14.24. **IDENTIFY:** The mechanical energy of the system is conserved. The maximum acceleration occurs at the maximum displacement and the motion is SHM.

SET UP: Energy conservation gives $\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2,\ T = 2\pi\sqrt{\frac{m}{k}},$ and $a_{\text{max}} = \frac{kA}{m}.$

EXECUTE: **(a)** From the graph, we read off $T = 16.0$ s and $A = 10.0$ cm = 0.100 m. $\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$ gives $v_{\text{max}} = A\sqrt{\frac{k}{m}}.$ $T = 2\pi\sqrt{\frac{m}{k}},$ so $\sqrt{\frac{k}{m}} = \frac{2\pi}{T}.$ Therefore

$$v_{\text{max}} = A\left(\frac{2\pi}{T}\right) = (0.100\ \text{m})\left(\frac{2\pi}{16.0\ \text{s}}\right) = 0.0393\ \text{m/s}.$$

(b) $a_{\text{max}} = \frac{kA}{m} = \left(\frac{2\pi}{T}\right)^2 A = \left(\frac{2\pi}{16.0\ \text{s}}\right)^2 (0.100\ \text{m}) = 0.0154\ \text{m/s}^2$

EVALUATE: The acceleration is much less than g.

14.25. **IDENTIFY:** The mechanical energy of the system is conserved. The maximum acceleration occurs at the maximum displacement and the motion is SHM.

SET UP: Energy conservation gives $\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$ and $a_{\text{max}} = \frac{kA}{m}.$

EXECUTE: $A = 0.165$ m. $\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$ gives $\frac{k}{m} = \left(\frac{v_{\text{max}}}{A}\right)^2 = \left(\frac{3.90\ \text{m/s}}{0.165\ \text{m}}\right)^2 = 558.7\ \text{s}^{-2}.$

$$a_{\text{max}} = \frac{kA}{m} = (558.7\ \text{s}^{-2})(0.165\ \text{m}) = 92.2\ \text{m/s}^2.$$

EVALUATE: The acceleration is much greater than g.

14.26. **IDENTIFY:** The mechanical energy of the system is conserved, Newton's second law applies and the motion is SHM.

SET UP: Energy conservation gives $\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2,\ F_x = ma_x,\ F_x = -kx,$ and the period is $T = 2\pi\sqrt{\frac{m}{k}}.$

EXECUTE: Solving $\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ for v_x gives $v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}.$ $T = 2\pi\sqrt{\frac{m}{k}},$ so $\sqrt{\frac{k}{m}} = \frac{2\pi}{T} = \frac{2\pi}{3.20\ \text{s}} = 1.963\ \text{s}^{-1}.$ $v_x = \pm(1.963\ \text{s}^{-1})\sqrt{(0.250\ \text{m})^2 - (0.160\ \text{m})^2} = \pm 0.377\ \text{m/s}.$

$$a_x = -\frac{kx}{m} = -(1.963\ \text{s}^{-1})^2\ (0.160\ \text{m}) = -0.617\ \text{m/s}^2.$$

EVALUATE: The block is on the positive side of the equilibrium position $(x = 0)$. If $v_x = +0.377$ m/s, the block is moving in the positive direction and slowing down since the acceleration is in the negative direction. If $v_x = -0.377$ m/s, the block is moving in the negative direction and speeding up.

14.27. **IDENTIFY** and **SET UP:** Use $E=\frac{1}{2}mv^2+\frac{1}{2}kx^2=\frac{1}{2}kA^2$. $x=\pm A$ when $v_x=0$ and $v_x=\pm v_{\max}$ when $x=0$.

EXECUTE: **(a)** $E=\frac{1}{2}mv^2+\frac{1}{2}kx^2$

$E=\frac{1}{2}(0.150\text{ kg})(0.400\text{ m/s})^2+\frac{1}{2}(300\text{ N/m})(0.012\text{ m})^2=0.0336\text{ J}.$

(b) $E=\frac{1}{2}kA^2$ so $A=\sqrt{2E/k}=\sqrt{2(0.0336\text{ J})/(300\text{ N/m})}=0.0150\text{ m}$

(c) $E=\frac{1}{2}mv_{\max}^2$ so $v_{\max}=\sqrt{2E/m}=\sqrt{2(0.0336\text{ J})/(0.150\text{ kg})}=0.669\text{ m/s}.$

EVALUATE: The total energy E is constant but is transferred between kinetic and potential energy during the motion.

14.28. **IDENTIFY** and **SET UP:** Use $E=\frac{1}{2}mv^2+\frac{1}{2}kx^2=\frac{1}{2}kA^2$ to relate K and U. U depends on x and K depends on v_x.

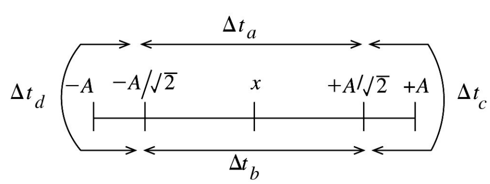
EXECUTE: **(a)** $U+K=E$, so $U=K$ says that $2U=E$

$2\left(\frac{1}{2}kx^2\right)=\frac{1}{2}kA^2$ and $x=\pm A/\sqrt{2}$; magnitude is $A/\sqrt{2}$

But $U=K$ also implies that $2K=E$

$2\left(\frac{1}{2}mv_x^2\right)=\frac{1}{2}kA^2$ and $v_x=\pm\sqrt{k/m}A/\sqrt{2}=\pm\omega A/\sqrt{2}$; magnitude is $\omega A/\sqrt{2}$.

(b) In one cycle x goes from A to 0 to $-A$ to 0 to $+A$. Thus $x=+A\sqrt{2}$ twice and $x=-A/\sqrt{2}$ twice in each cycle. Therefore, $U=K$ four times each cycle. The time between $U=K$ occurrences is the time Δt_a for $x_1=+A/\sqrt{2}$ to $x_2=-A\sqrt{2}$, time Δt_b for $x_1=-A/\sqrt{2}$ to $x_2=+A/\sqrt{2}$, time Δt_c for $x_1=+A/\sqrt{2}$ to $x_2=+A\sqrt{2}$, or the time Δt_d for $x_1=-A/\sqrt{2}$ to $x_2=-A/\sqrt{2}$, as shown in Figure 14.28.



$\Delta t_a=\Delta t_b$

$\Delta t_c=\Delta t_d$

Figure 14.28

Calculation of Δt_a:

Specify x in $x=A\cos\omega t$ (choose $\phi=0$ so $x=A$ at $t=0$) and solve for t.

$x_1=+A/\sqrt{2}$ implies $A/\sqrt{2}=A\cos(\omega t_1)$

$\cos\omega t_1=1/\sqrt{2}$ so $\omega t_1=\arccos(1/\sqrt{2})=\pi/4$ rad

$t_1=\pi/4\omega$

$x_2=-A/\sqrt{2}$ implies $-A/\sqrt{2}=A\cos(\omega t_2)$

$\cos\omega t_2=-1/\sqrt{2}$ so $\omega t_1=3\pi/4$ rad

$t_2=3\pi/4\omega$

$\Delta t_a=t_2-t_1=3\pi/4\omega-\pi/4\omega=\pi/2\omega$ (Note that this is $T/4$, one-fourth period.)

Calculation of Δt_d:

$x_1=-A/\sqrt{2}$ implies $t_1=3\pi/4\omega$

$x_2=-A/\sqrt{2}$, t_2 is the *next* time after t_1 that gives $\cos\omega t_2=-1/\sqrt{2}$

Thus $\omega t_2=\omega t_1+\pi/2=5\pi/4$ and $t_2=5\pi/4\omega$

$\Delta t_d = t_2 - t_1 = 5\pi/4\omega - 3\pi/4\omega = \pi/2\omega,$ so is the same as Δt_a.

Therfore the occurrences of $K = U$ are equally spaced in time, with a time interval between them of $\pi/2\omega$.

Evaluate: This is one-fourth *T*, as it must be if there are 4 equally spaced occurrences each period.

(c) Execute: $x = A/2$ and $U + K = E$

$$K = E - U = \tfrac{1}{2}kA^2 - \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 - \tfrac{1}{2}k(A/2)^2 = \tfrac{1}{2}kA^2 - \tfrac{1}{8}kA^2 = 3kA^2/8$$

Then $\dfrac{K}{E} = \dfrac{3kA^2/8}{\frac{1}{2}kA^2} = \dfrac{3}{4}$ and $\dfrac{U}{E} = \dfrac{\frac{1}{8}kA^2}{\frac{1}{2}kA^2} = \dfrac{1}{4}$

Evaluate: At $x = 0$ all the energy is kinetic and at $x = \pm A$ all the energy is potiential. But $K = U$ does not occur at $x = \pm A/2,$ since *U* is not linear in *x*.

14.29. **Identify:** Velocity and position are related by $E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2$. Acceleration and position are related by $-kx = ma_x$.

Set Up: The maximum speed is at $x = 0$ and the maximum magnitude of acceleration is at $x = \pm A$.

Execute: **(a)** For $x = 0,$ $\tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}kA^2$ and $v_{\text{max}} = A\sqrt{\dfrac{k}{m}} = (0.040\ \text{m})\sqrt{\dfrac{450\ \text{N/m}}{0.500\ \text{kg}}} = 1.20\ \text{m/s}$

(b) $v_x = \pm\sqrt{\dfrac{k}{m}}\sqrt{A^2 - x^2} = \pm\sqrt{\dfrac{450\ \text{N/m}}{0.500\ \text{kg}}}\sqrt{(0.040\ \text{m})^2 - (0.015\ \text{m})^2} = \pm 1.11\ \text{m/s}.$

The speed is $v = 1.11$ m/s.

(c) For $x = \pm A,$ $a_{\text{max}} = \dfrac{k}{m}A = \left(\dfrac{450\ \text{N/m}}{0.500\ \text{kg}}\right)(0.040\ \text{m}) = 36\ \text{m/s}^2$

(d) $a_x = -\dfrac{kx}{m} = -\dfrac{(450\ \text{N/m})(-0.015\ \text{m})}{0.500\ \text{kg}} = +13.5\ \text{m/s}^2$

(e) $E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(450\ \text{N/m})(0.040\ \text{m})^2 = 0.360\ \text{J}$

Evaluate: The speed and acceleration at $x = -0.015$ m are less than their maximum values.

14.30. **Identify:** The block moves in SHM attached to a spring.

Set Up: We are looking at the block's average speed and maximum speed. $v_{\text{av}} = d/t$, $v_{\text{max}} = A\omega$, $\omega = \sqrt{\dfrac{k}{m}}$ and $T = 2\pi\sqrt{\dfrac{m}{k}}$.

Execute: **(a)** During one cycle the block moves through a total distance of $d = 4A$ in time *T*, so

$$v_{\text{av}} = \frac{d}{t} = \frac{4A}{T} = \frac{4A}{2\pi\sqrt{\frac{m}{k}}} = \frac{2A}{\pi}\sqrt{\frac{k}{m}}.$$

(b) $v_{\text{max}} = A\omega = A\sqrt{\dfrac{k}{m}}$. Using this fact and our result from part (a), we have

$$v_{\text{av}} = \frac{2A}{\pi}\sqrt{\frac{k}{m}} = \left(A\sqrt{\frac{k}{m}}\right)\left(\frac{2}{\pi}\right) = \frac{2}{\pi}v_{\text{max}}.$$

(c) $\dfrac{v_{\text{av}}}{\frac{1}{2}v_{\text{max}}} = \dfrac{\frac{2}{\pi}v_{\text{max}}}{\frac{1}{2}v_{\text{max}}} = \dfrac{4}{\pi} > 1,$ so $v_{\text{av}} > \dfrac{v_{\text{max}}}{2}$. This suggests that the block spends more time traveling at speeds greater than $v_{\text{max}}/2$.

Evaluate: The answer in (c) is reasonable because $v_{\text{av}} = (2/\pi)v_{\text{max}} < v_{\text{max}}$.

14.31. **IDENTIFY:** A block moves in SHM on a spring.

SET UP: $x(t) = A\cos(\omega t + \phi)$, $v(t) = dx/dt$. We want to know how far the block is from the origin when its speed is one-half of its maximum speed.

EXECUTE: First find $v(t)$: $v(t) = dx/dt = \dfrac{d\left(A\cos(\omega t + \phi)\right)}{dt} = -A\omega\sin(\omega t + \phi)$. Now find the time when $v = v_{max}/2$. We can drop the minus sign since we are interested only in the speed, not its direction. We also realize that $A\omega = v_{max}$. This gives $\dfrac{v_{max}}{2} = v_{max}\sin(\omega t + \phi)$, so $\sin(\omega t + \phi) = \dfrac{1}{2}$, which gives $(\omega t + \phi) = 30° = \dfrac{\pi}{6}$. At this time $x = A\cos(\omega t + \phi) = A\cos\dfrac{\pi}{6} = A\dfrac{\sqrt{3}}{2} \approx 0.866A$. The distance is $x \approx 0.866A$, which is greater than $A/2$.

EVALUATE: The block changes its speed most rapidly when it is near $x = A$ because a_x is greatest then.

14.32. **IDENTIFY:** Newton's second law applies to the system, and mechanical energy is conserved.

SET UP: $\Sigma F_x = ma_x$, $K_1 + U_1 = K_2 + U_2$, $U = \frac{1}{2}kx^2$, $F_{spring} = -kx$.

EXECUTE: **(a)** $\Sigma F_x = ma_x$ gives $ma_x = -kx$, which gives

$(0.300 \text{ kg})(-12.0 \text{ m/s}^2) = -k(0.240 \text{ m})$.

Solving for k gives $k = 15.0$ N/m.

(b) Applying $K_1 + U_1 = K_2 + U_2$ gives $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$.

Putting in the numbers gives

$(0.300 \text{ kg})(4.00 \text{ m/s})^2 + (15.0 \text{ N/m})(0.240 \text{ m})^2 = (15.0 \text{ N/m})A^2$,

so $A = 0.61449$ m, which rounds to 0.614 m.

(c) The kinetic energy is maximum when the potential energy is zero, which is when $x = 0$. Therefore $\frac{1}{2}kA^2 = \frac{1}{2}mv^2$, which gives

$(15.0 \text{ N/m})(0.61449 \text{ m})^2 = (0.300 \text{ kg})v^2$

$v = 4.345$ m/s which rounds to 4.35 m/s.

(d) The maximum force occurs when $x = A$, so Newton's second law gives $F_{max} = ma_{max} = kA$.

$(15.0 \text{ N/m})(0.61449 \text{ m}) = (0.300 \text{ kg})a_{max}$, which gives $a_{max} = 30.7 \text{ m/s}^2$.

EVALUATE: It is frequently necessary to use a combination of energy conservation and Newton's laws.

14.33. **IDENTIFY:** Conservation of energy says $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ and Newton's second law says $-kx = ma_x$.

SET UP: Let $+x$ be to the right. Let the mass of the object be m.

EXECUTE: $k = -\dfrac{ma_x}{x} = -m\left(\dfrac{-8.40 \text{ m/s}^2}{0.600 \text{ m}}\right) = (14.0 \text{ s}^{-2})m.$

$A = \sqrt{x^2 + (m/k)v^2} = \sqrt{(0.600 \text{ m})^2 + \left(\dfrac{m}{(14.0 \text{ s}^{-2})m}\right)(2.20 \text{ m/s})^2} = 0.840 \text{ m}$. The object will therefore travel $0.840 \text{ m} - 0.600 \text{ m} = 0.240 \text{ ma}$ to the right before stopping at its maximum amplitude.

EVALUATE: The acceleration is not constant and we cannot use the constant acceleration kinematic equations.

14.34. **IDENTIFY:** The mechanical energy (the sum of the kinetic energy and potential energy) is conserved.

SET UP: $K + U = E$, with $E = \frac{1}{2}kA^2$ and $U = \frac{1}{2}kx^2$

EXECUTE: $U = K$ says $2U = E$. This gives $2(\frac{1}{2}kx^2) = \frac{1}{2}kA^2$, so $x = A/\sqrt{2}$.

EVALUATE: When $x = A/2$ the kinetic energy is three times the elastic potential energy.

14.35. IDENTIFY and SET UP: Velocity, position, and total energy are related by $E = \frac{1}{2}kA^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$. Acceleration and position are related by $-kx = ma_x$. The maximum magnitude of acceleration is at $x = \pm A$.

EXECUTE: **(a)** $E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}(2.00 \text{ kg})(-4.00 \text{ m/s})^2 + \frac{1}{2}(315 \text{ N/m})(+0.200 \text{ m})^2$.

$E = 16.0 \text{ J} + 6.3 \text{ J} = 22.3 \text{ J}$. $E = \frac{1}{2}kA^2$ and

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(22.3 \text{ J})}{315 \text{ N/m}}} = 0.376 \text{ m}.$$

(b) $a_{\max} = \dfrac{k}{m}A = \left(\dfrac{315 \text{ N/m}}{2.00 \text{ kg}}\right)(0.376 \text{ m}) = 59.2 \text{ m/s}^2$

(c) $F_{\max} = ma_{\max} = (2.00 \text{ kg})(59.2 \text{ m/s}^2) = 118 \text{ N}$. Or, $F_x = -kx$ gives $F_{\max} = kA = (315 \text{ N/m})(0.376 \text{ m}) = 118 \text{ N}$, which checks.

EVALUATE: The maximum force and maximum acceleration occur when the displacement is maximum and the velocity is zero.

14.36. IDENTIFY: Use the amount the spring is stretched by the weight of the fish to calculate the force constant *k* of the spring. $T = 2\pi\sqrt{m/k}$. $v_{\max} = \omega A = 2\pi fA$.

SET UP: When the fish hangs at rest the upward spring force $|F_x| = kx$ equals the weight *mg* of the fish. $f = 1/T$. The amplitude of the SHM is 0.0500 m.

EXECUTE: **(a)** $mg = kx$ so $k = \dfrac{mg}{x} = \dfrac{(65.0 \text{ kg})(9.80 \text{ m/s}^2)}{0.180 \text{ m}} = 3.54 \times 10^3 \text{ N/m}$.

(b) $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{65.0 \text{ kg}}{3.54 \times 10^3 \text{ N/m}}} = 0.8514 \text{ s}$ which rounds to 0.851 s.

(c) $v_{\max} = 2\pi fA = \dfrac{2\pi A}{T} = \dfrac{2\pi(0.0500 \text{ m})}{0.8514 \text{ s}} = 0.369 \text{ m/s}$.

EVALUATE: Note that *T* depends only on *m* and *k* and is independent of the distance the fish is pulled down. But $v_{\max}$ does depend on this distance.

14.37. IDENTIFY: Initially part of the energy is kinetic energy and part is potential energy in the stretched spring. When $x = \pm A$ all the energy is potential energy and when the glider has its maximum speed all the energy is kinetic energy. The total energy of the system remains constant during the motion.

SET UP: Initially $v_x = \pm 0.815 \text{ m/s}$ and $x = \pm 0.0300 \text{ m}$.

EXECUTE: **(a)** Initially the energy of the system is

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.175 \text{ kg})(0.815 \text{ m/s})^2 + \frac{1}{2}(155 \text{ N/m})(0.0300 \text{ m})^2 = 0.128 \text{ J}$. $\frac{1}{2}kA^2 = E$ and

$A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(0.128 \text{ J})}{155 \text{ N/m}}} = 0.0406 \text{ m} = 4.06 \text{ cm}$.

(b) $\frac{1}{2}mv_{\max}^2 = E$ and $v_{\max} = \sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2(0.128 \text{ J})}{0.175 \text{ kg}}} = 1.21 \text{ m/s}$.

(c) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{155 \text{ N/m}}{0.175 \text{ kg}}} = 29.8 \text{ rad/s}$.

EVALUATE: The amplitude and the maximum speed depend on the total energy of the system but the angular frequency is independent of the amount of energy in the system and just depends on the force constant of the spring and the mass of the object.

14.38. **IDENTIFY:** The torsion constant κ is defined by $\tau_z = -\kappa\theta$. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}}$ and $T = 1/f$.

$\theta(t) = \Theta\cos(\omega t + \phi)$.

SET UP: For the disk, $I = \frac{1}{2}MR^2$. $\tau_z = -FR$. At $t = 0$, $\theta = \Theta = 3.34° = 0.0583$ rad, so $\phi = 0$.

EXECUTE: **(a)** $\kappa = -\dfrac{\tau_z}{\theta} = -\dfrac{-FR}{0.0583\text{ rad}} = +\dfrac{(4.23\text{ N})(0.120\text{ m})}{0.0583\text{ rad}} = 8.71\text{ N}\cdot\text{m/rad}$

(b) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}} = \dfrac{1}{2\pi}\sqrt{\dfrac{2\kappa}{MR^2}} = \dfrac{1}{2\pi}\sqrt{\dfrac{2(8.71\text{ N}\cdot\text{m/rad})}{(6.50\text{ kg})(0.120\text{ m})^2}} = 2.17\text{ Hz}$. $T = 1/f = 0.461$ s.

(c) $\omega = 2\pi f = 13.6$ rad/s. $\theta(t) = (3.34°)\cos([13.6\text{ rad/s}]t)$.

EVALUATE: The frequency and period are independent of the initial angular displacement, so long as this displacement is small.

14.39. **IDENTIFY:** $K = \frac{1}{2}mv^2$, $U_{\text{grav}} = mgy$ and $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: At the lowest point of the motion, the spring is stretched an amount $2A$.

EXECUTE: **(a)** At the top of the motion, the spring is unstretched and so has no potential energy, the cat is not moving and so has no kinetic energy, and the gravitational potential energy relative to the bottom is $2mgA = 2(4.00\text{ kg})(9.80\text{ m/s}^2)(0.050\text{ m}) = 3.92$ J. This is the total energy, and is the same total for each part.

(b) $U_{\text{grav}} = 0, K = 0,$ so $U_{\text{spring}} = 3.92$ J.

(c) At equilibrium the spring is stretched half as much as it was for part (a), and so $U_{\text{spring}} = \frac{1}{4}(3.92\text{ J}) = 0.98$ J, $U_{\text{grav}} = \frac{1}{2}(3.92\text{ J}) = 1.96$ J, and so $K = 0.98$ J.

EVALUATE: During the motion, work done by the forces transfers energy among the forms kinetic energy, gravitational potential energy and elastic potential energy.

14.40. **IDENTIFY:** $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}}$ and $T = 1/f$ says $T = 2\pi\sqrt{\dfrac{I}{\kappa}}$.

SET UP: $I = \frac{1}{2}mR^2$.

EXECUTE: Solving $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}}$ for κ in terms of the period,

$$\kappa = \left(\frac{2\pi}{T}\right)^2 I = \left(\frac{2\pi}{1.00\text{ s}}\right)^2\left[\frac{1}{2}(2.00\times10^{-3}\text{ kg})(2.20\times10^{-2}\text{ m})^2\right] = 1.91\times10^{-5}\text{ N}\cdot\text{m/rad}.$$

EVALUATE: The longer the period, the smaller the torsion constant.

14.41. **IDENTIFY and SET UP:** The number of ticks per second tells us the period and therefore the frequency. We can use a formula from Table 9.2 to calculate *I*. Then $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}}$ allows us to calculate the torsion constant κ.

EXECUTE: Ticks four times each second implies 0.25 s per tick. Each tick is half a period, so $T = 0.50$ s and $f = 1/T = 1/0.50\text{ s} = 2.00$ Hz.

(a) Thin rim implies $I = MR^2$ (from Table 9.2).
$I = (0.900\times10^{-3}\text{ kg})(0.55\times10^{-2}\text{ m})^2 = 2.7\times10^{-8}\text{ kg}\cdot\text{m}^2$

(b) $T = 2\pi\sqrt{I/\kappa}$ so $\kappa = I(2\pi/T)^2 = (2.7\times10^{-8}\text{ kg}\cdot\text{m}^2)(2\pi/0.50\text{ s})^2 = 4.3\times10^{-6}\text{ N}\cdot\text{m/rad}$

EVALUATE: Both *I* and κ are small numbers.

14.42. **IDENTIFY:** $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}}$.

SET UP: $f = 165/(265\text{ s})$, the number of oscillations per second.

EXECUTE: $I = \dfrac{\kappa}{(2\pi f)^2} = \dfrac{0.450\text{ N}\cdot\text{m/rad}}{[2\pi(165)/(265\text{ s})]^2} = 0.0294\text{ kg}\cdot\text{m}^2.$

EVALUATE: For a larger I, f is smaller.

14.43. **IDENTIFY:** $T = 2\pi\sqrt{L/g}$ is the time for one complete swing.

SET UP: The motion from the maximum displacement on either side of the vertical to the vertical position is one-fourth of a complete swing.

EXECUTE: **(a)** To the given precision, the small-angle approximation is valid. The highest speed is at the bottom of the arc, which occurs after a quarter period, $\dfrac{T}{4} = \dfrac{\pi}{2}\sqrt{\dfrac{L}{g}} = 0.25\text{ s}.$

(b) The same as calculated in (a), 0.25 s. The period is independent of amplitude.

EVALUATE: For small amplitudes of swing, the period depends on L and g.

14.44. **IDENTIFY:** We are looking at a real pendulum for which the simplified formulas are not completely accurate.

SET UP: Equation (14.35), using only the first correction term, is $T = 2\pi\sqrt{\dfrac{L}{g}}\left(1 + \dfrac{1}{4}\sin^2\dfrac{\Theta}{2}\right)$. We want to find Θ so that the simplified formula $T = 2\pi\sqrt{\dfrac{L}{g}}$ is in error by 2.0%, which means that $\dfrac{\Delta T}{T} = 0.020$.

EXECUTE: $\dfrac{\Delta T}{T} = \dfrac{T_{\text{large }\Theta} - T_{\text{small }\Theta}}{T_{\text{large }\Theta}} = 1 - \dfrac{T_{\text{small }\Theta}}{T_{\text{large }\Theta}} = 1 - \dfrac{2\pi\sqrt{\dfrac{L}{g}}}{2\pi\sqrt{\dfrac{L}{g}}\left(1 + \dfrac{1}{4}\sin^2\dfrac{\Theta}{2}\right)}$, which simplifies to

$0.020 = \dfrac{\dfrac{1}{4}\sin^2\dfrac{\Theta}{2}}{1 + \dfrac{1}{4}\sin^2\dfrac{\Theta}{2}}$. Solving for $\sin^2\dfrac{\Theta}{2}$ gives $\sin^2\dfrac{\Theta}{2} = 8.163\times 10^{-2}$, so $\Theta = 33°$.

EVALUATE: For a 1% error, the maximum angle could be $\Theta = 23°$. These results show that it is adequate to use the simplified formula in a large number of cases.

14.45. **IDENTIFY:** Since the cord is much longer than the height of the object, the system can be modeled as a simple pendulum. We will assume the amplitude of swing is small, so that $T = 2\pi\sqrt{\dfrac{L}{g}}$.

SET UP: The number of swings per second is the frequency $f = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$.

EXECUTE: $f = \dfrac{1}{2\pi}\sqrt{\dfrac{9.80\text{ m/s}^2}{1.50\text{ m}}} = 0.407$ swings per second.

EVALUATE: The period and frequency are both independent of the mass of the object.

14.46. **IDENTIFY:** Use $T = 2\pi\sqrt{L/g}$ to relate the period to g.

SET UP: Let the period on earth be $T_E = 2\pi\sqrt{L/g_E}$, where $g_E = 9.80\text{ m/s}^2$, the value on earth.

Let the period on Mars be $T_M = 2\pi\sqrt{L/g_M}$, where $g_M = 3.71\text{ m/s}^2$, the value on Mars.

We can eliminate L, which we don't know, by taking a ratio:

EXECUTE: $\dfrac{T_M}{T_E} = 2\pi\sqrt{\dfrac{L}{g_M}}\dfrac{1}{2\pi}\sqrt{\dfrac{g_E}{L}} = \sqrt{\dfrac{g_E}{g_M}}.$

$T_M = T_E\sqrt{\dfrac{g_E}{g_M}} = (1.60\text{ s})\sqrt{\dfrac{9.80\text{ m/s}^2}{3.71\text{ m/s}^2}} = 2.60\text{ s}.$

EVALUATE: Gravity is weaker on Mars so the period of the pendulum is longer there.

14.47. IDENTIFY: Apply $T = 2\pi\sqrt{L/g}$

SET UP: The period of the pendulum is $T = (136\text{ s})/100 = 1.36\text{ s}.$

EXECUTE: $g = \dfrac{4\pi^2 L}{T^2} = \dfrac{4\pi^2(0.500\text{ m})}{(1.36\text{ s})^2} = 10.7\text{ m/s}^2.$

EVALUATE: The same pendulum on earth, where g is smaller, would have a larger period.

14.48. IDENTIFY and SET UP: The period is for the time for one cycle. The angular amplitude is the maximum value of θ.

EXECUTE: **(a)** From the graph with the problem,

$T = 1.60\text{ s}.$ $f = \dfrac{1}{T} = 0.625\text{ Hz}.$ $\omega = 2\pi f = 3.93\text{ rad/s}.$ From the graph we also determine that the amplitude is 6 degrees.

(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ so $L = g\left(\dfrac{T}{2\pi}\right)^2 = (9.80\text{ m/s}^2)\left(\dfrac{1.60\text{ s}}{2\pi}\right)^2 = 0.635\text{ m}.$

(c) No. The graph is unchanged if the mass of the bob is changed while the length of the pendulum and amplitude of swing are kept constant. The period is independent of the mass of the bob.

EVALUATE: The amplitude of the graph in the problem does not decrease over the time shown, so there must be little or no friction in this pendulum.

14.49. IDENTIFY: $a_{tan} = L\alpha$, $a_{rad} = L\omega^2$ and $a = \sqrt{a_{tan}^2 + a_{rad}^2}$. Use energy conservation in parts (b) and (c).

SET UP: Just after the sphere is released, $\omega = 0$ and $a_{rad} = 0$. When the rod is vertical, $a_{tan} = 0$.

EXECUTE: **(a)** The forces and acceleration are shown in Figure 14.49(a). $a_{rad} = 0$ so $a = a_{tan} = g\sin\theta$.

(b) The forces and acceleration are shown in Figure 14.49(b). In this case, the sphere has radial and tangential acceleration, so we need to use $a = \sqrt{a_{tan}^2 + a_{rad}^2}$. Use energy conservation, calling point 1 the instant that the sphere is released from from rest at angle θ and point 2 the instant the rod makes an angle ϕ with the vertical. This gives $U_1 = U_2 + K_2$, so $mgL(1-\cos\theta) = mgL(1-\cos\phi) + \dfrac{1}{2}mv^2$. Solving for v^2 gives $v^2 = 2gL(\cos\phi - \cos\theta)$. Therefore $a_{rad} = \dfrac{v^2}{L} = 2g(\cos\phi - \cos\theta)$. To find a_{tan}, apply $\Sigma\vec{\tau} = I\alpha$: $mgL\sin\phi = mL^2\alpha = mL(L\alpha) = mL\tan\alpha$, which gives $a_{tan} = g\sin\phi$. The magnitude of the acceleration is $a = \sqrt{a_{tan}^2 + a_{rad}^2} = \sqrt{(g\sin\phi)^2 + [2g(\cos\phi - \cos\theta)]^2}$, which simplifies to $a = g\sqrt{\sin^2\phi + 4(\cos\phi - \cos\theta)^2}$. In this case $\phi = \theta/2$, so the acceleration is $a = g\sqrt{\sin^2(\theta/2) + 4[\cos(\theta/2) - \cos\theta]^2}$.

(c) The forces and acceleration are shown in Figure 14.49(c). Calling point 2 the lowest part of the swing, $U_1 = K_2$ gives $mgL(1-\cos\theta) = \frac{1}{2}mv^2$ and $v = \sqrt{2gL(1-\cos\theta)}$. Using the formula derived in part (b) with $\phi = 0°$, the acceleration is $a = g\sqrt{\sin^2 0° + 4(\cos 0° - \cos\theta)^2} = 2g(1-\cos\theta)$.

EVALUATE: As the rod moves toward the vertical, v increases, a_{rad} increases and a_{tan} decreases. The result in (c) agrees with the fact that $a_{\text{tan}} = 0$ when the rod is vertical because in that case $a = a_{\text{rad}} =$ $\dfrac{v^2}{L} = \dfrac{\left(\sqrt{2gL(1-\cos\theta)}\right)^2}{L} = 2g(1-\cos\theta),$ which is what we found in part (c).

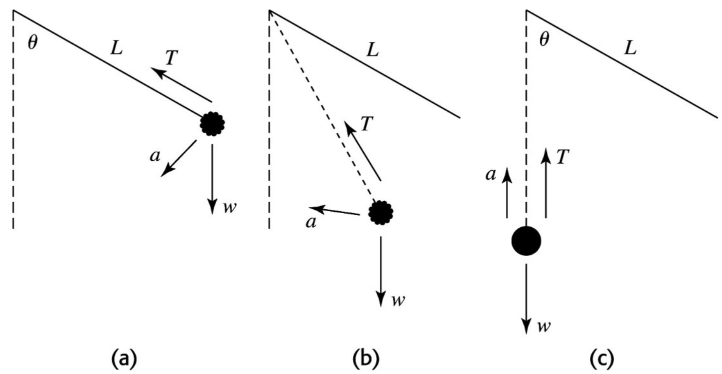


Figure 14.49

14.50. IDENTIFY: $T = 2\pi\sqrt{I/mgd}$

SET UP: From the parallel axis theorem, the moment of inertia of the hoop about the nail is $I = MR^2 + MR^2 = 2MR^2$. $d = R$.

EXECUTE: Solving for R, $R = gT^2/8\pi^2 = 0.496$ m.

EVALUATE: A simple pendulum of length $L = R$ has period $T = 2\pi\sqrt{R/g}$. The hoop has a period that is larger by a factor of $\sqrt{2}$.

14.51. IDENTIFY: Pendulum A can be treated as a simple pendulum. Pendulum B is a physical pendulum.

SET UP: For pendulum B the distance d from the axis to the center of gravity is $3L/4$. $I = \frac{1}{3}(m/2)L^2$ for a bar of mass $m/2$ and the axis at one end. For a small ball of mass $m/2$ at a distance L from the axis, $I_{\text{ball}} = (m/2)L^2$.

EXECUTE: Pendulum A: $T_A = 2\pi\sqrt{\dfrac{L}{g}}$.

Pendulum B: $I = I_{\text{bar}} + I_{\text{ball}} = \frac{1}{3}(m/2)L^2 + (m/2)L^2 = \frac{2}{3}mL^2$.

$T_B = 2\pi\sqrt{\dfrac{I}{mgd}} = 2\pi\sqrt{\dfrac{\frac{2}{3}mL^2}{mg(3L/4)}} = 2\pi\sqrt{\dfrac{L}{g}}\sqrt{\dfrac{2}{3}\cdot\dfrac{4}{3}} = \sqrt{\dfrac{8}{9}}\left(2\pi\sqrt{\dfrac{L}{g}}\right) = 0.943T_A$. The period is longer for pendulum A.

EVALUATE: Example 14.9 shows that for the bar alone, $T = \sqrt{\dfrac{2}{3}}T_A = 0.816T_A$. Adding the ball of equal mass to the end of the rod increases the period compared to that for the rod alone.

14.52. IDENTIFY: Apply $T = 2\pi\sqrt{\dfrac{I}{mgd}}$ to calculate I and conservation of energy to calculate the maximum angular speed, Ω_{max}.

SET UP: $d = 0.250$ m. In part (b), $y_{\text{i}} = d(1-\cos\Theta)$, with $\Theta = 0.400$ rad and $y_{\text{f}} = 0$.

EXECUTE: **(a)** Solving $T = 2\pi\sqrt{\dfrac{I}{mgd}}$ for *I*, we get

$$I = \left(\frac{T}{2\pi}\right)^2 mgd = \left(\frac{0.940\ \text{s}}{2\pi}\right)^2 (1.80\ \text{kg})(9.80\ \text{m/s}^2)(0.250\ \text{m}) = 0.0987\ \text{kg}\cdot\text{m}^2.$$

(b) The small-angle approximation will not give three-figure accuracy for $\Theta = 0.400$ rad. From energy considerations, $mgd(1-\cos\Theta) = \frac{1}{2}I\Omega_{\text{max}}^2$. Expressing Ω_{max} in terms of the period of small-angle oscillations, this becomes

$$\Omega_{\text{max}} = \sqrt{2\left(\frac{2\pi}{T}\right)^2(1-\cos\Theta)} = \sqrt{2\left(\frac{2\pi}{0.940\ \text{s}}\right)^2[1-\cos(0.400\ \text{rad})]} = 2.66\ \text{rad/s}.$$

EVALUATE: The time for the motion in part (b) is $t = T/4$, so $\Omega_{\text{av}} = \Delta\theta/\Delta t = (0.400\ \text{rad})/(0.235\ \text{s}) =$ 1.70 rad/s. Ω increases during the motion and the final Ω is larger than the average Ω.

14.53. **IDENTIFY:** Pendulum *A* can be treated as a simple pendulum. Pendulum *B* is a physical pendulum. Use the parallel-axis theorem to find the moment of inertia of the ball in *B* for an axis at the top of the string.
SET UP: For pendulum *B* the center of gravity is at the center of the ball, so $d = L$. For a solid sphere with an axis through its center, $I_{\text{cm}} = \frac{2}{5}MR^2$. $R = L/2$ and $I_{\text{cm}} = \frac{1}{10}ML^2$.

EXECUTE: Pendulum *A*: $T_A = 2\pi\sqrt{\dfrac{L}{g}}$.

Pendulum *B*: The parallel-axis theorem says $I = I_{\text{cm}} + ML^2 = \frac{11}{10}ML^2$.

$T = 2\pi\sqrt{\dfrac{I}{mgd}} = 2\pi\sqrt{\dfrac{11ML^2}{10MgL}} = \sqrt{\dfrac{11}{10}}\left(2\pi\sqrt{\dfrac{L}{g}}\right) = \sqrt{\dfrac{11}{10}}T_A = 1.05T_A$. It takes pendulum *B* longer to completea swing.

EVALUATE: The center of the ball is the same distance from the top of the string for both pendulums, but the mass is distributed differently and *I* is larger for pendulum *B*, even though the masses are the same.

14.54. **IDENTIFY:** The ornament is a physical pendulum, so $T = 2\pi\sqrt{I/mgd}$. *T* is the target variable.

SET UP: $I = 5MR^2/3$, the moment of inertia about an axis at the edge of the sphere. *d* is the distance from the axis to the center of gravity, which is at the center of the sphere, so $d = R$.

EXECUTE: $T = 2\pi\sqrt{5/3}\sqrt{R/g} = 2\pi\sqrt{5/3}\sqrt{0.050\ \text{m}/(9.80\ \text{m/s}^2)} = 0.58\ \text{s}.$

EVALUATE: A simple pendulum of length $R = 0.050$ m has period 0.45 s; the period of the physical pendulum is longer.

14.55. **IDENTIFY:** The object is moving with damped SHM.
SET UP: We want to know the angular frequency ω' if the damping constant is one-half the critical value. $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$, where *b* is the damping constant and $b_{\text{crit}} = 2\sqrt{km}$. If there is no damping $(b = 0)$, then $\omega = \sqrt{k/m}$.

EXECUTE: If $b = \frac{1}{2}b_{\text{crit}} = \frac{1}{2}\left(2\sqrt{km}\right)$, the angular frequency becomes

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{k}{m} - \frac{\left[\frac{1}{2}\left(2\sqrt{km}\right)\right]^2}{4m^2}} = \sqrt{\frac{k}{m} - \frac{k}{4m}} = \sqrt{\frac{3}{4}\frac{k}{m}} = \omega\frac{\sqrt{3}}{2}.$$

EVALUATE: Notice that if $b = \frac{1}{2}b_{\text{crit}}$, it does *not* follow that $\omega' = \frac{\omega}{2}$.

14.56. IDENTIFY: From a small damping force, $A_2 = A_1 e^{-(b/2m)t}$.

SET UP: $\ln(e^{-x}) = -x$

EXECUTE: $b = \frac{2m}{t}\ln\left(\frac{A_1}{A_2}\right) = \frac{2(0.050\text{ kg})}{(5.00\text{ s})}\ln\left(\frac{0.300\text{ m}}{0.100\text{ m}}\right) = 0.0220\text{ kg/s}.$

EVALUATE: As a check, note that the oscillation frequency is the same as the undamped frequency to $4.8\times10^{-3}\%$, so our assumption of a small damping force is valid.

14.57. IDENTIFY and SET UP: Use $\omega' = \sqrt{(k/m)-(b^2/4m^2)}$ to calculate ω', and then $f' = \omega'/2\pi$.

(a) EXECUTE: $\omega' = \sqrt{(k/m)-(b^2/4m^2)} = \sqrt{\frac{2.50\text{ N/m}}{0.300\text{ kg}} - \frac{(0.900\text{ kg/s})^2}{4(0.300\text{ kg})^2}} = 2.47\text{ rad/s}$

$f' = \omega'/2\pi = (2.47\text{ rad/s})/2\pi = 0.393\text{ Hz}$

(b) IDENTIFY and SET UP: The condition for critical damping is $b = 2\sqrt{km}$.

EXECUTE: $b = 2\sqrt{(2.50\text{ N/m})(0.300\text{ kg})} = 1.73\text{ kg/s}$

EVALUATE: The value of b in part (a) is less than the critical damping value found in part (b). With no damping, the frequency is $f = 0.459\text{ Hz}$; the damping reduces the oscillation frequency.

14.58. IDENTIFY: The graph with the problem shows that the amplitude of vibration is decreasing, so the system must be losing mechanical energy.

SET UP: The mechanical energy is $E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$.

EXECUTE: **(a)** When $|x|$ is a maximum and the tangent to the curve is horizontal the speed of the mass is zero. This occurs at $t = 0$, $t = 1.0\text{ s}$, $t = 2.0\text{ s}$, $t = 3.0\text{ s}$ and $t = 4.0\text{ s}$.

(b) At $t = 0$, $v_x = 0$ and $x = 7.0\text{ cm}$ so $E_0 = \frac{1}{2}kx^2 = \frac{1}{2}(225\text{ N/m})(0.070\text{ m})^2 = 0.55\text{ J}$.

(c) At $t = 1.0\text{ s}$, $v_x = 0$ and $x = -6.0\text{ cm}$ so $E_1 = \frac{1}{2}kx^2 = \frac{1}{2}(225\text{ N/m})(-0.060\text{ m})^2 = 0.405\text{ J}$. At $t = 4.0\text{ s}$, $v_x = 0$ and $x = 3.0\text{ cm}$ so $E_4 = \frac{1}{2}kx^2 = \frac{1}{2}(225\text{ N/m})(0.030\text{ m})^2 = 0.101\text{ J}$. The mechanical energy "lost" is $E_1 - E_4 = 0.30\text{ J}$. The mechanical energy lost was converted to other forms of energy by nonconservative forces, such as friction, air resistance, and other dissipative forces.

EVALUATE: After a while the mass will come to rest and then all its initial mechanical energy will have been "lost" because it will have been converted to other forms of energy by nonconservative forces.

14.59. IDENTIFY: Apply $A = \frac{F_{\text{max}}}{\sqrt{\left(k - m\omega_d^2\right)^2 + b^2\omega_d^2}}$.

SET UP: $\omega_d = \sqrt{k/m}$ corresponds to resonance, and in this case $A = \frac{F_{\text{max}}}{\sqrt{\left(k - m\omega_d^2\right)^2 + b^2\omega_d^2}}$ reduces to $A = F_{\text{max}}/b\omega_d$.

EXECUTE: **(a)** $A_1/3$

(b) $2A_1$

EVALUATE: Note that the resonance frequency is independent of the value of b. (See Figure 14.28 in the textbook).

14.60. **IDENTIFY:** We are dealing with forced oscillations and resonance.

SET UP: Equation (14.46) gives $A = \dfrac{F_{\max}}{\sqrt{\left(k - m\omega_{\mathrm{d}}^2\right)^2 + b^2\omega_{\mathrm{d}}^2}}$, the amplitude A of a forced oscillator where $\omega = \sqrt{k/m}$ and $F_{\max}$ is the maximum value of the driving force. We want to investigate how varying the damping constant b affects the amplitude.

EXECUTE: **(a)** In this case, $b = 0.20\sqrt{km}$ and $\omega_{\mathrm{d}} = \omega = \sqrt{k/m}$, so A is

$$A = \frac{F_{\max}}{\sqrt{\left(k - m(k/m)\right)^2 + \left(0.20\sqrt{km}\right)^2\left(\sqrt{k/m}\right)^2}} = \frac{F_{\max}}{0.20k} = \frac{5F_{\max}}{k}.$$

(b) In this case, $b = 0.40\sqrt{km}$ and $\omega_{\mathrm{d}} = \omega = \sqrt{k/m}$, so A is

$$A = \frac{F_{\max}}{\sqrt{\left(0.40\sqrt{km}\right)^2 (k/m)}} = \frac{F_{\max}}{0.40k} = \frac{2.5F_{\max}}{k}.$$

(c) For $b = 0.20\sqrt{km}$ and $\omega_{\mathrm{d}} = \dfrac{\omega}{2} = \dfrac{1}{2}\sqrt{k/m}$, A is $A = \dfrac{F_{\max}}{\sqrt{\left(k - m(\omega^2/4)\right)^2 + \left(0.20\sqrt{km}\right)^2\left(\dfrac{1}{2}\sqrt{k/m}\right)^2}}$

$$= \frac{F_{\max}}{\sqrt{\left(k - m(k/4m)\right)^2 + (0.040km)(k/4m)}} = \frac{F_{\max}}{\sqrt{(3k/4)^2 + (0.010)k^2}} = \frac{F_{\max}}{0.20k} = \frac{1.3F_{\max}}{k}.$$

$$\frac{A_\omega}{A_{\omega/2}} = \frac{5F_{\max}/k}{1.32F_{\max}k} = 3.8.$$

For $b = 0.40\sqrt{km}$ and $\omega_{\mathrm{d}} = \dfrac{\omega}{2} = \dfrac{1}{2}\sqrt{k/m}$, $A = \dfrac{F_{\max}}{\sqrt{(3k/4)^2 + \left(0.40\sqrt{km}\right)^2\left(\dfrac{1}{2}\sqrt{k/m}\right)^2}} =$

$$\frac{F_{\max}}{\sqrt{(3k/4)^2 + (0.040)k^2}} = \frac{0.13F_{\max}}{k}.\quad \frac{A_\omega}{A_{\omega/2}} = \frac{2.5F_{\max}/k}{0.13F_{\max}k} = 1.9.$$

EVALUATE: The amplitude increases by a larger factor for $b = 0.20\sqrt{km}$ than for $b = 0.40\sqrt{km}$, which agrees with Fig. 14.28.

14.61. **IDENTIFY:** Two objects are in SHM on different springs, and we want to compare their maximum speed and maximum acceleration.

SET UP: We know that $v_{\max} = A\omega = A\sqrt{k/m}$ and $a_{\max} = \omega^2 A = (k/m)A.$

EXECUTE: **(a)** $\dfrac{v_{\max,A}}{v_{\max,B}} = \dfrac{\sqrt{\dfrac{k_A}{m_A}}A_A}{\sqrt{\dfrac{k_B}{m_B}}A_B} = \sqrt{\left(\dfrac{k_A}{k_B}\right)\left(\dfrac{m_B}{m_A}\right)}\left(\dfrac{A_A}{A_B}\right) = \sqrt{\left(\dfrac{9k_B}{k_B}\right)\left(\dfrac{4m_A}{m_A}\right)}\left(\dfrac{2A_B}{A_B}\right) = 12.$

(b) $\dfrac{a_{\max,A}}{a_{\max,B}} = \dfrac{\dfrac{k_A}{m_A}A_A}{\dfrac{k_B}{m_B}A_B} = \left(\dfrac{k_A}{k_B}\right)\left(\dfrac{m_B}{m_A}\right)\left(\dfrac{A_A}{A_B}\right) = \left(\dfrac{9k_B}{k_B}\right)\left(\dfrac{4m_A}{m_A}\right)\left(\dfrac{2A_B}{A_B}\right) = 72.$

EVALUATE: The ratio of the accelerations is considerably greater than that of the speeds because the acceleration depends on k/m while the speed depends on $\sqrt{k/m}$.

14.62. **IDENTIFY:** Apply $x(t) = A\cos(\omega t + \phi)$

SET UP: $x = A$ at $t = 0$, so $\phi = 0$. $A = 6.00$ cm. $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.300 \text{ s}} = 20.9$ rad/s, so $x(t) = (6.00 \text{ cm})\cos[(20.9 \text{ rad/s})t]$.

EXECUTE: $t = 0$ at $x = 6.00$ cm. $x = -1.50$ cm when $-1.50 \text{ cm} = (6.00 \text{ cm})\cos[(20.9 \text{ rad/s})t]$.

$$t = \left(\frac{1}{20.9 \text{ rad/s}}\right)\arccos\left(\frac{1.50 \text{ cm}}{6.00 \text{ cm}}\right) = 0.0872 \text{ s}.$$ It takes 0.0872 s.

EVALUATE: It takes $t = T/4 = 0.075$ s to go from $x = 6.00$ cm to $x = 0$ and 0.150 s to go from $x = +6.00$ cm to $x = -6.00$ cm. Our result is between these values, as it should be.

14.63. IDENTIFY and **SET UP:** Calculate x using $x = A\cos(\omega t + \phi)$. Use T to find ω and x_0 to calculate ϕ.

EXECUTE: At $t = 0$, $x = 0$ and the object is traveling in the $-x$-direction, so $\phi = \pi/2$ rad.

Thus $x = A\cos(\omega t + \pi/2)$.

$T = 2\pi/\omega$ so $\omega = 2\pi/T = 2\pi/1.20 \text{ s} = 5.236$ rad/s

$x = (0.600 \text{ m})\cos[(5.236 \text{ rad/s})(0.480 \text{ s}) + \pi/2] = -0.353$ m.

The distance of the object from the equilibrium position is 0.353 m.

EVALUATE: It takes the object time $t = T/2 = 0.600$ s to return to $x = 0$, so at $t = 0.480$ s it is still at negative x.

14.64. IDENTIFY: $T = 2\pi\sqrt{\dfrac{m}{k}}$. The period changes when the mass changes.

SET UP: M is the mass of the empty car and the mass of the loaded car is $M + 250$ kg.

EXECUTE: The period of the empty car is $T_\text{E} = 2\pi\sqrt{\dfrac{M}{k}}$. The period of the loaded car is

$$T_\text{L} = 2\pi\sqrt{\frac{M + 250 \text{ kg}}{k}}. \quad k = \frac{(250 \text{ kg})(9.80 \text{ m/s}^2)}{4.00\times10^{-2} \text{ m}} = 6.125\times10^4 \text{ N/m}$$

$$M = \left(\frac{T_\text{L}}{2\pi}\right)^2 k - 250 \text{ kg} = \left(\frac{1.92 \text{ s}}{2\pi}\right)^2 (6.125\times10^4 \text{ N/m}) - 250 \text{ kg} = 5.469\times10^3 \text{ kg}.$$

$$T_\text{E} = 2\pi\sqrt{\frac{5.469\times10^3 \text{ kg}}{6.125\times10^4 \text{ N/m}}} = 1.88 \text{ s}.$$

EVALUATE: When the mass decreases, the period decreases.

14.65. IDENTIFY: An object is executing SHM on a spring.

SET UP: We want to change the spring so that the amplitude A_2 is half the original amplitude A_1 and the mechanical energy E_2 is 4 times its original value E_1. Our target variables are the new force constant k_2 and the new maximum speed $v_{2,\max}$ of the object. We know that $U_{\max} = \frac{1}{2}kA^2$, $E = K + U = K_{\max} = E_{\max}$, and $v_{\max} = A\omega = A\sqrt{k/m}$.

EXECUTE: (a) We want to relate k_2 to k_1. The original and final mechanical energies are $E_1 = \frac{1}{2}k_1A_1^2$ and $E_2 = \frac{1}{2}k_2A_2^2 = 4E_1 = 4\left(\frac{1}{2}k_1A_1^2\right)$, so $k_2 = 4\left(\dfrac{A_1}{A_2}\right)^2 k_1 = 4\left(\dfrac{2A_2}{A_2}\right)^2 k_1 = 16k_1$.

(b) Using $v_{\max} = A\omega = A\sqrt{k/m}$ and taking the ratio of the maximum speeds gives

$$\frac{v_{2,\max}}{v_{1,\max}} = \frac{\sqrt{\frac{k_2}{m}}A_2}{\sqrt{\frac{k_1}{m}}A_1} = \sqrt{\frac{k_2}{k_1}}\left(\frac{A_2}{A_1}\right) = \sqrt{\frac{16k_1}{k_1}}\left(\frac{A_2}{2A_2}\right) = 2,$$ so $v_{2,\max} = 2v_{1,\max}$.

EVALUATE: $E = \frac{1}{2}mv_{max}^2$, so if we increase E by a factor of 4, v_{max}^2 must increase by a factor of 4, so v_{max} must increase by a factor of 2, which is what we found in part (b).

14.66. IDENTIFY: In SHM, $a_{max} = \frac{k}{m_{tot}}A$. Apply $\sum \vec{F} = m\vec{a}$ to the top block.

SET UP: The maximum acceleration of the lower block can't exceed the maximum acceleration that can be given to the other block by the friction force.

EXECUTE: For block m, the maximum friction force is $f_s = \mu_s n = \mu_s mg$. $\sum F_x = ma_x$ gives $\mu_s mg = ma$ and $a = \mu_s g$. Then treat both blocks together and consider their simple harmonic motion. $a_{max} = \left(\frac{k}{M+m}\right)A$. Set $a_{max} = a$ and solve for A: $\mu_s g = \left(\frac{k}{M+m}\right)A$ and $A = \frac{\mu_s g(M+m)}{k}$.

EVALUATE: If A is larger than this the spring gives the block with mass M a larger acceleration than friction can give the other block, and the first block accelerates out from underneath the other block.

14.67. IDENTIFY: A block is moving with SHM on a spring. Using measurements of its maximum speed and mass, we will use graphical interpretation.

SET UP: $v_{max} = A\omega = A\sqrt{k/m}$. Our target variable is the force constant of the spring. The data is plotted as v_{max}^2 versus $1/m$, so we need to find a relation between these quantities to interpret the graph.

EXECUTE: Solving $v_{max} = A\omega = A\sqrt{k/m}$ gives $v_{max}^2 = (A^2k)(1/m)$, so a graph of v_{max}^2 versus $1/m$ should be a straight line having slope equal to A^2k. Therefore A^2k = slope = 8.62 $\text{N}\cdot\text{m}$, which gives k = (8.62 $\text{N}\cdot\text{m}$)/(0.120 m)^2 = 599 N/m.

EVALUATE: Converting 599 N/m to lb/in gives k = 3.42 lb/in. which is reasonable for a rather stiff spring.

14.68. IDENTIFY: Two blocks oscillate on a spring in SHM. One of them is accelerated only by static friction, so we'll need to use Newton's second law in addition to the principles of SHM.

SET UP: The maximum friction force on the upper block occurs when its acceleration is a maximum, and that occurs at $a_{max} = \omega^2 A$ for the SHM. For the block not to slip, $f_s = \mu_s n$. The target variable is the coefficient of static friction between the two blocks if the maximum distance that the spring can move from its equilibrium position without causing slipping is d = 8.8 cm. We know that $a_{max} = \omega^2 A$ and $T = 2\pi\sqrt{m/k}$ and will need to use $\sum F_x = ma_x$.

EXECUTE: **(a)** $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4.50\text{ kg}}{150\text{ N/m}}} = 1.09\text{ s}.$

(b) At d = 8.8 cm, f_s is at its maximum value, so $f_s = \mu_s n = \mu_s mg$. Only friction is accelerating the upper block, so $\sum F_x = ma_x$ gives $\mu_s mg = ma_x$. Now look at the system of two blocks (of total mass $m + M$) that are oscillating together. For the system, $\omega = \sqrt{k/m_{total}} = \sqrt{k/(m+M)}$, so $a_{max} = \omega^2 A = \left(\sqrt{k/(m+M)}\right)^2 d = \frac{kd}{m+M}$. Going back to $\sum F_x = ma_x$ gives $\mu_s mg = ma_{max} = m\left(\frac{kd}{m+M}\right)$. Which gives $\mu_s = \frac{kd}{g(m+M)} = \frac{(150\text{ N/m})(0.088\text{ m})}{(9.80\text{ m/s}^2)(4.50\text{ kg})} = 0.30.$

EVALUATE: According to Table 5.1, this value is about the same as for rubber on wet concrete, so it is reasonable.

14.69. IDENTIFY: The largest downward acceleration the ball can have is g whereas the downward acceleration of the tray depends on the spring force. When the downward acceleration of the tray is greater than g, then the ball leaves the tray. $y(t) = A\cos(\omega t + \phi)$.

SET UP: The downward force exerted by the spring is $F = kd$, where d is the distance of the object above the equilibrium point. The downward acceleration of the tray has magnitude $\frac{F}{m} = \frac{kd}{m}$, where m is the total mass of the ball and tray. $x = A$ at $t = 0$, so the phase angle ϕ is zero and $+x$ is downward.

EXECUTE: **(a)** $\frac{kd}{m} = g$ gives $d = \frac{mg}{k} = \frac{(1.775\text{ kg})(9.80\text{ m/s}^2)}{185\text{ N/m}} = 9.40\text{ cm}$. This point is 9.40 cm above the equilibrium point so is $9.40\text{ cm} + 15.0\text{ cm} = 24.4\text{ cm}$ above point A.

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{185\text{ N/m}}{1.775\text{ kg}}} = 10.2\text{ rad/s}$. The point in (a) is above the equilibrium point so $x = -9.40\text{ cm}$.

$x = A\cos(\omega t)$ gives $\omega t = \arccos\left(\frac{x}{A}\right) = \arccos\left(\frac{-9.40\text{ cm}}{15.0\text{ cm}}\right) = 2.25\text{ rad}$. $t = \frac{2.25\text{ rad}}{10.2\text{ rad/s}} = 0.221\text{ s}$.

(c) $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$ gives $v = \sqrt{\frac{k}{m}(A^2 - x^2)} = \sqrt{\frac{185\text{ N/m}}{1.775\text{ kg}}([0.150\text{ m}]^2 - [-0.0940\text{ m}]^2)} = 1.19\text{ m/s}$.

EVALUATE: The period is $T = 2\pi\sqrt{\frac{m}{k}} = 0.615\text{ s}$. To go from the lowest point to the highest point takes time $T/2 = 0.308\text{ s}$. The time in (b) is less than this, as it should be.

14.70. **IDENTIFY:** Apply conservation of linear momentum to the collision and conservation of energy to the motion after the collision. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and $T = \frac{1}{f}$.

SET UP: The object returns to the equilibrium position in time $T/2$.

EXECUTE: **(a)** Momentum conservation during the collision: $mv_0 = (2m)V$.

$$V = \frac{1}{2}v_0 = \frac{1}{2}(2.00\text{ m/s}) = 1.00\text{ m/s}.$$

Energy conservation after the collision: $\frac{1}{2}MV^2 = \frac{1}{2}kx^2$.

$$x = \sqrt{\frac{MV^2}{k}} = \sqrt{\frac{(20.0\text{ kg})(1.00\text{ m/s})^2}{170.0\text{ N/m}}} = 0.343\text{ m (amplitude)}$$

$\omega = 2\pi f = \sqrt{k/M}$. $f = \frac{1}{2\pi}\sqrt{k/M} = \frac{1}{2\pi}\sqrt{\frac{170.0\text{ N/m}}{20.0\text{ kg}}} = 0.464\text{ Hz}$. $T = \frac{1}{f} = \frac{1}{0.464\text{ Hz}} = 2.16\text{ s}$.

(b) It takes $1/2$ period to first return: $\frac{1}{2}(2.16\text{ s}) = 1.08\text{ s}$.

EVALUATE: The total mechanical energy of the system determines the amplitude. The frequency and period depend only on the force constant of the spring and the mass that is attached to the spring.

14.71. **IDENTIFY** and **SET UP:** The bounce frequency is given by $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and the pendulum frequency by $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$. Use the relation between these two frequencies that is specified in the problem to calculate the equilibrium length L of the spring, when the apple hangs at rest on the end of the spring.

EXECUTE: Vertical SHM: $f_b = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

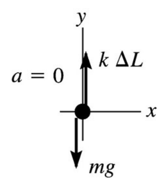
Pendulum motion (small amplitude): $f_p = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

The problem specifies that $f_{\text{p}} = \frac{1}{2} f_{\text{b}}$, so $\frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2}\frac{1}{2\pi}\sqrt{\frac{k}{m}}$. Thus $g/L = k/4m$, which gives $L = 4gm/k = 4w/k = 4(1.00 \text{ N})/1.50 \text{ N/m} = 2.67 \text{ m}$.

EVALUATE: This is the *stretched* length of the spring, its length when the apple is hanging from it. (Note: Small angle of swing means *v* is small as the apple passes through the lowest point, so a_{rad} is small and the component of *mg* perpendicular to the spring is small. Thus the amount the spring is stretched changes very little as the apple swings back and forth.)

IDENTIFY: Use Newton's second law to calculate the distance the spring is stretched from its unstretched length when the apple hangs from it.

SET UP: The free-body diagram for the apple hanging at rest on the end of the spring is given in Figure 14.71.



EXECUTE: $\sum F_y = ma_y$

$k\Delta L - mg = 0$

$\Delta L = mg/k = w/k = 1.00 \text{ N}/1.50 \text{ N/m} = 0.667 \text{ m}$.

Thus the unstretched length of the spring is

$$2.67 \text{ m} - 0.67 \text{ m} = 2.00 \text{ m}.$$

Figure 14.71

EVALUATE: The spring shortens to its unstretched length when the apple is removed.

14.72. **IDENTIFY:** The vertical forces on the floating object must sum to zero. The buoyant force *B* applied to the object by the liquid is given by Archimedes's principle. The motion is SHM if the net force on the object is of the form $F_y = -ky$ and then $T = 2\pi\sqrt{m/k}$.

SET UP: Take $+y$ to be downward.

EXECUTE: **(a)** $V_{\text{submerged}} = LA$, where *L* is the vertical distance from the surface of the liquid to the bottom of the object. Archimedes's principle states $\rho gLA = Mg$, so $L = \frac{M}{\rho A}$.

(b) The buoyant force is $\rho gA(L + y) = Mg + F$, where *y* is the additional distance the object moves downward. Using the result of part (a) and solving for *y* gives $y = \frac{F}{\rho gA}$.

(c) The net force is $F_{\text{net}} = Mg - \rho gA(L + y) = -\rho gAy$. $k = \rho gA$, and the period of oscillation is

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M}{\rho gA}}.$$

EVALUATE: The force *F* determines the amplitude of the motion but the period does not depend on how much force was applied.

14.73. **IDENTIFY:** The object oscillates as a physical pendulum, so $f = \frac{1}{2\pi}\sqrt{\frac{m_{\text{object}} gd}{I}}$. Use the parallel-axis theorem, $I = I_{\text{cm}} + Md^2$, to find the moment of inertia of each stick about an axis at the hook.

SET UP: The center of mass of the square object is at its geometrical center, so its distance from the hook is $L\cos 45° = L/\sqrt{2}$. The center of mass of each stick is at its geometrical center. For each stick, $I_{\text{cm}} = \frac{1}{12} mL^2$.

EXECUTE: The parallel-axis theorem gives I for each stick for an axis at the center of the square to be $\frac{1}{12}mL^2 + m(L/2)^2 = \frac{1}{3}mL^2$ and the total I for this axis is $\frac{4}{3}mL^2$. For the entire object and an axis at the hook, applying the parallel-axis theorem again to the object of mass $4m$ gives
$I = \frac{4}{3}mL^2 + 4m(L/\sqrt{2})^2 = \frac{10}{3}mL^2$.

$$f = \frac{1}{2\pi}\sqrt{\frac{m_{\text{object}}gd}{I}} = \frac{1}{2\pi}\sqrt{\frac{4m_{\text{object}}gL/\sqrt{2}}{\frac{10}{3}m_{\text{object}}L^2}} = \sqrt{\frac{6}{5\sqrt{2}}}\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right) = 0.921\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right).$$

EVALUATE: Just as for a simple pendulum, the frequency is independent of the mass. A simple pendulum of length L has frequency $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ and this object has a frequency that is slightly less than this.

14.74. IDENTIFY: Conservation of energy says $K + U = E$.

SET UP: $U = \frac{1}{2}kx^2$ and $E = U_{\max} = \frac{1}{2}kA^2$.

EXECUTE: **(a)** The graph is given in Figure 14.74. The following answers are found algebraically, to be used as a check on the graphical method.

(b) $A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.200\text{ J})}{(10.0\text{ N/m})}} = 0.200\text{ m}$.

(c) $\frac{E}{4} = 0.050\text{ J}$.

(d) $U = \frac{1}{2}E.\;\; x = \frac{A}{\sqrt{2}} = 0.141\text{ m}$.

(e) From Eq. (14.18), using $v_0 = \sqrt{\frac{2K_0}{m}}$ and $x_0 = -\sqrt{\frac{2U_0}{k}}$, $-\frac{v_0}{\omega x_0} = \frac{\sqrt{(2K_0/m)}}{\sqrt{(k/m)}\sqrt{(2U_0/k)}} = \sqrt{\frac{K_0}{U_0}} = \sqrt{0.429}$

and $\phi = \arctan(\sqrt{0.429}) = 3.72\text{ rad}$.

EVALUATE: The dependence of U on x is not linear and $U = \frac{1}{2}U_{\max}$ does not occur at $x = \frac{1}{2}x_{\max}$.

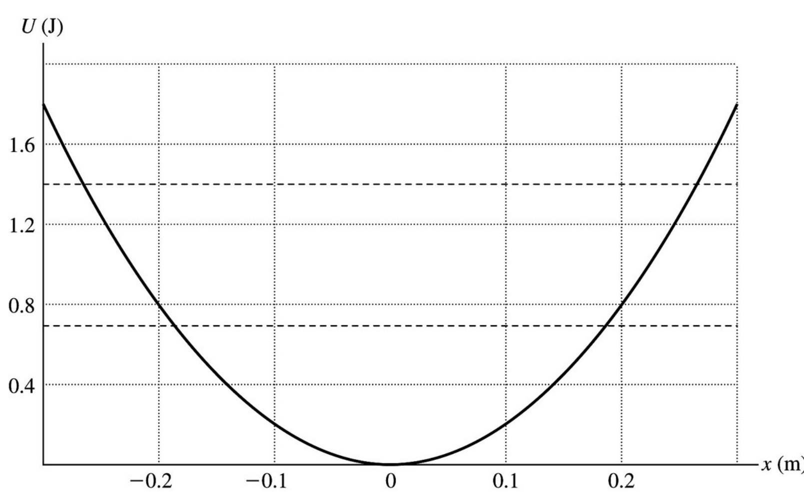


Figure 14.74

14.75. IDENTIFY: $T = 2\pi\sqrt{\frac{m}{k}}$ so the period changes because the mass changes.

SET UP: $\dfrac{dm}{dt} = -2.00\times10^{-3}$ kg/s. The rate of change of the period is $\dfrac{dT}{dt}$.

EXECUTE: **(a)** When the bucket is half full, $m = 7.00$ kg. $T = 2\pi\sqrt{\dfrac{7.00\text{ kg}}{450\text{ N/m}}} = 0.784$ s.

(b) $\dfrac{dT}{dt} = \dfrac{2\pi}{\sqrt{k}}\dfrac{d}{dt}(m^{1/2}) = \dfrac{2\pi}{\sqrt{k}}\tfrac{1}{2}m^{-1/2}\dfrac{dm}{dt} = \dfrac{\pi}{\sqrt{mk}}\dfrac{dm}{dt}$.

$\dfrac{dT}{dt} = \dfrac{\pi}{\sqrt{(7.00\text{ kg})(450\text{ N/m})}}(-2.00\times10^{-3}\text{ kg/s}) = -1.12\times10^{-4}$ s per s. $\dfrac{dT}{dt}$ is negative, so the period is getting shorter.

(c) The shortest period is when all the water has leaked out and $m = 2.00$ kg. In that case, $T = 2\pi\sqrt{m/k} = 0.419$ s.

EVALUATE: The rate at which the period changes is not constant but instead increases in time, even though the rate at which the water flows out is constant.

14.76. **IDENTIFY:** We are looking at molecular vibrations of the HI molecule.

SET UP: The classical frequency is $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$, where $m \approx m_{\text{proton}} = 1.6726\times10^{-27}$ kg.

EXECUTE: **(a)** We want the force constant of the HI molecule, assuming that only the H atom moves significantly. Solve $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ for *k*, giving $k = 4\pi^2 mf^2$ from which we have

$k = 4\pi^2(1.6726\times10^{-27}\text{ kg})(7\times10^{13}\text{ Hz})^2$ = 300k N/m.

(b) $E_{\text{vibr}} = \dfrac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2E_{\text{vibr}}}{m}} = \sqrt{\dfrac{2(5\times10^{-20}\text{ J})}{1.6726\times10^{-27}\text{ kg}}} = 7700\text{ m/s} - 8\text{ km/s}$.

(c) $E = \dfrac{1}{2}kA^2$ so $A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(5\times10^{-20}\text{ J})}{300\text{ N/m}}} \approx 2\times10^{-11}$ m. This result is a little more than 1/10 the equilibrium distance.

EVALUATE: These results are not bad for a rough calculation.

14.77. **IDENTIFY** and **SET UP:** Measure *x* from the equilibrium position of the object, where the gravity and spring forces balance. Let $+x$ be downward.

(a) Use conservation of energy $\tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2$ to relate v_x and *x*. Use $T = 2\pi\sqrt{\dfrac{m}{k}}$ to relate *T* to *k/m*.

EXECUTE: $\tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2$

For $x = 0, \tfrac{1}{2}mv_x^2 = \tfrac{1}{2}kA^2$ and $v = A\sqrt{k/m}$, just as for horizontal SHM. We can use the period to calculate $\sqrt{k/m} : T = 2\pi\sqrt{m/k}$ implies $\sqrt{k/m} = 2\pi/T$. Thus $v = 2\pi A/T = 2\pi(0.100\text{ m})/4.20\text{ s} = 0.150\text{ m/s}$.

(b) **IDENTIFY** and **SET UP:** Use $a_x = -\dfrac{k}{m}x$ to relate a_x and *x*.

EXECUTE: $ma_x = -kx$ so $a_x = -(k/m)x$

+*x*-direction is downward, so here $x = -0.050$ m

$a_x = -(2\pi/T)^2(-0.050\text{ m}) = +(2\pi/4.20\text{ s})^2(0.050\text{ m}) = 0.112\text{ m/s}^2$ (positive, so direction is downward)

(c) **IDENTIFY** and **SET UP:** Use $x = A\cos(\omega t + \phi)$ to relate *x* and *t*. The time asked for is twice the time it takes to go from $x = 0$ to $x = +0.050$ m.

EXECUTE: $x(t) = A\cos(\omega t + \phi)$

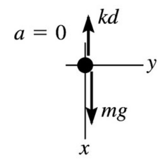
Let $\phi = -\pi/2$, so $x = 0$ at $t = 0$. Then $x = A\cos(\omega t - \pi/2) = A\sin\omega t = A\sin(2\pi t/T)$. Find the time t that gives $x = +0.050$ m: $0.050 \text{ m} = (0.100 \text{ m}) \sin(2\pi t/T)$

$2\pi t/T = \arcsin(0.50) = \pi/6$ and $t = T/12 = 4.20 \text{ s}/12 = 0.350$ s

The time asked for in the problem is twice this, 0.700 s.

(d) IDENTIFY: The problem is asking for the distance d that the spring stretches when the object hangs at rest from it. Apply Newton's second law to the object.

SET UP: The free-body diagram for the object is given in Figure 14.77.



EXECUTE: $\sum F_x = ma_x$

$mg - kd = 0$

$d = (m/k)g$

Figure 14.77

But $\sqrt{k/m} = 2\pi/T$ (part (a)) and $m/k = (T/2\pi)^2$

$$d = \left(\frac{T}{2\pi}\right)^2 g = \left(\frac{4.20 \text{ s}}{2\pi}\right)^2 (9.80 \text{ m/s}^2) = 4.38 \text{ m.}$$

EVALUATE: When the displacement is upward (part (b)), the acceleration is downward. The mass of the partridge is never entered into the calculation. We used just the ratio k/m, that is determined from T.

14.78. IDENTIFY: The rod is a physical pendulum. We will need to do graphical analysis.

SET UP: The period of a physical pendulum is $T = 2\pi\sqrt{\dfrac{I}{mgd}}$, where d is the distance from the rotation axis to the center of gravity of the rod. We want to find I_{cm} about the center of mass of the rod, which is at its center. Therefore we need to derive a relation between I_{cm} and d so we can interpret the graph.

EXECUTE: In the formula $T = 2\pi\sqrt{\dfrac{I}{mgd}}$, I is the moment of inertia of the rod about the rotation axis, which is a distance d from the center of the rod. But we want I_{cm} about the center of gravity of the rod. Using the parallel-axis theorem from Chapter 9, we have $I = I_{cm} + md^2$. Therefore the period is $T = 2\pi\sqrt{\dfrac{I_{cm} + md^2}{mgd}}$. Solving for T^2 gives $T^2 = 4\pi^2\left(\dfrac{I_{cm} + md^2}{mgd}\right) = \dfrac{4\pi^2 I_{cm}}{mgd} + \dfrac{4\pi^2 d}{g}$. Rearranging gives $T^2 - \dfrac{4\pi^2 d}{g} = \left(\dfrac{4\pi^2 I_{cm}}{mg}\right)\dfrac{1}{d}$. Now we can see that a graph of $T^2 - \dfrac{4\pi^2 d}{g}$ versus $1/d$ should be a straight line having slope equal to $\dfrac{4\pi^2 I_{cm}}{mg}$. Therefore $\dfrac{4\pi^2 I_{cm}}{mg} = \text{slope}$. Solving for I_{cm} gives us

$I_{cm} = mg(\text{slope})/4\pi^2 = (0.400 \text{ kg})(9.80 \text{ m/s}^2)(0.320 \text{ m}\cdot\text{s}^2)/(4\pi^2) = 0.0318 \text{ kg}\cdot\text{m}^2$.

EVALUATE: If this rod were uniform, its moment of inertia about its center of gravity would be $I = \dfrac{1}{12}ML^2 = \dfrac{1}{12}(0.400 \text{ kg})(0.800 \text{ m})^2 = 0.0213 \text{ kg}\cdot\text{m}^2$. This is less than we found above, so the nonuniform rod must have more mass toward its ends than a uniform rod. Its density must increase with distance from the center.

14.79. **IDENTIFY:** Apply conservation of linear momentum to the collision between the steak and the pan. Then apply conservation of energy to the motion after the collision to find the amplitude of the subsequent SHM. Use $T = 2\pi\sqrt{\frac{m}{k}}$ to calculate the period.

(a) SET UP: First find the speed of the steak just before it strikes the pan. Use a coordinate system with $+y$ downward.

$v_{0y} = 0$ (released from the rest); $y - y_0 = 0.40$ m; $a_y = +9.80\ \text{m/s}^2$; $v_y = ?$

$v_y^2 = v_{0_y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_y = +\sqrt{2a_y(y - y_0)} = +\sqrt{2(9.80\ \text{m/s}^2)(0.40\ \text{m})} = +2.80\ \text{m/s}$

SET UP: Apply conservation of momentum to the collision between the steak and the pan. After the collision the steak and the pan are moving together with common velocity v_2. Let *A* be the steak and *B* be the pan. The system before and after the collision is shown in Figure 14.79.

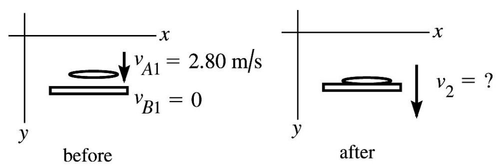


Figure 14.79

EXECUTE: P_y conserved: $m_A v_{A1y} + m_B v_{B1y} = (m_A + m_B)v_{2y}$

$m_A v_{A1} = (m_A + m_B)v_2$

$$v_2 = \left(\frac{m_A}{m_A + m_B}\right)v_{A1} = \left(\frac{2.2\ \text{kg}}{2.2\ \text{kg} + 0.20\ \text{kg}}\right)(2.80\ \text{m/s}) = 2.57\ \text{m/s}$$

(b) SET UP: Conservation of energy applied to the SHM gives: $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kA^2$ where v_0 and x_0 are the initial speed and displacement of the object and where the displacement is measured from the equilibrium position of the object.

EXECUTE: The weight of the steak will stretch the spring an additional distance *d* given by $kd = mg$ so $d = \frac{mg}{k} = \frac{(2.2\ \text{kg})(9.80\ \text{m/s}^2)}{400\ \text{N/m}} = 0.0539\ \text{m}.$ So just after the steak hits the pan, before the pan has had time to move, the steak plus pan is 0.0539 m above the equilibrium position of the combined object. Thus $x_0 = 0.0539$ m. From part (a) $v_0 = 2.57$ m/s, the speed of the combined object just after the collision. Then $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kA^2$ gives

$$A = \sqrt{\frac{mv_0^2 + kx_0^2}{k}} = \sqrt{\frac{2.4\ \text{kg}(2.57\ \text{m/s})^2 + (400\ \text{N/m})(0.0539\ \text{m})^2}{400\ \text{N/m}}} = 0.21\ \text{m}$$

(c) $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{\frac{2.4\ \text{kg}}{400\ \text{N/m}}} = 0.49\ \text{s}$

EVALUATE: The amplitude is less than the initial height of the steak above the pan because mechanical energy is lost in the inelastic collision.

14.80. **IDENTIFY:** $F_x = -kx$ allows us to calculate *k*. $T = 2\pi\sqrt{m/k}$. $x(t) = A\cos(\omega t + \phi)$. $F_{\text{net}} = -kx$.

SET UP: Let $\phi = \pi/2$ so $x(t) = A\sin(\omega t)$. At $t = 0$, $x = 0$ and the object is moving downward. When the object is below the equilibrium position, F_{spring} is upward.

EXECUTE: **(a)** Solving $T = 2\pi\sqrt{m/k}$ for m, and using $k = \dfrac{F}{\Delta l}$ gives

$$m = \left(\frac{T}{2\pi}\right)^2 \frac{F}{\Delta l} = \left(\frac{1.00 \text{ s}}{2\pi}\right)^2 \frac{40.0 \text{ N}}{0.250 \text{ m}} = 4.05 \text{ kg}.$$

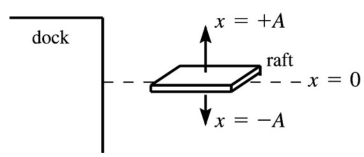
(b) $t = (0.35)T,$ and so $x = -A\sin[2\pi(0.35)] = -0.0405$ m. Since $t > T/4,$ the mass has already passed the lowest point of its motion, and is on the way up.

(c) Taking upward forces to be positive, $F_{\text{spring}} - mg = -kx,$ where x is the displacement from equilibrium, so $F_{\text{spring}} = -(160 \text{ N/m})(-0.030 \text{ m}) + (4.05 \text{ kg})(9.80 \text{ m/s}^2) = 44.5 \text{ N}.$

EVALUATE: When the object is below the equilibrium position the net force is upward and the upward spring force is larger in magnitude than the downward weight of the object.

14.81. **IDENTIFY**: Use $x = A\cos(\omega t + \phi)$ to relate x and t. $T = 3.5$ s.

SET UP: The motion of the raft is sketched in Figure 14.81.



Let the raft be at $x = +A$ when $t = 0.$ Then $\phi = 0$ and $x(t) = A\cos\omega t.$

Figure 14.81

EXECUTE: Calculate the time it takes the raft to move from $x = +A = +0.200$ m to $x =$ $A - 0.100 \text{ m} = 0.100$ m.

Write the equation for $x(t)$ in terms of T rather than ω: $\omega = 2\pi/T$ gives that $x(t) = A\cos(2\pi t/T)$

$x = A$ at $t = 0$

$x = 0.100$ m implies $0.100 \text{ m} = (0.200 \text{ m}) \cos(2\pi t/T)$

$\cos\ (2\pi t/T) = 0.500$ so $2\pi t/T = \arccos(0.500) = 1.047$ rad

$t = (T/2\pi)(1.047 \text{ rad}) = (3.5 \text{ s}/2\pi)(1.047 \text{ rad}) = 0.583$ s

This is the time for the raft to move down from $x = 0.200$ m to $x = 0.100$ m. But people can also get off while the raft is moving up from $x = 0.100$ m to $x = 0.200$ m, so during each period of the motion the time the people have to get off is $2t = 2(0.583 \text{ s}) = 1.17$ s.

EVALUATE: The time to go from $x = 0$ to $x = A$ and return is $T/2 = 1.75$ s. The time to go from $x = A/2$ to A and return is less than this.

14.82. **IDENTIFY**: $T = 2\pi/\omega.$ $F_r(r) = -kr$ to determine k.

SET UP: Example 13.10 derives $F_r(r) = -\dfrac{GM_{\text{E}}m}{R_{\text{E}}^3}r.$

EXECUTE: $F_r(r) = -\dfrac{GM_{\text{E}}m}{R_{\text{E}}^3}r$ is in the form of $F = -kx$, with x replaced by r, so the motion is simple harmonic. $k = \dfrac{GM_{\text{E}}m}{R_{\text{E}}^3}.$ $\omega^2 = \dfrac{k}{m} = \dfrac{GM_{\text{E}}}{R_{\text{E}}^3} = \dfrac{g}{R_{\text{E}}}.$ The period is then

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R_{\text{E}}}{g}} = 2\pi\sqrt{\frac{6.37\times 10^6 \text{ m}}{9.80 \text{ m/s}^2}} = 5070 \text{ s, or } 84.5 \text{ min.}$$

EVALUATE: The period is independent of the mass of the object but does depend on R_{E}, which is also the amplitude of the motion.

14.83. **IDENTIFY**: During the collision, linear momentum is conserved. After the collision, mechanical energy is conserved and the motion is SHM.

SET UP: The linear momentum is $p_x = mv_x$, the kinetic energy is $\frac{1}{2}mv^2$, and the potential energy is $\frac{1}{2}kx^2$. The period is $T = 2\pi\sqrt{\frac{m}{k}}$, which is the target variable.

EXECUTE: Apply conservation of linear momentum to the collision:

$(8.00\times10^{-3}\text{ kg})(280\text{ m/s}) = (1.00\text{ kg})v.$ $v = 2.24\text{ m/s}.$ This is v_{max} for the SHM. $A = 0.150\text{ m}$ (given).So $\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2.$ $k = \left(\frac{v_{\text{max}}}{A}\right)^2 m = \left(\frac{2.24\text{ m/s}}{0.150\text{ m}}\right)^2(1.00\text{ kg}) = 223.0\text{ N/m}.$

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.00\text{ kg}}{223.0\text{ N/m}}} = 0.421\text{ s}.$

EVALUATE: This block would weigh about 2 pounds, which is rather heavy, but the spring constant is large enough to keep the period within an easily observable range.

14.84. **IDENTIFY:** Newton's second law, in both its linear and rotational form, applies to this system. The motion is SHM.

SET UP: $\sum F = ma_{\text{cm}}$ and $\sum\tau = I\alpha$, where $I = \frac{2}{5}MR^2$ for a solid sphere, and $R\alpha = a_{\text{cm}}$ with no slipping.

EXECUTE: For each sphere, $f_s R = \left(\frac{2}{5}MR^2\right)\alpha.$ $R\alpha = a_{\text{cm}}.$ $f_s = \frac{2}{5}Ma_{\text{cm}}.$ For the system of two spheres,

$2f_s - kx = -2Ma_{\text{cm}}.$ $\frac{4}{5}Ma_{\text{cm}} - kx = -2Ma_{\text{cm}}.$ $kx = \frac{14}{5}Ma_{\text{cm}}$ and $a_{\text{cm}} = \frac{5}{14}\left(\frac{k}{M}\right)x.$ $a_x = -\frac{5}{14}\left(\frac{k}{M}\right)x.$

$a_x = -\omega^2 x$ so $\omega = \sqrt{\frac{5k}{14M}}.$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{14M}{5k}} = 2\pi\sqrt{\frac{14(0.800\text{ kg})}{5(160\text{ N/m})}} = 0.743\text{ s}.$

EVALUATE: If the surface were smooth, there would be no rolling, but the presence of friction provides the torque to cause the spheres to rotate.

14.85. **IDENTIFY**: Apply conservation of energy to the motion before and after the collision. Apply conservation of linear momentum to the collision. After the collision the system moves as a simple pendulum. If the maximum angular displacement is small, $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}.$

SET UP: In the motion before and after the collision there is energy conversion between gravitational potential energy mgh, where h is the height above the lowest point in the motion, and kinetic energy.

EXECUTE: Energy conservation during downward swing: $m_2gh_0 = \frac{1}{2}m_2v^2$ and

$v = \sqrt{2gh_0} = \sqrt{2(9.8\text{ m/s}^2)(0.100\text{ m})} = 1.40\text{ m/s}.$

Momentum conservation during collision: $m_2v = (m_2 + m_3)V$ and

$V = \frac{m_2v}{m_2 + m_3} = \frac{(2.00\text{ kg})(1.40\text{ m/s})}{5.00\text{ kg}} = 0.560\text{ m/s}.$

Energy conservation during upward swing: $Mgh_f = \frac{1}{2}MV^2$ and

$h_f = V^2/2g = \frac{(0.560\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 0.0160\text{ m} = 1.60\text{ cm}.$

Figure 14.85 shows how the maximum angular displacement is calculated from h_{f}. $\cos\theta = \dfrac{48.4 \text{ cm}}{50.0 \text{ cm}}$ and $\theta = 14.5°$. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}} = \dfrac{1}{2\pi}\sqrt{\dfrac{9.80 \text{ m/s}^2}{0.500 \text{ m}}} = 0.705 \text{ Hz}.$

EVALUATE: $14.5° = 0.253$ rad. $\sin(0.253 \text{ rad}) = 0.250$. $\sin\theta \approx \theta$ and the equation $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$ is accurate.

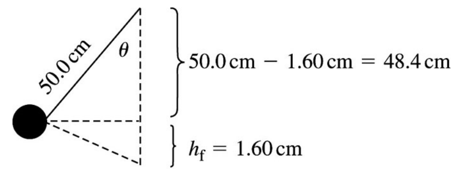


Figure 14.85

14.86. **IDENTIFY** and **SET UP:** $T = 2\pi\sqrt{\dfrac{I}{mgd}}$ gives the period for the bell and $T = 2\pi\sqrt{L/g}$ gives the period for the clapper.
EXECUTE: The bell swings as a physical pendulum so its period of oscillation is given by
$T = 2\pi\sqrt{I/mgd} = 2\pi\sqrt{18.0 \text{ kg}\cdot\text{m}^2/(34.0 \text{ kg})(9.80 \text{ m/s}^2)(0.60 \text{ m})} = 1.885 \text{ s}.$
The clapper is a simple pendulum so its period is given by $T = 2\pi\sqrt{L/g}$.
Thus $L = g(T/2\pi)^2 = (9.80 \text{ m/s}^2)(1.885 \text{ s}/2\pi)^2 = 0.88 \text{ m}.$
EVALUATE: If the cm of the bell were at the geometrical center of the bell, the bell would extend 1.20 m from the pivot, so the clapper is well inside the bell.

14.87. **IDENTIFY**: The motion is simple harmonic if the equation of motion for the angular oscillations is of the form $\dfrac{d^2\theta}{dt^2} = -\dfrac{\kappa}{I}\theta$, and in this case the period is $T = 2\pi\sqrt{I/\kappa}$.

SET UP: For a slender rod pivoted about its center, $I = \frac{1}{12}ML^2$.

EXECUTE: The torque on the rod about the pivot is $\tau = -\left(k\dfrac{L}{2}\theta\right)\dfrac{L}{2}$. $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$ gives

$\dfrac{d^2\theta}{dt^2} = -k\dfrac{L^2/4}{I}\theta = -\dfrac{3k}{M}\theta.$ $\dfrac{d^2\theta}{dt^2}$ is proportional to θ and the motion is angular SHM. $\dfrac{\kappa}{I} = \dfrac{3k}{M}$,
$T = 2\pi\sqrt{\dfrac{M}{3k}}.$

EVALUATE: The expression we used for the torque, $\tau = -\left(k\dfrac{L}{2}\theta\right)\dfrac{L}{2}$, is valid only when θ is small enough for $\sin\theta \approx \theta$ and $\cos\theta \approx 1$.

14.88. **IDENTIFY**: The object oscillates as a physical pendulum, with $f = \dfrac{1}{2\pi}\sqrt{\dfrac{Mgd}{I}}$, where $M = 2m$ is the total mass of the object.
SET UP: The moment of inertia about the pivot is $2(1/3)ML^2 = (2/3)\,ML^2$, and the center of gravity when balanced is a distance $d = L/(2\sqrt{2})$ below the pivot.

EXECUTE: The frequency is $f = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{6g}{4\sqrt{2}L}} = \dfrac{1}{4\pi}\sqrt{\dfrac{6g}{\sqrt{2}L}}.$

EVALUATE: If $f_{\text{sp}} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$ is the frequency for a simple pendulum of length L,

$f = \dfrac{1}{2}\sqrt{\dfrac{6}{\sqrt{2}}} f_{\text{sp}} = 1.03 f_{\text{sp}}.$

14.89. IDENTIFY: The velocity is a sinusoidal function. From the graph we can read off the period and use it to calculate the other quantities.

SET UP: The period is the time for 1 cycle; after time T the motion repeats. The graph shows that $T = 1.60$ s and $v_{\max} = 20.0$ cm/s. Mechanical energy is conserved, so $\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$, and Newton's second law applies to the mass.

EXECUTE: **(a)** $T = 1.60$ s (from the graph with the problem).

(b) $f = \dfrac{1}{T} = 0.625$ Hz.

(c) $\omega = 2\pi f = 3.93$ rad/s.

(d) $v_x = v_{\max}$ when $x = 0$ so $\frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2$. $A = v_{\max}\sqrt{\dfrac{m}{k}}$. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ so $A = v_{\max}/(2\pi f)$. From the graph in the problem, $v_{\max} = 0.20$ m/s, so $A = \dfrac{0.20\text{ m/s}}{2\pi(0.625\text{ Hz})} = 0.051\text{ m} = 5.1\text{ cm}$. The mass is at $x = \pm A$ when $v_x = 0$, and this occurs at $t = 0.4$ s, 1.2 s, and 1.8 s.

(e) Newton's second law gives $-kx = ma_x$, so

$a_{\max} = \dfrac{kA}{m} = (2\pi f)^2 A = (4\pi^2)(0.625\text{ Hz})^2(0.051\text{ m}) = 0.79\text{ m/s}^2 = 79\text{ cm/s}^2$. The acceleration is maximum when $x = \pm A$ and this occurs at the times given in (d).

(f) $T = 2\pi\sqrt{\dfrac{m}{k}}$ so $m = k\left(\dfrac{T}{2\pi}\right)^2 = (75\text{ N/m})\left(\dfrac{1.60\text{ s}}{2\pi}\right)^2 = 4.9$ kg.

EVALUATE: The speed is maximum at $x = 0$, when $a_x = 0$. The magnitude of the acceleration is maximum at $x = \pm A$, where $v_x = 0$.

14.90. IDENTIFY and SET UP: As stated in the problem, $T = 2\pi\sqrt{\dfrac{m + m_{\text{eff}}}{k}}$.

EXECUTE: **(a)** The graph of T^2 versus m is shown in Figure 14.90. Note that the times given in the table with the problem are for 10 oscillations, so we must divide each of them by 10 to get the period.

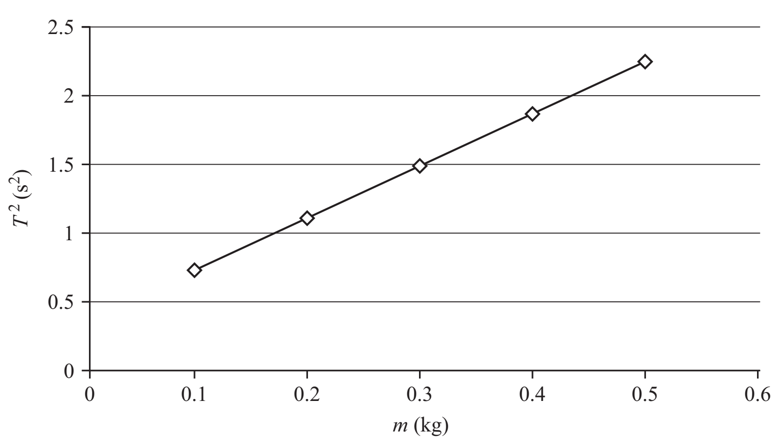


Figure 14.90

(b) Squaring the equation $T = 2\pi\sqrt{\frac{m+m_{\text{eff}}}{k}}$ and solving for T^2 in terms of m, we get

$$T^2 = \left(\frac{4\pi^2}{k}\right)m + \frac{4\pi^2 m_{\text{eff}}}{k}.$$

In the graph of T^2 versus m, the slope is $4\pi^2/k$ and the vertical intercept is $4\pi^2 m_{\text{eff}}/k$. The best-fit equation to our plotted points is $T^2 = (3.878\ \text{s}^2/\text{kg})m + 0.3492\ \text{s}^2$. Therefore we have
slope = $4\pi^2/k = 3.876\ \text{s}^2/\text{kg}$
$k = 4\pi^2/(3.876\ \text{s}^2/\text{kg}) = 10.19\ \text{kg/s}^2$ which rounds to $10.2\ \text{kg/s}^2 = 10.2$ N/m.
(c) Using the vertical intercept, we have
$4\pi^2 m_{\text{eff}}/k = 0.3492\ \text{s}^2$
$m_{\text{eff}} = (0.3492\ \text{s}^2)(10.19\ \text{kg/s}^2)/(4\pi^2) = 0.0901$ kg.
(d) The mass of the spring is 0.250 kg, so
$m_{\text{eff}}/m_{\text{spring}} = (0.0901\ \text{kg})/(0.250\ \text{kg}) = 0.360$.
Thus m_{eff} is 36% the mass of the spring.
(e) $T = 2\pi\sqrt{\frac{m+m_{\text{eff}}}{k}} = 2\pi\sqrt{\frac{0.450\ \text{kg} + 0.0901\ \text{kg}}{10.19\ \text{kg/s}^2}} = 1.45$ s.
$f = 1/T = 1/(1.45\ \text{s}) = 0.691$ Hz.
$\omega = 2\pi f = 2\pi(0.691\ \text{H}) = 4.34$ rad/s.
EVALUATE: If the mass of a spring is comparable to the mass of the object oscillating from it, the mass of the spring can have a significant effect on the period. The result we found, that the effective mass is 0.36 times the mass of the spring, is in very good agreement with the result of challenge problem 14.93(c), which shows that the effective mass is 1/3 the mass of the spring.

14.91. **IDENTIFY** and **SET UP**: For small-amplitude oscillations, the period of a simple pendulum is $T = 2\pi\sqrt{L/g}$.

EXECUTE: **(a)** The graph of T^2 versus L is shown in Figure 14.91a. Using $T = 2\pi\sqrt{L/g}$, we solve for T^2 in terms of L, which gives $T^2 = \left(\frac{4\pi^2}{g}\right)L$. The graph of T^2 versus L should be a straight line having slope $4\pi^2/g$. The best-fit line for our data has the equation $T^2 = (3.9795\ \text{s}^2/\text{m})L + 0.6674\ \text{s}^2$. The quantity $4\pi^2/g = 4\pi^2/(9.80\ \text{m/s}^2) = 4.03\ \text{s}^2/\text{m}$. Our line has slope 3.98 s^2/m, which is in very close agreement with the expected slope.

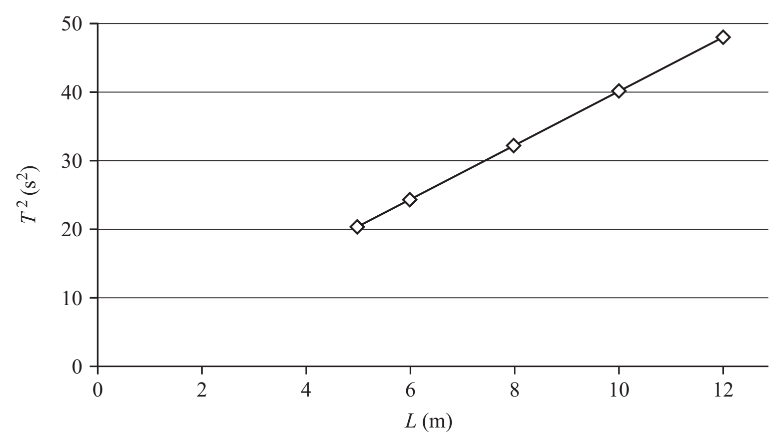


Figure 14.91a

(b) As L decreases, the angle the string makes with the vertical increases because the metal sphere is always released when it is touching the vertical wall. The formula $T = 2\pi\sqrt{L/g}$ is valid only for small angles. Figure 14.91b shows the graph of T/T_0 versus L.

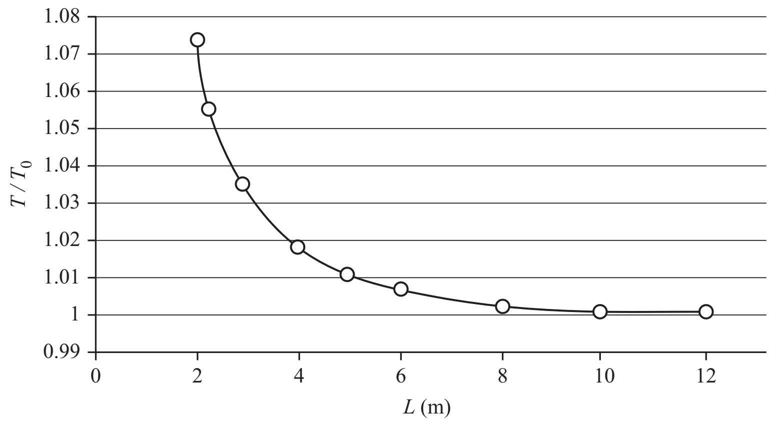


Figure 14.91b

(c) Since $T > T_0$, if T_0 is in error by 5%, $T/T_0 = 1.05$. From the graph in Figure 14.91b, that occurs for $L \approx 2.5$ m. In that case, $\sin\theta = (2.0\text{ m})/(2.5\text{ m}) = 0.80$, which gives $\theta = 53°$.

EVALUATE: Even for an angular amplitude of 53°, the error in using the formula $T = 2\pi\sqrt{L/g}$ is only 5%, so this formula is very useful in most situations. But for very large angular amplitudes it is not reliable.

14.92. **IDENTIFY**: In each situation, imagine the mass moves a distance Δx, the springs move distances Δx_1 and Δx_2, with forces $F_1 = -k_1\Delta x_1, F_2 = -k_2\Delta x_2$.

SET UP: Let Δx_1 and Δx_2 be positive if the springs are stretched, negative if compressed.

EXECUTE: **(a)** $\Delta x = \Delta x_1 = \Delta x_2, F = F_1 + F_2 = -(k_1 + k_2)\Delta x,$ so $k_{\text{eff}} = k_1 + k_2$.

(b) Despite the orientation of the springs, and the fact that one will be compressed when the other is extended, $\Delta x = \Delta x_1 - \Delta x_2$ and both spring forces are in the same direction. The above result is still valid; $k_{\text{eff}} = k_1 + k_2$.

(c) For massless springs, the force on the block must be equal to the tension in any point of the spring combination, and $F = F_1 = F_2$. $\Delta x_1 = -\dfrac{F}{k_1}, \Delta x_2 == -\dfrac{F}{k_2}$, $\Delta x = -\left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right)F = -\dfrac{k_1 + k_2}{k_1 k_2}F$ and

$$k_{\text{eff}} = \frac{k_1 k_2}{k_1 + k_2}.$$

(d) The result of part (c) shows that when a spring is cut in half, the effective spring constant doubles, and so the frequency increases by a factor of $\sqrt{2}$. Therefore $f_1/f_2 = 1/\sqrt{2}$.

EVALUATE: In cases (a) and (b) the effective force constant is greater than either k_1 or k_2 and in case (c) it is less.

14.93. **IDENTIFY:** Follow the procedure specified in the hint.
SET UP: Denote the position of a piece of the spring by l; $l=0$ is the fixed point and $l=L$ is the moving end of the spring. Then the velocity of the point corresponding to l, denoted u, is $u(l)=v\dfrac{l}{L}$ (when the spring is moving, l will be a function of time, and so u is an implicit function of time).

EXECUTE: **(a)** $dm=\dfrac{M}{L}dl$, and so $dK=\dfrac{1}{2}dm\,u^2=\dfrac{1}{2}\dfrac{Mv^2}{L^3}l^2\,dl$ and $K=\int dK=\dfrac{Mv^2}{2L^3}\int_0^L l^2\,dl=\dfrac{Mv^2}{6}$.

(b) $mv\dfrac{dv}{dt}+kx\dfrac{dx}{dt}=0$, or $ma+kx=0$, which is Eq. (14.4).

(c) m is replaced by $\dfrac{M}{3}$, so $\omega=\sqrt{\dfrac{3k}{M}}$ and $M'=\dfrac{M}{3}$.

EVALUATE: The effective mass of the spring is only one-third of its actual mass.

14.94. **IDENTIFY** and **SET UP**: The frequency is $f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

EXECUTE: Use $f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ to solve for the mass:
$m = k/(2\pi f)^2 = (1000\text{ N/m})/[2\pi(100\times10^3\text{ Hz})]^2 = 2.5\times10^{-9}\text{ kg} = 2.5\times10^{-6}\text{ g} = 2.5\ \mu\text{g}$, which is choice (c).

EVALUATE: This is a much smaller mass than we've dealt with in the previous problems, but we are looking at vibrations at the molecular level.

14.95. **IDENTIFY** and **SET UP**: The energy is constant, so it is equal to the potential energy when the speed is zero, so $E=\frac{1}{2}kA^2$.

EXECUTE: $E=\frac{1}{2}(1000\text{ N/m})(0.050\times10^{-9}\text{ m})^2 = 1.25\times10^{-18}$ J, which is closest to choice (a).

EVALUATE: This is a much smaller energy than we've dealt with in the previous problems, but we are looking at vibrations at the molecular level.

14.96. **IDENTIFY** and **SET UP**: The frequency is $f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$, and we want $\dfrac{\Delta f}{f}$.

EXECUTE: $\dfrac{\Delta f}{f}=\dfrac{f-f_0}{f_0}$. Using $f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ and calling k the new force constant, we have

$$\frac{\Delta f}{f}=\frac{(1/2\pi)\sqrt{k/m}-(1/2\pi)\sqrt{k_0/m}}{(1/2\pi)\sqrt{k_0/m}}=\frac{\sqrt{k}-\sqrt{k_0}}{\sqrt{k_0}}=\sqrt{\frac{k}{k_0}}-1=\sqrt{\frac{1005\text{ N/m}}{1000\text{ N/m}}}-1$$

$\dfrac{\Delta f}{f}=2.5\times10^{-3}=0.25\%$, which is choice (b).

EVALUATE: Since k_{surf} = 5 N/m is only 5/1000 = 0.5% of the original force constant, the effect on the frequency is even less, at 0.25%. This is reasonable because the frequency is proportional to the square root of the force constant.

15

MECHANICAL WAVES

VP15.3.1. **IDENTIFY:** We are dealing with general characteristics of waves.

SET UP: We know that $f = 1/T$, $v = f\lambda$, $\omega = 2\pi f$, and $k = 2\pi/\lambda$.

EXECUTE: **(a)** $f = 1/T = 1/(5.10\text{ s}) = 0.196\text{ Hz}$.

(b) $v = f\lambda = (0.196\text{ Hz})(30.5\text{ m}) = 5.98\text{ m/s}$.

(c) $\omega = 2\pi f = 2\pi(0.196\text{ Hz}) = 1.23\text{ rad/s}$.

(d) $k = 2\pi/\lambda = 2\pi/(0.206\text{ m}) = 0.206\text{ m}^{-1}$.

EVALUATE: Caution! The wave number k is *not* the number of waves. It is simply defined as $k = 2\pi/\lambda$.

VP15.3.2. **IDENTIFY:** This problem involves the characteristics of a sound wave on Mars.

SET UP: $T = 1/f$, $\omega = 2\pi f$, and $v = f\lambda$.

EXECUTE: **(a)** $T = 1/f = 1/(125\text{ Hz}) = 8.00\times 10^{-3}\text{ s}$.

Solve $v = f\lambda$ for λ: $\lambda = v/f = (245\text{ m/s})/(125\text{ Hz}) = 1.96\text{ m}$.

(b) Solve $v = f\lambda$ for f: $f = v/\lambda = (245\text{ m/s})/(3.00\text{ m}) = 81.7\text{ Hz}$.

$\omega = 2\pi f = 2\pi(81.7\text{ Hz}) = 513\text{ rad/s}$.

EVALUATE: On Earth the wavelength would be almost twice as long as on Mars for the same frequency because the speed of sound is about twice as great as on Mars.

VP15.3.3. **IDENTIFY:** We are dealing with a traveling wave on a string.

SET UP: $y(x,t) = A\cos(kx - \omega t)$, $\omega = 2\pi f$, $v = f\lambda$, $k = 2\pi/\lambda$, and $v = \sqrt{\dfrac{F}{\mu}}$. Call the $+x$-axis the direction in which the wave is traveling.

EXECUTE: **(a)** $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{810\text{ N}}{0.0650\text{ kg/m}}} = 112\text{ m/s}$.

(b) Solve $v = f\lambda$ for λ: $\lambda = v/f = (112\text{ m/s})/(25.0\text{ Hz}) = 4.47\text{ m}$.

(c) Use $y(x,t) = A\cos(kx - \omega t)$ with $x = 2.50\text{ m}$, $k = 2\pi/\lambda = 2\pi/(4.47\text{ m}) = 1.41\text{ m}^{-1}$, $A = 5.00\text{ mm}$, and $\omega = 2\pi f = 2\pi(25.0\text{ Hz}) = 157\text{ rad/s}$. Using these numbers gives

$y(2.50\text{ m},t) = (5.00\text{ mm})\cos[3.52 - (157\text{ rad/s})t]$.

EVALUATE: This wave is traveling in the $+x$ direction. The equation for reflected waves having the same amplitude would be $y(2.50\text{ m},t) = (5.00\text{ mm})\cos[3.52 + (157\text{ rad/s})t]$.

VP15.3.4. **IDENTIFY:** We are dealing with a traveling wave on a string.

SET UP: $v = f\lambda$, $v = \sqrt{\dfrac{F}{\mu}}$. The 0.400 m is the distance from a crest to the adjacent trough, which is one-half of a complete wave. Therefore $\lambda = 2(0.400\text{ m}) = 0.800\text{ m}$.

EXECUTE: **(a)** $v = f\lambda$ = (45.0 Hz)(0.800 m) = 36.0 m/s.

(b) Solve $v = \sqrt{\dfrac{F}{\mu}}$ for μ: $v = \sqrt{\dfrac{F}{\mu}}$ = 0.193 kg/m.

EVALUATE: A linear mass density of 0.193 kg/m is a bit large but not unreasonable for a string.

VP15.5.1. **IDENTIFY:** We are dealing with a traveling wave on a rope and the average power it transfers.

SET UP: $P_{\text{av}} = \dfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$, $\omega = 2\pi f$, and $f = 1/T$. The target variable is the average power delivered by the wave.

EXECUTE: **(a)** $\omega = 2\pi f$ = 2π/T = 2π/(0.575 s) = 10.9 rad/s.

(b) $P_{\text{av}} = \dfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \dfrac{1}{2}\sqrt{\dfrac{m}{L}F}\,\omega^2 A^2 = \dfrac{1}{2}\sqrt{\left(\dfrac{2.50\text{ kg}}{50.0\text{ m}}\right)(600\text{ N})}\,(10.9\text{ rad/s})^2(0.0300\text{ m})^2 = 0.294\text{ W}.$

EVALUATE: Note that the power depends on the *square* of the frequency and the amplitude. This power is much less than that of an ordinary 60 W light bulb.

VP15.5.2. **IDENTIFY:** We are dealing with the maximum power carried by a traveling wave on a piano wire.

SET UP: $P_{\max} = \sqrt{\mu F}\,\omega^2 A^2$, $\omega = 2\pi f$. We know all the quantities except the amplitude, and our target variable is the amplitude of the wave.

EXECUTE: Solve $P_{\max} = \sqrt{\mu F}\,\omega^2 A^2$ for A: $A = \sqrt{\dfrac{P_{\max}}{\omega^2\sqrt{\mu F}}}$. Using $P_{\max}$ = 5.20 W, F = 185 N, μ = 5.55×10^{-4} kg/m, and $\omega = 2\pi f$ = 2π(256 Hz) = 512π Hz, we get $A = 2.50\times 10^{-3}$ m = 2.50 mm.

EVALUATE: An amplitude of 2.50 mm on a piano string is large enough to see readily.

VP15.5.3. **IDENTIFY:** We are investigating the power output of a sound speaker.

SET UP: $I = P/A$ where $A = 4\pi r^2$.

EXECUTE: **(a)** $I = P/A$ = (8.00 W)/[4π(2.00 m)^2] = 0.159 W/m^2.

(b) Solve $I = P/(4\pi r^2)$ for r: $r = \sqrt{\dfrac{P}{4\pi I}} = \sqrt{\dfrac{8.00\text{ W}}{4\pi(0.045\text{ W/m}^2)}} = 3.76$ m.

EVALUATE: The distance from the speaker in part (b) is nearly twice as great as in part (a), which is reasonable because the intensity in (b) is considerably less than in (a).

VP15.5.4. **IDENTIFY:** We are dealing with the sonic power received by a frog's "ears."

SET UP: $I = P/4\pi r^2$ and the energy E received by an area A during a time t is $E = IAt$. Our target variable is the amount of energy the frog's membrane receives in one second.

EXECUTE: $I = P/4\pi r^2$ where r is the distance from the source of sound and P is the power the source is emitting. The area A is $A = \pi R^2$, where R is the radius of the membrane. Combining these quantities gives $E = IAt = \left(\dfrac{P}{4\pi r^2}\right)\left(\pi R^2\right)t$. Using $P = 2.00\times 10^{-6}$ W, r = 1.50 m, t = 1.00 s, and $R = 5.00\times 10^{-3}$ m, we get $E = 5.56\times 10^{-12}$ J.

EVALUATE: This is a very small amount of energy, so the frog must have very sensitive "ears"!

VP15.8.1. **IDENTIFY:** We are dealing with a standing wave on a string.

SET UP: The equation for the standing wave is $y(x,t) = A_{\text{SW}}\sin k_n x \cos \omega_n t$, where $\omega = 2\pi f$. The speed of the wave is $v = f\lambda$; it is a constant and directed along the string. The transverse velocity is $v_y(x,t) = \dfrac{\partial y}{\partial t}$; it is different for different points on the string and is directed perpendicular to the string.

EXECUTE: **(a)** The distance of 0.125 m between nodes is one-half a wavelength, so λ = 0.250 m. Therefore $v = f\lambda$ = (256 Hz)(0.250 m) = 64.0 m/s.

(b) $v_y = \dfrac{\partial y}{\partial t} = \dfrac{\partial\left((A_{SW}\sin k_n x)\sin\omega_n t\right)}{\partial t} = A_{SW}\omega_n \sin k_n x\,\cos\omega_n t$. The maximum speed is $A_{SW}\omega_n$, so $v_{y\text{-max}}$ $= (1.40\times10^{-3}\text{ m})[2\pi(256\text{ Hz})] = 2.25\text{ m/s}$.

(c) $a_y = \dfrac{\partial v_y}{\partial t} = -A_{SW}\omega_n^2 \sin k_n x\,\sin\omega_n t$. The $= 2L$ maximum acceleration is $A_{SW}\omega_n^2$, so

$a_{y\text{-max}} = (1.40\times10^{-3}\text{ m})[2\pi(256\text{ Hz})]^2 = 3620\text{ m/s}^2$.

EVALUATE: For a very taut string or wire, the maximum acceleration can be extremely large.

VP15.8.2. **IDENTIFY:** This problem involves a standing wave on the G string of a guitar string.

SET UP: $v = \sqrt{\dfrac{F}{\mu}}$, $v = f\lambda$, $\lambda_n = \dfrac{2L}{n}$, and for the fundamental mode $n = 1$ so $\lambda_1 = \dfrac{2L}{1} = 2L$.

EXECUTE: **(a)** We want the wave speed. $v = f_1\lambda_1 = f_1(2L) = (196\text{ Hz})(2)(0.641\text{ m}) = 251\text{ m/s}$.

(b) We want the tension in the string, so solve $v = \sqrt{\dfrac{F}{\mu}}$ for F, giving $F = \mu v^2$. Therefore

$F = (2.29\times10^{-3}\text{ kg/m})(251\text{ m/s})^2 = 145\text{ N}$.

EVALUATE: This is a very fast wave since the tension is large.

VP15.8.3. **IDENTIFY:** Standing waves form on the cable that is fixed at both ends.

SET UP: $\lambda_n = \dfrac{2L}{n}$, $v = f\lambda$, $v = \sqrt{\dfrac{F}{\mu}}$. Since there are 5 antinodes on the cable, it is vibrating in its 5th harmonic, so $n = 5$. We want to know the wavelength and frequency of the waves and the linear mass density of the cable.

EXECUTE: **(a)** $\lambda_n = \dfrac{2L}{n}$ so $\lambda_5 = \dfrac{2L}{5} = \dfrac{6.00\text{ m}}{5} = 1.20\text{ m}$.

(b) Solve $v = f\lambda$ for f, giving $f = \dfrac{v}{\lambda} = \dfrac{96.0\text{ m/s}}{1.20\text{ m}} = 80.0\text{ Hz}$.

(c) Solve $v = \sqrt{\dfrac{F}{\mu}}$ for μ giving $\mu = \dfrac{F}{v^2} = \dfrac{175\text{ N}}{(96.0\text{ m/s})^2} = 1.90\times10^{-2}\text{ kg/m}$.

EVALUATE: This density is 19 kg/m, which is not unreasonable for a thin cable.

VP15.8.4. **IDENTIFY:** A string fixed at both ends is vibrating in its second harmonic of a standing wave pattern. This vibration produces a sound wave.

SET UP: $v = \sqrt{\dfrac{F}{\mu}}$, $\lambda_n = \dfrac{2L}{n}$, $f_n = nf_1$, $v = f\lambda$. For the second harmonic, $n = 2$. The target variables are the tension in the string and the wavelength of the sound wave the vibrating string produces.

EXECUTE: **(a)** We want the tension in the string. First find the wavelength and use it to find the speed of the wave. Then use $v = \sqrt{\dfrac{F}{\mu}}$ to find the tension. For the string in the second harmonic,

$\lambda_2 = 2L/2 = L = 0.500\text{ m}$. The wave speed is $v = f\lambda = (512\text{ Hz})(0.500\text{ m}) = 256\text{ m/s}$. Now solve

$v = \sqrt{\dfrac{F}{\mu}}$ for F giving $F = \mu v^2 = (1.17\times10^{-3}\text{ kg/m})(256\text{ m/s})^2 = 76.7\text{ N}$.

(b) We want the wavelength of the sound wave this string produces when vibrating in its fundamental harmonic. Each cycle of the string produces a cycle of sound waves, so the frequency of the sound will be the same as the frequency of the string. The string is now vibrating in its *fundamental* frequency, not its second harmonic. Using $f_n = nf_1$, we have $f_2 = 2f_1 = 512$ Hz, so its frequency is $f_1 = (512\text{ Hz})/2 = 256$ Hz. Now solve $v_s = f\lambda$ for λ, where v_s is the speed of sound. This gives $\lambda = \dfrac{v_s}{f} = \dfrac{344\text{ m/s}}{256\text{ Hz}} = 1.34\text{ m}$.

EVALUATE: The wavelength of the sound wave is very different from the wavelength of the waves on the string, even though both have the same frequency. This difference is due to the difference in the speeds of the two waves.

15.1. IDENTIFY: $v = f\lambda$. $T = 1/f$ is the time for one complete vibration.

SET UP: The frequency of the note one octave higher is 1568 Hz.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{784\text{ Hz}} = 0.439\text{ m}$. $T = \dfrac{1}{f} = 1.28\text{ ms}$.

(b) $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{1568\text{ Hz}} = 0.219\text{ m}$.

EVALUATE: When f is doubled, λ is halved.

15.2. IDENTIFY: $f\lambda = v$.

SET UP: $1.0\text{ mm} = 0.0010\text{ m}$.

EXECUTE: $f = \dfrac{v}{\lambda} = \dfrac{1500\text{ m/s}}{0.0010\text{ m}} = 1.5\times10^6\text{ Hz}$.

EVALUATE: The frequency is much higher than the upper range of human hearing.

15.3. IDENTIFY: $v = f\lambda = \lambda/T$.

SET UP: $1.0\text{ h} = 3600\text{ s}$. The crest to crest distance is λ.

EXECUTE: $v = \dfrac{800\times10^3\text{ m}}{3600\text{ s}} = 220\text{ m/s}$. $v = \dfrac{800\text{ km}}{1.0\text{ h}} = 800\text{ km/h}$.

EVALUATE: Since the wave speed is very high, the wave strikes with very little warning.

15.4. IDENTIFY: The fisherman observes the amplitude, wavelength, and period of the waves.

SET UP: The time from the highest displacement to lowest displacement is $T/2$. The distance from highest displacement to lowest displacement is $2A$. The distance between wave crests is λ, and the speed of the waves is $v = f\lambda = \lambda/T$.

EXECUTE: **(a)** $T = 2(2.5\text{ s}) = 5.0\text{ s}$. $\lambda = 4.8\text{ m}$. $v = \dfrac{4.8\text{ m}}{5.0\text{ s}} = 0.96\text{ m/s}$.

(b) $A = (0.53\text{ m})/2 = 0.265\text{ m}$ which rounds to 0.27 m.

(c) The amplitude becomes 0.15 m but the wavelength, period and wave speed are unchanged.

EVALUATE: The wavelength, period and wave speed are independent of the amplitude of the wave.

15.5. IDENTIFY: We want to relate the wavelength and frequency for various waves.

SET UP: For waves $v = f\lambda$.

EXECUTE: **(a)** $v = 344\text{ m/s}$. For $f = 20{,}000\text{ Hz}$, $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{20{,}000\text{ Hz}} = 1.7\text{ cm}$. For $f = 20\text{ Hz}$,

$\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{20\text{ Hz}} = 17\text{ m}$. The range of wavelengths is 1.7 cm to 17 m.

(b) $v = c = 3.00\times10^8\text{ m/s}$. For $\lambda = 700\text{ nm}$, $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{700\times10^{-9}\text{ m}} = 4.3\times10^{14}\text{ Hz}$. For $\lambda = 400\text{ nm}$,

$f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{400\times10^{-9}\text{ m}} = 7.5\times10^{14}\text{ Hz}$. The range of frequencies for visible light is $4.3\times10^{14}\text{ Hz}$ to $7.5\times10^{14}\text{ Hz}$.

(c) $v = 344\text{ m/s}$. $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{23\times10^3\text{ Hz}} = 1.5\text{ cm}$.

(d) $v = 1480\text{ m/s}$. $\lambda = \dfrac{v}{f} = \dfrac{1480\text{ m/s}}{23\times10^3\text{ Hz}} = 6.4\text{ cm}$.

EVALUATE: For a given v, a larger f corresponds to smaller λ. For the same f, λ increases when v increases.

15.6. **IDENTIFY:** The string is vibrating in SHM. The maximum force occurs when the acceleration of the bead is a maximum.

SET UP: The equation for the wave is $y(x,t) = A\cos(kx - \omega t)$. The maximum force on the bead occurs when the transverse acceleration is a maximum, so use $\sum F_y = ma_y$, where $a_y = \dfrac{\partial^2 y}{\partial t^2}$. The target variable is the maximum force on the bead.

EXECUTE: Find $a_y = \dfrac{\partial^2 y}{\partial t^2}$ for $y(x,t) = A\cos(kx - \omega t)$. $\dfrac{\partial y}{\partial t} = A\omega \sin(kx - \omega t)$, so $a_y = \dfrac{\partial^2 y}{\partial t^2} = -A\omega^2 \cos(kx - \omega t)$. The maximum magnitude acceleration of the bead is $a_{\max} = A\omega^2$, so the maximum force on the bead is $F_{\max} = ma_{\max} = mA\omega^2$. Putting in the numbers gives $F_{\max} = (0.00400\ \text{kg})(8.00 \times 10^{-3}\ \text{m})[2\pi(20.0\ \text{Hz})]^2 = 0.505\ \text{N}$.

EVALUATE: The weight of the bead is (0.00400 kg)(9.80 m/s^2) = 0.0392 N, so it is small enough to be neglected with reasonable accuracy.

15.7. **IDENTIFY:** Use $v = f\lambda$ to calculate v. $T = 1/f$ and k is defined by $k = 2\pi/\lambda$. The general form of the wave function is given by $y(x, t) = A\cos 2\pi(x/\lambda + t/T)$, which is the equation for the transverse displacement.

SET UP: $v = 8.00\ \text{m/s}$, $A = 0.0700\ \text{m}$, $\lambda = 0.320\ \text{m}$

EXECUTE: **(a)** $v = f\lambda$ so $f = v/\lambda = (8.00\ \text{m/s})/(0.320\ \text{m}) = 25.0\ \text{Hz}$

$T = 1/f = 1/25.0\ \text{Hz} = 0.0400\ \text{s}$

$k = 2\pi/\lambda = 2\pi\ \text{rad}/0.320\ \text{m} = 19.6\ \text{rad/m}$

(b) For a wave traveling in the $-x$-direction,

$y(x, t) = A\cos 2\pi(x/\lambda + t/T)$

At $x = 0$, $y(0, t) = A\cos 2\pi(t/T)$, so $y = A$ at $t = 0$. This equation describes the wave specified in the problem.

Substitute in numerical values:

$y(x, t) = (0.0700\ \text{m})\cos\left[2\pi\left(x/(0.320\ \text{m}) + t/(0.0400\ \text{s})\right)\right]$.

Or, $y(x, t) = (0.0700\ \text{m})\cos\left[(19.6\ \text{m}^{-1})x + (157\ \text{rad/s})t\right]$.

(c) From part (b), $y = (0.0700\ \text{m})\cos[2\pi(x/0.320\ \text{m} + t/0.0400\ \text{s})]$.

Plug in $x = 0.360\ \text{m}$ and $t = 0.150\ \text{s}$:

$y = (0.0700\ \text{m})\cos[2\pi(0.360\ \text{m}/0.320\ \text{m} + 0.150\ \text{s}/0.0400\ \text{s})]$

$y = (0.0700\ \text{m})\cos[2\pi(4.875\ \text{rad})] = +0.0495\ \text{m} = +4.95\ \text{cm}$

(d) In part (c) $t = 0.150\ \text{s}$.

$y = A$ means $\cos[2\pi(x/\lambda + t/T)] = 1$

$\cos\theta = 1$ for $\theta = 0, 2\pi, 4\pi, \ldots = n(2\pi)$ or $n = 0, 1, 2, \ldots$

So $y = A$ when $2\pi(x/\lambda + t/T) = n(2\pi)$ or $x/\lambda + t/T = n$

$t = T(n - x/\lambda) = (0.0400\ \text{s})(n - 0.360\ \text{m}/0.320\ \text{m}) = (0.0400\ \text{s})(n - 1.125)$

For $n = 4$, $t = 0.1150\ \text{s}$ (before the instant in part (c))

For $n = 5$, $t = 0.1550\ \text{s}$ (the first occurrence of $y = A$ after the instant in part (c)). Thus the elapsed time is $0.1550\ \text{s} - 0.1500\ \text{s} = 0.0050\ \text{s}$.

EVALUATE: Part (d) says $y = A$ at 0.115 s and next at 0.155 s; the difference between these two times is 0.040 s, which is the period. At $t = 0.150\ \text{s}$ the particle at $x = 0.360\ \text{m}$ is at $y = 4.95\ \text{cm}$ and traveling upward. It takes $T/4 = 0.0100$ s for it to travel from $y = 0$ to $y = A$, so our answer of 0.0050 s is reasonable.

15.8. **IDENTIFY:** Compare $y(x, t)$ given in the problem to the general form $y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$. $f = 1/T$ and $v = f\lambda$

SET UP: The comparison gives $A = 6.50$ mm, $\lambda = 28.0$ cm and $T = 0.0360$ s.

EXECUTE: **(a)** 6.50 mm

(b) 28.0 cm

(c) $f = \frac{1}{0.0360\text{ s}} = 27.8\text{ Hz}$

(d) $v = (0.280\text{ m})(27.8\text{ Hz}) = 7.78\text{ m/s}$

(e) Since there is a minus sign in front of the t/T term, the wave is traveling in the $+x$-direction.

EVALUATE: The speed of propagation does not depend on the amplitude of the wave.

15.9. **IDENTIFY:** Evaluate the partial derivatives and see if the wave equation $\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$ is satisfied.

SET UP: $\frac{\partial}{\partial x}\cos(kx + \omega t) = -k\sin(kx + \omega t)$. $\frac{\partial}{\partial t}\cos(kx + \omega t) = -\omega\sin(kx + \omega t)$. $\frac{\partial}{\partial x}\sin(kx + \omega t) = k\cos(kx + \omega t)$. $\frac{\partial}{\partial t}\sin(kx + \omega t) = \omega\cos(kx + \omega t)$.

EXECUTE: **(a)** $\frac{\partial^2 y}{\partial x^2} = -Ak^2\cos(kx + \omega t)$. $\frac{\partial^2 y}{\partial t^2} = -A\omega^2\cos(kx + \omega t)$. The wave equation is satisfied, if $v = \omega/k$.

(b) $\frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(kx + \omega t)$. $\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(kx + \omega t)$. The wave equation is satisfied, if $v = \omega/k$.

(c) $\frac{\partial y}{\partial x} = -kA\sin(kx)$. $\frac{\partial^2 y}{\partial x^2} = -k^2A\cos(kx)$. $\frac{\partial y}{\partial t} = -\omega A\sin(\omega t)$. $\frac{\partial^2 y}{\partial t^2} = -\omega^2 A\cos(\omega t)$. The wave equation is not satisfied.

(d) $v_y = \frac{\partial y}{\partial t} = \omega A\cos(kx + \omega t)$. $a_y = \frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(kx + \omega t)$

EVALUATE: The functions $\cos(kx + \omega t)$ and $\sin(kx + \omega t)$ differ only in phase.

15.10. **IDENTIFY:** The general form of the wave function for a wave traveling in the $-x$-direction is given by $y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)$. The time for one complete cycle to pass a point is the period T and the number that pass per second is the frequency f. The speed of a crest is the wave speed v and the maximum speed of a particle in the medium is $v_{max} = \omega A$.

SET UP: Comparison to $y(x,t) = A\cos(kx + \omega t)$ gives $A = 2.75$ cm, $k = 0.410$ rad/cm and $\omega = 6.20$ rad/s.

EXECUTE: **(a)** $T = \frac{2\pi\text{ rad}}{\omega} = \frac{2\pi\text{ rad}}{6.20\text{ rad/s}} = 1.0134\text{ s}$ which rounds to 1.01 s. In one cycle a wave crest travels a distance $\lambda = \frac{2\pi\text{ rad}}{k} = \frac{2\pi\text{ rad}}{0.410\text{ rad/cm}} = 15.325\text{ cm} = 0.153\text{ m}$.

(b) $k = 0.410$ rad/cm. $f = 1/T = 0.9868\text{ Hz} = 0.987$ waves/second.

(c) $v = f\lambda = (0.9868\text{ Hz})(0.15325\text{ m}) = 0.151\text{ m/s}$.

$v_{max} = \omega A = (6.20\text{ rad/s})(2.75\text{ cm}) = 17.1\text{ cm/s} = 0.171\text{ m/s}$.

EVALUATE: The transverse velocity of the particles in the medium (water) is not the same as the velocity of the wave.

15.11. **IDENTIFY and SET UP:** Read A and T from the graph. Apply $y(x,t) = A\cos 2\pi\left(\dfrac{x}{\lambda} - \dfrac{t}{T}\right)$ to determine λ and then use $v = f\lambda$ to calculate v.

EXECUTE: **(a)** The maximum y is 4 mm (read from graph).
(b) For either x the time for one full cycle is 0.040 s; this is the period.
(c) Since $y = 0$ for $x = 0$ and $t = 0$ and since the wave is traveling in the $+x$-direction then $y(x, t) = A\sin[2\pi(t/T - x/\lambda)]$. (The phase is different from the wave described by $y(x,t) = A\cos 2\pi\left(\dfrac{x}{\lambda} - \dfrac{t}{T}\right)$; for that wave $y = A$ for $x = 0,\ t = 0$.) From the graph, if the wave is traveling in the $+x$-direction and if $x = 0$ and $x = 0.090$ m are within one wavelength the peak at $t = 0.01$ s for $x = 0$ moves so that it occurs at $t = 0.035$ s (read from graph so is approximate) for $x = 0.090$ m. The peak for $x = 0$ is the first peak past $t = 0$ so corresponds to the first maximum in $\sin[2\pi(t/T - x/\lambda)]$ and hence occurs at $2\pi(t/T - x/\lambda) = \pi/2$. If this same peak moves to $t_1 = 0.035$ s at $x_1 = 0.090$ m, then

$2\pi(t/T - x/\lambda) = \pi/2.$

Solve for λ: $t_1/T - x_1/\lambda = 1/4$

$x_1/\lambda = t_1/T - 1/4 = 0.035\text{ s}/0.040\text{ s} - 0.25 = 0.625$

$\lambda = x_1/0.625 = 0.090\text{ m}/0.625 = 0.14\text{ m}.$

Then $v = f\lambda = \lambda/T = 0.14\text{ m}/0.040\text{ s} = 3.5\text{ m/s}.$

(d) If the wave is traveling in the $-x$-direction, then $y(x, t) = A\sin(2\pi(t/T + x/\lambda))$ and the peak at $t = 0.050$ s for $x = 0$ corresponds to the peak at $t_1 = 0.035$ s for $x_1 = 0.090$ m. This peak at $x = 0$ is the second peak past the origin so corresponds to $2\pi(t/T + x/\lambda) = 5\pi/2$. If this same peak moves to $t_1 = 0.035$ s for $x_1 = 0.090$ m, then $2\pi(t_1/T + x_1/\lambda) = 5\pi/2.$

$t_1/T + x_1/\lambda = 5/4$

$x_1/\lambda = 5/4 - t_1/T = 5/4 - 0.035\text{ s}/0.040\text{ s} = 0.375$

$\lambda = x_1/0.375 = 0.090\text{ m}/0.375 = 0.24\text{ m}.$

Then $v = f\lambda = \lambda/T = 0.24\text{ m}/0.040\text{ s} = 6.0\text{ m/s}.$

EVALUATE: **(e)** No. Wouldn't know which point in the wave at $x = 0$ moved to which point at $x = 0.090$ m.

15.12. **IDENTIFY:** $v_y = \dfrac{\partial y}{\partial t}$. $v = f\lambda = \lambda/T.$

SET UP: $\dfrac{\partial}{\partial t}A\cos\left(\dfrac{2\pi}{\lambda}(x - vt)\right) = +A\left(\dfrac{2\pi v}{\lambda}\right)\sin\left(\dfrac{2\pi}{\lambda}(x - vt)\right)$

EXECUTE: **(a)** $A\cos 2\pi\left(\dfrac{x}{\lambda} - \dfrac{t}{T}\right) = +A\cos\dfrac{2\pi}{\lambda}\left(x - \dfrac{\lambda}{T}t\right) = +A\cos\dfrac{2\pi}{\lambda}(x - vt)$ where $\dfrac{\lambda}{T} = \lambda f = v$ has been used.

(b) $v_y = \dfrac{\partial y}{\partial t} = \dfrac{2\pi v}{\lambda}A\sin\dfrac{2\pi}{\lambda}(x - vt).$

(c) The speed is the greatest when the sine is 1, and that speed is $2\pi vA/\lambda$. This will be equal to v if $A = \lambda/2\pi$, less than v if $A < \lambda/2\pi$ and greater than v if $A > \lambda/2\pi$.

EVALUATE: The propagation speed applies to all points on the string. The transverse speed of a particle of the string depends on both x and t.

15.13. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: For λ and x in cm, v in cm/s and t in s, the argument of the cosine is in radians.

EXECUTE: **(a)** $t = 0$:

x(cm)	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
y(cm)	0.300	0.212	0	–0.212	–0.300	–0.212	0	0.212	0.300

The graph is shown in Figure 15.13a.

(b) (i) $t = 0.400$ s:

x(cm)	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
y(cm)	–0.221	–0.0131	0.203	0.300	0.221	0.0131	–0.203	–0.300	–0.221

The graph is shown in Figure 15.13b.

(ii) $t = 0.800$ s:

x(cm)	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
y(cm)	0.0262	–0.193	–0.300	–0.230	–0.0262	0.193	0.300	0.230	0.0262

The graph is shown in Figure 15.13c.

(iii) The graphs show that the wave is traveling in the $+x$-direction.

EVALUATE: We know that $y(x,t) = A\cos 2\pi f\left(\dfrac{x}{v} - t\right)$ is for a wave traveling in the $+x$-direction, and $y(x, t)$ is derived from this. This is consistent with the direction of propagation we deduced from our graph.

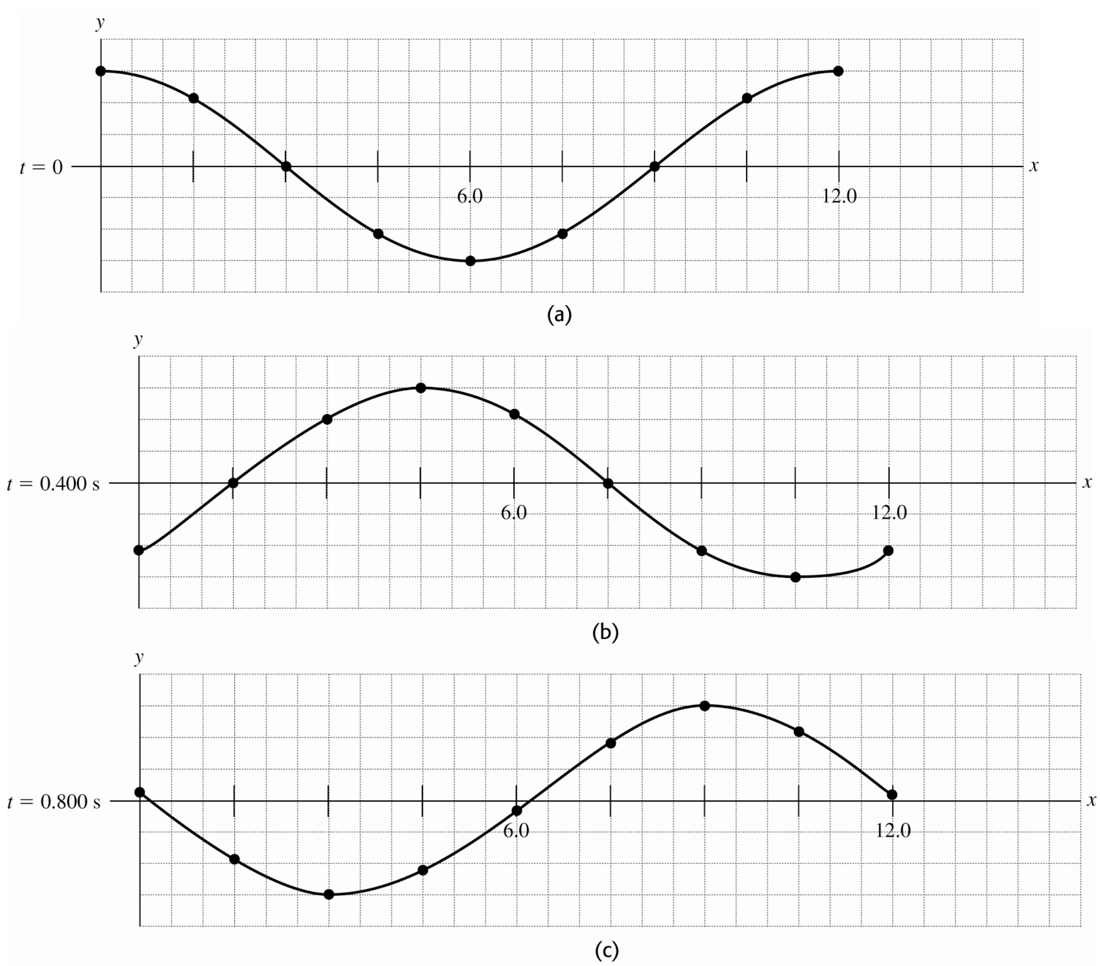


Figure 15.13

15.14. **IDENTIFY:** We are dealing the tension in a taut vibrating string.

SET UP: $v=\sqrt{\dfrac{F}{\mu}}$ and $v=f\lambda$. We want to know what change in the tension is required to double the frequency of the fundamental mode.

EXECUTE: Combining $v=\sqrt{\dfrac{F}{\mu}}$ and $v=f\lambda$ gives $f\lambda = \sqrt{\dfrac{F}{\mu}}$. Solving for F gives $F=\mu\lambda^2 f^2$. From this result we see that to double the frequency, F we would have to increase by a factor of 4.

EVALUATE: This is *not* a good idea because such a large increase in tension could damage the instrument or cause the spring to break.

15.15. **IDENTIFY** and **SET UP:** Use $v=\sqrt{F/\mu}$ to calculate the wave speed. Then use $v=f\lambda$ to calculate the wavelength.

EXECUTE: **(a)** The tension F in the rope is the weight of the hanging mass:

$F=mg=(1.50\ \text{kg})(9.80\ \text{m/s}^2)=14.7\ \text{N}$

$v=\sqrt{F/\mu}=\sqrt{14.7\ \text{N}/(0.0480\ \text{kg/m})}=17.5\ \text{m/s}.$

(b) $v=f\lambda$ so $\lambda=v/f=(17.5\ \text{m/s})/120\ \text{Hz}=0.146\ \text{m}.$

(c) EVALUATE: $v=\sqrt{F/\mu}$, where $F=mg$. Doubling m increases v by a factor of $\sqrt{2}$. $\lambda=v/f$. f remains 120 Hz and v increases by a factor of $\sqrt{2}$, so λ increases by a factor of $\sqrt{2}$.

15.16. **IDENTIFY:** The frequency and wavelength determine the wave speed and the wave speed depends on the tension.

SET UP: $v=\sqrt{\dfrac{F}{\mu}}$. $\mu=m/L$. $v=f\lambda$.

EXECUTE: $F=\mu v^2=\mu(f\lambda)^2=\dfrac{0.120\ \text{kg}}{2.50\ \text{m}}([40.0\ \text{Hz}][0.750\ \text{m}])^2=43.2\ \text{N}$

EVALUATE: If the frequency is held fixed, increasing the tension will increase the wavelength.

15.17. **IDENTIFY:** The speed of the wave depends on the tension in the wire and its mass density. The target variable is the mass of the wire of known length.

SET UP: $v=\sqrt{\dfrac{F}{\mu}}$ and $\mu=m/L$.

EXECUTE: First find the speed of the wave: $v=\dfrac{3.80\ \text{m}}{0.0492\ \text{s}}=77.24\ \text{m/s}$. $v=\sqrt{\dfrac{F}{\mu}}$. $\mu=\dfrac{F}{v^2}=$

$\dfrac{(54.0\ \text{kg})(9.8\ \text{m/s}^2)}{(77.24\ \text{m/s})^2}=0.08870\ \text{kg/m}$. The mass of the wire is

$m=\mu L=(0.08870\ \text{kg/m})(3.80\ \text{m})=0.337\ \text{kg}.$

EVALUATE: This mass is 337 g, which is a bit large for a wire 3.80 m long. It must be fairly thick.

15.18. **IDENTIFY:** For transverse waves on a string, $v=\sqrt{F/\mu}$. The general form of the equation for waves traveling in the $+x$-direction is $y(x,t)=A\cos(kx-\omega t)$. For waves traveling in the $-x$-direction it is $y(x,t)=A\cos(kx+\omega t)$. $v=\omega/k$.

SET UP: Comparison to the general equation gives $A=8.50\ \text{mm}$, $k=172\ \text{rad/m}$ and $\omega=4830\ \text{rad/s}$. The string has mass 0.00128 kg and $\mu=m/L=0.000850\ \text{kg/m}$.

EXECUTE: **(a)** $v=\dfrac{\omega}{k}=\dfrac{4830\ \text{rad/s}}{172\ \text{rad/m}}=28.08\ \text{m/s}$. $t=\dfrac{d}{v}=\dfrac{1.50\ \text{m}}{28.08\ \text{m/s}}=0.0534\ \text{s} = 53.4\ \text{ms}.$

(b) $W=F=\mu v^2=(0.000850\ \text{kg/m})(28.08\ \text{m/s})^2=0.670\ \text{N}.$

(c) $\lambda = \dfrac{2\pi \text{ rad}}{k} = \dfrac{2\pi \text{ rad}}{172 \text{ rad/m}} = 0.0365 \text{ m}$. The number of wavelengths along the length of the string is $\dfrac{1.50 \text{ m}}{0.0365 \text{ m}} = 41.1.$

(d) For a wave traveling in the opposite direction, $y(x,t) = (8.50 \text{ mm})\cos([172 \text{ rad/m}]x + [4830 \text{ rad/s}]t).$

EVALUATE: We have assumed that the tension in the string is constant and equal to W. This is reasonable since $W \gg 0.0125$ N, so the weight of the string has a negligible effect on the tension.

15.19. **IDENTIFY:** For transverse waves on a string, $v = \sqrt{F/\mu}$. $v = f\lambda$.

SET UP: The wire has $\mu = m/L = (0.0165 \text{ kg})/(0.750 \text{ m}) = 0.0220 \text{ kg/m}$.

EXECUTE: **(a)** $v = f\lambda = (625 \text{ Hz})(3.33\times 10^{-2} \text{ m}) = 20.813 \text{ m/s}$. The tension is $F = \mu v^2 = (0.0220 \text{ kg/m})(20.813 \text{ m/s})^2 = 9.53 \text{ N}$.

(b) $v = 20.8$ m/s

EVALUATE: If λ is kept fixed, the wave speed and the frequency increase when the tension is increased.

15.20. **IDENTIFY:** The rope is heavy, so the tension at any point in it must support not only the weight attached but the weight of the rope below that point. Assume that the rope is uniform.

SET UP: $v = \sqrt{F/\mu}$ and $\mu = m/L$ = [(29.4 N)/(9.80 m/s^2)]/(6.00 m) = 0.500 kg/m.

EXECUTE: **(a)** At the bottom, the rope supports only the 0.500-kg object, so $T = mg$ = (0.500 kg)(9.80 m/s^2) = 4.90 N. Now use $v = \sqrt{F/\mu}$ find v.

v = [(4.90 N)/(0.500 kg/m)]$^{1/2}$ = 3.13 m/s.

(b) At the middle, the tension supports the 0.500-kg object plus half the weight of the rope, so T = (29.4 N)/2 + 4.90 N = 19.6 N. Therefore v = [(19.6 N)/(0.500 kg/m)]$^{1/2}$ = 6.26 m/s.

(c) At the top, the tension supports the entire rope plus the object, so T = 29.4 N + 4.90 N = 34.3 N. Therefore v = [(34.3 N)/(0.500 kg/m)]$^{1/2}$ = 8.28 m/s.

(d) T_{middle}= 19.6 N. $T_{\text{av}} = (T_{\text{top}} + T_{\text{bot}})/2$ = (34.3 N + 4.90 N)/2 = 19.6 N, which is equal to T_{middle}. v_{middle} = 6.26 m/s. v_{av} = (8.28 m/s + 3.13 m/s)/2 = 5.71 m/s, which is not equal to v_{middle}.

EVALUATE: The average speed is not equal to the speed at the middle because the speed depends on the square root of the tension. So even though the tension at the middle is the average of the top and bottom tensions, that is not true of the wave speed.

15.21. **IDENTIFY:** We are dealing with the power carried by a wave on a vibrating string.

SET UP: Estimates: The amplitude is the length of an arm ≈ 65 cm = 0.65 m. The maximum tension is $T_{\max}$ ≈ 25 lb ≈ 110 N. The time to complete each pulse: do about 2 per second, so the time per pulse is about 0.50 s. We want to know about the power we supply. Use $\omega = 2\pi / T$ and $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$.

EXECUTE: **(a)** $\omega = 2\pi / T$ = 2π/(0.50 s) = 4π rad/s. The average power is given by $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$

$= \frac{1}{2}\sqrt{(0.500 \text{ kg/m})(110 \text{ N})}(4\pi \text{ rad/s})^2(0.65 \text{ m})^2 = 250 \text{ W}.$

(b) $P_{\text{av}} \propto (A\omega)^2$, so if we halve A and double ω, the effects cancel out so there is no change in the average power.

EVALUATE: With this power you could keep around four 60-W light bulbs burning, but it would be difficult to keep it up for very long!

15.22. **IDENTIFY:** Apply $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$. d

SET UP: $\omega = 2\pi f$. $\mu = m/L$.

EXECUTE: **(a)** Using $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$, we get $P_{av} = \frac{1}{2}\sqrt{\left(\frac{3.00\times10^{-3}\text{ kg}}{0.80\text{ m}}\right)(25.0\text{ N})}\left[2\pi(120.0\text{ Hz})\right]^2$ $(1.6\times10^{-3}\text{ m})^2 = 0.223\text{ W}$ or 0.22 W to two significant figures.

(b) P_{av} is proportional to A^2, so halving the amplitude quarters the average power, to 0.056 W.

EVALUATE: The average power is also proportional to the square of the frequency.

15.23. **IDENTIFY:** The average power carried by the wave depends on the mass density of the wire and the tension in it, as well as on the square of both the frequency and amplitude of the wave (the target variable).

SET UP: $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$, $v = \sqrt{\frac{F}{\mu}}$.

EXECUTE: Solving $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$ for A gives $A = \left(\frac{2P_{av}}{\omega^2\sqrt{\mu F}}\right)^{1/2}$. $P_{av} = 0.365$ W. $\omega = 2\pi f =$ $2\pi(69.0\text{ Hz}) = 433.5\text{ rad/s}$. The tension is $F = 94.0$ N and $v = \sqrt{\frac{F}{\mu}}$ so $\mu = \frac{F}{v^2} = \frac{94.0\text{ N}}{(406\text{ m/s})^2} =$ 5.703×10^{-4} kg/m. $A = \left(\frac{2(0.365\text{ W})}{(433.5\text{ rad/s})^2\sqrt{(5.703\times10^{-4}\text{ kg/m})(94.0\text{ N})}}\right)^{1/2} = 4.10\times10^{-3}\text{ m} = 4.10\text{ mm}$.

EVALUATE: Vibrations of strings and wires normally have small amplitudes, which this wave does.

15.24. **IDENTIFY:** Apply $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$.

SET UP: $I_1 = 0.11\text{ W/m}^2$. $r_1 = 7.5$ m. Set $I_2 = 1.0\text{ W/m}^2$ and solve for r_2.

EXECUTE: $r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (7.5\text{ m})\sqrt{\frac{0.11\text{ W/m}^2}{1.0\text{ W/m}^2}} = 2.5$ m, so it is possible to move $r_1 - r_2 = 7.5\text{ m} - 2.5\text{ m} = 5.0$ m closer to the source.

EVALUATE: I increases as the distance r of the observer from the source decreases.

15.25. **IDENTIFY:** For a point source, $I = \frac{P}{4\pi r^2}$ and $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$.

SET UP: $1\,\mu\text{W} = 10^{-6}$ W

EXECUTE: **(a)** $r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (30.0\text{ m})\sqrt{\frac{10.0\text{ W/m}^2}{1\times10^{-6}\text{ W/m}^2}} = 95$ km

(b) $\frac{I_2}{I_3} = \frac{r_3^2}{r_2^2}$, with $I_2 = 1.0\,\mu\text{W/m}^2$ and $r_3 = 2r_2$. $I_3 = I_2\left(\frac{r_2}{r_3}\right)^2 = I_2/4 = 0.25\,\mu\text{W/m}^2$.

(c) $P = I(4\pi r^2) = (10.0\text{ W/m}^2)(4\pi)(30.0\text{ m})^2 = 1.1\times10^5$ W

EVALUATE: These are approximate calculations, that assume the sound is emitted uniformly in all directions and that ignore the effects of reflection, for example reflections from the ground.

15.26. **IDENTIFY:** The tension and mass per unit length of the rope determine the wave speed. Compare $y(x,t)$ given in the problem to the general form given in $y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)$. $v = \omega/k$. The average power is given by $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$.

SET UP: Comparison with $y(x,t) = A\cos(kx - \omega t)$ gives $A = 2.30$ mm, $k = 6.98$ rad/m and $\omega = 742$ rad/s.

EXECUTE: **(a)** $A = 2.30$ mm

(b) $f = \frac{\omega}{2\pi} = \frac{742 \text{ rad/s}}{2\pi} = 118 \text{ Hz}$

(c) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{6.98 \text{ rad/m}} = 0.90 \text{ m}$

(d) $v = \frac{\omega}{k} = \frac{742 \text{ rad/s}}{6.98 \text{ rad/m}} = 106 \text{ m/s}$

(e) The wave is traveling in the $-x$-direction because the phase of $y(x, t)$ has the form $kx + \omega t$.

(f) The linear mass density is $\mu = (3.38\times10^{-3} \text{ kg})/(1.35 \text{ m}) = 2.504\times10^{-3} \text{ kg/m}$, so the tension is $F = \mu v^2 = (2.504\times10^{-3} \text{ kg/m})(106.3 \text{ m/s})^2 = 28.3 \text{ N}$.

(g) $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2 = \frac{1}{2}\sqrt{(2.50\times10^{-3} \text{ kg/m})(28.3 \text{ N})}(742 \text{ rad/s})^2(2.30\times10^{-3} \text{ m})^2 = 0.39 \text{ W}$

EVALUATE: In part (d) we could also calculate the wave speed as $v = f\lambda$ and we would obtain the same result.

15.27. **IDENTIFY** and **SET UP:** Apply $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$ and $I = \frac{P}{4\pi r^2}$ to relate I and r.

Power is related to intensity at a distance r by $P = I(4\pi r^2)$. Energy is power times time.

EXECUTE: **(a)** $I_1 r_1^2 = I_2 r_2^2$

$I_2 = I_1(r_1/r_2)^2 = (0.026 \text{ W/m}^2)(4.3 \text{ m}/3.1 \text{ m})^2 = 0.050 \text{ W/m}^2$

(b) $P = 4\pi r^2 I = 4\pi(4.3 \text{ m})^2(0.026 \text{ W/m}^2) = 6.04 \text{ W}$

$\text{Energy} = Pt = (6.04 \text{ W})(3600 \text{ s}) = 2.2\times10^4 \text{ J}$

EVALUATE: We could have used $r = 3.1$ m and $I = 0.050 \text{ W/m}^2$ in $P = 4\pi r^2 I$ and would have obtained the same P. Intensity becomes less as r increases because the radiated power spreads over a sphere of larger area.

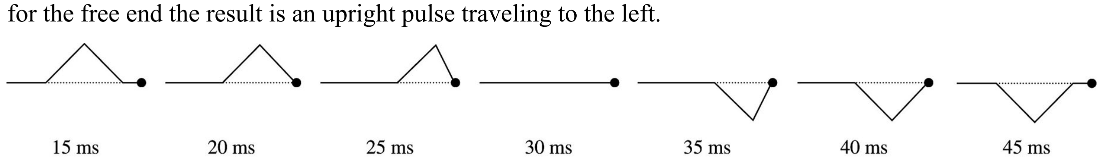
15.28. **IDENTIFY:** The distance the wave shape travels in time t is vt. The wave pulse reflects at the end of the string, at point O.

SET UP: The reflected pulse is inverted when O is a fixed end and is not inverted when O is a free end.

EXECUTE: **(a)** The wave form for the given times, respectively, is shown in Figure 15.28(a).

(b) The wave form for the given times, respectively, is shown in Figure 15.28(b).

EVALUATE: For the fixed end the result of the reflection is an inverted pulse traveling to the left and for the free end the result is an upright pulse traveling to the left.



(a)

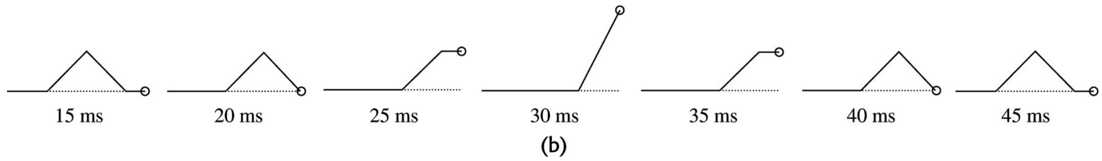


Figure 15.28

15.29. **IDENTIFY:** The distance the wave shape travels in time t is vt. The wave pulse reflects at the end of the string, at point O.

SET UP: The reflected pulse is inverted when O is a fixed end and is not inverted when O is a free end.

EXECUTE: **(a)** The wave form for the given times, respectively, is shown in Figure 15.29(a).

(b) The wave form for the given times, respectively, is shown in Figure 15.29(b).

EVALUATE: For the fixed end the result of the reflection is an inverted pulse traveling to the right and for the free end the result is an upright pulse traveling to the right.

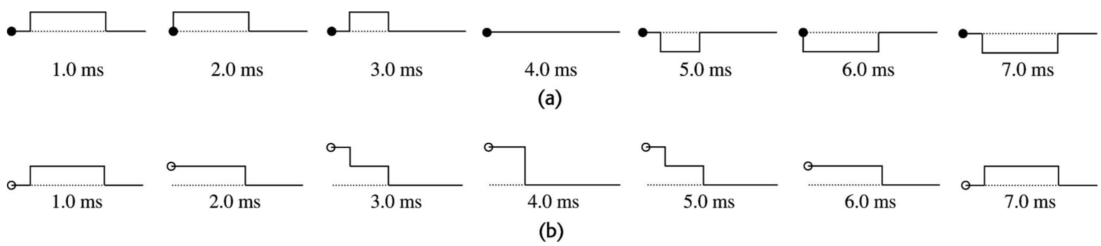


Figure 15.29

15.30. **IDENTIFY:** Apply the principle of superposition.

SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.

EXECUTE: The shape of the string at each specified time is shown in Figure 15.30.

EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

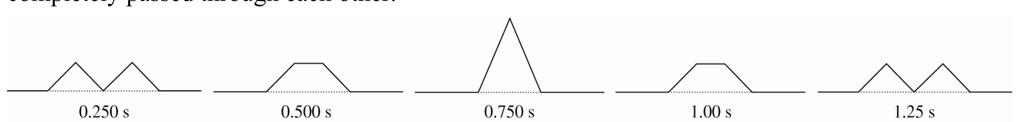


Figure 15.30

15.31. **IDENTIFY:** Apply the principle of superposition.

SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.

EXECUTE: The shape of the string at each specified time is shown in Figure 15.31.

EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

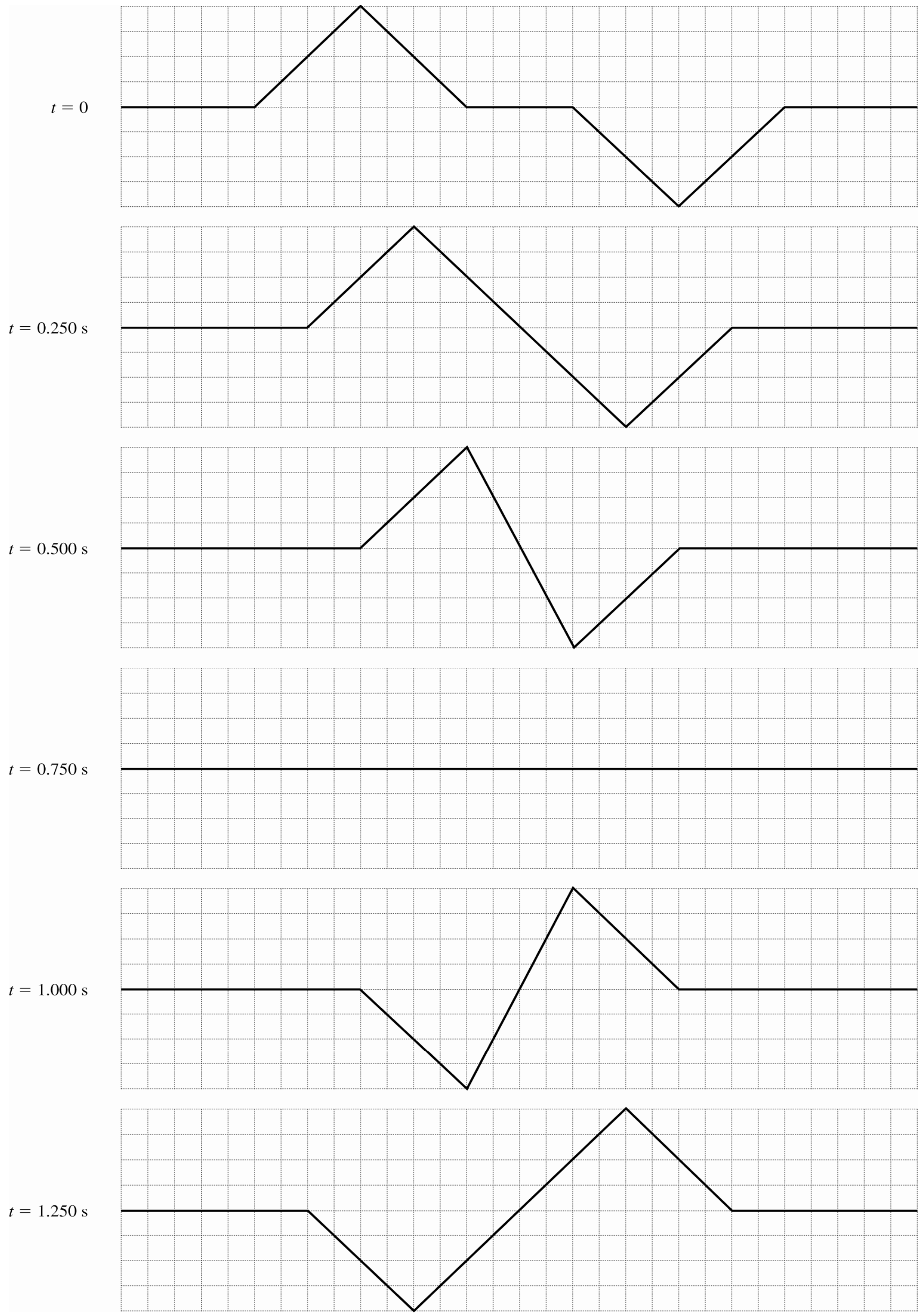


Figure 15.31

15.32. IDENTIFY: Apply the principle of superposition.

SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.

EXECUTE: The shape of the string at each specified time is shown in Figure 15.32.

EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

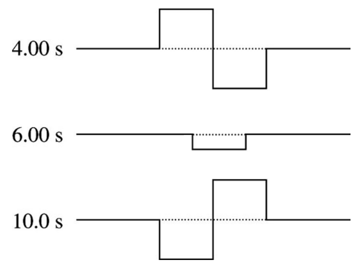


Figure 15.32

15.33. **IDENTIFY:** We are investigating standing waves on a violin string that is fixed at both ends.

SET UP: Estimate: The portion of the violin string that is free to vibrate is about 30 cm = 0.30 m long. For the bass viol, the free portion is 1.0 m long. We use $v = \sqrt{\dfrac{F}{\mu}}$ and $v = f\lambda$.

EXECUTE: **(a)** We want the speed of the wave, so $v = f\lambda$ = (659 Hz)(0.30 m) = 200 m/s.

(b) We want to compare the wavelength of the wave on the string to the wavelength of the sound wave it produces. Both waves have the same frequency but different speeds. In air we have $\lambda_{air} = v_{air}/f$ = (344 m/s)/(659 Hz) = 0.522 m. On the string λ_{string} = 0.30 m, so $\lambda_{air} > \lambda_{string}$.

(c) $v = f\lambda$ = (98 Hz)(1.0 m) = 98 m/s (on the string).

$\lambda_{air} = v_{air}/f$ = (344 m/s)/(98 Hz) = 3.5 m (in air).

So $\lambda_{air} > \lambda_{string}$.

EVALUATE: Each cycle of vibration of the string produces one cycle of sound, so the frequency is the same for the sound and for the string. But the sound wave travels at a different speed than the wave on the string, so the wavelength of the sound is not the same as the wavelength of the wave on the string.

15.34. **IDENTIFY:** Apply $y(x,t) = (A_{SW} \sin kx)\sin \omega t$ and $v = f\lambda$. At an antinode, $y(t) = A_{SW} \sin \omega t$. k and ω for the standing wave have the same values as for the two traveling waves.

SET UP: $A_{SW} = 0.850$ cm. The antinode to antinode distance is $\lambda/2$, so $\lambda = 30.0$ cm. $v_y = \partial y/\partial t$.

EXECUTE: **(a)** The node to node distance is $\lambda/2 = 15.0$ cm.

(b) λ is the same as for the standing wave, so $\lambda = 30.0$ cm. $A = \frac{1}{2}A_{SW} = 0.425$ cm.

$$v = f\lambda = \frac{\lambda}{T} = \frac{0.300\text{ m}}{0.0750\text{ s}} = 4.00\text{ m/s}.$$

(c) $v_y = \dfrac{\partial y}{\partial t} = A_{SW}\,\omega \sin kx \cos \omega t$. At an antinode $\sin kx = 1$, so $v_y = A_{SW}\,\omega \cos \omega t$. $v_{max} = A_{SW}\omega$.

$$\omega = \frac{2\pi\text{ rad}}{T} = \frac{2\pi\text{ rad}}{0.0750\text{ s}} = 83.8\text{ rad/s}.\quad v_{max} = (0.850\times10^{-2}\text{ m})(83.8\text{ rad/s}) = 0.712\text{ m/s}.\quad v_{min} = 0.$$

(d) The distance from a node to an adjacent antinode is $\lambda/4 = 7.50$ cm.

EVALUATE: The maximum transverse speed for a point at an antinode of the standing wave is twice the maximum transverse speed for each traveling wave, since $A_{SW} = 2A$.

15.35. **IDENTIFY** and **SET UP:** Nodes occur where $\sin kx = 0$ and antinodes are where $\sin kx = \pm 1$.

EXECUTE: Use $y = (A_{SW} \sin kx)\sin \omega t$:

(a) At a node $y = 0$ for all t. This requires that $\sin kx = 0$ and this occurs for $kx = n\pi$, $n = 0, 1, 2, \ldots$

$$x = n\pi/k = \frac{n\pi}{0.750\pi\text{ rad/m}} = (1.33\text{ m})n,\ n = 0,\ 1,\ 2, \ldots$$

(b) At an antinode $\sin kx = \pm 1$ so y will have maximum amplitude. This occurs when $kx = (n+\frac{1}{2})\pi$, $n = 0, 1, 2, \ldots$

$$x = (n+\tfrac{1}{2})\pi/k = (n+\tfrac{1}{2})\frac{\pi}{0.750\pi \text{ rad/m}} = (1.33 \text{ m})(n+\tfrac{1}{2}), n = 0, 1, 2, \ldots$$

EVALUATE: $\lambda = 2\pi/k = 2.66$ m. Adjacent nodes are separated by $\lambda/2$, adjacent antinodes are separated by $\lambda/2$, and the node to antinode distance is $\lambda/4$.

15.36. **IDENTIFY:** For a string fixed at both ends, $\lambda_n = \dfrac{2L}{n}$ and $f_n = n\left(\dfrac{v}{2L}\right)$.

SET UP: For the fundamental, $n = 1$. For the second overtone, $n = 3$. For the fourth harmonic, $n = 4$.

EXECUTE: **(a)** $\lambda_1 = 2L = 3.00$ m. $f_1 = \dfrac{v}{2L} = \dfrac{(62.0 \text{ m/s})}{2(1.50 \text{ m})} = 20.7$ Hz.

(b) $\lambda_3 = \lambda_1/3 = 1.00$ m. $f_3 = 3f_1 = 62.0$ Hz.

(c) $\lambda_4 = \lambda_1/4 = 0.75$ m. $f_4 = 4f_1 = 82.7$ Hz.

EVALUATE: As n increases, λ decreases and f increases.

15.37. **IDENTIFY:** Use $v = f\lambda$ for v and $v = \sqrt{F/\mu}$ for the tension F. $v_y = \partial y/\partial t$ and $a_y = \partial v_y/\partial t$.

(a) SET UP: The fundamental standing wave is sketched in Figure 15.37.

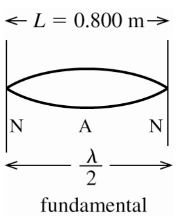


Figure 15.37

$f = 60.0$ Hz

From the sketch,
$\lambda/2 = L$ so
$\lambda = 2L = 1.60$ m

EXECUTE: $v = f\lambda = (60.0 \text{ Hz})(1.60 \text{ m}) = 96.0$ m/s

(b) The tension is related to the wave speed by $v = \sqrt{F/\mu}$:

$v = \sqrt{F/\mu}$ so $F = \mu v^2$.

$\mu = m/L = 0.0400 \text{ kg}/0.800 \text{ m} = 0.0500$ kg/m

$F = \mu v^2 = (0.0500 \text{ kg/m})(96.0 \text{ m/s})^2 = 461$ N.

(c) $\omega = 2\pi f = 377$ rad/s and $y(x, t) = A_{SW} \sin kx \sin \omega t$

$v_y = \omega A_{SW} \sin kx \cos \omega t$; $a_y = -\omega^2 A_{SW} \sin kx \sin \omega t$

$(v_y)_{max} = \omega A_{SW} = (377 \text{ rad/s})(0.300 \text{ cm}) = 1.13$ m/s.

$(a_y)_{max} = \omega^2 A_{SW} = (377 \text{ rad/s})^2(0.300 \text{ cm}) = 426 \text{ m/s}^2$.

EVALUATE: The transverse velocity is different from the wave velocity. The wave velocity and tension are similar in magnitude to the values in the examples in the text. Note that the transverse acceleration is quite large.

15.38. **IDENTIFY:** The fundamental frequency depends on the wave speed, and that in turn depends on the tension.

SET UP: $v = \sqrt{\dfrac{F}{\mu}}$ where $\mu = m/L$. $f_1 = \dfrac{v}{2L}$. The nth harmonic has frequency $f_n = nf_1$.

EXECUTE: **(a)** $v = \sqrt{\dfrac{F}{m/L}} = \sqrt{\dfrac{FL}{m}} = \sqrt{\dfrac{(800 \text{ N})(0.400 \text{ m})}{3.00\times10^{-3} \text{ kg}}} = 327$ m/s. $f_1 = \dfrac{v}{2L} = \dfrac{327 \text{ m/s}}{2(0.400 \text{ m})} = 409$ Hz.

(b) $n = \dfrac{10{,}000 \text{ Hz}}{f_1} = 24.4.$ The 24th harmonic is the highest that could be heard.

EVALUATE: In part (b) we use the fact that a standing wave on the wire produces a sound wave in air of the same frequency.

15.39. IDENTIFY: Compare $y(x,t)$ given in the problem to $y(x,t) = (A_{SW} \sin kx)\sin \omega t$. From the frequency and wavelength for the third harmonic find these values for the eighth harmonic.

(a) SET UP: The third harmonic standing wave pattern is sketched in Figure 15.39.

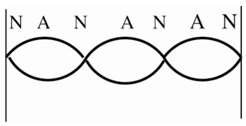


Figure 15.39

EXECUTE: **(b)** Use the general equation for a standing wave on a string:

$y(x,t) = (A_{SW} \sin kx)\sin \omega t$

$A_{SW} = 2A$, so $A = A_{SW}/2 = (5.60 \text{ cm})/2 = 2.80 \text{ cm}$

(c) The sketch in part (a) shows that $L = 3(\lambda/2)$. $k = 2\pi/\lambda$, $\lambda = 2\pi/k$

Comparison of $y(x,t)$ given in the problem to $y(x,t) = (A_{SW} \sin kx)\sin \omega t$ gives $k = 0.0340 \text{ rad/cm}$.

So, $\lambda = 2\pi/(0.0340 \text{ rad/cm}) = 184.8 \text{ cm}$

$L = 3(\lambda/2) = 277 \text{ cm}$

(d) $\lambda = 185 \text{ cm}$, from part (c)

$\omega = 50.0 \text{ rad/s}$ so $f = \omega/2\pi = 7.96 \text{ Hz}$

period $T = 1/f = 0.126 \text{ s}$ $v = f\lambda = 1470 \text{ cm/s}$

(e) $v_y = \partial y/\partial t = \omega A_{SW} \sin kx \cos \omega t$

$v_{y,\,\max} = \omega A_{SW} = (50.0 \text{ rad/s})(5.60 \text{ cm}) = 280 \text{ cm/s}$

(f) $f_3 = 7.96 \text{ Hz} = 3f_1$, so $f_1 = 2.65 \text{ Hz}$ is the fundamental

$f_8 = 8f_1 = 21.2 \text{ Hz}$; $\omega_8 = 2\pi f_8 = 133 \text{ rad/s}$

$\lambda = v/f = (1470 \text{ cm/s})/(21.2 \text{ Hz}) = 69.3 \text{ cm}$ and $k = 2\pi/\lambda = 0.0906 \text{ rad/cm}$

$y(x,t) = (5.60 \text{ cm})\sin\left[(0.0906 \text{ rad/cm})x\right]\sin[(133 \text{ rad/s})t]$.

EVALUATE: The wavelength and frequency of the standing wave equals the wavelength and frequency of the two traveling waves that combine to form the standing wave. In the eighth harmonic the frequency and wave number are larger than in the third harmonic.

15.40. IDENTIFY: Compare the $y(x,t)$ specified in the problem to the general form of $y(x,t) = (A_{SW} \sin kx)\sin \omega t$.

SET UP: The comparison gives $A_{SW} = 4.44 \text{ mm}$, $k = 32.5 \text{ rad/m}$ and $\omega = 754 \text{ rad/s}$.

EXECUTE: **(a)** $A = \frac{1}{2}A_{SW} = \frac{1}{2}(4.44 \text{ mm}) = 2.22 \text{ mm}$.

(b) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{32.5 \text{ rad/m}} = 0.193 \text{ m}$.

(c) $f = \dfrac{\omega}{2\pi} = \dfrac{754 \text{ rad/s}}{2\pi} = 120 \text{ Hz}$.

(d) $v = \dfrac{\omega}{k} = \dfrac{754 \text{ rad/s}}{32.5 \text{ rad/m}} = 23.2 \text{ m/s}$.

(e) If the wave traveling in the +x-direction is written as $y_1(x,t) = A\cos(kx - \omega t)$, then the wave traveling in the $-x$-direction is $y_2(x,t) = -A\cos(kx + \omega t)$, where $A = 2.22 \text{ mm}$ from part (a), $k = 32.5 \text{ rad/m}$ and $\omega = 754 \text{ rad/s}$.

(f) The harmonic cannot be determined because the length of the string is not specified.
EVALUATE: The two traveling waves that produce the standing wave are identical except for their direction of propagation.

15.41. **IDENTIFY:** Standing waves are formed on a string fixed at both ends.
SET UP: The distance between adjacent nodes is one-half of a wavelength. The second overtone is the third harmonic ($n = 3$). $\lambda_n = 2L/n$.
EXECUTE: **(a)** We want the length of the string. In the third harmonic $\lambda_3 = 2L/3$, so $L = 3\lambda_3/2$. The distance between adjacent nodes is $\lambda/2$ = 8.00 cm, so λ = 16.0 cm. Therefore $L = 3\lambda_3/2$ = 24.0 cm = 0.240 m.
(b) For the 4th harmonic, $n = 4$, so $\lambda_4 = 2L/4$ = (48.0 cm)/4 = 12.0 cm = 0.120 m. The node-to-node distance is $\lambda/2$ = (12.0 cm)/2 = 6.00 cm = 0.0600 m.
EVALUATE: The node-to-node distance decreases with higher harmonics because the wavelength decreases, which is consistent with your results.

15.42. **IDENTIFY:** $v = \sqrt{F/\mu}$. $v = f\lambda$. The standing waves have wavelengths $\lambda_n = \dfrac{2L}{n}$ and frequencies $f_n = nf_1$. The standing wave on the string and the sound wave it produces have the same frequency.
SET UP: For the fundamental $n = 1$ and for the second overtone $n = 3$. The string has $\mu = m/L = (8.75\times10^{-3}\text{ kg})/(0.750\text{ m}) = 1.17\times10^{-2}\text{ kg/m}$.
EXECUTE: **(a)** $\lambda = 2L/3 = 2(0.750\text{ m})/3 = 0.500\text{ m}$. The sound wave has frequency

$$f = \frac{v}{\lambda} = \frac{344\text{ m/s}}{0.765\text{ m}} = 449.7\text{ Hz}.$$ For waves on the string,

$v = f\lambda = (449.7\text{ Hz})(0.500\text{ m}) = 224.8\text{ m/s}$. The tension in the string is

$F = \mu v^2 = (1.17\times10^{-2}\text{ kg/m})(224.8\text{ m/s})^2 = 591\text{ N}$.

(b) $f_1 = f_3/3 = (449.7\text{ Hz})/3 = 150\text{ Hz}$.

EVALUATE: The waves on the string have a much longer wavelength than the sound waves in the air because the speed of the waves on the string is much greater than the speed of sound in air.

15.43. **IDENTIFY** and **SET UP:** Use the information given about the A_4 note to find the wave speed that depends on the linear mass density of the string and the tension. The wave speed isn't affected by the placement of the fingers on the bridge. Then find the wavelength for the D_5 note and relate this to the length of the vibrating portion of the string.
EXECUTE: **(a)** $f = 440\text{ Hz}$ when a length $L = 0.600\text{ m}$ vibrates; use this information to calculate the speed v of waves on the string. For the fundamental $\lambda/2 = L$ so $\lambda = 2L = 2(0.600\text{ m}) = 1.20\text{ m}$. Then $v = f\lambda = (440\text{ Hz})(1.20\text{ m}) = 528\text{ m/s}$. Now find the length $L = x$ of the string that makes $f = 587\text{ Hz}$.

$$\lambda = \frac{v}{f} = \frac{528\text{ m/s}}{587\text{ Hz}} = 0.900\text{ m}$$

$L = \lambda/2 = 0.450\text{ m}$, so $x = 0.450\text{ m} = 45.0\text{ cm}$.

(b) No retuning means same wave speed as in part (a). Find the length of vibrating string needed to produce $f = 392\text{ Hz}$.

$$\lambda = \frac{v}{f} = \frac{528\text{ m/s}}{392\text{ Hz}} = 1.35\text{ m}$$

$L = \lambda/2 = 0.675\text{ m}$; string is shorter than this. No, not possible.

EVALUATE: Shortening the length of this vibrating string increases the frequency of the fundamental.

15.44. **IDENTIFY:** $y(x,t) = (A_{\text{SW}}\sin kx)\sin\omega t$. $v_y = \partial y/\partial t$. $a_y = \partial^2 y/\partial t^2$.

SET UP: $v_{\text{max}} = (A_{\text{SW}}\sin kx)\omega$. $a_{\text{max}} = (A_{\text{SW}}\sin kx)\omega^2$.

EXECUTE: (a) (i) $x = \dfrac{\lambda}{2}$ is a node, and there is no motion.

(ii) $x = \dfrac{\lambda}{4}$ is an antinode, and $v_{\text{max}} = \omega A = 2\pi fA$, $a_{\text{max}} = \omega v_{\text{max}} = (2\pi f)v_{\text{max}} = 4\pi^2 f^2 A.$

(iii) $\cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$ and this factor multiplies the results of (ii), so $v_{\text{max}} = \sqrt{2}\pi fA$, $a_{\text{max}} = 2\sqrt{2}\pi^2 f^2 A.$

(b) The amplitude is $2A\sin kx$, or (i) 0, (ii) $2A$, (iii) $2A/\sqrt{2}$.

(c) The time between the extremes of the motion is the same for any point on the string (although the period of the zero motion at a node might be considered indeterminate) and is $1/2f$.

EVALUATE: Any point in a standing wave moves in SHM. All points move with the same frequency but have different amplitude.

15.45. IDENTIFY: We are dealing with a traveling sinusoidal wave on a string.
SET UP: The equation $y(x,t) = A\cos(kx - \omega t)$ describes this wave. We know the maximum transverse speed and acceleration. We use $v_{y\text{-max}} = A\omega$, $a_{y\text{-max}} = A\omega^2$, and $v = f\lambda$. The target variables are the speed v of the wave and its amplitude A.
EXECUTE: Take the ratio of the maximum acceleration to the maximum speed, giving $\dfrac{a_{y\text{-max}}}{v_{y\text{-max}}} = \dfrac{A\omega^2}{A\omega} = \omega$. Using the known values gives $\omega = \dfrac{8.50\times10^4 \text{ m/s}^2}{3.00 \text{ m/s}} = 2.8333\times10^4$ rad/s. Now use

$$v = f\lambda = (\omega/2\pi)\lambda = \left(\frac{2.8333\times10^4 \text{ rad/s}}{2\pi}\right)(0.400 \text{ m}) = 1.80\times10^3 \text{ m/s}.$$

From the velocity equation $v_{y\text{-max}} = A\omega$ we have $A = \dfrac{v_{y\text{-max}}}{\omega}$, which gives

$$A = \frac{3.00 \text{ m/s}}{2.8333\times10^4 \text{ rad/s}} = 1.06\times10^{-4} \text{ m}.$$

EVALUATE: The propagation speed v is constant. But the transverse speed $v_y = \dfrac{\partial y}{\partial t}$ is *not* constant; it varies with time and from place to place on the string.

15.46. IDENTIFY: Compare $y(x,t)$ given in the problem to the general form $y(x,t) = A\cos(kx - \omega t)$.
SET UP: The comparison gives $A = 0.750$ cm, $k = 0.400\pi$ rad/cm and $\omega = 250\pi$ rad/s.

EXECUTE: (a) $A = 0.750$ cm, $\lambda = \dfrac{2}{0.400 \text{ rad/cm}} = 5.00$ cm, $f = 125$ Hz, $T = \frac{1}{f} = 0.00800$ s and $v = \lambda f = 6.25$ m/s.

(b) The sketches of the shape of the rope at each time are given in Figure 15.46.
(c) To stay with a wavefront as t increases, x decreases and so the wave is moving in the $-x$-direction.
(d) From $v = \sqrt{F/\mu}$, the tension is $F = \mu v^2 = (0.050 \text{ kg/m})(6.25 \text{ m/s})^2 = 1.95$ N.

(e) $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2 = 5.42$ W.

EVALUATE: The argument of the cosine is $(kx + \omega t)$ for a wave traveling in the $-x$-direction, and that is the case here.

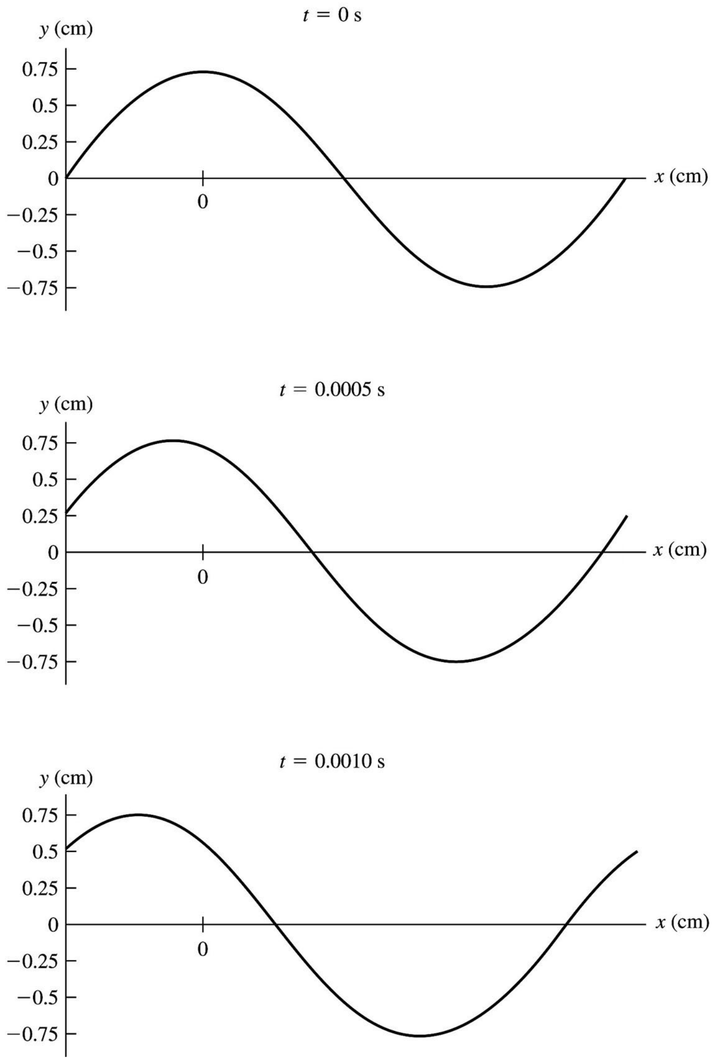


Figure 15.46

15.47. **IDENTIFY** and **SET UP:** Calculate v, ω, and k from $v = f\lambda$, $\omega = vk$, $k = 2\pi/\lambda$. Then apply $y(x,t) = A\cos(kx - \omega t)$ to obtain $y(x,t)$.

$A = 2.50\times10^{-3}$ m, $\lambda = 1.80$ m, $v = 36.0$ m/s

EXECUTE: **(a)** $v = f\lambda$ so $f = v/\lambda = (36.0 \text{ m/s})/1.80 \text{ m} = 20.0 \text{ Hz}$

$\omega = 2\pi f = 2\pi(20.0 \text{ Hz}) = 126 \text{ rad/s}$

$k = 2\pi/\lambda = 2\pi \text{ rad}/1.80 \text{ m} = 3.49 \text{ rad/m}$

(b) For a wave traveling to the right, $y(x,t) = A\cos(kx - \omega t)$. This equation gives that the $x = 0$ end of the string has maximum upward displacement at $t = 0$.

Put in the numbers: $y(x,t) = (2.50\times10^{-3} \text{ m})\cos[(3.49 \text{ rad/m})x - (126 \text{ rad/s})t]$.

(c) The left-hand end is located at $x = 0$. Put this value into the equation of part (b):

$y(0,t) = +(2.50\times10^{-3} \text{ m})\cos((126 \text{ rad/s})t)$.

(d) Put $x = 1.35$ m into the equation of part (b):

$y(1.35 \text{ m}, t) = (2.50\times10^{-3} \text{ m})\cos((3.49 \text{ rad/m})(1.35 \text{ m}) - (126 \text{ rad/s})t)$.

$y(1.35 \text{ m}, t) = (2.50\times10^{-3} \text{ m})\cos(4.71 \text{ rad} - (126 \text{ rad/s})t)$

$4.71 \text{ rad} = 3\pi/2$ and $\cos(\theta) = \cos(-\theta)$, so $y(1.35 \text{ m}, t) = (2.50\times 10^{-3} \text{ m})\cos((126 \text{ rad/s})t - 3\pi/2 \text{ rad})$

(e) $y = A\cos(kx - \omega t)$ (part (b))

The transverse velocity is given by $v_y = \dfrac{\partial y}{\partial t} = A\dfrac{\partial}{\partial t}\cos(kx - \omega t) = +A\omega\sin(kx - \omega t)$.

The maximum v_y is $A\omega = (2.50\times 10^{-3} \text{ m})(126 \text{ rad/s}) = 0.315 \text{ m/s}$.

(f) $y(x,t) = (2.50\times 10^{-3} \text{ m})\cos((3.49 \text{ rad/m})x - (126 \text{ rad/s})t)$

$t = 0.0625$ s and $x = 1.35$ m gives

$y = (2.50\times 10^{-3} \text{ m})\cos((3.49 \text{ rad/m})(1.35 \text{ m}) - (126 \text{ rad/s})(0.0625 \text{ s})) = -2.50\times 10^{-3}$ m.

$v_y = +A\omega\sin(kx - \omega t) = +(0.315 \text{ m/s})\sin((3.49 \text{ rad/m})x - (126 \text{ rad/s})t)$

$t = 0.0625$ s and $x = 1.35$ m gives

$v_y = (0.315 \text{ m/s})\sin((3.49 \text{ rad/m})(1.35 \text{ m}) - (126 \text{ rad/s})(0.0625 \text{ s})) = 0.0$

EVALUATE: The results of part (f) illustrate that $v_y = 0$ when $y = \pm A$, as we saw from SHM in Chapter 14.

15.48. **IDENTIFY:** Apply $\Sigma\tau_z = 0$ to find the tension in each wire. Use $v = \sqrt{F/\mu}$ to calculate the wave speed for each wire and then $t = L/v$ is the time for each pulse to reach the ceiling, where $L = 1.25$ m.

SET UP: The wires have $\mu = \dfrac{m}{L} = \dfrac{0.290 \text{ N}}{(9.80 \text{ m/s}^2)(1.25 \text{ m})} = 0.02367 \text{ kg/m}$. The free-body diagram for the beam is given in Figure 15.48. Take the axis to be at the end of the beam where wire A is attached.

EXECUTE: $\Sigma\tau_z = 0$ gives $T_B L = w(L/3)$ and $T_B = w/3 = 583$ N. $T_A + T_B = 1750$ N, so $T_A = 1167$ N.

$v_A = \sqrt{\dfrac{T_A}{\mu}} = \sqrt{\dfrac{1167 \text{ N}}{0.02367 \text{ kg/m}}} = 222$ m/s. $t_A = \dfrac{1.25 \text{ m}}{222 \text{ m/s}} = 0.00563 \text{ s} = 5.63$ ms.

$v_B = \sqrt{\dfrac{583 \text{ N}}{0.02367 \text{ kg/m}}} = 156.9$ m/s. $t_B = \dfrac{1.25 \text{ m}}{156.9 \text{ m/s}} = 0.007965 \text{ s} = 7.965$ ms.

$\Delta t = t_B - t_A = 7.965 \text{ ms} - 5.63 \text{ ms} = 2.34$ ms.

EVALUATE: The wave pulse travels faster in wire A, since that wire has the greater tension, so the pulse in wire A arrives first.

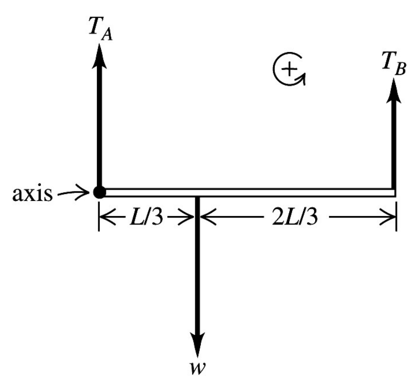


Figure 15.48

15.49. **IDENTIFY:** The block causes tension in the wire supporting the rod. A standing wave pattern exists on the wire, and the torques on the rod balance.

SET UP: The graph is a plot of the square of the frequency f versus the mass m hanging from the rod. The target variable is the mass of the wire. To interpret the graph, we need to find a relationship between f^2 and m. We apply $\sum\tau_z = 0$ about the hinge and use $v = \sqrt{\dfrac{F}{\mu}}$ and $v = f\lambda$.

EXECUTE: $\sum\tau_z = 0$: $TL\sin\theta - mg\dfrac{L}{2} = 0$ gives $T = \dfrac{mg}{2\sin\theta}$. Now use $v = \sqrt{\dfrac{F}{\mu}}$ and $v = f\lambda$. Equating these velocity equations gives $f\lambda = \sqrt{\dfrac{F}{\mu}}$, so $f^2 = \dfrac{1}{\lambda^2}\left(\dfrac{F}{\mu}\right)$. Using the result for the tension gives $f^2 = \dfrac{1}{\mu\lambda^2}\left(\dfrac{mg}{2\sin\theta}\right) = \left(\dfrac{g}{2\mu\lambda^2\sin\theta}\right)m$. The wire is vibrating in its fundamental mode so $\lambda = 2L$. The linear mass density of the wire is $\mu = \dfrac{m_w}{L}$. Therefore the final equation becomes $f^2 = \left(\dfrac{g}{8m_w L\sin\theta}\right)m$. From this we see that a graph of f^2 versus m should be a straight line with slope equal to $\dfrac{g}{8m_w L\sin\theta}$.

Thus $m_w = \dfrac{g}{(\text{slope})8L\sin\theta}$ which gives

$$m_w = \frac{9.80\ \text{m/s}^2}{(20.4\ \text{kg}^{-1}\cdot\text{s}^{-2})(8)(2.00\ \text{m})(\sin 30.0°)} = 0.0600\ \text{kg} = 60.0\ \text{g}.$$

EVALUATE: A mass of 60 g for a 2.00-m wire is not unreasonable.

15.50. IDENTIFY: The maximum vertical acceleration must be at least g.

SET UP: $a_{max} = \omega^2 A$

EXECUTE: $g = \omega^2 A_{min}$ and thus $A_{min} = g/\omega^2$. Using $\omega = 2\pi f = 2\pi v/\lambda$ and $v = \sqrt{F/\mu}$, this becomes $A_{min} = \dfrac{g\lambda^2\mu}{4\pi^2 F}$.

EVALUATE: When the amplitude of the motion increases, the maximum acceleration of a point on the rope increases.

15.51. IDENTIFY: Calculate the speed of the wave and use that to find the length of the wire since we know how long it takes the wave to travel the length of the wire.

SET UP: $v = \sqrt{F/\mu}$, $x = v_x t$, and $\mu = m/L$.

EXECUTE: **(a)** $\mu = m/L = (14.5\times10^{-9}\ \text{kg})/(0.0200\ \text{m}) = 7.25\times10^{-7}\ \text{kg/m}$. Now combine $v = \sqrt{F/\mu}$ and $x = v_x t$: $vt = L$, so $L = t\sqrt{F/\mu} = (26.7\times10^{-3}\ \text{s})\sqrt{\dfrac{(0.400\ \text{kg})(9.80\ \text{m/s}^2)}{7.25\times10^{-7}\ \text{kg/m}}} = 62.1\ \text{m}$.

(b) The mass of the wire is $m = \mu L = (7.25\times10^{-7}\ \text{kg/m})(62.1\ \text{m}) = 4.50\times10^{-5}\ \text{kg} = 0.0450\ \text{g}$.

EVALUATE: The mass of the wire is negligible compared to the 0.400-kg object hanging from the wire.

15.52. IDENTIFY: The frequencies at which a string vibrates depend on its tension, mass density and length.

SET UP: $f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{2L}$, where $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{FL}{m}}$. F is the tension in the string, L is its length and m is its mass.

EXECUTE: **(a)** $f_1 = \dfrac{v}{2L} = \dfrac{1}{2L}\sqrt{\dfrac{FL}{m}} = \dfrac{1}{2}\sqrt{\dfrac{F}{Lm}}$. Solving for F gives

$$F = (2f_1)^2 Lm = 4(262\ \text{Hz})^2(0.350\ \text{m})(8.00\times10^{-3}\ \text{kg}) = 769\ \text{N}.$$

(b) $m = \dfrac{F}{L(2f_1)^2} = \dfrac{769 \text{ N}}{(0.350 \text{ m})(4)(466 \text{ Hz})^2} = 2.53 \text{ g}.$

(c) For S_1, $\mu = \dfrac{8.00\times10^{-3} \text{ kg}}{0.350 \text{ m}} = 0.0229 \text{ kg/m}$. $F = 769 \text{ N}$ and $v = \sqrt{F/\mu} = 183 \text{ m/s}$. $f_1 = \dfrac{v}{2L}$ gives $L = \dfrac{v}{2f_1} = \dfrac{183 \text{ m/s}}{2(277 \text{ Hz})} = 33.0 \text{ cm}$. $x = 35.0 \text{ cm} - 33.0 \text{ cm} = 2.00 \text{ cm}$.

(d) For S_2, $\mu = \dfrac{2.53\times10^{-3} \text{ kg}}{0.350 \text{ m}} = 7.23\times10^{-3} \text{ kg/m}$. $F = 769 \text{ N}$ and $v = \sqrt{F/\mu} = 326 \text{ m/s}$. $L = 0.330 \text{ m}$ and $f_1 = \dfrac{v}{2L} = \dfrac{326 \text{ m/s}}{2(0.330 \text{ m})} = 494 \text{ Hz}$.

EVALUATE: If the tension is the same in the strings, the mass densities must be different to produce sounds of different pitch.

15.53. **IDENTIFY:** Apply $\Sigma\tau_z = 0$ to one post and calculate the tension in the wire. $v = \sqrt{F/\mu}$ for waves on the wire. $v = f\lambda$. The standing wave on the wire and the sound it produces have the same frequency. For standing waves on the wire, $\lambda_n = \dfrac{2L}{n}$.

SET UP: For the fifth overtone, $n = 6$. The wire has $\mu = m/L = (0.732 \text{ kg})/(5.00 \text{ m}) = 0.146 \text{ kg/m}$. The free-body diagram for one of the posts is given in Figure 15.53. Forces at the pivot aren't shown. We take the rotation axis to be at the pivot, so forces at the pivot produce no torque.

EXECUTE: $\Sigma\tau_z = 0$ gives $w\left(\dfrac{L}{2}\cos 57.0°\right) - T(L\sin 57.0°) = 0$. $T = \dfrac{w}{2\tan 57.0°} = \dfrac{235 \text{ N}}{2\tan 57.0°} = 76.3 \text{ N}$.

For waves on the wire, $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{76.3 \text{ N}}{0.146 \text{ kg/m}}} = 22.9 \text{ m/s}$. For the fifth overtone standing wave on the wire, $\lambda = \dfrac{2L}{6} = \dfrac{2(5.00 \text{ m})}{6} = 1.67 \text{ m}$. $f = \dfrac{v}{\lambda} = \dfrac{22.9 \text{ m/s}}{1.67 \text{ m}} = 13.7 \text{ Hz}$. The sound waves have frequency 13.7 Hz and wavelength $\lambda = \dfrac{344 \text{ m/s}}{13.7 \text{ Hz}} = 25.0 \text{ m}$.

EVALUATE: The frequency of the sound wave is just below the lower limit of audible frequencies. The wavelength of the standing wave on the wire is much less than the wavelength of the sound waves, because the speed of the waves on the wire is much less than the speed of sound in air.

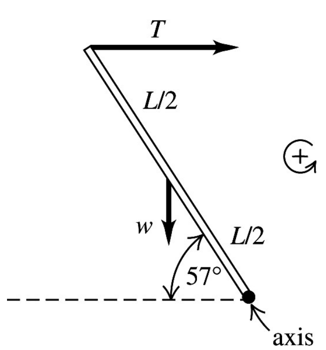


Figure 15.53

15.54. **IDENTIFY:** The mass of the planet (the target variable) determines g at its surface, which in turn determines the weight of the lead object hanging from the string. The weight is the tension in the string, which determines the speed of a wave pulse on that string.

SET UP: At the surface of the planet $g = G\dfrac{m_{\rm p}}{R_{\rm p}^2}$. The pulse speed is $v = \sqrt{\dfrac{F}{\mu}}$.

EXECUTE: On earth, $v = \dfrac{4.00\ \text{m}}{0.0390\ \text{s}} = 1.0256\times 10^2\ \text{m/s}$. $\mu = \dfrac{0.0280\ \text{kg}}{4.00\ \text{m}} = 7.00\times 10^{-3}\ \text{kg/m}^3$. $F = Mg$, so $v = \sqrt{\dfrac{Mg}{\mu}}$ and the mass of the lead weight is

$M = \left(\dfrac{\mu}{g}\right)v^2 = \left(\dfrac{7.00\times 10^{-3}\ \text{kg/m}}{9.8\ \text{m/s}^2}\right)(1.0256\times 10^2\ \text{m/s})^2 = 7.513\ \text{kg}$. On the planet,

$v = \dfrac{4.00\ \text{m}}{0.0685\ \text{s}} = 58.394\ \text{m/s}$. Therefore $g = \left(\dfrac{\mu}{M}\right)v^2 = \left(\dfrac{7.00\times 10^{-3}\ \text{kg/m}}{7.513\ \text{kg}}\right)(58.394\ \text{m/s})^2 = 3.1770\ \text{m/s}^2$.

$g = G\dfrac{m_{\rm p}}{R_{\rm p}^2}$ and $m_{\rm p} = \dfrac{gR_{\rm p}^2}{G} = \dfrac{(3.1770\ \text{m/s}^2)(7.20\times 10^7\ \text{m})^2}{6.6743\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 2.47\times 10^{26}\ \text{kg}$.

EVALUATE: This mass is about 41 times that of earth, but its radius is about 10 times that of earth, so the result is reasonable.

15.55. **IDENTIFY:** The wavelengths of standing waves depend on the length of the string (the target variable), which in turn determine the frequencies of the waves.

SET UP: $f_n = nf_1$ where $f_1 = \dfrac{v}{2L}$.

EXECUTE: $f_n = nf_1$ and $f_{n+1} = (n+1)f_1$. We know the wavelengths of two adjacent modes, so $f_1 = f_{n+1} - f_n = 630\ \text{Hz} - 525\ \text{Hz} = 105\ \text{Hz}$. Solving $f_1 = \dfrac{v}{2L}$ for L gives $L = \dfrac{v_1}{2f} = \dfrac{384\ \text{m/s}}{2(105\ \text{Hz})} = 1.83\ \text{m}$.

EVALUATE: The observed frequencies are both audible which is reasonable for a string that is about a half meter long.

15.56. **IDENTIFY:** A standing wave exists on a string fixed at both ends.

SET UP: The graph plots the number n of nodes (*not* antinodes) versus the frequency f_n. Therefore we look for a relationship between f_n and n so we can interpret the slope of the graph. The target variable is the speed v of waves on the string. We use $v = f\lambda$.

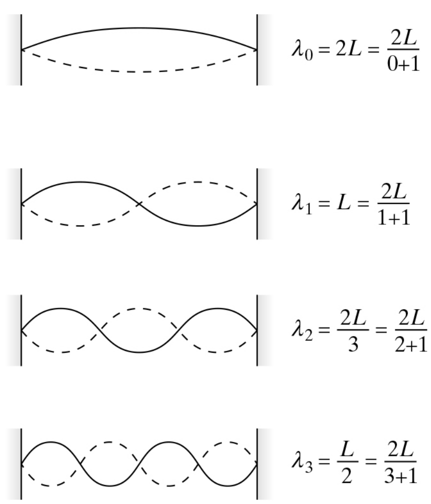


Figure 15.56

EXECUTE: We cannot use $\lambda_n = 2L/n$ because in that formula n is the number of *antinodes* and we need a relationship involving the number of *nodes* (not counting the nodes at the ends of the string). Fig. 15.56 helps to find this relationship. We already know that the wavelengths of the harmonics are $2L$, L, $2L/3$, $L/2$, … . Letting n be the number of antinodes between the ends, from the figure we see that $\lambda_0 = 2L$, $\lambda_1 = L = 2L/2$, $\lambda_2 = 2L/3$, $\lambda_3 = L/2 = 2L/4$, … . Notice that the denominator is 1 more than n, so the general formula is $\lambda_n = \dfrac{2L}{n+1}$, where n = 0, 1, 2, … . Now we need to relate this result to the frequency f_n. Using $v = f\lambda$ gives $f_n\lambda_n = v$. Using $\lambda_n = \dfrac{2L}{n+1}$, we get $f_n\left(\dfrac{2L}{n+1}\right) = v$. Solving for n gives $n = \dfrac{2L}{v}f_n - 1$. From this equation we see that a graph of n versus f_n should be a straight line having slope $2L/v$, so $v = \dfrac{2L}{\text{slope}} = \dfrac{2(0.800\text{ m})}{7.30\times10^{-3}\text{ s}} = 219\text{ m/s}$.

EVALUATE: The method used here would be useful if the string could not be removed to measure its mass and length to use $v = \sqrt{\dfrac{F}{\mu}}$.

15.57. **IDENTIFY:** The tension in the wires along with their lengths determine the fundamental frequency in each one (the target variables). These frequencies are different because the wires have different linear mass densities. The bar is in equilibrium, so the forces and torques on it balance.

SET UP: $T_a + T_c = w$, $\Sigma\tau_z = 0$, $v = \sqrt{\dfrac{F}{\mu}}$, $f_1 = v/2L$ and $\mu = \dfrac{m}{L}$, where $m = \rho V = \rho\pi r^2 L$. The densities of copper and aluminum are given in a table in the text.

EXECUTE: Using the subscript "a" for aluminum and "c" for copper, we have $T_a + T_c = w = 638\text{ N}$. $\Sigma\tau_z = 0$, with the axis at left-hand end of bar, gives $T_c(1.40\text{ m}) = w(0.90\text{ m})$, so $T_c = 410.1\text{ N}$.

$T_a = 638\text{ N} - 410.1\text{ N} = 227.9\text{ N}$. $f_1 = \dfrac{v}{2L}$. $\mu = \dfrac{m}{L} = \dfrac{\rho\pi r^2 L}{L} = \rho\pi r^2$.

<u>For the copper wire</u>: $F = 410.1\text{ N}$ and $\mu = (8.90\times10^3\text{ kg/m}^3)\pi(0.280\times10^{-3}\text{ m})^2 = 2.19\times10^{-3}\text{ kg/m}$, so

$v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{410.1\text{ N}}{2.19\times10^{-3}\text{ kg/m}}} = 432.7\text{ m/s}$. $f_1 = \dfrac{v}{2L} = \dfrac{432.7\text{ m/s}}{2(0.600\text{ m})} = 361\text{ Hz}$.

<u>For the aluminum wire</u>: $F = 227.9\text{ N}$ and

$\mu = (2.70\times10^3\text{ kg/m}^3)\pi(0.280\times10^{-3}\text{ m})^2 = 6.65\times10^{-4}\text{ kg/m}$, so

$v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{227.9\text{ N}}{6.65\times10^{-4}\text{ kg/m}}} = 585.4\text{ m/s}$, which gives $f_1 = \dfrac{585.4\text{ m/s}}{2(0.600\text{ m})} = 488\text{ Hz}$.

EVALUATE: The wires have different fundamental frequencies because they have different tensions and different linear mass densities.

15.58. **IDENTIFY:** We are dealing with a standing wave on a string fixed at both ends.

SET UP: We want to know the maximum transverse speed at two points on the string. The equation for the displacement $y(x,t)$ of the string is $y(x,t) = A_{SW}\sin kx \sin\omega t$. The transverse velocity is $v_y = \dfrac{\partial y}{\partial t}$, $v = f\lambda$, $k = 2\pi/\lambda$, and $\omega = 2\pi f$. The first antinode occurs at x = 0.150 m, which is ¼ of a wavelength from the end, so λ = 0.600 m.

EXECUTE: **(a)** The maximum transverse speed occurs at the antinodes. Using $v_y = \dfrac{\partial y}{\partial t}$, we get $v_y = \omega A_{SW}\sin kx\cos\omega t$. The maximum speed is ωA_{SW}. Now use $v = f\lambda$ and $\omega = 2\pi f$ to find ω:

$v = f\lambda = (\omega / 2\pi)\lambda$, so $\omega = 2\pi v / \lambda$. Thus $v_{y,\max} = \omega A_{\rm SW} = \dfrac{2\pi v A_{\rm SW}}{\lambda}$. Putting in the numbers gives

$v_{y,\max} = \dfrac{2\pi(260 \text{ m/s})(0.00180 \text{ m})}{0.600 \text{ m}} = 4.90 \text{ m/s}.$

(b) Use $v_y = \omega A_{\rm SW} \sin kx \cos \omega t$ at $x = 0.075$ m. The maximum speed occurs when $\cos \omega t = \pm 1$, so

$v_{y,\max} = (4.90 \text{ m/s}) \sin\left[(10.472 \text{ m}^{-1})(0.075 \text{ m})\right] = 3.5 \text{ m/s}.$

EVALUATE: Notice that the wave speed (260 m/s) moves much faster than any point on the string.

15.59. IDENTIFY: The distance between adjacent nodes is one-half the wavelength. The second overtone is the third harmonic ($n = 3$).

SET UP: The wavelengths are $\lambda_n = 2L/n$, $f = 1/T$, $v = \sqrt{F/\mu}$, $v = f\lambda$, and $\mu = m/L$.

EXECUTE: **(a)** The node-to-node distance is $\lambda/2$, so $\lambda = 2(6.28 \text{ cm}) = 12.56 \text{ cm} = 0.1256$ m. In the third harmonic, $\lambda_n = 2L/n$ gives $0.1256 \text{ m} = 2L/3$, so $L = 0.1884$ m which rounds to 0.188 m.

(b) The time to go from top to bottom is one-half the period, $T/2$, so $T = 2(8.40 \text{ ms}) = 16.8 \text{ ms} = 0.0168$ s.

$f = 1/T = 1/(0.0168 \text{ s}) = 59.52$ Hz. Combining $v = \sqrt{F/\mu}$, $\mu = m/L$, and $v = f\lambda$ gives

$(f\lambda)^2 = \dfrac{F}{\mu} \to \mu = \dfrac{F}{(f\lambda)^2} = \dfrac{m}{L} \to m = \dfrac{FL}{(f\lambda)^2}$. Putting in the numbers gives

$m = (5.00 \text{ N})(0.1884 \text{ m})/[(59.52 \text{ Hz})(0.1256 \text{ m})]^2 = 0.0169 \text{ kg} = 16.9$ g.

EVALUATE: In a standing wave pattern, the nodes, we well as the antinodes, are spaced one-half a wavelength apart.

15.60. IDENTIFY: The wavelengths of the standing waves on the wire are given by $\lambda_n = \dfrac{2L}{n}$. When the ball is changed the wavelength changes because the length of the wire changes; $\Delta l = \dfrac{Fl_0}{AY}$.

SET UP: For the third harmonic, $n = 3$. For copper, $Y = 11 \times 10^{10}$ Pa. The wire has cross-sectional area $A = \pi r^2 = \pi(0.512 \times 10^{-3} \text{ m})^2 = 8.24 \times 10^{-7} \text{ m}^2$.

EXECUTE: **(a)** $\lambda_3 = \dfrac{2(1.20 \text{ m})}{3} = 0.800 \text{ m}$

(b) The increase in length when the 100.0 N ball is replaced by the 500.0 N ball is given by

$\Delta l = \dfrac{(\Delta F)l_0}{AY}$, where $\Delta F = 400.0$ N is the increase in the force applied to the end of the wire.

$\Delta l = \dfrac{(400.0 \text{ N})(1.20 \text{ m})}{(8.24 \times 10^{-7} \text{ m}^2)(11 \times 10^{10} \text{ Pa})} = 5.30 \times 10^{-3}$ m. The change in wavelength is $\Delta\lambda = \frac{2}{3}\Delta l = 3.5$ mm.

EVALUATE: The change in tension changes the wave speed and that in turn changes the frequency of the standing wave, but the problem asks only about the wavelength.

15.61. IDENTIFY and SET UP: The average power is given by $P_{\rm av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$. Rewrite this expression in terms of v and λ in place of F and ω.

EXECUTE: **(a)** $P_{\rm av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$

$v = \sqrt{F/\mu}$ so $\sqrt{F} = v\sqrt{\mu}$

$\omega = 2\pi f = 2\pi(v/\lambda)$

Using these two expressions to replace $\sqrt{F}$ and ω gives $P_{\rm av} = 2\mu\pi^2 v^3 A^2/\lambda^2$;

$\mu = (6.00 \times 10^{-3} \text{ kg})/(8.00 \text{ m})$

$$A=\left(\frac{2\lambda^2 P_{av}}{4\pi^2 v^3 \mu}\right)^{1/2}=7.07\text{ cm}$$

(b) EVALUATE: $P_{av}\sim v^3$ so doubling v increases P_{av} by a factor of 8.

$P_{av}=8(50.0\text{ W})=400.0\text{ W}$

15.62. **IDENTIFY:** The time between positions 1 and 5 is equal to $T/2$. $v=f\lambda$. The velocity of points on the string is given by $v_y(x,t)=\omega A\sin(kx-\omega t)$.

SET UP: Four flashes occur from position 1 to position 5, so the elapsed time is $4\left(\frac{60\text{ s}}{5000}\right)=0.048\text{ s}$.

The figure in the problem shows that $\lambda=L=0.500\text{ m}$. At point P the amplitude of the standing wave is 1.5 cm.

EXECUTE: **(a)** $T/2=0.048\text{ s}$ and $T=0.096\text{ s}$. $f=1/T=10.4\text{ Hz}$. $\lambda=0.500\text{ m}$.

(b) The fundamental standing wave has nodes at each end and no nodes in between. This standing wave has one additional node. This is the first overtone and second harmonic.

(c) $v=f\lambda=(10.4\text{ Hz})(0.500\text{ m})=5.20\text{ m/s}$.

(d) In position 1, point P is at its maximum displacement and its speed is zero. In position 3, point P is passing through its equilibrium position and its speed is

$v_{max}=\omega A=2\pi fA=2\pi(10.4\text{ Hz})(0.015\text{ m})=0.980\text{ m/s}$.

(e) $v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{FL}{m}}$ and $m=\frac{FL}{v^2}=\frac{(1.00\text{ N})(0.500\text{ m})}{(5.20\text{ m/s})^2}=18.5\text{ g}$.

EVALUATE: The standing wave is produced by traveling waves moving in opposite directions. Each point on the string moves in SHM, and the amplitude of this motion varies with position along the string.

15.63. **IDENTIFY:** The chain of beads vibrates in a standing wave pattern.

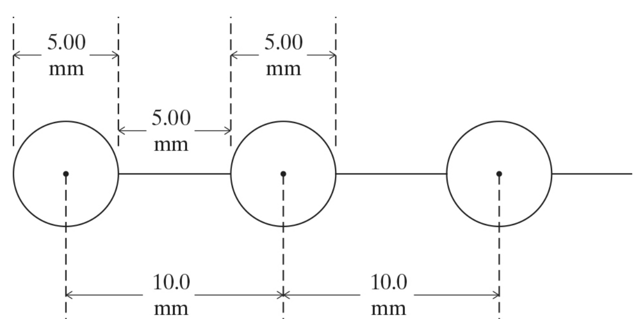


Figure 15.63

SET UP: First find the number of beads on a string 1.005 m long. Fig. 15.63 shows the first few beads. Starting at the left end, the string is made up of units each consisting of one bead of diameter 5.00 mm plus a space of 5.00 mm, so each unit is 10.0 mm long. To make a chain 1.005 m long, we need 100 of these units plus 5.00 mm for the end bead on the right. So the total mass of the 101 beads is 101 g = 0.101 kg. We use $y(x,t)=A_{SW}\sin kx\sin\omega t$, $v=\sqrt{\frac{F}{\mu}}$, $v=f\lambda$, $v_y=\frac{\partial y}{\partial t}$, $\lambda_n=2L/n$, $k=2\pi/\lambda$, $\omega=2\pi f$, and $F=kx$ (this k is the force constant).

EXECUTE: **(a)** $\mu=m/L$ = (0.101 kg)/(1.50 m) = 0.0673 kg/m.

(b) $T=kx$ = (28.8 N/m)(1.50 m – 1.005 m) = 14.3 N.

(c) $v=\sqrt{\dfrac{F}{\mu}}=\sqrt{\dfrac{14.3\text{ N}}{0.0673\text{ kg/m}}}=14.6\text{ m/s}.$

(d) We want the wave speed v. The wave pattern has 4 antinodes, so the chain is vibrating in its 4th harmonic, so $n = 4$. $\lambda_n = 2L/n$ = (3.00 m)/4 = 0.750 m. Using $v = f\lambda$ gives $f = v/\lambda$ = (14.6 m/s)/(0.750 m) = 19.5 Hz.

(e) The motionless beads are at the nodes. In the 4th harmonic, nodes occur at the ends of the chain, in the middle, and ¼ and ¾ of the way along the chain. The beads at these locations are #1, 26, 51, 76, and 101.

(f) Bead #26 is at a node, so bead #13 is at an antinode, so its maximum speed is $v_{y,\max} = A\omega$. Using $\omega = 2\pi f$, we get $A=\dfrac{v_{y,\max}}{\omega}=\dfrac{v_{y,\max}}{2\pi f}=\dfrac{7.54\text{ m/s}}{2\pi(19.5\text{ Hz})}=0.0615\text{ m}=6.15\text{ cm}.$

(g) When the chain is stretched, the beads are 15.0 mm apart center-to-center, with the origin at the center of the first bead. The first few coordinates are:

x_1 = 0 mm = (1 – 1)(15.0 mm)

x_2 = 10.0 mm = (2 – 1)(15.0 mm)

x_3 = 20.0 mm = (3 – 1)(15.0 mm)

x_4 = 30.0 mm = (4 – 1)(15.0 mm)

From this pattern we recognize that the n^{th} bead is at $x_n = (n - 1)(15.0\text{ mm})$, where n = 1, 2, … .

(h) The 30th bead is at x_{30} = (30 – 1)(15.0 mm) = 435 mm = 0.435 m. The standing wave equation is $y(x,t) = A_{\text{SW}}\sin kx \sin\omega t$. Thus $v_y=\dfrac{\partial y}{\partial t}=\omega A_{\text{SW}}\sin kx\cos\omega t$, so at any position x, $v_{y\text{-max}} = \omega A_{\text{SW}}\sin kx = 2\pi f A_{\text{SW}}\sin kx$. From part (f) A_{SW} = 0.0615 m, and $k = 2\pi/\lambda$ = 2π/(0.750 m) = 8.378 m^{-1}. Therefore $v_{y,\max}$ = 2π(19.5 Hz)(0.0615 m) sin[(8.3778 m^{-1})(0.435 m)] = –3.63 m/s, so the maximum *speed* is 3.63 m/s.

EVALUATE: The 30th bead has a smaller maximum speed than the 13th bead. This is reasonable since the 13th bead is at an antinode but the 30th bead is not.

15.64. **IDENTIFY** and **SET UP:** $v=\sqrt{F/\mu}$ is the wave speed and $v_y=\dfrac{\partial y}{\partial t}$ is the transverse speed of a point on the string. $v = f\lambda$, $\lambda_n = 2L/n$, $\mu = m/L$, $v_{\max} = \omega A$ (maximum v_y), $a_{\max} = \omega^2 A$ (maximum a_y).

EXECUTE: **(a)** $\mu = m/L$ = (0.00300 kg)/(2.20 m) = 0.0013636 kg/m. In the fundamental mode, n = 1, so $\lambda_n = 2L/n$ = 2L. Combining $v = f\lambda$ and $v=\sqrt{F/\mu}$, we get $f\lambda=\sqrt{F/\mu}$. Putting in the numbers get f(2)(2.20 m) = [(330 N)/(0.0013636 kg/m)]$^{1/2}$, which gives f = 111.8 Hz. Now use $v_{\max} = \omega A$ to get $A = v_{\max}/(2\pi f)$ = (9.00 m/s)/[2π(111.8 Hz)] = 0.0128 m = 1.28 cm.

(b) $a_{\max} = \omega^2 A = (2\pi f)^2 A$ = [2π(111.8 Hz)]^2(0.0128 m) = 6320 m/s^2.

EVALUATE: It is important to distinguish between the transverse velocity of a point on the string, $v_y=\dfrac{\partial y}{\partial t}$, and speed of the wave, $v=\sqrt{F/\mu}$. The wave speed is constant in time, but the transverse speed is not. The maximum acceleration of a point on the string is 645g!

15.65. **IDENTIFY** and **SET UP:** $v=\sqrt{F/\mu}$ is the wave speed and $v_y=\dfrac{\partial y}{\partial t}$ is the transverse speed of a point on the string. $v = f\lambda$, $\lambda_n = 2L/n$, $\mu = m/L$, $v_{\max} = \omega A$ (maximum v_y), $a_{\max} = \omega^2 A$ (maximum a_y). The first overtone is the second harmonic (n = 2).

EXECUTE: **(a)** $v_{\max} = \omega A = 2\pi f A$, which gives 28.0 m/s = (0.0350 m)(2πf), so f = 127.32 Hz. $\lambda_n = 2L/n = 2L/2 = L$ = 2.50 m. Using these results to get v gives $v = f\lambda$ = (127.32 Hz)(2.50 m) = 318.3 m/s. Now combine $v=\sqrt{F/\mu}$ and $\mu = m/L$ to find m, giving

$m = LF/v^2 = (2.50\text{ m})(90.0\text{ N})/(318.3\text{ m/s})^2 = 0.00222\text{ kg} = 2.22\text{ g}.$

(b) $a_{\max} = \omega^2 A = A(2\pi f)^2 = (0.0350\text{ m})[2\pi(127.32\text{ Hz})]^2 = 22{,}400\text{ m/s}^2.$

EVALUATE: It is important to distinguish between the transverse velocity of a point on the string, $v_y = \dfrac{\partial y}{\partial t}$, and speed of the wave, $v = \sqrt{F/\mu}$. The wave speed is constant in time, but the transverse speed is not. The maximum acceleration of a point on the string is about 2300g!

15.66. **IDENTIFY:** The displacement of the string at any point is $y(x,t) = (A_{\text{SW}} \sin kx)\sin \omega t$. For the fundamental mode $\lambda = 2L$, so at the midpoint of the string $\sin kx = \sin(2\pi/\lambda)(L/2) = 1$, and $y = A_{\text{SW}} \sin \omega t$. The transverse velocity is $v_y = \partial y/\partial t$ and the transverse acceleration is $a_y = \partial v_y/\partial t$.

SET UP: Taking derivatives gives $v_y = \dfrac{\partial y}{\partial t} = \omega A_{\text{SW}} \cos \omega t$, with maximum value $v_{y,\max} = \omega A_{\text{SW}}$, and $a_y = \dfrac{\partial v_y}{\partial t} = -\omega^2 A_{\text{SW}} \sin \omega t$, with maximum value $a_{y,\max} = \omega^2 A_{\text{SW}}$.

EXECUTE: **(a)** $\omega = a_{y,\max}/v_{y,\max} = (8.40\times10^3\text{ m/s}^2)/(3.80\text{ m/s}) = 2.21\times10^3\text{ rad/s}$, and then $A_{\text{SW}} = v_{y,\max}/\omega = (3.80\text{ m/s})/(2.21\times10^3\text{ rad/s}) = 1.72\times10^{-3}\text{ m}$.

(b) $v = \lambda f = (2L)(\omega/2\pi) = L\omega/\pi = (0.386\text{ m})(2.21\times10^3\text{ rad/s})/\pi = 272\text{ m/s}$.

EVALUATE: The maximum transverse velocity and acceleration will have different (smaller) values at other points on the string.

15.67. **IDENTIFY:** The standing wave frequencies are given by $f_n = n\left(\dfrac{v}{2L}\right)$. $v = \sqrt{F/\mu}$. Use the density of steel to calculate μ for the wire.

SET UP: For steel, $\rho = 7.8\times10^3\text{ kg/m}^3$. For the first overtone standing wave, $n = 2$.

EXECUTE: $v = \dfrac{2Lf_2}{2} = (0.550\text{ m})(311\text{ Hz}) = 171\text{ m/s}$. The volume of the wire is $V = (\pi r^2)L$. $m = \rho V$ so $\mu = \dfrac{m}{L} = \dfrac{\rho V}{L} = \rho\pi r^2 = (7.8\times10^3\text{ kg/m}^3)\pi(0.57\times10^{-3}\text{ m})^2 = 7.96\times10^{-3}\text{ kg/m}$. The tension is $F = \mu v^2 = (7.96\times10^{-3}\text{ kg/m})(171\text{ m/s})^2 = 233\text{ N}$.

EVALUATE: The tension is not large enough to cause much change in length of the wire.

15.68. **IDENTIFY:** The standing wave is given by $y(x,t) = (A_{\text{SW}} \sin kx)\sin \omega t$.

SET UP: At an antinode, $\sin kx = 1$. $v_{y,\max} = \omega A$. $a_{y,\max} = \omega^2 A$.

EXECUTE: **(a)** $\lambda = v/f = (192.0\text{ m/s})/(240.0\text{ Hz}) = 0.800\text{ m}$, and the wave amplitude is $A_{\text{SW}} = 0.400\text{ cm}$. The amplitude of the motion at the given points is

(i) $(0.400\text{ cm})\sin(\pi) = 0$ (a node) (ii) $(0.400\text{ cm})\sin(\pi/2) = 0.400\text{ cm}$ (an antinode)

(iii) $(0.400\text{ cm})\sin(\pi/4) = 0.283\text{ cm}$

(b) The time is half of the period, or $1/(2f) = 2.08\times10^{-3}\text{ s}$.

(c) In each case, the maximum velocity is the amplitude multiplied by $\omega = 2\pi f$ and the maximum acceleration is the amplitude multiplied by $\omega^2 = 4\pi^2 f^2$:

(i) 0, 0; (ii) 6.03 m/s, $9.10\times10^3\text{ m/s}^2$; (iii) 4.27 m/s, $6.43\times10^3\text{ m/s}^2$.

EVALUATE: The amplitude, maximum transverse velocity, and maximum transverse acceleration vary along the length of the string. But the period of the simple harmonic motion of particles of the string is the same at all points on the string.

15.69. **IDENTIFY:** When the rock is submerged in the liquid, the buoyant force on it reduces the tension in the wire supporting it. This in turn changes the frequency of the fundamental frequency of the vibrations of the wire. The buoyant force depends on the density of the liquid (the target variable). The vertical forces on the rock balance in both cases, and the buoyant force is equal to the weight of the liquid displaced by the rock (Archimedes's principle).

SET UP: The wave speed is $v = \sqrt{\dfrac{F}{\mu}}$ and $v = f\lambda$. $B = \rho_{\text{liq}} V_{\text{rock}} g$. $\Sigma F_y = 0$.

EXECUTE: $\lambda = 2L = 6.00$ m. In air, $v = f\lambda = (42.0\text{ Hz})(6.00\text{ m}) = 252$ m/s. $v = \sqrt{\dfrac{F}{\mu}}$ so

$\mu = \dfrac{F}{v^2} = \dfrac{164.0\text{ N}}{(252\text{ m/s})^2} = 0.002583$ kg/m. In the liquid, $v = f\lambda = (28.0\text{ Hz})(6.00\text{ m}) = 168$ m/s.

$F = \mu v^2 = (0.002583\text{ kg/m})(168\text{ m/s})^2 = 72.90$ N. $F + B - mg = 0$.

$B = mg - F = 164.0\text{ N} - 72.9\text{ N} = 91.10$ N. For the rock,

$V = \dfrac{m}{\rho} = \dfrac{(164.0\text{ N}/9.8\text{ m/s}^2)}{3200\text{ kg/m}^3} = 5.230 \times 10^{-3}\text{ m}^3$. $B = \rho_{\text{liq}} V_{\text{rock}} g$ and

$\rho_{\text{liq}} = \dfrac{B}{V_{\text{rock}} g} = \dfrac{91.10\text{ N}}{(5.230 \times 10^{-3}\text{ m}^3)(9.8\text{ m/s}^2)} = 1.78 \times 10^3\text{ kg/m}^3$.

EVALUATE: This liquid has a density 1.78 times that of water, which is rather dense but not impossible.

15.70. **IDENTIFY:** We model a vibrating stretched rubber band as a standing wave pattern, fixed at both ends.

SET UP: **(a)** Estimate: Tension is about 1 lb ≈ 4.5 N. We use $v = \sqrt{\dfrac{F}{\mu}}$ and $v = f\lambda$. Our target variable is the frequency of vibration.

EXECUTE: **(b)** $\mu = m/L = (0.10\text{ g})/10\text{ cm}) = 1.0 \times 10^{-3}$ kg/m.

(c) If we simply pluck the rubber band, it vibrates in its fundamental mode, so $\lambda = 2L = 0.20$ m.

Combining $v = \sqrt{\dfrac{F}{\mu}}$ and $v = f\lambda$ gives $f = \dfrac{1}{\lambda}\sqrt{\dfrac{F}{\mu}} = \dfrac{1}{0.20\text{ m}}\sqrt{\dfrac{4.5\text{ N}}{1.0 \times 10^{-3}\text{ kg/m}}} = 340$ Hz.

EVALUATE: This result seems somewhat high, but our model is just a rough approximation.

15.71. **IDENTIFY:** Compute the wavelength from the length of the string. Use $v = f\lambda$ to calculate the wave speed and then apply $v = \sqrt{F/\mu}$ to relate this to the tension.

(a) SET UP: The tension F is related to the wave speed by $v = \sqrt{F/\mu}$, so use the information given to calculate v.

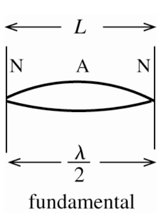


Figure 15.71

EXECUTE: $\lambda/2 = L$

$\lambda = 2L = 2(0.600\text{ m}) = 1.20$ m

$v = f\lambda = (65.4\text{ Hz})(1.20\text{ m}) = 78.5$ m/s

$\mu = m/L = 14.4 \times 10^{-3}\text{ kg}/0.600\text{ m} = 0.024$ kg/m

Then $F = \mu v^2 = (0.024\text{ kg/m})(78.5\text{ m/s})^2 = 148$ N.

(b) SET UP: $F = \mu v^2$ and $v = f\lambda$ give $F = \mu f^2 \lambda^2$.

μ is a property of the string so is constant.

λ is determined by the length of the string so stays constant.

μ, λ constant implies $F/f^2 = \mu\lambda^2 = \text{constant}$, so $F_1/f_1^2 = F_2/f_2^2$.

EXECUTE: $F_2 = F_1\left(\dfrac{f_2}{f_1}\right)^2 = (148\text{ N})\left(\dfrac{73.4\text{ Hz}}{65.4\text{ Hz}}\right)^2 = 186\text{ N}.$

The percent change in F is $\dfrac{F_2 - F_1}{F_1} = \dfrac{186\text{ N} - 148\text{ N}}{148\text{ N}} = 0.26 = 26\%.$

EVALUATE: The wave speed and tension we calculated are similar in magnitude to values in the examples. Since the frequency is proportional to $\sqrt{F}$, a 26% increase in tension is required to produce a 13% increase in the frequency.

15.72. IDENTIFY: The mass and breaking stress determine the length and radius of the string. $f_1 = \dfrac{v}{2L}$, with $v = \sqrt{\dfrac{F}{\mu}}$.

SET UP: The tensile stress is $F/\pi r^2$.

EXECUTE: **(a)** The breaking stress is $\dfrac{F}{\pi r^2} = 7.0\times 10^8\text{ N/m}^2$ and the maximum tension is $F = 900$ N, so solving for r gives the minimum radius $r = \sqrt{\dfrac{900\text{ N}}{\pi(7.0\times 10^8\text{ N/m}^2)}} = 6.4\times 10^{-4}$ m. The mass and density are fixed, $\rho = \dfrac{M}{\pi r^2 L}$. so the minimum radius gives the maximum length

$$L = \frac{M}{\pi r^2 \rho} = \frac{4.0\times 10^{-3}\text{ kg}}{\pi(6.4\times 10^{-4}\text{ m})^2(7800\text{ kg/m}^3)} = 0.40\text{ m}.$$

(b) The fundamental frequency is $f_1 = \dfrac{1}{2L}\sqrt{\dfrac{F}{\mu}} = \dfrac{1}{2L}\sqrt{\dfrac{F}{M/L}} = \dfrac{1}{2}\sqrt{\dfrac{F}{ML}}$. Assuming the maximum length of the string is free to vibrate, the highest fundamental frequency occurs when $F = 900$ N and

$$f_1 = \frac{1}{2}\sqrt{\frac{900\text{ N}}{(4.0\times 10^{-3}\text{ kg})(0.40\text{ m})}} = 375\text{ Hz}.$$

EVALUATE: If the radius was any smaller the breaking stress would be exceeded. If the radius were greater, so the stress was less than the maximum value, then the length would be less to achieve the same total mass.

15.73. IDENTIFY and SET UP: Assume that the mass M is large enough so that there no appreciable motion of the string at the pulley or at the oscillator. For a string fixed at both ends, $\lambda_n = 2L/n$. The node-to-node distance d is $\lambda/2$, so $d = \lambda/2$. $v = f\lambda = \sqrt{F/\mu}$.

EXECUTE: **(a)** Because it is essentially fixed at its ends, the string can vibrate in only wavelengths for which $\lambda_n = 2L/n$, so $d = \lambda/2 = L/n$, where $n = 1, 2, 3, \ldots$.

(b) $f\lambda = \sqrt{F/\mu}$ and $\lambda = 2d$. Combining these two conditions and squaring gives $f^2(4d^2) = T/\mu = Mg/\mu$. Solving for μd^2 gives $\mu d^2 = \left(\dfrac{g}{4f^2}\right)M$. Therefore the graph of μd^2 versus M should be a straight line having slope equal to $g/4f^2$. Figure 15.73 shows this graph.

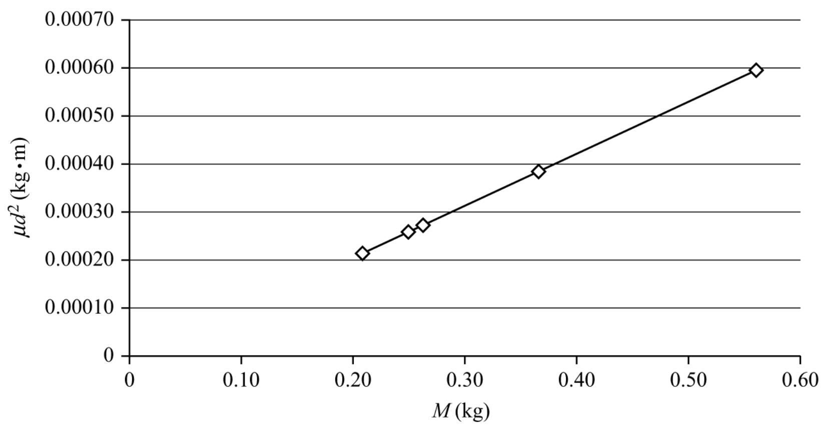


Figure 15.73

(c) The best fit straight line for the data has the equation μd^2 = (0.001088 m)M – 0.00009074 kg·m. The slope is $g/4f^2$, so $g/4f^2$ = 0.001088 m. Solving for f gives f = 47.5 Hz.

(d) For string A, μ = 0.0260 g/cm = 0.00260 kg/m. We want the mass M for λ = 48.0 cm. Using $f\lambda = \sqrt{F/\mu}$ where $F = Mg$, squaring and solving for M, we get $M = \dfrac{\mu(f\lambda)^2}{g}$. Putting in the numbers gives M = (0.00260 kg/m)[(47.5 Hz)(0.480 m)]^2/(9.80 m/s^2) = 0.138 kg = 138 g.

EVALUATE: In part (d), if the string is vibrating in its fundamental mode, n = 1, so $d = L$ = 48.0 cm. The mass of the string in that case would be $m = \mu L$ = (0.00260 kg/m)(0.48 m) = 0.00125 kg = 1.25 g, so the string would be much lighter than the 138-g weight attached to it.

15.74. **IDENTIFY** and **SET UP:** We have a standing wave in its fundamental mode on each string of the guitar. $v = \sqrt{F/\mu}$, $v = f\lambda$, $\lambda_n = 2L/n$, 25.5 in. = 64.77 cm = 0.6477 m.

EXECUTE: **(a)** Each string has the same length, so the wavelength in the fundamental mode is $\lambda_n = 2L/n$ = $2L$ = 2(0.6477 m) = 1.2954 m.

Combining $v = \sqrt{F/\mu}$ and $v = f\lambda$ and solving for μ gives $\mu = F/(f\lambda)^2$.

For the E2 string, we have μ_{E2} = (78.0 N)/[(82.4 Hz)(1.2954 m)]^2 = 0.006846 kg/m = 0.0685 g/cm. Using similar calculations for the other strings, we get μ_{G3} = 0.0121 g/cm and μ_{E4} = 0.00428 g/cm.

(b) From $f\lambda = \sqrt{F/\mu}$, we get $F = \mu(f\lambda)^2$ = (0.006846 kg/m)[(196.0 Hz)(1.2954 m)]^2 = 441 N.

EVALUATE: A tension of 441 N is nearly 100 lb, which is why a string will snap violently if it happens to break.

15.75. **IDENTIFY** and **SET UP:** $P_{av} = \dfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$, $v = \sqrt{F/\mu}$, and $\omega = 2\pi f$.

EXECUTE: Combining $P_{av} = \dfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$ and $\omega = 2\pi f$ gives $P_{av} = 2\pi^2\sqrt{\mu F}\,A^2 f^2$. Therefore a graph of P_{av} versus f^2 should be a straight line having slope $2\pi^2\sqrt{\mu F}\,A^2$.

(b) Calculate the slope from the graph shown in the problem. Estimating the points (40,000 Hz^2, 12.2 W) and (10,000 Hz^2, 3 W), we get a slope of (9.2 W)/(30,000 Hz^2) = 3.07×10^{-4} W/Hz^2. (Answers here will vary, depending on the accuracy in reading the graph.) We can get F from the slope of the graph and then use $v = \sqrt{F/\mu}$ to calculate v. Using our measured slope, we have $2\pi^2\sqrt{\mu F}\,A^2$ = slope = 3.07×10^{-4} W/Hz^2. Solving for F gives $F = \dfrac{(\text{slope})^2}{4\pi^4 A^4 \mu}$. Putting this result into $v = \sqrt{F/\mu}$ gives us

$v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{(\text{slope})^2}{4\pi^4 A^4 \mu^2}} = \dfrac{\text{slope}}{2\pi^2 A^2 \mu}$. This gives $v = (3.07\times 10^{-4}\ \text{W/Hz}^2)/[2\pi^2(0.0040\ \text{m})^2(0.0035\ \text{kg/m})] =$ 280 m/s.

(c) From the graph, $P = 10.0$ W corresponds to $f^2 = 33{,}000\ \text{Hz}^2$, so

$\omega = 2\pi f = 2\pi\sqrt{33{,}000\ \text{Hz}^2} = 1100\ \text{rad/s}.$

EVALUATE: At 280 m/s and with an angular frequency of 1100 rad/s, the string is moving too fast to follow individual waves.

15.76. IDENTIFY: The membrane is stretched and caused to vibrate in a standing wave pattern. We can visualize this membrane as n tiny strings, each of length L, vibrating together.

SET UP: We want to investigate the characteristics of its vibrational motion. We use $v = \sqrt{\dfrac{F}{\mu}}$, $v = f\lambda$, $k = 2\pi/\lambda$, $P = \sqrt{\mu F}\,\omega^2 A^2 \sin^2(kx - \omega t)$, and $P_{\text{av}} = \dfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$.

EXECUTE: **(a)** The tension F is equally divided among the n tiny strings, as is the mass M. Therefore the speed of the wave on each string is $v = \sqrt{\dfrac{F/n}{\mu/n}} = \sqrt{\dfrac{F}{\mu}}$. Using $\mu = M/L$, we get $v = \sqrt{\dfrac{FL}{M}}$. For the values here, we have $v = \sqrt{\dfrac{(81.0\ \text{N})(4.00\ \text{m})}{4.00\ \text{kg}}} = 9.00\ \text{m/s}$.

(b) We want k, so first find λ. Solving $v = f\lambda$ for λ gives $\lambda = v/f$. Therefore we get

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{v/f} = \dfrac{2\pi f}{v} = \dfrac{2\pi(1.00\ \text{Hz})}{9.00\ \text{m/s}} = 0.698\ \text{m}^{-1}.$

(c) $P = \sqrt{\mu F}\,\omega^2 A^2 \sin^2(kx - \omega t) = \sqrt{(M/L)F}\,(2\pi f)^2 A^2 \sin^2(kx - \omega t)$, which we can write as

$P = \sqrt{\dfrac{MF}{L}}\,4\pi^2 f^2 A^2 \sin^2(kx - \omega t).$

(d) $E = P_{\text{av}} t = \dfrac{1}{2}\sqrt{\dfrac{MF}{L}}\,4\pi^2 f^2 A^2 t = 2\pi^2 f^2 A^2 \sqrt{\dfrac{MF}{L}}\,t.$ For the numbers here, we get

$E = 2\pi^2(1.0\ \text{Hz})^2(0.100\ \text{m})^2\sqrt{\dfrac{(4.00\ \text{kg})(81.0\ \text{N})}{4.00\ \text{m}}}(1.00\ \text{s}) = 1.78\ \text{J}.$

EVALUATE: Think of the membrane as a very wide string.

15.77. IDENTIFY: Apply $\Sigma F_y = 0$ to segments of the cable. The forces are the weight of the diver, the weight of the segment of the cable, the tension in the cable and the buoyant force on the segment of the cable and on the diver.

SET UP: The buoyant force on an object of volume V that is completely submerged in water is $B = \rho_{\text{water}} V g$.

EXECUTE: **(a)** The tension is the difference between the diver's weight and the buoyant force,

$F = (m - \rho_{\text{water}} V)g = \left[120\ \text{kg} - (1000\ \text{kg/m}^3)(0.0800\ \text{m}^3)\right](9.80\ \text{m/s}^2) = 392\ \text{N}.$

(b) The increase in tension will be the weight of the cable between the diver and the point at x, minus the buoyant force. This increase in tension is then

$\left[\mu x - \rho(Ax)\right]g = \left[1.10\ \text{kg/m} - (1000\ \text{kg/m}^3)\pi(1.00\times 10^{-2}\ \text{m})^2\right](9.80\ \text{m/s}^2)x = (7.70\ \text{N/m})x.$ The tension as a function of x is then $F(x) = (392\ \text{N}) + (7.70\ \text{N/m})x.$

(c) Denote the tension as $F(x)=F_0+ax$, where $F_0=392$ N and $a=7.70$ N/m. Then the speed of transverse waves as a function of x is $v=\dfrac{dx}{dt}=\sqrt{(F_0+ax)/\mu}$ and the time t needed for a wave to reach the surface is found from $t=\int dt=\int \dfrac{dx}{dx/dt}=\int \dfrac{\sqrt{\mu}}{\sqrt{F_0+ax}}\,dx.$

Let the length of the cable be L, so $t=\sqrt{\mu}\int_0^L \dfrac{dx}{\sqrt{F_0+ax}}=\sqrt{\mu}\dfrac{2}{a}\sqrt{F_0+ax}\Big|_0^L=\dfrac{2\sqrt{\mu}}{a}\left(\sqrt{F_0+aL}-\sqrt{F_0}\right).$

$$t=\frac{2\sqrt{1.10\text{ kg/m}}}{7.70\text{ N/m}}(\sqrt{392\text{ N}+(7.70\text{ N/m})(100\text{ m})}-\sqrt{392\text{ N}})=3.89\text{ s}.$$

EVALUATE: If the weight of the cable and the buoyant force on the cable are neglected, then the tension would have the constant value calculated in part (a). Then $v=\sqrt{\dfrac{F}{\mu}}=\sqrt{\dfrac{392\text{ N}}{1.10\text{ kg/m}}}=18.9\text{ m/s}$ and $t=\dfrac{L}{v}=5.29$ s. The weight of the cable increases the tension along the cable and the time is reduced from this value.

15.78. IDENTIFY and **SET UP:** Apply $v=f\lambda$.

EXECUTE: $v=f\lambda$ gives (125 Hz) λ = 3.75 m/s, so λ = 0.030 m = 3 cm, which is choice (d).

EVALUATE: These are waves on the vocal cords, not sound waves in air.

15.79. IDENTIFY and **SET UP:** Using the figure shown with the problem in the text, the wave is traveling in the +z-direction, so it must be of the form $A\sin(kz-\omega t)$.

EXECUTE: The required wave is of the form $A\sin(kz-\omega t)$. Putting this in terms of f and v gives

$$A\sin(kz-\omega t)=\sin[-(\omega t-kz)]=\sin[-(2\pi ft-2\pi z/\lambda)]=\sin[-2\pi f(t-z/f\lambda)]=-\sin[2\pi f(t-z/v)],$$

which is of the form of choice (b).

EVALUATE: A wave traveling in the opposite direction would be of the form $\sin[2\pi f(t+z/v)]$.

15.80. IDENTIFY and **SET UP:** The graph of v versus f is a straight line that appears to pass through the origin, so v is directly proportional to v. $v=f\lambda$.

EXECUTE: $v=f\lambda$ gives λ = v/f. From the graph of v versus f, v is proportional to f, so $v = Kf$, where K is a constant. Thus λ = v/f = Kf/f = K = constant, which is choice (c).

EVALUATE: Normally we expect the wavelength to decrease as the frequency increases, but that is only true if the wave speed is consant. In this case the speed depends on the frequency, so it is possible for the wavelength to remain constant.

16

SOUND AND HEARING

VP16.9.1. **IDENTIFY:** We want to know the pressure amplitude of a sound wave.

SET UP: $I = \dfrac{p_{\max}^2}{2\rho v}$

EXECUTE: **(a)** Solve for $p_{\max}$ giving $p_{\max} = \sqrt{2\rho v I}$. Putting in the numbers we get $p_{\max} = \sqrt{2(1.20 \text{ kg/m}^3)(344 \text{ m/s})(5.50\times10^{-8} \text{ W/m}^2)} = 6.74\times10^{-3}$ Pa.

(b) Neither ρ nor v is affected by the frequency change, so $p_{\max}$ remain unchanged.

EVALUATE: Changing the air density would affect the pressure amplitude.

VP16.9.2. **IDENTIFY:** This problem deals with sound intensity and intensity level.

SET UP: $\beta = (10 \text{ dB}) \log\dfrac{I}{I_0}$

EXECUTE: **(a)** We know the sound level is 85.0 dB and want the sound intensity. Using $\beta = (10 \text{ dB}) \log\dfrac{I}{I_0}$ gives $85.0\,\text{dB} = (10 \text{ dB}) \log\dfrac{I}{I_0}$. Solving for I gives

$I = 10^{8.5} I_0 = 10^{8.5}10^{-12} \text{ W/m}^2 = 10^{-3.50} \text{ W/m}^2 = 3.16\times10^{-4} \text{ W/m}^2$.

(b) Solve $\beta = (10 \text{ dB}) \log\dfrac{I}{I_0}$ for I, giving $I = I_0 10^{\beta/(10 \text{ dB})}$. Now take the ratio of the two intensities.

$\dfrac{I_{85}}{I_{67}} = \dfrac{I_0 10^{(85.0\,\text{dB})/(10 \text{ dB})}}{I_0 10^{(67.0\,\text{dB})/(10 \text{ dB})}} = \dfrac{10^{8.50}}{10^{6.70}} = 63.1$, so I_{85} is 63.1 times greater than I_{67}.

EVALUATE: The ratio of the sound intensity levels is 85/67 = 1.27, but the intensity ratio is *much* greater.

VP16.9.3. **IDENTIFY:** This problem involves sound intensity and intensity level.

SET UP: We know the sound intensity level of the lion's roar is 114 at 1.00 m, and we want to know what it is at 4.00 m and 15.8 m from the lion. The sound intensity obeys an inverse-square law, but the sound intensity level does not. We use $\beta = (10 \text{ dB}) \log\dfrac{I}{I_0}$ and $I_2 = I_1\left(\dfrac{r_1}{r_2}\right)^2$.

EXECUTE: **(a)** <u>At 1.00 m</u>: $\beta_1 = (10 \text{ dB}) \log\dfrac{I_1}{I_0} = 114$ dB. Solve for I_1: $I_1 = 10^{11.4} I_0$.

<u>At 4.00 m</u>: $I_2 = I_1\left(\dfrac{r_1}{r_2}\right)^2$ gives $I_4 = I_1\left(\dfrac{1.00 \text{ m}}{4.00 \text{ m}}\right)^2 = \dfrac{I_1}{16.0} = \dfrac{10^{11.4} I_0}{16.0}$. Now find the sound intensity level:

$\beta_4 = (10 \text{ dB}) \log\dfrac{I_4}{I_0} = (10 \text{ dB}) \log\left(\dfrac{10^{11.4} I_0}{16.0 I_0}\right) = 102$ dB.

(b) At 15.8 m we follow the same procedure as above, giving

$$\beta_{15.8} = (10\text{ dB})\log\frac{I_{16}}{I_0} = (10\text{ dB})\log\left(\frac{10^{11.4}I_0}{(15.8)^2 I_0}\right) = 90.0\text{ dB}.$$

EVALUATE: The sound intensity obeys an inverse-square law, but sound intensity level does not.

VP16.9.4. **IDENTIFY:** We are investigating the relationship between sound intensity level and the pressure amplitude of a sound wave.

SET UP: We know that $I = \dfrac{p_{\max}^2}{2\rho v}$, $\beta = (10\text{ dB})\log\dfrac{I}{I_0}$, and $I_0 = 10^{-12}$ W/m^2. We want to find the pressure amplitude of the sound wave and then see how changing it would affect the sound intensity level.

EXECUTE: **(a)** First use $\beta = (10\text{ dB})\log\dfrac{I}{I_0}$ to find the intensity I. $\beta = (10\text{ dB})\log\dfrac{I}{I_0}$ = 66.0 dB, so

$I = 10^{6.60} I_0 = 3.9811\times10^{-6}$ W/m^2. Now use I to find $p_{\max}$. Solving $I = \dfrac{p_{\max}^2}{2\rho v}$ for $p_{\max}$ gives

$p_{\max} = \sqrt{2\rho v I}$, which gives $p_{\max} = \sqrt{2(0.920\text{ kg/m}^3)(344\text{ m/s})(3.9811\times10^{-6}\text{ W/m}^2)} = 5.02\times10^{-2}$ Pa.

(b) Since $I \propto p_{\max}^2$, increasing $p_{\max}$ by a factor of 10.0 will increase I by a factor of $10.0^2 = 100$, so $I_2 = 100I_1$. Taking the ratio of the sound intensity levels gives

$\dfrac{\beta_2}{\beta_1} = \dfrac{(10\text{ dB})\log(I_2/I_0)}{(10\text{ dB})\log(I_1/I_0)} = \dfrac{\log(100I_1/I_0)}{\log(I_1/I_0)} = \dfrac{2+\log(I_1/I_0)}{\log(I_1/I_0)}$. But $\beta_1 = (10\text{ dB})\log(I_1/I_0)$, so

$\log(I_1/I_0) = \dfrac{\beta_1}{10} = \dfrac{66.0\text{ dB}}{10} = 6.60$. Therefore $\dfrac{\beta_2}{\beta_1} = \dfrac{2+\dfrac{\beta_1}{10}}{\dfrac{\beta_1}{10}} = \dfrac{2+6.60}{6.60} = 1.303$. So we have

$\beta_2 = 1.303\beta_1 = (1.303)(66.0\text{ dB}) = 86.0$ dB.

EVALUATE: Careful in handling logarithms because $\log(100I_1/I_0)$ is *not* equal to $100\log I_1/I_0$.

VP16.12.1. **IDENTIFY:** We are dealing with standing sound waves in open and stopped pipes.

SET UP: For an open pipe, $f_n = nv/2L$ (n = 1, 2, 3, …), and for a stopped pipe $f_n = nv/4L$ (n = 1, 3, 5, …).

EXECUTE: **(a)** We want the length L of this open pipe. For the fundamental frequency, $f_1 = v/2L$, so the length is L = (344 m/s)/[2(220 Hz)] = 0.782 m.

(b) For the open pipe in its 3rd harmonic, n = 3, so $f_3 = \dfrac{3v}{2L_3}$.

For the stopped pipe in its fundamental frequency, $f_1 = \dfrac{v}{4L_1}$.

The two frequencies are the same, so $\dfrac{3v}{2L_3} = \dfrac{v}{4L_1}$, so $L_1 = L_3/6$ = (0.782 m)/6 = 0.130 m.

EVALUATE: Careful on stopped pipes: n must be an *odd* integer, but on open pipes n can be odd and even integers.

VP16.12.2. **IDENTIFY:** We are investigating the harmonics of a stopped and an open organ pipe. We want to find what harmonics of a stopped pipe will resonate at the same frequency as the third harmonic of an open pipe.

SET UP: For an open pipe $f_n = nv/2L$ (n = 1, 2, 3, …), and for a stopped pipe $f_n = nv/4L$ (n = 1, 3, 5, …). We know that $f_3 = 3v/2L_o$ for the open pipe. We want to find values of n for the stopped pipe so that it will have the same frequency as the third harmonic of the open pipe. That is f_n(stopped) = f_3(open), where we want to find n for the stopped pipe. Equating the frequencies gives $\frac{nv}{4L_s} = \frac{3v}{2L_o}$, so $n = 6\frac{L_s}{L_o}$.

EXECUTE: **(a)** In this case $L_s = L_o/6$ so $n = 6\frac{L_s}{L_o} = 6\left(\frac{L_o}{6L_o}\right) = 1$.

(b) In this case, $L_s = L_o/2$, so $n = 6\frac{L_s}{L_o} = 6\left(\frac{L_o}{2L_o}\right) = 3$.

(c) In this case, $L_s = L_o/3$, so $n = 6\frac{L_s}{L_o} = 6\left(\frac{L_o}{3L_o}\right) = 2$. But n is only *odd* for a stopped pipe, so there are no harmonics of the stopped pipe having a frequency that is equal to the 3rd harmonic frequency of the open pipe.

EVALUATE: We cannot always make two pipes resonate at a given frequency.

VP16.12.3. **IDENTIFY:** A string vibrating in its fundamental frequency causes a nearby stopped organ pipe to vibrate in its fundamental frequency. So we are dealing with standing waves on a string and in a stopped pipe.

SET UP: For a string fixed at its ends, $f_n = nv_{str}/2L$ and $\lambda_n = 2L/n$ (n = 1, 2, 3, …), and for a stopped pipe $f_n = nv/4L$ (n = 1, 3, 5, …). For any wave $v = f\lambda$.

EXECUTE: **(a)** For the fundamental mode of the string, $\lambda_1 = 2L$ so $v_{str} = f\lambda = f(2L) =$ (165 Hz)(2)(0.680 m) = 224 m/s.

(b) The organ pipe in its fundamental mode is vibrating at the same frequency as the string. Using $f_n = nv/4L$ and solving for L gives $L = \frac{v}{4f_1} = \frac{344 \text{ m/s}}{4(165 \text{ Hz})} = 0.521$ m.

EVALUATE: The frequency of the sound wave is the same as the frequency at which the string is vibrating, but the wavelengths of the two waves are *not* the same because they have different speeds.

VP16.12.4. **IDENTIFY:** A stopped pipe 1.00 m long that is filled with helium is vibrating in its third harmonic, which causes a nearby open pipe to vibrate in its fifth harmonic.

SET UP: We want to know the frequency and wavelength of the sound in both pipes and the length of the open pipe. The speed of sound in the helium is v_{He} = 999 m/s, $v = f\lambda$, $\lambda_n = 4L/n$ (n = 1, 3, 4, …) for a stopped pipe, and $f_n = nv/2L$ (n = 1, 2, 3 …) for an open pipe.

EXECUTE: **(a)** In the stopped pipe, $\lambda_3 = 4L/3$ = 4(1.00 m)/3 = 1.33 m.

Using $v = f\lambda$ gives $f_3 = \frac{v_{He}}{\lambda_3} = \frac{999 \text{ m/s}}{1.33 \text{ m}} =$ 749 Hz.

(b) In the open pipe, we know that f_5 = 749 Hz.

Using $v = f\lambda$ gives $\lambda_5 = \frac{v_{air}}{f_5} = \frac{344 \text{ m/s}}{749 \text{ Hz}} =$ 0.459 m.

(c) Using $f_n = nv/2L$ in the open pipe gives $f_5 = \frac{5v_{air}}{2L}$. Solving for L gives

$$L = \frac{5v_{air}}{2f_5} = \frac{5(344 \text{ m/s})}{2(749 \text{ Hz})} = 1.15 \text{ m}.$$

EVALUATE: The speed of sound is different in the helium than it is in air because the density of the helium is not the same as that of air.

VP16.18.1. **IDENTIFY:** You do not hear the same frequency that the siren is emitting due to the motion of the ambulance. This is due to the Doppler effect.

SET UP: We have a moving source of sound and a stationary listener. The target variables are the frequency and wavelength of the sound heard by the listener, so use $f_{\text{L}} = \left(\dfrac{v+v_{\text{L}}}{v+v_{\text{S}}}\right) f_{\text{S}}$ and $v = f\lambda$.

EXECUTE: **(a)** Listener in front of source: v_{S} = –26.0 m/s. $f_{\text{L}} = \left(\dfrac{v+v_{\text{L}}}{v+v_{\text{S}}}\right) f_{\text{S}}$ gives

$$f_{\text{L}} = \left(\frac{340\text{ m/s} + 0}{340\text{ m/s} - 26.0\text{ m/s}}\right)(2.80\times10^3\text{ Hz}) = 3.03\times10^3\text{ Hz}.$$

The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{340\text{ m/s}}{3.03\times10^3\text{ Hz}} = 0.112\text{ m}.$

(b) Listener behind source: v_{S} = +26.0 m/s. $f_{\text{L}} = \left(\dfrac{340\text{ m/s} + 0}{340\text{ m/s} + 26.0\text{ m/s}}\right)(2.80\times10^3\text{ Hz}) = 2.60\times10^3\text{ Hz}.$

The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{340\text{ m/s}}{2.60\times10^3\text{ Hz}} = 0.131\text{ m}.$

EVALUATE: When the source moves toward the listener, the wave crests are closer together so the frequency is increased. When the source moves away from the listener, the distance between wave crests is greater so the frequency is decreased.

VP16.18.2. **IDENTIFY:** The bike rider hears a sound different from what the bagpiper is emitting due to the bike's motion. This is due to the Doppler effect.

SET UP: The bagpiper is the stationary source and the bike rider is the moving observer. We want the frequency and wavelength of the sound the bike rider hears, so we use $v = f\lambda$ and $f_{\text{L}} = \left(\dfrac{v+v_{\text{L}}}{v+v_{\text{S}}}\right) f_{\text{S}}$ with $v_{\text{S}} = 0$.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{340\text{ m/s}}{440\text{ Hz}} = 0.773\text{ m}.$

(b) <u>Biker approaching source</u>: v_{L} = +10.0 m/s. Using $f_{\text{L}} = \left(\dfrac{v+v_{\text{L}}}{v+v_{\text{S}}}\right) f_{\text{S}}$ gives

$$f_{\text{L}} = \left(\frac{340\text{ m/s} + 10.0\text{ m/s}}{340\text{ m/s} + 0}\right)(440\text{ Hz}) = 453\text{ Hz}.$$

Now use $v = f\lambda$, but v is the speed of sound relative to the listener, which is 340 m/s + 10.0 m/s = 350 m/s. Therefore $\lambda = \dfrac{v}{f} = \dfrac{350\text{ m/s}}{453\text{ Hz}} = 0.773\text{ m}.$

<u>Biker moving away from source</u>: v_{L} = –10.0 m/s. Now we get

$$f_{\text{L}} = \left(\frac{340\text{ m/s} - 10.0\text{ m/s}}{340\text{ m/s}}\right)(440\text{ Hz}) = 427\text{ Hz}.$$

Now use $v = f\lambda$, where v = 340 m/s – 10.0 m/s = 330 m/s. $\lambda = \dfrac{v}{f} = \dfrac{330\text{ m/s}}{453\text{ Hz}} = 0.773\text{ m}.$

EVALUATE: The frequency varies due to the speed of the listener because she runs into the waves at a higher (or lower) rate due to her motion. But the wavelength is just the distance between wave crests, so it is not affected by her motion and we get the same answer in both cases.

VP16.18.3. **IDENTIFY:** The police car and the sports car are both moving, so both of their motions affect the frequency that the listener receives. We need to use the Doppler effect.

SET UP: Our target variable is the frequency f_{L} of sound that the listener receives. We use $f_{\text{L}} = \left(\dfrac{v+v_{\text{L}}}{v+v_{\text{S}}}\right) f_{\text{S}}$. Figure VP16.18.3 shows both situations.

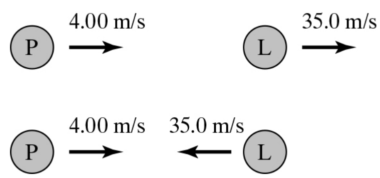


Figure VP16.18.3

EXECUTE: **(a)** The police car is moving in the direction of the listener and the listener is moving away from the police car, so $v_S = -40.0$ m/s and $v_L = -35.0$ m/s. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ gives

$$f_L = \left(\frac{340\text{ m/s} - 35.0\text{ m/s}}{340\text{ m/s} - 40.0\text{ m/s}}\right)(1200\text{ Hz}) = 1220\text{ Hz}.$$

(b) The police car is moving in the direction of the listener and the listener is in the direction of the police car, so $v_S = -40.0$ m/s and $v_L = +35.0$ m/s. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ gives

$$f_L = \left(\frac{340\text{ m/s} + 35.0\text{ m/s}}{340\text{ m/s} - 40.0\text{ m/s}}\right)(1200\text{ Hz}) = 1500\text{ Hz}.$$

EVALUATE: In part (a), the motion of the police car causes a higher frequency but the motion of the sports car causes a lower frequency, so there is a small change in the frequency heard by the listener compared to the emitted frequency. In part (b) both of their motions increase the frequency, so there is a large frequency change.

VP16.18.4. **IDENTIFY:** The motion of the car increases the sound frequency in front of it, and this motion also increases the frequency of the sound the driver hears reflected from the wall. There are two Doppler effects to consider.

SET UP: We need to break this problem into two steps: (1) the car is a moving source and the wall is a stationary listener and (2) the wall is a stationary source and the car is a moving listener. The wall reflects the same frequency it receives from the car. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. In part (a) we want to find the frequency that the driver receives reflected from the wall. In part (b) we want to find how fast he most drive to hear sound of frequency 495 Hz coming from the wall.

EXECUTE: **(a)** Car as moving source and wall as stationary listener: $v_L = 0$ (the wall), $v_S = -25.0$ m/s. Use $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ to find the frequency f_w that the wall receives.

$$f_w = \left(\frac{340\text{ m/s}}{340\text{ m/s} - 25.0\text{ m/s}}\right)(415\text{ Hz}) = 447.94\text{ Hz}.$$

Car as moving listener and wall as stationary source: $v_S = 0$ (the wall), $v_L = +25.0$ m/s, and $f_S = 447.94$ Hz. Use $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ to find the frequency f_d that the driver receives coming back from the wall.

$$f_d = \left(\frac{340\text{ m/s} + 25.0\text{ m/s}}{340\text{ m/s}}\right)(447.94\text{ Hz}) = 481\text{ Hz}.$$

(b) We want to find v_S so that $f_c = 495$ Hz. We follow the same steps as in part (a).

Car as listener and wall as source: $v_L = v_c = ?$ (where v_c is the *magnitude* of the car's speed), $f_S = f_w = ?$, $f_L = f_c = 495$ Hz, $v_S = 0$ (wall). Using $f_L = \left(\frac{v+v_L}{v+v_S}\right) f_S$ gives $495\text{ Hz} = \left(\frac{340\text{ m/s} + v_c}{340\text{ m/s}}\right) f_w$, so $f_w = \frac{(495\text{ Hz})(340\text{ m/s})}{340\text{ m/s} + v_c}$.

Car as source and wall as listener: $v_S = -v_c$, $v_L = 0$, $f_S = 415$ Hz, $f_L = f_w = ?$ so $f_L = \left(\frac{v+v_L}{v+v_S}\right) f_S$ gives $f_w = \left(\frac{340\text{ m/s}}{340\text{ m/s} - v_c}\right)(415\text{ Hz})$. Now use the result for f_w that we just found in the previous step.

Equating these two equations for f_w and solving for v_c gives us $\frac{(495\text{ Hz})(340\text{ m/s})}{340\text{ m/s} + v_c} = \left(\frac{340\text{ m/s}}{340\text{ m/s} - v_c}\right)(415\text{ Hz})$, so $v_c = 29.9$ m/s.

EVALUATE: Our result in part (b) gives $v_c = 29.9$ m/s which is greater than the speed of 25.0 m/s in part (a). This result is reasonable because the frequency change in (b) is greater than the one in (a).

16.1. **IDENTIFY** and **SET UP:** $v = f\lambda$ gives the wavelength in terms of the frequency. Use $p_{\text{max}} = BkA$ to relate the pressure and displacement amplitudes.

EXECUTE: **(a)** $\lambda = v/f = (344\text{ m/s})/1000\text{ Hz} = 0.344\text{ m}$.

(b) $p_{\text{max}} = BkA$ and Bk is constant gives $p_{\text{max}1}/A_1 = p_{\text{max}2}/A_2$

$$A_2 = A_1\left(\frac{p_{\text{max}2}}{p_{\text{max}1}}\right) = 1.2\times10^{-8}\text{ m}\left(\frac{30\text{ Pa}}{3.0\times10^{-2}\text{ Pa}}\right) = 1.2\times10^{-5}\text{ m}.$$

(c) $p_{\text{max}} = BkA = 2\pi BA/\lambda$

$$p_{\text{max}}\lambda = 2\pi BA = \text{constant so } p_{\text{max}1}\lambda_1 = p_{\text{max}2}\lambda_2 \text{ and } \lambda_2 = \lambda_1\left(\frac{p_{\text{max}1}}{p_{\text{max}2}}\right) = (0.344\text{ m})\left(\frac{3.0\times10^{-2}\text{ Pa}}{1.5\times10^{-3}\text{ Pa}}\right)$$

$= 6.9$ m

$f = v/\lambda = (344\text{ m/s})/6.9\text{ m} = 50\text{ Hz}$.

EVALUATE: The pressure amplitude and displacement amplitude are directly proportional. For the same displacement amplitude, the pressure amplitude decreases when the frequency decreases and the wavelength increases.

16.2. **IDENTIFY:** Apply $p_{\text{max}} = BkA$. $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v}$, so $p_{\text{max}} = \frac{2\pi fBA}{v}$.

SET UP: $v = 344$ m/s

EXECUTE: $f = \frac{vp_{\text{max}}}{2\pi BA} = \frac{(344\text{ m/s})(10.0\text{ Pa})}{2\pi(1.42\times10^5\text{ Pa})(1.00\times10^{-6}\text{ m})} = 3.86\times10^3\text{ Hz}$

EVALUATE: Audible frequencies range from about 20 Hz to about 20,000 Hz, so this frequency is audible.

16.3. **IDENTIFY:** Use $p_{\text{max}} = BkA$ to relate the pressure and displacement amplitudes.

SET UP: As stated in Example 16.1 the adiabatic bulk modulus for air is $B = 1.42\times10^5$ Pa. Use $v = f\lambda$ to calculate λ from f, and then $k = 2\pi/\lambda$.

EXECUTE: **(a)** $f = 150$ Hz

Need to calculate k: $\lambda = v/f$ and $k = 2\pi/\lambda$ so $k = 2\pi f/v = (2\pi\text{ rad})(150\text{ Hz})/344\text{ m/s} = 2.74\text{ rad/m}$.

Then $p_{\text{max}} = BkA = (1.42\times10^5\text{ Pa})(2.74\text{ rad/m})(0.0200\times10^{-3}\text{ m}) = 7.78\text{ Pa}$. This is below the pain threshold of 30 Pa.

(b) *f* is larger by a factor of 10 so $k = 2\pi f/v$ is larger by a factor of 10, and $p_{\text{max}} = BkA$ is larger by a factor of 10. $p_{\text{max}} = 77.8\text{ Pa},$ above the pain threshold.

(c) There is again an increase in *f*, *k*, and p_{max} of a factor of 10, so $p_{\text{max}} = 778\text{ Pa},$ far above the pain threshold.

EVALUATE: When *f* increases, λ decreases so *k* increases and the pressure amplitude increases.

16.4. **IDENTIFY** and **SET UP:** Use the relation $v = f\lambda$ to find the wavelength or frequency of various sounds.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{1531\text{ m/s}}{17\text{ Hz}} = 90\text{ m}.$

(b) $f = \dfrac{v}{\lambda} = \dfrac{1531\text{ m/s}}{0.015\text{ m}} = 102\text{ kHz}.$

(c) $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{25\times10^3\text{ Hz}} = 1.4\text{ cm}.$

(d) For $f = 78\text{ kHz},$ $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{78\times10^3\text{ Hz}} = 4.4\text{ mm}.$ For $f = 39\text{ kHz},$ $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{39\times10^3\text{ Hz}} = 8.8\text{ mm}.$

The range of wavelengths is 4.4 mm to 8.8 mm.

(e) $\lambda = 0.25\text{ mm}$ so $f = \dfrac{v}{\lambda} = \dfrac{1550\text{ m/s}}{0.25\times10^{-3}\text{m}} = 6.2\text{ MHz}.$

EVALUATE: Nonaudible (to human) sounds cover a wide range of frequencies and wavelengths.

16.5. **IDENTIFY** and **SET UP:** Use $t = \text{distance/speed}.$ Calculate the time it takes each sound wave to travel the $L = 60.0\text{ m}$ length of the pipe. Use $v = \sqrt{\dfrac{Y}{\rho}}$ to calculate the speed of sound in the brass rod.

EXECUTE: Wave in air: $t = (60.0\text{ m})/(344\text{ m/s}) = 0.1744\text{ s}.$

Wave in the metal: $v = \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{9.0\times10^{10}\text{ Pa}}{8600\text{ kg/m}^3}} = 3235\text{ m/s},$ so $t = \dfrac{60.0\text{ m}}{3235\text{ m/s}} = 0.01855\text{ s}.$

The time interval between the two sounds is $\Delta t = 0.1744\text{ s} - 0.01855\text{ s} = 0.156\text{ s}.$

EVALUATE: The restoring forces that propagate the sound waves are much greater in solid brass than in air, so *v* is much larger in brass.

16.6. **IDENTIFY:** $v = f\lambda.$ Apply $v = \sqrt{\dfrac{B}{\rho}}$ for the waves in the liquid and $v = \sqrt{\dfrac{Y}{\rho}}$ for the waves in the metal bar.

SET UP: In part (b) the wave speed is $v = \dfrac{d}{t} = \dfrac{1.50\text{ m}}{3.90\times10^{-4}\text{ s}}.$

EXECUTE: **(a)** Using $v = \sqrt{\dfrac{B}{\rho}},$ we have $B = v^2\rho = (\lambda f)^2\rho,$ so

$B = [(8\text{ m})(400\text{ Hz})]^2(1300\text{ kg/m}^3) = 1.33\times10^{10}\text{ Pa}.$

(b) Using $v = \sqrt{\dfrac{Y}{\rho}},$ we have

$Y = v^2\rho = (L/t)^2\rho = [(1.50\text{ m})/(3.90\times10^{-4}\text{ s})]^2(6400\text{ kg / m}^3) = 9.47\times10^{10}\text{ Pa}.$

EVALUATE: In the liquid, $v = 3200\text{ m/s}$ and in the metal, $v = 3850\text{ m/s}.$ Both these speeds are much greater than the speed of sound in air.

16.7. **IDENTIFY:** $d = vt$ for the sound waves in air and in water.

SET UP: Use $v_{\text{water}} = 1482\text{ m/s}$ at $20°\text{C},$ as given in Table 16.1. In air, $v = 344\text{ m/s}.$

EXECUTE: Since along the path to the diver the sound travels 1.2 m in air, the sound wave travels in water for the same time as the wave travels a distance $22.0\text{ m} - 1.20\text{ m} = 20.8\text{ m}$ in air. The depth of the diver is $(20.8\text{ m})\dfrac{v_{\text{water}}}{v_{\text{air}}} = (20.8\text{ m})\dfrac{1482\text{ m/s}}{344\text{ m/s}} = 89.6\text{ m}.$ This is the depth of the diver; the distance from the horn is $90.8\text{ m}.$

EVALUATE: The time it takes the sound to travel from the horn to the person on shore is $t_1 = \dfrac{22.0\text{ m}}{344\text{ m/s}} = 0.0640\text{ s}.$ The time it takes the sound to travel from the horn to the diver is $t_2 = \dfrac{1.2\text{ m}}{344\text{ m/s}} + \dfrac{89.6\text{ m}}{1482\text{ m/s}} = 0.0035\text{ s} + 0.0605\text{ s} = 0.0640\text{ s}.$ These times are indeed the same. For three figure accuracy the distance of the horn above the water can't be neglected.

16.8. IDENTIFY: Apply $v = \sqrt{\dfrac{\gamma RT}{M}}$ to each gas.

SET UP: In each case, express M in units of kg/mol. For H_2, $\gamma = 1.41.$ For He and Ar, $\gamma = 1.67.$

EXECUTE: **(a)** $v_{H_2} = \sqrt{\dfrac{(1.41)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(2.02\times10^{-3}\text{ kg/mol})}} = 1.32\times10^3\text{ m/s}$

(b) . $v_{\text{He}} = \sqrt{\dfrac{(1.67)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(4.00\times10^{-3}\text{ kg/mol})}} = 1.02\times10^3\text{ m/s}$.

(c) $v_{\text{Ar}} = \sqrt{\dfrac{(1.67)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(39.9\times10^{-3}\text{ kg/mol})}} = 323\text{ m/s}.$

(d) Repeating the calculation of Example 16.4 at $T = 300.15\text{ K}$ gives $v_{\text{air}} = 348\text{ m/s},$ and so $v_{H_2} = 3.80 v_{\text{air}}$, $v_{\text{He}} = 2.94 v_{\text{air}}$ and $v_{\text{Ar}} = 0.928 v_{\text{air}}$.

EVALUATE: v is larger for gases with smaller M.

16.9. IDENTIFY: $v = f\lambda.$ The relation of v to gas temperature is given by $v = \sqrt{\dfrac{\gamma RT}{M}}.$

SET UP: Let $T = 22.0°\text{C} = 295.15\text{ K}.$

EXECUTE: At $22.0°\text{C}$, $\lambda = \dfrac{v}{f} = \dfrac{325\text{ m/s}}{1250\text{ Hz}} = 0.260\text{ m} = 26.0\text{ cm}.$ $\lambda = \dfrac{v}{f} = \dfrac{1}{f}\sqrt{\dfrac{\gamma RT}{M}}.$ $\dfrac{\lambda}{\sqrt{T}} = \dfrac{1}{f}\sqrt{\dfrac{\gamma R}{M}},$ which is constant, so $\dfrac{\lambda_1}{\sqrt{T_1}} = \dfrac{\lambda_2}{\sqrt{T_2}}.$ $T_2 = T_1\left(\dfrac{\lambda_2}{\lambda_1}\right)^2 = (295.15\text{ K})\left(\dfrac{28.5\text{ cm}}{26.0\text{ cm}}\right)^2 = 354.6\text{ K} = 81.4°\text{C}.$

EVALUATE: When T increases v increases and for fixed f, λ increases. Note that we did not need to know either γ or M for the gas.

16.10. IDENTIFY: $v = \sqrt{\dfrac{\gamma RT}{M}}.$ Take the derivative of v with respect to T. In part (b) replace dv by Δv and dT by ΔT in the expression derived in part (a).

SET UP: $\dfrac{d(x^{1/2})}{dx} = \tfrac{1}{2}x^{-1/2}.$ In $v = \sqrt{\dfrac{\gamma RT}{M}},$ T must be in kelvins. $20°\text{C} = 293\text{ K}.$ $\Delta T = 1\text{ C}° = 1\text{ K}.$

EXECUTE: **(a)** $\dfrac{dv}{dT} = \sqrt{\dfrac{\gamma R}{M}}\dfrac{dT^{1/2}}{dT} = \sqrt{\dfrac{\gamma R}{M}}\tfrac{1}{2}T^{-1/2} = \dfrac{1}{2T}\sqrt{\dfrac{\gamma RT}{M}} = \dfrac{v}{2T}.$ Rearranging gives $\dfrac{dv}{v} = \dfrac{1}{2}\dfrac{dT}{T},$ the desired result.

(b) $\dfrac{\Delta v}{v} = \dfrac{1}{2}\dfrac{\Delta T}{T}.$ $\Delta v = \dfrac{v}{2}\dfrac{\Delta T}{T} = \left(\dfrac{344\text{ m/s}}{2}\right)\left(\dfrac{1\text{ K}}{293\text{ K}}\right) = 0.59\text{ m/s}.$

EVALUATE: Since $\frac{\Delta T}{T} = 3.4\times10^{-3}$ and $\frac{\Delta v}{v}$ is one-half this, replacing dT by ΔT and dv by Δv is accurate. Using the result from part (a) is much simpler than calculating v for $20°\text{C}$ and for $21°\text{C}$ and subtracting, and is not subject to round-off errors.

16.11. IDENTIFY and **SET UP:** Sound delivers energy (and hence power) to the ear. For a whisper, $I = 1\times10^{-10}\ \text{W/m}^2$. The area of the tympanic membrane is $A = \pi r^2$, with $r = 4.2\times10^{-3}\ \text{m}$. Intensity is energy per unit time per unit area.

EXECUTE: **(a)** $E = IAt = (1\times10^{-10}\ \text{W/m}^2)\pi(4.2\times10^{-3}\ \text{m})^2(1\ \text{s}) = 5.5\times10^{-15}\ \text{J}$.

(b) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.5\times10^{-15}\ \text{J})}{2.0\times10^{-6}\ \text{kg}}} = 7.4\times10^{-5}\ \text{m/s} = 0.074\ \text{mm/s}$.

EVALUATE: Compared to the energy of ordinary objects, it takes only a very small amount of energy for hearing. As part (b) shows, a mosquito carries a lot more energy than is needed for hearing.

16.12. IDENTIFY: The sound intensity level decreases by 13.0 dB, and from this we can find the change in the intensity.

SET UP: $\beta = 10\log(I/I_0)$. $\Delta\beta = 13.0$ dB.

EXECUTE: **(a)** $\Delta\beta = \beta_2 - \beta_1 = 10\ \text{dB}\log(I_2/I_0) - 10\ \text{dB}\log(I_1/I_0) = 10\ \text{dB}\log(I_2/I_1) = 13.0\ \text{dB}$, so $1.3 = \log(I_2/I_1)$ which gives $I_2/I_1 = 20.0$.

(b) EVALUATE: According to the equation in part (a) the difference in two sound intensity levels is determined by the ratio of the sound intensities. So you don't need to know I_1, just the ratio I_2/I_1.

16.13. IDENTIFY and **SET UP:** We want the sound intensity level to increase from 20.0 dB to 60.0 dB. The previous problem showed that $\beta_2 - \beta_1 = (10\ \text{dB})\log\left(\frac{I_2}{I_1}\right)$. We also know that $\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$.

EXECUTE: Using $\beta_2 - \beta_1 = (10\ \text{dB})\log\left(\frac{I_2}{I_1}\right)$, we have $\Delta\beta = +40.0\ \text{dB}$. Therefore $\log\left(\frac{I_2}{I_1}\right) = 4.00$, so $\frac{I_2}{I_1} = 1.00\times10^4$. Using $\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$ and solving for r_2, we get $r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (15.0\ \text{m})\sqrt{\frac{1}{1.00\times10^4}} = 15.0\ \text{cm}$.

EVALUATE: A change of 10^2 in distance gives a change of 10^4 in intensity. Our analysis assumes that the sound spreads from the source uniformly in all directions.

16.14. IDENTIFY: This problem deals with sound intensity and intensity level.

SET UP: The sound intensity level is $\beta = (10\ \text{dB})\log\frac{I}{I_0}$ and the intensity is $I = \frac{P}{A} = \frac{P}{4\pi r^2}$. We want to know by what factor the power P should increase so that β will increase by 5.00 dB at 20.0 m from the source. We know that $\beta_2 - \beta_1 = 5.00\ \text{dB}$ and we want to find P_2/P_1.

EXECUTE: $\beta_1 = (10\ \text{dB})\log\frac{I_1}{I_0}$ and $\beta_2 = (10\ \text{dB})\log\frac{I_2}{I_0}$. We know that $\beta_2 - \beta_1 = 5.00\ \text{dB}$, so we have $\beta_2 - \beta_1 = (10\ \text{dB})\log(I_2/I_0) - (10\ \text{dB})\log(I_1/I_0)$. Using the properties of logarithms gives $\beta_2 - \beta_1 = (10\ \text{dB})\left[\log I_2 - \log I_0 - (\log I_1 - \log I_0)\right] = (10\ \text{dB})\log(I_2/I_1)$. Now use $I = \frac{P}{4\pi r^2}$, giving $\beta_2 - \beta_1 = (10\ \text{dB})\log\left(\frac{P_2/4\pi r^2}{P_1/4\pi r^2}\right) = (10\ \text{dB})\log(P_2/P_1)$. This gives us $5.00\ \text{dB} = (10\ \text{dB})\log(P_2/P_1)$, so $P_2/P_1 = 10^{1/2} = \sqrt{10} = 3.16$.

EVALUATE: Our result does not depend on the distance from the source, since the r divided out. Therefore the sound intensity level at all points would increase by 5.00 dB if the power of the source increased by a factor of 3.16.

16.15. IDENTIFY and SET UP: Apply $p_{\max} = BkA$, $I = \frac{1}{2}B\omega kA^2$, and $\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right)$.

EXECUTE: (a) $\omega = 2\pi f = (2\pi\text{ rad})(320\text{ Hz}) = 2011\text{ rad/s}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v} = \frac{2011\text{ rad/s}}{344\text{ m/s}} = 5.84\text{ rad/m}$$

$B = 1.42\times10^5\text{ Pa}$ (Example 16.1)

Then $p_{\max} = BkA = (1.42\times10^5\text{ Pa})(5.84\text{ rad/m})(5.00\times10^{-6}\text{ m}) = 4.14\text{ Pa}$.

(b) Using $I = \frac{1}{2}\omega BkA^2$ gives

$I = \frac{1}{2}(2011\text{ rad/s})(1.42\times10^5\text{ Pa})(5.84\text{ rad/m})(5.00\times10^{-6}\text{ m})^2 = 2.08\times10^{-2}\text{ W/m}^2$.

(c) $\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right)$: $\beta = (10\text{ dB})\log(I/I_0)$, with $I_0 = 1\times10^{-12}\text{ W/m}^2$.

$\beta = (10\text{ dB})\log[(2.08\times10^{-2}\text{ W/m}^2)/(1\times10^{-12}\text{ W/m}^2)] = 103\text{ dB}$.

EVALUATE: Even though the displacement amplitude is very small, this is a very intense sound. Compare the sound intensity level to the values in Table 16.2.

16.16. IDENTIFY: Changing the sound intensity level will decrease the rate at which energy reaches the ear.

SET UP: Example 16.9 shows that $\beta_2 - \beta_1 = (10\text{ dB})\log\left(\frac{I_2}{I_1}\right)$.

EXECUTE: (a) $\Delta\beta = -30\text{ dB}$ so $\log\left(\frac{I_2}{I_1}\right) = -3$ and $\frac{I_2}{I_1} = 10^{-3} = 1/1000$.

(b) $I_2/I_1 = \frac{1}{2}$ so $\Delta\beta = 10\log\left(\frac{1}{2}\right) = -3.0\text{ dB}$.

EVALUATE: Because of the logarithmic relationship between the intensity and intensity level of sound, a small change in the intensity level produces a large change in the intensity.

16.17. IDENTIFY: Use $I = \frac{vp_{\max}^2}{2B}$ to relate I and $p_{\max}$. $\beta = (10\text{ dB})\log(I/I_0)$. The equation $p_{\max} = BkA$ says the pressure amplitude and displacement amplitude are related by $p_{\max} = BkA = B\left(\frac{2\pi f}{v}\right)A$.

SET UP: At $20°\text{C}$ the bulk modulus for air is $1.42\times10^5\text{ Pa}$ and $v = 344\text{ m/s}$. $I_0 = 1\times10^{-12}\text{ W/m}^2$.

EXECUTE: (a) $I = \frac{vp_{\max}^2}{2B} = \frac{(344\text{ m/s})(6.0\times10^{-5}\text{ Pa})^2}{2(1.42\times10^5\text{ Pa})} = 4.4\times10^{-12}\text{ W/m}^2$

(b) $\beta = (10\text{ dB})\log\left(\frac{4.4\times10^{-12}\text{ W/m}^2}{1\times10^{-12}\text{ W/m}^2}\right) = 6.4\text{ dB}$

(c) $A = \frac{vp_{\max}}{2\pi fB} = \frac{(344\text{ m/s})(6.0\times10^{-5}\text{ Pa})}{2\pi(400\text{ Hz})(1.42\times10^5\text{ Pa})} = 5.8\times10^{-11}\text{ m}$

EVALUATE: This is a very faint sound and the displacement and pressure amplitudes are very small. Note that the displacement amplitude depends on the frequency but the pressure amplitude does not.

16.18. IDENTIFY and SET UP: Apply the relation $\beta_2 - \beta_1 = (10\text{ dB})\log(I_2/I_1)$ that is derived in Example 16.9.

EXECUTE: (a) $\Delta\beta = (10\text{ dB})\log\left(\frac{4I}{I}\right) = 6.0\text{ dB}$

(b) The total number of crying babies must be multiplied by four, for an increase of 12 kids.
EVALUATE: For $I_2 = \alpha I_1$, where α is some factor, the increase in sound intensity level is $\Delta\beta = (10\text{ dB})\log\alpha$. For $\alpha = 4$, $\Delta\beta = 6.0$ dB.

16.19. **IDENTIFY** and **SET UP:** Let 1 refer to the mother and 2 to the father. Use the result derived in Example 16.9 for the difference in sound intensity level for the two sounds. Relate intensity to distance from the source using $I_1/I_2 = r_2^2/r_1^2$.
EXECUTE: From Example 16.9, $\beta_2 - \beta_1 = (10\text{ dB})\log(I_2/I_1)$
Using $I_1/I_2 = r_2^2/r_1^2$ gives us
$\Delta\beta = \beta_2 - \beta_1 = (10\text{ dB})\log(I_2/I_1) = (10\text{ dB})\log(r_1/r_2)^2 = (20\text{ dB})\log(r_1/r_2)$
$\Delta\beta = (20\text{ dB})\log(1.50\text{ m}/0.30\text{ m}) = 14.0\text{ dB}.$
EVALUATE: The father is 5 times closer so the intensity at his location is 25 times greater.

16.20. **IDENTIFY:** We must use the relationship between intensity and sound level.
SET UP: Example 16.9 shows that $\beta_2 - \beta_1 = (10\text{ dB})\log\left(\dfrac{I_2}{I_1}\right)$.
EXECUTE: **(a)** $\Delta\beta = 5.00$ dB gives $\log\left(\dfrac{I_2}{I_1}\right) = 0.5$ and $\dfrac{I_2}{I_1} = 10^{0.5} = 3.16.$
(b) $\dfrac{I_2}{I_1} = 100$ gives $\Delta\beta = 10\log(100) = 20$ dB.
(c) $\dfrac{I_2}{I_1} = 2$ gives $\Delta\beta = 10\log 2 = 3.0$ dB.
EVALUATE: Every doubling of the intensity increases the decibel level by 3.0 dB.

16.21. **IDENTIFY:** The intensity of sound obeys an inverse square law.
SET UP: $\dfrac{I_2}{I_1} = \dfrac{r_1^2}{r_2^2}$. $\beta = (10\text{ dB})\log\left(\dfrac{I}{I_0}\right)$, with $I_0 = 1\times10^{-12}\text{ W/m}^2$.
EXECUTE: **(a)** $\beta = 53$ dB gives $5.3 = \log\left(\dfrac{I}{I_0}\right)$ and $I = (10^{5.3})I_0 = 2.0\times10^{-7}\text{ W/m}^2$.
(b) $r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (3.0\text{ m})\sqrt{\dfrac{4}{1}} = 6.0$ m.
(c) $\beta = \dfrac{53\text{ dB}}{4} = 13.25$ dB gives $1.325 = \log\left(\dfrac{I}{I_0}\right)$ and $I = 2.1\times10^{-11}\text{ W/m}^2$.
$r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (3.0\text{ m})\sqrt{\dfrac{2.0\times10^{-7}\text{ W/m}^2}{2.1\times10^{-11}\text{ W/m}^2}} = 290\text{ m}.$
EVALUATE: **(d)** Intensity obeys the inverse square law but noise level does not.

16.22. **IDENTIFY:** We are looking at the standing wave pattern in a pipe. The pattern has displacement antinodes at both ends of the pipe.
SET UP: For an open pipe, $\lambda_n = 2L/n$, and for any wave $v = f\lambda$. Table 16.1 tells us that at 20°C v_{air} = 344 m/s and v_{He} = 999 m/s.
EXECUTE: **(a)** The fact that displacement nodes occur at both ends of the pipe tells us that it must be an open pipe.
(b) The pattern shows 5 nodes, so it is the 5th harmonic.
(c) $\lambda_n = 2L/n$, so $\lambda_5 = 2L/5$, which gives $L = 5\lambda_5/2$. Now get λ_5 which we can use to find L.
$\lambda_5 = v/f_5 = (344\text{ m/s})/(1710\text{ Hz}) = 0.2012$ m. Therefore $L = 5\lambda_5/2 = 5(0.2012\text{ m})/2 = 0.503$ m.

(d) L remains the same, so $\lambda_1 = 2L$. $f_1\lambda_1 = v_{\text{He}}$, so $f_1 = \dfrac{v_{\text{He}}}{\lambda_1} = \dfrac{v_{\text{He}}}{2L} = \dfrac{999\text{ m/s}}{2(0.503\text{ m})} = 993\text{ Hz}.$

EVALUATE: Replacing the air with helium changes the harmonic frequencies. But it does not change the wavelengths because they are determined by the length of the pipe.

16.23. **IDENTIFY** and **SET UP:** An open end is a displacement antinode and a closed end is a displacement node. Sketch the standing wave pattern and use the sketch to relate the node-to-antinode distance to the length of the pipe. A displacement node is a pressure antinode and a displacement antinode is a pressure node.

EXECUTE: **(a)** The placement of the displacement nodes and antinodes along the pipe is as sketched in Figure 16.23a. The open ends are displacement antinodes.

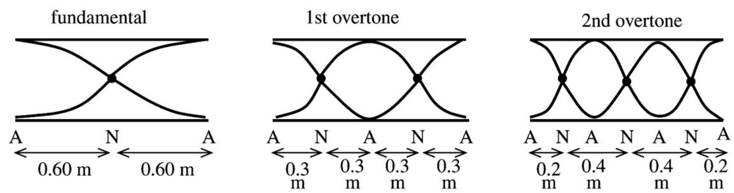


Figure 16.23a

Location of the displacement nodes (N) measured from the left end:
fundamental 0.60 m
1st overtone 0.30 m, 0.90 m
2nd overtone 0.20 m, 0.60 m, 1.00 m

Location of the pressure nodes (displacement antinodes (A)) measured from the left end:
fundamental 0, 1.20 m
1st overtone 0, 0.60 m, 1.20 m
2nd overtone 0, 0.40 m, 0.80 m, 1.20 m

(b) The open end is a displacement antinode and the closed end is a displacement node. The placement of the displacement nodes and antinodes along the pipe is sketched in Figure 16.23b.

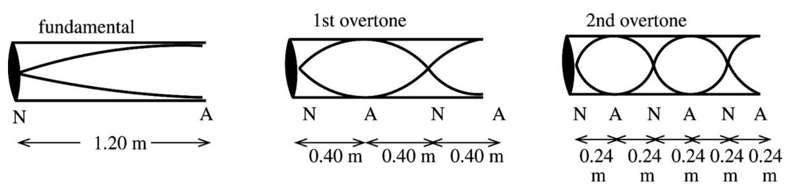


Figure 16.23b

Location of the displacement nodes (N) measured from the closed end:
fundamental 0
1st overtone 0, 0.80 m
2nd overtone 0, 0.48 m, 0.96 m

Location of the pressure nodes (displacement antinodes (A)) measured from the closed end:
fundamental 1.20 m
1st overtone 0.40 m, 1.20 m
2nd overtone 0.24 m, 0.72 m, 1.20 m

EVALUATE: The node-to-node or antinode-to-antinode distance is $\lambda/2$. For the higher overtones the frequency is higher and the wavelength is smaller.

16.24. **IDENTIFY:** For an open pipe, $f_1 = \frac{v}{2L}$. For a stopped pipe, $f_1 = \frac{v}{4L}$. $v = f\lambda$.

SET UP: $v = 344$ m/s. For a pipe, there must be a displacement node at a closed end and an antinode at the open end.

EXECUTE: **(a)** $L = \frac{v}{2f_1} = \frac{344 \text{ m/s}}{2(524 \text{ Hz})} = 0.328$ m.

(b) There is a node at one end, an antinode at the other end and no other nodes or antinodes in between, so $\frac{\lambda_1}{4} = L$ and $\lambda_1 = 4L = 4(0.328 \text{ m}) = 1.31$ m.

(c) $f_1 = \frac{v}{4L} = \frac{1}{2}\left(\frac{v}{2L}\right) = \frac{1}{2}(524 \text{ Hz}) = 262$ Hz.

EVALUATE: We could also calculate f_1 for the stopped pipe as $f_1 = \frac{v}{\lambda_1} = \frac{344 \text{ m/s}}{1.31 \text{ m}} = 262$ Hz, which agrees with our result in part (c).

16.25. **IDENTIFY:** For a stopped pipe, the standing wave frequencies are given by $f_n = nv/4L$.
SET UP: The first three standing wave frequencies correspond to $n = 1$, 3, and 5.

EXECUTE: $f_1 = \frac{(344 \text{ m/s})}{4(0.17 \text{ m})} = 506$ Hz, $f_3 = 3f_1 = 1517$ Hz, $f_5 = 5f_1 = 2529$ Hz.

EVALUATE: All three of these frequencies are in the audible range, which is about 20 Hz to 20,000 Hz.

16.26. **IDENTIFY:** The vocal tract is modeled as a stopped pipe, open at one end and closed at the other end, so we know the wavelength of standing waves in the tract.

SET UP: For a stopped pipe, $\lambda_n = 4L/n$ $(n = 1, 3, 5, \ldots)$ and $v = f\lambda$, so $f_1 = \frac{v}{4L}$ with $f_1 = 220$ Hz.

EXECUTE: $L = \frac{v}{4f_1} = \frac{344 \text{ m/s}}{4(220 \text{ Hz})} = 39.1$ cm. This result is a reasonable value for the mouth to diaphragm distance for a typical adult.
EVALUATE: 1244 Hz is not an integer multiple of the fundamental frequency of 220 Hz; it is 5.65 times the fundamental. The production of sung notes is more complicated than harmonics of an air column of fixed length.

16.27. **IDENTIFY:** A pipe open at one end and closed at the other is a stopped pipe.
SET UP: For an open pipe, the fundamental is $f_1 = v/2L$, and for a stopped pipe, it is $f_1 = v/4L$.

EXECUTE: **(a)** For an open pipe, $f_1 = \frac{v}{2L} = \frac{344 \text{ m/s}}{2(4.88 \text{ m})} = 35.2$ Hz.

(b) For a stopped pipe, $f_1 = \frac{v}{4L} = \frac{35.2 \text{ Hz}}{2} = 17.6$ Hz.

EVALUATE: Even though the pipes both have the same length, their fundamental frequencies are very different, depending on whether they are open or closed at their ends.

16.28. **IDENTIFY:** There must be a node at each end of the pipe. For the fundamental there are no additional nodes and each successive overtone has one additional node. $v = f\lambda$.

SET UP: $v = 344$ m/s. The node to node distance is $\lambda/2$.

EXECUTE: **(a)** $\frac{\lambda_1}{2} = L$ so $\lambda_1 = 2L$. Each successive overtone adds an additional $\lambda/2$ along the pipe, so $n\left(\frac{\lambda_n}{2}\right) = L$ and $\lambda_n = \frac{2L}{n}$, where $n = 1, 2, 3, \ldots$ $f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$.

(b) $f_1 = \dfrac{v}{2L} = \dfrac{344\text{ m/s}}{2(2.50\text{ m})} = 68.8\text{ Hz}.$ $f_2 = 2f_1 = 138\text{ Hz}.$ $f_3 = 3f_1 = 206\text{ Hz}.$ All three of these frequencies are audible.

EVALUATE: A pipe of length L closed at both ends has the same standing wave wavelengths, frequencies and nodal patterns as for a string of length L that is fixed at both ends.

16.29. **IDENTIFY:** We are looking at the standing wave pattern in a pipe. The pattern has a displacement node at one end and a displacement antinode at the other end of the pipe.

SET UP: For a stopped pipe $\lambda_n = 4L/n$ (n an odd integer). $v = f\lambda$

EXECUTE: **(a)** With a displacement node at one end and an antinode at the other, this must be a stopped pipe.

(b) The n^{th} harmonic has $\dfrac{n+1}{2}$ nodes, so for 5 nodes, $5 = \dfrac{n+1}{2}$, which gives $n = 5$. This pipe is resonating in its 5^{th} harmonic.

(c) Using $v = f\lambda$ gives $\lambda_9 = \dfrac{v}{f_9} = \dfrac{344\text{ m/s}}{1710\text{ Hz}} = 0.20117\text{ m}.$ Using $\lambda_n = 4L/n$ gives

$$L = \frac{n\lambda_n}{4} = \frac{9(0.20117\text{ m})}{4} = 0.453\text{ m}.$$

(d) $f_9 = 9f_1$, so $f_1 = (1710\text{ Hz})/9 = 190\text{ Hz}$.

EVALUATE: A stopped pipe does not have even harmonics because it has a displacement node at the closed end and an antinode at the open end.

16.30. **IDENTIFY:** The wire will vibrate in its second overtone with frequency f_3^{wire} when $f_3^{\text{wire}} = f_1^{\text{pipe}}$. For a stopped pipe, $f_1^{\text{pipe}} = \dfrac{v}{4L_{\text{pipe}}}$. The second overtone standing wave frequency for a wire fixed at both ends is $f_3^{\text{wire}} = 3\left(\dfrac{v_{\text{wire}}}{2L_{\text{wire}}}\right)$. $v_{\text{wire}} = \sqrt{F/\mu}$.

SET UP: The wire has $\mu = \dfrac{m}{L_{\text{wire}}} = \dfrac{7.25\times10^{-3}\text{ kg}}{0.620\text{ m}} = 1.169\times10^{-2}\text{ kg/m}$. The speed of sound in air is $v = 344\text{ m/s}$.

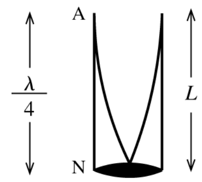
EXECUTE: $v_{\text{wire}} = \sqrt{\dfrac{4110\text{ N}}{1.169\times10^{-2}\text{ kg/m}}} = 592.85\text{ m/s}.$ $f_3^{\text{wire}} = f_1^{\text{pipe}}$ gives $3\dfrac{v_{\text{wire}}}{2L_{\text{wire}}} = \dfrac{v}{4L_{\text{pipe}}}$.

$$L_{\text{pipe}} = \frac{2L_{\text{wire}}v}{12v_{\text{wire}}} = \frac{2(0.620\text{ m})(344\text{ m/s})}{12(592.85\text{ m/s})} = 0.0600\text{ m} = 6.00\text{ cm}.$$

EVALUATE: The fundamental for the pipe has the same frequency as the third harmonic of the wire. But the wave speeds for the two objects are different and the two standing waves have different wavelengths.

16.31. **IDENTIFY** and **SET UP:** Use the standing wave pattern to relate the wavelength of the standing wave to the length of the air column and then use $v = f\lambda$ to calculate f. There is a displacement antinode at the top (open) end of the air column and a node at the bottom (closed) end, as shown in Figure 16.31.

EXECUTE: **(a)**



$$\lambda / 4 = L$$
$$\lambda = 4L = 4(0.140 \text{ m}) = 0.560 \text{ m}$$
$$f = \frac{v}{\lambda} = \frac{344 \text{ m/s}}{0.560 \text{ m}} = 614 \text{ Hz}$$

Figure 16.31

(b) Now the length L of the air column becomes $\frac{1}{2}(0.140 \text{ m}) = 0.070 \text{ m}$ and $\lambda = 4L = 0.280 \text{ m}$.

$$f = \frac{v}{\lambda} = \frac{344 \text{ m/s}}{0.280 \text{ m}} = 1230 \text{ Hz}$$

EVALUATE: Smaller L means smaller λ which in turn corresponds to larger *f*.

16.32. **IDENTIFY** and **SET UP:** The path difference for the two sources is *d*. For destructive interference, the path difference is a half-integer number of wavelengths. For constructive interference, the path difference is an integer number of wavelengths. $v = f\lambda$.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{344 \text{ m/s}}{725 \text{ Hz}} = 0.474 \text{ m}$. Destructive interference will first occur when $d = \lambda/2 = 0.237 \text{ m}$.

(b) Destructive interference will next occur when $d = 3\lambda/2 = 0.711 \text{ m}$.

(c) Constructive interference will first occur when $d = \lambda = 0.474 \text{ m}$.

EVALUATE: Constructive interference should first occur midway between the first two points where destructive interference occurs. This midpoint is (0.237 m + 0.711 m)/2 = 0.474 m, which is just what we found in part (c).

16.33. **(a) IDENTIFY** and **SET UP:** Path difference from points *A* and *B* to point *Q* is $3.00 \text{ m} - 1.00 \text{ m} = 2.00 \text{ m}$, as shown in Figure 16.33. Constructive interference implies path difference $= n\lambda$, $n = 1, 2, 3, \ldots$

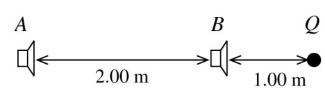


Figure 16.33

EXECUTE: $2.00 \text{ m} = n\lambda$ so $\lambda = 2.00 \text{ m}/n$

$$f = \frac{v}{\lambda} = \frac{nv}{2.00 \text{ m}} = \frac{n(344 \text{ m/s})}{2.00 \text{ m}} = n(172 \text{ Hz}), \quad n = 1, 2, 3, \ldots$$

The lowest frequency for which constructive interference occurs is 172 Hz.

(b) IDENTIFY and **SET UP:** Destructive interference implies path difference $= (n/2)\lambda$, $n = 1, 3, 5, \ldots$

EXECUTE: $2.00 \text{ m} = (n/2)\lambda$ so $\lambda = 4.00 \text{ m}/n$

$$f = \frac{v}{\lambda} = \frac{nv}{4.00 \text{ m}} = \frac{n(344 \text{ m/s})}{(4.00 \text{ m})} = n(86 \text{ Hz}), \quad n = 1, 3, 5, \ldots.$$

The lowest frequency for which destructive interference occurs is 86 Hz.

EVALUATE: As the frequency is slowly increased, the intensity at *Q* will fluctuate, as the interference changes between destructive and constructive.

16.34. **IDENTIFY:** Constructive interference occurs when the difference of the distances of each source from point *P* is an integer number of wavelengths. The interference is destructive when this difference of path lengths is a half integer number of wavelengths.

SET UP: The wavelength is $\lambda = v/f = (344\text{ m/s})/(206\text{ Hz}) = 1.67\text{ m}$. Since P is between the speakers, x must be in the range 0 to L, where $L = 2.00\text{ m}$ is the distance between the speakers.
EXECUTE: The difference in path length is $\Delta l = (L - x) - x = L - 2x$, or $x = (L - \Delta l)/2$. For destructive interference, $\Delta l = (n + (1/2))\lambda$, and for constructive interference, $\Delta l = n\lambda$.
(a) Destructive interference: $n = 0$ gives $\Delta l = 0.835\text{ m}$ and $x = 0.58\text{ m}$. $n = -1$ gives $\Delta l = -0.835\text{ m}$ and $x = 1.42\text{ m}$. No other values of n place P between the speakers.
(b) Constructive interference: $n = 0$ gives $\Delta l = 0$ and $x = 1.00\text{ m}$. $n = 1$ gives $\Delta l = 1.67\text{ m}$ and $x = 0.17\text{ m}$. $n = -1$ gives $\Delta l = -1.67\text{ m}$ and $x = 1.83\text{ m}$. No other values of n place P between the speakers.
(c) Treating the speakers as point sources is a poor approximation for these dimensions, and sound reaches these points after reflecting from the walls, ceiling and floor.
EVALUATE: Points of constructive interference are a distance $\lambda/2$ apart, and the same is true for the points of destructive interference.

16.35. **IDENTIFY:** For constructive interference the path difference is an integer number of wavelengths and for destructive interference the path difference is a half-integer number of wavelengths.
SET UP: $\lambda = v/f = (344\text{ m/s})/(688\text{ Hz}) = 0.500\text{ m}$
EXECUTE: To move from constructive interference to destructive interference, the path difference must change by $\lambda/2$. If you move a distance x toward speaker B, the distance to B gets shorter by x and the distance to A gets longer by x so the path difference changes by $2x$. $2x = \lambda/2$ and $x = \lambda/4 = 0.125\text{ m}$.
EVALUATE: If you walk an additional distance of 0.125 m farther, the interference again becomes constructive.

16.36. **IDENTIFY:** Destructive interference occurs when the path difference is a half integer number of wavelengths.
SET UP: $v = 344\text{ m/s}$, so $\lambda = v/f = (344\text{ m/s})/(172\text{ Hz}) = 2.00\text{ m}$. If $r_A = 8.00\text{ m}$ and r_B are the distances of the person from each is $r_B - r_A = \left(n + \tfrac{1}{2}\right)\lambda$, where n is any integer.
EXECUTE: Requiring $r_B = r_A + \left(n + \tfrac{1}{2}\right)\lambda > 0a$ gives $n + \tfrac{1}{2} > -r_A/\lambda = 0 - (8.00\text{ m})/(2.00\text{ m}) = -4$, so the smallest value of r_B occurs when $n = -4$, and the closest distance to B is
$r_B = 8.00\text{ m} + \left(-4 + \tfrac{1}{2}\right)(2.00\text{ m}) = 1.00\text{ m}$.
EVALUATE: For $r_B = 1.00\text{ m}$, the path difference is $r_A - r_B = 7.00\text{ m}$. This is 3.5λ.

16.37. **IDENTIFY:** For constructive interference, the path difference is an integer number of wavelengths. For destructive interference, the path difference is a half-integer number of wavelengths.
SET UP: One speaker is 4.50 m from the microphone and the other is 4.92 m from the microphone, so the path difference is 0.42 m. $f = v/\lambda$.
EXECUTE: **(a)** $\lambda = 0.42\text{ m}$ gives $f = \dfrac{v}{\lambda} = 820\text{ Hz}$; $2\lambda = 0.42\text{ m}$ gives $\lambda = 0.21\text{ m}$ and $f = \dfrac{v}{\lambda} = 1640\text{ Hz}$; $3\lambda = 0.42\text{ m}$ gives $\lambda = 0.14\text{ m}$ and $f = \dfrac{v}{\lambda} = 2460\text{ Hz}$, and so on. The frequencies for constructive interference are $n(820\text{ Hz})$, $n = 1, 2, 3, \ldots$.
(b) $\lambda/2 = 0.42\text{ m}$ gives $\lambda = 0.84\text{ m}$ and $f = \dfrac{v}{\lambda} = 410\text{ Hz}$; $3\lambda/2 = 0.42\text{ m}$ gives $\lambda = 0.28\text{ m}$ and $f = \dfrac{v}{\lambda} = 1230\text{ Hz}$; $5\lambda/2 = 0.42\text{ m}$ gives $\lambda = 0.168\text{ m}$ and $f = \dfrac{v}{\lambda} = 2050\text{ Hz}$, and so on. The frequencies for destructive interference are $(2n+1)(410\text{ Hz})$, $n = 0, 1, 2, \ldots$.
EVALUATE: The frequencies for constructive interference lie midway between the frequencies for destructive interference.

16.38. **IDENTIFY:** $f_{\text{beat}} = |f_1 - f_2|$. $v = f\lambda$.

SET UP: $v = 344$ m/s. Let $\lambda_1 = 64.8$ cm and $\lambda_2 = 65.2$ cm. $\lambda_2 > \lambda_1$ so $f_1 > f_2$.

EXECUTE: $f_1 - f_2 = v\left(\dfrac{1}{\lambda_1} - \dfrac{1}{\lambda_2}\right) = \dfrac{v(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2} = \dfrac{(344\text{ m/s})(0.04\times10^{-2}\text{ m})}{(0.648\text{ m})(0.652\text{ m})} = 0.33$ beats/s, which rounds to 0.3 beats/s.

EVALUATE: We could have calculated f_1 and f_2 and subtracted, but doing it this way we would have to be careful to retain enough figures in intermediate calculations to avoid round-off errors.

16.39. **IDENTIFY:** The beat is due to a difference in the frequencies of the two sounds.

SET UP: $f_{\text{beat}} = f_1 - f_2$. Tightening the string increases the wave speed for transverse waves on the string and this in turn increases the frequency.

EXECUTE: **(a)** If the beat frequency increases when she raises her frequency by tightening the string, it must be that her frequency is 433 Hz, 3 Hz above concert *A*.

(b) She needs to lower her frequency by loosening her string.

EVALUATE: The beat would only be audible if the two sounds are quite close in frequency. A musician with a good sense of pitch can come very close to the correct frequency just from hearing the tone.

16.40. **IDENTIFY:** $f_{\text{beat}} = |f_a - f_b|$. For a stopped pipe, $f_1 = \dfrac{v}{4L}$.

SET UP: $v = 344$ m/s. Let $L_a = 1.14$ m and $L_b = 1.16$ m. $L_b > L_a$ so $f_{1a} > f_{1b}$.

EXECUTE: $f_{1a} - f_{1b} = \dfrac{v}{4}\left(\dfrac{1}{L_a} - \dfrac{1}{L_b}\right) = \dfrac{v(L_b - L_a)}{4L_aL_b} = \dfrac{(344\text{ m/s})(2.00\times10^{-2}\text{ m})}{4(1.14\text{ m})(1.16\text{ m})} = 1.3$ Hz. There are 1.3 beats per second.

EVALUATE: Increasing the length of the pipe increases the wavelength of the fundamental and decreases the frequency.

16.41. **IDENTIFY:** Apply the Doppler shift equation $f_{\text{L}} = \left(\dfrac{v + v_{\text{L}}}{v + v_{\text{S}}}\right)f_{\text{S}}$.

SET UP: The positive direction is from listener to source. $f_{\text{S}} = 1200$ Hz. $f_{\text{L}} = 1240$ Hz.

EXECUTE: $v_{\text{L}} = 0$. $v_{\text{S}} = -25.0$ m/s. $f_{\text{L}} = \left(\dfrac{v}{v + v_{\text{S}}}\right)f_{\text{S}}$ gives

$$v = \frac{v_{\text{S}}f_{\text{L}}}{f_{\text{S}} - f_{\text{L}}} = \frac{(-25\text{ m/s})(1240\text{ Hz})}{1200\text{ Hz} - 1240\text{ Hz}} = 780\text{ m/s}.$$

EVALUATE: $f_{\text{L}} > f_{\text{S}}$ since the source is approaching the listener.

16.42. **IDENTIFY** and **SET UP:** Apply $\lambda = \dfrac{v - v_{\text{S}}}{f_{\text{S}}}$ and $\lambda = \dfrac{v + v_{\text{S}}}{f_{\text{S}}}$ for the wavelengths in front of and behind the source. Then $f = v/\lambda$. When the source is at rest $\lambda = \dfrac{v}{f_{\text{S}}} = \dfrac{344\text{ m/s}}{400\text{ Hz}} = 0.860$ m.

EXECUTE: **(a)** $\lambda = \dfrac{v - v_{\text{S}}}{f_{\text{S}}} = \dfrac{344\text{ m/s} - 25.0\text{ m/s}}{400\text{ Hz}} = 0.798$ m

(b) $\lambda = \dfrac{v + v_{\text{S}}}{f_{\text{S}}} = \dfrac{344\text{ m/s} + 25.0\text{ m/s}}{400\text{ Hz}} = 0.922$ m

(c) $f_{\text{L}} = v/\lambda$ (since $v_{\text{L}} = 0$), so $f_{\text{L}} = (344\text{ m/s})/0.798\text{ m} = 431$ Hz

(d) $f_{\text{L}} = v/\lambda = (344\text{ m/s})/0.922\text{ m} = 373$ Hz

EVALUATE: In front of the source (source moving toward listener) the wavelength is decreased and the frequency is increased. Behind the source (source moving away from listener) the wavelength is increased and the frequency is decreased.

16.43. IDENTIFY: Apply the Doppler shift equation $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S$.

SET UP: The positive direction is from listener to source. $f_S = 392$ Hz.

EXECUTE: **(a)** $v_S = 0$. $v_L = -15.0$ m/s. $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{344 \text{ m/s} - 15.0 \text{ m/s}}{344 \text{ m/s}}\right)(392 \text{ Hz}) = 375 \text{ Hz}$

(b) $v_S = +35.0$ m/s. $v_L = +15.0$ m/s. $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{344 \text{ m/s} + 15.0 \text{ m/s}}{344 \text{ m/s} + 35.0 \text{ m/s}}\right)(392 \text{ Hz}) = 371 \text{ Hz}$

(c) $f_{beat} = f_1 - f_2 = 4$ Hz

EVALUATE: The distance between whistle A and the listener is increasing, and for whistle A $f_L < f_S$. The distance between whistle B and the listener is also increasing, and for whistle B $f_L < f_S$.

16.44. IDENTIFY: Apply $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S$.

SET UP: $f_S = 1000$ Hz. The positive direction is from the listener to the source. $v = 344$ m/s.

EXECUTE: **(a)** $v_S = -(344 \text{ m/s})/2 = -172$ m/s, $v_L = 0$.

$$f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{344 \text{ m/s}}{344 \text{ m/s} - 172 \text{ m/s}}\right)(1000 \text{ Hz}) = 2000 \text{ Hz}$$

(b) $v_S = 0$, $v_L = +172$ m/s. $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{344 \text{ m/s} + 172 \text{ m/s}}{344 \text{ m/s}}\right)(1000 \text{ Hz}) = 1500 \text{ Hz}$

EVALUATE: The answer in (b) is much less than the answer in (a). It is the velocity of the source and listener relative to the air that determines the effect, not the relative velocity of the source and listener relative to each other.

16.45. IDENTIFY: The distance between crests is λ. In front of the source $\lambda = \frac{v - v_S}{f_S}$ and behind the source $\lambda = \frac{v + v_S}{f_S}$. $f_S = 1/T$.

SET UP: $T = 1.6$ s. $v = 0.32$ m/s. The crest to crest distance is the wavelength, so $\lambda = 0.12$ m.

EXECUTE: **(a)** $f_S = 1/T = 0.625$ Hz. $\lambda = \frac{v - v_S}{f_S}$ gives

$v_S = v - \lambda f_S = 0.32 \text{ m/s} - (0.12 \text{ m})(0.625 \text{ Hz}) = 0.25 \text{ m/s}$.

(b) $\lambda = \frac{v + v_S}{f_S} = \frac{0.32 \text{ m/s} + 0.25 \text{ m/s}}{0.625 \text{ Hz}} = 0.91 \text{ m}$

EVALUATE: If the duck was held at rest but still paddled its feet, it would produce waves of wavelength $\lambda = \frac{0.32 \text{ m/s}}{0.625 \text{ Hz}} = 0.51$ m. In front of the duck the wavelength is decreased and behind the duck the wavelength is increased. The speed of the duck is 78% of the wave speed, so the Doppler effects are large.

16.46. **IDENTIFY:** Apply the Doppler effect formula $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S$.

(a) SET UP: The positive direction is from the listener toward the source, as shown in Figure 16.46a.

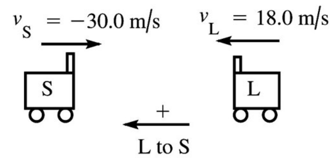


Figure 16.46a

EXECUTE: $f_L = \left(\frac{v+v_L}{v+v_S}\right) f_S = \left(\frac{344\text{ m/s}+18.0\text{ m/s}}{344\text{ m/s}-30.0\text{ m/s}}\right)(352\text{ Hz}) = 406\text{ Hz}.$

EVALUATE: Listener and source are approaching and $f_L > f_S$.

(b) SET UP: See Figure 16.46b.

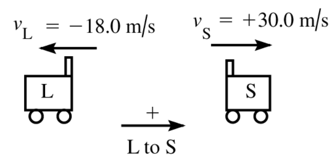


Figure 16.46b

EXECUTE: $f_L = \left(\frac{v+v_L}{v+v_S}\right) f_S = \left(\frac{344\text{ m/s}-18.0\text{ m/s}}{344\text{ m/s}+30.3\text{ m/s}}\right)(352\text{ Hz}) = 307\text{ Hz}.$

EVALUATE: Listener and source are moving away from each other and $f_L < f_S$.

16.47. **IDENTIFY:** Apply $f_L = \left(\frac{v+v_L}{v+v_S}\right) f_S$.

SET UP: The positive direction is from the motorcycle toward the car. The car is stationary, so $v_S = 0$.

EXECUTE: $f_L = \frac{v+v_L}{v+v_S} f_S = (1 + v_L/v) f_S$, which gives

$v_L = v\left(\frac{f_L}{f_S} - 1\right) = (344\text{ m/s})\left(\frac{490\text{ Hz}}{520\text{ Hz}} - 1\right) = -19.8\text{ m/s}.$ You must be traveling at 19.8 m/s.

EVALUATE: $v_L < 0$ means that the listener is moving away from the source.

16.48. **IDENTIFY:** We have a moving source and a stationary observer. The beat frequency is due to interference between the Doppler-shifted sound from the horn in the moving car and the horn in the stationary car. The beat frequency is equal to the difference between these two frequencies.

SET UP: Apply $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S = \left(\frac{v}{v + v_S}\right) f_S$, where $f_S = 260$ Hz. Since the source is moving toward you, you will hear the moving car horn at a higher pitch than your horn, and the beat frequency will be given by $f_{beat} = f_L - f_S$.

EXECUTE: We can determine f_L from the beat frequency: $f_{beat} = 6.0 \text{ Hz} = f_L - f_S = f_L - 260 \text{ Hz}$; thus, $f_L = 266 \text{ Hz}$. Assuming that $v = 344 \text{ m/s}$, we obtain $266 \text{ Hz} = \left(\dfrac{344 \text{ m/s}}{344 \text{ m/s} + v_S}\right)(260 \text{ Hz})$. Solving for v_S we obtain $v_S = \left(\dfrac{260 \text{ Hz}}{266 \text{ Hz}} - 1\right)(344 \text{ m/s}) = -7.8 \text{ m/s}$. Thus, your friend is moving at 7.8 m/s toward you.

EVALUATE: What frequency will your friend hear? In this case, you have a stationary source (your horn) and a moving observer (your friend). The positive direction points from listener to source. Thus, we have $f_L = \left(\dfrac{344 \text{ m/s} + 7.8 \text{ m/s}}{344 \text{ m/s}}\right)(260 \text{ Hz}) = 265.8 \text{ Hz} \approx 266 \text{ Hz}$. At low speeds, there is little difference in the Doppler shift of a moving source or that of a moving observer.

16.49. **IDENTIFY:** The source of sound is moving and so is the listener, so we are dealing with the Doppler effect.

SET UP: We use $v = f\lambda$, where v is the speed relative to the listener, and $f_L = \left(\dfrac{v + v_L}{v + v_S}\right)f_S$.

EXECUTE: **(a)** You and the sound waves are moving toward each other, so the speed of sound that you measure is $v = 340 \text{ m/s} + 15 \text{ m/s} = 355 \text{ m/s}$.
(b) The wavelength of the sound that you receive is the same as the wavelength that the police car would measure behind the car. The speed of those waves relative to the police car is $v = 340 \text{ m/s} + 15 \text{ m/s} = 355 \text{ m/s}$ because the sound and police car are moving away from each other. The wavelength of those waves is $\lambda = \dfrac{v}{f} = \dfrac{355 \text{ m/s}}{500 \text{ Hz}} = 0.710 \text{ m}$, which is also the wavelength you observe.
(c) The speed of the sound waves relative to you is $v = 340 \text{ m/s} + 15 \text{ m/s} = 355 \text{ m/s}$ since you are moving toward the waves, and the wavelength you measure is 0.710 m. So the frequency you measure is $f = \dfrac{v}{\lambda} = \dfrac{355 \text{ m/s}}{0.710 \text{ m}} = 500 \text{ Hz}$. This is the same frequency the police siren is emitting.

EVALUATE: We can check using the Doppler effect formula $f_L = \left(\dfrac{v + v_L}{v + v_S}\right)f_S$, where $v_L = +15 \text{ m/s}$ and $v_S = +15 \text{ m/s}$. Doing so gives $f_L = f_S = 500 \text{ Hz}$.

16.50. **IDENTIFY:** There is a Doppler shift due to the motion of the fire engine as well as due to the motion of the truck, which reflects the sound waves.

SET UP: We use the Doppler shift equation $f_L = \left(\dfrac{v + v_L}{v + v_S}\right)f_S$.

EXECUTE: **(a)** First consider the truck as the listener, as shown in Figure 16.50(a).

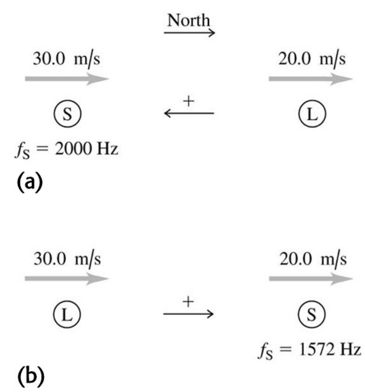


Figure 16.50

$f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{344\text{ m/s} - 20.0\text{ m/s}}{344\text{ m/s} - 30.0\text{ m/s}}\right)(2000\text{ Hz}) = 2064\text{ Hz}.$ Now consider the truck as a source, with $f_S = 2064\text{ Hz},$ and the fire engine driver as the listener, as shown in Figure 16.50(b). $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{344\text{ m/s} + 30.0\text{ m/s}}{344\text{ m/s} + 20.0\text{ m/s}}\right)(2064\text{ Hz}) = 2120\text{ Hz}.$ The objects are getting closer together so the frequency is increased.

(b) The driver detects a frequency of 2120 Hz and the waves returning from the truck move past him at 344 m/s + 30.0 m/s, so the wavelength he measures is $\lambda = \frac{344\text{ m/s} + 30\text{ m/s}}{2120\text{ Hz}} = 0.176\text{ m}.$ The wavelength of waves emitted by the fire engine when it is stationary is $\lambda = \frac{344\text{ m/s}}{2000\text{ Hz}} = 0.172\text{ m}.$

EVALUATE: In (a) the objects are getting closer together so the frequency is increased. In (b), the quantities to use in the equation $v = f\lambda$ are measured *relative to the observer*.

16.51. **IDENTIFY:** Apply the Doppler shift formulas. We first treat the stationary police car as the source and then as the observer as he receives his own sound reflected from the on-coming car.

SET UP: $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S.$

EXECUTE: (a) Since the frequency is increased the moving car must be approaching the police car. Let v_c be the speed of the moving car. The speed v_p of the police car is zero. First consider the moving car as the listener, as shown in Figure 16.51(a).

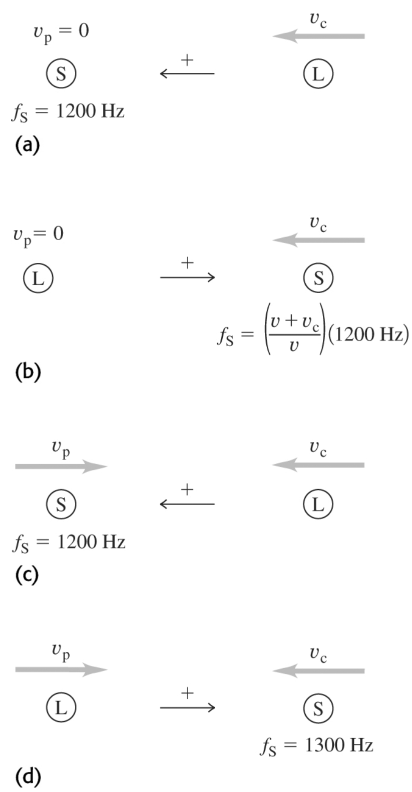


Figure 16.51

$$f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{v+v_c}{v}\right)(1200\ \text{Hz})$$

Then consider the moving car as the source and the police car as the listener, as shown in Figure 16.51(b):

$$f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S \text{ gives } 1250\ \text{Hz} = \left(\frac{v}{v-v_c}\right)\left(\frac{v+v_c}{v}\right)(1200\ \text{Hz}).$$

Solving for v_c gives

$$v_c = \left(\frac{50}{2450}\right)v = \left(\frac{50}{2450}\right)(344\ \text{m/s}) = 7.02\ \text{m/s}$$

(**b**) Repeat the calculation of part (a), but now $v_p = 20.0\ \text{m/s},$ toward the other car.

Waves received by the car (Figure 16.51(c)):

$$f_L = \left(\frac{v+v_c}{v-v_p}\right)f_S = \left(\frac{344\ \text{m/s}+7\ \text{m/s}}{344\ \text{m/s}-20\ \text{m/s}}\right)(1200\ \text{Hz}) = 1300\ \text{Hz}$$

Waves reflected by the car and received by the police car (Figure 16.51(d)):

$$f_L = \left(\frac{v+v_p}{v-v_c}\right)f_S = \left(\frac{344\ \text{m/s}+20\ \text{m/s}}{344\ \text{m/s}-7\ \text{m/s}}\right)(1300\ \text{Hz}) = 1404\ \text{Hz}$$

EVALUATE: The cars move toward each other with a greater relative speed in (b) and the increase in frequency is much larger there.

16.52. **IDENTIFY:** A sound detector is moving toward a stationary source, so we are dealing with the Doppler effect.

SET UP: The graph we must interpret plots f_L versus v_L, so we need to find a relationship between those quantities to be able to interpret the graph. We use $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$.

EXECUTE: Solve $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ for f_L explicitly in terms of v_L, giving $f_L = \dfrac{f_S v_L}{v+v_S} + \dfrac{v f_S}{v+v_S}$. From this equation, we see that the graph should be a straight line having slope $\dfrac{f_S}{v+v_S}$ and y-intercept $\dfrac{v f_S}{v+v_S}$.

From this we see that (y-intercept) = (slope)v. Solving for v gives

$v = \dfrac{y\text{-intercept}}{\text{slope}} = \dfrac{600.0\ \text{Hz}}{1.75\ \text{m}^{-1}} = 3.43\,\text{m/s}.$

EVALUATE: This result is very close to the value v = 344 m/s in Table 16.1.

16.53. **IDENTIFY:** Apply $\sin\alpha = v/v_S$ to calculate α. Use the method of Example 16.19 to calculate t.

SET UP: Mach 1.70 means $v_S/v = 1.70$.

EXECUTE: **(a)** In $\sin\alpha = v/v_S$, $v/v_S = 1/1.70 = 0.588$ and $\alpha = \arcsin(0.588) = 36.0°$.

(b) As in Example 16.19, $t = \dfrac{1250\ \text{m}}{(1.70)(344\ \text{m/s})(\tan\ 36.0°)} = 2.94\ \text{s}.$

EVALUATE: The angle α decreases when the speed v_S of the plane increases.

16.54. **IDENTIFY:** Apply $\sin\alpha = v/v_S$.

SET UP: The Mach number is the value of v_S/v, where v_S is the speed of the shuttle and v is the speed of sound at the altitude of the shuttle.

EXECUTE: **(a)** $\dfrac{v}{v_S} = \sin\alpha = \sin 58.0° = 0.848$. The Mach number is $\dfrac{v_S}{v} = \dfrac{1}{0.848} = 1.18.$

(b) $v_S = \dfrac{v}{\sin\alpha} = \dfrac{331\ \text{m/s}}{\sin 58.0°} = 390\ \text{m/s}$

(c) $\dfrac{v_S}{v} = \dfrac{390\ \text{m/s}}{344\ \text{m/s}} = 1.13$. The Mach number would be 1.13. $\sin\alpha = \dfrac{v}{v_S} = \dfrac{344\ \text{m/s}}{390\ \text{m/s}}$ and $\alpha = 61.9°$.

EVALUATE: The smaller the Mach number, the larger the angle of the shock-wave cone.

16.55. **IDENTIFY:** This problem involves the speed of sound in a gas.

SET UP: Eq. 16.10: $v = \sqrt{\dfrac{\gamma RT}{M}}$. Information from Example 16.4: M = 0.0288 kg/mol, γ = 1.40. If T is in K, $T_C = T - 273$, so $T = T_C + 273$. We want to determine the speed of sound at 20°C.

EXECUTE: **(a)** $v = \sqrt{\dfrac{\gamma RT}{M}} = \sqrt{\dfrac{\gamma R(T_C+273)}{M}} = \sqrt{\dfrac{\gamma R}{M}}\sqrt{T_C+273} = \sqrt{\dfrac{273\gamma R}{M}}\sqrt{\dfrac{T_C}{273}+1}$. If T_C is much less than 273°C, $T_C/273 \ll 1$, so we can use the approximation $\sqrt{1+x} \approx 1+\dfrac{x}{2}$, which gives

$v \approx \sqrt{\dfrac{273\gamma R}{M}}\left[1+\dfrac{T_C}{2(273)}\right] = \sqrt{\dfrac{273\gamma R}{M}}\left[1+\dfrac{T_C}{546}\right]$. Putting in M = 0.0288 kg/mol, γ = 1.40, and R = 8.314 J/mol·K gives v = (332 m/s)(1 + T_C/546).

(b) At 20°C we have v = (332 m/s)(1 + 20/546) = 344 m/s.

(c) When T_C = 120°C, T_C/273 = 0.440 is not much much greater than 1, so the approximation is less valid. So the answer is no.

EVALUATE: For part (c), compare the second and third terms of the power series. The second term is $\frac{1}{2}\left(\frac{120}{273}\right) = 0.220$. The third term is $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}\left(\frac{120}{273}\right)^2 = -0.024$, which is large enough to affect the sum.

16.56. IDENTIFY: Use the equations that relate intensity level and intensity, intensity and pressure amplitude, pressure amplitude and displacement amplitude, and intensity and distance.

(a) SET UP: Use the intensity level β to calculate I at this distance. $\beta = (10\text{ dB})\log(I/I_0)$

EXECUTE: $52.0\text{ dB} = (10\text{ dB})\log[I/(10^{-12}\text{ W/m}^2)]$

$\log[I/(10^{-12}\text{ W/m}^2)] = 5.20$ implies $I = 1.585\times10^{-7}\text{ W/m}^2$

SET UP: Then use $I = \dfrac{p_{\max}^2}{2\rho v}$ to calculate $p_{\max}$:

$I = \dfrac{p_{\max}^2}{2\rho v}$ so $p_{\max} = \sqrt{2\rho v I}$

From Example 16.5, $\rho = 1.20\text{ kg/m}^3$ for air at $20°\text{C}$.

EXECUTE: $p_{\max} = \sqrt{2\rho v I} = \sqrt{2(1.20\text{ kg/m}^3)(344\text{ m/s})(1.585\times10^{-7}\text{ W/m}^2)} = 0.0114\text{ Pa}$

(b) SET UP: Use $p_{\max} = BkA$ so $A = \dfrac{p_{\max}}{Bk}$

For air $B = 1.42\times10^5\text{ Pa}$ (Example 16.1).

EXECUTE: $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{v} = \dfrac{(2\pi\text{ rad})(587\text{ Hz})}{344\text{ m/s}} = 10.72\text{ rad/m}$

$A = \dfrac{p_{\max}}{Bk} = \dfrac{0.0114\text{ Pa}}{(1.42\times10^5\text{ Pa})(10.72\text{ rad/m})} = 7.49\times10^{-9}\text{ m}$

(c) SET UP: $\beta_2 - \beta_1 = (10\text{ dB})\log(I_2/I_1)$ (Example 16.9).

Inverse-square law: $I_1/I_2 = r_2^2/r_1^2$ so $I_2/I_1 = r_1^2/r_2^2$

EXECUTE: $\beta_2 - \beta_1 = (10\text{ dB})\log(r_1/r_2)^2 = (20\text{ dB})\log(r_1/r_2)$.

$\beta_2 = 52.0\text{ dB}$ and $r_2 = 5.00\text{ m}$. Then $\beta_1 = 30.0\text{ dB}$ and we need to calculate r_1.

$52.0\text{ dB} - 30.0\text{ dB} = (20\text{ dB})\log(r_1/r_2)$

$22.0\text{ dB} = (20\text{ dB})\log(r_1/r_2)$

$\log(r_1/r_2) = 1.10$ so $r_1 = 12.6r_2 = 63.0\text{ m}$.

EVALUATE: The decrease in intensity level corresponds to a decrease in intensity, and this means an increase in distance. The intensity level uses a logarithmic scale, so simple proportionality between r and β doesn't apply.

16.57. IDENTIFY: We are investigating how the speed of sound depends on the air temperature as we look at higher and higher altitudes.

SET UP: Eq. 16.10: $v = \sqrt{\dfrac{\gamma RT}{M}}$, where $M = 0.0288$ kg/mol and $\gamma = 1.40$. First use the given information to sketch the graph of T versus y, as in Fig. 16.57. We use the slope-intercept equation of a straight line $T = my + b$. For this graph, $m = -\dfrac{20.0\text{ K}}{300.0\text{ m}} = -0.06667$ K/m and $b = 278$ K. Using these results, the speed v of sound as a function of altitude y is $v = \sqrt{\dfrac{\gamma R}{M}(my+b)}$. We want to find the time for the sound to travel 300.0 m straight upward, and we know $v_y = dy/dt$.

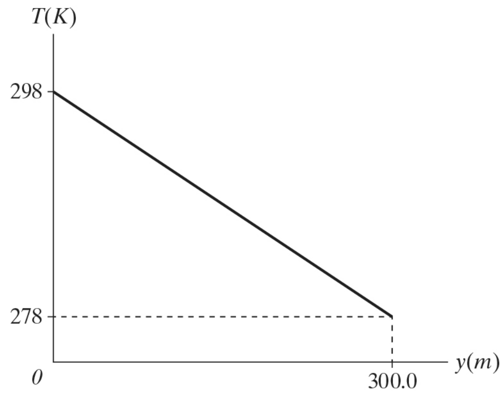


Figure 16.57

EXECUTE: **(a)** $v_y = dy/dt$ gives $dt = \dfrac{dy}{v}$, so we integrate, giving $\int_0^{t_{300}} dt = \int_0^{300\text{ m}} \dfrac{dy}{v}$. The time integral gives simply t_{300}, but the y integral requires a bit more effort. Using our equation for v from above, the y integral becomes $\int_0^{300\text{ m}} \dfrac{dy}{v} = \int_0^{300\text{ m}} \dfrac{dy}{\sqrt{\dfrac{\gamma R}{M}}\sqrt{my+b}} = \sqrt{\dfrac{M}{\gamma R}} \int_0^{300\text{ m}} \dfrac{dy}{(my+b)^{1/2}}$. To do the integral, let $u = my + b$, so $du = m\,dy$. The integral now becomes $\int \dfrac{(1/m)du}{u^{1/2}} = \dfrac{2}{m}u^{1/2}$. Returning to the original variables, the integral is $\sqrt{\dfrac{M}{\gamma R}} \int_0^{300\text{ m}} \dfrac{dy}{(my+b)^{1/2}} = \sqrt{\dfrac{M}{\gamma R}} \dfrac{2}{m}(my+b)^{1/2}\Big|_0^{300\text{ m}} = t_{300}$. Putting in the numbers for m, b, M, R, and γ gives $t_{300} = 0.879$ s.

(b) At 300.0 m, $T = 5.00°\text{C} = 298$ K, so using these numbers gives $v = \sqrt{\dfrac{\gamma RT}{M}} = 335.2$ m/s. The horizontal distance the sound travels is $x = vt_{300} = (335.2\text{ m/s})(0.879\text{ s}) = 295$ m.

EVALUATE: Since $v \propto \sqrt{T}$ (with T in K units), there is not much difference between the surface temperature and the temperature at 300 m, so v at 300 m is not very much different from v at the surface, as we see.

16.58. IDENTIFY: $f_{\text{beat}} = |f_A - f_B|$. $f_1 = \dfrac{v}{2L}$ and $v = \sqrt{\dfrac{FL}{m}}$ gives $f_1 = \dfrac{1}{2}\sqrt{\dfrac{F}{mL}}$. Apply $\Sigma\tau_z = 0$ to the bar to find the tension in each wire.

SET UP: For $\Sigma\tau_z = 0$ take the pivot at wire A and let counterclockwise torques be positive. The free-body diagram for the bar is given in Figure 16.58. Let L be the length of the bar.

EXECUTE: $\Sigma\tau_z = 0$ gives $F_B L - w_{\text{lead}}(3L/4) - w_{\text{bar}}(L/2) = 0$.

$F_B = 3w_{\text{lead}}/4 + w_{\text{bar}}/2 = 3(185\text{ N})/4 + (165\text{ N})/2 = 221\text{ N}$. $F_A + F_B = w_{\text{bar}} + w_{\text{lead}}$ so

$F_A = w_{\text{bar}} + w_{\text{lead}} - F_B = 165\text{ N} + 185\text{ N} - 221\text{ N} = 129\text{ N}$.

$f_{1A} = \dfrac{1}{2}\sqrt{\dfrac{129\text{ N}}{(5.50\times10^{-3}\text{ kg})(0.750\text{ m})}} = 88.4\text{ Hz}$. $f_{1B} = f_{1A}\sqrt{\dfrac{221\text{ N}}{129\text{ N}}} = 115.7\text{ Hz}$.

$f_{\text{beat}} = f_{1B} - f_{1A} = 27.3\text{ Hz}$.

EVALUATE: The frequency increases when the tension in the wire increases.

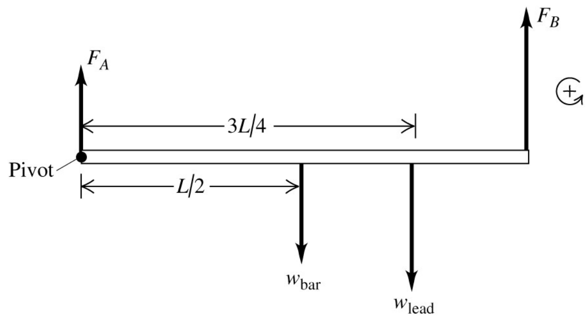


Figure 16.58

16.59. **IDENTIFY** and **SET UP:** The frequency of any harmonic is an integer multiple of the fundamental. For a stopped pipe only odd harmonics are present. For an open pipe, all harmonics are present. See which pattern of harmonics fits to the observed values in order to determine which type of pipe it is. Then solve for the fundamental frequency and relate that to the length of the pipe.

EXECUTE: **(a)** For an open pipe the successive harmonics are $f_n = nf_1,\ n = 1, 2, 3, \ldots$ For a stopped pipe the successive harmonics are $f_n = nf_1,\ n = 1, 3, 5, \ldots.$ If the pipe is open and these harmonics are successive, then $f_n = nf_1 = 1372\text{ Hz}$ and $f_{n+1} = (n+1)f_1 = 1764\text{ Hz}.$ Subtract the first equation from the second: $(n+1)f_1 - nf_1 = 1764\text{ Hz} - 1372\text{ Hz}.$ This gives $f_1 = 392\text{ Hz}.$ Then $n = \dfrac{1372\text{ Hz}}{392\text{ Hz}} = 3.5.$ But n must be an integer, so the pipe can't be open. If the pipe is stopped and these harmonics are successive, then $f_n = nf_1 = 1372\text{ Hz}$ and $f_{n+2} = (n+2)f_1 = 1764\text{ Hz}$ (in this case successive harmonics differ in n by 2). Subtracting one equation from the other gives $2f_1 = 392\text{ Hz}$ and $f_1 = 196\text{ Hz}.$ Then $n = 1372\text{ Hz}/f_1 = 7$ so $1372\text{ Hz} = 7f_1$ and $1764\text{ Hz} = 9f_1.$ The solution gives integer n as it should; the pipe is stopped.

(b) From part (a) these are the seventh and ninth harmonics.

(c) From part (a) $f_1 = 196\text{ Hz}.$

For a stopped pipe $f_1 = \dfrac{v}{4L}$ and $L = \dfrac{v}{4f_1} = \dfrac{344\text{ m/s}}{4(196\text{ Hz})} = 0.439\text{ m}.$

EVALUATE: It is essential to know that these are successive harmonics and to realize that 1372 Hz is not the fundamental. There are other lower frequency standing waves; these are just two successive ones.

16.60. **IDENTIFY:** This problem involves standing waves in a tube and the dependence of the speed of sound on temperature.

SET UP: A Kundt's tube is a stopped pipe, and the node-to-node distance is $\lambda/2$, so $\dfrac{\lambda}{2} = 47.0\text{ cm}$ which gives $\lambda = 0.940\text{ m}.$ We know that $v = f\lambda$, $f = 1200\text{ Hz}$, and $v = \sqrt{\dfrac{\gamma RT}{M}}.$ At 20°C the speed of sound in helium is 999 m/s. Our target variable is the temperature T of the helium in the tube.

EXECUTE: First find v_T in the tube at temperature T: $v_T = f\lambda = (1200\text{ Hz})(0.940\text{ m}) = 1128\text{ m/s}.$ Now solve $v_T = \sqrt{\dfrac{\gamma RT}{M}}$ for T, which gives $T = \dfrac{Mv_T^2}{\gamma R}.$ We don't know M and γ, but we do know that at 20°C (which is 293 K), $v_{293} = 999\text{ m/s}$ for helium. Using $v = \sqrt{\dfrac{\gamma RT}{M}}$ and solving for T_{293} gives $T_{293} = \dfrac{Mv_{293}^2}{\gamma R}.$

Now take the ratio T/T_{293}, which gives $\dfrac{T}{T_{293}} = \dfrac{\dfrac{Mv_{\text{T}}^2}{\gamma R}}{\dfrac{Mv_{293}^2}{\gamma R}} = \left(\dfrac{v_{\text{T}}}{v_{293}}\right)^2 = \left(\dfrac{1128\text{ m/s}}{999\text{ m/s}}\right)^2 = 1.275.$ This gives T = (293 K)(1.275) = 374 K = 101°C.

EVALUATE: This type of experiment relies on very simple measurements: the frequency of the tuning fork (or electronic oscillator) and the visible node-to-node distance of the standing wave pattern in the tube. We do not to know anything about the gas since γ and M divide out.

16.61. IDENTIFY: Destructive interference occurs when the path difference is a half-integer number of wavelengths. Constructive interference occurs when the path difference is an integer number of wavelengths.

SET UP: $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{784\text{ Hz}} = 0.439\text{ m}$

EXECUTE: **(a)** If the separation of the speakers is denoted h, the condition for destructive interference is $\sqrt{x^2 + h^2} - x = \beta\lambda$, where β is an odd multiple of one-half. Adding x to both sides, squaring, canceling the x^2 term from both sides, and solving for x gives $x = \dfrac{h^2}{2\beta\lambda} - \dfrac{\beta}{2}\lambda.$ Using $\lambda = 0.439$ m and $h = 2.00$ m yields 9.01 m for $\beta = \frac{1}{2}$, 2.71 m for $\beta = \frac{3}{2}$, 1.27 m for $\beta = \frac{5}{2}$, 0.53 m for $\beta = \frac{7}{2}$, and 0.026 m for $\beta = \frac{9}{2}$. These are the only allowable values of β that give positive solutions for x.

(b) Repeating the above for integral values of β, constructive interference occurs at 4.34 m, 1.84 m, 0.86 m, 0.26 m. Note that these are between, but not midway between, the answers to part (a).

(c) If $h = \lambda/2$, there will be destructive interference at speaker *B*. If $\lambda/2 > h$, the path difference can never be as large as $\lambda/2$. (This is also obtained from the above expression for x, with $x = 0$ and $\beta = \frac{1}{2}$.) The minimum frequency is then $v/2h = (344\text{ m/s})/(4.0\text{ m}) = 86\text{ Hz}$.

EVALUATE: When f increases, λ is smaller and there are more occurrences of points of constructive and destructive interference.

16.62. IDENTIFY: Apply $f_{\text{L}} = \left(\dfrac{v + v_{\text{L}}}{v + v_{\text{S}}}\right) f_{\text{S}}$. The wall first acts as a listener and then as a source.

SET UP: The positive direction is from listener to source. The bat is moving toward the wall so the Doppler effect increases the frequency and the final frequency received, f_{L2}, is greater than the original source frequency, f_{S1}. $f_{\text{S1}} = 1700$ Hz. $f_{\text{L2}} - f_{\text{S1}} = 8.00$ Hz.

EXECUTE: The wall receives the sound: $f_{\text{S}} = f_{\text{S1}}$. $f_{\text{L}} = f_{\text{L1}}$. $v_{\text{S}} = -v_{\text{bat}}$ and $v_{\text{L}} = 0$. $f_{\text{L}} = \left(\dfrac{v + v_{\text{L}}}{v + v_{\text{S}}}\right) f_{\text{S}}$ gives $f_{\text{L1}} = \left(\dfrac{v}{v - v_{\text{bat}}}\right) f_{\text{S1}}$. The wall receives the sound: $f_{\text{S2}} = f_{\text{L1}}$. $v_{\text{S}} = 0$ and $v_{\text{L}} = +v_{\text{bat}}$.

$$f_{\text{L2}} = \left(\frac{v + v_{\text{bat}}}{v}\right) f_{\text{S2}} = \left(\frac{v + v_{\text{bat}}}{v}\right)\left(\frac{v}{v - v_{\text{bat}}}\right) f_{\text{S1}} = \left(\frac{v + v_{\text{bat}}}{v - v_{\text{bat}}}\right) f_{\text{S1}}.$$

$$f_{\text{L2}} - f_{\text{S1}} = \Delta f = \left(\frac{v + v_{\text{bat}}}{v - v_{\text{bat}}} - 1\right) f_{\text{S1}} = \left(\frac{2v_{\text{bat}}}{v - v_{\text{bat}}}\right) f_{\text{S1}}.$$

$$v_{\text{bat}} = \frac{v\Delta f}{2f_{\text{S1}} + \Delta f} = \frac{(344\text{ m/s})(8.00\text{ Hz})}{2(1700\text{ Hz}) + 8.00\text{ Hz}} = 0.808\text{ m/s}.$$

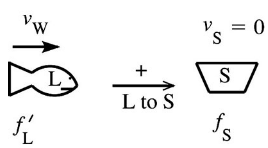
EVALUATE: $f_{S1} < \Delta f$, so we can write our result as the approximate but accurate expression $\Delta f = \left(\dfrac{2v_{bat}}{v}\right) f_{S1}$.

16.63. **(a) IDENTIFY** and **SET UP:** Use $v = f\lambda$ to calculate λ.

EXECUTE: $\lambda = \dfrac{v}{f} = \dfrac{1482 \text{ m/s}}{18.0\times10^3 \text{ Hz}} = 0.0823 \text{ m}.$

(b) IDENTIFY: Apply the Doppler effect equation, $f_L = \left(\dfrac{v+v_L}{v}\right) f_S = \left(1+\dfrac{v_L}{v}\right) f_S$. The frequency of the directly radiated waves is $f_S = 18{,}000$ Hz. The moving whale first plays the role of a moving listener, receiving waves with frequency f_L'. The whale then acts as a moving source, emitting waves with the same frequency, $f_S' = f_L'$ with which they are received. Let the speed of the whale be v_W.

SET UP: Whale receives waves: (Figure 16.63a)

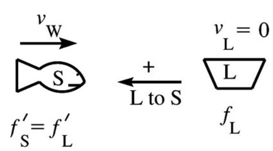


EXECUTE: $v_L = +v_W$

$$f_L' = f_S\left(\frac{v+v_L}{v+v_S}\right) = f_S\left(\frac{v+v_W}{v}\right)$$

Figure 16.63a

SET UP: Whale re-emits the waves: (Figure 16.63b)



EXECUTE: $v_S = -v_W$

$$f_L = f_S\left(\frac{v+v_L}{v+v_S}\right) = f_S'\left(\frac{v}{v-v_W}\right)$$

Figure 16.63b

But $f_S' = f_L'$ so $f_L = f_S\left(\dfrac{v+v_W}{v}\right)\left(\dfrac{v}{v-v_W}\right) = f_S\left(\dfrac{v+v_W}{v-v_W}\right)$.

Then $\Delta f = f_S - f_L = f_S\left(1-\dfrac{v+v_W}{v-v_W}\right) = f_S\left(\dfrac{v-v_W-v-v_W}{v-v_W}\right) = \dfrac{-2f_S v_W}{v-v_W}$.

$$\Delta f = \frac{-2(1.80\times10^4 \text{ Hz})(4.95 \text{ m/s})}{1482 \text{ m/s} - 4.95 \text{ m/s}} = -120 \text{ Hz}.$$

EVALUATE: Δf is negative, which means that $f_L > f_S$. This is reasonable because the listener and source are moving toward each other so the frequency is raised.

16.64. **IDENTIFY:** This problem deals with sound intensity and sound intensity level.

SET UP and EXECUTE: **(a)** Estimate: 5.0 s delay between the flash and sound of thunder.

(b) $x = vt = (344 \text{ m/s})(5.0 \text{ s}) = 1700$ m.

(c) Estimate: 70 dB. $\beta = (10 \text{ dB})\log\dfrac{I}{I_0} = 70$ dB, so $I = 10^7 I_0 = 10^7\ 10^{-12} \text{ W/m}^2 = 10^{-5} \text{ W/m}^2$.

(d) $I = P/A$, so $P_{av} = IA = I(4\pi r^2) = (10^{-5} \text{ W/m}^2)(4\pi)(1700 \text{ m})^2 = 370$ W.

(e) Estimate: 4.0 s duration. $E = P_{av}t = (370 \text{ W})(4.0 \text{ s}) = 1500$ J.

EVALUATE: The power due to a lightning strike is much greater than 370 W because the strike lasts only a fraction of a second and can create a great deal of heat. We are looking only at the *sound* energy.

16.65. **IDENTIFY:** This problem deals with sound intensity and sound intensity level.

SET UP and EXECUTE: **(a)** Estimate: Somewhere between busy traffic and an elevated train, so use 85 dB. $\beta = (10\text{ dB})\log\dfrac{I}{I_0} = 85$ dB, so $I = 10^{8.5}I_0 = 10^{8.5}\ 10^{-12}\ \text{W/m}^2 = 3.2\times10^{-4}\ \text{W/m}^2$.

(b) $I = P/A$, so $P_{av} = IA = I(4\pi r^2) = (3.2\times10^{-4}\ \text{W/m}^2)(4\pi)(18\text{ m})^2 = 1.3$ W.

(c) Estimate: 2.0 s for the crashing sound. $E = P_{av}t = (1.3\text{ W})(2.0\text{ s}) = 2.6$ J.

(d) Estimate: 4.0 s to travel down the alley. $v = x/t = (18\text{ m})/(4.0\text{ s}) = 4.5$ m/s.

(e) $K = \dfrac{1}{2}mv_{cm}^2 + \dfrac{1}{2}I\omega^2$, with $\omega = v_{cm}/R$ for rolling and $I = \dfrac{2}{5}MR^2$. Using these gives

$$K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{cm}}{R}\right)^2 = \frac{7}{10}mv_{cm}^2 = \frac{7}{10}(6.4\text{ kg})(4.5\text{ m/s})^2 = 91\text{ J}.$$

(f) $\dfrac{2.6\text{ J}}{91\text{ J}} = 0.029 \approx 3\%$.

EVALUATE: Just from watching the heavy pins scatter, it is clear that most of the ball's kinetic energy is transferred to them, with little left over for sound energy. The 3% is not unreasonable.

16.66. **IDENTIFY:** Apply $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. The heart wall first acts as the listener and then as the source.

SET UP: The positive direction is from listener to source. The heart wall is moving toward the receiver so the Doppler effect increases the frequency and the final frequency received, f_{L2}, is greater than the source frequency, f_{S1}. $f_{L2} - f_{S1} = 72$ Hz.

EXECUTE: Heart wall receives the sound: $f_S = f_{S1}$. $f_L = f_{L1}$. $v_S = 0$. $v_L = -v_{wall}$. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ gives $f_{L1} = \left(\dfrac{v-v_{wall}}{v}\right)f_{S1}$.

Heart wall emits the sound: $f_{S2} = f_{L1}$. $v_S = +v_{wall}$. $v_L = 0$.

$$f_{L2} = \left(\frac{v}{v+v_{wall}}\right)f_{S2} = \left(\frac{v}{v+v_{wall}}\right)\left(\frac{v-v_{wall}}{v}\right)f_{S1} = \left(\frac{v-v_{wall}}{v+v_{wall}}\right)f_{S1}.$$

$f_{L2} - f_{S1} = \left(1 - \dfrac{v-v_{wall}}{v+v_{wall}}\right)f_{S1} = \left(\dfrac{2v_{wall}}{v+v_{wall}}\right)f_{S1}$. $v_{wall} = \dfrac{(f_{L2}-f_{S1})v}{2f_{S1} - (f_{L2}-f_{S1})}$. $f_{S1} \gg f_{L2} - f_{S1}$ and

$$v_{wall} = \frac{(f_{L2}-f_{S1})v}{2f_{S1}} = \frac{(72\text{ Hz})(1500\text{ m/s})}{2(2.00\times10^6\text{ Hz})} = 0.0270\text{ m/s} = 2.70\text{ cm/s}.$$

EVALUATE: $f_{S1} = 2.00\times10^6$ Hz and $f_{L2} - f_{S1} = 72$ Hz, so the approximation we made is very accurate. Within this approximation, the frequency difference between the reflected and transmitted waves is directly proportional to the speed of the heart wall.

16.67. **IDENTIFY:** Follow the method of Example 16.18 and apply the Doppler shift formula twice, once with the insect as the listener and again with the insect as the source.

SET UP: Let v_{bat} be the speed of the bat, v_{insect} be the speed of the insect, and f_i be the frequency with which the sound waves both strike and are reflected from the insect. The positive direction in each application of the Doppler shift formula is from the listener to the source.

EXECUTE: The frequencies at which the bat sends and receives the signals are related by $f_L = f_i\left(\dfrac{v+v_{bat}}{v-v_{insect}}\right) = f_S\left(\dfrac{v+v_{insect}}{v-v_{bat}}\right)\left(\dfrac{v+v_{bat}}{v-v_{insect}}\right)$. Solving for v_{insect},

$$v_{\text{insect}} = v\left[\frac{1-\dfrac{f_S}{f_L}\left(\dfrac{v+v_{\text{bat}}}{v-v_{\text{bat}}}\right)}{1+\dfrac{f_S}{f_L}\left(\dfrac{v+v_{\text{bat}}}{v-v_{\text{bat}}}\right)}\right] = v\left[\frac{f_L(v-v_{\text{bat}})-f_S(v+v_{\text{bat}})}{f_L(v-v_{\text{bat}})+f_S(v+v_{\text{bat}})}\right].$$

Letting $f_L = f_{\text{refl}}$ and $f_S = f_{\text{bat}}$ gives the result.

(b) If $f_{\text{bat}} = 80.7$ kHz, $f_{\text{refl}} = 83.5$ kHz, and $v_{\text{bat}} = 3.9$ m/s, then $v_{\text{insect}} = 2.0$ m/s.

EVALUATE: $f_{\text{refl}} > f_{\text{bat}}$ because the bat and insect are approaching each other.

16.68. IDENTIFY: Apply the Doppler effect formula $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. In the SHM the source moves toward and away from the listener, with maximum speed $\omega_p A_p$.

SET UP: The direction from listener to source is positive.

EXECUTE: **(a)** The maximum velocity of the siren is $\omega_P A_P = 2\pi f_P A_P$. You hear a sound with frequency $f_L = f_{\text{siren}} v/(v+v_S)$, where v_S varies between $+2\pi f_P A_P$ and $-2\pi f_P A_P$.

$f_{L-\max} = f_{\text{siren}} v/(v-2\pi f_P A_P)$ and $f_{L-\min} = f_{\text{siren}} v/(v+2\pi f_P A_P)$.

(b) The maximum (minimum) frequency is heard when the platform is passing through equilibrium and moving up (down).

EVALUATE: When the platform is moving upward the frequency you hear is greater than f_{siren} and when it is moving downward the frequency you hear is less than f_{siren}. When the platform is at its maximum displacement from equilibrium its speed is zero and the frequency you hear is f_{siren}.

16.69. IDENTIFY: The sound from the speaker moving toward the listener will have an increased frequency, while the sound from the speaker moving away from the listener will have a decreased frequency. The difference in these frequencies will produce a beat.

SET UP: The greatest frequency shift from the Doppler effect occurs when one speaker is moving away and one is moving toward the person. The speakers have speed $v_0 = r\omega$, where $r = 0.75$ m.

$f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$, with the positive direction from the listener to the source. $v = 344$ m/s.

EXECUTE: **(a)** $f = \dfrac{v}{\lambda} = \dfrac{344\text{ m/s}}{0.313\text{ m}} = 1100$ Hz. $\omega = (75\text{ rpm})\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 7.85$ rad/s and

$v_0 = (0.75\text{ m})(7.85\text{ rad/s}) = 5.89$ m/s.

For speaker A, moving toward the listener: $f_{LA} = \left(\dfrac{v}{v-5.89\text{ m/s}}\right)(1100\text{ Hz}) = 1119$ Hz.

For speaker B, moving toward the listener: $f_{LB} = \left(\dfrac{v}{v+5.89\text{ m/s}}\right)(1100\text{ Hz}) = 1081$ Hz.

$f_{\text{beat}} = f_1 - f_2 = 1119\text{ Hz} - 1081\text{ Hz} = 38$ Hz.

(b) A person can hear individual beats only up to about 7 Hz and this beat frequency is much larger than that.

EVALUATE: As the turntable rotates faster the beat frequency at this position of the speakers increases.

16.70. IDENTIFY and SET UP: Assuming that the gas is nearly ideal, the speed of sound in it is given by $v = \sqrt{\dfrac{\gamma RT}{M}}$, where T is in absolute (Kelvin) units and M is the molar mass of the gas.

EXECUTE: **(a)** Squaring $v=\sqrt{\dfrac{\gamma RT}{M}}$, gives $v^2=\left(\dfrac{\gamma R}{M}\right)T$. On a graph of v^2 versus T, we would expect a straight line with slope equal to $\dfrac{\gamma R}{M}$. Figure 16.70 shows the graph of the data given in the problem.

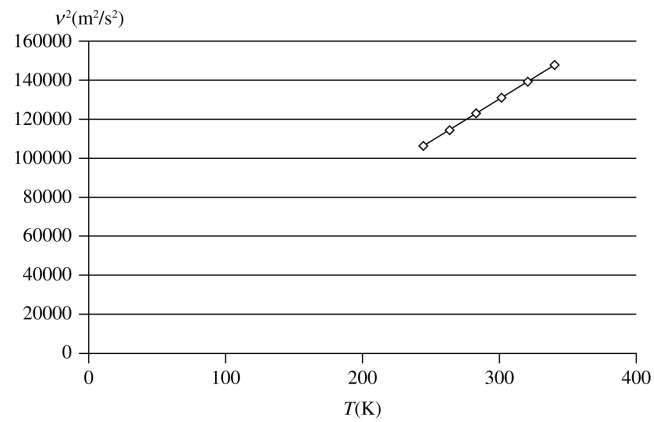


Figure 16.70

(b) The best-fit equation for the graph in Figure 16.70 is $v^2 = (416.47\ \text{m}^2/\text{K}\cdot\text{s}^2)\,T + 296.65\ \text{m}^2/\text{s}^2$. Solving our expression for the slope for M gives $M = \gamma R$ /slope. Putting in the numbers gives $M = (1.40)(8.3145\ \text{J/mol}\cdot\text{K})/(416.47\ \text{m}^2/\text{K}\cdot\text{s}^2) = 0.0279$ kg/mol = 27.9 g/mol.

EVALUATE: Nitrogen N_2 is a diatomic gas with a molecular mass of 28.0 g/mol, so the gas is probably nitrogen.

16.71. **IDENTIFY** and **SET UP:** There is a node at the piston, so the distance the piston moves is the node to node distance, $\lambda/2$. Use $v = f\lambda$ to calculate v and $v=\sqrt{\dfrac{\gamma RT}{M}}$ to calculate γ from v.

EXECUTE: **(a)** $\lambda/2 = 37.5$ cm, so $\lambda = 2(37.5\text{ cm}) = 75.0\text{ cm} = 0.750$ m.
$v = f\lambda = (500\text{ Hz})(0.750\text{ m}) = 375$ m/s

(b) Solve $v=\sqrt{\gamma RT/M}$ for γ: $\gamma = \dfrac{Mv^2}{RT} = \dfrac{(28.8\times10^{-3}\ \text{kg/mol})(375\ \text{m/s})^2}{(8.3145\ \text{J/mol}\cdot\text{K})(350\ \text{K})} = 1.39.$

(c) EVALUATE: There is a node at the piston so when the piston is 18.0 cm from the open end the node is inside the pipe, 18.0 cm from the open end. The node to antinode distance is $\lambda/4 = 18.8$ cm, so the antinode is 0.8 cm beyond the open end of the pipe.
The value of γ we calculated agrees with the value given for air in Example 16.4.

16.72. **IDENTIFY** and **SET UP:** We know from the problem that $f_R = f_S\left(1-\dfrac{v}{c}\right)^{1/2}\left(1+\dfrac{v}{c}\right)^{1/2}$. The radius of the nebula is $R = vt$, where t is the time since the supernova explosion. When the source and receiver are moving toward each other, v is negative and $f_R > f_S$. The light from the explosion reached earth 960 years ago, so that is the amount of time the nebula has expanded. $1\text{ ly} = 9.461\times10^{15}$ m.

EXECUTE: **(a)** According to the binomial theorem, $(1 \pm x)^n \approx 1 \pm nx$ if $|x| << 1$. Applying this to the two square roots, where $n = \pm\dfrac{1}{2}$ and $x = v/c$, the equation $f_R = f_S\left(1-\dfrac{v}{c}\right)^{1/2}\left(1+\dfrac{v}{c}\right)^{1/2}$ becomes

$$f_R \approx f_S\left(1-\frac{1}{2}\frac{v}{c}\right)\left(1-\frac{1}{2}\frac{v}{c}\right) \approx f_S\left(1-\frac{1}{2}\frac{v}{c}\right)^2 \approx f_S\left[1-2\left(\frac{1}{2}\frac{v}{c}\right)\right] \approx f_S\left(1-\frac{v}{c}\right).$$

(b) Solving the equation we derived in part (a) for v gives

$v = c\dfrac{f_S - f_R}{f_S} = (3.00\times10^8\text{ m/s})\dfrac{-0.018\times10^{14}\text{ Hz}}{4.568\times10^{14}\text{ Hz}} = -1.2\times10^6\text{ m/s},$ with the minus sign indicating that the gas is approaching the earth, as is expected since $f_R > f_S$.

(c) As of 2014, the supernova occurred 960 years ago. The diameter D is therefore

$D = 2(960\text{ y})(3.156\times10^7\text{ s/y})(1.2\times10^6\text{ m/s}) = 7.15\times10^{16}\text{ m} = 7.6\text{ ly}.$

(d) The ratio of the width of the nebula to 2π times the distance from the earth is the ratio of the angular width (taken as 5 arc minutes) to an entire circle, which is 60×360 arc minutes. The distance to the nebula is then $\left(\dfrac{2}{2\pi}\right)(3.75\text{ ly})\dfrac{(60)(360)}{5} = 5.2\times10^3$ ly. The time it takes light to travel this distance is 5200 yr, so the explosion actually took place 5200 yr before 1054 C.E., or about 4100 B.C.E.

EVALUATE: $\left|\dfrac{v}{c}\right| = 4.0\times10^{-3}$, so even though $|v|$ is very large the approximation required for $v = c\dfrac{\Delta f}{f}$ is accurate.

16.73. **IDENTIFY:** The wire vibrating in its fundamental causes the tube to resonate with the same frequency in its fundamental. We are dealing with standing waves on a string and in an open pipe.

SET UP: Fig. 16.73a illustrates the information in the problem. The information we have is as follows:

<u>wire</u>: μ = 1.40 g/m = 0.00140 kg/m, vibrating in its fundamental f_1.

<u>pole</u>: M = 8.00 kg, L = 1.56 m

<u>tube</u>: 39.0 cm long, m = 4.00 kg, hollow (open pipe), vibrating in its fundamental f_1

We want to find the frequency f_1 at which the wire and tube are vibrating and the height h in Fig. 16.73a. The equations we use for the tube are $f_n = nv_s/2L_t$, $\lambda_n = 2L_t/n$, $\lambda_1 = 2L_t$ and for the wire $f_n = nv/2L_w$, $\lambda_n = 2L_w/n$, $\lambda_1 = 2L_w$. We also use $v = f\lambda$.

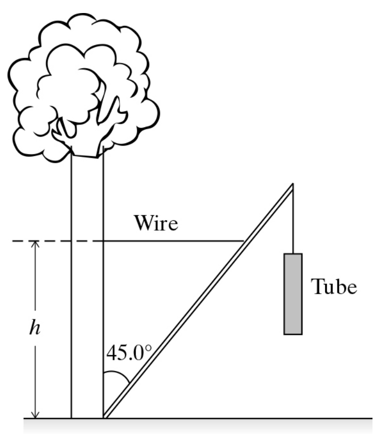


Figure 16.73a

EXECUTE: **(a)** For the tube in its fundamental mode, we have $\lambda_1 = 2L_t$ = 2(0.390 m) = 0.780 m. The frequency is $f_1 = \dfrac{v_s}{\lambda_t} = \dfrac{344\text{ m/s}}{0.780\text{ m}} = 441$ Hz. This is the frequency at which the wire and the tube are resonating.

(b) We want to find h in Fig. 16.73a. Now look at the wire. $f_w \lambda_w = v_w = \sqrt{\frac{F}{\mu}}$. In its fundamental mode $\lambda_1 = 2L_w$. From Fig. 16.73a we see that $L_w = h\tan 45.0° = h$, so $\lambda_1 = 2h$. From part (a) we know that f_1 = 441 Hz, so $f_w \lambda_w = v_w = \sqrt{\frac{F}{\mu}}$ gives $f_1(2h) = (441\text{ Hz})(2h) = \sqrt{\frac{F}{\mu}}$, which becomes

$(882\text{ Hz})h = \sqrt{\frac{F}{\mu}}$. (Eq. 1)

We need to find F in order to find h. Make a free-body diagram of the pole as in Fig. 16.73b and apply $\sum \tau_z = 0$ about the hinge. Using Fig. 16.73 as a guide, we get

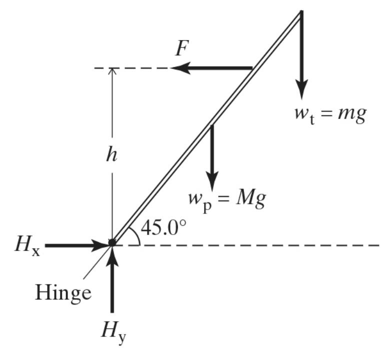


Figure 16.73b

$Fh - Mg\frac{L}{2}\cos 45.0° - mgL\cos 45.0° = 0,$ which simplifies to $Fh = \left(\frac{M}{2} + m\right)gL\cos 45.0°.$ Putting in M =8.00 kg, m = 4.00 kg, and L = 1.56 m, and solving for F, we get $F = \frac{86.48\text{ N}\cdot\text{m}}{h}$. Now return to Eq. 1. Square the equation and substitute for F, giving $(882\text{ Hz})^2 h^2 = \frac{F}{\mu} = \frac{86.48\text{ N}\cdot\text{m}}{h(0.00140\text{ kg/m})}$. Solving for h gives h = 0.430 m.

EVALUATE: Changing h would change the length of the wire as well as the tension in it. These changes would affect the fundamental frequency of the wire.

16.74. **IDENTIFY:** The sound from the two speakers travels different distances to reach the aisle. Therefore interference occurs along the aisle.

SET UP: Start with a figure to illustrate the information given, as in Fig. 16.74. The path difference d between sound from speakers A and B is $d = r - x$. For destructive interference to occur, the path difference must be $d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \cdots = (2n-1)\frac{\lambda}{2}$ $(n = 1,2,3,...)$. From the figure, we can see that $d = \sqrt{x^2 + (15.0\text{ m})^2} - x$, so $d = \sqrt{x^2 + (15.0\text{ m})^2} - x = (2n-1)\frac{\lambda}{2}$ $(n = 1,2,3,...)$. The wavelength of the sound from both speakers is $\lambda = \frac{v}{f} = \frac{344\text{ m/s}}{440\text{ Hz}} = 0.7818\text{ m}.$

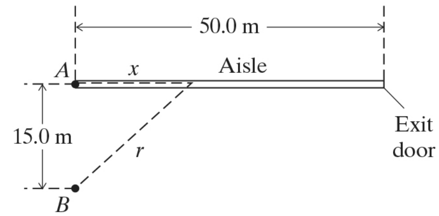


Figure 16.74

EXECUTE: **(a)** The path difference d increases as we get closer to speaker A in the figure. The limit would be right at the speaker, in which case $x = 0$, $r = 15.0$ m, and $d = 15.0$ m. Therefore we have $d = 15.0\text{ m} = (2n-1)\dfrac{\lambda}{2} = (2n-1)\dfrac{0.7818\text{ m}}{2}$. Solving for n gives $n = 19.69$. But n must be an integer and cannot exceed 19.69, so $n = 19$. This is the total number of points along the aisle where the waves from the two speakers would cancel each other. But aisle is only 50.0 m long, so the theater may not be deep enough for all these points to occur. The smallest path difference would occur at the back door 50.0 m from the stage. The path difference at that point is d_{min} which is $d_{\text{min}} = \sqrt{(50.0\text{ m})^2 + (15.0\text{ m})^2} - 50.0\text{ m} = 2.2015\text{ m}$. Now use this distance to find out how may points of cancellation will fit inside the theater. $2.2015\text{ m} = (2n-1)\left(\dfrac{0.7818\text{ m}}{2}\right) = (0.3909\text{ m})(2n-1)$. Solving for n gives $n = 3.316$, which suggests $n = 3$. However for $n = 3$, $d = (2n-1)\dfrac{\lambda}{2} = \dfrac{5\lambda}{2} = \dfrac{5(0.7818\text{ m})}{2} = 1.955\text{ m}$. But this distance is shorter than the minimum path difference of 2.2015 m that we calculated earlier. So we would have to be farther than 50.0 m from the stage to experience it, which is outside the theater. Thus the $n = 3$ point does not occur, and neither do the $n = 2$ and $n = 1$ points. Therefore we have found that a total of 19 cancellation points are possible, but 3 of them would occur outside the theater, so the number that occur inside the theater are 19 – 3 = 16.

(b) We have just seen that the $n = 3$ interference point does not occur, so the farthest point at which cancellation does occur would be for the $n = 4$ point for which $d = \dfrac{7}{2}\lambda = \dfrac{7}{2}(0.7818\text{ m}) = 2.7363\text{ m}$. Using this value of d and solving for the distance x from the stage gives $\sqrt{x^2 + (15.0\text{ m})^2} - x = 2.7363\text{ m}$. Squaring and solving for x gives $x = 39.746$ m. The distance from the exit door is 50.0 m – 39.746 m = 10.3 m.

(c) The second frequency must be such that it cancels at the same points as the original frequency (and it may cancel at other points also). This frequency must be some multiple of the original frequency. Call the original wavelength λ and the second frequency λ', and realize that λ' must be less than λ, so we can write it as $\lambda' = \lambda / k$. For cancellation of the sound at both frequencies to occur, d must be odd multiples of a half wavelength for each frequency f and f'. That is, $d = N(\lambda/2)$ and $d = N'(\lambda'/2)$, where both N and N' must be odd integers. Substituting $\lambda' = \lambda / k$ into $d = N(\lambda/2)$ gives $d = N\left(\dfrac{k\lambda'}{2}\right) = Nk\dfrac{\lambda'}{2}$. This result tells us that Nk must be an odd integer. Since N is already an odd integer, k must also be an odd integer for the product Nk to be odd. The frequency f' of the second sound is $f' = \dfrac{v}{\lambda'} = \dfrac{v}{\lambda/k} = k(v/\lambda) = kf$, where k is an odd integer. We want the smallest possible frequency for f', so k must have its smallest possible value. The smallest is $k = 1$, but in that case the second frequency is the same as the first one, which is not what we want. So k must be 3, which makes the second wavelength $\lambda' = \frac{\lambda}{3} = \frac{0.7818\text{ m}}{3} = 0.2606\text{ m}$ and the second frequency $f' = 3f = 3(440\text{ Hz}) = 1320\text{ Hz}.$

(d) We follow the same procedure as in part (a), but this time for f' and λ'. For destructive interference, the path difference must be $d = (2N-1)\frac{\lambda'}{2}$, N = 1, 2, 3, … . The greatest path difference occurs closest to speaker *A*, with the limit being d = 15.0 m. In this case we have $15.0\text{ m} = (2N-1)\frac{\lambda'}{2} = (2N-1)\frac{0.2606\text{ m}}{2}$. Solving for N gives N = 58. This means that destructive interference can occur at a maximum of 58 points. However the hall may not be deep enough for all of them to occur. As we saw in part (a), the minimum path difference d_{min} that can occur in the hall is at the back door, and at that point d_{min} = 2.2015 m. Using this information to find the minimum value of N for points inside the hall gives $2.2015\text{ m} = (2N-1)\frac{\lambda'}{2} = (2N-1)\frac{0.2606\text{ m}}{2}$. Solving for N we get N = 8.95, so we use N = 8. But first check to see if d is shorter than d_{min} for this result. Doing so gives $d = (2N-1)\frac{\lambda'}{2} = (15)\frac{0.2606\text{ m}}{2} = 1.955\text{ m}$, which is less than 2.2015 m. Therefore the N = 8 point does not occur within the hall, and neither do all the others for N = 7, 6, etc. So the total number of points within the hall at which destructive interference occurs is 58 – 8 = 50. But 16 of those points occur at the same places as the 440-Hz sound, so the *additional* number of points is 50 – 16 = 34 points.

(e) At the point closest to the speaker, N = 58 from part (d). The path difference at this point is $d = (2N-1)\frac{\lambda'}{2} = (115)\frac{0.2606\text{ m}}{2} = 14.9845\text{ m}.$ Using $\sqrt{x^2 + (15.0\text{ m})^2} - x = 14.9845\text{ m}$, we solve for x, giving x = 0.0155 m = 1.55 cm.

EVALUATE: The sound cannot cancel completely at the points of destructive interference because the intensity from the two speakers is different due to the difference in distance from them. But the loudness will definitely reach a minimum at the points of destructive interference.

16.75. **IDENTIFY:** The phase of the wave is determined by the value of $x - vt$, so t increasing is equivalent to x decreasing with t constant. The pressure fluctuation and displacement are related by the equation $p(x,t) = -B\frac{\partial y(x,t)}{\partial x}$.

SET UP: $y(x,t) = -\frac{1}{B}\int p(x,t)dx.$ If $p(x,t)$ versus x is a straight line, then $y(x,t)$ versus x is a parabola. For air, $B = 1.42\times10^5\text{ Pa}.$

EXECUTE: **(a)** The graph is sketched in Figure 16.75a.

(b) From $p(x,t) = BkA\sin(kx - \omega t)$, the function that has the given $p(x,0)$ at $t = 0$ is given graphically in Figure 16.75b. Each section is a parabola, not a portion of a sine curve. The period is $\lambda/v = (0.200\text{ m})/(344\text{ m/s}) = 5.81\times10^{-4}\text{ s}$ and the amplitude is equal to the area under the p versus x curve between $x = 0$ and $x = 0.0500$ m divided by B, or 7.04×10^{-6} m.

(c) Assuming a wave moving in the $+x$-direction, $y(0,t)$ is as shown in Figure 16.75c.

(d) The maximum velocity of a particle occurs when a particle is moving through the origin, and the particle speed is $v_y = -\frac{\partial y}{\partial x}v = \frac{pv}{B}$. The maximum velocity is found from the maximum pressure, and $v_{y\text{max}} = (40\text{ Pa})(344\text{ m/s})/(1.42\times10^5\text{ Pa}) = 9.69\text{ cm/s}.$ The maximum acceleration is the maximum pressure gradient divided by the density,

$$a_{\text{max}} = \frac{(80.0\text{ Pa})/(0.100\text{ m})}{(1.20\text{ kg/m}^3)} = 6.67\times10^2\text{ m/s}^2.$$

(e) The speaker cone moves with the displacement as found in part (c); the speaker cone alternates between moving forward and backward with constant magnitude of acceleration (but changing sign). The acceleration as a function of time is a square wave with amplitude $667\ \text{m/s}^2$ and frequency $f = v/\lambda = (344\ \text{m/s})/(0.200\ \text{m}) = 1.72\ \text{kHz}$.

EVALUATE: We can verify that $p(x,t)$ versus x has a shape proportional to the slope of the graph of $y(x,t)$ versus x. The same is also true of the graphs versus t.

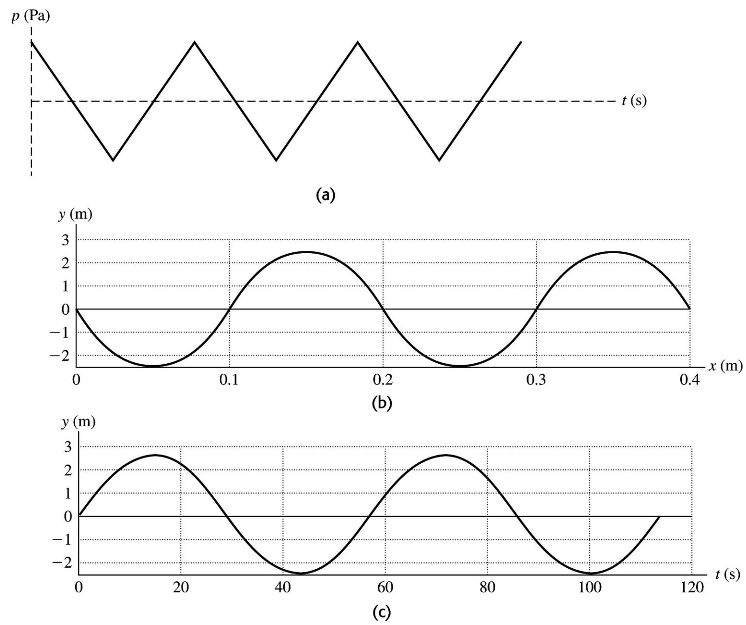


Figure 16.75

16.76. **IDENTIFY** and **SET UP:** Consider the derivation of the speed of a longitudinal wave in Section 16.2.
EXECUTE: **(a)** The quantity of interest is the change in force per fractional length change. The force constant k' is the change in force per length change, so the force change per fractional length change is $k'L$, the applied force at one end is $F = (k'L)(v_y/v)$ and the longitudinal impulse when this force is applied for a time t *is* $k'Ltv_y/v$. The change in longitudinal momentum is $((vt)m/L)v_y$ and equating the expressions, canceling a factor of t and solving for v gives $v^2 = L^2k'/m$.
(b) $v = (2.00\ \text{m})\sqrt{(1.50\ \text{N/m})/(0.250\ \text{kg})} = 4.90\ \text{m/s}$
EVALUATE: *A* larger k' corresponds to a stiffer spring and for a stiffer spring the wave speed is greater.

16.77. **IDENTIFY** and **SET UP:** The time between pulses is limited by the time for the wave to travel from the transducer to the structure and then back again. Use $x = v_xt$ and $f = 1/T$.
EXECUTE: **(a)** The wave travels 10 cm in and 10 cm out, so $t = x/v_x = (0.20\ \text{m})/(1540\ \text{m/s}) = 0.13 \times 10^{-3}\ \text{s} = 0.13\ \text{ms}$. The period can be no shorter than this, so the highest pulse frequency is $f = 1/t = 1/(0.13\ \text{ms}) = 7700\ \text{Hz}$, which is choice (b).
EVALUATE: The pulse frequency is not the same thing as the frequency of the ultrasound waves, which is around 1.0 MHz.

16.78. **IDENTIFY** and **SET UP:** Call S the intensity level of the beam. The beam attenuates by 100 dB per meter, so in 10 cm (0.10 m) it would attenuate by 1/10 of this amount. Therefore $\Delta S = -10$ dB. $S = 10\ \text{dB}\log(I/I_0)$.

EXECUTE: **(a)** $\Delta S = S_2 - S_1 = 10 \text{ dB} \log(I_2/I_0) - 10 \text{ dB} \log(I_1/I_0) = 10 \text{ dB} \log(I_2/I_0)$ since $I_1 = I_0$. Therefore $-10 \text{ dB} = 10 \text{ dB} \log(I_2/I_0)$, which gives $I_2/I_0 = 10^{-1}$, so $I_2 = 1/10\ I_0$, which is choice (d).
EVALUATE: In the next 10 cm, the beam would attenuate by another factor of 1/10, so it would be 1/100 of the initial intensity.

16.79. **IDENTIFY** and **SET UP:** The beam goes through 5.0 cm of tissue and 2.0 cm of bone. Use $d = vt$ to calculate the total time in this case and compare it with the time to travel 7.0 cm through only tissue.
EXECUTE: $d = vt$ gives $t = x/v$. Calculate the time to go through 2.0 cm of bone and 5.0 cm of tissue and then get the total time t_{tot}. $t_T = x_T/v_T$ and $t_B = x_B/v_B$, so $t_{tot} = x_T/v_T + x_B/v_B$. Putting in the numbers gives
$t_{tot} = (0.050 \text{ m})/(1540 \text{ m/s}) + (0.020 \text{ m})/(3080 \text{ m/s}) = 3.896 \times 10^{-5}$ s. If the wave went through only tissue during this time, it would have traveled $x = v_T t_{tot} = (1540 \text{ m/s})(3.896 \times 10^{-5} \text{ s}) = 6.0 \times 10^{-2} \text{ m} =$ 6.0 cm. So the beam traveled 7.0 cm, but you think it traveled 6.0 cm, so the structure is actually 1.0 cm deeper than you think, which makes choice (a) the correct one.
EVALUATE: A difference of 1.0 cm when a structure is 7.0 below the surface can be very significant.

16.80. **IDENTIFY** and **SET UP:** In a standing wave pattern, the distance between antinodes is one-half the wavelength of the waves. Use $v = f\lambda$ to find the wavelength.
EXECUTE: $\lambda = v/f = (1540 \text{ m/s})/(1.00 \text{ MHz}) = 1.54 \times 10^{-3} \text{ m} = 1.54$ mm. The distance D between antinodes is $D = \lambda/2 = (1.54 \text{ mm})/2 = 0.77 \text{ mm} \approx 0.75$ mm, which is choice (b).
EVALUATE: Decreasing the frequency could reduce the distance between antinodes if this is desired.

16.81. **IDENTIFY** and **SET UP:** The antinode spacing is $\lambda/2$. Use $v = f\lambda$.
EXECUTE: **(a)** The antinode spacing d is $\lambda/2$. Using $v = f\lambda$, we have $d = \lambda/2 = v/2f$. For the numbers in this problem, we have $d = (1540 \text{ m/s})/[2(1.0 \text{ kHz})] = 0.77 \text{ m} = 77$ cm. The cranium is much smaller than 77 cm, so there will be no standing waves within it at 1.0 kHz, which is choice (b).
EVALUATE: Using 1.0 MHz waves, the distance between antinodes would be 1000 times smaller, or 0.077 cm, so there could certainly be standing waves within the cranium.

17

TEMPERATURE AND HEAT

VP17.4.1. **IDENTIFY:** The rods expand (or contract) when their temperature is changed, so we are dealing with thermal expansion.

SET UP: $\Delta L = \alpha L_0 \Delta T$

EXECUTE: **(a)** We want to find α, the coefficient of linear expansion. Using $\Delta L = \alpha L_0 \Delta T$ gives $2.20\times10^{-4}\ \text{m} = \alpha(0.500\ \text{m})(37.0°\text{C} - 15.0°\text{C})$, so $\alpha = 2.00\times10^{-5}\ (\text{C}°)^{-1} = 2.00\times10^{-5}\ \text{K}^{-1}$.

(b) We want the change in length. Using $\Delta L = \alpha L_0 \Delta T$ gives

$\Delta L = \left[2.00\times10^{-5}\ (\text{C}°)^{-1}\right](0.300\ \text{m})(-20.0°\text{C} - 25.0°\text{C}) = -2.7\times10^{-4}\ \text{m} = -0.27\ \text{mm}$. Its length would *decrease* by 0.27 mm.

EVALUATE: Increasing temperature causes thermal expansion, while decreasing temperature causes thermal contraction. In both cases, the relative change in length is normally very small.

VP17.4.2. **IDENTIFY:** The mug and ethanol both decrease in volume as their temperature is decreased, so we are dealing with thermal contraction.

SET UP: We are dealing with volume contraction, so we use $\Delta V = \beta V_0 \Delta T$. If the volume of the ethanol and the mug decreased by the same amount, the ethanol would continue to fill the mug. In that case we could not add any additional ethanol to the mug. But $\beta_{\text{ethanol}} > \beta_{\text{mug}}$, so the ethanol contracts more than the mug, leaving room to add additional ethanol. The amount of empty space after the contraction will be $V_{\text{empy}} = \Delta V_{\text{ethanol}} - \Delta V_{\text{mug}}$, which is what we want to find. Table 17.2 tells us that $\beta_{\text{Cu}} = 5.1\times10^{-5}(\text{C}°)^{-1}$ and $\beta_{\text{ethanol}} = 75\times10^{-5}(\text{C}°)^{-1}$.

EXECUTE: $V_{\text{empy}} = \Delta V_{\text{ethanol}} - \Delta V_{\text{mug}} = \beta_{\text{e}} V_0 \Delta T - \beta_{\text{c}} V_0 \Delta T = (\beta_{\text{e}} - \beta_{\text{c}}) V_0 \Delta T$. Using the coefficients of volume expansion from Table 17.2, $V_0 = 250\ \text{cm}^3$, and $\Delta T = -70.0°\text{C}$, we get $V_{\text{empty}} = -12\ \text{cm}^3$. This is the net change in volume, and it is negative since the volumes have decreased. The available volume is $12\ \text{cm}^3$.

EVALUATE: If β_{ethanol} were less than β_{mug}, ethanol would spill out of the mug as the temperature fell because the mug would contract more than the ethanol.

VP17.4.3. **IDENTIFY:** The thermal stress is due to the contraction of the brass rod as its temperature is decreased.

SET UP: Thermal stress is $\dfrac{F}{A} = -Y\alpha\Delta T$. Table 17.1 gives $\alpha_{\text{brass}} = 2.0\times10^{-5}(\text{C}°)^{-1}$ and Y_{brass} is given in the problem. The target variable is the thermal stress.

EXECUTE: The force is $F = -AY\alpha\Delta T = -\pi r^2 Y\alpha\Delta T$. Using the given values for Y and α, $r = 5.00\times10^{-3}\ \text{m}$, and $\Delta T = 13.0°\text{C} - 25.0°\text{C} = -12°\text{C}$, we get $F = 1700\ \text{N}$.

EVALUATE: The bar tends to shrink when cooled, so the force prevent this. Therefore the forces are *tensile.*

VP17.4.4. **IDENTIFY:** The rods increase in length when their temperature is raised, so we are dealing with thermal expansion.

SET UP: The change in length of the combined rod is the sum of the changes of the two rods. So we know that $L = L_A + L_B$ and $\Delta L = \Delta L_A + \Delta L_B$, and we use $\Delta L = \alpha L_0 \Delta T$ for each rod. We want to solve for L_A in terms of the other quantities.

EXECUTE: $\Delta L = \Delta L_A + \Delta L_B = \alpha_A L_A \Delta T + \alpha_B (L - L_A)\Delta T$. Solve for L_A: $L_A = \dfrac{\alpha_B L - \dfrac{\Delta L}{\Delta T}}{\alpha_B - \alpha_A}$.

EVALUATE: The units may not look right. But realize that $\Delta L = \alpha L_0 \Delta T$ tells us that $\dfrac{\Delta L}{\Delta T} = \alpha L_0$ so the units in the numerator are those of αL, so the overall units are those of $\dfrac{\alpha L}{\alpha}$, which are units of length.

VP17.9.1. **IDENTIFY:** The heat lost by the aluminum is equal to the heat gained by the water, so the net heat transfer for the system is zero.

SET UP: The aluminum is hotter, so it loses heat and the water gains that heat. Use $Q = mc\Delta T$ for each substance. The target variable is the mass of the aluminum. From Table 17.3 we have $c_{\text{Al}} = 910\ \text{J/kg}\cdot\text{K}$ and $c_{\text{w}} = 4190\ \text{J/kg}\cdot\text{K}$.

EXECUTE: Use $Q_{\text{Al}} + Q_{\text{w}} = 0$: $m_{\text{Al}} c_{\text{Al}} \Delta T_{\text{Al}} + m_{\text{w}} c_{\text{w}} \Delta T_{\text{w}} = 0$. Using the numbers gives $m_{\text{Al}}(910\ \text{J/kg}\cdot\text{K})(-228.0\ \text{K}) + (5.00\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(2.0°\text{C}) = 0$, so $m_{\text{Al}} = 0.20$ kg.

EVALUATE: It is *not true* that $T_C = T_K$. But it *is true* that $\Delta T_C = \Delta T_K$ because the size of the Kelvin degree is the same as the size of the Celsius degree.

VP17.9.2. **IDENTIFY:** Heat is transferred to the copper cube to the ice cube, and this heat melts the ice. We are dealing with temperature changes and a change of phase for the ice from solid to liquid.

SET UP: The net heat transfer between the ice and copper is zero because the heat lost by the copper is gained by the ice cube. For temperature changes we use $Q = mc\Delta T$ and for the phase change we use $Q = mL_f$. The target variable is the initial temperature of the copper.

EXECUTE: Use $Q_{\text{ice}} + Q_{\text{Cu}} = 0$ and call T the initial temperature of the copper. This gives $m_{\text{Cu}} c_{\text{Cu}} \Delta T_{\text{Cu}} + m_{\text{i}} L_{\text{i}} = 0$. Using c_{Cu} from Table 17.3 and L_f from Table 17.4, we have $(0.460\ \text{kg})(390\ \text{J/kg}\cdot\text{K})(0.00°\text{C} - T) + (7.50\times10^{-3}\ \text{kg})(334\times10^{3}\ \text{J/kg}) = 0$, so $T = 14.0°$C.

EVALUATE: Just the melting a small ice cube can produce a large temperature change for the copper because c_{Cu} is small and L_f is large for water.

VP17.9.3. **IDENTIFY:** We mix ice and ethanol and wait until the mixture reaches an equilibrium temperature. The heat lost by the ethanol is equal to the heat gained by the ice, so the net heat transferred is zero.

SET UP: The target variable is the mass of the ice, m_i. For temperature changes we use $Q = mc\Delta T$ and for the phase change we use $Q = mL_f$. The heat transferred to the ice causes three changes: (1) increase in the ice temperature from –5.00°C to 0.00°C, (2) melt the ice at 0.00°C, and (3) increase the temperature of the melted ice from 0.00°C to 10.0°C.

EXECUTE: Use $Q_{\text{ice}} + Q_{\text{ethanol}} = 0$ and include the three changes for the ice listed above. This gives $m_{\text{i}} c_{\text{i}} \Delta T_{\text{i}} + m_{\text{i}} L_{\text{i}} + m_{\text{i}} c_{\text{w}} \Delta T_{\text{w}} + m_{\text{e}} c_{\text{e}} \Delta T_{\text{e}} = 0$. We know that $m_e = 1.60$ kg, and from Tables 17.3 and 17.4, we have $c_i = 2100\ \text{J/kg}\cdot\text{K}$, $c_w = 4190\ \text{J/kg}\cdot\text{K}$, $c_e = 2428\ \text{J/kg}\cdot\text{K}$, and $L_f = 334\times10^{3}\ \text{J/kg}$. The temperature changes are $\Delta T_{\text{i}} = 5.00°\text{C}$, $\Delta T_{\text{w}} = 10.0°\text{C}$, and $\Delta T_{\text{e}} = -18.0°\text{C}$. Putting in the numbers and solving for m_i gives $m_i = 0.181$ kg.

EVALUATE: We must treat the ice in three stages. In addition to melting at 0°C, the ice and liquid water have different specific heats so we cannot treat the ice change from –5.00°C to 10.0°C in a single step.

VP17.9.4. **IDENTIFY:** The ice cools a silver ingot. Several possible outcomes are possible depending on the relative masses of the ice and silver and their initial temperatures: the ice could remain ice but at a higher temperature, the ice could partially melt and remain at 0.00°C, the ice could all melt and remain at 0.00°C , or the ice could all melt and increase its temperature above 0.00°C.

SET UP: For temperature changes use $Q = mc\Delta T$ and for the phase change use $Q = mL_f$. First see if there is enough heat in the silver to bring the ice up to its melting point temperature of 0.00°C and to melt it at 0.00°C. Use quantities from Tables 17.3 and 17.4 for water, ice, and silver. The target variable is the final equilibrium temperature T of the system.

EXECUTE: The heat to cool the silver down to 0.00°C is $Q = mc\Delta T$, which gives $Q = (1.25\text{ kg})(234\ \text{J/kg}\cdot\text{K})(315°\text{C}) = 9.214\times10^4\ \text{J}$. The heat needed to melt all the ice is $Q = mc\Delta T$ + $mL_f = (0.250\text{ kg})(2100\ \text{J/kg}\cdot\text{K})(8.00°\text{C}) + (0.250\text{ kg})(334\times10^3\ \text{J/kg}) = 8.77\times10^4\ \text{J}$. As we see, there is enough heat in the silver ingot to melt all the ice. Now we use the same procedure as in the previous problem except we have silver instead of ethanol. This gives $m_i c_i \Delta T_i + m_i L_i + m_i c_w \Delta T_w + m_s c_s \Delta T_s = 0$. The temperature changes are $\Delta T_i = 8.00°\text{C}$, $\Delta T_w = T - 00.0°\text{C}$, and $\Delta T_s = T - 315°\text{C}$, $m_i = 0.250$ kg, and $m_s = 1.25$ kg. The result is $T = 3.31°\text{C}$, and all the ice melts.

EVALUATE: We must treat the ice in three stages. In addition to melting at 0°C, the ice and liquid water have different specific heats so we cannot treat the ice change from –8.00°C to 3.31°C in a single step.

VP17.15.1. **IDENTIFY:** The heat flows through the pane of glass, so we are dealing with heat conduction.

SET UP: The rate of heat flow is $H = kA\dfrac{T_H - T_C}{L}$.

EXECUTE: **(a)** We want the thermal conductivity k of the glass. Solve $H = kA\dfrac{T_H - T_C}{L}$ for k, which gives $k = \dfrac{LH}{A(T_H - T_C)} = \dfrac{(0.00600\text{ m})(1100\text{ W})}{(0.500\text{ m})^2(35.0\text{ C°})} = 0.754\ \text{W/m}\cdot\text{K}$.

(b) $H \propto 1/L$, so $\dfrac{H_9}{H_6} = \dfrac{1/9}{1/6} = \dfrac{2}{3}$, which gives $H_9 = \dfrac{2}{3}(1100\text{ W}) = 733\text{ W}$.

EVALUATE: Other ways to decrease the heat current would be to decrease the area of the pane or to use a double-pane window which traps a layer of air between two panes of glass. The air has a *much* lower thermal conductivity than glass.

VP17.15.2. **IDENTIFY:** Heat current flows through the two end-to-end rods (brass and lead), so we are dealing with heat conduction.

SET UP: Use $H = kA\dfrac{T_H - T_C}{L}$. Table 17.5 gives $k_{\text{brass}} = 109.0\ \text{W/m}\cdot\text{K}$ and $k_{\text{lead}} = 34.7\ \text{W/m}\cdot\text{K}$. The heat current is the same (6.00 W) in both rods. The target variables are the temperatures of the free ends of the two-rod system.

EXECUTE: **(a)** Brass: Using $H = kA\dfrac{T_H - T_C}{L}$ and putting in the numbers gives

$$6.00\text{ W} = (109.0\text{ W/m}\cdot\text{K})(2.00\times10^{-4}\text{ m})\left(\frac{T_{\text{brass}} - 185°\text{C}}{0.250\text{ m}}\right)$$, so $T_{\text{brass}} = 254°\text{C}$.

(b) Lead: We follow the same procedure as for brass, giving

$$6.00\text{ W} = (34.7\text{ W/m}\cdot\text{K})(2.00\times10^{-4}\text{ m})\left(\frac{185°\text{C} - T_{\text{lead}}}{0.250\text{ m}}\right)$$, so $T_{\text{lead}} = –31°\text{C}$.

EVALUATE: From $H = kA\frac{T_H - T_C}{L}$ we see that $\Delta T \propto \frac{1}{k}$. So, if all else is the same, an object with a small k should have a large temperature difference compared to one with a large k. Our results agree with this since $\Delta T_{lead} > \Delta T_{brass}$ and $k_{lead} < k_{brass}$.

VP17.15.3. IDENTIFY: We are dealing the radiation from a star.

SET UP: $H = Ae\sigma T^4$, where T must be in K, $e = 1$ for the star, and $A = 4\pi R^2$.

EXECUTE: $H = Ae\sigma T^4 = 4\pi R^2\sigma T^4$, so solve for R, giving $R = \sqrt{\frac{H}{4\pi\sigma T^4}} = \frac{1}{T^2}\sqrt{\frac{H}{4\pi\sigma}}$. Putting in the numbers gives $R = \frac{1}{(9940\text{ K})^2}\sqrt{\frac{9.7\times10^{27}\text{ W}}{4\pi\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)}} = 1.2\times10^9\text{ m}$.

$\frac{R_{Sirius}}{R_{sun}} = \frac{1.2\times10^9\text{ m}}{6.96\times10^8\text{ m}} = 1.7$.

EVALUATE: The surface area of Sirius is $(1.7)^2 = 2.9$ times as great as our sun, so Sirius is a luminous star.

VP17.15.4. IDENTIFY: The building radiates heat into the air, but the hot air also radiates heat back into the building. So we are looking for the *net* radiation, which will be *into* the building because its surface is less hot than the outside air.

SET UP: $H_{net} = Ae\sigma\left(T^4 - T_s^4\right)$, where temperatures must be in K and $e = 0.91$.

EXECUTE: $H_{net} = (525\text{ m}^2)(0.91)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(293\text{ K})^4 - (308\text{ K})^4\right] = -4.4\times10^4\text{ W}$. The minus sign tells us that net heat flows *into* the building from the hot desert air at a rate of 4.4×10^4 W = 44 kW.

EVALUATE: The heat flow into the building would be even greater during the day when the outside temperature could be 40°C or even higher.

17.1. IDENTIFY and SET UP: $T_F = \frac{9}{5}T_C + 32°$.

EXECUTE: **(a)** $T_F = (9/5)(-62.8) + 32 = -81.0°\text{F}$.

(b) $T_F = (9/5)(56.7) + 32 = 134.1°\text{F}$.

(c) $T_F = (9/5)(31.1) + 32 = 88.0°\text{F}$.

EVALUATE: Fahrenheit degrees are smaller than Celsius degrees, so it takes more F° than C° to express the difference of a temperature from the ice point.

17.2. IDENTIFY and SET UP: To convert a temperature between °C and K use $T_C = T_K - 273.15$. To convert from °F to °C, subtract 32° and multiply by 5/9. To convert from °C to °F, multiply by 9/5 and add 32°. To convert a temperature difference, use that Celsius and Kelvin degrees are the same size and that $9\text{ F}° = 5\text{ C}°$.

EXECUTE: **(a)** $T_C = T_K - 273.15 = 310 - 273.15 = 36.9°\text{C}$; $T_F = \frac{9}{5}T_C + 32° = \frac{9}{5}(36.9°) + 32° = 98.4°\text{F}$.

(b) $T_K = T_C + 273.15 = 40 + 273.15 = 313\text{ K}$; $T_F = \frac{9}{5}T_C + 32° = \frac{9}{5}(40°) + 32° = 104°\text{F}$.

(c) 7 C° = 7 K; 7 C° = (7 C°)(9 F°/5 C°) = 13 F°.

(d) 4.0°C: $T_F = \frac{9}{5}T_C + 32° = \frac{9}{5}(4.0°) + 32° = 39.2°\text{F}$; $T_K = T_C + 273.15 = 4.0 + 273.15 = 277\text{ K}$.

$-160°\text{C}$: $T_F = \frac{9}{5}T_C + 32° = \frac{9}{5}(-160°) + 32° = -256°\text{F}$; $T_K = T_C + 273.15 = -160 + 273.15 = 113\text{ K}$.

(e) $T_C = \frac{5}{9}(T_F - 32°) = \frac{5}{9}(105° - 32°) = 41°\text{C}$; $T_K = T_C + 273.15 = 41 + 273.15 = 314\text{ K}$.

EVALUATE: Celsius-Fahrenheit conversions do not involve simple proportions due to the additive constant of 32°, but Celsius-Kelvin conversions require only simple addition/subtraction of 273.15.

17.3. **IDENTIFY:** Convert ΔT between different scales.

SET UP: ΔT is the same on the Celsius and Kelvin scales. $180\ \text{F}° = 100\ \text{C}°$, so $1\ \text{C}° = \frac{9}{5}\text{F}°$.

EXECUTE: **(a)** $\Delta T = 49.0\ \text{F}°$. $\Delta T = (49.0\ \text{F}°)\left(\dfrac{1\ \text{C}°}{\frac{9}{5}\ \text{F}°}\right) = 27.2\ \text{C}°$.

(b) $\Delta T = -100\ \text{F}°$. $\Delta T = (-100.0\ \text{F}°)\left(\dfrac{1\ \text{C}°}{\frac{9}{5}\ \text{F}°}\right) = -55.6\ \text{C}°$

EVALUATE: The magnitude of the temperature change is larger in F° than in C°.

17.4. **IDENTIFY:** This problem requires conversion of temperature scales. We want T_K as a function of T_F and T_F as a function of T_K.

SET UP and EXECUTE: We know that $T_K = T_C + 273.15$ and $T_C = \dfrac{5}{9}(T_F - 32°)$. Using these gives

$T_K = T_C + 273.15 = \dfrac{5}{9}(T_F - 32°) + 273.15$. This reduces to $T_K = \dfrac{5}{9}T_F + 255.37$.

Solve this result for T_F, giving $T_F = \dfrac{9}{5}T_K - 459.67$.

EVALUATE: The fractions 5/9 and 5/9 occur because the Kelvin degree is not the same size as the Fahrenheit degree; the Kelvin degree is larger.

17.5. **IDENTIFY:** Convert ΔT in kelvins to C° and to F°.

SET UP: $1\ \text{K} = 1\ \text{C}° = \frac{9}{5}\text{F}°$

EXECUTE: **(a)** $\Delta T_F = \frac{9}{5}\Delta T_C = \frac{9}{5}(-10.0\ \text{C}°) = -18.0\ \text{F}°$

(b) $\Delta T_C = \Delta T_K = -10.0\ \text{C}°$

EVALUATE: Kelvin and Celsius degrees are the same size. Fahrenheit degrees are smaller, so it takes more of them to express a given ΔT value.

17.6. **IDENTIFY:** Set $T_C = T_F$ and $T_F = T_K$.

SET UP: $T_F = \frac{9}{5}T_C + 32°\text{C}$ and $T_K = T_C + 273.15 = \frac{5}{9}(T_F - 32°) + 273.15$

EXECUTE: **(a)** $T_F = T_C = T$ gives $T = \frac{9}{5}T + 32°$ and $T = -40°$; $-40°\text{C} = -40°\text{F}$.

(b) $T_F = T_K = T$ gives $T = \frac{5}{9}(T - 32°) + 273.15$ and $T = \frac{9}{4}(-(\frac{5}{9})(32°) + 273.15) = 575°$; $575°\text{F} = 575\ \text{K}$.

EVALUATE: Since $T_K = T_C + 273.15$ there is no temperature at which Celsius and Kelvin thermometers agree.

17.7. **IDENTIFY:** When the volume is constant, $\dfrac{T_2}{T_1} = \dfrac{p_2}{p_1}$, for T in kelvins.

SET UP: $T_{\text{triple}} = 273.16\ \text{K}$. Figure 17.7 in the textbook gives that the temperature at which CO_2 solidifies is $T_{CO_2} = 195\ \text{K}$.

EXECUTE: $p_2 = p_1\left(\dfrac{T_2}{T_1}\right) = (1.35\ \text{atm})\left(\dfrac{195\ \text{K}}{273.16\ \text{K}}\right) = 0.964\ \text{atm}$

EVALUATE: The pressure decreases when T decreases.

17.8. **IDENTIFY:** Convert T_K to T_C and then convert T_C to T_F.

SET UP: $T_K = T_C + 273.15$ and $T_F = \frac{9}{5}T_C + 32°$.

EXECUTE: **(a)** $T_C = 400 - 273.15 = 127°\text{C}$, $T_F = (9/5)(126.85) + 32 = 260°\text{F}$

(b) $T_C = 95 - 273.15 = -178°\text{C}$, $T_F = (9/5)(-178.15) + 32 = -289°\text{F}$

(c) $T_C = 1.55 \times 10^7 - 273.15 = 1.55 \times 10^7 °\text{C}$, $T_F = (9/5)(1.55 \times 10^7) + 32 = 2.79 \times 10^7 °\text{F}$

EVALUATE: All temperatures on the Kelvin scale are positive. T_C is negative if the temperature is below the freezing point of water.

17.9. **IDENTIFY** and **SET UP:** Fit the data to a straight line for $p(T)$ and use this equation to find T when $p=0$.

EXECUTE: **(a)** If the pressure varies linearly with temperature, then $p_2 = p_1 + \gamma(T_2 - T_1)$.

$$\gamma = \frac{p_2 - p_1}{T_2 - T_1} = \frac{6.50\times10^4\ \text{Pa} - 4.80\times10^4\ \text{Pa}}{100°\text{C} - 0.01°\text{C}} = 170.0\ \text{Pa/C°}$$

Apply $p = p_1 + \gamma(T - T_1)$ with $T_1 = 0.01°\text{C}$ and $p = 0$ to solve for T.

$$0 = p_1 + \gamma(T - T_1)$$

$$T = T_1 - \frac{p_1}{\gamma} = 0.01°\text{C} - \frac{4.80\times10^4\ \text{Pa}}{170\ \text{Pa/C°}} = -282°\text{C}.$$

(b) Let $T_1 = 100°\text{C}$ and $T_2 = 0.01°\text{C}$; use $T_2/T_1 = p_2/p_1$ to calculate p_2, where T is in kelvins.

$$p_2 = p_1\left(\frac{T_2}{T_1}\right) = 6.50\times10^4\ \text{Pa}\left(\frac{0.01+273.15}{100+273.15}\right) = 4.76\times10^4\ \text{Pa};$$ this differs from the 4.80×10^4 Pa that was measured so $T_2/T_1 = p_2/p_1$ is not precisely obeyed.

EVALUATE: The answer to part (a) is in reasonable agreement with the accepted value of $-273°\text{C}$.

17.10. **IDENTIFY:** Apply $T = T_{\text{triple}}\dfrac{p}{p_{\text{triple}}} = (273.16\ \text{K})\dfrac{p}{p_{\text{triple}}}$ and solve for p.

SET UP: $p_{\text{triple}} = 325$ mm of mercury

EXECUTE: $p = (325.0\ \text{mm of mercury})\left(\dfrac{373.15\ \text{K}}{273.16\ \text{K}}\right) = 444$ mm of mercury

EVALUATE: mm of mercury is a unit of pressure. Since $T = T_{\text{triple}}\dfrac{p}{p_{\text{triple}}} = (273.16\ \text{K})\dfrac{p}{p_{\text{triple}}}$ involves a ratio of pressures, it is not necessary to convert the pressure to units of Pa.

17.11. **IDENTIFY:** $\Delta L = L_0\alpha\Delta T$

SET UP: For steel, $\alpha = 1.2\times10^{-5}\ (\text{C°})^{-1}$

EXECUTE: $\Delta L = (1.2\times10^{-5}(\text{C°})^{-1})(1410\ \text{m})(18.0°\text{C} - (-5.0°\text{C})) = +0.39\ \text{m}$

EVALUATE: The length increases when the temperature increases. The fractional increase is very small, since $\alpha\Delta T$ is small.

17.12. **IDENTIFY:** Apply $\Delta L = \alpha L_0 \Delta T$ and calculate ΔT. Then $T_2 = T_1 + \Delta T$, with $T_1 = 15.5°\text{C}$.

SET UP: Table 17.1 gives $\alpha = 1.2\times10^{-5}\ (\text{C°})^{-1}$ for steel.

EXECUTE: $\Delta T = \dfrac{\Delta L}{\alpha L_0} = \dfrac{0.471\ \text{ft}}{[1.2\times10^{-5}\ (\text{C°})^{-1}][1671\ \text{ft}]} = 23.5\ \text{C°}$. $T_2 = 15.5°\text{C} + 23.5\ \text{C°} = 39.0°\text{C}$.

EVALUATE: Since then the lengths enter in the ratio $\Delta L/L_0$, we can leave the lengths in ft.

17.13. **IDENTIFY:** Apply $L = L_0(1+\alpha\Delta T)$ to the diameter D of the penny.

SET UP: $1\ \text{K} = 1\ \text{C°}$, so we can use temperatures in °C.

EXECUTE: Death Valley: $\alpha D_0\Delta T = [2.6\times10^{-5}\ (\text{C°})^{-1}](1.90\ \text{cm})(28.0\ \text{C°}) = 1.4\times10^{-3}$ cm, so the diameter is 1.9014 cm. Greenland: $\alpha D_0 \Delta T = -3.6\times10^{-3}$ cm, so the diameter is 1.8964 cm.

EVALUATE: When T increases the diameter increases and when T decreases the diameter decreases.

17.14. IDENTIFY: Apply $L = L_0(1+\alpha\Delta T)$ to the diameter d of the rivet.

SET UP: For aluminum, $\alpha = 2.4\times10^{-5}\ (\text{C}°)^{-1}$. Let d_0 be the diameter at –78.0°C and d be the diameter at 23.0°C.

EXECUTE: $d = d_0 + \Delta d = d_0(1+\alpha\Delta T) = (0.4500\ \text{cm})(1+(2.4\times10^{-5}(\text{C}°)^{-1}[23.0°\text{C} - (-78.0°\text{C})])$.
$d = 0.4511\ \text{cm} = 4.511\ \text{mm}$.

EVALUATE: We could have let d_0 be the diameter at 23.0°C and d be the diameter at −78.0°C. Then $\Delta T = -78.0°\text{C} - 23.0°\text{C}$.

17.15. IDENTIFY: Apply $\Delta V = V_0\beta\Delta T$.

SET UP: For copper, $\beta = 5.1\times10^{-5}\ (\text{C}°)^{-1}$. $\Delta V/V_0 = 0.150\times10^{-2}$.

EXECUTE: $\Delta T = \dfrac{\Delta V/V_0}{\beta} = \dfrac{0.150\times10^{-2}}{5.1\times10^{-5}\ (\text{C}°)^{-1}} = 29.4\ \text{C}°$. $T_\text{f} = T_\text{i} + \Delta T = 49.4°\text{C}$.

EVALUATE: The volume increases when the temperature increases.

17.16. IDENTIFY: $\Delta V = \beta V_0\Delta T$. Use the diameter at −15°C to calculate the value of V_0 at that temperature.

SET UP: For a hemisphere of radius R, the volume is $V = \frac{2}{3}\pi R^3$. Table 17.2 gives $\beta = 7.2\times10^{-5}\ (\text{C}°)^{-1}$ for aluminum.

EXECUTE: $V_0 = \frac{2}{3}\pi R^3 = \frac{2}{3}\pi(27.5\ \text{m})^3 = 4.356\times10^4\ \text{m}^3$.
$\Delta V = (7.2\times10^{-5}\ (\text{C}°)^{-1})(4.356\times10^4\ \text{m}^3)[35°\text{C} - (-15°\text{C})] = 160\ \text{m}^3$.

EVALUATE: We could also calculate $R = R_0(1+\alpha\Delta T)$ and calculate the new V from R. The increase in volume is $V - V_0$, but we would have to be careful to avoid round-off errors when two large volumes of nearly the same size are subtracted.

17.17. IDENTIFY: Apply $\Delta V = V_0\beta\Delta T$ to the volume of the flask and to the mercury. When heated, both the volume of the flask and the volume of the mercury increase.

SET UP: For mercury, $\beta_\text{Hg} = 18\times10^{-5}\ (\text{C}°)^{-1}$.

8.95 cm^3 of mercury overflows, so $\Delta V_\text{Hg} - \Delta V_\text{glass} = 8.95\ \text{cm}^3$.

EXECUTE: $\Delta V_\text{Hg} = V_0\beta_\text{Hg}\Delta T = (1000.00\ \text{cm}^3)(18\times10^{-5}\ (\text{C}°)^{-1})(55.0\ \text{C}°) = 9.9\ \text{cm}^3$.

$\Delta V_\text{glass} = \Delta V_\text{Hg} - 8.95\ \text{cm}^3 = 0.95\ \text{cm}^3$. $\beta_\text{glass} = \dfrac{\Delta V_\text{glass}}{V_0\Delta T} = \dfrac{0.95\ \text{cm}^3}{(1000.00\ \text{cm}^3)(55.0\ \text{C}°)} = 1.7\times10^{-5}\ (\text{C}°)^{-1}$.

EVALUATE: The coefficient of volume expansion for the mercury is larger than for glass. When they are heated, both the volume of the mercury and the inside volume of the flask increase. But the increase for the mercury is greater and it no longer all fits inside the flask.

17.18. IDENTIFY: Apply $\Delta V = V_0\beta\Delta T$ to the tank and to the ethanol.

SET UP: For ethanol, $\beta_\text{e} = 75\times10^{-5}\ (\text{C}°)^{-1}$. For steel, $\beta_\text{s} = 3.6\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: The volume change for the tank is
$\Delta V_\text{s} = V_0\beta_\text{s}\Delta T = (1.90\ \text{m}^3)(3.6\times10^{-5}(\text{C}°)^{-1})(-14.0\ \text{C}°) = -9.576\times10^{-4}\ \text{m}^3 = -0.9576\ \text{L}$.
The volume change for the ethanol is
$\Delta V_\text{e} = V_0\beta_\text{e}\Delta T = (1.90\ \text{m}^3)(75\times10^{-5}\ (\text{C}°)^{-1})(-14.0\ \text{C}°) = -1.995\times10^{-2}\ \text{m}^3 = -19.95\ \text{L}$.
The empty volume in the tank is $\Delta V_\text{e} - \Delta V_\text{s} = -19.95\ \text{L} - (-0.9576\ \text{L}) = -19.0\ \text{L}$. So 19.0 L of ethanol can be added to the tank.

EVALUATE: Both volumes decrease. But $\beta_e > \beta_s$, so the magnitude of the volume decrease for the ethanol is greater than it is for the tank.

17.19. IDENTIFY and SET UP: Use $\Delta A = 2\alpha A_0 \Delta T$ to calculate ΔA for the plate, and then $A = A_0 + \Delta A$.

EXECUTE: **(a)** $A_0 = \pi r_0^2 = \pi\left(\dfrac{1.350\text{ cm}}{2}\right)^2 = 1.431\text{ cm}^2$.

(b) Using $\Delta A = 2\alpha A_0 \Delta T$, we have

$\Delta A = 2(1.2\times10^{-5}\text{ C}^{\circ -1})(1.431\text{ cm}^2)(175°\text{C} - 25°\text{C}) = 5.15\times10^{-3}\text{ cm}^2$ $A = A_0 + \Delta A = 1.436\text{ cm}^2$.

EVALUATE: A hole in a flat metal plate expands when the metal is heated just as a piece of metal the same size as the hole would expand.

17.20. IDENTIFY: We are looking at the thermal expansion of the area of a metal plate.

SET UP: For length and width, we use $\Delta L = \alpha L_0 \Delta T$. We want to find a comparable formula for the change in the area of the plate. Fig. 17.20 shows the changes in the plate.

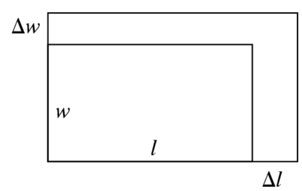


Figure 17.20

EXECUTE: The original area is $A_0 = \ell w$. The length expands by an amount $\Delta\ell = \alpha\ell\Delta T$ and width expands by $\Delta w = \alpha w \Delta T$. The expanded area is $A_2 = (\ell + \Delta\ell)(w + \Delta w) = \ell w + \ell\Delta w + w\Delta\ell + \Delta\ell\Delta w$. The change in area is $\Delta A = A_2 - A_0 = (\ell w + \ell\Delta w + w\Delta\ell + \Delta\ell\Delta w) - \ell w = \ell\Delta w + w\Delta\ell + \Delta\ell\Delta w$. Using $A_0 = \ell w$, $\Delta\ell = \alpha\ell\Delta T$, and $\Delta w = \alpha w\Delta T$, we get $\Delta A = \ell w\alpha\Delta T + w\ell\alpha\Delta T + \ell\alpha\Delta T w\alpha\Delta T = 2\alpha A_0\Delta T + \ell\alpha\Delta T w\alpha\Delta T$. The last term contains α^2, which is much smaller than α because α is typically very small. So we can drop the last term, leaving $\Delta A = 2\alpha A_0 \Delta T$.

EVALUATE: This result has the same form as linear and volume expansion. For volume expansion the coefficient of volume expansion is $\beta = 3\alpha$, and for area expansion the coefficient of area expansion is 2α.

17.21. IDENTIFY: Apply $\Delta L = L_0\alpha\Delta T$ and $\text{stress} = F/A = -Y\alpha\Delta T$.

SET UP: For steel, $\alpha = 1.2\times10^{-5}\ (\text{C}^\circ)^{-1}$ and $Y = 2.0\times10^{11}$ Pa.

EXECUTE: **(a)** $\Delta L = L_0\alpha\Delta T = (12.0\text{ m})(1.2\times10^{-5}\ (\text{C}^\circ)^{-1})(42.0\text{ C}^\circ) = 0.0060\text{ m} = 6.0\text{ mm}$.

(b) $\text{stress} = -Y\alpha\Delta T = -(2.0\times10^{11}\text{ Pa})(1.2\times10^{-5}\ (\text{C}^\circ)^{-1})(42.0\text{ C}^\circ) = -1.0\times10^{8}\text{ Pa}$. The minus sign means the stress is compressive.

EVALUATE: Commonly occurring temperature changes result in very small fractional changes in length but very large stresses if the length change is prevented from occurring.

17.22. IDENTIFY: Apply $\text{stress} = F/A = -Y\alpha\Delta T$ and solve for F.

SET UP: For brass, $Y = 0.9\times10^{11}$ Pa and $\alpha = 2.0\times10^{-5}\ (\text{C}^\circ)^{-1}$.

EXECUTE: $F = -Y\alpha\Delta T A = -(0.9\times10^{11}\text{ Pa})(2.0\times10^{-5}(\text{C}^\circ)^{-1})(-110\text{ C}^\circ)(2.01\times10^{-4}\text{ m}^2) = 4.0\times10^{4}\text{ N}$

EVALUATE: A large force is required. ΔT is negative and a positive tensile force is required.

17.23. **IDENTIFY:** We are dealing with the thermal expansion of two rods.

SET UP: Use $\Delta L = \alpha L_0 \Delta T$. From Table 17.1 we get the coefficients of linear expansion for aluminum and Invar. We know that $\Delta L_{\text{Al}} = 2\Delta L_{\text{Invar}}$ and $\Delta T_{\text{Al}} = \frac{1}{3}\Delta T_{\text{Invar}}$. We want $L_{\text{Al}}/L_{\text{Invar}}$.

EXECUTE: Apply $\Delta L = \alpha L_0 \Delta T$ to each bar and take the ratio of the length changes, giving $\frac{\Delta L_{\text{Al}}}{\Delta L_{\text{Invar}}} = \frac{\alpha_{\text{Al}} L_{\text{Al}} \Delta T_{\text{Al}}}{\alpha_{\text{Invar}} L_{\text{Invar}} \Delta T_{\text{Invar}}}$. Solving for the length ratio gives $\frac{L_{\text{Al}}}{L_{\text{Invar}}} = \frac{\alpha_{\text{Invar}} \Delta L_{\text{Al}} \Delta T_{\text{Invar}}}{\alpha_{\text{Al}} \Delta L_{\text{Invar}} \Delta T_{\text{Al}}} = \left(\frac{0.09}{2.4}\right)\left(\frac{\Delta L_{\text{Al}}}{\Delta L_{\text{Invar}}}\right)\left(\frac{3\Delta T_{\text{Al}}}{\Delta T_{\text{Al}}}\right) = 0.225$, which we round to 0.2. This also gives $\frac{L_{\text{Invar}}}{L_{\text{Al}}} = \frac{1}{0.225} = 4.$

EVALUATE: The coefficient of linear expansion for Invar is much less than that of aluminum, so the Invar rod needs to be much longer than the aluminum rod to expand half as much with only 1/3 the temperature change.

17.24. **IDENTIFY:** The heat required is $Q = mc\Delta T$. $P = 200\text{ W} = 200\text{ J/s}$, which is energy divided by time.

SET UP: For water, $c = 4.19\times10^3\text{ J/kg}\cdot\text{K}$.

EXECUTE: **(a)** $Q = mc\Delta T = (0.320\text{ kg})(4.19\times10^3\text{ J/kg}\cdot\text{K})(60.0\text{ C}°) = 8.04\times10^4\text{ J}$

(b) $t = \dfrac{8.04\times10^4\text{ J}}{200.0\text{ J/s}} = 402\text{ s} = 6.7\text{ min}$

EVALUATE: 0.320 kg of water has volume 0.320 L. The time we calculated in part (b) is consistent with our everyday experience.

17.25. **IDENTIFY** and **SET UP:** Apply $Q = mc\,\Delta T$ to the kettle and water.

EXECUTE: <u>kettle</u>

$Q = mc\Delta T$, $c = 910\text{ J/kg}\cdot\text{K}$ (from Table 17.3)

$Q = (1.10\text{ kg})(910\text{ J/kg}\cdot\text{K})(85.0°\text{C} - 20.0°\text{C}) = 6.5065\times10^4\text{ J}$

<u>water</u>

$Q = mc\Delta T$, $c = 4190\text{ J/kg}\cdot\text{K}$ (from Table 17.3)

$Q = (1.80\text{ kg})(4190\text{ J/kg}\cdot\text{K})(85.0°\text{C} - 20.0°\text{C}) = 4.902\times10^5\text{ J}$

Total $Q = 6.5065\times10^4\text{ J} + 4.902\times10^5\text{ J} = 5.55\times10^5\text{ J}.$

EVALUATE: Water has a much larger specific heat capacity than aluminum, so most of the heat goes into raising the temperature of the water.

17.26. **IDENTIFY** and **SET UP:** Use $Q = mc\,\Delta T$.

EXECUTE: **(a)** $Q = mc\Delta T$

$m = \frac{1}{2}(1.3\times10^{-3}\text{ kg}) = 0.65\times10^{-3}\text{ kg}$

$Q = (0.65\times10^{-3}\text{ kg})(1020\text{ J/kg}\cdot\text{K})\big(37°\text{C} - (-20°\text{C})\big) = 38\text{ J}.$

(b) 20 breaths/min $(60\text{ min/1 h}) = 1200$ breaths/h

So $Q = (1200)(38\text{ J}) = 4.6\times10^4\text{ J}.$

EVALUATE: The heat loss rate is $Q/t = 13\text{ W}$.

17.27. **IDENTIFY:** Apply $Q = mc\Delta T$ to find the heat that would raise the temperature of the student's body 7 C°.

SET UP: $1\text{ W} = 1\text{ J/s}$

EXECUTE: Find Q to raise the body temperature from 37°C to 44°C.

$Q = mc\Delta T = (70\text{ kg})(3480\text{ J/kg}\cdot\text{K})(7\text{ C}°) = 1.7\times10^6\text{ J}.$

$$t = \frac{1.7\times 10^6 \text{ J}}{1200 \text{ J/s}} = 1400 \text{ s} = 23 \text{ min.}$$

EVALUATE: Heat removal mechanisms are essential to the well-being of a person.

17.28. **IDENTIFY:** The heat input increases the temperature of 2.5 gal/min of water from 10°C to 49°C.
SET UP: 1.00 L of water has a mass of 1.00 kg, so
$9.46 \text{ L/min} = (9.46 \text{ L/min})(1.00 \text{ kg/L})(1 \text{ min}/60 \text{ s}) = 0.158 \text{ kg/s}$. For water, $c = 4190 \text{ J/kg}\cdot\text{C°}$.
EXECUTE: $Q = mc\Delta T$ so $H = (Q/t) = (m/t)c\,\Delta T$. Putting in the numbers gives

$H = (0.158 \text{ kg/s})(4190 \text{ J/kg}\cdot\text{C°})(49\text{°C} - 10\text{°C}) = 2.6\times 10^4 \text{ W} = 26 \text{ kW}$.

EVALUATE: The power requirement is large, the equivalent of 260 100-watt light bulbs, but this large power is needed only for short periods of time. The rest of the time, the unit uses no energy, unlike a conventional water heater, which continues to replace lost heat even when hot water is not needed.

17.29. **IDENTIFY:** Apply $Q = mc\Delta T$. $m = w/g$.
SET UP: The temperature change is $\Delta T = 18.0$ K.

EXECUTE: $c = \dfrac{Q}{m\Delta T} = \dfrac{gQ}{w\Delta T} = \dfrac{(9.80 \text{ m/s}^2)(1.25\times 10^4 \text{ J})}{(28.4 \text{ N})(18.0 \text{ K})} = 240 \text{ J/kg}\cdot\text{K}$.

EVALUATE: The value for c is similar to that for silver in Table 17.3, so it is a reasonable result.

17.30. **IDENTIFY:** The work done by the brakes equals the initial kinetic energy of the train. Use the volume of the air to calculate its mass. Use $Q = mc\Delta T$ applied to the air to calculate ΔT for the air.

SET UP: $K = \frac{1}{2}mv^2$. $m = \rho V$.

EXECUTE: The initial kinetic energy of the train is $K = \frac{1}{2}(25{,}000 \text{ kg})(15.5 \text{ m/s})^2 = 3.00\times 10^6$ J.

Therefore, Q for the air is 3.00×10^6 J.
$m = \rho V = (1.20 \text{ kg/m}^3)(65.0 \text{ m})(20.0 \text{ m})(12.0 \text{ m}) = 1.87\times 10^4$ kg. $Q = mc\Delta T$ gives

$$\Delta T = \frac{Q}{mc} = \frac{3.00\times 10^6 \text{ J}}{(1.87\times 10^4 \text{ kg})(1020 \text{ J/kg}\cdot\text{K})} = 0.157 \text{ C°}.$$

EVALUATE: The mass of air in the station is comparable to the mass of the train and the temperature rise is small.

17.31. **IDENTIFY** and **SET UP:** Set the change in gravitational potential energy equal to the quantity of heat added to the water.
EXECUTE: The change in mechanical energy equals the decrease in gravitational potential energy, $\Delta U = -mgh$; $|\Delta U| = mgh$. $Q = |\Delta U| = mgh$ implies $mc\Delta T = mgh$

$\Delta T = gh/c = (9.80 \text{ m/s}^2)(225 \text{ m})/(4190 \text{ J/kg}\cdot\text{K}) = 0.526 \text{ K} = 0.526 \text{ C°}$

EVALUATE: Note that the answer is independent of the mass of the object. Note also the small change in temperature that corresponds to this large change in height!

17.32. **IDENTIFY:** Set the energy delivered to the nail equal to $Q = mc\Delta T$ for the nail and solve for ΔT.

SET UP: For aluminum, $c = 0.91\times 10^3 \text{ J/kg}\cdot\text{K}$. $K = \frac{1}{2}mv^2$.

EXECUTE: The kinetic energy of the hammer before it strikes the nail is
$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.80 \text{ kg})(7.80 \text{ m/s})^2 = 54.8$ J. Each strike of the hammer transfers $0.60(54.8 \text{ J}) = 32.9$ J, and with 10 strikes $Q = 329$ J. $Q = mc\Delta T$ and

$$\Delta T = \frac{Q}{mc} = \frac{329 \text{ J}}{(8.00\times 10^{-3} \text{ kg})(0.91\times 10^3 \text{ J/kg}\cdot\text{K})} = 45.2 \text{ C°}.$$

EVALUATE: This agrees with our experience that hammered nails get noticeably warmer.

17.33. IDENTIFY: Some of the kinetic energy of the bullet is transformed through friction into heat, which raises the temperature of the water in the tank.

SET UP: Set the loss of kinetic energy of the bullet equal to the heat energy Q transferred to the water. $Q = mc\Delta T$. From Table 17.3, the specific heat of water is 4.19×10^3 J/kg·C°.

EXECUTE: The kinetic energy lost by the bullet is

$K_i - K_f = \frac{1}{2}m(v_i^2 - v_f^2) = \frac{1}{2}(15.0\times10^{-3}\text{ kg})[(865\text{ m/s})^2 - (534\text{ m/s})^2] = 3.47\times10^3\text{ J}$, so for the water

$Q = 3.47\times10^3$ J. $Q = mc\Delta T$ gives $\Delta T = \dfrac{Q}{mc} = \dfrac{3.47\times10^3\text{ J}}{(13.5\text{ kg})(4.19\times10^3\text{ J/kg}\cdot\text{C°})} = 0.0613\text{ C°}$.

EVALUATE: The heat energy required to change the temperature of ordinary-size objects is very large compared to the typical kinetic energies of moving objects.

17.34. IDENTIFY: The amount of heat lost by the boiling water is equal to the amount of heat gained by the water in the beaker.

SET UP: Calculate Q for each mass of water and set their algebraic sum equal to zero. Let the water you add have mass m. The 750 g of cold water has a temperature change of $+65$ C° and a heat flow Q_c. The mass m of water has a temperature change of -25 C° and a heat flow Q_h. $Q = mc\,\Delta T$.

EXECUTE: $Q_c = mc\,\Delta T = (0.750\text{ kg})(4190\text{ J/kg}\cdot\text{C°})(65\text{ C°}) = 2.043\times10^5\text{ J}$

$Q_h = mc\,\Delta T = m(4190\text{ J/kg}\cdot\text{C°})(-25\text{ C°}) = -(1.048\times10^5\text{ J/kg})m$

$Q_h + Q_c = 0$ so $2.043\times10^5\text{ J} + (-1.048\times10^5\text{ J/kg})m = 0$ and $m = 1.95\text{ kg} = 1950\text{ g}$.

EVALUATE: The amount of water we need to add (1950 g) is considerably greater than the water already in the beaker (750 g). This is reasonable because we want the final temperature to be closer to the 100°C temperature of the boiling water than to the original 10.0°C temperature of the original water.

17.35. IDENTIFY and SET UP: Heat comes out of the metal and into the water. The final temperature is in the range $0 < T < 100°\text{C}$, so there are no phase changes. $Q_{\text{system}} = 0$.

(a) EXECUTE: $Q_{\text{water}} + Q_{\text{metal}} = 0$

$m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} + m_{\text{metal}}c_{\text{metal}}\Delta T_{\text{metal}} = 0$

$(1.00\text{ kg})(4190\text{ J/kg}\cdot\text{K})(2.0\text{ C°}) + (0.500\text{ kg})(c_{\text{metal}})(-78.0\text{ C°}) = 0$

$c_{\text{metal}} = 215\text{ J/kg}\cdot\text{K}$

(b) EVALUATE: Water has a larger specific heat capacity so stores more heat per degree of temperature change.

(c) If some heat went into the styrofoam then Q_{metal} should actually be larger than in part (a), so the true c_{metal} is larger than we calculated; the value we calculated would be smaller than the true value.

17.36. IDENTIFY: The heat that comes out of the person goes into the ice-water bath and causes some of the ice to melt.

SET UP: Normal body temperature is 98.6°F = 37.0°C, so for the person $\Delta T = -5$ C°. The ice-water bath stays at 0°C. A mass m of ice melts and $Q_{\text{ice}} = mL_f$. From Table 17.4, for water $L_f = 334\times10^3$ J/kg.

EXECUTE: $Q_{\text{person}} = mc\Delta T = (70.0\text{ kg})(3480\text{ J/kg}\cdot\text{C°})(-5.0\text{ C°}) = -1.22\times10^6$ J. Therefore, the amount of heat that goes into the ice is 1.22×10^6 J. $m_{\text{ice}}L_f = 1.22\times10^6$ J and $m_{\text{ice}} = \dfrac{1.22\times10^6\text{ J}}{334\times10^3\text{ J/kg}} = 3.7\text{ kg}$.

EVALUATE: If less ice than this is used, all the ice melts and the temperature of the water in the bath rises above 0°C.

17.37. **IDENTIFY:** The amount of heat lost by the iron is equal to the amount of heat gained by the water. The water must first be heated to 100°C and then vaporized.

SET UP: The relevant equations are $Q = mc\Delta T$ and $Q = L_v m.$ The specific heat of iron is $c_{\text{iron}} = 0.47\times10^3\ \text{J/(kg}\cdot\text{K)},$ the specific heat of water is $c_{\text{water}} = 4.19\times10^3\ \text{J/(kg}\cdot\text{K)},$ and the heat of vaporization of water is $L_v = 2256\times10^3\ \text{J/kg}.$

EXECUTE: The iron cools: $Q_{\text{iron}} = m_i c_i \Delta T_i.$

The water warms and vaporizes: $Q_{\text{water}} = c_w m_w \Delta T_w + m_w L_{v_w} = m_w(c_w \Delta T_w + L_{v_w}).$

Assume that all of the heat lost by the iron is gained by the water so that $Q_{\text{water}} = -Q_{\text{iron}}.$ Equating the respective expressions for each Q and solving for m_w we obtain

$$m_w = \frac{-m_i c_i \Delta T_i}{c_w \Delta T_w + L_{v_w}} = \frac{-(1.20\ \text{kg})(0.47\times10^3\ \text{J/kg}\cdot\text{K})(120.0^\circ\text{C} - 650.0^\circ\text{C})}{(4.19\times10^3\ \text{J/kg}\cdot\text{K})(100.0^\circ\text{C} - 15.0^\circ\text{C}) + 2256\times10^3\ \text{J/kg}} = 0.114\ \text{kg}.$$

EVALUATE: Note that only a relatively small amount of water is required to cause a very large temperature change in the iron. This is due to the high heat of vaporization and specific heat of water, and the relatively low specific heat capacity of iron.

17.38. **IDENTIFY:** The initial temperature of the ice and water mixture is $0.0°\text{C}.$ Assume all the ice melts. We will know that assumption is incorrect if the final temperature we calculate is less than $0.0°\text{C}.$ The net Q for the system (can, water, ice and lead) is zero.

SET UP: For copper, $c_c = 390\ \text{J/kg}\cdot\text{K}.$ For lead, $c_l = 130\ \text{J/kg}\cdot\text{K}.$ For water, $c_w = 4.19\times10^3\ \text{J/kg}\cdot\text{K}$ and $L_f = 3.34\times10^5\ \text{J/kg}.$

EXECUTE: For the copper can, $Q_c = m_c c_c \Delta T_c = (0.100\ \text{kg})(390\ \text{J/kg}\cdot\text{K})(T - 0.0°\text{C}) = (39.0\ \text{J/K})T.$

For the water, $Q_w = m_w c_w \Delta T_w = (0.160\ \text{kg})(4.19\times10^3\ \text{J/kg}\cdot\text{K})(T - 0.0°\text{C}) = (670.4\ \text{J/K})T.$

For the ice, $Q_i = m_i L_f + m_i c_w \Delta T_w$

$Q_i = (0.018\ \text{kg})(3.34\times10^5\ \text{J/kg}) + (0.018\ \text{kg})(4.19\times10^3\ \text{J/kg}\cdot\text{K})(T - 0.0°\text{C}) = 6012\ \text{J} + (75.4\ \text{J/K})T$

For the lead, $Q_l = m_l c_l \Delta T_l = (0.750\ \text{kg})(130\ \text{J/kg}\cdot\text{K})(T - 255°\text{C}) = (97.5\ \text{J/K})T - 2.486\times10^4\ \text{J}$

$\Sigma Q = 0$ gives $(39.0\ \text{J/K})T + (670.4\ \text{J/K})T + 6012\ \text{J} + (75.4\ \text{J/K})T + (97.5\ \text{J/K})T - 2.486\times10^4\ \text{J} = 0.$

$$T = \frac{1.885\times10^4\ \text{J}}{882.3\ \text{J/K}} = 21.4°\text{C}.$$

EVALUATE: $T > 0.0°\text{C},$ which confirms that all the ice melts.

17.39. **IDENTIFY:** The heat lost by the cooling copper is absorbed by the water and the pot, which increases their temperatures.

SET UP: For copper, $c_c = 390\ \text{J/kg}\cdot\text{K}.$ For iron, $c_i = 470\ \text{J/kg}\cdot\text{K}.$ For water, $c_w = 4.19\times10^3\ \text{J/kg}\cdot\text{K}.$

EXECUTE: For the copper pot,

$Q_c = m_c c_c\, \Delta T_c = (0.500\ \text{kg})(390\ \text{J/kg}\cdot\text{K})(T - 20.0°\text{C}) = (195\ \text{J/K})T - 3900\ \text{J}.$ For the block of iron,

$Q_i = m_i c_i\, \Delta T_i = (0.250\ \text{kg})(470\ \text{J/kg}\cdot\text{K})(T - 85.0°\text{C}) = (117.5\ \text{J/K})T - 9988\ \text{J}.$ For the water,

$Q_w = m_w c_w\, \Delta T_w = (0.170\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(T - 20.0°\text{C}) = (712.3\ \text{J/K})T - 1.425\times10^4\ \text{J}.$ $\Sigma Q = 0$ gives

$(195\ \text{J/K})T - 3900\ \text{J} + (117.5\ \text{J/K})T - 9988\ \text{J} + (712.3\ \text{J/K})T - 1.425\times10^4\ \text{J}.$

$$T = \frac{2.814\times10^4\ \text{J}}{1025\ \text{J/K}} = 27.5°\text{C}.$$

EVALUATE: The basic principle behind this problem is conservation of energy: no energy is lost; it is only transferred.

17.40. IDENTIFY: By energy conservation, the heat lost by the water is gained by the ice. This heat must first increase the temperature of the ice from −40.0°C to the melting point of 0.00°C, then melt the ice, and finally increase its temperature to 28.0°C. The target variable is the mass of the water m.

SET UP: $Q_{\text{ice}} = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{w}}\Delta T_{\text{melted ice}}$ and $Q_{\text{water}} = mc_{\text{w}}\Delta T_{\text{w}}$.

EXECUTE: Using $Q_{\text{ice}} = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{w}}\Delta T_{\text{melted ice}}$, with the values given in the table in the text, we have $Q_{\text{ice}} = (0.200\text{ kg})[2100\text{ J/(kg}\cdot\text{C°)}](40.0\text{C°}) + (0.200\text{ kg})(3.34\times10^5\text{ J/kg})$ $+(0.200\text{ kg})[4190\text{ J/(kg}\cdot\text{C°)}](28.0\text{C°}) = 1.071\times10^5\text{ J}$.

$Q_{\text{water}} = mc_{\text{w}}\Delta T_{\text{w}} = m[4190\text{ J/(kg}\cdot\text{C°)}](28.0\text{°C} - 80.0\text{°C}) = -(217{,}880\text{ J/kg})m$. $Q_{\text{ice}} + Q_{\text{water}} = 0$ gives $1.071\times10^5\text{ J} = (217{,}880\text{ J/kg})m$. $m = 0.491\text{ kg}$.

EVALUATE: There is about twice as much water as ice because the water must provide the heat not only to melt the ice but also to increase its temperature.

17.41. IDENTIFY: By energy conservation, the heat lost by the copper is gained by the ice. This heat must first increase the temperature of the ice from –20.0°C to the melting point of 0.00°C, then melt some of the ice. At the final thermal equilibrium state, there is ice and water, so the temperature must be 0.00°C. The target variable is the initial temperature of the copper.

SET UP: For temperature changes, $Q = mc\Delta T$ and for a phase change from solid to liquid $Q = mL_{\text{F}}$.

EXECUTE: For the ice,

$Q_{\text{ice}} = (2.00\text{ kg})[2100\text{ J/(kg}\cdot\text{C°)}](20.0\text{C°}) + (0.80\text{ kg})(3.34\times10^5\text{ J/kg}) = 3.512\times10^5\text{ J}$. For the copper, using the specific heat from the table in the text gives

$Q_{\text{copper}} = (6.00\text{ kg})[390\text{ J/(kg}\cdot\text{C°)}](0\text{°C} - T) = -(2.34\times10^3\text{ J/C°})T$. Setting the sum of the two heats equal to zero gives $3.512\times10^5\text{ J} = (2.34\times10^3\text{ J/C°})T$, which gives $T = 150\text{°C}$.

EVALUATE: Since the copper has a smaller specific heat than that of ice, it must have been quite hot initially to provide the amount of heat needed.

17.42. IDENTIFY: For a temperature change $Q = mc\Delta T$ and for the liquid to solid phase change $Q = -mL_{\text{f}}$.

SET UP: For water, $c = 4.19\times10^3\text{ J/kg}\cdot\text{K}$ and $L_{\text{f}} = 3.34\times10^5\text{ J/kg}$.

EXECUTE:

$Q = mc\Delta T - mL_{\text{f}} = (0.290\text{ kg})[(4.19\times10^3\text{ J/kg}\cdot\text{K})(-18.0\text{ C°}) - 3.34\times10^5\text{ J/kg}] = -1.187\times10^5\text{ J}$, which rounds to -1.19×10^5 J.

The minus sign says 1.19×10^5 J must be removed from the water.

$$(1.187\times10^5\text{ J})\left(\frac{1\text{ cal}}{4.186\text{ J}}\right) = 2.84\times10^4\text{ cal} = 28.4\text{ kcal}.$$

$$(1.187\times10^5\text{ J})\left(\frac{1\text{ Btu}}{1055\text{ J}}\right) = 113\text{ Btu}.$$

EVALUATE: $Q < 0$ when heat comes out of an object. The equation $Q = mc\Delta T$ puts in the correct sign automatically, from the sign of $\Delta T = T_{\text{f}} - T_{\text{i}}$. But in $Q = \pm mL$ we must select the correct sign.

17.43. IDENTIFY and SET UP: Use $Q = mc\,\Delta T$ for the temperature changes and $Q = mL$ for the phase changes.

EXECUTE: Heat must be added to do the following:

ice at $-10.0\text{°C} \rightarrow$ ice at 0°C

$Q_{\text{ice}} = mc_{\text{ice}}\Delta T = (18.0\times10^{-3}\text{ kg})(2100\text{ J/kg}\cdot\text{K})(0\text{°C} - (-10.0\text{°C})) = 378\text{ J}$

phase transition ice (0°C) $\rightarrow$ liquid water (0°C)(melting)

$Q_{\text{melt}} = +mL_{\text{f}} = (18.0\times10^{-3}\text{ kg})(334\times10^3\text{ J/kg}) = 6.012\times10^3\text{ J}$

water at 0°C (from melted ice) $\rightarrow$ water at 100°C

$Q_{\text{water}} = mc_{\text{water}}\Delta T = (18.0\times10^{-3}\text{ kg})(4190\text{ J/kg}\cdot\text{K})(100°\text{C} - 0°\text{C}) = 7.542\times10^{3}\text{ J}$

phase transition water $(100°\text{C}) \rightarrow$ steam $(100°\text{C})$(boiling)

$Q_{\text{boil}} = +mL_{\text{v}} = (18.0\times10^{-3}\text{ kg})(2256\times10^{3}\text{ J/kg}) = 4.0608\times10^{4}\text{ J}$

The total Q is $Q = 378\text{ J} + 6.012\times10^{3}\text{ J} + 7.542\times10^{3}\text{ J} + 4.068\times10^{4}\text{ J} = 5.45\times10^{4}\text{ J}$

$(5.45\times10^{4}\text{ J})(1\text{ cal}/4.186\text{ J}) = 1.30\times10^{4}\text{ cal}$

$(5.45\times10^{4}\text{ J})(1\text{ Btu}/1055\text{ J}) = 51.7\text{ Btu}.$

EVALUATE: Q is positive and heat must be added to the material. Note that more heat is needed for the liquid to gas phase change than for the temperature changes.

17.44. **IDENTIFY:** $Q = mc\Delta T$ for a temperature change and $Q = +mL_{\text{f}}$ for the solid to liquid phase transition. The ice starts to melt when its temperature reaches $0.0°\text{C}$. The system stays at $0.00°\text{C}$ until all the ice has melted.

SET UP: For ice, $c = 2.10\times10^{3}\text{ J/kg}\cdot\text{K}$. For water, $L_{\text{f}} = 3.34\times10^{5}\text{ J/kg}$.

EXECUTE: **(a)** Q to raise the temperature of ice to $0.00°\text{C}$:

$Q = mc\Delta T = (0.550\text{ kg})(2.10\times10^{3}\text{ J/kg}\cdot\text{K})(15.0\text{ C}°) = 1.73\times10^{4}\text{ J}.$ $t = \dfrac{1.73\times10^{4}\text{ J}}{800.0\text{ J/min}} = 21.7\text{ min}.$

(b) To melt all the ice requires $Q = mL_{\text{f}} = (0.550\text{ kg})(3.34\times10^{5}\text{ J/kg}) = 1.84\times10^{5}\text{ J}.$

$t = \dfrac{1.84\times10^{5}\text{ J}}{800.0\text{ J/min}} = 230\text{ min}.$ The total time after the start of the heating is 252 min.

(c) A graph of T versus t is sketched in Figure 17.44.

EVALUATE: It takes much longer for the ice to melt than it takes the ice to reach the melting point.

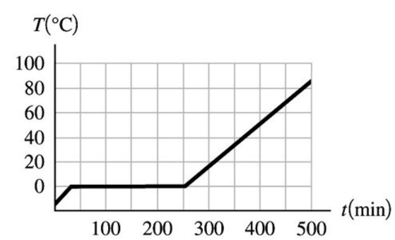


Figure 17.44

17.45. **IDENTIFY** and **SET UP:** The heat that must be added to a lead bullet of mass m to melt it is $Q = mc\Delta T + mL_{\text{f}}$ ($mc\Delta T$ is the heat required to raise the temperature from 25°C to the melting point of 327.3°C; mL_{f} is the heat required to make the solid $\rightarrow$ liquid phase change.) The kinetic energy of the bullet if its speed is v is $K = \frac{1}{2}mv^{2}$.

EXECUTE: $K = Q$ says $\frac{1}{2}mv^{2} = mc\Delta T + mL_{\text{f}}$

$v = \sqrt{2(c\Delta T + L_{\text{f}})}$

$v = \sqrt{2[(130\text{ J/kg}\cdot\text{K})(327.3°\text{C} - 25°\text{C}) + 24.5\times10^{3}\text{ J/kg}]} = 357\text{ m/s}$

EVALUATE: This is a typical speed for a rifle bullet. A bullet fired into a block of wood does partially melt, but in practice not all of the initial kinetic energy is converted to heat that remains in the bullet.

17.46. **IDENTIFY:** For a temperature change, $Q = mc\Delta T$. For the vapor $\rightarrow$ liquid phase transition, $Q = -mL_{\text{v}}$.

SET UP: For water, $L_{\text{v}} = 2.256\times10^{6}\text{ J/kg}$ and $c = 4.19\times10^{3}\text{ J/kg}\cdot\text{K}$.

EXECUTE: **(a)** $Q = +m(-L_\text{v} + c\Delta T)$

$Q = +(25.0\times10^{-3}\ \text{kg})(-2.256\times10^{6}\ \text{J/kg} + [4.19\times10^{3}\ \text{J/kg}\cdot\text{K}][-66.0\ \text{C}°]) = -6.33\times10^{4}\ \text{J}$

(b) $Q = mc\Delta T = (25.0\times10^{-3}\ \text{kg})(4.19\times10^{3}\ \text{J/kg}\cdot\text{K})(-66.0\ \text{C}°) = -6.91\times10^{3}\ \text{J}.$

(c) The total heat released by the water that starts as steam is nearly a factor of ten larger than the heat released by water that starts at $100°\text{C}$. Steam burns are much more severe than hot-water burns.

EVALUATE: For a given amount of material, the heat for a phase change is typically much more than the heat for a temperature change.

17.47. **IDENTIFY:** Use $Q = Mc\Delta T$ to find Q for a temperature rise from $34.0°\text{C}$ to $40.0°\text{C}$. Set this equal to $Q = mL_\text{v}$ and solve for m, where m is the mass of water the camel would have to drink.

SET UP: $c = 3480\ \text{J/kg}\cdot\text{K}$ and $L_\text{v} = 2.42\times10^{6}\ \text{J/kg}$. For water, 1.00 kg has a volume 1.00 L. $M = 400$ kg is the mass of the camel.

EXECUTE: The mass of water that the camel saves is

$$m = \frac{Mc\Delta T}{L_\text{v}} = \frac{(400\ \text{kg})(3480\ \text{J/kg}\cdot\text{K})(6.0\ \text{K})}{(2.42\times10^{6}\ \text{J/kg})} = 3.45\ \text{kg}$$ which is a volume of 3.45 L.

EVALUATE: This is nearly a gallon of water, so it is an appreciable savings.

17.48. **IDENTIFY:** For a temperature change, $Q = mc\Delta T$. For the liquid $\to$ vapor phase change, $Q = +mL_\text{v}$.

SET UP: The density of water is $1000\ \text{kg/m}^3$.

EXECUTE: **(a)** The heat that goes into mass m of water to evaporate it is $Q = +mL_\text{v}$. The heat flow for the man is $Q = m_\text{man}c\Delta T$, where $\Delta T = -1.00\ \text{C}°$. $\Sigma Q = 0$ so $mL_\text{v} + m_\text{man}c\Delta T = 0$ and

$$m = -\frac{m_\text{man}c\Delta T}{L_\text{v}} = -\frac{(70.0\ \text{kg})(3480\ \text{J/kg}\cdot\text{K})(-1.00\ \text{C}°)}{2.42\times10^{6}\ \text{J/kg}} = 0.101\ \text{kg} = 101\ \text{g}.$$

(b) $V = \dfrac{m}{\rho} = \dfrac{0.101\ \text{kg}}{1000\ \text{kg/m}^3} = 1.01\times10^{-4}\ \text{m}^3 = 101\ \text{cm}^3$. This is about 35% of the volume of a soft-drink can.

EVALUATE: Fluid loss by evaporation from the skin can be significant.

17.49. **IDENTIFY:** The asteroid's kinetic energy is $K = \frac{1}{2}mv^2$. To boil the water, its temperature must be raised to $100.0°\text{C}$ and the heat needed for the phase change must be added to the water.

SET UP: For water, $c = 4190\ \text{J/kg}\cdot\text{K}$ and $L_\text{v} = 2256\times10^{3}\ \text{J/kg}$.

EXECUTE: $K = \frac{1}{2}(2.60\times10^{15}\ \text{kg})(32.0\times10^{3}\ \text{m/s})^2 = 1.33\times10^{24}\ \text{J}$. $Q = mc\Delta T + mL_\text{v}$.

$$m = \frac{Q}{c\Delta T + L_\text{v}} = \frac{1.33\times10^{22}\ \text{J}}{(4190\ \text{J/kg}\cdot\text{K})(90.0\ \text{K}) + 2256\times10^{3}\ \text{J/kg}} = 5.05\times10^{15}\ \text{kg}.$$

EVALUATE: The mass of water boiled is 2.5 times the mass of water in Lake Superior.

17.50. **IDENTIFY:** $Q = mc\Delta T$ for a temperature change. The net Q for the system (sample, can and water) is zero.

SET UP: For water, $c_\text{w} = 4.19\times10^{3}\ \text{J/kg}\cdot\text{K}$. For copper, $c_\text{c} = 390\ \text{J/kg}\cdot\text{K}$.

EXECUTE: For the water, $Q_\text{w} = m_\text{w}c_\text{w}\Delta T_\text{w} = (0.200\ \text{kg})(4.19\times10^{3}\ \text{J/kg}\cdot\text{K})(7.1\ \text{C}°) = 5.95\times10^{3}\ \text{J}$.

For the copper can, $Q_\text{c} = m_\text{c}c_\text{c}\Delta T_\text{c} = (0.150\ \text{kg})(390\ \text{J/kg}\cdot\text{K})(7.1\ \text{C}°) = 415\ \text{J}$.

For the sample, $Q_\text{s} = m_\text{s}c_\text{s}\Delta T_\text{s} = (0.085\ \text{kg})c_\text{s}(-73.9\ \text{C}°)$.

$\Sigma Q = 0$ gives $(0.085\ \text{kg})(-73.9\ \text{C}°)c_\text{s} + 415\ \text{J} + 5.95\times10^{3}\ \text{J} = 0$. $c_\text{s} = 1.01\times10^{3}\ \text{J/kg}\cdot\text{K}$.

EVALUATE: Heat comes out of the sample and goes into the water and the can. The value of c_s we calculated is consistent with the values in Table 17.3.

17.51. **IDENTIFY** and **SET UP:** Heat flows out of the water and into the ice. The net heat flow for the system is zero. The ice warms to 0°C, melts, and then the water from the melted ice warms from 0°C to the final temperature.

EXECUTE: $Q_{\text{system}} = 0$; calculate Q for each component of the system: (Beaker has small mass says that $Q = mc\Delta T$ for beaker can be neglected.)

0.250 kg of water: cools from 75.0°C to 40.0°C

$Q_{\text{water}} = mc\Delta T = (0.250\text{ kg})(4190\text{ J/kg}\cdot\text{K})(40.0°\text{C} - 75.0°\text{C}) = -3.666\times10^4\text{ J}.$

ice: warms to 0°C; melts; water from melted ice warms to 40.0°C

$Q_{\text{ice}} = mc_{\text{ice}}\Delta T + mL_{\text{f}} + mc_{\text{water}}\Delta T.$

$Q_{\text{ice}} = m[(2100\text{ J/kg}\cdot\text{K})(0°\text{C} - (-20.0°\text{C})) + 334\times10^3\text{ J/kg} + (4190\text{ J/kg}\cdot\text{K})(40.0°\text{C} - 0°\text{C})].$

$Q_{\text{ice}} = (5.436\times10^5\text{ J/kg})m.$ $Q_{\text{system}} = 0$ says $Q_{\text{water}} + Q_{\text{ice}} = 0.$ $-3.666\times10^4\text{ J} + (5.436\times10^5\text{ J/kg})m = 0.$

$$m = \frac{3.666\times10^4\text{ J}}{5.436\times10^5\text{ J/kg}} = 0.0674\text{ kg}.$$

EVALUATE: Since the final temperature is 40.0°C we know that all the ice melts and the final system is all liquid water. The mass of ice added is much less than the mass of the 75°C water; the ice requires a large heat input for the phase change.

17.52. **IDENTIFY** and **SET UP:** Large block of ice implies that ice is left, so $T_2 = 0°\text{C}$ (final temperature). Heat comes out of the ingot and into the ice. The net heat flow is zero. The ingot has a temperature change and the ice has a phase change.

EXECUTE: $Q_{\text{system}} = 0$; calculate Q for each component of the system:

ingot

$Q_{\text{ingot}} = mc\Delta T = (4.00\text{ kg})(234\text{ J/kg}\cdot\text{K})(0°\text{C} - 750°\text{C}) = -7.02\times10^5\text{ J}$

ice

$Q_{\text{ice}} = +mL_{\text{f}}$, where m is the mass of the ice that changes phase (melts)

$Q_{\text{system}} = 0$ says $Q_{\text{ingot}} + Q_{\text{ice}} = 0$

$-7.02\times10^5\text{ J} + m(334\times10^3\text{ J/kg}) = 0$

$$m = \frac{7.02\times10^5\text{ J}}{334\times10^3\text{ J/kg}} = 2.10\text{ kg}$$

EVALUATE: The liquid produced by the phase change remains at 0°C since it is in contact with ice.

17.53. **IDENTIFY:** We mix liquids at different temperatures, so this is a problem in calorimetry.

SET UP: The unknown liquid cools down from 30.0°C to 14.0°C. The ice all melts and the resulting water increases from 0.0°C to 14.0°C. The heat gained by the ice all comes from the unknown liquid, so the net heat change for the mixture is zero. The ice goes through two changes: melting at 0.0°C followed by a temperature increase to 14.0°C. We use $Q = mc\Delta T$ and want to find the specific heat c of the unknown liquid.

EXECUTE: $Q_{\text{ice}} = Q_{\text{melt}} + Q_{\text{increase temp}} = m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{water}}\Delta T_{\text{water}}$ and $Q_{\text{unknown}} = mc\Delta T_{\text{unknown}}$. Using $Q_{\text{ice}} + Q_{\text{unknown}} = 0$ gives $m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{water}}\Delta T_{\text{water}} + mc\Delta T_{\text{unknown}} = 0$. Using the given masses and temperatures as well as $L_{\text{f}} = 334\times10^3$ J/kg and $c_{\text{water}} = 4190\text{ J/kg}\cdot\text{K}$, we get $c = 2370\text{ J/kg}\cdot\text{K}$.

EVALUATE: From Table 17.3 we see that this value is close to the specific heats of ethylene and glycol, so it is a reasonable result.

17.54. IDENTIFY: At steady state, the rate of heat flow is the same throughout both rods, as well as out of the boiling water and into the ice-water mixture. The heat that flows into the ice-water mixture goes only into melting ice since the temperature remains at 0.00°C.

SET UP: For steady state heat flow, $\frac{Q}{t}=\frac{kA\Delta T}{L}$. The heat to melt ice is $Q=mL_{\text{f}}$.

EXECUTE: **(a)** $\frac{Q}{t}=\frac{kA\Delta T}{L}$ is the same for both of the rods. Using the physical properties of brass and copper from the tables in the text, we have

$$\frac{[109.0\text{ W/(m}\cdot\text{K)}](100.0°\text{C}-T)}{0.300\text{ m}}=\frac{[385.0\text{ W/(m}\cdot\text{K)}](T-0.0°\text{C})}{0.800\text{ m}}.$$

Solving for T gives $T=43.0°\text{C}$.

(b) The heat entering the ice-water mixture is

$$Q=\frac{kAt\Delta T}{L}=\frac{[109.0\text{ W/(m}\cdot\text{K)}](0.00500\text{ m}^2)(300.0\text{ s})(100.0°\text{C}-43.0°\text{C})}{0.300\text{ m}}.\ Q=3.1065\times10^4\text{ J. Then}$$

$$Q=mL_{\text{f}}\text{ so } m=\frac{3.1065\times10^4\text{ J}}{3.34\times10^5\text{ J/kg}}=0.0930\text{ kg}=93.0\text{ g}.$$

EVALUATE: The temperature of the interface between the two rods is between the two extremes (0°C and 100°C), but not midway between them.

17.55. IDENTIFY: A heat current flows through the two bars, so we are dealing with thermal conduction.

SET UP: Both bars have the same length and cross-sectional area, and we want to find the thermal conductivity of the unknown metal. The graph plots T versus T_{H}, so we need to relate these quantities to interpret the graph. For this we use $H=kA\frac{T_{\text{H}}-T_{\text{C}}}{L}$. At steady state, H is the same in both bars.

EXECUTE: For the copper bar $H=k_{\text{Cu}}A\frac{T_{\text{H}}-T}{L}$, and for the unknown bar $H=kA\frac{T-0.0°\text{C}}{L}$.

Equating the heat currents and simplifying gives $k_{\text{Cu}}(T_{\text{H}}-T)=kT$. Solving for T gives $T=\frac{k_{\text{Cu}}}{k+k_{\text{Cu}}}T_{\text{H}}$,

so a graph of T versus T_{H} should be a straight line having slope $\frac{k_{\text{Cu}}}{k+k_{\text{Cu}}}$. Solving for k gives

$k=k_{\text{Cu}}\left(\frac{1}{\text{slope}}-1\right)$. From Table 17.5, we have $k_{\text{Cu}}=385\text{ W/mol}\cdot\text{K}$, so

$$k=(385\text{ W/m}\cdot\text{K})\left(\frac{1}{0.710}-1\right)=157\text{ W/m}\cdot\text{K}.$$

EVALUATE: From Table 17.5, $k=157\text{ W/m}\cdot\text{K}$ is between that of brass and aluminum, so our result is reasonable.

17.56. IDENTIFY: For a melting phase transition, $Q=mL_{\text{f}}$. The rate of heat conduction is $\frac{Q}{t}=\frac{kA(T_{\text{H}}-T_{\text{C}})}{L}$.

SET UP: For water, $L_{\text{f}}=3.34\times10^5\text{ J/kg}$.

EXECUTE: The heat conducted by the rod in 10.0 min is

$$Q=mL_{\text{f}}=(8.50\times10^{-3}\text{ kg})(3.34\times10^5\text{ J/kg})=2.84\times10^3\text{ J. }\frac{Q}{t}=\frac{2.84\times10^3\text{ J}}{600\text{ s}}=4.73\text{ W}.$$

$$k=\frac{(Q/t)L}{A(T_{\text{H}}-T_{\text{C}})}=\frac{(4.73\text{ W})(0.600\text{ m})}{(1.25\times10^{-4}\text{ m}^2)(100\text{ C}°)}=227\text{ W/m}\cdot\text{K}.$$

EVALUATE: The heat conducted by the rod is the heat that enters the ice and produces the phase change.

17.57. **IDENTIFY** and **SET UP:** Call the temperature at the interface between the wood and the styrofoam T. The heat current in each material is given by $H = kA(T_H - T_C)/L$.

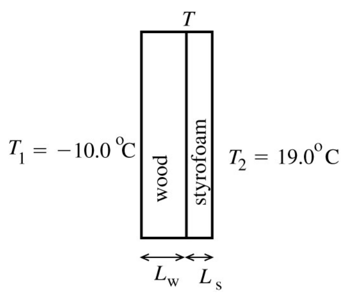


Figure 17.57

See Figure 17.57.
Heat current through the wood: $H_w = k_w A(T - T_1)L_w$
Heat current through the styrofoam: $H_s = k_s A(T_2 - T)/L_s$

In steady-state heat does not accumulate in either material. The same heat has to pass through both materials in succession, so $H_w = H_s$.

EXECUTE: **(a)** This implies $k_w A(T - T_1)/L_w = k_s A(T_2 - T)/L_s$

$k_w L_s (T - T_1) = k_s L_w (T_2 - T)$

$$T = \frac{k_w L_s T_1 + k_s L_w T_2}{k_w L_s + k_s L_w} = \frac{-0.0176\ \text{W}\cdot{}^\circ\text{C/K} + 0.01539\ \text{W}\cdot{}^\circ\text{C/K}}{0.00257\ \text{W/K}} = -0.86^\circ\text{C}.$$

EVALUATE: The temperature at the junction is much closer in value to T_1 than to T_2. The styrofoam has a very small k, so a larger temperature gradient is required for than for wood to establish the same heat current.

(b) IDENTIFY and **SET UP:** Heat flow per square meter is $\dfrac{H}{A} = k\left(\dfrac{T_H - T_C}{L}\right)$. We can calculate this either for the wood or for the styrofoam; the results must be the same.

EXECUTE: <u>Wood</u>: $\dfrac{H_w}{A} = k_w \dfrac{T - T_1}{L_w} = (0.080\ \text{W/m}\cdot\text{K})\dfrac{-0.86^\circ\text{C} - (-10.0^\circ\text{C})}{0.030\ \text{m}} = 24\ \text{W/m}^2.$

<u>Styrofoam</u>: $\dfrac{H_s}{A} = k_s \dfrac{T_2 - T}{L_s} = (0.027\ \text{W/m}\cdot\text{K})\dfrac{19.0^\circ\text{C} - (-0.86^\circ\text{C})}{0.022\ \text{m}} = 24\ \text{W/m}^2.$

EVALUATE: H must be the same for both materials and our numerical results show this. Both materials are good insulators and the heat flow is very small.

17.58. **IDENTIFY:** This problem is about heat flow, so we use $\dfrac{Q}{t} = \dfrac{kA(T_H - T_C)}{L}$.

SET UP: $T_H - T_C = 175^\circ\text{C} - 35^\circ\text{C}$. $1\ \text{K} = 1\ \text{C}^\circ$, so there is no need to convert the temperatures to kelvins.

EXECUTE: **(a)** $\dfrac{Q}{t} = \dfrac{(0.040\ \text{W/m}\cdot\text{K})(1.40\ \text{m}^2)(175^\circ\text{C} - 35^\circ\text{C})}{4.0\times10^{-2}\ \text{m}} = 196\ \text{W}.$

(b) The power input must be 196 W, to replace the heat conducted through the walls.

EVALUATE: The heat current is small because k is small for fiberglass.

17.59. **IDENTIFY:** We compare the thermal conductivity of several materials.

SET UP and EXECUTE: **(a)** Touch metal, glass, and wood. The result is that the metal is the coldest and the wood the warmest.

(b) From Table 17.5: $k_{Cu} = 385\ \text{W/m}\cdot\text{K}$, k_{wood} is between $0.12\ \text{W/m}\cdot\text{K}$ and $0.04\ \text{W/m}\cdot\text{K}$, and $k_{glass} = 0.8\ \text{W/m}\cdot\text{K}$. Our ranking agrees with these thermal conductivities.

EVALUATE: The large variation in the thermal conductivity of wood is due to the density of the wood. The more air that is trapped within its fibers, the poorer it is at conducting heat because $k_{\text{air}} = 0.024$ $\text{W/m}\cdot\text{K}$, making a very poor conductor of heat (but therefore an excellent insulator).

17.60. IDENTIFY: $\dfrac{Q}{t} = \dfrac{kA\Delta T}{L}$. Q/t is the same for both sections of the rod.

SET UP: For copper, $k_{\text{c}} = 385\ \text{W/m}\cdot\text{K}$. For steel, $k_{\text{s}} = 50.2\ \text{W/m}\cdot\text{K}$.

EXECUTE: **(a)** For the copper section, $\dfrac{Q}{t} = \dfrac{(385\ \text{W/m}\cdot\text{K})(4.00\times10^{-4}\ \text{m}^2)(100°\text{C} - 65.0°\text{C})}{1.00\ \text{m}} = 5.39\ \text{J/s}.$

(b) For the steel section, $L = \dfrac{kA\Delta T}{(Q/t)} = \dfrac{(50.2\ \text{W/m}\cdot\text{K})(4.00\times10^{-4}\ \text{m}^2)(65.0°\text{C} - 0°\text{C})}{5.39\ \text{J/s}} = 0.242\ \text{m}.$

EVALUATE: The thermal conductivity for steel is much less than that for copper, so for the same ΔT and A, a smaller L for steel would be needed for the same heat current as in copper.

17.61. IDENTIFY and SET UP: The heat conducted through the bottom of the pot goes into the water at 100°C to convert it to steam at 100°C. We can calculate the amount of heat flow from the mass of material that changes phase. Then use $H = kA(T_{\text{H}} - T_{\text{C}})/L$ to calculate T_{H}, the temperature of the lower surface of the pan.

EXECUTE: $Q = mL_{\text{v}} = (0.390\ \text{kg})(2256\times10^3\ \text{J/kg}) = 8.798\times10^5\ \text{J}$

$H = Q/t = 8.798\times10^5\ \text{J}/180\ \text{s} = 4.888\times10^3\ \text{J/s}$

Then $H = kA(T_{\text{H}} - T_{\text{C}})/L$ says that $T_{\text{H}} - T_{\text{C}} = \dfrac{HL}{kA} = \dfrac{(4.888\times10^3\ \text{J/s})(8.50\times10^{-3}\ \text{m})}{(50.2\ \text{W/m}\cdot\text{K})(0.150\ \text{m}^2)} = 5.52\ \text{C°}$

$T_{\text{H}} = T_{\text{C}} + 5.52\ \text{C°} = 100°\text{C} + 5.52\ \text{C°} = 105.5°\text{C}.$

EVALUATE: The larger $T_{\text{H}} - T_{\text{C}}$ is the larger H is and the faster the water boils.

17.62. IDENTIFY: Apply $H = kA(T_{\text{H}} - T_{\text{C}})/L$ and solve for A.

SET UP: The area of each circular end of a cylinder is related to the diameter D by $A = \pi R^2 = \pi(D/2)^2$. For steel, $k = 50.2\ \text{W/m}\cdot\text{K}$. The boiling water has $T = 100°\text{C}$, so $\Delta T = 300\ \text{K}$.

EXECUTE: $\dfrac{Q}{t} = kA\dfrac{\Delta T}{L}$ and $190\ \text{J/s} = (50.2\ \text{W/m}\cdot\text{K})A\left(\dfrac{300\ \text{K}}{0.500\ \text{m}}\right)$. This gives $A = 6.308\times10^{-3}\ \text{m}^2$, and $D = \sqrt{4A/\pi} = \sqrt{4(6.308\times10^{-3}\ \text{m}^2)/\pi} = 8.96\times10^{-2}\ \text{m} = 8.96\ \text{cm}.$

EVALUATE: H increases when A increases.

17.63. IDENTIFY: Assume the temperatures of the surfaces of the window are the outside and inside temperatures. Use the concept of thermal resistance. For part (b) use the fact that when insulating materials are in layers, the R values are additive.

SET UP: From Table 17.5, $k = 0.8\ \text{W/m}\cdot\text{K}$ for glass. $R = L/k$.

EXECUTE: **(a)** For the glass, $R_{\text{glass}} = \dfrac{5.20\times10^{-3}\ \text{m}}{0.8\ \text{W/m}\cdot\text{K}} = 6.50\times10^{-3}\ \text{m}^2\cdot\text{K/W}.$

$$H = \frac{A(T_{\text{H}} - T_{\text{C}})}{R} = \frac{(1.40\ \text{m})(2.50\ \text{m})(39.5\ \text{K})}{6.50\times10^{-3}\ \text{m}^2\cdot\text{K/W}} = 2.1\times10^4\ \text{W}$$

(b) For the paper, $R_{\text{paper}} = \dfrac{0.750\times10^{-3}\ \text{m}}{0.05\ \text{W/m}\cdot\text{K}} = 0.015\ \text{m}^2\cdot\text{K/W}$. The total R is

$R = R_{\text{glass}} + R_{\text{paper}} = 0.0215\ \text{m}^2\cdot\text{K/W}$. $H = \dfrac{A(T_{\text{H}} - T_{\text{C}})}{R} = \dfrac{(1.40\ \text{m})(2.50\ \text{m})(39.5\ \text{K})}{0.0215\ \text{m}^2\cdot\text{K/W}} = 6.4\times10^3\ \text{W}.$

EVALUATE: The layer of paper decreases the rate of heat loss by a factor of about 3.

17.64. IDENTIFY: The rate of energy radiated per unit area is $\frac{H}{A} = e\sigma T^4$.

SET UP: A perfect blackbody has $e = 1$.

EXECUTE: **(a)** $\frac{H}{A} = (1)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(273\ \text{K})^4 = 315\ \text{W/m}^2$

(b) $\frac{H}{A} = (1)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(2730\ \text{K})^4 = 3.15\times10^6\ \text{W/m}^2$

EVALUATE: When the Kelvin temperature increases by a factor of 10 the rate of energy radiation increases by a factor of 10^4.

17.65. IDENTIFY: Use $H = Ae\sigma T^4$ to calculate A.

SET UP: $H = Ae\sigma T^4$ so $A = H/e\sigma T^4$

150-W and all electrical energy consumed is radiated says $H = 150$ W.

EXECUTE: $A = \dfrac{150\ \text{W}}{(0.35)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(2450\ \text{K})^4} = 2.1\times10^{-4}\ \text{m}^2(1\times10^4\ \text{cm}^2/1\ \text{m}^2) = 2.1\ \text{cm}^2$

EVALUATE: Light-bulb filaments are often in the shape of a tightly wound coil to increase the surface area; larger A means a larger radiated power H.

17.66. IDENTIFY: The net heat current is $H = Ae\sigma(T^4 - T_s^4)$. A power input equal to H is required to maintain constant temperature of the sphere.

SET UP: The surface area of a sphere is $4\pi r^2$.

EXECUTE: $H = 4\pi(0.0150\ \text{m})^2(0.35)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(3000\ \text{K})^4 - (290\ \text{K})^4] = 4.54\times10^3\ \text{W}$.

EVALUATE: Since $3000\ \text{K} > 290\ \text{K}$ and H is proportional to T^4, the rate of emission of heat energy is much greater than the rate of absorption of heat energy from the surroundings.

17.67. IDENTIFY: Apply $H = Ae\sigma T^4$ and calculate A.

SET UP: For a sphere of radius R, $A = 4\pi R^2$. $\sigma = 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4$. The radius of the earth is $R_E = 6.37\times10^6$ m, the radius of the sun is $R_{sun} = 6.96\times10^8$ m, and the distance between the earth and the sun is $r = 1.50\times10^{11}$ m.

EXECUTE: The radius is found from $R = \sqrt{\dfrac{A}{4\pi}} = \sqrt{\dfrac{H/(\sigma T^4)}{4\pi}} = \sqrt{\dfrac{H}{4\pi\sigma}\dfrac{1}{T^2}}$.

(a) $R_a = \sqrt{\dfrac{(2.7\times10^{32}\ \text{W})}{4\pi(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}}\dfrac{1}{(11{,}000\ \text{K})^2} = 1.61\times10^{11}\ \text{m}$

(b) $R_b = \sqrt{\dfrac{(2.10\times10^{23}\ \text{W})}{4\pi(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}}\dfrac{1}{(10{,}000\ \text{K})^2} = 5.43\times10^6\ \text{m}$

EVALUATE: **(c)** The radius of Procyon B is comparable to that of the earth, and the radius of Rigel is comparable to the earth-sun distance.

17.68. IDENTIFY: This problem deals with the thermal expansion for water in the range of 0°C to 10°C.

SET UP: Use Fig. 17.12 in the textbook. From analytic geometry, a parabola with vertex at the point (h,k) and opening upward is $(x-h)^2 = 4p(y-k)$ with $p > 0$. In this case, $h = 4.0$°C and $k = 1.00003$ cm^3. We want to find the equation for $V(T)$ with T in degrees Celsius.

EXECUTE: **(a)** Using the equation for a parabola gives $(T - 4.0°\text{C})^2 = 4p(V - 1.00003\ \text{cm}^3)$. We need to find 4$p$. From the graph, $V = 1.00015$ cm^3 when $T = 0$°C, so $(-4.0°\text{C})^2 = 4p(1.00015 - 1.00003)\text{cm}^3$, which gives $4p = 1.33\times10^5\ (\text{C}°)^2/\text{cm}^3$. Therefore

$(T - 4.0°\text{C})^2 = \left[1.33\times10^5 (\text{C}°)^2 / \text{cm}^3\right](V - 1.00003\,\text{cm}^3)$. Now solve for V which gives

$V = \left[7.5\times10^{-6}\text{cm}^3 / (\text{C}°)^2\right](T - 4.0°\text{C})^2 + 1.00003\ \text{cm}^3$. From this result, we see that A = 1.00003 cm^3 and $B = 7.5\times10^{-6}\text{cm}^3 / (\text{C}°)^2$.

(b) Use $dV = \beta(T_\text{C})Vdt$, where $V = A + B(T - 4.0°\text{C})^2$ and $V \approx 1$ cm^3 (because the graph in the Fig. 17.12 is for 1 gram of water), which gives $dV = d\left[A + B(T - 4.0°\text{C})^2\right] = 2B(T - 4°\text{C})dT$. Therefore $\beta(T) = 2B(T - 4°\text{C})$.

At 1.0°C: $\beta = 2\left[7.5\times10^{-6}\text{cm}^3 / (\text{C}°)^2\right](1.0°\text{C} - 4.0°\text{C}) = -4.5\times10^{-5} / \text{C}°$.

At 4.0°C: $\beta = 0$.

At 7.0°C: $\beta = 4.5\times10^{-5} / \text{C}°$.

At 10.0°C: $\beta = 9.0\times10^{-5} / \text{C}°$.

EVALUATE: With V = 1.0 cm^3, $\beta = dV/dT$ is the slope of the graph in Fig. 17.12. From this graph, we can see that the slope is zero when T = 4.0°C. Also when T = 1.0°C and 7.0°C (3C° within 4.0°C), the slope has the same magnitude but different sign. This also agrees with our result.

17.69. **IDENTIFY:** Use $\Delta L = L_0 \alpha \Delta T$ to find the change in diameter of the sphere and the change in length of the cable. Set the sum of these two increases in length equal to 2.00 mm.

SET UP: $\alpha_\text{brass} = 2.0\times10^{-5}\ \text{K}^{-1}$ and $\alpha_\text{steel} = 1.2\times10^{-5}\ \text{K}^{-1}$.

EXECUTE: $\Delta L = (\alpha_\text{brass} L_{0,\text{brass}} + \alpha_\text{steel} L_{0,\text{steel}})\Delta T$.

$$\Delta T = \frac{2.00\times10^{-3}\ \text{m}}{(2.0\times10^{-5}\ \text{K}^{-1})(0.350\ \text{m}) + (1.2\times10^{-5}\ \text{K}^{-1})(10.5\ \text{m})} = 15.0\ \text{C}°.\quad T_2 = T_1 + \Delta T = 35.0°\text{C}.$$

EVALUATE: The change in diameter of the brass sphere is 0.10 mm. This is small, but should not be neglected.

17.70. **IDENTIFY:** The tension in the wire is related to the speed of waves on the wire, and it also related to the thermal stress in the wire.

SET UP: The thermal stress is $\dfrac{F}{A} = -Y\alpha\Delta T$, $v = f\lambda$, and $v = \sqrt{\dfrac{F}{\mu}}$. In the fundamental mode $\lambda = 2L$.

We know that L = 0.400 m and at 20.0°C, f_1 = 440 Hz.

EXECUTE: **(a)** We want the tension F in the wire when f_1 = 440 Hz at 20.0°C. We know that $\lambda = 2L$ = 0.800 m and $v = \sqrt{\dfrac{F}{\mu}}$, where $\mu = m/L$ = (0.00250 kg)/(0.400 m) = 6.25×10^{-3} kg/m. Solving $v = \sqrt{\dfrac{F}{\mu}}$ for F and using $v = f\lambda$ gives $F = \mu(f\lambda)^2$. For the numbers here we have

$F = (6.25\times10^{-3}\ \text{kg/m})\left[(440\ \text{Hz})(0.800\ \text{m})\right]^2 = 774$ N.

(b) Now f_1 = 460 Hz and we want to find the temperature of the wire. The thermal stress is $\dfrac{F}{A} = -Y\alpha\Delta T$, so we need F and A to find ΔT. The wavelength is the same as in part (a) because the wire has not changed length, but the frequency has changed. As before, we use $v = f\lambda$ and $v = \sqrt{\dfrac{F}{\mu}}$ to get F, giving $F = \mu(f\lambda)^2$. The numbers are the same as before except that f = 460 Hz instead of 440 Hz. The result is F = 846.4 N. (Note that the tension has *increased*.) Now find the cross-sectional area A of the wire. We know its mass, length, and density, so we use $m = \rho V = \rho AL$ which gives $A = \dfrac{m}{\rho L}$

$= \dfrac{0.00250 \text{ kg}}{(7800 \text{ kg/m}^3)(0.400 \text{ m})} = 8.013\times 10^{-7} \text{ m}^2$. We are now ready to use $\dfrac{F}{A} = -Y\alpha\Delta T$ to find ΔT. The F in that equation is not the tension. Rather it is the *difference* in tension when the temperature is changed by ΔT. It is the force needed to maintain the length of the wire. The wire would tend to contract so the tension prevents this. Therefore F = 846.4 N – 774 N = 72 N. Solving $\dfrac{F}{A} = -Y\alpha\Delta T$ for ΔT gives $\Delta T = -\dfrac{F}{AY\alpha}$. For the numbers here we get

$\Delta T = -\dfrac{72 \text{ N}}{(8.013\times 10^{-7} \text{m}^2)(20\times 10^{10} \text{ Pa})(1.2\times 10^{-5} \text{ K})} = -37$ C°. The new temperature is $T = T_0 + \Delta T =$ 20.0°C + (–37 C°) = –17°C.

EVALUATE: Heating the wire would have caused it to expand which would have decreased the tension and hence lowered the fundamental frequency.

17.71. IDENTIFY: We are dealing the thermal expansion of a volume of liquid when heat is added to it.

SET UP: Use $Q = mc\Delta T$, $\Delta V = \beta L_0 \Delta T$, and $m = \rho V$.

EXECUTE: Using the above equations gives $Q = mc\Delta T = \rho Vc\Delta T$. From $\Delta V = \beta L_0 \Delta T$ we have $\Delta T = \dfrac{\Delta V}{\beta\rho}$, so $Q = \rho Vc\dfrac{\Delta V}{\beta V}$. Solving for c gives $c = \dfrac{Q\beta}{\rho\Delta V}$.

EVALUATE: Our result indicates that if c is large, a large amount of heat Q will be needed to change its volume. This is a reasonable result.

17.72. IDENTIFY: In this problem, we are dealing with the effect of thermal expansion on the period of a simple pendulum.

SET UP: At 20.0°C the length L of the copper wire is 3.00 m. We want to find the percent change in the period if the temperature is increased to 220°C. We know that $T = 2\pi\sqrt{\dfrac{L}{g}}$ and $\Delta L = \alpha L_0 \Delta T$, and we want $\dfrac{\Delta T}{T_1}$.

EXECUTE: At 20.0°C: $T_1 = 2\pi\sqrt{\dfrac{L}{g}}$. At 22.0°C: $T_2 = 2\pi\sqrt{\dfrac{L_2}{g}} = 2\pi\sqrt{\dfrac{L+\Delta L}{g}}$. Using $\Delta L = \alpha L_0 \Delta T$ gives $T_2 = 2\pi\sqrt{\dfrac{L+\Delta L}{g}} = 2\pi\sqrt{\dfrac{L+\alpha L\Delta T}{g}} = 2\pi\sqrt{\dfrac{L}{g}}\sqrt{1+\alpha\Delta T}$. We want $\dfrac{\Delta T}{T_1}$ which is $\dfrac{\Delta T}{T_1}$ =

$$\frac{T_2 - T_1}{T_1} = \frac{T_2}{T_1} - 1 = \frac{2\pi\sqrt{\dfrac{L}{g}}\sqrt{1+\alpha\Delta T}}{2\pi\sqrt{\dfrac{L}{g}}} - 1 = \sqrt{1+\alpha\Delta T} - 1.$$ For copper we have

$\alpha\Delta T = (1.7\times 10^{-5} \text{ K}^{-1})(200 \text{ K}) = 3.4\times 10^{-3}$, which is much less than 1. Therefore we can use the approximation $(1 + x)^n \approx 1 + nx$, which in this case is $\sqrt{1+x} = (1 + x)^{1/2} \approx 1 + x/2$, where $x = \alpha\Delta T$.

This gives $\dfrac{\Delta T}{T_1} = \sqrt{1+\alpha\Delta T} - 1 \approx \dfrac{\alpha\Delta T}{2} = \dfrac{3.4\times 10^{-3}}{2} = 0.0017 = 0.17\%$.

EVALUATE: Even for a temperature change as large as 200°C, the percent change is only 0.17%. This illustrates that thermal expansion typically produces very small fractional length changes. Note that the fact that the pendulum bob was fused quartz played no role in the solution; it was extraneous information.

17.73. **IDENTIFY** and **SET UP:** Use the temperature difference in M° and in C° between the melting and boiling points of mercury to relate M° to C°. Also adjust for the different zero points on the two scales to get an equation for T_{M} in terms of T_{C}.

(a) EXECUTE: normal melting point of mercury: $-39°\text{C} = 0.0°\text{M}$

normal boiling point of mercury: $357°\text{C} = 100.0°\text{M}$

$100.0\ \text{M}° = 396\ \text{C}°$ so $1\ \text{M}° = 3.96\ \text{C}°$

Zero on the M scale is -39 on the C scale, so to obtain T_{C} multiply T_{M} by 3.96 and then subtract 39°:

$T_{\text{C}} = 3.96T_M - 39°$

Solving for T_{M} gives $T_{\text{M}} = \frac{1}{3.96}(T_{\text{C}} + 39°)$

The normal boiling point of water is 100°C; $T_{\text{M}} = \frac{1}{3.96}(100° + 39°) = 35.1°\text{M}$.

(b) $10.0\ \text{M}° = 39.6\ \text{C}°$

EVALUATE: A M° is larger than a C° since it takes fewer of them to express the difference between the boiling and melting points for mercury.

17.74. **IDENTIFY:** $v = \sqrt{F/\mu} = \sqrt{FL/m}$. For the fundamental, $\lambda = 2L$ and $f = \dfrac{v}{\lambda} = \dfrac{1}{2}\sqrt{\dfrac{F}{mL}}$. F, v and λ change when T changes because L changes. $\Delta L = L\alpha\Delta T$, where L is the original length.

SET UP: For copper, $\alpha = 1.7\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: **(a)** We can use differentials to find the frequency change because all length changes are small percents. $\Delta f \approx \dfrac{\partial f}{\partial L}\Delta L$ (only L changes due to heating).

$$\Delta f = \tfrac{1}{2}\tfrac{1}{2}(F/mL)^{-1/2}(F/m)(-1/L^2)\Delta L = -\tfrac{1}{2}\left(\tfrac{1}{2}\sqrt{\frac{F}{mL}}\right)\frac{\Delta L}{L} = -\tfrac{1}{2}f\frac{\Delta L}{L}.$$

$\Delta f = -\frac{1}{2}(\alpha\Delta T)f = -\frac{1}{2}(1.7\times10^{-5}\ (\text{C}°)^{-1})(40\ \text{C}°)(440\ \text{Hz}) = -0.15\ \text{Hz}$. The frequency decreases since the length increases.

(b) $\Delta v = \dfrac{\partial v}{\partial L}\Delta L.$

$$\frac{\Delta v}{v} = \frac{\frac{1}{2}(FL/m)^{-1/2}(F/m)\Delta L}{\sqrt{FL/m}} = \frac{\Delta L}{2L} = \frac{\alpha\Delta T}{2} = \frac{1}{2}(1.7\times10^{-5}\ (\text{C}°)^{-1})(40\ \text{C}°) = 3.4\times10^{-4} = 0.034\%.$$

(c) $\lambda = 2L$ so $\Delta\lambda = 2\Delta L \rightarrow \dfrac{\Delta\lambda}{\lambda} = \dfrac{2\Delta L}{2L} = \dfrac{\Delta L}{L} = \alpha\Delta T.$

$\dfrac{\Delta\lambda}{\lambda} = (1.7\times10^{-5}\ (\text{C}°)^{-1})(40\ \text{C}°) = 6.8\times10^{-4} = 0.068\%$. λ increases.

EVALUATE: The wave speed and wavelength increase when the length increases and the frequency decreases. The percentage change in the frequency is -0.034%. The fractional change in all these quantities is very small.

17.75. **IDENTIFY** and **SET UP:** Use $\Delta V = V_0\beta\Delta T$ for the volume expansion of the oil and of the cup. Both the volume of the cup and the volume of the olive oil increase when the temperature increases, but β is larger for the oil so it expands more. When the oil starts to overflow, $\Delta V_{\text{oil}} = \Delta V_{\text{glass}} + (3.00\times10^{-3}\ \text{m})A$, where A is the cross-sectional area of the cup.

EXECUTE: $\Delta V_{\text{oil}} = V_{0,\text{oil}}\beta_{\text{oil}}\Delta T = (9.7\ \text{cm})A\beta_{\text{oil}}\Delta T$. $\Delta V_{\text{glass}} = V_{0,\text{glass}}\beta_{\text{glass}}\Delta T = (10.0\ \text{cm})A\beta_{\text{glass}}\Delta T$.

$(9.7\ \text{cm})A\beta_{\text{oil}}\Delta T = (10.0\ \text{cm})A\beta_{\text{glass}}\Delta T + (0.300\ \text{cm})A$. The A divides out. Solving for ΔT gives $\Delta T = 47.4\ \text{C}°$. $T_2 = T_1 + \Delta T = 69.4°\text{C}$.

EVALUATE: If the expansion of the cup is neglected, the olive oil will have expanded to fill the cup when $(0.300\text{ cm})A = (9.7\text{ cm})A\beta_{\text{oil}}\Delta T$, so $\Delta T = 45.5\text{ C}°$ and $T_2 = 77.5°\text{C}$. Our result is somewhat higher than this. The cup also expands but not as much since $\beta_{\text{glass}} << \beta_{\text{oil}}$.

17.76. **IDENTIFY:** As the tape changes temperature, the distances between the markings will increase, thus making the readings inaccurate.

SET UP: For steel, $\alpha = 1.2\times10^{-5}\,(\text{C}°)^{-1}$. The two points that match the length of the object are 25.970 m apart at $20.0°\text{C}$. Find the distance between them at $5.00°\text{C}$. For linear expansion, $L = L_0(1+\alpha\Delta T)$.

EXECUTE: $L = L_0(1+\alpha\Delta T) = (25.970\text{ m})[1+(1.2\times10^{-5}\,(\text{C}°)^{-1})(5.00°\text{C} - 20.0°\text{C})] = 25.965\text{ m}$. The true distance between the points is 25.965 m.

EVALUATE: The error in measurement is 25.970 m – 25.965 m = 0.005 m = 5 mm. This is not likely to be a very serious error in a measurement of nearly 30 m. If greater precision is needed, some sort of laser measuring device would probably be used.

17.77. **IDENTIFY** and **SET UP:** Call the metals *A* and *B*. Use the data given to calculate α for each metal.

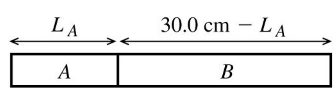
EXECUTE: $\Delta L = L_0\alpha\Delta T$ so $\alpha = \Delta L/(L_0\Delta T)$

metal *A*: $\alpha_A = \dfrac{\Delta L}{L_0\Delta T} = \dfrac{0.0650\text{ cm}}{(30.0\text{ cm})(100\text{ C}°)} = 2.167\times10^{-5}\ (\text{C}°)^{-1}$

metal *B*: $\alpha_B = \dfrac{\Delta L}{L_0\Delta T} = \dfrac{0.0350\text{ cm}}{(30.0\text{ cm})(100\text{ C}°)} = 1.167\times10^{-5}\ (\text{C}°)^{-1}$

EVALUATE: L_0 and ΔT are the same, so the rod that expands the most has the larger α.

IDENTIFY and **SET UP:** Now consider the composite rod (Figure 17.77). Apply $\Delta L = L_0\alpha\Delta T$. The target variables are L_A and L_B, the lengths of the metals *A* and *B* in the composite rod.



$\Delta T = 100\text{ C}°$
$\Delta L = 0.058\text{ cm}$

Figure 17.77

EXECUTE: $\Delta L = \Delta L_A + \Delta L_B = (\alpha_A L_A + \alpha_B L_B)\Delta T$

$\Delta L/\Delta T = \alpha_A L_A + \alpha_B(0.300\text{ m} - L_A)$

$$L_A = \frac{\Delta L/\Delta T - (0.300\text{ m})\alpha_B}{\alpha_A - \alpha_B} = \frac{(0.058\times10^{-2}\text{ m})/(100\text{ C}°) - (0.300\text{ m})(1.167\times10^{-5}(\text{C}°)^{-1})}{1.00\times10^{-5}\ (\text{C}°)^{-1}} = 23.0\text{ cm}$$

$L_B = 30.0\text{ cm} - L_A = 30.0\text{ cm} - 23.0\text{ cm} = 7.0\text{ cm}$

EVALUATE: The expansion of the composite rod is similar to that of rod *A*, so the composite rod is mostly metal *A*.

17.78. **IDENTIFY:** The copper sphere is radiating energy and is therefore cooling down. So we are dealing with heat capacity and thermal radiation.

SET UP: The rate at which heat is radiated is $H = Ae\sigma T^4$ and $Q = mc\Delta T$. For this sphere e = 1.00 and $c = 390\text{ J/kg}\cdot\text{K}$, and $m = \rho V$. The radiated energy in time *t* is *Ht*.

EXECUTE: **(a)** We want the time *t* that it takes the sphere to cool by 1.00 K due to the radiation, assuming that *H* remains constant. The heat for the copper sphere to cool down by 1 K is $Q = mc\Delta T$ = $\rho Vc\Delta T$, so *Ht* = *Q*. This gives $A\sigma T^4 t = \rho Vc\Delta T$. Using $A = 4\pi R^2$ and $V = \frac{4}{3}\pi R^3$, the previous equation becomes $\sigma T^4 t = \dfrac{\rho Rc\Delta T}{3}$. Solving for *t* using *T* = 300 K, $c = 390\text{ J/kg}\cdot\text{K}$, $\Delta T = 1\text{ K}$, ρ = 8900 kg/m^3, and *R* = 0.0500 m, we get *t* = 126 s = 2.10 min.

(b) $\dfrac{\Delta H}{H} = \dfrac{H_2 - H_1}{H_1} = \dfrac{H_2}{H_1} - 1 = \dfrac{A\sigma T_2^4}{A\sigma T_1^4} - 1 = \left(\dfrac{299\text{ K}}{300\text{ K}}\right)^4 - 1 = -0.0133.$

EVALUATE: The fractional change in H is so small that neglecting it was a reasonable option. The fractional change is negative because the sphere is decreasing in temperature so the rate of radiation is decreasing.

17.79. **IDENTIFY:** The change in length due to heating is $\Delta L_T = L_0\alpha\Delta T$ and this need not equal ΔL. The change in length due to the tension is $\Delta L_F = \dfrac{FL_0}{AY}$. Set $\Delta L = \Delta L_F + \Delta L_T$.

SET UP: $\alpha_{\text{brass}} = 2.0\times10^{-5}\ (\text{C}°)^{-1}$. $\alpha_{\text{steel}} = 1.5\times10^{-5}\ (\text{C}°)^{-1}$. $Y_{\text{steel}} = 20\times10^{10}$ Pa.

EXECUTE: **(a)** The change in length is due to the tension and heating. $\dfrac{\Delta L}{L_0} = \dfrac{F}{AY} + \alpha\Delta T$. Solving for F/A, $\dfrac{F}{A} = Y\left(\dfrac{\Delta L}{L_0} - \alpha\Delta T\right)$.

(b) The brass bar is given as "heavy" and the wires are given as "fine," so it may be assumed that the stress in the bar due to the fine wires does not affect the amount by which the bar expands due to the temperature increase. This means that ΔL is not zero, but is the amount $\alpha_{\text{brass}}L_0\Delta T$ that the brass expands, and so

$\dfrac{F}{A} = Y_{\text{steel}}(\alpha_{\text{brass}} - \alpha_{\text{steel}})\Delta T = (20\times10^{10}\text{ Pa})(2.0\times10^{-5}\ (\text{C}°)^{-1} - 1.2\times10^{-5}\ (\text{C}°)^{-1})(120\text{ C}°) = 1.92\times10^{8}\text{ Pa}.$

EVALUATE: The length of the brass bar increases more than the length of the steel wires. The wires remain taut and are under tension when the temperature of the system is raised above 20°C.

17.80. **IDENTIFY** and **SET UP:** $v = \sqrt{F/\mu}$. The coefficient of linear expansion α is defined by $\Delta L = L_0\alpha\Delta T$. This can be combined with $Y = \dfrac{F/A}{\Delta L/L_0}$ to give $\Delta F = -Y\alpha A\Delta T$ for the change in tension when the temperature changes by ΔT. Combine the two equations and solve for α.

EXECUTE: $v_1 = \sqrt{F/\mu}$, $v_1^2 = F/\mu$ and $F = \mu v_1^2$

The length and hence μ stay the same but the tension decreases by $\Delta F = -Y\alpha A\Delta T$.

$v_2 = \sqrt{(F + \Delta F)/\mu} = \sqrt{(F - Y\alpha A\Delta T)/\mu}$

$v_2^2 = F/\mu - Y\alpha A\,\Delta T/\mu = v_1^2 - Y\alpha A\,\Delta T/\mu$

And $\mu = m/L$ so $A/\mu = AL/m = V/m = 1/\rho$. ($A$ is the cross-sectional area of the wire, V is the volume of a length L.) Thus $v_1^2 - v_2^2 = \alpha(Y\Delta T/\rho)$ and $\alpha = \dfrac{v_1^2 - v_2^2}{(Y/\rho)\,\Delta T}$.

EVALUATE: When T increases the tension decreases and v decreases.

17.81. **(a) IDENTIFY** and **SET UP:** The diameter of the ring undergoes linear expansion (increases with T) just like a solid steel disk of the same diameter as the hole in the ring. Heat the ring to make its diameter equal to 2.5020 in.

EXECUTE: $\Delta L = \alpha L_0\Delta T$ so $\Delta T = \dfrac{\Delta L}{L_0\alpha} = \dfrac{0.0020\text{ in.}}{(2.5000\text{ in.})(1.2\times10^{-5}(\text{C}°)^{-1})} = 66.7\text{ C}°$

$T = T_0 + \Delta T = 20.0°\text{C} + 66.7\text{ C}° = 87°\text{C}$

(b) IDENTIFY and **SET UP:** Apply the linear expansion equation to the diameter of the brass shaft and to the diameter of the hole in the steel ring.

EXECUTE: $L = L_0(1 + \alpha\Delta T)$

Want L_s (steel) $= L_b$ (brass) for the same ΔT for both materials: $L_{0s}(1+\alpha_s\Delta T) = L_{0b}(1+\alpha_b\Delta T)$ so

$L_{0s} + L_{0s}\alpha_s\Delta T = L_{0b} + L_{0b}\alpha_b\Delta T$

$$\Delta T = \frac{L_{0b} - L_{0s}}{L_{0s}\alpha_s - L_{0b}\alpha_b} = \frac{2.5020 \text{ in.} - 2.5000 \text{ in.}}{(2.5000 \text{ in.})(1.2\times10^{-5}(\text{C}°)^{-1}) - (2.5020 \text{ in.})(2.0\times10^{-5}(\text{C}°)^{-1})}$$

$$\Delta T = \frac{0.0020}{3.00\times10^{-5} - 5.00\times10^{-5}} \text{ C}° = -100 \text{ C}°$$

$T = T_0 + \Delta T = 20.0°\text{C} - 100 \text{ C}° = -80°\text{C}$

Evaluate: Both diameters decrease when the temperature is lowered but the diameter of the brass shaft decreases more since $\alpha_b > \alpha_s$; $|\Delta L_b| - |\Delta L_s| = 0.0020$ in.

17.82. **Identify:** Calculate the total food energy value for one doughnut. $K = \frac{1}{2}mv^2$.

Set Up: 1 cal = 4.186 J

Execute: **(a)** The energy is $(2.0 \text{ g})(4.0 \text{ kcal/g}) + (17.0 \text{ g})(4.0 \text{ kcal/g}) + (7.0 \text{ g})(9.0 \text{ kcal/g}) = 139$ kcal. The time required is $(139 \text{ kcal})/(510 \text{ kcal/h}) = 0.273 \text{ h} = 16.4$ min.

(b) $v = \sqrt{2K/m} = \sqrt{2(139\times10^3 \text{ cal})(4.186 \text{ J/cal})/(60 \text{ kg})} = 139 \text{ m/s} = 501 \text{ km/h}$.

Evaluate: When we set $K = Q$, we must express Q in J, so we can solve for v in m/s.

17.83. **Identify:** We mix an unknown liquid at 90.0°C with water at 0.0°C and measure the final equilibrium temperature T. The heat lost by the unknown liquid is equal to the heat gained by the water, so $Q_{\text{net}} = 0$.

Set Up: $Q_{\text{net}} = Q_{\text{water}} + Q_{\text{unknown}}$. We need to relate the mass of the water m_w to T in order to interpret the graph of m_w versus T^{-1}. Let x refer to the unknown liquid. We want to find c_x. For temperature changes, we use $Q = mc\Delta T$.

Execute: Using $Q_{\text{net}} = Q_{\text{water}} + Q_{\text{unknown}} = 0$ gives $m_w c_w \Delta T_w + m_x c_x \Delta T_x = 0$. Solve for m_w as a function of T^{-1}. $m_w c_w (T - 0.0°\text{C}) = m_x c_x (90.0°\text{C} - T)$, so $m_w = \left(\frac{m_x c_x}{c_w}\right)\left(\frac{90.0°\text{C}}{T} - 1\right)$. The graph of m_w versus T^{-1} should be a straight line having slope equal to $\frac{m_x c_x 90.0°\text{C}}{c_w}$. Solving for c_x gives

$$c_x = \frac{c_w(\text{slope})}{m_x(90.0°\text{C})} = \frac{(4190 \text{ J/kg}\cdot\text{K})(2.15 \text{ kg}\cdot\text{C}°)}{(0.050 \text{ kg})(90.0°\text{C})} = 2000 \text{ J/kg}\cdot\text{K}.$$

Evaluate: From Table 17.3 we see that c_x is about half that of water, but close to that of ice, ethanol, and ethylene glycol, so we get a reasonable result.

17.84. **Identify:** $Q_{\text{system}} = 0$. Assume that the normal melting point of iron is above 745°C so the iron initially is solid.

Set Up: For water, $c = 4190$ J/kg·K and $L_v = 2256\times10^3$ J/kg. For solid iron, $c = 470$ J/kg·K.

Execute: The heat released when the iron slug cools to 100°C is $Q = mc\Delta T =$ $(0.1000 \text{ kg})(470 \text{ J/kg}\cdot\text{K})(645 \text{ K}) = 3.03\times10^4$ J. The heat absorbed when the temperature of the water is raised to 100°C is $Q = mc\Delta T = (0.0850 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(80.0 \text{ K}) = 2.85\times10^4$ J. This is less than the heat released from the iron and $3.03\times10^4 \text{ J} - 2.85\times10^4 \text{ J} = 1.81\times10^3$ J of heat is available for converting some of the liquid water at 100°C to vapor. The mass m of water that boils is

$$m = \frac{1.81\times10^3 \text{ J}}{2256\times10^3 \text{ J/kg}} = 8.01\times10^{-4} \text{ kg} = 0.801 \text{ g}.$$

(a) The final temperature is 100°C.

(b) There is $85.0 \text{ g} - 0.801 \text{ g} = 84.2 \text{ g}$ of liquid water remaining, so the final mass of the iron and remaining water is 184.2 g.

EVALUATE: If we ignore the phase change of the water and write $m_{\text{iron}}c_{\text{iron}}(T - 745°\text{C}) + m_{\text{water}}c_{\text{water}}(T - 20.0°\text{C}) = 0$, when we solve for T we will get a value slightly larger than 100°C. That result is unphysical and tells us that some of the water changes phase.

17.85. IDENTIFY and SET UP: To calculate Q, use $Q = mc\,\Delta T$ in the form $dQ = nC\,dT$ and integrate, using $C(T)$ given in the problem. C_{av} is obtained from $C = \frac{1}{n}\frac{dQ}{dT}$ using the finite temperature range instead of an infinitesimal dT.

EXECUTE: (a) $dQ = nCdT$

$$Q = n\int_{T_1}^{T_2} C\,dT = n\int_{T_1}^{T_2} k(T^3/\theta^3)dT = (nk/\theta^3)\int_{T_1}^{T_2} T^3\,dt = (nk/\theta^3)\left(\tfrac{1}{4}T^4\Big|_{T_1}^{T_2}\right)$$

$$Q = \frac{nk}{4\theta^3}\left(T_2^4 - T_1^4\right) = \frac{(1.50\text{ mol})(1940\text{ J/mol}\cdot\text{K})}{4(281\text{ K})^3}\left[(40.0\text{ K})^4 - (10.0\text{ K})^4\right] = 83.6\text{ J}.$$

(b) $C_{\text{av}} = \dfrac{1}{n}\dfrac{\Delta Q}{\Delta T} = \dfrac{1}{1.50\text{ mol}}\left(\dfrac{83.6\text{ J}}{40.0\text{ K} - 10.0\text{ K}}\right) = 1.86\text{ J/mol}\cdot\text{K}$

(c) $C = k(T/\theta)^3 = (1940\text{ J/mol}\cdot\text{K})(40.0\text{ K}/281\text{ K})^3 = 5.60\text{ J/mol}\cdot\text{K}$

EVALUATE: C is increasing with T, so C at the upper end of the temperature integral is larger than its average value over the interval.

17.86. IDENTIFY: We are looking at the thermal radiation from the logs in a campfire.

SET UP and EXECUTE: The net radiation rate is $H_{\text{net}} = Ae\sigma\left(T^4 - T_s^4\right)$.

(a) Estimate: Log diameter is 25 cm = 0.25 m and length is 0.50 m.

(b) The total area is that of the two faces plus the side: $A = 2A_{\text{face}} + A_{\text{side}} = 2\pi R^2 + 2\pi RL$. For our log, we have $A = 2\pi(0.125\text{ m})^2 + 2\pi(0.125\text{ m})(0.50\text{ m}) = 0.49\text{ m}^2$.

(c) We want the net radiated power for $T = 700°\text{C} = 973\text{ K}$, $T_s = 20.0°\text{C} = 293\text{ K}$, and $e = 1$. Therefore $H_{\text{net}} = Ae\sigma\left(T^4 - T_s^4\right)$ = $(0.49\text{ m}^2)(5.67\times10\text{ W/m}^2\cdot\text{K}^4)\left[(973\text{ K})^4 - (293\text{ K})^4\right]$ so we get H_{net} = 2.5×10^4 W.

EVALUATE: On a chilly night when the temperature is 0°C (273 K) the rate of radiation would be even greater. As any camper knows, campfires can get very hot and radiate a lot of energy, as we have discovered.

17.87. IDENTIFY: Apply $Q = mc\Delta T$ to the air in the room.

SET UP: The mass of air in the room is $m = \rho V = (1.20\text{ kg/m}^3)(3200\text{ m}^3) = 3840\text{ kg}$. $1\text{ W} = 1\text{ J/s}$.

EXECUTE: (a) $Q = (3000\text{ s})(140\text{ students})(100\text{ J/s}\cdot\text{student}) = 4.20\times10^7\text{ J}$.

(b) $Q = mc\Delta T$. $\Delta T = \dfrac{Q}{mc} = \dfrac{4.20\times10^7\text{ J}}{(3840\text{ kg})(1020\text{ J/kg}\cdot\text{K})} = 10.7\text{ C°}$.

(c) $\Delta T = (10.7\text{ C°})\left(\dfrac{280\text{ W}}{100\text{ W}}\right) = 30.0\text{ C°}$.

EVALUATE: In the absence of a cooling mechanism for the air, the air temperature would rise significantly.

17.88. IDENTIFY: $dQ = nCdT$ so for the temperature change $T_1 \to T_2$, $Q = n\int_{T_1}^{T_2} C(T)dT$.

SET UP: $\int dT = T$ and $\int TdT = \frac{1}{2}T^2$. Express T_1 and T_2 in kelvins: $T_1 = 300\text{ K}$, $T_2 = 500\text{ K}$.

EXECUTE: Denoting C by $C = a + bT$, a and b independent of temperature, integration gives

$$Q = n\left[a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2)\right].$$

$Q = (3.00 \text{ mol})[(29.5 \text{ J/mol}\cdot\text{K})(500 \text{ K} - 300 \text{ K}) + (4.10\times10^{-3} \text{ J/mol}\cdot\text{K}^2)((500 \text{ K})^2 - (300 \text{ K})^2)].$

$Q = 1.97\times10^4$ J.

EVALUATE: If C is assumed to have the constant value 29.5 J/mol·K, then $Q = 1.77\times10^4$ J for this temperature change. At $T_1 = 300$ K, $C = 32.0$ J/mol·K and at $T_2 = 500$ K, $C = 33.6$ J/mol·K. The average value of C is 32.8 J/mol·K. If C is assumed to be constant and to have this average value, then $Q = 1.97\times10^4$ J, which is equal to the correct value.

17.89. **IDENTIFY:** The energy generated in the body is used to evaporate water, which prevents the body from overheating.

SET UP: Energy is (power)(time); calculate the heat energy Q produced in one hour. The mass m of water that vaporizes is related to Q by $Q = mL_v$. 1.0 kg of water has a volume of 1.0 L.

EXECUTE: **(a)** $Q = (0.80)(500 \text{ W})(3600 \text{ s}) = 1.44\times10^6$ J. The mass of water that evaporates each hour is $m = \dfrac{Q}{L_v} = \dfrac{1.44\times10^6 \text{ J}}{2.42\times10^6 \text{ J/kg}} = 0.60$ kg.

(b) (0.60 kg/h)(1.0 L/kg) = 0.60 L/h. The number of bottles of water is

$\dfrac{0.60 \text{ L/h}}{0.750 \text{ L/bottle}} = 0.80$ bottles/h.

EVALUATE: It is not unreasonable to drink 8/10 of a bottle of water per hour during vigorous exercise.

17.90. **IDENTIFY:** If it cannot be gotten rid of in some way, the metabolic energy transformed to heat will increase the temperature of the body.

SET UP: From Problem 17.89, $Q = 1.44\times10^6$ J and $m = 70$ kg. $Q = mc\Delta T$. Convert the temperature change in C° to F° using that 9 F° = 5 C°.

EXECUTE: **(a)** $Q = mc\Delta T$ so $\Delta T = \dfrac{Q}{mc} = \dfrac{1.44\times10^6 \text{ J}}{(70 \text{ kg})(3480 \text{ J/kg}\cdot\text{C}°)} = 5.9$ C°.

(b) $\Delta T = (5.9°\text{C})\left(\dfrac{9 \text{ F}°}{5 \text{ C}°}\right) = 10.6°\text{F}$. $T = 98.6°\text{F} + 10.6 \text{ F}° = 109°\text{F}$.

EVALUATE: A temperature this high can cause heat stroke and be lethal.

17.91. **IDENTIFY** and **SET UP:** The heat produced from the reaction is $Q_{\text{reaction}} = mL_{\text{reaction}}$, where L_{reaction} is the heat of reaction of the chemicals.

$Q_{\text{reaction}} = W + \Delta U_{\text{spray}}$

EXECUTE: For a mass m of spray, $W = \frac{1}{2}mv^2 = \frac{1}{2}m(19 \text{ m/s})^2 = (180.5 \text{ J/kg})m$ and

$\Delta U_{\text{spray}} = Q_{\text{spray}} = mc\Delta T = m(4190 \text{ J/kg}\cdot\text{K})(100°\text{C} - 20°\text{C}) = (335{,}200 \text{ J/kg})m.$

Then $Q_{\text{reaction}} = (180 \text{ J/kg} + 335{,}200 \text{ J/kg})m = (335{,}380 \text{ J/kg})m$ and $Q_{\text{reaction}} = mL_{\text{reaction}}$ implies $mL_{\text{reaction}} = (335{,}380 \text{ J/kg})m.$

The mass m divides out and $L_{\text{reaction}} = 3.4\times10^5$ J/kg.

EVALUATE: The amount of energy converted to work is negligible for the two significant figures to which the answer should be expressed. Almost all of the energy produced in the reaction goes into heating the compound.

17.92. **IDENTIFY:** The bike peddler generates energy, 70% of which goes into melting the ice. We are dealing with a change of phase of the ice and the power generated by turning the bicycle wheel.

SET UP: The bike peddler generates 25.0 N·m of torque while peddling to turn the bike wheel at 30.0 rev/min = 3.142 rad/s. The power due to this torque is $P = \tau\omega$, the energy E it produces in time t is $E = Pt$, the heat Q to melt ice is $Q = mL_f$, and the heat to raise the water temperature is $Q = mc\Delta T$.

EXECUTE: **(a)** The target variable is the time to melt 3.00 kg of ice at 0.0°C. All the ice is at 0.0°C so all the energy from peddling is used only to melt the ice. This heat is $Q = mL_f =$ $(3.00\text{ kg})(334\times10^3\text{ J/kg}) = 1.00\times10^6\text{ J}$. The peddling power is $P = \tau\omega = (25.0\text{ N}\cdot\text{m})(3.142\text{ rad/s}) =$ 78.55 W. Only 70% of this power is used to melt ice, so the available power is (0.70)(78.55 W) = 55.0 W. The heat from this power in time t is $Q = Pt$, and this must be $1.00\times10^6\text{ J}$ to melt the ice. Therefore $1.00\times10^6\text{ J} = (55.0\text{ W})t$, so $t = 1.8\times10^4\text{ s} = 5.1\text{ h}$.

(b) The additional heat is required to raise the water temperature from 0.0°C to 10.5°C. We use $Q = mc\Delta T$ where m is the mass of 6.0 L of water plus 3.00 kg of melted ice. One liter is 6000 cm^3, and each cm^3 has a mass of 1.00 g, so 6.0 L of water has a mass of 6000 g = 6.0 kg. The total water mass is 9.0 kg. The heat is $Q = mc\Delta T = (9.00\text{ kg})(4190\text{ J/kg}\cdot\text{K})(10.5\text{ C}°) = 3.96\times10^5\text{ J}$. The cyclist is providing 55.0 W of power, so $(55.0\text{ W})t = 3.96\times10^5\text{ J}$, which gives $t = 7200\text{ s} = 2.0\text{ h}$.

EVALUATE: It takes about 2½ times as long to melt the ice than to heat all the water by 10.5°C. This difference in time occurs because L_f is large for water.

17.93. **IDENTIFY:** The heat lost by the water is equal to the amount of heat gained by the ice. First calculate the amount of heat the water could give up if it is cooled to 0.0°C. Then see how much heat it would take to melt all of the ice. If the heat to melt the ice is less than the heat the water would give up, the ice all melts and then the resulting water is heated to some final temperature.

SET UP: $Q = mc\,\Delta T$ and $Q = mL_f$.

EXECUTE: **(a)** Heat from water if cooled to 0.0°C: $Q = mc\,\Delta T$

$Q = mc\,\Delta T = (1.50\text{ kg})(4190\text{ J/kg}\cdot\text{K})(28.0\text{ K}) = 1.760\times10^5\text{ J}$

Heat to melt all of the ice: $Q = mc\,\Delta T + mL_f = m(c\Delta T + L_f)$

$Q = (0.600\text{ kg})[(2100\text{ J/kg}\cdot\text{K})(22.0\text{ K}) + 3.34\times10^5\text{ J/kg}] = 2.276\times10^5\text{ J}$

Since the heat required to melt all the ice is greater than the heat available by cooling the water to 0.0°C, not all the ice will melt.

(b) Since not all the ice melts, the final temperature of the water (and ice) will be 0.0°C. So the heat from the water will melt only part of the ice. Call m the mass of the melted ice. Therefore $Q_{\text{from water}} = 1.760\times10^5\text{ J} = (0.600\text{ kg})(2100\text{ J/kg}\cdot\text{K})(22.0\text{ K}) + m(3.34\times10^5\text{ J/kg})$, which gives $m =$ 0.444 kg, which is the amount of ice that melts. The mass of ice remaining is 0.600 kg – 0.444 kg = 0.156 kg. The final temperature will be 0.0°C since some ice remains in the water.

EVALUATE: An alternative approach would be to assume that all the ice melts and find the final temperature of the water in the container. This actually comes out to be negative, which is not possible if all the ice melts. Therefore not all the ice could have melted. Once you know this, proceed as in part (b).

17.94. **IDENTIFY:** The amount of heat lost by the soft drink and mug is equal to the heat gained by the ice. The ice must first be heated to 0.0°C, then melted, and finally the resulting water heated to the final temperature of the system.

SET UP: Assume that all the ice melts. If we calculate $T_f < 0$, we will know this assumption is incorrect. For aluminum, $c_a = 910\text{ J/kg}\cdot\text{K}$. For water, $L_f = 3.35\times10^5\text{ J/kg}$ and $c_w = 4.19\times10^3\text{ J/kg}\cdot\text{K}$. For ice, $c_i = 2100\text{ J/kg}\cdot\text{K}$. The density of water is $1.00\times10^3\text{ kg/m}^3$, so 1.00 L of water has mass 1.00 kg.

EXECUTE: For the soft drink:

$Q_w = m_w c_w\,\Delta T_w = (2.00\text{ kg})(4.19\times10^3\text{ J/kg}\cdot\text{K})(T - 20.0°\text{C}) = (8380\text{ J/K})T - 1.676\times10^5\text{ J}.$

For the mug: $Q_a = m_a c_a\,\Delta T_a = (0.257\text{ kg})(910\text{ J/kg}\cdot\text{K})(T - 20.0°\text{C}) = (234\text{ J/K})T - 4.68\times10^3\text{ J}.$

For the ice: $Q_i = m_i c_i\,\Delta T_i + m_i L_f + m_i c_w\,\Delta T_w$.

$Q_i = (0.120\text{ kg})[(2100\text{ J/kg}\cdot\text{K})(15.0\text{ C°}) + 3.34\times10^5\text{ J/kg} + (4.19\times10^3\text{ J/kg}\cdot\text{K})(T - 0\text{ C°})].$

$Q_i = 4.386\times10^4\text{ J} + (503\text{ J/K})T.$

$\sum Q = 0$ gives

$(8380\text{ J/K})T - 1.676\times10^5\text{ J} + (234\text{ J/K})T - 4.68\times10^3\text{ J} + 4.386\times10^4\text{ J} + (503\text{ J/K})T = 0.$

$$T = \frac{1.284\times10^5\text{ J}}{9117\text{ J/K}} = 14.1°\text{C}.$$

EVALUATE: $T > 0°\text{C}$, so our assumption that all the ice melts is correct. Note that the ice and the water from the melted ice have different specific heat capacities.

17.95. **IDENTIFY** and **SET UP:** Assume that all the ice melts and that all the steam condenses. If we calculate a final temperature T that is outside the range 0°C to 100°C then we know that this assumption is incorrect. Calculate Q for each piece of the system and then set the total $Q_{\text{system}} = 0$.

EXECUTE: **(a)** Copper can (changes temperature from 0.0° to T; no phase change):

$Q_{\text{can}} = mc\Delta T = (0.446\text{ kg})(390\text{ J/kg}\cdot\text{K})(T - 0.0°\text{C}) = (173.9\text{ J/K})T$

Ice (melting phase change and then the water produced warms to T):

$Q_{\text{ice}} = +mL_f + mc\Delta T = (0.0950\text{ kg})(334\times10^3\text{ J/kg}) + (0.0950\text{ kg})(4190\text{ J/kg}\cdot\text{K})(T - 0.0°\text{C})$

$Q_{\text{ice}} = 3.173\times10^4\text{ J} + (398.0\text{ J/K})T.$

Steam (condenses to liquid and then water produced cools to T):

$Q_{\text{steam}} = -mL_v + mc\Delta T = -(0.0350\text{ kg})(2256\times10^3\text{ J/kg}) + (0.0350\text{ kg})(4190\text{ J/kg}\cdot\text{K})(T - 100.0°\text{C})$

$Q_{\text{steam}} = -7.896\times10^4\text{ J} + (146.6\text{ J/K})T - 1.466\times10^4\text{ J} = -9.362\times10^4\text{ J} + (146.6\text{ J/K})T$

$Q_{\text{system}} = 0$ implies $Q_{\text{can}} + Q_{\text{ice}} + Q_{\text{steam}} = 0.$

$(173.9\text{ J/K})T + 3.173\times10^4\text{ J} + (398.0\text{ J/K})T - 9.362\times10^4\text{ J} + (146.6\text{ J/K})T = 0$

$(718.5\text{ J/K})T = 6.189\times10^4\text{ J}$

$$T = \frac{6.189\times10^4\text{ J}}{718.5\text{ J/K}} = 86.1°\text{C}.$$

(b) No ice, no steam, and $0.0950\text{ kg} + 0.0350\text{ kg} = 0.130\text{ kg}$ of liquid water.

EVALUATE: This temperature is between 0°C and 100°C so our assumptions about the phase changes being complete were correct.

17.96. **IDENTIFY:** The final amount of ice is less than the initial mass of water, so water remains and the final temperature is 0°C. The ice added warms to 0°C and heat comes out of water to convert that water to ice. Conservation of energy says $Q_i + Q_w = 0,$ where Q_i and Q_w are the heat flows for the ice that is added and for the water that freezes.

SET UP: Let m_i be the mass of ice that is added and m_w is the mass of water that freezes. The mass of ice increases by 0.434 kg, so $m_i + m_w = 0.434\text{ kg}.$ For water, $L_f = 334\times10^3\text{ J/kg}$ and for ice $c_i = 2100\text{ J/kg}\cdot\text{K}.$ Heat comes out of the water when it freezes, so $Q_w = -mL_f.$

EXECUTE: $Q_i + Q_w = 0$ gives $m_i c_i(15.0\text{ C°}) + (-m_w L_f) = 0,$ $m_w = 0.434\text{ kg} - m_i,$ so $m_i c_i(15.0\text{ C°}) + (-0.434\text{ kg} + m_i)L_f = 0.$

$$m_i = \frac{(0.434\text{ kg})L_f}{c_i(15.0\text{ C°}) + L_f} = \frac{(0.434\text{ kg})(334\times10^3\text{ J/kg})}{(2100\text{ J/kg}\cdot\text{K})(15.0\text{ K}) + 334\times10^3\text{ J/kg}} = 0.397\text{ kg}.$$ 0.397 kg of ice was added.

EVALUATE: The mass of water that froze when the ice at -15.0C° was added was $0.884\text{ kg} - 0.450\text{ kg} - 0.397\text{ kg} = 0.037\text{ kg}.$

17.97. **IDENTIFY** and **SET UP:** Heat comes out of the steam when it changes phase and heat goes into the water and causes its temperature to rise. $Q_{\text{system}} = 0$. First determine what phases are present after the system has come to a uniform final temperature.

EXECUTE: (a) Heat that must be removed from steam if all of it condenses is

$Q = -mL_{\text{v}} = -(0.0400\text{ kg})(2256\times10^3\text{ J/kg}) = -9.02\times10^4\text{ J}$

Heat absorbed by the water if it heats all the way to the boiling point of 100°C:

$Q = mc\Delta T = (0.200\text{ kg})(4190\text{ J/kg}\cdot\text{K})(50.0\text{ C°}) = 4.19\times10^4\text{ J}$

(b) Mass of steam that condenses is $m = Q/L_{\text{v}} = 4.19\times10^4\text{ J}/2256\times10^3\text{ J/kg} = 0.0186\text{ kg}$.

Thus there is $0.0400\text{ kg} - 0.0186\text{ kg} = 0.0214\text{ kg}$ of steam left. The amount of liquid water is $0.0186\text{ kg} + 0.200\text{ kg} = 0.219\text{ kg}$.

EVALUATE: The water can't absorb enough heat for all the steam to condense. Steam is left and the final temperature then must be 100°C.

17.98. **IDENTIFY:** Heat is conducted out of the body. At steady state, the rate of heat flow is the same in both layers (fat and fur).

SET UP: Since the model is only a crude approximation to a bear, we will make the simplifying assumption that the surface area of each layer is constant and given by the surface area of a sphere of radius 1.5 m. Let the temperature of the fat-air boundary be T. A section of the two layers is sketched in Figure 17.98. A Kelvin degree is the same size as a Celsius degree, so $\text{W/m}\cdot\text{K}$ and $\text{W/m}\cdot\text{C°}$ are equivalent units. At steady state the heat current through each layer is equal to 50 W. The area of each layer is $A = 4\pi r^2$, with $r = 0.75$ m. Use $H = kA\dfrac{T_{\text{H}} - T_{\text{C}}}{L}$.

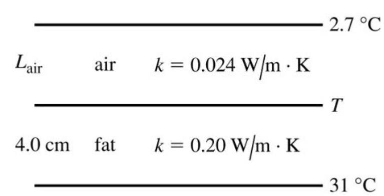


Figure 17.98

EXECUTE: **(a)** Apply $H = kA\dfrac{T_{\text{H}} - T_{\text{C}}}{L}$ to the fat layer and solve for $T_{\text{C}} = T$. For the fat layer

$$T_{\text{H}} = 31°\text{C}.\quad T = T_{\text{H}} - \frac{HL}{kA} = 31°\text{C} - \frac{(50\text{ W})(4.0\times10^{-2}\text{ m})}{(0.20\text{ W/m}\cdot\text{K})(4\pi)(0.75\text{ m})^2} = 31°\text{C} - 1.4°\text{C} = 29.6°\text{C}.$$

(b) Apply $H = kA\dfrac{T_{\text{H}} - T_{\text{C}}}{L}$ to the air layer and solve for $L = L_{\text{air}}$. For the air layer $T_{\text{H}} = T = 29.6°\text{C}$ and

$$T_{\text{C}} = 2.7°\text{C}.\quad L = \frac{kA(T_{\text{H}} - T_{\text{C}})}{H} = \frac{(0.024\text{ W/m}\cdot\text{K})(4\pi)(0.75\text{ m})^2(29.6°\text{C} - 2.7°\text{C})}{50\text{ W}} = 9.1\text{ cm}.$$

EVALUATE: The thermal conductivity of air is much lass than the thermal conductivity of fat, so the temperature gradient for the air must be much larger to achieve the same heat current. So, most of the temperature difference is across the air layer.

17.99. **IDENTIFY:** Apply $H = kA\dfrac{T_{\text{H}} - T_{\text{C}}}{L}$.

SET UP: For the glass use $L = 12.45$ cm, to account for the thermal resistance of the air films on either side of the glass.

EXECUTE: **(a)** $H = (0.120 \text{ W/m}\cdot\text{K})\ (2.00\times 0.95 \text{ m}^2)\left(\dfrac{28.0 \text{ C}°}{5.0\times 10^{-2} \text{ m} + 1.8\times 10^{-2} \text{ m}}\right) = 93.9 \text{ W}.$

(b) The heat flow through the wood part of the door is reduced by a factor of $1 - \dfrac{(0.50)^2}{(2.00\times 0.95)} = 0.868,$

so it becomes 81.5 W. The heat flow through the glass is

$H_{\text{glass}} = (0.80 \text{ W/m}\cdot\text{K})(0.50 \text{ m})^2\left(\dfrac{28.0 \text{ C}°}{12.45\times 10^{-2} \text{ m}}\right) = 45.0 \text{ W},$ and so the ratio is $\dfrac{81.5 + 45.0}{93.9} = 1.35.$

EVALUATE: The single-pane window produces a significant increase in heat loss through the door. (See Problem 17.101).

17.100. **IDENTIFY:** This problem deals with thermal stress.

SET UP: The metal bar tends to expand as its temperature rises from 0°C to T, but it cannot do this because it is confined by the concrete slab which does not expand appreciably. The force F on the bar due to this thermal stress is the normal force for the static friction force that supports the bar and the container it is lifting. Our target variable is the mass m of the heaviest container this system can support, and the thermal stress is $F / A = -Y\alpha\Delta T$.

EXECUTE: $f_S = w_{\text{bar}} + w_{\text{container}}$. The force magnitude is $F = AY\alpha\Delta T = AY\alpha T$. For the heaviest container, static friction is a maximum, so $f_S = \mu_S n$, where $n = F$. The area A is the area of contact between the slab and the cylindrical bar, so $A = 2\pi rd$. Using M for the mass of the bar and m for the mass of the container, we have $\mu_S(2\pi rd)Y\alpha T = Mg + mg$. Solving for m gives

$m = M + 2\pi\mu_S rdY\alpha T / g$.

EVALUATE: The slab would need to be very strong to support a reasonably heavy container (including its contents).

17.101. **IDENTIFY** and **SET UP:** Use H written in terms of the thermal resistance R: $H = A\Delta T/R$, where $R = L/k$ and $R = R_1 + R_2 + \ldots$ (additive).

EXECUTE: <u>single pane:</u> $R_s = R_{\text{glass}} + R_{\text{film}}$, where $R_{\text{film}} = 0.15 \text{ m}^2\cdot\text{K/W}$ is the combined thermal resistance of the air films on the room and outdoor surfaces of the window.

$R_{\text{glass}} = L/k = (4.2\times 10^{-3} \text{ m})/(0.80 \text{ W/m}\cdot\text{K}) = 0.00525 \text{ m}^2\cdot\text{K/W}$

Thus $R_s = 0.00525 \text{ m}^2\cdot\text{K/W} + 0.15 \text{ m}^2\cdot\text{K/W} = 0.1553 \text{ m}^2\cdot\text{K/W}.$

<u>double pane:</u> $R_d = 2R_{\text{glass}} + R_{\text{air}} + R_{\text{film}}$, where R_{air} is the thermal resistance of the air space between the panes. $R_{\text{air}} = L/k = (7.0\times 10^{-3} \text{ m})/(0.024 \text{ W/m}\cdot\text{K}) = 0.2917 \text{ m}^2\cdot\text{K/W}$

Thus $R_d = 2(0.00525 \text{ m}^2\cdot\text{K/W}) + 0.2917 \text{ m}^2\cdot\text{K/W} + 0.15 \text{ m}^2\cdot\text{K/W} = 0.4522 \text{ m}^2\cdot\text{K/W}$

$H_s = A\Delta T/R_s,\ H_d = A\Delta T/R_d$, so $H_s/H_d = R_d/R_s$ (since A and ΔT are same for both)

$H_s/H_d = (0.4522 \text{ m}^2\cdot\text{K/W})/(0.1553 \text{ m}^2\cdot\text{K/W}) = 2.9$

EVALUATE: The heat loss is about a factor of 3 less for the double-pane window. The increase in R for a double pane is due mostly to the thermal resistance of the air space between the panes.

17.102. **IDENTIFY:** Apply $H = \dfrac{kA\Delta T}{L}$ to each rod. Conservation of energy requires that the heat current through the copper equals the sum of the heat currents through the brass and the steel.

SET UP: Denote the quantities for copper, brass and steel by 1, 2, and 3, respectively, and denote the temperature at the junction by T_0.

EXECUTE: (a) $H_1 = H_2 + H_3$. Using $H = kA(T_H - T_C)/L$ and dividing by the common area gives

$\frac{k_1}{L_1}(100°\text{C} - T_0) = \frac{k_2}{L_2}T_0 + \frac{k_3}{L_3}T_0$. Solving for T_0 gives $T_0 = \frac{(k_1/L_1)}{(k_1/L_1)+(k_2/L_2)+(k_3/L_3)}(100°\text{C})$.
Substitution of numerical values gives $T_0 = 78.4°\text{C}$.

(b) Using $H = \frac{kA}{L}\Delta T$ for each rod, with $\Delta T_1 = 21.6\ \text{C}°$, $\Delta T_2 = \Delta T_3 = 78.4\ \text{C}°$ gives

$H_1 = 12.8\ \text{W}$, $H_2 = 9.50\ \text{W}$ and $H_3 = 3.30\ \text{W}$.

EVALUATE: In part (b), H_1 is seen to be the sum of H_2 and H_3.

17.103. **IDENTIFY:** The jogger radiates heat but the air radiates heat back into the jogger.
SET UP: The emissivity of a human body is taken to be 1.0. In the equation for the radiation heat current, $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$, the temperatures must be in kelvins.

EXECUTE: **(a)** $P_{\text{jog}} = (0.80)(1300\ \text{W}) = 1.04 \times 10^3\ \text{J/s}$.

(b) $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$, which gives

$H_{\text{net}} = (1.85\ \text{m}^2)(1.00)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)([306\ \text{K}]^4 - [313\ \text{K}]^4) = -87.1\ \text{W}$. The person gains 87.1 J of heat each second by radiation.
(c) The total excess heat per second is $1040\ \text{J/s} + 87\ \text{J/s} = 1130\ \text{J/s}$.

(d) In $1\ \text{min} = 60\ \text{s}$, the runner must dispose of $(60\ \text{s})(1130\ \text{J/s}) = 6.78 \times 10^4\ \text{J}$. If this much heat goes to evaporate water, the mass *m* of water that evaporates in one minute is given by $Q = mL_v$, so

$$m = \frac{Q}{L_v} = \frac{6.78 \times 10^4\ \text{J}}{2.42 \times 10^6\ \text{J/kg}} = 0.028\ \text{kg} = 28\ \text{g}.$$

(e) In a half-hour, or 30 minutes, the runner loses $(30\ \text{min})(0.028\ \text{kg/min}) = 0.84\ \text{kg}$. The runner must drink 0.84 L, which is $\frac{0.84\ \text{L}}{0.750\ \text{L/bottle}} = 1.1$ bottles.

EVALUATE: The person *gains* heat by radiation since the air temperature is greater than his skin temperature.

17.104. **IDENTIFY:** The nonmechanical part of the basal metabolic rate (i.e., the heat) leaves the body by radiation from the surface.
SET UP: In the radiation equation, $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$, the temperatures must be in kelvins; $e = 1.0$, $T = 30°\text{C} = 303\ \text{K}$, and $T_s = 18°\text{C} = 291\ \text{K}$. Call the basal metabolic rate BMR.

EXECUTE: **(a)** $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$.

$H_{\text{net}} = (2.0\ \text{m}^2)(1.0)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)([303\ \text{K}]^4 - [291\ \text{K}]^4) = 140\ \text{W}$.
(b) $(0.80)\text{BMR} = 140\ \text{W}$, so $\text{BMR} = 180\ \text{W}$.

EVALUATE: If the emissivity of the skin were less than 1.0, the body would radiate less so the BMR would have to be lower than we found in (b).

17.105. **(a) IDENTIFY** and **EXECUTE:** Heat must be conducted from the water to cool it to 0°C and to cause the phase transition. The entire volume of water is not at the phase transition temperature, just the upper surface that is in contact with the ice sheet.
(b) IDENTIFY: The heat that must leave the water in order for it to freeze must be conducted through the layer of ice that has already been formed.
SET UP: Consider a section of ice that has area *A*. At time *t* let the thickness be *h*. Consider a short time interval *t* to $t + dt$. Let the thickness that freezes in this time be *dh*. The mass of the section that freezes in the time interval *dt* is $dm = \rho\, dV = \rho A\, dh$. The heat that must be conducted away from this mass of water to freeze it is $dQ = dmL_f = (\rho A L_f)dh$. $H = dQ/dt = kA(\Delta T/h)$, so the heat *dQ* conducted in time

dt throughout the thickness h that is already there is $dQ = kA\left(\frac{T_H - T_C}{h}\right)dt$. Solve for dh in terms of dt and integrate to get an expression relating h and t.

EXECUTE: Equate these expressions for dQ.

$$\rho A L_f dh = kA\left(\frac{T_H - T_C}{h}\right)dt$$

$$h\,dh = \left(\frac{k(T_H - T_C)}{\rho L_f}\right)dt$$

Integrate from $t = 0$ to time t. At $t = 0$ the thickness h is zero.

$$\int_0^h h\,dh = [k(T_H - T_C)/\rho L_f]\int_0^t dt$$

$$\tfrac{1}{2}h^2 = \frac{k(T_H - T_C)}{\rho L_f}t \text{ and } h = \sqrt{\frac{2k(T_H - T_C)}{\rho L_f}}\sqrt{t}$$

The thickness after time t is proportional to $\sqrt{t}$.

(c) The expression in part (b) gives

$$t = \frac{h^2 \rho L_f}{2k(T_H - T_C)} = \frac{(0.25\text{ m})^2(920\text{ kg/m}^3)(334\times10^3\text{ J/kg})}{2(1.6\text{ W/m}\cdot\text{K})(0°\text{C} - (-10°\text{C}))} = 6.0\times10^5\text{ s}$$

$t = 170$ h.

(d) Find t for $h = 40$ m. t is proportional to h^2, so $t = (40\text{ m}/0.25\text{ m})^2(6.00\times10^5\text{ s}) = 1.5\times10^{10}$ s. This is about 500 years. With our current climate this will not happen.

EVALUATE: As the ice sheet gets thicker, the rate of heat conduction through it decreases. Part (d) shows that it takes a very long time for a moderately deep lake to totally freeze.

17.106. **IDENTIFY:** $I_1 r_1^2 = I_2 r_2^2$. Apply $H = Ae\sigma T^4$ to the sun.

SET UP: $I_1 = 1.50\times10^3\text{ W/m}^2$ when $r = 1.50\times10^{11}$ m.

EXECUTE: **(a)** The energy flux at the surface of the sun is

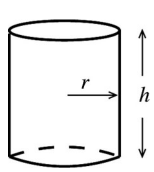
$$I_2 = (1.50\times10^3\text{ W/m}^2)\left(\frac{1.50\times10^{11}\text{ m}}{6.96\times10^8\text{ m}}\right)^2 = 6.97\times10^7\text{ W/m}^2.$$

(b) Solving $H = Ae\sigma T^4$ with $e = 1$, $T = \left[\frac{H}{A}\frac{1}{\sigma}\right]^{\frac{1}{4}} = \left[\frac{6.97\times10^7\text{ W/m}^2}{5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}\right]^{\frac{1}{4}} = 5920$ K.

EVALUATE: The total power output of the sun is $P = 4\pi r_2^2 I_2 = 4\times10^{26}$ W.

17.107. **IDENTIFY** and **SET UP:** Use $H_{net} = e\sigma A(T^4 - T_s^4)$ to find the net heat current into the can due to radiation. Use $Q = Ht$ to find the heat that goes into the liquid helium, set this equal to mL and solve for the mass m of helium that changes phase.

EXECUTE: Calculate the net rate of radiation of heat from the can. $H_{net} = Ae\sigma(T^4 - T_s^4)$.



The surface area of the cylindrical can is $A = 2\pi rh + 2\pi r^2$. (See Figure 17.107.)

Figure 17.107

$A = 2\pi r(h + r) = 2\pi(0.045\text{ m})(0.250\text{ m} + 0.045\text{ m}) = 0.08341\text{ m}^2.$

$H_{\text{net}} = (0.08341\text{ m}^2)(0.200)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)((4.22\text{ K})^4 - (77.3\text{ K})^4)$

$H_{\text{net}} = -0.0338\text{ W}$ (the minus sign says that the net heat current is into the can). The heat that is put into the can by radiation in one hour is $Q = -(H_{\text{net}})t = (0.0338\text{ W})(3600\text{ s}) = 121.7\text{ J}$. This heat boils a mass m of helium according to the equation $Q = mL_{\text{f}}$, so $m = \dfrac{Q}{L_{\text{f}}} = \dfrac{121.7\text{ J}}{2.09\times10^4\text{ J/kg}} = 5.82\times10^{-3}\text{ kg} = 5.82\text{ g}.$

EVALUATE: In the expression for the net heat current into the can the temperature of the surroundings is raised to the fourth power. The rate at which the helium boils away increases by about a factor of $(293/77)^4 = 210$ if the walls surrounding the can are at room temperature rather than at the temperature of the liquid nitrogen.

17.108. **IDENTIFY:** We have blackbody radiation. The sphere at 41.0°C (314 K) radiates into the box, but the box at 30.0°C (303 K) radiates back into the sphere. All temperatures must be in kelvins.

SET UP: The rate at which heat is radiated by a blackbody in surroundings is $H_{\text{net}} = e\sigma A(T^4 - T_{\text{s}}^4)$. The target variable is the emissivity e for part (a). We know that $H_{\text{net}} = 0.660$ J/s for the sphere, and $A = 4\pi r^2$ for a sphere.

EXECUTE: **(a)** Using $H_{\text{net}} = e\sigma A(T^4 - T_{\text{s}}^4)$, we put in the numbers and solve for the emissivity e. Using $H_{\text{net}} = 0.660$ J/s, $A = 4\pi r^2 = 4\pi(0.0320\text{ m})^2 = 0.012868\text{ m}^2$, $T = 314$ K, $T_{\text{S}} = 303$ K, and $\sigma = 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4$, we get $e = 0.700$. Note that this number has no units.

(b) Using $H_{\text{net}} = e\sigma A(T^4 - T_{\text{s}}^4)$, we get

$H = (0.700)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(0.012868\text{ m}^2)[(355\text{ K})^4 - (303\text{ K})^4] = 3.81\text{ J/s} = 3.81\text{ W}.$

EVALUATE: The ratio of powers is (3.81 W)/(0.660 W) = 5.77. This is much less than 2^4, which is 16. The temperature must be in kelvins, so a temperature of 82°C (355 K) is not double 41°C (314 K). The temperature ratio is only 355/314 = 1.13.

17.109. **IDENTIFY:** The latent heat of fusion L_{f} is defined by $Q = mL_{\text{f}}$ for the solid $\rightarrow$ liquid phase transition. For a temperature change, $Q = mc\Delta T$.

SET UP: At $t = 1$ min the sample is at its melting point and at $t = 2.5$ min all the sample has melted.

EXECUTE: **(a)** It takes 1.5 min for all the sample to melt once its melting point is reached and the heat input during this time interval is $(1.5\text{ min})(10.0\times10^3\text{ J/min}) = 1.50\times10^4\text{ J}$. $Q = mL_{\text{f}}$.

$$L_{\text{f}} = \frac{Q}{m} = \frac{1.50\times10^4\text{ J}}{0.500\text{ kg}} = 3.00\times10^4\text{ J/kg}.$$

(b) The liquid's temperature rises 30 C° in 1.5 min. $Q = mc\Delta T$.

$$c_{\text{liquid}} = \frac{Q}{m\Delta T} = \frac{1.50\times10^4\text{ J}}{(0.500\text{ kg})(30\text{ C°})} = 1.00\times10^3\text{ J/kg}\cdot\text{K}.$$

The solid's temperature rises 15 C° in 1.0 min. $c_{\text{solid}} = \dfrac{Q}{m\Delta T} = \dfrac{1.00\times10^4\text{ J}}{(0.500\text{ kg})(15\text{ C°})} = 1.33\times10^3\text{ J/kg}\cdot\text{K}.$

EVALUATE: The specific heat capacities for the liquid and solid states are different. The values of c and L_{f} that we calculated are within the range of values in Tables 17.3 and 17.4.

17.110. **IDENTIFY:** The heat lost by the water is equal to the heat gained by the liquid and the cup. The specific heat capacities do not change over the temperature ranges involved. No phase changes are involved.

SET UP: $Q = mc\,\Delta T$, and $Q_{\text{L}} + Q_{\text{m}} + Q_{\text{w}} = 0$, where L is for the liquid, m is for the metal, and w is for water.

EXECUTE: For the first experiment, 0.500 kg of the liquid are used. $Q_{\text{L}} + Q_{\text{m}} + Q_{\text{w}} = 0$ gives

$0 = (0.500\text{ kg})c_{\text{L}}(58.1\text{°C} - 20.0\text{°C}) + (0.200\text{ kg})c_{\text{m}}(58.1\text{°C} - 20.0\text{°C})$

$+ (0.500 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(58.1°\text{C} - 80.0°\text{C})$

For the second experiment, 1.00 kg of the liquid is used. $Q_L + Q_m + Q_w = 0$ gives
$0 = (1.00 \text{ kg})c_L(49.3°\text{C} - 20.0°\text{C}) + (0.200 \text{ kg})c_m(49.3°\text{C} - 20.0°\text{C})$
$+ (0.500 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(49.3°\text{C} - 80.0°\text{C})$

The two equations from the two experiments simplify to
$19.050c_L + 7.620c_m = 45{,}880.5$
$29.3c_L + 5.86c_m = 64{,}316.5$
Solving these two equations simultaneously gives $c_m = 1067 \text{ J/kg}\cdot\text{K}$, which rounds to $1070 \text{ J/kg}\cdot\text{K}$, and $c_L = 1981.6 \text{ J/kg}\cdot\text{K}$, which rounds to $1980 \text{ J/kg}\cdot\text{K}$.

EVALUATE: The liquid has about half the specific heat capacity of water. The metal has a specific heat capacity of $1070 \text{ J/kg}\cdot\text{K}$, which is a bit more than that of aluminum. So both answers are physically reasonable.

17.111. **IDENTIFY:** At steady state, the heat current in both bars is the same when they are connected end-to-end. The heat to melt the ice is the heat conducted through the bars.

SET UP: $Q = mL$ and $H = kA\dfrac{T_H - T_C}{L}$.

EXECUTE: With bar A alone: 0.109 kg of ice melts in 45.0 min = (45.0)(60) s. Therefore the heat current is $H = mL_f/t = (0.109 \text{ kg})(334 \times 10^5 \text{ J/kg})/[(45.0)(60) \text{ s}] = 13.48 \text{ J/s} = 13.48 \text{ W}$. Applying this result to the heat flow in bar A gives $H = kA\dfrac{T_H - T_C}{L}$. Solving for k_A gives $k_A = HL/A(T_H - T_C)$. Numerically we get $k_A = (13.48 \text{ W})(0.400 \text{ m})/[(2.50 \times 10^{-4} \text{ m}^2)(100 \text{ C}°)] = 215.7 \text{ W/m}\cdot\text{K}$, which rounds to $216 \text{ W/m}\cdot\text{K}$.
With the two bars end-to-end: The heat current is the same in both bars, so $H_A = H_B$. Using $H = kA\dfrac{T_H - T_C}{L}$ for each bar, we get $\dfrac{k_A A(100°\text{C} - 62.4°\text{C})}{L} = \dfrac{k_B A(62.4°\text{C} - 0°\text{C})}{L}$. Using our result for k_A and canceling A and L, we get $k_B = 130 \text{ W/m}\cdot\text{K}$.
EVALUATE: $k_A = 216 \text{ W/m}\cdot\text{K}$, which is slightly larger than that of aluminum, and $k_B = 130 \text{ W/m}\cdot\text{K}$, which is between that of aluminum and brass. Therefore these results are physically reasonable.

17.112. **IDENTIFY:** The heat of combustion of propane is used to melt ice, so a change of state is involved.
SET UP: The heat to melt ice is $Q = mL_f$. This heat comes from the combustion of propane, which releases 25.6 MJ/L. Only 30% of this heat is used to melt the ice. As the ice melts it is drawn off at 500 mL/min, so that is the rate at which the ice is melting. The propane tank holds 18 L of propane. One liter of water has a mass of 1.00 kg. We want to find out how long the propane tank can keep burning fuel before it runs out.
EXECUTE: The water melts at 500 L/min, which is 0.500 kg/min or 0.008333 kg/s. Therefore the rate at which the propane is providing heat to the ice is $Q/t = mL_f$. This gives
$Q/t = (0.008333 \text{ kg/s})(334\times10^3 \text{ J/kg}) = 2.783\times10^3 \text{ J/s}$. This heat comes from the combustion of propane. The total heat available from an 18-L tank is $(18 \text{ L})(25.6\times10^6 \text{ J/L})$, but only 30% of this heat is available to melt ice. So the available heat is $(0.30)(18 \text{ L})(25.6\times10^6 \text{ J/L}) = 1.3824\times10^8 \text{ J}$. To use up all the propane in the tank, $(Q/t)t = E_{\text{available}}$. Putting in the values we just found gives $(2.783\times10^3 \text{ J/s})t = 1.3824\times10^8 \text{ J}$, so $t = 5.0\times10^4 \text{s} = 830 \text{ min} = 14 \text{ h}$.
EVALUATE: The tank would melt even more ice if more than 30% of its heat went to melting ice.

17.113. **IDENTIFY:** Heat flows through the frustrum of a cone shown in the figure in the textbook. We want to derive an expression for the heat current H through this cone.

SET UP and EXECUTE: Start with $H = kA\dfrac{T_H - T_C}{L}$ and follow the suggestions in the problem.

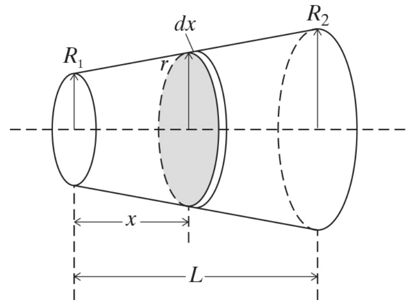


Figure 17.113

Figure 17.113 shown here shows the cone oriented along the x-axis. We break it up into tiny slabs of radius r and thickness dx, as shown. We write the heat current formula for differential differences in temperature and thickness. We also realize that at steady state H is the same at all the slabs, so it is constant. The equation for H becomes $H = kA\dfrac{dT}{dx}$ for the heat current through a slab of thickness dx. The cross-sectional area is $A = \pi r^2$, but r changes with x. So we need to relate r to x. To do so, realize that the side of the cone is a straight line and its equation will give r as a function of x. Using the slope-intercept form of a straight-line equation, $r = Bx + C$. When $x = 0$, $r = R_1$ and when $x = L$, $r = R_2$. The first of these conditions tells us that $C = R_1$, and the second says that $B = \dfrac{R_2 - R_1}{L}$. We can now write the heat flow equation as $H = k\pi(Bx + C)^2\dfrac{dT}{dx}$. Separating variables gives $\dfrac{Hdx}{(Bx+C)^2} = k\pi dT$. Integrating gives $H\displaystyle\int_0^L \frac{dx}{(Bx+C)^2} = \int_{T_H}^{T_C} k\pi dT$. The temperature integral just gives $k\pi\Delta T$ but we need to complete the left-hand-side integral. To do so, let $Bx + C = u$, so $du = Bdx$. The integral becomes $\displaystyle\int\frac{(1/B)du}{u^2} = -\frac{1}{B}u^{-1} = -\frac{1}{Bu}$. Returning to the original variables, we have $H\displaystyle\int_0^L \frac{dx}{(Bx+C)^2} = H\left(-\frac{1}{B(Bx+C)}\right)\Bigg|_0^L = -\frac{H}{B}\left[\frac{1}{BL+C} - \frac{1}{C}\right]$. Carrying out the algebra and putting in the values for B and C gives $HL\left(\dfrac{1}{R_1R_2}\right) = k\pi\Delta T$. Solving for H gives $H = \dfrac{k\pi R_1R_2}{L}(T_H - T_C)$.

EVALUATE: If the cone were reduced to a solid cylinder of radius R_1, H would be $H_1 = k\pi R_1^2\dfrac{\Delta T}{L}$, and if it were a solid cylinder of radius R_2, H would be $H_2 = k\pi R_2^2\dfrac{\Delta T}{L}$. The value we found $H = \dfrac{k\pi R_1R_2}{L}(T_H - T_C)$ is between these two extremes, so it is plausible.

17.114. IDENTIFY: The rate in (iv) is given by $H_{net} = e\sigma A(T^4 - T_s^4)$, with $T = 309$ K and $T_s = 320$ K. The heat absorbed in the evaporation of water is $Q = mL$.

SET UP: $m = \rho V$, so $\dfrac{m}{V} = \rho$.

EXECUTE: **(a)** The rates are: (i) 280 W,

(ii) $(54\ \text{J/h}\cdot\text{C}°\cdot\text{m}^2)(1.5\ \text{m}^2)(11\ \text{C}°)/(3600\ \text{s/h}) = 0.248$ W,

(iii) $(1400\ \text{W/m}^2)(1.5\ \text{m}^2) = 2.10\times10^3$ W,

(iv) $(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.5\ \text{m}^2)((320\ \text{K})^4-(309\ \text{K})^4)=116\ \text{W}.$

The total is 2.50 kW, with the largest portion due to radiation from the sun.

(b) $\dfrac{P}{\rho L_\text{v}}=\dfrac{2.50\times10^3\ \text{W}}{(1000\ \text{kg/m}^3)(2.42\times10^6\ \text{J/kg}\cdot\text{K})}=1.03\times10^{-6}\ \text{m}^3/\text{s}.$ This is equal to 3.72 L/h.

(c) Redoing the above calculations with $e=0$ and the decreased area gives a power of 945 W and a corresponding evaporation rate of 1.4 L/h. Wearing reflective clothing helps a good deal. Large areas of loose-weave clothing also facilitate evaporation.

EVALUATE: The radiant energy from the sun absorbed by the area covered by clothing is assumed to be zero, since $e\approx0$ for the clothing and the clothing reflects almost all the radiant energy incident on it. For the same reason, the exposed skin area is the area used in applying $H_\text{net}=e\sigma A(T^4-T_\text{s}^4)$.

17.115. IDENTIFY: Apply the equation $H=kA(T_\text{H}-T_\text{C})/L$. For a cylindrical surface, the area A in this equation is a function of the distance r from the central axis, and the material must be considered as a series of shells with thickness dr and a temperature difference dT between the inside and outside of the shell. The heat current will be a constant, and must be found by integrating a differential equation.

SET UP: The surface area of the curved side of a cylinder is $2\pi rL$. When $x \ll 1$, $\ln(1+x)\approx x$.

EXECUTE: **(a)** For a cylindrical shell, $H=kA(T_\text{H}-T_\text{C})/L$ becomes $H=k(2\pi rL)\dfrac{dT}{dr}$ or $\dfrac{Hdr}{2\pi r}=kLdT$.

Between the limits $r=a$ and $r=b$, this integrates to $\dfrac{H}{2\pi}\ln(b/a)=kL(T_2-T_1)$, or $H=\dfrac{2\pi kL(T_2-T_1)}{\ln(b/a)}$.

(b) Using $H=k(2\pi rL)\dfrac{dT}{dr}$ from part (a), we integrate: $\int_a^r\dfrac{Hdr}{k2\pi rL}=-\int_{T_2}^T dT$, which gives

$\dfrac{H}{2\pi kL}\ln(r/a)=-(T-T_2)=T_2-T$. Solving for T and using H from part (a) gives

$T=T_2-\dfrac{H}{2\pi kL}\ln(r/a)=\dfrac{(T_2-T_1)2\pi kL\ln(r/a)}{2\pi kL\ln(b/a)}=T_2-\dfrac{(T_2-T_1)\ln(r/a)}{\ln(b/a)}$, which can also be written as

$T(r)=T_2+(T_1-T_2)\dfrac{\ln(r/a)}{\ln(b/a)}$.

(c) For a thin-walled cylinder, $a\approx b$, so $\dfrac{b-a}{a}<<1$. We can write $\dfrac{b}{a}=1+\dfrac{b-a}{a}$, and the log term

becomes $\ln\left(\dfrac{b}{a}\right)=\ln\left(1+\dfrac{b-a}{a}\right)\approx\dfrac{b-a}{a}$ using $\ln(1+x)\approx x$ for $x<<1$. Therefore the rate of heat flow

becomes $H=\dfrac{(T_2-T_1)2\pi kL}{\dfrac{b-a}{a}}=k(2\pi aL)\dfrac{T_2-T_1}{b-a}$. This is equivalent to Eq. (17.21) in which $A=2\pi aL$, which is the surface area of the thin cylindrical shell of radius a and length L.

(d) For steady-state heat flow, the rate of flow through the cork is the same as through the Styrofoam. Using our result from part (a), we have $H_\text{C}=H_\text{S}$, which gives

$(140°\text{C}-T)2\pi k_\text{C}L/(\ln 4/2)=(T-15°\text{C})2\pi k_\text{S}L/(\ln 6/4)$

Cancelling $2\pi L$ and using the given values for k_C and k_S, we get $T=73°\text{C}$.

(e) Use the result of part (a) for H.

$$H=\frac{(140°\text{C}-73°\text{C})2\pi(0.0400\ \text{W/m}\cdot\text{K})(2.00\ \text{m})}{\ln(4/2)}=49\ \text{W}.$$

EVALUATE: As a check, calculate H in the Styrofoam. $\Delta T=73°\text{C}-15°\text{C}=58\ \text{C}°$.

$H_\text{S}=\dfrac{2\pi(2.00\ \text{m})(0.027\ \text{W/m}\cdot\text{K})}{\ln(6.00/4.00)}(58\ \text{C}°)=49\ \text{W}.$ This is the same as we just found, which it should be for steady-state flow.

17.116. **IDENTIFY:** The cryoprotectant must cool from 22°C to –20°C, then it must freeze, and finally it must cool from –20°C to 77 K (–196°C).

SET UP: We use $Q = mc\,\Delta T$ and $Q = mL_f$. $Q_{tot} = Q_1 + Q_2 + Q_3$.

EXECUTE: $Q_1 = mc\,\Delta T = (35 \times 10^{-3}\text{ kg})(4500\text{ J/kg}\cdot\text{K})(42\text{ C°}) = 6.6\text{ kJ}$

$Q_2 = mL_f = (35 \times 10^{-3}\text{ kg})(2.80 \times 10^5\text{ J/kg}) = 9.8\text{ kJ}$

$Q_3 = mc\,\Delta T = (35 \times 10^{-3}\text{ kg})(2000\text{ J/kg}\cdot\text{K})[-196°\text{C} - (-20°\text{C})] = 12.3\text{ kJ}$

$Q_{tot} = 6.6\text{ kJ} + 9.8\text{ kJ} + 12.3\text{ kJ} = 28.7\text{ kJ} \approx 29\text{ kJ} = 2.9 \times 10^4\text{ J}$, which is choice (b).

EVALUATE: This is a large amount of heat for such a small sample, but it must change its temperature from 22°C to –196°C and also freeze.

17.117. **IDENTIFY** and **SET UP:** The graph shows that the specific heat of the solid decreases with temperature, so its average value is less than the $2.0 \times 10^3\text{ J/kg}\cdot\text{K}$ shown in the table.

EXECUTE: Since the average specific heat is less than the value in the table, less heat will need to come out of it to bring it into equilibrium with the cold plate. Therefore a shorter time will be needed for it to come to equilibrium with the cold plate, which is choice (a).

EVALUATE: The average value of the specific heat is about $1500\text{ J/kg}\cdot\text{K}$ between –20°C and –200°C, so the difference in time could be important enough to be concerned about. As we saw in the previous problem, the heat to cool the cryoprotectant from –20°C down to –196°C was the largest contribution to the total heat.

17.118. **IDENTIFY:** The rate of heat conduction should be the same in both cases, but the area in the area in the second case is 4 times the area in the first case since the linear dimensions are both doubled.

SET UP: $H = kA\dfrac{T_H - T_C}{L}$, $H_1 = H_2$, $A_2 = 4A_1$, k and $T_H - T_C$ are the same in both cases.

EXECUTE: Calculate H_1 and H_2 and equate them: $kA_1\dfrac{\Delta T}{L_1} = kA_2\dfrac{\Delta T}{L_2} = k(4A_1)\dfrac{\Delta T}{L_2}$. Cancelling common factors gives $L_2 = 4L_1$, so the layer should be 4 times as thick as it was for the smaller plates, which is choice (d).

EVALUATE: If we doubled only one dimension, then L_2 would have to be only twice as thick as for the smaller plates.

17.119. **IDENTIFY** and **SET UP:** Heat from the cryoprotectant *and* the environment enters the cold plate. You measure the amount of heat that enters the cold plate, and assume that it all came from the cryoprotectant.

EXECUTE: You think that more heat entered the plate from the cryoprotectant than actually did so. This will make you think that the specific heat of the cryoprotectant is greater than it actually is, which is choice (a).

EVALUATE: Heat from the environment could also be entering the cryoprotectant, but since the cold plate is on average colder than the cryoprotectant, more heat will enter the cold plate than will enter the cryoprotectant, so the two effects will not cancel each other out. You will still measure a specific heat that is greater than the actual value.

18

THERMAL PROPERTIES OF MATTER

VP18.4.1. **IDENTIFY:** An ideal gas inside a tire undergoes a temperature change at constant volume, so the ideal gas law applies.

SET UP: Use $pV = nRT$, with $T_1 = 30.0°\text{C} = 303\text{ K}$ and $T_2 = 10.0°\text{C} = 283\text{ K}$.

EXECUTE: **(a)** We want the new pressure p_2. At fixed volume, p/T is constant, so $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ which gives

$$p_2 = p_1\frac{T_2}{T_1} = (5.00\times10^5\text{ Pa})\left(\frac{283\text{ K}}{303\text{ K}}\right) = 4.67\times10^5\text{ Pa}.$$

(b) We want the number n of moles in the tire. Solve $pV = nRT$ for n, giving $n = pV/RT$. Using $p = 5.00\times10^5\text{ Pa}$, $T = 303\text{ K}$, and $V = 1.40\times10^{-3}\text{ m}^3$ gives $n = 0.278$ mol.

EVALUATE: The temperature T must *always* be in Kelvins when using the ideal gas law.

VP18.4.2. **IDENTIFY:** As the balloon rises, the temperature and pressure decrease and the volume increases. The ideal gas law applies.

SET UP: Use $pV = nRT$. Since n remains constant, pV/T is constant.

<u>At sea level</u>: $T_1 = 15.0°\text{C} = 288\text{ K}$, $p_1 = 1.01\times10^5\text{ Pa}$, $V_1 = 13.0\text{ m}^3$.

<u>At 32.0 km</u>: $T_2 = -44.5°\text{C} = 228.5\text{ K}$, $p_2 = 868\text{ Pa}$, $V_2 = ?$

EXECUTE: **(a)** We want the volume at 32.0 km altitude. Since pV/T is constant, we have $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$,

so $V_2 = V_1\frac{T_2}{T_1}\frac{p_1}{p_2}$. Putting in the numbers shown above, we get $V_2 = 1.20\times10^3\text{ m}^3$.

(b) We want the ratio of the radii R_2/R_1 of the balloons. Using the volume of a sphere gives us

$$\frac{V_2}{V_1} = \frac{\frac{4}{3}\pi R_2^3}{\frac{4}{3}\pi R_1^3} = \left(\frac{R_2}{R_1}\right)^3, \text{ so } \frac{R_2}{R_1} = \left(\frac{V_2}{V_1}\right)^{1/3} = \left(\frac{1.20\times10^3\text{ m}^3}{13.0\text{ m}^3}\right)^{1/3} = 4.52,$$ so the radius is 4.52 times greater at 32.0 km than it is at sea level.

EVALUATE: The volume at 32.0 km is 1200/13 = 92 times as great as at sea level.

VP18.4.3. **IDENTIFY:** The pressure and temperature of Pluto's atmosphere are much lower than they are in Earth's atmosphere. The ideal gas law applies.

SET UP: Use $pV = nRT$, where $T = 42$ K and $p = 1.0$ Pa on Pluto. The molar mass of N_2 is 0.0028 kg/mol.

EXECUTE: **(a)** We want the molar density of Pluto's atmosphere in mol/m^3. Using $pV = nRT$ gives $n/V = p/RT$. Using $p = 1.0$ Pa and $T = 42$ K, we get $n/V = 2.9\times10^{-3}\text{ mol/m}^3$.

(b) We want the mass density of Pluto's atmosphere in kg/m³. Convert the molar density to mass density using the fact that the molar density of N_2 is 0.028 kg/mol. This gives $(2.9\times10^{-3}\ \text{mol/m})(0.028\ \text{kg/mol}) = 8.0\times10^{-5}\ \text{kg/m}^3$.

EVALUATE: Compare our results on Pluto to those on Earth. $\dfrac{n/V)_\text{P}}{n/V)_\text{E}} = \dfrac{2.9\times10^{-3}\ \text{mol/m}^3}{42\ \text{mol/m}^3} = 6.8\times10^{-5}$.

$\dfrac{m/V)_\text{P}}{m/V)_\text{E}} = \dfrac{8.0\times10^{-5}\ \text{kg/m}^3}{1.2\ \text{kg/m}^3} = 6.7\times10^{-5}$. The ratios are nearly the same, and both are very small due to Pluto's extremely low pressure.

VP18.4.4. **IDENTIFY:** We are investigating the effects of changes in an ideal gas using the ideal gas law.

SET UP: $pV = nRT$, and for constant n we have pV/T is constant. $\rho = m/V$ so $V = m/\rho$. We are given that $\rho_2 = \rho_1(p_2/p_1)^{3/5}$.

EXECUTE: **(a)** We want T_2. $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$, so $T_2 = T_1\dfrac{V_2}{V_1}\dfrac{p_2}{p_1} = T_1\dfrac{p_2}{p_1}\dfrac{m/\rho_1}{m/\rho_2} = T_1\dfrac{p_2}{p_1}\dfrac{\rho_1}{\rho_2}$. Using

$\rho_2 = \rho_1(p_2/p_1)^{3/5}$ gives $T_2 = T_1\dfrac{p_2}{p_1}\dfrac{\rho_1}{\rho_1\left(\dfrac{p_2}{p_1}\right)^{3/5}} = T_1\left(\dfrac{p_2}{p_1}\right)\left(\dfrac{p_1}{p_2}\right)^{3/5} = T_1\left(\dfrac{p_2}{p_1}\right)^{2/5}$.

(b) We want ρ_2/ρ_1 and T_2/T_1 when $p_2 = 0.500p_1$. Using $\rho_2 = \rho_1(p_2/p_1)^{3/5}$ we get

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{3/5} = \left(\frac{0.500p_1}{p_1}\right)^{3/5} = 0.660.$$

Using our result from part (a) gives $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{2/5} = \left(\dfrac{0.500p_1}{p_1}\right)^{2/5} = 0.758$.

(c) We want ρ_2/ρ_1 and T_2/T_1 when $p_2 = 2.00p_1$. Using $\rho_2 = \rho_1(p_2/p_1)^{3/5}$ we get

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{3/5} = \left(\frac{2.00p_1}{p_1}\right)^{3/5} = 1.52.$$

Using our result from part (a) gives $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{2/5} = \left(\dfrac{2.00p_1}{p_1}\right)^{2/5} = 1.32$.

EVALUATE: In part (b) our result says that decreasing p decreases ρ, and in part (c) it says that increasing p increases ρ. These are reasonable results.

VP18.7.1. **IDENTIFY:** We are comparing molecular speeds of different molecules.

SET UP and EXECUTE: Use $v_\text{rms} = \sqrt{\dfrac{3RT}{M}}$.

N_2 molecule at 20.0°C: $v_\text{rms} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3R(293\ \text{K})}{28.0\ \text{g/mol}}}$.

He atom with the same v_rms: $v_\text{rms} = \sqrt{\dfrac{3RT}{4.00\ \text{g/mol}}}$

Equate the speeds and solve for T: $\sqrt{\dfrac{3RT}{4.00\ \text{g/mol}}} = \sqrt{\dfrac{3R(293\ \text{K})}{28.0\ \text{g/mol}}}$, $T = 41.86\text{K} = -231°\text{C}$.

EVALUATE: It was not necessary to use R or convert g to kg since they divided out. It is best to save numerical calculations until the end to minimize tedious work.

VP18.7.2. **IDENTIFY:** We are looking at atomic kinetic energy and speed.

SET UP: $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$, $K_{\text{tr}} = \dfrac{3}{2}kT$. We want the average kinetic energy of an atom and the rms speed.

EXECUTE: **(a)** $K_{\text{av}} = \dfrac{3}{2}kT = \dfrac{3}{2}(1.381\times10^{-23}\text{ J/K})(3000\text{ K}) = 6.2\times10^{-20}\text{ J}$.

(b) Use $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$ with T = 3000 K and $m = 1.67\times10^{-27}$ kg, giving v_{rms} = 8600 m/s.

EVALUATE: The speed in part (b) is about 19,000 mph!

VP18.7.3. **IDENTIFY:** We want to find out how many molecules are in a typical room and how much kinetic energy they have.

SET UP: Use $pV = nRT$, $K_{\text{tr}} = \dfrac{3}{2}kT$, $K_{\text{tot}} = NK_{\text{tr}}$, and $N = nN_A$. The target variables are N and K_{tr}.

EXECUTE: **(a)** Solve $pV = nRT$ to get $n = pV/RT$. Using V = (5.0 m)(5.0 m)(2.4 m) = 612 m^3, p = 1.01×10^5 Pa , and T = 293 K gives n = 2488 mol. The number of molecules in 2488 mol is $N = nN_A$ = (2488 mol)(6.022×10^{23} molecules/mol) = 1.5×10^{27} molecules.

(b) $K_{\text{tot}} = NK_{\text{tr}} = N\dfrac{3}{2}kT$. Using T = 293 K and $N = 1.5\times10^{27}$ molecules gives $K_{\text{tot}} = 9.1\times10^6$ J.

(c) We want the speed of the car so its kinetic energy would be 9.1×10^6 J. $K_{\text{car}} = K_{\text{tot}}$ gives

$$\frac{1}{2}mv^2 = K_{\text{tot}},\ \text{so}\ v = \sqrt{\frac{2K_{\text{tot}}}{m}} = \sqrt{\frac{2(9.1\times10^6\text{ J})}{1500\text{ kg}}} = 110\text{ m/s}.$$

EVALUATE: The speed in part (c) is about 250 mph or 400 km/h.

VP18.7.4. **IDENTIFY:** We want to compare the average, average of squares, and rms for the first 10 integers.

SET UP and EXECUTE: **(a)** The average is $n_{\text{av}} = \dfrac{1+2+3+\cdots+10}{10} = \dfrac{55}{10} = 5.5.$

(b) The average of the squares is $\left(n^2\right)_{\text{av}} = \dfrac{1^2+2^2+3^2+\cdots+10^2}{10} = \dfrac{385}{10} = 38.5.$

(c) The rms value is $n_{\text{rms}} = \sqrt{(n^2)_{\text{av}}} = \sqrt{38.5} = 6.20.$

EVALUATE: Note that the rms value is *not* the same as the average value.

VP18.8.1. **IDENTIFY:** We are comparing the mean free path for molecules on Mars with those on Earth.

SET UP: We use $\lambda = \dfrac{kT}{4\pi\sqrt{2}r^2 p}$, with T_M = –63°C = 210 K and T_E = 15°C = 288 K, $p_M = 6.0\times10^2$ Pa .The molecular radius is 2.0×10^{10} m.

EXECUTE: **(a)** We want the mean free path on Mars. Using $\lambda = \dfrac{kT}{4\pi\sqrt{2}r^2 p}$ with values given above we get $\lambda_M = 6.8\times10^{-6}$ m.

(b) We want to compare the mean free path on Mars to that on Earth by finding λ_M / λ_E. Taking the ratio gives $\dfrac{\lambda_M}{\lambda_E} = \dfrac{T_M / p_M}{T_E / p_E} = \left(\dfrac{T_M}{T_E}\right)\left(\dfrac{p_E}{p_M}\right) = \left(\dfrac{210\text{ K}}{288\text{ K}}\right)\left(\dfrac{1.01\times10^5\text{ Pa}}{600\text{ Pa}}\right) = 120.$ The mean free path on Mars is 120 times greater than it is on Earth.

EVALUATE: The low pressure on Mars means that molecules are far apart compared to those on Earth, so they can travel much farther on Mars before running into each other.

VP18.8.2. **IDENTIFY:** We are investigating the mean free path of molecules.

SET UP: Use $\lambda = \dfrac{kT}{4\pi\sqrt{2}r^2 p}$ and $pV = nRT$.

EXECUTE: **(a)** We want to find the pressure so that the mean free path is 1.00 m. Solve $\lambda = \dfrac{kT}{4\pi\sqrt{2}r^2 p}$ for p, giving $p = \dfrac{kT}{4\pi\sqrt{2}r^2 \lambda}$. Using T = 293 K, $r = 2.0\times10^{-10}$ m, and λ = 1.00 m, we get p = 5.7×10^{-3} Pa.

(b) Our target variable is the number of moles n inside the box. Solving $pV = nRT$ for n gives $n = pV/RT$. Using the pressure from part (a) and the same values for the other variables gives $n = 2.3\times10^{-6}$ mol.

EVALUATE: Even at this low pressure, the number of air molecules in the box is $N = nN_A$ = $(2.3\times10^{-6}\text{ mol})(6.022\times10^{23}\text{ molecules/mol}) = 1.4\times10^{18}$ molecules.

VP18.8.3. **IDENTIFY:** We want to find the mean free path and mean free time for helium in a cylindrical tank.

SET UP: We use $\lambda = \dfrac{kT}{4\pi\sqrt{2}r^2 p}$, $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$, and $pV = nRT$. $v_{\text{rms}}t_{\text{mean}} = \lambda$. We know that T = 27.0°C = 300.0 K, $V = 5.00\times10^{-2}\text{ m}^3$, n = 400 mol, and $r = 3.1\times10^{-11}$ m, so $\lambda = 1.2\times10^{-8}$ m.

EXECUTE: **(a)** We want the mean free path. First use $pV = nRT$ to find p, so $p = nRT/V$. Using this result gives $\lambda = \dfrac{kT}{4\pi\sqrt{2}r^2 p} = \dfrac{kT}{4\pi\sqrt{2}r^2\left(\dfrac{nRT}{V}\right)} = \dfrac{kV}{4\pi\sqrt{2}r^2 nR}$. Using the given values for V, n, and r gives $\lambda = 1.2\times10^{-8}$ m.

(b) We want the mean free time. $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$ and $t_{\text{mean}} = \lambda / v_{\text{rms}}$. Combining these equations gives $t_{\text{mean}} = \dfrac{\lambda}{v_{\text{rms}}} = \dfrac{\lambda}{\sqrt{\dfrac{3RT}{M}}} = \lambda\sqrt{\dfrac{M}{3RT}}$. Using $M = 4.00\times10^{-3}$ kg/mol, $\lambda = 1.2\times10^{-8}$ m, and T = 300 K, we get $t_{\text{mean}} = 8.9\times10^{-12}$ s.

EVALUATE: The number of collisions per second would be $1/t_{\text{mean}} = 1.1\times10^{-11}$ collision/s.

VP18.8.4. **IDENTIFY:** We want to find the mean free time for gas molecules.

SET UP: We use $\lambda = \dfrac{kT}{4\pi\sqrt{2}r^2 p}$, $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$, $v_{\text{rms}}t_{\text{mean}} = \lambda$.

EXECUTE: **(a)** Combining the above equations gives $t_{\text{mean}} = \dfrac{\lambda}{v_{\text{rms}}} = \dfrac{\dfrac{kT}{4\pi\sqrt{2}r^2 p}}{\sqrt{\dfrac{3kT}{m}}}$, which simplifies to $t_{\text{mean}} = \dfrac{1}{4\pi r^2 p}\sqrt{\dfrac{mkT}{6}}$.

(b) From part (a), we see that $t_{\text{mean}} \propto \dfrac{\sqrt{T}}{r^2 p}$. From this result we can see the following.

Doubling p would cut t_{mean} in half.

Doubling T would increase t_{mean} by a factor of $\sqrt{2}$.

Doubling r would decrease t_{mean} by a factor of $\dfrac{1}{2^2} = \dfrac{1}{4}$.

Therefore doubling r would have the greatest effect on the mean free time.

EVALUATE: If we doubled all of these quantities at once, the effect would reduce t_{mean} by a factor of $\frac{\sqrt{2}}{8} \approx 0.177$.

18.1. **IDENTIFY:** **(a)** We are asked about a single state of the system.

SET UP: Use $m_{\text{total}} = nM$ to calculate the number of moles and then apply the ideal-gas equation.

EXECUTE: $n = \dfrac{m_{\text{tot}}}{M} = \dfrac{4.86\times10^{-4}\ \text{kg}}{4.00\times10^{-3}\ \text{kg/mol}} = 0.122\ \text{mol}.$

(b) $pV = nRT$ implies $p = nRT/V$. T must be in kelvins, so $T = (18+273)\ \text{K} = 291\ \text{K}$.

$$p = \frac{(0.122\ \text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(291\ \text{K})}{20.0\times10^{-3}\ \text{m}^3} = 1.47\times10^4\ \text{Pa}.$$

$$p = (1.47\times10^4\ \text{Pa})(1.00\ \text{atm}/1.013\times10^5\ \text{Pa}) = 0.145\ \text{atm}.$$

EVALUATE: The tank contains about 1/10 mole of He at around standard temperature, so a pressure around 1/10 atmosphere is reasonable.

18.2. **IDENTIFY:** $pV = nRT$.

SET UP: $T_1 = 41.0°\text{C} = 314\ \text{K}$. $R = 0.08206\ \text{L}\cdot\text{atm/mol}\cdot\text{K}$.

EXECUTE: n and R are constant so $\dfrac{pV}{T} = nR$ is constant. $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right) = (314\ \text{K})(2)(2) = 1.256\times10^3\ \text{K} = 983°\text{C}.$$

(b) $n = \dfrac{pV}{RT} = \dfrac{(0.180\ \text{atm})(3.20\ \text{L})}{(0.08206\ \text{L}\cdot\text{atm/mol}\cdot\text{K})(314\ \text{K})} = 0.02235\ \text{mol}.$

$$m_{\text{tot}} = nM = (0.02235\ \text{mol})(4.00\ \text{g/mol}) = 0.0894\ \text{g}.$$

EVALUATE: T is directly proportional to p and to V, so when p and V are each doubled the Kelvin temperature increases by a factor of 4.

18.3. **IDENTIFY:** $pV = nRT$.

SET UP: T is constant.

EXECUTE: nRT is constant so $p_1V_1 = p_2V_2$.

$$p_2 = p_1\left(\frac{V_1}{V_2}\right) = (0.355\ \text{atm})\left(\frac{0.110\ \text{m}^3}{0.390\ \text{m}^3}\right) = 0.100\ \text{atm}.$$

EVALUATE: For T constant, p decreases as V increases.

18.4. **IDENTIFY:** $pV = nRT$.

SET UP: $T_1 = 20.0°\text{C} = 293\ \text{K}$.

EXECUTE: **(a)** n, R and V are constant. $\dfrac{p}{T} = \dfrac{nR}{V} = \text{constant}$. $\dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}$.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right) = (293\ \text{K})\left(\frac{1.00\ \text{atm}}{3.00\ \text{atm}}\right) = 97.7\ \text{K} = -175°\text{C}.$$

(b) $p_2 = 1.00\ \text{atm}$, $V_2 = 3.00\ \text{L}$. $p_3 = 3.00\ \text{atm}$. n, R and T are constant so $pV = nRT = \text{constant}$.

$p_2V_2 = p_3V_3$.

$$V_3 = V_2\left(\frac{p_2}{p_3}\right) = (3.00\ \text{L})\left(\frac{1.00\ \text{atm}}{3.00\ \text{atm}}\right) = 1.00\ \text{L}.$$

EVALUATE: The final volume is one-third the initial volume. The initial and final pressures are the same, but the final kelvin temperature is one-third the initial kelvin temperature.

18.5. **IDENTIFY:** This problem compares the ideal gas law to the van der Waals equation for carbon dioxide gas.

SET UP and EXECUTE: The ideal gas law is $pV = nRT$ and the van der Waals equation is $\left(p+\dfrac{an^2}{V^2}\right)(V-nb)=nRT$. We have n = 1 mol, $V = 4.48\times10^{-4}\ \text{m}^3$, and T = 273.0 K.

(a) Using these numbers, $p = nRT/V = 5.7\times10^6$ Pa.

(b) Using the van der Waals equation and solving for p gives $p=\dfrac{nRT}{V-nb}-\dfrac{an^2}{V^2}$. Using the same values as for part (a), and in addition for CO_2 $a=0.364\ \text{J}\cdot\text{m}^3/\text{mol}^2$ and $b=4.27\times10^{-5}\ \text{m}^3/\text{mol}$, we get p = 3.79×10^6 Pa.

EVALUATE: The percent difference is $\dfrac{\Delta p}{p}=\dfrac{3.79-5.07}{5.07}=-0.252=-25.2\%$, which is not very good agreement.

18.6. **IDENTIFY:** $pV=nRT$ and the mass of the gas is $m_{\text{tot}}=nM$.

SET UP: The temperature is $T=22.0°\text{C}=295.15$ K. The average molar mass of air is $M=28.8\times10^{-3}$ kg/mol. For helium $M=4.00\times10^{-3}$ kg / mol.

EXECUTE: **(a)** $m_{\text{tot}}=nM=\dfrac{pV}{RT}M=\dfrac{(1.00\ \text{atm})(0.900\ \text{L})(28.8\times10^{-3}\ \text{kg/mol})}{(0.08206\ \text{L}\cdot\text{atm/mol}\cdot\text{K})(295.15\ \text{K})}=1.07\times10^{-3}$ kg.

(b) $m_{\text{tot}}=nM=\dfrac{pV}{RT}M=\dfrac{(1.00\ \text{atm})(0.900\ \text{L})(4.00\times10^{-3}\ \text{kg/mol})}{(0.08206\ \text{L}\cdot\text{atm/mol}\cdot\text{K})(295.15\ \text{K})}=1.49\times10^{-4}$ kg.

EVALUATE: $n=\dfrac{N}{N_\text{A}}=\dfrac{pV}{RT}$ says that in each case the balloon contains the same number of molecules. The mass is greater for air since the mass of one molecule is greater than for helium.

18.7. **IDENTIFY:** We are asked to compare two states. Use the ideal gas law to obtain T_2 in terms of T_1 and ratios of pressures and volumes of the gas in the two states.

SET UP: $pV=nRT$ and n, R constant implies $pV/T=nR=$ constant and $p_1V_1/T_1=p_2V_2/T_2$.

EXECUTE: $T_1=(27+273)\text{K}=300$ K

$p_1=1.01\times10^5$ Pa

$p_2=2.72\times10^6\ \text{Pa}+1.01\times10^5\ \text{Pa}=2.82\times10^6$ Pa (in the ideal gas equation the pressures must be absolute, not gauge, pressures)

$$T_2=T_1\left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right)=300\ \text{K}\left(\frac{2.82\times10^6\ \text{Pa}}{1.01\times10^5\ \text{Pa}}\right)\left(\frac{46.2\ \text{cm}^3}{499\ \text{cm}^3}\right)=776\ \text{K}$$

$T_2=(776-273)°\text{C}=503°\text{C}$.

EVALUATE: The units cancel in the V_2/V_1 volume ratio, so it was not necessary to convert the volumes in cm^3 to m^3. It was essential, however, to use T in kelvins.

18.8. **IDENTIFY:** $pV=nRT$ and $m=nM$.

SET UP: We must use absolute pressure in $pV=nRT$. $p_1=4.01\times10^5$ Pa, $p_2=2.81\times10^5$ Pa. $T_1=310$ K, $T_2=295$ K.

EXECUTE: **(a)** $n_1=\dfrac{p_1V_1}{RT_1}=\dfrac{(4.01\times10^5\ \text{Pa})(0.075\ \text{m}^3)}{(8.315\ \text{J/mol}\cdot\text{K})(310\ \text{K})}=11.7$ mol.

$m=nM=(11.7\ \text{mol})(32.0\ \text{g/mol})=374$ g.

(b) $n_2 = \dfrac{p_2V_2}{RT_2} = \dfrac{(2.81\times10^5\ \text{Pa})(0.075\ \text{m}^3)}{(8.315\ \text{J/mol}\cdot\text{K})(295\ \text{K})} = 8.59\ \text{mol}.\quad m = 275\ \text{g}.$

The mass that has leaked out is $374\ \text{g} - 275\ \text{g} = 99\ \text{g}$.

EVALUATE: In the ideal gas law we must use absolute pressure, expressed in Pa, and T must be in kelvins.

18.9. **IDENTIFY:** $pV = nRT$.

SET UP: $T_1 = 300\ \text{K},\ T_2 = 430\ \text{K}$.

EXECUTE: **(a)** n, R are constant so $\dfrac{pV}{T} = nR = \text{constant}.\ \dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$.

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = (7.50\times10^3\ \text{Pa})\left(\frac{0.750\ \text{m}^3}{0.410\ \text{m}^3}\right)\left(\frac{430\ \text{K}}{300\ \text{K}}\right) = 1.97\times10^4\ \text{Pa}.$$

EVALUATE: Since the temperature increased while the volume decreased, the pressure must have increased. In $pV = nRT$, T must be in kelvins, even if we use a ratio of temperatures.

18.10. **IDENTIFY:** Use the ideal-gas equation to calculate the number of moles, n. The mass m_{total} of the gas is $m_{\text{total}} = nM$.

SET UP: The volume of the cylinder is $V = \pi r^2 l$, where $r = 0.450$ m and $l = 1.50$ m.

$T = 22.0°\text{C} = 293.15\ \text{K}.\ 1\ \text{atm} = 1.013\times10^5\ \text{Pa}.\ M = 32.0\times10^{-3}\ \text{kg/mol}.\ R = 8.314\ \text{J/mol}\cdot\text{K}.$

EXECUTE: **(a)** $pV = nRT$ gives

$$n = \frac{pV}{RT} = \frac{(21.0\ \text{atm})(1.013\times10^5\ \text{Pa/atm})\pi(0.450\ \text{m})^2(1.50\ \text{m})}{(8.314\ \text{J/mol}\cdot\text{K})(295.15\ \text{K})} = 827\ \text{mol}.$$

(b) $m_{\text{total}} = (827\ \text{mol})(32.0\times10^{-3}\ \text{kg/mol}) = 26.5\ \text{kg}$

EVALUATE: In the ideal-gas law, T must be in kelvins. Since we used R in units of $\text{J/mol}\cdot\text{K}$ we had to express p in units of Pa and V in units of m^3.

18.11. **IDENTIFY:** We are asked to compare two states. Use the ideal-gas law to obtain V_1 in terms of V_2 and the ratio of the temperatures in the two states.

SET UP: $pV = nRT$ and n, R, p are constant so $V/T = nR/p = \text{constant}$ and $V_1/T_1 = V_2/T_2$.

EXECUTE: $T_1 = (19 + 273)\ \text{K} = 292\ \text{K}$ (T must be in kelvins)

$V_2 = V_1(T_2/T_1) = (0.600\ \text{L})(77.3\ \text{K}/292\ \text{K}) = 0.159\ \text{L}.$

EVALUATE: p is constant so the ideal-gas equation says that a decrease in T means a decrease in V.

18.12. **IDENTIFY:** The ideal gas law applies. We should be able to identify the gas from its molecular weight.

SET UP: $pV = nRT$, $\rho = m/V = (m/\text{mole})n/V$. $R = 8.314\ \text{J/mol}\cdot\text{K}$.

EXECUTE: $\rho = m/V = (m/\text{mole})n/V$, so $m/\text{mole} = \dfrac{\rho V}{n} = \rho\dfrac{RT}{p}$, using $pV = nRT$.

$\rho = 1.33\times10^{-3}\ \text{kg/m}^3$, $p = 101.3$ Pa, $T = 293.15$ K.

Therefore $m/\text{mole} = (1.33\times10^{-3}\ \text{kg/m}^3)(8.314\ \text{J/mol}\cdot\text{K})(293.15\ \text{K})/(101.3\ \text{Pa}) = 0.0320\ \text{kg/mol} = 32.0$ g/mol. The gas is probably diatomic oxygen, O_2.

EVALUATE: There are few gases having small molecular weights, so we are quite certain that O_2 is the correct gas.

18.13. **IDENTIFY:** We know the volume of the gas at STP on the earth and want to find the volume it would occupy on Venus where the pressure and temperature are much greater.

SET UP: STP is $T = 273$ K and $p = 1$ atm. Set up a ratio using $pV = nRT$ with nR constant.

$T_{\text{V}} = 1003 + 273 = 1276\ \text{K}.$

EXECUTE: $pV = nRT$ gives $\frac{pV}{T} = nR = \text{constant}$, so $\frac{p_E V_E}{T_E} = \frac{p_V V_V}{T_V}$.

$$V_V = V_E\left(\frac{p_E}{p_V}\right)\left(\frac{T_V}{T_E}\right) = V\left(\frac{1\text{ atm}}{92\text{ atm}}\right)\left(\frac{1276\text{ K}}{273\text{ K}}\right) = 0.0508V.$$

EVALUATE: Even though the temperature on Venus is higher than it is on earth, the pressure there is much greater than on earth, so the volume of the gas on Venus is only about 5% what it is on earth.

18.14. IDENTIFY: $pV = nRT$.

SET UP: $T_1 = 277$ K. $T_2 = 296$ K. Assume the number of moles of gas in the bubble remains constant.

EXECUTE: **(a)** n, R are constant so $\frac{pV}{T} = nR = \text{constant}$. $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ and

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right) = \left(\frac{3.50\text{ atm}}{1.00\text{ atm}}\right)\left(\frac{296\text{ K}}{277\text{ K}}\right) = 3.74.$$

(b) This increase in volume of air in the lungs would be dangerous.

EVALUATE: The large decrease in pressure results in a large increase in volume.

18.15. IDENTIFY: We are looking at the gauge pressure in a gas, so we apply the ideal gas law.

SET UP: $pV = nRT$, $p_{\text{tot}} = p_{\text{atm}} + p_{\text{gauge}}$. The pressure we need to use is the *total* pressure since that is the pressure the gas exerts in the container, so $p_{\text{tot}} = 0.876$ atm + 1.000 atm = 1.876 atm.

EXECUTE: Using the numbers given in the problem, $n = pV/RT$. Putting in the numbers gives

$$n = \frac{(1.876\text{ atm})(5.43\text{ L})}{(0.08206\text{ L}\cdot\text{atm/mol}\cdot\text{K})(295.2\text{ K})} = 0.421\text{ mol}.$$

EVALUATE: We use the gauge pressure because that is the actual pressure in the tank. A pressure gauge reads the gauge pressure, but that is not the tank pressure since it is only the pressure above one atmosphere.

18.16. IDENTIFY: $F = pA$ and $pV = nRT$.

SET UP: For a cube, $V / A = L$.

EXECUTE: **(a)** The force of any side of the cube is $F = pA = (nRT / V)A = (nRT) / L,$ since the ratio of area to volume is $A/V = 1/L$. For $T = 20.0°\text{C} = 293.15$ K, so

$$F = \frac{nRT}{L} = \frac{(3\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(293.15\ K)}{0.300\text{ m}} = 2.44\times 10^4\text{ N}.$$

(b) For $T = 100.00°\text{C} = 373.15$ K, so

$$F = \frac{nRT}{L} = \frac{(3\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(373.15\text{ K})}{0.300\text{ m}} = 3.10\times 10^4\text{ N}.$$

EVALUATE: When the temperature increases while the volume is kept constant, the pressure increases and therefore the force increases. The force increases by the ratop T_2/T_1 of Kelvin temperatures.

18.17. IDENTIFY: We know the volume, pressure, and temperature of the gas and want to find its mass and density.

SET UP: $V = 3.00\times 10^{-3}\text{ m}^3$. $T = 295$ K. $p = 2.03\times 10^{-8}$ Pa. The ideal gas law, $pV = nRT$, applies.

EXECUTE: **(a)** $pV = nRT$ gives

$$n = \frac{pV}{RT} = \frac{(2.03\times 10^{-8}\text{ Pa})(3.00\times 10^{-3}\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(295\text{ K})} = 2.48\times 10^{-14}\text{ mol}.$$ The mass of this amount of gas is

$$m = nM = (2.48\times 10^{-14}\text{ mol})(28.0\times 10^{-3}\text{ kg/mol}) = 6.95\times 10^{-16}\text{ kg}.$$

(b) $\rho = \frac{m}{V} = \frac{6.95\times 10^{-16}\text{ kg}}{3.00\times 10^{-3}\text{ m}^3} = 2.32\times 10^{-13}\text{ kg/m}^3.$

EVALUATE: The density at this level of vacuum is 13 orders of magnitude less than the density of air at STP, which is 1.20 kg/m^3.

18.18. IDENTIFY: Use $\rho = pM / RT$ and solve for p.

SET UP: $\rho = pM / RT$ and $p = RT\rho / M$

$T = (-56.5 + 273.15)\ \text{K} = 216.6\ \text{K}$

For air $M = 28.8 \times 10^{-3}$ kg/mol (Example 18.3).

EXECUTE: $p = \dfrac{(8.3145\ \text{J/mol}\cdot\text{K})(216.6\ \text{K})(0.364\ \text{kg/m}^3)}{28.8 \times 10^{-3}\ \text{kg/mol}} = 2.28 \times 10^4\ \text{Pa}.$

EVALUATE: The pressure is about one-fifth the pressure at sea-level.

18.19. IDENTIFY: $n = \dfrac{m}{M} = \dfrac{N}{N_A}.$

SET UP: $N_A = 6.022 \times 10^{23}$ molecules/mol. For water, $M = 18 \times 10^{-3}$ kg/mol.

EXECUTE: $n = \dfrac{m}{M} = \dfrac{1.00\ \text{kg}}{18 \times 10^{-3}\ \text{kg/mol}} = 55.6\ \text{mol}.$

$N = nN_A = (55.6\ \text{mol})(6.022 \times 10^{23}\ \text{molecules/mol}) = 3.35 \times 10^{25}\ \text{molecules}.$

EVALUATE: Note that we converted M to kg/mol.

18.20. IDENTIFY: The molar mass is $M = N_A m$, where m is the mass of one molecule.

SET UP: $N_A = 6.02 \times 10^{23}$ molecules/mol.

EXECUTE: $M = N_A m = (6.02 \times 10^{23}\ \text{molecules/mol})(1.41 \times 10^{-21}\ \text{kg/molecule}) = 849\ \text{kg/mol}.$

EVALUATE: For a carbon atom, $M = 12 \times 10^{-3}$ kg/mol. If this molecule is mostly carbon, so the average mass of its atoms is the mass of carbon, the molecule would contain $\dfrac{849\ \text{kg/mol}}{12 \times 10^{-3}\ \text{kg/mol}} = 71{,}000$ atoms.

18.21. IDENTIFY: Use $pV = nRT$ to calculate the number of moles and then the number of molecules would be $N = nN_A$.

SET UP: $1\ \text{atm} = 1.013 \times 10^5\ \text{Pa}$. $1.00\ \text{cm}^3 = 1.00 \times 10^{-6}\ \text{m}^3$. $N_A = 6.022 \times 10^{23}$ molecules/mol.

EXECUTE: (a) $n = \dfrac{pV}{RT} = \dfrac{(9.00 \times 10^{-14}\ \text{atm})(1.013 \times 10^5\ \text{Pa/atm})(1.00 \times 10^{-6}\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(300.0\ \text{K})} = 3.655 \times 10^{-18}\ \text{mol}.$

$N = nN_A = (3.655 \times 10^{-18}\ \text{mol})(6.022 \times 10^{23}\ \text{molecules/mol}) = 2.20 \times 10^6\ \text{molecules}.$

(b) $N = \dfrac{pVN_A}{RT}$ so $\dfrac{N}{p} = \dfrac{VN_A}{RT} = \text{constant}$ and $\dfrac{N_1}{p_1} = \dfrac{N_2}{p_2}.$

$N_2 = N_1\left(\dfrac{p_2}{p_1}\right) = (2.20 \times 10^6\ \text{molecules})\left(\dfrac{1.00\ \text{atm}}{9.00 \times 10^{-14}\ \text{atm}}\right) = 2.44 \times 10^{19}\ \text{molecules}.$

EVALUATE: The number of molecules in a given volume is directly proportional to the pressure. Even at the very low pressure in part (a) the number of molecules in $1.00\ \text{cm}^3$ is very large.

18.22. IDENTIFY: We are asked about a single state of the system.

SET UP: Use the ideal-gas law. Write n in terms of the number of molecules N.

(a) EXECUTE: $pV = nRT$, $n = N / N_A$ so $pV = (N / N_A)RT$

$$p = \left(\frac{N}{V}\right)\left(\frac{R}{N_A}\right)T$$

$$p = \left(\frac{80 \text{ molecules}}{1\times10^{-6}\ \text{m}^3}\right)\left(\frac{8.3145\ \text{J/mol}\cdot\text{K}}{6.022\times10^{23}\ \text{molecules/mol}}\right)(7500\ \text{K}) = 8.28\times10^{-12}\ \text{Pa}$$

$p = 8.2\times10^{-17}$ atm. This is much lower than the laboratory pressure of 9×10^{-14} atm in Exercise 18.21.

(b) EVALUATE: The Lagoon Nebula is a very rarefied low pressure gas. The gas would exert *very* little force on an object passing through it.

18.23. IDENTIFY: Use $pV = nRT$ and $n = \dfrac{N}{N_A}$ with $N = 1$ to calculate the volume V occupied by 1 molecule. The length l of the side of the cube with volume V is given by $V = l^3$.

SET UP: $T = 27°\text{C} = 300$ K. $p = 1.00\ \text{atm} = 1.013\times10^5$ Pa. $R = 8.314$ J/mol·K.

$N_A = 6.022\times10^{23}$ molecules/mol.

The diameter of a typical molecule is about 10^{-10} m. $0.3\ \text{nm} = 0.3\times10^{-9}$ m.

EXECUTE: **(a)** $pV = nRT$ and $n = \dfrac{N}{N_A}$ gives

$$V = \frac{NRT}{N_A p} = \frac{(1.00)(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}{(6.022\times10^{23}\ \text{molecules/mol})(1.013\times10^5\ \text{Pa})} = 4.09\times10^{-26}\ \text{m}^3.\quad l = V^{1/3} = 3.45\times10^{-9}\ \text{m}.$$

(b) The distance in part (a) is about 10 times the diameter of a typical molecule.

(c) The spacing is about 10 times the spacing of atoms in solids.

EVALUATE: There is space between molecules in a gas whereas in a solid the atoms are closely packed together.

18.24. IDENTIFY: We are dealing with an ideal monatomic gas. We are removing gas from a tank and doubling the pressure and rms speed at the same time.

SET UP and EXECUTE: We want to know how many moles we need to remove from the tank. The volume stays the same (rigid walls), but n, p, and T can change. We want $p_2 = 2p_1$ and $v_{\text{rms},2} = 2v_{\text{rms},1}$.

We know that $v_{\text{rms}} = \sqrt{\dfrac{2kT}{m}}$ and $pV = nRT$. We want to know about the number of moles, so for the original and final states, the ideal gas law gives $n = \dfrac{p_1V}{RT_1}$ and $n_2 = \dfrac{p_2V}{RT_2}$. Squaring $v_{\text{rms}} = \sqrt{\dfrac{2kT}{m}}$ gives $v_{\text{rms}}^2 = \dfrac{2kT}{m}$. This tells us that if we square v_{rms}, the temperature increases by a factor of 4, which means that $T_2 = 4T_1$. Putting this and $p_2 = 2p_1$ into the ideal gas law gives $n_2 = \dfrac{p_2V}{RT_2} = \dfrac{2p_1V}{R(4T_1)} = \dfrac{1}{2}\dfrac{p_1V}{RT_1} = \dfrac{1}{2}n.$

Therefore we take out $\dfrac{1}{2}n$ moles.

EVALUATE: Just taking out half the moles would cut the pressure in half, and only doubling the rms speed of the molecules would increase the temperature a factor of 4. Doing both together increases the pressure by a factor of 2.

18.25. IDENTIFY: The ideal gas law applies. The translational kinetic energy of a gas depends on its absolute temperature.

SET UP: $pV = nRT$, $K_{\text{tr}} = 3/2\ nRT$, $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** From $pV = nRT$, we have $n = pV/RT$. Putting this into $K_{\text{tr}} = 3/2\ nRT$, we have

$K_{\text{tr}} = 3/2\ (pV/RT)(RT) = 3/2\ pV = (3/2)(1.013\times10^5\ \text{Pa})(8.00\ \text{m})(12.00\ \text{m})(4.00\ \text{m}) = 5.83\times10^7$ J.

(b) $K = \frac{1}{2}mv^2$: $\frac{1}{2}(2000\ \text{kg})v^2 = 5.83\times10^7$ J, gives $v = 242$ m/s.

EVALUATE: No automobile can travel this fast! Obviously the molecules in the room have a great deal of kinetic energy because there are so many of them.

18.26. **IDENTIFY:** $K_{av} = \frac{3}{2}kT$. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$.

SET UP: $M_{Ne} = 20.180$ g/mol, $M_{Kr} = 83.80$ g/mol, and $M_{Rn} = 222$ g/mol.

EXECUTE: **(a)** $K_{av} = \frac{3}{2}kT$ depends only on the temperature so it is the same for each species of atom in the mixture.

(b) $\dfrac{v_{rms,Ne}}{v_{rms,Kr}} = \sqrt{\dfrac{M_{Kr}}{M_{Ne}}} = \sqrt{\dfrac{83.80 \text{ g/mol}}{20.18 \text{ g/mol}}} = 2.04$. $\dfrac{v_{rms,Ne}}{v_{rms,Rn}} = \sqrt{\dfrac{M_{Rn}}{M_{Ne}}} = \sqrt{\dfrac{222 \text{ g/mol}}{20.18 \text{ g/mol}}} = 3.32$.

$\dfrac{v_{rms,Kr}}{v_{rms,Rn}} = \sqrt{\dfrac{M_{Rn}}{M_{Kr}}} = \sqrt{\dfrac{222 \text{ g/mol}}{83.80 \text{ g/mol}}} = 1.63$.

EVALUATE: The average kinetic energies are the same. The gas atoms with smaller mass have larger v_{rms}.

18.27. **IDENTIFY:** We make several measurements of the pressure and temperature of a gas. Using a graph of the pressure versus the temperature, we want to determine the number N of gas molecules in the container.

SET UP: We use the ideal gas law to find a relationship between p and T_C so we can interpret the graph. The gas volume is $V = 80.0 \text{ cm}^3 = 0.0800$ L. We use $pV = nRT$ and $N = nN_A$. The target variable is N.

EXECUTE: $pV = nRT = nR(T_C + 273)$, so $p = \dfrac{nR(T_C + 273)}{V} = \dfrac{nR}{V}T_C + \dfrac{273nR}{V}$. Therefore a graph of p versus T_C should be a straight line having slope equal to nR/V. Putting this in terms of N gives slope = $\dfrac{nR}{V} = \dfrac{(N/N_A)R}{V}$, so $N = \dfrac{N_A V(\text{slope})}{R}$. Putting in the appropriate numbers gives

$$N = \frac{(6.022\times10^{23} \text{ molec/mol})(0.0800 \text{ L})(1.10 \text{ atm/C°})}{0.08206 \text{ L}\cdot\text{atm/mol}\cdot\text{K}} = 6.46\times10^{23} \text{ molecules.}$$

EVALUATE: The number of moles is $\dfrac{6.46\times10^{23}}{6.022\times10^{23}} = 1.07$ mol.

18.28. **IDENTIFY:** We can relate the temperature to the rms speed and the temperature to the pressure using the ideal gas law. The target variable is the pressure.

SET UP: $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ and $pV = nRT$, where $n = m/M$.

EXECUTE: Use v_{rms} to calculate T: $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ so

$T = \dfrac{Mv_{rms}^2}{3R} = \dfrac{(28.014\times10^{-3} \text{ kg/mol})(182 \text{ m/s})^2}{3(8.314 \text{ J/mol}\cdot\text{K})} = 37.20$ K. The ideal gas law gives $p = \dfrac{nRT}{V}$.

$n = \dfrac{m}{M} = \dfrac{0.226\times10^{-3} \text{ kg}}{28.014\times10^{-3} \text{ kg/mol}} = 8.067\times10^{-3}$ mol. Solving for p gives

$$p = \frac{(8.067\times10^{-3} \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(37.20 \text{ K})}{1.64\times10^{-3} \text{ m}^3} = 1.52\times10^3 \text{ Pa.}$$

EVALUATE: This pressure is around 1.5% of atmospheric pressure, which is not unreasonable since we have only around 1% of a mole of gas.

18.29. **IDENTIFY:** $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$

SET UP: The mass of a deuteron is $m = m_{\text{p}} + m_{\text{n}} = 1.673\times10^{-27}\ \text{kg} + 1.675\times10^{-27}\ \text{kg} = 3.35\times10^{-27}\ \text{kg}$. $c = 3.00\times10^{8}\ \text{m/s}$. $k = 1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K}$.

EXECUTE: **(a)** $v_{\text{rms}} = \sqrt{\dfrac{3(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})(300\times10^{6}\ \text{K})}{3.35\times10^{-27}\ \text{kg}}} = 1.93\times10^{6}\ \text{m/s}$. $\dfrac{v_{\text{rms}}}{c} = 6.43\times10^{-3}$.

(b) $T = \left(\dfrac{m}{3k}\right)(v_{\text{rms}})^2 = \left(\dfrac{3.35\times10^{-27}\ \text{kg}}{3(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})}\right)(3.0\times10^{7}\ \text{m/s})^2 = 7.3\times10^{10}\ \text{K}$.

EVALUATE: Even at very high temperatures and for this light nucleus, v_{rms} is a small fraction of the speed of light.

18.30. **IDENTIFY:** $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$, where T is in kelvins. $pV = nRT$ gives $\dfrac{n}{V} = \dfrac{p}{RT}$.

SET UP: $R = 8.314\ \text{J/mol}\cdot\text{K}$. $M = 44.0\times10^{-3}\ \text{kg/mol}$.

EXECUTE: **(a)** For $T = 0.0°\text{C} = 273.15\ \text{K}$, $v_{\text{rms}} = \sqrt{\dfrac{3(8.314\ \text{J/mol}\cdot\text{K})(273.15\ \text{K})}{44.0\times10^{-3}\ \text{kg/mol}}} = 393\ \text{m/s}$. For $T = -100.0°\text{C} = 173\ \text{K}$, $v_{\text{rms}} = 313\ \text{m/s}$. The range of speeds is 393 m/s to 313 m/s.

(b) For $T = 273.15\ \text{K}$, $\dfrac{n}{V} = \dfrac{650\ \text{Pa}}{(8.314\ \text{J/mol}\cdot\text{K})(273.15\ \text{K})} = 0.286\ \text{mol/m}^3$. For $T = 173.15\ \text{K}$,

$\dfrac{n}{V} = 0.452\ \text{mol/m}^3$. The range of densities is $0.286\ \text{mol/m}^3$ to $0.452\ \text{mol/m}^3$.

EVALUATE: When the temperature decreases the rms speed decreases and the density increases.

18.31. **IDENTIFY** and **SET UP:** Apply the analysis of Section 18.3.

EXECUTE: **(a)** $\tfrac{1}{2}m(v^2)_{\text{av}} = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38\times10^{-23}\ \text{J/molecule}\cdot\text{K})(300\ \text{K}) = 6.21\times10^{-21}\ \text{J}$.

(b) We need the mass m of one molecule:

$$m = \frac{M}{N_{\text{A}}} = \frac{32.0\times10^{-3}\ \text{kg/mol}}{6.022\times10^{23}\ \text{molecules/mol}} = 5.314\times10^{-26}\ \text{kg/molecule}.$$

Then $\tfrac{1}{2}m(v^2)_{\text{av}} = 6.21\times10^{-21}\ \text{J}$ (from part (a)) gives

$$(v^2)_{\text{av}} = \frac{2(6.21\times10^{-21}\ \text{J})}{m} = \frac{2(6.21\times10^{-21}\ \text{J})}{5.314\times10^{-26}\ \text{kg}} = 2.34\times10^{5}\ \text{m}^2/\text{s}^2.$$

(c) $v_{\text{rms}} = \sqrt{(v^2)_{\text{rms}}} = \sqrt{2.34\times10^{4}\ \text{m}^2/\text{s}^2} = 484\ \text{m/s}$.

(d) $p = mv_{\text{rms}} = (5.314\times10^{-26}\ \text{kg})(484\ \text{m/s}) = 2.57\times10^{-23}\ \text{kg}\cdot\text{m/s}$.

(e) Time between collisions with one wall is $t = \dfrac{0.20\ \text{m}}{v_{\text{rms}}} = \dfrac{0.20\ \text{m}}{484\ \text{m/s}} = 4.13\times10^{-4}\ \text{s}$.

In a collision $\vec{v}$ changes direction, so $\Delta p = 2mv_{\text{rms}} = 2(2.57\times10^{-23}\ \text{kg}\cdot\text{m/s}) = 5.14\times10^{-23}\ \text{kg}\cdot\text{m/s}$

$F = \dfrac{dp}{dt}$ so $F_{\text{av}} = \dfrac{\Delta p}{\Delta t} = \dfrac{5.14\times10^{-23}\ \text{kg}\cdot\text{m/s}}{4.13\times10^{-4}\ \text{s}} = 1.24\times10^{-19}\ \text{N}$.

(f) pressure $= F/A = 1.24\times10^{-19}\ \text{N}/(0.10\ \text{m})^2 = 1.24\times10^{-17}\ \text{Pa}$ (due to one molecule).

(g) pressure $= 1\ \text{atm} = 1.013\times10^{5}\ \text{Pa}$.

Number of molecules needed is $1.013\times10^{5}\ \text{Pa}/(1.24\times10^{-17}\ \text{Pa/molecule}) = 8.17\times10^{21}$ molecules.

(h) $pV = NkT$ (Eq. 18.18), so $N = \dfrac{pV}{kT} = \dfrac{(1.013\times10^5\ \text{Pa})(0.10\ \text{m})^3}{(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})(300\ \text{K})} = 2.45\times10^{22}$ molecules.

(i) From the factor of $\frac{1}{3}$ in $(v_x^2)_{\text{av}} = \frac{1}{3}(v^2)_{\text{av}}$.

EVALUATE: This exercise shows that the pressure exerted by a gas arises from collisions of the molecules of the gas with the walls.

18.32. IDENTIFY: Apply $\lambda = \dfrac{kT}{4\pi\sqrt{2}r^2 p}$ and calculate λ.

SET UP: $1\ \text{atm} = 1.013\times10^5$ Pa, so $p = 3.55\times10^{-8}$ Pa. $r = 2.0\times10^{-10}$ m and $k = 1.38\times10^{-23}$ J/K.

EXECUTE: $\lambda = \dfrac{kT}{4\pi\sqrt{2}r^2 p} = \dfrac{(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}{4\pi\sqrt{2}(2.0\times10^{-10}\ \text{m})^2(3.55\times10^{-8}\ \text{Pa})} = 1.6\times10^5$ m

EVALUATE: At this very low pressure the mean free path is very large. If $v = 484$ m/s, as in Example 18.8, then $t_{\text{mean}} = \dfrac{\lambda}{v} = 330$ s. Collisions are infrequent.

18.33. IDENTIFY and SET UP: Use equal v_{rms} to relate T and M for the two gases. $v_{\text{rms}} = \sqrt{3RT/M}$, so $v_{\text{rms}}^2/3R = T/M$, where T must be in kelvins. Same v_{rms} so same T/M for the two gases and $T_{N_2}/M_{N_2} = T_{H_2}/M_{H_2}$.

EXECUTE: $T_{N_2} = T_{H_2}\left(\dfrac{M_{N_2}}{M_{H_2}}\right) = [(20+273)\text{K}]\left(\dfrac{28.014\ \text{g/mol}}{2.016\ \text{g/mol}}\right) = 4.071\times10^3$ K

$T_{N_2} = (4071 - 273)°\text{C} = 3800°\text{C}.$

EVALUATE: A N_2 molecule has more mass so N_2 gas must be at a higher temperature to have the same v_{rms}.

18.34. IDENTIFY: $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}.$

SET UP: $k = 1.381\times10^{-23}$ J/molecule·K.

EXECUTE: **(a)** $v_{\text{rms}} = \sqrt{\dfrac{3(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})(300\ \text{K})}{3.00\times10^{-16}\ \text{kg}}} = 6.44\times10^{-3}\ \text{m/s} = 6.44\ \text{mm/s}$

EVALUATE: **(b)** No. The rms speed depends on the average kinetic energy of the particles. At this T, H_2 molecules would have larger v_{rms} than the typical air molecules but would have the same average kinetic energy and the average kinetic energy of the smoke particles would be the same.

18.35. IDENTIFY: We add heat energy to a gas in a sealed rigid container and want to find the new rms speed of the molecules due to this added energy.

SET UP: We know that at constant volume $Q = nC_V\Delta T$ where $C_V = \dfrac{3}{2}R$ for a monatomic gas, and that $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$. First use $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$ to get the original temperature. Then use $Q = nC_V\Delta T$ to get ΔT, and finally use $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$ again to get the new v_{rms}.

EXECUTE: Get the original temperature T_1. Square $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$ to get $T_1 = \dfrac{Mv_{\text{rms}}^2}{3R}$. Using the numbers gives $T_1 = \dfrac{(0.00400\ \text{kg/mol})(900\ \text{m/s})^2}{3(8.314\ \text{J/mol}\cdot\text{K})} = 129.9$ K. Now get the new temperature T_2. Using

$Q = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$ gives $2400\text{ J} = (3.00\text{ mol})(3/2)(8.314\text{ J/mol}\cdot\text{K})\Delta T$, so we have ΔT = 64.15 K. Thus $T_2 = T_1 + \Delta T$ = 129.9 K + 64.15 K = 194.05 K. Finally use $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ to calculate the new rms speed using T_2 = 194.05 K and M = 0.00400 kg/mol. The result is v_{rms} = 1100 m/s.

EVALUATE: Since $v_{\text{rms}} \propto \sqrt{T}$, when our fractional increase in T is $\frac{194.05}{129.9} = 1.494$, the fractional increase in v_{rms} is $\sqrt{1.494} = 1222$. Thus we would expect that $v_2 = 1.222v_1 = (1.222)(900\text{ m/s}) = 1100$ m/s, which is exactly what we found.

18.36. **IDENTIFY:** We add 6000 J of heat energy to a monatomic gas in a rigid container and want to find the new pressure.

SET UP: We know that $Q = nC_V\Delta T$ where $C_V = \frac{3}{2}R$ for a monatomic gas, and $pV = nRT$. Our target variable is the final pressure, and we know that the volume remains constant.

EXECUTE: Use $Q = nC_V\Delta T$ = 6000 J to find ΔT and from this get the new temperature.

$$\Delta T = \frac{Q}{nC_V} = \frac{Q}{n\left(\frac{3}{2}R\right)} = \frac{6000\text{ J}}{(4.00\text{ mol})(3/2)(8.314\text{ J/mol}\cdot\text{K})} = 120.3\text{ K},\ \text{so } T_2 = 300\text{ K} + 120\text{ K} = 420\text{ K}.$$

Since V and n are constant, $pV = nRT$ gives T/p = constant, so

$$p_2 = p_1\frac{T_2}{T_1} = (6.00\times10^4\text{ Pa})\frac{420\text{ K}}{300\text{ K}} = 8.41\times10^4\text{ Pa}.$$

EVALUATE: The temperature ratio is $\frac{420}{300} = 1.40$ and the pressure ratio is $\frac{8.41}{6.00} = 1.40$, so they agree.

18.37. **IDENTIFY:** Use $dQ = nC_VdT$ applied to a finite temperature change.

SET UP: $C_V = 5R/2$ for a diatomic ideal gas and $C_V = 3R/2$ for a monatomic ideal gas.

EXECUTE: **(a)** $Q = nC_V\Delta T = n\left(\frac{5}{2}R\right)\Delta T$. $Q = (1.80\text{ mol})\left(\frac{5}{2}\right)(8.314\text{ J/mol}\cdot\text{K})(50.0\text{ K}) = 1870\text{ J}$.

(b) $Q = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$. $Q = (1.80\text{ mol})\left(\frac{3}{2}\right)(8.314\text{ J/mol}\cdot\text{K})(50.0\text{ K}) = 1120\text{ J}$.

EVALUATE: More heat is required for the diatomic gas; not all the heat that goes into the gas appears as translational kinetic energy, some goes into energy of the internal motion of the molecules (rotations).

18.38. **IDENTIFY:** The heat Q added is related to the temperature increase ΔT by $Q = nC_V\Delta T$.

SET UP: For ideal H_2 (a diatomic gas), $C_{V,H_2} = 5/2\,R$, and for ideal Ne (a monatomic gas), $C_{V,\text{Ne}} = 3/2\,R$.

EXECUTE: $C_V\Delta T = \frac{Q}{n}$ = constant, so $C_{V,H_2}\Delta T_{H_2} = C_{V,\text{Ne}}\Delta T_{\text{Ne}}$.

$$\Delta T_{\text{Ne}} = \left(\frac{C_{V,H_2}}{C_{V,\text{Ne}}}\right)\Delta T_{H_2} = \left(\frac{5/2\,R}{3/2\,R}\right)(2.50\text{ C}^\circ) = 4.17\text{ C}^\circ = 4.17\text{ K}.$$

EVALUATE: The same amount of heat causes a smaller temperature increase for H_2 since some of the energy input goes into the internal degrees of freedom.

18.39. **IDENTIFY:** $C = Mc$, where C is the molar heat capacity and c is the specific heat capacity.

$$pV = nRT = \frac{m}{M}RT.$$

SET UP: $M_{N_2} = 2(14.007\text{ g/mol}) = 28.014\times10^{-3}$ kg/mol. For water, $c_w = 4190$ J/kg·K. For N_2, $C_V = 20.76$ J/mol·K.

EXECUTE: **(a)** $c_{N_2} = \dfrac{C}{M} = \dfrac{20.76 \text{ J/mol} \cdot \text{K}}{28.014 \times 10^{-3} \text{ kg/mol}} = 741 \text{ J/kg} \cdot \text{K}$. $\dfrac{c_w}{c_{N_2}} = 5.65$; c_w is over five time larger.

(b) To warm the water, $Q = mc_w \Delta T = (1.00 \text{ kg})(4190 \text{ J/mol} \cdot \text{K})(10.0 \text{ K}) = 4.19 \times 10^4 \text{ J}$. For air,

$$m = \frac{Q}{c_{N_2} \Delta T} = \frac{4.19 \times 10^4 \text{ J}}{(741 \text{ J/kg} \cdot \text{K})(10.0 \text{ K})} = 5.65 \text{ kg}.$$

$$V = \frac{mRT}{Mp} = \frac{(5.65 \text{ kg})(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})}{(28.014 \times 10^{-3} \text{ kg/mol})(1.013 \times 10^5 \text{ Pa})} = 4.85 \text{ m}^3 = 4850 \text{ L}.$$

EVALUATE: c is smaller for N_2, so less heat is needed for 1.0 kg of N_2 than for 1.0 kg of water.

18.40. **(a) IDENTIFY** and **SET UP:** $\frac{1}{2}R$ contribution to C_V for each degree of freedom. The molar heat capacity C is related to the specific heat capacity c by $C = Mc$.

EXECUTE: $C_V = 6\left(\frac{1}{2}R\right) = 3R = 3(8.3145 \text{ J/mol} \cdot \text{K}) = 24.9 \text{ J/mol} \cdot \text{K}$. The specific heat capacity is $c_V = C_V / M = (24.9 \text{ J/mol} \cdot \text{K})/(18.0 \times 10^{-3} \text{ kg/mol}) = 1380 \text{ J/kg} \cdot \text{K}$.

(b) For water vapor the specific heat capacity is $c = 2000 \text{ J/kg} \cdot \text{K}$. The molar heat capacity is $C = Mc = (18.0 \times 10^{-3} \text{ kg/mol})(2000 \text{ J/kg} \cdot \text{K}) = 36.0 \text{ J/mol} \cdot \text{K}$.

EVALUATE: The difference is $36.0 \text{ J/mol} \cdot \text{K} - 24.9 \text{ J/mol} \cdot \text{K} = 11.1 \text{ J/mol} \cdot \text{K}$, which is about $2.7\left(\frac{1}{2}R\right)$; the vibrational degrees of freedom make a significant contribution.

18.41. **IDENTIFY:** Apply $v_{mp} = \sqrt{2kT/m}$, $v_{av} = \sqrt{8kT/\pi m}$, and $v_{rms} = \sqrt{3kT/m}$.

SET UP: Note that $\dfrac{k}{m} = \dfrac{R/N_A}{M/N_A} = \dfrac{R}{M}$. $M = 44.0 \times 10^{-3}$ kg/mol.

EXECUTE: **(a)** $v_{mp} = \sqrt{2(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})/(44.0 \times 10^{-3} \text{ kg/mol})} = 3.37 \times 10^2$ m/s.

(b) $v_{av} = \sqrt{8(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})/(\pi(44.0 \times 10^{-3} \text{ kg/mol}))} = 3.80 \times 10^2$ m/s.

(c) $v_{rms} = \sqrt{3(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})/(44.0 \times 10^{-3} \text{ kg/mol})} = 4.12 \times 10^2$ m/s.

EVALUATE: The average speed is greater than the most probable speed and the rms speed is greater than the average speed.

18.42. **IDENTIFY:** Table 18.2 gives the value of v / v_{rms} for which 94.7% of the molecules have a smaller value of v / v_{rms}. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$.

SET UP: For N_2, $M = 28.0 \times 10^{-3}$ kg/mol. $v / v_{rms} = 1.60$.

EXECUTE: $v_{rms} = \dfrac{v}{1.60} = \sqrt{\dfrac{3RT}{M}}$, so the temperature is

$$T = \frac{Mv^2}{3(1.60)^2 R} = \frac{(28.0 \times 10^{-3} \text{ kg/mol})}{3(1.60)^2 (8.3145 \text{ J/mol} \cdot \text{K})} v^2 = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2) v^2.$$

(a) $T = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2)(1500 \text{ m/s})^2 = 987$ K.

(b) $T = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2)(1000 \text{ m/s})^2 = 438$ K.

(c) $T = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2)(500 \text{ m/s})^2 = 110$ K.

EVALUATE: As T decreases the distribution of molecular speeds shifts to lower values.

18.43. **IDENTIFY:** We want to compare the speed of sound in a gas to the rms speed and average speed of its molecules.

SET UP: $v=\sqrt{\dfrac{\gamma RT}{M}}$ (for sound), $v_{\text{rms}}=\sqrt{\dfrac{3RT}{M}}$, and $v_{\text{av}}=\sqrt{\dfrac{8kT}{\pi m}}$. Using $k/m = R/M$ gives $v_{\text{av}}=\sqrt{\dfrac{8RT}{\pi M}}$.

EXECUTE: **(a)** Compare v_{rms} to v: $\dfrac{v_{\text{rms}}}{v}=\dfrac{\sqrt{3RT/M}}{\sqrt{\gamma RT/M}}=\sqrt{3/\gamma}=\sqrt{3/1.67}=1.34.$

(b) Compare v_{av} to v: $\dfrac{v_{\text{av}}}{v}=\dfrac{\sqrt{8RT/\pi M}}{\sqrt{\gamma RT/M}}=\sqrt{\dfrac{8}{\pi\gamma}}=\sqrt{\dfrac{8}{\pi(1.67)}}=1.23.$

EVALUATE: The average speed is closer to the speed of sound, but both are greater than it.

18.44. **IDENTIFY:** This problem concerns the distribution of molecular speeds.

SET UP and EXECUTE: We want to find the number of atoms within 20% of the rms speed. This means that the limits on v are $v = v_{\text{rms}} \pm 0.20v_{\text{rms}} = \begin{cases} v_{\text{rms}}+0.20v_{\text{rms}}=1.20v_{\text{rms}} \\ v_{\text{rms}}-0.20v_{\text{rms}}=0.80v_{\text{rms}} \end{cases}$. Table 18.2 tells us that if we have N atoms (or molecules), $0.771N$ have speeds less than $1.20v_{\text{rms}}$ and $0.411N$ have speeds less than $0.80v_{\text{rms}}$. Therefore the number with speeds between $0.80v_{\text{rms}}$ and $1.20v_{\text{rms}}$ is $0.771N-0.411N=0.360N$. In 0.0345 mol of gas, the number of molecules is (0.0345 mol)N_A = (0.0345 mol)(6.022×10^{23} molecules/mol) = 2.0776×10^{22} molecules. In this gas the number of molecules with speeds between $0.80v_{\text{rms}}$ and $1.20v_{\text{rms}}$ is (0.360)(2.0776×10^{22}) = 7.48×10^{21}.

EVALUATE: Careful! We do not want the number of molecules that have a speed of 20% the rms speed. We want the number *within* 20% of the rms speed.

18.45. **IDENTIFY:** Refer to the phase diagram in Figure 18.24 in the textbook.

SET UP: For water the triple-point pressure is 610 Pa and the critical-point pressure is 2.212×10^7 Pa.

EXECUTE: **(a)** To observe a solid to liquid (melting) phase transition the pressure must be greater than the triple-point pressure, so $p_1=610$ Pa. For $p<p_1$ the solid to vapor (sublimation) phase transition is observed.

(b) No liquid to vapor (boiling) phase transition is observed if the pressure is greater than the critical-point pressure. $p_2=2.212\times10^7$ Pa. For $p_1<p<p_2$ the sequence of phase transitions is solid to liquid and then liquid to vapor.

EVALUATE: Normal atmospheric pressure is approximately 1.0×10^5 Pa, so the solid to liquid to vapor sequence of phase transitions is normally observed when the material is water.

18.46. **IDENTIFY:** Apply the definition of relative humidity given in the problem. $pV=nRT=\dfrac{m_{\text{tot}}}{M}RT$.

SET UP: $M=18.0\times10^{-3}$ kg/mol.

EXECUTE: **(a)** The pressure due to water vapor is $(0.60)(2.34\times10^3\text{ Pa})=1.40\times10^3$ Pa.

(b) $m_{\text{tot}}=\dfrac{MpV}{RT}=\dfrac{(18.0\times10^{-3}\text{ kg/mol})(1.40\times10^3\text{ Pa})(1.00\text{ m}^3)}{(8.3145\text{ J/mol}\cdot\text{K})(293.15\text{ K})}=10\text{ g}.$

EVALUATE: The vapor pressure of water vapor at this temperature is much less than the total atmospheric pressure of 1.0×10^5 Pa.

18.47. **IDENTIFY:** We are investigating characteristics of the gas in a party balloon.
SET UP: Estimate: The sound intensity level is 95 dB. The sound lasts 100 ms = 0.10 s. We use $I = P/A = P/4\pi r^2$ and $\beta = (10 \text{ dB})\log\dfrac{I}{I_0}$.

EXECUTE: **(a)** $95 \text{ dB} = (10 \text{ dB})\log\dfrac{I}{I_0}$, so $I = 10^{9.5} I_0 = 10^{9.5}\ 10^{-12} \text{ W/m}^2 = 3.2\times10^{-3} \text{ W/m}^2$. The intensity is $I = P/A$, so $P_{av} = IA = (3.2\times10^{-3} \text{ W/m}^2)(4\pi)(2.0 \text{ m})^2 = 0.16$ W. The energy E in this sound is $E = P_{av}t = (0.16 \text{ W})(0.10 \text{ s}) = 0.016$ J.
(b) $T = E/A = E/(4\pi r^2) = (0.016 \text{ J})/[4\pi(0.075 \text{ m})^2] = 0.23 \text{ J/m}^2$.
(c) $p_{gauge} = T/R = (0.23 \text{ J/m}^2)/(0.075 \text{ m}) = 3.0$ Pa.
EVALUATE: These results are only a rough approximation for the gauge pressure.

18.48. **IDENTIFY:** The ideal gas law will tell us the number of moles of gas in the room, which we can use to find the number of molecules.
SET UP: $pV = nRT$, $N = nN_A$, and $m = nM$.
EXECUTE: **(a)** $T = 27.0°\text{C} + 273 = 300$ K. $p = 1.013\times10^5$ Pa.

$$n = \frac{pV}{RT} = \frac{(1.013\times10^5 \text{ Pa})(216 \text{ m}^3)}{(8.314 \text{ J/mol}\cdot\text{K})(300 \text{ K})} = 8773 \text{ mol}.$$

$N = nN_A = (8773 \text{ mol})(6.022\times10^{23} \text{ molecules/mol}) = 5.28\times10^{27}$ molecules.

(b) $V = (216 \text{ m}^3)(1 \text{ cm}^3/10^{-6} \text{ m}^3) = 2.16\times10^8 \text{ cm}^3$. The particle density is

$$\frac{5.28\times10^{27} \text{ molecules}}{2.16\times10^8 \text{ cm}^3} = 2.45\times10^{19} \text{ molecules/cm}^3.$$

(c) $m = nM = (8773 \text{ mol})(28.014\times10^{-3} \text{ kg/mol}) = 246$ kg.

EVALUATE: A cubic centimeter of air (about the size of a sugar cube) contains around 10^{19} molecules, and the air in the room weighs about 500 lb!

18.49. **IDENTIFY:** We can model the atmosphere as a fluid of constant density, so the pressure depends on the depth in the fluid, as we saw in Section 12.2.
SET UP: The pressure difference between two points in a fluid is $\Delta p = \rho gh$, where h is the difference in height of two points.
EXECUTE: **(a)** $\Delta p = \rho gh = (1.2 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1000 \text{ m}) = 1.18\times10^4$ Pa.
(b) At the bottom of the mountain, $p = 1.013\times10^5$ Pa. At the top, $p = 8.95\times10^4$ Pa.

$$pV = nRT = \text{constant so } p_bV_b = p_tV_t \text{ and } V_t = V_b\left(\frac{p_b}{p_t}\right) = (0.50 \text{ L})\left(\frac{1.013\times10^5 \text{ Pa}}{8.95\times10^4 \text{ Pa}}\right) = 0.566 \text{ L}.$$

EVALUATE: The pressure variation with altitude is affected by changes in air density and temperature and we have neglected those effects. The pressure decreases with altitude and the volume increases. You may have noticed this effect: bags of potato chips "puff up" when taken to the top of a mountain.

18.50. **IDENTIFY:** As the pressure on the bubble changes, its volume will change. As we saw in Section 12.2, the pressure in a fluid depends on the depth.
SET UP: The pressure at depth h in a fluid is $p = p_0 + \rho gh$, where p_0 is the pressure at the surface. $p_0 = p_{air} = 1.013\times10^5$ Pa. The density of water is $\rho = 1000 \text{ kg/m}^3$.

EXECUTE: $p_1 = p_0 + \rho gh = 1.013\times10^5$ Pa + (1000 kg/m^3)(9.80 m/s^2)(25 m) $= 3.463\times10^5$ Pa.

$p_2 = p_{\text{air}} = 1.013\times10^5$ Pa. $V_1 = 1.0\text{ mm}^3$. n, R and T are constant so $pV = nRT =$ constant. $p_1V_1 = p_2V_2$ and $V_2 = V_1\left(\dfrac{p_1}{p_2}\right) = (1.0\text{ mm}^3)\left(\dfrac{3.463\times10^5\text{ Pa}}{1.013\times10^5\text{ Pa}}\right) = 3.4\text{ mm}^3$.

EVALUATE: This is a large change and would have serious effects.

18.51. IDENTIFY: The buoyant force on the balloon must be equal to the weight of the load plus the weight of the gas.

SET UP: The buoyant force is $F_{\text{B}} = \rho_{\text{air}}Vg$. A lift of 290 kg means $\dfrac{F_{\text{B}}}{g} - m_{\text{hot}} = 290\text{ kg}$, where m_{hot} is the mass of hot air in the balloon. $m = \rho V$.

EXECUTE: $m_{\text{hot}} = \rho_{\text{hot}}V$. $\dfrac{F_{\text{B}}}{g} - m_{\text{hot}} = 290\text{ kg}$ gives $(\rho_{\text{air}} - \rho_{\text{hot}})V = 290\text{ kg}$.

Solving for ρ_{hot} gives $\rho_{\text{hot}} = \rho_{\text{air}} - \dfrac{290\text{ kg}}{V} = 1.23\text{ kg/m}^3 - \dfrac{290\text{ kg}}{500.0\text{ m}^3} = 0.65\text{ kg/m}^3$. $\rho_{\text{hot}} = \dfrac{pM}{RT_{\text{hot}}}$.

$\rho_{\text{air}} = \dfrac{pM}{RT_{\text{air}}}$. $\rho_{\text{hot}}T_{\text{hot}} = \rho_{\text{air}}T_{\text{air}}$ so

$$T_{\text{hot}} = T_{\text{air}}\left(\frac{\rho_{\text{air}}}{\rho_{\text{hot}}}\right) = (288\text{ K})\left(\frac{1.23\text{ kg/m}^3}{0.65\text{ kg/m}^3}\right) = 545\text{ K} = 272°\text{C}.$$

EVALUATE: This temperature is well above normal air temperatures, so the air in the balloon would need considerable heating.

18.52. IDENTIFY: The ideal gas law applies. The gas pressure supports the disk and whatever is on it.
SET UP: For constant temperature, $pV = nRT$ gives $p_1V_1 = p_2V_2$. Call A the area of the disk, m the mass of the disk, M the mass of the lead brick, and h_2 the final height of the disk. $p = F_\perp/A$.

EXECUTE: $p_1 = mg/A$, $p_2 = (m + M)g/A$. For constant temperature, $p_1V_1 = p_2V_2$, which gives $p_2 = p_1(V_1/V_2) = (mg/A)(Ah/Ah_2) = (m + M)g/A$. Solving for h_2 gives $h_2 = mh/(m + M)$, which gives $h_2 = (3.00\text{ kg})(4.00\text{ m})/(12.0\text{ kg}) = 1.00\text{ m}$.
EVALUATE: Since $p \to 4p$, $V \to \frac{1}{4}V$, so it is reasonable that $h \to \frac{1}{4}h$, as we found.

18.53. IDENTIFY: We are asked to compare two states. Use the ideal-gas law to obtain m_2 in terms of m_1 and the ratio of pressures in the two states. Apply $pV = \dfrac{m_{\text{total}}}{M}RT$ to the initial state to calculate m_1.

SET UP: $pV = nRT$ can be written $pV = (m/M)RT$

T, V, M, R are all constant, so $p/m = RT/MV =$ constant.

So $p_1/m_1 = p_2/m_2$, where m is the mass of the gas in the tank.

EXECUTE: $p_1 = 1.30\times10^6\text{ Pa} + 1.01\times10^5\text{ Pa} = 1.40\times10^6\text{ Pa}$

$p_2 = 3.40\times10^5\text{ Pa} + 1.01\times10^5\text{ Pa} = 4.41\times10^5\text{ Pa}$

$m_1 = p_1VM/RT$; $V = hA = h\pi r^2 = (1.00\text{ m})\pi(0.060\text{ m})^2 = 0.01131\text{ m}^3$

$$m_1 = \frac{(1.40\times10^6\text{ Pa})(0.01131\text{ m}^3)(44.1\times10^{-3}\text{ kg/mol})}{(8.3145\text{ J/mol}\cdot\text{K})((22.0 + 273.15)\text{K})} = 0.2845\text{ kg}$$

Then $m_2 = m_1\left(\dfrac{p_2}{p_1}\right) = (0.2845\text{ kg})\left(\dfrac{4.41\times10^5\text{ Pa}}{1.40\times10^6\text{ Pa}}\right) = 0.0896\text{ kg}$.

m_2 is the mass that remains in the tank. The mass that has been used is

$m_1 - m_2 = 0.2845\text{ kg} - 0.0896\text{ kg} = 0.195\text{ kg}$.

EVALUATE: Note that we have to use absolute pressures. The absolute pressure decreases by a factor of approximately 3 and the mass of gas in the tank decreases by a factor of approximately 3.

18.54. IDENTIFY: Apply $pV = nRT$ to the air inside the diving bell. The pressure p at depth y below the surface of the water is $p = p_{\text{atm}} + \rho gy$.

SET UP: $p = 1.013\times10^5$ Pa. $T = 300.15$ K at the surface and $T' = 280.15$ K at the depth of 13.0 m.

EXECUTE: **(a)** The height h' of the air column in the diving bell at this depth will be proportional to the volume, and hence inversely proportional to the pressure and proportional to the Kelvin temperature:

$$h' = h\frac{p}{p'}\frac{T'}{T} = h\frac{p_{\text{atm}}}{p_{\text{atm}} + \rho gy}\frac{T'}{T}.$$

$$h' = (2.30\text{ m})\frac{(1.013\times10^5\text{ Pa})}{(1.013\times10^5\text{ Pa}) + (1030\text{ kg/m}^3)(9.80\text{ m/s}^2)(73.0\text{ m})}\left(\frac{280.15\text{ K}}{300.15\text{ K}}\right) = 0.26\text{ m}.$$

The height of the water inside the diving bell is $h - h' = 2.04$ m.

(b) The necessary gauge pressure is the term ρgy from the above calculation, $p_{\text{gauge}} = 7.37\times10^5$ Pa.

EVALUATE: The gauge pressure required in part (b) is about 7 atm.

18.55. IDENTIFY: We are investigating the movement of air above a campfire. The air is heated by the fire and expands so the number of molecules in a given volume decreases.

SET UP and EXECUTE: The mass of a volume of air is $m = nM_{\text{air}}$, where M is the molar mass and n is the number of moles. If m_{out} is the the mass of gas out of the heated region and m_{in} is the mass within that region, the upward force F_{up} on a parcel of air above the fire can be roughly approximated by $F_{\text{up}} = (m_{\text{out}} - m_{\text{in}})g$. We can use $pV = nRT$ and $\sum F_y = ma_y$.

(a) We want the acceleration a of the air parcel as a function of $T_{\text{out}}/T_{\text{in}}$. Using $m = nM_{\text{air}}$ we have $F_{\text{up}} = (m_{\text{out}} - m_{\text{in}})g = (n_{\text{out}}M_{\text{air}} - n_{\text{in}}M_{\text{air}})g = M_{\text{air}}(n_{\text{out}} - n_{\text{in}})g$. Now solve for n in $pV = nRT$ to get $n = pV/RT$ and use this in the equation for F_{up}, which gives $F_{\text{up}} = \frac{M_{\text{air}}g}{R}\left(\frac{p_{\text{out}}V_{\text{out}}}{T_{\text{out}}} - \frac{p_{\text{in}}V_{\text{in}}}{T_{\text{in}}}\right)$. As stated in the problem, the pressure above the fire is the same as the ambient pressure, so $p_0 = p_1 = p_{\text{amb}}$. This gives $F_{\text{up}} = \frac{M_{\text{air}}gp_{\text{amb}}}{R}\left(\frac{V_{\text{out}}}{T_{\text{out}}} - \frac{V_{\text{in}}}{T_{\text{in}}}\right)$. Now apply $\sum F_y = ma_y$ where m is the mass of air in the air parcel over the fire. This mass is $m = n_{\text{in}}M_{\text{air}}$, so $F_{\text{up}} = \frac{M_{\text{air}}gp_{\text{amb}}}{R}\left(\frac{V_{\text{out}}}{T_{\text{out}}} - \frac{V_{\text{in}}}{T_{\text{in}}}\right) = n_{\text{in}}M_{\text{air}}a$. Substituting for n_{in} gives $\frac{M_{\text{air}}gp_{\text{amb}}}{R}\left(\frac{V_{\text{out}}}{T_{\text{out}}} - \frac{V_{\text{in}}}{T_{\text{in}}}\right) = \left(\frac{p_{\text{in}}V_{\text{in}}}{RT_{\text{in}}}\right)M_{\text{air}}a$. We are told that $p_{\text{in}} = p_{\text{amb}}$ and the masses are for similar volumes of air, so $V_{\text{in}} = V_{\text{out}}$, this equation simplifies to $F_{\text{up}} = g\left(\frac{1}{T_0} - \frac{1}{T_1}\right) = \left(\frac{1}{T_{\text{in}}}\right)a$. Solving for a we get $a = \left(\frac{T_{\text{in}}}{T_{\text{out}}} - 1\right)g$.

(b) Solving for T_{in} as a function of a gives $T_{\text{in}} = T_{\text{out}}\left(\frac{a}{g} + 1\right)$.

(c) Estimate: $a \approx 1.5g$.

(d) $T_{\text{out}} = 15°\text{C} = 288$ K, so $T_{\text{in}} = (288\text{ K})\left(\frac{1.5g}{g} + 1\right) = 720\text{ K}$ � 450°C.

EVALUATE: This would be a roaring fire at 840°F!

18.56. **IDENTIFY:** For constant temperature, the variation of pressure with altitude is calculated in Example 18.4 to be $p = p_0 e^{-Mgy/RT}$. $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$.

SET UP: $g_{\text{earth}} = 9.80\text{ m/s}^2$. $T = 460°\text{C} = 733\text{ K}$. $M = 44.0\text{ g/mol} = 44.0\times10^{-3}\text{ kg/mol}$.

EXECUTE: **(a)** $\dfrac{Mgy}{RT} = \dfrac{(44.0\times10^{-3}\text{ kg/mol})(0.894)(9.80\text{ m/s}^2)(1.00\times10^3\text{ m})}{(8.314\text{ J/mol}\cdot\text{K})(733\text{ K})} = 0.06326.$

$p = p_0 e^{-Mgy/RT} = (92\text{ atm})e^{-0.06326} = 86\text{ atm}$. The pressure is 86 earth-atmospheres, or 0.94 Venus-atmospheres.

(b) $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3(8.314\text{ J/mol}\cdot\text{K})(733\text{ K})}{44.0\times10^{-3}\text{ kg/mol}}} = 645\text{ m/s}$. v_{rms} has this value both at the surface and at an altitude of 1.00 km.

EVALUATE: v_{rms} depends only on T and the molar mass of the gas. For Venus compared to earth, the surface temperature, in kelvins, is nearly a factor of three larger and the molecular mass of the gas in the atmosphere is only about 50% larger, so v_{rms} for the Venus atmosphere is larger than it is for the earth's atmosphere.

18.57. **IDENTIFY:** Air and water can be pumped in and out of a diving bell.

SET UP and EXECUTE: The average density can be changed by pumping water in and out of the bell. We use $\rho = m/V$, $p = p_0 + \rho gh$, and $pV = nRT$.

(a) We want to know what volume of seawater to add to the bell so it is neutrally buoyant. The average density of the bell and all its contents must be equal to the density of seawater. Call m_b the mass of the bell, m_d the mass of the diver, and m_w the mass of the added water. Using $\rho_{\text{bell}} = \rho_{\text{seawater}}$ gives

$\dfrac{m_b + m_d + m_w}{V_{\text{bell}}} = \rho_{\text{seawater}}$. Using $V_{\text{bell}} = \pi r^2 L$ and solving for m_w gives $m_w = \pi r^2 L\rho_{\text{seawater}} - m_b - m_d$.

Using $\rho_{\text{seawater}} = 1025\text{ kg/m}^3$, $r = 0.750$ m, $L = 2.50$ m, $m_b = 4350$ kg, and $m_d = 80.0$ kg, we get $m_w =$ 98.31 kg. The volume of this water is $V = m/\rho$ so $V = \dfrac{98.31\text{ kg}}{1025\text{ kg/m}^3} = 0.0959\text{ m}^3 = 95.9\text{ L}$.

(b) We want to know the rate at which we should release air from the tank to maintain a constant pressure as the bell descends at a steady 1.0 m/s, so we want dn/dt. Combining $p = p_0 + \rho gh$ and $pV = nRT$ gives $p_0 + \rho gh = (RT/V)n$. Taking the time derivative of both sides of this equation gives

$\rho g\dfrac{dy}{dt} = \left(\dfrac{RT}{V}\right)\dfrac{dn}{dt}$. Solving for dn/dt gives $\dfrac{dn}{dt} = \dfrac{\rho gV}{RT}\dfrac{dy}{dt} = \dfrac{\rho gVv}{RT}$. V is the volume of air in the bell which is $V = V_b - V_w = \pi r^2 L - V_w = \pi(0.750\text{ m})^2(2.50\text{ m}) - 0.0959\text{ m}^3 = 4.322\text{ m}^3$. Using this value for V plus $\rho = 1025\text{ kg/m}^3$, $T = 293$ K, $v = 1.00$ m/s, $R = 8.314\text{ J/mol}\cdot\text{K}$, and $L = 2.50$ m, we get $dn/dt =$ 17.8 mol/s.

(c) The tank contains 600 ft^3 of gas, which is 1.6992×10^4 L. It was loaded at standard conditions of 1.0 atm at 0°C = 273 K, so $n = pV/RT$. Putting in the numbers gives $n = 758.5$ mol. The tank releases gas at a rate of 17.8 mol/s, so (17.8 mol/s)t = 758.5 mol, which gives $t = 42.6$ s. During this time the bell is descending at a steady 1.00 m/s, so the distance it travels is (1.00 m/s)(42.6 s) = 42.6 m.

EVALUATE: At 42.6 m the water pressure would be $p = p_0 + \rho gh = p_0 + (1025\text{ kg/m}^3)(g)(42.6\text{ m}) =$ 5.3 atm, so the diving bell should not have any leaks!

18.58. **IDENTIFY:** In part (a), apply $pV = nRT$ to the ethane in the flask. The volume is constant once the stopcock is in place. In part (b) apply $pV = \dfrac{m_{\text{tot}}}{M}RT$ to the ethane at its final temperature and pressure.

SET UP: $1.50\text{ L} = 1.50\times10^{-3}\text{ m}^3$. $M = 30.1\times10^{-3}\text{ kg/mol}$. Neglect the thermal expansion of the flask.

EXECUTE: (a) $p_2 = p_1(T_2/T_1) = (1.013\times10^5\ \text{Pa})(300\ \text{K}/550\ \text{K}) = 5.525\times10^4\ \text{Pa},$ which rounds to 5.53×10^4 Pa.

(b) $m_{\text{tot}} = \left(\dfrac{p_2V}{RT_2}\right)M = \left(\dfrac{(5.525\times10^4\ \text{Pa})(1.50\times10^{-3}\ \text{m}^3)}{(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})}\right)(30.1\times10^{-3}\ \text{kg/mol}) = 1.00\ \text{g}.$

EVALUATE: We could also calculate m_{tot} with $p = 1.013\times10^5$ Pa and $T = 550$ K, and we would obtain the same result. Originally, before the system was warmed, the mass of ethane in the flask was $m = (1.00\ \text{g})\left(\dfrac{1.013\times10^5\ \text{Pa}}{5.525\times10^4\ \text{Pa}}\right) = 1.83\ \text{g}.$

18.59. (a) IDENTIFY: Consider the gas in one cylinder. Calculate the volume to which this volume of gas expands when the pressure is decreased from $(1.20\times10^6\ \text{Pa} + 1.01\times10^5\ \text{Pa}) = 1.30\times10^6\ \text{Pa}$ to 1.01×10^5 Pa. Apply the ideal-gas law to the two states of the system to obtain an expression for V_2 in terms of V_1 and the ratio of the pressures in the two states.

SET UP: $pV = nRT$

n, R, T constant implies $pV = nRT = \text{constant}$, so $p_1V_1 = p_2V_2$.

EXECUTE: $V_2 = V_1(p_1/p_2) = (1.90\ \text{m}^3)\left(\dfrac{1.30\times10^6\ \text{Pa}}{1.01\times10^5\ \text{Pa}}\right) = 24.46\ \text{m}^3$

The number of cylinders required to fill a $750\ \text{m}^3$ balloon is $750\ \text{m}^3 / 24.46\ \text{m}^3 = 30.7$ cylinders.

EVALUATE: The ratio of the volume of the balloon to the volume of a cylinder is about 400. Fewer cylinders than this are required because of the large factor by which the gas is compressed in the cylinders.

(b) IDENTIFY: The upward force on the balloon is given by Archimedes's principle: $B =$ weight of air displaced by balloon $= \rho_{\text{air}}Vg$. Apply Newton's second law to the balloon and solve for the weight of the load that can be supported. Use the ideal-gas equation to find the mass of the gas in the balloon.

SET UP: The free-body diagram for the balloon is given in Figure 18.59.

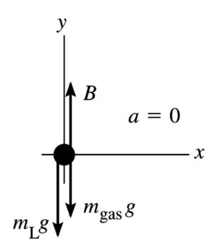


Figure 18.59

m_{gas} is the mass of the gas that is inside the balloon; m_{L} is the mass of the load that is supported by the balloon.

EXECUTE: $\sum F_y = ma_y$

$B - m_{\text{L}}g - m_{\text{gas}}g = 0$

$\rho_{\text{air}}Vg - m_{\text{L}}g - m_{\text{gas}}g = 0$

$m_{\text{L}} = \rho_{\text{air}}V - m_{\text{gas}}$

Calculate m_{gas}, the mass of hydrogen that occupies $750\ \text{m}^3$ at 15°C and $p = 1.01\times10^5$ Pa.

$pV = nRT = (m_{\text{gas}}/M)RT$

gives $m_{\text{gas}} = pVM/RT = \dfrac{(1.01\times10^5\ \text{Pa})(750\ \text{m}^3)(2.02\times10^{-3}\ \text{kg/mol})}{(8.3145\ \text{J/mol}\cdot\text{K})(288\ \text{K})} = 63.9\ \text{kg}.$

Then $m_{\text{L}} = (1.23\ \text{kg/m}^3)(750\ \text{m}^3) - 63.9\ \text{kg} = 859\ \text{kg},$ and the weight that can be supported is $w_{\text{L}} = m_{\text{L}}g = (859\ \text{kg})(9.80\ \text{m/s}^2) = 8420\ \text{N}.$

(c) $m_L = \rho_{air}V - m_{gas}$

$m_{gas} = pVM / RT = (63.9 \text{ kg})((4.00 \text{ g/mol}) / (2.02 \text{ g/mol})) = 126.5 \text{ kg}$ (using the results of part (b)).

Then $m_L = (1.23 \text{ kg/m}^3)(750 \text{ m}^3) - 126.5 \text{ kg} = 796 \text{ kg}.$

$w_L = m_L g = (796 \text{ kg})(9.80 \text{ m/s}^2) = 7800 \text{ N}.$

EVALUATE: A greater weight can be supported when hydrogen is used because its density is less.

18.60. IDENTIFY: The upward force exerted by the gas on the piston must equal the piston's weight. Use $pV = nRT$ to calculate the volume of the gas, and from this the height of the column of gas in the cylinder.

SET UP: $F = pA = p\pi r^2$, with $r = 0.100 \text{ m}$ and $p = 0.300 \text{ atm} = 3.039\times10^4 \text{ Pa}$. For the cylinder, $V = \pi r^2 h.$

EXECUTE: **(a)** $p\pi r^2 = mg$ and $m = \dfrac{p\pi r^2}{g} = \dfrac{(3.039\times10^4 \text{ Pa})\pi(0.100 \text{ m})^2}{9.80 \text{ m/s}^2} = 97.4 \text{ kg}.$

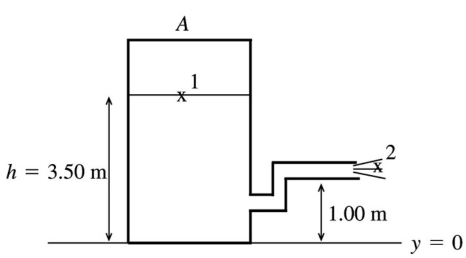
(b) $V = \pi r^2 h$ and $V = nRT/p$. Combining these equations gives $h = nRT/\pi r^2 p$, which gives

$$h = \frac{(1.80 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(293.15 \text{ K})}{\pi(0.100 \text{ m})^2(3.039\times10^4 \text{ Pa})} = 4.60 \text{ m}.$$

EVALUATE: The calculation assumes a vacuum $(p = 0)$ in the tank above the piston.

18.61. IDENTIFY: Apply Bernoulli's equation to relate the efflux speed of water out the hose to the height of water in the tank and the pressure of the air above the water in the tank. Use the ideal-gas equation to relate the volume of the air in the tank to the pressure of the air.

SET UP: Points 1 and 2 are shown in Figure 18.61.



$p_1 = 4.20\times10^5 \text{ Pa}$

$p_2 = p_{air} = 1.00\times10^5 \text{ Pa}$

large tank implies $v_1 \approx 0$

Figure 18.61

EXECUTE: **(a)** $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$\frac{1}{2}\rho v_2^2 = p_1 - p_2 + \rho g(y_1 - y_2)$

$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$

$v_2 = 26.2 \text{ m/s}.$

(b) $\underline{h = 3.00 \text{ m}}$

The volume of the air in the tank increases so its pressure decreases. $pV = nRT = \text{constant}$, so

$pV = p_0V_0$ (p_0 is the pressure for $h_0 = 3.50 \text{ m}$ and p is the pressure for $h = 3.00 \text{ m}$)

$p(4.00 \text{ m} - h)A = p_0(4.00 \text{ m} - h_0)A$

$$p = p_0\left(\frac{4.00 \text{ m} - h_0}{4.00 \text{ m} - h}\right) = (4.20\times10^5 \text{ Pa})\left(\frac{4.00 \text{ m} - 3.50 \text{ m}}{4.00 \text{ m} - 3.00 \text{ m}}\right) = 2.10\times10^5 \text{ Pa}.$$

Repeat the calculation of part (a), but now $p_1 = 2.10\times10^5 \text{ Pa}$ and $y_1 = 3.00 \text{ m}.$

$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$

$v_2 = 16.1$ m/s

$\underline{h = 2.00 \text{ m}}$

$$p = p_0\left(\frac{4.00 \text{ m} - h_0}{4.00 \text{ m} - h}\right) = (4.20\times10^5 \text{ Pa})\left(\frac{4.00 \text{ m} - 3.50 \text{ m}}{4.00 \text{ m} - 2.00 \text{ m}}\right) = 1.05\times10^5 \text{ Pa}$$

$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$

$v_2 = 5.44$ m/s.

(c) $v_2 = 0$ means $(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2) = 0$

$p_1 - p_2 = -\rho g(y_1 - y_2)$

$y_1 - y_2 = h - 1.00$ m

$p = p_0\left(\frac{0.50 \text{ m}}{4.00 \text{ m} - h}\right) = (4.20\times10^5 \text{ Pa})\left(\frac{0.50 \text{ m}}{4.00 \text{ m} - h}\right)$. This is p_1, so

$$(4.20\times10^5 \text{ Pa})\left(\frac{0.50 \text{ m}}{4.00 \text{ m} - h}\right) - 1.00\times10^5 \text{ Pa} = (9.80 \text{ m/s}^2)(1000 \text{ kg/m}^3)(1.00 \text{ m} - h)$$

$(210/(4.00 - h)) - 100 = 9.80 - 9.80h,$ with h in meters.

$210 = (4.00 - h)(109.8 - 9.80h)$

$9.80h^2 - 149h + 229.2 = 0$ and $h^2 - 15.20h + 23.39 = 0$

quadratic formula: $h = \frac{1}{2}\left(15.20 \pm \sqrt{(15.20)^2 - 4(23.39)}\right) = (7.60 \pm 5.86)$ m

h must be less than 4.00 m, so the only acceptable value is $h = 7.60 \text{ m} - 5.86 \text{ m} = 1.74$ m.

EVALUATE: The flow stops when $p + \rho g(y_1 - y_2)$ equals air pressure. For $h = 1.74$ m, $p = 9.3\times10^4$ Pa and $\rho g(y_1 - y_2) = 0.7\times10^4$ Pa, so $p + \rho g(y_1 - y_2) = 1.0\times10^5$ Pa, which is air pressure.

18.62. IDENTIFY: The vertical forces on the plastic sphere must balance. Archimedes's principle and the ideal gas law both apply.

SET UP: $pV = nRT$, $\rho = m/V$. Call V the volume of the sphere, m its mass, and ρ its volume. Let ρ_a be the density of the air and F the tension in the thread. The buoyant force B is equal to the weight of the air displaced by the sphere, so $B = \rho_a Vg$.

EXECUTE: **(a)** Balancing vertical forces gives $F + B = mg$. $B = \rho_a Vg$ and $V = m/\rho$.

For the air: ρ_a = (mass of n moles)/(volume of n moles) = Mn/V, where M is the molar mass of air, which is 28.8 g/mol = 0.0288 kg/mol. From $pV = nRT$ we have $n/V = p/RT$, so the density of air is $\rho_a = M(n/V) = Mp/RT$.

$\rho_a = (0.0288 \text{ kg/mol})(1.013\times10^5 \text{ Pa})/[(8.314 \text{ J/mol}\cdot\text{K})(298.15 \text{ K})] = 1.177 \text{ kg/m}^3$.

Therefore $B = \rho_a Vg = \rho_a (m/\rho)g$

$B = (1.177 \text{ kg/m}^3)(0.00900 \text{ kg})(9.80 \text{ m/s}^2)/(4.00 \text{ kg/m}^3) = 0.02595$ N.

$F = mg - B = (0.00900 \text{ kg})(9.80 \text{ m/s}^2) - 0.02595 \text{ N} = 0.0622$ N.

(b) If the buoyant force increases, the tension decreases, so $\Delta F = -\Delta B = -\frac{mg}{\rho}\Delta\rho_a$. This gives us

$\Delta F = -\frac{mg}{\rho}(\rho_{a,2} - \rho_{a,1})$. Using $\rho_a = pM/RT$, this equation becomes

$\Delta F = -\frac{mgpM}{\rho R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right) = \frac{mgpM}{\rho R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$. Putting in the numbers gives

$$\Delta F = \frac{(0.00900 \text{ kg})(9.80 \text{ m/s}^2)(1.013\times10^5 \text{ Pa})(0.0288 \text{ kg/mol})}{(4.00 \text{ kg/m}^3)(8.314 \text{ J/mol}\cdot\text{K})}\left(\frac{1}{278.15 \text{ K}} - \frac{1}{308.15 \text{ K}}\right)$$

$\Delta F = 2.71\times10^{-3}$ N = 2.71 mN. The positive sign tells us that the tension increases.

EVALUATE: As the air temperature increased, its density decreased, so the buoyant force it exerted decreased. So it is reasonable that the tension should increase.

18.63. **IDENTIFY** and **SET UP:** Apply $m_{\text{total}} = nM$ to find *n* and then use Avogadro's number to find the number of molecules.

EXECUTE: Calculate the number of water molecules *N*.

Number of moles: $n = \dfrac{m_{\text{tot}}}{M} = \dfrac{50 \text{ kg}}{18.0\times10^{-3} \text{ kg/mol}} = 2.778\times10^{3} \text{ mol}$

$N = nN_{\text{A}} = (2.778\times10^{3} \text{ mol})(6.022\times10^{23} \text{ molecules/mol}) = 1.7\times10^{27} \text{ molecules}$

Each water molecule has three atoms, so the number of atoms is $3(1.7\times10^{27}) = 5.1\times10^{27}$ atoms.

EVALUATE: We could also use the masses in Example 18.5 to find the mass *m* of one H_2O molecule: $m = 2.99\times10^{-26}$ kg. Then $N = m_{\text{tot}}/m = 1.7\times10^{27}$ molecules, which checks.

18.64. **IDENTIFY:** Use the ideal gas law to find the number of moles of air taken in with each breath and from this calculate the number of oxygen molecules taken in. Then find the pressure at an elevation of 2000 m and repeat the calculation.

SET UP: The number of molecules in a mole is $N_{\text{A}} = 6.022\times10^{23}$ molecules/mol. $R = 0.08206 \text{ L}\cdot\text{atm/mol}\cdot\text{K}$. Example 18.4 shows that the pressure variation with altitude *y*, when constant temperature is assumed, is $p = p_0 e^{-Mgy/RT}$. For air, $M = 28.8\times10^{-3}$ kg/mol.

EXECUTE: **(a)** $pV = nRT$ gives $n = \dfrac{pV}{RT} = \dfrac{(1.00 \text{ atm})(0.50 \text{ L})}{(0.08206 \text{ L}\cdot\text{atm/mol}\cdot\text{K})(293.15 \text{ K})} = 0.0208 \text{ mol.}$

$N = (0.210)nN_{\text{A}} = (0.210)(0.0208 \text{ mol})(6.022\times10^{23} \text{ molecules/mol}) = 2.63\times10^{21} \text{ molecules.}$

(b) $\dfrac{Mgy}{RT} = \dfrac{(28.8\times10^{-3} \text{ kg/mol})(9.80 \text{ m/s}^2)(2000 \text{ m})}{(8.314 \text{ J/mol}\cdot\text{K})(293.15 \text{ K})} = 0.2316.$

$p = p_0 e^{-Mgy/RT} = (1.00 \text{ atm})e^{-0.2316} = 0.793 \text{ atm.}$

N is proportional to *n*, which is in turn proportional to *p*, so

$N = \left(\dfrac{0.793 \text{ atm}}{1.00 \text{ atm}}\right)(2.63\times10^{21} \text{ molecules}) = 2.09\times10^{21} \text{ molecules.}$

(c) Less O_2 is taken in with each breath at the higher altitude, so the person must take more breaths per minute.

EVALUATE: A given volume of gas contains fewer molecules when the pressure is lowered and the temperature is kept constant.

18.65. **IDENTIFY:** The mass of one molecule is the molar mass, *M*, divided by the number of molecules in a mole, N_{A}. The average translational kinetic energy of a single molecule is $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT$. Use $pV = NkT$ to calculate *N*, the number of molecules.

SET UP: $k = 1.381\times10^{-23} \text{ J/molecule}\cdot\text{K}$. $M = 28.0\times10^{-3}$ kg/mol. $T = 295.15$ K. The volume of the balloon is $V = \frac{4}{3}\pi(0.250 \text{ m})^3 = 0.0654 \text{ m}^3$. $p = 1.25 \text{ atm} = 1.27\times10^{5}$ Pa.

EXECUTE: **(a)** $m = \dfrac{M}{N_{\text{A}}} = \dfrac{28.0\times10^{-3} \text{ kg/mol}}{6.022\times10^{23} \text{ molecules/mol}} = 4.65\times10^{-26} \text{ kg.}$

(b) $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT = \frac{3}{2}(1.381\times10^{-23} \text{ J/molecule}\cdot\text{K})(295.15 \text{ K}) = 6.11\times10^{-21} \text{ J.}$

(c) $N = \dfrac{pV}{kT} = \dfrac{(1.27\times10^{5} \text{ Pa})(0.0654 \text{ m}^3)}{(1.381\times10^{-23} \text{ J/molecule}\cdot\text{K})(295.15 \text{ K})} = 2.04\times10^{24} \text{ molecules.}$

(d) The total average translational kinetic energy is
$N\left(\frac{1}{2}m(v^2)_{\text{av}}\right) = (2.04\times10^{24}\text{ molecules})(6.11\times10^{-21}\text{ J/molecule}) = 1.25\times10^4\text{ J}.$

EVALUATE: The number of moles is $n = \dfrac{N}{N_\text{A}} = \dfrac{2.04\times10^{24}\text{ molecules}}{6.022\times10^{23}\text{ molecules/mol}} = 3.39\text{ mol}.$

$K_\text{tr} = \frac{3}{2}nRT = \frac{3}{2}(3.39\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(295.15\text{ K}) = 1.25\times10^4\text{ J},$ which agrees with our results in part (d).

18.66. **IDENTIFY:** $pV = nRT = \dfrac{N}{N_\text{A}}RT.$ Deviations will be noticeable when the volume V of a molecule is on the order of 1% of the volume of gas that contains one molecule.

SET UP: The volume of a sphere of radius r is $V = \dfrac{4}{3}\pi r^3.$

EXECUTE: The volume of gas per molecule is $\dfrac{RT}{N_\text{A}p},$ and the volume of a molecule is about

$V_0 = \dfrac{4}{3}\pi(2.0\times10^{-10}\text{ m})^3 = 3.4\times10^{-29}\text{ m}^3.$ Denoting the ratio of these volumes as

$f,\ p = f\dfrac{RT}{N_\text{A}V_0} = f\dfrac{(8.3145\text{ J / mol}\cdot\text{K})(300\text{ K})}{(6.022\times10^{23}\text{ molecules / mol})(3.4\times10^{-29}\text{ m}^3)} = (1.2\times10^8\text{ Pa})f.$

"Noticeable deviations" is a subjective term, but f on the order of 1.0% gives a pressure of 10^6 Pa.

EVALUATE: The forces between molecules also cause deviations from ideal-gas behavior.

18.67. **IDENTIFY** and **SET UP:** At equilibrium $F(r) = 0.$ The work done to increase the separation from r_2 to ∞ is $U(\infty) - U(r_2).$

(a) EXECUTE: $U(r) = U_0[(R_0 / r)^{12} - 2(R_0 / r)^6]$

From Chapter 14: $F(r) = 12(U_0/R_0)[(R_0/r)^{13} - (R_0/r)^7].$ The graphs are given in Figure 18.67.

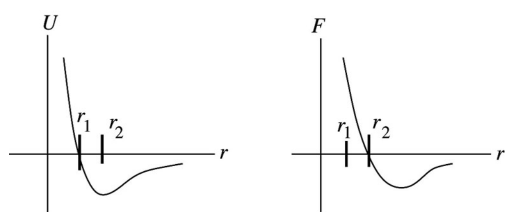


Figure 18.67

(b) Equilibrium requires $F = 0;$ occurs at point $r_2.$ r_2 is where U is a minimum (stable equilibrium).

(c) $U = 0$ implies $(R_0/r)^{12}2(R_0/r)^6 = 0$

$(r_1/R_0)^6 = 1/2$ and $r_1 = R_0/(2)^{1/6}.$

$F = 0$ implies $(R_0/r)^{13} - (R_0/r)^7 = 0$

$(r_2/R_0)^6 = 1$ and $r_2 = R_0.$

Then $r_1/r_2 = (R_0/2^{1/6})/R_0 = 2^{-1/6}.$

(d) $W_\text{other} = \Delta U.$

At $r \to \infty,$ $U = 0,$ so $W = -U(R_0) = -U_0[(R_0/R_0)^{12} - 2(R_0/R_0)^6] = +U_0.$

EVALUATE: The answer to part (d), $U_0,$ is the depth of the potential well shown in the graph of $U(r).$

18.68. **IDENTIFY:** Use $pV = nRT$ to calculate the number of moles, *n*. Then $K_{\text{tr}} = \frac{3}{2}nRT$. The mass of the gas, m_{tot}, is given by $m_{\text{tot}} = nM$.

SET UP: $5.00\ \text{L} = 5.00\times10^{-3}\ \text{m}^3$.

EXECUTE: **(a)** $n = \dfrac{pV}{RT} = \dfrac{(1.01\times10^5\ \text{Pa})(5.00\times10^{-3}\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})} = 0.2025\ \text{moles}$

$K_{\text{tr}} = \frac{3}{2}(0.2025\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K}) = 758\ \text{J}$.

(b) $m_{\text{tot}} = nM = (0.2025\ \text{mol})(2.016\times10^{-3}\ \text{kg/mol}) = 4.08\times10^{-4}\ \text{kg}$. The kinetic energy due to the speed of the jet is $K = \frac{1}{2}mv^2 = \frac{1}{2}(4.08\times10^{-4}\ \text{kg})(300.0\ \text{m/s})^2 = 18.4\ \text{J}$. The total kinetic energy is $K_{\text{tot}} = K + K_{\text{tr}} = 18.4\ \text{J} + 758\ \text{J} = 776\ \text{J}$. The percentage increase is

$\dfrac{K}{K_{\text{tot}}}\times100\% = \dfrac{18.4\ \text{J}}{776\ \text{J}}\times100\% = 2.37\%$.

(c) No. The temperature is associated with the random translational motion, and that hasn't changed.

EVALUATE: The equation $pV = \dfrac{2}{3}K_{\text{tr}}$ gives $K_{\text{tr}} = \frac{3}{2}pV = \frac{3}{2}(1.01\times10^5\ \text{Pa})(5.00\times10^{-3}\ \text{m}^3) = 758\ \text{J}$, which agrees with our result in part (a). $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}} = 1.93\times10^3\ \text{m/s}$. v_{rms} is a lot larger than the speed of the jet, so the percentage increase in the total kinetic energy, calculated in part (b), is small.

18.69. **IDENTIFY:** The equipartition principle says that each atom has an average kinetic energy of $\frac{1}{2}kT$ for each degree of freedom. There is an equal average potential energy.

SET UP: The atoms in a three-dimensional solid have three degrees of freedom and the atoms in a two-dimensional solid have two degrees of freedom.

EXECUTE: **(a)** In the same manner that $C_V = 3R$ was obtained, the heat capacity of the two-dimensional solid would be $2R = 16.6\ \text{J/mol}\cdot\text{K}$.

(b) The heat capacity would behave qualitatively like those in Figure 18.21 in the textbook, and the heat capacity would decrease with decreasing temperature.

EVALUATE: At very low temperatures the equipartition theorem doesn't apply. Most of the atoms remain in their lowest energy states because the next higher energy level is not accessible.

18.70. **IDENTIFY:** $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$.

SET UP: $M = 1.99\times10^{30}\ \text{kg}$, $R = 6.96\times10^8\ \text{m}$ and $G = 6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}} = \sqrt{\dfrac{3(1.38\times10^{-23}\ \text{J/K})(5800\ \text{K})}{(1.67\times10^{-27}\ \text{kg})}} = 1.20\times10^4\ \text{m/s}$.

(b) $v_{\text{escape}} = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(6.96\times10^8\ \text{m})}} = 6.18\times10^5\ \text{m/s}$.

EVALUATE: **(c)** The escape speed is about 50 times the rms speed, and either Figure 18.23 in the textbook, the Maxwell-Boltzmann distribution, or Table 18.2 will indicate that there is a negligibly small fraction of molecules with the escape speed.

18.71. **(a) IDENTIFY** and **SET UP:** Apply conservation of energy $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $U = -Gmm_{\text{p}}/r$. Let point 1 be at the surface of the planet, where the projectile is launched, and let point 2 be far from the earth. Just barely escapes says $v_2 = 0$.

EXECUTE: Only gravity does work says $W_{\text{other}} = 0$.

$U_1 = -Gmm_{\text{p}}/R_{\text{p}}$; $r_2 \to \infty$ so $U_2 = 0$; $v_2 = 0$ so $K_2 = 0$.

The conservation of energy equation becomes $K_1 - Gmm_\text{p}/R_\text{p} = 0$ and $K_1 = Gmm_\text{p}/R_\text{p}$.

But $g = Gm_\text{p}/R_\text{p}^2$ so $Gm_\text{p}/R_\text{p} = R_\text{p}g$ and $K_1 = mgR_\text{p}$, as was to be shown.

EVALUATE: The greater gR_p is, the more initial kinetic energy is required for escape.

(b) IDENTIFY and SET UP: Set K_1 from part (a) equal to the average kinetic energy of a molecule as given by the equation $\frac{1}{2}m(v^2)_\text{av} = \frac{3}{2}kT$. $\frac{1}{2}m(v^2)_\text{av} = mgR_\text{p}$ (from part (a)). But also, $\frac{1}{2}m(v^2)_\text{av} = \frac{3}{2}kT$, so $mgR_\text{p} = \frac{3}{2}kT$.

EXECUTE: $T = \dfrac{2mgR_\text{p}}{3k}$

Nitrogen:

$m_{\text{N}_2} = (28.0\times10^{-3}\text{ kg/mol})/(6.022\times10^{23}\text{ molecules/mol}) = 4.65\times10^{-26}\text{ kg/molecule}$

$$T = \frac{2mgR_\text{p}}{3k} = \frac{2(4.65\times10^{-26}\text{ kg/molecule})(9.80\text{ m/s}^2)(6.37\times10^6\text{ m})}{3(1.381\times10^{-23}\text{ J/molecule}\cdot\text{K})} = 1.40\times10^5\text{ K}$$

Hydrogen:

$m_{\text{H}_2} = (2.02\times10^{-3}\text{ kg/mol})/(6.022\times10^{23}\text{ molecules/mol}) = 3.354\times10^{-27}\text{ kg/molecule}$

$$T = \frac{2mgR_\text{p}}{3k} = \frac{2(3.354\times10^{-27}\text{ kg/molecule})(9.80\text{ m/s}^2)(6.37\times10^6\text{ m})}{3(1.381\times10^{-23}\text{ J/molecule}\cdot\text{K})} = 1.01\times10^4\text{ K}$$

(c) $T = \dfrac{2mgR_\text{p}}{3k}$

Nitrogen:

$$T = \frac{2(4.65\times10^{-26}\text{ kg/molecule})(1.63\text{ m/s}^2)(1.74\times10^6\text{ m})}{3(1.381\times10^{-23}\text{ J/molecule}\cdot\text{K})} = 6370\text{ K}$$

Hydrogen:

$$T = \frac{2(3.354\times10^{-27}\text{ kg/molecule})(1.63\text{ m/s}^2)(1.74\times10^6\text{ m})}{3(1.381\times10^{-23}\text{ J/molecule}\cdot\text{K})} = 459\text{ K}$$

(d) EVALUATE: The "escape temperatures" are much less for the moon than for the earth. For the moon a larger fraction of the molecules at a given temperature will have speeds in the Maxwell-Boltzmann distribution larger than the escape speed. After a long time most of the molecules will have escaped from the moon.

18.72. IDENTIFY: The ideal gas law applies. The rms speed depends on the Kelvin temperature of the gas.

SET UP: $pV = nRT$, $v_\text{rms} = \sqrt{3kT/m}$. For constant volume, the ideal gas law gives $p_2/T_2 = p_1/T_1$. All temperatures must be in kelvins.

EXECUTE: Solving for p_2 gives $p_2 = p_1(T_2/T_1)$. Solving $v_\text{rms} = \sqrt{3kT/m}$ for T gives $T = \dfrac{m}{3k}v_\text{rms}^2$. Applying this result to the pressure equation gives

$$p_2 = p_1\frac{(m/3k)v_{\text{rms},2}^2}{(m/3k)v_{\text{rms},1}^2} = (2.00\text{ atm})\left(\frac{276\text{ m/s}}{176\text{ m/s}}\right)^2 = 4.92\text{ atm}.$$ The change in pressure is

4.92 atm – 2.00 atm = 2.92 atm.

EVALUATE: An increase in speed of (100 m/s)/(176 m/s) = 57% produced an increase in pressure of (2.92 atm)/(2.00 atm) = 146%. The large increase in pressure for a much smaller increase in molecular speed is due to the fact that the temperature, and hence the pressure, depends on the *square* of the molecular speed.

18.73. **IDENTIFY** and **SET UP:** Evaluate the integral as specified in the problem.

EXECUTE: $\int_0^\infty v^2 f(v)\,dv = 4\pi(m/2\pi kT)^{3/2}\int_0^\infty v^4 e^{-mv^2/2kT}\,dv.$

The integral formula with $n=2$ gives $\int_0^\infty v^4 e^{-av^2}\,dv = (3/8a^2)\sqrt{\pi/a}.$

Apply with $a = m/2kT,$

$\int_0^\infty v^2 f(v)\,dv = 4\pi(m/2\pi kT)^{3/2}(3/8)(2kT/m)^2\sqrt{2\pi kT/m} = (3/2)(2kT/m) = 3kT/m.$

EVALUATE: $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT$ says $\frac{1}{2}m(v^2)_{\text{av}} = 3kT/2,$ so $(v^2)_{\text{av}} = 3kT/m,$ in agreement with our calculation.

18.74. **IDENTIFY:** The equipartition principle says that each molecule has average kinetic energy of $\frac{1}{2}kT$ for each degree of freedom. $I = 2m(L/2)^2,$ where L is the distance between the two atoms in the molecule. $K_{\text{rot}} = \frac{1}{2}I\omega^2.$ $\omega_{\text{rms}} = \sqrt{(\omega^2)_{\text{av}}}.$

SET UP: The mass of one atom is $m = M/N_{\text{A}} = (16.0\times10^{-3}\ \text{kg/mol})/(6.022\times10^{23}\ \text{molecules/mol}) =$ 2.66×10^{-26} kg.

EXECUTE: **(a)** The two degrees of freedom associated with the rotation for a diatomic molecule account for two-fifths of the total kinetic energy, so $K_{\text{rot}} = nRT = (1.00\ \text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K}) = 2.49\times10^3$ J.

(b) $I = 2m(L/2)^2 = 2\left(\dfrac{16.0\times10^{-3}\ \text{kg/mol}}{6.022\times10^{23}\,\text{molecules/mol}}\right)(6.05\times10^{-11}\ \text{m})^2 = 1.94\times10^{-46}\ \text{kg}\cdot\text{m}^2.$

(c) Since the result in part (b) is for one mole, the rotational kinetic energy for one atom is $K_{\text{rot}}/N_{\text{A}}$ and $\omega_{\text{rms}} = \sqrt{\dfrac{2K_{\text{rot}}/N_{\text{A}}}{I}} = \sqrt{\dfrac{2(2.49\times10^3\ \text{J})}{(1.94\times10^{-46}\ \text{kg}\cdot\text{m}^2)(6.022\times10^{23}\ \text{molecules/mol})}} = 6.52\times10^{12}\ \text{rad/s}.$ This is much larger than the typical value for a piece of rotating machinery.

EVALUATE: The average rotational period, $T = \dfrac{2\pi\ \text{rad}}{\omega_{\text{rms}}},$ for molecules is very short.

18.75. **IDENTIFY:** $f(v)dv$ is the probability that a particle has a speed between v and $v+dv.$ The equation for the Maxwell-Boltzmann distribution gives $f(v).$ v_{mp} is given by $v_{\text{mp}} = \sqrt{2kT/m}.$

SET UP: For O_2, the mass of one molecule is $m = M/N_{\text{A}} = 5.32\times10^{-26}$ kg.

EXECUTE: **(a)** $f(v)dv$ is the fraction of the particles that have speed in the range from v to $v+dv.$ The number of particles with speeds between v and $v+dv$ is therefore $dN = Nf(v)dv$ and

$\Delta N = N\int_v^{v+\Delta v} f(v)dv.$

(b) Setting $v = v_{\text{mp}} = \sqrt{\dfrac{2kT}{m}}$ in $f(v)$ gives $f(v_{\text{mp}}) = 4\pi\left(\dfrac{m}{2\pi kT}\right)^{3/2}\left(\dfrac{2kT}{m}\right)e^{-1} = \dfrac{4}{e\sqrt{\pi}v_{\text{mp}}}.$ For oxygen gas at 300 K, $v_{\text{mp}} = 3.95\times10^2$ m/s and $f(v)\Delta v = 0.0421.$

(c) Increasing v by a factor of 7 changes f by a factor of 7^2e^{-48}, and $f(v)\Delta v = 2.94\times10^{-21}.$

(d) Multiplying the temperature by a factor of 2 increases the most probable speed by a factor of $\sqrt{2},$ and the answers are decreased by $\sqrt{2}$: 0.0297 and $2.08\times10^{-21}.$

(e) Similarly, when the temperature is one-half what it was in parts (b) and (c), the fractions increase by $\sqrt{2}$ to 0.0595 and $4.15\times10^{-21}.$

EVALUATE: **(f)** At lower temperatures, the distribution is more sharply peaked about the maximum (the most probable speed), as is shown in Figure 18.23a in the textbook.

18.76. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: If $v^2 = x$, then $dx = 2v dv$.

EXECUTE: $\int_0^\infty v f(v) dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^3 e^{-mv^2/2kT} dv$. Making the suggested change of variable,

$v^2 = x$. $2v dv = dx$, $v^3 dv = (1/2) x\, dx$, and the integral

becomes $\int_0^\infty v f(v) dv = 2\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty x e^{-mx/2kT}\, dx = 2\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^2 = \frac{2}{\sqrt{\pi}} \sqrt{\frac{2kT}{m}} = \sqrt{\frac{8kT}{\pi m}}$

which is the equation $v_{av} = \sqrt{8kT/\pi m}$.

EVALUATE: The integral $\int_0^\infty v f(v) dv$ is the definition of v_{av}.

18.77. **IDENTIFY:** At equilibrium the net upward force of the gas on the piston equals the weight of the piston. When the piston moves upward the gas expands, the pressure of the gas drops and there is a net downward force on the piston. For simple harmonic motion the net force has the form $F_y = -ky$, for a displacement y from equilibrium, and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

SET UP: $pV = nRT$. T is constant.

EXECUTE: **(a)** The difference between the pressure, inside and outside the cylinder, multiplied by the area of the piston, must be the weight of the piston. The pressure in the trapped gas is $p_0 + \frac{mg}{A} = p_0 + \frac{mg}{\pi r^2}$.

(b) When the piston is a distance $h + y$ above the cylinder, the pressure in the trapped gas is $\left(p_0 + \frac{mg}{\pi r^2}\right)\left(\frac{h}{h+y}\right)$ and for values of y small compared to h, $\frac{h}{h+y} = \left(1 + \frac{y}{h}\right)^{-1} \sim 1 - \frac{y}{h}$. The net force, taking the positive direction to be upward, is then

$$F_y = \left[\left(p_0 + \frac{mg}{\pi r^2}\right)\left(1 - \frac{y}{h}\right) - p_0\right](\pi r^2) - mg = -\left(\frac{y}{h}\right)(p_0 \pi r^2 + mg).$$

This form shows that for positive h, the net force is down; the trapped gas is at a lower pressure than the equilibrium pressure, and so the net force tends to restore the piston to equilibrium.

(c) The angular frequency of small oscillations would be given by

$$\omega^2 = \frac{(p_0 \pi r^2 + mg)/h}{m} = \frac{g}{h}\left(1 + \frac{p_0 \pi r^2}{mg}\right).\quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{h}}\left(1 + \frac{p_0 \pi r^2}{mg}\right)^{1/2}.$$

If the displacements are not small, the motion is not simple harmonic. This can be seen be considering what happens if $y \sim -h$; the gas is compressed to a very small volume, and the force due to the pressure of the gas would become unboundedly large for a finite displacement, which is not characteristic of simple harmonic motion. If $y >> h$ (but not so large that the piston leaves the cylinder), the force due to the pressure of the gas becomes small, and the restoring force due to the atmosphere and the weight would tend toward a constant, and this is not characteristic of simple harmonic motion.

EVALUATE: The assumption of small oscillations was made when $\frac{h}{h+y}$ was replaced by $1 - y/h$; this is accurate only when y/h is small.

18.78. **IDENTIFY** and **SET UP:** For an ideal gas, its pressure approaches zero as its temperature approaches absolute zero. For volume expansion, $\Delta V = V_0 \beta \Delta T$.

EXECUTE: **(a)** Graph the pressure p versus the temperature T. This graph is shown in Figure 18.78.

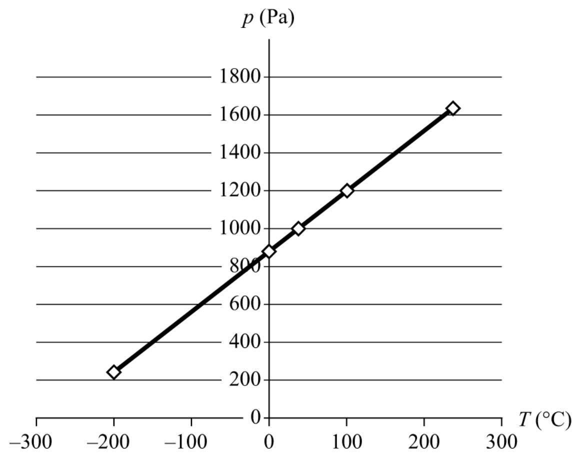


Figure 18.78

The best-fit equation for the graph is $p = (3.2289\ \text{Pa/C°})T + 888.81\ \text{Pa}$. The temperature when $p = 0$ is given by $T = -\dfrac{888.81\ \text{Pa}}{3.2289\ \text{Pa/C°}} = -275°\text{C}$, so this temperature is our determined value for absolute zero.

(b) Solve $\Delta V = V_0 \beta \Delta T$ for the fractional change in volume:

$$\frac{\Delta V}{V} = \beta \Delta T = (3.6\times10^{-5}\ \text{K}^{-1})(232°\text{C} + 195.8°\text{C}) = 1.5\times10^{-2} = 1.5\%.$$

EVALUATE: If the temperature range in an experiment is much less than the extremes in the table, it may be acceptable to ignore the change in volume of the cylinder. But for the range shown in the table, ignoring the volume change of the cylinder would cause a small but significant error.

18.79. IDENTIFY: The measurement gives the dew point. Relative humidity is defined in Problem 18.46, and the vapor pressure table is given with the problem in the text.

SET UP: relative humidity $= \dfrac{\text{partial pressure of water vapor at temperature } T}{\text{vapor pressure of water at temperature } T}$. At $28.0°\text{C}$ the vapor pressure of water is 3.78×10^3 Pa.

EXECUTE: **(a)** The experiment shows that the dew point is $16.0°\text{C}$, so the partial pressure of water vapor at $30.0°\text{C}$ is equal to the vapor pressure at $16.0°\text{C}$, which is 1.81×10^3 Pa.

Thus the relative humidity $= \dfrac{1.81\times10^3\ \text{Pa}}{4.25\times10^3\ \text{Pa}} = 0.426 = 42.6\%$.

(b) For a relative humidity of 35%, the partial pressure of water vapor is $(0.35)(3.78\times10^3\ \text{Pa}) = 1.323\times10^3\ \text{Pa}$. This is close to the vapor pressure at $12°\text{C}$, which would be at an altitude $(30°\text{C} - 12°\text{C})/(0.6\ \text{C°}/100\ \text{m}) = 3\ \text{km}$ above the ground.

(c) For a relative humidity of 80%, the vapor pressure will be the same as the water pressure at around $24°\text{C}$, corresponding to an altitude of about 1 km.

EVALUATE: The lower the dew point is compared to the air temperature, the smaller the relative humidity. Clouds form at a lower height when the relative humidity at the surface is larger.

18.80. IDENTIFY and **SET UP:** With the multiplicity of each score denoted by n_i, the average score is $\left(\dfrac{1}{150}\right)\sum n_i x_i$ and the rms score is $\left[\left(\dfrac{1}{150}\right)\sum n_i x_i^2\right]^{1/2}$. Read the numbers from the bar graph in the problem.

EXECUTE: **(a)** For example, $n_1x_1 = (11)(10)$, $n_2x_2 = (12)(20)$, $n_3x_3 = (24)(30)$, etc. The result is 54.6.

(b) Using the same data, the result is 61.1.

EVALUATE: (c) The rms score is higher than the average score since the rms calculation gives more weight to the higher scores.

18.81. IDENTIFY: We are looking at collisions between nitrogen molecules N_2. Each molecule contains two nitrogen atoms, each of mass $m = 2.3\times10^{-26}$ kg and length $2r$ = 188 pm = 188×10^{-12} m.

SET UP: For a point mass, $I = mr^2$. We also know that $K_{tot} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$, $L = I\omega$, and for a point mass $L = mvr$. Angular momentum is conserved during the collision. Since it is an elastic collision, kinetic energy is also conserved.

EXECUTE: **(a)** We want the moment of inertia of a N_2 molecule. Calling m the mass of a single nitrogen atom, we have $I = 2(mr^2) = 2(2.3\times10^{-26}\text{ kg})(94.0\times10^{-12}\text{ m})^2 = 4.1\times10^{-46}\text{ kg}\cdot\text{m}^2$.

(b) and (c) We want to look at conservation of energy and angular momentum during the collision. The mass of each molecule is $2m$.

<u>Energy conservation</u>: $\frac{1}{2}(2m)v_1^2 + \frac{1}{2}(2m)v_1^2 = \frac{1}{2}(2m)v_2^2 + \frac{1}{2}(2m)v_2^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}I\omega^2$. Using $I = 2mr^2$, this equation becomes $v_1^2 = v_2^2 + r^2\omega^2$, giving $v_f = \sqrt{v_i^2 - r^2\omega^2}$.

<u>Angular momentum conservation</u>: Using $L = mvr$ for a translating point mass gives $mv_1(2r) + mv_1(2r) = 2I\omega$. Using $I = 2mr^2$ and simplifying gives $4mv_1r = 4mr^2\omega$, so $\omega = v_i / r$. Combining this result with $v_f = \sqrt{v_i^2 - r^2\omega^2}$ gives $v_f = \sqrt{v_i^2 - v_i^2} = 0$. The final results are: $\omega = v_i / r$ and $v_f = 0$.

(d) The target variable is ω when $v_i = v_{rms}$. Use $v_{rms} = \sqrt{\frac{3kT}{m}}$ with T = 293 K and m = $2(2.3)\times10^{-26}\text{ kg} = 4.6\times10^{-26}\text{ kg}$. This gives v_{rms} = 514 m/s. From part (c) we have $\omega = v_i / r$ so

$$\omega = \frac{514\text{ m/s}}{94\times10^{-12}\text{ m}} = 5.5\times10^{12}\text{ rad/s}.$$

EVALUATE: The molecules stop translational motion but continue to rotate, so all the initial translational kinetic energy is transformed into rotational kinetic energy. We also should note that linear momentum must be conserved during the collision. The initial momentum is zero since the molecules have equal but opposite velocities. The final momentum is zero because they both stop, so momentum is conserved.

18.82. IDENTIFY: The gas pressure in a pneumatic lift is used to support heavy loads.

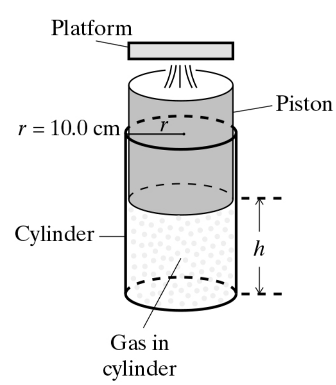


Figure 18.82

SET UP: Fig. 18.82 shows this pneumatic life. The mass of the piston and platform together is 50.0 kg, the gas temperature is a constant 20.0°C = 293 K, and the outside pressure is 1.00 atm. We use $pV = nRT$ and $p = F/A$.

EXECUTE: **(a)** Our target variable is h in the figure. There is 1.00 mol of gas in the cylinder and no load on the platform. The cylinder is airtight, so the only pressure inside is due to the gas that is there. The force due to the pressure on the bottom of the piston supports the weight of the piston and the platform and it also balances the force on the top of the piston due to atmospheric pressure (remember the cylinder is airtight). Using $F = pA$, we have $F = pA = mg + p_{\text{atm}}A$, which becomes $p = \dfrac{mg}{A} + p_{\text{atm}}$.

Using $pV = nRT$, where $V = Ah$ is the volume of gas in the cylinder, we get $\left(\dfrac{mg}{A} + p_{\text{atm}}\right)Ah = nRT$.

Solving for h gives $h = \dfrac{nRT}{mg + p_{\text{atm}}A}$. Using $A = \pi R^2$ with $R = 0.100$ m, $p_{\text{atm}} = 1.01\times10^5$ Pa, $n = 1.00$ mol, $m = 50.0$ kg, and $T = 293$ K, we get $h = 0.665$ m.

(b) The *change* in pressure supports the *added* weight, and this change is due to the change in volume. Since $p = \dfrac{F}{A}$, it follows that $\Delta p = \dfrac{\Delta F}{A}$, where ΔF is the added weight of $(200\text{ kg})(9.80\text{ m/s}^2) = 1960$ N. Using $pV = nRT$ we have

$\Delta p = p_2 - p_1 = nRT\left(\dfrac{1}{V_2} - \dfrac{1}{V_1}\right) = nRT\left(\dfrac{1}{Ah_2} - \dfrac{1}{Ah_1}\right)$. Solving for $1/h_2$ gives

$\dfrac{1}{h_2} = \dfrac{A\Delta p}{nRT} + \dfrac{1}{h_1} = \dfrac{A\left(\dfrac{\Delta F}{A}\right)}{nRT} + \dfrac{1}{h_1} = \dfrac{\Delta F}{nRT} + \dfrac{1}{h_1}$. Putting in $h_1 = 0.665$ m, $\Delta F = 1960$ N, $T = 293$ K, and $n = 1.00$ mol, we have $h_2 = 0.4332$ m. The distance platform drops is $h_1 - h_2 = 0.665\text{ m} - 0.433\text{ m} = 0.232$ m.

(c) The piston and load rise back to the original position, so they rise by a distance $\Delta h = 0.232$ m from part (b). We want to find the amount of gas Δn that enters the cylinder. The pressure must remain the same to support the load, so $p = \dfrac{(m_{\text{piston}} + m_{\text{load}})g}{A} + p_{\text{atm}}$. Using $pV = nRT$ with constant p and T, we have $p\Delta V = RT\Delta n$. Combining this result with $\Delta V = A\Delta h$ and our equation for p, we get

$\left[\dfrac{(m_{\text{piston}} + m_{\text{load}})g}{A} + p_{\text{atm}}\right]A\Delta h = RT\Delta n$. Solving for Δn using $m_{\text{piston}} + m_{\text{load}} = 250$ kg, $\Delta h = 0.232$ m, $T = 293$ K, and $A = \pi R^2$ with $R = 0.100$ m, we get $\Delta n = 0.536$ mol.

(d) We want to find Δn to raise the platform an additional 2.00 m. Everything is the same as in part (c) except that $\Delta h = 2.00$ m instead of 0.232 m. To save a lot of arithmetic, take the ratio of Δn for the two cases and divide out all the common factors. This leaves $\dfrac{\Delta n_2}{\Delta n_1} = \dfrac{\Delta h_2}{\Delta h_1}$, so

$$\Delta n_2 = \Delta n_1 \frac{\Delta h_2}{\Delta h_1} = (0.536\text{ mol})\left(\frac{2.00\text{ m}}{0.232\text{ m}}\right) = 4.62\text{ mol}.$$

(e) We want the rate $\Delta n/\Delta t$ at which air should be introduced so that the platform rises at a steady 10.0 cm/s = 0.100 m/s. From part (c), we have $\left[\dfrac{(m_{\text{piston}} + m_{\text{load}})g}{A} + p_{\text{atm}}\right]A\Delta h = RT\Delta n$. Solving for Δn gives $\Delta n = \left[\dfrac{(m_{\text{piston}} + m_{\text{load}})g}{A} + p_{\text{atm}}\right]\dfrac{A\Delta h}{RT}$. Divide by Δt and realize that $\Delta h/\Delta t = v = 0.100$ m/s, giving $\Delta n = \left[\frac{(m_{\text{piston}} + m_{\text{load}})g}{A} + p_{\text{atm}}\right]\frac{Av}{RT}$. Putting in $v = 0.100$ m/s along with the same numbers as before, we get $\Delta n / \Delta t = 0.231$ mol/s = 231 millimol/s.

EVALUATE: In most cases we've dealt with, the amount of gas has remained constant. But in this case the pressure was increased in parts (c), (d) and (e) by adding gas to the cylinder. This is how you inflate your car tires.

18.83. **IDENTIFY:** The equation $\lambda = \frac{V}{4\pi\sqrt{2}r^2N}$ gives the mean free path λ. In the equation $t_{\text{mean}} = \frac{V}{4\pi\sqrt{2}r^2vN}$, use $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ in place of v. $pV = nRT = NkT$. The escape speed is $v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$.

SET UP: For atomic hydrogen, $M = 1.008\times10^{-3}$ kg/mol.

EXECUTE: **(a)** From $\lambda = \frac{V}{4\pi\sqrt{2}r^2N}$, we have

$\lambda = [4\pi\sqrt{2}r^2(N/V)]^{-1} = [4\pi\sqrt{2}(5.0\times10^{-11}\text{ m})^2(50\times10^6\text{ m}^{-3})]^{-1} = 4.5\times10^{11}\text{ m}.$

(b) $v_{\text{rms}} = \sqrt{3RT/M} = \sqrt{3(8.3145\text{ J/mol}\cdot\text{K})(20\text{ K})/(1.008\times10^{-3}\text{ kg/mol})} = 703\text{ m/s}$, and the time between collisions is then $(4.5\times10^{11}\text{ m})/(703\text{ m/s}) = 6.4\times10^8\text{ s}$, about 20 yr. Collisions are not very important.

(c) $p = (N/V)kT = (50/1.0\times10^{-6}\text{ m}^3)(1.381\times10^{-23}\text{ J/K})(20\text{ K}) = 1.4\times10^{-14}\text{ Pa}.$

(d) $v_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G(Nm/V)(4\pi R^3/3)}{R}} = \sqrt{(8\pi/3)G(N/V)mR^2}$

$v_{\text{escape}} = \sqrt{(8\pi/3)(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(50\times10^6\text{ m}^{-3})(1.67\times10^{-27}\text{ kg})(10\times9.46\times10^{15}\text{ m})^2}$

$v_{\text{escape}} = 650\text{ m/s}$. This is lower than v_{rms} and the cloud would tend to evaporate.

(e) In equilibrium (clearly not *thermal* equilibrium), the pressures will be the same; from $pV = NkT$, $kT_{\text{ISM}}(N/V)_{\text{ISM}} = kT_{\text{nebula}}(N/V)_{\text{nebula}}$ and the result follows.

(f) With the result of part (e),

$$T_{\text{ISM}} = T_{\text{nebula}}\left(\frac{(N/V)_{\text{nebula}}}{(N/V)_{\text{ISM}}}\right) = (20\text{ K})\left(\frac{50\times10^6\text{ m}^3}{(200\times10^{-6}\text{ m}^3)^{-1}}\right) = 2\times10^5\text{ K},$$

more than three times the temperature of the sun. This indicates a high average kinetic energy, but the thinness of the ISM means that a ship would not burn up.

EVALUATE: The temperature of a gas is determined by the average kinetic energy per atom of the gas. The energy density for the gas also depends on the number of atoms per unit volume, and this is very small for the ISM.

18.84. **IDENTIFY:** Follow the procedure of Example 18.4, but use $T = T_0 - \alpha y$.

SET UP: $\ln(1+x) \approx x$ when x is very small.

EXECUTE: **(a)** $\frac{dp}{dy} = -\frac{pM}{RT}$, which in this case becomes $\frac{dp}{p} = -\frac{Mg}{R}\frac{dy}{T_0 - \alpha y}$. This integrates to

$$\ln\left(\frac{p}{p_0}\right) = \frac{Mg}{R\alpha}\ln\left(1 - \frac{\alpha y}{T_0}\right),\ \text{or}\ p = p_0\left(1 - \frac{\alpha y}{T_0}\right)^{Mg/R\alpha}.$$

(b) For sufficiently small α, $\ln\left(1 - \frac{\alpha y}{T_0}\right) \approx -\frac{\alpha y}{T_0}$, and this gives the expression derived in Example 18.4.

(c) $\left(1-\dfrac{(0.6\times10^{-2}\ \text{C}°/\text{m})(8863\ \text{m})}{(288\ \text{K})}\right)=0.8154,\ \dfrac{Mg}{R\alpha}=\dfrac{(28.8\times10^{-3})(9.80\ \text{m/s}^2)}{(8.3145\ \text{J/mol}\cdot\text{K})(0.6\times10^{-2}\ \text{C}°/\text{m})}=5.6576$ and

$p_0(0.8154)^{5.6576}=0.315\ \text{atm}$, which is 0.95 of the result found in Example 18.4.

EVALUATE: The pressure is calculated to decrease more rapidly with altitude when we assume that T also decreases with altitude.

18.85. **IDENTIFY** and **SET UP:** Noble gases are monatomic, but nitrogen and oxygen are diatomic, and k is proportional to C_V.

EXECUTE: For a monatomic gas, $C_V = 3/2\ R$, but for a diatomic gas $C_V = 5/2\ R$. A small C_V will give a small k, which means that that a monatomic gas has a smaller thermal conductivity than a diatomic gas. This is true because rotational modes are not present for a monatomic gas, but they are present for a diatomic gas. This makes choice (a) the correct one.

EVALUATE: An added advantage of noble gases is that they are less reactive than other gases and therefore would not react with the window materials.

18.86. **IDENTIFY** and **SET UP:** $k \propto \dfrac{C_V}{r^2\sqrt{M}}$.

EXECUTE: Take the ratio of the thermal conductivities for xenon and helium.

$\dfrac{k_{\text{Xe}}}{k_{\text{He}}}=\sqrt{\dfrac{M_{\text{He}}}{M_{\text{Xe}}}}\left(\dfrac{r_{\text{He}}}{r_{\text{Xe}}}\right)^2=\sqrt{\dfrac{4.0\ \text{g/mol}}{131\ \text{g/mol}}}\left(\dfrac{0.13\ \text{nm}}{0.22\ \text{nm}}\right)^2=0.061$, which is choice (b).

EVALUATE: Since both He and Xe are monatomic, they have the same C_V so this cancels out in the ratio.

18.87. **IDENTIFY** and **SET UP:** The rate of effusion is proportional to v_{rms} and $v_{\text{rms}}=\sqrt{3kT/m}$, so the rate $R = Cv_{\text{rms}} = C\sqrt{3kT/m}$, where C is a constant.

EXECUTE: Take the ratio of the rates for helium and xenon:

$\dfrac{R_{\text{He}}}{R_{\text{Xe}}}=\dfrac{C\sqrt{3kT/M_{\text{He}}}}{C\sqrt{3kT/M_{\text{Xe}}}}=\sqrt{\dfrac{M_{\text{Xe}}}{M_{\text{He}}}}=\sqrt{\dfrac{131}{4.0}}=5.7\approx 6$, which makes choice (c) correct.

EVALUATE: At a given temperature, helium atoms will be moving faster than xenon atoms, so they will more easily move through any small openings (leaks) in the window seal.

19

THE FIRST LAW OF THERMODYNAMICS

VP19.5.1. **IDENTIFY:** We investigate several thermodynamic processes and apply the first law of thermodynamics.

SET UP: The first law of thermodynamics is $\Delta U = Q - W$.

EXECUTE: **(a)** $\Delta U = Q - W$: 2.50×10^3 J $= Q -$ 3.25×10^3 J . $Q =$ 5.75×10^3 J .

(b) $\Delta U = Q - W$: $\Delta U = -7.00\times10^3$ J $- 2.50\times10^4$ J $= -3.20\times10^4$ J.

(c) $\Delta U = Q - W$: 4.25×10^3 J $= 2.40\times10^3$ J $- W$. $W = -1.85\times10^3$ J.

EVALUATE: Careful! Q is the heat put *into* the gas and W is the work done *by* the gas. If heat comes *out* of the gas, Q will be negative. If work is done *on* the gas, W will be negative.

VP19.5.2. **IDENTIFY:** We want to calculate the work that a gas does in various thermodynamic processes.

SET UP: $W = p\Delta V$ if pressure is constant. In general $W = \int p dV$. The work done by a gas is equal to the area under the curve on a pV-diagram.

EXECUTE: **(a)** The pressure is constant, so we use $W = p\Delta V = p(V_2 - V_1) =$

$(6.20\times10^4\ \text{Pa})(4.50\times10^{-3}\ \text{m}^3 - 2.00\times10^{-3}\ \text{m}^3) = 155$ J.

(b) $\Delta V = 0$ so $W = 0$.

(c) Pressure is constant so $W = p\Delta V$ $= (1.25\times10^5\ \text{Pa})(-3.00\times10^{-3}\ \text{m}^3) = -375$ J.

(d) First sketch a pV-diagram of the process, as shown in Fig. VP19.5.2. The work done by the gas is the area under the curve. This area is made up of a triangle above a rectangle. The work is $W = A_{\text{triangle}} + A_{\text{rectangle}}$ which gives

$$W = \frac{1}{2}(2.50\times10^{-3}\ \text{m}^3)(4.00\times10^5\ \text{Pa}) + (2.50\times10^{-3}\ \text{m}^3)(1.50\times10^5\ \text{Pa}) = 875\ \text{J}.$$

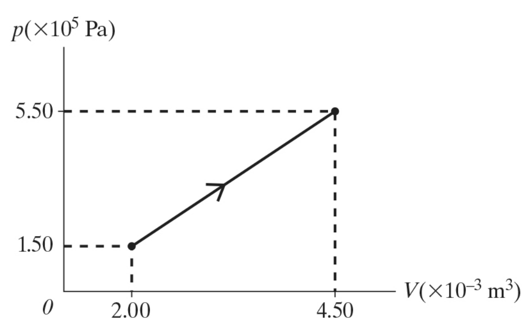


Figure VP19.5.2

EVALUATE: For more complicated curves, the area under the curve can be replaced by the definite integral $W = \int pdV$.

VP19.5.3. IDENTIFY: A gas undergoes various thermodynamic changes. We want to determine the work it does and the heat energy put into it. The ideal gas law and the first law of thermodynamics apply.

SET UP: We apply $\Delta U = Q - W$, $W = p\Delta V$ for constant pressure, and $pV = nRT$. We know that $p_1 = 2.00\times10^5$ Pa and $V_1 = 3.60\times10^{-3}$ m^3.

EXECUTE: **(a)** We want the work the gas does and its internal energy change. First find the work W done by the gas during the two processes 1 and 2. $W = W_1 + W_2$. This gives

$W = p_1\Delta V_1 + p\Delta V_2 = p_1\Delta V_1 + 0 = (2.00\times10^5 \text{ Pa})(-1.20\times10^{-3} \text{ m}^3) = -2.40\times10^2$ J.

Now use $\Delta U = Q - W$ = 1560 J – (–240 J) = 1.80×10^3 J.

(b) We want to find the work the gas does and how much heat flows into the gas. As before,

$W = W_1 + W_2 = p_1\Delta V_1 + p_2\Delta V_2 = 0 + p_2\Delta V_2 = (6.00\times10^5 \text{ Pa})(-1.20\times10^{-3} \text{ m}^3) = -7.20\times10^2$ J.

Now apply $\Delta U = Q - W$ but we first need to find ΔU.

For the process in (a): $p_1V_1 = (2.00\times10^5 \text{ Pa})(3.60\times10^{-3} \text{ m}^3)$ = 720 J

$p_2V_2 = (6.00\times10^5 \text{ Pa})(2.40\times10^{-3} \text{ m}^3)$ = 1440 J

For the process in (b): $p_1V_1 = (2.00\times10^5 \text{ Pa})(3.60\times10^{-3} \text{ m}^3)$ = 720 J

$p_2V_2 = (6.00\times10^5 \text{ Pa})(2.40\times10^{-3} \text{ m}^3)$ = 1440 J

In both cases, the *change* in pV is $\Delta(pV)$ = 720 J. From $pV = nRT$ we see that $nR\Delta T = \Delta(pV)$. Therefore for these two processes, ΔT is the same because $\Delta(pV)$ is the same, and if ΔT is the same, ΔU must also be the same. So $\Delta U = 1.80\times10^3$ J as we found in part (a). Now we use $\Delta U = Q - W$: 1.80×10^3 J $= Q - (-7.20\times10^2$ J), which gives $Q = 1.08\times10^3$ J.

EVALUATE: Note that ΔU depends only on ΔT, so if ΔT is the same ΔU must be the same, no matter what the process. ΔU is independent of path.

VP19.5.4. IDENTIFY: Heat flows into the solid CO_2 causing the gas to do work and change its internal energy. The first law of thermodynamics applies.

SET UP: $\Delta U = Q - W$, for sublimation $Q = mL_{sub}$, where $L_{sub} = 5.71\times10^5$ J/kg. At constant pressure $W = p\Delta V$.

EXECUTE: **(a)** $Q = mL_{sub} = (6.0 \text{ kg})(5.71\times10^5 \text{ J/kg}) = 3.4\times10^6$ J.

(b) We want the work. The pressure is constant so the work done is

$W = p\Delta V = (1.01\times10^5 \text{ Pa})(3.4 \text{ m}^3 - 0.0040 \text{ m}^3) = 3.4\times10^5$ J.

(c) We want the change in internal energy. $\Delta U = Q - W = 3.4\times10^6 \text{ J} - 3.4\times10^5 \text{ J} = 3.1\times10^6$ J.

EVALUATE: The internal energy change is due to an increase in the electrical potential energy of CO_2 molecules.

VP19.6.1. IDENTIFY: This problem involves molar heat capacity and internal energy for the monatomic gas neon.

SET UP: $Q = nC_V\Delta T$, $C_V = \frac{3}{2}R$ for a monatomic gas, $\Delta U = Q - W$. The target variable is the change in internal energy ΔU.

EXECUTE: **(a)** At constant volume, $W = 0$, so $\Delta U = Q = nC_V\Delta T = n\frac{3}{2}R\Delta T$. Using n = 4.00 mol and ΔT = 20.0 K gives ΔU = 998 J.

(b) Since ΔU is independent of path, it is the same for constant volume and constant pressure processes (or any other process) if ΔT is the same. Therefore $\Delta U = Q_V = nC_V\Delta T = n\frac{3}{2}R\Delta T$. Using n = 4.00 mol and ΔT = 15.0 K gives ΔU = 748 J.

(c) Use the same reasoning as in part (b) since ΔU is independent of path. We have $\Delta U = n\frac{3}{2}R\Delta T$, and using n = 4.00 mol and ΔT = 12.0 K gives ΔU = 599 J.

EVALUATE: For *any* ideal gas processes, if ΔT is the same, ΔU is the same.

VP19.6.2. **IDENTIFY:** We are investigating the behavior of ideal N_2 gas. The ideal gas law and the first law of thermodynamics both apply.

SET UP: Use $pV = nRT$, $\Delta U = Q - W$, $Q = nC_V\Delta T$, and $C_V = \frac{3}{2}R$.

EXECUTE: **(a)** We want the initial volume of the gas. Solving $pV = nRT$ for V gives $V = nRT/p$. Using n = 2.10 mol, T = 300 K, and $p = 1.00\times10^5$ Pa gives $V = 5.24\times10^{-2}\ \text{m}^3$.

(b) (i) We want T_2 and ΔU when the pressure doubles at constant volume. For constant volume, $pV = nRT$ tells us that if p doubles, T also doubles, so T_2 = 600 K = 327°C.

For constant volume, W = 0, so $\Delta U = Q = nC_V\Delta T = n\frac{5}{2}R\Delta T$. Using n = 2.10 mol and ΔT = 300 K gives $\Delta U = 1.31\times10^4$ J.

(ii) We want T_2 and ΔU when the volume doubles at constant pressure. For constant pressure, $pV = nRT$ tells us that if V doubles, T also doubles, so T_2 = 600 K = 327°C. By the4 same reasoning as in part (ii), $\Delta U = 1.31\times10^4$ J.

(iii) We want T_2 and ΔU when both pressure and volume double. From $pV = nRT$ we see that if *both* p and V double, the product pV increases by a factor of 4. Therefore T also increases by a factor of 4, so $T_2 = 4T_1$ = 4(300 K) = 1200 K = 927°C.

Now ΔT = 1200 K – 300 K = 900 K, which is 3 times the change in parts (i) and (ii), so ΔU is 3 times as large as before. Therefore $\Delta U = 3(1.31\times10^4\ \text{J}) = 3.93\times10^4$ J.

EVALUATE: Note that if we quadruple the Kelvin temperature we do *not* quadruple the Celsius temperature.

VP19.6.3. **IDENTIFY:** Ideal monatomic argon gas goes through changes in its temperature and pressure. We want to find the resulting change in its internal energy. The ideal gas law and first law of thermodynamics both apply.

SET UP: We use $pV = nRT$, $\Delta U = Q - W$, $\Delta U = nC_V\Delta T$ for *any* process, and $C_V = \frac{3}{2}R$ for a monatomic gas. $pV = nRT$ tells us that $\Delta(pV) = nR\Delta T$, so $\Delta T = \frac{\Delta(pV)}{nR}$. This gives $\Delta U = nC_V\Delta T = n\left(\frac{3}{2}R\right)\frac{\Delta(pV)}{nR} = \frac{3}{2}\Delta(pV) = \frac{3}{2}(p_2V_2 - p_1V_1)$. We want to find ΔU for each process. We know that $p_1 = 1.20\times10^5$ Pa and $V_1 = 0.250\ \text{m}^3$ for each process.

EXECUTE: **(a)** $V_1 = V_2 = 0.250\ \text{m}^3$ and $p_2 = 2.4\times10^5$ Pa. Use $\Delta U = \frac{3}{2}(p_2V_2 - p_1V_1)$ which becomes

$$\Delta U = \frac{3}{2}V_1(p_2 - p_1) = \frac{3}{2}(0.250\ \text{m}^3)(1.20\times10^5\ \text{Pa}) = 4.50\times10^4\ \text{J}.$$

(b) $p_1 = p_2 = 1.20\times10^5$ Pa and $V_2 = 0.125$ m^3. Use $\Delta U = \frac{3}{2}(p_2V_2 - p_1V_1)$ which becomes ΔU = $\frac{3}{2}p_1(V_2 - V_1) = \frac{3}{2}(1.20\times10^5 \text{ Pa})(0.125 \text{ m}^3 - 0.250 \text{ m}^3) = -2.25\times10^4$ J.

(c) $p_2 = 1.80\times10^5$ Pa and $V_2 = 0.600$ m^3. Use $\Delta U = \frac{3}{2}(p_2V_2 - p_1V_1)$ which gives $\Delta U = 1.17\times10^5$ J.

EVALUATE: Even though only one of the processes was at constant volume, we could use $\Delta U = nC_V\Delta T$ to calculate ΔU because ΔU is the same *as if* the process were at constant volume. Remember that ΔU is independent of path.

VP19.6.4. **IDENTIFY:** We increase the temperature of a gas from T to $2T$ by isochoric, isobaric, and adiabatic processes. In each case we want to find the change in the internal energy of the gas, the heat flow into it, and work done by the gas. The first law of thermodynamics applies.

SET UP: $\Delta U = Q - W$, $\Delta U = nC_V\Delta T$, at constant pressure $Q = nC_p\Delta T$, and for a diatomic gas $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. The target variable is ΔU for each process.

EXECUTE: **(a)** The *volume* is constant for an isochoric process, so $W = 0$. Therefore $\Delta U = Q - W$ gives $\Delta U = Q = nC_V\Delta T$. Since the temperature increases from T to $2T$, $\Delta T = 2T - T = T$. Thus ΔU = $Q = nC_V\Delta T = n\left(\frac{5}{2}R\right)T = \frac{5}{2}nRT.$

(b) The *pressure* is constant for an isobaric process. $Q = nC_p\Delta T = n\left(\frac{7}{2}R\right)T = \frac{7}{2}nRT$. $\Delta U = nC_V\Delta T$ = $\frac{5}{2}nRT$, the same as in part (a). From $\Delta U = Q - W$ we have $W = Q - \Delta U = \frac{7}{2}nRT - \frac{5}{2}nRT = nRT$.

(c) For an adiabatic process, no heat enters or leaves the gas, so $Q = 0$. From part (a) we know that $\Delta U = nC_V\Delta T = \frac{5}{2}nRT$. $\Delta U = Q - W$ gives $W = Q - \Delta U = 0 - \Delta U = -\frac{5}{2}nRT$.

EVALUATE: As we have seen here, doubling the gas temperature can have different results depending on how the process is carried out.

VP19.7.1. **IDENTIFY:** We are investigating monatomic argon gas during an adiabatic expansion.

SET UP: pV^γ is constant during an adiabatic process, $\gamma = C_p / C_V$, $W = \frac{C_V}{R}(p_1V_1 - p_2V_2)$.

EXECUTE: **(a)** $\gamma = \frac{C_p}{C_V} = \frac{5/2R}{3/2R} = 5/3.$

(b) We want the final pressure p_2. Since pV^γ is constant, we have $p_1V_1^\gamma = p_2V_2^\gamma$ which gives $p_2 = p_1\left(\frac{V_1}{V_2}\right)^\gamma$. Using $p_1 = 4.00\times10^5$ Pa, $V_1 = 2.00\times10^{-3}$ m^3, $V_2 = 6.00\times10^{-3}$ m^3, and $\gamma = 5/3$, we have $p_2 = 6.41\times10^4$ Pa.

(c) We want the work W. $W = \frac{C_V}{R}(p_1V_1 - p_2V_2) = \frac{3/2R}{R}(p_1V_1 - p_2V_2) = \frac{3}{2}(p_1V_1 - p_2V_2)$. Using the pressures and volumes above, we get $W = 623$ J.

EVALUATE: Careful! An adiabatic process is *not* the same as an isothermal process. The temperature can change during an adiabatic process but not during an isothermal process.

VP19.7.2. **IDENTIFY:** We are investigating diatomic oxygen O_2 gas during an adiabatic compression.

SET UP: $TV^{\gamma-1}$ and pV^{γ} are constant during an adiabatic process, $\gamma = C_p / C_V$,

$W = \dfrac{1}{\gamma-1}(p_1V_1 - p_2V_2)$.

EXECUTE: **(a)** $\gamma = \dfrac{C_p}{C_V} = \dfrac{7/2R}{5/2R} = 7/5$.

(b) We want the final volume V_2. Since $TV^{\gamma-1}$ is constant, we have $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ which gives $V_2^{\gamma-1} = V_1^{\gamma-1}\dfrac{T_1}{T_2}$. Taking the $\gamma-1$ root gives $V_2 = V_1\left(\dfrac{T_1}{T_2}\right)^{1/(\gamma-1)} = V_1\left(\dfrac{T_1}{T_2}\right)^{5/2}$. Using $p_1 = 1.00\times10^5$ Pa, $V_1 = 6.50\times10^{-3}\ \text{m}^3$, $T_1 = 325$ K, and $T_2 = 855$ K, we have $V_2 = 5.79\times10^{-4}\ \text{m}^3$.

(c) We want the final pressure p_2. pV^{γ} is constant so $p_1V_1^{\gamma} = p_2V_2^{\gamma}$. Solving for p_2 gives $p_2 = p_1\left(\dfrac{V_1}{V_2}\right)^{\gamma}$. Using $p_1 = 1.00\times10^5$ Pa, $V_1 = 6.50\times10^{-3}\ \text{m}^3$, and $V_2 = 5.79\times10^{-4}\ \text{m}^3$, we get $p_2 = 2.95\times10^6$ Pa.

(d) We want the work done by the gas during this compression. Use $W = \dfrac{1}{\gamma-1}(p_1V_1 - p_2V_2)$ and put in the numbers from above, we get $W = -2650$ J. The minus sign tells us that work is done *on* the gas to compress it.

EVALUATE: Regardless of the process, the gas must still obey the ideal gas law, so pV/T should be the same for states 1 and 2. Using the values for the pressure, volume, and temperature of each state, we get $p_1V_1/T_1 = 2.00$ J/K and $p_2V_2/T_2 = 2.00$ J/K, so our results are consistent with the ideal gas law.

VP19.7.3. **IDENTIFY:** Monatomic neon gas expands adiabatically.

SET UP: $TV^{\gamma-1}$ and pV^{γ} are constant during an adiabatic process and $\gamma = C_p / C_V = 5/3$. We also have $\Delta U = Q - W$, $\Delta U = nC_V\Delta T$, $pV = nRT$. We know $T_1 = 305$ K, $V_1 = 0.0400\ \text{m}^3$, and $V_2 = 0.0900\ \text{m}^3$, and $n = 5.00$ mol. $Q = 0$ for an adiabatic process.

EXECUTE: **(a)** We want the initial pressure p_1. Solving $pV = nRT$ gives $p_1 = nRT_1/V_1$. Using the numbers above gives $p_1 = 3.17\times10^5$ Pa.

(b) We want the final pressure p_2. Since pV^{γ} is constant, $p_1V_1^{\gamma} = p_2V_2^{\gamma}$. Solving for p_2 gives $p_2 = p_1\left(\dfrac{V_1}{V_2}\right)^{\gamma}$. Using $\gamma = 5/3$ and the values for the volumes and pressure, we get $p_2 = 8.20\times10^4$ Pa.

(c) We want the final temperature T_2. Since $TV^{\gamma-1}$ is constant, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$. Solving for T_2 gives $T_2 = T_1\left(\dfrac{V_1}{V_2}\right)^{\gamma-1}$. Using $T_1 = 305$ K and the values for the volumes, we have $T_2 = 178$ K.

(d) We want the work done by the gas. For an adiabatic process, $Q = 0$, so $\Delta U = Q - W$ gives $W = -\Delta U = -nC_V\Delta T = -n\left(\dfrac{3}{2}R\right)\Delta T$. Using $\Delta T = 178\ \text{K} - 305\ \text{K}$ and $n = 5.00$ mol, we get $W = 7940$ J.

EVALUATE: We can check the answer in part (d) by using $W = \dfrac{1}{\gamma-1}(p_1V_1 - p_2V_2)$. Putting in the numbers from above gives $W = 7950$ J. The slight difference in the last digit is from rounding.

VP19.7.4. **IDENTIFY:** Diatomic hydrogen gas H_2 expands adiabatically.

SET UP: $TV^{\gamma-1}$ and pV^{γ} are constant during an adiabatic process and $\gamma = C_p / C_V = 7/5$. We also have $\Delta U = Q - W$, $\Delta U = nC_V\Delta T$, $pV = nRT$. We know that $T_1 = 325$ K, $T_2 = 195$ K, and $n = 1.25$ mol. $Q = 0$ for an adiabatic process.

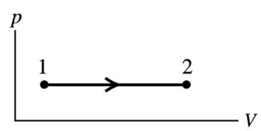
EXECUTE: **(a)** We want the work done by the gas. Using $\Delta U = Q - W$ with $Q = 0$ gives $W = -\Delta U = -nC_V\Delta T = -n\left(\frac{5}{2}R\right)\Delta T = -(1.25\text{ mol})(5/2)(8.314\text{ J/mol}\cdot\text{K})(130\text{ K})$, so $W = -3.38\times10^3$ J.

(b) We want V_2/V_1. Since $TV^{\gamma-1}$ is constant, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, which gives Using the numbers gives $\frac{V_2}{V_1} = \left(\frac{325\text{ K}}{195\text{ K}}\right)^{5/2} = 3.59.$

(c) We want p_2/p_1. Solving $p_1V_1^{\gamma} = p_2V_2^{\gamma}$ for p_2/p_1 gives $\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = \left(\frac{1}{3.59}\right)^{7/5} = 0.167.$

EVALUATE: The gas must also obey the ideal gas law, so we check the ratio p_2/p_1 ratio. Using $pV = nRT$ this ratio is $\frac{p_2}{p_1} = \frac{nRT_2/V_2}{nRT_1/V_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right) = \left(\frac{195\text{ K}}{325\text{ K}}\right)\left(\frac{1}{3.59}\right) = 0.167,$ which agrees with our result in part (c).

19.1. **(a) IDENTIFY** and **SET UP:** The pressure is constant and the volume increases.



The pV-diagram is sketched in Figure 19.1.

Figure 19.1

(b) $W = \int_{V_1}^{V_2} p\,dV$. Since p is constant, $W = p\int_{V_1}^{V_2} dV = p(V_2 - V_1)$. The problem gives T rather than p and V, so use the ideal gas law to rewrite the expression for W.

EXECUTE: $pV = nRT$ so $p_1V_1 = nRT_1$, $p_2V_2 = nRT_2$; subtracting the two equations gives $p(V_2 - V_1) = nR(T_2 - T_1)$. Thus the work is $W = nR(T_2 - T_1)$ during a constant pressure process for an ideal gas. Then $W = nR(T_2 - T_1) = (2.00\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(107°\text{C} - 27°\text{C}) = +1330$ J.

EVALUATE: The gas expands when heated and does positive work.

19.2. **IDENTIFY:** At constant pressure, $W = p\Delta V = nR\Delta T$. Since the gas is doing work, it must be expanding, so ΔV is positive, which means that ΔT must also be positive.

SET UP: $R = 8.3145$ J/mol·K. ΔT has the same numerical value in kelvins and in C°.

EXECUTE: $\Delta T = \frac{W}{nR} = \frac{2.40\times10^3\text{ J}}{(6\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})} = 48.1\text{ K}$. $\Delta T_K = \Delta T_C$ and $T_2 = 27.0°\text{C} + 48.1\text{ C}° = 75.1°\text{C}$.

EVALUATE: When $W > 0$ the gas expands. When p is constant and V increases, T increases.

19.3. **IDENTIFY:** For an isothermal process $W = nRT\ln(p_1/p_2)$. $pV = nRT$ says V decreases when p increases and T is constant.

SET UP: $T = 65.0 + 273.15 = 338.15$ K. $p_2 = 3p_1$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.3.

(b) $W = (2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(338.15\text{ K})\ln\left(\dfrac{p_1}{3p_1}\right) = -6180\text{ J}.$

EVALUATE: Since V decreases, W is negative.

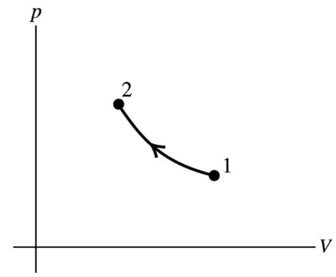


Figure 19.3

19.4. **IDENTIFY:** The work done in a cycle is the area enclosed by the cycle in a pV diagram.
SET UP: **(a)** 1 mm of Hg $= 133.3$ Pa. $p_{\text{gauge}} = p - p_{\text{air}}$. In calculating the enclosed area only changes in pressure enter and you can use gauge pressure. $1\text{ L} = 10^{-3}\text{ m}^3$.
(b) Since $pV = nRT$ and T is constant, the maximum number of moles of air in the lungs is when pV is a maximum. In the ideal gas law the absolute pressure $p = p_{\text{gauge}} + p_{\text{air}}$ must be used.
$p_{\text{air}} = 760$ mm of Hg. 1 mm of Hg = 1 torr.
EXECUTE: **(a)** By counting squares and noting that the area of 1 square is (1 mm of Hg)(0.1 L), we estimate that the area enclosed by the cycle is about $7.5\text{ (mm of Hg)}\cdot\text{L} = 1.00\text{ N}\cdot\text{m}$. The net work done is positive.
(b) The maximum pV is when $p = 11\text{ torr} + 760\text{ torr} = 771\text{ torr} = 1.028\times10^5\text{ Pa}$ and $V = 1.4\text{ L} = 1.4\times10^{-3}\text{ m}^3$. The maximum pV is $(pV)_{\text{max}} = 144\text{ N}\cdot\text{m}$. $pV = nRT$ so

$$n_{\text{max}} = \frac{(pV)_{\text{max}}}{RT} = \frac{144\text{ N}\cdot\text{m}}{(8.315\text{ J/mol}\cdot\text{K})(293\text{ K})} = 0.059\text{ mol}.$$

EVALUATE: While inhaling the gas does positive work on the lungs, but while exhaling the lungs do work on the gas, so the net work is positive.

19.5. **IDENTIFY:** For an isothermal process $W = nRT\ln(p_1/p_2)$. Solve for p_1.
SET UP: For a compression (V decreases) W is negative, so $W = -392$ J. $T = 295.15$ K.
EXECUTE: **(a)** $\dfrac{W}{nRT} = \ln\left(\dfrac{p_1}{p_2}\right)$. $\dfrac{p_1}{p_2} = e^{W/nRT}$.

$$\frac{W}{nRT} = \frac{-392\text{ J}}{(0.305\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(295.15\text{ K})} = -0.5238.$$

$p_1 = p_2 e^{W/nRT} = (1.76\text{ atm})e^{-0.5238} = 1.04\text{ atm}.$
(b) In the process the pressure increases and the volume decreases. The pV-diagram is sketched in Figure 19.5.
EVALUATE: W is the work done by the gas, so when the surroundings do work on the gas, W is negative. The gas was compressed at constant temperature, so its pressure must have increased, which means that $p_1 < p_2$, which is what we found.

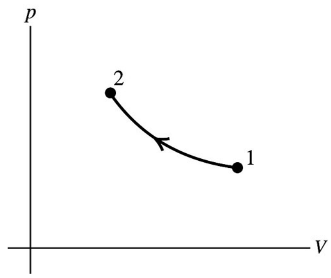


Figure 19.5

19.6. **(a) IDENTIFY** and **SET UP:** The pV-diagram is sketched in Figure 19.6.

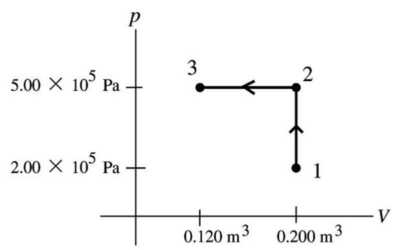


Figure 19.6

(b) Calculate W for each process, using the expression for W that applies to the specific type of process.
EXECUTE: $1 \to 2: \ \Delta V = 0,$ so $W = 0$

$2 \to 3:$ Since p is constant, $W = p\,\Delta V = (5.00\times10^5 \text{ Pa})(0.120 \text{ m}^3 - 0.200 \text{ m}^3) = -4.00\times10^4 \text{ J}$ (W is negative since the volume decreases in the process.)

$W_{\text{tot}} = W_{1\to2} + W_{2\to3} = -4.00\times10^4 \text{ J}.$

EVALUATE: The volume decreases so the total work done is negative.

19.7. **IDENTIFY:** Calculate W for each step using the appropriate expression for each type of process.
SET UP: When p is constant, $W = p\Delta V$. When $\Delta V = 0,\ W = 0.$

EXECUTE: **(a)** $W_{13} = p_1(V_2 - V_1), W_{32} = 0, W_{24} = p_2(V_1 - V_2)$ and $W_{41} = 0.$ The total work done by the system is $W_{13} + W_{32} + W_{24} + W_{41} = (p_1 - p_2)(V_2 - V_1),$ which is the area in the pV plane enclosed by the loop.

(b) For the process in reverse, the pressures are the same, but the volume changes are all the negatives of those found in part (a), so the total work is negative of the work found in part (a).
EVALUATE: When $\Delta V > 0,\ W > 0$ and when $\Delta V < 0,\ W < 0.$

19.8. **IDENTIFY:** An ideal gas undergoes a process during which its pressure is directly proportional to its volume.
SET UP: We want the work done by the gas. We know that $p = \alpha V$ where $\alpha > 0$. The work done by (or on) the gas is the area under the curve in a pV-diagram. Start by making such a diagram in Fig. 19.8.

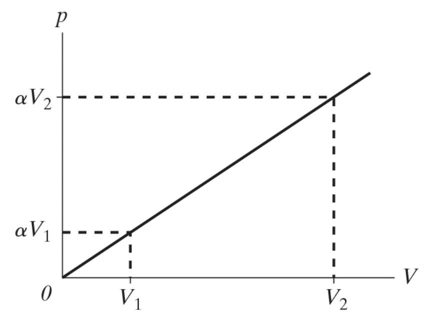


Figure 19.8

EXECUTE: The region from V_1 to V_2 is composed of a triangle plus a rectangle, so the total area is $W = A = A_{\text{tri}} + A_{\text{rect}} = \frac{1}{2}(V_2 - V_1)(\alpha V_2 - \alpha V_1) + (V_2 - V_1)(\alpha V_1) = \frac{\alpha}{2}\left(V_2^2 - V_1^2\right).$

EVALUATE: Since $V_2 > V_1$, the work is positive so the gas does work.

19.9. **IDENTIFY:** $\Delta U = Q - W$. For a constant pressure process, $W = p\Delta V$.

SET UP: $Q = +1.15\times10^5$ J, since heat enters the gas.

EXECUTE: **(a)** $W = p\Delta V = (1.65\times10^5 \text{ Pa})(0.320 \text{ m}^3 - 0.110 \text{ m}^3) = 3.47\times10^4$ J.

(b) $\Delta U = Q - W = 1.15\times10^5 \text{ J} - 3.47\times10^4 \text{ J} = 8.04\times10^4$ J.

EVALUATE: **(c)** $W = p\Delta V$ for a constant pressure process and $\Delta U = Q - W$ both apply to any material. The ideal gas law wasn't used and it doesn't matter if the gas is ideal or not.

19.10. **IDENTIFY:** The type of process is not specified. We can use $\Delta U = Q - W$ because this applies to all processes. Calculate ΔU and then from it calculate ΔT.

SET UP: Q is positive since heat goes into the gas; $Q = +1500$ J.

W is positive since gas expands; $W = +2100$ J.

EXECUTE: $\Delta U = 1500 \text{ J} - 2100 \text{ J} = -600$ J.

We can also use $\Delta U = n(\frac{3}{2}R)\,\Delta T$ since this is true for any process for an ideal gas.

$$\Delta T = \frac{2\,\Delta U}{3nR} = \frac{2(-600 \text{ J})}{3(5.00 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})} = -9.62 \text{ C}^\circ$$

$T_2 = T_1 + \Delta T = 127^\circ\text{C} + (-9.62 \text{ C}^\circ) = 117^\circ\text{C}.$

EVALUATE: More energy leaves the gas in the expansion work than enters as heat. The internal energy therefore decreases, and for an ideal gas this means the temperature decreases. We didn't have to convert ΔT to kelvins since ΔT is the same on the Kelvin and Celsius scales.

19.11. **IDENTIFY:** Part *ab* is isochoric, but *bc* is not any of the familiar processes.

SET UP: $pV = nRT$ determines the Kelvin temperature of the gas. The work done in the process is the area under the curve in the *pV* diagram. Q is positive since heat goes into the gas.

$1 \text{ atm} = 1.013\times10^5$ Pa. $1 \text{ L} = 1\times10^{-3} \text{ m}^3$. $\Delta U = Q - W$.

EXECUTE: **(a)** The lowest T occurs when pV has its smallest value. This is at point a, and

$$T_a = \frac{p_a V_a}{nR} = \frac{(0.20 \text{ atm})(1.013\times10^5 \text{ Pa/atm})(2.0 \text{ L})(1.0\times10^{-3} \text{ m}^3/\text{L})}{(0.0175 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})} = 278 \text{ K}.$$

(b) *a to b*: $\Delta V = 0$ so $W = 0$.

b to c: The work done by the gas is positive since the volume increases. The magnitude of the work is the area under the curve so $W = \frac{1}{2}(0.50 \text{ atm} + 0.30 \text{ atm})(6.0 \text{ L} - 2.0 \text{ L})$ and

$W = (1.6 \text{ L}\cdot\text{atm})(1\times10^{-3} \text{ m}^3/\text{L})(1.013\times10^5 \text{ Pa/atm}) = 162$ J.

(c) For *abc*, $W = 162$ J. $\Delta U = Q - W = 215 \text{ J} - 162 \text{ J} = 53$ J.

EVALUATE: 215 J of heat energy went into the gas. 53 J of energy stayed in the gas as increased internal energy and 162 J left the gas as work done by the gas on its surroundings.

19.12. IDENTIFY and SET UP: Calculate W using the equation for a constant pressure process. Then use $\Delta U = Q - W$ to calculate Q.

EXECUTE: **(a)** $W = \int_{V_1}^{V_2} p\, dV = p(V_2 - V_1)$ for this constant pressure process.

$W = (1.80\times10^5 \text{ Pa})(1.20 \text{ m}^3 - 1.70 \text{ m}^3) = -9.00\times10^4 \text{ J}$. (The volume decreases in the process, so W is negative.)

(b) $\Delta U = Q - W$. $Q = \Delta U + W = -1.40\times10^5 \text{ J} + (-9.00\times10^4 \text{ J}) = -2.30\times10^5 \text{ J}$. Negative Q means heat flows out of the gas.

(c) EVALUATE: $W = \int_{V_1}^{V_2} p\, dV = p(V_2 - V_1)$ (constant pressure) and $\Delta U = Q - W$ apply to *any* system, not just to an ideal gas. We did not use the ideal gas equation, either directly or indirectly, in any of the calculations, so the results are the same whether the gas is ideal or not.

19.13. IDENTIFY: We read values from the pV-diagram and use the ideal gas law, as well as the first law of thermodynamics.

SET UP: Use $pV = nRT$ to calculate T at each point. The work done in a process is the area under the curve in the pV diagram. $\Delta U = Q - W$ for all processes.

EXECUTE: **(a)** $pV = nRT$ so $T = \dfrac{pV}{nR}$.

Point a: $T_a = \dfrac{(2.0\times10^5 \text{ Pa})(0.010 \text{ m}^3)}{(0.450 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})} = 535 \text{ K}$.

Point b: $T_b = \dfrac{(5.0\times10^5 \text{ Pa})(0.070 \text{ m}^3)}{(0.450 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})} = 9350 \text{ K}$.

Point c: $T_c = \dfrac{(8.0\times10^5 \text{ Pa})(0.070 \text{ m}^3)}{(0.450 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})} = 15{,}000 \text{ K}$.

(b) The work done by the gas is positive since the volume increases. The magnitude of the work is the area under the curve: $W = \frac{1}{2}(2.0\times10^5 \text{ Pa} + 5.0\times10^5 \text{ Pa})(0.070 \text{ m}^3 - 0.010 \text{ m}^3) = 2.1\times10^4 \text{ J}$.

(c) $\Delta U = Q - W$ so $Q = \Delta U + W = 15{,}000 \text{ J} + 2.1\times10^4 \text{ J} = 3.6\times10^4 \text{ J}$.

EVALUATE: Q is positive so heat energy goes into the gas.

19.14. IDENTIFY: $\Delta U = Q - W$. For a constant pressure process, $W = p\Delta V$.

SET UP: $Q = +2.20\times10^6 \text{ J}$; $Q > 0$ since this amount of heat goes into the water.

$p = 2.00 \text{ atm} = 2.03\times10^5 \text{ Pa}$.

EXECUTE: **(a)** $W = p\Delta V = (2.03\times10^5 \text{ Pa})(0.824 \text{ m}^3 - 1.00\times10^{23} \text{ m}^3) = 1.67\times10^5 \text{ J}$

(b) $\Delta U = Q - W = 2.20\times10^6 \text{ J} - 1.67\times10^5 \text{ J} = 2.03\times10^6 \text{ J}$.

EVALUATE: $2.20\times10^6 \text{ J}$ of energy enters the water. $1.67\times10^5 \text{ J}$ of energy leaves the materials through expansion work and the remainder stays in the material as an increase in internal energy.

19.15. IDENTIFY: For a certain thermondynamics process, $|Q|$ = 100 J and $|W|$ = 300 J. The first law of thermodynamics applies.

SET UP and EXECUTE: We use $\Delta U = Q - W$ and want to find Q and W for each value of ΔU.

(a) ΔU = +400 J: 400 J = $Q - W$, so Q = +100 J and W = –300 J.
(b) ΔU = +200 J: 200 J = $Q - W$, so Q = –100 J and W = –300 J.
(c) ΔU = –200 J: –200 J = $Q - W$, so Q = +100 J and W = +300 J.

(d) $\Delta U = -400$ J: -400 J $= Q - W$, so $Q = -100$ J and $W = +300$ J.

EVALUATE: All the quantities in $\Delta U = Q - W$ can be positive and negative.

19.16. **IDENTIFY:** $\Delta U = Q - W$.

SET UP: $Q < 0$ when heat leaves the gas.

EXECUTE: For an isothermal process, $\Delta U = 0$, so $W = Q = -410$ J.

EVALUATE: In a compression the volume decreases and $W < 0$.

19.17. **IDENTIFY:** For a constant pressure process, $W = p\Delta V$, $Q = nC_p\Delta T$, and $\Delta U = nC_V\Delta T$. $\Delta U = Q - W$ and $C_p = C_V + R$. For an ideal gas, $p\Delta V = nR\Delta T$.

SET UP: From Table 19.1, $C_V = 28.46$ J/mol·K.

EXECUTE: **(a)** The pV diagram is shown in Figure 19.17 (next page).

(b) $W = pV_2 - pV_1 = nR(T_2 - T_1) = (0.250 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})(100.0 \text{ K}) = 208$ J.

(c) The work is done on the piston.

(d) Since $\Delta U = nC_V\Delta T$ holds for any process, we have

$\Delta U = nC_V\Delta T = (0.250 \text{ mol})(28.46 \text{ J/mol}\cdot\text{K})(100.0 \text{ K}) = 712$ J.

(e) Either $Q = nC_p\Delta T$ or $Q = \Delta U + W$ gives $Q = 920$ J to three significant figures.

(f) The lower pressure would mean a correspondingly larger volume, and the net result would be that the work done would be the same as that found in part (b).

EVALUATE: $W = nR\Delta T$, so W, Q and ΔU all depend only on ΔT. When T increases at constant pressure, V increases and $W > 0$. ΔU and Q are also positive when T increases.

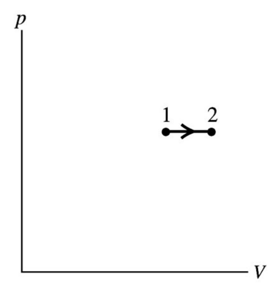


Figure 19.17

19.18. **IDENTIFY:** For constant volume $Q = nC_V\Delta T$. For constant pressure, $Q = nC_p\Delta T$. For any process of an ideal gas, $\Delta U = nC_V\Delta T$.

SET UP: $R = 8.315$ J/mol·K. For helium, $C_V = 12.47$ J/mol·K and $C_p = 20.78$ J/mol·K.

EXECUTE: **(a)** $Q = nC_V\Delta T = (0.0100 \text{ mol})(12.47 \text{ J/mol}\cdot\text{K})(40.0 \text{ C}°) = 4.99$ J. The pV-diagram is sketched in Figure 19.18a.

(b) $Q = nC_p\Delta T = (0.0100 \text{ mol})(20.78 \text{ J/mol}\cdot\text{K})(40.0 \text{ C}°) = 8.31$ J. The pV-diagram is sketched in Figure 19.18b.

(c) More heat is required for the constant pressure process. ΔU is the same in both cases. For constant volume $W = 0$ and for constant pressure $W > 0$. The additional heat energy required for constant pressure goes into expansion work.

(d) $\Delta U = nC_V\Delta T = 4.99$ J for both processes. ΔU is path independent and for an ideal gas depends only on ΔT.

EVALUATE: $C_p = C_V + R$, so $C_p > C_V$.

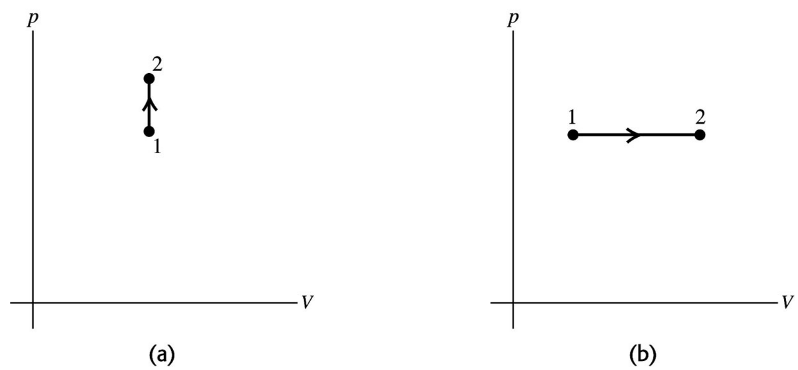


Figure 19.18

19.19. **IDENTIFY:** For constant volume, $Q = nC_V\Delta T$. For constant pressure, $Q = nC_p\Delta T$.

SET UP: From Table 19.1 in the text, $C_V = 20.76 \text{ J/mol}\cdot\text{K}$ and $C_p = 29.07 \text{ J/mol}\cdot\text{K}$.

EXECUTE: **(a)** Using $Q = nC_V\Delta$, $\Delta T = \dfrac{Q}{nC_V} = \dfrac{645 \text{ J}}{(0.185 \text{ mol})(20.76 \text{ J/mol}\cdot\text{K})} = 167.9 \text{ K}$ and $T = 948 \text{ K}$.

The pV-diagram is sketched in Figure 19.19a.

(b) Using $Q = nC_p\Delta T$, $\Delta T = \dfrac{Q}{nC_p} = \dfrac{645 \text{ J}}{(0.185 \text{ mol})(29.07 \text{ J/mol}\cdot\text{K})} = 119.9 \text{ K}$ and $T = 900 \text{ K}$.

The pV-diagram is sketched in Figure 19.19b.

EVALUATE: At constant pressure some of the heat energy added to the gas leaves the gas as expansion work and the internal energy change is less than if the same amount of heat energy is added at constant volume. ΔT is proportional to ΔU.

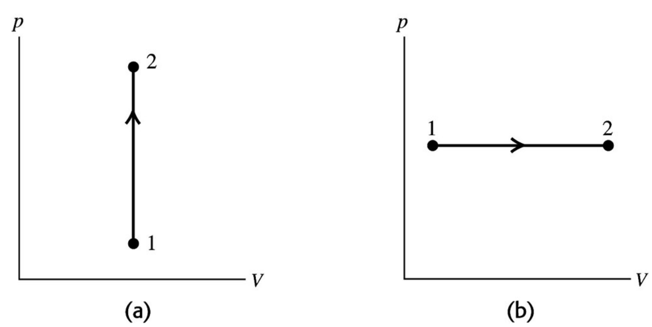


Figure 19.19

19.20. IDENTIFY: A monatomic ideal gas goes between points a and b by two different paths.

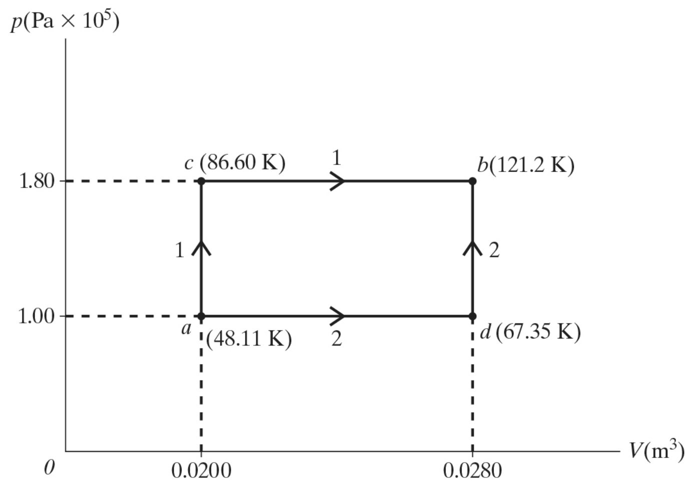


Figure 19.20

SET UP: We want to calculate $Q - W$ for each of the two paths shown in the figure. For a monatomic ideal gas $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$. We also have $Q_V = nC_V\Delta T$, $Q_p = nC_p\Delta T$, $\Delta U = Q - W$, and $pV = nRT$. All of the processes in these paths are either at constant volume or constant pressure, so we first find the temperature at each of the four points and add them to the pV-diagram.

EXECUTE: From $pV = nRT$ we get $T = pV/nR$. Using the values on the pV-diagram for point a, we have $p_a = 1.00\times10^5$ Pa, $V_a = 0.0200\ \text{m}^3$ and $n = 5.00$ mol. This gives $T_a = 48.11$ K. We could do the same thing for the other three points. Or we could use the fact that, for example, during segment ac the volume is constant, so T/p is constant. This gives us $\frac{T_c}{T_a} = \frac{p_c}{p_a} = \frac{1.80\times10^5\ \text{Pa}}{1.00\times10^5\ \text{Pa}} = 1.8$, so $T_c = 1.8T_a = (1.8)(48.11\ \text{K}) = 86.60$ K. We could apply this type of procedure to the other path segments. The final result is $T_b = 121.2$ K and $T_d = 67.35$ K.

(a) Segment ac: Volume is constant, so $Q_{ac} = nC_V\Delta T_{ac} = n\left(\frac{3}{2}R\right)\Delta T_{ac}$. Using $n = 5.00$ mol and $\Delta T = T_c - T_a = 86.60\ \text{K} - 48.11\ \text{K} = 38.49\ \text{K}$ gives $Q_{ac} = 2400$ J.

$W_{ac} = 0$ because volume is constant.

Segment cb: Pressure is constant, so $Q_{cb} = nC_p\Delta T_{cb} = n\left(\frac{5}{2}R\right)\Delta T_{cb}$. Using the same value for n and $\Delta T = 121.2\ \text{K} - 86.60\ \text{K} = 34.6\ \text{K}$ gives $Q_{cb} = 3595.8$ J.

W_{cb} is the area under the curve in the pV-diagram, which gives us

$W_{cb} = (0.0280\ \text{m}^3 - 0.0200\ \text{m}^3)(1.80\times10^5\ \text{Pa}) = 1440$ J.

Path 1: $Q_1 = Q_{ac} + Q_{cb} = 2400\ \text{J} + 3595.8\ \text{J} = 6000$ J.

(b) Follow the same procedure as for path 1. The results are

Segment ad: Pressure is constant. $Q_{ad} = 2000$ J. W_{ad} = area under the curve = 800 J.

Segment db: Volume is constant. $Q_{db} = 3358$ J. $W_{db} = 0$ because the volume is constant.

Path 2: $Q_2 = 2000\ \text{J} + 3358\ \text{J} = 5360$ J. $W_2 = 800$ J.

(c) Path 1: $Q_1 - W_1 = 6000\ \text{J} - 1440\ \text{J} = 4560$ J.

Path 2: $Q_2 - W_1 = 5360\ \text{J} - 800\ \text{J} = 4560$ J.

The quantity $Q - W$ is the same for both paths.

EVALUATE: Recognize from the first law of thermodynamics that $Q - W$ is equal to ΔU. Since ΔU for an ideal gas depends only on ΔT, and ΔT is the same for paths 1 and 2 since it is $T_b - T_a$ for both of them, it follows that ΔU is the same for both paths. Therefore $Q - W$ must be the same.

19.21. IDENTIFY: $\Delta U = Q - W$. For an ideal gas, $\Delta U = C_V\,\Delta T,$ and at constant pressure, $W = p\Delta V = nR\Delta T$.

SET UP: $C_V = \frac{3}{2}R$ for a monatomic gas.

EXECUTE: $\Delta U = n\left(\frac{3}{2}R\right)\Delta T = \frac{3}{2}p\Delta V = \frac{3}{2}W$. Then $Q = \Delta U + W = \frac{5}{2}W$, so $W/Q = \frac{2}{5}$.

EVALUATE: For diatomic or polyatomic gases, C_V is a different multiple of R and the fraction of Q that is used for expansion work is different.

19.22. IDENTIFY: Apply $pV = nRT$ to calculate T. For this constant pressure process, $W = p\Delta V$. $Q = nC_p\Delta T$. Use $\Delta U = Q - W$ to relate Q, W, and ΔU.

SET UP: $2.50\text{ atm} = 2.53\times10^5\text{ Pa}$. For a monatomic ideal gas, $C_V = 12.47\text{ J/mol}\cdot\text{K}$ and $C_p = 20.78\text{ J/mol}\cdot\text{K}$.

EXECUTE: **(a)** $T_1 = \dfrac{pV_1}{nR} = \dfrac{(2.53\times10^5\text{ Pa})(3.20\times10^{-2}\text{ m}^3)}{(3.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = 325\text{ K}.$

$$T_2 = \frac{pV_2}{nR} = \frac{(2.53\times10^5\text{ Pa})(4.50\times10^{-2}\text{ m}^3)}{(3.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = 456\text{ K}.$$

(b) $W = p\Delta V = (2.53\times10^5\text{ Pa})(4.50\times10^{-2}\text{ m}^3 - 3.20\times10^{-2}\text{ m}^3) = 3.29\times10^3\text{ J}.$

(c) $Q = nC_p\Delta T = (3.00\text{ mol})(20.78\text{ J/mol}\cdot\text{K})(456\text{ K} - 325\text{ K}) = 8.17\times10^3\text{ J}.$

(d) $\Delta U = Q - W = 4.88\times10^3\text{ J}.$

EVALUATE: We could also calculate ΔU as $\Delta U = nC_V\Delta T = (3.00\text{ mol})(12.47\text{ J/mol}\cdot\text{K})(456\text{ K} - 325\text{ K}) = 4.90\times10^3\text{ J}$, which agrees with the value we calculated in part (d).

19.23. IDENTIFY: $\Delta U = Q - W$. Apply $Q = nC_p\Delta T$ to calculate C_p. Apply $\Delta U = nC_V\Delta T$ to calculate C_V. $\gamma = C_p/C_V$.

SET UP: $\Delta T = 15.0\text{ C}° = 15.0\text{ K}$. Since heat is added, $Q = +970\text{ J}$.

EXECUTE: **(a)** $\Delta U = Q - W = 1970\text{ J} - 223\text{ J} = 747\text{ J}.$

(b) $C_p = \dfrac{Q}{n\Delta T} = \dfrac{970\text{ J}}{(1.75\text{ mol})(15.0\text{ K})} = 37.0\text{ J/mol}\cdot\text{K}.$ $C_V = \dfrac{\Delta U}{n\Delta T} = \dfrac{747\text{ J}}{(1.75\text{ mol})(15.0\text{ K})} = 28.5\text{ J/mol}\cdot\text{K}.$

$$\gamma = \frac{C_p}{C_V} = \frac{37.0\text{ J/mol}\cdot\text{K}}{28.5\text{ J/mol}\cdot\text{K}} = 1.30.$$

EVALUATE: The value of γ we calculated is similar to the values given in Tables 19.1 for polyatomic gases.

19.24. IDENTIFY: A monatomic gas is compressed at constant pressure, so this is an isobaric compression.

SET UP: We want to find the work W, heat flow Q, and internal energy change ΔU. We know that $pV = nRT$, $\Delta U = Q - W$, $W = p\Delta V$ when pressure is constant, $Q_p = nC_p\Delta T$, and $\Delta U = nC_V\Delta T$. For an ideal monatomic gas $C_V = \dfrac{3}{2}R$ and $C_p = \dfrac{5}{2}R$.

EXECUTE: **(a)** We want the work done by (or on) the gas. $W = p\Delta V = p(V_2 - V_1)$, which gives $W = (1.80\times10^4\text{ Pa})(0.0500\text{ m}^3 - 0.0800\text{ m}^3) = -540\text{ J}$. The minus sign tells us that work is done *on* the gas.

(b) At constant pressure we have $Q_p = nC_p\Delta T = n\left(\frac{5}{2}R\right)\Delta T$. At constant pressure, $pV = nRT$ tells us that $p\Delta V = nR\Delta T$, so $\Delta T = \frac{p\Delta V}{nR} = \frac{W}{nR}$. Therefore $Q_p = n\left(\frac{5}{2}R\right)\left(\frac{W}{nR}\right) = \frac{5}{2}W = \frac{5}{2}(-540\text{ J}) = -1350\text{ J}$.
The minus sign tells us that heat *leaves* the gas.
(c) $\Delta U = Q - W = -1350\text{ J} - (-540\text{ J}) = -810\text{ J}$. The internal energy *decreases*.

EVALUATE: As a check in part (c), use $\Delta U = nC_V\Delta T = Q_p = n\left(\frac{3}{2}R\right)\left(\frac{W}{nR}\right) = \frac{3}{2}W$, so $Q_p = \frac{3}{2}(-540\text{ J}) = -810\text{ J}$, which agrees with our result.

19.25. IDENTIFY: Calculate W and ΔU and then use the first law to calculate Q.
(a) SET UP: $W = \int_{V_1}^{V_2} p\,dV$

$pV = nRT$ so $p = nRT/V$

$W = \int_{V_1}^{V_2} (nRT/V)\,dV = nRT\int_{V_1}^{V_2} dV/V = nRT\ln(V_2/V_1)$ (work done during an isothermal process).

EXECUTE: $W = (0.150\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(350\text{ K})\ln(0.25V_1/V_1) = (436.5\text{ J})\ln(0.25) = -605\text{ J}$.
EVALUATE: W for the gas is negative, since the volume decreases.
(b) SET UP: $\Delta U = nC_V\Delta T$ for any ideal gas process.
EXECUTE: $\Delta T = 0$ (isothermal) so $\Delta U = 0$.
EVALUATE: $\Delta U = 0$ for any ideal gas process in which T doesn't change.
(c) SET UP: $\Delta U = Q - W$
EXECUTE: $\Delta U = 0$ so $Q = W = -605\text{ J}$. (Q is negative; the gas liberates 605 J of heat to the surroundings.)
EVALUATE: $Q = nC_V\Delta T$ is only for a constant volume process, so it doesn't apply here.
$Q = nC_p\Delta T$ is only for a constant pressure process, so it doesn't apply here.

19.26. IDENTIFY: We want to investigate how much heat input goes into work for the expansion of liquid ethanol.
SET UP: We are looking at the work W by expanding ethanol due to heat input Q. We use $W = p\Delta V$ when pressure is constant, $\rho = m/V$, $Q = mc\Delta T$ and $\Delta V = \beta V_0\Delta T$.
EXECUTE: **(a)** m = (46.1 g/mol)(5.00 mol) = 0.231 kg.
(b) We want the heat Q that enters the ethanol. $Q = mc\Delta T$ = (0.231 kg)(2428 J/kg·K)(50 K) = 2.80×10^4 J.
(c) We want the work done by the ethanol $W = p\Delta V$ and $\Delta V = \beta V_0\Delta T$. Combining these gives $\Delta V = p\beta V_0\Delta T$. We know the mass and density of the ethanol, so use $\rho = m/V$, which gives $V_0 = m/\rho$. Therefore $W = p\beta(m/\rho)\Delta T$. Using $p = 1.01\times10^5$ Pa, $\beta = 75\times10^{-5}\text{ K}^{-1}$, $\Delta T = 50$ K, ρ = 810 kg/m^3, and m = 0.231 kg gives W = 1.08 J which rounds to W = 1.1 J.
(d) $\frac{W}{Q} = \frac{1.08\text{ J}}{2.80\times10^4\text{ J}} = 3.8\times10^{-5}$.
EVALUATE: The work is a very small fraction of the heat input. Most of that heat goes into increasing the internal energy and therefore the temperature of the ethanol.

19.27. **IDENTIFY:** For an adiabatic process of an ideal gas, $p_1V_1^\gamma = p_2V_2^\gamma$, $W = \dfrac{1}{\gamma-1}(p_1V_1 - p_2V_2)$, and $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$.

SET UP: For a monatomic ideal gas $\gamma = 5/3$.

EXECUTE: **(a)** $p_2 = p_1\left(\dfrac{V_1}{V_2}\right)^\gamma = (1.50\times10^5\ \text{Pa})\left(\dfrac{0.0800\ \text{m}^3}{0.0400\ \text{m}^3}\right)^{5/3} = 4.76\times10^5\ \text{Pa}.$

(b) This result may be substituted into $W = \dfrac{1}{\gamma-1}p_1V_1(1-(V_1/V_2)^{\gamma-1})$, or, substituting the above form for p_2, $W = \dfrac{1}{\gamma-1}p_1V_1(1-(V_1/V_2)^{\gamma-1}) = \dfrac{3}{2}(1.50\times10^5\ \text{Pa})(0.0800\ \text{m}^3)\left(1-\left(\dfrac{0.0800}{0.0400}\right)^{2/3}\right) = -1.06\times10^4\ \text{J}.$

(c) From $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, $(T_2/T_1) = (V_2/V_1)^{\gamma-1} = (0.0800/0.0400)^{2/3} = 1.59$, and since the final temperature is higher than the initial temperature, the gas is heated.

EVALUATE: In an adiabatic compression $W < 0$ since $\Delta V < 0$. $Q = 0$ so $\Delta U = -W$. $\Delta U > 0$ and the temperature increases.

19.28. **IDENTIFY:** For an adiabatic process of an ideal gas, $p_1V_1^\gamma = p_2V_2^\gamma$, no heat enters or leaves the gas. The ideal gas law still applies.

SET UP: $p_1V_1^\gamma = p_2V_2^\gamma$, $pV = nRT$, $W = \int_{V_1}^{V_2} p\,dV$. For an ideal monatomic gas, $\gamma = 5/3$.

EXECUTE: Solving $p_1V_1^\gamma = p_2V_2^\gamma$ for p_2 and rearranging gives $p_2 = p_1\left(\dfrac{V_1}{V_2}\right)^\gamma$, so we need to find V_2.

Applying $W = \int_{V_1}^{V_2} p\,dV$ to an adiabatic process, we use the fact that pV^γ = constant. In this case, the constant is $p_1V_1^\gamma$ (since we know p_1 and V_1), and we'll call it K for the time being. This tells us that $p = K/V^\gamma$. Using this in the integral, we get $W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} K/V^\gamma\,dV = \dfrac{K}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$.

$K = p_1V_1^\gamma = (2500\ \text{Pa})(2.10\ \text{m}^3)^{5/3} = 8609\ \text{N}\cdot\text{m}^3$, $W = 1480$ J, and $\gamma = 5/3$. Putting in these numbers and solving for V_2 gives $V_2 = 2.8697\ \text{m}^3$. Putting this value into $p_2 = p_1\left(\dfrac{V_1}{V_2}\right)^\gamma$ gives

$$p_2 = (2500\ \text{Pa})\left(\frac{2.10\ \text{m}^3}{2.8697\ \text{m}^3}\right)^{5/3} = 1490\ \text{Pa} = 1.49\ \text{kPa}.$$

EVALUATE: The pressure dropped because the gas expanded adiabatically and did work, so our result is reasonable. An alternative approach is the following: We know that $Q = 0$ and $\Delta U = n\left(\dfrac{3}{2}R\right)\Delta T$. We have $W = -\Delta U$, so $\Delta U = -1480$ J. Therefore $-1480\ \text{J} = n\left(\dfrac{3}{2}R\right)\Delta T$, which gives $\Delta T = -23.73$ K.

The ideal gas law gives $T_1 = p_1V_1/nR = (2500\ \text{Pa})(2.10\ \text{m}^3)/[(5.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})] = 126.3$ K.
Therefore $T_2 = T_1 + \Delta T = +102.6$ K. Using $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ gives

$V_2^{2/3} = V_1^{2/3}\left(\dfrac{T_1}{T_2}\right) = (2.10\ \text{m}^3)\left(\dfrac{126.3\ \text{K}}{102.6\ \text{K}}\right) = 2.019\ \text{m}^2$, so $V_2 = (2.019\ \text{m}^2)^{3/2} = 2.869\ \text{m}^3$. Therefore

$p_2 = nRT_2/V_2 = (5.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(102.6\ \text{K})/(2.869\ \text{m}^3) = 1490\ \text{Pa} = 1.49\ \text{kPa}.$

19.29. **(a) IDENTIFY and SET UP:** In the expansion the pressure decreases and the volume increases. The pV-diagram is sketched in Figure 19.29.

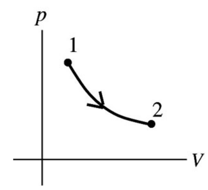


Figure 19.29

(b) Adiabatic means $Q = 0$.

Then $\Delta U = Q - W$ gives $W = -\Delta U = -nC_V\,\Delta T = nC_V(T_1 - T_2)$.

$C_V = 12.47\ \text{J/mol}\cdot\text{K}$ (Table 19.1).

EXECUTE: $W = (0.450\ \text{mol})(12.47\ \text{J/mol}\cdot\text{K})(66.0°\text{C} - 10.0°\text{C}) = +314\ \text{J}$.

W positive for $\Delta V > 0$ (expansion)

(c) $\Delta U = -W = -314\ \text{J}$.

EVALUATE: There is no heat energy input. The energy for doing the expansion work comes from the internal energy of the gas, which therefore decreases. For an ideal gas, when T decreases, U decreases.

19.30. **IDENTIFY:** Assume the expansion is adiabatic. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ relates V and T. Assume the air behaves as an ideal gas, so $\Delta U = nC_V\Delta T$. Use $pV = nRT$ to calculate n.

SET UP: For air, $C_V = 29.76\ \text{J/mol}\cdot\text{K}$ and $\gamma = 1.40$. $V_2 = 0.800V_1$. $T_1 = 293.15\ \text{K}$.

$p_1 = 2.026\times10^5\ \text{Pa}$. For a sphere, $V = \frac{4}{3}\pi r^3$.

EXECUTE: **(a)** $T_2 = T_1\left(\dfrac{V_1}{V_2}\right)^{\gamma-1} = (293.15\ \text{K})\left(\dfrac{V_1}{0.800V_1}\right)^{0.40} = 320.5\ \text{K} = 47.4°\text{C}$.

(b) $V_1 = \frac{4}{3}\pi r^3 = \dfrac{4\pi}{3}(0.1195\ \text{m})^3 = 7.15\times10^{-3}\ \text{m}^3$.

$$n = \frac{p_1V_1}{RT_1} = \frac{(2.026\times10^5\ \text{Pa})(7.15\times10^{-3}\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(293.15\ \text{K})} = 0.594\ \text{mol}.$$

$\Delta U = nC_V\Delta T = (0.594\ \text{mol})(20.76\ \text{J/mol}\cdot\text{K})(321\ \text{K} - 293\ \text{K}) = 345\ \text{J}$.

EVALUATE: We could also use $\Delta U = -W = -\dfrac{1}{\gamma-1}(p_1V_1 - p_2V_2)$ to calculate ΔU, if we first found p_2 from $pV = nRT$.

19.31. **IDENTIFY:** Combine $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ with $pV = nRT$ to obtain an expression relating T and p for an adiabatic process of an ideal gas.

SET UP: $T_1 = 299.15\ \text{K}$.

EXECUTE: $V = \dfrac{nRT}{p}$ so $T_1\left(\dfrac{nRT_1}{p_1}\right)^{\gamma-1} = T_2\left(\dfrac{nRT_2}{p_2}\right)^{\gamma-1}$ and $\dfrac{T_1^\gamma}{p_1^{\gamma-1}} = \dfrac{T_2^\gamma}{p_2^{\gamma-1}}$.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = (299.15\ \text{K})\left(\frac{0.850\times10^5\ \text{Pa}}{1.01\times10^5\ \text{Pa}}\right)^{0.4/1.4} = 284.8\ \text{K} = 11.6°\text{C}.$$

EVALUATE: For an adiabatic process of an ideal gas, when the pressure decreases the temperature decreases.

19.32. IDENTIFY: $pV = nRT$ For an adiabatic process, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$.

SET UP: For an ideal monatomic gas, $\gamma = 5/3$.

EXECUTE: **(a)** $T = \dfrac{pV}{nR} = \dfrac{(1.00\times10^5 \text{ Pa})(2.50\times10^{-3} \text{ m}^3)}{(0.1 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})} = 301 \text{ K}.$

(b) (i) Isothermal: If the expansion is *isothermal*, the process occurs at constant temperature and the final temperature is the same as the initial temperature, namely 301 K.

$p_2 = p_1(V_1/V_2) = \frac{1}{2}p_1 = 5.00\times10^4$ Pa.

(ii) Isobaric: $\Delta p = 0$ so $p_2 = 1.00\times10^5$ Pa. $T_2 = T_1(V_2/V_1) = 2T_1 = 602$ K.

(iii) Adiabatic: Using $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, $T_2 = \dfrac{T_1V_1^{\gamma-1}}{V_2^{\gamma-1}} = \dfrac{(301 \text{ K})(V_1)^{0.67}}{(2V_1)^{0.67}} = (301 \text{ K})(\tfrac{1}{2})^{0.67} = 189 \text{ K}.$ Then

$pV = nRT$ gives $p_2 = 3.14\times10^4$ Pa.

EVALUATE: In an isobaric expansion, T increases. In an adiabatic expansion, T decreases.

19.33. IDENTIFY: As helium undergoes an adiabatic process it's Kelvin temperature doubles.

SET UP: Treat helium as an ideal monatomic gas, so $\gamma = 5/3$. We also have $pV = nRT$ and $p_1V_1^\gamma = p_2V_2^\gamma$, and we know that $T_2 = 2T_1$. We want the factor by which p changes during this process, so we want p_2/p_1.

EXECUTE: From $pV = nRT$ we have $V = nRT/p$. Taking the ratio of the volumes gives $\dfrac{V_1}{V_2} = \dfrac{nRT_1/p_1}{nRT_2/p_2} = \left(\dfrac{T_1}{T_2}\right)\left(\dfrac{p_2}{p_1}\right)$. Now use $p_1V_1^\gamma = p_2V_2^\gamma$, which gives $\dfrac{p_2}{p_1} = \left(\dfrac{V_1}{V_2}\right)^\gamma$. Use the previous result for the volume ration to get $\dfrac{p_2}{p_1} = \left(\dfrac{T_1}{T_2}\cdot\dfrac{p_2}{p_1}\right)^\gamma$, which can be arranged to get $\dfrac{p_2}{p_1}\left(\dfrac{p_2}{p_1}\right)^{-\gamma} = \left(\dfrac{T_1}{T_2}\right)^\gamma$. Using $\gamma = 5/3$ and $T_1/T_2 = ½$, this becomes $\dfrac{p_2}{p_1} = \left(\dfrac{1}{2}\right)^{-5/2} = 4\sqrt{2}$.

EVALUATE: Look at the volume ratio V_2/V_1, which is $\dfrac{V_2}{V_1} = \dfrac{nRT_2/p_2}{nRT_1/p_1} = \left(\dfrac{T_2}{T_1}\right)\left(\dfrac{p_1}{p_2}\right)$, so $\dfrac{V_2}{V_1} = (2)\left(\dfrac{1}{4\sqrt{2}}\right) = \dfrac{\sqrt{2}}{4} \approx 0.354$, which means that the volume decreases. This is reasonable because the only way to increase the gas temperature adiabatically is to compress it.

19 34. IDENTIFY: We are looking at an adiabatic compression of an ideal diatomic gas in which the temperature increases.

SET UP: We want to find the change ΔU in the internal energy of the gas. $\Delta U = nC_V\Delta T$ for any process, and $C_V = \dfrac{5}{2}R$ for an ideal diatomic gas.

EXECUTE: $\Delta U = n\left(\dfrac{5}{2}R\right)\Delta T$. We need n. We know that 1 mol of a gas contains N_A molecules so the number N of molecules is nN_A. Therefore $n = \dfrac{N}{N_A} = \dfrac{3.01\times10^{20} \text{ molecules}}{6.022\times10^{23} \text{ molecules/mol}} = 4.99\times10^{-4}$ mol.

Now we use $\Delta U = n\left(\dfrac{5}{2}R\right)\Delta T$ with $\Delta T = 35.0$ K to get $\Delta U = 0.364$ J.

EVALUATE: For an adiabatic process, $\Delta U = Q - W = 0 - W$, which tells us that $W = -0.364$ J. Work was done *on* the gas which compressed it and increased its internal energy and therefore its temperature.

19.35. **IDENTIFY** and **SET UP:** For an ideal gas, $pV = nRT$. The work done is the area under the path in the pV-diagram.

EXECUTE: **(a)** The product pV increases and this indicates a temperature increase.

(b) The work is the area in the pV plane bounded by the blue line representing the process and the verticals at V_a and V_b. The area of this trapezoid is

$\frac{1}{2}(p_b + p_a)(V_b - V_a) = \frac{1}{2}(2.40\times10^5 \text{ Pa})(0.0400 \text{ m}^3) = 4800 \text{ J}.$

EVALUATE: The work done is the average pressure, $\frac{1}{2}(p_1 + p_2)$, times the volume increase.

19.36. **IDENTIFY:** Steam from boiling water slightly lifts the lid of a pot. This does work on the lid and therefore cools down the steam slightly. We want to find out the decrease in temperature of the steam.

SET UP and EXECUTE: For an adiabatic process, $Q = 0$ so $\Delta U = Q - W = -W$. $\Delta U = nC_V\Delta T$ where $C_V = \dfrac{5}{2}R$ for an ideal diatomic gas. Treat the air as N_2, which is diatomic.

(a) Estimate: Mass of the lie is 250 g = 0.250 kg.

(b) Estimate: Lid rises 0.50 cm = 0.0050 m.

(c) $W = mgh = (0.250 \text{ kg})(9.80 \text{ m/s}^2)(0.0050 \text{ m}) = 0.12 \text{ J}$.

(d) The expansion is adiabatic so $W = -\Delta U = -nC_V\Delta T = -n\left(\dfrac{5}{2}R\right)\Delta T$. Therefore

$\Delta T = -\dfrac{2W}{5nR} = -\dfrac{2(0.12 \text{ J})}{5\left(\dfrac{1}{11.2} \text{ mol}\right)(8.314 \text{ J/mol}\cdot\text{K})} = -0.06 \text{ K}$. The minus sign tells us that the temperature has dropped by 0.06 K = 0.06 C° ≈ 0.1 F.

EVALUATE: This is a very small temperature change, but based upon experience it seems reasonable.

19.37. **IDENTIFY:** We can read the values from the pV-diagram and apply the ideal gas law and the first law of thermodynamics.

SET UP: At each point $pV = nRT$, with $T = 85 \text{ K} + 273 \text{ K} = 358 \text{ K}$. For an isothermal process of an ideal gas, $W = nRT \ln(V_2/V_1)$. $\Delta U = nC_V\,\Delta T$ for any ideal gas process.

EXECUTE: **(a)** At point b, $p = 0.200 \text{ atm} = 2.026\times10^4 \text{ Pa}$ and $V = 0.100 \text{ m}^3$.

$$n = \frac{pV}{RT} = \frac{(2.026\times10^4 \text{ Pa})(0.100 \text{ m}^3)}{(8.315 \text{ J/mol}\cdot\text{K})(358 \text{ K})} = 0.681 \text{ moles.}$$

(b) n, R, and T are constant so $p_aV_a = p_bV_b$.

$$V_a = V_b\left(\frac{p_b}{p_a}\right) = (0.100 \text{ m}^3)\left(\frac{0.200 \text{ atm}}{0.600 \text{ atm}}\right) = 0.0333 \text{ m}^3.$$

(c) $W = nRT \ln(V_b/V_a) = (0.681 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(358 \text{ K})\ln\left(\dfrac{0.100 \text{ m}^3}{0.0333 \text{ m}^3}\right) = 2230 \text{ J} = 2.23 \text{ kJ}.$

W is positive and corresponds to work done by the gas.

(d) $\Delta U = nC_V\,\Delta T$ so for an isothermal process $(\Delta T = 0)$, $\Delta U = 0$.

EVALUATE: W is positive when the volume increases, so the area under the curve is positive. For *any* isothermal process, $\Delta U = 0$.

19.38. **IDENTIFY:** Segment ab is isobaric, bc is isochoric, and ca is isothermal.

SET UP: He is a monatomic gas so $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$. For any process of an ideal gas, $\Delta U = nC_V\Delta T$. For an isothermal process of an ideal gas, $\Delta U = 0$ so $Q = W = nRT\ln(V_2/V_1)$.

EXECUTE: **(a)** Apply $pV = nRT$ to states *a* and *c*. $T_a = T_c$ so *nRT* is constant and $p_aV_a = p_cV_c$.

$$p_a = p_c\left(\frac{V_c}{V_a}\right) = (2.0\times10^5\ \text{Pa})\left(\frac{0.040\ \text{m}^3}{0.010\ \text{m}^3}\right) = 8.0\times10^5\ \text{Pa}.$$

(b) $T_a = \dfrac{p_aV_a}{nR} = \dfrac{(8.0\times10^5\ \text{Pa})(0.010\ \text{m}^3)}{(3.25\ \text{mol})(8.315\ \text{J/mol}\cdot\text{K})} = 296\ \text{K};$

$$T_b = \frac{p_bV_b}{nR} = \frac{(8.0\times10^5\ \text{Pa})(0.040\ \text{m}^3)}{(3.25\ \text{mol})(8.315\ \text{J/mol}\cdot\text{K})} = 1184\ \text{K};$$

$$T_c = \frac{p_cV_c}{nR} = \frac{(2.0\times10^5\ \text{Pa})(0.040\ \text{m}^3)}{(3.25\ \text{mol})(8.315\ \text{J/mol}\cdot\text{K})} = 296\ \text{K} = T_a.$$

(c) <u>*ab*</u>: $Q = nC_p\Delta T = (3.25\ \text{mol})(\tfrac{5}{2})(8.315\ \text{J/mol}\cdot\text{K})(1184\ \text{K} - 296\ \text{K}) = 6.00\times10^4\ \text{J}$; heat enters the gas.

<u>*bc*</u>: $Q = nC_V\Delta T = (3.25\ \text{mol})(\tfrac{3}{2})(8.315\ \text{J/mol}\cdot\text{K})(296\ \text{K} - 1184\ \text{K}) = -3.60\times10^4\ \text{J}$; heat leaves the gas.

<u>*ca*</u>: $Q = nRT\ln\left(\dfrac{V_a}{V_c}\right) = (3.25\ \text{mol})(8.315\ \text{J/mol}\cdot\text{K})(296\ \text{K})\ln\left(\dfrac{0.010\ \text{m}^3}{0.040\ \text{m}^3}\right) = -1.11\times10^4\ \text{J}$; heat leaves the gas.

(d) <u>*ab*</u>: $\Delta U = nC_V\Delta T = (3.25\ \text{mol})(\tfrac{3}{2})(8.315\ \text{J/mol}\cdot\text{K})(1184\ \text{K} - 296\ \text{K}) = 3.60\times10^4\ \text{J}$; the internal energy increased.

<u>*bc*</u>: $\Delta U = nC_V\Delta T = (3.25\ \text{mol})(\tfrac{3}{2})(8.315\ \text{J/mol}\cdot\text{K})(296\ \text{K} - 1184\ \text{K}) = -3.60\times10^4\ \text{J}$; the internal energy decreased.

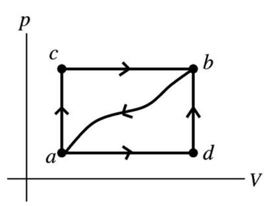
<u>*ca*</u>: $\Delta T = 0$ so $\Delta U = 0$.

EVALUATE: As we saw in (d), for any closed path on a *pV* diagram, $\Delta U = 0$ because we are back at the same values of *P*, *V*, and *T*.

19.39. **IDENTIFY:** Use $\Delta U = Q - W$ and the fact that ΔU is path independent.

$W > 0$ when the volume increases, $W < 0$ when the volume decreases, and $W = 0$ when the volume is constant. $Q > 0$ if heat flows into the system.

SET UP: The paths are sketched in Figure 19.39.



$Q_{acb} = +90.0\ \text{J}$ (positive since heat flows in)

$W_{acb} = +60.0\ \text{J}$ (positive since $\Delta V > 0$)

Figure 19.39

EXECUTE: **(a)** $\Delta U = Q - W$

ΔU is path independent; *Q* and *W* depend on the path.

$\Delta U = U_b - U_a$

This can be calculated for any path from *a* to *b*, in particular for path *acb*:

$\Delta U_{a\to b} = Q_{acb} - W_{acb} = 90.0\ \text{J} - 60.0\ \text{J} = 30.0\ \text{J}.$

Now apply $\Delta U = Q - W$ to path *adb*; $\Delta U = 30.0\ \text{J}$ for this path also.

$W_{adb} = +15.0\ \text{J}$ (positive since $\Delta V > 0$)

$\Delta U_{a\to b} = Q_{adb} - W_{adb}$ so $Q_{adb} = \Delta U_{a\to b} + W_{adb} = 30.0\ \text{J} + 15.0\ \text{J} = +45.0\ \text{J}.$

(b) Apply $\Delta U = Q - W$ to path *ba*: $\Delta U_{b\to a} = Q_{ba} - W_{ba}$

$W_{ba} = -35.0\text{ J}$ (negative since $\Delta V < 0$)

$\Delta U_{b\to a} = U_a - U_b = -(U_b - U_a) = -\Delta U_{a\to b} = -30.0\text{ J}$

Then $Q_{ba} = \Delta U_{b\to a} + W_{ba} = -30.0\text{ J} - 35.0\text{ J} = -65.0\text{ J}$.

($Q_{ba} < 0;$ the system liberates heat.)

(c) $U_a = 0,\ U_d = 8.0\text{ J}$

$\Delta U_{a\to b} = U_b - U_a = +30.0\text{ J}$, so $U_b = +30.0\text{ J}$.

<u>Process $a \to d$</u>:

$\Delta U_{a\to d} = Q_{ad} - W_{ad}$

$\Delta U_{a\to d} = U_d - U_a = +8.0\text{ J}$

$W_{adb} = +15.0\text{ J}$ and $W_{adb} = W_{ad} + W_{db}$. But the work W_{db} for the process $d \to b$ is zero since $\Delta V = 0$ for that process. Therefore $W_{ad} = W_{adb} = +15.0\text{ J}$.

Then $Q_{ad} = \Delta U_{a\to d} + W_{ad} = +8.0\text{ J} + 15.0\text{ J} = +23.0\text{ J}$ (positive implies heat absorbed).

<u>Process $d \to b$</u>:

$\Delta U_{d\to b} = Q_{db} - W_{db}$

$W_{db} = 0,$ as already noted.

$\Delta U_{d\to b} = U_b - U_d = 30.0\text{ J} - 8.0\text{ J} = +22.0\text{ J}$.

Then $Q_{db} = \Delta U_{d\to b} + W_{db} = +22.0\text{ J}$ (positive; heat absorbed).

EVALUATE: The signs of our calculated Q_{ad} and Q_{db} agree with the problem statement that heat is absorbed in these processes.

19.40. **IDENTIFY:** $\Delta U = Q - W$.

SET UP: $W = 0$ when $\Delta V = 0$.

EXECUTE: For each process, $Q = \Delta U + W$. No work is done in the processes *ab* and *dc*, and so $W_{bc} = W_{abc} = 450\text{ J}$ and $W_{ad} = W_{adc} = 120\text{ J}$. The heat flow for each process is: for *ab*, $Q = 90\text{ J}$. For *bc*, $Q = 440\text{ J} + 450\text{ J} = 890\text{ J}$. For *ad*, $Q = 180\text{ J} + 120\text{ J} = 300\text{ J}$. For *dc*, $Q = 350\text{ J}$. Heat is absorbed in each process. Note that the arrows representing the processes all point in the direction of increasing temperature (increasing *U*).

EVALUATE: ΔU is path independent so is the same for paths *adc* and *abc*.
$Q_{adc} = 300\text{ J} + 350\text{ J} = 650\text{ J}$. $Q_{abc} = 90\text{ J} + 890\text{ J} = 980\text{ J}$. *Q* and *W* are path dependent and are different for these two paths.

19.41. **IDENTIFY:** Use $pV = nRT$ to calculate T_c/T_a. Calculate ΔU and *W* and use $\Delta U = Q - W$ to obtain *Q*.

SET UP: For path *ac*, the work done is the area under the line representing the process in the *pV*-diagram.

EXECUTE: **(a)** $\dfrac{T_c}{T_a} = \dfrac{p_cV_c}{p_aV_a} = \dfrac{(1.0\times 10^5\text{ J})(0.060\text{ m}^3)}{(3.0\times 10^5\text{ J})(0.020\text{ m}^3)} = 1.00.\ T_c = T_a$.

(b) Since $T_c = T_a$, $\Delta U = 0$ for process *abc*. For *ab*, $\Delta V = 0$ and $W_{ab} = 0$. For *bc*, *p* is constant and $W_{bc} = p\Delta V = (1.0\times 10^5\text{ Pa})(0.040\text{ m}^3) = 4.0\times 10^3\text{ J}$. Therefore, $W_{abc} = +4.0\times 10^3\text{ J}$. Since $\Delta U = 0$, $Q = W = +4.0\times 10^3\text{ J}$. $4.0\times 10^3\text{ J}$ of heat flows into the gas during process *abc*.

(c) $W = \frac{1}{2}(3.0\times 10^5\text{ Pa} + 1.0\times 10^5\text{ Pa})(0.040\text{ m}^3) = +8.0\times 10^3\text{ J}$. $Q_{ac} = W_{ac} = +8.0\times 10^3\text{ J}$.

EVALUATE: The work done is path dependent and is greater for process *ac* than for process *abc*, even though the initial and final states are the same.

19.42. **IDENTIFY:** Apply the appropriate expression for W for each type of process. $pV = nRT$ and $C_p = C_V + R.$

SET UP: $R = 8.315$ J/mol·K.

EXECUTE: Path ac has constant pressure, so $W_{ac} = p\Delta V = nR\Delta T,$ and

$W_{ac} = nR(T_c - T_a) = (3\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(492\text{ K} - 300\text{ K}) = 4.789\times 10^3\text{ J}.$

Path cb is adiabatic $(Q = 0)$, so $W_{cb} = Q - \Delta U = -\Delta U = -nC_V\Delta T,$ and using $C_V = C_p - R,$

$W_{cb} = -n(C_p - R)(T_b - T_c) = -(3\text{ mol})(29.1\text{ J/mol}\cdot\text{K} - 8.3145\text{ J/mol}\cdot\text{K})(600\text{ K} - 492\text{ K}) = -6.735\times 10^3\text{ J}.$

Path ba has constant volume, so $W_{ba} = 0.$ So the total work done is

$$W = W_{ac} + W_{cb} + W_{ba} = 4.789\times 10^3\text{ J} - 6.735\times 10^3\text{ J} + 0 = -1.95\times 10^3\text{ J}.$$

EVALUATE: $W > 0$ when $\Delta V > 0,$ $W < 0$ when $\Delta V < 0$ and $W = 0$ when $\Delta V = 0.$

19.43. **IDENTIFY:** Segment ab is isochoric, bc is isothermal, and ca is isobaric.

SET UP: For bc, $\Delta T = 0,$ $\Delta U = 0,$ and $Q = W = nRT\ln(V_c/V_b).$ For ideal H_2 (diatomic), $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R.$ $\Delta U = nC_V\ \Delta T$ for any process of an ideal gas.

EXECUTE: **(a)** $T_b = T_c.$ For states b and c, $pV = nRT = \text{constant}$ so $p_bV_b = p_cV_c$ and

$$V_c = V_b\left(\frac{p_b}{p_c}\right) = (0.20\text{ L})\left(\frac{2.0\text{ atm}}{0.50\text{ atm}}\right) = 0.80\text{ L}.$$

(b) $T_a = \dfrac{p_aV_a}{nR} = \dfrac{(0.50\text{ atm})(1.013\times 10^5\text{ Pa/atm})(0.20\times 10^{-3}\text{ m}^3)}{(0.0040\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = 305\text{ K}.$ $V_a = V_b$ so for states a and b,

$$\frac{T}{p} = \frac{V}{nR} = \text{constant so } \frac{T_a}{p_a} = \frac{T_b}{p_b}.\ T_b = T_c = T_a\left(\frac{p_b}{p_a}\right) = (305\text{ K})\left(\frac{2.0\text{ atm}}{0.50\text{ atm}}\right) = 1220\text{ K};\ T_c = 1220\text{ K}.$$

(c) <u>ab</u>: $Q = nC_V\ \Delta T = n(\frac{5}{2}R)\ \Delta T,$ which gives

$Q = (0.0040\text{ mol})(\frac{5}{2})(8.315\text{ J/mol}\cdot\text{K})(1220\text{ K} - 305\text{ K}) = +76\text{ J}.$ Q is positive and heat goes into the gas.

<u>ca</u>: $Q = nC_p\ \Delta T = n(\frac{7}{2}R)\ \Delta T,$ which gives

$Q = (0.0040\text{ mol})(\frac{7}{2})(8.315\text{ J/mol}\cdot\text{K})(305\text{ K} - 1220\text{ K}) = -107\text{ J}.$ Q is negative and heat comes out of the gas.

<u>bc</u>: $Q = W = nRT\ln(V_c/V_b),$ which gives

$Q = (0.0040\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(1220\text{ K})\ln(0.80\text{ L}/0.20\text{ L}) = 56\text{ J}.$ Q is positive and heat goes into the gas.

(d) <u>ab</u>: $\Delta U = nC_V\Delta T = n(\frac{5}{2}R)\Delta T,$ which gives

$\Delta U = (0.0040\text{ mol})(\frac{5}{2})(8.315\text{ J/mol}\cdot\text{K})(1220\text{ K} - 305\text{ K}) = +76\text{ J}.$ The internal energy increased.

<u>bc</u>: $\Delta T = 0$ so $\Delta U = 0.$ The internal energy does not change.

<u>ca</u>: $\Delta U = nC_V\Delta T = n(\frac{5}{2}R)\Delta T,$ which gives

$\Delta U = (0.0040\text{ mol})(\frac{5}{2})(8.315\text{ J/mol}\cdot\text{K})(305\text{ K} - 1220\text{ K}) = -76\text{ J}.$ The internal energy decreased.

EVALUATE: The net internal energy change for the complete cycle $a \to b \to c \to a$ is $\Delta U_{\text{tot}} = +76\text{ J} + 0 + (-76\text{ J}) = 0.$ For any complete cycle the final state is the same as the initial state and the net internal energy change is zero. For the cycle the net heat flow is $Q_{\text{tot}} = +76\text{ J} + (-107\text{ J}) + 56\text{ J} = +25\text{ J}.$ $\Delta U_{\text{tot}} = 0$ so $Q_{\text{tot}} = W_{\text{tot}}.$ The net work done in the cycle is positive and this agrees with our result that the net heat flow is positive.

19.44 **IDENTIFY:** The segments *ab* and *bc* are not any of the familiar ones, such as isothermal, isobaric, or isochoric, but *ac* is isobaric.

SET UP: For helium, $C_V = 12.47\text{ J/mol}\cdot\text{K}$ and $C_p = 20.78\text{ J/mol}\cdot\text{K}$. $\Delta U = Q - W$. *W* is the area under the *p* versus *V* curve. $\Delta U = nC_V\Delta T$ for any process of an ideal gas.

EXECUTE: **(a)** $W = \frac{1}{2}(1.0\times10^5\text{ Pa} + 3.5\times10^5\text{ Pa})(0.0060\text{ m}^3 - 0.0020\text{ m}^3)$

$+\frac{1}{2}(1.0\times10^5\text{ Pa} + 3.5\times10^5\text{ Pa})(0.0100\text{ m}^3 - 0.0060\text{ m}^3) = 1800\text{ J}.$

Find $\Delta T = T_c - T_a$. *p* is constant so $\Delta T = \dfrac{p\Delta V}{nR} = \dfrac{(1.0\times10^5\text{ Pa})(0.0100\text{ m}^3 - 0.0020\text{ m}^3)}{\left(\frac{1}{3}\text{ mol}\right)(8.315\text{ J/mol}\cdot\text{K})} = 289\text{ K}.$

Then

$\Delta U = nC_V\Delta T = (\frac{1}{3}\text{ mol})(12.47\text{ J/mol}\cdot\text{K})(289\text{ K}) = 1.20\times10^3\text{ J}.$

$Q = \Delta U + W = 1.20\times10^3\text{ J} + 1800\text{ J} = 3.00\times10^3\text{ J}$. $Q > 0$, so this heat is transferred into the gas.

(b) This process is isobaric, so $Q = nC_p\Delta T = \left(\frac{1}{3}\text{ mol}\right)(20.78\text{ J/mol}\cdot\text{K})(289\text{ K}) = 2.00\times10^3\text{ J}$. $Q > 0$, so this heat is transferred into the gas.

(c) Q is larger in part (a).

EVALUATE: ΔU is the same in parts (a) and (b) because the initial and final states are the same, but in (a) more work is done.

19.45. **IDENTIFY** and **SET UP:** We have information on the pressure and volume of the gas during the process, but we know almost nothing else about the gas. We do know that the first law of thermodynamics must apply to the gas during this process, so $Q = \Delta U + W$, and the work done by the g as is $W = \int_{V_1}^{V_2} p\,dV$. If *W* is positive, the gas does work, but if *W* is negative, work is done on the gas.

EXECUTE: **(a)** Figure 19.45 shows the *pV*-diagram for this process. On the *pV*-diagram, we see that the graph is a closed figure; the gas begins and ends in the same state.

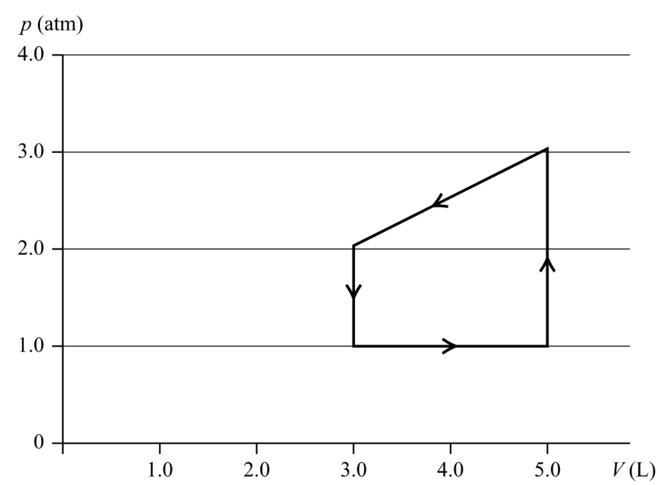


Figure 19.45

(b) Applying $Q = \Delta U + W$, we see that $\Delta U = 0$ because the gas ends up at the same state from which it began. Therefore $Q = W$. $W = \int_{V_1}^{V_2} p\,dV$, so the work is the area under the curve on a *pV*-diagram. For a closed cycle such as this one, the work is the area enclosed within the diagram. We calculate this work geometrically: $|W|$ = area (rectangle) + area (triangle) = (2.0 L)(1.0 atm) + $\frac{1}{2}$ (2.0 L)(1.0 atm) = 3.0

$\text{L}\cdot\text{atm} = 300$ J. But the net work is negative, so $Q = -3.0\ \text{L}\cdot\text{atm} = -300$ J. Since Q is negative, heat flows out of the gas.

EVALUATE: We know that the work is negative because in the upper part of the diagram, the volume is decreasing, which means that the gas is being compressed.

19.46. IDENTIFY: For a constant pressure process, $Q = nC_p\Delta T$. $\Delta U = Q - W$. $\Delta U = nC_V\Delta T$ for any ideal gas process.

SET UP: For N_2, $C_V = 20.76\ \text{J/mol}\cdot\text{K}$ and $C_p = 29.07\ \text{J/mol}\cdot\text{K}$. $Q < 0$ if heat comes out of the gas.

EXECUTE: **(a)** $n = \dfrac{Q}{C_p\Delta T} = \dfrac{-2.5\times10^4\ \text{J}}{(29.07\ \text{J/mol}\cdot\text{K})(-40.0\ \text{K})} = 21.5\ \text{mol}.$

(b) $\Delta U = nC_V\Delta T = Q(C_V/C_p) = (-2.5\times10^4\ \text{J})(20.76/29.07) = -1.79\times10^4\ \text{J}.$

(c) $W = Q - \Delta U = -7.15\times10^3\ \text{J}.$

(d) ΔU is the same for both processes, and if $\Delta V = 0$, $W = 0$ and $Q = \Delta U = -1.79\times10^4$ J.

EVALUATE: For a given ΔT, Q is larger in magnitude when the pressure is constant than when the volume is constant.

19.47. IDENTIFY: $pV = nRT$. For an isothermal process $W = nRT\ln(V_2/V_1)$. For a constant pressure process, $W = p\Delta V$.

SET UP: $1\ \text{L} = 10^{-3}\ \text{m}^3$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.47.

(b) At constant temperature, the product pV is constant, so

$V_2 = V_1(p_1/p_2) = (1.5\ \text{L})\left(\dfrac{1.00\times10^5\ \text{Pa}}{2.50\times10^4\ \text{Pa}}\right) = 6.00\ \text{L}.$ The final pressure is given as being the same as $p_3 = p_2 = 2.5\times10^4$ Pa. The final volume is the same as the initial volume, so $T_3 = T_1(p_3/p_1) = 75.0\ \text{K}.$

(c) Treating the gas as ideal, the work done in the first process is $W = nRT\ln(V_2/V_1) = p_1V_1\ln(p_1/p_2)$.

$$W = (1.00\times10^5\ \text{Pa})(1.5\times10^{-3}\ \text{m}^3)\ln\left(\frac{1.00\times10^5\ \text{Pa}}{2.50\times10^4\ \text{Pa}}\right) = 208\ \text{J}.$$

For the second process, $W = p_2(V_3 - V_2) = p_2(V_1 - V_2) = p_2V_1[1 - (p_1/p_2)]$.

$$W = (2.50\times10^4\ \text{Pa})(1.5\times10^{-3}\ \text{m}^3)\left(1 - \frac{1.00\times10^5\ \text{Pa}}{2.50\times10^4\ \text{Pa}}\right) = -113\ \text{J}.$$

The total work done is $208\ \text{J} - 113\ \text{J} = 95\ \text{J}$.

(d) Heat at constant volume. No work would be done by the gas or on the gas during this process.

EVALUATE: When the volume increases, $W > 0$. When the volume decreases, $W < 0$.

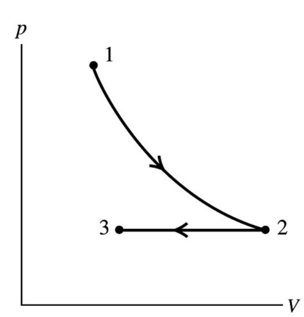


Figure 19.47

19.48. **IDENTIFY:** We are looking at the effect of gas compression on the buoyant force as a gas container descends in the ocean.

SET UP: As the inverted drum descends in the ocean, the water pressure increases and compresses the air in the drum. The pressure at a depth y in the ocean is $p = p_0 + \rho g y$. The temperature decreases linearly with depth from 23.0°C at the water surface to 3.0°C at a depth of 1000 m. The graph in Fig. 19.48 shows the temperature T as a function of depth y. This graph shows a line with a slope of –(20.0 C°)/(1000 m) = –0.0200 C°/m. The line intercepts the T axis at 23.0°C (y = 0 at the surface). Using the slope-intercept form of the equation, the equation is $T = T_0 - By$, where T_0 = 23.0°C = 296 K and B = 0.0200 C°/m.

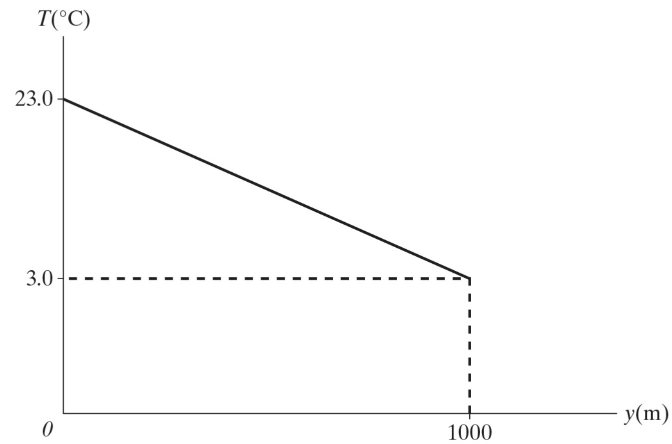


Figure 19.48

EXECUTE: **(a)** and **(b)** We want to find the depth for neutral buoyancy of the drum and the volume of the air in the drum at that depth. For neutral buoyancy, the buoyant force B on the drum must be equal to its weight mg, so $B = mg$. By Archimedes's principle, the buoyant force is equal to the weight of water displaced by the drum. At depth y the air in the drum has volume V_y, so $\rho g V_y = mg$. In this equation ρ is the density of seawater (1025 kg/m^3) and V_y is the volume of water displaced by the drum at depth y, which is the same as the volume of air in the drum. From $\rho g V_y = mg$ we get

$$V_y = \frac{m}{\rho} = \frac{17.3\text{ kg}}{1025\text{ kg/m}^3} = 0.0169\text{ m}^3.$$

We now want to find the depth y at which the air volume in the drum is 0.0169 m^3. At the water surface the volume of the air is the volume of the drum, which is $V_0 = \pi r^2 L$, which gives $V_0 = \pi(0.305\text{ m})^2(0.880\text{ m}) = 0.2572\text{ m}^3$. Therefore at this depth y we know $\frac{V_y}{V_0} = \frac{0.0169\text{ m}^3}{0.2572\text{ m}^3} = 0.06571$. We now look at the gas pressure at depth y. We do this because that pressure must be equal to the water pressure at the same depth. Using $pV = nRT$ gives

$$\frac{p_y}{p_0} = \left(\frac{T_y}{T_0}\right)\left(\frac{V_0}{V_y}\right) = \left(\frac{T_0 - By}{T_0}\right)\left(\frac{1}{0.06571}\right) = 15.219\left(\frac{T_0 - By}{T_0}\right),$$ from which we get

$$p_y = (15.219)p_0\left(\frac{T_0 - By}{T_0}\right).$$ Now equate this pressure to the water pressure at depth y, which is

$p = p_0 + \rho g y$, giving $p_0 + \rho g y = (15.219)p_0\left(\frac{T_0 - By}{T_0}\right)$. This gives $y = \frac{14.219 p_0}{\rho g + 15.219 p_0 B / T_0}$. Using $p_0 = 101\times10^3$ Pa, T_0 = 296 K, B = 0.0200 K/m and ρ = 1025 kg/m^3, we get y = 142 m.

EVALUATE: The compression of the air in the drum is *not* adiabatic because the air is in contact with the water so heat can flow out of the air as it moves to colder water at depth. In this process, none of the variables (p, V, and T) remained constant.

19.49. IDENTIFY: For an adiabatic process of an ideal gas, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$. $pV = nRT$.

SET UP: For air, $\gamma = 1.40 = \frac{7}{5}$.

EXECUTE: **(a)** As the air moves to lower altitude its density increases; under an adiabatic compression, the temperature rises. If the wind is fast-moving, Q is not as likely to be significant, and modeling the process as adiabatic (no heat loss to the surroundings) is more accurate.

(b) $V = \dfrac{nRT}{p}$, so $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ gives $T_1^{\gamma}p_1^{1-\gamma} = T_2^{\gamma}p_2^{1-\gamma}$. The temperature at the higher pressure is

$T_2 = T_1(p_1/p_2)^{(\gamma-1)/\gamma} = (258.15\text{ K})\left[(8.12\times10^4\text{ Pa})/(5.60\times10^4\text{ Pa})\right]^{2/7} = 287.1\text{ K} = 13.9°\text{C}$ so the temperature would rise by $11.9\ \text{C}°$.

EVALUATE: In an adiabatic compression, $Q = 0$ but the temperature rises because of the work done on the gas.

19.50. IDENTIFY: The process is adiabatic. Apply $p_1V_1^{\gamma} = p_2V_2^{\gamma}$ and $pV = nRT$. $Q = 0$ so

$$\Delta U = -W = -\frac{1}{\gamma-1}(p_1V_1 - p_2V_2).$$

SET UP: For ideal monatomic helium, $\gamma = 5/3 = 1.667$. $p_1 = 1.00\text{ atm} = 1.013\times10^5\text{ Pa}$.

$V_1 = 2.00\times10^3\text{ m}^3$. $p_2 = 0.900\text{ atm} = 9.117\times10^4\text{ Pa}$. $T_1 = 288.15\text{ K}$.

EXECUTE: **(a)** $V_2^{\gamma} = V_1^{\gamma}\left(\dfrac{p_1}{p_2}\right)$. $V_2 = V_1\left(\dfrac{p_1}{p_2}\right)^{1/\gamma} = (2.00\times10^3\text{ m}^3)\left(\dfrac{1.00\text{ atm}}{0.900\text{ atm}}\right)^{1/1.67} = 2.13\times10^3\text{ m}^3$.

(b) $pV = nRT$ gives $\dfrac{T_1}{p_1V_1} = \dfrac{T_2}{p_2V_2}$.

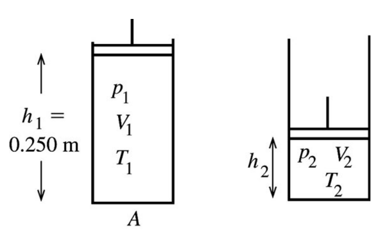
$$T_2 = T_1\left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right) = (288.15\text{ K})\left(\frac{0.900\text{ atm}}{1.00\text{ atm}}\right)\left(\frac{2.13\times10^3\text{ m}^3}{2.00\times10^3\text{ m}^3}\right) = 276.2\text{ K} = 3.0°\text{C}.$$

(c) $\Delta U = -\dfrac{(1.013\times10^5\text{ Pa})(2.00\times10^3\text{ m}^3) - (9.117\times10^4\text{ Pa})(2.13\times10^3\text{ m}^3)}{1.667-1} = -1.25\times10^7\text{ J}$.

EVALUATE: The internal energy decreases when the temperature decreases.

19.51. IDENTIFY: Assume that the gas is ideal and that the process is adiabatic. Apply $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ and $p_1V_1^{\gamma} = p_2V_2^{\gamma}$ to relate pressure and volume and temperature and volume. The distance the piston moves is related to the volume of the gas. Use $W = nC_V(T_1 - T_2)$ to calculate W.

(a) SET UP: $\gamma = C_p/C_V = (C_V + R)/C_V = 1 + R/C_V = 1.40$. The two positions of the piston are shown in Figure 19.51.



$p_1 = 1.01\times10^5\text{ Pa}$

$p_2 = 3.80\times10^5\text{ Pa} + p_{\text{air}} = 4.81\times10^5\text{ Pa}$

$V_1 = h_1A$

$V_2 = h_2A$

Figure 19.51

EXECUTE: For an adiabatic process of an ideal gas, $p_1V_1^\gamma = p_2V_2^\gamma$.

$p_1h_1^\gamma A^\gamma = p_2h_2^\gamma A^\gamma$

$$h_2 = h_1\left(\frac{p_1}{p_2}\right)^{1/\gamma} = (0.250\text{ m})\left(\frac{1.01\times10^5\text{ Pa}}{4.81\times10^5\text{ Pa}}\right)^{1/1.40} = 0.08199\text{ m}$$

The piston has moved a distance $h_1 - h_2 = 0.250\text{ m} - 0.08199\text{ m} = 0.168\text{ m}$.

(b) SET UP: $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_1h_1^{\gamma-1}A^{\gamma-1} = T_2h_2^{\gamma-1}A^{\gamma-1}$

EXECUTE: $T_2 = T_1\left(\dfrac{h_1}{h_2}\right)^{\gamma-1} = 300.1\text{ K}\left(\dfrac{0.250\text{ m}}{0.08199\text{ m}}\right)^{0.40} = 468.7\text{ K} = 196°\text{C}$.

(c) SET UP and **EXECUTE:** $W = nC_V(T_1 - T_2)$ gives

$W = (20.0\text{ mol})(20.8\text{ J/mol}\cdot\text{K})(300.1\text{ K} - 468.7\text{ K}) = -7.01\times10^4\text{ J} = -70.1\text{ kJ}$. This is the work done *by* the gas. The work done *on* the gas by the pump is +70.1 kJ.

EVALUATE: In an adiabatic compression of an ideal gas the temperature increases. In any compression the work W done by the gas is negative.

19.52. IDENTIFY: For constant pressure, $W = p\Delta V$. For an adiabatic process of an ideal gas,

$W = \dfrac{C_V}{R}(p_1V_1 - p_2V_2)$ and $p_1V_1^\gamma = p_2V_2^\gamma$.

SET UP: $\gamma = \dfrac{C_p}{C_V} = \dfrac{C_p + C_V}{C_V} = 1 + \dfrac{R}{C_V}$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.52.

(b) The work done is $W = p_0(2V_0 - V_0) + \dfrac{C_V}{R}(p_0(2V_0) - p_3(4V_0))$. $p_3 = p_0(2V_0/4V_0)^\gamma$ and so

$W = p_0V_0\left[1 + \dfrac{C_V}{R}(2 - 2^{2-\gamma})\right]$. Note that p_0 is the absolute pressure.

(c) The most direct way to find the temperature is to find the ratio of the final pressure and volume to the original and treat the air as an ideal gas. $p_3 = p_2\left(\dfrac{V_2}{V_3}\right)^\gamma = p_1\left(\dfrac{V_2}{V_3}\right)^\gamma$, since $p_1 = p_2$. Then

$$T_3 = T_0\frac{p_3V_3}{p_1V_1} = T_0\left(\frac{V_2}{V_3}\right)^\gamma\left(\frac{V_3}{V_1}\right) = T_0\left(\frac{1}{2}\right)^\gamma 4 = T_0(2)^{2-\gamma}.$$

(d) Since $n = \dfrac{p_0V_0}{RT_0}, Q = \dfrac{p_0V_0}{RT_0}(C_V + R)(2T_0 - T_0) = p_0V_0\left(\dfrac{C_V}{R} + 1\right)$. This amount of heat flows into the gas, since $Q > 0$.

EVALUATE: In the isobaric expansion the temperature doubles and in the adiabatic expansion the temperature decreases. If the gas is diatomic, with $\gamma = \frac{7}{5}$, $2 - \gamma = \frac{3}{5}$ and $T_3 = 1.52T_0$, $W = 2.21p_0V_0$, and $Q = 3.50p_0V_0$. $\Delta U = 1.29p_0V_0$. $\Delta U > 0$ and this is consistent with an increase in temperature.

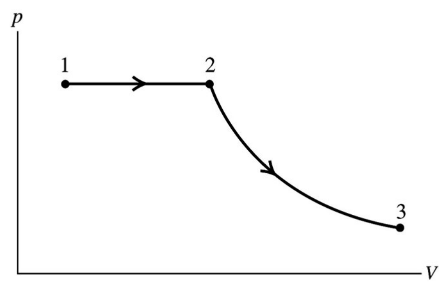


Figure 19.52

19.53. IDENTIFY: In each case calculate either ΔU or Q for the specific type of process and then apply the first law.

(a) SET UP: <u>Isothermal</u>: $(\Delta T = 0)$ $\Delta U = Q - W$; $W = +450$ J. For any process of an ideal gas, $\Delta U = nC_V \Delta T$.

EXECUTE: Therefore, for an ideal gas, if $\Delta T = 0$ then $\Delta U = 0$ and $Q = W = +450$ J.

(b) SET UP: <u>Adiabatic</u>: $(Q = 0)$

$\Delta U = Q - W;$ $W = +450$ J.

EXECUTE: $Q = 0$ says $\Delta U = -W = -450$ J.

(c) SET UP: <u>Isobaric</u>: $\Delta p = 0$

Use W to calculate ΔT and then calculate Q.

EXECUTE: $W = p\Delta V = nR\Delta T$; $\Delta T = W/nR$

$Q = nC_p \Delta T$ and for a monatomic ideal gas $C_p = \frac{5}{2}R$.

Thus $Q = n\frac{5}{2}R\Delta T = (5Rn/2)(W/nR) = 5W/2 = +1125$ J.

$\Delta U = nC_V \Delta T$ for any ideal gas process and $C_V = C_p - R = \frac{3}{2}R$.

Thus $\Delta U = 3W/2 = +675$ J.

EVALUATE: 450 J of energy leaves the gas when it performs expansion work. In the isothermal process this energy is replaced by heat flow into the gas and the internal energy remains the same. In the adiabatic process the energy used in doing the work decreases the internal energy. In the isobaric process 1125 J of heat energy enters the gas, 450 J leaves as the work done and 675 J remains in the gas as increased internal energy.

19.54. IDENTIFY: $pV = nRT$. For the isobaric process, $W = p\Delta V = nR\Delta T$. For the isothermal process, $W = nRT\ln\left(\frac{V_2}{V_1}\right)$.

SET UP: $R = 8.315$ J/mol·K.

EXECUTE: **(a)** The pV diagram for these processes is sketched in Figure 19.54.

(b) Find T_2. For process $1 \to 2$, n, R and p are constant so $\frac{T}{V} = \frac{p}{nR} = \text{constant}$. $\frac{T_1}{V_1} = \frac{T_2}{V_2}$ and

$T_2 = T_1\left(\frac{V_2}{V_1}\right) = (355\text{ K})(2) = 710\text{ K}.$

(c) The maximum pressure is for state 3. For process $2 \to 3$, n, R and T are constant. $p_2V_2 = p_3V_3$ and

$p_3 = p_2\left(\frac{V_2}{V_3}\right) = (2.40\times 10^5\text{ Pa})(2) = 4.80\times 10^5\text{ Pa}.$

(d) <u>Proces $1 \to 2$</u>: $W = p\Delta V = nR\Delta T = (0.250\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(710\text{ K} - 355\text{ K}) = 738\text{ K}.$

<u>Proces 2 → 3</u>: $W = nRT\ln\left(\frac{V_3}{V_2}\right) = (0.250 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(710 \text{ K})\ln\left(\frac{1}{2}\right) = -1023 \text{ J}.$

<u>Proces 3 → 1</u>: $\Delta V = 0$ and $W = 0.$

The total work done is $738 \text{ J} + (-1023 \text{ J}) = -285 \text{ J}.$ This is the work done by the gas. The work done on the gas is 285 J.

EVALUATE: The final pressure and volume are the same as the initial pressure and volume, so the final state is the same as the initial state. For the cycle, $\Delta U = 0$ and $Q = W = -285 \text{ J}.$ During the cycle, 285 J of heat energy must leave the gas.

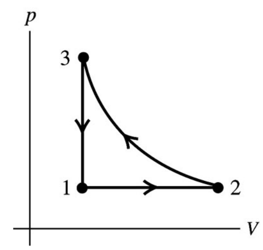


Figure 19.54

19.55. **IDENTIFY** and **SET UP:** Use the ideal gas law, the first law of thermodynamics, and expressions for Q and W for specific types of processes.

EXECUTE: **(a)** initial expansion (state 1 → state 2)

$p_1 = 2.40\times10^5 \text{ Pa},\ T_1 = 355 \text{ K},\ p_2 = 2.40\times10^5 \text{ Pa},\ V_2 = 2V_1$

$pV = nRT;\ T/V = p/nR = \text{constant, so } T_1/V_1 = T_2/V_2 \text{ and } T_2 = T_1(V_2/V_1) = 355 \text{ K}(2V_1/V_1) = 710 \text{ K}$

$\Delta p = 0$ so $W = p\Delta V = nR\Delta T = (0.250 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})(710 \text{ K} - 355 \text{ K}) = +738 \text{ J}$

$Q = nC_p\Delta T = (0.250 \text{ mol})(29.17 \text{ J/mol}\cdot\text{K})(710 \text{ K} - 355 \text{ K}) = +2590 \text{ J}$

$\Delta U = Q - W = 2590 \text{ J} - 738 \text{ J} = 1850 \text{ J}$

(b) At the beginning of the final cooling process (cooling at constant volume), $T = 710$ K. The gas returns to its original volume and pressure, so also to its original temperature of 355 K.

$\Delta V = 0$ so $W = 0$

$Q = nC_V\Delta T = (0.250 \text{ mol})(20.85 \text{ J/mol}\cdot\text{K})(355 \text{ K} - 710 \text{ K}) = -1850 \text{ J}$

$\Delta U = Q - W = -1850 \text{ J}.$

(c) For any ideal gas process $\Delta U = nC_V\Delta T.$ For an isothermal process $\Delta T = 0,$ so $\Delta U = 0.$

EVALUATE: The three processes return the gas to its initial state, so $\Delta U_{\text{total}} = 0;$ our results agree with this.

19.56. **IDENTIFY:** $pV = nRT.$ For an adiabatic process of an ideal gas, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}.$

SET UP: For N_2, $\gamma = 1.40.$

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.56.

(b) At constant pressure, halving the volume halves the Kelvin temperature, and the temperature at the beginning of the adiabatic expansion is 150 K. The volume doubles during the adiabatic expansion, and from Eq. (19.22), the temperature at the end of the expansion is $(150 \text{ K})(1/2)^{0.40} = 114 \text{ K}.$

(c) The minimum pressure occurs at the end of the adiabatic expansion (state 3). During the final heating the volume is held constant, so the minimum pressure is proportional to the Kelvin temperature, $p_{\min} = (1.80\times10^5 \text{ Pa})(114\text{K}/300 \text{ K}) = 6.82\times10^4 \text{ Pa}.$

EVALUATE: In the adiabatic expansion the temperature decreases.

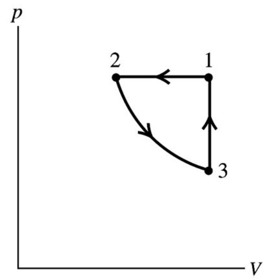


Figure 19.56

19.57. **IDENTIFY:** Use the appropriate expressions for Q, W, and ΔU for each type of process. $\Delta U = Q - W$ can also be used.

SET UP: For N_2, $C_V = 20.76 \text{ J/mol} \cdot \text{K}$ and $C_p = 29.07 \text{ J/mol} \cdot \text{K}$.

EXECUTE: **(a)** $W = p\Delta V = nR\Delta T = (0.150 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(-150 \text{ K}) = -187 \text{ J}$,

$Q = nC_p\Delta T = (0.150 \text{ mol})(29.07 \text{ mol} \cdot \text{K})(-150 \text{ K}) = -654 \text{ J}$, $\Delta U = Q - W = -467 \text{ J}$.

(b) From Eq. (19.26), using the expression for the temperature found in Problem 19.56,

$W = \dfrac{1}{0.40}(0.150 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(150 \text{ K})[1 - (1/2^{0.40})] = 113 \text{ J}$. $Q = 0$ for an adiabatic process,

and $\Delta U = Q - W = -W = -113 \text{ J}$.

(c) $\Delta V = 0$, so $W = 0$. Using the temperature change as found in Problem 19.66 part (b),

$Q = nC_V\Delta T = (0.150 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 113.7 \text{ K}) = 580 \text{ J}$ and $\Delta U = Q - W = Q = 580 \text{ J}$.

EVALUATE: For each process we could also use $\Delta U = nC_V\Delta T$ to calculate ΔU.

19.58. **IDENTIFY:** Use the appropriate expression for W for each type of process.

SET UP: For a monatomic ideal gas, $\gamma = 5/3$ and $C_V = 3R/2$.

EXECUTE: **(a)** $W = nRT \ln(V_2/V_1) = nRT \ln(3) = 3.29 \times 10^3 \text{ J}$.

(b) $Q = 0$ so $W = -\Delta U = -nC_V\Delta T$. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ gives $T_2 = T_1(1/3)^{2/3}$. Then

$W = nC_VT_1(1 - (1/3^{2/3})) = 2.33 \times 10^3 \text{ J}$.

(c) $V_2 = 3V_1$, so $W = p\Delta V = 2pV_1 = 2nRT_1 = 6.00 \times 10^3 \text{ J}$.

(d) Each process is shown in Figure 19.58. The most work done is in the isobaric process, as the pressure is maintained at its original value. The least work is done in the adiabatic process.

(e) The isobaric process involves the most work and the largest temperature increase, and so requires the most heat. Adiabatic processes involve no heat transfer, and so the magnitude is zero.

(f) The isobaric process doubles the Kelvin temperature, and so has the largest change in internal energy. The isothermal process necessarily involves no change in internal energy.

EVALUATE: The work done is the area under the path for the process in the pV-diagram. Figure 19.58 shows that the work done is greatest in the isobaric process and least in the adiabatic process.

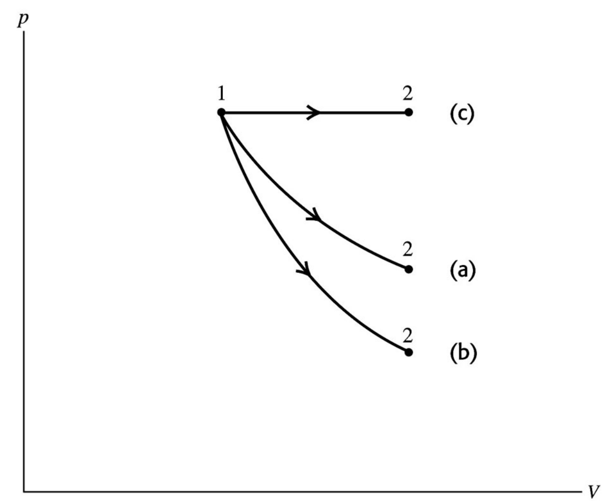


Figure 19.58

19.59. **IDENTIFY:** For an adiabatic process, no heat enters or leaves the gas. An isochoric process takes place at constant volume, and an isoboric process takes place at constant pressure. The first law of thermodynamics applies.

SET UP: For any process, including an isochoric process, $Q = nC_V\Delta T$, and for an isobaric process, $Q = nC_p\Delta T$. $Q = \Delta U + W$.

EXECUTE: **(a)** Process *a* is adiabatic since no heat goes into or out of the system. In processes *b* and *c*, the temperature change is the same, but more heat goes into the gas for process *c*. Since the change in internal energy is the same for both *b* and *c*, some of the heat in *c* must be doing work, but not in *b*. Therefore *b* is isochoric and *c* is isobaric. To summarize: *a* is adiabatic, *b* is isochoric, *c* is isobaric.

(b) $Q_b = nC_V\Delta T$ and $Q_c = nC_p\Delta T$. Subtracting gives

$Q_c - Q_b = nC_p\ \Delta T - nC_V\Delta T = n(C_p - C_V)\ \Delta T = nR\Delta T = 20$ J. Solving for ΔT gives

$\Delta T = (20\text{ J})/nR = (20\text{ J})/[(0.300\text{ mol})(8.314\text{ J/mol}\cdot\text{K})] = 8.0\ \text{C}°$, so $T_2 = 20.0°\text{C} + 8.0°\text{C} = 28.0°\text{C}$.

(c) $\dfrac{Q_c}{Q_b} = \dfrac{nC_p\Delta T}{nC_V\Delta T} = \dfrac{C_p}{C_V} = \gamma = \dfrac{50\text{ J}}{30\text{ J}} = \dfrac{5}{3}$. Since $\gamma = 5/3$, the gas must be monatomic, in which case we have $C_V = 3/2\ R$ and $C_p = 5/2\ R$. Therefore

Process *a*: $Q = \Delta U + W$ gives $0 = nC_V\Delta T + W$.

$W = -n(3/2\ R)\ \Delta T = -(0.300\text{ mol})(3/2)(8.314\text{ J/mol}\cdot\text{K})(8.0\text{ K}) = -30$ J.

Process *b*: The volume is constant, so $W = 0$.

Process *c*: $Q = \Delta U + W$. ΔU is the same as for process *a* because ΔT is the same, so we have $50\text{ J} = 30\text{ J} + W$, which gives $W = 20$ J.

(d) The greatest work has the greatest volume change. Using the results of part (c), process *a* has the greatest amount of work and hence the greatest volume change.

(e) The volume is increasing if *W* is positive. Therefore

Process *a*: *W* is negative, so the volume decreases.

Process *b*: $W = 0$ so the volume stays the same.

Process *c*: *W* is positive, so the volume increases.

EVALUATE: In Process *a*, no heat enters the gas, yet its temperature increases. This means that work must have been done on the gas, as we found.

19.60. **IDENTIFY** and **SET UP:** The cylinder is insulated, so no heat can go into or out of the gas, which makes this an adiabatic process. For an adiabatic process, $pV^\gamma = \text{constant}$, and the ideal gas law, $pV = nRT$, also applies.

EXECUTE: **(a)** The graph of log p versus log V is shown in Figure 19.60.

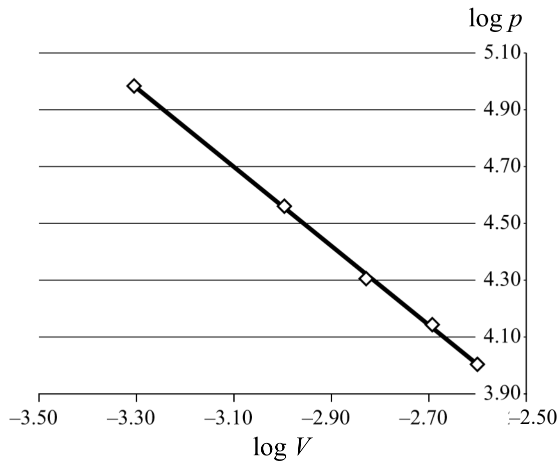


Figure 19.60

For an adiabatic process, $pV^{\gamma} = \text{constant}$. Taking logs of both sides of this equation gives $\log(pV^{\gamma}) = \log p + \log V^{\gamma} = \log p + \gamma \log V = \log(\text{constant})$. Solving for log p gives $\log p = -\gamma \log V + \log(\text{constant})$. Therefore a graph of log p versus log V should be a straight line having a slope equal to $-\gamma$.

(b) The equation of the best-fit line in the graph is log p = –1.3946 log$V$ + 0.381, so $-\gamma$ = –1.3946, so γ = 1.4. This is the adiabatic constant for a diatomic gas, so this gas must be diatomic.

(c) The ideal gas law gives $pV = nRT$, so $nR = pV/T$. Using the first set of data points in the table gives $nR = (0.101 \text{ atm})(2.50 \text{ L})/(293.15 \text{ K}) = 8.61 \times 10^{-4} \text{ L} \cdot \text{atm/K}$, so $1/nR = 1160 \text{ K/L} \cdot \text{atm}$. Using this number, we can calculate T for the rest of the pairs of points in the table. For example, for the next set of points, we have $T = pV/nR = (1160 \text{ K/L} \cdot \text{atm})(2.02 \text{ L})(0.139 \text{ atm}) = 326 \text{ K}$. We do likewise for the other pairs of points. The results are:

For 1.48 L, 0.202 atm: T = 347 K.

For 1.01 L, 0.361 atm: T = 423 K.

For 0.50 L, 0.952 atm: T = 553 K.

As the volume decreases, the temperature is increasing, so the temperature is increasing during compression.

EVALUATE: This result confirms that during an adiabatic compression, the gas temperature increases because work is being done on the gas.

19.61. **IDENTIFY:** The air in a cylinder can be compressed by a moveable piston at one end. It goes through a cycle that ends in the same state at which it began. The compression ratio v is defined as $V_{max}/V_{min} = v$.

SET UP: Summarize the steps of the cycle and make a pV-diagram of the process, shown in Fig. 19.61.

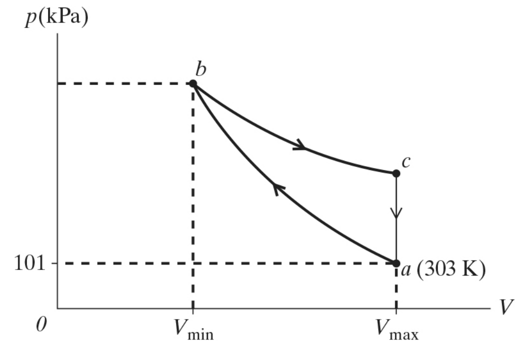


Figure 19.61

<u>Segment *ab*</u>: Beginning at ambient temperature, an adiabatic compression quickly increases the air temperature.
<u>Segment *bc*</u>: Isorthermal expansion to maximum volume
<u>Segment *ca*</u>: Isochoric cooling back to ambient temperature
We know the following things about this system: For diatomic air $C_V = 20.8\ \text{J/mol}\cdot\text{K}$ and $\gamma = 1.40$. The volume of the air in the cylinder is $V = AL$, where A is the area of the faces and L is the length of the gas-containing part of the cylinder. $L_{\max} = 30.0\text{ cm} = 0.300\text{ m}$ and $L_{\min} = L_{\max}/v = (0.300\text{ m})/v$. The cycle starts at point a with $T_a = 30.0°\text{C} = 303\text{ K}$, $p_a = 101\text{ kPa}$, and $V_a = L_{\max}A = (0.300\text{ m})A$, where $A = \pi r^2$.

EXECUTE: (a) We want the work done by the air for a complete cycle *abc*. We need to break this process up into its three segments, so $W_{\text{tot}} = W_{ab} + W_{bc} + W_{ca}$. Segment *ab* is an adiabatic compression, so $W_{ab} = \dfrac{1}{\gamma-1}\left(p_aV_a - p_bV_b\right)$. Using $p_aV_a^\gamma = p_bV_b^\gamma$ with $V_b = V_a/v$, this becomes

$$W_{ab} = \frac{1}{\gamma-1}\left[p_aV_a - p_aV_b\left(\frac{V_a}{V_b}\right)^\gamma\right] = \frac{p_a}{\gamma-1}\left[V_a - v^\gamma(V_a/v)\right] = \frac{p_aV_a}{\gamma-1}\left(1-v^\gamma/v\right) = \frac{p_aV_a}{\gamma-1}\left(1-v^{\gamma-1}\right).$$

Segment *bc* is isothermal, so T = constant = T_b. Using $pV = nRT$ and $W_{bc} = \int_b^c p\,dV$, we have $W_{bc} = \int_b^c \frac{nRT_b}{V}dV = nRT_b\ln\left(V_c/V_b\right)$. But $V_c = V_a$ and $V_b = V_a/v$, so $V_c/V_b = v$, so W_{bc} is $W_{bc} = nRT_b\ln v = p_bV_b\ln v$. From $p_aV_a^\gamma = p_bV_b^\gamma$ we have $p_b = p_a\left(\frac{V_a}{V_b}\right)^\gamma$, and we also know that $V_b = V_a/v$, so $W_{bc} = p_a\left(\frac{V_a}{V_b}\right)^\gamma\frac{V_a}{v}\ln v = p_aV_av^\gamma\frac{1}{v}\ln v = p_aV_av^{\gamma-1}\ln v$. The work during segment *ca* is 0 because the volume is constant. $V_a = (0.300\text{ m})A = (0.300\text{ m})\pi(0.0150\text{ m})^2 = 2.1205\times10^{-4}\text{ m}^3$. Using the values for p_a, V_a, and γ we get $W_{ab} = (53.5\text{ J})\left(1-v^{0.40}\right)$ and $W_{bc} = (21.4\text{ J})v^{0.40}\ln v$.

$$W_{\text{tot}} = 53.5\text{ J} + v^{0.40}\left[(21.42\text{ J})\ln v - 53.5\text{ J}\right].$$

(b) We want the maximum integer value of v so that maximum temperature is no greater than 400°C. The maximum temperature occurs along segment *bc*, so the maximum that T_b can be is 400°C = 673 K. Using $T_bV_b^{\gamma-1} = T_aV_a^{\gamma-1}$ gives $T_b = T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = T_a\left(\frac{vV_b}{V_b}\right)^{\gamma-1} = T_av^{\gamma-1}$. This gives $v^{0.40} = 673/303$, so $v = (673/303)^{1/0.400} = 7.35$. Since v must be an integer, $v_{\max} = 7$. The temperature at b for this value of v is $T_b = T_av^{\gamma-1} = (303\text{ K})7^{0.40} = 660\text{ K} = 387°\text{C}$. (Note that this is *not* 400°C, but the requirement was that the maximum temperature be *no greater than* 400°C, not that it be *equal to* 400°C.)

(c) We want the minimum value of v so that the gas does at least 25.0 J of work per cycle. Using $W_{\text{tot}} = 53.5\text{ J} + v^{0.40}\left[(21.42\text{ J})\ln v - 53.5\text{ J}\right] = 25.0\text{ J}$ gives $-28.55\text{ J} = v^{0.40}\left[(21.42\text{ J})\ln v - 53.5\text{ J}\right]$. The LHS of this equation is negative, so the RHS must also be negative. The only way this is possible is for $\left[(21.42\text{ J})\ln v - 53.5\text{ J}\right]$ to be negative. This tells us that ln v < 53.5/21.42, so v < 12.18. Since v must be an integer, v < 12. But in part (b) we saw that $v \le 7$. We try $v = 7$ to see how much work is done. Using $v = 7$ in $W_{\text{tot}} = 53.5\text{ J} + v^{0.40}\left[(21.42\text{ J})\ln v - 53.5\text{ J}\right]$ gives $= 27.7$ J, which is acceptable since it is more than 25.0 J. Now try $v = 6$, which gives $W_{\text{tot}} = 22.5$ J. This is not acceptable since it is less than 25.0 J, and no smaller integers would be acceptable either. Therefore $v_{\text{min}} = 7$.

(d) $v \ge 7$ and $v \le 7$, so $v = 7$.

(e) We want the heat that leaves the gas during the isochoric segment, which is *ca*. The work is zero, so $\Delta U = Q - W = Q$. Therefore $Q = \Delta U = nC_V\Delta T$. First we need n. Using $pV = nRT$ gives $n = p_aV_a/RT_a$, which gives $n = 8.50\times10^{-3}$ mol. Now we can use $Q = \Delta U = nC_V\Delta T$. This gives $Q =$ $(8.50\times10^{-3}\text{ mol})(20.8\text{ J/mol}\cdot\text{K})(303\text{ K} - 660\text{ K}) = -63.1$ J. The minus sign tells us that the heat comes *out of* the air.

EVALUATE: This device does 27.7 J of work per cycle operating between a high temperature of 660 K (387°C) and low temperature of 303 K (30°C).

19.62. IDENTIFY: $m = \rho V$. The density of air is given by $\rho = \dfrac{pM}{RT}$. For an adiabatic process, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$. $pV = nRT$.

SET UP: Using $V = \dfrac{nRT}{p}$ in $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ gives $T_1p_1^{1-\gamma} = T_2p_2^{1-\gamma}$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.62.

(b) The final temperature is the same as the initial temperature, and the density is proportional to the absolute pressure. The mass needed to fill the cylinder is then

$$m = \rho_0 V\frac{p}{p_{\text{air}}} = (1.23\text{ kg/m}^3)(575\times10^{-6}\text{ m}^3)\frac{1.45\times10^5\text{ Pa}}{1.01\times10^5\text{ Pa}} = 1.02\times10^{-3}\text{ kg}.$$

Without the turbocharger or intercooler the mass of air at $T = 15.0°\text{C}$ and $p = 1.01\times10^5$ Pa in a cylinder is $m = \rho_0 V = 7.07\times10^{-4}$ kg. The increase in power is proportional to the increase in mass of air in the cylinder; the percentage increase is $\dfrac{1.02\times10^{-3}\text{ kg}}{7.07\times10^{-4}\text{ kg}} - 1 = 0.44 = 44\%$.

(c) The temperature after the adiabatic process is $T_2 = T_1\left(\dfrac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$. The density becomes

$$\rho = \rho_0\left(\frac{T_1}{T_2}\right)\left(\frac{p_2}{p_1}\right) = \rho_0\left(\frac{p_2}{p_1}\right)^{(1-\gamma)/\gamma}\left(\frac{p_2}{p_1}\right) = \rho_0\left(\frac{p_2}{p_1}\right)^{1/\gamma}.$$ The mass of air in the cylinder is

$$m = (1.23\text{ kg/m}^3)(575\times10^{-6}\text{ m}^3)\left(\frac{1.45\times10^5\text{ Pa}}{1.01\times10^5\text{ Pa}}\right)^{1/1.40} = 9.16\times10^{-4}\text{ kg},$$

The percentage increase in power is $\dfrac{9.16\times10^{-4}\text{ kg}}{7.07\times10^{-4}\text{ kg}} - 1 = 0.30 = 30\%$.

EVALUATE: The turbocharger and intercooler each have an appreciable effect on the engine power.

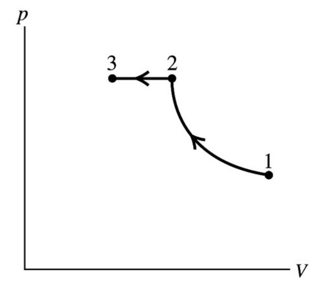


Figure 19.62

19.63. **IDENTIFY and SET UP:** The gas is cooled at constant volume. The ideal gas law applies, so $pV = nRT$.

At constant volume, this becomes $\frac{p_1}{T_1} = \frac{p_2}{T_2}$.

EXECUTE: Solving $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ for p_2 gives $p_2 = p_1\frac{T_2}{T_1} = (2000 \text{ psi})\frac{268 \text{ K}}{293 \text{ K}} = 1830 \text{ psi}$, which is choice (c).

EVALUATE: As the temperature decreases, the pressure decreases, as expected.

19.64. **IDENTIFY and SET UP:** The rapid expansion of the gas is an adiabatic process.
EXECUTE: The work the gas does comes from its internal energy, so its temperature decreases, causing some of it to condense. Therefore choice (d) is correct.
EVALUATE: This is the same principle used by snow-making machines.

19.65. **IDENTIFY and SET UP:** The gas is initially a gauge pressure of 2000 psi (absolute pressure of 2014.7 psi). It will continue to flow out until it is at the same absolute pressure as the outside air, which is 1.0 atm, or 14.7 psi. So we need to find the volume the gas would occupy at 1.0 atm of absolute pressure. The ideal gas law, $pV = nRT$, applies to the gas, and the temperature is constant during this process.

EXECUTE: For an isothermal process, T is constant, so $pV = nRT$ can be put into the form

$V_2 = V_1\frac{p_1}{p_2} = (500 \text{ L})\frac{2014.7 \text{ psi}}{14.7 \text{ psi}} = 6.85\times10^4 \text{ L}$. The volume of gas lost is therefore

$6.85\times10^4 \text{ L} - 500 \text{ L} = 6.80\times10^4 \text{ L}$. The gas flows at a constant rate of 8.2 L/min, so

$(8.2 \text{ L/min})t = 6.80\times10^4 \text{ L}$, which gives t = 8300 min = 140 h, which is choice (d).

EVALUATE: The rate of flow might not be uniform as the gas approaches 1.0 atm, but for most of the time under high pressure, it should be reasonable to assume that the flow rate can be held constant.

19.66. **IDENTIFY and SET UP:** The oxygen and the N_2O are at the same temperature in the same container. Therefore to have a 50%/50% mixture by volume, they should have equal numbers of moles. $pV = nRT$ applies.

EXECUTE: The molecular mass of N_2O is (28 + 16) g/mol = 44 g/mol. The amount present is 1.7 kg = 1700 g, which is 1700/44 = 38.64 mol. Therefore the O_2 must also contain 38.64 mol. The temperature is 20°C = 293 K. The pressure is 50 psi + 14.7 psi = 64.7 psi. Since 1.0 atm = 14.7 psi = 1.01×10^5 Pa, converting gives 64.7 psi = 4.445×10^5 Pa. Using $pV = nRT$, we have

$V = nRT/p = (38.64 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(293 \text{ K})/(4.445 \times 10^5 \text{ Pa}) = 0.21 \text{ m}^3$, which is choice (a).

EVALUATE: The mixture is 50%/50% by volume, but not by weight, since N_2O is heavier than O_2.

20

THE SECOND LAW OF THERMODYNAMICS

VP20.1.1. **IDENTIFY:** We are dealing the efficiency of a diesel engine.

SET UP: Efficiency = $e = W/Q_H$ and $Q_H = W + Q_C$.

EXECUTE: **(a)** We want the heat taken in, which is Q_H. Solve $e = W/Q_H$ for Q_H giving

$$Q_H = \frac{W}{e} = \frac{1.24\times10^4 \text{ J}}{0.180} = 6.89\times10^4 \text{ J}.$$

(b) $Q_C = W + Q_H = 6.89\times10^4 \text{ J} - 1.24\times10^4 \text{ J} = 5.65\times10^4 \text{ J}$.

EVALUATE: Only 18% of the heat taken in is converted to work. This is around the efficiency of automobile engines.

VP20.1.2. **IDENTIFY:** We are dealing with the efficiency of a heat engine.

SET UP and EXECUTE: $e = 1 - \left|\frac{Q_C}{Q_H}\right| = 1 - \frac{3.16\times10^4 \text{ J}}{3.82\times10^4 \text{ J}} = 0.173.$

EVALUATE: Only 17.3% of the heat input is converted to work; the rest is rejected from the engine as waste heat.

VP20.1.3. **IDENTIFY:** We are dealing with the efficiency of a gasoline heat engine.

SET UP: The efficiency is $e = 0.196$ and $Q_C = 4.96\times10^8 \text{ J}$ in 20 min.

EXECUTE: **(a)** We want to find Q_H. $e = 1 - \left|\frac{Q_C}{Q_H}\right| = 1 - \frac{4.96\times10^8 \text{ J}}{Q_H} = 0.196$. $Q_H = 6.17\times10^8 \text{ J}$.

(b) We want the work W. $W = Q_H - Q_C = 6.17\times10^8 \text{ J} - 4.96\times10^8 \text{ J} = 1.21\times10^8 \text{ J}$.

EVALUATE: The power output is $P = W/t = (1.21\times10^8 \text{ J})/[(20)(60) \text{ s}] = 1.01\times10^5 \text{ W} = 101 \text{ kW}$.

VP20.1.4. **IDENTIFY:** A heat engine has a power output of 1.10×10^5 W and gets its heat input from the combustion of gasoline.

SET UP and EXECUTE: The heat of combustion of gasoline is 5.0×10^7 J/kg.

(a) We want to see how much work this engine does in one hour. The power output is the rate at which the engine does work, so the work it does in one hour is $W = Pt = (1.10\times10^5 \text{ W})(3600 \text{ s}) = 3.96\times10^8 \text{ J}$.

(b) We want the heat input. $Q_H = (34 \text{ kg})(5.0\times10^7 \text{ J/kg}) = 1.7\times10^9 \text{ J}$.

(c) We want the efficiency. $e = \frac{W}{Q_H} = \frac{3.96\times10^8 \text{ J}}{1.7\times10^9 \text{ J}} = 0.23.$

EVALUATE: The heat wasted each hour is $Q_H - W = 1.7\times10^9 \text{ J} - 3.96\times10^8 \text{ J} = 1.3\times10^9 \text{ J}$.

VP20.4.1. **IDENTIFY:** Th3is problem deals with a Carnot heat engine.

SET UP: For any heat engine $e = W/Q_H$ and $Q_H = W + Q_C$, and for a Carnot engine $e = 1 - \dfrac{T_C}{T_H}$.

EXECUTE: **(a)** We want the efficiency. $e = \dfrac{W}{Q_H} = \dfrac{1.68\times 10^4 \text{ J}}{8.00\times 10^4 \text{ J}} = 0.210.$

(b) We want Q_C. $Q_H = W + Q_C$, so $Q_C = 8.00\times 10^4 \text{ J} - 1.68\times 10^4 \text{ J} = 6.32\times 10^4 \text{ J}$.

(c) We want T_H. $e = 1 - \dfrac{T_C}{T_H}$ gives $0.210 = 1 - \dfrac{298 \text{ K}}{T_H}$. $T_H = 377$ K = 104°C.

EVALUATE: The temperature must be in Kelvins to use these formulas.

VP20.4.2. **IDENTIFY:** We are analyzing a Carnot heat engine.

SET UP and EXECUTE: **(a)** We want the efficiency. $e = 1 - \dfrac{T_C}{T_H}$ = 1 – (200 K)/(500 K) = 0.60.

(b) Segment ab is the same as in Example 20.3, so W_{ab} = 576 J.
$W_{bc} = nC_V(T_H - T_C)$ = (0.200 mol)(20.8 J/mol·K) (500 K – 200 K) = 1250 J.
$W_{cd} = nRT_C\ln(V_d/V_c)$ = (0.200 mol)(8.314 J/mol·K)(200 K) ln(1/2) = –231 J.
$W_{da} = nC_V(T_C - T_H)$ = (0.200 mol)(20.8 J/mol·K) (200 K – 500 K) = –1250 J.

EVALUATE: The total work is 576 J + 1250 J – 231 J – 1250 J = 345 J. This is much more work than in Example 20.3 due to the greater temperature difference between the hot and cold reservoirs. We get much more work from the same heat input of 576 J.

VP20.4.3. **IDENTIFY:** This problem is about a refrigerator operating on a Carnot cycle. T_C = –10.0°C = 263.15 K and T_H = 25.0°C = 298.15 K.

SET UP and EXECUTE: **(a)** We want the coefficient of performance K.

$$K = \frac{T_C}{T_H - T_C} = \frac{263.15 \text{ K}}{298 \text{ K} - 263 \text{ K}} = 7.52.$$

(b) We want the work input. $K = \dfrac{|Q_C|}{|Q_H| - |Q_C|} = \dfrac{|Q_C|}{W}$. $W = \dfrac{|Q_C|}{K} = \dfrac{4.00\times 10^6 \text{ J}}{7.52} = 5.32\times 10^5 \text{ J}$.

EVALUATE: $Q_H = W + |Q_C| = 5.32\times 10^5 \text{ J} + 4.00\times 10^6 \text{ J} = 4.53\times 10^6 \text{ J}$. Each cycle this refrigerator discharges 4.53×10^6 J of heat into the room.

VP20.4.4. **IDENTIFY:** We are dealing with a Carnot heat engine. We know that $V_b = 2V_a$.

SET UP and EXECUTE: **(a)** We want the work done during the isothermal expansion ab.

$$W_{ab} = \int_a^b p\,dV = \int_a^b \frac{nRT}{V}dV = nRT_H \ln(V_b/V_a) = nRT_H \ln 2.$$

(b) We want the work done during the adiabatic expansion bc. $Q = 0$, so $W_{bc} = Q_{bc}$. So

$$W_{bc} = nC_V\Delta T_{bc} = n\left(\frac{3}{2}R\right)(T_H - T_C) = \frac{3}{2}nR(T_H - T_C).$$

(c) Given that $W_{ab} = W_{bc}$, we want e and the ratio of the cold to hot temperatures. Using the results from parts (a) and (b) gives $nRT_H \ln 2 = \dfrac{3}{2}nR(T_H - T_C)$, which gives $\dfrac{T_C}{T_H} = 1 - \dfrac{2}{3}\ln 2$. The efficiency is

$$e = 1 - \frac{T_C}{T_H} = 1 - \left(1 - \frac{2}{3}\ln 2\right) = \frac{2}{3}\ln 2 = 0.462.$$

EVALUATE: This is quite an efficient engine. It is not necessarily true that $W_{ab} = W_{bc}$ for any Carnot engine.

VP20.10.1. **IDENTIFY:** We want to calculate the entropy changes due to melting, boiling, and temperature change.

SET UP and EXECUTE: For constant temperature $\Delta S = \dfrac{Q}{T}$, and for changing temperature $\Delta S = \int \dfrac{dQ}{T}$.

We want the entropy change in each case.

(a) The temperature stays constant, so $\Delta S = \dfrac{Q}{T} = \dfrac{mL_\text{f}}{T} = \dfrac{(1.00\text{ kg})(1.042\times10^5\text{ J/kg})}{159\text{ K}} = 655\text{ J/K}.$

(b) The temperature increases, so $\Delta S = \int \dfrac{dQ}{T} = \int_{T_1}^{T_2} \dfrac{mcdT}{T} = mc\ln\left(T_2/T_1\right) =$ 1920 J/K using T_1 = 159 K, T_2 = 159 K, m = 1.00 kg, and $c = 2428\text{ J/kg}\cdot\text{K}$.

(c) The temperature stays constant, so $\Delta S = \dfrac{Q}{T} = \dfrac{mL_\text{v}}{T} = \dfrac{(1.00\text{ kg})(8.54\times10^5\text{ J/kg})}{351\text{ K}} = 2430\text{ J/K}.$

EVALUATE: The greatest increase is for boiling because $L_\text{v} >> L_\text{f}$.

VP20.10.2. **IDENTIFY:** This problem involves the entropy change due to helium expansion by two different processes. We want to compare the entropy change in the two cases.

SET UP and EXECUTE: For constant temperature $\Delta S = \dfrac{Q}{T}$, and for changing temperature $\Delta S = \int \dfrac{dQ}{T}$.

(a) For an isothermal expansion we use $\Delta S = \dfrac{Q}{T}$. Since T is constant, $\Delta U = 0$, so $Q = W$. This gives $Q = W = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} \dfrac{nRT}{V} dV = nRT\ln(V_2/V_1).$ $\Delta S = \dfrac{Q}{T} = \dfrac{nRT\ln(V_2/V_1)}{T} = nR\ln(V_2/V_1).$ Putting in the numbers gives $\Delta S = (5.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})\ln\left(\dfrac{0.360\text{ m}^3}{0.120\text{ m}^3}\right) = 45.7\text{ J/K}.$

(b) Start with a pV-diagram showing the process (Fig. VP20.10.2). Break the process into its two segments: ab is isobaric and bc is isochoric. We know that $T_a = T_c = 20.0°\text{C}$ and $V_c = V_b = 3V_a$.

Segment ab (constant pressure): The temperature is not constant, so we use $\Delta S = \int \dfrac{dQ}{T}$. The first law of thermodynamics (in its differential form) gives $dQ = dU + dW$. The pressure is constant, so $pV = nRT$ gives $pdV = nRdT$. Using this in the entropy calculation gives

$dS_{ab} = \dfrac{dQ}{T} = \dfrac{dU + dW}{T} = \dfrac{nC_V dT}{T} + \dfrac{pdV}{T} = \dfrac{nC_V dT}{T} + \dfrac{nRdT}{T}.$ Integrating gives

$$\Delta S_{ab} = \int_{T_a}^{T_b} \left(\frac{nC_V}{T} + \frac{nR}{T}\right) dT = nC_V\ln(T_b/T_a) + nR\ln(T_b/T_a).$$

Segment bc (constant volume): Since the volume is constant, $dW = 0$, so $dQ = dU = nC_V dT$. Therefore

$$\Delta S_{bc} = \int \frac{dQ}{T} = \int \frac{dU}{T} = \int_{T_b}^{T_c} \frac{nC_V dT}{T} = nC_V\ln(T_c/T_b).$$

The total entropy change is $\Delta S_{ac} = \Delta S_{ab} + \Delta S_{bc} = nC_V\ln(T_b/T_a) + nR\ln(T_b/T_a) + nC_V\ln(T_c/T_b)$. But $T_a = T_c = 20.0°\text{C}$, so $T_c/T_b = T_a/T_b$ which means that the first and third terms in the last sum are equal but have opposite signs, so they cancel. This leaves $\Delta S_{ac} = nR\ln(T_b/T_a)$. Using $pV = nRT$ for constant pressure tells us that $\dfrac{T_b}{T_a} = \dfrac{V_b}{V_a} = 3.$ Thus the total entropy change is

$$\Delta S_{ac} = nR\ln 3 = (5.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})\ln 3 = 45.7\text{ J/K}.$$

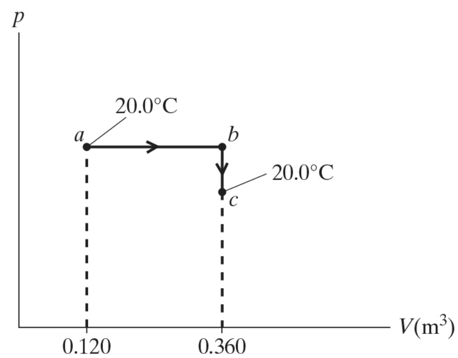


Figure VP20 10.2

EVALUATE: Notice that the entropy change is the same for both paths between the same initial and final states.

VP20.10.3. **IDENTIFY:** We are dealing the entropy change of a heat engine that is *not* using a Carnot cycle.

SET UP: For a constant temperature process $\Delta S = \frac{Q}{T}$. For one cycle, we want the entropy changes of the gas in the engine, the hot reservoir, and the cold reservoir. The two reservoirs are assumed to be so large that they do not change their temperatures as the engine runs. T_H = 260°C = 533.15 K and T_C = 20°C = 293.15 K.

EXECUTE: **(a)** $\Delta S = 0$ for a reversible cycle.

(b) The engine takes a quantity of heat Q_H *out* of the hot reservoir, so

$$\Delta S_{\text{hot res}} = \frac{Q}{T} = \frac{-Q_H}{T_H} = \frac{-8.00\times 10^4\ \text{J}}{533.15\ \text{K}} = -150\ \text{J/K}.$$

(b) The engine transfers a quantity of heat Q_C *into* the hot reservoir, so

$$\Delta S_{\text{cold res}} = \frac{Q}{T} = \frac{+Q_C}{T_H} = \frac{+6.40\times 10^4\ \text{J}}{293.15\ \text{K}} = +218\ \text{J/K}.$$

(c) $\Delta S_{\text{tot}} = \Delta S_{\text{engine}} + \Delta S_{\text{hot res}} + \Delta S_{\text{cold res}} = 0 - 150\ \text{J/K} + 218\ \text{J/K} = +68\ \text{J/K}$. The entropy of the system *increases*.

EVALUATE: The entropy of the universe (that is, the total entropy) increases.

VP20.10.4. **IDENTIFY:** We are looking at the entropy change for an *irreversible* process.

SET UP: For ice melting at constant temperature $\Delta S = \frac{Q}{T}$, and for water changing temperature $\Delta S = \int \frac{dQ}{T}$. We want the entropy change of the ice and of the water and the total entropy change for the entire system. T_1 = 0.0°C = 273 K and T_2 = 95.0°C = 368 K.

EXECUTE: **(a)** $\Delta S_{\text{ice}} = \frac{Q}{T} = \frac{mL_f}{T} = \frac{(1.00\ \text{kg})(3.34\times 10^5\ \text{J/kg})}{273.15\ \text{K}} = +1220\ \text{J/K}.$

(b) $\Delta S_{\text{water}} = \int \frac{dQ}{T} = \int_{368\ \text{K}}^{273\ \text{K}} \frac{mcdT}{T} = (0.839\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})\ln\left(\frac{273\ \text{K}}{368\ \text{K}}\right) = -1050\ \text{J/K}.$

(c) $\Delta S_{\text{system}} =$ +1220 J/K – 1050 J/K = +170 J/K. The entropy increases.

EVALUATE: This is an irreversible process so the entropy should increase, as we've found.

20.1. **IDENTIFY:** For a heat engine, $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$.

SET UP: $W = 2200$ J. $|Q_C| = 4300$ J.

EXECUTE: (a) $Q_H = W + |Q_C| = 6500\ \text{J}.$

(b) $e = \dfrac{2200\ \text{J}}{6500\ \text{J}} = 0.34 = 34\%.$

EVALUATE: Since the engine operates on a cycle, the net Q equal the net W. But to calculate the efficiency we use the heat energy input, Q_H.

20.2. **IDENTIFY:** For a heat engine, $W = |Q_H| - |Q_C|$. $e = \dfrac{W}{Q_H}$. $Q_H > 0,\ Q_C < 0.$

SET UP: $|Q_H| = 9000\ \text{J}$. $|Q_C| = 6400\ \text{J}.$

EXECUTE: (a) $W = 9000\ \text{J} - 6400\ \text{J} = 2600\ \text{J}.$

(b) $e = \dfrac{W}{Q_H} = \dfrac{2600\ \text{J}}{9000\ \text{J}} = 0.29 = 29\%.$

EVALUATE: Since the engine operates on a cycle, the net Q equal the net W. But to calculate the efficiency we use the heat energy input, Q_H.

20.3. **IDENTIFY** and **SET UP:** The problem deals with a heat engine. $W = +3700\ \text{W}$ and $Q_H = +16{,}100\ \text{J}.$
Use $e = \dfrac{W}{Q_H} = 1 - \left|\dfrac{Q_C}{Q_H}\right|$ to calculate the efficiency e and $W = |Q_H| - |Q_C|$ to calculate $|Q_C|$.
Power $= W/t$.

EXECUTE: (a) $e = \dfrac{\text{work output}}{\text{heat energy input}} = \dfrac{W}{Q_H} = \dfrac{3700\ \text{J}}{16{,}100\ \text{J}} = 0.23 = 23\%.$

(b) $W = Q = |Q_H| - |Q_C|$
Heat discarded is $|Q_C| = |Q_H| - W = 16{,}100\ \text{J} - 3700\ \text{J} = 12{,}400\ \text{J}.$

(c) Q_H is supplied by burning fuel; $Q_H = mL_c$ where L_c is the heat of combustion.

$m = \dfrac{Q_H}{L_c} = \dfrac{16{,}100\ \text{J}}{4.60\times10^4\ \text{J/g}} = 0.350\ \text{g}.$

(d) $W = 3700$ J per cycle
In $t = 1.00\ \text{s}$ the engine goes through 60.0 cycles.
$P = W/t = 60.0(3700\ \text{J})/1.00\ \text{s} = 222\ \text{kW}$

$P = (2.22\times10^5\ \text{W})(1\ \text{hp}/746\ \text{W}) = 298\ \text{hp}$

EVALUATE: $Q_C = -12{,}400\ \text{J}$. In one cycle $Q_{tot} = Q_C + Q_H = 3700\ \text{J}$. This equals W_{tot} for one cycle.

20.4. **IDENTIFY:** $W = |Q_H| - |Q_C|$. $e = \dfrac{W}{Q_H}$. $Q_H > 0,\ Q_C < 0.$

SET UP: For 1.00 s, $W = 180\times10^3\ \text{J}.$

EXECUTE: (a) $Q_H = \dfrac{W}{e} = \dfrac{180\times10^3\ \text{J}}{0.280} = 6.43\times10^5\ \text{J}.$

(b) $|Q_C| = |Q_H| - W = 6.43\times10^5\ \text{J} - 1.80\times10^5\ \text{J} = 4.63\times10^5\ \text{J}.$

EVALUATE: Of the 6.43×10^5 J of heat energy supplied to the engine each second, 1.80×10^5 J is converted to mechanical work and the remaining 4.63×10^5 J is discarded into the low temperature reservoir.

20.5. **IDENTIFY:** This cycle involves adiabatic (ab), isobaric (bc), and isochoric (ca) processes.
SET UP: ca is at constant volume, ab has $Q = 0$, and bc is at constant pressure. For a constant pressure process $W = p\Delta V$ and $Q = nC_p\,\Delta T$. $pV = nRT$ gives $n\Delta T = \dfrac{p\Delta V}{R}$, so $Q = \left(\dfrac{C_p}{R}\right)p\Delta V$. If $\gamma = 1.40$

the gas is diatomic and $C_p = \frac{7}{2}R$. For a constant volume process $W = 0$ and $Q = nC_V\ \Delta T$. $pV = nRT$ gives $n\Delta T = \dfrac{V\Delta p}{R}$, so $Q = \left(\dfrac{C_V}{R}\right)V\Delta p$. For a diatomic ideal gas $C_V = \frac{5}{2}R$. $1\text{ atm} = 1.013\times10^5$ Pa.

EXECUTE: **(a)** $V_b = 9.0\times10^{-3}\text{ m}^3$, $p_b = 1.5$ atm and $V_a = 2.0\times10^{-3}\text{ m}^3$. For an adiabatic process $p_aV_a^\gamma = p_bV_b^\gamma$. $p_a = p_b\left(\dfrac{V_b}{V_a}\right)^\gamma = (1.5\text{ atm})\left(\dfrac{9.0\times10^{-3}\text{ m}^3}{2.0\times10^{-3}\text{ m}^3}\right)^{1.4} = 12.3$ atm.

(b) Heat enters the gas in process ca, since T increases.

$Q = \left(\dfrac{C_V}{R}\right)V\Delta p = \left(\dfrac{5}{2}\right)(2.0\times10^{-3}\text{ m}^3)(12.3\text{ atm} - 1.5\text{ atm})(1.013\times10^5\text{ Pa/atm}) = 5470\text{ J}$. $Q_\text{H} = 5470$ J.

(c) Heat leaves the gas in process bc, since T decreases.

$Q = \left(\dfrac{C_p}{R}\right)p\Delta V = \left(\dfrac{7}{2}\right)(1.5\text{ atm})(1.013\times10^5\text{ Pa/atm})(-7.0\times10^{-3}\text{ m}^3) = -3723\text{ J}$. $Q_\text{C} = -3723$ J.

(d) $W = Q_\text{H} + Q_\text{C} = +5470\text{ J} + (-3723\text{ J}) = 1747\text{ J}$.

(e) $e = \dfrac{W}{Q_\text{H}} = \dfrac{1747\text{ J}}{5470\text{ J}} = 0.319 = 31.9\%$.

EVALUATE: We did not use the number of moles of the gas.

20.6. **IDENTIFY and SET UP:** This problem deals with the work done by a heat engine. We want to find the work done during 100 cycles. $|Q_\text{H}| = W + |Q_\text{C}|$. $Q_\text{C} = mL_\text{f}$. We know that $Q_\text{H} = 8000$ J.

EXECUTE: $Q_\text{C} = mL_\text{f} = (0.0180\text{ kg})(334{,}000\text{ J/kg}) = 6012$ J. $W = |Q_\text{H}| - |Q_\text{C}| = 8000\text{ J} - 6012\text{ J} = 1990$ J.

EVALUATE: The efficiency of this engine is $e = W/Q_\text{H} = (1990\text{ J})/(8000\text{ J}) = 0.249$, which is typical of many engines.

20.7. **IDENTIFY and SET UP:** We are dealing with an engine operating on the Otto cycle for which $\gamma = 7/5$ and $e = 0.63$. The compression ration r, which we want to find, is given by $\dfrac{1}{r^{\gamma-1}} = 1 - e$.

EXECUTE: **(a)** We want r. Solving $\dfrac{1}{r^{\gamma-1}} = 1 - e$ for r gives $r^{\gamma-1} = \dfrac{1}{1-e}$, so $r^{7/5-1} = \dfrac{1}{1-0.63}$, which gives $r = 12$.

(b) We want the rejected and absorbed heat if the engine does 12.6 kJ of work per cycle. Solving $e = W/Q_\text{H}$ for Q_H gives $Q_\text{H} = W/e = (12.6\text{ kJ})/(0.63) = 20$ kJ.

Using $|Q_\text{H}| = W + |Q_\text{C}|$ gives $Q_\text{C} = Q_\text{H} - W = 20\text{ kJ} - 12.6\text{ kJ} = 7.4$ kJ.

EVALUATE: This engine is indeed very efficient at 63%.

20.8. **IDENTIFY:** Apply $e = 1 - \dfrac{1}{r^{\gamma-1}}$. $e = 1 - \dfrac{|Q_\text{C}|}{|Q_\text{H}|}$.

SET UP: In part (b), $Q_\text{H} = 10{,}000$ J. The heat discarded is $|Q_\text{C}|$.

EXECUTE: **(a)** $e = 1 - \dfrac{1}{9.50^{0.40}} = 0.594 = 59.4\%$.

(b) $|Q_\text{C}| = |Q_\text{H}|(1-e) = (10{,}000\text{ J})(1 - 0.594) = 4060\text{ J}$.

EVALUATE: The work output of the engine is $W = |Q_\text{H}| - |Q_\text{C}| = 10{,}000\text{ J} - 4060\text{ J} = 5940\text{ J}$.

20.9. **IDENTIFY:** For the Otto-cycle engine, $e = 1 - r^{1-\gamma}$.

SET UP: r is the compression ratio.

EXECUTE: **(a)** $e = 1 - (8.8)^{-0.40} = 0.581$, which rounds to 58%.

(b) $e = 1 - (9.6)^{20.40} = 0.595$ an increase of 1.4%.

EVALUATE: An increase in *r* gives an increase in *e*.

20.10. **IDENTIFY:** $|Q_\text{H}| = |Q_\text{C}| + |W|$. $K = \dfrac{|Q_\text{C}|}{W}$.

SET UP: For water, $c_\text{w} = 4190\ \text{J/kg} \cdot \text{K}$ and $L_\text{f} = 3.34 \times 10^5\ \text{J/kg}$. For ice, $c_\text{ice} = 2100\ \text{J/kg} \cdot \text{K}$.

EXECUTE: **(a)** $Q = mc_\text{ice}\Delta T_\text{ice} - mL_\text{f} + mc_\text{w}\Delta T_\text{w}$.

$$Q = (1.80\ \text{kg})\left[(2100\ \text{J/kg} \cdot \text{K})(-5.0\ \text{C}°) - 3.34 \times 10^5\ \text{J/kg} + (4190\ \text{J/kg} \cdot \text{K})(-25.0\ \text{C}°)\right] = -8.09 \times 10^5\ \text{J}$$

$Q = -8.09 \times 10^5$ J. Q is negative for the water since heat is removed from it.

(b) $|Q_\text{C}| = 8.09 \times 10^5$ J. $W = \dfrac{|Q_\text{C}|}{K} = \dfrac{8.09 \times 10^5\ \text{J}}{2.40} = 3.37 \times 10^5$ J.

(c) $|Q_\text{H}| = 8.09 \times 10^5\ \text{J} + 3.37 \times 10^5\ \text{J} = 1.15 \times 10^6$ J.

EVALUATE: For this device, $Q_\text{C} > 0$ and $Q_\text{H} < 0$. More heat is rejected to the room than is removed from the water.

20.11. **IDENTIFY:** The heat $Q = mc\Delta T$ that comes out of the water to cool it to 5.0°C is Q_C for the refrigerator.

SET UP: For water 1.0 L has a mass of 1.0 kg and $c = 4.19 \times 10^3\ \text{J/kg} \cdot \text{C}°$. $P = \dfrac{|W|}{t}$. The coefficient of performance is $K = \dfrac{|Q_\text{C}|}{|W|}$.

EXECUTE: $Q = mc\Delta T = (12.0\ \text{kg})(4.19 \times 10^3\ \text{J/kg} \cdot \text{C}°)(5.0°\text{C} - 31°\text{C}) = -1.31 \times 10^6$ J. $|Q_\text{C}| = 1.31 \times 10^6$ J.

$$K = \frac{|Q_\text{C}|}{|W|} = \frac{|Q_\text{C}|}{Pt} \text{ so } t = \frac{|Q_\text{C}|}{PK} = \frac{1.31 \times 10^6\ \text{J}}{(135\ \text{W})(2.25)} = 4313\ \text{s} = 71.88\ \text{min} = 1.20\ \text{h}.$$

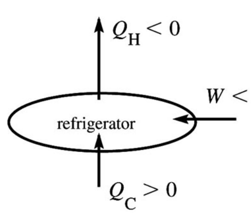
EVALUATE: 1.2 h seems like a reasonable time to cool down the dozen bottles.

20.12. **IDENTIFY** and **SET UP:** For the refrigerator $K = 2.10$ and $Q_\text{C} = +3.1 \times 10^4$ J. Use $K = Q_\text{C}/|W|$ to calculate $|W|$ and then $W = Q_\text{C} + Q_\text{H}$ to calculate Q_H.

EXECUTE: **(a)** Coefficient of performance: $K = Q_\text{C}/|W|$.

$|W| = Q_\text{C}/K = 3.10 \times 10^4\ \text{J}/2.10 = 1.48 \times 10^4$ J.

(b) The operation of the device is illustrated in Figure 20.12.



$W = Q_\text{C} + Q_\text{H}$

$Q_\text{H} = W - Q_\text{C}$

$Q_\text{H} = -1.48 \times 10^4\ \text{J} - 3.10 \times 10^4\ \text{J} = -4.58 \times 10^4$ J (negative because heat goes out of the system)

Figure 20.12

EVALUATE: $|Q_\text{H}| = |W| + |Q_\text{C}|$. The heat $|Q_\text{H}|$ delivered to the high temperature reservoir is greater than the heat taken in from the low temperature reservoir.

20.13. **IDENTIFY:** The hot reservoir of a Carnot engine is 72.0 C° higher than the cold reservoir and this engine is 12.5% efficient. We want the temperature of the two reservoirs.

SET UP: For a Carnot engine $e = 1 - \dfrac{T_\text{C}}{T_\text{H}}$. $T_\text{C} = T_\text{H} - 72.0$ K.

EXECUTE: $e = 1 - \dfrac{T_C}{T_H} = 1 - \dfrac{T_H - 72.0\text{ K}}{T_H} = 0.125$ gives $T_H = 576$ K, so $T_C = 576\text{ K} - 72\text{ K} = 504$ K.

EVALUATE: Careful! The 72.0 C° is a temperature *difference*. Since Kelvin and Celsius degrees are the same size, so $\Delta T_K = \Delta T_C$. We should *not* convert the 72.0 C° to get 345 K.

20.14. **IDENTIFY:** $|W| = |Q_H| - |Q_C|$. $Q_C < 0$, $Q_H > 0$. $e = \dfrac{W}{Q_H}$. For a Carnot cycle, $\dfrac{Q_C}{Q_H} = -\dfrac{T_C}{T_H}$.

SET UP: $T_C = 300$ K, $T_H = 520$ K. $|Q_H| = 6.45\times10^3$ J.

EXECUTE: **(a)** $Q_C = -Q_H\left(\dfrac{T_C}{T_H}\right) = -(6.45\times10^3\text{ J})\left(\dfrac{300\text{ K}}{520\text{ K}}\right) = -3.72\times10^3$ J.

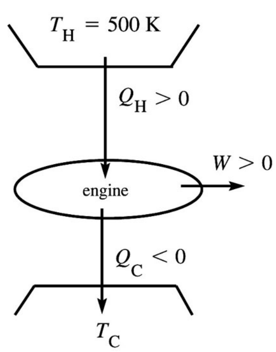
(b) $|W| = |Q_H| - |Q_C| = 6.45\times10^3\text{ J} - 3.72\times10^3\text{ J} = 2.73\times10^3$ J.

(c) $e = \dfrac{W}{Q_H} = \dfrac{2.73\times10^3\text{ J}}{6.45\times10^3\text{ J}} = 0.423 = 42.3\%$.

EVALUATE: We can verify that $e = 1 - T_C/T_H$ also gives $e = 42.3\%$.

20.15. **IDENTIFY** and **SET UP:** Use $W = Q_C + Q_H$ to calculate $|W|$. Since it is a Carnot device we can use $\dfrac{|Q_C|}{|Q_H|} = \dfrac{T_C}{T_H}$ to relate the heat flows out of the reservoirs. The reservoir temperatures can be used in $e_{\text{Carnot}} = 1 - \dfrac{T_C}{T_H}$ to calculate e.

EXECUTE: **(a)** The operation of the device is sketched in Figure 20.15.



$W = Q_C + Q_H$
$W = -335\text{ J} + 550\text{ J} = 215\text{ J}$

Figure 20.15

(b) For a Carnot cycle, $\dfrac{|Q_C|}{|Q_H|} = \dfrac{T_C}{T_H}$, which gives

$$T_C = T_H\frac{|Q_C|}{|Q_H|} = 620\text{ K}\left(\frac{335\text{ J}}{550\text{ J}}\right) = 378\text{ K}.$$

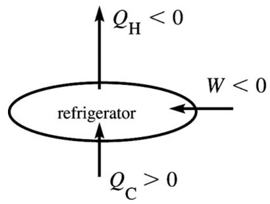
(c) $e_{\text{Carnot}} = 1 - T_C/T_H = 1 - 378\text{ K}/620\text{ K} = 0.390 = 39.0\%$.

EVALUATE: We could use the fundamental definition of e, $e = \dfrac{W}{Q_H}$:

$e = W/Q_H = (215\text{ J})/(550\text{ J}) = 39\%$, which checks.

20.16. **IDENTIFY** and **SET UP:** The device is a Carnot refrigerator.
We can use $W = Q_C + Q_H$ and $|Q_C|/|Q_H| = T_C/T_H$.

(a) The operation of the device is sketched in Figure 20.16.



$T_H = 24.0°C = 297$ K
$T_C = 0.0°C = 273$ K

Figure 20.16

The amount of heat taken out of the water to make the liquid $\rightarrow$ solid phase change is $Q = -mL_f = -(85.0\text{ kg})(334\times10^3\text{ J/kg}) = -2.84\times10^7$ J. This amount of heat must go into the working substance of the refrigerator, so $Q_C = +2.84\times10^7$ J. For Carnot cycle $|Q_C|/|Q_H| = T_C/T_H$.

EXECUTE: $|Q_H| = |Q_C|(T_H/T_C) = 2.84\times10^7\text{ J}(297\text{ K}/273\text{ K}) = 3.09\times10^7$ J.

(b) $W = Q_C + Q_H = +2.84\times10^7\text{ J} - 3.09\times10^7\text{ J} = -2.5\times10^6$ J

EVALUATE: W is negative because this much energy must be supplied to the refrigerator rather than obtained from it. Note that in $W = Q_C + Q_H$ we must use Kelvin temperatures.

20.17. IDENTIFY: $e = \dfrac{W}{Q_H}$ for any engine. For the Carnot cycle, $\dfrac{Q_C}{Q_H} = -\dfrac{T_C}{T_H}$.

SET UP: $T_C = 20.0°C + 273.15\text{ K} = 293.15$ K

EXECUTE: **(a)** $Q_H = \dfrac{W}{e} = \dfrac{2.5\times10^4\text{ J}}{0.66} = 3.79\times10^4$ J, which rounds to 3.8×10^4 J.

(b) $W = Q_H + Q_C$ so $Q_C = W - Q_H = 2.5\times10^4\text{ J} - 3.79\times10^4\text{ J} = -1.29\times10^4$ J.

$$T_H = -T_C\frac{Q_H}{Q_C} = -(293.15\text{ K})\left(\frac{3.79\times10^4\text{ J}}{-1.29\times10^4\text{ J}}\right) = 861\text{ K} = 590°C.$$

EVALUATE: For a heat engine, $W > 0$, $Q_H > 0$ and $Q_C < 0$.

20.18. IDENTIFY: The theoretical maximum performance coefficient is a $K_{\text{Cannot}} = \dfrac{T_C}{T_H - T_C}$. $K = \dfrac{|Q_C|}{|W|}$. $|Q_C|$ is the heat removed from the water to convert it to ice. For the water, $|Q| = mc_w\Delta T + mL_f$.

SET UP: $T_C = -5.0°C = 268$ K. $T_H = 20.0°C = 293$ K. $c_w = 4190\text{ J/kg}\cdot\text{K}$ and $L_f = 334\times10^3$ J/kg.

EXECUTE: **(a)** In one year the freezer operates $(5\text{ h/day})(365\text{ days}) = 1825$ h.

$$P = \frac{730\text{ kWh}}{1825\text{ h}} = 0.400\text{ kW} = 400\text{ W}.$$

(b) $K_{\text{Carnot}} = \dfrac{268\text{ K}}{293\text{ K} - 268\text{ K}} = 10.7.$

(c) $|W| = Pt = (400\text{ W})(3600\text{ s}) = 1.44\times10^6$ J. $|Q_C| = K|W| = 1.54\times10^7$ J. $|Q| = mc_w\Delta T + mL_f$ gives

$$m = \frac{|Q_C|}{c_w\Delta T + L_f} = \frac{1.54\times10^7\text{ J}}{(4190\text{ J/kg}\cdot\text{K})(20.0\text{ K}) + 334\times10^3\text{ J/kg}} = 36.9\text{ kg}.$$

EVALUATE: For any actual device, $K < K_{\text{Carnot}}$, $|Q_C|$ is less than we calculated and the freezer makes less ice in one hour than the mass we calculated in part (c).

20.19. IDENTIFY: $|Q_H| = |W| + |Q_C|$. $Q_H < 0$, $Q_C > 0$. $K = \dfrac{|Q_C|}{|W|}$. For a Carnot cycle, $\dfrac{Q_C}{Q_H} = -\dfrac{T_C}{T_H}$.

SET UP: $T_C = 270$ K, $T_H = 320$ K. $|Q_C| = 415$ J.

EXECUTE: **(a)** $Q_H = -\left(\frac{T_H}{T_C}\right)Q_C = -\left(\frac{320\text{ K}}{270\text{ K}}\right)(415\text{ J}) = -492\text{ J}.$

(b) For one cycle, $|W| = |Q_H| - |Q_C| = 492\text{ J} - 415\text{ J} = 77\text{ J}.$ $P = \frac{(165)(77\text{ J})}{60\text{ s}} = 212\text{ W}.$

(c) $K = \frac{|Q_C|}{|W|} = \frac{415\text{ J}}{77\text{ J}} = 5.4.$

EVALUATE: The amount of heat energy $|Q_H|$ delivered to the high-temperature reservoir is greater than the amount of heat energy $|Q_C|$ removed from the low-temperature reservoir.

20.20. **IDENTIFY:** $W = Q_C + Q_H$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$. For the ice to liquid water phase transition, $Q = mL_f$.

SET UP: For water, $L_f = 334\times10^3$ J/kg.

EXECUTE: $Q_C = -mL_f = -(0.0400\text{ kg})(334\times10^3\text{ J/kg}) = -1.336\times10^4\text{ J}.$ $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ gives

$Q_H = -(T_H/T_C)Q_C = -(-1.336\times10^4\text{ J})\left[(373.15\text{ K})/(273.15\text{ K})\right] = +1.825\times10^4\text{ J}.$

$W = Q_C + Q_H = 4.89\times10^3\text{ J}.$

EVALUATE: For a heat engine, Q_C is negative and Q_H is positive. The heat that comes out of the engine $(Q < 0)$ goes into the ice $(Q > 0)$.

20.21. **IDENTIFY and SET UP:** We are looking at two refrigerators, *A* and *B*, operating on a Carnot cycle. The coefficient of performance of a Carnot refrigerator is $K_{\text{Carnot}} = \frac{T_C}{T_H - T_C} = \frac{T_C}{\Delta T}$. We know that $K_A = K_B + 0.16K_B = 1.16K_B$, $\Delta T_B = \Delta T_A + 0.30\Delta T_A = 1.30\Delta T_A$, and $T_C(B)$ = 180 K. We want to find the cold reservoir temperature for *A*, $T_C(A)$.

EXECUTE: $K_A = \frac{T_C(A)}{\Delta T_A}$ and $K_B = \frac{T_C(B)}{\Delta T_B}$, so $\frac{K_A}{K_B} = \frac{\frac{T_C(A)}{\Delta T_A}}{\frac{T_C(B)}{\Delta T_B}}$. Using the given information, we have

$\frac{1.16K_B}{K_B} = \frac{T_C(A)}{T_C(B)}\frac{\Delta T_B}{\Delta T_A} = \frac{T_C(A)}{T_C(B)}\frac{1.30\Delta T_A}{\Delta T_A}$. This gives $1.16 = \frac{1.30T_C(A)}{180\text{ K}}$, so $T_C(A)$ = 161 K.

EVALUATE: All temperatures must be in Kelvins.

20.22. **IDENTIFY:** Apply $Q_{\text{system}} = 0$ to calculate the final temperature. $Q = mc\Delta T$. $\Delta S = mc\ln(T_2/T_1)$ when an object undergoes a temperature change.

SET UP: For water $c = 4190\text{ J/kg}\cdot\text{K}$. Boiling water has $T = 100.0°\text{C} = 373\text{ K}$.

EXECUTE: **(a)** The heat transfer between 100°C water and 30°C water occurs over a finite temperature difference and the process is irreversible.

(b) $(195\text{ kg})c(T_2 - 30.0°\text{C}) + (5.00\text{ kg})c(T_2 - 100°\text{C}) = 0.$ $T_2 = 31.75°\text{C} = 304.90\text{ K}.$

(c) $\Delta S = (195\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln\left(\frac{304.90\text{ K}}{303.15\text{ K}}\right) + (5.00\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln\left(\frac{304.90\text{ K}}{373.15\text{ K}}\right) = 471\text{ J/K}.$

EVALUATE: $\Delta S_{\text{system}} > 0$, as it should for an irreversible process.

20.23. **IDENTIFY:** $\Delta S = \frac{Q}{T}$ for each object, where *T* must be in kelvins. The temperature of each object remains constant.

SET UP: For water, $L_f = 3.34\times10^5$ J/kg.

EXECUTE: **(a)** The heat flow into the ice is $Q = mL_f = (0.350\text{ kg})(3.34\times10^5\text{ J/kg}) = 1.17\times10^5\text{ J}$. The heat flow occurs at $T = 273$ K, so $\Delta S = \dfrac{Q}{T} = \dfrac{1.17\times10^5\text{ J}}{273\text{ K}} = 429$ J/K. Q is positive and ΔS is positive.

(b) $Q = -1.17\times10^5$ J flows out of the heat source, at $T = 298$ K. $\Delta S = \dfrac{Q}{T} = \dfrac{-1.17\times10^5\text{ J}}{298\text{ K}} = -393$ J/K. Q is negative and ΔS is negative.

(c) $\Delta S_{\text{tot}} = 429\text{ J/K} + (-393\text{ J/K}) = +36\text{ J/K}$.

EVALUATE: For the total isolated system, $\Delta S > 0$ and the process is irreversible.

20.24. IDENTIFY: $Q = mc\Delta T$ for the water. $\Delta S = mc\ln(T_2/T_1)$ when an object undergoes a temperature change. $\Delta S = Q/T$ for an isothermal process.

SET UP: For water, $c = 4190\text{ J/kg}\cdot\text{K}$. $85.0°\text{C} = 358.2$ K. $20.0°\text{C} = 293.2$ K.

EXECUTE: **(a)** $\Delta S = mc\ln\left(\dfrac{T_2}{T_1}\right) = (0.250\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln\left(\dfrac{293.2\text{ K}}{358.2\text{ K}}\right) = -210$ J/K. Heat comes out of the water and its entropy decreases.

(b) $Q = mc\Delta T = (0.250)(4190\text{ J/kg}\cdot\text{K})(-65.0\text{ K}) = -6.81\times10^4$ J. The amount of heat that goes into the air is $+6.81\times10^4$ J. For the air, $\Delta S = \dfrac{Q}{T} = \dfrac{+6.81\times10^4\text{ J}}{293.1\text{ K}} = +232$ J/K.

$\Delta S_{\text{system}} = -210\text{ J/K} + 232\text{ J/K} = +22\text{ J/K}$.

EVALUATE: $\Delta S_{\text{system}} > 0$ and the process is irreversible.

20.25. IDENTIFY: The process is at constant temperature, so $\Delta S = \dfrac{Q}{T}$. $\Delta U = Q - W$.

SET UP: For an isothermal process of an ideal gas, $\Delta U = 0$ and $Q = W$. For a compression, $\Delta V < 0$ and $W < 0$.

EXECUTE: $Q = W = -1850$ J. $\Delta S = \dfrac{-1850\text{ J}}{293\text{ K}} = -6.31$ J/K.

EVALUATE: The entropy change of the gas is negative. Heat must be removed from the gas during the compression to keep its temperature constant and therefore the gas is not an isolated system.

20.26. IDENTIFY and SET UP: The initial and final states are at the same temperature, at the normal boiling point of 4.216 K. Calculate the entropy change for the irreversible process by considering a reversible isothermal process that connects the same two states, since ΔS is path independent and depends only on the initial and final states. For the reversible isothermal process we can use $\Delta S = Q/T$.

The heat flow for the helium is $Q = -mL_v$, negative since in condensation heat flows out of the helium.

The heat of vaporization L_v is given in Table 17.4 and is $L_v = 20.9\times10^3$ J/kg.

EXECUTE: $Q = -mL_v = -(0.130\text{ kg})(20.9\times10^3\text{ J/kg}) = -2717$ J

$\Delta S = Q/T = -2717\text{ J}/4.216\text{ K} = -644$ J/K.

EVALUATE: The system we considered is the 0.130 kg of helium; ΔS is the entropy change of the helium. This is not an isolated system since heat must flow out of it into some other material. Our result that $\Delta S < 0$ doesn't violate the second law since it is not an isolated system. The material that receives the heat that flows out of the helium would have a positive entropy change and the total entropy change would be positive.

20.27. IDENTIFY: Each phase transition occurs at constant temperature and $\Delta S = \dfrac{Q}{T}$. $Q = mL_{\text{v}}$.

SET UP: For vaporization of water, $L_{\text{v}} = 2256\times10^3$ J/kg.

EXECUTE: **(a)** $\Delta S = \dfrac{Q}{T} = \dfrac{mL\text{v}}{T} = \dfrac{(1.00\text{ kg})(2256\times10^3\text{ J/kg})}{(373.15\text{ K})} = 6.05\times10^3$ J/K. Note that this is the change of entropy of the water as it changes to steam.

(b) The magnitude of the entropy change is roughly five times the value found in Example 20.5.

EVALUATE: Water is less ordered (more random) than ice, but water is far less random than steam; a consideration of the density changes indicates why this should be so.

20.28. IDENTIFY and SET UP: **(a)** The velocity distribution of the Maxwell-Boltzmann distribution, $f(v) = 4\pi\left(\dfrac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$, depends only on T, so in an isothermal process it does not change.

(b) EXECUTE: Calculate the change in the number of available microscopic states and apply Eq. (20.23). Following the reasoning of Example 20.11, the number of possible positions available to each molecule is altered by a factor of 3 (becomes larger). Hence the number of microscopic states the gas occupies at volume $3V$ is $w_2 = (3)^N w_1$, where N is the number of molecules and w_1 is the number of possible microscopic states at the start of the process, where the volume is V. Then, using $\Delta S = k\ln(w_2/w_1)$, we have $\Delta S = k\ln(w_2/w_1) = k\ln(3)^N = Nk\ln(3) = nN_{\text{A}}k\ln(3) = nR\ln(3)$.

$\Delta S = (2.00\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})\ln(3) = +18.3$ J/K.

(c) IDENTIFY and SET UP: For an isothermal reversible process $\Delta S = Q/T$.

EXECUTE: Calculate W and then use the first law to calculate Q.

$\Delta T = 0$ implies that $\Delta U = 0$, since system is an ideal gas.

Then by $\Delta U = Q - W$, $Q = W$.

For an isothermal process, $W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} (nRT/V)\,dV = nRT\ln(V_2/V_1)$.

Thus $Q = nRT\ln(V_2/V_1)$ and $\Delta S = Q/T = nR\ln(V_2/V_1)$.

$\Delta S = (2.00\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})\ln(3V_1/V_1) = +18.3$ J/K.

EVALUATE: This is the same result as obtained in part (b).

20.29. IDENTIFY: For a free expansion, $\Delta S = nR\ln(V_2/V_1)$.

SET UP: $V_1 = 2.40\text{ L} = 2.40\times10^{-3}\text{ m}^3$.

EXECUTE: $\Delta S = (0.100\text{ mol})(8.314\text{ J/mol}\cdot\text{K})\ln\left(\dfrac{425\text{ m}^3}{2.40\times10^{-3}\text{ m}^3}\right) = 10.0$ J/K.

EVALUATE: $\Delta S_{\text{system}} > 0$ and the free expansion is irreversible.

20.30. IDENTIFY: For a Carnot engine, $\dfrac{Q_{\text{C}}}{Q_{\text{H}}} = -\dfrac{T_{\text{C}}}{T_{\text{H}}}$. $e_{\text{Carnot}} = 1 - \dfrac{T_{\text{C}}}{T_{\text{H}}}$. $|W| = |Q_{\text{H}}| - |Q_{\text{C}}|$. $Q_{\text{H}} > 0$, $Q_{\text{C}} < 0$. $pV = nRT$.

SET UP: The work done by the engine each cycle is $mg\Delta y$, with $m = 15.0$ kg and $\Delta y = 2.00$ m. $T_{\text{H}} = 773$ K. $Q_{\text{H}} = 500$ J.

EXECUTE: **(a)** The pV diagram is sketched in Figure 20.30.

(b) $W = mg\Delta y = (15.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 294$ J. $|Q_{\text{C}}| = |Q_{\text{H}}| - |W| = 500\text{ J} - 294\text{ J} = 206$ J, and $Q_{\text{C}} = -206$ J.

$$T_{\text{C}} = -T_{\text{H}}\left(\frac{Q_{\text{C}}}{Q_{\text{H}}}\right) = -(773\text{ K})\left(\frac{-206\text{ J}}{500\text{ J}}\right) = 318\text{ K} = 45°\text{C}.$$

(c) $e = 1 - \dfrac{T_\text{C}}{T_\text{H}} = 1 - \dfrac{318\text{ K}}{773\text{ K}} = 0.589 = 58.9\%.$

(d) $|Q_\text{C}| = 206\text{ J}.$

(e) The maximum pressure is for state a. This is also where the volume is a minimum, so $V_a = 5.00\text{ L} = 5.00\times10^{-3}\text{ m}^3.$ $T_a = T_\text{H} = 773\text{ K}.$

$$p_a = \frac{nRT_a}{V_a} = \frac{(2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(773\text{ K})}{5.00\times10^{-3}\text{ m}^3} = 2.57\times10^6\text{ Pa}.$$

EVALUATE: We can verify that $e = \dfrac{W}{Q_\text{H}}$ gives the same value for e as calculated in part (c).

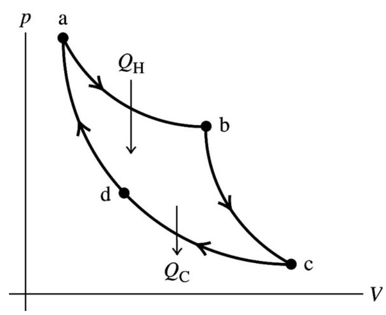


Figure 20.30

20.31. **IDENTIFY:** The total work that must be done is $W_\text{tot} = mg\Delta y$. $|W| = |Q_\text{H}| - |Q_\text{C}|$. $Q_\text{H} > 0$, $W > 0$ and $Q_\text{C} < 0$. For a Carnot cycle, $\dfrac{Q_\text{C}}{Q_\text{H}} = -\dfrac{T_\text{C}}{T_\text{H}}$.

SET UP: $T_\text{C} = 373\text{ K}$, $T_\text{H} = 773\text{ K}$. $|Q_\text{H}| = 250\text{ J}$.

EXECUTE: **(a)** $Q_\text{C} = -Q_\text{H}\left(\dfrac{T_\text{C}}{T_\text{H}}\right) = -(250\text{ J})\left(\dfrac{373\text{ K}}{773\text{ K}}\right) = -121\text{ J}.$

(b) $|W| = 250\text{ J} - 121\text{ J} = 129\text{ J}$. This is the work done in one cycle.

$W_\text{tot} = (500\text{ kg})(9.80\text{ m/s}^2)(100\text{ m}) = 4.90\times10^5\text{ J}$. The number of cycles required is

$\dfrac{W_\text{tot}}{|W|} = \dfrac{4.90\times10^5\text{ J}}{129\text{ J/cycle}} = 3.80\times10^3$ cycles.

EVALUATE: In $\dfrac{Q_\text{C}}{Q_\text{H}} = -\dfrac{T_\text{C}}{T_\text{H}}$, the temperatures must be in kelvins.

20.32. **IDENTIFY:** The same amount of heat that enters the person's body also leaves the body, but these transfers of heat occur at different temperatures, so the person's entropy changes.

SET UP: We are asked to find the entropy change of the person. The person is not an isolated system. In 1.0 s, $0.80(80\text{ J}) = 64\text{ J}$ of heat enters the person's body at $37°\text{C} = 310\text{ K}$. This amount of heat leaves the person at a temperature of $30°\text{C} = 303\text{ K}$. $\Delta S = \dfrac{Q}{T}$.

EXECUTE: For the person, $\Delta S = \dfrac{+64\text{ J}}{310\text{ K}} + \dfrac{-64\text{ J}}{303\text{ K}} = -4.8\times10^{-3}\text{ J/K}.$

EVALUATE: The entropy of the person can decrease without violating the second law of thermodynamics because the person isn't an isolated system.

20.33. **IDENTIFY:** We know the efficiency of this Carnot engine, the heat it absorbs at the hot reservoir and the temperature of the hot reservoir.

SET UP: For a heat engine $e = \dfrac{W}{|Q_H|}$ and $Q_H + Q_C = W$. For a Carnot cycle, $\dfrac{Q_C}{Q_H} = -\dfrac{T_C}{T_H}$. $Q_C < 0$, $W >$ 0, and $Q_H > 0$. $T_H = 135°\text{C} = 408\text{ K}$. In each cycle, $|Q_H|$ leaves the hot reservoir and $|Q_C|$ enters the cold reservoir. The work done on the water equals its increase in gravitational potential energy, *mgh*.

EXECUTE: **(a)** $e = \dfrac{W}{Q_H}$ so $W = eQ_H = (0.220)(410\text{ J}) = 90.2\text{ J}$.

(b) $Q_C = W - Q_H = 90.2\text{ J} - 410\text{ J} = -319.85\text{ J}$, which rounds to –320 J.

(c) $\dfrac{Q_C}{Q_H} = -\dfrac{T_C}{T_H}$ so $T_C = -T_H\left(\dfrac{Q_C}{Q_H}\right) = -(408\text{ K})\left(\dfrac{-319.8\text{ J}}{410\text{ J}}\right) = 318\text{ K} = 45°\text{C}$.

(d) $\Delta S = \dfrac{-|Q_H|}{T_H} + \dfrac{|Q_C|}{T_C} = \dfrac{-410\text{ J}}{408\text{ K}} + \dfrac{319.8\text{ J}}{318\text{ K}} = 0$. The Carnot cycle is reversible and $\Delta S = 0$.

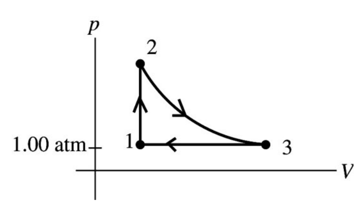
(e) $W = mgh$ so $m = \dfrac{W}{gh} = \dfrac{90.2\text{ J}}{(9.80\text{ m/s}^2)(35.0\text{ m})} = 0.263\text{ kg} = 263\text{ g}$.

EVALUATE: The Carnot cycle is reversible so $\Delta S = 0$ for the world. However some parts of the world gain entropy while other parts lose it, making the sum equal to zero.

20.34. **IDENTIFY:** Use the ideal gas law to calculate *p* and *V* for each state. Use the first law and specific expressions for *Q*, *W*, and ΔU for each process. Use $e = \dfrac{W}{Q_H} = 1 - \left|\dfrac{Q_C}{Q_H}\right|$ to calculate *e*. Q_H is the net heat flow into the gas.

SET UP: $\gamma = 1.40$

$C_V = R/(\gamma - 1) = 20.79\text{ J/mol}\cdot\text{K}$; $C_p = C_V + R = 29.10\text{ J/mol}\cdot\text{K}$. The cycle is sketched in Figure 20.34.



$T_1 = 300\text{ K}$
$T_2 = 600\text{ K}$
$T_3 = 492\text{ K}$

Figure 20.34

EXECUTE: **(a)** Point 1:

$p_1 = 1.00\text{ atm} = 1.013\times10^5\text{ Pa}$ (given); $pV = nRT$;

$$V_1 = \frac{nRT_1}{p_1} = \frac{(0.350\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(300\text{ K})}{1.013\times10^5\text{ Pa}} = 8.62\times10^{-3}\text{ m}^3$$

Point 2:

Process $1 \to 2$ is at constant volume so $V_2 = V_1 = 8.62\times10^{-3}\text{ m}^3$

$pV = nRT$ and *n*, *R*, *V* constant implies $p_1/T_1 = p_2/T_2$

$p_2 = p_1(T_2/T_1) = (1.00\text{ atm})(600\text{ K}/300\text{ K}) = 2.00\text{ atm} = 2.03\times10^5\text{ Pa}$

Point 3:

Consider the process $3 \to 1$, since it is simpler than $2 \to 3$.

Process $3 \to 1$ is at constant pressure so $p_3 = p_1 = 1.00\text{ atm} = 1.013\times10^5\text{ Pa}$

$pV = nRT$ and n, R, p constant implies $V_1/T_1 = V_3/T_3$

$V_3 = V_1(T_3/T_1) = (8.62\times10^{-3}\ \text{m}^3)(492\ \text{K}/300\ \text{K}) = 14.1\times10^{-3}\ \text{m}^3$

(b) Process $1\to2$:

Constant volume $(\Delta V = 0)$

$Q = nC_V\Delta T = (0.350\ \text{mol})(20.79\ \text{J/mol}\cdot\text{K})(600\ \text{K} - 300\ \text{K}) = 2180\ \text{J}$

$\Delta V = 0$ and $W = 0$. Then $\Delta U = Q - W = 2180\ \text{J}$

Process $2\to3$:

Adiabatic means $Q = 0$.

$\Delta U = nC_V\Delta T$ (any process), so

$\Delta U = (0.350\ \text{mol})(20.79\ \text{J/mol}\cdot\text{K})(492\ \text{K} - 600\ \text{K}) = -780\ \text{J}$

Then $\Delta U = Q - W$ gives $W = Q - \Delta U = 1780\ \text{J}$. (It is correct for W to be positive since ΔV is positive.)

Process $3\to1$:

For constant pressure

$W = p\Delta V = (1.013\times10^5\ \text{Pa})(8.62\times10^{-3}\ \text{m}^3 - 14.1\times10^{-3}\ \text{m}^3) = -560\ \text{J}$

or $W = nR\Delta T = (0.350\ \text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K} - 492\ \text{K}) = -560\ \text{J}$, which checks. (It is correct for W to be negative, since ΔV is negative for this process.)

$Q = nC_p\Delta T = (0.350\ \text{mol})(29.10\ \text{J/mol}\cdot\text{K})(300\ \text{K} - 492\ \text{K}) = -1960\ \text{J}$

$\Delta U = Q - W = -1960\ \text{J} - (-560\ \text{K}) = -1400\ \text{J}$

or $\Delta U = nC_V\Delta T = (0.350\ \text{mol})(20.79\ \text{J/mol}\cdot\text{K})(300\ \text{K} - 492\ \text{K}) = -1400\ \text{J}$, which checks.

(c) $W_{\text{net}} = W_{1\to2} + W_{2\to3} + W_{3\to1} = 0 + 780\ \text{J} - 560\ \text{J} = +220\ \text{J}$

(d) $Q_{\text{net}} = Q_{1\to2} + Q_{2\to3} + Q_{3\to1} = 2180\ \text{J} + 0 - 1960\ \text{J} = +220\ \text{J}$

(e) $e = \dfrac{\text{work output}}{\text{heat energy input}} = \dfrac{W}{Q_\text{H}} = \dfrac{220\ \text{J}}{2180\ \text{J}} = 0.101 = 10.1\%.$

$e(\text{Carnot}) = 1 - T_\text{C}/T_\text{H} = 1 - 300\ \text{K}/600\ \text{K} = 0.500.$

EVALUATE: For a cycle $\Delta U = 0$, so by $\Delta U = Q - W$ it must be that $Q_{\text{net}} = W_{\text{net}}$ for a cycle. We can also check that $\Delta U_{\text{net}} = 0$: $\Delta U_{\text{net}} = \Delta U_{1\to2} + \Delta U_{2\to3} + \Delta U_{3\to1} = 2180\ \text{J} - 1050\ \text{J} - 1130\ \text{J} = 0$

$e < e(\text{Carnot})$, as it must.

20.35. **IDENTIFY:** The same amount of heat that enters the person's body also leaves the body, but these transfers of heat occur at different temperatures, so the person's entropy changes.

SET UP: 1 food calorie = 1000 cal = 4186 J. The heat enters the person's body at 37°C = 310 K and leaves at a temperature of 30°C = 303 K. $\Delta S = \dfrac{Q}{T}$.

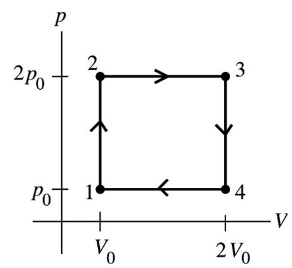
EXECUTE: $|Q| = (0.80)(2.50\ \text{g})(9.3\ \text{food calorie/g})\left(\dfrac{4186\ \text{J}}{1\ \text{food calorie}}\right) = 7.79\times10^4\ \text{J}.$

$\Delta S = \dfrac{+7.79\times10^4\ \text{J}}{310\ \text{K}} + \dfrac{-7.79\times10^4\ \text{J}}{303\ \text{K}} = -5.8\ \text{J/K}.$ Your body's entropy decreases.

EVALUATE: The entropy of your body can decrease without violating the second law of thermodynamics because you are not an isolated system.

20.36. **IDENTIFY:** Use $e = \dfrac{W}{Q_\text{H}}$ to calculate e.

SET UP: The cycle is sketched in Figure 20.36.



$C_V = 5R/2$

for an ideal gas $C_p = C_V + R = 7R/2$

Figure 20.36

SET UP: Calculate Q and W for each process.

Process $1 \to 2$:

$\Delta V = 0$ implies $W = 0$

$\Delta V = 0$ implies $Q = nC_V \Delta T = nC_V (T_2 - T_1)$

But $pV = nRT$ and V constant says $p_1 V = nRT_1$ and $p_2 V = nRT_2$.

Thus $(p_2 - p_1)V = nR(T_2 - T_1)$; $V\Delta p = nR\Delta T$ (true when V is constant).

Then $Q = nC_V \Delta T = nC_V (V\Delta p/nR) = (C_V/R)V\Delta p = (C_V/R)V_0(2p_0 - p_0) = (C_V/R)p_0V_0$. ($Q > 0$; heat is absorbed by the gas.)

Process $2 \to 3$:

$\Delta p = 0$ so $W = p\Delta V = p(V_3 - V_2) = 2p_0(2V_0 - V_0) = 2p_0V_0$ (W is positive since V increases.)

$\Delta p = 0$ implies $Q = nC_p \Delta T = nC_p(T_2 - T_1)$.

But $pV = nRT$ and p constant says $pV_1 = nRT_1$ and $pV_2 = nRT_2$.

Thus $p(V_2 - V_1) = nR(T_2 - T_1)$; $p\Delta V = nR\Delta T$ (true when p is constant).

Then $Q = nC_p \Delta T = nC_p(p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)2p_0(2V_0 - V_0) = (C_p/R)2p_0V_0$. ($Q > 0$; heat is absorbed by the gas.)

Process $3 \to 4$:

$\Delta V = 0$ implies $W = 0$

$\Delta V = 0$ so

$Q = nC_V \Delta T = nC_V(V\Delta p/nR) = (C_V/R)(2V_0)(p_0 - 2p_0) = -2(C_V/R)p_0V_0$

($Q < 0$ so heat is rejected by the gas.)

Process $4 \to 1$:

$\Delta p = 0$ so $W = p\Delta V = p(V_1 - V_4) = p_0(V_0 - 2V_0) = -p_0V_0$ (W is negative since V decreases)

$\Delta p = 0$ so $Q = nC_p \Delta T = nC_p(p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)p_0(V_0 - 2V_0) = -(C_p/R)p_0V_0$ ($Q < 0$ so heat is rejected by the gas.)

Total work performed by the gas during the cycle:

$W_{\text{tot}} = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + W_{4\to 1} = 0 + 2p_0V_0 + 0 - p_0V_0 = p_0V_0$

(Note that W_{tot} equals the area enclosed by the cycle in the pV-diagram.)

Total heat absorbed by the gas during the cycle (Q_{H}):

Heat is absorbed in processes $1 \to 2$ and $2 \to 3$.

$$Q_{\text{H}} = Q_{1\to 2} + Q_{2\to 3} = \frac{C_V}{R}p_0V_0 + 2\frac{C_p}{R}p_0V_0 = \left(\frac{C_V + 2C_p}{R}\right)p_0V_0$$

But $C_p = C_V + R$ so $Q_{\text{H}} = \dfrac{C_V + 2(C_V + R)}{R}p_0V_0 = \left(\dfrac{3C_V + 2R}{R}\right)p_0V_0$.

Total heat rejected by the gas during the cycle (Q_{C}):

Heat is rejected in processes $3 \to 4$ and $4 \to 1$.

$$Q_{\text{C}} = Q_{3\to4} + Q_{4\to1} = -2\frac{C_V}{R}p_0V_0 - \frac{C_p}{R}p_0V_0 = -\left(\frac{2C_V + C_p}{R}\right)p_0V_0$$

But $C_p = C_V + R$ so $Q_{\text{C}} = -\dfrac{2C_V + (C_V + R)}{R}p_0V_0 = -\left(\dfrac{3C_V + R}{R}\right)p_0V_0.$

Efficiency: $e = \dfrac{W}{Q_{\text{H}}} = \dfrac{p_0V_0}{[(3C_V + 2R)/R](p_0V_0)} = \dfrac{R}{3C_V + 2R} = \dfrac{R}{3(5R/2) + 2R} = \dfrac{2}{19}.$ $e = 0.105 = 10.5\%.$

EVALUATE: As a check on the calculations note that

$Q_{\text{C}} + Q_{\text{H}} = -\left(\dfrac{3C_V + R}{R}\right)p_0V_0 + \left(\dfrac{3C_V + 2R}{R}\right)p_0V_0 = p_0V_0 = W,$ as it should.

20.37. **IDENTIFY:** $pV = nRT$, so pV is constant when T is constant. Use the appropriate expression to calculate Q and W for each process in the cycle. $e = \dfrac{W}{Q_{\text{H}}}.$

SET UP: For an ideal diatomic gas, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R.$

EXECUTE: **(a)** $p_aV_a = 2.0\times10^3$ J. $p_bV_b = 2.0\times10^3$ J. $pV = nRT$ so $p_aV_a = p_bV_b$ says $T_a = T_b.$

(b) For an isothermal process, $Q = W = nRT\ln(V_2/V_1)$. ab is a compression, with $V_b < V_a$, so $Q < 0$ and heat is rejected. bc is at constant pressure, so $Q = nC_p\Delta T = \dfrac{C_p}{R}p\Delta V.$ ΔV is positive, so $Q > 0$ and heat is absorbed. ca is at constant volume, so $Q = nC_V\Delta T = \dfrac{C_V}{R}V\Delta p.$ Δp is negative, so $Q < 0$ and heat is rejected.

(c) $T_a = \dfrac{p_aV_a}{nR} = \dfrac{2.0\times10^3\text{ J}}{(1.00)(8.314\text{ J/mol}\cdot\text{K})} = 241\text{ K}.$ $T_b = \dfrac{p_bV_b}{nR} = T_a = 241\text{ K}.$

$$T_c = \frac{p_cV_c}{nR} = \frac{4.0\times10^3\text{ J}}{(1.00)(8.314\text{ J/mol}\cdot\text{K})} = 481\text{ K}.$$

(d) $Q_{ab} = nRT\ln\left(\dfrac{V_b}{V_a}\right) = (1.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(241\text{ K})\ln\left(\dfrac{0.0050\text{ m}^3}{0.010\text{ m}^3}\right) = -1.39\times10^3\text{ J}.$

$$Q_{bc} = nC_p\Delta T = (1.00)\left(\frac{7}{2}\right)(8.314\text{ J/mol}\cdot\text{K})(241\text{ K}) = 7.01\times10^3\text{ J}.$$

$$Q_{ca} = nC_V\Delta T = (1.00)\left(\frac{5}{2}\right)(8.314\text{ J/mol}\cdot\text{K})(-241\text{ K}) = -5.01\times10^3\text{ J}.$$

$Q_{\text{net}} = Q_{ab} + Q_{bc} + Q_{ca} = 610$ J. $W_{\text{net}} = Q_{\text{net}} = 610$ J.

(e) $e = \dfrac{W}{Q_{\text{H}}} = \dfrac{610\text{ J}}{7.01\times10^3\text{ J}} = 0.087 = 8.7\%.$

EVALUATE: We can calculate W for each process in the cycle. $W_{ab} = Q_{ab} = -1.39\times10^3$ J. $W_{bc} = p\Delta V = (4.0\times10^5\text{ Pa})(0.0050\text{ m}^3) = 2.00\times10^3$ J. $W_{ca} = 0.$ $W_{\text{net}} = W_{ab} + W_{bc} + W_{ca} = 610$ J, which does equal Q_{net}.

20.38. **IDENTIFY and SET UP:** This problem involves the entropy change of ice water and the surrounding air in a room. Estimate: The cup is ¼ ice and ¾ water all at 0°C when we go to bed. Convert pints to liters and cubic meters: 1 gal = 3.788 L and 8 pints = 1 gal, so 1 pint = (3.788 L)/8 = 0.474 L = $4.74\times10^{-4}\text{ m}^3$. From Table 12.1, $\rho_{\text{w}} = 1000\text{ kg/m}^3$ and $\rho_{\text{ice}} = 920\text{ kg/m}^3$, so the masses are m_{ice} = (920 kg/m³)(¼)($4.74\times10^{-4}\text{ m}^3$) = 0.109 kg, m_{w} = (1000 kg/m³)(¾)($4.74\times10^{-4}\text{ m}^3$) = 0.355 kg. In the morning, the cup is full of liquid water at 22.2°C. During the night, the water-ice mixture went from 0.0°C to 22.2°C. $\Delta S = Q/T$ (for constant temperature) and $\Delta S = \int \frac{dQ}{T}$ (if the temperature varies).

EXECUTE: **(a)** We want the entropy change of the ice-water mixture. It starts at 0.0°C and ends up at 22.2°C. $\Delta S_{\text{tot}} = \Delta S_{\text{ice}} + \Delta S_{\text{water}}$. The entropy change of the original ice is the sum of the entropy change as it melts and the entropy change of the resulting water as it increases from 0.0°C to 22.2°C.

$$\Delta S_{\text{melt ice}} = \frac{Q}{T} = \frac{mL_{\text{f}}}{T} = \frac{(0.109\text{ kg})(334\times10^3\text{ J/kg})}{273\text{ K}} = +133.2\text{ J/kg}.$$

$\Delta S_{\text{melted ice}} = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{m_{\text{i}}c_{\text{w}}dT}{T} = m_{\text{i}}c_{\text{w}} \ln(T_2/T_1)$. Using m_{i} = 0.109 kg, c_{w} = 4190 J/kg · K , T_1 = 0.0°C = 273.15 K, T_2 = 22.2°C = 295.35 K, we get $\Delta S_{\text{melted ice}} = 35.7$ J/K. The total entropy change of the ice is $\Delta S_{\text{ice}} =$ 133.2 J/K + 35.7 J/K = 169 J/K. The entropy change of the original water in the glass is

$\Delta S_{\text{water}} = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{m_{\text{w}}c_{\text{w}}dT}{T} = m_{\text{w}}c_{\text{w}} \ln T_2/T_1$. Using T_1 = 0.0°C = 273.15 K, T_2 = 22.2°C = 295.35 K, m_{w} = 0.355 kg, we get $\Delta S_{\text{water}} =$ 116.2 J/K. The total entropy change is $\Delta S_{\text{tot}} = \Delta S_{\text{ice}} + \Delta S_{\text{water}}$ = 169 J/K + 116 J/K = +285 J/K.

(b) The air remains at a constant 22.2°C = 295.35 K, so $\Delta S_{\text{air}} = Q/T_{\text{air}}$. Q is the *total* heat that flows out of the air into the ice-water mixture in the plastic cup. This heat melts the ice and then increases the temperature of *all* the water from 0.0°C to 22.2°C. Therefore $Q = Q_{\text{melt ice}} + Q_{\text{heat all water to 22.2°C}} = m_{\text{i}}L_{\text{f}} + m_{\text{all water}}c_{\text{w}}\Delta T_{\text{w}}$. This gives

Q = (0.109 kg + 0.355 kg)(4190 J/kg · K)(22.2°C) = 7.96×10^4 J. This heat comes *out* of the air, so

$\Delta S_{\text{air}} = \frac{Q}{T_{\text{air}}} = \frac{-7.96\times10^4\text{ J}}{285.35\text{ K}} = -279$ J/K. The total entropy change is –279 J/K + 285 J/K = +6 J/K, a positive change.

EVALUATE: The heating of the ice/water mixture is irreversible, so $\Delta S > 0$.

20.39. **IDENTIFY:** $T_b = T_c$ and is equal to the maximum temperature. Use the ideal gas law to calculate T_a. Apply the appropriate expression to calculate Q for each process. $e = \frac{W}{Q_{\text{H}}}$. $\Delta U = 0$ for a complete cycle and for an isothermal process of an ideal gas.

SET UP: For helium, $C_V = 3R/2$ and $C_p = 5R/2$. The maximum efficiency is for a Carnot cycle, and $e_{\text{Carnot}} = 1 - T_{\text{C}}/T_{\text{H}}$.

EXECUTE: **(a)** $Q_{\text{in}} = Q_{ab} + Q_{bc}$. $Q_{\text{out}} = Q_{ca}$. $T_{\max} = T_b = T_c = 327°\text{C} = 600\text{ K}$.

$$\frac{p_aV_a}{T_a} = \frac{p_bV_b}{T_b} \to T_a = \frac{p_a}{p_b}T_b = \frac{1}{3}(600\text{ K}) = 200\text{ K}.$$

$$p_bV_b = nRT_b \to V_b = \frac{nRT_b}{p_b} = \frac{(2\text{ moles})(8.31\text{ J/mol}\cdot\text{K})(600\text{ K})}{3.0\times10^5\text{ Pa}} = 0.0332\text{ m}^3.$$

$$\frac{p_bV_b}{T_b} = \frac{p_cV_c}{T_c} \to V_c = V_b\frac{p_b}{p_c} = (0.0332\text{ m}^3)\left(\frac{3}{1}\right) = 0.0997\text{ m}^3 = V_a.$$

$$Q_{ab} = nC_V\Delta T_{ab} = (2\text{ mol})\left(\frac{3}{2}\right)(8.31\text{ J/mol}\cdot\text{K})(400\text{ K}) = 9.97\times10^3\text{ J}.$$

$$Q_{bc} = W_{bc} = \int_b^c p\,dV = \int_b^c \frac{nRT_b}{V}dV = nRT_b \ln\frac{V_c}{V_b} = nRT_b \ln 3.$$

$Q_{bc} = (2.00\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(600\text{ K})\ln 3 = 1.10\times10^4\text{ J}$. $Q_{\text{in}} = Q_{ab} + Q_{bc} = 2.10\times10^4\text{ J}$.

$$Q_{\text{out}} = Q_{ca} = nC_p\Delta T_{ca} = (2.00\text{ mol})\left(\frac{5}{2}\right)(8.31\text{ J/mol}\cdot\text{K})(400\text{ K}) = 1.66\times10^4\text{ J}.$$

(b) $Q = \Delta U + W = 0 + W \to W = Q_{\text{in}} - Q_{\text{out}} = 2.10\times10^4\text{ J} - 1.66\times10^4\text{ J} = 4.4\times10^3\text{ J}$.

$$e = W/Q_{\text{in}} = \frac{4.4\times10^3\text{ J}}{2.10\times10^4\text{ J}} = 0.21 = 21\%.$$

(c) $e_{\text{max}} = e_{\text{Carnot}} = 1 - \dfrac{T_C}{T_H} = 1 - \dfrac{200\text{ k}}{600\text{ k}} = 0.67 = 67\%$.

Evaluate: The thermal efficiency of this cycle is about one-third of the efficiency of a Carnot cycle that operates between the same two temperatures.

20.40. **Identify:** Since there is temperature difference between the inside and outside of your body, you can use it as a heat engine.

Set Up: For a heat engine $e = \dfrac{W}{Q_H}$. For a Carnot engine $e = 1 - \dfrac{T_C}{T_H}$. Gravitational potential energy is $U_{\text{grav}} = mgh$. 1 food calorie = 1000 cal = 4186 J.

Execute: **(a)** $e = 1 - \dfrac{T_C}{T_H} = 1 - \dfrac{303\text{ K}}{310\text{ K}} = 0.0226 = 2.26\%$. This engine has a very low thermal efficiency.

(b) $U_{\text{grav}} = mgh = (2.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}) = 29.4\text{ J}$. This equals the work output of the engine.

$e = \dfrac{W}{Q_H}$ so $Q_H = \dfrac{W}{e} = \dfrac{29.4\text{ J}}{0.0226} = 1.30\times10^3\text{ J}$.

(c) Since 80% of food energy goes into heat, you must eat food with a food energy of $\dfrac{1.30\times10^3\text{ J}}{0.80} = 1.63\times10^3\text{ J}$. Each candy bar gives $(350\text{ food calorie})(4186\text{ J/food calorie}) = 1.47\times10^6\text{ J}$.

The number of candy bars required is $\dfrac{1.63\times10^3\text{ J}}{1.47\times10^6\text{ J/candy bar}} = 1.11\times10^{-3}$ candy bars.

Evaluate: A large amount of mechanical work must be done to use up the energy from one candy bar.

20.41. **Identify:** $e_{\text{max}} = e_{\text{Carnot}} = 1 - T_C/T_H$. $e = \dfrac{W}{Q_H} = \dfrac{W/t}{Q_H/t}$. $W = Q_H + Q_C$ so $\dfrac{W}{t} = \dfrac{Q_C}{t} + \dfrac{Q_H}{t}$. For a temperature change $Q = mc\Delta T$.

Set Up: $T_H = 300.15\text{ K}$, $T_C = 279.15\text{ K}$. For water, $\rho = 1000\text{ kg/m}^3$, so a mass of 1 kg has a volume of 1 L. For water, $c = 4190\text{ J/kg}\cdot\text{K}$.

Execute: **(a)** $e = 1 - \dfrac{279.15\text{K}}{300.15\text{K}} = 7.0\%$.

(b) $\dfrac{Q_H}{t} = \dfrac{P_{\text{out}}}{e} = \dfrac{210\text{ kW}}{0.070} = 3.0\text{ MW}$. $\dfrac{|Q_C|}{t} = \dfrac{Q_H}{t} - \dfrac{W}{t} = 3.0\text{ MW} - 210\text{ kW} = 2.8\text{ MW}$.

(c) $\dfrac{m}{t} = \dfrac{|Q_C|/t}{c\Delta T} = \dfrac{(2.8\times10^6\text{ W})(3600\text{ s/h})}{(4190\text{ J/kg}\cdot\text{K})(4\text{ K})} = 6\times10^5\text{ kg/h} = 6\times10^5\text{ L/h}$.

Evaluate: The efficiency is small since T_C and T_H don't differ greatly.

20.42. **IDENTIFY:** Use $\Delta U = Q - W$ and the appropriate expressions for Q, W and ΔU for each type of process. $pV = nRT$ relates ΔT to p and V values. $e = \dfrac{W}{Q_H}$, where Q_H is the heat that enters the gas during the cycle.

SET UP: For a monatomic ideal gas, $C_p = \frac{5}{2}R$ and $C_V = \frac{3}{2}R$.

(a) *ab*: The temperature changes by the same factor as the volume, and so

$$Q = nC_p\Delta T = \frac{C_p}{R}p_a(V_a - V_b) = (2.5)(3.00\times10^5 \text{ Pa})(0.300 \text{ m}^3) = 2.25\times10^5 \text{ J}.$$

The work $p\Delta V$ is the same except for the factor of $\frac{5}{2}$, so $W = 0.90\times10^5$ J.

$\Delta U = Q - W = 1.35\times10^5$ J.

bc: The temperature now changes in proportion to the pressure change, and $Q = \frac{3}{2}(p_c - p_b)V_b = (1.5)(-2.00\times10^5 \text{ Pa})(0.800 \text{ m}^3) = -2.40\times10^5$ J, and the work is zero $(\Delta V = 0)$. $\Delta U = Q - W = -2.40\times10^5$ J.

ca: The easiest way to do this is to find the work done first; W will be the negative of area in the *p-V* plane bounded by the line representing the process *ca* and the verticals from points *a* and *c*. The area of this trapezoid is $\frac{1}{2}(3.00\times10^5 \text{ Pa} + 1.00\times10^5 \text{ Pa})(0.800 \text{ m}^3 - 0.500 \text{ m}^3) = 6.00\times10^4$ J and so the work is -0.60×10^5 J. ΔU must be 1.05×10^5 J (since $\Delta U = 0$ for the cycle, anticipating part (b)), and so Q must be $\Delta U + W = 0.45\times10^5$ J.

(b) See above; $Q = W = 0.30\times10^5$ J, $\Delta U = 0$.

(c) The heat added, during process *ab* and *ca*, is $2.25\times10^5 \text{ J} + 0.45\times10^5 \text{ J} = 2.70\times10^5$ J and the efficiency is $e = \dfrac{W}{Q_H} = \dfrac{0.30\times10^5}{2.70\times10^5} = 0.111 = 11.1\%$.

EVALUATE: For any cycle, $\Delta U = 0$ and $Q = W$.

20.43. **IDENTIFY:** Use $pV = nRT$. Apply the expressions for Q and W that apply to each type of process. $e = \dfrac{W}{Q_H}$.

SET UP: For O_2, $C_V = 20.85$ J/mol·K and $C_p = 29.17$ J/mol·K.

EXECUTE: **(a)** $p_1 = 2.00$ atm, $V_1 = 4.00$ L, $T_1 = 300$ K.

$p_2 = 2.00$ atm. $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$. $V_2 = \left(\dfrac{T_2}{T_1}\right)V_1 = \left(\dfrac{450 \text{ K}}{300 \text{ K}}\right)(4.00 \text{ L}) = 6.00$ L.

$V_3 = 6.00$ L. $\dfrac{p_2}{T_2} = \dfrac{p_3}{T_3}$. $p_3 = \left(\dfrac{T_3}{T_2}\right)p_2 = \left(\dfrac{250 \text{ K}}{450 \text{ K}}\right)(2.00 \text{ atm}) = 1.11$ atm.

$V_4 = 4.00$ L. $p_3V_3 = p_4V_4$. $p_4 = p_3\left(\dfrac{V_3}{V_4}\right) = (1.11 \text{ atm})\left(\dfrac{6.00 \text{ L}}{4.00 \text{ L}}\right) = 1.67$ atm.

These processes are shown in Figure 20.43.

(b) $n = \dfrac{p_1V_1}{RT_1} = \dfrac{(2.00 \text{ atm})(4.00 \text{ L})}{(0.08206 \text{ L}\cdot\text{atm/mol}\cdot\text{K})(300 \text{ K})} = 0.325$ mol

Process $1 \to 2$: $W = p\Delta V = nR\Delta T = (0.325 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(150 \text{ K}) = 405$ J.

$Q = nC_p\Delta T = (0.325 \text{ mol})(29.17 \text{ J/mol}\cdot\text{K})(150 \text{ K}) = 1422$ J.

Process $2 \to 3$: $W = 0$. $Q = nC_V\Delta T = (0.325 \text{ mol})(20.85 \text{ J/mol}\cdot\text{K})(-200 \text{ K}) = -1355$ J.

Process $3 \to 4$: $\Delta U = 0$ and

$$Q = W = nRT_3 \ln\left(\frac{V_4}{V_3}\right) = (0.325 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(250 \text{ K})\ln\left(\frac{4.00 \text{ L}}{6.00 \text{ L}}\right) = -274 \text{ J}.$$

Process $4 \to 1$: $W = 0$. $Q = nC_V\Delta T = (0.325 \text{ mol})(20.85 \text{ J/mol}\cdot\text{K})(50 \text{ K}) = 339 \text{ J}$.

(c) $W = 405 \text{ J} - 274 \text{ J} = 131 \text{ J}$.

(d) $e = \dfrac{W}{Q_\text{H}} = \dfrac{131 \text{ J}}{1422 \text{ J} + 339 \text{ J}} = 0.0744 = 7.44\%$.

$e_\text{Carnot} = 1 - \dfrac{T_\text{C}}{T_\text{H}} = 1 - \dfrac{250 \text{ K}}{450 \text{ K}} = 0.444 = 44.4\%$; e_Carnot is much larger.

Evaluate: $Q_\text{tot} = +1422 \text{ J} + (-1355 \text{ J}) + (-274 \text{ J}) + 339 \text{ J} = 132 \text{ J}$. This is equal to W_tot, apart from a slight difference due to rounding. For a cycle, $W_\text{tot} = Q_\text{tot}$, since $\Delta U = 0$.

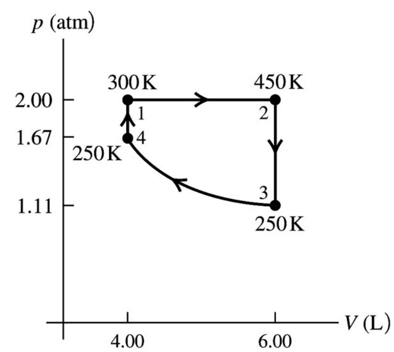


Figure 20.43

20.44. Identify: $e = \dfrac{W}{Q_\text{H}}$. 1 day $= 8.64\times10^4$ s. For the river water, $Q = mc\Delta T$, where the heat that goes into the water is the heat Q_C rejected by the engine. The density of water is 1000 kg/m^3. When an object undergoes a temperature change, $\Delta S = mc\ln(T_2/T_1)$.

Set Up: $18.0°\text{C} = 291.1 \text{ K}$. $18.5°\text{C} = 291.6 \text{ K}$.

Execute: **(a)** $Q_\text{H} = \dfrac{W}{e}$ so $P_\text{H} = \dfrac{P_\text{W}}{e} = \dfrac{1000 \text{ MW}}{0.40} = 2.50\times10^3 \text{ MW}$.

(b) The heat input in one day is $(2.50\times10^9 \text{ W})(8.64\times10^4 \text{ s}) = 2.16\times10^{14} \text{ J}$. The mass of coal used per day is $\dfrac{2.16\times10^{14} \text{ J}}{2.65\times10^7 \text{ J/kg}} = 8.15\times10^6 \text{ kg}$.

(c) $|Q_\text{H}| = |W| + |Q_\text{C}|$. $|Q_\text{C}| = |Q_\text{H}| - |W|$. $P_\text{C} = P_\text{H} - P_\text{W} = 2.50\times10^3 \text{ MW} - 1000 \text{ MW} = 1.50\times10^3 \text{ MW}$.

(d) The heat input to the river is 1.50×10^9 J/s. $Q = mc\Delta T$ and $\Delta T = 0.5 \text{ C°}$ gives

$m = \dfrac{Q}{c\Delta T} = \dfrac{1.50\times10^9 \text{ J}}{(4190 \text{ J/kg}\cdot\text{K})(0.5 \text{ K})} = 7.16\times10^5 \text{ kg}$. $V = \dfrac{m}{\rho} = 716 \text{ m}^3$. The river flow rate must be $716 \text{ m}^3/\text{s}$.

(e) In one second, 7.16×10^5 kg of water goes from 291.1 K to 291.6 K.

$$\Delta S = mc\ln\left(\frac{T_2}{T_1}\right) = (7.16\times10^5 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})\ln\left(\frac{291.6 \text{ K}}{291.1 \text{ K}}\right) = 5.1\times10^6 \text{ J/K}.$$

Evaluate: The entropy of the river increases because heat flows into it. The mass of coal used per second is huge.

20.45. IDENTIFY: The efficiency of the composite engine is $e_{12} = \dfrac{W_1 + W_2}{Q_{\text{H1}}}$, where Q_{H1} is the heat input to the first engine and W_1 and W_2 are the work outputs of the two engines. For any heat engine, $W = Q_\text{C} + Q_\text{H}$, and for a Carnot engine, $\dfrac{Q_{\text{low}}}{Q_{\text{high}}} = -\dfrac{T_{\text{low}}}{T_{\text{high}}}$, where Q_{low} and Q_{high} are the heat flows at the two reservoirs that have temperatures T_{low} and T_{high}.

SET UP: $Q_{\text{high,2}} = -Q_{\text{low,1}}$. $T_{\text{low,1}} = T'$, $T_{\text{high,1}} = T_\text{H}$, $T_{\text{low,2}} = T_\text{C}$ and $T_{\text{high,2}} = T'$.

EXECUTE: $e_{12} = \dfrac{W_1 + W_2}{Q_{\text{H1}}} = \dfrac{Q_{\text{high,1}} + Q_{\text{low,1}} + Q_{\text{high,2}} + Q_{\text{low,2}}}{Q_{\text{high,1}}}$. Since $Q_{\text{high,2}} = -Q_{\text{low,1}}$, this reduces to

$e_{12} = 1 + \dfrac{Q_{\text{low,2}}}{Q_{\text{high,1}}}$. $Q_{\text{low,2}} = -Q_{\text{high,2}}\dfrac{T_{\text{low,2}}}{T_{\text{high,2}}} = Q_{\text{low,1}}\dfrac{T_\text{C}}{T'} = -Q_{\text{high,1}}\left(\dfrac{T_{\text{low,1}}}{T_{\text{high,1}}}\right)\dfrac{T_\text{C}}{T'} = -Q_{\text{high,1}}\left(\dfrac{T'}{T_\text{H}}\right)\dfrac{T_\text{C}}{T'}$. This gives

$e_{12} = 1 - \dfrac{T_\text{C}}{T_\text{H}}$. The efficiency of the composite system is the same as that of the original engine.

EVALUATE: The overall efficiency is independent of the value of the intermediate temperature T'.

20.46. IDENTIFY: We are dealing with the efficiency of an automobile engine.

SET UP: **(a)** Estimate: 45 miles/gallon (a Prius).

EXECUTE: **(b)** The combustion releases 120 MJ/gal. At 40 mph the engine is turning at 3000 rpm which is 3000 cycles/min. We want to know how many joules/cycle are released. At 40 mph, the time to travel 45 mi is $t = x/v$ = (45 mi)/(40 mi/h) = 1.125 h = 67.5 min. The energy released during this time is 1 gal = 120 MJ. In 67.5 min the number of cycles the engine turns through is (3000 cyles/min)(67.5 min) = 2.025×10^5 cycles. The energy per cycle is $\dfrac{120 \text{ MJ}}{2.025\times10^5 \text{ cycles}} = 593$ J/cycle.

(c) We want the power the car engine supplies at 40 mph. The power P is the energy E divided by the time t, so $P = E/t$. At 20% efficiency, E = (0.20)(593 J/cycle) = 118.5 J/cycle and 3000 cycles/min = 50 cycles/s, so the time for one cycle is t = 1/50 s. So $P = E/t$ = (118.5 J)/(1/50 s) = 5900 W.

(d) Convert to horsepower: (5900 W)(1 hp/746 W) = 7.9 hp.

EVALUATE: Based on the "power climb" in Example 6.10, the car power is about $\dfrac{5900 \text{ W}}{241 \text{ W}}$ = 24 times as great as the power climber.

20.47. (a) IDENTIFY and SET UP: Calculate e from $e = 1 - 1/(r^{\gamma-1})$, Q_C from $e = (Q_\text{H} + Q_\text{C})/Q_\text{H}$, and then W from $W = Q_\text{C} + Q_\text{H}$.

EXECUTE: $e = 1 - 1/(r^{\gamma-1}) = 1 - 1/(10.6^{0.4}) = 0.6111$

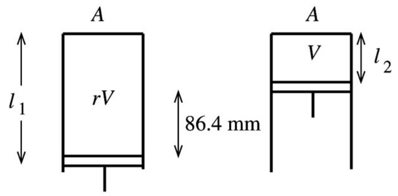
$e = (Q_\text{H} + Q_\text{C})/Q_\text{H}$ and we are given $Q_\text{H} = 200$ J; calculate Q_C.

$Q_\text{C} = (e-1)Q_\text{H} = (0.6111 - 1)(200\text{ J}) = -78$ J. (negative, since corresponds to heat leaving)

Then $W = Q_\text{C} + Q_\text{H} = -78\text{ J} + 200\text{ J} = 122$ J. (positive, in agreement with Figure 20.6 in the text)

EVALUATE: Q_H, $W > 0$, and $Q_\text{C} < 0$ for an engine cycle.

(b) IDENTIFY and SET UP: The stoke times the bore equals the change in volume. The initial volume is the final volume V times the compression ratio r. Combining these two expressions gives an equation for V. For each cylinder of area $A = \pi(d/2)^2$ the piston moves 0.0864 m and the volume changes from rV to V, as shown in Figure 20.47a.



$l_1 A = rV$

$l_2 A = V$

and $l_1 - l_2 = 86.4\times10^{-3}$ m

Figure 20.47a

EXECUTE: $l_1 A - l_2 A = rV - V$ and $(l_1 - l_2)A = (r-1)V$

$$V = \frac{(l_1 - l_2)A}{r-1} = \frac{(86.4\times10^{-3}\ \text{m})\pi(41.25\times10^{-3}\ \text{m})^2}{10.6-1} = 4.811\times10^{-5}\ \text{m}^3$$

At point a the volume is $rV = 10.6(4.811\times10^{-5}\ \text{m}^3) = 5.10\times10^{-4}\ \text{m}^3$.

(c) IDENTIFY and **SET UP:** The processes in the Otto cycle are either constant volume or adiabatic. Use the Q_H that is given to calculate ΔT for process bc. Use $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ and $pV = nRT$ to relate p, V and T for the adiabatic processes ab and cd.

EXECUTE: <u>Point a</u>: $T_a = 300$ K, $p_a = 8.50\times10^4$ Pa and $V_a = 5.10\times10^{-4}\ \text{m}^3$.

<u>Point b</u>: $V_b = V_a/r = 4.81\times10^{-5}\ \text{m}^3$. Process $a \to b$ is adiabatic, so $T_aV_a^{\gamma-1} = T_bV_b^{\gamma-1}$.

$T_a(rV)^{\gamma-1} = T_bV^{\gamma-1}$

$T_b = T_ar^{\gamma-1} = 300\ \text{K}(10.6)^{0.4} = 771$ K

$pV = nRT$ so $pV/T = nR = \text{constant}$, so $p_aV_a/T_a = p_bV_b/T_b$

$p_b = p_a(V_a/V_b)(T_b/T_a) = (8.50\times10^4\ \text{Pa})(rV/V)(771\ \text{K}/300\ \text{K}) = 2.32\times10^6$ Pa

<u>Point c</u>: Process $b \to c$ is at constant volume, so $V_c = V_b = 4.81\times10^{25}\ \text{m}^3$

$Q_\text{H} = nC_V\Delta T = nC_V(T_c - T_b)$. The problem specifies $Q_\text{H} = 200$ J; use to calculate T_c. First use the p, V, T values at point a to calculate the number of moles n.

$$n = \frac{pV}{RT} = \frac{(8.50\times10^4\ \text{Pa})(5.10\times10^{-4}\ \text{m}^3)}{(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})} = 0.01738\ \text{mol}$$

Then $T_c - T_b = \dfrac{Q_\text{H}}{nC_V} = \dfrac{200\ \text{J}}{(0.01738\ \text{mol})(20.5\ \text{J/mol}\cdot\text{K})} = 561.3$ K, and

$T_c = T_b + 561.3\ \text{K} = 771\ \text{K} + 561\ \text{K} = 1332$ K

$p/T = nR/V = \text{constant}$ so $p_b/T_b = p_c/T_c$

$p_c = p_b(T_c/T_b) = (2.32\times10^6\ \text{Pa})(1332\ \text{K}/771\ \text{K}) = 4.01\times10^6$ Pa

<u>Point d</u>: $V_d = V_a = 5.10\times10^{-4}\ \text{m}^3$

Process $c \to d$ is adiabatic, so $T_dV_d^{\gamma-1} = T_cV_c^{\gamma-1}$

$T_d(rV)^{\gamma-1} = T_cV^{\gamma-1}$

$T_d = T_c/r^{\gamma-1} = 1332\ \text{K}/10.6^{0.4} = 518$ K

$p_cV_c/T_c = p_dV_d/T_d$

$p_d = p_c(V_c/V_d)(T_d/T_c) = (4.01\times10^6\ \text{Pa})(V/rV)(518\ \text{K}/1332\ \text{K}) = 1.47\times10^5$ Pa

EVALUATE: Can look at process $d \to a$ as a check.

$Q_\text{C} = nC_V(T_a - T_d) = (0.01738\ \text{mol})(20.5\ \text{J/mol}\cdot\text{K})(300\ \text{K} - 518\ \text{K}) = -78$ J, which agrees with part (a).

The cycle is sketched in Figure 20.47b.

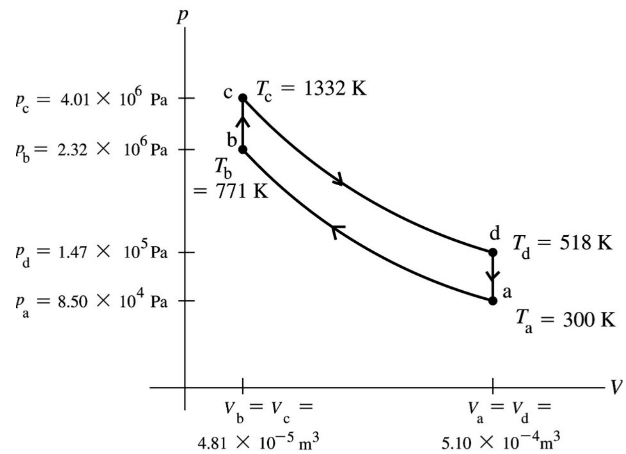


Figure 20.47b

(d) IDENTIFY and **SET UP:** The Carnot efficiency is given by $e_{\text{Carnot}} = 1 - \dfrac{T_\text{C}}{T_\text{H}}$. T_H is the highest temperature reached in the cycle and T_C is the lowest.

EXECUTE: From part (a) the efficiency of this Otto cycle is $e = 0.611 = 61.1\%$.
The efficiency of a Carnot cycle operating between 1332 K and 300 K is
$e_{\text{Carnot}} = 1 - T_\text{C}/T_\text{H} = 1 - 300\text{ K}/1332\text{ K} = 0.775 = 77.5\%$, which is larger.

EVALUATE: The second law of thermodynamics requires that $e \le e_{\text{Carnot}}$, and our result obeys this law.

20.48. **IDENTIFY:** We want to estimate the entropy of the air in a typical room.
SET UP: **(a)** Estimate: My office volume is $V = (12\text{ ft})(15\text{ ft})(8.0\text{ ft}) = 1440\text{ ft}^3 = 41\text{ m}^3$.
EXECUTE: **(b)** $n/(41\text{ m}^3) = (1\text{ mol})/(22.4\text{ L})$, which gives $n = 1800$ mol.
(c) $N = nN_\text{A} = (1800\text{ mol})(6.022\times10^{23}\text{ molecules/mol}) = 1.1\times10^{27}$ molecules.
(d) $S = k \ln w$. In this case, $N^N = w$, so $S = k \ln N^N = kN \ln N$. For the numbers here, we get
$S = (1.38\times10^{-23}\text{ J/K})(1.1\times10^{27})\ln(1.1\times10^{27}) = 9.3\times10^5\text{ J/K}$.
EVALUATE: As N increases, there are more possible states, so S also increases.

20.49. **IDENTIFY** and **SET UP:** A refrigerator is like a heat engine run in reverse. In the pV-diagram shown with the figure, heat enters the gas during parts *ab* and *bc* of the cycle, and leaves during *ca*. Treating H_2 as a diatomic gas, we know that $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. Segment *bc* is isochoric, so $Q_{bc} = nC_V\Delta T$. Segment *ca* is isobaric, so $Q_{ca} = nC_p\Delta T$. Segment *ab* is isothermal, so $Q_{ab} = nRT\ln(V_b/V_a)$. The coefficient of performance of a refrigerator is $K = \dfrac{|Q_\text{C}|}{|W|} = \dfrac{|Q_\text{C}|}{|Q_\text{H}| - |Q_\text{C}|}$, and $pV = nRT$ applies.

Calculate the values for Q_C and Q_H and use the definition of K. Use 1000 L = 1 m^3 and work in units of $\text{L}\cdot\text{atm}$.
EXECUTE: Use $pV = nRT$ to find p_b. Since *ab* is isothermal, $p_aV_a = p_bV_b$, which gives
$p_b = (0.700\text{ atm})(0.0300\text{ m}^3)/(0.100\text{ m}^3) = 0.210$ atm.
$Q_\text{C} = Q_{ab} + Q_{bc}$, so we need to calculate these quantities.
$Q_{ab} = nRT\ln(V_b/V_a) = p_aV_a\ln(V_b/V_a) = (0.700\text{ atm})(30.0\text{ L})\ln[(100\text{ L})/(30\text{ L})] = 25.2834\ \text{L}\cdot\text{atm}$.
$Q_{bc} = nC_V\Delta T_{bc} = n(\frac{5}{2}R)\ \Delta T_{bc} = \frac{5}{2} V_b\Delta p_{bc} = (5/2)(100\text{ L})(0.700\text{ atm} - 0.210\text{ atm}) = 122.5\ \text{L}\cdot\text{atm}$.
Therefore $Q_\text{C} = Q_{ab} + Q_{bc} = 25.2834\ \text{L}\cdot\text{atm} + 122.500\ \text{L}\cdot\text{atm} = 147.7834\ \text{L}\cdot\text{atm}$.

$Q_H = Q_{ca} = nC_p\Delta T_{ca} = n\left(\frac{7}{2}R\right)\Delta T_{ca} = \frac{7}{2}p_c\Delta T_{ca}$ = (7/2)(0.700 atm)(30.0 L – 100.0 L) =–171.500 L · atm.

Now get K: $K = \dfrac{|Q_C|}{|Q_H| - |Q_C|} = \dfrac{147.7834 \text{ L}\cdot\text{atm}}{(171.500 \text{ L}\cdot\text{atm} - 147.7834 \text{ L}\cdot\text{atm}} = 6.23.$

EVALUATE: K is greater than 1, which it must be. Efficiencies are less than 1.

20.50. **IDENTIFY** and **SET UP:** A person radiates heat from the surface of her body which is at a constant temperature of T = 30.0°C into air at T_s = 20.0°C. In 1.0 s, heat $Ae\sigma tT^4$ flows from the person into the room and heat $Ae\sigma tT_s^4$ flows out of the room into the person. The heat flows into and out of the room occur at a temperature of T_s. At constant temperature, $\Delta S = Q/T$.

EXECUTE: For the room,

$$\Delta S = \frac{Ae\sigma tT^4}{T_s} - \frac{Ae\sigma tT_s^4}{T_s} = \frac{Ae\sigma t(T^4 - T_s^4)}{T_s}$$

$$\Delta S = \frac{(1.85 \text{ m}^2)(1.00)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(1.0)[(303 \text{ K})^4 - (293 \text{ K})^4]}{293 \text{ K}} = 0.379 \text{ J/K}$$

EVALUATE: The entropy change is positive since the air in the room becomes more disordered. In addition, this process is irreversible; heat from the cool room will not spontaneously flow into the warmer person and increase her temperature even further. Heat flows only from hot to cold.

20.51. **IDENTIFY:** Use $\Delta S = mc\ln(T_2/T_1)$ for an isothermal process. For the value of T for which ΔS is a maximum, $d(\Delta S)/dT = 0$.

SET UP: The heat flow for a temperature change is $Q = mc\Delta T$.

EXECUTE: **(a)** As in Example 20.10, the entropy change of the first object is $m_1c_1\ln(T/T_1)$ and that of the second is $m_2c_2\ln(T'/T_2)$, and so the net entropy change is as given. Neglecting heat transfer to the surroundings, $Q_1 + Q_2 = 0, m_1c_1(T - T_1) + m_2c_2(T' - T_2) = 0,$ which is the given expression.

(b) Solving the energy-conservation relation for T' and substituting into the expression for ΔS gives $\Delta S = m_1c_1\ln\left(\frac{T}{T_1}\right) + m_2c_2\ln\left(1 - \frac{m_1c_1}{m_2c_2}\left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right)$. Differentiating with respect to T and setting the derivative equal to 0 gives $0 = \dfrac{m_1c_1}{T} + \dfrac{(m_2c_2)(m_1c_1/m_2c_2)(-1/T_2)}{\left(1 - (m_1c_1/m_2c_2)\left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right)}$. This may be solved for

$T = \dfrac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$. Using this value for T in the conservation of energy expression in part (a) and solving for T' gives $T' = \dfrac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$. Therefore, $T = T'$ when ΔS is a maximum.

EVALUATE: **(c)** The final state of the system will be that for which no further entropy change is possible. If $T < T'$, it is possible for the temperatures to approach each other while increasing the total entropy, but when $T = T'$, no further spontaneous heat exchange is possible.

20.52. **IDENTIFY:** Use the expression derived in Example 20.6 for the entropy change in a temperature change.

SET UP: For water, $c = 4190$ J/kg · K. 20°C = 293.15 K, 78°C = 351.15 K and 120°C = 393.15 K.

EXECUTE:

(a) $\Delta S = mc\ln(T_2/T_1) = (250\times10^{-3} \text{ kg})(4190 \text{ J/kg}\cdot\text{K})\ln(351.15 \text{ K}/293.15 \text{ K}) = 189 \text{ J/K}.$

(b) $\Delta S = \dfrac{-mc\Delta T}{T_{\text{element}}} = \dfrac{-(250\times10^{-3} \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(351.15 \text{ K} - 293.15 \text{ K})}{393.15 \text{ K}} = -155 \text{ J/K}.$

(c) The sum of the result of parts (a) and (b) is $\Delta S_{\text{system}} = 34.6$ J/K. (Carry extra figures when subtraction is involved.)

EVALUATE: **(d)** Heating a liquid is not reversible. Whatever the energy source for the heating element, heat is being delivered at a higher temperature than that of the water, and the entropy loss of the source will be less in magnitude than the entropy gain of the water. The net entropy change is positive.

20.53. **IDENTIFY** and **SET UP:** The most efficient heat engine operating between any two given temperatures is the Carnot engine, and its efficiency is $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$.

EXECUTE: **(a)** For prototype A, $e_{max} = e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ = 1 – (320 K)/(450 K) = 0.289 = 28.9%. By similar calculations, we get the following:

A: e_{max} = 0.289 = 28.9%
B: e_{max} = 0.383 = 38.3%
C: e_{max} = 0.538 = 53.8%
D: e_{max} = 0.244 = 24.4%

(b) Engine C claims a maximum efficiency of 56%, which is greater than the maximum possible for its temperature range, so it is impossible.

(c) We get the following ratios:

A: $e_{claimed}/e_{max}$ = 0.21/0.289 = 0.73
B: $e_{claimed}/e_{max}$ = 0.35/0.383 = 0.90
D: $e_{claimed}/e_{max}$ = 0.20/0.244 = 0.82

In decreasing order, we have B, D, A.

EVALUATE: Engine B is not only the most efficient, it is also closest to its maximum possible efficiency for its temperature range. Engines A and D have nearly the same efficiency, but D comes somewhat closer to its theoretical maximum than does A.

20.54. **IDENTIFY** and **SET UP:** In terms of power, $K = \frac{|Q_C|/t}{|W|/t}$. 1 Btu = 1055 J and $1\ \text{W}\cdot\text{h} = 3600$ J.

EXECUTE: **(a)** $1\ \text{W}\cdot\text{h}$ = (3600 J)(1 Btu/1055 J) = 3.412 Btu, so EER = $(3.412\ \text{Btu/W}\cdot\text{h})K$.

(b) T_H = 95°F = 35°C = 308.1 K; T_C = 80°F = 26.7°C = 299.8 K.

For a Carnot air conditioner, $K = T_C/(T_H - T_C)$ = (299.8 K)/(308.1 K – 299.8 K) = 36. The corresponding EER is

EER = $(3.412\ \text{Btu/W}\cdot\text{h})(36) = 120\ \text{Btu/W}\cdot\text{h}$.

(c) K = EER/3.412 = 10.9/3.412 = 3.195

$|W| = |Q_C|/K = (1.9\times10^{10}\ \text{J})/(3.195) = 5.95\times10^{9}\ \text{J}\ [(1\ \text{kWh})/(3.6\times10^{6}\ \text{J})] = 1650$ kWh. The cost is $0.153(1650 kWh) = $253.

(d) Using the same approach as in (c) gives the following values:

K = 4.279, W = 1233 kWh, cost = $189, savings = $253 – $189 = $64 per year.

EVALUATE: Whether it is worth it to replace your air conditioner depends on what it costs. In 10 years you would save $640 (probably more since utility rates tend to rise over time). But would the unit last10 years?

20.55. **IDENTIFY** and **SET UP:** The cycle consists of two isochoric processes (*ab* and *cd*) and two isobaric processes (*bc* and *da*). Use $Q = nC_V\Delta T$ and $Q = nC_p\Delta T$ for these processes. For an ideal monatomic gas (argon), $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$. Use $R = 8.3145\ \text{J/mol}\cdot\text{K}$.

EXECUTE: **(a)** Using the equations listed above, the heat transfers are as follows:

Q_{ab} = (3/2)(4.00 mol)(8.3145 J/mol·K)(300.0 K – 250.0 K) = 2.494 kJ

Q_{bc} = (5/2)(4.00 mol)(8.3145 J/mol·K)(380.0 K – 300.0 K) = 6.652 kJ

Q_{cd} = (3/2)(4.00 mol) (8.3145 J/mol · K) (316.7 K – 380.0 K) = –3.158 kJ

Q_{da} = (5/2)(4.00 mol) (8.3145 J/mol · K) (250.0 K – 316.7 K) = –5.546 kJ

The efficiency of this cycle is $e = \dfrac{W}{|Q_{\text{in}}|} = \dfrac{|Q_{\text{in}}| - |Q_{\text{out}}|}{|Q_{\text{in}}|} = 1 - \dfrac{|Q_{\text{out}}|}{|Q_{\text{in}}|}$. This gives

$e = 1 - \dfrac{3.158\text{ kJ} + 5.546\text{ kJ}}{2.494\text{ kJ} + 6.652\text{ kJ}} = 0.0483 = 4.83\%.$

(b) If we double the number of moles, all the values of Q will double, but the factor of 2 cancels out, so the efficiency remains the same.

(c) Using the same procedure as in (a), the revised numbers are

Q_{ab} = 2.494 kJ (unchanged)

Q_{bc} = 38.247 kJ

Q_{cd} = –6.316 kJ

Q_{da} = –31.878 kJ

As in part (a), the efficiency of this cycle is $e = 1 - \dfrac{|Q_{\text{out}}|}{|Q_{\text{in}}|}$, which gives

$e = 1 - \dfrac{31.878\text{ kJ} + 6.316\text{ kJ}}{38.247\text{ kJ} + 2.494\text{ kJ}} = 0.0625 = 6.25\%.$

(d) In symbolic form, we have Q_{ab} = +2.494 kJ (unchanged)

Q_{bc} = (5/2)(4.00 mol) (8.3145 J/mol · K) (T_c – 300.0 K), which is positive.

Q_{cd} = (3/2)(4.00 mol) (8.3145 J/mol · K) (T_d – T_c), which is negative.

Q_{da} = (5/2)(4.00 mol) (8.3145 J/mol · K) (250.0 K – T_d), which is positive.

Using these values, the efficiency becomes $e = 1 - \dfrac{3(T_c - T_d) + 5(T_d - 250.0\text{ K})}{150 + 5(T_c - 300.0\text{ K})}$. Using the fact that T_c = 1.20T_d and simplifying, we get $e = \dfrac{0.40T_d - 100\text{ K}}{6.00T_d - 1350\text{ K}}$. As $T_d \to \infty$, $e \to$ 0.40/6.00 = 0.0667 = 6.67%.

EVALUATE: In (c), the Carnot efficiency for the temperature extremes given would be $e_{\text{Carnot}} = 1 - \dfrac{T_{\text{C}}}{T_{\text{H}}} =$ 1 – (250 K)/(760 K) = 0.67 = 67%, which is 10 times the maximum efficiency of your engine. Maybe you need a new design!

20.56. **IDENTIFY:** We are dealing with an ideal Stirling heat engine.

SET UP: First make a pV-diagram, as shown in Fig. 20.56.

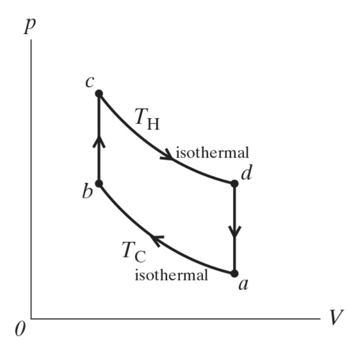


Figure 20.56

EXECUTE: **(a)** Both ab and cd are isothermal, and both bc and da are isochoric. Using $pV = nRT$ and taking ratios gives

<u>bc</u>: $\dfrac{p_cV_c}{p_bV_b} = \dfrac{T_c}{T_b} = \dfrac{T_H}{T_C}$. Since $V_b = V_c$, we have $\dfrac{p_c}{p_b} = \dfrac{T_H}{T_C}$.

<u>da</u>: Since $V_d = V_a$, we get $\dfrac{p_d}{p_a} = \dfrac{T_H}{T_C}$.

Equating the two expressions for T_H/T_C gives $\dfrac{p_d}{p_a} = \dfrac{p_c}{p_b}$. Rearranging gives $\dfrac{p_b}{p_a} = \dfrac{p_c}{p_d} =$ CR.

(b) We want the work done in segment *cd*. $W_{cd} = \int_c^d p\,dV = \int_c^d \dfrac{nRT_H}{V} dV = nRT_H \ln(V_d / V_c)$. Using $pV = nRT$ we have $\dfrac{V_d}{V_c} = \dfrac{nRT_H / p_d}{nRT_H / p_c} = \dfrac{p_c}{p_d} = \text{CR}$. Using this result we get $W_{cd} = nRT_H \ln(\text{CR})$.

(c) We want W_{ab}. $W_{ab} = \int_a^b p\,dV = \int_a^b \dfrac{nRT_C}{V} dV = nRT_C \ln(V_b / V_a)$. As in part (b), we take ratios giving $\dfrac{V_b}{V_a} = \dfrac{nRT_b / p_b}{nRT_a / p_a} = \dfrac{p_a}{p_b} = \dfrac{1}{\text{CR}}$. So $W_{ab} = nRT_C \ln(1/\text{CR}) = -nRT_C \ln(\text{CR})$.

(d) We want the power output of the engine, which is the rate at which it does work, so $P = W/t$. With a frequency of operation of 100 Hz, each cycle takes 1/100 s. Work is done only in segments *ab* and *cd* since the other segments are isochoric. Using our results from parts (b) and (c) gives $W_{tot} = W_{cd} + W_{ab} = nRT_H \ln(\text{CR}) - nRT_C \ln(\text{CR}) = nR \ln(\text{CR})(T_H - T_C)$, so the power output is $P = W/t = nR \ln(\text{CR})(T_H - T_C)/t$. Using $n = 1$ mol, CR = 10, $T_C = 20°\text{C} = 293$ K, $T_H = 100°\text{C} = 373$ K, and $t = 1/100$ s, we get $P = 1.53\times10^5$ W = 153 kW.

EVALUATE: If this were are a Carnot engine operating between the same temperature extremes, its efficiency would be $e = 1 - (293\text{ K})/(373\text{ K}) = 0.214$. For the Stirling engine, heat enters the gas during segments *bc* and *cd*, so $Q_{in} = Q_{bc} + Q_{cd}$. Using the numbers for part (c), we have $Q_{bc} = nC_V(T_H - T_C) = n(3/2\,R)(T_H - T_C) = 997.7$ J. During segment *cd*, *T* is constant so the internal energy *U* is constant. By the first law of thermodynamics, this means that $Q_{in} = Q_{cd} = W_{cd} = nRT_H \ln(\text{CR}) = 7141$ J. Thus during one cycle $Q_{in} = 997.7\text{ J} + 7141\text{ J} = 8138$ J. The work during one cycle is $W = Pt = (153\text{ kW})(1/100\text{ s}) = 1530$ J. So the efficiency is $e = W/Q_{in} = \dfrac{1530\text{ J}}{8138\text{ J}} = 0.188$. As expected, this engine is not as efficient as a Carnot engine operating between the same temperature extremes.

20.57. **IDENTIFY and SET UP:** Once you cut through all the extraterrestrial description, this problem boils down to an ordinary Carnot heat engine. First list all the known information and then summarize it on a *pV*-diagram as shown in Fig. 20.57. We know the following information:

$T_d = 123$ K = ambient temperature = T_a

$p_b = 20.3$ kPa

p_d = ambient pressure because the gauge pressure is zero at *d*

Trigger volume is V_a.

Combustion starts at point *b*.

$r_a = 1/3\, r_d \;\rightarrow\; r_d = 3r_a \rightarrow V_d = 27V_a$

$V_b = ½\, V_a \;\rightarrow\; V_a = 2V_b$

$\gamma = 1.30$

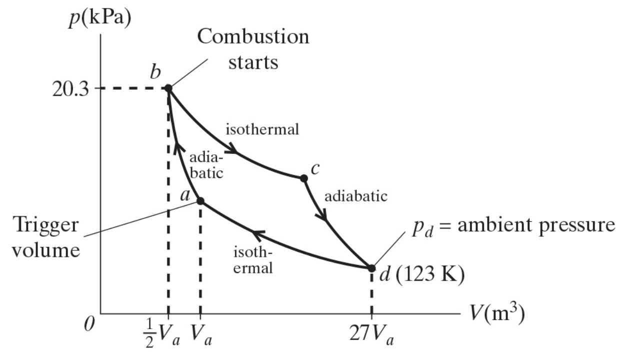


Figure 20.57

EXECUTE: **(a)** We want the combustion temperature, which is T_b. Since da is isothermal, $T_a = T_d =$ 123 K. Since ab is adiabatic, we know that $T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1}$. Using $V_b = ½ V_a$, we have

$$T_b = T_a(V_a/V_b)^{\gamma-1} = (123\text{ K})\left(\frac{2V_b}{V_b}\right)^{1.30-1} = 151\text{ K}.$$

(b) We want the ambient pressure, which is p_d. Since ab is adiabatic, we use $p_a V_a^{\gamma} = p_b V_b^{\gamma}$ to get p_a. Then use $p_aV_a = p_dV_d$ to find p_d since da is isothermal.

Find p_a: $p_a = p_b(V_b/V_a)^{\gamma} = (20.3\text{ kPa})\left(\frac{V_b}{2V_b}\right)^{1.30} = 8.244\text{ kPa}.$

Find p_d: $p_d = p_a\left(\frac{V_a}{V_d}\right) = (8.244\text{ kPa})\left(\frac{V_a}{27V_a}\right) = 0.305\text{ kPa} = 305\text{ Pa}.$ At point d the gauge pressure is zero, so 305 Pa is the ambient pressure.

(c) We want the heat input Q_{in} each minute. The work that is done is W = 60 kJ/h = 1.00 kJ/min. For a Carnot engine $e = \frac{W}{Q_{in}} = 1 - \frac{T_C}{T_H} = 1 - \frac{T_d}{T_b}$. Using the result from part (a) and the given information, we have $\frac{1.00\text{ kJ}}{Q_{in}} = 1 - \frac{123\text{ K}}{151\text{ K}}$, so Q_{in} = 5.39 kJ.

(d) We want the heat rejected Q_{out} each minute. $Q_{in} = W + Q_{out}$, so 5.39 kJ = 1.00 kJ + Q_{out}, which gives Q_{out} = 4.39 kJ.

EVALUATE: The efficiency of this engine is $e = W/Q_{in}$ = (1.00 kJ)/(5.39 kJ) = 0.185. Sporons are about as efficient at doing work as automobile engines on Earth.

20.58. **IDENTIFY:** Calculate Q_C and Q_H in terms of p and V at each point. Use the ideal gas law and the pressure-volume relation for adiabatic processes for an ideal gas. $e = 1 - \frac{|Q_C|}{|Q_H|}$.

SET UP: For an ideal gas, $C_p = C_V + R$, and taking air to be diatomic, $C_p = \frac{7}{2}R, C_V = \frac{5}{2}R$, and $\gamma = \frac{7}{5}$.

EXECUTE: **(a)** Referring to Figure 20.7 in the textbook, $Q_H = n\frac{7}{2}R(T_c - T_b) = \frac{7}{2}(p_cV_c - p_bV_b)$. Similarly, $Q_C = n\frac{5}{2}R(p_aV_a - p_dV_d)$. What needs to be done is to find the relations between the product of the pressure and the volume at the four points. For an ideal gas, $\frac{p_cV_c}{T_c} = \frac{p_bV_b}{T_b}$ so $p_cV_c = p_aV_a\left(\frac{T_c}{T_a}\right)$. For a compression ratio r, and given that for the Diesel cycle the process ab is adiabatic,

$p_bV_b = p_aV_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = p_aV_ar^{\gamma-1}$. Similarly, $p_dV_d = p_cV_c\left(\frac{V_c}{V_a}\right)^{\gamma-1}$. Note that the last result uses the fact that process da is isochoric, and $V_d = V_a$; also, $p_c = p_b$ (process bc is isobaric), and so $V_c = V_b\left(\dfrac{T_c}{T_a}\right)$.

Then,

$$\frac{V_c}{V_a} = \frac{T_c}{T_b}\cdot\frac{V_b}{V_a} = \frac{T_b}{T_a}\cdot\frac{T_a}{T_b}\cdot\frac{V_a}{V_b} = \frac{T_c}{T_a}\cdot\left(\frac{T_a V_a^{\gamma-1}}{T_b V_b^{\gamma-1}}\right)\left(\frac{V_a}{V_b}\right)^{-\gamma} = \frac{T_c}{T_a}r^{\gamma}$$

Combining the above results, $p_d V_d = p_a V_a\left(\dfrac{T_c}{T_a}\right)^{\gamma} r^{\gamma-\gamma^2}$. Substitution of the above results into

$e = 1 - \dfrac{|Q_{\text{C}}|}{|Q_{\text{H}}|}$ gives $e = 1 - \dfrac{5}{7}\left[\dfrac{\left(\dfrac{T_c}{T_a}\right)^{\gamma} r^{\gamma-\gamma^2} - 1}{\left(\dfrac{T_c}{T_a}\right) - r^{\gamma-1}}\right]$.

(b) $e = 1 - \dfrac{1}{1.4}\left[\dfrac{(5.002)r^{-0.56} - 1}{(3.167) - r^{0.40}}\right]$, where $\dfrac{T_c}{T_a} = 3.167$ and $\gamma = 1.40$ have been used. Substitution of $r = 21.0$ yields $e = 0.708 = 70.8\%$.

EVALUATE: The efficiency for an Otto cycle with $r = 21.0$ and $\gamma = 1.40$ is $e = 1 - r^{1-\gamma} = 1 - (21.0)^{-0.40} = 70.4\%$. This efficiency is very close to the value for the Diesel cycle.

20.59. **IDENTIFY** and **SET UP:** The Carnot efficiency is $e_{\text{Carnot}} = 1 - \dfrac{T_{\text{C}}}{T_{\text{H}}}$. Solve for T_{C} to get the temperature for the desired efficiency. Then use the graph to find the depth at which the water is at that temperature.

EXECUTE: Solving $e_{\text{Carnot}} = 1 - \dfrac{T_{\text{C}}}{T_{\text{H}}}$ for T_{C} gives $T_{\text{C}} = T_{\text{H}}(1 - e) = (300\text{ K})(1 - 0.065) = 280.5\text{ K} = 7.5°\text{C}$. From the graph, we see that this temperature occurs at a depth of about 400 m, which is choice (b).

EVALUATE: This depth is over 1200 ft, so deep water is essential for such a power plant.

20.60. **IDENTIFY** and **SET UP:** This power plant produces energy (i.e., does work) at a rate of 10 MW and is 6.5% efficient. This means that the 10 MW is 6.5% of the heat input, Q_{H}, per second.

EXECUTE: $W/t = 0.065Q_{\text{H}}/t$, so $Q_{\text{H}}/t = (10\text{ MW})/(0.065)$. The entropy change per second is $[Q/T_{\text{H}}]/t = [(10\text{ MW})/(0.065)]/(300\text{ K}) = 5.1 \times 10^5$ J/K, which is choice (b).

EVALUATE: The entropy increases because the ammonia gets more disordered as it vaporizes, so our answer is plausible.

20.61. **IDENTIFY** and **SET UP:** Both the warm and cold reservoirs are so large that they do not change temperature as heat is added or lost, so $\Delta S = Q/T$. For a Carnot engine, $|Q_{\text{C}}/Q_{\text{H}}| = |T_{\text{C}}/T_{\text{H}}|$.

EXECUTE: $\Delta S_{\text{warm}} = Q_{\text{H}}/T_{\text{H}}$ and $\Delta S_{\text{cold}} = Q_{\text{C}}/T_{\text{C}}$. But for a Carnot engine, $|Q_{\text{H}}/T_{\text{H}}| = |Q_{\text{C}}/T_{\text{C}}|$. Therefore $\Delta S_{\text{warm}} = -\Delta S_{\text{cold}}$, which is choice (d).

EVALUATE: Our result means that the net entropy change for a cycle is zero. An ideal Carnot engine is reversible, so the entropy change in a cycle is zero, which agrees with our result.

20.62. **IDENTIFY** and **SET UP:** For a cycle, $Q_{\text{H}} = W + Q_{\text{C}}$ and $e = W/Q_{\text{H}}$.

EXECUTE: For this engine, W/t =10 MW and $Q_{\text{C}}/t = 165$ MW, so $Q_{\text{H}}/t = W/t + Q_{\text{C}}/t = 175$ W. The efficiency is $e = (W/t)/(Q_{\text{C}}/t) = (10\text{ MW})/(175\text{ MW}) = 0.057 = 5.7\%$, choice (a).

EVALUATE: An actual heat engine is always less efficient than the theoretical limit due to friction as well as internal effects in the gas which prevent it from behaving exactly like an ideal gas.

21

ELECTRIC CHARGE AND ELECTRIC FIELD

VP21.4.1. **IDENTIFY:** In this problem we use Coulomb's law to calculate the electric force between charges.

SET UP: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|qQ|}{r^2}$. We want the total force on q_3. Start with a sketch showing the charge arrangement, as in Fig. VP21.4.1a.

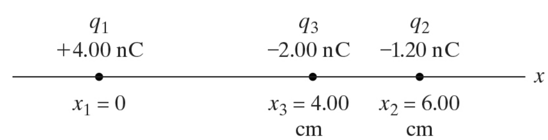


Figure VP21.4.1a

EXECUTE: We know that q_1 attracts q_3 because they have opposite signs, and q_2 repels q_3 because they have the same sign (both are negative). Sketch the forces on q_3 due to the other charges (see Fig. VP21.4.1b). Using $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|qQ|}{r^2}$, we add the forces since they are both in the same direction. $F_{\text{tot}} =$ $F_{1\text{ on }3} + F_{2\text{ on }3} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1q_3|}{r_{13}^2} + \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2q_3|}{r_{23}^2} = \dfrac{|q_3|}{4\pi\epsilon_0}\left(\dfrac{|q_1|}{r_{13}^2} + \dfrac{|q_2|}{r_{23}^2}\right)$. Using the given charges, $r_{13} =$ 4.00 cm = 0.0400 m, and r_{23} = 2.00 cm = 0.0200 m gives $F_{\text{tot}} = 9.89\times10^{-5}$ N = 98.9 μN. This force is in the $-x$ direction, so we can express it as $98.9\ \mu\text{N}\hat{\boldsymbol{i}}$.

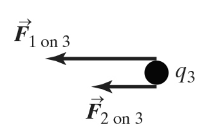


Figure VP21.4.1b

EVALUATE: It is always very helpful to make sketches like those shown here to get charge arrangements, distances, and force directions clear.

VP21.4.2. **IDENTIFY:** This problem requires the calculation of the electric force between charges, so we use Coulomb's law.

SET UP: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|qQ|}{r^2}$. Start with a sketch showing the charge arrangement, as in Fig. VP21.4.2a.

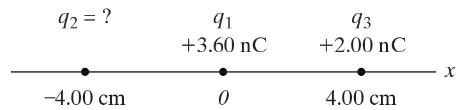


Figure VP21.4.2a

EXECUTE: **(a)** We want the force that q_1 exerts on q_3.
$F_{1\text{ on }3} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_3|}{r_{13}^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(3.60\text{ nC})(2.00\text{ nC})}{(0.0400\text{ m})^2} = 40.5\ \mu\text{N}$. The direction is to the right (+x). Now sketch the forces on q_3 (Fig. 21.4.2b).

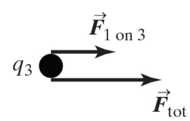


Figure VP21.4.2b

(b) We want the force that q_2 exerts on q_3. $F_{2\text{ on }3} = 54.0\ \mu\text{N} - 40.5\ \mu\text{N} = 13.5\ \mu\text{N}$ in the +x direction.
(c) We want q_2. $F_{2\text{ on }3} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2 q_3|}{r_{23}^2}$. Since q_2 repels q_3, q_2 must be positive. Solving for q_2 and using $r_{23} = 8.00\text{ cm} = 0.0800\text{ m}$, $q_3 = 2.00\text{ nC}$, and the result from part (b) gives $q_2 = +4.82\text{ nC}$.
EVALUATE: Always make sketches like those shown here to get charge arrangements, distances, and force directions clear.

VP21.4.3. **IDENTIFY:** This problem requires the calculation of the electric force between charges, so we use Coulomb's law.
SET UP: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|qQ|}{r^2}$. Start with a sketch showing the charge arrangement, as in Fig. VP21.4.3a.

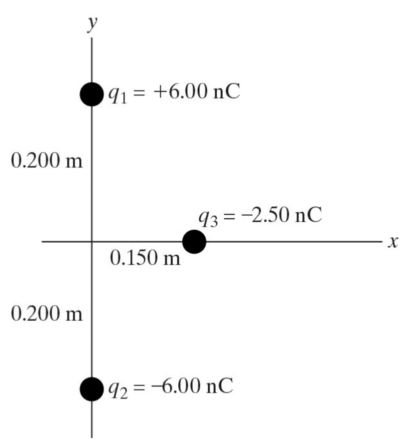


Figure VP21.4.3a

EXECUTE: Start with a careful sketch showing the forces on q_3 shown in Fig. VP21.4.3b. Since q_1 and q_2 have the same magnitude, the figure shows that the x components cancel and the y components add. So $F_{tot} = 2F_{1 \text{ on } 3}\sin\theta = \dfrac{2}{4\pi\epsilon_0}\dfrac{|q_1q_3|}{r_{13}^2}\sin\theta.$ $\theta = \arctan\left(\dfrac{0.200 \text{ m}}{0.150 \text{ m}}\right) = 53.13°.$

$r_{13}^2 = (0.150 \text{ m})^2 + (0.200 \text{ m})^2 = 0.625 \text{ m}^2.$ Using these results, $q_1 = 6.00$ nC, and $q_3 = -2.50$ nC gives $F_{tot} = 3.45$ N in the $+y$ direction.

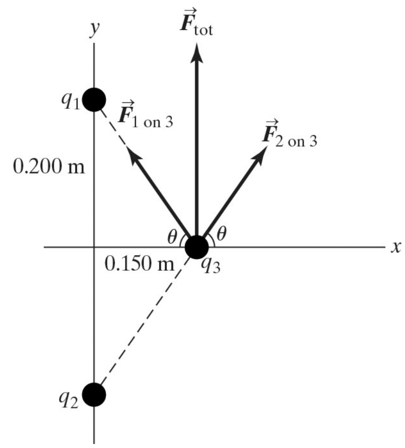


Figure VP21.4.3b

EVALUATE: Before doing any calculations, always make a vector drawing as in the figure. It can save a lot of unnecessary arithmetic and algebra.

VP21.4.4. **IDENTIFY:** We use Coulomb's law to calculate electric forces between point charges. Begin with a clear sketch of the charge arrangement, as in Fig. VP21.4.4a.

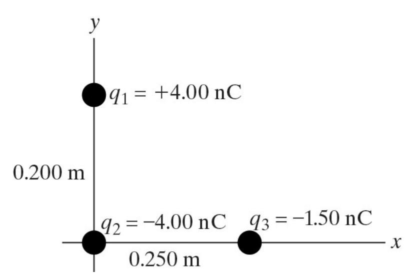


Figure VP21.4.4a

SET UP: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|qQ|}{r^2}$. We want the components of the total electric force on q_3. Fig. VP21.4.4b shows the two forces on q_3.

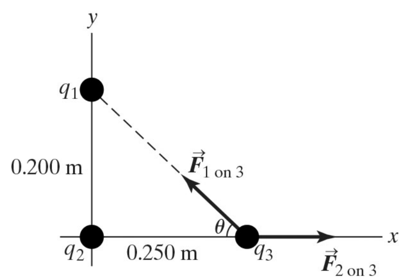


Figure VP21.4.4b

EXECUTE: $F_x = F_{2\text{ on }3} - F_{1\text{ on }3}\cos\theta$. Using Coulomb's law gives

$F_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2q_3|}{r_{23}^2} - \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1q_3|}{r_{13}^2}\cos\theta$. $r_{13}^2 = (0.250\text{ m})^2 + (0.200\text{ m})^2 = 0.1025\text{ m}^2$.

$\theta = \arctan\left(\dfrac{0.200\text{ m}}{0.250\text{ m}}\right) = 38.66°$. Using $r_{23} = 0.250$ m and the given charges, gives $F_x = 0.451\ \mu$N.

$F_y = F_{1\text{ on }3}\sin\theta = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1q_3|}{r_{13}^2}\sin\theta = 0.329\ \mu\text{N}$.

EVALUATE: Always start with careful sketches showing the charges and the force vectors.

VP21.10.1 **IDENTIFY:** We want to find the total electric field due to two point charges.

SET UP: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$.

EXECUTE: **(a)** First sketch the charge arrangement and the electric fields at the point (0, 0.100 m) (Fig. VP21.10.1a). Both fields point in the $-y$ direction, so $E_y = E_1 + E_2$ and $E_x = 0$. Using $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$

gives $E_y = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} - \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2}$. Using the given charges and $r_1 = r_2 = 0.100$ m gives

$E_y = -8090$ N/C.

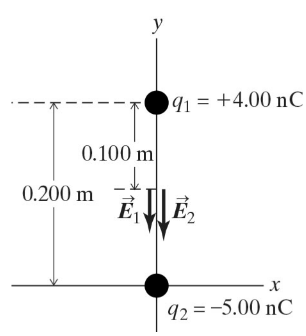


Figure VP21.10.1a

(b) Sketch the electric fields at the point (0, 0.400 m) as in Fig. VP21.10.1b. As in part (a), $E_x = 0$. Now the fields point in opposite directions, so $E_y = E_1 - E_2$. This gives $E_y = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} - \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2}$.

Using $r_1 = 0.200$ m and $r_2 = 0.400$ m, we get $E_y = 618$ N/C.

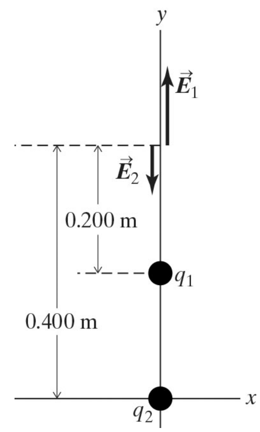


Figure VP21.10.1b

(c) Sketch the electric fields at the point (0.200 m, 0) as in Fig. VP21.10.1c. From this figure, we can see that $E_x = E_{1x} - |E_{2x}| = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2}\cos\theta - \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2}$. $\theta = \arctan\left(\dfrac{0.200\text{ m}}{0.20\text{ m}}\right) = 45.0°$.

$r_1^2 = (0.200\text{ m})^2 + (0.200\text{ m})^2 = 0.0800\text{ m}^2$. Using these results and $r_2 = 0.200$ m, we get $E_x = -806$ N/C. From the figure we see that $E_y = -|E_{1y}| = -E_1\sin\theta = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2}\sin\theta$. Using $r_1 =$ 0.0800 m gives $E_y = -318$ N/C.

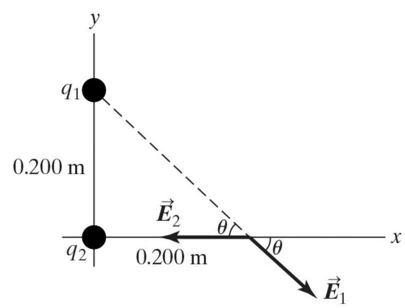


Figure VP21.10.1c

EVALUATE: Careful sketches are especially important when working with two-dimensional charge configurations.

VP21.10.2. **IDENTIFY:** We are dealing with the electric field due to two point charges.

SET UP: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$. First sketch the charge arrangement, as in Fig. 21.10.2a.

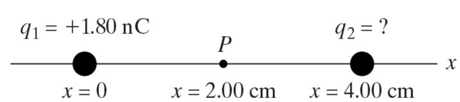


Figure VP21.10.2a

EXECUTE: **(a)** We want the field due to q_1 at point P. Using $q_1 = 1.80$ nC and $r_1 = 0.0200$ m, $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2}$ gives $E_1 = 4.05\times10^4$ N/C in the $+x$ direction.

(c) Fig. VP21.10b shows the known electric fields at P. We want to find E_2. $E_{tot} = E_1 - E_2$, which gives $E_2 = E_{tot} - E_1 = 6.75\times10^4\ \text{N/C} - 4.05\times10^4\ \text{N/C} = 2.70\times10^4\ \text{N/C}$ in the $+x$ direction.

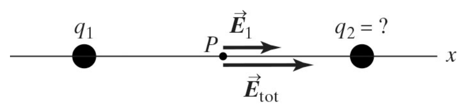


Figure VP21.10.2b

(c) The field E_2 must point in the $+x$ direction, which is toward q_2, so q_2 must be negative.
$E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2}$. Solve for $|q_2|$, use $r_2 = 0.0200$ m and the result from part (b), so $q_2 = -1.20$ nC.

EVALUATE: Careful when finding electric fields. The components can be negative but the magnitude cannot be negative.

VP21.10.3. **IDENTIFY:** We view the hydrogen atom by modeling the orbital electron as a ring of charge centered on the proton.

SET UP: $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. The total electric field is the field of the electron and the proton. We want the total field at $x = a$.

EXECUTE: **(a)** Electron: $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}} = \dfrac{1}{4\pi\epsilon_0}\dfrac{ea}{(a^2+a^2)^{3/2}} = \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{2^{3/2}a^2}$. The electron is negative, so this field points *toward* the proton, which gives $E_x = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e}{2^{3/2}a^2}$.

Proton: $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{a^2}$ away from the proton.

$E_{tot} = E_p + E_e = E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{a^2} - \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{2^{3/2}a^2} = \dfrac{e}{4\pi\epsilon_0 a^2}\left(1-\dfrac{1}{2\sqrt{2}}\right)$.

(b) $1-\dfrac{1}{2\sqrt{2}} > 0$, so the total field points *away from* the proton.

EVALUATE: The total field is less than the proton's field because the electron partially cancel it.

VP21.10.4. **IDENTIFY:** We want the electric field caused by a charged rod. We need calculus to do this calculation.
SET UP: Fig. VP21.10.4 shows the set up of the rod along the x-axis. The uniform linear charge density of the rod is Q/L.

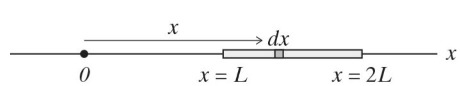


Figure VP21.10.4

EXECUTE: **(a)** $dq = (Q/L)dx$.
(b) $dE_x = -\dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{x^2} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/L)dx}{x^2}$. $dE_y = 0$.
(c) $E_x = \int dE_x = \int_L^{2L} -\dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/L)dx}{x^2} = -\dfrac{Q}{8\pi\epsilon_0 L^2}$.

EVALUATE: If we treat the rod as a point charge at its center, we get $E_x = -\dfrac{Q}{4\pi\epsilon_0 (3L/2)^2} = -\dfrac{Q}{9\pi\epsilon_0 L^2}$, which is *not* the same as our result. So we cannot simplify the rod as a point charge.

VP21.14.1. **IDENTIFY:** This problem involves an electric dipole in an external electric field.

SET UP: We want the torque on the dipole and its potential energy.

EXECUTE: **(a)** $\tau = pE\sin\phi = (6.13\times10^{-30}\,\text{C}\cdot\text{m})(3.00\times10^5\ \text{N/C})\sin 50.0° = 1.41\times10^{-24}\ \text{N}\cdot\text{m}$.

(b) $U = -pE\cos\phi = -1.18\times10^{-24}$ J, using the same values as in part (a).

EVALUATE: These results are very small, but a molecule consists of very small charges that are very close together.

VP21.14.2. **IDENTIFY** This problem involves an electric dipole in an external electric field.

SET UP: We want the charges that make up the dipole. $\tau = pE\sin\phi$ where $p = qd$.

EXECUTE: Solve for q: $q = \dfrac{\tau}{dE\sin\phi} = \dfrac{6.60\times10^{-26}\ \text{N}\cdot\text{m}}{(1.10\times10^{-10}\ \text{m})(8.50\times10^4\ \text{N/C})(\sin 90°)} = 7.06\times10^{-21}$ C.

EVALUATE: Each end of the dipole has charge of this magnitude but opposite signs.

VP21.14.3. **IDENTIFY:** We are dealing with the potential energy of an electric dipole in an external electric field.

SET UP: $U = -pE\cos\phi$.

EXECUTE: We want p. $\Delta U = U_2 - U_1 = -pE\cos 0° = (-pE\cos 180°) = -2pE$, so the work required is $W = +2pE$. Therefore $p = \dfrac{W}{2E} = \dfrac{4.60\times10^{-25}\ \text{J}}{2(1.20\times10^5\ \text{N/C})} = 1.92\times10^{-30}\ \text{C}\cdot\text{m}$.

EVALUATE: If the charges are 0.50×10^{-11} m apart (about 1/10 the radius of an atom), the charges are each $q = p/d =$ $(1.92\times10^{-30}\ \text{C}\cdot\text{m})/(0.50\times10^{-11}\ \text{m}) = 3.8\times10^{-19}$ C.

VP21.14.4. **IDENTIFY:** We are looking at a KBr dipole having a dipole moment of $p = 3.50\times10^{-29}\ \text{C}\cdot\text{m}$.

SET UP and EXECUTE: **(a)** We want the distance between the ions. $p = qd = ed$, so $d = p/e$. Using the given p gives $d = 2.19\times10^{-10}$ m.

(b) We want to find the point on the dipole axis where the electric field is $E = 8.00\times10^4$ N/C. There are two possibilities to consider: the point is between the ions or it is outside the dipole. Between the ions: The field is weakest midway between the ions. At that point

$$E_{\text{min}} = 2\left(\frac{1}{4\pi\epsilon_0}\frac{e}{(d/2)^2}\right) = 2.40\times10^{11}\ \text{N/C}$$

This is less than 8.00×10^4 N/C, so the point must be *outside* the dipole.

Outside the dipole: In this case the fields due to the two ions point in opposite directions. Call x the distance from the center of the dipole to the desired point and $2a$ the length of the dipole. The total field is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{(x-a)^2} - \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{(x+a)^2} = \dfrac{e}{4\pi\epsilon_0}\dfrac{4ax}{(x+a)^2(x-a)^2}$. The electric field at this point is 8.00×10^4 N/C, which is much much less than E_{min} inside the dipole. Therefore this point must be very far from the center of the dipole, so $x >> a$. In this case, $x + a \approx x$ and $x - a \approx x$, so the equation for E simplifies to $E \approx \dfrac{e}{4\pi\epsilon_0}\dfrac{4ax}{x^4} = \dfrac{ea}{\pi\epsilon_0 x^3}$. Using $a = d/2$ and solving for x gives $x = \left(\dfrac{ed/2}{\pi\epsilon_0 E}\right)^{1/3}$. Using $d = 2.19\times10^{-10}$ m and $E = 8.00\times10^4$ N/C gives $x = 1.99\times10^{-8}$ m.

EVALUATE: Between the ions of the dipole the field is very strong since we are very close to the charges and the fields point in the same direction. Outside the dipole the fields are in opposite directions and partially cancel each other so the net field is much weaker. Our approximation in part (b) that $x <<$ a is reasonable $x \approx 100d$.

21.1. **(a) IDENTIFY and SET UP:** Use the charge of one electron $(-1.602\times10^{-19}\text{ C})$ to find the number of electrons required to produce the net charge.

EXECUTE: The number of excess electrons needed to produce net charge q is

$$\frac{q}{-e} = \frac{-3.20\times10^{-9}\text{ C}}{-1.602\times10^{-19}\text{ C/electron}} = 2.00\times10^{10}\text{ electrons.}$$

(b) IDENTIFY and SET UP: Use the atomic mass of lead to find the number of lead atoms in 8.00×10^{-3} kg of lead. From this and the total number of excess electrons, find the number of excess electrons per lead atom.

EXECUTE: The atomic mass of lead is $207\times10^{-3}\text{ kg/mol}$, so the number of moles in 8.00×10^{-3} kg is $n = \dfrac{m_{\text{tot}}}{M} = \dfrac{8.00\times10^{-3}\text{ kg}}{207\times10^{-3}\text{ kg/mol}} = 0.03865\text{ mol.}$ N_A (Avogadro's number) is the number of atoms in 1 mole, so the number of lead atoms is

$N = nN_A = (0.03865\text{ mol})(6.022\times10^{23}\text{ atoms/mol}) = 2.328\times10^{22}\text{ atoms.}$ The number of excess electrons per lead atom is $\dfrac{2.00\times10^{10}\text{ electrons}}{2.328\times10^{22}\text{ atoms}} = 8.59\times10^{-13}$.

EVALUATE: Even this small net charge corresponds to a large number of excess electrons. But the number of atoms in the sphere is much larger still, so the number of excess electrons per lead atom is very small.

21.2. **IDENTIFY:** The charge that flows is the rate of charge flow times the duration of the time interval.

SET UP: The charge of one electron has magnitude $e = 1.60\times10^{-19}$ C.

EXECUTE: The rate of charge flow is $20{,}000\text{ C/s}$ and $t = 100\ \mu\text{s} = 1.00\times10^{-4}\text{ s}$.

$Q = (20{,}000\text{ C/s})(1.00\times10^{-4}\text{ s}) = 2.00\text{ C}$. The number of electrons is $n_e = \dfrac{Q}{1.60\times10^{-19}\text{ C}} = 1.25\times10^{19}$.

EVALUATE: This is a very large amount of charge and a large number of electrons.

21.3. **IDENTIFY and SET UP:** A proton has charge $+e$ and an electron has charge $-e$, with $e = 1.60\times10^{-19}$ C. The force between them has magnitude $F = k\dfrac{|q_1q_2|}{r^2} = k\dfrac{e^2}{r^2}$ and is attractive since the charges have opposite sign. A proton has mass $m_p = 1.67\times10^{-27}$ kg and an electron has mass 9.11×10^{-31} kg. The acceleration is related to the net force $\vec{F}$ by $\vec{F} = m\vec{a}$.

EXECUTE: $F = k\dfrac{e^2}{r^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\text{ C})^2}{(2.0\times10^{-10}\text{ m})^2} = 5.75\times10^{-9}\text{ N.}$

Proton: $a_p = \dfrac{F}{m_p} = \dfrac{5.75\times10^{-9}\text{ N}}{1.67\times10^{-27}\text{ kg}} = 3.4\times10^{18}\text{ m/s}^2$.

Electron: $a_e = \dfrac{F}{m_e} = \dfrac{5.75\times10^{-9}\text{ N}}{9.11\times10^{-31}\text{ kg}} = 6.3\times10^{21}\text{ m/s}^2$

The proton has an initial acceleration of $3.4\times10^{18}\text{ m/s}^2$ toward the electron and the electron has an initial acceleration of $6.3\times10^{21}\text{ m/s}^2$ toward the proton.

EVALUATE: The force the electron exerts on the proton is equal in magnitude to the force the proton exerts on the electron, but the accelerations of the two particles are very different because their masses are very different.

21.4. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_2.

SET UP: $\vec{F}_{2\text{ on }1}$ is in the $+y$-direction.

EXECUTE: $F_{2\text{ on }1} = \dfrac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-6}\ \text{C})(2.0\times10^{-6}\ \text{C})}{(0.60\ \text{m})^2} = 0.100\ \text{N}$. $(F_{2\text{ on }1})_x = 0$ and $(F_{2\text{ on }1})_y = +0.100\ \text{N}$. $F_{Q\text{ on }1}$ is equal and opposite to $F_{1\text{ on }Q}$ (Example 21.4), so $(F_{Q\text{ on }1})_x = -0.23\ \text{N}$ and . $(F_{Q\text{ on }1})_y = 0.17\ \text{N}$. . $F_x = (F_{2\text{ on }1})_x + (F_{Q\text{ on }1})_x = -0.23\ \text{N}$.
$F_y = (F_{2\text{ on }1})_y + (F_{Q\text{ on }1})_y = 0.100\ \text{N} + 0.17\ \text{N} = 0.27\ \text{N}$. The magnitude of the total force is $F = \sqrt{(0.23\ \text{N})^2 + (0.27\ \text{N})^2} = 0.35\ \text{N}$. $\tan^{-1}\dfrac{0.23}{0.27} = 40°$, so $\vec{F}$ is $40°$ counterclockwise from the $+y$-axis, or $130°$ counterclockwise from the $+x$- axis.

EVALUATE: Both forces on q_1 are repulsive and are directed away from the charges that exert them.

21.5. **IDENTIFY:** Each ion carries charge as it enters the axon.

SET UP: The total charge Q is the number N of ions times the charge of each one, which is e. So $Q = Ne$, where $e = 1.60\times10^{-19}\ \text{C}$.

EXECUTE: The number N of ions is $N = (5.6\times10^{11}\ \text{ions/m})(1.5\times10^{-2}\ \text{m}) = 8.4\times10^9$ ions. The total charge Q carried by these ions is $Q = Ne = (8.4\times10^9)(1.60\times10^{-19}\ \text{C}) = 1.3\times10^{-9}\ \text{C} = 1.3\ \text{nC}$.

EVALUATE: The amount of charge is small, but these charges are close enough together to exert large forces on nearby charges.

21.6. **IDENTIFY:** Apply Coulomb's law and calculate the net charge q on each sphere.

SET UP: The magnitude of the charge of an electron is $e = 1.60\times10^{-19}\ \text{C}$.

EXECUTE: $F = k\dfrac{|q_1q_2|}{r^2}$ gives

$|q| = \sqrt{4\pi\epsilon_0\, Fr^2} = \sqrt{4\pi\epsilon_0\,(3.33\times10^{-21}\ \text{N})(0.200\ \text{m})^2} = 1.217\times10^{-16}\ \text{C}$. Therefore, the total number of electrons required is $n = |q|/e = (1.217\times10^{-16}\ \text{C})/(1.60\times10^{-19}\ \text{C/electron}) = 760$ electrons.

EVALUATE: Each sphere has 760 excess electrons and each sphere has a net negative charge. The two like charges repel.

21.7. **IDENTIFY:** Apply $F = \dfrac{k|q_1q_2|}{r^2}$ and solve for r.

SET UP: $F = 650\ \text{N}$.

EXECUTE: $r = \sqrt{\dfrac{k|q_1q_2|}{F}} = \sqrt{\dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\ \text{C})^2}{650\ \text{N}}} = 3.7\times10^3\ \text{m} = 3.7\ \text{km}$

EVALUATE: Charged objects typically have net charges much less than 1 C.

21.8. **IDENTIFY:** Use the mass of a sphere and the atomic mass of aluminum to find the number of aluminum atoms in one sphere. Each atom has 13 electrons. Apply Coulomb's law and calculate the magnitude of charge $|q|$ on each sphere.

SET UP: $N_\text{A} = 6.02\times10^{23}$ atoms/mol. $|q| = n'_\text{e}e$, where n'_e is the number of electrons removed from one sphere and added to the other.

EXECUTE: **(a)** The total number of electrons on each sphere equals the number of protons.

$$n_\text{e} = n_\text{p} = (13)(N_\text{A})\left(\frac{0.0250\ \text{kg}}{0.026982\ \text{kg/mol}}\right) = 7.25\times10^{24}\ \text{electrons}.$$

(b) For a force of 1.00×10^4 N to act between the spheres, $F = 1.00\times10^4\ \text{N} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2}$. This gives

$|q| = \sqrt{4\pi\epsilon_0\,(1.00\times10^4\ \text{N})(0.800\ \text{m})^2} = 8.43\times10^{-4}$ C. The number of electrons removed from one sphere and added to the other is $n'_e = |q|/e = 5.27\times10^{15}$ electrons.

(c) $n'_e/n_e = 7.27\times10^{-10}$.

EVALUATE: When ordinary objects receive a net charge, the fractional change in the total number of electrons in the object is very small.

21.9. IDENTIFY: Apply Coulomb's law.

SET UP: Consider the force on one of the spheres.

EXECUTE: **(a)** $q_1 = q_2 = q$ and $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1q_2|}{r^2} = \dfrac{q^2}{4\pi\epsilon_0 r^2}$, so

$$q = r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = 0.150\ \text{m}\sqrt{\frac{0.220\ \text{N}}{8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 7.42\times10^{-7}\ \text{C (on each)}.$$

(b) $q_2 = 4q_1$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{4q_1^2}{4\pi\epsilon_0 r^2}\ \text{so}\ q_1 = r\sqrt{\frac{F}{4(1/4\pi\epsilon_0)}} = \frac{1}{2}r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = \frac{1}{2}(7.42\times10^{-7}\ \text{C}) = 3.71\times10^{-7}\ \text{C}.$$

And then $q_2 = 4q_1 = 1.48\times10^{-6}$ C.

EVALUATE: The force on one sphere is the same magnitude as the force on the other sphere, whether the spheres have equal charges or not.

21.10. IDENTIFY: Apply $F = k\dfrac{|qq'|}{r^2}$ to each pair of charges. The net force is the vector sum of the forces due to q_1 and q_2.

SET UP: Like charges repel and unlike charges attract. The charges and their forces on q_3 are shown in Figure 21.10.

EXECUTE: $F_1 = k\dfrac{|q_1q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(4.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 5.394\times10^{-6}$ N.

$$F_2 = k\frac{|q_2q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(5.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = 2.997\times10^{-6}\ \text{N}.$$

$F_x = F_{1x} + F_{2x} = +F_1 - F_2 = 2.40\times10^{-6}$ N. The net force has magnitude 2.40×10^{-6} N and is in the $+x$-direction.

EVALUATE: Each force is attractive, but the forces are in opposite directions because of the placement of the charges. Since the forces are in opposite directions, the net force is obtained by subtracting their magnitudes.

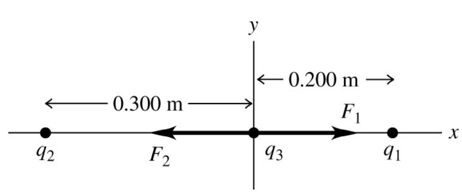


Figure 21.10

21.11. IDENTIFY: In a space satellite, the only force accelerating the free proton is the electrical repulsion of the other proton.

SET UP: Coulomb's law gives the force, and Newton's second law gives the acceleration: $a = F/m = (1/4\pi\epsilon_0)(e^2/r^2)/m.$

EXECUTE:

(a) $a = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2/[(0.00250\ \text{m})^2(1.67\times10^{-27}\ \text{kg})] = 2.21\times10^4\ \text{m/s}^2.$

(b) The graphs are sketched in Figure 21.11.

EVALUATE: The electrical force of a single stationary proton gives the moving proton an initial acceleration about 20,000 times as great as the acceleration caused by the gravity of the entire earth. As the protons move farther apart, the electrical force gets weaker, so the acceleration decreases. Since the protons continue to repel, the velocity keeps increasing, but at a decreasing rate.

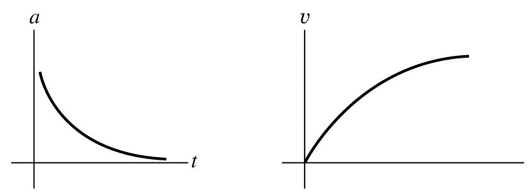


Figure 21.11

21.12. IDENTIFY: Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: **(a)** $F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2}$ gives $0.600\ \text{N} = \frac{1}{4\pi\epsilon_0}\frac{(0.550\times10^{-6}\ \text{C})|q_2|}{(0.30\ \text{m})^2}$ and

$|q_2| = +1.09\times10^{-5}\ \text{C} = 10.9\ \mu\text{C}$. The force is attractive and $q_1 < 0$, so $q_2 = +1.09\times10^{-5}\ \text{C} = +10.9\mu\text{C}$.

(b) $F = 0.600\ \text{N}$. The force is attractive, so is downward.

EVALUATE: The forces between the two charges obey Newton's third law.

21.13. IDENTIFY: Apply Coulomb's law. The two forces on q_3 must have equal magnitudes and opposite directions.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: The force $\vec{F}_2$ that q_2 exerts on q_3 has magnitude $F_2 = k\frac{|q_2q_3|}{r_2^2}$ and is in the +x-direction. $\vec{F}_1$ must be in the –x-direction, so q_1 must be positive. $F_1 = F_2$ gives $k\frac{|q_1||q_3|}{r_1^2} = k\frac{|q_2||q_3|}{r_2^2}$.

$$|q_1| = |q_2|\left(\frac{r_1}{r_2}\right)^2 = (3.00\ \text{nC})\left(\frac{2.00\ \text{cm}}{4.00\ \text{cm}}\right)^2 = 0.750\ \text{nC}.$$

EVALUATE: The result for the magnitude of q_1 doesn't depend on the magnitude of q_3.

21.14. IDENTIFY: Apply Coulomb's law and find the vector sum of the two forces on Q.

SET UP: The force that q_1 exerts on Q is repulsive, as in Example 21.4, but now the force that q_2 exerts is attractive.

EXECUTE: The x-components cancel. We only need the y-components, and each charge contributes equally. $F_{1y} = F_{2y} = -\frac{1}{4\pi\epsilon_0}\frac{(2.0\times10^{-6}\ \text{C})(4.0\times10^{-6}\ \text{C})}{(0.500\ \text{m})^2}\sin\alpha = -0.173\ \text{N}$ (since $\sin\alpha = 0.600$).

Therefore, the total force is $2F = 0.35\ \text{N}$, in the $-y$-direction.

EVALUATE: If q_1 is $-2.0\ \mu\text{C}$ and q_2 is $+2.0\ \mu\text{C}$, then the net force is in the $+y$-direction.

21.15. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_1.

SET UP: Like charges repel and unlike charges attract, so $\vec{F}_2$ and $\vec{F}_3$ are both in the $+x$-direction.

EXECUTE: $F_2 = k\dfrac{|q_1q_2|}{r_{12}^2} = 6.749\times10^{-5}\text{ N}$, $F_3 = k\dfrac{|q_1q_3|}{r_{13}^2} = 1.124\times10^{-4}\text{ N}$. $F = F_2 + F_3 = 1.8\times10^{-4}\text{ N}$.

$F = 1.8\times10^{-4}$ N and is in the $+x$-direction.

EVALUATE: Comparing our results to those in Example 21.3, we see that $\vec{F}_{1\text{ on }3} = -\vec{F}_{3\text{ on }1}$, as required by Newton's third law.

21.16. **IDENTIFY** and **SET UP:** Apply Coulomb's law to calculate the force exerted by q_2 and q_3 on q_1. Add these forces as vectors to get the net force. The target variable is the x-coordinate of q_3.

EXECUTE: $\vec{F}_2$ is in the x-direction.

$F_2 = k\dfrac{|q_1q_2|}{r_{12}^2} = 3.37\text{ N}$, so $F_{2x} = +3.37\text{ N}$

$F_x = F_{2x} + F_{3x}$ and $F_x = -7.00\text{ N}$

$F_{3x} = F_x - F_{2x} = -7.00\text{ N} - 3.37\text{ N} = -10.37\text{ N}$

For F_{3x} to be negative, q_3 must be on the $-x$-axis.

$F_3 = k\dfrac{|q_1q_3|}{x^2}$, so $|x| = \sqrt{\dfrac{k|q_1q_3|}{F_3}} = 0.144\text{ m}$, so $x = -0.144\text{ m}$

EVALUATE: q_2 attracts q_1 in the $+x$-direction so q_3 must attract q_1 in the $-x$-direction, and q_3 is at negative x.

21.17. **IDENTIFY:** Apply Coulomb's law to calculate the force each of the two charges exerts on the third charge. Add these forces as vectors.

SET UP: The three charges are placed as shown in Figure 21.17a.

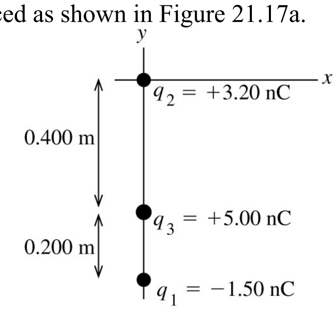
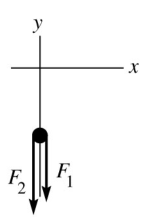


Figure 21.17a

EXECUTE: Like charges repel and unlike attract, so the free-body diagram for q_3 is as shown in Figure 21.17b.



$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_3|}{r_{13}^2}$$

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2q_3|}{r_{23}^2}$$

Figure 21.17b

$$F_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.50\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 1.685\times10^{-6}\ \text{N}$$

$$F_2 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.400\ \text{m})^2} = 8.988\times10^{-7}\ \text{N}$$

The resultant force is $\vec{R} = \vec{F}_1 + \vec{F}_2$.

$R_x = 0$.

$R_y = -(F_1 + F_2) = -(1.685\times10^{-6}\ \text{N} + 8.988\times10^{-7}\ \text{N}) = -2.58\times10^{-6}\ \text{N}$.

The resultant force has magnitude $2.58\times10^{-6}\ \text{N}$ and is in the $-y$-direction.

EVALUATE: The force between q_1 and q_3 is attractive and the force between q_2 and q_3 is repulsive.

21.18. **IDENTIFY:** We use Coulomb's law to find each electrical force and combine these forces to find the net force.

SET UP: In the O-H-N combination the O^- is 0.170 nm from the H^+ and 0.280 nm from the N^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. Like charges repel and unlike charges attract. The net force is the vector sum of the individual forces. The force due to each pair of charges is $F = k\frac{|q_1 q_2|}{r^2} = k\frac{e^2}{r^2}$.

EXECUTE: **(a)** $F = k\frac{|q_1 q_2|}{r^2} = k\frac{e^2}{r^2}$.

O-H-N:

O^-- H^+: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{(0.170\times10^{-9}\ \text{m})^2} = 7.96\times10^{-9}\ \text{N}$, attractive

O^-- N^-: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{(0.280\times10^{-9}\ \text{m})^2} = 2.94\times10^{-9}\ \text{N}$, repulsive

N-H-N:

N^-- H^+: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{(0.190\times10^{-9}\ \text{m})^2} = 6.38\times10^{-9}\ \text{N}$, attractive

N^-- N^-: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{(0.300\times10^{-9}\ \text{m})^2} = 2.56\times10^{-9}\ \text{N}$, repulsive

The total attractive force is $1.43\times10^{-8}\ \text{N}$ and the total repulsive force is $5.50\times10^{-9}\ \text{N}$. The net force is attractive and has magnitude $1.43\times10^{-8}\ \text{N} - 5.50\times10^{-9}\ \text{N} = 8.80\times10^{-9}\ \text{N}$.

(b) $F = k\frac{e^2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{(0.0529\times10^{-9}\ \text{m})^2} = 8.22\times10^{-8}\ \text{N}$.

EVALUATE: The bonding force of the electron in the hydrogen atom is a factor of 10 larger than the bonding force of the adenine-thymine molecules.

21.19. **IDENTIFY:** We use Coulomb's law to find each electrical force and combine these forces to find the net force.

SET UP: In the O-H-O combination the O^- is 0.180 nm from the H^+ and 0.290 nm from the other O^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. In the O-H-N combination the O^- is 0.180 nm from the H^+ and 0.290 nm from the other N^-. Like charges repel and unlike charges attract. The net force is the vector sum of the individual forces. The force due to each pair of charges is $F = k\frac{|q_1 q_2|}{r^2} = k\frac{e^2}{r^2}$.

EXECUTE: Using $F = k\frac{|q_1 q_2|}{r^2} = k\frac{e^2}{r^2}$, we find that the attractive forces are: $O^- - H^+$, 7.10×10^{-9} N; $N^- - H^+$, 6.37×10^{-9} N; $O^- - H^+$, 7.10×10^{-9} N. The total attractive force is 2.06×10^{-8} N. The repulsive forces are: $O^- - O^-$, 2.74×10^{-9} N; $N^- - N^-$, 2.56×10^{-9} N; $O^- - N^-$, 2.74×10^{-9} N. The total repulsive force is 8.04×10^{-9} N. The net force is attractive and has magnitude 1.26×10^{-8} N.

EVALUATE: The net force is attractive, as it should be if the molecule is to stay together.

21.20. IDENTIFY: Apply constant acceleration equations to the motion of the proton. $E = F/|q|$.

SET UP: A proton has mass $m_p = 1.67\times10^{-27}$ kg and charge $+e$. Let $+x$ be in the direction of motion of the proton.

EXECUTE: **(a)** $v_{0x} = 0$. $a = \frac{eE}{m_p}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = \frac{1}{2}a_x t^2 = \frac{1}{2}\frac{eE}{m_p}t^2$. Solving for E gives $E = \frac{2(0.0160\text{ m})(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(3.20\times10^{-6}\text{ s})^2} = 32.6\text{ N/C}$.

(b) $v_x = v_{0x} + a_x t = \frac{eE}{m_p}t = \frac{e}{m_p}\left(\frac{2(x-x_0)m_p}{et^2}\right)t = \frac{2(x-x_0)}{t} = \frac{2(0.0160\text{ m})}{3.20\times10^{-6}\text{ s}} = 1.00\times10^4\text{ m/s}$.

EVALUATE: The electric field is directed from the positively charged plate toward the negatively charged plate and the force on the proton is also in this direction.

21.21. IDENTIFY: $F = |q|E$. Since the field is uniform, the force and acceleration are constant and we can use a constant acceleration equation to find the final speed.

SET UP: A proton has charge $+e$ and mass 1.67×10^{-27} kg.

EXECUTE: **(a)** $F = (1.60\times10^{-19}\text{ C})(2.75\times10^3\text{ N/C}) = 4.40\times10^{-16}\text{ N}$.

(b) $a = \frac{F}{m} = \frac{4.40\times10^{-16}\text{ N}}{1.67\times10^{-27}\text{ kg}} = 2.63\times10^{11}\text{ m/s}^2$.

(c) $v_x = v_{0x} + a_x t$ gives $v = (2.63\times10^{11}\text{ m/s}^2)(1.00\times10^{-6}\text{ s}) = 2.63\times10^5\text{ m/s}$.

EVALUATE: The acceleration is very large and the gravity force on the proton can be ignored.

21.22. IDENTIFY: For a point charge, $E = k\frac{|q|}{r^2}$.

SET UP: $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: **(a)** The field is toward the negative charge so is downward.

$$E = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\text{ C}}{(0.250\text{ m})^2} = 719\text{ N/C}.$$

(b) $r = \sqrt{\frac{k|q|}{E}} = \sqrt{\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-9}\text{ C})}{12.0\text{ N/C}}} = 1.94\text{ m}$.

EVALUATE: At different points the electric field has different directions, but it is always directed toward the negative point charge.

21.23. IDENTIFY: The acceleration that stops the charge is produced by the force that the electric field exerts on it. Since the field and the acceleration are constant, we can use the standard kinematics formulas to find acceleration and time.

(a) SET UP: First use kinematics to find the proton's acceleration. $v_x = 0$ when it stops. Then find the electric field needed to cause this acceleration using the fact that $F = qE$.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$. $0 = (4.50\times10^6\ \text{m/s})^2 + 2a(0.0320\ \text{m})$ and $a = 3.16\times10^{14}\ \text{m/s}^2$. Now find the electric field, with $q = e$. $eE = ma$ and

$E = ma/e = (1.67\times10^{-27}\ \text{kg})(3.16\times10^{14}\ \text{m/s}^2)/(1.60\times10^{-19}\ \text{C}) = 3.30\times10^6\ \text{N/C}$, to the left.

(b) SET UP: Kinematics gives $v = v_0 + at$, and $v = 0$ when the electron stops, so $t = v_0/a$.

EXECUTE: $t = v_0/a = (4.50\times10^6\ \text{m/s})/(3.16\times10^{14}\ \text{m/s}^2) = 1.42\times10^{-8}\ \text{s} = 14.2\ \text{ns}$.

(c) SET UP: In part (a) we saw that the electric field is proportional to *m*, so we can use the ratio of the electric fields. $E_e/E_p = m_e/m_p$ and $E_e = (m_e/m_p)E_p$.

EXECUTE: $E_e = [(9.11\times10^{-31}\ \text{kg})/(1.67\times10^{-27}\ \text{kg})](3.30\times10^6\ \text{N/C}) = 1.80\times10^3\ \text{N/C}$, to the right.

EVALUATE: Even a modest electric field, such as the ones in this situation, can produce enormous accelerations for electrons and protons.

21.24. IDENTIFY: Use constant acceleration equations to calculate the upward acceleration *a* and then apply $\vec{F} = q\vec{E}$ to calculate the electric field.

SET UP: Let +*y* be upward. An electron has charge $q = -e$.

EXECUTE: **(a)** $v_{0y} = 0$ and $a_y = a$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = \frac{1}{2}at^2$. Then

$$a = \frac{2(y - y_0)}{t^2} = \frac{2(4.50\ \text{m})}{(3.00\times10^{-6}\ \text{s})^2} = 1.00\times10^{12}\ \text{m/s}^2.$$

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11\times10^{-31}\ \text{kg})(1.00\times10^{12}\ \text{m/s}^2)}{1.60\times10^{-19}\ \text{C}} = 5.69\ \text{N/C}$$

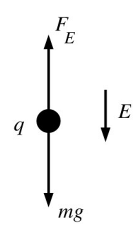
The force is up, so the electric field must be *downward* since the electron has negative charge.

(b) The electron's acceleration is $\sim 10^{11}g$, so gravity must be negligibly small compared to the electrical force.

EVALUATE: Since the electric field is uniform, the force it exerts is constant and the electron moves with constant acceleration.

21.25. IDENTIFY: The equation $\vec{F} = q\vec{E}$ relates the electric field, charge of the particle, and the force on the particle. If the particle is to remain stationary the net force on it must be zero.

SET UP: The free-body diagram for the particle is sketched in Figure 21.25. The weight is *mg*, downward. For the net force to be zero the force exerted by the electric field must be upward. The electric field is downward. Since the electric field and the electric force are in opposite directions the charge of the particle is negative.



$mg = |q|E$

Figure 21.25

EXECUTE: **(a)** $|q| = \dfrac{mg}{E} = \dfrac{(1.45\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{650\ \text{N/C}} = 2.19\times10^{-5}\ \text{C}$ and $q = -21.9\ \mu\text{C}$.

(b) SET UP: The electrical force has magnitude $F_E = |q|E = eE$. The weight of a proton is $w = mg$. $F_E = w$ so $eE = mg$.

EXECUTE: $E = \dfrac{mg}{e} = \dfrac{(1.673\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)}{1.602\times10^{-19}\ \text{C}} = 1.02\times10^{-7}\ \text{N/C}.$

This is a very small electric field.

EVALUATE: In both cases $|q|E = mg$ and $E = (m/|q|)g$. In part (b) the $m/|q|$ ratio is much smaller $(\sim10^{-8})$ than in part (a) $(\sim10^{2})$ so E is much smaller in (b). For subatomic particles gravity can usually be ignored compared to electric forces.

21.26. IDENTIFY: The net force on each charge must be zero.

SET UP: The force diagram for the $-6.50\ \mu\text{C}$ charge is given in Figure 21.26. *FE* is the force exerted on the charge by the uniform electric field. The charge is negative and the field is to the right, so the force exerted by the field is to the left. F_q is the force exerted by the other point charge. The two charges have opposite signs, so the force is attractive. Take the +*x*-axis to be to the right, as shown in the figure.

EXECUTE: **(a)** $F_E = |q|E = (6.50\times10^{-6}\ \text{C})(1.85\times10^{8}\ \text{N/C}) = 1.20\times10^{3}\ \text{N}$

$$F_q = k\frac{|q_1q_2|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6.50\times10^{26}\ \text{C})(8.75\times10^{-6}\ \text{C})}{(0.0250\ \text{m})^2} = 8.18\times10^2\ \text{N}$$

$\sum F_x = 0$ gives $T + F_q - F_E = 0$ and $T = F_E - F_q = 382\ \text{N}.$

(b) Now F_q is to the left, since like charges repel.

$\sum F_x = 0$ gives $T - F_q - F_E = 0$ and $T = F_E + F_q = 2.02\times10^{3}\ \text{N}.$

EVALUATE: The tension is much larger when both charges have the same sign, so the force one charge exerts on the other is repulsive.

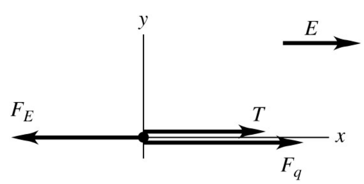
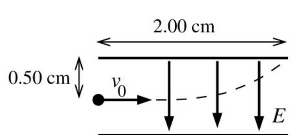


Figure 21.26

21.27. IDENTIFY: The equation $\vec{F} = q\vec{E}$ gives the force on the particle in terms of its charge and the electric field between the plates. The force is constant and produces a constant acceleration. The motion is similar to projectile motion; use constant acceleration equations for the horizontal and vertical components of the motion.

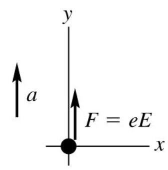
SET UP: The motion is sketched in Figure 21.27a.



For an electron $q = -e$.

Figure 21.27a

$\vec{F} = q\vec{E}$ and q negative gives that $\vec{F}$ and $\vec{E}$ are in opposite directions, so $\vec{F}$ is upward. The free-body diagram for the electron is given in Figure 21.27b.



EXECUTE: **(a)** $\sum F_y = ma_y$

$eE = ma$

Figure 21.27b

Solve the kinematics to find the acceleration of the electron: Just misses upper plate says that $x - x_0 = 2.00\text{ cm}$ when $y - y_0 = +0.500\text{ cm}$.

<u>x-component</u>:

$v_{0x} = v_0 = 1.60\times10^6\text{ m/s},\ a_x = 0,\ x - x_0 = 0.0200\text{ m},\ t = ?$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.0200\text{ m}}{1.60\times10^6\text{ m/s}} = 1.25\times10^{-8}\text{ s}$$

In this same time t the electron travels 0.0050 m vertically.

<u>y-component</u>:

$t = 1.25\times10^{-8}\text{ s},\ v_{0y} = 0,\ y - y_0 = +0.0050\text{ m},\ a_y = ?$

$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$

$$a_y = \frac{2(y - y_0)}{t^2} = \frac{2(0.0050\text{ m})}{(1.25\times10^{-8}\text{ s})^2} = 6.40\times10^{13}\text{ m/s}^2.$$

(This analysis is very similar to that used in Chapter 3 for projectile motion, except that here the acceleration is upward rather than downward.) This acceleration must be produced by the electric-field force: $eE = ma$.

$$E = \frac{ma}{e} = \frac{(9.109\times10^{-31}\text{ kg})(6.40\times10^{13}\text{ m/s}^2)}{1.602\times10^{-19}\text{ C}} = 364\text{ N/C}$$

Note that the acceleration produced by the electric field is <u>much</u> larger than g, the acceleration produced by gravity, so it is perfectly ok to neglect the gravity force on the electron in this problem.

(b) $a = \dfrac{eE}{m_\text{p}} = \dfrac{(1.602\times10^{-19}\text{ C})(364\text{ N/C})}{1.673\times10^{-27}\text{ kg}} = 3.49\times10^{10}\text{ m/s}^2.$

This is much less than the acceleration of the electron in part (a) so the vertical deflection is less and the proton won't hit the plates. The proton has the same initial speed, so the proton takes the same time $t = 1.25\times10^{-8}\text{ s}$ to travel horizontally the length of the plates. The force on the proton is downward (in the same direction as $\vec{E}$, since q is positive), so the acceleration is downward and $a_y = -3.49\times10^{10}\text{ m/s}^2$. $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}(-3.49\times10^{10}\text{ m/s}^2)(1.25\times10^{-8}\text{ s})^2 = -2.73\times10^{-6}\text{ m}.$

The displacement is $2.73\times10^{-6}\text{ m}$, downward.

EVALUATE: **(c)** The displacements are in opposite directions because the electron has negative charge and the proton has positive charge. The electron and proton have the same magnitude of charge, so the force the electric field exerts has the same magnitude for each charge. But the proton has a mass larger by a factor of 1836 so its acceleration and its vertical displacement are smaller by this factor.

(d) In each case $a \gg g$ and it is reasonable to ignore the effects of gravity.

21.28. **IDENTIFY:** Apply constant acceleration equations to the motion of the electron.

SET UP: Let $+x$ be to the right and let $+y$ be downward. The electron moves 2.00 cm to the right and 0.50 cm downward.

EXECUTE: Use the horizontal motion to find the time when the electron emerges from the field. $x - x_0 = 0.0200$ m, $a_x = 0$, $v_{0x} = 1.60\times10^6$ m/s. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = 1.25\times10^{-8}$ s. Since $a_x = 0$, $v_x = 1.60\times10^6$ m/s. $y - y_0 = 0.0050$ m, $v_{0y} = 0$, $t = 1.25\times10^{-8}$ s. $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = 8.00\times10^5$ m/s. Then $v = \sqrt{v_x^2 + v_y^2} = 1.79\times10^6$ m/s.

EVALUATE: $v_y = v_{0y} + a_y t$ gives $a_y = 6.4\times10^{13}$ m/s^2. The electric field between the plates is $E = \dfrac{ma_y}{e} = \dfrac{(9.11\times10^{-31}\text{ kg})(6.4\times10^{13}\text{ m/s}^2)}{1.60\times10^{-19}\text{ C}} = 364$ N/C. This is not a very large field.

21.29. **IDENTIFY:** Find the angle θ that $\hat{\boldsymbol{r}}$ makes with the $+x$-axis. Then $\hat{\boldsymbol{r}} = (\cos\theta)\hat{\boldsymbol{i}} + (\sin\theta)\hat{\boldsymbol{j}}$.

SET UP: $\tan\theta = y/x$.

EXECUTE: **(a)** $\tan^{-1}\left(\dfrac{-1.35}{0}\right) = -\dfrac{\pi}{2}$ rad. $\hat{\boldsymbol{r}} = -\hat{\boldsymbol{j}}$.

(b) $\tan^{-1}\left(\dfrac{12}{12}\right) = \dfrac{\pi}{4}$ rad. $\hat{\boldsymbol{r}} = \dfrac{\sqrt{2}}{2}\hat{\boldsymbol{i}} + \dfrac{\sqrt{2}}{2}\hat{\boldsymbol{j}}$.

(c) $\tan^{-1}\left(\dfrac{2.6}{+1.10}\right) = 1.97\text{ rad} = 112.9°$. $\hat{\boldsymbol{r}} = -0.39\hat{\boldsymbol{i}} + 0.92\hat{\boldsymbol{j}}$ (Second quadrant).

EVALUATE: In each case we can verify that $\hat{\boldsymbol{r}}$ is a unit vector, because $\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{r}} = 1$.

21.30. **IDENTIFY** and **SET UP:** Use $\vec{E}$ in $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate $\vec{F}$, $\vec{F} = m\vec{a}$ to calculate $\vec{a}$, and a constant acceleration equation to calculate the final velocity. Let $+x$ be east.

(a) EXECUTE: $F_x = |q|E = (1.602\times10^{-19}\text{ C})(1.50\text{ N/C}) = 2.403\times10^{-19}$ N.

$a_x = F_x/m = (2.403\times10^{-19}\text{ N})/(9.109\times10^{-31}\text{ kg}) = +2.638\times10^{11}\text{ m/s}^2$.

$v_{0x} = +4.50\times10^5$ m/s, $a_x = +2.638\times10^{11}$ m/s^2, $x - x_0 = 0.375$ m, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 6.33\times10^5$ m/s.

EVALUATE: $\vec{E}$ is west and q is negative, so $\vec{F}$ is east and the electron speeds up.

(b) EXECUTE: $F_x = -|q|E = -(1.602\times10^{-19}\text{ C})(1.50\text{ N/C}) = -2.403\times10^{-19}$ N.

$a_x = F_x/m = (-2.403\times10^{-19}\text{ N})/(1.673\times10^{-27}\text{ kg}) = -1.436\times10^8\text{ m/s}^2$.

$v_{0x} = +1.90\times10^4$ m/s, $a_x = -1.436\times10^8$ m/s^2, $x - x_0 = 0.375$ m, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 1.59\times10^4$ m/s.

EVALUATE: $q > 0$ so $\vec{F}$ is west and the proton slows down.

21.31. **IDENTIFY:** We want to find the force that a charged sphere exerts on a line of charge. By Newton's third law, this is also the force that the line exerts on the sphere, which is much easier to calculate. Fig. 21.31 shows the arrangement of the objects involved.

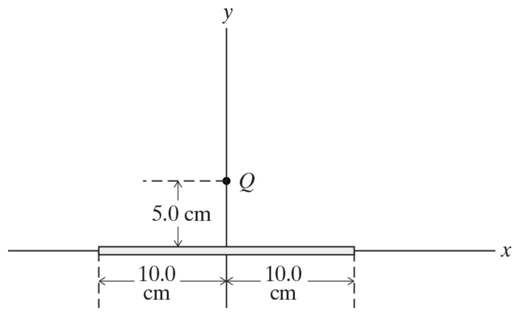


Figure 21.31

SET UP and EXECUTE: The small sphere is equivalent to a point charge. From the textbook we know that the electric field due to the line is $E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{x\sqrt{(x/a)^2+1}}$. The magnitude of the force that the line exerts on the sphere is $F_y = QE_y$. We know that $Q = -2.00\ \mu\text{C}$, $a = 10.0\text{ cm} = 0.100\text{ m}$, $x = 5.00\text{ cm} = 0.0500\text{ m}$, and $\lambda = 4.80\text{ nC/m}$. Using $F = \frac{Q}{2\pi\epsilon_0} \frac{\lambda}{x\sqrt{(x/a)^2+1}}$ for the given quantities, we get $F_y = 3.09 \times 10^{-3}$ N. The sphere is negative and the line is positive, so the sphere attracts the line, which means that the direction of the force is in the $+y$ direction.

EVALUATE: This is a small force, but for a very light line (such as a very thin wire) it could readily be observed.

21.32. **IDENTIFY:** The net electric field is the vector sum of the fields due to the individual charges.

SET UP: The electric field points toward negative charge and away from positive charge.

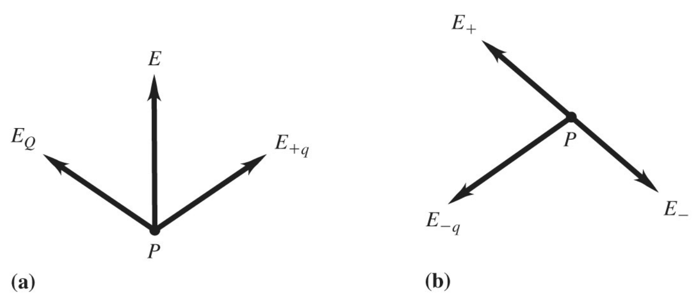


Figure 21.32

EXECUTE: **(a)** Figure 21.32(a) shows $\vec{E}_Q$ and $\vec{E}_{+q}$ at point P. $\vec{E}_Q$ must have the direction shown, to produce a resultant field in the specified direction. $\vec{E}_Q$ is toward Q, so Q is negative. In order for the horizontal components of the two fields to cancel, Q and q must have the same magnitude.

(b) No. If the lower charge were negative, its field would be in the direction shown in Figure 21.32(b). The two possible directions for the field of the upper charge, when it is positive ($\vec{E}_+$) or negative ($\vec{E}_-$), are shown. In neither case is the resultant field in the direction shown in the figure in the problem.

EVALUATE: When combining electric fields, it is always essential to pay attention to their directions.

21.33. **IDENTIFY and SET UP:** Two very long lines of charge are parallel to each other. At any point, the total electric field is the vector sum of the two fields. We want to find where the resultant field is zero. The resultant field can be zero only where the two fields have equal magnitudes but opposite directions.

Since the lines have opposite sign charge densities, in the region between the lines both fields point in the $+y$ direction, so they cannot cancel. Below the x-axis ($y < 0$) the fields are in opposite directions, but the field due to the lower line is always greater than the field due to the upper line, so they cannot cancel. For $y > 10.0$ cm, the fields are again in opposite directions. If we are close enough to the upper line, its field can equal in magnitude the field due to the lower line, so they can cancel. Call P the point on the y-axis at which the two field magnitudes are equal. Calling 1 the lower line and 2 the upper line, we have $|E_1| = |E_2|$ at point P. The field due to a very long line of charge is $E = \dfrac{\lambda}{2\pi\epsilon_0 x}$.

EXECUTE: Call y the y coordinate of P (and recalling that P is above the upper line), $|E_1| = |E_2|$ gives $\dfrac{\lambda_1}{2\pi\epsilon_0 y} = \dfrac{|\lambda_2|}{2\pi\epsilon_0 (y - 10.0\text{ cm})}$, which gives $\dfrac{8.00\,\mu\text{C/m}}{y} = \dfrac{4.00\,\mu\text{C/m}}{y - 10.0\text{ cm}}$, which gives $y = 20.0$ cm.

EVALUATE: If both lines had the same sign charge, the point of cancellation would be in the region between the lines.

21.34. **IDENTIFY:** Add the individual electric fields to obtain the net field.
SET UP: The electric field points away from positive charge and toward negative charge. The electric fields $\vec{E}_1$ and $\vec{E}_2$ add to form the net field $\vec{E}$.
EXECUTE: **(a)** The electric field is toward A at points B and C and the field is zero at A.
(b) The electric field is away from A at B and C. The field is zero at A.
(c) The field is horizontal and to the right at points A, B, and C.
EVALUATE: Compare your results to the field lines shown in Figure 21.28a and b in the textbook.

21.35. **IDENTIFY:** $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate the force, using the electric field calculated in part (a).
SET UP: The placement of charges is sketched in Figure 21.35a.

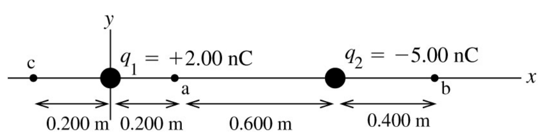


Figure 21.35a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$, where r is the distance between the point where the field is calculated and the point charge.
(a) EXECUTE: (i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.35b.

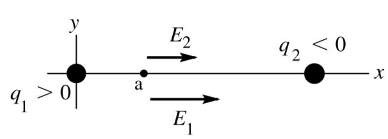


Figure 21.35b

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.600\ \text{m})^2} = 124.8\ \text{N/C}.$$

$E_{1x} = 449.4$ N/C, $E_{1y} = 0$.

$E_{2x} = 124.8$ N/C, $E_{2y} = 0$.

$E_x = E_{1x} + E_{2x} = +449.4\ \text{N/C} + 124.8\ \text{N/C} = +574.2\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point a has magnitude 574 N/C and is in the $+x$-direction.

(ii) At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.35c.

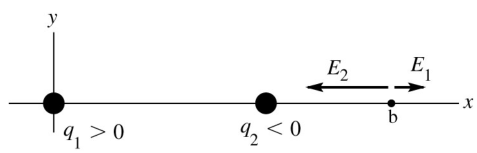


Figure 21.35c

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(1.20\ \text{m})^2} = 12.5\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 280.9\ \text{N/C}.$$

$E_{1x} = 12.5$ N/C, $E_{1y} = 0$.

$E_{2x} = -280.9$ N/C, $E_{2y} = 0$.

$E_x = E_{1x} + E_{2x} = +12.5\ \text{N/C} - 280.9\ \text{N/C} = -268.4\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 268 N/C and is in the $-x$-direction.

(iii) At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.35d.

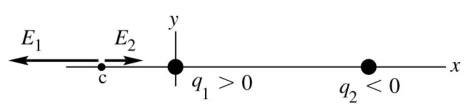


Figure 21.35d

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(1.00\ \text{m})^2} = 44.9\ \text{N/C}.$$

$E_{1x} = -449.4$ N / C, $E_{1y} = 0$.

$E_{2x} = +44.9$ N / C, $E_{2y} = 0$.

$E_x = E_{1x} + E_{2x} = -449.4\ \text{N/C} + 44.9\ \text{N/C} = -404.5\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 404 N/C and is in the $-x$-direction.

(b) Set Up: Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

Execute: (i) $F = (1.602\times10^{-19}\ \text{C})(574.2\ \text{N/C}) = 9.20\times10^{-17}\ \text{N},\ -x\text{-direction}.$

(ii) $F = (1.602\times10^{-19}\ \text{C})(268.4\ \text{N/C}) = 4.30\times10^{-17}\ \text{N},\ +x\text{-direction}.$

(iii) $F = (1.602\times10^{-19}\ \text{C})(404.5\ \text{N/C}) = 6.48\times10^{-17}\ \text{N},\ +x\text{-direction}.$

Evaluate: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the $+x$- or $-x$-direction depends on where the field point is relative to the charge that produces the field. In part (a), for (i) the field magnitudes were added because the fields were in the same direction and in (ii) and (iii) the field magnitudes were subtracted because the two fields were in opposite directions. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.36. Identify: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate the force, using the electric field calculated in part (a).

(a) Set Up: The placement of charges is sketched in Figure 21.36a.

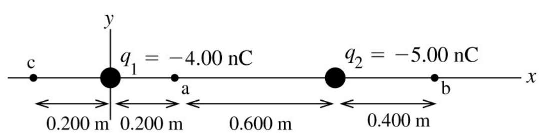


Figure 21.36a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$, where r is the distance between the point where the field is calculated and the point charge.

(i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.36b.

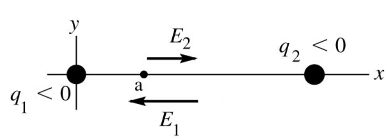


Figure 21.36b

Execute: $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 898.8\ \text{N/C}.$

$E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-9}\ \text{C}}{(0.600\ \text{m})^2} = 124.8\ \text{N/C}.$

$E_{1x} = 898.8\ \text{N/C},\ E_{1y} = 0.$

$E_{2x} = 124.8\ \text{N/C},\ E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = -898.8\ \text{N/C} + 124.8\ \text{N/C} = -774\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point a has magnitude 774 N/C and is in the –x-direction.

(ii) **SET UP:** At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.36c.

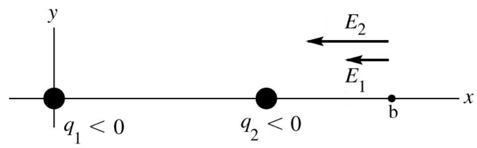


Figure 21.36c

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4.00\times10^{-9}\ \text{C}}{(1.20\ \text{m})^2} = 24.97\ \text{N/C}.$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 280.9\ \text{N/C}.$$

$E_{1x} = -24.97\ \text{N/C},\ E_{1y} = 0.$

$E_{2x} = -280.9\ \text{N/C},\ E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = -24.97\ \text{N/C} - 280.9\ \text{N/C} = -305.9\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 306 N/C and is in the $-x$-direction.

(iii) **SET UP:** At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.36d.

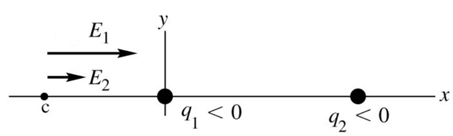


Figure 21.36d

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 898.8\ \text{N/C}.$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(1.00\ \text{m})^2} = 44.9\ \text{N/C}.$$

$E_{1x} = +898.8\ \text{N/C},\ E_{1y} = 0.$

$E_{2x} = +44.9\ \text{N/C},\ E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = +898.8\ \text{N/C} + 44.9\ \text{N/C} = +943.7\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 944 N/C and is in the $+x$-direction.

(b) SET UP: Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

EXECUTE: (i) $F = (1.602\times10^{-19}\ \text{C})(774\ \text{N/C}) = 1.24\times10^{-16}\ \text{N}$, +$x$-direction.

(ii) $F = (1.602\times10^{-19}\ \text{C})(305.9\ \text{N/C}) = 4.90\times10^{-17}\ \text{N}$, $+x$-direction.

(iii) $F = (1.602\times10^{-19}\ \text{C})(943.7\ \text{N/C}) = 1.51\times10^{-16}\ \text{N}$, $-x$-direction.

EVALUATE: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the +*x*- or −*x*-direction depends on where the field point is relative to the charge that produces the field. In part (a), for (i) the field magnitudes were subtracted because the fields were in opposite directions and in (ii) and (iii) the field magnitudes were added because the two fields were in the same direction. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.37. **IDENTIFY:** $E = k\dfrac{|q|}{r^2}$. The net field is the vector sum of the fields due to each charge.

SET UP: The electric field of a negative charge is directed toward the charge. Label the charges $q_1, q_2,$ and $q_3,$ as shown in Figure 21.37(a). This figure also shows additional distances and angles. The electric fields at point *P* are shown in Figure 21.37(b). This figure also shows the *xy*-coordinates we will use and the *x*- and *y*-components of the fields $\vec{E}_1,$ $\vec{E}_2,$ and $\vec{E}_3$.

EXECUTE: $E_1 = E_3 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-6}\ \text{C}}{(0.100\ \text{m})^2} = 4.49\times10^6\ \text{N/C}.$

$E_2 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.00\times10^{-6}\ \text{C}}{(0.0600\ \text{m})^2} = 4.99\times10^6\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} + E_{3y} = 0$ and $E_x = E_{1x} + E_{2x} + E_{3x} = E_2 + 2E_1\cos 53.1° = 1.04\times10^7\ \text{N/C}.$

$E = 1.04\times10^7\ \text{N/C},$ toward the $-2.00\ \mu\text{C}$ charge.

EVALUATE: The *x*-components of the fields of all three charges are in the same direction.

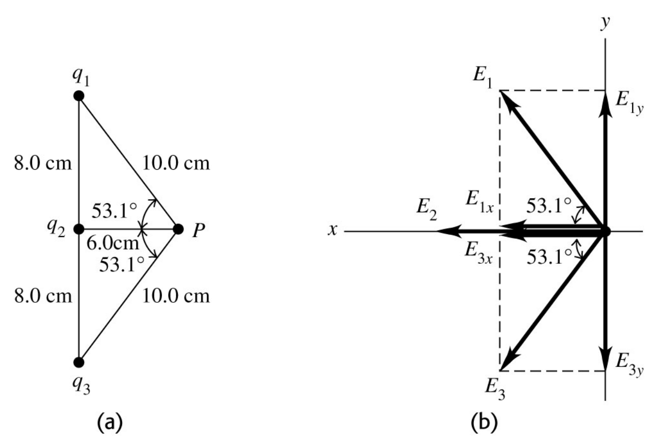


Figure 21.37

21.38. **IDENTIFY:** The net electric field is the vector sum of the individual fields.

SET UP: The distance from a corner to the center of the square is $r = \sqrt{(a/2)^2 + (a/2)^2} = a/\sqrt{2}$. The magnitude of the electric field due to each charge is the same and equal to $E_q = \dfrac{kq}{r^2} = 2\dfrac{kq}{a^2}$. All four *y*-components add and the *x*-components cancel.

EXECUTE: Each *y*-component is equal to $E_{qy} = -E_q\cos 45° = -\dfrac{E_q}{\sqrt{2}} = \dfrac{-2kq}{\sqrt{2}a^2} = -\dfrac{\sqrt{2}kq}{a^2}$. The resultant field is $\dfrac{4\sqrt{2}kq}{a^2}$, in the $-y$-direction.

EVALUATE: We must add the *y*-components of the fields, not their magnitudes.

21.39. **IDENTIFY:** For a point charge, $E = k\frac{|q|}{r^2}$. The net field is the vector sum of the fields produced by each charge. A charge q in an electric field $\vec{E}$ experiences a force $\vec{F} = q\vec{E}$.

SET UP: The electric field of a negative charge is directed toward the charge. Point A is 0.100 m from q_2 and 0.150 m from q_1. Point B is 0.100 m from q_1 and 0.350 m from q_2.

EXECUTE: **(a)** The electric fields at point A due to the charges are shown in Figure 21.39(a).

$$E_1 = k\frac{|q_1|}{r_{A1}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2} = 2.50\times10^3\ \text{N/C}.$$

$$E_2 = k\frac{|q_2|}{r_{A2}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\ \text{C}}{(0.100\ \text{m})^2} = 1.124\times10^4\ \text{N/C}.$$

Since the two fields are in opposite directions, we subtract their magnitudes to find the net field.
$E = E_2 - E_1 = 8.74\times10^3\ \text{N/C},$ to the right.

(b) The electric fields at point B are shown in Figure 21.39(b).

$$E_1 = k\frac{|q_1|}{r_{B1}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\ \text{C}}{(0.100\ \text{m})^2} = 5.619\times10^3\ \text{N/C}.$$

$$E_2 = k\frac{|q_2|}{r_{B2}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\ \text{C}}{(0.350\ \text{m})^2} = 9.17\times10^2\ \text{N/C}.$$

Since the fields are in the same direction, we add their magnitudes to find the net field.
$E = E_1 + E_2 = 6.54\times10^3\ \text{N/C},$ to the right.

(c) At A, $E = 8.74\times10^3\ \text{N/C},$ to the right. The force on a proton placed at this point would be
$F = qE = (1.60\times10^{-19}\ \text{C})(8.74\times10^3\ \text{N/C}) = 1.40\times10^{-15}\ \text{N},$ to the right.

EVALUATE: A proton has positive charge so the force that an electric field exerts on it is in the same direction as the field.

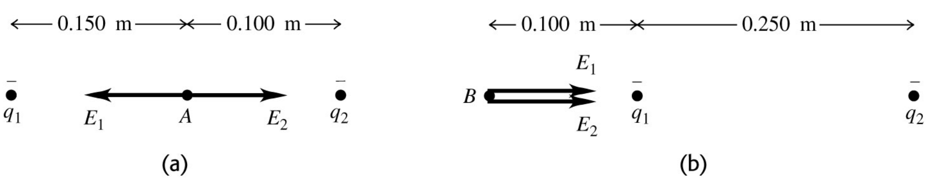


Figure 21.39

21.40. **IDENTIFY:** Apply $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$ to calculate the electric field due to each charge and add the two field vectors to find the resultant field.

SET UP: For q_1, $\hat{r} = \hat{j}$. For q_2, $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$, where θ is the angle between $\vec{E}_2$ and the $+x$-axis.

EXECUTE: **(a)** $\vec{E}_1 = \frac{q_1}{4\pi\epsilon_0 r_1^2}\hat{j} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-5.00\times10^{-9}\ \text{C})}{(0.0400\ \text{m})^2}\hat{j} = (-2.813\times10^4\ \text{N/C})\ \hat{j}.$

$\left|\vec{E}_2\right| = \frac{q_2}{4\pi\epsilon_0 r_2^2} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\ \text{C})}{(0.0300\ \text{m})^2 + (0.0400\ \text{m})^2} = 1.080\times10^4\ \text{N/C}.$ The angle of $\vec{E}_2$, measured from the x-axis, is $180° - \tan^{-1}\left(\frac{4.00\ \text{cm}}{3.00\ \text{cm}}\right) = 126.9°$ Thus

$$\vec{E}_2 = (1.080\times10^4\ \text{N/C})(\hat{i}\cos126.9° + \hat{j}\sin126.9°) = (-6.485\times10^3\ \text{N/C})\hat{i} + (8.64\times10^3\ \text{N/C})\hat{j}.$$

(b) The resultant field is $\vec{E}_1 + \vec{E}_2 = (-6.485\times10^3\ \text{N/C})\hat{i} + (-2.813\times10^4\ \text{N/C} + 8.64\times10^3\ \text{N/C})\hat{j}$.

$$\vec{E}_1 + \vec{E}_2 = (-6.485\times10^3\ \text{N/C})\hat{i} - (1.95\times10^4\ \text{N/C})\hat{j}.$$

EVALUATE: $\vec{E}_1$ is toward q_1 since q_1 is negative. $\vec{E}_2$ is directed away from q_2, since q_2 is positive.

21.41. IDENTIFY: The forces the charges exert on each other are given by Coulomb's law. The net force on the proton is the vector sum of the forces due to the electrons.

SET UP: $q_e = -1.60\times10^{-19}$ C. $q_p = +1.60\times10^{-19}$ C. The net force is the vector sum of the forces exerted by each electron. Each force has magnitude $F = k\dfrac{|q_1 q_2|}{r^2} = k\dfrac{e^2}{r^2}$ and is attractive so is directed toward the electron that exerts it.

EXECUTE: Each force has magnitude

$F_1 = F_2 = k\dfrac{|q_1 q_2|}{r^2} = k\dfrac{e^2}{r^2} = \dfrac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(1.50\times10^{-10}\ \text{m})^2} = 1.023\times10^{-8}$ N. The vector force diagram is shown in Figure 21.41.

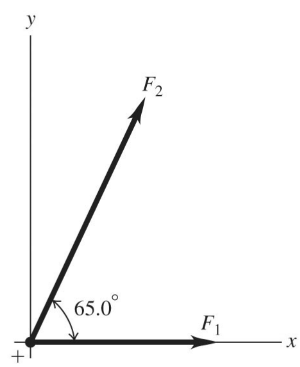


Figure 21. 41

Taking components, we get $F_{1x} = 1.023\times10^{-8}$ N; $F_{1y} = 0$. $F_{2x} = F_2 \cos 65.0° = 4.32\times10^{-9}$ N;

$F_{2y} = F_2 \sin 65.0° = 9.27\times10^{-9}$ N. $F_x = F_{1x} + F_{2x} = 1.46\times10^{-8}$ N; $F_y = F_{1y} + F_{2y} = 9.27\times10^{-9}$ N.

$F = \sqrt{F_x^2 + F_y^2} = 1.73\times10^{-8}$ N. $\tan\theta = \dfrac{F_y}{F_x} = \dfrac{9.27\times10^{-9}\ \text{N}}{1.46\times10^{-8}\ \text{N}} = 0.6349$ which gives

$\theta = 32.4°$. The net force is 1.73×10^{-8} N and is directed toward a point midway between the two electrons.

EVALUATE: Note that the net force is less than the algebraic sum of the individual forces.

21.42. IDENTIFY: We can model a segment of the axon as a point charge.

SET UP: If the axon segment is modeled as a point charge, its electric field is $E = k\dfrac{q}{r^2}$. The electric field of a point charge is directed away from the charge if it is positive.

EXECUTE: **(a)** 5.6×10^{11} Na^+ ions enter per meter so in a $0.10\ \text{mm} = 1.0\times10^{-4}$ m section, 5.6×10^7 Na^+ ions enter. This number of ions has charge $q = (5.6\times10^7)(1.60\times10^{-19}\ \text{C}) = 9.0\times10^{-12}$ C.

(b) $E = k\dfrac{|q|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{9.0\times10^{-12}\ \text{C}}{(5.00\times10^{-2}\ \text{m})^2} = 32$ N/C, directed away from the axon.

(c) $r = \sqrt{\dfrac{k|q|}{E}} = \sqrt{\dfrac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(9.0\times10^{-12}\ \text{C})}{1.0\times10^{-6}\ \text{N/C}}} = 280\ \text{m}.$

EVALUATE: The field in (b) is considerably smaller than ordinary laboratory electric fields.

21.43. **IDENTIFY:** The electric field of a positive charge is directed radially outward from the charge and has magnitude $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$. The resultant electric field is the vector sum of the fields of the individual charges.

SET UP: The placement of the charges is shown in Figure 21.43a.

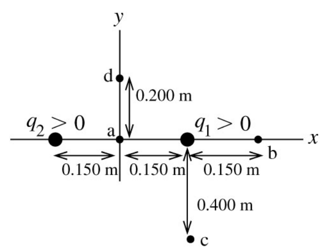
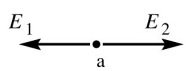


Figure 21.43a

EXECUTE: **(a)** The directions of the two fields are shown in Figure 21.43b.

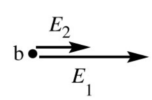


$E_1 = E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ with $r = 0.150$ m.

$E = E_2 - E_1 = 0;\ E_x = 0,\ E_y = 0.$

Figure 21.43b

(b) The two fields have the directions shown in Figure 21.43c.



$E = E_1 + E_2$, in the $+x$-direction.

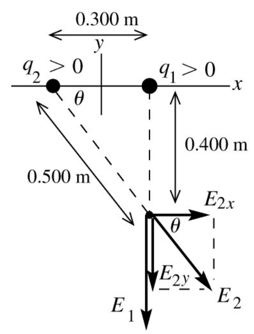
Figure 21.43c

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2} = 2396.8\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.450\ \text{m})^2} = 266.3\ \text{N/C}.$$

$E = E_1 + E_2 = 2396.8\ \text{N/C} + 266.3\ \text{N/C} = 2660\ \text{N/C};\ E_x = +2660\ \text{N/C},\ E_y = 0.$

(c) The two fields have the directions shown in Figure 21.43d.



$$\sin\theta = \frac{0.400\text{ m}}{0.500\text{ m}} = 0.800.$$

$$\cos\theta = \frac{0.300\text{ m}}{0.500\text{ m}} = 0.600.$$

Figure 21.43d

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 337.1\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.500\ \text{m})^2} = 215.7\ \text{N/C}.$$

$E_{1x} = 0$, $E_{1y} = -E_1 = -337.1$ N/C.

$E_{2x} = +E_2\cos\theta = +(215.7\ \text{N/C})(0.600) = +129.4$ N/C.

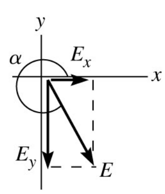
$E_{2y} = -E_2\sin\theta = -(215.7\ \text{N/C})(0.800) = -172.6$ N/C.

$E_x = E_{1x} + E_{2x} = +129$ N/C.

$E_y = E_{1y} + E_{2y} = -337.1\ \text{N/C} - 172.6\ \text{N/C} = -510$ N/C.

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(129\ \text{N/C})^2 + (-510\ \text{N/C})^2} = 526\ \text{N/C}.$$

$\vec{E}$ and its components are shown in Figure 21.43e.



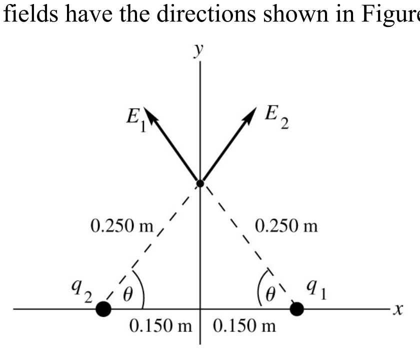
$$\tan\alpha = \frac{E_y}{E_x}.$$

$$\tan\alpha = \frac{-510\ \text{N/C}}{+129\ \text{N/C}} = -3.953.$$

$\alpha = 284°$, counterclockwise from $+x$-axis.

Figure 21.43e

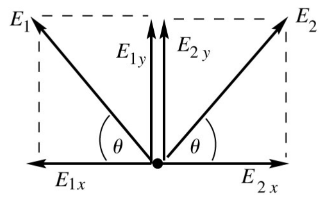
(d) The two fields have the directions shown in Figure 21.43f.



$$\sin\theta = \frac{0.200\text{ m}}{0.250\text{ m}} = 0.800.$$

Figure 21.43f

The components of the two fields are shown in Figure 21.43g.



$E_1 = E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$.

$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{6.00\times10^{-9}\ \text{C}}{(0.250\ \text{m})^2}$.

$E_1 = E_2 = 862.8$ N/C.

Figure 21.43g

$E_{1x} = -E_1\cos\theta,\ E_{2x} = +E_2\cos\theta.$

$E_x = E_{1x} + E_{2x} = 0.$

$E_{1y} = +E_1\sin\theta,\ E_{2y} = +E_2\sin\theta.$

$E_y = E_{1y} + E_{2y} = 2E_{1y} = 2E_1\sin\theta = 2(862.8\ \text{N/C})(0.800) = 1380\ \text{N/C}.$

$E = 1380$ N/C, in the $+y$-direction.

EVALUATE: Point a is symmetrically placed between identical charges, so symmetry tells us the electric field must be zero. Point b is to the right of both charges and both electric fields are in the $+x$-direction and the resultant field is in this direction. At point c both fields have a downward component and the field of q_2 has a component to the right, so the net $\vec{E}$ is in the fourth quadrant. At point d both fields have an upward component but by symmetry they have equal and opposite x-components so the net field is in the $+y$-direction. We can use this sort of reasoning to deduce the general direction of the net field before doing any calculations.

21.44. **IDENTIFY:** Apply $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ to calculate the field due to each charge and then calculate the vector sum of those fields.

SET UP: The fields due to q_1 and to q_2 are sketched in Figure 21.44.

EXECUTE: $\vec{E}_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{(6.00\times10^{-9}\ \text{C})}{(0.6\ \text{m})^2}(-\hat{i}) = -150\hat{i}\ \text{N/C}.$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}(4.00\times10^{-9}\ \text{C})\left(\frac{1}{(1.00\ \text{m})^2}(0.600)\hat{i} + \frac{1}{(1.00\ \text{m})^2}(0.800)\hat{j}\right) = (21.6\hat{i} + 28.8\hat{j})\text{N/C}.$$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4\ \text{N/C})\hat{i} + (28.8\ \text{N/C})\hat{j}$. $E = \sqrt{(128.4\ \text{N/C})^2 + (28.8\ \text{N/C})^2} = 131.6\ \text{N/C}$ at $\theta = \tan^{-1}\left(\dfrac{28.8}{128.4}\right) = 12.6°$ above the $-x$-axis and therefore $167.4°$ counterclockwise from the $+x$-axis.

EVALUATE: $\vec{E}_1$ is directed toward q_1 because q_1 is negative and $\vec{E}_2$ is directed away from q_2 because q_2 is positive.

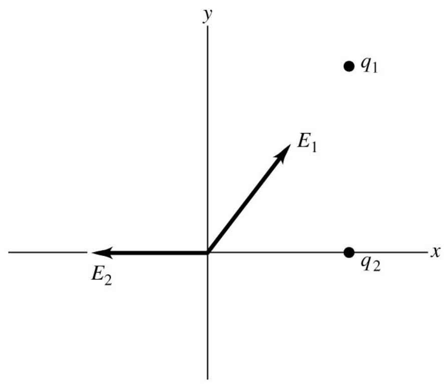


Figure 21.44

21.45. **IDENTIFY and SET UP:** We are dealing with the net electric field produced by three very large sheets of charge. The magnitude of the field due to each sheet is $E = \sigma / 2\epsilon_0$ and is independent of the distance from the sheet. $\vec{E}_A$ points away from sheet A, $\vec{E}_B$ points toward sheet B, and $\vec{E}_C$ points away from sheet C. Draw the sheet arrangement, locate point P midway between B and C, and show the fields at P (see Fig. 21.45). All the fields are along the x-axis.

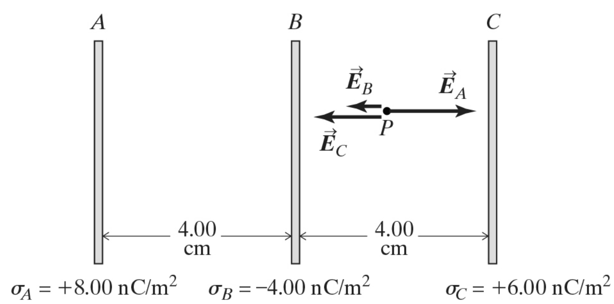


Figure 21.45

EXECUTE: Using the directions shown in the figure, we have $E_{Px} = \dfrac{|\sigma_A|}{2\epsilon_0} - \dfrac{|\sigma_B|}{2\epsilon_0} - \dfrac{|\sigma_C|}{2\epsilon_0} =$

$\dfrac{8.00 \text{ nC/m}^2 - 4.00 \text{ nC/m}^2 - 6.00 \text{ nC/m}^2}{2\epsilon_0} = -113$ N/C. The magnitude is 113 N/C and the direction is in the $-x$ direction (to the left).

EVALUATE: Since the electric fields are independent of distance from the sheet, the resultant field would would be the same *anywhere* between sheets B and C.

21.46. **IDENTIFY and SET UP:** We are dealing with the electric fields of two point charges. First sketch the charge arrangement as in Fig. 21.46. We want to find q_2 so that the resultant electric field at point P is zero. The magnitude of the field due to a point charge is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$. The two fields at P cancel, so $E_1 = E_2$.

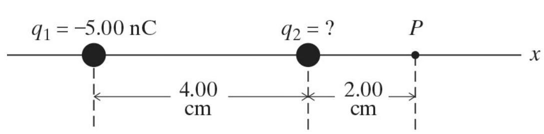


Figure 21.46

EXECUTE: $\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2}=\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2}$ gives $\dfrac{5.00\text{ nC}}{(6.00\text{ cm})^2}=\dfrac{q_2}{(2.00\text{ cm})^2}$, so $q_2 = +0.556$ nC. At point P E_1 points to the left, so E_2 must point to the right, which tells us that q_2 is positive.

EVALUATE: There was no need to convert to standard SI units because most of the quantities cancel.

21.47. IDENTIFY: We are dealing with the resultant electric field of two point charges. Sketch the charge arrangement as in Fig. 21.47.

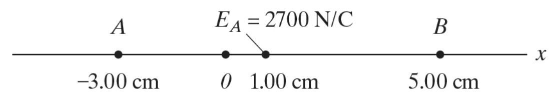


Figure 21.47

SET UP: We know that $|q_A| = 2|q_B|$, $E_A = 2700$ N/C (magnitude only) at $x = 1.00$ cm, and $E=\dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$. In each case, we want to find the resultant electric field at the origin.

EXECUTE: **(a)** Both A and B are positive. First find q_A: $E_A=\dfrac{1}{4\pi\epsilon_0}\dfrac{q_A}{r^2}=2700\text{ N/C}$, and $r = 4.00$ cm = 0.0400 m, which gives $q_A = 0.48053$ nC. Therefore $q_B = 2q_A = 0.96106$ nC. At the origin, the two fields point in opposite directions, so $E = E_A - E_B = \dfrac{1}{4\pi\epsilon_0}\left[\dfrac{0.48053\text{ nC}}{(0.0300\text{ m})^2}-\dfrac{0.96106\text{ nC}}{(0.0500\text{ m})^2}\right] = 1340\text{ N/C}$, in the $+x$ direction.

(b) Both A and B are negative. The calculation is the same as in part (a) except the field is in the opposite direction: $E = 1340$ N/C in the $-x$ direction.

(c) A is positive and B is negative. Both fields point in the $+x$ direction, so the magnitudes add, giving $E = E_A + E_B = \dfrac{1}{4\pi\epsilon_0}\left[\dfrac{0.48053\text{ nC}}{(0.0300\text{ m})^2}+\dfrac{0.96106\text{ nC}}{(0.0500\text{ m})^2}\right] = 8260$ N/C in the $+x$ direction.

(d) A is negative and B is positive. Both fields point in the $-x$ direction but E has the same magnitude as in part (c), so $E = 8260$ N/C in the $-x$ direction.

EVALUATE: It is always best to sketch the charge arrangement to visualize the direction of the electric fields.

21.48. IDENTIFY: For a long straight wire, $E=\dfrac{\lambda}{2\pi\epsilon_0 r}$.

SET UP: $\dfrac{1}{2\pi\epsilon_0}=1.80\times10^{10}\ \text{N}\cdot\text{m}^2/\text{C}^2$.

EXECUTE: Solve $E=\dfrac{\lambda}{2\pi\epsilon_0 r}$ for r: $r=\dfrac{3.20\times10^{-10}\text{ C/m}}{2\pi\epsilon_0\,(2.50\text{ N/C})}=2.30\text{ m}$.

EVALUATE: For a point charge, E is proportional to $1/r^2$. For a long straight line of charge, E is proportional to $1/r$.

21.49. IDENTIFY: For a ring of charge, the magnitude of the electric field is given by $E_x=\dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. Use $\vec{F}=q\vec{E}$. In part (b) use Newton's third law to relate the force on the ring to the force exerted by the ring.

SET UP: $Q=0.125\times10^{-9}$ C, $a=0.025$ m and $x=0.400$ m.

EXECUTE: **(a)** $\vec{E}=\dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}\hat{i}=(7.0\text{ N/C})\hat{i}$.

(b) $\vec{F}_{\text{on ring}} = -\vec{F}_{\text{on q}} = -q\vec{E} = -(-2.50\times10^{-6}\ \text{C})(7.0\ \text{N/C})\hat{i} = (1.75\times10^{-5}\ \text{N})\hat{i}$.

EVALUATE: Charges q and Q have opposite sign, so the force that q exerts on the ring is attractive.

21.50. **(a) IDENTIFY:** The field is caused by a finite uniformly charged wire.

SET UP: The field for such a wire a distance x from its midpoint is

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x/a)^2+1}} = 2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\lambda}{x\sqrt{(x/a)^2+1}}.$$

EXECUTE: $E = \dfrac{(18.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(175\times10^{-9}\ \text{C/m})}{(0.0600\ \text{m})\sqrt{\left(\dfrac{6.00\ \text{cm}}{4.25\ \text{cm}}\right)^2+1}} = 3.03\times10^4\ \text{N/C},\ \text{directed upward.}$

(b) IDENTIFY: The field is caused by a uniformly charged circular wire.

SET UP: The field for such a wire a distance x from its midpoint is $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. We first find the radius a of the circle using $2\pi a = l$.

EXECUTE: Solving for a gives $a = l/2\pi = (8.50\ \text{cm})/2\pi = 1.353\ \text{cm}$.

The charge on this circle is $Q = \lambda l = (175\ \text{nC/m})(0.0850\ \text{m}) = 14.88\ \text{nC}$.

The electric field is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}} = \frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(14.88\times10^{-9}\ \text{C/m})(0.0600\ \text{m})}{\left[(0.0600\ \text{m})^2+(0.01353\ \text{m})^2\right]^{3/2}}$$

$$E = 3.45\times10^4\ \text{N/C},\ \text{upward.}$$

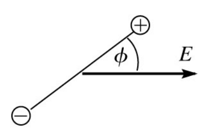
EVALUATE: In both cases, the fields are of the same order of magnitude, but the values are different because the charge has been bent into different shapes.

21.51. **(a) IDENTIFY and SET UP:** Use $p = qd$ to relate the dipole moment to the charge magnitude and the separation d of the two charges. The direction is from the negative charge toward the positive charge.

EXECUTE: $p = qd = (4.5\times10^{-9}\ \text{C})(3.1\times10^{-3}\ \text{m}) = 1.4\times10^{-11}\ \text{C}\cdot\text{m}$. The direction of $\vec{p}$ is from q_1 toward q_2.

(b) IDENTIFY and SET UP: Use $\tau = pE\sin\phi$ to relate the magnitudes of the torque and field.

EXECUTE: $\tau = pE\sin\phi$, with ϕ as defined in Figure 21.51, so



$$E = \frac{\tau}{p\sin\phi}.$$

$$E = \frac{7.2\times10^{-9}\ \text{N}\cdot\text{m}}{(1.4\times10^{-11}\ \text{C}\cdot\text{m})\sin36.9^\circ} = 860\ \text{N/C}.$$

Figure 21. 51

EVALUATE: The equation $\tau = pE\sin\phi$ gives the torque about an axis through the center of the dipole. But the forces on the two charges form a couple and the torque is the same for any axis parallel to this one. The force on each charge is $|q|E$ and the maximum moment arm for an axis at the center is $d/2$, so the maximum torque is $2(|q|E)(d/2) = 1.2\times10^{-8}\ \text{N}\cdot\text{m}$. The torque for the orientation of the dipole in the problem is less than this maximum.

21.52. **(a) IDENTIFY:** The potential energy is given by $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi.$

SET UP: $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, where ϕ is the angle between $\vec{p}$ and $\vec{E}$.

EXECUTE: parallel: $\phi = 0$ and $U(0°) = -pE$.
perpendicular: $\phi = 90°$ and $U(90°) = 0$.

$\Delta U = U(90°) - U(0°) = pE = (5.0\times10^{-30}\ \text{C}\cdot\text{m})(1.6\times10^{6}\ \text{N/C}) = 8.0\times10^{-24}\ \text{J}$.

(b) $\frac{3}{2}kT = \Delta U$ so $T = \dfrac{2\Delta U}{3k} = \dfrac{2(8.0\times10^{-24}\ \text{J})}{3(1.381\times10^{-23}\ \text{J/K})} = 0.39\ \text{K}$.

EVALUATE: Only at very low temperatures are the dipoles of the molecules aligned by a field of this strength. A much larger field would be required for alignment at room temperature.

21.53. **IDENTIFY:** The torque on a dipole in an electric field is given by $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}}\times\vec{\boldsymbol{E}}$.

SET UP: $\tau = pE\sin\phi$, where ϕ is the angle between the direction of $\vec{\boldsymbol{p}}$ and the direction of $\vec{\boldsymbol{E}}$.

EXECUTE: **(a)** The torque is zero when $\vec{\boldsymbol{p}}$ is aligned either in the *same* direction as $\vec{\boldsymbol{E}}$ or in the *opposite* direction, as shown in Figure 21.53(a).
(b) The stable orientation is when $\vec{\boldsymbol{p}}$ is aligned in the *same* direction as $\vec{\boldsymbol{E}}$. In this case a small rotation of the dipole results in a torque directed so as to bring $\vec{\boldsymbol{p}}$ back into alignment with $\vec{\boldsymbol{E}}$. When $\vec{\boldsymbol{p}}$ is directed opposite to $\vec{\boldsymbol{E}}$, a small displacement results in a torque that takes $\vec{\boldsymbol{p}}$ farther from alignment with $\vec{\boldsymbol{E}}$.
(c) Field lines for E_{dipole} in the stable orientation are sketched in Figure 21.53(b).

EVALUATE: The field of the dipole is directed from the + charge toward the − charge.

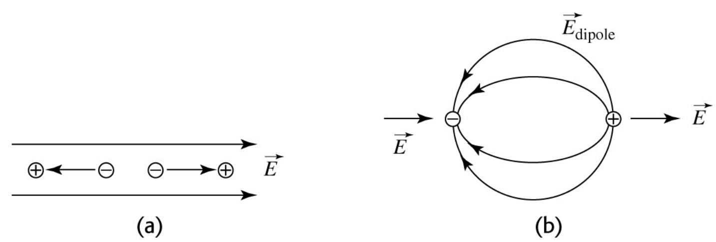


Figure 21. 53

21.54. **IDENTIFY:** Calculate the electric field due to the dipole and then apply $\vec{\boldsymbol{F}} = q\vec{\boldsymbol{E}}$.

SET UP: The field of a dipole is $E_{\text{dipole}}(x) = \dfrac{p}{2\pi\epsilon_0 x^3}$.

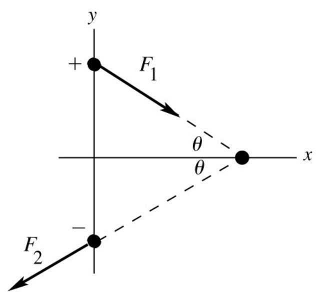
EXECUTE: $E_{\text{dipole}} = \dfrac{6.17\times10^{-30}\ \text{C}\cdot\text{m}}{2\pi\epsilon_0 (3.0\times10^{-9}\ \text{m})^3} = 4.11\times10^{6}\ \text{N/C}$. The electric force is

$F = qE = (1.60\times10^{-19}\ \text{C})(4.11\times10^{6}\ \text{N/C}) = 6.58\times10^{-13}\ \text{N}$ and is toward the water molecule (negative x-direction).

EVALUATE: $\vec{\boldsymbol{E}}_{\text{dipole}}$ is in the direction of $\vec{\boldsymbol{p}}$, so is in the $+x$-direction. The charge q of the ion is negative, so $\vec{\boldsymbol{F}}$ is directed opposite to $\vec{\boldsymbol{E}}$ and is therefore in the $-x$-direction.

21.55. **(a) IDENTIFY:** Use Coulomb's law to calculate each force and then add them as vectors to obtain the net force. Torque is force times moment arm.

SET UP: The two forces on each charge in the dipole are shown in Figure 21.55a.



$\sin\theta = 1.50/2.00$ so $\theta = 48.6°$.

Opposite charges attract and like charges repel.

$F_x = F_{1x} + F_{2x} = 0.$

Figure 21. 55a

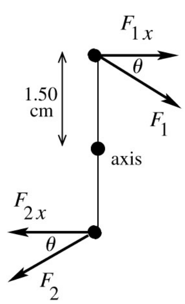
EXECUTE: $F_1 = k\dfrac{|qq'|}{r^2} = k\dfrac{(5.00\times10^{-6}\ \text{C})(10.0\times10^{-6}\ \text{C})}{(0.0200\ \text{m})^2} = 1.124\times10^3\ \text{N}.$

$F_{1y} = -F_1\sin\theta = -842.6\ \text{N}.$

$F_{2y} = -842.6\ \text{N}$ so $F_y = F_{1y} + F_{2y} = -1680\ \text{N}$ (in the direction from the $+5.00$-μC charge toward the -5.00-μC charge).

EVALUATE: The x-components cancel and the y-components add.

(b) SET UP: Refer to Figure 21.55b.



The y-components have zero moment arm and therefore zero torque.

F_{1x} and F_{2x} both produce clockwise torques.

Figure 21. 55b

EXECUTE: $F_{1x} = F_1\cos\theta = 743.1\ \text{N}.$

$\tau = 2(F_{1x})(0.0150\ \text{m}) = 22.3\ \text{N}\cdot\text{m}$, clockwise.

EVALUATE: The electric field produced by the $-10.00\ \mu$C charge is not uniform so $\tau = pE\sin\phi$ does not apply.

21.56. IDENTIFY: An electric dipole is in an external electric field. We want to know about the torque on this dipole and its electric potential energy due to this field.

SET UP: The torque is $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}}\times\vec{\boldsymbol{E}}$ and the potential energy is $U = -\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{E}} = -pE\cos\phi$.

EXECUTE: **(a)** We want the orientation of the dipole that will produce the maximum torque into the paper. Fig. 21.56a shows the orientation so that the torque will be into the page. The magnitude of the torque is $\tau = pE\sin\phi$, which is a maximum when $\phi = 90°$. Fig. 21.56b shows this orientation, for which $\vec{\boldsymbol{p}}$ is downward. The potential energy at this angle is $U = -pE\cos\phi = -pE\cos 90° = 0$.

Figures 21.56a and 21.56b

(b) We want the orientation so that the torque is zero and the potential energy is a maximum. The magnitude of the torque is $\tau = pE\sin\phi$, which is zero for $\phi = 0°$ or 180°.

For $\phi = 0°$: $U = -pE \cos 0° = -pE$.

For $\phi = 180°$: $U = -pE \cos 180° = +pE$.

As we can see, the orientation for maximum potential energy is $\phi = 180°$. At this angle, $\vec{p}$ points opposite to the electric field. To find the type of equilibrium, imagine displacing the dipole a small angle from the $\phi = 180°$ equilibrium position. Fig. 21.56c shows the dipole and the electric forces on the dipole. As the figure shows, the torques tend to rotate the dipole *away from* the equilibrium position, so this is an *unstable* equilibrium.

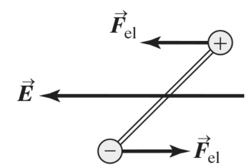
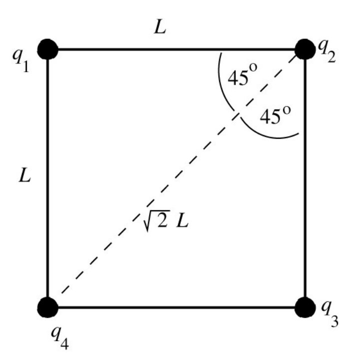


Figure 21.56c

EVALUATE: A system is in a stable equilibrium state when its potential energy is a minimum (like a ball in a valley) and an unstable state when the potential energy is a maximum (like a ball at the top of a peak). In our case, $U = +pE$ (a maximum) when $\phi = 180°$, so this is an unstable equilibrium. The state $U = -pE$ (a minimum) when $\phi = 0°$ is a stable state. To show this, use the procedure we followed with Fig. 21.56c except displace the dipole a small angle from the $\phi = 0°$ state to see which way the torques tend to turn the dipole.

21.57. **IDENTIFY:** Apply Coulomb's law to calculate the force exerted on one of the charges by each of the other three and then add these forces as vectors.

SET UP: The charges are placed as shown in Figure 21.57a.



$q_1 = q_2 = q_3 = q_4 = Q$

Figure 21.57a

Consider forces on q_4. The free-body diagram is given in Figure 21.57b. Take the y-axis to be parallel to the diagonal between q_2 and q_4 and let $+y$ be in the direction away from q_2. Then $\vec{F}_2$ is in the $+y$-direction.

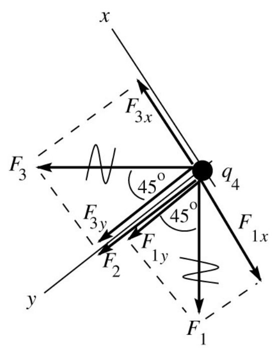


Figure 21.57b

EXECUTE: **(a)** $F_3 = F_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{L^2}$.

$F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{2L^2}$.

$F_{1x} = -F_1 \sin 45° = -F_1/\sqrt{2}$.

$F_{1y} = +F_1 \cos 45° = +F_1/\sqrt{2}$.

$F_{3x} = +F_3 \sin 45° = +F_3/\sqrt{2}$.

$F_{3y} = +F_3 \cos 45° = +F_3/\sqrt{2}$.

$F_{2x} = 0, F_{2y} = F_2$.

(b) $R_x = F_{1x} + F_{2x} + F_{3x} = 0$.

$R_y = F_{1y} + F_{2y} + F_{3y} = (2/\sqrt{2})\dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{L^2} + \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{2L^2} = \dfrac{Q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2})$.

$R = \dfrac{Q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2})$. Same for all four charges.

EVALUATE: In general the resultant force on one of the charges is directed away from the opposite corner. The forces are all repulsive since the charges are all the same. By symmetry the net force on one charge can have no component perpendicular to the diagonal of the square.

21.58. **IDENTIFY:** Apply $F = \dfrac{k|qq'|}{r^2}$ to find the force of each charge on $+q$. The net force is the vector sum of the individual forces.

SET UP: Let $q_1 = +2.50\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$. The charge $+q$ must be to the left of q_1 or to the right of q_2 in order for the two forces to be in opposite directions. But for the two forces to have equal magnitudes, $+q$ must be closer to the charge q_1, since this charge has the smaller magnitude. Therefore, the two forces can combine to give zero net force only in the region to the left of q_1. Let $+q$ be a distance d to the left of q_1, so it is a distance $d + 0.600$ m from q_2.

EXECUTE: $F_1 = F_2$ gives $\dfrac{kq|q_1|}{d^2} = \dfrac{kq|q_2|}{(d+0.600\text{ m})^2}$.

$d = \pm\sqrt{\dfrac{|q_1|}{|q_2|}}(d + 0.600\text{ m}) = \pm(0.8452)(d + 0.600\text{ m})$. d must be positive, so

$d = \dfrac{(0.8452)(0.600\text{ m})}{1-0.8452} = 3.27$ m. The net force would be zero when $+q$ is at $x = -3.27$ m.

EVALUATE: When $+q$ is at $x = -3.27$ m, $\vec{F}_1$ is in the $-x$-direction and $\vec{F}_2$ is in the $+x$-direction.

21.59. **IDENTIFY:** Apply $F = k\dfrac{|qq'|}{r^2}$ for each pair of charges and find the vector sum of the forces that q_1 and q_2 exert on q_3.

SET UP: Like charges repel and unlike charges attract. The three charges and the forces on q_3 are shown in Figure 21.59.

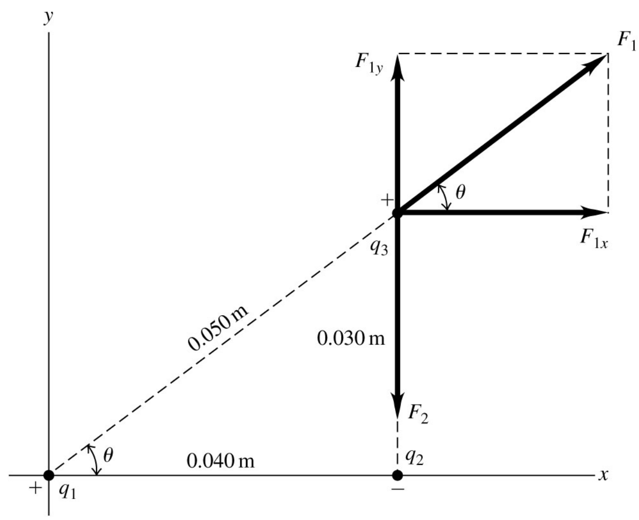


Figure 21.59

EXECUTE: **(a)** $F_1 = k\dfrac{|q_1 q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(5.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.0500\ \text{m})^2} = 1.079\times10^{-4}\ \text{N}.$

$\theta = 36.9°$. $F_{1x} = +F_1\cos\theta = 8.63\times10^{-5}\ \text{N}$. $F_{1y} = +F_1\sin\theta = 6.48\times10^{-5}\ \text{N}$.

$F_2 = k\dfrac{|q_2 q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.0300\ \text{m})^2} = 1.20\times10^{-4}\ \text{N}.$

$F_{2x} = 0$, $F_{2y} = -F_2 = -1.20\times10^{-4}\ \text{N}$. $F_x = F_{1x} + F_{2x} = 8.63\times10^{-5}\ \text{N}$.

$F_y = F_{1y} + F_{2y} = 6.48\times10^{-5}\ \text{N} + (-1.20\times10^{-4}\ \text{N}) = -5.52\times10^{-5}\ \text{N}.$

(b) $F = \sqrt{F_x^2 + F_y^2} = 1.02\times10^{-4}\ \text{N}$. $\tan\phi = \left|\dfrac{F_y}{F_x}\right| = 0.640$. $\phi = 32.6°$, below the +x-axis.

EVALUATE: The individual forces on q_3 are computed from Coulomb's law and then added as vectors, using components.

21.60. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.60. F_e is the repulsive Coulomb force between the spheres. For small θ, $\sin\theta \approx \tan\theta$.

EXECUTE: $\sum F_x = T\sin\theta - F_\text{e} = 0$ and $\sum F_y = T\cos\theta - mg = 0$. So $\dfrac{mg\sin\theta}{\cos\theta} = F_\text{e} = \dfrac{kq^2}{d^2}$. But

$\tan\theta \approx \sin\theta = \dfrac{d}{2L}$, so $d^3 = \dfrac{2kq^2L}{mg}$ and $d = \left(\dfrac{q^2 L}{2\pi\epsilon_0\, mg}\right)^{1/3}$.

EVALUATE: d increases when q increases.

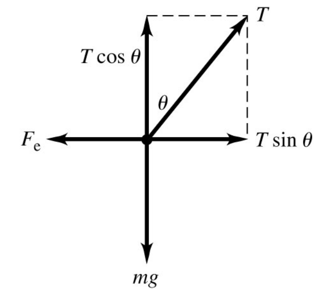


Figure 21.60

21.61. **IDENTIFY:** Use Coulomb's law for the force that one sphere exerts on the other and apply the first condition of equilibrium to one of the spheres.
SET UP: The placement of the spheres is sketched in Figure 21.61a.

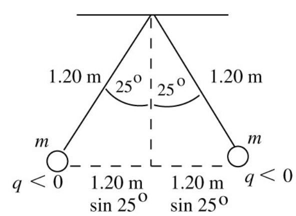


Figure 21.61a

EXECUTE: **(a)** The free-body diagrams for each sphere are given in Figure 21.61b.

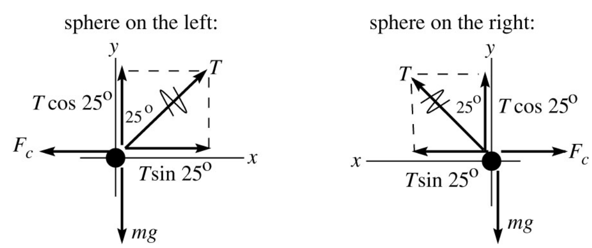


Figure 21.61b

F_c is the repulsive Coulomb force exerted by one sphere on the other.
(b) From either force diagram in part (a): $\sum F_y = ma_y$.

$T\cos 25.0° - mg = 0$ and $T = \dfrac{mg}{\cos 25.0°}$.

$\sum F_x = ma_x$.

$T\sin 25.0° - F_c = 0$ and $F_c = T\sin 25.0°$.

Use the first equation to eliminate T in the second: $F_c = (mg/\cos 25.0°)(\sin 25.0°) = mg\tan 25.0°$.

$$F_c = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{[2(1.20\text{ m})\sin 25.0°]^2}.$$

Combine this with $F_c = mg\tan 25.0°$ and get $mg\tan 25.0° = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{[2(1.20\text{ m})\sin 25.0°]^2}$.

$q = (2.40\text{ m})\sin 25.0°\sqrt{\dfrac{mg\tan 25.0°}{(1/4\pi\epsilon_0)}}.$

$q = (2.40\text{ m})\sin 25.0°\sqrt{\dfrac{(15.0\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan 25.0°}{8.988\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 2.80\times 10^{-6}\text{ C}.$

(c) The separation between the two spheres is given by $2L\sin\theta$. $q = 2.80\mu\text{C}$ as found in part (b).
$F_c = (1/4\pi\epsilon_0)q^2/(2L\sin\theta)^2$ and $F_c = mg\tan\theta$. Thus $(1/4\pi\epsilon_0)q^2/(2L\sin\theta)^2 = mg\tan\theta$.

$(\sin\theta)^2\tan\theta = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{4L^2mg} =$

$(8.988\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2.80\times 10^{-6}\text{ C})^2}{4(0.600\text{ m})^2(15.0\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)} = 0.3328.$

Solve this equation by trial and error. This will go quicker if we can make a good estimate of the value of θ that solves the equation. For θ small, $\tan\theta \approx \sin\theta$. With this approximation the equation becomes $\sin^3\theta = 0.3328$ and $\sin\theta = 0.6930$, so $\theta = 43.9°$. Now refine this guess:

θ	$\sin^2\theta\tan\theta$	
45.0°	0.5000	
40.0°	0.3467	
39.6°	0.3361	
39.5°	0.3335	
39.4°	0.3309	so $\theta = 39.5°$.

EVALUATE: The expression in part (c) says $\theta \to 0$ as $L \to \infty$ and $\theta \to 90°$ as $L \to 0$. When L is decreased from the value in part (a), θ increases.

21.62. IDENTIFY and SET UP: The horizontal electric field exerts a force on the moving sphere. The field is in the same direction as the velocity of the sphere, so it will increase the sphere's speed as the sphere falls. We want to know the magnitude E of the electric field. It points toward the east, so it does not affect the vertical velocity of the sphere. It does, however, give the sphere horizontal acceleration. We know the initial and final speeds of the sphere.

EXECUTE: First find the time to fall 60.0 cm from rest. Then use that time to find the vertical velocity v_y just as the sphere reaches the ground. Using $y = \frac{1}{2}gt^2$ gives $t = \sqrt{\dfrac{2y}{g}} = \sqrt{\dfrac{2(0.600\text{ m})}{9.80\text{ m/s}^2}} = 0.3499\text{ s}$. $v_y = gt = (9.80\text{ m/s}^2)(0.3499\text{ s}) = 3.429\text{ m/s}$. Now find v_x just as the sphere reaches the ground. Using $\Sigma F_x = ma_x$ and $F_x = qE$ gives $a_x = qE/m$. The horizontal velocity is $v_x = v_{0x} + a_xt = v_0 + (qE/m)t$. At ground level $v = 5.00$ m/s, so $\sqrt{v_x^2 + v_y^2} = 5.00\text{ m/s}$. Squaring gives $v_x^2 + v_y^2 = 25.0\text{ m}^2/\text{s}^2$. Using our results for v_x and v_y, this becomes $\left(v_0 + qEt/m\right)^2 + v_y^2 = 25.0\text{ m}^2/\text{s}^2$. Solving for E gives

$E = \dfrac{m}{qt}\left(\sqrt{25.0\text{ m}^2/\text{s}^2 - v_y^2} - v_0\right)$. Using $m = 0.500\text{ g} = 5.00\times 10^{-4}\text{ kg}$, $q = 5.00\ \mu\text{C}$, $t = 0.3499$ s, $v_y = 3.429$ m/s, and $v_0 = 2.00$ m/s, we get $E = 468\text{ N/C}$.

EVALUATE: We can check using the work-energy theorem. $W_{tot} = W_g + W_E$. $W_g = mgy = (0.500\text{ g})(9.80\text{ m/s}^2)(0.600\text{ m}) = 2.94\times 10^{-3}\text{ J}$. $W_E = qEx$, where $x = v_0t + \frac{1}{2}a_xt^2$. Using $a_x = qE/m$ and putting in $q = 5.00\ \mu$C, $E = 468$ N/C, and $m = 0.500$ g, we get $x = 0.986$ m. Therefore $W_E = (0.500\ \mu\text{C})(468\text{ N/C})(0.986\text{ m}) = 2.308\times 10^{-3}\text{ J}$. Adding gives $W_{tot} = 5.25\times 10^{-3}\text{ J}$. The kinetic energy change is $K_2 - K_1 = \dfrac{1}{2}mv^2 - \dfrac{1}{2}mv_0^2 = \dfrac{1}{2}m\left(v^2 - v_0^2\right)$, which gives

$\Delta K = \frac{1}{2}(0.500\text{ g})\left[(5.00\text{ m/s})^2 - (2.00\text{ m/s})^2\right] = 5.25\times10^{-3}$ J. Our result is consistent with the work-energy theorem.

21.63. **IDENTIFY:** The electric field exerts a horizontal force away from the wall on the ball. When the ball hangs at rest, the forces on it (gravity, the tension in the string, and the electric force due to the field) add to zero.

SET UP: The ball is in equilibrium, so for it $\sum F_x = 0$ and $\sum F_y = 0$. The force diagram for the ball is given in Figure 21.63. F_E is the force exerted by the electric field. $\vec{F} = q\vec{E}$. Since the electric field is horizontal, $\vec{F}_E$ is horizontal. Use the coordinates shown in the figure. The tension in the string has been replaced by its x- and y-components.

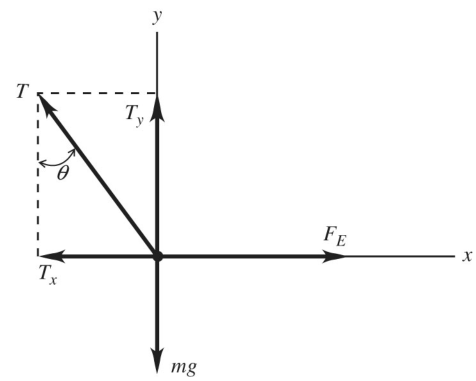


Figure 21.63

EXECUTE: $\sum F_y = 0$ gives $T_y - mg = 0$. $T\cos\theta - mg = 0$ and $T = \dfrac{mg}{\cos\theta}$. $\sum F_x = 0$ gives $F_E - T_x = 0$. $F_E - T\sin\theta = 0$. Combing the equations and solving for F_E gives

$F_E = \left(\dfrac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta = (12.3\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(\tan 17.4°) = 3.78\times10^{-2}$ N. $F_E = |q|E$ so

$E = \dfrac{F_E}{|q|} = \dfrac{3.78\times10^{-2}\text{ N}}{1.11\times10^{-6}\text{ C}} = 3.41\times10^4$ N/C. Since q is negative and $\vec{F}_E$ is to the right, $\vec{E}$ is to the left in the figure.

EVALUATE: The larger the electric field E the greater the angle the string makes with the wall.

21.64. **IDENTIFY:** A charged sphere is released in a vertical electric field and accelerates upward.

SET UP: We want the time it takes the sphere to travel upward a distance d. The forces on it are mg downward and qE upward. $\sum F_y = ma_y$ applies to the sphere.

EXECUTE: The vertical distance it travels in time t is $d = \frac{1}{2}at^2$. Using $\sum F_y = ma_y$ we get $qE - mg = ma$, so $a = \dfrac{qE - mg}{m}$. Therefore $d = \frac{1}{2}at^2 = \frac{1}{2}\left(\dfrac{qE - mg}{m}\right)t^2$. Solving for t gives $t = \sqrt{\dfrac{2md}{qE - mg}}$.

EVALUATE: We must have $qE > mg$ or the sphere would not travel upward.

21.65. **IDENTIFY and SET UP:** We model the interaction as being due to a negative charge on the balloon and an equal positive charge in the ceiling due to the positive charge left behind when the negative charge on the comb repels electrons in the ceiling. We are assuming that the negative charge in the ceiling is far enough away to be ignored. Furthermore, we model both of these charges as point charges separated by 500 μm and apply Coulomb's law to find the magnitude of the electric force. $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_2|}{r^2}$.

EXECUTE: **(a)** $F_{\text{el}} = mg$. $\dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2} = mg$. $q = \sqrt{\dfrac{mgr^2}{1/4\pi\epsilon_0}}$. Using m = 0.004 kg and r = 500 μm gives q = 1 nC.

(b) $Q = 10q = 10.0$ nC $= Ne$, so $N = Q/e$ (10 nC)/ (1.6×10^{-19} C) $\approx 6\times 10^{10}$ electrons.

EVALUATE: This is a very rough approximation because the charges in the ceiling and balloon are not concentrated to a point as we assumed in our model, and the negative charges in the ceiling are not really far enough away to be ignored.

21.66. **IDENTIFY:** We want to estimate the amount of charge in a penny made of zinc with a mass of 2.5 g.

SET UP: The gram atomic mass of zinc is 65.38 g/mol, and a zinc atom contains 30 electrons.

EXECUTE: **(a)** $(2.5\text{ g})\left(\dfrac{1\text{ mol}}{65.38\text{ g}}\right)\left(\dfrac{6.022\times 10^{23}\text{ atoms}}{\text{mol}}\right)\left(\dfrac{30\text{ electrons}}{\text{atom}}\right) = 6.9\times 10^{23}$ electrons.

(b) $q = (6.9\times 10^{23}\text{ electrons})\left(\dfrac{1.60\times 10^{-19}\text{ C}}{\text{electron}}\right) = 1.1\times 10^{5}\text{ C}.$

(c) Coulomb's law: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(1.1\times 10^{5}\text{ C})^2}{(0.020\text{ m})^2} = 2.7\times 10^{23}\text{ N}.$

(d) Estimate: 2 million leaves.

(e) There would be 6.9×10^{23} leaves and each tree would contain 2×10^{6} leaves, so the number of trees in the forest would be $(6.9\times 10^{23}\text{ leaves})\left(\dfrac{1\text{ tree}}{2\times 10^{6}\text{ leaves}}\right) = 3.5\times 10^{17}$ trees. Think of each tree as being at the center of a 10 m by 10 m square, so the area for each tree would be 100 m^2. The area of this forest would be $(3.5\times 10^{17}\text{ trees})\left(\dfrac{100\text{ m}^2}{1\text{ tree}}\right) = 3.5\times 10^{19}\text{ m}^2$.

(f) $\dfrac{A_{\text{trees}}}{A_{\text{earth}}} = \dfrac{3.5\times 10^{19}\text{ m}^2}{4\pi(6.37\times 10^{6}\text{ m}^2)} \approx 70{,}000$. The trees would occupy an area about 70,000 times the surface area of the earth.

EVALUATE: The penny contains about 110,000 C of negative charge, but it also contains 110,000 C of positive charge. That is why it doesn't blow itself apart.

21.67. **IDENTIFY:** For a point charge, $E = k\dfrac{|q|}{r^2}$. For the net electric field to be zero, $\vec{E}_1$ and $\vec{E}_2$ must have equal magnitudes and opposite directions.

SET UP: Let $q_1 = +0.500$ nC and $q_2 = +8.00$ nC. $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: The two charges and the directions of their electric fields in three regions are shown in Figure 21.67. Only in region II are the two electric fields in opposite directions. Consider a point a distance x from q_1 so a distance $1.20\text{ m} - x$ from q_2. $E_1 = E_2$ gives $k\dfrac{0.500\text{ nC}}{x^2} = k\dfrac{8.00\text{ nC}}{(1.20\text{ m} - x)^2}$.

$16x^2 = (1.20\text{ m} - x)^2$. $4x = \pm(1.20\text{ m} - x)$ and $x = 0.24$ m is the positive solution. The electric field is zero at a point between the two charges, 0.24 m from the 0.500 nC charge and 0.96 m from the 8.00 nC charge.

EVALUATE: There is only one point along the line connecting the two charges where the net electric field is zero. This point is closer to the charge that has the smaller magnitude.

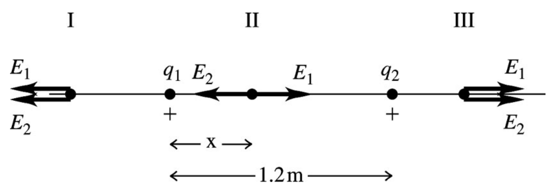


Figure 21.67

21.68. IDENTIFY: The net electric field at the origin is the vector sum of the fields due to the two charges.

SET UP: $E = k\dfrac{|q|}{r^2}$. $\vec{E}$ is toward a negative charge and away from a positive charge. At the origin, $\vec{E}_1$ due to the -3.00 nC charge is in the $+x$-direction, toward the charge.

EXECUTE: (a)

$E_1 = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(3.00\times 10^{-9}\ \text{C})}{(1.20\ \text{m})^2} = 18.73\ \text{N/C}$, so $E_{1x} = +18.73\ \text{N/C}$. $E_x = E_{1x} + E_{2x}$.

$E_x = +45.0\ \text{N/C}$, so $E_{2x} = E_x - E_{1x} = +45.0\ \text{N/C} - 18.73\ \text{N/C} = 26.27\ \text{N/C}$. $\vec{E}$ is away from Q so Q is positive. Using $E_2 = k\dfrac{|Q|}{r^2}$ gives $|Q| = \dfrac{E_2 r^2}{k} = \dfrac{(26.27\ \text{N/C})(0.600\ \text{m})^2}{8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.05\times 10^{-9}\ \text{C} = 1.05\ \text{nC}$. Since Q is positive, $Q = +1.05$ nC.

(b) $E_x = -45.0\ \text{N/C}$, so $E_{2x} = E_x - E_{1x} = -45.0\ \text{N/C} - 18.73\ \text{N/C} = -63.73\ \text{N/C}$. $\vec{E}$ is toward Q so Q is negative. $|Q| = \dfrac{E_2 r^2}{k} = \dfrac{(63.73\ \text{N/C})(0.600\ \text{m})^2}{8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 2.55\times 10^{-9}\ \text{C} = 2.55\ \text{nC}$. Since Q is negative, we have $Q = -2.55$ nC.

EVALUATE: The equation $E = k\dfrac{|q|}{r^2}$ gives only the *magnitude* of the electric field. When combining fields, you still must figure out whether to add or subtract the magnitudes depending on the direction in which the fields point.

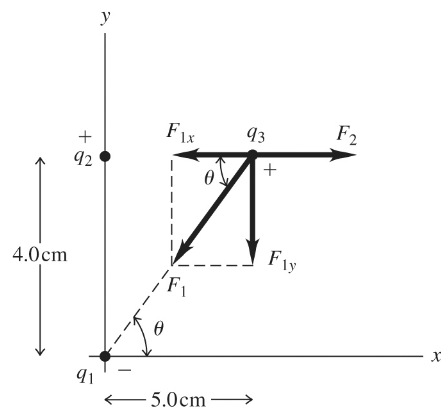
21.69. IDENTIFY: For equilibrium, the forces must balance. The electrical force is given by Coulomb's law.

SET UP: Set up axes so that the charge $+Q$ is located at $x = 0$, the charge $+4Q$ is located at $x = d$, and the unknown charge that is required to produce equilibrium, q, is located at a position $x = a$. Apply $F = k\dfrac{|q_1 q_2|}{r^2}$ to each pair of charges to obtain eqilibrium.

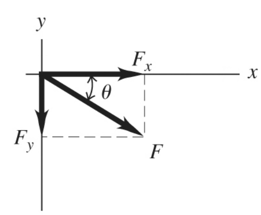
EXECUTE: For a charge q to be in equilbrium, it must be placed between the two given positive charges $(0 < a < d)$ and the magnitude of the force between q and $+Q$ must be equal to the magnitude of the force between q and $+4Q$: $k\dfrac{|q|Q}{a^2} = k\dfrac{4|q|Q}{(d-a)^2}$. Solving for a we obtain $(d-a) = \pm 4a$, which has $a = \dfrac{d}{3}$ as its only root in the required interval $(0 < a < d)$. Furthermore, to conteract the repulsive force between $+Q$ and $+4Q$ the charge q must be negative $(q = -|q|)$. The condition that $+Q$ is in equilibrium gives us $k\dfrac{-qQ}{(d/3)^2} = k\dfrac{4Q^2}{d^2}$. Solving for q we obtain $q = -\dfrac{4}{9}Q$.

EVALUATE: We have shown that both q and $+Q$ are in equilibrium provided that $a = \frac{d}{3}$ and $q = -\frac{4}{9}Q$. To make sure that the problem is well posed, we should check that these conditions also place the charge $+4Q$ is in equilbrium. We can do this by showing that $k\frac{-4qQ}{(d-a)^2}$ is equal to $k\frac{4Q^2}{d^2}$ when the given values for both a and q are substituted.

21.70. **IDENTIFY** and **SET UP:** Like charges repel and unlike charges attract, and Coulomb's law applies. The positions of the three charges are sketched in Figure 21.70(a) and each force acting on q_3 is shown. The distance between q_1 and q_3 is 5.00 cm.



(a)



(b)

Figure 21.70

EXECUTE: **(a)** $F_1 = k\frac{|q_1 q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.00\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(5.00\times10^{-2}\ \text{m})^2} = 5.394\times10^{-5}\ \text{N}.$

$F_{1x} = -F_1\cos\theta = -(5.394\times10^{-5}\ \text{N})(0.600) = -3.236\times10^{-5}\ \text{N}.$

$F_{1y} = -F_1\sin\theta = -(5.394\times10^{-5}\ \text{N})(0.800) = -4.315\times10^{-5}\ \text{N}.$

$F_2 = k\frac{|q_2 q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.00\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(3.00\times10^{-2}\ \text{m})^2} = 9.989\times10^{-5}\ \text{N}.$

$F_{2x} = 9.989\times10^{-5}\ \text{N};\ F_{2y} = 0.$

$F_x = F_{1x} + F_{2x} = 9.989\times10^{-5}\ \text{N} + (-3.236\times10^{-5}\ \text{N}) = 6.75\times10^{-5}\ \text{N};$

$F_y = F_{1y} + F_{2y} = -4.32\times10^{-5}\ \text{N}.$

(b) $\vec{F}$ and its components are shown in Figure 21.70(b).

$F=\sqrt{F_x^2+F_y^2}=8.01\times10^{-5}$ N. $\tan\theta=\left|\dfrac{F_y}{F_x}\right|=0.640$ and $\theta=32.6°$. $\vec{F}$ is 327° counterclockwise from the $+x$-axis.

EVALUATE: The equation $F=k\dfrac{|q_1q_2|}{r^2}$ gives only the magnitude of the force. We must find the direction by deciding if the force between the charges is attractive or repulsive.

21.71. **IDENTIFY:** Use Coulomb's law to calculate the forces between pairs of charges and sum these forces as vectors to find the net charge.

(a) SET UP: The forces are sketched in Figure 21.71a.

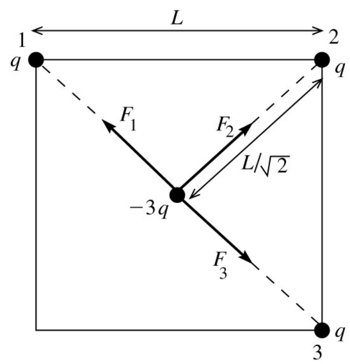


Figure 21. 71a

EXECUTE: $\vec{F}_1+\vec{F}_3=0$, so the net force is $\vec{F}=\vec{F}_2$.

$$F=\frac{1}{4\pi\epsilon_0}\frac{q(3q)}{(L/\sqrt{2})^2}=\frac{6q^2}{4\pi\epsilon_0 L^2},$$ away from the vacant corner.

(b) SET UP: The forces are sketched in Figure 21.71b.

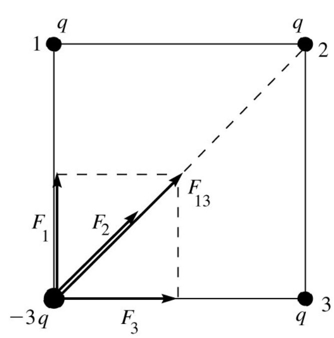


Figure 21. 71b

EXECUTE: $$F_2=\frac{1}{4\pi\epsilon_0}\frac{q(3q)}{(\sqrt{2}L)^2}=\frac{3q^2}{4\pi\epsilon_0(2L^2)}.$$

$$F_1=F_3=\frac{1}{4\pi\epsilon_0}\frac{q(3q)}{L^2}=\frac{3q^2}{4\pi\epsilon_0 L^2}.$$

The vector sum of F_1 and F_3 is $F_{13}=\sqrt{F_1^2+F_3^2}$.

$F_{13}=\sqrt{2}F_1=\dfrac{3\sqrt{2}q^2}{4\pi\epsilon_0 L^2}$; $\vec{F}_{13}$ and $\vec{F}_2$ are in the same direction.

$F=F_{13}+F_2=\dfrac{3q^2}{4\pi\epsilon_0 L^2}\left(\sqrt{2}+\dfrac{1}{2}\right)$, and is directed toward the center of the square.

EVALUATE: By symmetry the net force is along the diagonal of the square. The net force is only slightly larger when the $-3q$ charge is at the center. Here it is closer to the charge at point 2 but the other two forces cancel.

21.72. **IDENTIFY:** For the acceleration (and hence the force) on Q to be upward, as indicated, the forces due to q_1 and q_2 must have equal strengths, so q_1 and q_2 must have equal magnitudes. Furthermore, for the force to be upward, q_1 must be positive and q_2 must be negative.

SET UP: Since we know the acceleration of Q, Newton's second law gives us the magnitude of the force on it. We can then add the force components using $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$. The electrical force on Q is given by Coulomb's law, $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|Qq_1|}{r^2}$ (for q_1) and likewise for q_2.

EXECUTE: First find the net force: $F = ma =$ $(0.00500\text{ kg})(324\text{ m/s}^2) = 1.62\text{ N}$. Now add the force components, calling θ the angle between the line connecting q_1 and q_2 and the line connecting q_1 and Q. $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$ and $F_{Qq_1} = \dfrac{F}{2\cos\theta} = \dfrac{1.62\text{ N}}{2\left(\dfrac{2.25\text{ cm}}{3.00\text{ cm}}\right)} = 1.08\text{ N}$. Now find the charges by solving for q_1 in Coulomb's law and use the fact that q_1 and q_2 have equal magnitudes but opposite signs. $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|Q|q_1}{r^2}$ and

$$q_1 = \frac{r^2 F_{Qq_1}}{\dfrac{1}{4\pi\epsilon_0}|Q|} = \frac{(0.0300\text{ m})^2(1.08\text{ N})}{(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.75\times10^{-6}\text{ C})} = 6.17\times10^{-8}\text{ C}.$$

$q_2 = -q_1 = -6.17\times10^{-8}$ C.

EVALUATE: Simple reasoning allows us first to conclude that q_1 and q_2 must have equal magnitudes but opposite signs, which makes the equations much easier to set up than if we had tried to solve the problem in the general case. As Q accelerates and hence moves upward, the magnitude of the acceleration vector will change in a complicated way.

21.73. **IDENTIFY:** The small bags of protons behave like point-masses and point-charges since they are extremely far apart.

SET UP: For point-particles, we use Newton's formula for universal gravitation $(F = Gm_1m_2/r^2)$ and Coulomb's law. The number of protons is the mass of protons in the bag divided by the mass of a single proton.

EXECUTE: **(a)** $(0.0010\text{ kg})/(1.67\times10^{-27}\text{ kg}) = 6.0\times10^{23}$ protons.

(b) Using Coulomb's law, where the separation is twice the radius of the earth, we have
$F_{\text{electrical}} = (9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.0\times10^{23}\times1.60\times10^{-19}\text{ C})^2/(2\times6.37\times10^6\text{ m})^2 = 5.1\times10^5\text{ N}.$
$F_{\text{grav}} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(0.0010\text{ kg})^2/(2\times6.37\times10^6\text{ m})^2 = 4.1\times10^{-31}\text{ N}.$

EVALUATE: **(c)** The electrical force (≈200,000 lb!) is certainly large enough to feel, but the gravitational force clearly is not since it is about 10^{36} times weaker.

21.74. **IDENTIFY:** The positive sphere will be deflected in the direction of the electric field but the negative sphere will be deflected in the direction opposite to the electric field. Since the spheres hang at rest, they are in equilibrium so the forces on them must balance. The external forces on each sphere are gravity, the tension in the string, the force due to the uniform electric field and the electric force due to the other sphere.

SET UP: The electric force on one sphere due to the other is $F_C = k\dfrac{|q^2|}{r^2}$ in the horizontal direction, the force on it due to the uniform electric field is $F_E = qE$ in the horizontal direction, the gravitational force is mg vertically downward and the force due to the string is T directed along the string. For equilibrium $\sum F_x = 0$ and $\sum F_y = 0$.

EXECUTE: **(a)** The positive sphere is deflected in the same direction as the electric field, so the one that is deflected to the left is positive.

(b) The separation between the two spheres is $2(0.530\text{ m})\sin 29.0^\circ = 0.5139\text{ m}$.

$F_C = k\dfrac{|q^2|}{r^2} = \dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(72.0\times10^{-9}\text{ C})^2}{(0.5139\text{ m})^2} = 1.765\times10^{-4}\text{ N}$. $F_E = qE$. $\sum F_y = 0$ gives

$T\cos 29.0^\circ - mg = 0$ so $T = \dfrac{mg}{\cos 29.0^\circ}$. $\sum F_x = 0$ gives $T\sin 29.0^\circ + F_C - F_E = 0$.

$mg\tan 29.0^\circ + F_C = qE$. Combining the equations and solving for E gives

$$E = \frac{mg\tan 29.0^\circ + F_C}{q} = \frac{(6.80\times10^{-6}\text{ kg})(9.80\text{ m/s}^2)\tan 29.0^\circ + 1.765\times10^{-4}\text{ N}}{72.0\times10^{-9}\text{ C}} = 2.96\times10^3\text{ N/C}.$$

EVALUATE: Since the charges have opposite signs, they attract each other, which tends to reduce the angle between the strings. Therefore if their charges were negligibly small, the angle between the strings would be greater than 58.0°.

21.75. **IDENTIFY:** The only external force acting on the electron is the electrical attraction of the proton, and its acceleration is toward the center of its circular path (that is, toward the proton). Newton's second law applies to the electron and Coulomb's law gives the electrical force on it due to the proton.

SET UP: Newton's second law gives $F_C = m\dfrac{v^2}{r}$. Using the electrical force for F_C gives $k\dfrac{e^2}{r^2} = m\dfrac{v^2}{r}$.

EXECUTE: Solving for v gives $v = \sqrt{\dfrac{ke^2}{mr}} = \sqrt{\dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(9.109\times10^{-31}\text{ kg})(5.29\times10^{-11}\text{ m})}} = 2.19\times10^6\text{ m/s}$.

EVALUATE: This speed is less than 1% the speed of light, so it is reasonably safe to use Newtonian physics.

21.76. **IDENTIFY:** A uniformly charged horizontal disk exerts an upward electric force on a small charged sphere. Gravity exerts a downward force on the sphere.

SET UP: The magnitude of the electric force on the sphere is $F = QE$, where E is the electric field a distance z above the center of the disk and is given by $E = \dfrac{\sigma}{2\epsilon_0}\left(1 - \dfrac{1}{\sqrt{(R/z)^2+1}}\right)$.

EXECUTE: **(a)** The net upward force is $F = F_{\text{el}} + F_{\text{g}} = \dfrac{Q\sigma}{2\epsilon_0}\left(1 - \dfrac{1}{\sqrt{(R/z)^2+1}}\right) - Mg$.

(b) We want the height h above the disk at which the sphere hovers. This will occur when the electric force is equal to the weight of the sphere, so $\dfrac{Q\sigma}{2\epsilon_0}\left(1 - \dfrac{1}{\sqrt{(R/z)^2+1}}\right) - Mg = 0$. Now solve for z.

Rearranging gives $1 - \dfrac{1}{\sqrt{(R/z)^2+1}} = \dfrac{2Mg\epsilon_0}{Q\sigma} \equiv v$, where we have used the v defined in the problem.

We now need to solve the equation $1 - v = \dfrac{1}{\sqrt{(R/h)^2+1}}$ for h, where h is the height at which the sphere hovers. Doing some algebra yields $h = \dfrac{R}{\sqrt{1/(1-v)^2 - 1}} = \dfrac{R(1-v)}{\sqrt{v(2-v)}}$.

(c) We want to find h if $M = 100$ g $= 0.100$ kg, $Q = 1.00\ \mu$C, $R = 5.00$ cm $= 0.0500$ m, and $\sigma = 10.0\ \text{nC/cm}^2 = 1.00\times10^{-4}\ \text{C/m}^2$. Using $v = \dfrac{2Mg\epsilon_0}{Q\sigma}$ and $h = \dfrac{R(1-v)}{\sqrt{v(2-v)}}$ with these numbers, we get $v = 0.1735$, which gives $h = 0.0734$ m $= 7.34$ cm.

EVALUATE: A 100-gram object hovering about 7 cm from the disk could easily be observed in a student laboratory.

21.77. **IDENTIFY:** $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ gives the force exerted by the electric field. This force is constant since the electric field is uniform and gives the proton a constant acceleration. Apply the constant acceleration equations for the x- and y-components of the motion, just as for projectile motion.

SET UP: The electric field is upward so the electric force on the positively charged proton is upward and has magnitude $F = eE$. Use coordinates where positive y is downward. Then applying $\sum\vec{F} = m\vec{a}$ to the proton gives that $a_x = 0$ and $a_y = -eE/m$. In these coordinates the initial velocity has components $v_x = +v_0\cos\alpha$ and $v_y = +v_0\sin\alpha$, as shown in Figure 21.77a.

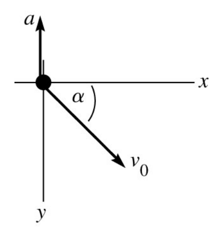


Figure 21.77a

EXECUTE: **(a)** Finding $h_{\max}$: At $y = h_{\max}$ the y-component of the velocity is zero.

$v_y = 0, v_{0y} = v_0\sin\alpha, a_y = -eE/m, y - y_0 = h_{\max} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0).$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y}.$$

$$h_{\max} = \frac{-v_0^2\sin^2\alpha}{2(-eE/m)} = \frac{mv_0^2\sin^2\alpha}{2eE}.$$

(b) Use the vertical motion to find the time t: $y - y_0 = 0, v_{0y} = v_0\sin\alpha, a_y = -eE/m, t = ?$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2.$$

With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(v_0\sin\alpha)}{-eE/m} = \dfrac{2mv_0\sin\alpha}{eE}.$

Then use the x-component motion to find d: $a_x = 0, v_{0x} = v_0\cos\alpha, t = 2mv_0\sin\alpha/eE, x - x_0 = d = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ gives $d = v_0\cos\alpha\left(\dfrac{2mv_0\sin\alpha}{eE}\right) = \dfrac{mv_0^2 2\sin\alpha\cos\alpha}{eE} = \dfrac{mv_0^2\sin2\alpha}{eE}.$

(c) The trajectory of the proton is sketched in Figure 21.77b.

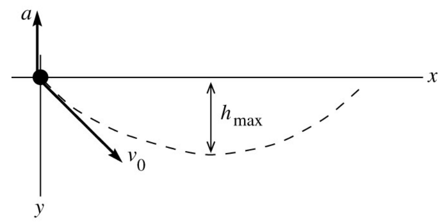


Figure 21.77b

(d) Use the expression in part (a): $h_{max} = \dfrac{[(4.00\times10^5 \text{ m/s})(\sin 30.0°)]^2(1.673\times10^{-27} \text{ kg})}{2(1.602\times10^{-19} \text{ C})(500 \text{ N/C})} = 0.418 \text{ m}.$

Use the expression in part (b): $d = \dfrac{(1.673\times10^{-27} \text{ kg})(4.00\times10^5 \text{ m/s})^2 \sin 60.0°}{(1.602\times10^{-19} \text{ C})(500 \text{ N/C})} = 2.89 \text{ m}.$

EVALUATE: In part (a), $a_y = -eE/m = -4.8\times10^{10} \text{ m/s}^2$. This is much larger in magnitude than g, the acceleration due to gravity, so it is reasonable to ignore gravity. The motion is just like projectile motion, except that the acceleration is upward rather than downward and has a much different magnitude. h_{max} and d increase when α or v_0 increase and decrease when E increases.

21.78. **IDENTIFY:** The electric field is vertically downward and the charged object is deflected downward, so it must be positively charged. While the object is between the plates, it is accelerated downward by the electric field. Once it is past the plates, it moves downward with a constant vertical velocity which is the same downward velocity it acquired while between the plates. Its horizontal velocity remains constant at v_0 throughout its motion. The forces on the object are all constant, so its acceleration is constant; therefore we can use the standard kinematics equations. Newton's second law applies to the object.

SET UP: Call the x-axis positive to the right and the y-axis positive downward. The equations $\vec{E} = \dfrac{\vec{F}_0}{q_0}$, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, $x = v_x t$, and $\Sigma F_y = ma_y$ all apply. $v_x = v_0$ = constant.

EXECUTE: *Time through the plates:* $t = x/v_x = x/v_0$ = (0.260 m)/(5000 m/s) = 5.20×10^{-5} s.

Vertical deflection between the plates: $\Delta y_1 = y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}a_y t^2 = \frac{1}{2}(qE/m)t^2$

$\Delta y_1 = \frac{1}{2}(800 \text{ N/C})(5.20 \times10^{-5} \text{ s})^2(q/m) = (1.0816 \times10^{-6} \text{ kg}\cdot\text{m/C})(q/m).$

v_y as the object just emerges from the plates:
$v_y = v_{0y} + a_y t = (qE/m)t = (q/m)(800 \text{ N/C})(5.20 \times10^{-5} \text{ s}) = (0.04160 \text{ kg}\cdot\text{m/C}\cdot\text{s})(q/m)$. (This is the initial vertical velocity for the next step.)
Time to travel 56.0 cm: $t = x/v_x$ = (0.560 m)/(5000 m/s) = 1.120×10^{-4} s.
Vertical deflection after leaving the plates:
$\Delta y_2 = v_{0y}t = (0.04160 \text{ kg}\cdot\text{m/C}\cdot\text{s})(q/m)(1.120 \times10^{-4} \text{ s}) = (4.6592 \times10^{-6} \text{ kg}\cdot\text{m/C})(q/m).$
Total vertical deflection:
$d = \Delta y_1 + \Delta y_2$.
1.25 cm = 0.0125 m = $(1.0816 \times10^{-6} \text{ kg}\cdot\text{m/C})(q/m) + (4.6592 \times10^{-6} \text{ kg}\cdot\text{m/C})(q/m)$.
q/m = 2180 C/kg.
EVALUATE: The charge on 1.0 kg is so huge that it could not be dealt with in a laboratory. But this is a tiny object, more likely with a mass in the range of 1.0 μg, so its charge would be (2180 C/kg)(10^{-9} kg) = 2.18×10^{-6} C ≈ 2 μC. That amount of charge could be used in an experiment.

21.79. **IDENTIFY:** Divide the charge distribution into infinitesimal segments of length dx'. Calculate E_x and E_y due to a segment and integrate to find the total field.

SET UP: The charge dQ of a segment of length dx' is $dQ = (Q/a)dx'$. The distance between a segment at x' and a point at x on the x-axis is $x - x'$ since $x > a$.

EXECUTE: **(a)** $dE_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{dQ}{(x-x')^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/a)dx'}{(x-x')^2}$. Integrating with respect to x' over the length of the charge distribution gives

$$E_x = \frac{1}{4\pi\epsilon_0}\int_0^a \frac{(Q/a)dx'}{(x-x')^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{1}{x-a}-\frac{1}{x}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\frac{a}{x(x-a)} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x(x-a)}.\ Ey = 0.$$

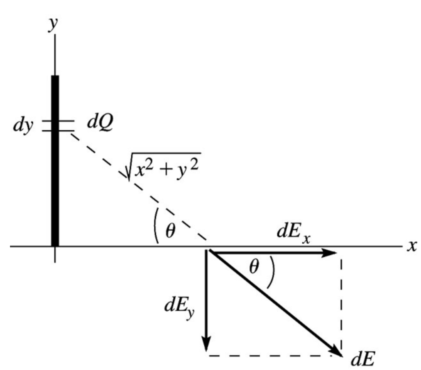
(b) At the location of the charge, $x = r + a$, so $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{(r+a)(r+a-a)} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r(r+a)}$.

Using $\vec{F} = q\vec{E}$, we have $\vec{F} = q\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{r(r+a)}\hat{\boldsymbol{i}}$.

EVALUATE: **(c)** For $r \gg a$, $r + a \to r$, so the magnitude of the force becomes $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{r^2}$. The charge distribution looks like a point charge from far away, so the force takes the form of the force between a pair of point charges.

21.80. **IDENTIFY:** Use $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.10. Use $\vec{E} = \dfrac{\vec{F}}{q_0}$ to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.80.



$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}.$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}.$$

Figure 21.80

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{dQ}{x^2+y^2}\right) = \dfrac{Q}{4\pi\epsilon_0 a}\left(\dfrac{dy}{x^2+y^2}\right)$.

$$dE_x = dE\cos\theta = \frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{(x^2+y^2)^{3/2}}\right).$$

$$dE_y = -dE\sin\theta = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{(x^2+y^2)^{3/2}}\right).$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a = \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}.$$

$$E_y = \int dEy = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right).$$

(b) $\vec{F} = q_0\vec{E}$.

$$F_x = -qE_x = \frac{-qQ}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}};\ F_y = -qE_y = \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right).$$

(c) For $x \gg a$, $\frac{1}{\sqrt{x^2+a^2}} = \frac{1}{x}\left(1+\frac{a^2}{x^2}\right)^{-1/2} = \frac{1}{x}\left(1-\frac{a^2}{2x^2}\right) = \frac{1}{x}-\frac{a^2}{2x^3}.$

$$F_x \approx -\frac{qQ}{4\pi\epsilon_0 x^2},\ F_y \approx \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{x}+\frac{a^2}{2x^3}\right) = \frac{qQa}{8\pi\epsilon_0 x^3}.$$

Evaluate: For $x \gg a$, $F_y \ll F_x$ and $F \approx |F_x| = \frac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x \gg a$ the charge distribution Q acts like a point charge.

21.81. Identify: Apply $E = \frac{\sigma}{2\epsilon_0}[1-(R^2/x^2+1)^{-1/2}]$.

Set Up: $\sigma = Q/A = Q/\pi R^2$. $(1+y^2)^{-1/2} \approx 1-y^2/2$, when $y^2 \ll 1$.

Execute: **(a)** $E = \frac{\sigma}{2\epsilon_0}[1-(R^2/x^2+1)^{-1/2}]$ gives

$$E = \frac{7.00\ \text{pC}/\pi(0.025\ \text{m})^2}{2\epsilon_0}\left[1-\left(\frac{(0.025\ \text{m})^2}{(0.200\ \text{m})^2}+1\right)^{-1/2}\right] = 1.56\ \text{N/C},\ \text{in the } +x\text{-direction.}$$

(b) For $x \gg R$, $E = \frac{\sigma}{2\epsilon_0}[1-(1-R^2/2x^2+\cdots)] \approx \frac{\sigma}{2\epsilon_0}\frac{R^2}{2x^2} = \frac{\sigma\pi R^2}{4\pi\epsilon_0 x^2} = \frac{Q}{4\pi\epsilon_0 x^2}.$

(c) The electric field of (a) is less than that of the point charge (0.90 N/C) since the first correction term to the point charge result is negative.

(d) For $x = 0.200$ m, the percent difference is $\frac{(1.58-1.56)}{1.56} = 0.01 = 1\%$. For $x = 0.100$ m,

$E_{\text{disk}} = 6.00$ N/C and $E_{\text{point}} = 6.30$ N/C, so the percent difference is $\frac{(6.30-6.00)}{6.30} = 0.047 \approx 5\%$.

Evaluate: The field of a disk becomes closer to the field of a point charge as the distance from the disk increases. At $x = 10.0$ cm, $R/x = 25\%$ and the percent difference between the field of the disk and the field of a point charge is 5%.

21.82. Identify: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere, with x horizontal and y vertical.

Set Up: The free-body diagram for the sphere is given in Figure 21.82. The electric field $\vec{E}$ of the sheet is directed away from the sheet and has magnitude $E = \frac{\sigma}{2\epsilon_0}$.

EXECUTE: $\sum F_y = 0$ gives $T\cos\alpha = mg$ and $T = \dfrac{mg}{\cos\alpha}$. $\sum F_x = 0$ gives $T\sin\alpha = \dfrac{q\sigma}{2\epsilon_0}$ and

$T = \dfrac{q\sigma}{2\epsilon_0 \sin\alpha}$. Combining these two equations we have $\dfrac{mg}{\cos\alpha} = \dfrac{q\sigma}{2\epsilon_0\sin\alpha}$ and $\tan\alpha = \dfrac{q\sigma}{2\epsilon_0 mg}$.

Therefore, $\alpha = \arctan\left(\dfrac{q\sigma}{2\epsilon_0 mg}\right)$.

EVALUATE: The electric field of the sheet, and hence the force it exerts on the sphere, is independent of the distance of the sphere from the sheet.

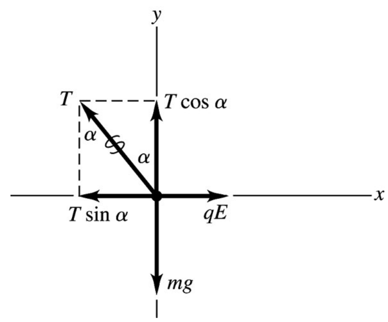
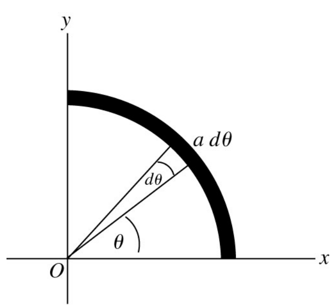


Figure 21.82

21.83. IDENTIFY: Divide the charge distribution into small segments, use the point charge formula for the electric field due to each small segment and integrate over the charge distribution to find the *x*- and *y*-components of the total field.

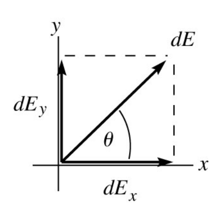
SET UP: Consider the small segment shown in Figure 21.83a.



EXECUTE: A small segment that subtends angle $d\theta$ has length $a\,d\theta$ and contains charge $dQ = \left(\dfrac{a d\theta}{\frac{1}{2}\pi a}\right)Q = \dfrac{2Q}{\pi}d\theta$. ($\frac{1}{2}\pi a$ is the total length of the charge distribution.)

Figure 21.83a

The charge is negative, so the field at the origin is directed toward the small segment. The small segment is located at angle θ as shown in the sketch. The electric field due to dQ is shown in Figure 21.83b, along with its components.



$$dE = \frac{1}{4\pi\epsilon_0}\frac{|dQ|}{a^2}.$$

$$dE = \frac{Q}{2\pi^2\epsilon_0 a^2}d\theta.$$

Figure 21.83b

$dE_x = dE\cos\theta = (Q/2\pi^2 \epsilon_0 a^2)\cos\theta d\theta.$

$E_x = \int dE_x = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}\int_0^{\pi/2}\cos\theta d\theta = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}(\sin\theta\Big|_0^{\pi/2}) = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}.$

$dE_y = dE\sin\theta = (Q/2\pi^2 \epsilon_0 a^2)\sin\theta d\theta.$

$E_y = \int dE_y = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}\int_0^{\pi/2}\sin\theta d\theta = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}(-\cos\theta\Big|_0^{\pi/2}) = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}.$

EVALUATE: Note that $E_x = E_y$, as expected from symmetry.

21.84. IDENTIFY: We must add the electric field components of the positive half and the negative half.

SET UP: From Problem 21.83, the electric field due to the quarter-circle section of positive charge has components $E_x = +\dfrac{Q}{2\pi^2 \epsilon_0 a^2}$, $E_y = -\dfrac{Q}{2\pi^2 \epsilon_0 a^2}$. The field due to the quarter-circle section of negative charge has components $E_x = +\dfrac{Q}{2\pi^2 \epsilon_0 a^2}$, $E_y = +\dfrac{Q}{2\pi^2 \epsilon_0 a^2}$.

EXECUTE: The components of the resultant field are the sum of the *x*- and *y*-components of the fields due to each half of the semicircle. The *y*-components cancel, but the *x*-components add, giving

$E_x = +\dfrac{Q}{\pi^2 \epsilon_0 a^2}$, in the $+x$-direction.

EVALUATE: Even though the net charge on the semicircle is zero, the field it produces is *not* zero because of the way the charge is arranged.

21.85. IDENTIFY: Each wire produces an electric field at *P* due to a finite wire. These fields add by vector addition.

SET UP: Each field has magnitude $\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}$. The field due to the negative wire points to the left, while the field due to the positive wire points downward, making the two fields perpendicular to each other and of equal magnitude. The net field is the vector sum of these two, which is $E_{\text{net}} = 2E_1\cos 45° = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}\cos 45°$. In part (b), the electrical force on an electron at *P* is *eE*.

EXECUTE: **(a)** The net field is $E_{\text{net}} = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}\cos 45°.$

$$E_{\text{net}} = \frac{2(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-6}\ \text{C})\cos 45°}{(0.600\ \text{m})\sqrt{(0.600\ \text{m})^2+(0.600\ \text{m})^2}} = 6.25\times10^4\ \text{N/C}.$$

The direction is 225° counterclockwise from an axis pointing to the right at point *P*.

(b) $F = eE = (1.60\times10^{-19}\ \text{C})(6.25\times10^4\ \text{N/C}) = 1.00\times10^{-14}\ \text{N}$, opposite to the direction of the electric field, since the electron has negative charge.

EVALUATE: Since the electric fields due to the two wires have equal magnitudes and are perpendicular to each other, we only have to calculate one of them in the solution.

21.86. IDENTIFY: Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\epsilon_0$, and the net field is the vector sum of these, $E_{\text{net}} = \dfrac{\sigma_B}{2\epsilon_0} \pm \dfrac{\sigma_A}{2\epsilon_0} = \dfrac{\sigma_B \pm \sigma_A}{2\epsilon_0}$, where we use the $+$ or $-$ sign depending on whether the fields are in the same or opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: **(a)** The two fields oppose and the field of *B* is stronger than that of *A*, so

$$E_{\text{net}}=\frac{\sigma_B}{2\epsilon_0}-\frac{\sigma_A}{2\epsilon_0}=\frac{\sigma_B-\sigma_A}{2\epsilon_0}=\frac{11.6\ \mu\text{C/m}^2-8.80\ \mu\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}=1.58\times10^5\ \text{N/C},\ \text{to the right.}$$

(b) The fields are now in the same direction, so their magnitudes add.

$$E_{\text{net}}=\ (11.6\ \mu\text{C/m}^2+8.80\ \mu\text{C/m}^2)/2\epsilon_0=1.15\times10^6\ \text{N/C},\ \text{to the right.}$$

(c) The fields add but now point to the left, so $E_{\text{net}}=1.15\times10^6$ N/C, to the left.

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

21.87. **IDENTIFY:** Apply the formula for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2-R_1^2)\sigma$. The electric field of a disk is

$$E=\frac{\sigma}{2\epsilon_0}\left[1-1/\sqrt{(R/x)^2+1}\right].$$

EXECUTE: **(a)** $Q=A\sigma=\pi(R_2^2-R_1^2)\sigma.$

(b) $\vec{\boldsymbol{E}}(x)=\dfrac{\sigma}{2\epsilon_0}\left(\left[1-1/\sqrt{(R_2/x)^2+1}\right]-\left[1-1/\sqrt{(R_1/x)^2+1}\right]\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}.$

$\vec{\boldsymbol{E}}(x)=\dfrac{\sigma}{2\epsilon_0}\left(1/\sqrt{(R_1/x)^2+1}-1/\sqrt{(R_2/x)^2+1}\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}.$ The electric field is in the $+x$-direction at points

above

the disk and in the $-x$-direction at points below the disk, and the factor $\dfrac{|x|}{x}\hat{\boldsymbol{i}}$ specifies these directions.

(c) Note that $1/\sqrt{(R_1/x)^2+1}=\dfrac{|x|}{R_1}(1+(x/R_1)^2)^{-1/2}\approx\dfrac{|x|}{R_1}.$ This gives

$\vec{\boldsymbol{E}}(x)=\dfrac{\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)\dfrac{|x|^2}{x}\hat{\boldsymbol{i}}=\dfrac{\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)x\hat{\boldsymbol{i}}.$ Sufficiently close means that $(x/R_1)^2\ll 1.$

(d) $F_x=-qE_x=-\dfrac{q\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)x.$ The force is in the form of Hooke's law: $F_x=-kx,$ with

$$k=\frac{q\sigma}{2\epsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right).\quad f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\epsilon_0 m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}.$$

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.

21.88. **IDENTIFY:** Apply constant acceleration equations to a drop to find the acceleration. Then use $F=ma$ to find the force and $F=|q|E$ to find $|q|$.

SET UP: Let $D=2.0$ cm be the horizontal distance the drop travels and $d=0.30$ mm be its vertical displacement. Let $+x$ be horizontal and in the direction from the nozzle toward the paper and let $+y$ be vertical, in the direction of the deflection of the drop. $a_x=0$ and call $a_y=a.$

EXECUTE: **(a)** Find the time of flight: $t=D/v=(0.020\ \text{m})/(50\ \text{m/s})=4.00\times10^{-4}$ s. $d=\dfrac{1}{2}at^2.$

$a=\dfrac{2d}{t^2}=\dfrac{2(3.00\times10^{-4}\ \text{m})}{(4.00\times10^{-4}\ \text{s})^2}=3750\ \text{m/s}^2.$ Then $a=F/m=qE/m$ gives

$q=ma/E=\dfrac{(1.4\times10^{-11}\ \text{kg})(3750\ \text{m/s}^2)}{8.00\times10^4\ \text{N/C}}=6.56\times10^{-13}$ C, which rounds to 6.6×10^{-13} s.

(b) Use the equations and calculations above: if $v \rightarrow v/2$, then $t \rightarrow 2t$, so $a \rightarrow a/4$, which means that $q \rightarrow q/4$, so $q = (6.56\times10^{-13}\text{ s})/4 = 1.64\times10^{-13}$ s, which rounds to 1.6×10^{-13} s.

EVALUATE: Since q is positive the vertical deflection is in the direction of the electric field.

21.89 **IDENTIFY:** The net force on the third sphere is the vector sum of the forces due to the other two charges. Coulomb's law gives the forces.

SET UP: $F = k\dfrac{|q_1 q_2|}{r^2}$.

EXECUTE: **(a)** Between the two fixed charges, the electric forces on the third sphere q_3 are in opposite directions and have magnitude 4.50 N in the $+x$-direction. Applying Coulomb's law gives
$4.50\text{ N} = k[q_1(4.00\ \mu\text{C})/(0.200\text{ m})^2 - q_2(4.00\ \mu\text{C})/(0.200\text{ m})^2]$.
Simplifying gives $q_1 - q_2 = 5.00\ \mu$C.
With q_3 at $x = +0.600$ m, the electric forces on q_3 are all in the $+x$-direction and add to 3.50 N. As before, Coulomb's law gives
$3.50\text{ N} = k[q_1(4.00\ \mu\text{C})/(0.600\text{ m})^2 + q_2(4.00\ \mu\text{C})/(0.200\text{ m})^2]$.
Simplifying gives $q_1 + 9q_2 = 35.0\ \mu$C.
Solving the two equations simultaneously gives $q_1 = 8.00\ \mu$C and $q_2 = 3.00\ \mu$C.
(b) Both forces on q_3 are in the $-x$-direction, so their magnitudes add. Factoring out common factors and using the values for q_1 and q_2 we just found, Coulomb's law gives
$F_{\text{net}} = kq_3\,[q_1/(0.200\text{ m})^2 + q_2/(0.600\text{ m})^2]$.
$F_{\text{net}} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,[(8.00\ \mu\text{C})/(0.200\text{ m})^2 + (3.00\ \mu\text{C})/(0.600\text{ m})^2] = 7.49$ N, and it is in the $-x$-direction.
(c) The forces are in opposite direction and add to zero, so
$0 = kq_1q_3/x^2 - kq_2q_3/(0.400\text{ m} - x)^2$.
$(0.400\text{ m} - x)^2 = (q_2/q_1)x^2$.
Taking square roots of both sides gives
$0.400\text{ m} - x = \pm x\sqrt{q_2/q_1} = \pm 0.6124x$.

Solving for x, we get two values: $x = 0.248$ m and $x = 1.03$ m. The charge q_3 must be between the other two charges for the forces on it to balance. Only the first value is between the two charges, so it is the correct one: $x = 0.248$ m.

EVALUATE: Check the answers in part (a) by substituting these values back into the original equations. $8.00\ \mu\text{C} - 3.00\ \mu\text{C} = 5.00\ \mu$C and $8.00\ \mu\text{C} + 9(3.00\ \mu\text{C}) = 35.0\ \mu$C, so the answers check in both equations. In part (c), the second root, $x = 1.03$ m, has some meaning. The condition we imposed to solve the problem was that the magnitudes of the two forces were equal. This happens at $x = 0.248$ mn, but it also happens at $x = 1.03$ m. However at the second root the forces are both in the $+x$-direction and therefore cannot cancel.

21.90. **IDENTIFY** and **SET UP:** The electric field E_x produced by a uniform ring of charge, for points on an axis perpendicular to the plane of the ring at its center, is $E_x = \dfrac{kQx}{(x^2+a^2)^{3/2}}$, where a is the radius of the ring, x is the distance from its center along the axis, and Q is the total charge on the ring.

EXECUTE: **(a)** Far from the ring, at large values of x, the ring can be considered as a point-charge, so its electric field would be $E = kQ/x^2$. Therefore $Ex^2 = kQ$, which is a constant. From the graph (a) in the problem, we read off that at large distances $Ex^2 = 45\ \text{N}\cdot\text{m}^2/\text{C}$, which is equal to kQ, so we have
$Q = (45\ \text{N}\cdot\text{m}^2/\text{C})/k = 5.0\times10^{-9}\text{ C} = 5.0$ nC.

(b) The electric field along the axis a distance x from the ring is $E_x = \dfrac{kQx}{(x^2+a^2)^{3/2}}$. Very close to the ring, $x^2 << a^2$, so the formula becomes $Ex = kQx/a^3$. Therefore $E/x = kQ/a^3$, which is a constant. From graph (b) in the problem, E/x approaches 700 N/C·m as x approaches zero. So $kQ/a^3 = 700$ N/C·m, which gives

$a = [kQ/(700\ \text{N/C}\cdot\text{m})]^{1/3} = [(45\ \text{N}\cdot\text{m}^2/\text{C})/(700\ \text{N/C}\cdot\text{m})]^{1/3} = 0.40\ \text{m} = 40\ \text{cm}.$

EVALUATE: It is physically reasonable that a ring 40 cm in radius could carry 5.0 nC of charge.

21.91. **IDENTIFY:** An infinite positively charged sheet has a round hole cut in it. A particle with mass m and charge $-q$ is dropped from the center of the hole and falls. The net force on it is the electric force upward and gravity downward.

SET UP and EXECUTE: **(a)** We can think of the sheet with the hole as a combination of an infinite positive sheet and a negative disk of radius R, both having the same magnitude charge density σ, so the net field E_z is $E_z = E_{\text{sheet}} + E_{\text{disk}}$. Calling $+z$ upward, the net field a distance Δ below the center of the hole is $E_z = -\dfrac{\sigma}{2\epsilon_0} + \dfrac{\sigma}{2\epsilon_0}\left[1 - \dfrac{1}{\sqrt{1+(R/\Delta)^2}}\right] = -\dfrac{\sigma}{2\epsilon_0}\left[\dfrac{1}{\sqrt{1+(R/\Delta)^2}}\right]$, and it points downward.

(b) As the particle falls the net electric field *increases* because the field of the negative hole decreases while that of the positive sheet increases. The maximum field occurs very far from the hole, and that field is just due to the sheet, which is $E_z = -\dfrac{\sigma}{2\epsilon_0}$. The maximum mass for which the particle will not fall indefinitely is given by $F_{\text{el}} = m_{\text{max}}g$, so $qE_z = m_{\text{max}}g$, and $\dfrac{q\sigma}{2\epsilon_0} = m_{\text{max}}g$. Thus $m_{\text{max}} = \dfrac{q\sigma}{2g\epsilon_0}$.

(c) We want the work done by the electric field as the particle drops from $z = 0$ to $z = -\Delta$ if $m < m_{\text{max}}$. The force on the particle is upward but its displacement is downward, so the work is negative. Therefore $W_{\text{E}} = -\int qE_z dz = -\int_0^{-\Delta} \dfrac{q\sigma}{2\epsilon_0\sqrt{1+(R/z)^2}}dz = -\dfrac{q\sigma}{2\epsilon_0}\int_0^{-\Delta}\dfrac{zdz}{\sqrt{z^2+R^2}}$. The integration can be done using tables or by letting $u = z^2 + R^2$ and $du = 2zdz$. Either way, the result is

$W_{\text{E}} = -\dfrac{q\sigma}{2\epsilon_0}\sqrt{z^2+R^2}\Big|_0^{-\Delta} = -\dfrac{q\sigma}{2\epsilon_0}\left(\sqrt{\Delta^2+R^2} - R\right).$

(d) Gravity is constant, so $W_{\text{g}} = +mg\Delta$.

(e) We want the depth to which the particle will fall. The work-energy theorem $W_{\text{tot}} = K_2 - K_1$ gives $W_{\text{g}} + W_{\text{E}} = K_2 - K_1$. $K_1 = 0$, so we have $mg\Delta - \dfrac{q\sigma}{2\epsilon_0}\left(\sqrt{\Delta^2+R^2} - R\right) = \dfrac{1}{2}mv^2$. At the maximum depth the particle remains at rest, so solve this result for Δ when $v = 0$ and call the result Δ_{max}. Doing so we have $mg\Delta - \dfrac{q\sigma}{2\epsilon_0}\left(\sqrt{\Delta^2+R^2} - R\right) = 0$. So $\Delta_{\text{max}} = \dfrac{mgq\sigma R}{\epsilon_0\left[(q\sigma/2\epsilon_0)^2 - (mg)^2\right]}$. Finally we express this result in terms of $\alpha = m/m_{\text{max}}$. Using $m_{\text{max}}g = \dfrac{q\sigma}{2\epsilon_0}$ gives $\alpha = \dfrac{m}{\dfrac{q\sigma}{2g\epsilon_0}} = \dfrac{2g\epsilon_0 m}{q\sigma}$. Thus $mg = \dfrac{q\sigma\alpha}{2\epsilon_0}$.

Putting this into our result for Δ_{max} gives $\Delta_{\text{max}} = \dfrac{mgq\sigma R}{\epsilon_0\left[(mg/\alpha)^2 - (mg)^2\right]}$. Substituting $mg = \dfrac{q\sigma\alpha}{2\epsilon_0}$ and simplifying gives $\Delta_{\text{max}} = \dfrac{2R\alpha}{1-\alpha^2}$.

(f) We want to find v as a function of Δ if $\Delta < \Delta_{\text{max}}$. Solve $mg\Delta - \dfrac{q\sigma}{2\epsilon_0}\left(\sqrt{\Delta^2 + R^2} - R\right) = \dfrac{1}{2}mv^2$ for v.

Using $m_{\text{max}}g = \dfrac{q\sigma}{2\epsilon_0}$ gives $mg\Delta - m_{\text{max}}g\left(\sqrt{\Delta^2 + R^2} - R\right) = \dfrac{1}{2}mv^2$. Dividing by m and using

$\alpha = m/m_{\text{max}}$ gives $v^2 = 2g\Delta - \dfrac{2g}{\alpha}\left(\sqrt{\Delta^2 + R^2} - R\right)$, so $v = \sqrt{2g\left(\Delta - \dfrac{\sqrt{\Delta^2 + R^2} - R}{\alpha}\right)}$.

(g) We want m_{max} and Δ_{max}. Using the given quantities, with $\sigma = 1.00\ \text{nC/cm}^2 = 1.00\times10^{-5}\ \text{C/m}^2$, the results from parts (b) and (e) give $m_{\text{max}} = 57.7$ g and $\Delta_{\text{max}} = 10.7$ cm.

EVALUATE: It may seem strange that the electric field gets *stronger* as we get farther from the sheet. But the closer we get to the sheet, the greater the effect of the hole. As we get farther from the sheet the effect of the hole diminishes.

21.92. IDENTIFY: The charges at the ends of the stationary rod exert electric forces on the charges at the ends of the second rod that is free to rotate. These forces produce torques, which cause the second rod to spin about its center.

SET UP: The magnitude of the electric force is $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1q_2|}{r^2}$.

EXECUTE: (a) We want the force $\vec{F}_1$ on the upper right charge of the movable rod due to the charge on the left side of the fixed rod. Fig. 21.92a shows this arrangement and the force $\vec{F}_1$. Use the law of cosines to find the distance r_1. $r_1^2 = (L/2)^2 + (L/2)^2 - 2(L/2)^2\cos(180° - \theta)$. This gives

$r_1^2 = (L^2/2)(1+\cos\theta)$, so $F_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1q_2|}{r^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{(L^2/2)(1+\cos\theta)} = \dfrac{2}{4\pi\epsilon_0}\dfrac{Q^2}{L^2(1+\cos\theta)}$. Using Fig. 21.92c, we see that the components of $\vec{F}_1$ are $F_{1x} = F_1\cos\phi = F_1\cos(\theta/2)$ and

$F_{1y} = F_1\sin\phi = F_1\sin(\theta/2)$. Therefore $\vec{F}_1 = \dfrac{Q^2}{2\pi\epsilon_0 L^2(1+\cos\theta)}\left[\cos(\theta/2)\hat{\boldsymbol{i}} + \sin(\theta/2)\hat{\boldsymbol{j}}\right]$.

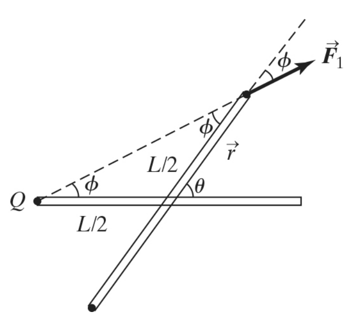


Figure 21.92a

(b) We want the force $\vec{F}_2$ on the upper right charge of the movable rod due to the charge on the right side of the fixed rod. Fig. 21.92b shows this arrangement and for force $\vec{F}_2$. The law of cosines gives r_2:

$r_2^2 = (L/2)^2 + (L/2)^2 - 2(L/2)^2\cos\theta = (L^2/2)(1-\cos\theta)$. Coulomb's law gives $F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1q_2|}{r^2}$

$= \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{(L^2/2)(1-\cos\theta)} = \dfrac{2}{4\pi\epsilon_0}\dfrac{Q^2}{L^2(1-\cos\theta)}$. Using Fig. 21.92d, we see that $2\phi + \theta = 180°$, so

$\phi = 180° - \theta/2$. Therefore the components of $\vec{F}_2$ are

$F_{2x} = -F_2 \cos\phi = -F_2 \cos(90° - \theta/2) = -F_2 \sin(\theta/2)$ and

$F_{2y} = F_2 \sin\phi = -F_2 \sin(90° - \theta/2) = F_2 \cos(\theta/2)$. In vector form, the force is

$$\vec{F}_2 = \frac{Q^2}{2\pi\epsilon_0 L^2(1-\cos\theta)}\left[-\sin(\theta/2)\hat{\boldsymbol{i}} + \cos(\theta/2)\hat{\boldsymbol{j}}\right].$$

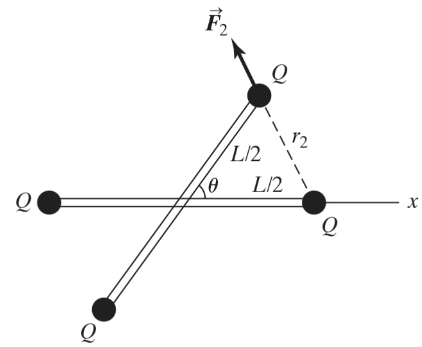


Figure 21.92b

(c) We want the torque due to $\vec{F}_1$. Use $\tau = rF\sin\phi$ for the quantities shown in Fig. 21.92c. $r = L/2$ and extends from the center of the rods to the upper end of the movable rod. From the triangle in the figure, we see that $2\phi + (180° - \theta) = 180°$, so $\phi = \theta/2$. Using $\tau = rF\sin\phi$, we have

$$\tau_1 = \frac{L}{2}\left(\frac{2}{4\pi\epsilon_0}\frac{Q^2}{L^2(1+\cos\theta)}\right)\sin(\theta/2) = \frac{1}{4\pi\epsilon_0}\frac{Q^2\sin(\theta/2)}{L(1+\cos\theta)}, \text{ clockwise.}$$

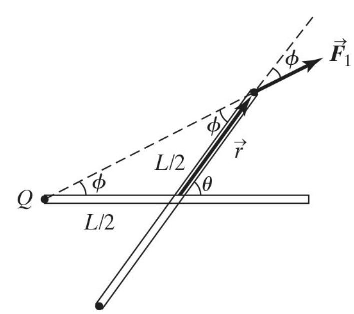


Figure 21.92c

(d) We want the torque due to $\vec{F}_2$. See Fig. 21.92d for the geometry. As in part (c), use $\tau = rF\sin\phi$, where $r = L/2$. From the figure, we see that $2\phi + \theta = 180°$, so $\phi = (180° - \theta)/2 = 90° - \theta/2$. So

$\sin\phi = \cos(\theta/2)$. The torque is $\tau_2 = \dfrac{L}{2}\left(\dfrac{2}{4\pi\epsilon_0}\dfrac{Q^2}{L^2(1-\cos\theta)}\right)\cos(\theta/2) = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2\cos(\theta/2)}{L(1-\cos\theta)}$,

counterclockwise.

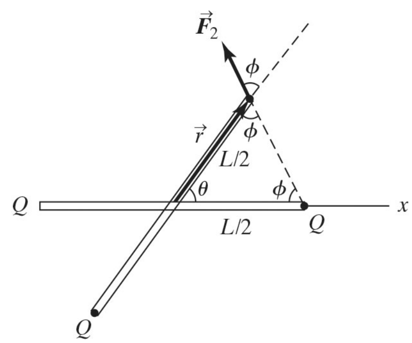


Figure 21.92d

(e) Torque τ_3 is on the lower charge of the movable rod due to the left charge on the fixed rod. This torque is the same as τ_2 and is also counterclockwise, so $\tau_3 = \tau_2$. Torque τ_4 is on the lower charge of the movable rod due to the right charge on the fixed rod. This torque is the same as τ_1 and is also clockwise, so $\tau_4 = \tau_1$. Calling counterclockwise positive, the total torque is

$\tau_{\text{tot}} = -\tau_1 + \tau_2 + \tau_3 - \tau_4 = 2\tau_2 - 2\tau_1 = 2(\tau_2 - \tau_1)$.

$$\tau_{\text{tot}} = 2\left[\frac{1}{4\pi\epsilon_0}\frac{Q^2\cos(\theta/2)}{L(1-\cos\theta)} - \frac{1}{4\pi\epsilon_0}\frac{Q^2\sin(\theta/2)}{L(1+\cos\theta)}\right] = \frac{Q^2}{2\pi\epsilon_0 L}\left[\frac{\cos(\theta/2)}{1-\cos\theta} - \frac{\sin(\theta/2)}{1+\cos\theta}\right].$$

(f) We want the net torque for small angular displacements from the equilibrium position of $\theta = \pi/2$ rad. Letting $\theta = \frac{\pi}{2} - \epsilon$, where $\epsilon \ll 1$ rad, we use $\tau_{\text{net}} = \frac{Q^2}{2\pi\epsilon_0 L}\left[\frac{\cos(\theta/2)}{1-\cos\theta} - \frac{\sin(\theta/2)}{1+\cos\theta}\right]$ with $\theta = \frac{\pi}{2} - \epsilon$ and keep only terms with powers lower than ϵ^2. Using the power series in Appendix B we have the following expansions.

$$\cos(\theta/2) \to \cos\left(\frac{\pi/2-\epsilon}{2}\right) = \cos\left(\frac{\pi}{4} - \frac{\epsilon}{2}\right) = \frac{1}{\sqrt{2}}\cos\frac{\epsilon}{2} + \frac{1}{\sqrt{2}}\sin\frac{\epsilon}{2} \approx \frac{1}{\sqrt{2}}\left(1 - \frac{\epsilon^2}{8}\right) + \frac{\epsilon}{2\sqrt{2}}.$$

$$\sin(\theta/2) \to \sin\left(\frac{\pi/2-\epsilon}{2}\right) = \sin\left(\frac{\pi}{4} - \frac{\epsilon}{2}\right) = \frac{1}{\sqrt{2}}\cos\frac{\epsilon}{2} - \frac{1}{\sqrt{2}}\sin\frac{\epsilon}{2} \approx \frac{1}{\sqrt{2}}\left(1 - \frac{\epsilon^2}{8}\right) - \frac{\epsilon}{2\sqrt{2}}.$$

Both denominators are approximately equal to 1 for small $\mathcal{E}$. Using these results for the torque gives $\tau \approx \frac{Q^2}{2\pi\epsilon_0 L}\left\{\frac{1}{\sqrt{2}}\left(1-\frac{\epsilon^2}{8}\right) + \frac{\epsilon}{2\sqrt{2}} - \left[\frac{1}{\sqrt{2}}\left(1-\frac{\epsilon^2}{8}\right) - \frac{\epsilon}{2\sqrt{2}}\right]\right\}$, which reduces to $\tau = \left(\frac{Q^2}{2\sqrt{2}\pi\epsilon_0 L}\right)\epsilon$.

(g) The torque tends to return the rod to the equilibrium position, so Newton's second law for rotation gives $I\frac{d^2\epsilon}{dt^2} = -\tau$. Using our result from part (f) gives $\frac{d^2\epsilon}{dt^2} = -\left(\frac{Q^2}{2\sqrt{2}\pi\epsilon_0 LI}\right)\epsilon$. From this we see that $\omega = \sqrt{\frac{Q^2}{2\sqrt{2}\pi\epsilon_0 LI}}$, so $f = \frac{1}{2\pi}\sqrt{\frac{Q^2}{2\sqrt{2}\pi\epsilon_0 LI}}$. In this result, I is the moment of inertia of the two small balls at the ends of the bar, which is $I = 2M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{2}$. Using this value for I gives

$$f = \frac{Q}{2\pi\sqrt{\pi\sqrt{2}\epsilon_0 ML^3}}.$$

EVALUATE: According to our result in (f), a large Q gives a high frequency while a large M or L gives a low frequency. These results are physically reasonable.

21.93. **IDENTIFY:** Two parallel rods of length L carry equal positive charge Q and are a distance a apart. We want to find the electric force that each one exerts on the other.

SET UP: We break the rods up into infinitesimal segments and integrate to find the total force that one of them exerts on the other. The force that dq_1 exerts on dq_2 is $dF = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq_1 dq_2}{r_{12}^2}$. Fig. 21.93 shows the charge elements.

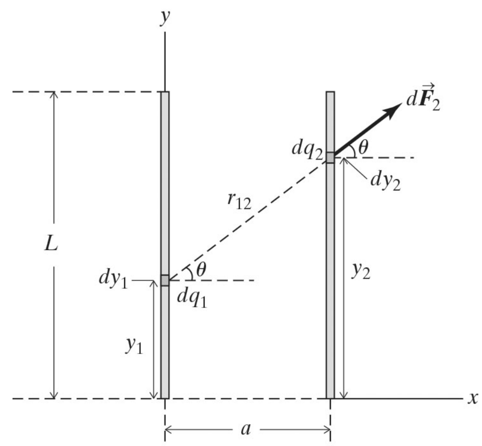


Figure 21.93

EXECUTE: (a) Because both rods have uniform charge distribution, every charge element above the midpoint of rod 1 will have an identical element below the midpoint. So the y components of the force on rod 2 cancel but the x components add.

(b) In light of the answer to part (a), we need only calculate the x components of for force on rod 2. We first find the force that dq_1 exerts on dq_2. Then we integrate over rod 1 to find the *total* force on dq_2 due rod 1. Using the geometry of the figure, we have $dF_{2x} = dF_2\cos\theta = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq_1 dq_2}{r_{12}^2}\cos\theta.$ Using

$dq_1 = \lambda dy_1$, $r_{12}^2 = (y_2 - y_1)^2 + a^2$, and $\cos\theta = \dfrac{a}{r_{12}} = \dfrac{a}{\sqrt{(y_2-y_1)^2+a^2}}$, we have

$dF_{2x} = \left(\dfrac{1}{4\pi\epsilon_0}\displaystyle\int_{y_1=0}^{L}\dfrac{\lambda dy_1\cos\theta}{(y_2-y_1)^2+a^2}\right)dq_2 = \left(\dfrac{\lambda}{4\pi\epsilon_0}\displaystyle\int_{y_1=0}^{L}\dfrac{a\,dy_1}{\left[(y_2-y_1)^2+a^2\right]^{3/2}}\right)dq_2$. The integral can be done by letting $u = y_2 - y_1$, $du = -dy_1$. It is then in the form $\displaystyle\int\dfrac{-du}{(u^2+a^2)^{3/2}} = -\dfrac{1}{a^2}\dfrac{u}{\sqrt{u^2+a^2}}$. Doing this gives us $dF_{2x} = \left(\dfrac{\lambda a}{4\pi\epsilon_0}\right)\left(-\dfrac{1}{a^2}\right)\dfrac{y_2-y_1}{\sqrt{(y_2-y_1)^2+a^2}}\Bigg|_{y_1=0}^{L} dq_2$. Evaluating at the limits and using $dq_2 = \lambda dy_2$ gives $dF_{2x} = \left(-\dfrac{\lambda}{4\pi\epsilon_0\, a}\right)\left[\dfrac{y_2-L}{\sqrt{(y_2-L)^2+a^2}} - \dfrac{y_2}{\sqrt{y_2^2+a^2}}\right]\lambda dy_2$. This result gives us the total x component of the force that rod 1 exerts on dq_2 in rod 2.

(c) We now integrate the result from part (b) over dy_2 to get the total x component of the force F_x on the full length of rod 2. $F_{2x} = \left(-\dfrac{\lambda^2}{4\pi\epsilon_0\, a}\right)\displaystyle\int_0^L\left[\dfrac{y_2-L}{\sqrt{(y_2-L)^2+a^2}} - \dfrac{y_2}{\sqrt{y_2^2+a^2}}\right]dy_2$. We do the integral in two parts. To do the first one, let $u = y_2 - L$, $du = dy_2$, so it becomes $\int \frac{u\,du}{\sqrt{u^2+a^2}} = \sqrt{u^2+a^2}$. Evaluating at the limits, this part of the integral is $a - \sqrt{L^2+a^2}$. The second part of the integral is similar to the first part, and it is equal to $\sqrt{L^2+a^2} - a$. Combining these two results gives $F_{2x} = -\frac{\lambda^2}{4\pi\epsilon_0 a}\left[a - \sqrt{L^2+a^2} - \left(\sqrt{L^2+a^2} - a\right)\right] = \frac{\lambda^2}{2\pi\epsilon_0 a}\left(\sqrt{L^2+a^2} - a\right)$. Using $\lambda = Q/L$ this becomes $F_{2x} = \frac{Q^2}{2\pi\epsilon_0 L^2 a}\left(\sqrt{L^2+a^2} - a\right) = \frac{Q^2}{2\pi\epsilon_0 L^2}\left(\sqrt{(L/a)^2+1} - 1\right)$. The force vector is $\vec{F} = \frac{Q^2}{2\pi\epsilon_0 L^2}\left(\sqrt{(L/a)^2+1} - 1\right)\hat{i}$.

(d) For $a >> L$: Use the result from part (c). $F_{2x} = \frac{Q^2}{2\pi\epsilon_0 L^2}\left(\sqrt{(L/a)^2+1} - 1\right)$. Since $a >> L$, $L/a << 1$, so we can use the approximation $\sqrt{1+x} \approx 1 + x/2$ where $x = (L/a)^2$. Doing so gives $F_{2x} \approx \frac{Q^2}{2\pi\epsilon_0 L^2}\left(1 + \frac{1}{2}(L/a)^2 - 1\right) = \frac{Q^2}{4\pi\epsilon_0 a^2}$, so $\vec{F} \approx \frac{Q^2}{4\pi\epsilon_0 a^2}\hat{i}$. This result is reasonable because the two finite lines behave as point charges if they are far enough away from each other.

(e) $W = \lim_{X\to\infty} \int_X^a F_x dx$. The work we do is the negative of the work done by the electric field. Using F_{2x} from part (c) with x replacing a in the formula, we have $W = -\frac{Q^2}{2\pi\epsilon_0 L^2}\int_X^a \frac{\sqrt{L^2+x^2} - x}{x}dx$.

$W = -\frac{Q^2}{2\pi\epsilon_0 L^2}\int_X^a \left(\frac{\sqrt{L^2+x^2}}{x} - 1\right)dx$. Using tables for the first part of the integral gives

$$W = -\frac{Q^2}{2\pi\epsilon_0 L^2}\left[\sqrt{x^2+L^2} - L\ln\left(\frac{L+\sqrt{x^2+L^2}}{x}\right) - x\right]\Bigg|_X^a$$, which gives

$$W = -\frac{Q^2}{2\pi\epsilon_0 L^2}\left\{\left[\sqrt{a^2+L^2} - L\ln\left(\frac{L+\sqrt{a^2+L^2}}{a}\right) - a\right] - \left[\sqrt{X^2+L^2} - L\ln\left(\frac{L+\sqrt{X^2+L^2}}{X}\right) - X\right]\right\}.$$

Taking the limit as $X \to \infty$ gives $W = \frac{Q^2}{2\pi\epsilon_0 L^2}\left[L\ln\left(\frac{L+\sqrt{a^2+L^2}}{a}\right) + a - \sqrt{a^2+L^2}\right]$.

(f) We want the *relative* speed of the rods when they have moved very far apart, in which case their potential energy is zero. Since the rods have equal masses, they will have equal speeds by conservation of momentum. Therefore $U(x=a) = K(x\to\infty) = K_1 + K_2 = 2K_1 = 2\left(\frac{1}{2}mv^2\right) = mv^2$. We can use the result from part (e). $\frac{Q^2}{2\pi\epsilon_0 L^2}\left[L\ln\left(\frac{L+\sqrt{a^2+L^2}}{a}\right) + a - \sqrt{a^2+L^2}\right] = mv^2$. Solving for v using $Q =$ 10.0 μC, L = 50.0 cm = 0.500 m, m = 500 g = 0.500 kg, a = 10.0 cm = 0.100 m, we get v = 10.735 m/s. This is the speed relative to the earth. The *relative speed* is the speed of one rod relative to the other. Since they are moving away from each other, their *relative* speed is 2(10.735 m/s) = 21.5 m/s.

EVALUATE: Evaluate our result in part (c) if $L \to 0$. For $x << 1$ we can use the approximation $\sqrt{1+x} \approx 1 + x/2$ where $x = (L/a)^2$. Doing so gives $F_{2x} \approx \frac{Q^2}{2\pi\epsilon_0 L^2}\left(1+\frac{1}{2}(L/a)^2 - 1\right) = \frac{Q^2}{4\pi\epsilon_0 a^2}$, so $\vec{F} \approx \frac{Q^2}{4\pi\epsilon_0 a^2}\hat{\boldsymbol{i}}$. This result is reasonable because the two rods behave like point charges if they are extremely short compared to the distance between them, so our result is reasonable. We can also check our result if the rods are extremely long ("infinite"). Using results from the text, we know that in that case $E_1 = \frac{\lambda}{2\pi\epsilon_0 r}$, so the force on rod 2 is $F_2 = E_1 Q_2 = E_1 \lambda L = \left(\frac{\lambda}{2\pi\epsilon_0 a}\right)\lambda L = \frac{\lambda^2 L}{2\pi\epsilon_0 a}$. Therefore the force per unit length on rod 2 is $\frac{F}{L} = \frac{\lambda^2}{2\pi\epsilon_0 a}$. Now look at our result $F = \frac{Q^2}{2\pi\epsilon_0 L^2}\left(\sqrt{(L/a)^2+1} - 1\right)$. $Q = \lambda L$ and for very large L the quantity in parentheses becomes L/a. Therefore the force becomes $F = \frac{(\lambda L)^2}{2\pi\epsilon_0 L^2}(L/a) = \frac{\lambda^2 L}{2\pi\epsilon_0 a}$, so the force per unit length is $\frac{F}{L} = \frac{\lambda^2}{2\pi\epsilon_0 a}$, which is what we just found. Therefore our result checks for very long rods.

21.94. IDENTIFY and SET UP: The electric field exerts a force (and hence a torque) on the charged ball at the end of the rod. This torque is always counterclockwise, so it causes the rod to increase its angular speed as the charged ball passes through the field.

EXECUTE: **(a)** From Fig. 21.94a, $r_x = a\cos\theta$ and $r_y = a\sin\theta$, so $\vec{r} = a\cos\theta\hat{\boldsymbol{i}} + a\sin\theta\hat{\boldsymbol{j}}$.

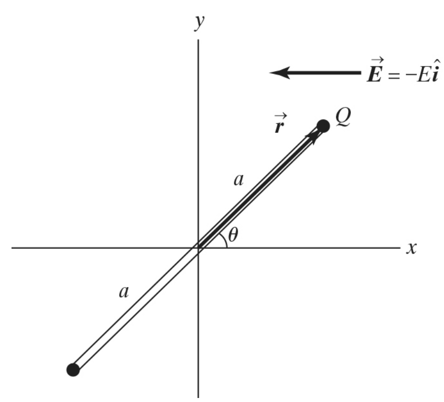


Figure 21.94a

(b) We want the torque on the rod when $0 \le \theta \le \pi$ (see Fig. 21.94b). The magnitude of the torque is $\tau = rF\sin\phi$. From the figure we see that $\phi = \pi - \theta$, so $\sin\phi = \sin(\pi - \theta) = \sin\theta$. $F = QE$ and $r = a$, so $\tau = aQE\sin\theta$. Using $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$ and the right-hand rule for the vector product, we see that the torque points along the +z-axis, so $\vec{\boldsymbol{\tau}} = aQE\sin\theta\hat{\boldsymbol{k}}$.

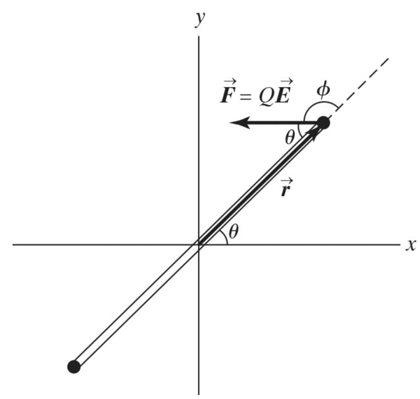


Figure 21.94b

(c) The torque is zero because the electric field is zero.
(d) $I_{\text{balls}} = 2Ma^2$, $I_{\text{rod}} = 0$ since it is very light.
(e) We want to use $\tau = -dU/d\theta$ to find the potential energy $U(\theta)$. Integrating and letting $U_0 = 0$ when $\theta = 0$ gives $U - U_0 = -\int \tau d\theta$.

$\underline{0 \le \theta \le \pi}$: $U - U_0 = U = -\int_0^\theta aQE\sin\theta d\theta = aQE\cos\theta\Big|_0^\theta = aQE(\cos\theta - 1)$.

$\underline{\pi \le \theta \le 2\pi}$: U = constant = $U(\pi) = -2aQE$.

$\underline{2\pi \le \theta \le 3\pi}$: $U - U_0 = -\int_{2\pi}^\theta aQE\sin\theta d\theta = aQE\cos\theta\Big|_{2\pi}^\theta = aQE(\cos\theta - 1)$, which gives $U = aQE(\cos\theta - 1) + (-2aQE) = aQE(\cos\theta - 3)$.

$\underline{3\pi \le \theta \le 4\pi}$: U = constant = $U(3\pi) = -4aQE$.

(f) Calling $\in$ the total energy, we have $\in = \frac{1}{2}I\omega^2 + U(\theta)$. We also know that $\tau = I\alpha = I\frac{d^2\theta}{dt^2} = I\frac{d\omega}{dt}$.

Combining this result with $\tau = -dU/d\theta$ gives $\frac{d\omega}{dt} = -\frac{1}{I}\frac{dU}{d\theta}$. Now look at the energy:

$\frac{d\in}{dt} = I\omega\frac{d\omega}{dt} + \frac{dU}{dt}$. The chain rule gives $\frac{dU}{dt} = \frac{dU}{d\theta}\frac{d\theta}{dt} = \omega\frac{dU}{d\theta}$. Using this and our result for $d\omega/dt$ gives $\frac{d\in}{dt} = I\omega\left(-\frac{1}{I}\frac{dU}{d\theta}\right) + \omega\frac{dU}{d\theta} = -\omega\frac{dU}{d\theta} + \omega\frac{dU}{d\theta} = 0$. Thus the energy is constant and therefore conserved.

(g) We want the angular speed ω at the n^{th} time that Q crosses the $-y$-axis. Doing calculations similar to those in part (e) gives us the following results:
At the first crossing: $\pi \le \theta \le 2\pi$: $U_1 = -2aQE$
At the second crossing: $3\pi \le \theta \le 4\pi$: $U_2 = -4aQE$
At the third crossing: $5\pi \le \theta \le 6\pi$: $U_3 = -6aQE$
At the fourth crossing: $7\pi \le \theta \le 8\pi$: $U_4 = -8aQE$

We can see that the pattern is $U_n = -2naQE$ (n = 1, 2, 3, …). The total energy is $\in = \frac{1}{2}I\omega^2 + U(\theta)$.

Solving for ω gives $\omega = \sqrt{\frac{2(\in - U)}{I}}$. At the n^{th} crossing, we found that $U_n = -2naQE$, so

$\omega_n = \sqrt{\dfrac{2(\epsilon - U_n)}{I}} = \sqrt{\dfrac{2(\epsilon + 2naQE)}{2Ma^2}} = \sqrt{\dfrac{\epsilon + 2naQE}{Ma^2}}$. But $\epsilon = K + U$ and $K_0 = 0$ and $U_0 = 0$, so $\epsilon = 0$.

Therefore $\omega_n = \sqrt{\dfrac{2nQE}{Ma}}$ $(n = 1, 2, 3, \cdots)$.

EVALUATE: The total energy is zero, but that does not mean that the rod stops moving. Remember that the potential energy is negative which cancels out the positive kinetic energy.

21.95. IDENTIFY: Apply Coulomb's law to calculate the forces that q_1 and q_2 exert on q_3, and add these force vectors to get the net force.

SET UP: Like charges repel and unlike charges attract. Let $+x$ be to the right and $+y$ be toward the top of the page.

EXECUTE: **(a)** The four possible force diagrams are sketched in Figure 21.95a.
Only the last picture can result in a net force in the $-x$-direction.

(b) $q_1 = -2.00\ \mu\text{C}, q_3 = +4.00\ \mu\text{C}$, and $q_2 > 0$.

(c) The forces $\vec{F}_1$ and $\vec{F}_2$ and their components are sketched in Figure 21.95b.

$F_y = 0 = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1||q_3|}{(0.0400\text{ m})^2}\sin\theta_1 + \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2||q_3|}{(0.0300\text{ m})^2}\sin\theta_2$. This gives

$q_2 = \dfrac{9}{16}|q_1|\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{9}{16}|q_1|\dfrac{3/5}{4/5} = \dfrac{27}{64}|q_1| = 0.843\ \mu\text{C}.$

(d) $F_x = F_{1x} + F_{2x}$ and $F_y = 0$, so $F = |q_3|\dfrac{1}{4\pi\epsilon_0}\left(\dfrac{|q_1|}{(0.0400\text{ m})^2}\dfrac{4}{5} + \dfrac{|q_2|}{(0.0300\text{ m})^2}\dfrac{3}{5}\right) = 56.2\text{ N}.$

EVALUATE: The net force $\vec{F}$ on q_3 is in the same direction as the resultant electric field at the location of q_3 due to q_1 and q_2.

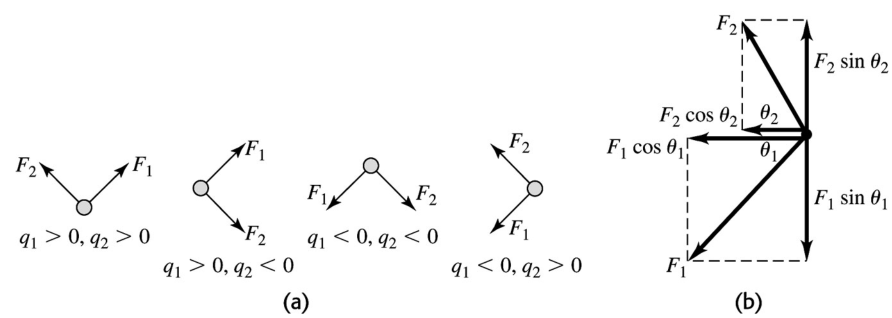


Figure 21.95

21.96. IDENTIFY: Calculate the electric field at *P* due to each charge and add these field vectors to get the net field.

SET UP: The electric field of a point charge is directed away from a positive charge and toward a negative charge. Let $+x$ be to the right and let $+y$ be toward the top of the page.

EXECUTE: **(a)** The four possible diagrams are sketched in Figure 21.96(a).
The first diagram is the only one in which the electric field must point in the negative *y*-direction.

(b) $q_1 = -3.00\ \mu\text{C}$, and $q_2 < 0$.

(c) The electric fields $\vec{E}_1$ and $\vec{E}_2$ and their components are sketched in Figure 21.96(b). $\cos\theta_1 = \dfrac{5}{13}$, $\sin\theta_1 = \dfrac{12}{13}$, $\cos\theta_2 = \dfrac{12}{13}$ and $\sin\theta_2 = \dfrac{5}{13}$. $E_x = 0 = -\dfrac{k|q_1|}{(0.050\text{ m})^2}\dfrac{5}{13} + \dfrac{k|q_2|}{(0.120\text{ m})^2}\dfrac{12}{13}$. This gives

$\dfrac{k|q_2|}{(0.120\text{ m})^2}=\dfrac{k|q_1|}{(0.050\text{ m})^2}\dfrac{5}{12}$. Solving for $|q_2|$ gives $|q_2|=7.2\ \mu\text{C}$, so $q_2=-7.2\ \mu\text{C}$. Then

$E_y=-\dfrac{k|q_1|}{(0.050\text{ m})^2}\dfrac{12}{13}-\dfrac{kq_2}{(0.120\text{ m})^2}\dfrac{5}{13}=-1.17\times10^7\text{ N/C}$. $E=1.17\times10^7\text{ N/C}$.

EVALUATE: With q_1 known, specifying the direction of $\vec{E}$ determines both q_2 and E.

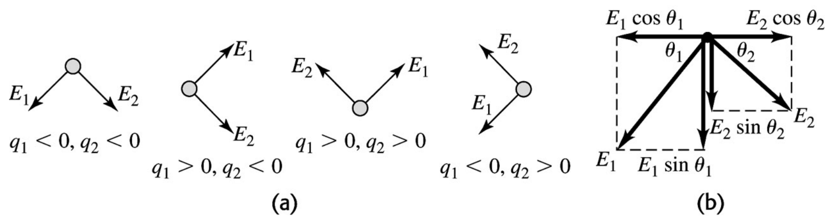


Figure 21.96

21.97. IDENTIFY: To find the electric field due to the second rod, divide that rod into infinitesimal segments of length dx, calculate the field dE due to each segment and integrate over the length of the rod to find the total field due to the rod. Use $d\vec{F}=dq\,\vec{E}$ to find the force the electric field of the second rod exerts on each infinitesimal segment of the first rod.

SET UP: An infinitesimal segment of the second rod is sketched in Figure 21.97. $dQ=(Q/L)dx'$.

EXECUTE: **(a)** $dE=\dfrac{k\,dQ}{(x+a/2+L-x')^2}=\dfrac{kQ}{L}\dfrac{dx'}{(x+a/2+L-x')^2}$.

$$E_x=\int_0^L dE_x=\frac{kQ}{L}\int_0^L\frac{dx'}{(x+a/2+L-x')^2}=\frac{kQ}{L}\left[\frac{1}{x+a/2+L-x'}\right]_0^L=\frac{kQ}{L}\left(\frac{1}{x+a/2}-\frac{1}{x+a/2+L}\right).$$

$$E_x=\frac{2kQ}{L}\left(\frac{1}{2x+a}-\frac{1}{2L+2x+a}\right).$$

(b) Now consider the force that the field of the second rod exerts on an infinitesimal segment dq of the first rod. This force is in the $+x$-direction. $dF=dq\,E$.

$$F=\int E\,dq=\int_{a/2}^{L+a/2}\frac{EQ}{L}\,dx=\frac{2kQ^2}{L^2}\int_{a/2}^{L+a/2}\left(\frac{1}{2x+a}-\frac{1}{2L+2x+a}\right)dx.$$

$$F=\frac{2kQ^2}{L^2}\frac{1}{2}\left(\left[\ln\,(a+2x)\right]_{a/2}^{L+a/2}-\left[\ln(2L+2x+a)\right]_{a/2}^{L+a/2}\right)=\frac{kQ^2}{L^2}\ln\left[\left(\frac{a+2L+a}{2a}\right)\left(\frac{2L+2a}{4L+2a}\right)\right].$$

$$F=\frac{kQ^2}{L^2}\ln\left(\frac{(a+L)^2}{a(a+2L)}\right).$$

(c) For $a\gg L$, $F=\dfrac{kQ^2}{L^2}\ln\left(\dfrac{a^2(1+L/a)^2}{a^2(1+2L/a)}\right)=\dfrac{kQ^2}{L^2}(2\ln\,(1+L/a)-\ln(1+2L/a))$.

For small z, $\ln(1+z)\approx z-\dfrac{z^2}{2}$. Therefore, for $a\gg L$,

$$F\approx\frac{kQ^2}{L^2}\left[2\left(\frac{L}{a}-\frac{L^2}{2a^2}+\cdots\right)-\left(\frac{2L}{a}-\frac{2L^2}{a^2}+\cdots\right)\right]\approx\frac{kQ^2}{a^2}.$$

EVALUATE: The distance between adjacent ends of the rods is a. When $a\gg L$ the distance between the rods is much greater than their lengths and they interact as point charges.

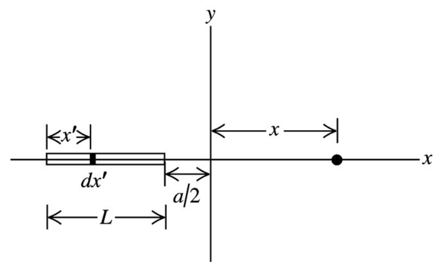


Figure 21.97

21.98. **IDENTIFY** and **SET UP:** The charge of n electrons is ne.
EXECUTE: The charge on the bee is $Q = ne$, so the number of missing electrons is
$n = Q/e = (30 \text{ pC})/e = (30 \times 10^{-12} \text{ C})/(1.60 \times 10^{-19} \text{ C}) = 1.88 \times 10^8 \approx 1.9 \times 10^8$ electrons, which makes choice (a) correct.
EVALUATE: This charge is due to around 190 million electrons.

21.99. **IDENTIFY** and **SET UP:** One charge exerts a force on another charge without being in contact.
EXECUTE: Even though the bee does not touch the stem, the positive charges on the bee attract negative charges (electrons normally) in the stem. This pulls electrons toward the bee, leaving positive charge at the opposite end of the stem, which polarizes it. Thus choice (c) is correct.
EVALUATE: Choice (b) cannot be correct because the bee is positive and would therefore not attract the positive charges in the stem.

21.100. **IDENTIFY** and **SET UP:** Electric field lines begin on positive charges and end on negative charges.
EXECUTE: The flower and bee are both positive, so no field lines can end on either of them. This makes the figure in choice (c) the correct one.
EVALUATE: The net electric field is the vector sum of the field due to the bee and the field due to the flower. Somewhere between the bee and flower the fields cancel, depending on the relative amounts of charge on the bee and flower.

21.101. **IDENTIFY** and **SET UP:** Assume that the charge remains at the end of the stem and that the bees approach to 15 cm from this end of the stem. The electric field is $E = k\dfrac{|q|}{r^2}$.
EXECUTE: Using the numbers given, we have
$E = k\dfrac{|q|}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(40 \times 10^{-12} \text{ C})/(0.15 \text{ m})^2 = 16 \text{ N/C}$, which is choice (b).
EVALUATE: Even if the charge spread out a bit over the stem, the result would be in the neighborhood of the value we calculated.

22

GAUSS'S LAW

VP22.4.1. **IDENTIFY:** We want to find the electric flux through the surface of a cube.

SET UP: $\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot\hat{n}A$, $\vec{E} = E_x\hat{i} + E_y\hat{j}$. $\hat{n}$ is the outward normal to the surface. Use $\Phi_E = E_xA_x + E_yA_y + E_zA_z$.

EXECUTE: **(a)** $\hat{n} = -\hat{i}$, so $A_x = -L^2$, $A_y = A_z = 0$. $\Phi_E = E_xA_x + E_yA_y + E_zA_z = E_1(-L^2) = -E_1L^2$.

(b) $\hat{n} = +\hat{i}$, so $\Phi_E = E_1A = E_1L^2$.

(c) $\hat{n} = -\hat{j}$, so $\Phi_E = E_2(-L^2) = -E_2L^2$.

(d) $\hat{n} = +\hat{j}$, so $\Phi_E = E_2A = E_2L^2$.

(e) $E_z = 0$, so $\Phi_E = 0$.

(f) $E_z = 0$, so $\Phi_E = 0$.

(g) $\Phi_E = -E_1L^2 + E_1L^2 - E_2L^2 + E_2L^2 = 0$.

EVALUATE: Every electric field line that enters the cube also goes out of the cube, so the total flux is zero.

VP22.4.2. **IDENTIFY:** This problem involves the electric flux through a hemisphere and Gauss's law.

SET UP: If the spherical surface were *not* but in half, half the flux would go through the upper half and half the flux would go through the lower half. Since the charge q is at the center of the sphere, each of these would be $\frac{1}{2}\left(\frac{q}{\epsilon_0}\right)$ by Gauss's law due to the symmetry of the electric field. Our target variable is the flux.

EXECUTE: **(a)** $\Phi_E = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right) = \frac{1}{2}\left(\frac{8.00\text{ nC}}{\epsilon_0}\right) = +452\text{ N}\cdot\text{m}^2/\text{C}.$

(b) $\Phi_E = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right) = \frac{1}{2}\left(\frac{-4.00\text{ nC}}{\epsilon_0}\right) = -226\text{ N}\cdot\text{m}^2/\text{C}.$

(c) The charge is the same as in part (b), so the answer is the same: $\Phi_E = -226\text{ N}\cdot\text{m}^2/\text{C}$.

EVALUATE: The flux depends only on the charge inside the surface, not on the size of the surface.

VP22.4.3. **IDENTIFY:** This problem involves the electric flux through a cube and Gauss's law.

SET UP: Use $\Phi_E = \dfrac{q}{\epsilon_0}$.

EXECUTE: **(a)** We want the flux. $\Phi_E = \dfrac{q}{\epsilon_0} = \dfrac{+6.00\text{ nC}}{\epsilon_0} = +678\,\text{N}\cdot\text{m}^2/\text{C}$.

(b) The charge is at the center of the cube, so the electric field is symmetric in all directions from the center. Therefore the flux through each of the six faces of the cube is the same.

$\Phi_E = \dfrac{1}{6}(+678\,\text{N}\cdot\text{m}^2/\text{C}) = +113\text{ N}\cdot\text{m}^2/\text{C}$.

EVALUATE: If the charge were inside the cube but not at its center, the answer to part (a) would be the same but the answer to (b) would be different because a different flux would pass through each face of the cube.

VP22.4.4. **IDENTIFY:** This problem involves the electric flux through a sphere and Gauss's law.

SET UP: Use $\Phi_E = \dfrac{q}{\epsilon_0}$. Only charge *within* the sphere contributes to the flux through the surface of the sphere.

EXECUTE: **(a)** q_1 is within the sphere, so $\Phi_E = \dfrac{q_1}{\epsilon_0} = \dfrac{+3.00\text{ nC}}{\epsilon_0} = +339\,\text{N}\cdot\text{m}^2/\text{C}$.

(b) q_2 is also within the sphere, so $q = q_1 + q_2$. $\Phi_E = \dfrac{q_1+q_2}{\epsilon_0} = \dfrac{+3.00\text{ nC} - 8.00\text{ nC}}{\epsilon_0} = -565\,\text{N}\cdot\text{m}^2/\text{C}$.

(c) The distance of q_3 from the origin is $r = \sqrt{(4.00\text{ cm})^2 + (-2.00\text{ cm})^2 + (3.00\text{ cm})^2} = 5.39\text{ cm}$, so q_3 is *not* inside the sphere. Therefore the charge within the sphere is –5.00 nC, as in part (b), so the flux is also the same: $-565\,\text{N}\cdot\text{m}^2/\text{C}$.

EVALUATE: When computing the flux, it doesn't matter *where* the charges are within the sphere, just so they are inside of it.

VP22.10.1. **IDENTIFY:** This problem involves the electric field due to a sphere of charge for points inside and outside the sphere. In one case, the charge is an insulator and in the other case it is a conductor. In each case, the target variable is the magnitude of the electric field.

SET UP and **EXECUTE:** **(a)** Solid insulator. At $r = 4.00$ cm: The point is inside the sphere. From Example 22.9 we know that $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qr}{R^3} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qr}{R^3} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(3.00\text{ nC})(0.0400\text{ m})}{(0.0500\text{ m})^3} = 8630\text{ N/C}$.

At $r = 6.00$ cm: The point is outside the sphere, so the electric field of the sphere is the same as that of a point charge at its center. $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{3.00\text{ nC}}{(0.0600\text{ m})^2} = 7490\text{ N/C}$.

(b) Solid conductor. At $r = 4.00$ cm: All the excess charge of the conductor is on its surface, so $E = 0$ inside the conductor.

At $r = 6.00$ cm: The field is the same as that of a point charge at the center, so $E = 7490$ N/C.

EVALUATE: A uniform sphere of charge behaves like a point charge only for points *outside* the sphere.

VP22.10.2. **IDENTIFY:** We want the electric force on a point charge close to a very long charged wire.

SET UP: $F = qE$ where $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$.

EXECUTE: $F = qE = \dfrac{q\lambda}{2\pi\epsilon_0 r} = \dfrac{(4.00\text{ nC})(3.00\text{ nC/m})}{2\pi\epsilon_0(0.0900\text{ m})} = 2.40\times10^{-6}\text{ N} = 2.40\ \mu\text{N}$. The wire is positive and the point charge is negative, so the wire attracts the point charge.

EVALUATE: Note that the electric field due to the wire is not an inverse square law.

VP22.10.3. **IDENTIFY:** This problem involves the electric fields due to a point charge and an infinite sheet of charge.

SET UP: Sheet: $\sigma/2\epsilon_0$, point charge: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$. The charge is 8.00 cm from the sheet and $E = 0$ midway between the charge and sheet. The target variable is the surface charge density σ on the sheet.

EXECUTE: At 4.00 cm from the point charge, the fields cancel, so $E_q = E_{\text{sheet}}$. Equate their magnitudes: $\dfrac{\sigma}{2\epsilon_0} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$. Solve for σ: $\sigma = \dfrac{q}{2\pi r^2} = \dfrac{1.80\text{ nC}}{2\pi(0.0400\text{ m})^2} = 179\text{ nC/m}^2$. The point charge is positive, so its electric field points toward the sheet at the point in question, so the field due to the sheet must point away from the sheet. Therefore the sheet must be positively charged.

EVALUATE: The field due to the sheet is independent of distance from the sheet. So if q were negative, the total field would cancel 4.00 cm from the point charge in a direction away from the sheet.

VP22.10.4. **IDENTIFY:** A point charge is located between two large conducting plates. We know the force on this charge and want to find the surface charge density on each sheet.

SET UP: The field due to the plates is $E = \sigma/\epsilon_0$ and the force on q is $F = qE$.

EXECUTE: $F = qE = q(\sigma/\epsilon_0)$. Solve for σ:

$$\sigma = \frac{\epsilon_0 F}{q} = \frac{\epsilon_0(22.0\ \mu\text{N})}{3.60\text{ nC}} = 5.41\times10^{-8}\text{ C/m}^2 = 54.1\text{ nC/m}^2.$$

EVALUATE: The plates have equal charge densities, so σ is the density on each plate.

VP22.12.1. **IDENTIFY:** A charge is within a cavity inside a copper conductor. We want the charges on the surface of the cavity and the outer surface of the copper block.

SET UP and **EXECUTE:** Do part (b) first. **(b)** Apply Gauss's law using a Gaussian surface that is totally within the copper and encloses the entire cavity. On this Gaussian surface, $E = 0$ inside a conductor, so $q_{\text{inside}} = 0$. Therefore +3.00 nC + $q_{\text{cavity}} = 0$, so $q_{\text{cavity}} = -3.00$ nC.

(a) The excess charge on a conductor is on its surface. The presence of the +3.00 nC charge inside the cavity induces a charge of –3.00 nC on the surface of the cavity, which induces +3.00 nC on the outer surface of the copper block. The net charge on the outer surface is –8.00 nC + 3.00 nC = –5.00 nC.

EVALUATE: It does not matter where the +3.00 nC charge is within the cavity for our answers to hold. If the charge inside the cavity were +8.00 nC, there would be no excess charge on the outer surface of the copper.

VP22.12.2. **IDENTIFY:** A charge is within a cavity inside a silver conductor. We want to find this charge.

SET UP and **EXECUTE:** **(a)** Apply Gauss's law using a Gaussian surface that is totally within the silver and encloses the entire cavity. On this Gaussian surface, $E = 0$ inside a conductor, so $q_{\text{inside}} = 0$. Therefore –2.00 nC + $q_{\text{inside}} = 0$, so $q_{\text{inside}} = +2.00$ nC.

(b) Moving the charge within the cavity would have no effect on the *amount* of charge on the cavity surface or on the outer surface of the silver. We know this because moving the charge would not transfer any charge to (or from) the silver.

EVALUATE: With the point charge at the center of the spherical cavity, the charge would be uniformly distributed over the cavity surface and the outer surface of the silver. This would not be true of the point charge were off center, even though the total amount of charge on each surface would not change.

VP22.12.3. IDENTIFY: We are looking at a charged spherical conducting shell.

SET UP: All the excess charge on a conductor lies on its surface.

EXECUTE: **(a)** All the excess charge is on the outer surface, so $E = 0$ inside the spherical shell.

(b) At the surface of the sphere, it is equivalent to a point charge at its center, so $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$.

Solving for q with $r = R$ gives $q = R^2 E 4\pi\epsilon_0$. Putting in the numbers gives us

$q = (0.90\text{ m})^2(3.0\times10^5\text{ N/C})4\pi\epsilon_0 = 2.7\times10^{-5}\text{ C} = 27\ \mu\text{C}.$

(c) $\sigma = \dfrac{Q}{A} = \dfrac{Q}{4\pi R^2} = \dfrac{R^2 E 4\pi\epsilon_0}{4\pi R^2} = E\epsilon_0 = (3.0\times10^5\text{ N/C})\epsilon_0 = 2.7\,\mu\text{C/m}^2.$

EVALUATE: It takes only 27 μC to create an electric field of 300,000 N/C at the sphere's surface, yet $E = 0$ inside the sphere.

VP22.12.4. IDENTIFY: A charge is within a cavity inside of a conductor.

SET UP and **EXECUTE:** **(a)** Apply Gauss's law using a Gaussian surface that is totally within the conductor and encloses the entire cavity. On this Gaussian surface, $E = 0$ inside a conductor, so $q_{\text{inside}} = 0$. Therefore +4.00 nC + $q_{\text{surface}} = 0$, so $q_{\text{surface}} = -4.00$ nC.

(b) The +4.00 nC induces a charge of –4.00 nC on the cavity surface which induces a charge of +4.00 nC on the outer surface of the sphere. So the total charge on the outer surface is +4.00 nC – 6.00 nC = –2.00 nC.

(c) Inside the cavity, the electric field is due only to the +4.00 nC charge, so

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{4.00\text{ nC}}{(0.120\text{ m})^2} = 2.50\times10^3\text{ N/C}.$$

(d) Outside the conductor, the field is due to –2.00 nC on its surface. The magnitude of the field is

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{2.00\text{ nC}}{(0.330\text{ m})^2} = 1.65\times10^2\text{ N/C}.$$

EVALUATE: The only excess charge on the conductor is –2.00 nC, so outside this conductor all the field is due to this charge.

22.1. IDENTIFY and **SET UP:** $\Phi_E = \int E\cos\phi\, dA$, where ϕ is the angle between the normal to the sheet $\hat{\boldsymbol{n}}$ and the electric field $\vec{\boldsymbol{E}}$.

(a) EXECUTE: In this problem E and $\cos\phi$ are constant over the surface so

$\Phi_E = E\cos\phi\int dA = E\cos\phi A = (14\text{ N/C})(\cos 60°)(0.250\text{ m}^2) = 1.8\text{ N}\cdot\text{m}^2/\text{C}.$

EVALUATE: **(b)** Φ_E is independent of the shape of the sheet as long as ϕ and E are constant at all points on the sheet.

(c) EXECUTE: (i) $\Phi_E = E\cos\phi A.$ Φ_E is largest for $\phi = 0°$, so $\cos\phi = 1$ and $\Phi_E = EA.$

(ii) Φ_E is smallest for $\phi = 90°$, so $\cos\phi = 0$ and $\Phi_E = 0.$

EVALUATE: Φ_E is 0 when the surface is parallel to the field so no electric field lines pass through the surface.

22.2. IDENTIFY: The field is uniform and the surface is flat, so use $\Phi_E = EA\cos\phi.$

SET UP: ϕ is the angle between the normal to the surface and the direction of $\vec{\boldsymbol{E}}$, so $\phi = 70°.$

EXECUTE: $\Phi_E = (90.0\text{ N/C})(0.400\text{ m})(0.600\text{ m})\cos 70° = 7.39\text{ N}\cdot\text{m}^2/\text{C}.$

EVALUATE: If the field were perpendicular to the surface the flux would be $\Phi_E = EA = 21.6\text{ N}\cdot\text{m}^2/\text{C}.$ The flux in this problem is much less than this because only the component of $\vec{\boldsymbol{E}}$ perpendicular to the surface contributes to the flux.

22.3. **IDENTIFY:** The electric flux through an area is defined as the product of the component of the electric field perpendicular to the area times the area.

(a) SET UP: In this case, the electric field is perpendicular to the surface of the sphere, so $\Phi_E = EA = E(4\pi r^2)$.

EXECUTE: Substituting in the numbers gives

$$\Phi_E = (1.25\times10^6\ \text{N/C})4\pi(0.150\ \text{m})^2 = 3.53\times10^5\ \text{N}\cdot\text{m}^2/\text{C}.$$

(b) IDENTIFY: We use the electric field due to a point charge.

SET UP: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$

EXECUTE: Solving for q and substituting the numbers gives

$$q = 4\pi\epsilon_0 r^2 E = \frac{1}{9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}(0.150\ \text{m})^2(1.25\times10^6\ \text{N/C}) = 3.13\times10^{-6}\ \text{C}.$$

EVALUATE: The flux would be the same no matter how large the sphere, since the area is proportional to r^2 while the electric field is proportional to $1/r^2$.

22.4. **IDENTIFY:** Use $\Phi_E = \int \vec{E}\cdot d\vec{A} = \int E\cos\phi dA$ to calculate the flux through the surface of the cylinder.

SET UP: The line of charge and the cylinder are sketched in Figure 22.4.

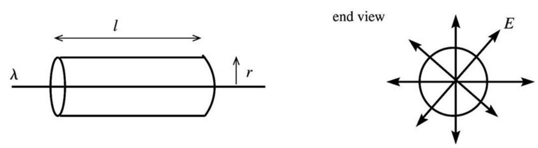


Figure 22.4

EXECUTE: **(a)** The area of the curved part of the cylinder is $A = 2\pi rl$.
The electric field is parallel to the end caps of the cylinder, so $\vec{E}\cdot\vec{A} = 0$ for the ends and the flux through the cylinder end caps is zero.
The electric field is normal to the curved surface of the cylinder and has the same magnitude $E = \lambda/2\pi\epsilon_0 r$ at all points on this surface. Thus $\phi = 0°$ and

$$\Phi_E = EA\cos\phi = EA = (\lambda/2\pi\epsilon_0 r)(2\pi rl) = \frac{\lambda l}{\epsilon_0} = \frac{(3.00\times10^{-6}\ \text{C/m})(0.400\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.36\times10^5\ \text{N}\cdot\text{m}^2/\text{C}.$$

(b) In the calculation in part (a) the radius r of the cylinder divided out, so the flux remains the same, $\Phi_E = 1.36\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$.

(c) $\Phi_E = \dfrac{\lambda l}{\epsilon_0} = \dfrac{(3.00\times10^{-6}\ \text{C/m})(0.800\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 2.71\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$, which is twice the flux calculated in parts (a) and (b).

EVALUATE: The flux depends on the number of field lines that pass through the surface of the cylinder.

22.5. **IDENTIFY:** We know the flux through a surface and want to know the magnitude of the electric field causing that flux.

SET UP: $\Phi_E = EA\cos\phi$. ϕ is the angle between the electric field and the normal to the surface, so $\phi = 30.0°$.

EXECUTE: Solve for E: $E = \dfrac{\Phi_E}{A\cos\phi} = \dfrac{4.44\ \text{N}\cdot\text{m}^2/\text{C}}{(6.66\times10^{-4}\ \text{m}^2)(\cos 30.0°)} = 7.70\times10^3\ \text{N/C}.$

EVALUATE: Careful! ϕ is not the angle between the flat surface and the electric field. It is the angle between the normal to the surface and the field.

22.6. **IDENTIFY:** Use $\Phi_E = \vec{E}\cdot\vec{A}$ to calculate the flux for each surface.

SET UP: $\Phi = \vec{E}\cdot\vec{A} = EA\cos\phi$ where $\vec{A} = A\hat{n}$.

EXECUTE: **(a)** $\hat{n}_{S_1} = -\hat{j}$ (left). $\Phi_{S_1} = -(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos(90° - 53.1°) = -32\ \text{N}\cdot\text{m}^2/\text{C}.$

$\hat{n}_{S_2} = +\hat{k}$ (top). $\Phi_{S_2} = -(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 90° = 0.$

$\hat{n}_{S_3} = +\hat{j}$ (right). $\Phi_{S_3} = +(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos(90° - 53.1°) = +32\ \text{N}\cdot\text{m}^2/\text{C}.$

$\hat{n}_{S_4} = -\hat{k}$ (bottom). $\Phi_{S_4} = (4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 90° = 0.$

$\hat{n}_{S_5} = +\hat{i}$ (front). $\Phi_{S_5} = +(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 53.1° = 24\ \text{N}\cdot\text{m}^2/\text{C}.$

$\hat{n}_{S_6} = -\hat{i}$ (back). $\Phi_{S_6} = -(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 53.1° = -24\ \text{N}\cdot\text{m}^2/\text{C}.$

EVALUATE: **(b)** The total flux through the cube must be zero; any flux entering the cube must also leave it, since the field is uniform. Our calculation gives the result; the sum of the fluxes calculated in part (a) is zero.

22.7. **IDENTIFY:** This problem involves Gauss's law.
SET UP: Gauss's law: $\Phi_E = q/\epsilon_0$. $\sigma = Q/A_{\text{sheet}}$ and $q = \sigma A_{\text{enclosed}}$.

EXECUTE: We want the area of the Gaussian surface. $\Phi_E = \dfrac{\sigma A_{\text{enclosed}}}{\epsilon_0} = \dfrac{(Q/A_{\text{sheet}})A_{\text{enclosed}}}{\epsilon_0}$.

$A_{\text{surface}} = \dfrac{\epsilon_0\Phi_E A_{\text{sheet}}}{Q}$. Using $\Phi_E = 5.00\ \text{N}\cdot\text{m}^2/\text{C}$, $A_{\text{sheet}} = 29.2\ \text{cm}^2 = 0.00292\ \text{m}^2$, and $Q = 87.6$ pC, we get $A_{\text{surf}} = 14.8\ \text{cm}^2$.

EVALUATE: The area of the Gaussian surface is less than the area of the sheet, which is reasonable.

22.8. **IDENTIFY:** Apply Gauss's law to each surface.
SET UP: Q_{encl} is the algebraic sum of the charges enclosed by each surface. Flux out of the volume is positive and flux into the enclosed volume is negative.
EXECUTE: **(a)** $\Phi_{S_1} = q_1/\epsilon_0 = (4.00\times10^{-9}\ \text{C})/\epsilon_0 = 452\ \text{N}\cdot\text{m}^2/\text{C}.$

(b) $\Phi_{S_2} = q_2/\epsilon_0 = (-7.80\times10^{-9}\ \text{C})/\epsilon_0 = -881\ \text{N}\cdot\text{m}^2/\text{C}.$

(c) $\Phi_{S_3} = (q_1 + q_2)/\epsilon_0 = ((4.00 - 7.80)\times10^{-9}\ \text{C})/\epsilon_0 = -429\ \text{N}\cdot\text{m}^2/\text{C}.$

(d) $\Phi_{S_4} = (q_1 + q_3)/\epsilon_0 = [(4.00 + 2.40)\times10^{-9}\ \text{C}]/\epsilon_0 = 723\ \text{N}\cdot\text{m}^2/\text{C}.$

(e) $\Phi_{S_5} = (q_1 + q_2 + q_3)/\epsilon_0 = ((4.00 - 7.80 + 2.40)\times10^{-9}\ \text{C})/\epsilon_0 = -158\ \text{N}\cdot\text{m}^2/\text{C}.$

EVALUATE: **(f)** All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

22.9. **IDENTIFY:** Apply the results in Example 22.5 for the field of a spherical shell of charge.

SET UP: Example 22.5 shows that $E = 0$ inside a uniform spherical shell and that $E = k\dfrac{|q|}{r^2}$ outside the shell.

EXECUTE: **(a)** $E = 0$.

(b) $r = 0.060\ \text{m}$ and $E = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{49.0\times10^{-6}\ \text{C}}{(0.060\ \text{m})^2} = 1.22\times10^8\ \text{N/C}$.

(c) $r = 0.110\ \text{m}$ and $E = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{49.0\times10^{-6}\ \text{C}}{(0.110\ \text{m})^2} = 3.64\times10^7\ \text{N/C}$.

EVALUATE: Outside the shell the electric field is the same as if all the charge were concentrated at the center of the shell. But inside the shell the field is not the same as for a point charge at the center of the shell, inside the shell the electric field is zero.

22.10. **IDENTIFY:** Apply Gauss's law to the spherical surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by the sphere.

EXECUTE: **(a)** No charge enclosed so $\Phi_E = 0$.

(b) $\Phi_E = \dfrac{q_2}{\epsilon_0} = \dfrac{-6.00\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -678\ \text{N}\cdot\text{m}^2/\text{C}$.

(c) $\Phi_E = \dfrac{q_1+q_2}{\epsilon_0} = \dfrac{(4.00-6.00)\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -226\ \text{N}\cdot\text{m}^2/\text{C}$.

EVALUATE: Negative flux corresponds to flux directed into the enclosed volume. The net flux depends only on the net charge enclosed by the surface and is not affected by any charges outside the enclosed volume.

22.11. **IDENTIFY:** Apply Gauss's law to a Gaussian surface that coincides with the cell boundary.

SET UP: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$.

EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{-8.65\times10^{-12}\ \text{C}}{8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = -0.977\ \text{N}\cdot\text{m}^2/\text{C}$. Q_{encl} is negative, so the flux is inward.

EVALUATE: If the cell were positive, the field would point outward, so the flux would be positive.

22.12. **IDENTIFY:** Apply the results of Examples 22.9 and 22.10.

SET UP: $E = k\dfrac{|q|}{r^2}$ outside the sphere. A proton has charge $+e$.

EXECUTE: **(a)** $E = k\dfrac{|q|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{92(1.60\times10^{-19}\ \text{C})}{(7.4\times10^{-15}\ \text{m})^2} = 2.4\times10^{21}\ \text{N/C}$.

(b) For $r = 1.0\times10^{-10}\ \text{m}$, $E = (2.4\times10^{21}\ \text{N/C})\left(\dfrac{7.4\times10^{-15}\ \text{m}}{1.0\times10^{-10}\ \text{m}}\right)^2 = 1.3\times10^{13}\ \text{N/C}$.

(c) $E = 0$, inside a spherical shell.

EVALUATE: The electric field in an atom is very large.

22.13. **IDENTIFY:** Each line lies in the electric field of the other line, and therefore each line experiences a force due to the other line.

SET UP: The field of one line at the location of the other is $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. For charge $dq = \lambda dx$ on one line, the force on it due to the other line is $dF = Edq$. The total force is $F = \int Edq = E\int dq = Eq$.

EXECUTE: $E = \dfrac{\lambda}{2\pi\epsilon_0 r} = \dfrac{5.20\times10^{-6}\ \text{C/m}}{2\pi(8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2))(0.300\ \text{m})} = 3.116\times10^5\ \text{N/C}.$ The force on one line due to the other is $F = Eq,$ where $q = \lambda(0.0500\ \text{m}) = 2.60\times10^{-7}\ \text{C}.$ The net force is

$F = Eq = (3.116\times10^5\ \text{N/C})(2.60\times10^{-7}\ \text{C}) = 0.0810\ \text{N}.$

EVALUATE: Since the electric field at each line due to the other line is uniform, each segment of line experiences the same force, so all we need to use is $F = Eq,$ even though the line is *not* a point charge.

22.14. IDENTIFY: Apply the results of Example 22.5.

SET UP: At a point 0.100 m outside the surface, $r = 0.550$ m.

EXECUTE: **(a)** $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(2.50\times10^{-10}\ \text{C})}{(0.550\ \text{m})^2} = 7.44\ \text{N/C}.$

(b) $E = 0$ inside of a conductor or else free charges would move under the influence of forces, violating our electrostatic assumptions (i.e., that charges aren't moving).

EVALUATE: Outside the sphere its electric field is the same as would be produced by a point charge at its center, with the same charge.

22.15. IDENTIFY: We want the electric field for several charge arrangements.

SET UP and EXECUTE: **(a)** Conducting sphere: Use $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ for $r > R$. Take rations:

$\dfrac{E_{4R}}{E_{2R}} = \dfrac{(1/4\pi\epsilon_0)Q/(4R)^2}{(1/4\pi\epsilon_0)Q/(2R)^2} = \dfrac{1}{4},$ so $E_{4R} = (1/4)E_{2R} = (1/4)(1400\ \text{N/C}) = 350\ \text{N/C}.$

(b) Conducting cylinder: Use $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r > R$ and take ratios. $\dfrac{E_{4R}}{E_{2R}} = \dfrac{(\lambda/2\pi\epsilon_0)/(4R)}{(\lambda/2\pi\epsilon_0)/(2R)} = \dfrac{1}{2},$ so

$E_{4R} = (1/2)E_{2R} = (1/2)(1400\ \text{N/C}) = 700\ \text{N/C}.$

(c) Large sheet of charge: $E = \sigma/2\epsilon_0$ which is independent of distance from the sheet, so

$E_{2d} = 1400\ \text{N/C}.$

EVALUATE: For a uniform sphere and point charges $E \propto 1/r^2$, for lines and cylinders $E \propto 1/r$, and for large sheets E is independent of r.

22.16. IDENTIFY: According to the problem, Mars's flux is negative, so its electric field must point toward the center of Mars. Therefore the charge on Mars must be negative. We use Gauss's law to relate the electric flux to the charge causing it.

SET UP: Gauss's law is $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}.$ The enclosed charge is negative, so the electric flux must also be negative. The flux is $\Phi_E = EA\cos\phi = -EA$ since $\phi = 180°$ and E is the magnitude of the electric field, which is positive.

EXECUTE: **(a)** Solving Gauss's law for q, putting in the numbers, and recalling that q is negative, gives $q = \epsilon_0\Phi_E = (-3.63\times10^{16}\ \text{N}\cdot\text{m}^2/\text{C})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = -3.21\times10^5\ \text{C}.$

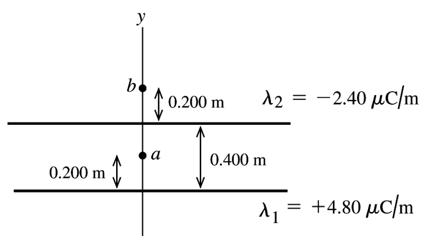
(b) Use the definition of electric flux to find the electric field. The area to use is the surface area of Mars. $E = \dfrac{\Phi_E}{A} = \dfrac{3.63\times10^{16}\ \text{N}\cdot\text{m}^2/\text{C}}{4\pi(3.39\times10^6\ \text{m})^2} = 2.51\times10^2\ \text{N/C}.$

(c) The surface charge density on Mars is therefore $\sigma = \dfrac{q}{A_{\text{Mars}}} = \dfrac{-3.21\times10^5\ \text{C}}{4\pi(3.39\times10^6\ \text{m})^2} = -2.22\times10^{-9}\ \text{C/m}^2.$

22.17. **EVALUATE:** Even though the charge on Mars is very large, it is spread over a large area, giving a small surface charge density.

IDENTIFY: Add the vector electric fields due to each line of charge. $E(r)$ for a line of charge is given by Example 22.6 and is directed toward a negative line of charge and away from a positive line.

SET UP: The two lines of charge are shown in Figure 22.17.



$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}.$$

Figure 22.17

EXECUTE: **(a)** At point a, $\vec{E}_1$ and $\vec{E}_2$ are in the $+y$-direction (toward negative charge, away from positive charge).

$E_1 = (1/2\pi\epsilon_0)[(4.80\times10^{-6}\ \text{C/m})/(0.200\ \text{m})] = 4.314\times10^5\ \text{N/C}.$

$E_2 = (1/2\pi\epsilon_0)[(2.40\times10^{-6}\ \text{C/m})/(0.200\ \text{m})] = 2.157\times10^5\ \text{N/C}.$

$E = E_1 + E_2 = 6.47\times10^5\ \text{N/C}$, in the y-direction.

(b) At point b, $\vec{E}_1$ is in the $+y$-direction and $\vec{E}_2$ is in the $-y$-direction.

$E_1 = (1/2\pi\epsilon_0)[(4.80\times10^{-6}\ \text{C/m})/(0.600\ \text{m})] = 1.438\times10^5\ \text{N/C}.$

$E_2 = (1/2\pi\epsilon_0)[(2.40\times10^{-6}\ \text{C/m})/(0.200\ \text{m})] = 2.157\times10^5\ \text{N/C}.$

$E = E_2 - E_1 = 7.2\times10^4\ \text{N/C}$, in the $-y$-direction.

EVALUATE: At point a the two fields are in the same direction and the magnitudes add. At point b the two fields are in opposite directions and the magnitudes subtract.

22.18. **IDENTIFY:** Apply Gauss's law.

SET UP: Draw a cylindrical Gaussian surface with the line of charge as its axis. The cylinder has radius 0.400 m and is 0.0200 m long. The electric field is then 840 N/C at every point on the cylindrical surface and is directed perpendicular to the surface.

EXECUTE: $\oint \vec{E}\cdot d\vec{A} = EA_{\text{cylinder}} = E(2\pi rL) = (840\ \text{N/C})(2\pi)(0.400\ \text{m})(0.0200\ \text{m}) = 42.2\ \text{N}\cdot\text{m}^2/\text{C}.$

The field is parallel to the end caps of the cylinder, so for them $\oint \vec{E}\cdot d\vec{A} = 0.$ From Gauss's law,

$q = \epsilon_0\Phi_E = (8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(42.2\ \text{N}\cdot\text{m}^2/\text{C}) = 3.74\times10^{-10}\ \text{C}.$

EVALUATE: We could have applied the result in Example 22.6 and solved for λ. Then $q = \lambda L.$

22.19. **IDENTIFY:** The electric field inside the conductor is zero, and all of its initial charge lies on its outer surface. The introduction of charge into the cavity induces charge onto the surface of the cavity, which induces an equal but opposite charge on the outer surface of the conductor. The net charge on the outer surface of the conductor is the sum of the positive charge initially there and the additional negative charge due to the introduction of the negative charge into the cavity.

(a) SET UP: First find the initial positive charge on the outer surface of the conductor using $q_i = \sigma A$, where A is the area of its outer surface. Then find the net charge on the surface after the negative charge has been introduced into the cavity. Finally, use the definition of surface charge density.

EXECUTE: The original positive charge on the outer surface is

$$q_i = \sigma A = \sigma(4\pi r^2) = (6.37\times10^{-6}\ \text{C/m}^2)4\pi(0.250\ \text{m})^2 = 5.00\times10^{-6}\ \text{C}.$$

After the introduction of $-0.500\ \mu\text{C}$ into the cavity, the outer charge is now

$$5.00\ \mu\text{C} - 0.500\ \mu\text{C} = 4.50\ \mu\text{C}.$$

The surface charge density is now $\sigma = \dfrac{q}{A} = \dfrac{q}{4\pi r^2} = \dfrac{4.50\times10^{-6}\ \text{C}}{4\pi(0.250\ \text{m})^2} = 5.73\times10^{-6}\ \text{C/m}^2.$

(b) SET UP: Using Gauss's law, the electric field is $E = \dfrac{\Phi_E}{A} = \dfrac{q}{\epsilon_0 A} = \dfrac{q}{\epsilon_0 4\pi r^2}.$

EXECUTE: Substituting numbers gives

$$E = \frac{4.50\times10^{-6}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi)(0.250\ \text{m})^2} = 6.47\times10^5\ \text{N/C}.$$

(c) SET UP: We use Gauss's law again to find the flux. $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}.$

EXECUTE: Substituting numbers gives

$$\Phi_E = \frac{-0.500\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -5.65\times10^4\ \text{N}\cdot\text{m}^2/\text{C}.$$

EVALUATE: The excess charge on the conductor is still $+5.00\ \mu\text{C}$, as it originally was. The introduction of the $-0.500\ \mu\text{C}$ inside the cavity merely induced equal but opposite charges (for a net of zero) on the surfaces of the conductor.

22.20. IDENTIFY: We want to find the electric field inside and outside an insulating sphere of charge.

SET UP: For $r \le R$: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|Q|r}{R^3}$, and for $r \ge R$: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$.

EXECUTE: $\dfrac{E_{2R}}{E_{R/2}} = \dfrac{(1/4\pi\epsilon_0)Q/(2R)^2}{(1/4\pi\epsilon_0)Q/\left(\dfrac{R/2}{R^3}\right)} = \dfrac{1}{2}$, so E_{2R} = (1/2)(800 N/C) = 400 N/C.

EVALUATE: Inside a sphere of charge the net field is not an inverse square field.

22.21. IDENTIFY: The magnitude of the electric field is constant at any given distance from the center because the charge density is uniform inside the sphere. We can use Gauss's law to relate the field to the charge causing it.

(a) SET UP: Gauss's law tells us that $EA = \dfrac{q}{\epsilon_0}$, and the charge density is given by $\rho = \dfrac{q}{V} = \dfrac{q}{(4/3)\pi R^3}$.

EXECUTE: Solving for q and substituting numbers gives
$q = EA\epsilon_0 = E(4\pi r^2)\epsilon_0 = (1750\ \text{N/C})(4\pi)(0.500\ \text{m})^2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 4.866\times10^{-8}\ \text{C}$. Using the formula for charge density we get $\rho = \dfrac{q}{V} = \dfrac{q}{(4/3)\pi R^3} = \dfrac{4.866\times10^{-8}\ \text{C}}{(4/3)\pi(0.355\ \text{m})^3} = 2.60\times10^{-7}\ \text{C/m}^3$.

(b) SET UP: Take a Gaussian surface of radius $r = 0.200$ m, concentric with the insulating sphere. The charge enclosed within this surface is $q_{\text{encl}} = \rho V = \rho\left(\dfrac{4}{3}\pi r^3\right)$, and we can treat this charge as a point-charge, using Coulomb's law $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_{\text{encl}}}{r^2}$. The charge beyond $r = 0.200$ m makes no contribution to the electric field.

EXECUTE: First find the enclosed charge:

$$q_{\text{encl}} = \rho\left(\frac{4}{3}\pi r^3\right) = (2.60\times10^{-7}\ \text{C/m}^3)\left[\frac{4}{3}\pi(0.200\ \text{m})^3\right] = 8.70\times10^{-9}\ \text{C}$$

Now treat this charge as a point-charge and use Coulomb's law to find the field:

$$E = (9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{8.70\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 1.96\times10^{3}\ \text{N/C}$$

EVALUATE: Outside this sphere, it behaves like a point-charge located at its center. Inside of it, at a distance *r* from the center, the field is due only to the charge between the center and *r*.

22.22. **IDENTIFY:** We apply Gauss's law, taking the Gaussian surface beyond the cavity but inside the solid.

SET UP: Because of the symmetry of the charge, Gauss's law gives us $E_1 = \dfrac{q_{\text{total}}}{\epsilon_0 A}$, where A is the surface area of a sphere of radius $R = 9.50$ cm centered on the point-charge, and q_{total} is the total charge contained within that sphere. This charge is the sum of the $-3.00\ \mu$C point charge at the center of the cavity plus the charge within the solid between $r = 6.50$ cm and $R = 9.50$ cm. The charge within the solid is $q_{\text{solid}} = \rho V = \rho[(4/3)\pi R^3 - (4/3)\pi r^3] = (4\pi/3)\rho(R^3 - r^3)$.

EXECUTE: First find the charge within the solid between $r = 6.50$ cm and $R = 9.50$ cm:

$q_{\text{solid}} = \dfrac{4\pi}{3}(7.35\times10^{-4}\ \text{C/m}^3)[(0.0950\ \text{m})^3 - (0.0650\ \text{m})^3] = 1.794\times10^{-6}\ \text{C}.$

Now find the total charge within the Gaussian surface:

$$q_{\text{total}} = q_{\text{solid}} + q_{\text{point}} = -3.00\ \mu\text{C} + 1.794\ \mu\text{C} = -1.206\ \mu\text{C}.$$

Now find the magnitude of the electric field from Gauss's law:

$$E = \frac{|q|}{\epsilon_0 A} = \frac{|q|}{\epsilon_0 4\pi r^2} = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.206\times10^{-6}\ \text{C})}{(0.0950\ \text{m})^2} = 1.20\times10^{6}\ \text{N/C}.$$

The fact that the charge is negative means that the electric field points radially inward.

EVALUATE: Because of the uniformity of the charge distribution, the charge beyond 9.50 cm does not contribute to the electric field.

22.23. **IDENTIFY:** The charged sheet exerts a force on the electron and therefore does work on it.

SET UP: The electric field is uniform so the force on the electron is constant during the displacement. The electric field due to the sheet is $E = \dfrac{\sigma}{2\epsilon_0}$ and the magnitude of the force the sheet exerts on the electron is $F = qE$. The work the force does on the electron is $W = Fs$. In (b) we can use the work-energy theorem, $W_{\text{tot}} = \Delta K = K_2 - K_1$.

EXECUTE: **(a)** $W = Fs$, where $s = 0.250$ m. $F = Eq$, where

$E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{2.90\times10^{-12}\ \text{C/m}^2}{2(8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2))} = 0.1638\ \text{N/C}.$ Therefore the force is

$F = (0.1638\ \text{N/C})(1.602\times10^{-19}\ \text{C}) = 2.624\times10^{-20}\ \text{N}.$ The work this force does is

$W = Fs = 6.56\times10^{-21}\ \text{J}.$

(b) Use the work-energy theorem: $W_{\text{tot}} = \Delta K = K_2 - K_1$. $K_1 = 0$. $K_2 = \frac{1}{2}mv_2^2$. So, $\frac{1}{2}mv_2^2 = W$, which gives $v_2 = \sqrt{\dfrac{2W}{m}} = \sqrt{\dfrac{2(6.559\times10^{-21}\ \text{J})}{9.109\times10^{-31}\ \text{kg}}} = 1.2\times10^{5}\ \text{m/s}.$

EVALUATE: If the field were not constant, we would have to integrate in (a), but we could still use the work-energy theorem in (b).

22.24. IDENTIFY: The charge distribution is uniform, so we can readily apply Gauss's law. Outside a spherically symmetric charge distribution, the electric field is equivalent to that of a point-charge at the center of the sphere.

SET UP: Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\epsilon_0}$, $E = k\dfrac{|q|}{r^2}$ outside the sphere.

EXECUTE: **(a)** Outside the sphere, $E = k\dfrac{|q|}{r^2}$, so $Q = Er^2/k$, which gives

$Q = (940\text{ N/C})(0.0800\text{ m})^2/(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 6.692\times10^{-10}$ C. The volume charge density is

$\rho = \dfrac{Q}{V} = \dfrac{Q}{\frac{4}{3}\pi R^3} = (6.692\times10^{-10}\text{ C})/(4\pi/3)(0.0400\text{ m})^3 = 2.50\times10^{-6}\text{ C/m}^3$.

(b) Apply Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\epsilon_0}$, with the Gaussian surface being a sphere of radius $r = 0.0200$ m centered on the sphere of charge. This gives

$E(4\pi r^2) = Q_{\text{encl}}/\epsilon_0$, where $Q_{\text{encl}} = 4/3\ \pi r^3\rho$. Solving for E and simplifying gives

$E = r\rho/3\epsilon_0 = (0.0200\text{ m})(2.50\times10^{-6}\text{ C/m}^3)/[3(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)] = 1880$ N/C.

EVALUATE: Outside the sphere of charge, the electric field obeys an inverse-square law, but inside the field is proportional to the distance from the center of the sphere.

22.25. IDENTIFY: Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that lies wholly within the conducting material.

EXECUTE: **(a)** Positive charge is attracted to the inner surface of the conductor by the charge in the cavity. Its magnitude is the same as the cavity charge: $q_{\text{inner}} = +6.00$ nC, since $E = 0$ inside a conductor and a Gaussian surface that lies wholly within the conductor must enclose zero net charge.

(b) On the outer surface the charge is a combination of the net charge on the conductor and the charge "left behind" when the +6.00 nC moved to the inner surface:

$$q_{\text{tot}} = q_{\text{inner}} + q_{\text{outer}} \Rightarrow q_{\text{outer}} = q_{\text{tot}} - q_{\text{inner}} = 5.00\text{ nC} - 6.00\text{ nC} = -1.00\text{ nC}.$$

EVALUATE: The electric field outside the conductor is due to the charge on its surface.

EVALUATE: Our result for the field between the plates agrees with the result stated in Example 22.8.

22.26 IDENTIFY: Close to a finite sheet the field is the same as for an infinite sheet. Very far from a finite sheet the field is that of a point charge.

SET UP: For an infinite sheet, $E = \dfrac{\sigma}{2\epsilon_0}$. For a positive point charge, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$.

EXECUTE: **(a)** At a distance of 0.100 mm from the center, the sheet appears "infinite," so

$$E \approx \frac{\sigma}{2\epsilon_0} = \frac{q}{2\epsilon_0 A} = \frac{4.50\times10^{-9}\text{ C}}{2\epsilon_0(0.800\text{ m})^2} = 397\text{ N/C}.$$

(b) At a distance of 100 m from the center, the sheet looks like a point, so:

$$E \approx \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{(4.50\times10^{-9}\text{ C})}{(100\text{ m})^2} = 4.05\times10^{-3}\text{ N/C}.$$

(c) There would be no difference if the sheet was a conductor. The charge would automatically spread out evenly over both faces, giving it half the charge density on either face as the insulator but the same electric field. Far away, they both look like points with the same charge.

EVALUATE: The sheet can be treated as infinite at points where the distance to the sheet is much less than the distance to the edge of the sheet. The sheet can be treated as a point charge at points for which the distance to the sheet is much greater than the dimensions of the sheet.

22.27. **IDENTIFY:** Apply Gauss's law to a Gaussian surface and calculate E.

(a) SET UP and **EXECUTE:** Consider the charge on a length l of the cylinder. This can be expressed as $q = \lambda l$. But since the surface area is $2\pi Rl$ it can also be expressed as $q = \sigma 2\pi Rl$. These two expressions must be equal, so $\lambda l = \sigma 2\pi Rl$ and $\lambda = 2\pi R\sigma$.

(b) SET UP: Apply Gauss's law to a Gaussian surface that is a cylinder of length l, radius r, and whose axis coincides with the axis of the charge distribution, as shown in Figure 22.27.

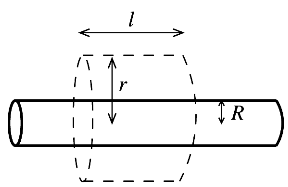


Figure 22.27

EXECUTE:

$Q_{\text{encl}} = \sigma(2\pi Rl)$

$\Phi_E = 2\pi rlE$

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $2\pi rlE = \dfrac{\sigma(2\pi Rl)}{\epsilon_0}$, so $E = \dfrac{\sigma R}{\epsilon_0 r}$.

EVALUATE: **(c)** Example 22.6 shows that the electric field of an infinite line of charge is $E = \lambda/2\pi\epsilon_0 r$. $\sigma = \dfrac{\lambda}{2\pi R}$, so $E = \dfrac{\sigma R}{\epsilon_0 r} = \dfrac{R}{\epsilon_0 r}\left(\dfrac{\lambda}{2\pi R}\right) = \dfrac{\lambda}{2\pi\epsilon_0 r}$, the same as for an infinite line of charge that is along the axis of the cylinder.

22.28. **IDENTIFY:** The net electric field is the vector sum of the fields due to each of the four sheets of charge.

SET UP: The electric field of a large sheet of charge is $E = \sigma/2\epsilon_0$. The field is directed away from a positive sheet and toward a negative sheet.

EXECUTE: **(a)** At A: $E_A = \dfrac{|\sigma_2|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_1|}{2\epsilon_0} = \dfrac{|\sigma_2| + |\sigma_3| + |\sigma_4| - |\sigma_1|}{2\epsilon_0}$.

$E_A = \dfrac{1}{2\epsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2) = 2.82\times10^5$ N/C to the left.

(b) $E_B = \dfrac{|\sigma_1|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} = \dfrac{|\sigma_1| + |\sigma_3| + |\sigma_4| - |\sigma_2|}{2\epsilon_0}$.

$E_B = \dfrac{1}{2\epsilon_0}(6\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2) = 3.95\times10^5$ N/C to the left.

(c) $E_C = \dfrac{|\sigma_4|}{2\epsilon_0} + \dfrac{|\sigma_1|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} - \dfrac{|\sigma_3|}{2\epsilon_0} = \dfrac{|\sigma_4| + |\sigma_1| - |\sigma_2| - |\sigma_3|}{2\epsilon_0}$.

$E_C = \dfrac{1}{2\epsilon_0}(4\ \mu\text{C/m}^2 + 6\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2 - 2\ \mu\text{C/m}^2) = 1.69\times10^5$ N/C to the left.

EVALUATE: The field at C is not zero. The pieces of plastic are not conductors.

22.29. **IDENTIFY:** The uniform electric field of the sheet exerts a constant force on the proton perpendicular to the sheet, and therefore does not change the parallel component of its velocity. Newton's second law allows us to calculate the proton's acceleration perpendicular to the sheet, and uniform-acceleration kinematics allows us to determine its perpendicular velocity component.

SET UP: Let $+x$ be the direction of the initial velocity and let $+y$ be the direction perpendicular to the sheet and pointing away from it. $a_x = 0$ so $v_x = v_{0x} = 9.70\times10^2$ m/s. The electric field due to the sheet is $E = \dfrac{\sigma}{2\epsilon_0}$ and the magnitude of the force the sheet exerts on the proton is $F = eE$.

EXECUTE: $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{2.34\times10^{-9}\ \text{C/m}^2}{2(8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2))} = 132.1\ \text{N/C}.$ Newton's second law gives

$a_y = \dfrac{Eq}{m} = \dfrac{(132.1\ \text{N/C})(1.602\times10^{-19}\ \text{C})}{1.673\times10^{-27}\ \text{kg}} = 1.265\times10^{10}\ \text{m/s}^2.$ Kinematics gives

$v_y = v_{0y} + a_y y = (1.265\times10^{10}\ \text{m/s}^2)(5.00\times10^{-8}\ \text{s}) = 632.7\ \text{m/s}.$ The speed of the proton is the magnitude of its velocity, so $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.70\times10^2\ \text{m/s})^2 + (632.7\ \text{m/s})^2} = 1.16\times10^3\ \text{m/s}.$

EVALUATE: We can use the constant-acceleration kinematics equations because the uniform electric field of the sheet exerts a constant force on the proton, giving it a constant acceleration. We could *not* use this approach if the sheet were replaced with a sphere, for example.

22.30. IDENTIFY: The sheet repels the charge electrically, slowing it down and eventually stopping it at its closest approach.

SET UP: Let $+y$ be in the direction toward the sheet. The electric field due to the sheet is $E = \dfrac{\sigma}{2\epsilon_0}$ and the magnitude of the force the sheet exerts on the object is $F = qE.$ Newton's second law, and the constant-acceleration kinematics formulas, apply to the object as it is slowing down.

EXECUTE: $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{5.90\times10^{-8}\ \text{C/m}^2}{2[8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)]} = 3.332\times10^3\ \text{N/C}.$

$a_y = -\dfrac{F}{m} = -\dfrac{Eq}{m} = -\dfrac{(3.332\times10^3\ \text{N/C})(6.50\times10^{-9}\ \text{C})}{8.20\times10^{-9}\ \text{kg}} = -2.641\times10^3\ \text{m/s}^2.$ Using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-2.64\times10^3\ \text{m/s}^2)(0.300\ \text{m})} = 39.8\ \text{m/s}.$

EVALUATE: We can use the constant-acceleration kinematics equations because the uniform electric field of the sheet exerts a constant force on the object, giving it a constant acceleration. We could *not* use this approach if the sheet were replaced with a sphere, for example.

22.31. IDENTIFY: First make a free-body diagram of the sphere. The electric force acts to the left on it since the electric field due to the sheet is horizontal. Since it hangs at rest, the sphere is in equilibrium so the forces on it add to zero, by Newton's first law. Balance horizontal and vertical force components separately.

SET UP: Call T the tension in the thread and E the electric field. Balancing horizontal forces gives $T\sin\theta = qE.$ Balancing vertical forces we get $T\cos\theta = mg.$ Combining these equations gives $\tan\theta = qE/mg,$ which means that $\theta = \arctan(qE/mg).$ The electric field for a sheet of charge is $E = \sigma/2\epsilon_0$.

EXECUTE: Substituting the numbers gives us

$E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{2.50\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.41\times10^2\ \text{N/C}.$ Then

$$\theta = \arctan\left[\frac{(5.00\times10^{-8}\ \text{C})(1.41\times10^2\ \text{N/C})}{(4.00\times10^{-6}\ \text{kg})(9.80\ \text{m/s}^2)}\right] = 10.2°.$$

EVALUATE: Increasing the field, or decreasing the mass of the sphere, would cause the sphere to hang at a larger angle.

22.32. IDENTIFY: Use $\Phi_E = \vec{E}\cdot\vec{A}$ to calculate the flux for each surface. Use $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ to calculate the total enclosed charge.

SET UP: $\vec{E} = (-5.00\ \text{N/C}\cdot\text{m})x\hat{\boldsymbol{i}} + (3.00\ \text{N/C}\cdot\text{m})z\hat{\boldsymbol{k}}$. The area of each face is L^2, where $L = 0.300$ m.

EXECUTE: **(a)** $\hat{\boldsymbol{n}}_{S_1} = -\hat{\boldsymbol{j}} \Rightarrow \Phi_1 = \vec{E}\cdot\hat{\boldsymbol{n}}_{S_1}A = 0$.

$\hat{\boldsymbol{n}}_{S_2} = +\hat{\boldsymbol{k}} \Rightarrow \Phi_2 = \vec{E}\cdot\hat{\boldsymbol{n}}_{S_2}A = (3.00\ \text{N/C}\cdot\text{m})(0.300\ \text{m})^2 z = (0.27\ (\text{N/C})\cdot\text{m})z$.

$\Phi_2 = (0.27\ (\text{N/C})\cdot\text{m})(0.300\ \text{m}) = 0.081\ (\text{N/C})\cdot\text{m}^2$.

$\hat{\boldsymbol{n}}_{S_3} = +\hat{\boldsymbol{j}} \Rightarrow \Phi_3 = \vec{E}\cdot\hat{\boldsymbol{n}}_{S_3}A = 0$.

$\hat{\boldsymbol{n}}_{S_4} = -\hat{\boldsymbol{k}} \Rightarrow \Phi_4 = \vec{E}\cdot\hat{\boldsymbol{n}}_{S_4}A = -(0.27\ (\text{N/C})\cdot\text{m})z = 0$ (since $z = 0$).

$\hat{\boldsymbol{n}}_{S_5} = +\hat{\boldsymbol{i}} \Rightarrow \Phi_5 = \vec{E}\cdot\hat{\boldsymbol{n}}_{S_5}A = (-5.00\ \text{N/C}\cdot\text{m})(0.300\ \text{m})^2 x = -(0.45\ (\text{N/C})\cdot\text{m})x$.

$\Phi_5 = -(0.45\ (\text{N/C})\cdot\text{m})(0.300\ \text{m}) = -(0.135\ (\text{N/C})\cdot\text{m}^2)$.

$\hat{\boldsymbol{n}}_{S_6} = -\hat{\boldsymbol{i}} \Rightarrow \Phi_6 = \vec{E}\cdot\hat{\boldsymbol{n}}_{S_6}A = +(0.45\ (\text{N/C})\cdot\text{m})x = 0$ (since $x = 0$).

(b) Total flux: $\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135)(\text{N/C})\cdot\text{m}^2 = -0.054\ \text{N}\cdot\text{m}^2/\text{C}$. Therefore,

$q = \epsilon_0\Phi = -4.78\times10^{-13}$ C.

EVALUATE: Flux is positive when $\vec{E}$ is directed out of the volume and negative when it is directed into the volume.

22.33. **IDENTIFY:** Use $\Phi_E = \vec{E}\cdot\vec{A}$ to calculate the flux through each surface and use Gauss's law to relate the net flux to the enclosed charge.

SET UP: Flux into the enclosed volume is negative and flux out of the volume is positive.

EXECUTE: **(a)** $\Phi = EA = (125\ \text{N/C})(6.0\ \text{m}^2) = 750\ \text{N}\cdot\text{m}^2/\text{C}$.

(b) Since the field is parallel to the surface, $\Phi = 0$.

(c) Choose the Gaussian surface to equal the volume's surface. Then $750\ \text{N}\cdot\text{m}^2/\text{C} - EA = q/\epsilon_0$ and

$E = \dfrac{1}{6.0\ \text{m}^2}(2.40\times10^{-8}\ \text{C}/\epsilon_0 + 750\ \text{N}\cdot\text{m}^2/\text{C}) = 577\ \text{N/C}$, in the positive x-direction. Since $q < 0$ we must have some net flux flowing *in* so the flux is $-|EA|$ on second face.

EVALUATE: **(d)** $q < 0$ but we have E pointing *away* from face I. This is due to an external field that does not affect the flux but affects the value of E. The electric field is produced by charges both inside and outside the slab.

22.34. **IDENTIFY:** The electric field is perpendicular to the square but varies in magnitude over the surface of the square, so we will need to integrate to find the flux.

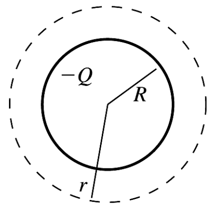
SET UP and **EXECUTE:** $\vec{E} = (964\ \text{N/C}\cdot\text{m})x\hat{\boldsymbol{k}}$. Consider a thin rectangular slice parallel to the y-axis and at coordinate x with width dx. $d\vec{A} = (Ldx)\hat{\boldsymbol{k}}$. $d\Phi_E = \vec{E}\cdot d\vec{A} = (964\ \text{N/C}\cdot\text{m})Lxdx$.

$$\Phi_E = \int_0^L d\Phi_E = (964\ \text{N/C}\cdot\text{m})L\int_0^L xdx = (964\ \text{N/C}\cdot\text{m})L\left(\frac{L^2}{2}\right).$$

$$\Phi_E = \frac{1}{2}(964\ \text{N/C}\cdot\text{m})(0.350\ \text{m})^3 = 20.7\ \text{N}\cdot\text{m}^2/\text{C}.$$

22.35. **IDENTIFY:** Use Gauss's law to find the electric field $\vec{E}$ produced by the shell for $r < R$ and $r > R$ and then use $\vec{F} = q\vec{E}$ to find the force the shell exerts on the point charge.

(a) SET UP: Apply Gauss's law to a spherical Gaussian surface that has radius $r > R$ and that is concentric with the shell, as sketched in Figure 22.35a.



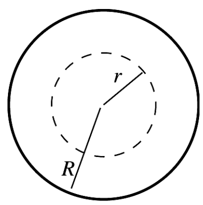
EXECUTE: $\Phi_E = -E(4\pi r^2)$.
$Q_{\text{encl}} = -Q$.

Figure 22.35a

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $-E(4\pi r^2) = \dfrac{-Q}{\epsilon_0}$.

The magnitude of the field is $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$ and it is directed toward the center of the shell. Then $F = qE = \dfrac{qQ}{4\pi\epsilon_0 r^2}$, directed toward the center of the shell. (Since q is positive, $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{F}}$ are in the same direction.)

(b) SET UP: Apply Gauss's law to a spherical Gaussian surface that has radius $r < R$ and that is concentric with the shell, as sketched in Figure 22.35b.



EXECUTE: $\Phi_E = E(4\pi r^2)$.
$Q_{\text{encl}} = 0$.

Figure 22.35b

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = 0$.

Then $E = 0$ so $F = 0$.

EVALUATE: Outside the shell the electric field and the force it exerts is the same as for a point charge $-Q$ located at the center of the shell. Inside the shell $E = 0$ and there is no force.

EVALUATE: To set up the integral, we take rectangular slices parallel to the *y*-axis (and not the *x*-axis) because the electric field is constant over such a slice. It would not be constant over a slice parallel to the *x*-axis.

22.36. **IDENTIFY:** The α particle feels no force where the net electric field due to the two distributions of charge is zero.

SET UP: The fields can cancel only in the regions *A* and *B* shown in Figure 22.36, because only in these two regions are the two fields in opposite directions.

EXECUTE: $E_{\text{line}} = E_{\text{sheet}}$ gives $\dfrac{\lambda}{2\pi\epsilon_0 r} = \dfrac{\sigma}{2\epsilon_0}$ and $r = \lambda/\pi\sigma = \dfrac{50\ \mu\text{C/m}}{\pi(100\ \mu\text{C/m}^2)} = 0.16\text{ m} = 16\text{ cm}$.

The fields cancel 16 cm from the line in regions *A* and *B*.

EVALUATE: The result is independent of the distance between the line and the sheet. The electric field of an infinite sheet of charge is uniform, independent of the distance from the sheet.

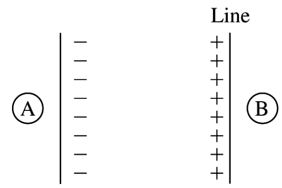


Figure 22.36

22.37. **(a) IDENTIFY:** Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $a < r < b$, and calculate E on the surface of the cylinder.

SET UP: The Gaussian surface is sketched in Figure 22.37a.

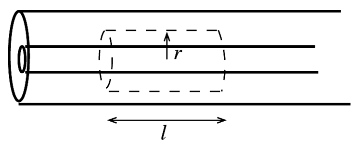


Figure 22.37a

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface).

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\epsilon_0}$.

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{\boldsymbol{E}}$ is radially outward.

(b) IDENTIFY and SET UP: Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $r > c$, as shown in Figure 22.37b.

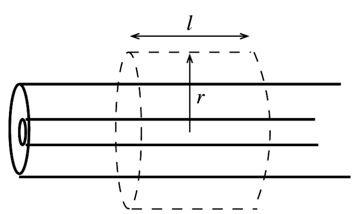


Figure 22.37b

EXECUTE: $\Phi_E = E(2\pi rl)$.

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface; the outer conductor carries no net charge).

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\epsilon_0}$.

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{\boldsymbol{E}}$ is radially outward.

(c) IDENTIFY and EXECUTE: $E = 0$ within a conductor. Thus $E = 0$ for $r < a$;

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $a < r < b$; $E = 0$ for $b < r < c$;

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r > c$. The graph of E versus r is sketched in Figure 22.37c.

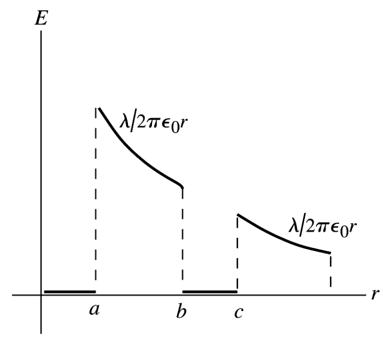


Figure 22.37c

EVALUATE: Inside either conductor $E = 0$. Between the conductors and outside both conductors the electric field is the same as for a line of charge with linear charge density λ lying along the axis of the inner conductor.

(d) IDENTIFY and **SET UP:** <u>inner surface:</u> Apply Gauss's law to a Gaussian cylinder with radius *r*, where $b < r < c$. We know *E* on this surface; calculate Q_{encl}.

EXECUTE: This surface lies within the conductor of the outer cylinder, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses charge λl on the inner conductor, so it must enclose charge $-\lambda l$ on the inner surface of the outer conductor. The charge per unit length on the inner surface of the outer cylinder is $-\lambda$.

<u>outer surface:</u> The outer cylinder carries no net charge. So if there is charge per unit length $-\lambda$ on its inner surface there must be charge per unit length $+\lambda$ on the outer surface.

EVALUATE: The electric field lines between the conductors originate on the surface charge on the outer surface of the inner conductor and terminate on the surface charges on the inner surface of the outer conductor. These surface charges are equal in magnitude (per unit length) and opposite in sign. The electric field lines outside the outer conductor originate from the surface charge on the outer surface of the outer conductor.

22.38. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius *r*, length *l* and that has the line of charge along its axis. The charge on a length *l* of the line of charge or of the tube is $q = \alpha l$.

EXECUTE: **(a)** (i) For $r < a$, Gauss's law gives $E(2\pi r l) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{\alpha l}{\epsilon_0}$ and $E = \dfrac{\alpha}{2\pi \epsilon_0 r}$.

(ii) The electric field is zero because these points are within the conducting material.

(iii) For $r > b$, Gauss's law gives $E(2\pi r l) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{2\alpha l}{\epsilon_0}$ and $E = \dfrac{\alpha}{\pi \epsilon_0 r}$.

The graph of *E* versus *r* is sketched in Figure 22.38.

(b) (i) The Gaussian cylinder with radius *r*, for $a < r < b$, must enclose zero net charge, so the charge per unit length on the inner surface is $-\alpha$. (ii) Since the net charge per length for the tube is $+\alpha$ and there is $-\alpha$ on the inner surface, the charge per unit length on the outer surface must be $+2\alpha$.

EVALUATE: For $r > b$ the electric field is due to the charge on the outer surface of the tube.

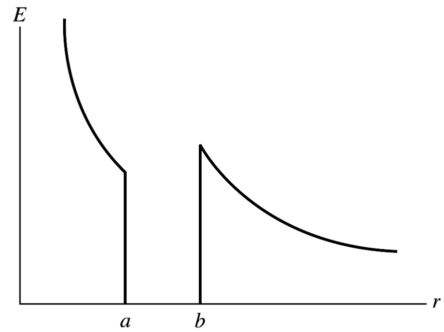


Figure 22.38

22.39. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r and length l, and that is coaxial with the cylindrical charge distributions. The volume of the Gaussian cylinder is $\pi r^2 l$ and the area of its curved surface is $2\pi rl$. The charge on a length l of the charge distribution is $q = \lambda l$, where $\lambda = \rho\pi R^2$.

EXECUTE: **(a)** For $r < R$, $Q_{\text{encl}} = \rho\pi r^2 l$ and Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{\rho\pi r^2 l}{\epsilon_0}$ and $E = \dfrac{\rho r}{2\epsilon_0}$, radially outward.

(b) For $r > R$, $Q_{\text{encl}} = \lambda l = \rho\pi R^2 l$ and Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{\rho\pi R^2 l}{\epsilon_0}$ and $E = \dfrac{\rho R^2}{2\epsilon_0 r} = \dfrac{\lambda}{2\pi\epsilon_0 r}$, radially outward.

(c) At $r = R$, the electric field for *both* regions is $E = \dfrac{\rho R}{2\epsilon_0}$, so they are consistent.

(d) The graph of E versus r is sketched in Figure 22.39.

EVALUATE: For $r > R$ the field is the same as for a line of charge along the axis of the cylinder.

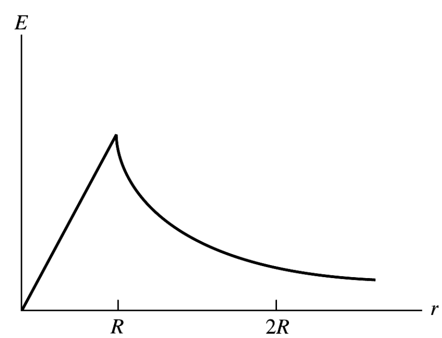


Figure 22.39

22.40. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the conducting spheres.

EXECUTE: **(a)** For $r < a, E = 0$, since these points are within the conducting material.

For $a < r < b, E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$, since there is $+q$ inside a radius r.

For $b < r < c, E = 0$, since these points are within the conducting material.

For $r > c, E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$, since again the total charge enclosed is $+q$.

(b) The graph of E versus r is sketched in Figure 22.40a.
(c) Since the Gaussian sphere of radius r, for $b < r < c$, must enclose zero net charge, the charge on the inner shell surface is $-q$.
(d) Since the hollow sphere has no net charge and has charge $-q$ on its inner surface, the charge on the outer shell surface is $+q$.
(e) The field lines are sketched in Figure 22.40b. Where the field is nonzero, it is radially outward.
EVALUATE: The net charge on the inner solid conducting sphere is on the surface of that sphere. The presence of the hollow sphere does not affect the electric field in the region $r < b$.

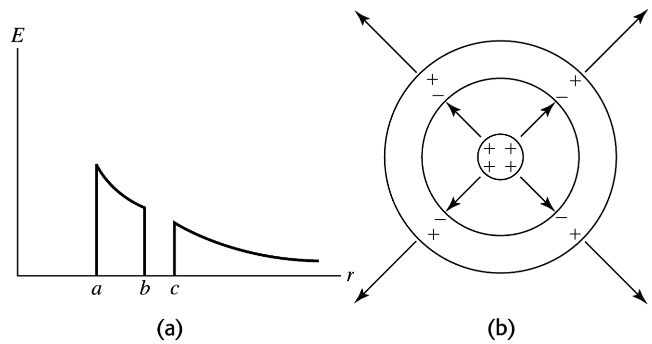


Figure 22.40

22.41. **IDENTIFY:** Apply Gauss's law.
SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charge distributions.
EXECUTE: **(a)** For $r < R$, $E = 0$, since these points are within the conducting material. For $R < r < 2R$, $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, since the charge enclosed is Q. The field is radially outward. For $r > 2R$, $E = \frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$ since the charge enclosed is $2Q$. The field is radially outward.
(b) The graph of E versus r is sketched in Figure 22.41.
EVALUATE: For $r < 2R$ the electric field is unaffected by the presence of the charged shell.

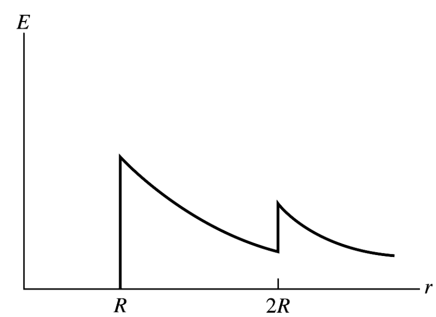


Figure 22.41

22.42. **IDENTIFY:** Apply Gauss's law and conservation of charge.
SET UP: Use a Gaussian surface that is a sphere of radius r and that has the point charge at its center.

EXECUTE: **(a)** For $r < a$, $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, radially outward, since the charge enclosed is Q, the charge of the point charge. For $a < r < b$, $E = 0$ since these points are within the conducting material. For $r > b$, $E = \frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$, radially inward, since the total enclosed charge is $-2Q$.

(b) Since a Gaussian surface with radius r, for $a < r < b$, must enclose zero net charge because $E = 0$ inside the conductor, the total charge on the inner surface is $-Q$ and the surface charge density on the inner surface is $\sigma = -\frac{Q}{4\pi a^2}$.

(c) Since the net charge on the shell is $-3Q$ and there is $-Q$ on the inner surface, there must be $-2Q$ on the outer surface. The surface charge density on the outer surface is $\sigma = -\frac{2Q}{4\pi b^2}$.

(d) The field lines and the locations of the charges are sketched in Figure 22.42a.

(e) The graph of E versus r is sketched in Figure 22.42b.

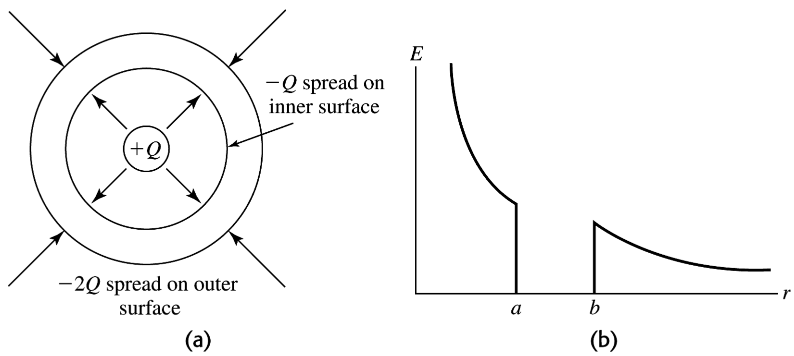
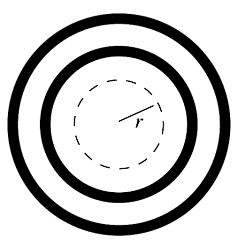


Figure 22.42

EVALUATE: For $r < a$ the electric field is due solely to the point charge Q. For $r > b$ the electric field is due to the charge $-2Q$ that is on the outer surface of the shell.

22.43. **IDENTIFY:** Apply Gauss's law to a spherical Gaussian surface with radius r. Calculate the electric field at the surface of the Gaussian sphere.

(a) SET UP: (i) $r < a$: The Gaussian surface is sketched in Figure 22.43a.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$.

$Q_{\text{encl}} = 0$; no charge is enclosed.

$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$ says

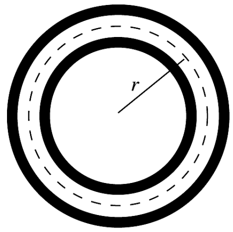
$E(4\pi r^2) = 0$ and $E = 0$.

Figure 22.43a

(ii) $a < r < b$: Points in this region are in the conductor of the small shell, so $E = 0$.

(iii) **SET UP:** $b < r < c$: The Gaussian surface is sketched in Figure 22.43b.

Apply Gauss's law to a spherical Gaussian surface with radius $b < r < c$.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$.

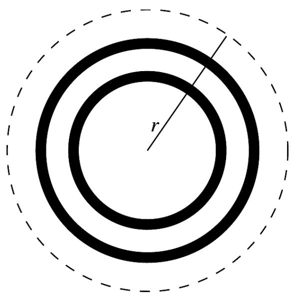
The Gaussian surface encloses all of the small shell and none of the large shell, so $Q_{\text{encl}} = +2q$.

Figure 22.43b

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{2q}{\epsilon_0}$ so $E = \dfrac{2q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

(iv) $c < r < d$: Points in this region are in the conductor of the large shell, so $E = 0$.

(v) **SET UP:** $r > d$: Apply Gauss's law to a spherical Gaussian surface with radius $r > d$, as shown in Figure 22.43c.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$.

The Gaussian surface encloses all of the small shell and all of the large shell, so $Q_{\text{encl}} = +2q + 4q = 6q$.

Figure 22.43c

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{6q}{\epsilon_0}$.

$E = \dfrac{6q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

The graph of E versus r is sketched in Figure 22.43d.

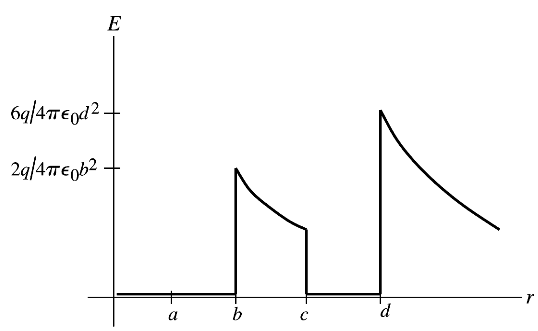


Figure 22.43d

(b) IDENTIFY and **SET UP:** Apply Gauss's law to a sphere that lies outside the surface of the shell for which we want to find the surface charge.

EXECUTE: (i) charge on inner surface of the small shell: Apply Gauss's law to a spherical Gaussian surface with radius $a < r < b$. This surface lies within the conductor of the small shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$, so there is zero charge on the inner surface of the small shell.
(ii) charge on outer surface of the small shell: The total charge on the small shell is $+2q$. We found in part (i) that there is zero charge on the inner surface of the shell, so all $+2q$ must reside on the outer surface.
(iii) charge on inner surface of large shell: Apply Gauss's law to a spherical Gaussian surface with radius $c < r < d$. The surface lies within the conductor of the large shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses the $+2q$ on the small shell so there must be charge $-2q$ on the inner surface of the large shell to make the total enclosed charge zero.
(iv) charge on outer surface of large shell: The total charge on the large shell is $+4q$. We showed in part (iii) that the charge on the inner surface is $-2q$, so there must be $+6q$ on the outer surface.
EVALUATE: The electric field lines for $b < r < c$ originate from the surface charge on the outer surface of the inner shell and all terminate on the surface charge on the inner surface of the outer shell. These surface charges have equal magnitude and opposite sign. The electric field lines for $r > d$ originate from the surface charge on the outer surface of the outer sphere.

22.44. IDENTIFY: Apply Gauss's law.
SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charged shells.
EXECUTE: **(a)** (i) For $r < a, E = 0$, since the charge enclosed is zero. (ii) For $a < r < b, E = 0$, since the points are within the conducting material. (iii) For $b < r < c, E = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$, outward, since the charge enclosed is $+2q$. (iv) For $c < r < d, E = 0$, since the points are within the conducting material. (v) For $r > d, E = 0$, since the net charge enclosed is zero. The graph of E versus r is sketched in Figure 22.44.
(b) (i) small shell inner surface: Since a Gaussian surface with radius r, for $a < r < b$, must enclose zero net charge, the charge on this surface is zero. (ii) small shell outer surface: $+2q$. (iii) large shell inner surface: Since a Gaussian surface with radius r, for $c < r < d$, must enclose zero net charge, the charge on this surface is $-2q$. (iv) large shell outer surface: Since there is $-2q$ on the inner surface and the total charge on this conductor is $-2q$, the charge on this surface is zero.
EVALUATE: The outer shell has no effect on the electric field for $r < c$. For $r > d$ the electric field is due only to the charge on the outer surface of the larger shell.

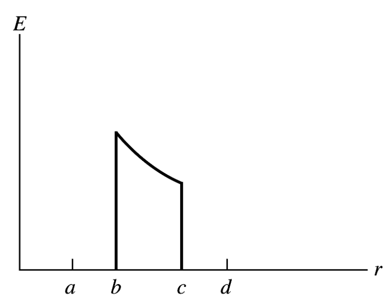


Figure 22.44

22.45. IDENTIFY: We apply Gauss's law in (a) and take a spherical Gaussian surface because of the spherical symmetry of the charge distribution. In (b), the net field is the vector sum of the field due to q and the field due to the sphere.

(a) SET UP: $\rho(r)=\frac{\alpha}{r}$, $dV=4\pi r^2 dr$, and $Q=\int_a^r \rho(r')dV$.

EXECUTE: For a Gaussian sphere of radius r, $Q_{\text{encl}}=\int_a^r \rho(r')dV=4\pi\alpha\int_a^r r'dr'=4\pi\alpha\frac{1}{2}(r^2-a^2)$.

Gauss's law says that $E(4\pi r^2)=\frac{2\pi\alpha(r^2-a^2)}{\epsilon_0}$, which gives $E=\frac{\alpha}{2\epsilon_0}\left(1-\frac{a^2}{r^2}\right)$.

(b) SET UP and EXECUTE: The electric field of the point charge is $E_q=\frac{q}{4\pi\epsilon_0 r^2}$. The total electric field is $E_{\text{total}}=\frac{\alpha}{2\epsilon_0}-\frac{\alpha}{2\epsilon_0}\frac{a^2}{r^2}+\frac{q}{4\pi\epsilon_0 r^2}$. For E_{total} to be constant, $-\frac{\alpha a^2}{2\epsilon_0}+\frac{q}{4\pi\epsilon_0}=0$ and $q=2\pi\alpha a^2$. The constant electric field is $\frac{\alpha}{2\epsilon_0}$.

EVALUATE: The net field is constant, but not zero.

22.46. IDENTIFY: Example 22.9 gives the expression for the electric field both inside and outside a uniformly charged sphere. Use $\vec{F}=-e\vec{E}$ to calculate the force on the electron.

SET UP: The sphere has charge $Q=+e$.

EXECUTE: **(a)** Only at $r=0$ is $E=0$ for the uniformly charged sphere.

(b) At points inside the sphere, $E_r=\frac{er}{4\pi\epsilon_0 R^3}$. The field is radially outward. $F_r=-eE=-\frac{1}{4\pi\epsilon_0}\frac{e^2 r}{R^3}$.

The minus sign denotes that F_r is radially inward. For simple harmonic motion, $F_r=-kr=-m\omega^2 r$, where $\omega=\sqrt{k/m}=2\pi f$. $F_r=-m\omega^2 r=-\frac{1}{4\pi\epsilon_0}\frac{e^2 r}{R^3}$ so $\omega=\sqrt{\frac{1}{4\pi\epsilon_0}\frac{e^2}{mR^3}}$ and $f=\frac{1}{2\pi}\sqrt{\frac{1}{4\pi\epsilon_0}\frac{e^2}{mR^3}}$.

(c) If $f=4.57\times10^{14}\text{ Hz}=\frac{1}{2\pi}\sqrt{\frac{1}{4\pi\epsilon_0}\frac{e^2}{mR^3}}$ then

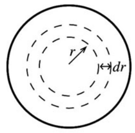
$R=\sqrt[3]{\frac{1}{4\pi\epsilon_0}\frac{(1.60\times10^{-19}\text{ C})^2}{4\pi^2(9.11\times10^{-31}\text{ kg})(4.57\times10^{14}\text{ Hz})^2}}=3.13\times10^{-10}\text{ m}$. The atom radius in this model is the correct order of magnitude.

(d) If $r>R$, $E_r=\frac{e}{4\pi\epsilon_0 r^2}$ and $F_r=-\frac{e^2}{4\pi\epsilon_0 r^2}$. The electron would still oscillate because the force is directed toward the equilibrium position at $r=0$. But the motion would not be simple harmonic, since F_r is proportional to $1/r^2$ and simple harmonic motion requires that the restoring force be proportional to the displacement from equilibrium.

EVALUATE: As long as the initial displacement is less than R the frequency of the motion is independent of the initial displacement.

22.47. (a) IDENTIFY: The charge density varies with r inside the spherical volume. Divide the volume up into thin concentric shells, of radius r and thickness dr. Find the charge dq in each shell and integrate to find the total charge.

SET UP: $\rho(r)=\rho_0(1-r/R)$ for $r\le R$ where $\rho_0=3Q/\pi R^3$. $\rho(r)=0$ for $r\ge R$. The thin shell is sketched in Figure 22.47a.



EXECUTE: The volume of such a shell is $dV = 4\pi r^2 dr$.

The charge contained within the shell is

$$dq = \rho(r)dV = 4\pi r^2 \rho_0 (1 - r/R)dr.$$

Figure 22.47a

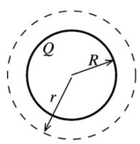
The total charge Q_{tot} in the charge distribution is obtained by integrating dq over all such shells into which the sphere can be subdivided:

$$Q_{\text{tot}} = \int dq = \int_0^R 4\pi r^2 \rho_0 (1 - r/R)dr = 4\pi\rho_0 \int_0^R (r^2 - r^3/R)dr$$

$Q_{\text{tot}} = 4\pi\rho_0 \left[\dfrac{r^3}{3} - \dfrac{r^4}{4R}\right]_0^R = 4\pi\rho_0 \left(\dfrac{R^3}{3} - \dfrac{R^4}{4R}\right) = 4\pi\rho_0 (R^3/12) = 4\pi(3Q/\pi R^3)(R^3/12) = Q,$ as was to be shown.

(b) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r > R$.

SET UP: The Gaussian surface is shown in Figure 22.47b.



EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$.

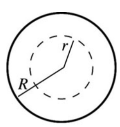
$$E(4\pi r^2) = \frac{Q}{\epsilon_0}.$$

Figure 22.47b

$E = \dfrac{Q}{4\pi \epsilon_0 r^2}$; same as for point charge of charge Q.

(c) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r < R$.

SET UP: The Gaussian surface is shown in Figure 22.47c.



EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$.

$$\Phi_E = E(4\pi r^2).$$

Figure 22.47c

To calculate the enclosed charge Q_{encl} use the same technique as in part (a), except integrate dq out to r rather than R. (We want the charge that is inside radius r.)

$$Q_{\text{encl}} = \int_0^r 4\pi r'^2 \rho_0 \left(1 - \frac{r'}{R}\right)dr' = 4\pi\rho_0 \int_0^r \left(r'^2 - \frac{r'^3}{R}\right)dr'.$$

$$Q_{\text{encl}} = 4\pi\rho_0 \left[\frac{r'^3}{3} - \frac{r'^4}{4R}\right]_0^r = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) = 4\pi\rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R}\right).$$

$\rho_0 = \dfrac{3Q}{\pi R^3}$ so $Q_{\text{encl}} = 12Q\dfrac{r^3}{R^3}\left(\dfrac{1}{3} - \dfrac{r}{4R}\right) = Q\left(\dfrac{r^3}{R^3}\right)\left(4 - 3\dfrac{r}{R}\right).$

Thus Gauss's law gives $E(4\pi r^2) = \dfrac{Q}{\epsilon_0}\left(\dfrac{r^3}{R^3}\right)\left(4 - 3\dfrac{r}{R}\right).$

$E = \dfrac{Qr}{4\pi\varepsilon_0 R^3}\left(4 - \dfrac{3r}{R}\right), r \le R.$

(d) The graph of E versus r is sketched in Figure 22.47d.

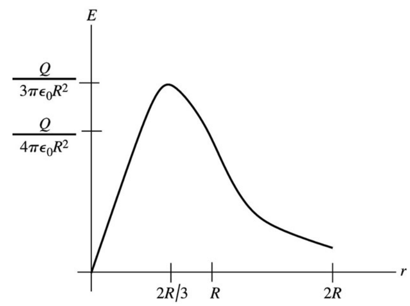


Figure 22.47d

(e) Where the electric field is a maximum, $\dfrac{dE}{dr} = 0.$ Thus

$\dfrac{d}{dr}\left(4r - \dfrac{3r^2}{R}\right) = 0$ so $4 - 6r/R = 0$ and $r = 2R/3.$

At this value of r, $E = \dfrac{Q}{4\pi\epsilon_0 R^3}\left(\dfrac{2R}{3}\right)\left(4 - \dfrac{3}{R}\dfrac{2R}{3}\right) = \dfrac{Q}{3\pi\epsilon_0 R^2}.$

EVALUATE: Our expressions for $E(r)$ for $r < R$ and for $r > R$ agree at $r = R.$ The results of part (e) for the value of r where $E(r)$ is a maximum agrees with the graph in part (d).

22.48. **IDENTIFY:** The method of Example 22.9 shows that the electric field outside the sphere is the same as for a point charge of the same charge located at the center of the sphere.

SET UP: The charge of an electron has magnitude $e = 1.60\times10^{-19}$ C.

EXECUTE: **(a)** $E = k\dfrac{|q|}{r^2}.$ For $r = R = 0.150$ m, $E = 1390$ N/C so

$|q| = \dfrac{Er^2}{k} = \dfrac{(1390 \text{ N/C})(0.150 \text{ m})^2}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 3.479\times10^{-9}$ C. The number of excess electrons is

$\dfrac{3.479\times10^{-9} \text{ C}}{1.60\times10^{-19} \text{ C/electron}} = 2.17\times10^{10}$ electrons.

(b) $r = R + 0.100 \text{ m} = 0.250$ m. $E = k\dfrac{|q|}{r^2} = (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{3.479\times10^{-9} \text{ C}}{(0.250 \text{ m})^2} = 5.00\times10^2$ N/C.

EVALUATE: The magnitude of the electric field decreases according to the square of the distance from the center of the sphere.

22.49. **IDENTIFY:** The charge density inside the cylinder is not uniform but depends on distance from the central axis. We want the electric field both inside and outside the cylinder.

SET UP and EXECUTE: <u>Inside the cylinder ($r \le R$)</u>: For the Gaussian surface, choose a cylinder of length L and radius $r < R$ that is coaxial with the charged cylinder. The electric field is perpendicular to the curved surface and parallel to the ends of this surface. The charge density inside the cylinder depends on r, so we must integrate to get the charge q within the Gaussian surface.

$q = \int \rho dV = \int \alpha\left(1-\frac{r}{R}\right)2\pi rLdr = 2\pi L\alpha r^2\left(\frac{1}{2}-\frac{r}{3R}\right)$. Now apply Gauss's law using the cylindrical Gaussian surface. $E(2\pi rL) = \frac{2\pi L\alpha r^2\left(\frac{1}{2}-\frac{r}{3R}\right)}{\epsilon_0}$. $E = \frac{\alpha r}{\epsilon_0}\left(\frac{1}{2}-\frac{r}{3R}\right)$.

Outside the cylinder ($r \geq R$): Use the same Gaussian surface as above except $r > R$. The enclosed charge is just the charge within the cylinder, not the full Gaussian surface. Use the same formula for q that we found above except use $r = R$. This gives $q = 2\pi L\alpha R^2\left(\frac{1}{2}-\frac{R}{3R}\right) = \pi L\alpha R^2/3$. Now apply Gauss's law.

$E(2\pi rL) = \frac{\pi L\alpha R^2}{3\epsilon_0}$. $E = \frac{\alpha R^2}{6\epsilon_0 r}$.

EVALUATE: Compare E at the surface of the cylinder using the two equations we derived. $E_{\text{inside}} = \frac{\alpha R}{\epsilon_0}\left(\frac{1}{2}-\frac{R}{3R}\right) = \frac{\alpha R}{6\epsilon_0}$. $E_{\text{outside}} = \frac{\alpha R^2}{6\epsilon_0 R} = \frac{\alpha R}{6\epsilon_0}$. Both equations give the same result, as they should.

22.50. **IDENTIFY:** The charge density inside the sphere is not uniform but depends on distance from the center. We want the electric field both inside and outside the sphere. The charge density inside the sphere is $\rho(r) = \rho_0\left(1-\frac{r}{R}\right)$. For Gaussian surfaces, choose a sphere of radius r concentric with the charged sphere.

SET UP and EXECUTE: **(a)** Inside the sphere ($r \leq R$): We need the charge contained within the Gaussian surface. $q = \int \rho dV = \int \rho_0\left(1-\frac{r}{R}\right)4\pi r^2 Ldr = 4\pi\rho_0\left(\frac{r^3}{3}-\frac{r^4}{4R}\right)$. Now use Gauss's law.

$E(4\pi r^2) = \frac{4\pi\rho_0}{\varepsilon_0}\left(\frac{r^3}{3}-\frac{r^4}{4R}\right)$. $E = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$.

(b) Outside the sphere ($r \geq R$): For q use the same result as in part (a) except let $r = R$, giving $q = \pi\rho_0 R^3/3$. Now use Gauss's law. $E(4\pi r^2) = \frac{\pi\rho_0 R}{3\epsilon_0}$. $E = \frac{\rho_0 R^3}{12\epsilon_0 r^2}$.

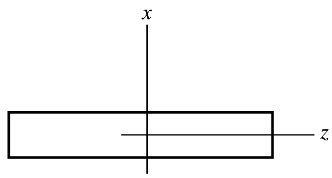
EVALUATE: The electric field should be continuous at the surface of the sphere. Evaluate our results above at $r = R$. $E_{\text{inside}} = \frac{\rho_0}{\epsilon_0}\left(\frac{R}{3}-\frac{R^2}{4R}\right) = \frac{\rho_0 R}{12\epsilon_0}$. $E_{\text{outside}} = \frac{\rho_0 R^3}{12\epsilon_0 R^2} = \frac{\rho_0 R}{12\epsilon_0}$. The field is continuous at the surface.

SET UP and EXECUTE: **(c)** Where is E a maximum? For a maximum, $dE/dr = 0$. Inside the sphere we have $\frac{dE}{dr} = \frac{\rho_0}{\epsilon_0}\left(\frac{1}{3}-\frac{2r}{4R}\right) = 0, r = \frac{2}{3}R$. E decreases after $r = 2R/3$ and is equal to E_{outside} at the surface.

After $r = R$, E_{out} decreases as r increases. So the maximum field occurs at $r = \frac{2}{3}R$.

EVALUATE: For $r > R$, we know that the field should be $E = \frac{Q}{4\pi\epsilon_0 r^2}$, but in part (b) we got $E = \frac{\rho_0 R^3}{12\epsilon_0 r^2}$. But $Q = \pi\rho_0 R^3/3$, so $E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{\pi\rho_0 R^3/3}{4\pi\epsilon_0 r^2} = \frac{\rho_0 R^3}{12\epsilon_0 r^2}$. Our result agrees with the expected equation.

22.51. **(a) IDENTIFY and SET UP:** Consider the direction of the field for x slightly greater than and slightly less than zero. The slab is sketched in Figure 22.51a.



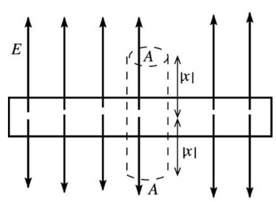
$$\rho(x) = \rho_0 (x/d)^2.$$

Figure 22.51a

EXECUTE: The charge distribution is symmetric about $x = 0$, so by symmetry $E(x) = E(-x)$. But for $x > 0$ the field is in the $+x$-direction and for $x < 0$ the field is in the $-x$-direction. At $x = 0$ the field can't be both in the $+x$- and $-x$-directions so must be zero. That is, $E_x(x) = -E_x(-x)$. At point $x = 0$ this gives $E_x(0) = -E_x(0)$ and this equation is satisfied only for $E_x(0) = 0$.

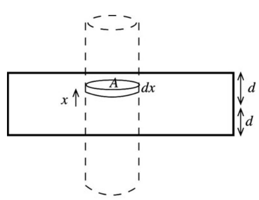
(b) IDENTIFY and SET UP: $\underline{|x| > d}$ (outside the slab).

Apply Gauss's law to a cylindrical Gaussian surface whose axis is perpendicular to the slab and whose end caps have area A and are the same distance $|x| > d$ from $x = 0$, as shown in Figure 22.51b.



EXECUTE: $\Phi_E = 2EA$.

Figure 22.51b



To find Q_{encl} consider a thin disk at coordinate x and with thickness dx, as shown in Figure 22.51c. The charge within this disk is $dq = \rho dV = \rho A dx = (\rho_0 A/d^2)\, x^2 dx$.

Figure 22.51c

The total charge enclosed by the Gaussian cylinder is

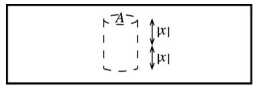
$$Q_{\text{encl}} = 2\int_0^d dq = (2\rho_0 A/d^2)\int_0^d x^2 dx = (2\rho_0 A/d^2)(d^3/3) = \tfrac{2}{3}\rho_0 A d.$$

Then $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $2EA = 2\rho_0 A d/3\epsilon_0$. This gives $E = \rho_0 d/3\epsilon_0$.

$\vec{E}$ is directed away from $x = 0$, so $\vec{E} = (\rho_0 d/3\epsilon_0)(x/|x|)\hat{\boldsymbol{i}}$.

(c) IDENTIFY and SET UP: $\underline{|x| < d}$ (inside the slab).

Apply Gauss's law to a cylindrical Gaussian surface whose axis is perpendicular to the slab and whose end caps have area A and are the same distance $|x| < d$ from $x = 0$, as shown in Figure 22.51d.



EXECUTE: $\Phi_E = 2EA.$

Figure 22.51d

Q_{encl} is found as above, but now the integral on dx is only from 0 to x instead of 0 do d.

$$Q_{\text{encl}} = 2\int_0^x dq = (2\rho_0 A/d^2)\int_0^x x^2 dx = (2\rho_0 A/d^2)(x^3/3).$$

Then $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $2EA = 2\rho_0 Ax^3/3\epsilon_0\, d^2$. This gives $E = \rho_0 x^3/3\epsilon_0\, d^2$.

$\vec{E}$ is directed away from $x = 0$, so $\vec{E} = (\rho_0 x^3/3\epsilon_0\, d^2)\hat{\boldsymbol{i}}$.

EVALUATE: Note that $E = 0$ at $x = 0$ as stated in part (a). Note also that the expressions for $|x| > d$ and $|x| < d$ agree for $x = d$.

22.52. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the spherical distribution of charge. The volume of a thin spherical shell of radius r and thickness dr is $dV = 4\pi r^2 dr$.

EXECUTE: **(a)** $Q = \int \rho(r)dV = 4\pi\int_0^\infty \rho(r)r^2 dr = 4\pi\rho_0\int_0^R \left(1 - \dfrac{4r}{3R}\right)r^2 dr = 4\pi\rho_0\left[\int_0^R r^2 dr - \dfrac{4}{3R}\int_0^R r^3 dr\right].$

$Q = 4\pi\rho_0\left[\dfrac{R^3}{3} - \dfrac{4}{3R}\dfrac{R^4}{4}\right] = 0.$ The total charge is zero.

(b) For $r \ge R$, $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\epsilon_0} = 0,$ so $E = 0.$

(c) For $r \le R$, $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{4\pi}{\epsilon_0}\int_0^r \rho(r')r'^2 dr'.$ $E4\pi r^2 = \dfrac{4\pi\rho_0}{\epsilon_0}\left[\int_0^r r'^2 dr' - \dfrac{4}{3R}\int_0^r r'^3 dr'\right]$ and

$E = \dfrac{\rho_0}{\epsilon_0}\dfrac{1}{r^2}\left[\dfrac{r^3}{3} - \dfrac{r^4}{3R}\right] = \dfrac{\rho_0}{3\epsilon_0}r\left[1 - \dfrac{r}{R}\right].$

(d) The graph of E versus r is sketched in Figure 22.52.

(e) Where E is a maximum, $\dfrac{dE}{dr} = 0.$ This gives $\dfrac{\rho_0}{3\epsilon_0} - \dfrac{2\rho_0 r_{\max}}{3\epsilon_0\, R} = 0$ and $r_{\max} = \dfrac{R}{2}.$ At this r,

$E = \dfrac{\rho_0}{3\epsilon_0}\dfrac{R}{2}\left[1 - \dfrac{1}{2}\right] = \dfrac{\rho_0 R}{12\epsilon_0}.$

EVALUATE: The result in part (b) for $r \le R$ gives $E = 0$ at $r = R$; the field is continuous at the surface of the charge distribution.

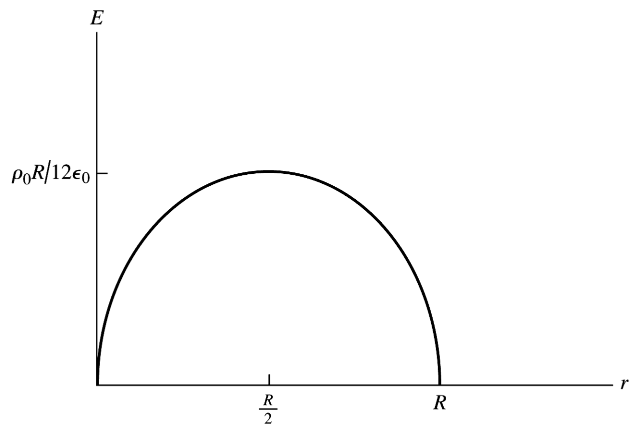


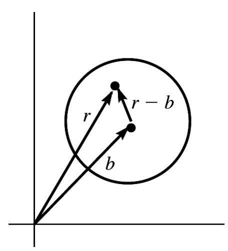
Figure 22.52

22.53. **(a) IDENTIFY:** Use $\vec{E}(\vec{r})$ from Example (22.9) (inside the sphere) and relate the position vector of a point inside the sphere measured from the origin to that measured from the center of the sphere.

SET UP: For an insulating sphere of uniform charge density ρ and centered at the origin, the electric field inside the sphere is given by $E = Qr'/4\pi\epsilon_0 R^3$ (Example 22.9), where $\vec{r}'$ is the vector from the center of the sphere to the point where E is calculated.

But $\rho = 3Q/4\pi R^3$ so this may be written as $E = \rho r/3\epsilon_0$. And $\vec{E}$ is radially outward, in the direction of $\vec{r}'$, so $\vec{E} = \rho\vec{r}'/3\epsilon_0$.

For a sphere whose center is located by vector $\vec{b}$, a point inside the sphere and located by $\vec{r}$ is located by the vector $\vec{r}' = \vec{r} - \vec{b}$ relative to the center of the sphere, as shown in Figure 22.53.



EXECUTE: Thus $\vec{E} = \dfrac{\rho(\vec{r} - \vec{b})}{3\epsilon_0}$.

Figure 22.53

EVALUATE: When $b = 0$ this reduces to the result of Example 22.9. When $\vec{r} = \vec{b}$, this gives $E = 0$, which is correct since we know that $E = 0$ at the center of the sphere.

(b) IDENTIFY: The charge distribution can be represented as a uniform sphere with charge density ρ and centered at the origin added to a uniform sphere with charge density $-\rho$ and centered at $\vec{r} = \vec{b}$.

SET UP: $\vec{E} = \vec{E}_{\text{uniform}} + \vec{E}_{\text{hole}}$, where $\vec{E}_{\text{uniform}}$ is the field of a uniformly charged sphere with charge density ρ and $\vec{E}_{\text{hole}}$ is the field of a sphere located at the hole and with charge density $-\rho$. (Within the spherical hole the net charge density is $+\rho - \rho = 0$.)

EXECUTE: $\vec{E}_{\text{uniform}} = \dfrac{\rho\vec{r}}{3\epsilon_0}$, where $\vec{r}$ is a vector from the center of the sphere.

$\vec{E}_{\text{hole}} = \dfrac{-\rho(\vec{r} - \vec{b})}{3\epsilon_0}$, at points inside the hole. Then $\vec{E} = \dfrac{\rho\vec{r}}{3\epsilon_0} + \left(\dfrac{-\rho(\vec{r} - \vec{b})}{3\epsilon_0}\right) = \dfrac{\rho\vec{b}}{3\epsilon_0}$.

EVALUATE: $\vec{E}$ is independent of $\vec{r}$ so is uniform inside the hole. The direction of $\vec{E}$ inside the hole is in the direction of the vector $\vec{b}$, the direction from the center of the insulating sphere to the center of the hole.

22.54. **IDENTIFY:** We first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields: that of a solid cylinder on-axis, and one off-axis, at the location of the hole.

SET UP: Let $\vec{r}$ locate a point within the hole, relative to the axis of the cylinder and let $\vec{r}'$ locate this point relative to the axis of the hole. Let $\vec{b}$ locate the axis of the hole relative to the axis of the cylinder. As shown in Figure 22.54, $\vec{r}' = \vec{r} - \vec{b}$. Problem 22.39 shows that at points within a long insulating cylinder, $\vec{E} = \dfrac{\rho\vec{r}}{2\epsilon_0}$.

EXECUTE: $\vec{E}_{\text{off-axis}} = \dfrac{\rho\vec{r}'}{2\epsilon_0} = \dfrac{\rho(\vec{r}-\vec{b})}{2\epsilon_0}$. $\vec{E}_{\text{hole}} = \vec{E}_{\text{cylinder}} - \vec{E}_{\text{off-axis}} = \dfrac{\rho\vec{r}}{2\epsilon_0} - \dfrac{\rho(\vec{r}-\vec{b})}{2\epsilon_0} = \dfrac{\rho\vec{b}}{2\epsilon_0}$.

Note that $\vec{E}$ is uniform.

EVALUATE: If the hole is coaxial with the cylinder, $b = 0$ and $E_{\text{hole}} = 0$.

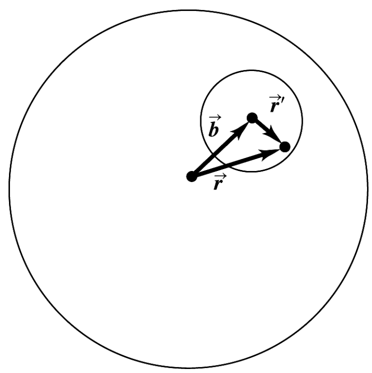


Figure 22.54

22.55. **IDENTIFY** and **SET UP:** For a uniformly charged sphere, $E = k\dfrac{|Q|}{r^2}$, so $Er^2 = k|Q|$ = constant. For a long uniform line of charge, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$, so $Er = \dfrac{\lambda}{2\pi\epsilon_0}$ = constant.

EXECUTE: **(a)** Figure 22.55a shows the graphs for data set A. We see that the graph of Er versus r is a horizontal line, which means that Er = constant. Therefore data set A is for a uniform straight line of charge.

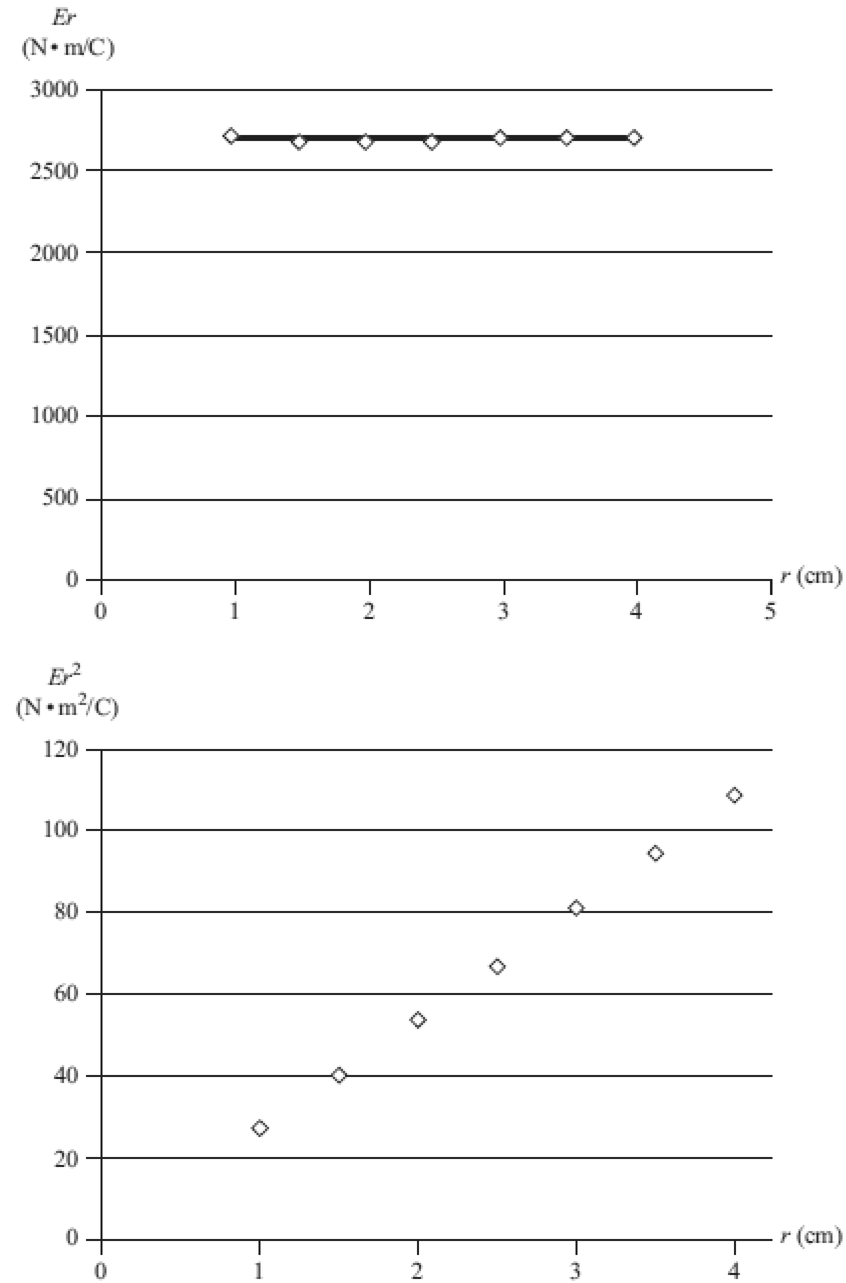
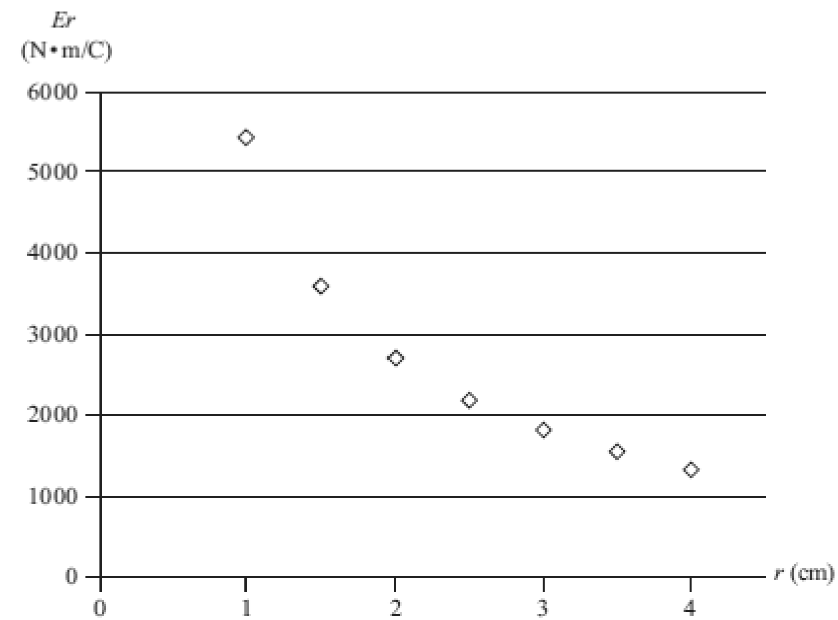


Figure 22.55a

Figure 22.55b shows the graphs for data set B. We see that the graph of Er^2 versus r is a horizontal line, so Er^2 = constant. Thus data set B is for a uniformly charged sphere.



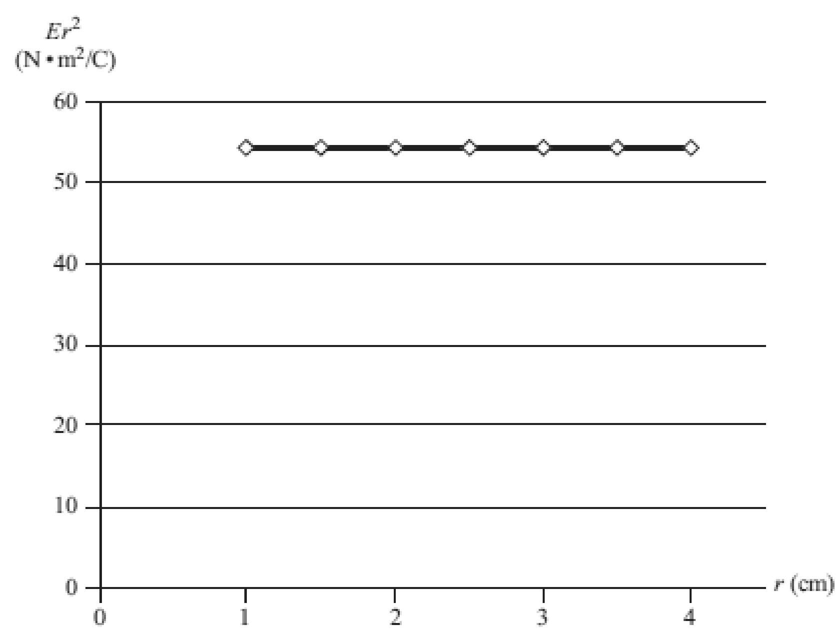


Figure 22.55b

(b) For A: $E = \dfrac{\lambda}{2\pi \epsilon_0 r}$, so $\lambda = 2\pi \epsilon_0 Er$. From our graph in Figure 22.55a, Er = constant = 2690 $\text{N} \cdot \text{m/C}$. Therefore

$\lambda = 2\pi \epsilon_0 Er = 2\pi(8.854 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2)\ (2690\ \text{N} \cdot \text{m/C}) = 1.50 \times 10^{-7}$ C/m = 0.150 μC/m.

For B: $E = k\dfrac{|Q|}{r^2}$, so $kQ = Er^2$ = constant, which means that Q = (constant)/k. From our graph in Figure 22.55b, Er^2 = constant = 54.1 $\text{N} \cdot \text{m}^2/\text{C}$. Therefore

$Q = (54.1\ \text{N} \cdot \text{m}^2/\text{C}) / (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2) = 6.0175 \times 10^{-9}$ C.

The charge density ρ is $\rho = \dfrac{Q}{V} = \dfrac{Q}{\frac{4}{3}\pi R^3}$ = (6.0175×10^{-9} C)/[(4π/3)(0.00800 m)^3 = 2.81×10^{-3} C/m^3.

EVALUATE: A linear charge density of 0.150 C/m and a volume charge density of 2.81×10^{-3} C/m^3 are both physically reasonable and could be achieved in a normal laboratory.

22.56. **IDENTIFY** and **SET UP:** The electric field inside a uniform sphere of charge does not follow an inverse-square law. Apply Gauss's law, $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\epsilon_0}$, to find the field.

SET UP: Apply $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\epsilon_0}$. As the Gaussian surface, use a sphere of radius r that is centered on the given sphere.

EXECUTE: Gauss's law gives $E(4\pi r^2) = \dfrac{\rho\left(\frac{4}{3}\pi r^3\right)}{\epsilon_0}$, from which we get $E = \dfrac{\rho}{3\epsilon_0} r$. Therefore in a graph of E versus r, the slope is $\dfrac{\rho}{3\epsilon_0}$. From the graph in the problem, the slope is

slope = $\dfrac{(6-3)\times 10^4\ \text{N/C}}{(8-4)\times 10^{-3}\ \text{m}} = 7.5\times 10^6\ \text{N/m}\cdot\text{C}$. Solving for ρ gives

ρ = (slope)($3\epsilon_0$) = $(7.5\times 10^6\ \text{N/m}\cdot\text{C})\,(3)\,(8.854\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)$ = 1.99×10^{-4} C/m^3.

EVALUATE: A sphere of volume 1.0 m^3 would have only 199 μC of charge, which is physically realistic.

22.57. **IDENTIFY** and **SET UP:** Apply Gauss's law, $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\epsilon_0}$. The enclosed charge is $Q_{\text{encl}} = \rho V$, where $V = \dfrac{4}{3}\pi r^3$ for a sphere of radius r. Read the charge densities from the graph in the problem.

EXECUTE: Apply Gauss's law $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\epsilon_0}$. As a Gaussian surface, use a sphere of radius r centered on the given sphere. This gives $E(4\pi r^2) = Q_{\text{encl}}/\epsilon_0$, so $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_{\text{encl}}}{r^2} = k\dfrac{Q_{\text{encl}}}{r^2}$. In each case, we must first use $Q_{\text{encl}} = \rho V$ to calculate Q_{encl} and then use that result to calculate E.

(i) First find Q_{encl}: $Q_{\text{encl}} = \rho V$ = (10.0×10^{-6} C/m^3)(4π/3)(0.00100 m)^3 = 4.19×10^{-14} C.

Now calculate E: $E = k\dfrac{Q_{\text{encl}}}{r^2}$ = (8.99×10^9 N·m^2/C^2)(4.19×10^{-14} C)/(0.00100 m)^2 = 377 N/C.

(ii) Q_{encl} = (10.0×10^{-6} C/m^3)(4π/3)(0.00200 m)^3 + (4.0×10^{-6} C/m^3)(4π/3)[(0.00300 m)^3 – (0.00200 m)^3]
Q_{encl} = 6.534×10^{-13} C.

$E = k\dfrac{Q_{\text{encl}}}{r^2}$ = (8.99×10^9 N·m^2/C^2)(6.534×10^{-13} C)/(0.00300 m)^2 = 653 N/C.

(iii) Q_{encl} = (10.0×10^{-6} C/m^3)(4π/3)(0.00200 m)^3 + (4.0×10^{-6} C/m^3)(4π/3)[(0.00400 m)^3 – (0.00200 m)^3] + (–2.0×10^{-6} C/m^3)(4π/3)[(0.00500 m)^3 – (0.00400 m)^3].
Q_{encl} = 7.624×10^{-13} C.

$E = k\dfrac{Q_{\text{encl}}}{r^2}$ = (8.99×10^9 N·m^2/C^2) (7.624×10^{-13} C)/(0.00500 m)^2 = 274 N/C.

(iv) Q_{encl} = 7.624×10^{-13} C + (–2.0×10^{-6} C/m^3)(4π/3)[(0.00600 m)^3 – (0.00500 m)^3] = 0, so E = 0.

EVALUATE: We found that E = 0 at r = 7.00 mm, but E is also zero at all points beyond r = 6.00 mm because the enclosed charge is zero for any Gaussian surface having a radius r > 6.00 mm.

22.58. **IDENTIFY:** Electrostatic forces affect the behavior of pollen in flowers.

SET UP and EXECUTE: **(a)** Estimate: Diameter of central disk is 1.0 cm.

(b) $q = (75{,}000 \text{ electrons})(1.60\times10^{-19} \text{ C/electron}) = 1.2\times10^{-14} \text{ C}$.

(c) We want the electric field 1.0 cm from the bee. The bee is a sphere, so it is equivalent to a point charge at the edge of the disk. Using $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ with $q = 1.2\times10^{-14}$ C and $r = 1.0$ cm $= 0.010$ m, we get $E = 0.27$ N/C.

(c) We want the charge on the pollen. The force on the pollen is due to the electric field of the bee, so $F = qE$, which gives $q = F/E = (10 \text{ pN})/(0.27 \text{ N/C}) = 3.7\times10^{-11}$ C.

EVALUATE: The charge on the pollen is very small, but a pollen grain is extremely tiny and needs only a small force to be pulled off the stamen.

22.59. **IDENTIFY:** The charge density inside the cylinder is not uniform but depends on distance from the central axis. It has the form $\rho(r) = \rho_0\left(1-\dfrac{r}{R}\right)$.

SET UP and EXECUTE: **(a)** We want the linear charge density λ of the cylinder. $\lambda = q/L$, so we need to find q in terms of L. $q = \int \rho dV = \int_0^R \rho_0\left(1-\dfrac{r}{R}\right)2\pi rL dr = \dfrac{\pi L\rho_0 R^2}{3}$. Therefore

$$\lambda = \frac{q}{L} = \frac{\pi L\rho_0 R^2/3}{L} = \frac{\pi\rho_0 R^2}{3}.$$

(b) We want the period T of the orbit. Use $\Sigma F = \dfrac{mv^2}{r} = mr\omega^2$ with $F = QE_{\text{cylinder}}$. For a very long cylinder $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$, so $F = QE = \dfrac{Q\lambda}{2\pi\epsilon_0 r}$. Using the λ we found in part (a) gives

$F = \dfrac{Q\left(\pi\rho_0 R^2/3\right)}{2\pi\epsilon_0 r} = \dfrac{Q\rho_0 R^2_{\text{cylinder}}}{6\epsilon_0 R_{\text{orbit}}}$. Using $\Sigma F = mr\omega^2$ gives $\dfrac{Q\rho_0 R^2_{\text{cylinder}}}{6\epsilon_0 R_{\text{orbit}}} = MR_{\text{orbit}}\omega^2$. Solving for ω and using $T = 2\pi/\omega$ gives $T = 2\pi\left(\dfrac{R_{\text{orbit}}}{R_{\text{cylinder}}}\right)\sqrt{\dfrac{6\epsilon_0 M}{Q\rho_0}}$.

EVALUATE: According to our results, if Q is large, T is small. This is reasonable because the force on the particle will be large resulting in fast motion. If M is large, T is small because the particle has more inertia. If R_{orbit} is large, T is large since the particle moves slower at a greater distance. All these results are reasonable.

22.60. **IDENTIFY:** This problem relates the electric flux through an object due to an external electric field and the force exerted on it by that field.

SET UP and EXECUTE: **(a)** Consider a flat charged surface of area A with charge Q in an external electric field of magnitude E. The force perpendicular to this surface is $F_\perp = QE_\perp$. The surface charge density is σ, so $Q = \sigma A$. Therefore $F_\perp = \sigma AE_\perp$. But $AE_\perp = \Phi_E$, so $F_\perp = \sigma\Phi_E$.

(b) We want the mass M for the hemisphere to remain stationary. The electric field due to the sheet is uniform and equal to $\dfrac{\sigma}{2\epsilon_0}$. We can reduce the hemisphere to a flat disk of radius R with charge Q, so $\sigma = Q/\pi R^2$. Using the result from part (a), we have

$F_{\text{el}} = \sigma_{\text{disk}}\Phi_{\text{through disk due to sheet}} = \sigma_{\text{disk}}E_{\text{sheet}}A_{\text{disk}} = \left(\dfrac{Q}{\pi R^2}\right)\left(\dfrac{\sigma}{2\epsilon_0}\right)\left(\pi R^2\right) = \dfrac{Q\sigma}{2\epsilon_0}$. The force of gravity and the electric force must be equal for balance, so $\dfrac{Q\sigma}{2\epsilon_0} = Mg$, giving $M = \dfrac{Q\sigma}{2\epsilon_0 g}$.

(c) We want the acceleration of the hemisphere. The charge density is the same for the sheet and the disk. From (b) we have $F_{\text{el}} = \dfrac{Q\sigma}{2\epsilon_0}$. $Q = \sigma(\pi R^2) = \pi R^2\sigma$. Therefore $F_{\text{el}} = \dfrac{\sigma^2\pi R^2}{2\epsilon_0}$. Using $\sum F_y = ma_y$ gives $a = \dfrac{F_{\text{el}} - Mg}{M} = \dfrac{\sigma^2\pi R^2}{2\epsilon_0 M} - g$. Using the given numbers for σ, R, and M gives $a = 6.17\ \text{m/s}^2$.

EVALUATE: The analysis in (b) and (c) is simplified because the electric field due to a very large sheet is independent of the distance from the sheet and is uniform.

22.61. **IDENTIFY:** A uniformly charged sphere is totally within a hollow cylinder. We want the electric flux through the rounded side of the cylinder and the two flat end caps.

SET UP: The flux through a surface is $\Phi_E = \int E_\perp dA$. The charged sphere is equivalent to a point charge at its center, so $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ where $|q| = Q$.

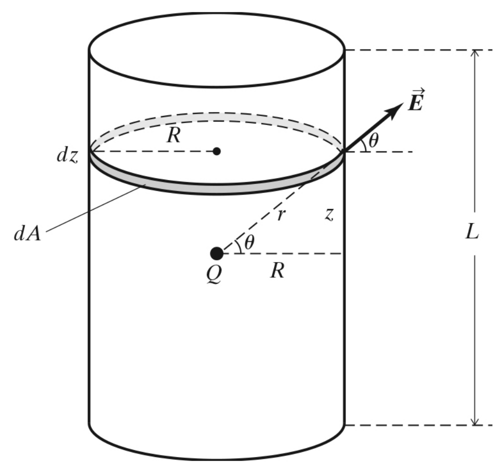


Figure 22.61a

EXECUTE: **(a)** We want the flux through the rounded side of the cylinder. Fig. 22.61a shows the set up of the integral with the central sphere shown as a point charge Q. $dA = 2\pi R dz$, $r^2 = z^2 + R^2$, $E_\perp = E\cos\theta$, and $\cos\theta = \dfrac{R}{r} = \dfrac{R}{\sqrt{z^2+R^2}}$. Calling $1/4\pi\epsilon_0 \equiv k$, we have

$\Phi_E = \int E_\perp dA = 2\int_{z=0}^{L/2} E\cos\theta dA = 2\int_0^{L/2} \dfrac{kQ}{z^2+R^2}\dfrac{R}{\sqrt{z^2+R^2}}2\pi R dz$. Using the integral tables in Appendix B

gives $\Phi_E = (4\pi QkR^2)\left(\dfrac{1}{R^2}\dfrac{z}{\sqrt{z^2+R^2}}\Bigg|_0^{L/2}\right) = \dfrac{2\pi QkL}{\sqrt{(L/2)^2+R^2}} = \dfrac{QL}{2\epsilon_0\sqrt{(L/2)^2+R^2}}$.

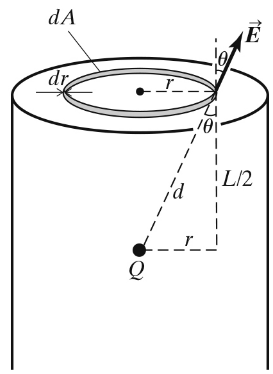


Figure 22.61b

(b) We want the flux through the upper cap. Fig. 22.61b shows the set up of the integral. Using $dA = 2\pi r dr$, $d^2 = r^2 + (L/2)^2$, $E = \dfrac{kQ}{d^2} = \dfrac{kQ}{r^2 + (L/2)^2}$, and $\cos\theta = \dfrac{L/2}{d} = \dfrac{L/2}{\sqrt{r^2 + (L/2)^2}}$, we have

$\Phi_E = \int E\cos\theta dA = \int_0^R \dfrac{kQ}{r^2 + (L/2)^2}\dfrac{L/2}{\sqrt{r^2 + (L/2)^2}} 2\pi r dr$. Using integral tables from Appendix B gives

$\Phi_E = (kQ\pi L)\left(-\dfrac{1}{\sqrt{r^2 + (L/2)^2}}\Bigg|_0^R\right) = kQ\pi L\left(\dfrac{2}{L} - \dfrac{1}{\sqrt{R^2 + (L/2)^2}}\right)$. Using $1/4\pi\epsilon_0 \equiv k$, we get

$\Phi_E = \dfrac{QL}{4\epsilon_0}\left(\dfrac{2}{L} - \dfrac{1}{\sqrt{R^2 + (L/2)^2}}\right)$.

(c) The solution is exactly the same as for part (b) and gives the same answer.

(d) $\Phi_E(\text{total}) = \Phi_{\text{sides}} + 2\Phi_{\text{cap}} = \dfrac{QL}{2\epsilon_0\sqrt{(L/2)^2 + R^2}} + 2\left[\dfrac{QL}{4\epsilon_0}\left(\dfrac{2}{L} - \dfrac{1}{\sqrt{R^2 + (L/2)^2}}\right)\right] = \dfrac{Q}{\epsilon_0}$.

EVALUATE: (e) Gauss's law states that the electric flux through any closed surface is equal to the total charge enclosed divided by epsilon-zero: $\Phi_E = \dfrac{Q}{\epsilon_0}$, which is what we just found. So our result is consistent with Gauss's law.

22.62. IDENTIFY: The charge in a spherical shell of radius r and thickness dr is $dQ = \rho(r)4\pi r^2\,dr$. Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r. Let Q_i be the charge in the region $r \le R/2$ and let Q_0 be the charge in the region where $R/2 \le r \le R$.

EXECUTE: (a) The total charge is $Q = Q_i + Q_0$, where $Q_i = 4\pi\int_0^{R/2}\dfrac{3\alpha r^3}{2R}dr = \dfrac{6\pi\alpha}{R}\dfrac{1}{4}\dfrac{R^4}{16} = \dfrac{3}{32}\pi\alpha R^3$ and

$Q_0 = 4\pi\alpha\int_{R/2}^{R}(1-(r/R)^2)r^2\,dr = 4\pi\alpha R^3\left(\dfrac{7}{24} - \dfrac{31}{160}\right) = \dfrac{47}{120}\pi\alpha R^3$. Therefore,

$Q = \left(\dfrac{3}{32} + \dfrac{47}{120}\right)\pi\alpha R^3 = \dfrac{233}{480}\pi\alpha R^3$ and $\alpha = \dfrac{480Q}{233\pi R^3}$.

(b) For $r \le R/2$, Gauss's law gives $E4\pi r^2 = \dfrac{4\pi}{\epsilon_0}\displaystyle\int_0^r \dfrac{3\alpha r'^3}{2R}dr' = \dfrac{3\pi\alpha r^4}{2\epsilon_0 R}$ and $E = \dfrac{6\alpha r^2}{16\epsilon_0 R} = \dfrac{180Qr^2}{233\pi\epsilon_0 R^4}$.

For $R/2 \le r \le R$, $E4\pi r^2 = \dfrac{Q_i}{\epsilon_0} + \dfrac{4\pi\alpha}{\epsilon_0}\displaystyle\int_{R/2}^r (1-(r'/R)^2)r'^2\, dr' = \dfrac{Q_i}{\epsilon_0} + \dfrac{4\pi\alpha}{\epsilon_0}\left(\dfrac{r^3}{3} - \dfrac{R^3}{24} - \dfrac{r^5}{5R^2} + \dfrac{R^3}{160}\right)$.

$E4\pi r^2 = \dfrac{3}{128}\dfrac{4\pi\alpha R^3}{\epsilon_0} + \dfrac{4\pi\alpha R^3}{\epsilon_0}\left(\dfrac{1}{3}\left(\dfrac{r}{R}\right)^3 - \dfrac{1}{5}\left(\dfrac{r}{R}\right)^5 - \dfrac{17}{480}\right)$ and

$E = \dfrac{480Q}{233\pi\epsilon_0 r^2}\left(\dfrac{1}{3}\left(\dfrac{r}{R}\right)^3 - \dfrac{1}{5}\left(\dfrac{r}{R}\right)^5 - \dfrac{23}{1920}\right)$. For $r \ge R$, $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$, since all the charge is enclosed.

(c) The fraction of Q between $R/2 \le r \le R$ is $\dfrac{Q_0}{Q} = \dfrac{47}{120}\dfrac{480}{233} = 0.807$.

(d) $E = \dfrac{180}{233}\dfrac{Q}{4\pi\epsilon_0 R^2}$ using either of the electric field expressions above, evaluated at $r = R/2$.

EVALUATE: **(e)** The force an electron would feel never is proportional to $-r$ which is necessary for simple harmonic oscillations. It is oscillatory since the force is always attractive, but it has the wrong power of r to be simple harmonic motion.

22.63. **IDENTIFY** and **SET UP:** Treat the sphere as a point-charge, so $E = k\dfrac{|q|}{r^2}$, so $|q| = Er^2/k$.

EXECUTE: $|q| = Er^2/k = (1\times10^6\ \text{N/C})(25\ \text{m})^2/(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 0.0695\ \text{C} \approx 0.07\ \text{C}$. The charge must be negative since the field is intended to repel negative electrons. Choice (a) is correct.

EVALUATE: 0.07 C is quite a large amount of charge, much larger than normally encountered in typical college physics laboratories.

22.64. **IDENTIFY** and **SET UP:** Treat the sphere as a point-charge, so $E = k\dfrac{|q|}{r^2}$. Use the result from the previous problem for the charge on the sphere.

EXECUTE: $E = k\dfrac{|q|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.0695\ \text{C})/(2.5\ \text{m})^2 = 1.0\times10^8\ \text{N/C}$, choice (d).

EVALUATE: The field strength at 2.5 m is 100 times what it is at 25 m. This is reasonable since the field strength obeys an inverse-square law. At 25 m, which is a distance 10 times as far as 2.5 m, the field strength is $[(2.5\ \text{m})/(25\ \text{m})]^2(1\times10^6\ \text{N/C}) = 1\times10^6\ \text{N/C}$, which was given in the previous problem.

22.65. **IDENTIFY** and **SET UP:** Electric field lines point away from positive charges and toward negative charges. For a point-charge, the lines radiated from (or to) the charge. For a uniform sphere of charge, the field lines look the same as those for a point-charge for points outside the sphere.

EXECUTE: The sphere is negative and equivalent to a negative point-charge, so at its surface the field lines are perpendicular to it and pointing inward, which is choice (b).

EVALUATE: The sphere behaves like a point-charge at or above its surface.

22.66. **IDENTIFY** and **SET UP:** All the charge is on the surface of a spherical shell.

EXECUTE: The field inside the sphere comes from any charge that is inside, but there is none. So the field is zero, choice (c).

EVALUATE: This result is true only if the surface of the sphere is uniformly charged.

23

ELECTRIC POTENTIAL

VP23.2.1. **IDENTIFY:** This problem involves the energy of a system consisting of an electron and a lead nucleus.

SET UP: The electric potential energy (relative to infinity) of two point charges is $U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$.

EXECUTE: **(a)** We want the work to move the electron. $W = U_2 - U_1$. Use $U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$:

$$W = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_2} - \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_1} = \frac{q_1q_2}{4\pi\epsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{(-e)(82e)}{4\pi\epsilon_0}\left(\frac{1}{5.00\times10^{-10}\ \text{m}} - \frac{1}{1.00\times10^{-10}\ \text{m}}\right).$$

$W = 1.51\times10^{-16}$ J.

(b) We want kinetic energy. Use energy conservation with $K_1 = 0$: $U_1 = U_2 + K_2$. Solving for K_2 gives

$$K_2 = U_1 - U_2 = \frac{q_1q_2}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{(-e)(82e)}{4\pi\epsilon_0}\left(\frac{1}{1.00\times10^{-10}\ \text{m}} - \frac{1}{8.00\times10^{-12}\ \text{m}}\right).\ \ K_2 = 2.17\times10^{-15}\ \text{J}.$$

EVALUATE: As the electron gets closer to the nucleus, its potential energy gets more and more negative, so it is losing potential energy and gaining kinetic energy.

VP23.2.2. **IDENTIFY:** We fire a proton at a nucleus and make use of electric potential energy of point charges.

SET UP: The potential energy is $U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$.

EXECUTE: **(a)** We want the potential energy. Energy conservation gives $K_{-1} = U_2$, where 1 is when the proton is very far from the nucleus and 2 is when it is at its closest point and has stopped moving.

$$U_2 = K_1 = \frac{1}{2}m_\text{p}v^2 = \frac{1}{2}(1.67\times10^{-27}\ \text{kg})(2.50\times10^6\ \text{m/s})^2 = 5.22\times10^{-15}\ \text{J}.$$

(b) Call Q the charge of the nucleus, which is our target variable. $U_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_2} = \frac{1}{4\pi\epsilon_0}\frac{eQ}{r}$. Solve for Q: $Q = \frac{U_2 r 4\pi\epsilon_0}{e} = \frac{(5.22\times10^{-15}\ \text{J})(5.29\times10^{-13}\ \text{m})(4\pi\epsilon_0)}{1.60\times10^{-19}\ \text{C}} = 1.92\times10^{-18}\ \text{C}.$

EVALUATE: $Q/e = 12$, so this nucleus has 12 protons, which means it must be magnesium (from Appendix D).

VP23.2.3. **IDENTIFY:** This problem involves the potential energy of a system of point charges.

SET UP: Fig. VP22.2.3 shows the three charges in their final arrangement. $U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$.

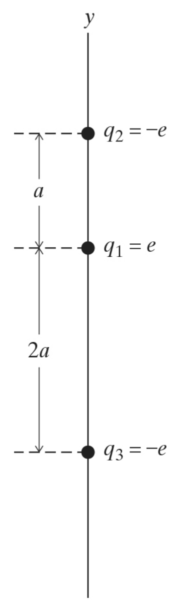


Figure VP23.2.3

EXECUTE: **(a)** The work needed to bring in the charge q_3 is equal to its final potential energy due to the q_1 and q_2. $W = U_{1,3} + U_{2,3} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{r_{23}} = \frac{-e}{4\pi\epsilon_0}\left(\frac{e}{2a} + \frac{-e}{3a}\right) = -\frac{e^2}{24\pi\epsilon_0 a}$.

(b) $U_{\text{tot}} = U_{1,2} + U_{1,3} + U_{2,3} = \frac{1}{4\pi\epsilon_0}\left(\frac{-e^2}{a} + \frac{-e^2}{3a} + \frac{e^2}{3a}\right) = -\frac{7e^2}{24\pi\epsilon_0 a}$.

EVALUATE: The total potential energy is negative since q_1 attracts q_2 and q_3. Only q_2 and q_3 repel each other.

VP23.2.4. **IDENTIFY:** This problem involves the potential energy of a system of point charges.

SET UP: Fig. VP22.2.4 shows the three charges with q_3 in place. $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$.

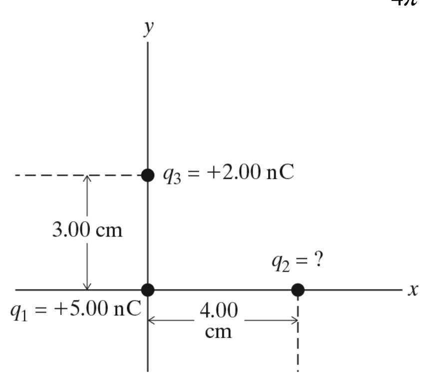


Figure VP23.2.4

EXECUTE: **(a)** We want q_2. The work you do is the potential energy of the q_1-q_2 system.

$W = U_{1,2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}$. Solve for q_2: $q_2 = \frac{4\pi\epsilon_0 W r_{12}}{q_1} = \frac{4\pi\epsilon_0 (8.10\times10^{-6}\ \text{J})(0.0400\ \text{m})}{5.00\ \text{nC}} = 7.21$ nC.

(b) The *additional* work is the added potential energy of the system, which is the potential energy of q_3 due to q_1 and q_2. $W = U_{1,3} + U_{2,3} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0}\frac{q_2q_3}{r_{23}} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$. Using the result from part (a) and the given numbers, we get $W = 5.59\times10^{-6}$ J.

EVALUATE: Even though two charges are already present, it takes less work to bring in q_3 than it did to bring in q_2 because $q_3 = (2/7.21)q_2 = 0.277q_2$.

VP23.7.1. **IDENTIFY:** We need to relate the potential to the electric field.

SET UP: Use $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$, $V(0,0,0) = V_0$, $\vec{E} = E_x\hat{i}$, where $E_x = 5.00\times10^2$ V/m. The target variable is the potential difference.

EXECUTE: **(a)** $V_0 - V_P = \int_0^P \vec{E}\cdot d\vec{l} = \int_0^{5.00\text{ cm}} E_x dx = (5.00\times10^2\text{ V/m})(0.0500\text{ m}) = +25.0\text{ V}.$

(b) Since $E_y = E_z = 0$, we have $V_0 - V_P = \int_0^{3.00\text{ cm}} E_x dx = (5.00\times10^2\text{ V/m})(0.0300\text{ m}) = +15.0\text{ V}.$

(c) Since $E_y = E_z = 0$ and $\Delta x = 0$, we have $V_0 - V_P = 0$.

(d) $V_0 - V_P = \int_0^P \vec{E}\cdot d\vec{l} = \int_0^{-5.00\text{ cm}} E_x dx = (5.00\times10^2\text{ V/m})(-0.0500\text{ m}) = -25.0\text{ V}.$

EVALUATE: In parts (a) and (b), $V_P < V_0$, in (c) $V_P = V_0$, and in (d) $V_P > V_0$.

VP23.7.2. **IDENTIFY:** This problem involves the electric potential and electric field of point charges.

SET UP: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ and $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$. We want the potential. First sketch the arrangement of charges as in Fig. 23.7.2.

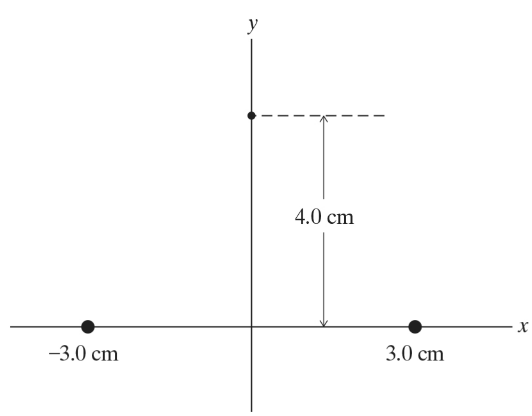


Figure VP23.7.2

EXECUTE: **(a)** The charges are equal and r = 3.0 cm = 0.030 m for each of them. So

$$V = V_1 + V_2 = 2\frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{2}{4\pi\epsilon_0}\frac{6.0\text{ nC}}{0.030\text{ m}} = 3600\text{ V} = 3.6\text{ kV}.$$

(b) Use the same procedure as in part (a) with r = 5.0 cm = 0.050 m, giving V = 2200 V = 2.2 kV.

(c) The point we are interested in lies on the x-axis at x = 8.0 cm. So r_1 = 11.0 cm for the left-hand charge and r_2 = 5.0 cm for the right-hand charge. The potential at this point is

$$V = V_1 + V_2 = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{(6.0\text{ nC})}{4\pi\epsilon_0}\left(\frac{1}{0.0500\text{ m}} + \frac{1}{0.110\text{ m}}\right) = 160\text{ V}.$$

(d) The net field can only be zero if the fields due to the two charges have equal magnitudes and are in opposite directions. These conditions are met only at the point (0,0,0), which is the point in part (a).

EVALUATE: Note an important point: In part (d) we saw that if $E = 0$ at a point, it does *not* necessarily follow that $V = 0$ at that point.

VP23.7.3. **IDENTIFY:** This problem involves the potential due to point charges.

SET UP: $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$. Fig. VP23.7.3 shows the charges and the given information.

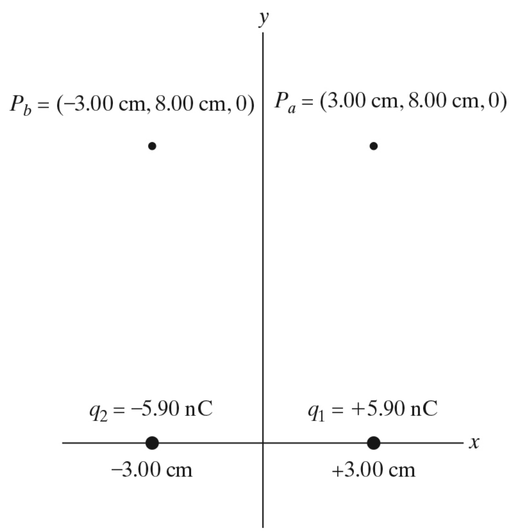


Figure VP23.7.3

EXECUTE: **(a)** We want V_a. $r_2 = \sqrt{(6.00\text{ cm})^2 + (8.00\text{ cm})^2} = 10.0\text{ cm} = 0.100\text{ m}$ and $r_1 = 8.00$ cm = 0.0800 m. $V_a = V_1 + V_2 = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2}\right) = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{5.90\text{ nC}}{0.0800\text{ m}} + \dfrac{-5.90\text{ nC}}{0.100\text{ m}}\right) = +133\text{ V}.$

(b) $V_b = V_1 + V_2 = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2}\right) = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{5.90\text{ nC}}{0.100\text{ m}} + \dfrac{-5.90\text{ nC}}{0.0800\text{ m}}\right) = -133\text{ V}.$

(c) We want the proton's speed at *a*. The proton starts at *b* and goes to *a*, so it is going from a low to a high potential. Therefore it is losing kinetic energy in the process. So $K_a = K_b - q(V_a - V_b)$. $\dfrac{1}{2}mv_a^2 = \dfrac{1}{2}mv_b^2 - e(V_a - V_b)$. $v_a = \sqrt{v_b^2 - 2e(V_a - V_b)/m}$. Using the given quantities and the results of parts (a) and (b), we get $v_a = 1.98\times10^5$ m/s.

EVALUATE: A positive charge (like a proton) will be accelerated to higher speed by the electric field going from high to low potential (just as a falling object is sped up by the gravitational field as it goes from high potential energy to low potential energy).

VP23.7.4. **IDENTIFY:** We want to get the potential difference by integrating the electric field. We'll need calculus for this.

SET UP: Use $V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{l}$. Call V_0 the potential at the center of the sphere and V_R the potential at the surface when $r = R$. $d\vec{l} = dr\,\hat{r}$ and $\vec{E} = \dfrac{Qr}{4\pi\epsilon_0 R^3}\hat{r}$ inside the sphere.

EXECUTE: **(a)** Using the above quantities gives

$$V_R - V_0 = -\int_0^R \frac{Qr}{4\pi\epsilon_0 R^3}dr = -\left(\frac{Q}{4\pi\epsilon_0 R^3}\right)\frac{r^2}{2}\bigg|_0^R = -\frac{Q}{8\pi\epsilon_0 R}.$$

(b) From part (a), we have $V_R = V_0 - \dfrac{Q}{8\pi\epsilon_0 R}$. This tells us that $V_R < V_0$, so the center is at a higher potential than the surface. Another way to see this is to note that the electric field does work on a positive charge as it goes from the center to the surface because the field points outward. Thus the center is at a higher potential then the surface.

EVOALUATE: Notice that $E = 0$ at the center of the sphere, but the potential is highest there and not zero.

VP23.12.1. **IDENTIFY:** We want to find the work using electric potential.

SET UP: $W = q\Delta V$ and for a point charge $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$.

EXECUTE: **(a)** <u>From $r = 5R$ to $r = 3R$</u>: $W = q_0(V_{3R} - V_{5R}) = q_0\left[\dfrac{q}{4\pi\epsilon_0}\left(\dfrac{1}{3R} - \dfrac{1}{5R}\right)\right] = \dfrac{qq_0}{30\pi\epsilon_0 R}$.

(b) <u>From $r = 2R$ to $r = 7R$</u>: $W = q_0(V_{7R} - V_{2R}) = \dfrac{q_0 q}{4\pi\epsilon_0}\left(\dfrac{1}{3R} - \dfrac{1}{5R}\right) = -\dfrac{5qq_0}{56\pi\epsilon_0 R}$.

(c) <u>From $r = 4R$ to $r = R/2$</u>: Inside the conducting sphere $E = 0$ so no work is done on q_0. So the work required is the work to go from $r = 4R$ to $r = R$, which is

$$W = q_0(V_{4R} - V_R) = \frac{q_0 q}{4\pi\epsilon_0}\left(\frac{1}{4R} - \frac{1}{R}\right) = \frac{3qq_0}{16\pi\epsilon_0 R}.$$

EVALUATE: In parts (a) and (c) the work we do is positive because we must push on q_0 against the electric field. In (b) the work is negative because negative because we must hold back on q_0 since the electric field pulls it in.

VP23.12.2. **IDENTIFY:** This problem involves two oppositely charged parallel plates.

SET UP: The electric field between the plates is uniform everywhere.

EXECUTE: **(a)** We want the potential at 3.0 mm above the lower plate. The electric field is uniform, so $V_{ab} = Ed$. At a distance x above the lower plate, the potential difference over that distance is Ex. Since E is uniform $\dfrac{V_x}{V_{ab}} = \dfrac{Ex}{Ed} = \dfrac{x}{d} = \dfrac{3.00\text{ mm}}{4.50\text{ mm}} = \dfrac{2}{3}$, so $V_x = \dfrac{2}{3}V_{ab} = \dfrac{2}{3}(+24.0\text{ V}) = +16.0\text{ V}$.

(b) $U = qV$, where V is relative to b. So $U = (2.00\text{ nC})(16.0\text{ V}) = 32\text{ nV} = 3.20\times10^{-8}\text{ J}$.

(c) Since it is positive, the particle will move in the direction of the electric field, which is toward the lower plate. Using $U_1 = K_2$ gives $\frac{1}{2}mv^2 = U$, so $v = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2(3.20\times10^{-8}\text{ J})}{5.00\times10^{-9}\text{ kg}}} = 3.58\text{ m/s}$.

EVALUATE: Note in part (c) that the positive particle moved from higher to lower potential.

VP23.12.3. IDENTIFY: We are dealing with the potential of a uniformly charged ring.

SET UP: We know that $U = qV$. Example 23.11 showed that for a ring $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$. We want the potential energy of the point charge, its kinetic energy, and its speed.

EXECUTE: (a) We want the potential energy of the charge at 8.00 cm from the center of the ring.

$$U_8 = qV = \frac{q}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+a^2}} = \frac{(3.00\text{ nC})}{4\pi\epsilon_0}\frac{5.00\text{ nC}}{\sqrt{(0.0800\text{ m})^2+(0.0250\text{ m})^2}} = 1.61\times10^{-6}\text{ J}.$$

(b) At the center of the ring, $x = 0$. Using the above equations gives $U_0 = 5.39\times10^{-6}$ J.

(c) Energy conservation: $K_8 + U_8 = K_0 + U_0 \rightarrow K_0 = K_8 + U_8 - U_0$. From this we get $K_0 = \frac{1}{2}mv_8^2 + U_8 - U_0$. Using the results from parts (a) and (b) as well as the given quantities, we get $K_0 = 3.42\times10^{-6}$ J. Solving $K_0 = \frac{1}{2}mv_8^2$ for v_8 gives $v_8 = \sqrt{\frac{2K_0}{m}}$. Using K_0 and the given quantities gives $v_8 = 41.3$ m/s.

EVALUATE: The point charge loses kinetic energy (and hence speed) because the electric field of the ring does negative work on it as it approaches the center of the ring.

VP23.12.4. IDENTIFY: We want to calculate the potential due to a uniformly charged rod.

SET UP: Break the rod into infinitesimal segments of length dx carrying charge dq. Applying $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ to a typical segment, the potential at the origin is $dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{x}$. Fig. VP23.12.4 shows the arrangement.

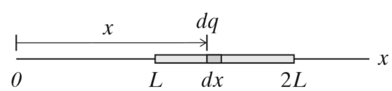


Figure VP23.12.4

EXECUTE: (a) $dq = \lambda dx = (Q/L)dx$.

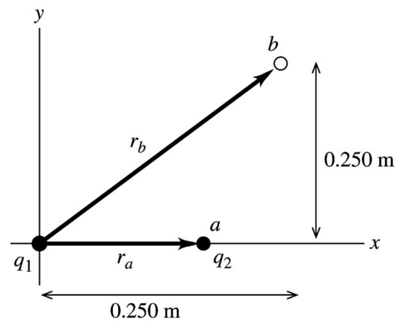
(b) Using Fig. VP23.12.4 and $dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{x}$, and integrating, we have

$$V = \int_0^L \frac{1}{4\pi\epsilon_0}\frac{dq}{x} = \int_0^L \frac{Q/L}{4\pi\epsilon_0}\frac{dx}{x} = \frac{Q}{4\pi\epsilon_0 L}\ln x\Big|_L^{2L} = \frac{Q}{4\pi\epsilon_0 L}\ln 2.$$

EVALUATE: We cannot treat the rod as being equivalent to a point charge at its center. If that were the case, the potential due to the rod at the origin would be $\frac{Q}{4\pi\epsilon_0(3L/2)} = \frac{Q}{6\pi\epsilon_0 L}$, which is *not* what we calculated.

23.1. IDENTIFY: Apply $W_{a\to b} = U_a - U_b$ to calculate the work. The electric potential energy of a pair of point charges is given by $U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$.

SET UP: Let the initial position of q_2 be point a and the final position be point b, as shown in Figure 23.1.



$r_a = 0.150$ m.

$r_b = \sqrt{(0.250\text{ m})^2 + (0.250\text{ m})^2}$.

$r_b = 0.3536$ m.

Figure 23.1

EXECUTE: $W_{a\to b} = U_a - U_b$.

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_a} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times10^{-6}\text{ C})(-4.30\times10^{-6}\text{ C})}{0.150\text{ m}}.$$

$U_a = -0.6184$ J.

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_b} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times10^{-6}\text{ C})(-4.30\times10^{-6}\text{ C})}{0.3536\text{ m}}.$$

$U_b = -0.2623$ J.

$$W_{a\to b} = U_a - U_b = -0.6184\text{ J} - (-0.2623\text{ J}) = -0.356\text{ J}.$$

EVALUATE: The attractive force on q_2 is toward the origin, so it does negative work on q_2 when q_2 moves to larger r.

23.2. **IDENTIFY:** Apply $W_{a\to b} = U_a - U_b$.

SET UP: $U_a = +5.4\times10^{-8}$ J. Solve for U_b.

EXECUTE:

$W_{a\to b} = -1.9\times10^{-8}\text{ J} = U_a - U_b$. $U_b = U_a - W_{a\to b} = +5.4\times10^{-8}\text{ J} - (-1.9\times10^{-8}\text{ J}) = 7.3\times10^{-8}$ J.

EVALUATE: When the electric force does negative work the electrical potential energy increases.

23.3. **IDENTIFY:** The work needed to assemble the nucleus is the sum of the electrical potential energies of the protons in the nucleus, relative to infinity.

SET UP: The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is $U = (1/4\pi\epsilon_0)(qq_0/r)$. Each charge is e and the charges are equidistant from each other, so the total potential energy is $U = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{e^2}{r} + \dfrac{e^2}{r} + \dfrac{e^2}{r}\right) = \dfrac{3e^2}{4\pi\epsilon_0 r}$.

EXECUTE: Adding the potential energies gives

$$U = \frac{3e^2}{4\pi\epsilon_0 r} = \frac{3(1.60\times10^{-19}\text{ C})^2(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}{2.00\times10^{-15}\text{ m}} = 3.46\times10^{-13}\text{ J} = 2.16\text{ MeV}.$$

EVALUATE: This is a small amount of energy on a macroscopic scale, but on the scale of atoms, 2 MeV is quite a lot of energy.

23.4. **IDENTIFY:** The work required is the change in electrical potential energy. The protons gain speed after being released because their potential energy is converted into kinetic energy.

(a) SET UP: Using the potential energy of a pair of point charges relative to infinity, $U = (1/4\pi\epsilon_0)(qq_0/r)$, we have $W = \Delta U = U_2 - U_1 = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{e^2}{r_2} - \dfrac{e^2}{r_1}\right)$.

EXECUTE: Factoring out the e^2 and substituting numbers gives

$$W = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2\left(\frac{1}{3.00\times10^{-15}\ \text{m}} - \frac{1}{2.00\times10^{-10}\ \text{m}}\right) = 7.68\times10^{-14}\ \text{J}$$

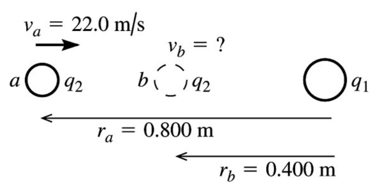
(b) SET UP: The protons have equal momentum, and since they have equal masses, they will have equal speeds and hence equal kinetic energy. $\Delta U = K_1 + K_2 = 2K = 2\left(\dfrac{1}{2}mv^2\right) = mv^2$.

EXECUTE: Solving for v gives $v = \sqrt{\dfrac{\Delta U}{m}} = \sqrt{\dfrac{7.68\times10^{-14}\ \text{J}}{1.67\times10^{-27}\ \text{kg}}} = 6.78\times10^6\ \text{m/s}$.

EVALUATE: The potential energy may seem small (compared to macroscopic energies), but it is enough to give each proton a speed of nearly 7 million m/s.

23.5. **(a) IDENTIFY:** Use conservation of energy: $K_a + U_a + W_{\text{other}} = K_b + U_b$. U for the pair of point charges is given by $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$.

SET UP:



Let point a be where q_2 is 0.800 m from q_1 and point b be where q_2 is 0.400 m from q_1, as shown in Figure 23.5a.

Figure 23.5a

EXECUTE: Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$.

$$K_a = \tfrac{1}{2}mv_a^2 = \tfrac{1}{2}(1.50\times10^{-3}\ \text{kg})(22.0\ \text{m/s})^2 = 0.3630\ \text{J}.$$

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_a} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.800\ \text{m}} = +0.2454\ \text{J}.$$

$$K_b = \tfrac{1}{2}mv_b^2.$$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_b} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.400\ \text{m}} = +0.4907\ \text{J}.$$

The conservation of energy equation then gives $K_b = K_a + (U_a - U_b)$.

$$\tfrac{1}{2}mv_b^2 = +0.3630\ \text{J} + (0.2454\ \text{J} - 0.4907\ \text{J}) = 0.1177\ \text{J}.$$

$$v_b = \sqrt{\frac{2(0.1177\ \text{J})}{1.50\times10^{-3}\ \text{kg}}} = 12.5\ \text{m/s}.$$

EVALUATE: The potential energy increases when the two positively charged spheres get closer together, so the kinetic energy and speed decrease.

(b) IDENTIFY: Let point *c* be where q_2 has its speed momentarily reduced to zero. Apply conservation of energy to points *a* and *c*: $K_a + U_a + W_{\text{other}} = K_c + U_c$.

SET UP: Points *a* and *c* are shown in Figure 23.5b.

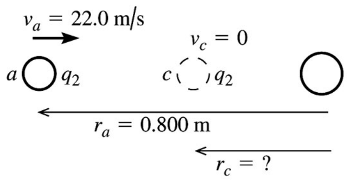


Figure 23.5b

EXECUTE: $K_a = +0.3630$ J (from part (a)).

$U_a = +0.2454$ J (from part (a)).

$K_c = 0$ (at distance of closest approach the speed is zero).

$$U_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_c}.$$

Thus conservation of energy $K_a + U_a = U_c$ gives $\dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r_c} = +0.3630\text{ J} + 0.2454\text{ J} = 0.6084\text{ J}.$

$$r_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{0.6084\text{ J}} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\text{ C})(-7.80\times10^{-6}\text{ C})}{+0.6084\text{ J}} = 0.323\text{ m}.$$

EVALUATE: $U \to \infty$ as $r \to 0$ so q_2 will stop no matter what its initial speed is.

23.6. IDENTIFY: The total potential energy is the scalar sum of the individual potential energies of each pair of charges.

SET UP: For a pair of point charges the electrical potential energy is $U = k\dfrac{qq'}{r}$. In the O-H-N combination the O^- is 0.170 nm from the H^+ and 0.280 nm from the N^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. *U* is positive for like charges and negative for unlike charges.

EXECUTE: (a) O-H-N:

$$\text{O}^-\text{- H}^+: U = -(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\text{ C})^2}{0.170\times10^{-9}\text{ m}} = -1.35\times10^{-18}\text{ J}.$$

$$\text{O}^-\text{-N}^-: U = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\text{ C})^2}{0.280\times10^{-9}\text{ m}} = +8.22\times10^{-19}\text{ J}.$$

N-H-N:

$$\text{N}^-\text{- H}^+: U = -(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\text{ C})^2}{0.190\times10^{-9}\text{ m}} = -1.21\times10^{-18}\text{ J}.$$

$$\text{N}^-\text{-N}^-: U = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\text{ C})^2}{0.300\times10^{-9}\text{ m}} = +7.67\times10^{-19}\text{ J}.$$

The total potential energy is

$U_{\text{tot}} = -1.35\times10^{-18}\text{ J} + 8.22\times10^{-19}\text{ J} - 1.21\times10^{-18}\text{ J} + 7.67\times10^{-19}\text{ J} = -9.71\times10^{-19}\text{ J}.$

(b) In the hydrogen atom the electron is 0.0529 nm from the proton.

$$U = -(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.0529\times10^{-9}\ \text{m}} = -4.35\times10^{-18}\ \text{J}.$$

EVALUATE: The magnitude of the potential energy in the hydrogen atom is about a factor of 4 larger than what it is for the adenine-thymine bond.

23.7. IDENTIFY: Use conservation of energy $U_a + K_a = U_b + K_b$ to find the distance of closest approach r_b. The maximum force is at the distance of closest approach, $F = k\dfrac{|q_1q_2|}{r_b^2}$.

SET UP: $K_b = 0$. Initially the two protons are far apart, so $U_a = 0$. A proton has mass 1.67×10^{-27} kg and charge $q = +e = +1.60\times10^{-19}$ C.

EXECUTE: $K_a = U_b$. $2(\frac{1}{2}mv_a^2) = k\dfrac{q_1q_2}{r_b}$. $mv_a^2 = k\dfrac{e^2}{r_b}$ and

$$r_b = \frac{ke^2}{mv_a^2} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(1.67\times10^{-27}\ \text{kg})(2.00\times10^{5}\ \text{m/s})^2} = 3.45\times10^{-12}\ \text{m}.$$

$$F = k\frac{e^2}{r_b^2} = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{(3.445\times10^{-12}\ \text{m})^2} = 1.94\times10^{-5}\ \text{N}.$$

EVALUATE: The acceleration $a = F/m$ of each proton produced by this force is extremely large.

23.8. IDENTIFY: Call the three charges 1, 2, and 3. $U = U_{12} + U_{13} + U_{23}$.

SET UP: $U_{12} = U_{23} = U_{13}$ because the charges are equal and each pair of charges has the same separation, 0.400 m.

EXECUTE: $U = \dfrac{3kq^2}{0.400\ \text{m}} = \dfrac{3k(1.2\times10^{-6}\ \text{C})^2}{0.400\ \text{m}} = 0.0971\ \text{J}.$

EVALUATE: When the three charges are brought in from infinity to the corners of the triangle, the repulsive electrical forces between each pair of charges do negative work and electrical potential energy is stored.

23.9. IDENTIFY: The protons repel each other and therefore accelerate away from one another. As they get farther and farther away, their kinetic energy gets greater and greater but their acceleration keeps decreasing. Conservation of energy and Newton's laws apply to these protons.

SET UP: Let *a* be the point when they are 0.750 nm apart and *b* be the point when they are very far apart. A proton has charge $+e$ and mass 1.67×10^{-27} kg. As they move apart the protons have equal kinetic energies and speeds. Their potential energy is $U = ke^2/r$ and $K = \frac{1}{2}mv^2$. $K_a + U_a = K_b + U_b$.

EXECUTE: **(a)** They have maximum speed when they are far apart and all their initial electrical potential energy has been converted to kinetic energy. $K_a + U_a = K_b + U_b$.

$K_a = 0$ and $U_b = 0$, so

$$K_b = U_a = k\frac{e^2}{r_a} = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.750\times10^{-9}\ \text{m}} = 3.07\times10^{-19}\ \text{J}.$$

$K_b = \frac{1}{2}mv_b^2 + \frac{1}{2}mv_b^2$, so $K_b = mv_b^2$ and $v_b = \sqrt{\dfrac{K_b}{m}} = \sqrt{\dfrac{3.07\times10^{-19}\ \text{J}}{1.67\times10^{-27}\ \text{kg}}} = 1.36\times10^{4}\ \text{m/s}.$

(b) Their acceleration is largest when the force between them is largest and this occurs at $r = 0.750$ nm, when they are closest.

$$F = k\frac{e^2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{1.60\times10^{-19}\ \text{C}}{0.750\times10^{-9}\ \text{m}}\right)^2 = 4.09\times10^{-10}\ \text{N}.$$

$$a = \frac{F}{m} = \frac{4.09\times10^{-10}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = 2.45\times10^{17}\ \text{m/s}^2.$$

EVALUATE: The acceleration of the protons decreases as they move farther apart, but the force between them is repulsive so they continue to increase their speeds and hence their kinetic energies.

23.10. **IDENTIFY:** The work done on the alpha particle is equal to the difference in its potential energy when it is moved from the midpoint of the square to the midpoint of one of the sides.

SET UP: We apply the formula $W_{a\to b} = U_a - U_b$. In this case, a is the center of the square and b is the midpoint of one of the sides. Therefore $W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}}$ is the work done by the Coulomb force. There are 4 electrons, so the potential energy at the center of the square is 4 times the potential energy of a single alpha-electron pair. At the center of the square, the alpha particle is a distance $r_1 = \sqrt{50}$ nm from each electron. At the midpoint of the side, the alpha is a distance $r_2 = 5.00$ nm from the two nearest electrons and a distance $r_3 = \sqrt{125}$ nm from the two most distant electrons. Using the formula for the potential energy (relative to infinity) of two point charges, $U = (1/4\pi\epsilon_0)(qq_0/r)$, the total work done by the Coulomb force is

$$W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}} = 4\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_1} - \left(2\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_2} + 2\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_3}\right).$$

Substituting $q_e = -e$ and $q_\alpha = 2e$ and simplifying gives

$$W_{\text{center}\to\text{side}} = -4e^2\frac{1}{4\pi\epsilon_0}\left[\frac{2}{r_1} - \left(\frac{1}{r_2} + \frac{1}{r_3}\right)\right].$$

EXECUTE: Substituting the numerical values into the equation for the work gives

$$W = -4(1.60\times10^{-19}\ \text{C})^2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[\frac{2}{\sqrt{50}\ \text{nm}} - \left(\frac{1}{5.00\ \text{nm}} + \frac{1}{\sqrt{125}\ \text{nm}}\right)\right] = 6.08\times10^{-21}\ \text{J}.$$

EVALUATE: Since the work done by the Coulomb force is positive, the system has more potential energy with the alpha particle at the center of the square than it does with it at the midpoint of a side. To move the alpha particle to the midpoint of a side and leave it there at rest an external force must do -6.08×10^{-21} J of work.

23.11. **IDENTIFY:** We want to find the potential difference by using the electric field.

SET UP and **EXECUTE:** Start with a sketch as shown in Fig. 23.11.

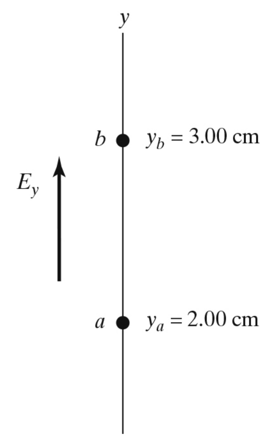


Figure 23.11

$V_a - V_b = \int_a^b E_y dy = \int_a^b (\alpha + \beta / y^2) dy = (\alpha y - \beta / y)\Big|_{y_a}^{y_b}$. Simplifying gives

$V_a - V_b = \alpha(y_b - y_a) + \beta\left(\frac{1}{y_a} - \frac{1}{y_b}\right)$. Using $y_a = 2.00$ cm, $y_b = 3.00$ cm, and the other values gives $V_a - V_b = 89.3$ V. $V_a = V_b + 89.3$ V, so V_a is higher than V_b.

EVALUATE: The electric field points from *a* to *b*, so a positive charge would gain kinetic energy as it was accelerated by the field from *a* to *b*, so the potential at *a* must be higher than at *b*, as we found.

23.12. **IDENTIFY**: Work is done on the object by the electric field, and this changes its kinetic energy, so we can use the work-energy theorem.

SET UP: $W_{A \to B} = \Delta K$ and $W_{A \to B} = q(V_A - V_B)$.

EXECUTE: **(a)** Applying the two equations above gives $W_{A \to B} = q(V_A - V_B) = K_B - 0 = K_B$.

$V_B = V_A - K_B/q = 30.0 \text{ V} - (3.00 \times 10^{-7} \text{ J})/(-6.00 \times 10^{-9} \text{ C}) = 80.0 \text{ V}$.

(b) The negative charge accelerates from *A* to *B*, so the electric field must point from *B* toward *A*. Since the field is uniform, we have $E = \frac{\Delta V}{\Delta x}$ = (50.0 V)/(0.500 m) = 100 V/m.

EVALUATE: A positive charge is accelerated from high to low potential, but a negative charge (as we have here) is accelerated from low to high potential.

23.13. **IDENTIFY** and **SET UP:** Apply conservation of energy to points *A* and *B*.

EXECUTE: $K_A + U_A = K_B + U_B$.

$U = qV$, so $K_A + qV_A = K_B + qV_B$.

$K_B = K_A + q(V_A - V_B) = 0.00250 \text{ J} + (-5.00 \times 10^{-6} \text{ C})(200 \text{ V} - 800 \text{ V}) = 0.00550 \text{ J}$.

$v_B = \sqrt{2K_B/m} = 7.42$ m/s.

EVALUATE: It is faster at *B*; a negative charge gains speed when it moves to higher potential.

23.14. **IDENTIFY**: The work-energy theorem says $W_{a \to b} = K_b - K_a$. $\frac{W_{a \to b}}{q} = V_a - V_b$.

SET UP: Point *a* is the starting point and point *b* is the ending point. Since the field is uniform, $W_{a \to b} = Fs\cos\phi = E|q|s\cos\phi$. The field is to the left so the force on the positive charge is to the left. The particle moves to the left so $\phi = 0°$ and the work $W_{a \to b}$ is positive.

EXECUTE: **(a)** $W_{a\to b} = K_b - K_a = 2.20\times10^{-6}\text{ J} - 0 = 2.20\times10^{-6}\text{ J}.$

(b) $V_a - V_b = \dfrac{W_{a\to b}}{q} = \dfrac{2.20\times10^{-6}\text{ J}}{4.20\times10^{-9}\text{ C}} = 524\text{ V}.$ Point a is at higher potential than point b.

(c) $E|q|s = W_{a\to b}$, so $E = \dfrac{W_{a\to b}}{|q|s} = \dfrac{V_a - V_b}{s} = \dfrac{524\text{ V}}{6.00\times10^{-2}\text{ m}} = 8.73\times10^{3}\text{ V/m}.$

EVALUATE: A positive charge gains kinetic energy when it moves to lower potential; $V_b < V_a$.

23.15. **IDENTIFY**: Apply $W_{a\to b} = q'\int_a^b \vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{l}}$. Use coordinates where $+y$ is upward and $+x$ is to the right. Then $\vec{\boldsymbol{E}} = E\hat{\boldsymbol{j}}$ with $E = 4.00\times10^{4}$ N/C.

SET UP: **(a)** The path is sketched in Figure 23.15a.

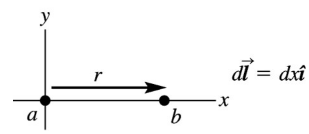


Figure 23.15a

EXECUTE: $\vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{l}} = (E\hat{\boldsymbol{j}})\cdot(dx\hat{\boldsymbol{i}}) = 0$ so $W_{a\to b} = q'\int_a^b \vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{l}} = 0.$

EVALUATE: The electric force on the positive charge is upward (in the direction of the electric field) and does no work for a horizontal displacement of the charge.

(b) SET UP: The path is sketched in Figure 23.15b.

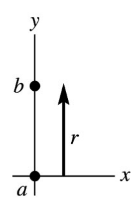


Figure 23.15b

EXECUTE: $\vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{l}} = (E\hat{\boldsymbol{j}})\cdot(dy\hat{\boldsymbol{j}}) = E\,dy.$

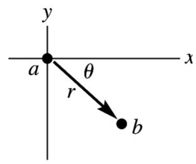
$$W_{a\to b} = q'\int_a^b \vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{l}} = q'E\int_a^b dy = q'E(y_b - y_a).$$

$y_b - y_a = +0.670$ m; it is positive since the displacement is upward and we have taken $+y$ to be upward.

$$W_{a\to b} = q'E(y_b - y_a) = (+28.0\times10^{-9}\text{ C})(4.00\times10^{4}\text{ N/C})(+0.670\text{ m}) = +7.50\times10^{-4}\text{ J}.$$

EVALUATE: The electric force on the positive charge is upward so it does positive work for an upward displacement of the charge.

(c) SET UP: The path is sketched in Figure 23.15c.



$y_a = 0.$

$y_b = -r\sin\theta = -(2.60\text{ m})\sin 45° = -1.838\text{ m}.$

The vertical component of the 2.60 m displacement is 1.838 m downward.

Figure 23.15c

EXECUTE: $d\vec{l} = dx\hat{i} + dy\hat{j}$ (The displacement has both horizontal and vertical components.)

$\vec{E}\cdot d\vec{l} = (E\hat{j})\cdot(dx\hat{i} + dy\hat{j}) = E\,dy$ (Only the vertical component of the displacement contributes to the work.)

$$W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l} = q'E\int_a^b dy = q'E(y_b - y_a).$$

$$W_{a\to b} = q'E(y_b - y_a) = (+28.0\times10^{-9}\text{ C})(4.00\times10^4\text{ N/C})(-1.838\text{ m}) = -2.06\times10^{-3}\text{ J}.$$

EVALUATE: The electric force on the positive charge is upward so it does negative work for a displacement of the charge that has a downward component.

23.16. IDENTIFY: Apply $K_a + U_a = K_b + U_b$.

SET UP: Let $q_1 = +3.00\text{ nC}$ and $q_2 = +2.00\text{ nC}$. At point *a*, $r_{1a} = r_{2a} = 0.250\text{ m}$. At point *b*, $r_{1b} = 0.100\text{ m}$ and $r_{2b} = 0.400\text{ m}$. The electron has $q = -e$ and $m_e = 9.11\times10^{-31}\text{ kg}$. $K_a = 0$ since the electron is released from rest.

EXECUTE: $-\dfrac{keq_1}{r_{1a}} - \dfrac{keq_2}{r_{2a}} = -\dfrac{keq_1}{r_{1b}} - \dfrac{keq_2}{r_{2b}} + \dfrac{1}{2}m_e v_b^2.$

$$E_a = K_a + U_a = k(-1.60\times10^{-19}\text{ C})\left(\frac{(3.00\times10^{-9}\text{ C})}{0.250\text{ m}} + \frac{(2.00\times10^{-9}\text{ C})}{0.250\text{ m}}\right) = -2.88\times10^{-17}\text{ J}.$$

$$E_b = K_b + U_b = k(-1.60\times10^{-19}\text{ C})\left(\frac{(3.00\times10^{-9}\text{ C})}{0.100\text{ m}} + \frac{(2.00\times10^{-9}\text{ C})}{0.400\text{ m}}\right) + \frac{1}{2}m_e v_b^2 = -5.04\times10^{-17}\text{ J} + \frac{1}{2}m_e v_b^2.$$

Setting $E_a = E_b$ gives $v_b = \sqrt{\dfrac{2}{9.11\times10^{-31}\text{ kg}}(5.04\times10^{-17}\text{ J} - 2.88\times10^{-17}\text{ J})} = 6.89\times10^6\text{ m/s}.$

EVALUATE: $V_a = V_{1a} + V_{2a} = 180\text{ V}$. $V_b = V_{1b} + V_{2b} = 315\text{ V}$. $V_b > V_a$. The negatively charged electron gains kinetic energy when it moves to higher potential.

23.17. IDENTIFY: The potential at any point is the scalar sum of the potentials due to individual charges.

SET UP: $V = kq/r$ and $W_{ab} = q(V_a - V_b)$.

EXECUTE: **(a)** $r_{a1} = r_{a2} = \dfrac{1}{2}\sqrt{(0.0300\text{ m})^2 + (0.0300\text{ m})^2} = 0.0212\text{ m}.$ $V_a = k\left(\dfrac{q_1}{r_{a1}} + \dfrac{q_2}{r_{a2}}\right) = 0.$

(b) $r_{b1} = 0.0424\text{ m}$, $r_{b2} = 0.0300\text{ m}$.

$$V_b = k\left(\frac{q_1}{r_{b1}} + \frac{q_2}{r_{b2}}\right) = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.00\times10^{-6}\text{ C}}{0.0424\text{ m}} + \frac{-2.00\times10^{-6}\text{ C}}{0.0300\text{ m}}\right) = -1.75\times10^5\text{ V}.$$

(c) $W_{ab} = q_3(V_a - V_b) = (-5.00\times10^{-6}\text{ C})[0 - (-1.75\times10^5\text{ V})] = -0.875\text{ J}.$

EVALUATE: Since $V_b < V_a$, a positive charge would be pulled by the existing charges from *a* to *b*, so they would do positive work on this charge. But they would repel a negative charge and hence do negative work on it, as we found in part (c).

23.18. **IDENTIFY:** The total potential is the *scalar* sum of the individual potentials, but the net electric field is the *vector* sum of the two fields.

SET UP: The net potential can only be zero if one charge is positive and the other is negative, since it is a scalar. The electric field can only be zero if the two fields point in opposite directions.

EXECUTE: **(a)** (i) Since both charges have the same sign, there are no points for which the potential is zero.

(ii) The two electric fields are in opposite directions only between the two charges, and midway between them the fields have equal magnitudes. So $E = 0$ midway between the charges, but V is never zero.

(b) (i) The two potentials have equal magnitude but opposite sign midway between the charges, so $V = 0$ midway between the charges, but $E \neq 0$ there since the fields point in the same direction.

(ii) Between the two charges, the fields point in the same direction, so E cannot be zero there. In the other two regions, the field due to the nearer charge is always greater than the field due to the more distant charge, so they cannot cancel. Hence E is not zero anywhere.

EVALUATE: It does *not* follow that the electric field is zero where the potential is zero, or that the potential is zero where the electric field is zero.

23.19. **IDENTIFY:** Apply $V = \dfrac{1}{4\pi\varepsilon_0}\sum_i \dfrac{q_i}{r_i}$.

SET UP: The locations of the charges and points A and B are sketched in Figure 23.19.

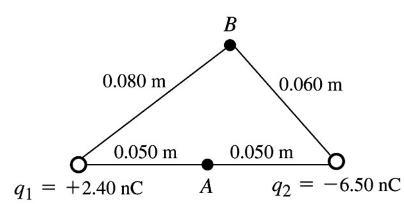


Figure 23.19

EXECUTE: **(a)** $V_A = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_{A1}} + \dfrac{q_2}{r_{A2}}\right)$.

$$V_A = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times10^{-9}\ \text{C}}{0.050\ \text{m}} + \frac{-6.50\times10^{-9}\ \text{C}}{0.050\ \text{m}}\right) = -737\ \text{V}.$$

(b) $V_B = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_{B1}} + \dfrac{q_2}{r_{B2}}\right)$.

$$V_B = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times10^{-9}\ \text{C}}{0.080\ \text{m}} + \frac{-6.50\times10^{-9}\ \text{C}}{0.060\ \text{m}}\right) = -704\ \text{V}.$$

(c) IDENTIFY and **SET UP:** Use $W_{a\to b} = q(V_a - V_b)$ and the results of parts (a) and (b) to calculate W.

EXECUTE: $W_{B\to A} = q(V_B - V_A) = (2.50\times10^{-9}\ \text{C})\left[-704\ \text{V} - (-737\ \text{V})\right] = +8.2\times10^{-8}\ \text{J}$.

EVALUATE: The electric force does positive work on the positive charge when it moves from higher potential (point B) to lower potential (point A).

23.20. **IDENTIFY** and **SET UP:** Apply conservation of energy: $K_a + U_a = K_b + U_b$. Use $V = U/q_0$ to express U in terms of V.

(a) EXECUTE: $K_1 + qV_1 = K_2 + qV_2$, $q(V_2 - V_1) = K_1 - K_2$; $q = -1.602\times10^{-19}$ C.

$K_1 = \frac{1}{2}m_e v_1^2 = 4.099\times10^{-18}$ J; $K_2 = \frac{1}{2}m_e v_2^2 = 2.915\times10^{-17}$ J. $\Delta V = V_2 - V_1 = \dfrac{K_1 - K_2}{q} = 156$ V.

EVALUATE: The electron gains kinetic energy when it moves to higher potential.

(b) EXECUTE: Now $K_1 = 2.915\times10^{-17}$ J, $K_2 = 0$. $V_2 - V_1 = \dfrac{K_1 - K_2}{q} = -182$ V.

EVALUATE: The electron loses kinetic energy when it moves to lower potential.

23.21. **IDENTIFY:** We want to relate the electric field and potential.
SET UP and **EXECUTE:** (a) $V_B > V_A$. The electric field points from higher to lower potential (from B to A), so E_y is negative.
(b) We want $|E_y|$. $|E_y|(y_B - y_A) = 12.0$ V, so $|E_y| = (12.0\text{ V})/(0.0700\text{ m}) = 171$ V/m.
(c) $V_C - V_B = E_y(y_C - y_B) = (-171\text{ V/m})(0.100\text{ m}) = -17.1$ V.
EVALUATE: The $V_C = V_B - 17.1$ V/m, so $V_B > V_C$.

23.22. **IDENTIFY:** For a point charge, $E = \dfrac{k|q|}{r^2}$ and $V = \dfrac{kq}{r}$.

SET UP: The electric field is directed toward a negative charge and away from a positive charge.

EXECUTE: **(a)** $V > 0$ so $q > 0$. $\dfrac{V}{E} = \dfrac{kq/r}{k|q|/r^2} = \left(\dfrac{kq}{r}\right)\left(\dfrac{r^2}{kq}\right) = r$. $r = \dfrac{4.98\text{ V}}{16.2\text{ V/m}} = 0.307$ m.

(b) $q = \dfrac{rV}{k} = \dfrac{(0.307\text{ m})(4.98\text{ V})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 1.70\times10^{-10}$ C.

(c) $q > 0$, so the electric field is directed away from the charge.

EVALUATE: The ratio of V to E due to a point charge increases as the distance r from the charge increases, because E falls off as $1/r^2$ and V falls off as $1/r$.

23.23. **(a) IDENTIFY** and **EXECUTE:** The direction of $\vec{E}$ is always from high potential to low potential so point b is at higher potential.

(b) IDENTIFY and **SET UP:** Apply $V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$ to relate $V_b - V_a$ to E.

EXECUTE: $V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\,dx = E(x_b - x_a)$.

$$E = \frac{V_b - V_a}{x_b - x_a} = \frac{+240\text{ V}}{0.90\text{ m} - 0.60\text{ m}} = 800\text{ V/m}$$

(c) SET UP and **EXECUTE:** $W_{b\to a} = q(V_b - V_a) = (-0.200\times10^{-6}\text{ C})(+240\text{ V}) = -4.80\times10^{-5}$ J.

EVALUATE: The electric force does negative work on a negative charge when the negative charge moves from high potential (point b) to low potential (point a).

23.24. **IDENTIFY:** This problem deals with the potential and energy of point charges.
SET UP and **EXECUTE:** (a) We want $V_A - V_B$. The negative sphere loses kinetic energy as it goes from A to B, so it is gaining potential energy. Thus it has higher potential energy at B than at A. A negative charge is accelerated from low to high potential by the field, so since $U_B > U_A$, it follows that $V_B < V_A$. Thus A has higher potential. Energy conservation gives $U_A + K_A = U_B + K_B$, so
$U_A - U_B = K_B - K_A = q(V_A - V_B)$. Using the given numbers gives $V_A - V_B = +100$ V.

(b) Since $V_A > V_B$, the electric field points from A to B, which is in the $+x$ direction. Thus $E_x \Delta x = \Delta V$.

So $E_x = \dfrac{\Delta V}{\Delta x} = \dfrac{100\text{ V}}{0.400\text{ m}} = +250\text{ V/m}.$

EVALUATE: Careful! Positive charges are accelerated from high to low potential, but negative charges are accelerated from low to high potential.

23.25. **IDENTIFY:** This problem involves the potential energy of point charges.

SET UP and **EXECUTE:** (a) We want the work. $W = -\Delta U = -Q\Delta V = -Q(V_0 - V\infty) = -QV_0$. Using $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$ with $q_1 = q_2 = q$ and $r_1 = r_2 = r$ gives $W = \dfrac{-Qq}{4\pi\epsilon_0}\left(\dfrac{1}{r} + \dfrac{1}{r}\right)$. Using $r = 0.400$ m and the given values gives $W = 0.0539$ J.

(b) We want the speed at the origin. $W = K = \dfrac{1}{2}mv^2$ gives $v = \sqrt{2W/m}$. Using W from part (a) and the given values gives $v = 3.00$ m/s.

EVALUATE: At the origin the force on the sphere is zero, but it does not stop. In fact, its kinetic energy would be converted back to electric potential energy and it would move out to infinity.

23.26. **IDENTIFY:** This problem involves the potential of a small charged sphere.

SET UP and **EXECUTE:** We want the charge on the sphere. We need to relate $V_A - V_B$ to $1/r$ to interpret the graph. $V_A - V_B = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{r_A} - \dfrac{q}{r}\right)$. The slope should be $-kq$ and the y-intercept $\dfrac{q}{4\pi\epsilon_0 r_A}$.

$q = -4\pi\epsilon_0 \text{ (slope)} = -4\pi\epsilon_0 (-18.0\text{ V}\cdot\text{m}) = +2.00\text{ nC}.$

EVALUATE: Since the y-intercept is $\dfrac{q}{4\pi\epsilon_0 r_A}$, we could use it as a check if we knew its value.

23.27. **IDENTIFY:** The potential at any point is the scalar sum of the potential due to each shell.

SET UP: $V = \dfrac{kq}{R}$ for $r \le R$ and $V = \dfrac{kq}{r}$ for $r > R$.

EXECUTE: **(a)** (i) $r = 0$. This point is inside both shells so

$$V = k\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right) = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{6.00\times10^{-9}\text{ C}}{0.0300\text{ m}} + \frac{-9.00\times10^{-9}\text{ C}}{0.0500\text{ m}}\right).$$

$$V = +1.798\times10^3\text{ V} + (-1.618\times10^3\text{ V}) = 180\text{ V}.$$

(ii) $r = 4.00$ cm. This point is outside shell 1 and inside shell 2.

$$V = k\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{6.00\times10^{-9}\text{ C}}{0.0400\text{ m}} + \frac{-9.00\times10^{-9}\text{ C}}{0.0500\text{ m}}\right).$$

$$V = +1.348\times10^3\text{ V} + (-1.618\times10^3\text{ V}) = -270\text{ V}.$$

(iii) $r = 6.00$ cm. This point is outside both shells.

$$V = k\left(\frac{q_1}{r} + \frac{q_2}{r}\right) = \frac{k}{r}(q_1 + q_2) = \frac{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}{0.0600\text{ m}}\left[6.00\times10^{-9}\text{ C} + (-9.00\times10^{-9}\text{ C})\right].\ V = -450\text{ V}.$$

(b) At the surface of the inner shell, $r = R_1 = 3.00$ cm. This point is inside the larger shell, so $V_1 = k\left(\dfrac{q_1}{R_1} + \dfrac{q_2}{R_2}\right) = 180$ V. At the surface of the outer shell, $r = R_2 = 5.00$ cm. This point is outside the smaller shell, so

$$V = k\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{6.00\times10^{-9}\ \text{C}}{0.0500\ \text{m}} + \frac{-9.00\times10^{-9}\ \text{C}}{0.0500\ \text{m}}\right).$$

$V_2 = +1.079\times10^3\ \text{V} + (-1.618\times10^3\ \text{V}) = -539\ \text{V}$. The potential difference is $V_1 - V_2 = 719\ \text{V}$. The inner shell is at higher potential. The potential difference is due entirely to the charge on the inner shell.

EVALUATE: Inside a uniform spherical shell, the electric field is zero so the potential is constant (but *not* necessarily zero).

23.28. **IDENTIFY** and **SET UP:** Outside a solid conducting sphere $V = k\frac{q}{r}$. Inside the sphere the potential is constant because $E = 0$, and it has the same value as at the surface of the sphere.

EXECUTE: **(a)** This is outside the sphere, so $V = \frac{kq}{r} = \frac{k(3.50\times10^{-9}\ \text{C})}{0.480\ \text{m}} = 65.6\ \text{V}$.

(b) This is at the surface of the sphere, so $V = \frac{k(3.50\times10^{-9}\ \text{C})}{0.240\ \text{m}} = 131\ \text{V}$.

(c) This is inside the sphere. The potential has the same value as at the surface, 131 V.

EVALUATE: All points of a conductor are at the same potential.

23.29. **(a) IDENTIFY** and **SET UP:** The electric field on the ring's axis is given by $E_x = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}$.

The magnitude of the force on the electron exerted by this field is given by $F = eE$.

EXECUTE: When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form $F = -kx$ so the oscillatory motion is not simple harmonic motion.

(b) IDENTIFY: Apply conservation of energy to the motion of the electron.

SET UP: $K_a + U_a = K_b + U_b$ with a at the initial position of the electron and b at the center of the ring.

From Example 23.11, $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$, where a is the radius of the ring.

EXECUTE: $x_a = 30.0\ \text{cm}, x_b = 0$.

$K_a = 0$ (released from rest), $K_b = \frac{1}{2}mv^2$.

Thus $\frac{1}{2}mv^2 = U_a - U_b$.

And $U = qV = -eV$ so $v = \sqrt{\frac{2e(V_b - V_a)}{m}}$.

$$V_a = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_a^2+a^2}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\ \text{C}}{\sqrt{(0.300\ \text{m})^2+(0.150\ \text{m})^2}}.$$

$$V_a = 643\ \text{V}.$$

$$V_b = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_b^2+a^2}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\ \text{C}}{0.150\ \text{m}} = 1438\ \text{V}.$$

$$v = \sqrt{\frac{2e(V_b - V_a)}{m}} = \sqrt{\frac{2(1.602\times10^{-19}\ \text{C})(1438\ \text{V} - 643\ \text{V})}{9.109\times10^{-31}\ \text{kg}}} = 1.67\times10^7\ \text{m/s}.$$

EVALUATE: The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

23.30. **IDENTIFY:** For an isolated conducting sphere, all the excess charge is on its outer surface. For points outside the sphere, it behaves like a point-charge at its center, and the electric field is zero inside the sphere.

SET UP: Use V at 1.20 m to find V at the surface. $V = k\dfrac{q}{r}$. We don't know the charge on the sphere, but we know the potential 1.20 m from its center.

EXECUTE: Take the ratio of the potentials: $\dfrac{V_{\text{surface}}}{V_{1.20\text{ m}}} = \dfrac{kq/(0.400\text{ m})}{kq(1.20\text{ m})} = \dfrac{1.20}{0.400} = 3.00,$ so

$V_{\text{surface}} = (3.00)(24.0\text{ V}) = 72.0\text{ V}$.

The electric field is zero inside the sphere, so the potential inside is constant and equal to the potential at the surface. So at the center $V = 72.0$ V.

EVALUATE: An alternative approach would be to use the given information to find the charge on the sphere. Then use that charge to calculate the potential at the surface. The potential is 72.0 V at *all* points inside the sphere, not just at the center. Careful! Just because the electric field inside the sphere is zero, it does not follow that the potential is zero there.

23.31. **IDENTIFY:** If the small sphere is to have its *minimum* speed, it must just stop at 8.00 cm from the surface of the large sphere. In that case, the initial kinetic energy of the small sphere is all converted to electrical potential energy at its point of closest approach.

SET UP: $K_1 + U_1 = K_2 + U_2$. $K_2 = 0$. $U_1 = 0$. Therefore, $K_1 = U_2$. Outside a spherical charge distribution the potential is the same as for a point charge at the location of the center of the sphere, so $U = kqQ/r. K = \frac{1}{2}mv^2$.

EXECUTE: $U_2 = \dfrac{kqQ}{r_2}$, with $r_2 = 12.0\text{ cm} + 8.0\text{ cm} = 0.200\text{ m}$. $\dfrac{1}{2}mv_1^2 = \dfrac{kqQ}{r_2}$.

$$v_1 = \sqrt{\frac{2kqQ}{mr_2}} = \sqrt{\frac{2(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\text{ C})(5.00\times10^{-6}\text{ C})}{(6.00\times10^{-5}\text{ kg})(0.200\text{ m})}} = 150\text{ m/s}.$$

EVALUATE: If the small sphere had enough initial speed to actually penetrate the surface of the large sphere, we could no longer treat the large sphere as a point charge once the small sphere was inside.

23.32. **IDENTIFY:** For a line of charge, $V_a - V_b = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$. Apply conservation of energy to the motion of the proton.

SET UP: Let point a be 18.0 cm from the line and let point b be at the distance of closest approach, where $K_b = 0$.

EXECUTE: **(a)** $K_a = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(3.50\times10^3\text{ m/s})^2 = 1.02\times10^{-20}\text{ J}$.

(b) $K_a + qV_a = K_b + qV_b$. $V_a - V_b = \dfrac{K_b - K_a}{q} = \dfrac{-1.02\times10^{-20}\text{ J}}{1.60\times10^{-19}\text{ C}} = -0.06397\text{ V}$.

$$\ln(r_b/r_a) = \left(\frac{2\pi\epsilon_0}{\lambda}\right)(-0.06397\text{ V}).$$

$$r_b = r_a\exp\left(\frac{2\pi\epsilon_0\,(-0.06397\text{ V})}{\lambda}\right) = (0.180\text{ m})\exp\left(-\frac{2\pi\epsilon_0\,(0.06397\text{ V})}{5.00\times10^{-12}\text{ C/m}}\right) = 0.0883\text{ m} = 8.83\text{ cm}.$$

EVALUATE: The potential increases with decreasing distance from the line of charge. As the positively charged proton approaches the line of charge it gains electrical potential energy and loses kinetic energy.

23.33. IDENTIFY: For points outside the cylinder, its electric field behaves like that of a line of charge. Since a voltmeter reads potential difference, that is what we need to calculate.

SET UP: The potential difference is $\Delta V = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$.

EXECUTE: **(a)** Substituting numbers gives

$$\Delta V = \frac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a) = (8.50\times10^{-6}\text{ C/m})(2\times9.00\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\ln\left(\frac{10.0\text{ cm}}{6.00\text{ cm}}\right).$$

$$\Delta V = 7.82\times10^{4}\text{ V} = 78{,}200\text{ V} = 78.2\text{ kV}.$$

(b) $E = 0$ inside the cylinder, so the potential is constant there, meaning that the voltmeter reads zero.

EVALUATE: Caution! The fact that the voltmeter reads zero in part (b) does not mean that $V = 0$ inside the cylinder. The electric field is zero, but the potential is constant and equal to the potential at the surface.

23.34. IDENTIFY: The voltmeter reads the potential difference between the two points where the probes are placed. Therefore we must relate the potential difference to the distances of these points from the center of the cylinder. For points outside the cylinder, its electric field behaves like that of a line of charge.

SET UP: Using $\Delta V = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$ and solving for r_b, we have $r_b = r_a e^{2\pi\epsilon_0\Delta V/\lambda}$.

EXECUTE: The exponent is $\dfrac{\left(\dfrac{1}{2\times8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2}\right)(175\text{ V})}{15.0\times10^{-9}\text{ C/m}} = 0.648$, which gives

$$r_b = (2.50\text{ cm})\,e^{0.648} = 4.78\text{ cm}.$$

The distance above the *surface* is $4.78\text{ cm} - 2.50\text{ cm} = 2.28\text{ cm}$.

EVALUATE: Since a voltmeter measures potential difference, we are actually given ΔV, even though that is not stated explicitly in the problem. We must also be careful when using the formula for the potential difference because each *r* is the distance from the *center* of the cylinder, not from the surface.

23.35. IDENTIFY: The electric field of the line of charge does work on the sphere, increasing its kinetic energy.

SET UP: $K_1 + U_1 = K_2 + U_2$ and $K_1 = 0$. $U = qV$ so $qV_1 = K_2 + qV_2$. $V = \dfrac{\lambda}{2\pi\epsilon_0}\ln\left(\dfrac{r_0}{r}\right)$.

EXECUTE: $V_1 = \dfrac{\lambda}{2\pi\epsilon_0}\ln\left(\dfrac{r_0}{r_1}\right)$. $V_2 = \dfrac{\lambda}{2\pi\epsilon_0}\ln\left(\dfrac{r_0}{r_2}\right)$.

$$K_2 = q(V_1 - V_2) = \frac{q\lambda}{2\pi\epsilon_0}\left(\ln\left(\frac{r_0}{r_1}\right) - \ln\left(\frac{r_0}{r_2}\right)\right) = \frac{\lambda q}{2\pi\epsilon_0}(\ln r_2 - \ln r_1) = \frac{\lambda q}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right).$$

$$K_2 = \frac{(3.00\times10^{-6}\text{ C/m})(8.00\times10^{-6}\text{ C})}{2\pi(8.854\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)}\ln\left(\frac{4.50}{1.50}\right) = 0.474\text{ J}.$$

EVALUATE: The potential due to the line of charge does *not* go to zero at infinity but is defined to be zero at an arbitrary distance r_0 from the line.

23.36. **IDENTIFY** and **SET UP:** For oppositely charged parallel plates, $E = \sigma/\epsilon_0$ between the plates and the potential difference between the plates is $V = Ed$.

EXECUTE: **(a)** $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{47.0\times10^{-9}\ \text{C/m}^2}{\epsilon_0} = 5310\ \text{N/C}.$

(b) $V = Ed = (5310\ \text{N/C})(0.0220\ \text{m}) = 117\ \text{V}.$

(c) The electric field stays the same if the separation of the plates doubles. The potential difference between the plates doubles.

EVALUATE: The electric field of an infinite sheet of charge is uniform, independent of distance from the sheet. The force on a test charge between the two plates is constant because the electric field is constant. The potential difference is the work per unit charge on a test charge when it moves from one plate to the other. When the distance doubles, the work, which is force times distance, doubles and the potential difference doubles.

23.37. **IDENTIFY** and **SET UP:** Use $\Delta V = Ed$ to relate the electric field between the plates to the potential difference between them and their separation. The magnitude of the force this field exerts on the particle is given by $F = qE$. Use $W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l}$ to calculate the work.

EXECUTE: **(a)** Using $\Delta V = Ed$ gives $E = \dfrac{V_{ab}}{d} = \dfrac{360\ \text{V}}{0.0450\ \text{m}} = 8000\ \text{V/m}.$

(b) $F = |q|E = (2.40\times10^{-9}\ \text{C})(8000\ \text{V/m}) = +1.92\times10^{-5}\ \text{N}.$

(c) The electric field between the plates is shown in Figure 23.37.

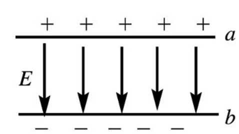


Figure 23.37

The plate with positive charge (plate *a*) is at higher potential. The electric field is directed from high potential toward low potential (or, $\vec{E}$ is from + charge toward − charge), so $\vec{E}$ points from *a* to *b*. Hence the force that $\vec{E}$ exerts on the positive charge is from *a* to *b*, so it does positive work.

$W = \int_a^b \vec{F}\cdot d\vec{l} = Fd$, where *d* is the separation between the plates.

$W = Fd = (1.92\times10^{-5}\ \text{N})(0.0450\ \text{m}) = +8.64\times10^{-7}\ \text{J}.$

(d) $V_a - V_b = +360\ \text{V}$ (plate *a* is at higher potential).

$\Delta U = U_b - U_a = q(V_b - V_a) = (2.40\times10^{-9}\ \text{C})(-360\ \text{V}) = -8.64\times10^{-7}\ \text{J}.$

EVALUATE: We see that $W_{a\to b} = -(U_b - U_a) = U_a - U_b$.

23.38. **IDENTIFY** and **SET UP:** $V_{ab} = Ed$ for parallel plates.

EXECUTE: $d = \dfrac{V_{ab}}{E} = \dfrac{1.5\ \text{V}}{1.0\times10^{-6}\ \text{V/m}} = 1.5\times10^{6}\ \text{m} = 1.5\times10^{3}\ \text{km}.$

EVALUATE: The plates would have to be nearly a thousand miles apart with only a AA battery across them! This is a small field!

23.39. **IDENTIFY:** The potential of a solid conducting sphere is the same at every point inside the sphere because $E = 0$ inside, and this potential has the value $V = q/4\pi\epsilon_0 R$ at the surface. Use the given value of *E* to find *q*.

SET UP: For negative charge the electric field is directed toward the charge.
For points outside this spherical charge distribution the field is the same as if all the charge were concentrated at the center.

EXECUTE: $E = \dfrac{|q|}{4\pi\epsilon_0 r^2}$ and $|q| = 4\pi\epsilon_0 Er^2 = \dfrac{(3800\ \text{N/C})(0.200\ \text{m})^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.69\times10^{-8}\ \text{C}.$

Since the field is directed inward, the charge must be negative. The potential of a point charge, taking ∞ as zero, is $V = \dfrac{q}{4\pi\epsilon_0 r} = \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-1.69\times10^{-8}\ \text{C})}{0.200\ \text{m}} = -760\ \text{V}$ at the surface of the sphere. Since the charge all resides on the surface of a conductor, the field inside the sphere due to this symmetrical distribution is zero. No work is therefore done in moving a test charge from just inside the surface to the center, and the potential at the center must also be –760 V.

EVALUATE: Inside the sphere the electric field is zero and the potential is constant.

23.40. IDENTIFY: The electric field is zero inside the sphere, so the potential is constant there. Thus the potential at the center must be the same as at the surface, where it is equivalent to that of a point-charge.

SET UP: At the surface, and hence also at the center of the sphere, the potential is that of a point-charge, $V = Q/(4\pi\epsilon_0 R)$.

EXECUTE: **(a)** Solving for Q and substituting the numbers gives

$Q = 4\pi\epsilon_0 RV = (0.125\ \text{m})(3750\ \text{V})/(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 5.21\times10^{-8}\ \text{C} = 52.1\ \text{nC}.$

(b) Since the potential is constant inside the sphere, its value at the surface must be the same as at the center, 3.75 kV.

EVALUATE: The electric field inside the sphere is zero, so the potential is constant but is not zero.

23.41. IDENTIFY and **SET UP:** For a solid metal sphere or for a spherical shell, $V = \dfrac{kq}{r}$ outside the sphere and $V = \dfrac{kq}{R}$ at all points inside the sphere, where R is the radius of the sphere. When the electric field is radial, $E = -\dfrac{\partial V}{\partial r}$.

EXECUTE: **(a)** (i) $r < r_a$: This region is inside both spheres. $V = \dfrac{kq}{r_a} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right).$

(ii) $r_a < r < r_b$: This region is outside the inner shell and inside the outer shell.

$V = \dfrac{kq}{r} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right).$

(iii) $r > r_b$: This region is outside both spheres and $V = 0$ since outside a sphere the potential is the same as for a point charge. Therefore the potential is the same as for two oppositely charged point charges at the same location. These potentials cancel.

(b) $V_a = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{r_a} - \dfrac{q}{r_b}\right)$ and $V_b = 0$, so $V_{ab} = \dfrac{1}{4\pi\epsilon_0} q\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right).$

(c) Between the spheres $r_a < r < r_b$ and $V = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right).$

$$E_r = -\frac{\partial V}{\partial r} = -\frac{q}{4\pi\epsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r} - \frac{1}{r_b}\right) = +\frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{V_{ab}}{\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}\frac{1}{r^2}.$$

(d) Since $E_r = -\frac{\partial V}{\partial r}$, $E = 0$, since V is constant (zero) outside the spheres.

(e) If the outer charge is different, then outside the outer sphere the potential is no longer zero but is $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} - \frac{1}{4\pi\epsilon_0}\frac{Q}{r} = \frac{1}{4\pi\epsilon_0}\frac{(q-Q)}{r}$. All potentials inside the outer shell are just shifted by an amount $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_b}$. Therefore relative potentials within the shells are not affected. Thus (b) and (c) do not change. However, now that the potential does vary outside the spheres, there is an electric field there: $E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{kq}{r} + \frac{-kQ}{r}\right) = \frac{kq}{r^2}\left(1 - \frac{Q}{q}\right) = \frac{k}{r^2}(q-Q)$.

EVALUATE: In part (a) the potential is greater than zero for all $r < r_b$.

23.42. **IDENTIFY:** By the definition of electric potential, if a positive charge gains potential along a path, then the potential along that path must have increased. The electric field produced by a very large sheet of charge is uniform and is independent of the distance from the sheet.

(a) SET UP: No matter what the reference point, we must do work on a positive charge to move it away from the negative sheet.

EXECUTE: Since we must do work on the positive charge, it gains potential energy, so the potential increases.

(b) SET UP: Since the electric field is uniform and is equal to $\sigma/2\epsilon_0$, we have $\Delta V = Ed = \frac{\sigma}{2\epsilon_0}d$.

EXECUTE: Solving for d gives

$$d = \frac{2\epsilon_0\,\Delta V}{\sigma} = \frac{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.00\ \text{V})}{6.00\times10^{-9}\ \text{C/m}^2} = 0.00295\ \text{m} = 2.95\ \text{mm}.$$

EVALUATE: Since the spacing of the equipotential surfaces (d = 2.95 mm) is independent of the distance from the sheet, the equipotential surfaces are planes parallel to the sheet and spaced 2.95 mm apart.

23.43. **IDENTIFY** and **SET UP:** Use $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, and $E_z = \frac{\partial V}{\partial z}$ to calculate the components of $\vec{E}$.

EXECUTE: $V = Axy - Bx^2 + Cy$.

(a) $E_x = -\frac{\partial V}{\partial x} = -Ay + 2Bx$.

$E_y = -\frac{\partial V}{\partial y} = -Ax - C$.

$E_z = \frac{\partial V}{\partial z} = 0$.

(b) $E = 0$ requires that $E_x = E_y = E_z = 0$.

$E_z = 0$ everywhere.

$E_y = 0$ at $x = -C/A$.

And E_x is also equal to zero for this x, any value of z and $y = 2Bx/A = (2B/A)(-C/A) = -2BC/A^2$.

EVALUATE: V doesn't depend on z so $E_z = 0$ everywhere.

23.44. IDENTIFY: Apply $E_x = -\frac{\partial V}{\partial x}$ and $E_y = -\frac{\partial V}{\partial y}$ to find the components of $\vec{E}$, then use them to find its magnitude and direction. $V(x, y) = Ax^2y - Bxy^2$.

SET UP: $E = \sqrt{E_x^2 + E_y^2}$ and $\tan\theta = E_y/E_x$.

EXECUTE: First find the components of $\vec{E}$: $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(Ax^2y - Bxy^2) = -(2Axy - By^2)$.

Now evaluate this result at the point $x = 2.00$ m, $y = 0.400$ m using the given values for A and B.

$E_x = -[2(5.00\ \text{V/m}^3)(2.00\ \text{m})(0.400\ \text{m}) - (8.00\ \text{V/m}^3)(0.400\ \text{m})^2] = -6.72\ \text{V/m}$.

$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(Ax^2y - Bxy^2) = -(Ax^2 - 2Bxy)$. At the point (2.00 m, 0.400 m), this is

$E_y = -[(5.00\ \text{V/m}^3)(2.00\ \text{m})^2 - 2(8.00\ \text{V/m}^3)(2.00\ \text{m})(0.400\ \text{m})] = -7.20\ \text{V/m}$.

Now use the components to find the magnitude and direction of $\vec{E}$.

$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-6.72\ \text{V/m})^2 + (-7.20\ \text{V/m})^2} = 9.85\ \text{V/m}$.

$\tan\theta = E_y/E_x = (-7.20\ \text{V/m})/(-6.72\ \text{V/m})$, which gives $\theta = 47.0°$. Since both components are negative, the vector lies in the third quadrant in the xy-plane and makes an angle of 47.0° + 180.0° = 227.0° with the $+x$-axis.

EVALUATE: V is a scalar but $\vec{E}$ is a vector and has components.

23.45. IDENTIFY: Exercise 23.41 shows that $V = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$ for $r < r_a$, $V = kq\left(\frac{1}{r} - \frac{1}{r_b}\right)$ for $r_a < r < r_b$ and

$V_{ab} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$.

SET UP: $E = \frac{kq}{r^2}$, radially outward, for $r_a \le r \le r_b$.

EXECUTE: **(a)** $V_{ab} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = 500\ \text{V}$ gives $q = \dfrac{500\ \text{V}}{k\left(\frac{1}{0.012\ \text{m}} - \frac{1}{0.096\ \text{m}}\right)} = 7.62\times10^{-10}\ \text{C}$

$= 0.762$ nC.

(b) $V_b = 0$ so $V_a = 500$ V. The inner metal sphere is an equipotential with $V = 500$ V. $\frac{1}{r} = \frac{1}{r_a} + \frac{V}{kq}$.

$V = 400$ V at $r = 1.45$ cm, $V = 300$ V at $r = 1.85$ cm, $V = 200$ V at $r = 2.53$ cm, $V = 100$ V at $r = 4.00$ cm, $V = 0$ at $r = 9.60$ cm. The equipotential surfaces are sketched in Figure 23.45.

EVALUATE: **(c)** The equipotential surfaces are concentric spheres and the electric field lines are radial, so the field lines and equipotential surfaces are mutually perpendicular. The equipotentials are closest at smaller r, where the electric field is largest.

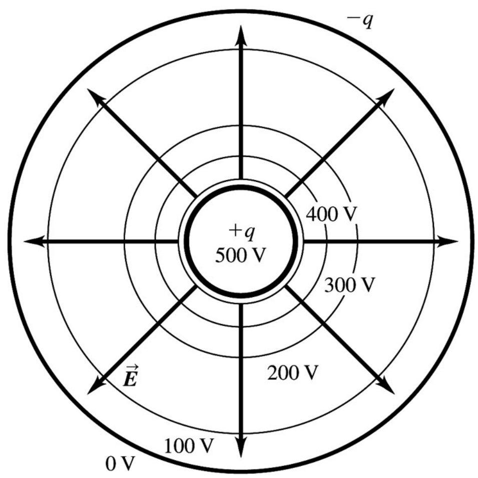


Figure 23.45

23.46. **IDENTIFY:** As the sphere approaches the point charge, the speed of the sphere decreases because it loses kinetic energy, but its acceleration increases because the electric force on it increases. Its mechanical energy is conserved during the motion, and Newton's second law and Coulomb's law both apply.

SET UP: $K_a + U_a = K_b + U_b$, $K = \frac{1}{2}mv^2$, $U = kq_1q_2/r$, $F = kq_1q_2/r^2$, and $F = ma$.

EXECUTE: Find the distance between the two charges when $v_2 = 25.0$ m/s.

$K_a + U_a = K_b + U_b$.

$$K_a = \frac{1}{2}mv_a^2 = \frac{1}{2}(4.00\times10^{-3}\text{ kg})(40.0\text{ m/s})^2 = 3.20\text{ J}.$$

$$K_b = \frac{1}{2}mv_b^2 = \frac{1}{2}(4.00\times10^{-3}\text{ kg})(25.0\text{ m/s})^2 = 1.25\text{ J}.$$

$$U_a = k\frac{q_1q_2}{r_a} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})(2.00\times10^{-6}\text{ C})}{0.0600\text{ m}} = 1.498\text{ J}.$$

$U_b = K_a + U_a - K_b = 3.20\text{ J} + 1.498\text{ J} - 1.25\text{ J} = 3.448\text{ J}$. $U_b = k\dfrac{q_1q_2}{r_b}$ and

$$r_b = \frac{kq_1q_2}{U_b} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})(2.00\times10^{-6}\text{ C})}{3.448\text{ J}} = 0.02607\text{ m}.$$

$$F_b = \frac{kq_1q_2}{r_b^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})(2.00\times10^{-6}\text{ C})}{(0.02607\text{ m})^2} = 132.3\text{ N}.$$

$$a = \frac{F}{m} = \frac{132.3\text{ N}}{4.00\times10^{-3}\text{ kg}} = 3.31\times10^4\text{ m/s}^2.$$

EVALUATE: As the sphere approaches the point charge, its speed decreases but its acceleration keeps increasing because the electric force on it keeps increasing.

23.47. **IDENTIFY:** $U = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_1q_3}{r_{13}} + \dfrac{q_2q_3}{r_{23}}\right)$.

SET UP: In part (a), $r_{12} = 0.200$ m, $r_{23} = 0.100$ m and $r_{13} = 0.100$ m. In part (b) let particle 3 have coordinate x, so $r_{12} = 0.200$ m, $r_{13} = x$ and $r_{23} = 0.200\text{ m} - x$.

EXECUTE: (a)

$$U = k\left(\frac{(4.00\text{ nC})(-3.00\text{ nC})}{(0.200\text{ m})} + \frac{(4.00\text{ nC})(2.00\text{ nC})}{(0.100\text{ m})} + \frac{(-3.00\text{ nC})(2.00\text{ nC})}{(0.100\text{ m})}\right) = -3.60\times10^{-7}\text{ J}.$$

(b) If $U = 0$, then $0 = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{x} + \frac{q_2q_3}{r_{12}-x}\right)$. Solving for x we find:

$0 = -60 + \frac{8}{x} - \frac{6}{0.2-x} \Rightarrow 60x^2 - 26x + 1.6 = 0 \Rightarrow x = 0.074\text{ m}, 0.360\text{ m}$. Therefore, $x = 0.074\text{ m}$ since it is the only value between the two charges.

EVALUATE: U_{13} is positive and both U_{23} and U_{12} are negative. If $U = 0$, then $|U_{13}| = |U_{23}| + |U_{12}|$. For $x = 0.074\text{ m}$, $U_{13} = +9.7\times10^{-7}\text{ J}$, $U_{23} = -4.3\times10^{-7}\text{ J}$ and $U_{12} = -5.4\times10^{-7}\text{ J}$. It is true that $U = 0$ at this x.

23.48. IDENTIFY: We are dealing with the electric field and potential of point charges.

SET UP: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ and $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$. Let $\frac{1}{4\pi\epsilon_0}$ be k.

EXECUTE: (a) We want to find where $E = 0$. To cancel, the fields must be in opposite directions and have equal magnitudes. This can happen only for $x > 0.200$ m. Call d the distance between the –4.00 nC charge and the point where $E = 0$. The magnitudes are equal so $\frac{k(8.00\text{ nC})}{(d+0.400\text{ m})^2} = \frac{k(4.00\text{ nC})}{d^2}$. Solving for d gives $d = 1.17$ m.

(b) We want to find where $V = 0$. V can be zero when the potentials have equal magnitude but opposite sign. This occurs only for points closer to the –4.00 nC charge than to the 8.00 nC charge, which is between the charges and for $x > 0.200$ m.

<u>Between the charges</u>: Call x the distance from the –4.00 nC charge. Equating the magnitudes of the potentials gives $\frac{k(8.00\text{ nC})}{0.400\text{ m}-x} = \frac{k(4.00\text{ nC})}{x}$. x =0.067 m.

<u>For $x > 0.200$ m</u>: Call d the distance between the –4.00 nC charge and the point of cancellation. $\frac{k(8.00\text{ nC})}{d+0.400\text{ m}} = \frac{k(4.00\text{ nC})}{d}$. $d = 0.400$ m. $x = 0.400\text{ m} + 0.200\text{ m} = 0.600$ m.

(c) E could be zero only for $x > 0.200$ m, which is at $x = 1.17$ m (as we found), so E is not equal to zero at any of these points.

EVALUATE: This problem is a good illustration of the need to analyze before using any equations. Doing so allows us to decide which regions to consider for E to be zero and for V to be zero before doing any unnecessary calculations.

23.49. IDENTIFY: This problem involves potential difference and electric fields.

SET UP: For a sheet of charge $E_\sigma = \sigma/2\epsilon_0$, and for a line of charge $E_\lambda = \lambda/2\pi\epsilon_0 r$. Make a sketch showing the fields, as in Fig. 23.49. $V_B - V_A$ is due to the two fields. Between the sheet and the line the fields are in opposite directions and on the other side of the line they are in the same direction, as shown in the figure. $V_A - V_B = \Delta V_\sigma + \Delta V_\lambda$.

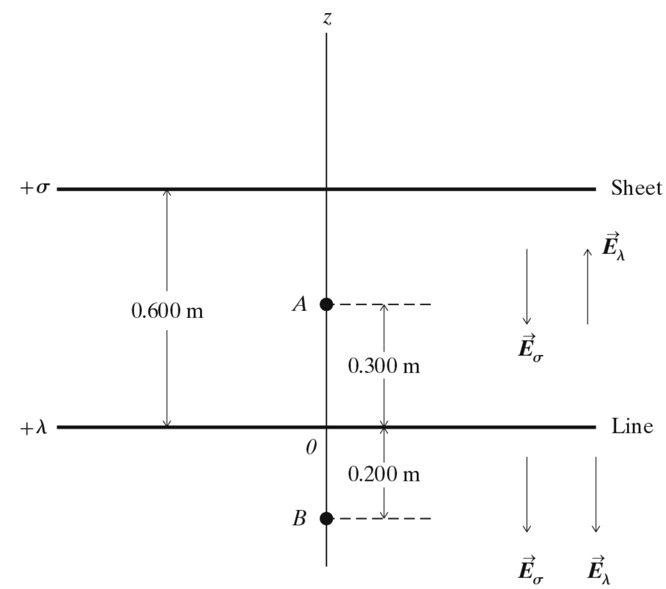


Figure 23.49

EXECUTE: <u>For the sheet</u>: E_σ is uniform, so

$$V_A - V_B = \Delta V_\sigma = E_\sigma \Delta z = \frac{\sigma}{2\epsilon_0}\Delta z = \frac{8.00\mu\text{C/m}^2}{2\epsilon_0}(0.500\text{ m}) = 2.260\times 10^5\text{ V.}$$

<u>For the line</u>: The field due to the line reverses direction as we cross the line. So the potential change in going from 0.200 m to the line is the opposite of the potential change in going from the line to 0.200 m on the other side. Thus the only potential change that does not cancel is in going from 0.300 m to 0.200 m on the upper side of the line. Therefore $\Delta V_\lambda = \int_{0.300\text{ m}}^{0.200\text{ m}} E_z dz = \int_{0.300\text{ m}}^{0.200\text{ m}} \frac{\lambda}{2\pi\epsilon_0 z} dz = \frac{\lambda}{2\pi\epsilon_0}\ln(2/3)$. This gives -3.645×10^4 V.

The total potential difference is $2.26\times 10^5\text{ V} - 3.645\times 10^4\text{ V} = +1.90\times 10^5\text{ V}$.

$V_A = V_B + 1.90\times 10^5$ V, so A is at a higher potential than B.

EVALUATE: It may appear that we skipped over an important point, namely that the potential becomes infinite if the line has no thickness. But any real line has some (but very small) thickness, so the potential difference between 0.200 m above the line can cancel the potential difference below the line without any infinities coming into play.

23.50. **IDENTIFY:** Two forces do work on the sphere as it falls: gravity and the electrical force due to the sheet. The energy of the sphere is conserved.

SET UP: The gravity force is mg, downward. The electric field of the sheet is $E = \frac{\sigma}{2\epsilon_0}$ upward, and the force it exerts on the sphere is $F = qE$. The sphere gains kinetic energy $K = \frac{1}{2}mv^2$ as it falls.

EXECUTE: $mg = 4.90\times10^{-6}$ N. $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{8.00\times10^{-12}\ \text{C/m}^2}{2(8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2))} = 0.4518$ N/C. The electric force is $qE = (7.00\times10^{-6}\ \text{C})(0.4518\ \text{N/C}) = 3.1626\times10^{-6}$ N, upward. The net force is downward, so the sphere moves downward when released. Let $y = 0$ at the sheet. $U_{\text{grav}} = mgy$. For the electric force, $\dfrac{W_{a\to b}}{q} = V_a - V_b$. Let point *a* be at the sheet and let point *b* be a distance *y* above the sheet. Take $V_a = 0$. The force on *q* is qE, upward, so $\dfrac{W_{a\to b}}{q} = Ey$ and $V_b = -Ey$. $U_b = -Eyq$. $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$.

$y_1 = 0.400$ m, $y_2 = 0.100$ m. $K_2 = U_1 - U_2 = mg(y_1 - y_2) - E(y_1 - y_2)q$.

$K_2 = (5.00\times10^{-7}\ \text{kg})(9.80\ \text{m/s}^2)(0.300\ \text{m}) - (0.4518\ \text{N/C})(0.300\ \text{m})(7.00\times10^{-6}\ \text{C})$.

$K_2 = 1.470\times10^{-6}\ \text{J} - 0.94878\times10^{-6}\ \text{J} = 0.52122\times10^{-6}$ J. $K_2 = \dfrac{1}{2}mv_2^2$ so

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.52122\times10^{-6}\ \text{J})}{5.00\times10^{-7}\ \text{kg}}} = 1.44\ \text{m/s}.$$

EVALUATE: Because the weight is greater than the electric force, the sphere will accelerate downward, but if it were light enough the electric force would exceed the weight. In that case it would never get closer to the sheet after being released. We could also solve this problem using Newton's second law and the constant-acceleration kinematics formulas. $a = F/m = (mg - qE)/m$ gives the acceleration. Then we use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $v_{0x} = 0$ to find *v*.

23.51. IDENTIFY and SET UP: Treat the gold nucleus as a point charge so that $V = k\dfrac{q}{r}$. According to conservation of energy we have $K_1 + U_1 = K_2 + U_2$, where $U = qV$.

EXECUTE: Assume that the alpha particle is at rest before it is accelerated and that it momentarily stops when it arrives at its closest approach to the surface of the gold nucleus. Thus we have $K_1 = K_2 = 0$, which implies that $U_1 = U_2$. Since $U = qV$ we conclude that the accelerating voltage must be equal to the voltage at its point of closest approach to the surface of the gold nucleus. Therefore

$$V_a = V_b = k\frac{q}{r} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{79(1.60\times10^{-19}\ \text{C})}{(7.3\times10^{-15}\ \text{m} + 2.0\times10^{-14}\ \text{m})} = 4.2\times10^6\ \text{V}.$$

EVALUATE: Although the alpha particle has kinetic energy as it approaches the gold nucleus this is irrelevant to our solution since energy is conserved for the whole process.

23.52. IDENTIFY: The charged particles repel each other and therefore accelerate away from one another, causing their speeds and kinetic energies to continue to increase. They do not have equal speeds because they have different masses. The mechanical energy and momentum of the system are conserved.

SET UP: The proton has charge $q_\text{p} = +e$ and mass $m_\text{p} = 1.67\times10^{-27}$ kg. The alpha particle has charge $q_\text{a} = +2e$ and mass $m_\text{a} = 4m_\text{p} = 6.68\times10^{-27}$ kg. We can apply both conservation of energy and conservation of linear momentum to the system. $a = \dfrac{F}{m}$, where $F = k\dfrac{|q_1q_2|}{r^2}$.

EXECUTE: *Acceleration:* The maximum force and hence the maximum acceleration occurs just after they are released, when $r = 0.225$ nm. $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(1.60\times10^{-19}\ \text{C})^2}{(0.225\times10^{-9}\ \text{m})^2} = 9.09\times10^{-9}$ N.

$a_{\rm p} = \dfrac{F}{m_{\rm p}} = \dfrac{9.09\times10^{-9}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = 5.44\times10^{18}\ \text{m/s}^2;\ a_{\rm a} = \dfrac{F}{m_{\rm a}} = \dfrac{9.09\times10^{-9}\ \text{N}}{6.68\times10^{-27}\ \text{kg}} = 1.36\times10^{18}\ \text{m/s}^2$. The acceleration of the proton is larger by a factor of $m_{\rm a}/m_{\rm p}$.

Speed: Conservation of energy says $U_1 + K_1 = U_2 + K_2$. $K_1 = 0$ and $U_2 = 0$, so $K_2 = U_1$.

$U_1 = k\dfrac{q_1 q_2}{r} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(1.60\times10^{-19}\ \text{C})^2}{0.225\times10^{-9}\ \text{m}} = 2.05\times10^{-18}\ \text{J}$, so the total kinetic energy of the two particles when they are far apart is $K_2 = 2.05\times10^{-18}$ J. Conservation of linear momentum says how this energy is divided between the proton and alpha particle. $p_1 = p_2$. $0 = m_{\rm p}v_{\rm p} - m_{\rm a}v_{\rm a}$ so

$v_{\rm a} = \left(\dfrac{m_{\rm p}}{m_{\rm a}}\right)v_{\rm p}$.

$K_2 = \frac{1}{2}m_{\rm p}v_{\rm p}^2 + \frac{1}{2}m_{\rm a}v_{\rm a}^2 = \frac{1}{2}m_{\rm p}v_{\rm p}^2 + \frac{1}{2}m_{\rm a}\left(\dfrac{m_{\rm p}}{m_{\rm a}}\right)^2 v_{\rm p}^2 = \frac{1}{2}m_{\rm p}v_{\rm p}^2\left(1+\dfrac{m_{\rm p}}{m_{\rm a}}\right)$.

$v_{\rm p} = \sqrt{\dfrac{2K_2}{m_{\rm p}(1+(m_{\rm p}/m_{\rm a}))}} = \sqrt{\dfrac{2(2.05\times10^{-18}\ \text{J})}{(1.67\times10^{-27}\ \text{kg})(1+\frac{1}{4})}} = 4.43\times10^4\ \text{m/s}$.

$v_{\rm a} = \left(\dfrac{m_{\rm p}}{m_{\rm a}}\right)v_{\rm p} = \frac{1}{4}(4.43\times10^4\ \text{m/s}) = 1.11\times10^4\ \text{m/s}$. The maximum acceleration occurs just after they are released. The maximum speed occurs after a long time.

EVALUATE: The proton and alpha particle have equal momenum, but proton has a greater acceleration and more kinetic energy.

23.53. **(a) IDENTIFY:** Apply the work-energy theorem.

SET UP: Points *a* and *b* are shown in Figure 23.53a.

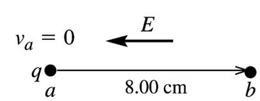


Figure 23.53a

EXECUTE: $W_{\text{tot}} = \Delta K = K_b - K_a = K_b = 4.35\times10^{-5}$ J.

The electric force F_E and the additional force F both do work, so that $W_{\text{tot}} = W_{F_E} + W_F$.

$$W_{F_E} = W_{\text{tot}} - W_F = 4.35\times10^{-5}\ \text{J} - 6.50\times10^{-5}\ \text{J} = -2.15\times10^{-5}\ \text{J}.$$

EVALUATE: The forces on the charged particle are shown in Figure 23.53b.

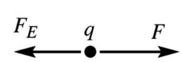


Figure 23.53b

The electric force is to the left (in the direction of the electric field since the particle has positive charge). The displacement is to the right, so the electric force does negative work. The additional force F is in the direction of the displacement, so it does positive work.

(b) IDENTIFY and **SET UP:** For the work done by the electric force, $W_{a\to b} = q(V_a - V_b)$.

EXECUTE: $V_a - V_b = \dfrac{W_{a\to b}}{q} = \dfrac{-2.15\times10^{-5}\ \text{J}}{7.60\times10^{-9}\ \text{C}} = -2.83\times10^{3}\ \text{V}.$

EVALUATE The starting point (point a) is at 2.83×10^{3} V lower potential than the ending point (point b). We know that $V_b > V_a$ because the electric field always points from high potential toward low potential.

(c) IDENTIFY: Calculate E from $V_a - V_b$ and the separation d between the two points.

SET UP: Since the electric field is uniform and directed opposite to the displacement $W_{a\to b} = -F_E d = -qEd$, where $d = 8.00$ cm is the displacement of the particle.

EXECUTE: $E = -\dfrac{W_{a\to b}}{qd} = -\dfrac{V_a - V_b}{d} = -\dfrac{-2.83\times10^{3}\ \text{V}}{0.0800\ \text{m}} = 3.54\times10^{4}\ \text{V/m}.$

EVALUATE: In part (a), W_{tot} is the total work done by both forces. In parts (b) and (c) $W_{a\to b}$ is the work done just by the electric force.

23.54. IDENTIFY: The net force on q_0 is the vector sum of the forces due to the two charges. Coulomb's law applies.

SET UP: $F = k\dfrac{|q_1 q_2|}{r^2}$, $W_{a\to b} = q(V_a - V_b)$, $V = k\dfrac{q}{r}$.

EXECUTE: **(a)** The magnitude of the force on q_0 due to each of the two charges at opposite corners of the square is $F = k\dfrac{|q_1 q_2|}{r^2}$ = k(5.00 μC)(3.00 μC)/(0.0800 m)^2 = 21.07 N. Adding the two forces vectorially gives the net force F_{net} = (21.07 N) $\sqrt{2}$ = 29.8 N. The direction is from A to B since both charges attract q_0. Figure 23.54 shows this force.

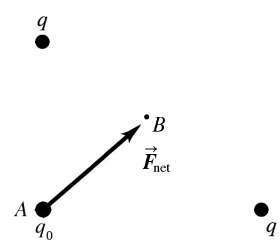


Figure 23.54

(b) At point B the two forces on q_0 are in opposite directions and have equal magnitudes, so they add to zero: $F_{\text{net}} = 0$.

(c) For each charge, $W_{A\to B} = q(V_A - V_B)$, so for both we must double this. Using $V = k\dfrac{q}{r}$ and simplifying we get $W_{A\to B} = 2q(V_A - V_B) = 2kqq_0\left(\dfrac{1}{r_A} - \dfrac{1}{r_B}\right)$. Putting in q_0 = –3.00 μC, q = 5.00 μC, r_A = 0.0800 m, and $r_b = 0.0400\sqrt{2}$ m, we get $W_{A\to B}$ = +1.40 J. The work done on q_0 by the electric field is positive since it this charge moves from A to B in the direction of the force. The charge loses potential energy as it gains kinetic energy. But since q_0 is negative, it moves to a point of higher potential.

EVALUATE: Positive charges accelerate toward lower potential, but negative charges accelerate toward higher potential.

23.55. **IDENTIFY** and **SET UP:** Calculate the components of $\vec{E}$ using $E_x = -\dfrac{\partial V}{\partial x}$, $E_y = -\dfrac{\partial V}{\partial y}$, and $E_z = -\dfrac{\partial V}{\partial z}$, and use $\vec{F} = q\vec{E}$.

EXECUTE: **(a)** $V = Cx^{4/3}$.

$$C = V/x^{4/3} = 240\text{ V}/(13.0\times10^{-3}\text{ m})^{4/3} = 7.85\times10^{4}\text{ V/m}^{4/3}.$$

(b) $E_x(x) = -\dfrac{\partial V}{\partial x} = -\dfrac{4}{3}Cx^{1/3} = -(1.05\times10^{5}\text{ V/m}^{4/3})x^{1/3}$.

The minus sign means that E_x is in the $-x$-direction, which says that $\vec{E}$ points from the positive anode toward the negative cathode.

(c) $\vec{F} = q\vec{E}$ so $F_x = -eE_x = \frac{4}{3}eCx^{1/3}$.

Halfway between the electrodes means $x = 6.50\times10^{-3}$ m.

$$F_x = \tfrac{4}{3}(1.602\times10^{-19}\text{ C})(7.85\times10^{4}\text{ V/m}^{4/3})(6.50\times10^{-3}\text{ m})^{1/3} = 3.13\times10^{-15}\text{ N}.$$

F_x is positive, so the force is directed toward the positive anode.

EVALUATE: V depends only on x, so $E_y = E_z = 0$. $\vec{E}$ is directed from high potential (anode) to low potential (cathode). The electron has negative charge, so the force on it is directed opposite to the electric field.

23.56. **IDENTIFY:** We model the finger as a parallel-plate capacitor.

SET UP and **EXECUTE:** **(a)** Estimate: 2 mm

(b) We want the field E. Assuming a uniform field, $V = Ed$ = (3 MV/m)(2 mm) = 6 kV.

(c) $E = \sigma/\epsilon_0$, so $\sigma = \epsilon_0 E = \epsilon_0$ (3 MV/m) $\approx$ 30 μC/m^2.

(d) Estimate: ½ cm by ½ cm = 0.25 cm^2.

(e) $q = \sigma A$ = (30 μC/m^2)(0.25 cm^2) = 0.02 pC.

(f) $q = ne$, so $n = q/e$ = (0.02 pC)/e = 10^5 electrons.

EVALUATE: A voltage of 6 kV seems dangerous, but only a small amount of charge is involved.

23.57. **IDENTIFY:** $U = \dfrac{kq_1q_2}{r}$.

SET UP: Eight charges means there are $8(8-1)/2 = 28$ pairs. There are 12 pairs of q and $-q$ separated by d, 12 pairs of equal charges separated by $\sqrt{2}d$ and 4 pairs of q and $-q$ separated by $\sqrt{3}d$.

EXECUTE: **(a)** $U = kq^2\left(-\dfrac{12}{d} + \dfrac{12}{\sqrt{2}d} - \dfrac{4}{\sqrt{3}d}\right) = -\dfrac{12kq^2}{d}\left(1 - \dfrac{1}{\sqrt{2}} + \dfrac{1}{3\sqrt{3}}\right) = -1.46q^2/\pi\epsilon_0\, d.$

EVALUATE: **(b)** The fact that the electric potential energy is less than zero means that it is energetically favorable for the crystal ions to be together.

23.58. **IDENTIFY:** We are modeling electrical power lines as conducting cylinders.

SET UP and **EXECUTE:** **(a)** Estimate: About 22 ft ≈ 7.0 m.

(b) We want the linear charge density λ. $\Delta V = \int E_r dr = \displaystyle\int_{0.02\text{ m}}^{7\text{ m}} \frac{\lambda}{2\pi\epsilon_0 r}dr = \frac{\lambda}{2\pi\epsilon_0}\ln(7/0.02).$ Solving for λ using V = 22 kV gives $\lambda \approx$ 210 nC/m which rounds to 200 nC/m.

(c) We want E. $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. Using λ =210 nC/m and r = 7 m gives $E \approx$ 540 V/m.

EVALUATE: The field in (c) is not very strong compared to many fields in physics labs.

23.59. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere. The electric force on the sphere is $F_e = qE$. The potential difference between the plates is $V = Ed$.

SET UP: The free-body diagram for the sphere is given in Figure 23.59.

EXECUTE: $T\cos\theta = mg$ and $T\sin\theta = F_e$ gives

$F_e = mg\tan\theta = (1.50\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan(30°) = 0.0085\text{ N}.$

$F_e = Eq = \dfrac{Vq}{d}$ and $V = \dfrac{Fd}{q} = \dfrac{(0.0085\text{ N})(0.0500\text{ m})}{8.90\times10^{-6}\text{ C}} = 47.8\text{ V}.$

EVALUATE: $E = V/d = 956$ V/m. $E = \sigma/\epsilon_0$ and $\sigma = E\epsilon_0 = 8.46\times10^{-9}\text{ C/m}^2$.

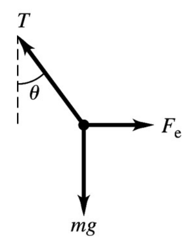


Figure 23.59

23.60. **IDENTIFY:** Outside a uniform spherical shell of charge, the electric field and potential are the same as for a point-charge at the center. Inside the shell, the electric field is zero so the potential is constant and equal to its value at the surface of the shell. The net potential is the scalar sum of the individual potentials.

SET UP: $V = k\dfrac{q}{r}$. Call V_1 the potential due to the inner shell and V_2 the potential due to the outer shell. $V_{net} = V_1 + V_2$.

EXECUTE: **(a)** At $r = 2.50$ cm, we are inside both shells. V_1 is the potential at the surface of the inner shell, so $V_1 = kq_1/R_1$; and V_2 is the potential at the surface of the outer shell, so $V_2 = kq_2/R_2$. The net potential is

$V_{net} = kq_1/R_1 + kq_2/R_2 = k(q_1/R_1 + q_2/R_2)$.

$V_{net} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)[(3.00\mu\text{C})/(0.0500\text{ m}) + (-5.00\,\mu\text{C})/(0.150\text{ m})] = 2.40\times10^5\text{ V} = 240\text{ kV}.$

(b) At $r = 10.0$ cm, we are outside the inner shell but still inside the outer shell. The inner shell now is equivalent to a point-charge at its center, so the net potential is

$V_{net} = kq_1/r + kq_2/R_2 = k(q_1/r + q_2/R_2)$.

$V_{net} = k[(3.00\,\mu\text{C})/(0.100\text{ m}) + (-5.00\,\mu\text{C})/(0.150\text{ m})] = -30.0$ kV.

(c) At $r = 20.0$ cm, we are outside both shells, so both are equivalent to point-charges at their center. So $V_{net} = kq_1/r + kq_2/r = k(q_1 + q_2)/r = k(-2.00\,\mu\text{C})/(0.200\text{ m}) = -89.9$ kV.

EVALUATE: $E = 0$ inside a spherically symmetric shell, but that does not necessarily mean that $V = 0$ there. It only means that $V_a - V_b = 0$ for any two points in side the shell, so V is constant.

23.61. **(a) IDENTIFY:** The potential at any point is the sum of the potentials due to each of the two charged conductors.

SET UP: For a conducting cylinder with charge per unit length λ the potential outside the cylinder is given by $V = (\lambda/2\pi\epsilon_0)\ln(r_0/r)$ where r is the distance from the cylinder axis and r_0 is the distance from the axis for which we take $V = 0$. Inside the cylinder the potential has the same value as on the cylinder surface. The electric field is the same for a solid conducting cylinder or for a hollow conducting tube so this expression for V applies to both. This problem says to take $r_0 = b$.

EXECUTE: For the hollow tube of radius b and charge per unit length $-\lambda$: outside $V = -(\lambda/2\pi\epsilon_0)\ln(b/r)$; inside $V = 0$ since $V = 0$ at $r = b$.

For the metal cylinder of radius a and charge per unit length λ:
outside $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$, inside $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$, the value at $r = a$.

(i) $r < a$; inside both $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

(ii) $a < r < b$; outside cylinder, inside tube $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$.

(iii) $r > b$; outside both the potentials are equal in magnitude and opposite in sign so $V = 0$.

(b) For $r = a$, $V_a = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

For $r = b$, $V_b = 0$.

Thus $V_{ab} = V_a - V_b = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

(c) IDENTIFY and **SET UP:** Use $E_r = -\dfrac{\partial V}{\partial r}$ to calculate E.

EXECUTE: $E = -\dfrac{\partial V}{\partial r} = -\dfrac{\lambda}{2\pi\epsilon_0}\dfrac{\partial}{\partial r}\ln\left(\dfrac{b}{r}\right) = -\dfrac{\lambda}{2\pi\epsilon_0}\left(\dfrac{r}{b}\right)\left(-\dfrac{b}{r^2}\right) = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}.$

(**d**) The electric field between the cylinders is due only to the inner cylinder, so V_{ab} is not changed, $V_{ab} = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

EVALUATE: The electric field is not uniform between the cylinders, so $V_{ab} \neq E(b-a)$.

23.62. IDENTIFY: The wire and hollow cylinder form coaxial cylinders. Problem 23.61 gives $E(r) = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$.

SET UP: $a = 145\times10^{-6}$ m, $b = 0.0180$ m.

EXECUTE: $E = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$ and

$V_{ab} = E\ln(b/a)r = (2.00\times10^4 \text{ N/C})(\ln(0.018 \text{ m}/145\times10^{-6} \text{ m}))0.012 \text{ m} = 1157 \text{ V}.$

EVALUATE: The electric field at any r is directly proportional to the potential difference between the wire and the cylinder.

23.63. IDENTIFY and **SET UP:** Use $\vec{F} = q\vec{E}$ to calculate $\vec{F}$ and then $\vec{F} = m\vec{a}$ gives $\vec{a}$. $E = V/d$.

EXECUTE: **(a)** $\vec{F}_E = q\vec{E}$. Since $q = -e$ is negative $\vec{F}_E$ and $\vec{E}$ are in opposite directions; $\vec{E}$ is upward so $\vec{F}_E$ is downward. The magnitude of E is $E = \dfrac{V}{d} = \dfrac{22.0 \text{ V}}{0.0200 \text{ m}} = 1.10\times10^3 \text{ V/m} = 1.10\times10^3 \text{ N/C}$. The magnitude of F_E is $F_E = |q|E = eE = (1.602\times10^{-19} \text{ C})(1.10\times10^3 \text{ N/C}) = 1.76\times10^{-16} \text{ N}$.

(b) Calculate the acceleration of the electron produced by the electric force:

$a = \dfrac{F}{m} = \dfrac{1.76\times10^{-16} \text{ N}}{9.109\times10^{-31} \text{ kg}} = 1.93\times10^{14} \text{ m/s}^2.$

EVALUATE: This acceleration is much larger than $g = 9.80 \text{ m/s}^2$, so the gravity force on the electron can be neglected. $\vec{F}_E$ is downward, so $\vec{a}$ is downward.

(c) IDENTIFY and **SET UP:** The acceleration is constant and downward, so the motion is like that of a projectile. Use the horizontal motion to find the time and then use the time to find the vertical displacement.

EXECUTE: x-component: $v_{0x} = 6.50\times10^6$ m/s; $a_x = 0$; $x - x_0 = 0.060$ m; $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and the a_x term is zero, so $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{0.060\text{ m}}{6.50\times10^6\text{ m/s}} = 9.231\times10^{-9}$ s.

y-component: $v_{0y} = 0$; $a_y = 1.93\times10^{14}$ m/s^2; $t = 9.231\times10^{-9}$ m/s; $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. $y - y_0 = \frac{1}{2}(1.93\times10^{14}\text{ m/s}^2)(9.231\times10^{-9}\text{ s})^2 = 0.00822\text{ m} = 0.822$ cm.

(d) IDENTIFY and SET UP: The velocity and its components as the electron leaves the plates are sketched in Figure 23.63.

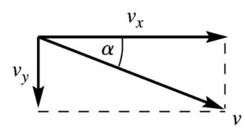


Figure 23.63

EXECUTE:

$v_x = v_{0x} = 6.50\times10^6$ m/s (since $a_x = 0$).

$v_y = v_{0y} + a_y t$.

$v_y = 0 + (1.93\times10^{14}\text{ m/s}^2)(9.231\times10^{-9}\text{ s})$.

$v_y = 1.782\times10^6$ m/s.

$\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{1.782\times10^6\text{ m/s}}{6.50\times10^6\text{ m/s}} = 0.2742$ so $\alpha = 15.3°$.

EVALUATE: The greater the electric field or the smaller the initial speed the greater the downward deflection.

(e) IDENTIFY and SET UP: Consider the motion of the electron after it leaves the region between the plates. Outside the plates there is no electric field, so $a = 0$. (Gravity can still be neglected since the electron is traveling at such high speed and the times are small.) Use the horizontal motion to find the time it takes the electron to travel 0.120 m horizontally to the screen. From this time find the distance downward that the electron travels.

EXECUTE: x-component: $v_{0x} = 6.50\times10^6$ m/s; $a_x = 0$; $x - x_0 = 0.120$ m; $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and the a_x term is term is zero, so $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{0.120\text{ m}}{6.50\times10^6\text{ m/s}} = 1.846\times10^{-8}$ s.

y-component: $v_{0y} = 1.782\times10^6$ m/s (from part (b)); $a_y = 0$; $t = 1.846\times10^{-8}$ m/s; $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.782\times10^6\text{ m/s})(1.846\times10^{-8}\text{ s}) = 0.0329\text{ m} = 3.29$ cm.

EVALUATE: The electron travels downward a distance 0.822 cm while it is between the plates and a distance 3.29 cm while traveling from the edge of the plates to the screen. The total downward deflection is 0.822 cm + 3.29 cm = 4.11 cm. The horizontal distance between the plates is half the horizontal distance the electron travels after it leaves the plates. And the vertical velocity of the electron increases as it travels between the plates, so it makes sense for it to have greater downward displacement during the motion after it leaves the plates.

23.64. IDENTIFY: The charge on the plates and the electric field between them depend on the potential difference across the plates.

SET UP: For two parallel plates, the potential difference between them is $V = Ed = \dfrac{\sigma}{\epsilon_0}d = \dfrac{Qd}{\epsilon_0 A}$.

EXECUTE: **(a)** Solving for Q gives $Q = \epsilon_0 AV/d = \dfrac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.030\text{ m})^2(25.0\text{ V})}{0.0050\text{ m}}$.

$Q = 3.98\times10^{-11}\text{C} = 39.8$ pC.

(b) $E = V/d = (25.0\text{ V})/(0.0050\text{ m}) = 5.00\times 10^3\text{ V/m}$.

(c) SET UP: Energy conservation gives $\frac{1}{2}mv^2 = eV$.

EXECUTE: Solving for v gives $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times 10^{-19}\text{C})(25.0\text{ V})}{9.11\times 10^{-31}\text{ kg}}} = 2.96\times 10^6\text{m/s}$.

EVALUATE: Typical voltages in student laboratory work run up to around 25 V, so typical reasonable values for the charge on the plates is about 40 pC and a reasonable value for the electric field is about 5000 V/m, as we found here. The electron speed would be about 3 million m/s.

23.65. **(a) IDENTIFY** and **SET UP:** Problem 23.61 derived that $E = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$, where a is the radius of the inner cylinder (wire) and b is the radius of the outer hollow cylinder. The potential difference between the two cylinders is V_{ab}. Use this expression to calculate E at the specified r.

EXECUTE: Midway between the wire and the cylinder wall is at a radius of $r = (a+b)/2 = (90.0\times 10^{-6}\text{ m} + 0.140\text{ m})/2 = 0.07004\text{ m}$.

$$E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r} = \frac{50.0\times 10^3\text{ V}}{\ln(0.140\text{ m}/90.0\times 10^{-6}\text{ m})(0.07004\text{ m})} = 9.71\times 10^4\text{ V/m}.$$

(b) IDENTIFY and **SET UP:** The magnitude of the electric force is given by $F = |q|E$. Set this equal to ten times the weight of the particle and solve for $|q|$, the magnitude of the charge on the particle.

EXECUTE: $F_E = 10mg$.

$$|q|E = 10mg \text{ and } |q| = \frac{10mg}{E} = \frac{10(30.0\times 10^{-9}\text{ kg})(9.80\text{ m/s}^2)}{9.71\times 10^4\text{ V/m}} = 3.03\times 10^{-11}\text{ C}.$$

EVALUATE: It requires only this modest net charge for the electric force to be much larger than the weight.

23.66. **(a) IDENTIFY:** Calculate the potential due to each thin ring and integrate over the disk to find the potential. V is a scalar so no components are involved.

SET UP: Consider a thin ring of radius y and width dy. The ring has area $2\pi y\,dy$ so the charge on the ring is $dq = \sigma(2\pi y\,dy)$.

EXECUTE: The result of Example 23.11 then says that the potential due to this thin ring at the point on the axis at a distance x from the ring is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{x^2+y^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{y\,dy}{\sqrt{x^2+y^2}}.$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+y^2}\right]_0^R = \frac{\sigma}{2\epsilon_0}(\sqrt{x^2+R^2} - x).$$

EVALUATE: For $x \gg R$ this result should reduce to the potential of a point charge with $Q = \sigma\pi R^2$.

$$\sqrt{x^2+R^2} = x(1+R^2/x^2)^{1/2} \approx x(1+R^2/2x^2) \text{ so } \sqrt{x^2+R^2} - x \approx R^2/2x.$$

Then $V \approx \dfrac{\sigma}{2\epsilon_0}\dfrac{R^2}{2x} = \dfrac{\sigma\pi R^2}{4\pi\epsilon_0 x} = \dfrac{Q}{4\pi\epsilon_0 x}$, as expected.

(b) IDENTIFY and **SET UP:** Use $E_x = -\dfrac{\partial V}{\partial x}$ to calculate E_x.

EXECUTE: $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\sigma}{2\epsilon_0}\left(\dfrac{x}{\sqrt{x^2+R^2}} - 1\right) = \dfrac{\sigma x}{2\epsilon_0}\left(\dfrac{1}{x} - \dfrac{1}{\sqrt{x^2+R^2}}\right).$

EVALUATE: Our result agrees with the results of Example 21.11.

23.67. **IDENTIFY:** We must integrate to find the total energy because the energy to bring in more charge depends on the charge already present.

SET UP: If ρ is the uniform volume charge density, the charge of a spherical shell of radius r and thickness dr is $dq = \rho 4\pi r^2\, dr$, and $\rho = Q/(4/3\ \pi R^3)$. The charge already present in a sphere of radius r is $q = \rho(4/3\ \pi r^3)$. The energy to bring the charge dq to the surface of the charge q is Vdq, where V is the potential due to q, which is $q/4\pi\epsilon_0\, r$.

EXECUTE: The total energy to assemble the entire sphere of radius R and charge Q is sum (integral) of the tiny increments of energy.

$$U = \int V dq = \int \frac{q}{4\pi\epsilon_0\, r}\, dq = \int_0^R \frac{\rho \frac{4}{3}\pi r^3}{4\pi\epsilon_0\, r}(\rho 4\pi r^2 dr) = \frac{3}{5}\left(\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R}\right)$$

where we have substituted $\rho = Q/(4/3\ \pi R^3)$ and simplified the result.

EVALUATE: For a point charge, $R \to 0$ so $U \to \infty$, which means that a point charge should have infinite self-energy. This suggests that either point charges are impossible, or that our present treatment of physics is not adequate at the extremely small scale, or both.

23.68. **IDENTIFY:** Divide the rod into infinitesimal segments with charge dq. The potential dV due to the segment is $dV = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{r}$. Integrate over the rod to find the total potential.

SET UP: $dq = \lambda dl$, with $\lambda = Q/\pi a$ and $dl = a\, d\theta$.

EXECUTE: $dV = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{r} = \dfrac{1}{4\pi\epsilon_0}\dfrac{\lambda\, dl}{a} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{\pi a}\dfrac{dl}{a} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q\, d\theta}{\pi a}.$

$V = \dfrac{1}{4\pi\epsilon_0}\displaystyle\int_0^\pi \dfrac{Q\, d\theta}{\pi a} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{a}.$

EVALUATE: All the charge of the ring is the same distance a from the center of curvature.

23.69. **IDENTIFY** and **SET UP:** The sphere no longer behaves as a point charge because we are inside of it. We know how the electric field varies with distance from the center of the sphere and want to use this to find the potential difference between the center and surface, which requires integration.

$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$. The electric field is radially outward, so $\vec{E}\cdot d\vec{l} = E\, dr$.

EXECUTE: For $r < R$: $E = \dfrac{kQr}{R^3}$. Integrating gives

$V = -\displaystyle\int_\infty^R \vec{E}\cdot d\vec{r}' - \int_R^r \vec{E}\cdot d\vec{r}' = \dfrac{kQ}{R} - \dfrac{kQ}{R^3}\int_R^r r' dr' = \dfrac{kQ}{R} - \dfrac{kQ}{R^3}\dfrac{1}{2}r'^2\Big|_R^r = \dfrac{kQ}{R} + \dfrac{kQ}{2R} - \dfrac{kQr^2}{2R^3} = \dfrac{kQ}{2R}\left[3 - \dfrac{r^2}{R^2}\right]$. At the center of the sphere, $r = 0$ and $V_1 = \dfrac{3kQ}{2R}$. At the surface of the sphere, $r = R$ and $V_2 = \dfrac{kQ}{R}$. The potential difference is $V_1 - V_2 = \dfrac{kQ}{2R} = \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.00\times10^{-6}\ \text{C})}{2(0.0500\ \text{m})} = 3.60\times10^5\ \text{V}.$

EVALUATE: To check our answer, we could actually do the integration. We can use the fact that $E = \frac{kQr}{R^3}$ so $V_1 - V_2 = \int_0^R E dr = \frac{kQ}{R^3}\int_0^R r dr = \frac{kQ}{R^3}\left(\frac{R^2}{2}\right) = \frac{kQ}{2R}.$

23.70. IDENTIFY: The potential at the surface of a uniformly charged sphere is $V = \frac{kQ}{R}.$

SET UP: For a sphere, $V = \frac{4}{3}\pi R^3$. When the raindrops merge, the total charge and volume are conserved.

EXECUTE: **(a)** $V = \frac{kQ}{R} = \frac{k(-3.60\times 10^{-12}\text{ C})}{6.50\times 10^{-4}\text{ m}} = -49.8\text{ V}.$

(b) The volume doubles, so the radius increases by the cube root of two: $R_{\text{new}} = \sqrt[3]{2}\,R = 8.19\times 10^{-4}$ m and the new charge is $Q_{\text{new}} = 2Q = -7.20\times 10^{-12}$ C. The new potential is

$V_{\text{new}} = \frac{kQ_{\text{new}}}{R_{\text{new}}} = \frac{k(-7.20\times 10^{-12}\text{ C})}{8.19\times 10^{-4}\text{ m}} = -79.0\text{ V}.$

EVALUATE: The charge doubles but the radius also increases and the potential at the surface increases by only a factor of $\frac{2}{2^{1/3}} = 2^{2/3} \approx 1.6.$

23.71. IDENTIFY: Slice the rod into thin slices and use $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ to calculate the potential due to each slice. Integrate over the length of the rod to find the total potential at each point.

(a) SET UP: An infinitesimal slice of the rod and its distance from point P are shown in Figure 23.71a.

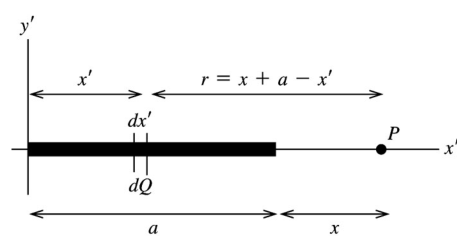


Figure 23.71a

Use coordinates with the origin at the left-hand end of the rod and one axis along the rod. Call the axes x' and y' so as not to confuse them with the distance x given in the problem.

EXECUTE: Slice the charged rod up into thin slices of width dx'. Each slice has charge $dQ = Q(dx'/a)$ and a distance $r = x + a - x'$ from point P. The potential at P due to the small slice dQ is

$$dV = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{dx'}{x+a-x'}\right).$$

Compute the total V at P due to the entire rod by integrating dV over the length of the rod ($x' = 0$ to $x' = a$):

$$V = \int dV = \frac{Q}{4\pi\epsilon_0\, a}\int_0^a \frac{dx'}{(x+a-x')} = \frac{Q}{4\pi\epsilon_0\, a}[-\ln(x+a-x')]_0^a = \frac{Q}{4\pi\epsilon_0\, a}\ln\left(\frac{x+a}{x}\right).$$

EVALUATE: As $x \to \infty$, $V \to \dfrac{Q}{4\pi \epsilon_0 a} \ln\left(\dfrac{x}{x}\right) = 0.$

(b) SET UP: An infinitesimal slice of the rod and its distance from point R are shown in Figure 23.71b.

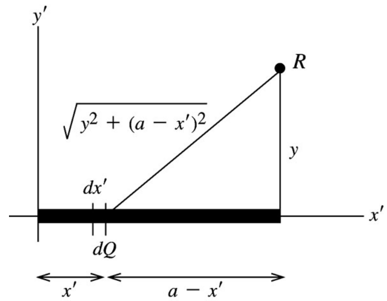


Figure 23.71b

$dQ = (Q/a)dx'$ as in part (a).

Each slice dQ is a distance $r = \sqrt{y^2 + (a - x')^2}$ from point R.

EXECUTE: The potential dV at R due to the small slice dQ is

$$dV = \frac{1}{4\pi \epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi \epsilon_0}\frac{Q}{a}\frac{dx'}{\sqrt{y^2 + (a - x')^2}}.$$

$$V = \int dV = \frac{Q}{4\pi \epsilon_0 a}\int_0^a \frac{dx'}{\sqrt{y^2 + (a - x')^2}}.$$

In the integral make the change of variable $u = a - x'$; $du = -dx'$

$$V = -\frac{Q}{4\pi \epsilon_0 a}\int_a^0 \frac{du}{\sqrt{y^2 + u^2}} = -\frac{Q}{4\pi \epsilon_0 a}\left[\ln\left(u + \sqrt{y^2 + u^2}\right)\right]_a^0.$$

$$V = -\frac{Q}{4\pi \epsilon_0 a}\left[\ln y - \ln(a + \sqrt{y^2 + a^2})\right] = \frac{Q}{4\pi \epsilon_0 a}\left[\ln\left(\frac{a + \sqrt{a^2 + y^2}}{y}\right)\right].$$

(The expression for the integral was found in Appendix B.)

EVALUATE: As $y \to \infty$, $V \to \dfrac{Q}{4\pi \epsilon_0 a} \ln\left(\dfrac{y}{y}\right) = 0.$

(c) SET UP: *part (a):* $V = \dfrac{Q}{4\pi \epsilon_0 a}\ln\left(\dfrac{x + a}{x}\right) = \dfrac{Q}{4\pi \epsilon_0 a}\ln\left(1 + \dfrac{a}{x}\right).$

From Appendix B, $\ln(1 + u) = u - u^2/2\ldots$, so $\ln(1 + a/x) = a/x - a^2/2x^2$ and this becomes a/x when x is large.

EXECUTE: Thus $V \to \dfrac{Q}{4\pi \epsilon_0 a}\left(\dfrac{a}{x}\right) = \dfrac{Q}{4\pi \epsilon_0 x}$. For large x, V becomes the potential of a point charge.

part (b): $V = \dfrac{Q}{4\pi \epsilon_0 a}\left[\ln\left(\dfrac{a + \sqrt{a^2 + y^2}}{y}\right)\right] = \dfrac{Q}{4\pi \epsilon_0 a}\ln\left(\dfrac{a}{y} + \sqrt{1 + \dfrac{a^2}{y^2}}\right).$

From Appendix B, $\sqrt{1 + a^2/y^2} = (1 + a^2/y^2)^{1/2} = 1 + a^2/2y^2 + \ldots$

Thus $a/y+\sqrt{1+a^2/y^2} \to 1+a/y+a^2/2y^2+\ldots \to 1+a/y$. And then using $\ln(1+u)\approx u$ gives

$$V \to \frac{Q}{4\pi\epsilon_0\, a}\ln(1+a/y) \to \frac{Q}{4\pi\epsilon_0\, a}\left(\frac{a}{y}\right)=\frac{Q}{4\pi\epsilon_0\, y}.$$

EVALUATE: For large y, V becomes the potential of a point charge.

23.72. **IDENTIFY:** We are dealing with a charged bar in an external electric field.

SET UP and **EXECUTE:** **(a)** We want V as a function of y. E is constant so $V_y - V_0 = -E_y y$, which gives $V(y) = V_0 - Ey$.

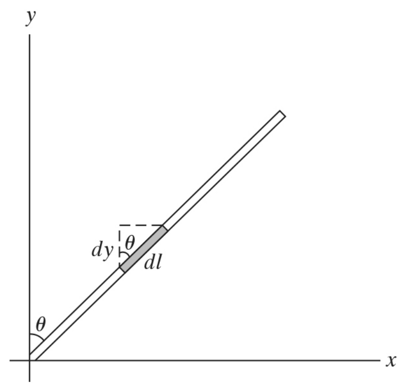


Figure 23.72

(b) We want the potential energy. Refer to Fig. 23.72. $U=\int V dq$. $dq=\lambda dl$ with $\lambda = Q/L$, and

$$dl\cos\theta = dy.\;\; U=\int_0^{L\cos\theta}(V_0-Ey)(Q/L)\frac{dy}{\cos\theta} = \left.\frac{\frac{Q}{L}\left(V_0 y-\frac{Ey^2}{2}\right)}{\cos\theta}\right|_0^{L\cos\theta} = Q\left(V_0-\frac{EL\cos\theta}{2}\right).$$

(c) We want V_0 so $U = 0$ when $\theta = 0°$. $Q\left(V_0-\frac{EL\cos\theta}{2}\right)=0$ gives $V_0 = EL/2$.

(d) We want the angular speed ω at $\theta = 0°$ if the bar is released from rest at $\theta = 90°$. Energy conservation gives $U_0 + K_0 = U_{90} + K_{90}$. We can neglect gravity, so $\frac{1}{2}I\omega^2 = QV_0$. Using the result from (c) and $I = 1/3\, ML^2$ gives $\omega=\sqrt{\frac{3QE}{ML}}$.

(e) We want the frequency f of oscillation. Apply $\sum\tau_z = I\alpha_z$. $-QE\frac{L}{2}\sin\theta=\frac{1}{3}ML^2\frac{d^2\theta}{dt^2}$. This gives $\frac{d^2\theta}{dt^2}=-\left(\frac{3QE}{2ML}\right)\sin\theta\approx-\left(\frac{3QE}{2ML}\right)\theta$ for small amplitude oscillations. $\omega=\sqrt{\frac{3QE}{2ML}}$, so $f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{3QE}{2ML}}$.

EVALUATE: This situation is very similar to an upside-down pendulum.

23.73. **IDENTIFY:** We are modeling forces in the atomic nucleus.

SET UP and **EXECUTE:** **(a)** We want the potential energy U. $U = U_{\text{spring}} + U_{\text{electric}}$, so
$U = \frac{1}{2}kd^2 + \frac{1}{4\pi\epsilon_0}\frac{e^2}{d}$.

(b) We want the equilibrium separation d_0. When U is a minimum, $dU/dd = 0$. Taking the derivative of the energy in (a) gives $kd - \frac{1}{4\pi\epsilon_0}\frac{e^2}{d^2} = 0$. Solve for $d = d_0$: $d_0 = \left(\frac{e^2}{4\pi\epsilon_0 k}\right)^{1/3}$.

(c) We want k. Solve the result of (b) for k and use d_0 = 1.00 fm = 1.00×10^{-15} m, giving $k = 2.30\times10^{17}$ N/m.

(d) We want the stored energy. $U_0 = \frac{1}{2}kd_0^2 + \frac{1}{4\pi\epsilon_0}\frac{e^2}{d_0}$. Using d_0 and k from above gives

$U_0 = 3.45\times10^{-13}$ J = 2.16 MeV.

(e) We want the speed v. $U_0 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$, so $v = \sqrt{U_0/m}$ = 1.44×10^7 m/s.

EVALUATE: $v/c = 1.44/3.0 = 0.048$, so $v \approx 5\%$ of the speed of light.

23.74. **IDENTIFY** and **SET UP:** For points outside of them, the spheres behave as though all the charge were concentrated at their centers. The charge initially on sphere 1 spreads between the two spheres such as to bring them to the same potential.

EXECUTE: **(a)** $E_1 = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{R_1^2}$, $V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{R_1} = R_1E_1$.

(b) Two conditions must be met:

1) Let q_1 and q_2 be the final charges of each sphere. Then $q_1 + q_2 = Q_1$ (charge conservation).

2) Let V_1 and V_2 be the final potentials of each sphere. All points of a conductor are at the same potential, so $V_1 = V_2$.

$V_1 = V_2$ requires that $\frac{1}{4\pi\epsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{R_2}$ and then $q_1/R_1 = q_2/R_2$.

$$q_1R_2 = q_2R_1 = (Q_1 - q_1)R_1.$$

This gives $q_1 = (R_1/[R_1 + R_2])Q_1$ and $q_2 = Q_1 - q_1 = Q_1(1 - R_1/[R_1 + R_2]) = Q_1(R_2/[R_1 + R_2])$.

(c) $V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{R_1} = \frac{Q_1}{4\pi\epsilon_0(R_1 + R_2)}$ and $V_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{R_2} = \frac{Q_1}{4\pi\epsilon_0(R_1 + R_2)}$, which equals V_1 as it should.

(d) $E_1 = \frac{V_1}{R_1} = \frac{Q_1}{4\pi\epsilon_0 R_1(R_1 + R_2)}$. $E_2 = \frac{V_2}{R_2} = \frac{Q_1}{4\pi\epsilon_0 R_2(R_1 + R_2)}$.

EVALUATE: Part (a) says $q_2 = q_1(R_2/R_1)$. The sphere with the larger radius needs more charge to produce the same potential at its surface. When $R_1 = R_2$, $q_1 = q_2 = Q_1/2$. The sphere with the larger radius has the smaller electric field at its surface.

23.75. **IDENTIFY:** Apply conservation of energy, $K_a + U_a = K_b + U_b$.

SET UP: Assume the particles initially are far apart, so $U_a = 0$. The alpha particle has zero speed at the distance of closest approach, so $K_b = 0$. $1\text{ eV} = 1.60\times10^{-19}$ J. The alpha particle has charge $+2e$ and the lead nucleus has charge $+82e$.

EXECUTE: Set the alpha particle's kinetic energy equal to its potential energy: $K_a = U_b$ gives

$$9.50\text{ MeV} = \frac{k(2e)(82e)}{r} \text{ and } r = \frac{k(164)(1.60\times10^{-19}\text{ C})^2}{(9.50\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 2.48\times10^{-14}\text{ m}.$$

EVALUATE: The calculation assumes that at the distance of closest approach the alpha particle is outside the radius of the lead nucleus.

23.76. **IDENTIFY** and **SET UP:** Apply $\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{\boldsymbol{i}} + \frac{\partial V}{\partial y}\hat{\boldsymbol{j}} + \frac{\partial V}{\partial z}\hat{\boldsymbol{k}}\right)$. $\frac{W_{a\to b}}{q_0} = V_a - V_b$ and $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$.

EXECUTE: **(a)** $\vec{E} = -\frac{\partial V}{\partial x}\hat{\boldsymbol{i}} - \frac{\partial V}{\partial y}\hat{\boldsymbol{j}} - \frac{\partial V}{\partial z}\hat{\boldsymbol{k}} = -2Ax\hat{\boldsymbol{i}} + 6Ay\hat{\boldsymbol{j}} - 2Az\hat{\boldsymbol{k}}.$

(b) A charge is moved in along the z-axis. The work done is given by

$W = q\int_{z_0}^{0} \vec{E}\cdot\hat{\boldsymbol{k}}dz = q\int_{z_0}^{0}(-2Az)dz = +(Aq)z_0^2$. Therefore,

$$A = \frac{W_{a\to b}}{qz_0^2} = \frac{6.00\times10^{-5}\text{ J}}{(1.5\times10^{-6}\text{ C})(0.250\text{ m})^2} = 640\text{ V/m}^2.$$

(c) $\vec{E}(0,0,0.250) = -2(640\text{ V/m}^2)(0.250\text{ m})\hat{\boldsymbol{k}} = -(320\text{ V/m})\hat{\boldsymbol{k}}.$

(d) In every plane parallel to the xz-plane, y is constant, so $V(x,y,z) = Ax^2 + Az^2 - C$, where $C = 3Ay^2$. $x^2 + z^2 = \frac{V+C}{A} = R^2$, which is the equation for a circle since R is constant as long as we have constant potential on those planes.

(e) $V = 1280$ V and $y = 2.00$ m, so $x^2 + z^2 = \frac{1280\text{ V} + 3(640\text{ V/m}^2)(2.00\text{ m})^2}{640\text{ V/m}^2} = 14.0\text{ m}^2$ and the radius of the circle is 3.74 m.

EVALUATE: In any plane parallel to the xz-plane, $\vec{E}$ projected onto the plane is radial and hence perpendicular to the equipotential circles.

23.77. **IDENTIFY** and **SET UP:** We know that the potential is of the mathematical form $V(x,y,z) = Ax^l + By^m + Cz^n + D$. We also know that $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, and $E_z = -\frac{\partial V}{\partial z}$. Various measurements are given in the table with the problem in the text.

EXECUTE: **(a)** First get A, B, C, and D using data from the table in the problem.

$V(0, 0, 0) = 10.0\text{ V} = 0 + 0 + 0 + D$, so $D = 10.0$ V.

$V(1.00, 0, 0) = A(1.00\text{ m})^l + 0 + 0 + 10.0\text{ V} = 4.00$ V, so $A = -6.0\text{ V}\cdot\text{m}^{-l}$.

$V(0, 1.00, 0) = B(1.00\text{ m})^m + 10.0\text{ V} = 6.0$ V, so $B = -4.0\text{ V}\cdot\text{m}^{-m}$.

$V(0, 0, 1.00\text{ m}) = C(1.00\text{ m})^n + 10.0\text{ V} = 8.0$ V, so $C = -2.0\text{ V}\cdot\text{m}^{-n}$.

Now get l, m, and n.

$E_x = -\frac{\partial V}{\partial x} = -lAx^{l-1}$, and from the table we know that $E_x(1.00, 0, 0) = 12.0$ V/m. Therefore

$-l(-6.0\text{ V}\cdot\text{m}^{-l})(1.00\text{ m})^{l-1} = 12.0$ V/m.

$l(6.0\text{ V}\cdot\text{m}^{-1}) = 12.0$ V/m.

$l = 2.0$.

$E_y = -\frac{\partial V}{\partial y} = -mBy^{m-1}$.

$E_y(0, 1.00, 0) = -m(-4.0\text{ V}\cdot\text{m}^{-m})(1.00\text{ m})^{m-1} = 12.0$ V/m.

$m = 3.0$.

$E_z = -\frac{\partial V}{\partial z} = -nCz^{n-1}$.

$E_z(0, 0, 1.00) = -n(-2.0\ \text{V}\cdot\text{m}^{-n})(1.00\ \text{m})^{n-1} = 12.0\ \text{V/m}$.
$n = 6.0$.
Now that we have l, m, and n, we see the units of A, B, and C, so
$A = -6.0\ \text{V/m}^2$.
$B = -4.0\ \text{V/m}^3$.
$C = -2.0\ \text{V/m}^6$.
Therefore the equation for $V(x,y,z)$ is
$V = (-6.0\ \text{V/m}^2)x^2 + (-4.0\ \text{V/m}^3)y^3 + (-2.0\ \text{V/m}^6)z^6 + 10.0\ \text{V}$.
(b) At (0, 0, 0): $V = 0$ and $E = 0$ (from the table with the problem).
At (0.50 m, 0.50 m, 0.50 m):
$V = (-6.0\ \text{V/m}^2)(0.50\ \text{m})^2 + (-4.0\ \text{V/m}^3)(0.50\ \text{m})^3 + (-2.0\ \text{V/m}^6)(0.50\ \text{m})^6 + 10.0\ \text{V} = 8.0\ \text{V}$.

$$E_x = -\frac{\partial V}{\partial x} = -(-12.0\ \text{V/m}^2)x = (12.0\ \text{V/m}^2)(0.50\ \text{m}) = 6.0\ \text{V/m}.$$

$$E_y = -\frac{\partial V}{\partial y} = -3(-4.0\ \text{V/m}^3)y^2 = (12\ \text{V/m}^3)(0.50\ \text{m})^2 = 3.0\ \text{V/m}.$$

$$E_z = -\frac{\partial V}{\partial z} = -(-12.0\ \text{V/m}^6)z^5 = (12.0\ \text{V/m}^6)(0.50\ \text{m})^5 = 0.375\ \text{V/m}.$$

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(6.0\ \text{V/m})^2 + (3.0\ \text{V/m})^2 + (0.375\ \text{V/m})^2} = 6.7\ \text{V/m}.$$

At (1.00 m, 1.00 m, 1.00 m):
Follow the same procedure as above. The results are $V = -2.0$ V, $E = 21$ V/m.
EVALUATE: We know that l, m, and n must be greater that 1 because the components of the electric field are all zero at (0, 0, 0).

23.78. IDENTIFY and SET UP: Energy is conserved and the potential energy is $U = k\frac{q_1 q_2}{r}$. $K_1 + U_1 = K_2 + U_2$.

EXECUTE: **(a)** Energy conservation gives $K_1 + 0 = K_2 + U_2$.

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + k\frac{qQ}{x} \quad\rightarrow\quad v^2 = v_0^2 - \frac{2kqQ}{m}\cdot\frac{1}{x}.$$

On a graph of v_2 versus $1/x$, the graph of this equation will be a straight line with y-intercept equal to v_0^2 and slope equal to $-\frac{2kqQ}{m}$.

(b) With the given equation of the line in the problem, we have $v^2 = 400\ \text{m}^2/\text{s}^2 - (15.75\ \text{m}^3/\text{s}^2)\frac{1}{x}$. As x gets very large, $1/x$ approaches zero, so $v_0 = \sqrt{400\,\text{m}^2/\text{s}^2} = 20$ m/s.

(c) The slope is $-\frac{2kqQ}{m} = -15.75\ \text{m}^3/\text{s}^2$, which gives

$$Q = -m(\text{slope})/2kq = -(4.00\times10^{-4}\ \text{kg})(-15.75\ \text{m}^3/\text{s}^2)/[2k(5.00\times10^{-8}\ \text{C})] = +7.01\times10^{-6}\ \text{C} = +7.01\ \mu\text{C}.$$

(d) The particle is closest when its speed is zero, so

$v^2 = 400\ \text{m}^2/\text{s}^2 - (15.75\ \text{m}^3/\text{s}^2)\frac{1}{x} = 0$, which gives $x = 3.94\times10^{-2}\ \text{m} = 3.94\ \text{cm}$.

EVALUATE: From the graph in the problem, we see that v^2 decreases as $1/x$ increases, so v^2 decreases as x decreases. This means that the positively charged particle is slowing down as it gets closer to the sphere, so the sphere is repelling it. Therefore the sphere must be positively charged, as we found.

23.79. IDENTIFY: When the oil drop is at rest, the upward force $|q|E$ from the electric field equals the downward weight of the drop. When the drop is falling at its terminal speed, the upward viscous force equals the downward weight of the drop.

SET UP: The volume of the drop is related to its radius r by $V = \frac{4}{3}\pi r^3$.

EXECUTE: **(a)** $F_g = mg = \frac{4\pi r^3}{3}\rho g$. $F_e = |q|E = |q|V_{AB}/d$. $F_e = F_g$ gives $|q| = \frac{4\pi}{3}\frac{\rho r^3 gd}{V_{AB}}$.

(b) $\frac{4\pi r^3}{3}\rho g = 6\pi\eta r v_t$ gives $r = \sqrt{\frac{9\eta v_t}{2\rho g}}$. Using this result to replace r in the expression in part (a) gives

$$|q| = \frac{4\pi}{3}\frac{\rho g d}{V_{AB}}\left[\sqrt{\frac{9\eta v_t}{2\rho g}}\right]^3 = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}.$$

(c) We use the values for V_{AB} and v_t given in the table in the problem and the formula

$|q| = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}$ from (c). For example, for drop 1 we get

$|q| = 18\pi\frac{1.00\times10^{-3}\text{ m}}{9.16\text{ V}}\sqrt{\frac{(1.81\times10^{-5}\text{ N}\cdot\text{s/m}^2)^3(2.54\times10^{-5}\text{ m/s})^3}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 4.79\times10^{-19}\text{ C}$. Similar calculations for the remaining drops gives the following results:

Drop 1: 4.79×10^{-19} C
Drop 2: 1.59×10^{-19} C
Drop 3: 8.09×10^{-19} C
Drop 4: 3.23×10^{-19} C

(d) Use $n = q/e_2$ to find the number of excess electrons on each drop. Since all quantities have a power of 10^{-19} C, this factor will cancel, so all we need to do is divide the coefficients of 10^{-19} C. This gives

Drop 1: $n = q_1/q_2 = 4.79/1.59 = 3$ excess electrons
Drop 2: $n = q_2/q_2 = 1$ excess electron
Drop 3: $n = q_3/q_2 = 8.09/1.59 = 5$ excess electrons
Drop 4: $n = q_4/q_2 = 3.23/1.59 = 2$ excess electrons

(e) Using $q = -ne$ gives $e = -q/n$. All the charges are negative, so e will come out positive. Thus we get

Drop 1: $e_1 = q_1/n_1 = (4.79\times10^{-19}\text{ C})/3 = 1.60\times10^{-19}$ C
Drop 2: $e_2 = q_2/n_2 = (1.59\times10^{-19}\text{ C})/1 = 1.59\times10^{-19}$ C
Drop 3: $e_3 = q_3/n_3 = (8.09\times10^{-19}\text{ C})/5 = 1.62\times10^{-19}$ C
Drop 4: $e_4 = q_4/n_4 = (3.23\times10^{-19}\text{ C})/2 = 1.61\times10^{-19}$ C

The average is

$e_{av} = (e_1 + e_2 + e_3 + e_4)/4 = [(1.60 + 1.59 + 1.62 + 1.61)\times10^{-19}\text{ C}]/4 = 1.61\times10^{-19}$ C.

EVALUATE: The result $e = 1.61\times10^{-19}$ C is very close to the well-established value of 1.60×10^{-19} C.

23.80. **IDENTIFY:** We want the potential due to an annulus. For this we need to use calculus.

SET UP: Fig. 23.80 shows the mathematical set up. $V = \int\frac{1}{4\pi\epsilon_0}\frac{dq}{D}$, where $D = \sqrt{z^2 + r^2}$, $dq = \sigma dA$, and $dA = 2\pi r dr$.

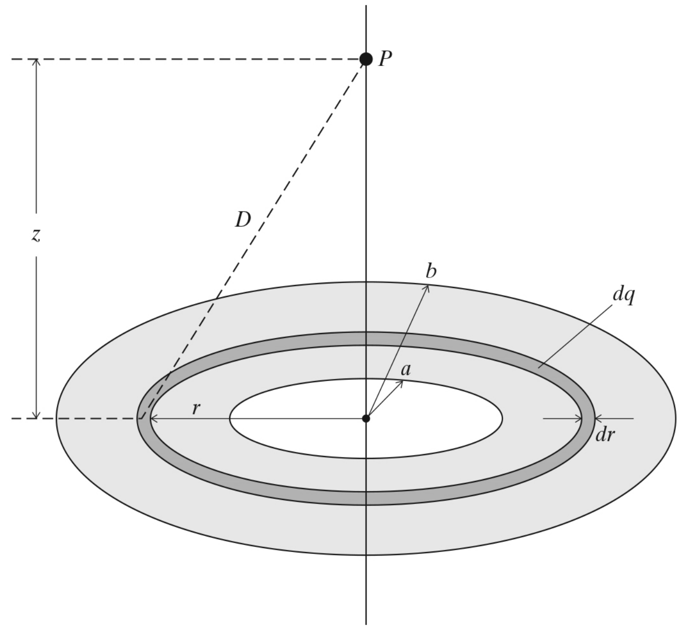


Figure 23.80

EXECUTE: **(a)** $V = \int \frac{1}{4\pi \epsilon_0} \frac{dq}{D} = \int_a^b \frac{\sigma 2\pi r dr}{4\pi \epsilon_0 \sqrt{z^2 + r^2}} = \frac{\sigma}{4\epsilon_0} \int_a^b \frac{2rdr}{\sqrt{z^2+r^2}}$. To integrate, let $u = z^2 + r^2$ so $du = 2rdr$. The integral is then of the form $\int \frac{du}{u^{1/2}} = 2u^{1/2}$, which gives $V = \frac{\sigma}{4\epsilon_0}\left(2\sqrt{z^2+r^2}\right)\Big|_a^b$.

$V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2+b^2} - \sqrt{z^2+a^2}\right)$.

(b) We want E_z. Use $E_z = -dV/dz$ for the result from (a), giving $\vec{E} = \frac{\sigma z}{2\epsilon_0}\left(\frac{1}{\sqrt{z^2+a^2}} - \frac{1}{\sqrt{z^2+b^2}}\right)\hat{k}$.

(c) Let $a \to 0$: $E_z \to \frac{\sigma z}{2\epsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{z^2+b^2}}\right)$.

Let $b \to \infty$: The second term approaches zero, so $E_z \to \frac{\sigma z}{2\epsilon_0}\left(\frac{1}{z}\right) = \frac{\sigma}{2\epsilon_0}$, which is the field for an infinite sheet of charge.

(d) We want the potential at the origin. At the origin, $z = 0$, so $V(0) = \frac{\sigma}{2\epsilon_0}(b-a)$.

$\sigma = \frac{Q}{A} = \frac{Q}{\pi b^2 - \pi a^2} = \frac{Q}{\pi(b^2-a^2)}$, so $V(0) = \frac{Q}{2\pi\epsilon_0 (b^2-a^2)}(b-a) = \frac{Q}{2\pi\epsilon_0 (b+a)}$.

Using $a = 5.00$ cm, $b = 10.0$ cm, and $Q = 1.00\ \mu$C gives $V(0) = 120$ kV.

(e) We want the speed v. Energy conservation gives $qV = \frac{1}{2}mv^2$, so $v = \sqrt{2qV/m}$. Using $V = 120$ kV and $m = 0.00100$ kg gives $v = 15.5$ m/s.

EVALUATE: The electric field is *not* an inverse square in *z* and the potential is *not* an inverse *z*.

23.81. IDENTIFY: We are dealing with a heart cell that is modeled as a cylindrical shell of charge.

SET UP: $E = \dfrac{\lambda}{2\pi \epsilon_0 r}$.

EXECUTE: **(a)** We want the charge Q. Using $E = \dfrac{\lambda}{2\pi \epsilon_0 r}$ with $\lambda = Q/L$, we have

$\Delta V = \int E_r dr = \int \dfrac{\lambda}{2\pi \epsilon_0} dr = \int_a^b \dfrac{Q/L}{2\pi \epsilon_0 r} dr = \dfrac{Q/L}{2\pi \epsilon_0} \ln(b/a)$. Solving for Q and using $\Delta V = 90.0$ mV, a = 9.0 μm, b = 10.0 μm, and L = 100 μm gives $Q = 4.75 \times 10^{-15}$ C.

(b) We want the electric field just inside the membrane at $r = a = 9.0$ μm. Using $E = \dfrac{Q/L}{2\pi \epsilon_0 a}$ gives E = 94.9 kV/m.

(c) We want to find out how much charged moved across the cell wall. The potential went from -90.0 mV to +20.0 mV, so $\Delta V = 110$ mV. Using information from (a) gives $Q = \dfrac{(2\pi \epsilon_0)(\Delta V)L}{\ln(b/a)}$.

With $\Delta V = 110$ mV and the given numbers, we get $Q = 5.81 \times 10^{-15}$ C.

(d) We want to know how many Na^+ ions crossed the boundary. $N = \dfrac{5.81 \times 10^{-15} \text{ C}}{1.60 \times 10^{-19} \text{ C/ion}} = 36{,}300.$

EVALUATE: Note how small the charges, fields, and potentials are in cellular electrical processes. By contrast the electric field of the proton in a hydrogen atom at the location of the electron is approximately 6×10^{11} V/m.

23.82. IDENTIFY: Consider the potential due to an infinitesimal slice of the cylinder and integrate over the length of the cylinder to find the total potential. The electric field is along the axis of the tube and is given by $E = -\dfrac{\partial V}{\partial x}$.

SET UP: Use the expression from Example 23.11 for the potential due to each infinitesimal slice. Let the slice be at coordinate z along the x-axis, relative to the center of the tube.

EXECUTE: **(a)** For an infinitesimal slice of the finite cylinder, we have the potential

$dV = \dfrac{k\,dQ}{\sqrt{(x-z)^2 + R^2}} = \dfrac{kQ}{L}\dfrac{dz}{\sqrt{(x-z)^2 + R^2}}$. Integrating gives

$V = \dfrac{kQ}{L}\int_{-L/2}^{L/2} \dfrac{dz}{\sqrt{(x-z)^2 + R^2}} = \dfrac{kQ}{L}\int_{-L/2-x}^{L/2-x} \dfrac{du}{\sqrt{u^2 + R^2}}$ where $u = x - z$. Therefore,

$V = \dfrac{kQ}{L}\ln\left[\dfrac{\sqrt{(L/2-x)^2 + R^2} + (L/2-x)}{\sqrt{(L/2+x)^2 + R^2} - L/2 - x}\right]$ on the cylinder axis.

(b) For $L << R$, $V \approx \dfrac{kQ}{L}\ln\left[\dfrac{\sqrt{(L/2-x)^2 + R^2} + L/2 - x}{\sqrt{(L/2+x)^2 + R^2} - L/2 - x}\right] \approx \dfrac{kQ}{L}\ln\left[\dfrac{\sqrt{x^2 - xL + R^2} + L/2 - x}{\sqrt{x^2 + xL + R^2} - L/2 - x}\right]$.

$$V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{1 - xL/(R^2 + x^2)} + (L/2 - x)/\sqrt{R^2 + x^2}}{\sqrt{1 + xL/(R^2 + x^2)} + (-L/2 - x)/\sqrt{R^2 + x^2}}\right] = \frac{kQ}{L}\ln\left[\frac{1 - xL/2(R^2 + x^2) + (L/2 - x)/\sqrt{R^2 + x^2}}{1 + xL/2(R^2 + x^2) + (-L/2 - x)/\sqrt{R^2 + x^2}}\right].$$

$$V \approx \frac{kQ}{L}\ln\left[\frac{1 + L/2\sqrt{R^2 + x^2}}{1 - L/2\sqrt{R^2 + x^2}}\right] = \frac{kQ}{L}\left(\ln\left[1 + \frac{L}{2\sqrt{R^2 + x^2}}\right] - \ln\left[1 - \frac{L}{2\sqrt{R^2 + x^2}}\right]\right).$$

$V \approx \frac{kQ}{L}\frac{2L}{2\sqrt{x^2+R^2}} = \frac{kQ}{\sqrt{x^2+R^2}}$, which is the same as for a ring.

(c) $E_x = -\frac{\partial V}{\partial x} = \frac{2kQ\left(\sqrt{(L-2x)^2+4R^2}-\sqrt{(L+2x)^2+4R^2}\right)}{\sqrt{(L-2x)^2+4R^2}\sqrt{(L+2x)^2+4R^2}}.$

EVALUATE: For $L << R$ the expression for E_x reduces to that for a ring of charge, $E_x = \frac{kQx}{(x^2+a^2)^{3/2}}$, as shown in Example 23.14.

23.83. **IDENTIFY:** Angular momentum and energy must be conserved.

SET UP: At the distance of closest approach the speed is not zero. $E = K + U$. $q_1 = 2e$, $q_2 = 82e$.

EXECUTE: $mv_1 b = mv_2 r_2$. $E_1 = E_2$ gives $E_1 = \frac{1}{2}mv_2{}^2 + \frac{kq_1q_2}{r_2}$. $E_1 = 11\text{ MeV} = 1.76\times10^{-12}$ J. r_2 is the distance of closest approach. Substituting in for $v_2 = v_1\left(\frac{b}{r_2}\right)$ we find $E_1 = E_1\frac{b^2}{r_2{}^2} + \frac{kq_1q_2}{r_2}$.

$(E_1)r_2^2 - (kq_1q_2)r_2 - E_1 b^2 = 0$. For $b = 10^{-12}$ m, $r_2 = 1.01\times10^{-12}$ m. For $b = 10^{-13}$ m, $r_2 = 1.11\times10^{-13}$ m. And for $b = 10^{-14}$ m, $r_2 = 2.54\times10^{-14}$ m.

EVALUATE: As b decreases the collision is closer to being head-on and the distance of closest approach decreases.

23.84. **IDENTIFY** and **SET UP:** The He ions are first accelerated toward the center and then accelerated away from the center, but always in the same direction. During the first acceleration, their charge is $-e$, and during the second acceleration it is $+2e$. The work-energy theorem gives $\Delta K = q\Delta V$. Call V the voltage at the center.

EXECUTE: **(a)** Toward the center: $\Delta K = q\Delta V = eV$.

Away from the center: $\Delta K = q\Delta V = 2eV$.

The ions gain 3.0 MeV of kinetic energy, so $eV + 2eV = 3.0$ MeV.

$3eV = 3.0$ MeV.

$V = +1.0$ MV, since the e cancels. This is choice (d).

EVALUATE: The negative He^- ions are accelerating to higher potential, and the positive He^{++} ions are accelerating toward lower potential.

23.85. **IDENTIFY** and **SET UP:** Conservation of energy gives $K = U_{\text{electric}} = k\frac{q_1q_2}{r}$.

EXECUTE: Solve for Q: $Q = rK/kq = (10\times10^{-15}\text{ m})(3.0\text{ MeV})/(2ek) = 1.67\times10^{-18}$ C. In terms of e, this is $Q = (1.67\times10^{-18}\text{ C})/(1.60\times10^{-19}\text{ C}) = 10.4e \approx 11e$, so choice (b) is best.

EVALUATE: If $Q = 11e$, the atom is sodium (Na), which has an atomic mass of 23, compared to 4 for He. So it is reasonable to assume that the nucleus does not move appreciably, since it is about 6 times more massive than the He.

23.86. **IDENTIFY** and **SET UP:** The potential changes by 6.0 MV over a distance of 12 m. $E_{\text{av}} = \frac{\Delta V}{\Delta x}$.

EXECUTE: $E_{\text{av}} = \frac{\Delta V}{\Delta x} = (6.0\text{ MV})/(12\text{ m}) = 0.50\times10^6\text{ V/m} = 500{,}000\text{ V/m}$, which is choice (c).

EVALUATE: The actual variation of the field may be somewhat complicated, but the average value gives a good idea of a typical electric field in such apparatus.

24

CAPACITANCE AND DIELECTRICS

VP24.4.1. **IDENTIFY:** We are dealing with a parallel-plate capacitor.

SET UP and **EXECUTE:** **(a)** We want the capacitance. $C = \dfrac{\epsilon_0 A}{d} = \dfrac{\epsilon_0 (2.75\text{ m}^2)}{0.00350\text{ m}} = 6.95\text{ nF}.$

(b) We want the potential difference between the plates. $V = Ed = (\sigma/\epsilon_0)d$. Using the given values gives $V = 5.54$ kV.

EVALUATE: $E = \sigma/\epsilon_0 = 1.58$ MV/m between the plates, so we get a large potential difference of over 5000 V.

VP24.4.2. **IDENTIFY:** We are dealing with a parallel-plate capacitor.

SET UP and **EXECUTE:** **(a)** We want the plate spacing. Solving $C = \dfrac{\epsilon_0 A}{d}$ for d gives $d = \dfrac{\epsilon_0 A}{C}$. Using the given numbers we get $d = 4.80$ mm.

(b) We want the charge on the plates and the potential difference between them when $V = 3.00$ kV. $Q = CV = (8.30\text{ nF})(3.00\text{ kV}) = 24.9\ \mu\text{C}$. $E = V/d = (3.00\text{ kV})/(4.80\text{ mm}) = 625$ kV/m.

EVALUATE: Use $E = \sigma/\epsilon_0 = \dfrac{Q/A}{\epsilon_0}$, which gives the same answer as in (b).

VP24.4.3. **IDENTIFY** and **SET UP:** We have a spherical capacitor. $C = 4\pi\epsilon_0\left(\dfrac{r_a r_b}{r_b - r_a}\right)$.

EXECUTE: **(a)** We want the outer radius of the inner shell, which is r_a. Solve $C = 4\pi\epsilon_0\left(\dfrac{r_a r_b}{r_b - r_a}\right)$ for r_a giving $r_a = \dfrac{r_b C}{C + 4\pi\epsilon_0 r_b}$. The given numbers yield $r_a = 8.33$ cm.

(b) We want the charge when $V = 355$ V. $Q = CV = (125\text{ pF})(355\text{ V}) = 44.4$ nC.

(c) We want the surface charge density at both surfaces. $\sigma = \dfrac{Q}{A} = \dfrac{Q}{4\pi r^2}$.

Outer surface: $\sigma_b = \dfrac{Q}{4\pi r_b^2} = \dfrac{-44.4\text{ nC}}{4\pi(0.0900\text{ m})^2} = -4.36\times10^{-7}\text{ C/m}^2.$

Inner surface: $\sigma_a = \dfrac{Q}{4\pi r_a^2} = \dfrac{+44.4\text{ nC}}{4\pi(0.0833\text{ m})^2} = +5.09\times10^{-7}\text{ C/m}^2.$

EVALUATE: The inner and outer surfaces carry the same magnitude charge, but the inner surface has a smaller area so its charge density should be greater than that of the outer surface. This agrees with our result.

VP24.4.4. **IDENTIFY** and **SET UP:** We have a cylindrical capacitor. From Example 24.4, $C/L = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$.

EXECUTE: **(a)** We want r_b/r_a. Solving $C/L = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ for r_b/r_a and putting in the given numbers we have $\ln(r_b/r_a) = \dfrac{2\pi\epsilon_0}{C/L} = 0.80524$. $r_b/r_a = e^{0.80524} = 2.24$.

(b) We want the potential difference. $C = \dfrac{Q}{V}$, so $C/L = \dfrac{Q/L}{V}$, giving

$V = \dfrac{Q/L}{C/L} = \dfrac{8.62\text{ nC/m}}{69.0\text{ pF/m}} = 125\text{ V}$. The inner conductor is positive, so the electric field between the cylinders does work on a positive charge in going from a to b, so the inner conductor is at a higher potential then the outer one.

EVALUATE: A kilometer of this cylinder would have a capacitance of only 69 nF.

VP24.9.1. **IDENTIFY:** We have capacitors in series and parallel.

SET UP: In series: $1/C_{\text{eq}} = 1/C_1 + 1/C_2 + \cdots$, in parallel: $C_{\text{eq}} = C_1 + C_2 + \text{L}$. We want the equivalent capacitance.

EXECUTE: **(a)** $1/C_{\text{eq}} = 1/(1.00\ \mu\text{F}) + 1/(2.50\ \mu\text{F}) + 1/(5.00\ \mu\text{F})$. $C_{\text{eq}} = 0.625\ \mu\text{F}$.

(b) $C_{\text{eq}} = 1.00\ \mu\text{F} + 2.50\ \mu\text{F} + 5.00\ \mu\text{F} = 8.50\ \mu\text{F}$.

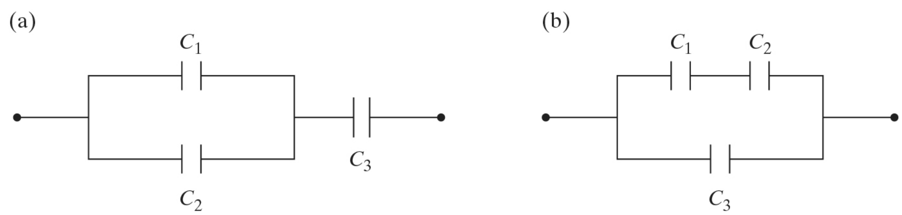


Figure VP24.9.1

(c) Figure VP24.9.1(a) shows the combination. The parallel combination is given by $C_{\text{p}} = 1.00\ \mu\text{F} + 2.50\ \mu\text{F} = 3.50\ \mu\text{F}$. The series combination is given by $1/C_S = 1/(3.50\ \mu\text{F}) + 1/(5.00\ \mu\text{F})$. This gives $C_S = 2.06\ \mu\text{F}$.

(d) Fig. VP24.9.1(b) shows the combination. For the series combination, $1/C_S = 1/(1.00\ \mu\text{F}) + 1/(2.50\ \mu\text{F})$. $C_S = 0.715\ \mu\text{F}$. The equivalent capacitance is $C_{\text{eq}} = 0.714\ \mu\text{F} + 5.00\ \mu\text{F} = 5.71\ \mu\text{F}$.

EVALUATE: Notice that for capacitors in series, the equivalent capacitance is less than the smallest capacitance, but when they are in parallel the equivalent capacitance is larger than the largest capacitance.

VP24.9.2. **IDENTIFY:** We are dealing with the energy stored in capacitors in combination.

SET UP: $U = \dfrac{Q^2}{2C} = \dfrac{1}{2}CV^2$. We want to find the stored energy.

EXECUTE: **(a)** $U_1 = \dfrac{Q^2}{2C_1} =$ 0.0675 J using the given numbers. Similarly $U_2 = 0.0169$ J. We see that C_1 stores more energy than C_2.

(b) $U_1 = \dfrac{1}{2}C_1V^2 = 0.0117$ J using the given numbers. Similarly $U_2 = 0.0469$ J. C_2 has more energy than C_1.

EVALUATE: Note that a larger capacitor does *not necessarily* store more energy than a small capacitor. It depends on how they are connected.

VP24.9.3. **IDENTIFY:** We are dealing with the energy stored in capacitors in combination.

SET UP: $U = \dfrac{Q^2}{2C} = \dfrac{1}{2}CV^2$. $1/C_{eq} = 1/C_1 + 1/C_2 = \dfrac{C_1 + C_2}{C_1 C_2}$. We want the stored energy.

EXECUTE: **(a)** Series: $U_S = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\left(\dfrac{C_1C_2}{C_1 + C_2}\right)V^2$ = 0.0115 J using the given numbers.

Parallel: $U_p = \dfrac{1}{2}(C_1 + C_2)V^2 =$ 0.0578 J using the given numbers. The parallel combination stores more energy than the series combination.

(b) Series: $U_S = \dfrac{Q^2}{2C_{eq}} = \dfrac{Q^2}{2}\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right) = 0.0733$ J using the given numbers.

Parallel: $U_p = \dfrac{Q^2}{2C_{eq}} = \dfrac{Q^2}{2(C_1 + C_2)} = 0.0145$ J using the given numbers. The series combination stores more energy.

EVALUATE: In part (a) the parallel combination stored more energy, but in (b) the series store more. The result depends on how the capacitors are connected in the circuit.

VP24.9.4. **IDENTIFY:** This problem involves a parallel-plate capacitor.

SET UP and **EXECUTE:** **(a)** We want the surface charge density on the plates. Using $\sigma = Q/A$ and the given numbers, we get σ = 5.10 μC/m^2.f

(b) We want the electric field. $E = \sigma/\epsilon_0$ = 576 kV/m.

(c) We want the energy density. $u = \dfrac{1}{2}\epsilon_0 E^2$ = 1.47 J/m^3 using the result from (b).

(d) We want the total energy. $U = u$(volume) $= uAd$ = (1.47 J/m^3)(2.45 m^2)(0.00140 m) = 5.04 mJ.

EVALUATE: As a check, use $U = \dfrac{Q^2}{2C}$ and $C = \epsilon_0 A/d$ to get $U = \dfrac{Q^2 d}{2\epsilon_0 A}$ = 5.04 mJ, which agrees with our result in (d).

VP24.11.1. **IDENTIFY:** We are dealing with a capacitor containing dielectric.

SET UP and **EXECUTE:** **(a)** We want C without dielectric. $C = \dfrac{\epsilon_0 A}{d}$ = 1.36 nF using the given numbers.

(b) We want the charge. $Q = CV$ = (1.36 nF)(4.00 kV) = 5.43 μC.

(c) We want the dielectric constant. $V = V_0/K$, so $K = V_0/V$ = (4.00 kV)/(2.50 kV) = 1.60.

(d) We want the capacitance. $C = KC_0$ = (1.60)(1.36 nF) = 2.17 nF.

(e) We want the induced charge. $Q_{\neg i} = Q(1 - 1/K)$ = (5.43 μC)(1 – 1/1.60) = 2.04 μC.

EVALUATE: The dielectric increases the capacitance. The presence of the induced charge decreases the potential difference between the plates. This charge produces and electric field opposite to the original field.

VP24.11.2. **IDENTIFY:** We are dealing with a capacitor containing dielectric.

SET UP and **EXECUTE:** **(a)** We want the capacitance. $C = \epsilon_0 A/d$ = 2.36 nF using the given numbers.

(b) We want the charge. $Q = swCV$ = (2.36 nF)(3.50 kV) = 8.26 μC.

(c) We want C with the dielectric. $C = KC_0$ = (2.50)(2.36 nF) = 5.90 nF.

(d) We want the charge. $Q = CV$ = (5.90 nF)(3.50 kV) = 20.7 μC.

(e) We want the induced charge. With the battery connected, V remains the same, so $E = V/d$ also remains the same. Thus there must be more charge on the plates than for an empty capacitor to make up for the electric field due to the induced charge of the dielectric. Using $E = \sigma/\epsilon_0$ gives

$\dfrac{\sigma_{plates}}{\epsilon_0} = \dfrac{\sigma_0}{\epsilon_0} + \dfrac{|\sigma_{induced}|}{\epsilon_0}$. Which gives $|Q_{induced}| = Q_{plates} - Q_0$ = 20.7 μC – 8.26 μC = 12.4 μC.

EVALUATE: If the battery were not left connected while the dielectric was inserted, V would *not* remain constant but the charge Q on the plates would stay constant as in problem VP24.11.1.

VP24.11.3. **IDENTIFY:** We are dealing with the energy stored in a capacitor with dielectric.

SET UP and **EXECUTE:** We want the energy stored before and after the dielectric is inserted.

(a) $U_0 = \frac{1}{2}CV_0^2 = 17.6$ J using the given numbers.

(b) $K = V_0/V$ and $C = KC_0$, so $U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)V^2 = \frac{1}{2}\left(\frac{V_0}{V}C_0\right)V^2 = \frac{1}{2}C_0V_0V$. Using $C_0 = 4.50\ \mu$F, $V_0 = 2.80$ kV, and $V = 1.20$ kV gives $U = 7.56$ J.

EVALUATE: The dielectric increased the capacitance but decreased the stored energy because V decreased.

VP24.11.4. **IDENTIFY:** We are dealing with the energy stored in a capacitor with dielectric.

SET UP and **EXECUTE:** We want the energy stored before and after the dielectric is inserted.

(a) $U_0 = \frac{1}{2}CV_0^2 = 6.30$ J using the given numbers.

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)V^2 = K\left(\frac{1}{2}C_0V^2\right) = KU_0 = (2.85)(6.30\text{ J}) = 18.0$ J.

EVALUATE: With the battery connected as the dielectric is inserted, V stays the same and C increases, so U increases by a factor of K.

24.1. **IDENTIFY:** The capacitance depends on the geometry (area and plate separation) of the plates.

SET UP: For a parallel-plate capacitor, $V_{ab} = Ed$, $E = \frac{Q}{\epsilon_0 A}$, and $C = \frac{Q}{V_{ab}}$.

EXECUTE: **(a)** $V_{ab} = Ed = (4.00\times10^6\text{ V/m})(2.50\times10^{-3}\text{ m}) = 1.00\times10^4$ V.

(b) Solving for the area gives

$$A = \frac{Q}{E\epsilon_0} = \frac{80.0\times10^{-9}\text{ C}}{(4.00\times10^6\text{ V/m})[8.854\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)]} = 2.26\times10^{-3}\text{ m}^2 = 22.6\text{ cm}^2.$$

(c) $C = \frac{Q}{V_{ab}} = \frac{80.0\times10^{-9}\text{ C}}{1.00\times10^4\text{ V}} = 8.00\times10^{-12}\text{ F} = 8.00$ pF.

EVALUATE: The capacitance is reasonable for laboratory capacitors, but the area is rather large.

24.2. **IDENTIFY** and **SET UP:** $C = \frac{\epsilon_0 A}{d}$, $C = \frac{Q}{V}$ and $V = Ed$.

EXECUTE: **(a)** $C = \epsilon_0 \frac{A}{d} = \epsilon_0 \frac{0.000982\text{ m}^2}{0.00328\text{ m}} = 2.65$ pF.

(b) $V = \frac{Q}{C} = \frac{4.35\times10^{-8}\text{ C}}{2.65\times10^{-12}\text{ F}} = 16.4$ kV.

(c) $E = \frac{V}{d} = \frac{16.4\times10^3\text{ V}}{0.00328\text{ m}} = 5.00\times10^6$ V/m.

EVALUATE: The electric field is uniform between the plates, at points that aren't close to the edges.

24.3. **IDENTIFY** and **SET UP:** It is a parallel-plate air capacitor, so we can apply the equations of Section 24.1.

EXECUTE: **(a)** $C = \dfrac{Q}{V_{ab}}$ so $V_{ab} = \dfrac{Q}{C} = \dfrac{0.148\times10^{-6}\text{ C}}{245\times10^{-12}\text{ F}} = 604\text{ V}.$

(b) $C = \dfrac{\epsilon_0 A}{d}$ so $A = \dfrac{Cd}{\epsilon_0} = \dfrac{(245\times10^{-12}\text{ F})(0.328\times10^{-3}\text{ m})}{8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 9.08\times10^{-3}\text{ m}^2 = 90.8\text{ cm}^2.$

(c) $V_{ab} = Ed$ so $E = \dfrac{V_{ab}}{d} = \dfrac{604\text{ V}}{0.328\times10^{-3}\text{ m}} = 1.84\times10^6\text{ V/m}.$

(d) $E = \dfrac{\sigma}{\epsilon_0}$ so $\sigma = E\epsilon_0 = (1.84\times10^6\text{ V/m})(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 1.63\times10^{-5}\text{ C/m}^2.$

EVALUATE: We could also calculate σ directly as Q/A. $\sigma = \dfrac{Q}{A} = \dfrac{0.148\times10^{-6}\text{ C}}{9.08\times10^{-3}\text{ m}^2} = 1.63\times10^{-5}\text{ C/m}^2,$ which checks.

24.4. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, enough charge flows onto the plates to make $V_{ab} = 12.0\text{ V}$.

EXECUTE: **(a)** 12.0 V.

(b) (i) When d is doubled, C is halved. $V_{ab} = \dfrac{Q}{C}$ and Q is constant, so V doubles. $V = 24.0\text{ V}$.

(ii) When r is doubled, A increases by a factor of 4. V decreases by a factor of 4 and $V = 3.0\text{ V}$.

EVALUATE: The electric field between the plates is $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{Q}{\epsilon_0 A}$. $V_{ab} = Ed$. When d is doubled E is unchanged and V doubles. When A is increased by a factor of 4, E decreases by a factor of 4 so V decreases by a factor of 4.

24.5. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, $V_{ab} = 12.0\text{ V}$.

EXECUTE: **(a)** $Q = CV_{ab} = (10.0\times10^{-6}\text{ F})(12.0\text{ V}) = 1.20\times10^{-4}\text{ C} = 120\ \mu\text{C}.$

(b) When d is doubled C is halved, so Q is halved. $Q = 60\ \mu\text{C}$.

(c) If r is doubled, A increases by a factor of 4. C increases by a factor of 4 and Q increases by a factor of 4. $Q = 480\ \mu\text{C}$.

EVALUATE: When the plates are moved apart, less charge on the plates is required to produce the same potential difference. With the separation of the plates constant, the electric field must remain constant to produce the same potential difference. The electric field depends on the surface charge density, σ. To produce the same σ, more charge is required when the area increases.

24.6. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $V_{ab} = Ed$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: We want $E = 1.00\times10^4\text{ N/C}$ when $V = 100\text{ V}$.

EXECUTE: **(a)** $d = \dfrac{V_{ab}}{E} = \dfrac{1.00\times10^2 \text{ V}}{1.00\times10^4 \text{ N/C}} = 1.00\times10^{-2} \text{ m} = 1.00 \text{ cm}.$

$A = \dfrac{Cd}{\epsilon_0} = \dfrac{(5.00\times10^{-12} \text{ F})(1.00\times10^{-2} \text{ m})}{8.854\times10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)} = 5.65\times10^{-3} \text{ m}^2.$ $A = \pi r^2$ so

$r = \sqrt{\dfrac{A}{\pi}} = 4.24\times10^{-2} \text{ m} = 4.24 \text{ cm}.$

(b) $Q = CV_{ab} = (5.00\times10^{-12} \text{ F})(1.00\times10^2 \text{ V}) = 5.00\times10^{-10} \text{ C} = 500 \text{ pC}.$

EVALUATE: $C = \dfrac{\epsilon_0 A}{d}$. We could have a larger d, along with a larger A, and still achieve the required C without exceeding the maximum allowed E.

24.7. **IDENTIFY:** The energy stored in a capacitor depends on its capacitance, which in turn depends on its geometry.

SET UP: $C = Q/V$ for any capacitor, and $C = \dfrac{\epsilon_0 A}{d}$ for a parallel-plate capacitor.

EXECUTE: **(a)** $C = \dfrac{Q}{V} = \dfrac{2.40\times10^{-10} \text{ C}}{42.0 \text{ V}} = 5.714\times10^{-12} \text{ F}.$ Using $C = \dfrac{\epsilon_0 A}{d}$ gives

$d = \dfrac{\epsilon_0 A}{C} = \dfrac{[8.854\times10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)](6.80\times10^{-4} \text{ m}^2)}{5.714\times10^{-12} \text{ F}} = 1.05 \text{ mm}.$

(b) $d = 2.10\times10^{-3}$ m. $C = \dfrac{\epsilon_0 A}{d} = \dfrac{5.714\times10^{-12} \text{ F}}{2} = 2.857\times10^{-12} \text{ F}.$ $V = \dfrac{Q}{C}$, so

$V = 2(42.0 \text{ V}) = 84.0 \text{ V}.$

EVALUATE: Doubling the plate separation halves the capacitance, so twice the potential difference is required to keep the same charge on the plates.

24.8. **IDENTIFY:** Capacitance depends on the geometry of the object.

(a) SET UP: The capacitance of a cylindrical capacitor is $C = \dfrac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$. Solving for r_b gives

$r_b = r_a e^{2\pi\epsilon_0 L/C}$.

EXECUTE: Substituting in the numbers for the exponent gives

$$\frac{2\pi(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(0.120 \text{ m})}{3.67\times10^{-11} \text{ F}} = 0.182.$$

Now use this value to calculate r_b: $r_b = r_a e^{0.182} = (0.250 \text{ cm})e^{0.182} = 0.300 \text{ cm}.$

(b) SET UP: For any capacitor, $C = Q/V$ and $\lambda = Q/L$. Combining these equations and substituting the numbers gives $\lambda = Q/L = CV/L$.

EXECUTE: Numerically we get

$$\lambda = \frac{CV}{L} = \frac{(3.67\times10^{-11} \text{ F})(125 \text{ V})}{0.120 \text{ m}} = 3.82\times10^{-8} \text{ C/m} = 38.2 \text{ nC/m}.$$

EVALUATE: The distance between the surfaces of the two cylinders would be only 0.050 cm, which is just 0.50 mm. These cylinders would have to be carefully constructed.

24.9. **IDENTIFY:** Apply the results of Example 24.4. $C = Q/V$.

SET UP: $r_a = 0.50$ mm, $r_b = 5.00$ mm.

EXECUTE: **(a)** $C = \dfrac{L 2\pi\epsilon_0}{\ln(r_b/r_a)} = \dfrac{(0.180 \text{ m})2\pi\epsilon_0}{\ln(5.00/0.50)} = 4.35\times10^{-12} \text{ F}.$

(b) $V = Q/C = (10.0\times10^{-12}\text{ C})/(4.35\times10^{-12}\text{ F}) = 2.30\text{ V}.$

EVALUATE: $\dfrac{C}{L} = 24.2\text{ pF}.$ This value is similar to those in Example 24.4. The capacitance is determined entirely by the dimensions of the cylinders.

24.10. **IDENTIFY** and **SET UP:** Use $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ which was derived in Example 24.4. Then use $Q = CV$ to calculate Q.

EXECUTE: **(a)** Using $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ gives

$$\frac{C}{L} = \frac{2\pi(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}{\ln[(3.5\text{ mm})/(2.2\text{ mm})]} = 1.2\times10^{-10}\text{ F/m} = 120\text{ pF/m}.$$

(b) $C = (1.20\times10^{-10}\text{ F/m})(2.8\text{ m}) = 3.355\times10^{-10}\text{ F}.$

$Q = CV = (3.355\times10^{-10}\text{ F})(350\times10^{-3}\text{ V}) = 1.2\times10^{-10}\text{ C} = 120\text{ pC}.$

The conductor at higher potential has the positive charge, so there is +120 pC on the inner conductor and −120 pC on the outer conductor.

EVALUATE: C depends only on the dimensions of the capacitor. Q and V are proportional.

24.11. **IDENTIFY:** We can use the definition of capacitance to find the capacitance of the capacitor, and then relate the capacitance to geometry to find the inner radius.

(a) SET UP: By the definition of capacitance, $C = Q/V$.

EXECUTE: $C = \dfrac{Q}{V} = \dfrac{3.30\times10^{-9}\text{ C}}{2.20\times10^{2}\text{ V}} = 1.50\times10^{-11}\text{ F} = 15.0\text{ pF}.$

(b) SET UP: The capacitance of a spherical capacitor is $C = 4\pi\epsilon_0\dfrac{r_a r_b}{r_b - r_a}.$

EXECUTE: Solve for r_a and evaluate using $C = 15.0\text{ pF}$ and $r_b = 4.00\text{ cm},$ giving $r_a = 3.09\text{ cm}.$

(c) SET UP: We can treat the inner sphere as a point charge located at its center and use Coulomb's law, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}.$

EXECUTE: $E = \dfrac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(3.30\times10^{-9}\text{ C})}{(0.0309\text{ m})^2} = 3.12\times10^{4}\text{ N/C}.$

EVALUATE: Outside the capacitor, the electric field is zero because the charges on the spheres are equal in magnitude but opposite in sign.

24.12. **IDENTIFY:** Apply the results of Example 24.3. $C = Q/V$.

SET UP: $r_a = 15.0\text{ cm}.$ Solve for r_b.

EXECUTE: **(a)** For two concentric spherical shells, the capacitance is $C = \dfrac{1}{k}\left(\dfrac{r_a r_b}{r_b - r_a}\right).$

$kCr_b - kCr_a = r_a r_b$ and $r_b = \dfrac{kCr_a}{kC - r_a} = \dfrac{k(116\times10^{-12}\text{ F})(0.150\text{ m})}{k(116\times10^{-12}\text{ F}) - 0.150\text{ m}} = 0.175\text{ m} = 17.5\text{ cm}.$

(b) $V = 220\text{ V}$ and $Q = CV = (116\times10^{-12}\text{ F})(220\text{ V}) = 2.55\times10^{-8}\text{ C} = 25.5\text{ nC}.$

EVALUATE: A parallel-plate capacitor with $A = 4\pi r_a r_b = 0.33\text{ m}^2$ and $d = r_b - r_a = 2.5\times10^{-2}\text{ m}$ has $C = \dfrac{\epsilon_0 A}{d} = 117\text{ pF},$ in excellent agreement with the value of C for the spherical capacitor.

24.13. **IDENTIFY:** This problem involves dielectrics and capacitors in series.
SET UP: First sketch the circuit as in Fig. 24.13. The graph plots V_2 versus V, so we need to find a relationship between these quantities so we can interpret the slope. We want to find C_1.

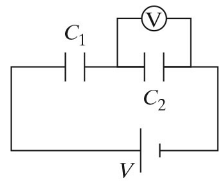


Figure 24.13

EXECUTE: For capacitors in series $Q_1 = Q_2 = Q = C_{eq}V$. $V_2 = Q/C_2 = C_{eq}V/C_2$. $C_{eq} = \dfrac{C_1C_2}{C_1+C_2}$, so $V_2 = \left(\dfrac{C_1C_2}{C_1+C_2}\right)\dfrac{V}{C_2} = \left(\dfrac{C_1}{C_1+C_2}\right)V$. Therefore the slope of the graph is $\dfrac{C_1}{C_1+C_2}$ = slope, which gives $C_1 = \dfrac{C_2(\text{slope})}{1-\text{slope}} = \dfrac{(3.00\,\mu\text{F})(0.650)}{1-0.650} = 5.57\ \mu\text{F}$.

EVALUATE: This seems an odd way to measure C_1, but potentials are easy to measure using simple voltmeters, so these measurements could easily be made.

24.14. **IDENTIFY:** Simplify the network by replacing series and parallel combinations of capacitors by their equivalents.

SET UP: For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$ For capacitors in parallel the voltages are the same and the charges add; $C_{eq} = C_1 + C_2 + \ldots$ $C = \dfrac{Q}{V}$.

EXECUTE: **(a)** The equivalent capacitance of the $5.0\ \mu\text{F}$ and $8.0\ \mu\text{F}$ capacitors in parallel is $13.0\ \mu\text{F}$. When these two capacitors are replaced by their equivalent we get the network sketched in Figure 24.14. The equivalent capacitance of these three capacitors in series is $3.47\ \mu\text{F}$.

(b) $Q_{tot} = C_{tot}V = (3.47\ \mu\text{F})(50.0\ \text{V}) = 174\ \mu\text{C}$.

(c) Q_{tot} is the same as Q for each of the capacitors in the series combination shown in Figure 24.22, so Q for each of the capacitors is $174\ \mu\text{C}$.

EVALUATE: The voltages across each capacitor in Figure 24.14 are $V_{10} = \dfrac{Q_{tot}}{C_{10}} = 17.4\ \text{V}$, $V_{13} = \dfrac{Q_{tot}}{C_{13}} = 13.4\ \text{V}$, and $V_9 = \dfrac{Q_{tot}}{C_9} = 19.3\ \text{V}$. $V_{10} + V_{13} + V_9 = 17.4\ \text{V} + 13.4\ \text{V} + 19.3\ \text{V} = 50.1\ \text{V}$. The sum of the voltages equals the applied voltage, apart from a small difference due to rounding.

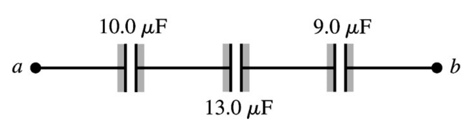


Figure 24.14

24.15. **IDENTIFY:** For capacitors in series the voltage across the combination equals the sum of the voltages in the individual capacitors. For capacitors in parallel the voltage across the combination is the same as the voltage across each individual capacitor.
SET UP and EXECUTE: **(a)** Connect the capacitors in series so their voltages will add.

(b) $V = V_1 + V_2 + V_3 + \ldots = NV_1,$ where N is the number of capacitors in the series combination, since the capacitors are identical. $N = \dfrac{V}{V_1} = \dfrac{500\text{ V}}{0.10\text{ V}} = 5000.$

EVALUATE: It requires many small cells to produce a large voltage surge.

24.16. **IDENTIFY:** The capacitors between b and c are in parallel. This combination is in series with the 15 pF capacitor.

SET UP: Let $C_1 = 15\text{ pF},\; C_2 = 9.0\text{ pF}$ and $C_3 = 11\text{ pF}.$

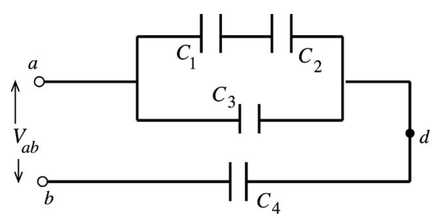
EXECUTE: **(a)** For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + \ldots$ so $C_{23} = C_2 + C_3 = 20\text{ pF}.$

(b) $C_1 = 15\text{ pF}$ is in series with $C_{23} = 20\text{ pF}.$ For capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$ so $\dfrac{1}{C_{123}} = \dfrac{1}{C_1} + \dfrac{1}{C_{23}}$ and $C_{123} = \dfrac{C_1 C_{23}}{C_1 + C_{23}} = \dfrac{(15\text{ pF})(20\text{ pF})}{15\text{ pF} + 20\text{ pF}} = 8.6\text{ pF}.$

EVALUATE: For capacitors in parallel the equivalent capacitance is larger than any of the individual capacitors. For capacitors in series the equivalent capacitance is smaller than any of the individual capacitors.

24.17. **IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

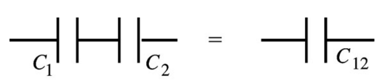
SET UP: Do parts (a) and (b) together. The capacitor network is drawn in Figure 24.17a.



$C_1 = C_2 = C_3 = C_4 = 4.00\;\mu\text{F}.$
$V_{ab} = 28.0\text{ V}.$

Figure 24.17a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_1 and C_2 are in series and are equivalent to C_{12} (Figure 24.17b).

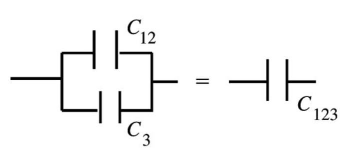


$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

Figure 24.17b

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4.00\times 10^{-6}\text{ F})(4.00\times 10^{-6}\text{ F})}{4.00\times 10^{-6}\text{ F} + 4.00\times 10^{-6}\text{ F}} = 2.00\times 10^{-6}\text{ F}.$$

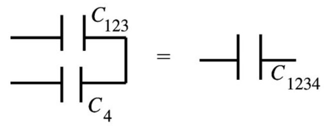
C_{12} and C_3 are in parallel and are equivalent to C_{123} (Figure 24.17c).



$C_{123} = C_{12} + C_3.$
$C_{123} = 2.00\times 10^{-6}\text{ F} + 4.00\times 10^{-6}\text{ F}.$
$C_{123} = 6.00\times 10^{-6}\text{ F}.$

Figure 24.17c

C_{123} and C_4 are in series and are equivalent to C_{1234} (Figure 24.17d).

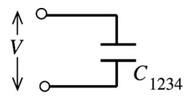


$$\frac{1}{C_{1234}}=\frac{1}{C_{123}}+\frac{1}{C_4}.$$

Figure 24.17d

$$C_{1234}=\frac{C_{123}C_4}{C_{123}+C_4}=\frac{(6.00\times10^{-6}\text{ F})(4.00\times10^{-6}\text{ F})}{6.00\times10^{-6}\text{ F}+4.00\times10^{-6}\text{ F}}=2.40\times10^{-6}\text{ F}.$$

The circuit is equivalent to the circuit shown in Figure 24.17e.

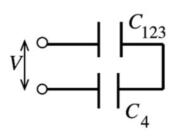


$V_{1234}=V=28.0\text{ V}.$

$Q_{1234}=C_{1234}V=(2.40\times10^{-6}\text{ F})(28.0\text{ V})=67.2\ \mu\text{C}.$

Figure 24.17e

Now build back up the original circuit, step by step. C_{1234} represents C_{123} and C_4 in series (Figure 24.17f).



$Q_{123}=Q_4=Q_{1234}=67.2\ \mu\text{C}$

(charge same for capacitors in series).

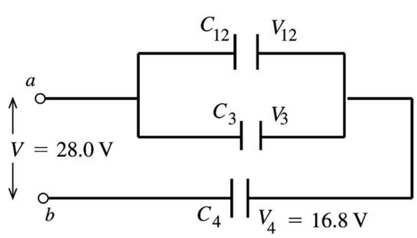
Figure 24.17f

Then $V_{123}=\dfrac{Q_{123}}{C_{123}}=\dfrac{67.2\ \mu\text{C}}{6.00\ \mu\text{F}}=11.2\text{ V}$.

$$V_4=\frac{Q_4}{C_4}=\frac{67.2\ \mu\text{C}}{4.00\ \mu\text{F}}=16.8\text{ V}.$$

Note that $V_4+V_{123}=16.8\text{ V}+11.2\text{ V}=28.0\text{ V}$, as it should.

Next consider the circuit as written in Figure 24.17g.



$V_3=V_{12}=28.0\text{ V}-V_4.$

$V_3=11.2\text{ V}.$

$Q_3=C_3V_3=(4.00\ \mu\text{F})(11.2\text{ V}).$

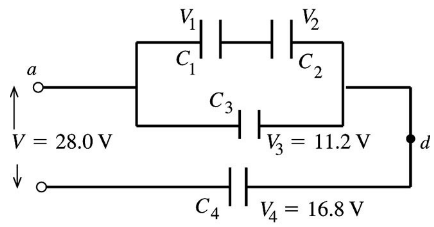
$Q_3=44.8\ \mu\text{C}.$

$Q_{12}=C_{12}V_{12}=(2.00\ \mu\text{F})(11.2\text{ V}).$

$Q_{12}=22.4\ \mu\text{C}.$

Figure 24.17g

Finally, consider the original circuit, as shown in Figure 24.17h.



$Q_1 = Q_2 = Q_{12} = 22.4\ \mu\text{C}$

(charge same for capacitors in series).

$$V_1 = \frac{Q_1}{C_1} = \frac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6\ \text{V}.$$

$$V_2 = \frac{Q_2}{C_2} = \frac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6\ \text{V}.$$

Figure 24.17h

Note that $V_1 + V_2 = 11.2$ V, which equals V_3 as it should.

Summary: $Q_1 = 22.4\ \mu\text{C},\ V_1 = 5.6\ \text{V}.$

$Q_2 = 22.4\ \mu\text{C},\ V_2 = 5.6\ \text{V}.$

$Q_3 = 44.8\ \mu\text{C},\ V_3 = 11.2\ \text{V}.$

$Q_4 = 67.2\ \mu\text{C},\ V_4 = 16.8\ \text{V}.$

(c) $V_{ad} = V_3 = 11.2$ V.

EVALUATE: $V_1 + V_2 + V_4 = V$, or $V_3 + V_4 = V$. $Q_1 = Q_2, Q_1 + Q_3 = Q_4$ and $Q_4 = Q_{1234}$.

24.18. **IDENTIFY:** The two capacitors are in series. The equivalent capacitance is given by $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

SET UP: For capacitors in series the charges are the same and the potentials add to give the potential across the network.

EXECUTE: **(a)** $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{(3.00\times10^{-6}\ \text{F})} + \dfrac{1}{(5.00\times10^{-6}\ \text{F})}$, so $C_{\text{eq}} = 1.875\times10^{-6}$ F. Then

$Q = VC_{\text{eq}} = (64.0\ \text{V})(1.875\times10^{-6}\ \text{F}) = 1.20\times10^{-4}\ \text{C} = 120\ \mu\text{C}$. Each capacitor has a charge of $1.20\times10^{-4}\ \text{C} = 120\ \mu\text{C}$.

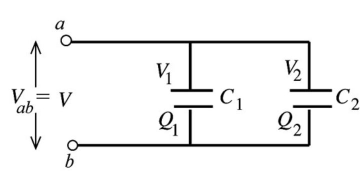
(b) $V_1 = Q/C_1 = (1.20\times10^{-4}\ \text{C})/(3.0\times10^{-6}\ \text{F}) = 40.0\ \text{V}.$

$V_2 = Q/C_2 = (1.20\times10^{-4}\ \text{C})/(5.0\times10^{-6}\ \text{F}) = 24.0\ \text{V}.$

EVALUATE: $V_1 + V_2 = 64.0$ V, which is equal to the applied potential V_{ab}. The capacitor with the smaller *C* has the larger *V*.

24.19. **IDENTIFY:** The two capacitors are in parallel so the voltage is the same on each, and equal to the applied voltage V_{ab}.

SET UP: Do parts (a) and (b) together. The network is sketched in Figure 24.19.



EXECUTE: $V_1 = V_2 = V$.

$V_1 = 52.0$ V.

$V_2 = 52.0$ V.

Figure 24.19

$C = Q/V$ so $Q = CV$.

$Q_1 = C_1V_1 = (3.00\ \mu\text{F})(52.0\ \text{V}) = 156\ \mu\text{C}$. $Q_2 = C_2V_2 = (5.00\ \mu\text{F})(52.0\ \text{V}) = 260\ \mu\text{C}$.

EVALUATE: To produce the same potential difference, the capacitor with the larger C has the larger Q.

24.20. IDENTIFY: For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. $C = Q/V$.

SET UP: C_1 and C_2 are in parallel and C_3 is in series with the parallel combination of C_1 and C_2.

EXECUTE: **(a)** C_1 and C_2 are in parallel and so have the same potential across them:

$V_1 = V_2 = \dfrac{Q_2}{C_2} = \dfrac{30.0\times10^{-6}\ \text{C}}{3.00\times10^{-6}\ \text{F}} = 10.0\ \text{V}$. Therefore, $Q_1 = V_1C_1 = (10.0\ \text{V})(6.00\times10^{-6}\ \text{F}) = 60.0\times10^{-6}\ \text{C}$.

Since C_3 is in series with the parallel combination of C_1 and C_2, its charge must be equal to their combined charge: $Q_3 = 30.0\times10^{-6}\ \text{C} + 60.0\times10^{-6}\ \text{C} = 90.0\times10^{-6}\ \text{C}$.

(b) The total capacitance is found from $\dfrac{1}{C_\text{eq}} = \dfrac{1}{C_{12}} + \dfrac{1}{C_3} = \dfrac{1}{9.00\times10^{-6}\ \text{F}} + \dfrac{1}{5.00\times10^{-6}\ \text{F}}$ and

$C_\text{eq} = 3.21\ \mu\text{F}$. $V_{ab} = \dfrac{Q_\text{tot}}{C_\text{eq}} = \dfrac{90.0\times10^{-6}\ \text{C}}{3.21\times10^{-6}\ \text{F}} = 28.0\ \text{V}$.

EVALUATE: $V_3 = \dfrac{Q_3}{C_3} = \dfrac{90.0\times10^{-6}\ \text{C}}{5.00\times10^{-6}\ \text{F}} = 18.0\ \text{V}$. $V_{ab} = V_1 + V_3 = 10.0\ \text{V} + 18.0\ \text{V} = 28.0\ \text{V}$, as we just found.

24.21. IDENTIFY: Three of the capacitors are in series, and this combination is in parallel with the other two capacitors.

SET UP: For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_\text{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$. For capacitors in parallel the voltages are the same and the charges add; $C_\text{eq} = C_1 + C_2 + \ldots$. $C = \dfrac{Q}{V}$.

EXECUTE: **(a)** The equivalent capacitance of the 18.0 nF, 30.0 nF and 10.0 nF capacitors in series is 5.29 nF. When these capacitors are replaced by their equivalent we get the network sketched in Figure 24.21. The equivalent capacitance of these three capacitors in parallel is 19.3 nF, and this is the equivalent capacitance of the original network.

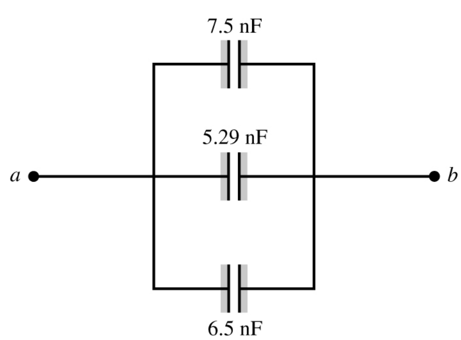


Figure 24.21

(b) $Q_{\text{tot}} = C_{\text{eq}}V = (19.3 \text{ nF})(25 \text{ V}) = 482 \text{ nC}.$

(c) The potential across each capacitor in the parallel network of Figure 24.21 is 25 V. $Q_{6.5} = C_{6.5}V_{6.5} = (6.5 \text{ nF})(25 \text{ V}) = 162 \text{ nC}.$

(d) 25 V.

EVALUATE: As with most circuits, we must go through a series of steps to simplify it as we solve for the unknowns.

24.22. IDENTIFY: Apply $u = \frac{1}{2}\epsilon_0 E^2$.

SET UP: Example 24.3 shows that $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$ between the conducting shells and that $\dfrac{Q}{4\pi\epsilon_0} = \left(\dfrac{r_a r_b}{r_b - r_a}\right)V_{ab}.$

EXECUTE: $E = \left(\dfrac{r_a r_b}{r_b - r_a}\right)\dfrac{V_{ab}}{r^2} = \left(\dfrac{(0.125 \text{ m})(0.148 \text{ m})}{0.148 \text{ m} - 0.125 \text{ m}}\right)\dfrac{120 \text{ V}}{r^2} = \dfrac{96.5 \text{ V}\cdot\text{m}}{r^2}.$

(a) For $r = 0.126$ m, $E = 6.08\times10^3$ V/m. $u = \frac{1}{2}\epsilon_0 E^2 = 1.64\times10^{-4} \text{ J/m}^3.$

(b) For $r = 0.147$ m, $E = 4.47\times10^3$ V/m. $u = \frac{1}{2}\epsilon_0 E^2 = 8.85\times10^{-5} \text{ J/m}^3.$

EVALUATE: **(c)** No, the results of parts (a) and (b) show that the energy density is not uniform in the region between the plates. *E* decreases as *r* increases, so *u* decreases also.

24.23. IDENTIFY and **SET UP:** The energy density is given by $u = \frac{1}{2}\epsilon_0 E^2$. Use $V = Ed$ to solve for *E*.

EXECUTE: Calculate *E*: $E = \dfrac{V}{d} = \dfrac{400 \text{ V}}{5.00\times10^{-3} \text{ m}} = 8.00\times10^4 \text{ V/m}.$

Then $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(8.00\times10^4 \text{ V/m})^2 = 0.0283 \text{ J/m}^3.$

EVALUATE: *E* is smaller than the value in Example 24.8 by about a factor of 6 so *u* is smaller by about a factor of $6^2 = 36$.

24.24. IDENTIFY: Apply $C = Q/V$. $C = \dfrac{\epsilon_0 A}{d}$. The work done to double the separation equals the change in the stored energy.

SET UP: $U = \dfrac{1}{2}CV^2 = \dfrac{Q^2}{2C}.$

EXECUTE: **(a)** $V = Q/C = (3.90 \,\mu\text{C})/(920\times10^{-12} \text{ F}) = 4240 \text{ V} = 4.24 \text{ kV}.$

(b) $C = \dfrac{\epsilon_0 A}{d}$ says that since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to 8480 V = 8.48 kV.

(c) $U_i = \dfrac{Q^2}{2C} = \dfrac{(3.90\times10^{-6} \text{ C})^2}{2(920\times10^{-12} \text{ F})} = 8.27\times10^{-3} \text{ J} = 8.27 \text{ mJ}.$ If the separation is doubled while *Q* stays the same, the capacitance halves, and the energy stored doubles to $2U_i$. The amount of work done to move the plates equals the difference in energy stored in the capacitor, so $\Delta U = U_f - U_i = 2U_i - U_i = U_i = 8.27 \text{ mJ}.$

EVALUATE: The oppositely charged plates attract each other so positive work must be done by an external force to pull them farther apart.

24.25. IDENTIFY: $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$. $V_{ab} = Ed$. The stored energy is $\frac{1}{2}QV$.

SET UP: $d = 1.50\times10^{-3}$ m. $1\,\mu\text{C} = 10^{-6}$ C

EXECUTE: **(a)** $C = \dfrac{0.0180\times10^{-6}\ \text{C}}{200\ \text{V}} = 9.00\times10^{-11}\ \text{F} = 90.0\ \text{pF}.$

(b) $C = \dfrac{\epsilon_0 A}{d}$ so $A = \dfrac{Cd}{\epsilon_0} = \dfrac{(9.00\times10^{-11}\ \text{F})(1.50\times10^{-3}\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = 0.0152\ \text{m}^2.$

(c) $V = Ed = (3.0\times10^6\ \text{V/m})(1.50\times10^{-3}\ \text{m}) = 4.5\times10^3\ \text{V} = 4.5\ \text{kV}.$

(d) Energy $= \frac{1}{2}QV = \frac{1}{2}(0.0180\times10^{-6}\ \text{C})(200\ \text{V}) = 1.80\times10^{-6}\ \text{J} = 1.80\ \mu\text{J}.$

EVALUATE: We could also calculate the stored energy as $\dfrac{Q^2}{2C} = \dfrac{(0.0180\times10^{-6}\ \text{C})^2}{2(9.00\times10^{-11}\ \text{F})} = 1.80\ \mu\text{J}.$

24.26. IDENTIFY: $C = \dfrac{\epsilon_0 A}{d}$. The stored energy can be expressed either as $\dfrac{Q^2}{2C}$ or as $\dfrac{CV^2}{2}$, whichever is more convenient for the calculation.

SET UP: Since d is halved, C doubles.

EXECUTE: **(a)** If the separation distance is halved while the charge is kept fixed, then the capacitance increases and the stored energy, which was 8.38 J, decreases since $U = Q^2/2C$. Therefore the new energy is 4.19 J.

(b) If the voltage is kept fixed while the separation is decreased by one half, then the doubling of the capacitance leads to a doubling of the stored energy to 16.8 J, using $U = CV^2/2$, when V is held constant throughout.

EVALUATE: When the capacitor is disconnected, the stored energy decreases because of the positive work done by the attractive force between the plates. When the capacitor remains connected to the battery, $Q = CV$ tells us that the charge on the plates increases. The increased stored energy comes from the battery when it puts more charge onto the plates.

24.27. IDENTIFY: Use the rules for series and for parallel capacitors to express the voltage for each capacitor in terms of the applied voltage. Express U, Q, and E in terms of the capacitor voltage.

SET UP: Let the applied voltage be V. Let each capacitor have capacitance C. $U = \frac{1}{2}CV^2$ for a single capacitor with voltage V.

EXECUTE: **(a)** Series: The voltage across each capacitor is $V/2$. The total energy stored is $U_\text{s} = 2(\frac{1}{2}C(V/2)^2) = \frac{1}{4}CV^2$.

Parallel: The voltage across each capacitor is V. The total energy stored is

$U_\text{p} = 2(\frac{1}{2}CV^2) = CV^2 \quad \rightarrow \quad U_\text{p} = 4U_\text{s}$.

(b) $Q = CV$ for a single capacitor with voltage V. $Q_\text{s} = 2[C(V/2)] = CV$; $Q_\text{p} = 2(CV) = 2CV$; $Q_\text{p} = 2Q_\text{s}$.

(c) $E = V/d$ for a capacitor with voltage V. $E_\text{s} = V/2d$; $E_\text{p} = V/d$; $E_\text{p} = 2E_\text{s}$.

EVALUATE: The parallel combination stores more energy and more charge since the voltage for each capacitor is larger for parallel. More energy stored and larger voltage for parallel means larger electric field in the parallel case.

24.28. IDENTIFY: The two capacitors are in series. $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$, $C = \frac{Q}{V}$, and $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same.

EXECUTE: **(a)** $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$ so $C_{\text{eq}} = \frac{C_1C_2}{C_1 + C_2} = \frac{(150\text{ nF})(120\text{ nF})}{150\text{ nF} + 120\text{ nF}} = 66.7\text{ nF}$.

$Q = CV = (66.7\text{ nF})(48\text{ V}) = 3.2\times10^{-6}\text{ C} = 3.2\ \mu\text{C}$.

(b) $Q = 3.2\ \mu\text{C}$ for each capacitor.

(c) $U = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(66.7\times10^{-9}\text{ F})(48\text{ V})^2 = 77\ \mu\text{J}$.

(d) We know C and Q for each capacitor so rewrite U in terms of these quantities.

$U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = Q^2/2C$.

150 nF: $U = \frac{(3.2\times10^{-6}\text{ C})^2}{2(150\times10^{-9}\text{ F})} = 34\ \mu\text{J}$.

120 nF: $U = \frac{(3.2\times10^{-6}\text{ C})^2}{2(120\times10^{-9}\text{ F})} = 43\ \mu\text{J}$.

Note that $34\ \mu\text{J} + 43\ \mu\text{J} = 77\ \mu\text{J}$, the total stored energy calculated in part (c).

(e) 150 nF: $V = \frac{Q}{C} = \frac{3.2\times10^{-6}\text{ C}}{150\times10^{-9}\text{ F}} = 21\text{ V}$.

120 nF: $V = \frac{Q}{C} = \frac{3.2\times10^{-6}\text{ C}}{120\times10^{-9}\text{ F}} = 27\text{ V}$.

Note that these two voltages sum to 48 V, the voltage applied across the network.

EVALUATE: Since Q is the same, the capacitor with smaller C stores more energy $\left(U = Q^2/2C\right)$ and has a larger voltage $(V = Q/C)$.

24.29. IDENTIFY: The two capacitors are in parallel. $C_{\text{eq}} = C_1 + C_2$. $C = \frac{Q}{V}$. $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in parallel, the voltages are the same and the charges add.

EXECUTE: **(a)** $C_{\text{eq}} = C_1 + C_2 = 35\text{ nF} + 75\text{ nF} = 110\text{ nF}$. $Q_{\text{tot}} = C_{\text{eq}}V = (110\times10^{-9}\text{ F})(220\text{ V}) = 24.2\ \mu\text{C}$

(b) $V = 220\text{ V}$ for each capacitor.

35 nF: $Q_{35} = C_{35}V = (35\times10^{-9}\text{ F})(220\text{ V}) = 7.7\ \mu\text{C}$; 75 nF:

$Q_{75} = C_{75}V = (75\times10^{-9}\text{ F})(220\text{ V}) = 16.5\ \mu\text{C}$. Note that $Q_{35} + Q_{75} = Q_{\text{tot}}$.

(c) $U_{\text{tot}} = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(110\times10^{-9}\text{ F})(220\text{ V})^2 = 2.66\text{ mJ}$.

(d) 35 nF: $U_{35} = \frac{1}{2}C_{35}V^2 = \frac{1}{2}(35\times10^{-9}\text{ F})(220\text{ V})^2 = 0.85\text{ mJ}$;

75 nF: $U_{75} = \frac{1}{2}C_{75}V^2 = \frac{1}{2}(75\times10^{-9}\text{ F})(220\text{ V})^2 = 1.81\text{ mJ}$. Since V is the same the capacitor with larger C stores more energy.

(e) 220 V for each capacitor.

EVALUATE: The capacitor with the larger C has the larger Q.

24.30. **IDENTIFY:** This problem involves dielectrics and capacitors in parallel.

SET UP: First sketch the circuit as in Fig. 24.30. $C_{eq} = C_1 + C_2$, $C = KC_0$, and $Q = CV$. We want the charge in both cases.

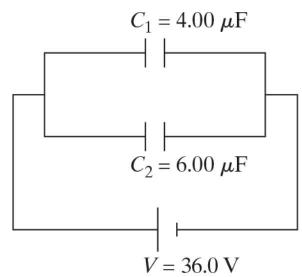


Figure 24.30

EXECUTE: **(a)** $Q = C_{eq}V = (C_1 + C_2)V = (10.0\ \mu F)(36.0\ V) = 360\ \mu C$.

(b) C_1 is now $KC_0 = (5.00)(4.00\ \mu F) = 20.0\ \mu F$. $Q = C_{eq}V = (26.0\ \mu F)(36.0\ V) = 936\ \mu C$.

EVALUATE: The total charge increases due to the insertion of the dielectric. The dielectric increases the equivalent capacitance which increases the stored charge.

24.31. **IDENTIFY:** $C = KC_0$. $U = \frac{1}{2}CV^2$.

SET UP: $C_0 = 12.5\ \mu F$ is the value of the capacitance without the dielectric present.

EXECUTE: **(a)** With the dielectric, $C = (3.75)(12.5\ \mu F) = 46.9\ \mu F$.

Before: $U = \frac{1}{2}C_0V^2 = \frac{1}{2}(12.5\times10^{-6}\ F)(24.0\ V)^2 = 3.60\ mJ$.

After: $U = \frac{1}{2}CV^2 = \frac{1}{2}(46.9\times10^{-6}\ F)(24.0\ V)^2 = 13.5\ mJ$.

(b) $\Delta U = 13.5\ mJ - 3.6\ mJ = 9.9\ mJ$. The energy increased.

EVALUATE: The power supply must put additional charge on the plates to maintain the same potential difference when the dielectric is inserted. $U = \frac{1}{2}QV$, so the stored energy increases.

24.32. **IDENTIFY:** $V = Ed$ and $C = Q/V$. With the dielectric present, $C = KC_0$.

SET UP: $V = Ed$ holds both with and without the dielectric.

EXECUTE: **(a)** $V = Ed = (3.00\times10^4\ V/m)(1.50\times10^{-3}\ m) = 45.0\ V$.

$Q = C_0V = (8.00\times10^{-12}\ F)(45.0\ V) = 3.60\times10^{-10}\ C = 360\ pC$.

(b) With the dielectric, $C = KC_0 = (2.70)(8.00\ pF) = 21.6\ pF$. V is still 45.0 V, so

$Q = CV = (21.6\times10^{-12}\ F)(45.0\ V) = 9.72\times10^{-10}\ C = 972\ pC$.

EVALUATE: The presence of the dielectric increases the amount of charge that can be stored for a given potential difference and electric field between the plates. Q increases by a factor of K.

24.33. **IDENTIFY** and **SET UP:** Q is constant so we can apply Eq. (24.14). The charge density on each surface of the dielectric is given by $\sigma_i = \sigma(1 - 1/K)$.

EXECUTE: $E = \dfrac{E_0}{K}$ so $K = \dfrac{E_0}{E} = \dfrac{3.20\times10^5\ V/m}{2.50\times10^5\ V/m} = 1.28$.

(a) $\sigma_i = \sigma(1 - 1/K)$.

$\sigma = \epsilon_0 E_0 = (8.854\times10^{-12}\ C^2/N\cdot m^2)(3.20\times10^5\ N/C) = 2.833\times10^{-6}\ C/m^2$.

$\sigma_i = (2.833\times10^{-6}\ C/m^2)(1 - 1/1.28) = 6.20\times10^{-7}\ C/m^2$.

(b) As calculated above, $K = 1.28$.

EVALUATE: The surface charges on the dielectric produce an electric field that partially cancels the electric field produced by the charges on the capacitor plates.

24.34. **IDENTIFY:** We are dealing with the energy in a capacitor with dielectric.

SET UP: $1/C_{eq} = 1/C_1 + 1/C_2$. $U = \frac{1}{2}CV^2$. In this case, $C_1 = C_2 = C$. We want the ratio of U/U_0.

EXECUTE: Without dielectric: $1/C_{eq} = 1/C + 1/C = 2/C$, so $C_{eq} = C/2$.

$$U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\left(\frac{C}{2}\right)V^2 = \frac{CV^2}{4}.$$

With dielectric: $1/C_{eq} = 1/KC + 1/C = KC/(1 + K)$. $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\left(\frac{KC}{1+K}\right)V^2 = \frac{CV^2}{4}$.

$$\frac{U}{U_0} = \frac{\frac{1}{2}\left(\frac{KC}{1+K}\right)V^2}{\frac{CV^2}{4}} = \frac{2K}{1+K}.$$

EVALUATE: $\frac{U}{U_0} = \frac{2K}{1+K}$. $K \geq 1$, so the smallest this ratio can be is 1 and the largest it can be is 2.

Therefore $U \geq U_0$, so the dielectric *increases* the stored energy.

24.35. **IDENTIFY:** This problem involves capacitors in series, with and without dielectric.
SET UP: $1/C_{eq} = 1/C_1 + 1/C_2$. $C = KC_0$. $Q = CV$. The charge is the same on capacitors in series. We want the charge on C_1 (the capacitor with dielectric).
EXECUTE: Without dielectric: $1/C_{eq} = 1/C + 1/C = 2/C$, so $C_{eq} = C/2$. $Q_0 = C_{eq}V = CV/2$.
With dielectric: $1/C_{eq} = 1/KC + 1/C$, so $C_{eq} = CK/(1 + K)$. $Q = CV_{eq} = \left(\frac{CK}{1+K}\right)V$. $Q_0 = CV/2$, so $V = 2Q_0/C$, which gives $Q = \left(\frac{CK}{1+K}\right)\left(\frac{2Q_0}{C}\right) = \frac{2KQ_0}{1+K}$.
EVALUATE: $K \geq 1$, so $Q_{min} = Q_0$ and $Q_{max} = 2Q_0$. Therefore Q *increases*.

24.36. **IDENTIFY:** We are dealing with a capacitors in series and in parallel with dielectric.
SET UP: $E = V/d$, series: $1/C_{eq} = 1/C_1 + 1/C_2$, parallel: $C_{eq} = C_1 + C_2$. $Q = CV$. We want the ratio E_2/E_{02}.

EXECUTE: **(a)** Without dielectric: $1/C_{eq} = 1/(3.00\ \mu\text{F}) + 1/(6.00\ \mu\text{F})$. $C_{eq} = 2.00\ \mu\text{F}$. For the combination $Q = C_{eq}V = (2.00\ \mu\text{F})V$, which is the same charge on each capacitor. For C_2, $V_2 = Q_2/C_2 = Q/C_2 = \frac{(2.00\mu\text{F})V}{6.00\mu\text{F}} = \frac{V}{3}$. $E_{02} = V_2/d = V/3d$.

With dielectric: $C_1 = KC_0 = (4)(3.00\ \mu\text{F}) = 12.0\ \mu\text{F}$. $1/C_{eq} = 1/(12.0\ \mu\text{F}) + 1/(6.00\ \mu\text{F})$, which gives $C_{eq} = 4.00\ \mu\text{F}$. $Q = C_{eq}V = (4.00\ \mu\text{F})V$. For C_2, $V_2 = Q/C_2 = \frac{(4.00\mu\text{F})V}{6.00\mu\text{F}} = \frac{2V}{3}$. $E_2 = \frac{V_2}{d} = \frac{2V/3}{d}$.

$$\frac{E_2}{E_{02}} = \frac{2V/3d}{V/3d} = 2.00.$$ The field in C_2 has *increased*.

(b) Without dielectric: V is the same for both capacitors, so $E_{02} = V/d$.
With dielectric: V is unchanged, so $E_2 = V/d$. $E_2/E_{02} = 1$. The dielectric in C_1 does not affect the electric field in C_2.
EVALUATE: When the capacitors were in series, the capacitor in one affected the field in the other one. But when they were in parallel, the dielectric had no effect on the other capacitor.

24.37. **IDENTIFY and SET UP:** For a parallel-plate capacitor with a dielectric we can use the equation $C = K\varepsilon_0 A/d$. Minimum A means smallest possible d. d is limited by the requirement that E be less than 1.60×10^7 V/m when V is as large as 5500 V.

EXECUTE: $V = Ed$ so $d = \dfrac{V}{E} = \dfrac{5500 \text{ V}}{1.60\times10^7 \text{ V/m}} = 3.44\times10^{-4}$ m.

Then $A = \dfrac{Cd}{K\epsilon_0} = \dfrac{(1.25\times10^{-9} \text{ F})(3.44\times10^{-4} \text{ m})}{(3.60)(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} = 0.0135 \text{ m}^2.$

EVALUATE: The relation $V = Ed$ applies with or without a dielectric present. A would have to be larger if there were no dielectric.

24.38. **IDENTIFY:** We can model the cell wall as a large sheet carrying equal but opposite charges, which makes it equivalent to a parallel-plate capacitor.

SET UP: With air between the layers, $E_0 = \dfrac{Q}{\epsilon_0 A} = \dfrac{\sigma}{\epsilon_0}$ and $V_0 = E_0 d$. The energy density in the electric field is $u = \frac{1}{2}\epsilon_0 E^2$. The volume of a shell of thickness t and average radius R is $4\pi R^2 t$. The volume of a solid sphere of radius R is $\frac{4}{3}\pi R^3$. With the dielectric present, $E = \dfrac{E_0}{K}$ and $V = \dfrac{V_0}{K}$.

EXECUTE: **(a)** $E_0 = \dfrac{\sigma}{\epsilon_0} = \dfrac{0.50\times10^{-3} \text{ C/m}^2}{8.854\times10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)} = 5.6\times10^7$ V/m.

(b) $V_0 = E_0 d = (5.6\times10^7 \text{ V/m})(5.0\times10^{-9} \text{ m}) = 0.28$ V. The outer wall of the cell is at higher potential, since it has positive charge.

(c) For the cell, $V_{\text{cell}} = \frac{4}{3}\pi R^3$, which gives $R = \left(\dfrac{3V_{\text{cell}}}{4\pi}\right)^{1/3} = \left(\dfrac{3(10^{-16} \text{ m}^3)}{4\pi}\right)^{1/3} = 2.9\times10^{-6}$ m. The volume of the cell wall is $V_{\text{wall}} = 4\pi R^2 t = 4\pi(2.9\times10^{-6} \text{ m})^2(5.0\times10^{-9} \text{ m}) = 5.3\times10^{-19} \text{ m}^3$. The energy density in the cell wall is $u_0 = \frac{1}{2}\epsilon_0 E_0^2 = \frac{1}{2}[8.854\times10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)](5.6\times10^7 \text{ V/m})^2 = 1.39\times10^4 \text{ J/m}^3$. The total electric-field energy in the cell wall is $(1.39\times10^4 \text{ J/m}^3)(5.3\times10^{-19} \text{ m}^3) = 7\times10^{-15}$ J.

(d) $E = \dfrac{E_0}{K} = \dfrac{5.6\times10^7 \text{ V/m}}{5.4} = 1.0\times10^7$ V/m and $V = \dfrac{V_0}{K} = \dfrac{0.28 \text{ V}}{5.4} = 0.052$ V.

EVALUATE: To a first approximation, many biological structures can be modeled as basic circuit elements.

24.39. **IDENTIFY:** $C = Q/V$. $C = KC_0$. $V = Ed$.

SET UP: Table 24.1 gives $K = 3.1$ for mylar.

EXECUTE: **(a)** $\Delta Q = Q - Q_0 = (K-1)Q_0 = (K-1)C_0V_0 = (2.1)(2.5\times10^{-7} \text{ F})(12 \text{ V}) = 6.3\times10^{-6}$ C.

(b) $\sigma_{\text{i}} = \sigma(1-1/K)$ so $Q_{\text{i}} = Q(1-1/K) = (9.3\times10^{-6} \text{ C})(1-1/3.1) = 6.3\times10^{-6}$ C.

(c) The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

EVALUATE: $E = V/d$ and V is constant so E doesn't change when the dielectric is inserted.

24.40. **IDENTIFY and SET UP:** The energy density is due to the electric field in the dielectric. $u = \frac{1}{2}\epsilon E^2$, where $\epsilon = K\epsilon_0$. $V = Ed$. In this case, $E = 0.800E_{\text{m}}$.

EXECUTE: **(a)** Using $u = \frac{1}{2}\epsilon E^2$ with $\epsilon = K\epsilon_0$, we have

$u = (1/2)(2.6)(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)[(0.800)(2.0\times10^7\ \text{V/m})]^2 = 2945\ \text{J/m}^3$, which rounds to 2900 J/m^3.

(b) First get the plate separation d: $V = Ed$ gives

$d = V/E = (500\ V)/[(0.800)(2.0\times10^7\ \text{V/m})] = 3.125\times10^{-5}\ \text{m}$.

The stored energy is $U = u\times\text{volume} = uAd$, so

$A = U/ud = (0.200\times10^{-3}\ \text{J})/[(2945\ \text{J/m}^3)(3.125\times10^{-5}\text{m})] = 2.2\times10^{-3}\ \text{m}^2 = 22\ \text{cm}^2$.

EVALUATE: If this capacitor has square plates, their dimensions would be $x = (22\ \text{cm}^2)^{1/2} = 4.7$ cm on each side. This is considerably larger than ordinary laboratory capacitors used in circuits.

24.41. **(a) IDENTIFY** and **SET UP:** Since the capacitor remains connected to the power supply the potential difference doesn't change when the dielectric is inserted. Use $U = \frac{1}{2}CV^2$ to calculate V and combine it with $K = C/C_0$ to obtain a relation between the stored energies and the dielectric constant and use this to calculate K.

EXECUTE: Before the dielectric is inserted $U_0 = \frac{1}{2}C_0V^2$ so $V = \sqrt{\dfrac{2U_0}{C_0}} = \sqrt{\dfrac{2(1.85\times10^{-5}\ \text{J})}{360\times10^{-9}\ \text{F}}} = 10.1\ \text{V}$.

(b) $K = C/C_0$.

$U_0 = \frac{1}{2}C_0V^2$, $U = \frac{1}{2}CV^2$ so $C/C_0 = U/U_0$.

$$K = \frac{U}{U_0} = \frac{1.85\times10^{-5}\ \text{J} + 2.32\times10^{-5}\ \text{J}}{1.85\times10^{-5}\ \text{J}} = 2.25.$$

EVALUATE: K increases the capacitance and then from $U = \frac{1}{2}CV^2$, with V constant an increase in C gives an increase in U.

24.42. **IDENTIFY:** $C = KC_0$. $C = Q/V$. $V = Ed$.

SET UP: Since the capacitor remains connected to the battery the potential between the plates of the capacitor doesn't change.

EXECUTE: **(a)** The capacitance changes by a factor of K when the dielectric is inserted. Since V is unchanged (the battery is still connected), $\dfrac{C_{\text{after}}}{C_{\text{before}}} = \dfrac{Q_{\text{after}}}{Q_{\text{before}}} = \dfrac{45.0\ \text{pC}}{25.0\ \text{pC}} = K = 1.80$.

(b) The area of the plates is $\pi r^2 = \pi(0.0300\ \text{m})^2 = 2.827\times10^{-3}\ \text{m}^2$ and the separation between them is thus $d = \dfrac{\epsilon_0 A}{C} = \dfrac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)}{12.5\times10^{-12}\ \text{F}} = 2.00\times10^{-3}\ \text{m}$. Before the dielectric is inserted, $C = \dfrac{\epsilon_0 A}{d} = \dfrac{Q}{V}$ and $V = \dfrac{Qd}{\epsilon_0 A} = \dfrac{(25.0\times10^{-12}\ \text{C})(2.00\times10^{-3}\ \text{m})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)} = 2.00\ \text{V}$. The battery remains connected, so the potential difference is unchanged after the dielectric is inserted.

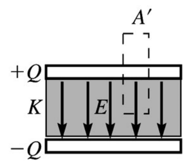
(c) Before the dielectric is inserted, $E = \dfrac{Q}{\epsilon_0 A} = \dfrac{25.0\times10^{-12}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)} = 1000\ \text{N/C}$.

Again, since the voltage is unchanged after the dielectric is inserted, the electric field is also unchanged.

EVALUATE: $E = \dfrac{V}{d} = \dfrac{2.00\ \text{V}}{2.00\times10^{-3}\ \text{m}} = 1000\ \text{N/C}$, whether or not the dielectric is present. This agrees with the result in part (c). The electric field has this value at any point between the plates. We need d to calculate E because V is the potential difference between points separated by distance d.

24.43. IDENTIFY: Apply $\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0}$ to calculate E. $V = Ed$ and $C = Q/V$ apply whether there is a dielectric between the plates or not.

(a) SET UP: Apply $\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0}$ to the dashed surface in Figure 24.43.



EXECUTE: $\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0}$.

$\oint K\vec{E}\cdot d\vec{A} = KEA'$.

since $E = 0$ outside the plates

$Q_{\text{encl-free}} = \sigma A' = (Q/A)A'$.

Figure 24.43

Thus $KEA' = \frac{(Q/A)A'}{\epsilon_0}$ and $E = \frac{Q}{\epsilon_0 AK}$.

SET UP and EXECUTE: **(b)** $V = Ed = \frac{Qd}{\epsilon_0 AK}$.

(c) $C = \frac{Q}{V} = \frac{Q}{Qd/\epsilon_0 AK} = K\frac{\epsilon_0 A}{d} = KC_0$.

EVALUATE: Our result shows that $K = C/C_0$, which is Eq. (24.12).

24.44. IDENTIFY: Gauss's law in dielectrics has the same form as in vacuum except that the electric field is multiplied by a factor of K and the charge enclosed by the Gaussian surface is the free charge. The capacitance of an object depends on its geometry.

(a) SET UP: The capacitance of a parallel-plate capacitor is $C = K\epsilon_0 A/d$ and the charge on its plates is $Q = CV$.

EXECUTE: First find the capacitance:

$$C = \frac{K\epsilon_0 A}{d} = \frac{(2.1)(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)}{1.00\times 10^{-3}\ \text{m}} = 4.18\times 10^{-10}\ \text{F}.$$

Now find the charge on the plates: $Q = CV = (4.18\times 10^{-10}\ \text{F})(12.0\ \text{V}) = 5.02\times 10^{-9}\ \text{C}$.

(b) SET UP: Gauss's law within the dielectric gives $KEA = Q_{\text{free}}/\epsilon_0$.

EXECUTE: Solving for E gives

$$E = \frac{Q_{\text{free}}}{KA\epsilon_0} = \frac{5.02\times 10^{-9}\ \text{C}}{(2.1)(0.0225\ \text{m}^2)(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.20\times 10^4\ \text{N/C}.$$

(c) SET UP: Without the Teflon and the voltage source, the charge is unchanged but the potential increases, so $C = \epsilon_0 A/d$ and Gauss's law now gives $EA = Q/\epsilon_0$.

EXECUTE: First find the capacitance:

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)}{1.00\times 10^{-3}\ \text{m}} = 1.99\times 10^{-10}\ \text{F}.$$

The potential difference is $V = \frac{Q}{C} = \frac{5.02\times 10^{-9}\ \text{C}}{1.99\times 10^{-10}\ \text{F}} = 25.2\ \text{V}$. From Gauss's law, the electric field is

$$E = \frac{Q}{\epsilon_0 A} = \frac{5.02\times 10^{-9}\ \text{C}}{(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)} = 2.52\times 10^4\ \text{N/C}.$$

Evaluate: The dielectric reduces the electric field inside the capacitor because the electric field due to the dipoles of the dielectric is opposite to the external field due to the free charge on the plates.

24.45. **Identify:** $P = E/t$, where E is the total light energy output. The energy stored in the capacitor is $U = \frac{1}{2}CV^2$.

Set Up: $E = 0.95U$.

Execute: **(a)** The power output is 2.70×10^5 W, and 95% of the original energy is converted, so $E = Pt = (2.70\times10^5 \text{ W})(1.48\times10^{-3} \text{ s}) = 400 \text{ J}$. $U = \dfrac{400 \text{ J}}{0.95} = 421 \text{ J}$.

(b) $U = \frac{1}{2}CV^2$ so $C = \dfrac{2U}{V^2} = \dfrac{2(421 \text{ J})}{(125 \text{ V})^2} = 0.054 \text{ F}$.

Evaluate: For a given V, the stored energy increases linearly with C.

24.46. **Identify** and **Set Up:** $C = \dfrac{\epsilon_0 A}{d}$. $C = Q/V$. $V = Ed$. $U = \frac{1}{2}CV^2$. With the battery disconnected, Q is constant. When the separation d is doubled, C is halved.

Execute: **(a)** $C = \dfrac{\epsilon_0 A}{d} = \dfrac{\epsilon_0 (0.12 \text{ m})^2}{3.7\times10^{-3} \text{ m}} = 3.446\times10^{-11} \text{ F}$, which rounds to 34 pF.

(b) $Q = CV = (3.446\times10^{-11} \text{ F})(12 \text{ V}) = 4.135\times10^{-10} \text{ C}$, which rounds to 410 pC.

(c) $E = V/d = (12 \text{ V})/(3.7\times10^{-3} \text{ m}) = 3200 \text{ V/m}$.

(d) $U = \frac{1}{2}CV^2 = \frac{1}{2}(3.446\times10^{-11} \text{ F})(12 \text{ V})^2 = 2.48\times10^{-9} \text{ J}$, which rounds to 2.5 nJ.

(e) If the battery is disconnected, so the charge remains constant, and the plates are pulled farther apart to 0.0074 m, then the calculations above can be carried out just as before, and we find:

(a) $C = 1.7\times10^{-11} \text{ F} = 17 \text{ pF}$.

(b) $Q = 4.1\times10^{-10} \text{ C} = 410 \text{ pC}$.

(c) $E = 3200 \text{ V/m}$.

(d) $U = \dfrac{Q^2}{2C} = \dfrac{(4.1\times10^{-10} \text{ C})^2}{2(1.7\times10^{-11} \text{ F})} = 5.0\times10^{-9} \text{ J} = 5.0 \text{ nJ}$.

Evaluate: Q is unchanged. $E = \dfrac{Q}{\epsilon_0 A}$ so E is therefore unchanged. U doubles because C is halved with Q unchanged. The additional stored energy comes from the work done by the force that pulled the plates apart.

24.47. **Identify:** $C = \dfrac{\epsilon_0 A}{d}$.

Set Up: $A = 4.2\times10^{-5} \text{ m}^2$. The original separation between the plates is $d = 0.700\times10^{-3}$ m. d' is the separation between the plates at the new value of C.

Execute: $C_0 = \dfrac{A\epsilon_0}{d} = \dfrac{(4.20\times10^{-5} \text{ m}^2)\epsilon_0}{7.00\times10^{-4} \text{ m}} = 5.31\times10^{-13} \text{ F}$. The new value of C is $C = C_0 + 0.25 \text{ pF} = 7.81\times10^{-13} \text{ F}$. But $C = \dfrac{A\epsilon_0}{d'}$, so $d' = \dfrac{A\epsilon_0}{C} = \dfrac{(4.20\times10^{-5} \text{ m}^2)\epsilon_0}{7.81\times10^{-13} \text{ F}} = 4.76\times10^{-4} \text{m}$.

Therefore the key must be depressed by a distance of $7.00\times10^{-4} \text{ m} - 4.76\times10^{-4} \text{ m} = 0.224 \text{ mm}$.

Evaluate: When the key is depressed, d decreases and C increases.

24.48. **Identify:** $C = KC_0 = K\epsilon_0 \dfrac{A}{d}$. $V = Ed$ for a parallel plate capacitor; this equation applies whether or not a dielectric is present.

SET UP: $A = 1.0 \text{ cm}^2 = 1.0\times10^{-4} \text{ m}^2$.

EXECUTE: **(a)** $C = (10)\dfrac{(8.85\times10^{-12} \text{ F/m})(1.0\times10^{-4} \text{ m}^2)}{7.5\times10^{-9} \text{ m}} = 1.18\,\mu\text{F per cm}^2$.

(b) $E = \dfrac{V}{d} = \dfrac{85 \text{ mV}}{7.5\times10^{-9} \text{ m}} = 1.13\times10^{7} \text{ V/m}$.

EVALUATE: The dielectric material increases the capacitance. If the dielectric were not present, the same charge density on the faces of the membrane would produce a larger potential difference across the membrane.

24.49. **IDENTIFY:** Some of the charge from the original capacitor flows onto the uncharged capacitor until the potential differences across the two capacitors are the same.

SET UP: $C = \dfrac{Q}{V_{ab}}$. Let $C_1 = 20.0\,\mu\text{F}$ and $C_2 = 10.0\,\mu\text{F}$. The energy stored in a capacitor is $\frac{1}{2}QV_{ab} = \frac{1}{2}CV_{ab}^2 = \dfrac{Q^2}{2C}$.

EXECUTE: **(a)** The initial charge on the $20.0\,\mu\text{F}$ capacitor is

$Q = C_1(800 \text{ V}) = (20.0\times10^{-6} \text{ F})(800 \text{ V}) = 0.0160 \text{ C}$.

(b) In the final circuit, charge Q is distributed between the two capacitors and $Q_1 + Q_2 = Q$. The final circuit contains only the two capacitors, so the voltage across each is the same, $V_1 = V_2$. $V = \dfrac{Q}{C}$ so $V_1 = V_2$ gives $\dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2}$. $Q_1 = \dfrac{C_1}{C_2}Q_2 = 2Q_2$. Using this in $Q_1 + Q_2 = 0.0160 \text{ C}$ gives $3Q_2 = 0.0160 \text{ C}$ and $Q_2 = 5.33\times10^{-3} \text{ C}$. $Q = 2Q_2 = 1.066\times10^{-2} \text{ C}$. $V_1 = \dfrac{Q_1}{C_1} = \dfrac{1.066\times10^{-2} \text{ C}}{20.0\times10^{-6} \text{ F}} = 533 \text{ V}$. $V_2 = \dfrac{Q_2}{C_2} = \dfrac{5.33\times10^{23} \text{ C}}{10.0\times10^{26} \text{ F}} = 533 \text{ V}$. The potential differences across the capacitors are the same, as they should be.

(c) $\text{Energy} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$ gives

$\text{Energy} = \frac{1}{2}(20.0\times10^{-6} \text{ F} + 10.0\times10^{-6} \text{ F})(533 \text{ V})^2 = 4.26 \text{ J}$.

(d) The $20.0\,\mu\text{F}$ capacitor initially has $\text{energy} = \frac{1}{2}C_1V^2 = \frac{1}{2}(20.0\times10^{-6} \text{ F})(800 \text{ V})^2 = 6.40 \text{ J}$. The decrease in stored energy that occurs when the capacitors are connected is $6.40 \text{ J} - 4.26 \text{ J} = 2.14 \text{ J}$.

EVALUATE: The decrease in stored energy is because of conversion of electrical energy to other forms during the motion of the charge when it becomes distributed between the two capacitors. Thermal energy is generated by the current in the wires and energy is emitted in electromagnetic waves.

24.50. **IDENTIFY:** We model a car as a spherical capacitor.

SET UP: $C = 4\pi\epsilon_0 \dfrac{r_a r_b}{r_b - r_a}$.

EXECUTE: **(a)** Using $C = 4\pi\epsilon_0 \dfrac{r_a r_b}{r_b - r_a}$ and let $r_b \to \infty$. $C \to 4\pi\epsilon_0 \dfrac{r_a r_b}{r_b} = 4\pi\epsilon_0\, r_a$.

Using a charged sphere, $C = Q/V = \dfrac{Q}{\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r_a}} = 4\pi\epsilon_0\, r_a$. Our results agree.

(b) Estimate: Care is about 4 paces long $\approx$ 4.0 m long, so $r \approx 2.0$ m.

(c) $C = 4\pi\epsilon_0\, r_a = 4\pi\epsilon_0\,(2.0 \text{ m})$ □ 220 pF.

(d) $Q = CV = (220 \text{ pF})(100 \text{ MV}) = 22 \text{ mC}$.

EVALUATE: This is quite a large charge, which why lightning strikes can be fatal!

24.51. **IDENTIFY:** Simplify the network by replacing series and parallel combinations by their equivalent. The stored energy in a capacitor is $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$ For capacitors in parallel the voltages are the same and the charges add; $C_{eq} = C_1 + C_2 + \ldots$ $C = \dfrac{Q}{V}$.

$U = \frac{1}{2}CV^2$.

EXECUTE: **(a)** Find C_{eq} for the network by replacing each series or parallel combination by its equivalent. The successive simplified circuits are shown in Figure 24.51.

$U_{tot} = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(2.19\times10^{-6}\text{ F})(12.0\text{ V})^2 = 1.58\times10^{-4}\text{ J} = 158\ \mu\text{J}$.

(b) From Figure 24.51c, $Q_{tot} = C_{eq}V = (2.19\times10^{-6}\text{ F})(12.0\text{ V}) = 2.63\times10^{-5}\text{ C}$. From Figure 24.51b,

$Q_{4.8} = 2.63\times10^{-5}\text{ C}$. $V_{4.8} = \dfrac{Q_{4.8}}{C_{4.8}} = \dfrac{2.63\times10^{-5}\text{ C}}{4.80\times10^{-6}\text{ F}} = 5.48\text{ V}$.

$U_{4.8} = \frac{1}{2}CV^2 = \frac{1}{2}(4.80\times10^{-6}\text{ F})(5.48\text{ V})^2 = 7.21\times10^{-5}\text{ J} = 72.1\ \mu\text{J}$.

This one capacitor stores nearly half the total stored energy.

EVALUATE: $U = \dfrac{Q^2}{2C}$. For capacitors in series the capacitor with the smallest C stores the greatest amount of energy.

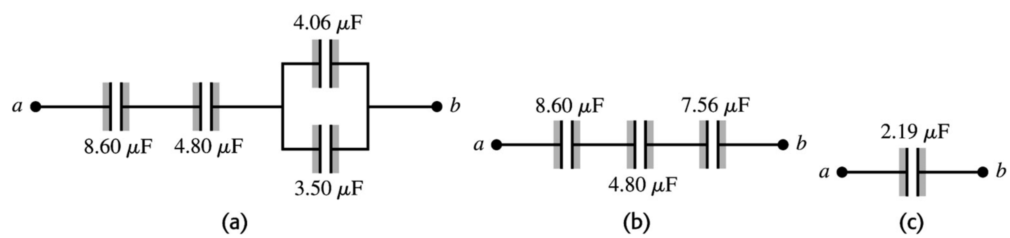


Figure 24.51

24.52. **IDENTIFY** and **SET UP:** The charge Q is the same on capacitors in series, and the potential V is the same for capacitors in parallel. C_1 is in series with C_2, and that combination is in parallel with C_3. The C_1-C_2-C_3 combination is in series with C_4. $V = Q/C$.

EXECUTE: **(a)** Since C_1 and C_2 are in series, and that combination is in parallel with C_3, the potential difference across the C_1-C_2 combination is the same as the potential difference across C_3, which is 40.0 V. Also, $Q_1 = Q_2 = Q$.

$V_1 + V_2 = 40.0$ V.

$Q/C_1 + Q/C_2 = 40.0$ V.

$Q/(6.00\ \mu\text{F}) + Q/(3.00\ \mu\text{F}) = 40.0$ V.

$Q = 80.0\ \mu\text{C}$.

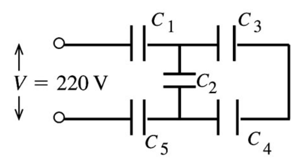
Therefore

$V_1 = Q/C_1 = (80.0\ \mu\text{C})/(6.00\ \mu\text{F}) = 13.3$ V.

$V_2 = Q/C_2 = (80.0\ \mu\text{C})/(3.00\ \mu\text{F}) = 26.7$ V.

(b) First get the charge Q_4 on C_4. We know that $Q_1 = Q_2 = Q = 80.0\ \mu\text{C}$. We also have
$Q_3 = C_3V_3 = (4.00\ \mu\text{F})(40.0\ \text{V}) = 160\ \mu\text{C}$.
$Q_4 = Q + Q_3 = 80.0\ \mu\text{C} + 160\ \mu\text{C} = 240\ \mu\text{C}$.
$V_4 = Q_4/C_4 = (240\ \mu\text{C})/(8.00\ \mu\text{F}) = 30.0\ \text{V}$.
(c) $V_{ab} = V_3 + V_4 = 40.0\ \text{V} + 30.0\ \text{V} = 70.0\ \text{V}$.
EVALUATE: C_3 and C_4 are *not* in parallel, so $V_3 \neq V_4$.

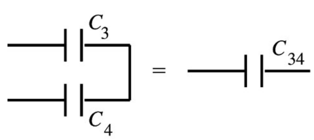
24.53. **(a) IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents.
SET UP: The network is sketched in Figure 24.53a.



$C_1 = C_5 = 8.4\ \mu\text{F}$.
$C_2 = C_3 = C_4 = 4.2\ \mu\text{F}$.

Figure 24.53a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents:
C_3 and C_4 are in series and can be replaced by C_{34} (Figure 24.53b):

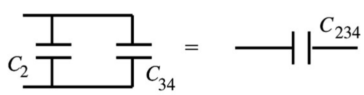


$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}.$$
$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3C_4}.$$

Figure 24.53b

$$C_{34} = \frac{C_3C_4}{C_3 + C_4} = \frac{(4.2\ \mu\text{F})(4.2\ \mu\text{F})}{4.2\ \mu\text{F} + 4.2\ \mu\text{F}} = 2.1\ \mu\text{F}.$$

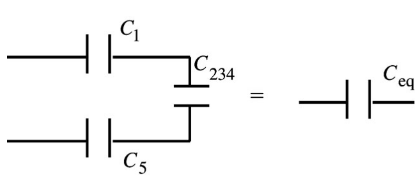
C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.53c):



$C_{234} = C_2 + C_{34}$.
$C_{234} = 4.2\ \mu\text{F} + 2.1\ \mu\text{F}$.
$C_{234} = 6.3\ \mu\text{F}$.

Figure 24.53c

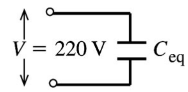
C_1, C_5, and C_{234} are in series and can be replaced by C_{eq} (Figure 24.53d):



$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}.$$
$$\frac{1}{C_{\text{eq}}} = \frac{2}{8.4\ \mu\text{F}} + \frac{1}{6.3\ \mu\text{F}}.$$
$$C_{\text{eq}} = 2.5\ \mu\text{F}.$$

Figure 24.53d

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.
(b) IDENTIFY and **SET UP:** In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.
EXECUTE: The equivalent circuit is drawn in Figure 24.53e.



$Q_{eq} = C_{eq}V.$
$Q_{eq} = (2.5\ \mu\text{F})(220\ \text{V}) = 550\ \mu\text{C}.$

Figure 24.53e

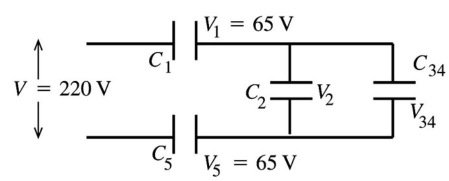
$Q_1 = Q_5 = Q_{234} = 550\ \mu\text{C}$ (capacitors in series have same charge).

$$V_1 = \frac{Q_1}{C_1} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}.$$

$$V_5 = \frac{Q_5}{C_5} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}.$$

$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550\ \mu\text{C}}{6.3\ \mu\text{F}} = 87\ \text{V}.$$

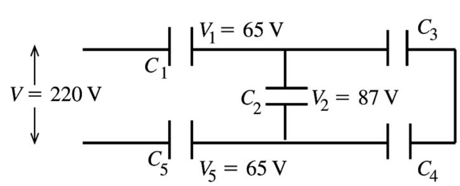
Now draw the network as in Figure 24.53f.



$V_2 = V_{34} = V_{234} = 87\ \text{V}$
capacitors in parallel have the same potential.

Figure 24.53f

$Q_2 = C_2V_2 = (4.2\ \mu\text{F})(87\ \text{V}) = 370\ \mu\text{C}.$
$Q_{34} = C_{34}V_{34} = (2.1\ \mu\text{F})(87\ \text{V}) = 180\ \mu\text{C}.$
Finally, consider the original circuit (Figure 24.53g).



$Q_3 = Q_4 = Q_{34} = 180\ \mu\text{C}$
capacitors in series have the same charge.

Figure 24.53g

$$V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}.$$

$$V_4 = \frac{Q_4}{C_4} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}.$$

Summary: $Q_1 = 550\ \mu\text{C},\ V_1 = 65\ \text{V}.$
$Q_2 = 370\ \mu\text{C},\ V_2 = 87\ \text{V}.$
$Q_3 = 180\ \mu\text{C},\ V_3 = 43\ \text{V}.$
$Q_4 = 180\ \mu\text{C},\ V_4 = 43\ \text{V}.$

$Q_5 = 550\ \mu\text{C},\ V_5 = 65\ \text{V}.$

EVALUATE: $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220\ \text{V}$ (apart from some small rounding error)

$Q_1 = Q_2 + Q_3$ and $Q_5 = Q_2 + Q_4$.

24.54. **IDENTIFY** and **SET UP:** We want to estimate the excess charge rubbed onto our head and the resulting voltage when we comb our hair. Treat the head as a sphere and model it as a spherical capacitor.

EXECUTE: **(a)** Estimate: $L \approx 15$ cm = 0.15 m long.

(b) m = (65 μg/cm)(15 cm) ≈ 975 μg.

(c) Estimate: N = 25 hairs.

(d) The electric force $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_2|}{r^2}$ is the force between the charges at both ends of a hair. $q_1 = q_2 = Q/N$, $r = L$, and the force is twice the weight of the hair, which is $2mg$. Therefore $2mg = \dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/N)^2}{L^2}$. Solve for Q: $Q = \sqrt{(2mg)(4\pi\epsilon_0)L^2 N}$. Using m = 975 μg, L = 0.15 m, and N = 25 gives Q = 0.35 μC. The total charge on your head is $2Q$, so Q_{head} = 0.70 μC.

(e) Estimate: Diameter ≈ 22 cm, so $R \approx 11$ cm. $C = 4\pi\epsilon_0 R = 4\pi\epsilon_0$ (0.11 m) = 12 pF.

(f) $V = Q/C$ = (0.35 μC)/(12 pF) = 29 kV.

EVALUATE: This is a large potential but it is not dangerous because of the small amount of charge.

24.55. **IDENTIFY:** Capacitors in series carry the same charge, while capacitors in parallel have the same potential difference across them.

SET UP: $V_{ab} = 150\ \text{V},\ Q_1 = 150\ \mu\text{C},\ Q_3 = 450\ \mu\text{C}$, and $V = Q/C$.

EXECUTE: $C_1 = 3.00\ \mu\text{F}$ so $V_1 = \dfrac{Q_1}{C_1} = \dfrac{150\ \mu\text{C}}{3.00\ \mu\text{F}} = 50.0\ \text{V}$ and $V_1 = V_2 = 50.0\ \text{V}$. $V_1 + V_3 = V_{ab}$ so $V_3 = 100\ \text{V}$. $C_3 = \dfrac{Q_3}{V_3} = \dfrac{450\ \mu\text{C}}{100\ \text{V}} = 4.50\ \mu\text{F}$. $Q_1 + Q_2 = Q_3$ so $Q_2 = Q_3 - Q_1 = 450\ \mu\text{C} - 150\ \mu\text{C} = 300\ \mu\text{C}$ and $C_2 = \dfrac{Q_2}{V_2} = \dfrac{300\ \mu\text{C}}{50.0\ \text{V}} = 6.00\ \mu\text{F}$.

EVALUATE: Capacitors in parallel only carry the same charge if they have the same capacitance.

24.56. **IDENTIFY:** Apply the rules for combining capacitors in series and in parallel.

SET UP: With the switch open, each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed, each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in parallel with each other and the two pairs are in series.

EXECUTE: **(a)** With the switch open $C_{\text{eq}} = \left(\left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1} + \left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1}\right) = 4.00\ \mu\text{F}.$

$Q_{\text{total}} = C_{\text{eq}}V = (4.00\ \mu\text{F})(210\ \text{V}) = 8.40\times10^{-4}\ \text{C}$. By symmetry, each capacitor carries $4.20\times10^{-4}\ \text{C}$. The voltages are then calculated via $V = Q/C$. This gives $V_{ad} = Q/C_3 = 140\ \text{V}$ and $V_{ac} = Q/C_6 = 70\ \text{V}$. $V_{cd} = V_{ad} - V_{ac} = 70\ \text{V}$.

(b) When the switch is closed, the points c and d must be at the same potential, so the equivalent capacitance is $C_{\text{eq}} = \left(\dfrac{1}{(3.00 + 6.00)\ \mu\text{F}} + \dfrac{1}{(3.00 + 6.00)\ \mu\text{F}}\right)^{-1} = 4.5\ \mu\text{F}.$

$Q_{\text{total}} = C_{\text{eq}}V = (4.50\ \mu\text{F})(210\ \text{V}) = 9.5\times10^{-4}\ \text{C}$, and each capacitor has the same potential difference of 105 V (again, by symmetry).

(c) Consider the $C_3 = 3.00\ \mu\text{F}$ and $C_6 = 6.00\ \mu\text{F}$ capacitors in the upper branch of the network. The only way for the net charge Q_{net} on the negative plate of C_3 and the positive plate of C_6 to change is by charge to flow through the switch. With the switch open all four capacitors have the same charge and $Q_{\text{net}} = 0$. With the switch closed the charge on C_3 is $Q_3 = (3.00\ \mu\text{F})(105\ \text{V}) = 315\ \mu\text{C}$ and the charge on C_6 is $Q_6 = (6.00\ \mu\text{F})(105\ \text{V}) = 630\ \mu\text{C}$ and $Q_{\text{net}} = Q_2 - Q_1 = 315\ \mu\text{C}$. Therefore, the change in Q_{net} is $315\ \mu\text{C}$ and this is the amount of charge that flowed through the switch when it was closed.

EVALUATE: When the switch is closed the charge must redistribute to make points *c* and *d* be at the same potential.

24.57. **(a) IDENTIFY:** Replace the three capacitors in series by their equivalent. The charge on the equivalent capacitor equals the charge on each of the original capacitors.

SET UP: The three capacitors can be replaced by their equivalent as shown in Figure 24.57a.

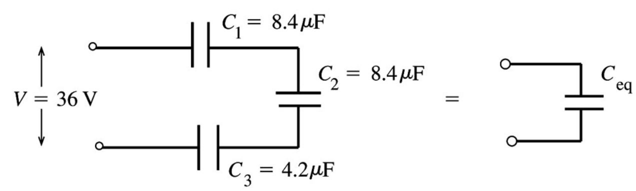


Figure 24.57a

EXECUTE: $C_3 = C_1/2$ so $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{4}{8.4\ \mu\text{F}}$ and $C_{\text{eq}} = 8.4\ \mu\text{F}/4 = 2.1\ \mu\text{F}$.

$Q = C_{\text{eq}}V = (2.1\ \mu\text{F})(36\ \text{V}) = 76\ \mu\text{C}$.

The three capacitors are in series so they each have the same charge: $Q_1 = Q_2 = Q_3 = 76\ \mu\text{C}$.

EVALUATE: The equivalent capacitance for capacitors in series is smaller than each of the original capacitors.

(b) IDENTIFY and **SET UP:** Use $U = \frac{1}{2}QV$. We know each Q and we know that $V_1 + V_2 + V_3 = 36\ \text{V}$.

EXECUTE: $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$.

But $Q_1 = Q_2 = Q_3 = Q$ so $U = \frac{1}{2}Q(V_1 + V_2 + V_3)$.

But also $V_1 + V_2 + V_3 = V = 36\ \text{V}$, so $U = \frac{1}{2}QV = \frac{1}{2}(76\ \mu\text{C})(36\ \text{V}) = 1.4\times10^{-3}\ \text{J}$.

EVALUATE: We could also use $U = Q^2/2C$ and calculate U for each capacitor.

(c) IDENTIFY: The charges on the plates redistribute to make the potentials across each capacitor the same.

SET UP: The capacitors before and after they are connected are sketched in Figure 24.57b.

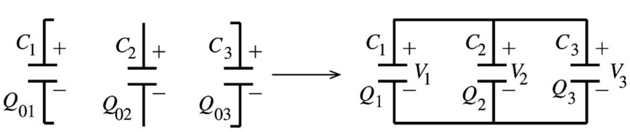


Figure 24.57b

EXECUTE: The total positive charge that is available to be distributed on the upper plates of the three capacitors is $Q_0 = Q_{01} + Q_{02} + Q_{03} = 3(76\ \mu\text{C}) = 228\ \mu\text{C}$. Thus $Q_1 + Q_2 + Q_3 = 228\ \mu\text{C}$. After the circuit is completed the charge distributes to make $V_1 = V_2 = V_3$. $V = Q/C$ and $V_1 = V_2$ so $Q_1/C_1 = Q_2/C_2$ and then $C_1 = C_2$ says $Q_1 = Q_2$. $V_1 = V_3$ says $Q_1/C_1 = Q_3/C_3$ and $Q_1 = Q_3(C_1/C_3) = Q_3(8.4\ \mu\text{F} / 4.2\ \mu\text{F}) = 2Q_3$.

Using $Q_2 = Q_1$ and $Q_1 = 2Q_3$ in the above equation gives $2Q_3 + 2Q_3 + Q_3 = 228\ \mu\text{C}$.

$5Q_3 = 228\ \mu\text{C}$ and $Q_3 = 45.6\ \mu\text{C}$, $Q_1 = Q_2 = 91.2\ \mu\text{C}$

Then $V_1 = \dfrac{Q_1}{C_1} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11\ \text{V}$, $V_2 = \dfrac{Q_2}{C_2} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11\ \text{V}$, and $V_3 = \dfrac{Q_3}{C_3} = \dfrac{45.6\ \mu\text{C}}{4.2\ \mu\text{F}} = 11\ \text{V}$.

The voltage across each capacitor in the parallel combination is 11 V.

(d) $U = \tfrac{1}{2}Q_1V_1 + \tfrac{1}{2}Q_2V_2 + \tfrac{1}{2}Q_3V_3$.

But $V_1 = V_2 = V_3$ so $U = \tfrac{1}{2}V_1(Q_1 + Q_2 + Q_3) = \tfrac{1}{2}(11\ \text{V})(228\ \mu\text{C}) = 1.3\times10^{-3}\ \text{J}$.

EVALUATE: This is less than the original energy of 1.4×10^{-3} J. The stored energy has decreased, as in Example 24.7.

24.58. IDENTIFY: $C = \dfrac{\epsilon_0 A}{d}$. $C = \dfrac{Q}{V}$. $V = Ed$. $U = \tfrac{1}{2}QV$.

SET UP: $d = 3.0\times10^3$ m. $A = \pi r^2$, with $r = 1.0\times10^3$ m.

EXECUTE: **(a)** $C = \dfrac{\epsilon_0 A}{d} = \dfrac{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\pi(1.0\times10^3\ \text{m})^2}{3.0\times10^3\ \text{m}} = 9.3\times10^{-9}\ \text{F}$.

(b) $V = \dfrac{Q}{C} = \dfrac{20\ \text{C}}{9.3\times10^{-9}\ \text{F}} = 2.2\times10^9\ \text{V}$.

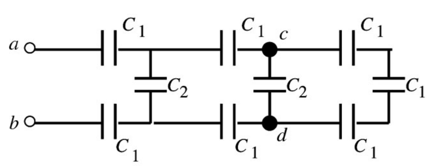
(c) $E = \dfrac{V}{d} = \dfrac{2.2\times10^9\ \text{V}}{3.0\times10^3\ \text{m}} = 7.3\times10^5\ \text{V/m}$.

(d) $U = \tfrac{1}{2}QV = \tfrac{1}{2}(20\ \text{C})(2.2\times10^9\ \text{V}) = 2.2\times10^{10}\ \text{J}$.

EVALUATE: Thunderclouds involve very large potential differences and large amounts of stored energy.

24.59. IDENTIFY: Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

(a) SET UP: The network is sketched in Figure 24.59a.

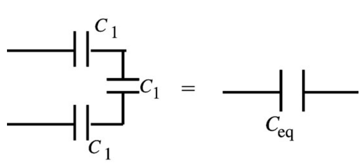


$C_1 = 6.9\ \mu\text{F}$.

$C_2 = 4.6\ \mu\text{F}$.

Figure 24.59a

EXECUTE: Simplify the network by replacing the capacitor combinations by their equivalents. Make the replacement shown in Figure 24.59b.

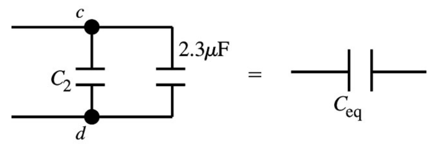


$\dfrac{1}{C_{eq}} = \dfrac{3}{C_1}$.

$C_{eq} = \dfrac{C_1}{3} = \dfrac{6.9\ \mu\text{F}}{3} = 2.3\ \mu\text{F}$.

Figure 24.59b

Next make the replacement shown in Figure 24.59c.

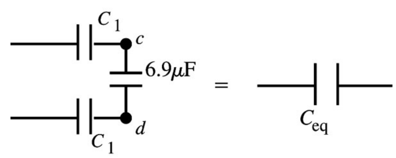


$C_{eq} = 2.3\ \mu\text{F} + C_2.$

$C_{eq} = 2.3\ \mu\text{F} + 4.6\ \mu\text{F} = 6.9\ \mu\text{F}.$

Figure 24.59c

Make the replacement shown in Figure 24.59d.

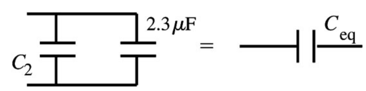


$$\frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{1}{6.9\ \mu\text{F}} = \frac{3}{6.9\ \mu\text{F}}.$$

$C_{eq} = 2.3\ \mu\text{F}.$

Figure 24.59d

Make the replacement shown in Figure 24.59e.

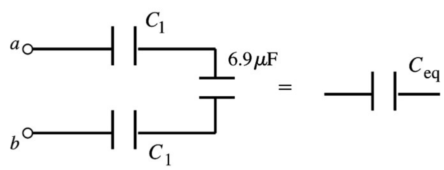


$C_{eq} = C_2 + 2.3\ \mu\text{F} = 4.6\ \mu\text{F} + 2.3\ \mu\text{F}.$

$C_{eq} = 6.9\ \mu\text{F}.$

Figure 24.59e

Make the replacement shown in Figure 24.59f.

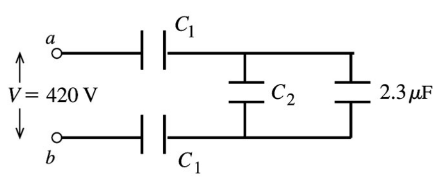


$$\frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{1}{6.9\ \mu\text{F}} = \frac{3}{6.9\ \mu\text{F}}.$$

$C_{eq} = 2.3\ \mu\text{F}.$

Figure 24.59f

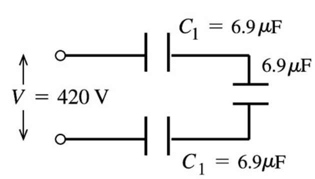
(b) SET UP and **EXECUTE**: Consider the network as drawn in Figure 24.59g.



From part (a) $2.3\ \mu\text{F}$ is the equivalent capacitance of the rest of the network.

Figure 24.59g

The equivalent network is shown in Figure 24.59h.

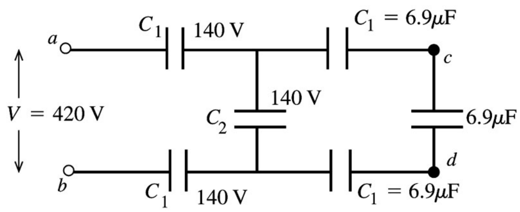


The capacitors are in series, so all three capacitors have the same Q.

Figure 24.59h

But here all three have the same C, so by $V = Q/C$ all three must have the same V. The three voltages must add to 420 V, so each capacitor has $V = 140$ V. The $6.9\ \mu\text{F}$ to the right is the equivalent of C_2 and the $2.3\ \mu\text{F}$ capacitor in parallel, so $V_2 = 140$ V. (Capacitors in parallel have the same potential difference.) Hence $Q_1 = C_1V_1 = (6.9\ \mu\text{F})(140\ \text{V}) = 9.7\times10^{-4}$ C and
$Q_2 = C_2V_2 = (4.6\ \mu\text{F})(140\ \text{V}) = 6.4\times10^{-4}$ C.

(c) From the potentials deduced in part (b) we have the situation shown in Figure 24.59i.



From part (a) $6.9\ \mu\text{F}$ is the equivalent capacitance of the rest of the network.

Figure 24.59i

The three right-most capacitors are in series and therefore have the same charge. But their capacitances are also equal, so by $V = Q/C$ they each have the same potential difference. Their potentials must sum to 140 V, so the potential across each is 47 V and $V_{cd} = 47$ V.

EVALUATE: In each capacitor network the rules for combining V for capacitors in series and parallel are obeyed. Note that $V_{cd} < V$, in fact $V - 2(140\ \text{V}) - 2(47\ \text{V}) = V_{cd}$.

24.60. **IDENTIFY:** This situation is analogous to having two capacitors C_1 in series, each with separation $\frac{1}{2}(d-a)$.

SET UP: For capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

EXECUTE: **(a)** $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_1}\right)^{-1} = \frac{1}{2}C_1 = \frac{1}{2}\dfrac{\epsilon_0 A}{(d-a)/2} = \dfrac{\epsilon_0 A}{d-a}$.

(b) $C = \dfrac{\epsilon_0 A}{d-a} = \dfrac{\epsilon_0 A}{d}\dfrac{d}{d-a} = C_0\dfrac{d}{d-a}$.

EVALUATE: **(c)** As $a \to 0$, $C \to C_0$. The metal slab has no effect if it is very thin. And as $a \to d$, $C \to \infty$. $V = Q/C$. $V = Ey$ is the potential difference between two points separated by a distance y parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large Q on the plates for a given potential difference. Since $Q = CV$ this corresponds to a very large C.

24.61. **IDENTIFY:** Capacitors in series carry the same charge, but capacitors in parallel have the same potential difference across them.

SET UP: $V_{ab} = 48.0$ V. $C = Q/V$ and $U = \dfrac{1}{2}CV^2$. For capacitors in parallel, $C = C_1 + C_2$, and for capacitors in series, $1/C = 1/C_1 + 1/C_2$.

EXECUTE: Using $U = \dfrac{1}{2}CV^2$ gives $C = \dfrac{2U}{V^2} = \dfrac{2(2.90\times10^{-3}\ \text{J})}{(48.0\ \text{V})^2} = 2.517\times10^{-6}$ F, which is the equivalent capacitance of the network. The equivalent capacitance for C_1 and C_2 in series is $C_{12} = \dfrac{1}{2}(4.00\ \mu\text{F}) = 2.00\ \mu\text{F}$. If C_{123} is the equivalent capacitance for C_{12} and C_3 in parallel, then

$\dfrac{1}{C_{123}}+\dfrac{1}{C_4}=\dfrac{1}{C}$. Solving for C_{123} gives

$\dfrac{1}{C_{123}}=\dfrac{1}{C}-\dfrac{1}{C_4}=\dfrac{1}{2.517\times10^{-6}\text{ F}}-\dfrac{1}{8.00\times10^{-6}\text{ F}}=2.722\times10^5\text{ F}^{-1}$, so $C_{123}=3.673\times10^{-6}$ F.

$C_{12}+C_3=C_{123}$. $C_3=C_{123}-C_{12}=3.673\ \mu\text{F}-2.00\ \mu\text{F}=1.67\ \mu\text{F}$.

EVALUATE: As with most circuits, it is necessary to solve them in a series of steps rather than using a single step.

24.62. **IDENTIFY:** The electric field energy density is $u=\frac{1}{2}\epsilon_0 E^2$. $U=\dfrac{Q^2}{2C}$.

SET UP: For this charge distribution, $E=0$ for $r<r_a$, $E=\dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r_a<r<r_b$ and $E=0$ for $r>r_b$.

Example 24.4 shows that $\dfrac{C}{L}=\dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

EXECUTE: **(a)** $u=\frac{1}{2}\epsilon_0 E^2=\frac{1}{2}\epsilon_0\left(\dfrac{\lambda}{2\pi\epsilon_0 r}\right)^2=\dfrac{\lambda^2}{8\pi^2\epsilon_0 r^2}$.

(b) $U=\int u\,dV=2\pi L\int ur\,dr=\dfrac{L\lambda^2}{4\pi\epsilon_0}\int_{r_a}^{r_b}\dfrac{dr}{r}$ and $\dfrac{U}{L}=\dfrac{\lambda^2}{4\pi\epsilon_0}\ln(r_b/r_a)$.

(c) $U=\dfrac{Q^2}{2C}=\dfrac{Q^2}{4\pi\epsilon_0 L}\ln(r_b/r_a)=\dfrac{\lambda^2 L}{4\pi\epsilon_0}\ln(r_b/r_a)$. This agrees with the result of part (b).

EVALUATE: We could have used the results of part (b) and $U=\dfrac{Q^2}{2C}$ to calculate C/L and would obtain the same result as in Example 24.4.

24.63. **IDENTIFY:** We are dealing with a parallel-plate capacitor.
SET UP: $E=E_0/K$ with dielectric, where $E_0=\sigma/\epsilon_0$.

EXECUTE: **(a)** We want the compressive force applied to the dielectric. $F_{\text{on }+Q}=QE_{-Q}=$ $Q\left(\dfrac{\sigma}{2\epsilon_0 K}\right)=Q\left(\dfrac{Q/A}{2\epsilon_0 K}\right)=\dfrac{Q^2}{2A\epsilon_0 K}$, and the force on $-Q$ is the same.

(b) We want the squeezing coefficient s. $d=d_0-sQ^2$, so $\Delta d=d_0-d=sQ^2$. Young's modulus is $Y=\dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$. Applied here, $\Delta\ell=\Delta d=sQ^2$ and $\ell_0=d_0$. Solving for s: $s=\dfrac{F_\perp d_0}{AQ^2Y}$. Using F from part (a) gives $s=\left(\dfrac{Q^2}{2A\epsilon_0 K}\right)\dfrac{d_0}{AQ^2Y}=\dfrac{d_0}{2A^2\epsilon_0 YK}$.

(c) Using A = 1.00 cm^2, d_0 = 0.400 mm, K = 3.00, and Y = 0.0100 GPa gives $s=7.53\times10^7$ m/C^2.

(d) We want the applied voltage. $V=Ed=\dfrac{E_0}{K}\left(d_0-sQ^2\right)=\dfrac{\sigma/\epsilon_0}{K}\left(d_0-sQ^2\right)=\dfrac{Q}{A\epsilon_0 K}\left(d_0-sQ^2\right)$.

Using Q = 0.700 μC and the other given numbers, we get V = 95.7 kV.
(e) We want the voltage if we double the charge. The same calculation but using Q = 1.40 μC leads to V = 133 kV.
EVALUATE: Doubling the charge did not double the potential.

24.64. **IDENTIFY:** The capacitor is equivalent to two capacitors in parallel, as shown in Figure 24.64.

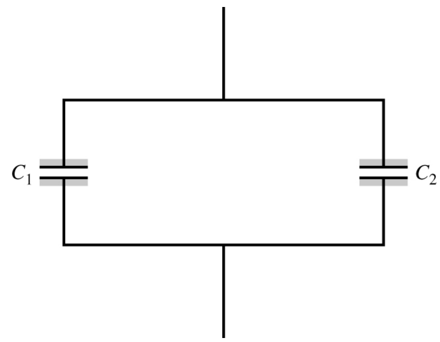


Figure 24.64

SET UP: Each of these two capacitors have plates that are 12.0 cm by 6.0 cm. For a parallel-plate capacitor with dielectric filling the volume between the plates, $C = K\epsilon_0 \dfrac{A}{d}$. For two capacitors in parallel, $C = C_1 + C_2$. The energy stored in a capacitor is $U = \tfrac{1}{2}CV^2$.

EXECUTE: **(a)** $C = C_1 + C_2$.

$$C_2 = \epsilon_0 \frac{A}{d} = \frac{(8.854\times10^{-12}\ \text{F/m})(0.120\ \text{m})(0.060\ \text{m})}{4.50\times10^{-3}\ \text{m}} = 1.42\times10^{-11}\ \text{F}.$$

$C_1 = KC_2 = (3.40)(1.42\times10^{-11}\ \text{F}) = 4.83\times10^{-11}\ \text{F}$. $C = C_1 + C_2 = 6.25\times10^{-11}\ \text{F} = 62.5\ \text{pF}$.

(b) $U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(6.25\times10^{-11}\ \text{F})(18.0\ \text{V})^2 = 1.01\times10^{-8}\ \text{J}$.

(c) Now $C_1 = C_2$ and $C = 2(1.42\times10^{-11}\ \text{F}) = 2.84\times10^{-11}\ \text{F}$.

$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(2.84\times10^{-11}\ \text{F})(18.0\ \text{V})^2 = 4.60\times10^{-9}\ \text{J}$.

EVALUATE: The plexiglass increases the capacitance and that increases the energy stored for the same voltage across the capacitor.

24.65. **IDENTIFY:** We are looking at the force between oppositely charged parallel plates.

SET UP: The plates are oppositely charged so they attract. The tension in the cable is equal to the force with which the plates attract each other.

EXECUTE: **(a)** We want the tension in the cable. The force on the upper plate is $F_{\text{upper}} = QE_{\text{lower}}$. The field between the plates is σ/ϵ_0, half of which is due to each plate, so $E_{\text{lower}} = \sigma/2\epsilon_0$. Using $\sigma = Q/A$ we get $F_{\text{electric}} = \dfrac{Q^2}{2\epsilon_0 A}$. For a parallel-plate capacitor with plates a distance z apart, $C = \epsilon_0 A/z$, where $A = \pi r^2$. $Q = CV = \dfrac{\epsilon_0 A}{z}V$, so $F_{\text{electric}} = \left(\dfrac{\epsilon_0 AV}{z}\right)\dfrac{1}{2\epsilon_0 A} = \dfrac{\pi\epsilon_0 r^2V^2}{2z^2}$, which is the tension in the cable.

(b) We want the work. $\displaystyle\int_d^{2d} F\,dz = \int_d^{2d} \frac{\pi\epsilon_0 r^2V^2}{2z^2}dz = \frac{\pi\epsilon_0 r^2V^2}{2}\left(-\frac{1}{z}\right)\Bigg|_d^{2d} = \frac{\pi\epsilon_0 r^2V^2}{4d}$.

(c) We want the initial energy stored in the electric field of the capacitor.

$$U_1 = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)V^2 = \frac{\pi\epsilon_0 r^2V^2}{2d}.$$

(d) We want the energy stored in the electric field of the capacitor after raising the plate.

$U_1 = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{2d}\right)V^2 = \frac{\pi\epsilon_0 r^2V^2}{4d}.$

EVALUATE: **(e)** Subtract the two energies, giving $\Delta U = -\frac{\pi\epsilon_0 r^2V^2}{4d}$. The work done in separating the plates is equal to the *magnitude* of the energy change in the plates. This does *not* mean that the work done is *equal* to the change in the energy stored in the plates. The work done on the plates is positive but the plates *lose* energy. The plates are connected to the battery, so the potential difference across them remains constant as they are separated. Therefore change is forced off of the plates through the battery, which does work on the battery. We have neglected gravity because it was much weaker than the electric force.

24.66. **IDENTIFY:** The system is equivalent to two capacitors in parallel. One of the capacitors has plate separation *d*, plate area $w(L-h)$ and air between the plates. The other has the same plate separation *d*, plate area *wh* and dielectric constant *K*.

SET UP: Define K_{eff} by $C_{\text{eq}} = \frac{K_{\text{eff}}\epsilon_0 A}{d}$, where $A = wL$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

EXECUTE: **(a)** The capacitors are in parallel, so $C = \frac{\epsilon_0 w(L-h)}{d} + \frac{K\epsilon_0 wh}{d} = \frac{\epsilon_0 wL}{d}\left(1 + \frac{Kh}{L} - \frac{h}{L}\right)$.

This gives $K_{\text{eff}} = \left(1 + \frac{Kh}{L} - \frac{h}{L}\right)$.

(b) For gasoline, with $K = 1.95$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \frac{L}{4}\right) = 1.24$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \frac{L}{2}\right) = 1.48$; $\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \frac{3L}{4}\right) = 1.71$.

(c) For methanol, with $K = 33$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \frac{L}{4}\right) = 9$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \frac{L}{2}\right) = 17$; $\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \frac{3L}{4}\right) = 25$.

(d) This kind of fuel tank sensor will work best for methanol since it has the greater range of K_{eff} values.

EVALUATE: When $h = 0$, $K_{\text{eff}} = 1$. When $h = L$, $K_{\text{eff}} = K$.

24.67. **IDENTIFY** and **SET UP:** For two capacitors in series, $\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{\text{eq}}}$, which gives $C_{\text{eq}} = \frac{C_1C_2}{C_1 + C_2}$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$. $C = Q/V$. The stored energy can be written as $U = \frac{1}{2}CV^2$ or $U = \frac{Q^2}{2C}$.

EXECUTE: **(a)** When connected in series, the stored energy is 0.0400 J, so

a = $\frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)V^2 = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(200.0 \text{ V})^2 = 0.0400$ J, which gives

$\frac{C_1C_2}{C_1 + C_2} = 2.00\ \mu\text{F}.$

When connected in parallel, the stored energy is 0.180 J, so

$U = \frac{1}{2}CV^2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(C_1 + C_2)(200.0 \text{ V})^2 = 0.180$ J.

$C_1 + C_2 = 9.00\ \mu$F.

Solving the two equations for C_1 and C_2 gives $C_1 = 6.00\ \mu$F and $C_2 = 3.00\ \mu$F.

(b) When the capacitors are in series, both have the same charge. The stored energy is $U = \dfrac{Q^2}{2C}$, so the capacitor with the *smaller* capacitance stores more energy, which is C_2.
(c) When the capacitors are in parallel, the potential across them is the same. The stored energy is $U = \frac{1}{2}CV^2$, so the capacitor with the *larger* capacitance stores the most energy, which is C_1.

EVALUATE: When the two capacitors are connected in parallel, they can store considerably more energy than when in series.

24.68. **IDENTIFY** and **SET UP:** The presence of the dielectric affects the charge and energy in the capacitor for a given potential difference. $V = Ed$, $Q = CV$, $K = C/C_0$, $U = \frac{1}{2}CV^2$. We use the values for K and E_{m} from Table 24.2. In this case, $E = 0.500E_{\text{m}}$ and $d = 2.50$ mm $= 0.00250$ m.

EXECUTE: **(a)** Using $U = \frac{1}{2}CV^2$, $C = KC_0$, $V = Ed$, and $E = 0.500E_{\text{m}}$, the stored energy is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}KC_0(Ed)^2 = \tfrac{1}{2}KC_0(0.500E_{\text{m}}d)^2.$$

For polycarbonate, $K = 2.8$ and $E_{\text{m}} = 3\times10^7$ V/m. Therefore the stored energy is

$U = (1/2)[(2.8)(6.00\times10^{-12}\text{ F})][(0.500)(3\times10^7\text{ V/m})(0.00250\text{ m})]^2 = 1.18\times10^{-2}\text{ J}$, which rounds to 12 mJ. Using similar calculations for the other materials, the results for U are:
12 mJ (polycarbonate)
56 mJ (polyester)
51 mJ (polypropylene)
4.9 mJ (polystyrene)
2.2 mJ (pyrex)
(b) $Q = CV = KC_0(Ed) = KC_0(0.500E_{\text{m}})d$.
For polycarbonate we have

$Q = (2.8)(6.00\times10^{-12}\text{ F})(0.500)(3\times10^7\text{ V/m})(0.00250\text{ m}) = 6.3\times10^{-7}\text{ C} = 0.63\ \mu\text{C}$.

Similar calculations for the other materials yield:
0.63 μC (polycarbonate)
1.5 μC (polyester)
1.2 μC (polypropylene)
0.39 μC (polystyrene)
0.35 μC (pyrex)
(c) $V = Ed = 0.500E_{\text{m}}d$. For polycarbonate this gives

$V = (0.500)(3\times10^7\text{ V/m})(0.00250\text{ m}) = 3.8\times10^4\text{ V} = 38\text{ kV}$.

Similar calculations for the other materials yield:
38 kV (polycarbonate)
75 kV (polyester)
88 kV (polypropylene)
25 kV (polystyrene)
13 kV (pyrex)

EVALUATE: **(d)** Polyester is best for maximum energy storage and maximum charge, but polypropylene is best for maximum voltage. No single material is best for all three categories. As so often occurs, the choice of materials is a trade-off.

24.69. **IDENTIFY** and **SET UP:** For a parallel-plate capacitor, $C = \dfrac{\epsilon_0 A}{d}$. The stored energy can be expressed as $U = \tfrac{1}{2}CV^2$ or $U = \dfrac{Q^2}{2C}$.

EXECUTE: **(a)** If the battery remains connected, V remains constant, so it is useful to write the energy in terms of V and C:

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)V^2 = \frac{\epsilon_0 AV^2}{2}\cdot\frac{1}{d}.$$

If the battery is disconnected, Q remains constant, so it is useful to write the energy in terms of Q and C:

$$U = \frac{Q^2}{2C} = \frac{Q^2}{2\left(\dfrac{\epsilon_0 A}{d}\right)} = \left(\frac{Q^2}{2\epsilon_0 A}\right)d.$$

The graph shows a linear relationship between U and $1/d$, so it must represent the case where the battery remains connected to the capacitor.

(b) In a graph of U versus $1/d$ for the equation $U = \dfrac{\epsilon_0 AV^2}{2}\cdot\dfrac{1}{d}$, the slope should be equal to $\dfrac{\epsilon_0 AV^2}{2}$.

Choosing points on the graph in the problem, the slope is $\dfrac{(73-18)\times 10^{-9}\ \text{J}}{20.0\ \text{cm}^{-1} - 5.0\ \text{cm}^{-1}} = 3.67\times 10^{-11}\ \text{J}\cdot\text{m}.$

Solving for A gives

$A = 2(\text{slope})/\epsilon_0 V^2 = 2(3.67\times 10^{-11}\ \text{J}\cdot\text{m})/(8.854\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(24.0\ \text{V})^2] = 0.014\ \text{m}^2 = 144\ \text{cm}^2.$

(c) <u>With the battery connected</u>: $U = \dfrac{\epsilon_0 AV^2}{2}\cdot\dfrac{1}{d}$, so as we increase d from 0.0500 cm to 0.400 cm, the energy *decreases* since V remains constant.

<u>With the battery disconnected</u>: $U = \left(\dfrac{Q^2}{2\epsilon_0 A}\right)d$, so as we increase d, the energy *increases* since Q does not change. Therefore there is more energy stored with the battery *disconnected* as d is increased.

EVALUATE: If this capacitor were square, its plates would be $12\ \text{cm}\times 12\ \text{cm}$. This is a reasonable size for a piece of apparatus for use in a laboratory and could easily be manufactured.

24.70. **IDENTIFY:** Two coaxial conducting shells with dielectric in the space between them form a cylindrical capacitor. We assume that there are no appreciable effects due to the part of the dielectric that is not within the region between the cylinders.

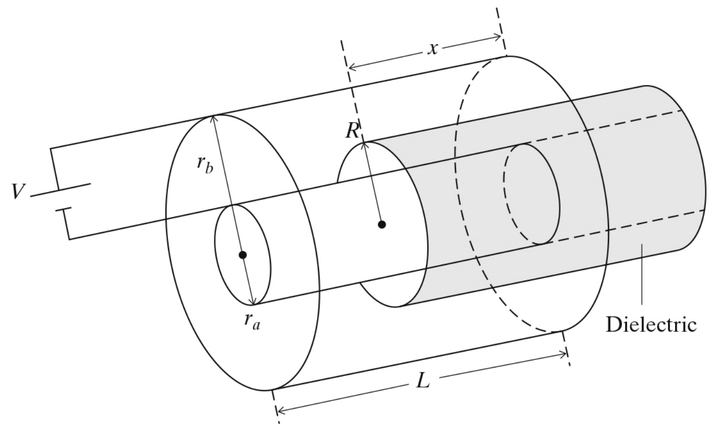


Figure 24.70a

SET UP: Refer to Fig. 24.70a. We can think of this combination as two capacitors in parallel. One capacitor is the section without dielectric and the other is the section with dielectric. They are in parallel because they share the same positive side and the same negative side. The section containing dielectric can be viewed as two cylindrical capacitors of length x in series, as shown in Fig. 24.70b. If R is the radius of the dielectric section, that part has an inner radius r_a and outer radius R. The part without dielectric has inner radius R and outer radius r_b. For a cylindrical capacitor of length ℓ with inner radius r_a and outer radius r_b, $C = \dfrac{2\pi \epsilon_0 \ell}{\ln(r_b / r_a)}$. We want the capacitance of this device.

EXECUTE: (a) The section containing no dielectric: Use $C = \dfrac{2\pi \epsilon_0 \ell}{\ln(r_b / r_a)}$ with $\ell = L - x$. Call this capacitance C_1 so $C_1 = \dfrac{2\pi \epsilon_0 (L-x)}{\ln(r_b / r_a)}$.

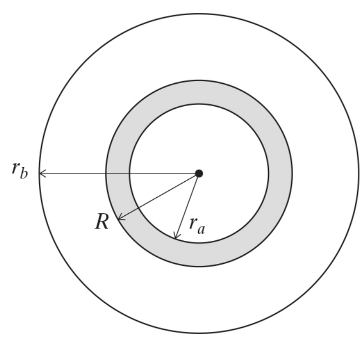


Figure 24.70b

The section partially filled with dielectric: Refer to Fig. 24.70b. Call the inner part (radii r_a and R) C_{aR}. Applying $C = \dfrac{2\pi \epsilon_0 \ell}{\ln(r_b / r_a)}$ with $\ell = x$ and allowing for the dielectric, we have $C_{aR} = \dfrac{K 2\pi \epsilon_0 x}{\ln(R / r_a)}$. Call the outer part (radii R and r_b) C_{Rb}. Applying the same formula gives $C_{Rb} = \dfrac{2\pi \epsilon_0 x}{\ln(r_b / R)}$. Call C_2 the equivalent capacitance of C_{aR} and C_{Rb}. Since they are in series, their equivalent capacitance is given by $\dfrac{1}{C_2} = \dfrac{1}{C_{aR}} + \dfrac{1}{C_{Rb}} = \dfrac{\ln(R / r_a)}{K 2\pi \epsilon_0 x} + \dfrac{\ln(r_b / R)}{2\pi \epsilon_0 x}$. This gives $C_2 = \dfrac{2\pi \epsilon_0 x}{\dfrac{\ln(R / r_a)}{K} + \ln(r_b / R)}$. Since C_1 and C_2 are in

series, the equivalent capacitance of this device is $C = C_1 + C_2$, which gives
$$C = \frac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)} + \frac{2\pi\epsilon_0 x}{\dfrac{\ln(R/r_a)}{K} + \ln(r_b/R)}.$$

(b) We want the equivalent capacitance when $x = 0$. Using our result from part (a) gives
$C = \dfrac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)}$. We recognize this as an ordinary air-filled cylindrical capacitor of length $L-x$.

Using $L = 10.0$ cm, $r_a = 1.00$ cm, $r_b = 4.00$ cm, and $K = 3.21$, we get $C = 4.01$ pF.

(c) We want C when $x = L$. Using the result from (a) with $K = 3.21$ gives $C = 8.83$ pF.

(d) We want x. First find C. $C = Q/V = (6.00\text{ nC})/(1.00\text{ kV}) = 6.00$ pF. Use our result from (a) and solve for x: $C = \dfrac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)} + \dfrac{2\pi\epsilon_0 x}{\dfrac{\ln(R/r_a)}{K} + \ln(r_b/R)} = 6.00$ pF. The result is $x = 4.12$ cm.

EVALUATE: A device like this one could be used to make a variable capacitor that one could easily vary as desired simply by sliding the dielectric in and out.

24.71. IDENTIFY: This problem involves a capacitor with dielectric inside.

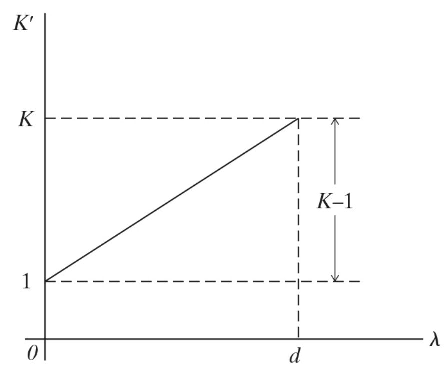


Figure 24.71a

SET UP: The dielectric constant K' is not uniform within the capacitor. We first need to find an equation for K'. At the first plate, the $K' = 1$, at the second plate $K' = K$, and it varies linearly. Call x the distance of any point from the first plate. Using this information, sketch the graph of K' versus x, as shown in Fig. 24.71a. Using the slope-intercept form, we see that the slope of this line is $(K-1)/d$ and the x-intercept is 1. The equation is $K' = \left(\dfrac{K-1}{d}\right)x + 1$.

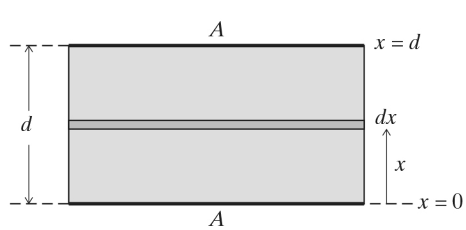


Figure 24.71b

EXECUTE: (a) We want to find the capacitance of this device. Since K' depends only on the distance x from the first plate, break the dielectric into thin slabs of area A and thickness dx as shown in Fig.

24.71b. Using the equation $C=\frac{\epsilon_0 AK}{d}$, the capacitance of a single slab is $dC=\frac{\epsilon_0 AK}{dx}$. These slabs are all in series with each other, so we apply the equation $1/C_{eq} = 1/C_1 + 1/C_2 + \ldots$ But in this case each capacitance to be added is infinitesimal, so we must integrate to get the sum. Doing so gives $\frac{1}{C}=\int\frac{1}{dC}=\int_0^d\frac{dx}{K'\epsilon_0 A}=\frac{1}{\epsilon_0 A}\int_0^d\frac{dx}{1+(K-1)x/d}=\frac{d}{\epsilon_0 A(K-1)}\ln\left[1+(K-1)x/d\right]\Big|_0^d$. Evaluating at the two limits and rearranging gives $C=\frac{\epsilon_0 A(K-1)}{d\ln K}$.

(b) We want to find C when $K = 1$. Putting $K = 1$ into our result gives $\frac{0}{0}$, which is indeterminate. So to evaluate the limit, we need to use L'Hopital's rule, which gives
$C=\lim_{K\to 1}\left[\frac{\epsilon_0 A(K-1)}{d\ln K}\right]=\lim_{K\to 1}\left[\frac{\epsilon_0 A}{d}\frac{d(K-1)/dK}{d(\ln K)/dK}\right]=\left(\frac{\epsilon_0 A}{d}\right)\left(\frac{1}{1/K}\right)=\frac{K\epsilon_0 A}{d}$. We recognize this result as the capacitance of a parallel-plate capacitor filled with uniform material having dielectric constant K.

EVALUATE: It would be plausible to construct a capacitor like this by filling the gap between the plates with thin layers of material have a progressively larger value of K.

24.72. **IDENTIFY:** The system can be considered to be two capacitors in parallel, one with plate area $L(L-x)$ and air between the plates and one with area Lx and dielectric filling the space between the plates.

SET UP: $C=\frac{K\epsilon_0 A}{d}$ for a parallel-plate capacitor with plate area A.

EXECUTE: **(a)** $C=\frac{\epsilon_0}{D}\left[(L-x)L+xKL\right]=\frac{\epsilon_0 L}{D}\left[L+(K-1)x\right]$.

(b) $dU=\frac{1}{2}(dC)V^2$, where $C=C_0+\frac{\epsilon_0 L}{D}(-dx+dxK)$, with $C_0=\frac{\epsilon_0 L}{D}\left[L+(K-1)x\right]$. This gives
$dU=\frac{1}{2}\left(\frac{\epsilon_0 Ldx}{D}(K-1)\right)V^2=\frac{(K-1)\epsilon_0 V^2 L}{2D}dx.$

(c) If the charge is kept constant on the plates, then $Q=\frac{\epsilon_0 LV}{D}\left[L+(K-1)x\right]$ and
$U=\frac{1}{2}CV^2=\frac{1}{2}C_0V^2\left(\frac{C}{C_0}\right)$. $U\approx\frac{C_0V^2}{2}\left(1-\frac{\epsilon_0 L}{DC_0}(K-1)dx\right)$ and $\Delta U=U-U_0=-\frac{(K-1)\epsilon_0 V^2 L}{2D}dx.$

(d) Since $dU=-Fdx=-\frac{(K-1)\epsilon_0 V^2 L}{2D}dx$, the force is in the opposite direction to the motion dx, meaning that the slab feels a force pushing it out.

EVALUATE: **(e)** When the plates are connected to the battery, the plates plus slab are not an isolated system. In addition to the work done on the slab by the charges on the plates, energy is also transferred between the battery and the plates. Comparing the results for dU in part (c) to $dU=-Fdx$ gives
$F=\frac{(K-1)\epsilon_0 V^2 L}{2D}.$

24.73. **IDENTIFY** and **SET UP:** The potential difference is V = 30 mV – (–70 mV) = 100 mV, and $Q = CV$.
EXECUTE: $Q = CV$ gives $Q/\text{cm}^2 = (C/\text{cm}^2)V = (1\ \mu\text{F/cm}^2)(100\ \text{mV})(1\ \text{mol}/10^5\ \text{C}) = 10^{-12}\ \text{mol/cm}^2$, which is choice (c).
EVALUATE: This charge produces a potential difference of 100 mV = 0.1 V, which is certainly measurable using ordinary laboratory meters.

24.74. **IDENTIFY** and **SET UP:** The change in concentration of Na^+ ions is equal to the added charge divided by the volume of the spherical egg. The original concentration of ions is given as 30 mmol/L. We use the answer from Problem 24.73 to get the added charge.

EXECUTE: The added charge is (10^{-12} mol/cm^2)(surface area of egg) = (10^{-12} mol/cm^2)($4\pi R^2$), and the original volume of the egg is $(4\pi/3)R^3$. Therefore the change in concentration is

$$(10^{-12}\ \text{mol/cm}^2)(4\pi R^2)/\left[(4\pi/3)R^3\right] = 3(10^{-12}\ \text{mol/cm}^2)/R = 3(10^{-12}\ \text{mol/cm}^2)/(100\times10^{-4}\,\text{cm}) =$$

$$3\times10^{10}\ \text{mol/cm}^3 = 3\times10^{-5}\ \text{mmol/L}.$$

The fractional change in the concentrations is $(3\times10^{-5}\ \text{mmol/L})/(30\ \text{mmol/L}) = 10^{-5}$, which is 1 part in 10^5. Therefore choice (b) is correct.

EVALUATE: As a percent, this change is 10^{-3}% = 0.001%, which is quite small yet certainly important for the organism.

24.75. **IDENTIFY** and **SET UP:** The calcium Ca^{2+} ions carry twice the charge of the Na^+ ions.

EXECUTE: The charge to produce the given voltage change would be the same as with Na^+, so we would need only half as many Ca^{2+} ions to accomplish this. Thus choice (a) is correct.

EVALUATE: Ca^{+2} ions are nearly twice as heavy as Na^+ ions, so they may not move as readily as the sodium ions.

24.76. **IDENTIFY** and **SET UP:** The energy is needed to change the potential from 30 mV to –70 mV. $U = \frac{1}{2}CV^2$. The capacitance is (1 μF/cm^2)(surface area of egg).

EXECUTE: For a spherical egg, the surface area is $4\pi R^2$, so the capacitance is

$$C = (1\ \mu\text{F/cm}^2)(4\pi R^2) = (1\ \mu\text{F/cm}^2)(4\pi)(100\times10^{-4}\ \text{cm})^2 = 1.26\times10^{-9}\ \text{F}.$$

The change in stored energy is

$$\Delta U = \tfrac{1}{2}CV_2^2 - \tfrac{1}{2}CV_1^2 = \tfrac{1}{2}C(V_2^2 - V_1^2).$$

$\Delta U = (1/2)(1.26\times10^{-9}\ \text{F})[(-70\times10^{-3}\ \text{V})^2 - (30\times10^{-3}\ \text{V})^2] = 2.5\times10^{-12}\ \text{J} = 2.5\ \text{pJ} \approx 3\ \text{pJ}$, which makes choice (d) the correct one.

EVALUATE: The actual energy required would probably be greater than 2.5 pJ, depending on the process by which the charging is accomplished, but our value is the minimum energy needed.

25

Current, Resistance, and Electromotive Force

VP25.3.1. **Identify:** We are looking at resistivity, resistance, and the electric field in a wire.

Set Up: $R = \dfrac{\rho L}{A}$, $V = EL$, $V = RI$.

Execute: **(a)** We want the electric field. $E = V/L = (1.45\text{ V})/(35.0\text{ m}) = 0.0414\text{ V/m}$.

(b) The resistance is the target variable. $R = V/I = (1.45\text{ V})/(1.20\text{ A}) = 1.21\ \Omega$.

(c) We want the cross-sectional area of the wire. Solve $R = \dfrac{\rho L}{A}$ for A, giving $A = \dfrac{\rho L}{R}$. Using the given numbers we get $A = 7.97\times10^{-7}\ \text{m}^2$.

Evaluate: Copper has a resistivity of 1.72×10^{-8} ž·m, so if this wire were made of copper, its resistance would be $[(1.72)/(2.75)](1.21\ \Omega) = 0.756\ \Omega$, which is roughly 60% that of the aluminum wire.

VP25.3.2. **Identify:** We are looking at resistivity and the resistance of a wire.

Set Up: $R = \dfrac{\rho L}{A}$, $V = EL$, $V = RI$.

Execute: **(a)** We want the resistance. Combine $V = EL$ and $V = RI$ to get $R = EL/I = 0.596\ \Omega$ using the given values.

(b) Resistivity is the target variable. Solve $R = \dfrac{\rho L}{A}$ for ρ, giving $\rho = \dfrac{RA}{L} = 2.44\times10^{-8}$ ž·m using the given or calculated values for R, A, and L.

Evaluate: According to Table 25.1, this resistivity is the same as that of gold. These wires must be really special!

VP25.3.3. **Identify:** We want to compare the electrical characteristics of two wires.

Set Up: Wire 1: radius r and length L. Wire 2: radius $2r$, length $2L$. The potential difference is the same across both wires.

Execute: **(a)** We want R_2/R_1. Use $R = \dfrac{\rho L}{A}$ and take the ratio of the resistances, which gives

$$\frac{R_2}{R_1} = \frac{\rho L_2 / A_2}{\rho L_1 / A_1} = \frac{L_2}{L_1}\frac{A_1}{A_2} = \frac{L_2}{L_1}\left(\frac{r_1}{r_2}\right)^2 = \frac{2L}{L}\left(\frac{r}{2r}\right)^2 = \frac{1}{2}.$$

(b) We want I_2/I_1. Use $I = V/R$ and take the ratio. $\dfrac{I_2}{I_1} = \dfrac{V/R_2}{V/R_1} = \dfrac{R_1}{R_2} = 2.$

(c) We want J_2/J_1. Use $J = I/A$ and take the ratio. $\dfrac{J_2}{J_1} = \dfrac{I_2/(\pi r_2^2)}{I_1/(\pi r_1^2)} = \dfrac{I_2}{I_1}\left(\dfrac{r_1}{r_2}\right)^2 = 2\left(\dfrac{r}{2r}\right)^2 = \dfrac{1}{2}.$

(d) We want E_2/E_1. Use $E = V/L$ and take the ratio. $\frac{E_2}{E_1} = \frac{V/L_2}{V/L_1} = \frac{L_1}{L_2} = \frac{L}{2L} = \frac{1}{2}$.

EVALUATE: As should now be clear, the *geometry* of a wire affects its electrical characteristics.

VP25.3.4. **IDENTIFY:** We are investigating the effect of temperatures on the resistance of a wire.

SET UP: $\rho(T) = \rho_0[1+\alpha(T-T_0)]$ so $R(T) = R_0[1+\alpha(T-T_0)]$. $V = RI$.

EXECUTE: **(a)** We want to find R at 20.0°C and 80.0°C.

At 20.0°C: $R = V/I$ = (1.50 V)/(2.40 A) = 0.625 Ω.

At 80.0°C: $R = V/I$ = (1.50 V)/(2.00 A) = 0.750 Ω.

(b) The target variable is α. Call T_0 = 20.0°C and T = 80.0°C and use $R(T) = R_0[1+\alpha(T-T_0)]$.

0.750 Ω = (0.625 Ω)$[1+\alpha(80.0°\text{C} - 20.0°\text{C})]$. α = 0.00333 (C°)$^{-1}$.

EVALUATE: As we see, resistance can vary considerably as the temperature changes.

VP25.5.1. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.

SET UP and EXECUTE: **(a)** We want the current. $I = \frac{\mathcal{E}}{R} = \frac{9.00\text{ V}}{15.3\ \Omega + 1.10\ \Omega} = 0.549\text{ A}$.

(b) We want V across the resistor. $V = RI$ = (15.3 Ω)(0.549 A) = 8.40 V.

(c) We want the terminal voltage. $\mathcal{E}_{\text{terminal}} = \mathcal{E} - rI =$ 9.00 V – (1.10 Ω)(0.549 A) = 8.40 V.

EVALUATE: The terminal voltage (8.40 V) is less than the internal emf (9.00 V) because there is a potential drop across the internal resistance.

VP25.5.2. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.

SET UP and EXECUTE: **(a)** We want the potential difference across the resistor.

$V = RI$ = (25.0 Ω)(0.480 A) = 12.0 V.

(b) We want the internal resistance. $\mathcal{E} = (R+r)I \quad \rightarrow \quad r = \frac{\mathcal{E}}{I} - R = \frac{12.6\text{ V}}{0.480\text{ A}} - 25.0\ \Omega = 1.3\ \Omega$.

EVALUATE: An internal resistance of 1.3 Ω is large enough to affect a circuit containing only small resistors.

VP25.5.3. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.

SET UP and EXECUTE: **(a)** We want the current. $I = \frac{\mathcal{E}_{\text{terminal}}}{R}$ = (1.30 V)/(20.0 Ω) = 0.0650 A.

(b) We want the internal resistance. $\mathcal{E} - \mathcal{E}_{\text{terminal}} = rI$. 1.50 V – 1.30 V = (0.0650 A)r. r = 3.1 Ω.

EVALUATE: An internal resistance of 3.1 Ω is large enough to considerably affect a circuit containing only small resistors.

VP25.5.4. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.

SET UP and EXECUTE: **(a)** We want the resistance. $\mathcal{E} = rI + RI$ so $R = \frac{\mathcal{E} - rI}{I}$. This gives

$$R = \frac{1.50\text{ V} - (0.160\ \Omega)(2.00\text{ A})}{2.00\text{ A}} = 0.590\ \Omega.$$

(b) We want the internal resistance. $\mathcal{E}_{\text{terminal}} = \mathcal{E} - rI$ = 1.50 V – (0.160 Ω)(2.00 A) = 1.18 V.

EVALUATE: A resistance smaller than 0.590 Ω would draw a current greater than 2.00 A which would run down the battery.

VP25.9.1. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.

SET UP and EXECUTE: **(a)** We want the current. $\mathcal{E} = rI + RI$. $I = \frac{\mathcal{E}}{R+r} = \frac{24.0\text{ V}}{18.0\ \Omega + 1.30\ \Omega} = 1.24\text{ A}$.

(b) We want the rate of energy conversion in the battery, which is the power. $P = I\mathcal{E}$ which gives P = (1.24 A)(24.0 V) = 29.8 W.
(c) We want the rate of energy dissipation in the battery. This takes place in the internal resistance. $P_{\text{int}} = I^2 r$ = (1.24 A)^2(1.30 Ω) = 2.01 W.
(d) We want the net power output of the battery. $P_{\text{net}} = P - P_{\text{int}}$ = 29.8 W – 2.01 W = 27.8 W.
(e) We want the power in the 18.0 ohm resistor. $P_{18} = I^2 R$ = (1.24 A)^2(18.0 Ω) = 27.8 W.
EVALUATE: Note that the net power output of the battery is equal to the power dissipated in the 18.0 ohm resistor. This result is consistent with the conservation of energy.

VP25.9.2. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.
SET UP and **EXECUTE:** **(a)** We want the current. Solve $P = I^2 R$ for I. $I = \sqrt{P/R}$ = 0.738 W using the given values.
(b) We want the net power output of the battery. This is the power dissipated in the resistor, which is given as P = 6.54 W.
(c) Target variable is the terminal voltage. $\mathcal{E}_{\text{terminal}} = RI$ = (0.738 A)(12.0 Ω) = 8.86 V.
(d) Target variable is the internal resistance. $\mathcal{E} = (R + r)I. r = \dfrac{\mathcal{E}}{I} - R.$ Using the numbers gives

$$r = \frac{9.00\text{ V}}{0.738\text{ A}} - 12.0\ \Omega = 0.191\ \Omega,\ \text{which rounds to } 0.2\ \Omega.$$

EVALUATE: The battery gives 6.54 W of power to the 12.0 Ω resistor, but the internal resistance dissipates energy at a rate of $I^2 r$ = (0.738 A)^2(0.191 Ω) = 0.10 W. This power is small compared to the power in the 12.0 Ω resistor.

VP25.9.3. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.
SET UP and **EXECUTE:** **(a)** We want the terminal voltage. $\mathcal{E}_{\text{terminal}} = RI$ = (16.0 Ω)(0.720 A) = 11.5 V.
(b) We want the internal resistance. $\mathcal{E} = (R + r)I. r = \dfrac{\mathcal{E}}{I} - R.$ $r = \dfrac{12.0\text{ V}}{0.720\text{ A}} - 16.0\ \Omega = 0.667\ \Omega,$ which rounds to 0.7 Ω.
(c) The target variable is the power in the resistor. $P = I^2 R$ = (0.720 A)^2(16.0 Ω) = 8.29 W.
(d) We want the power generated in the battery. $P = I\mathcal{E} =$ (0.720 A)(12.0 V) = 8.64 W.
(e) We want the rate of energy dissipation within the battery. This power is due to the internal resistance, so $P = I^2 r$ = (0.720 A)^2(0.667 Ω) = 0.346 W, which rounds to 0.3 W.
EVALUATE: The internal resistance is much smaller than the external resistance so most of the power is dissipated in the external resistor.

VP25.9.4. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.
SET UP and **EXECUTE:** **(a)** We want I_2/I_1.

<u>First circuit</u>: $\mathcal{E} = (R_1 + r)I_1. I_1 = \dfrac{\mathcal{E}}{R_1 + r} = \dfrac{\mathcal{E}}{R + r}.$

<u>Second circuit</u>: $\mathcal{E} = (R_2 + r)I_2. I_2 = \dfrac{\mathcal{E}}{R_2 + r} = \dfrac{\mathcal{E}}{2R + r}.$

$$\frac{I_2}{I_1} = \frac{\dfrac{\mathcal{E}}{2R + r}}{\dfrac{\mathcal{E}}{R + r}} = \frac{R + r}{2R + r}.$$

(b) We want P_2/P_1.

<u>First circuit</u>: $P_1 = I_1^2 R_1 = \left(\dfrac{\mathcal{E}}{R + r}\right)^2 R.$

<u>Second circuit</u>: $P_2 = I_2^2 R_2 = \left(\dfrac{\mathcal{E}}{2R + r}\right)^2 2R.$

$$\frac{P_2}{P_1}=\frac{\left(\frac{\mathcal{E}}{2R+r}\right)^2 2R}{\left(\frac{\mathcal{E}}{R+r}\right)^2 R}=2\left(\frac{R+r}{2R+r}\right)^2.$$

(c) We want to compare the currents and the power dissipation. $\frac{I_2}{I_1}=\frac{R+r}{2R+r}$. Since $2R+r>R+r$, $\frac{R+r}{2R+r}<1$, so the current is greater in R_1. $\frac{P_2}{P_1}=2\left(\frac{R+r}{2R+r}\right)^2$. Since $R>r$, $0<r\le R$. If $r=R$, $\frac{P_2}{P_1}=2\left(\frac{2R}{3R}\right)^2=\frac{8}{9}$, so $P_1>P_2$. If $r=0$, $\frac{P_2}{P_1}=2\left(\frac{R}{2R}\right)^2=\frac{1}{2}$, so $P_1>P_2$. In all cases, $P_1>P_2$, so the power dissipation is greatest in R_1.

EVALUATE: Our result tells us that increase the resistance decreases the current (which is reasonable) and decreases the power dissipation (which is also reasonable).

25.1. IDENTIFY and SET UP: The lightning is a current that lasts for a brief time. $I=\frac{\Delta Q}{\Delta t}$.

EXECUTE: $\Delta Q=I\Delta t=(25{,}000\text{ A})(40\times10^{-6}\text{ s})=1.0\text{ C}$.

EVALUATE: Even though it lasts for only 40 μs, the lightning carries a huge amount of charge since it is an enormous current.

25.2. IDENTIFY: $I=Q/t$. Use $I=n|q|v_\text{d}A$ to calculate the drift velocity v_d.

SET UP: $n=5.8\times10^{28}\text{ m}^{-3}$. $|q|=1.60\times10^{-19}\text{ C}$.

EXECUTE: **(a)** $I=\frac{Q}{t}=\frac{420\text{ C}}{80(60\text{ s})}=8.75\times10^{-2}\text{ A}$.

(b) $I=n|q|v_\text{d}A$. This gives $v_\text{d}=\frac{I}{n|q|A}=\frac{8.75\times10^{-2}\text{ A}}{(5.8\times10^{28})(1.60\times10^{-19}\text{ C})(\pi(1.3\times10^{-3}\text{ m})^2)}=1.78\times10^{-6}\text{ m/s}$.

EVALUATE: v_d is smaller than in Example 25.1, because I is smaller in this problem.

25.3. IDENTIFY: $I=Q/t$. $J=I/A$. $J=n|q|v_\text{d}$.

SET UP: $A=(\pi/4)D^2$, with $D=2.05\times10^{-3}$ m. The charge of an electron has magnitude $+e=1.60\times10^{-19}$ C.

EXECUTE: **(a)** $Q=It=(5.00\text{ A})(1.00\text{ s})=5.00\text{ C}$. The number of electrons is $\frac{Q}{e}=3.12\times10^{19}$.

(b) $J=\frac{I}{(\pi/4)D^2}=\frac{5.00\text{ A}}{(\pi/4)(2.05\times10^{-3}\text{ m})^2}=1.51\times10^6\text{ A/m}^2$.

(c) $v_\text{d}=\frac{J}{n|q|}=\frac{1.51\times10^6\text{ A/m}^2}{(8.5\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})}=1.11\times10^{-4}\text{ m/s}=0.111\text{ mm/s}$.

EVALUATE: **(d)** If I is the same, $J=I/A$ would decrease and v_d would decrease. The number of electrons passing through the light bulb in 1.00 s would not change.

25.4. (a) IDENTIFY: By definition, $J=I/A$ and radius is one-half the diameter.

SET UP: Solve for the current: $I=JA=J\pi(D/2)^2$

EXECUTE: $I = (3.20\times10^6\ \text{A/m}^2)(\pi)[(0.00102\ \text{m})/2]^2 = 2.61\ \text{A}.$

EVALUATE: This is a realistic current.

(b) IDENTIFY: The current density is $J = n|q|v_\text{d}$.

SET UP: Solve for the drift velocity: $v_\text{d} = J/n|q|$

EXECUTE: We use the value of *n* for copper, giving

$v_\text{d} = (3.20\times10^6\ \text{A/m}^2)/[(8.5\times10^{28}/\text{m}^3)(1.60\times10^{-19}\ \text{C})] = 2.4\times10^{-4}\ \text{m/s} = 0.24\ \text{mm/s}.$

EVALUATE: This is a typical drift velocity for ordinary currents and wires.

25.5. **IDENTIFY:** This problem is about the drift speed in a current-carrying wire.

SET UP: $\rho = E/J$ and $J = nqv_\text{d}$.

EXECUTE: **(a)** The drift velocity is the target variable. Combine $\rho = E/J$ and $J = nqv_\text{d}$. $J = E/\rho$ = nqv_d gives $v_\text{d} = \dfrac{E}{nq\rho} = \dfrac{E}{ne\rho}$. Using the given numbers gives v_d = 0.26 mm/s.

(b) We want the potential difference. $V = Ed$ = (0.0600 N/C)(0.200 m) = 0.0120 N/C.

EVALUATE: Note how small v_d, *E*, and *V* are for metals.

25.6. **IDENTIFY:** The resistance depends on the length, cross-sectional area, and material of the wires.

SET UP: $R = \dfrac{\rho L}{A}$, $A = \pi r^2 = d^2/4$. The resistivities come from Table 25.1.

EXECUTE: **(a)** Combining $R = \dfrac{\rho L}{A}$ and $A = \pi d^2/4$, gives $R = \dfrac{\rho L}{\frac{\pi}{4}d^2} = \dfrac{4\rho L}{\pi d^2}$. Solving for *L* gives

$L = \dfrac{R\pi d^2}{4\rho}$. Using this formula gives the length of each type of metal.

<u>Gold</u>: $L = \dfrac{(1.00\ \Omega)\pi(1.00\times10^{-3}\ \text{m})^2}{4(2.44\times10^{-8}\ \Omega\cdot\text{m})} = 32.2\ \text{m}.$

<u>Copper</u>: Using $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$ we get L = 45.7 m.

<u>Aluminum</u>: Using $\rho = 2.75\times10^{-8}\ \Omega\cdot\text{m}$, we get L = 28.6 m.

(b) The mass of the gold is the product of its mass density and its volume, so

$m = (density)(\pi d^2/4)L = (1.93\times10^4\ \text{kg/m}^3)\pi(1.00\times10^{-3}\ \text{m})^2(32.2\ \text{m})/4 = 0.488\ \text{kg} = 488\ g.$

If gold is currently worth \$40 per gram, the cost of the gold wire would be (\$40/g)(488 g) = \$19,500. At this price, you wouldn't want to wire your house with gold wires!

EVALUATE: The resistivities of the three metals are all fairly close to each other, so it is reasonable to expect that the lengths of the wires would also be fairly close to each other, which is just what we find.

25.7. **IDENTIFY** and **SET UP:** Apply $I = \dfrac{dQ}{dt}$ to find the charge dQ in time dt. Integrate to find the total charge in the whole time interval.

EXECUTE: **(a)** $dQ = I\,dt$.

$$Q = \int_0^{8.0\text{s}} (55\ \text{A} - (0.65\ \text{A/s}^2)t^2)dt = \Big[(55\ \text{A})t - (0.217\ \text{A/s}^2)t^3\Big]_0^{8.0\ \text{s}}.$$

$Q = (55\ \text{A})(8.0\ \text{s}) - (0.217\ \text{A/s}^2)(8.0\ \text{s})^3 = 330\ \text{C}.$

(b) $I = \dfrac{Q}{t} = \dfrac{330\ \text{C}}{8.0\ \text{s}} = 41\ \text{A}.$

EVALUATE: The current decreases from 55 A to 13.4 A during the interval. The decrease is not linear and the average current is not equal to $(55\text{A} + 13.4\ \text{A})/2$.

25.8. IDENTIFY: $I = Q/t$. Positive charge flowing in one direction is equivalent to negative charge flowing in the opposite direction, so the two currents due to Cl^- and Na^+ are in the same direction and add.

SET UP: Na^+ and Cl^- each have magnitude of charge $|q| = +e$.

EXECUTE: **(a)** $Q_{total} = (n_{Cl} + n_{Na})e = (3.92\times10^{16} + 2.68\times10^{16})(1.60\times10^{-19}\text{ C}) = 0.0106\text{ C}$. Then $I = \dfrac{Q_{total}}{t} = \dfrac{0.0106\text{ C}}{1.00\text{ s}} = 0.0106\text{ A} = 10.6\text{ mA}$.

(b) Current flows, by convention, in the direction of positive charge. Thus, current flows with Na^+ toward the negative electrode.

EVALUATE: The Cl^- ions have negative charge and move in the direction opposite to the conventional current direction.

25.9. IDENTIFY and SET UP: The number of ions that enter gives the charge that enters the axon in the specified time. $I = \dfrac{\Delta Q}{\Delta t}$.

EXECUTE: $\Delta Q = (5.6\times10^{11}\text{ ions})(1.60\times10^{-19}\text{ C/ion}) = 9.0\times10^{-8}\text{ C}$. $I = \dfrac{\Delta Q}{\Delta t} = \dfrac{9.0\times10^{-8}\text{ C}}{10\times10^{-3}\text{ s}} = 9.0\ \mu\text{A}$.

EVALUATE: This current is much smaller than household currents but are comparable to many currents in electronic equipment.

25.10. IDENTIFY: This problem deals with free-electron density.

SET UP and EXECUTE: First find the number of silver atoms per cubic meter, then use that to get the number of free electrons per cubic meter.

$$n = \left(10.5\times10^3\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{1\text{ mol}}{108\text{ g}}\right)\left(\frac{1000\text{ g}}{1\text{ kg}}\right)\left(\frac{6.022\times10^{23}\text{atoms}}{1\text{ mol}}\right)\left(\frac{1\text{ free electron}}{1\text{ atom}}\right) = 5.85\times10^{28}\frac{\text{el}}{\text{m}^3}.$$ Now

compare this result to the value in Example 25.1. $n_{Ag}/n_{Cu} = 5.85/8.5 = 0.69$, so n_{Ag} is $0.69n_{Cu}$.

EVALUATE: Copper has more free electrons per cubic meter than silver does even though silver is denser than copper.

25.11. IDENTIFY: We want to find the resistivity of the metal.

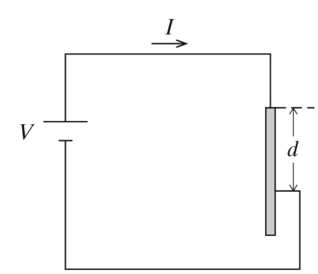


Figure 25.11

SET UP and EXECUTE: First sketch the circuit as in Fig. 25.11. Combine $I = V/R$ and $R = \dfrac{\rho d}{A}$ to relate I to $1/d$. $I = \dfrac{V}{R} = \dfrac{V}{\rho d / A} = \left(\dfrac{VA}{\rho}\right)\dfrac{1}{d}$. A graph of I versus $1/d$ should be a straight line having slope equal to VA/ρ. Thus $\rho = \dfrac{VA}{\text{slope}} = \dfrac{(12.0\text{ V})\pi(0.800\text{ mm})^2}{600\text{ A}\cdot\text{m}} = 4.02\times10^{-8}\ \Omega\cdot\text{m}$.

EVALUATE: From Table 25.1 we see that this resistivity is between that of aluminum and tungsten, so our result is physically reasonable.

25.12. **(a) IDENTIFY:** Start with the definition of resistivity and solve for E.

SET UP: $E = \rho J = \rho I/\pi r^2$.

EXECUTE: $E = (1.72\times10^{-8}\ \Omega\cdot\text{m})(4.50\ \text{A})/[\pi(0.001025\ \text{m})^2] = 2.345\times10^{-2}\ \text{V/m}$, which rounds to 0.0235 V/m.

EVALUATE: The field is quite weak, since the potential would drop only a volt in 43 m of wire.

(b) IDENTIFY: Take the ratio of the field in silver to the field in copper.

SET UP: Take the ratio and solve for the field in silver: $E_S = E_C(\rho_S/\rho_C)$.

EXECUTE: $E_S = (0.02345\ \text{V/m})[(1.47)/(1.72)] = 2.00\times10^{-2}\ \text{V/m}$.

EVALUATE: Since silver is a better conductor than copper, the field in silver is smaller than the field in copper.

25.13. **IDENTIFY:** First use Ohm's law to find the resistance at 20.0°C; then calculate the resistivity from the resistance. Finally use the dependence of resistance on temperature to calculate the temperature coefficient of resistance.

SET UP: Ohm's law is $R = V/I$, $R = \rho L/A$, $R = R_0[1+\alpha(T-T_0)]$, and the radius is one-half the diameter.

EXECUTE: **(a)** At 20.0°C, $R = V/I = (15.0\ \text{V})/(18.5\ \text{A}) = 0.811\ \Omega$. Using $R = \rho L/A$ and solving for ρ gives $\rho = RA/L = R\pi(D/2)^2/L = (0.811\ \Omega)\pi[(0.00500\ \text{m})/2]^2/(1.50\ \text{m}) = 1.06\times10^{-5}\ \Omega\cdot\text{m}$.

(b) At 92.0°C, $R = V/I = (15.0\ \text{V})/(17.2\ \text{A}) = 0.872\ \Omega$. Using $R = R_0[1+\alpha(T-T_0)]$ with T_0 taken as 20.0°C, we have $0.872\ \Omega = (0.811\ \Omega)[1+\alpha(92.0°\text{C} - 20.0°\text{C})]$. This gives $\alpha = 0.00105\ (\text{C}°)^{-1}$.

EVALUATE: The results are typical of ordinary metals.

25.14. **IDENTIFY:** $E = \rho J$, where $J = I/A$. The drift velocity is given by $I = n|q|v_d A$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $n = 8.5\times10^{28}/\text{m}^3$.

EXECUTE: **(a)** $J = \dfrac{I}{A} = \dfrac{3.6\ \text{A}}{(2.3\times10^{-3}\ \text{m})^2} = 6.81\times10^5\ \text{A/m}^2$.

(b) $E = \rho J = (1.72\times10^{-8}\ \Omega\cdot\text{m})(6.81\times10^5\ \text{A/m}^2) = 0.012\ \text{V/m}$.

(c) The time to travel the wire's length l is

$$t = \frac{l}{v_d} = \frac{ln|q|A}{I} = \frac{(4.0\ \text{m})(8.5\times10^{28}/\text{m}^3)(1.6\times10^{-19}\ \text{C})(2.3\times10^{-3}\ \text{m})^2}{3.6\ \text{A}} = 8.0\times10^4\ \text{s}.$$

$t = 1333\ \text{min} \approx 22\ \text{hrs!}$

EVALUATE: The currents propagate very quickly along the wire but the individual electrons travel very slowly.

25.15. **IDENTIFY:** Knowing the resistivity of a metal, its geometry and the current through it, we can use Ohm's law to find the potential difference across it.

SET UP: $V = IR$. For copper, Table 25.1 gives that $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$ and for silver, $\rho = 1.47\times10^{-8}\ \Omega\cdot\text{m}$. $R = \dfrac{\rho L}{A}$.

EXECUTE: **(a)** $R = \dfrac{\rho L}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.00\ \text{m})}{\pi(0.814\times10^{-3}\ \text{m})^2} = 1.65\times10^{-2}\ \Omega$.

$V = (12.5\times10^{-3}\ \text{A})(1.65\times10^{-2}\ \Omega) = 2.06\times10^{-4}\ \text{V}$.

(b) $V = \dfrac{I\rho L}{A}$. $\dfrac{V}{\rho} = \dfrac{IL}{A} = \text{constant}$, so $\dfrac{V_s}{\rho_s} = \dfrac{V_c}{\rho_c}$.

$$V_s = V_c\left(\frac{\rho_s}{\rho_c}\right) = (2.06\times10^{-4}\ \text{V})\left(\frac{1.47\times10^{-8}\ \Omega\cdot\text{m}}{1.72\times10^{-8}\ \Omega\cdot\text{m}}\right) = 1.76\times10^{-4}\ \text{V}.$$

EVALUATE: The potential difference across a 2-m length of wire is less than 0.2 mV, so normally we do not need to worry about these potential drops in laboratory circuits.

25.16. IDENTIFY: The resistivity of the wire should identify what the material is.
SET UP: $R = \rho L/A$ and the radius of the wire is half its diameter.
EXECUTE: Solve for ρ and substitute the numerical values.

$$\rho = AR/L = \pi(D/2)^2 R/L = \frac{\pi([0.00205\ \text{m}]/2)^2(0.0290\ \Omega)}{6.50\ \text{m}} = 1.47\times10^{-8}\ \Omega\cdot\text{m}$$

EVALUATE: This result is the same as the resistivity of silver, which implies that the material is silver.

25.17. IDENTIFY: We want to compare the electric field in two metals.

SET UP and **EXECUTE:** $\rho = E/J$ and $J = I/A$. $E = \rho J = \rho\left(\dfrac{I}{A}\right) = \dfrac{\rho I}{\pi r^2}$. The current I is the same for both wires, and we get the resistivities from Table 25.1.

$$\frac{E_{\text{Cu}}}{E_{\text{Ag}}} = \frac{\rho_{\text{Cu}} I/\pi r_{\text{Cu}}^2}{\rho_{\text{Ag}} I/\pi r_{\text{Ag}}^2} = \left(\frac{\rho_{\text{Cu}}}{\rho_{\text{Ag}}}\right)\left(\frac{r_{\text{Ag}}}{r_{\text{Cu}}}\right)^2 = \left(\frac{1.72}{1.47}\right)\left(\frac{0.500}{0.800}\right)^2 = 0.457.$$

EVALUATE: The field in copper is about half the field in silver.

25.18. IDENTIFY: The geometry of the wire is changed, so its resistance will also change.

SET UP: $R = \dfrac{\rho L}{A}$. $L_{\text{new}} = 3L$. The volume of the wire remains the same when it is stretched.

EXECUTE: Volume $= LA$ so $LA = L_{\text{new}}A_{\text{new}}$. $A_{\text{new}} = \dfrac{L}{L_{\text{new}}}A = \dfrac{A}{3}$.

$$R_{\text{new}} = \frac{\rho L_{\text{new}}}{A_{\text{new}}} = \frac{\rho(3L)}{A/3} = 9\frac{\rho L}{A} = 9R.$$

EVALUATE: When the length increases the resistance increases and when the area decreases the resistance increases.

25.19. IDENTIFY: $R = \dfrac{\rho L}{A}$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $A = \pi r^2$.

EXECUTE: $R = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(24.0\ \text{m})}{\pi(1.025\times10^{-3}\ \text{m})^2} = 0.125\ \Omega.$

EVALUATE: The resistance is proportional to the length of the piece of wire.

25.20. IDENTIFY: $R = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi d^2/4}$.

SET UP: For aluminum, $\rho_{\text{al}} = 2.75\times10^{-8}\ \Omega\cdot\text{m}$. For copper, $\rho_c = 1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: $\dfrac{\rho}{d^2} = \dfrac{R\pi}{4L} =$ constant, so $\dfrac{\rho_{\text{al}}}{d_{\text{al}}^2} = \dfrac{\rho_c}{d_c^2}$.

$$d_c = d_{\text{al}}\sqrt{\frac{\rho_c}{\rho_{\text{al}}}} = (2.14\ \text{mm})\sqrt{\frac{1.72\times10^{-8}\ \Omega\cdot\text{m}}{2.75\times10^{-8}\ \Omega\cdot\text{m}}} = 1.69\ \text{mm}.$$

EVALUATE: Copper has a smaller resistivity, so the copper wire has a smaller diameter in order to have the same resistance as the aluminum wire.

25.21. IDENTIFY and SET UP: The equation $\rho = E/J$ relates the electric field that is given to the current density. $V = EL$ gives the potential difference across a length L of wire and $V = IR$ allows us to calculate R.

EXECUTE: (a) $\rho = E/J$ so $J = E/\rho$.

From Table 25.1 the resistivity for gold is $2.44\times10^{-8}\ \Omega\cdot\text{m}$.

$$J = \frac{E}{\rho} = \frac{0.49\ \text{V/m}}{2.44\times10^{-8}\ \Omega\cdot\text{m}} = 2.008\times10^{7}\ \text{A/m}^2.$$

$I = JA = J\pi r^2 = (2.008\times10^{7}\ \text{A/m}^2)\pi(0.42\times10^{-3}\ \text{m})^2 = 11\ \text{A}.$

(b) $V = EL = (0.49\ \text{V/m})(6.4\ \text{m}) = 3.1\ \text{V}.$

(c) We can use Ohm's law: $V = IR$.

$$R = \frac{V}{I} = \frac{3.1\ \text{V}}{11\ \text{A}} = 0.28\ \Omega.$$

EVALUATE: We can also calculate R from the resistivity and the dimensions of the wire:

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{(2.44\times10^{-8}\ \Omega\cdot\text{m})(6.4\ \text{m})}{\pi(0.42\times10^{-3}\ \text{m})^2} = 0.28\ \Omega,\ \text{which checks.}$$

25.22. IDENTIFY: Apply $R = \dfrac{\rho L}{A}$ and $V = IR$.

SET UP: $A = \pi r^2$.

EXECUTE: $\rho = \dfrac{RA}{L} = \dfrac{VA}{IL} = \dfrac{(4.50\ \text{V})\pi(6.54\times10^{-4}\ \text{m})^2}{(17.6\ \text{A})(2.50\ \text{m})} = 1.37\times10^{-7}\ \Omega\cdot\text{m}.$

EVALUATE: Our result for ρ shows that the wire is made of a metal with resistivity greater than that of good metallic conductors such as copper and aluminum.

25.23. IDENTIFY: Apply $R = R_0[1+\alpha(T-T_0)]$ to calculate the resistance at the second temperature.

(a) SET UP: $\alpha = 0.0004\ (\text{C}°)^{-1}$ (Table 25.2). Let T_0 be 0.0°C and T be 11.5°C.

EXECUTE: $R_0 = \dfrac{R}{1+\alpha(T-T_0)} = \dfrac{100.0\ \Omega}{1+(0.0004\ (\text{C}°)^{-1}(11.5\ \text{C}°))} = 99.54\ \Omega.$

(b) SET UP: $\alpha = -0.0005\ (\text{C}°)^{-1}$ (Table 25.2). Let $T_0 = 0.0°\text{C}$ and $T = 25.8°\text{C}$.

EXECUTE: $R = R_0[1+\alpha(T-T_0)] = 0.0160\ \Omega[1+(-0.0005\ (\text{C}°)^{-1})(25.8\ \text{C}°)] = 0.0158\ \Omega.$

EVALUATE: Nichrome, like most metallic conductors, has a positive α and its resistance increases with temperature. For carbon, α is negative and its resistance decreases as T increases.

25.24. IDENTIFY: $R_T = R_0[1+\alpha(T-T_0)]$.

SET UP: $R_0 = 217.3\ \Omega$. $R_T = 215.8\ \Omega$. For carbon, $\alpha = -0.00050(\text{C}°)^{-1}$.

EXECUTE: $T - T_0 = \dfrac{(R_T/R_0)-1}{\alpha} = \dfrac{(215.8\ \Omega/217.3\ \Omega)-1}{-0.00050\ (\text{C}°)^{-1}} = 13.8\ \text{C}°.$ $T = 13.8\ \text{C}° + 4.0°\text{C} = 17.8°\text{C}.$

EVALUATE: For carbon, α is negative so R decreases as T increases.

25.25. IDENTIFY: Use $R = \dfrac{\rho L}{A}$ to calculate R and then apply $V = IR$. $P = VI$ and energy $= Pt$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $A = \pi r^2$, where $r = 0.050$ m.

EXECUTE: **(a)** $R = \dfrac{\rho L}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(100\times10^{3}\text{m})}{\pi(0.050\ \text{m})^{2}} = 0.219\ \Omega.$

$V = IR = (125\ \text{A})(0.219\ \Omega) = 27.4\ \text{V}.$

(b) $P = VI = (27.4\ \text{V})(125\ \text{A}) = 3422\ \text{W} = 3422\ \text{J/s}$ and energy $= Pt = (3422\ \text{J/s})(3600\ \text{s}) = 1.23\times10^{7}\ \text{J}.$

EVALUATE: The rate of electrical energy loss in the cable is large, over 3 kW.

25.26. **IDENTIFY:** When current passes through a battery in the direction from the $-$ terminal toward the + terminal, the terminal voltage V_{ab} of the battery is $V_{ab} = \mathcal{E} - Ir.$ Also, $V_{ab} = IR,$ the potential across the circuit resistor.

SET UP: $\mathcal{E} = 24.0\ \text{V}.$ $I = 4.00\ \text{A}.$

EXECUTE: **(a)** $V_{ab} = \mathcal{E} - Ir$ gives $r = \dfrac{\mathcal{E} - V_{ab}}{I} = \dfrac{24.0\ \text{V} - 21.2\ \text{V}}{4.00\ \text{A}} = 0.700\ \Omega.$

(b) $V_{ab} - IR = 0$ so $R = \dfrac{V_{ab}}{I} = \dfrac{21.2\ \text{V}}{4.00\ \text{A}} = 5.30\ \Omega.$

EVALUATE: The voltage drop across the internal resistance of the battery causes the terminal voltage of the battery to be less than its emf. The total resistance in the circuit is $R + r = 6.00\ \Omega.$

$I = \dfrac{24.0\ \text{V}}{6.00\ \Omega} = 4.00\ \text{A},$ which agrees with the value specified in the problem.

25.27. **IDENTIFY:** The terminal voltage of the battery is $V_{ab} = \mathcal{E} - Ir.$ The voltmeter reads the potential difference between its terminals.

SET UP: An ideal voltmeter has infinite resistance.

EXECUTE: **(a)** Since an ideal voltmeter has infinite resistance, so there would be NO current through the $2.0\ \Omega$ resistor.

(b) $V_{ab} = \mathcal{E} = 5.0\ \text{V};$ Since there is no current there is no voltage lost over the internal resistance.

(c) The voltmeter reading is therefore 5.0 V since with no current flowing there is no voltage drop across either resistor.

EVALUATE: This not the proper way to connect a voltmeter. If we wish to measure the terminal voltage of the battery in a circuit that does not include the voltmeter, then connect the voltmeter across the terminals of the battery.

25.28. **IDENTIFY:** The *idealized* ammeter has no resistance so there is no potential drop across it. Therefore it acts like a short circuit across the terminals of the battery and removes the $4.00\text{-}\Omega$ resistor from the circuit. Thus the only resistance in the circuit is the $2.00\text{-}\Omega$ internal resistance of the battery.

SET UP: Use Ohm's law: $I = \mathcal{E}/r.$

EXECUTE: **(a)** $I = (10.0\ \text{V})/(2.00\ \Omega) = 5.00\ \text{A}.$

(b) The zero-resistance ammeter is in parallel with the $4.00\text{-}\Omega$ resistor, so all the current goes through the ammeter. If no current goes through the $4.00\text{-}\Omega$ resistor, the potential drop across it must be zero.

(c) The terminal voltage is zero since there is no potential drop across the ammeter.

EVALUATE: An ammeter should *never* be connected this way because it would seriously alter the circuit!

25.29. **IDENTIFY:** The voltmeter reads the potential difference V_{ab} between the terminals of the battery.

SET UP: open circuit: $I = 0.$ The circuit is sketched in Figure 25.29a.

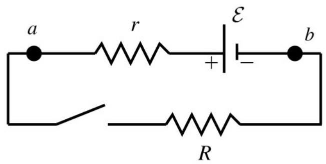


Figure 25.29a

EXECUTE: $V_{ab} = \mathcal{E} = 3.08$ V.

SET UP: switch closed: The circuit is sketched in Figure 25.29b.

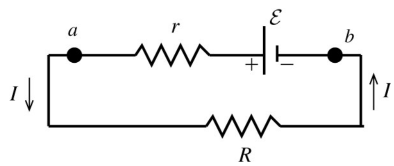


Figure 25.29b

EXECUTE: $V_{ab} = \mathcal{E} - Ir = 2.97$ V.

$r = \dfrac{\mathcal{E} - 2.97\text{ V}}{I}.$

$r = \dfrac{3.08\text{ V} - 2.97\text{ V}}{1.65\text{ A}} = 0.067\ \Omega.$

And $V_{ab} = IR$ so $R = \dfrac{V_{ab}}{I} = \dfrac{2.97\text{ V}}{1.65\text{ A}} = 1.80\ \Omega.$

EVALUATE: When current flows through the battery there is a voltage drop across its internal resistance and its terminal voltage V is less than its emf.

25.30. **IDENTIFY:** The sum of the potential changes around the circuit loop is zero. Potential decreases by IR when going through a resistor in the direction of the current and increases by $\mathcal{E}$ when passing through an emf in the direction from the $-$ to $+$ terminal.

SET UP: The current is counterclockwise, because the 16-V battery determines the direction of current flow.

EXECUTE: $+16.0\text{ V} - 8.0\text{ V} - I(1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega) = 0.$

$$I = \frac{16.0\text{ V} - 8.0\text{ V}}{1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega} = 0.47\text{ A}.$$

(b) $V_b + 16.0\text{ V} - I(1.6\ \Omega) = V_a$, so $V_a - V_b = V_{ab} = 16.0\text{ V} - (1.6\ \Omega)(0.47\text{ A}) = 15.2\text{ V}.$

(c) $V_c + 8.0\text{ V} + I(1.4\ \Omega + 5.0\ \Omega) = V_a$ so $V_{ac} = (5.0\ \Omega)(0.47\text{ A}) + (1.4\ \Omega)(0.47\text{ A}) + 8.0\text{ V} = 11.0\text{ V}.$

(d) The graph is sketched in Figure 25.30.

EVALUATE: $V_{cb} = (0.47\text{ A})(9.0\ \Omega) = 4.2\text{ V}.$ The potential at point b is 15.2 V below the potential at point a and the potential at point c is 11.0 V below the potential at point a, so the potential of point c is $15.2\text{ V} - 11.0\text{ V} = 4.2\text{ V}$ above the potential of point b.

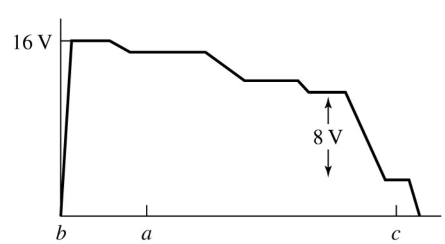


Figure 25.30

25.31. **(a) IDENTIFY and SET UP:** Assume that the current is clockwise. The circuit is sketched in Figure 25.31a.

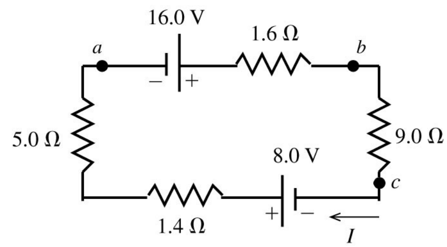


Figure 25.31a

Add up the potential rises and drops as travel clockwise around the circuit.

EXECUTE: $16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) + 8.0\text{ V} - I(1.4\ \Omega) - I(5.0\ \Omega) = 0.$

$$I = \frac{16.0\text{ V} + 8.0\text{ V}}{9.0\ \Omega + 1.4\ \Omega + 5.0\ \Omega + 1.6\ \Omega} = \frac{24.0\text{ V}}{17.0\ \Omega} = 1.41\text{ A, clockwise.}$$

EVALUATE: The 16.0-V battery and the 8.0-V battery both drive the current in the same direction.

(b) IDENTIFY and SET UP: Start at point *a* and travel through the battery to point *b*, keeping track of the potential changes. At point *b* the potential is V_b.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) = V_b.$

$V_a - V_b = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega).$

$V_{ab} = -16.0\text{ V} + 2.3\text{ V} = -13.7\text{ V}$ (point *a* is at lower potential; it is the negative terminal). Therefore, $V_{ba} = 13.7\text{V}.$

EVALUATE: Could also go counterclockwise from *a* to *b:*

$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} + (1.41\text{ A})(9.0\ \Omega) = V_b.$

$V_{ab} = -13.7\text{ V},$ which checks.

(c) IDENTIFY and SET UP: Start at point *a* and travel through the battery to point *c*, keeping track of the potential changes.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) = V_c.$

$V_a - V_c = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega + 9.0\ \Omega).$

$V_{ac} = -16.0\text{ V} + 15.0\text{ V} = -1.0\text{ V}$ (point *a* is at lower potential than point *c*).

EVALUATE: Could also go counterclockwise from *a* to *c*:

$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} = V_c.$

$V_{ac} = -1.0\text{ V},$ which checks.

(d) Call the potential zero at point *a*. Travel clockwise around the circuit. The graph is sketched in Figure 25.31b.

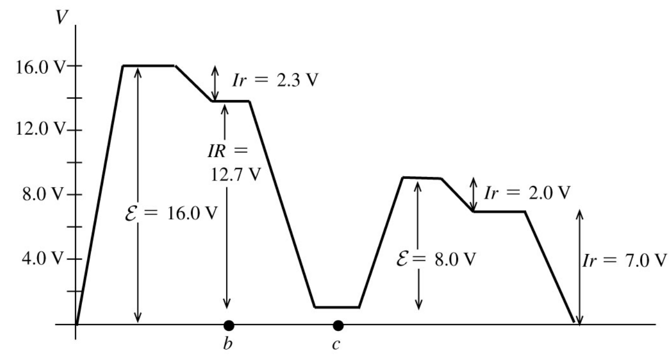


Figure 25.31b

25.32. **IDENTIFY:** This problem involves the internal resistance of a battery.

SET UP: $\mathcal{E} = (R+r)I, I = V_R / R$. The internal resistance is the target variable.

EXECUTE: $I = V_R/R = (27.0\text{ V})/(9.00\ \Omega) = 3.00\text{ A}$. Solve for r: $r = \dfrac{\mathcal{E}}{I} - R$.

$$r = \frac{30.0\text{ V}}{3.00\text{ A}} - 9.00\ \Omega = 1.0\ \Omega.$$

EVALUATE: If there were no internal resistance the current would be (30.0 V)/(9.00 Ω) = 3.33 A and V_R would be 30.0 V. The internal resistance makes a significant difference in the circuit.

25.33. **IDENTIFY:** This problem involves the internal resistance of a battery.

SET UP: $\mathcal{E} = (R+r)I,\ P = I^2R$. The target variable is the external resistance R.

EXECUTE: $P = I^2R = \left(\dfrac{\mathcal{E}}{R+r}\right)^2 R$. $\mathcal{E}^2R = P(R^2 + 2rR + r^2)$. Putting in the given values and solving this quadratic equation for R gives $R = 0.429\ \Omega$ and $R = 21.0\ \Omega$.

EVALUATE: To check, apply $P = I^2R = \left(\dfrac{\mathcal{E}}{R+r}\right)^2 R$ for both resistances.

$$P = I^2R = \left(\frac{24.0\text{ V}}{3.429\ \Omega}\right)^2 (0.429\ \Omega) = 21\text{ W} \text{ and } P = \left(\frac{24.0\text{ V}}{24.0\ \Omega}\right)^2 (21.0\ \Omega) = 21\text{ W}.$$ Both answers check.

25.34. **IDENTIFY** and **SET UP:** The resistance is the same in both cases, and $P = V^2/R$.

EXECUTE: **(a)** Solving $P = V^2/R$ for R, gives $R = V^2/P$. Since the resistance is the same in both cases, we have $\dfrac{V_1^2}{P_1} = \dfrac{V_2^2}{P_2}$. Solving for P_2 gives $P_2 = P_1(V_2/V_1)^2 = (0.0625\text{ W})[(12.5\text{ V})/(1.50\text{ V})]^2 = 4.41$ W.

(b) Solving for V_2 gives $V_2 = V_1\sqrt{\dfrac{P_2}{P_1}} = (1.50\text{ V})\sqrt{\dfrac{5.00\text{ W}}{0.0625\text{ W}}} = 13.4\text{ V}$.

EVALUATE: These calculations are correct assuming that the resistor obeys Ohm's law throughout the range of currents involved.

25.35. **IDENTIFY:** The bulbs are each connected across a 120-V potential difference.

SET UP: Use $P = V^2/R$ to solve for R and Ohm's law $(I = V/R)$ to find the current.

EXECUTE: **(a)** $R = V^2/P = (120\text{ V})^2/(100\text{ W}) = 144\ \Omega$.

(b) $R = V^2/P = (120\text{ V})^2/(60\text{ W}) = 240\ \Omega$.

(c) For the 100-W bulb: $I = V/R = (120\text{ V})/(144\ \Omega) = 0.833\text{ A}$.

For the 60-W bulb: $I = (120\ \text{V})/(240\ \Omega) = 0.500\ \text{A}$.

EVALUATE: The 60-W bulb has *more* resistance than the 100-W bulb, so it draws less current.

25.36. **IDENTIFY:** Across 120 V, a 75-W bulb dissipates 75 W. Use this fact to find its resistance, and then find the power the bulb dissipates across 220 V.

SET UP: $P = V^2/R$, so $R = V^2/P$.

EXECUTE: Across 120 V: $R = (120\ \text{V})^2/(75\ \text{W}) = 192\ \Omega$. Across a 220-V line, its power will be $P = V^2/R = (220\ \text{V})^2/(192\ \Omega) = 252\ \text{W}$.

EVALUATE: The bulb dissipates much more power across 220 V, so it would likely blow out at the higher voltage. An alternative solution to the problem is to take the ratio of the powers.

$\dfrac{P_{220}}{P_{120}} = \dfrac{V_{220}^2/R}{V_{120}^2/R} = \left(\dfrac{V_{220}}{V_{120}}\right)^2 = \left(\dfrac{220}{120}\right)^2$. This gives $P_{220} = (75\ \text{W})\left(\dfrac{220}{120}\right)^2 = 252\ \text{W}$.

25.37. **IDENTIFY:** A "100-W" European bulb dissipates 100 W when used across 220 V.

(a) SET UP: Take the ratio of the power in the U.S. to the power in Europe, as in the alternative method for Problem 25.36, using $P = V^2/R$.

EXECUTE: $\dfrac{P_{\text{US}}}{P_{\text{E}}} = \dfrac{V_{\text{US}}^2/R}{V_{\text{E}}^2/R} = \left(\dfrac{V_{\text{US}}}{V_{\text{E}}}\right)^2 = \left(\dfrac{120\ \text{V}}{220\ \text{V}}\right)^2$. This gives $P_{\text{US}} = (100\ \text{W})\left(\dfrac{120\ \text{V}}{220\ \text{V}}\right)^2 = 29.8\ \text{W}$.

(b) SET UP: Use $P = IV$ to find the current.

EXECUTE: $I = P/V = (29.8\ \text{W})/(120\ \text{V}) = 0.248\ \text{A}$.

EVALUATE: The bulb draws considerably less power in the U.S., so it would be much dimmer than in Europe.

25.38. **IDENTIFY:** $P = VI$. Energy $= Pt$.

SET UP: $P = (9.0\ \text{V})(0.13\ \text{A}) = 1.17\ \text{W}$.

EXECUTE: Energy $= (1.17\ \text{W})(30\ \text{min})(60\ \text{s/min}) = 2100\ \text{J}$.

EVALUATE: The energy consumed is proportional to the voltage, to the current and to the time.

25.39. **IDENTIFY:** Calculate the current in the circuit. The power output of a battery is its terminal voltage times the current through it. The power dissipated in a resistor is I^2R.

SET UP: The sum of the potential changes around the circuit is zero.

EXECUTE: **(a)** $I = \dfrac{8.0\ \text{V}}{17\ \Omega} = 0.47\ \text{A}$. Then $P_{5\Omega} = I^2R = (0.47\ \text{A})^2(5.0\ \Omega) = 1.1\ \text{W}$ and $P_{9\Omega} = I^2R = (0.47\ \text{A})^2(9.0\ \Omega) = 2.0\ \text{W}$, so the total is 3.1 W.

(b) $P_{16\text{V}} = \mathcal{E}I - I^2r = (16\ \text{V})(0.47\ \text{A}) - (0.47\ \text{A})^2(1.6\ \Omega) = 7.2\ \text{W}$.

(c) $P_{8\text{V}} = \mathcal{E}I + Ir^2 = (8.0\ \text{V})(0.47\ \text{A}) + (0.47\ \text{A})^2(1.4\ \Omega) = 4.1\ \text{W}$.

EVALUATE: **(d)** (b) = (a) + (c). The rate at which the 16.0-V battery delivers electrical energy to the circuit equals the rate at which it is consumed in the 8.0-V battery and the 5.0-Ω and 9.0-Ω resistors.

25.40. **IDENTIFY:** Knowing the current and potential difference, we can find the power.

SET UP: $P = VI$ and energy is the product of power and time.

EXECUTE: $P = (500\ \text{V})(80\times10^{-3}\ \text{A}) = 40\ \text{W}$.

Energy $= Pt = (40\ \text{W})(10\times10^{-3}\ \text{s}) = 0.40\ \text{J}$.

EVALUATE: The energy delivered depends not only on the voltage and current but also on the length of the pulse. The pulse is short but the voltage is large.

25.41. **IDENTIFY:** We know the current, voltage and time the current lasts, so we can calculate the power and the energy delivered.

SET UP: Power is energy per unit time. The power delivered by a voltage source is $P = V_{ab}I$.

EXECUTE: **(a)** $P = (25\text{ V})(12\text{ A}) = 300\text{ W}$.

(b) $\text{Energy} = Pt = (300\text{ W})(3.0\times 10^{-3}\text{ s}) = 0.90\text{ J}$.

EVALUATE: The energy is not very great, but it is delivered in a short time (3 ms) so the power is large, which produces a short shock.

25.42. **IDENTIFY** and **SET UP:** The average power delivered by the battery can be calculated in two different ways: $P = \dfrac{\text{energy}}{\text{time}}$ or $P = VI$. The time is 5.25 h, which in seconds is

$5.25\text{ h} = (5.25\text{ h})(3600\text{ s/h}) = 1.89\times 10^4\text{ s}$.

EXECUTE: The average power delivered by the battery is $P = \dfrac{\text{energy}}{\text{time}} = \dfrac{3.15\times 10^4\text{ J}}{1.89\times 10^4\text{ s}} = 1.6667\text{ W}$. Thus, the current must be $I = \dfrac{P}{V} = \dfrac{1.6667\text{ W}}{3.70\text{ V}} = 0.450\text{ A}$.

EVALUATE: The energy stored in the battery can be expressed in joules or watt-hours. The energy is equal to *Pt*, so we can express the stored energy as either $3.15\times 10^4\text{ J}$ or (1.6667 W)(5.25 h) = $8.75\text{ W}\cdot\text{h}$.

25.43. **(a) IDENTIFY** and **SET UP:** $P = VI$ and $\text{energy} = (\text{power})\times(\text{time})$.

EXECUTE: $P = VI = (12\text{ V})(60\text{ A}) = 720\text{ W}$.

The battery can provide this for 1.0 h, so the energy the battery has stored is

$U = Pt = (720\text{ W})(3600\text{ s}) = 2.6\times 10^6\text{ J}$.

(b) IDENTIFY and **SET UP:** For gasoline the heat of combustion is $L_c = 46\times 10^6\text{ J/kg}$. Solve for the mass *m* required to supply the energy calculated in part (a) and use density $\rho = m/V$ to calculate *V*.

EXECUTE: The mass of gasoline that supplies $2.6\times 10^6\text{ J}$ is $m = \dfrac{2.6\times 10^6\text{ J}}{46\times 10^6\text{ J/kg}} = 0.0565\text{ kg}$.

The volume of this mass of gasoline is

$$V = \frac{m}{\rho} = \frac{0.0565\text{ kg}}{900\text{ kg/m}^3} = 6.3\times 10^{-5}\text{ m}^3\left(\frac{1000\text{ L}}{1\text{ m}^3}\right) = 0.063\text{ L}.$$

(c) IDENTIFY and **SET UP:** $\text{Energy} = (\text{power})\times(\text{time})$; the energy is that calculated in part (a).

EXECUTE: $U = Pt,\ t = \dfrac{U}{P} = \dfrac{2.6\times 10^6\text{ J}}{450\text{ W}} = 5800\text{ s} = 97\text{ min} = 1.6\text{ h}$.

EVALUATE: The battery discharges at a rate of 720 W (for 1.0 h) and is charged at a rate of 450 W (for 1.6 h), so it takes longer to charge than to discharge.

25.44. **IDENTIFY:** This problem involves the internal resistance of a battery.

SET UP: $\mathcal{E} = (R + r)I$, $P = I^2R$. The target variable is the emf of the battery.

EXECUTE: Solve $P = I^2R$ for *I*: $I = \sqrt{P/R} = \sqrt{\dfrac{96.0\text{ J/s}}{12.0\ \Omega}} = 2.8284\text{ A}$. Now find the emf:

$\mathcal{E} = (R + r)I =$ (14.0 Ω)(2.8284 A) = 39.6 V.

EVALUATE: The power the battery produces is $P = I\mathcal{E}$ = (2.8284 A)(39.6 V) = 112 W. So the power lost in the internal resistance is 112 W – 96 W = 16 W. We can also get this using $P = I^2R$ = (2.8284 A)^2(2.00 Ω) = 16 W.

25.45. **IDENTIFY:** Some of the power generated by the internal emf of the battery is dissipated across the battery's internal resistance, so it is not available to the bulb.

SET UP: Use $P = I^2R$ and take the ratio of the power dissipated in the internal resistance r to the total power.

EXECUTE: $\dfrac{P_r}{P_{\text{Total}}} = \dfrac{I^2r}{I^2(r+R)} = \dfrac{r}{r+R} = \dfrac{3.5\ \Omega}{28.5\ \Omega} = 0.123 = 12.3\%.$

EVALUATE: About 88% of the power of the battery goes to the bulb. The rest appears as heat in the internal resistance.

25.46. **IDENTIFY:** The power delivered to the bulb is I^2R. Energy $= Pt$.

SET UP: The circuit is sketched in Figure 25.46. r_{total} is the combined internal resistance of both batteries.

EXECUTE: **(a)** $r_{\text{total}} = 0$. The sum of the potential changes around the circuit is zero, so $1.5\ \text{V} + 1.5\ \text{V} - I(17\ \Omega) = 0$. $I = 0.1765\ \text{A}$. $P = I^2R = (0.1765\ \text{A})^2(17\ \Omega) = 0.530\ \text{W}$. This is also $(3.0\ \text{V})(0.1765\ \text{A})$.

(b) Energy $= (0.530\ \text{W})(5.0\ \text{h})(3600\ \text{s/h}) = 9540\ \text{J}$.

(c) $P = \dfrac{0.530\ \text{W}}{2} = 0.265\ \text{W}$. $P = I^2R$ so $I = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{0.265\ \text{W}}{17\ \Omega}} = 0.125\ \text{A}$.

The sum of the potential changes around the circuit is zero, so $1.5\ \text{V} + 1.5\ \text{V} - IR - Ir_{\text{total}} = 0$.

$r_{\text{total}} = \dfrac{3.0\ \text{V} - (0.125\ \text{A})(17\ \Omega)}{0.125\ \text{A}} = 7.0\ \Omega.$

EVALUATE: When the power to the bulb has decreased to half its initial value, the total internal resistance of the two batteries is nearly half the resistance of the bulb. Compared to a single battery, using two identical batteries in series doubles the emf but also doubles the total internal resistance.

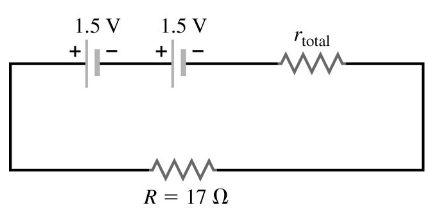


Figure 25.46

25.47. **IDENTIFY:** Solve for the current I in the circuit. Apply $P = VI = I^2R$ to the specified circuit elements to find the rates of energy conversion.

SET UP: The circuit is sketched in Figure 25.47.

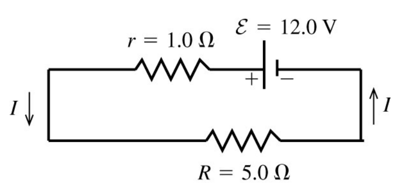


Figure 25.47

EXECUTE: Compute I:

$\mathcal{E} - Ir - IR = 0.$

$I = \dfrac{\mathcal{E}}{r+R} = \dfrac{12.0\ \text{V}}{1.0\ \Omega + 5.0\ \Omega} = 2.00\ \text{A}.$

(a) The rate of conversion of chemical energy to electrical energy in the emf of the battery is $P = \mathcal{E}I = (12.0\ \text{V})(2.00\ \text{A}) = 24.0\ \text{W}$.

(b) The rate of dissipation of electrical energy in the internal resistance of the battery is $P = I^2 r = (2.00\ \text{A})^2 (1.0\ \Omega) = 4.0\ \text{W}$.

(c) The rate of dissipation of electrical energy in the external resistor R is $P = I^2 R = (2.00\ \text{A})^2 (5.0\ \Omega) = 20.0\ \text{W}$.

EVALUATE: The rate of production of electrical energy in the circuit is 24.0 W. The total rate of consumption of electrical energy in the circuit is $4.00\ \text{W} + 20.0\ \text{W} = 24.0\ \text{W}$. Equal rates of production and consumption of electrical energy are required by energy conservation.

25.48. **IDENTIFY:** $P = I^2 R = \dfrac{V^2}{R} = VI$. $V = IR$.

SET UP: The heater consumes 540 W when $V = 120$ V. Energy $= Pt$.

EXECUTE: **(a)** $P = \dfrac{V^2}{R}$ so $R = \dfrac{V^2}{P} = \dfrac{(120\ \text{V})^2}{540\ \text{W}} = 26.7\ \Omega$.

(b) $P = VI$ so $I = \dfrac{P}{V} = \dfrac{540\ \text{W}}{120\ \text{V}} = 4.50\ \text{A}$.

(c) Assuming that R remains $26.7\ \Omega$, $P = \dfrac{V^2}{R} = \dfrac{(110\ \text{V})^2}{26.7\ \Omega} = 453\ \text{W}$. P is smaller by a factor of $(110/120)^2$.

EVALUATE: **(d)** With the lower line voltage the current will decrease and the operating temperature will decrease. R will be less than $26.7\ \Omega$ and the power consumed will be greater than the value calculated in part (c).

25.49. **IDENTIFY:** The resistivity is $\rho = \dfrac{m}{ne^2\tau}$.

SET UP: For silicon, $\rho = 2300\ \Omega \cdot \text{m}$.

EXECUTE: **(a)** $\tau = \dfrac{m}{ne^2\rho} = \dfrac{9.11\times10^{-31}\ \text{kg}}{(1.0\times10^{16}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})^2(2300\ \Omega\cdot\text{m})} = 1.55\times10^{-12}\ \text{s}$.

EVALUATE: **(b)** The number of free electrons in copper $(8.5\times10^{28}\ \text{m}^{-3})$ is much larger than in pure silicon $(1.0\times10^{16}\ \text{m}^{-3})$. A smaller density of current carriers means a higher resistivity.

25.50. **IDENTIFY:** We are investigating a cell phone.

SET UP and **EXECUTE:** **(a)** Charge capacity = 2600 mAh, energy = 9.88 Wh, and potential rating = 3.8 V.

(b) 2600 mAh = (2.6 A)(3600 s) = 9360 C.

(c) (9.88 Wh)(9.88 J/s)(3600 s) = 35.6 kJ.

(d) qV = (9360 C)(3.8 V) = 35.6 kJ. They are equivalent.

(e) $C = Q/V$ = (9360 C)/(3.8 V) = 2460 F.

(f) $U = mc\Delta T \rightarrow \Delta T = U / mc$. m = 1.0 kg. $\Delta T = \dfrac{35.6\ \text{kJ}}{(1.0\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})} = 8.5\ \text{C}°$.

EVALUATE: A capacitance of 2460 F would be very large!

25.51. **(a) IDENTIFY** and **SET UP:** Use $V_{ab} = \mathcal{E} - rI$.

EXECUTE: $\rho = \dfrac{RA}{L} = \dfrac{(0.104\ \Omega)\pi(1.25\times10^{-3}\ \text{m})^2}{14.0\ \text{m}} = 3.65\times10^{-8}\ \Omega\cdot\text{m}$.

EVALUATE: This value is similar to that for good metallic conductors in Table 25.1.

(b) IDENTIFY and **SET UP:** Use $V = EL$ to calculate E and then Ohm's law gives I.

EXECUTE: $V = EL = (1.28\ \text{V/m})(14.0\ \text{m}) = 17.9\ \text{V}$.

$I = \dfrac{V}{R} = \dfrac{17.9\text{ V}}{0.104\ \Omega} = 172\text{ A}.$

EVALUATE: We could do the calculation another way:

$E = \rho J$ so $J = \dfrac{E}{\rho} = \dfrac{1.28\text{ V/m}}{3.65\times10^{-8}\ \Omega\cdot\text{m}} = 3.51\times10^{7}\ \text{A/m}^2.$

$I = JA = (3.51\times10^{7}\ \text{A/m}^2)\pi(1.25\times10^{-3}\ \text{m})^2 = 172\text{ A},$ which checks.

(c) IDENTIFY and SET UP: Calculate $J = I/A$ or $J = E/\rho$ and then use Eq. (25.3) for the target variable v_d.

EXECUTE: $J = n|q|v_\text{d} = nev_\text{d}.$

$v_\text{d} = \dfrac{J}{ne} = \dfrac{3.51\times10^{7}\ \text{A/m}^2}{(8.5\times10^{28}\ \text{m}^{-3})(1.602\times10^{-19}\ \text{C})} = 2.58\times10^{-3}\ \text{m/s} = 2.58\ \text{mm/s}.$

EVALUATE: Even for this very large current the drift speed is small.

25.52. **IDENTIFY:** We are investigating a hot water heater.

SET UP and EXECUTE: **(a)** Estimate: 50 gal = (50)(3.788 L) = 190 L.

(b) $Q = mc\Delta T$ = (190 kg)(4190 J/kg · K)(25 C°) = 20 MJ.

(c) $P = Q/t$ = (20 MJ)/[(1.5)(3600 s)] = 3.7 kW.

(d) $I = P/V$ = (3.7 kW)/(220 V) = 17 A.

(e) $R = V/I$ = (220 V)/(17 A) = 13 Ω.

EVALUATE: Since $P = V^2/R$, for a given voltage we need a small resistance to have a large power.

25.53. **IDENTIFY and SET UP:** With the voltmeter connected across the terminals of the battery there is no current through the battery and the voltmeter reading is the battery emf; $\mathcal{E} = 12.6\text{ V}.$

With a wire of resistance R connected to the battery current I flows and $\mathcal{E} - Ir - IR = 0,$ where r is the internal resistance of the battery. Apply this equation to each piece of wire to get two equations in the two unknowns.

EXECUTE: Call the resistance of the 20.0-m piece R_1; then the resistance of the 40.0-m piece is $R_2 = 2R_1.$

$\mathcal{E} - I_1 r - I_1 R_1 = 0;\quad 12.6\text{ V} - (7.00\text{ A})r - (7.00\text{ A})R_1 = 0.$

$\mathcal{E} - I_2 r - I_2(2R_2) = 0;\quad 12.6\text{ V} - (4.20\text{ A})r - (4.20\text{ A})(2R_1) = 0.$

Solving these two equations in two unknowns gives $R_1 = 1.20\ \Omega.$ This is the resistance of 20.0 m, so the resistance of one meter is $[1.20\ \Omega/(20.0\text{ m})](1.00\text{ m}) = 0.060\ \Omega.$

EVALUATE: We can also solve for r and we get $r = 0.600\ \Omega.$ When measuring small resistances, the internal resistance of the battery has a large effect.

25.54. **IDENTIFY:** As the resistance R varies, the current in the circuit also varies, which causes the potential drop across the internal resistance of the battery to vary. The largest current will occur when $R = 0,$ and the smallest current will occur when $R \to \infty.$ The largest terminal voltage will occur when the current is zero ($R \to \infty$) and the smallest terminal voltage will be when the current is a maximum ($R = 0$).

SET UP: If $\mathcal{E}$ is the internal emf of the battery and r is its internal resistance, then $V_{ab} = \mathcal{E} - rI.$

EXECUTE: **(a)** As $R \to \infty,$ $I \to 0,$ so $V_{ab} \to \mathcal{E} = 15.0\text{ V},$ which is the largest reading of the voltmeter. When $R = 0,$ the current is largest at $(15.0\text{ V})/(4.00\ \Omega) = 3.75\text{ A},$ so the smallest terminal voltage is $V_{ab} = \mathcal{E} - rI = 15.0\text{ V} - (4.00\ \Omega)(3.75\text{ A}) = 0.$

(b) From part (a), the maximum current is 3.75 A when $R = 0,$ and the minimum current is 0.00 A when $R \to \infty.$

(c) The graphs are sketched in the Figure 25.54.

EVALUATE: Increasing the resistance R increases the terminal voltage, but at the same time it decreases the current in the circuit.

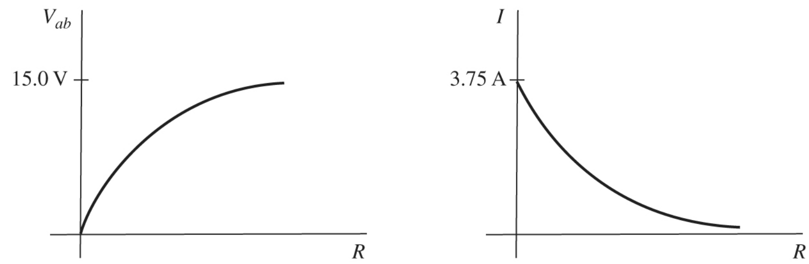


Figure 25.54

25.55. **IDENTIFY:** Conservation of charge requires that the current be the same in both sections of the wire. $E = \rho J = \dfrac{\rho I}{A}$. For each section, $V = IR = JAR = \left(\dfrac{EA}{\rho}\right)\left(\dfrac{\rho L}{A}\right) = EL$. The voltages across each section add.

SET UP: $A = (\pi/4)D^2$, where D is the diameter.

EXECUTE: **(a)** The current must be the same in both sections of the wire, so the current in the thin end is 2.5 mA.

(b) $E_{1.6\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.5\times10^{-3}\ \text{A})}{(\pi/4)(1.6\times10^{-3}\ \text{m})^2} = 2.14\times10^{-5}\ \text{V/m}.$

(c) $E_{0.8\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.5\times10^{-3}\ \text{A})}{(\pi/4)(0.80\times10^{-3}\ \text{m})^2} = 8.55\times10^{-5}\ \text{V/m}.$ This is $4E_{1.6\text{mm}}$.

(d)
$V = E_{1.6\text{mm}}L_{1.6\text{ mm}} + E_{0.8\text{ mm}}L_{0.8\text{ mm}}.$ $V = (2.14\times10^{-5}\ \text{V/m})(1.20\ \text{m}) + (8.55\times10^{-5}\ \text{V/m})(1.80\ \text{m}) = 1.80\times10^{-4}\ \text{V}.$

EVALUATE: The currents are the same but the current density is larger in the thinner section and the electric field is larger there.

25.56. **IDENTIFY** and **SET UP:** The voltage is the same at both temperatures since the same battery is used. The power is $P = V^2/R$ and $R = R_0(1+\alpha\Delta T)$.

EXECUTE: Since the voltage is the same, we have $V^2 = P_{80}R_{80} = P_{150}R_{150}$. Therefore $P_{80}R_0[1+\alpha(T_{80}-T_0)] = P_{150}R_0[1+\alpha(T_{150}-T_0)]$. Solving for P_{150} and putting in the numbers gives

$$P_{150} = P_{80}\frac{1+\alpha(T_{80}-T_0)}{1+\alpha(T_{150}-T_0)} = (480\ \text{W})\frac{1 + (0.0045\ \text{K}^{-1})(80°\text{C} - 20°\text{C})}{1 + (0.0045\ \text{K}^{-1})(150°\text{C} - 20°\text{C})} = 385\ \text{W}.$$

EVALUATE: This result assumes that α is the same at all the temperatures.

25.57. **IDENTIFY:** Knowing the current and the time for which it lasts, plus the resistance of the body, we can calculate the energy delivered.

SET UP: Electric energy is deposited in his body at the rate $P = I^2R$. Heat energy Q produces a temperature change ΔT according to $Q = mc\Delta T$, where $c = 4190\ \text{J/kg}\cdot\text{C°}$.

EXECUTE: **(a)** $P = I^2R = (25{,}000\ \text{A})^2(1.0\ \text{k}\Omega) = 6.25\times10^{11}\ \text{W}$. The energy deposited is $Pt = (6.15\times10^{11}\ \text{W})(40\times10^{-6}\ \text{s}) = 2.5\times10^{7}\ \text{J}$. Find ΔT when $Q = 2.5\times10^7\ \text{J}$.

$$\Delta T = \frac{Q}{mc} = \frac{2.5\times10^7\ \text{J}}{(75\ \text{kg})(4190\ \text{J/kg}\cdot\text{C°})} = 80\ \text{C°}.$$

(b) An increase of only $63\ \text{C°}$ brings the water in the body to the boiling point; part of the person's body will be vaporized.

EVALUATE: Even this approximate calculation shows that being hit by lightning is very dangerous.

25.58. **IDENTIFY:** The current in the circuit depends on R and on the internal resistance of the battery, as well as the emf of the battery. It is only the current in R that dissipates energy in the resistor R.

SET UP: $I = \dfrac{\mathcal{E}}{R+r}$, where $\mathcal{E}$ is the emf of the battery, and $P = I^2 R$.

EXECUTE: $P = I^2 R = \dfrac{\mathcal{E}^2}{(R+r)^2} R$, which gives $\mathcal{E}^2 R = (R^2 + 2Rr + r^2)P$.

$$R^2 + \left(2r - \frac{\mathcal{E}^2}{P}\right)R + r^2 = 0. \quad R = \frac{1}{2}\left[\left(\frac{\mathcal{E}^2}{P} - 2r\right) \pm \sqrt{\left(\frac{\mathcal{E}^2}{P} - 2r\right)^2 - 4r^2}\right].$$

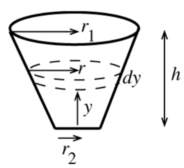
$$R = \frac{1}{2}\left[\left(\frac{(12.0\text{ V})^2}{80.0\text{ W}} - 2(0.40\,\Omega)\right) \pm \sqrt{\left(\frac{(12.0\text{ V})^2}{80.0\text{ W}} - 2(0.40\,\Omega)\right)^2 - 4(0.40\,\Omega)^2}\right].$$

$R = 0.50\,\Omega \pm 0.30\,\Omega$. $R = 0.20\,\Omega$ and $R = 0.80\,\Omega$.

EVALUATE: There are two values for R because there are two ways for the power dissipated in R to be 80 W. The power is $P = I^2 R$, so we can have a small $R(0.20\,\Omega)$ and large current, or a larger $R(0.80\,\Omega)$ and a smaller current.

25.59. **(a) IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ to calculate the resistance of each thin disk and then integrate over the truncated cone to find the total resistance.

SET UP:



EXECUTE: The radius of a truncated cone a distance y above the bottom is given by $r = r_2 + (y/h)(r_1 - r_2) = r_2 + y\beta$ with $\beta = (r_1 - r_2)/h$.

Figure 25.59

Consider a thin slice a distance y above the bottom. The slice has thickness dy and radius r (see Figure 25.59.) The resistance of the slice is $dR = \dfrac{\rho dy}{A} = \dfrac{\rho dy}{\pi r^2} = \dfrac{\rho dy}{\pi(r_2 + \beta y)^2}$.

The total resistance of the cone if obtained by integrating over these thin slices:

$$R = \int dR = \frac{\rho}{\pi}\int_0^h \frac{dy}{(r_2 + \beta y)^2} = \frac{\rho}{\pi}\left[-\frac{1}{\beta}(r_2 + y\beta)^{-1}\right]_0^h = -\frac{\rho}{\pi\beta}\left[\frac{1}{r_2 + h\beta} - \frac{1}{r_2}\right].$$

But $r_2 + h\beta = r_1$.

$$R = \frac{\rho}{\pi\beta}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{\rho}{\pi}\left(\frac{h}{r_1 - r_2}\right)\left(\frac{r_1 - r_2}{r_1 r_2}\right) = \frac{\rho h}{\pi r_1 r_2}.$$

(b) EVALUATE: Let $r_1 = r_2 = r$. Then $R = \rho h/\pi r^2 = \rho L/A$ where $A = \pi r^2$ and $L = h$. This agrees with $R = \dfrac{\rho L}{A}$.

25.60. **IDENTIFY:** Divide the region into thin spherical shells of radius r and thickness dr. The total resistance is the sum of the resistances of the thin shells and can be obtained by integration.

SET UP: $I = V/R$ and $J = I/4\pi r^2$, where $4\pi r^2$ is the surface area of a shell of radius r.

EXECUTE: **(a)** $dR = \dfrac{\rho dr}{4\pi r^2} \Rightarrow R = \dfrac{\rho}{4\pi}\displaystyle\int_a^b \dfrac{dr}{r^2} = -\dfrac{\rho}{4\pi}\dfrac{1}{r}\Bigg|_a^b = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right) = \dfrac{\rho}{4\pi}\left(\dfrac{b-a}{ab}\right).$

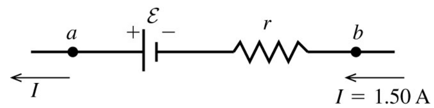
(b) $I = \dfrac{V_{ab}}{R} = \dfrac{V_{ab}4\pi ab}{\rho(b-a)}$ and $J = \dfrac{I}{A} = \dfrac{V_{ab}4\pi ab}{\rho(b-a)4\pi r^2} = \dfrac{V_{ab}ab}{\rho(b-a)r^2}.$

(c) If the thickness of the shells is small, then $4\pi ab \approx 4\pi a^2$ is the surface area of the conducting material. $R = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right) = \dfrac{\rho(b-a)}{4\pi ab} \approx \dfrac{\rho L}{4\pi a^2} = \dfrac{\rho L}{A},$ where $L = b-a.$

EVALUATE: The current density in the material is proportional to $1/r^2$.

25.61. **IDENTIFY:** In each case write the terminal voltage in terms of $\mathcal{E}$, I, and r. Since I is known, this gives two equations in the two unknowns $\mathcal{E}$ and r.

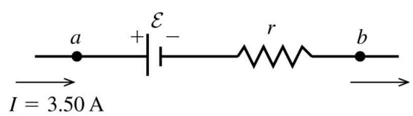
SET UP: The battery with the 1.50-A current is sketched in Figure 25.61a.



$V_{ab} = 8.40$ V.
$V_{ab} = \mathcal{E} - Ir.$
$\mathcal{E} - (1.50\text{ A})r = 8.40$ V.

Figure 25.61a

The battery with the 3.50-A current is sketched in Figure 25.61b.



$V_{ab} = 10.2$ V.
$V_{ab} = \mathcal{E} + Ir.$
$\mathcal{E} + (3.50\text{ A})r = 10.2$ V.

Figure 25.61b

EXECUTE: **(a)** Solve the first equation for $\mathcal{E}$ and use that result in the second equation:
$\mathcal{E} = 8.40\text{ V} + (1.50\text{ A})r.$

$8.40\text{ V} + (1.50\text{ A})r + (3.50\text{ A})r = 10.2\text{ V}.$

$(5.00\text{ A})r = 1.8\text{ V}$ so $r = \dfrac{1.8\text{ V}}{5.00\text{ A}} = 0.36\ \Omega.$

(b) Then $\mathcal{E} = 8.40\text{ V} + (1.50\text{ A})r = 8.40\text{ V} + (1.50\text{ A})(0.36\ \Omega) = 8.94\text{ V}.$

EVALUATE: When the current passes through the emf in the direction from $-$ to $+$, the terminal voltage is less than the emf and when it passes through from $+$ to $-$, the terminal voltage is greater than the emf.

25.62. **IDENTIFY:** Consider the potential changes around the circuit. For a complete loop the sum of the potential changes is zero.

SET UP: There is a potential drop of IR when you pass through a resistor in the direction of the current.

EXECUTE: **(a)** $I = \dfrac{8.0\text{ V} - 4.0\text{ V}}{24.0\ \Omega} = 0.167\text{ A}.$ $V_d + 8.00\text{ V} - I(0.50\ \Omega + 8.00\ \Omega) = V_a,$ so

$V_{ad} = 8.00\text{ V} - (0.167\text{ A})(8.50\ \Omega) = 6.58\text{ V}.$

(b) The terminal voltage is $V_{bc} = V_b - V_c.$ $V_c + 4.00\text{ V} + I(0.50\ \Omega) = V_b$ and

$V_{bc} = +4.00\text{ V} + (0.167\text{ A})(0.50\ \Omega) = +4.08\text{ V}.$

(c) Adding another battery at point d in the opposite sense to the 8.0-V battery produces a counterclockwise current with magnitude $I = \dfrac{10.3\ \text{V} - 8.0\ \text{V} + 4.0\ \text{V}}{24.5\ \Omega} = 0.257\ \text{A}$. Then

$V_c + 4.00\ \text{V} - I(0.50\ \Omega) = V_b$ and $V_{bc} = 4.00\ \text{V} - (0.257\ \text{A})(0.50\ \Omega) = 3.87\ \text{V}$.

EVALUATE: When current enters the battery at its negative terminal, as in part (c), the terminal voltage is less than its emf. When current enters the battery at the positive terminal, as in part (b), the terminal voltage is greater than its emf.

25.63. **IDENTIFY:** $R = \dfrac{\rho L}{A}$. $V = IR$. $P = I^2 R$.

SET UP: The area of the end of a cylinder of radius r is πr^2.

EXECUTE: **(a)** $R = \dfrac{(5.0\ \Omega \cdot \text{m})(1.6\ \text{m})}{\pi (0.050\ \text{m})^2} = 1.0 \times 10^3\ \Omega$.

(b) $V = IR = (100 \times 10^{-3}\ \text{A})(1.0 \times 10^3\ \Omega) = 100\ \text{V}$.

(c) $P = I^2 R = (100 \times 10^{-3}\ \text{A})^2 (1.0 \times 10^3\ \Omega) = 10\ \text{W}$.

EVALUATE: The resistance between the hands when the skin is wet is about a factor of ten less than when the skin is dry (Problem 25.64).

25.64. **IDENTIFY:** $V = IR$. $P = I^2 R$.

SET UP: The total resistance is the resistance of the person plus the internal resistance of the power supply.

EXECUTE: **(a)** $I = \dfrac{V}{R_{\text{tot}}} = \dfrac{14 \times 10^3\ \text{V}}{10 \times 10^3\ \Omega + 2000\ \Omega} = 1.17\ \text{A}$.

(b) $P = I^2 R = (1.17\ \text{A})^2 (10 \times 10^3\ \Omega) = 1.37 \times 10^4\ \text{J} = 13.7\ \text{kJ}$.

(c) $R_{\text{tot}} = \dfrac{V}{I} = \dfrac{14 \times 10^3\ \text{V}}{1.00 \times 10^{-3}\ \text{A}} = 14 \times 10^6\ \Omega$. The resistance of the power supply would need to be $14 \times 10^6\ \Omega - 10 \times 10^3\ \Omega = 14 \times 10^6\ \Omega = 14\ \text{M}\Omega$.

EVALUATE: The current through the body in part (a) is large enough to be fatal.

25.65. **IDENTIFY:** The cost of operating an appliance is proportional to the amount of energy consumed. The energy depends on the power the item consumes and the length of time for which it is operated.

SET UP: At a constant power, the energy is equal to Pt, and the total cost is the cost per kilowatt-hour (kWh) times the energy (in kWh).

EXECUTE: **(a)** Use the fact that $1.00\ \text{k Wh} = (1000\ \text{J/s})(3600\ \text{s}) = 3.60 \times 10^6\ \text{J}$, and one year contains $3.156 \times 10^7\ \text{s}$.

$$(75\ \text{J/s})\left(\frac{3.156 \times 10^7\ \text{s}}{1\ \text{yr}}\right)\left(\frac{\$0.120}{3.60 \times 10^6\ \text{J}}\right) = \$78.90.$$

(b) At $8\ \text{h/day}$, the refrigerator runs for 1/3 of a year. Using the same procedure as above gives

$$(400\ \text{J/s})\left(\frac{1}{3}\right)\left(\frac{3.156 \times 10^7\ \text{s}}{1\ \text{yr}}\right)\left(\frac{\$0.120}{3.60 \times 10^6\ \text{J}}\right) = \$140.27.$$

EVALUATE: Electric lights can be a substantial part of the cost of electricity in the home if they are left on for a long time!

25.66. **(a) IDENTIFY:** The rate of heating (power) in the cable depends on the potential difference across the cable and the resistance of the cable.

SET UP: The power is $P = V^2/R$ and the resistance is $R = \rho L/A$. The diameter D of the cable is twice its radius. $P = \dfrac{V^2}{R} = \dfrac{V^2}{(\rho L/A)} = \dfrac{AV^2}{\rho L} = \dfrac{\pi r^2 V^2}{\rho L}$. The electric field in the cable is equal to the potential difference across its ends divided by the length of the cable: $E = V/L$.

EXECUTE: Solving for r and using the resistivity of copper gives

$$r = \sqrt{\frac{P\rho L}{\pi V^2}} = \sqrt{\frac{(90.0\text{ W})(1.72\times 10^{-8}\ \Omega\cdot\text{m})(1500\text{ m})}{\pi(220.0\text{ V})^2}} = 1.236\times 10^{-4}\text{ m} = 0.1236\text{ mm}.$$

$D = 2r = 0.247$ mm.

(b) IDENTIFY and **SET UP:** $E = V/L$.

EXECUTE: $E = (220\text{ V})/(1500\text{ m}) = 0.147$ V/m.

EVALUATE: This would be an extremely thin (and hence fragile) cable.

25.67. **(a) IDENTIFY:** Since the resistivity is a function of the position along the length of the cylinder, we must integrate to find the resistance.

SET UP: The resistance of a cross-section of thickness dx is $dR = \rho dx/A$.

EXECUTE: Using the given function for the resistivity and integrating gives

$$R = \int \frac{\rho dx}{A} = \int_0^L \frac{(a + bx^2)dx}{\pi r^2} = \frac{aL + bL^3/3}{\pi r^2}.$$

Now get the constants a and b: $\rho(0) = a = 2.25\times 10^{-8}\ \Omega\cdot\text{m}$ and $\rho(L) = a + bL^2$ gives $8.50\times 10^{-8}\ \Omega\cdot\text{m} = 2.25\times 10^{-8}\ \Omega\cdot\text{m} + b(1.50\text{ m})^2$ which gives $b = 2.78\times 10^{-8}\ \Omega/\text{m}$. Now use the above result to find R.

$$R = \frac{(2.25\times 10^{-8}\ \Omega\cdot\text{m})(1.50\text{ m}) + (2.78\times 10^{-8}\ \Omega/\text{m})(1.50\text{ m})^3/3}{\pi(0.0110\text{ m})^2} = 1.71\times 10^{-4}\ \Omega = 171\ \mu\Omega.$$

(b) IDENTIFY: Use the definition of resistivity to find the electric field at the midpoint of the cylinder, where $x = L/2$.

SET UP: $E = \rho J$. Evaluate the resistivity, using the given formula, for $x = L/2$.

EXECUTE: At the midpoint, $x = L/2$, giving $E = \dfrac{\rho I}{\pi r^2} = \dfrac{[a + b(L/2)^2]I}{\pi r^2}$.

$$E = \frac{[2.25\times 10^{-8}\ \Omega\cdot\text{m} + (2.78\times 10^{-8}\ \Omega/\text{m})(0.750\text{ m})^2](1.75\text{ A})}{\pi(0.0110\text{ m})^2} = 1.76\times 10^{-4}\text{ V/m} = 176\ \mu\text{V/m}$$

(c) IDENTIFY: For the first segment, the result is the same as in part (a) except that the upper limit of the integral is $L/2$ instead of L.

SET UP: Integrating using the upper limit of $L/2$ gives $R_1 = \dfrac{a(L/2) + (b/3)(L^3/8)}{\pi r^2}$.

EXECUTE: Substituting the numbers gives

$$R_1 = \frac{(2.25\times 10^{-8}\ \Omega\cdot\text{m})(0.750\text{ m}) + (2.78\times 10^{-8}\ \Omega/\text{m})/3((1.50\text{ m})^3/8)}{\pi(0.0110\text{ m})^2} = 5.47\times 10^{-5}\ \Omega = 54.7\ \mu\Omega.$$

The resistance R_2 of the second half is equal to the total resistance minus the resistance of the first half.

$R_2 = R - R_1 = 1.71\times 10^{-4}\ \Omega - 5.47\times 10^{-5}\ \Omega = 1.16\times 10^{-4}\ \Omega = 116\ \mu\Omega.$

EVALUATE: The second half has a greater resistance than the first half because the resistance increases with distance along the cylinder.

25.68. **IDENTIFY:** Compact fluorescent bulbs draw much less power than incandescent bulbs and last much longer. Hence they cost less to operate.

SET UP: A kWh is power of 1 kW for a time of 1 h. $P = \dfrac{V^2}{R}$.

EXECUTE: **(a)** In 3.0 yr the bulbs are on for $(3.0\text{ yr})(365.24\text{ days/yr})(4.0\text{ h/day}) = 4.38\times10^3\text{ h}$.

Compact bulb: The energy used is $(23\text{ W})(4.38\times10^3\text{ h}) = 1.01\times10^5\text{ Wh} = 101\text{ kWh}$. The cost of this energy is $(\$0.080/\text{kWh})(101\text{ kWh}) = \$8.08$. One bulb will last longer than this. The bulb cost is \$11.00, so the total cost is \$19.08.

Incandescent bulb: The energy used is $(100\text{ W})(4.38\times10^3\text{ h}) = 4.38\times10^5\text{ Wh} = 438\text{ kWh}$. The cost of this energy is $(\$0.080/\text{kWh})(438\text{ kWh}) = \$35.04$. Six bulbs will be used during this time and the bulb cost will be \$4.50. The total cost will be \$39.54.

(b) The compact bulb will save $\$39.54 - \$19.08 = \$20.46$.

(c) $R = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{23\text{ W}} = 626\ \Omega$.

EVALUATE: The initial cost of the bulb is much greater for the compact fluorescent bulb but the savings soon repay the cost of the bulb. The compact bulb should last for over six years, so over a 6-year period the savings per year will be even greater. The cost of compact fluorescent bulbs has come down dramatically, so the savings today would be considerably greater than indicated here.

25.69. **IDENTIFY:** This problem involves capacitance, dielectrics, and resistance.

SET UP: $C = \dfrac{\epsilon_0 A}{d}$, $R = \dfrac{\rho L}{A}$, $P = I^2R$, $\rho = (s_0/s)\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** We want the salinity s when the ammeter reads 484 mA. Using $V = (R + R_{\text{can}})I$ gives 10.0 V = (15.0 Ω + R_{can}) (0.484 A). R_{can} = 5.66 Ω. $R_{\text{can}} = \dfrac{\rho L}{A} = \left(\dfrac{s_0}{s}\right)\dfrac{L}{\pi r^2}$, so $s = \dfrac{s_0 L}{\pi r^2 R_{\text{can}}} =$

$$\frac{(6.30\text{ ppt }\Omega\cdot\text{m})(0.0300\text{ m})}{\pi(0.0500\text{ m})^2(5.66\ \Omega)} = 4.25\text{ ppt}.$$

(b) We want the charge on the left capacitor. Treat the can like an ideal parallel-plate capacitor, so $C = \dfrac{\epsilon_0 AK}{d} = \dfrac{\epsilon_0 \pi r^2 K}{L}$. $V_{\text{can}} = R_{\text{can}}I$ = (5.66 Ω)(0.484 A) = 2.739 V. Now use $Q = CV_{\text{can}}$.

$$Q = \left(\frac{\epsilon_0 \pi r^2 K}{L}\right)V_{\text{can}} = \left(\frac{\epsilon_0 \pi(0.0500\text{ m})^2(80.4)}{0.0300\text{ m}}\right)(2.739\text{ V}) = 510\text{ pC}.$$

(c) We want the power generated in the saline solution. $P = I^2R_{\text{can}} = (0.484\text{ A})^2(5.66\ \Omega) = 1.33\text{ W}$.

(d) We want the salinity level s. $P = V^2/R$, so if R and R_{can} each dissipate half the power of the battery, they must have equal resistances. Therefore $R_{\text{can}} = R = 15.0\ \Omega$. From part (a) we have

$$s = \frac{s_0 L}{\pi r^2 R_{\text{can}}} = \frac{(6.30\text{ ppt }\Omega\cdot\text{m})(0.0300\text{ m})}{\pi(0.0500\text{ m})^2(15.0\ \Omega)} = 1.60\text{ ppt}.$$

EVALUATE: The measurements needed to calculate the salinity s (V, I, and R) are easily made with simple meters, so s could easily be determined by this method.

25.70. **IDENTIFY:** No current flows to the capacitors when they are fully charged.

SET UP: $V_R = RI$ and $V_C = Q/C$.

EXECUTE: **(a)** $V_{C_1} = \dfrac{Q_1}{C_1} = \dfrac{18.0\ \mu\text{C}}{3.00\ \mu\text{F}} = 6.00\text{ V}$. $V_{C_2} = V_{C_1} = 6.00\text{ V}$.

$Q_2 = C_2V_{C_2} = (6.00\ \mu\text{F})(6.00\text{ V}) = 36.0\ \mu\text{C}$.

(b) No current flows to the capacitors when they are fully charged, so $\mathcal{E} = IR_1 + IR_2$.

$V_{R_2} = V_{C_1} = 6.00\text{ V}$. $I = \dfrac{V_{R_2}}{R_2} = \dfrac{6.00\text{ V}}{2.00\ \Omega} = 3.00\text{ A}$.

$$R_1 = \frac{\mathcal{E} - IR_2}{I} = \frac{72.0\text{ V} - 6.00\text{ V}}{3.00\text{ A}} = 22.0\ \Omega.$$

EVALUATE: When a capacitor is fully charged, it acts like an open circuit and prevents any current from flowing though it.

25.71. IDENTIFY: No current flows through the capacitor when it is fully charged.

SET UP: With the capacitor fully charged, $I = \dfrac{\mathcal{E}}{R_1 + R_2}$. $V_R = IR$ and $V_C = Q/C$.

EXECUTE: $V_C = \dfrac{Q}{C} = \dfrac{36.0\ \mu\text{C}}{9.00\ \mu\text{F}} = 4.00\ \text{V}$. $V_{R_1} = V_C = 4.00\ \text{V}$ and $I = \dfrac{V_{R_1}}{R_1} = \dfrac{4.00\ \text{V}}{6.00\ \Omega} = 0.667\ \text{A}$.

$V_{R_2} = IR_2 = (0.667\ \text{A})(4.00\ \Omega) = 2.668\ \text{V}$. $\mathcal{E} = V_{R_1} + V_{R_2} = 4.00\ \text{V} + 2.668\ \text{V} = 6.67\ \text{V}$.

EVALUATE: When a capacitor is fully charged, it acts like an open circuit and prevents any current from flowing though it.

25.72. IDENTIFY and SET UP: Ohm's law applies. The terminal voltage V_{ab} is less than the internal emf $\mathcal{E}$ due to voltage losses in the internal resistance *r* of the battery when current *I* is flowing in the circuit. $V_{ab} = \mathcal{E} - rI$.

EXECUTE: **(a)** The equation $V_{ab} = \mathcal{E} - rI$ applies to this circuit, so a graph of V_{ab} versus *I* should be a straight line with a slope equal to $-r$ and a *y*-intercept equal to $\mathcal{E}$. Using points where the graph crosses grid lines, the slope is: $\text{slope} = \dfrac{22.0\ \text{V} - 30.0\ \text{V}}{7.00\ \text{A} - 3.00\ \text{A}} = -2.00\ \text{V/A}$. Therefore $r = -(-2.00\ \text{V/A}) = 2.00\ \Omega$.

The equation of the graph is $V_{ab} = \mathcal{E} - rI$, so we can solve for $\mathcal{E}$ and use a point on the graph to calculate $\mathcal{E}$. This gives

$\mathcal{E} = V_{ab} + rI = 30.0\ \text{V} + (2.00\ \Omega)(3.00\ \text{A}) = 36.0\ \text{V}$.

(b) $R = V_{ab}/I$ and $I = \dfrac{\mathcal{E} - V_{ab}}{r}$, so $R = \dfrac{V_{ab}}{\dfrac{\mathcal{E} - V_{ab}}{r}} = \dfrac{rV_{ab}}{\mathcal{E} - V_{ab}}$. Putting in the numbers gives

$R = (2.00\ \Omega)(0.800)(36.0\ \text{V})/[36.0\ \text{V} - (0.800)(36.0\ \text{V})] = 8.00\ \Omega$.

EVALUATE: For large currents, the terminal voltage can be much less than the internal emf, as shown by the graph with the problem.

25.73. IDENTIFY: According to Ohm's law, $R = \dfrac{V_{ab}}{I}$ = constant, and a graph of V_{ab} versus *I* will be a straight line with positive slope passing through the origin.

SET UP and EXECUTE: **(a)** Figure 25.73a shows the graphs of V_{ab} versus *I* and *R* versus *I* for resistor A.

Figure 25.73b shows these graphs for resistor B.

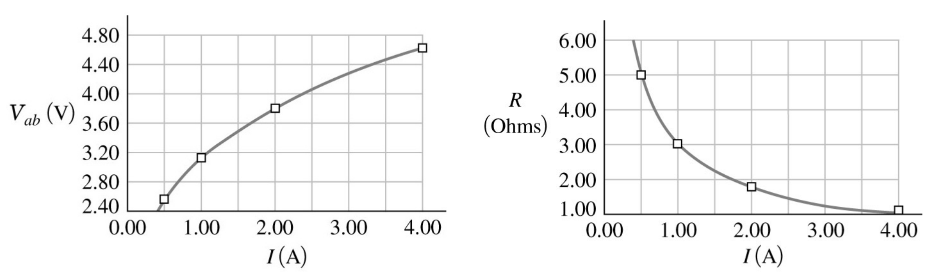


Figure 25.73a

(b) In Figure 25.73a, the graph of V_{ab} versus *I* is not a straight line so resistor A does not obey Ohm's law. In the graph of *R* versus *I*, *R* is not constant; it decreases as *I* increases.

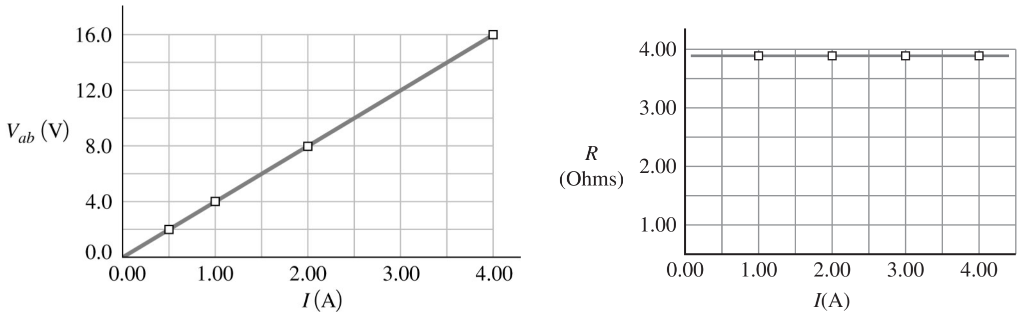


Figure 25.73b

(c) In Figure 25.73b, the graph of V_{ab} versus I is a straight line with positive slope passing through the origin, so resistor B obeys Ohm's law. The graph of R versus I is a horizontal line. This means that R is constant, which is consistent with Ohm's law.

(d) We use $P = IV$. From the graph of V_{ab} versus I in Figure 25.73a, we read that $I = 2.35$ A when $V = 4.00$ V. Therefore $P = IV = (2.35\text{ A})(4.00\text{ V}) = 9.40$ W.

(e) We use $P = V^2/R$. From the graph of R versus I in Figure 25.73b, we find that $R = 3.88\ \Omega$. Thus $P = V^2/R = (4.00\text{ V})^2/(3.88\ \Omega) = 4.12$ W.

EVALUATE: Since resistor B obeys Ohm's law $V_{ab} = RI$, R is the slope of the graph of V_{ab} versus I in Figure 25.73b. The given data points lie on the line, so we use them to calculate the slope.

slope = $R = \dfrac{15.52\text{ V} - 1.94\text{ V}}{4.00\text{ A} - 0.50\text{ A}} = 3.88\ \Omega.$ This value is the same as the one we got from the graph of R versus I in Figure 25.73b, so our results agree.

25.74. IDENTIFY: The power supplied to the house is $P = VI$. The rate at which electrical energy is dissipated in the wires is I^2R, where $R = \dfrac{\rho L}{A}$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** The line voltage, current to be drawn, and wire diameter are what must be considered in household wiring.

(b) $P = VI$ gives $I = \dfrac{P}{V} = \dfrac{4200\text{ W}}{120\text{ V}} = 35\text{ A},$ so the 8-gauge wire is necessary, since it can carry up to 40 A.

(c) $P = I^2R = \dfrac{I^2\rho L}{A} = \dfrac{(35\text{ A})^2\,(1.72\times10^{-8}\ \Omega\cdot\text{m})(42.0\text{ m})}{(\pi/4)(0.00326\text{ m})^{\,2}} = 106\text{ W}.$

(d) If 6-gauge wire is used, $P = \dfrac{I^2\rho L}{A} = \dfrac{(35\text{ A})^2\,(1.72\times10^{-8}\ \Omega\cdot\text{m})\,(42\text{ m})}{(\pi/4)\,(0.00412\text{ m})^2} = 66\text{ W}.$ The decrease in energy consumption is $\Delta E = \Delta P t = (40\text{ W})(365\text{ days/yr})\,(12\text{ h/day}) = 175\text{ kWh/yr}$ and the savings is $(175\text{ kWh/yr})(\$0.11/\text{kWh}) = \$19.25$ per year.

EVALUATE: The cost of the 4200 W used by the appliances is \$2020. The savings is about 1%.

25.75. IDENTIFY: This problem involves resistivity.

SET UP: $E = c/r$, $R = \dfrac{\rho\ell}{A}$.

EXECUTE: **(a)** The outer conductor is at a higher potential than the inner conductor, so the electric field points inward toward the central axis.

(b) The target variable is c, where $E = c/r$. $V = \int_{r_{\text{inner}}}^{r_{\text{outer}}} E_r dr = \int_{r_{\text{inner}}}^{r_{\text{outer}}} \frac{c}{r} dr = c\ln\left(r_{\text{outer}} / r_{\text{inner}}\right)$, which gives

$$c = \frac{V}{\ln\left(r_{\text{outer}} / r_{\text{inner}}\right)}.$$

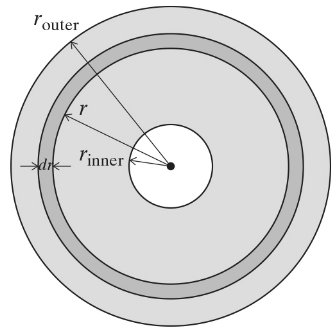


Figure 25.75

(c) We want the resistance of this device. Break the material into infinitesimal coaxial cylindrical shells of radius r, length L, and thickness dr as shown in Fig. 25.75. Apply $R = \frac{\rho \ell}{A}$ where $\ell = dr$ and $A = 2\pi rL$. Doing so gives $R = \int_{r_{\text{inner}}}^{r_{\text{outer}}} \frac{\rho dr}{2\pi rL} = \frac{\rho}{2\pi L}\ln\left(r_{\text{outer}} / r_{\text{inner}}\right)$.

(d) We want the resistivity so that $R = 6.80$ kΩ. Solve the result from (c) for ρ and put in the given quantities, giving $\rho = \frac{2\pi LR}{\ln\left(r_{\text{outer}} / r_{\text{inner}}\right)} = 616\ \Omega \cdot \text{m}$.

EVALUATE: From Table 25.1 we see that this resistivity would be about ¼ that of pure silicon but much less than for insulators and much greater than for conductors.

25.76. **IDENTIFY:** In this problem we investigate the resistance of a light bulb filament as it varies with temperature.

SET UP: $R = \frac{\rho L}{A}$, $R(T) = R_0\left[1 + \alpha\left(T - T_0\right)\right]$, $P = I^2R$.

EXECUTE: **(a)** The target variable is the resistance at 20.0°C. Use $R = \frac{\rho L}{A}$ with ρ at its 20.0°C value. Putting in the given numbers gives $R_{20} = 18.3\ \Omega$.

(b) The target variable is the current when $V = 120$ V. First find T as a function of I. The graph of T versus I is a straight line passing through (0 A, 20.0°C) and (1.00 A, 2520°C), so its slope is 2500°C/A and its T-intercept is 20.0°C. Using the slope-intercept equation of a straight line, the equation is $T(I) = (2500°\text{C/A})I + 20.0°\text{C}$. Using $T_0 = 20.0°\text{C}$, the resistance as a function of temperature is $R(T) = R_{20}\left[1 + \alpha\left(T - 20.0°\text{C}\right)\right]$ and a comparable equation hold for the resistivity. Now combine our equation for $T(I)$ with $R(T)$ to find R as a function of I. $R(I) = R_{20}\left\{1 + \alpha\left[\left(2500°\text{C/A}\right)I + 20.0°\text{C}\right]\right\}$.

Using $V = RI$ gives $V = R_{20}\left\{1 + \alpha\left[\left(2500°\text{C/A}\right)I + 20.0°\text{C}\right]\right\}I$. Now solve for I when $V = 120$ V.

$120\ \text{V} = (18.3\ \Omega)\left\{1 + (0.0045 / \text{C°})\left[\left(2500°\text{C/A}\right)I + 20.0°\text{C}\right]\right\}I$. The positive solution to this quadratic equation is $I = 716$ mA which rounds to 720 mA.

(c) We want R when V is 120 V. $R = V/I = (120\ \text{V})/(0.716\ \text{A}) = 167\ \Omega$.

(d) We want the energy dissipated during 1 minute. $U = Pt = I^2Rt$. Using the results from (b) and (c) gives $U = (0.716\text{ A})^2(167\ \Omega)(60\text{ s}) = 5140\text{ J} = 5.14\text{ kJ}$.
(e) We want the energy when $V = 60$ V. Use $P = IV$ where $V = 60$ V. Get I using the method of part (b) with $V = 60$ V, which gives $I = 0.49355$ A. $R = V/I = (60\text{ V})/(0.49355\text{ A}) = 121.6\ \Omega$. The energy is $U = Pt = I^2Rt = (0.49355\text{ A})^2(121.6\ \Omega)(60\text{ s}) = 1.78$ kJ.
EVALUATE: Our results show that temperature variation can have significant effects on circuits.

25.77. **IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ to find the resistance of a thin slice of the rod and integrate to find the total R. $V = IR$. Also find $R(x)$, the resistance of a length x of the rod.
SET UP: $E(x) = \rho(x)J$

EXECUTE: **(a)** $dR = \dfrac{\rho dx}{A} = \dfrac{\rho_0 \exp[-x/L]\,dx}{A}$ so

$R = \dfrac{\rho_0}{A}\int_0^L \exp\left[-x/L\right] dx = \dfrac{\rho_0}{A}[-L\exp(-x/L)]_0^L = \dfrac{\rho_0 L}{A}(1-e^{-1})$ and $I = \dfrac{V_0}{R} = \dfrac{V_0 A}{\rho_0 L(1-e^{-1})}$. With an upper limit of x rather than L in the integration, $R(x) = \dfrac{\rho_0 L}{A}(1-e^{-x/L})$.

(b) $E(x) = \rho(x)J = \dfrac{I\,\rho_0 e^{-x/L}}{A} = \dfrac{V_0 e^{-x/L}}{L(1-e^{-1})}$.

(c) $V = V_0 - IR(x)$. $V = V_0 - \left(\dfrac{V_0 A}{\rho_0 L[1-e^{-1}]}\right)\left(\dfrac{\rho_0 L}{A}\right)(1-e^{-x/L}) = V_0\dfrac{(e^{-x/L}-e^{-1})}{(1-e^{-1})}$.

(d) Graphs of resistivity, electric field, and potential from $x = 0$ to L are given in Figure 25.77. Each quantity is given in terms of the indicated unit.
EVALUATE: The current is the same at all points in the rod. Where the resistivity is larger the electric field must be larger, in order to produce the same current density.

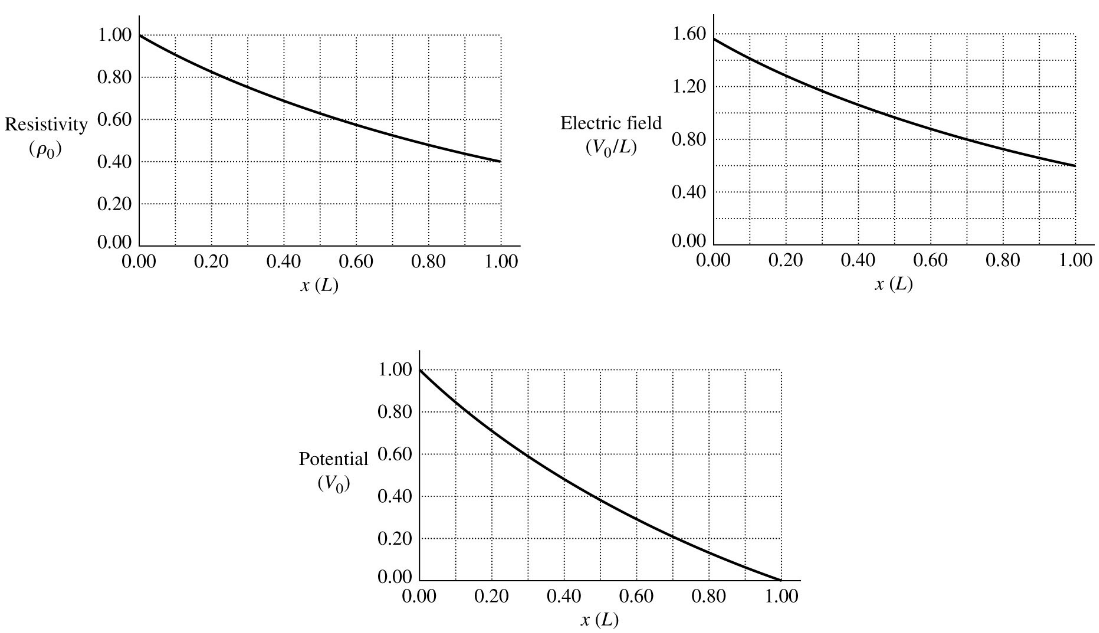


Figure 25.77

25.78. **IDENTIFY** and **SET UP:** The power output P of the source is the power delivered to the resistor R, so P is the power output of the internal emf $\mathcal{E}$ minus the power consumed by the internal resistance r. Therefore $P = \mathcal{E}I - I^2r$. For the entire circuit, $\mathcal{E} = (R + r)I$.

EXECUTE: **(a)** Combining $P = I^2/R$ and $\mathcal{E} = (R + r)I$ gives $P = \left(\dfrac{\mathcal{E}}{R+r}\right)^2 R = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$. From this result, we can see that as $R \to 0,\quad P \to 0.$

(b) Using the same equation as in (a), we see that as $R \to \infty,\quad P \to \dfrac{\mathcal{E}^2}{R} \to 0.$

(c) In (a) we showed that $P = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$. For maximum power, $dP/dR = 0$.

$$\frac{dP}{dR} = \mathcal{E}^2\left[-\frac{2R}{(R+r)^3} + \frac{1}{(R+r)^2}\right] = 0 \qquad \to \qquad \frac{2R}{R+r} = 1 \qquad \to \qquad R = r.$$

The maximum power is therefore

$$P_{\max} = \left.\frac{R\mathcal{E}^2}{(R+r)^2}\right|_{R=r} = \frac{r\mathcal{E}^2}{(2r)^2} = \frac{\mathcal{E}^2}{4r}.$$

(d) Use $P = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$ to calculate P.

<u>For $R = 2.00\ \Omega$</u>: $P_2 = (64.0\text{ V})^2(2.00\ \Omega)/(6.00\ \Omega)^2 = 228$ W.
<u>For $R = 4.00\ \Omega$</u>: $P_4 = (64.0\text{ V})^2(4.00\ \Omega)/(8.00\ \Omega)^2 = 256$ W.
<u>For $R = 6.00\ \Omega$</u>: $P_6 = (64.0\text{ V})^2(6.00\ \Omega)/(10.0\ \Omega)^2 = 246$ W.

EVALUATE: The maximum power in (d) occurred when $R = r = 4.00\ \Omega$, so it is consistent with the result from (c). The equation we found, $P_{\max} = \dfrac{\mathcal{E}^2}{4r}$, gives $P_{\max} = (64.0\text{ V})^2/[4(4.00\ \Omega)] = 256$ W, which agrees with our calculation in (d). When R is smaller than r, I is large and the I^2r losses in the battery are large. When R is larger than r, I is small and the power output $\mathcal{E}I$ of the battery emf is small.

25.79. **IDENTIFY** and **SET UP:** $R = \dfrac{\rho L}{A}$.

EXECUTE: From the equation $R = \dfrac{\rho L}{A}$, if we double the length of a resistor and change nothing else, the resistance will double. But from the data table given in the problem, we see that doubling the length of the thread causes its resistance to do much more than double. For example, at 5 mm the resistance is $9\times10^9\ \Omega$ and at 11 mm (approximately double) the resistance is $63\times10^9\ \Omega$, which is much more than twice the resistance at 5 mm. Therefore as the thread stretches, its coating gets thinner, which decreases its cross-sectional area. This decreased area contributes significantly to the increase in resistance. Therefore choice (c) is correct.

EVALUATE: The cross-sectional area of the coating depends on the square of the radius of the thread, so a decrease in the radius has a very large effect on the resistance.

25.80. **IDENTIFY** and **SET UP:** Use data from the table for 5 mm and 13 mm to compare the resistance. $R = \dfrac{\rho L}{A}$.

EXECUTE: $\dfrac{R_{13}}{R_5} = \dfrac{102}{9} = \dfrac{\dfrac{\rho(13\text{ mm})}{A_5}}{\dfrac{\rho(5\text{ mm})}{A_{13}}} = \dfrac{13A_5}{5A_{13}}$. Solving for A_{13} gives

$$A_{13} = A_5\left(\frac{13}{5}\right)\left(\frac{9}{102}\right) = 0.23 \approx \frac{1}{4},$$ which is choice (b).

EVALUATE: It is reasonable that $A_{13} < A_5$ because the thread and its coating stretch out and get thinner.

25.81. **IDENTIFY** and **SET UP:** Apply Ohm's law, $V = RI$. The minimum resistance will give the maximum current. Get data from the table in the problem.

EXECUTE: $I_{max} = V/R_{min} = (9\text{ V})/(9\times10^9\ \Omega) = 1\times10^{-9}\text{ A} = 1\text{ nA}$, which is choice (d).

EVALUATE: This is a very small current, but the thread of a spider web is very thin.

25.82. **IDENTIFY** and **SET UP:** An electrically neutral conductor contains equal amounts of positive and negative charge, and these charges can move if a charged object comes near to them.

EXECUTE: If a positively charged object comes near to the web, it attracts negative charges in the web. The attraction between these negative charges in the web and the positive charges in the charged object pull the web toward the object. If a negatively charged object comes near the web, it repels negative charges in the web, leaving the web positively charged near the object. The attraction between the negatively charged object and the positive side of the web pulls the web toward the object. This is best explained by choice (d).

EVALUATE: This is similar to the principle of charging by induction. The amounts of charge are small, but the web is moved because it is extremely light.

26

DIRECT-CURRENT CIRCUITS

VP26.2.1. **IDENTIFY:** We have resistors in series and parallel.

SET UP: Parallel: $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots$, series: $R_{eq} = R_1 + R_2 + \cdots$. We want the equivalent resistance in each combination.

EXECUTE: **(a)** Series: $R_{eq} = 1.00\ \Omega + 2.00\ \Omega + 4.00\ \Omega = 7.00\ \Omega$.

(b) Parallel: $1/R_{eq} = 1/(1.00\ \Omega) + 1/(2.00\ \Omega) + 1/(4.00\ \Omega)$. $R_{eq} = 0.571\ \Omega$.

(c) Series/parallel combination: R_2 and R_3 in parallel. $1/R_p = 1/(2.00\ \Omega) + 1/(4.00\ \Omega)$. $R_p = 1.33\ \Omega$. $R_{eq} = R_1 + R_p = 1.00\ \Omega + 1.33\ \Omega = 2.33\ \Omega$.

(d) Series/parallel combination: R_2 and R_3 in series: $R_s = 2.00\ \Omega + 4.00\ \Omega = 6.00\ \Omega$. The parallel combination: $1/R_{eq} = 1/R_1 + 1/R_s = 1/(1.00\ \Omega) + 1/(6.00\ \Omega)$. $R_{eq} = 0.857\ \Omega$.

EVALUATE: Note that for a parallel combination the equivalent resistance is *less than* the *smallest* resistance. For a series combination, R_{eq} is *greater than* the *largest* resistance.

VP26.2.2. **IDENTIFY:** We have resistors in series and parallel and a battery.

SET UP: Parallel: $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots$, series: $R_{eq} = R_1 + R_2 + \cdots$. We want the currents.

EXECUTE: **(a)** We want the current through the battery. First find R_{eq} for the circuit. R_2 and R_3 are in series, so $R_s = 11.0\ \Omega$. This combination is in parallel with R_1 so $1/R_{eq} = 1/R_1 + 1/R_s$. This gives $1/R_{eq} = 1/(4.00\ \Omega) + 1/(11.0\ \Omega)$. $R_{eq} = 2.933\ \Omega$. $= (24.0\ \text{V})/(2.933\ \Omega) = 8.18\ \text{A}$.

(b) We want the current through R_1. $I_1 = \mathcal{E}/R_1 = (24.0\ \text{V})/(4.00\ \Omega) = 6.00\ \text{A}$.

(c) We want the current through R_2. $I_2 = \mathcal{E}/(R_2 + R_3) = (24.0\ \text{V})/(11.0\ \Omega) = 2.18\ \text{A}$.

(d) We want the current through R_3. The resistors are in series so $I_3 = I_2 = 2.18\ \text{A}$.

EVALUATE: Check: $I_1 + I_2 = 6.00\ \text{A} + 2.18\ \text{A} = 8.18\ \text{A}$, which is the current from the battery as it should be.

VP26.2.3. **IDENTIFY:** We have three resistors in parallel across a battery. We want the power in each circuit element.

SET UP: $P = I^2R = V^2/R = IV$. $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots$ The potential difference is 12.0 V across each resistor.

EXECUTE: **(a)** We want $P_{battery}$. First get the equivalent resistance to find the current the battery puts out. $1/R_{eq} = 1/(7.00\ \Omega) + 1/(8.00\ \Omega) + 1/(9.00\ \Omega)$. $R_{eq} = 2.64\ \Omega$. Now find I. $I = \mathcal{E}/R_{eq} = (12.0\ \text{V})/(2.64\ \Omega) = 4.548\ \text{A}$. $P_{battery} = I\mathcal{E} = (4.548\ \text{A})(12.0\ \text{V}) = 54.6\ \text{W}$.

(b) We want the power dissipated in R_1. $P_1 = \mathcal{E}^2/R_1 = (12.0\ \text{V})^2/(7.00\ \Omega) = 20.6\ \text{W}$.

(c) $P_2 = (24.0\ \text{V})^2/(8.00\ \Omega) = 18.0\ \text{W}$.

(d) $P_3 = (12.0\ \text{V})^2/(9.00\ \Omega) = 16.0\ \text{W}$.

EVALUATE: The total power dissipated in the resistors is 20.6 W + 18.0 W + 16.0 W = 54.6 W, which is the power output of the battery. This agrees with energy conservation.

VP26.2.4. **IDENTIFY:** We have a series/parallel resistor combination across a battery. We want the power in each circuit element.

SET UP: $P = I^2R = V^2/R = IV$. $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots$. $V = RI$.

EXECUTE: **(a)** We want $P_{battery}$. First get the equivalent resistance of the circuit. For the parallel part: $1/R_p = 1/(6.00\ \Omega) + 1/(7.00\ \Omega)$. $R_p = 3.231\ \Omega$. $R_{eq} = 5.00\ \Omega + 3.231\ \Omega = 8.23\ \Omega$. $I = \mathcal{E}/R_{eq} = (9.00\ \text{V})/(8.23\ \Omega) = 1.0935$ A. $P_{battery} = I\mathcal{E} = (1.0935\ \text{A})(9.00\ \text{V}) = 9.84$ W.

(b) We want P_1. $P_1 = I_1^2R_1 = I^2R_1 = (1.0935\ \text{A})^2(5.00\ \Omega) = 5.98$ W.

(c) We want P_2. $V_2 = \mathcal{E} - V_1 = \mathcal{E} - R_1I_1 = 9.00\ \text{V} - (5.00\ \Omega)(1.0935\ \text{A}) = 3.5325$ V. $P_2 = V_2^2/R_2 = (3.5325\ \text{V})^2/(6.00\ \Omega) = 2.08$ W.

(d) $P_3 = (3.5325\ \text{V})^2/(7.00\ \Omega) = 1.78$ W.

EVALUATE: Check: $P_1 + P_2 + P_3 = 5.98\ \text{W} + 2.08\ \text{W} + 1.78\ \text{W} = 9.84$ W, which is the battery power, as it should be by energy conservation.

VP26.7.1. **IDENTIFY:** We have a circuit with two batteries. Kirchhoff's rules apply.

SET UP: $P = IV = I^2R$.

EXECUTE: **(a)** We want the current. Make a loop using the same path as in Fig. 26.10(a) in the text. This gives: $12\ \text{V} - (2\ \Omega)I - (3\ \Omega)I - (4\ \Omega)I + 4\ \text{V} - (7\ \Omega)I = 0$. $I = 1$ A.

(b) $V_{ab} = V_a - V_b = (7\ \Omega)(1\ \text{A}) - 4\ \text{V} + (4\ \Omega)(1\ \text{A}) = +7$ V.

(c) We want the power. $P_4 = I\mathcal{E}_4 = (1\ \text{A})(4\ \text{V}) = 4$ W. $P_{12} = (1\ \text{A})(12\ \text{V}) = 12$ W.

EVALUATE: Check: Power dissipated in the resistors is $I^2R = (1\ \text{A})^2(16\ \Omega) = 16$ W. The power output of the batteries is 4W + 12 W = 16 W. They agree, as they should by energy conservation.

VP26.7.2. **IDENTIFY:** This problem requires Kirchhoff's rules.

SET UP: Refer to Fig. 26.6(a) in the text.

EXECUTE: **(a)** We want $V_{ab} = V_a - V_b$. $I_R = I_1 + I_2 = 1.55$ A. Go from b to a through R. This gives $V_{ab} = (5.00\ \Omega)(1.55\ \text{A}) = +7.75$ V.

(b) We want r_1. $V_{ac} = V_{ab} = 7.75$ V. Go from $c \to a$: $8.00\ \text{V} - r_1(0.200\ \text{A}) = 7.75$ V. $r_1 = 1.25\ \Omega$.

(c) We want r_2. $V_{ab} = 9.00\ \text{V} - r_2(1.35\ \text{A}) = 7.75$ V. $r_2 = 0.926\ \Omega$.

EVALUATE: Check for r_1: Make a loop from $c \to a \to b \to c$ through R, giving $+8.00\ \text{V} - r_1(0.200\ \text{A}) - (5.00\ \Omega)(1.55\ \text{A}) = 0$, which gives $r_1 = 1.25\ \Omega$, in agreement with our result.

VP26.7.3. **IDENTIFY:** We need to use Kirchhoff's rules and want to find the power in each resistor.

SET UP: Refer to Fig. 26.12 in the text and the equations in Example 26.6. We first must find the currents, as in the example in the text. Using the same loops, we find that Equations (1), (3), and $(1')$ are the same as in the example. Equation (2) becomes: $-I_2(1\ \Omega) - (I_2 + I_3)(3\ \Omega) + 13\ \text{V} = 0$. Equation $(2')$ becomes: $13\ \text{V} = I_1(4\ \Omega) + I_3(7\ \Omega)$. Solve these equations as was shown in the example. The results are: $I_1 = 5.778$ A, $I_2 = 4.333$ A, and $I_3 = -1.445$ A. Now we can find the powers.

EXECUTE: **(a)** $P_{ca} = I_1^2(1\ \Omega) = (5.778\ \text{A})^2(1\ \Omega) = 33.4$ W.

(b) $P_{cb} = I_2^2(1\ \Omega) = (4.333\ \text{A})^2(1\ \Omega) = 18.8$ W.

(c) $P_{ab} = I_3^2(1\ \Omega) = (-1.445\ \text{A})^2(1\ \Omega) = 2.09$ W.

(d) $P_{ad} = (I_1 - I_3)^2(1\ \Omega) = [5.778\ \text{A} - (-1.445\ \text{A})](1\ \Omega) = 2.09$ W.

(e) $P_{bd} = (I_2 + I_3)^2(3\ \Omega) = (4.333\ \text{A} - 1.445\ \text{A})^2(3\ \Omega) = 25.0$ W.

EVALUATE: Check: $P_R = 33.4\ \text{W} + 18.8\ \text{W} + 2.09\ \text{W} + 52.2\ \text{W} + 25.0\ \text{W} = 131$ W. The power the battery produces is $P_{battery} = I\mathcal{E} = (I_1 + I_2)\mathcal{E} = (5.778\ \text{A} + 4.333\ \text{A})(13\ \text{V}) = 131$ W. Our results are consistent with the conservation of energy.

VP26.7.4. **IDENTIFY:** Kirchhoff's rules apply to this circuit. We want the currents.

SET UP: Refer to Fig. VP26.7.4. Use the same approach as shown in Fig. 26.9(a) in the text.

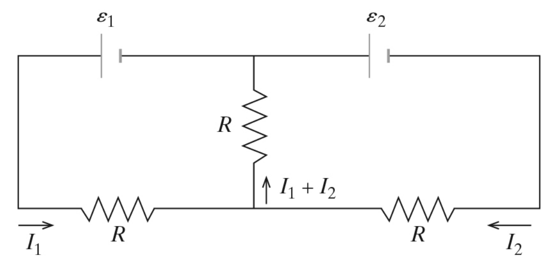


Figure VP26.7.4

EXECUTE: Take a counterclockwise loop in the left half of the circuit, which gives $\mathcal{E}_1 - RI_1 - (I_1 + I_2)R = 0.$ This simplifies to s $\mathcal{E}_1 = 2RI_1 + RI_2$ (Eq. 1)

Take a counterclockwise loop in the right half of the circuit, which gives $\mathcal{E}_2 + R(I_1 + I_2) + I_2R = 0.$ This simplifies to $\mathcal{E}_2 = -RI_1 - 2RI_2$ (Eq. 2)

The junction rule gives $I_3 = I_1 + I_2$ (Eq. 3)

Solving these three equations by substitution (or any other method) gives the following results:

(a) $I_1 = \dfrac{2\mathcal{E}_1 + \mathcal{E}_2}{3R}.$

(b) $I_2 = -\dfrac{\mathcal{E}_1 + 2\mathcal{E}_2}{3R}.$

(c) $I_3 = -\dfrac{\mathcal{E}_1 - \mathcal{E}_2}{3R}.$

EVALUATE: It is not possible to solve a circuit like this with simple series/parallel reduction.

VP26.13.1. **IDENTIFY:** This circuit is a charging capacitor in an *R-C* circuit.

SET UP: $q = Q_f(1 - e^{-t/RC}),\ i = I_0 e^{-t/RC},\ \tau = RC.$

EXECUTE: **(a)** We want the final charge. The current has stopped, so $\mathcal{E} = V_C = Q_f/C$. Therefore $Q_f = C\mathcal{E} = (3.20\ \mu\text{F})(9.00\ \text{V}) = 28.8\ \mu\text{C}$.

(b) We want the initial current. Initially $q_C = 0$, so $\mathcal{E} = V_R = RI_0$. $I_0 = \mathcal{E}/R = (9.00\ \text{V})/(10.0\ \text{M}\Omega) = 0.900\ \mu\text{A}$.

(c) We want the time constant. $\tau = RC = (10.0\ \text{M}\Omega)(3.20\ \mu\text{F}) = 32.0\ \text{s}$.

(d) We want q/Q_f at $t = 18.0$ s. Using $q = Q_f(1 - e^{-t/RC})$ gives $q/Q_f = 1 - e^{-(18.0\ \text{s})/(32.0\ \text{s})} = 0.430$.

(e) We want i/I_0 at 18.0 s. Using $i = I_0 e^{-t/RC}$ gives $i/I_0 = e^{-(18.0\ \text{s})/(32.0\ \text{s})} = 0.570$.

EVALUATE: As the current in the circuit decreases the charge on the capacitor increases. Both of them follow a form of exponential change.

VP26.13.2. **IDENTIFY:** This circuit is a discharging capacitor in an *R-C* circuit.

SET UP: $i = I_0 e^{-t/RC}$, $q = Q_0 e^{-t/RC}$

EXECUTE: **(a)** We want the time when $q = 1.20\ \mu\text{C}$. Solve $q = Q_0 e^{-t/RC}$ for t by taking logarithms, giving $t = -RC \ln(q/Q_0) = -(4.00\ \text{M}\Omega)(2.20\ \mu\text{C}) \ln(1.20/4.20) = 11.0$ s.

(b) We want the current at $t = 11.0$ s. $i = I_0 e^{-t/RC}$. From (a), $e^{-t/RC} = q/Q_0 = 1.20/4.20 = 0.2857$. Find I_0:

$$I_0 = \frac{V_0}{R} = \frac{Q_0/C}{R} = \frac{4.20\ \mu\text{C}}{(4.00\ \text{M}\Omega)(2.20\ \mu\text{F})} = 4.77\times10^{-7}\ \text{A}.\ i = (4.77\times10^{-7}\ \text{A})(0.2857) = 1.36\times10^{-7}\ \text{A}.$$

EVALUATE: For a discharging capacitor, both the current and the charge on the capacitor plates decrease exponentially.

VP26.13.3. **IDENTIFY:** We have a discharging capacitor.

SET UP: $i = I_0 e^{-t/RC}$, $q = Q_0 e^{-t/RC}$

EXECUTE: **(a)** We want the resistance. Solving $q = Q_0 e^{-t/RC}$ for R gives

$$R = -\frac{t}{C\ln(q/Q_0)} = -\frac{17.0 \text{ s}}{(8.00\ \mu\text{F})\ln(1.10/5.50)} = 1.32\ \text{M}\Omega.$$

(b) We want I_0. $I_0 = \dfrac{V_0}{R} = \dfrac{Q_0/C}{R} = \dfrac{5.50\ \mu\text{C}}{(1.32\ \text{M}\Omega)(8.00\ \mu F)} = 5.21\times10^{-7}$ A.

(c) We want the current at t = 17.0 s. $i = I_0 e^{-t/RC} = 1.04\times10^{-7}$ A using the given and calculated values for t, R, C and I_0.

EVALUATE: At t = 17.0 s, check V_C and V_R. $V_C = q/C$ = (1.10 μC)/(8.00 μF) = 0.1375 V. $V_R = Ri$ = (1.32 MΩ) (1.04×10^{-7} A) = 0.1374 V. They agree, as they should. The tiny difference is due to rounding.

VP26.13.4. **IDENTIFY:** This problem involves a capacitor that charges and then discharges.

SET UP and **EXECUTE:** **(a)** We want the charge at 51.0 s. We have a charging capacitor so $q = C\mathcal{E}(1-e^{-t/RC})$. Using t = 51.0 s, R = 5.00 MΩ, and the other given values, we get q = 61.2 μC.

(b) We want q 70.0 s after closing the switch. We have a discharging capacitor with Q_0 = 61.2 μC. Using $q = Q_0 e^{-t/RC}$ with t = 70.0 s, R = 6.00 MΩ and the other given values, we get q = 9.89 μC.

EVALUATE: The charging time constant is different from the discharging time constant because the resistance is different in the two cases.

26.1. **IDENTIFY:** The newly-formed wire is a combination of series and parallel resistors.

SET UP: Each of the three linear segments has resistance $R/3$. The circle is two $R/6$ resistors in parallel.

EXECUTE: The resistance of the circle is $R/12$ since it consists of two $R/6$ resistors in parallel. The equivalent resistance is two $R/3$ resistors in series with an $R/12$ resistor, giving $R_{\text{equiv}} = R/3 + R/3 + R/12 = 3R/4.$

EVALUATE: The equivalent resistance of the original wire has been reduced because the circle's resistance is less than it was as a linear wire.

26.2. **IDENTIFY:** It may appear that the meter measures X directly. But note that X is in parallel with three other resistors, so the meter measures the equivalent parallel resistance between ab.

SET UP: We use the formula for resistors in parallel.

EXECUTE: $1/(2.00\ \Omega) = 1/X + 1/(15.0\ \Omega) + 1/(5.0\ \Omega) + 1/(10.0\ \Omega)$, so $X = 7.5\ \Omega$.

EVALUATE: X is *greater* than the equivalent parallel resistance of $2.00\ \Omega$.

26.3. **IDENTIFY:** The emf of the battery remains constant, but changing the resistance across it changes its power output.

SET UP: The power consumption in a resistor is $P = \dfrac{V^2}{R}$.

EXECUTE: With just R_1, $P_1 = \dfrac{V^2}{R_1}$ and $V = \sqrt{P_1R_1} = \sqrt{(36.0\ \text{W})(25.0\ \Omega)} = 30.0\ \text{V}$ is the battery voltage. With R_2 added, $R_{\text{tot}} = 40.0\ \Omega$. $P = \dfrac{V^2}{R_{\text{tot}}} = \dfrac{(30.0\ \text{V})^2}{40.0\ \Omega} = 22.5\ \text{W}$.

EVALUATE: The two resistors in series dissipate electrical energy at a smaller rate than R_1 alone.

26.4. **IDENTIFY:** For resistors in parallel the voltages are the same and equal to the voltage across the equivalent resistance.

SET UP: $V = IR.$ $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}.$

EXECUTE: **(a)** $R_{eq} = \left(\dfrac{1}{42\ \Omega} + \dfrac{1}{20\ \Omega}\right)^{-1} = 13.548\ \Omega,$ which rounds to 13 Ω.

(b) $I = \dfrac{V}{R_{eq}} = \dfrac{240\ \text{V}}{13.548\ \Omega} = 17.7\ \text{A},$ which rounds to 18 A.

(c) $I_{42\Omega} = \dfrac{V}{R} = \dfrac{240\ \text{V}}{42\ \Omega} = 5.7\ \text{A};$ $I_{20\Omega} = \dfrac{V}{R} = \dfrac{240\ \text{V}}{20\ \Omega} = 12\ \text{A}.$

EVALUATE: More current flows through the resistor that has the smaller R.

26.5. **IDENTIFY:** The equivalent resistance will vary for the different connections because the series-parallel combinations vary, and hence the current will vary.

SET UP: First calculate the equivalent resistance using the series-parallel formulas, then use Ohm's law $(V = RI)$ to find the current.

EXECUTE: **(a)** $1/R = 1/(15.0\ \Omega) + 1/(30.0\ \Omega)$ gives $R = 10.0\ \Omega.$ $I = V/R = (35.0\ \text{V})/(10.0\ \Omega) = 3.50\ \text{A}.$

(b) $1/R = 1/(10.0\ \Omega) + 1/(35.0\ \Omega)$ gives $R = 7.78\ \Omega.$ $I = (35.0\ \text{V})/(7.78\ \Omega) = 4.50\ \text{A}.$

(c) $1/R = 1/(20.0\ \Omega) + 1/(25.0\ \Omega)$ gives $R = 11.11\ \Omega,$ so $I = (35.0\ \text{V})/(11.11\ \Omega) = 3.15\ \text{A}.$

(d) From part (b), the resistance of the triangle alone is 7.78 Ω. Adding the 3.00-Ω internal resistance of the battery gives an equivalent resistance for the circuit of 10.78 Ω. Therefore the current is $I = (35.0\text{V})/(10.78\ \Omega) = 3.25\ \text{A}.$

EVALUATE: It makes a big difference how the triangle is connected to the battery.

26.6. **IDENTIFY:** The potential drop is the same across the resistors in parallel, and the current into the parallel combination is the same as the current through the 45.0-Ω resistor.

(a) SET UP: Apply Ohm's law in the parallel branch to find the current through the 45.0-Ω resistor. Then apply Ohm's law to the 45.0-Ω resistor to find the potential drop across it.

EXECUTE: The potential drop across the 25.0-Ω resistor is $V_{25} = (25.0\ \Omega)(1.25\ \text{A}) = 31.25\ \text{V}.$ The potential drop across each of the parallel branches is 31.25 V. For the 15.0-Ω resistor: $I_{15} = (31.25\text{V})/(15.0\ \Omega) = 2.083\ \text{A}.$ The resistance of the $10.0\text{-}\Omega + 15.0\text{-}\Omega$ combination is $25.0\ \Omega,$ so the current through it must be the same as the current through the upper 25.0-Ω resistor: $I_{10+15} = 1.25\ \text{A}.$ The sum of currents in the parallel branch will be the current through the 45.0-Ω resistor.

$$I_{Total} = 1.25\ \text{A} + 2.083\ \text{A} + 1.25\ \text{A} = 4.58\ \text{A}.$$

Apply Ohm's law to the 45.0-Ω resistor: $V_{45} = (4.58\ \text{A})(45.0\ \Omega) = 206\ \text{V}.$

(b) SET UP: First find the equivalent resistance of the circuit and then apply Ohm's law to it.

EXECUTE: The resistance of the parallel branch is $1/R = 1/(25.0\ \Omega) + 1/(15.0\ \Omega) + 1/(25.0\ \Omega),$ so $R = 6.82\ \Omega.$ The equivalent resistance of the circuit is $6.82\ \Omega + 45.0\ \Omega + 35.00\ \Omega = 86.82\ \Omega.$ Ohm's law gives $V_{Bat} = (86.62\ \Omega)(4.58\ \text{A}) = 398\ \text{V}.$

EVALUATE: The emf of the battery is the sum of the potential drops across each of the three segments (parallel branch and two series resistors).

26.7. **IDENTIFY:** First do as much series-parallel reduction as possible.

SET UP: The 45.0-Ω and 15.0-Ω resistors are in parallel, so first reduce them to a single equivalent resistance. Then find the equivalent series resistance of the circuit.

EXECUTE: $1/R_p = 1/(45.0\ \Omega) + 1/(15.0\ \Omega)$ and $R_p = 11.25\ \Omega.$ The total equivalent resistance is $18.0\ \Omega + 11.25\ \Omega + 3.26\ \Omega = 32.5\ \Omega.$ Ohm's law gives $I = (25.0\ \text{V})/(32.5\ \Omega) = 0.769\ \text{A}.$

EVALUATE: The circuit appears complicated until we realize that the 45.0-Ω and 15.0-Ω resistors are in parallel.

26.8. IDENTIFY: We are measuring an unknown resistance using Ohm's law.
SET UP and **EXECUTE:** We want R_1. For resistors in parallel $1/R_{\text{eq}} = 1/R_1 + 1/R_2 + \cdots$ The graph plots I versus V, so we need to find a relationship between those variables. Ohm's law gives $I = \dfrac{V}{R_{\text{eq}}} = V\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$. So a graph of I versus V should be a straight line having slope equal to $\dfrac{1}{R_1} + \dfrac{1}{R_2}$. Solve for R_1: $R_1 = \dfrac{1}{\text{slope} - 1/R_2} = \dfrac{1}{0.208\ \Omega^{-1} - 1/(8.00\ \Omega)} = 12.0\ \Omega.$
EVALUATE: This procedure might be useful if R_1 were not accessible to direct measurement, or if one lacked a working ohmmeter.

26.9. IDENTIFY: We have a combination of resistors in a circuit.
SET UP and **EXECUTE:** **(a)** We want the current. The potential difference is the same across all the resistors in parallel, so $I = \dfrac{\mathcal{E}}{R}$ for each of the resistors since they are equal.

(b) We want the current. $R_{\text{eq}} = R_1 + R_2 + \cdots = 6R$. $I = \dfrac{\mathcal{E}}{6R}$. The same current flows through all of the series resistors.

(c) Parallel: $P_{\text{p}} = \dfrac{\mathcal{E}^2}{R}$ in each resistor.

Series: $P_{\text{s}} = I^2 R = \left(\dfrac{\mathcal{E}}{6R}\right)^2 R = \dfrac{\mathcal{E}^2}{36R}.$

The power is greatest in the parallel connection.

EVALUATE: An alternate approach in (c): $P_{\text{s}} = \dfrac{V_R^2}{R} = \dfrac{(\mathcal{E}/6)^2}{R} = \dfrac{\mathcal{E}^2}{36R}$, which agrees with our result.

26.10. IDENTIFY: The current, and hence the power, depends on the potential difference across the resistor.
SET UP: $P = V^2/R$.
EXECUTE: **(a)** $V = \sqrt{PR} = \sqrt{(5.0\ \text{W})(15{,}000\ \Omega)} = 274\ \text{V}$.

(b) $P = V^2/R = (120\ \text{V})^2/(9{,}000\ \Omega) = 1.6\ \text{W}$.

(c) SET UP: If the larger resistor generates 2.00 W, the smaller one will generate less and hence will be safe. Therefore the maximum power in the larger resistor must be 2.00 W. Use $P = I^2 R$ to find the maximum current through the series combination and use Ohm's law to find the potential difference across the combination.
EXECUTE: $P = I^2 R$ gives $I = \sqrt{P/R} = \sqrt{(2.00\ \text{W})/(150\ \Omega)} = 0.115\ \text{A}$. The same current flows through both resistors, and their equivalent resistance is $250\ \Omega$. Ohm's law gives $V = IR = (0.115\ \text{A})(250\ \Omega) = 28.8\ \text{V}$. Therefore $P_{150} = 2.00\ \text{W}$ and $P_{100} = I^2 R = (0.115\ \text{A})^2 (100\ \Omega) = 1.32\ \text{W}$.
EVALUATE: If the resistors in a series combination all have the same power rating, it is the *largest* resistance that limits the amount of current.

26.11. IDENTIFY and **SET UP:** Ohm's law applies to the resistors, the potential drop across resistors in parallel is the same for each of them, and at a junction the currents in must equal the currents out.
EXECUTE: **(a)** $V_2 = I_2 R_2 = (4.00\ \text{A})(6.00\ \Omega) = 24.0\ \text{V}$. $V_1 = V_2 = 24.0\ \text{V}$.

$I_1 = \dfrac{V_1}{R_1} = \dfrac{24.0\ \text{V}}{3.00\ \Omega} = 8.00\ \text{A}.\ \ I_3 = I_1 + I_2 = 4.00\ \text{A} + 8.00\ \text{A} = 12.0\ \text{A}.$

(b) $V_3 = I_3 R_3 = (12.0\ \text{A})(5.00\ \Omega) = 60.0\ \text{V}.\ \ \mathcal{E} = V_1 + V_3 = 24.0\ \text{V} + 60.0\ \text{V} = 84.0\ \text{V}.$

EVALUATE: Series/parallel reduction was not necessary in this case.

26.12. **IDENTIFY** and **SET UP:** Ohm's law applies to the resistors, and at a junction the currents in must equal the currents out.

EXECUTE: $V_1 = I_1 R_1 = (1.50\ \text{A})(5.00\ \Omega) = 7.50\ \text{V}.\ \ V_2 = 7.50\ \text{V}.\ \ I_1 + I_2 = I_3$ so

$I_2 = I_3 - I_1 = 4.50\ \text{A} - 1.50\ \text{A} = 3.00\ \text{A}.\ \ R_2 = \dfrac{V_2}{I_2} = \dfrac{7.50\ \text{V}}{3.00\ \text{A}} = 2.50\ \Omega.$

$V_3 = \mathcal{E} - V_1 = 35.0\ \text{V} - 7.50\ \text{V} = 27.5\ \text{V}.\ \ R_3 = \dfrac{V_3}{I_3} = \dfrac{27.5\ \text{V}}{4.50\ \text{A}} = 6.11\ \Omega.$

EVALUATE: Series/parallel reduction was not necessary in this case.

26.13. **IDENTIFY:** For resistors in parallel, the voltages are the same and the currents add. $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ so

$R_{\text{eq}} = \dfrac{R_1 R_2}{R_1 + R_2}$, For resistors in series, the currents are the same and the voltages add. $R_{\text{eq}} = R_1 + R_2$.

SET UP: The rules for combining resistors in series and parallel lead to the sequences of equivalent circuits shown in Figure 26.13.

EXECUTE: In Figure 26.13c, $I = \dfrac{60.0\ \text{V}}{5.00\ \Omega} = 12.0\ \text{A}$. This is the current through each of the resistors in Figure 26.13b. $V_{12} = IR_{12} = (12.0\ \text{A})(2.00\ \Omega) = 24.0\ \text{V}.\ \ V_{34} = IR_{34} = (12.0\ \text{A})(3.00\ \Omega) = 36.0\ \text{V}$. Note that $V_{12} + V_{34} = 60.0\ \text{V}$. V_{12} is the voltage across R_1 and across R_2, so $I_1 = \dfrac{V_{12}}{R_1} = \dfrac{24.0\ \text{V}}{3.00\ \Omega} = 8.00\ \text{A}$ and V_{34} is the voltage across R_3 and across R_4, so $I_3 = \dfrac{V_{34}}{R_3} = \dfrac{36.0\ \text{V}}{12.0\ \Omega} = 3.00\ \text{A}$ and

$I_4 = \dfrac{V_{34}}{R_4} = \dfrac{36.0\ \text{V}}{4.00\ \Omega} = 9.00\ \text{A}.$

EVALUATE: Note that $I_1 + I_2 = I_3 + I_4$.

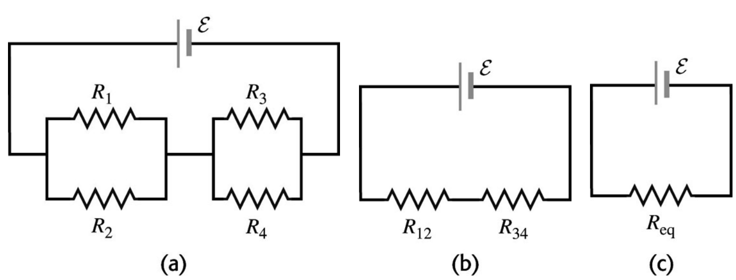


Figure 26.13

26.14. **IDENTIFY:** Replace the series combinations of resistors by their equivalents. In the resulting parallel network the battery voltage is the voltage across each resistor.

SET UP: The circuit is sketched in Figure 26.14a.

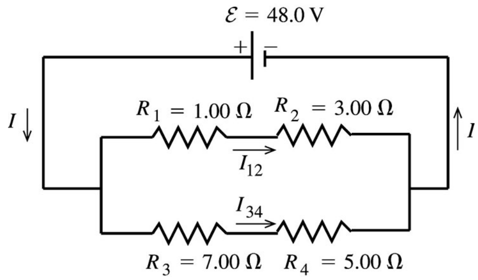


Figure 26.14a

EXECUTE: R_1 and R_2 in series have an equivalent resistance of $R_{12} = R_1 + R_2 = 4.00\ \Omega$. R_3 and R_4 in series have an equivalent resistance of $R_{34} = R_3 + R_4 = 12.0\ \Omega$.

The circuit is equivalent to the circuit sketched in Figure 26.14b.

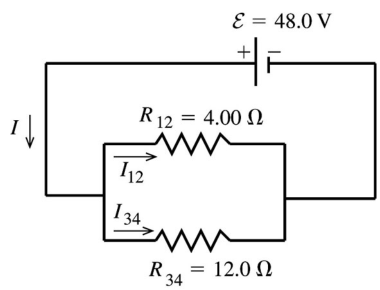


Figure 26.14b

R_{12} and R_{34} in parallel are equivalent to R_{eq} given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_{12}} + \dfrac{1}{R_{34}} = \dfrac{R_{12}+R_{34}}{R_{12}R_{34}}$.

$R_{eq} = \dfrac{R_{12}R_{34}}{R_{12}+R_{34}}$. $R_{eq} = \dfrac{(4.00\ \Omega)(12.0\ \Omega)}{4.00\ \Omega + 12.0\ \Omega} = 3.00\ \Omega$.

The voltage across each bred belowanch of the parallel combination is $\mathcal{E}$, so $\mathcal{E} - I_{12}R_{12} = 0$.

$$I_{12} = \frac{\mathcal{E}}{R_{12}} = \frac{48.0\ \text{V}}{4.00\ \Omega} = 12.0\ \text{A}.$$

$$\mathcal{E} - I_{34}R_{34} = 0 \text{ so } I_{34} = \frac{\mathcal{E}}{R_{34}} = \frac{48.0\ \text{V}}{12.0\ \Omega} = 4.0\ \text{A}.$$

The current is 12.0 A through the 1.00-Ω and 3.00-Ω resistors, and it is 4.0 A through the 7.00-Ω and 5.00-Ω resistors.

EVALUATE: The current through the battery is $I = I_{12} + I_{34} = 12.0\ \text{A} + 4.0\ \text{A} = 16.0\ \text{A}$, and this is equal to $\mathcal{E}/R_{eq} = 48.0\ \text{V}/3.00\ \Omega = 16.0\ \text{A}$.

26.15. **IDENTIFY:** In both circuits, with and without R_4, replace series and parallel combinations of resistors by their equivalents. Calculate the currents and voltages in the equivalent circuit and infer from this the currents and voltages in the original circuit. Use $P = I^2R$ to calculate the power dissipated in each bulb.
(a) SET UP: The circuit is sketched in Figure 26.15a.

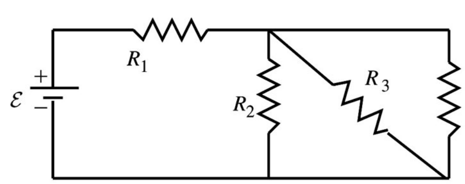


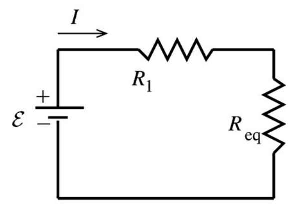
Figure 26.15a

EXECUTE: R_2, R_3, and R_4 are in parallel, so their equivalent resistance R_{eq} is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}.$$

$\frac{1}{R_{eq}} = \frac{3}{4.50\ \Omega}$ and $R_{eq} = 1.50\ \Omega.$

The equivalent circuit is drawn in Figure 26.15b.



$\mathcal{E} - I(R_1 + R_{eq}) = 0.$

$I = \frac{\mathcal{E}}{R_1 + R_{eq}}.$

Figure 26.15b

$I = \frac{9.00\ \text{V}}{4.50\ \Omega + 1.50\ \Omega} = 1.50\ \text{A}$ and $I_1 = 1.50\ \text{A}.$

Then $V_1 = I_1 R_1 = (1.50\ \text{A})(4.50\ \Omega) = 6.75\ \text{V}.$

$I_{eq} = 1.50\ \text{A},\ V_{eq} = I_{eq} R_{eq} = (1.50\ \text{A})(1.50\ \Omega) = 2.25\ \text{V}.$

For resistors in parallel the voltages are equal and are the same as the voltage across the equivalent resistor, so $V_2 = V_3 = V_4 = 2.25\ \text{V}.$

$I_2 = \frac{V_2}{R_2} = \frac{2.25\ \text{V}}{4.50\ \Omega} = 0.500\ \text{A},\ I_3 = \frac{V_3}{R_3} = 0.500\ \text{A},\ I_4 = \frac{V_4}{R_4} = 0.500\ \text{A}.$

EVALUATE: Note that $I_2 + I_3 + I_4 = 1.50\ \text{A},$ which is I_{eq}. For resistors in parallel the currents add and their sum is the current through the equivalent resistor.

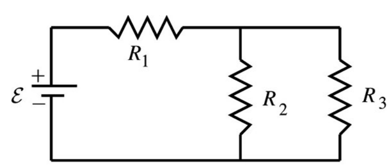
(b) SET UP: $P = I^2 R.$

EXECUTE: $P_1 = (1.50\ \text{A})^2 (4.50\ \Omega) = 10.1\ \text{W}.$

$P_2 = P_3 = P_4 = (0.500\ \text{A})^2 (4.50\ \Omega) = 1.125\ \text{W},$ which rounds to 1.12 W. R_1 glows brightest.

EVALUATE: Note that $P_2 + P_3 + P_4 = 3.37\ \text{W}$. This equals $P_{eq} = I_{eq}^2 R_{eq} = (1.50\ \text{A})^2 (1.50\ \Omega) = 3.37\ \text{W},$ the power dissipated in the equivalent resistor.

(c) SET UP: With R_4 removed the circuit becomes the circuit in Figure 26.15c.

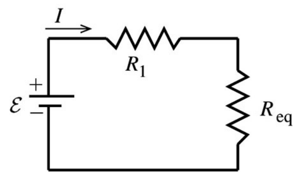


EXECUTE: R_2 and R_3 are in parallel and their equivalent resistance R_{eq} is given by

$\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{2}{4.50\ \Omega}$ and $R_{eq} = 2.25\ \Omega.$

Figure 26.15c

The equivalent circuit is shown in Figure 26.15d.



$$\mathcal{E} - I(R_1 + R_{eq}) = 0.$$

$$I = \frac{\mathcal{E}}{R_1 + R_{eq}}.$$

$$I = \frac{9.00\text{ V}}{4.50\ \Omega + 2.25\ \Omega} = 1.333\text{ A}.$$

Figure 26.15d

$I_1 = 1.33$ A, $V_1 = I_1R_1 = (1.333\text{ A})(4.50\ \Omega) = 6.00$ V.

$I_{eq} = 1.33$ A, $V_{eq} = I_{eq}R_{eq} = (1.333\text{ A})(2.25\ \Omega) = 3.00$ V and $V_2 = V_3 = 3.00$ V.

$I_2 = \dfrac{V_2}{R_2} = \dfrac{3.00\text{ V}}{4.50\ \Omega} = 0.667$ A, $I_3 = \dfrac{V_3}{R_3} = 0.667$ A.

(d) SET UP: $P = I^2R$.

EXECUTE: $P_1 = (1.333\text{ A})^2(4.50\ \Omega) = 8.00$ W.

$P_2 = P_3 = (0.667\text{ A})^2(4.50\ \Omega) = 2.00$ W.

EVALUATE: **(e)** When R_4 is removed, P_1 decreases and P_2 and P_3 increase. Bulb R_1 glows less brightly and bulbs R_2 and R_3 glow more brightly. When R_4 is removed the equivalent resistance of the circuit increases and the current through R_1 decreases. But in the parallel combination this current divides into two equal currents rather than three, so the currents through R_2 and R_3 increase. Can also see this by noting that with R_4 removed and less current through R_1 the voltage drop across R_1 is less so the voltage drop across R_2 and across R_3 must become larger.

26.16. **IDENTIFY:** Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: From Ohm's law, the voltage drop across the $6.00\text{-}\Omega$ resistor is $V = IR = (4.00\text{ A})(6.00\ \Omega) = 24.0$ V. The voltage drop across the $8.00\text{-}\Omega$ resistor is the same, since these two resistors are wired in parallel. The current through the $8.00\text{-}\Omega$ resistor is then $I = V/R = 24.0\text{ V}/8.00\ \Omega = 3.00$ A. The current through the $25.0\text{-}\Omega$ resistor is the sum of the current through these two resistors: 7.00 A. The voltage drop across the $25.0\text{-}\Omega$ resistor is $V = IR = (7.00\text{ A})(25.0\ \Omega) = 175$ V, and total voltage drop across the top branch of the circuit is $175\text{ V} + 24.0\text{ V} = 199$ V, which is also the voltage drop across the $20.0\text{-}\Omega$ resistor. The current through the $20.0\text{-}\Omega$ resistor is then $I = V/R = 199\text{ V}/20\ \Omega = 9.95$ A.

EVALUATE: The total current through the battery is $7.00\text{ A} + 9.95\text{ A} = 16.95$ A. Note that we did not need to calculate the emf of the battery.

26.17. **IDENTIFY:** Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: The current through the $2.00\text{-}\Omega$ resistor is 6.00 A. Current through the $1.00\text{-}\Omega$ resistor also is 6.00 A and the voltage is 6.00 V. Voltage across the $6.00\text{-}\Omega$ resistor is $12.0\text{ V} + 6.0\text{ V} = 18.0$ V. Current through the $6.00\text{-}\Omega$ resistor is $(18.0\text{ V})/(6.00\ \Omega) = 3.00$ A. The battery emf is 18.0 V.

EVALUATE: The current through the battery is $6.00\text{ A} + 3.00\text{ A} = 9.00\text{ A}$. The equivalent resistor of the resistor network is $2.00\ \Omega$, and this equals $(18.0\text{ V})/(9.00\text{ A})$.

26.18. **IDENTIFY:** Ohm's law applies to each resistor. In one case, the resistors are connected in series, and in the other case they are in parallel.

SET UP: $V = RI$, $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dots$ (in parallel), $R_{\text{eq}} = R_1 + R_2 + \dots$ (in series). Figure 26.18 shows the equivalent circuit when S is open and when S is closed.

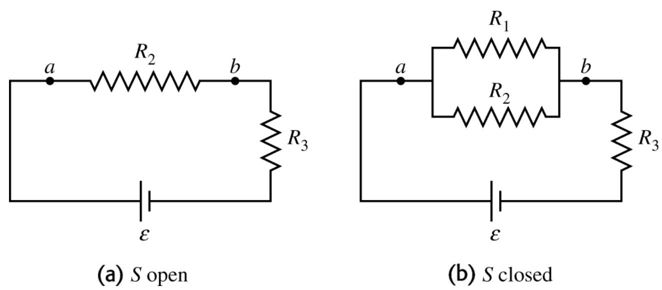


Figure 26.18

EXECUTE: **(a)** <u>S open</u>: We use the circuit in Figure 26.18a. R_2 and R_3 are in series. Ohm's law gives $\mathcal{E} = (R_2 + R_3)I$.

$I = \mathcal{E}/(R_2 + R_3) = (36.0\ V)/(9.00\ \Omega) = 4.00\text{ A}$.

$V_{ab} = R_2I = (6.00\ \Omega)(4.00\text{ A}) = 24.0\text{ V}$.

<u>S closed</u>: We use the circuit in Figure 26.18b. R_1 and R_2 are in parallel, and this combination is in series with R_3. For the parallel branch

$\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dots = 1/(4.00\ \Omega) + 1/(6.00\ \Omega)$, which gives $R_{\text{eq}} = 2.40\ \Omega$. The equivalent resistance R of the circuit is $2.40\ \Omega + 3.00\ \Omega = 5.40\ \Omega$. The current is $I = \mathcal{E}/R = (36.0\text{ V})/(5.40\ \Omega) = 6.667\text{ A}$. Therefore $V_{ab} = IR_{\text{eq}} = (6.667\text{ A})(2.40\ \Omega) = 16.0\text{ V}$.

(b) <u>S open</u>: From part (a), we know that $I_2 = 4.00\text{ A}$ through R_2. Since S is open, no current can flow through R_1, so $I_1 = 0$, $I_2 = I_3 = 4.00\text{ A}$.

<u>S closed</u>: $I_1 = V_{ab}/R_1 = (16.0\text{ V})/(4.00\ \Omega) = 4.00\text{ A}$. $I_2 = V_{ab}/R_2 = (16.0\text{ V})/(6.00\ \Omega) = 2.67\text{ A}$.

$I_3 = I_1 + I_2 = 4.00\text{ A} + 2.67\text{ A} = 6.67\text{ A}$.

I_1 increased from 0 to 4.00 A.

I_2 decreased from 4.00 A to 2.67 A.

I_3 increased from 4.00 A to 6.67 A.

EVALUATE: With S closed, $V_{ab} + V_3 = 16.0\text{ V} + (3.00\ \Omega)(6.67\text{ A}) = 36.0\text{ V}$, which is equal to $\mathcal{E}$, as it should be.

26.19. **IDENTIFY** and **SET UP:** Replace series and parallel combinations of resistors by their equivalents until the circuit is reduced to a single loop. Use the loop equation to find the current through the $20.0\text{-}\Omega$ resistor. Set $P = I^2R$ for the $20.0\text{-}\Omega$ resistor equal to the rate Q/t at which heat goes into the water and set $Q = mc\Delta T$.

EXECUTE: Replace the network by the equivalent resistor, as shown in Figure 26.19.

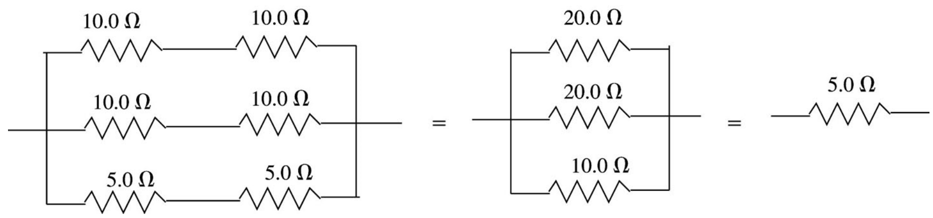


Figure 26.19

$30.0\text{ V} - I(20.0\,\Omega + 5.0\,\Omega + 5.0\,\Omega) = 0;\ I = 1.00\text{ A}.$

For the $20.0\text{-}\Omega$ resistor thermal energy is generated at the rate $P = I^2R = 20.0\text{ W}.$

$Q = Pt$ and $Q = mc\Delta T$ gives $t = \dfrac{mc\Delta T}{P} = \dfrac{(0.100\text{ kg})(4190\text{ J/kg}\cdot\text{ K})(48.0\text{ C}°)}{20.0\text{ W}} = 1.01\times 10^3\text{ s}.$

EVALUATE: The battery is supplying heat at the rate $P = \mathcal{E}I = 30.0\text{ W}.$ In the series circuit, more energy is dissipated in the larger resistor $(20.0\,\Omega)$ than in the smaller ones $(5.00\,\Omega).$

26.20. IDENTIFY: $P = I^2R$ determines R_1. $R_1, R_2,$ and the $10.0\text{-}\Omega$ resistor are all in parallel so have the same voltage. Apply the junction rule to find the current through R_2.

SET UP: $P = I^2R$ for a resistor and $P = \mathcal{E}I$ for an emf. The emf inputs electrical energy into the circuit and electrical energy is removed in the resistors.

EXECUTE: **(a)** $P_1 = I_1^2R_1$. $15.0\text{ W} = (2.00\text{ A})^2 R_1$ so $R_1 = 3.75\ \Omega.$ R_1 and $10.0\,\Omega$ are in parallel, so $(10.0\,\Omega)I_{10} = (3.75\,\Omega)(2.00\text{ A})$ so $I_{10} = 0.750\text{ A}.$ So $I_2 = 3.50\text{ A} - I_1 - I_{10} =$ 3.50 A – 2.00 A – 0.750 A = 0.750 A. R_1 and R_2 are in parallel, so $(0.750\text{ A})R_2 = (2.00\text{ A})(3.75\,\Omega)$ which gives $R_2 = 10.0\,\Omega.$

(b) $\mathcal{E} = V_1 = (2.00\text{ A})(3.75\ \Omega) = 7.50\text{ V}.$

(c) From part (a), $I_2 = 0.750\text{ A}, I_{10} = 0.750\text{ A}.$

(d) $P_1 = 15.0\text{ W}$ (given). $P_2 = I_2^2R_2 = (0.750\text{ A})^2(10.0\,\Omega) = 5.625\text{ W},$ which rounds to 5.63 W.

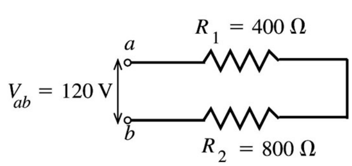
$P_{10} = I_{10}^2R_{10} = (0.750\text{ A})^2(10.0\,\Omega) = 5.625\text{ W}.$ The total rate at which the resistors remove electrical energy is $P_{\text{Resist}} = 15.0\text{ W} + 5.625\text{ W} + 5.625\text{ W} = 26.25\text{ W},$ which rounds to 26.3 W.

The total rate at which the battery inputs electrical energy is $P_{\text{Battery}} = I\mathcal{E} = (3.50\text{ A})(7.50\text{ V}) =$ 26.3 W. Therefore $P_{\text{Resist}} = P_{\text{Battery}}$, which agrees with conservation of energy.

EVALUATE: The three resistors are in parallel, so the voltage for each is the battery voltage, 7.50 V. The currents in the three resistors add to give the current in the battery.

26.21. IDENTIFY: For resistors in series, the voltages add and the current is the same. For resistors in parallel, the voltages are the same and the currents add. $P = I^2R.$

(a) SET UP: The circuit is sketched in Figure 26.21a.



For resistors in series the current is the same through each.

Figure 26.21a

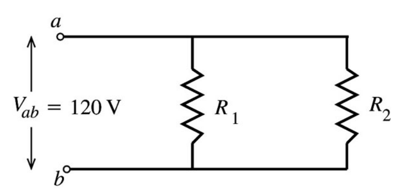
EXECUTE: $R_{eq} = R_1 + R_2 = 1200\ \Omega$. $I = \dfrac{V}{R_{eq}} = \dfrac{120\text{ V}}{1200\ \Omega} = 0.100\text{ A}$. This is the current drawn from the line.

(b) $P_1 = I_1^2 R_1 = (0.100\text{ A})^2 (400\ \Omega) = 4.0\text{ W}$.

$P_2 = I_2^2 R_2 = (0.100\text{ A})^2 (800\ \Omega) = 8.0\text{ W}$.

(c) $P_{out} = P_1 + P_2 = 12.0\text{ W}$, the total power dissipated in both bulbs. Note that $P_{in} = V_{ab} I = (120\text{ V})(0.100\text{ A}) = 12.0\text{ W}$, the power delivered by the potential source, equals P_{out}.

(d) SET UP: The circuit is sketched in Figure 26.21b.



For resistors in parallel the voltage across each resistor is the same.

Figure 26.21b

EXECUTE: $I_1 = \dfrac{V_1}{R_1} = \dfrac{120\text{ V}}{400\ \Omega} = 0.300\text{ A}, I_2 = \dfrac{V_2}{R_2} = \dfrac{120\text{ V}}{800\ \Omega} = 0.150\text{ A}$.

EVALUATE: Note that each current is larger than the current when the resistors are connected in series.

EXECUTE: **(e)** $P_1 = I_1^2 R_1 = (0.300\text{ A})^2 (400\ \Omega) = 36.0\text{ W}$.

$P_2 = I_2^2 R_2 = (0.150\text{ A})^2 (800\ \Omega) = 18.0\text{ W}$.

(f) $P_{out} = P_1 + P_2 = 54.0\text{ W}$.

EVALUATE: Note that the total current drawn from the line is $I = I_1 + I_2 = 0.450\text{ A}$. The power input from the line is $P_{in} = V_{ab} I = (120\text{ V})(0.450\text{ A}) = 54.0\text{ W}$, which equals the total power dissipated by the bulbs.

(g) The bulb that is dissipating the most power glows most brightly. For the series connection the currents are the same and by $P = I^2 R$ the bulb with the larger *R* has the larger *P*; the $800\text{-}\Omega$ bulb glows more brightly. For the parallel combination the voltages are the same and by $P = V^2/R$ the bulb with the smaller *R* has the larger *P*; the $400\text{-}\Omega$ bulb glows more brightly.

(h) The total power output P_{out} equals $P_{in} = V_{ab} I$, so P_{out} is larger for the parallel connection where the current drawn from the line is larger (because the equivalent resistance is smaller.)

26.22. IDENTIFY: This circuit cannot be reduced using series/parallel combinations, so we apply Kirchhoff's rules. The target variables are the currents in each segment.

SET UP: Assume the unknown currents have the directions shown in Figure 26.22. We have used the junction rule to write the current through the 10.0 V battery as $I_1 + I_2$. There are two unknowns, I_1 and I_2, so we will need two equations. Three possible circuit loops are shown in the figure.

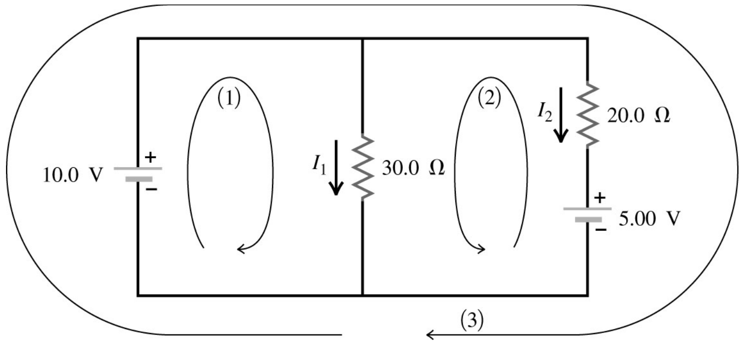


Figure 26.22

EXECUTE: **(a)** Apply the loop rule to loop (1), going around the loop in the direction shown: $+10.0\text{ V} - (30.0\ \Omega)I_1 = 0$ and $I_1 = 0.333$ A.

(b) Apply the loop rule to loop (3): $+10.0\text{ V} - (20.0\ \Omega)I_2 - 5.00\text{ V} = 0$ and $I_2 = 0.250$ A.

(c) $I_1 + I_2 = 0.333\text{ A} + 0.250\text{ A} = 0.583$ A.

EVALUATE: For loop (2) we get
$+5.00\text{ V} + I_2(20.0\ \Omega) - I_1(30.0\ \Omega) = 5.00\text{ V} + (0.250\text{ A})(20.0\ \Omega) - (0.333\text{ A})(30.0\ \Omega) =$
$5.00\text{ V} + 5.00\text{ V} - 10.0\text{ V} = 0$, so that with the currents we have calculated the loop rule is satisfied for this third loop.

26.23. **IDENTIFY:** Apply Kirchhoff's junction rule at point *a* to find the current through *R*. Apply Kirchhoff's loop rule to loops (1) and (2) shown in Figure 26.23a to calculate *R* and $\mathcal{E}$. Travel around each loop in the direction shown.
SET UP:

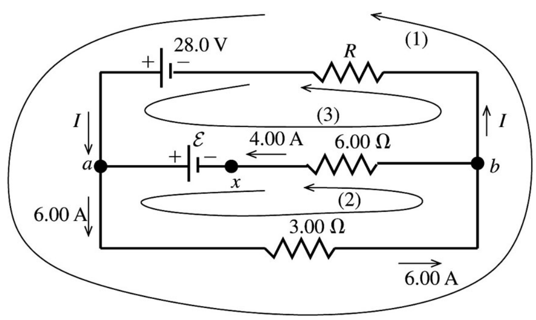


Figure 26.23a

EXECUTE: **(a)** Apply Kirchhoff's junction rule to point *a*: $\sum I = 0$ so $I + 4.00\text{ A} - 6.00\text{ A} = 0$
$I = 2.00$ A (in the direction shown in the diagram).

(b) Apply Kirchhoff's loop rule to loop (1): $-(6.00\text{ A})(3.00\ \Omega) - (2.00\text{ A})R + 28.0\text{ V} = 0$

$-18.0\text{ V} - (2.00\ \Omega)R + 28.0\text{ V} = 0$.

$$R = \frac{28.0\text{ V} - 18.0\text{ V}}{2.00\text{ A}} = 5.00\ \Omega.$$

(c) Apply Kirchhoff's loop rule to loop (2): $-(6.00\text{ A})(3.00\ \Omega) - (4.00\text{ A})(6.00\ \Omega) + \varepsilon = 0$.

$\mathcal{E} = 18.0\text{ V} + 24.0\text{ V} = 42.0$ V.

EVALUATE: We can check that the loop rule is satisfied for loop (3), as a check of our work:
$28.0\text{ V} - \mathcal{E} + (4.00\text{ A})(6.00\ \Omega) - (2.00\text{ A})R = 0$.

$28.0\text{ V} - 42.0\text{ V} + 24.0\text{ V} - (2.00\text{ A})(5.00\ \Omega) = 0$.

$52.0\text{ V} = 42.0\text{ V} + 10.0\text{ V}.$

$52.0\text{ V} = 52.0\text{ V},$ so the loop rule is satisfied for this loop.

(d) IDENTIFY: If the circuit is broken at point *x* there can be no current in the $6.00\text{-}\Omega$ resistor. There is now only a single current path and we can apply the loop rule to this path.

SET UP: The circuit is sketched in Figure 26.23b.

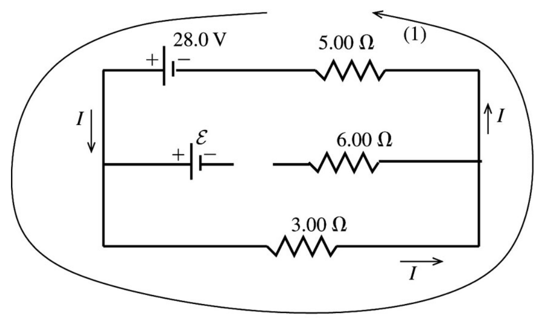


Figure 26.23b

EXECUTE: $+28.0\text{ V} - (3.00\,\Omega)I - (5.00\,\Omega)I = 0.$

$I = \dfrac{28.0\text{ V}}{8.00\,\Omega} = 3.50\text{ A}.$

EVALUATE: Breaking the circuit at *x* removes the 42.0-V emf from the circuit and the current through the $3.00\text{-}\Omega$ resistor is reduced.

26.24. IDENTIFY: Apply Kirchhoff's loop rule and junction rule.

SET UP: The circuit diagram is given in Figure 26.24. The junction rule has been used to find the magnitude and direction of the current in the middle branch of the circuit. There are no remaining unknown currents.

EXECUTE: The loop rule applied to loop (1) gives:

$+20.0\text{V} - (1.00\text{ A})(1.00\,\Omega) + (1.00\text{ A})(4.00\,\Omega) + (1.00\text{ A})(1.00\,\Omega) - \mathcal{E}_1 - (1.00\text{ A})(6.00\,\Omega) = 0.$

$\mathcal{E}_1 = 20.0\text{ V} - 1.00\text{ V} + 4.00\text{ V} + 1.00\text{ V} - 6.00\text{ V} = 18.0\text{ V}.$ The loop rule applied to loop (2) gives:

$+20.0\text{ V} - (1.00\text{ A})(1.00\,\Omega) - (2.00\text{ A})(1.00\,\Omega) - \mathcal{E}_2 - (2.00\text{ A})(2.00\,\Omega) - (1.00\text{ A})(6.00\,\Omega) = 0.$

$\mathcal{E}_2 = 20.0\text{ V} - 1.00\text{ V} - 2.00\text{ V} - 4.00\text{ V} - 6.00\text{ V} = 7.0\text{ V}.$ Going from *b* to *a* along the lower branch,

$V_b + (2.00\text{ A})(2.00\,\Omega) + 7.0\text{ V} + (2.00\text{ A})(1.00\,\Omega) = V_a \cdot V_b - V_a = -13.0\text{ V};$ point *b* is at 13.0 V lower potential than point *a*.

EVALUATE: We can also calculate $V_b - V_a$ by going from *b* to *a* along the upper branch of the circuit.

$V_b - (1.00\text{ A})(6.00\,\Omega) + 20.0\text{ V} - (1.00\text{ A})(1.00\,\Omega) = V_a$ and $V_b - V_a = -13.0\text{ V}.$ This agrees with $V_b - V_a$ calculated along a different path between *b* and *a*.

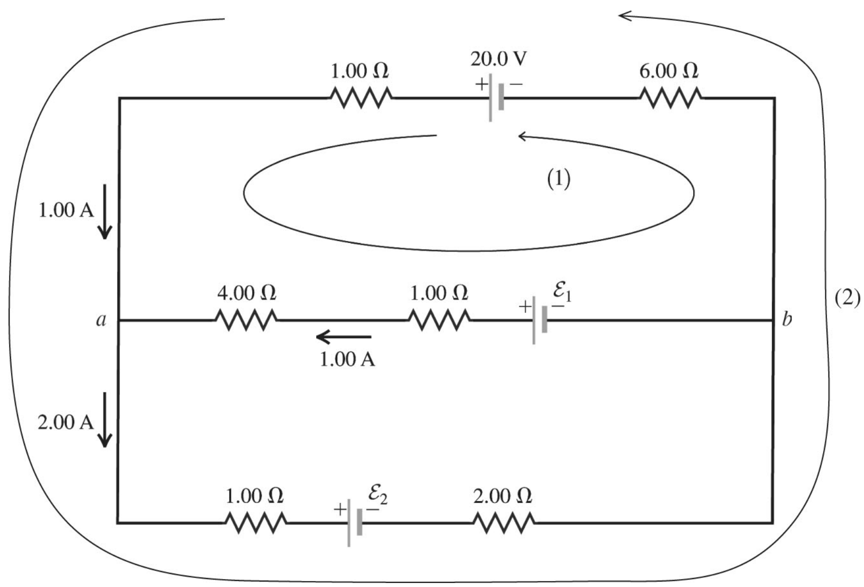


Figure 26.24

26.25. **IDENTIFY:** Apply Kirchhoff's junction rule at points *a*, *b*, *c*, and *d* to calculate the unknown currents. Then apply the loop rule to three loops to calculate $\mathcal{E}_1, \mathcal{E}_2$, and *R*.

SET UP: The circuit is sketched in Figure 26.25.

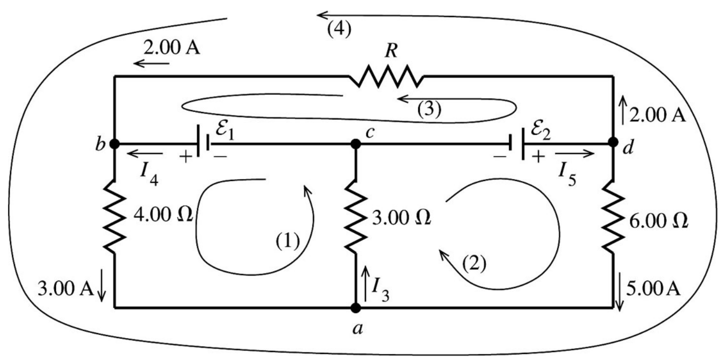


Figure 26.25

(a) EXECUTE: Apply the junction rule to point *a*: $3.00\text{ A} + 5.00\text{ A} - I_3 = 0$.

$I_3 = 8.00\text{ A}$.

Apply the junction rule to point *b*: $2.00\text{ A} + I_4 - 3.00\text{ A} = 0$.

$I_4 = 1.00\text{ A}$.

Apply the junction rule to point *c*: $I_3 - I_4 - I_5 = 0$.

$I_5 = I_3 - I_4 = 8.00\text{ A} - 1.00\text{ A} = 7.00\text{ A}$.

EVALUATE: As a check, apply the junction rule to point *d*: $I_5 - 2.00\text{ A} - 5.00\text{ A} = 0$.

$I_5 = 7.00\text{ A}$.

(b) EXECUTE: Apply the loop rule to loop (1): $\mathcal{E}_1 - (3.00\text{ A})(4.00\,\Omega) - I_3(3.00\,\Omega) = 0$.

$\mathcal{E}_1 = 12.0\text{ V} + (8.00\text{ A})(3.00\,\Omega) = 36.0\text{ V}$.

Apply the loop rule to loop (2): $\mathcal{E}_2 - (5.00\text{ A})(6.00\,\Omega) - I_3(3.00\,\Omega) = 0$.

$\mathcal{E}_2 = 30.0\text{ V} + (8.00\text{ A})(3.00\ \Omega) = 54.0\text{ V}.$

(c) EXECUTE: Apply the loop rule to loop (3): $-(2.00\text{ A})R - \mathcal{E}_1 + \mathcal{E}_2 = 0.$

$$R = \frac{\mathcal{E}_2 - \mathcal{E}_1}{2.00\text{ A}} = \frac{54.0\text{ V} - 36.0\text{ V}}{2.00\text{ A}} = 9.00\ \Omega.$$

EVALUATE: Apply the loop rule to loop (4) as a check of our calculations:
$-(2.00\text{ A})R - (3.00\text{ A})(4.00\ \Omega) + (5.00\text{ A})(6.00\ \Omega) = 0.$

$-(2.00\text{ A})(9.00\ \Omega) - 12.0\text{ V} + 30.0\text{ V} = 0.$

$-18.0\text{ V} + 18.0\text{ V} = 0.$

26.26. **IDENTIFY:** Use Kirchhoff's rules to find the currents.
SET UP: Since the 10.0-V battery has the larger voltage, assume I_1 is to the left through the 10-V battery, I_2 is to the right through the 5-V battery, and I_3 is to the right through the 10-Ω resistor. Go around each loop in the counterclockwise direction.
EXECUTE: **(a)** Upper loop: $10.0\text{ V} - (2.00\ \Omega + 3.00\ \Omega)I_1 - (1.00\ \Omega + 4.00\ \Omega)I_2 - 5.00\text{ V} = 0.$ This gives $5.0\text{ V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0,$ and $\Rightarrow I_1 + I_2 = 1.00\text{ A}.$

Lower loop: $5.00\text{ V} + (1.00\ \Omega + 4.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0.$ This gives $5.00\text{ V} + (5.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0,$ and $I_2 - 2I_3 = -1.00\text{ A}.$

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:
$I_1 = 0.800\text{ A}, I_2 = 0.200\text{ A}, I_3 = 0.600\text{ A}.$

(b) $V_{ab} = -(0.200\text{ A})(4.00\ \Omega) - (0.800\text{ A})(3.00\ \Omega) = -3.20\text{ V}.$

EVALUATE: Traveling from *b* to *a* through the 4.00-Ω and 3.00-Ω resistors you pass through the resistors in the direction of the current and the potential decreases. Therefore point *b* is at higher potential than point *a*.

26.27. **IDENTIFY:** Apply the junction rule to reduce the number of unknown currents. Apply the loop rule to two loops to obtain two equations for the unknown currents I_1 and I_2.
(a) SET UP: The circuit is sketched in Figure 26.27.

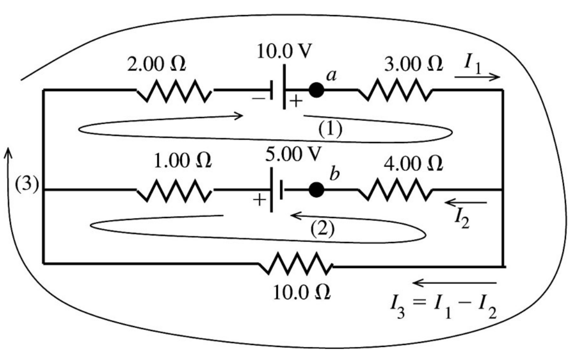


Figure 26.27

Let I_1 be the current in the 3.00-Ω resistor and I_2 be the current in the 4.00-Ω resistor and assume that these currents are in the directions shown. Then the current in the 10.0-Ω resistor is $I_3 = I_1 - I_2$, in the direction shown, where we have used Kirchhoff's junction rule to relate I_3 to I_1 and I_2. If we get a negative answer for any of these currents we know the current is actually in the opposite direction to what we have assumed. Three loops and directions to travel around the loops are shown in the circiut diagram in Figure 26.27. Apply Kirchhoff's loop rule to each loop.
EXECUTE: Loop (1):
$+10.0\text{ V} - I_1(3.00\ \Omega) - I_2(4.00\ \Omega) + 5.00\text{ V} - I_2(1.00\ \Omega) - I_1(2.00\ \Omega) = 0.$

$15.00\text{ V}-(5.00\ \Omega)I_1-(5.00\ \Omega)I_2=0.$

$3.00\text{ A}-I_1-I_2=0.$

Loop (2):

$+5.00\text{ V}-I_2(1.00\ \Omega)+(I_1-I_2)10.0\ \Omega-I_2(4.00\ \Omega)=0.$

$5.00\text{ V}+(10.0\ \Omega)I_1-(15.0\ \Omega)I_2=0.$

$1.00\text{ A}+2.00I_1-3.00I_2=0.$

The first equation says $I_2=3.00\text{ A}-I_1.$

Use this in the second equation: $1.00\text{ A}+2.00I_1-9.00\text{ A}+3.00I_1=0.$

$5.00I_1=8.00\text{ A}, I_1=1.60\text{ A}.$

Then $I_2=3.00\text{ A}-I_1=3.00\text{ A}-1.60\text{ A}=1.40\text{ A}.$

$I_3=I_1-I_2=1.60\text{ A}-1.40\text{ A}=0.20\text{ A}.$

EVALUATE: Loop (3) can be used as a check.

$+10.0\text{ V}-(1.60\text{ A})(3.00\ \Omega)-(0.20\text{ A})(10.00\ \Omega)-(1.60\text{ A})(2.00\ \Omega)=0.$

$10.0\text{ V}=4.8\text{ V}+2.0\text{ V}+3.2\text{ V}.$

$10.0\text{ V}=10.0\text{ V}.$

We find that with our calculated currents the loop rule is satisfied for loop (3). Also, all the currents came out to be positive, so the current directions in the circuit diagram are correct.

(b) IDENTIFY and **SET UP:** To find $V_{ab}=V_a-V_b$ start at point *b* and travel to point *a*. Many different routes can be taken from *b* to *a* and all must yield the same result for V_{ab}.

EXECUTE: Travel through the $4.00\text{-}\Omega$ resistor and then through the $3.00\text{-}\Omega$ resistor:

$V_b+I_2(4.00\ \Omega)+I_1(3.00\ \Omega)=V_a.$

$V_a-V_b=(1.40\text{ A})(4.00\ \Omega)+(1.60\text{ A})(3.00\ \Omega)=5.60\text{ V}+4.8\text{ V}=10.4\text{ V}$ (point *a* is at higher potential than point *b*).

EVALUATE: Alternatively, travel through the 5.00-V emf, the $1.00\text{-}\Omega$ resistor, the $2.00\text{-}\Omega$ resistor, and the 10.0-V emf.

$V_b+5.00\text{ V}-I_2(1.00\ \Omega)-I_1(2.00\ \Omega)+10.0\text{ V}=V_a.$

$V_a-V_b=15.0\text{ V}-(1.40\text{ A})(1.00\ \Omega)-(1.60\text{ A})(2.00\ \Omega)=15.0\text{ V}-1.40\text{ V}-3.20\text{ V}=10.4\text{ V},$ the same as before.

26.28. IDENTIFY: Use Kirchhoff's rules to find the currents.

SET UP: Since the 15.0-V battery has the largest voltage, assume I_1 is to the right through the 10.0-V battery, I_2 is to the left through the 15.0-V battery, and I_3 is to the right through the $10.00\text{-}\Omega$ resistor. Go around each loop in the counterclockwise direction.

EXECUTE: (a) Upper loop: $10.0\text{ V}+(2.00\ \Omega+3.00\ \Omega)I_1+(1.00\ \Omega+4.00\ \Omega)I_2-15.00\text{ V}=0.$

$-5.00\text{ V}+(5.00\ \Omega)I_1+(5.00\ \Omega)I_2=0,$ so $I_1+I_2=+1.00\text{A}.$

Lower loop: $15.00\text{ V}-(1.00\ \Omega+4.00\ \Omega)I_2-(10.0\ \Omega)I_3=0.$

$15.00\text{ V}-(5.00\ \Omega)I_2-(10.0\ \Omega)I_3=0,$ so $I_2+2I_3=3.00\text{ A}.$

Along with $I_2=I_1+I_3,$ we can solve for the three currents and find

$I_1=0.00\text{ A}, I_2=+1.00\text{ A (to the left)}, I_3=+1.00\text{ A (to the right)}.$

(b) $V_{ab}=I_2(4.00\ \Omega)+I_1(3.00\ \Omega)=(1.00\text{ A})(4.00\ \Omega)+(0.00\text{ A})(3.00\ \Omega)=4.00\text{ V}.$

EVALUATE: Traveling from *b* to *a* through the $4.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors you pass through each resistor opposite to the direction of the current and the potential increases; point *a* is at higher potential than point *b*.

26.29. **(a) IDENTIFY:** With the switch open, the circuit can be solved using series-parallel reduction.
SET UP: Find the current through the unknown battery using Ohm's law. Then use the equivalent resistance of the circuit to find the emf of the battery.
EXECUTE: The $30.0\text{-}\Omega$ and $50.0\text{-}\Omega$ resistors are in series, and hence have the same current. Using Ohm's law $I_{50} = (15.0\text{ V})/(50.0\ \Omega) = 0.300\text{ A} = I_{30}$. The potential drop across the $75.0\text{-}\Omega$ resistor is the same as the potential drop across the $80.0\text{-}\Omega$ series combination. We can use this fact to find the current through the $75.0\text{-}\Omega$ resistor using Ohm's law: $V_{75} = V_{80} = (0.300\text{ A})(80.0\ \Omega) = 24.0\text{ V}$ and
$I_{75} = (24.0\text{ V})/(75.0\ \Omega) = 0.320\text{ A}$.
The current through the unknown battery is the sum of the two currents we just found:
$I_{\text{Total}} = 0.300\text{ A} + 0.320\text{ A} = 0.620\text{ A}$.
The equivalent resistance of the resistors in parallel is $1/R_{\text{p}} = 1/(75.0\ \Omega) + 1/(80.0\ \Omega)$. This gives
$R_{\text{p}} = 38.7\ \Omega$. The equivalent resistance "seen" by the battery is $R_{\text{equiv}} = 20.0\ \Omega + 38.7\ \Omega = 58.7\ \Omega$.
Applying Ohm's law to the battery gives $\mathcal{E} = R_{\text{equiv}} I_{\text{Total}} = (58.7\ \Omega)(0.620\text{ A}) = 36.4\text{ V}$.
(b) IDENTIFY: With the switch closed, the 25.0-V battery is connected across the $50.0\text{-}\Omega$ resistor.
SET UP: Take a loop around the right part of the circuit.
EXECUTE: Ohm's law gives $I = (25.0\text{ V})/(50.0\ \Omega) = 0.500\text{ A}$.
EVALUATE: The current through the $50.0\text{-}\Omega$ resistor, and the rest of the circuit, depends on whether or not the switch is open.

26.30. **IDENTIFY:** We need to use Kirchhoff's rules.
SET UP: Take a loop around the outside of the circuit, apply the junction rule at the upper junction, and then take a loop around the right side of the circuit.
EXECUTE: The outside loop gives $75.0\text{ V} - (12.0\ \Omega)(1.50\text{ A}) - (48.0\ \Omega)I_{48} = 0$, so $I_{48} = 1.188\text{ A}$. At a junction we have $1.50\text{A} = I_{\mathcal{E}} + 1.188\text{ A}$, and $I_{\mathcal{E}} = 0.313\text{ A}$. A loop around the right part of the circuit gives $\mathcal{E} - (48\ \Omega)(1.188\text{ A}) + (15.0\ \Omega)(0.313\text{ A})$. $\mathcal{E} = 52.3\text{ V}$, with the polarity shown in the figure in the problem.
EVALUATE: The unknown battery has a smaller emf than the known one, so the current through it goes against its polarity.

26.31. **(a) IDENTIFY:** With the switch open, we have a series circuit with two batteries.
SET UP: Take a loop to find the current, then use Ohm's law to find the potential difference between *a* and *b*.
EXECUTE: Taking the loop: $I = (40.0\text{ V})/(175\ \Omega) = 0.229\text{ A}$. The potential difference between *a* and *b* is $V_b - V_a = +15.0\text{ V} - (75.0\ \Omega)(0.229\text{ A}) = -2.14\text{ V}$.
EVALUATE: The minus sign means that *a* is at a higher potential than *b*.
(b) IDENTIFY: With the switch closed, the ammeter part of the circuit divides the original circuit into two circuits. We can apply Kirchhoff's rules to both parts.
SET UP: Take loops around the left and right parts of the circuit, and then look at the current at the junction.
EXECUTE: The left-hand loop gives $I_{100} = (25.0\text{ V})/(100.0\ \Omega) = 0.250\text{ A}$. The right-hand loop gives $I_{75} = (15.0\text{ V})/(75.0\ \Omega) = 0.200\text{ A}$. At the junction just above the switch we have $I_{100} = 0.250\text{ A}$ (in) and $I_{75} = 0.200\text{ A (out)}$, so $I_{\text{A}} = 0.250\text{ A} - 0.200\text{ A} = 0.050\text{ A}$, downward. The voltmeter reads zero because the potential difference across it is zero with the switch closed.
EVALUATE: The ideal ammeter acts like a short circuit, making *a* and *b* at the same potential. Hence the voltmeter reads zero.

26.32. IDENTIFY: We first reduce the parallel combination of the $20.0\text{-}\Omega$ resistors and then apply Kirchhoff's rules.

SET UP: $P = I^2R$ so the power consumption of the $6.0\text{-}\Omega$ resistor allows us to calculate the current through it. Unknown currents I_1, I_2, and I_3 are shown in Figure 26.32. The junction rule says that $I_1 = I_2 + I_3$. In Figure 26.34 the two $20.0\text{-}\Omega$ resistors in parallel have been replaced by their equivalent $(10.0\ \Omega)$.

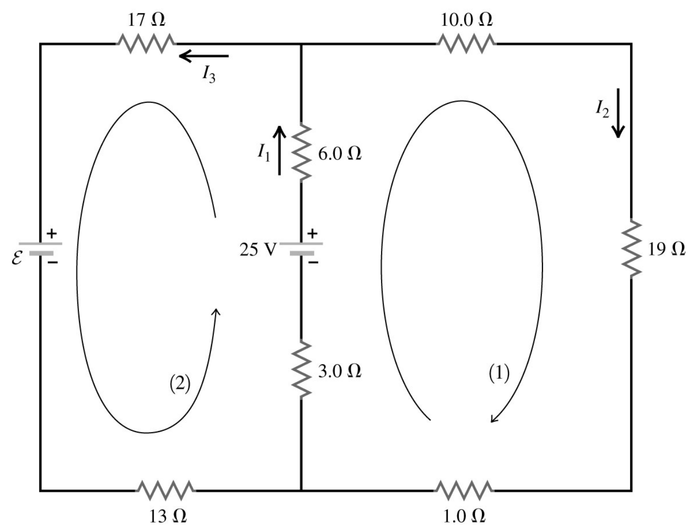


Figure 26.32

EXECUTE: (a) $P = I^2R$ gives $I_1 = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{24\ \text{J/s}}{6.0\ \Omega}} = 2.0\ \text{A}$. The loop rule applied to loop (1) gives:

$-(2.0\ \text{A})(3.0\ \Omega) - (2.0\ \text{A})(6.0\ \Omega) + 25\ \text{V} - I_2(10.0\ \Omega + 19.0\ \Omega + 1.0\ \Omega) = 0.$

$I_2 = \dfrac{25\ \text{V} - 18\ \text{V}}{30.0\ \Omega} = 0.233\ \text{A}.$

(b) $I_3 = I_1 - I_2 = 2.0\ \text{A} - 0.233\ \text{A} = 1.77\ \text{A}$. The loop rule applied to loop (2) gives:

$-(2.0\ \text{A})(3.0\ \Omega + 6.0\ \Omega) + 25\ \text{V} - (1.77\ \text{A})(17\ \Omega) - \mathcal{E} - (1.77\ \text{A})(13\ \Omega) = 0.$

$\mathcal{E} = 25\ \text{V} - 18\ \text{V} - 53.1\ \text{V} = -46.1\ \text{V}$. The emf is 46.1 V.

EVALUATE: Because of the minus sign for the emf, the polarity of the battery is opposite to what is shown in the figure in the problem; the + terminal is adjacent to the $13\text{-}\Omega$ resistor.

26.33. IDENTIFY: To construct an ammeter, add a shunt resistor in parallel with the galvanometer coil. To construct a voltmeter, add a resistor in series with the galvanometer coil.

SET UP: The full-scale deflection current is $500\ \mu\text{A}$ and the coil resistance is $25.0\ \Omega$.

EXECUTE: (a) For a 20-mA ammeter, the two resistances are in parallel and the voltages across each are the same. $V_c = V_s$ gives $I_cR_c = I_sR_s$. $(500\times10^{-6}\ \text{A})(25.0\ \Omega) = (20\times10^{-3}\ \text{A} - 500\times10^{-6}\ \text{A})R_s$ and $R_s = 0.641\ \Omega$.

(b) For a 500-mV voltmeter, the resistances are in series and the current is the same through each:

$V_{ab} = I(R_c + R_s)$ and $R_s = \dfrac{V_{ab}}{I} - R_c = \dfrac{500\times10^{-3}\ \text{V}}{500\times10^{-6}\ \text{A}} - 25.0\ \Omega = 975\ \Omega.$

EVALUATE: The equivalent resistance of the voltmeter is $R_{eq} = R_s + R_c = 1000\ \Omega$. The equivalent resistance of the ammeter is given by $\frac{1}{R_{eq}} = \frac{1}{R_{sh}} + \frac{1}{R_c}$ and $R_{eq} = 0.625\ \Omega$. The voltmeter is a high-resistance device and the ammeter is a low-resistance device.

26.34. IDENTIFY: The galvanometer is represented in the circuit as a resistance R_G. Use the junction rule to relate the current through the galvanometer and the current through the shunt resistor. The voltage drop across each parallel path is the same; use this to write an equation for the resistance *R*.

SET UP: The circuit is sketched in Figure 26.34.

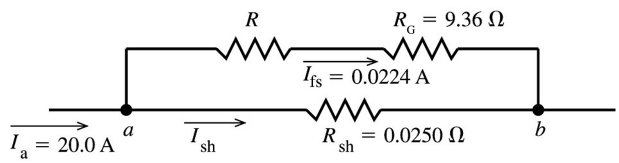


Figure 26.34

We want that $I_a = 20.0$ A in the external circuit to produce $I_{fs} = 0.0224$ A through the galvanometer coil.

EXECUTE: Applying the junction rule to point *a* gives $I_a - I_{fs} - I_{sh} = 0$.

$I_{sh} = I_a - I_{fs} = 20.0\text{ A} - 0.0224\text{ A} = 19.98\text{ A}$.

The potential difference V_{ab} between points *a* and *b* must be the same for both paths between these two points: $I_{fs}(R + R_G) = I_{sh}R_{sh}$.

$$R = \frac{I_{sh}R_{sh}}{I_{fs}} - R_G = \frac{(19.98\text{ A})(0.0250\ \Omega)}{0.0224\text{ A}} - 9.36\ \Omega = 22.30\ \Omega - 9.36\ \Omega = 12.9\ \Omega.$$

EVALUATE: $R_{sh} << R + R_G$; most of the current goes through the shunt. Adding *R* decreases the fraction of the current that goes through R_G.

26.35. IDENTIFY: The meter introduces resistance into the circuit, which affects the current through the $5.00\text{-k}\Omega$ resistor and hence the potential drop across it.

SET UP: Use Ohm's law to find the current through the $5.00\text{-k}\Omega$ resistor and then the potential drop across it.

EXECUTE: **(a)** The parallel resistance with the voltmeter is $3.33\text{ k}\Omega$, so the total equivalent resistance across the battery is $9.33\text{ k}\Omega$, giving $I = (50.0\text{ V})/(9.33\text{ k}\Omega) = 5.36\text{ mA}$. Ohm's law gives the potential drop across the $5.00\text{-k}\Omega$ resistor: $V_{5\text{ k}\Omega} = (3.33\text{ k}\Omega)(5.36\text{ mA}) = 17.9\text{ V}$.

(b) The current in the circuit is now $I = (50.0\text{ V})/(11.0\text{ k}\Omega) = 4.55\text{ mA}$.

$V_{5\text{ k}\Omega} = (5.00\text{ k}\Omega)(4.55\text{ mA}) = 22.7\text{ V}$.

(c) % error $= (22.7\text{ V} - 17.9\text{ V})/(22.7\text{ V}) = 0.214 = 21.4\%$. (We carried extra decimal places for accuracy since we had to subtract our answers.)

EVALUATE: The presence of the meter made a very large percent error in the reading of the "true" potential across the resistor.

26.36. IDENTIFY: We are measuring a capacitor in a circuit. We measure the time $T_{1/2}$ for the voltage to decrease to $V_0/2$ and then graph $T_{1/2}$ versus *R*.

SET UP: We need to find a relationship between $T_{1/2}$ and *R* to interpret the graph.

EXECUTE: For discharging $V = V_0 e^{-t/RC}$. $V_{1/2} = \frac{1}{2}V_0 = V_0 e^{-T_{1/2}/RC}$. Solve for $T_{1/2}$ by taking logarithms.

$T_{1/2} = (C \ln 2)R$, so a graph of $T_{1/2}$ versus R should be a straight line having slope equal to $C \ln 2$. C = (slope)/(ln 2) = (5.00 μF)/(ln 2) = 7.21 μF.

EVALUATE: Note that $T_{1/2}$ is *not* the time constant.

26.37. **IDENTIFY:** The capacitor discharges exponentially through the voltmeter. Since the potential difference across the capacitor is directly proportional to the charge on the plates, the voltage across the plates decreases exponentially with the same time constant as the charge.

SET UP: The reading of the voltmeter obeys the equation $V = V_{0e}^{-t/RC}$, where RC is the time constant.

EXECUTE: **(a)** Solving for C and evaluating the result when $t = 4.00\text{ s}$ gives

$$C = \frac{t}{R\ln(V/V_0)} = \frac{4.00\text{ s}}{(3.40\times10^6\ \Omega)\ln\left(\dfrac{12.0\text{ V}}{3.00\text{ V}}\right)} = 8.49\times10^{-7}\text{ F}.$$

(b) $\tau = RC = (3.40\times10^6\ \Omega)(8.49\times10^{-7}\ \text{F}) = 2.89\text{ s}.$

EVALUATE: In most laboratory circuits, time constants are much shorter than this one.

26.38. **IDENTIFY:** When S is closed, charge starts to flow and charge the capacitor until the potential difference across the capacitor is equal to the emf of the battery.

SET UP: $V_R = RI$, $V_C = \mathcal{E}(1 - e^{-t/RC})$, and $U_C = Q^2/2C$.

EXECUTE: **(a)** Kirchhoff's loop rule gives $V_C + V_R = \mathcal{E}$,

so $I = (\mathcal{E} - V_C)/R$ = (36.0 V – 8.00 V)/(120 Ω) = 0.2333 A, which rounds to 0.233 A.

(b) From $V_C = \mathcal{E}(1 - e^{-t/RC})$, we get $e^{-t/RC} = 1 - V_C/\mathcal{E}$. Taking logs gives $-t/RC = \ln(1 - V_C/\mathcal{E})$. Solving for t gives t = –(120 Ω)(5.00 μF) ln[1 – (8.00 V)/(36.0 V)] = 151 μs.

(c) $U_C = Q^2/2C$, so $P_C = dU_C/dt = (Q/C)\,dQ/dt = V_C I$ = (8.00 V)(0.2333 A) = 1.87 W.

EVALUATE: $P_C + P_R = P_C + I^2R$ = 1.87 W + (0.2333 A)^2(120 Ω) = 8.40 W. $P_{\mathcal{E}} = I\mathcal{E}$ = (0.2333 A)(36.0 V) = 8.40 W. These results for the power agree, as they should by conservation of energy.

26.39. **IDENTIFY:** An uncharged capacitor is placed into a circuit. Apply the loop rule at each time.

SET UP: The voltage across a capacitor is $V_C = q/C$.

EXECUTE: **(a)** At the instant the circuit is completed, there is no voltage across the capacitor, since it has no charge stored.

(b) Since s the full battery voltage appears across the resistor $V_R = \mathcal{E} = 245\text{ V}$.

(c) There is no charge on the capacitor.

(d) The current through the resistor is $i = \dfrac{\mathcal{E}}{R_{\text{total}}} = \dfrac{245\text{ V}}{7500\ \Omega} = 0.0327\text{ A} = 32.7\text{ mA}.$

(e) After a long time has passed the full battery voltage is across the capacitor and $i = 0$. The voltage across the capacitor balances the emf: $V_C = 245\text{ V}$. The voltage across the resistor is zero. The capacitor's charge is $q = CV_C = (4.60\times10^{-6}\text{ F})(245\text{ V}) = 1.13\times10^{-3}\text{ C}$. The current in the circuit is zero.

EVALUATE: The current in the circuit starts at 0.0327 A and decays to zero. The charge on the capacitor starts at zero and rises to $q = 1.13\times10^{-3}\text{ C}$.

26.40. **IDENTIFY:** Once the switch S is closed, current starts to flow and charge the capacitor.

SET UP: $P = IV$, $V_R = RI$, $U_C = Q^2/2C$, $Q = C\mathcal{E}(1 - e^{-t/RC})$, $(1 - e^{-t/RC})$, and $I = (\mathcal{E}/R)\,e^{-t/RC}$.

EXECUTE: **(a)** $\mathcal{E} = V_R + V_C = IR + Q/C$ = (3.00 A)(12.0 Ω) + (40.0 μC)/(5.00 μF) = 44.0 V.

(b) The current is $I = (\mathcal{E}/R)\,e^{-t/RC}$. The current is 3.00 A when Q = 40.0 μC, so

$3.00\text{ A} = [(44.0\ V)/(12.0\ \Omega)]e^{-t/RC}$. Taking logs and solving for t gives

$-t/RC = \ln(36.0/44.0)$.

$t = -(12.0\ \Omega)(5.00\ \mu\text{F})\ \ln(36.0/44.0) = 12.0\ \mu\text{s}$.

(c) (i) The power in the capacitor is $P_C = dU/dt = d(Q^2/2C)/dt = (Q/C)\ dQ/dt = QI/C$, so

$P_C = (40.0\ \mu C)(3.00\ A)/(5.00\ \mu F) = 24.0$ W.

(ii) $P_{\mathcal{E}} = I\mathcal{E} = (3.00\ \text{A})(44.0\ \text{V}) = 132$ W.

EVALUATE: In (c), when $I = 3.00$ A, $P_R = I^2R = (3.00\ A)^2(12.0\ \Omega) = 108\ W$. Therefore $P_R + P_C = 108$ W + 24.0 W = 132 W, which is equal to $P_{\mathcal{E}}$, as it should be by energy conservation. In (b), we can use the equation $Q = C\mathcal{E}(1 - e^{-t/RC})$ to calculate Q when $t = 12.0\ \mu$s; it should be 40.0 μC. We have $Q = (44.0\ \text{V})(5.00\ \mu\text{F})(1 - e^{-(12.0\ \mu s)/[(12.0\ \Omega)(5.00\ \mu F)]}) = 40.0\ \mu\text{C}$, as expected.

26.41. **IDENTIFY:** The capacitors, which are in parallel, will discharge exponentially through the resistors.

SET UP: Since V is proportional to Q, V must obey the same exponential equation as Q, $V = V_0 e^{-t/RC}$. The current is $I = (V_0/R)\ e^{-t/RC}$.

EXECUTE: **(a)** Solve for time when the potential across each capacitor is 10.0 V:

$t = -RC\ \ln(V/V_0) = -(80.0\ \Omega)(35.0\ \mu\text{F})\ \ln(10/45) = 4210\ \mu\text{s} = 4.21$ ms.

(b) $I = (V_0/R)\ e^{-t/RC}$. Using the above values, with $V_0 = 45.0$ V, gives $I = 0.125$ A.

EVALUATE: Since the current and the potential both obey the same exponential equation, they are both reduced by the same factor (0.222) in 4.21 ms.

26.42. **IDENTIFY:** For a charging capacitor $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$ and $i(t) = \dfrac{\mathcal{E}}{R}e^{-t/\tau}$.

SET UP: The time constant is $RC = (0.895\times10^6\ \Omega)\ (12.4\times10^{-6}\ \text{F}) = 11.1$ s.

EXECUTE: **(a)** At $t = 0$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = 0$.

At $t = 5$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(5.0\ \text{s})/(11.1\ \text{s})}) = 2.70\times10^{-4}$ C.

At $t = 10$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(10.0\ \text{s})/(11.1\ \text{s})}) = 4.42\times10^{-4}$ C.

At $t = 20$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(20.0\ \text{s})/(11.1\ \text{s})}) = 6.21\times10^{-4}$ C.

At $t = 100$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(100\ \text{s})/(11.1\ \text{s})}) = 7.44\times10^{-4}$ C.

(b) The current at time t is given by: $i = \dfrac{\mathcal{E}}{R}e^{-t/RC}$.

At $t = 0$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-0/11.1} = 6.70\times10^{-5}$ A.

At $t = 5$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-5/11.1} = 4.27\times10^{-5}$ A.

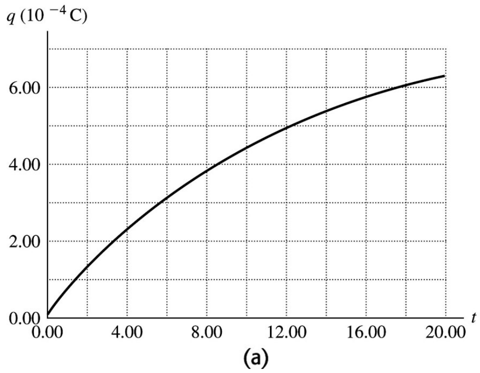
At $t = 10$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-10/11.1} = 2.72\times10^{-5}$ A.

At $t = 20$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-20/11.1} = 1.11\times10^{-5}$ A.

At $t = 100$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-100/11.1} = 8.20\times10^{-9}$ A.

(c) The graphs of $q(t)$ and $i(t)$ are given in Figure 26.42a and b.

EVALUATE: The charge on the capacitor increases in time as the current decreases.



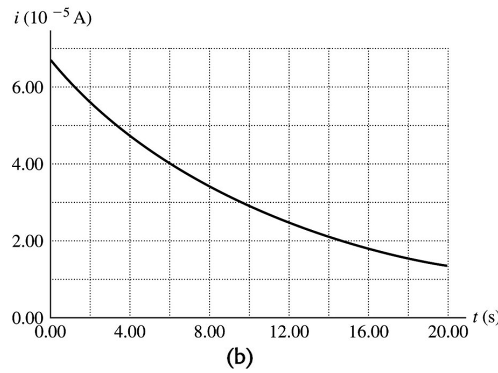


Figure 26.42

26.43. **IDENTIFY** and **SET UP:** Apply Kirchhoff's loop rule. The voltage across the resistor depends on the current through it and the voltage across the capacitor depends on the charge on its plates.

EXECUTE: $\mathcal{E} - V_R - V_C = 0.$

$\mathcal{E} = 120$ V, $V_R = IR = (0.900 \text{ A})(80.0\,\Omega) = 72$ V, so $V_C = 48$ V.

$Q = CV = (4.00\times 10^{-6} \text{ F})(48 \text{ V}) = 192\,\mu\text{C}.$

EVALUATE: The initial charge is zero and the final charge is $C\mathcal{E} = 480\,\mu\text{C}$. Since current is flowing at the instant considered in the problem the capacitor is still being charged and its charge has not reached its final value.

26.44. **IDENTIFY:** We have two capacitors charging through a resistor.

SET UP: We want the time constant if the capacitors are in series and if they are in parallel. $\tau = RC$. $1/C_{\text{eq}} = 1/C_1 + 1/C_2 + \cdots$ (series) and $C_{\text{eq}} = C_1 + C_2 + \cdots$ (parallel).

EXECUTE: **(a)** Series: $1/C_{\text{eq}} = 1/C_1 + 1/C_2 + \cdots = 1/C + 1/C = 2/C$. $C_{\text{eq}} = C/2$. $\tau = RC_{\text{eq}} = RC/2$.

(b) Parallel: $C_{\text{eq}} = C_1 + C_2 + \cdots = C + C = 2C$. $\tau = RC_{\text{eq}} = R(2C) = 2RC$.

(c) The series connection has the shorter time constant so the current decreases faster. This means that V_{-R} also decreases faster since $V_R = IR$. So the answer is *series*.

EVALUATE: There is a factor of 4 difference in the time constants. The series circuit changes much faster than the parallel circuit.

26.45. **IDENTIFY:** The stored energy is proportional to the square of the charge on the capacitor, so it will obey an exponential equation, but not the same equation as the charge.

SET UP: The energy stored in the capacitor is $U = Q^2/2C$ and the charge on the plates is $Q_0 e^{-t/RC}$. The current is $I = I_0 e^{-t/RC}$.

EXECUTE: $U = Q^2/2C = (Q_0 e^{-t/RC})^2/2C = U_0 e^{-2t/RC}$. When the capacitor has lost 80% of its stored energy, the energy is 20% of the initial energy, which is $U_0/5. U_0/5 = U_0\ e^{-2t/RC}$ gives $t = (RC/2)\ \ln 5 = (25.0\ \Omega)(4.62 \text{ pF})(\ln 5)/2 = 92.9$ ps.

At this time, the current is $I = I_0\ e^{-t/RC} = (Q_0/RC)\ e^{-t/RC}$, so

$$I = (3.5 \text{ nC})/[(25.0\ \Omega)(4.62 \text{ pF})]\ e^{-(92.9 \text{ ps})/[(25.0\ \Omega)(4.62 \text{ pF})]} = 13.6 \text{ A}.$$

EVALUATE: When the energy is reduced by 80%, neither the current nor the charge are reduced by that percent.

26.46. **IDENTIFY:** The charge is increasing while the current is decreasing. Both obey exponential equations, but they are not the same equation.

SET UP: The charge obeys the equation $Q = Q_{\max}(1 - e^{-t/RC})$, but the equation for the current is $I = I_{\max}e^{-t/RC}$.

EXECUTE: When the charge has reached $\frac{1}{4}$ of its maximum value, we have $Q_{\max}/4 = Q_{\max}(1 - e^{-t/RC})$, which says that the exponential term has the value $e^{-t/RC} = \frac{3}{4}$. The current at this time is $I = I_{\max}e^{-t/RC} = I_{\max}(3/4) = (3/4)[(10.0\ \text{V})/(12.0\ \Omega)] = 0.625\ \text{A}$.

EVALUATE: Notice that the current will be $\frac{3}{4}$, not $\frac{1}{4}$, of its maximum value when the charge is $\frac{1}{4}$ of its maximum. Although current and charge both obey exponential equations, the equations have different forms for a charging capacitor.

26.47. **IDENTIFY:** In both cases, simplify the complicated circuit by eliminating the appropriate circuit elements. The potential across an uncharged capacitor is initially zero, so it behaves like a short circuit. A fully charged capacitor allows no current to flow through it.

(a) SET UP: Just after closing the switch, the uncharged capacitors all behave like short circuits, so any resistors in parallel with them are eliminated from the circuit.

EXECUTE: The equivalent circuit consists of $50\ \Omega$ and $25\ \Omega$ in parallel, with this combination in series with $75\ \Omega$, $15\ \Omega$, and the 100-V battery. The equivalent resistance is $90\ \Omega + 16.7\ \Omega = 106.7\ \Omega$, which gives $I = (100\ \text{V})/(106.7\ \Omega) = 0.937\ \text{A}$.

(b) SET UP: Long after closing the switch, the capacitors are essentially charged up and behave like open circuits since no charge can flow through them. They effectively eliminate any resistors in series with them since no current can flow through these resistors.

EXECUTE: The equivalent circuit consists of resistances of $75\ \Omega$, $15\ \Omega$, and three 25-Ω resistors, all in series with the 100-V battery, for a total resistance of $165\ \Omega$. Therefore $I = (100\text{V})/(165\ \Omega) = 0.606\ \text{A}$.

EVALUATE: The initial and final behavior of the circuit can be calculated quite easily using simple series-parallel circuit analysis. Intermediate times would require much more difficult calculations!

26.48. **IDENTIFY:** Both the charge and energy decay exponentially, but not with the same time constant since the energy is proportional to the *square* of the charge.

SET UP: The charge obeys the equation $Q = Q_0e^{-t/RC}$ but the energy obeys the equation $U = Q^2/2C = (Q_0e^{-t/RC})^2/2C = U_0e^{-2t/RC}$.

EXECUTE: **(a)** The charge is reduced by half: $Q_0/2 = Q_0e^{-t/RC}$. This gives $t = RC\ \ln 2 = (225\ \Omega)(12.0\ \mu\text{F})(\ln 2) = 1.871\ \text{ms}$, which rounds to 1.87 ms.

(b) The energy is reduced by half: $U_0/2 = U_0e^{-2t/RC}$. This gives $t = (RC\ \ln 2)/2 = (1.871\ \text{ms})/2 = 0.936\ \text{ms}$.

EVALUATE: The energy decreases faster than the charge because it is proportional to the square of the charge.

26.49. **IDENTIFY:** When the capacitor is fully charged the voltage V across the capacitor equals the battery emf and $Q = CV$. For a charging capacitor, $q = Q(1 - e^{-t/RC})$.

SET UP: $\ln e^x = x$.

EXECUTE: **(a)** $Q = CV = (5.90\times10^{-6}\ \text{F})(28.0\ \text{V}) = 1.65\times10^{-4}\ \text{C} = 165\ \mu\text{C}$.

(b) $q = Q(1 - e^{-t/RC})$, so $e^{-t/RC} = 1 - \dfrac{q}{Q}$ and $R = \dfrac{-t}{C\ln(1 - q/Q)}$. After

$$t = 3\times10^{-3}\ \text{s}: R = \frac{-3\times10^{-3}\ \text{s}}{(5.90\times10^{-6}\ \text{F})(\ln(1 - 110/165))} = 463\ \Omega.$$

(c) If the charge is to be 99% of final value: $\frac{q}{Q} = (1 - e^{-t/RC})$ gives

$t = -RC \ln(1 - q/Q) = -(463\ \Omega)(5.90\times10^{-6}\ \text{F})\ln(0.01) = 0.0126\ \text{s} = 12.6\ \text{ms}.$

EVALUATE: The time constant is $\tau = RC = 2.73$ ms. The time in part (b) is a bit more than one time constant and the time in part (c) is about 4.6 time constants.

26.50. **IDENTIFY:** We have a capacitor discharging through a resistor.

SET UP: $i = I_0 e^{-t/RC}$ and $q = Q_0 e^{-t/RC}$. When the current is 0.180 A, we want the charge and time.

EXECUTE: **(a)** $V_C = V_R$, so $q/C = RI$. $q = RIC = (185\ \Omega)(0.180\ \text{A})(6.00\ \mu\text{F}) = 200\ \mu\text{C}$.

(b) $i = I_0 e^{-t/RC}$, where $I_0 = V_0/R = (50.0\ \text{V})/(185\ \Omega) = 0.2703$ A. Use logarithms to solve for t.

$t = -RC\ln(i/I_0) = -(185\ \Omega)(6.00\ \mu\text{F})\ln(0.180/0.2703) = 451\ \mu\text{s}.$

EVALUATE: Check: At the time in (b), q should be 200 μC. Putting the numbers into $q = Q_0 e^{-t/RC}$ gives 200 μC, so our result checks.

26.51. **IDENTIFY** and **SET UP:** The heater and hair dryer are in parallel so the voltage across each is 120 V and the current through the fuse is the sum of the currents through each appliance. As the power consumed by the dryer increases, the current through it increases. The maximum power setting is the highest one for which the current through the fuse is less than 20 A.

EXECUTE: Find the current through the heater. $P = VI$ so $I = P/V = (1500\ \text{W})/(120\ \text{V}) = 12.5$ A. The maximum total current allowed is 20 A, so the current through the dryer must be less than $20\ \text{A} - 12.5\ \text{A} = 7.5$ A. The power dissipated by the dryer if the current has this value is $P = VI = (120\ \text{V})(7.5\ \text{A}) = 900$ W. For P at this value or larger the circuit breaker trips.

EVALUATE: $P = V^2/R$ and for the dryer V is a constant 120 V. The higher power settings correspond to a smaller resistance R and larger current through the device.

26.52. **IDENTIFY:** $P = VI = I^2R$

SET UP: Problem 25.76 says that for 12-gauge wire the maximum safe current is 25 A.

EXECUTE: **(a)** $I = \frac{P}{V} = \frac{4100\ \text{W}}{240\ \text{V}} = 17.1$ A. So we need at least 14-gauge wire (good up to 18 A). 12-gauge is also ok (good up to 25 A).

(b) $P = \frac{V^2}{R}$ and $R = \frac{V^2}{P} = \frac{(240\ \text{V})^2}{4100\ \text{W}} = 14\ \Omega.$

(c) At 11¢ per kWh, for 1 hour the cost is $(11¢/\text{kWh})(1\ \text{h})(4.1\ \text{kW}) = 45¢$.

EVALUATE: The cost to operate the device is proportional to its power consumption.

26.53. **IDENTIFY:** We have a capacitor charging through a resistor.

SET UP: $i = I_0 e^{-t/RC}$, $U_C = \frac{1}{2}CV^2$, $P_{\mathcal{E}} = i\mathcal{E}$, and $P = i^2R$. Our target variable is the energy.

EXECUTE: **(a)** We want the energy in the capacitor. When fully charged, $i = 0$ so $V_C = \mathcal{E}$. Thus $U_C = \frac{1}{2}C\mathcal{E}^2$.

(b) $P_{\mathcal{E}} = i\mathcal{E}$. Since i is variable, we need to integrate to find the energy.

$U_{\mathcal{E}} = \int_0^\infty i\mathcal{E}dt = \int_0^\infty I_0 e^{-t/RC}dt = I_0\mathcal{E}(-RC)e^{-t/RC}\Big|_0^\infty = I_0\mathcal{E}RC.\ \ I_0 = \frac{\mathcal{E}}{R}$ so $U_{\mathcal{E}} = C\mathcal{E}^2$.

(c) $P_R = i^2R$, so $U_R = \int_0^\infty i^2Rdt = \int_0^\infty \left(I_0 e^{-t/RC}\right)^2 Rdt = I_0^2R\int_0^\infty e^{-2t/RC}dt = \left(I_0^2R\right)\left(\frac{RC}{2}\right).\ \ I_0 = \frac{\mathcal{E}}{R}$ so $U_R = \frac{1}{2}C\mathcal{E}^2$.

(d) $\dfrac{U_C}{U_\mathcal{E}}=\dfrac{\frac{1}{2}C\mathcal{E}^2}{C\mathcal{E}^2}=\dfrac{1}{2}$. $\dfrac{U_R}{U_\mathcal{E}}=\dfrac{\frac{1}{2}C\mathcal{E}^2}{C\mathcal{E}^2}=\dfrac{1}{2}$. Half the energy is stored in the capacitor and half is dissipated in the resistor.

EVALUATE: The result in (d) is compatible with energy conservation.

26.54. IDENTIFY: We need to do series/parallel reduction to solve this circuit.

SET UP: $P=\dfrac{\mathcal{E}^2}{R}$, where R is the equivalent resistance of the network. For resistors in series, $R_{\text{eq}}=R_1+R_2$, and for resistors in parallel $1/R_{\text{P}}=1/R_1+1/R_2$.

EXECUTE: $R=\dfrac{\mathcal{E}^2}{P}=\dfrac{(48.0\text{ V})^2}{295\text{ W}}=7.810\ \Omega$. $R_{12}=R_1+R_2=8.00\ \Omega$. $R=R_{123}+R_4$.

$R_{123}=R-R_4=7.810\ \Omega-3.00\ \Omega=4.810\ \Omega$. $\dfrac{1}{R_{12}}+\dfrac{1}{R_3}=\dfrac{1}{R_{123}}$. $\dfrac{1}{R_3}=\dfrac{1}{R_{123}}-\dfrac{1}{R_{12}}=\dfrac{R_{12}-R_{123}}{R_{123}R_{12}}$.

$R_3=\dfrac{R_{123}R_{12}}{R_{12}-R_{123}}=\dfrac{(4.810\ \Omega)(8.00\ \Omega)}{8.00\ \Omega-4.810\ \Omega}=12.1\ \Omega$.

EVALUATE: The resistance R_3 is greater than R, since the equivalent parallel resistance is less than any of the resistors in parallel.

26.55. IDENTIFY: This problem requires Kirchhoff's rules. The target variables are the currents.

SET UP: Refer to Fig. 26.6(a) in the textbook and use the same loops shown there. For the currents choose I_1 downward through r_1, I_2 upward through r_2, and I_R downward through R. Do all the loops in a counterclockwise sense, as in the textbook. Now apply Kirchhoff's rules.

EXECUTE: <u>Junction rule</u>: $I_2 = I_1 + I_R$.

<u>Loop s</u>: $-r_2I_2-\mathcal{E}_1+RI_R=0 \quad\rightarrow\quad -(2.00\ \Omega)I_1 - 24.0\text{ V} + (20.0\ \Omega)I_R = 0$.

<u>Loop 2</u>: $\mathcal{E}_2-r_2I_2-r_1I_1-\mathcal{E}_1=0 \quad\rightarrow\quad 36.0\text{ V} - (2.00\ \Omega)I_2 - (2.00\ \Omega)I_1 - 24.0\text{ V} = 0$.

<u>Loop 3</u>: $+r_2I_2-\mathcal{E}_2+RI_R=0 \quad\rightarrow\quad (2.00\ \Omega)I_2 - 36.0\text{ V} + (20.0\ \Omega)I_R = 0$.

Solve these equations by substitution (or any other method) and obtain the following answers.

(a) $I–_1 = 2.29$ A.

(b) $I_2 = 3.71$ A.

(c) $I_R = 1.43$ A.

EVALUATE: Check: $I_1 + I_R = 2.29\text{ A} + 1.43\text{ A} = 3.72$ A. This agrees with our answer in (b).The slight difference is due to rounding during calculations.

26.56. IDENTIFY: Half the current flows through each parallel resistor and the full current flows through the third resistor, that is in series with the parallel combination. Therefore, only the series resistor will be at its maximum power.

SET UP: $P=I^2R$.

EXECUTE: The maximum allowed power is when the total current is the maximum allowed value of Then half the current flows through the parallel resistors and the maximum power is

$P_{\max}=(I/2)^2R+(I/2)^2R+I^2R=\frac{3}{2}I^2R=\frac{3}{2}(4.47\text{ A})^2(2.4\ \Omega)=72\text{ W}$.

EVALUATE: If all three resistors were in series or all three were in parallel, then the maximum power would be $3(48\text{ W})=144\text{ W}$. For the network in this problem, the maximum power is half this value.

26.57. (a) IDENTIFY: Break the circuit between points a and b means no current in the middle branch that contains the 3.00-Ω resistor and the 10.0-V battery. The circuit therefore has a single current path. Find the current, so that potential drops across the resistors can be calculated. Calculate V_{ab} by traveling from a to b, keeping track of the potential changes along the path taken.

SET UP: The circuit is sketched in Figure 26.57a.

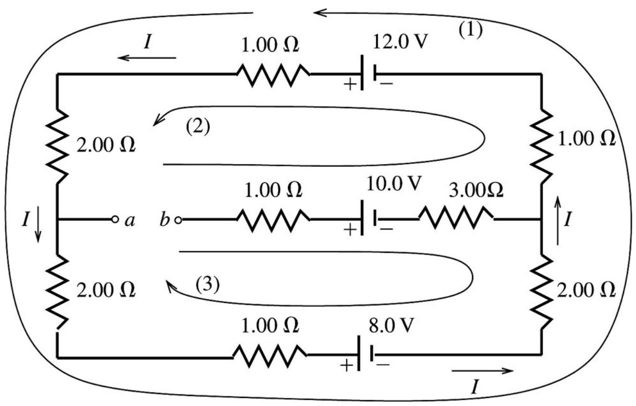


Figure 26.57a

EXECUTE: Apply Kirchhoff's loop rule to loop (1).
$+12.0\text{ V} - I(1.00\,\Omega + 2.00\,\Omega + 2.00\,\Omega + 1.00\,\Omega) - 8.0\text{ V} - I(2.00\,\Omega + 1.00\,\Omega) = 0.$

$I = \dfrac{12.0\text{ V} - 8.0\text{ V}}{9.00\,\Omega} = 0.4444\text{ A}.$

To find V_{ab} start at point b and travel to a, adding up the potential rises and drops. Travel on path (2) shown on the diagram. The $1.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors in the middle branch have no current through them and hence no voltage across them. Therefore,

$V_b - 10.0\text{ V} + 12.0\text{ V} - I(1.00\,\Omega + 1.00\,\Omega + 2.00\,\Omega) = V_a$; thus

$V_a - V_b = 2.0\text{ V} - (0.4444\text{ A})(4.00\,\Omega) = +0.22\text{ V}$ (point a is at higher potential).

EVALUATE: As a check on this calculation we also compute V_{ab} by traveling from b to a on path (3).

$V_b - 10.0\text{ V} + 8.0\text{ V} + I(2.00\,\Omega + 1.00\,\Omega + 2.00\,\Omega) = V_a.$

$V_{ab} = -2.00\text{ V} + (0.4444\text{ A})(5.00\,\Omega) = +0.22\text{ V}$, which checks.

(b) IDENTIFY and **SET UP:** With points a and b connected by a wire there are three current branches, as shown in Figure 26.57b.

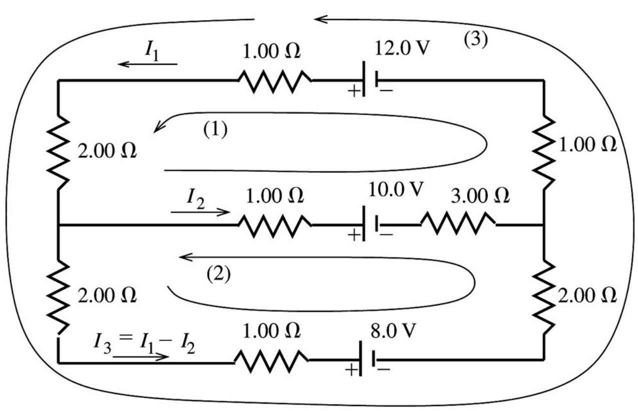


Figure 26.57b

The junction rule has been used to write the third current (in the 8.0-V battery) in terms of the other currents. Apply the loop rule to loops (1) and (2) to obtain two equations for the two unknowns I_1 and I_2.

EXECUTE: Apply the loop rule to loop (1).

$12.0\text{ V} - I_1(1.00\,\Omega) - I_1(2.00\,\Omega) - I_2(1.00\,\Omega) - 10.0\text{ V} - I_2(3.00\,\Omega) - I_1(1.00\,\Omega) = 0$

$2.0\text{ V} - I_1(4.00\,\Omega) - I_2(4.00\,\Omega) = 0$

$(2.00\,\Omega)I_1 + (2.00\,\Omega)I_2 = 1.0\text{ V}$ eq. (1)

Apply the loop rule to loop (2).

$-(I_1 - I_2)(2.00\,\Omega) - (I_1 - I_2)(1.00\,\Omega) - 8.0\text{ V} - (I_1 - I_2)(2.00\,\Omega) + I_2(3.00\,\Omega) + 10.0\text{ V} + I_2(1.00\,\Omega) = 0$

$2.0\text{ V} - (5.00\,\Omega)I_1 + (9.00\,\Omega)I_2 = 0$ eq. (2)

Solve eq. (1) for I_2 and use this to replace I_2 in eq. (2).

$I_2 = 0.50\text{ A} - I_1$

$2.0\text{ V} - (5.00\,\Omega)I_1 + (9.00\,\Omega)(0.50\text{ A} - I_1) = 0$

$(14.0\,\Omega)I_1 = 6.50\text{ V}$ so $I_1 = (6.50\text{ V})/(14.0\,\Omega) = 0.464\text{ A}$

$I_2 = 0.500\text{ A} - 0.464\text{ A} = 0.036\text{ A}.$

The current in the 12.0-V battery is $I_1 = 0.464\text{ A}$

EVALUATE: We can apply the loop rule to loop (3) as a check.

$+12.0\text{ V} - I_1(1.00\,\Omega + 2.00\,\Omega + 1.00\,\Omega) - (I_1 - I_2)(2.00\,\Omega + 1.00\,\Omega + 2.00\,\Omega) - 8.0\text{ V} = 4.0\text{ V} - 1.86\text{ V} - 2.14\text{ V} = 0,$ as it should.

26.58. **IDENTIFY:** Heat, which is generated in the resistor, melts the ice.

SET UP: Find the rate at which heat is generated in the $20.0\text{-}\Omega$ resistor using $P = V^2/R$. Then use the heat of fusion of ice to find the rate at which the ice melts. The heat dH to melt a mass of ice dm is $dH = L_F\ dm$, where L_F is the latent heat of fusion. The rate at which heat enters the ice, dH/dt, is the power P in the resistor, so $P = L_F\ dm/dt$. Therefore the rate of melting of the ice is $dm/dt = P/L_F$.

EXECUTE: The equivalent resistance of the parallel branch is $5.00\ \Omega$, so the total resistance in the circuit is $35.0\ \Omega$. Therefore the total current in the circuit is $I_{\text{Total}} = (45.0\text{ V})/(35.0\ \Omega) = 1.286\text{ A}$. The potential difference across the $20.0\text{-}\Omega$ resistor in the ice is the same as the potential difference across the parallel branch: $V_{\text{ice}} = I_{\text{Total}}R_p = (1.286\text{ A})(5.00\ \Omega) = 6.429\text{ V}$. The rate of heating of the ice is

$P_{\text{ice}} = V_{\text{ice}}^2/R = (6.429\text{ V})^2/(20.0\ \Omega) = 2.066\text{ W}$. This power goes into to heat to melt the ice, so

$dm/dt = P/L_F = (2.066\text{ W})/(3.34\times10^5\text{ J/kg}) = 6.19\times10^{-6}\text{ kg/s} = 6.19\times10^{-3}\text{ g/s}.$

EVALUATE: The melt rate is about 6 mg/s, which is not much. It would take 1000 s to melt just 6 g of ice.

25.59. **IDENTIFY:** Apply Kirchhoff's junction rule to express the currents through the $5.00\text{-}\Omega$ and $8.00\text{-}\Omega$ resistors in terms of I_1, I_2, and I_3. Apply the loop rule to three loops to get three equations in the three unknown currents.

SET UP: The circuit is sketched in Figure 26.59.

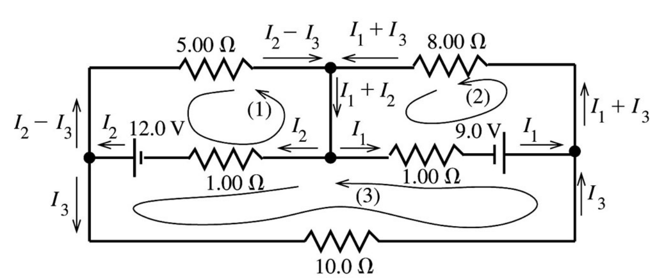


Figure 26.59

The current in each branch has been written in terms of I_1, I_2, and I_3 such that the junction rule is satisfied at each junction point.

EXECUTE: Apply the loop rule to loop (1).

$-12.0\text{ V} + I_2(1.00\,\Omega) + (I_2 - I_3)(5.00\,\Omega) = 0$

$I_2(6.00\,\Omega) - I_3(5.00\,\Omega) = 12.0\text{ V}$ eq. (1)

Apply the loop rule to loop (2).

$-I_1(1.00\,\Omega) + 9.00\text{ V} - (I_1 + I_3)(8.00\,\Omega) = 0$

$I_1(9.00\,\Omega) + I_3(8.00\,\Omega) = 9.00\text{ V}$ eq. (2)

Apply the loop rule to loop (3).

$-I_3(10.0\,\Omega) - 9.00\text{ V} + I_1(1.00\,\Omega) - I_2(1.00\,\Omega) + 12.0\text{ V} = 0$

$-I_1(1.00\,\Omega) + I_2(1.00\,\Omega) + I_3(10.0\,\Omega) = 3.00\text{ V}$ eq. (3)

Eq. (1) gives $I_2 = 2.00\text{ A} + \frac{5}{6}I_3$; eq. (2) gives $I_1 = 1.00\text{ A} - \frac{8}{9}I_3$.

Using these results in eq. (3) gives

$-(1.00\text{ A} - \frac{8}{9}I_3)(1.00\,\Omega) + (2.00\text{ A} + \frac{5}{6}I_3)(1.00\,\Omega) + I_3(10.0\,\Omega) = 3.00\text{ V}.$

$(\frac{16+15+180}{18})I_3 = 2.00\text{ A};\ I_3 = \frac{18}{211}(2.00\text{ A}) = 0.171\text{ A}.$

Then $I_2 = 2.00\text{ A} + \frac{5}{6}I_3 = 2.00\text{ A} + \frac{5}{6}(0.171\text{ A}) = 2.14\text{ A}$ and

$I_1 = 1.00\text{ A} - \frac{8}{9}I_3 = 1.00\text{ A} - \frac{8}{9}(0.171\text{ A}) = 0.848\text{ A}.$

EVALUATE: We could check that the loop rule is satisfied for a loop that goes through the $5.00\text{-}\Omega$, $8.00\text{-}\Omega$ and $10.0\text{-}\Omega$ resistors. Going around the loop clockwise:

$-(I_2 - I_3)(5.00\,\Omega) + (I_1 + I_3)(8.00\,\Omega) + I_3(10.0\,\Omega) = -9.85\text{ V} + 8.15\text{ V} + 1.71\text{ V}$, which does equal zero, apart from rounding.

26.60. IDENTIFY: Apply the junction rule and the loop rule to the circuit.

SET UP: Because of the polarity of each emf, the current in the $7.00\text{-}\Omega$ resistor must be in the direction shown in Figure 26.60a. Let I be the current in the 24.0-V battery.

EXECUTE: The loop rule applied to loop (1) gives: $+24.0\text{ V} - (1.80\text{ A})(7.00\,\Omega) - I(3.00\,\Omega) = 0.$

$I = 3.80\text{ A}.$ The junction rule then says that the current in the middle branch is 2.00 A, as shown in Figure 26.64b. The loop rule applied to loop (2) gives: $+\mathcal{E} - (1.80\text{ A})(7.00\,\Omega) + (2.00\text{ A})(2.00\,\Omega) = 0$ and $\mathcal{E} = 8.6\text{ V}.$

EVALUATE: We can check our results by applying the loop rule to loop (3) in Figure 26.60b:

$+24.0\text{ V} - \mathcal{E} - (2.00\text{ A})(2.00\,\Omega) - (3.80\text{ A})(3.00\,\Omega) = 0$ and $\mathcal{E} = 24.0\text{ V} - 4.0\text{ V} - 11.4\text{ V} = 8.6\text{ V}$, which agrees with our result from loop (2).

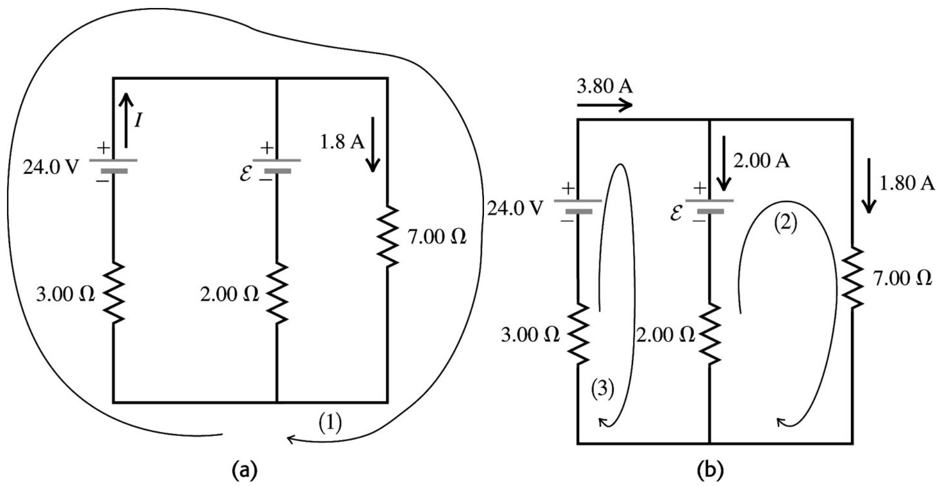


Figure 26.60

26.61. **IDENTIFY:** This problem involves resistivity and a resistor in a circuit.

SET UP and **EXECUTE:** $R = \dfrac{\rho L}{A}$.

(a) Estimate: Diameter = 0.50 mm.

(b) Solve $R = \dfrac{\rho L}{A}$ for R/L giving $\dfrac{R}{L} = \dfrac{\rho}{A} = \dfrac{\rho}{\pi r^2} = \dfrac{1.72 \times 10^{-8}\ \Omega \cdot \text{m}}{\pi (0.25\ \text{mm})^2} = 0.088\ \Omega/\text{m}.$

(c) $IR = 1.0$ V, so $(1.0\ \text{A})R = 1.0$ V, which gives $R = 1.0\ \Omega$. $(R/L)L = R$, so $(0.088\ \Omega/\text{m})L = 1.0\ \Omega$. This gives $L = 11$ m.

EVALUATE: The wire must be tightly wound to be 11 m (≈35 ft) long.

26.62. **IDENTIFY:** Apply the loop and junction rules.

SET UP: Use the currents as defined on the circuit diagram in Figure 26.62 and obtain three equations to solve for the currents.

EXECUTE: **(a)** Left loop: $14 - I_1 - 2(I_1 - I_2) = 0$ and $3I_1 - 2I_2 = 14$.

Top loop: $-2(I - I_1) + I_2 + I_1 = 0$ and $-2I + 3I_1 + I_2 = 0$.

Bottom loop: $-(I - I_1 + I_2) + 2(I_1 - I_2) - I_2 = 0$ and $-I + 3I_1 - 4I_2 = 0$.

Solving these equations for the currents we find: $I = I_{\text{battery}} = 10.0$ A; $I_1 = I_{R_1} = 6.0$ A; $I_2 = I_{R_3} = 2.0$ A.

So the other currents are: $I_{R_2} = I - I_1 = 4.0$ A; $I_{R_4} = I_1 - I_2 = 4.0$ A; $I_{R_5} = I - I_1 + I_2 = 6.0$ A.

(b) $R_{\text{eq}} = \dfrac{V}{I} = \dfrac{14.0\ \text{V}}{10.0\ \text{A}} = 1.40\ \Omega.$

EVALUATE: It isn't possible to simplify the resistor network using the rules for resistors in series and parallel. But the equivalent resistance is still defined by $V = IR_{\text{eq}}$.

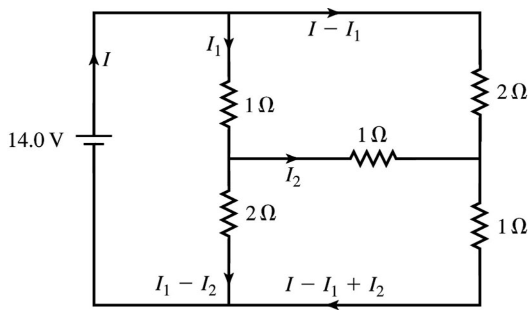


Figure 26.62

26.63. **IDENTIFY:** Simplify the resistor networks as much as possible using the rule for series and parallel combinations of resistors. Then apply Kirchhoff's laws.
SET UP: First do the series/parallel reduction. This gives the circuit in Figure 26.63. The rate at which the $10.0\text{-}\Omega$ resistor generates thermal energy is $P = I^2 R$.
EXECUTE: **(a)** Apply Kirchhoff's laws and solve for $\mathcal{E}$.
$\Delta V_{\text{adefa}} = 0\text{: } -(20\ \Omega)(2\text{ A}) - 5\text{ V} - (20\ \Omega)I_2 = 0.$
This gives $I_2 = -2.25\text{ A}$. Then $I_1 + I_2 = 2\text{ A}$ gives $I_1 = 2\text{ A} - (-2.25\text{ A}) = 4.25\text{ A}$.
$\Delta V_{\text{abcdefa}} = 0\text{: } (15\ \Omega)(4.25\text{ A}) + \mathcal{E} - (20\ \Omega)(-2.25\text{ A}) = 0$. This gives $\mathcal{E} = -109\text{ V}$. Since $\mathcal{E}$ is calculated to be negative, its polarity should be reversed.
(b) The parallel network that contains the $10.0\text{-}\Omega$ resistor in one branch has an equivalent resistance of $10\ \Omega$. The voltage across each branch of the parallel network is $V_{\text{par}} = RI = (10\ \Omega)(2\text{A}) = 20\text{ V}$. The current in the upper branch is $I = \dfrac{V}{R} = \dfrac{20\text{ V}}{30\ \Omega} = \dfrac{2}{3}\text{ A}$. $Pt = E$, so $I^2\ Rt = E$, where $E = 60.0\text{ J}$.

$\left(\tfrac{2}{3}\text{A}\right)^2 (10\ \Omega)t = 60\text{ J}$, and $t = 13.5\text{ s}$.

EVALUATE: For the $10.0\text{-}\Omega$ resistor, $P = I^2 R = 4.44\text{ W}$. The total rate at which electrical energy is inputted to the circuit in the emf is $(5.0\text{ V})(2.0\text{ A}) + (109\text{ V})(4.25\text{ A}) = 473\text{ J}$. Only a small fraction of the energy is dissipated in the $10.0\text{-}\Omega$ resistor.

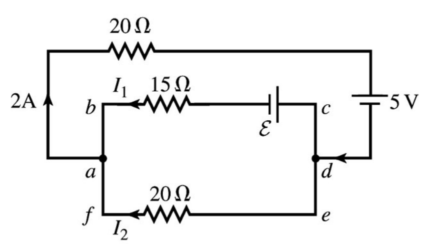


Figure 26.63

26.64. **IDENTIFY:** The resistor R_2 can vary between $3.00\ \Omega$ and $24.0\ \Omega$. R_2 is in parallel with R_1, so as R_2 is changed it affects the current in R_1 and hence the power dissipated in R_1. Ohm's law and Kirchhoff's rules apply.
SET UP: $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$, $V_R = IR$, $P_R = I^2R$.

EXECUTE: $P_1 = V_1^2/R_1$, so P_1 is largest when V_1 is largest. By Kirchhoff's loop rule,

$\mathcal{E} - V_1 - V_3 = 0$, so $V_1 = \mathcal{E} - V_3$, which means that V_1 is largest when V_3 is smallest.
$V_3 = IR_3 = \mathcal{E}/(R_{eq} + R_3)$, where R_{eq} is the equivalent resistance of the R_1-R_2 combination. Since they are in parallel, $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$, which gives $R_{eq} = \dfrac{R_1R_2}{R_1 + R_2}$. The smallest V_3 is for the smallest I, which occurs for the largest $R_{eq} = \dfrac{R_1R_2}{R_1 + R_2} = \dfrac{R_1}{\dfrac{R_1}{R_2} + 1}$.

As we can see, the largest R_{eq} occurs when R_2 is largest, which is $R_2 = 24.0\ \Omega$.
The equivalent parallel resistance is then
$R_{eq} = \dfrac{R_1R_2}{R_1 + R_2} = (6.00\ \Omega)(24.0\ \Omega)/(6.00\ \Omega + 24.0\ \Omega) = 4.80\ \Omega$.
The current I is then
$I = \mathcal{E}/(R_{eq} + R_3) = (24.0\ \text{V})/(4.80\ \Omega + 12.0\ \Omega) = 1.429\ \text{A}$.
$V_3 = IR_3 = (1.429\ \text{A})(12.0\ \Omega) = 17.148\ \text{V}$.
The potential difference across R_1 is
$V_1 = \mathcal{E} - V_3 = 24.0\ \text{V} - 17.148\ \text{V} = 6.852\ \text{V}$.
The power dissipated in R_1 is
$P_1 = V_1^2/R_1 = (6.852\ \text{V})^2/(6.00\ \Omega) = 7.83\ \text{W}$.

EVALUATE: Since all the circuit elements except for R_2 are fixed, varying R_2 affects the current in the circuit as well as the current through R_1.

26.65. **IDENTIFY** and **SET UP:** We want to estimate the cost to use some electrical appliances. The estimates and calculated results are shown in the table in part (a).

EXECUTE: **(a)** See the accompanying table.

Appliance	Voltage V	Current I	Time T	Power $P = IV$	Energy $U = PT$
Refrigerator	120 V	7.2 A	24 h	864 W	21 kWh
Water heater	240 V	20 A	4 h	4800 W	19 kWh
Electric oven	240 V	16 A	1 h	3840 W	3.8 kWh
Dishwasher	120 V	10 A	2 h	1200 W	2.4 kWh
Clothes washer	120 V	10 A	2/15 h	1200 W	0.2 kWh
Clothes dryer	120 V	26 A	¼ h	3120 W	0.8 kWh
15 light bulbs	120 V	7.5 A	7 h	900 W	6.3 kWh
Stereo system	120 V	0.2	2 h	24 W	0.05 kWh

The total energy is about 54 kWh.
(b) (54 kWh/day)(30 days/mo)($0.12/kWh) = $194/month.

EVALUATE: The refrigerator and water heater are by far the most costly to use, but listening to music costs hardly anything!

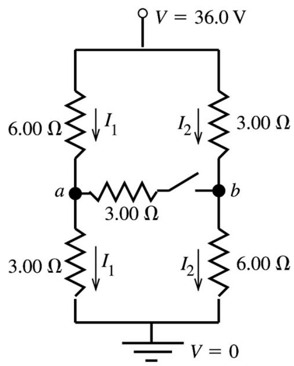
26.66. **IDENTIFY:** The current through the 40.0-Ω resistor equals the current through the emf, and the current through each of the other resistors is less than or equal to this current. So, set $P_{40} = 2.00\ \text{W}$, and use this to solve for the current I through the emf. If $P_{40} = 2.00\ \text{W}$, then P for each of the other resistors is less than 2.00 W.

SET UP: Use the equivalent resistance for series and parallel combinations to simplify the circuit.

EXECUTE: $I^2R = P$ gives $I^2(40\ \Omega) = 2.00\ \text{W}$, and $I = 0.2236\ \text{A}$. Now use series/parallel reduction to simplify the circuit. The upper parallel branch is $6.38\ \Omega$ and the lower one is $25\ \Omega$. The series sum is now $126\ \Omega$. Ohm's law gives $\mathcal{E} = (126\ \Omega)(0.2236\ \text{A}) = 28.2\ \text{V}$.

EVALUATE: The power input from the emf is $\mathcal{E}I = 6.30\text{ W}$, so nearly one-third of the total power is dissipated in the $40.0\text{-}\Omega$ resistor.

26.67. **(a) IDENTIFY** and **SET UP:** The circuit is sketched in Figure 26.67a.



With the switch open there is no current through it and there are only the two currents I_1 and I_2 indicated in the sketch.

Figure 26.67a

The potential drop across each parallel branch is 36.0 V. Use this fact to calculate I_1 and I_2. Then travel from point *a* to point *b* and keep track of the potential rises and drops in order to calculate V_{ab}.

EXECUTE: $-I_1(6.00\,\Omega + 3.00\,\Omega) + 36.0\text{ V} = 0.$

$$I_1 = \frac{36.0\text{ V}}{6.00\ \Omega + 3.00\,\Omega} = 4.00\text{ A.}$$

$$-I_2(3.00\,\Omega + 6.00\,\Omega) + 36.0\text{ V} = 0.$$

$$I_2 = \frac{36.0\text{ V}}{3.00\ \Omega + 6.00\,\Omega} = 4.00\text{ A.}$$

To calculate $V_{ab} = V_a - V_b$ start at point *b* and travel to point *a*, adding up all the potential rises and drops along the way. We can do this by going from *b* up through the $3.00\text{-}\Omega$ resistor:

$V_b + I_2(3.00\,\Omega) - I_1(6.00\,\Omega) = V_a$.

$V_a - V_b = (4.00\text{ A})(3.00\,\Omega) - (4.00\text{ A})(6.00\,\Omega) = 12.0\text{ V} - 24.0\text{ V} = -12.0\text{ V}.$

$V_{ab} = -12.0\text{ V}$ (point *a* is 12.0 V lower in potential than point *b*).

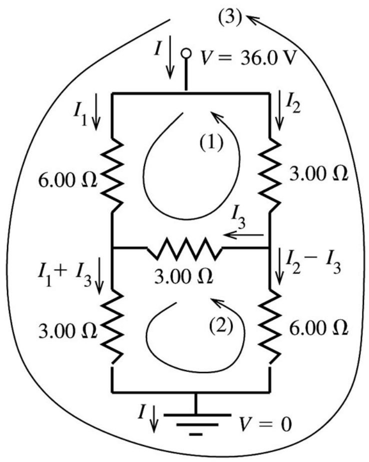
EVALUATE: Alternatively, we can go from point *b* down through the $6.00\text{-}\Omega$ resistor.

$V_b - I_2(6.00\,\Omega) + I_1(3.00\,\Omega) = V_a$.

$V_a - V_b = -(4.00\text{ A})(6.00\,\Omega) + (4.00\text{ A})(3.00\,\Omega) = -24.0\text{ V} + 12.0\text{ V} = -12.0\text{ V}$, which checks.

(b) IDENTIFY: Now there are multiple current paths, as shown in Figure 26.67b. Use the junction rule to write the current in each branch in terms of three unknown currents I_1, I_2, and I_3. Apply the loop rule to three loops to get three equations for the three unknowns. The target variable is I_3, the current through the switch. R_{eq} is calculated from $V = IR_{\text{eq}}$, where *I* is the total current that passes through the network.

SET UP:



The three unknown currents $I_1, I_2,$ and I_3 are labeled on Figure 26.67b.

Figure 26.67b

EXECUTE: Apply the loop rule to loops (1), (2) and (3).

Loop (1): $-I_1(6.00\ \Omega)+I_3(3.00\ \Omega)+I_2(3.00\ \Omega)=0$

$I_2 = 2I_1 - I_3$ eq. (1)

Loop (2): $-(I_1+I_3)(3.00\ \Omega)+(I_2-I_3)(6.00\ \Omega)-I_3(3.00\ \Omega)=0$

$6I_2 - 12I_3 - 3I_1 = 0$ so $2I_2 - 4I_3 - I_1 = 0$

Use eq (1) to replace I_2:

$4I_1 - 2I_3 - 4I_3 - I_1 = 0$

$3I_1 = 6I_3$ and $I_1 = 2I_3$ eq. (2)

Loop (3): This loop is completed through the battery (not shown), in the direction from the $-$ to the $+$ terminal.

$-I_1(6.00\ \Omega)-(I_1+I_3)(3.00\ \Omega)+36.0\text{ V}=0$

$9I_1 + 3I_3 = 36.0\text{ A}$ and $3I_1 + I_3 = 12.0\text{ A}$ eq. (3)

Use eq. (2) in eq. (3) to replace I_1:

$3(2I_3)+I_3 = 12.0\text{ A}$

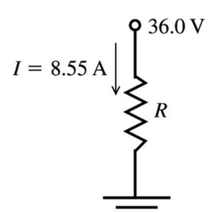
$I_3 = 12.0\text{ A}/7 = 1.71\text{ A}$

$I_1 = 2I_3 = 3.42\text{ A}$

$I_2 = 2I_1 - I_3 = 2(3.42\text{ A}) - 1.71\text{ A} = 5.13\text{ A}$

The current through the switch is $I_3 = 1.71\text{ A}$.

(c) SET UP and **EXECUTE:** From the results in part (a) the current through the battery is $I = I_1 + I_2 = 3.42\text{ A} + 5.13\text{ A} = 8.55\text{ A}$. The equivalent circuit is a single resistor that produces the same current through the 36.0-V battery, as shown in Figure 26.67c.



$-IR + 36.0\text{ V} = 0.$

$$R = \frac{36.0\text{ V}}{I} = \frac{36.0\text{ V}}{8.55\text{ A}} = 4.21\ \Omega.$$

Figure 26.67c

EVALUATE: With the switch open (part a), point b is at higher potential than point a, so when the switch is closed the current flows in the direction from b to a. With the switch closed the circuit cannot be simplified using series and parallel combinations but there is still an equivalent resistance that represents the network.

26.68. **IDENTIFY:** $P_{\text{tot}} = \dfrac{V^2}{R_{\text{eq}}}$.

SET UP: Let R be the resistance of each resistor.

EXECUTE: When the resistors are in series, $R_{\text{eq}} = 3R$ and $P_{\text{s}} = \dfrac{V^2}{3R}$. When the resistors are in parallel,

$R_{\text{eq}} = R/3$. $P_{\text{p}} = \dfrac{V^2}{R/3} = 3\dfrac{V^2}{R} = 9P_{\text{s}} = 9(45.0\ \text{W}) = 405\ \text{W}$.

EVALUATE: In parallel, the voltage across each resistor is the full applied voltage V. In series, the voltage across each resistor is $V/3$ and each resistor dissipates less power.

26.69. **IDENTIFY:** We have an R-C circuit with a discharging capacitor.

SET UP: $q = Q_0 e^{-t/RC}$, $C = \dfrac{\epsilon_0 AK}{d}$, $R = \dfrac{\rho L}{A}$, $\tau = RC$.

EXECUTE: **(a)** We want the time constant. When S is opened, we have a discharging capacitor.

$\tau = RC = \left(\dfrac{\rho d}{A}\right)\left(\dfrac{\epsilon_0 AK}{d}\right) = \epsilon_0 \rho K$.

(b) We want Q_0 if $V = 5.00$ V. $Q_0 = CV = \dfrac{\epsilon_0 AKV}{d} = 1.28$ nC using the given numbers.

(c) At what time will q be $Q_0/2$? Solve $q = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$ for t using logarithms. At this time $q = Q_0/2$. $t = \tau \ln 2 = \epsilon_0 \rho K \ln 2 = 165$ s using the given numbers.

(d) We want the current. $V_R = V_C$ so $RI = \dfrac{q}{C} = \dfrac{Q_0}{2C}$. $I = \dfrac{Q_0}{2RC} = \dfrac{Q_0}{2\epsilon_0 K\rho}$ using the result from (a).

Putting in the given numbers gives $I = 2.68$ pA.

EVALUATE: The current in (d) is very small, but the dielectric has a very large resistivity so the result is reasonable. The leakage current is the reason that capacitors in electrical devices eventually discharge if the device has been turned off for a long time.

26.70. **IDENTIFY** and **SET UP:** Just after the switch is closed the charge on the capacitor is zero, the voltage across the capacitor is zero and the capacitor can be replaced by a wire in analyzing the circuit. After a long time the current to the capacitor is zero, so the current through R_3 is zero. After a long time the capacitor can be replaced by a break in the circuit.

EXECUTE: **(a)** Ignoring the capacitor for the moment, the equivalent resistance of the two parallel resistors is $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{6.00\ \Omega} + \dfrac{1}{3.00\ \Omega} = \dfrac{3}{6.00\ \Omega}$; $R_{\text{eq}} = 2.00\ \Omega$. In the absence of the capacitor, the total current in the circuit (the current through the $8.00\text{-}\Omega$ resistor) would be

$i = \dfrac{\mathcal{E}}{R} = \dfrac{42.0\ \text{V}}{8.00\ \Omega + 2.00\ \Omega} = 4.20\ \text{A}$, of which 2/3, or 2.80 A, would go through the $3.00\text{-}\Omega$ resistor and 1/3, or 1.40 A, would go through the 6.00-Ω resistor. Since the current through the capacitor is given by $i = \dfrac{V}{R}e^{-t/RC}$, at the instant $t = 0$ the circuit behaves as through the capacitor were not present, so the currents through the various resistors are as calculated above.

(b) Once the capacitor is fully charged, no current flows through that part of the circuit. The 8.00-Ω and the 6.00-Ω resistors are now in series, and the current through them is $i = \mathcal{E}/R = (42.0\ \text{V})/(8.00\ \Omega +$

$6.00\ \Omega) = 3.00$ A. The voltage drop across both the $6.00\text{-}\Omega$ resistor and the capacitor is thus $V = iR = (3.00\ \text{A})(6.00\ \Omega) = 18.0$ V. (There is no current through the $3.00\text{-}\Omega$ resistor and so no voltage drop across it.) The charge on the capacitor is $Q = CV = (4.00\times10^{-6}\ \text{F})(18.0\ \text{V}) = 7.2\times10^{-5}$ C.

EVALUATE: The equivalent resistance of R_2 and R_3 in parallel is less than R_3, so initially the current through R_1 is larger than its value after a long time has elapsed.

26.71. **IDENTIFY:** We have a capacitor that contains a dielectric and is in a series circuit with a resistor and a battery.

SET UP and **EXECUTE:** **(a)** We want the charge. $Q_0 = CV_0$. $C = \dfrac{\epsilon_0 A}{d}$ = 1.18 pF using the given A and d. Q_0 = (1.18 pF)(10.0 V) = 11.8 pC.

(b) The target variable is the current. $V_C = Ed = (E_0/K)d = V_0/K$. For the complete circuit $\mathcal{E} = RI + V_C = RI + V_0/K = RI + \mathcal{E}/K$. $I = \dfrac{\mathcal{E}}{R}\left(1-\dfrac{1}{K}\right) = \dfrac{10.0\ \text{V}}{10.0\ \Omega}\left(1-\dfrac{1}{12.0}\right) = 0.917$ A.

(c) We want the initial energy in the capacitor. $U_C = \dfrac{1}{2}CV_C^2 = \dfrac{1}{2}(KC_0)\left(\dfrac{V_0}{K}\right)^2 = \dfrac{U_0}{K} = 4.92$ pJ. (This result also tells us that the stored energy before the dielectric was inserted was U_0 = (12.0)(4.92 pJ) = 59.0 pJ.)

(d) We want the final energy in the capacitor. $U_\text{f} = \dfrac{1}{2}CV_\text{f}^2 = \dfrac{1}{2}(KC_0)\mathcal{E}^2 = KU_0$.

$\Delta U = KU_0 - U_0$ = (12.0)(59.0 pJ) – 4.92 pJ = 703 pJ.

(e) We want the total energy supplied by the battery. $U_\mathcal{E} = \int P_\mathcal{E}dt = \int i\mathcal{E}dt$. $i = I_0e^{-t/RC} = I_0e^{-t/RKC_0}$.

Therefore $U_\mathcal{E} = \int_0^\infty I_0e^{-t/RKC_0}\mathcal{E}dt = I_0KRC_0\mathcal{E} = (0.917\ \text{A})(12.0)(10.0\ \Omega)(1.18\ \text{pF})(10.0\ \text{V})$ = 1298 pJ, which rounds to 1300 pJ.

(f) We want the energy dissipated in the resistor. $U_R = \int P_R dt = \int i^2 R dt \int_0^\infty \left(I_0e^{-t/RKC_0}\right)^2 dt = \dfrac{I_0^2R^2C_0K}{2} =$ 595 pJ using the given numbers.

EVALUATE: Check: $U_C + U_R$ = 703 pJ + 595 pJ = 1298 pJ = $U_\mathcal{E}$, which is consistent with energy conservation.

26.72. **IDENTIFY** and **SET UP:** $P_R = i^2R$, $\mathcal{E} - iR - \dfrac{q}{C} = 0$, and $U_C = \dfrac{q^2}{2C}$.

EXECUTE: $P_R = i^2R$ so $i = \sqrt{\dfrac{P_R}{R}} = \sqrt{\dfrac{300\ \text{W}}{5.00\ \Omega}} = 7.746$ A. $\mathcal{E} - iR - \dfrac{q}{C} = 0$ so

$q = C(\mathcal{E} - iR) = (6.00\times10^{-6}\ \text{F})[50.0\ \text{V} - (7.746\ \text{A})(5.00\ \Omega)] = 6.762\times10^{-5}$ C.

$$U_C = \frac{q^2}{2C} = \frac{(6.762\times10^{-5}\ \text{C})^2}{2(6.00\times10^{-6}\ \text{F})} = 3.81\times10^{-4}\ \text{J}.$$

EVALUATE: The energy stored in the capacitor can be returned to a circuit as current, but the energy dissipated in a resistor cannot.

26.73. **IDENTIFY:** Connecting the voltmeter between point b and ground gives a resistor network and we can solve for the current through each resistor. The voltmeter reading equals the potential drop across the 200-kΩ resistor.

SET UP: For two resistors in parallel, $\dfrac{1}{R_\text{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$. For two resistors in series, $R_\text{eq} = R_1 + R_2$.

EXECUTE: **(a)** $R_{\text{eq}} = 100\text{ k}\Omega + \left(\dfrac{1}{200\text{ k}\Omega} + \dfrac{1}{50\text{ k}\Omega}\right)^{-1} = 140\text{ k}\Omega$. The total current is

$I = \dfrac{0.400\text{ kV}}{140\text{ k}\Omega} = 2.86\times10^{-3}\text{ A}$. The voltage across the $200\text{-k}\Omega$ resistor is

$V_{200\text{ k}\Omega} = IR = (2.86\times10^{-3}\text{ A})\left(\dfrac{1}{200\text{ k}\Omega} + \dfrac{1}{50\text{ k}\Omega}\right)^{-1} = 114.4\text{ V}.$

(b) If the resistance of the voltmeter is $5.00\times10^{6}\ \Omega$, then we carry out the same calculations as above to find $R_{\text{eq}} = 292\text{ k}\Omega$, $I = 1.37\times10^{-3}\text{ A}$ and $V_{200\text{ k}\Omega} = 263\text{ V}$.

(c) If the resistance of the voltmeter is infinite, then we find $R_{\text{eq}} = 300\text{ k}\Omega$, $I = 1.33\times10^{-3}\text{ A}$ and $V_{200\text{k}\Omega} = 266\text{ V}$.

EVALUATE: When a voltmeter of finite resistance is connected to a circuit, current flows through the voltmeter and the presence of the voltmeter alters the currents and voltages in the original circuit. The effect of the voltmeter on the circuit decreases as the resistance of the voltmeter increases.

26.74. **IDENTIFY** and **SET UP:** Zero current through the galvanometer means the current I_1 through *N* is also the current through *M* and the current I_2 through *P* is the same as the current through *X*. And it means that points *b* and *c* are at the same potential, so $I_1N = I_2P$.

EXECUTE: **(a)** The voltage between points *a* and *d* is $\mathcal{E}$, so $I_1 = \dfrac{\mathcal{E}}{N+M}$ and $I_2 = \dfrac{\mathcal{E}}{P+X}$. Using these expressions in $I_1N = I_2P$ gives $\dfrac{\mathcal{E}}{N+M}N = \dfrac{\mathcal{E}}{P+X}P$. $N(P+X) = P(N+M)$. $NX = PM$ and $X = MP/N$.

(b) $X = \dfrac{MP}{N} = \dfrac{(850.0\ \Omega)(33.48\ \Omega)}{15.00\ \Omega} = 1897\ \Omega$

EVALUATE: The measurement of *X* does not require that we know the value of the emf.

26.75. **IDENTIFY:** With *S* open and after equilibrium has been reached, no current flows and the voltage across each capacitor is 18.0 V. When *S* is closed, current *I* flows through the $6.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors.

SET UP: With the switch closed, *a* and *b* are at the same potential and the voltage across the $6.00\text{-}\Omega$ resistor equals the voltage across the $6.00\text{-}\mu\text{F}$ capacitor and the voltage is the same across the $3.00\text{-}\mu\text{F}$ capacitor and $3.00\text{-}\Omega$ resistor.

EXECUTE: **(a)** With an open switch: $V_{ab} = \varepsilon = 18.0\text{ V}$.

(b) Point *a* is at a higher potential since it is directly connected to the positive terminal of the battery.

(c) When the switch is closed $18.0\text{ V} = I(6.00\ \Omega + 3.00\ \Omega)$. $I = 2.00\text{ A}$ and $V_b = (2.00\text{ A})(3.00\ \Omega) = 6.00\text{ V}$.

(d) Initially the capacitor's charges were $Q_3 = CV = (3.00\times10^{-6}\text{ F})(18.0\text{ V}) = 5.40\times10^{-5}\text{ C}$ and $Q_6 = CV = (6.00\times10^{-6}\text{ F})(18.0\text{ V}) = 1.08\times10^{-4}\text{ C}$. After the switch is closed $Q_3 = CV = (3.00\times10^{-6}\text{ F})(18.0\text{ V} - 12.0\text{ V}) = 1.80\times10^{-5}\text{ C}$ and $Q_6 = CV = (6.00\times10^{-6}\text{ F})(18.0\text{ V} - 6.0\text{ V}) = 7.20\times10^{-5}\text{ C}$. Both capacitors lose $3.60\times10^{-5}\text{ C}$ $= 36.0\ \mu\text{C}$.

EVALUATE: The voltage across each capacitor decreases when the switch is closed, because there is then current through each resistor and therefore a potential drop across each resistor.

26.76. IDENTIFY: The energy stored in a capacitor is $U = q^2/2C$. The electrical power dissipated in the resistor is $P = i^2R$.

SET UP: For a discharging capacitor, $i = -\dfrac{q}{RC}$.

EXECUTE: **(a)** $U_0 = \dfrac{Q_0^2}{2C} = \dfrac{(0.0069\text{ C})^2}{2(4.62\times 10^{-6}\text{ F})} = 5.15\text{ J}.$

(b) $P_0 = I_0^2R = \left(\dfrac{Q_0}{RC}\right)^2 R = \dfrac{(0.0069\text{ C})^2}{(850\ \Omega)(4.62\times 10^{-6}\text{ F})^2} = 2620\text{ W}.$

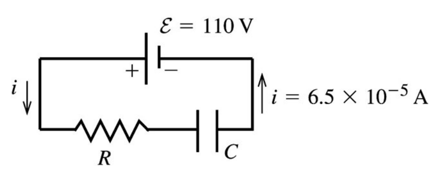
(c) Since $U = q^2/2C$, when $U \to U_0/2, q \to Q_0/\sqrt{2}$. Since $q = Q_0e^{-t/RC}$, this means that $e^{-t/RC} = 1/\sqrt{2}$. Therefore the current is $i = i_0e^{-t/RC} = i_0/\sqrt{2}$. Therefore

$P_R = \left(\dfrac{i_0}{\sqrt{2}}\right)^2 R = \dfrac{1}{2}\left(\dfrac{V_0}{R}\right)^2 R = \dfrac{1}{2}\left(\dfrac{Q_0}{RC}\right)^2 R = \dfrac{1}{RC}\left(\dfrac{Q_0^2}{2C}\right) = \dfrac{U_0}{RC}$. Putting in the numbers gives

$P_R = \dfrac{5.15\text{ J}}{(850\ \Omega)(4.62\ \mu\text{F})} = 1310\text{ W}.$

EVALUATE: All the energy originally stored in the capacitor is eventually dissipated as current flows through the resistor.

26.77. IDENTIFY: Apply the loop rule to the circuit. The initial current determines *R*. We can then use the time constant to calculate *C*.
SET UP: The circuit is sketched in Figure 26.77.



Initially, the charge of the capacitor is zero, so by $V = q/C$ the voltage across the capacitor is zero.

Figure 26.77

EXECUTE: The loop rule therefore gives $\mathcal{E} - iR = 0$ and $R = \dfrac{\mathcal{E}}{i} = \dfrac{110\text{ V}}{6.5\times 10^{-5}\text{ A}} = 1.7\times 10^6\ \Omega.$

The time constant is given by $\tau = RC$, so $C = \dfrac{\tau}{R} = \dfrac{5.2\text{ s}}{1.7\times 10^6\ \Omega} = 3.1\ \mu\text{F}.$

EVALUATE: The resistance is large so the initial current is small and the time constant is large.

26.78. IDENTIFY and SET UP: When the switch *S* is closed, current begins to flow as the capacitor plates discharge. The current in the circuit is $i = (Q_0/RC)e^{-t/RC}$.

EXECUTE: **(a)** Taking logs of the equation for *i* gives ln(*i*) = ln(Q_0/*RC*) – *t*/*RC*. A graph of ln(*i*) versus *t* will be a straight line with slope equal to –1/*RC*.
(b) Using the points (1.50 ms, –3.0) and (3.00 ms, –4.0) on the graph in the problem, the slope is
slope = $\dfrac{-4.0-(-3.0)}{3.00\text{ ms} - 1.50\text{ ms}} = -0.667\ (\text{ms})^{-1} = -667\text{ s}^{-1}$. Therefore
$-1/RC = -667\text{ s}^{-1}$.
$C = 1/[(196\ \Omega)(667\text{ s}^{-1})] = 7.65\times 10^{-6}$ F, which rounds to 7.7 μF.
Using point (1.50 ms, –3.0) on the graph, the equation of the graph gives
–3.0 = ln(Q_0/*RC*) – (1.50 ms)/*RC*.
Simplifying and rearranging gives
–2.0 = ln(Q_0/*RC*).

$Q_0 = RC\ e^{-2.0} = (196\ \Omega)(7.65\ \mu F)\ e^{-2.0} = 203\ \mu C,$ which rounds to 200 μC.

(c) Taking a loop around the circuit gives

$V_R + V_C = 0.$

$-IR + Q/C = 0.$

$Q = RCI = (196\ \Omega)(7.65\ \mu\text{F})(0.0500\ \text{A}) = 75\mu C.$

(d) From (c), we have $Q = RCI$, so $I = Q/RC = (500\mu C)/[(196\ \Omega)(7.65\ \mu\text{F})] = 0.33$ A.

EVALUATE: The accuracy of the answers depends on how well we can get information from the graph with the problem, so answers may differ slightly from those given here.

26.79. **IDENTIFY** and **SET UP:** Kirchhoff's rules apply to the circuit. Taking a loop around the circuit gives $\mathcal{E} - Ri - q/C = 0.$

EXECUTE: **(a)** Solving the loop equation for q gives $q = q = \mathcal{E}C - RCi.$ A graph of q as a function of i should be a straight line with slope equal to $-RC$ and y-intercept equal to $\mathcal{E}C.$ Figure 26.79 shows this graph.

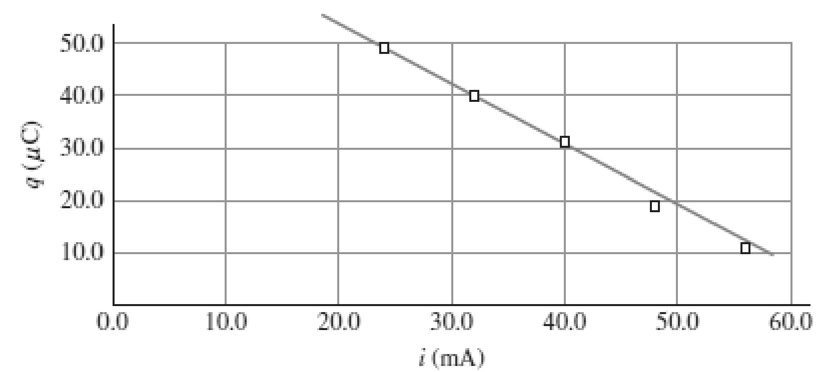


Figure 26.79

The best-fit slope of this graph is -1.233×10^{-3} C/A, and the y-intercept is 7.054×10^{-5} C.

(b) $RC = -\text{slope} = -(-1.233\times10^{-3}\ \text{C/A})$, which gives

$R = (-1.233\times10^{-3}\ \text{C/A})/(5.00\times10^{-6}\ \text{F}) = 246.6\ \Omega$, which rounds to 247 Ω.

The y-intercept is $\mathcal{E}C$, so

$7.054 \times 10^{-5}\ \text{C} = \mathcal{E}\ (5.00 \times 10^{-6}\ \text{F}).$

$\mathcal{E} = 15.9$ V.

(c) $V_C = \mathcal{E}(1 - e^{-t/RC}).$

$V_C/\mathcal{E} = 1 - e^{-t/RC} = (10.0\ \text{V})(15.9\ \text{V}).$

Solving for t gives

$t = (247\ \Omega)(5.00\ \mu\text{F})\ \ln(0.3714) = 1223\ \mu\text{s}$, which rounds to 1.22 ms.

(d) $V_R = \mathcal{E} - V_C = 15.9\ \text{V} - 4.00\ \text{V} = 11.9$ V.

EVALUATE: As time increases, the potential difference across the capacitor increases as it gets charged, but the potential difference across the resistor decreases as the current decreases.

26.80. **IDENTIFY** and **SET UP:** When connected in series across a 48.0-V battery, R_1 and R_2 dissipate 48.0 W of power, and when in parallel across the same battery, they dissipate a total of 256 W. $PR = I^2R = V^2/R$.

EXECUTE: **(a)** In series: $I = \mathcal{E}/(R_1 + R_2).$

$P_s = I^2(R_1 + R_2) = [\mathcal{E}/(R_1 + R_2)]2(R_1 + R_2) = \mathcal{E}^2/(R_1 + R_2).$

$48.0\ \text{W} = (48.0\ \text{V})^2/(R_1 + R_2).$

$R_1 + R_2 = 48.0\ \Omega.$

In parallel: $P_p = I_1^2 R_1 + I_2^2 R_2 = \dfrac{\mathcal{E}^2}{R_1^2} R_1 + \dfrac{\mathcal{E}^2}{R_2^2} R_2 = \mathcal{E}^2 \left(\dfrac{1}{R_1} + \dfrac{1}{R_2} \right) = \mathcal{E}^2 \left(\dfrac{R_1 + R_2}{R_1 R_2} \right) = 256 \text{ W}.$

Therefore $(48.0 \text{ V})^2 \left(\dfrac{R_1 R_2}{R_1 + R_2} \right) = 256 \text{ W}.$ Using $R_1 + R_2 = 48.0\ \Omega,$ this becomes $R_1 R_2 = 432\ \Omega^2.$

Solving the two equations for R_1 and R_2 simultaneously, we get two sets of answers: $R_1 = 36.0\ \Omega,$ $R_2 = 12.0\ \Omega$ and $R_1 = 12.0\ \Omega,$ $R_2 = 36.0\ \Omega.$ But we are told that that $R_1 > R_2,$ so the solution to use is $R_1 = 36.0\ \Omega,$ $R_2 = 12.0\ \Omega.$

(b) In series, both resistors have the same current. $P = I^2 R,$ so the larger resistor, which is $R_1,$ consumes more power.

(c) In parallel, the potential difference across both resistors is the same. $P = V^2 R,$ so the smaller resistor, which is $R_2,$ consumes more power.

EVALUATE: If we did not know which resistor was larger, we would know that one resistor was 12.0 Ω and the other was 36.0 Ω, but we would not know which one was the larger of the two.

26.81. **IDENTIFY:** This problem requires Kirchhoff's rules.

SET UP: Refer to Fig. 26.81 with the problem in the text. The given relations are:
I: $I_C + I_B = I_E$. II: $V_e = V_b - 0.60$ V. III: $I_C = \beta I_B$ ($\beta << 1$).

Use the following loops:
Loop 1: Clockwise through the small circuit containing V_{in} and the 100 Ω resistor.
Loop 2: Clockwise around the outside of the full circuit.

EXECUTE: **(a)** We want V_{out} in terms of V_{in}. If $\beta \to \infty, I_B = I_C/\beta \to 0,$ so $I_C = I_E.$ Apply Kirchhoff's rules.

Loop 1: $V_{in} - 0.60 \text{ V} - I_E(100\ \Omega) = 0$
Loop 2: $15 \text{ V} - I_C(1 \text{ kV}) - V_{out} = 0$
Using $I_C = I_E$ we get $I_E = (V_{in} - 0.60 \text{ V})/(100\ \Omega)$. Combining and solving gives $V_{out} = 21 \text{ V} - 10V_{in}$.

(b) We want V_{in} so V_{out} is 7.5 V. From (a): $V_{in} = (21 \text{ V} - V_{out})/10 = (21 \text{ V} - 7.5 \text{ V})/10 = 1.35$ V.

(c) The target variable is G. If V_{in} were just 15.0 V, then V_{out} would be $V_{out} = 21 \text{ V} - 10(15.0 \text{ V}) = -129$ V. But with the small $v(t)$, $V_{out} = 21 \text{ V} - 10(V_{in} + v_{in}) = 21 \text{ V} - 10(15.0 \text{ V} + v_{in})$. The coefficient of v_{in} is -10, so $G = -10$.

EVALUATE: Kirchhoff's rules apply to any type of circuit.

26.82. **IDENTIFY:** This problem involves a capacitor in an R-C circuit. We need to use Kirchhoff's rules.

SET UP: Refer to Fig. 26.82 with the problem in the textbook.

EXECUTE: **(a)** We want V_{out}. After a long time, the capacitor is fully charged, so $V_{out} = V_R = IR$.
$I = \mathcal{E}/5R$, so $V_{out} = R(\mathcal{E}/5R) = \mathcal{E}/5 = (15 \text{ V})/5 = 3.0$ V.

(b) We want the time constant τ_{ch} during charging, which is with S open. The resistance in the circuit is $4R$, so $\tau_{ch} = 4RC$.

(c) We want the time constant τ_d during discharging, which is with S closed. Apply Kirchhoff's rules. The current choices are: I_1 is downward through R; I_2 is upward through C, and I_4 is downward through $4R$.

Loop 1: Clockwise through the small circuit with R and C: $I_1 R = q/C$.
Loop 2: Clockwise around the outside of the circuit: $\mathcal{E} - 4RI_4 - q/C = 0.$

Junction rule: $I_4 = I_1 - I_2$.
The capacitor is discharging, so $I_2 = -dq/dt$.

Combining these equations gives $\dfrac{dq}{dt} = \dfrac{\mathcal{E}}{4R} - \dfrac{q}{4RC/5}$. From this result we see that $\tau_d = 4RC/5.$

(d) During the charging-discharging cycle, we want the time between successive 10.0 V output voltages across the capacitor. In one complete cycle, the potential difference across the capacitor discharges from 10.0 V to 5.0 V and then recharges from 5.0 V back to 10.0 V. The time constants in the two parts of the cycle are *not* the same.

Discharging: Using the result of part (c), solve $\frac{dq}{dt} = \frac{\mathcal{E}}{4R} - \frac{q}{4RC/5}$. The circuit discharges from 10.0 V to 5.0 V, so the initial voltage across the capacitor is $V_{0,d} = 10.0$ V. Separate variables and integrate.

$\int \frac{dq}{\mathcal{E}/4R - q/(4RC/5)} = \int dt$ gives $\ln\left(\frac{\mathcal{E}}{4R} - \frac{q}{4RC/5}\right) = t + K$, where K is a constant of integration.

Putting this result into exponential form gives $K' e^{-5t/4RC} = \frac{\mathcal{E}}{4R} - \frac{q}{4RC/5}$, where K' is a constant. When $t = 0$, $q = Q_0$, which gives $K' = \frac{\mathcal{E}}{4R} - \frac{Q_0}{4RC/5}$. Using this result, $V = q/C$, and $V_{0,d} = Q_0/C$ gives

$\left(\frac{\mathcal{E}}{4R} - \frac{V_{0,d}}{4R/5}\right) e^{-5t/4RC} = \frac{\mathcal{E}}{4R} - \frac{V}{4R/5}$. Solving for V (and calling it V_d) and simplifying, we get

$V_d = \frac{\mathcal{E}}{5} + \left(V_{0,d} - \frac{\mathcal{E}}{5}\right) e^{-5t/4RC}$.

(Check: At $t = 0$, $\mathcal{E} = 15$ V and $V_{0,d} = 10.0$ V, which gives $V_d = 10$ V, as it should. For $t \to \infty$ we have $V_d = 3.0$ V, which agrees with our result in part (a).)

Charging: The circuit is a simple series circuit containing the battery, the capacitor, and a resistance $4R$. It charges from 5.0 V to 10.0 V, so $V_{0,ch} = 5.0$ V. Applying Kirchhoff's loop rule gives $\frac{\mathcal{E}}{4R} - \frac{q}{4RC} = \frac{dq}{dt}$.

We solve this differential equation as we did for discharging. Separate variables and integrate, using the initial condition that $V = V_{0,ch}$ when $t = 0$. Carrying out these steps and solving for V_{ch} gives

$V_{ch} = \mathcal{E} - \left(\mathcal{E} - V_{0,ch}\right) e^{-t/4RC}$. (Check: At $t = 0$, $V_{ch} = 15$ V – (15 V – 5.0 V) = 5.0 V, as we should get. As $t \to \infty$, $V_{ch} \to \mathcal{E}$ as it should.) Now we find the time to charge the capacitor from 5.0 V to 10.0 V and to discharge it from 10.0 V to 5.0 V.

Charging from 5.0 V to 10.0 V: Using $V_{ch} = \mathcal{E} - \left(\mathcal{E} - V_{0,ch}\right) e^{-t/4RC}$ with $\mathcal{E} = 15$ V, $V_{0,ch} = 5.0$ V, and $V_{ch} = 10.0$ V, we have 10.0 V = 15 V – (15 V – 5.0 V)$e^{-t/4RC}$. Solving for the charging time t_{ch} gives

$t_{ch} = 4RC \ln 2$.

Discharging from 10.0 V to 5.0 V: Use $V_d = \frac{\mathcal{E}}{5} + \left(V_{0,d} - \frac{\mathcal{E}}{5}\right) e^{-5t/4RC}$ with $\mathcal{E} = 15$ V, $V_{0,d} = 10.0$ V, and $V_d = 5.0$ V, we have 5.0 V = 3.0 V + (10.0 V – 3.0 V)$e^{-5t/4RC}$. Solving for the discharge time t_d gives

$t_d = \frac{4RC}{5} \ln(7/2)$.

The total time T for one cycle is $T = t_d + t_{ch} = \frac{4RC}{5} \ln(7/2) + 4RC \ln 2$. Simplifying gives

$T = \frac{4RC}{5}\left(\ln 7 + 4 \ln 2\right)$.

(e) We want the frequency f of operation. Using $f = 1/T$ with $R = 10.0$ kΩ and $C = 10.0$ μF, we have $T =$ 4(10.0 kΩ)(10.0 μF)(ln 7 + 4 ln 2)/5 = 0.3775 s. $f = 1/T = 1/(0.3775$ s$) = 2.65$ Hz.

EVALUATE: The charging and discharging times are different because the time constants are different.

26.83. IDENTIFY: Consider one segment of the network attached to the rest of the network.

SET UP: We can re-draw the circuit as shown in Figure 26.83.

EXECUTE: $R_T = 2R_1 + \left(\dfrac{1}{R_2} + \dfrac{1}{R_T}\right)^{-1} = 2R_1 + \dfrac{R_2R_T}{R_2 + R_T}$. $R_T{}^2 - 2R_1R_T - 2R_1R_2 = 0$.

$R_T = R_1 \pm \sqrt{R_1{}^2 + 2R_1R_2}$. $R_T > 0$, so $R_T = R_1 + \sqrt{R_1{}^2 + 2R_1R_2}$.

EVALUATE: Even though there are an infinite number of resistors, the equivalent resistance of the network is finite.

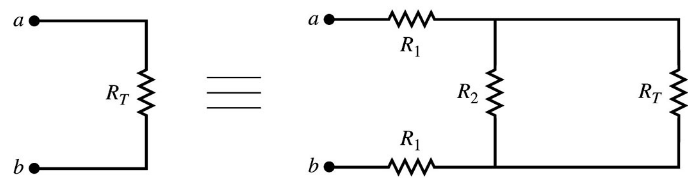


Figure 26.83

26.84. **IDENTIFY:** Assume a voltage V applied between points a and b and consider the currents that flow along each path between a and b.

SET UP: The currents are shown in Figure 26.84.

EXECUTE: Let current I enter at a and exit at b. At a there are three equivalent branches, so current is $I/3$ in each. At the next junction point there are two equivalent branches so each gets current $I/6$. Then at b there are three equivalent branches with current $I/3$ in each. The voltage drop from a to b then is $V = \left(\dfrac{I}{3}\right)R + \left(\dfrac{I}{6}\right)R + \left(\dfrac{I}{3}\right)R = \frac{5}{6}IR$. This must be the same as $V = IR_{\text{eq}}$, so $R_{\text{eq}} = \frac{5}{6}R$.

EVALUATE: The equivalent resistance is less than R, even though there are 12 resistors in the network.

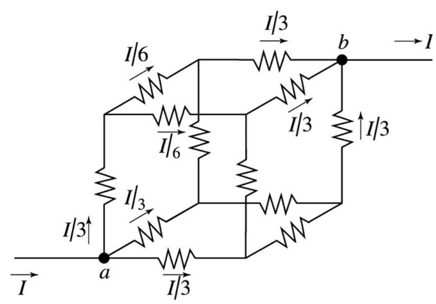


Figure 26.84

26.85. **IDENTIFY:** The network is the same as the one in Challenge Problem 26.83, and that problem shows that the equivalent resistance of the network is $R_T = \sqrt{R_1^2 + 2R_1R_2}$.

SET UP: The circuit can be redrawn as shown in Figure 26.85.

EXECUTE: **(a)** $V_{cd} = V_{ab}\dfrac{R_{\text{eq}}}{2R_1 + R_{\text{eq}}} = V_{ab}\dfrac{1}{2R_1/R_{\text{eq}} + 1}$ and $R_{\text{eq}} = \dfrac{R_2R_T}{R_2 + R_T}$. But $\beta = \dfrac{2R_1(R_T + R_2)}{R_TR_2} = \dfrac{2R_1}{R_{\text{eq}}}$, so $V_{cd} = V_{ab}\dfrac{1}{1+\beta}$.

(b) $V_1 = \dfrac{V_0}{(1+\beta)} \Rightarrow V_2 = \dfrac{V_1}{(1+\beta)} = \dfrac{V_0}{(1+\beta)^2} \Rightarrow V_n = \dfrac{V_{n-1}}{(1+\beta)} = \dfrac{V_0}{(1+\beta)^n}$.

If $R_1 = R_2$, then $R_T = R_1 + \sqrt{R_1^{\ 2} + 2R_1R_1} = R_1(1+\sqrt{3})$ and $\beta = \dfrac{2(2+\sqrt{3})}{1+\sqrt{3}} = 2.73$. So, for the nth segment to have 1% of the original voltage, we need: $\dfrac{1}{(1+\beta)^n} = \dfrac{1}{(1+2.73)^n} \le 0.01$. This says $n = 4$, and then $V_4 = 0.005V_0$.

(c) $R_T = R_1 + \sqrt{R_1^{\ 2} + 2R_1R_2}$ gives $R_T = 6400\ \Omega + \sqrt{(6400\ \Omega)^2 + 2(6400\ \Omega)(8.0\times10^8\ \Omega)} = 3.2\times10^6\ \Omega$ and $\beta = \dfrac{2(6400\ \Omega)(3.2\times10^6\ \Omega + 8.0\times10^8\ \Omega)}{(3.2\times10^6\ \Omega)(8.0\times10^8\ \Omega)} = 4.0\times10^{-3}$.

(d) Along a length of 2.0 mm of axon, there are 2000 segments each 1.0 μm long. The voltage therefore attenuates by $V_{2000} = \dfrac{V_0}{(1+\beta)^{2000}}$, so $\dfrac{V_{2000}}{V_0} = \dfrac{1}{(1+4.0\times10^{-3})^{2000}} = 3.4\times10^{-4}$.

(e) If $R_2 = 3.3\times10^{12}\ \Omega$, then $R_T = 2.1\times10^8\ \Omega$ and $\beta = 6.2\times10^{-5}$. This gives $\dfrac{V_{2000}}{V_0} = \dfrac{1}{(1+6.2\times10^{-5})^{2000}} = 0.88$.

EVALUATE: As R_2 increases, β decreases and the potential difference decrease from one section to the next is less.

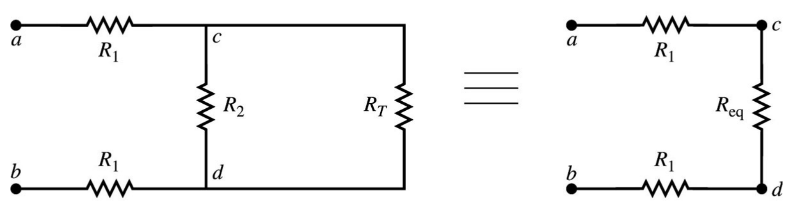


Figure 26.85

26.86. IDENTIFY and SET UP: $R = \dfrac{\rho L}{A}$.

EXECUTE: Solve for ρ: $\rho = \dfrac{AR}{L} = \dfrac{\pi r^2 R}{L} = \dfrac{\pi(0.3\ \text{nm})^2(1\times10^{11}\ \Omega)}{12\ \text{nm}} =\ 2.4\ \Omega\cdot\text{m}\ \Omega \approx 2\ \Omega\cdot\text{m}$, which is choice (c).

EVALUATE: According to the information in Table 25.1, this resistivity is much greater than that of conductors but much less than that of insulators. It is closer to that of semiconductors.

26.87. IDENTIFY and SET UP: The channels are all in parallel. For n identical resistors R in parallel, $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots = \dfrac{1}{R} + \dfrac{1}{R} + \ldots = \dfrac{n}{R}$, so $R_{\text{eq}} = R/n$. $I = jA$.

EXECUTE: $I = jA = V/\text{R}_{\text{eq}} = V/(R/n) = nV/R$.

$jR/V = n/A = (5\ \text{mA/cm}^2)(10^{11}\ \Omega)/(50\ \text{mV}) = 10^{10}/\text{cm}^2 = 100/\mu\text{m}^2$, which is choice (d).

EVALUATE: A density of 100 per μm^2 seems plausible, since these are microscopic structures.

26.88. IDENTIFY and SET UP: $\tau = RC$. The resistance is $1\times10^{11}\ \Omega$. C is the capacitance per area divided by the number density of channels, which is 100/μm^2 from Problem 26.87.

EXECUTE: $C = (1\ \mu\text{F/cm}^2)/(100/\mu\text{m}^2) = 10^{-16}$ F. The time constant is

$\tau = RC = (1\times10^{11}\ \Omega)(10^{-16}\ \text{F}) = 1\times10^{-5}\ \text{s} = 10\ \mu s$, which is choice (b).

EVALUATE: This time constant is comparable to that of typical laboratory RC circuits.

27

MAGNETIC FIELD AND MAGNETIC FORCES

VP27.1.1. **IDENTIFY:** We want the magnitude and direction of the magnetic force on a moving electron.
SET UP: The magnitude is $F = |q|vB\sin\phi$ and the right-hand rule gives the direction.

EXECUTE: **(a)** $F = |q|vB\sin\phi = e(220\text{ km/s})(1.55\text{ T})\sin(50.0°) = 4.18\times10^{-14}$ N.

(b) If the charge were positive, the direction of the force would be the same as the direction of the cross product $\vec{F} = q\vec{v}\times\vec{B}$, which is in the $+z$-direction. But electrons are negative, so the force is in the opposite direction, which is the $-z$-direction.

EVALUATE: If the electron were replaced by a proton, the force would have the same magnitude but would point in the $+z$-direction.

VP27.1.2. **IDENTIFY:** We are dealing with the magnetic force on a moving particle.
SET UP: $F = |q|vB\sin\phi$. We want the charge of the particle.

EXECUTE: Solve for q: Using the given quantities gives $q = \dfrac{F}{vB\sin\phi} = 2.91\times10^{-15}$ C.

EVALUATE: We do not know the direction of the force so we cannot determine the sign of the charge.

VP27.1.3. **IDENTIFY:** This problem involves the magnetic force on a moving particle.
SET UP: $\vec{F} = q\vec{v}\times\vec{B}$, $F = |q|vB\sin\phi$. We want the magnitude and direction of the magnetic field.

EXECUTE: **(a)** Solve $F = |q|vB\sin\phi$ for B and use the given quantities. $B = \dfrac{F}{|q|v\sin\phi} = 0.0175$ T.

(b) Using the right-hand rule for the cross-product, we see that the force is in the $+z$-direction since q is positive.

EVALUATE: An electron would experience a force in the $-y$-direction since it is negatively charged.

VP27.1.4. **IDENTIFY:** This problem involves the magnetic force on a moving particle.
SET UP: $\vec{F} = q\vec{v}\times\vec{B}$, $F = |q|vB\sin\phi$, and the right-hand rule gives the direction of the force. We want the magnitude of the magnetic field and the direction of the force.

EXECUTE: **(a)** Solve for B and use the given numbers with $\phi = 53.1°$. $B = \dfrac{F}{|q|v\sin\phi} = 0.189$ T.

(b) The cross product points in the $-y$-direction, so that is the force direction since the ions are positive.

EVALUATE: Negatively charged ions would feel a force in the $+y$-direction.

VP27.6.1. **IDENTIFY:** The proton makes a circular path in the magnetic field.
SET UP: We want the radius of the path, the angular speed of the proton, and the frequency of its motion. $R = mv/|q|B$, $v = R\omega$, and $f = \omega/2\pi$.

EXECUTE: **(a)** Using $R = mv/|q|B$ and the given numbers gives $R = 0.435$ mm.

(b) Use $\omega = v/R$ and the given numbers to get $\omega = 2.87\times10^7$ rad/s.

(c) Use $f = \omega/2\pi =$ $(2.87\times10^7$ rad/s$)/2\pi = 4.57\times10^6$ Hz.

EVALUATE: Electrons would have a radius much smaller than that of the protons because they have the same magnitude charge as protons but much smaller mass.

VP27.6.2. **IDENTIFY:** We are dealing with the helical motion of a proton in a magnetic field.
SET UP: See Fig. VP27.6.2. $R = mv_x/|q|B$.

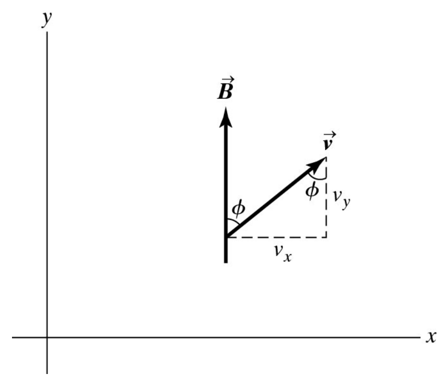


Figure VP27.6.2

EXECUTE: **(a)** We want the radius. $R = mv_x/|q|B = 0.348$ mm using the given numbers.

(b) We want the distance the proton moves along the helix in one revolution.

$\omega = v_x/R = 2\pi/T. T = 2\pi R/v_x$. $y = v_yT = v_y\left(\dfrac{2\pi R}{v_x}\right)$. Using the components given and $R = 0.348$ mm gives $y = 1.64$ mm.

(c) We want the magnitude of the magnetic force. $F = ev_xB = 4.80\times10^{-16}$ N using the given quantities.

EVALUATE: Only the velocity component perpendicular to the magnetic field causes circular motion. The component parallel to the field causes the path to be a helix rather than simply a circle.

VP27.6.3. **IDENTIFY:** This problem involves a velocity selector.
SET UP: $F_{magn} = |q|vB\sin\phi$ and $F_{el} = |q|E$.

EXECUTE: **(a)** We want the speed. $|q|E = |q|vB\sin\phi$. Using the numbers with $\phi = 90°$ gives $v = 8.00\times10^5$ m/s.

(b) We want the direction of motion. The particle should travel perpendicular to both fields so that the electric and magnetic forces are in opposite directions. So it should travel in the $-y$-direction.

(c) We want the direction it would deflect. The magnetic force is now greater than the electric force, so it will deflect in the direction of the magnetic force, which is in the $-x$-direction.

EVALUATE: Careful! The direction of the magnetic *force* is not the same as the direction of the magnetic *field*.

VP27.6.4. **IDENTIFY:** This problem is about a mass spectrometer.
SET UP and **EXECUTE:** **(a)** We want the radius for ^{16}O ions. $R_{16} = m_{16}v/|q|B = 6.65$ cm using the given numbers.

(b) We want the radius for ^{18}O ions. $R_{18} = m_{18}v/|q|B = 7.48$ cm using the given numbers.

EVALUATE: The distance between the two ions at the detector is $d = 2R_{18} - 2R_{16} =$ $2(7.48\text{ cm} - 6.65\text{ cm}) = 1.66$ cm. This separation would be very easy to measure.

VP27.7.1. **IDENTIFY:** This problem involves the magnetic force on a current-carrying wire.

SET UP: $\vec{F} = I\vec{l} \times \vec{B}$, $F = IlB\sin\phi$.

EXECUTE: **(a)** We want the magnitude of the force. Use $F = IlB\sin\phi$ with $\phi = 160°$ and the other given numbers, giving $F = 2.44$ mN.

(b) We want the direction of the force. Apply the right-hand rule for $\vec{l} \times \vec{B}$, which tells us that the force is in the $+z$-direction.

EVALUATE: The force is small, but it would be readily observable with a thin light wire.

VP27.7.2. **IDENTIFY:** This problem involves the magnetic force on a current-carrying wire.

SET UP: $\vec{F} = I\vec{l} \times \vec{B}$, $F = IlB\sin\phi$. We want the magnitude and direction of the current. First sketch the force and magnetic field as in Fig. VP27.7.2.

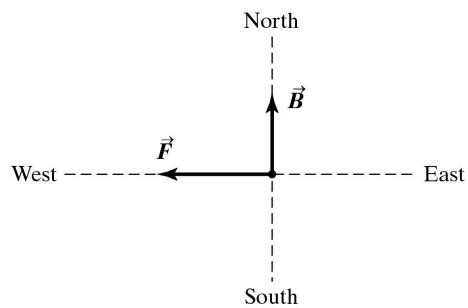


Figure VP27.7.2

EXECUTE: **(a)** Use $F = IlB\sin\phi$ with $\phi = 90°$ and the other numbers, giving $I = 1.60$ A.

(b) $\vec{l} \times \vec{B}$ points to the west, so $\vec{l}$ must point vertically downward, which means that the current I is traveling vertically *upward*.

EVALUATE: If I were downward, the magnetic force would be toward the east.

VP27.7.3. **IDENTIFY:** This problem involves the magnetic force on a current-carrying wire.

SET UP: $\vec{F} = I\vec{l} \times \vec{B}$. We want the components and magnitude of the force. First write the force in terms of its components and the unit vectors. $\vec{F} = I\vec{l} \times \vec{B} = I(l_x\hat{\boldsymbol{i}} + l_y\hat{\boldsymbol{j}}) \times B_z\hat{\boldsymbol{k}}$. Using the cross products of the unit vectors, this reduces to $\vec{F} = I\vec{l} \times \vec{B} = IB_z(l_y\hat{\boldsymbol{i}} - l_x\hat{\boldsymbol{j}})$.

EXECUTE: **(a)** $F_x = IB_z l_y = (1.20\text{ A})(0.0175\text{ T})(-0.120\text{ m}) = -2.52$ mN.

(b) $F_y = -IB_z l_x = -(1.20\text{ A})(0.0175\text{ T})(0.200\text{ m}) = -4.20$ mN.

(c) $F_z = 0$.

(d) $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = 4.90$ mN using the components we just found.

EVALUATE: When working in three dimensions, it is usually easier to use components than to try to visualize the vectors.

VP27.7.4. **IDENTIFY:** This problem involves the magnetic force on a current-carrying wire.

SET UP: $F = IlB\sin\phi$. We want the *two* possible angles between direction of the current and the magnetic field.

EXECUTE: Solve for ϕ: $\phi = \text{a}\left(\dfrac{F}{IlB}\right) = \arcsin\left(\dfrac{0.0250\text{ N}}{(3.40\text{ A})(0.280\text{ m})(0.0400\text{ T})}\right) = 41.0°$. But $\sin(180° - 41.0°) = \sin 41.0°$, so the two angles are 41.0° and 139.0°.

EVALUATE: Physically the two angles correspond to the current running in opposite directions in the same magnetic field.

27.1. **IDENTIFY** and **SET UP:** Apply $\vec{F} = q\vec{v} \times \vec{B}$ to calculate $\vec{F}$. Use the cross products of unit vectors from Chapter 1. $\vec{v} = (+4.19 \times 10^4 \text{ m/s})\hat{i} + (-3.85 \times 10^4 \text{ m/s})\hat{j}$.

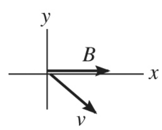
(a) EXECUTE: $\vec{B} = (1.40 \text{ T})\hat{i}$.

$\vec{F} = q\vec{v} \times \vec{B} = (-1.24 \times 10^{-8} \text{ C})(1.40 \text{ T})[(4.19 \times 10^4 \text{ m/s})\hat{i} \times \hat{i} - (3.85 \times 10^4 \text{ m/s})\hat{j} \times \hat{i}]$.

$\hat{i} \times \hat{i} = 0$, $\hat{j} \times \hat{i} = -\hat{k}$.

$\vec{F} = (-1.24 \times 10^{-8} \text{ C})(1.40 \text{ T})(-3.85 \times 10^4 \text{ m/s})(-\hat{k}) = (-6.68 \times 10^{-4} \text{ N})\hat{k}$.

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1a.



The right-hand rule gives that $\vec{v} \times \vec{B}$ is directed out of the paper ($+z$-direction). The charge is negative so $\vec{F}$ is opposite to $\vec{v} \times \vec{B}$.

Figure 27.1a

$\vec{F}$ is in the $-z$-direction. This agrees with the direction calculated with unit vectors.

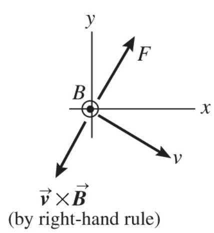
(b) EXECUTE: $\vec{B} = (1.40 \text{ T})\hat{k}$.

$\vec{F} = q\vec{v} \times \vec{B} = (-1.24 \times 10^{-8} \text{ C})(1.40 \text{ T})[(+4.19 \times 10^4 \text{ m/s})\hat{i} \times \hat{k} - (3.85 \times 10^4 \text{ m/s})\hat{j} \times \hat{k}]$.

$\hat{i} \times \hat{k} = -\hat{j}$, $\hat{j} \times \hat{k} = \hat{i}$.

$\vec{F} = (-7.27 \times 10^{-4} \text{ N})(-\hat{j}) + (6.68 \times 10^{-4} \text{ N})\hat{i} = [(6.68 \times 10^{-4} \text{ N})\hat{i} + (7.27 \times 10^{-4} \text{ N})\hat{j}]$.

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1b.



The direction of $\vec{F}$ is opposite to $\vec{v} \times \vec{B}$ since q is negative. The direction of $\vec{F}$ computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.

Figure 27.1b

27.2. **IDENTIFY:** The net force must be zero, so the magnetic and gravity forces must be equal in magnitude and opposite in direction.

SET UP: The gravity force is downward so the force from the magnetic field must be upward. The charge's velocity and the forces are shown in Figure 27.2. Since the charge is negative, the magnetic force is opposite to the right-hand rule direction. The minimum magnetic field is when the field is perpendicular to $\vec{v}$. The force is also perpendicular to $\vec{B}$, so $\vec{B}$ is either eastward or westward.

EXECUTE: If $\vec{B}$ is eastward, the right-hand rule direction is into the page and $\vec{F}_B$ is out of the page, as required. Therefore, $\vec{B}$ is eastward. $mg = |q|vB\sin\phi$. $\phi = 90°$ and

$$B = \frac{mg}{v|q|} = \frac{(0.195 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{(4.00 \times 10^4 \text{ m/s})(2.50 \times 10^{-8} \text{ C})} = 1.91 \text{ T}.$$

EVALUATE: The magnetic field could also have a component along the north-south direction, that would not contribute to the force, but then the field wouldn't have minimum magnitude.

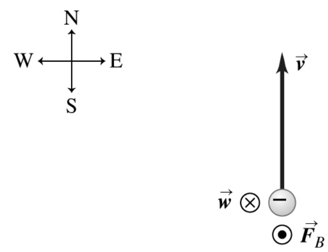


Figure 27.2

27.3. **IDENTIFY:** The force $\vec{F}$ on the particle is in the direction of the deflection of the particle. Apply the right-hand rule to the directions of $\vec{v}$ and $\vec{B}$. See if your thumb is in the direction of $\vec{F}$, or opposite to that direction. Use $F = |q|vB\sin\phi$ with $\phi = 90°$ to calculate F.

SET UP: The directions of $\vec{v}$, $\vec{B}$, and $\vec{F}$ are shown in Figure 27.3.

EXECUTE: **(a)** When you apply the right-hand rule to $\vec{v}$ and $\vec{B}$, your thumb points east. $\vec{F}$ is in this direction, so the charge is positive.

(b) $F = |q|vB\sin\phi = (8.50\times10^{-6}\text{ C})(4.75\times10^{3}\text{ m/s})(1.25\text{ T})\sin 90° = 0.0505\text{ N}$

EVALUATE: If the particle had negative charge and $\vec{v}$ and $\vec{B}$ are unchanged, the particle would be deflected toward the west.

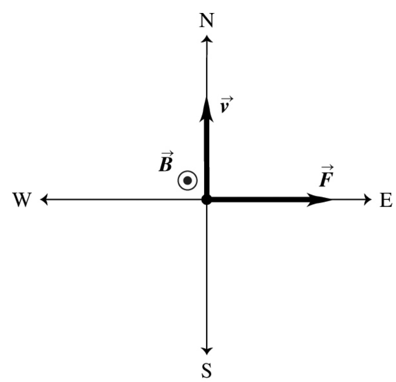


Figure 27.3

27.4. **IDENTIFY:** Apply Newton's second law, with the force being the magnetic force.
SET UP: $\hat{\boldsymbol{j}}\times\hat{\boldsymbol{i}} = -\hat{\boldsymbol{k}}$.

EXECUTE: $\vec{F} = m\vec{a} = q\vec{v}\times\vec{B}$ gives $\vec{a} = \dfrac{q\vec{v}\times\vec{B}}{m}$ and

$$\vec{a} = \frac{(1.22\times10^{-8}\text{ C})(3.0\times10^{4}\text{ m/s})(1.63\text{ T})(\hat{\boldsymbol{j}}\times\hat{\boldsymbol{i}})}{1.81\times10^{-3}\text{ kg}} = -(0.330\text{ m/s}^2)\hat{\boldsymbol{k}}.$$

EVALUATE: The acceleration is in the $-z$-direction and is perpendicular to both $\vec{v}$ and $\vec{B}$.

27.5. **IDENTIFY:** This problem involves the magnetic force on a moving charge.
SET UP: $\vec{F} = q\vec{v}\times\vec{B}$, $F = |q|vB\sin\phi$. We want the velocity.
EXECUTE: Solve $F = |q|vB\sin\phi$ for v. Using the given numbers with $\phi = 90°$ and $q = e$ we get $v = \dfrac{F}{eB}$ = 172 m/s. By the right-hand rule, the velocity would have to be in the $+x$-direction for a positive charge, so for an electron it must be in the $-x$-direction.
EVALUATE: The force on a proton would have the same magnitude but opposite direction.

27.6. **IDENTIFY:** Apply Newton's second law and $F = |q|vB\sin\phi.$

SET UP: ϕ is the angle between the direction of $\vec{v}$ and the direction of $\vec{B}$.

EXECUTE: **(a)** The smallest possible acceleration is zero, when the motion is parallel to the magnetic field. The greatest acceleration is when the velocity and magnetic field are at right angles:

$$a = \frac{|q|vB}{m} = \frac{(1.6\times10^{-19}\text{ C})(1.40\times10^{6}\text{ m/s})(7.4\times10^{-2}\text{ T})}{(9.11\times10^{-31}\text{ kg})} = 1.82\times10^{16}\text{ m/s}^2.$$

(b) If $a = \frac{1}{4}(1.82\times10^{16}\text{ m/s}^2) = \dfrac{|q|vB\sin\phi}{m}$, then $\sin\phi = 0.25$ and $\phi = 14.5°$.

EVALUATE: The force and acceleration decrease as the angle ϕ approaches zero.

27.7. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$ to the force on the proton and to the force on the electron. Solve for the components of $\vec{B}$ and use them to find its magnitude and direction.

SET UP: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. Since the force on the proton is in the $+y$-direction, $B_y = 0$ and $\vec{B} = B_x\hat{i} + B_z\hat{k}$. For the proton, $\vec{v}_\text{p} = (1.50\text{ km/s})\hat{i} = v_\text{p}\hat{i}$ and $\vec{F}_\text{p} = (2.25\times10^{-16}\text{ N})\hat{j} = F_\text{p}\hat{j}$. For the electron, $\vec{v}_\text{e} = -(4.75\text{ km/s})\hat{k} = -v_\text{e}\hat{k}$ and $\vec{F}_\text{e} = (8.50\times10^{-16}\text{ N})\hat{j} = F_\text{e}\hat{j}$. The magnetic force is $\vec{F} = q\vec{v}\times\vec{B}$.

EXECUTE: **(a)** For the proton, $\vec{F}_\text{p} = q\vec{v}_\text{p}\times\vec{B}$ gives $F_\text{p}\hat{j} = ev_\text{p}\hat{i}\times(B_x\hat{i} + B_z\hat{k}) = -ev_\text{p}B_z\hat{j}$. Solving for B_z gives $B_z = -\dfrac{F_\text{p}}{ev_\text{p}} = -\dfrac{2.25\times10^{-16}\text{ N}}{(1.60\times10^{-19}\text{ C})(1500\text{ m/s})} = -0.9375\text{ T}$. For the electron, $\vec{F}_\text{e} = -e\vec{v}_\text{e}\times\vec{B}$, which gives $F_\text{e}\hat{j} = (-e)(-v_\text{e}\hat{k})\times(B_x\hat{i} + B_z\hat{k}) = ev_\text{e}B_x\hat{j}$. Solving for B_x gives

$B_x = \dfrac{F_\text{e}}{ev_\text{e}} = \dfrac{8.50\times10^{-16}\text{ N}}{(1.60\times10^{-19}\text{ C})(4750\text{ m/s})} = 1.118\text{ T}$. Therefore $\vec{B} = 1.118\text{ T}\hat{i} - 0.9375\text{ T}\hat{k}$. The magnitude of the field is $B = \sqrt{B_x^2 + B_z^2} = \sqrt{(1.118\text{ T})^2 + (-0.9375\text{ T})^2} = 1.46\text{ T}$. Calling θ the angle that the magnetic field makes with the $+x$-axis, we have $\tan\theta = \dfrac{B_z}{B_x} = \dfrac{-0.9375\text{ T}}{1.118\text{ T}} = -0.8386$, so $\theta = -40.0°$.

Therefore the magnetic field is in the xz-plane directed at 40.0° from the $+x$-axis toward the $-z$-axis, having a magnitude of 1.46 T.

(b) $\vec{B} = B_x\hat{i} + B_z\hat{k}$ and $\vec{v} = (3.2\text{ km/s})(-\hat{j})$.

$\vec{F} = q\vec{v}\times\vec{B} = (-e)(3.2\text{ km/s})(-\hat{j})\times(B_x\hat{i} + B_z\hat{k}) = e(3.2\times10^{3}\text{ m/s})[B_x(-\hat{k}) + B_z\hat{i}]$.

$\vec{F} = e(3.2\times10^{3}\text{ m/s})(-1.118\text{ T}\hat{k} - 0.9375\text{ T}\hat{i}) = -4.80\times10^{-16}\text{ N}\hat{i} - 5.724\times10^{-16}\text{ N}\hat{k}$.

$F = \sqrt{F_x^2 + F_z^2} = 7.47\times10^{-16}\text{ N}$. Calling θ the angle that the force makes with the $-x$-axis, we have $\tan\theta = \dfrac{F_z}{F_x} = \dfrac{-5.724\times10^{-16}\text{ N}}{-4.800\times10^{-16}\text{ N}}$, which gives $\theta = 50.0°$. The force is in the xz-plane and is directed at 50.0° from the $-x$-axis toward either the $-z$-axis.

EVALUATE: The force on the electrons in parts (a) and (b) are comparable in magnitude because the electron speeds are comparable in both cases.

27.8. **IDENTIFY** and **SET UP:** $\vec{F} = q\vec{v}\times\vec{B} = qB_z[v_x(\hat{i}\times\hat{k}) + v_y(\hat{j}\times\hat{k}) + v_z(\hat{k}\times\hat{k})] = qB_z[v_x(-\hat{j}) + v_y(\hat{i})]$.

EXECUTE: **(a)** Set the expression for $\vec{F}$ equal to the given value of $\vec{F}$ to obtain:

$$v_x = \frac{F_y}{-qB_z} = \frac{(7.40\times10^{-7}\text{ N})}{-(-5.60\times10^{-9}\text{ C})(-1.25\text{ T})} = -106\text{ m/s}.$$

$$v_y = \frac{F_x}{qB_z} = \frac{-(3.40\times10^{-7}\text{ N})}{(-5.60\times10^{-9}\text{ C})(-1.25\text{ T})} = -48.6\text{ m/s.}$$

(b) v_z does not contribute to the force, so is not determined by a measurement of $\vec{F}$.

(c) $\vec{v}\cdot\vec{F} = v_xF_x + v_yF_y + v_zF_z = \frac{F_y}{-qB_z}F_x + \frac{F_x}{qB_z}F_y = 0;\ \theta = 90°.$

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{B}\cdot\vec{F}$ is also zero.

27.9. IDENTIFY and **SET UP:** $\Phi_B = \int \vec{B}\cdot d\vec{A}$.

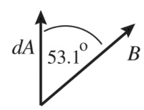
Circular area in the xy-plane, so $A = \pi r^2 = \pi(0.0650\text{ m})^2 = 0.01327\text{ m}^2$ and $d\vec{A}$ is in the z-direction. Use Eq. (1.18) to calculate the scalar product.

EXECUTE: (a) $\vec{B} = (0.230\text{ T})\hat{k}$; $\vec{B}$ and $d\vec{A}$ are parallel $(\phi = 0°)$ so $\vec{B}\cdot d\vec{A} = B\,dA$.

B is constant over the circular area so

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\,dA = B\int dA = BA = (0.230\text{ T})(0.01327\text{ m}^2) = 3.05\times10^{-3}\text{ Wb.}$$

(b) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.9a.



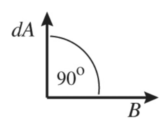
$\vec{B}\cdot d\vec{A} = B\cos\phi\, dA$
with $\phi = 53.1°$.

Figure 27.9a

B and ϕ are constant over the circular area so $\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\cos\phi\, dA = B\cos\phi \int dA = B\cos\phi A$

$$\Phi_B = (0.230\text{ T})\cos 53.1°(0.01327\text{ m}^2) = 1.83\times10^{-3}\text{ Wb.}$$

(c) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.9b.



$\vec{B}\cdot d\vec{A} = 0$ since $d\vec{A}$ and $\vec{B}$ are perpendic
$\Phi_B = \int \vec{B}\cdot d\vec{A} = 0.$

Figure 27.9b

EVALUATE: Magnetic flux is a measure of how many magnetic field lines pass through the surface. It is maximum when $\vec{B}$ is perpendicular to the plane of the loop (part a) and is zero when $\vec{B}$ is parallel to the plane of the loop (part c).

27.10. IDENTIFY: Knowing the area of a surface and the magnetic field it is in, we want to calculate the flux through it.

SET UP: $d\vec{A} = dA\hat{k}$, so $d\Phi_B = \vec{B}\cdot d\vec{A} = B_z dA$.

EXECUTE: $\Phi_B = B_zA = (-0.500\text{ T})(0.0340\text{ m})^2 = -5.78\times10^{-4}\text{ T}\cdot\text{m}^2$. $|\Phi_B| = 5.78\times10^{-4}$ Wb.

EVALUATE: Since the field is uniform over the surface, it is not necessary to integrate to find the flux.

27.11. IDENTIFY: The total flux through the bottle is zero because it is a closed surface.

SET UP: The total flux through the bottle is the flux through the plastic plus the flux through the open cap, so the sum of these must be zero. $\Phi_{\text{plastic}} + \Phi_{\text{cap}} = 0$.

$$\Phi_{\text{plastic}} = -\Phi_{\text{cap}} = -BA\cos\phi = -B(\pi r^2)\cos\phi.$$

EXECUTE: Substituting the numbers gives $\Phi_{\text{plastic}} = -(1.75\text{ T})\pi(0.0125\text{ m})^2\cos 25° = -7.8\times10^{-4}$ Wb.

EVALUATE: It would be very difficult to calculate the flux through the plastic directly because of the complicated shape of the bottle, but with a little thought we can find this flux through a simple calculation.

27.12. IDENTIFY: Knowing the area of a surface and the magnetic flux through it, we want to find the magnetic field needed to produce this flux.
SET UP: $\Phi_B = BA\cos\phi$ where $\phi = 60.0°$.
EXECUTE: Solving $\Phi_B = BA\cos\phi$ for B gives

$$B = \frac{\Phi_B}{A\cos\phi} = \frac{3.10\times10^{-4}\ \text{Wb}}{(0.0280\ \text{m})(0.0320\ \text{m})\cos 60.0°} = 0.692\ \text{T}.$$

EVALUATE: This is a fairly strong magnetic field, but not impossible to achieve in modern laboratories.

27.13. IDENTIFY: When $\vec{B}$ is uniform across the surface, $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\phi.$

SET UP: $\vec{A}$ is normal to the surface and is directed outward from the enclosed volume. For surface *abcd*, $\vec{A} = -A\hat{i}$. For surface *befc*, $\vec{A} = -A\hat{k}$. For surface *aefd*, $\cos\phi = 3/5$ and the flux is positive.

EXECUTE: (a) $\Phi_B(abcd) = \vec{B}\cdot\vec{A} = 0.$

(b) $\Phi_B(befc) = \vec{B}\cdot\vec{A} = -(0.128\ \text{T})(0.300\ \text{m})(0.300\ \text{m}) = -0.0115\ \text{Wb}.$

(c) $\Phi_B(aefd) = \vec{B}\cdot\vec{A} = BA\cos\phi = \frac{3}{5}(0.128\ \text{T})(0.500\ \text{m})(0.300\ \text{m}) = +0.0115\ \text{Wb}.$

(d) The net flux through the rest of the surfaces is zero since they are parallel to the *x*-axis. The total flux is the sum of all parts above, which is zero.
EVALUATE: The total flux through any closed surface, that encloses a volume, is zero.

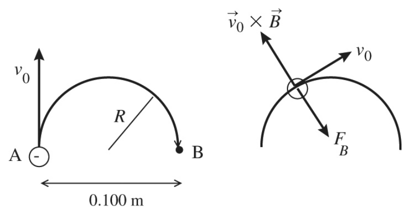
27.14. IDENTIFY: Newton's second law gives $|q|vB = mv^2/R.$ The speed v is constant and equals v_0. The direction of the magnetic force must be in the direction of the acceleration and is toward the center of the semicircular path.
SET UP: A proton has $q = +1.60\times10^{-19}$ C and $m = 1.67\times10^{-27}$ kg. The direction of the magnetic force is given by the right-hand rule.

EXECUTE: (a) $B = \dfrac{mv}{qR} = \dfrac{(1.67\times10^{-27}\ \text{kg})(1.41\times10^{6}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.0500\ \text{m})} = 0.294\ \text{T}.$

The direction of the magnetic field is out of the page (the charge is positive), in order for $\vec{F}$ to be directed to the right at point *A*.
(b) The time to complete half a circle is $t = \pi R/v_0 = 1.11\times10^{-7}$ s.

EVALUATE: The magnetic field required to produce this path for a proton has a different magnitude (because of the different mass) and opposite direction (because of opposite sign of the charge) than the field required to produce the path for an electron.

27.15. (a) IDENTIFY: Apply $\vec{F} = q\vec{v}\times\vec{B}$ to relate the magnetic force $\vec{F}$ to the directions of $\vec{v}$ and $\vec{B}$. The electron has negative charge so $\vec{F}$ is opposite to the direction of $\vec{v}\times\vec{B}$. For motion in an arc of a circle the acceleration is toward the center of the arc so $\vec{F}$ must be in this direction. $a = v^2/R.$
SET UP:



As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of $\vec{v}_0 \times \vec{B}$ at a point along the path is shown in Figure 27.15.

Figure 27.15

EXECUTE: For circular motion the acceleration of the electron $\vec{a}_{\text{rad}}$ is directed in toward the center of the circle. Thus the force $\vec{F}_B$ exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since q is negative, $\vec{F}_B$ is opposite to the direction given by the right-hand rule for $\vec{v}_0 \times \vec{B}$. Thus $\vec{B}$ is directed into the page. Apply Newton's second law to calculate the magnitude of $\vec{B}$: $\sum \vec{F} = m\vec{a}$ gives $\sum F_{\text{rad}} = ma \quad F_B = m(v^2/R)$.

$F_B = |q| vB\sin\phi = |q| vB$, so $|q| vB = m(v^2/R)$.

$$B = \frac{mv}{|q|R} = \frac{(9.109\times10^{-31}\text{ kg})(1.41\times10^{6}\text{ m/s})}{(1.602\times10^{-19}\text{ C})(0.050\text{ m})} = 1.60\times10^{-4}\text{ T}.$$

(b) IDENTIFY and **SET UP:** The speed of the electron as it moves along the path is constant. ($\vec{F}_B$ changes the direction of $\vec{v}$ but not its magnitude.) The time is given by the distance divided by v_0.

EXECUTE: The distance along the semicircular path is πR, so $t = \dfrac{\pi R}{v_0} = \dfrac{\pi(0.050\text{ m})}{1.41\times10^{6}\text{ m/s}} = 1.11\times10^{-7}$ s.

EVALUATE: The magnetic field required increases when v increases or R decreases and also depends on the mass to charge ratio of the particle.

27.16. **IDENTIFY:** Since the particle moves perpendicular to the uniform magnetic field, the radius of its path is $R = \dfrac{mv}{|q|B}$. The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$.

SET UP: The alpha particle has charge $q = +2e = 3.20\times10^{-19}$ C.

EXECUTE: **(a)** $R = \dfrac{(6.64\times10^{-27}\text{ kg})(35.6\times10^{3}\text{ m/s})}{(3.20\times10^{-19}\text{ C})(1.80\text{ T})} = 4.104\times10^{-4}\text{ m} = 0.4104\text{ mm}.$ The alpha particle moves in a circular arc of diameter $2R = 2(0.4104\text{ mm}) = 0.821$ mm.

(b) For a very short time interval the displacement of the particle is in the direction of the velocity. The magnetic force is always perpendicular to this direction so it does no work. The work-energy theorem therefore says that the kinetic energy of the particle, and hence its speed, is constant.

(c) The acceleration is

$a = \dfrac{F_B}{m} = \dfrac{|q|vB\sin\phi}{m} = \dfrac{(3.20\times10^{-19}\text{ C})(35.6\times10^{3}\text{ m/s})(1.80\text{ T})\sin 90^\circ}{6.64\times10^{-27}\text{ kg}} = 3.09\times10^{12}\text{ m/s}^2.$ We can also use $a = \dfrac{v^2}{R}$ and the result of part (a) to calculate $a = \dfrac{(35.6\times10^{3}\text{ m/s})^2}{4.104\times10^{-4}\text{ m}} = 3.09\times10^{12}\text{ m/s}^2$, the same result. The acceleration is perpendicular to $\vec{v}$ and $\vec{B}$ and so is horizontal, toward the center of curvature of the particle's path.

EVALUATE: **(d)** The unbalanced force $(\vec{F}_B)$ is perpendicular to $\vec{v}$, so it changes the direction of $\vec{v}$ but not its magnitude, which is the speed.

27.17. **IDENTIFY** and **SET UP:** Use conservation of energy to find the speed of the ball when it reaches the bottom of the shaft. The right-hand rule gives the direction of $\vec{F}$ and $F = |q|v B \sin\phi$ gives its magnitude. The number of excess electrons determines the charge of the ball.

EXECUTE: $q = (4.00\times 10^{8})(-1.602\times 10^{-19}\text{ C}) = -6.408\times 10^{-11}\text{ C}.$

speed at bottom of shaft: $\frac{1}{2}mv^2 = mgy;\ v = \sqrt{2gy} = 49.5\text{ m/s}.$

$\vec{v}$ is downward and $\vec{B}$ is west, so $\vec{v}\times\vec{B}$ is north. Since $q < 0$, $\vec{F}$ is south.

$F = |q|vB\sin\theta = (6.408\times 10^{-11}\text{ C})(49.5\text{ m/s})(0.250\text{ T})\sin 90° = 7.93\times 10^{-10}\text{ N}.$

EVALUATE: Both the charge and speed of the ball are relatively small so the magnetic force is small, much less than the gravity force of 1.5 N.

27.18. **IDENTIFY:** The magnetic field acts perpendicular to the velocity, causing the ion to move in a circular path but not changing its speed.

SET UP: $R = \dfrac{mv}{|q|B}$ and $K = \frac{1}{2}mv^2$. $K = 5.0\text{ MeV} = 8.0\times 10^{-13}\text{ J}.$

EXECUTE: **(a)** Solving $K = \frac{1}{2}mv^2$ for v gives $v = \sqrt{2K/m}.$

$v = [2(8.0\times 10^{-13}\text{ J})/(1.67\times 10^{-27}\text{ kg})]^{1/2} = 3.095\times 10^{7}\text{ m/s},$ which rounds to $3.1\times 10^{7}\text{ m/s}.$

(b) Using $R = \dfrac{mv}{|q|B} = (1.67\times 10^{-27}\text{ kg})(3.095\times 10^{7}\text{ m/s})/[(1.602\times 10^{-19}\text{ C})(1.9\text{ T})] = 0.17\text{ m} = 17\text{ cm}.$

EVALUATE: If the hydride ions were accelerated to 20 MeV, which is 4 times the value used here, their speed would be twice as great, so the radius of their path would also be twice as great.

27.19. **IDENTIFY:** For motion in an arc of a circle, $a = \dfrac{v^2}{R}$ and the net force is radially inward, toward the center of the circle.

SET UP: The direction of the force is shown in Figure 27.19. The mass of a proton is 1.67×10^{-27} kg.

EXECUTE: **(a)** $\vec{F}$ is opposite to the right-hand rule direction, so the charge is negative. $\vec{F} = m\vec{a}$ gives $|q|vB\sin\phi = m\dfrac{v^2}{R}$. $\phi = 90°$ and $v = \dfrac{|q|BR}{m} = \dfrac{3(1.60\times 10^{-19}\text{ C})(0.250\text{ T})(0.475\text{ m})}{12(1.67\times 10^{-27}\text{ kg})} = 2.84\times 10^{6}\text{ m/s}.$

(b) $F_B = |q|vB\sin\phi = 3(1.60\times 10^{-19}\text{ C})(2.84\times 10^{6}\text{ m/s})(0.250\text{ T})\sin 90° = 3.41\times 10^{-13}\text{ N}.$

$w = mg = 12(1.67\times 10^{-27}\text{ kg})(9.80\text{ m/s}^2) = 1.96\times 10^{-25}\text{ N}.$ The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.

EVALUATE: **(c)** The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

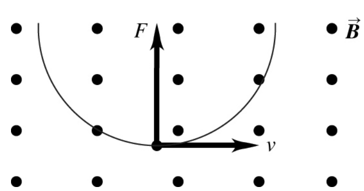


Figure 27.19

27.20. **(a) IDENTIFY** and **SET UP:** Apply Newton's second law, with $a = v^2/R$ since the path of the particle is circular.

EXECUTE: $\sum\vec{F} = m\vec{a}$ says $|q|vB = m(v^2/R).$

$v = \dfrac{|q|BR}{m} = \dfrac{(1.602\times10^{-19}\text{ C})(2.50\text{ T})(6.96\times10^{-3}\text{ m})}{3.34\times10^{-27}\text{ kg}} = 8.35\times10^{5}\text{ m/s.}$

(b) IDENTIFY and SET UP: The speed is constant so $t = \text{distance}/v$.

EXECUTE: $t = \dfrac{\pi R}{v} = \dfrac{\pi(6.96\times10^{-3}\text{ m})}{8.35\times10^{5}\text{ m/s}} = 2.62\times10^{-8}\text{ s.}$

(c) IDENTIFY and SET UP: kinetic energy gained = electric potential energy lost.

EXECUTE: $\frac{1}{2}mv^2 = |q|V.$

$V = \dfrac{mv^2}{2|q|} = \dfrac{(3.34\times10^{-27}\text{ kg})(8.35\times10^{5}\text{ m/s})^2}{2(1.602\times10^{-19}\text{ C})} = 7.27\times10^{3}\text{ V} = 7.27\text{ kV.}$

EVALUATE: The deutron has a much larger mass to charge ratio than an electron so a much larger B is required for the same v and R. The deutron has positive charge so gains kinetic energy when it goes from high potential to low potential.

27.21. IDENTIFY: When a particle of charge $-e$ is accelerated through a potential difference of magnitude V, it gains kinetic energy eV. When it moves in a circular path of radius R, its acceleration is $\dfrac{v^2}{R}$.

SET UP: An electron has charge $q = -e = -1.60\times10^{-19}\text{ C}$ and mass $9.11\times10^{-31}\text{ kg}$.

EXECUTE: $\frac{1}{2}mv^2 = eV$ and $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(2.00\times10^{3}\text{ V})}{9.11\times10^{-31}\text{ kg}}} = 2.65\times10^{7}\text{ m/s.}$ $\vec{F} = m\vec{a}$ gives $|q|vB\sin\phi = m\dfrac{v^2}{R}$. $\phi = 90°$ and $B = \dfrac{mv}{|q|R} = \dfrac{(9.11\times10^{-31}\text{ kg})(2.65\times10^{7}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.180\text{ m})} = 8.38\times10^{-4}\text{ T.}$

EVALUATE: The smaller the radius of the circular path, the larger the magnitude of the magnetic field that is required.

27.22. IDENTIFY: The magnetic force on the beam bends it through a quarter circle.
SET UP: The distance that particles in the beam travel is $s = R\theta$, and the radius of the quarter circle is $R = mv/qB$.

EXECUTE: Solving for R gives $R = s/\theta = s/(\pi/2) = 1.18\text{ cm}/(\pi/2) = 0.751\text{ cm}$. Solving for the magnetic field: $B = mv/qR = (1.67\times10^{-27}\text{ kg})(1200\text{ m/s})/[(1.60\times10^{-19}\text{ C})(0.00751\text{ m})] = 1.67\times10^{-3}\text{ T.}$

EVALUATE: This field is about 10 times stronger than the earth's magnetic field, but much weaker than many laboratory fields.

27.23. IDENTIFY: For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.
SET UP: $v = E/B$ for no deflection.

EXECUTE: To pass undeflected in both cases, $E = vB = (5.85\times10^{3}\text{ m/s})(1.35\text{ T}) = 7898\text{ N/C}$.

(a) If $q = 0.640\times10^{-9}\text{ C}$, the electric field direction is given by $-(\hat{\boldsymbol{j}}\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since it must point in the opposite direction to the magnetic force.
(b) If $q = -0.320\times10^{-9}\text{ C}$, the electric field direction is given by $((-\hat{\boldsymbol{j}})\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since the electric force must point in the opposite direction as the magnetic force. Since the particle has negative charge, the electric force is opposite to the direction of the electric field and the magnetic force is opposite to the direction it has in part (a).
EVALUATE: The same configuration of electric and magnetic fields works as a velocity selector for both positively and negatively charged particles.

27.24. **IDENTIFY:** For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection. With only the magnetic force, $|q|vB = mv^2/R$.

EXECUTE: **(a)** $v = E/B = (1.56\times10^4\text{ V/m})/(4.62\times10^{-3}\text{ T}) = 3.38\times10^6\text{ m/s}$.

(b) The directions of the three vectors $\vec{v}$, $\vec{E}$, and $\vec{B}$ are sketched in Figure 27.24.

(c) $R = \dfrac{mv}{|q|B} = \dfrac{(9.11\times10^{-31}\text{ kg})(3.38\times10^6\text{ m/s})}{(1.60\times10^{-19}\text{ C})(4.62\times10^{-3}\text{ T})} = 4.17\times10^{-3}\text{ m}$.

$T = \dfrac{2\pi m}{|q|B} = \dfrac{2\pi R}{v} = \dfrac{2\pi(4.17\times10^{-3}\text{ m})}{(3.38\times10^6\text{ m/s})} = 7.74\times10^{-9}\text{ s}$.

EVALUATE: For the field directions shown in Figure 27.24, the electric force is toward the top of the page and the magnetic force is toward the bottom of the page.

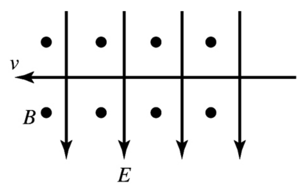


Figure 27.24

27.25. **IDENTIFY:** For the alpha particles to emerge from the plates undeflected, the magnetic force on them must exactly cancel the electric force. The battery produces an electric field between the plates, which acts on the alpha particles.

SET UP: First use energy conservation to find the speed of the alpha particles as they enter the region between the plates: $qV = 1/2\ mv^2$. The electric field between the plates due to the battery is $E = V_\text{b}/d$. For the alpha particles not to be deflected, the magnetic force must cancel the electric force, so $qvB = qE$, giving $B = E/v$.

EXECUTE: Solve for the speed of the alpha particles just as they enter the region between the plates. Their charge is $2e$.

$$v_\alpha = \sqrt{\frac{2(2e)V}{m}} = \sqrt{\frac{4(1.60\times10^{-19}\text{ C})(1750\text{ V})}{6.64\times10^{-27}\text{ kg}}} = 4.11\times10^5\text{ m/s}.$$

The electric field between the plates, produced by the battery, is
$E = V_\text{b}/d = (150\text{ V})/(0.00820\text{ m}) = 18{,}300\text{ V/m}$.

The magnetic force must cancel the electric force:
$B = E/v_\alpha = (18{,}300\text{ V/m})/(4.11\times10^5\text{ m/s}) = 0.0445\text{ T}$.

The magnetic field is perpendicular to the electric field. If the charges are moving to the right and the electric field points upward, the magnetic field is out of the page.

EVALUATE: The sign of the charge of the alpha particle does not enter the problem, so negative charges of the same magnitude would also not be deflected.

27.26. **IDENTIFY** and **SET UP:** For a velocity selector, $E = vB$. For parallel plates with opposite charge, $V = Ed$.

EXECUTE: **(a)** $E = vB = (1.82\times10^6\text{ m/s})(0.510\text{ T}) = 9.28\times10^5\text{ V/m}$.

(b) $V = Ed = (9.28\times10^5\text{ V/m})(5.20\times10^{-3}\text{ m}) = 4.83\text{ kV}$.

EVALUATE: Any charged particle with $v = 1.82\times10^6\text{ m/s}$ will pass through undeflected, regardless of the sign and magnitude of its charge.

27.27. **IDENTIFY:** The velocity selector eliminates all ions not having the desired velocity. Then the magnetic field bends the ions in a circular arc.

SET UP: In a velocity selector, $E = vB$. For motion in a circular arc in a magnetic field of magnitude B, $R = \dfrac{mv}{|q|B}$. The ion has charge $+e$.

EXECUTE: **(a)** $v = \dfrac{E}{B} = \dfrac{155\ \text{V/m}}{0.0315\ \text{T}} = 4.92\times10^3\ \text{m/s}$.

(b) $m = \dfrac{R|q|B}{v} = \dfrac{(0.175\ \text{m})(1.60\times10^{-19}\ \text{C})(0.0175\ \text{T})}{4.92\times10^3\ \text{m/s}} = 9.96\times10^{-26}\ \text{kg}$.

EVALUATE: Ions with larger ratio $\dfrac{m}{|q|}$ will move in a path of larger radius.

27.28. **IDENTIFY:** A mass spectrometer separates ions by mass. Since ^{14}N and ^{15}N have different masses they will be separated and the relative amounts of these isotopes can be determined.

SET UP: $R = \dfrac{mv}{|q|B}$. For $m = 1.99\times10^{-26}\ \text{kg}$ (^{12}C), $R_{12} = 12.5\ \text{cm}$. The separation of the isotopes at the detector is $2(R_{15} - R_{14})$.

EXECUTE: Since $R = \dfrac{mv}{|q|B}$, $\dfrac{R}{m} = \dfrac{v}{|q|B} = \text{constant}$. Therefore $\dfrac{R_{14}}{m_{14}} = \dfrac{R_{12}}{m_{12}}$ which gives

$R_{14} = R_{12}\left(\dfrac{m_{14}}{m_{12}}\right) = (12.5\ \text{cm})\left(\dfrac{2.32\times10^{-26}\ \text{kg}}{1.99\times10^{-26}\ \text{kg}}\right) = 14.6\ \text{cm}$ and

$R_{15} = R_{12}\left(\dfrac{m_{15}}{m_{12}}\right) = (12.5\ \text{cm})\left(\dfrac{2.49\times10^{-26}\ \text{kg}}{1.99\times10^{-26}\ \text{kg}}\right) = 15.6\ \text{cm}$. The separation of the isotopes at the detector is $2(R_{15} - R_{14}) = 2(15.6\ \text{cm} - 14.6\ \text{cm}) = 2.0\ \text{cm}$.

EVALUATE: The separation is large enough to be easily detectable. Since the diameter of the ion path is large, about 30 cm, the uniform magnetic field within the instrument must extend over a large area.

27.29. **IDENTIFY:** Apply $F = IlB\sin\phi$.

SET UP: Label the three segments in the field as a, b, and c. Let x be the length of segment a. Segment b has length 0.300 m and segment c has length $0.600\ \text{m} - x$. Figure 27.29a shows the direction of the force on each segment. For each segment, $\phi = 90°$. The total force on the wire is the vector sum of the forces on each segment.

EXECUTE: $F_a = IlB = (4.50\ \text{A})x(0.240\ \text{T})$. $F_c = (4.50\ \text{A})(0.600\ \text{m} - x)(0.240\ \text{T})$. Since $\vec{F}_a$ and $\vec{F}_c$ are in the same direction their vector sum has magnitude $F_{ac} = F_a + F_c = (4.50\ \text{A})(0.600\ \text{m})(0.240\ \text{T}) = 0.648\ \text{N}$ and is directed toward the bottom of the page in Figure 27.29a. $F_b = (4.50\ \text{A})(0.300\ \text{m})(0.240\ \text{T}) = 0.324\ \text{N}$ and is directed to the right. The vector addition diagram for $\vec{F}_{ac}$ and $\vec{F}_b$ is given in Figure 27.29b.

$F = \sqrt{F_{ac}^2 + F_b^2} = \sqrt{(0.648\ \text{N})^2 + (0.324\ \text{N})^2} = 0.724\ \text{N}$. $\tan\theta = \dfrac{F_{ac}}{F_b} = \dfrac{0.648\ \text{N}}{0.324\ \text{N}}$ and $\theta = 63.4°$. The net force has magnitude 0.724 N and its direction is specified by $\theta = 63.4°$ in Figure 27.29b.

EVALUATE: All three current segments are perpendicular to the magnetic field, so $\phi = 90°$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.

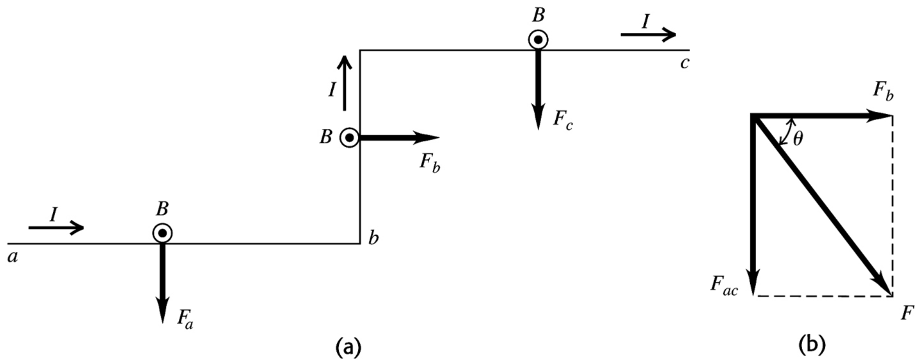


Figure 27.29

27.30. **IDENTIFY:** The earth's magnetic field exerts a force on the moving charges in the wire.

SET UP: $F = IlB\sin\phi$. The direction of $\vec{F}$ is determined by applying the right-hand rule to the directions of I and $\vec{B}$. $1 \text{ gauss} = 10^{-4} \text{ T}$.

EXECUTE: **(a)** The directions of I and $\vec{B}$ are sketched in Figure 27.30a. $\phi = 90°$ so $F = (1.5 \text{ A})(2.5 \text{ m})(0.55\times10^{-4} \text{ T}) = 2.1\times10^{-4} \text{ N}$. The right-hand rule says that $\vec{F}$ is directed out of the page, so it is upward.

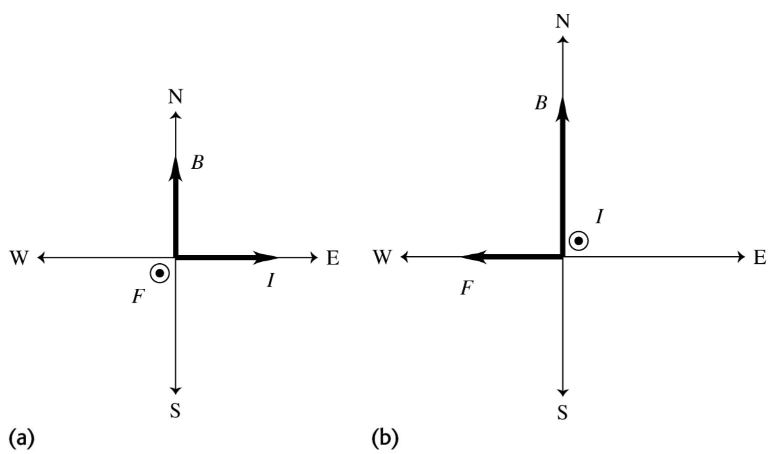


Figure 27.30

(b) The directions of I and $\vec{B}$ are sketched in Figure 27.30b. $\phi = 90°$ and $F = 2.1\times10^{-4} \text{ N}$. $\vec{F}$ is directed east to west.

(c) $\vec{B}$ and the direction of the current are antiparallel. $\phi = 180°$ so $F = 0$.

(d) The magnetic force of $2.1\times10^{-4} \text{ N}$ is not large enough to cause significant effects.

EVALUATE: The magnetic force is a maximum when the directions of I and $\vec{B}$ are perpendicular and it is zero when the current and magnetic field are either parallel or antiparallel.

27.31. **IDENTIFY** and **SET UP:** The magnetic force is given by $F = IlB\sin\phi$. $F_I = mg$ when the bar is just ready to levitate. When I becomes larger, $F_I > mg$ and $F_I - mg$ is the net force that accelerates the bar upward. Use Newton's second law to find the acceleration.

EXECUTE: **(a)** $IlB = mg$, $I = \dfrac{mg}{lB} = \dfrac{(0.750 \text{ kg})(9.80 \text{ m/s}^2)}{(0.500 \text{ m})(0.450 \text{ T})} = 32.67 \text{ A}$.

$V = IR = (32.67 \text{ A})(25.0\ \Omega) = 817 \text{ V}$.

(b) $R = 2.0\ \Omega, I = \mathcal{E}/R = (816.7\ \text{V})/(2.0\ \Omega) = 408\ \text{A}.$

$F_I = IlB = 92\ \text{N}.$

$a = (F_I - mg)/m = 113\ \text{m/s}^2.$

EVALUATE: I increases by over an order of magnitude when R changes to $F_I >> mg$ and a is an order of magnitude larger than g.

27.32. **IDENTIFY:** Apply $F = IlB\sin\phi.$

SET UP: $l = 0.0500\ \text{m}$ is the length of wire in the magnetic field. Since the wire is perpendicular to $\vec{B}$, $\phi = 90°.$

EXECUTE: $F = IlB = (10.8\ \text{A})(0.0500\ \text{m})(0.550\ \text{T}) = 0.297\ \text{N}.$

EVALUATE: The force per unit length of wire is proportional to both B and I.

27.33. **IDENTIFY:** The magnetic force $\vec{F}_B$ must be upward and equal to mg. The direction of $\vec{F}_B$ is determined by the direction of I in the circuit.

SET UP: $F_B = IlB\sin\phi$, with $\phi = 90°$. $I = \dfrac{V}{R}$, where V is the battery voltage.

EXECUTE: **(a)** The forces are shown in Figure 27.33. The current I in the bar must be to the right to produce $\vec{F}_B$ upward. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F_B = mg.\ \ IlB = mg.\ \ m = \dfrac{IlB}{g} = \dfrac{VlB}{Rg} = \dfrac{(175\ \text{V})(0.600\ \text{m})(1.50\ \text{T})}{(5.00\ \Omega)(9.80\ \text{m/s}^2)} = 3.21\ \text{kg}.$

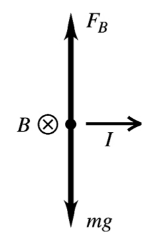


Figure 27.33

EVALUATE: If the battery had opposite polarity, with point a as the negative terminal, then the current would be clockwise and the magnetic force would be downward.

27.34. **IDENTIFY** and **SET UP:** $F = IlB\sin\phi$. The direction of $\vec{F}$ is given by applying the right-hand rule to the directions of I and $\vec{B}$.

EXECUTE: **(a)** The current and field directions are shown in Figure 27.34a. The right-hand rule gives that $\vec{F}$ is directed to the south, as shown. $\phi = 90°$ and

$F = (2.60\ \text{A})(1.00 \times 10^{-2}\ \text{m})(0.588\ \text{T}) = 0.0153\ \text{N}.$

(b) The right-hand rule gives that $\vec{F}$ is directed to the west, as shown in Figure 27.34b. $\phi = 90°$ and $F = 0.0153\ \text{N}$, the same as in part (a).

(c) The current and field directions are shown in Figure 27.34c. The right-hand rule gives that $\vec{F}$ is $60.0°$ north of west. $\phi = 90°$ so $F = 0.0153\ \text{N}$, the same as in part (a).

EVALUATE: In each case the current direction is perpendicular to the magnetic field. The magnitude of the magnetic force is the same in each case but its direction depends on the direction of the magnetic field.

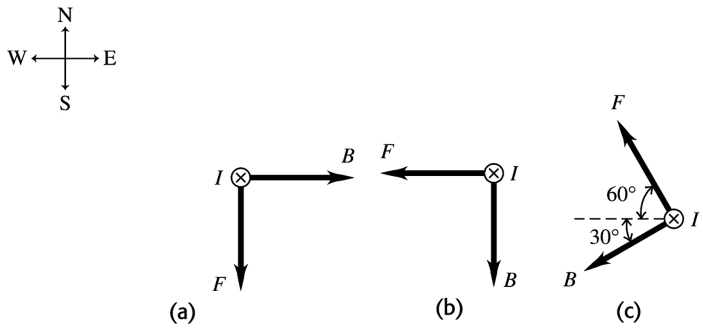


Figure 27.34

27.35. **IDENTIFY:** We are dealing with the magnetic dipole moment and torque.

SET UP: $\vec{\mu} = I\vec{A}$, $\mu = INA$, $\vec{\tau} = \vec{\mu} \times \vec{B}$.

EXECUTE: **(a)** We want *N*. Use $\mu = INA$ with the given numbers. $N = \mu/IA = 55$ turns.

(b) We want the direction of the current. If the fingers of the right hand curl counterclockwise, the thumb points toward the observer, so the current is *counterclockwise*.

(c) We want the torque. $\tau = \mu B \sin\phi = (0.194\ \text{A} \cdot \text{m}^2)(45.0\ \text{T})(\sin 90°) = 8.73\ \text{N} \cdot \text{m}$. $\vec{\tau} = \vec{\mu} \times \vec{B}$ is perpendicular to $\vec{\mu}$ and points *upward*.

EVALUATE: Even for a very large magnetic field like this one, the torque is small because the area is very small.

27.36. **IDENTIFY:** $\tau = IAB\sin\phi$. The magnetic moment of the loop is $\mu = IA$.

SET UP: Since the plane of the loop is parallel to the field, the field is perpendicular to the normal to the loop and $\phi = 90°$.

EXECUTE: **(a)** $\tau = IAB = (6.2\ \text{A})(0.050\ \text{m})(0.080\ \text{m})(0.19\ \text{T}) = 4.7 \times 10^{-3}\ \text{N} \cdot \text{m}$.

(b) $\mu = IA = (6.2\ \text{A})(0.050\ \text{m})(0.080\ \text{m}) = 0.025\ \text{A} \cdot \text{m}^2$.

(c) Maximum area is when the loop is circular. $R = \dfrac{0.050\ \text{m} + 0.080\ \text{m}}{\pi} = 0.0414\ \text{m}$.

$A = \pi R^2 = 5.38 \times 10^{-3}\ \text{m}^2$ and $\tau = (6.2\ \text{A})(5.38 \times 10^{-3}\ \text{m}^2)(0.19\ \text{T}) = 6.34 \times 10^{-3}\ \text{N} \cdot \text{m}$.

EVALUATE: The torque is a maximum when the field is in the plane of the loop and $\phi = 90°$.

27.37. **IDENTIFY:** The wire segments carry a current in an external magnetic field. Only segments *ab* and *cd* will experience a magnetic force since the other two segments carry a current parallel (and antiparallel) to the magnetic field. Only the force on segment *cd* will produce a torque about the hinge.

SET UP: $F = IlB\sin\phi$. The direction of the magnetic force is given by the right-hand rule applied to the directions of *I* and $\vec{B}$. The torque due to a force equals the force times the moment arm, the perpendicular distance between the axis and the line of action of the force.

EXECUTE: **(a)** The direction of the magnetic force on each segment of the circuit is shown in Figure 27.37. For segments *bc* and *da* the current is parallel or antiparallel to the field and the force on these segments is zero.

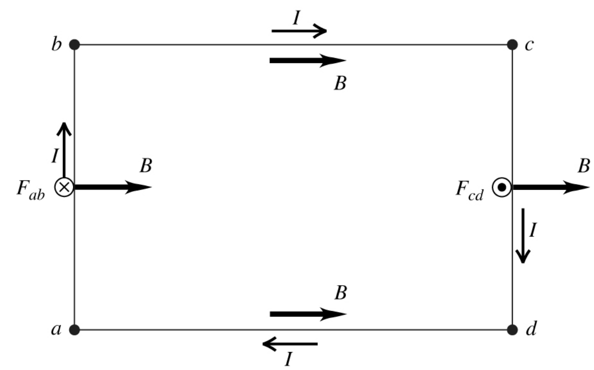


Figure 27.37

(b) $\vec{F}_{ab}$ acts at the hinge and therefore produces no torque. $\vec{F}_{cd}$ tends to rotate the loop about the hinge so it does produce a torque about this axis. $F_{cd} = IlB\sin\phi = (5.00\text{ A})(0.200\text{ m})(1.20\text{ T})\sin 90° = 1.20\text{ N}$

(c) $\tau = Fl = (1.20\text{ N})(0.350\text{ m}) = 0.420\text{ N}\cdot\text{m}.$

EVALUATE: The torque is directed so as to rotate side *cd* out of the plane of the page in Figure 27.37.

27.38. IDENTIFY: $\tau = IAB\sin\phi$, where ϕ is the angle between $\vec{B}$ and the normal to the loop.

SET UP: The coil as viewed along the axis of rotation is shown in Figure 27.38a for its original position and in Figure 27.38b after it has rotated $30.0°$.

EXECUTE: **(a)** The forces on each side of the coil are shown in Figure 27.38a. $\vec{F}_1 + \vec{F}_2 = 0$ and $\vec{F}_3 + \vec{F}_4 = 0$. The net force on the coil is zero. $\phi = 0°$ and $\sin\phi = 0$, so $\tau = 0$. The forces on the coil produce no torque.

(b) The net force is still zero. $\phi = 30.0°$ and the net torque is $\tau = (1)(1.95\text{ A})(0.220\text{ m})(0.350\text{ m})(1.50\text{ T})\sin 30.0° = 0.113\text{ N}\cdot\text{m}$. The net torque is clockwise in Figure 27.38b and is directed so as to increase the angle ϕ.

EVALUATE: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.

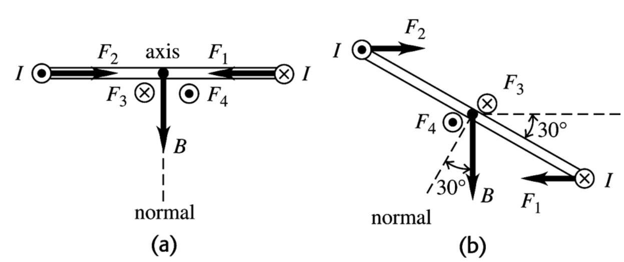


Figure 27.38

27.39. IDENTIFY: The magnetic field exerts a torque on the current-carrying coil, which causes it to turn. We can use the rotational form of Newton's second law to find the angular acceleration of the coil.

SET UP: The magnetic torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$, and the rotational form of Newton's second law is $\sum\tau = I\alpha$. The magnetic field is parallel to the plane of the loop.

EXECUTE: **(a)** The coil rotates about axis A_2 because the only torque is along top and bottom sides of the coil.

(b) To find the moment of inertia of the coil, treat the two 1.00-m segments as point-masses (since all the points in them are 0.250 m from the rotation axis) and the two 0.500-m segments as thin uniform bars rotated about their centers. Since the coil is uniform, the mass of each segment is proportional to its fraction of the total perimeter of the coil. Each 1.00-m segment is 1/3 of the total perimeter, so its mass is $(1/3)(210\text{ g}) = 70\text{ g} = 0.070\text{ kg}$. The mass of each 0.500-m segment is half this amount, or 0.035 kg.

The result is

$I = 2(0.070\text{ kg})(0.250\text{ m})^2 + 2\tfrac{1}{12}(0.035\text{ kg})(0.500\text{ m})^2 = 0.0102\text{ kg}\cdot\text{m}^2.$

The torque is

$|\vec{\boldsymbol{\tau}}| = |\vec{\boldsymbol{\mu}}\times\vec{\boldsymbol{B}}| = IAB\sin 90° = (2.00\text{A})(0.500\text{m})(1.00\text{m})(3.00\text{T}) = 3.00\text{ N}\cdot\text{m}.$

Using the above values, the rotational form of Newton's second law gives

$\alpha = \dfrac{\tau}{I} = 290\text{ rad/s}^2.$

EVALUATE: This angular acceleration will not continue because the torque changes as the coil turns.

27.40. **IDENTIFY** and **SET UP:** Both coils A and B have the same area A and N turns, but they carry current in opposite directions in a magnetic field. The torque is $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\boldsymbol{B}}$ and the potential energy is $U = -\mu B\cos\phi$. The magnetic moment is $\vec{\mu} = I\vec{\boldsymbol{A}}$.

EXECUTE: **(a)** Using the right-hand rule for the magnetic moment, $\vec{\mu}$ points in the –z-direction (into the page) for coil A and in the +z-direction (out of the page) for coil B.

(b) The torque is $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\boldsymbol{B}}$ which has magnitude $\tau = \mu B\sin\phi$. For coil A, ϕ = 180°, and for coil B, ϕ = 0°. In both cases, sin ϕ = 0, making the torque zero.

(c) <u>For coil A</u>: $U_A = -\mu B\cos\phi = -NIAB\cos 180° = NIAB.$

<u>For coil B</u>: $U_B = -\mu B\cos\phi = -NIAB\cos 0° = -NIAB.$

(d) If coil A is rotated slightly from its equilibrium position, the magnetic field will flip it 180°, so its equilibrium is unstable. But if the same thing it done to coil B, the magnetic field will return it to its original equilibrium position, which makes its equilibrium stable.

EVALUATE: For the stable equilibrium (coil B), its potential energy is a minimum, while for the unstable equilibrium (coil A), its potential energy is a maximum.

27.41. **IDENTIFY:** $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\boldsymbol{B}}$ and $U = -\mu B\cos\phi$, where $\mu = NIA$. $\tau = \mu B\sin\phi$.

SET UP: ϕ is the angle between $\vec{\boldsymbol{B}}$ and the normal to the plane of the loop.

EXECUTE: **(a)** $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $\hat{\boldsymbol{k}}\times\hat{\boldsymbol{j}} = -\hat{\boldsymbol{i}}$. $U = -\mu B\cos\phi = 0$.

(b) $\phi = 0$. $\tau = NIAB\sin(0) = 0$, no direction. $U = -\mu B\cos\phi = -NIAB$.

(c) $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $-\hat{\boldsymbol{k}}\times\hat{\boldsymbol{j}} = \hat{\boldsymbol{i}}$. $U = -\mu B\cos\phi = 0$.

(d) $\phi = 180°$: $\tau = NIAB\sin(180°) = 0$, no direction, $U = -\mu B\cos(180°) = NIAB$.

EVALUATE: When τ is maximum, $U = 0$. When $|U|$ is maximum, $\tau = 0$.

27.42. **IDENTIFY** and **SET UP:** The potential energy is given by $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}}$. The scalar product depends on the angle between $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$.

EXECUTE: For $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$ parallel, $\phi = 0°$ and $\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}} = \mu B\cos\phi = \mu B$. For $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$ antiparallel, $\phi = 180°$ and $\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}} = \mu B\cos\phi = -\mu B$.

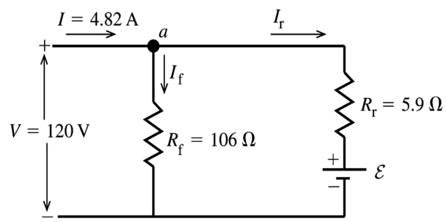
$U_1 = +\mu B,\ U_2 = -\mu B.$

$\Delta U = U_2 - U_1 = -2\mu B = -2(1.45\text{ A}\cdot\text{m}^2)(0.835\text{ T}) = -2.42\text{ J}.$

EVALUATE: U is maximum when $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$ are antiparallel and minimum when they are parallel. When the coil is rotated as specified its magnetic potential energy decreases.

27.43. **IDENTIFY:** The circuit consists of two parallel branches with the potential difference of 120 V applied across each. One branch is the rotor, represented by a resistance R_r and an induced emf that opposes the applied potential. Apply the loop rule to each parallel branch and use the junction rule to relate the currents through the field coil and through the rotor to the 4.82 A supplied to the motor.

SET UP: The circuit is sketched in Figure 27.43.



$\mathcal{E}$ is the induced emf developed by the motor. It is directed so as to oppose the current through the rotor.

Figure 27.43

EXECUTE: **(a)** The field coils and the rotor are in parallel with the applied potential difference V, so $V = I_f R_f$. $I_f = \dfrac{V}{R_f} = \dfrac{120\text{ V}}{106\ \Omega} = 1.13\text{ A}.$

(b) Applying the junction rule to point *a* in the circuit diagram gives $I - I_f - I_r = 0$.

$I_r = I - I_f = 4.82\text{ A} - 1.13\text{ A} = 3.69\text{ A}.$

(c) The potential drop across the rotor, $I_r R_r + \mathcal{E}$, must equal the applied potential difference V: $V = I_r R_r + \mathcal{E}$

$\mathcal{E} = V - I_r R_r = 120\text{ V} - (3.69\text{ A})(5.9\ \Omega) = 98.2\text{ V}$

(d) The mechanical power output is the electrical power input minus the rate of dissipation of electrical energy in the resistance of the motor:

electrical power input to the motor

$P_{in} = IV = (4.82\text{ A})(120\text{ V}) = 578\text{ W}.$

electrical power loss in the two resistances

$P_{loss} = I_f^2 R_f + I_r^2 R_r = (1.13\text{ A})^2(106\ \Omega) + (3.69\text{ A})^2(5.9\ \Omega) = 216\text{ W}.$

mechanical power output

$P_{out} = P_{in} - P_{loss} = 578\text{ W} - 216\text{ W} = 362\text{ W}.$

The mechanical power output is the power associated with the induced emf $\mathcal{E}$.

$P_{out} = P_{\mathcal{E}} = \mathcal{E} I_r = (98.2\text{ V})(3.69\text{ A}) = 362\text{ W}$, which agrees with the above calculation.

EVALUATE: The induced emf reduces the amount of current that flows through the rotor. This motor differs from the one described in Example 27.11. In that example the rotor and field coils are connected in series and in this problem they are in parallel.

27.44. **IDENTIFY:** Apply $V_{ab} = \mathcal{E} + Ir$ in order to calculate *I*. The power drawn from the line is $P_{supplied} = IV_{ab}$. The mechanical power is the power supplied minus the $I^2 r$ electrical power loss in the internal resistance of the motor.

SET UP: $V_{ab} = 120\text{ V}$, $\mathcal{E} = 105\text{ V}$, and $r = 3.2\ \Omega$.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} + Ir \Rightarrow I = \dfrac{V_{ab} - \mathcal{E}}{r} = \dfrac{120\text{ V} - 105\text{ V}}{3.2\ \Omega} = 4.7\text{ A}.$

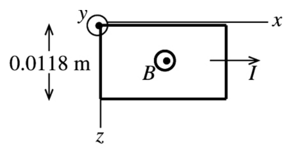
(b) $P_{supplied} = IV_{ab} = (4.7\text{ A})(120\text{ V}) = 564\text{ W}.$

(c) $P_{mech} = IV_{ab} - I^2 r = 564\text{ W} - (4.7\text{ A})^2(3.2\ \Omega) = 493\text{ W}.$

EVALUATE: If the rotor isn't turning, when the motor is first turned on or if the rotor bearings fail, then $\mathcal{E}=0$ and $I=\dfrac{120\text{V}}{3.2\,\Omega}=37.5$ A. This large current causes large I^2r heating and can trip the circuit breaker.

27.45. IDENTIFY: The drift velocity is related to the current density by $J_x = n|q|v_\text{d}$. The electric field is determined by the requirement that the electric and magnetic forces on the current-carrying charges are equal in magnitude and opposite in direction.

SET UP and **EXECUTE:** **(a)** The section of the silver ribbon is sketched in Figure 27.45a.



$J_x = n|q|v_\text{d}$.

so $v_\text{d} = \dfrac{J_x}{n|q|}$.

Figure 27.45a

EXECUTE: $J_x = \dfrac{I}{A} = \dfrac{I}{y_1 z_1} = \dfrac{120\text{ A}}{(0.23\times10^{-3}\text{ m})(0.0118\text{ m})} = 4.42\times10^{7}\text{ A/m}^2.$

$$v_\text{d} = \frac{J_x}{n|q|} = \frac{4.42\times10^{7}\text{ A/m}^2}{(5.85\times10^{28}/\text{m}^3)(1.602\times10^{-19}\text{ C})} = 4.7\times10^{-3}\text{ m/s} = 4.7\text{ mm/s}.$$

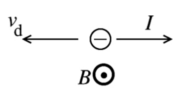
(b) magnitude of $\vec{E}$:

$|q|E_z = |q|v_\text{d}B_y$.

$E_z = v_\text{d}B_y = (4.7\times10^{-3}\text{ m/s})(0.95\text{ T}) = 4.5\times10^{-3}\text{ V/m}.$

direction of $\vec{E}$:

The drift velocity of the electrons is in the opposite direction to the current, as shown in Figure 27.45b.

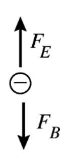


$\vec{v}\times\vec{B}\uparrow$.

$\vec{F}_B = q\vec{v}\times\vec{B} = -e\vec{v}\times\vec{B}\downarrow$.

Figure 27.45b

The directions of the electric and magnetic forces on an electron in the ribbon are shown in Figure 27.45c.



$\vec{F}_E$ must oppose $\vec{F}_B$ so $\vec{F}_E$ is in the $-z$-direction.

Figure 27.45c

$\vec{F}_E = q\vec{E} = -e\vec{E}$ so $\vec{E}$ is opposite to the direction of $\vec{F}_E$ and thus $\vec{E}$ is in the $+z$-direction.

(c) The Hall emf is the potential difference between the two edges of the strip (at $z=0$ and $z=z_1$) that results from the electric field calculated in part (b). $\mathcal{E}_\text{Hall} = Ez_1 = (4.5\times10^{-3}\text{ V/m})(0.0118\text{ m}) = 53\,\mu\text{V}.$

EVALUATE: Even though the current is quite large the Hall emf is very small. Our calculated Hall emf is more than an order of magnitude larger than in Example 27.12. In this problem the magnetic field and current density are larger than in the example, and this leads to a larger Hall emf.

27.46. **IDENTIFY:** Apply $qn = \dfrac{-J_x B_y}{E_z}$.

SET UP: $A = y_1 z_1$. $E = \mathcal{E}/z_1$. $|q| = e$.

EXECUTE: $n = \dfrac{J_x B_y}{|q|E_z} = \dfrac{IB_y}{A|q|E_z} = \dfrac{IB_y z_1}{A|q|\mathcal{E}} = \dfrac{IB_y}{y_1|q|\mathcal{E}}$.

$$n = \frac{(78.0\text{ A})(2.29\text{ T})}{(2.3\times10^{-4}\text{ m})(1.6\times10^{-19}\text{ C})(1.31\times10^{-4}\text{ V})} = 3.7\times10^{28}\text{ electrons/m}^3.$$

EVALUATE: The value of n for this metal is about one-third the value of n calculated in Example 27.12 for copper.

27.47. **IDENTIFY:** This problem involves the magnetic and electric forces on a moving charged particle.
SET UP: First sketch the situation, as in Fig. 27.47a.

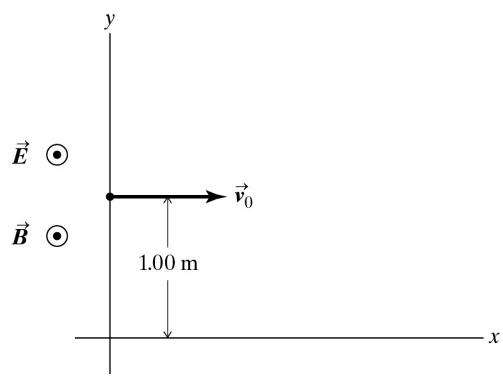


Figure 27.47a

EXECUTE: **(a)** We want the coordinates at t = 0.0200 s. Applying $\sum F_z = qE = ma_z$ gives $a_z = qE/m$.
$z = \frac{1}{2}a_z t^2 = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$ = 9.00 cm using the given quantities. The magnetic field deflects the particle in the $-y$-direction in a circle of radius $R = mv_0/qB$. Using the given numbers gives R = 1.000 m. The circle is centered at the origin. The time T for one circle is $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi m}{qB} =$ 0.2094 s. In 0.0200 s, it has gone through $\dfrac{0.0200}{0.2094} = 0.09549$ of a complete revolution, which is an angle of 34.38°. Fig. 27.47b shows the path of the particle and it x and y coordinates. From this figure, we see that
$x = (1.00\text{ m})\sin\theta = (1.00\text{ m})\sin(34.38°) = 0.565\text{ m}$ and
$y = (1.00\text{ m})\cos\theta = (1.00\text{ m})\cos(34.38°) = 0.825\text{ m}$. Therefore the coordinates of the particle are
x = 0.565 m, y = 0.825 m, z = 0.0900 m.

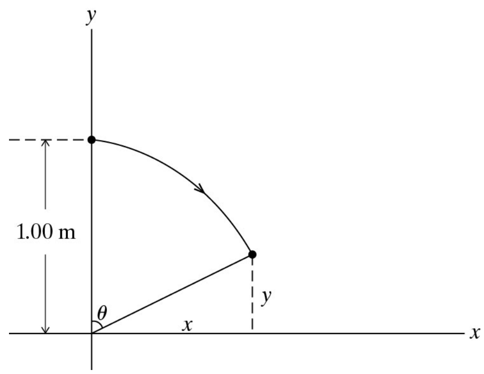


Figure 27.47b

(b) We want the speed at 0.0200 s. Only the electric field changes the speed. Using a_z from (a) gives $v_z = a_z t = \left(\frac{qE}{m}\right)t = 9.00$ m/s. v_x and v_y also change, but $v_x^2 + v_y^2 = v_0^2$, so $v = \sqrt{v_z^2 + v_0^2} =$ 31.3 m/s.

EVALUATE: The z component of velocity keeps increasing. The x and y components change but always combine to give $v_x^2 + v_y^2 = v_0^2$.

27.48. **IDENTIFY:** Apply $\vec{F} = q\vec{v} \times \vec{B}$.

SET UP: $B_x = 0.650$ T. $B_y = 0$ and $B_z = 0$.

EXECUTE: $F_x = q(v_y B_z - v_z B_y) = 0$.

$F_y = q(v_z B_x - v_x B_z) = (7.26\times10^{-8}\ \text{C})(5.85\times10^4\ \text{m/s})(0.650\ \text{T}) = 2.76\times10^{-3}$ N.

$F_z = q(v_x B_y - v_y B_x) = -(7.26\times10^{-8}\ \text{C})(-3.11\times10^4\ \text{m/s})(0.650\ \text{T}) = 1.47\times10^{-3}$ N.

EVALUATE: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. We can verify that $\vec{F}\cdot\vec{v} = 0$. Since $\vec{B}$ is along the x-axis, v_x does not affect the force components.

27.49. **IDENTIFY:** In part (a), apply conservation of energy to the motion of the two nuclei. In part (b) apply $|q|vB = mv^2/R$.

SET UP: In part (a), let point 1 be when the two nuclei are far apart and let point 2 be when they are at their closest separation.

EXECUTE: **(a)** $K_1 + U_1 = K_2 + U_2$. $U_1 = K_2 = 0$, so $K_1 = U_2$. There are two nuclei having equal kinetic energy, so $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = ke^2/r$. Solving for v gives

$$v = e\sqrt{\frac{k}{mr}} = (1.602\times10^{-19}\ \text{C})\sqrt{\frac{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{(3.34\times10^{-27}\ \text{kg})(1.0\times10^{-15}\ \text{m})}} = 8.3\times10^6\ \text{m/s}.$$

(b) $\sum\vec{F} = m\vec{a}$ gives $qvB = mv^2/r$. $B = \frac{mv}{qr} = \frac{(3.34\times10^{-27}\ \text{kg})(8.3\times10^6\ \text{m/s})}{(1.602\times10^{-19}\ \text{C})(1.25\ \text{m})} = 0.14$ T.

EVALUATE: The speed calculated in part (a) is large, nearly 3% of the speed of light.

27.50. **IDENTIFY:** The period is $T = 2\pi r/v$, the current is Q/t and the magnetic moment is $\mu = IA$.

SET UP: The electron has charge $-e$. The area enclosed by the orbit is πr^2.

EXECUTE: **(a)** $T = 2\pi r/v = 1.5\times10^{-16}$ s.

(b) Charge $-e$ passes a point on the orbit once during each period, so $I = Q/t = e/t = 1.1$ mA.

(c) $\mu = IA = I\pi r^2 = 9.3\times10^{-24}\ \text{A}\cdot\text{m}^2$.

EVALUATE: Since the electron has negative charge, the direction of the current is opposite to the direction of motion of the electron.

27.51. **IDENTIFY:** We want to determine q/m for a particle using a graphical method.
SET UP and **EXECUTE:** Since we graphed R^2 versus ΔV, we need to relate these variables. If we accelerate the particle through a potential difference ΔV, we have $q\,\Delta V = \dfrac{1}{2}mv^2$. Therefore $q/m = \dfrac{v^2}{2\Delta V}$. In the magnetic field, $R = mv/qB$, so $v = RqB/m$. Combining these results gives $q/m = \dfrac{\left(\dfrac{RqB}{m}\right)^2}{2\Delta V} = \dfrac{R^2(q/m)^2B^2}{2\Delta V}$. Solving for R^2 gives $R^2 = \left(\dfrac{2}{B^2(q/m)}\right)\Delta V$. The graph of R^2 versus ΔV should be a straight line having slope $\dfrac{2}{B^2(q/m)}$. Thus $q/m = \dfrac{2}{B^2(\text{slope})}$. Putting in the numbers with the measured slope gives $q/m = 4.81\times10^7$ C/kg.

EVALUATE: For a proton $q/m = e/m_p = 9.58\times10^7$ C/kg, which is about twice our result. So our answer is reasonable.

27.52. **IDENTIFY** and **SET UP:** The maximum radius of the orbit determines the maximum speed v of the protons. Use Newton's second law and $a_{\text{rad}} = v^2/R$ for circular motion to relate the variables. The energy of the particle is the kinetic energy $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** $\sum\vec{F} = m\vec{a}$ gives $|q|vB = m(v^2/R)$.

$$v = \frac{|q|BR}{m} = \frac{(1.60\times10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{1.67\times10^{-27}\text{ kg}} = 3.257\times10^7\text{ m/s}.$$ The kinetic energy of a proton moving with this speed is $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(3.257\times10^7\text{ m/s})^2 = 8.9\times10^{-13}\text{ J} = 5.5\text{ MeV}$.

(b) The time for one revolution is the period $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(0.40\text{ m})}{3.257\times10^7\text{ m/s}} = 7.7\times10^{-8}$ s.

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2 = \frac{1}{2}\dfrac{|q|^2B^2R^2}{m}$. Or, $B = \dfrac{\sqrt{2Km}}{|q|R}$. B is proportional to $\sqrt{K}$, so if K is increased by a factor of 2 then B must be increased by a factor of $\sqrt{2}$. $B = \sqrt{2}(0.85\text{ T}) = 1.2$ T.

(d) $v = \dfrac{|q|BR}{m} = \dfrac{(3.20\times10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{6.65\times10^{-27}\text{ kg}} = 1.636\times10^7$ m/s

$K = \frac{1}{2}mv^2 = \frac{1}{2}(6.65\times10^{-27}\text{ kg})(1.636\times10^7\text{ m/s})^2 = 8.9\times10^{-13}\text{ J} = 5.5\text{ MeV}$, the same as the maximum energy for protons.

EVALUATE: We can see that the maximum energy must be approximately the same as follows: From part (c), $K = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2$. For alpha particles $|q|$ is larger by a factor of 2 and m is larger by a factor of 4 (approximately). Thus $|q|^2/m$ is unchanged and K is the same.

27.53. **IDENTIFY:** For the velocity selector, $E = vB$. For circular motion in the field B', $R = \dfrac{mv}{|q|B'}$.

SET UP: $B = B' = 0.682$ T.

EXECUTE: $v = \dfrac{E}{B} = \dfrac{1.88\times10^4 \text{ N/C}}{0.682 \text{ T}} = 2.757\times10^4$ m/s. $R = \dfrac{mv}{qB'}$, so

$$R_{82} = \frac{82(1.66\times10^{-27}\text{ kg})(2.757\times10^4\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.682\text{ T})} = 0.0344\text{ m} = 3.44\text{ cm}.$$

$$R_{84} = \frac{84(1.66\times10^{-27}\text{ kg})(2.757\times10^4\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.682\text{ T})} = 0.0352\text{ m} = 3.52\text{ cm}.$$

$$R_{86} = \frac{86(1.66\times10^{-27}\text{ kg})(2.757\times10^4\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.682\text{ T})} = 0.0361\text{ m} = 3.61\text{ cm}.$$

The distance between two adjacent lines is $2\Delta R = 2(3.52\text{ cm} - 3.44\text{ cm}) = 0.16\text{ cm} = 1.6\text{ mm}$.

EVALUATE: The distance between the ^{82}Kr line and the ^{84}Kr line is 1.6 mm and the distance between the ^{84}Kr line and the ^{86}Kr line is 1.6 mm. Adjacent lines are equally spaced since the ^{82}Kr versus ^{84}Kr and ^{84}Kr versus ^{86}Kr mass differences are the same.

27.54. IDENTIFY: Apply conservation of energy to the acceleration of the ions and Newton's second law to their motion in the magnetic field.

SET UP: The singly ionized ions have $q = +e$. A ^{12}C ion has mass 12 u and a ^{14}C ion has mass 14 u, where $1\text{ u} = 1.66\times10^{-27}$ kg.

EXECUTE: **(a)** During acceleration of the ions, $qV = \frac{1}{2}mv^2$ and $v = \sqrt{\dfrac{2qV}{m}}$. In the magnetic field,

$$R = \frac{mv}{qB} = \frac{m\sqrt{2qV/m}}{qB} \text{ and } m = \frac{qB^2R^2}{2V}.$$

(b) $V = \dfrac{qB^2R^2}{2m} = \dfrac{(1.60\times10^{-19}\text{ C})(0.150\text{ T})^2(0.500\text{ m})^2}{2(12)(1.66\times10^{-27}\text{ kg})} = 2.26\times10^4$ V.

(c) The ions are separated by the differences in the diameters of their paths. $D = 2R = 2\sqrt{\dfrac{2Vm}{qB^2}}$, so

$$\Delta D = D_{14} - D_{12} = 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{14} - 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{12} = 2\sqrt{\frac{2V(1\text{ u})}{qB^2}}(\sqrt{14} - \sqrt{12}).$$

$\Delta D = 2\sqrt{\dfrac{2(2.26\times10^4\text{ V})(1.66\times10^{-27}\text{ kg})}{(1.6\times10^{-19}\text{ C})(0.150\text{ T})^2}}(\sqrt{14} - \sqrt{12}) = 8.01\times10^{-2}$ m. This is about 8 cm and is easily distinguishable.

EVALUATE: The speed of the ^{12}C ion is $v = \sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(2.26\times10^4\text{V})}{12(1.66\times10^{-27}\text{ kg})}} = 6.0\times10^5$ m/s. This is very fast, but well below the speed of light, so relativistic mechanics is not needed.

27.55. IDENTIFY: The force exerted by the magnetic field is given by $F = IlB\sin\phi$. The net force on the wire must be zero.

SET UP: For the wire to remain at rest the force exerted on it by the magnetic field must have a component directed up the incline. To produce a force in this direction, the current in the wire must be directed from right to left in the figure with the problem in the textbook. Or, viewing the wire from its left-hand end the directions are shown in Figure 27.55a.

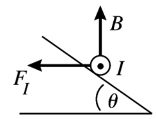


Figure 27.55a

The free-body diagram for the wire is given in Figure 27.55b.

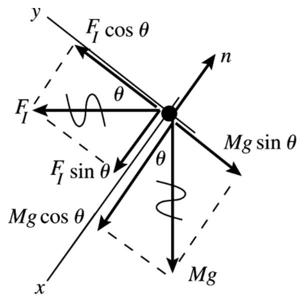


Figure 27.55b

EXECUTE: $\sum F_y = 0.$

$F_I \cos\theta - Mg\sin\theta = 0.$

$F_I = ILB\sin\phi.$

$\phi = 90°$ since $\vec{B}$ is perpendicular to the current direction.

Thus $(ILB)\ \cos\theta - Mg\sin\theta = 0$ and $I = \dfrac{Mg\tan\theta}{LB}.$

EVALUATE: The magnetic and gravitational forces are in perpendicular directions so their components parallel to the incline involve different trig functions. As the tilt angle θ increases there is a larger component of Mg down the incline and the component of F_I up the incline is smaller; I must increase with θ to compensate. As $\theta \to 0$, $I \to 0$ and as $\theta \to 90°$, $I \to \infty$.

27.56. **IDENTIFY:** In the figure shown with the problem in the text, the current in the bar is toward the bottom of the page, so the magnetic force is toward the right. Newton's second law gives the acceleration. The bar is in parallel with the $10.0\text{-}\Omega$ resistor, so we must use circuit analysis to find the initial current through the bar.

SET UP: First find the current. The equivalent resistance across the battery is $30.0\ \Omega$, so the total current is $4.00\,\text{A}$, half of which goes through the bar. Applying Newton's second law to the bar gives $\sum F = ma = F_B = ILB.$

EXECUTE: Equivalent resistance of the $10.0\text{-}\Omega$ resistor and the bar is $5.0\,\Omega$. Current through the $25.0\text{-}\Omega$ resistor is $I_{\text{tot}} = \dfrac{120.0\ \text{V}}{30.0\,\Omega} = 4.00\ \text{A}.$ The current in the bar is 2.00 A, toward the bottom of the page. The force $\vec{F}_I$ that the magnetic field exerts on the bar has magnitude $F_I = IlB$ and is directed to the right. $a = \dfrac{F_I}{m} = \dfrac{IlB}{m} = \dfrac{(2.00\ \text{A})(0.850\ \text{m})(1.60\ \text{T})}{(2.60\ \text{N})/(9.80\ \text{m/s}^2)} = 10.3\ \text{m/s}^2.$ $\vec{a}$ is directed to the right.

EVALUATE: Once the bar has acquired a non-zero speed there will be an induced emf (Chapter 29) and the current and acceleration will start to decrease.

27.57. **IDENTIFY:** $R = \dfrac{mv}{|q|B}.$

SET UP: After completing one semicircle the separation between the ions is the difference in the diameters of their paths, or $2(R_{13} - R_{12})$. A singly ionized ion has charge $+e$.

EXECUTE: **(a)** $B = \dfrac{mv}{|q|R} = \dfrac{(1.99\times10^{-26}\ \text{kg})(8.50\times10^{3}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.125\ \text{m})} = 8.46\times10^{-3}\ \text{T}.$

(b) The only difference between the two isotopes is their masses. $\dfrac{R}{m}=\dfrac{v}{|q|B}=\text{constant}$ and $\dfrac{R_{12}}{m_{12}}=\dfrac{R_{13}}{m_{13}}$.

$R_{13}=R_{12}\left(\dfrac{m_{13}}{m_{12}}\right)=(12.5\text{ cm})\left(\dfrac{2.16\times10^{-26}\text{ kg}}{1.99\times10^{-26}\text{ kg}}\right)=13.6\text{ cm}.$ The diameter is 27.2 cm.

(c) The separation is $2(R_{13}-R_{12})=2(13.6\text{ cm}-12.5\text{ cm})=2.2\text{ cm}.$ This distance can be easily observed.

EVALUATE: Decreasing the magnetic field increases the separation between the two isotopes at the detector.

27.58. **IDENTIFY:** Turning the charged loop creates a current, and the external magnetic field exerts a torque on that current.

SET UP: The current is $I=q/T=q/(1/f)=qf=q(\omega/2\pi)=q\omega/2\pi.$ The torque is $\tau=\mu B\sin\phi.$

EXECUTE: In this case, $\phi=90°$ and $\mu=IA,$ giving $\tau=IAB.$ Combining the results for the torque and current and using $A=\pi r^2$ gives $\tau=\left(\dfrac{q\omega}{2\pi}\right)\pi r^2B=\tfrac{1}{2}q\omega r^2B.$

EVALUATE: Any moving charge is a current, so turning the loop creates a current causing a magnetic force.

27.59. **IDENTIFY:** The force exerted by the magnetic field is $F=ILB\sin\phi.$ $a=F/m$ and is constant. Apply a constant acceleration equation to relate v and d.

SET UP: $\phi=90°.$ The direction of $\vec{F}$ is given by the right-hand rule.

EXECUTE: **(a)** $F=ILB,$ to the right.

(b) $v_x^2=v_{0x}^2+2a_x(x-x_0)$ gives $v^2=2ad$ and $d=\dfrac{v^2}{2a}=\dfrac{v^2m}{2ILB}.$

(c) $d=\dfrac{(1.12\times10^4\text{ m/s})^2(25\text{ kg})}{2(2000\text{ A})(0.50\text{ m})(0.80\text{ T})}=1.96\times10^6\text{ m}=1960\text{ km}.$

EVALUATE: $a=\dfrac{ILB}{m}=\dfrac{(2.0\times10^3\text{ A})(0.50\text{ m})(0.80\text{ T})}{25\text{ kg}}=32\text{ m/s}^2.$ The acceleration due to gravity is not negligible. Since the bar would have to travel nearly 2000 km, this would not be a very effective launch mechanism using the numbers given.

27.60. **IDENTIFY:** Apply $\vec{F}=I\vec{l}\times\vec{B}.$

SET UP: $\vec{l}=l\hat{k}.$

EXECUTE: **(a)** $\vec{F}=I(l\hat{k})\times\vec{B}=Il[(-B_y)\hat{i}+(B_x)\hat{j}].$ This gives

$F_x=-IlB_y=-(7.40\text{ A})(0.250\text{ m})(-0.985\text{ T})=1.82\text{ N}$ and

$F_y=IlB_x=(7.40\text{ A})(0.250\text{ m})(-0.242\text{ T})=-0.448\text{ N}.$ $F_z=0,$ since the wire is in the z-direction.

(b) $F=\sqrt{F_x^2+F_y^2}=\sqrt{(1.82\text{ N})^2+(0.448\text{ N})^2}=1.88\text{ N}.$

EVALUATE: $\vec{F}$ must be perpendicular to the current direction, so $\vec{F}$ has no z-component.

27.61. **IDENTIFY:** We are dealing with the magnetic force on a curved current-carrying wire.

SET UP and **EXECUTE:** **(a)** Use Example 27.8 as a guide except integrate from $\theta=0$ to $\pi/2$. This gives $F_x=F_y=IRB$, so $F=\sqrt{(IRB)^2+(IRB)^2}=IRB\sqrt{2}=(5.00\text{ A})(0.200\text{ m})(0.800\text{ T})\sqrt{2}=1.13\text{ N}.$ Using the right-hand rule for the magnetic force on a current-carrying wire, we see that the direction of the force is toward the origin.

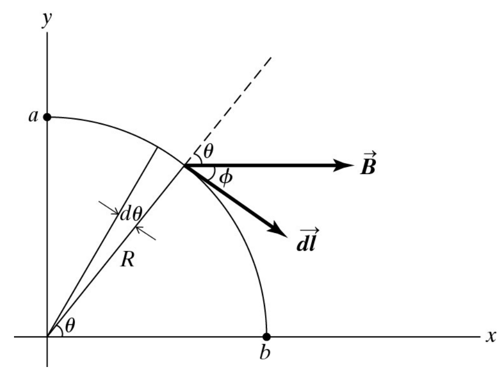


Figure 27.61

(b) Refer to Fig. 27.61. $dF = IBdl\sin\phi = IBdl\cos\theta$. $dl = Rd\theta$. $F = \int_0^{\pi/2} IB\cos\theta R d\theta = IBR$. Using the numbers gives F = (5.00 A)(0.800 T)(0.200 m) = 0.800 N. By the right-hand rule, the force is in the +z-direction.

EVALUATE: Just by changing the direction of the field, the force can change direction *and* magnitude.

27.62. **IDENTIFY:** This problem involves the torque on a current-carrying coil in a magnetic field.

SET UP and **EXECUTE:** **(a)** Estimate: 15 cm in diameter.

(b) We want the rotor diameter. 1/3 of the hub circumference is (1/3)(π)(15 cm) = 15.7 cm. 15.7 cm = (12)(diameter of one rotor) = 12D, so D = 1.3 cm.

(c) We want the magnetic moment. $\tau = \mu B\sin\phi$ for each rotor. Thus $\frac{1.4\ \text{N}\cdot\text{m}}{12} = \mu(1.0\ \text{T})\sin 90°$ which gives $\mu = 0.12\ \text{A}\cdot\text{m}^2$.

(d) We want N. $\mu = NIA = NI\pi r^2$. Using $\mu = 0.12\ \text{A}\cdot\text{m}^2$, r = 0.0065 m, and I = (0.75 A)/12 gives N = 14,000 turns.

EVALUATE: This wire must be extremely thin to fit 14,000 turns into the small volume of the central hub.

27.63. **IDENTIFY:** This problem involves the magnetic force on a current-carrying wire.

SET UP and **EXECUTE:** **(a)** Estimate the current. $F_{mag} = F_{grav}$, so $ILB_{horiz} = mg = \rho\pi r^2 Lg$.

$I = \frac{\rho\pi r^2 g}{B\cos 45°}$ = 48,000 A using the given quantities.

(b) 48,000 A > 900 A so this is *not feasible.*

(c) We want the minimum magnetic field. For B to be a minimum, I would have to have its maximum possible value of 900 A. Using our analysis in (a) but with the bar perpendicular to the field, we get

$B_{min} = \frac{\rho\pi r^2 g}{I}$ = 1.9 mT = 19 G.

(d) We want the mass we could support. $F_{mag} = (m_{bar} + M)g$. $ILB = \rho\pi r^2 Lg + Mg$. Solving for M gives $M = L\left(\frac{IB}{g} - \rho\pi r^2\right)$. Using I = 10 A, B =1.0 T, and the usual quantities for the other variables, we get M = 0.085 kg, so w = 0.83 N.

EVALUATE: Magnetic levitation the earth's magnetic field is obviously not feasible in most cases.

27.64. IDENTIFY: The torque exerted by the magnetic field is $\vec{\tau} = \vec{\mu} \times \vec{B}$. The torque required to hold the loop in place is $-\vec{\tau}$.

SET UP: $\mu = IA$. $\vec{\mu}$ is normal to the plane of the loop, with a direction given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. $\tau = IAB\sin\phi$, where ϕ is the angle between the normal to the loop and the direction of $\vec{B}$.

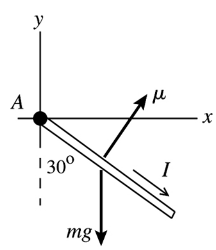
EXECUTE: (a) $\tau = IAB \sin 60° = (15.0\text{ A})(0.060\text{ m})(0.080\text{ m})(0.48\text{ T})\sin 60° = 0.030\text{ N}\cdot\text{m}$, in the $-\hat{j}$-direction. To keep the loop in place, you must provide a torque in the $+\hat{j}$-direction.

(b) $\tau = IAB \sin 30° = (15.0\text{ A})(0.60\text{ m})(0.080\text{ m})(0.48\text{ T})\sin 30° = 0.017\text{ N}\cdot\text{m}$, in the $+\hat{j}$-direction. You must provide a torque in the $-\hat{j}$-direction to keep the loop in place.

EVALUATE: (c) If the loop was pivoted through its center, then there would be a torque on both sides of the loop parallel to the rotation axis. However, the lever arm is only half as large, so the total torque in each case is identical to the values found in parts (a) and (b).

27.65. IDENTIFY: For the loop to be in equilibrium the net torque on it must be zero. Use $\vec{\tau} = \vec{\mu} \times \vec{B}$ to calculate the torque due to the magnetic field and $\tau_{mg} = mgr\sin\phi$ for the torque due to gravity.

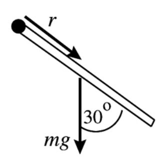
SET UP: See Figure 27.65a.



Use $\sum \tau_A = 0$, where point A is at the origin.

Figure 27.65a

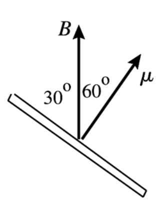
EXECUTE: See Figure 27.65b.



$\tau_{mg} = mgr\sin\phi = mg(0.400\text{ m})\sin 30.0°$.
The torque is clockwise; $\vec{\tau}_{mg}$ is directed into the paper.

Figure 27.65b

For the loop to be in equilibrium the torque due to $\vec{B}$ must be counterclockwise (opposite to $\vec{\tau}_{mg}$) and it must be that $\tau_B = \tau_{mg}$. See Figure 27.65c.



$\vec{\tau}_B = \vec{\mu} \times \vec{B}$. For this torque to be counter-clockwise ($\vec{\tau}_B$ directed out of the paper), $\vec{B}$ must be in the $+y$-direction.

Figure 27.65c

$\tau_B = \mu B \sin\phi = IAB\sin 60.0°$.

$\tau_B = \tau_{mg}$ gives $IAB\sin 60.0° = mg(0.0400\text{ m})\sin 30.0°$.

$m = (0.15\text{ g/cm})2(8.00\text{ cm} + 6.00\text{ cm}) = 4.2\text{ g} = 4.2\times 10^{-3}\text{ kg}$.

$A = (0.0800\text{ m})(0.0600\text{ m}) = 4.80\times 10^{-3}\text{ m}^2$.

$$B = \frac{mg(0.0400\text{ m})(\sin 30.0°)}{IA\sin 60.0°}.$$

$$B = \frac{(4.2\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.0400\text{ m})\sin 30.0°}{(8.2\text{ A})(4.80\times 10^{-3}\text{ m}^2)\sin 60.0°} = 0.024\text{ T}.$$

EVALUATE: As the loop swings up the torque due to $\vec{B}$ decreases to zero and the torque due to mg increases from zero, so there must be an orientation of the loop where the net torque is zero.

27.66. **IDENTIFY** and **SET UP:** The force on a current-carrying bar of length l is $F = IlB$ if the field is perpendicular to the bar. The torque is $\tau_z = \mu B\sin\phi$.

EXECUTE: **(a)** The force on the infinitesimal segment is $dF = IBdl = IBdx$. The torque about point a is $d\tau_z = xdF\sin\phi = xIBdx$. In this case, $\sin\phi = 1$ because the force is perpendicular to the bar.

(b) We integrate to get the total torque: $\tau_z = \int_0^L xIBdx = \frac{1}{2}IBL^2$.

(c) For $F = ILB$ at the center of the bar, the torque is $\tau_z = F\left(\frac{L}{2}\right) = ILB\left(\frac{L}{2}\right) = \frac{1}{2}IBL^2$, which is the same result we got by integrating.

EVALUATE: We can think of the magnetic force as all acting at the center of the bar because the magnetic field is uniform. This is the same reason we can think of gravity acting at the center of a uniform bar.

27.67. **IDENTIFY:** Apply $\vec{F} = I\vec{l}\times\vec{B}$ to calculate the force on each side of the loop.

SET UP: The net force is the vector sum of the forces on each side of the loop.

EXECUTE: **(a)** $F_{PQ} = (5.00\text{ A})(0.600\text{ m})(3.00\text{ T})\sin(0°) = 0\text{ N}$.

$F_{RP} = (5.00\text{ A})(0.800\text{ m})(3.00\text{ T})\sin(90°) = 12.0\text{ N}$, into the page.

$F_{QR} = (5.00\text{ A})(1.00\text{ m})(3.00\text{ T})(0.800/1.00) = 12.0\text{ N}$, out of the page.

(b) The net force on the triangular loop of wire is zero.

(c) For calculating torque on a straight wire we can assume that the force on a wire is applied at the wire's center. Also, note that we are finding the torque with respect to the PR-axis (not about a point), and consequently the lever arm will be the distance from the wire's center to the x-axis. $\tau = rF\sin\phi$ gives $\tau_{PQ} = r(0\text{ N}) = 0$, $\tau_{RP} = (0\text{ m})F\sin\phi = 0$ and $\tau_{QR} = (0.300\text{ m})(12.0\text{ N})\sin(90°) = 3.60\text{ N}\cdot\text{m}$. The net torque is $3.60\text{ N}\cdot\text{m}$.

(d) Using $\tau = NIAB\sin\phi$ gives

$\tau = NIAB\sin\phi = (1)(5.00\text{ A})\left(\frac{1}{2}\right)(0.600\text{ m})(0.800\text{ m})(3.00\text{ T})\sin(90°) = 3.60\text{ N}\cdot\text{m}$, which agrees with our result in part (c).

(e) Since F_{QR} is out of the page and since this is the force that produces the net torque, the point Q will be rotated out of the plane of the figure.

EVALUATE: In the expression $\tau = NIAB\sin\phi$, ϕ is the angle between the plane of the loop and the direction of $\vec{B}$. In this problem, $\phi = 90°$.

27.68. **IDENTIFY:** We are dealing with magnetic force and torque.

SET UP and EXECUTE: We want the force and torque. $F = IlB\sin\phi$ and $\tau = \mu B\sin\phi$.

(a) We want for force on each segment. PQ: $F = IlB\sin\phi$ = (5.00 A)(0.600 m)(3.00 T)(1) = 9.00 N. By the right-hand rule, the direction is to the *left*.

QR: F = (5.00 A)(1.00 m)(3.00 T)(1) = 15.0 N. The direction is perpendicular to *QR* pointing upward and to the right, making a 53.1° angle with *PR*.

RP: F = (5.00 A)(0.800 m)(3.00 T)(1) = 12.0 N, downward perpendicular to *PR*.

(b) We want the net force. F_x = –9.00 N + (15.0 N) cos(53.1°) = 0. F_y = –12.0 N + (15.0 N) sin 53.1° = 0. The net force is zero.

(c) We want the torque about *PR*. $\tau_{PR} = 0$, $\tau_{PQ} = 0$, and $\tau_{QR} = 0$. The net torque is zero.

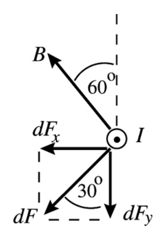
(d) From (c), the net torque is zero. Use $\tau = \mu B\sin\phi$. $\vec{\mu}$ and $\vec{B}$ both point into the page, so ϕ = 0, so the torque is zero. Both methods agree.

(e) The torque is zero, so the field would cause no rotation.

EVALUATE: The magnetic field does exert torque on the wire segments, but not about the *PR* axis.

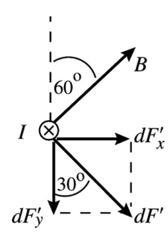
27.69. **IDENTIFY:** Use $dF = Idl\,B\sin\phi$ to calculate the force on a short segment of the coil and integrate over the entire coil to find the total force.

SET UP: See Figures 27.69a and 27.69b. The two sketches show that the *x*-components cancel and that the *y*-components add. This is true for all pairs of short segments on opposite sides of the coil. The net magnetic force on the coil is in the *y* direction and its magnitude is given by $F = \int dF_y$.



Consider the force $d\vec{F}$ on a short segment *dl* at the left-hand side of the coil, as viewed in the figure with the problem in the textbook. The current at this point is directed out of the page. $d\vec{F}$ is perpendicular both to $\vec{B}$ and to the direction of *I*.

Figure 27.69a



Consider also the force $d\vec{F}'$ on a short segment on the opposite side of the coil, at the right-hand side of the coil in the figure with the problem in the textbook. The current at this point is directed into the page.

Figure 27.69b

EXECUTE: $dF = Idl\,B\sin\phi$. But $\vec{B}$ is perpendicular to the current direction so $\phi = 90°$.

$dF_y = dF\cos 30.0 = IB\cos 30.0°dl.$

$F = \int dF_y = IB\cos 30.0° \int dl.$

But $\int dl = N(2\pi r)$, the total length of wire in the coil.

$F = IB\cos 30.0° N(2\pi r) = (0.950\text{ A})(0.220\text{ T})(\cos 30.0°)(50)2\pi(0.0078\text{ m}) = 0.444\text{ N}$ and

$\vec{F} = -(0.444\text{ N})\hat{j}$

EVALUATE: The magnetic field makes a constant angle with the plane of the coil but has a different direction at different points around the circumference of the coil so is not uniform. The net force is proportional to the magnitude of the current and reverses direction when the current reverses direction.

27.70. IDENTIFY: For rotational equilibrium, the torques due to gravity and the magnetic field must balance around point a.

SET UP: From Problem 27.66 we have $\tau_z = \frac{1}{2}IBL^2$.

EXECUTE: **(a)** Balancing the two torques gives: $mg\frac{L}{2}\cos\theta = \frac{1}{2}IBL^2$. Simplifying gives Putting in the numbers gives

$I(0.150\text{ T})(0.300\text{ m}) = (0.0120\text{ kg})(9.80\text{ m/s}^2)\cos(30.0°)$, so $I = 2.26$ A.

(b) Gravity tends to rotate the bar clockwise about point a, so the magnetic force must be upward and to the left to tend to rotate the bar clockwise. Therefore the current must flow from a to b.

EVALUATE: If the current were from b to a, the bar could not balance.

27.71. IDENTIFY: Apply $d\vec{F} = Id\vec{l} \times \vec{B}$ to each side of the loop.

SET UP: For each side of the loop, $d\vec{l}$ is parallel to that side of the loop and is in the direction of I. Since the loop is in the xy-plane, $z = 0$ at the loop and $B_y = 0$ at the loop.

EXECUTE: **(a)** The magnetic field lines in the yz-plane are sketched in Figure 27.71.

(b) Side 1, that runs from (0,0) to (0,L): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 y\, dy}{L}\hat{i} = \frac{1}{2}B_0 LI\hat{i}$.

Side 2, that runs from (0,L) to (L,L): $\vec{F} = \int_{0,y=L}^{L} Id\vec{l} \times \vec{B} = I\int_{0,y=L}^{L} \frac{B_0 y\, dx}{L}\hat{j} = -IB_0 L\hat{j}$.

Side 3, that runs from (L,L) to (L,0): $\vec{F} = \int_{L,x=L}^{0} Id\vec{l} \times \vec{B} = I\int_{L,x=L}^{0} \frac{B_0 y\, dy}{L}(-\hat{i}) = -\frac{1}{2}IB_0 L\hat{i}$.

Side 4, that runs from (L,0) to (0,0): $\vec{F} = \int_{L,y=0}^{0} Id\vec{l} \times \vec{B} = I\int_{L,y=0}^{0} \frac{B_0 y\, dx}{L}\hat{j} = 0$.

(c) The sum of all forces is $\vec{F}_{\text{total}} = -IB_0 L\hat{j}$.

EVALUATE: The net force on sides 1 and 3 is zero. The force on side 4 is zero, since $y = 0$ and $z = 0$ at that side and therefore $B = 0$ there. The net force on the loop equals the force on side 2.

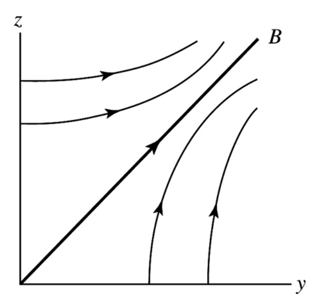


Figure 27.71

27.72. IDENTIFY and SET UP: The rod is in rotational equilibrium, so the torques must balance. Take torques about point P and use $\tau_z = \frac{1}{2}IBL^2$ from Problem 27.66.

EXECUTE: Balancing torques gives $mg\frac{L}{2}\cos\theta + \frac{1}{2}IBL^2 = T\sin\theta L$, where L is the length of the bar and T is the tension in the string. Solving for T and putting in the numbers gives

$T = [(0.0840\text{ kg})(9.80\text{ m/s}^2)\cos(53.0°) + (12.0\text{ A})(0.120\text{ T})(0.180\text{ m})]/[2\sin(53.0°)] = 0.472$ N.

Evaluate: If the current were reversed, the tension would be less than 0.472 N.

27.73. **Identify:** This problem involves the torque on a current loop due to a magnetic field.
Set Up: $\tau = \mu B \sin\phi$.

Execute: **(a)** We want the minimum current so that $h > 0$. Refer to the figure included with the problem in the textbook. The torques balance, so $\tau_{\text{tension}} = \tau_{\text{gravity}}$. $\mu B \sin\phi = MgR$. $IANB \sin\phi = MgR$.

Using $A = 2RW$ and solving for I gives $I = \dfrac{Mg}{2WNB \sin\phi}$. The minimum I occurs for maximum $\sin\phi$, which is 1 when $\phi = 90°$. So $I_{\text{min}} = \dfrac{Mg}{2WNB}$.

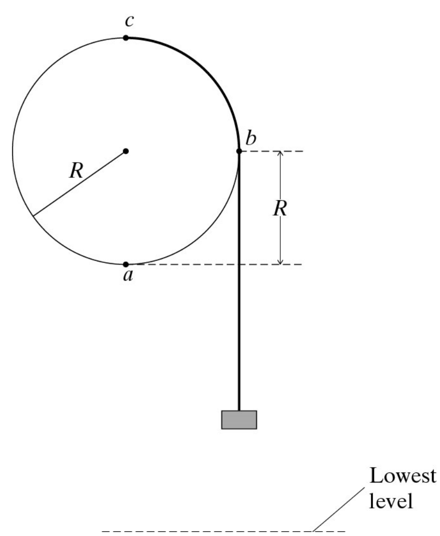


Figure 27.73a

(b) Refer to Fig. 27.73a. In one-half turn, the mass first rises a distance R from a to b. In going from b to c, the length of cable that is pulled up is ¼ of a circumference because after b the cable wraps around the rim. So the total rise is $R + (2\pi R)/4 = R(1 + \pi/2) = h_{\text{top}}$.

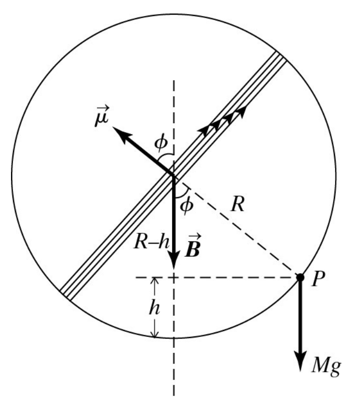


Figure 27.73b

(c) We want the net torque if $0 \le h \le R$. Refer to Fig. 27.73b. $\tau_{\text{net}} = \tau_{\text{mag}} + \tau_{\text{grav}}$. $\tau_{\text{grav}} = MgR \sin\phi$.
$\tau_{\text{mag}} = \mu B \sin\phi = 2IWRNB \sin\phi$. So $\tau_{\text{net}} = (2IWRNB - MgR)\sin\phi$. Since the cable is attached to the rim

of the cylinder at point P on the axis of the coil, $\vec{\mu}$ as shown in Fig. 27.73 in the text always points directly away from P. To find $\sin\phi$, look at Fig. 27.73a which shows that

$\sin\phi = \dfrac{\sqrt{R^2-(R-h)^2}}{R} = \dfrac{\sqrt{h(2R-h)}}{R}$. Therefore $\tau_{\text{net}} = (2IWNB - Mg)\sqrt{h(2R-h)}$. Using $\sigma = \dfrac{2NIWB}{Mg}$ as defined in the problem, we can write the net torque as

$$\tau_{\text{net}} = Mg\left(\frac{2IWNB}{Mg} - 1\right)\sqrt{h(2R-h)} = Mg(\sigma-1)\sqrt{h(2R-h)}.$$

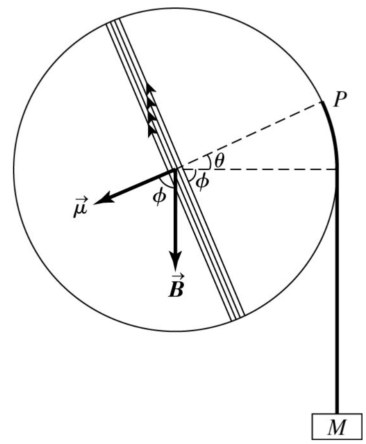


Figure 27.73c

(d) We want the net torque for $R \le h \le h_{\text{top}}$. Refer to Fig. 27.73c. Use an approach similar to that in part (a). h is equal to R plus the length of cable turned through θ. With θ in radians, this gives $h = R + R\theta$, so $\theta = \dfrac{h}{R} - 1$. From Fig. 27.73c, $\sin\phi = \cos\theta$. Therefore

$\tau_{\text{net}} = \tau_{\text{mag}} + \tau_{\text{grav}} = \mu B\sin\phi - MgR = 2INWRB\cos\theta - MgR$. Regrouping gives

$$\tau_{\text{net}} = \left(\frac{2INWB}{Mg}\cos\theta - 1\right)MgR = \left[\sigma\cos\left(\frac{h}{R} - 1\right) - 1\right]MgR.$$

(e) We want the potential energy for $0 \le h \le R$. $U = U_{\text{grav}} + U_{\text{mag}} + U_0$. $U_{\text{grav}} = Mgh$.
$U_{\text{mag}} = -\vec{\mu}\cdot\vec{B} = -\mu B\cos(180° - \phi) = \mu B\sin\phi$. Define $U(h)$ so that $U(0) = 0$. When $h = 0$, $\phi = 0$ and $\cos\phi = 1$, so U_0 must be $-\mu B$. Thus $U(h) = \mu B\cos\phi + Mgh - \mu B\cos 0° = \mu B(\cos\phi - 1) + Mgh$. Using our expression for μ, we can write this as $U(h) = 2INRWB(\cos\phi - 1) + Mgh$. From Fig. 27.73b, we see that $\cos\phi = \dfrac{R-h}{R} = 1 - \dfrac{h}{R}$, so $U(h)$ becomes

$$U(h) = 2INRWB(1 - h/R - 1) + Mgh = -2INWBh + Mgh = \left(-\frac{2INWB}{Mg} + 1\right)Mgh = (1-\sigma)Mgh.$$

(f) We want U when $R \le h \le h_{\text{top}}$. As we have seen, $U(h) = U_{\text{mag}} + U_{\text{grav}} - \mu B$. See Fig. 27.73c and the procedure for part (d). We have shown: $h = R(1+\theta)$, $\theta = \dfrac{h}{R} - 1$, $\cos\phi = \sin\theta = \sin\left(\dfrac{h}{R} - 1\right)$. Therefore $U(h) = -\mu B\cos\phi + Mgh - \mu B = -\mu B(\cos\phi + 1) + Mgh$. Continuing we get

$U(h) = -\mu B\left[\sin\left(\frac{h}{R}-1\right)+1\right]+Mgh.$ Putting in for μ and expressing in terms of σ finally gives

$U(h) = \left\{\frac{h}{R}-\sigma\left[\sin\left(\frac{h}{R}-1\right)+1\right]\right\}(MgR).$

(g) If $\sigma > 1$, what h will suspend M motionless? The net torque will be zero. There are two possibilities: $0 \le h \le R$ and $R \le h \le h_{\text{top}}$. Consider each one separately.

$\underline{0 \le h \le R}$: $\tau_{\text{net}} = Mg(\sigma-1)\sqrt{h(2R-h)} = 0.$ Either h = 0 or h = 2R. If h = 0, the object just hangs down from the rim at point P so it is not really suspended. Since h cannot be greater than 2R, neither of these answers is possible.

$\underline{R \le h \le h_{\text{top}}}$: $\tau_{\text{net}} = \left[\sigma\cos\left(\frac{h}{R}-1\right)-1\right]MgR = 0.$ $\sigma\cos\left(\frac{h}{R}-1\right)-1=0.$ $\cos(h/R-1) = 1/\sigma.$

$\frac{h}{R}-1 = \arccos(1/\sigma)$, which gives $h = R[1+\arccos(1/\sigma)].$

(h) What is σ such that $U(h_{\text{top}}) > 0$? For $R \le h \le h_{\text{top}}$ we have from part (f)

$U(h) = \left\{\frac{h}{R}-\sigma\left[\sin\left(\frac{h}{R}-1\right)+1\right]\right\}(MgR).$ From (b) we have $h_{\text{top}} = R(1 + \pi/2)$. Combining these gives

$U(h_{\text{top}}) = \frac{R(1+\pi/2)}{R} - \sigma\left[\sin(1+\pi/2-1)+1\right]MgR = 1+\pi/2-2\sigma > 0.$ This gives $\sigma < \frac{1}{2}\left(1+\frac{\pi}{2}\right).$

EVALUATE: When in doubt, check units!

27.74. **IDENTIFY:** We have a positive particle moving in a magnetic field.

SET UP and **EXECUTE:** $\vec{F} = q\vec{v}\times\vec{B}$. **(a)** We want a_x and a_y at time t. $F_x = qBv_y$,

so $a_x = \frac{qv_yB}{m} = \left(\frac{qB}{m}\right)v_y.$ $F_y = mg - qv_xB$, so $a_y = g - \left(\frac{qB}{m}\right)v_x.$

(b) $\frac{da_y}{dt} = -\left(\frac{qB}{m}\right)\frac{dv_x}{dt} = -\left(\frac{qB}{m}\right)a_x.$ Using $a_x = \left(\frac{qB}{m}\right)v_y$ and $\frac{da_y}{dt} = \frac{d^2v_y}{dt^2}$ gives

$\frac{d^2v_y}{dt^2} = -\left(\frac{qB}{m}\right)\left(\frac{qB}{m}\right)v_y = -\left(\frac{qB}{m}\right)^2 v_y.$

(c) The result in (b) is of the form $\frac{d^2x}{dt^2} = -\frac{k}{m}x,$ which has solution $x = A\cos(\omega t+\phi),$ where $\omega = \sqrt{k/m}.$ Applying this here we have $k/m \to (qB/m)^2$, so $\omega = qB/m.$ The equation for $v_y(t)$ is $v_y(t) = A\cos(\omega t+\phi).$ We need to determine A and ϕ. The initial conditions are: at t =0, v_y = 0 and a_y = g. $v_y(0) = A\cos\phi = 0$, so $\phi = \pi/2$. Thus $v_y(t) = A\cos(\omega t+\pi/2) = -A\sin\omega t.$

$a_y = \frac{dv_y}{dt} = -A\omega\cos\omega t.$ At t = 0, we have $-A\omega = g,$ so $-A = g/\omega.$ Thus $v_y(t) = \frac{g}{\omega}\sin\omega t,$ where $\omega = qB/m.$

(d) $a_x = \frac{dv_x}{dt} = \frac{qB}{m}v_y$ and $\frac{dv_x}{dt} = \frac{qB}{m}\left(\frac{g}{\omega}\right)\sin\omega t = \frac{qB}{m}\left(\frac{g}{qB/m}\right)\sin\omega t = g\sin\omega t.$

$v_x(t) = \int g\sin\omega t\,dt = -\frac{g}{\omega}\cos\omega t + C.$ When t = 0, v_x = 0, so $C = g/\omega.$ This gives $v_x(t) = \frac{g}{\omega}(1-\cos\omega t).$

(e) $x(t) = \int\frac{g}{\omega}(1-\cos\omega t)\,dt = \frac{g}{\omega}\left(t-\frac{1}{\omega}\sin\omega t\right)+C.$ When t = 0, x = 0, so C = 0. Thus

$x(t) = \frac{gt}{\omega} - \frac{g}{\omega^2}\sin\omega t.$ Now find $y(t)$.

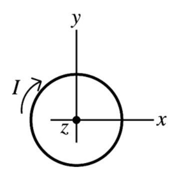
$y(t) = \int \frac{g}{\omega}\sin\omega t = -\frac{g}{\omega^2}\cos\omega t + C$. When $t = 0$, $y = 0$, so $C = g/\omega^2$. Thus $y(t) = \frac{g}{\omega^2}(1-\cos\omega t)$.

(f) We want the maximum vertical distance. From (e) we have $y(t) = \frac{g}{\omega^2}(1-\cos\omega t)$. y is a maximum when $\cos\omega t = -1$. $\omega = qB/m = (19.6\ \mu\text{C})(10.0\ \text{T})/(1.00\ \text{mg}) = 196$ rad/s. So $y_{\max} = 2g/\omega^2 = 2g/(196\ \text{rad/s})^2 = 0.510$ mm.

EVALUATE: The complete motion is quite complicated so it is easiest to deal with components as we did here.

27.75. **IDENTIFY:** Use $U = -\vec{\mu}\cdot\vec{B}$ to relate $U, \mu,$ and $\vec{B}$ and use $\vec{\tau} = \vec{\mu}\times\vec{B}$ to relate $\vec{\tau}, \vec{\mu},$ and $\vec{B}$. We also know that $B_0^2 = B_x^2 + B_y^2 + B_z^2$. This gives three equations for the three components of $\vec{B}$.

SET UP: The loop and current are shown in Figure 27.75.



$\vec{\mu}$ is into the plane of the paper, in the $-z$-direction.

Figure 27.75

EXECUTE: **(a)** $\vec{\mu} = -\mu\hat{k} = -IA\hat{k}$.

(b) $\vec{\tau} = D(+4\hat{i} - 3\hat{j})$, where $D > 0$.

$\vec{\mu} = -IA\hat{k}, \vec{B} = B_x\hat{i} + B_y\hat{j} + B_y\hat{k}$.

$\vec{\tau} = \vec{\mu}\times\vec{B} = (-IA)(B_x\hat{k}\times\hat{i} + B_y\hat{k}\times\hat{j} + B_z\hat{k}\times\hat{k}) = IAB_y\hat{i} - IAB_x\hat{j}$.

Compare this to the expression given for $\vec{\tau}$: $IAB_y = 4D$ so $B_y = 4D/IA$ and $-IAB_x = -3D$ so $B_x = 3D/IA$. B_z doesn't contribute to the torque since $\vec{\mu}$ is along the z direction. But $B = B_0$ and $B_x^2 + B_y^2 + B_z^2 = B_0^2$; with $B_0 = 13D/IA$. Thus

$B_z = \pm\sqrt{B_0^2 - B_x^2 - B_y^2} = \pm(D/IA)\sqrt{169 - 9 - 16} = \pm 12(D/IA)$.

That $U = -\vec{\mu}\,.\,\vec{B}$ is negative determines the sign of

B_z: $U = -\vec{\mu}\cdot\vec{B} = -(-IA\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) = +IAB_z$.

So U negative says that B_z is negative, and thus $B_z = -12D/IA$.

EVALUATE: $\vec{\mu}$ is along the z-axis so only B_x and B_y contribute to the torque. B_x produces a y-component of $\vec{\tau}$ and B_y produces an x-component of $\vec{\tau}$. Only B_z affects U, and U is negative when $\vec{\mu}$ and $\vec{B}_z$ are parallel.

27.76. **IDENTIFY:** The ions are accelerated from rest. When they enter the magnetic field, they are bent into a circular path. Newton's second law applies to the ions in the magnetic field.

SET UP: $K = \frac{1}{2}mv^2 = qV$. $R = \frac{mv}{qB}$, where q is the magnitude of the charge.

EXECUTE: **(a)** As the ions are accelerated through the potential difference V, we have $K = \frac{1}{2}mv^2 = qV$, which gives $v = \sqrt{\frac{2qV}{m}}$. In the magnetic field, $R = \frac{mv}{qB}$. Using the v we just found gives

$R = \dfrac{mv}{qB} = \dfrac{m}{qB}\sqrt{\dfrac{2qV}{m}} = \sqrt{\dfrac{m}{q}}\dfrac{\sqrt{2V}}{B} = \dfrac{1}{B}\sqrt{\dfrac{2m}{q}}\sqrt{V}$. From this result we see that a graph of R versus $\sqrt{V}$ should be a straight line with a slope equal to $\dfrac{1}{B}\sqrt{\dfrac{2m}{q}}$.

(b) The graph of R versus $\sqrt{V}$ is shown in Figure 27.76. The slope of the best-fit line is $(6.355\text{ cm})/\sqrt{\text{kV}} = (0.06355\text{ m})/\sqrt{1000\text{ V}} = 0.00201\text{ m}\cdot\text{V}^{-1/2}$. We know that $\dfrac{1}{B}\sqrt{\dfrac{2m}{q}}$ = slope, so

$\dfrac{q}{m} = \dfrac{2}{[B(\text{slope})]^2} = \dfrac{2}{[(0.250\text{ T})(0.00201\text{ m}\cdot\text{V}^{-1/2}]^2} = 7.924\times10^{-6}\text{ C/kg}$, which rounds to $7.92\times10^{6}\text{ C/kg}$.

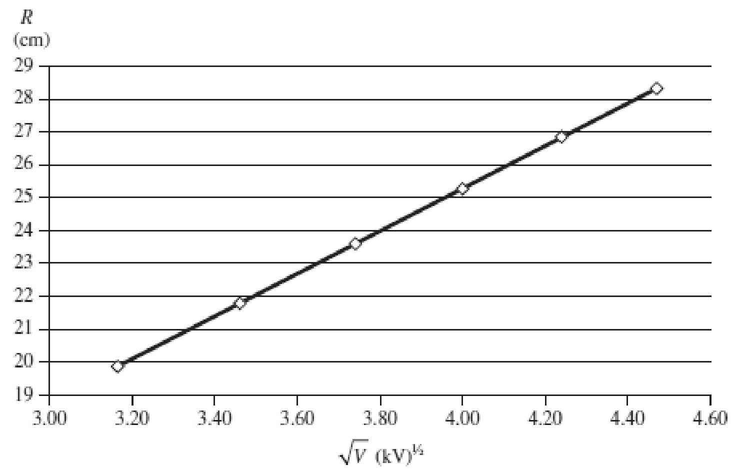


Figure 27.76

(c) Use our result for q/m: $v = \sqrt{\dfrac{2qV}{m}} = \sqrt{2(20.0\times10^{3}\text{ V})(7.924\times10^{6}\text{ C/kg})} = 5.63\times10^{5}\text{ m/s}$.

(d) Since $R = \dfrac{1}{B}\sqrt{\dfrac{2m}{q}}\sqrt{V}$, doubling q means that R is smaller by a factor of $\sqrt{2}$. Therefore $R = (21.1\text{ cm})/\sqrt{2} = 15.0\text{ cm}$.

EVALUATE: Besides the approach we have taken, the equation $R = \dfrac{1}{B}\sqrt{\dfrac{2m}{q}}\sqrt{V}$ can be graphed in other ways to obtain a straight line. For example, we could graph R^2 versus V, or even $\log R$ versus $\log V$. Ideally they should all give the same result for q/m. But differences can arise because we are dealing with less-than-ideal data points.

27.77. **IDENTIFY and SET UP:** The analysis in the text of the Thomson e/m experiment gives $\dfrac{e}{m} = \dfrac{E^2}{2VB^2}$. For a particle of charge e and mass m accelered through a potential V, $eV = \frac{1}{2}mv^2$.

EXECUTE: **(a)** Solving the equation $\dfrac{e}{m} = \dfrac{E^2}{2VB^2}$ for E^2 gives $E^2 = 2\left(\dfrac{e}{m}\right)B^2V$. Therefore a graph of E^2 versus V should be a straight line with slope equal to $2(e/m)B^2$.

(b) We can find the slope using two easily-read points on the graph. Using (100, 200) and (300, 600), we get $\dfrac{600\times10^{8}\text{ V}^2/\text{m}^2 - 200\times10^{8}\text{ V}^2/\text{m}^2}{300\text{ V} - 100\text{ V}} = 2.00\times10^{8}\text{ V/m}^2$ for the slope. This gives

e/m = (slope)/$2B^2$ = $(2.00\times10^8\ \text{V/m}^2)/[2(0.340\ \text{T})^2] = 8.65\times10^8$ C/kg, which gives

$m = 1.85\times10^{-28}$ kg.

(c) $V = Ed = (2.00\times10^5\ \text{V/m})\,(0.00600\ \text{m}) = 1.20$ kV.

(d) Using $eV = \frac{1}{2}mv^2$ to find the muon speed gives

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{2(8.65\times10^8\ \text{C/kg})(400\ \text{V})} = 8.32\times10^5\ \text{m/s}.$$

EVALUATE: Results may vary due to inaccuracies in determining the slope of the graph.

27.78. IDENTIFY and SET UP: If q is the magnitude of the charge, the cyclotron frequency is $\omega = \frac{qB}{m}$, where $\omega = 2\pi f$, and $R = mv/qB$.

EXECUTE: (a) Combining $\omega = \frac{qB}{m}$ and $\omega = 2\pi f$ gives $f = \left(\frac{1}{2\pi}\frac{q}{m}\right)B$. Therefore a graph of f versus B should be a straight line having slope equal to $q/2\pi m = (2e)/2\pi m = e/\pi m$. Solving for m gives $m = \frac{e}{\pi(\text{slope})}$. We use two points on the graph to calculate the slope, giving 7.667×10^6 Hz/T.

Therefore $m = \frac{e}{\pi(\text{slope})} = e/[\pi(7.667\times10^6\ \text{Hz/T})] = 6.65\times10^{-27}$ kg.

(b) Apply $f = \left(\frac{1}{2\pi}\frac{q}{m}\right)B = qB/2\pi m$ to the electron and the proton.

Electron: $f_e = (1.602\times10^{-19}\ \text{C})(0.300\ \text{T})/[2\pi(9.11\times10^{-31}\ \text{kg})] = 8.40\times10^9\ \text{Hz} = 8.40$ GHz.

Proton: $f_p = (1.602\times10^{-19}\ \text{C})(0.300\ \text{T})/[2\pi(1.67\times10^{-27}\ \text{kg})] = 4.58\times10^6\ \text{Hz} = 4.58$ MHz.

For an alpha particle, $q = 2e$ and $m \approx 4m_p$, so q/m for an alpha particle is $(2e)/(4m_p) = \frac{1}{2}$ of what it is for a proton. Therefore $f_\alpha = \frac{1}{2}f_p = 2.3$ MHz.

For an alpha particle, $q = 2e$ and $m = 4(1836)m_e$, so q/m for an alpha particle is $2/[4(1836)] = 1/[2(1836)]$ what it is for an electron. Therefore $f_\alpha = \frac{1}{2(1836)}f_e = \frac{1}{3672}f_e = 2.3$ MHz.

(c) $R = mv/qB$ gives $v = RqB/m = (0.120\ \text{m})\,(3.2\times10^{-19}\ \text{C})(0.300\ \text{T})/(6.65\times10^{-27}\ \text{kg}) = 1.73\times10^6$ m/s.

$K = \frac{1}{2}mv^2 = (1/2)(6.65\times10^{-27}\ \text{kg})(1.73\times10^6\ \text{m/s})^2 = 1.0\times10^{-14}\ \text{J} = 6.25\times10^5\ \text{eV} = 625\ \text{keV} = 0.625$ MeV.

EVALUATE: We could use $v = R\omega$ to find v in part (c), where $\omega = 2\pi f$.

27.79. IDENTIFY: We want the magnetic moment of a spinning spherical shell.

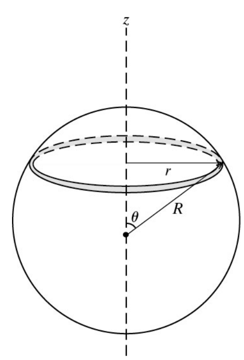


Figure 27.79

SET UP and **EXECUTE:** **(a)** See Fig. 27.79. Note that $r = R\sin\theta$.

(b) $dI = \sigma v dW = \sigma v R d\theta$. $v = r\omega$ so $dI = \sigma v R d\theta = \sigma(R\sin\theta)\omega R d\theta = \sigma\omega R^2 \sin\theta d\theta$.

(c) $d\mu = AdI$, where $A = \pi r^2 = \pi(R\sin\theta)^2$ and dI is what we found in part (b). The result is $d\mu = AdI = \pi(R\sin\theta)^2\left(\sigma\omega R^2 \sin\theta d\theta\right) = \pi R^4 \sigma\omega \sin^3\theta d\theta$.

(d) $\mu = \int_0^\pi \pi\sigma\omega R^4 \sin^3\theta d\theta = \pi\sigma\omega R^4 \int_0^\pi \sin\theta(1-\cos^2\theta)d\theta$. The integrals are fairly straightforward. The result is $\mu = \frac{4}{3}\pi\sigma\omega R^4$. Substituting $\sigma = Q/4\pi R^2$ gives $\mu = \frac{Q\omega R^2}{3}$. The right-hand rule tells us that it points in the $+z$-direction, so $\vec{\boldsymbol{\mu}} = \frac{Q\omega R^2}{3}\hat{\boldsymbol{k}}$.

(e) $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{B} = \frac{Q\omega R^2}{3}\hat{\boldsymbol{k}} \times \left(\sin\alpha\hat{\boldsymbol{i}} + \cos\alpha\hat{\boldsymbol{j}}\right)B = \frac{Q\omega BR^2}{3}\left(\sin\alpha\hat{\boldsymbol{j}} - \cos\alpha\hat{\boldsymbol{i}}\right)B$.

EVALUATE It helps to visualize a spinning spherical shell as a series of parallel concentric current loops of different sizes.

27.80. **IDENTIFY:** This problem involves the magnetic force on an electron moving in the earth's magnetic field.

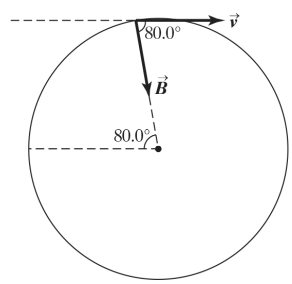


Figure 27.80a

SET UP: Fig. 27.80a shows the geometry of the motion and magnetic field.

EXECUTE: **(a)** We want the radius of the helix. The circular motion is due to the component of the velocity perpendicular to the magnetic field, which is $v_\perp = v\sin\phi$ with $\phi = 80.0°$. Therefore $R = \frac{mv_\perp}{|q|B} = \frac{mv\sin\phi}{eB} =$ 15.0 cm using the given numbers.

(b) We want the speed at which the electron approaches the surface of the earth. This is $v\cos\phi =$ (400 km/s)(cos 80.0°) = 69.5 km/s.

(c) By the right-hand rule, a positive charge would deflect clockwise, but electrons are negative so they deflect in a *counterclockwise* direction, as shown in Fig. 27.80b.

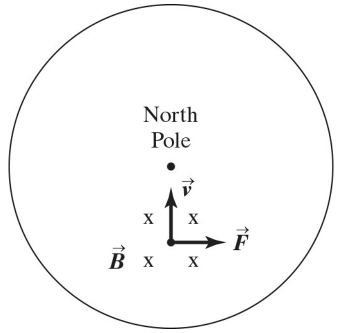


Figure 27.80b

(d) We want the frequency. Using the given numbers we get $f = \dfrac{qB}{2\pi m} = 419$ kHz.

(e) We want the new speed. Energy conservation gives $K_1 + W_{\text{E-field}} = K_2$. This gives $\frac{1}{2}mv_1^2 + eEd = \frac{1}{2}mv_2^2$. Solving for v_2 and using $E = 0.020$ V/m, $d = 100$ km, and $v_1 = 400$ km/s, we get $v_2 = \sqrt{v_1^2 + \dfrac{2eEd}{m}} = 2.65\times10^4$ km/s.

(f) $\dfrac{K_2}{K_1} = \dfrac{1/2\,mv_2^2}{1/2\,mv_1^2} = \left(\dfrac{v_2}{v_1}\right)^2 = \left(\dfrac{26{,}500 \text{ km/s}}{4000 \text{ km/s}}\right)^2 = 4390.$

EVALUATE: The electrons and protons trapped by Earth's magnetic field form the Van Allen radiation belts.

27.81. **IDENTIFY** and **SET UP:** In the magnetic field, $R = \dfrac{mv}{qB}$. Once the particle exits the field it travels in a straight line. Throughout the motion the speed of the particle is constant.

EXECUTE: **(a)** $R = \dfrac{mv}{qB} = \dfrac{(3.20\times10^{-11}\text{ kg})(1.45\times10^5\text{ m/s})}{(2.15\times10^{-6}\text{ C})(0.420\text{ T})} = 5.14$ m.

(b) See Figure 27.81. The distance along the curve, d, is given by $d = R\theta$. $\sin\theta = \dfrac{0.25\text{ m}}{5.14\text{ m}}$, so $\theta = 2.79° = 0.0486$ rad. $d = R\theta = (5.14\text{ m})(0.0486\text{ rad}) = 0.25$ m. And $t = \dfrac{d}{v} = \dfrac{0.25\text{ m}}{1.45\times10^5\text{ m/s}} = 1.72\times10^{-6}$s.

(c) $\Delta x_1 = d\tan(\theta/2) = (0.25\text{ m})\tan(2.79°/2) = 6.08\times10^{-3}$ m.

(d) $\Delta x = \Delta x_1 + \Delta x_2$, where Δx_2 is the horizontal displacement of the particle from where it exits the field region to where it hits the wall. $\Delta x_2 = (0.50\text{ m})\tan 2.79° = 0.0244$ m. Therefore, $\Delta x = 6.08\times10^{-3}\text{ m} + 0.0244\text{ m} = 0.0305$ m.

EVALUATE: d is much less than R, so the horizontal deflection of the particle is much smaller than the distance it travels in the y direction.

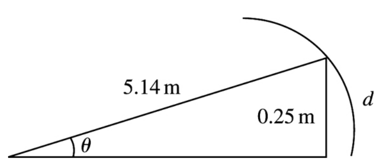


Figure 27.81

27.82. **IDENTIFY:** The electric and magnetic fields exert forces on the moving charge. The work done by the electric field equals the change in kinetic energy. At the top point, $a_y = \frac{v^2}{R}$ and this acceleration must correspond to the net force.

SET UP: The electric field is uniform so the work it does for a displacement y in the y direction is $W = Fy = qEy$. At the top point, $\vec{F}_B$ is in the $-y$-direction and $\vec{F}_E$ is in the $+y$-direction.

EXECUTE: **(a)** The maximum speed occurs at the top of the cycloidal path, and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y = 0$, the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y-direction of the particle, leading to the repeated motion.

(b) $W = qEy = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{2qEy}{m}}$.

(c) At the top, $F_y = qE - qvB = -\frac{mv^2}{R} = -\frac{m}{2y}\frac{2qEy}{m} = -qE$. $2qE = qvB$ and $v = \frac{2E}{B}$.

EVALUATE: The speed at the top depends on B because B determines the y-displacement and the work done by the electric force depends on the y-displacement.

27.83. **IDENTIFY** and **SET UP:** The torque on a magnetic moment is $\tau = \mu B \sin\phi$.

EXECUTE: $\tau = \mu B \sin\phi = (1.4\times10^{-26}\text{ J/T})(2\text{ T})(\sin 90°) = 2.8\times10^{-26}\text{ N}\cdot\text{m}$, which is choice (c).

EVALUATE: The value we have found is the maximum torque. It could be less, depending on the orientation of the proton relative to the magnetic field.

27.84. **IDENTIFY** and **SET UP:** For the nucleus to have a net magnetic moment, it must have an odd number of protons and neutrons.

EXECUTE: Only $^{31}\text{P}_{15}$ has an odd number of protons and neutrons, so choice (d) is correct.

EVALUATE: All the other choices have an even number of protons and an even number of neutrons.

27.85. **IDENTIFY** and **SET UP:** Model the nerve as a current-carrying bar in a magnetic field. The resistance of the nerve is $R = \frac{\rho L}{A}$, the current through it is $I = V/R$ (by Ohm's law), and the maximum magnetic force on it is $F = ILB$.

EXECUTE: The resistance is $R = \frac{\rho L}{A} = (0.6\ \Omega\cdot\text{m})(0.001\text{ m})/[\pi(0.0015/2\text{ m})^2] = 340\ \Omega$.

The current is $I = V/R = (0.1\text{ V})/(340\ \Omega) = 2.9\times10^{-4}\text{ A}$.

The maximum force is $F = ILB = (2.9\times10^{-4}\text{ A})(0.001\text{ m})(2\text{ T}) = 5.9\times10^{-7}\text{ N} \approx 6\times10^{-7}\text{ N}$, which is choice (a).

EVALUATE: This is the force on a 1-mm segment of nerve. The force on the entire nerve would be somewhat larger, depending on the length of the nerve.

28

SOURCES OF MAGNETIC FIELD

VP28.2.1. **IDENTIFY:** We want to calculate the magnetic field due to a moving charged particle.

SET UP: $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$, $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$. Our target variable is the magnetic field at various points.

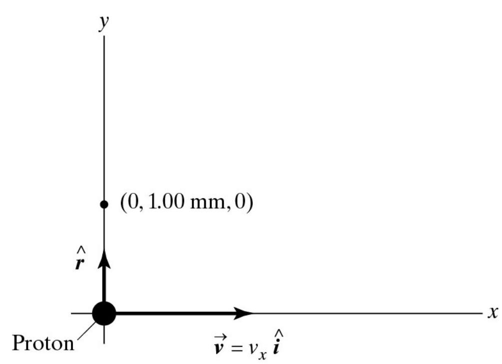


Figure VP28.2.1a

EXECUTE: **(a)** At (0, 1.00 mm, 0). See Fig. 28.2.1a. Use $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$ with $q = e$, $\phi = 90°$, and the given values for v and r. By the right-hand rule, the direction is along the $+z$ axis. The result is $\vec{B} = 3.20\times10^{-15}\ \text{T}\ \hat{i}$.

(b) At (0, 0, 2.00 mm). The approach is the same as in (a) except r = 2.00 mm. The cross product is in the $-y$-direction, so we get $\vec{B} = -8.00\times10^{-16}\ \text{T}\ \hat{j}$.

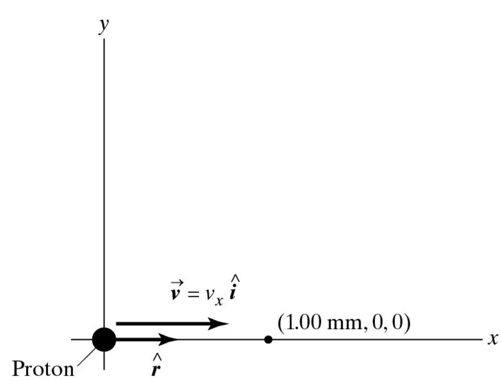


Figure VP28.2.1b

(c) At (1.00 mm, 0, 0). As Fig. VP28.2.1b shows, $\phi = 0$, so $B = 0$.

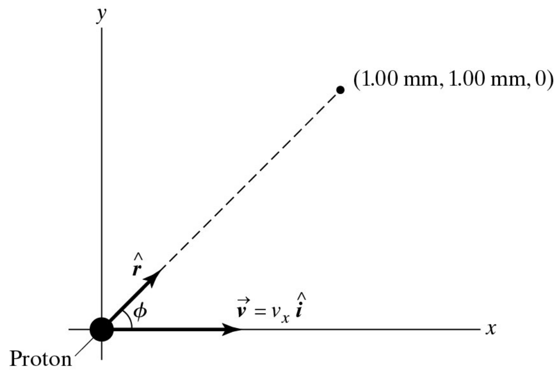


Figure VP28.2.1c

(d) At (1.00 mm, 1.00 mm, 0). See Fig. VP28.2.1c. Use $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$ with $\phi = 45.0°$ and $r = \sqrt{2}$ mm. The direction is along the $+z$-axis, so $\vec{B} = 1.13\times10^{-15}\ \text{T}\ \hat{k}$.

EVALUATE: As we see, the magnetic fields produced by individual charges are very small.

VP28.2.2. **IDENTIFY:** We want to find the magnetic force between two moving charged particles.

SET UP: Our target variable is the force that the proton exerts on the electron and that the electron exerts on the proton. $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$, $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$, $\vec{F} = q\vec{v}\times\vec{B}$, $F = |q|vB\sin\phi$.

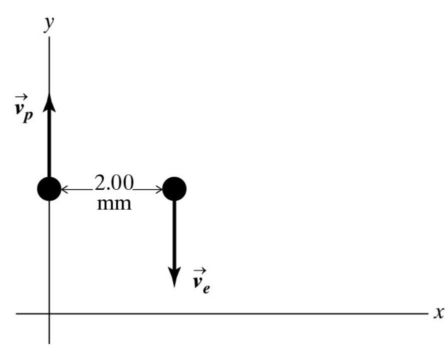


Figure VP28.2.2

EXECUTE: **(a)** We want the force that the proton exerts on the electron. See Fig. VP28.2.2. $\vec{F}_{\text{on e}} = q_e\vec{v}_e\times\vec{B}_p$, where $\vec{B}_p = \dfrac{\mu_0}{4\pi}\dfrac{q_p\vec{v}_p\times\hat{r}}{r^2}$. Putting these quantities together gives $F_{\text{on e}} = (ev)\left(\dfrac{\mu_0 ve}{4\pi r^2}\right) = \dfrac{\mu_0}{4\pi}\dfrac{v^2e^2}{r^2}$. The field due to the proton points in the $-z$-direction by the right-hand rule. The velocity of the electron is in the $-y$-direction, so the force on the electron (which is *negatively charged*) is in the $-x$-direction. Using $r = 2.00$ mm, $\phi = 90°$, and $v_e = 420$ km/s gives $F_{\text{on e}} = 1.13\times10^{-28}$ N in the $-x$-direction, or $-\hat{i}$ direction.

(b) We want the force that the electron exerts on the proton. Follow exactly the same approach as in part (a). The force on the proton has the same magnitude as the force on the electron but it points in the opposite direction. So $F_{\text{on p}} = 1.13\times10^{-28}$ N in the $+\hat{i}$ direction.

EVALUATE: The two forces are equal but opposite, which agrees with Newton's third law (action-reaction). Also note that the two charges attract each other, but this is *not* the same as the Coulomb force. That force would be $F = \dfrac{1}{4\pi \epsilon_0}\dfrac{e^2}{r^2} = \; = 2.30\times 10^{-22}$ N, which is roughly 2 million times stronger than the magnetic attraction.

VP28.2.3. **IDENTIFY:** We are looking at the magnetic field due to a very small wire segment.

SET UP: $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\hat{l}\times\hat{r}}{r^2}$.

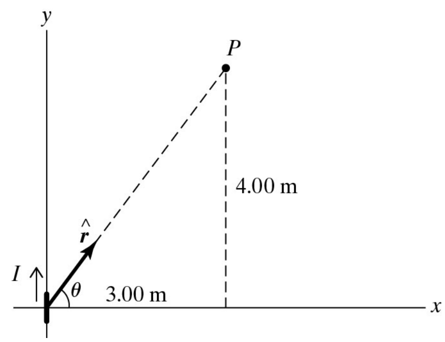


Figure VP28.2.3

EXECUTE: **(a)** We want $\hat{r}$. See Fig. VP28.2.3. We see that $\theta = \arctan(4.00/3.00) = 53.13°$. Also $|\hat{r}| = 1$. So $\hat{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j} = \; = \cos 53.13°\,\hat{i} + \sin 53.13°\,\hat{j} = \; 0.600\,\hat{i} + 0.800\,\hat{j}$.

(b) We want $\vec{B}$. Use $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\hat{l}\times\hat{r}}{r^2}$. $d\vec{l} = dl\hat{j} = (0.00200\text{ m})\hat{j}$. Using $\hat{r}$ and $d\vec{l}$ and taking the cross product with the given numbers, we get $d\vec{B} = -2.88\times 10^{-11}\text{ T}\,\hat{k}$.

EVALUATE: This is a very small field, but the wire segment is also very small.

VP28.2.4. **IDENTIFY:** We want to find the magnetic force on a moving charged particle due to a tiny current-carrying wire segment.

SET UP: $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\hat{l}\times\hat{r}}{r^2}$, $\vec{F} = q\vec{v}\times\vec{B}$, $F = |q|vB\sin\phi$. Sketch the situation as in Fig. VP28.2.4.

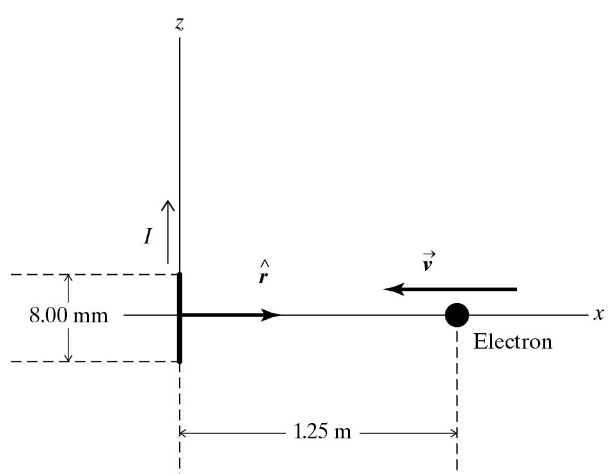


Figure VP28.2.4

EXECUTE: $\vec{F}_{\text{one}} = q_e \vec{v}_e \times \vec{B}_{\text{wire}}$. At the location of the electron, the field due to the wire points into the paper, which is in the –y-direction. Therefore the magnetic force on the electron is in the +z-direction by the right-hand rule (remember that the electron is *negative*). Combining $F = |q|vB\sin\phi$ and $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l}\times\hat{r}}{r^2}$ gives $\vec{F}_{\text{one}} = \dfrac{\mu_0}{4\pi}\dfrac{evdl}{r^2}\hat{k}$. Using the given numbers, we have $\vec{F} = 9.83\times 10^{-23}\text{ N }\hat{k}$.

EVALUATE: Magnetic forces are typically very small compared to electrostatic forces, but not always.

VP28.5.1. IDENTIFY and **SET UP:** We want to find the magnetic field due to a long current-carrying wire. $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$.

EXECUTE: Solve for I and use the given numbers. $I = \dfrac{2\pi rB}{\mu_0} = 2.24\text{ A}$.

EVALUATE: This result is typical of many household currents, so it is clear that they produce small magnetic fields. This one is about 30 μT.

VP28.5.2. IDENTIFY: We want the net magnetic field $\vec{B}$ produced at various points by two long current-carrying wires.

SET UP: $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$. The net field is the vector sum of the two individual fields $\vec{B}_1$ and $\vec{B}_4$. Let the paper be the xy-plane with the currents coming out of the paper, as shown in Fig. VP28.5.2.

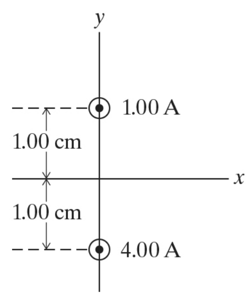


Figure VP28.5.2

EXECUTE: **(a)** At (0, 0, 0). This point is midway between the wires. $\vec{B}_1$ points in the +x-direction and $\vec{B}_4$ points in the –x-direction. $B = B_1 - B_4$. Using $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ we get $B = \dfrac{\mu_0}{2\pi r}(I_1 - I_4)$. Using the given numbers gives $\vec{B} = -6.00\times 10^{-5}\text{ T }\hat{i}$.

(b) At (0, 2.00 cm, 0). This point is on the y-axis 1.00 cm above the 1.00 A wire. Both fields point in the –x-direction. $B = B_1 + B_4$. Using $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ for each wire gives $B = \dfrac{\mu_0}{2\pi}\left(\dfrac{I_1}{r_1} + \dfrac{I_4}{r_4}\right)$ which gives $\vec{B} = -4.67\times 10^{-5}\text{ T }\hat{i}$.

(c) At (0, –2.00 cm, 0). This point is 1.00 cm below the 4.00 A wire. Both fields point in the +x-direction, so $B = B_1 + B_4$ and $B = \dfrac{\mu_0}{2\pi}\left(\dfrac{I_1}{r_1} + \dfrac{I_4}{r_4}\right)$. Using the numbers gives $\vec{B} = 8.67\times 10^{-5}\text{ T }\hat{i}$.

EVALUATE: The net field in (c) is the greatest of the three since this point is closer to the larger current, so this is a reasonable result.

VP28.5.3. **IDENTIFY:** We use the magnetic field due to a very long current-carrying wire.

SET UP: $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$. The net field $\vec{B}$ is the vector sum of the two individual fields $\vec{B}_1$ and $\vec{B}_2$. We know $I_1 = 2.00$ A and the net field, and we want to find I_2. See Fig. VP28.5.3.

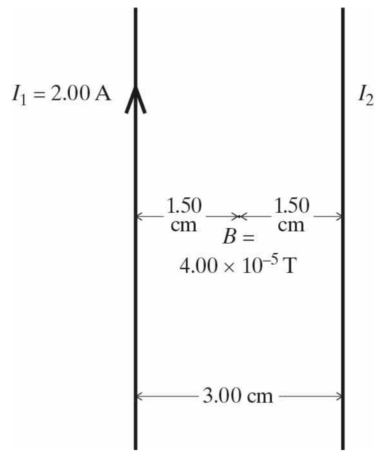


Figure VP28.5.3

EXECUTE: **(a)** Both currents are in the same direction. Their magnetic fields point in opposite directions in the region between the wires, so $B = B_2 - B_1$. Using $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ for each field and solving for I_2 gives $I_2 = \dfrac{Br}{\mu_0/2\pi} + I_1$. Using $r = 1.50$ cm and the other given numbers, we get $I_2 = 5.00$ A.

(b) The currents are in opposite directions. Now both fields point into the paper, so $B = B_2 + B_1$. Using $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ for each field and solving for I_2 gives $I_2 = \dfrac{Br}{\mu_0/2\pi} - I_1$. Using $r = 1.50$ cm and the other given numbers, gives $I_2 = 1.00$ A.

EVALUATE: A clear sketch is important to help decided which way the fields point and if their magnitudes should be added or subtracted.

VP28.5.4. **IDENTIFY:** This problem involves the magnetic force between two long current-carrying wires.

SET UP: $F/L = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{r}$. The target variable is I_2.

EXECUTE: The wires attract each other so their currents must be in the *same* direction, which is *upward*. Solving $F/L = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{r}$ for I_2 gives $I_2 = \dfrac{r(F/L)}{I_1(\mu_0/2\pi)}$. Using the given quantities with $r = 0.900$ m gives $I_2 = 3.29\times 10^4$ A.

EVALUATE: Like currents (i.e. in the same direction) attract while unlike currents (opposite direction) repel.

VP28.10.1. **IDENTIFY:** This problem deals with the magnetic field inside a current-carrying conductor.

SET UP: The target variable is the magnetic field at various distances from the axis. Inside $B = \dfrac{\mu_0 I}{2\pi}\dfrac{r}{R^2}$ and outside $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$.

EXECUTE: **(a)** Inside: Use $B=\frac{\mu_0 I}{2\pi}\frac{r}{R^2}$ with the given quantities, giving I = 4.94 μT.

(b) At the surface: Use either formula with $r = R$. $B=\frac{\mu_0}{2\pi}\frac{I}{R}$ = 8.89 μT.

(c) Outside: Use $B=\frac{\mu_0}{2\pi}\frac{I}{r}$ = 6.67 μT.

EVALUATE: Using $B=\frac{\mu_0 I}{2\pi}\frac{r}{R^2}$ at the surface gives $B=\frac{\mu_0 I}{2\pi}\frac{R}{R^2}=\frac{\mu_0 I}{2\pi R}$, which is the equation for outside the cylinder evaluated when $r = R$. Note that B is largest at the *surface* of the cylinder.

VP28.10.2. **IDENTIFY:** We are dealing with the magnetic field produced by a hollow cylindrical conductor. We will need to use Ampere's law to find the field.

SET UP: Ampere's law is $\oint \vec{B}\cdot d\hat{l}=\mu_0 I_{\text{encl}}$, $J = I/A$. We want to find B at several locations.

EXECUTE: **(a)** For $r < R_1$: Use $\oint \vec{B}\cdot d\hat{l}=\mu_0 I_{\text{encl}}$ and as an integration path use a circle of radius $r < R_1$. From Ampere's law this gives $B(2\pi r)=\mu_0 I_{\text{encl}}=0$, which means that $B = 0$

For $R_1 < r < R_2$: Use the same path as above except $R_1 < r < R_2$. $I_{\text{encl}} = JA_{\text{encl}} =$ $J\left(\pi r^2-\pi R_1^2\right)=J\pi\left(r^2-R_1^2\right)$. Ampere's law becomes $B(2\pi r)=\mu_0 J\pi\left(r^2-R_1^2\right)$. This gives

$B=\frac{\mu_0 J}{2r}\left(r^2-R_1^2\right)$.

For $r > R_2$: Use the same path as before except $r > R_2$. $I_{\text{encl}}=J\pi\left(R_2^2-R_1^2\right)$, so

$B(2\pi r)=\mu_0 J\pi\left(R_2^2-R_1^2\right)$. This gives $B=\frac{\mu_0 J\left(R_2^2-R_1^2\right)}{2r}$.

(b) Where is B a maximum? For $r < R_1$, $B = 0$ so it can't be in that region. For $r > R_2$, B decreases as r increases so it cannot be outside. Investigate $R_1 < r < R_2$. For a maximum, $dB/dr = 0$. Using $B=\frac{\mu_0 J}{2r}\left(r^2-R_1^2\right)$, $\frac{dB}{dr}=\frac{\mu_0 J}{2}\left(1+\frac{R_1^2}{r^2}\right)=0$. There are no real solutions to this equation, so there is no *relative* maximum in this region. But at $r = 0$, $B = 0$ and at $r = R_2$, $B=\frac{\mu_0 J}{2}\left(R_2-\frac{R_1^2}{R_2}\right)$ which is greater than zero. So in this region B keeps increasing as r increases and reaches its maximum value at $r = R_2$. For $r > R_2$, B decreases with r, so B_{max} occurs at $r = R_2$.

EVALUATE: Our result is physically reasonable. As r increases from R_1 to R_2, more and more current contributes to the field, so B increases.

VP28.10.3. **IDENTIFY** and **SET UP:** We have the magnetic field inside a solenoid. $B=\mu_0 nI$. The current is the target variable.

EXECUTE: Solve for I and use the given numbers. $I=\frac{B}{\mu_0 n}$ = 3.18 A.

EVALUATE: This is a reasonable amount of current.

VP28.10.4. **IDENTIFY** and **SET UP:** We have a toroidal solenoid. $B=\frac{\mu_0 NI}{2\pi r}$.

EXECUTE: **(a)** We want N. Solve for N and use the given values. $N=\frac{Br}{I(\mu_0/2\pi)}$ = 833 turns.

(b) We want to find the maximum and minimum field inside the toroidal solenoid. $B_{\max}$ occurs at $r = 6.00$ cm. Evaluate $B = \dfrac{\mu_0 NI}{2\pi r}$ when $r = 6.00$ cm, giving $B_{\max} = 2.33$ mT. $B_{\min}$ occurs when $r = 8.00$ cm. Using this value gives $B_{\min} = 1.75$ mT.

EVALUATE: For an ideal toroidal solenoid, $B = 0$ outisde of it ($r > 8.00$ cm) and inside the opening ($r < 6.00$ cm). For a *real* toroidal solenoid this is not quite true.

28.1. IDENTIFY and **SET UP:** Use $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$ to calculate $\vec{B}$ at each point.

$\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\vec{r}}{r^3}$, since $\hat{r} = \dfrac{\vec{r}}{r}$.

$\vec{v} = (8.00\times 10^6\ \text{m/s})\hat{j}$ and $\vec{r}$ is the vector from the charge to the point where the field is calculated.

EXECUTE: (a) $\vec{r} = (0.500\ \text{m})\hat{i},\ r = 0.500\ \text{m}$.

$\vec{v}\times\vec{r} = vr\hat{j}\times\hat{i} = -vr\hat{k}$.

$\vec{B} = -\dfrac{\mu_0}{4\pi}\dfrac{qv}{r^2}\hat{k} = -(1\times 10^{-7}\ \text{T}\cdot\text{m/A})\dfrac{(6.00\times 10^{-6}\ \text{C})(8.00\times 10^6\ \text{m/s})}{(0.500\ \text{m})^2}\hat{k}$.

$\vec{B} = -(1.92\times 10^{-5}\ \text{T})\hat{k}$.

(b) $\vec{r} = -(0.500\ \text{m})\hat{j},\ r = 0.500\ \text{m}$.

$\vec{v}\times\vec{r} = -vr\hat{j}\times\hat{j} = 0$ and $\vec{B} = 0$.

(c) $\vec{r} = (0.500\ \text{m})\hat{k},\ r = 0.500\ \text{m}$.

$\vec{v}\times\vec{r} = vr\hat{j}\times\hat{k} = vr\hat{i}$.

$\vec{B} = (1\times 10^{-7}\ \text{T}\cdot\text{m/A})\dfrac{(6.00\times 10^{-6}\ \text{C})(8.00\times 10^6\ \text{m/s})}{(0.500\ \text{m})^2}\hat{i} = +(1.92\times 10^{-5}\ \text{T})\hat{i}$.

(d) $\vec{r} = -(0.500\ \text{m})\hat{j} + (0.500\ \text{m})\hat{k},\ r = \sqrt{(0.500\ \text{m})^2 + (0.500\ \text{m})^2} = 0.7071\ \text{m}$.

$\vec{v}\times\vec{r} = v(0.500\ \text{m})(-\hat{j}\times\hat{j} + \hat{j}\times\hat{k}) = (4.00\times 10^6\ \text{m}^2/\text{s})\hat{i}$.

$\vec{B} = (1\times 10^{-7}\ \text{T}\cdot\text{m/A})\dfrac{(6.00\times 10^{-6}\ \text{C})(4.00\times 10^6\ \text{m}^2/\text{s})}{(0.7071\ \text{m})^3}\hat{i} = +(6.79\times 10^{-6}\ \text{T})\hat{i}$.

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{r}$. $B = 0$ along the direction of $\vec{v}$.

28.2. IDENTIFY: A moving charge creates a magnetic field as well as an electric field.

SET UP: The magnetic field caused by a moving charge is $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$, and its electric field is

$E = \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{r^2}$ since $q = e$.

EXECUTE: Substitute the appropriate numbers into the above equations.

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2} = \frac{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times 10^{-19}\text{C})(2.2\times 10^6\text{m/s})\sin 90^\circ}{(5.3\times 10^{-11}\text{m})^2} = 13\ \text{T},\ \text{out of the page.}$$

$$E = \frac{1}{4\pi_0}\frac{e}{r^2} = \frac{(9.00\times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19}\,\text{C})}{(5.3\times 10^{-11}\text{m})^2} = 5.1\times 10^{11}\,\text{N/C},\ \text{toward the electron.}$$

EVALUATE: There are enormous fields within the atom!

28.3. IDENTIFY: A moving charge creates a magnetic field.

SET UP: The magnetic field due to a moving charge is $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$.

EXECUTE: Substituting numbers into the above equation gives

(a) $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2} = \dfrac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\dfrac{(1.6\times10^{-19}\text{C})(3.0\times10^7\,\text{m/s})\sin30°}{(2.00\times10^{-6}\text{m})^2}.$

$B = 6.00\times10^{-8}$ T, out of the paper, and it is the same at point B.

(b) $B = (1.00\times10^{-7}\ \text{T}\cdot\text{m/A})(1.60\times10^{-19}\ \text{C})(3.00\times10^7\ \text{m/s})/(2.00\times10^{-6}\ \text{m})^2.$

$B = 1.20\times10^{-7}$ T, out of the page.

(c) $B = 0$ T since $\sin(180°) = 0$.

EVALUATE: Even at high speeds, these charges produce magnetic fields much less than the earth's magnetic field.

28.4. **IDENTIFY:** Both moving charges produce magnetic fields, and the net field is the vector sum of the two fields.

SET UP: Both fields point out of the paper, so their magnitudes add, giving

$$B = B_{\text{alpha}} + B_{\text{el}} = \frac{\mu_0 v}{4\pi r^2}(e\sin40° + 2e\sin140°).$$

EXECUTE: Factoring out an e and putting in the numbers gives

$$B = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times10^{-19}\ \text{C})(2.50\times10^5\ \text{m/s})}{(8.65\times10^{-9}\ \text{m})^2}(\sin40° + 2\sin140°).$$

$B = 1.03\times10^{-4}\ \text{T} = 0.103$ mT, out of the page.

EVALUATE: At distances very close to the charges, the magnetic field is strong enough to be important.

28.5. **IDENTIFY:** Apply $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\vec{r}}{r^3}$.

SET UP: Since the charge is at the origin, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

EXECUTE: **(a)** $\vec{v} = v\vec{i}$, $\vec{r} = r\hat{i}$; $\vec{v}\times\vec{r} = 0$, $B = 0$.

(b) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{j}$; $\vec{v}\times\vec{r} = vr\hat{k}$, $r = 0.500$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^5\ \text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}.$$

q is negative, so $\vec{B} = -(1.31\times10^{-6}\ \text{T})\hat{k}$.

(c) $\vec{v} = v\hat{i}$, $\vec{r} = (0.500\ \text{m})(\hat{i} + \hat{j})$; $\vec{v}\times\vec{r} = (0.500\ \text{m})v\hat{k}$, $r = 0.7071$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(|q||\vec{v}\times\vec{r}|/r^3\right) = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(0.500\ \text{m})(6.80\times10^5\ \text{m/s})}{(0.7071\ \text{m})^3}.$$

$B = 4.62\times10^{-7}$ T. $\vec{B} = -(4.62\times10^{-7}\ \text{T})\hat{k}$.

(d) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{k}$; $\vec{v}\times\vec{r} = -vr\hat{j}$, $r = 0.500$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^5\ \text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}.$$

$\vec{B} = (1.31\times10^{-6}\ \text{T})\hat{j}$.

EVALUATE: In each case, $\vec{B}$ is perpendicular to both $\vec{r}$ and $\vec{v}$.

28.6. IDENTIFY: Apply $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\vec{r}}{r^3}$. For the magnetic force, apply the results of Example 28.1, except here the two charges and velocities are different.

SET UP: In part (a), $r = d$ and $\vec{r}$ is perpendicular to $\vec{v}$ in each case, so $\dfrac{|\vec{v}\times\vec{r}|}{r^3} = \dfrac{v}{d^2}$. For calculating the force between the charges, $r = 2d$.

EXECUTE: **(a)** $B_{\text{total}} = B + B' = \dfrac{\mu_0}{4\pi}\left(\dfrac{qv}{d^2} + \dfrac{q'v'}{d^2}\right)$.

$$B = \frac{\mu_0}{4\pi}\left(\frac{(8.0\times10^{-6}\ \text{C})(4.5\times10^6\ \text{m/s})}{(0.120\ \text{m})^2} + \frac{(3.0\times10^{-6}\ \text{C})(9.0\times10^6\ \text{m/s})}{(0.120\ \text{m})^2}\right) = 4.38\times10^{-4}\ \text{T}.$$

The direction of $\vec{B}$ is into the page.

(b) Following Example 28.1 we can find the magnetic force between the charges:

$$F_B = \frac{\mu_0}{4\pi}\frac{qq'vv'}{r^2} = (10^{-7}\ \text{T}\cdot\text{m/A})\frac{(8.00\times10^{-6}\ \text{C})(3.00\times10^{-6}\ \text{C})(4.50\times10^6\ \text{m/s})(9.00\times10^6\ \text{m/s})}{(0.240\ \text{m})^2}.$$

$F_B = 1.69\times10^{-3}$ N. The force on the upper charge points up and the force on the lower charge points down. The Coulomb force between the charges is

$F_C = k\dfrac{q_1q_2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(8.0\times10^{-6}\ \text{C})(3.0\times10^{-6}\ \text{C})}{(0.240\ \text{m})^2} = 3.75\ \text{N}$. The force on the upper charge points up and the force on the lower charge points down. The ratio of the Coulomb force to the magnetic force is $\dfrac{F_C}{F_B} = \dfrac{c^2}{v_1v_2} = \dfrac{3.75\ \text{N}}{1.69\times10^{-3}\ \text{N}} = 2.22\times10^3$; the Coulomb force is much larger.

(c) The magnetic forces are reversed in direction when the direction of only one velocity is reversed but the magnitude of the force is unchanged.

EVALUATE: When two charges have the same sign and move in opposite directions, the force between them is repulsive. When two charges of the same sign move in the same direction, the force between them is attractive.

28.7. IDENTIFY: We want the magnetic field due to a moving charge.

SET UP: $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$, $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$.

EXECUTE: **(a)** The unit vector points from the proton to point *P*, which is to the left.

(b) We want *B*. $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$ and ϕ is either 0 or 180°. In either case $\sin\phi = 0$, so $B = 0$.

(c) We want $\vec{B}$. $\phi = 90°$ and $q = e$. Using the given quantities $B = \dfrac{\mu_0}{4\pi}\dfrac{ev}{r^2} = 1.21$ pT. By the right-hand rule, the cross product in $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$ is out of the page, so $\vec{B}$ is out of the page.

(d) For an electron, only the charge and mass are different, so *B* would be the same but the direction would be reversed. So $B = 1.21$ pT, into the page.

EVALUATE: The magnetic field due to a negative charge is opposite to the direction of $\vec{v}\times\hat{r}$.

28.8. IDENTIFY: Both moving charges create magnetic fields, and the net field is the vector sum of the two. The magnetic force on a moving charge is $F_{\text{mag}} = qvB\sin\phi$ and the electrical force obeys Coulomb's law.

SET UP: The magnetic field due to a moving charge is $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$.

EXECUTE: **(a)** Both fields are into the page, so their magnitudes add,

giving $B = B_e + B_p = \dfrac{\mu_0}{4\pi}\left(\dfrac{ev}{r_e^2} + \dfrac{ev}{r_p^2}\right)\sin 90°.$

$$B = \frac{\mu_0}{4\pi}(1.60\times10^{-19}\,\text{C})(735{,}000\ \text{m/s})\left[\frac{1}{(5.00\times10^{-9}\,\text{m})^2} + \frac{1}{(4.00\times10^{-9}\,\text{m})^2}\right].$$

$$B = 1.21\times10^{-3}\ \text{T} = 1.21\ \text{mT, into the page.}$$

(b) Using $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$, where $r = \sqrt{41}$ nm and $\phi = 180° - \arctan(5/4) = 128.7°$, we get

$$B = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.6\times10^{-19}\,\text{C})(735{,}000\ \text{m/s})\sin128.7°}{(\sqrt{41}\times10^{-9}\,\text{m})^2} = 2.24\times10^{-4}\ \text{T, into the page.}$$

(c) $F_{\text{mag}} = qvB\sin90° = (1.60\times10^{-19}\ \text{C})(735{,}000\ \text{m/s})(2.24\times10^{-4}\ \text{T}) = 2.63\times10^{-17}\ \text{N}$, in the $+x$-direction.

$$F_{\text{elec}} = (1/4\pi\epsilon_0)e^2/r^2 = \frac{(9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{C})^2}{(\sqrt{41}\times10^{-9}\,\text{m})^2} = 5.62\times10^{-12}\ \text{N, at } 129°$$

counterclockwise from the $+x$-axis.

EVALUATE: The electric force is over 200,000 times as strong as the magnetic force.

28.9. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\phi}{r^2}$.

EXECUTE: Applying the law of Biot and Savart gives

(a) $dB = \dfrac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\dfrac{(10.0\ \text{A})(0.00110\ \text{m})\sin90°}{(0.0500\ \text{m})^2} = 4.40\times10^{-7}\ \text{T}$, out of the paper.

(b) The same as above, except $r = \sqrt{(5.00\ \text{cm})^2 + (14.0\ \text{cm})^2}$ and $\phi = \arctan(5/14) = 19.65°$, giving $dB = 1.67\times10^{-8}\ \text{T}$, out of the page.

(c) $dB = 0$ since $\phi = 0°$.

EVALUATE: This is a very small field, but it comes from a very small segment of current.

28.10. **IDENTIFY:** Apply the Biot-Savart law.

SET UP: Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\,d\vec{l}\times\vec{r}}{r^3}$. $r = \sqrt{(-0.730\ \text{m})^2 + (0.390\ \text{m})^2} = 0.8276\ \text{m}$.

EXECUTE:

$d\vec{l}\times\vec{r} = [0.500\times10^{-3}\ \text{m}]\hat{j}\times[(-0.730\ \text{m})\hat{i} + (0.390\ \text{m})\hat{k}] = (+3.65\times10^{-4}\ \text{m}^2)\hat{k} + (+1.95\times10^{-4}\ \text{m}^2)\hat{i}.$

$$d\vec{B} = (1.00\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{5.40\ \text{A}}{(0.8276\ \text{m})^3}[(3.65\times10^{-4}\ \text{m}^2)\hat{k} + (1.95\times10^{-4}\ \text{m}^2)\hat{i}].$$

$$d\vec{B} = (1.86\times10^{-10}\ \text{T})\hat{i} + (3.48\times10^{-10}\ \text{T})\hat{k}.$$

EVALUATE: The magnetic field lies in the xz-plane.

28.11. **IDENTIFY and SET UP:** The magnetic field produced by an infinitesimal current element is given by $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\vec{l}\times\hat{r}}{r^2}$.

As in Example 28.2, use $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\vec{l}\times\hat{r}}{r^2}$ for the finite 0.500-mm segment of wire since the $\Delta l = 0.500\text{-mm}$ length is much smaller than the distances to the field points.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l} \times \vec{r}}{r^3}$$

I is in the $+z$-direction, so $\Delta\vec{l} = (0.500\times10^{-3}\ \text{m})\hat{k}$.

EXECUTE: **(a)** The field point is at $x = 2.00$ m, $y = 0$, $z = 0$ so the vector $\vec{r}$ from the source point (at the origin) to the field point is $\vec{r} = (2.00\ \text{m})\hat{i}$.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\ \text{m})(2.00\ \text{m})\hat{k}\times\hat{i} = +(1.00\times10^{-3}\ \text{m}^2)\hat{j}$.

$$\vec{B} = \frac{(1\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})(1.00\times10^{-3}\ \text{m}^2)}{(2.00\ \text{m})^3}\hat{j} = (5.00\times10^{-11}\ \text{T})\hat{j}.$$

(b) $\vec{r} = (2.00\ \text{m})\hat{j}$, $r = 2.00$ m.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\ \text{m})(2.00\ \text{m})\hat{k}\times\hat{j} = -(1.00\times10^{-3}\ \text{m}^2)\hat{i}$.

$$\vec{B} = \frac{(1\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})(-1.00\times10^{-3}\ \text{m}^2)}{(2.00\ \text{m})^3}\hat{i} = -(5.00\times10^{-11}\ \text{T})\hat{i}.$$

(c) $\vec{r} = (2.00\ \text{m})(\hat{i}+\hat{j})$, $r = \sqrt{2}(2.00\ \text{m})$.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\ \text{m})(2.00\ \text{m})\hat{k}\times(\hat{i}+\hat{j}) = (1.00\times10^{-3}\ \text{m}^2)(\hat{j}-\hat{i})$.

$$\vec{B} = \frac{(1\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})(1.00\times10^{-3}\ \text{m}^2)}{\left[\sqrt{2}(2.00\ \text{m})\right]^3}(\hat{j}-\hat{i}) = (-1.77\times10^{-11}\ \text{T})(\hat{i}-\hat{j}).$$

(d) $\vec{r} = (2.00\ \text{m})\hat{k}$, $r = 2.00$ m.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\ \text{m})(2.00\ \text{m})\hat{k}\times\hat{k} = 0$; $\vec{B} = 0$.

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{r}$ and $\Delta\vec{l}$. $B = 0$ along the length of the wire.

28.12. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\phi}{r^2}$.

Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the law of Biot and Savart for the 12.0-A current

gives $dB = \dfrac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\dfrac{(12.0\ \text{A})(0.00150\ \text{m})\left(\dfrac{2.50\ \text{cm}}{8.00\ \text{cm}}\right)}{(0.0800\ \text{m})^2} = 8.79\times10^{-8}\ \text{T}.$

The field from the 24.0-A segment is twice this value, so the total field is 2.64×10^{-7} T, into the page.

EVALUATE: The rest of each wire also produces field at P. We have calculated just the field from the two segments that are indicated in the problem.

28.13. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\phi}{r^2}$. Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the Biot and Savart law, where $r = \frac{1}{2}\sqrt{(3.00\ \text{cm})^2 + (3.00\ \text{cm})^2} = 2.121\ \text{cm}$, we have $dB = 2\dfrac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\dfrac{(28.0\ \text{A})(0.00200\ \text{m})\sin 45.0^\circ}{(0.02121\ \text{m})^2} = 1.76\times10^{-5}\ \text{T}$, into the paper.

EVALUATE: Even though the two wire segments are at right angles, the magnetic fields they create are in the same direction.

28.14. **IDENTIFY:** This problem involves the magnetic field due to a long wire and the force on a charge moving in that field.

SET UP: $B = \frac{\mu_0}{2\pi}\frac{I}{r}$, $\vec{F} = q\vec{v}\times\vec{B}$, $\sum F_x = ma_x$, $\sum F_y = ma_y$. We want the velocity components of the particle. The given acceleration is so large that we can neglect the effects of gravity.

EXECUTE: The magnetic field at the location of the particle (call this point P) points in the $+z$-direction. At that point we use the given quantities and $B = \frac{\mu_0}{2\pi}\frac{I}{r}$ to find $B = 0.150$ mT.

Applying $\sum F_x = ma_x$ gives $qv_yB = ma_x$, so $v_y = \frac{ma_x}{qB}$. Using the given values we get $v_y = -12.5$ km/s. Now apply $\sum F_y = ma_y$. $qv_xB = ma_y$, which gives $v_x = \frac{ma_y}{qB}$. Using the given values gives $v_x = -22.5$ km/s.

EVALUATE: Notice that the x component of the velocity affects the y component of the acceleration and likewise v_y affects a_x.

28.15. **IDENTIFY:** We can model the lightning bolt and the household current as very long current-carrying wires.

SET UP: The magnetic field produced by a long wire is $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: Substituting the numerical values gives

(a) $B = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(20{,}000\ \text{A})}{2\pi(5.0\ \text{m})} = 8\times10^{-4}\ \text{T}$.

(b) $B = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(10\ \text{A})}{2\pi(0.050\ \text{m})} = 4.0\times10^{-5}\ \text{T}$.

EVALUATE: The field from the lightning bolt is about 20 times as strong as the field from the household current.

28.16. **IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: First find the current: $I = (8.20\times10^{18}\ \text{el/s})(1.60\times10^{-19}\ \text{C/el}) = 1.312\ \text{A}$.

Now find the magnetic field: $\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1.312\ \text{A})}{2\pi(0.0400\ \text{m})} = 6.56\times10^{-6}\ \text{T} = 6.56\ \mu\text{T}$.

Since electrons are negative, the conventional current runs from east to west, so the magnetic field above the wire points toward the north.

EVALUATE: This magnetic field is much less than that of the earth, so any experiments involving such a current would have to be shielded from the earth's magnetic field, or at least would have to take it into consideration.

28.17. **IDENTIFY:** We can model the current in the heart as that of a long straight wire. It produces a magnetic field around it.

SET UP: For a long straight wire, $B = \frac{\mu_0 I}{2\pi r}$. $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$. $1\ \text{gauss} = 10^{-4}\ \text{T}$.

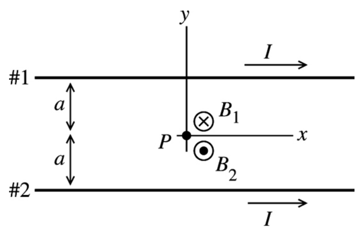
EXECUTE: Solving for the current gives

$$I = \frac{2\pi rB}{\mu_0} = \frac{2\pi(0.050\ \text{m})(1.0\times10^{-9}\ \text{T})}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} = 25\times10^{-5}\ \text{A} = 250\ \mu\text{A}.$$

EVALUATE: By household standards, this is a very small current. But the magnetic field around the heart $(\approx 10\ \mu\text{G})$ is also very small.

28.18. **IDENTIFY:** For each wire $B = \dfrac{\mu_0 I}{2\pi r}$, and the direction of $\vec{B}$ is given by the right-hand rule (Figure 28.6 in the textbook). Add the field vectors for each wire to calculate the total field.

(a) SET UP: The two fields at this point have the directions shown in Figure 28.18a.

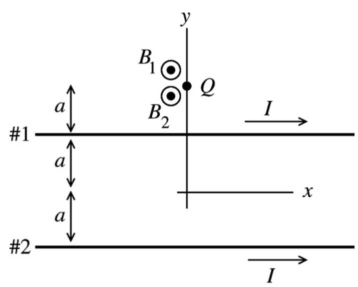


EXECUTE: At point P midway between the two wires the fields $\vec{B}_1$ and $\vec{B}_2$ due to the two currents are in opposite directions, so $B = B_2 - B_1$.

Figure 28.18a

But $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi a}$, so $B = 0$.

(b) SET UP: The two fields at this point have the directions shown in Figure 28.18b.



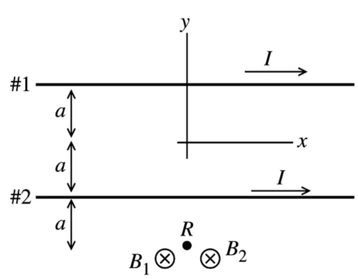
EXECUTE: At point Q above the upper wire $\vec{B}_1$ and $\vec{B}_2$ are both directed out of the page (+z-direction), so $B = B_1 + B_2$.

Figure 28.18b

$$B_1 = \frac{\mu_0 I}{2\pi a},\ B_2 = \frac{\mu_0 I}{2\pi(3a)}.$$

$$B = \frac{\mu_0 I}{2\pi a}\left(1 + \tfrac{1}{3}\right) = \frac{2\mu_0 I}{3\pi a};\ \vec{B} = \frac{2\mu_0 I}{3\pi a}\hat{k}.$$

(c) SET UP: The two fields at this point have the directions shown in Figure 28.18c.



EXECUTE: At point R below the lower wire $\vec{B}_1$ and $\vec{B}_2$ are both directed into the page ($-z$-direction), so $B = B_1 + B_2$.

Figure 28.18c

$$B_1 = \frac{\mu_0 I}{2\pi(3a)},\ B_2 = \frac{\mu_0 I}{2\pi a}.$$

$B_1 = \dfrac{\mu_0 I}{2\pi a}\left(1+\tfrac{1}{3}\right) = \dfrac{2\mu_0 I}{3\pi a}$; $\vec{\boldsymbol{B}} = -\dfrac{2\mu_0 I}{3\pi a}\hat{\boldsymbol{k}}$.

EVALUATE: In the figures we have drawn, $\vec{\boldsymbol{B}}$ due to each wire is out of the page at points above the wire and into the page at points below the wire. If the two field vectors are in opposite directions the magnitudes subtract.

28.19. **IDENTIFY:** The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field.

SET UP: For the wire, $B_{\text{wire}} = \dfrac{\mu_0 I}{2\pi r}$ and the direction of B_{wire} is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook. $\vec{\boldsymbol{B}}_0 = (1.50\times 10^{-6}\ \text{T})\hat{\boldsymbol{i}}$.

EXECUTE: **(a)** At (0, 0, 1 m), $\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_0 - \dfrac{\mu_0 I}{2\pi r}\hat{\boldsymbol{i}} = (1.50\times 10^{-6}\ \text{T})\hat{\boldsymbol{i}} - \dfrac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{\boldsymbol{i}} = -(1.0\times 10^{-7}\ \text{T})\hat{\boldsymbol{i}}$.

(b) At (1 m, 0, 0), $\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_0 + \dfrac{\mu_0 I}{2\pi r}\hat{\boldsymbol{k}} = (1.50\times 10^{-6}\ \text{T})\hat{\boldsymbol{i}} + \dfrac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{\boldsymbol{k}}$.

$\vec{\boldsymbol{B}} = (1.50\times 10^{-6}\ \text{T})\hat{\boldsymbol{i}} + (1.6\times 10^{-6}\ \text{T})\hat{\boldsymbol{k}} = 2.19\times 10^{-6}\ \text{T}$, at $\theta = 46.8°$ from x to z.

(c) At $(0, 0, -0.25\ \text{m})$, $\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_0 + \dfrac{\mu_0 I}{2\pi r}\hat{\boldsymbol{i}} = (1.50\times 10^{-6}\ \text{T})\hat{\boldsymbol{i}} + \dfrac{\mu_0(8.00\ \text{A})}{2\pi(0.25\ \text{m})}\hat{\boldsymbol{i}} = (7.9\times 10^{-6}\ \text{T})\hat{\boldsymbol{i}}$.

EVALUATE: At point c the two fields are in the same direction and their magnitudes add. At point a they are in opposite directions and their magnitudes subtract. At point b the two fields are perpendicular.

28.20. **IDENTIFY:** The magnetic field is that of a long current-carrying wire.

SET UP: $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{(2.0\times 10^{-7}\ \text{T}\cdot\text{m/A})(150\ \text{A})}{8.0\ \text{m}} = 3.8\times 10^{-6}\ \text{T}$. This is 7.5% of the earth's field.

EVALUATE: Since this field is much smaller than the earth's magnetic field, it would be expected to have less effect than the earth's field.

28.21. **IDENTIFY:** $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{\boldsymbol{B}}$ is given by the right-hand rule.

SET UP: Call the wires a and b, as indicated in Figure 28.21. The magnetic fields of each wire at points P_1 and P_2 are shown in Figure 28.21a. The fields at point 3 are shown in Figure 28.21b.

EXECUTE: **(a)** At P_1, $B_a = B_b$ and the two fields are in opposite directions, so the net field is zero.

(b) $B_a = \dfrac{\mu_0 I}{2\pi r_a}$. $B_b = \dfrac{\mu_0 I}{2\pi r_b}$. $\vec{\boldsymbol{B}}_a$ and $\vec{\boldsymbol{B}}_b$ are in the same direction so

$$B = B_a + B_b = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_a} + \frac{1}{r_b}\right) = \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})}{2\pi}\left[\frac{1}{0.300\ \text{m}} + \frac{1}{0.200\ \text{m}}\right] = 6.67\times 10^{-6}\ \text{T}.$$

$\vec{\boldsymbol{B}}$ has magnitude $6.67\ \mu\text{T}$ and is directed toward the top of the page.

(c) In Figure 28.21b, $\vec{\boldsymbol{B}}_a$ is perpendicular to $\vec{\boldsymbol{r}}_a$ and $\vec{\boldsymbol{B}}_b$ is perpendicular to $\vec{\boldsymbol{r}}_b$. $\tan\theta = \dfrac{5\ \text{cm}}{20\ \text{cm}}$ and $\theta = 14.04°$. $r_a = r_b = \sqrt{(0.200\ \text{m})^2 + (0.050\ \text{m})^2} = 0.206\ \text{m}$ and $B_a = B_b$.

$$B = B_a\cos\theta + B_b\cos\theta = 2B_a\cos\theta = 2\left(\frac{\mu_0 I}{2\pi r_a}\right)\cos\theta = \frac{2(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(4.0\ \text{A})\cos 14.04°}{2\pi(0.206\ \text{m})} = 7.54\ \mu\text{T}$$

B has magnitude $7.53\ \mu\text{T}$ and is directed to the left.

EVALUATE: At points directly to the left of both wires the net field is directed toward the bottom of the page.

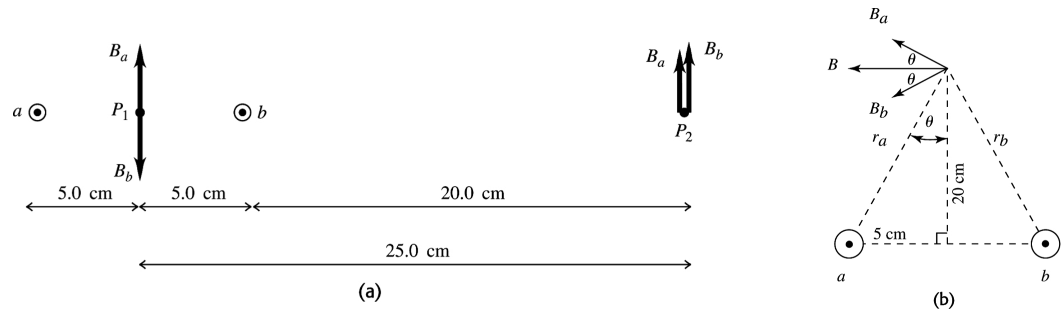


Figure 28.21

28.22. **IDENTIFY:** Each segment of the rectangular loop creates a magnetic field at the center of the loop, and all these fields are in the same direction.

SET UP: The field due to each segment is $B = \dfrac{\mu_0 I}{4\pi}\dfrac{2a}{x\sqrt{x^2+a^2}}$. $\vec{B}$ is into paper so I is clockwise around the loop.

EXECUTE: <u>Long sides:</u> $a = 4.75$ cm. $x = 2.10$ cm. For the two long sides,

$$B = 2(1.00\times10^{-7}\ \text{T}\cdot\text{m/A})I\frac{2(4.75\times10^{-2}\ \text{m})}{(2.10\times10^{-2}\ \text{m})\sqrt{(0.0210\ \text{m})^2+(0.0475\ \text{m})^2}} = (1.742\times10^{-5}\ \text{T/A})I.$$

<u>Short sides</u>: $a = 2.10$ cm. $x = 4.75$ cm. For the two short sides,

$$B = 2(1.00\times10^{-7}\ \text{T}\cdot\text{m/A})I\frac{2(2.10\times10^{-2}\ \text{m})}{(4.75\times10^{-2}\ \text{m})\sqrt{(0.0475\ \text{m})^2+(0.0210\ \text{m})^2}} = (3.405\times10^{-6}\ \text{T/A})I.$$

Using the known field, we have $B = (2.082\times10^{-5}\ \text{T/A})I = 5.50\times10^{-5}$ T, which gives $I = 2.64$ A.

EVALUATE: This is a typical household current, yet it produces a magnetic field which is about the same as the earth's magnetic field.

28.23. **IDENTIFY:** The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

SET UP: For each wire, $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook.

EXECUTE: **(a)** and **(b)** $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

(c) The fields due to each wire are sketched in Figure 28.23.

$$B = B_a\cos45° + B_b\cos45° + B_c\cos45° + B_d\cos45° = 4B_a\cos45° = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos45°.$$

$r = \sqrt{(10\ \text{cm})^2+(10\ \text{cm})^2} = 10\sqrt{2}\ \text{cm} = 0.10\sqrt{2}$ m, so

$$B = 4\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(100\ \text{A})}{2\pi(0.10\sqrt{2}\ \text{m})}\cos45° = 4.0\times10^{-4}\ \text{T, to the left.}$$

EVALUATE: In part (c), if all four currents are reversed in direction, the net field at the center of the square would be to the right.

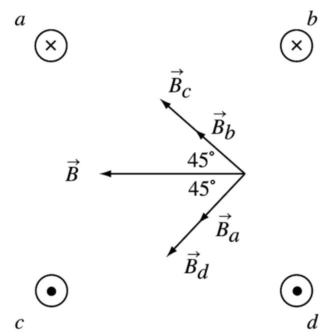
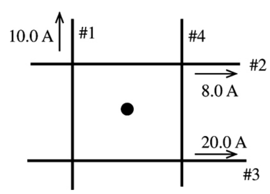


Figure 28.23

28.24. **IDENTIFY:** Use $B = \dfrac{\mu_0 I}{2\pi r}$ and the right-hand rule to determine the field due to each wire. Set the sum of the four fields equal to zero and use that equation to solve for the field and the current of the fourth wire.

SET UP: The three known currents are shown in Figure 28.24.



$\vec{B}_1 \otimes, \vec{B}_2 \otimes, \vec{B}_3 \odot$

$B = \dfrac{\mu_0 I}{2\pi r}$; $r = 0.200$ m for each wire.

Figure 28.24

EXECUTE: Let $\odot$ be the positive z-direction. $I_1 = 10.0\text{ A}, I_2 = 8.0\text{ A}, I_3 = 20.0\text{ A}$. Then

$B_1 = 1.00\times10^{-5}\text{ T},\ B_2 = 0.80\times10^{-5}\text{ T},\ \text{and}\ B_3 = 2.00\times10^{-5}\text{ T}.$

$B_{1z} = -1.00\times10^{-5}\text{ T}, B_{2z} = -0.80\times10^{-5}\text{ T}, B_{3z} = +2.00\times10^{-5}\text{ T}.$

$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0.$

$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0\times10^{-6}\text{ T}.$

To give $\vec{B}_4$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r} \text{ so } I_4 = \frac{rB_4}{(\mu_0/2\pi)} = \frac{(0.200\text{ m})(2.0\times10^{-6}\text{ T})}{(2\times10^{-7}\text{ T}\cdot\text{m/A})} = 2.0\text{ A}.$$

EVALUATE: The fields of wires #2 and #3 are in opposite directions and their net field is the same as due to a current $20.0\text{ A} - 8.0\text{ A} = 12.0\text{ A}$ in one wire. The field of wire #4 must be in the same direction as that of wire #1, and $10.0\text{ A} + I_4 = 12.0\text{ A}$.

28.25. **IDENTIFY:** The net magnetic field at any point is the vector sum of the magnetic fields of the two wires.

SET UP: For each wire $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is determined by the right-hand rule described in the text. Let the wire with 12.0 A be wire 1 and the wire with 10.0 A be wire 2.

EXECUTE: **(a)** Point Q: $B_1 = \dfrac{\mu_0 I_1}{2\pi r_1} = \dfrac{(4\pi\times10^{-7}\text{T}\cdot\text{m/A})(12.0\text{ A})}{2\pi(0.15\text{ m})} = 1.6\times10^{-5}\text{ T}.$

The direction of $\vec{B}_1$ is out of the page. $B_2 = \dfrac{\mu_0 I_2}{2\pi r_2} = \dfrac{(4\pi\times10^{-7}\text{T}\cdot\text{m/A})(10.0\text{ A})}{2\pi(0.080\text{ m})} = 2.5\times10^{-5}\text{ T}.$

The direction of $\vec{B}_2$ is out of the page. Since $\vec{B}_1$ and $\vec{B}_2$ are in the same direction,

$B = B_1 + B_2 = 4.1\times10^{-5}\text{ T}$ and $\vec{B}$ is directed out of the page.

Point P: $B_1 = 1.6\times10^{-5}$ T, directed into the page. $B_2 = 2.5\times10^{-5}$ T, directed into the page.

$B = B_1 + B_2 = 4.1\times10^{-5}$ T and $\vec{B}$ is directed into the page.

(b) $\vec{B}_1$ is the same as in part (a), out of the page at Q and into the page at P. The direction of $\vec{B}_2$ is reversed from what it was in (a) so is into the page at Q and out of the page at P.

Point Q: $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions so $B = B_2 - B_1 = 2.5\times10^{-5}\text{ T} - 1.6\times10^{-5}\text{ T} = 9.0\times10^{-6}$ T and $\vec{B}$ is directed into the page.

Point P: $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions so $B = B_2 - B_1 = 9.0\times10^{-6}$ T and $\vec{B}$ is directed out of the page.

EVALUATE: Points P and Q are the same distances from the two wires. The only difference is that the fields point in either the same direction or in opposite directions.

28.26. **IDENTIFY:** This problem involves the magnetic field of a short current-carrying wire.

SET UP: $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ (infinite wire), $B = \dfrac{\mu_0 I}{2\pi}\dfrac{2a}{x\sqrt{x^2+a^2}}$ (finite wire). We want the magnetic field this wire produces at point P(0, 5.00 cm).

EXECUTE: **(a)** Finite length: Use $B = \dfrac{\mu_0 I}{2\pi}\dfrac{2a}{x\sqrt{x^2+a^2}}$ with x = 5.00 cm and a = 10.0 cm. This gives $B = 28.6\ \mu$T.

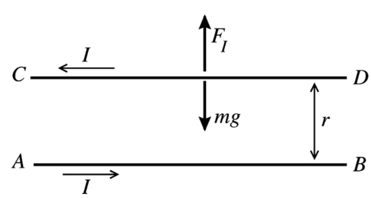
(b) Infinite length: Use $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ with r = 5.00 cm, giving $B = 32.0\ \mu$T.

$$\frac{\Delta B}{B} = \frac{32.0\ \mu\text{T} - 28.6\ \mu\text{T}}{28.6\ \mu\text{T}} = 0.119 = 11.9\%.$$

EVALUATE: Using the less accurate infinite-length approximation gives an easier calculation but a less accurate answer.

28.27. **IDENTIFY:** The wire CD rises until the upward force F_I due to the currents balances the downward force of gravity.

SET UP: The forces on wire CD are shown in Figure 28.27.



Currents in opposite directions so the force is repulsive and F_I is upward, as shown.

Figure 28.27

$\dfrac{F}{L} = \dfrac{\mu_0 I'I}{2\pi r}$ says $F_I = \dfrac{\mu_0 I^2 L}{2\pi h}$ where L is the length of wire CD and h is the distance between the wires.

EXECUTE: $mg = \lambda L g$.

Thus $F_I - mg = 0$ says $\dfrac{\mu_0 I^2 L}{2\pi h} = \lambda L g$ and $h = \dfrac{\mu_0 I^2}{2\pi g \lambda}$.

EVALUATE: The larger I is or the smaller λ is, the larger h will be.

28.28. **IDENTIFY:** Apply $\dfrac{F}{L} = \dfrac{\mu_0 I'I}{2\pi r}$ for the force from each wire.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: On the top wire $\frac{F}{L}=\frac{\mu_0 I^2}{2\pi}\left(\frac{1}{d}-\frac{1}{2d}\right)=\frac{\mu_0 I^2}{4\pi d}$, upward. On the middle wire, the magnetic forces cancel so the net force is zero. On the bottom wire $\frac{F}{L}=\frac{\mu_0 I^2}{2\pi}\left(\frac{1}{d}-\frac{1}{2d}\right)=\frac{\mu_0 I^2}{4\pi d}$, downward.

EVALUATE: The net force on the middle wire is zero because at the location of the middle wire the net magnetic field due to the other two wires is zero.

28.29. IDENTIFY: Apply $\frac{F}{L}=\frac{\mu_0 I'I}{2\pi r}$.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $F=\frac{\mu_0 I_1 I_2 L}{2\pi r}=\frac{\mu_0(5.00\text{ A})(2.00\text{ A})(1.20\text{ m})}{2\pi(0.400\text{ m})}=6.00\times10^{-6}\text{ N}$, and the force is repulsive since the currents are in opposite directions.

(b) Doubling the currents makes the force increase by a factor of four to w

EVALUATE: Doubling the current in a wire doubles the magnetic field of that wire. For fixed magnetic field, doubling the current in a wire doubles the force that the magnetic field exerts on the wire.

28.30. IDENTIFY: Apply $\frac{F}{L}=\frac{\mu_0 I'I}{2\pi r}$.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $\frac{F}{L}=\frac{\mu_0 I_1 I_2}{2\pi r}$ gives $I_2=\frac{F}{L}\frac{2\pi r}{\mu_0 I_1}=(4.0\times10^{-5}\text{ N/m})\frac{2\pi(0.0250\text{ m})}{\mu_0(0.60\text{ A})}=8.33\text{ A}$.

(b) The two wires repel so the currents are in opposite directions.

EVALUATE: The force between the two wires is proportional to the product of the currents in the wires.

28.31. IDENTIFY: We can model the current in the brain as a ring. Since we know the magnetic field at the center of the ring, we can calculate the current.

SET UP: At the center of a ring, $B=\frac{\mu_0 I}{2R}$. In this case, t.

EXECUTE: Solving for I gives $I=\frac{2RB}{\mu_0}=\frac{2(8\times10^{-2}\text{ m})(3.0\times10^{-12}\text{ T})}{4\times10^{-7}\text{ T}\cdot\text{m/A}}=3.8\times10^{-7}\text{ A}$.

EVALUATE: This current is about a third of a microamp, which is a very small current by household standards. However, the magnetic field in the brain is a very weak field, about a hundreth of the earth's magnetic field.

28.32. IDENTIFY: The magnetic field at the center of a circular loop is $B=\frac{\mu_0 I}{2R}$. By symmetry each segment of the loop that has length Δl contributes equally to the field, so the field at the center of a semicircle is $\frac{1}{2}$ that of a full loop.

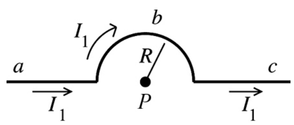
SET UP: Since the straight sections produce no field at P, the field at P is $B=\frac{\mu_0 I}{4R}$.

EXECUTE: $B=\frac{\mu_0 I}{4R}$. The direction of $\vec{B}$ is given by the right-hand rule: $\vec{B}$ is directed into the page.

EVALUATE: For a quarter-circle section of wire the magnetic field at its center of curvature is $B=\frac{\mu_0 I}{8R}$.

28.33. **IDENTIFY:** Calculate the magnetic field vector produced by each wire and add these fields to get the total field.

SET UP: First consider the field at P produced by the current I_1 in the upper semicircle of wire. See Figure 28.33a.

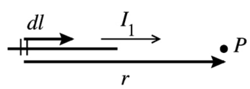


Consider the three parts of this wire:
a: long straight section
b: semicircle
c: long, straight section

Figure 28.33a

Apply the Biot-Savart law $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$ to each piece.

EXECUTE: <u>Part a</u>: See Figure 28.33b.

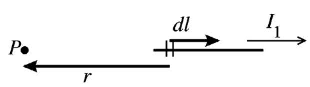


$d\vec{l}\times\vec{r} = 0$,
so $dB = 0$.

Figure 28.33b

The same is true for all the infinitesimal segments that make up this piece of the wire, so $B = 0$ for this piece.

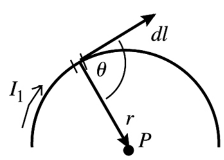
<u>Part c</u>: See Figure 28.33c.



$d\vec{l}\times\vec{r} = 0$,
so $dB = 0$ and $B = 0$ for this piece.

Figure 28.33c

<u>Part b</u>: See Figure 28.33d.



$d\vec{l}\times\vec{r}$ is directed into the paper for all infinitesimal segments that make up this semicircular piece, so $\vec{B}$ is directed into the paper and $B = \int dB$ (the vector sum of the $d\vec{B}$ is obtained by adding their magnitudes since they are in the same direction).

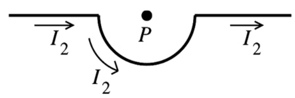
Figure 28.33d

$\left|d\vec{l}\times\vec{r}\right| = rdl\sin\theta$. The angle θ between $d\vec{l}$ and $\vec{r}$ is 90° and $r = R$, the radius of the semicircle. Thus $\left|d\vec{l}\times\vec{r}\right| = R\,dl$.

$$dB = \frac{\mu_0}{4\pi}\frac{I\left|d\vec{l}\times\vec{r}\right|}{r^3} = \frac{\mu_0 I_1}{4\pi}\frac{R}{R^3}dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)dl.$$

$$B = \int dB = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)\int dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)(\pi R) = \frac{\mu_0 I_1}{4R}.$$

(We used that $\int dl$ is equal to πR, the length of wire in the semicircle.) We have shown that the two straight sections make zero contribution to $\vec{B}$, so $B_1 = \mu_0 I_1/4R$ and is directed into the page.



For current in the direction shown in Figure 28.33e, a similar analysis gives $B_2 = \mu_0 I_2/4R$, out of the paper.

Figure 28.33e

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions, so the magnitude of the net field at P is $B = |B_1 - B_2| = \dfrac{\mu_0 |I_1 - I_2|}{4R}$.

EVALUATE: When $I_1 = I_2, B = 0$.

28.34. **IDENTIFY:** Apply $B_x = \dfrac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}$.

SET UP: At the center of the coil, $x = 0$. a is the radius of the coil, 0.0240 m.

EXECUTE: **(a)** $B_x = \mu_0 NI/2a$, so $I = \dfrac{2aB_x}{\mu_0 N} = \dfrac{2(0.024\text{ m})(0.0770\text{ T})}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(800)} = 3.68\text{ A}$.

(b) At the center, $B_c = \mu_0 NI/2a$. At a distance x from the center,

$B_x = \dfrac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}} = \left(\dfrac{\mu_0 NI}{2a}\right)\left(\dfrac{a^3}{(x^2 + a^2)^{3/2}}\right) = B_c\left(\dfrac{a^3}{(x^2 + a^2)^{3/2}}\right)$. $B_x = \frac{1}{2}B_c$ says $\dfrac{a^3}{(x^2 + a^2)^{3/2}} = \frac{1}{2}$, and

$(x^2 + a^2)^3 = 4a^6$. Since $a = 0.024$ m, $x = 0.0184\text{ m} = 1.84\text{ cm}$.

EVALUATE: As shown in Figure 28.14 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

28.35. **IDENTIFY:** The field at the center of the loops is the vector sum of the field due to each loop. They must be in opposite directions in order to add to zero.

SET UP: Let wire 1 be the inner wire with diameter 20.0 cm and let wire 2 be the outer wire with diameter 30.0 cm. To produce zero net field, the fields $\vec{B}_1$ and $\vec{B}_2$ of the two wires must have equal magnitudes and opposite directions. At the center of a wire loop $B = \dfrac{\mu_0 I}{2R}$. The direction of $\vec{B}$ is given by the right-hand rule applied to the current direction.

EXECUTE: $B_1 = \dfrac{\mu_0 I}{2R_1}$, $B_2 = \dfrac{\mu_0 I}{2R_2}$. $B_1 = B_2$ gives $\dfrac{\mu_0 I_1}{2R_1} = \dfrac{\mu_0 I_2}{2R_2}$. Solving for I_2 gives

$I_2 = \left(\dfrac{R_2}{R_1}\right)I_1 = \left(\dfrac{15.0\text{ cm}}{10.0\text{ cm}}\right)(12.0\text{ A}) = 18.0\text{ A}$. The directions of I_1 and of its field are shown in Figure 28.35. Since $\vec{B}_1$ is directed into the page, $\vec{B}_2$ must be directed out of the page and I_2 is counterclockwise.

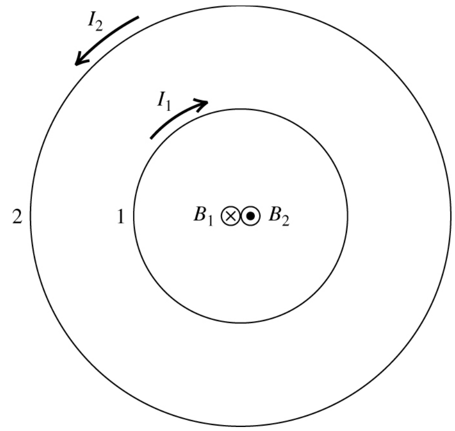


Figure 28.35

EVALUATE: The outer current, I_2, must be larger than the inner current, I_1, because the outer ring is larger than the inner ring, which makes the outer current farther from the center than the inner current is.

28.36. **IDENTIFY** and **SET UP:** The magnetic field at a point on the axis of N circular loops is given by $B_x = \dfrac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$. Solve for N and set $x = 0.0600$ m.

EXECUTE: $N = \dfrac{2B_x(x^2+a^2)^{3/2}}{\mu_0 I a^2} = \dfrac{2(6.39\times10^{-4}\ \text{T})\left[(0.0600\ \text{m})^2+(0.0600\ \text{m})^2\right]^{3/2}}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.50\ \text{A})(0.0600\ \text{m})^2} = 69.$

EVALUATE: At the center of the coil the field is $B_x = \dfrac{\mu_0 N I}{2a} = 1.8\times10^{-3}$ T. The field 6.00 cm from the center is a factor of $1/2^{3/2}$ times smaller.

28.37. **IDENTIFY:** Apply Ampere's law.

SET UP: $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$.

EXECUTE: **(a)** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} = 3.83\times10^{-4}\ \text{T}\cdot\text{m}$ and $I_{\text{encl}} = 305$ A.

(b) $-3.83\times10^{-4}\ \text{T}\cdot\text{m}$ since at each point on the curve the direction of $d\vec{l}$ is reversed.

EVALUATE: The line integral $\oint \vec{B}\cdot d\vec{l}$ around a closed path is proportional to the net current that is enclosed by the path.

28.38. **IDENTIFY:** Apply Ampere's law.

SET UP: From the right-hand rule, when going around the path in a counterclockwise direction currents out of the page are positive and currents into the page are negative.

EXECUTE: Path a: $I_{\text{encl}} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{l} = 0.$

Path b: $I_{\text{encl}} = -I_1 = -4.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = -\mu_0(4.0\ \text{A}) = -5.03\times10^{-6}\ \text{T}\cdot\text{m}.$

Path c: $I_{\text{encl}} = -I_1 + I_2 = -4.0\ \text{A} + 6.0\ \text{A} = 2.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0(2.0\ \text{A}) = 2.51\times10^{-6}\ \text{T}\cdot\text{m}$

Path d: $I_{\text{encl}} = -I_1 + I_2 + I_3 = 4.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = +\mu_0(4.0\ \text{A}) = 5.03\times10^{-6}\ \text{T}\cdot\text{m}.$

EVALUATE: If we instead went around each path in the clockwise direction, the sign of the line integral would be reversed.

28.39. **IDENTIFY:** Apply Ampere's law.

SET UP: To calculate the magnetic field at a distance r from the center of the cable, apply Ampere's law to a circular path of radius r. By symmetry, $\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$ for such a path.

EXECUTE: **(a)** For $a < r < b, I_{\text{encl}} = I \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}.$

(b) For $r > c,$ the enclosed current is zero, so the magnetic field is also zero.

EVALUATE: A useful property of coaxial cables for many applications is that the current carried by the cable doesn't produce a magnetic field outside the cable.

28.40. **IDENTIFY** and **SET UP:** At the center of a long solenoid $B = \mu_0 n I = \mu_0 \dfrac{N}{L} I.$

EXECUTE: $I = \dfrac{BL}{\mu_0 N} = \dfrac{(0.150\text{ T})(0.550\text{ m})}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(4000)} = 16.4\text{ A}.$

EVALUATE: The magnetic field inside the solenoid is independent of the radius of the solenoid, if the radius is much less than the length, as is the case here.

28.41. **IDENTIFY:** Apply Ampere's law to calculate $\vec{B}$.

(a) SET UP: For $a < r < b$ the end view is shown in Figure 28.41a.

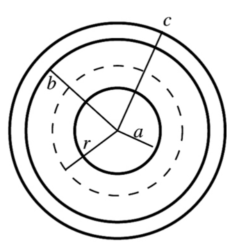


Figure 28.41a

Apply Ampere's law to a circle of radius r, where $a < r < b.$ Take currents I_1 and I_2 to be directed into the page. Take this direction to be positive, so go around the integration path in the clockwise direction.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}.$

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r), I_{\text{encl}} = I_1.$

Thus $B(2\pi r) = \mu_0 I_1$ and $B = \dfrac{\mu_0 I_1}{2\pi r}.$

(b) SET UP: $\underline{r > c}$: See Figure 28.41b.

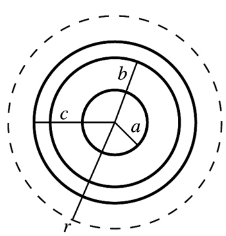


Figure 28.41b

Apply Ampere's law to a circle of radius r, where $r > c.$ Both currents are in the positive direction.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}.$

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r), I_{\text{encl}} = I_1 + I_2.$

Thus $B(2\pi r) = \mu_0 (I_1 + I_2)$ and $B = \dfrac{\mu_0 (I_1 + I_2)}{2\pi r}.$

EVALUATE: For $a < r < b$ the field is due only to the current in the central conductor. For $r > c$ both currents contribute to the total field.

28.42. **IDENTIFY:** $B = \mu_0 nI = \dfrac{\mu_0 NI}{L}.$

SET UP: $L = 0.150$ m.

EXECUTE: $B = \dfrac{\mu_0\,(600)(8.00\text{ A})}{(0.150\text{ m})} = 0.0402\text{ T}.$

EVALUATE: The field near the center of the solenoid is independent of the radius of the solenoid, as long as the radius is much less than the length, as it is here.

28.43. **IDENTIFY** and **SET UP:** The magnetic field near the center of a long solenoid is given by $B = \mu_0 nI$.

EXECUTE: **(a)** Turns per unit length $n = \dfrac{B}{\mu_0 I} = \dfrac{0.0270\text{ T}}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(12.0\text{ A})} = 1790\text{ turns/m}.$

(b) $N = nL = (1790\text{ turns/m})(0.400\text{ m}) = 716\text{ turns}.$

Each turn of radius R has a length $2\pi R$ of wire. The total length of wire required is

$N(2\pi R) = (716)(2\pi)(1.40\times10^{-2}\text{ m}) = 63.0\text{ m}.$

EVALUATE: A large length of wire is required. Due to the length of wire the solenoid will have appreciable resistance.

28.44. **IDENTIFY:** Outside an ideal toroidal solenoid there is no magnetic field and inside it the magnetic field is given by $B = \dfrac{\mu_0 NI}{2\pi r}.$

SET UP: The torus extends from $r_1 = 15.0$ cm to $r_2 = 18.0$ cm.

EXECUTE: **(a)** $r = 0.12$ m, which is outside the torus, so $B = 0$.

(b) $r = 0.16$ m, so $B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\mu_0(250)(8.50\text{ A})}{2\pi(0.160\text{ m})} = 2.66\times10^{-3}\text{ T}.$

(c) $r = 0.20$ m, which is outside the torus, so $B = 0$.

EVALUATE: The magnetic field inside the torus is proportional to $1/r$, so it varies somewhat over the cross-section of the torus.

28.45. **IDENTIFY** and **SET UP:** Use the appropriate expression for the magnetic field produced by each current configuration.

EXECUTE: **(a)** $B = \dfrac{\mu_0 I}{2\pi r}$ so $I = \dfrac{2\pi rB}{\mu_0} = \dfrac{2\pi(2.00\times10^{-2}\text{ m})(37.2\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 3.72\times10^{6}\text{ A} = 3.72\text{ MA}.$

(b) $B = \dfrac{N\mu_0 I}{2R}$ so $I = \dfrac{2RB}{N\mu_0} = \dfrac{2(0.420\text{ m})(37.2\text{ T})}{(100)(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = 2.49\times10^{5}\text{ A} = 249\text{ kA}.$

(c) $B = \mu_0\dfrac{N}{L}I$ so $I = \dfrac{BL}{\mu_0 N} = \dfrac{(37.2\text{ T})(0.320\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(40{,}000)} = 237\text{ A}.$

EVALUATE: Much less current is needed for the solenoid, because of its large number of turns per unit length.

28.46. **IDENTIFY:** Use $B = \dfrac{\mu_0 NI}{2\pi r}$, with μ_0 replaced by $\mu = K_m\mu_0$, with $K_m = 80$.

SET UP: The contribution from atomic currents is the difference between B calculated with μ and B calculated with μ_0.

EXECUTE: **(a)** $B = \dfrac{\mu NI}{2\pi r} = \dfrac{K_m\mu_0 NI}{2\pi r} = \dfrac{\mu_0(80)(400)(0.25\text{ A})}{2\pi(0.060\text{ m})} = 0.0267\text{ T}.$

(b) The amount due to atomic currents is $B' = \frac{79}{80}B = \frac{79}{80}(0.0267\text{ T}) = 0.0263\text{ T}.$

EVALUATE: The presence of the core greatly enhances the magnetic field produced by the solenoid.

28.47. IDENTIFY: The magnetic field from the solenoid alone is $B_0 = \mu_0 nI$. The total magnetic field is $B = K_m B_0$. M is given by $\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$.

SET UP: $n = 6000$ turns/m.

EXECUTE: **(a)** (i) $B_0 = \mu_0 nI = \mu_0(6000\text{ m}^{-1})(0.15\text{ A}) = 1.13\times10^{-3}\text{ T}.$

(ii) $M = \frac{K_m - 1}{\mu_0}B_0 = \frac{5199}{\mu_0}(1.13\times10^{-3}\text{ T}) = 4.68\times10^6\text{ A/m}.$

(iii) $B = K_m B_0 = (5200)(1.13\times10^{-3}\text{ T}) = 5.88\text{ T}.$

(b) The directions of $\vec{B}$, $\vec{B}_0$ and $\vec{M}$ are shown in Figure 28.47. Silicon steel is paramagnetic and $\vec{B}_0$ and $\vec{M}$ are in the same direction.

EVALUATE: The total magnetic field is much larger than the field due to the solenoid current alone.

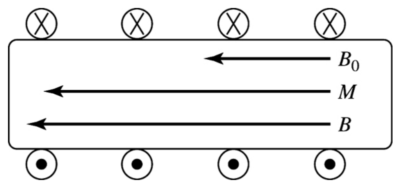


Figure 28.47

28.48. IDENTIFY: Apply $B = \frac{K_m \mu_0 NI}{2\pi r}$.

SET UP: K_m is the relative permeability and $\chi_m = K_m - 1$ is the magnetic susceptibility.

EXECUTE: **(a)** $K_m = \frac{2\pi rB}{\mu_0 NI} = \frac{2\pi(0.2500\text{ m})(1.940\text{ T})}{\mu_0(500)(2.400\text{ A})} = 2021.$

(b) $\chi_m = K_m - 1 = 2020.$

EVALUATE: Without the magnetic material the magnetic field inside the windings would be $B/2021 = 9.6\times10^{-4}\text{ T}$. The presence of the magnetic material greatly enhances the magnetic field inside the windings.

28.49. IDENTIFY: Moving charges create magnetic fields. The net field is the vector sum of the two fields. A charge moving in an external magnetic field feels a force.

(a) SET UP: The magnitude of the magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2}$. Both fields are into the paper, so their magnitudes add, giving $B_{net} = B + B' = \frac{\mu_0}{4\pi}\left(\frac{|q|v\sin\phi}{r^2} + \frac{|q'|v'\sin\phi'}{r'^2}\right)$.

EXECUTE: Substituting numbers gives

$$B_{net} = \frac{\mu_0}{4\pi}\left[\frac{(8.00\ \mu\text{C})(9.00\times10^4\text{ m/s})\sin 90^\circ}{(0.300\text{ m})^2} + \frac{(5.00\ \mu\text{C})(6.50\times10^4\text{ m/s})\sin 90^\circ}{(0.400\text{ m})^2}\right].$$

$B_{net} = 1.00\times10^{-6}\text{ T} = 1.00\mu\text{T}$, into the paper.

(b) SET UP: The magnetic force on a moving charge is $\vec{F} = q\vec{v}\times\vec{B}$, and the magnetic field of charge q' at the location of charge q is into the page. The force on q is

$$\vec{F} = q\vec{v}\times\vec{B}' = (qv)\hat{i}\times\frac{\mu_0}{4\pi}\frac{q\vec{v}'\times\hat{r}}{r^2} = (qv)\hat{i}\times\left(\frac{\mu_0}{4\pi}\frac{qv'\sin\phi}{r^2}\right)(-\hat{k}) = \left(\frac{\mu_0}{4\pi}\frac{qq'vv'\sin\phi}{r^2}\right)\hat{j}$$

where ϕ is the angle between $\vec{v}'$ and $\hat{r}'$.

EXECUTE: Substituting numbers gives

$$\vec{F} = \frac{\mu_0}{4\pi}\left[\frac{(8.00\times10^{-6}\ \text{C})(5.00\times10^{-6}\ \text{C})(9.00\times10^{4}\ \text{m/s})(6.50\times10^{4}\ \text{m/s})}{(0.500\ \text{m})^2}\left(\frac{0.400}{0.500}\right)\right]\hat{j}.$$

$\vec{F} = (7.49\times10^{-8}\,\text{N})\hat{j}.$

EVALUATE: These are small fields and small forces, but if the charge has small mass, the force can affect its motion.

28.50. **IDENTIFY:** Charge q_1 creates a magnetic field due to its motion. This field exerts a magnetic force on q_2, which is moving in that field.

SET UP: Find $\vec{B}_1$, the field produced by q_1 at the location of q_2. $\vec{B}_1 = \dfrac{\mu_0}{4\pi}\dfrac{q_1\vec{v}_1\times\vec{r}_{1\to2}}{r_{1\to2}^3}$, since $\hat{r} = \vec{r}/r$.

EXECUTE: $\vec{r}_{1\to2} = (0.150\ \text{m})\hat{i} + (-0.250\ \text{m})\hat{j}$, so $r_{1\to2} = 0.2915$ m.

$\vec{v}_1\times\vec{r}_{1\to2} = [(9.20\times10^5\ \text{m/s})\hat{i}]\times[(0.150\ \text{m})\hat{i} + (-0.250\ \text{m})\hat{j}] = (9.20\times10^5\ \text{m/s})(-0.250\ \text{m})\hat{k}$.

$$\vec{B}_1 = (1.00\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{(4.80\times10^{-6}\ \text{C})(9.20\times10^5\ \text{m/s})(-0.250\ \text{m})}{(0.2915\ \text{m})^3}\hat{k} = -(4.457\times10^{-6}\ \text{T})\hat{k}.$$

The force that $\vec{B}_1$ exerts on q_2 is

$F_2 = q_2\vec{v}_2\times\vec{B}_1 = (-2.90\times10^{-6}\ \text{C})(-5.30\times10^5\ \text{m/s})(-4.457\times10^{-6}\ \text{T})\hat{j}\times\hat{k} = -(6.85\times10^{-6}\ \text{N})\hat{i}$.

EVALUATE: If we think of the moving charge q_1 as a current, we can use the right-hand rule for the direction of the magnetic field due to a current to find the direction of the magnetic field it creates in the vicinity of q_2. Then we can use the cross product right-hand rule to find the direction of the force this field exerts on q_2, which is in the $-x$-direction, in agreement with our result.

28.51. **IDENTIFY:** This problem involves the magnetic field due to two long wires. We want to know where the fields cancel completely.

SET UP: $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$. The fields completely cancel only if they are in opposite directions and of the same magnitude. Both conditions can be met only in the region between the wires.

EXECUTE: Call y the coordinate of the point where $B = 0$. At that point $B_1 = B_2$. This gives $\dfrac{\mu_0}{2\pi}\dfrac{I_1}{r_1} = \dfrac{\mu_0}{2\pi}\dfrac{I_2}{r_2}$, so $\dfrac{2}{r_1} = \dfrac{6}{r_2}$. $r_1 = y$ and $r_2 = 0.800\ \text{m} - y$, so $\dfrac{1}{y} = \dfrac{3}{0.800\ \text{m} - y}$. Solving for y gives us $y = 0.200$ m.

EVALUATE: The point where $B = 0$ must be closer to the smaller current than to the larger one. $y_1 = 0.200$ m and $y_2 = 0.600$ m, which agrees with this condition.

28.52. **IDENTIFY:** This problem involves the magnetic field due to two long wires. We want to know where the fields cancel completely.

SET UP: $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$. The fields completely cancel only if they are in opposite directions and of the same magnitude. Both conditions can be met only in the region below the 2.00 A current wire.

EXECUTE: Call y the coordinate of the point where $B = 0$. At that point $B_1 = B_2$. This gives $\dfrac{\mu_0}{2\pi}\dfrac{I_1}{r_1} = \dfrac{\mu_0}{2\pi}\dfrac{I_2}{r_2}$, so $\dfrac{2}{r_1} = \dfrac{6}{r_2}$. $r_1 = y$ and $r_2 = 0.800\ \text{m} + y$, so $\dfrac{1}{y} = \dfrac{3}{0.800\ \text{m} + y}$. Solving for y gives us $y = -0.400$ m.

EVALUATE: The point where $B = 0$ must be closer to the smaller current than to the larger one. $y_1 = 0.400$ m and $y_2 = 1.20$ m, which agrees with this condition.

28.53. **IDENTIFY:** In this problem, we are dealing with the magnetic force on a current-carrying wire.

SET UP: $F = IlB\sin\phi$, $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ (long wire), $B = \dfrac{\mu_0 I}{2a}$ (center of circular loop).

EXECUTE: **(a)** We want the force. Combine $F = IlB\sin\phi$ and $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ with ϕ = 90°. This gives

$F = IlB = I(2\pi R)\left(\dfrac{\mu_0 I}{2\pi d}\right) = \mu_0 I^2 R/d.$

(b) We want the force. At the center of the loop we use $B = \dfrac{\mu_0 I}{2a}$. In this case, $a = R$. Solve for I:

$I = \dfrac{2RB}{\mu_0}$. Express the area A in terms of R: $A = \pi R^2 \rightarrow R = \sqrt{A/\pi}$. Use the result from (a):

$F = \dfrac{\mu_0 I^2 R}{d} = \dfrac{\mu_0 (2RB/\mu_0)^2 \sqrt{A/\pi}}{d}$, which reduces to $F = \dfrac{4B^2}{\mu_0 d}\left(\dfrac{A}{\pi}\right)^{3/2}$.

(c) Solve for B: $B = \dfrac{1}{2}\sqrt{\mu_0 F d}\left(\dfrac{\pi}{A}\right)^{3/4}$.

(d) Estimate: F = 5 N.

(e) Estimate: A = 2 cm by 4 cm = 8 cm^2.

(f) $B = \dfrac{1}{2}\sqrt{\mu_0 F d}\left(\dfrac{\pi}{A}\right)^{3/4}$. Using the estimates and d = 25 μm, we get 3 mT.

EVALUATE: This field is about 3 mT and the earth's magnetic field is about 0.05 mT, so the magnet's field is roughly 60 times that of the earth.

28.54. **IDENTIFY:** The wire creates a magnetic field near it, and the moving electron feels a force due to this field.

SET UP: The magnetic field due to the wire is $B = \dfrac{\mu_0 I}{2\pi r}$, and the force on a moving charge is $F = |q| vB\sin\phi$.

EXECUTE: $F = |q| vB\sin\phi = (ev\mu_0 I \sin\phi)/2\pi r$. Substituting numbers gives

$F = (1.60\times10^{-19}\text{ C})(6.00\times10^4\text{ m/s})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(8.60\text{ A})(\sin 90°)/[2\pi(0.0450\text{ m})]$.

$F = 3.67\times10^{-19}$ N. From the right-hand rule for the cross product, the direction of $\vec{v}\times\vec{B}$ is opposite to the current, but since the electron is negative, the force is in the same direction as the current.

EVALUATE: This force is small at an everyday level, but it would give the electron an acceleration of over 10^{11} m/s^2.

28.55. **IDENTIFY:** Find the force that the magnetic field of the wire exerts on the electron.

SET UP: The force on a moving charge has magnitude $F = |q| vB\sin\phi$ and direction given by the right-hand rule. For a long straight wire, $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** $a = \dfrac{F}{m} = \dfrac{|q| vB\sin\phi}{m} = \dfrac{ev}{m}\left(\dfrac{\mu_0 I}{2\pi r}\right)$. Substituting numbers gives

$a = \dfrac{(1.6\times10^{-19}\text{C})(2.50\times10^5\text{ m/s})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(13.0\text{ A})}{(9.11\times10^{-31}\text{ kg})(2\pi)(0.0200\text{ m})} = 5.7\times10^{12}\text{ m/s}^2$, away from the wire.

(b) The electric force must balance the magnetic force. $eE = evB$, and

$E = vB = v\dfrac{\mu_0 I}{2\pi r} = \dfrac{(250{,}000\text{ m/s})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(13.0\text{ A})}{2\pi(0.0200\text{ m})} = 32.5\text{ N/C}$. The magnetic force is directed away from the wire so the force from the electric field must be toward the wire. Since the charge of the electron is negative, the electric field must be directed away from the wire to produce a force in the desired direction.

EVALUATE: **(c)** $mg = (9.11\times10^{-31}\text{ kg})(9.8\text{ m/s}^2) \approx 10^{-29}\text{ N}.$

$F_{\text{el}} = eE = (1.6\times10^{-19}\text{ C})(32.5\text{ N/C}) \approx 5\times10^{-18}\text{ N}.$ $F_{\text{el}} \approx 5\times10^{11}\,F_{\text{grav}},$ so we can neglect gravity.

28.56. **IDENTIFY:** The current in the wire creates a magnetic field, and that field exerts a force on the moving electron.

SET UP: The magnetic field due to the current in the wire is $B = \dfrac{\mu_0 I}{2\pi r}$. The force the field exerts on the electron is $\vec{F} = q\vec{v}\times\vec{B},$ where $q = -e$. The magnitude of a vector is $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$. The electron is on the $+y$-axis. The current is in the $-x$-direction so, by the right-hand rule, the magnetic field it produces at the location of the electron is in the $-z$-direction, so $\vec{B} = -\dfrac{\mu_0 I}{2\pi r}\,\hat{k}.$

EXECUTE: The magnitude of the magnetic field is $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{\mu_0(9.00\text{ A})}{2\pi(0.200\text{ m})} = 9.00\times10^{-6}\text{ T},$ so

$\vec{B} = -9.00\times10^{-6}\text{ T}\,\hat{k}$. The force on the electron is $\vec{F} = q\vec{v}\times\vec{B},$ so

$\vec{F} = q\vec{v}\times\vec{B} = -e(5.00\times10^4\text{ m/s}\,\hat{i} - 3.00\times10^4\text{ m/s}\,\hat{j})\times(-9.00\times10^{-6}\text{ T}\,\hat{k}).$

Taking out common factors gives $\vec{F} = (9\times10^{-2}e\text{ T}\cdot\text{m/s})(5\hat{i} - 3\hat{j})\times\hat{k}.$ Using the fact that $\boldsymbol{i}\times\boldsymbol{k} = -\boldsymbol{j}$ and $\boldsymbol{j}\times\boldsymbol{k} = \boldsymbol{i},$ we get $\vec{F} = (9\times10^{-2}e\text{ T}\cdot\text{m/s})(-5\hat{j} - 3\hat{i}).$ Using $e = 1.60\times10^{-19}\text{ C}$ gives

$\vec{F} = -4.32\times10^{-20}\text{ N}\,\hat{i} - 7.20\times10^{-20}\text{ N}\,\hat{j}.$

The magnitude of this force is

$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(-4.32\times10^{-20}\text{ N})^2 + (-7.20\times10^{-20}\text{ N})^2} = 8.40\times10^{-20}\text{ N}.$

EVALUATE: This is a small force on an everyday scale, but it would give the electron an acceleration of $a = F/m = (8.40\times10^{-20}\text{ N})/(9.11\times10^{-31}\text{ kg}) \approx 9\times10^{10}\text{ m/s}^2.$

28.57. **IDENTIFY:** This problem deals with the magnetic field in a cell phone.

SET UP and **EXECUTE:** **(a)** $P = 1.5$ W.

(b) $I = P/V = (1.5\text{ W})/(1.5\text{ V}) = 1.0$ A.

(c) Width = 20 cm.

(d) Estimate: Diameter = 3.0 cm. Treat the phone as a circular current loop. The field is given by $B = \dfrac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$, where a is the radius of the loop and x is the distance from its center along its axis. The estimated diameter is 3.0 cm, so $a = 1.5$ cm, and x is the distance from the phone to the middle of your head, which is 10 cm from our estimate in (c). Using these values,

$B = \dfrac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} = 0.14\ \mu\text{T}.$

(e) $\dfrac{B_{\text{phone}}}{B_{\text{earth}}} = \dfrac{0.14\ \mu\text{T}}{50\ \mu\text{T}} \approx 3\times10^{-3} \approx 0.3\%.$ The magnetic field of the cell phone at the location of your brain is about 0.3% of the earth's magnetic field.

EVALUATE: This magnetic field is extremely weak compared to the earth's field which we experience all day every day.

28.58. **IDENTIFY:** Find the vector sum of the magnetic fields due to each wire.

SET UP: For a long straight wire $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule and is perpendicular to the line from the wire to the point where the field is calculated.

EXECUTE: **(a)** The magnetic field vectors are shown in Figure 28.58a.

(b) At a position on the x-axis $B_{\text{net}} = 2\dfrac{\mu_0 I}{2\pi r}\sin\theta = \dfrac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\dfrac{a}{\sqrt{x^2+a^2}} = \dfrac{\mu_0 I a}{\pi(x^2+a^2)}$, in the positive x-direction.

(c) The graph of B versus x/a is given in Figure 28.58b.

EVALUATE: **(d)** The magnetic field is a maximum at the origin, $x = 0$.

(e) When $x \gg a$, $B \approx \dfrac{\mu_0 I a}{\pi x^2}$.

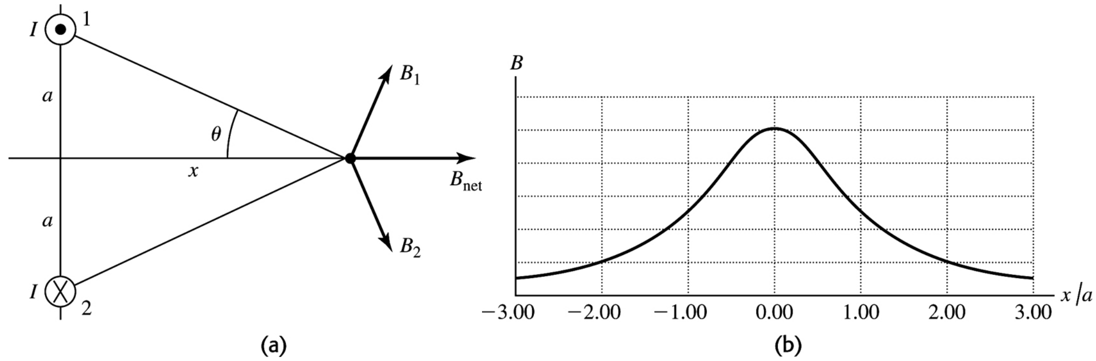
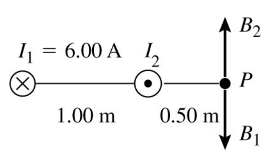


Figure 28.58

28.59. **IDENTIFY:** Use $B = \dfrac{\mu_0 I}{2\pi r}$ and the right-hand rule to calculate the magnitude and direction of the magnetic field at P produced by each wire. Add these two field vectors to find the net field.

(a) SET UP: The directions of the fields at point P due to the two wires are sketched in Figure 28.59a.



EXECUTE: $\vec{B}_1$ and $\vec{B}_2$ must be equal and opposite for the resultant field at P to be zero. $\vec{B}_2$ is to the upward so I_2 is out of the page.

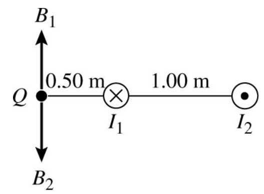
Figure 28.59a

$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0}{2\pi}\left(\frac{6.00\ \text{A}}{1.50\ \text{m}}\right) \qquad B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.50\ \text{m}}\right).$$

$$B_1 = B_2 \text{ says } \frac{\mu_0}{2\pi}\left(\frac{6.00\ \text{A}}{1.50\ \text{m}}\right) = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.50\ \text{m}}\right).$$

$$I_2 = \left(\frac{0.50\ \text{m}}{1.50\ \text{m}}\right)(6.00\ \text{A}) = 2.00\ \text{A}.$$

(b) SET UP: The directions of the fields at point Q are sketched in Figure 28.59b.



EXECUTE: $B_1 = \dfrac{\mu_0 I_1}{2\pi r_1}$.

$$B_1 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{6.00\ \text{A}}{0.50\ \text{m}}\right) = 2.40\times10^{-6}\ \text{T}.$$

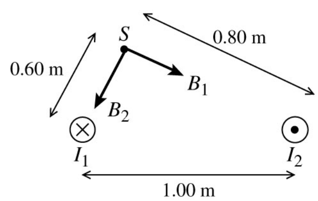
$$B_2 = \frac{\mu_0 I_2}{2\pi r_2}.$$

$$B_2 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{2.00\ \text{A}}{1.50\ \text{m}}\right) = 2.67\times10^{-7}\ \text{T}.$$

Figure 28.59b

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions and $B_1 > B_2$ so $B = B_1 - B_2 = 2.40\times10^{-6}\ \text{T} - 2.67\times10^{-7}\ \text{T} = 2.13\times10^{-6}\ \text{T}$, and $\vec{B}$ is upward.

(c) SET UP: The directions of the fields at point *S* are sketched in Figure 28.59c.



EXECUTE: $B_1 = \dfrac{\mu_0 I_1}{2\pi r_1}$.

$$B_1 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{6.00\ \text{A}}{0.60\ \text{m}}\right) = 2.00\times10^{-6}\ \text{T}.$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2}.$$

$$B_2 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{2.00\ \text{A}}{0.80\ \text{m}}\right) = 5.00\times10^{-7}\ \text{T}.$$

Figure 28.59c

$\vec{B}_1$ and $\vec{B}_2$ are right angles to each other, so the magnitude of their resultant is given by

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(2.00\times10^{-6}\ \text{T})^2 + (5.00\times10^{-7}\ \text{T})^2} = 2.06\times10^{-6}\ \text{T}.$$

EVALUATE: The magnetic field lines for a long, straight wire are concentric circles with the wire at the center. The magnetic field at each point is tangent to the field line, so $\vec{B}$ is perpendicular to the line from the wire to the point where the field is calculated.

28.60. IDENTIFY: Consider the forces on each side of the loop.

SET UP: The forces on the left and right sides cancel. The forces on the top and bottom segments of the loop are in opposite directions, so the magnitudes subtract.

EXECUTE: $F = F_\text{t} - F_\text{b} = \left(\dfrac{\mu_0 I_\text{wire}}{2\pi}\right)\left(\dfrac{Il}{r_\text{t}} - \dfrac{Il}{r_\text{b}}\right) = \dfrac{\mu_0 I l I_\text{wire}}{2\pi}\left(\dfrac{1}{r_\text{t}} - \dfrac{1}{r_\text{b}}\right)$.

$F = \dfrac{\mu_0 (5.00\ \text{A})(0.200\ \text{m})(14.0\ \text{A})}{2\pi}\left(-\dfrac{1}{0.100\ \text{m}} + \dfrac{1}{0.026\ \text{m}}\right) = 7.97\times10^{-5}\ \text{N}$. The force on the top segment is toward the wire, so the net force is toward the wire.

EVALUATE: The net force on a current loop in a uniform magnetic field is zero, but the magnetic field of the wire is not uniform; it is stronger closer to the wire.

28.61. **IDENTIFY:** Apply $\sum \vec{F} = 0$ to one of the wires. The force one wire exerts on the other depends on I so $\sum \vec{F} = 0$ gives two equations for the two unknowns T and I.

SET UP: The force diagram for one of the wires is given in Figure 28.61.

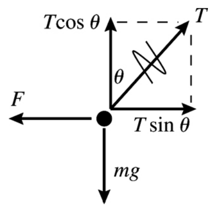


Figure 28.61

The force one wire exerts on the other is $F = \left(\dfrac{\mu_0 I^2}{2\pi r}\right)L,$ where $r = 2(0.040\ \text{m})\sin\theta = 8.362\times10^{-3}\ \text{m}$ is the distance between the two wires.

EXECUTE: $\sum F_y = 0$ gives $T\cos\theta = mg$ and $T = mg/\cos\theta$.

$\sum F_x = 0$ gives $F = T\sin\theta = (mg/\cos\theta)\sin\theta = mg\tan\theta$.

And $m = \lambda L$, so $F = \lambda Lg\tan\theta$.

$$\left(\frac{\mu_0 I^2}{2\pi r}\right)L = \lambda Lg\tan\theta.$$

$$I = \sqrt{\frac{\lambda gr\tan\theta}{(\mu_0/2\pi)}}.$$

$$I = \sqrt{\frac{(0.0125\ \text{kg/m})(9.80\ \text{m/s}^2)(\tan 6.00°)(8.362\times10^{-3}\ \text{m})}{2\times10^{-7}\ \text{T}\cdot\text{m/A}}} = 23.2\ \text{A}.$$

EVALUATE: Since the currents are in opposite directions the wires repel. When I is increased, the angle θ from the vertical increases; a large current is required even for the small displacement specified in this problem.

28.62. **IDENTIFY:** Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l}\times\hat{r}}{r^2}$.

SET UP: The two straight segments produce zero field at P. The field at the center of a circular loop of radius R is $B = \dfrac{\mu_0 I}{2R}$, so the field at the center of curvature of a semicircular loop is $B = \dfrac{\mu_0 I}{4R}$.

EXECUTE: The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B = B_a - B_b = \dfrac{1}{2}\left(\dfrac{\mu_0 I}{2}\right)\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{\mu_0 I}{4a}\left(1 - \dfrac{a}{b}\right)$, out of page.

EVALUATE: If $a = b$, $B = 0$.

28.63. **IDENTIFY:** Apply Ampere's law to a circle of radius r.

SET UP: The current within a radius r is $I = \int \vec{J}\cdot d\vec{A}$, where the integration is over a disk of radius r.

EXECUTE: **(a)** $I_0 = \int \vec{J}\cdot d\vec{A} = \int\left(\dfrac{b}{r}e^{(r-a)/\delta}\right)rdrd\theta = 2\pi b\int_0^a e^{(r-a)/\delta}dr = 2\pi b\delta\, e^{(r-a)/\delta}\Big|_0^a = 2\pi b\delta(1 - e^{-a/\delta})$.

$I_0 = 2\pi(600\ \text{A/m})(0.025\ \text{m})(1 - e^{(0.050/0.025)}) = 81.5\ \text{A}.$

(b) For $r \ge a$, r$\vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0$ and $B = \dfrac{\mu_0 I_0}{2\pi r}$.

(c) For $r \le a$, $I(r) = \int \vec{J}\cdot d\vec{A} = \int\left(\dfrac{b}{r'}e^{(r'-a)/\delta}\right)r'dr'd\theta = 2\pi b\int_0^r e^{(r-a)/\delta}dr = 2\pi b\delta e^{(r'-a)/\delta}\Big|_0^r$.

$I(r)=2\pi b\delta(e^{(r-a)/\delta}-e^{-a/\delta})=2\pi b\delta e^{-a/\delta}(e^{r/\delta}-1)$ and $I(r)=I_0\dfrac{(e^{r/\delta}-1)}{(e^{a/\delta}-1)}$.

(d) For $r\le a$, $\oint \vec{B}\cdot d\vec{l}=B(r)2\pi r=\mu_0 I_{\text{encl}}=\mu_0 I_0\dfrac{(e^{r/\delta}-1)}{(e^{a/\delta}-1)}$ and $B=\dfrac{\mu_0 I_0(e^{r/\delta}-1)}{2\pi r(e^{a/\delta}-1)}$.

(e) At $r=\delta=0.025$ m, $B=\dfrac{\mu_0 I_0(e-1)}{2\pi\delta(e^{a/\delta}-1)}=\dfrac{\mu_0(81.5\text{ A})}{2\pi(0.025\text{ m})}\dfrac{(e-1)}{(e^{0.050/0.025}-1)}=1.75\times10^{-4}$ T.

At $r=a=0.050$ m, $B=\dfrac{\mu_0 I_0}{2\pi a}\dfrac{(e^{a/\delta}-1)}{(e^{a/\delta}-1)}=\dfrac{\mu_0(81.5\text{ A})}{2\pi(0.050\text{ m})}=3.26\times10^{-4}$ T.

At $r=2a=0.100$ m, $B=\dfrac{\mu_0 I_0}{2\pi r}=\dfrac{\mu_0(81.5\text{ A})}{2\pi(0.100\text{ m})}=1.63\times10^{-4}$ T.

EVALUATE: At points outside the cylinder, the magnetic field is the same as that due to a long wire running along the axis of the cylinder.

28.64. IDENTIFY: Both arcs produce magnetic fields at point *P* perpendicular to the plane of the page. The field due to arc *DA* points into the page, and the field due to arc *BC* points out of the page. The field due to *DA* has a greater magnitude than the field due to arc *BC*. The net field is the sum of these two fields.

SET UP: The magnitude field at the center of a circular loop of radius a is $B=\dfrac{\mu_0 I}{2\pi a}$. Each arc is 120°/360° = 1/3 of a complete loop, so the field due to each of them is $B=\dfrac{1}{3}\dfrac{\mu_0 I}{2\pi a}=\dfrac{\mu_0 I}{6\pi a}$.

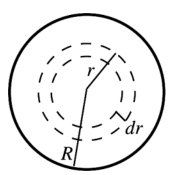
EXECUTE: The net field is

$B_{\text{net}}=B_{20}-B_{30}=\dfrac{\mu_0(12.0\text{ A})}{6\pi}\left(\dfrac{1}{0.200\text{ m}}-\dfrac{1}{0.300\text{ m}}\right)=4.19\times10^{-6}\text{ T}=4.19\ \mu\text{T}$. Since $B_{20}>B_{30}$, the net field points into the page at *P*.

EVALUATE: The current in segments *CD* and *AB* produces no magnetic field at *P* because its direction is directly toward (or away from) point *P*.

28.65. (a) IDENTIFY: Consider current density *J* for a small concentric ring and integrate to find the total current in terms of α and *R*.

SET UP: We can't say $I=JA=J\pi R^2$, since *J* varies across the cross section.



To integrate *J* over the cross section of the wire, divide the wire cross section up into thin concentric rings of radius *r* and width *dr*, as shown in Figure 28.65.

Figure 28.65

EXECUTE: The area of such a ring is *dA*, and the current through it is $dI=J\,dA$; $dA=2\pi r dr$ and $dI=J\,dA=\alpha r(2\pi r\,dr)=2\pi\alpha r^2 dr$.

$I=\int dI=2\pi\alpha\int_0^R r^2 dr=2\pi\alpha(R^3/3)$ so $\alpha=\dfrac{3I}{2\pi R^3}$.

(b) IDENTIFY and **SET UP:** (i) $r\le R$.

Apply Ampere's law to a circle of radius $r<R$. Use the method of part (a) to find the current enclosed by Ampere's law path.

EXECUTE: $\oint \vec{B}\cdot d\vec{l}=\oint B\,dl=B\oint dl=B(2\pi r)$, by the symmetry and direction of $\vec{B}$. The current passing through the path is $I_{\text{encl}}=\int dl$, where the integration is from 0 to *r*.

$I_{\text{encl}} = 2\pi\alpha\int_0^r r^2 dr = \dfrac{2\pi\alpha r^3}{3} = \dfrac{2\pi}{3}\left(\dfrac{3I}{2\pi R^3}\right)r^3 = \dfrac{Ir^3}{R^3}$. Thus $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives

$B(2\pi r) = \mu_0\left(\dfrac{Ir^3}{R^3}\right)$ and $B = \dfrac{\mu_0 I r^2}{2\pi R^3}$.

(ii) **IDENTIFY** and **SET UP:** $r \ge R$.

Apply Ampere's law to a circle of radius $r > R$.

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$.

$I_{\text{encl}} = I$; all the current in the wire passes through this path. Thus $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives

$B(2\pi r) = \mu_0 I$ and $B = \dfrac{\mu_0 I}{2\pi r}$.

EVALUATE: Note that at $r = R$ the expression in (i) (for $r \le R$) gives $B = \dfrac{\mu_0 I}{2\pi R}$. At $r = R$ the expression in (ii) (for $r \ge R$) gives $B = \dfrac{\mu_0 I}{2\pi R}$, which is the same.

28.66. **IDENTIFY:** Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I d\vec{l}\times\hat{r}}{r^2}$.

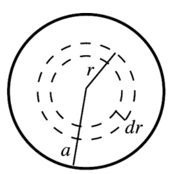
SET UP: The horizontal wire yields zero magnetic field since $d\vec{l}\times\vec{r} = 0$. The vertical current provides the magnetic field of half of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

EXECUTE: $B = \frac{1}{2}\left(\dfrac{\mu_0 I}{2\pi a}\right) = \dfrac{\mu_0 I}{4\pi a}$ and is directed out of the page.

EVALUATE: In the equation preceding Eq. (28.8) the limits on the integration are 0 to a rather than $-a$ to a and this introduces a factor of $\frac{1}{2}$ into the expression for B.

28.67. **IDENTIFY:** Use the current density J to find dI through a concentric ring and integrate over the appropriate cross section to find the current through that cross section. Then use Ampere's law to find $\vec{B}$ at the specified distance from the center of the wire.

(a) SET UP:

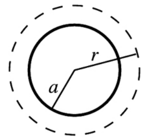


Divide the cross section of the cylinder into thin concentric rings of radius r and width dr, as shown in Figure 28.67a. The current through each ring is $dI = J\,dA = J2\pi r\,dr$.

Figure 28.67a

EXECUTE: $dI = \dfrac{2I_0}{\pi a^2}\left[1-(r/a)^2\right]2\pi r\,dr = \dfrac{4I_0}{a^2}\left[1-(r/a)^2\right]r\,dr$. The total current I is obtained by integrating dI over the cross section $I = \int_0^a dI = \left(\dfrac{4I_0}{a^2}\right)\int_0^a (1-r^2/a^2)r\,dr = \left(\dfrac{4I_0}{a^2}\right)\left[\dfrac{1}{2}r^2 - \dfrac{1}{4}r^4/a^2\right]_0^a = I_0$, as was to be shown.

(b) SET UP: Apply Ampere's law to a path that is a circle of radius $r > a$, as shown in Figure 28.67b.



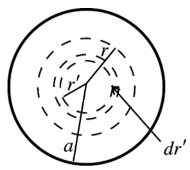
$\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$.

$I_{\text{encl}} = I_0$ (the path encloses the entire cylinder).

Figure 28.67b

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 I_0$ and $B = \dfrac{\mu_0 I_0}{2\pi r}$.

(c) SET UP:



Divide the cross section of the cylinder into concentric rings of radius r' and width dr', as was done in part (a). See Figure 28.67c. The current dI through each ring is

$$dI = \frac{4I_0}{a^2}\left[1 - \left(\frac{r'}{a}\right)^2\right] r'\,dr'.$$

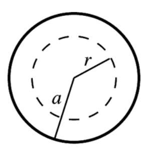
Figure 28.67c

EXECUTE: The current I is obtained by integrating dI from $r' = 0$ to $r' = r$:

$$I = \int dI = \frac{4I_0}{a^2}\int_0^r \left[1 - \left(\frac{r'}{a}\right)^2\right] r'\,dr' = \frac{4I_0}{a^2}\left[\tfrac{1}{2}(r')^2 - \tfrac{1}{4}(r')^4/a^2\right]_0^r.$$

$$I = \frac{4I_0}{a^2}(r^2/2 - r^4/4a^2) = \frac{I_0 r^2}{a^2}\left(2 - \frac{r^2}{a^2}\right).$$

(d) SET UP: Apply Ampere's law to a path that is a circle of radius $r < a$, as shown in Figure 28.67d.



$\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$.

$I_{\text{encl}} = \dfrac{I_0 r^2}{a^2}\left(2 - \dfrac{r^2}{a^2}\right)$ (from part (c)).

Figure 28.67d

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 \dfrac{I_0 r^2}{a^2}(2 - r^2/a^2)$ and $B = \dfrac{\mu_0 I_0}{2\pi}\dfrac{r}{a^2}(2 - r^2/a^2)$.

EVALUATE: Result in part (b) evaluated at $r = a$: $B = \dfrac{\mu_0 I_0}{2\pi a}$. Result in part (d) evaluated at $r = a$: $B = \dfrac{\mu_0 I_0}{2\pi}\dfrac{a}{a^2}(2 - a^2/a^2) = \dfrac{\mu_0 I_0}{2\pi a}$. The two results, one for $r > a$ and the other for $r < a$, agree at $r = a$.

28.68. IDENTIFY: The net field is the vector sum of the fields due to the circular loop and to the long straight wire.

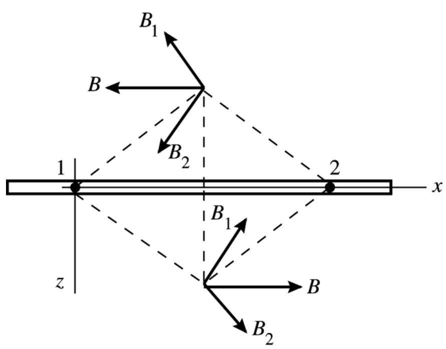
SET UP: For the long wire, $B = \dfrac{\mu_0 I_1}{2\pi D}$, and for the loop, $B = \dfrac{\mu_0 I_2}{2R}$.

EXECUTE: At the center of the circular loop the current I_2 generates a magnetic field that is into the page, so the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude: $\dfrac{\mu_0 I_1}{2\pi D} = \dfrac{\mu_0 I_2}{2R}$. Thus, $I_1 = \dfrac{\pi D}{R} I_2$.

EVALUATE: If I_1 is to the left the two fields add.

28.69. **IDENTIFY:** Use what we know about the magnetic field of a long, straight conductor to deduce the symmetry of the magnetic field. Then apply Ampere's law to calculate the magnetic field at a distance a above and below the current sheet.

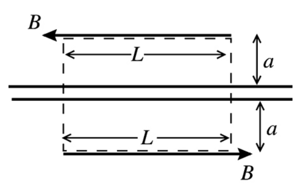
SET UP: Do parts (a) and (b) together.



Consider the individual currents in pairs, where the currents in each pair are equidistant on either side of the point where $\vec{B}$ is being calculated. Figure 28.69a shows that for each pair the z-components cancel, and that above the sheet the field is in the $-x$-direction and that below the sheet it is in the $+x$-direction.

Figure 28.69a

Also, by symmetry the magnitude of $\vec{B}$ a distance a above the sheet must equal the magnitude of $\vec{B}$ a distance a below the sheet. Now that we have deduced the symmetry of $\vec{B}$, apply Ampere's law. Use a path that is a rectangle, as shown in Figure 28.69b.



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}.$$

Figure 28.69b

I is directed out of the page, so for I to be positive the integral around the path is taken in the counterclockwise direction.

EXECUTE: Since $\vec{B}$ is parallel to the sheet, on the sides of the rectangle that have length $2a$, $\oint \vec{B} \cdot d\vec{l} = 0.$ On the long sides of length L, $\vec{B}$ is parallel to the side, in the direction we are integrating around the path, and has the same magnitude, B, on each side. Thus $\oint \vec{B} \cdot d\vec{l} = 2BL.$ n conductors per unit length and current I out of the page in each conductor gives $I_{\text{encl}} = InL.$ Ampere's law then gives $2BL = \mu_0 InL$ and $B = \frac{1}{2}\mu_0 In.$

EVALUATE: Note that B is independent of the distance a from the sheet. Compare this result to the electric field due to an infinite sheet of charge in Chapter 22.

28.70. **IDENTIFY:** Find the vector sum of the fields due to each sheet.

SET UP: Problem 28.69 shows that for an infinite sheet $B = \frac{1}{2}\mu_0 In.$ If I is out of the page, $\vec{B}$ is to the left above the sheet and to the right below the sheet. If I is into the page, $\vec{B}$ is to the right above the sheet and to the left below the sheet. B is independent of the distance from the sheet. The directions of the two fields at points P, R and S are shown in Figure 28.70.

EXECUTE: **(a)** Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).

(b) In between the sheets the two fields add up to yield $B = \mu_0 nI$, to the right.

(c) Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

EVALUATE: The two sheets with currents in opposite directions produce a uniform field between the sheets and zero field outside the two sheets. This is analogous to the electric field produced by large parallel sheets of charge of opposite sign.

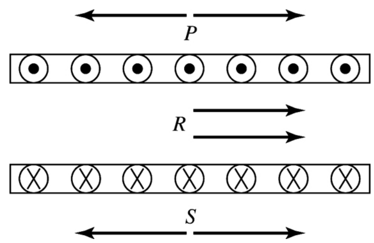


Figure 28.70

28.71. **IDENTIFY:** A charged cylindrical shell is rotating. This motion produces a magnetic field which exerts a torque on a very small disk at its midpoint.

SET UP and **EXECUTE:** **(a)** We want the current. $I = \dfrac{\Delta Q}{\Delta t}$. In one full rotation, charge Q_1 passes through in time T_1 which is the period of rotation of the cylinder. Thus $\Delta Q = Q_1$ and $\Delta T = T_1 = 2\pi/\omega_1$. So $I = \dfrac{Q_1}{(2\pi/\omega_1)} = \dfrac{Q_1\omega_1}{2\pi}$.

(b) We want B. Apply Ampere's law: $\oint \vec{B}\cdot d\hat{l} = \mu_0 I_{\text{encl}}$. The rectangular path of integration is similar to the one used in Example 28.9. The inner segment ab is on the axis of the cylinder, is equidistant from its ends, and has length $l << H$. $B = 0$ outside the cylinder, and the field is perpendicular to bc and da. Using the current from part (a), $\oint \vec{B}\cdot d\hat{l} = \mu_0 I_{\text{encl}}$ gives us $Bl = \mu_0 I \dfrac{l}{H} = \mu_0 \dfrac{Q_1\omega_1}{2\pi}\dfrac{l}{H}$. Solving for B and realizing that the field points along the $+z$-axis, we have $\vec{B} = \dfrac{\mu_0\omega_1 Q_1}{2\pi H}\hat{k}$.

(c) We want the torque on the disk. $\tau = \mu B\sin\phi$. Since the disk is very small, we can treat the magnetic field as uniform over its surface and equal to the field at the center of the cylinder. Using the given magnetic moment, our result from (b), and $\phi = \theta$, we get

$$\tau = \left(\frac{1}{4}Q_2\omega_2 R_2^2\right)\left(\frac{\mu_0 Q_1\omega_1}{2\pi H}\right)\sin\theta = \frac{\mu_0 Q_1 Q_2\omega_1\omega_2 R_2^2}{8\pi H}\sin\theta.$$

(d) We want the angular momentum of the disk. $L = I\omega = \dfrac{1}{2}MR_2^2\omega_2$.

(e) We want the precession rate. From Section 10.7 we have

$$\Omega = \frac{\tau_z}{L_z} = \frac{\dfrac{\mu_0}{8\pi H}Q_1Q_2\omega_1\omega_2 R_2^2\sin\theta}{\dfrac{1}{2}MR_2^2\omega_2} = \frac{\mu_0}{4\pi H}\frac{Q_1Q_2\omega_1\sin\theta}{M}.$$

EVALUATE: The spinning charged cylinder behaves like a solenoid.

28.72. **IDENTIFY** and **SET UP:** We assume that both solenoids are ideal, in which case the field due to each one is given by $B = \mu_0 nI = \mu_0 \dfrac{N}{L}I$. The net field inside is the sum of both the fields.

EXECUTE: **(a)** The net field is $B=\mu_0\frac{N_1}{L}I_1+\mu_0\frac{N_2}{L}I_2=\frac{\mu_0}{L}[N_1I_1+N_2I_2]$. For the numbers in this problem, we have $BL/\mu_0=(0.00200\text{ A})N_1+N_2I_2$. Therefore a graph of BL/μ_0 versus I_2 should be a straight line with slope equal to N_2 and *y*-intercept equal to (0.00200 A)N_1.

(b) Using the graph given with the problem, we calculate the slope using the points (5.00 mA, 16.00 A) and (2.00 mA, 8.00 A), which gives slope = (16.00 A – 8.00 A)/(5.00 mA – 2.00 mA) = 2667. Therefore N_2 = 2667 turns, which rounds to 2670 turns. To find the *y*-intercept, we use the point (5.00 mA, 16.00 A) and the slope to deduce the equation of the line. This gives $\frac{y-16.00\text{ A}}{x-0.00500\text{ A}}=2667$, which simplifies to y = 2667x + 2.67. When x = 0, y = 2.67 A. As we saw, the *y*-intercept is equal to (0.00200 A)N_1, so N_1 = (2.67 A)/(0.00200 A) = 1335 turns, which rounds to 1340 turns.

(c) Now the fields are in opposite directions, so $B=\mu_0\frac{N_1}{L}I_1-\mu_0\frac{N_2}{L}I_2=\frac{\mu_0}{L}[N_1I_1-N_2I_2]$.

B = [(μ_0)/(0.400 m)][(0.00200 A)(1335) – (0.00500 A)(2667)] $=-3.35\times10^{-5}$ T. The minus sign just tells us that the field due to I_2 is stronger than the field due to I_1. So the magnitude of the net field is $B=3.35\times0^{-5}\text{ T}=33.5\ \mu\text{T}$.

EVALUATE: As a check for N_1 in part (b), we could use a ruler to extrapolate the graph in the textbook back to its intersection with the *y*-axis to find the *y*-intercept. This method is not particularly accurate, but it should give reasonable agreement with the result for N_1 from part (b).

28.73. **IDENTIFY** and **SET UP:** The magnitude of the magnetic a distance r from the center of a very long current-carrying wire is $B=\frac{\mu_0 I}{2\pi r}$. In this case, the measured quantity x is the distance from the *surface* of the cable, not from the center.

EXECUTE: **(a)** Multiplying the quantities given in the table in the problem, we get the following values for Bx in units of $\text{T}\cdot\text{cm}$, starting with the first pair: 0.812, 1.00, 1.09, 1.13, 1.16. As we can see, these values are not constant. However the last three values are nearly constant. Therefore Bx is not truly constant. The reason for this is that x is the distance from the *surface* of the cable, not from the center. In the formula $B=\frac{\mu_0 I}{2\pi r}$, r is the distance from the center of the cable. In that case, we would expect Br to be constant. For the last three points, it does appear that Bx is nearly constant. The reason for this is that the proper formula for the magnetic field for this cable is $B=\frac{\mu_0 I}{2\pi(R+x)}$, where R is the radius of the cable. As x gets large compared to R, $r\approx x$ and the magnitude approaches $\frac{\mu_0 I}{2\pi r}$.

(b) Using the equation appropriate for the cable and solving for x gives $x=(\mu_0 I/2\pi)\frac{1}{B}-R$. A graph of x versus $1/B$ should have a slope equal to $\mu_0 I/2\pi$ and a *y*-intercept equal to $-R$. Figure 28.73 shows the graph of x versus $1/B$.

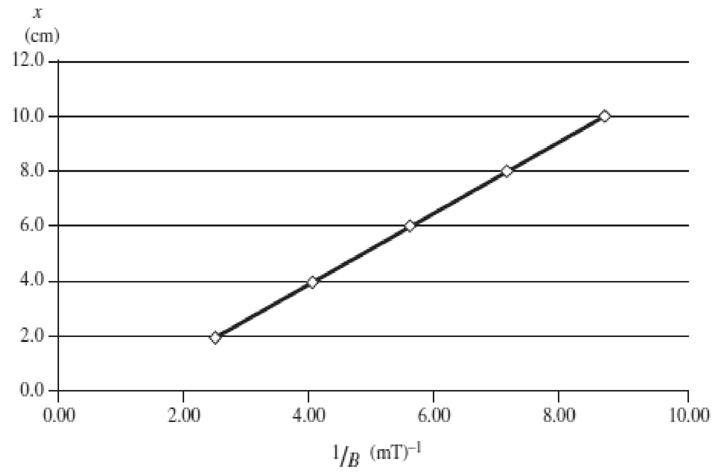


Figure 28.73

(c) The best-fit equation for this graph is $x = (1.2981\ \text{mT}\cdot\text{cm})\dfrac{1}{B} - 1.1914$ cm. The slope is $1.2981\ \text{mT}\cdot\text{cm} = 1.2981\times10^{-5}\ \text{T}\cdot\text{m}$. Since the slope is equal to $\mu_0 I/2\pi$, we have $\mu_0 I/2\pi$ = slope, which gives $I = 2\pi(\text{slope})/\mu_0 = 2\pi(1.2981\times10^{-5}\ \text{T}\cdot\text{m})/\mu_0 = 64.9\ \text{A}$, which rounds to 65 A. The *y*-intercept is $-R$, so $R = -(-1.1914\ \text{cm}) = 1.2$ cm.

EVALUATE: As we can see, the field within 2 cm or so of the surface of the cable would vary considerably from the value given by $B = \dfrac{\mu_0 I}{2\pi r}$.

28.74. **IDENTIFY** and **SET UP:** The wires repel each other since they carry currents in opposite directions, so the wires will move away from each other until the magnetic force is just balanced by the force due to the spring. The force per unit length between two parallel current-carrying wires of equal length and separation r is $\dfrac{F}{L} = \dfrac{\mu_0}{2\pi}\dfrac{I'I}{r}$. In this case, the currents are the same and the distance between the wires is $l_0 + x$, where x is the distance the spring stretches. Therefore the force is $F = \dfrac{\mu_0 I^2 L}{2\pi(l_0 + x)}$. The magnitude of the force that each spring exerts is $F = kx$, by Hooke's law. On each wire, $F_{\text{spr}} = F_{\text{mag}}$, and there are *two* spring forces on each wire. Therefore $\dfrac{\mu_0 I^2 L}{2\pi(l_0 + x)} = 2kx$.

EXECUTE: **(a)** We are given two cases with values for I and x, and each one leads to an equation involving l_0 and k. If we take the ratio of these two equations, common factors such as L will cancel. This gives us

$$\frac{(13.1\ \text{A})^2(l_0 + 0.40\ \text{m})}{(8.05\ \text{A})^2(l_0 + 0.80\ \text{m})} = \frac{0.80\ \text{cm}}{0.40\ \text{cm}} = 2.0.$$ Solving for l_0 gives $l_0 = 0.834$ cm, which rounds to 0.83 cm.

Now we can solve for k using this value for l_0 using $\dfrac{\mu_0 I^2 L}{2\pi(l_0 + x)} = 2kx$.

$$\frac{\mu_0(13.1\text{ A})^2(0.50\text{ m})}{2\pi(0.0080\text{ m}+0.00834\text{ m})}=2k(0.0080\text{ m}).$$ $k = 0.0656$ N/m, which rounds to 0.066 N/m.

(b) For a 12.0-A current, we have $\frac{\mu_0(12.0\text{ A})^2(0.50\text{ m})}{2\pi(x+0.00834\text{ m})}=2(0.0656\text{ N/m})x.$ Carrying out the multiplication and division and simplifying we get the quadratic equation
$x^2 + (0.00834\text{ m})x - 1.097\times10^{-4}\text{ m}^2 = 0.$

Using the quadratic formula and taking the positive solution gives $x = 0.0071$ m = 0.71 cm.

(c) To stretch the spring by 1.00 cm, the current must satisfy the equation

$$\frac{\mu_0 I^2(0.50\text{ m})}{2\pi(0.0100\text{ m}+0.00834\text{ m})}=2(0.0656\text{ N/m})(0.0100\text{ m}).$$ This gives $I = 15.5$ A, which rounds to 16 A.

EVALUATE: The spring force in part (c) is kx =(0.0656 N/m)(0.0100 m) = 6.56×10^{-4} N. This is a very small force resulting from a rather large 16-A current. This tells us that magnetic forces between parallel wires, such as extension cords, are not very significant for typical household currents.

28.75. **IDENTIFY:** The moving charges in the plasma cause a magnetic field.
SET UP and **EXECUTE:** **(a)** We want the charge density. There are n ions per unit volume, each with charge q, so $\rho = nq$.

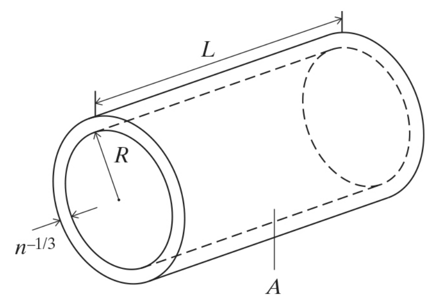


Figure 28.75a

(b) We want the surface charge density. See Fig. 28.75a. The "surface" has thickness equal to $n^{-1/3}$, so its volume is $V = 2\pi Rn^{-1/3}L.$ $\sigma A = \rho V,$ so $\sigma=\frac{\rho V}{A}=\frac{\rho 2\pi R n^{-1/3}L}{2\pi RL}=\rho n^{-1/3}=(nq)n^{-1/3}=n^{2/3}q.$

(c) We want B at the surface. Use Ampere's law, taking an integration path around the surface. The current density is $J = \rho v.$ This gives $B2\pi R=\mu_0 I=\mu_0(JA)=\mu_0\rho vA=\mu_0\rho v\pi R^2$. Solve for B and using $\rho = nq.$ $B=\frac{1}{2}\mu_0 nqvR.$ By the right-hand rule, its direction is tangent to the circle. So we can express the field as $\vec{B}=\frac{1}{2}\mu_0 nqvR\hat{\phi}.$

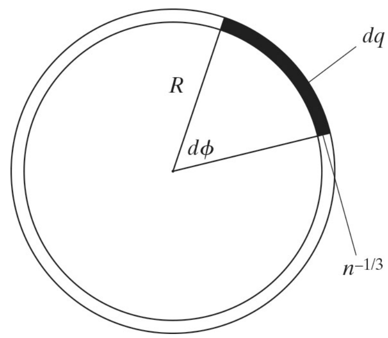


Figure 28.75b

(d) We want dI. Fig. 28.75b shows the geometric configuration. We know $\rho = nq$, $J = \rho v$, and $I = JA$. $dA = (Rd\phi)\left(n^{-1/3}\right)$. $dI = JdA = \rho v dA = \rho v\left(n^{-1/3}Rd\phi\right) = (qn)vn^{-1/3}Rd\phi = n^{2/3}qvRd\phi.$

(e) We want dF. $dF = dI(BL) = BL\left(n^{2/3}qvRd\phi\right) = \left(\frac{\mu_0}{2}nqvR\right)(L)\left(n^{2/3}qvRd\phi\right)$. This simplifies to $dF = \frac{\mu_0}{2}n^{5/3}q^2v^2R^2Ld\phi.$

(f) We want the force F and the pressure p. $F = \int_0^{2\pi} dF = \pi\mu_0 n^{5/3}q^2v^2R^2L$. The direction is inward toward the axis of the cylinder. $p = F/A$. Using F from above and $A = 2\pi RL$ gives $p = \frac{\mu_0}{2}n^{5/3}q^2v^2R.$

(g) We want the pressure. Using the numbers given in the problem, we get $p = 1.9\times10^{-5}$ Pa.

EVALUATE: This is a small pressure but it acts on very small objects (atoms).

28.76. IDENTIFY: We want to find the magnetic field inside a spinning charged cylinder.

SET UP and **EXECUTE:** **(a)** We want the surface charge density. $\sigma = \frac{Q}{A} = \frac{Q}{2\pi RW}.$

(b) We want dI. In one period T, the charge dq that passes along dx is $dq = \sigma dA = \sigma 2\pi Rdx$. The time for this charge to pass by is $T = 2\pi/\omega$, so $dI = \frac{dq}{T} = \frac{\sigma 2\pi Rdx}{2\pi/\omega} = \sigma\omega Rdx.$

(c) We want dB_x at the origin. Eq. (28.15): $B_x = \frac{\mu_0 Ia^2}{2\left(x^2+a^2\right)^{3/2}}$. Let a circular strip be at a distance x from the origin. In this case, $I \to dI, a \to R, x \to x$. Using this information and our results from parts (a) and (b), we get $dB_x = \frac{\mu_0 dI\,R^2}{2\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0\sigma\omega RdxR^2}{2\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0}{2}\left(\frac{Q}{2\pi RW}\right)\frac{\omega R^3dx}{\left(x^2+R^2\right)^{3/2}}$, which reduces to $dB_x = \frac{\mu_0 Q\omega R^2dx}{4\pi W\left(x^2+R^2\right)^{3/2}}$. By the right-hand rule, the direction is along the $+x$ axis.

(d) Integrate to find B_x. $B_x = 2\int_0^{W/2} dB_x = 2\int_0^{W/2}\frac{\mu_0 Q\omega R^2dx}{4\pi W\left(x^2+R^2\right)^{3/2}}$. Using the integral tables in Appendix B we find $B_x = \frac{\mu_0}{2\pi}\frac{Q\omega}{\sqrt{W^2+4R^2}}$. The full field is $\vec{B} = \frac{\mu_0}{2\pi}\frac{Q\omega}{\sqrt{W^2+4R^2}}\hat{\imath}.$

EVALUATE: The rotating cylinder is quite similar to a solenoid.

28.77. **IDENTIFY:** A spinning spherical shell produces a magnetic moment.

SET UP and **EXECUTE:** $\vec{\mu} = \gamma \vec{L}$ where γ is the gyromagnetic ratio. $\gamma = g\dfrac{Q}{2M}$ where g is the g-factor.

(a) We want dI. Refer to Fig. 28.77 with the problem in the textbook. $dI = dq/T$, where $T =$ $T = 2\pi/\omega$. dq is the charge in one complete turn in time T. $q = \sigma dA = \left(\dfrac{Q}{4\pi R^2}\right)(Rd\theta)2\pi r$.

$r = R\sin\theta$. So $dI = \dfrac{\left(\dfrac{Q}{4\pi R^2}\right)(Rd\theta)(2\pi)(R\sin\theta)}{2\pi/\omega} = \dfrac{Q\omega}{4\pi}\sin\theta d\theta.$

(b) We want $d\mu$. $d\mu = AdI = \pi r^2 dI = \pi(R\sin\theta)^2\left(\dfrac{Q\omega}{4\pi}\sin\theta d\theta\right) = \dfrac{1}{4}Q\omega R^2\sin^3\theta\, d\theta$. The direction is $+z$.

(c) We want $\vec{\mu}$. $\mu = \int d\mu = \int_0^\pi \dfrac{Q\omega R^2}{4}\sin^3\theta\, d\theta = \dfrac{Q\omega R^2}{4}\int_0^\pi \sin\theta(1-\cos^2\theta)d\theta = \dfrac{Q\omega R^2}{3}$. The direction is $+z$.

(d) We want the angular momentum. $L = I\omega = \dfrac{2}{3}MR^2\omega.$

(e) We want γ. $\gamma = \dfrac{\mu}{L} = \dfrac{Q\omega R^2/3}{2MR^2\omega/3} = \dfrac{Q}{2M}$.

(f) We want g. $\gamma = g\dfrac{Q}{2M}$, $L = \dfrac{\mu}{\gamma}$, and $L = \dfrac{2}{3}MR^2\omega$. Equate the two expressions for L and solve for g, which gives $g = 1$.

EVALUATE: If $g = 1$, $\gamma = \dfrac{Q}{2M}$ so $\mu = QL/2M$.

28.78. **IDENTIFY:** We are dealing with the magnetic field inside a rotating charged cylinder.

SET UP: $\vec{B} = \dfrac{\mu_0}{4\pi}\int \dfrac{\sigma\vec{v}\times\hat{r}}{r^2}dA$. Refer to the Fig. 28.78 with the problem in the textbook.

EXECUTE: **(a)** Find σ. $\sigma = Q/A = \dfrac{Q}{2\pi RW}$.

(b) We want $\vec{r}$. ϕ is the angle with the $+y$-axis in the yz-plane. $\vec{r}$ is the vector from point (x, y, z) to the origin (0, 0, 0). So $r_x = -x$, $r_{y-} = -R\cos\phi$, and $r_z = -R\sin\phi$. Therefore

$\vec{r} = -(x\hat{i} + R\cos\phi\hat{j} + R\sin\phi\hat{k})$.

(c) We want $\vec{v}$. The velocity is in the yz-plane. v_y is negative when $0 < \phi \le \pi$. So $v_x = 0$, $v_y = -R\omega\sin\phi$, $v_z = R\omega\cos\phi$. So $\vec{v} = -R\omega\sin\phi\hat{j} + R\omega\cos\phi\hat{k}$.

(d) We want $\vec{v}\times\hat{r}$. $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{\vec{r}}{\sqrt{x^2+R^2}}$. Use this fact, along with the results from (b) and (c), to take the cross product $\vec{v}\times\hat{r}$. Combining and simplifying gives

$\vec{v}\times\hat{r} = \dfrac{R\omega}{\sqrt{x^2+R^2}}\left[R\hat{i} - x\left(\cos\phi\hat{j} + \sin\phi\hat{k}\right)\right]$.

(e) Integrate to find $\vec{B}$. $\vec{B} = \dfrac{\mu_0}{4\pi}\int \dfrac{\sigma\vec{v}\times\hat{r}}{r^2}dA$ where $dA = R\,dx\,d\phi$. Use

$\vec{v}\times\hat{r} = \dfrac{R\omega}{\sqrt{x^2+R^2}}\left[R\hat{i} - x\left(\cos\phi\hat{j} + \sin\phi\hat{k}\right)\right]$ from part (d). The integral is

$\vec{B} = \dfrac{\mu_0 \sigma}{4\pi} \int_{-W/2}^{W/2} \int_{\phi=0}^{2\pi} \dfrac{R\omega}{(x^2+R^2)\sqrt{x^2+R^2}} \left[R\hat{i} - x(\cos\phi \hat{j} + \sin\phi \hat{k}) \right] R\,dx\,d\phi.$ Carrying out the integration

(which takes some time) yields $\vec{B} = \dfrac{\mu_0}{2\pi} \dfrac{Q\omega}{\sqrt{W^2+4R^2}} \hat{i}.$

EVALUATE: **(f)** Our answer agrees with the result of problem 28.76.

28.79. **IDENTIFY:** The current-carrying wires repel each other magnetically, causing them to accelerate horizontally. Since gravity is vertical, it plays no initial role.

SET UP: The magnetic force per unit length is $\dfrac{F}{L} = \dfrac{\mu_0}{2\pi} \dfrac{I^2}{d}$, and the acceleration obeys the equation $F/L = m/L\ a$. The rms current over a short discharge time is $I_0/\sqrt{2}$.

EXECUTE: **(a)** First get the force per unit length: $\dfrac{F}{L} = \dfrac{\mu_0}{2\pi} \dfrac{I^2}{d} = \dfrac{\mu_0}{2\pi d}\left(\dfrac{I_0}{\sqrt{2}}\right)^2 = \dfrac{\mu_0}{4\pi d}\left(\dfrac{V}{R}\right)^2 = \dfrac{\mu_0}{4\pi d}\left(\dfrac{Q_0}{RC}\right)^2.$

Now apply Newton's second law using the result above: $\dfrac{F}{L} = \dfrac{m}{L} a = \lambda a = \dfrac{\mu_0}{4\pi d}\left(\dfrac{Q_0}{RC}\right)^2$. Solving for a gives $a = \dfrac{\mu_0 Q_0^2}{4\pi\lambda R^2 C^2 d}$. From the kinematics equation $v_x = v_{0x} + a_x t$, we have $v_0 = at = aRC = \dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}.$

(b) Conservation of energy gives $\frac{1}{2}mv_0^2 = mgh$ and $h = \dfrac{v_0^2}{2g} = \dfrac{\left(\dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2}{2g} = \dfrac{1}{2g}\left(\dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2.$

EVALUATE: Once the wires have swung apart, we would have to consider gravity in applying Newton's second law.

28.80. **IDENTIFY:** Approximate the moving belt as an infinite current sheet.

SET UP: Problem 28.69 shows that $B = \frac{1}{2}\mu_0 In$ for an infinite current sheet. Let L be the width of the sheet, so $n = 1/L$.

EXECUTE: The amount of charge on a length Δx of the belt is $\Delta Q = L\Delta x\sigma$, so $I = \dfrac{\Delta Q}{\Delta t} = L\dfrac{\Delta x}{\Delta t}\sigma = Lv\sigma$. Approximating the belt as an infinite sheet $B = \dfrac{\mu_0 I}{2L} = \dfrac{\mu_0 v\sigma}{2}$. $\vec{B}$ is directed out of the page, as shown in Figure 28.80.

EVALUATE: The field is uniform above the sheet, for points close enough to the sheet for it to be considered infinite.

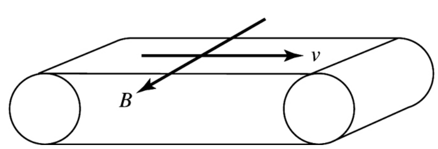


Figure 28.80

28.81. **IDENTIFY** and **SET UP:** This solenoid is not ideal since its width is fairly large compared to its length. But we can get a rough estimate using the ideal formula, $B = \mu_0 nI$.

EXECUTE: $B = \mu_0 nI = \mu_0 (1000\ \text{m}^{-1})I = 150\times10^{-6}\ \text{T}$, which gives $I = 0.12$ A, choice (b).

EVALUATE: This is a reasonable laboratory current of 120 mA.

28.82. **IDENTIFY** and **SET UP:** The magnetic field of an ideal solenoid is $B = \mu_0 nI$.

EXECUTE: Both solenoids have the same current, the same length, and the same number of turns, so the magnetic field inside both of them should be the same, which is choice (c).

EVALUATE: This answer is somewhat of an approximation. Even though both solenoids have the same current and same length and number of turns, the second (larger) solenoid is even farther from the ideal case than the first one. Therefore there would be some difference in the magnetic fields inside.

28.83. **IDENTIFY** and **SET UP:** The enclosure is no longer present to shield the solenoid from the earth's magnetic field of $50\ \mu\text{T}$, so net field inside is a sum of the solenoid field and the earth's field. Whether the earth's field adds or subtracts from the solenoid's field depends on the orientation of the solenoid. The magnetic field due to the solenoid is $150\ \mu\text{T}$.

EXECUTE: When the solenoid field is parallel to the earth's field, the net field is $150\ \mu\text{T} + 50\ \mu\text{T} = 200\ \mu\text{T}$. When the field's are antiparallel (opposite), the net field is $150\ \mu\text{T} - 50\ \mu\text{T} = 100\ \mu\text{T}$. So the field that the bacteria experience is between $100\ \mu\text{T}$ and $200\ \mu\text{T}$, which is choice (c).

EVALUATE: Since the earth's field is quite appreciable compared to the solenoid's field, it is important to shield the solenoid from external fields, such as that of the earth. The earth's field can make a difference of up to a factor of 2 in the field experienced by the bacteria.

29

ELECTROMAGNETIC INDUCTION

VP29.8.1. **IDENTIFY:** This problem is about magnetic flux and induced emfs. It requires use of Faraday's law.

SET UP: $\Phi_B = BA\cos\phi$, $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$.

EXECUTE: **(a)** We want the flux. $A = \pi r^2$. $\Phi_B = BA\cos\phi$.

At $t = 0$: $\Phi_B = (0.140\text{ T})\pi(0.0240\text{ m})^2\cos 0° = 2.53\times10^{-4}\text{ T}\cdot\text{m}^2 = 2.53\times10^{-4}\text{ Wb}$.

At $t = 2.00$ s: $\Phi_B = (0.110\text{ T})\pi(0.0240\text{ m})^2\cos 180° = -1.99\times10^{-4}\text{ T}\cdot\text{m}^2 = -1.99\times10^{-4}\text{ Wb}$.

(b) We want the emf. Since this is a steady rate of change, and we only want the magnitude of the emf, we can use $\mathcal{E} = \dfrac{\Delta\Phi_B}{\Delta t} = \dfrac{\left|-1.99\times10^{-4}\text{ T}\cdot\text{m}^2 - 2.53\times10^{-4}\text{ T}\cdot\text{m}^2\right|}{2.00\text{ s}} = 0.226\text{ mV}$.

EVALUATE: Careful! Flux can be negative as well as positive.

VP29.8.2. **IDENTIFY:** This problem is about magnetic flux and induced emfs. It requires use of Faraday's law.

SET UP: $\Phi_B = BA\cos\phi$, $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$.

EXECUTE: **(a)** We want the emf. Since this is a steady rate of change, and we only want the magnitude of the emf, we can use $\mathcal{E} = N\dfrac{\Delta\Phi_B}{\Delta t}$. $\Phi_B = BA\cos\phi = B\pi r^2\cos\phi$.

$\mathcal{E} = N\dfrac{\Delta\Phi_B}{\Delta t} = N\dfrac{\Delta BA\cos\phi}{\Delta t} = NA\cos\phi\dfrac{\Delta B}{\Delta t} = N\pi r^2\cos\phi\dfrac{\Delta B}{\Delta t}$. Using $\phi = 20.0°$, $N = 455$ turns, $\dfrac{\Delta B}{\Delta t} = -3.00\times10^{-3}$ T/s, and $r = 0.0500$ m, we get for the magnitude $\mathcal{E} = 10.1$ mV.

(b) We want the current. $I = \mathcal{E}/R = (10.1\text{ mV})/(14.5\ \Omega) = 0.695$ mA.

EVALUATE: It is not the strength of the magnetic field that induces an emf. It is the *rate* at which the field changes that does it.

VP29.8.3. **IDENTIFY:** This problem is about changing flux so we need to use Faraday's law.

SET UP: $\Phi_B = \vec{B}\bullet\vec{A}$, $\mathcal{E} = N\dfrac{\Delta\Phi_B}{\Delta t}$.

EXECUTE: **(a)** We want the magnitude of the emf. $\Phi_B = \vec{B}\bullet\vec{A} = B_zA_z$, so $\mathcal{E} = N\dfrac{\Delta\Phi_B}{\Delta t}$ gives

$$\mathcal{E} = N\frac{A_z\Delta B_z}{\Delta t} = (875)\frac{\left|(0.0400\text{ m})^2(-0.200\text{ T} - 0.150\text{ T})\right|}{3.00\text{ s}} = 0.163\text{ V}.$$

(b) B_z is decreasing so the flux through the coil is decreasing. Initially B_z was positive and after 3.00 s it was negative, so the flux went from positive to negative. Therefore the induced magnetic field must be *counterclockwise* to oppose this change.

EVALUATE: It is the *change* in flux that determines the direction of the current, not the field direction alone.

VP29.8.4. **IDENTIFY:** This problem requires the use of Faraday's law.

SET UP: $\Phi_B = BA\cos\phi$, $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$.

EXECUTE: **(a)** We want the emf. $\mathcal{E} = -N\dfrac{d(BA)}{dt} = -NA\dfrac{d(B_0 \sin\omega t)}{dt} = -NA\omega B_0 \cos\omega t$.

(b) At $t = 0$, $B = 0$ but increasing in the $+z$-direction, so the induced current is in the *clockwise* direction to oppose the flux increase. At $t = \pi/\omega$, $B = 0$ but increasing in the $-z$-direction. So the induced current is *counterclockwise* to oppose the flux increase.

EVALUATE: It is not the flux that is important, but rather the *rate of change* of the flux.

VP29.9.1. **IDENTIFY:** The movement causes a motional emf.

SET UP: $\mathcal{E} = vBL$, $E = V/d$.

EXECUTE: **(a)** We want the emf. $\mathcal{E} = vBL$ = (3.90 m/s)(0.600 T)(0.0800 m) = 0.187 V.

(b) We want electric field. $E = V/d = (vBL)/L = vB$ = (3.90 m/s)(0.600 T) = 2.34 V/m. The force on a positive charge is $\vec{F} = q\vec{v}\times\vec{B}$ in the $+x$-direction, so $\vec{E} = 2.34\ \text{V/m}\,\hat{i}$.

EVALUATE: The electric field is due to the separation of charges in the conducting rod by the magnetic force.

VP29.9.2. **IDENTIFY:** The movement causes a motional emf.

SET UP: $\mathcal{E} = vBL$.

EXECUTE: $E = vBL = RI$, so $I = vBL/R$ = (2.40 m/s)(0.150 T)(0.500 m)/(0.0200 Ω) = 0.900 A. Positive charges in the rod experience a magnetic force $\vec{F} = q\vec{v}\times\vec{B}$ downward, so the current is *clockwise* around the circuit.

EVALUATE: The flux is decreasing so the current flows to increase it, which is clockwise.

VP29.9.3. **IDENTIFY:** A motional emf is caused by the movement of the rod.

SET UP: $\mathcal{E} = \left(\vec{v}\times\vec{B}\right)\cdot\vec{L}$.

EXECUTE: **(a)** We want the emf. Use the given vectors for $\vec{v}$ and $\vec{B}$ to find $\vec{v}\times\vec{B}$. Then do the dot product of that with $\vec{L}$. $\vec{v}\times\vec{B} = 0.300\ \text{T}\cdot\text{m/s}\,\hat{i} - 2.25\,\text{T}\cdot\text{m/s}\,\hat{j} + 2.40\,\text{T}\cdot\text{m/s}\,\hat{k}$ and $\vec{L} = 0.0800\ \text{m}\,\hat{i}$.

$\mathcal{E} = \left(\vec{v}\times\vec{B}\right)_x \vec{L}_x$ = (0.300 T · m/s)(0.0800 m) = 0.0240 V = 24.0 mV.

(b) The x component of $\vec{v}\times\vec{B}$ is positive, so F_x is positive, so positive charges move toward b and negative ones toward a. Therefore b is at a higher potential than a.

EVALUATE: The right-hand rule for $\vec{F} = q\vec{v}\times\vec{B}$ shows us that F_x is toward the $+x$-direction, which supports our result.

VP29.9.4. **IDENTIFY:** A motional emf is caused by the movement of the rod.

SET UP: $\mathcal{E} = vBL$ and $F = ILB$. The speed of the rod is the target variable.

EXECUTE: $\mathcal{E} = vBL = RI$. $F = ILB$, so $I = F/LB$. Combining these equations gives $vBL = RF/LB$. Solving for v gives $v = RF/(LB)^2$. Using the given numbers gives v = 4.12 m/s.

EVALUATE: Check: The current at this instant is $I = F/LB$ = 0.5742 A. $v = RI/LB$ = 4.12 m/s, which agrees with our result.

VP29.11.1. **IDENTIFY:** This problem involves an induced emf in a solenoid. We need to use Faraday's law.

SET UP: $B = \mu_0 nI$, $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$, $E = V/d$. We need only magnitudes.

EXECUTE: **(a)** We want n. $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \dfrac{d(BA)}{dt} = \dfrac{d(\mu_0 InA)}{dt} = \mu_0 nA\dfrac{dI}{dt}$. Solve for n and use the given numbers. $n = \dfrac{\mathcal{E}}{\mu_0 A(dI/dt)}$ = 1810 turns/m.

(b) We want E within the loop. $E = V/d = V/2\pi r = (15.0\ \mu\text{V})/[2\pi(0.0310\ \text{m})] = 77.0\ \mu\text{V/m}$.

EVALUATE: 1810 turns/m is a reasonable turn density since wires are rather thin. The electric field is small compared to typical laboratory fields.

VP29.11.2. **IDENTIFY:** The changing magnetic field causes a changing flux which induces an emf in the loop. Faraday's law applies.

SET UP: $\mathcal{E} = Ed$, $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$. The target variable is the magnitude of the induced electric field in the wire.

EXECUTE: $\mathcal{E} = Ed = E(2\pi r)$. $\mathcal{E} = \dfrac{d\Phi_B}{dt} = \dfrac{d(BA)}{dt} = A\dfrac{dB}{dt} = \pi r^2\dfrac{dB}{dt}$. Equate the two expressions for $\mathcal{E}$ and solve for E. $\mathcal{E} = \dfrac{r}{2}\dfrac{dB}{dt} = [(0.00360\ \text{m})/2](0.0150\ \text{T/s}) = 27.0\ \mu\text{V/m}$.

EVALUATE: The electric field is small compared to typical lab fields, but after all, B is changing quite slowly.

VP29.11.3. **IDENTIFY:** The changing magnetic field inside the solenoid causes a changing flux through the wire loop surrounding the solenoid. This induces an emf in the loop. Faraday's law applies.

SET UP: $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$, $B = \mu_0 nI$, $E = V/d$. The target variable is dI/dt in the *solenoid*. We must be very careful to distinguish between quantities that pertain to the solenoid and those that pertain to the loop. To do so, we use subscripts L for the loop and S for the solenoid. Fig. VP29.11.3 illustrates the loop inside the solenoid. Let r be the radius of the loop. Recall that the magnetic field of the solenoid is zero outside of it.

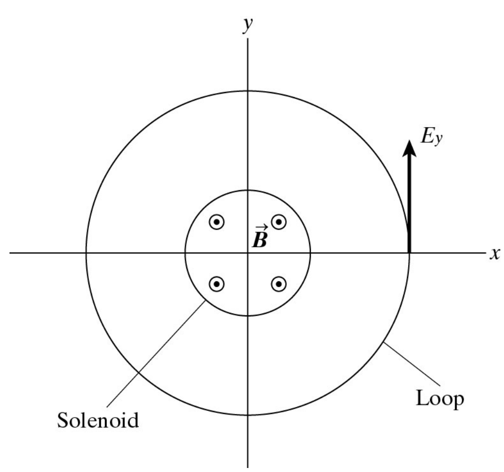


Figure VP29.11.3

EXECUTE: (a) $E_y = +1.20\times10^{-5}$ V/m. $\mathcal{E}_L = \dfrac{d\Phi_L}{dt} = \dfrac{d(B_S A_S)}{dt} = \dfrac{d(\mu_0 I_S n A_S)}{dt} = \mu_0 n A_S \dfrac{dI_S}{dt}$.

Using $E = V/d$, we have $E_y = \dfrac{\mathcal{E}_L}{2\pi r}$. This gives $E_y = \dfrac{\mathcal{E}_L}{2\pi r} = \dfrac{\mu_0 n A_S \dfrac{dI_S}{dt}}{2\pi r}$. Solving for dI_S/dt gives

$\dfrac{dI_S}{dt} = \dfrac{2\pi r E_y}{\mu_0 n A_S} = \dfrac{2\pi(0.0500\text{ m})(1.20\times10^{-5}\text{ V/m})}{\mu_0(965\text{ turns/m})(4.00\times10^{-4}\text{ m}^2)} = 7.77\text{ A/s}$. As shown in Fig. VP29.11.3, E_y is positive at the point (5.00 cm, 0, 0), so the current in the loop is *counterclockwise*. That current produces a magnetic field inside the solenoid that is in the $+z$-direction—the same as the field of the solenoid. This means that the flux through the solenoid must be decreasing, so I_S is decreasing, which means that dI_S/dt is negative, so $dI_s/dt = -7.77$ A/s.

(b) In this case, $E_y = -1.80\times10^{-5}$ V/m. Use the same approach as in part (a) except that E_y is negative and has a different numerical value. Since E_y is negative, I_S must be increasing, so dI_S/dt is positive. Using the new value for E_y, the result is $dI_S/dt = +11.7$ A/s.

EVALUATE: Only E_y changes, so it should follow that $\dfrac{dI_S/dt_{part\ a}}{dI_S/dt_{part\ b}} = \dfrac{E_a}{E_b} = \dfrac{1.20}{1.80} = 0.667$. Our result gives $\dfrac{7.77}{11.7} = 0.664$. The two results agree; the small difference is due to rounding during the numerical calculations.

VP29.11.4. IDENTIFY: The changing magnetic field inside the solenoid causes a changing flux through the wire loop surrounding the solenoid. This induces an emf in the loop. Faraday's law applies.

SET UP: $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$, $B = \mu_0 n I$. The target variable is the current in the loop at $t = 13.9$ s. We must be very careful to distinguish between quantities that pertain to the solenoid and those that pertain to the loop. To do so, we use subscripts L for the loop and S for the solenoid. Let r be the radius of the loop. Recall that the magnetic field of the solenoid is zero outside of it.

EXECUTE: $\mathcal{E}_L = \dfrac{d\Phi_L}{dt} = \dfrac{d(B_S A_S)}{dt} = \dfrac{d(\mu_0 I_S n A_S)}{dt} = \mu_0 n A_S \dfrac{dI_S}{dt}$. $I_L = \dfrac{\mathcal{E}_L}{R} = \dfrac{\mu_0 n A_S \dfrac{dI_S}{dt}}{R}$. At time $t = 13.9$ s, $dI_S/dt = d\left[\left(0.600\text{ A/s}^2\right)t^2\right]/dt = (1.20\text{ A/s}^2)t = (1.20\text{ A/s}^2)(13.9\text{ s}) = 16.68$ A/s. Using this quantity and the other given quantities, we get $I_L = 5.52\ \mu$A.

EVALUATE: We use the cross-sectional area of the solenoid to find the flux through the loop because B_S is essentially zero outside the solenoid.

29.1. IDENTIFY: The changing magnetic field causes a changing magnetic flux through the loop. This induces an emf in the loop which causes a current to flow in it.

SET UP: $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right|$, $\Phi_B = BA\cos\phi$, $\phi = 0°$. A is constant and B is changing.

EXECUTE: (a) $|\mathcal{E}| = A\dfrac{dB}{dt} = (0.0900\text{ m}^2)(0.190\text{ T/s}) = 0.0171\text{ V}$.

(b) $I = \dfrac{\mathcal{E}}{R} = \dfrac{0.0171\text{V}}{0.600\,\Omega} = 0.0285\text{ A}$.

EVALUATE: These are small emfs and currents by everyday standards.

29.2. IDENTIFY: $|\mathcal{E}| = N\left|\dfrac{d\Phi_B}{dt}\right|$. $\Phi_B = BA\cos\phi$. Φ_B is the flux through each turn of the coil.

SET UP: $\phi_\text{i} = 0°$. $\phi_\text{f} = 90°$.

EXECUTE: **(a)** $\Phi_{B,\text{i}} = BA\cos 0° = (6.0\times10^{-5}\ \text{T})(12\times10^{-4}\ \text{m}^2)(1) = 7.2\times10^{-8}\ \text{Wb}$.

$\Phi_{B,\text{f}} = BA\cos 90° = 0.$ So $\Phi_B = 7.2\times10^{-8}$ Wb.

(b) $|\mathcal{E}_\text{av}| = N\left|\dfrac{\Phi_\text{i}-\Phi_\text{f}}{\Delta t}\right| = 200\left(\dfrac{7.2\times10^{-8}\ \text{Wb}}{0.040\ \text{s}}\right) = \dfrac{1.44\times10^{-5}\ \text{Wb}}{0.040\ \text{s}} = 3.6\times10^{-4}\ \text{V} = 0.36\ \text{mV}.$

EVALUATE: The average induced emf depends on how rapidly the flux changes.

29.3. IDENTIFY: The changing flux through the coil induces an emf in it. Faraday's law applies.

SET UP: $\Phi_\text{B} = BA\cos\phi$, $\mathcal{E} = -\dfrac{d\Phi_\text{B}}{dt}$.

EXECUTE: **(a)** We want the units. Magnetic flux has units of webers (Wb). Therefore α has units of Wb/s and β has units of Wb/s^3.

(b) We want to relate α and β. $\mathcal{E} = -\dfrac{d\Phi_\text{B}}{dt} = -\dfrac{d(\alpha t - \beta t^3)}{dt} = -\alpha + 3\beta t^2$. At t = 0.500 s, $\mathcal{E}$ = 0, so

$-\alpha + 3\beta(0.500\ \text{s})^2 = 0.$ So $\alpha = (0.750\ \text{s}^2)\beta$.

(c) We want the emf at t = 0.250 s. <u>At t = 0</u>: β = –1.60 V, so –1.60 V = $-\alpha + 3\beta(0)$, so $\alpha = 1.60$ V.

$\alpha = (0.750\ \text{s}^2)\beta$ so 1.60 V = $(0.750\ \text{s}^2)\beta$. β = 2.13 V/s^2.

<u>At t = 0.250 s</u>: $\mathcal{E} = -\alpha + 3\beta t^2$ = –1.60 V + 3(2.13 V/s^2)(0.250 s)^2 = –1.20 V.

EVALUATE: Comparing the units from (a) and (c) shows that Wb/s = V and Wb/s^3 = V/s^2, and these results are consistent with each other.

29.4. IDENTIFY: We are dealing with an induced emf due to changing magnetic flux.

SET UP and **EXECUTE:** **(a)** We want current. $\mathcal{E}_\text{av} = N\dfrac{\Delta\Phi_\text{B}}{\Delta t} = N\dfrac{BA}{\Delta t} = \dfrac{NAB}{\Delta t}$. $I_\text{av} = \dfrac{\mathcal{E}_\text{av}}{R} = \dfrac{NAB}{R\Delta t}$.

(b) We want the charge. $Q = I_\text{av}\Delta t = \dfrac{NAB}{R}$.

(c) We want the charge. Use $Q = \dfrac{NAB}{R}$ with the given numbers. Q = 0.450 mC.

EVALUATE: The *rate* of flux change affects the current but not the charge that flows.

29.5. IDENTIFY: Apply Faraday's law.

SET UP: Let $+z$ be the positive direction for $\vec{A}$. Therefore, the initial flux is positive and the final flux is zero.

EXECUTE: **(a)** and **(b)** $\mathcal{E} = -\dfrac{\Delta\Phi_B}{\Delta t} = -\dfrac{0-(1.5\ \text{T})\pi(0.120\ \text{m})^2}{2.0\times10^{-3}\ \text{s}} = +34\ \text{V}$. Since $\mathcal{E}$ is positive and $\vec{A}$ is toward us, the induced current is counterclockwise.

EVALUATE: The shorter the removal time, the larger the average induced emf.

29.6. IDENTIFY: Apply $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$ and $I = \mathcal{E}/R$.

SET UP: $d\Phi_B/dt = AdB/dt$.

EXECUTE: **(a)** $|\mathcal{E}| = \dfrac{Nd\Phi_B}{dt} = NA\dfrac{d}{dt}(B) = NA\dfrac{d}{dt}((0.012\ \text{T/s})t + (3.00\times10^{-5}\ \text{T/s}^4)t^4)$.

$|\mathcal{E}| = NA((0.012\ \text{T/s}) + (1.2\times10^{-4}\ \text{T/s}^4)t^3) = 0.0302\ \text{V} + (3.02\times10^{-4}\ \text{V/s}^3)t^3$.

(b) At $t = 5.00$ s, $|\mathcal{E}| = 0.0302 \text{ V} + (3.02\times10^{-4} \text{ V/s}^3)(5.00 \text{ s})^3 = 0.0680 \text{ V}$.

$I = \dfrac{\mathcal{E}}{R} = \dfrac{0.0680 \text{ V}}{600\ \Omega} = 1.13\times10^{-4} \text{ A}$.

EVALUATE: The rate of change of the flux is increasing in time, so the induced current is not constant but rather increases in time.

29.7. **IDENTIFY:** Calculate the flux through the loop and apply Faraday's law.

SET UP: To find the total flux integrate $d\Phi_B$ over the width of the loop. The magnetic field of a long straight wire, at distance *r* from the wire, is $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** $B = \dfrac{\mu_0 i}{2\pi r}$, into the page.

(b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r} Ldr$.

(c) $\Phi_B = \int_a^b d\Phi_B = \dfrac{\mu_0 iL}{2\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 iL}{2\pi}\ln(b/a)$.

(d) $|\mathcal{E}| = \dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 L}{2\pi}\ln(b/a)\dfrac{di}{dt}$.

(e) $|\mathcal{E}| = \dfrac{\mu_0 (0.240 \text{ m})}{2\pi}\ln(0.360/0.120)(9.60 \text{ A/s}) = 5.06\times10^{-7} \text{ V}$.

EVALUATE: The induced emf is proportional to the rate at which the current in the long straight wire is changing

29.8. **IDENTIFY:** Apply Faraday's law.

SET UP: Let $\vec{A}$ be upward in Figure E29.8 in the textbook.

EXECUTE: **(a)** $|\mathcal{E}_{\text{ind}}| = \left|\dfrac{d\Phi_B}{dt}\right| = \left|\dfrac{d}{dt}(B_\perp A)\right|$.

$|\mathcal{E}_{\text{ind}}| = A\sin 60°\left|\dfrac{dB}{dt}\right| = A\sin 60°\left|\dfrac{d}{dt}\left((1.4 \text{ T})e^{-(0.057\text{s}^{-1})t}\right)\right| = (\pi r^2)(\sin 60°)(1.4 \text{ T})(0.057 \text{ s}^{-1})e^{-(0.057 \text{ s}^{-1})t}$.

$|\mathcal{E}_{\text{ind}}| = \pi(0.75 \text{ m})^2(\sin 60°)(1.4 \text{ T})(0.057 \text{ s}^{-1})e^{-(0.057 \text{ s}^{-1})t} = (0.12 \text{ V})\ e^{-(0.057 \text{ s}^{-1})t}$.

(b) $\mathcal{E} = \frac{1}{10}\mathcal{E}_0 = \frac{1}{10}(0.12 \text{ V})$. $\frac{1}{10}(0.12 \text{ V}) = (0.12 \text{ V})\ e^{-(0.057 \text{ s}^{-1})t}$. $\ln(1/10) = -(0.057 \text{ s}^{-1})t$ and $t = 40.4$ s.

(c) $\vec{B}$ is in the direction of $\vec{A}$ so Φ_B is positive. *B* is getting weaker, so the magnitude of the flux is decreasing and $d\Phi_B/dt < 0$. Faraday's law therefore says $\mathcal{E} > 0$. Since $\mathcal{E} > 0$, the induced current must flow *counterclockwise* as viewed from above.

EVALUATE: The flux changes because the magnitude of the magnetic field is changing.

29.9. **IDENTIFY** and **SET UP:** Use Faraday's law to calculate the emf (magnitude and direction). The direction of the induced current is the same as the direction of the emf. The flux changes because the area of the loop is changing; relate dA/dt to dc/dt, where *c* is the circumference of the loop.

(a) EXECUTE: $c = 2\pi r$ and $A = \pi r^2$ so $A = c^2/4\pi$.

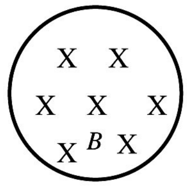
$\Phi_B = BA = (B/4\pi)c^2$.

$|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = \left(\dfrac{B}{2\pi}\right)c\left|\dfrac{dc}{dt}\right|$.

At $t = 9.0$ s, $c = 1.650 \text{ m} - (9.0 \text{ s})(0.120 \text{ m/s}) = 0.570 \text{ m}$.

$|\mathcal{E}| = (0.500 \text{ T})(1/2\pi)(0.570 \text{ m})(0.120 \text{ m/s}) = 5.44 \text{ mV}$.

(b) SET UP: The loop and magnetic field are sketched in Figure 29.9.



Take into the page to be the positive direction for $\vec{A}$.
Then the magnetic flux is positive.

Figure 29.9

EXECUTE: The positive flux is decreasing in magnitude; $d\Phi_B/dt$ is negative and $\mathcal{E}$ is positive. By the right-hand rule, for $\vec{A}$ into the page, positive $\mathcal{E}$ is clockwise.
EVALUATE: Even though the circumference is changing at a constant rate, dA/dt is not constant and $|\mathcal{E}|$ is not constant. Flux $\otimes$ is decreasing so the flux of the induced current is $\otimes$ and this means that I is clockwise, which checks.

29.10. **IDENTIFY:** Rotating the coil changes the angle between it and the magnetic field, which changes the magnetic flux through it. This change induces an emf in the coil.
SET UP: $\mathcal{E}_{\text{av}} = N\left|\frac{\Delta\Phi_B}{\Delta t}\right|$, $\Phi_B = BA\cos\phi$. ϕ is the angle between the normal to the loop and $\vec{B}$, so $\phi_{\text{i}} = 90.0° - 37.0° = 53.0°$ and $\phi_{\text{f}} = 0°$.
EXECUTE: $\mathcal{E}_{\text{av}} = \frac{NBA|\cos\phi_{\text{f}} - \cos\phi_{\text{i}}|}{\Delta t} = \frac{(80)(1.70\text{ T})(0.250\text{ m})(0.400\text{ m})}{0.0600\text{ s}}|\cos 0° - \cos 53.0°| = 90.3\text{ V}.$
EVALUATE: The flux changes because the orientation of the coil relative to the magnetic field changes, even though the field remains constant.

29.11. **IDENTIFY:** We are dealing with an induced emf due to changing magnetic flux.
SET UP: $\mathcal{E} = -\frac{d\Phi_{\text{B}}}{dt}$, $\mathcal{E} = RI$, $B = B_0 e^{-t/\tau}$. The target variable is the current.
EXECUTE: **(a)** $\mathcal{E} = \frac{dBA}{dt} = \frac{dB_0 e^{-t/\tau} A}{dt} = -\frac{B_0 A}{\tau} e^{-t/\tau}$. $|I| = \frac{\mathcal{E}}{R} = \frac{B_0 A}{R\tau} e^{-t/\tau}$. I is a maximum when $t = 0$.
Using $A = \pi r^2$ and the given values, we get $I_{\max} = 12.6$ mA.
(b) At $t = 1.50$ s, we use the result from (a) for the emf with $t = 1.50$ s and $I_{\max} = 12.6$ mA. This gives $I = 0.626$ mA $= 626\ \mu$A.
EVALUATE: With exponential decay such as this, the current initially decreases rapidly. Note that $e^{-1.5/0.5} = e^{-3} \approx 0.05$.

29.12. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.
SET UP: The flux through a coil is $\Phi_B = NBA\cos\phi$ and the induced emf is $\mathcal{E} = -d\Phi_B/dt$.
EXECUTE: The flux is constant in each case, so the induced emf is zero in all cases.
EVALUATE: Even though the coil is moving within the magnetic field and has flux through it, this flux is not *changing*, so no emf is induced in the coil.

29.13. **IDENTIFY:** Apply the results of Example 29.3.
SET UP: $\mathcal{E}_{\max} = NBA\omega$.
EXECUTE: $\omega = \frac{\mathcal{E}_{\max}}{NBA} = \frac{2.40\times10^{-2}\text{ V}}{(120)(0.0750\text{ T})(0.016\text{ m})^2} = 10.4\text{ rad/s}.$
EVALUATE: We may also express ω as 99.3 rev/min or 1.66 rev/s.

29.14. **IDENTIFY:** The changing flux through the loop due to the changing magnetic field induces a current in the wire. Energy is dissipated by the resistance of the wire due to the induced current in it.

SET UP: The magnitude of the induced emf is $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = \pi r^2 \left|\dfrac{dB}{dt}\right|$, $P = I^2 R$, $I = \mathcal{E}/R$.

EXECUTE: **(a)** $\vec{B}$ is out of page and Φ_B is decreasing, so the field of the induced current is directed out of the page inside the loop and the induced current is counterclockwise.

(b) $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = \pi r^2 \left|\dfrac{dB}{dt}\right|$. The current due to the emf is

$I = \dfrac{|\mathcal{E}|}{R} = \dfrac{\pi r^2}{R}\left|\dfrac{dB}{dt}\right| = \dfrac{\pi(0.0480\ \text{m})^2}{0.160\ \Omega}(0.680\ \text{T/s}) = 0.03076\ \text{A}$. The rate of energy dissipation is

$P = I^2 R = (0.03076\ \text{A})^2(0.160\ \Omega) = 1.51\times10^{-4}\ \text{W}$.

EVALUATE: Both the current and resistance are small, so the power is also small.

29.15. **IDENTIFY** and **SET UP:** The field of the induced current is directed to oppose the change in flux.
EXECUTE: **(a)** The field is into the page and is increasing so the flux is increasing. The field of the induced current is out of the page. To produce field out of the page the induced current is counterclockwise.
(b) The field is into the page and is decreasing so the flux is decreasing. The field of the induced current is into the page. To produce field into the page the induced current is clockwise.
(c) The field is constant so the flux is constant and there is no induced emf and no induced current.
EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.16. **IDENTIFY** and **SET UP:** Use Lenz's law. The induced current flows so as to oppose the flux change that is inducing it. The magnetic field due to I is out of the page for loops A and C and into the page for loops B and D. The field is constant since I is constant, so any flux change is due to the motion of the loops.
EXECUTE: **(a)** *A:* The loop is moving away from the wire, so the magnetic field through the loop is getting weaker. This results in decreasing flux through the loop. Since the field is out of the page, the induced current flows in a direction so that its magnetic field inside the loop will be out of the page, which is a counterclockwise direction.
B: The flux through the loop is decreasing with the magnetic field into the page, so the induced current is clockwise.
C: The flux through the loop is constant, so there is no induced current.
D: The flux through the loop is increasing with the field into the page, so the induced current is counterclockwise.
(b) *A:* The flux is decreasing, so the loop is pulled toward the wire to increase the flux through the loop.
B: The flux is decreasing, so the loop is pulled toward the wire to increase the flux through the loop.
C: No current is induced, so there is no force.
D: The flux is increasing, so the loop is repelled by the wire to decrease the flux through the loop.
EVALUATE: In part (b), look at the direction of the force on the segment of each loop closest to the wire. For A and B, the induced current is in the same direction as I, so the wire attracts these loops. For D the induced current is opposite to I, so the wire repels the loop. For C there is no induced current, so there is no force.

29.17. **IDENTIFY** and **SET UP:** Use the right-hand rule to find the direction of the magnetic field due to the long wire at the location of each loop. Lenz's law says that the magnetic field of the induced current is directed to oppose the change in flux through the circuit. Since the current I is decreasing, the flux through each coil is decreasing, so the induced current flows to oppose this flux decrease.

EXECUTE: **(a)** The magnetic field of the long wire is directed out of the page at *C* and into the page at *A*. When the current decreases, the magnetic field decreases. Therefore, the magnetic field of the induced current in loop *C* is directed out of the page inside the loop, to oppose the decrease in flux out of the page due to the current in the long wire. To produce magnetic field in this direction, the induced current in *C* is counterclockwise. The magnetic field of the induced current in loop *A* is directed into the page inside the loop, to oppose the decrease in flux into the page due to the current in the long wire. To produce a magnetic field in this direction, the induced current in *A* is clockwise.
(b) The through both coils A and C is decreasing, so they will be pulled toward the long wire to oppose this decrease.
EVALUATE: As a check on the answer in (b), look at the current in the section of each loop that is nearest to the wire. For both loops, this induced current is in the same direction as the current *I* in the wire. When two parallel wires carry current in the same direction, they attract each other, which agrees with our answer in (b).

29.18. **IDENTIFY:** By Lenz's law, the induced current flows to oppose the flux change that caused it.
SET UP and **EXECUTE:** The magnetic field is outward through the round coil and is decreasing, so the magnetic field due to the induced current must also point outward to oppose this decrease. Therefore the induced current is counterclockwise.
EVALUATE: Careful! Lenz's law does not say that the induced current flows to oppose the magnetic flux. Instead it says that the current flows to oppose the *change* in flux.

29.19. **IDENTIFY** and **SET UP:** Apply Lenz's law, in the form that states that the flux of the induced current tends to oppose the change in flux.
EXECUTE: **(a)** With the switch closed the magnetic field of coil *A* is to the right at the location of coil *B*. When the switch is opened the magnetic field of coil *A* goes away. Hence by Lenz's law the field of the current induced in coil *B* is to the right, to oppose the decrease in the flux in this direction. To produce magnetic field that is to the right the current in the circuit with coil *B* must flow through the resistor in the direction *a* to *b*.
(b) With the switch closed the magnetic field of coil *A* is to the right at the location of coil *B*. This field is stronger at points closer to coil *A* so when coil *B* is brought closer the flux through coil *B* increases. By Lenz's law the field of the induced current in coil *B* is to the left, to oppose the increase in flux to the right. To produce magnetic field that is to the left the current in the circuit with coil *B* must flow through the resistor in the direction *b* to *a*.
(c) With the switch closed the magnetic field of coil *A* is to the right at the location of coil *B*. The current in the circuit that includes coil *A* increases when *R* is decreased and the magnetic field of coil *A* increases when the current through the coil increases. By Lenz's law the field of the induced current in coil *B* is to the left, to oppose the increase in flux to the right. To produce magnetic field that is to the left the current in the circuit with coil *B* must flow through the resistor in the direction *b* to *a*.
EVALUATE: In parts (b) and (c) the change in the circuit causes the flux through circuit *B* to increase and in part (a) it causes the flux to decrease. Therefore, the direction of the induced current is the same in parts (b) and (c) and opposite in part (a).

29.20. **IDENTIFY:** Apply Lenz's law.
SET UP: The field of the induced current is directed to oppose the change in flux in the secondary circuit.
EXECUTE: **(a)** The magnetic field in *A* is to the left and is increasing. The flux is increasing so the field due to the induced current in *B* is to the right. To produce magnetic field to the right, the induced current flows through *R* from right to left.
(b) The magnetic field in *A* is to the right and is decreasing. The flux is decreasing so the field due to the induced current in *B* is to the right. To produce magnetic field to the right the induced current flows through *R* from right to left.

(c) The magnetic field in A is to the right and is increasing. The flux is increasing so the field due to the induced current in B is to the left. To produce magnetic field to the left the induced current flows through R from left to right.

EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.21. IDENTIFY and **SET UP:** Lenz's law requires that the flux of the induced current opposes the change in flux.

EXECUTE: **(a)** The magnetic field is out of the page and increasing, so the induced current should flow so that its field is into the page, so the induced current is clockwise.

(b) The current reaches a constant value so Φ_B is constant. $d\Phi_B/dt = 0$ and there is no induced current.

(c) The magnetic field is out of the page and is decreasing, so the induced current should flow that its magnetic field is out of the page. Thus the induced current is counterclockwise.

EVALUATE: Only a change in flux produces an induced current. The induced current is in one direction when the current in the outer ring is increasing and is in the opposite direction when that current is decreasing.

29.22. IDENTIFY: The changing flux through the loop due to the changing magnetic field induces a current in the wire.

SET UP: The magnitude of the induced emf is $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$, $I = \mathcal{E}/R$.

EXECUTE: $\vec{B}$ is into the page and Φ_B is increasing, so the field of the induced current is directed out of the page inside the loop and the induced current is counterclockwise.

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right| = \pi(0.0250\text{ m})^2(0.380\text{ T/s}^3)(3t^2) = (2.238\times10^{-3}\text{ V/s}^2)t^2.$$

$I = \frac{|\mathcal{E}|}{R} = (5.739\times10^{-3}\text{ A/s}^2)t^2$. When $B = 1.33\text{ T}$, we have $1.33\text{ T} = (0.380\text{ T/s}^3)t^3$, which gives $t = 1.518\text{ s}$. At this t, $I = (5.739\times10^{-3}\text{ A/s}^2)(1.518\text{ s})^2 = 0.0132\text{ A}$.

EVALUATE: As the field changes, the current will also change.

29.23. IDENTIFY: The movement of the wire causes a motional emf.

SET UP: $\mathcal{E} = \left(\vec{v}\times\vec{B}\right)\bullet\vec{L}$. We want the emf.

EXECUTE: **(a)** $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, $\vec{B} = 0.080\text{T}\,\hat{j}$ and $\vec{L} = L\hat{k}$. The result of the vector products is $\mathcal{E} = \left(\vec{v}\times\vec{B}\right)\bullet\vec{L} = Lv_xB_y$ = (0.50 m)(18 m/s)(0.080 T) = 0.72 V.

(b) $\vec{v}\times\vec{B}$ has no y component, so its dot product with $L\hat{k}$ is zero, so $\mathcal{E} = 0$.

EVALUATE: It is visualize 3-dimensional problems, but working with components makes the solutions easier to do.

29.24. IDENTIFY: The magnetic flux through the loop is decreasing, so an emf will be induced in the loop, which will induce a current in the loop. The magnetic field will exert a force on the loop due to this current.

SET UP: The motional $\mathcal{E}$ is $\mathcal{E} = vBL$, $I = \mathcal{E}/R$, and $F_B = ILB$.

EXECUTE: Use $I = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$ and $F_B = ILB$.

$$F_B = ILB = v\frac{B^2L^2}{R} = \frac{3.00\text{ m/s}}{0.600\ \Omega}(2.40\text{ T})^2(0.0150\text{ m})^2 = 6.48\times10^{-3}\text{ N} = 6.48\text{ mN}.$$

$\vec{B}$ is into the page and Φ_B is decreasing, so the field of the induced current is into the page inside the loop and the induced current is clockwise. Using $\vec{F} = I\vec{l} \times \vec{B}$, we see that the force on the left-hand end of the loop to be to the left.

EVALUATE: The force is very small by everyday standards.

29.25. **IDENTIFY:** A conductor moving in a magnetic field may have a potential difference induced across it, depending on how it is moving.

SET UP: The induced emf is $\mathcal{E} = vBL \sin\phi$, where ϕ is the angle between the velocity and the magnetic field.

EXECUTE: **(a)** $\mathcal{E} = vBL \sin\phi = (5.00\text{ m/s})(0.450\text{ T})(0.300\text{ m})(\sin 90°) = 0.675\text{ V}$

(b) The positive charges are moved to end b, so b is at the higher potential.

(c) $E = V/L = (0.675\text{ V})/(0.300\text{ m}) = 2.25\text{ V/m}$. The direction of $\vec{E}$ is from b to a.

(d) The positive charges are pushed to b, so b has an excess of positive charge.

(e) (i) If the rod has no appreciable thickness, $L = 0$, so the emf is zero. (ii) The emf is zero because no magnetic force acts on the charges in the rod since it moves parallel to the magnetic field.

EVALUATE: The motional emf is large enough to have noticeable effects in some cases.

29.26. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi_B = NBA\cos\phi$ and the induced emf is $\mathcal{E} = -d\Phi_B/dt$.

EXECUTE: **(a)** and **(c)** The magnetic flux is constant, so the induced emf is zero.

(b) The area inside the field is changing. If we let x be the length (along the 30.0-cm side) in the field, then

$A = (0.400\text{ m})x.\ \Phi_B = BA = B(0.400\text{ m})x.$

$|\mathcal{E}| = |d\Phi_B/dt| = B\, d[(0.400\text{ m})x]/dt = B(0.400\text{ m})dx/dt = B(0.400\text{ m})v.$

$\mathcal{E} = (1.25\text{ T})(0.400\text{ m})(0.0200\text{ m/s}) = 0.0100\text{ V}.$

EVALUATE: It is not *flux* that induces an emf, but rather a *rate of change* of the flux. The induced emf in part (b) is small enough to be ignored in many instances.

29.27. **IDENTIFY** and **SET UP:** $\mathcal{E} = vBL$. Use Lenz's law to determine the direction of the induced current. The force F_{ext} required to maintain constant speed is equal and opposite to the force F_I that the magnetic field exerts on the rod because of the current in the rod.

EXECUTE: **(a)** $\mathcal{E} = vBL = (7.50\text{ m/s})(0.800\text{ T})(0.500\text{ m}) = 3.00\text{ V}.$

(b) $\vec{B}$ is into the page. The flux increases as the bar moves to the right, so the magnetic field of the induced current is out of the page inside the circuit. To produce magnetic field in this direction the induced current must be counterclockwise, so from b to a in the rod.

(c) $I = \dfrac{\mathcal{E}}{R} = \dfrac{3.00\text{ V}}{1.50\ \Omega} = 2.00\text{ A}.$ $F_I = ILB\sin\phi = (2.00\text{ A})(0.500\text{ m})(0.800\text{ T})\sin 90° = 0.800\text{ N}.$ $\vec{F}_I$ is to the left. To keep the bar moving to the right at constant speed an external force with magnitude $F_{\text{ext}} = 0.800\text{ N}$ and directed to the right must be applied to the bar.

(d) The rate at which work is done by the force F_{ext} is $F_{\text{ext}}v = (0.800\text{ N})(7.50\text{ m/s}) = 6.00\text{ W}$. The rate at which thermal energy is developed in the circuit is $I^2R = (2.00\text{ A})^2(1.50\ \Omega) = 6.00\text{ W}$. These two rates are equal, as is required by conservation of energy.

EVALUATE: The force on the rod due to the induced current is directed to oppose the motion of the rod. This agrees with Lenz's law.

29.28. **IDENTIFY:** Use the three approaches specified in the problem for determining the direction of the induced current. $I = \mathcal{E}/R$. The induced potential across a moving bar is $\mathcal{E} = vBL$.

SET UP: Let $\vec{A}$ be directed into the figure, so a clockwise emf is positive.

EXECUTE: **(a)** $\mathcal{E} = vBL = (5.0\text{ m/s})(0.750\text{ T})(0.650\text{ m}) = 2.438\text{ V}$, which rounds to 2.4 V.

(b) (i) Let q be a positive charge in the moving bar, as shown in Figure 29.28a. The magnetic force on this charge is $\vec{F} = q\vec{v} \times \vec{B}$, which points *upward*. This force pushes the current in a *counterclockwise* direction through the circuit.

(ii) Φ_B is positive and is increasing in magnitude, so $d\Phi_B/dt > 0$. Then by Faraday's law $\mathcal{E} < 0$ and the emf and induced current are counterclockwise.

(iii) The flux through the circuit is increasing, so the induced current must cause a magnetic field out of the paper to oppose this increase. Hence this current must flow in a *counterclockwise sense*, as shown in Figure 29.28b.

(c) $\mathcal{E} = RI$. $I = \dfrac{\mathcal{E}}{R} = \dfrac{2.438\ \text{V}}{25.0\ \Omega} = 0.09752\ \text{A}$, which rounds to 98 mA.

EVALUATE: All three methods agree on the direction of the induced current.

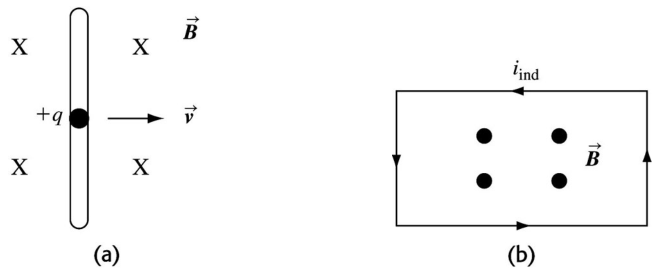


Figure 29.28

29.29. IDENTIFY: The motion of the bar due to the applied force causes a motional emf to be induced across the ends of the bar, which induces a current through the bar. The magnetic field exerts a force on the bar due to this current.

SET UP: The applied force is to the left and equal to $F_{\text{applied}} = F_B = ILB$. $\mathcal{E} = BvL$ and $I = \dfrac{\mathcal{E}}{R} = \dfrac{BvL}{R}$.

EXECUTE: **(a)** $\vec{B}$ out of page and Φ_B decreasing, so the field of the induced current is out of the page inside the loop and the induced current is counterclockwise.

(b) Combining $F_{\text{applied}} = F_B = ILB$ and $\mathcal{E} = BvL$, we have $I = \dfrac{\mathcal{E}}{R} = \dfrac{BvL}{R}$. $F_{\text{applied}} = \dfrac{vB^2L^2}{R}$. The rate at which this force does work is

$$P_{\text{applied}} = F_{\text{applied}}v = \frac{(vBL)^2}{R} = \frac{[(5.90\ \text{m/s})(0.650\ \text{T})(0.360\ \text{m})]^2}{45.0\ \Omega} = 0.0424\ \text{W}.$$

EVALUATE: The power is small because the magnetic force is usually small compared to everyday forces.

29.30. IDENTIFY: The motion of the bar due to the applied force causes a motional emf to be induced across the ends of the bar, which induces a current through the bar and through the resistor. This current dissipates energy in the resistor.

SET UP: $P_R = I^2R$, $\mathcal{E} = BvL = IR$.

EXECUTE: **(a)** $\vec{B}$ is out of the page and Φ_B is increasing, so the field of the induced current is into the page inside the loop and the induced current is clockwise.

(b) $P_R = I^2R$ so $I = \sqrt{\dfrac{P_R}{R}} = \sqrt{\dfrac{0.840\ \text{W}}{45.0\ \Omega}} = 0.1366\ \text{A}$. $I = \dfrac{\text{emf}}{R} = \dfrac{BvL}{R}$.

$$v = \frac{IR}{BL} = \frac{(0.1366\ \text{A})(45.0\ \Omega)}{(0.650\ \text{T})(0.360\ \text{m})} = 26.3\ \text{m/s}.$$

EVALUATE: This speed is around 60 mph, so it would not be very practical to generate energy this way.

29.31. IDENTIFY: The motion of the bar causes an emf to be induced across its ends, which induces a current in the circuit.
SET UP: $\mathcal{E} = BvL, I = \mathcal{E}/R.$
EXECUTE: $\vec{F}_B$ on the bar is to the left so $\vec{v}$ is to the right. Using $\mathcal{E} = BvL$ and $I = \mathcal{E}/R,$ we have
$I = \dfrac{BvL}{R}.$ $v = \dfrac{IR}{BL} = \dfrac{(1.75\text{ A})(6.00\ \Omega)}{(1.20\text{ T})(0.250\text{ m})} = 35.0\text{ m/s}.$
EVALUATE: This speed is greater than 60 mph!

29.32. IDENTIFY: A motional emf is induced across the blood vessel.
SET UP and **EXECUTE:** **(a)** Each slab of flowing blood has maximum width *d* and is moving perpendicular to the field with speed *v*. $\mathcal{E} = vBL$ becomes $\mathcal{E} = vBd$.

(b) $B = \dfrac{\mathcal{E}}{vd} = \dfrac{1.0\times10^{-3}\text{ V}}{(0.15\text{ m/s})(5.0\times10^{-3}\text{ m})} = 1.3\text{ T}.$

(c) The blood vessel has cross-sectional area $A = \pi d^2/4.$ The volume of blood that flows past a cross section of the vessel in time *t* is $\pi(d^2/4)vt.$ The volume flow rate is $\text{volume/time} = R = \pi d^2 v/4.$
$v = \dfrac{\mathcal{E}}{Bd}$
so $R = \dfrac{\pi d^2}{4}\left(\dfrac{\mathcal{E}}{Bd}\right) = \dfrac{\pi\mathcal{E}d}{4B}.$

EVALUATE: A very strong magnetic field (1.3 T) is required to produce a small potential difference of only 1 mV.

29.33. IDENTIFY: While the circuit is entering and leaving the region of the magnetic field, the flux through it will be changing. This change will induce an emf in the circuit.
SET UP: When the loop is entering or leaving the region of magnetic field the flux through it is changing and there is an induced emf. The magnitude of this induced emf is $\mathcal{E} = BLv.$ The length *L* is 0.750 m. When the loop is totally within the field the flux through the loop is not changing so there is no induced emf. The induced current has magnitude $I = \dfrac{\mathcal{E}}{R}$ and direction given by Lenz's law.

EXECUTE: **(a)** $I = \dfrac{\mathcal{E}}{R} = \dfrac{BLv}{R} = \dfrac{(1.25\text{ T})(0.750\text{ m})(3.0\text{ m/s})}{12.5\ \Omega} = 0.225\text{ A}.$ The magnetic field through the loop is directed out of the page and is increasing, so the magnetic field of the induced current is into the page inside the loop and the induced current is clockwise.
(b) The flux is not changing so $\mathcal{E}$ and *I* are zero.
(c) $I = \dfrac{\mathcal{E}}{R} = 0.225\text{ A}.$ The magnetic field through the loop is directed out of the page and is decreasing, so the magnetic field of the induced current is out of the page inside the loop and the induced current is counterclockwise.

(d) Let clockwise currents be positive. At $t = 0$ the loop is entering the field. It is totally in the field at time t_a and beginning to move out of the field at time $t_b.$ The graph of the induced current as a function of time is sketched in Figure 29.33.

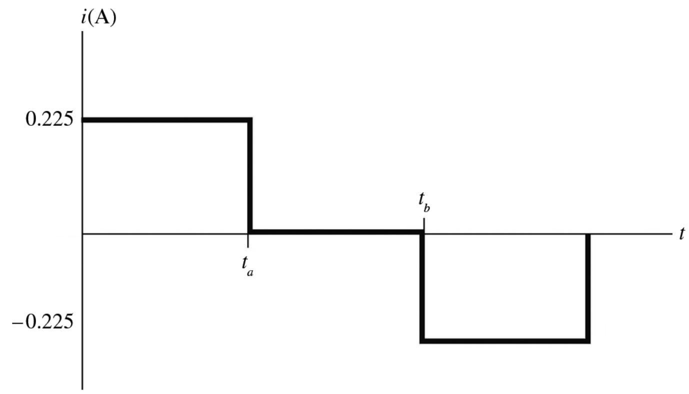


Figure 29.33

EVALUATE: Even though the circuit is moving throughout all parts of this problem, an emf is induced in it only when the flux through it is changing. While the coil is entirely within the field, the flux is constant, so no emf is induced.

29.34. **IDENTIFY:** A changing magnetic flux through a coil induces an emf in that coil, which means that an electric field is induced in the material of the coil.

SET UP: According to Faraday's law, the induced electric field obeys the equation $r\vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$.

EXECUTE: **(a)** For the magnitude of the induced electric field, Faraday's law gives

$$E2\pi r = d(B\pi r^2)/dt = \pi r^2\ dB/dt.$$

$$E = \frac{r}{2}\frac{dB}{dt} = \frac{0.0225\text{ m}}{2}(0.250\text{ T/s}) = 2.81\times10^{-3}\text{ V/m}.$$

(b) The field points toward the south pole of the magnet and is decreasing, so the induced current is counterclockwise.

EVALUATE: This is a very small electric field compared to most others found in laboratory equipment.

29.35. **IDENTIFY:** Apply $E = \dfrac{1}{2\pi r}\left|\dfrac{d\Phi_B}{dt}\right|$ with $\Phi_B = \mu_0 niA.$

SET UP: $A = \pi r^2$, where $r = 0.0110$ m. In $E = \dfrac{1}{2\pi r}\left|\dfrac{d\Phi_B}{dt}\right|$, $r = 0.0350$ m.

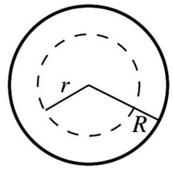
EXECUTE: $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = \left|\dfrac{d}{dt}(BA)\right| = \left|\dfrac{d}{dt}(\mu_0 niA)\right| = \mu_0 nA\left|\dfrac{di}{dt}\right|$ and $|\mathcal{E}| = E(2\pi r)$. Therefore, $\left|\dfrac{di}{dt}\right| = \dfrac{E2\pi r}{\mu_0 nA}$.

$$\left|\frac{di}{dt}\right| = \frac{(8.00\times10^{-6}\text{ V/m})2\pi(0.0350\text{ m})}{\mu_0(400\text{ m}^{-1})\pi(0.0110\text{ m})^2} = 9.21\text{ A/s}.$$

EVALUATE: Outside the solenoid the induced electric field decreases with increasing distance from the axis of the solenoid.

29.36. **IDENTIFY:** Use $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to calculate the induced electric field E at a distance r from the center of the solenoid. Away from the ends of the solenoid, $B = \mu_0 nI$ inside and $B = 0$ outside.

SET UP: The end view of the solenoid is sketched in Figure 29.36.



Let R be the radius of the solenoid.

Figure 29.36

Apply $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to an integration path that is a circle of radius r, where $r < R$. We need to calculate just the magnitude of E so we can take absolute values.

EXECUTE: **(a)** $\oint \left| \vec{E} \cdot d\vec{l} \right| = E(2\pi r)$.

$\Phi_B = B\pi r^2, \left| -\dfrac{d\Phi_B}{dt} \right| = \pi r^2 \left| \dfrac{dB}{dt} \right|$.

$\oint \left| \mathrm{r}\vec{E} \cdot d\vec{l} \right| = \left| -\dfrac{d\Phi_B}{dt} \right|$ implies $E(2\pi r) = \pi r^2 \left| \dfrac{dB}{dt} \right|$.

$E = \frac{1}{2} r \left| \dfrac{dB}{dt} \right|$.

$B = \mu_0 nI$, so $\dfrac{dB}{dt} = \mu_0 n \dfrac{dI}{dt}$.

Thus $E = \frac{1}{2} r \mu_0 n \dfrac{dI}{dt} = \frac{1}{2}(0.00500\text{ m})(4\pi \times 10^{-7}\text{ T} \cdot \text{m/A})(900\text{ m}^{-1})(36.0\text{ A/s}) = 1.02 \times 10^{-4}\text{ V/m}$.

(b) $r = 0.0100\text{ cm}$ is still inside the solenoid so the expression in part (a) applies.

$E = \frac{1}{2} r \mu_0 n \dfrac{dI}{dt} = \frac{1}{2}(0.0100\text{ m})(4\pi \times 10^{-7}\text{ T} \cdot \text{m/A})(900\text{ m}^{-1})(36.0\text{ A/s}) = 2.04 \times 10^{-4}\text{ V/m}$.

EVALUATE: Inside the solenoid E is proportional to r, so E doubles when r doubles.

29.37. **IDENTIFY:** Apply Faraday's law in the form $|\mathcal{E}_{\text{av}}| = N \left| \dfrac{\Delta\Phi_B}{\Delta t} \right|$.

SET UP: The magnetic field of a large straight solenoid is $B = \mu_0 nI$ inside the solenoid and zero outside. $\Phi_B = BA$, where A is 8.00 cm^2, the cross-sectional area of the long straight solenoid.

EXECUTE: $|\mathcal{E}_{\text{av}}| = N \left| \dfrac{\Delta\Phi_B}{\Delta t} \right| = \left| \dfrac{NA(B_\text{f} - B_\text{i})}{\Delta t} \right| = \dfrac{NA\mu_0 nI}{\Delta t}$.

$\mathcal{E}_{\text{av}} = \dfrac{\mu_0(12)(8.00 \times 10^{-4}\text{ m}^2)(9000\text{ m}^{-1})(0.350\text{ A})}{0.0400\text{ s}} = 9.50 \times 10^{-4}\text{ V}$.

EVALUATE: An emf is induced in the second winding even though the magnetic field of the solenoid is zero at the location of the second winding. The changing magnetic field induces an electric field outside the solenoid and that induced electric field produces the emf.

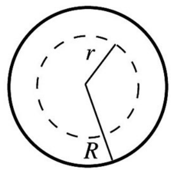
29.38. **IDENTIFY** and **SET UP:** The equations $i_\text{C} = \dfrac{dq}{dt} = \mathcal{E}\dfrac{d\Phi_E}{dt}$ and $i_\text{D} = \mathcal{E}\dfrac{d\Phi_E}{dt}$ show that $i_\text{C} = i_\text{D}$ and also relate i_D to the rate of change of the electric field flux between the plates. Use this to calculate dE/dt and apply the generalized form of Ampere's law to calculate B.

EXECUTE: **(a)** $i_\text{C} = i_\text{D}$, so $j_\text{D} = \dfrac{i_\text{D}}{A} = \dfrac{i_\text{C}}{A} = \dfrac{0.520\text{ A}}{\pi r^2} = \dfrac{0.520\text{ A}}{\pi(0.0400\text{ m})^2} = 103\text{ A/m}^2$.

(b) $j_\text{D} = \epsilon_0 \dfrac{dE}{dt}$ so $\dfrac{dE}{dt} = \dfrac{j_\text{D}}{\epsilon_0} = \dfrac{103\text{ A/m}^2}{8.854 \times 10^{-12}\text{ C}^2/\text{N} \cdot \text{m}^2} = 1.16 \times 10^{13}\text{ V/m} \cdot \text{s}$.

SET UP and EXECUTE: **(c)** Apply Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_C + i_D)_{encl}$ to a circular path with radius $r = 0.0200$ m.

An end view of the solenoid is given in Figure 29.38.



By symmetry the magnetic field is tangent to the path and constant around it.

Figure 29.38

Thus $\oint \vec{B} . d\vec{l} = \text{r}Bdl = B\int dl = B(2\pi r).$

$i_C = 0$ (no conduction current flows through the air space between the plates)

The displacement current enclosed by the path is $j_D \pi r^2$.

Thus $B(2\pi r) = \mu_0 (j_D \pi r^2)$ and

$B = \frac{1}{2}\mu_0 j_D r = \frac{1}{2}(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(103 \text{ A/m}^2)(0.0200 \text{ m}) = 1.30 \times 10^{-6} \text{ T} = 1.30\ \mu\text{T}.$

(d) $B = \frac{1}{2}\mu_0 j_D r$. Now r is $\frac{1}{2}$ the value in (c), so B is also $\frac{1}{2}$ its value in (c):

$B = \frac{1}{2}(1.30 \times 10^{-6} \text{ T}) = 0.650 \times 10^{-7} \text{ T} = 0.650\ \mu\text{T}.$

EVALUATE: The definition of displacement current allows the current to be continuous at the capacitor. The magnetic field between the plates is zero on the axis $(r = 0)$ and increases as r increases.

29.39. **IDENTIFY:** $q = CV$. For a parallel-plate capacitor, $C = \dfrac{\mathcal{E}A}{d}$, where $\mathcal{E} = K\mathcal{E}_0$. $i_C = dq/dt$. $j_D = \mathcal{E}\dfrac{dE}{dt}$.

SET UP: $E = q/\mathcal{E}A$ so $dE/dt = i_C/\mathcal{E}A$.

EXECUTE: **(a)** $q = CV = \left(\dfrac{\mathcal{E}A}{d}\right)V = \dfrac{(4.70)\epsilon_0 (3.00 \times 10^{-4} \text{ m}^2)(120 \text{ V})}{2.50 \times 10^{-3} \text{ m}} = 5.99 \times 10^{-10} \text{ C}.$

(b) $\dfrac{dq}{dt} = i_C = 6.00 \times 10^{-3}$ A.

(c) $j_D = \mathcal{E}\dfrac{dE}{dt} = K\epsilon_0 \dfrac{i_C}{K\varepsilon_0 A} = \dfrac{i_C}{A} = j_C$, so $i_D = i_C = 6.00 \times 10^{-3}$ A.

EVALUATE: $i_D = i_C$, so Kirchhoff's junction rule is satisfied where the wire connects to each capacitor plate.

29.40. **IDENTIFY and SET UP:** Use $i_C = q/t$ to calculate the charge q that the current has carried to the plates in time t. The equations $V = Ed$ and $E = \dfrac{\sigma}{\epsilon_0}$ relate q to the electric field E and the potential difference between the plates. The displacement current density is $j_D = \mathcal{E}\dfrac{dE}{dt}$.

EXECUTE: **(a)** $i_C = 1.80 \times 10^{-3}$ A.

$q = 0$ at $t = 0$.

The amount of charge brought to the plates by the charging current in time t is

$q = i_C t = (1.80 \times 10^{-3} \text{ A})(0.500 \times 10^{-6} \text{ s}) = 9.00 \times 10^{-10} \text{ C}.$

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} = \frac{9.00 \times 10^{-10} \text{ C}}{(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(5.00 \times 10^{-4} \text{ m}^2)} = 2.03 \times 10^5 \text{ V/m}.$$

$V = Ed = (2.03\times10^5\ \text{V/m})(2.00\times10^{-3}\,\text{m}) = 406\ \text{V}.$

(b) $E = q/\epsilon_0 A.$

$$\frac{dE}{dt} = \frac{dq/dt}{\epsilon_0 A} = \frac{i_C}{\epsilon_0 A} = \frac{1.80\times10^{-3}\ \text{A}}{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00\times10^{-4}\ \text{m}^2)} = 4.07\times10^{11}\ \text{V/m}\cdot\text{s}.$$

Since i_C is constant dE/dt does not vary in time.

(c) $j_D = \epsilon_0 \dfrac{dE}{dt}$ (with ϵ replaced by ϵ_0 since there is vacuum between the plates).

$j_D = (8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.07\times10^{11}\ \text{V/m}\cdot\text{s}) = 3.60\ \text{A/m}^2.$

$i_D = j_D A = (3.60\ \text{A/m}^2)(5.00\times10^{-4}\ \text{m}^2) = 1.80\times10^{-3}\ \text{A};\ i_D = i_C.$

EVALUATE: $i_C = i_D$. The constant conduction current means the charge q on the plates and the electric field between them both increase linearly with time and i_D is constant.

29.41. **IDENTIFY:** We are dealing with displacement current.

SET UP: $I_d = \mathcal{E}\dfrac{d\Phi_E}{dt}$, $\mathcal{E} = K\epsilon_0$.

EXECUTE: **(a)** We want I_d at t = 1.5 s. $I_d = \mathcal{E}\dfrac{d\Phi_E}{dt} = \mathcal{E}\dfrac{d\left[(4.0\ \text{V}\cdot\text{m/s}^5)t^5\right]}{dt} = K\epsilon_0\,(20\ \text{V}\cdot\text{m/s}^5)t^4$. At t = 1.5 s using K = 2.5 we get I_d = 2.2 nA.

(b) We want the time when I_d = 1/16 as much. Solve the result in (a) when I_d = (1/16)(2.2 nA), giving t = 0.75 s.

EVALUATE: The displacement current is much smaller than typical household currents or even many lab currents.

29.42. **IDENTIFY:** Apply $\vec{B} = \vec{B}_0 + \mu_0\vec{M}$.

SET UP: For magnetic fields less than the critical field, there is no internal magnetic field. For fields greater than the critical field, $\vec{B}$ is very nearly equal to $\vec{B}_0$.

EXECUTE: **(a)** The external field is less than the critical field, so inside the superconductor $\vec{B} = 0$ and $\vec{M} = -\dfrac{\vec{B}_0}{\mu_0} = -\dfrac{(0.130\ \text{T})\hat{i}}{\mu_0} = -(1.03\times10^5\ \text{A/m})\hat{i}$. Outside the superconductor, $\vec{B} = \vec{B}_0 = (0.130\ \text{T})\hat{i}$ and $\vec{M} = 0$.

(b) The field is greater than the critical field and $\vec{B} = \vec{B}_0 = (0.260\ \text{T})\hat{i}$, both inside and outside the superconductor.

EVALUATE: Below the critical field the external field is expelled from the superconducting material.

29.43. **IDENTIFY:** We are dealing with induced current and Faraday's law.

SET UP and EXECUTE: $\mathcal{E}_{av} = N\dfrac{\Delta\Phi_B}{\Delta t}$. **(a)** We want the charge. Q = 70 A·h = (70 C/s)(3600 s) = 2.5×10^5 C.

(b) We want the peak current.

Headlights: 2(20 A) = 40 A

Radiator fan: 10 A

Windshield wipers (2 front, 1 back): 3(5 A) = 15 A

Peak current is 65 A.

(c) We want the magnetic field. Stator coil: 42 windings, d = 5.0 cm, f = 400 Hz, $\mathcal{E}$ = 14 V.

The magnetic field reverses once per cycle but the flux changes twice for each direction of the field, so $\Delta\Phi_B = 4\Phi_B$. The time for a cycle is $1/f$. Therefore $\Phi_B = BA$. $\dfrac{\Delta\Phi_B}{\Delta t} = \dfrac{4BA}{\Delta t}$.

$\mathcal{E}_{\text{av}} = N\dfrac{\Delta\Phi_B}{\Delta t} = N\dfrac{4AB}{1/f} = 4NABf$. Putting in the numbers: 14 V = 4(42)π(0.025 m)^2 (500 Hz)B, which gives B = 0.11 T.

EVALUATE: The field needed could be decreased by adding more windings or spinning the coils faster.

29.44. IDENTIFY: The 4.00-cm long left side of the loop is a bar moving in a magnetic field, so an emf is induced across its ends. This emf causes current to flow through the loop, and the external magnetic field exerts a force on the bar due to the current in it. Ohm's law applies to the circuit and Newton's second law applies to the loop.

SET UP: The induced potential across the left-end side is $\mathcal{E} = vBL$, the magnetic force on the bar is $F_{\text{mag}} = ILB$, and Ohm's law is $\mathcal{E} = IR$. Newton's second law is $\Sigma\vec{F} = m\vec{a}$. The flux through the loop is decreasing, so the induced current is clockwise. Alternatively, the magnetic force on positive charge in the moving left-end bar is upward, by the right-hand rule, which also gives a clockwise current. Therefore the magnetic force on the 4.00-cm segment is to the left, opposite to $\vec{F}_{\text{ext}}$.

EXECUTE: **(a)** Combining the equations discussed in the set up, the magnetic force on the 4.00-cm bar (and on the loop) is

$F_{\text{mag}} = ILB = (\mathcal{E}/R)LB = (vBL/R)LB = v(BL)^2/R.$

Newton's second law gives

$F_{\text{ext}} - F_{\text{mag}} = ma.$

$ma = F_{\text{ext}} - v(BL)^2/R.$

(0.0240 kg)a = 0.180 N – (0.0300 m/s)[(2.90 T)(0.0400 m)]^2/(0.00500 Ω).

a = 4.14 m/s^2.

(b) At terminal speed v_T, $F_{\text{mag}} = F_{\text{ext}}$.

$v_T(BL)^2/R = F_{\text{ext}}$.

$v_T = RF_{\text{ext}}/(BL)^2$ = (0.00500 Ω)(0.180 N)/[(2.90 T)(0.0400 m)]^2 = 0.0669 m/s = 6.69 cm/s. The speed is constant thereafter, so the acceleration is zero.

(c) $a = F_{\text{ext}}/m$ = (0.180 N)/(0.0240 kg) = 7.50 m/s^2.

EVALUATE: The acceleration is acceleration is constant once the loop is out of the magnetic field. But while it is partly in the field, the acceleration is not constant because the current changes as the speed changes and this causes the magnetic force to vary.

29.45. IDENTIFY: Apply Faraday's law and Lenz's law.

SET UP: For a discharging RC circuit, $i(t) = \dfrac{V_0}{R}e^{-t/RC}$, where V_0 is the initial voltage across the capacitor. The resistance of the small loop is $(25)(0.600\text{ m})(1.0\ \Omega/\text{m}) = 15.0\ \Omega$.

EXECUTE: **(a)** The large circuit is an RC circuit with a time constant of $\tau = RC = (10\ \Omega)(20\times10^{-6}\text{ F}) = 200\ \mu\text{s}$. Thus, the current as a function of time is $i = ((100\text{ V})/(10\ \Omega))\,e^{-t/200\,\mu\text{s}}$. At $t = 200\ \mu s$, we obtain $i = (10\text{ A})(e^{-1}) = 3.7$ A.

(b) Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop and referring to the solution of Exercise 29.7 we obtain

$\Phi_B = \int_c^{c+a}\dfrac{\mu_0 ib}{2\pi r}dr = \dfrac{\mu_0 ib}{2\pi}\ln\left(1+\dfrac{a}{c}\right)$. Therefore, the emf induced in the small loop at $t = 200\ \mu$s is

$\mathcal{E} = -N\dfrac{d\Phi_B}{dt} = -\dfrac{N\mu_0 b}{2\pi}\ln\left(1+\dfrac{a}{c}\right)\dfrac{di}{dt}.$

$$\mathcal{E} = -\frac{(25)(4\pi\times10^{-7}\ \text{Wb/A}\cdot\text{m}^2)(0.200\ \text{m})}{2\pi}\ln(3.0)\left(-\frac{3.7\ \text{A}}{200\times10^{-6}\ \text{s}}\right) = +20.0\ \text{mV}.$$ Thus, the induced current in the small loop is $i' = \frac{\mathcal{E}}{R} = \frac{20.0\ \text{mV}}{15.0\ \Omega} = 1.33\ \text{mA}.$

(c) The magnetic field from the large loop is directed out of the page within the small loop.The induced current will act to oppose the decrease in flux from the large loop. Thus, the induced current flows counterclockwise.

EVALUATE: **(d)** Three of the wires in the large loop are too far away to make a significant contribution to the flux in the small loop—as can be seen by comparing the distance *c* to the dimensions of the large loop.

29.46 **IDENTIFY:** The changing current in the large *RC* circuit produces a changing magnetic flux through the small circuit, which induces an emf in the small circuit. This emf causes a current in the small circuit.

SET UP: For a charging *RC* circuit, $i(t) = \frac{\mathcal{E}}{R}e^{-t/RC}$, where $\mathcal{E}$ is the emf (90.0 V) added to the large circuit. Exercise 29.7 shows that $\Phi_B = \frac{\mu_0 ib}{2\pi}\ln(1 + a/c)$ for each turn of the small circuit, and $\mathcal{E}_{\text{induced}} = -\frac{d\Phi_B}{dt}$.

EXECUTE: $\frac{d\Phi_B}{dt} = \frac{\mu_0 b}{2\pi}\ln(1 + a/c)\frac{di}{dt}$. $\frac{di}{dt} = -\frac{\mathcal{E}}{R^2C}e^{-t/RC}$ and

$\left|\mathcal{E}_{\text{induced}}\right| = N\left|\frac{d\Phi_B}{dt}\right| = \frac{N\mu_0 b}{2\pi}\ln(1 + a/c)\frac{\varepsilon}{R^2C}e^{-t/RC} = \frac{N\mu_0 b}{2\pi}\ln(1 + a/c)\frac{1}{RC}i.$ The resistance of the small loop is $(25)(0.600\ \text{m})(1.0\ \Omega/\text{m}) = 15\ \Omega.$

$$\left|\mathcal{E}_{\text{induced}}\right| = (25)(2.00\times10^{-7}\ \text{T}\cdot\text{m/A})(0.200\ \text{m})\ln(1 + 10.0/5.0)\frac{1}{(10\ \Omega)(20\times10^{-6}\ \text{F})}(5.00\ \text{A}).$$

$\left|\mathcal{E}_{\text{induced}}\right| = 0.02747\ \text{V}.$ The induced current is $\frac{\left|\mathcal{E}_{\text{induced}}\right|}{R} = \frac{0.02747\ \text{V}}{15\ \Omega} = 1.83\times10^{-3}\ \text{A} = 1.83\ \text{mA},$ which rounds to 1.8 mA. The current in the large loop is counterclockwise. The magnetic field through the small loop is into the page and the flux is decreasing, so the magnetic field due to the induced current in the small loop is into the page and the induced current in the small loop is clockwise.

EVALUATE: The answer is actually independent of *N* because the emf induced in the small coil is proportional to *N* and the resistance of that coil is also proportional to *N*. Since $I = \mathcal{E}/R$, the *N* will cancel out.

29.47. **IDENTIFY:** The changing current in the solenoid will cause a changing magnetic field (and hence changing flux) through the secondary winding, which will induce an emf in the secondary coil.

SET UP: The magnetic field of the solenoid is $B = \mu_0 ni$, and the induced emf is $\left|\mathcal{E}\right| = N\left|\frac{d\Phi_B}{dt}\right|$.

EXECUTE: $B = \mu_0 ni = (4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(90.0\times10^2\ \text{m}^{-1})(0.160\ \text{A/s}^2)t^2 = (1.810\times10^{-3}\ \text{T/s}^2)t^2$. The total flux through secondary winding is $(5.0)B(2.00\times10^{-4}\ \text{m}^2) = (1.810\times10^{-6}\ \text{Wb/s}^2)t^2$.

$\left|\mathcal{E}\right| = N\left|\frac{d\Phi_B}{dt}\right| = (3.619\times10^{-6}\ \text{V/s})t.$ $i = 3.20\ \text{A}$ says $3.20\ \text{A} = (0.160\ \text{A/s}^2)t^2$ and $t = 4.472\ \text{s}.$ This gives $\left|\mathcal{E}\right| = (3.619\times10^{-6}\ \text{V/s})(4.472\ \text{s}) = 1.62\times10^{-5}\ \text{V}.$

EVALUATE: This a very small voltage, about $16\ \mu\text{V}$.

29.48. IDENTIFY: Apply Faraday's law.

SET UP: For rotation about the y-axis the situation is the same as in Examples 29.3 and 29.4 and we can apply the results from those examples.

EXECUTE: **(a)** Rotating about the y-axis: the flux is given by $\Phi_B = BA\cos\phi$ and

$\mathcal{E}_{\text{max}} = \omega BA = (35.0 \text{ rad/s})(0.320 \text{ T})(6.00\times10^{-2} \text{ m}^2) = 0.672 \text{ V}.$

(b) Rotating about the x-axis: $\dfrac{d\Phi_B}{dt} = 0$ and $\mathcal{E} = 0.$

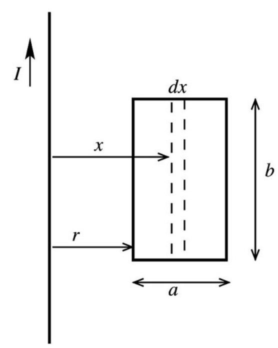
(c) Rotating about the z-axis: the flux is given by $\Phi_B = BA\cos\phi$ and

$\mathcal{E}_{\text{max}} = \omega BA = (35.0 \text{ rad/s})(0.320 \text{ T})(6.00\times10^{-2} \text{ m}^2) = 0.672 \text{ V}.$

EVALUATE: The maximum emf is the same if the loop is rotated about an edge parallel to the z-axis as it is when it is rotated about the z-axis.

29.49. (a) IDENTIFY: (i) $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right|$. The flux is changing because the magnitude of the magnetic field of the wire decreases with distance from the wire. Find the flux through a narrow strip of area and integrate over the loop to find the total flux.

SET UP:



Consider a narrow strip of width dx and a distance x from the long wire, as shown in Figure 29.49a. The magnetic field of the wire at the strip is $B = \mu_0 I/2\pi x$. The flux through the strip is $d\Phi_B = Bb\,dx = (\mu_0 Ib/2\pi)(dx/x)$.

Figure 29.49a

EXECUTE: The total flux through the loop is $\Phi_B = \int d\Phi_B = \left(\dfrac{\mu_0 Ib}{2\pi}\right)\int_r^{r+a}\dfrac{dx}{x}.$

$$\Phi_B = \left(\frac{\mu_0 Ib}{2\pi}\right)\ln\left(\frac{r+a}{r}\right).$$

$$\frac{d\Phi_B}{dt} = \frac{d\Phi_B}{dr}\frac{dr}{dt} = \frac{\mu_0 Ib}{2\pi}\left(-\frac{a}{r(r+a)}\right)v.$$

$$|\mathcal{E}| = \frac{\mu_0 Iabv}{2\pi r(r+a)}.$$

(ii) **IDENTIFY:** $\mathcal{E} = Bvl$ for a bar of length l moving at speed v perpendicular to a magnetic field B. Calculate the induced emf in each side of the loop, and combine the emfs according to their polarity.

SET UP: The four segments of the loop are shown in Figure 29.49b.

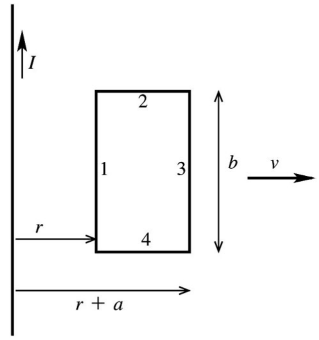


Figure 29.49b

EXECUTE: The emf in each side of the loop is

$$\mathcal{E}_1 = \left(\frac{\mu_0 I}{2\pi r}\right)vb,\quad \mathcal{E}_3 = \left(\frac{\mu_0 I}{2\pi(r+a)}\right)vb,\quad \mathcal{E}_2 = \mathcal{E}_4 = 0.$$

Both emfs $\mathcal{E}_1$ and $\mathcal{E}_3$ are directed toward the top of the loop so oppose each other. The net emf is

$$\mathcal{E} = \mathcal{E}_1 - \mathcal{E}_3 = \frac{\mu_0 Ivb}{2\pi}\left(\frac{1}{r} - \frac{1}{r+a}\right) = \frac{\mu_0 Iabv}{2\pi r(r+a)}.$$

This expression agrees with what was obtained in (i) using Faraday's law.

(b) (i) **IDENTIFY** and **SET UP:** The flux of the induced current opposes the change in flux.

EXECUTE: $\vec{B}$ is $\otimes$. Φ_B is decreasing, so the flux Φ_{ind} of the induced current is $\otimes$ and the current is clockwise.

(ii) **IDENTIFY** and **SET UP:** Use the right-hand rule to find the force on the positive charges in each side of the loop. The forces on positive charges in segments 1 and 3 of the loop are shown in Figure 29.49c.

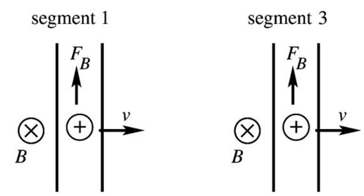


Figure 29.49c

EXECUTE: B is larger at segment 1 since it is closer to the long wire, so F_B is larger in segment 1 and the induced current in the loop is clockwise. This agrees with the direction deduced in (i) using Lenz's law.

(c) EVALUATE: When $v = 0$ the induced emf should be zero; the expression in part (a) gives this. When $a \to 0$ the flux goes to zero and the emf should approach zero; the expression in part (a) gives this. When $r \to \infty$ the magnetic field through the loop goes to zero and the emf should go to zero; the expression in part (a) gives this.

29.50. **IDENTIFY:** We are dealing with induced current and Faraday's law.

SET UP and **EXECUTE:** $\mathcal{E}_{\text{av}} = N\dfrac{\Delta\Phi_B}{\Delta t}$ s.

EXECUTE: **(a)** We want the flux. $\Phi_B = BA\cos\phi$ = (5 mT)(8 cm^2)(1) = 4 μWb.

(b) We want $\dfrac{\Delta\Phi_B}{\Delta t}$ and $\mathcal{E}$. Δt = x/v = (4 cm)/(2 cm/s) = 2 s. $\dfrac{\Delta\Phi_B}{\Delta t}$ = (4 μWb)/(2 s) = 2 μWb/s.

$\mathcal{E}_{\text{av}} = \dfrac{\Delta\Phi_B}{\Delta t}$ = 2 μV.

EVALUATE: This is a small voltage, but the motion is very slow.

29.51. **IDENTIFY:** Apply Faraday's law in the form $\mathcal{E}_{\text{av}} = -N\dfrac{\Delta\Phi_B}{\Delta t}$ to calculate the average emf. Apply Lenz's law to calculate the direction of the induced current.

SET UP: $\Phi_B = BA.$ The flux changes because the area of the loop changes.

EXECUTE: **(a)** $\mathcal{E}_{\text{av}} = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = B\left|\dfrac{\Delta A}{\Delta t}\right| = B\dfrac{\pi r^2}{\Delta t} = (1.35\text{ T})\dfrac{\pi(0.0650/2\text{ m})^2}{0.250\text{ s}} = 0.0179\text{ V} = 17.9\text{ mV}.$

(b) Since the magnetic field is directed into the page and the magnitude of the flux through the loop is decreasing, the induced current must produce a field that goes into the page. Therefore the current flows from point *a* through the resistor to point *b*.

EVALUATE: Faraday's law can be used to find the direction of the induced current. Let $\vec{A}$ be into the page. Then Φ_B is positive and decreasing in magnitude, so $d\Phi_B/dt < 0.$ Therefore $\mathcal{E} > 0$ and the induced current is clockwise around the loop.

29.52. **IDENTIFY:** The movement of the rod causes an emf to be induced across its ends, which causes a current to flow through the circuit. The magnetic field exerts a force on this current.

SET UP: The magnetic force is $F_{\text{mag}} = ILB,$ the induced emf is $\mathcal{E} = vBL.$ $\sum F = ma$ applies to the rod, and $a = dv/dt.$

EXECUTE: The net force on the rod is $F - iLB = ma.$ $i = \dfrac{vBL}{R}.$ $F - \dfrac{vB^2L^2}{R} = ma.$ $F - \dfrac{vB^2L^2}{R} = m\dfrac{dv}{dt}.$

Integrating to find the time gives $\dfrac{F}{m}\displaystyle\int_0^t dt' = \int_0^v \dfrac{dv'}{1 - \dfrac{v'B^2L^2}{FR}},$ which gives $\dfrac{Ft}{m} = -\dfrac{FR}{B^2L^2}\ln\left(1 - \dfrac{vB^2L^2}{FR}\right).$

Solving for *t* and putting in the numbers gives

$$t = -\frac{Rm}{B^2L^2}\ln\left(1 - \frac{vB^2L^2}{FR}\right) = -(0.120\text{ kg})(888.9\text{ s/kg})\ln\left(1 - \frac{25.0\text{ m/s}}{(1.90\text{ N})(888.9\text{ s/kg})}\right) = 1.59\text{ s}.$$

EVALUATE: We cannot use the constant-acceleration kinematics formulas because as the speed *v* of the rod changes, the magnetic force on it also changes. Therefore the acceleration of the rod is not constant.

29.53. **IDENTIFY:** Find the magnetic field at a distance *r* from the center of the wire. Divide the rectangle into narrow strips of width *dr*, find the flux through each strip and integrate to find the total flux.

SET UP: Example 28.8 uses Ampere's law to show that the magnetic field inside the wire, a distance r from the axis, is $B(r) = \mu_0 Ir/2\pi R^2.$

EXECUTE: Consider a small strip of length *W* and width *dr* that is a distance *r* from the axis of the wire, as shown in Figure 29.53. The flux through the strip is $d\Phi_B = B(r)W\,dr = \dfrac{\mu_0 IW}{2\pi R^2}r\,dr.$ The total flux through the rectangle is $\Phi_B = \displaystyle\int d\Phi_B = \left(\frac{\mu_0 IW}{2\pi R^2}\right)\int_0^R r\,dr = \frac{\mu_0 IW}{4\pi}.$

EVALUATE: Note that the result is independent of the radius *R* of the wire.

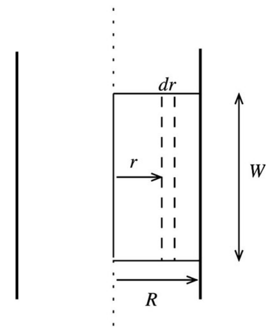


Figure 29.53

29.54. **IDENTIFY:** Apply Newton's second law to the bar. The bar will experience a magnetic force due to the induced current in the loop. Use $a = dv/dt$ to solve for v. At the terminal speed, $a = 0$.

SET UP: The induced emf in the loop has a magnitude BLv. The induced emf is counterclockwise, so it opposes the voltage of the battery, $\mathcal{E}$.

EXECUTE: **(a)** The net current in the loop is $I = \dfrac{\mathcal{E} - BLv}{R}$. The acceleration of the bar is $a = \dfrac{F}{m} = \dfrac{ILB\sin(90°)}{m} = \dfrac{(\mathcal{E} - BLv)LB}{mR}$. To find $v(t)$, set $\dfrac{dv}{dt} = a = \dfrac{(\mathcal{E} - BLv)LB}{mR}$ and solve for v using the method of separation of variables:

$\int_0^v \dfrac{dv}{(\mathcal{E} - BLv)} = \int_0^t \dfrac{LB}{mR}dt \rightarrow v = \dfrac{\mathcal{E}}{BL}(1 - e^{-B^2L^2t/mR}) = (14 \text{ m/s})(1 - e^{-t/6.0 \text{ s}})$. The graph of v versus t is sketched in Figure 29.54. Note that the graph of this function is similar in appearance to that of a charging capacitor.

(b) Just after the switch is closed, $v = 0$ and $I = \mathcal{E}/R = 2.4$ A, $F = ILB = 2.074$ N, and $a = F/m = 2.3 \text{ m/s}^2$.

(c) When $v = 2.0$ m/s, $a = \dfrac{[12 \text{ V} - (2.4 \text{ T})(0.36 \text{ m})(2.0 \text{ m/s})](0.36 \text{ m})(2.4 \text{ T})}{(0.90 \text{ kg})(5.0\ \Omega)} = 2.0 \text{ m/s}^2$.

(d) Note that as the speed increases, the acceleration decreases. The speed will asymptotically approach the terminal speed $\dfrac{\mathcal{E}}{BL} = \dfrac{12 \text{ V}}{(2.4 \text{ T})(0.36 \text{ m})} = 14$ m/s, which makes the acceleration zero.

EVALUATE: The current in the circuit is clockwise and the magnetic force on the bar is to the right. The energy that appears as kinetic energy of the moving bar is supplied by the battery.

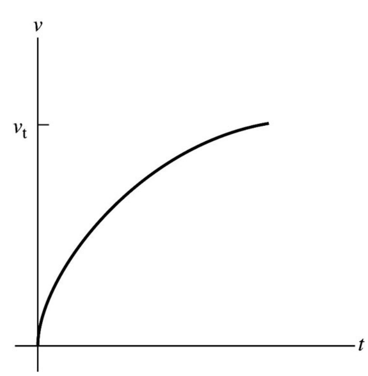
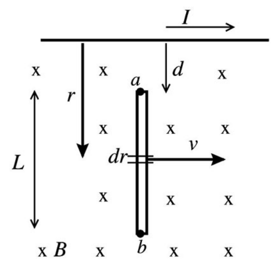


Figure 29.54

29.55. **(a)** and **(b)** **IDENTIFY** and **SET UP:**



The magnetic field of the wire is given by $B = \dfrac{\mu_0 I}{2\pi r}$ and varies along the length of the bar. At every point along the bar $\vec{B}$ has direction into the page. Divide the bar up into thin slices, as shown in Figure 29.55a.

Figure 29.55a

EXECUTE: The emf $d\mathcal{E}$ induced in each slice is given by $d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{l}$. $\vec{v} \times \vec{B}$ is directed toward the wire, so $d\mathcal{E} = -vB\,dr = -v\left(\dfrac{\mu_0 I}{2\pi r}\right)dr.$ The total emf induced in the bar is

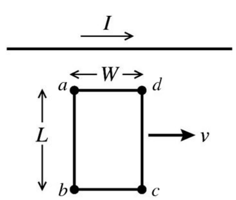
$$V_{ba} = \int_a^b d\mathcal{E} = -\int_d^{d+L}\left(\frac{\mu_0 Iv}{2\pi r}\right)dr = -\frac{\mu_0 Iv}{2\pi}\int_d^{d+L}\frac{dr}{r} = -\frac{\mu_0 Iv}{2\pi}\left[\ln(r)\right]_d^{d+L}.$$

$$V_{ba} = -\frac{\mu_0 Iv}{2\pi}(\ln(d+L) - \ln(d)) = -\frac{\mu_0 Iv}{2\pi}\ \ln(1 + L/d).$$

EVALUATE: The minus sign means that V_{ba} is negative, point *a* is at higher potential than point *b*. (The force $\vec{F} = q\vec{v} \times \vec{B}$ on positive charge carriers in the bar is towards *a*, so *a* is at higher potential.) The potential difference increases when *I* or *v* increase, or *d* decreases.

(c) **IDENTIFY:** Use Faraday's law to calculate the induced emf.

SET UP: The wire and loop are sketched in Figure 29.55b.



EXECUTE: As the loop moves to the right the magnetic flux through it doesn't change.

Thus $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = 0$ and $I = 0$.

Figure 29.55b

EVALUATE: This result can also be understood as follows. The induced emf in section *ab* puts point *a* at higher potential; the induced emf in section *dc* puts point *d* at higher potential. If you travel around the loop then these two induced emf's sum to zero. There is no emf in the loop and hence no current.

29.56. **IDENTIFY:** Apply Faraday's law to calculate the magnitude and direction of the induced emf.

SET UP: Let $\vec{A}$ be directed out of the page in the figure with the problem in the textbook. This means that counterclockwise emf is positive.

EXECUTE: **(a)** $\Phi_B = BA = B_0 \pi r_0^2\left[1 - 3(t/t_0)^2 + 2(t/t_0)^3\right].$

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -B_0 \pi r_0^2 \dfrac{d}{dt}\left[1 - 3(t/t_0)^2 + 2(t/t_0)^3\right] = -\dfrac{B_0 \pi r_0^2}{t_0}\left[-6(t/t_0) + 6(t/t_0)^2\right].$

$\mathcal{E} = -\dfrac{6\, B_0 \pi r_0^2}{t_0}\left(\left(\dfrac{t}{t_0}\right)^2 - \left(\dfrac{t}{t_0}\right)\right).$ At $t = 5.0 \times 10^{-3}$ s,

$$\mathcal{E} = -\frac{6B_0\pi(0.0420\text{ m})^2}{0.010\text{ s}}\left(\left(\frac{5.0\times10^{-3}\text{ s}}{0.010\text{ s}}\right)^2 - \left(\frac{5.0\times10^{-3}\text{ s}}{0.010\text{ s}}\right)\right) = 0.0665\text{ V}.$$ $\mathcal{E}$ is positive so it is counterclockwise.

(c) $I = \dfrac{\mathcal{E}}{R_{\text{total}}} \Rightarrow R_{\text{total}} = r + R = \dfrac{\mathcal{E}}{I} \Rightarrow r = \dfrac{0.0665\text{ V}}{3.0\times10^{-3}\text{ A}} - 12\ \Omega = 10.2\ \Omega.$

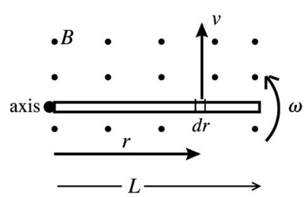
(d) Evaluating the emf at $t = 1.21\times10^{-2}$ s and using the equations of part (b), $\mathcal{E} = -0.0676$ V, and the current flows clockwise, from b to a through the resistor.

(e) $\mathcal{E} = 0$ when $0 = \left(\left(\dfrac{t}{t_0}\right)^2 - \left(\dfrac{t}{t_0}\right)\right)$. $1 = \dfrac{t}{t_0}$ and $t = t_0 = 0.010$ s.

EVALUATE: At $t = t_0$, $B = 0$. At $t = 5.00\times10^{-3}$ s, $\vec{B}$ is in the $+\hat{k}$-direction and is decreasing in magnitude. Lenz's law therefore says $\mathcal{E}$ is counterclockwise. At $t = 0.0121$ s, $\vec{B}$ is in the $+\hat{k}$-direction and is increasing in magnitude. Lenz's law therefore says $\mathcal{E}$ is clockwise. These results for the direction of $\mathcal{E}$ agree with the results we obtained from Faraday's law.

29.57. **IDENTIFY:** Use the expression for motional emf to calculate the emf induced in the rod.

SET UP: **(a)** The rotating rod is shown in Figure 29.57a.



The emf induced in a thin slice is $d\mathcal{E} = \vec{v}\times\vec{B}\cdot d\vec{l}$.

Figure 29.57a

EXECUTE: Assume that $\vec{B}$ is directed out of the page. Then $\vec{v}\times\vec{B}$ is directed radially outward and $dl = dr$, so $\vec{v}\times\vec{B}\cdot d\vec{l} = vB\,dr$.

$v = r\omega$ so $d\mathcal{E} = \omega Br\,dr$.

The $d\mathcal{E}$ for all the thin slices that make up the rod are in series so they add:

$\mathcal{E} = \int d\mathcal{E} = \int_0^L \omega Br\,dr = \frac{1}{2}\omega BL^2 = \frac{1}{2}(8.80\text{ rad/s})(0.650\text{ T})(0.240\text{ m})^2 = 0.165\text{ V}.$

EVALUATE: $\mathcal{E}$ increases with $\omega, B,$ or L^2.

(b) SET UP and **EXECUTE:** No current flows so there is no IR drop in potential. Thus the potential difference between the ends equals the emf of 0.165 V calculated in part (a).

(c) SET UP: The rotating rod is shown in Figure 29.57b.

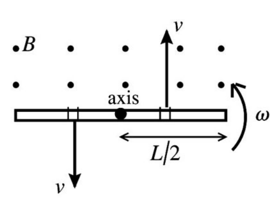


Figure 29.57b

EXECUTE: The emf between the center of the rod and each end is $\mathcal{E}=\frac{1}{2}\omega B(L/2)^2=\frac{1}{4}(0.165\text{ V})=0.0412\text{ V},$ with the direction of the emf from the center of the rod toward each end. The emfs in each half of the rod thus oppose each other and there is no net emf between the ends of the rod.

EVALUATE: ω and B are the same as in part (a) but L of each half is $\frac{1}{2}L$ for the whole rod. $\mathcal{E}$ is proportional to L^2, so is smaller by a factor of $\frac{1}{4}$.

29.58. **IDENTIFY:** Since the bar is straight and the magnetic field is uniform, integrating $d\mathcal{E}=\vec{v}\times\vec{B}\cdot d\vec{l}$ along the length of the bar gives $\mathcal{E}=(\vec{v}\times\vec{B})\cdot\vec{L}.$

SET UP: $\vec{v}=(6.80\text{ m/s})\hat{i}.$ $\vec{L}=(0.250\text{ m})(\cos 36.9°\hat{i}+\sin 36.9°\hat{j}).$

EXECUTE: **(a)** $\mathcal{E}=(\vec{v}\times\vec{B})\cdot\vec{L}=(6.80\text{ m/s})\hat{i}\times\left[(0.120\text{ T})\hat{i}-(0.220\text{ T})\hat{j}-(0.0900\text{ T})\hat{k}\right]\cdot\vec{L}.$

$\mathcal{E}=\left[(0.612\text{ V/m})\hat{j}-(1.496\text{ V/m})\hat{k}\right]\cdot\left[(0.250\text{ m})(\cos 36.9°\hat{i}+\sin 36.9°\hat{j})\right].$

$\mathcal{E}=(0.612\text{ V/m})(0.250\text{ m})\sin 36.9°=0.0919\text{ V}=91.9\text{ mV}.$

(b) The higher potential end is the end to which positive charges in the rod are pushed by the magnetic force. $\vec{v}\times\vec{B}$ has a positive y-component, so the end of the rod marked + in Figure 29.58 is at higher potential.

EVALUATE: Since $\vec{v}\times\vec{B}$ has nonzero $\hat{j}$- and $\hat{k}$-components, and $\vec{L}$- has nonzero $\hat{i}$- and $\hat{j}$-components, only the $\hat{k}$-component of $\vec{B}$ contributes to $\mathcal{E}$. In fact, $|\mathcal{E}|=|v_xB_zL_y|(6.80\text{ m/s})(0.0900\text{ T})(0.250\text{ m})\sin 36.9°=0.0919\text{ V}=91.9\text{ mV}.$

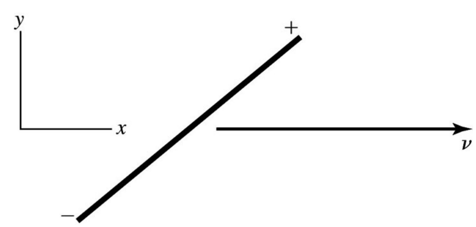


Figure 29.58

29.59. **(a) IDENTIFY:** Use Faraday's law to calculate the induced emf, Ohm's law to calculate I, and $\vec{F}=I\vec{l}\times\vec{B}$ to calculate the force on the rod due to the induced current.

SET UP: The force on the wire is shown in Figure 29.59.

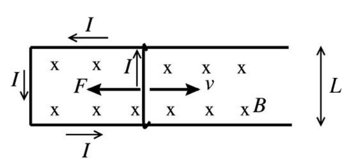


Figure 29.59

EXECUTE: When the wire has speed v the induced emf is $\mathcal{E}=BvL$ and the induced current is $I=\mathcal{E}/R=\dfrac{BvL}{R}.$

The induced current flows upward in the wire as shown, so the force $\vec{F}=I\vec{l}\times\vec{B}$ exerted by the magnetic field on the induced current is to the left. $\vec{F}$ opposes the motion of the wire, as it must by Lenz's law. The magnitude of the force is $F=ILB=B^2L^2v/R.$

(b) IDENTIFY and **SET UP:** Apply $\sum \vec{F} = m\vec{a}$ to the wire. Take $+x$ to be toward the right and let the origin be at the location of the wire at $t = 0$, so $x_0 = 0$.

EXECUTE: $\sum F_x = ma_x$ says $-F = ma_x$.

$$a_x = -\frac{F}{m} = -\frac{B^2L^2v}{mR}.$$

Use this expression to solve for $v(t)$:

$$a_x = \frac{dv}{dt} = -\frac{B^2L^2v}{mR} \text{ and } \frac{dv}{v} = -\frac{B^2L^2}{mR}dt.$$

$$\int_{v_0}^{v} \frac{dv'}{v'} = -\frac{B^2L^2}{mR}\int_0^t dt'.$$

$$\ln(v) - \ln(v_0) = -\frac{B^2L^2t}{mR}.$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{B^2L^2t}{mR} \text{ and } v = v_0 e^{-B^2L^2t/mR}.$$

Note: At $t = 0, v = v_0$ and $v \to 0$ when $t \to \infty$.

Now solve for $x(t)$:

$$v = \frac{dx}{dt} = v_0 e^{-B^2L^2t/mR} \text{ so } dx = v_0 e^{-B^2L^2t/mR}dt.$$

$$\int_0^x dx' = \int_0^t v_0 e^{-B^2L^2t'/mR}dt'.$$

$$x = v_0\left(-\frac{mR}{B^2L^2}\right)\left[e^{-B^2L^2t'/mR}\right]_0^t = \frac{mRv_0}{B^2L^2}(1 - e^{-B^2L^2t/mR}).$$

Comes to rest implies $v = 0$. This happens when $t \to \infty$.

$t \to \infty$ gives $x = \dfrac{mRv_0}{B^2L^2}$. Thus this is the distance the wire travels before coming to rest.

EVALUATE: The motion of the slide wire causes an induced emf and current. The magnetic force on the induced current opposes the motion of the wire and eventually brings it to rest. The force and acceleration depend on v and are constant. If the acceleration were constant, not changing from its initial value of $a_x = -B^2L^2v_0/mR$, then the stopping distance would be $x = -v_0^2/2a_x = mRv_0/2B^2L^2$. The actual stopping distance is twice this.

29.60. IDENTIFY: This problem involves Faraday's law, Lenz's law, and an *R-C* circuit.

SET UP: $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$, $i = I_0 e^{-t/RC}$, $q = Q_0 e^{-t/RC}$, $B = \dfrac{\mu_0 I a^2 N}{2(x^2 + a^2)^{3/2}}$.

EXECUTE: **(a)** We want I_2 at $t = 0$. $I_1 = I_0 e^{-t/R_1C}$. $R_1 = (0.0100\ \Omega/\text{m})(2\pi a)(N_1) = 125.7\ \Omega$ so $I_0 = Q_0/R_1C = 79.6$ mA.

(b) We want Φ_2 at $t = 0$. Apply $B_1 = \dfrac{\mu_0 I_1 a^2 N_1}{2(x^2 + a^2)^{3/2}}$ for $x = 0$: $B_1 = \dfrac{\mu_0 I_1 N_1}{2a}$. $\Phi_2 = B_1 A_2$ which gives

$\Phi_2 = \left(\dfrac{\mu_0 I_1 N_1}{2a}\right)(\pi b^2)$. Using the given numbers we get $\Phi_2 =$ 314 μWb.

(c) We want the direction of I_2. Just after S is closed, I_1 is increasing to its maximum value and runs counterclockwise through the outer loop. This produces an increasing field in the inner circuit pointing out of the paper. I_2 flows to oppose this increase, so it flows *clockwise.*

(d) We want the direction of I_2. At this time, I_1 is decreasing, so B_1 is out of the paper but decreasing. So I_2 flows to oppose this decrease, which is *counterclockwise*.

(e) We want I_1 at $t = 1.26$ ms. $I_1 = \dfrac{Q_0}{R_1C}e^{-t/R_1C}.\ \dfrac{dI_1}{dt} = \dfrac{Q_0}{(R_1C)^2}e^{-t/R_1C}.\ \ \Phi_2 = \left(\dfrac{\mu_0 I_1 N_1}{2a}\right)(\pi b^2)$.

$B_1 = \dfrac{\mu_0 I_1 N_1}{2a}.\ \ \mathcal{E}_2 = N_2\dfrac{d\Phi_2}{dt} = \left(\dfrac{\mu_0 N_1 N_2 \pi b^2}{2a}\right)\dfrac{dI_1}{dt} = \left(\dfrac{\mu_0 N_1 N_2 \pi b^2}{2a}\right)\dfrac{Q_0}{(R_1C)^2}e^{-t/R_1C}$.

$I_2 = \dfrac{\mathcal{E}_2}{R_2} = \dfrac{\mathcal{E}_2}{N_2(0.0100\ \Omega/\text{m})(2\pi b)}$. Substituting our expression above for $\mathcal{E}_2$ into our equation for I_2 and dividing out common factors gives $I_2 = \dfrac{\mu_0 N_1 b Q_0 e^{-t/R_1C}}{4a(R_1C)^2(0.0100\ \Omega/\text{m})}$. Putting in the numbers gives $I_2 = 365$ mA.

EVALUATE: Note that N_2 does not affect the final result for I_2 because it is a factor in R_2 and in $\mathcal{E}_2$ so it cancels out.

29.61. IDENTIFY: Use $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to calculate the induced electric field at each point and then use $\vec{F} = q\vec{E}$.

SET UP:

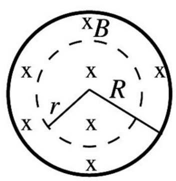


Figure 29.61a

Apply $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to a concentric circle of radius r, as shown in Figure 29.61a. Take $\vec{A}$ to be into the page, in the direction of $\vec{B}$.

EXECUTE: B increasing then gives $\dfrac{d\Phi_B}{dt} > 0$, so $\oint \vec{E}\cdot d\vec{l}$ is negative. This means that E is tangent to the circle in the counterclockwise direction, as shown in Figure 29.61b.

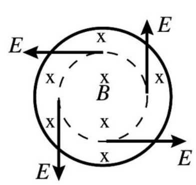


Figure 29.61b

$$\oint \vec{E}\cdot d\vec{l} = -E(2\pi r)$$

$$\frac{d\Phi_B}{dt} = \pi r^2\frac{dB}{dt}$$

$-E(2\pi r) = -\pi r^2\dfrac{dB}{dt}$ so $E = \frac{1}{2}r\dfrac{dB}{dt}$.

Point *a*: The induced electric field and the force on q are shown in Figure 29.61c.

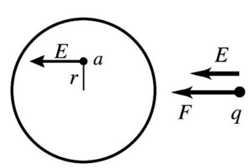
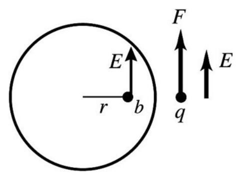


Figure 29.61c

$F = qE = \frac{1}{2}qr\dfrac{dB}{dt}$.

$\vec{F}$ is to the left ($\vec{F}$ is in the same direction as $\vec{E}$ since q is positive).

Point *b*: The induced electric field and the force on *q* are shown in Figure 29.61d.



$$F = qE = \tfrac{1}{2}qr\frac{dB}{dt}.$$

$\vec{F}$ is toward the top of the page.

Figure 29.61d

Point *c*: $r = 0$ here, so $E = 0$ and $F = 0$.

EVALUATE: If there were a concentric conducting ring of radius *r* in the magnetic field region, Lenz's law tells us that the increasing magnetic field would induce a counterclockwise current in the ring. This agrees with the direction of the force we calculated for the individual positive point charges.

29.62. **IDENTIFY:** This problem involves motional emf and damped harmonic motion.
SET UP: Eq. (14.41): $-kx - bv_x = ma_x$ (or $-kx - bdx/dt = m\, d^2x/dt^2$). $\mathcal{E} = vBL = BL\, dx/dt$. $F = ILB$. $\mathcal{E} = RI$.
EXECUTE: **(a)** We want the damping constant. Apply Newton's second law to the bar when it is a distance *x* beyond the equilibrium position and released, as shown in Fig. P29.62 in the textbook. This gives $F_{\text{spr}} + F_{\text{mag}} = m\dfrac{d^2x}{dt^2}$. $F_{\text{spr}} = -kx$. $F_{\text{mag}} = ILB = \dfrac{\mathcal{E}}{R}LB = \left(\dfrac{vBL}{R}\right)LB = \dfrac{(BL)^2}{R}v = \dfrac{(BL)^2}{R}\dfrac{dx}{dt}$. When released the bar moves to the left, which forces a downward current in the bar. The magnetic force on the bar is in the +*x*-direction, which is opposite to the velocity, so $F_{\text{mag}} = -\dfrac{(BL)^2}{R}\dfrac{dx}{dt}$. Newton's second law now becomes $-kx - \dfrac{(BL)^2}{R}\dfrac{dx}{dt} = m\dfrac{d^2x}{dt^2}$. Comparing with Eq. (14.41) tells us that

$$b = \frac{(BL)^2}{R} = \frac{[(1.00\text{ T})(0.400\text{ m})]^2}{0.500\ \Omega} = 0.320\text{ kg/s}.$$

(b) We want the frequency. For damped harmonic motion $f' = \dfrac{\omega'}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$. Putting in the given values we get $f' = 1.38$ Hz. (This also gives $\omega' = 8.67$ rad/s.)

(c) We want the amplitude at $t = 5.00$ s. For damped harmonic motion, the amplitude is $A(t) = A_0 e^{-(b/2m)t}$. $A_0 = x_0 = 10.0$ cm, so using the numbers we get $A(5.00\text{ s}) = 5.13$ cm.

(d) We want the current when the bar passes its equilibrium position. For damped harmonic motion we have $x(t) = A(t)\cos(\omega' t + \phi)$. If the bar starts from rest, $v = 0$ when $x = x_0$, which makes $\phi = 0$. When $x = 0$ for the first time, $x = 0$ so $\cos(\omega' t) = 0$, which means that $\omega' t = \pi/2$.

$\mathcal{E} = vBL = BL\dfrac{dx}{dt} = BL\dfrac{d[A(t)\cos(\omega' t)]}{dt}$. Carry out this derivative of a product using the fact that $\omega' t = \pi/2$. The result is $\mathcal{E} = A\omega' BL$. Now evaluate $A(t) = A_0 e^{-(b/2m)t}$ using $t = \pi/2\omega'$ and $\omega' = 8.67$ rad/s from part (b). The result is $A = 9.76$ cm. The current is $I = \dfrac{\mathcal{E}}{R} = \dfrac{A\omega' BL}{R}$. Using these known quantities gives $I = 0.677$ A.

(e) The bar is moving to the left so the magnetic force on a positive charge in the bar is downward in Fig. P29.62, so the current is *counterclockwise* in the circuit.

EVALUATE: The amplitude of the motion of the bar decreases with time, so the motional emf and the induced current also decrease in amplitude with time.

29.63. **IDENTIFY:** Apply $i_D = \mathcal{E}\dfrac{d\Phi_E}{dt}$.

SET UP: $\mathcal{E} = 3.5\times10^{-11}$ F/m.

EXECUTE: $i_D = \mathcal{E}\dfrac{d\Phi_E}{dt} = (3.5\times10^{-11}\text{ F/m})(24.0\times10^3\text{ V}\cdot\text{m/s}^3)t^2$. $i_D = 21\times10^{-6}$ A gives $t = 5.0$ s.

EVALUATE: i_D depends on the rate at which Φ_E is changing.

29.64. **IDENTIFY:** Faraday's law and Ohm's law both apply. The flux change is due to the changing magnetic field.

SET UP: $\mathcal{E} = \left|\dfrac{d\Phi_B}{dt}\right|$ and $V = IR$, where $V = \mathcal{E}$ since it is caused by the changing flux. Since the flux change is due only to the change in B, we have $\mathcal{E} = \left|\dfrac{d\Phi_B}{dt}\right| = AN\left|\dfrac{dB}{dt}\right|$, where N is the number of turns.

EXECUTE: **(a)** Combining Ohm's law and Faraday's law and dropping the absolute value signs gives
$\dfrac{dB}{dt} = \dfrac{\varepsilon}{AN} = \dfrac{RI}{AN} \quad \rightarrow dB = \dfrac{RI}{AN}dt.$

Integrating gives $\Delta B_{0\rightarrow2} = \dfrac{R}{AN}\int_0^{2.00\text{ s}} Idt$. The integral is the area under the curve in the i-versus-t graph shown with the problem. We can get that using simple geometry on the graph.
area = integral = (1/2)(2.00 s)(3.00 mA) = 0.00300 A·s.
The field starts out with zero magnitude, so at 2.00 s it is
$B = R(\text{integral})/AN = (0.250\ \Omega)(0.00300\text{ A}\cdot\text{s})/[\pi(0.00800\text{ m})^2(4)] = 0.9325$ T, which rounds to 0.933 T.
(b) We use the same geometric approach as in part (a).
$\Delta B_{2\rightarrow5} = R(\text{area from 2.00 s to 5.00 s})/AN = (0.250\ \Omega)(3.00\text{ mA})(3.0\text{ s})/[\pi(0.00800\text{ m})^2(4)] = 2.798$ T.
$B_5 = B_2 + \Delta B_{2\rightarrow5} = 0.9325\text{ T} + 2.798\text{ T} = 3.73$ T.
(c) The area under the curve from 5.00 s to 6.00 s is half the area from 0.00 s to 2.00 s, so
$\Delta B_{5\rightarrow6} = \tfrac{1}{2}\,\Delta B_{0\rightarrow2} = (0.9325\text{ T})/2 = 0.46625$ T.
$B_6 = B_5 + \Delta B_{5\rightarrow6} = 3.73\text{ T} + 0.46625\text{ T} = 4.20$ T.

EVALUATE: Careful! Just because the current i is constant between 2.0 s and 5.0 s does *not* mean that B is constant since i is induced by a changing B. A constant i just means that B is changing at a constant rate.

29.65. **IDENTIFY:** An emf is induced across the moving metal bar, which causes current to flow in the circuit. The magnetic field exerts a force on the moving bar due to the current in it, which causes acceleration of the bar. Newton's second law applies to the accelerating bar. Ohm's law applies to the resistor in the circuit.
SET UP: The induced potential across the moving bar is $\mathcal{E} = vBL$, the magnetic force on the bar is $F_{mag} = ILB$, and Ohm's law is $\mathcal{E} = IR$. Newton's second law is $\Sigma\vec{F} = m\vec{a}$, and $a_x = dv_x/dt$. The flux through the loop is increasing, so the induced current is counterclockwise. Alternatively, the magnetic force $\vec{F} = q\vec{v}\times\vec{B}$ on positive charge in the moving bar is upward, by the right-hand rule, which also gives a counterclockwise current. So the magnetic force on the bar is to the left, opposite to the velocity of the bar.
EXECUTE: **(a)** Combining the equations discussed in the set up, the magnetic force on the moving bar is $F_{mag} = ILB = (\mathcal{E}/R)LB = (vBL/R)LB = v(BL)^2/R$. Newton's second law gives

$F_{mag} = ma.$
$ma = v(BL)^2/R.$
$$a = \frac{(BL)^2}{mR}v.$$
A graph of a versus v should be a straight line having slope equal to $(BL)^2/mR$. The graph of a versus v is shown in Figure 29.65. The best-fit slope of this graph is $0.3071\ \text{s}^{-1}$.

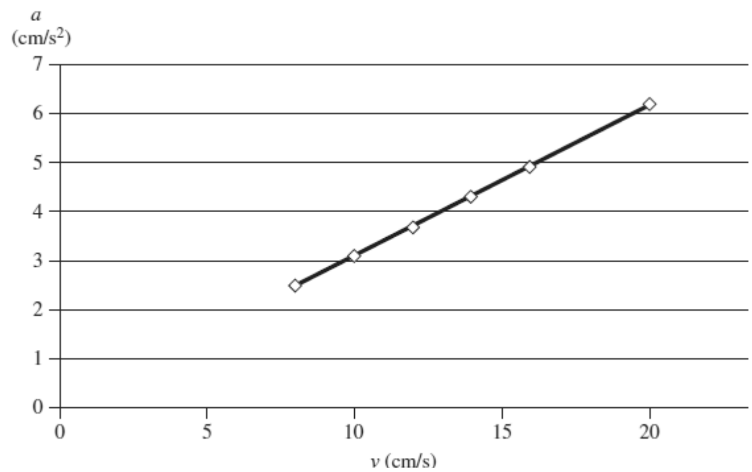


Figure 29.65

(b) $(BL)^2/mR$ = slope, so $B = \sqrt{\frac{(\text{slope})mR}{L^2}} = \sqrt{\frac{(0.3071\ \text{s}^{-1})(0.200\ \text{kg})(0.800\ \Omega)}{(0.0600\ \text{m})^2}} = 3.69\ \text{T}.$

(c) The current flows in a counterclockwise direction in the circuit. Therefore the charges lose potential energy as they pass through the resistor R from a to b, which makes point a at a higher potential than b.
(d) We know that $a_x = dv_x/dt$, and in part (a) we found that the magnitude of the acceleration is $a = \frac{(BL)^2}{mR}v.$ We also saw that a is opposite to v, so $a_x = -\frac{(BL)^2}{mR}v.$ Therefore $\frac{dv}{dt} = -\frac{(BL)^2}{mR}v.$
Separating variables and integrating gives
$$\int_{20.0\ \text{cm/s}}^{10.0\ \text{cm/s}} \frac{dv}{v} = -\int_0^t \frac{(BL)^2}{mR}dt'.$$
$$\ln\left(\frac{10}{20}\right) = -\frac{(BL)^2}{mR}t.$$
$$t = -\frac{mR}{(BL)^2}\ln(1/2) = -(0.200\ \text{kg})(0.800\ \Omega)(\ln ½)/[(3.69\ \text{T})(0.0600\ \text{m})]^2 = 2.26\ \text{s}.$$
EVALUATE: We cannot use the standard kinematics formulas because the acceleration is not constant.

29.66. **IDENTIFY:** The 8.00-cm long left side of the loop is a bar moving in a magnetic field, so an emf is induced across its ends. This emf causes current to flow through the loop, and the external magnetic field exerts a force on this bar due to the current in it. Ohm's law applies to the circuit and Newton's second law applies to the loop.
SET UP: The induced potential across the left-end side is $\mathcal{E} = vBL,$ the magnetic force on the 8.00-cm bar is $F_{mag} = ILB$, and Ohm's law is $\mathcal{E} = IR$. Newton's second law is $\Sigma\vec{F} = m\vec{a}.$ The flux through the loop is decreasing, so the induced current is counterclockwise to oppose this decrease. Alternatively, the magnetic force on positive charge in the moving left-end segment is downward, by the right-hand rule, which also gives a counterclockwise current. Therefore the magnetic force on the 8.00-cm segment is to

the left, opposite to the velocity and the external $\vec{F}$. Since the speed of the loop is constant, the external force is equal in magnitude to the magnetic force, so $F_{mag} = F$.

EXECUTE: **(a)** Combining the equations discussed in the set up, the magnetic force on the 8.00-cm bar (and on the loop) is $F = F_{mag} = ILB = (\mathcal{E}/R)LB = (vBL/R)LB = v(BL)^2/R$, so $F = v(BL)^2/R$. Therefore a graph of F versus v should be a straight line having slope equal to $(BL)^2/R$. Figure 29.66 shows a graph of F versus v. The best-fit slope of the line in this graph is 0.0520 N/(cm/s) = 5.20 N·s/m.

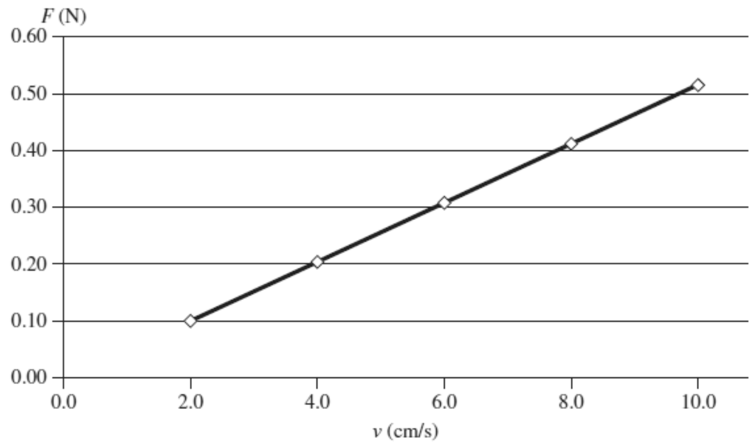


Figure 29.66

(b) Since $(BL)^2/R$ = slope, we solve for B and have

$$B = \sqrt{\frac{R(\text{slope})}{L^2}} = \sqrt{\frac{(0.00400\ \Omega)(5.20\ \text{N}\cdot\text{s/m})}{(0.0800\ \text{m})^2}} = 1.80\ \text{T}.$$

(c) The magnetic flux is decreasing through the loop, so the induced current must flow counterclockwise to oppose the decrease.

(d) $P = Fv = \dfrac{(BL)^2 v}{R}v = \dfrac{(BLv)^2}{R}$ = [(1.80 T)(0.0800 m)(0.0500 m/s)]^2/(0.00400 Ω) = 0.0130 W = 13.0 mW.

EVALUATE: For (d) we could use $P = I^2R = (vBL/R)^2/R = (vBL)^2/R$, the same result we got.

29.67. **IDENTIFY:** We are dealing with induced emf and Faraday's law.

SET UP: Refer to Fig. P29.67 with the problem in the textbook.

EXECUTE: **(a)** We want the velocity of a point on the sphere. As it is described, the particle is moving in the $-x$-direction with speed $v = r\omega$, where $r = R\sin\theta$. So $\vec{v} = R\omega\sin\theta\,\hat{i}$. Using $\omega = 2\pi f$ gives $\vec{v} = -(1.26\ \text{m/s})\sin\theta\,\hat{i}$.

(b) We want $\vec{v}\times\vec{B}$. $\vec{B}$ is in the $-z$-direction and $\vec{v}$ is in the $-x$-direction, so $\vec{v}\times\vec{B} = -(-1.26\ \text{m/s}\ \sin\theta)(-1.00\ \text{T})\,\hat{j} = -1.26\sin\theta\ \text{T}\cdot\text{m/s}\,\hat{j}$.

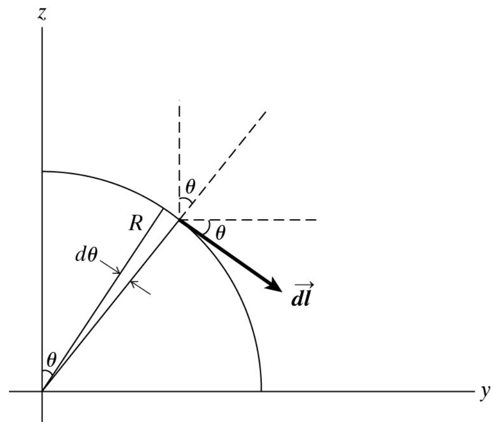


Figure 29.67

(c) We want $d\hat{l}$. See Fig. 29.67. We see that $dl = Rd\theta$ and $d\hat{l} = dl\cos\theta\,\hat{j} - dl\sin\theta\,\hat{k}$ $= Rd\theta(\cos\theta\,\hat{j} - \sin\theta\,\hat{k}) = (10.0\text{ cm})(\cos\theta\,\hat{j} - \sin\theta\,\hat{k})d\theta.$

(d) We want the current in the wire. Using our results in (b) and (c) gives $\mathcal{E} = \int(\vec{v}\times\vec{B})\bullet d\vec{L}$ $= \int_0^{60.0°}(-1.26\sin\theta\text{ T}\cdot\text{m/s}\,\hat{j})\bullet(10.0\text{ cm})(\cos\theta\,\hat{j} - \sin\theta\,\hat{k})d\theta.$ Doing the integration gives $\mathcal{E} = 4.73$ mA.

The current is $I = \dfrac{\mathcal{E}}{R} = \dfrac{4.73\text{ mA}}{10.0\ \Omega} = 4.73$ mA.

EVALUATE: (e) The segment of spherical surface from the upper rod to the lower rod behaves like a rotating curved bar and develops motional emf between its ends. The magnetic force on charges in this section of the surface forces positive charges to flow *upward* in the vertical bar.

29.68. IDENTIFY: This problem involves displacement current and Faraday's law.

SET UP: $I_d = \epsilon_0\dfrac{d\Phi_E}{dt}$, $\oint\vec{B}\cdot d\hat{l} = \mu_0(I + I_d)$, $\Phi_B = \int\vec{B}\bullet d\vec{A}$, $E = \eta t^2$.

EXECUTE: (a) We want the displacement current. $I_d = \epsilon_0\dfrac{d\Phi_E}{dt} = \epsilon_0\dfrac{d(E\pi r^2)}{dt} = \epsilon_0\pi r^2\dfrac{dE}{dt}$. Using $E = \eta t^2$ we get $dE/dt = 2\eta t$, so $I_d = 2\pi\epsilon_0\eta r^2 t.$

(b) We want $B(r)$. $\oint\vec{B}\cdot d\hat{l} = \mu_0(I + I_d)$. $I = 0$ and we have I_d from (a). Using a path or radius r we get $B2\pi r = 2\mu_0\pi\epsilon_0\eta r^2 t.$ Solving for B gives $B = \mu_0\epsilon_0\eta rt.$

(c) We want the magnetic flux. $\Phi_B = \int\vec{B}\bullet d\vec{A}$. Use $dA = bdr$. $\Phi_B = \int_0^a\mu_0\epsilon_0\eta rtbdr = \dfrac{1}{2}\mu_0\epsilon_0\eta bta^2.$

(d) We want the current. $I = \dfrac{\mathcal{E}}{R} = \dfrac{1}{R}\dfrac{d\Phi_B}{dt} = \dfrac{1}{R}\dfrac{d(\mu_0\epsilon_0\eta bta^2/2)}{dt} = \dfrac{\mu_0\epsilon_0\eta ba^2}{2R}.$

(e) The magnetic flux through the circuit is increasing. This means that the induced current flows to oppose this increase, so the current is *counterclockwise*.

EVALUATE: A changing magnetic field induces an electric field and a changing electric field induces a magnetic field.

29.69. IDENTIFY: The motion of the bar produces an induced current and that results in a magnetic force on the bar.

SET UP: $\vec{F}_B$ is perpendicular to $\vec{B}$, so is horizontal. The vertical component of the normal force equals $mg\cos\phi$, so the horizontal component of the normal force equals $mg\tan\phi$.

EXECUTE: **(a)** As the bar starts to slide, the flux is decreasing, so the current flows to increase the flux, which means it flows from a to b.

$$F_B = iLB = \frac{LB}{R}\mathcal{E} = \frac{LB}{R}\frac{d\Phi_B}{dt} = \frac{LB}{R}(B\cos\phi)\frac{dA}{dt} = \frac{LB^2}{R}(vL\cos\phi) = \frac{vL^2B^2}{R}\cos\phi.$$

(b) At the terminal speed the horizontal forces balance, so $mg\tan\phi = \frac{v_t L^2B^2}{R}\cos\phi$ and $v_t = \frac{Rmg\tan\phi}{L^2B^2\cos\phi}$.

(c) $i = \frac{\mathcal{E}}{R} = \frac{1}{R}\frac{d\Phi_B}{dt} = \frac{1}{R}(B\cos\phi)\frac{dA}{dt} = \frac{B}{R}(v_t L\cos\phi) = \frac{v_t LB\cos\phi}{R} = \frac{mg\tan\phi}{LB}$.

(d) $P = i^2R = \frac{Rm^2g^2\tan^2\phi}{L^2B^2}$.

(e) $P_g = Fv_t\cos(90° - \phi) = mg\left(\frac{Rmg\tan\phi}{L^2B^2\cos\phi}\right)\sin\phi$ and $P_g = \frac{Rm^2g^2\tan^2\phi}{L^2B^2}$.

EVALUATE: The power in part (e) equals that in part (d), as is required by conservation of energy.

29.70. **IDENTIFY:** A current is induced in the loop because of its motion and because of this current the magnetic field exerts a torque on the loop.

SET UP: Each side of the loop has mass $m/4$ and the center of mass of each side is at the center of each side. The flux through the loop is $\Phi_B = BA\cos\phi$.

EXECUTE: **(a)** $\vec{\boldsymbol{\tau}}_g = \sum \vec{\boldsymbol{r}}_{cm} \times m\vec{\boldsymbol{g}}$ summed over each leg.

$$\tau_g = \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\sin(90° - \phi) + \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\sin(90° - \phi) + (L)\left(\frac{m}{4}\right)g\sin(90° - \phi).$$

$$\tau_g = \frac{mgL}{2}\cos\phi \text{ (clockwise).}$$

$$\tau_B = \left|\vec{\boldsymbol{\tau}} \times \vec{\boldsymbol{B}}\right| = IAB\sin\phi \text{ (counterclockwise).}$$

$I = \frac{\mathcal{E}}{R} = -\frac{BA}{R}\frac{d}{dt}\cos\phi = \frac{BA}{R}\frac{d\phi}{dt}\sin\phi = \frac{BA\omega}{R}\sin\phi$. The current is going counterclockwise looking to the $-\hat{\boldsymbol{k}}$-direction. Therefore, $\tau_B = \frac{B^2A^2\omega}{R}\sin^2\phi = \frac{B^2L^4\omega}{R}\sin^2\phi$. The net torque is

$\tau = \frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi$, opposite to the direction of the rotation.

(b) $\tau = I\alpha$ (I being the moment of inertia). About this axis $I = \frac{5}{12}mL^2$. Therefore,

$$\alpha = \frac{12}{5}\frac{1}{mL^2}\left[\frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi\right] = \frac{6g}{5L}\cos\phi - \frac{12B^2L^2\omega}{5mR}\sin^2\phi.$$

EVALUATE: **(c)** The magnetic torque slows down the fall (since it opposes the gravitational torque). **(d)** Some energy is lost through heat from the resistance of the loop.

29.71. **IDENTIFY** and **SET UP:** Apply Lenz's law to determine the direction of the induced current. The figure shows the current pulse in the coil is in the counterclockwise direction as viewed from above. Also, the figure shows that direction-1 for the induced current is clockwise and direction-2 is counterclockwise.

EXECUTE: As the current pulse increases, it produces an increasing upward magnetic field in the brain. To oppose the increasing flux, the induced current must flow clockwise (direction-1). As the current pulse decreases its upward magnetic field decreases and the induced current must flow counterclockwise (direction-2) to oppose this. The correct choice is (c).

EVALUATE: Although the brain is made up of tissue, in some ways it behaves like a resistor and allows current to flow in it.

29.72. **IDENTIFY** and **SET UP:** Apply Faraday's law, $\mathcal{E} = \left|\frac{d\Phi_B}{dt}\right|$.

EXECUTE: $\mathcal{E} = \left|\frac{d\Phi_B}{dt}\right|$ $= d(BA)/dt = A\ dB/dt$. The greater the area, the greater the flux and hence the greater the rate of change of the flux in a given time. Therefore the largest area will have the greatest induced emf, and this is the periphery of the dashed line, which is choice (b).

EVALUATE: Only the field is changing, but the flux depends on the field *and* the area.

29.73. **IDENTIFY** and **SET UP:** Faraday's law gives $\mathcal{E} = \left|\frac{d\Phi_B}{dt}\right| = \frac{d(B_{\text{av}}A)}{dt} = A\frac{dB_{\text{av}}}{dt}$. $d(B_{\text{av}}A)/dt = A\ dB_{\text{av}}/dt$.

The quantity dB_{av}/dt is the slope in a B-versus-t graph, so the induced emf is greatest when the slope is steepest. Ohm's law gives $\mathcal{E} = IR$, so the current will be greatest when $\mathcal{E}$ is the greatest, which is where the slope of the B-versus-t graph is the greatest.

EXECUTE: We need to compare the slopes of graphs A and B with the slope of the graph in part (b) of the introduction to this set of passage problems. The graph in part (b) rises to 4 T in about 0.15 ms. In Figure P29.73, graph A rises to 4 T in less than 0.1 ms, and graph B also reaches 4 T in less than 0.1 ms. Therefore both graphs A and B have steeper slopes than the graph in part (b), so both of them would achieve a larger current than the process shown by the graph in part (b). This makes choice (c) correct.

EVALUATE: It is not the magnitude of the magnetic field that induces potential, but rather the *rate* at which the field changes.

29.74. **IDENTIFY** and **SET UP:** Faraday's law gives $\mathcal{E} = -\frac{d\Phi_B}{dt} - \frac{d(B_{\text{av}}A)}{dt} = -A\frac{dB_{\text{av}}}{dt}$. Ohm's law gives $\mathcal{E} = IR$, so the current is proportional to the rate at which the magnetic field is changing. That is, the current is proportional to the slope of the B-t graph.

EXECUTE: From the graph in part (b) of the figure shown with the introduction to the passage problems, we see that the magnetic field first increases rapidly as the graph has a positive slope. It then reaches a maximum value at around 0.15 ms, and then gradually decreases and the graph has a negative slope that approaches zero. Since the current is proportional to the slope of the B-t graph, the current is initially positive, then curves down to zero when the B-t graph is a maximum and becomes negative as the slope becomes negative, and it then gradually approaches zero as the slope approaches zero. Graph C most closely describes this behavior, so (c) it is the best choice.

EVALUATE: From Faraday's law, we see that the current depends on the rate at which B changes, not on the magnitude of B.

30

INDUCTANCE

VP30.4.1. **IDENTIFY:** This problem is about mutual inductance.

SET UP: $M = -\dfrac{N_2\Phi_2}{i_1}$, $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$.

EXECUTE: **(a)** We want the mutual inductance. $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$. Solve for M, giving $M = \dfrac{\mathcal{E}_2}{di_1/dt}$ = (0.130 mV)/(2.00 A/s) = 65.0 μH.

(b) We want the flux due to i_1. $M = -\dfrac{N_2\Phi_2}{i_1}$. Using the answer to (a) and the given quantities gives $\Phi_2 = 1.56\times10^{-3}$ Wb.

EVALUATE: Each coil induces an emf in the other one. That's why it's called *mutual* inductance.

VP30.4.2. **IDENTIFY:** This problem is about mutual inductance.

SET UP: $M = -\dfrac{N_2\Phi_2}{i_1}$, $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$. We want the magnitude of $\mathcal{E}_2$.

EXECUTE: **(a)** $\mathcal{E}_2 = M\dfrac{di_1}{dt}$ = (48.0 μH)(7.00 A/s) = 0.336 mV.

(b) Since di/dt = 0, $\mathcal{E}_2$ = 0.

(c) $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$ = (48.0 μH)(3.00 A/s) = 0.144 mV.

EVALUATE: The signs of $\mathcal{E}_2$ would be reversed in (a) and (c) because di/dt has opposite signs.

VP30.4.3. **IDENTIFY:** This problem involves the self-inductance of a toroidal solenoid.

SET UP: $L = N\dfrac{\Phi_B}{I}$, $\mathcal{E} = L\dfrac{di}{dt}$.

EXECUTE: **(a)** We want the flux when i = 12.0 A. Solve $L = N\dfrac{\Phi_B}{I}$ for Φ_B. This gives $\Phi_B = Li/N$ = (76.0 μH)(12.0 A)/465 = 1.96 μWb.

(b) We want the magnitude of the emf. $\mathcal{E} = L\dfrac{di}{dt}$ = (76.0 μH)(55.0 A/s) = 4.18 mV.

EVALUATE: It is not the amount of flux that causes an induced emf, but rather the *rate of change* of the flux.

VP30.4.4. **IDENTIFY:** This problem is about self-inductance.

SET UP: $\mathcal{E} = -L\dfrac{di}{dt}$.

EXECUTE: **(a)** We want L. $\mathcal{E} = L\frac{di}{dt}$. $L = \frac{\mathcal{E}}{di/dt} =$ (3.70 mV)/(145 A/s) = 25.5 μH.

(b) We want $\mathcal{E}$ at t = 2.00 s. i = (225 A/s^2)t^2, so di/dt = (450 A/s)t. Now evaluate $\mathcal{E} = -L\frac{di}{dt}$ at t = 2.00 s. $\mathcal{E} = -L\frac{di}{dt}$ = –(25.5 μH)[(450 A/s)(2.00 s)] = –23.0 mV. The minus sign means that its direction is opposite that of the current.

EVALUATE: The current is increasing, so the induced emf opposes this increase, which means that it opposes the current.

VP30.7.1. **IDENTIFY:** This is about an R-L circuit.

SET UP: $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$.

EXECUTE: **(a)** Resistance is the target variable. When t = 80.0 μs, i = 0.750i_{max}. $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$:

0.750i_{max} = $i_{max}\left(1 - e^{-(R/L)(80.0\ \mu s)}\right)$. Use logarithms to solve for R giving

$R = -\frac{L\ln(0.250)}{t} = -\frac{(4.90\text{ mH})\ln(0.250)}{80.0\mu\text{s}}$ = 84.9 Ω.

(b) We want the current 80.0 μs after closing the switch. At 80.0 μs the current is 75.0% of its maximum value, so $i = (0.750)\frac{\mathcal{E}}{R} = (0.750)\frac{24.0\text{ V}}{84.9\ \Omega} = 0.212$ A.

EVALUATE: As a check we can use the equation for the current. Putting all these values into $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$, we get i = 0.212. which is in agreement with our result in (b).

VP30.7.2. **IDENTIFY:** This problem is about an R-L circuit.

SET UP: $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$, $U_L = \frac{1}{2}Li^2$. We want the current, di/dt, and stored energy in the inductor at time t = 0.700 ms.

EXECUTE: **(a)** Current: Use $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$ with the given quantities and t = 0.700 ms. The result is i = 0.190 A.

(b) di/dt: Take the derivative of $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$. $di/dt = \frac{\mathcal{E}}{L}e^{-(R/L)t}$. Now evaluate it at t = 0.700 ms, giving di/dt = +229 A/s.

(c) Energy: Evaluate $U_L = \frac{1}{2}Li^2 = \frac{1}{2}$(37.5 μH)(0.190 A)^2 = 0.679 mJ.

EVALUATE: The energy does not remain in the inductor if the current decreases. Unlike a capacitor, an inductor cannot store energy if you remove it from a circuit.

VP30.7.3. **IDENTIFY:** This is an R-L circuit.

SET UP: $i = I_0 e^{-(R/L)t}$, $U_L = \frac{1}{2}Li^2$. We want the current, di/dt, and the energy in the inductor at t = 0.700 ms.

EXECUTE: **(a)** Current: Evaluate $i = I_0 e^{-(R/L)t}$ at t = 0.700 ms, giving i = 0.476 A.

(b) di/dt: Take the derivative of $i = I_0 e^{-(R/L)t}$ and evaluate it at t = 0.700 ms. This gives $di/dt = -\frac{\mathcal{E}}{L}e^{-(R/L)t}$ = –229 A/s.

(c) Energy: Evaluate $U_L = \frac{1}{2}Li^2 = \frac{1}{2}(37.5\ \mu\text{H})(0.476\ \text{A})^2 = 4.26\ \text{mJ}$.

EVALUATE: *di/dt* is negative because the current is decreasing with time. U_L decreases as the current decreases.

VP30.7.4. **IDENTIFY:** We have an *R-L* circuit.

SET UP: $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$, $\mathcal{E} = -L\frac{di}{dt}$, $V_R = Ri$.

EXECUTE: **(a)** We want the time when $V_L = V_R$. $V_L = L\frac{di}{dt}$ and $V_R = Ri$. Equate these potentials and solve for *t*. Take *di/dt* to get V_L, giving $L\frac{di}{dt} = \mathcal{E}e^{-(R/L)t}$. Equating potentials gives $R\left(\frac{\mathcal{E}}{R}\right)\left(1 - e^{-(R/L)t}\right) = \mathcal{E}e^{-(R/L)t}$. Use logarithms to solve for *t*, giving $t = (L/R)\ln 2$.

(b) We want the current at the time in (a). $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$. During the solution in (a), we found that $e^{-(R/L)t} = \frac{1}{2}$. Therefore $i = \frac{\mathcal{E}}{2R}$.

(c) We want *di/dt* at this time. When $V_{-L} = V_R$, each is $\mathcal{E}/2$, so $L\frac{di}{dt} = \frac{\mathcal{E}}{2}$. Thus $\frac{di}{dt} = \frac{\mathcal{E}}{2L}$.

EVALUATE: Check for (b): When $V_L = V_R$, $V_R = \mathcal{E}/2 = Ri$, so $i = \frac{\mathcal{E}}{2R}$ as we found.

VP30.10.1. **IDENTIFY:** This is an *L-C* circuit.

SET UP: $\omega = 1/\sqrt{LC}$, $U_L = \frac{1}{2}Li^2$, $U_C = \frac{Q^2}{2C}$.

EXECUTE: **(a)** We want *C*. $2\pi f = \omega = 1/\sqrt{LC}$. Solve for *C* and use the given values. $C = \frac{1}{L(2\pi f)^2}$ $= 5.12\ \text{mF}$.

(b) We want the maximum current. By energy conservation $U_{L,\max} = U_{C,\max}$. $\frac{1}{2}Li_{\max}^2 = \frac{Q_{\max}^2}{2C}$. Solve for $i_{\max}$ and use the given quantities. $i_{\max} = \frac{Q_{\max}}{\sqrt{LC}} = 14.0\ \text{A}$.

EVALUATE: The maximum charge occurs when there is no current in the circuit because there is no energy in the inductor at that instant.

VP30.10.2. **IDENTIFY:** This is an *L-C* circuit.

SET UP: $2\pi f = \omega = 1/\sqrt{LC}$, $T = 1/f$, $U_L = \frac{1}{2}Li^2$, $U_C = \frac{Q^2}{2C}$.

EXECUTE: **(a)** We want the time when the capacitor first discharges. This time is $\frac{1}{4}$ of a period. $T = 1/f = 2\pi/\omega$. Using $\omega = 1/\sqrt{LC}$, we get $t = \frac{1}{4}T = \frac{1}{4}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{2}\sqrt{LC}$. Using the values for *L* and *C* gives $t = 0.220\ \text{ms}$.

(b) We want the current when the capacitor is uncharged. By energy conservation $U_{L,\max} = U_{C,\max}$. $\frac{1}{2}Li_{\max}^2 = \frac{Q_{\max}^2}{2C}$. Solve for $i_{\max}$ and use the given quantities. $i_{\max} = \frac{Q_{\max}}{\sqrt{LC}} = 2.85\ \text{A}$.

EVALUATE: The current in (b) is the maximum current.

VP30.10.3. IDENTIFY: This is an L-C circuit.

SET UP: $U_L = \frac{1}{2}Li^2,\ U_C = \frac{Q^2}{2C},\ \omega = 1/\sqrt{LC}$. $U_L + U_C = 0.800$ J. When $U_L = U_C$, $Q = 5.30$ mC and $i = 8.00$ A.

EXECUTE: **(a)** We want C. Solve $U_C = \frac{Q^2}{2C}$ for C. $C = \frac{Q^2}{2U_C} = \frac{(5.30\text{ mC})^2}{2(0.400\text{ J})} = 35.1\,\mu\text{F}$.

(b) We want L. Solve $U_L = \frac{1}{2}Li^2$ for L. $L = \frac{2U_L}{i^2} = \frac{2(0.400\text{ mC})}{(8.00\text{ A})^2} = 12.5\text{ mH}$.

(c) We want ω. Use $\omega = 1/\sqrt{LC}$ = 1510 rad/s using the values of L and C from parts (a) and (b).

EVALUATE: If this circuit has absolutely no resistance, the sum $U_L + U_C$ will always be 0.800 J.

VP30.10.4. IDENTIFY: This is an L-R-C circuit, so we have damped oscillations.

SET UP: $\omega_0 = 1/\sqrt{LC},\ \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$.

EXECUTE: **(a)** We want C. Use $\omega_0 = 1/\sqrt{LC}$ and solve for C, giving

$$C = \frac{1}{L\omega_0^2} = \frac{1}{(42.0\text{ mH})(624\text{ rad/s})^2} = 61.1\,\mu\text{F}.$$

(b) We want R. For damped oscillations, $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \sqrt{\omega_0^2 - \frac{R^2}{4L^2}}$. Solve for R:

$R = 2L\sqrt{\omega_0^2 - \omega'^2}$. Using $\omega' = 208$ rad/s, $\omega_0 = 624$ rad/s, and $L = 42.0$ mH, we get $R = 49.4\ \Omega$.

EVALUATE: As we see, the resistance makes a very large difference in the angular frequency.

30.1. IDENTIFY and SET UP: Apply $|\mathcal{E}_2| = M\left|\frac{di_1}{dt}\right|$ and $|\mathcal{E}_1| = M\left|\frac{di_2}{dt}\right|$.

EXECUTE: **(a)** $|\mathcal{E}_2| = M\left|\frac{di_1}{dt}\right| = (3.25\times10^{-4}\text{ H})(830\text{ A/s}) = 0.270\text{ V}$; yes, it is constant.

(b) $|\mathcal{E}_1| = M\left|\frac{di_2}{dt}\right|$; M is a property of the pair of coils so is the same as in part (a). Thus $|\mathcal{E}_1| = 0.270$ V.

EVALUATE: The induced emf is the same in either case. A constant di/dt produces a constant emf.

30.2. IDENTIFY: $\mathcal{E}_1 = M\left|\frac{\Delta i_2}{\Delta t}\right|$ and $\mathcal{E}_2 = M\left|\frac{\Delta i_1}{\Delta t}\right|$. $M = \left|\frac{N_2\Phi_{B2}}{i_1}\right|$, where Φ_{B2} is the flux through one turn of the second coil.

SET UP: M is the same whether we consider an emf induced in coil 1 or in coil 2.

EXECUTE: **(a)** $M = \frac{\mathcal{E}_2}{|\Delta i_1/\Delta t|} = \frac{1.65\times10^{-3}\text{ V}}{0.242\text{ A/s}} = 6.82\times10^{-3}\text{ H} = 6.82\text{ mH}$.

(b) $\Phi_{B2} = \frac{Mi_1}{N_2} = \frac{(6.82\times10^{-3}\text{ H})(1.20\text{ A})}{25} = 3.27\times10^{-4}\text{ Wb}$.

(c) $\mathcal{E}_1 = M\left|\frac{\Delta i_2}{\Delta t}\right| = (6.82\times10^{-3}\text{ H})(0.360\text{ A/s}) = 2.46\times10^{-3}\text{ V} = 2.46\text{ mV}$.

EVALUATE: We can express M either in terms of the total flux through one coil produced by a current in the other coil, or in terms of the emf induced in one coil by a changing current in the other coil.

30.3. **IDENTIFY** and **SET UP:** Apply $M = \dfrac{N_2\Phi_{B2}}{i_1}$.

EXECUTE: **(a)** $M = \dfrac{N_2\Phi_{B2}}{i_1} = \dfrac{400(0.0320 \text{ Wb})}{6.52 \text{ A}} = 1.96 \text{ H}.$

(b) $M = \dfrac{N_1\Phi_{B1}}{i_2}$ so $\Phi_{B1} = \dfrac{Mi_2}{N_1} = \dfrac{(1.96 \text{ H})(2.54 \text{ A})}{700} = 7.11\times10^{-3} \text{ Wb}.$

EVALUATE: M relates the current in one coil to the flux through the other coil. Eq. (30.5) shows that M is the same for a pair of coils, no matter which one has the current and which one has the flux.

30.4. **IDENTIFY:** Changing flux from one object induces an emf in another object.

(a) SET UP: The magnetic field due to a solenoid is $B = \mu_0 nI$.

EXECUTE: The above formula gives

$$B_1 = \frac{(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(300)(0.120 \text{ A})}{0.250 \text{ m}} = 1.81\times10^{-4} \text{ T}.$$

The average flux through each turn of the inner solenoid is therefore

$$\Phi_B = B_1 A = (1.81\times10^{-4} \text{ T})\pi(0.0100 \text{ m})^2 = 5.68\times10^{-8} \text{ Wb}.$$

(b) SET UP: The flux is the same through each turn of both solenoids due to the geometry, so

$$M = \frac{N_2\Phi_{B,2}}{i_1} = \frac{N_2\Phi_{B,1}}{i_1}.$$

EXECUTE: $M = \dfrac{(25)(5.68\times10^{-8} \text{ Wb})}{0.120 \text{ A}} = 1.18\times10^{-5} \text{ H}.$

(c) SET UP: The induced emf is $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$.

EXECUTE: $\mathcal{E}_2 = -(1.18\times10^{-5} \text{ H})(1750 \text{ A/s}) = -0.0207 \text{ V}.$

EVALUATE: A mutual inductance around 10^{-5} H is not unreasonable.

30.5. **IDENTIFY:** We can relate the known self-inductance of the toroidal solenoid to its geometry to calculate the number of coils it has. Knowing the induced emf, we can find the rate of change of the current.

SET UP: Example 30.3 shows that the self-inductance of a toroidal solenoid is $L = \dfrac{\mu_0 N^2 A}{2\pi r}$. The voltage across the coil is related to the rate at which the current in it is changing by $\mathcal{E} = L\left|\dfrac{di}{dt}\right|$.

EXECUTE: **(a)** Solving $L = \dfrac{\mu_0 N^2 A}{2\pi r}$ for N gives

$$N = \sqrt{\frac{2\pi rL}{\mu_0 A}} = \sqrt{\frac{2\pi(0.0600 \text{ m})(2.50\times10^{-3} \text{ H})}{(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(2.00\times10^{-4} \text{ m}^2)}} = 1940 \text{ turns}.$$

(b) $\left|\dfrac{di}{dt}\right| = \dfrac{\mathcal{E}}{L} = \dfrac{2.00 \text{ V}}{2.50\times10^{-3} \text{ H}} = 800 \text{ A/s}.$

EVALUATE: The inductance is determined solely by how the coil is constructed. The induced emf depends on the rate at which the current through the coil is changing.

30.6. **IDENTIFY:** A changing current in an inductor induces an emf in it.

(a) SET UP: The self-inductance of a toroidal solenoid is $L = \dfrac{\mu_0 N^2 A}{2\pi r}$.

EXECUTE: $L = \dfrac{(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(500)^2 (6.25 \times 10^{-4}\ \text{m}^2)}{2\pi (0.0400\ \text{m})} = 7.81 \times 10^{-4}\ \text{H}.$

(b) SET UP: The magnitude of the induced emf is $\mathcal{E} = L\left|\dfrac{di}{dt}\right|$.

EXECUTE: $\mathcal{E} = (7.81 \times 10^{-4}\ \text{H})\left(\dfrac{5.00\ \text{A} - 2.00\ \text{A}}{3.00 \times 10^{-3}\ \text{s}}\right) = 0.781\ \text{V}.$

(c) The current is decreasing, so the induced emf will be in the same direction as the current, which is from a to b, making b at a higher potential than a.

EVALUATE: This is a reasonable value for self-inductance, in the range of a mH.

30.7. **IDENTIFY:** $\mathcal{E} = L\left|\dfrac{\Delta i}{\Delta t}\right|$ and $L = \dfrac{N\Phi_B}{i}$.

SET UP: $\dfrac{\Delta i}{\Delta t} = 0.0640\ \text{A/s}.$

EXECUTE: **(a)** $L = \dfrac{\mathcal{E}}{|\Delta i/\Delta t|} = \dfrac{0.0160\ \text{V}}{0.0640\ \text{A/s}} = 0.250\ \text{H}.$

(b) The average flux through each turn is $\Phi_B = \dfrac{Li}{N} = \dfrac{(0.250\ \text{H})(0.720\ \text{A})}{400} = 4.50 \times 10^{-4}\ \text{Wb}.$

EVALUATE: The self-induced emf depends on the rate of change of flux and therefore on the rate of change of the current, not on the value of the current.

30.8. **IDENTIFY:** Combine the two expressions for L: $L = N\Phi_B/i$ and $L = \mathcal{E}/|di/dt|$.

SET UP: Φ_B is the average flux through one turn of the solenoid.

EXECUTE: Solving for N we have $N = \mathcal{E}i/\Phi_B|di/dt| = \dfrac{(12.6 \times 10^{-3}\ \text{V})(1.40\ \text{A})}{(0.00285\ \text{Wb})(0.0260\ \text{A/s})} = 238$ turns.

EVALUATE: The induced emf depends on the time rate of change of the total flux through the solenoid.

30.9. **IDENTIFY** and **SET UP:** Apply $|\mathcal{E}| = L|di/dt|$. Apply Lenz's law to determine the direction of the induced emf in the coil.

EXECUTE: **(a)** $|\mathcal{E}| = L|di/dt| = (0.260\ \text{H})(0.0180\ \text{A/s}) = 4.68 \times 10^{-3}\ \text{V}.$

(b) Terminal a is at a higher potential since the coil pushes current through from b to a and if replaced by a battery it would have the $+$ terminal at a.

EVALUATE: The induced emf is directed so as to oppose the decrease in the current.

30.10. **IDENTIFY:** Apply $\mathcal{E} = -L\dfrac{di}{dt}$.

SET UP: The induced emf points from low potential to high potential across the inductor.

EXECUTE: **(a)** The induced emf points from b to a, in the direction of the current. Therefore, the current is decreasing and the induced emf is directed to oppose this decrease.

(b) $|\mathcal{E}| = L|di/dt|$, so $|di/dt| = V_{ab}/L = (1.04\ \text{V})/(0.260\ \text{H}) = 4.00\ \text{A/s}.$ In 2.00 s the decrease in i is 8.00 A and the current at 2.00 s is 12.0 A − 8.0 A = 4.0 A.

EVALUATE: When the current is decreasing the end of the inductor where the current enters is at the lower potential. This agrees with our result and with Figure 30.6d in the textbook.

30.11. IDENTIFY: Use the definition of inductance and the geometry of a solenoid to derive its self-inductance.

SET UP: The magnetic field inside a solenoid is $B = \mu_0 \frac{N}{l} i$, and the definition of self-inductance is

$L = \frac{N\Phi_B}{i}$.

EXECUTE: **(a)** $B = \mu_0 \frac{N}{l} i$, $L = \frac{N\Phi_B}{i}$, and $\Phi_B = \frac{\mu_0 NAi}{l}$. Combining these expressions gives

$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 A}{l}$.

(b) $L = \frac{\mu_0 N^2 A}{l}$. $A = \pi r^2 = \pi(0.0750\times10^{-2}\text{ m})^2 = 1.767\times10^{-6}\text{ m}^2$.

$$L = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(50)^2(1.767\times10^{-6}\text{ m}^2)}{5.00\times10^{-2}\text{ m}} = 1.11\times10^{-7}\text{ H} = 0.111\ \mu\text{H}.$$

EVALUATE: This is a physically reasonable value for self-inductance.

30.12. IDENTIFY: The changing current induces an emf in the solenoid.

SET UP: By definition of self-inductance, $L = \frac{N\Phi_B}{i}$. The magnitude of the induced emf is $|\mathcal{E}| = L\left|\frac{di}{dt}\right|$.

EXECUTE: $L = \frac{N\Phi_B}{i} = \frac{(800)(3.25\times10^{-3}\text{ Wb})}{2.90\text{ A}} = 0.8966\text{ H}.$

$$\left|\frac{di}{dt}\right| = \frac{|\mathcal{E}|}{L} = \frac{6.20\times10^{-3}\text{ V}}{0.8966\text{ H}} = 6.92\times10^{-3}\text{ A/s} = 6.92\text{ mA/s}.$$

EVALUATE: An inductance of nearly a henry is rather large. For ordinary laboratory inductors, which are around a few millihenries, the current would have to be changing much faster to induce 6.2 mV.

30.13. IDENTIFY: This problem deals with a solenoid and Faraday's law.

SET UP: $B = \mu_0 nI$, $L = \mu_0 AN^2/l$, $\mathcal{E} = -L\frac{di}{dt}$. We want B. Use the given information to find n, and then use that result to find B. We only need to deal with magnitudes.

EXECUTE: $\mathcal{E}_L = L\frac{di}{dt} = \left(\mu_0 AN^2/l\right)\frac{di}{dt} = \left(\mu_0 ANn\right)\frac{di}{dt}$. Using the given numbers for $\mathcal{E}$, A, N, and di/dt gives $n = 0.0100/\mu_0$ turns/m. $B = \mu_0 nI = \mu_0\left(\frac{0.0100\text{ turns/m}}{\mu_0}\right)(3.00\text{ A}) = 0.0300\text{ T}$.

EVALUATE: By planning a solution, one can sometimes avoid unnecessary arithmetic. In this case, we did not need to calculate with μ_0 because it cancels in the final step.

30.14. IDENTIFY and SET UP: The stored energy is $U = \frac{1}{2}LI^2$. The rate at which thermal energy is developed is $P = I^2 R$.

EXECUTE: **(a)** $U = \frac{1}{2}LI^2 = \frac{1}{2}(12.0\text{ H})(0.500\text{ A})^2 = 1.50\text{ J}$.

(b) $P = I^2 R = (0.500\text{ A})^2(180\ \Omega) = 45.0\text{ W} = 45.0\text{ J/s}$.

EVALUATE: **(c)** No. If I is constant then the stored energy U is constant. The energy being consumed by the resistance of the inductor comes from the emf source that maintains the current; it does not come from the energy stored in the inductor.

30.15. IDENTIFY and SET UP: Use $U_L = \frac{1}{2}LI^2$ to relate the energy stored to the inductance. Example 30.3 gives the inductance of a toroidal solenoid to be $L = \dfrac{\mu_0 N^2 A}{2\pi r}$, so once we know L we can solve for N.

EXECUTE: $U = \frac{1}{2}LI^2$ so $L = \dfrac{2U}{I^2} = \dfrac{2(0.390\text{ J})}{(12.0\text{ A})^2} = 5.417\times 10^{-3}\text{ H}.$

$$N = \sqrt{\frac{2\pi r L}{\mu_0 A}} = \sqrt{\frac{2\pi(0.150\text{ m})(5.417\times 10^{-3}\text{ H})}{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(5.00\times 10^{-4}\text{ m}^2)}} = 2850.$$

EVALUATE: L and hence U increase according to the square of N.

30.16. IDENTIFY: A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) SET UP: The magnetic field inside a toroidal solenoid is $B = \dfrac{\mu_0 NI}{2\pi r}$.

EXECUTE: $B = \dfrac{\mu_0(300)(5.00\text{ A})}{2\pi(0.120\text{ m})} = 2.50\times 10^{-3}\text{ T} = 2.50\text{ mT}.$

(b) SET UP: The self-inductance of a toroidal solenoid is $L = \dfrac{\mu_0 N^2 A}{2\pi r}$.

EXECUTE: $L = \dfrac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(300)^2(4.00\times 10^{-4}\text{ m}^2)}{2\pi(0.120\text{ m})} = 6.00\times 10^{-5}\text{ H}.$

(c) SET UP: The energy stored in an inductor is $U_L = \frac{1}{2}LI^2$.

EXECUTE: $U_L = \frac{1}{2}(6.00\times 10^{-5}\text{ H})(5.00\text{ A})^2 = 7.50\times 10^{-4}\text{ J}.$

(d) SET UP: The energy density in a magnetic field is $u = \dfrac{B^2}{2\mu_0}$.

EXECUTE: $u = \dfrac{(2.50\times 10^{-3}\text{ T})^2}{2(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})} = 2.49\text{ J/m}^3.$

EVALUATE: **(e)** $u = \dfrac{\text{energy}}{\text{volume}} = \dfrac{\text{energy}}{2\pi r A} = \dfrac{7.50\times 10^{-4}\text{ J}}{2\pi(0.120\text{ m})(4.00\times 10^{-4}\text{ m}^2)} = 2.49\text{ J/m}^3.$

An inductor stores its energy in the magnetic field inside of it.

30.17. IDENTIFY: A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) SET UP: The magnetic field inside a solenoid is $B = \mu_0 nI$.

EXECUTE: $B = \dfrac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(400)(80.0\text{ A})}{0.250\text{ m}} = 0.161\text{ T}.$

(b) SET UP: The energy density in a magnetic field is $u = \dfrac{B^2}{2\mu_0}$.

EXECUTE: $u = \dfrac{(0.161\text{ T})^2}{2(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})} = 1.03\times 10^4\text{ J/m}^3.$

(c) SET UP: The total stored energy is $U = uV$.

EXECUTE: $U = uV = u(lA) = (1.03\times 10^4\text{ J/m}^3)(0.250\text{ m})(0.500\times 10^{-4}\text{ m}^2) = 0.129\text{ J}.$

(d) SET UP: The energy stored in an inductor is $U = \frac{1}{2}LI^2$.

EXECUTE: Solving for L and putting in the numbers gives

$$L = \frac{2U}{I^2} = \frac{2(0.129\ \text{J})}{(80.0\ \text{A})^2} = 4.02\times10^{-5}\ \text{H}.$$

EVALUATE: An inductor stores its energy in the magnetic field inside of it.

30.18. **IDENTIFY:** Energy $= Pt$. $U = \frac{1}{2}LI^2$.

SET UP: $P = 150\ \text{W} = 150\ \text{J/s}$.

EXECUTE: **(a)** Energy $= (150\ \text{W})(24\ \text{h})(3600\ \text{s/h}) = 1.296\times10^7\ \text{J}$, which rounds to $1.30\times10^7\ \text{J} = 13.0\ \text{MJ}$.

(b) $L = \dfrac{2U}{I^2} = \dfrac{2(1.296\times10^7\ \text{J})}{(80.0\ \text{A})^2} = 4.05\times10^3\ \text{H} = 4.05\ \text{kH}$.

EVALUATE: A large value of L and a large current would be required, just for one light bulb. Also, the resistance of the inductor would have to be very small, to avoid a large $P = I^2R$ rate of electrical energy loss.

30.19. **IDENTIFY:** The energy density depends on the strength of the magnetic field, and the energy depends on the volume in which the magnetic field exists.

SET UP: The energy density is $u = \dfrac{B^2}{2\mu_0}$.

EXECUTE: First find the energy density: $u = \dfrac{B^2}{2\mu_0} = \dfrac{(4.80\ \text{T})^2}{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})} = 9.167\times10^6\ \text{J/m}^3$. The energy U in a volume V is $U = uV = (9.167\times10^6\ \text{J/m}^3)(10.0\times10^{-6}\ \text{m}^3) = 91.7\ \text{J}$.

EVALUATE: A field of 4.8 T is very strong, so this is a high energy density for a magnetic field.

30.20. **IDENTIFY:** This problem is about the energy density in electric and magnetic fields.

SET UP: $u_E = \dfrac{\epsilon_0 E^2}{2}$, $u_B = \dfrac{B^2}{2\mu_0}$.

EXECUTE: **(a)** We want E/B when $u_E = u_B$. Equate the energy densities. $\dfrac{\epsilon_0 E^2}{2} = \dfrac{B^2}{2\mu_0}$.

$$\frac{E}{B} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3.00\times10^8\ \text{V/m}\cdot\text{T}.$$

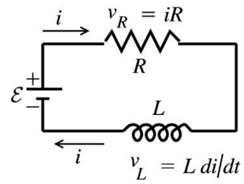
(b) We want B. Use the result from (a) giving $B = 1.67\ \mu\text{T}$.

EVALUATE: Note the units of E/B: $\dfrac{\text{V}}{\text{m}\cdot\text{T}} = \dfrac{\text{J/C}}{\text{m}\left(\dfrac{\text{N}}{\text{C}\cdot\text{m/s}}\right)} =$ m/s. Therefore,

$$\frac{E}{B} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3.00\times10^8\ \text{V/m}\cdot\text{T} = 3.00\times10^8\ \text{m/s},$$ which is the speed of light in vacuum.

30.21. **IDENTIFY:** Apply Kirchhoff's loop rule to the circuit. $i(t)$ is given by $i = \dfrac{\mathcal{E}}{R}(1 - e^{-(R/L)t})$.

SET UP: The circuit is sketched in Figure 30.21.



$\frac{di}{dt}$ is positive as the current increases from its initial value of zero.

Figure 30.21

EXECUTE: $\mathcal{E} - v_R - v_L = 0.$

$\mathcal{E} - iR - L\frac{di}{dt} = 0$ so $i = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t}).$

(a) Initially $(t = 0)$, $i = 0$ so $\mathcal{E} - L\frac{di}{dt} = 0.$

$$\frac{di}{dt} = \frac{\mathcal{E}}{L} = \frac{6.00\text{ V}}{2.50\text{ H}} = 2.40\text{ A/s}.$$

(b) $\mathcal{E} - iR - L\frac{di}{dt} = 0.$ (Use this equation rather than $\frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-(R/L)t}$ since i rather than t is given.)

Thus $\frac{di}{dt} = \frac{\mathcal{E} - iR}{L} = \frac{6.00\text{ V} - (0.500\text{ A})(8.00\ \Omega)}{2.50\text{ H}} = 0.800\text{ A/s}.$

(c) $i = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t}) = \left(\frac{6.00\text{ V}}{8.00\ \Omega}\right)(1 - e^{-(8.00\ \Omega/2.50\text{ H})(0.250\text{ s})}) = 0.750\text{ A}(1 - e^{-0.800}) = 0.413\text{ A}.$

(d) Final steady state means $t \to \infty$ and $\frac{di}{dt} \to 0$, so $\mathcal{E} - iR = 0.$

$$i = \frac{\mathcal{E}}{R} = \frac{6.00\text{ V}}{8.00\ \Omega} = 0.750\text{ A}.$$

EVALUATE: Our results agree with Figure 30.12 in the textbook. The current is initially zero and increases to its final value of $\mathcal{E}/R$. The slope of the current in the figure, which is *di/dt*, decreases with *t*.

30.22. IDENTIFY: With S_1 closed and S_2 open, the current builds up to a steady value. Then with S_1 open and S_2 closed, the current decreases exponentially.

SET UP: The decreasing current is $i = I_0 e^{-(R/L)t}$.

EXECUTE: **(a)** $i = I_0 e^{-(R/L)t} = \frac{\mathcal{E}}{R}e^{-(R/L)t}$. $e^{-(R/L)t} = \frac{iR}{\mathcal{E}} = \frac{(0.280\text{ A})(15.0\ \Omega)}{6.30\text{ V}} = 0.6667.$

$$\frac{Rt}{L} = -\ln(0.6667).\quad L = -\frac{Rt}{\ln(0.6667)} = -\frac{(15.0\ \Omega)(2.00\times10^{-3}\text{ s})}{\ln(0.6667)} = 0.0740\text{ H} = 74.0\text{ mH}.$$

(b) $\frac{i}{I_0} = e^{-(R/L)t}$. $e^{-(R/L)t} = 0.0100.$ $\frac{Rt}{L} = -\ln(0.0100).$

$$t = -\frac{\ln(0.0100)L}{R} = -\frac{\ln(0.0100)(0.0740\text{ H})}{15.0\ \Omega} = 0.0227\text{ s} = 22.7\text{ ms}.$$

EVALUATE: Typical *LR* circuits change rapidly compared to human time scales, so 22.7 ms is not unusual.

30.23. IDENTIFY: $i = \mathcal{E}/R(1 - e^{-t/\tau})$, with $\tau = L/R$. The energy stored in the inductor is $U = \frac{1}{2}Li^2$.

SET UP: The maximum current occurs after a long time and is equal to $\mathcal{E}/R$.

EXECUTE: **(a)** $i_{max} = \mathcal{E}/R$ so $i = i_{max}/2$ when $(1 - e^{-t/\tau}) = \frac{1}{2}$ and $e^{-t/\tau} = \frac{1}{2}$. $-t/\tau = \ln(\frac{1}{2})$.

$$t = \frac{L\ln 2}{R} = \frac{(\ln 2)(1.25\times10^{-3}\text{ H})}{50.0\ \Omega} = 17.3\ \mu\text{s}.$$

(b) $U = \frac{1}{2}U_{\max}$ when $i = i_{\max}/\sqrt{2}$. $1 - e^{-t/\tau} = 1/\sqrt{2}$, so $e^{-t/\tau} = 1 - 1/\sqrt{2} = 0.2929$.
$t = -L\ln(0.2929)/R = 30.7\ \mu\text{s}$.

EVALUATE: $\tau = L/R = 2.50 \times 10^{-5}\ \text{s} = 25.0\ \mu s$. The time in part (a) is 0.692τ and the time in part (b) is 1.23τ.

30.24. **IDENTIFY:** We have an *R*-*L* circuit.

SET UP: $\mathcal{E}_L = -L\dfrac{di}{dt}$, $i = I_{\max}\left(1 - e^{-(R/L)t}\right)$. We want the time *T* when *i* = 5.00 A. First find *R* and *L*.

EXECUTE: After a long time, $i = I_{\max}$ = 15.0 A, so $R = \mathcal{E}/I_{\max}$ = (240 V)/(15.0 A) = 16.0 Ω. At time *T*, $V_R = Ri$ = (16.0 Ω)(5.00 A) = 80.0 V, so $\mathcal{E}_L$ = 240 V – 80.0 V = 160 V, so $L\dfrac{di}{dt}$ = 160 V. This gives *L*(20.0 A/s) = 160 V, so *L* = 8.00 H. Now find the current. Use $i = I_{\max}\left(1 - e^{-(R/L)t}\right)$, and solve for *T* using the known quantities. Using logarithms gives *T* = –(*L/R*) ln(2/3) = 0.203 s.

EVALUATE: $\tau = L/R$ = (8.00 H)/(16.0 Ω) = 0.500 s, so *T* is *not* the time constant for this circuit.

30.25. **IDENTIFY:** We have an *R*-*L* circuit.

SET UP: $U_L = \dfrac{1}{2}Li^2$. We want the voltage across the inductor when it contains 0.400 J of energy.

EXECUTE: Solve $U_L = \dfrac{1}{2}Li^2$ for *i*, giving $i = \sqrt{2U_L/L}$. $V_L = \mathcal{E} - V_R = \mathcal{E} - RI = \mathcal{E} - R\sqrt{2U_L/L}$. Using the given quantities gives V_L = 15.4 V.

EVALUATE: As time increases, *i* increases so U_L increases to a maximum value when $i = \mathcal{E}/R$ = 1.67 A.

30.26. **IDENTIFY:** This is an *R*-*L* circuit.

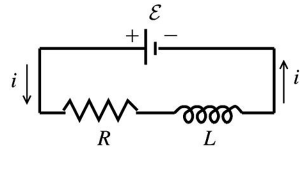
SET UP: We want the time *T* for the current to decrease from 12.0 A to 6.00 A. $i = I_0 e^{-(R/L)t}$, $\mathcal{E}_L = -L\dfrac{di}{dt}$.

EXECUTE: After we close S_2 the current is $i = I_0 e^{-(R/L)t}$. At time *T* the current is half its initial value, so $\dfrac{1}{2}I_0 = I_0 e^{-(R/L)T}$. Use logarithms to solve for *T*, giving *T* = (*L/R*) ln 2. We see that we need to find *L/R*. At *t* = 0, $V_L = V_R$, so $-L\dfrac{di}{dt} = -Ri_0$. $L/R = i_0/(di/dt)$ = (12.0 A)/(36.0 A/s) = 0.333 s. Therefore *T* = (0.333 s) ln 2 = 0.231 s.

EVALUATE: The time constant *L/R* is the same with S_1 closed and S_2 open as it is with S_1 open and S_2 closed. But in the first case, the current increases with time but in the second case it decreases with time.

30.27. **IDENTIFY:** Apply the concepts of current decay in an *R*-*L* circuit. Apply the loop rule to the circuit. $i(t)$ is given by $i = I_0 e^{-(R/L)t}$. The voltage across the resistor depends on *i* and the voltage across the inductor depends on di/dt.

SET UP: The circuit with S_1 closed and S_2 open is sketched in Figure 30.27a.



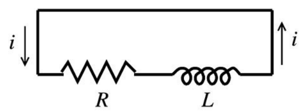
$$\mathcal{E} - iR - L\frac{di}{dt} = 0.$$

Figure 30.27a

Constant current established means $\frac{di}{dt} = 0.$

$$i = \frac{\mathcal{E}}{R} = \frac{60.0\ \text{V}}{240\ \Omega} = 0.250\ \text{A}.$$

EXECUTE: **(a)** The circuit with S_2 closed and S_1 open is shown in Figure 30.27b.



$i = I_0 e^{-(R/L)t}$.

At $t = 0, i = I_0 = 0.250$ A.

Figure 30.27b

The inductor prevents an instantaneous change in the current; the current in the inductor just after S_2 is closed and S_1 is opened equals the current in the inductor just before this is done.

(b) $i = I_0 e^{-(R/L)t} = (0.250\ \text{A})e^{-(240\,\Omega/0.160\,\text{H})(4.00\times10^{-4}\,\text{s})} = (0.250\ \text{A})e^{-0.600} = 0.137\ \text{A}.$

(c) See Figure 30.27c.

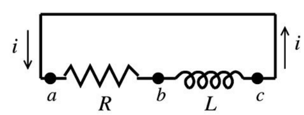


Figure 30.27c

If we trace around the loop in the direction of the current the potential falls as we travel through the resistor so it must rise as we pass through the inductor: $v_{ab} > 0$ and $v_{bc} < 0$. So point *c* is at a higher potential than point *b*.

$v_{ab} + v_{bc} = 0$ and $v_{bc} = -v_{ab}$.

Or, $v_{cb} = v_{ab} = iR = (0.137\ \text{A})(240\ \Omega) = 32.9\ \text{V}.$

(d) $i = I_0 e^{-(R/L)t}$.

$i = \frac{1}{2}I_0$ says $\frac{1}{2}I_0 = I_0 e^{-(R/L)t}$ and $\frac{1}{2} = e^{-(R/L)t}$.

Taking natural logs of both sides of this equation gives $\ln(\frac{1}{2}) = -Rt/L$.

$$t = \left(\frac{0.160\ \text{H}}{240\ \Omega}\right)\ln 2 = 4.62\times10^{-4}\ \text{s}.$$

EVALUATE: The current decays, as shown in Figure 30.13 in the textbook. The time constant is $\tau = L/R = 6.67\times10^{-4}$ s. The values of *t* in the problem are less than one time constant. At any instant the potential drop across the resistor (in the direction of the current) equals the potential rise across the inductor.

30.28. **IDENTIFY:** Apply $i = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t})$.

SET UP: $v_{ab} = iR$. $v_{bc} = L\frac{di}{dt}$. The current is increasing, so di/dt is positive.

EXECUTE: **(a)** At $t = 0$, $i = 0$. $v_{ab} = 0$ and $v_{bc} = 60$ V.

(b) As $t \to \infty$, $i \to \mathcal{E}/R$ and $di/dt \to 0$. $v_{ab} \to 60$ V and $v_{bc} \to 0$.

(c) When $i = 0.150$ A, $v_{ab} = iR = 36.0$ V and $v_{bc} = 60.0\ \text{V} - 36.0\ \text{V} = 24.0$ V.

EVALUATE: At all times, $\mathcal{E} = v_{ab} + v_{bc}$, as required by the loop rule.

30.29. **IDENTIFY:** With S_1 closed and S_2 open, the current builds up to a steady value.

SET UP: Applying Kirchhoff's loop rule gives $\mathcal{E} - iR - L\dfrac{di}{dt} = 0.$

EXECUTE: $v_R = \mathcal{E} - L\dfrac{di}{dt} = 18.0\text{ V} - (0.380\text{ H})(7.20\text{ A/s}) = 15.3\text{ V}.$

EVALUATE: The rest of the 18.0 V of the emf is across the inductor.

30.30. **IDENTIFY and SET UP:** The inductor opposes changes in current through it. $P = iV$, $P_R = i^2R$, $V = Ri$, $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** The inductor prevents an instantaneous build up of current, so the initial current is zero. The power supplied by the battery is $P = i\mathcal{E} = 0$ since $i_0 = 0$.

(b) The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$ and $i = \mathcal{E}/R$, so

$U_L = (1/2)(2.50\text{ H})[(6.00\text{ V})/(8.00\ \Omega)]^2 = 0.703\text{ J}.$
$P_R = i^2R = [(6.00\text{ V})/(8.00\ \Omega)]^2(8.00\ \Omega) = 4.50\text{ W}.$
The power supplied by the battery is
$P_{\mathcal{E}} = i\mathcal{E} = (\mathcal{E}/R)\mathcal{E} = (6.00\text{ V})^2/(8.00\ \Omega) = 4.50\text{ W}.$

EVALUATE: At steady-state the current is not changing so the potential difference across the inductor is zero. Therefore the power supplied by the battery is all consumed in the resistor, as we found.

30.31. **IDENTIFY:** This is an *R-L* circuit.
SET UP: $P = iV$. We want the power in the inductor.
EXECUTE: **(a)** At $t = 0$: $P_L = iV_L = 0$ because $i = 0$.
(b) As $t \to \infty$: The current is not changing, so $V_L = 0$. Therefore $P_L = iV_L = 0$.
(c) When $i = \mathcal{E}/2R$: $V_R = Ri = R\left(\dfrac{\mathcal{E}}{2R}\right) = \dfrac{\mathcal{E}}{2}$, so $V_L = \dfrac{\mathcal{E}}{2}$. $P_L = iV_L = \left(\dfrac{\mathcal{E}}{2R}\right)\left(\dfrac{\mathcal{E}}{2}\right) = \dfrac{\mathcal{E}^2}{4R}.$

EVALUATE: As $t \to \infty$, P_L becomes a maximum but its rate of change approaches zero.

30.32. **IDENTIFY:** An L-C circuit oscillates, with the energy going back and forth between the inductor and capacitor.

(a) SET UP: The frequency is $f = \dfrac{\omega}{2\pi}$ and $\omega = \dfrac{1}{\sqrt{LC}}$, giving $f = \dfrac{1}{2\pi\sqrt{LC}}$.

EXECUTE: $f = \dfrac{1}{2\pi\sqrt{(0.280\times10^{-3}\text{ H})(15.0\times10^{-6}\text{ F})}} = 2.456\times10^3\text{ Hz},$ which rounds to 2.46 kHz.

(b) SET UP: The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: $U = \frac{1}{2}(15.0\times10^{-6}\text{ F})(150.0\text{ V})^2 = 0.169\text{ J}.$

(c) SET UP: The current in the circuit is $i = -\omega Q\sin\omega t$, and the energy stored in the inductor is $U = \frac{1}{2}Li^2$.

EXECUTE: First find ω and Q. $\omega = 2\pi f = 2\pi(2456\text{ Hz}) = 1.543\times10^4\text{ rad/s}.$

$Q = CV = (15.0\times10^{-6}\text{ F})(150.0\text{ V}) = 2.25\times10^{-3}\text{ C}.$ Now calculate the current:

$i = -(1.543\times10^4\text{ rad/s})(2.25\times10^{-3}\text{ C})\sin[(1.543\times10^4\text{ rad/s})(1.30\times10^{-3}\text{ s})].$ Notice that the argument of the sine is in *radians*, so convert it to degrees if necessary. The result is $i = -32.48$ A.
Now find the energy in the inductor: $U = \frac{1}{2}Li^2 = \frac{1}{2}(0.280\times10^{-3}\text{ H})(-32.48\text{ A})^2 = 0.148\text{ J}.$

EVALUATE: At the end of 1.30 ms, more of the energy is now in the inductor than in the capacitor.

30.33. **IDENTIFY:** Apply $\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{Q^2}{2C}$.

SET UP: $q = Q$ when $i = 0$. $i = i_{\text{max}}$ when $q = 0$. $1/\sqrt{LC} = 1917\ \text{s}^{-1}$.

EXECUTE: **(a)** $\frac{1}{2}Li_{\text{max}}^2 = \frac{Q^2}{2C}$.

$Q = i_{\text{max}}\sqrt{LC} = (0.850\times10^{-3}\ \text{A})\sqrt{(0.0850\ \text{H})(3.20\times10^{-6}\ \text{F})} = 4.43\times10^{-7}\ \text{C}$.

(b) $q = \sqrt{Q^2 - LCi^2} = \sqrt{(4.43\times10^{-7}\ \text{C})^2 - \left(\frac{5.00\times10^{-4}\,\text{A}}{1917\ \text{s}^{-1}}\right)^2} = 3.58\times10^{-7}\ \text{C}$.

EVALUATE: The value of q calculated in part (b) is less than the maximum value Q calculated in part (a).

30.34. **IDENTIFY:** The energy moves back and forth between the inductor and capacitor.

(a) SET UP: The period is $T = \frac{1}{f} = \frac{1}{\omega/2\pi} = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$.

EXECUTE: Solving for L gives

$$L = \frac{T^2}{4\pi^2 C} = \frac{(8.60\times10^{-5}\ \text{s})^2}{4\pi^2(7.50\times10^{-9}\ \text{F})} = 2.50\times10^{-2}\ \text{H} = 25.0\ \text{mH}.$$

(b) SET UP: The charge on a capacitor is $Q = CV$.

EXECUTE: $Q = CV = (7.50\times10^{-9}\ \text{F})(12.0\ \text{V}) = 9.00\times10^{-8}\ \text{C}$.

(c) SET UP: The stored energy is $U = Q^2/2C$.

EXECUTE: $U = \frac{(9.00\times10^{-8}\ \text{C})^2}{2(7.50\times10^{-9}\ \text{F})} = 5.40\times10^{-7}\ \text{J}$.

(d) SET UP: The maximum current occurs when the capacitor is discharged, so the inductor has all the initial energy. $U_L + U_C = U_{\text{Total}}$. $\frac{1}{2}LI^2 + 0 = U_{\text{Total}}$.

EXECUTE: Solve for the current:

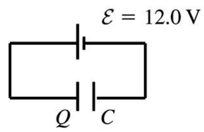
$$I = \sqrt{\frac{2U_{\text{Total}}}{L}} = \sqrt{\frac{2(5.40\times10^{-7}\ \text{J})}{2.50\times10^{-2}\ \text{H}}} = 6.58\times10^{-3}\ \text{A} = 6.58\ \text{mA}.$$

EVALUATE: The energy oscillates back and forth forever. However, if there is any resistance in the circuit, no matter how small, all this energy will eventually be dissipated as thermal energy in the resistor.

30.35. **IDENTIFY and SET UP:** The angular frequency is given by $\omega = \frac{1}{\sqrt{LC}}$. $q(t)$ and $i(t)$ are given by $q = Q\cos(\omega t + \phi)$ and $i = -\omega Q\sin(\omega t + \phi)$. The energy stored in the capacitor is $U_C = \frac{1}{2}CV^2 = q^2/2C$. The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.50\ \text{H})(6.00\times10^{-5}\ \text{F})}} = 105.4\ \text{rad/s}$, which rounds to $105\ \text{rad/s}$. The period is given by $T = \frac{2\pi}{\omega} = \frac{2\pi}{105.4\ \text{rad/s}} = 0.0596\ \text{s}$.

(b) The circuit containing the battery and capacitor is sketched in Figure 30.35.



$\mathcal{E} - \dfrac{Q}{C} = 0.$

$Q = \mathcal{E}C = (12.0\text{ V})(6.00\times10^{-5}\text{ F}) = 7.20\times10^{-4}\text{ C}.$

Figure 30.35

(c) $U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(6.00\times10^{-5}\text{ F})(12.0\text{ V})^2 = 4.32\times10^{-3}\text{ J}.$

(d) $q = Q\cos(\omega t + \phi)$ (Eq. 30.21).

$q = Q$ at $t = 0$ so $\phi = 0$.

$q = Q\cos\omega t = (7.20\times10^{-4}\text{ C})\cos([105.4\text{ rad/s}][0.0230\text{ s}]) = -5.42\times10^{-4}\text{ C}.$

The minus sign means that the capacitor has discharged fully and then partially charged again by the current maintained by the inductor; the plate that initially had positive charge now has negative charge and the plate that initially had negative charge now has positive charge.

(e) The current is $i = -\omega Q\sin(\omega t + \phi)$.

$i = -(105\text{ rad/s})(7.20\times10^{-4}\text{ C})\sin[(105.4\text{ rad/s})(0.0230\text{ s})] = -0.050\text{ A}.$

The negative sign means the current is counterclockwise in Figure 30.15 in the textbook.

or

$\tfrac{1}{2}Li^2 + \dfrac{q^2}{2C} = \dfrac{Q^2}{2C}$ gives $i = \pm\sqrt{\dfrac{1}{LC}}\sqrt{Q^2 - q^2}$ (Eq. 30.26).

$i = \pm(105\text{ rad/s})\sqrt{(7.20\times10^{-4}\text{ C})^2 - (-5.42\times10^{-4}\text{ C})^2} = \pm0.050\text{ A},$ which checks.

(f) $U_C = \dfrac{q^2}{2C} = \dfrac{(-5.42\times10^{-4}\text{ C})^2}{2(6.00\times10^{-5}\text{ F})} = 2.45\times10^{-3}\text{ J}.$

$U_L = \tfrac{1}{2}Li^2 = \tfrac{1}{2}(1.50\text{ H})(0.050\text{ A})^2 = 1.87\times10^{-3}\text{ J}.$

Evaluate: Note that $U_C + U_L = 2.45\times10^{-3}\text{ J} + 1.87\times10^{-3}\text{ J} = 4.32\times10^{-3}\text{ J}.$

This agrees with the total energy initially stored in the capacitor,

$U = \dfrac{Q^2}{2C} = \dfrac{(7.20\times10^{-4}\text{ C})^2}{2(6.00\times10^{-5}\text{ F})} = 4.32\times10^{-3}\text{ J}.$

Energy is conserved. At some times there is energy stored in both the capacitor and the inductor. When $i = 0$ all the energy is stored in the capacitor and when $q = 0$ all the energy is stored in the inductor. But at all times the total energy stored is the same.

30.36. Identify: $\omega = \dfrac{1}{\sqrt{LC}} = 2\pi f.$

Set Up: ω is the angular frequency in rad/s and *f* is the corresponding frequency in Hz.

Execute: **(a)** $L = \dfrac{1}{4\pi^2 f^2 C} = \dfrac{1}{4\pi^2(1.6\times10^6\text{ Hz})^2(4.18\times10^{-12}\text{ F})} = 2.37\times10^{-3}\text{ H}.$

(b) The maximum capacitance corresponds to the minimum frequency.

$C_{\max} = \dfrac{1}{4\pi^2 f_{\min}^2 L} = \dfrac{1}{4\pi^2(5.40\times10^5\text{ Hz})^2(2.37\times10^{-3}\text{ H})} = 3.67\times10^{-11}\text{ F} = 36.7\text{ pF}.$

Evaluate: To vary *f* by a factor of three (approximately the range in this problem), *C* must be varied by a factor of nine.

30.37. Identify: Apply energy conservation and $\omega = \dfrac{1}{\sqrt{LC}}$ and $i = -\omega Q\sin(\omega t + \phi)$.

Set Up: If *I* is the maximum current, $\tfrac{1}{2}LI^2 = \dfrac{Q^2}{2C}$. For the inductor, $U_L = \tfrac{1}{2}Li^2$.

EXECUTE: (a) $\frac{1}{2}LI^2 = \dfrac{Q^2}{2C}$ gives $Q = I\sqrt{LC} = (0.750\text{ A})\sqrt{(0.0800\text{ H})(1.25\times10^{-9}\text{ F})} = 7.50\times10^{-6}\text{ C}.$

(b) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.0800\text{ H})(1.25\times10^{-9}\text{ F})}} = 1.00\times10^5\text{ rad/s}.$ $f = \dfrac{\omega}{2\pi} = 1.59\times10^4\text{ Hz}.$

(c) $q = Q$ at $t = 0$ means $\phi = 0$. $i = -\omega Q\sin(\omega t)$, so

$i = -(1.00\times10^5\text{ rad/s})(7.50\times10^{-6}\text{ C})\sin[(1.00\times10^5\text{ rad/s})(2.50\times10^{-3}\text{ s})] = 0.7279\text{ A}.$

$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(0.0800\text{ H})(0.7279\text{ A})^2 = 0.0212\text{ J}.$

EVALUATE: The total energy of the system is $\frac{1}{2}LI^2 = 0.0225\text{ J}$. At $t = 2.50\text{ ms}$, the current is close to its maximum value and most of the system's energy is stored in the inductor.

30.38. IDENTIFY: The presence of resistance in an *L-R-C* circuit affects the frequency of oscillation and causes the amplitude of the oscillations to decrease over time.

(a) SET UP: The frequency of damped oscillations is $\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}.$

EXECUTE: $\omega' = \sqrt{\dfrac{1}{(22\times10^{-3}\text{ H})(15.0\times10^{-9}\text{ F})} - \dfrac{(75.0\ \Omega)^2}{4(22\times10^{-3}\text{ H})^2}} = 5.5\times10^4\text{ rad/s}.$

The frequency *f* is $f = \dfrac{\omega}{2\pi} = \dfrac{5.50\times10^4\text{ rad/s}}{2\pi} = 8.76\times10^3\text{ Hz} = 8.76\text{ kHz}.$

(b) SET UP: The amplitude decreases as $A(t) = A_0\, e^{-(R/2L)t}$.

EXECUTE: Solving for *t* and putting in the numbers gives:

$$t = \frac{-2L\ln(A/A_0)}{R} = \frac{-2(22.0\times10^{-3}\text{ H})\ln(0.100)}{75.0\ \Omega} = 1.35\times10^{-3}\text{ s} = 1.35\text{ ms}.$$

(c) SET UP: At critical damping, $R = \sqrt{4L/C}$.

EXECUTE: $R = \sqrt{\dfrac{4(22.0\times10^{-3}\text{ H})}{15.0\times10^{-9}\text{ F}}} = 2420\ \Omega.$

EVALUATE: The frequency with damping is almost the same as the resonance frequency of this circuit $(1/\sqrt{LC})$, which is plausible because the $75\text{-}\Omega$ resistance is considerably less than the $2420\ \Omega$ required for critical damping.

30.39. IDENTIFY: Evaluate $\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}.$

SET UP: The angular frequency of the circuit is ω'.

EXECUTE: (a) When $R = 0$, $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.450\text{ H})(2.50\times10^{-5}\text{ F})}} = 298\text{ rad/s}.$

(b) We want $\dfrac{\omega'}{\omega_0} = 0.95$, so $\dfrac{(1/LC - R^2/4L^2)}{1/LC} = 1 - \dfrac{R^2C}{4L} = (0.95)^2$. This gives

$R = \sqrt{\dfrac{4L}{C}(1-(0.95)^2)} = \sqrt{\dfrac{4(0.450\text{ H})(0.0975)}{(2.50\times10^{-5}\text{ F})}} = 83.8\ \Omega.$

EVALUATE: When *R* increases, the angular frequency decreases and approaches zero as $R \to 2\sqrt{L/C}$.

30.40. IDENTIFY and SET UP: Eq. (30.28) is $q = Ae^{-(R/2L)t}\cos\left(\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}t + \phi\right)$. We first find *A* and ϕ using the given information.

EXECUTE: **(a)** The charge is a maximum at $t = 0$, so $A = q_0 = 2.80\times10^{-4}$ C and $\phi = 0$.

(b) At the end of the first oscillation, $\sqrt{\dfrac{1}{LC}-\dfrac{R^2}{4L^2}}\,t = 2\pi$. Solving for t gives

$$t=\frac{2\pi}{\sqrt{\dfrac{1}{LC}-\dfrac{R^2}{4L^2}}}=\frac{2\pi}{\sqrt{\dfrac{1}{(0.400\text{ H})(7.00\ \mu\text{F})}-\dfrac{(320\ \Omega)^2}{4(0.400\text{ H})^2}}}=0.0142\text{ s}=14.2\text{ ms}.$$

(c) In Eq. (30.28), at the end of the first oscillation, the cosine factor is equal to 1, so the charge is $q = q_0 e^{-(R/2L)t} = (2.80\times10^{-4}\text{ C})\,e^{-(320\,\Omega)(0.0142\text{ s})/[2(0.400\text{ H})]} = 9.75\times10^{-7}\text{ C}$.

EVALUATE: The charge on the capacitor is only 0.35% of its initial value.

30.41. **IDENTIFY:** This problem is about a solenoid inductor.

SET UP and **EXECUTE:** **(a)** Estimate: Diameter = 7.0 mm.

(b) Estimate: With no overlap of the wires, $N(0.812\text{ mm}) = 4.0$ cm. $N = (40\text{ mm})/(0.812\text{ mm}) = 49$, which we round to 50 turns.

(c) If B is constant, $L = \mu_0 AN^2/l = \mu_0\,[\pi(0.406\text{ mm})^2](50^2)/(4.0\text{ cm}) = 3.0\ \mu\text{H}$.

(d) $U_L = \dfrac{1}{2}Li^2 = (1/2)(3.0\ \mu\text{H})(1.0\text{ A})^2 = 1.5\ \mu\text{J}$.

EVALUATE: Some lab inductors have an inductance of a few microhenries.

30.42. **IDENTIFY:** This is an R-L circuit and $i(t)$ is given by $i = \dfrac{\mathcal{E}}{R}(1-e^{-(R/L)t})$.

SET UP: When $t\to\infty$, $i \to i_f = V/R$.

EXECUTE: **(a)** $R = \dfrac{V}{i_f} = \dfrac{16.0\text{ V}}{6.45\times10^{-3}\text{ A}} = 2481\ \Omega$, which rounds to 2480 Ω.

(b) $i = i_f(1-e^{-(R/L)t})$ so $\dfrac{Rt}{L} = -\ln(1-i/i_f)$ and $L = \dfrac{-Rt}{\ln(1-i/i_f)} = \dfrac{-(2481\ \Omega)(9.40\times10^{-4}\text{ s})}{\ln(1-(4.86/6.45))} = 1.67$ H.

EVALUATE: The current after a long time depends only on R and is independent of L. The value of R/L determines how rapidly the final value of i is reached.

30.43. **IDENTIFY:** This problem involves electromagnetic induction and self-inductance.

SET UP and **EXECUTE:** $L = \dfrac{\Phi_B}{i}$, $B = \dfrac{\mu_0 I}{2r}$, $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$.. **(a)** We want the self-inductance.

$$\Phi_B = BA = \left(\frac{\mu_0 I}{2r}\right)(\pi r^2) = \frac{\mu_0\pi ri}{2}.\quad L = \frac{\Phi_B}{i} = \frac{\mu_0\pi ri/2}{i} = \frac{\mu_0\pi r}{2} = \frac{\mu_0\pi(3.0\text{ cm})}{2} = 59\text{ nH}.$$

(b) We want the maximum emf. $|\mathcal{E}| = \dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}\left(\dfrac{\mu_0\pi ri}{2}\right) = \dfrac{\mu_0\pi r}{2}\dfrac{di}{dt}$. $i(t) = I_0\sin(2\pi ft)$ gives

$\mathcal{E} = \mu_0\pi^2 rI_0 f\cos(2\pi ft)$. $\mathcal{E}_{\text{max}} = \mu_0\pi^2 rI_0 f = \ = \mu_0\pi^2(0300\text{ m})(1.20\text{ A})(60.0\text{ Hz}) = 26.8\ \mu\text{V}$.

EVALUATE: This only an estimate but it suggests a rather small induced voltage.

30.44. **IDENTIFY:** Apply $\mathcal{E} = -L\dfrac{di}{dt}$ and $Li = N\Phi_B$.

SET UP: Φ_B is the flux through one turn.

EXECUTE: **(a)** $\mathcal{E} = -L\dfrac{di}{dt} = -(7.50\times10^{-3}\text{ H})\dfrac{d}{dt}\{(0.680\text{ A})\cos[\pi t/(0.0250\text{ s})]\}.$

$\mathcal{E} = (7.50\times10^{-3}\text{ H})(0.680\text{ A})\dfrac{\pi}{0.0250\text{ s}}\sin[\pi t/(0.0250\text{ s})].$ Therefore,

$\mathcal{E}_{\text{max}} = (7.50\times10^{-3}\text{ H})(0.680\text{ A})\dfrac{\pi}{0.0250\text{ s}} = 0.641\text{ V}.$

(b) $\Phi_{B\max} = \dfrac{Li_{\text{max}}}{N} = \dfrac{(7.50\times10^{-3}\text{ H})(0.680\text{ A})}{400} = 1.28\times10^{-5}\text{ Wb} = 12.8\ \mu\text{Wb}.$

(c) $\mathcal{E}(t) = -L\dfrac{di}{dt} = (7.50\times10^{-3}\text{ H})(0.680\text{ A})(\pi/0.0250\text{ s})\sin[\pi t/(0.0250\text{ s})].$

$\mathcal{E}(t) = (0.641\text{ V})\sin[(125.6\text{ s}^{-1})t].$ Therefore, at $t = 0.0180$ s,

$\mathcal{E}(0.0180\text{ s}) = (0.641\text{ V})\sin[(125.6\text{ s}^{-1})(0.0180\text{ s})] = 0.494\text{ V}.$ The magnitude of the induced emf is 0.494 V.

EVALUATE: The maximum emf is when $i = 0$ and at this instant $\Phi_B = 0$.

30.45. **IDENTIFY:** Set $U_B = K$, where $K = \frac{1}{2}mv^2$.

SET UP: The energy density in the magnetic field is $u_B = B^2/2\mu_0$. Consider volume $V = 1\text{ m}^3$ of sunspot material.

EXECUTE: The energy density in the sunspot is $u_B = B^2/2\mu_0 = 6.366\times10^4\text{ J/m}^3$. The total energy stored in volume *V* of the sunspot is $U_B = u_B V$. The mass of the material in volume *V* of the sunspot is $m = \rho V$. $K = U_B$ so $\frac{1}{2}mv^2 = U_B$. $\frac{1}{2}\rho V v^2 = u_B V$. The volume divides out, and $v = \sqrt{2u_B/\rho} = 2\times10^4$ m/s.

EVALUATE: The speed we calculated is about 30 times smaller than the escape speed.

30.46. **IDENTIFY:** Follow the steps outlined in the problem.

SET UP: The energy stored is $U = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B2\pi r = \mu_0 i \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}.$

(b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r}ldr.$

(c) $\Phi_B = \displaystyle\int_a^b d\Phi_B = \frac{\mu_0 il}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 il}{2\pi}\ln(b/a).$

(d) $L = \dfrac{N\Phi_B}{i} = l\dfrac{\mu_0}{2\pi}\ln(b/a).$

(e) $U = \dfrac{1}{2}Li^2 = \dfrac{1}{2}l\dfrac{\mu_0}{2\pi}\ln(b/a)i^2 = \dfrac{\mu_0 li^2}{4\pi}\ln(b/a).$

EVALUATE: The magnetic field between the conductors is due only to the current in the inner conductor.

30.47. **IDENTIFY:** $U = \frac{1}{2}LI^2$. The self-inductance of a solenoid is found in Exercise 30.15 to be $L = \dfrac{\mu_0 AN^2}{l}$.

SET UP: The length *l* of the solenoid is the number of turns divided by the turns per unit length.

EXECUTE: **(a)** $L = \dfrac{2U}{I^2} = \dfrac{2(10.0\text{ J})}{(2.00\text{ A})^2} = 5.00\text{ H}.$

(b) $L = \dfrac{\mu_0 AN^2}{l}$. If α is the number of turns per unit length, then $N = \alpha l$ and $L = \mu_0 A\alpha^2 l$. For this coil $\alpha = 10$ coils/mm $= 10\times10^3$ coils/m. Solving for l gives

$$l = \frac{L}{\mu_0 A\alpha^2} = \frac{5.00\text{ H}}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})\pi(0.0200\text{ m})^2(10\times10^3\text{ coils/m})^2} = 31.7\text{ m}.$$ This is not a practical length for laboratory use.

EVALUATE: The number of turns is $N = (31.7\text{ m})(10\times10^3\text{ coils/m}) = 3.17\times10^5$ turns. The length of wire in the solenoid is the circumference C of one turn times the number of turns. $C = \pi d = \pi(4.00\times10^{-2}\text{ m}) = 0.126$ m. The length of wire is $(0.126\text{ m})(3.17\times10^5) = 4.0\times10^4\text{ m} = 40$ km. This length of wire will have a large resistance and I^2R electrical energy loses will be very large.

30.48. IDENTIFY and **SET UP:** Eq. (30.14) is $i = \dfrac{\mathcal{E}}{R}(1-e^{-Rt/L})$, $P_R = i^2R$, $\mathcal{E}_L = -L\dfrac{di}{dt}$.

EXECUTE: **(a)** Using Eq. (30.14) in the power consumed in the resistor gives

$$P_R = i^2R = \left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right]^2 R = \frac{\mathcal{E}^2}{R}(1-2e^{-Rt/L}+e^{-2Rt/L}).$$

After a long time, that is $t\to\infty$, the exponential terms all go to zero and the power approaches its maximum value of $\dfrac{\mathcal{E}^2}{R}$.

(b) The power in the inductor is

$$P_L = i\mathcal{E}_L = i\left(L\frac{di}{dt}\right) = \left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right]L\frac{d}{dt}\left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right] = \left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right]L\left(\frac{\mathcal{E}}{L}\right)e^{-Rt/L}.$$

$$P_L = \frac{\mathcal{E}^2}{R}(1-e^{-Rt/L})e^{-Rt/L}.$$

(c) $P_L(0) = 0$ since $1-e^0 = 0$. $P_L(t\to\infty) = 0$ since $e^{-Rt/L}\to 0$ as $t\to\infty$.

(d) P_L is a maximum when $dP_L/dt = 0$. Taking the time derivative of P_L from (b), we have

$$\frac{\mathcal{E}^2}{R}\left[\left(\frac{R}{L}e^{-Rt/L}\right)(e^{-Rt/L}) - \frac{R}{L}(1-e^{-Rt/L})(e^{-Rt/L})\right] = \frac{\mathcal{E}^2}{L}e^{-Rt/L}(e^{-Rt/L}-1+e^{-Rt/L}) = 0.$$

$2e^{-Rt/L} = 1.$

$t = -(L/R)\ln(1/2) = (L/R)\ln 2.$

At this instant, $e^{-Rt/L} = \frac{1}{2}$. Using the result from (b), we have

$$P_L = \frac{\mathcal{E}^2}{R}(1-e^{-Rt/L})e^{-Rt/L} = \frac{\mathcal{E}^2}{R}\left(1-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}\frac{\mathcal{E}^2}{R}.$$

(e) $P_{\varepsilon} = i\mathcal{E} = \dfrac{\mathcal{E}^2}{R}(1-e^{-Rt/L})$. The maximum power is $\dfrac{\mathcal{E}^2}{R}$ as $t\to\infty$.

EVALUATE: As time gets very large, current approaches a steady-state value, so the potential across an inductor approaches zero since the current through it is not changing.

30.49. IDENTIFY and **SET UP:** Use $U_C = \frac{1}{2}CV_C^2$ (energy stored in a capacitor) to solve for C. Then use $\omega = \dfrac{1}{\sqrt{LC}}$ and $\omega = 2\pi f$ to solve for the L that gives the desired current oscillation frequency.

EXECUTE: $V_C = 12.0$ V; $U_C = \frac{1}{2}CV_C^2$ so $C = 2U_C/V_C^2 = 2(0.0160\text{ J})/(12.0\text{ V})^2 = 222\ \mu$F.

$f = \dfrac{1}{2\pi\sqrt{LC}}$ so $L = \dfrac{1}{(2\pi f)^2 C}$.

$f = 3500$ Hz gives $L = 9.31\ \mu$H.

EVALUATE: f is in Hz and ω is in rad/s; we must be careful not to confuse the two.

30.50. IDENTIFY and SET UP: Apply $\mathcal{E}_L = -L\dfrac{di}{dt}$ and $V_R = Ri$. An inductor opposes a change in current through it. Kirchhoff's rules apply.

EXECUTE: **(a)** At the instant the switch is closed, the inductor will not allow any current through it, so all the current goes through R_1. So $i_1 = i_2 = \mathcal{E}/R_1 = (96.0\text{ V})/(12.0\ \Omega) = 8.00$ A. $i_3 = 0$.

(b) After a long time, steady-state is reached, so $di_3/dt = 0$ and $\mathcal{E}_L = -L\dfrac{di_3}{dt} = 0$. In this case, the potential across R_1 and across R_2 is 96.0 V. Therefore

$i_2 = (96.0\text{ V})/(12.0\ \Omega) = 8.00$ A.

$i_3 = (96.0\text{ V})/(16.0\ \Omega) = 6.00$ A.

$i_1 = i_2 + i_3 = 8.00\text{ A} + 6.00\text{ A} = 14.00$ A.

(c) Apply Kirchhoff's loop rule, giving

$$\mathcal{E} - i_3R_2 - L\frac{di_3}{dt} = 0.$$

Separating variables and integrating gives

$$\int_0^t -\frac{R_2}{L}dt' = \int_0^{i_3}\frac{1}{i_3' - \mathcal{E}/R_2}di_3'.$$

Carrying out the integration and solving for t gives

$$-\frac{R_2}{L}t = \ln\left(\frac{i_3 - \mathcal{E}/R_2}{-\mathcal{E}/R_2}\right).$$

$$t = \frac{L}{R_2}\ln\left(\frac{\mathcal{E}/R_2}{\mathcal{E}/R_2 - i_3}\right) = \frac{0.300\text{ H}}{16.0\ \Omega}\ln\left(\frac{96.0\text{ V}}{96.0\text{ V} - (3.00\text{ A})(16.0\ \Omega)}\right) = 0.0130\text{ s} = 13.0\text{ ms}.$$

(d) $i_2 = \mathcal{E}/R_1 = (96.0\text{ V})/(12.0\ \Omega) = 8.00$ A. $i_1 = i_2 + i_3 = 8.00\text{ A} + 3.00\text{ A} = 11.0$ A.

EVALUATE: At steady-state, the potential drop across an inductor is zero if it has no resistance. Initially the inductor acts like an open circuit because it will not allow current to flow through it.

30.51. IDENTIFY: This problem involves induction and magnetic torque.

SET UP and EXECUTE: **(a)** We want the force the bar exerts on the spool. The torques due to the force of the bar and the magnetic force balance so $Fa = \mu B\sin\theta = i(\pi a^2)NB\sin\theta$. This gives $F = \pi aiNB\sin\theta = \pi(0.0500\text{ m})(1.00\text{ A})(500)(2.00\text{ T})(\sin 45°) = 111$ N.

(b) We want the time when the force is zero. At this time, the magnetic torque equals the counter torque, so $i\pi a^2NB\sin\theta = \tau$. The current is $i = i_0e^{-(R/L)t}$, so $i_0e^{-(R/L)t}\ \pi a^2NB\sin\theta = \tau$. Isolate the exponential: $e^{-(R/L)t} = \dfrac{\tau}{i_0\pi a^2NB\sin\theta}$. Using the given numerical values gives $e^{-(R/L)t} = 0.09005$. Solving using logarithms gives $t = -(L/R)\ln(0.09005) = 37.1$ ms.

(c) We want the angular acceleration after a long time. Using $\Sigma\tau = I\alpha$, we have $\tau_{\text{mag}} = I\alpha$. $\tau_{\text{mag}} = \mu BN\sin\theta = i\pi a^2BN\sin\theta$. $I = \dfrac{1}{2}Ma^2$. So $\tau_{\text{mag}} = I\alpha$ gives $i\pi a^2BN\sin\theta = \dfrac{1}{2}Ma^2\alpha$.

$\alpha = \dfrac{2i\pi BN\sin\theta}{M}$. After a long time, $i = 1.00$ A, as in part (a). Using the given quantities we have $\alpha = 4000\text{ rad/s}^2$.

EVALUATE: Be careful not to confuse μ (the magnetic moment) with μ_0 (the magnetic constant).

30.52. **IDENTIFY** and **SET UP:** Apply Kirchhoff's rules. $V_R = Ri$ and $\mathcal{E}_L = -L\dfrac{di}{dt}$.

EXECUTE: **(a)** Immediately after the switch is closed, the inductor will not allow any current in it, so all the current flows through R_2. At that instant, the equivalent circuit consists of R_1 and R_2 in series with each other and connected across the terminals of the battery. Ohm's law gives
$i_1 = i_2 = \mathcal{E}/(R_1 + R_2) = (48.0\text{ V})/(14\ \Omega) = 3.43\text{ A}$.

The current through the inductor is zero, so $i_3 = 0$.
(b) After a long time, steady-state has been achieved, so the potential across the inductor is zero. Therefore it acts like a short circuit, so no current flows through R_2. The equivalent circuit consists of R_3 connected across the terminals of the battery. By Ohm's law
$i_1 = i_3 = \mathcal{E}/R_1 = (48.0\text{ V})/(8.00\ \Omega) = 6.00\text{ A}$. $i_2 = 0$.
(c) Use Kirchhoff's loop rule. A loop around the left-hand section of the circuit gives
$\mathcal{E} - i_1R_1 - i_2R = 0$ (Eq. 1).
A loop around the right-hand section of the circuit gives
$-i_2R_2 + L\,di_3/dt = 0$ (Eq. 2).
Kirchhoff's junction rule gives
$i_1 = i_2 + i_3$ (Eq. 3).

Combining Eq. 1 and Eq. 3 and rearranging gives $i_2 = \dfrac{\mathcal{E} - i_3R_1}{R_1 + R_2}$. Putting this result into Eq. 2 gives

$-\left(\dfrac{\mathcal{E} - i_3R_1}{R_1 + R_2}\right)R_2 + L\dfrac{di_3}{dt} = 0$. Separating variables and integrating gives

$$\int_0^{i_3} \frac{di_3'}{i_3' - \mathcal{E}/R_1} = -\int_0^t \frac{R_1R_2}{L(R_1 + R_2)}\,dt'.$$

$$\ln\left(\frac{i_3 - \mathcal{E}/R_1}{-\mathcal{E}/R_1}\right) = -\frac{R_1R_2}{L(R_1 + R_2)}t.$$

$$i_3 = \frac{\mathcal{E}}{R_1}(1 - e^{-R_1R_2t/L(R_1+R_2)}).$$

(d) Solve for t when $i_3 = (\mathcal{E}/R_1)/2$.

$$t = \frac{L(R_1 + R_2)}{R_1R_2}\ln\left(\frac{\mathcal{E}/R_1}{\mathcal{E}/R_1 - \mathcal{E}/2R_1}\right) = \frac{L(R_1 + R_2)}{R_1R_2}\ln 2 = \frac{(0.200\text{ H})(14.00\ \Omega)}{(8.00\ \Omega)(6.00\ \Omega)}\ln 2 = 0.0404\text{ s}.$$

(e) $i_2 = \dfrac{\mathcal{E} - i_3R_1}{R_1 + R_2} = \dfrac{48.0\text{ V} - (3.00\text{ A})(8.00\ \Omega)}{14.00\ \Omega} = 1.71\text{ A}$.

$i_1 = i_2 + i_3 = 1.71\text{ A} + 3.00\text{ A} = 4.71\text{ A}$.
EVALUATE: The inductor initially acted like an open circuit, but at steady-state it acted like a short circuit. If it had resistance, it would not have behaved like a short circuit at steady-state.

30.53. **IDENTIFY:** In this problem we treat damping in an L-R-C circuit.

SET UP and **EXECUTE:** **(a)** We want the charge that flows onto the capacitor. Apply $\mathcal{E}_{\text{av}} = N\dfrac{\Delta\Phi_B}{\Delta t}$.

$\Delta B = B - 0 = B$, so $\dfrac{\Delta\Phi_B}{\Delta t} = \dfrac{BA}{\Delta t}$. $\mathcal{E}_{\text{av}} = N\dfrac{BA}{\Delta t} = IR$. $I = \dfrac{NBA}{R\Delta t}$. $Q = I\Delta t = \left(\dfrac{NBA}{R\Delta t}\right)\Delta t = \dfrac{NBA}{R}$. $A = \pi r^2$. R $= (0.0333\ \Omega/\text{m})(2\pi r)N = 0.6277\ \Omega$. Using the area, this R, and the other given quantities, we get $Q =$ 15.0 mC.
(b) We want the time for the capacitor to fully discharge for the first time. Apply Kirchhoff's loop rule, giving $L\dfrac{di}{dt} + Ri + \dfrac{q}{C} = 0$, which we can express as $\dfrac{d^2q}{dt^2} + \dfrac{R}{L}\dfrac{dq}{dt} + \dfrac{q}{LC} = 0$. The solution to this

differential equation is $q(t) = Ae^{-(R/2L)t}\cos\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\,t+\phi\right)$. At $t = 0$, $q = Q_0$ and $i = 0$, so $A = Q_0$ and $\phi = \pi/2$. When the capacitor discharges for the first time, $q = 0$, which occurs when $\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\,t$ $= \pi/2$. Therefore $t = \dfrac{\pi}{2\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}}$. We know that $R = 0.6277\ \Omega$ and $C = 10.0\ \mu\text{F}$, so we need L. Using $L = \mu_0 AN^2/l$ with the given quantities we get $L = 355.3\ \mu\text{H}$. Using these values gives $t = 93.8\ \mu\text{S}$.

(c) We want the frequency f'. Using $f' = \dfrac{\omega'}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}-\dfrac{R^2}{4L^2}}$ gives $f' = 2.67$ kHz.

(d) We want the energy in the capacitor when $t = 0$. $U_0 = \dfrac{Q_0^2}{2C} = 11.3$ J using $Q_0 = 15.0$ mC from (a).

(e) We want the time for the maximum capacitor energy to be 10.0% of its initial energy. First find the charge for which the energy is 10.0% of the initial energy. $\dfrac{U}{U_0} = \dfrac{Q^2/2C}{Q_0^2/2C} = \left(\dfrac{Q}{Q_0}\right)^2 = 0.100$, so $Q = Q_0\sqrt{0.100}$. Now use the equation for $q(t)$. When q is a maximum, $t = 0$ so the cosine factor is equal to one. Using $Q = Q_0\sqrt{0.100}$ gives $Q_0 e^{-(R/2L)t} = Q_0\sqrt{0.100}$. Solve for t giving $t = -\dfrac{2L}{R}\ln\left(\sqrt{0.100}\right)$ $= 1.30$ ms.

EVALUATE: The charge and current continue to oscillate but with decreasing amplitude.

30.54. **IDENTIFY:** The initial energy stored in the capacitor is shared between the inductor and the capacitor.

SET UP: The potential across the capacitor and inductor is always the same, so $\dfrac{q}{C} = L\left|\dfrac{di}{dt}\right|$. The capacitor energy is $U_C = \dfrac{q^2}{2C} = \dfrac{1}{2}CV^2$, and the inductor energy is $U_L = \dfrac{1}{2}Li^2$.

EXECUTE: **(a)** The initial energy in the capacitor is $U_0 = \dfrac{1}{2}Cv_0^2 = \dfrac{1}{2}(6.40\ \text{nF})(24.0\ \text{V})^2 = 1.84\ \mu\text{J}$. This energy is shared between the inductor and the capacitor. The energy in the capacitor at this time is

$$U_C = \frac{q^2}{2C} = \frac{(0.0800\ \mu\text{C})^2}{2(6.40\ \text{nF})} = 0.500\ \mu\text{J}.$$

The energy remaining in the inductor is
$U_L = U_0 - U_C = 1.84\ \mu\text{J} - 0.500\ \mu\text{J} = 1.34\ \mu\text{J}$.

The energy in the inductor is $U_L = \dfrac{1}{2}Li^2$, so

$$i = \sqrt{\frac{2U_L}{L}} = \sqrt{\frac{2(1.34\times10^{-6}\ \text{J})}{0.0660\ \text{H}}} = 6.37\times10^{-3}\ \text{A} = 6.37\ \text{mA}.$$

(b) When the capacitor charge is 0.0800 μC, we found that the energy stored in the capacitor is 0.500 μJ.

$$U_C = \frac{1}{2}Cv_C^2 \;\rightarrow\; v_C = \sqrt{\frac{2U_0}{C}} = \sqrt{\frac{2(0.500\ \mu\text{J})}{6.40\ \text{nF}}} = 12.5\ \text{V}.$$

The potential across the inductor and capacitor is the same, so $v_L = 12.5$ V.

Using $v_L = \mathcal{E}_L = \left|L\dfrac{di}{dt}\right|$, we have $\dfrac{di}{dt} = \dfrac{v_L}{L} = \dfrac{12.5\ \text{V}}{0.0660\ \text{H}} = 189\ \text{A/s}.$

EVALUATE: When the capacitor contains 0.0800 μC of charge, the energy in the inductor is 1.34 μJ. Therefore the current is $U_L = \frac{1}{2}Li^2$, so $i = \sqrt{\frac{2U_L}{L}} = \sqrt{\frac{2(1.34\ \mu\text{J})}{0.0660\ \text{H}}} = 6.4$ mA. The current is only 6.4 mA but is changing at a rate of 189 A/s. However, it only changes at that rate for a tiny fraction of a second.

30.55. **IDENTIFY:** Apply energy conservation to the circuit.

SET UP: For a capacitor $V = q/C$ and $U = q^2/2C$. For an inductor $U = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $V_{\text{max}} = \frac{Q}{C} = \frac{6.00\times10^{-6}\ \text{C}}{2.50\times10^{-4}\ \text{F}} = 0.0240\ \text{V}.$

(b) $\frac{1}{2}Li_{\text{max}}^2 = \frac{Q^2}{2C}$, so $i_{\text{max}} = \frac{Q}{\sqrt{LC}} = \frac{6.00\times10^{-6}\ \text{C}}{\sqrt{(0.0600\ \text{H})(2.50\times10^{-4}\ \text{F})}} = 1.55\times10^{-3}\ \text{A}.$

(c) $U_{\text{max}} = \frac{1}{2}Li_{\text{max}}^2 = \frac{1}{2}(0.0600\ \text{H})(1.55\times10^{-3}\text{A})^2 = 7.21\times10^{-8}\ \text{J}.$

(d) If $i = \frac{1}{2}i_{\text{max}}$ then $U_L = \frac{1}{4}U_{\text{max}} = 1.80\times10^{-8}$ J and $U_C = \frac{3}{4}U_{\text{max}} = \frac{(\sqrt{(3/4)}Q)^2}{2C} = \frac{q^2}{2C}$. This gives $q = \sqrt{\frac{3}{4}}Q = 5.20\times10^{-6}$ C.

EVALUATE: $U_{\text{max}} = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}$ for all times.

30.56. **IDENTIFY:** The total energy is shared between the inductor and the capacitor.

SET UP: The potential across the capacitor and inductor is always the same, so $\frac{q}{C} = L\left|\frac{di}{dt}\right|$. The capacitor energy is $U_C = \frac{q^2}{2C}$ and the inductor energy is $U_L = \frac{1}{2}Li^2$.

EXECUTE: The total energy is $\frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{Q_{\text{max}}^2}{2C} = \frac{1}{2}CV_{\text{max}}^2$.

$$q = LC\left|\frac{di}{dt}\right| = (0.330\ \text{H})(5.90\times10^{-4}\ \text{F})(73.0\ \text{A/s}) = 1.421\times10^{-2}\ \text{C}.$$

$$\frac{1}{2}CV_{\text{max}}^2 = \frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{(1.421\times10^{-2}\ \text{C})^2}{2(5.90\times10^{-4}\ \text{F})} + \frac{1}{2}(0.330\ \text{H})(2.50\ \text{A})^2 = 1.202\ \text{J}.$$

$$V_{\text{max}} = \sqrt{\frac{2(1.202\ \text{J})}{5.90\times10^{-4}\ \text{F})}} = 63.8\ \text{V}.$$

EVALUATE: By energy conservation, the maximum energy stored in the inductor will be 1.202 J, and this will occur at the instants when the capacitor is uncharged.

30.57. **IDENTIFY:** The current through an inductor doesn't change abruptly. After a long time the current isn't changing and the voltage across each inductor is zero.

SET UP: For part (c) combine the inductors.

EXECUTE: **(a)** Just after the switch is closed there is no current in the inductors. There is no current in the resistors so there is no voltage drop across either resistor. A reads zero and V reads 20.0 V.

(b) After a long time the currents are no longer changing, there is no voltage across the inductors, and the inductors can be replaced by short-circuits. The circuit becomes equivalent to the circuit shown in Figure 30.57a. $I = (20.0\ \text{V})/(75.0\ \Omega) = 0.267$ A. The voltage between points a and b is zero, so the voltmeter reads zero.

(c) Combine the inductor network into its equivalent, as shown in Figure 30.57b. $R = 75.0\ \Omega$ is the equivalent resistance. The current is $i = (\mathcal{E}/R)(1 - e^{-t/\tau})$ with $\tau = L/R = (10.8\text{ mH})/(75.0\ \Omega) = 0.144\text{ ms}$. $\mathcal{E} = 20.0\text{ V}$, $R = 75.0\ \Omega$, $t = 0.115\text{ ms}$ so $i = 0.147\text{ A}$. $V_R = iR = (0.147\text{ A})(75.0\ \Omega) = 11.0\text{ V}$. $20.0\text{ V} - V_R - V_L = 0$ and $V_L = 20.0\text{ V} - V_R = 9.0\text{ V}$. The ammeter reads 0.147 A and the voltmeter reads 9.0 V.

EVALUATE: The current through the battery increases from zero to a final value of 0.267 A. The voltage across the inductor network drops from 20.0 V to zero.

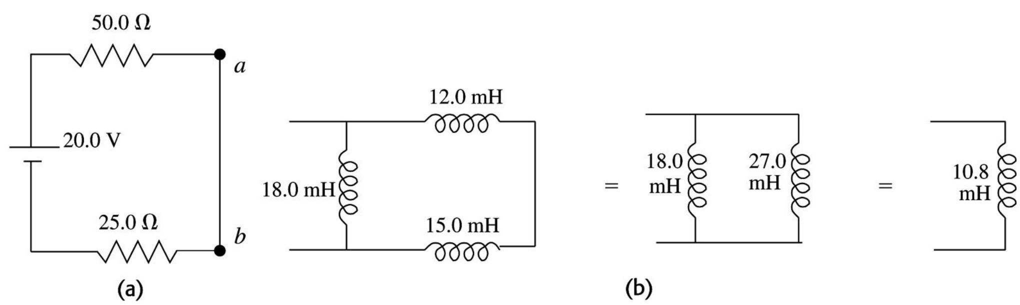


Figure 30.57

30.58. IDENTIFY: At $t = 0$, $i = 0$ through each inductor. At $t \to \infty$, the voltage is zero across each inductor.
SET UP: In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \to \infty$ replace each inductor by a wire.
EXECUTE: **(a)** Initially the inductor blocks current through it, so the simplified equivalent circuit is shown in Figure 30.58a. $i = \dfrac{\mathcal{E}}{R} = \dfrac{50\text{ V}}{150\ \Omega} = 0.333\text{ A}$. $V_1 = (100\ \Omega)(0.333\text{ A}) = 33.3\text{ V}$.
$V_4 = (50\ \Omega)(0.333\text{ A}) = 16.7\text{ V}$. $V_3 = 0$ since no current flows through it. $V_2 = V_4 = 16.7\text{ V}$, since the inductor is in parallel with the 50-Ω resistor. $A_1 = A_3 = 0.333\text{ A}$, $A_2 = 0$.
(b) Long after S is closed, steady state is reached, so the inductor has no potential drop across it. The simplified circuit is sketched in Figure 30.58b. $i = \mathcal{E}/R = \dfrac{50\text{ V}}{130\ \Omega} = 0.385\text{ A}$.
$V_1 = (100\ \Omega)(0.385\text{ A}) = 38.5\text{ V}$; $V_2 = 0$; $V_3 = V_4 = 50\text{ V} - 38.5\text{ V} = 11.5\text{ V}$.
$i_1 = 0.385\text{ A}$; $i_2 = \dfrac{11.5\text{ V}}{75\ \Omega} = 0.153\text{ A}$; $i_3 = \dfrac{11.5\text{ V}}{50\ \Omega} = 0.230\text{ A}$.
EVALUATE: Just after the switch is closed the current through the battery is 0.333 A. After a long time the current through the battery is 0.385 A. After a long time there is an additional current path, the equivalent resistance of the circuit is decreased and the current has increased.

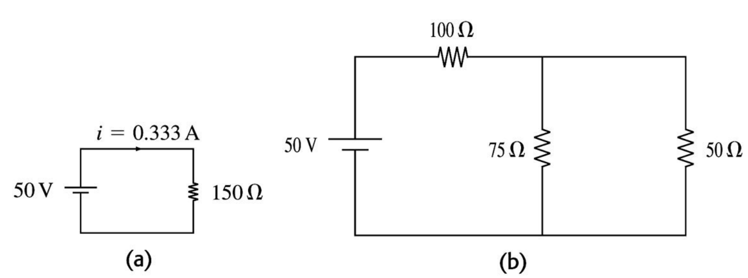


Figure 30.58

30.59. IDENTIFY and SET UP: The current in an *R-L* circuit is given by $i = i_0 e^{-Rt/L}$, where *R* is the total resistance. In our measurements, the current is one-half the initial current, so $i = i_0/2$.

EXECUTE: (a) Taking natural logarithms of the current equation, with $R = R_L + R_{ext}$ and $i = i_0/2$, we get

$\ln(i/i_0) = -Rt/L.$

$\ln(1/2) = -(R_L + R_{ext})t_{half}/L.$

$\ln 2 = t_{half}(R_L + R_{ext})/L.$

where t_{half} is the time for the current to decrease to half its initial value. Solving for $1/t_{half}$ gives $\frac{1}{t_{half}} = \frac{R_{ext}}{L\ln 2} + \frac{R_L}{L\ln 2}$. Therefore a graph of $1/t_{half}$ versus R_{ext} should be a straight line having a slope equal to $1/(L \ln 2)$ and a *y*-intercept equal to $R_L/(L \ln 2)$. Figure 30.59 shows this graph.

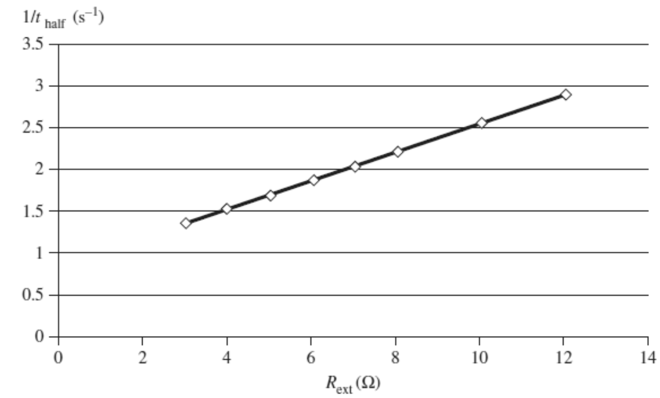


Figure 30.59

(b) The best-fit equation for the line in the graph is $\frac{1}{t_{half}} = 0.1692\ (\Omega\cdot\text{s})^{-1}R_{ext} + 0.8524\ \text{s}^{-1}$. Using the slope and solving for *L* gives $L = \frac{1}{(\text{slope})\ln 2} = \frac{1}{[0.1692\ (\Omega\cdot\text{s})^{-1}]\ln 2} = 8.53\ \text{H}$, which rounds to 8.5 H.

Now use the *y*-intercept and solve for R_L.

$\frac{R_L}{L\ln 2} = y\text{-intercept}$, so $R_L = (y\text{-intercept})(L\ln 2) = (0.8524\ \text{s}^{-1})(8.53\ \text{H})\ln 2 = 5.04\ \Omega$, which rounds to 5.0 Ω.

(c) $U_L = \frac{1}{2}Li^2 = (1/2)(8.53\ \text{H})(20.0\ \text{A})^2 = 1.7\times10^3\ \text{J} = 1.7\ \text{kJ}.$

$P_R = i^2R = (20.0\ \text{A})^2(5.04\ \Omega) = 2.0\times10^3\ \text{W} = 2.0\ \text{kW}.$

EVALUATE: Whether the 5.0-Ω resistance of this inductor would be significant would depend on the external resistance in the circuit. For the data of this problem, the solenoid resistance would definitely be significant for the external resistances used.

30.60. IDENTIFY: Closing S_2 and simultaneously opening S_1 produces an *L-C* circuit with initial current through the inductor of 3.50 A. When the current is a maximum the charge *q* on the capacitor is zero and when the charge *q* is a maximum the current is zero. Conservation of energy says that the maximum energy $\frac{1}{2}Li_{max}^2$ stored in the inductor equals the maximum energy $\frac{1}{2}\frac{q_{max}^2}{C}$ stored in the capacitor.

SET UP: $i_{max} = 3.50$ A, the current in the inductor just after the switch is closed.

EXECUTE: (a) $\frac{1}{2}Li_{max}^2 = \frac{1}{2}\frac{q_{max}^2}{C}.$

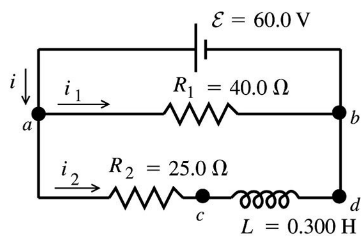
$q_{max} = (\sqrt{LC})i_{max} = \sqrt{(2.0\times10^{-3}\text{ H})(5.0\times10^{-6}\text{ F})}(3.50\text{ A}) = 3.50\times10^{-4}\text{ C} = 0.350\text{ mC}.$

(b) When q is maximum, $i = 0.$

EVALUATE: In the final circuit the current will oscillate.

30.61. **IDENTIFY:** Apply the loop rule to each parallel branch. The voltage across a resistor is given by iR and the voltage across an inductor is given by $L|di/dt|$. The rate of change of current through the inductor is limited.

SET UP: With S closed the circuit is sketched in Figure 30.61a.



The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is closed the current in the inductor has not had time to increase from zero, so $i_2 = 0.$

Figure 30.61a

EXECUTE : **(a)** $\mathcal{E} - v_{ab} = 0$, so $v_{ab} = 60.0$ V.

(b) The voltage drops across R, as we travel through the resistor in the direction of the current, so point a is at higher potential.

(c) $i_2 = 0$ so $v_{R_2} = i_2R_2 = 0.$

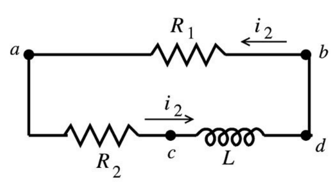
$\mathcal{E} - v_{R_2} - v_L = 0$ so $v_L = \mathcal{E} = 60.0$ V.

(d) The voltage rises when we go from b to a through the emf, so it must drop when we go from a to b through the inductor. Point c must be at higher potential than point d.

(e) After the switch has been closed a long time, $\frac{di_2}{dt} \to 0$ so $v_L = 0.$ Then $\mathcal{E} - v_{R_2} = 0$ and $i_2R_2 = \mathcal{E}$

so $i_2 = \frac{\mathcal{E}}{R_2} = \frac{60.0\text{ V}}{25.0\ \Omega} = 2.40$ A.

SET UP: The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is opened again the current through the inductor hasn't had time to change and is still $i_2 = 2.40$ A. The circuit is sketched in Figure 30.61b.



EXECUTE: The current through R_1 is $i_2 = 2.40$ A in the direction b to a.
Thus $v_{ab} = -i_2R_1 = -(2.40\text{ A})(40.0\ \Omega).$
$v_{ab} = -96.0$ V.

Figure 30.61b

(f) Point where current enters resistor is at higher potential; point b is at higher potential.

(g) $v_L - v_{R_1} - v_{R_2} = 0.$

$v_L = v_{R_1} + v_{R_2}.$

$v_{R_1} = -v_{ab} = 96.0$ V; $v_{R_2} = i_2R_2 = (2.40\text{ A})(25.0\ \Omega) = 60.0$ V.

Then $v_L = v_{R_1} + v_{R_2} = 96.0\text{ V} + 60.0\text{ V} = 156$ V.

As you travel counterclockwise around the circuit in the direction of the current, the voltage drops across each resistor, so it must rise across the inductor and point *d* is at higher potential than point *c*. The current is decreasing, so the induced emf in the inductor is directed in the direction of the current. Thus, $v_{cd} = -156\text{ V}$.

(h) Point *d* is at higher potential.

EVALUATE: The voltage across R_1 is constant once the switch is closed. In the branch containing R_2, just after S is closed the voltage drop is all across *L* and after a long time it is all across R_2. Just after S is opened the same current flows in the single loop as had been flowing through the inductor and the sum of the voltage across the resistors equals the voltage across the inductor. This voltage dies away, as the energy stored in the inductor is dissipated in the resistors.

30.62. IDENTIFY: Apply the loop rule to the two loops. The current through the inductor doesn't change abruptly.

SET UP: For the inductor $|\mathcal{E}| = L\left|\dfrac{di}{dt}\right|$ and $\mathcal{E}$ is directed to oppose the change in current.

EXECUTE: **(a)** Switch is closed, then at some later time

$$\frac{di}{dt} = 50.0\text{ A/s} \Rightarrow v_{cd} = L\frac{di}{dt} = (0.300\text{ H})(50.0\text{ A/s}) = 15.0\text{ V}.$$

<u>The top circuit loop:</u> $60.0\text{ V} = i_1 R_1 \Rightarrow i_1 = \dfrac{60.0\text{ V}}{40.0\ \Omega} = 1.50\text{ A}.$

<u>The bottom loop:</u> $60.0\text{ V} - i_2 R_2 - 15.0\text{ V} = 0 \Rightarrow i_2 = \dfrac{45.0\text{ V}}{25.0\ \Omega} = 1.80\text{ A}.$

(b) After a long time: $i_2 = \dfrac{60.0\text{ V}}{25.0\ \Omega} = 2.40\text{ A},$ and immediately when the switch is opened, the inductor maintains this current, so $i_1 = i_2 = 2.40\text{ A}.$

EVALUATE: The current through R_1 changes abruptly when the switch is closed.

30.63. IDENTIFY and SET UP: The circuit is sketched in Figure 30.63a. Apply the loop rule. Just after S_1 is closed, $i = 0$. After a long time *i* has reached its final value and $di/dt = 0$. The voltage across a resistor depends on *i* and the voltage across an inductor depends on di/dt.

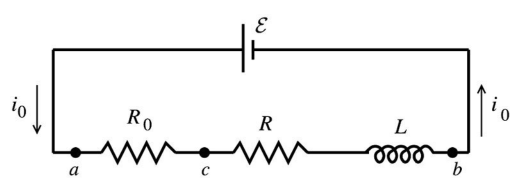


Figure 30.63a

EXECUTE: **(a)** At time $t = 0$, $i_0 = 0$ so $v_{ac} = i_0 R_0 = 0$. By the loop rule $\mathcal{E} - v_{ac} - v_{cb} = 0$ so $v_{cb} = \mathcal{E} - v_{ac} = \mathcal{E} = 36.0\text{ V}$. ($i_0 R = 0$ so this potential difference of 36.0 V is across the inductor and is an induced emf produced by the changing current.)

(b) After a long time $\dfrac{di_0}{dt} \to 0$ so the potential $-L\dfrac{di_0}{dt}$ across the inductor becomes zero. The loop rule gives $\mathcal{E} - i_0(R_0 + R) = 0.$

$$i_0 = \frac{\mathcal{E}}{R_0 + R} = \frac{36.0\text{ V}}{50.0\ \Omega + 150\ \Omega} = 0.180\text{ A}.$$

$v_{ac} = i_0 R_0 = (0.180\text{ A})(50.0\ \Omega) = 9.0\text{ V}.$

Thus $v_{cb} = i_0 R + L\dfrac{di_0}{dt} = (0.180\text{ A})(150\ \Omega) + 0 = 27.0\text{ V}.$ (Note that $v_{ac} + v_{cb} = \mathcal{E}$.)

(c) $\mathcal{E} - v_{ac} - v_{cb} = 0.$

$$\mathcal{E} - iR_0 - iR - L\frac{di}{dt} = 0.$$

$$L\frac{di}{dt} = \mathcal{E} - i(R_0 + R) \text{ and } \left(\frac{L}{R+R_0}\right)\frac{di}{dt} = -i + \frac{\mathcal{E}}{R+R_0}.$$

$$\frac{di}{-i + \mathcal{E}/(R+R_0)} = \left(\frac{R+R_0}{L}\right)dt.$$

Integrate from $t = 0$, when $i = 0$, to t, when $i = i_0$:

$$\int_0^{i_0} \frac{di}{-i + \mathcal{E}/(R+R_0)} = \frac{R+R_0}{L}\int_0^t dt = -\ln\left[-i + \frac{\mathcal{E}}{R+R_0}\right]_0^{i_0} = \left(\frac{R+R_0}{L}\right)t, \text{ so}$$

$$\ln\left(-i_0 + \frac{\mathcal{E}}{R+R_0}\right) - \ln\left(\frac{\mathcal{E}}{R+R_0}\right) = -\left(\frac{R+R_0}{L}\right)t.$$

$$\ln\left(\frac{-i_0 + \mathcal{E}/(R+R_0)}{\mathcal{E}/(R+R_0)}\right) = -\left(\frac{R+R_0}{L}\right)t.$$

Taking exponentials of both sides gives $\dfrac{-i_0 + \mathcal{E}/(R+R_0)}{\mathcal{E}/(R+R_0)} = e^{-(R+R_0)t/L}$ and $i_0 = \dfrac{\mathcal{E}}{R+R_0}(1 - e^{-(R+R_0)t/L}).$

Substituting in the numerical values gives $i_0 = \dfrac{36.0\text{ V}}{50\ \Omega + 150\ \Omega}(1 - e^{-(200\ \Omega/4.00\text{ H})t}) = (0.180\text{ A})(1 - e^{-t/0.020\text{ s}}).$

At $t \to 0, i_0 = (0.180\text{ A})(1-1) = 0$ (agrees with part (a)). At $t \to \infty, i_0 = (0.180\text{ A})(1-0) = 0.180\text{ A}$ (agrees with part (b)).

$$v_{ac} = i_0 R_0 = \frac{\mathcal{E}R_0}{R+R_0}(1 - e^{-(R+R_0)t/L}) = 9.0\text{ V}(1 - e^{-t/0.020\text{ s}}).$$

$$v_{cb} = \mathcal{E} - v_{ac} = 36.0\text{ V} - 9.0\text{ V}(1 - e^{-t/0.020\text{ s}}) = 9.0\text{ V}(3.00 + e^{-t/0.020\text{ s}}).$$

At $t \to 0, v_{ac} = 0, v_{cb} = 36.0\text{ V}$ (agrees with part (a)). At $t \to \infty, v_{ac} = 9.0\text{ V}, v_{cb} = 27.0\text{ V}$ (agrees with part (b)). The graphs are given in Figure 30.63b.

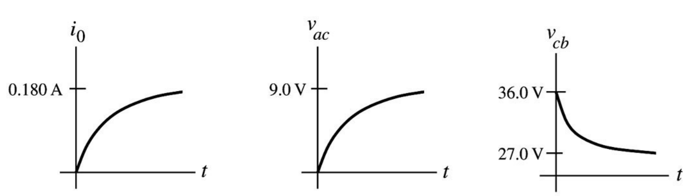


Figure 30.63b

EVALUATE: The expression for $i(t)$ we derived becomes $i = \dfrac{\mathcal{E}}{R}(1 - e^{-(R/L)t})$ if the two resistors R_0 and R in series are replaced by a single equivalent resistance $R_0 + R$.

30.64. **IDENTIFY:** Apply the loop rule. The current through the inductor doesn't change abruptly.

SET UP: With S_2 closed, v_{cb} must be zero.

EXECUTE: **(a)** Immediately after S_2 is closed, the inductor maintains the current $i = 0.180\text{ A}$ through R. The loop rule around the outside of the circuit yields

$\mathcal{E} + \mathcal{E}_L - iR - i_0 R_0 = 36.0\text{ V} + (0.18\text{ A})(150\ \Omega) - (0.18\text{ A})(150\ \Omega) - i_0(50\ \Omega) = 0.$ $i_0 = \dfrac{36\text{ V}}{50\ \Omega} = 0.720\text{ A}.$

$v_{ac} = (0.72\text{ A})(50\ \Omega) = 36.0\text{ V}$ and $v_{cb} = 0.$

(b) After a long time, $v_{ac} = 36.0\text{ V},$ and $v_{cb} = 0.$ Thus $i_0 = \dfrac{\mathcal{E}}{R_0} = \dfrac{36.0\text{ V}}{50\ \Omega} = 0.720\text{ A},$ $i_R = 0$ and

$i_{s2} = 0.720\text{ A}.$

(c) $i_0 = 0.720\text{ A},$ $i_R(t) = \dfrac{\mathcal{E}}{R_{\text{total}}} e^{-(R/L)t}$ and $i_R(t) = (0.180\text{ A})e^{-(37.5\text{ s}^{-1})t}.$

$i_{s2}(t) = (0.720\text{ A}) - (0.180\text{ A})e^{-(37.5\text{ s}^{-1})t} = (0.180\text{ A})(4 - e^{-(37.5\text{ s}^{-1})t}).$ The graphs of the currents are given in Figure 30.64.

EVALUATE: R_0 is in a loop that contains just $\mathcal{E}$ and $R_0,$ so the current through R_0 is constant. After a long time the current through the inductor isn't changing and the voltage across the inductor is zero. Since v_{cb} is zero, the voltage across *R* must be zero and i_R becomes zero.

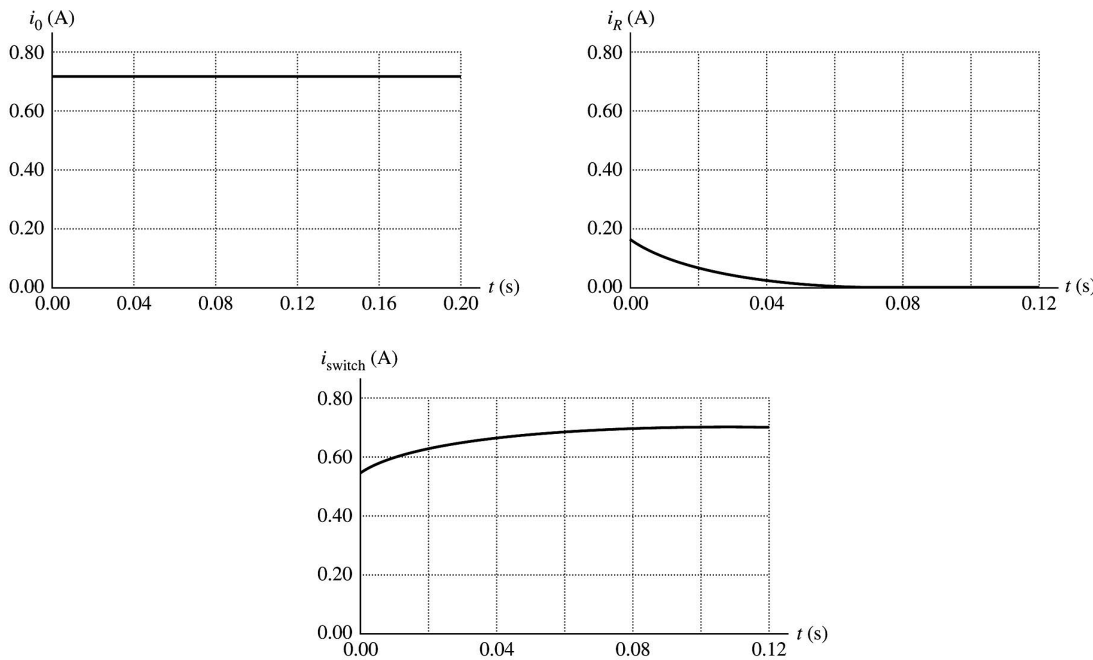


Figure 30.64

30.65. **IDENTIFY:** At $t = 0,$ $i = 0$ through each inductor. At $t \to \infty,$ the voltage is zero across each inductor.

SET UP: In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \to \infty$ replace each inductor by a wire.

EXECUTE: **(a)** Just after the switch is closed there is no current through either inductor and they act like breaks in the circuit. The current is the same through the $40.0\text{-}\Omega$ and $15.0\text{-}\Omega$ resistors and is equal to $(25.0\text{ V})/(40.0\ \Omega + 15.0\ \Omega) = 0.455\text{ A}.$ $A_1 = A_4 = 0.455\text{ A};$ $A_2 = A_3 = 0.$

(b) After a long time the currents are constant, there is no voltage across either inductor, and each inductor can be treated as a short-circuit. The circuit is equivalent to the circuit sketched in Figure 30.65. $I = (25.0\text{ V})/(42.73\ \Omega) = 0.585\text{ A}.$ A_1 reads $0.585\text{ A}.$ The voltage across each parallel branch is $25.0\text{ V} - (0.585\text{ A})(40.0\ \Omega) = 1.60\text{ V}.$ A_2 reads $(1.60\text{ V})/(5.0\ \Omega) = 0.320\text{ A}.$ A_3 reads $(1.60\text{ V})/(10.0\ \Omega) = 0.160\text{ A}.$ A_4 reads $(1.60\text{ V})/(15.0\ \Omega) = 0.107\text{ A}.$

EVALUATE: Just after the switch is closed the current through the battery is 0.455 A. After a long time the current through the battery is 0.585 A. After a long time there are additional current paths, the equivalent resistance of the circuit is decreased and the current has increased.

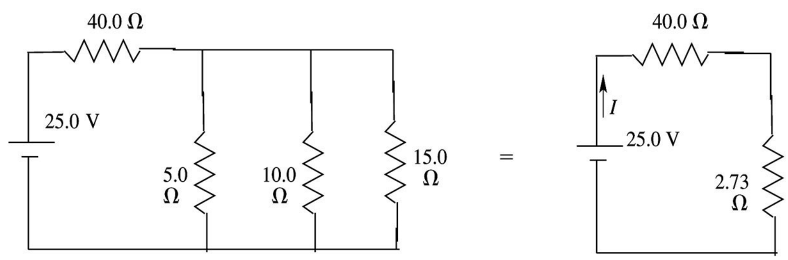


Figure 30.65

30.66. IDENTIFY: At steady state with the switch in position 1, no current flows to the capacitors and the inductors can be replaced by wires. Apply conservation of energy to the circuit with the switch in position 2.

SET UP: Replace the series combinations of inductors and capacitors by their equivalents.

EXECUTE: **(a)** At steady state $i = \dfrac{\mathcal{E}}{R} = \dfrac{75.0\text{ V}}{125\ \Omega} = 0.600\text{ A}.$

(b) The equivalent circuit capacitance of the two capacitors is given by $\dfrac{1}{C_s} = \dfrac{1}{25\ \mu\text{F}} + \dfrac{1}{35\ \mu\text{F}}$ and

$C_s = 14.6\ \mu\text{F}.$ $L_s = 15.0\text{ mH} + 5.0\text{ mH} = 20.0\text{ mH}.$ The equivalent circuit is sketched in Figure 30.66a.

Energy conservation: $\dfrac{q^2}{2C} = \dfrac{1}{2}Li_0^2.$ $q = i_0\sqrt{LC} = (0.600\text{ A})\sqrt{(20\times10^{-3}\text{ H})(14.6\times10^{-6}\text{ F})} = 3.24\times10^{-4}\text{ C}.$

As shown in Figure 30.66b, the capacitors have their maximum charge at $t = T/4$.

$t = \frac{1}{4}T = \frac{1}{4}(2\pi\sqrt{LC}) = \dfrac{\pi}{2}\sqrt{LC} = \dfrac{\pi}{2}\sqrt{(20\times10^{-3}\text{ H})(14.6\times10^{-6}\text{ F})} = 8.49\times10^{-4}\text{ s}.$

EVALUATE: With the switch closed the battery stores energy in the inductors. This then is the energy in the *L*-*C* circuit when the switch is in position 2.

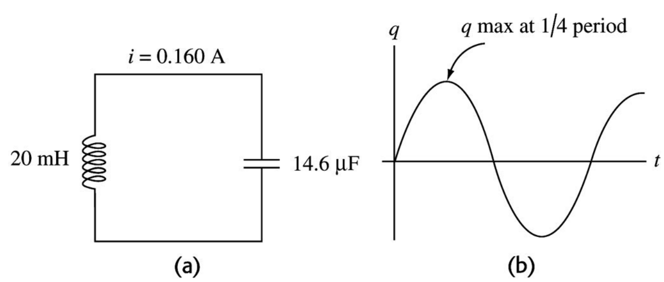


Figure 30.66

30.67. IDENTIFY and **SET UP:** Kirchhoff's loop rule applies, the emf across an inductor is $\mathcal{E}_L = -L\dfrac{di}{dt}$, the potential across a resistor is $V = Ri$, and the time constant for an *L*-*R* circuit is $\tau = L/R$.

EXECUTE: **(a)** First find the current as a function of time. The inductor has a resistance R_L which is in series with the 10.0-Ω resistor *R*. Apply Kirchhoff's loop rule to the circuit: $\mathcal{E} - iR - iR_L - L\dfrac{di}{dt} = 0.$

Now separate variables and integrate.

$$\int_0^t -\frac{R+R_L}{L}dt' = \int_0^i \frac{di'}{i' - \varepsilon/(R+R_L)}.$$

$$-\frac{R+R_L}{L}t = \ln\left(\frac{i-\mathcal{E}/(R+R_L)}{-\mathcal{E}/(R+R_L)}\right).$$

$$i = \frac{\mathcal{E}}{R+R_L}(1-e^{-(R+R_L)t/L}).$$

The potential across the inductor is the sum of the potential due to the resistance and the potential due to the inductance, so $v_L = iR_L + L\,di/dt$. Using the equation we just found for the current i and taking $L\,di/dt$, we get $v_L = iR_L + L\frac{di}{dt} = \left[\frac{\mathcal{E}}{R+R_L}(1-e^{-(R+R_L)t/L})\right]R_L + \mathcal{E}e^{-(R+R_L)t/L}$.

Collecting terms and taking out common factors, the result is $v_L = \frac{\mathcal{E}}{R+R_L}(R_L + Re^{-(R+R_L)t/L})$.

(b) Initially there is no current in the circuit due to the inductor, so the potential across the resistance R is zero. Therefore the potential across the inductor is equal to the emf of the battery.

$v_L(0) = \frac{\mathcal{E}}{R+R_L}(R_L+R) = \mathcal{E} = 50.0$ V.

(c) As $t \to \infty$, we know that $v_L = 20.0$ V. So $v_R = \mathcal{E} - v_L = 50.0\text{ V} - 20.0\text{ V} = 30.0$ V. The current in R is therefore $i = (30.0\text{ V})/(10.0\ \Omega) = 3.00$ A, which is also the current in the circuit.

(d) As $t \to \infty$, the potential across the inductor is due only to its resistance R_L, the potential across it is 20.0 V, and the current through it is 3.00 A. Therefore $R_L = (20.0\text{ V})/(3.00\text{ A}) = 6.67\ \Omega$.

(e) The time constant for this circuit is $\tau = L/(R+R_L)$. Using the equation derived in (a) for v_L, at the end of one time constant v_L is $v_L = \frac{\mathcal{E}}{R+R_L}(R_L + Re^{-1}) = \frac{50.0\text{ V}}{16.67\ \Omega}\left[6.67\,\Omega + (10.0\,\Omega)e^{-1}\right] = 31.0\text{ V}.$

From the graph shown with the problem in the textbook, we read that $t = 2.4$ ms when $v_L = 31.0$ V. So the time constant is 2.4 ms. Solving $\tau = L/(R+R_L)$ for L gives

$L = \tau(R+R_L) = (2.4\text{ ms})(10.0\ \Omega + 6.67\ \Omega) = 40$ mH.

EVALUATE: In this case, the resistance of the inductor is close to the external resistance in the circuit, so it is significant and cannot be ignored.

30.68. **IDENTIFY** and **SET UP:** The current grows in the circuit after the switch is closed. In an R-L circuit the full emf initially is across the inductance and after a long time is totally across the resistance because the inductor opposes changes in the current through it. A solenoid in a circuit is represented as a resistance in series with an inductance. Apply the loop rule to the circuit; the voltage across a resistance is given by Ohm's law, and emf across an inductor is $\mathcal{E}_L = -L\frac{di}{dt}$.

EXECUTE: **(a)** In the R-L circuit the voltage across the resistor starts at zero and increases to the battery voltage. The voltage across the solenoid (inductor) starts at the battery voltage and decreases to zero. As $t \to \infty$ the current in the circuit approaches its final, steady-state value. The final voltage across the solenoid is iR_L, where I is the final current in the circuit. The potential across the external resistor R is 25.0 V after the switch has been closed for a very long time, which is when steady-state has been achieved. Using $V_R = iR$ and $i = \frac{\mathcal{E}}{R+R_L}$ gives $V_R = \frac{\mathcal{E}}{R+R_L}R$. Solving for R_L gives

$R_L = R\left(\frac{\mathcal{E}}{V_R} - 1\right) = (50.0\ \Omega)\left(\frac{25.0\text{ V}}{25.0\text{ V}} - 1\right) = 0.$ The solenoid has no appreciable resistance.

(b) Kirchhoff's loop rule gives $\mathcal{E} - iR - L\frac{di}{dt} = 0.$ Separating variables and integrating gives

$$\int_0^i \frac{di'}{i' - \mathcal{E}/R} = -\int_0^t \frac{R}{L}dt' \qquad \rightarrow \qquad \ln\left(\frac{i - \mathcal{E}/R}{-\mathcal{E}/R}\right) = -\frac{R}{L}t.$$

$$i = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L}) \qquad \rightarrow \qquad v_R = \mathcal{E}(1 - e^{-Rt/L}).$$

(c) At the end of the first time constant, we have $\mathcal{E}(1 - e^{-Rt/L}) = \mathcal{E}(1 - e^{-1})$ = (24.0 V)(1 – 1/*e*) = 15.8 V.

From the graph with the problem in the text, we determine that when v_R = 15.8 V, t = 8.0 ms, so the time constant is τ = 8.0 ms. Using $\tau = L/R$, we have $L = \tau R$ = (8.0 ms)(50.0 Ω) = 0.40 H.

(d) At steady-state the current is (25.0 V)/(50.0 Ω) = 0.500 A. The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$ = (1/2)(0.40 H)(0.500 A)^2 = 0.050 J = 50 mJ.

EVALUATE: We found that the resistance of the inductor is zero. However that really just means that it is much less than the external resistance of 50.0 Ω, so it does not affect the measurements. In reality every inductor has *some* resistance since it is made out of real metal.

30.69. IDENTIFY: This problem deals with a solenoid that is within another solenoid.

SET UP: $B = \mu_0 nI$, $\Phi_B = BA\cos\phi = BA$ since the fields are uniform within an ideal solenoid. Refer to Fig. P30.69 in the textbook for the quantities involved.

EXECUTE: **(a)** We want the flux through the inner coil. The magnetic fields due to both solenoids point in the same direction, so $\Phi_{\text{in}} = B_1A_2 - B_2A_2 = \mu_0 In_1\pi a^2 - \mu_0 In_2\pi a^2$. Using $n = N/\lambda$ this reduces to $\Phi_{\text{in}} = \dfrac{\mu_0 I(N_1 - N_2)\pi a^2}{\lambda}$.

(b) We want the flux through the outer coil. $\Phi_{\text{out}} = B_1A_1 - B_2A_2 = \mu_0 In_1\pi b^2 - \mu_0 In_2\pi a^2$, which reduces to $\Phi_{\text{out}} = \dfrac{\mu_0 I\pi}{\lambda}(N_1b^2 - N_2a^2)$.

(c) We want L. $\Phi_{\text{in}} = B_1A_2 - B_2A_2 = \mu_0 In_1\pi a^2 - \mu_0 In_2\pi a^2$ and $\Phi_{\text{out}} = B_1A_1 - B_2A_2 = \mu_0 In_1\pi b^2 - \mu_0 In_2\pi a^2$, which reduce to $\Phi_{\text{in}} = \dfrac{\mu_0 I(N_1 - N_2)\pi a^2}{\lambda}$ and $\Phi_{\text{out}} = \dfrac{\mu_0 I\pi}{\lambda}(N_1b^2 - N_2a^2)$. The self-inductance is $L = \dfrac{\Phi_B}{I} = \dfrac{N_1\Phi_{\text{out}} + N_2\Phi_{\text{in}}}{I}$. Using the fluxes and simplifying gives $L = \dfrac{\mu_0\pi}{\lambda}(N_1^2b^2 - N_2^2a^2)$.

(d) We want the self-inductance if the inner current is reversed. In this case, B_2 reverses direction. $L = \dfrac{\Phi_B}{I} = \dfrac{N_1\Phi_{\text{out}} + N_2\Phi_{\text{in}}}{I}$. Substituting the fluxes with B_2 reversed and simplifying gives

$$L = \frac{\mu_0\pi}{\lambda}(N_1^2b^2 + 2N_1N_2a^2 + N_2^2a^2).$$

(e) We want L for the original configuration. Using $L = \dfrac{\mu_0\pi}{\lambda}(N_1^2b^2 - N_2^2a^2)$ with the same numbers gives L = 10.3 mH.

(f) We want L for the reversed configuration. Using $L = \dfrac{\mu_0\pi}{\lambda}(N_1^2b^2 + 2N_1N_2a^2 + N_2^2a^2)$ with λ = 20.0 cm, a = 1.00 cm, b = 2.00 cm, N_1 = 1200 turns, and N_2 = 750 turns, we get L = 16.0 mH.

EVALUATE: In case (d) if a = 0 (no inner coil), $L = \dfrac{\mu_0\pi}{\lambda}(N_1^2b^2) = \dfrac{\mu_0\pi b^2N_1^2}{\lambda}$, which is just the formula for the self-inductance of a single solenoid.

30.70. **IDENTIFY:** Apply $L = \dfrac{N\Phi_B}{i}$ to calculate L.

SET UP: In the air the magnetic field is $B_{\text{Air}} = \dfrac{\mu_0 Ni}{W}$. In the liquid, $B_{\text{L}} = \dfrac{\mu Ni}{W}$.

EXECUTE: **(a)** $\Phi_B = BA = B_{\text{L}}A_{\text{L}} + B_{\text{Air}}A_{\text{Air}} = \dfrac{\mu_0 Ni}{W}[(D-d)W] + \dfrac{K\mu_0 Ni}{W}(dW) = \mu_0 Ni[(D-d)+Kd]$.

$$L = \frac{N\Phi_B}{i} = \mu_0 N^2[(D-d)+Kd] = L_0 - L_0\frac{d}{D} + L_{\text{f}}\frac{d}{D} = L_0 + \left(\frac{L_{\text{f}}-L_0}{D}\right)d.$$

$$d = \left(\frac{L-L_0}{L_{\text{f}}-L_0}\right)D, \text{ where } L_0 = \mu_0 N^2 D, \text{ and } L_{\text{f}} = K\mu_0 N^2 D.$$

(b) and **(c)** Using $K = \chi_{\text{m}} + 1$ we can find the inductance for any height $L = L_0\left(1 + \chi_{\text{m}}\dfrac{d}{D}\right)$.

Height of Fluid	Inductance of Liquid Oxygen	Inductance of Mercury
$d = D/4$	0.63024 H	0.63000 H
$d = D/2$	0.63048 H	0.62999 H
$d = 3D/4$	0.63072 H	0.62999 H
$d = D$	0.63096 H	0.62998 H

The values $\chi_{\text{m}}(O_2) = 1.52\times10^{-3}$ and $\chi_{\text{m}}(\text{Hg}) = -2.9\times10^{-5}$ have been used.

EVALUATE: **(d)** The volume gauge is much better for the liquid oxygen than the mercury because there is an easily detectable spread of values for the liquid oxygen, but not for the mercury.

30.71. **IDENTIFY:** Apply Kirchhoff's loop rule to the to and bottom branches of the circuit.

SET UP: Just after the switch is closed the current through the inductor is zero and the charge on the capacitor is zero.

EXECUTE: **(a)** $\mathcal{E} - i_1R_1 - L\dfrac{di_1}{dt} = 0 \Rightarrow i_1 = \dfrac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t})$.

$$\mathcal{E} - i_2R_2 - \frac{q_2}{C} = 0 \Rightarrow -\frac{di_2}{dt}R_2 - \frac{i_2}{C} = 0 \Rightarrow i_2 = \frac{\mathcal{E}}{R_2}e^{-(1/R_2C)t}.$$

$$q_2 = \int_0^t i_2 dt' = -\frac{\mathcal{E}}{R_2}R_2Ce^{-(1/R_2C)t'}\Big|_0^t = \mathcal{E}C(1 - e^{-(1/R_2C)t}).$$

(b) $i_1(0)\dfrac{\mathcal{E}}{R_1}(1-e^0) = 0, i_2 = \dfrac{\mathcal{E}}{R_2}e^0 = \dfrac{48.0\text{ V}}{5000\ \Omega} = 9.60\times10^{-3}$ A.

(c) As $t \to \infty$: $i_1(\infty) = \dfrac{\mathcal{E}}{R_1}(1-e^{-\infty}) = \dfrac{\mathcal{E}}{R_1} = \dfrac{48.0\text{ V}}{25.0\ \Omega} = 1.92$ A, $i_2 = \dfrac{\mathcal{E}}{R_2}e^{-\infty} = 0$. A good definition of a "long time" is many time constants later.

(d) $i_1 = i_2 \Rightarrow \dfrac{\mathcal{E}}{R_1}(1-e^{-(R_1/L)t}) = \dfrac{\mathcal{E}}{R_2}e^{-(1/R_2C)t} \Rightarrow (1-e^{-(R_1/L)t}) = \dfrac{R_1}{R_2}e^{-(1/R_2C)t}$. Expanding the exponentials like $e^x = 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3!} + \ldots$, we find: $\dfrac{R_1}{L}t - \dfrac{1}{2}\left(\dfrac{R_1}{L}\right)^2 t^2 + \ldots = \dfrac{R_1}{R_2}\left(1 - \dfrac{t}{RC} + \dfrac{t^2}{2R^2C^2} - \ldots\right)$ and

$t\left(\dfrac{R_1}{L} + \dfrac{R_1}{R_2{}^2C}\right) + O(t^2) + \ldots = \dfrac{R_1}{R_2}$, if we have assumed that $t << 1$. Therefore:

$$t \approx \frac{1}{R_2}\left(\frac{1}{(1/L)+(1/R_2^2C)}\right) = \left(\frac{LR_2C}{L+R_2^2C}\right) = \left(\frac{(8.0\text{ H})(5000\ \Omega)(2.0\times10^{-5}\text{ F})}{8.0\text{ H} + (5000\ \Omega)^2(2.0\times10^{-5}\text{ F})}\right) = 1.6\times10^{-3}\text{s}.$$

(e) At $t = 1.57\times10^{-3}$ s: $i_1 = \dfrac{\mathcal{E}}{R_1}(1-e^{-(R_1/L)t}) = \dfrac{48\text{ V}}{25\ \Omega}(1-e^{-(25/8)t}) = 9.4\times10^{-3}$ A.

(f) We want to know when the current is half its final value. We note that the current i_2 is very small to begin with, and just gets smaller, so we ignore it and find:

$$i_{1/2} = 0.960\text{ A} = i_1 = \frac{\mathcal{E}}{R_1}(1-e^{-(R_1/L)t}) = (1.92\text{ A})(1-e^{-(R_1/L)t}).$$

$$e^{-(R_1/L)t} = 0.500 \Rightarrow t = -\frac{L}{R_1}\ln(0.5) = -\frac{8.0\text{ H}}{25\ \Omega}\ln(0.5) = 0.22\text{ s}.$$

EVALUATE: i_1 is initially zero and rises to a final value of 1.92 A. i_2 is initially 9.60 mA and falls to zero, q_2 is initially zero and rises to $q_2 = \mathcal{E}C = 960\ \mu$C.

30.72. **IDENTIFY** and **SET UP:** Apply $L = \dfrac{N\Phi_B}{i}$ to calculate L, then solve for the number of turns N. Treat the solenoid as being ideal.

EXECUTE: $L = \dfrac{N\Phi}{i} = \dfrac{N\mu_0 inA}{i} = \dfrac{\mu_0 N^2 A}{l}$. $N = \sqrt{\dfrac{Ll}{\mu_0 A}} = \sqrt{\dfrac{(4.4\text{ H})(2\text{ m})}{\mu_0\pi(0.5\text{ m})^2}} = 3000$, which is choice (b).

EVALUATE: This solenoid is far from ideal since its diameter is half its length, but we can get a rough estimate of the number of coils.

30.73. **IDENTIFY** and **SET UP:** The current in the circuit is $i = i_0 e^{-Rt/L}$. Solve for the time t_{half} for the current to reach one-half its original value.

EXECUTE: $i_0/2 = i_0 e^{-Rt/L}$, so $t_{\text{half}} = (L/R)\ln 2 = [(4.4\text{ H})/(0.005\ \Omega)]\ln 2 = 610\text{ s} \approx 10$ min, choice (b).

EVALUATE: This result is true if no more of the magnet loses its superconductivity. If more of it does so, the time will be less than this because R will be greater.

30.74. **IDENTIFY** and **SET UP:** In Problem 30.73, we saw that $t_{\text{half}} = (L/R)\ln 2$.

EXECUTE: The resistance is increasing, and t_{half} is inversely proportional to R, so the time will be shorter, which is choice (a).

EVALUATE: How much shorter the time will be will depend on how fast the magnet is losing its superconductivity.

30.75. **IDENTIFY:** The magnetic energy stored in the magnet is converted into thermal energy which evaporates the liquid helium.

SET UP: The magnetic energy is $U_L = \frac{1}{2}Li^2$. The heat Q to evaporate a mass m of liquid is $Q = mL_v$.

EXECUTE: $\frac{1}{2}Li^2 = mL_v$. Solving for m gives

$m = Li^2/2L_v = (4.4\text{ H})(750\text{ A})^2/[2(20900\text{ J/kg})] = 59\text{ kg} \approx 60$ kg, which is choice (c).

EVALUATE: This is a lot of liquid helium! It is important to avoid quenches!

31

ALTERNATING CURRENT

VP31.3.1. **IDENTIFY:** We are investigating an inductor in an ac circuit.

SET UP: $X_L = \omega L.$

EXECUTE: **(a)** We want the inductive reactance. $X_L = \omega L = 2\pi fL$ = 2π(108 Hz)(2.50 mH) = 1.70 MΩ.

(b) We want the current amplitude. $I_L = \dfrac{V_L}{X_L} =$ (4.20 kV)/(1.70 MΩ) = 2.48 mA.

EVALUATE: The answer in (b) is the current amplitude, but the current does not always have this value since it varies sinusoidally.

VP31.3.2. **IDENTIFY:** This is an ac circuit containing a resistor and a capacitor.

SET UP: The current is of the form $i(t) = I\cos\omega t,\ \ X_C = 1/\omega C.$

EXECUTE: **(a)** We want $v_R(t)$. $v_R = Ri = R\,I\cos\omega t$ = (125 V)(2.40 mA) cos(1750 rad/s t) = (0.300 V)cos(1750 rad/s t).

(b) We want X_C. $X_C = 1/\omega C$ = 1/[(1750 rad/s)(7.00 μF)] = 81.6 Ω.

(c) We want $v_C(t)$. v_C lags the current by π/2, so

$v_C = V_C\cos(\omega t - \pi/2) = IX_C\cos(\omega t - \pi/2) = (2.40\text{ mA})(81.6\ \Omega)\cos(1750\text{ rad/s }t - \pi/2)$

$= (0.196\text{ V})\cos[(1750\text{ rad/s})t - \pi/2].$

EVALUATE: There is a π/2 phase different between v_C and i because when i is a maximum the capacitor is uncharged.

VP31.3.3. **IDENTIFY:** We have a resistor and capacitor in an ac circuit.

SET UP: $X_C = 1/\omega C,\ \ \omega = 2\pi f$, and $X_C = R$ in this case.

EXECUTE: **(a)** We want the frequency. $X_C = R$, so $X_C = 1/\omega C = R.$ $\omega = 2\pi f$, so $f = \dfrac{1}{2\pi RC}$. This gives f = 1/[2π(155 Ω)(8.00 μF)] = 128 Hz.

(b) We want the amplitudes. $V_R = RI$ = (155 Ω)(4.00 mA) = 0.620 V. $V_C = IX_C$ = 0.620 V since $X_C = R$

(c) v_C lags v_R by π/2 so when v_R is a maximum, v_C = 0. When v_C is a maximum, v_R = 0 because of the π/2 phase difference.

EVALUATE: Minimum current mean s no potential across R but maximum potential across C.

VP31.3.4. **IDENTIFY:** We have a resistor and capacitor in an ac circuit.

SET UP and **EXECUTE:** **(a)** We want the current amplitude. $I = V/R$ = (2.45 V)/(115 Ω) = 21.3 mA.

(b) What is I at t = 3.50 ms? i is a maximum at t = 0, so $i = I\cos\omega t$. Using I = 21.3 mA and the given quantities gives i = 11.5 mA.

(c) We want V_C. When $i = 0$, $v_R = 0$, so $v_C = V_C$ and is equal to V_R which is 2.45 V.

(d) We want C. $V_C = IX_C = I/C$. $C = \dfrac{I}{\omega V_C}$ = (21.3 mA)/[(5100 rad/s)(2.45 V)] = 1.71 μF.

EVALUATE: When $v_R = 0$, $v_C = V_C$ and when $v_C = 0$, $v_R = V_R$ and $V_R = V_C$ but $v_R \neq v_C$.

VP31.5.1. **IDENTIFY:** We have an *L-R-C* series ac circuit.

SET UP: $X_L = \omega L$, $X_C = 1/\omega C$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $\tan\phi = \dfrac{X_L - X_C}{R}$, $\omega = 2\pi f$.

EXECUTE: **(a)** We want f. $X_L = \omega L = 2\pi fL = R$. $f = R/2\pi L$ = (255 Ω)[2π(4.50 mH)] = 9020 Hz.

(b) We want the impedance. Use $Z = \sqrt{R^2 + (X_L - X_C)^2}$ with $X_L = R$, $X_C = 1/\omega C$, and $\omega = 2\pi f$ = 56,670 rad/s. This gives X_C = 6.64 Ω. Put these values into the equation for Z giving Z = 356 Ω.

(c) We want the current amplitude. $I = V/Z$ = (55.0 V)/(356 Ω) = 0.154 A.

(d) We want ϕ. $\tan\phi = \dfrac{X_L - X_C}{R}$ = (255 Ω – 6.64 Ω)/(255 Ω), so ϕ = 44.2°.

(e) Since ϕ is positive, the voltage *leads* the current.

EVALUATE: The resistance never changes but the impedance depends on the frequency.

VP31.5.2. **IDENTIFY:** We have an *L-R-C* series ac circuit.

SET UP: $X_L = \omega L$, $X_C = 1/\omega C$, $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$.

EXECUTE: **(a)** We want R. Calculate ωL and $1/\omega C$ from the given quantities, which gives ωL = 35.1 Ω and $1/\omega C$ = 308.6 Ω. Solve $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$ for R using the known Z.

The result is R = 387 Ω.

(b) We want the current amplitude. $I = V/Z$ = (20.0 V)/(474 Ω) = 0.0422 A = 42.2 mA.

(c) We want V_R. $V_R = RI$ = (387 Ω)(0.0422 A) = 16.3 V.

(d) We want V_L. $V_L = IX_L$ = (0.0422 A)(35.1 Ω) = 1.48 V.

(e) We want V_C. $V_C = IX_C$ = (0.0422 A)(308.6 Ω) = 13.0 V.

EVALUATE: Note that the amplitudes do *not* add up to the voltage amplitude. This is not a problem because these amplitudes do not all occur at the same time due to phase differences.

VP31.5.3. **IDENTIFY:** We are dealing with an *L-R-C* series ac circuit.

SET UP: $\tan\phi = \dfrac{X_L - X_C}{R}$, $X_L = \omega L$, $X_C = 1/\omega C$.

EXECUTE: **(a)** We want $X_L - X_C$. Solve $\tan\phi = \dfrac{X_L - X_C}{R}$. $X_L - X_C$ = (65.0 Ω)tan 15.0° = 17.4 Ω.

(b) We want L. X_L = 17.4 Ω + X_C. Use $X_L = \omega L$ and $X_C = 1/\omega C$ and solve for L, giving $L = \dfrac{17.4\ \Omega\ + 1/\omega C}{\omega}$. Using the given values gives L = 8.65 mH.

EVALUATE: If we wanted the current to lead the voltage, we would need $X_L - X_C$ to be negative, so we would need $X_C > X_L$.

VP31.5.4. **IDENTIFY:** This problem involves an *L-R-C* series ac circuit.

SET UP and **EXECUTE:** **(a)** We want the voltage amplitudes.

$V_R = IR$ = (0.120 A)(95.0 Ω) = 11.4 V.

$V_L = IX_L = I\omega L$ = (0.120 A)(8000 rad/s)(6.50 mH) = 6.24 V.

$V_C = IX_C = I/\omega C$ = (0.120 A)/[(8000 rad/s)(0.440 μF)] = 34.1 V.

(b) We want the instantaneous voltages at $t = 0.305$ ms. The current is a maximum at $t = 0$, so $i(t) = I\cos\omega t$. Use the voltage amplitudes from part (a).

$v_R = Ri = RI\cos\omega t = (11.4\text{ V})\cos[(8000\text{ rad/s})(0.305\text{ ms})] = -8.71$ V.

v_L leads the current by $\pi/2$ so $v_L = V_L\cos(\omega t + \pi/2)$ = (6.24 V) cos[(8000 rad/s)(0.305 s) + π/2)] = –4.03 V.

v_C lags the current by $\pi/2$ so $v_C = V_C\cos(\omega t - \pi/2)$ = 22.0 V.

EVALUATE: Note that the voltage amplitudes add upto 51.7 V while the instantaneous voltages add up to 9.26 V. The instantaneous voltages all occur at the same time (0.305 s), but the voltage amplitudes do not.

VP31.7.1. **IDENTIFY:** We are dealing with power in an ac circuit.

SET UP and **EXECUTE:** The toaster is a pure resistor. **(a)** We want the average power. $P_{av} = V_{rms}I_{rms}$ = (120 V)(3.95 A) = 474 W.

(b) We want the maximum power. $P_{max} = 2P_{av}$ = 948 W.

(c) We want R. $P_{av} = I_{rms}^2 R$, so $R = P_{av}/I_{rms}^2$ = (474 W)/(3.95 A)^2 = 30.4 Ω.

EVALUATE: Check: $P_{max} = I_{max}V_{max} = \left(I_{rms}\sqrt{2}\right)\left(V_{rms}\sqrt{2}\right) = 2I_{rms}V_{rms} = 2P_{av}$. It's OK.

VP31.7.2. **IDENTIFY:** We have an L-R-C series ac circuit.

SET UP: $X_L = \omega L$, $X_C = 1/\omega C$, $\tan\phi = \dfrac{X_L - X_C}{R}$, power factor = $\cos\phi$.

EXECUTE: **(a)** We want X_L and X_C. $X_L = \omega L$ = (1300 rad/s)(82.3 mH) = 107 Ω.

$X_C = 1/\omega C$ = 1/[(1300 rad/s)(1.10 μF)] = 699 Ω.

(b) We want ϕ. $\phi = \arctan\left(\dfrac{X_L - X_C}{R}\right)$ = (107 Ω – 699 Ω)/(275 Ω) = –65.1°.

(c) We want the power factor. cos ϕ = cos(–65.1°) = 0.421.

EVALUATE: Since ϕ is negative, the voltage lags the current by 65.1°.

VP31.7.3. **IDENTIFY:** We have an L-R-C series ac circuit.

SET UP: $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $X_L = \omega L$, $X_C = 1/\omega C$.

EXECUTE: **(a)** We want the impedance. $X_L = \omega L$ = (1300 rad/s)(1.10 μF) = 107 Ω.

$X_C = 1/\omega C$ = 1/[(1300 rad/s)(1.10 μF)] = 699 Ω. Now use $Z = \sqrt{R^2 + (X_L - X_C)^2}$ with the above values, giving Z = 653 Ω.

(b) We want the current amplitude. $I = V/Z$ = (35.0 V)/(653 Ω) = 53.6 mA.

(c) We want the average power in the resistor. $P_{av} = \dfrac{1}{2}IV_R = \dfrac{1}{2}I^2R$ = (0.0536 A)^2(275 Ω)/2 = 0.395 W.

EVALUATE: The average power is *not* I^2R because the current is not equal to its maximum value I all the time; it is usually less than this value. That's where the factor of ½ comes from in part (c).

VP31.7.4. **IDENTIFY:** This problem involves power in an ac series circuit.

SET UP: $X_L = \omega L$, $X_C = 1/\omega C$, $\tan\phi = \dfrac{X_L - X_C}{R}$, power factor = $\cos\phi$.

EXECUTE: **(a)** We want the reactances. Using the given quantities, the results are $X_C = 1/\omega C$ = 85.5 Ω, $X_L = \omega L$ = 97.5 Ω.

(b) We want ϕ. $\cos\phi$ = power factor = 0.800, so ϕ = 36.9°.

(c) We want R. Solve $\tan\phi = \dfrac{X_L - X_C}{R}$ for R, giving $R = \dfrac{X_L - X_C}{\tan\phi}$ = (97.8 Ω – 85.5 Ω)/tan 36.9° = 16.0 Ω.

EVALUATE: Since $X_L > X_C$, ϕ is positive.

31.1. **IDENTIFY:** The maximum current is the current amplitude, and it must not ever exceed 1.50 A.

SET UP: $I_{\text{rms}} = I/\sqrt{2}$. I is the current amplitude, the maximum value of the current.

EXECUTE: $I = 1.50\text{A}$ gives $I_{\text{rms}} = \dfrac{1.50\text{ A}}{\sqrt{2}} = 1.06\text{ A}.$

EVALUATE: The current amplitude is larger than the root-mean-square current.

31.2. **IDENTIFY** and **SET UP:** Apply $V_{\text{rms}} = \dfrac{V}{\sqrt{2}}.$

EXECUTE: **(a)** $V_{\text{rms}} = \dfrac{V}{\sqrt{2}} = \dfrac{45.0\text{ V}}{\sqrt{2}} = 31.8\text{ V}.$

(b) Since the voltage is sinusoidal, the average is zero.

EVALUATE: The voltage amplitude is larger than V_{rms}.

31.3. **IDENTIFY:** We want the phase angle for the source voltage relative to the current, and we want the inductance if we know the current amplitude.

SET UP: $X_L = \dfrac{V}{I}$ and $X_L = 2\pi fL.$

EXECUTE: **(a)** $\phi = +90°.$ The source voltage leads the current by $90°.$

(b) $X_L = \dfrac{V}{I} = \dfrac{45.0\text{ V}}{3.90\text{ A}} = 11.54\;\Omega.$ Solving $X_L = 2\pi fL$ for f gives

$$f = \frac{X_L}{2\pi L} = \frac{11.54\;\Omega}{2\pi(9.50\times10^{-3}\text{ H})} = 193\text{ Hz}.$$

EVALUATE: The angular frequency is about $1200\text{ rad/s}.$

31.4. **IDENTIFY:** We want the phase angle for the source voltage relative to the current, and we want the capacitance if we know the current amplitude.

SET UP: $X_C = \dfrac{V}{I}$ and $X_C = \dfrac{1}{2\pi fC}.$

EXECUTE: **(a)** $\phi = -90°.$ The source voltage lags the current by $90°.$

(b) $X_C = \dfrac{V}{I} = \dfrac{60.0\text{ V}}{5.30\text{ A}} = 11.3\;\Omega.$ Solving $X_C = \dfrac{1}{2\pi fC}$ for C gives

$$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi(80.0\text{ Hz})(11.3\;\Omega)} = 1.76\times10^{-4}\text{ F}.$$

EVALUATE: This is a $176\text{-}\mu\text{F}$ capacitor, which is not unreasonable.

31.5. **IDENTIFY** and **SET UP:** Use $X_L = \omega L$ and $X_C = \dfrac{1}{\omega C}.$

EXECUTE: **(a)** $X_L = \omega L = 2\pi fL = 2\pi(80.0\text{ Hz})(3.00\text{ H}) = 1510\;\Omega.$

(b) $X_L = 2\pi fL$ gives $L = \dfrac{X_L}{2\pi f} = \dfrac{120\;\Omega}{2\pi(80.0\text{ Hz})} = 0.239\text{ H}.$

(c) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(80.0\text{ Hz})(4.00\times10^{-6}\text{ F})} = 497\;\Omega.$

(d) $X_C = \dfrac{1}{2\pi fC}$ gives $C = \dfrac{1}{2\pi fX_C} = \dfrac{1}{2\pi(80.0\text{ Hz})(120\ \Omega)} = 1.66\times10^{-5}$ F.

EVALUATE: X_L increases when L increases; X_C decreases when C increases.

31.6. **IDENTIFY:** The reactance of capacitors and inductors depends on the angular frequency at which they are operated, as well as their capacitance or inductance.

SET UP: The reactances are $X_C = 1/\omega C$ and $X_L = \omega L$.

EXECUTE: **(a)** Equating the reactances gives $\omega L = \dfrac{1}{\omega C} \Rightarrow \omega = \dfrac{1}{\sqrt{LC}}$.

(b) Using the numerical values we get $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(5.00\text{ mH})(3.50\ \mu\text{F})}} = 7560$ rad/s.

$$X_C = X_L = \omega L = (7560\text{ rad/s})(5.00\text{ mH}) = 37.8\ \Omega.$$

EVALUATE: At other angular frequencies, the two reactances could be very different.

31.7. **IDENTIFY:** The reactance of an inductor is $X_L = \omega L = 2\pi fL$. The reactance of a capacitor is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}$.

SET UP: The frequency f is in Hz.

EXECUTE: **(a)** At 60.0 Hz, $X_L = 2\pi(60.0\text{ Hz})(0.450\text{ H}) = 170\ \Omega$. X_L is proportional to f so at 600 Hz, $X_L = 1700\ \Omega$.

(b) At 60.0 Hz, $X_C = \dfrac{1}{2\pi(60.0\text{ Hz})(2.50\times10^{-6}\text{ F})} = 1.06\times10^{3}\ \Omega$. X_C is proportional to $1/f$, so at 600 Hz, $X_C = 106\ \Omega$.

(c) $X_L = X_C$ says $2\pi fL = \dfrac{1}{2\pi fC}$ and $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.450\text{ H})(2.50\times10^{-6}\text{ F})}} = 150$ Hz.

EVALUATE: X_L increases when f increases. X_C decreases when f increases.

31.8. **IDENTIFY:** $V_L = I\omega L$.

SET UP: ω is the angular frequency, in rad/s. $f = \dfrac{\omega}{2\pi}$ is the frequency in Hz.

EXECUTE: $V_L = I\omega L = 2\pi fIL$, so $f = \dfrac{V_L}{2\pi IL} = \dfrac{(12.0\text{ V})}{2\pi(1.80\times10^{-3}\text{ A})(4.50\times10^{-4}\text{ H})} = 2.36\times10^{6}$ Hz = 2.36 MHz.

EVALUATE: When f is increased, I decreases.

31.9. **IDENTIFY:** In an L-R ac circuit, we want to find out how the voltage across a resistor varies with time if we know how the voltage varies across the inductor.

SET UP: $v_L = -I\omega L\sin\omega t$ and $v_R = V_R\cos(\omega t)$.

EXECUTE: **(a)** $v_L = -I\omega L\sin\omega t$. $\omega = 480$ rad/s. $I\omega L = 12.0$ V.

$I = \dfrac{12.0\text{ V}}{\omega L} = \dfrac{12.0\text{ V}}{(480\text{ rad/s})(0.180\text{ H})} = 0.1389$ A. $V_R = IR = (0.1389\text{ A})(90.0\ \Omega) = 12.5$ V.

$v_R = V_R\cos(\omega t) = (12.5\text{ V})\cos[(480\text{ rad/s})t]$.

(b) $v_R = (12.5\text{ V})\cos[(480\text{ rad/s})(2.00\times10^{-3}\text{ s})] = 7.17$ V.

EVALUATE: The instantaneous voltage (7.17 V) is less than the voltage amplitude (12.5 V).

31.10. IDENTIFY: Compare v_C that is given in the problem to the general form $v_C = \dfrac{I}{\omega C}\sin\omega t$ and determine ω.

SET UP: $X_C = \dfrac{1}{\omega C}$. $v_R = iR$ and $i = I\cos\omega t$.

EXECUTE: **(a)** $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(120\text{ rad/s})(4.80\times10^{-6}\text{ F})} = 1736\ \Omega$.

(b) $I = \dfrac{V_C}{X_C} = \dfrac{7.60\text{ V}}{1736\ \Omega} = 4.378\times10^{-3}\text{ A}$ and $i = I\cos\omega t = (4.378\times10^{-3}\text{ A})\cos[(120\text{ rad/s})t]$. Then

$v_R = iR = (4.38\times10^{-3}\text{ A})(250\ \Omega)\cos[(120\text{ rad/s})t] = (1.10\text{ V})\cos[(120\text{ rad/s})t]$.

EVALUATE: The voltage across the resistor has a different phase than the voltage across the capacitor.

31.11. IDENTIFY and SET UP: The voltage and current for a resistor are related by $v_R = iR$. Deduce the frequency of the voltage and use this in $X_L = \omega L$ to calculate the inductive reactance. The equation $v_L = I\omega L\cos(\omega t + 90°)$ gives the voltage across the inductor.

EXECUTE: **(a)** $v_R = (3.80\text{ V})\cos[(720\text{ rad/s})t]$.

$v_R = iR$, so $i = \dfrac{v_R}{R} = \left(\dfrac{3.80\text{ V}}{150\ \Omega}\right)\cos[(720\text{ rad/s})t] = (0.0253\text{ A})\cos[(720\text{ rad/s})t]$.

(b) $X_L = \omega L$.

$\omega = 720$ rad/s, $L = 0.250$ H, so $X_L = \omega L = (720\text{ rad/s})(0.250\text{ H}) = 180\ \Omega$.

(c) If $i = I\cos\omega t$ then $v_L = V_L\cos(\omega t + 90°)$ (from Eq. 31.10).

$V_L = I\omega L = IX_L = (0.02533\text{ A})(180\ \Omega) = 4.56$ V.

$v_L = (4.56\text{ V})\cos[(720\text{ rad/s})t + 90°]$.

But $\cos(a + 90°) = -\sin a$ (Appendix B), so $v_L = -(4.56\text{ V})\sin[(720\text{ rad/s})t]$.

EVALUATE: The current is the same in the resistor and inductor and the voltages are $90°$ out of phase, with the voltage across the inductor leading.

31.12. IDENTIFY: Calculate the reactance of the inductor and of the capacitor. Calculate the impedance and use that result to calculate the current amplitude.

SET UP: With no capacitor, $Z = \sqrt{R^2 + X_L^2}$ and $\tan\phi = \dfrac{X_L}{R}$. $X_L = \omega L$. $I = \dfrac{V}{Z}$. $V_L = IX_L$ and $V_R = IR$. For an inductor, the voltage leads the current.

EXECUTE: **(a)** $X_L = \omega L = (250\text{ rad/s})(0.400\text{ H}) = 100\ \Omega$. $Z = \sqrt{(200\ \Omega)^2 + (100\ \Omega)^2} = 224\ \Omega$.

(b) $I = \dfrac{V}{Z} = \dfrac{30.0\text{ V}}{224\ \Omega} = 0.134$ A.

(c) $V_R = IR = (0.134\text{ A})(200\ \Omega) = 26.8$ V. $V_L = IX_L = (0.134\text{ A})(100\ \Omega) = 13.4$ V.

(d) $\tan\phi = \dfrac{X_L}{R} = \dfrac{100\ \Omega}{200\ \Omega}$ and $\phi = +26.6°$. Since ϕ is positive, the source voltage leads the current.

(e) The phasor diagram is sketched in Figure 31.12.

EVALUATE: Note that $V_R + V_L$ is greater than V. The loop rule is satisfied at each instance of time but the voltages across R and L reach their maxima at different times.

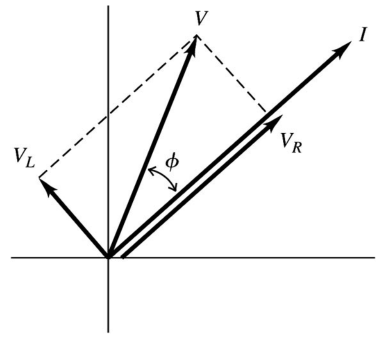


Figure 31.12

31.13. **IDENTIFY:** Apply the equations in Section 31.3.

SET UP: $\omega = 250$ rad/s, $R = 200\ \Omega$, $L = 0.400$ H, $C = 6.00\ \mu$F and $V = 30.0$ V.

EXECUTE: **(a)** $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

$Z = \sqrt{(200\ \Omega)^2 + ((250\text{ rad/s})(0.400\text{ H}) - 1/((250\text{ rad/s})(6.00\times 10^{-6}\text{ F})))^2} = 601\ \Omega.$

(b) $I = \dfrac{V}{Z} = \dfrac{30\text{ V}}{601\ \Omega} = 0.0499$ A.

(c) $\phi = \arctan\left(\dfrac{\omega L - 1/\omega C}{R}\right) = \arctan\left(\dfrac{100\ \Omega - 667\ \Omega}{200\ \Omega}\right) = -70.6°$, and the voltage lags the current.

(d) $V_R = IR = (0.0499\text{ A})(200\ \Omega) = 9.98$ V; $V_L = I\omega L = (0.0499\text{ A})(250\text{ rad/s})(0.400\text{ H}) = 4.99$ V;

$V_C = \dfrac{I}{\omega C} = \dfrac{(0.0499\text{ A})}{(250\text{ rad/s})(6.00\times 10^{-6}\text{ F})} = 33.3$ V.

EVALUATE: **(e)** At any instant, $v = v_R + v_C + v_L$. But v_C and v_L are 180° out of phase, so v_C can be larger than v at a value of t, if $v_L + v_R$ is negative at that t.

31.14. **IDENTIFY:** For an *L-R-C* series ac circuit, we want to find the voltages and voltage amplitudes across all the circuit elements.

SET UP: $X_C = \dfrac{1}{\omega C}$, $X_L = \omega L$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $I = \dfrac{V}{Z}$ and $\tan\phi = \dfrac{X_L - X_C}{R}$. The instantaneous voltages are $v_R = V_R\cos(\omega t) = IR\cos(\omega t)$, $v_L = -V_L\sin(\omega t) = -IX_L\sin(\omega t)$, $v_C = V_C\sin(\omega t) = IX_C\sin(\omega t)$ and $v = V\cos(\omega t + \phi)$.

EXECUTE: $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(250\text{ rad/s})(6.00\times 10^{-6}\text{ F})} = 666.7\ \Omega.$

$X_L = \omega L = (250\text{ rad/s})(0.900\text{ H}) = 225\ \Omega.$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (225\ \Omega - 666.7\ \Omega)^2} = 484.9\ \Omega.$

$I = \dfrac{V}{Z} = \dfrac{30.0\text{ V}}{484.9\ \Omega} = 0.06187\text{ A} = 61.87\text{ mA}.$

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{225\ \Omega - 666.7\ \Omega}{200\ \Omega} = -2.2085$ and $\phi = -1.146$ rad.

(a) $v_R = V_R\cos(\omega t) = IR\cos(\omega t) = (0.06187\text{ A})(200\ \Omega)\cos[(250\text{ rad/s})(20.0\times 10^{-3}\text{ s})] = 3.51$ V.

$v_L = -V_L\sin(\omega t) = -IX_L\sin(\omega t) = -(0.06187\text{ A})(225\ \Omega)\sin[(250\text{ rad/s})(20.0\times 10^{-3}\text{ s})] = 13.35$ V.

$v_C = V_C \sin(\omega t) = IX_C \sin(\omega t) = (0.06187\text{ A})(666.7\ \Omega)\sin[(250\text{ rad/s})(20.0\times10^{-3}\text{ s})] = -39.55\text{ V}.$

$v = V\cos(\omega t + \phi) = (30.0\text{ V})\cos[(250\text{ rad/s})(20.0\times10^{-3}\text{ s}) - 1.146\text{ rad}] = -22.70\text{ V}.$

$v_R + v_L + v_C = 3.51\text{ V} + 13.35\text{ V} + (-39.55\text{ V}) = -22.7\text{ V}.$ $v_R + v_L + v_C$ is equal to v.

(b) $V_R = IR = (0.06187\text{ A})(200\ \Omega) = 12.4\text{ V}$, $V_L = IX_L = (0.06187\text{ A})(225\ \Omega) = 13.9\text{ V}$, and $V_C = IX_C = (0.06187\text{ A})(666.7\ \Omega) = 41.2\text{ V}$.

$V_R + V_C + V_L = 12.4\text{ V} + 41.2\text{ V} + 13.9\text{ V} = 67.5\text{ V}.$ $V_R + V_C + V_L$ is not equal to V.

EVALUATE: The instantaneous voltages do add up to v because they all occur at the same time, so they must add to v by Kirchhoff's loop rule. The amplitudes do not add to V because the maxima do not occur at the same time due to phase differences between the inductor, capacitor and resistor.

31.15. **IDENTIFY** and **SET UP:** Use the equation that preceeds Eq. (31.20): $V^2 = V_R^2 + (V_L - V_C)^2$.

EXECUTE: $V = \sqrt{(30.0\text{ V})^2 + (50.0\text{ V} - 90.0\text{ V})^2} = 50.0\text{ V}.$

EVALUATE: The equation follows directly from the phasor diagrams of Fig. 31.13 (b or c) in the textbook. Note that the voltage amplitudes do not simply add to give 170.0 V for the source voltage.

31.16. **IDENTIFY:** This is an L-R-C series ac circuit.

SET UP: The voltage across the resistor is $v_R(t) = V_R\cos\omega t$, so $v_L(t) = -V_L\sin\omega t$.

EXECUTE: Using $v_L(t) = -V_L\sin\omega t$, we get $\sin\omega t = -\dfrac{v_L}{V_L} = -\dfrac{80.0\text{ mV}}{180\text{ mV}} = -0.4444.$ So $\omega t = -26.39°$.

$v_R(t) = V_R\cos\omega t = (160\text{ V})\cos(-26.39°) = 143\text{ V}.$

$v_C(t) = V_C\sin\omega t = (120\text{ V})\sin(26.39°) = 53.3\text{ V}.$

EVALUATE: The instantaneous voltages add to 80.0 V + 143 V + 53.3 V = 276 V, but the voltage amplitudes add to 180 V + 120 V + 160 V = 440 V. At any time the instantaneous voltages all add to the same value. But the voltage amplitudes do not add to that value because they do not all occur at the same time.

31.17. **IDENTIFY:** We are dealing with an L-R-C series ac circuit.

SET UP: $\tan\phi = \dfrac{V_L - V_C}{V_R}$, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$. The target variable is ϕ. Solve $V = \sqrt{V_R^2 + (V_L - V_C)^2}$ for V_R. Then use $\tan\phi = \dfrac{V_L - V_C}{V_R}$ to find ϕ.

EXECUTE: Solve for V_R: $V_R = \sqrt{V^2 - (V_L - V_C)^2} = \sqrt{(240\text{ V})^2 - (310\text{ V} - 180\text{ V})^2} = 201.7\text{ V}.$ Now find ϕ. $\phi = \arctan\left(\dfrac{V_L - V_C}{V_R}\right) = \arctan\left(\dfrac{310\text{ V} - 180\text{ V}}{201.7\text{ V}}\right) = 32.8°.$

EVALUATE: The angle of the voltage with respect to the current is 32.8°.

31.18. **IDENTIFY:** For an L-R ac circuit, we want to use the resistance, voltage amplitude of the source and power in the resistor to find the impedance, the voltage amplitude across the inductor and the power factor.

SET UP: $P_{\text{av}} = \dfrac{1}{2}I^2R$, $Z = \dfrac{V}{I}$, $V_R = IR$, and $\tan\phi = \dfrac{X_L}{R}$.

EXECUTE: **(a)** $P_{\text{av}} = \dfrac{1}{2}I^2R.$ $I = \sqrt{\dfrac{2P_{\text{av}}}{R}} = \sqrt{\dfrac{2(286\text{ W})}{300\ \Omega}} = 1.381\text{ A}.$ $Z = \dfrac{V}{I} = \dfrac{500\text{ V}}{1.381\text{ A}} = 362\ \Omega.$

(b) $V_R = IR = (1.381\text{ A})(300\ \Omega) = 414\text{ V}.$ $V_L = \sqrt{V^2 - V_R^2} = \sqrt{(500\text{ V})^2 - (414\text{ V})^2} = 280\text{ V}.$

(c) $\tan\phi = \dfrac{X_L}{R} = \dfrac{V_L}{V_R} = \dfrac{280\text{ V}}{414\text{ V}}$ gives $\phi = 34.1°$. The power factor is $\cos\phi = 0.828$.

EVALUATE: The voltage amplitude across the resistor cannot exceed the voltage amplitude (500 V) of the ac source.

31.19. **IDENTIFY:** For a pure resistance, $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}} = I_{\text{rms}}^2 R.$

SET UP: 20.0 W is the average power P_{av}.

EXECUTE: **(a)** The average power is one-half the maximum power, so the maximum instantaneous power is 40.0 W.

(b) $I_{\text{rms}} = \dfrac{P_{\text{av}}}{V_{\text{rms}}} = \dfrac{20.0\text{ W}}{120\text{ V}} = 0.167\text{ A}.$

(c) $R = \dfrac{P_{\text{av}}}{I_{\text{rms}}^2} = \dfrac{20.0\text{ W}}{(0.167\text{ A})^2} = 720\ \Omega.$

EVALUATE: We can also calculate the average power as $P_{\text{av}} = \dfrac{V_{R,\text{rms}}^2}{R} = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{(120\text{ V})^2}{720\ \Omega} = 20.0\text{ W}.$

31.20. **IDENTIFY:** The average power supplied by the source is $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi.$ The power consumed in the resistance is $P_{\text{av}} = I_{\text{rms}}^2 R.$

SET UP: $\omega = 2\pi f = 2\pi(1.25\times10^3\text{ Hz}) = 7.854\times10^3\text{ rad/s}.$ $X_L = \omega L = 157\ \Omega.$ $X_C = \dfrac{1}{\omega C} = 909\ \Omega.$

EXECUTE: **(a)** First, let us find the phase angle between the voltage and the current:
$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{157\ \Omega - 909\ \Omega}{350\ \Omega}$ and $\phi = -65.04°$. The impedance of the circuit is
$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(350\ \Omega)^2 + (-752\ \Omega)^2} = 830\ \Omega.$ The average power provided by the generator is then $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos(\phi) = \dfrac{V_{\text{rms}}^2}{Z}\cos(\phi) = \dfrac{(120\text{ V})^2}{830\ \Omega}\cos(-65.04°) = 7.32\text{ W}.$

(b) The average power dissipated by the resistor is $P_R = I_{\text{rms}}^2 R = \left(\dfrac{120\text{ V}}{830\ \Omega}\right)^2 (350\ \Omega) = 7.32\text{ W}.$

EVALUATE: Conservation of energy requires that the answers to parts (a) and (b) are equal.

31.21. **IDENTIFY:** We are dealing with power in an *L-R-C* series ac circuit.
SET UP and **EXECUTE:** **(a)** From Fig. 31.13(b) in the textbook, $\cos\phi = V_R/V = IR/V$. Thus $V\cos\phi = IR.$ From Eq. (31.31), $P_{\text{av}} = \dfrac{1}{2}IV\cos\phi = \dfrac{1}{2}I(IR) = \dfrac{1}{2}I^2R.$

(b) We want *R*. Use our results from part (a). $P_{\text{av}} = \dfrac{1}{2}I^2R = \dfrac{1}{2}\dfrac{(IR)^2}{R} = \dfrac{1}{2}\dfrac{(V\cos\phi)^2}{R}$. Solve for *R*.

$R = \dfrac{(V\cos\phi)^2}{2P_{\text{av}}} = \dfrac{[(120\text{ V})(\cos 53.1°)]^2}{2(80.0\text{ W})} = 32.4\ \Omega.$

EVALUATE: The current amplitude is $I = \sqrt{2P_{av}/R} = 2.22\ A.$

31.22. **IDENTIFY:** This is an *R-C* ac circuit.

SET UP and **EXECUTE:** **(a)** We want V_R. Solve $V = \sqrt{V_R^2 + V_C^2}$ for V_R.

$V_R = \sqrt{V^2 - V_C^2} = \sqrt{(900\text{ V})^2 - (500\text{ V})^2} = 748\text{ V}.$

(b) We want C. $\dfrac{V_R}{V_C} = \dfrac{IR}{IX_C} = \dfrac{R}{1/\omega C} = R\omega C.C = \dfrac{V_R/V_C}{R\omega}$. Using the numbers gives $C = 249\ \mu$F.

(c) $\tan\phi = \dfrac{-1/\omega C}{R} = -\dfrac{1}{R\omega C}$. Since ϕ is negative, the voltage *lags* the current.

(d) $P_{\text{av}} = \dfrac{1}{2} IV\cos\phi = \dfrac{1}{2} V\left(\dfrac{V_R}{R}\right)\cos\phi = \left(\dfrac{VV_R}{2R}\right)\dfrac{R}{\sqrt{R^2 + (1/\omega C)^2}}$. Using the given numbers, this gives

$P_{\text{av}} = 932$ W.

EVALUATE: The power is all used up in the resistor.

31.23. IDENTIFY and SET UP: Use the equations of Section 31.3 to calculate ϕ, Z, and V_{rms}. The average power delivered by the source is given by $P_{\text{av}} = I_{\text{rms}}V_{\text{rms}}\cos\phi$ and the average power dissipated in the resistor is $I_{\text{rms}}^2 R$.

EXECUTE: (a) $X_L = \omega L = 2\pi fL = 2\pi(400\text{ Hz})(0.120\text{ H}) = 301.6\ \Omega$.

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(400\text{ Hz})(7.3\times10^{-6}\text{ F})} = 54.51\ \Omega$.

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{301.6\ \Omega - 54.41\ \Omega}{240\ \Omega}$, so $\phi = +45.8°$. The power factor is $\cos\phi = +0.697$.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(240\ \Omega)^2 + (301.6\ \Omega - 54.51\ \Omega)^2} = 344\ \Omega$.

(c) $V_{\text{rms}} = I_{\text{rms}}Z = (0.450\text{ A})(344\ \Omega) = 155$ V.

(d) $P_{\text{av}} = I_{\text{rms}}V_{\text{rms}}\cos\phi = (0.450\text{ A})(155\text{ V})(0.697) = 48.6$ W.

(e) $P_{\text{av}} = I_{\text{rms}}^2 R = (0.450\text{ A})^2(240\ \Omega) = 48.6$ W.

EVALUATE: The average electrical power delivered by the source equals the average electrical power consumed in the resistor.

(f) All the energy stored in the capacitor during one cycle of the current is released back to the circuit in another part of the cycle. There is no net dissipation of energy in the capacitor.

(g) The answer is the same as for the capacitor. Energy is repeatedly being stored and released in the inductor, but no net energy is dissipated there.

31.24. IDENTIFY and SET UP: $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi$. $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}$. $\cos\phi = \dfrac{R}{Z}$.

EXECUTE: $I_{\text{rms}} = \dfrac{80.0\text{ V}}{105\ \Omega} = 0.762$ A. $\cos\phi = \dfrac{75.0\ \Omega}{105\ \Omega} = 0.714$.

$P_{\text{av}} = (80.0\text{ V})(0.762\text{ A})(0.714) = 43.5$ W.

EVALUATE: Since the average power consumed by the inductor and by the capacitor is zero, we can also calculate the average power as $P_{\text{av}} = I_{\text{rms}}^2 R = (0.762\text{ A})^2(75.0\ \Omega) = 43.5$ W.

31.25. IDENTIFY: The angular frequency and the capacitance can be used to calculate the reactance X_C of the capacitor. The angular frequency and the inductance can be used to calculate the reactance X_L of the inductor. Calculate the phase angle ϕ and then the power factor is $\cos\phi$. Calculate the impedance of the circuit and then the rms current in the circuit. The average power is $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi$. On the average no power is consumed in the capacitor or the inductor, it is all consumed in the resistor.

SET UP: The source has rms voltage $V_{\text{rms}} = \dfrac{V}{\sqrt{2}} = \dfrac{45\text{ V}}{\sqrt{2}} = 31.8$ V.

EXECUTE: **(a)** $X_L = \omega L = (360\ \text{rad/s})(15\times10^{-3}\ \text{H}) = 5.4\ \Omega.$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{(360\ \text{rad/s})(3.5\times10^{-6}\ \text{F})} = 794\ \Omega.$ $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{5.4\ \Omega - 794\ \Omega}{250\ \Omega}$ and $\phi = -72.4°.$

The power factor is $\cos\phi = 0.302.$

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(250\ \Omega)^2 + (5.4\ \Omega - 794\ \Omega)^2} = 827\ \Omega.$ $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{31.8\ \text{V}}{827\ \Omega} = 0.0385\ \text{A}.$

$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi = (31.8\ \text{V})(0.0385\ \text{A})(0.302) = 0.370\ \text{W}.$

(c) The average power delivered to the resistor is $P_{\text{av}} = I_{\text{rms}}^2 R = (0.0385\ \text{A})^2(250\ \Omega) = 0.370\ \text{W}.$ The average power delivered to the capacitor and to the inductor is zero.

EVALUATE: On average the power delivered to the circuit equals the power consumed in the resistor. The capacitor and inductor store electrical energy during part of the current oscillation but each return the energy to the circuit during another part of the current cycle.

31.26. **IDENTIFY:** At resonance in an *L-R-C* ac circuit, we know the reactance of the capacitor and the voltage amplitude across it. From this information, we want to find the voltage amplitude of the source.

SET UP: At resonance, $Z = R.$ $V_C = IX_C.$

EXECUTE: $I = \dfrac{V}{X_C} = \dfrac{600\ \text{V}}{200\ \Omega} = 3.00\ \text{A}.$ $Z = R = 300\ \Omega.$ $V = IZ = (3.00\ \text{A})(300\ \Omega) = 900\ \text{V}.$

EVALUATE: At resonance, $Z = R,$ but X_C is not zero.

31.27. **IDENTIFY** and **SET UP:** The current is largest at the resonance frequency. At resonance, $X_L = X_C$ and $Z = R.$ For part (b), calculate Z and use $I = V/Z.$

EXECUTE: **(a)** $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 113\ \text{Hz}.$ $I = V/R = 15.0\ \text{mA}.$

(b) $X_C = 1/\omega C = 500\ \Omega.$ $X_L = \omega L = 160\ \Omega.$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (160\ \Omega - 500\ \Omega)^2} = 394.5\ \Omega.$ $I = V/Z = 7.61\ \text{mA}.$ $X_C > X_L$ so the source voltage lags the current.

EVALUATE: $\omega_0 = 2\pi f_0 = 710\ \text{rad/s}.$ $\omega = 400\ \text{rad/s}$ and is less than ω_0. When $\omega < \omega_0,$ $X_C > X_L.$ Note that I in part (b) is less than I in part (a).

31.28. **IDENTIFY:** The impedance and individual reactances depend on the angular frequency at which the circuit is driven.

SET UP: The impedance is $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2},$ the current amplitude is $I = V/Z$ and the instantaneous values of the potential and current are $v = V\cos(\omega t + \phi),$ where $\tan\phi = (X_L - X_C)/R,$ and $i = I\cos\omega t.$

EXECUTE: **(a)** Z is a minimum when $\omega L = \dfrac{1}{\omega C},$ which gives

$\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(8.00\ \text{mH})(12.5\ \mu\text{F})}} = 3162\ \text{rad/s},$ which rounds to 3160 rad/s. $Z = R = 175\ \Omega.$

(b) $I = V/Z = (25.0\ \text{V})/(175\ \Omega) = 0.143\ \text{A}.$

(c) $i = I\cos\omega t = I/2,$ so $\cos\omega t = \frac{1}{2},$ which gives $\omega t = 60° = \pi/3\ \text{rad}.$ $v = V\cos(\omega t + \phi),$ where $\tan\phi = (X_L - X_C)/R = 0/R = 0.$ So, $v = (25.0\ \text{V})\cos\omega t = (25.0\ \text{V})(1/2) = 12.5\ \text{V}.$

$v_R = Ri = (175\ \Omega)(1/2)(0.143\ \text{A}) = 12.5\ \text{V}.$

$v_C = V_C \cos(\omega t - 90°) = IX_C \cos(\omega t - 90°) = \dfrac{0.143 \text{ A}}{(3162 \text{ rad/s})(12.5\ \mu\text{F})} \cos(60° - 90°) = +3.13 \text{ V}.$

$v_L = V_L \cos(\omega t + 90°) = IX_L \cos(\omega t + 90°) = (0.143 \text{ A})(3162 \text{ rad/s})(8.00 \text{ mH}) \cos(60° + 90°).$

$v_L = -3.13 \text{ V}.$

(d) $v_R + v_L + v_C = 12.5 \text{ V} + (-3.13 \text{ V}) + 3.13 \text{ V} = 12.5 \text{ V} = v_{\text{source}}.$

EVALUATE: The instantaneous potential differences across all the circuit elements always add up to the value of the source voltage at that instant. In this case (resonance), the potentials across the inductor and capacitor have the same magnitude but are $180°$ out of phase, so they add to zero, leaving all the potential difference across the resistor.

31.29. IDENTIFY and SET UP: At the resonance frequency, $Z = R$. Use that $V = IZ$, $V_R = IR$, $V_L = IX_L$, and $V_C = IX_C$. P_{av} is given by $P_{\text{av}} = \frac{1}{2} VI \cos\phi$.

EXECUTE: **(a)** $V = IZ = IR = (0.500 \text{ A})(300\ \Omega) = 150 \text{ V}.$

(b) $V_R = IR = 150 \text{ V}.$

$X_L = \omega L = L(1/\sqrt{LC}) = \sqrt{L/C} = 2582\ \Omega;\ V_L = IX_L = 1290 \text{ V}.$

$X_C = 1/(\omega C) = \sqrt{L/C} = 2582\ \Omega;\ V_C = IX_C = 1290 \text{ V}.$

(c) $P_{\text{av}} = \frac{1}{2} VI \cos\phi = \frac{1}{2} I^2 R$, since $V = IR$ and $\cos\phi = 1$ at resonance.

$P_{\text{av}} = \frac{1}{2}(0.500 \text{ A})^2 (300\ \Omega) = 37.5 \text{ W}.$

EVALUATE: At resonance $V_L = V_C$. Note that $V_L + V_C > V$. However, at any instant $v_L + v_C = 0$.

31.30. IDENTIFY: The current is maximum at the resonance frequency, so choose C such that $\omega = 50.0$ rad/s is the resonance frequency. At the resonance frequency $Z = R$.

SET UP: $V_L = I\omega L.$

EXECUTE: **(a)** The amplitude of the current is given by $I = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$. Thus, the current will have a maximum amplitude when $\omega L = \dfrac{1}{\omega C}$. Therefore,

$$C = \frac{1}{\omega^2 L} = \frac{1}{(50.0 \text{ rad/s})^2 (3.00 \text{ H})} = 1.33 \times 10^{-4} \text{ F} = 133\ \mu\text{F}.$$

(b) With the capacitance calculated above we find that $Z = R$, and the amplitude of the current is $I = \dfrac{V}{R} = \dfrac{120 \text{ V}}{400\ \Omega} = 0.300 \text{ A}$. Thus, the amplitude of the voltage across the inductor is

$V_L = I(\omega L) = (0.300 \text{ A})(50.0 \text{ rad/s})(3.00 \text{ H}) = 45.0 \text{ V}.$

EVALUATE: For the value of C found in part (a), the resonance angular frequency is 50.0 rad/s.

31.31. IDENTIFY and SET UP: At resonance $X_L = X_C$, $\phi = 0$ and $Z = R$. $R = 150\ \Omega$, $L = 0.750$ H, $C = 0.0180\ \mu\text{F}$, $V = 150$ V

EXECUTE: **(a)** At the resonance frequency $X_L = X_C$ and from $\tan\phi = \dfrac{X_L - X_C}{R}$ we have that $\phi = 0°$ and the power factor is $\cos\phi = 1.00$.

(b) $P_{\text{av}} = \frac{1}{2} VI \cos\phi$ (Eq. 31.31).

At the resonance frequency $Z = R$, so $I = \dfrac{V}{Z} = \dfrac{V}{R}$.

$$P_{av} = \frac{1}{2}V\left(\frac{V}{R}\right)\cos\phi = \frac{1}{2}\frac{V^2}{R} = \frac{1}{2}\frac{(150\text{ V})^2}{150\ \Omega} = 75.0\text{ W}.$$

EVALUATE: (c) When *C* and *f* are changed but the circuit is kept on resonance, nothing changes in $P_{av} = V^2/(2R)$, so the average power is unchanged: $P_{av} = 75.0$ W. The resonance frequency changes but since $Z = R$ at resonance the current doesn't change.

31.32. **IDENTIFY:** This problem is about resonance in an *L-R-C* series ac circuit.

SET UP and **EXECUTE:** At resonance, $Z = R$ and $P = V^2/2R = (80.0\text{ V})^2/[2(400\ \Omega)] = 8.00$ W.

EVALUATE: At resonance, $\cos\phi = 1$.

31.33. **IDENTIFY:** At resonance $Z = R$ and $X_L = X_C$.

SET UP: $\omega_0 = \dfrac{1}{\sqrt{LC}}$. $V = IZ$. $V_R = IR$, $V_L = IX_L$ and $V_C = V_L$.

EXECUTE: (a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.280\text{ H})(4.00\times10^{-6}\text{ F})}} = 945$ rad/s.

(b) $I = 1.70$ A at resonance, so $R = Z = \dfrac{V}{I} = \dfrac{120\text{ V}}{1.70\text{ A}} = 70.6\ \Omega$.

(c) At resonance, $V_R = 120$ V, $V_L = V_C = I\omega L = (1.70\text{ A})(945\text{ rad/s})(0.280\text{ H}) = 450$ V.

EVALUATE: At resonance, $V_R = V$ and $V_L - V_C = 0$.

31.34. **IDENTIFY:** Let I_1, V_1 and I_2, V_2 be rms values for the primary and secondary. A transformer transforms voltages according to $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$. The effective resistance of a secondary circuit of resistance *R* is $R_{eff} = \dfrac{R}{(N_2/N_1)^2}$. Resistance *R* is related to P_{av} and V_{rms} by $P_{av} = \dfrac{V_{rms}^2}{R}$. Conservation of energy requires $P_{av,1} = P_{av,2}$ so $V_1I_1 = V_2I_2$.

SET UP: Let $V_1 = 240$ V and $V_2 = 120$ V, so $P_{2,av} = 1600$ W. These voltages are rms.

EXECUTE: (a) $V_1 = 240$ V and we want $V_2 = 120$ V, so use a step-down transformer with $N_2/N_1 = \frac{1}{2}$.

(b) $P_{av} = V_1I_1$, so $I_1 = \dfrac{P_{av}}{V_1} = \dfrac{1600\text{ W}}{240\text{ V}} = 6.67$ A.

(c) The resistance *R* of the blower is $R = \dfrac{V_2^2}{P_{av}} = \dfrac{(120\text{ V})^2}{1600\text{ W}} = 9.00\ \Omega$. The effective resistance of the blower is $R_{eff} = \dfrac{9.00\ \Omega}{(1/2)^2} = 36.0\ \Omega$.

EVALUATE: $I_2 = V_2/R_2 = (120\text{ V})/(9.00\ \Omega) = 13.3$ A, so $I_2V_2 = (13.3\text{ A})(120\text{ V}) = 1600$ W. Energy is provided to the primary at the same rate that it is consumed in the secondary. Step-down transformers step up resistance and the current in the primary is less than the current in the secondary.

31.35. **IDENTIFY** and **SET UP:** The equation $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$ relates the primary and secondary voltages to the number of turns in each. $I = V/R$ and the power consumed in the resistive load is $I_{rms}^2 = V_{rms}^2/R$. Let I_1, V_1 and I_2, V_2 be rms values for the primary and secondary.

EXECUTE: (a) $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$ so $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{120\text{ V}}{12.0\text{ V}} = 10$.

(b) $I_2 = \dfrac{V_2}{R} = \dfrac{12.0\text{ V}}{5.00\ \Omega} = 2.40\text{ A}.$

(c) $P_{\text{av}} = I_2^2 R = (2.40\text{ A})^2(5.00\ \Omega) = 28.8\text{ W}.$

(d) The power drawn from the line by the transformer is the 28.8 W that is delivered by the load.

$P_{\text{av}} = \dfrac{V_1^2}{R}$ so $R = \dfrac{V_1^2}{P_{\text{av}}} = \dfrac{(120\text{ V})^2}{28.8\text{ W}} = 500\ \Omega.$

And $\left(\dfrac{N_1}{N_2}\right)^2 (5.00\ \Omega) = (10)^2(5.00\ \Omega) = 500\ \Omega,$ as was to be shown.

EVALUATE: The resistance is "transformed." A load of resistance R connected to the secondary draws the same power as a resistance $(N_1/N_2)^2 R$ connected directly to the supply line, without using the transformer.

31.36. IDENTIFY: $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}}$ and $P_{\text{av},1} = P_{\text{av},2}$. $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2}$. Let I_1, V_1 and I_2, V_2 be rms values for the primary and secondary.

SET UP: $V_1 = 120\text{ V}$. $V_2 = 13{,}000\text{ V}$.

EXECUTE: **(a)** $\dfrac{N_2}{N_1} = \dfrac{V_2}{V_1} = \dfrac{13{,}000\text{ V}}{120\text{ V}} = 108.$

(b) $P_{\text{av}} = V_2 I_2 = (13{,}000\text{ V})(8.50\times 10^{-3}\text{ A}) = 110\text{ W}.$

(c) $I_1 = \dfrac{P_{\text{av}}}{V_1} = \dfrac{110\text{ W}}{120\text{ V}} = 0.917\text{ A}.$

EVALUATE: Since the power supplied to the primary must equal the power delivered by the secondary, in a step-up transformer the current in the primary is greater than the current in the secondary.

31.37. IDENTIFY and **SET UP:** Use $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$ to relate L and R to ϕ. The voltage across the coil leads the current in it by $52.3°$, so $\phi = +52.3°$.

EXECUTE: $\tan\phi = \dfrac{X_L - X_C}{R}$. But there is no capacitance in the circuit so $X_C = 0$. Thus

$\tan\phi = \dfrac{X_L}{R}$ and $X_L = R\tan\phi = (48.0\ \Omega)\tan 52.3° = 62.1\ \Omega.$

$X_L = \omega L = 2\pi f L$ so $L = \dfrac{X_L}{2\pi f} = \dfrac{62.1\ \Omega}{2\pi(80.0\text{ Hz})} = 0.124\text{ H}.$

EVALUATE: $\phi > 45°$ when $(X_L - X_C) > R$, which is the case here.

31.38. IDENTIFY: We have an *L-R-C* series ac circuit.

SET UP: $\tan\phi = \dfrac{X_L - X_C}{R}$, $P_{\text{av}} = \dfrac{1}{2} I^2 R$, $P_{\text{av}} = \dfrac{1}{2} IV\cos\phi.$

EXECUTE: **(a)** We want X_L. Use $\tan\phi = \dfrac{X_L - X_C}{R}$. $X_L = R\tan\phi + X_C$. Using the numbers gives

$X_L = (300\ \Omega)\tan(-53.0°) + 500\ \Omega = 102\ \Omega.$

(b) We want I. Use $P_{\text{av}} = \dfrac{1}{2} I^2 R$ and solve for I. $I = \sqrt{2P_{\text{av}}/R}$ gives $I = 0.730$ A.

(c) We want V. Use $P_{\text{av}} = \frac{1}{2}IV\cos\phi$ and solve for V. $V = \frac{2P_{\text{av}}}{I\cos\phi} = 364\text{ V}$.

EVALUATE: The circuit is not close to resonance because X_L is very different from X_C (102 Ω compared to 500 Ω).

31.39. **IDENTIFY:** An L-R-C ac circuit operates at resonance. We know L, C, and V and want to find R.

SET UP: At resonance, $Z = R$ and $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$. $X_C = \frac{1}{\omega C}$, $I = V/Z$.

EXECUTE: $\omega = \frac{1}{\sqrt{LC}} = 633.0\text{ rad/s}$ $X_C = \frac{1}{\omega C} = \frac{1}{(633\text{ rad/s})(4.80\times10^{-6}\text{ F})} = 329.1\,\Omega.$

$I = \frac{V_C}{X_C} = \frac{80.0\text{ V}}{329.1\,\Omega} = 0.2431\text{ A}$. At resonance $Z = R$, so $I = \frac{V}{R}$. $R = \frac{V}{I} = \frac{56.0\text{ V}}{0.2431\text{ A}} = 230\,\Omega.$

EVALUATE: At resonance, the impedance is a minimum.

31.40. **IDENTIFY:** $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$. $V_{\text{rms}} = I_{\text{rms}}R$. $V_{C,\text{rms}} = I_{\text{rms}}X_C$. $V_{L,\text{rms}} = I_{\text{rms}}X_L$.

SET UP: $V_{\text{rms}} = \frac{V}{\sqrt{2}} = \frac{30.0\text{ V}}{\sqrt{2}} = 21.2\text{ V}.$

EXECUTE: **(a)** $\omega = 200\text{ rad/s}$, so $X_L = \omega L = (200\text{ rad/s})(0.400\text{ H}) = 80.0\,\Omega$ and

$X_C = \frac{1}{\omega C} = \frac{1}{(200\text{ rad/s})(6.00\times10^{-6}\text{ F})} = 833\,\Omega$. $Z = \sqrt{(200\,\Omega)^2 + (80.0\,\Omega - 833\,\Omega)^2} = 779\,\Omega.$

$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{21.2\text{ V}}{779\,\Omega} = 0.0272\text{ A}$. V_1 reads $V_{R,\text{rms}} = I_{\text{rms}}R = (0.0272\text{ A})(200\,\Omega) = 5.44\text{ V}.$

V_2 reads $V_{L,\text{rms}} = I_{\text{rms}}X_L = (0.0272\text{ A})(80.0\,\Omega) = 2.18\text{ V}.$

V_3 reads $V_{C,\text{rms}} = I_{\text{rms}}X_C = (0.0272\text{ A})(833\,\Omega) = 22.7\text{ V}.$

V_4 reads $\left|\frac{V_L - V_C}{\sqrt{2}}\right| = |V_{L,\text{rms}} - V_{C,\text{rms}}| = |2.18\text{ V} - 22.7\text{ V}| = 20.5\text{ V}.$

V_5 reads $V_{\text{rms}} = 21.2\text{ V}.$

(b) $\omega = 1000\text{ rad/s}$ so $X_L = \omega L = (5)(80.0\,\Omega) = 400\,\Omega$ and $X_C = \frac{1}{\omega C} = \frac{833\,\Omega}{5} = 167\,\Omega.$

$Z = \sqrt{(200\,\Omega)^2 + (400\,\Omega - 167\,\Omega)^2} = 307\,\Omega$. $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{21.2\text{ V}}{307\,\Omega} = 0.0691\text{ A}.$

V_1 reads $V_{R,\text{rms}} = 13.8\text{ V}$. V_2 reads $V_{L,\text{rms}} = 27.6\text{ V}$. V_3 reads $V_{C,\text{rms}} = 11.5\text{ V}.$

V_4 reads $|V_{L,\text{rms}} - V_{C,\text{rms}}| = |27.6\text{ V} - 11.5\text{ V}| = 16.1\text{ V}$. V_5 reads $V_{\text{rms}} = 21.2\text{ V}.$

EVALUATE: The resonance frequency for this circuit is $\omega_0 = \frac{1}{\sqrt{LC}} = 645\text{ rad/s}$. 200 rad/s is less than the resonance frequency and $X_C > X_L$. 1000 rad/s is greater than the resonance frequency and $X_L > X_C$.

31.41. **IDENTIFY:** We can use geometry to calculate the capacitance and inductance, and then use these results to calculate the resonance angular frequency.

SET UP: The capacitance of an air-filled parallel plate capacitor is $C = \frac{\epsilon_0 A}{d}$. The inductance of a long solenoid is $L = \frac{\mu_0 A N^2}{l}$. The inductor has $N = (125\text{ coils/cm})(9.00\text{ cm}) = 1125$ coils. The resonance frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$. $\epsilon_0 = 8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2$. $\mu_0 = 4\pi\times10^{-7}\text{ T}\cdot\text{m/A}$.

EXECUTE: $C = \dfrac{\epsilon_0 A}{d} = \dfrac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.50\times10^{-2}\ \text{m})^2}{8.00\times10^{-3}\ \text{m}} = 2.24\times10^{-12}\ \text{F}.$

$$L = \frac{\mu_0 AN^2}{l} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})\pi(0.250\times10^{-2}\ \text{m})^2(1125)^2}{9.00\times10^{-2}\ \text{m}} = 3.47\times10^{-4}\ \text{H}.$$

$$\omega_0 = \frac{1}{\sqrt{(3.47\times10^{-4}\ \text{H})(2.24\times10^{-12}\ \text{F})}} = 3.59\times10^7\ \text{rad/s}.$$

EVALUATE: The result is a rather high angular frequency.

31.42. **IDENTIFY:** Use geometry to calculate the self-inductance of the toroidal solenoid. Then find its reactance and use this to find the impedance, and finally the current amplitude, of the circuit.

SET UP: $L = \dfrac{\mu_0 N^2 A}{2\pi r}$, $X_L = 2\pi f L$, $Z = \sqrt{R^2 + X_L^2}$, and $I = V/Z$.

EXECUTE: $L = \dfrac{\mu_0 N^2 A}{2\pi r} = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\dfrac{(2900)^2(0.450\times10^{-4}\ \text{m}^2)}{9.00\times10^{-2}\ \text{m}} = 8.41\times10^{-4}\ \text{H}.$

$X_L = 2\pi f L = (2\pi)(495\ \text{Hz})(8.41\times10^{-4}\ \text{H}) = 2.616\ \Omega.$ $Z = \sqrt{R^2 + X_L^2} = 3.832\ \Omega.$

$$I = \frac{V}{Z} = \frac{24.0\ \text{V}}{3.832\ \Omega} = 6.26\ \text{A}.$$

EVALUATE: The inductance is physically reasonable.

31.43. **IDENTIFY** and **SET UP:** Source voltage lags current so it must be that $X_C > X_L$.

EXECUTE: **(a)** We must add an inductor in series with the circuit. When $X_C = X_L$ the power factor has its maximum value of unity, so calculate the additional L needed to raise X_L to equal X_C.

(b) Power factor $\cos\phi$ equals 1 so $\phi = 0$ and $X_C = X_L$. Calculate the present value of $X_C - X_L$ to see how much more X_L is needed: $R = Z\cos\phi = (60.0\ \Omega)(0.720) = 43.2\ \Omega$

$\tan\phi = \dfrac{X_L - X_C}{R}$ so $X_L - X_C = R\tan\phi$.

$\cos\phi = 0.720$ gives $\phi = -43.95°$ (ϕ is negative since the voltage lags the current).

Then $X_L - X_C = R\tan\phi = (43.2\ \Omega)\tan(-43.95°) = -41.64\ \Omega$.

Therefore need to add $41.64\ \Omega$ of X_L.

$X_L = \omega L = 2\pi f L$ and $L = \dfrac{X_L}{2\pi f} = \dfrac{41.64\ \Omega}{2\pi(50.0\ \text{Hz})} = 0.133\ \text{H}$, amount of inductance to add.

EVALUATE: From the information given we can't calculate the original value of L in the circuit, just how much to add. When this L is added the current in the circuit will increase.

31.44. **IDENTIFY:** We are dealing with a transformer as an ac adapter.

SET UP and **EXECUTE:** **(a)** Voltage = 19.5 V, current = 6.7 A.

(b) Power = 130 W. IV = (19.5 V)(6.7 A) = 131 W, so $P = IV$.

(c) Primary: 120 V rms, 200 turns. The full-wave rectifier following the secondary coil maintains a voltage amplitude V = 19.5 V. We want the number of turns in the secondary. $V_1 = V_{\text{rms}}\sqrt{2}$. The secondary output should be V_2 = 19.5 V. $N_2 = \dfrac{V_2}{V_1}N_1 = \dfrac{19.5\ \text{V}}{120\sqrt{2}\ \text{V}}(200) = 23$ turns.

(d) We want I_1. $P = IV$ gives 130 W = I_1(120 V), so I_1 = 1.1 A.

(e) Estimate the size: Outside: 5 cm by 5 cm by 1.5 cm. Inside: 3 cm by 3 cm by 1 cm. One coil: 4 cm. 200 coils: 800 cm = 8.0 m.

(f) We want B inside the core. Apply Ampere's law. Use permeability μ instead of μ_0, where $\mu = K_m\mu_0$, with K_m– being the relative permeability. $\oint \vec{B} \cdot d\hat{l} = K_m\mu_0 I_{\text{encl}}$. Use a rectangular path with one side of length l = 3 cm inside the core enclosing all the loops. $Bl = K\mu_0 N_1 I_{\text{encl}}$.

$B = K\mu_0 N_1 I_{\text{encl}}/l$ = (5000) μ_0 (200)(1.1 A)/(0.030 m) = 46 T.

EVALUATE: The very large field in the core is due to the large permeability of the metal. If it were air-filled, B would be only about 9 mT.

31.45. **IDENTIFY:** We know the impedances and the average power consumed. From these we want to find the power factor and the rms voltage of the source.

SET UP: $P = I_{\text{rms}}^2 R$. $\cos\phi = \dfrac{R}{Z}$. $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $V_{\text{rms}} = I_{\text{rms}} Z$.

EXECUTE: **(a)** $I_{\text{rms}} = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{60.0\ \text{W}}{300\ \Omega}} = 0.447\ \text{A}$. $Z = \sqrt{(300\ \Omega)^2 + (500\ \Omega - 300\ \Omega)^2} = 361\ \Omega$.

$\cos\phi = \dfrac{R}{Z} = \dfrac{300\ \Omega}{361\ \Omega} = 0.831$.

(b) $V_{\text{rms}} = I_{\text{rms}} Z = (0.447\ \text{A})(361\ \Omega) = 161\ \text{V}$.

EVALUATE: The voltage amplitude of the source is $V_{\text{rms}}\sqrt{2} = 228\ \text{V}$.

31.46. **IDENTIFY** and **SET UP:** $X_C = \dfrac{1}{\omega C}$. $X_L = \omega L$.

EXECUTE: **(a)** $\dfrac{1}{\omega_1 C} = \omega_1 L$ and $LC = \dfrac{1}{\omega_1^2}$. At angular frequency ω_2,

$\dfrac{X_L}{X_C} = \dfrac{\omega_2 L}{1/\omega_2 C} = \omega_2^2 LC = (2\omega_1)^2 \dfrac{1}{\omega_1^2} = 4$. $X_L > X_C$.

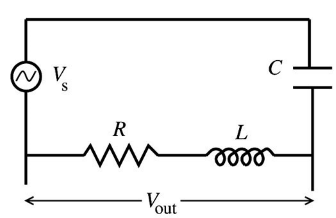
(b) At angular frequency ω_3, $\dfrac{X_L}{X_C} = \omega_3^2 LC = \left(\dfrac{\omega_1}{3}\right)^2 \left(\dfrac{1}{\omega_1^2}\right) = \dfrac{1}{9}$. $X_C > X_L$.

(c) The resonance angular frequency ω_0 is the value of ω for which $X_C = X_L$, so $\omega_0 = \omega_1$.

EVALUATE: When ω increases, X_L increases and X_C decreases. When ω decreases, X_L decreases and X_C increases.

31.47. **IDENTIFY** and **SET UP:** Express Z and I in terms of ω, L, C, and R. The voltages across the resistor and the inductor are 90° out of phase, so $V_{\text{out}} = \sqrt{V_R^2 + V_L^2}$.

EXECUTE: The circuit is sketched in Figure 31.47.



$$X_L = \omega L,\ X_C = \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$I = \frac{V_s}{Z} = \frac{V_s}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Figure 31.47

$$V_{\text{out}} = I\sqrt{R^2 + X_L^2} = I\sqrt{R^2 + \omega^2 L^2} = V_{\text{s}}\sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

$$\frac{V_{\text{out}}}{V_{\text{s}}} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

$\underline{\omega \text{ small}}$:

As ω gets small, $R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2 \to \dfrac{1}{\omega^2 C^2}$, $R^2 + \omega^2 L^2 \to R^2$.

Therefore, $\dfrac{V_{\text{out}}}{V_{\text{s}}} \to \sqrt{\dfrac{R^2}{(1/\omega^2 C^2)}} = \omega RC$ as ω becomes small.

$\underline{\omega \text{ large}}$:

As ω gets large, $R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2 \to R^2 + \omega^2 L^2 \to \omega^2 L^2$, $R^2 + \omega^2 L^2 \to \omega^2 L^2$.

Therefore, $\dfrac{V_{\text{out}}}{V_{\text{s}}} \to \sqrt{\dfrac{\omega^2 L^2}{\omega^2 L^2}} = 1$ as ω becomes large.

EVALUATE: $V_{\text{out}}/V_{\text{s}} \to 0$ as ω becomes small, so there is V_{out} only when the frequency ω of V_{s} is large. If the source voltage contains a number of frequency components, only the high frequency ones are passed by this filter.

31.48. IDENTIFY: $V = V_C = IX_C$. $I = V/Z$.

SET UP: $X_L = \omega L$, $X_C = \dfrac{1}{\omega C}$.

EXECUTE: $V_{\text{out}} = V_C = \dfrac{I}{\omega C} \Rightarrow \dfrac{V_{\text{out}}}{V_{\text{s}}} = \dfrac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

If ω is large: $\dfrac{V_{\text{out}}}{V_{\text{s}}} = \dfrac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \approx \dfrac{1}{\omega C\sqrt{(\omega L)^2}} = \dfrac{1}{(LC)\omega^2}$.

If ω is small: $\dfrac{V_{\text{out}}}{V_{\text{s}}} \approx \dfrac{1}{\omega C\sqrt{(1/\omega C)^2}} = \dfrac{\omega C}{\omega C} = 1$.

EVALUATE: When ω is large, X_C is small and X_L is large so Z is large and the current is small. Both factors in $V_C = IX_C$ are small. When ω is small, X_C is large and the voltage amplitude across the capacitor is much larger than the voltage amplitudes across the resistor and the inductor.

31.49. IDENTIFY: $I = V/Z$ and $P_{\text{av}} = \frac{1}{2}I^2 R$.

SET UP: $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

EXECUTE: **(a)** $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

(b) $P_{av} = \frac{1}{2}I^2R = \frac{1}{2}\left(\frac{V}{Z}\right)^2 R = \frac{V^2R/2}{R^2 + (\omega L - 1/\omega C)^2}$.

(c) The average power and the current amplitude are both greatest when the denominator is smallest, which occurs for $\omega_0 L = \frac{1}{\omega_0 C}$, so $\omega_0 = \frac{1}{\sqrt{LC}}$.

(d) The average power is

$$P_{av} = \frac{(100\text{ V})^2(200\ \Omega)/2}{(200\ \Omega)^2 + \left[\omega(2.00\text{ H}) - 1/[\omega(0.500\times 10^{-6}\text{ F})]\right]^2} = \frac{1{,}000{,}000\,\omega^2}{40{,}000\omega^2 + (2\omega^2 - 2{,}000{,}000\text{ s}^{-2})^2}\text{ W}.$$

The graph of P_{av} versus ω is sketched in Figure 31.49.

EVALUATE: Note that as the angular frequency goes to zero, the power and current are zero, just as they are when the angular frequency goes to infinity. This graph exhibits the same strongly peaked nature as the light purple curve in Figure 31.19 in the textbook.

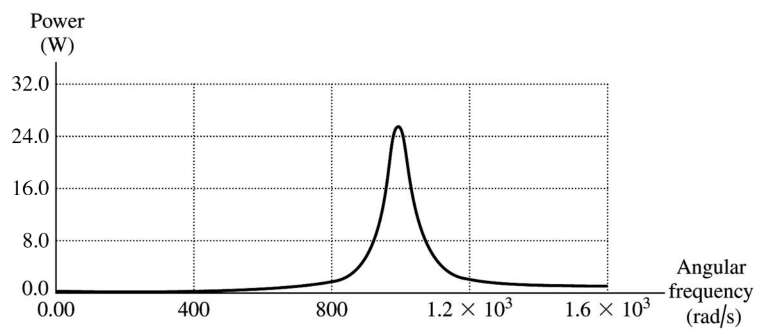


Figure 31.49

31.50. IDENTIFY: $V_L = I\omega L$ and $V_C = \frac{I}{\omega C}$.

SET UP: Problem 31.49 shows that $I = \frac{V}{\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$.

EXECUTE: **(a)** $V_L = I\omega L = \frac{V\omega L}{\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$.

(b) $V_C = \frac{I}{\omega C} = \frac{V}{\omega C\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$.

(c) The graphs are given in Figure 31.50.

EVALUATE: **(d)** When the angular frequency is zero, the inductor has zero voltage while the capacitor has voltage of 100 V (equal to the total source voltage). At very high frequencies, the capacitor voltage goes to zero, while the inductor's voltage goes to 100 V. At resonance, $\omega_0 = \frac{1}{\sqrt{LC}} = 1000$ rad/s, the two voltages are equal, and are a maximum, 1000 V.

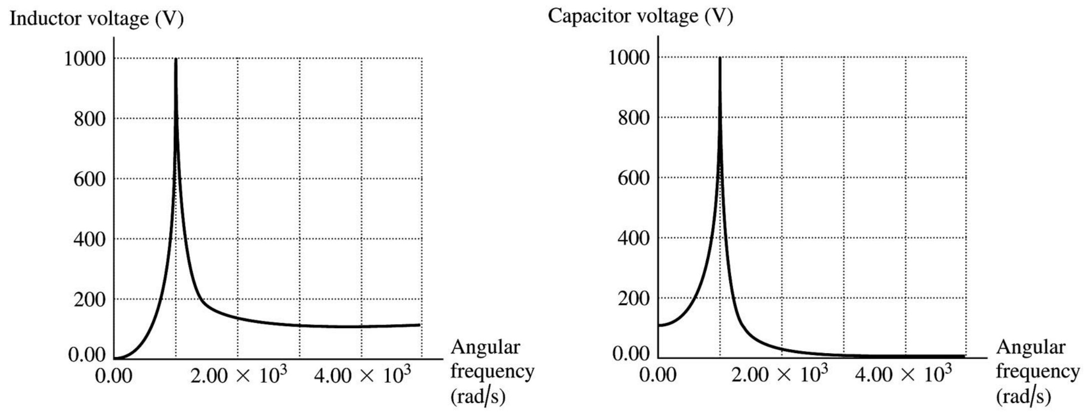


Figure 31.50

31.51. IDENTIFY: We know R, X_C, and ϕ so $\tan\phi = \dfrac{X_L - X_C}{R}$ tells us X_L. Use $P_{av} = I_{rms}^2 R$ to calculate I_{rms}. Then calculate Z and use $V_{rms} = I_{rms}Z$ to calculate V_{rms} for the source.

SET UP: Source voltage lags current so $\phi = -54.0°$. $X_C = 350\ \Omega$, $R = 180\ \Omega$, $P_{av} = 140$ W.

EXECUTE: (a) $\tan\phi = \dfrac{X_L - X_C}{R}$.

$X_L = R\tan\phi + X_C = (180\ \Omega)\tan(-54.0°) + 350\ \Omega = -248\ \Omega + 350\ \Omega = 102\ \Omega$.

(b) $P_{av} = V_{rms}I_{rms}\cos\phi = I_{rms}^2 R$ (Exercise 31.22). $I_{rms} = \sqrt{\dfrac{P_{av}}{R}} = \sqrt{\dfrac{140\text{ W}}{180\ \Omega}} = 0.882$ A.

(c) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(180\ \Omega)^2 + (102\ \Omega - 350\ \Omega)^2} = 306\ \Omega$.

$V_{rms} = I_{rms}Z = (0.882\text{ A})(306\ \Omega) = 270$ V.

EVALUATE: We could also use $P_{av} = V_{rms}I_{rms}\cos\phi$.

$V_{rms} = \dfrac{P_{av}}{I_{rms}\cos\phi} = \dfrac{140\text{ W}}{(0.882\text{ A})\cos(-54.0°)} = 270$ V, which agrees. The source voltage lags the current when $X_C > X_L$, and this agrees with what we found.

31.52. IDENTIFY: We have an *L-R-C* series ac circuit.

SET UP and EXECUTE: (a) To detect a 4.0n GHz signal, a relevant characteristic is the resonance angular frequency. $\omega_0 = 1/\sqrt{LC}$. Solve for LC. $LC = 1/\omega_0^2 = 1/(2\pi f_0)^2 = 1/[2\pi(4\text{ GHz})]^2$ $= 1.6\times10^{-21}\text{ s}^2$.

(b) Use the result from (a). $L(1.0\times10^{-15}\text{ F}) = 1.6\times10^{-21}\text{ s}^2$. $L = 1.6\ \mu$H.

(c) Estimate: Thickness of phone = 2 mm; area = 4 mm^2.

(d) Use $L = \dfrac{\mu_0 N^2 A}{2\pi r}$, solve for N, and use the given numbers. $N = \sqrt{\dfrac{2\pi rL}{\mu_0 A}}$ = 160 turns.

EVALUATE: I would be difficult for a do-it-yourselfer to actually make such a small inductor.

31.53. IDENTIFY: We have a variable capacitor in an *L-R-C* series ac circuit.

SET UP and EXECUTE: (a) We want C. Use $C = \dfrac{\epsilon_0 A}{d}$. $d = g/2$ and A is the area in common. The maximum area in common is $A_{max} = \pi a^2/2$ when $\theta = \pi$. So $A = A_{max}(\theta/\pi) = (\pi a^2/2)(\theta/\pi) = a^2\theta/2$. The capacitance at any θ is $C = \dfrac{\epsilon_0 \left(a^2\theta/2\right)}{g/2} = \epsilon_0 a^2\theta/g$. There are 5 sets of capacitors, all in parallel, so the total capacitance is $C = 5\epsilon_0 a^2\theta/g$.

(b) We want θ to receive a 1180 kHz signal. This frequency should be the resonance frequency f_0 of the circuit. Solve $1/\sqrt{LC} = 2\pi f_0$ for C and use f_0 = 1180 kHz and L = 100 μH. This gives $C = 1/L(2\pi f_0)^2 =$ 182 pF. Now find the angle θ to get that capacitance. Solve $C = 5\epsilon_0 a^2\theta/g$ for θ, giving $\theta = \dfrac{gC}{5\epsilon_0 a^2}$ = 2.29 rad = 131°.

(c) We want the amplitude of the output voltage. The output voltage is the amplitude V_C, so $V_{\text{out}} = V_C = IX_C$. At resonance $I = V/R$, so $V_{\text{out}} = IX_C = \left(\dfrac{V}{R}\right)X_C = \left(\dfrac{V}{R}\right)\left(\dfrac{1}{\omega C}\right) = \dfrac{V}{R\omega C} = 7.41$ V.

(d) We want f when θ = 120°. $\omega_0 = 1/\sqrt{LC}$ where $C = 5\epsilon_0 a^2\theta/g$ and θ = 120° = $2\pi/3$ rad. First find C with these quantities, which gives $C = 1.668\times10^{-10}$ F. Now use $1/\sqrt{LC} = 2\pi f_0$ to find f_0. With the numbers this gives $f_0 = \dfrac{1}{2\pi\sqrt{LC}} =$ 1230 kHz.

(e) We want the amplitude of the output voltage at this frequency. $V_{\text{out}} = \dfrac{V}{R\omega C} = 7.74$ V.

Evaluate: A device like this allows continuous tuning to signals within its range. Simply turn a dial to rotate the capacitor plates. This design was actually used for tuning equipment.

31.54. Identify: At any instant of time the same rules apply to the parallel ac circuit as to the parallel dc circuit: the voltages are the same and the currents add.

Set Up: For a resistor the current and voltage in phase. For an inductor the voltage leads the current by 90° and for a capacitor the voltage lags the current by 90°.

Execute: **(a)** The parallel L-R-C circuit must have equal potential drops over the capacitor, inductor and resistor, so $v_R = v_L = v_C = v$. Also, the sum of currents entering any junction must equal the current leaving the junction. Therefore, the sum of the currents in the branches must equal the current through the source: $i = i_R + i_L + i_C$.

(b) $i_R = \dfrac{v}{R}$ is always in phase with the voltage. $i_L = \dfrac{v}{\omega L}$ lags the voltage by 90°, and $i_C = v\omega C$ leads the voltage by 90°. The phasor diagram is sketched in Figure 31.54.

(c) From the diagram, $I^2 = I_R^2 + (I_C - I_L)^2 = \left(\dfrac{V}{R}\right)^2 + \left(V\omega C - \dfrac{V}{\omega L}\right)^2$.

(d) From part (c): $I = V\sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}$. But $I = \dfrac{V}{Z}$, so $\dfrac{1}{Z} = \sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}$.

Evaluate: For large ω, $Z \to \dfrac{1}{\omega C}$. The current in the capacitor branch is much larger than the current in the other branches. For small ω, $Z \to \omega L$. The current in the inductive branch is much larger than the current in the other branches.

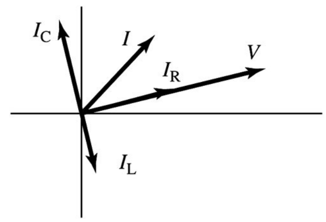


Figure 31.54

31.55. **IDENTIFY:** Apply the expression for $1/Z$ from Problem 31.54.

SET UP: From Problem 31.54, $\frac{1}{Z}=\sqrt{\frac{1}{R^2}+\left(\omega C-\frac{1}{\omega L}\right)^2}$.

EXECUTE: **(a)** Using $\frac{1}{Z}=\sqrt{\frac{1}{R^2}+\left(\omega C-\frac{1}{\omega L}\right)^2}$, we see that the impedance Z is a maximum when the square root is a minimum, and that occurs when $\omega C-\frac{1}{\omega L}=0$. But that occurs when $\omega=\frac{1}{\sqrt{LC}}$, which is the resonance angular frequency $\omega_0=\frac{1}{\sqrt{LC}}$. Since $I = V/Z$, the current is then a minimum when Z is a maximum.

(b) Using the result from part (a) gives $\omega=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{(0.300\text{ H})(0.100\times10^{-6}\text{ F})}}=5770\text{ rad/s}$.

(c) At resonance, $Z = R = 100\ \Omega$, so $I = V/R = (240\text{ V})/(100\ \Omega) = 2.40\text{ A}$.

(d) At resonance, the amplitude of the current in the resistor is $I = V/R = (240\text{ V})/(100\ \Omega) = 2.40\text{ A}$.

(e) At resonance, $X_L = \omega L = (5770\text{ rad/s})(0.300\text{ H}) = 1730\ \Omega$, which is also X_C. The amplitude of the maximum current through the inductor is $I = V/X_L = (240\text{ V})/(1730\ \Omega) = 0.139\text{ A}$.

(f) Since we are at resonance, $X_L = X_C = 1730$ A. Therefore $I = V/X_C = (240\text{ V})/(1730\ \Omega) = 0.139\text{ A}$.

EVALUATE: The parallel circuit is sketched in Figure 31.55. At resonance, $|i_C|=|i_L|$ and at any instant of time these two currents are in opposite directions. Therefore, the net current between a and b is always zero. If the inductor and capacitor each have some resistance, and these resistances aren't the same, then it is no longer true that $i_C+i_L=0$. The result in part (a) for a parallel L-R-C circuit at resonance that the impedance is a maximum and the current is a minimum is the opposite of the behavior of a series L-R-C circuit at resonance.

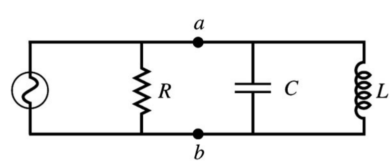


Figure 31.55

31.56. **IDENTIFY:** Refer to the results and the phasor diagram in Problem 31.54. The source voltage is applied across each parallel branch.

SET UP: $V=\sqrt{2}V_{\text{rms}}=254.6\text{ V}$.

EXECUTE: **(a)** $I_R=\frac{V}{R}=\frac{254.6\text{ V}}{400\ \Omega}=0.636\text{ A}$.

(b) $I_C = V\omega C = (254.6\ \text{V})(360\ \text{rad/s})(6.00\times10^{-6}\ \text{F}) = 0.550\ \text{A}.$

(c) $\phi = \arctan\left(\dfrac{I_C}{I_R}\right) = \arctan\left(\dfrac{0.550\ \text{A}}{0.636\ \text{A}}\right) = 40.8°.$

(d) $I = \sqrt{I_R^2 + I_C^2} = \sqrt{(0.636\ \text{A})^2 + (0.550\ \text{A})^2} = 0.841\ \text{A}.$

(e) Leads since $\phi > 0$.

EVALUATE: The phasor diagram shows that the current in the capacitor always leads the source voltage.

31.57. IDENTIFY: The average power depends on the phase angle ϕ.

SET UP: The average power is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$, and the impedance is $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$.

EXECUTE: **(a)** $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi = \frac{1}{2}(V_{\text{rms}} I_{\text{rms}})$, which gives $\cos\phi = \frac{1}{2}$, so $\phi = \pi/3 = 60°$. $\tan\phi = (X_L - X_C)/R$, which gives $\tan 60° = (\omega L - 1/\omega C)/R$. Using $R = 75.0\ \Omega$, $L = 5.00$ mH and $C = 2.50\ \mu\text{F}$ and solving for ω we get $\omega = 28760$ rad/s $= 28{,}800$ rad/s.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L = (28{,}760\ \text{rad/s})(5.00\ \text{mH}) = 144\ \Omega$ and $X_C = 1/\omega C = 1/[(28{,}760\ \text{rad/s})(2.50\ \mu\text{F})] = 13.9\ \Omega$, giving $Z = \sqrt{(75\ \Omega)^2 + (144\ \Omega - 13.9\ \Omega)^2} = 150\ \Omega$; $I = V/Z = (15.0\ \text{V})/(150\ \Omega) = 0.100\ \text{A}$ and $P_{\text{av}} = \frac{1}{2} VI\cos\phi = \frac{1}{2}(15.0\ \text{V})(0.100\ \text{A})(1/2) = 0.375\ \text{W}.$

EVALUATE: All this power is dissipated in the resistor because the average power delivered to the inductor and capacitor is zero.

31.58. IDENTIFY and **SET UP:** The maximum energy in the inductor depends on the current amplitude in the inductor. $U_L = \frac{1}{2}LI^2$ and $U_C = \frac{1}{2}CV^2$. The impedance of a series *L-R-C* circuit is

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2},\quad X_C = \frac{1}{\omega C}.$$

EXECUTE: **(a)** Use $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$ to find the impedance of the circuit.

$$Z = \sqrt{(60.0\ \Omega)^2 + \left[(120\ \text{rad/s})(0.800\ \text{H}) - \frac{1}{(120\ \text{rad/s})(3.00\times10^{-4}\ \text{F})}\right]^2} = 90.85\,\Omega.$$

The amplitude of the current is therefore $I = V/Z = (90.0\ \text{V})/(90.85\ \Omega) = 0.9906$ A, so the maximum energy stored in the inductor is $U_L = \frac{1}{2}LI^2 = (1/2)(0.800\ \text{H})(0.9906\ \text{A})^2 = 0.393$ J.

(b) The energy stored in the capacitor is $U_C = \frac{1}{2}CV^2$, but the capacitor voltage is 90° out of phase with the current. Thus when the current is a maximum, the voltage across the capacitor is zero, so the energy stored in the capacitor is also zero.

(c) The capacitor stores its maximum energy when it is at maximum voltage, which is

$V_C = IX_C = I\dfrac{1}{\omega C} = (0.9906\ \text{A})\left[\dfrac{1}{(120\ \text{rad/s})(3.00\times10^{-4}\ \text{F})}\right] = 27.52$ V. The maximum energy in the capacitor at this time is $U_C = \frac{1}{2}CV^2 = (1/2)(3.00\times10^{-4}\ \text{F})(27.52\ \text{V})^2 = 0.114$ J.

EVALUATE: The maximum energy stored in the inductor is not the same as in the capacitor due to the presence of resistance.

31.59. IDENTIFY and SET UP: The equation $V_C = IX_C$ allows us to calculate I and then $V = IZ$ gives Z. Solve $Z=\sqrt{R^2+(X_L-X_C)^2}$ for X_L.

EXECUTE: (a) $V_C = IX_C$ so $I=\dfrac{V_C}{X_C}=\dfrac{360\text{ V}}{480\ \Omega}=0.750\text{ A}.$

(b) $V = IZ$ so $Z=\dfrac{V}{I}=\dfrac{120\text{ V}}{0.750\text{ A}}=160\ \Omega.$

(c) $Z^2=R^2+(X_L-X_C)^2.$

$X_L-X_C=\pm\sqrt{Z^2-R^2}$, so

$X_L=X_C\pm\sqrt{Z^2-R^2}=480\ \Omega\pm\sqrt{(160\ \Omega)^2-(80.0\ \Omega)^2}=480\ \Omega\pm139\ \Omega.$

$X_L=619\ \Omega$ or $341\ \Omega.$

EVALUATE: (d) $X_C=\dfrac{1}{\omega C}$ and $X_L=\omega L.$ At resonance, $X_C=X_L.$ As the frequency is lowered below the resonance frequency X_C increases and X_L decreases. Therefore, for $\omega<\omega_0,\ X_L<X_C.$ So for $X_L=341\ \Omega$ the angular frequency is less than the resonance angular frequency. ω is greater than ω_0 when $X_L=619\ \Omega.$ But at these two values of $X_L,$ the magnitude of X_L-X_C is the same so Z and I are the same. In one case $(X_L=691\ \Omega)$ the source voltage leads the current and in the other $(X_L=341\ \Omega)$ the source voltage lags the current.

31.60. IDENTIFY and SET UP: The capacitive reactance is $X_C=\dfrac{1}{\omega C},$ the inductive reactance is $X_L=\omega L,$ and the impedance of an L-R-C series circuit is $Z=\sqrt{R^2+(X_L-X_C)^2}.$

EXECUTE: (a) The current amplitude is $I = V/R = (135\text{ V})/(90.0\ \Omega) = 1.50\text{ A}$.
(b) The voltage amplitude across the inductor is $V_L = IX_L = (1.50\ \Omega)(320\ \Omega) = 480\text{ V}$.
(c) The impedance is $Z = V/I = (240\text{ V})/(1.50\text{ A}) = 160\ \Omega$. We also know that the impedance is $Z=\sqrt{R^2+(X_L-X_C)^2}.$ We know that $X_{-L} = 320\ \Omega$, so we can find X_C.

$160\ \Omega=\sqrt{(90.0\ \Omega)^2+(320\ \Omega-X_C)^2}.$ Squaring and solving for X_C gives two values, $X_C = 188\ \Omega$ and $X_C = 452\ \Omega$.

(d) At resonance, $\omega L=\dfrac{1}{\omega C}.$ $X_C < X_L$ for $\omega>\omega_{\text{res}}$ and $X_C > X_L$ for $\omega<\omega_{\text{res}}$. In this circuit, $X_L = 320\ \Omega$, so $\omega<\omega_{\text{res}}$ for $X_C = 452\ \Omega$.

EVALUATE: Due to the square of $(X_L - X_C)$ in the impedance, we get two possibilities in (c).

31.61. IDENTIFY: At resonance, $Z=R.$ $I=V/R.$ $V_R=IR,$ $V_C=IX_C$ and $V_L=IX_L.$ $U_C=\tfrac{1}{2}CV_C^2$ and $U_L=\tfrac{1}{2}LI^2.$

SET UP: The amplitudes of each time-dependent quantity correspond to the maximum values of those quantities.

EXECUTE: (a) $I=\dfrac{V}{Z}=\dfrac{V}{\sqrt{R^2+(\omega L-1/\omega C)^2}}.$ At resonance $\omega L=\dfrac{1}{\omega C}$ and $I_{\max}=\dfrac{V}{R}.$

(b) $V_C=IX_C=\dfrac{V}{R\omega_0 C}=\dfrac{V}{R}\sqrt{\dfrac{L}{C}}.$

(c) $V_L = IX_L = \dfrac{V}{R}\omega_0 L = \dfrac{V}{R}\sqrt{\dfrac{L}{C}}.$

(d) $U_C = \dfrac{1}{2}CV_C^2 = \dfrac{1}{2}C\dfrac{V^2}{R^2}\dfrac{L}{C} = \dfrac{1}{2}L\dfrac{V^2}{R^2}.$

(e) $U_L = \dfrac{1}{2}LI^2 = \dfrac{1}{2}L\dfrac{V^2}{R^2}.$

EVALUATE: At resonance $V_C = V_L$ and the maximum energy stored in the inductor equals the maximum energy stored in the capacitor.

31.62. **IDENTIFY** and **SET UP:** We use $I = V/Z$, $X_L = \omega L$, and $Z = \sqrt{R^2 + X_L^2}$.

EXECUTE: $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + X_L^2}}$. Squaring and rearranging gives $\dfrac{1}{I^2} = \left(\dfrac{L}{V}\right)^2 \omega^2 + \left(\dfrac{R}{V}\right)^2$. Therefore a graph of $1/I^2$ versus ω^2 should be a straight line having slope equal to $(L/V)^2$ and y-intercept equal to $(R/V)^2$. We can find the slope using two convenient points on the graph, giving

slope = $\dfrac{(21.0 - 9.0)\ \text{A}^{-2}}{(3500 - 500)\ \text{rad}^2/\text{s}^2} = 4.00\times 10^{-3}\ \text{s}^2/\text{rad}^2 \cdot \text{A}^2.$

Solving for L gives $L = V\sqrt{\text{slope}} = (12.0\ \text{V})\sqrt{4.00\times 10^{-3}\ \text{s}^2/\text{rad}^2 \cdot \text{A}^2} = 0.759\ \text{H}.$

Extending the line, we find the y-intercept is 7.0 A^{-2}. Using this value to solve for R gives

$R = V\sqrt{y\text{-intercept}} = (12.0\ \text{V})\sqrt{7.00\,\text{A}^{-2}} = 32\ \Omega.$

EVALUATE: These are reasonable values for L and R for a large solenoid, so we're confident in the results.

31.63. **IDENTIFY** and **SET UP:** For an L-R-C series circuit, the maximum current occurs at resonance, and the resonance angular frequency is $\omega_{\text{res}} = \dfrac{1}{\sqrt{LC}}.$

EXECUTE: At resonance, the angular frequency is $\omega_{\text{res}} = \dfrac{1}{\sqrt{LC}}$. Squaring gives $\omega_{\text{res}}^2 = \dfrac{1}{L}\cdot\dfrac{1}{C}$, so a graph of ω_{res}^2 versus $1/C$ should be a straight line with a slope equal to $1/L$. Using two convenient points on the graph, we find the slope to be $\dfrac{(25.0 - 1.00)\times 10^4\ \text{rad}^2/\text{s}^2}{(4.50 - 1.75)\ \times 10^3\ \text{F}^{-1}} = 5.455\ \text{F/s}^2$. Solving for L gives $L = (\text{slope})^{-1} = (5.455\ \text{F/s}^2)^{-1} = 0.183$ H, which rounds to 0.18 H, since we cannot determine the slope of the graph in the text with anything better than 2 significant figures. To find R, we realize that at resonance $Z = R$, so $R = V/I = (90.0\ \text{V})/(4.50\ \text{A}) = 20.0\ \Omega$.

EVALUATE: These are reasonable values for L and R for a laboratory solenoid.

31.64. **IDENTIFY** and **SET UP:** For an L-R-C series circuit, $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$ and the power factor is $\cos\phi = R/Z$.

EXECUTE: **(a)** $\cos\phi = R/Z$, so $R = Z\cos\phi$.

At 80 Hz: $R = (15\ \Omega)\cos(-71°) = 4.88\ \Omega$.

At 160 Hz: $R = (13\ \Omega)\cos(67°) = 5.08\ \Omega$.

The average resistance is $(4.88\ \Omega + 5.08\ \Omega)/2 = 5.0\ \Omega$.

(b) We use $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$ with $R = 5.0\ \Omega$ from part (a).

At 80 Hz: $\tan(-71°) = \dfrac{2\pi(80\text{ Hz})L - \dfrac{1}{2\pi(80\text{ Hz})C}}{5.0\ \Omega}$.

$-14.52 = 160\pi$ Hz $L - 1/[(160\pi\text{ Hz})C]$. Eq. (1)

At 160 Hz: $\tan(67°) = \dfrac{2\pi(160\text{ Hz})L - \dfrac{1}{2\pi(160\text{ Hz})C}}{5.0\ \Omega}$.

$11.78 = 320\pi$ Hz $L - 1/[(320\pi\text{ Hz})C]$. Eq. (2)

Multiply Eq. (1) by –2 and add it to Eq. (2), giving

$2(14.52) + 11.78 = (1/C)(1/80\pi - 1/320\pi)$.

$C = 7.31\times10^{-5}$ F, which rounds to $C = 73\ \mu$F.

Substituting this result into either Eq. (1) or Eq. (2) gives $L = 25.3$ mH, which rounds to $L = 25$ mH.

(c) The resonance angular frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$, so the resonance frequency is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(2.53\times10^{-2}\text{ H})(73.1\times10^{-6}\text{ F})}} = 117\text{ Hz}.$$

At resonance, $Z = R = 5.0\ \Omega$ and $\phi = 0$.

EVALUATE: It is only at resonance that $Z = R$, not at the other frequencies.

31.65. IDENTIFY: We have an *L-R* ac circuit.

SET UP: $v_{in} = V_{in}\cos\omega t$, $G = (20\text{ dB})\ln(V_{out}/V_{in})$, $I = V_{in}/Z$, $Z = \sqrt{R^2 + (\omega L)^2}$, $\phi = \arctan\left(\dfrac{\omega L}{R}\right)$.

EXECUTE: **(a)** We want the current. $I = \dfrac{V_{in}}{Z} = \dfrac{V_{in}}{\sqrt{R^2 + (\omega L)^2}}$.

(b) We want ϕ. $\phi = \arctan\left(\dfrac{\omega L}{R}\right)$.

(c) We want V_{out}/V_{in}. $\dfrac{V_{out}}{V_{in}} = \dfrac{V_L}{V_{in}} = \dfrac{IX_L}{V_{in}} = \dfrac{\dfrac{V_{in}}{\sqrt{R^2+(\omega L)^2}}\cdot \omega L}{V_{in}} = \dfrac{\omega L}{\sqrt{R^2 + (\omega L)^2}} = \dfrac{1}{\sqrt{1 + (R/\omega L)^2}}$.

(d) We want *f* so $G = -3.0$ dB. Use $G = (20\text{ dB})\ln(V_{out}/V_{in})$. $-3.0\text{ dB} = (20\text{ dB})\ln(V_{out}/V_{in})$.

$-3.0/20 = \ln(V_{out}/V_{in})$. Write this result in terms of exponents and use the result from (c).

$10^{-3.0/20} = 0.708 = \dfrac{V_{out}}{V_{in}} = \dfrac{1}{\sqrt{1+(R/\omega L)^2}}$. Use $\omega = 2\pi f$ and solve for *f*, giving $f = \dfrac{R}{2.00\pi L}$.

(e) We want *L*. Solve the result in (d) when $f = 10.0$ kHz and $R = 100\ \Omega$, giving $L = 1.6$ mH.

EVALUATE: By varying the ration *R/L* we can tune to any desired frequency.

31.66. IDENTIFY: We have an *L-R-C* series ac circuit.

SET UP: The output is across the capacitor. $v_{in} = V_{in}\cos\omega t$, $v_{out} = V_{out}\cos(\omega t + \theta)$,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2},\quad \phi = \arctan\left(\frac{\omega L - 1/\omega C}{R}\right),\quad \omega_0 = 1/\sqrt{LC}.$$

EXECUTE: **(a)** We want V_{out}. $V_{\text{out}} = V_C = IX_C = \dfrac{V_{\text{in}}}{Z} X_C = \dfrac{V_{\text{in}}}{\omega C\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$, which we can express as $V_{\text{out}} = \dfrac{V_{\text{in}}}{\sqrt{(R\omega C)^2 + \left(\omega^2 LC - 1\right)^2}}$.

(b) We want the phase angle for v_{out}. The output voltage is across the capacitor, so the phase angle θ is the same as for the capacitor. The capacitor voltage lags the current by $\pi/2$, so the phase angle is $\theta = \phi - \pi/2$. $\phi = \arctan\left(\dfrac{\omega L - 1/\omega C}{R}\right) = \arctan\left(\dfrac{\omega^2 LC - 1}{\omega RC}\right)$. So $\theta = \arctan\left(\dfrac{\omega^2 LC - 1}{\omega RC}\right) - \dfrac{\pi}{2}$.

(c) We want θ at resonance. At resonance $\omega_0 = 1/\sqrt{LC}$, so $\phi = 0$. Thus $\theta = -\pi/2$.

(d) We want C. Solving $\omega_0 = 1/\sqrt{LC}$ for C and using the given values, we get

$C = \dfrac{1}{\omega_0^2 L} = \dfrac{1}{(2\pi f_0)^2 L} = 2.53\ \mu\text{F}$.

(e) We want V_{out}. $V_{\text{out}} = V_C$. Use the result from part (a) at resonance. $V_{\text{out}} = \dfrac{V_{\text{in}}}{R\omega C} = \dfrac{V_{\text{in}}}{2\pi fRC}$. Using the numbers gives $V_{\text{out}} = 62.9$ mV.

EVALUATE: Simply by varying the frequency the output voltage amplitude changes.

31.67. **IDENTIFY:** $p_R = i^2 R$. $p_L = iL\dfrac{di}{dt}$. $p_C = \dfrac{q}{C}i$.

SET UP: $i = I\cos\omega t$.

EXECUTE: **(a)** $p_R = i^2 R = I^2\cos^2(\omega t)R = V_R I\cos^2(\omega t) = \dfrac{1}{2}V_R I(1+\cos(2\omega t))$.

$P_{\text{av}}(R) = \dfrac{1}{T}\int_0^T p_R dt = \dfrac{V_R I}{2T}\int_0^T [1+\cos(2\omega t)]dt = \dfrac{V_R I}{2T}[t]_0^T = \tfrac{1}{2}V_R I$.

(b) $p_L = Li\dfrac{di}{dt} = -\omega L I^2\cos(\omega t)\sin(\omega t) = -\tfrac{1}{2}V_L I\sin(2\omega t)$. But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{\text{av}}(L) = 0$.

(c) $p_C = \dfrac{q}{C}i = v_C i = V_C I\sin(\omega t)\cos(\omega t) = \tfrac{1}{2}V_C I\sin(2\omega t)$. But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{\text{av}}(C) = 0$.

(d) $p = p_R + p_L + p_c = V_R I\cos^2(\omega t) - \tfrac{1}{2}V_L I\sin(2\omega t) + \tfrac{1}{2}V_C I\sin(2\omega t)$ and

$p = I\cos\omega t(V_R\cos\omega t - V_L\sin\omega t + V_C\sin\omega t)$. But $\cos\phi = \dfrac{V_R}{V}$ and $\sin\phi = \dfrac{V_L - V_C}{V}$, so

$p = VI\cos\omega t(\cos\phi\cos\omega t - \sin\phi\sin\omega t)$, at any instant of time.

EVALUATE: At an instant of time the energy stored in the capacitor and inductor can be changing, but there is no net consumption of electrical energy in these components.

31.68. **IDENTIFY:** $V_L = IX_L$. $\dfrac{dV_L}{d\omega} = 0$ at the ω where V_L is a maximum. $V_C = IX_C$. $\dfrac{dV_C}{d\omega} = 0$ at the ω where V_C is a maximum.

SET UP: Problem 31.49 shows that $I = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

EXECUTE: **(a)** $V_R = \text{maximum}$ when $V_C = V_L \Rightarrow \omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$.

(b) $V_L = \text{maximum}$ when $\dfrac{dV_L}{d\omega} = 0.$ Therefore: $\dfrac{dV_L}{d\omega} = 0 = \dfrac{d}{d\omega}\left(\dfrac{V\omega L}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right).$

$$0 = \frac{VL}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V\omega^2 L(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{\left[R^2 + (\omega L - 1/\omega C)^2\right]^{3/2}}.\quad R^2 + (\omega L - 1/\omega C)^2 = \omega^2(L^2 - 1/\omega^4 C^2).$$

$$R^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} = -\frac{1}{\omega^2 C^2}.\quad \frac{1}{\omega^2} = LC - \frac{R^2 C^2}{2} \text{ and } \omega = \frac{1}{\sqrt{LC - R^2 C^2/2}}.$$

(c) $V_C = \text{maximum}$ when $\dfrac{dV_C}{d\omega} = 0.$ Therefore: $\dfrac{dV_C}{d\omega} = 0 = \dfrac{d}{d\omega}\left(\dfrac{V}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right).$

$$0 = -\frac{V}{\omega^2 C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{C(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}.\quad R^2 + (\omega L - 1/\omega C)^2 = -\omega^2(L^2 - 1/\omega^4 C^2).$$

$$R^2 + \omega^2 L^2 - \frac{2L}{C} = -\omega^2 L^2 \text{ and } \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}.$$

EVALUATE: V_L is maximum at a frequency greater than the resonance frequency and V_C is a maximum at a frequency less than the resonance frequency. These frequencies depend on R, as well as on L and on C.

31.69. **IDENTIFY** and **SET UP:** We are told that the platinum electrode behaves like an ideal capacitor in series with the resistance of the fluid. The impedance of an *R*-*C* circuit is $Z = \sqrt{R^2 + X_C^2}$, where $X_C = \dfrac{1}{\omega C}.$

EXECUTE: For a dc signal we have $\omega = 2\pi f = 0.$ Using $X_C = \dfrac{1}{\omega C}$ we see that as $\omega \to 0$ we have $X_C \to \infty$, and so $Z \to \infty$. The correct choice is (b).

EVALUATE: The oscillation period of such a circuit is $T = 1/f$, so $T \to \infty$ as $\omega \to 0.$

31.70. **IDENTIFY** and **SET UP:** We are told that the platinum electrode behaves like an ideal capacitor in series with the resistance of the fluid, which is given by $R_A = \rho/(10a)$, where $\rho = 100\ \Omega\cdot\text{cm} = 1\ \Omega\cdot\text{m}$ and $d = 2a = 20\ \mu\text{m}$. We know that $X_C = \dfrac{1}{\omega C}$, where we are given $C = 10\ \text{nF} = 10^{-8}\ \text{F}$ and $\omega = 2\pi f = 2\pi[(5000/\pi)\text{Hz}] = 10^4\ \text{rad/s}.$ For an *R*-*C* circuit we know that the impedance is given by s

EXECUTE: $R_A = \rho/(10a) = (1\ \Omega\cdot\text{m})/[10(10^{-5}\ \text{m})] = 10^4\ \Omega.$ The capacitive reactance is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(10^4\ \text{rad/s})(10^{-8}\ \text{F})} = 10^4\ \Omega.$ Thus the impedance is

$Z = \sqrt{R^2 + X_C^2} = \sqrt{(10^4\ \Omega)^2 + (10^4\ \Omega)^2} = \sqrt{2}\times(10^4\ \Omega)$, so the correct choice is (c).

EVALUATE: In this case, the capacitance contributes as much to the impedance as the resistance does.

31.71. **IDENTIFY** and **SET UP:** We know that $V_{\text{rms}} = \dfrac{V}{\sqrt{2}}$, where V is the amplitude (peak value) of the voltage. According to the problem, the peak-to-peak voltage V_{pp} is the difference between the two extreme values of voltage.

EXECUTE: Since the voltage oscillates between $+V$ and $-V$ the peak-to-peak voltage is $V_{\text{pp}} = V - (-V) = 2V = 2\sqrt{2}V_{\text{rms}}.$ Thus, the correct answer is (d).

EVALUATE: The voltage amplitude is half the peak-to-peak voltage.

31.72. **IDENTIFY** and **SET UP:** The impedance of an *R*-*C* circuit is $Z = \sqrt{R^2 + X_C^2}$, where $X_C = \dfrac{1}{\omega C}$.

EXECUTE: As the frequency of oscillation gets very large, X_C gets very small, so the impedance approaches the access resistance *R*. So the impedance approaches a constant but nonzero value, which is choice (c).

EVALUATE: For high oscillation frequency, the access resistance has more effect on the circuit than the capacitance does.

32

ELECTROMAGNETIC WAVES

VP32.2.1. **IDENTIFY:** This problem is about the properties of an electromagnetic wave.

SET UP: $E = cB$, the wave propagates in the direction of $\vec{E}\times\vec{B}$.

EXECUTE: **(a)** We want E_{max}. $E_{max} = cB_{max} = c(4.30 \text{ mT}) = 1.29$ MV/m.

(b) We want the wavelength. $\lambda = 2\pi/k = 2\pi/(2.50 \text{ Mrad/m}) = 2.51\ \mu\text{m}$.

(c) We want the frequency. $f = c/\lambda = c/(2.51\ \mu\text{m}) = 1.19\times10^{14}$ Hz.

(d) We want the direction. The direction is the same as $\vec{E}\times\vec{B}$, which is $+z$.

EVALUATE: This wave is not in the visible part of the spectrum.

VP32.2.2. **IDENTIFY:** This problem is about the properties of an electromagnetic wave.

SET UP: $E = cB$, the wave propagates in the direction of $\vec{E}\times\vec{B}$, $\omega/k = c$.

EXECUTE: **(a)** We want λ. $\lambda = 2\pi/k = 2\pi/(6.50 \text{ Mrad/m}) = 967$ nm.

(b) We want ω. $\omega/k = c$, so $\omega = ck = c(6.50 \text{ Mrad/m}) = 1.95\times10^{14}$ rad/s.

(c) Direction of travel? E is of the form $E = E_{max}\cos(kx - \omega t)$, so the wave is traveling in the $+y$-direction.

(d) We want B. $B = E/c = (2.46 \text{ MV/m})/c = 8.20$ mT. The wave travels in the direction of $\vec{E}\times\vec{B}$. We know that $\vec{E}$ is in the $+x$-direction and $\vec{E}\times\vec{B}$ is in the $+y$-direction. So by the right-hand rule for the cross product, $\vec{B}$ must be in the –z-direction. The full equation for $\vec{B}$ is therefore

$\vec{B} = -\hat{k}(8.20\times10^{-3}\text{ T})\cos[(6.50\times10^{6}\text{ rad/m})y - (1.95\times10^{15}\text{ rad/s})t]$.

EVALUATE: With a wavelength of 967 nm, this is not visible light.

VP32.2.3. **IDENTIFY:** This is an electromagnetic wave.

SET UP: $E = cB$, the wave propagates in the direction of $\vec{E}\times\vec{B}$.

EXECUTE: **(a)** We want $\vec{B}$. $B = E/c = (4.20 \text{ MV/m})/c = 14.0$ mT. The direction of $\vec{E}$ is $+y$ and $\vec{E}\times\vec{B}$ is $+x$, so by the right-hand rule for the cross product, $\vec{B}$ must point in the $+z$-direction.

(b) We want $\vec{E}$ at $x = 1.85\ \mu$m. $E_y = E_{max}\cos kx$. $k = 2\pi/\lambda = 2\pi/(2.94\ \mu\text{m}) = 2.14$ Mrad/m. So $E_y = (4.20 \text{ MV/m})\cos[(2.137 \text{ Mrad/m})(1.85\ \mu\text{m})] = -2.89$ MV/m. The magnitude is 2.89 MV/m and the direction is in the $-y$-direction.

(c) We want $\vec{B}$ at $x = 1.85\ \mu$m. $B = E/c = (2.89 \text{ MV/m})/c = 9.63$ mT. The wave travels in the $+x$-direction and $\vec{E}$ is in the $-y$-direction, so $\vec{B}$ must be in the $-z$-direction by the right-hand rule for $\vec{E}\times\vec{B}$.

EVALUATE: Note that the wave has a large-amplitude electric field but a small-amplitude magnetic field. This is typical.

VP32.2.4. **IDENTIFY:** This problem deals with an electromagnetic wave in matter.

SET UP and **EXECUTE:** **(a)** We want *f* in vacuum. $f = c/\lambda = c/(1.16\ \mu\text{m}) = 2.59\times10^{14}$ Hz.

(b) We want *f* in the material. The frequency does not change, only the wavelength and speed change, so $f = 2.59\times10^{14}$ Hz.

(c) We want the speed in the material. Let *v* indicate vacuum. $\dfrac{v}{c} = \dfrac{f\lambda}{f\lambda_v} = \dfrac{\lambda}{\lambda_v}$.

$v = c\dfrac{\lambda}{\lambda_v} = c\dfrac{0.635\ \mu\text{m}}{1.16\ \mu\text{m}} = 1.64\times10^8$ m/s.

(d) We want *K*. $c/v = \sqrt{K} \rightarrow K = (c/v)^2$. Using *v* from (c) gives $K = 3.34$.

EVALUATE: The light slows down and the wavelength decreases as the light enters matter.

VP32.4.1. **IDENTIFY:** We are dealing with the energy in an electromagnetic wave.

SET UP: $E = cB$, $S = \dfrac{1}{\mu_0}EB$, $u = \epsilon_0 E^2$, $S = 11.0\ \text{W/m}^2$.

EXECUTE: **(a)** We want the magnitudes of the fields. Combine $S = \dfrac{1}{\mu_0}EB$ and $E = cB$ to obtain $S = \dfrac{c}{\mu_0}B^2$. Solve for *B* and using the given numbers. $B = \sqrt{\mu_0 S/c} = 0.215\ \mu\text{T}$. $E = cB = 64.4$ V/m using the *B* we just found.

(b) We want the energy density. $u = \epsilon_0 E^2 = 36.7\ \text{nJ/m}^3$ using the value of *E* that we just found.

EVALUATE: This energy density is typical of many electromagnetic waves.

VP32.4.2. **IDENTIFY:** This problem deals with the Poynting vector $\vec{S}$.

SET UP: $\vec{S} = \dfrac{1}{\mu_0}\vec{E}\times\vec{B}$, $S = \dfrac{1}{\mu_0}EB$, $k = 2\pi/\lambda$. We want $\vec{S}$ at the following times.

EXECUTE: **(a)** At $x = 0$ and $t = 0$: $\vec{S} = \dfrac{E_{\max}B_{\max}}{\mu_0}\hat{i}$.

(b) At $x = \lambda/4$, $t = 0$: $E = E_{\max}\cos(kx - 0) = E_{\max}\cos\left[\left(\dfrac{2\pi}{\lambda}\right)\left(\dfrac{\lambda}{4}\right)\right] = E_{\max}\cos(\pi/2) = 0$. So it follows that $S = 0$.

(c) At $x = \lambda/4$, $t = \pi/4\omega$:

$E = E_{\max}\cos(kx - \omega t) = E_{\max}\cos\left[\left(\dfrac{2\pi}{\lambda}\right)\left(\dfrac{\lambda}{4}\right) - \omega\left(\dfrac{\pi}{4\omega}\right)\right] = E_{\max}\cos\left(\dfrac{\pi}{2} - \dfrac{\pi}{4}\right) = E_{\max}\dfrac{1}{\sqrt{2}}$. And likewise $B = B_{\max}\dfrac{1}{\sqrt{2}}$. Multiplying these two magnitudes together gives $\vec{S} = \dfrac{E_{\max}B_{\max}}{2\mu_0}\hat{i}$.

EVALUATE: It is clear that *S* also varies as the fields vary.

VP32.4.3. **IDENTIFY:** We are dealing with the power in electromagnetic waves.

SET UP: $E = cB$, $I = \dfrac{1}{2}\epsilon_0 cE_{\max}^2$

EXECUTE: **(a)** We want *B* and *I*. Magnetic field: $B = E/c = (0.360\ \text{V/m})/c = 1.20$ nT.

Intensity: Use $I = \dfrac{1}{2}\epsilon_0 cE_{\max}^2$ with $E_{\max} = 0.360$ V/m, giving $I = 172\ \mu\text{W/m}^2$.

(b) We want the power radiated. $P = IA = I[(4\pi r^2)/2] = 156$ kW.

EVALUATE: The answer in (b) assumes that none of the energy is transformed into other types of energy (such as kinetic energy of air molecules) as the waves travel through the air.

VP32.4.4. **IDENTIFY:** This problem is about the energy in electromagnetic waves.

SET UP: $I = \dfrac{E_{\max}^2}{2\mu_0 c}$, $E = cB$.

EXECUTE: **(a)** We want the amplitudes of the fields. Solve $I = \dfrac{E_{\max}^2}{2\mu_0 c}$ for $E_{\max}$ and use $I = 1.36\text{ kW/m}^2$. $E_{\max} = \sqrt{2\mu_0 cI} = 1.01\text{ kV/m}$. $B_{\max} = (1.01\text{ kV/m})/c = 3.38\ \mu\text{T}$.

(b) We want the total power radiated by the sun. Use I from (a) and $r = 1.50\times 10^{11}$ m. $P = IA = I(4\pi r^2) = 3.85\times 10^{26}$ W.

EVALUATE: At half the earth-sun distance the intensity would be 4 times as great, so a planet there would be much hotter than the earth.

VP32.7.1. **IDENTIFY:** We are dealing with standing electromagnetic waves.

SET UP: $S = \dfrac{1}{\mu_0}EB$. We want the Poynting vector in the standing wave at the following times.

EXECUTE: **(a)** We want the maximum the Poynting vector can be. Using Eq. (32.34) and (32.35), we have $S = \dfrac{(2E_{\max}\sin kx\cos\omega t)(2B_{\max}\cos kx\sin\omega t)}{\mu_0}$. The largest that sin kx can be is 1, but then cos kx = 0, and likewise with $\sin\omega t$. The largest product is when sin kx = cos kx, which is when kx = π/4, and likewise for the $\sin\omega t$ factors. In this case, each factor is equal to $1/\sqrt{2}$. Since we have 4 such factors, the final result is $S_{\max} = \dfrac{1}{\mu_0}E_{\max}B_{\max}$. Using the given amplitudes gives $S_{\max} = 0.215\text{ W/m}^2$.

(b) At $t = 0$: $\sin\omega t = 0$, so $S = 0$.

(c) At $x = 0.125$ mm, $t = 3.15\times 10^{-13}$ s: As in Example 32.6, $S = \dfrac{E_{\max}B_{\max}\sin 2kx\sin 2\omega t)}{\mu_0}$. Using the given values for x, t, k, etc., we get $\vec{S} = 0.0907\text{ W/m}^2\,\hat{\boldsymbol{i}}$.

(d) Same approach except $t = 9.45\times 10^{-13}$ s. $\vec{S} = -0.207\text{ W/m}^2\,\hat{\boldsymbol{i}}$.

EVALUATE: Note that $\vec{S}$ can have negative components.

VP32.7.2. **IDENTIFY:** This problem is about a standing electromagnetic wave.
SET UP: $E = cB$, $f\lambda = c$.

EXECUTE: **(a)** We want the wavelength. The nearest nodal plane is $\lambda/4$ from the conductor, so $\lambda/4$ = 3.60 mm. λ = 14.4 mm.
(b) We want the frequency. $f = c/\lambda = c/(14.4\text{ mm}) = 2.08\times 10^{10}$ Hz.

(c) We want the amplitude of E in the first nodal plane of B. The nodal planes of B are antinodal planes of E, so at this point $E = E_{\max} = cB_{\max} = c(0.120\ \mu\text{T}) = 36.0$ V/m.
EVALUATE: The electric and magnetic fields are out of phase by π/2 in this polarized light.

VP32.7.3. **IDENTIFY:** We are looking at standing electromagnetic waves in a cavity.
SET UP: The walls are at $x = 0$ and $x = 4.50$ cm, $\lambda_n = 2L/n$, $f\lambda = c$.

EXECUTE: **(a)** We want the frequencies and wavelengths. Use $\lambda_n = 2L/n$.

$\lambda_1 = 2L/1 = 2L = 9.00$ cm. $f_1 = c/(9.00\text{ cm}) = 3.33$ GHz.

$\lambda_2 = L = 4.50$ cm. $f_2 = c/(4.50\text{ cm}) = 6.67$ GHz.

$\lambda_3 = 2L/3 = 3.00$ cm. $f_3 = c/(3.00\text{ cm}) = 10.0$ GHz.

(b) We want the nodal planes for E. The nodal planes are $\lambda/2$ apart.

For λ_1 : 0, 4.50 cm.

For λ_2 : 0, 2.25 cm, 4.50 cm.

For λ_3 : 0, 1.50 cm, 3.00 cm, 4.50 cm.

EVALUATE: The antinodal planes for E are midway between the nodal planes.

VP32.7.4. **IDENTIFY:** We are looking at standing electromagnetic waves in a cavity.

SET UP: The walls are 48.8 cm apart, λ = 12.2 cm. $\lambda_n = 2L/n$, $f\lambda = c$

EXECUTE: **(a)** What is the frequency? $f = c/\lambda$ = c/(12.2 cm) = 2.46 GHz.

(b) How many antinodal planes between the walls? Nodes occur when $\lambda_n = 2L/n$. $n_{\max} = 2L/\lambda$ = 2(48.8 cm)/(12.2 cm) = 8. The antinodal planes are between the nodal planes. There are 9 nodes including the node at x = 0, so there are 8 antinodal planes.

EVALUATE: If nodal (and antinodal) planes are too far apart, there could be cold (and hot) spots in the oven.

32.1. **IDENTIFY:** Since the speed is constant, distance $x = ct$.

SET UP: The speed of light is $c = 3.00\times10^8$ m/s. $1\text{ y} = 3.156\times10^7$ s.

EXECUTE: **(a)** $t = \dfrac{x}{c} = \dfrac{3.84\times10^8\text{ m}}{3.00\times10^8\text{ m/s}} = 1.28\text{ s}.$

(b) $x = ct = (3.00\times10^8\text{ m/s})(8.61\text{ y})(3.156\times10^7\text{ s/y}) = 8.15\times10^{16}\text{ m} = 8.15\times10^{13}\text{ km}.$

EVALUATE: The speed of light is very great. The distance between stars is very large compared to terrestrial distances.

32.2. **IDENTIFY:** Find the direction of propagation of an electromagnetic wave if we know the directions of the electric and magnetic fields.

SET UP: The direction of propagation of an electromagnetic wave is in the direction of $\vec{E}\times\vec{B}$, which is related to the directions of $\vec{E}$ and $\vec{B}$ according to the right-hand rule for the cross product. The directions of $\vec{E}$ and $\vec{B}$ in each case are shown in Figure 32.2.

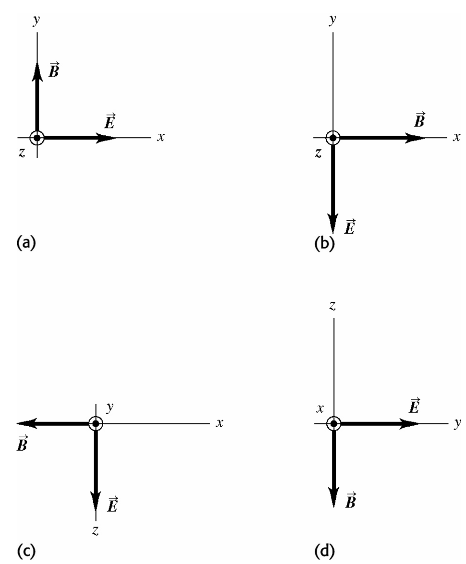


Figure 32.2

EXECUTE: **(a)** The wave is propagating in the $+z$-direction.

(b) $+z$-direction.

(c) $-y$-direction.

(d) $-x$-direction.

EVALUATE: In each case, the direction of propagation is perpendicular to the plane of $\vec{E}$ and $\vec{B}$.

32.3. IDENTIFY: $E_{max} = cB_{max}$. $\vec{E}\times\vec{B}$ is in the direction of propagation.

SET UP: $c = 3.00\times10^8$ m/s. $E_{max} = 4.00$ V/m.

EXECUTE: $B_{max} = E_{max}/c = 1.33\times10^{-8}$ T. For $\vec{E}$ in the $+x$-direction, $\vec{E}\times\vec{B}$ is in the $+z$-direction when $\vec{B}$ is in the $+y$-direction.

EVALUATE: $\vec{E}$, $\vec{B}$, and the direction of propagation are all mutually perpendicular.

32.4. IDENTIFY and **SET UP:** The direction of propagation is given by $\vec{E}\times\vec{B}$.

EXECUTE: **(a)** $\hat{S} = \hat{i}\times(-\hat{j}) = -\hat{k}$.

(b) $\hat{S} = \hat{j}\times\hat{i} = -\hat{k}$.

(c) $\hat{S} = (-\hat{k})\times(-\hat{i}) = \hat{j}$.

(d) $\hat{S} = \hat{i}\times(-\hat{k}) = \hat{j}$.

EVALUATE: In each case the directions of $\vec{E}$, $\vec{B}$, and the direction of propagation are all mutually perpendicular.

32.5. IDENTIFY: Knowing the wavelength and speed of x rays, find their frequency, period, and wave number. All electromagnetic waves travel through vacuum at the speed of light.

SET UP: $c = 3.00\times10^8$ m/s. $c = f\lambda$. $T = \dfrac{1}{f}$. $k = \dfrac{2\pi}{\lambda}$.

EXECUTE: $f = \dfrac{c}{\lambda} = \dfrac{3.0\times10^8 \text{ m/s}}{0.10\times10^{-9} \text{ m}} = 3.0\times10^{18}$ Hz,

$$T = \frac{1}{f} = \frac{1}{3.0\times10^{18} \text{ Hz}} = 3.3\times10^{-19} \text{ s}, \quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.10\times10^{-9} \text{ m}} = 6.3\times10^{10} \text{ m}^{-1}.$$

EVALUATE: The frequency of the x rays is much higher than the frequency of visible light, so their period is much shorter.

32.6. IDENTIFY: $c = f\lambda$ and $k = \dfrac{2\pi}{\lambda}$.

SET UP: $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda}$. UVA: 7.50×10^{14} Hz to 9.38×10^{14} Hz. UVB: 9.38×10^{14} Hz to 1.07×10^{15} Hz.

(b) $k = \dfrac{2\pi}{\lambda}$. UVA: 1.57×10^7 rad/m to 1.96×10^7 rad/m. UVB: 1.96×10^7 rad/m to 2.24×10^7 rad/m.

EVALUATE: Larger λ corresponds to smaller f and k.

32.7. IDENTIFY: Electromagnetic waves propagate through air at essentially the speed of light. Therefore, if we know their wavelength, we can calculate their frequency or vice versa.

SET UP: The wave speed is $c = 3.00\times10^8$ m/s. $c = f\lambda$.

EXECUTE: **(a)** (i) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^3 \text{ m}} = 6.0\times10^4$ Hz.

(ii) $f = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^{-6} \text{ m}} = 6.0\times10^{13}$ Hz.

(iii) $f = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^{-9} \text{ m}} = 6.0\times10^{16}$ Hz.

(b) (i) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{6.50\times10^{21} \text{ Hz}} = 4.62\times10^{-14} \text{ m} = 4.62\times10^{-5}$ nm.

(ii) $\lambda = \dfrac{3.00\times10^8 \text{ m/s}}{590\times10^3 \text{ Hz}} = 508 \text{ m} = 5.08\times10^{11} \text{ nm}.$

EVALUATE: Electromagnetic waves cover a huge range in frequency and wavelength.

32.8. IDENTIFY: $c = f\lambda$. $E_{\text{max}} = cB_{\text{max}}$. Apply Eqs. (32.17) and (32.19).

SET UP: The speed of the wave is $c = 3.00\times10^8$ m/s.

EXECUTE: (a) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{435\times10^{-9} \text{ m}} = 6.90\times10^{14} \text{ Hz}.$

(b) $B_{\text{max}} = \dfrac{E_{\text{max}}}{c} = \dfrac{2.70\times10^{-3} \text{ V/m}}{3.00\times10^8 \text{ m/s}} = 9.00\times10^{-12} \text{ T}.$

(c) $k = \dfrac{2\pi}{\lambda} = 1.44\times10^7$ rad/m. $\omega = 2\pi f = 4.34\times10^{15}$ rad/s. If $\vec{E}(z,t) = \hat{i}E_{\text{max}}\cos(kz+\omega t)$, then $\vec{B}(z,t) = -\hat{j}B_{\text{max}}\cos(kz+\omega t)$, so that $\vec{E}\times\vec{B}$ will be in the $-\hat{k}$-direction.

$\vec{E}(z,t) = \hat{i}(2.70\times10^{-3} \text{ V/m})\cos[(1.44\times10^7 \text{ rad/m})z + (4.34\times10^{15} \text{ rad/s})t]$ and

$\vec{B}(z,t) = -\hat{j}(9.00\times10^{-12} \text{ T})\cos[(1.44\times10^7 \text{ rad/m})z + (4.34\times10^{15} \text{ rad/s})t].$

EVALUATE: The directions of $\vec{E}$ and $\vec{B}$ and of the propagation of the wave are all mutually perpendicular. The argument of the cosine is $kz+\omega t$ since the wave is traveling in the $-z$-direction. Waves for visible light have very high frequencies.

32.9. IDENTIFY and SET UP: Compare the $\vec{E}(y,t)$ given in the problem to the general form given by Eq. (32.17). Use the direction of propagation and of $\vec{E}$ to find the direction of $\vec{B}$.

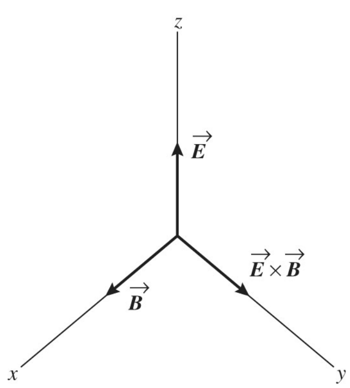
EXECUTE: (a) The equation for the electric field contains the factor $\cos(ky-\omega t)$ so the wave is traveling in the $+y$-direction.

(b) $\vec{E}(y,t) = (3.10\times10^5 \text{ V/m})\hat{k}\cos[ky - (12.65\times10^{12} \text{ rad/s})t].$

Comparing to Eq. (32.17) gives $\omega = 12.65\times10^{12}$ rad/s

$\omega = 2\pi f = \dfrac{2\pi c}{\lambda}$ so $\lambda = \dfrac{2\pi c}{\omega} = \dfrac{2\pi(2.998\times10^8 \text{ m/s})}{(12.65\times10^{12} \text{ rad/s})} = 1.49\times10^{-4} \text{ m}.$

(c)



$\vec{E}\times\vec{B}$ must be in the $+y$-direction (the direction in which the wave is traveling). When $\vec{E}$ is in the $+z$-direction then $\vec{B}$ must be in the $+x$-direction, as shown in Figure 32.9.

Figure 32.9

$k = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{c} = \dfrac{12.65\times10^{12} \text{ rad/s}}{2.998\times10^8 \text{ m/s}} = 4.22\times10^4 \text{ rad/m}.$

$E_{\text{max}} = 3.10\times10^5$ V/m.

Then $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{3.10\times10^5 \text{ V/m}}{2.998\times10^8 \text{ m/s}} = 1.03\times10^{-3} \text{ T}.$

Using Eq. (32.17) and the fact that $\vec{B}$ is in the $+\hat{\imath}$-direction when $\vec{E}$ is in the $+\hat{k}$-direction,

$\vec{B} = +(1.03\times10^{-3} \text{ T})\hat{\imath}\cos[(4.22\times10^4 \text{ rad/m})y - (12.65\times10^{12} \text{ rad/s})t].$

EVALUATE: $\vec{E}$ and $\vec{B}$ are perpendicular and oscillate in phase.

32.10. **IDENTIFY:** For an electromagnetic wave propagating in the negative x-direction, $E = E_{\max}\cos(kx+\omega t)$. $\omega = 2\pi f$ and $k = \dfrac{2\pi}{\lambda}$. $T = \dfrac{1}{f}$. $E_{\max} = cB_{\max}$.

SET UP: $E_{\max} = 375 \text{ V/m}$, $k = 1.99\times10^7 \text{ rad/m}$, and $\omega = 5.97\times10^{15} \text{ rad/s}$.

EXECUTE: **(a)** $c = \omega/k = (5.97\times10^{15} \text{ rad/s})/(1.99\times10^7 \text{ rad/m}) = 3.00\times10^8 \text{ m/s}$. This is what the wave speed should be for an electromagnetic wave propagating in vacuum.

(b) $E_{\max} = 375$ V/m, the amplitude of the given cosine function for E. $B_{\max} = \dfrac{E_{\max}}{c} = 1.25\ \mu\text{T}$.

(c) $f = \dfrac{\omega}{2\pi} = 9.50\times10^{14} \text{ Hz}$. $\lambda = \dfrac{2\pi}{k} = 3.16\times10^{-7} \text{ m} = 316 \text{ nm}$. $T = \dfrac{1}{f} = 1.05\times10^{-15} \text{ s}$. This wavelength is too short to be visible.

EVALUATE: $c = f\lambda = \left(\dfrac{\omega}{2\pi}\right)\left(\dfrac{2\pi}{k}\right) = \dfrac{\omega}{k}$ is an alternative expression for the wave speed.

32.11. **IDENTIFY** and **SET UP:** $c = f\lambda$ allows calculation of λ. $k = 2\pi/\lambda$ and $\omega = 2\pi f$. $E_{\max} = cB_{\max}$ relates the electric and magnetic field amplitudes.

EXECUTE: **(a)** $c = f\lambda$ so $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^8 \text{ m/s}}{830\times10^3 \text{ Hz}} = 361 \text{ m}.$

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi \text{ rad}}{361 \text{ m}} = 0.0174 \text{ rad/m}.$

(c) $\omega = 2\pi f = (2\pi)(830\times10^3 \text{ Hz}) = 5.22\times10^6 \text{ rad/s}.$

(d) Eq. (32.18): $E_{\max} = cB_{\max} = (2.998\times10^8 \text{ m/s})(4.82\times10^{-11} \text{ T}) = 0.0144 \text{ V/m}.$

EVALUATE: This wave has a very long wavelength; its frequency is in the AM radio broadcast band. The electric and magnetic fields in the wave are very weak.

32.12. **IDENTIFY:** Apply $v = \dfrac{c}{\sqrt{KK_{\text{m}}}}$. $E_{\max} = cB_{\max}$. $v = f\lambda$.

SET UP: $K = 3.64$. $K_{\text{m}} = 5.18$.

EXECUTE: **(a)** $v = \dfrac{c}{\sqrt{KK_{\text{m}}}} = \dfrac{(3.00\times10^8 \text{ m/s})}{\sqrt{(3.64)(5.18)}} = 6.91\times10^7 \text{ m/s}.$

(b) $\lambda = \dfrac{v}{f} = \dfrac{6.91\times10^7 \text{ m/s}}{65.0 \text{ Hz}} = 1.06\times10^6 \text{ m}.$

(c) $B_{\max} = \dfrac{E_{\max}}{v} = \dfrac{7.20\times10^{-3} \text{ V/m}}{6.91\times10^7 \text{ m/s}} = 1.04\times10^{-10} \text{ T}.$

EVALUATE: The wave travels slower in this material than in air.

32.13. **IDENTIFY** and **SET UP:** $v = f\lambda$ relates frequency and wavelength to the speed of the wave. Use $n = \sqrt{KK_{\text{m}}} \approx \sqrt{K}$ to calculate n and K.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{2.17\times10^8 \text{ m/s}}{5.70\times10^{14} \text{ Hz}} = 3.81\times10^{-7} \text{ m}.$

(b) $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\text{ m/s}}{5.70\times10^{14}\text{ Hz}} = 5.26\times10^{-7}\text{ m}.$

(c) $n = \dfrac{c}{v} = \dfrac{2.998\times10^{8}\text{ m/s}}{2.17\times10^{8}\text{ m/s}} = 1.38.$

(d) $n = \sqrt{KK_{\text{m}}} \approx \sqrt{K}$ so $K = n^2 = (1.38)^2 = 1.90.$

EVALUATE: In the material $v < c$ and f is the same, so λ is less in the material than in air. $v < c$ always, so n is always greater than unity.

32.14. IDENTIFY: We want to find the amount of energy given to each receptor cell and the amplitude of the magnetic field at the cell.

SET UP: Intensity is average power per unit area and power is energy per unit time. $I = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2$, $I = P/A$, and $E_{\text{max}} = cB_{\text{max}}$.

EXECUTE: **(a)** For the beam, the energy is $U = Pt = (2.0\times10^{12}\text{ W})(4.0\times10^{-9}\text{ s}) = 8.0\times10^{3}\text{ J} = 8.0\text{ kJ}.$ This energy is spread uniformly over 100 cells, so the energy given to each cell is 80 J.

(b) The cross-sectional area of each cell is $A = \pi r^2$, with $r = 2.5\times10^{-6}$ m.

$I = \dfrac{P}{A} = \dfrac{2.0\times10^{12}\text{ W}}{(100)\pi(2.5\times10^{-6}\text{ m})^2} = 1.0\times10^{21}\text{ W/m}^2.$

(c) $E_{\text{max}} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(1.0\times10^{21}\text{ W/m}^2)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{8}\text{ m/s})}} = 8.7\times10^{11}\text{ V/m}.$

$B_{\text{max}} = \dfrac{E_{\text{max}}}{c} = 2.9\times10^{3}\text{ T}.$

EVALUATE: Both the electric field and magnetic field are very strong compared to ordinary fields.

32.15. IDENTIFY: $I = P/A$. $I = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2$. $E_{\text{max}} = cB_{\text{max}}$.

SET UP: The surface area of a sphere of radius r is $A = 4\pi r^2$. $\epsilon_0 = 8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2$.

EXECUTE: **(a)** $I = \dfrac{P}{A} = \dfrac{(0.05)(75\text{ W})}{4\pi(3.0\times10^{-2}\text{ m})^2} = 330\text{ W/m}^2.$

(b) $E_{\text{max}} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(330\text{ W/m}^2)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{8}\text{ m/s})}} = 500\text{ V/m}.$

$B_{\text{max}} = \dfrac{E_{\text{max}}}{c} = 1.7\times10^{-6}\text{ T} = 1.7\ \mu\text{T}.$

EVALUATE: At the surface of the bulb the power radiated by the filament is spread over the surface of the bulb. Our calculation approximates the filament as a point source that radiates uniformly in all directions.

32.16. IDENTIFY: The intensity of the electromagnetic wave is given by $I = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2 = \epsilon_0 cE_{\text{rms}}^2$. The total energy passing through a window of area A during a time t is IAt.

SET UP: $\epsilon_0 = 8.85\times10^{-12}$ F/m.

EXECUTE: Use the fact that energy $= \epsilon_0 cE_{\text{rms}}^2 At$.

$\text{Energy} = (8.85\times10^{-12}\text{ F/m})(3.00\times10^{8}\text{ m/s})(0.0400\text{ V/m})^2(0.500\text{ m}^2)(30.0\text{ s}) = 6.37\times10^{-5}\text{ J} = 63.7\ \mu\text{J}.$

EVALUATE: The intensity is proportional to the square of the electric field amplitude.

32.17. IDENTIFY: $I = P_{\text{av}}/A$.

SET UP: At a distance r from the star, the radiation from the star is spread over a spherical surface of area $A = 4\pi r^2$.

EXECUTE: $P_{av} = I(4\pi r^2) = (5.0\times10^3\ \text{W/m}^2)(4\pi)(2.0\times10^{10}\ \text{m})^2 = 2.5\times10^{25}\ \text{W}.$

EVALUATE: The intensity decreases with distance from the star as $1/r^2$.

32.18. **IDENTIFY** and **SET UP:** $I = \frac{1}{2}\epsilon_0 c E_{max}{}^2$. $E_{max} = cB_{max}$. At the earth the power radiated by the sun is spread over an area of $4\pi r^2$, where $r = 1.50\times10^{11}$ m is the distance from the earth to the sun. $P = IA$.

EXECUTE: **(a)** $E_{max} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(1.4\times10^3\ \text{W/m}^2)}{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 1.03\times10^3\ \text{N/C}.$

$B_{max} = \dfrac{E_{max}}{c} = \dfrac{1.03\times10^3\ \text{N/C}}{3.00\times10^8\ \text{m/s}} = 3.43\times10^{-6}\ \text{T}.$

(b) $P = I(4\pi r^2) = (1.4\times10^3\ \text{W/m}^2)(4\pi)(1.50\times10^{11}\ \text{m})^2 = 4.0\times10^{26}\ \text{W}.$

EVALUATE: The intensity of the magnetic field of the light waves from the sun is about 1/10 the earth's magnetic field.

32.19. **IDENTIFY:** This problem is about the power carried by an electromagnetic wave.

SET UP: $I = \dfrac{1}{2}\epsilon_0 c E_{max}^2$, $I = P_{av}/A$. We want the average power output of the source.

EXECUTE: The data is plotted as E_{max} versus $1/r$, so we need to relate those quantities to interpret the graph. We know that $I = P_{av}/A$ and $I = \dfrac{1}{2}\epsilon_0 c E_{max}^2$. Equate the intensities and solve for E_{max}.

$\dfrac{1}{2}\epsilon_0 c E_{max}^2 = \dfrac{P_{av}}{A} = \dfrac{P_{av}}{4\pi r^2}$. $E_{max} = \sqrt{\dfrac{2P_{av}}{4\pi\epsilon_0 c}}\dfrac{1}{r}$. A graph of E_{max} versus $1/r$ should be a straight line having slope equal to $\sqrt{\dfrac{2P_{av}}{4\pi\epsilon_0 c}}\dfrac{1}{r}$. Solve for P_{av}. $P_{av} = \dfrac{4\pi\epsilon_0 c}{2}(\text{slope})^2$. Using the given slope, we get $P_{av} = 93.8$ W.

EVALUATE: This power is similar to an ordinary 100-W light bulb.

32.20. **IDENTIFY** and **SET UP:** $c = f\lambda$, $E_{max} = cB_{max}$ and $I = E_{max}B_{max}/2\mu_0$.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\ \text{m/s}}{0.354\ \text{m}} = 8.47\times10^8\ \text{Hz}.$

(b) $B_{max} = \dfrac{E_{max}}{c} = \dfrac{0.0540\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 1.80\times10^{-10}\ \text{T}.$

(c) $I = S_{av} = \dfrac{E_{max}B_{max}}{2\mu_0} = \dfrac{(0.0540\ \text{V/m})(1.80\times10^{-10}\ \text{T})}{2\mu_0} = 3.87\times10^{-6}\ \text{W/m}^2.$

EVALUATE: Alternatively, $I = \frac{1}{2}\epsilon_0 c E_{max}^2$.

32.21. **IDENTIFY:** $P_{av} = IA$ and $I = E_{max}^2/2\mu_0 c$

SET UP: The surface area of a sphere is $A = 4\pi r^2$.

EXECUTE: $P_{av} = S_{av}A = \left(\dfrac{E_{max}^2}{2c\mu_0}\right)(4\pi r^2)$. $E_{max} = \sqrt{\dfrac{P_{av}c\mu_0}{2\pi r^2}} = \sqrt{\dfrac{(60.0\ \text{W})(3.00\times10^8\ \text{m/s})\mu_0}{2\pi(5.00\ \text{m})^2}} = 12.0\ \text{V/m}.$

$B_{max} = \dfrac{E_{max}}{c} = \dfrac{12.0\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 4.00\times10^{-8}\ \text{T}.$

EVALUATE: E_{max} and B_{max} are both inversely proportional to the distance from the source.

32.22. **IDENTIFY:** The intensity and the energy density of an electromagnetic wave depend on the amplitudes of the electric and magnetic fields.

SET UP: Intensity is $I = P_{av}/A$, and the average radiation pressure is $P_{av} = 2I/c$, where $I = \frac{1}{2}\epsilon_0 cE_{max}^2$. The energy density is $u = \epsilon_0 E^2$.

EXECUTE: **(a)** $I = P_{av}/A = \dfrac{777{,}000\ \text{W}}{2\pi(5000\ \text{m})^2} = 0.004947\ \text{W/m}^2$.

$$p_{rad} = 2I/c = \frac{2(0.004947\ \text{W/m}^2)}{3.00\times10^8\ \text{m/s}} = 3.30\times10^{-11}\ \text{Pa}.$$

(b) $I = \frac{1}{2}\epsilon_0 cE_{max}^2$ gives

$$E_{max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(0.004947\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 1.93\ \text{N/C}.$$

$$B_{max} = E_{max}/c = (1.93\ \text{N/C})/(3.00\times10^8\ \text{m/s}) = 6.43\times10^{-9}\ \text{T}.$$

(c) $u = \epsilon_0 E^2$, so $u_{av} = \epsilon_0 (E_{rms})^2$ and $E_{rms} = \dfrac{E_{max}}{\sqrt{2}}$, so

$$u_{av} = \frac{\epsilon_0 E_{max}^2}{2} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.93\ \text{N/C})^2}{2} = 1.65\times10^{-11}\ \text{J/m}^3.$$

(d) As was shown in Section 32.4, the energy density is the same for the electric and magnetic fields, so each one has 50% of the energy density.

EVALUATE: Compared to most laboratory fields, the electric and magnetic fields in ordinary radiowaves are extremely weak and carry very little energy.

32.23. **IDENTIFY:** We know the greatest intensity that the eye can safely receive.

SET UP: $I = \dfrac{P}{A}$. $I = \frac{1}{2}\epsilon_0 cE_{max}^2$. $E_{max} = cB_{max}$.

EXECUTE: **(a)** $P = IA = (1.0\times10^2\ \text{W/m}^2)\pi(0.75\times10^{-3}\ \text{m})^2 = 1.8\times10^{-4}\ \text{W} = 0.18\ \text{mW}$.

(b) $E = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(1.0\times10^2\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 274\ \text{V/m}$. $B_{max} = \dfrac{E_{max}}{c} = 9.13\times10^{-7}\ \text{T}$.

(c) $P = 0.18\ \text{mW} = 0.18\ \text{mJ/s}$.

(d) $I = (1.0\times10^2\ \text{W/m}^2)\left(\dfrac{1\ \text{m}}{10^2\ \text{cm}}\right)^2 = 0.010\ \text{W/cm}^2$.

EVALUATE: Both the electric and magnetic fields are quite weak compared to normal laboratory fields.

32.24. **IDENTIFY:** Apply $p_{rad} = \dfrac{I}{c}$ and $p_{rad} = \dfrac{2I}{c}$. The average momentum density is given by $\dfrac{dp}{dV} = \dfrac{S_{av}}{c^2}$ with S replaced by $S_{av} = I$.

SET UP: $1\ \text{atm} = 1.013\times10^5\ \text{Pa}$.

EXECUTE: **(a)** Absorbed light: $p_{rad} = \dfrac{I}{c} = \dfrac{2500\ \text{W/m}^2}{3.0\times10^8\ \text{m/s}} = 8.33\times10^{-6}\ \text{Pa}$. Then

$$p_{rad} = \frac{8.33\times10^{-6}\ \text{Pa}}{1.013\times10^5\ \text{Pa/atm}} = 8.23\times10^{-11}\ \text{atm}.$$

(b) Reflecting light: $p_{rad} = \dfrac{2I}{c} = \dfrac{2(2500\ \text{W/m}^2)}{3.0\times10^8\ \text{m/s}} = 1.67\times10^{-5}\ \text{Pa}$. Then

$$p_{rad} = \frac{1.67\times10^{-5}\ \text{Pa}}{1.013\times10^5\ \text{Pa/atm}} = 1.65\times10^{-10}\ \text{atm}.$$

(c) The momentum density is $\dfrac{dp}{dV}=\dfrac{S_{\text{av}}}{c^2}=\dfrac{2500\ \text{W/m}^2}{(3.0\times10^8\ \text{m/s})^2}=2.78\times10^{-14}\ \text{kg/m}^2\cdot\text{s}.$

EVALUATE: The factor of 2 in p_{rad} for the reflecting surface arises because the momentum vector totally reverses direction upon reflection. Thus the *change* in momentum is twice the original momentum.

32.25. IDENTIFY: We know the wavelength and power of the laser beam, as well as the area over which it acts.

SET UP: $P = IA$. $A=\pi r^2$. $E_{\text{max}}=cB_{\text{max}}$. The intensity $I=S_{\text{av}}$ is related to the maximum electric field by $I=\frac{1}{2}\epsilon_0 cE_{\text{max}}^2$. The average energy density u_{av} is related to the intensity I by $I=u_{\text{av}}c$.

EXECUTE: **(a)** $I=\dfrac{P}{A}=\dfrac{0.500\times10^{-3}\ \text{W}}{\pi(0.500\times10^{-3}\ \text{m})^2}=637\ \text{W/m}^2.$

(b) $E_{\text{max}}=\sqrt{\dfrac{2I}{\epsilon_0 c}}=\sqrt{\dfrac{2(637\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}}=693\ \text{V/m}.$ $B_{\text{max}}=\dfrac{E_{\text{max}}}{c}=2.31\ \mu\text{T}.$

(c) $u_{\text{av}}=\dfrac{I}{c}=\dfrac{637\ \text{W/m}^2}{3.00\times10^8\ \text{m/s}}=2.12\times10^{-6}\ \text{J/m}^3.$

EVALUATE: The fields are very weak, so a cubic meter of space contains only about $2\ \mu\text{J}$ of energy.

32.26. IDENTIFY: This problem deals with radiation pressure.

SET UP: $p_{\text{rad}}=\dfrac{2I}{c}$, $I = P_{\text{av}}/A$. We want the average laser output power. Let subscripts L refer to the laser and D refer to the disk. Also let d be the thickness of the disk, r be the radius of the disk and R the radius of the laser beam. Fig. 32.26 illustrates the arrangement.

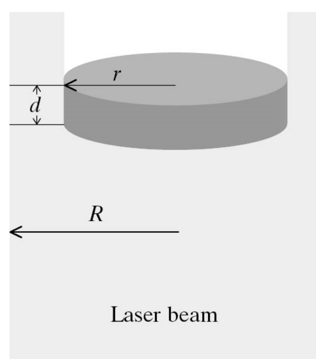


Figure 32.26

EXECUTE: **(a)** The force that the laser exerts on the disk must equal the weight of the disk. The laser force is due to radiation pressure p_{rad}, so $p_{\text{rad}}A_D = p_{\text{rad}}(\pi r^2) = mg = \rho V_D g=\rho(\pi r^2 d)g$. $p_{\text{rad}}=\rho g d$. Using $p_{\text{rad}}=\dfrac{2I}{c}$ gives $\dfrac{2I}{c}=\rho g d$. Using $I = P_{\text{av}}/A_L$ with $A_L = \pi R^2$, we get $\dfrac{2}{c}\left(\dfrac{P_{\text{av}}}{\pi R^2}\right)=\rho g d$. Solving for P_{av} and using the given values, we get $P_{\text{av}}=\dfrac{\pi R^2\rho g c d}{2}=33.3\ \text{W}.$

(b) The power does not depend on the radius of the disk, so the answer is the same as in part (a).

EVALUATE: The result in (b) may be surprising. But doubling r increases the laser force on the disk by a factor of $2^2 = 4$ and it also increase the weight of the disk by the same factor. This result would *not* be true if $r \geq 1.00$ mm, however, because the disk radius would be greater than the laser beam radius. In that case, doubling r would increase the weight of the disk but would not increase the laser force.

32.27. **IDENTIFY:** The nodal and antinodal planes are each spaced one-half wavelength apart.

SET UP: $2\frac{1}{2}$ wavelengths fit in the oven, so $(2\frac{1}{2})\lambda = L$, and the frequency of these waves obeys the equation $f\lambda = c$.

EXECUTE: **(a)** Since $(2\frac{1}{2})\lambda = L$, we have $L = (5/2)(12.2 \text{ cm}) = 30.5$ cm.

(b) Solving for the frequency gives $f = c/\lambda = (3.00\times10^8 \text{ m/s})/(0.122 \text{ m}) = 2.46\times10^9$ Hz.

(c) $L = 35.5$ cm in this case. $(2\frac{1}{2})\lambda = L$, so $\lambda = 2L/5 = 2(35.5 \text{ cm})/5 = 14.2$ cm.

$f = c/\lambda = (3.00\times10^8 \text{ m/s})/(0.142 \text{ m}) = 2.11\times10^9$ Hz.

EVALUATE: Since microwaves have a reasonably large wavelength, microwave ovens can have a convenient size for household kitchens. Ovens using radiowaves would need to be far too large, while ovens using visible light would have to be microscopic.

32.28. **IDENTIFY:** The nodal planes of $\vec{E}$ and $\vec{B}$ are located by Eqs. (32.26) and (32.27).

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{75.0\times10^6 \text{ Hz}} = 4.00$ m.

EXECUTE: **(a)** $\Delta x = \dfrac{\lambda}{2} = 2.00$ m.

(b) The distance between the electric and magnetic nodal planes is one-quarter of a wavelength, so is $\dfrac{\lambda}{4} = \dfrac{\Delta x}{2} = \dfrac{2.00 \text{ m}}{2} = 1.00$ m.

EVALUATE: The nodal planes of $\vec{B}$ are separated by a distance $\lambda/2$ and are midway between the nodal planes of $\vec{E}$.

32.29. **IDENTIFY:** We are looking at standing electromagnetic waves in a cavity.

SET UP: Nodal planes are a half wavelength apart. $\lambda_n = 2L/n$. We want the distance L between the walls.

EXECUTE: Use $\lambda_n = 2L/n$. For the two frequencies, the values of n differ by 1. $\dfrac{1}{2}\lambda_n = \dfrac{1}{2}\left(\dfrac{2L}{n}\right) =$ 1.50 cm. $\dfrac{1}{2}\lambda_{n+1} = \dfrac{1}{2}\left(\dfrac{2L}{n+1}\right)$ = 1.25 cm. Solving for L gives L = 7.50 cm.

EVALUATE: The antinodal planes have the same spacing as the nodal planes.

32.30. **IDENTIFY:** Evaluate the partial derivatives of the expressions for $E_y(x,t)$ and $B_z(x,t)$.

SET UP: $\dfrac{\partial}{\partial x}\cos(kx-\omega t) = -k\sin(kx-\omega t)$, $\dfrac{\partial}{\partial t}\cos(kx-\omega t) = \omega\sin(kx-\omega t)\cdot$

$\dfrac{\partial}{\partial x}\sin(kx-\omega t) = k\cos(kx-\omega t)$, $\dfrac{\partial}{\partial t}\sin(kx-\omega t) = -\omega\cos(kx-\omega t)$.

EXECUTE: Assume $\vec{E} = E_{\max}\hat{j}\cos(kx-\omega t)$ and $\vec{B} = B_{\max}\hat{k}\cos(kx-\omega t+\phi)$, with $-\pi < \phi < \pi$. Eq. (32.12) is $\dfrac{\partial E_y}{\partial x} = -\dfrac{\partial B_z}{\partial t}$. This gives $kE_{\max}\sin(kx-\omega t) = +\omega B_{\max}\sin(kx-\omega t+\phi)$, so $\phi = 0$, and $kE_{\max} = \omega B_{\max}$, so $E_{\max} = \dfrac{\omega}{k}B_{\max} = \dfrac{2\pi f}{2\pi/\lambda}B_{\max} = f\lambda B_{\max} = cB_{\max}$. Similarly for Eq. (32.14), $-\dfrac{\partial B_z}{\partial x} = \epsilon_0\mu_0\dfrac{\partial E_y}{\partial t}$ gives $kB_{\max}\sin(kx-\omega t+\phi) = \epsilon_0\mu_0\omega E_{\max}\sin(kx-\omega t)$, so $\phi = 0$ and $kB_{\max} = \epsilon_0\mu_0\omega E_{\max}$, so $B_{\max} = \dfrac{\epsilon_0\mu_0\omega}{k}E_{\max} = \dfrac{2\pi f}{c^2 2\pi/\lambda}E_{\max} = \dfrac{f\lambda}{c^2}E_{\max} = \dfrac{1}{c}E_{\max}$.

EVALUATE: The $\vec{E}$ and $\vec{B}$ fields must oscillate in phase.

32.31. **IDENTIFY:** We know the wavelength and power of a laser beam as well as the area over which it acts and the duration of a pulse.

SET UP: The energy is $U = Pt$. For absorption the radiation pressure is $\frac{I}{c}$, where $I = \frac{P}{A}$. The wavelength in the eye is $\lambda = \frac{\lambda_0}{n}$. $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$ and $E_{\max} = cB_{\max}$.

EXECUTE: **(a)** $U = Pt = (250\times10^{-3}\ \text{W})(1.50\times10^{-3}\ \text{s}) = 3.75\times10^{-4}\ \text{J} = 0.375\ \text{mJ}$.

(b) $I = \frac{P}{A} = \frac{250\times10^{-3}\ \text{W}}{\pi(255\times10^{-6}\ \text{m})^2} = 1.22\times10^6\ \text{W/m}^2$. The average pressure is

$\frac{I}{c} = \frac{1.22\times10^6\ \text{W/m}^2}{3.00\times10^8\ \text{m/s}} = 4.08\times10^{-3}\ \text{Pa}$.

(c) $\lambda = \frac{\lambda_0}{n} = \frac{810\ \text{nm}}{1.34} = 604\ \text{nm}$. $f = \frac{v}{\lambda} = \frac{c}{\lambda_0} = \frac{3.00\times10^8\ \text{m/s}}{810\times10^{-9}\ \text{m}} = 3.70\times10^{14}\ \text{Hz}$; f is the same in the air and in the vitreous humor.

(d) $E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(1.22\times10^6\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 3.03\times10^4\ \text{V/m}$.

$B_{\max} = \frac{E_{\max}}{c} = 1.01\times10^{-4}\ \text{T}$.

EVALUATE: The intensity of the beam is high, as it must be to weld tissue, but the pressure it exerts on the retina is only around 10^{-8} that of atmospheric pressure. The magnetic field in the beam is about twice that of the earth's magnetic field.

32.32. **IDENTIFY:** This problem involves the energy of the waves in a microwave oven.

SET UP and **EXECUTE:** **(a)** Estimate: 1 minute.

(b) We want the heat. Use $Q = mc\Delta T$. T_1 (room temperature) = 20°C, T_2 = 100°C, m = mass of 237 mL of water = 237 g = 0.237 kg, c = 4190 J/kg·K. Q = 79 kJ.

(c) We want the power. $P = Q/t$ = (79 kJ)/(60 s) = 1.3 kW.

(d) We want the average intensity. P = 2.6 kW. $I_{av} = P_{av}/A$. Estimate: Cup diameter = 7.5 cm, so r = 3.75 cm. I_{av} = (2.6 kW)/[π(0.0375 m)^2] = 600 kW/m^2.

(e) We want $E_{\max}$. Use $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$, solve for $E_{\max}$, and use I = 600 kW/m^2. This gives $E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}}$ = 21 kV/m.

EVALUATE: Since all the estimates are reasonable values, our result for $E_{\max}$ should be fairly reasonable.

32.33. **IDENTIFY:** The intensity of an electromagnetic wave depends on the amplitude of the electric and magnetic fields. Such a wave exerts a force because it carries energy.

SET UP: The intensity of the wave is $I = P_{av}/A = \frac{1}{2}\epsilon_0 cE_{\max}^2$, and the force is $F = p_{rad}A$ where $p_{rad} = I/c$.

EXECUTE: **(a)** $I = P_{av}/A = (25{,}000\ \text{W})/[4\pi(5.75\times10^5\ \text{m})^2] = 6.02\times10^{-9}\ \text{W/m}^2$.

(b) $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$, so $E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(6.02\times10^{-9}\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 2.13\times10^{-3}\ \text{N/C}$.

$B_{\max} = E_{\max}/c = (2.13\times10^{-3}\ \text{N/C})/(3.00\times10^8\ \text{m/s}) = 7.10\times10^{-12}\ \text{T}$.

(c) $F = p_{rad}A = (I/c)A = (6.02\times10^{-9}\ \text{W/m}^2)(0.150\ \text{m})(0.400\ \text{m})/(3.00\times10^8\ \text{m/s}) = 1.20\times10^{-18}\ \text{N}$.

EVALUATE: The fields are very weak compared to ordinary laboratory fields, and the force is hardly worth worrying about!

32.34. **IDENTIFY:** We are dealing with electromagnetic waves from a moving magnet. In one cycle, the magnet starts right at the coil, then moves 10 cm away, and then moves back to where it started.

SET UP and **EXECUTE:** **(a)** One cycle lasts ½ second. In ½ cycle, $\Delta B = B$ and $\Delta T = \frac{1}{2}\left(\frac{1}{2}\text{ s}\right) = \frac{1}{4}$ s.
Using these results and the given numbers gives $\frac{|\Delta\Phi_B|}{\Delta t} = \frac{A\Delta B}{\Delta t} = \frac{AB}{\Delta t} = \frac{\pi r^2 B}{\Delta t}$ = 50 μWb/s.

(b) We want the average magnitude of E within the loop. Using $\oint E dl = \frac{d\Phi_B}{dt}$ gives $E 2\pi r = \frac{|\Delta\Phi_B|}{\Delta t}$.

Solving for E and using the result from part (a) with r = 2.0 cm gives $E = \frac{1}{2\pi r}\frac{|\Delta\Phi_B|}{\Delta t}$ = 400 μV/m.

(c) We want the intensity. Eq. (32.28): $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$. Using $B = E/c$ this becomes $S = \frac{E^2}{\mu_0 c}$. Using E = 400 μV/m, this gives S = 0.42 nW/m^2.

(d) We want the total power. $P = SA = S(2\pi rl)$ = (0.42 nW/m^2)(2π)(0.0200 m)(0.10 m) = 5.3 pW.

EVALUATE: The radiated energy is extremely small because the fields are weak and the back-and-forth motion is very slow.

32.35. **IDENTIFY:** $I = P_{av}/A$. For an absorbing surface, the radiation pressure is $p_{rad} = \frac{I}{c}$.

SET UP: Assume the electromagnetic waves are formed at the center of the sun, so at a distance r from the center of the sun $I = P_{av}/(4\pi r^2)$.

EXECUTE: **(a)** At the sun's surface: $I = \frac{P_{av}}{4\pi R^2} = \frac{3.9\times10^{26}\text{ W}}{4\pi(6.96\times10^8\text{ m})^2} = 6.4\times10^7\text{ W/m}^2$ and
$p_{rad} = \frac{I}{c} = \frac{6.4\times10^7\text{ W/m}^2}{3.00\times10^8\text{ m/s}} = 0.21\text{ Pa}.$
Halfway out from the sun's center, the intensity is 4 times more intense, and so is the radiation pressure: $I = 2.6\times10^8\text{ W/m}^2$ and $p_{rad} = 0.85$ Pa. At the top of the earth's atmosphere, the measured sunlight intensity is 1400 W/m^2 and $p_{rad} = 5\times10^{-6}$ Pa, which is about 100,000 times less than the values above.

EVALUATE: **(b)** The gas pressure at the sun's surface is 50,000 times greater than the radiation pressure, and halfway out of the sun the gas pressure is believed to be about 6×10^{13} times greater than the radiation pressure. Therefore it is reasonable to ignore radiation pressure when modeling the sun's interior structure.

32.36. **(a) IDENTIFY** and **SET UP:** Calculate I and then use $I = \frac{E_{max}^2}{2\mu_0 c}$ to calculate E_{max} and $E_{max} = cB_{max}$ to calculate B_{max}.

EXECUTE: The intensity is power per unit area: $I = \frac{P}{A} = \frac{5.80\times10^{-3}\text{ W}}{\pi(1.25\times10^{-3}\text{ m})^2} = 1182\text{ W/m}^2.$

$I = \frac{E_{max}^2}{2\mu_0 c}$, so $E_{max} = \sqrt{2\mu_0 cI}$. $E_{max} = \sqrt{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.998\times10^8\text{ m/s})(1182\text{ W/m}^2)} = 943.5\text{ V/m},$

which rounds to 943 V/m.

$B_{max} = \frac{E_{max}}{c} = \frac{943.5\text{ V/m}}{2.998\times10^8\text{ m/s}} = 3.148\times10^{-6}\text{ T},$ which rounds to 3.15 μT.

EVALUATE: The magnetic field amplitude is quite small compared to laboratory fields.

(b) IDENTIFY and **SET UP:** $u_E = \frac{1}{2}\epsilon_0 E^2$ and $u_B = \frac{B^2}{2\mu_0}$ give the energy density in terms of the electric and magnetic field values at any time. For sinusoidal fields average over E^2 and B^2 to get the average energy densities.

EXECUTE: The energy density in the electric field is $u_E = \frac{1}{2}\epsilon_0 E^2$. $E = E_{\max}\cos(kx - \omega t)$ and the average value of $\cos^2(kx - \omega t)$ is $\frac{1}{2}$. The average energy density in the electric field then is $u_{E,\text{av}} = \frac{1}{4}\epsilon_0 E_{\max}^2 = \frac{1}{4}(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(943.5\ \text{V/m})^2 = 1.97\times10^{-6}\ \text{J/m}^3 = 1.97\ \mu\text{J/m}^3$. The energy density in the magnetic field is $u_B = \dfrac{B^2}{2\mu_0}$. The average value is

$$u_{B,\text{av}} = \frac{B_{\max}^2}{4\mu_0} = \frac{(3.148\times10^{-6}\ \text{T})^2}{4(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})} = 1.97\times10^{-6}\ \text{J/m}^3 = 1.97\ \mu\text{J/m}^3.$$

EVALUATE: Our result agrees with the statement in Section 32.4 that the average energy density for the electric field is the same as the average energy density for the magnetic field.

(c) IDENTIFY and **SET UP:** The total energy in this length of beam is the average energy density $(u_{\text{av}} = u_{E,\text{av}} + u_{B,\text{av}} = 3.94\times10^{-6}\ \text{J/m}^3)$ times the volume of this part of the beam.

EXECUTE: $U = u_{\text{av}}LA = (3.94\times10^{-6}\ \text{J/m}^3)(1.00\ \text{m})\pi(1.25\times10^{-3}\ \text{m})^2 = 1.93\times10^{-11}\ \text{J}.$

EVALUATE: This quantity can also be calculated as the power output times the time it takes the light to travel $L = 1.00$ m: $U = P\left(\dfrac{L}{c}\right) = (5.80\times10^{-3}\ \text{W})\left(\dfrac{1.00\ \text{m}}{2.998\times10^8\ \text{m/s}}\right) = 1.93\times10^{-11}\ \text{J}$, which checks.

32.37. **IDENTIFY:** The same intensity light falls on both reflectors, but the force on the reflecting surface will be twice as great as the force on the absorbing surface. Therefore there will be a net torque about the rotation axis.

SET UP: For a totally absorbing surface, $F = p_{\text{rad}}A = (I/c)A$, while for a totally reflecting surface the force will be twice as great. The intensity of the wave is $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$. Once we have the torque, we can use the rotational form of Newton's second law, $\tau_{\text{net}} = I\alpha$, to find the angular acceleration.

EXECUTE: The force on the absorbing reflector is $F_{\text{abs}} = p_{\text{rad}}A = (I/c)A = \dfrac{\frac{1}{2}\epsilon_0 cE_{\max}^2 A}{c} = \frac{1}{2}\epsilon_0 AE_{\max}^2$.

For a totally reflecting surface, the force will be twice as great, which is $\epsilon_0 cE_{\max}^2$. The net torque is therefore $\tau_{\text{net}} = F_{\text{refl}}(L/2) - F_{\text{abs}}(L/2) = \epsilon_0 AE_{\max}^2 L/4$.

Newton's second law for rotation gives $\tau_{\text{net}} = I\alpha$. $\epsilon_0 AE_{\max}^2 L/4 = 2m(L/2)^2\alpha$.

Solving for α gives

$$\alpha = \epsilon_0 AE_{\max}^2/(2mL) = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0150\ \text{m})^2(1.25\ \text{N/C})^2}{(2)(0.00400\ \text{kg})(1.00\ \text{m})} = 3.89\times10^{-13}\ \text{rad/s}^2.$$

EVALUATE: This is an extremely small angular acceleration. To achieve a larger value, we would have to greatly increase the intensity of the light wave or decrease the mass of the reflectors.

32.38. **IDENTIFY:** The intensity of the wave, not the electric field strength, obeys an inverse-square distance law.

SET UP: The intensity is inversely proportional to the distance from the source, and it depends on the amplitude of the electric field by $I = \text{S}_{\text{av}} = \frac{1}{2}\epsilon_0 cE_{\max}^2$.

EXECUTE: Since $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$, $E_{\max} \propto \sqrt{I}$. A point at 20.0 cm (0.200 m) from the source is 50 times closer to the source than a point that is 10.0 m from it. Since $I \propto 1/r^2$ and $(0.200\ \text{m})/(10.0\ \text{m}) = 1/50$, we have $I_{0.20} = 50^2 I_{10}$. Since $E_{\max} \propto \sqrt{I}$, we have $E_{0.20} = 50E_{10} = (50)(3.50\ \text{N/C}) = 175\ \text{N/C}$.

EVALUATE: While the intensity increases by a factor of $50^2 = 2500$, the amplitude of the wave only increases by a factor of 50. Recall that the intensity of *any* wave is proportional to the *square* of its amplitude.

32.39. **IDENTIFY** and **SET UP:** In the wire the electric field is related to the current density by $\vec{E} = \rho\vec{J}$. Use Ampere's law to calculate $\vec{B}$. The Poynting vector is given by $\vec{S} = \dfrac{1}{\mu_0}\vec{E}\times\vec{B}$ and $\vec{P} = \oint\vec{S}\cdot d\vec{A}$ relates the energy flow through a surface to $\vec{S}$.

EXECUTE: **(a)** The direction of $\vec{E}$ is parallel to the axis of the cylinder, in the direction of the current. $E = \rho J = \rho I/\pi a^2$. ($E$ is uniform across the cross section of the conductor.)

(b) A cross-sectional view of the conductor is given in Figure 32.39a; take the current to be coming out of the page.

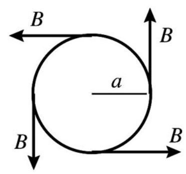


Figure 32.39a

Apply Ampere's law to a circle of radius a.

$\oint\vec{B}\cdot d\vec{l} = B(2\pi ra)$

$I_{\text{encl}} = I$.

$\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi a) = \mu_0 I$ and $B = \dfrac{\mu_0 I}{2\pi a}$.

The direction of $\vec{B}$ is counterclockwise around the circle.

(c) The directions of $\vec{E}$ and $\vec{B}$ are shown in Figure 32.39b.

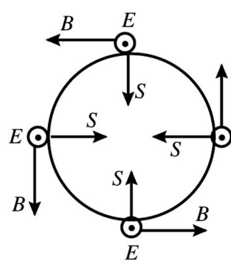


Figure 32.39b

The direction of $\vec{S} = \dfrac{1}{\mu_0}\vec{E}\times\vec{B}$.

is radially inward.

$$S = \frac{1}{\mu_0}EB = \frac{1}{\mu_0}\left(\frac{\rho I}{\pi a^2}\right)\left(\frac{\mu_0 I}{2\pi a}\right).$$

$$S = \frac{\rho I^2}{2\pi^2 a^3}.$$

EVALUATE: **(d)** Since S is constant over the surface of the conductor, the rate of energy flow P is given by S times the surface of a length l of the conductor: $P = SA = S(2\pi al) = \dfrac{\rho I^2}{2\pi^2 a^3}(2\pi al) = \dfrac{\rho l I^2}{\pi a^2}$. But $R = \dfrac{\rho l}{\pi a^2}$, so the result from the Poynting vector is $P = RI^2$. This agrees with $P_R = I^2 R$, the rate at which electrical energy is being dissipated by the resistance of the wire. Since $\vec{S}$ is radially inward at the surface of the wire and has magnitude equal to the rate at which electrical energy is being dissipated in the wire, this energy can be thought of as entering through the cylindrical sides of the conductor.

32.40. **IDENTIFY:** The changing magnetic field of the electromagnetic wave produces a changing flux through the wire loop, which induces an emf in the loop. The wavelength of the wave is much greater than the diameter of the loop, so we can treat the magnetic field as being uniform over the area of the loop.

SET UP: $\Phi_B = B\pi r^2 = \pi r^2 B_{\text{max}}\cos(kx - \omega t)$, taking x for the direction of propagation of the wave. Faraday's law says $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right|$. The intensity of the wave is $I = \dfrac{E_{\text{max}}B_{\text{max}}}{2\mu_0} = \dfrac{c}{2\mu_0}B_{\text{max}}^2$, and $f = \dfrac{c}{\lambda}$.

EXECUTE: $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = \omega B_{\max}\sin(kx-\omega t)\pi r^2$. $|\mathcal{E}|_{\max} = 2\pi f B_{\max}\pi r^2$.

$f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\ \text{m/s}}{6.90\ \text{m}} = 4.348\times10^7\ \text{Hz}$. Solving $I = \dfrac{E_{\max}B_{\max}}{2\mu_0} = \dfrac{c}{2\mu_0}B_{\max}^2$ for $B_{\max}$ gives

$B_{\max} = \sqrt{\dfrac{2\mu_0 I}{c}} = \sqrt{\dfrac{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.0275\ \text{W/m}^2)}{3.00\times10^8\ \text{m/s}}} = 1.518\times10^{-8}\ \text{T}$.

$|\mathcal{E}|_{\max} = 2\pi(4.348\times10^7\ \text{Hz})(1.518\times10^{-8}\ \text{T})\pi(0.075\ \text{m})^2 = 7.33\times10^{-2}\ \text{V} = 73.3\ \text{mV}$.

EVALUATE: This voltage is quite small compared to everyday voltages, so it normally would not be noticed. But in very delicate laboratory work, it could be large enough to take into consideration.

32.41. **IDENTIFY:** The nodal planes are one-half wavelength apart.
SET UP: The nodal planes of B are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots,$ which are $\lambda/2$ apart.

EXECUTE: **(a)** The wavelength is $\lambda = c/f = (2.998\times10^8\ \text{m/s})/(110.0\times10^6\ \text{Hz}) = 2.725\ \text{m}$. So the nodal planes are at $(2.725\ \text{m})/2 = 1.363\ \text{m}$ apart.

(b) For the nodal planes of E, we have $\lambda_n = 2L/n$, so $L = n\lambda/2 = (8)(2.725\ \text{m})/2 = 10.90\ \text{m}$.

EVALUATE: Because radiowaves have long wavelengths, the distances involved are easily measurable using ordinary metersticks.

32.42. **IDENTIFY:** This problem involves an L-R-C ac circuit, electromagnetic waves, and Faraday's law.

SET UP: $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$, $B = E/c$, $k = \omega/c$. B_x must have the same mathematical form as E_y.

EXECUTE: **(a)** We want the flux. $dA = adz$. $\Phi_B = \int_{-a/2}^{a/2} B_x a dz = aB_{\max}\int_{-a/2}^{a/2}\cos(kx-\omega t)dz$. This gives

$\Phi_B = \dfrac{aB_{\max}}{k}\left[\sin(ka/2-\omega t)+\sin(ka/2+\omega t)\right] = \dfrac{aE_{\max}}{\omega}\left[2\sin(ka/2)\cos\omega t\right]$.

(b) We want the magnitude of the emf. Using the result from part (a) we get $\mathcal{E} = N\dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}\left(\dfrac{aE_{\max}}{\omega}\left[2\sin(ka/2)\cos\omega t\right]\right) = 2NaE_{\max}\sin(ka/2)\sin\omega t$.

(c) We want C. At resonance, $2\pi f_0 = \omega_0 = 1/\sqrt{LC}$. Solve for C and put in the given numbers using f_0 = 4.00 MHz and L = 78.0 μH. The result is $C = \dfrac{1}{L(2\pi f_0)^2}$ = 20.3 pF.

(d) We want the rms current. The circuit is at resonance, so $i = V/Z = V/R$. (Note that we are using i instead of I for the current amplitude so as not to confuse it with the intensity I.) Using the result from part (b) gives $V = \mathcal{E}_{\max} = 2NaE_{\max}$. Now find $E_{\max}$ using the intensity $I = \dfrac{1}{2}\epsilon_0 cE_{\max}^2$. This gives

$E_{\max} = \sqrt{\dfrac{2I}{\epsilon_0 c}}$, so $V = 2NaE_{\max} = 2Na\sqrt{\dfrac{2I}{\epsilon_0 c}}$. Thus $i = \dfrac{V}{R} = \dfrac{2Na}{R}\sqrt{\dfrac{2I}{\epsilon_0 c}}$. Finally $i_{\text{rms}} = i/\sqrt{2}$, so we get

$i_{\text{rms}} = \dfrac{2Na}{R\sqrt{2}}\sqrt{\dfrac{2I}{\epsilon_0 c}} = \dfrac{2Na}{R}\sqrt{\dfrac{I}{\epsilon_0 c}}$. Using the numbers in the problem gives i_{rms} = 19.4 A.

EVALUATE: At resonance, Z is a minimum so the current amplitude is a maximum.

32.43. **IDENTIFY:** The orbiting satellite obeys Newton's second law of motion. The intensity of the electromagnetic waves it transmits obeys the inverse-square distance law, and the intensity of the waves depends on the amplitude of the electric and magnetic fields.

SET UP: Newton's second law applied to the satellite gives $mv^2/r = GmM/r^2$, where M is the mass of the earth and m is the mass of the satellite. The intensity I of the wave is $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 c{E_{\max}}^2$, and by definition, $I = P_{\text{av}}/A$.

EXECUTE: **(a)** The period of the orbit is 12 hr. Applying Newton's second law to the satellite gives $mv^2/r = GmM/r^2$, which gives $\dfrac{m(2\pi r/T)^2}{r} = \dfrac{GmM}{r^2}$. Solving for r, we get

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(12\times3600\ \text{s})^2}{4\pi^2}\right]^{1/3} = 2.66\times10^7\ \text{m}.$$

The height above the surface is $h = 2.66\times10^7\ \text{m} - 6.37\times10^6\ \text{m} = 2.02\times10^7\ \text{m}$. The satellite only radiates its energy to the lower hemisphere, so the area is 1/2 that of a sphere. Thus, from the definition of intensity, the intensity at the ground is

$$I = P_{\text{av}}/A = P_{\text{av}}/(2\pi h^2) = (25.0\ \text{W})/[2\pi(2.02\times10^7\ \text{m})^2] = 9.75\times10^{-15}\ \text{W/m}^2$$

(b) $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2$, so

$$E_{\text{max}} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(9.75\times10^{-15}\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 2.71\times10^{-6}\ \text{N/C}.$$

$$B_{\text{max}} = E_{\text{max}}/c = (2.71\times10^{-6}\ \text{N/C})/(3.00\times10^8\ \text{m/s}) = 9.03\times10^{-15}\ \text{T}.$$

$$t = d/c = (2.02\times10^7\ \text{m})/(3.00\times10^8\ \text{m/s}) = 0.0673\ \text{s}.$$

(c) $p_{\text{rad}} = I/c = (9.75\times10^{-15}\ \text{W/m}^2)/(3.00\times10^8\ \text{m/s}) = 3.25\times10^{-23}\ \text{Pa}.$

(d) $\lambda = c/f = (3.00\times10^8\ \text{m/s})/(1575.42\times10^6\ \text{Hz}) = 0.190\ \text{m}.$

EVALUATE: The fields and pressures due to these waves are very small compared to typical laboratory quantities.

32.44. IDENTIFY: For a totally reflective surface the radiation pressure is $\dfrac{2I}{c}$. Find the force due to this pressure and express the force in terms of the power output P of the sun. The gravitational force of the sun is $F_{\text{g}} = G\dfrac{mM_{\text{sun}}}{r^2}$.

SET UP: The mass of the sun is $M_{\text{sun}} = 1.99\times10^{30}\ \text{kg}$. $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** The sail should be reflective, to produce the maximum radiation pressure.

(b) $F_{\text{rad}} = \left(\dfrac{2I}{c}\right)A$, where A is the area of the sail. $I = \dfrac{P}{4\pi r^2}$, where r is the distance of the sail from the sun. $F_{\text{rad}} = \left(\dfrac{2A}{c}\right)\left(\dfrac{P}{4\pi r^2}\right) = \dfrac{PA}{2\pi r^2 c}\cdot F_{\text{rad}} = F_{\text{g}}$ so $\dfrac{PA}{2\pi r^2 c} = G\dfrac{mM_{\text{sun}}}{r^2}$.

$$A = \frac{2\pi cGmM_{\text{sun}}}{P} = \frac{2\pi(3.00\times10^8\ \text{m/s})(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(10{,}000\ \text{kg})(1.99\times10^{30}\ \text{kg})}{3.9\times10^{26}\ \text{W}}.$$

$A = 6.42\times10^6\ \text{m}^2 = 6.42\ \text{km}^2.$

(c) Both the gravitational force and the radiation pressure are inversely proportional to the square of the distance from the sun, so this distance divides out when we set $F_{\text{rad}} = F_{\text{g}}$.

EVALUATE: A very large sail is needed, just to overcome the gravitational pull of the sun.

32.45. **IDENTIFY and SET UP:** The gravitational force is given by $F_g = G\dfrac{mM}{r^2}$. Express the mass of the particle in terms of its density and volume. The radiation pressure is given by $p_{rad} = \dfrac{I}{c}$; relate the power output L of the sun to the intensity at a distance r. The radiation force is the pressure times the cross-sectional area of the particle.

EXECUTE: **(a)** The gravitational force is $F_g = G\dfrac{mM}{r^2}$. The mass of the dust particle is $m = \rho V = \rho \frac{4}{3}\pi R^3$. Thus $F_g = \dfrac{4\rho G\pi M R^3}{3r^2}$.

(b) For a totally absorbing surface $p_{rad} = \dfrac{I}{c}$. If L is the power output of the sun, the intensity of the solar radiation a distance r from the sun is $I = \dfrac{L}{4\pi r^2}$. Thus $p_{rad} = \dfrac{L}{4\pi c r^2}$. The force F_{rad} that corresponds to p_{rad} is in the direction of propagation of the radiation, so $F_{rad} = p_{rad}A_\perp$, where $A_\perp = \pi R^2$ is the component of area of the particle perpendicular to the radiation direction. Thus

$$F_{rad} = \left(\frac{L}{4\pi c r^2}\right)(\pi R^2) = \frac{LR^2}{4cr^2}.$$

(c) $F_g = F_{rad}$.

$$\frac{4\rho G\pi M R^3}{3r^2} = \frac{LR^2}{4cr^2}.$$

$$\left(\frac{4\rho G\pi M}{3}\right)R = \frac{L}{4c} \text{ and } R = \frac{3L}{16c\rho G\pi M}.$$

$$R = \frac{3(3.9\times10^{26}\text{ W})}{16(2.998\times10^{8}\text{ m/s})(3000\text{ kg/m}^3)(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\pi(1.99\times10^{30}\text{ kg})}.$$

$R = 1.9\times10^{-7}\text{ m} = 0.19\ \mu\text{m}$.

EVALUATE: The gravitational force and the radiation force both have a r^{-2} dependence on the distance from the sun, so this distance divides out in the calculation of R.

(d) $\dfrac{F_{rad}}{F_g} = \left(\dfrac{LR^2}{4cr^2}\right)\left(\dfrac{3r^2}{4\rho G\pi M R^3}\right) = \dfrac{3L}{16c\rho G\pi MR}$. F_{rad} is proportional to R^2 and F_g is proportional to R^3, so this ratio is proportional to $1/R$. If $R < 0.20\ \mu\text{m}$ then $F_{rad} > F_g$ and the radiation force will drive the particles out of the solar system.

32.46. **IDENTIFY and SET UP:** The intensity of an electromagnetic wave can be expressed in many ways, including $I = \dfrac{P}{A} = \frac{1}{2}\epsilon_0 c E_{max}^2 = \dfrac{cp_{rad}}{2}$, with the last way valid at a totally reflecting surface. In addition, the average energy density u in a wave is $u = \frac{1}{2}\epsilon_0 E_{max}^2$. Also, $B = E/c$ and $p = \dfrac{F_\perp}{A}$.

EXECUTE: For each laser, we calculate the beam intensity using formula that is appropriate for the information we know about the beam.

Laser A: $I = P/A = (2.6\text{ W})/[\pi(1.3\times10^{-3}\text{ m})^2] = 4.9\times10^5\text{ W/m}^2$.

Laser B: $I = \frac{1}{2}\epsilon_0 c E_{max}^2 = (1/2)(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\text{ m/s})(480\text{ V/m})^2 = 310\text{ W/m}^2$.

Laser C: Combining $I = \frac{1}{2}\epsilon_0 c E_{max}^2$ and $B_{max} = E_{max}/c$, we get

$I = \frac{1}{2}\epsilon_0 c^3 B_{max}^2 = (1/2)(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\text{ m/s})^3(8.7\times10^{-6}\text{ T})^2 = 9000\text{ W/m}^2$.

Laser D: The surface is totally reflecting, so

$I = \dfrac{cp_{rad}}{2} = \dfrac{cF_\perp}{2A} = (3.00\times10^8\text{ m/s})(6.0\times10^{-8}\text{ N})/[2\pi(0.90\times10^{-3}\text{ m})^2] = 3.5\times10^7\text{ W/m}^2$.

Laser E: Combining $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$ and $u = \frac{1}{2}\epsilon_0 E_{\max}^2$ gives $I = \frac{1}{2}\epsilon_0 c\left(\frac{2u}{\epsilon_0}\right) = cu,$ so

$I = (3.0\times10^8 \text{ m/s})(3.0\times10^{-7} \text{ J/m}^3) = 90 \text{ W/m}^2$.

In order of increasing intensity, we have E, B, C, A, D.

EVALUATE: The laser intensities vary a great deal. But even the least intense one is around 10 times as intense as a 100-W lightbulb viewed at 1 m, if the 100 W all went into light (which it certainly does *not*).

32.47. IDENTIFY and SET UP: The intensity of the light beam is $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$.

EXECUTE: **(a)** A graph of I versus $E_{\max}^2$ should be a straight line having slope equal to $\frac{1}{2}\epsilon_0 c$.

(b) Using the slope of the graph given with the problem, we have $\frac{1}{2}\epsilon_0 c = 1.33\times10^{-3} \text{ J/(V}^2\cdot\text{s)}$.

Solving for c gives $c = 2[1.33\times10^{-3} \text{ J/(V}^2\cdot\text{s)}]/(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2) = 3.00\times10^8$ m/s.

EVALUATE: This result is nearly identical to the speed of light in vacuum.

32.48. IDENTIFY and SET UP: The spacing between antinodes is $\lambda/2$, and $f\lambda = c$.

EXECUTE: The antinode spacing is $d = \lambda/2 = \frac{c}{2}\cdot\frac{1}{f}$. Therefore a graph of d versus $1/f$ should be a straight line having a slope equal to $c/2$. Figure 32.48 shows the graph of d versus $1/f$.

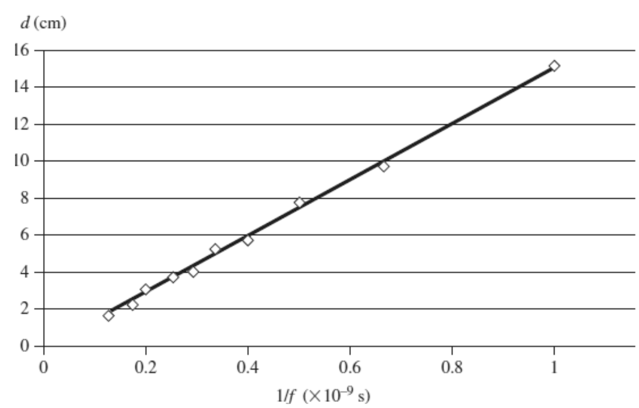


Figure 32.48

The slope of the best-fit line is 15.204×10^9 cm/s = 15.204×10^7 m/s, so $c/2 = 15.204\times10^7$ m/s, which gives $c = 3.0\times10^8$ m/s.

EVALUATE: This result is *very* close to the well-established value for the speed of light in vacuum.

32.49. IDENTIFY: This problem involves radiation pressure.

SET UP: Part of the incident beam is absorbed and part is reflected. The intensity of the incident beam is I_0, that of the transmitted beam is $eI\!-_0$, the that of the reflected beam is $(1 - e)I_0$ as shown in Fig. 32.49.

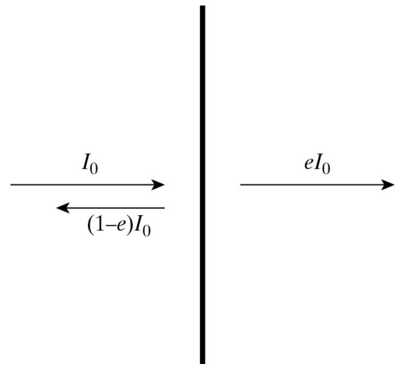


Figure 32.49

EXECUTE: (a) The radiation pressure due to the transmitted part of the beam is $p_{\text{tr}} = I/c$ and the pressure due to the reflected part is $p_{\text{ab}} = 2I/c$. In terms of the incident intensity, these pressures are $p_{\text{tr}} = 2eI_0/c$ and $p_{\text{ab}} = 2(1-e)I_0/c$. The total radiation pressure is the sum of these, which is

$$p_{\text{rad}} = \frac{I_0}{c}(2-e)$$

EVALUATE: Check: For a perfect absorber all the radiation is absorbed, so $e = 1$. In this case, $p_{\text{rad}} = \frac{I_0}{c}(2-e) = = \frac{I_0}{c}(2-1) = \frac{I_0}{c}$, as we have seen in the textbook. For a perfect reflector none of the incident beam is absorbed, so $e = 0$ and $p_{\text{rad}} = \frac{I_0}{c}(2-0) = \frac{2I_0}{c}$, as we have seen. Both extremes agree with the results in the textbook.

EXECUTE: (b) We want the force due to radiation. $F_{\text{rad}} = p_{\text{rad}}A$ and $p_{\text{rad}} = \frac{I_0}{c}(2-e)$.
$F_{\text{rad}} = \frac{I_0}{c}(2-e)\pi r^2$. Using $I_0 = 1.4\ \text{kW/m}^2$ and $r = 4.0\ \mu\text{m}$, we get $F_{\text{rad}} = 3.3\times 10^{-16}$ N.
(c) We want $F_{\text{rad}}/F_{\text{grav}}$. $\frac{F_{\text{rad}}}{F_{\text{grav}}} = \frac{F_{\text{rad}}}{GmM_{\text{sun}}/r^2}$. Using $m = 1.0\times 10^{-13}$ kg, $M_{\text{sun}} = 1.99\times 10^{30}$ kg,

$r = 1.5\times 10^{11}$ m, and the result from part (b), we get $F_{\text{rad}}/F_{\text{grav}} = 0.55$.

EVALUATE: The force of the sun's radiation is about half as great as its gravitational force. Light particles of the same size would feel the same pressure but less gravity, so they could be blown away by the sun. Also closer to the sun the radiation is more intense, so the radiation pressure could blow more particles away from the sun.

32.50. IDENTIFY: This problem is about electromagnetic waves.
SET UP and **EXECUTE: (a)** Evaluate $\oint \vec{E}\cdot d\hat{l}$. $E_z = 0$ and the path is in the xz-plane, so only E_x contributes to the integral. Therefore $\oint \vec{E}\cdot d\hat{l} = \int_{-h/2}^{h/2} E_x(z=0)dx + \int_{h/2}^{-h/2} E_x(z=\lambda/2)dx$. Using the given waves gives $\oint \vec{E}\cdot d\hat{l} = \int_{-h/2}^{h/2} E\cos(0-\omega t)dx + \int_{h/2}^{-h/2} E\cos(k\lambda/2-\omega t)dx$. Integrating gives $\oint \vec{E}\cdot d\hat{l}$ $= hE\left[\cos\omega t - \left(\cos(k\lambda/2)\cos\omega t\right) + \sin(k\lambda/2)\sin\omega t\right]$. $\frac{k\lambda}{2} = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right) = \pi$. $\sin\pi = 0$ and $\cos\pi = -1$, so our result reduces to $\oint \vec{E}\cdot d\hat{l} = 2hE\cos\omega t$.

(b) We want the flux. $dA = h\,dz$. $BdA = h\,dz$. B_y is positive and A_y is negative, so the flux is negative. As in (a), we make use of $k\lambda/2 = \pi$.

$$\Phi_B = -\int B_y dA = -\int_0^{\lambda/2} B\cos(kz-\omega t)h dz = -\frac{Bh}{k}\left[\sin\left(k\lambda/2-\omega t\right) - \sin(-\omega t)\right] = -\frac{2Bh}{k}\sin\omega t.$$

(c) We want to find dB in terms of E and c. Using the result of (b), $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \dfrac{2Bh}{k}\cos\omega t$. In part (a) we found that $\mathcal{E} = \oint \vec{E}\cdot d\hat{l} = 2hE\cos\omega t$. Equating these two expressions gives $E = cB$.

(d) Follow the directions in the problem: reverse the sign of the sine function and add the two terms. For $\vec{E}$ this gives

$\vec{E} = \vec{E}_R + \vec{E}_L = E\left[\cos(kz-\omega t)\hat{\boldsymbol{i}} + \sin(kz-\omega t)\hat{\boldsymbol{j}}\right] + E\left[\cos(kz-\omega t)\hat{\boldsymbol{i}} - \sin(kz-\omega t)\hat{\boldsymbol{j}}\right]$, which reduces to $\vec{E} = 2E\cos(kz-\omega t)\hat{\boldsymbol{i}}$. The same procedure for $\vec{B}$ leads to $\vec{B} = 2B\cos(kz-\omega t)\hat{\boldsymbol{j}}$.

(e) We want the E and B. $S = \dfrac{E_{\max}^2}{\mu_0 c} = I$. Solve for $E_{\max}$ and use the given intensity.

$E_{\max} = \sqrt{\mu_0 cI} = \sqrt{\mu_0 c(100\ \text{W/m}^2)} = 194\ \text{V/m}$. $B_{\max} = E_{\max}/c = (194\ \text{V/m})/c = 0.647\ \mu\text{T}$.

EVALUATE: Note that B is a weak magnetic field but E is a considerably stronger electric field.

32.51. IDENTIFY: The orbiting particle has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $K = \frac{1}{2}mv^2$. An electron has mass $m_e = 9.11\times10^{-31}$ kg and a proton has mass $m_p = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\left[\dfrac{q^2a^2}{6\pi\epsilon_0 c^3}\right] = \dfrac{\text{C}^2(\text{m/s}^2)^2}{(\text{C}^2/\text{N}\cdot\text{m}^2)(\text{m/s})^3} = \dfrac{\text{N}\cdot\text{m}}{\text{s}} = \dfrac{\text{J}}{\text{s}} = \text{W} = \left[\dfrac{dE}{dt}\right]$.

(b) For a proton moving in a circle, the acceleration is

$a = \dfrac{v^2}{R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \dfrac{2(6.00\times10^6\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})}{(1.67\times10^{-27}\ \text{kg})(0.75\ \text{m})} = 1.53\times10^{15}\ \text{m/s}^2$. The rate at which it emits energy because of its acceleration is

$\dfrac{dE}{dt} = \dfrac{q^2a^2}{6\pi\epsilon_0 c^3} = \dfrac{(1.6\times10^{-19}\ \text{C})^2(1.53\times10^{15}\ \text{m/s}^2)^2}{6\pi\epsilon_0(3.0\times10^8\ \text{m/s})^3} = 1.33\times10^{-23}\ \text{J/s} = 8.32\times10^{-5}\ \text{eV/s}$.

Therefore, the fraction of its energy that it radiates every second is

$\dfrac{(dE/dt)(1\ \text{s})}{E} = \dfrac{8.32\times10^{-5}\ \text{eV}}{6.00\times10^6\ \text{eV}} = 1.39\times10^{-11}$.

(c) Carry out the same calculations as in part (b), but now for an electron at the same speed and radius. That means the electron's acceleration is the same as the proton, and thus so is the rate at which it emits energy, since they also have the same charge. However, the electron's initial energy differs from the proton's by the ratio of their masses: $E_e = E_p\dfrac{m_e}{m_p} = (6.00\times10^6\ \text{eV})\dfrac{(9.11\times10^{-31}\ \text{kg})}{(1.67\times10^{-27}\ \text{kg})} = 3273\ \text{eV}$.

Therefore, the fraction of its energy that it radiates every second is

$\dfrac{(dE/dt)(1\ \text{s})}{E} = \dfrac{8.32\times10^{-5}\ \text{eV}}{3273\ \text{eV}} = 2.54\times10^{-8}$.

EVALUATE: The proton has speed $v = \sqrt{\dfrac{2E}{m_p}} = \sqrt{\dfrac{2(6.0\times10^6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.67\times10^{-27}\ \text{kg}}} = 3.39\times10^7\ \text{m/s}$.

The electron has the same speed and kinetic energy 3.27 keV. The particles in the accelerator radiate at a much smaller rate than the electron in Problem 32.52 does, because in the accelerator the orbit radius is very much larger than in the atom, so the acceleration is much less.

32.52. **IDENTIFY:** The electron has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $1\ \text{eV} = 1.60\times10^{-19}$ C. An electron has $|q| = e = 1.60\times10^{-19}$ C.

EXECUTE: For the electron in the classical hydrogen atom, its acceleration is

$a = \dfrac{v^2}{R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \dfrac{2(13.6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(9.11\times10^{-31}\ \text{kg})(5.29\times10^{-11}\ \text{m})} = 9.03\times10^{22}\ \text{m/s}^2$. Then using the formula for the rate of energy emission given in Problem 32.51:

$\dfrac{dE}{dt} = \dfrac{q^2a^2}{6\pi\epsilon_0 c^3} = \dfrac{(1.60\times10^{-19}\ \text{C})^2(9.03\times10^{22}\ \text{m/s}^2)^2}{6\pi\epsilon_0(3.00\times10^8\ \text{m/s})^3} = 4.64\times10^{-8}\ \text{J/s} = 2.89\times10^{11}\ \text{eV/s}$. This large value of $\dfrac{dE}{dt}$ would mean that the electron would almost immediately lose all its energy!

EVALUATE: The classical physics result in Problem 32.51 must not apply to electrons in atoms.

32.53. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** $E_y(x,t) = E_{\max}e^{-k_C x}\cos(k_C x - \omega t)$.

$$\frac{\partial E_y}{\partial x} = E_{\max}(-k_C)e^{-k_C x}\cos(k_C x - \omega t) + E_{\max}(-k_C)e^{-k_C x}\sin(k_C x - \omega t).$$

$$\frac{\partial^2 E_y}{\partial x^2} = E_{\max}(+k_C^2)e^{-k_C x}\cos(k_C x - \omega t) + E_{\max}(+k_C^2)e^{-k_C x}\sin(k_C x - \omega t).$$

$$+E_{\max}(+k_C^2)e^{-k_C x}\sin(k_C x - \omega t) + E_{\max}(-k_C^2)e^{-k_C x}\cos(k_C x - \omega t).$$

$\dfrac{\partial^2 E_y}{\partial x^2} = +2E_{\max}k_C^2 e^{-k_C x}\sin(k_C x - \omega t)$. $\dfrac{\partial E_y}{\partial t} = +E_{\max}e^{-k_C x}\,\omega\sin(k_C x - \omega t)$.

Setting $\dfrac{\partial^2 E_y}{\partial x^2} = \dfrac{\mu \partial E_y}{\rho \partial t}$ gives $2E_{\max}k_C^2 e^{-k_C x}\sin(k_C x - \omega t) = \mu/\rho E_{\max}e^{-k_C x}\,\omega\sin(k_C x - \omega t)$. This will only be true if $\dfrac{2k_C^2}{\omega} = \dfrac{\mu}{\rho}$, or $k_C = \sqrt{\dfrac{\omega\mu}{2\rho}}$.

(b) The energy in the wave is dissipated by the i^2R heating of the conductor.

(c) $E_y = \dfrac{E_{y0}}{e} \Rightarrow k_C x = 1$, $x = \dfrac{1}{k_C} = \sqrt{\dfrac{2\rho}{\omega\mu}} = \sqrt{\dfrac{2(1.72\times10^{-8}\ \Omega\cdot\text{m})}{2\pi(1.0\times10^6\ \text{Hz})\mu_0}} = 6.60\times10^{-5}\,\text{m}$.

EVALUATE: The lower the frequency of the waves, the greater is the distance they can penetrate into a conductor. A dielectric (insulator) has a much larger resistivity and these waves can penetrate a greater distance in these materials.

32.54. **IDENTIFY** and **SET UP:** Since 60 Hz is in the range 25 Hz to 3 kHz, we use the formula $E_{\max} = \dfrac{350}{f}$ V/m, where f is in kHz. The intensity is $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$.

EXECUTE: The maximum electric field is $E_{\max} = \dfrac{350}{f}$ V/m $= \dfrac{350}{0.060}$ V/m = 5800 V/m. Now find the intensity for the maximum field.

$I = \frac{1}{2}\epsilon_0 cE_{\max}^2 = (1/2)(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\ \text{m/s})(5800\ \text{V/m})^2 = 4.5\times10^4\ \text{W/m}^2$

$= 45\ \text{kW/m}^2$, which is choice (c).

EVALUATE: At higher frequencies the intensity would be less because the maximum electric field, which is inversely proportional to the frequency, would be smaller.

32.55. **IDENTIFY and SET UP:** The maximum electric field is proportional to $1/f$, and the intensity is proportional to E_{max}^2.

EXECUTE: Since E_{max} is proportional to $1/f$, doubling f decreases the maximum field by $\frac{1}{2}$. Because the intensity is proportional to E_{max}^2, decreasing E_{max} by a factor of $\frac{1}{2}$ will decrease the intensity by a factor of $(\frac{1}{2})^2 = \frac{1}{4}$, which is choice (d).

EVALUATE: Higher frequencies could be more harmful, so we tolerate lower fields at higher frequency.

32.56. **IDENTIFY and SET UP:** In the frequency range 25 Hz to 3 kHz, for a given frequency the maximum electric field is $E_{\text{max}} = 350/f$ and the maximum electric field is $B_{\text{max}} = 5/f$. $B = cE$.

EXECUTE: For the electric field, the maximum intensity at a frequency f is $I_{\text{max}} = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2$. Since $E_{\text{max}} = 350/f$, the intensity is $I_{\text{max}}^E = \frac{1}{2}\epsilon_0 c\left(\frac{350}{f}\right)^2$.

The intensity in terms of the magnetic field is $I = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2 = \frac{1}{2}\epsilon_0 c(B_{\text{max}}c)^2 = \frac{1}{2}\epsilon_0 c^3 B_{\text{max}}^2$, where we have used $E_{\text{max}} = cB_{\text{max}}$. The maximum magnetic field is $B_{\text{max}} = 5/f$, so the maximum intensity for this magnetic field is $I_{\text{max}}^B = \frac{1}{2}\epsilon_0 c^3\left(\frac{5}{f}\right)^2$. Taking the ratio of the two intensities gives

$$\frac{I_{\text{max}}^E}{I_{\text{max}}^B} = \frac{\frac{1}{2}\epsilon_0 c\left(\frac{350}{f}\right)^2}{\frac{1}{2}\epsilon_0 c^3\left(\frac{5}{f}\right)^2} = \frac{1}{c^2}\left(\frac{350}{5}\right)^2 = 5.4\times10^{-14}.$$ The allowed intensity using the electric field limitation is *much* less than the allowed intensity using the magnetic field limitation, which is choice (b).

EVALUATE: The magnetic force on a charge due to an electromagnetic wave is normally much less than the electric force, so the intensity allowed for the electric field is much less than for the magnetic field.

33

THE NATURE AND PROPAGATION OF LIGHT

VP33.2.1. **IDENTIFY:** We have reflection and refraction. Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: **(a)** For reflected rays, the angle of reflection is equal to the angle of incidence. So the angle of reflection is 70.0°.

(b) Apply Snell's law. $n_a \sin\theta_a = n_b \sin\theta_b$. $(1.00)\sin 70.0° = (1.80)\sin\theta_b$. $\theta_b = 31.5°$.

EVALUATE: The light is bent toward the normal in the glass.

VP33.2.2. **IDENTIFY:** We have refraction at a flat surface, so Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$, $n = c/v$.

EXECUTE: **(a)** We want n. (1.33) sin 55.0° = $n$sin37.0°. n = 1.81.

(b) We want speed of light in the glass. $v = c/n = c/1.81 = 1.66\times10^8$ m/s.

EVALUATE: The light is bent toward the normal in the glass because $n_{glass} > n_{water}$.

VP33.2.3. **IDENTIFY:** We have refraction at a flat surface, so Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$, $\lambda_n = \dfrac{\lambda_0}{n}$, λ_{air} is almost the same as λ_{vacuum} .

EXECUTE: **(a)** We want n. $\lambda_n = \dfrac{\lambda_0}{n}$. $n = \lambda_0/\lambda_n$ = (635 nm)/(508 nm) = 1.25.

(b) We want θ_b (in the liquid). Apply $n_a \sin\theta_a = n_b \sin\theta_b$. $(1.00)\sin 35.0° = (1.25)\sin\theta_b$. $\theta_b = 27.3°$.

(c) We want the frequency in air. $f_a = c/\lambda_a$ = c/(635 nm) = 4.72×10^{14} Hz.

(d) We want the frequency in the liquid. The frequency does not change, so it is the same as in part (c): $f_a = 4.72\times10^{14}$ Hz.

EVALUATE: Our answer to (a) gives n = 1.25, which is reasonable because n is always greater than one.

VP33.2.4. **IDENTIFY:** We have refraction at a flat surface, so Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$, $\lambda_n = \dfrac{\lambda_0}{n}$, λ_{air} is almost the same as λ_{vacuum} .

EXECUTE: **(a)** We want θ_a (in ethanol). Apply $n_a \sin\theta_a = n_b \sin\theta_b$. $(1.36)\sin\theta_a = (1.309)\sin 85.0°$. $\theta_a = 73.5°$.

(b) We want $\lambda_{ice}/\lambda_{ethanol}$. Use $\lambda_n = \dfrac{\lambda_0}{n}$. $\lambda_{ice}/\lambda_{ethanol} = \dfrac{\dfrac{\lambda_0}{n_i}}{\dfrac{\lambda_0}{n_e}} = \dfrac{n_e}{n_i} = \dfrac{1.36}{1.309} = 1.04.$

EVALUATE: Since $n_i < n_e$, we expect that $\theta_i > \theta_e$, which is what we have found.

VP33.5.1. **IDENTIFY:** This problem deals with polarized light, so Malus's law applies.

SET UP: $I = I_0 \cos^2 \phi$.

EXECUTE: We want the intensity. $I = I_0 \cos^2 \phi$ = (255 W/m^2) $\cos^2$ 15.0° = 238 W/m^2.

EVALUATE: The filter axis and the polarization direction of the beam are closely aligned, so most of the light should get through, which our result shows.

VP33.5.2. **IDENTIFY:** This problem involves a polarizing filter, so Malus's law applies.

SET UP: Fig. VP33.5.2 illustrates the situation. $I = I_0 \cos^2 \phi$.

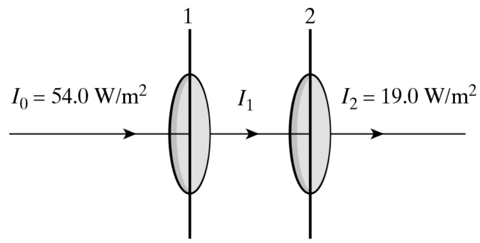


Figure VP33.5.2

EXECUTE: **(a)** We want the intensity after the first polarizer. A polarizer absorbs half the unpolarized light, so I_1 = (52.0 W/m^2)/2 = 27.0 W/m^2.

(b) We want ϕ . $I_2 = I_1 \cos^2 \phi$. 19.0 W/m^2 = (27.0 W/m^2) $\cos^2 \phi$. ϕ = 33.0°.

EVALUATE: The first polarizer reduced the intensity of the incident light by a greater percent than the second filter did.

VP33.5.3. **IDENTIFY:** This problem involves a polarizing filter, so Malus's law applies.

SET UP: Fig. VP33.5.3 illustrates the arrangement. $I = I_0 \cos^2 \phi$.

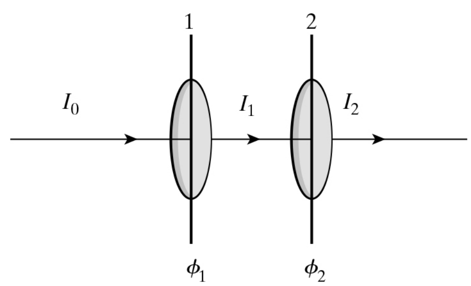


Figure VP33.5.3

EXECUTE: **(a)** We want I_1. $I_1 = I_0 \cos^2 \phi_1$ = (60.0 W/m^2) $\cos^2$ 25.0° = 49.3 W/m^2.

(b) We want I_2. I_1 is polarized at 25.0° from the *y* direction, so ϕ_2 = 50.0° – 25.0° = 25.0°.

$I_2 = I_1 \cos^2 \phi_2$ = (49.3 W/m^2) $\cos^2$ 25.0° = 40.5 W/m^2.

(c) We want I_2. ϕ = 50.0°. I_2 = (60.0 W/m^2) $\cos^2$ 50.0° = 25.8 W/m^2.

EVALUATE: Surprisingly we remove more of the light with just one filter, even though it is at the same angle as it was in the first case.

VP33.5.4. **IDENTIFY:** We are dealing with polarized light, so Malus's law applies.

SET UP: $I = I_0 \cos^2 \phi$. Two beams, one polarized and the other unpolarized, pass through the same filter simultaneously. We want the intensity of the emerging light.

EXECUTE: The emerging light has intensity $I = I_p + I_u$. $I_p = I_0 \cos^2\phi = I_0 \cos^2 30° = 0.750 I_0$.

$I_u = 0.500 I_0$. $I = 0.750I_0 + 0.500I_0 = 1.25I_0 = \dfrac{5}{4}I_0$.

EVALUATE: If there were no filter, the emerging intensity would be $2I_0$. With the filter, it is $1.25I_0$. So the final intensity is $1.25I_0/2.00I_0 = 0.625 = 62.5\%$ of the initial intensity. Therefore the filter has removed 37.5% of the light.

VP33.6.1. **IDENTIFY:** We are looking at polarization by reflection, so Brewster's law applies.
SET UP: $\tan\theta_p = n_b/n_a$.

EXECUTE: **(a)** We want the angle of incidence so the reflected light will be totally polarized. This is true at Brewster's angle $\tan\theta_p = n_b/n_a$, where $n_a = 1.00$ (air) and $n_b = 1.73$ (solid). This gives

$\tan\theta_p = 1.73/1.00$, so $\theta_p = 60.0°$.

(b) The reflected light is 100% polarized, but not all the light with that angle of polarization is reflected – some goes into the solid. So the transmitted light is only *partially* polarized.
EVALUATE: If water ($n = 1.33$) were on the solid, the polarizing angle would be arctan(1.73/1.33) = 52.4°.

VP33.6.2. **IDENTIFY:** This problem involves polarization by reflection, so Brewster's law applies as well as Snell's law.
SET UP: $\tan\theta_p = n_b/n_a$, $n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: **(a)** We want n for the glass. Since the reflected light is completely polarized, the angle of incidence must have Brewster's angle. Use $\tan\theta_p = n_b/n_a$ and solve for n.

$n = n_a \tan\theta_p = (1.00)\tan 57.0° = 1.54$.

(b) We want the angle of refraction. Use $n_a \sin\theta_a = n_b \sin\theta_b$. $(1.00)\sin 57.0° = (1.54)\sin\theta_b$. $\theta_b = 33.0°$.

EVALUATE: Since $n_{glass} > n_{air}$, the light should have been bent toward the normal in the glass, which in fact it was, so our results are reasonable.

VP33.6.3. **IDENTIFY:** This problem involves polarization by reflection, so Brewster's law applies.
SET UP: $\tan\theta_p = n_b/n_a$. We want the angle of incidence so no light reflects from the glass.

EXECUTE: **(a)** The incident light is already polarized perpendicular to the plane of incidence, so there is *no angle* at which no light reflects.
(b) At Brewster's angle, all the reflected light is polarized perpendicular to the plane of incidence. But for this light, none of it is polarized that way. So at Brewster's angle, none of it reflects. $\tan\theta_p = n_b/n_a$ = 1.66/1.00 = 1.66, so $\theta_p = 58.9°$.

EVALUATE: In part (b) all the light enters the glass.

VP33.6.4. **IDENTIFY:** This problem involves polarization by reflection, so Brewster's law applies.
SET UP: $\tan\theta_p = n_b/n_a$. At Brewster's angle, the refracted light is perpendicular to the reflected light.

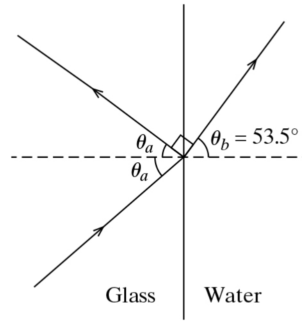


Figure VP33.6.4

EXECUTE: (a) We want θ_p. For 100% of the reflected light to be polarized, the angle of incidence must be Brewster's angle. The reflected and refracted rays are perpendicular, so $\theta_b + 90° + \theta_a = 180°$, which gives $\theta_a = 180° - 90° - \theta_b = 90° - 53.5° = 36.5°$.

EVALUATE: $n_{\text{glass}} > n_{\text{water}}$ so the light should be bent away from the normal in the water, which agrees with our result since θ_b (53.5°) > θ_a (36.5°).

33.1. IDENTIFY: For reflection, $\theta_r = \theta_a$.

SET UP: The desired path of the ray is sketched in Figure 33.1.

EXECUTE: $\tan\phi = \dfrac{14.0 \text{ cm}}{11.5 \text{ cm}}$, so $\phi = 50.6°$. $\theta_r = 90° - \phi = 39.4°$ and $\theta_r = \theta_a = 39.4°$.

EVALUATE: The angle of incidence is measured from the normal to the surface.

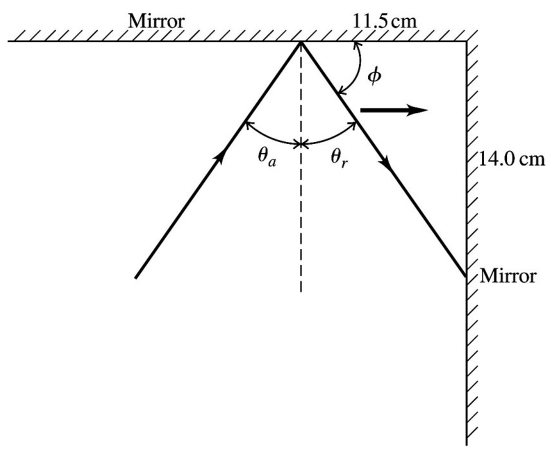


Figure 33.1

33.2. IDENTIFY: The speed and the wavelength of the light will be affected by the vitreous humor, but not the frequency.

SET UP: $n = \dfrac{c}{v}$. $v = f\lambda$. $\lambda = \dfrac{\lambda_0}{n}$.

EXECUTE: (a) $\lambda_v = \dfrac{\lambda_{0,v}}{n} = \dfrac{380 \text{ nm}}{1.34} = 284 \text{ nm}$. $\lambda_r = \dfrac{\lambda_{0,r}}{n} = \dfrac{750 \text{ nm}}{1.34} = 560 \text{ nm}$. The range is 284 nm to 560 nm.

(b) Calculate the frequency in air, where $v = c = 3.00\times10^8$ m/s.

$f_r = \dfrac{c}{\lambda_r} = \dfrac{3.00\times10^8 \text{ m/s}}{750\times10^{-9} \text{ m}} = 4.00\times10^{14}$ Hz. $f_v = \dfrac{c}{\lambda_v} = \dfrac{3.00\times10^8 \text{ m/s}}{380\times10^{-9} \text{ m}} = 7.89\times10^{14}$ Hz. The range is 4.00×10^{14} Hz to 7.89×10^{14} Hz.

(c) $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.34} = 2.24\times10^8$ m/s.

EVALUATE: The frequency range in air is the same as in the vitreous humor.

33.3. **IDENTIFY** and **SET UP:** Use $n = \dfrac{c}{v}$ and $\lambda = \dfrac{\lambda_0}{n}$ to calculate v and λ in the liquid.

EXECUTE: **(a)** $n = \dfrac{c}{v}$ so $v = \dfrac{c}{n} = \dfrac{2.998\times10^8 \text{ m/s}}{1.47} = 2.04\times10^8$ m/s.

(b) $\lambda = \dfrac{\lambda_0}{n} = \dfrac{650 \text{ nm}}{1.47} = 442$ nm.

EVALUATE: Light is slower in the liquid than in vacuum. By $v = f\lambda$, when v is smaller, λ is smaller.

33.4. **IDENTIFY:** In air, $c = f\lambda_0$. In glass, $\lambda = \dfrac{\lambda_0}{n}$.

SET UP: $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $\lambda_0 = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{5.80\times10^{14} \text{ Hz}} = 517$ nm.

(b) $\lambda = \dfrac{\lambda_0}{n} = \dfrac{517 \text{ nm}}{1.52} = 340$ nm.

EVALUATE: In glass the light travels slower than in vacuum and the wavelength is smaller.

33.5. **IDENTIFY:** This problem involves refraction at a glass-liquid boundary. Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$, $n = c/v$, $\lambda_n = \dfrac{\lambda_0}{n}$. In the liquid we want the speed, wavelength, and frequency of the light. Let subscript g denote glass and L denote the liquid.

EXECUTE: Find n_g: $n_g = c/v = c/(1.85\times10^8 \text{ m/s}) = 1.622$.

Find n_L: Use $n_a \sin\theta_a = n_b \sin\theta_b$. $(1.622)\sin 38.0° = n_L \sin 44.7°$. $n_L = 1.419$.

$v_L = c/n_L = c/(1.419) = 2.11\times10^8$ m/s. $\lambda_L = \dfrac{\lambda_0}{n_L}$. $\lambda_g = \dfrac{\lambda_0}{n_g}$, so $\lambda_0 = n_g\lambda_g$. Combining gives

$$\lambda_L = \frac{\lambda_g n_g}{n_L} = \frac{(365 \text{ nm})(1.622)}{1.419} = 417 \text{ nm}.$$

The frequency does not change, so $f_L = f_g = 5.07\times10^{14}$ Hz.

EVALUATE: The light was bent away from the normal in the liquid, so it should be true that $n_L < n_g$. We found that $n_g = 1.622$ and $n_L = 1.419$, so our result is reasonable.

33.6. **IDENTIFY:** $\lambda = \dfrac{\lambda_0}{n}$.

SET UP: From Table 33.1, $n_{\text{water}} = 1.333$ and $n_{\text{benzene}} = 1.501$.

EXECUTE: $\lambda_{\text{water}} n_{\text{water}} = \lambda_{\text{benzene}} n_{\text{benzene}} = \lambda_0$. $\lambda_{\text{benzene}} = \lambda_{\text{water}}\left(\dfrac{n_{\text{water}}}{n_{\text{benzene}}}\right) = (526 \text{ nm})\left(\dfrac{1.333}{1.501}\right) = 467$ nm.

EVALUATE: λ is smallest in benzene, since n is largest for benzene.

33.7. **IDENTIFY:** Apply the law of reflection and Snell's law to calculate θ_r and θ_b. The angles in these equations are measured with respect to the normal, not the surface.

SET UP: The incident, reflected and refracted rays are shown in Figure 33.7. The law of reflection is $\theta_r = \theta_a$, and Snell's law is $n_a \sin\theta_a = n_b \sin\theta_b$.

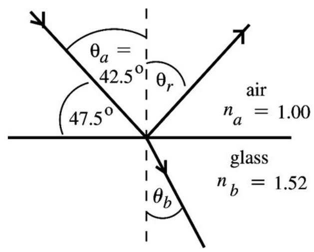


Figure 33.7

EXECUTE: **(a)** $\theta_r = \theta_a = 42.5°$ The reflected ray makes an angle of $90.0° - \theta_r = 47.5°$ with the surface of the glass.

(b) $n_a \sin\theta_a = n_b \sin\theta_b$, where the angles are measured from the normal to the interface.

$$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.00)(\sin 42.5°)}{1.66} = 0.4070.$$

$\theta_b = 24.0°$.

The refracted ray makes an angle of $90.0° - \theta_b = 66.0°$ with the surface of the glass.

EVALUATE: The light is bent toward the normal when the light enters the material of larger refractive index.

33.8. **IDENTIFY:** The time delay occurs because the beam going through the transparent material travels slower than the beam in air.

SET UP: $v = \dfrac{c}{n}$ in the material, but $v = c$ in air.

EXECUTE: The time for the beam traveling in air to reach the detector is

$t = \dfrac{d}{c} = \dfrac{2.50 \text{ m}}{3.00\times10^8 \text{ m/s}} = 8.33\times10^{-9}$ s. The light traveling in the block takes time

$t = 8.33\times10^{-9} \text{ s} + 6.25\times10^{-9} \text{ s} = 1.46\times10^{-8}$ s. The speed of light in the block is

$v = \dfrac{d}{t} = \dfrac{2.50 \text{ m}}{1.46\times10^{-8} \text{ s}} = 1.71\times10^8$ m/s. The refractive index of the block is $n = \dfrac{c}{v} = \dfrac{3.00\times10^8 \text{ m/s}}{1.71\times10^8 \text{ m/s}} = 1.75.$

EVALUATE: $n > 1$, as it must be, and 1.75 is a reasonable index of refraction for a transparent material such as plastic.

33.9. **IDENTIFY** and **SET UP:** Use Snell's law to find the index of refraction of the plastic and then use $n = \dfrac{c}{v}$ to calculate the speed v of light in the plastic.

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$.

$$n_b = n_a\left(\frac{\sin\theta_a}{\sin\theta_b}\right) = 1.00\left(\frac{\sin 62.7°}{\sin 48.1°}\right) = 1.194.$$

$n = \dfrac{c}{v}$ so $v = \dfrac{c}{n} = (3.00\times10^8 \text{ m/s})/1.194 = 2.51\times10^8$ m/s.

EVALUATE: Light is slower in plastic than in air. When the light goes from air into the plastic it is bent toward the normal.

33.10. IDENTIFY: Apply Snell's law at both interfaces.

SET UP: The path of the ray is sketched in Figure 33.10. Table 33.1 gives $n = 1.329$ for the methanol.

EXECUTE: **(a)** At the air-glass interface $(1.00)\sin 41.3° = n_{\text{glass}} \sin\alpha$. At the glass-methanol interface $n_{\text{glass}} \sin\alpha = (1.329)\sin\theta$. Combining these two equations gives $\sin 41.3° = 1.329 \sin\theta$ and $\theta = 29.8°$.

(b) The same figure applies apply as for part (a), except $\theta = 20.2°$. $(1.00)\sin 41.3° = n\sin 20.2°$ and $n = 1.91$.

EVALUATE: The angle α is 25.2°. The index of refraction of methanol is less than that of the glass and the ray is bent away from the normal at the glass $\rightarrow$ methanol interface. The unknown liquid has an index of refraction greater than that of the glass, so the ray is bent toward the normal at the glass $\rightarrow$ liquid interface.

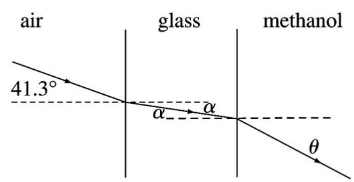


Figure 33.10

33.11. IDENTIFY: The figure shows the angle of incidence and angle of refraction for light going from the water into material *X*. Snell's law applies at the air-water and water-*X* boundaries.

SET UP: Snell's law says $n_a \sin\theta_a = n_b \sin\theta_b$. Apply Snell's law to the refraction from material *X* into the water and then from the water into the air.

EXECUTE: **(a)** Material *X* to water: $n_a = n_X$, $n_b = n_{\text{w}} = 1.333$. $\theta_a = 25°$ and $\theta_b = 48°$.

$$n_a = n_b\left(\frac{\sin\theta_b}{\sin\theta_a}\right) = (1.333)\left(\frac{\sin 48°}{\sin 25°}\right) = 2.34.$$

(b) Water to air: As Figure 33.11 shows, $\theta_a = 48°$. $n_a = 1.333$ and $n_b = 1.00$.

$$\sin\theta_b = \left(\frac{n_a}{n_b}\right)\sin\theta_a = (1.333)\sin 48° = 82°.$$

EVALUATE: $n > 1$ for material *X*, as it must be.

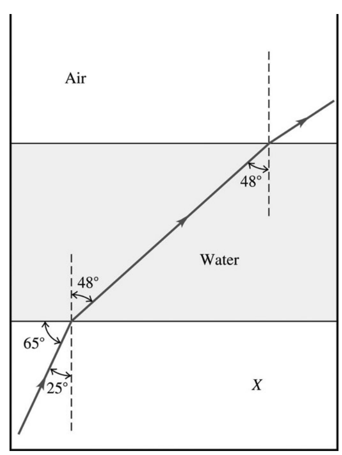


Figure 33.11

33.12. **IDENTIFY:** Apply Snell's law to the refraction at each interface.

SET UP: $n_{\text{air}} = 1.00$. $n_{\text{water}} = 1.333$.

EXECUTE: **(a)** $\theta_{\text{water}} = \arcsin\left(\dfrac{n_{\text{air}}}{n_{\text{water}}}\sin\theta_{\text{air}}\right) = \arcsin\left(\dfrac{1.00}{1.333}\sin 35.0^\circ\right) = 25.5^\circ$.

EVALUATE: **(b)** This calculation has no dependence on the glass because we can omit that step in the chain: $n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{glass}}\sin\theta_{\text{glass}} = n_{\text{water}}\sin\theta_{\text{water}}$.

33.13. **IDENTIFY:** This problem involves refraction at a boundary, so Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$, $\lambda_n = \dfrac{\lambda_0}{n}$. Let the subscript s denote the solid and g denote the glass. We want θ_s.

EXECUTE: Snell's law gives $n_g \sin\theta_g = n_s \sin\theta_s$, so see that we need n_g and n_s. $\lambda_g = \dfrac{\lambda_0}{n_g}$ and $\lambda_s = \dfrac{\lambda_0}{n_s}$.

$\dfrac{\lambda_g}{\lambda_s} = \dfrac{447\text{ nm}}{315\text{ nm}} = 1.419 = \dfrac{\lambda_0/n_g}{\lambda_0/n_s} = \dfrac{n_s}{n_g}$, so $n_g = 1.419n_g$. Thus $n_g \sin 62.0^\circ = 1.419 n_g \sin\theta_s$, so $\theta_s = 38.5^\circ$.

EVALUATE: $n_s > n_g$, so the light should be bent toward the normal, which agrees with our result.

33.14. **IDENTIFY:** The wavelength of the light depends on the index of refraction of the material through which it is traveling, and Snell's law applies at the water-glass interface.

SET UP: $\lambda_0 = \lambda n$ so $\lambda_{\text{w}} n_{\text{w}} = \lambda_{\text{gl}} n_{\text{gl}}$. Snell's law gives $n_{\text{gl}}\sin\theta_{\text{gl}} = n_{\text{w}}\sin\theta_{\text{w}}$.

EXECUTE: $n_{\text{gl}} = n_{\text{w}}\left(\dfrac{\lambda_{\text{w}}}{\lambda_{\text{gl}}}\right) = (1.333)\left(\dfrac{726\text{ nm}}{544\text{ nm}}\right) = 1.779$. Now apply $n_{\text{gl}}\sin\theta_{\text{gl}} = n_{\text{w}}\sin\theta_{\text{w}}$.

$\sin\theta_{\text{gl}} = \left(\dfrac{n_{\text{w}}}{n_{\text{gl}}}\right)\sin\theta_{\text{w}} = \left(\dfrac{1.333}{1.779}\right)\sin 56.0^\circ = 0.6212$. $\theta_{\text{gl}} = 38.4^\circ$.

EVALUATE: $\theta_{\text{gl}} < \theta_{\text{w}}$ because $n_{\text{gl}} > n_{\text{w}}$.

33.15. **IDENTIFY:** The critical angle for total internal reflection is θ_a that gives $\theta_b = 90^\circ$ in Snell's law.

SET UP: In Figure 33.15 the angle of incidence θ_a is related to angle β by $\theta_a + \beta = 90^\circ$.

EXECUTE: **(a)** Calculate θ_a that gives $\theta_b = 90^\circ$. $n_a = 1.60$, $n_b = 1.00$ so $n_a \sin\theta_a = n_b \sin\theta_b$ gives $(1.60)\sin\theta_a = (1.00)\sin 90^\circ$. $\sin\theta_a = \dfrac{1.00}{1.60}$ and $\theta_a = 38.7^\circ$. $\beta = 90^\circ - \theta_a = 51.3^\circ$.

(b) $n_a = 1.60$, $n_b = 1.333$. $(1.60)\sin\theta_a = (1.333)\sin 90^\circ$. $\sin\theta_a = \dfrac{1.333}{1.60}$ and $\theta_a = 56.4^\circ$. $\beta = 90^\circ - \theta_a = 33.6^\circ$.

EVALUATE: The critical angle increases when the ratio $\dfrac{n_a}{n_b}$ decreases.

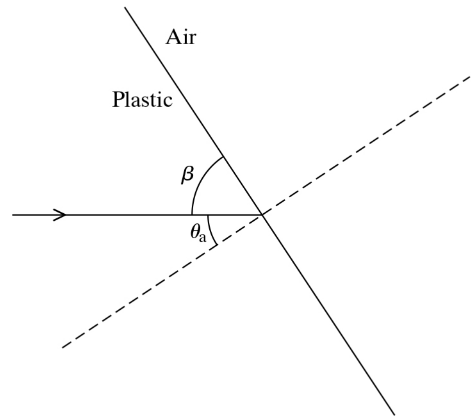


Figure 33.15

33.16. **IDENTIFY:** No light will enter the water if total internal reflection occurs at the glass-water boundary. Snell's law applies at the boundary.

SET UP: Find n_g, the refractive index of the glass. Then apply Snell's law at the boundary.

$n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: $n_g \sin 36.2° = n_w \sin 49.8°$. $n_g = (1.333)\left(\dfrac{\sin 49.8°}{\sin 36.2°}\right) = 1.724$. Now find θ_{crit} for the glass to water refraction. $n_g \sin\theta_{crit} = n_w \sin 90.0°$. $\sin\theta_{crit} = \dfrac{1.333}{1.724}$ and $\theta_{crit} = 50.6°$.

EVALUATE: For $\theta > 50.6°$ at the glass-water boundary, no light is refracted into the water.

33.17. **IDENTIFY:** Use the critical angle to find the index of refraction of the liquid.

SET UP: Total internal reflection requires that the light be incident on the material with the larger n, in this case the liquid. Apply $n_a \sin\theta_a = n_b \sin\theta_b$ with a = liquid and b = air, so $n_a = n_{liq}$ and $n_b = 1.0$.

EXECUTE: $\theta_a = \theta_{crit}$ when $\theta_b = 90°$, so $n_{liq} \sin\theta_{crit} = (1.0)\sin 90°$.

$$n_{liq} = \frac{1}{\sin\theta_{crit}} = \frac{1}{\sin 42.5°} = 1.48.$$

(a) $n_a \sin\theta_a = n_b \sin\theta_b$ (a = liquid, b = air).

$$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.48)\sin 35.0°}{1.0} = 0.8489 \text{ and } \theta_b = 58.1°.$$

(b) Now $n_a \sin\theta_a = n_b \sin\theta_b$ with a = air, b = liquid.

$$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.0)\sin 35.0°}{1.48} = 0.3876 \text{ and } \theta_b = 22.8°.$$

EVALUATE: Light traveling from liquid to air is bent away from the normal. Light traveling from air to liquid is bent toward the normal.

33.18. **IDENTIFY:** Since the refractive index of the glass is greater than that of air or water, total internal reflection will occur at the cube surface if the angle of incidence is greater than or equal to the critical angle.

SET UP: At the critical angle θ_{crit}, Snell's law gives $n_{glass} \sin\theta_{crit} = n_{air} \sin 90°$ and likewise for water.

EXECUTE: **(a)** At the critical angle θ_{crit}, $n_{glass} \sin\theta_{crit} = n_{air} \sin 90°$.

$1.62 \sin\theta_{crit} = (1.00)(1)$ and $\theta_{crit} = 38.1°$.

(b) Using the same procedure as in part (a), we have $1.62 \sin\theta_{crit} = 1.333 \sin 90°$ and $\theta_{crit} = 55.4°$.

EVALUATE: Since the refractive index of water is closer to the refractive index of glass than the refractive index of air is, the critical angle for glass-to-water is greater than for glass-to-air.

33.19. IDENTIFY and **SET UP:** For glass $\rightarrow$ water, $\theta_{\text{crit}} = 48.7°$. Apply Snell's law with $\theta_a = \theta_{\text{crit}}$ to calculate the index of refraction n_a of the glass.

EXECUTE: $n_a \sin\theta_{\text{crit}} = n_b \sin 90°$, so $n_a = \dfrac{n_b}{\sin\theta_{\text{crit}}} = \dfrac{1.333}{\sin 48.7°} = 1.77$

EVALUATE: For total internal reflection to occur the light must be incident in the material of larger refractive index. Our results give $n_{\text{glass}} > n_{\text{water}}$, in agreement with this.

33.20. IDENTIFY: The largest angle of incidence for which any light refracts into the air is the critical angle for water $\rightarrow$ air.

SET UP: Figure 33.20 shows a ray incident at the critical angle and therefore at the edge of the circle of light. The radius of this circle is r and $d = 10.0$ m is the distance from the ring to the surface of the water.

EXECUTE: From the figure, $r = d\tan\theta_{\text{crit}}$. θ_{crit} is calculated from $n_a \sin\theta_a = n_b \sin\theta_b$ with $n_a = 1.333$, $\theta_a = \theta_{\text{crit}}$, $n_b = 1.00$, and $\theta_b = 90°$. $\sin\theta_{\text{crit}} = \dfrac{(1.00)\sin 90°}{1.333}$ and $\theta_{\text{crit}} = 48.6°$.

$r = (10.0\text{ m})\tan 48.6° = 11.3\text{ m}.$

$A = \pi r^2 = \pi(11.3\text{ m})^2 = 401\text{ m}^2.$

EVALUATE: When the incident angle in the water is larger than the critical angle, no light refracts into the air.

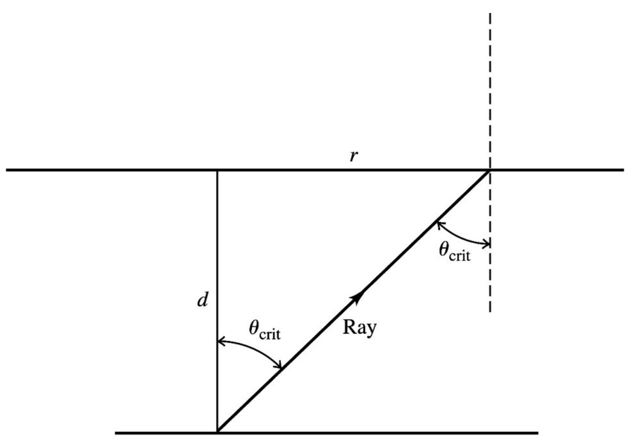


Figure 33.20

33.21. IDENTIFY: If no light refracts out of the glass at the glass to air interface, then the incident angle at that interface is θ_{crit}.

SET UP: The ray has an angle of incidence of $0°$ at the first surface of the glass, so enters the glass without being bent, as shown in Figure 33.21. The figure shows that $\alpha + \theta_{\text{crit}} = 90°$.

EXECUTE: **(a)** For the glass-air interface $\theta_a = \theta_{\text{crit}}$, $n_a = 1.52$, $n_b = 1.00$, and $\theta_b = 90°$.

$n_a \sin\theta_a = n_b \sin\theta_b$ gives $\sin\theta_{\text{crit}} = \dfrac{(1.00)(\sin 90°)}{1.52}$ and $\theta_{\text{crit}} = 41.1°$. $\alpha = 90° - \theta_{\text{crit}} = 48.9°$.

(b) Now the second interface is glass $\rightarrow$ water and $n_b = 1.333$. $n_a \sin\theta_a = n_b \sin\theta_b$ gives

$\sin\theta_{\text{crit}} = \dfrac{(1.333)(\sin 90°)}{1.52}$ and $\theta_{\text{crit}} = 61.3°$. $\alpha = 90° - \theta_{\text{crit}} = 28.7°$.

EVALUATE: The critical angle increases when the air is replaced by water.

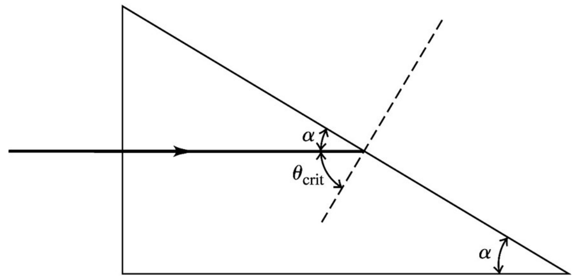


Figure 33.21

33.22. **IDENTIFY:** This problem involves Snell's law and total internal reflection.
SET UP: We want the wavelength in the plastic. Let subscript p denote the plastic and g denote the glass. $n_a \sin\theta_a = n_b \sin\theta_b$, $\lambda_n = \dfrac{\lambda_0}{n}$.

EXECUTE: At the critical angle $n_g \sin 48.6° = n_p \sin 90° = n_p$. $\lambda_p = \dfrac{\lambda_0}{n_p} = \dfrac{\lambda_0}{\lambda_g \sin 48.6°}$. $\lambda_g = \dfrac{\lambda_0}{n_g}$, so $\lambda_0 = n_g \lambda_g$. Combining gives $\lambda_p = \dfrac{n_g \lambda_g}{n_g \sin 48.6°} = \dfrac{\lambda_g}{\sin 48.6°} = \dfrac{350 \text{ nm}}{\sin 48.6°} = 467$ nm.

EVALUATE: Total internal reflection occurs only for light going from a high-n material to a low-n material, such as glass to water or water to air.

33.23. **IDENTIFY:** The index of refraction depends on the wavelength of light, so the light from the red and violet ends of the spectrum will be bent through different angles as it passes into the glass. Snell's law applies at the surface.
SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$. From the graph in Figure 33.17 in the textbook, for $\lambda = 400$ nm (the violet end of the visible spectrum), $n = 1.67$ and for $\lambda = 700$ nm (the red end of the visible spectrum), $n = 1.62$. The path of a ray with a single wavelength is sketched in Figure 33.23.

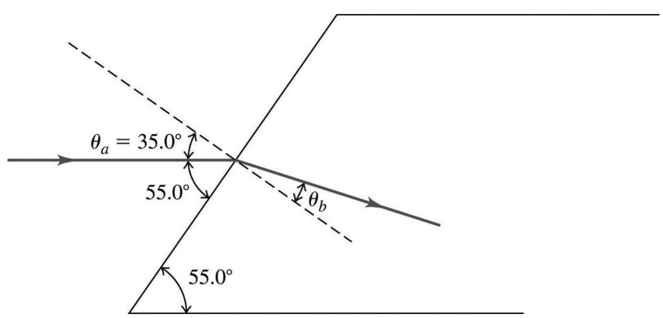


Figure 33.23

EXECUTE: For $\lambda = 400$ nm, $\sin\theta_b = \dfrac{n_a}{n_b}\sin\theta_a = \dfrac{1.00}{1.67}\sin 35.0°$, so $\theta_b = 20.1°$. For $\lambda = 700$ nm, $\sin\theta_b = \dfrac{1.00}{1.62}\sin 35.0°$, so $\theta_b = 20.7°$. $\Delta\theta$ is about $0.6°$.

EVALUATE: This angle is small, but the separation of the beams could be fairly large if the light travels through a fairly large slab.

33.24. **IDENTIFY:** The red and violet light will be bent through different angles in the glass because they have slightly different indexes of refraction. Use Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$.

SET UP: Apply Snell's law twice: the first time use the index of refraction for red light $(n = 2.41)$ and the second time use the index of refraction for violet light $(n = 2.46)$. Assume that the index of refraction for air is $n = 1.00$.

EXECUTE: For red light Snell's law gives $(1.00)\sin 53.5^\circ = (2.41)\sin\theta_{\text{red}}$. Solving this equation we find $\theta_{\text{red}} = 19.48^\circ$. For violet light Snell's law gives $(1.00)\sin 53.5^\circ = (2.46)\sin\theta_{\text{violet}}$. Solving this equation we find $\theta_{\text{violet}} = 19.07^\circ$. From these two values we can calculate the angle between the two initially coincident rays: $\Delta\theta = \theta_{\text{red}} - \theta_{\text{violet}} = 19.48° - 19.07° = 0.41^\circ$.

EVALUATE: Violet light is refracted more than red light since it has the larger index of refraction. Although the angular separation between the red and the blue rays is small, it is easily noticeable under the right circumstances.

33.25. **IDENTIFY:** Snell's law is $n_a \sin\theta_a = n_b \sin\theta_b$. $v = \dfrac{c}{n}$.

SET UP: $a = \text{air}$, $b = \text{glass}$.

EXECUTE: **(a)** red: $n_b = \dfrac{n_a \sin\theta_a}{\sin\theta_b} = \dfrac{(1.00)\sin 57.0°}{\sin 38.1°} = 1.36$. violet: $n_b = \dfrac{(1.00)\sin 57.0°}{\sin 36.7°} = 1.40$.

(b) red: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.36} = 2.21\times10^8 \text{ m/s}$; violet: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.40} = 2.14\times10^8 \text{ m/s}$.

EVALUATE: n is larger for the violet light and therefore this light is bent more toward the normal, and the violet light has a smaller speed in the glass than the red light.

33.26. **IDENTIFY:** This problem involves birefringence and a quarter-wave plate.

SET UP: For calcite, n = 1.658 and 1.486 for λ = 589 nm in vacuum, $v = c/n$, and $\lambda = cT$. For this light, what should be the minimum thickness of the quarter-wave plate?

EXECUTE: To cancel, the difference in time between the two waves must be ¼ of the period T. The two waves differ because there are two indices of refraction. $t_1 = d/v_1$ and $t_2 = d/v_2$.

$\Delta t = t_2 - t_1 = \dfrac{d}{v_2} - \dfrac{d}{v_1} = \dfrac{1}{4}T$. Using $v = c/n$ and $\lambda = cT$ gives $\dfrac{d}{c/n_2} - \dfrac{d}{c/n_1} = \dfrac{1}{4}T = \dfrac{1}{4}\left(\dfrac{\lambda}{c}\right)$. Simplifying and solving for d gives $d = \dfrac{\lambda}{4(n_2 - n_2)} = \dfrac{589 \text{ nm}}{4(1.658 - 1.486)} = 856 \text{ nm}$.

EVALUATE: This is the minimum thickness. If the time difference is $T + T/4$, $2T + T/4$, … , cancellation will also occur.

33.27. **IDENTIFY:** The first polarizer filters out half the incident light. The fraction filtered out by the second polarizer depends on the angle between the axes of the two filters.

SET UP: Use Malus's law: $I = I_0 \cos^2\phi$.

EXECUTE: After the first filter, $I = \frac{1}{2}I_0$. After the second filter, $I = (\frac{1}{2}I_0)\cos^2\phi$, which gives $I = \left(\frac{1}{2}I_0\right)\cos^2 30.0° = 0.375I_0$.

EVALUATE: The only variable that affects the answer is the angle between the axes of the two polarizers.

33.28. IDENTIFY: This problem involves polarization by reflection and Snell's law.

SET UP: $\tan\theta_p = n_b/n_a$, $n_a \sin\theta_a = n_b \sin\theta_b$. We want to find polarizing angles. For light traveling from *a* to *b*, the polarizing angle is given by $\tan\theta_p = n_b/n_a$. At this angle, 100% of the reflected light is polarized. Use the critical angle to relate the indices of refraction. Let subscript *L* denote the liquid and *s* the solid.

EXECUTE: **(a)** $n_s \sin 38.7° = 0.6252 n_s = n_L \sin 90° = n_L$. Now use Brewster's law. $\tan\theta_p = n_L/n_s = 0.6252 n_s/n_s = 0.6252$. $\theta_p = 32.0°$.

(b) If we reverse the rays, we get $\tan\theta_p = n_s/n_L = n_s/(0.6252 n_s) = 1.599$. $\theta_p = 58.0°$.

EVALUATE: Of course we would not have total internal reflection for the rays in part (b) since that only occurs for light going from high-*n* to low-*n* materials. However polarization would still occur in the reflected beam.

33.29. IDENTIFY: When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity I_{max} is incident on a polarizer, the transmitted intensity is $I = I_{max}\cos^2\phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

SET UP: For the second polarizer $\phi = 60°$. For the third polarizer, $\phi = 90° - 60° = 30°$.

EXECUTE: **(a)** At point *A* the intensity is $I_0/2$ and the light is polarized along the vertical direction. At point *B* the intensity is $(I_0/2)(\cos 60°)^2 = 0.125 I_0$, and the light is polarized along the axis of the second polarizer. At point *C* the intensity is $(0.125 I_0)(\cos 30°)^2 = 0.0938 I_0$.

(b) Now for the last filter $\phi = 90°$ and $I = 0$.

EVALUATE: Adding the middle filter increases the transmitted intensity.

33.30. IDENTIFY: Set $I = I_0/10$, where *I* is the intensity of light passed by the second polarizer.

SET UP: When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity I_{max} is incident on a polarizer, the transmitted intensity is $I = I_{max}\cos^2\phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

EXECUTE: **(a)** After the first filter $I = \dfrac{I_0}{2}$ and the light is polarized along the vertical direction. After the second filter we want $I = \dfrac{I_0}{10}$, so $\dfrac{I_0}{10} = \left(\dfrac{I_0}{2}\right)(\cos\phi)^2$. $\cos\phi = \sqrt{2/10}$ and $\phi = 63.4°$.

(b) Now the first filter passes the full intensity I_0 of the incident light. For the second filter $\dfrac{I_0}{10} = I_0(\cos\phi)^2$. $\cos\phi = \sqrt{1/10}$ and $\phi = 71.6°$.

EVALUATE: When the incident light is polarized along the axis of the first filter, ϕ must be larger to achieve the same overall reduction in intensity than when the incident light is unpolarized.

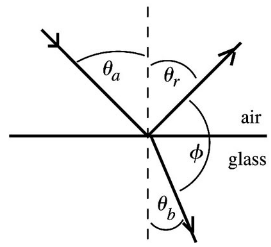
33.31. IDENTIFY and SET UP: Reflected beam completely linearly polarized implies that the angle of incidence equals the polarizing angle, so $\theta_p = 54.5°$. Use Brewster's law, $\tan\theta_p = \dfrac{n_b}{n_a}$, to calculate the refractive index of the glass. Then use Snell's law to calculate the angle of refraction. See Figure 33.29.

EXECUTE: **(a)** $\tan\theta_p = \dfrac{n_b}{n_a}$ gives $n_{glass} = n_{air}\tan\theta_p = (1.00)\tan 54.5° = 1.40$.

(b) $n_a \sin\theta_a = n_b \sin\theta_b$.

$\sin\theta_b = \dfrac{n_a \sin\theta_a}{n_b} = \dfrac{(1.00)\sin 54.5°}{1.40} = 0.5815$ and $\theta_b = 35.5°$.

EVALUATE:



Note: $\phi = 180.0° - \theta_r - \theta_b$ and $\theta_r = \theta_a$. Thus $\phi = 180.0° - 54.5° - 35.5° = 90.0°$; the reflected ray and the refracted ray are perpendicular to each other. This agrees with Figure 33.28 in the textbook.

Figure 33.29

33.32. **IDENTIFY:** The reflected light is completely polarized when the angle of incidence equals the polarizing angle θ_p, where $\tan\theta_\text{p} = \dfrac{n_b}{n_a}$.

SET UP: $n_b = 1.66$.

EXECUTE: **(a)** $n_a = 1.00$. $\tan\theta_\text{p} = \dfrac{1.66}{1.00}$ and $\theta_\text{p} = 58.9°$.

(b) $n_a = 1.333$. $\tan\theta_\text{p} = \dfrac{1.66}{1.333}$ and $\theta_\text{p} = 51.2°$.

EVALUATE: The polarizing angle depends on the refractive indicies of both materials at the interface.

33.33. **IDENTIFY:** When unpolarized light of intensity I_0 is incident on a polarizing filter, the transmitted light has intensity $\frac{1}{2}I_0$ and is polarized along the filter axis. When polarized light of intensity I_0 is incident on a polarizing filter the transmitted light has intensity $I_0 \cos^2\phi$.

SET UP: For the second filter, $\phi = 62.0° - 25.0° = 37.0°$.

EXECUTE: After the first filter the intensity is $\frac{1}{2}I_0 = 10.0\ \text{W/cm}^2$ and the light is polarized along the axis of the first filter. The intensity after the second filter is $I = I_0\cos^2\phi$, where $I_0 = 10.0\ \text{W/cm}^2$ and $\phi = 37.0°$. This gives $I = 6.38\ \text{W/cm}^2$.

EVALUATE: The transmitted intensity depends on the angle between the axes of the two filters.

33.34. **IDENTIFY:** Use the transmitted intensity when all three polarizers are present to solve for the incident intensity I_0. Then repeat the calculation with only the first and third polarizers.

SET UP: For unpolarized light incident on a filter, $I = \frac{1}{2}I_0$ and the light is linearly polarized along the filter axis. For polarized light incident on a filter, $I = I_\text{max}(\cos\phi)^2$, where I_max is the intensity of the incident light, and the emerging light is linearly polarized along the filter axis.

EXECUTE: With all three polarizers, if the incident intensity is I_0 the transmitted intensity is

$I = \frac{1}{2}I_0 \cos^2 23.0° \cos^2(62.0° - 23.0°) = 0.2559 I_0$. $I_0 = \dfrac{I}{0.2559} = \dfrac{55.0\ \text{W/cm}^2}{0.2559} = 215\ \text{W/cm}^2$. With only the first and third polarizers, $I = \frac{1}{2}I_0 \cos^2 62.0° = 0.110 I_0 = (0.110)(215\ \text{W/cm}^2) = 23.7\ \text{W/cm}^2$.

EVALUATE: The transmitted intensity is greater when all three filters are present.

33.35. IDENTIFY: The shorter the wavelength of light, the more it is scattered. The intensity is inversely proportional to the fourth power of the wavelength.

SET UP: The intensity of the scattered light is proportional to $1/\lambda^4$; we can write it as $I = (\text{constant})/\lambda^4$.

EXECUTE: **(a)** Since I is proportional to $1/\lambda^4$, we have $I = (\text{constant})/\lambda^4$. Taking the ratio of the intensity of the red light to that of the green light gives

$$\frac{I_\text{R}}{I} = \frac{(\text{constant})/\lambda_\text{R}^4}{(\text{constant})/\lambda_\text{G}^4} = \left(\frac{\lambda_\text{G}}{\lambda_\text{R}}\right)^4 = \left(\frac{532\text{ nm}}{685\text{ nm}}\right)^4 = 0.364, \text{ so } I_\text{R} = 0.364I.$$

(b) Following the same procedure as in part (a) gives $\dfrac{I_\text{V}}{I} = \left(\dfrac{\lambda_\text{G}}{\lambda_\text{V}}\right)^4 = \left(\dfrac{532\text{ nm}}{415\text{ nm}}\right)^4 = 2.70$, so $I_\text{V} = 2.70I$.

EVALUATE: In the scattered light, the intensity of the short-wavelength violet light is about 7 times as great as that of the red light, so this scattered light will have a blue-violet color.

33.36. IDENTIFY: We use measurements to determine the index of refraction for water. Snell's law applies. $n_a \sin\theta_a = n_b \sin\theta_b$

SET UP and EXECUTE: **(a)** Estimate: s = 8.0 mm.

(b) Estimate: d = 25 mm.

(c) Using Fig. P33.36 in the textbook, we have $\tan\theta_a = \dfrac{d}{H}$, $\tan\theta_b = \dfrac{s+d}{H}$.

(d) Apply Snell's law. $n_a \sin\theta_a = (1.00)\sin\theta_b$. $\sin\theta_a \approx \tan\theta_a$. $n\dfrac{d}{H} = \dfrac{s+d}{H}$. $n = \dfrac{s+d}{d} = 1 + \dfrac{s}{d}$.

(e) $n = 1 + \dfrac{8.0\text{ mm}}{25\text{ mm}} = 1.3$.

EVALUATE: This result is very close to the accepted value of 1.33, which is surprising given the rather crude estimates involved.

33.37. IDENTIFY: Snell's law applies to the sound waves in the heart.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$. If θ_a is the critical angle then $\theta_b = 90°$. For air, $n_\text{air} = 1.00$. For heart muscle, $n_\text{mus} = \dfrac{344\text{ m/s}}{1480\text{ m/s}} = 0.2324$.

EXECUTE: **(a)** $n_a \sin\theta_a = n_b \sin\theta_b$ gives $(1.00)\sin(9.73°) = (0.2324)\sin\theta_b$. $\sin\theta_b = \dfrac{\sin(9.73°)}{0.2324}$ so $\theta_b = 46.7°$.

(b) $(1.00)\sin\theta_\text{crit} = (0.2324)\sin 90°$ gives $\theta_\text{crit} = 13.4°$.

EVALUATE: To interpret a sonogram, it should be important to know the true direction of travel of the sound waves within muscle. This would require knowledge of the refractive index of the muscle.

33.38. IDENTIFY: Use the change in transit time to find the speed v of light in the slab, and then apply $n = \dfrac{c}{v}$ and $\lambda = \dfrac{\lambda_0}{n}$.

SET UP: It takes the light an additional 4.2 ns to travel 0.840 m after the glass slab is inserted into the beam.

EXECUTE: $\dfrac{0.840\text{ m}}{c/n} - \dfrac{0.840\text{ m}}{c} = (n-1)\dfrac{0.840\text{ m}}{c} = 4.2\text{ ns}$. We can now solve for the index of refraction: $n = \dfrac{(4.2\times10^{-9}\text{ s})(3.00\times10^{8}\text{ m/s})}{0.840\text{ m}} + 1 = 2.50$. The wavelength inside of the glass is $\lambda = \dfrac{490\text{ nm}}{2.50} = 196\text{ nm}$.

EVALUATE: Light travels slower in the slab than in air and the wavelength is shorter.

33.39. IDENTIFY: The angle of incidence at *A* is to be the critical angle. Apply Snell's law at the air to glass refraction at the top of the block.

SET UP: The ray is sketched in Figure 33.39.

EXECUTE: For glass $\rightarrow$ air at point *A*, Snell's law gives $(1.38)\sin\theta_{\text{crit}} = (1.00)\sin 90°$ and $\theta_{\text{crit}} = 46.4°$. $\theta_b = 90° - \theta_{\text{crit}} = 43.6°$. Snell's law applied to the refraction from air to glass at the top of the block gives $(1.00)\sin\theta_a = (1.38)\sin(43.6°)$ and $\theta_a = 72.1°$.

EVALUATE: If θ_a is larger than $72.1°$ then the angle of incidence at point *A* is less than the initial critical angle and total internal reflection doesn't occur.

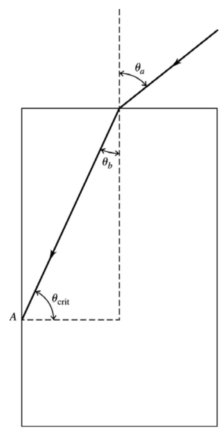


Figure 33.39

33.40. IDENTIFY: As the light crosses the glass-air interface along *AB*, it is refracted and obeys Snell's law.

SET UP: Snell's law is $n_a \sin\theta_\text{a} = n_b \sin\theta_\text{b}$ and $n = 1.000$ for air. At point *B* the angle of the prism is $30.0°$.

EXECUTE: Apply Snell's law at *AB*. The prism angle at *A* is 60.0°, so for the upper ray, the angle of refraction at *AB* is $60.0° + 12.0° = 72.0°$. Using this value gives $n_1 \sin 60.0° = \sin 72.0°$ and $n_1 = 1.10$. For the lower ray, the angle of refraction at *AB* is $60.0° + 12.0° + 8.50° = 80.5°$, giving $n_2 \sin 60.0° = \sin 80.5°$ and $n_2 = 1.14$.

EVALUATE: The lower ray is deflected more than the upper ray because that wavelength has a slightly greater index of refraction than the upper ray.

33.41. IDENTIFY: For total internal reflection, the angle of incidence must be at least as large as the critical angle.

SET UP: The angle of incidence for the glass-oil interface must be the critical angle, so $\theta_b = 90°$. $n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$ gives $(1.52)\sin 57.2° = n_{\text{oil}} \sin 90°$. $n_{\text{oil}} = (1.52)\sin 57.2° = 1.28$.

EVALUATE: $n_{\text{oil}} > 1$, which it must be, and 1.28 is a reasonable value for an oil.

33.42. **IDENTIFY:** Because the prism is a right-angle prism, the normals at point A and at surface BC are perpendicular to each other (see Figure 33.42). Therefore the angle of incidence at A is 50.0°, and this is the critical angle at that surface. Apply Snell's law at A and at surface BC. For light incident at the critical angle, the angle of refraction is 90°.

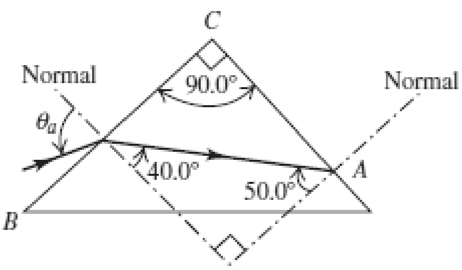


Figure 33.42

SET UP: Apply Snell's law: $n_a \sin\theta_a = n_b \sin\theta_b$. Use n = 1.00 for air, and let n be the index of refraction of the glass.

EXECUTE: Apply Snell's law at point A.

n sin(50.0°) = (1.00) sin(90°) = 1.00.

n = 1.305.

Now apply Snell's law at surface BC.

(1.00) $\sin\theta$ = (1.305) sin(40.0°).

θ = 57.0°.

EVALUATE: The critical angle at A would not be 50.0° if the prism were not a right-angle prism.

33.43. **IDENTIFY:** Apply $\lambda = \frac{\lambda_0}{n}$. The number of wavelengths in a distance d of a material is $\frac{d}{\lambda}$ where λ is the wavelength in the material.

SET UP: The distance in glass is $d_{\text{glass}} = 0.00250\text{ m}$. The distance in air is $d_{\text{air}} = 0.0180\text{ m} - 0.00250\text{ m} = 0.0155\text{ m}$.

EXECUTE: number of wavelengths = number in air + number in glass.

$$\text{number of wavelengths} = \frac{d_{\text{air}}}{\lambda} + \frac{d_{\text{glass}}}{\lambda}n = \frac{0.0155\text{ m}}{5.40\times 10^{-7}\text{ m}} + \frac{0.00250\text{ m}}{5.40\times 10^{-7}\text{ m}}(1.40) = 3.52\times 10^4.$$

EVALUATE: Without the glass plate the number of wavelengths between the source and screen is $\frac{0.0180\text{ m}}{5.40\times 10^{-3}\text{ m}} = 3.33\times 10^4$. The wavelength is shorter in the glass so there are more wavelengths in a distance in glass than there are in the same distance in air.

33.44. **IDENTIFY:** Apply Snell's law to the refraction of the light as it passes from water into air.

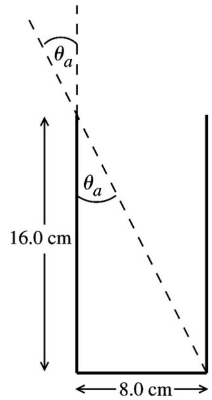
SET UP: $\theta_a = \arctan\left(\frac{1.5\text{ m}}{1.2\text{ m}}\right) = 51°$. $n_a = 1.00$. $n_b = 1.333$.

EXECUTE: $\theta_b = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.00}{1.333}\sin 51°\right) = 36°$. Therefore, the distance along the bottom of the pool from directly below where the light enters to where it hits the bottom is $x = (4.0\text{ m})\tan\theta_b = (4.0\text{ m})\tan 36° = 2.9\text{ m}$. $x_{\text{total}} = 1.5\text{ m} + x = 1.5\text{ m} + 2.9\text{ m} = 4.4\text{ m}$.

EVALUATE: The light ray from the flashlight is bent toward the normal when it refracts into the water.

33.45. **IDENTIFY:** Use Snell's law to determine the effect of the liquid on the direction of travel of the light as it enters the liquid.

SET UP: Use geometry to find the angles of incidence and refraction. Before the liquid is poured in, the ray along your line of sight has the path shown in Figure 33.45a.

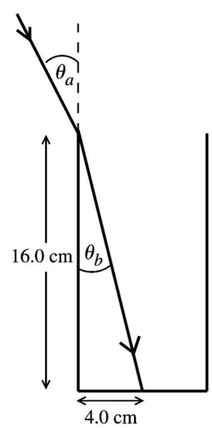


$$\tan\theta_a = \frac{8.0\text{ cm}}{16.0\text{ cm}} = 0.500.$$

$$\theta_a = 26.57°.$$

Figure 33.45a

After the liquid is poured in, θ_a is the same and the refracted ray passes through the center of the bottom of the glass, as shown in Figure 33.45b.



$$\tan\theta_b = \frac{4.0\text{ cm}}{16.0\text{ cm}} = 0.250.$$

$$\theta_b = 14.04°.$$

Figure 33.45b

EXECUTE: Use Snell's law to find n_b, the refractive index of the liquid:

$$n_a \sin\theta_a = n_b \sin\theta_b.$$

$$n_b = \frac{n_a \sin\theta_a}{\sin\theta_b} = \frac{(1.00)(\sin 26.57°)}{\sin 14.04°} = 1.84.$$

EVALUATE: When the light goes from air to liquid (larger refractive index) it is bent toward the normal.

33.46. **IDENTIFY:** Apply Snell's law. For light incident at the critical angle, the angle of refraction is 90°.

SET UP: Apply $n_a \sin\theta_a = n_b \sin\theta_b$ and use n = 1.00 for air.

EXECUTE: **(a)** Apply Snell's law at the interface between the cladding and the core. At that surface, the angle of incidence is the critical angle.

$n_1 \sin\theta_{\text{crit}} = n_2 \sin(90°) = n_2$.

$1.465 \sin\theta_{\text{crit}} = 1.450.$

$\theta_{\text{crit}} = 81.8°.$

(b) Apply Snell's law at the flat end of the cable and then at the core-cladding interface. Call θ the angle of refraction at the flat end, and α the angle of incidence at the core-cladding interface. Because the flat end is perpendicular to the surface at the core-cladding interface, $\sin\alpha = \cos\theta$. (See Figure 33.46.)

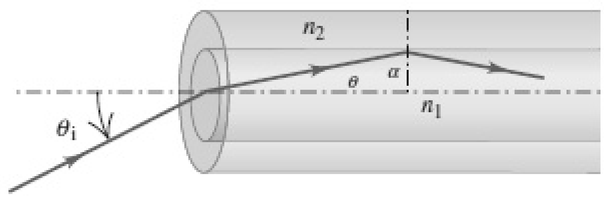


Figure 33.46

At the flat end of the cable: $(1.00)\sin\theta_i = n_1 \sin\theta \quad \rightarrow \quad \sin\theta = \dfrac{\sin\theta_i}{n_1}.$

At the core-cladding interface: $n_1 \sin\alpha = n_2 \sin(90°) = n_2 \quad \rightarrow \quad n_1 \cos\theta = n_2 \rightarrow \cos\theta = n_2/n_1.$

Using the fact that $\sin^2\theta + \cos^2\theta = 1$, we get $\left(\dfrac{\sin\theta_i}{n_1}\right)^2 + \left(\dfrac{n_2}{n_1}\right)^2 = 1.$ Solving for $\sin\theta_i$ gives

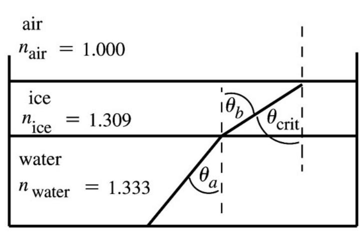
$\sin\theta_i = \sqrt{n_1^2 - n_2^2}.$

(c) Using the formula we just derived gives $\sin\theta_i = \sqrt{1.465^2 - 1.450^2} = 0.20911$, so $\theta_i = 12.1°$.

EVALUATE: If $n_2 > n_1$, the square root in (b) is not a real number, so there is no solution for θ_i. This is reasonable since total internal reflection will not occur unless $n_2 < n_1$.

33.47. **IDENTIFY:** Apply Snell's law to the water $\rightarrow$ ice and ice $\rightarrow$ air interfaces.

(a) SET UP: Consider the ray shown in Figure 33.47.



We want to find the incident angle θ_a at the water-ice interface that causes the incident angle at the ice-air interface to be the critical angle.

Figure 33.47

EXECUTE: ice-air interface: $n_{\text{ice}} \sin\theta_{\text{crit}} = 1.0 \sin 90°.$

$n_{\text{ice}} \sin\theta_{\text{crit}} = 1.0$ so $\sin\theta_{\text{crit}} = \dfrac{1}{n_{\text{ice}}}.$

But from the diagram we see that $\theta_b = \theta_{\text{crit}}$, so $\sin\theta_b = \dfrac{1}{n_{\text{ice}}}.$

water-ice interface: $n_{\text{w}} \sin\theta_a = n_{\text{ice}} \sin\theta_b.$

But $\sin\theta_b = \dfrac{1}{n_{\text{ice}}}$ so $n_w \sin\theta_a = 1.0$. $\sin\theta_a = \dfrac{1}{n_{\text{w}}} = \dfrac{1}{1.333} = 0.7502$ and $\theta_a = 48.6°.$

EVALUATE: **(b)** The angle calculated in part (a) is the critical angle for a water-air interface; the answer would be the same if the ice layer wasn't there!

33.48. **IDENTIFY:** This problem is about polarizing filters, so Malus's law applies.

SET UP: $I = I_0 \cos^2 \phi$. We are looking for the intensity after light has pass through *N* polarizers, each with its polarizing axis turned slightly from the one before it.

EXECUTE: **(a)** Each time turn the polarizers are turned through an angle ϕ = 90°/*N* from each other.

After the first polarizer: $I = I_0 \cos^2 (90° / N)$.

After the second polarizer: $I = \left[I_0 \cos^2 (90°/N)\right] \cos^2 (90°/N) = I_0 \cos^4 (90°/N)$.

After the third polarizer: $I = \left[I_0 \cos^4 (90°/N)\right] \cos^2 (90°/N) = I_0 \cos^6 (90°/N)$.

As we can see, the emerging pattern is $I = I_0 \cos^{2N} (90°/N)$.

(b) We want the minimum *N* so that the light is rotated by 90° yet retains more than 90° of its intensity. Use $I = I_0 \cos^{2N} (90°/N) = 0.90 I_0$. We want I/I_0 to be 0.90 or greater. Using a calculator, try several values to zero in on the appropriate value of *N*. For example:

$2N = 10$: $I/I_0 = \cos^{10}(90°/5) = 0.605$.

$2N = 30$: $I/I_0 = \cos^{30}(90°/15) = 0.85$.

$2N = 50$: $I/I_0 = \cos^{50}(90°/25) = 0.91$.

We want the minimum intensity ratio to be 0.90, so try $2N = 48$:

$2N = 48$: $I/I_0 = \cos^{48}(90°/24) = 0.90$.

$2N = 48$ gives the required intensity, so $N = 24$ polarizers.

(c) Use the same procedure as in (b). In both cases we want the *minimum N*. For $I/I_0 = 0.95$, $2N = 98$, so $N = 49$ polarizers. For $I/I_0 = 0.99$, $2N = 492$, so $N = 246$ polarizers.

EVALUATE: It takes a great number of polarizers to retain a high intensity. If the material of which the polarizers are made absorbs light due to impurities, it could be self-defeating to use so many of them unless they are extremely pure.

33.49. **IDENTIFY:** Apply Snell's law to the refraction of each ray as it emerges from the glass. The angle of incidence equals the angle $A = 25.0°$.

SET UP: The paths of the two rays are sketched in Figure 33.49.

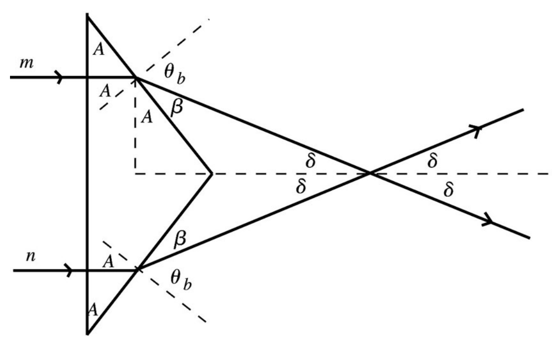


Figure 33.49

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$.

$n_{\text{glass}} \sin 25.0° = 1.00 \sin\theta_b$.

$\sin\theta_b = n_{\text{glass}} \sin 25.0°$.

$\sin\theta_b = 1.66 \sin 25.0° = 0.7015$.

$\theta_b = 44.55°$.

$\beta = 90.0° - \theta_b = 45.45°.$

Then $\delta = 90.0° - A - \beta = 90.0° - 25.0° - 45.45° = 19.55°.$ The angle between the two rays is $2\delta = 39.1°.$

EVALUATE: The light is incident normally on the front face of the prism so the light is not bent as it enters the prism.

33.50. IDENTIFY: The ray shown in the figure that accompanies the problem is to be incident at the critical angle.

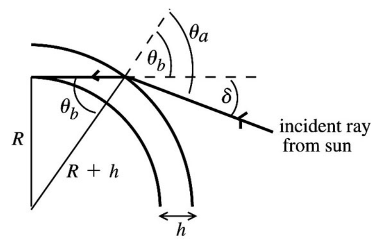
SET UP: $\theta_b = 90°.$ The incident angle for the ray in the figure is $60°.$

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$ gives $n_b = \left(\dfrac{n_a \sin\theta_a}{\sin\theta_b}\right) = \left(\dfrac{1.56 \sin 60°}{\sin 90°}\right) = 1.35.$

EVALUATE: Total internal reflection occurs only when the light is incident in the material of the greater refractive index.

33.51. IDENTIFY: Apply Snell's law to the refraction of the light as it enters the atmosphere.

SET UP: The path of a ray from the sun is sketched in Figure 33.51.



$\delta = \theta_a - \theta_b.$

From the diagram $\sin\theta_b = \dfrac{R}{R+h}.$

$\theta_b = \arcsin\left(\dfrac{R}{R+h}\right).$

Figure 33.51

EXECUTE: **(a)** Apply Snell's law to the refraction that occurs at the top of the atmosphere: $n_a \sin\theta_a = n_b \sin\theta_b$ (a = vacuum of space, refractive index 1.0; b = atmosphere, refractive index n).

$\sin\theta_a = n\sin\theta_b = n\left(\dfrac{R}{R+h}\right)$ so $\theta_a = \arcsin\left(\dfrac{nR}{R+h}\right).$

$\delta = \theta_a - \theta_b = \arcsin\left(\dfrac{nR}{R+h}\right) - \arcsin\left(\dfrac{R}{R+h}\right).$

(b) $\dfrac{R}{R+h} = \dfrac{6.38\times10^6 \text{ m}}{6.38\times10^6 \text{ m} + 20\times10^3 \text{ m}} = 0.99688.$

$\dfrac{nR}{R+h} = 1.0003(0.99688) = 0.99718.$

$\theta_b = \arcsin\left(\dfrac{R}{R+h}\right) = 85.47°.$

$\theta_a = \arcsin\left(\dfrac{nR}{R+h}\right) = 85.70°.$

$\delta = \theta_a - \theta_b = 85.70° - 85.47° = 0.23°.$

EVALUATE: The calculated δ is about the same as the angular radius of the sun.

33.52. IDENTIFY: Apply Snell's law to each refraction.

SET UP: Refer to the angles and distances defined in the figure that accompanies the problem.

EXECUTE: **(a)** For light in air incident on a parallel-faced plate, Snell's Law yields:
$n\sin\theta_a = n'\sin\theta_b' = n'\sin\theta_b = n\sin\theta_a' \Rightarrow \sin\theta_a = \sin\theta_a' \Rightarrow \theta_a = \theta_a'.$

(b) Adding more plates just adds extra steps in the middle of the above equation that always cancel out. The requirement of parallel faces ensures that the angle $\theta'_n = \theta_n$ and the chain of equations can continue.

(c) The lateral displacement of the beam can be calculated using geometry:

$d = L\sin(\theta_a - \theta'_b)$ and $L = \dfrac{t}{\cos\theta'_b} \Rightarrow d = \dfrac{t\sin(\theta_a - \theta'_b)}{\cos\theta'_b}$.

(d) $\theta'_b = \arcsin\left(\dfrac{n\sin\theta_a}{n'}\right) = \arcsin\left(\dfrac{\sin 66.0°}{1.80}\right) = 30.5°$ and $d = \dfrac{(2.40\text{ cm})\sin(66.0° - 30.5°)}{\cos 30.5°} = 1.62\text{ cm}.$

EVALUATE: The lateral displacement in part (d) is large, of the same order as the thickness of the plate.

33.53. IDENTIFY: The reflected light is totally polarized when light strikes a surface at Brewster's angle.

SET UP: At the plastic wall, Brewster's angle obeys the equation $\tan\theta_\text{p} = n_b/n_a$, and Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$, applies at the air-water surface.

EXECUTE: To be totally polarized, the reflected sunlight must have struck the wall at Brewster's angle. $\tan\theta_\text{p} = n_b/n_a = (1.61)/(1.00)$ and $\theta_\text{p} = 58.15°$.

This is the angle of incidence at the wall. A little geometry tells us that the angle of incidence at the water surface is $90.00° - 58.15° = 31.85°$. Applying Snell's law at the water surface gives $(1.00)\sin 31.85° = 1.333\sin\theta$ and $\theta = 23.3°$.

EVALUATE: We have two different principles involved here: Reflection at Brewster's angle at the wall and Snell's law at the water surface.

33.54. IDENTIFY: We are dealing with circularly polarized electromagnetic waves.

SET UP: $\tan\theta_\text{p} = n_b/n_a$, $n_a\sin\theta_a = n_b\sin\theta_b$

EXECUTE: **(a)** We want the angle of refraction. First find θ_p. $\tan\theta_\text{p} = n_b/n_a$ = (1.62)/(1.00) = 1.62, so θ_p = 58.31°. Now use Snell's law. $(1.00)\sin 58.31° = (1.62)\sin\theta_b$. θ_b = 31.7°.

(b) We want the reflected intensity. The reflecting surface is perpendicular to the xz-plane, so the component of the incident electric field parallel to that is E_y. Using Fresnel's equation gives

$\dfrac{E_\text{refl}}{E_{\text{inc},y}} = \dfrac{\sin(\theta_\text{p} - \theta_b)}{\sin(\theta_\text{p} + \theta_b)} = \dfrac{\sin(58.32° - 31.7°)}{\sin(58.32° + 31.7°)} = 0.448.$ $E_{\text{inc},y} = \dfrac{E_\text{inc}}{\sqrt{2}}$, so $E_\text{refl} = (0.448)\dfrac{E_\text{inc}}{\sqrt{2}} = 0.3168E_\text{inc}.$

The intensity is proportional to E^2, so $\dfrac{I_\text{refl}}{I_\text{inc}} = \dfrac{E^2_\text{refl}}{E^2_\text{inc}} = \dfrac{(0.3168E_\text{inc})^2}{E^2_\text{inc}} = 0.1004.$ Therefore we have

$I_\text{refl} = 0.1004I_\text{inc} =$ (0.1004)(150 W/m^2) = 15.1 W/m^2.

(c) and **(d)** We want $I_\parallel$. $I_\text{refr} = I_\text{inc} - I_\text{refl}$ = 150 W/m^2 – 15.1 W/m^2 = 134.9 W/m^2. In the incident light, $E_x = E_y$, so the intensity is the same for both of them, which is (150 W/m^2)/2 = 75.0 W/m^2. After reflection, none of the E_x light is reflected since the reflected light is 100% polarized in the y direction. Therefore E_x is the same in the transmitted (refracted) light, so that intensity is 75.0 W/m^2. The remainder of the light in the refracted beam is 134.9 W/m^2 – 75.0 W/m^2 = 59.9 W/m^2. Therefore the final answers are: (c) 59.9 W/m^2 and (d) 75.0 W/m^2.

(e) We want e. $e = \sqrt{1 - (E_1/E_2)^2}$. I is proportional to E^2, so $e = \sqrt{1 - I_1/I_2} = \sqrt{1 - \dfrac{59.9\text{ W/m}^2}{75.0\text{ W/m}^2}} = 0.449.$

EVALUATE: In the transmitted light E_x and E_y have different amplitudes so the light is elliptically polarized.

33.55. IDENTIFY: Apply Snell's law in part (a). In part (b), we know from Chapter 32 that in a dielectric, $n^2 = KK_\text{m}$. In this case, we are told that K_m is very close to 1, so $n^2 \approx K$, where K is the dielectric constant of the material.

SET UP: Use $n_a\sin\theta_a = n_b\sin\theta_b$ (Snell's law) in (a) and $n^2 \approx K$ in (b). Use n = 1.00 for air. $f\lambda = c$.

EXECUTE: **(a)** For each liquid, apply $n_a \sin\theta_a = n_b \sin\theta_b$ using the data in the table with the problem. The angle of incidence is 60.0° in each case.

$\sin(60.0°) = n \sin\theta_b$, which gives $n = \dfrac{\sin(60.0°)}{\sin\theta_b}$. Apply this formula for each liquid and then use the information in Table 33.1 to identify the liquids.

Liquid A: $n_\text{A} = \dfrac{\sin(60.0°)}{\sin(36.4°)} = 1.46$ (carbon tetrachloride).

Liquid B: $n_\text{B} = \dfrac{\sin(60.0°)}{\sin(40.5°)} = 1.33$ (water).

Liquid C: $n_\text{C} = \dfrac{\sin(60.0°)}{\sin(32.1°)} = 1.63$ (carbon disulfide).

Liquid D: $n_\text{D} = \dfrac{\sin(60.0°)}{\sin(35.2°)} = 1.50$ (benzene).

(b) Use $K = n^2$ for each liquid.

$K_\text{A} = (1.46)^2 = 2.13$.

$K_\text{B} = (1.33)^2 = 1.77$.

$K_\text{C} = (1.63)^2 = 2.66$.

$K_\text{D} = (1.50)^2 = 2.25$.

(c) Use $f\lambda = c$: $f = c/\lambda = (3.00\times10^8\ \text{m/s})/(589\times10^{-9}\ \text{m}) = 5.09\times10^{14}$ Hz. This is the frequency in air and also in each liquid.

EVALUATE: The indexes of refraction are accurate, but the dielectric constants are less so because $n^2 \approx K$ is an approximation.

33.56. **IDENTIFY:** Apply Snell's law at the air-glass interface and also at the glass-liquid interface. For light incident at the critical angle, the angle of refraction is 90°.

SET UP: Use Snell's law: $n_a \sin\theta_a = n_b \sin\theta_b$. If θ_b is the angle of refraction at the air-glass interface and θ_c is the angle of incidence at the glass-liquid interface, then $\sin\theta_c = \cos\theta_b$. This is true because the normal at the air-glass interface and the glass-liquid interface are perpendicular to each other. We also know that θ_c is the critical angle for the glass-liquid boundary. Use 1.00 for the index of refraction of air, and call n the index of refraction for the liquid.

EXECUTE: First apply Snell's law at the air-glass boundary to find θ_b, and then use that result to find $\sin\theta_c$. Finally use Snell's law at the glass-liquid boundary to find n for the liquid.

Liquid A: At the air-glass boundary we have

$(1.00)\sin(52.0°) = (1.52)\sin\theta_b$, which gives $\theta_b = 31.226°$ and $\cos\theta_b = 0.85512 = \sin\theta_c$.

At the glass-liquid boundary we have

$(1.52)\sin\theta_c = n\sin(90°) = n$.

$n_\text{A} = (1.52)(0.85512) = 1.30$.

Liquid B: At the air-glass boundary we have

$(1.00)\sin(44.3°) = (1.52)\sin\theta_b$, so $\theta_b = 27.3538°$, so $\cos\theta_b = 0.8882 = \sin\theta_c$.

At the glass-liquid boundary we have

$n_\text{B} = (1.52)(0.8882) = 1.35$.

Liquid C: Air-glass boundary: $(1.00)\sin(36.3°) = (1.52)\sin\theta_b$, $\theta_b = 22.922°$, $\cos\theta_b = 0.9210 = \sin\theta_c$.

Glass-liquid boundary: $n_\text{C} = (1.52)(0.9210) = 1.40$.

EVALUATE: The indexes of refraction would be slightly different at wavelengths other than 638 nm since n depends on the wavelength of the light. All the values for n are greater than 1, which they must be.

33.57. **IDENTIFY** and **SET UP**: The polarizer passes $\frac{1}{2}$ of the intensity of the unpolarized component, independent of α. Malus's law tells us that out of the intensity I_p of the polarized component, the polarizer passes intensity $I_p \cos^2(\alpha - \theta)$, where $\alpha - \theta$ is the angle between the plane of polarization and the axis of the polarizer.

EXECUTE: **(a)** Use the angle where the transmitted intensity is maximum or minimum to find θ. See Figure 33.57.

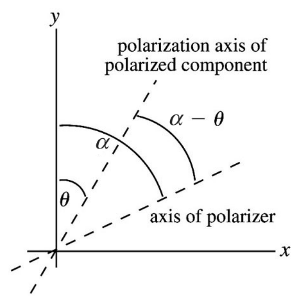


Figure 33.57

The total transmitted intensity is $I = \frac{1}{2} I_0 + I_p \cos^2(\alpha - \theta)$. This is maximum when $\theta = \alpha$, and from the graph in the problem this occurs when α is approximately 35°, so $\theta = 35°$. Alternatively, the total transmitted intensity is minimum when $\alpha - \theta = 90°$ and from the graph this occurs for $\alpha = 125°$. Thus, $\theta = \alpha - 90° = 125° - 90° = 35°$, which is in agreement with what we just found.

(b) For the equation, $I = \frac{1}{2} I_0 + I_p \cos^2(\alpha - \theta)$, we use data at two values of α to determine I_0 and I_p. It is easiest to use data where *I* is a maximum and a minimum. From the graph, we see that these extremes are 25 W/m^2 at α = 35° and 5.0 W/m^2 at $\alpha = 125°$.

At $\alpha = 125°$ the net intensity is 5.0 W/m^2, so we have

$5.0 \text{ W/m}^2 = \frac{1}{2} I_0 + I_p \cos^2(125° - 35°) = \frac{1}{2} I_0 + I_p \cos^2(90°) = \frac{1}{2} I_0 \;\rightarrow\; I_0 = 10 \text{ W/m}^2.$

At $\alpha = 35°$ the net intensity is 25 W/m^2, so we have

$25 \text{ W/m}^2 = \frac{1}{2} I_0 + I_p \cos^2 0° = \frac{1}{2} I_0 + I_p = 5 \text{ W/m}^2 + I_p \;\rightarrow\; I_p = 20 \text{ W/m}^2.$

EVALUATE: Now that we have I_0, I_p, and θ we can verify that $I = \frac{1}{2} I_0 + I_p \cos^2(\phi - \theta)$ describes the data in the graph.

33.58. **IDENTIFY:** This problem involves refraction at a spherical surface.

SET UP: Refer to Fig. P33.58 in the textbook and follow the items requested. Snell's law applies: $n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: **(a)** We want $\sin\theta_a$. The hypotenuse is *R*, so $\sin\theta_a = r/R$.

(b) We want $\sin\theta_b$. From the figure we see that $\sin\theta_b = r'/R$.

(c) Apply Snell's law at the surface of the sphere, giving $n \sin\theta_a = (1.00)\sin\theta_b$. Use the results from parts (a) and (b). $n\left(\frac{r}{R}\right) = \frac{r'}{R}$, which gives $r' = nr$.

(d) The diameter is $D = 2r' = 2nr$ = (1.53)(45.0 mm) = 68.9 mm.

EVALUATE: The seed head will be magnified by a factor of 68.9/45.0 = 1.53 times.

33.59. **IDENTIFY:** This problem deals with total internal reflection and Snell's law.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$. Follow the items requested in the parts of the problem.

EXECUTE: **(a)** We want θ. Follow the suggestion in the problem and refer to Fig. P33.59 in the textbook. $\sin\theta = \frac{bc}{ac} = \frac{R-d/2}{R+d/2} = \frac{2R-d}{2R+d}.$

(b) We want *R*. θ should be the critical angle, so $n_1 \sin\theta = n_2 \sin 90° = n_2$. Now use the result from part (a). $n_1\left(\frac{2R-d}{2R+d}\right) = n_2$. Solve for *R*, giving $R = \frac{d(n_1+n_2)}{2(n_1-n_2)}$.

(c) Using the given values, we get *R* = 2.07 cm.

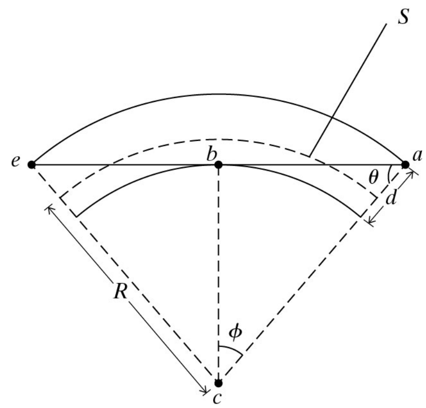


Figure 33.59

(d) The pattern shown in Fig. 33.59 keeps repeating as light goes down the cable. The dashed curve represents the path without reflections, which is a total of 1.00 km. In Fig. 33.59, the light follows the path *ae*, but without reflections it would follow the dashed curve. The angle θ is the critical angle. First find the distance *ae*. Referring to Fig. 33.59, we can see the following. *ae* = 2*ab*. $(ab)^2 + (R-d/2)^2 = (R+d/2)^2$. Solving for *ab* gives $(ab)^2 = 2Rd$, so $ab = \sqrt{2Rd}$. *ae* = 2*ab*, so $ae = 2\sqrt{2Rd} = 2\sqrt{2(0.02065 \text{ m})(50.0\ \mu\text{m})}$ = 2.8740 mm. Now get the length *S* of the dashed curved path. Since θ is the critical angle, $\sin\theta = n_2/n_1$ = 1.4440/1.4475, which gives θ = 86.015°. The length of *S* is the length of the curved path of radius *R* that subtends an angle 2ϕ. With ϕ in radians, we get $S = R(2\phi) = 2R\phi$. $\phi = 90° - \theta = 90° - 86.015° = 3.985° = 0.06955$ rad. Therefore $S = 2R\phi =$ 2(0.02065 m)(0.06955 rad) = 2.8726 mm. So while the unreflected ray has traveled 2.8726 mm, the reflected ray has traveled 2.8740 mm. Now find the distance the reflected ray travels (call it *L*) while the unreflected ray has traveled 1.00 km. Using proportionality, we have $\frac{L}{1.00 \text{ km}} = \frac{ae}{S}$, so $L = (1.00 \text{ km})\frac{ae}{S}$. We want the *difference* in distance between these two rays, which is *L* – 1.00 km. This is $L - 1.00 \text{ km} = (1.00 \text{ km})\frac{ae}{S} - 1.00 \text{ km} = (1.00 \text{ km})\left(\frac{ae}{S} - 1\right)$. Using the results we just found for *ae* and *S* gives $L - 1.00 \text{ km} = (1.00 \text{ km})\left(\frac{2.8740 \text{ mm}}{2.8726 \text{ mm}} - 1\right) = 48.7 \text{ cm},$ which is about 50 cm. (Note that this part involves the subtraction of two numbers that are nearly equal. Therefore the answer is heavily dependent on the amount of rounding in the intermediate numerical calculations.)

(e) From part (d), we saw that the extra distance is only around 50 cm, so that makes a negligible time difference for the two rays. However there is a time difference because the ray in an air-filled cable travels at the speed of light c while one in the filled cable travels at speed c/n. Calling x the length of the cable, we have $\Delta t = t_{\text{cable}} - t_{\text{air}} = \dfrac{x}{c/n} - \dfrac{x}{c} = \dfrac{x}{c}(n-1) = \dfrac{1.00\text{ km}}{c}(1.4475-1) = 1.49\,\mu\text{s}$.

EVALUATE: Compare the times: $t_{\text{cable}}/t_{\text{air}} = (nx/c)/(x/c) = n = 1.744$. So the time through the cable is over 1.7 times as long as through the air. The difference is small but the *fractional* difference is large.

33.60. **IDENTIFY:** Apply Snell's law to each refraction.

SET UP: Refer to the figure that accompanies the problem.

EXECUTE: **(a)** By the symmetry of the triangles, $\theta_b^A = \theta_a^B$, and $\theta_a^C = \theta_r^B = \theta_a^B = \theta_b^A$. Therefore,

$\sin\theta_b^C = n\sin\theta_a^C = n\sin\theta_b^A = \sin\theta_a^A = \theta_b^C = \theta_a^A$.

(b) The total angular deflection of the ray is $\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_a^B + \theta_b^C - \theta_a^C = 2\theta_a^A - 4\theta_b^A + \pi$.

(c) From Snell's law, $\sin\theta_a^A = n\sin\theta_b^A \Rightarrow \theta_b^A = \arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)$.

$$\Delta = 2\theta_a^A - 4\theta_b^A + \pi = 2\theta_a^A - 4\arcsin\left(\frac{1}{n}\sin\theta_a^A\right) + \pi.$$

(d) $$\frac{d\Delta}{d\theta_a^A} = 0 = 2 - 4\frac{d}{d\theta_a^A}\left(\arcsin\left(\frac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \frac{4}{\sqrt{1-\dfrac{\sin^2\theta_1}{n^2}}}\cdot\left(\frac{\cos\theta_1}{n}\right)\cdot 4\left(1-\frac{\sin^2\theta_1}{n^2}\right) = \left(\frac{16\cos^2\theta_1}{n^2}\right).$$

$$4\cos^2\theta_1 = n^2 - 1 + \cos^2\theta_1.\ 3\cos^2\theta_1 = n^2 - 1.\ \cos^2\theta_1 = \frac{1}{3}(n^2-1).$$

(e) For violet: $\theta_1 = \arccos\left(\sqrt{\tfrac{1}{3}(n^2-1)}\right) = \arccos\left(\sqrt{\tfrac{1}{3}(1.342^2-1)}\right) = 58.89°$.

$\Delta_{\text{violet}} = 139.2° \Rightarrow \theta_{\text{violet}} = 40.8°$.

For red: $\theta_1 = \arccos\left(\sqrt{\tfrac{1}{3}(n^2-1)}\right) = \arccos\left(\sqrt{\tfrac{1}{3}(1.330^2-1)}\right) = 59.58°$.

$\Delta_{\text{red}} = 137.5° \Rightarrow \theta_{\text{red}} = 42.5°$.

EVALUATE: The angles we have calculated agree with the values given in Figure 33.19d in the textbook. θ_1 is larger for red than for violet, so red in the rainbow is higher above the horizon.

33.61. **IDENTIFY:** Follow similar steps to Challenge Problem 33.60.

SET UP: Refer to Figure 33.19e in the textbook.

EXECUTE: **(a)** The total angular deflection of the ray is

$\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_b^A + \pi - 2\theta_b^A + \theta_a^A - \theta_b^A = 2\theta_a^A - 6\theta_b^A + 2\pi$, where we have used the fact from the previous problem that all the internal angles are equal and the two external equals are equal. Also using the Snell's law relationship,

we have: $\theta_b^A = \arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)$. $\Delta = 2\theta_a^A - 6\theta_b^A + 2\pi = 2\theta_a^A - 6\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right) + 2\pi$.

(b) $$\frac{d\Delta}{d\theta_a^A} = 0 = 2 - 6\frac{d}{d\theta_a^A}\left(\arcsin\left(\frac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \frac{6}{\sqrt{1-\dfrac{\sin\theta_2}{n^2}}}\cdot\left(\frac{\cos\theta_2}{n}\right).$$

$$n^2\left(1-\frac{\sin^2\theta_2}{n^2}\right) = (n^2 - 1 + \cos^2\theta_2) = 9\cos^2\theta_2.\ \cos^2\theta_2 = \frac{1}{8}(n^2-1).$$

(c) For violet, $\theta_2 = \arccos\left(\sqrt{\frac{1}{8}(n^2-1)}\right) = \arccos\left(\sqrt{\frac{1}{8}(1.342^2-1)}\right) = 71.55°$. $\Delta_{\text{violet}} = 233.2°$ and $\theta_{\text{violet}} = 53.2°$.

For red, $\theta_2 = \arccos\left(\sqrt{\frac{1}{8}(n^2-1)}\right) = \arccos\left(\sqrt{\frac{1}{8}(1.330^2-1)}\right) = 71.94°$. $\Delta_{\text{red}} = 230.1°$ and $\theta_{\text{red}} = 50.1°$.

EVALUATE: The angles we calculated agree with those given in Figure 33.19e in the textbook. The color that appears higher above the horizon is violet. The colors appear in reverse order in a secondary rainbow compared to a primary rainbow.

33.62. **IDENTIFY** and **SET UP:** Light polarized at 45° with the horizontal has both a horizontal component and a vertical component to its electric field.
EXECUTE: Since the light has both horizontal and vertical components, both H-type and V-type cells will be able to detect it, which makes choice (a) correct.
EVALUATE: Since the light is polarized at 45° with the horizontal, its horizontal and vertical components will be equal. So both types of cells should respond to it equally.

33.63. **IDENTIFY:** Light reflected from a glass surface is polarized to varying degrees, depending on the angle of incidence. At Brewster's angle the reflected light is 100% polarized parallel to the surface.
SET UP: Brewster's angle is given by $\tan\theta_p = n_b/n_a$. $n = 1.5$ for glass and $n = 1.0$ for air.
EXECUTE: For reflection from glass, $\tan\theta_p = n_b/n_a = (1.5)/(1.0) = 1.5$, so $\theta_p = 56°$. This is the angle with the normal to the glass. The light in this case makes an angle of 35° with the plane of the glass, so its angle of incidence is 55°, which is very close to Brewster's angle. Therefore the reflected light is almost totally polarized horizontally (since the glass is horizontal). Thus H cells will respond much more strongly to this light than will V cells, which is choice (d).
EVALUATE: The incident light is not *exactly* at Brewster's angle, so the reflected light will not be 100% horizontally polarized. Therefore the V cells will respond slightly to the reflect light.

33.64. **IDENTIFY** and **SET UP:** A polarizer reduces the intensity of unpolarized light by 50%.
EXECUTE: The first polarizer, with a vertical transmission axis, decreases the light intensity by half and leaves the transmitted light vertically polarized, so the intensity I after the first polarizer is $I = I_0/2$. The second polarizer removed none of the light, so it must have had a vertical transmission axis. Therefore the light emerging from both polarizers is vertically polarized. Thus only the V cells of the insect will detect this light, which is choice (b).
EVALUATE: If the second polarizer were rotated by 90°, no light would have emerged from the system.

34

GEOMETRIC OPTICS

VP34.4.1. **IDENTIFY:** We have a concave mirror.

SET UP: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$, $m=-s'/s$.

EXECUTE: **(a)** We want f. $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$ = 1/(15.0 cm) + 1/(450 cm). f = 14.5 cm.

(b) We want m. $m=-s'/s$ = –(450 cm)/(15.0 cm) = –30.0.

EVALUATE: The image is real and inverted.

VP34.4.2. **IDENTIFY:** We have a concave mirror.

SET UP: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$, $m=-s'/s$, $f = R/2$. We want the location and characteristics of the image.

EXECUTE: **(a)** s = 11.0 cm. $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}$ = 1/(18.5 cm) – 1/(1/11.0 cm). s' = –27.1 cm. $m=-s'/s$ = –(–27.1 cm)/(11.0 cm) = +2.47. Since m is positive and greater than 1, the image is erect and larger than the object. s' is negative, so the image is virtual.

(b) s = 31.0 cm. $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}$ = 1/(18.5 cm) – 1/(31.0 cm). s' = +45.9 cm. $m=-s'/s$ = –(45.9 cm)/(31.0 cm) = –1.48. s' is positive, so the image is real. m is negative, so the image is inverted, and it is larger than the object since $|m| > 1$.

(c) s = 55.0 cm. $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}$ = 1/(18.5 cm) – 1/(55.0 cm). s' = +27.9 cm. $m=-s'/s$ = –(27.9 cm)/(55.0 cm) = –0.507. s' is positive so the image is real. m is negative so the image is inverted, and $|m| < 1$ so the image is smaller than the object.

EVALUATE: Notice the variation in the type of image a lens can produce, depending on where the object is placed.

VP34.4.3. **IDENTIFY:** We have a curved mirror.

SET UP: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$, $m=-s'/s$.

EXECUTE: **(a)** $f = R/2$ = (–44.0 cm)/2 = –22.0 cm. Since f is negative, this is a *convex* mirror.

(b) We want s. $\frac{1}{s}=\frac{1}{f}-\frac{1}{s'}$ = 1/(–22.0 cm) – 1/(–18.0 cm). s = 99.0 cm.

(c) We want m. $m=-s'/s$ = –(–18.0 cm)/(99.0 cm) = +0.182. The image is virtual (s' is negative), smaller than the eye ($|m| < 1$), and erect (m is positive).

EVALUATE: For a single convex mirror, $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}$ tells us that s' is always negative because f is negative and s is positive.

VP34.4.4. **IDENTIFY:** We have a convex mirror.

SET UP: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$, $m=-s'/s$, $f = R/2 = -18.5$ cm. We want s' and the characteristics of the image.

EXECUTE: **(a)** $s = 11.0$ cm. $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}$ = 1/(–18.5 cm) – 1/(11.0 cm). s' = –6.90 cm. $m=-s'/s$ = –(–6.90 cm)/(11.0 cm) = +0.627. The image is virtual, erect, and smaller than the object.

(b) $s = 31.0$ cm. $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}$ = 1/(–18.5 cm) – 1/(31.0 cm). s' = –11.6 cm. $m=-s'/s$ = $-(-11.6$ cm) / (31.0 cm) = +0.374. The image is virtual, erect, and smaller than the object.

(c) $s = 55.0$ cm. $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}$ = 1/(–18.5 cm) – 1/(55.0 cm). s' = –13.8 cm. $m=-s'/s$ = $-(-13.8$ cm) / (55.0 cm) = +0.252. The image is virtual, erect, and smaller than the object.

EVALUATE: Compare these results with those of problem VP34.4.2 to see the big differences when a mirror is changed from concave to convex.

VP34.8.1. **IDENTIFY:** This problem involves a comvex lens and the lensmaker's equation.

SET UP: $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$, $|R_1| = |R_2|$, $f = +30.0$ cm, $n = 1.65$. We want R_1 and R_2.

EXECUTE: **(a)** R_1 is positive and R_2 is negative, so $R_1 = R$ and $R_2 = -R$. $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ becomes $\frac{1}{f}=(n-1)\left(\frac{1}{R}-\frac{1}{-R}\right)=(n-1)\frac{2}{R}$. $R_1 = R = 2(n-1)f = 2(0.65)(30.0 \text{ cm}) = 39$ cm.

(b) $R_2 = -R = -39$ cm.

EVALUATE: Careful! The radii of curvature have signs and can be negative.

VP34.8.2. **IDENTIFY:** We are dealing with a thin lens and the lensmaker's equation.

SET UP: $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$. Carefully sketch the lens showing the radii of curvature and C_1 and C_2. Fig. VP34.8.2 shows the lens.

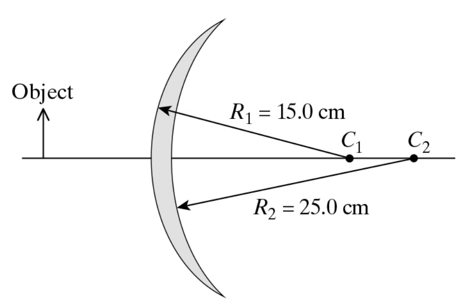


Figure VP34.8.2

EXECUTE: **(a)** As Fig. VP34.8.2 shows, this lens is thicker at its center than at the edges.

(b) We want *f*. $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1.55-1)\left(\frac{1}{15.0\text{ cm}}-\frac{1}{25.0\text{ cm}}\right)$. $f = +68$ cm.

(c) f is positive, so this is a *converging* lens.

EVALUATE: The fact that the lens is thicker in the middle than at the ends is consistent with its being a converging lens with a positive focal length.

VP34.8.3. **IDENTIFY:** We are dealing with a thin lens and the lensmaker's equation.

SET UP: $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$, both radii of curvature are positive since C_1 and C_2 are on the outgoing side of the lens. Carefully sketch the lens showing the radii of curvature and C_1 and C_2. Fig. VP34.8.3 shows the lens.

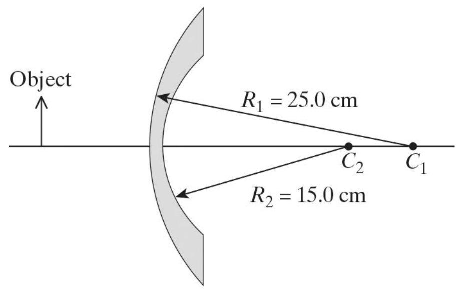


Figure VP34.8.3

EXECUTE: **(a)** As Fig. VP34.8.3 shows, this lens is thicker at its edges than at the center.

(b) We want *f*. $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1.55-1)\left(\frac{1}{25.0\text{ cm}}-\frac{1}{15.0\text{ cm}}\right)$. $f = -68$ cm.

EVALUATE: Compare this result with that of VP34.8.2 to see the difference when the radii of curvature are reversed.

VP34.8.4. **IDENTIFY:** We are dealing the lensmaker's equation.

SET UP: $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$, $R_1 = +28.0$ cm, $f = 14.0$ cm. We want R_2.

EXECUTE: **(a)** $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ gives $\frac{1}{14.0\text{ cm}}=(1.70-1)\left(\frac{1}{28.0\text{ cm}}-\frac{1}{R_2}\right)$. $R_2=-15$ cm.

(b) Since f is positive, the lens is *convex*.

EVALUATE: Since R_2 is negative, the lens will be thicker in the middle than at its edges, which is consistent with a converging (convex) lens.

VP34.10.1. **IDENTIFY:** This problem is about image formation by a converging lens.

SET UP: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$, $m=-s'/s$.

EXECUTE: **(a)** We want s'. $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}$ = 1/(25.0 cm) – 1/(15.0 cm). s' = –37.5 cm. The image is on the same side as the strawberry.

(b) We want m. $m=-s'/s$ = –(–37.5 cm)/(15.0 cm) = +2.50.

(c) s' is negative, so the image is virtual. m is positive so the image is erect, and $|m| > 1$, so it is larger than the strawberry.

EVALUATE: An object within the focal point of a converging lens always forms a virtual erect image on the same side of the les as the object, as we have seen here.

VP34.10.2. **IDENTIFY:** This problem is about image formation by a thin lens.

SET UP: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$, $m=-s'/s$.

EXECUTE: **(a)** We want *f*. $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$ = 1/(28.0 cm) + 1/(42.0 cm). f = +16.8 cm. Since f is positive, this is a *converging* lens.

(b) We want *m*. $m=-s'/s$ = –(42.0 cm)/(28.0 cm) = –1.50.

(c) Since s' is positive, the image is real. *m* is negative, so the image is inverted, and $|m| > 1$ so it is larger than the eraser.

EVALUATE: $s > f$, so the object is *outside* the focal point of the lens. Therefore the image should be real and inverted, as we have found.

VP34.10.3. **IDENTIFY:** This problem is about image formation by a thin lens.

SET UP: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$, $m=-s'/s$, $h_{\text{image}} = h_{\text{object}}$.

EXECUTE: **(a)** This lens forms an image on the opposite side from the object, so it is a *converging* lens.

(b) We want s'. $m=-s'/s$ = –1, so s' = s = +48.0 cm.

(c) $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$ = 1/(48.0 cm) 1/(48.0 cm), so f = 24.0 cm.

EVALUATE: In general for a converging lens, if $s = 2f$, then s' = s = $2f$ and m = +1.

VP34.10.4. **IDENTIFY:** This problem is about image formation by a thin lens.

SET UP: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$, $m=-s'/s$.

EXECUTE: **(a)** We want *m*. $m = h_{\text{image}}/h_{\text{object}}$ = (4.00 cm)/(20.00 cm) = +0.200.

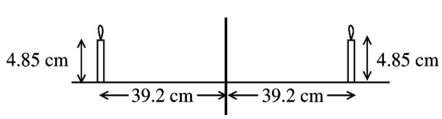
(b) We want s'. Use the result from part (a). $m=-s'/s$ = – s'/(250 cm) = +0.200. This gives s' = –50.0 cm.

(c) We want *f*. $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$ = 1/(250 cm) 1/(–50.0 cm), so f = –62.5 cm.

EVALUATE: For such a device to be of any use, it must *always* produce an erect image on the same side as the object. A diverging lens always gives this result. A converging lens would do it only if the object is within the focal point of the lens, which would not always be the case if someone outside were standing back from the door.

34.1. **IDENTIFY** and **SET UP:** Plane mirror: $s=-s'$ and $m=y'/y=-s'/s=+1$. We are given s and y and are asked to find s' and y'.

EXECUTE: The object and image are shown in Figure 34.1.



$s'=-s=-39.2$ cm.

$|y'|=|m||y|=(+1)(4.85\text{ cm})$.

$|y'|=4.85$ cm.

Figure 34.1

The image is 39.2 cm to the right of the mirror and is 4.85 cm tall.

EVALUATE: For a plane mirror the image is always the same distance behind the mirror as the object is in front of the mirror. The image always has the same height as the object.

34.2. **IDENTIFY:** Similar triangles say $\dfrac{h_{\text{tree}}}{h_{\text{mirror}}} = \dfrac{d_{\text{tree}}}{d_{\text{mirror}}}$.

SET UP: $d_{\text{mirror}} = 0.350$ m, $h_{\text{mirror}} = 0.0400$ m, and $d_{\text{tree}} = 28.0 \text{ m} + 0.350 \text{ m}$.

EXECUTE: $h_{\text{tree}} = h_{\text{mirror}} \dfrac{d_{\text{tree}}}{d_{\text{mirror}}} = 0.040 \text{ m} \dfrac{28.0 \text{ m} + 0.350 \text{ m}}{0.350 \text{ m}} = 3.24 \text{ m}.$

EVALUATE: The image of the tree formed by the mirror is 28.0 m behind the mirror and is 3.24 m tall.

34.3. **IDENTIFY** and **SET UP:** The virtual image formed by a plane mirror is the same size as the object and the same distance from the mirror as the object.

EXECUTE: $s' = -s$. The image of the tip is 12.0 cm behind the mirror surface and the image of the end of the eraser is 21.0 cm behind the mirror surface. The length of the image is 9.0 cm, the same as the length of the object. The image of the tip of the lead is the closest to the mirror surface.

EVALUATE: The same result would hold no matter how far the pencil was from the mirror.

34.4. **IDENTIFY:** $f = R/2$.

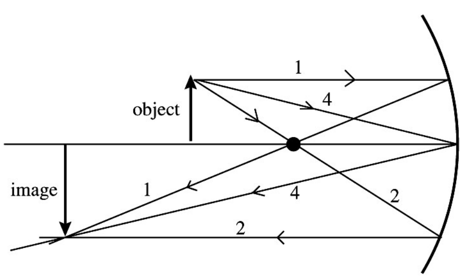
SET UP: For a concave mirror $R > 0$.

EXECUTE: **(a)** $f = \dfrac{R}{2} = \dfrac{34.0 \text{ cm}}{2} = 17.0 \text{ cm}.$

EVALUATE: **(b)** The image formation by the mirror is determined by the law of reflection and that is unaffected by the medium in which the light is traveling. The focal length remains 17.0 cm.

34.5. **IDENTIFY** and **SET UP:** Use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to calculate s' and use $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$ to calculate y'. The image is real if s' is positive and is erect if $m > 0$. Concave means R and f are positive, $R = +22.0$ cm; $f = R/2 = +11.0$ cm.

EXECUTE: **(a)**



Three principal rays, numbered as in Section 34.2, are shown in Figure 34.5. The principal-ray diagram shows that the image is real, inverted, and enlarged.

Figure 34.5

(b) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}.$

$\dfrac{1}{s'} = \dfrac{1}{f} - \dfrac{1}{s} = \dfrac{s-f}{sf}$ so $s' = \dfrac{sf}{s-f} = \dfrac{(16.5 \text{ cm})(11.0 \text{ cm})}{16.5 \text{ cm} - 11.0 \text{ cm}} = +33.0 \text{ cm}.$

$s' > 0$ so real image, 33.0 cm to left of mirror vertex.

$m = -\dfrac{s'}{s} = -\dfrac{33.0 \text{ cm}}{16.5 \text{ cm}} = -2.00$ ($m < 0$ means inverted image) $|y'| = |m||y| = 2.00(0.600 \text{ cm}) = 1.20 \text{ cm}.$

EVALUATE: The image is 33.0 cm to the left of the mirror vertex. It is real, inverted, and is 1.20 cm tall (enlarged). The calculation agrees with the image characterization from the principal-ray diagram. A concave mirror used alone always forms a real, inverted image if $s > f$ and the image is enlarged if $f < s < 2f$.

34.6. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$.

SET UP: For a convex mirror, $R<0$. $R=-22.0\text{ cm}$ and $f=\frac{R}{2}=-11.0\text{ cm}$.

EXECUTE: **(a)** The principal-ray diagram is sketched in Figure 34.6.

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $s'=\frac{sf}{s-f}=\frac{(16.5\text{ cm})(-11.0\text{ cm})}{16.5\text{ cm}-(-11.0\text{ cm})}=-6.6\text{ cm}$. $m=-\frac{s'}{s}=-\frac{-6.6\text{ cm}}{16.5\text{ cm}}=+0.400$.

$|y'|=|m||y|=(0.400)(0.600\text{ cm})=0.240\text{ cm}$. The image is 6.6 cm to the right of the mirror. It is 0.240 cm tall. $s'<0$, so the image is virtual. $m>0$, so the image is erect.

EVALUATE: The calculated image properties agree with the image characterization from the principal-ray diagram.

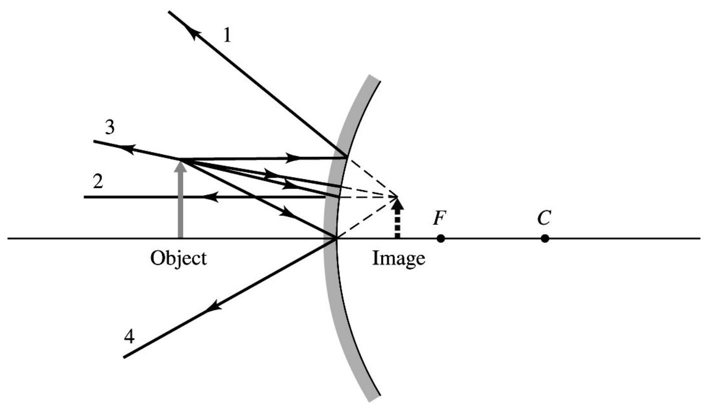


Figure 34.6

34.7. IDENTIFY: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $m=-\frac{s'}{s}$. $|m|=\frac{|y'|}{y}$. Find m and calculate y'.

SET UP: $f=+1.75\text{ m}$.

EXECUTE: $s \gg f$ so $s'=f=1.75\text{ m}$.

$$m=-\frac{s'}{s}=-\frac{1.75\text{ m}}{5.58\times10^{10}\text{ m}}=-3.14\times10^{-11}.$$

$|y'|=|m||y|=(3.14\times10^{-11})(6.794\times10^{6}\text{ m})=2.13\times10^{-4}\text{ m}=0.213\text{ mm}$.

EVALUATE: The image is real and is 1.75 m in front of the mirror.

34.8. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$.

SET UP: The mirror surface is convex so $R=-3.00\text{ cm}$. $s=18.0\text{ cm}-3.00\text{ cm}=15.0\text{ cm}$.

EXECUTE: $f = \frac{R}{2} = -1.50 \text{ cm}.$ $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}.$ $s' = \frac{sf}{s - f} = \frac{(15.0 \text{ cm})(-1.50 \text{ cm})}{15.0 \text{ cm} - (-1.50 \text{ cm})} = -1.3636 \text{ cm},$ which rounds to –1.36 cm. The image is 1.36 cm behind the surface so it is $3.00 \text{ cm} - 1.36 \text{ cm} = 1.64 \text{ cm}$ from the center of the ornament, on the same side of the center as the object.

$m = -\frac{s'}{s} = -\frac{-1.3636 \text{ cm}}{15.0 \text{ cm}} = +0.0909.$

EVALUATE: The image is virtual, upright and much smaller than the object.

34.9. IDENTIFY: The shell behaves as a spherical mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f},$ and its magnification is given by $m = -\frac{s'}{s}.$

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} = \frac{2}{-18.0 \text{ cm}} - \frac{1}{-6.00 \text{ cm}} \Rightarrow s = 18.0 \text{ cm}$ from the vertex.

$m = -\frac{s'}{s} = -\frac{-6.00 \text{ cm}}{18.0 \text{ cm}} = \frac{1}{3} \Rightarrow y' = \frac{1}{3}(1.5 \text{ cm}) = 0.50 \text{ cm}.$ The image is 0.50 cm tall, erect and virtual.

EVALUATE: Since the magnification is less than one, the image is smaller than the object.

34.10. IDENTIFY: The bottom surface of the bowl behaves as a spherical convex mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f},$ and its magnification is given by $m = -\frac{s'}{s}.$

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{-2}{35 \text{ cm}} - \frac{1}{60 \text{ cm}} \Rightarrow s' = -13.5 \text{ cm},$ which rounds to 14 cm behind the bowl.

$m = -\frac{s'}{s} = \frac{13.5 \text{ cm}}{60 \text{ cm}} = 0.225 \Rightarrow y' = (0.225)(5.0 \text{ cm}) = 1.1 \text{ cm}.$ The image is 1.1 cm tall, erect and virtual.

EVALUATE: Since the magnification is less than one, the image is smaller than the object.

34.11. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = -\frac{s'}{s}.$

SET UP: For a concave mirror, $R > 0.$ $R = 32.0 \text{ cm}$ and $f = \frac{R}{2} = 16.0 \text{ cm}.$

EXECUTE: (a) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}.$ $s' = \frac{sf}{s - f} = \frac{(12.0 \text{ cm})(16.0 \text{ cm})}{12.0 \text{ cm} - 16.0 \text{ cm}} = -48.0 \text{ cm}.$

$m = -\frac{s'}{s} = -\frac{-48.0 \text{ cm}}{12.0 \text{ cm}} = +4.00.$

(b) $s' = -48.0 \text{ cm},$ so the image is 48.0 cm to the right of the mirror. $s' < 0$ so the image is virtual.

(c) The principal-ray diagram is sketched in Figure 34.11. The rules for principal rays apply only to paraxial rays. Principal ray 2, which travels to the mirror along a line that passes through the focus, makes a large angle with the optic axis and is not described well by the paraxial approximation. Therefore, principal ray 2 is not included in the sketch.

EVALUATE: A concave mirror forms a virtual image whenever $s < f.$

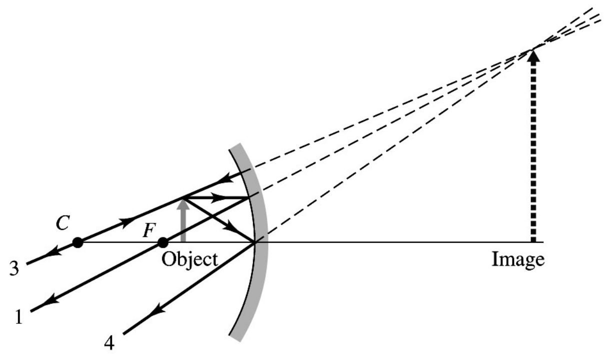


Figure 34.11

34.12. IDENTIFY and SET UP: For a spherical mirror, we have $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and the magnification is $m=-\frac{s'}{s}$. For a real image, $s' > 0$, so m is negative. The image height is the same as the object height, so $s' = s$.

EXECUTE: Using $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, with $s' = s$, we have $\frac{1}{f}=\frac{1}{s}+\frac{1}{s}=\frac{2}{s}=\frac{1}{18.0\text{ cm}}$, so $s = 36.0$ cm.

EVALUATE: The radius of curvature of the mirror is $R = 2f = 2(18.0\text{ cm}) = 36.0$ cm, which is the same as s. Therefore the object is at the center of curvature of the concave mirror.

34.13. IDENTIFY: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=\frac{y'}{y}=-\frac{s'}{s}$.

SET UP: $m=+2.00$ and $s=1.25$ cm. An erect image must be virtual.

EXECUTE: **(a)** $s'=\frac{sf}{s-f}$ and $m=-\frac{f}{s-f}$. For a concave mirror, m can be larger than 1.00. For a convex mirror, $|f|=-f$ so $m=+\frac{|f|}{s+|f|}$ and m is always less than 1.00. The mirror must be concave $(f>0)$.

(b) $\frac{1}{f}=\frac{s'+s}{ss'}$. $f=\frac{ss'}{s+s'}$. $m=-\frac{s'}{s}=+2.00$ and $s'=-2.00s$. $f=\frac{s(-2.00s)}{s-2.00s}=+2.00s=+2.50$ cm. $R=2f=+5.00$ cm.

(c) The principal-ray diagram is drawn in Figure 34.13.

EVALUATE: The principal-ray diagram agrees with the description from the equations.

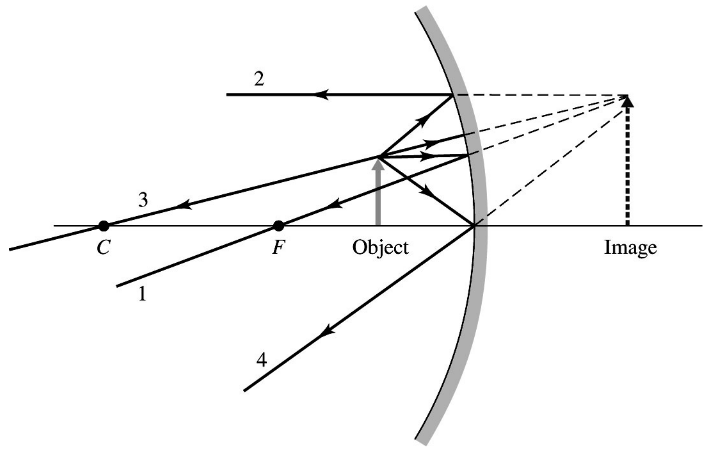


Figure 34.13

34.14. **IDENTIFY** and **SET UP:** For a spherical mirror, we have $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and the magnification is $m=-\frac{s'}{s}$. For a convex mirror, the image is virtual, so $s' < 0$, so m is positive. The image height is $\frac{1}{2}$ the same as the object height, so $m = +\frac{1}{2}$. Therefore $+\frac{1}{2}=-\frac{s'}{s}$, which gives $s' = -s/2$.

EXECUTE: Using $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, we have $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}-\frac{2}{s}=-\frac{1}{s}=\frac{1}{-12.0\text{ cm}}$, so $s = +12.0$ cm.

EVALUATE: $s' = -s/2 = -6.00$ cm, so the image is virtual, erect, and 6.0 cm from the vertex of the mirror on the side opposite the object.

34.15. **IDENTIFY:** In part (a), the shell is a concave mirror, but in (b) it is a convex mirror. The magnitude of its focal length is the same in both cases, but the sign reverses.

SET UP: For the orientation of the shell shown in the figure in the problem, $R = +12.0$ cm. When the glass is reversed, so the seed faces a convex surface, $R = -12.0$ cm. $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$ and $m=\frac{y'}{y}=-\frac{s'}{s}$.

EXECUTE: **(a)** $R = +12.0$ cm. $\frac{1}{s'}=\frac{2}{R}-\frac{1}{s}=\frac{2s-R}{Rs}$ and $s'=\frac{Rs}{2s-R}=\frac{(12.0\text{ cm})(15.0\text{ cm})}{30.0\text{ cm}-12.0\text{ cm}}=+10.0$ cm.

$m=-\frac{s'}{s}=-\frac{10.0\text{ cm}}{15.0\text{ cm}}=-0.667$. $y'=my=-2.20$ mm. The image is 10.0 cm to the left of the shell vertex and is 2.20 mm tall.

(b) $R=-12.0$ cm. $s'=\frac{(-12.0\text{ cm})(15.0\text{ cm})}{30.0\text{ cm}+12.0\text{ cm}}=-4.29$ cm. $m=-\frac{-4.29\text{ cm}}{15.0\text{ cm}}=+0.286$.

$y'=my=0.944$ mm. The image is 4.29 cm to the right of the shell vertex and is 0.944 mm tall.

EVALUATE: In (a), $s > R/2$ and the mirror is concave, so the image is real. In (b) the image is virtual because a convex mirror always forms a virtual image.

34.16. **IDENTIFY:** We have a concave mirror.

SET UP: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$, $m=-s'/s$, $f = R/2$. We want the radius of curvature R.

EXECUTE: Since $R = 2f$, we need to find f. First use the magnification. For this mirror, $m = -h_i/h_o$ $= -(2.50\text{ cm})/(0.600\text{ cm}) = -4.167$. Using $m = -s'/s$, we have $-4.167 = -s'/s$, so $s' = 4.167s =$ $(4.167)(24.0\text{ cm}) = 100\text{ cm}$. Now find f. $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$ $= 1/(24.0\text{ cm}) + 1/(100\text{ cm})$. $f = 19.4$ cm. Finally R $= 2f = 38.7$ cm.

EVALUATE: The image is on the same side of the mirror as the object and is real.

34.17. **IDENTIFY:** A spoon forms a concave mirror.

SET UP: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$, $m = -s'/s$, $f = R/2$.

EXECUTE: **(a)** Upside down.
(b) Real image.
(c) Estimate: Height = 22 cm.
(d) Estimate: Image height ≈ 2 cm.
(e) $m = -h_{\text{image}}/h_{\text{object}} = -(2\text{ cm})/(22\text{ cm}) = -0.09$.
(f) We want R. First find f and then use $R = 2f$. $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{25\text{ cm}} + \dfrac{1}{s'}$. $m = -s'/s = -0.09$. This gives $s' = (0.09)(25\text{ cm}) = 2.3$ cm. $\dfrac{1}{f} = \dfrac{1}{25\text{ cm}} + \dfrac{1}{2.3\text{ cm}}$. $f = 2.1$ cm. $R = 2(2.1\text{ cm}) \approx 4$ cm.
(g) Image is right-side up.
(h) Virtual.

EVALUATE: This is a easy way to get fairly reasonable results.

34.18. **IDENTIFY:** The surface is flat so $R \to \infty$ and $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0$.

SET UP: The light travels from the fish to the eye, so $n_a = 1.333$ and $n_b = 1.00$. When the fish is viewed, $s = 7.0$ cm. The fish is $20.0\text{ cm} - 7.0\text{ cm} = 13.0\text{ cm}$ above the mirror, so the image of the fish is 13.0 cm below the mirror and $20.0\text{ cm} + 13.0\text{ cm} = 33.0\text{ cm}$ below the surface of the water. When the image is viewed, $s = 33.0$ cm.

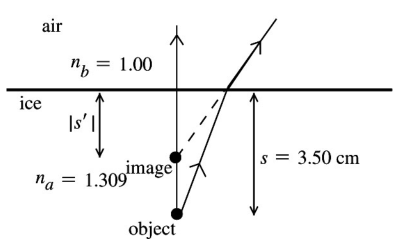
EXECUTE: **(a)** $s' = -\left(\dfrac{n_b}{n_a}\right)s = -\left(\dfrac{1.00}{1.333}\right)(7.0\text{ cm}) = -5.25\text{ cm}$. The apparent depth is 5.25 cm.

(b) $s' = -\left(\dfrac{n_b}{n_a}\right)s = -\left(\dfrac{1.00}{1.333}\right)(33.0\text{ cm}) = -24.8\text{ cm}$. The apparent depth of the image of the fish in the mirror is 24.8 cm.

EVALUATE: In each case the apparent depth is less than the actual depth of what is being viewed.

34.19. **IDENTIFY:** Apply $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}$, with $R \to \infty$. $|s'|$ is the apparent depth.

SET UP: The image and object are shown in Figure 34.19.



$\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}$;
$R \to \infty$ (flat surface), so
$\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0$.

Figure 34.19

EXECUTE: $s' = -\dfrac{n_b s}{n_a} = -\dfrac{(1.00)(3.50 \text{ cm})}{1.309} = -2.67 \text{ cm}.$

The apparent depth is 2.67 cm.

EVALUATE: When the light goes from ice to air (larger to smaller n), it is bent away from the normal and the virtual image is closer to the surface than the object is.

34.20. **IDENTIFY:** The concave end of a glass rod forms an image inside the glass of an outside object.

SET UP: $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}$, $R = -15.0$ cm (concave surface), a is the air and b is the glass. We want the image location s'.

EXECUTE: The distant object is very far away, so $s = \infty$. This gives

$s' = \dfrac{Rn_b}{n_b - n_a} = \dfrac{(-15.0 \text{ cm})(1.50)}{1.50 - 1.00} = -4.50 \text{ cm}.$ Since s' is negative the image is in the air.

EVALUATE: This image has been formed by *refraction*, not reflection.

34.21. **IDENTIFY:** Think of the surface of the water as a section of a sphere having an infinite radius of curvature.

SET UP: $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0.$ $n_a = 1.00.$ $n_b = 1.333.$

EXECUTE: The image is $5.20 \text{ m} - 0.80 \text{ m} = 4.40 \text{ m}$ above the surface of the water, so $s' = -4.40$ m.

$s = -\dfrac{n_a}{n_b}s' = -\left(\dfrac{1.00}{1.333}\right)(-4.40 \text{ m}) = +3.30 \text{ m}.$

EVALUATE: The diving board is closer to the water than it looks to the swimmer.

34.22. **IDENTIFY:** Think of the surface of the water as a section of a sphere having an infinite radius of curvature.

SET UP: $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0.$ $n_a = 1.333.$ $n_b = 1.00.$

EXECUTE: The image is 4.00 m below surface of the water, so $s' = -4.00$ m.

$s = -\dfrac{n_a}{n_b}s' = -\left(\dfrac{1.333}{1.00}\right)(-4.00 \text{ m}) = 5.33 \text{ m}.$

EVALUATE: The water is 1.33 m deeper than it appears to the person.

34.23. **IDENTIFY:** $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}.$ $m = -\dfrac{n_a s'}{n_b s}.$ Light comes from the fish to the person's eye.

SET UP: $R = -14.0$ cm. $s = +14.0$ cm. $n_a = 1.333$ (water). $n_b = 1.00$ (air). Figure 34.23 shows the object and the refracting surface.

EXECUTE: **(a)** $\dfrac{1.333}{14.0 \text{ cm}} + \dfrac{1.00}{s'} = \dfrac{1.00 - 1.333}{-14.0 \text{ cm}}.$ $s' = -14.0$ cm. $m = -\dfrac{(1.333)(-14.0 \text{ cm})}{(1.00)(14.0 \text{ cm})} = +1.33.$

The fish's image is 14.0 cm to the left of the bowl surface so is at the center of the bowl and the magnification is 1.33.

(b) The focal point is at the image location when $s \to \infty$. $\dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}.$ $n_a = 1.00.$ $n_b = 1.333.$

$R = +14.0$ cm. $\dfrac{1.333}{s'} = \dfrac{1.333 - 1.00}{14.0 \text{ cm}}.$ $s' = +56.0$ cm. s' is greater than the diameter of the bowl, so the surface facing the sunlight does not focus the sunlight to a point inside the bowl. The focal point is outside the bowl and there is no danger to the fish.

EVALUATE: In part (b) the rays refract when they exit the bowl back into the air so the image we calculated is not the final image.

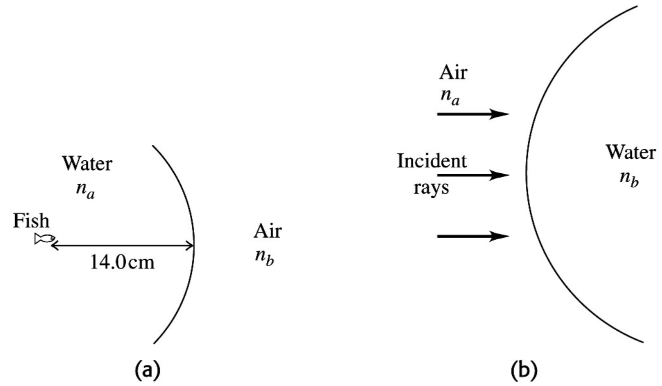


Figure 34.23

34.24. IDENTIFY: Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$.

SET UP: For a convex surface, $R>0$. $R=+3.00$ cm. $n_a=1.00$, $n_b=1.60$.

EXECUTE: **(a)** $s\to\infty$. $\frac{n_b}{s'}=\frac{n_b-n_a}{R}$. $s'=\left(\frac{n_b}{n_b-n_a}\right)R=\left(\frac{1.60}{1.60-1.00}\right)(+3.00\text{ cm})=+8.00\text{ cm}$. The image is 8.00 cm to the right of the vertex.

(b) $s=12.0$ cm. $\frac{1.00}{12.0\text{ cm}}+\frac{1.60}{s'}=\frac{1.60-1.00}{3.00\text{ cm}}$. $s'=+13.7$ cm. The image is 13.7 cm to the right of the vertex.

(c) $s=2.00$ cm. $\frac{1.00}{2.00\text{ cm}}+\frac{1.60}{s'}=\frac{1.60-1.00}{3.00\text{ cm}}$. $s'=-5.33$ cm. The image is 5.33 cm to the left of the vertex.

EVALUATE: The image can be either real $(s'>0)$ or virtual $(s'<0)$, depending on the distance of the object from the refracting surface.

34.25. IDENTIFY: The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and oil.

SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the equation $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$.

EXECUTE: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.45}{s}+\frac{1.60}{1.20\text{ m}}=\frac{0.15}{0.0300\text{ m}}\Rightarrow s=39.5\text{ cm}$.

EVALUATE: The presence of the oil changes the location of the image.

34.26. IDENTIFY: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$. $m=-\frac{n_a s'}{n_b s}$.

SET UP: $R=+4.00$ cm. $n_a=1.00$. $n_b=1.60$. $s=24.0$ cm.

EXECUTE: $\frac{1}{24.0\text{ cm}}+\frac{1.60}{s'}=\frac{1.60-1.00}{4.00\text{ cm}}$. $s'=+14.8$ cm. $m=-\frac{(1.00)(14.8\text{ cm})}{(1.60)(24.0\text{ cm})}=-0.385$.

$|y'|=|m||y|=(0.385)(1.50\text{ mm})=0.578\text{ mm}$. The image is 14.8 cm to the right of the vertex and is 0.578 mm tall. $m<0$, so the image is inverted.

EVALUATE: The image is real.

34.27. **IDENTIFY:** We have image formation by a spherical mirror.

SET UP: $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$, $m = -s'/s$, $R = 2f$.

EXECUTE: **(a)** We want R and h_{image} if the image is real. If the image is real, the mirror must be concave. $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$ = 1/(20.0 cm) + 1/(60.0 cm). f = 15.0 cm, so $R = 2f$ = 30.0 cm. $m = -s'/s$ = –(60.0 cm)/(20.0 cm) = –3.00. h_{image} = (3.00)(3.20 mm) = 9.60 mm, and it is inverted.

(b) Same as (a) except the image is virtual. In this case, s' = –60.0 cm.

$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$ = 1/(20.0 cm) + 1/(–60.0 cm). f = +30.0 cm, so $R = 2f$ = 60.0 cm.

$m = -s'/s$ = –(–60.0 cm)/(20.0 cm) = +3.00. h_{image} = (3.00)(3.20 mm) = 9.60 mm, and it is upright.

EVALUATE: We get the same magnitude magnification in both cases because s and s' have the same magnitudes in both cases, but they differ in sign.

34.28. **IDENTIFY:** We are trying to find the focal length of a converging lens.

SET UP: Since the graph plots $1/h'$ versus s, we need to relate these quantities so we can interpret the graph. $m = -s'/s$, $m = h'/h$, $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$. We only need the magnitude because only magnitudes are on the graph.

EXECUTE: $h' = mh = (s'/s)h$, so $\frac{1}{h'} = \frac{s}{hs'} = \frac{s}{h} \cdot \frac{1}{s'}$. Using $\frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$, we get $\frac{1}{h'} = \frac{s}{h}\left(\frac{1}{f} - \frac{1}{s}\right) = \frac{s}{hf} - \frac{1}{h}$.

Therefore a graph of $1/h'$ versus s should be a straight line having slope equal to $1/hf$, so $f = \frac{1}{h(\text{slope})}$.

$f = 1/[(4.00 \text{ mm})(0.208 \text{ cm}^{-2})] = 12.0$ cm.

EVALUATE: Since only the heights were measured, we used only the magnitude of the magnification.

34.29. **IDENTIFY:** Use the lensmaker's equation $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ to calculate f. Then apply the thin-lens equation $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$.

SET UP: $R_1 \to \infty$. $R_2 = -13.0$ cm. If the lens is reversed, $R_1 = +13.0$ cm and $R_2 \to \infty$.

EXECUTE: **(a)** $\frac{1}{f} = (0.70)\left(\frac{1}{\infty} - \frac{1}{-13.0 \text{ cm}}\right) = \frac{0.70}{13.0 \text{ cm}}$ and $f = 18.6$ cm. $\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf}$.

$s' = \frac{sf}{s-f} = \frac{(22.5 \text{ cm})(18.6 \text{ cm})}{22.5 \text{ cm} - 18.6 \text{ cm}} = 107$ cm. $m = -\frac{s'}{s} = -\frac{107 \text{ cm}}{22.5 \text{ cm}} = -4.76$.

$y' = my = (-4.76)(3.75 \text{ mm}) = -17.8$ mm. The image is 107 cm to the right of the lens and is 17.8 mm tall. The image is real and inverted.

(b) $\frac{1}{f} = (n-1)\left(\frac{1}{13.0 \text{ cm}} - \frac{1}{\infty}\right)$ and $f = 18.6$ cm. The image is the same as in part (a).

EVALUATE: Reversing a lens does not change the focal length of the lens.

34.30. **IDENTIFY:** $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. The sign of f determines whether the lens is converging or diverging.

SET UP: $s = 16.0$ cm. $s' = -12.0$ cm.

EXECUTE: **(a)** $f = \frac{ss'}{s+s'} = \frac{(16.0 \text{ cm})(-12.0 \text{ cm})}{16.0 \text{ cm} + (-12.0 \text{ cm})} = -48.0$ cm. $f < 0$ and the lens is diverging.

(b) $m = -\dfrac{s'}{s} = -\dfrac{-12.0\text{ cm}}{16.0\text{ cm}} = +0.750.\ |y'| = |m|\, y = (0.750)(8.50\text{ mm}) = 6.38\text{ mm}.$ $m > 0$ and the image is erect.

(c) The principal-ray diagram is sketched in Figure 34.30.

EVALUATE: A diverging lens always forms an image that is virtual, erect, and reduced in size.

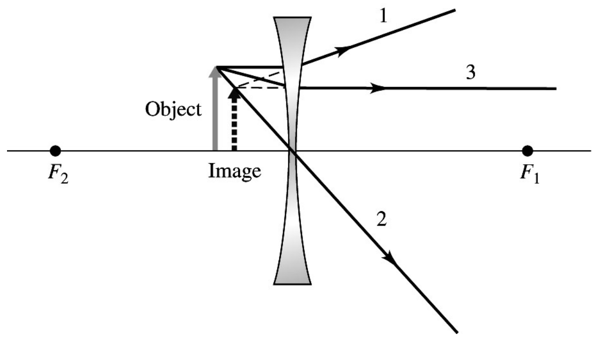


Figure 34.30

34.31. **IDENTIFY:** Use the lensmaker's equation and the thin-lens equation.

SET UP: Combine the lensmaker's equation and the thin-lens equation to get $s = 14.2\text{ cm}: m = -\dfrac{s'}{s} = -\dfrac{14.2}{3.80} = -3.74.$ and use the fact that the magnification of the lens is $m = -\dfrac{s'}{s}.$

EXECUTE: **(a)** $\dfrac{1}{s} + \dfrac{1}{s'} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) \Rightarrow \dfrac{1}{24.0\text{ cm}} + \dfrac{1}{s'} = (1.52-1)\left(\dfrac{1}{-7.00\text{ cm}} - \dfrac{1}{-4.00\text{ cm}}\right)$

$\Rightarrow s' = 71.2\text{ cm},$ to the right of the lens.

(b) $m = -\dfrac{s'}{s} = -\dfrac{71.2\text{ cm}}{24.0\text{ cm}} = -2.97.$

EVALUATE: Since the magnification is negative, the image is inverted.

34.32. **IDENTIFY:** Apply $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$ to relate s' and s and then use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}.$

SET UP: Since the image is inverted, $y' < 0$ and $m < 0.$

EXECUTE: $m = \dfrac{y'}{y} = \dfrac{-4.50\text{ cm}}{3.20\text{ cm}} = -1.406.$ $m = -\dfrac{s'}{s}$ gives $s' = +1.406s.$ $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ gives $\dfrac{1}{s} + \dfrac{1}{1.406s} = \dfrac{1}{70.0\text{ cm}}$ so $s = 119.8\text{ cm},$ which rounds to 120 cm. $s' = (1.406)(119.8\text{ cm}) = 168\text{ cm}.$ The object is 120 cm to the left of the lens. The image is 168 cm to the right of the lens and is real.

EVALUATE: For a single lens an inverted image is always real.

34.33. **IDENTIFY:** The thin-lens equation applies in this case.

SET UP: The thin-lens equation is $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f},$ and the magnification is $m = -\dfrac{s'}{s} = \dfrac{y'}{y}.$

EXECUTE: $m = \dfrac{y'}{y} = \dfrac{34.0\text{ mm}}{8.00\text{ mm}} = 4.25 = -\dfrac{s'}{s} = -\dfrac{-12.0\text{ cm}}{s} \Rightarrow s = 2.82\text{ cm}.$ The thin-lens equation gives $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow f = 3.69\text{ cm}.$

EVALUATE: Since the focal length is positive, this is a converging lens. The image distance is negative because the object is inside the focal point of the lens.

34.34. **IDENTIFY:** Apply $m = -\dfrac{s'}{s}$ to relate s and s'. Then use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$.

SET UP: Since the image is to the right of the lens, $s' > 0$. $s' + s = 6.00$ m.

EXECUTE: **(a)** $s' = 80.0s$ and $s + s' = 6.00$ m gives $81.00s = 6.00$ m and $s = 0.0741$ m. $s' = 5.93$ m.

(b) The image is inverted since both the image and object are real $(s' > 0, s > 0)$.

(c) $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{0.0741\text{ m}} + \dfrac{1}{5.93\text{ m}} \Rightarrow f = 0.0732$ m, and the lens is converging.

EVALUATE: The object is close to the lens and the image is much farther from the lens. This is typical for slide projectors.

34.35. **IDENTIFY:** Apply $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$.

SET UP: For a distant object the image is at the focal point of the lens. Therefore, $f = 1.87$ cm. For the double-convex lens, $R_1 = +R$ and $R_2 = -R$, where $R = 2.50$ cm.

EXECUTE: $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R} - \dfrac{1}{-R}\right) = \dfrac{2(n-1)}{R}$. $n = \dfrac{R}{2f} + 1 = \dfrac{2.50\text{ cm}}{2(1.87\text{ cm})} + 1 = 1.67$.

EVALUATE: $f > 0$ and the lens is converging. A double-convex lens surrounded by air is always converging.

34.36. **IDENTIFY:** We know the focal length and magnification and are asked to find the locations of the object and image.

SET UP: $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$. Since the image is erect, $y' > 0$ and $m > 0$. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$.

EXECUTE: $m = \dfrac{y'}{y} = \dfrac{1.30\text{ cm}}{0.400\text{ cm}} = +3.25$. $0.375t = 0.30$ cm gives $s' = -3.25s$. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ gives $\dfrac{1}{s} + \dfrac{1}{-3.25s} = \dfrac{1}{9.00\text{ cm}}$ so $s = 6.23$ cm. $s' = -(3.25)(6.23\text{ cm}) = -20.2$ cm. The object is 6.23 cm to the left of the lens. The image is 20.2 cm to the left of the lens and is virtual.

EVALUATE: The image is virtual because the object distance is less than the focal length.

34.37. **IDENTIFY:** This problem involves the lensmaker's equation and a thin lens.

SET UP: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$, $m = -s'/s$, $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$. We use $R_1 = \infty$ and want the height h' of the image.

EXECUTE: First find the focal length. $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = (1.50-1)\left(0 - \dfrac{1}{-20.0\text{ cm}}\right)$. This gives f = 40.0 cm. Now use $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$ and $m = -s'/s$ to find h'.

Get s': $\dfrac{1}{s'} = \dfrac{1}{f} - \dfrac{1}{s}$ = 1/(40.0 cm) – 1/(20.0 cm), so s' = –40.0 cm.

Now find m: $m = -s'/s$ = –(–40.0 cm)/(20.0 cm) = + 2.00.

$h' = |m|h$ = (2.00)(6.00 mm) = 12.0 mm.

EVALUATE: The image is erect and virtual.

34.38. **IDENTIFY:** Apply the lensmaker's formula to calculate the radii of the surfaces.

SET UP: $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$, where n = 1.55 and f = 20.0 cm.

EXECUTE: Since $f > 0$ we choose $R_1 = R$ and $R_2 = -R$, where R is the magnitude of the radius of curvature. Thus we have $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R} - \dfrac{1}{-R}\right) = \dfrac{2(n-1)}{R}$. Solving for R we obtain $R = 2(n-1)f = 2(1.55-1)(20.0\text{ cm}) = 22\text{ cm}.$

EVALUATE: For identical convex surfaces, the relation between f and R is $f = \dfrac{1}{n-1} \cdot \dfrac{R}{2}$. This is reminiscent of the relation for spherical mirrors, which is $f = \dfrac{R}{2}$.

34.39. IDENTIFY: This problem involves the lensmaker's equation and a thin lens.

SET UP: $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$. One side of the lens is flat, and we want the radius R of the curved side. For the flat side, $R_2 = \infty$.

EXECUTE: $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = (n-1)\left(\dfrac{1}{R} - \dfrac{1}{\infty}\right) = \dfrac{n-1}{R}$. Solving for R gives $R = (n-1)f =$ $(1.50 - 1)(-24.0\text{ cm}) = -12\text{ cm}.$

EVALUATE: R should be negative because the center of curvature for the first surface is on the side of the incoming light, and this agrees with our result. Also f is negative for diverging lenses, as we have found.

34.40. IDENTIFY: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$.

SET UP: $f = +12.0\text{ cm}$ and $s' = -17.0\text{ cm}.$

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s} = \dfrac{1}{12.0\text{ cm}} - \dfrac{1}{-17.0\text{ cm}} \Rightarrow s = 7.0\text{ cm}.$

$m = -\dfrac{s'}{s} = -\dfrac{(-17.0)}{7.0} = +2.4 \Rightarrow y = \dfrac{y'}{m} = \dfrac{0.800\text{ cm}}{+2.4} = +0.34\text{ cm},$ so the object is 0.34 cm tall, erect, same side as the image. The principal-ray diagram is sketched in Figure 34.40. The image is erect.

EVALUATE: When the object is inside the focal point, a converging lens forms a virtual, enlarged image.

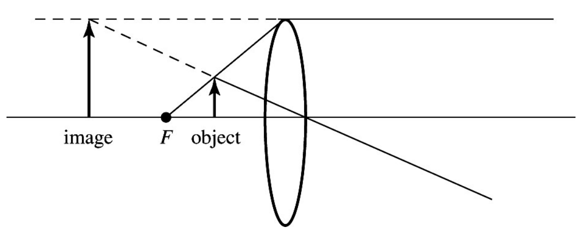


Figure 34.40

34.41. IDENTIFY: Use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to calculate the object distance s. m calculated from $m = -\dfrac{s'}{s}$ determines the size and orientation of the image.

SET UP: $f = -48.0\text{ cm}.$ Virtual image 17.0 cm from lens so $s' = -17.0\text{ cm}.$

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, so $\dfrac{1}{s} = \dfrac{1}{f} - \dfrac{1}{s'} = \dfrac{s'-f}{s'f}$.

$$s = \frac{s'f}{s'-f} = \frac{(-17.0\text{ cm})(-48.0\text{ cm})}{-17.0\text{ cm} - (-48.0\text{ cm})} = +26.3\text{ cm}.$$

$$m = -\frac{s'}{s} = -\frac{-17.0\text{ cm}}{+26.3\text{ cm}} = +0.646.$$

$$m = \frac{y'}{y} \text{ so } |y| = \frac{|y'|}{|m|} = \frac{8.00\text{ mm}}{0.646} = 12.4\text{ mm}.$$

The principal-ray diagram is sketched in Figure 34.41.

EVALUATE: Virtual image, real object $(s > 0)$ so image and object are on same side of lens. $m > 0$ so image is erect with respect to the object. The height of the object is 12.4 mm.

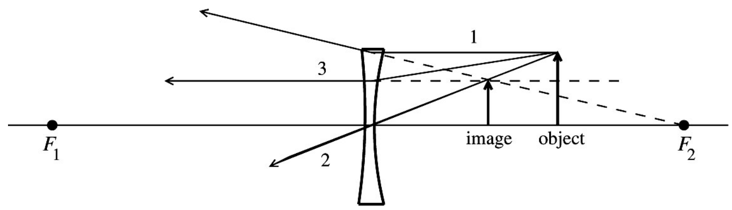


Figure 34.41

34.42. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

SET UP: The sign of f determines whether the lens is converging or diverging. $s = 16.0$ cm. $s' = +36.0$ cm. Use $m = -\frac{s'}{s}$ to find the size and orientation of the image.

EXECUTE: **(a)** $f = \frac{ss'}{s+s'} = \frac{(16.0\text{ cm})(36.0\text{ cm})}{16.0\text{ cm} + 36.0\text{ cm}} = 11.1\text{ cm}.$ $f > 0$ and the lens is converging.

(b) $m = -\frac{s'}{s} = -\frac{36.0\text{ cm}}{16.0\text{ cm}} = -2.25.$ $|y'| = |m|\, y = (2.25)(8.00\text{ mm}) = 18.0\text{ mm}.$ $m < 0$ so the image is inverted.

(c) The principal-ray diagram is sketched in Figure 34.42.

EVALUATE: The image is real so the lens must be converging.

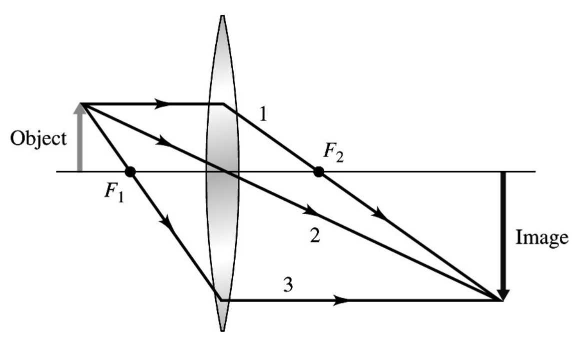


Figure 34.42

34.43. IDENTIFY: The first lens forms an image that is then the object for the second lens.

SET UP: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to each lens. $m_1 = \frac{y_1'}{y_1}$ and $m_2 = \frac{y_2'}{y_2}$.

EXECUTE: **(a)** Lens 1: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s_1'=\frac{s_1f_1}{s_1-f_1}=\frac{(50.0\text{ cm})(40.0\text{ cm})}{50.0\text{ cm}-40.0\text{ cm}}=+200\text{ cm}.$

$m_1=-\frac{s_1'}{s_1}=-\frac{200\text{ cm}}{50\text{ cm}}=-4.00.$ $y_1'=m_1y_1=(-4.00)(1.20\text{ cm})=-4.80\text{ cm}.$ The image I_1 is 200 cm to the right of lens 1, is 4.80 cm tall and is inverted.

(b) Lens 2: $y_2=-4.80$ cm. The image I_1 is $300\text{ cm}-200\text{ cm}=100\text{ cm}$ to the left of lens 2, so $s_2=+100\text{ cm}.$ $s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(100\text{ cm})(60.0\text{ cm})}{100\text{ cm}-60.0\text{ cm}}=+150\text{ cm}.$ $m_2=-\frac{s_2'}{s_2}=-\frac{150\text{ cm}}{100\text{ cm}}=-1.50.$

$y_2'=m_2y_2=(-1.50)(-4.80\text{ cm})=+7.20\text{ cm}.$ The image is 150 cm to the right of the second lens, is 7.20 cm tall, and is erect with respect to the original object.

EVALUATE: The overall magnification of the lens combination is $m_{\text{tot}}=m_1m_2$.

34.44. **IDENTIFY:** The first lens forms an image that is then the object for the second lens. We follow the same general procedure as in Problem 34.43.

SET UP: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. $m_1=\frac{y_1'}{y_1}$ and $m_2=\frac{y_2'}{y_2}$. For a diverging lens, $f<0$.

EXECUTE: **(a)** $f_1=+40.0$ cm. I_1 is the same as in Problem 34.41. For lens 2,

$s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(100\text{ cm})(-60.0\text{ cm})}{100\text{ cm}-(-60.0\text{ cm})}=-37.5\text{ cm}.$ $m_2=-\frac{s_2'}{s_2}=-\frac{-37.5\text{ cm}}{100\text{ cm}}=+0.375.$

$y_2'=m_2y_2=(+0.375)(-4.80\text{ cm})=-1.80\text{ cm}.$ The final image is 37.5 cm to the left of the second lens (262.5 cm to the right of the first lens). The final image is inverted and is 1.80 cm tall.

(b) $f_1=-40.0$ cm. $s_1'=\frac{s_1f_1}{s_1-f_1}=\frac{(50.0\text{ cm})(-40.0\text{ cm})}{50.0\text{ cm}-(-40.0\text{ cm})}=-22.2\text{ cm}.$ $m_1=-\frac{s_1'}{s_1}=-\frac{-22.2\text{ cm}}{50.0\text{ cm}}=+0.444.$

$y_1'=m_1y_1=(0.444)(1.20\text{ cm})=0.533\text{ cm}.$ The image I_1 is 22.2 cm to the left of lens 1 so is $22.2\text{ cm}+300\text{ cm}=322.2\text{ cm}$ to the left of lens 2 and $s_2=+322.2\text{ cm}.$ $y_2=y_1'=0.533\text{ cm}.$

$s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(322.2\text{ cm})(60.0\text{ cm})}{322.2\text{ cm}-60.0\text{ cm}}=+73.7\text{ cm}.$ $m_2=-\frac{s_2'}{s_2}=-\frac{73.7\text{ cm}}{322.2\text{ cm}}=-0.229.$

$y_2'=m_2y_2=(-0.229)(0.533\text{ cm})=-0.122\text{ cm}.$ The final image is 73.7 cm to the right of the second lens, is inverted and is 0.122 cm tall.

(c) $f_1=-40.0$ cm. $f_2=-60.0$ cm. $\frac{f-f_0}{f_0}=-0.02\Rightarrow\frac{f}{f_0}=0.98$ so $2-\frac{1}{\cos\theta}=0.98.$ is as calculated in part (b). $s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(322.2\text{ cm})(-60.0\text{ cm})}{322.2\text{ cm}-(-60.0\text{ cm})}=-50.6\text{ cm}.$ $m_2=-\frac{s_2'}{s_2}=-\frac{-50.6\text{ cm}}{322.2\text{ cm}}=+0.157.$

$y_2'=m_2y_2=(0.157)(0.533\text{ cm})=0.0837\text{ cm}.$ The final image is 50.6 cm to the left of the second lens (249.4 cm to the right of the first lens), is upright and is 0.0837 cm tall.

EVALUATE: The overall magnification of the lens combination is $m_{\text{tot}}=m_1m_2$.

34.45. **IDENTIFY:** The first lens forms an image that is then the object for the second lens. We follow the same general procedure as in Problem 34.43.

SET UP: $m_{\text{tot}}=m_1m_2$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s'=\frac{sf}{s-f}$.

EXECUTE: **(a)** Lens 1: $f_1=-12.0$ cm, $s_1=20.0$ cm. $s'=\frac{(20.0\text{ cm})(-12.0\text{ cm})}{20.0\text{ cm}+12.0\text{ cm}}=-7.5\text{ cm}.$

$m_1=-\frac{s_1'}{s_1}=-\frac{-7.5\text{ cm}}{20.0\text{ cm}}=+0.375.$

Lens 2: The image of lens 1 is 7.5 cm to the left of lens 1 so is $7.5\text{ cm} + 9.00\text{ cm} = 16.5\text{ cm}$ to the left of lens 2. $s_2 = +16.5\text{ cm}$. $f_2 = +12.0\text{ cm}$. $s_2' = \dfrac{(16.5\text{ cm})(12.0\text{ cm})}{16.5\text{ cm} - 12.0\text{ cm}} = 44.0\text{ cm}$.

$m_2 = -\dfrac{s_2'}{s_2} = -\dfrac{44.0\text{ cm}}{16.5\text{ cm}} = -2.67$. The final image is 44.0 cm to the right of lens 2 so is 53.0 cm to the right of the first lens.

(b) $s_2' > 0$ so the final image is real.

(c) $m_{\text{tot}} = m_1 m_2 = (+0.375)(-2.67) = -1.00$. The image is 2.50 mm tall and is inverted.

EVALUATE: The light travels through the lenses in the direction from left to right. A real image for the second lens is to the right of that lens and a virtual image is to the left of the second lens.

34.46. IDENTIFY: We have a diverging lens.

SET UP: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$, $m = -s'/s$. We want *f* and the image height.

EXECUTE: **(a)** The image is to the left of the lens because a diverging lens forms a virtual image on the object side of the lens.

(b) $\dfrac{1}{s} = \dfrac{1}{f} - \dfrac{1}{s'}$ = 1/–25.0 cm) – 1/(–18.0 cm). s = 64.3 cm.

(c) $|m| = s'/s$ = (18.0 cm)/(64.3 cm) = 0.280. $|m| < 1$, so the image is shorter than the object.

EVALUATE: The image is virtual and upright but shorter than the object.

34.47. IDENTIFY: We have a thin converging lens.

SET UP: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$, $m = -s'/s$. We want the image location and the height of the image.

EXECUTE: **(a)** Eq. (34.16): $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$. $s = 2f/3$, so $\dfrac{1}{s'} = \dfrac{1}{f} - \dfrac{1}{2f/3} = \dfrac{1}{f}\left(1 - \dfrac{3}{2}\right) = -\dfrac{1}{2f}$, so $s' = -2f$.

Since s' is negative, the image is *virtual* and to the *left* of the lens.

(b) $|m| = h'/h = s'/s = \dfrac{2f}{2f/3} = 3$. So $h' = 3h$. $m = -s'/s$ and s' is negative, so *m* is positive. Therefore the image is *upright.*

EVALUATE: Looking at Fig. 34.37 in the text, we see that our results agree with that figure.

34.48. IDENTIFY: We have a thin lens.

SET UP: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$, $m = -s'/s$. We want *f.*

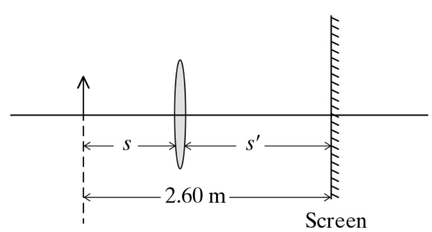


Figure 34.48

EXECUTE: **(a)** The image is on the outgoing side of the lens and is real, so it must be *inverted*. $m = -s'/s$ is negative because s' is positive.

(b) $m = -s'/s = -h'/h = -2.50$, so $s' = 2.50s$. From Fig. 34.38, we see that $s + s' = 2.60$ m. Therefore s + (2.50s) = 2.60 m, so s = 74.29 cm. Thus s' = (2.50)(74.29 cm) = 185.7 cm. $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$ = 1/(74.29 cm) + 1/(185.7 cm), so f = +53.1 cm. Since f is positive, the lens is *converging*.

EVALUATE: The lens would have to form a real image for it to be viewed on a screen. Only a converging lens will do that if used alone.

34.49. **IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$.

SET UP: $s = 3.90$ m. $f = 0.085$ m.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{3.90\text{ m}} + \frac{1}{s'} = \frac{1}{0.085\text{ m}} \Rightarrow s' = 0.0869$ m.

$y' = -\frac{s'}{s}y = -\frac{0.0869}{3.90}1750\text{ mm} = -39.0\text{ mm}$, so it will not fit on the 24-mm$\times$36-mm sensor.

EVALUATE: The image is just outside the focal point and $s' \approx f$. To have $|y'| = 36$ mm, so that the image will fit on the sensor, $s = -\frac{s'y}{y'} \approx -\frac{(0.085\text{ m})(1.75\text{ m})}{-0.036\text{ m}} = 4.1$ m. The person would need to stand about 4.1 m from the lens.

34.50. **IDENTIFY:** The projector lens can be modeled as a thin lens.

SET UP: The thin-lens equation is $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, and the magnification of the lens is $m = -\frac{s'}{s}$.

EXECUTE: **(a)** $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{0.150\text{ m}} + \frac{1}{9.00\text{ m}} \Rightarrow f = 147.5$ mm, so use a $f = 148$ mm lens.

(b) $m = -\frac{s'}{s} \Rightarrow |m| = 60 \Rightarrow$ Area $= 1.44\text{ m} \times 2.16\text{ m}$.

EVALUATE: The lens must produce a real image to be viewed on the screen. Since the magnification comes out negative, the slides to be viewed must be placed upside down in the tray.

34.51. **IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to each lens. The image of the first lens serves as the object for the second lens.

SET UP: For a distant object, $s \to \infty$.

EXECUTE: **(a)** $s_1 = \infty \Rightarrow s_1' = f_1 = 12$ cm to the right of the converging lens.

(b) $s_2 = 4.0\text{ cm} - 12\text{ cm} = -8\text{ cm}$.

(c) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{-8\text{ cm}} + \frac{1}{s_2'} = \frac{1}{-12\text{ cm}} \Rightarrow s_2' = 24$ cm, to the right of the diverging lens. This result agrees with Fig. 34.43a.

(d) $s_1 = \infty \Rightarrow s_1' = f_1 = 12$ cm to the right of the converging lens. $s_2 = 8.0\text{ cm} - 12\text{ cm} = -4\text{ cm}$.

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{-4\text{ cm}} + \frac{1}{s_2'} = \frac{1}{-12\text{ cm}} \Rightarrow s_2' = 6$ cm to the right of the diverging lens. This result agrees with Fig. 34.43b.

EVALUATE: In each case the image of the first lens serves as a virtual object for the second lens, and $s_2 < 0$.

34.52. **IDENTIFY:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

SET UP: $n_a = 1.00$, $n_b = 1.40$. $s = 40.0$ cm, $s' = 2.60$ cm.

EXECUTE: $\dfrac{1}{40.0\text{ cm}}+\dfrac{1.40}{2.60\text{ cm}}=\dfrac{0.40}{R}$ and $R=0.710$ cm.

EVALUATE: The cornea presents a convex surface to the object, so $R>0$.

34.53. **(a) IDENTIFY:** The purpose of the corrective lens is to take an object 25 cm from the eye and form a virtual image at the eye's near point. Use $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ to solve for the image distance when the object distance is 25 cm.

SET UP: $\dfrac{1}{f}=+2.75$ diopters means $f=+\dfrac{1}{2.75}\text{ m}=+0.3636\text{ m}$ (converging lens)

$f=36.36$ cm; $s=25$ cm; $s'=?$

EXECUTE: $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ so

$$s'=\frac{sf}{s-f}=\frac{(25\text{ cm})(36.36\text{ cm})}{25\text{ cm}-36.36\text{ cm}}=-80.0\text{ cm}.$$

The eye's near point is 80.0 cm from the eye.

(b) IDENTIFY: The purpose of the corrective lens is to take an object at infinity and form a virtual image of it at the eye's far point. Use $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ to solve for the image distance when the object is at infinity.

SET UP: $\dfrac{1}{f}=-1.30$ diopters means $f=-\dfrac{1}{1.30}\text{ m}=-0.7692\text{ m}$ (diverging lens).

$f=-76.92$ cm; $s=\infty$; $s'=?$

EXECUTE: $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ and $s=\infty$ says $\dfrac{1}{s'}=\dfrac{1}{f}$ and $s'=f=-76.9$ cm. The eye's far point is 76.9 cm from the eye.

EVALUATE: In each case a virtual image is formed by the lens. The eye views this virtual image instead of the object. The object is at a distance where the eye can't focus on it, but the virtual image is at a distance where the eye can focus.

34.54. **IDENTIFY** and **SET UP:** For an object 25.0 cm from the eye, the corrective lens forms a virtual image at the near point of the eye. $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$. $P(\text{in diopters})=1/f$ (in m).

EXECUTE: **(a)** The person is farsighted.

(b) A converging lens is needed.

(c) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$. $f=\dfrac{ss'}{s+s'}=\dfrac{(25.0\text{ cm})(-45.0\text{ cm})}{25.0\text{ cm}-45.0\text{ cm}}=+56.2$ cm. The power is $\dfrac{1}{0.562\text{ m}}=+1.78$ diopters.

EVALUATE: The object is inside the focal point of the lens, so it forms a virtual image.

34.55. **IDENTIFY** and **SET UP:** For an object 25.0 cm from the eye, the corrective lens forms a virtual image at the near point of the eye. The distances from the corrective lens are $s=23.0$ cm and $s'=-43.0$ cm. $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$. $P(\text{in diopters})=1/f$ (in m).

EXECUTE: Solving $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ for f gives $f=\dfrac{ss'}{s+s'}=\dfrac{(23.0\text{ cm})(-43.0\text{ cm})}{23.0\text{ cm}-43.0\text{ cm}}=+49.4$ cm. The power is $\dfrac{1}{0.494\text{ m}}=2.02$ diopters.

EVALUATE: In Problem 34.54 the contact lenses have power 1.78 diopters. The power of the lenses is different for ordinary glasses versus contact lenses.

34.56. **IDENTIFY** and **SET UP:** For an object very far from the eye, the corrective lens forms a virtual image at the far point of the eye. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $P(\text{in diopters}) = 1/f\,(\text{in m})$.

EXECUTE: **(a)** The person is nearsighted.

(b) A diverging lens is needed.

(c) In $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, $s \to \infty$, so $f = s' = -75.0$ cm. The power is $\frac{1}{-0.750\text{ m}} = -1.33$ diopters.

EVALUATE: A diverging lens is needed to form a virtual image of a distant object. A converging lens could not do this since distant objects cannot be inside its focal point.

34.57. **IDENTIFY:** We are dealing with the eye and want to find her far point and near point.

SET UP: For distant vision (upper half of the lens), f = 1/(–0.500 diopters) = –2.00 m = –200 cm. For close vision (lower half of the lens), f = 1/(+2.00 diopters) = 0.500 m = 50.0 cm. $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$.

EXECUTE: **(a)** Far point: A very distant object (i.e., at infinity) is placed at her far point. $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$, so $\frac{1}{-200\text{ cm}} = \frac{1}{\infty}+\frac{1}{s'}$, so s' = –200 cm. The image is 200 cm in front of the glasses, which is 202 cm in front of her eye. So her far point is 202 cm from her eye.

Near point: An object at 25 cm from her eye (23 cm from the glasses) is placed at her near point. $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$ gives $\frac{1}{50.0\text{ cm}}=\frac{1}{23\text{ cm}}+\frac{1}{s'}$, so s' = –42.6 cm. The image is 42.6 cm in front of the glasses, which is 44.6 cm from her eye. So her near point is at 44.6 cm.

(b) The image for the closest object she can see will be at her near point, which is 42.6 cm from the lens. $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$ gives $\frac{1}{-200\text{ cm}}=\frac{1}{s}+\frac{1}{-42.6\text{ cm}}$, s = 54.1 cm. The distance from her eye is 56.1 cm.

EVALUATE: Without glasses this woman can clearly see objects between 44.6 cm and 202 cm from her eye. A typical person with excellent vision can see between 25 cm and infinity.

34.58. **IDENTIFY:** When the object is at the focal point, $M = \frac{25.0\text{ cm}}{f}$. In part (b), apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate s for $s' = -25.0$ cm.

SET UP: Our calculation assumes the near point is 25.0 cm from the eye.

EXECUTE: **(a)** Angular magnification $M = \frac{25.0\text{ cm}}{f} = \frac{25.0\text{ cm}}{6.00\text{ cm}} = 4.17$.

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{s}+\frac{1}{-25.0\text{ cm}}=\frac{1}{6.00\text{ cm}} \Rightarrow s = 4.84$ cm.

EVALUATE: In part (b), $\theta' = \frac{y}{s}$, $\theta = \frac{y}{25.0\text{ cm}}$, and $M = \frac{25.0\text{ cm}}{s} = \frac{25.0\text{ cm}}{4.84\text{ cm}} = 5.17$. M is greater when the image is at the near point than when the image is at infinity.

34.59. **IDENTIFY:** Use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$ to calculate s and y'.

SET UP: $f = 8.00$ cm; $s' = -25.0$ cm; $s = ?$

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, so $\frac{1}{s}=\frac{1}{f}-\frac{1}{s'}=\frac{s'-f}{s'f}$.

$$s=\frac{s'f}{s'-f}=\frac{(-25.0\text{ cm})(+8.00\text{ cm})}{-25.0\text{ cm}-8.00\text{ cm}}=+6.06\text{ cm}.$$

(b) $m=-\frac{s'}{s}=-\frac{-25.0\text{ cm}}{6.06\text{ cm}}=+4.125.$

$|m|=\frac{|y'|}{|y|}$ so $|y'|=|m||y|=(4.125)(1.00\text{ mm})=4.12\text{ mm}.$

EVALUATE: The lens allows the object to be much closer to the eye than the near point. The lens allows the eye to view an image at the near point rather than the object.

34.60. **IDENTIFY:** For a thin lens, $-\frac{s'}{s}=\frac{y'}{y}$, so $\left|\frac{y'}{s'}\right|=\left|\frac{y}{s}\right|$, and the angular size of the image equals the angular size of the object.

SET UP: The object has angular size $\theta=\frac{y}{f}$, with θ in radians.

EXECUTE: $\theta=\frac{y}{f}\Rightarrow f=\frac{y}{\theta}=\frac{2.00\text{ mm}}{0.032\text{ rad}}=62.5\text{ mm}=6.25\text{ cm}$, which rounds to 6.3 cm.

EVALUATE: If the insect were at the near point of a normal human eye, its angular size would be $\frac{2.00\text{ mm}}{250\text{ mm}}=0.0080\text{ rad}.$

34.61. **(a) IDENTIFY** and **SET UP:** Use $M=-\frac{f_1}{f_2}$, with $f_1=95.0\text{ cm}$ (objective) and $f_2=15.0\text{ cm}$ (eyepiece).

EXECUTE: $M=-\frac{f_1}{f_2}=-\frac{95.0\text{ cm}}{15.0\text{ cm}}=-6.33.$

(b) IDENTIFY: Use $m=\frac{y'}{y}=-\frac{s'}{s}$ to calculate y'.

SET UP: $s=3.00\times10^3\text{ m}.$
$s'=f_1=95.0\text{ cm}$ (since s is very large, $s'\approx f$).

EXECUTE: $m=-\frac{s'}{s}=-\frac{0.950\text{ m}}{3.00\times10^3\text{ m}}=-3.167\times10^{-4}.$

$|y'|=|m||y|=(3.167\times10^{-4})(60.0\text{ m})=0.0190\text{ m}=1.90\text{ cm}.$

(c) IDENTIFY and **SET UP:** Use $M=\frac{\theta'}{\theta}$ and the angular magnification M obtained in part (a) to calculate θ'. The angular size θ of the image formed by the objective (object for the eyepiece) is its height divided by its distance from the objective.

EXECUTE: The angular size of the object for the eyepiece is $\theta=\frac{0.0190\text{ m}}{0.950\text{ m}}=0.0200\text{ rad}.$

(Note that this is also the angular size of the object for the objective: $\theta=\frac{60.0\text{ m}}{3.00\times10^3\text{ m}}=0.0200\text{ rad}.$ For a thin lens the object and image have the same angular size and the image of the objective is the object for the eyepiece.) $M=\frac{\theta'}{\theta}$, so the angular size of the image is $\theta'=M\theta=-(6.33)(0.0200\text{ rad})=$ -0.127 rad. (The minus sign shows that the final image is inverted.)

EVALUATE: The lateral magnification of the objective is small; the image it forms is much smaller than the object. But the total angular magnification is larger than 1.00; the angular size of the final image viewed by the eye is 6.33 times larger than the angular size of the original object, as viewed by the unaided eye.

34.62. **IDENTIFY:** For a telescope, $M = -\dfrac{f_1}{f_2}$.

SET UP: $f_2 = 9.0$ cm. The distance between the two lenses equals $f_1 + f_2$.

EXECUTE: $f_1 + f_2 = 1.20 \text{ m} \Rightarrow f_1 = 1.20 \text{ m} - 0.0900 \text{ m} = 1.11 \text{ m}$. $M = -\dfrac{f_1}{f_2} = -\dfrac{111 \text{ cm}}{9.00 \text{ cm}} = -12.3$.

EVALUATE: For a telescope, $f_1 \gg f_2$.

34.63. **IDENTIFY:** $f = R/2$ and $M = -\dfrac{f_1}{f_2}$.

SET UP: For object and image both at infinity, $f_1 + f_2$ equals the distance *d* between the eyepiece and the mirror vertex. $f_2 = 1.10$ cm. $R_1 = 1.30$ m.

EXECUTE: **(a)** $f_1 = \dfrac{R_1}{2} = 0.650 \text{ m} \Rightarrow d = f_1 + f_2 = 0.661 \text{ m}$.

(b) $|M| = \dfrac{f_1}{f_2} = \dfrac{0.650 \text{ m}}{0.011 \text{ m}} = 59.1$.

EVALUATE: For a telescope, $m = -\dfrac{s'}{s} = -\dfrac{-3.00 \text{ cm}}{8.00 \text{ cm}} = +0.375$.

34.64. **IDENTIFY:** This is a compound microscope.

SET UP: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$, $m_1 = -s'/s$, $M_2 = (25 \text{ cm})/f_2$, $M = m_1 M_2$. Fig. 34.64 shows the arrangement of the lenses. I_2 is at infinity, so I_1 is at F_2.

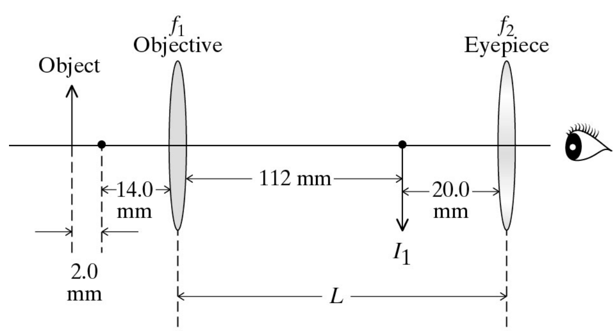


Figure 34.64

EXECUTE: **(a)** We want the length L. First image (I–$_1$): $\dfrac{1}{f_1} = \dfrac{1}{s_1} + \dfrac{1}{s_1'}$. $\dfrac{1}{14.0 \text{ mm}} = \dfrac{1}{16.0 \text{ mm}} + \dfrac{1}{s_1'}$.

$s_1' = 112$ mm. $m_1 = -s_1'/s_1 = -(112 \text{ mm})/(16 \text{ mm}) = -7.00$.

Second image (I–$_2$): $s_2' = \infty$. $\dfrac{1}{f_2} = \dfrac{1}{s_2} + \dfrac{1}{s_2'} = \dfrac{1}{s_2}$, so $s_2 = f_2 = 20.0$ mm.

$M_2 = (25 \text{ cm})/f_2 = (250 \text{ mm})/(20.0 \text{ mm}) = 12.5$. $L = 112 \text{ mm} + 20.0 \text{ mm} = 132$ mm.

(b) We want the magnification. $M = m_1 M_2$. $m_1 = -s_1'/s_1 = -(112 \text{ mm})/(16 \text{ mm}) = -7.00$. Therefore $M = (-7.00)(250 \text{ mm})/(20 \text{ mm}) = -87.5$.

(c) If we use $s_1 \approx f_1$, we have $M = -[(112\text{ mm})/(14.0\text{ mm})][(250\text{ mm})/(20.0\text{ mm})] = -100$. The percent difference is $(100 - 87.5)/(87.5) = 0.143 = 14.3\%$.

EVALUATE: Using $f_1 \approx s_1$ is convenient, but as we have seen, doing so can lead to significant error compared to the true magnification.

34.65. **IDENTIFY:** We have a compound microscope.

SET UP: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$, $m_1 = -s'/s$, $M_2 = (25\text{ cm})/f_2$, $M = m_1 M_2$. Fig. 34.65 shows the arrangement of the lenses. I_2 is at infinity, so I_1 is at F_2. We want f_1.

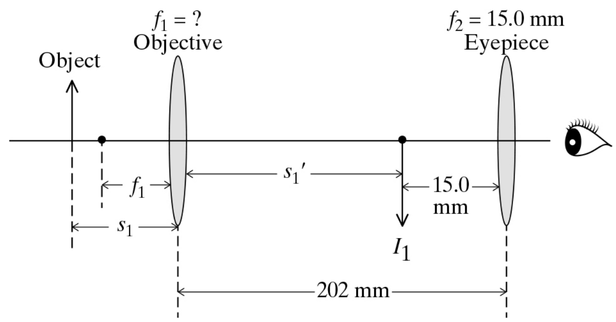


Figure 34.65

EXECUTE: From the figure, we see that $s_1' = 202\text{ mm} - 15.0\text{ mm} = 187\text{ mm}$. Use $\dfrac{1}{f_1} = \dfrac{1}{s_1} + \dfrac{1}{s_1'}$.

$\dfrac{1}{f_1} = \dfrac{1}{s_1} + \dfrac{1}{187\text{ mm}}$. Use $M = m_1 M_2$ to find s_1. $M = m_1 M_2 = -\dfrac{s_1'}{s_1}\dfrac{25\text{ cm}}{f_2}$. $178 = \dfrac{187\text{ mm}}{s_1}\left(\dfrac{250\text{ mm}}{15.0\text{ mm}}\right)$, so $1/s_1 = 0.05711\text{ mm}^{-1}$. Use this result in the lens equation. $\dfrac{1}{f_1} = 005711\text{ mm}^{-1} + \dfrac{1}{187\text{ mm}}$. $f_1 = 16.0$ mm.

EVALUATE: If we had used the approximation $s_1 \approx f_1$, the magnification we would have calculated would have been $M = -\dfrac{s_1'}{s_1}\dfrac{25\text{ cm}}{f_2} = \dfrac{187\text{ mm}}{16.0\text{ mm}}\left(\dfrac{250\text{ mm}}{15.0\text{ mm}}\right) = -195$, which is significantly different from the true magnification of -178.

34.66. **IDENTIFY:** Combine $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R}$ and $m = -\dfrac{s'}{s}$.

SET UP: $m = +2.50$. $R > 0$.

EXECUTE: $m = -\dfrac{s'}{s} = +2.50$. $s' = -2.50s$. $\dfrac{1}{s} + \dfrac{1}{-2.50s} = \dfrac{2}{R}$. $\dfrac{0.600}{s} = \dfrac{2}{R}$ and $s = 0.300R$.

$s' = -2.50s = (-2.50)(0.300R) = -0.750R$. The object is a distance of $0.300R$ in front of the mirror and the image is a distance of $0.750R$ behind the mirror.

EVALUATE: For a single mirror an erect image is always virtual.

34.67. **IDENTIFY:** We are given the image distance, the image height, and the object height. Use $m = -\dfrac{s'}{s}$ to calculate the object distance s. Then use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R}$ to calculate R.

SET UP: The image is to be formed on screen so it is a real image; $s' > 0$. The mirror-to-screen distance is 8.00 m, so $s' = +800$ cm. $m = -\dfrac{s'}{s} < 0$ since both s and s' are positive.

EXECUTE: **(a)** $|m| = \dfrac{|y'|}{|y|} = \dfrac{24.0\text{ cm}}{0.600\text{ cm}} = 40.0$, so $m = -40.0$. Then $m = -\dfrac{s'}{s}$ gives

$s = -\dfrac{s'}{m} = -\dfrac{800\text{ cm}}{-40.0} = +20.0\text{ cm}.$

(b) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R}$, so $\dfrac{2}{R} = \dfrac{s+s'}{ss'}$. $R = 2\left(\dfrac{ss'}{s+s'}\right) = 2\left(\dfrac{(20.0\text{ cm})(800\text{ cm})}{20.0\text{ cm} + 800\text{ cm}}\right) = 39.0\text{ cm}.$

EVALUATE: R is calculated to be positive, which is correct for a concave mirror. Also, in part (a) s is calculated to be positive, as it should be for a real object.

34.68. **IDENTIFY:** Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R}$ and $m = -\dfrac{s'}{s}$.

SET UP: Since the image is projected onto the wall it is real and $s' > 0$. $m = -\dfrac{s'}{s}$ so m is negative and $m = -3.50$. The object, mirror and wall are sketched in Figure 34.68. This sketch shows that $s' - s = 3.00$ m = 300 cm.

EXECUTE: $m = -3.50 = -\dfrac{s'}{s}$ so $s' = 3.50s$. $s' - s = 3.50s - s = 300\text{ cm}$ so $s = 120\text{ cm}$.

$s' = 300\text{ cm} + 120\text{ cm} = 420\text{ cm}$. The mirror should be 4.20 m from the wall. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R}$.

$\dfrac{1}{120\text{ cm}} + \dfrac{1}{420\text{ cm}} = \dfrac{2}{R}$. $R = 187\text{ cm} = 1.87\text{ m}.$

EVALUATE: The focal length of the mirror is $f = R/2 = 93.5\text{ cm}$ and $s > f$, as it must if the image is to be real.

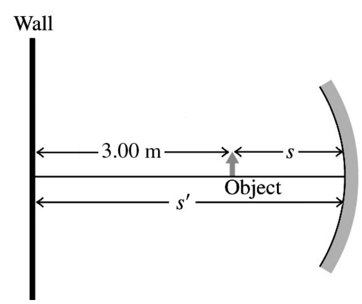


Figure 34.68

34.69. **IDENTIFY:** Since the truck is moving toward the mirror, its image will also be moving toward the mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, where $f = R/2$.

EXECUTE: Since the mirror is convex, $f = R/2 = (-1.50\text{ m})/2 = -0.75\text{ m}$. Applying the equation for a spherical mirror gives $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow s' = \dfrac{fs}{s-f}$. Using the chain rule from calculus and the fact that $v = ds/dt$, we have $v' = \dfrac{ds'}{dt} = \dfrac{ds'}{ds}\dfrac{ds}{dt} = v\dfrac{f^2}{(s-f)^2}$. Solving for v gives

$v = v'\left(\dfrac{s-f}{f}\right)^2 = (1.9\text{ m/s})\left[\dfrac{2.0\text{ m} - (-0.75\text{ m})}{-0.75\text{ m}}\right]^2 = 25.5\text{ m/s}$. This is the velocity of the truck relative to the mirror, so the truck is approaching the mirror at 25.5 m/s. You are traveling at 25 m/s, so the truck must be traveling at $25 \text{ m/s} + 25.5 \text{ m/s} = 51 \text{ m/s}$ relative to the highway.

EVALUATE: Even though the truck and car are moving at constant speed, the image of the truck is *not* moving at constant speed because its location depends on the distance from the mirror to the truck.

34.70. IDENTIFY: Apply $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}$, with $R \to \infty$ since the surfaces are flat.

SET UP: The image formed by the first interface serves as the object for the second interface.

EXECUTE: For the water-benzene interface, we get the apparent water depth:

$\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0 \Rightarrow \dfrac{1.33}{5.70 \text{ cm}} + \dfrac{1.50}{s'} = 0 \Rightarrow s' = -6.429 \text{ cm}$. For the benzene-air interface, we get the total apparent distance to the bottom: $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0 \Rightarrow \dfrac{1.50}{(6.429 \text{ cm} + 4.20 \text{ cm})} + \dfrac{1}{s'} = 0 \Rightarrow s' = -7.09 \text{ cm}$.

EVALUATE: At the water-benzene interface the light refracts into material of greater refractive index but at the benzene-air interface it refracts into material of smaller refractive index. The overall effect is that the apparent depth is less than the actual depth.

34.71. IDENTIFY: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to calculate s' and then use $m = -\dfrac{s'}{s} = \dfrac{y'}{y}$ to find the height of the image.

SET UP: For a convex mirror, $R < 0$, so $R = -18.0 \text{ cm}$ and $f = \dfrac{R}{2} = -9.00 \text{ cm}$.

EXECUTE: **(a)** $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$. $s' = \dfrac{sf}{s-f} = \dfrac{(900 \text{ cm})(-9.00 \text{ cm})}{900 \text{ cm} - (-9.00 \text{ cm})} = -8.91 \text{ cm}$.

$m = -\dfrac{s'}{s} = -\dfrac{-8.91 \text{ cm}}{900 \text{ cm}} = 9.90 \times 10^{-3}$. $|y'| = |m|\, y = (9.90 \times 10^{-3})(1.5 \text{ m}) = 0.0149 \text{ m} = 1.49 \text{ cm}$.

(b) The height of the image is much less than the height of the car, so the car appears to be farther away than its actual distance.

EVALUATE: A plane mirror would form an image the same size as the car. Since the image formed by the convex mirror is smaller than the car, the car appears to be farther away compared to what it would appear using a plane mirror.

34.72. IDENTIFY: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and the concept of principal rays.

SET UP: $s = 10.0 \text{ cm}$. If extended backward the ray comes from a point on the optic axis 18.0 cm from the lens and the ray is parallel to the optic axis after it passes through the lens.

EXECUTE: **(a)** The ray is bent toward the optic axis by the lens so the lens is converging.

(b) The ray is parallel to the optic axis after it passes through the lens so it comes from the focal point; $f = 18.0 \text{ cm}$.

(c) The principal-ray diagram is drawn in Figure 34.72. The diagram shows that the image is 22.5 cm to the left of the lens.

(d) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ gives $s' = \dfrac{sf}{s-f} = \dfrac{(10.0 \text{ cm})(18.0 \text{ cm})}{10.0 \text{ cm} - 18.0 \text{ cm}} = -22.5 \text{ cm}$. The calculated image position agrees with the principal-ray diagram.

EVALUATE: The image is virtual. A converging lens produces a virtual image when the object is inside the focal point.

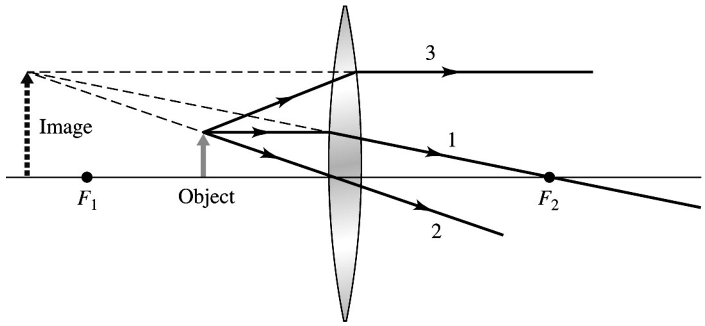


Figure 34.72

34.73. IDENTIFY and **SET UP:** Rays that pass through the hole are undeflected. All other rays are blocked. $m = -\dfrac{s'}{s}$.

EXECUTE: (a) The ray diagram is drawn in Figure 34.73. The ray shown is the only ray from the top of the object that reaches the film, so this ray passes through the top of the image. An inverted image is formed on the far side of the box, no matter how far this side is from the pinhole and no matter how far the object is from the pinhole.

(b) $s = 1.5\text{ m}$. $s' = 20.0\text{ cm}$. $m = -\dfrac{s'}{s} = -\dfrac{20.0\text{ cm}}{150\text{ cm}} = -0.133$. $y' = my = (-0.133)(18\text{ cm}) = -2.4\text{ cm}$.

The image is 2.4 cm tall.

EVALUATE: A defect of this camera is that not much light energy passes through the small hole each second, so long exposure times are required.

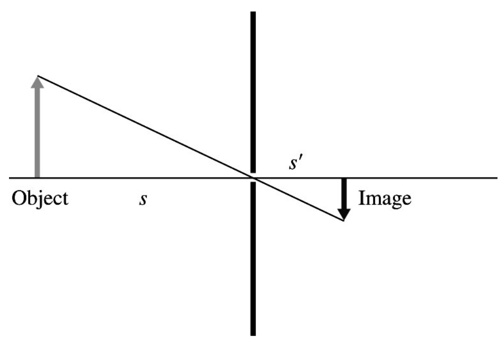


Figure 34.73

34.74. IDENTIFY: We have a combination of two thin lenses.

SET UP: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$, $m = -s'/s$, $M = m_1 m_2$. Fig. 34.74 shows the arrangement of the lenses. The first lens L_1 forms an image I_1 which is then the object for the second lens L_2, and L_2 forms the final image I_2. We want f_2 and the distance between the object and the final image I_2.

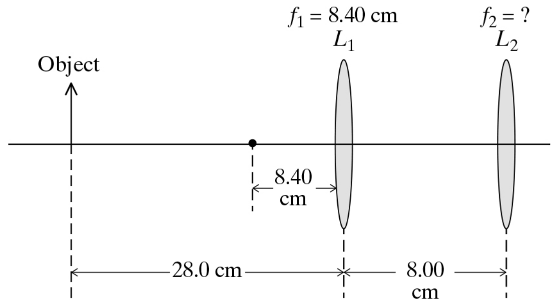


Figure 34.74

EXECUTE: **(a)** We want f_2. Locate the image formed by lens L_1. $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$ gives $\frac{1}{8.40 \text{ cm}} = \frac{1}{28.0 \text{ cm}} + \frac{1}{s'}$, so $s' = 12.0$ cm. $m_1 = -s_1'/s_1 = -(12.0 \text{ cm})/(28.0 \text{ cm}) = -0.4286$. Now look at lens L_2. I_1 is (or would be) formed 4.00 cm to the right of L_2. Thus it is a *virtual object* for that lens (see Fig. 34.37(f) in the textbook). So $s_2 = -4.00$ cm for L_2. $m_2 = -s_2'/s_2$, so $M = m_{-1}m_2 = (-0.4286)\left(-\frac{s_2'}{s_2}\right)$.

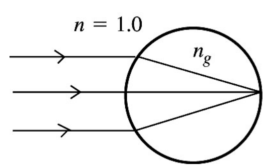
We also know that $M = -h_2/h = -(5.60 \text{ mm})/(4.00 \text{ mm}) = -1.40$. Use this and our latest result to find s_2'.
$-1.40 = (-0.4286)\left(-\frac{s_2'}{-4.00 \text{ cm}}\right)$, so $s_2' = +13.07$ cm. Now get f_2. $\frac{1}{f_2} = \frac{1}{-4.00 \text{ cm}} + \frac{1}{13.07 \text{ cm}}$, so $f_2 = -5.76$ cm.

(b) $s_2' = +13.1$ cm, so I_2 is 13.1 cm to the right of L_2. The object is 28.0 cm + 8.00 cm = 36.0 cm to the left of L_2, so the distance between the object and the final image is 36.0 cm + 13.1 cm = 49.1 cm.

EVALUATE: Careful of virtual objects: for them the object distance is *negative*.

34.75. **IDENTIFY:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ to the image formed by refraction at the front surface of the sphere.

SET UP: Let n_g be the index of refraction of the glass. The image formation is shown in Figure 34.75.



$s = \infty$.
$s' = +2r$, where r is the radius of the sphere.
$n_a = 1.00, n_b = n_g, R = +r$.

Figure 34.75

EXECUTE: $\frac{1}{\infty} + \frac{n_g}{2r} = \frac{n_g - 1.00}{r}$.

$\frac{n_g}{2r} = \frac{n_g}{r} - \frac{1}{r}$; $\frac{n_g}{2r} = \frac{1}{r}$ and $n_g = 2.00$.

EVALUATE: The required refractive index of the glass does not depend on the radius of the sphere.

34.76. **IDENTIFY:** This problem involves the lateral magnification by a curved surface.

SET UP: Eq. (343.11): $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$, Eq. (34.12): $m = -\frac{n_a s'}{n_b s}$. a is water and b is air.

EXECUTE: **(a)** Yes, it widens.

(b) Estimate: Lateral magnification is about 1 ½ = 1.5.

(c) We want m. $n_a = n$ (water) and $n_b = 1$ (air). Eqs. (34.11) and (34.12) become $\frac{n}{s}+\frac{1}{s'}=\frac{1-n}{R}$ and $m=-\frac{ns'}{s}$. Because R is negative for the glass, we can write the first equation as $\frac{n}{s}+\frac{1}{s'}=\frac{n-1}{|R|}$.

Combine these equations and solve for m, giving $m=\dfrac{1}{1-\frac{s}{|R|}(1-1/n)}$.

(d) We want n. Since the image is at the back of the glass, $s = 2|R|$. Combining this with the result of part (c) gives $m=\dfrac{1}{1+2(1-1/n)}$. Solve for n: $n=\dfrac{2}{1+1/m}$.

(e) $n=\dfrac{2}{1+1/m}=\dfrac{2}{1+1/1.5}=1.2.$

EVALUATE: This result is reasonably close to $n = 1.33$ considering the rough estimates involved.

34.77. **IDENTIFY:** We know the magnitude of the focal length is 35.0 cm and that it produces an image that is twice the height of the object. In part (a) the image is real, and in part (b) it is virtual. In each case we want to know the distance from the object to the lens and if the lens is converging or diverging. The thin-lens formula applies in both cases.

SET UP: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ with $f=\pm 35.0$ cm. We know that the magnification is $m=-\frac{s'}{s}$.

EXECUTE: **(a)** We want the size of the image to be twice that of the object, so we must have $m=\pm 2$. Since the image is real we know that $s'>0$, which implies that $m=-2=-\frac{s'}{s}$. Thus we conclude that $s'=2s$. Now we can determine the location of the object: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}+\frac{1}{2s}=\frac{3}{2s}=\frac{1}{f}$. Solving for s we get $s=\frac{3}{2}f$. Since we know that $s>0$ we must have that $f=+35.0$ cm, and thus $s=\frac{3}{2}f=\frac{3}{2}(35.0\text{ cm})=$ 52.5 cm. The lens is a converging lens, and the object must be placed 52.5 cm in front of the lens.

(b) We again want the image to be twice the size as the object; however, in this case we have a virtual image so $s'<0$ and $m=+2=-\frac{s'}{s}$. Thus, we have $s'=-2s$. Now we can determine the location of the object: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}+\frac{1}{-2s}=\frac{1}{2s}=\frac{1}{f}$. Solving for s we obtain $s=\frac{1}{2}f$. Since we know that $s>0$ we must have that $f=+35.0$ cm and thus $s=\frac{1}{2}(35.0\text{ cm})=17.5$ cm. The lens is a converging lens, and the object must be placed 17.5 cm in front of the lens.

EVALUATE: For a diverging lens we have $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}<0$. This can only occur if $\frac{1}{s'}$ is negative and larger in magnitude than $\frac{1}{s}$. Thus we have $|m|=\left|-\frac{s'}{s}\right|<1$. It follows that the image is always smaller than the object for a diverging lens. In this exercise $|m|=2>1$, so only a converging lens will work.

34.78. **IDENTIFY:** The lens forms an image of the object. That image (I_1) is reflected in the plane mirror, and its image (I_2) is just as far behind the mirror as I_1 is in front of the mirror. The image I_2 in the mirror then acts as the object for the lens which forms an image I_3 on the screen.

SET UP: The thin-lens equation, $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, applies to the lens.

EXECUTE: **(a)** Figure 34.78 shows the arrangement of the screen, object, lens, and mirror.

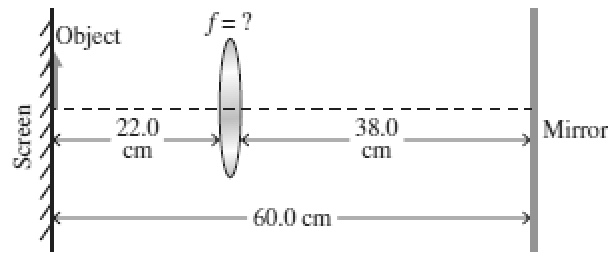


Figure 34.78

(b) First image formed by the lens (I_1) : Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ at the lens. The object distance s is 22.0 cm.

$$\frac{1}{22.0\text{ cm}}+\frac{1}{s'}=\frac{1}{f}. \qquad \text{Eq. (1)}$$

Image I_2 in the mirror : The image I_1 is a distance s' from the lens, so its distance from the mirror is 38.0 cm $-\ s'$. So its image I_2 in the mirror is a distance 38.0 cm $-\ s'$ behind the mirror.

Second image formed by the lens (I_3) : I_2 serves as the object for the lens, and its distance s from the lens is

s = 38.0 cm + (38.0 cm $-\ s'$) = 76.0 cm $-\ s'$. The lens forms the image I_3 on the screen, so the image distance is s' = 22.0 cm. Applying the thin-lens equation again gives

$$\frac{1}{76.0\text{ cm}-s'}+\frac{1}{22.0\text{ cm}}=\frac{1}{f}. \qquad \text{Eq. (2)}$$

Equating the two expressions for $1/f$ from Equations (1) and (2) gives

$\frac{1}{22.0\text{ cm}}+\frac{1}{s'}=\frac{1}{76.0\text{ cm}-s'}+\frac{1}{22.0\text{ cm}}$, which simplifies to $\frac{1}{s'}=\frac{1}{76.0\text{ cm}-s'}$. Solving for s' gives s' = 38.0 cm. Putting this into Eq. (1) gives $\frac{1}{f}=\frac{1}{22.0\text{ cm}}+\frac{1}{38.0\text{ cm}}$, so f = 13.9 cm.

EVALUATE: The image I_1 is *at* the mirror.

34.79. **IDENTIFY:** We know that the image is real, is 214 cm from the *object* (*not* from the lens), and is 5/3 times the height of the object. We want to find the type of lens and its focal length. The thin-lens equation applies.

SET UP: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ with the conditions that $s + s'$ = ±214 cm and $m=-\frac{s'}{s}$.

EXECUTE: Since the size of the image is greater than the size of the object, we know that the image must be farther from the lens than the object. This implies that the focal length of the lens is positive and the lens is converging. We know that the image is real, so $s'>0$. In this case we have $s+s'=+214$ cm and $m = -\dfrac{s'}{s} = -\dfrac{5}{3}$. Thus, we may write $s' = \frac{5}{3}s$ and $s + s' = \frac{8}{3}s = +214$ cm. Solving for s and s' we obtain $s = 80.25$ cm and $s' = 133.75$ cm. This gives $f = \dfrac{ss'}{s+s'} = \dfrac{(80.25\text{ cm})(133.75\text{ cm})}{214\text{ cm}} = 50.2$ cm.

EVALUATE: For a diverging lens we have $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} < 0$. This can only occur if $\dfrac{1}{s'}$ is negative and larger in magnitude than $\dfrac{1}{s}$. Thus we have $|m| = \left|-\dfrac{s'}{s}\right| < 1$. It follows that the image is always smaller than the object for a diverging lens. In this exercise $|m| = \dfrac{5}{3} > 1$, so only a converging lens will work.

34.80. IDENTIFY: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$. The type of lens determines the sign of *f*. The sign of s' determines whether the image is real or virtual.

SET UP: $s = +8.00$ cm. $s' = -3.00$ cm. s' is negative because the image is on the same side of the lens as the object.

EXECUTE: **(a)** $\dfrac{1}{f} = \dfrac{s+s'}{ss'}$ and $f = \dfrac{ss'}{s+s'} = \dfrac{(8.00\text{ cm})(-3.00\text{ cm})}{8.00\text{ cm} - 3.00\text{ cm}} = -4.80$ cm. *f* is negative so the lens is diverging.

(b) $m = -\dfrac{s'}{s} = -\dfrac{-3.00\text{ cm}}{8.00\text{ cm}} = +0.375$. $y' = my = (0.375)(6.50\text{ mm}) = 2.44$ mm. $s' < 0$ and the image is virtual.

EVALUATE: A converging lens can also form a virtual image, if the object distance is less than the focal length. But in that case $|s'| > s$ and the image would be farther from the lens than the object is.

34.81. IDENTIFY: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$. The type of lens determines the sign of *f*. $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$. The sign of s' depends on whether the image is real or virtual. $s = 16.0$ cm.

SET UP: $s' = -22.0$ cm; s' is negative because the image is on the same side of the lens as the object.

EXECUTE: **(a)** $\dfrac{1}{f} = \dfrac{s+s'}{ss'}$ and $f = \dfrac{ss'}{s+s'} = \dfrac{(16.0\text{ cm})(-22.0\text{ cm})}{16.0\text{ cm} - 22.0\text{ cm}} = +58.7$ cm. *f* is positive so the lens is converging.

(b) $m = -\dfrac{s'}{s} = -\dfrac{-22.0\text{ cm}}{16.0\text{ cm}} = 1.38$. $y' = my = (1.38)(3.25\text{ mm}) = 4.48$ mm. $s' < 0$ and the image is virtual.

EVALUATE: A converging lens forms a virtual image when the object is closer to the lens than the focal point.

34.82. IDENTIFY: Apply $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}$. Use the image distance when viewed from the flat end to determine the refractive index *n* of the rod.

SET UP: When viewing from the flat end, $n_a = n$, $n_b = 1.00$ and $R \to \infty$. When viewing from the curved end, $n_a = n$, $n_b = 1.00$, and $R = -10.0$ cm.

EXECUTE: When viewed from the flat end of the rod:

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \Rightarrow \frac{n}{15.0\text{ cm}} + \frac{1}{-8.20\text{ cm}} = 0 \Rightarrow n = \frac{15.0}{8.20} = 1.829.$$

When viewed from the curved end of the rod:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{n}{s}+\frac{1}{s'}=\frac{1-n}{R}\Rightarrow\frac{1.829}{15.0\text{ cm}}+\frac{1}{s'}=\frac{-0.829}{-10.0\text{ cm}},\text{ so } s'=-25.6\text{ cm}.$$

The image is 25.6 cm within the rod from the curved end.

EVALUATE: In each case the image is virtual and on the same side of the surface as the object.

34.83. IDENTIFY: The image formed by refraction at the surface of the eye is located by $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$.

SET UP: $n_a=1.00$, $n_b=1.35$. $R>0$. For a distant object, $s\approx\infty$ and $\frac{1}{s}\approx 0$.

EXECUTE: **(a)** $s\approx\infty$ and $s'=2.5$ cm: $\frac{1.35}{2.5\text{ cm}}=\frac{1.35-1.00}{R}$ and $R=0.648\text{ cm}=6.48\text{ mm}$.

(b) $R=0.648$ cm and $s=25$ cm: $\frac{1.00}{25\text{ cm}}+\frac{1.35}{s'}=\frac{1.35-1.00}{0.648}$. $\frac{1.35}{s'}=0.500$ and $s'=2.70\text{ cm}=27.0\text{ mm}$. The image is formed behind the retina.

(c) Calculate s' for $s\approx\infty$ and $R=0.50$ cm: $\frac{1.35}{s'}=\frac{1.35-1.00}{0.50\text{ cm}}$. $s'=1.93\text{ cm}=19.3\text{ mm}$. The image is formed in front of the retina.

EVALUATE: The cornea alone cannot achieve focus of both close and distant objects.

34.84. IDENTIFY and SET UP: Use the lensmaker's equation $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ to calculate the focal length of the lenses. The image formed by the first lens serves at the object for the second lens. $m_{\text{tot}}=m_1m_2$. The thin-lens formula $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s'=\frac{sf}{s-f}$.

EXECUTE: **(a)** $\frac{1}{f}=(0.60)\left(\frac{1}{12.0\text{ cm}}-\frac{1}{28.0\text{ cm}}\right)$ and $f=+35.0$ cm.

<u>Lens 1:</u> $f_1=+35.0$ cm. $s_1=+45.0$ cm. $s_1'=\frac{s_1f_1}{s_1-f_1}=\frac{(45.0\text{ cm})(35.0\text{ cm})}{45.0\text{ cm}-35.0\text{ cm}}=+158\text{ cm}.$

$m_1=-\frac{s_1'}{s_1}=-\frac{158\text{ cm}}{45.0\text{ cm}}=-3.51$. $|y_1'|=|m_1|y_1=(3.51)(5.00\text{ mm})=17.6\text{ mm}$. The image of the first lens is 158 cm to the right of lens 1 and is 17.6 mm tall.

(b) The image of lens 1 is $315\text{ cm}-158\text{ cm}=157\text{ cm}$ to the left of lens 2. $f_2=+35.0$ cm.

$s_2=+157$ cm. $s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(157\text{ cm})(35.0\text{ cm})}{157\text{ cm}-35.0\text{ cm}}=+45.0\text{ cm}$. $m_2=-\frac{s_2'}{s_2}=-\frac{45.0\text{ cm}}{157\text{ cm}}=-0.287$.

$m_{\text{tot}}=m_1m_2=(-3.51)(-0.287)=+1.00$. The final image is 45.0 cm to the right of lens 2. The final image is 5.00 mm tall. $m_{\text{tot}}>0$ and the final image is erect.

EVALUATE: The final image is real. It is erect because each lens produces an inversion of the image, and two inversions return the image to the orientation of the object.

34.85. IDENTIFY: We are dealing with the image formed by a spherical refracting surface.

SET UP: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$, $m=-\frac{n_as'}{n_bs}$. The object is in the liquid and the image is formed in air, so a is the liquid ($n_a=n=1.627$), b is air ($n_b=1.00$). Using these symbols, the equations become $\frac{n}{s}+\frac{1}{s'}=\frac{1-n}{R}$ and $m=-\frac{ns'}{s}$. Refer to Fig. P34.85 with the problem in the textbook.

EXECUTE: **(a)** We want the minimum *H*. For the device to function, s' must be positive to form a real image in the air. So $+\frac{1}{s'} = \frac{1-n}{R} - \frac{n}{s} = \frac{1-1.627}{-2.50 \text{ cm}} - \frac{1.627}{s} \geq 0.$ For the limit, we used the equality, which gives $s = 6.49$ cm. If *s* is less than this value, s' is negative, so this is the *minimum* height *H*.

(b) We want the range of object distances. s' varies from 50.0 cm to 1.00 m.

For s' = 50.0 cm: $\frac{1.627}{s} + \frac{1}{50.0 \text{ cm}} = \frac{1-1.627}{-2.50 \text{ cm}} = 0.2508 \text{ cm}^{-1},$ so s = 7.05 cm.

For s' = 1.00 m: The same procedure gives s = 6.76 cm.

Therefore the range is 6.76 cm $\leq s \leq$ 7.05 cm.

(c) We want s'. The object is halfway between the extremes in (b), so s = 6.90 cm. Using $\frac{n}{s} + \frac{1}{s'} = \frac{1-n}{R}$ gives $\frac{1}{s'} = 0.2508 \text{ cm}^{-1} - \frac{1.627}{6.90 \text{ cm}}$, so s' = 66.2 cm.

(d) We want the velocity of the image when s = 6.90 cm (at which time s' = 66.2 cm). The velocity of the image is ds'/dt. Take the time derivative of $\frac{n}{s} + \frac{1}{s'} = \frac{1-n}{R}$, which gives $\frac{ds'}{dt} = -n\left(\frac{s'}{s}\right)\frac{ds}{dt}.$ We know that $s = A\sin\omega t,$ so $ds/dt = \omega A\cos\omega t,$ which gives $\frac{ds'}{dt} = -n\left(\frac{s'}{s}\right)\omega A\cos\omega t.$ The motion varies between 6.76 cm and 7.05 cm, so the amplitude is 0.145 cm. Using $\omega = 2\pi f$ and the numerical quantities gives $\frac{ds'}{dt} = -(1.627)\left(\frac{66.2 \text{ cm}}{6.90 \text{ cm}}\right)(2\pi \text{ Hz})(0.145 \text{ cm})\cos\omega t.$ When the object is at its midpoint, $\sin\omega t = 0$, so its velocity is –1.36 m/s in the vertical direction.

(d) We want the maximum diameter of the image. $|m| = \frac{ns'}{s}.$ *m* is a maximum when s' is greatest and *s* is least. The maximum s' is 1.00 m, at which time s = 6.76 cm (from (b)), so

$$h'_{\text{max}} = hm_{\text{max}} = h\frac{ns'}{s} = (1.00 \text{ cm})(1.627)\left(\frac{100 \text{ cm}}{6.76 \text{ cm}}\right) = 24.1 \text{ cm}.$$

EVALUATE: A very small amplitude for the object motion produces a much larger amplitude for the image motion.

34.86. **IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = -\frac{s'}{s}.$

SET UP: $s + s' = 22.0$ cm.

EXECUTE: **(a)** $\frac{1}{22.0 \text{ cm} - s'} + \frac{1}{s'} = \frac{1}{3.00 \text{ cm}}.$ $(s')^2 - (22.0 \text{ cm})s' + 66.0 \text{ cm}^2 = 0$ so $s' = 18.42$ cm or 3.58 cm. $s = 3.58$ cm or 18.42 cm, so the lens must either be 3.58 cm or 18.4 cm from the object.

(b) $s = 3.58$ cm and s' = 18.42 cm gives $m = -\frac{s'}{s} = -\frac{18.42}{3.58} = -5.15.$

$s = 18.42$ cm and s' = 3.58 cm gives $m = -\frac{s'}{s} = -\frac{3.58}{18.42} = -0.914.$

EVALUATE: Since the image is projected onto the screen, the image is real and s' is positive. We assumed this when we wrote the condition $s + s' = 22.0$ cm.

34.87. **IDENTIFY** and **SET UP:** The person's eye cannot focus on anything closer than 85.0 cm. The problem asks us to find the location of an object such that his old lenses produce a virtual image 85.0 cm from his eye. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}.$ $P(\text{in diopters}) = 1/f\,(\text{in m}).$

EXECUTE: (a) $\dfrac{1}{f} = 2.25$ diopters so $f = 44.4$ cm. The image is 85.0 cm from his eye so is 83.0 cm from the eyeglass lens. Solving $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ for s gives $s = \dfrac{s'f}{s'-f} = \dfrac{(-83.0\text{ cm})(44.4\text{ cm})}{-83.0\text{ cm} - 44.4\text{ cm}} = +28.9$ cm.
The object is 28.9 cm from the eyeglasses so is 30.9 cm from his eyes.

(b) Now $s' = -85.0$ cm. $s = \dfrac{s'f}{s'-f} = \dfrac{(-85.0\text{ cm})(44.4\text{ cm})}{-85.0\text{ cm} - 44.4\text{ cm}} = +29.2$ cm.

EVALUATE: The old glasses allow him to focus on objects as close as about 30 cm from his eyes. This is much better than a closest distance of 85 cm with no glasses, but his current glasses probably allow him to focus as close as 25 cm.

34.88. IDENTIFY and SET UP: The thin-lens equation, $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$, applies. The lens forms an image of the object on the screen, so the distance from the lens to the screen is the image distance s'. The distance from the object to the lens is s, so $s + s' = d$.

EXECUTE: We combine $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ and $s + s' = d$ to solve for d.

$s + s' = d \quad \rightarrow \quad s' = d - s.$

$\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f} \quad \rightarrow \quad s' = \dfrac{sf}{s-f} \quad \rightarrow \quad d - s = \dfrac{sf}{s-f}.$

$ds - s^2 - df + sf = sf \quad \rightarrow \quad s^2 - ds + df = 0 \quad \rightarrow \quad s = \dfrac{1}{2}(d \pm \sqrt{d^2 - 4df}).$

If $4df > d^2$, there is no real solution, so we must have $d^2 \geq 4df$. The smallest that d can be is if $d^2 = 4df$, in which case $d = 4f$.

EVALUATE: Larger values of d are possible, but we want only the smallest one.

34.89. IDENTIFY: $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ gives $s' = \dfrac{sf}{s-f}$, for both the mirror and the lens.

SET UP: For the second image, the image formed by the mirror serves as the object for the lens. For the mirror, $f_{\text{m}} = +10.0$ cm. For the lens, $f = 32.0$ cm. The center of curvature of the mirror is $R = 2f_{\text{m}} = 20.0$ cm to the right of the mirror vertex.

EXECUTE: (a) The principal-ray diagrams from the two images are sketched in Figure 34.89. In Figure 34.89b, only the image formed by the mirror is shown. This image is at the location of the candle so the principal-ray diagram that shows the image formation when the image of the mirror serves as the object for the lens is analogous to that in Figure 34.89a and is not drawn.

(b) Image formed by the light that passes directly through the lens: The candle is 85.0 cm to the left of the lens. $s' = \dfrac{sf}{s-f} = \dfrac{(85.0\text{ cm})(32.0\text{ cm})}{85.0\text{ cm} - 32.0\text{ cm}} = +51.3$ cm. $m = -\dfrac{s'}{s} = -\dfrac{51.3\text{ cm}}{85.0\text{ cm}} = -0.604$. This image is 51.3 cm to the right of the lens. $s' > 0$ so the image is real. $m < 0$ so the image is inverted. Image formed by the light that first reflects off the mirror: First consider the image formed by the mirror. The candle is 20.0 cm to the right of the mirror, so $s = +20.0$ cm.

$s' = \dfrac{sf}{s-f} = \dfrac{(20.0\text{ cm})(10.0\text{ cm})}{20.0\text{ cm} - 10.0\text{ cm}} = 20.0$ cm. $m_1 = -\dfrac{s_1'}{s_1} = -\dfrac{20.0\text{ cm}}{20.0\text{ cm}} = -1.00$. The image formed by the mirror is at the location of the candle, so $s_2 = +85.0$ cm and $s_2' = 51.3$ cm. $m_2 = -0.604$.

$m_{\text{tot}} = m_1 m_2 = (-1.00)(-0.604) = 0.604$. The second image is 51.3 cm to the right of the lens. $s_2' > 0$, so the final image is real. $m_{\text{tot}} > 0$, so the final image is erect.

EVALUATE: The two images are at the same place. They are the same size. One is erect and one is inverted.

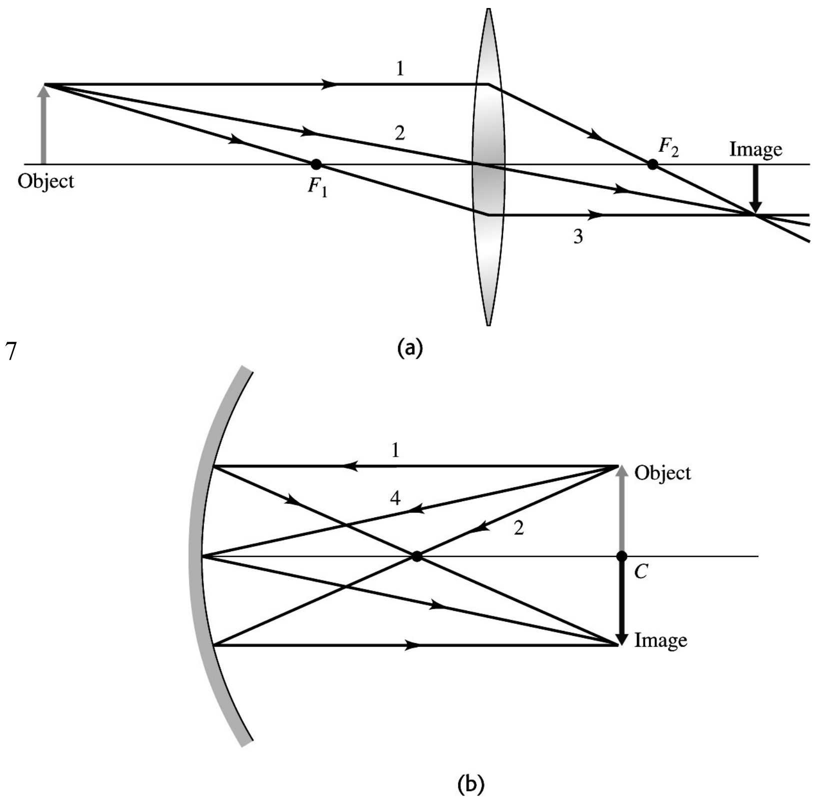


Figure 34.89

34.90. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. The image formed by the first lens serves as the object for the second lens. The focal length of the lens combination is defined by $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}$. In part (b) use $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ to calculate f for the meniscus lens and for the CCl_4, treated as a thin lens.

SET UP: With two lenses of different focal length in contact, the image distance from the first lens becomes exactly minus the object distance for the second lens.

EXECUTE: **(a)** $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow\frac{1}{s_1'}=\frac{1}{f_1}-\frac{1}{s_1}$ and $\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{-s_1'}+\frac{1}{s_2'}=\left(\frac{1}{s_1}-\frac{1}{f_1}\right)+\frac{1}{s_2'}=\frac{1}{f_2}$. But overall for the lens system, $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}\Rightarrow\frac{1}{f}=\frac{1}{f_2}+\frac{1}{f_1}$.

(b) With carbon tetrachloride sitting in a meniscus lens, we have two lenses in contact. All we need in order to calculate the system's focal length is calculate the individual focal lengths, and then use the formula from part (a). For the meniscus lens

$$\frac{1}{f_m}=(n_b-n_a)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(0.55)\left(\frac{1}{4.50\text{ cm}}-\frac{1}{9.00\text{ cm}}\right)=0.061\text{ cm}^{-1}\text{ and }f_m=16.4\text{ cm}.$$

For the CCl_4: $\dfrac{1}{f_w} = (n_b - n_a)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = (0.46)\left(\dfrac{1}{9.00\text{ cm}} - \dfrac{1}{\infty}\right) = 0.051\text{ cm}^{-1}$ and $f_w = 19.6$ cm.

$\dfrac{1}{f} = \dfrac{1}{f_w} + \dfrac{1}{f_m} = 0.112\text{ cm}^{-1}$ and $f = 8.93$ cm.

EVALUATE: $f = \dfrac{f_1 f_2}{f_1 + f_2}$, so f for the combination is less than either f_1 or f_2.

34.91. **IDENTIFY:** Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$.

SET UP: The image formed by the converging lens is 30.0 cm from the converging lens, and becomes a virtual object for the diverging lens at a position 15.0 cm to the right of the diverging lens. The final image is projected $15\text{ cm} + 19.2\text{ cm} = 34.2\text{ cm}$ from the diverging lens.

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{-15.0\text{ cm}} + \dfrac{1}{34.2\text{ cm}} = \dfrac{1}{f} \Rightarrow f = -26.7$ cm.

EVALUATE: Our calculation yields a negative value of f, which should be the case for a diverging lens.

34.92. **IDENTIFY:** Start with the two formulas right after the beginning of the section in the textbook on the lensmaker's equation: $\dfrac{n_a}{s_1} + \dfrac{n_b}{s_1'} = \dfrac{n_b - n_a}{R_1}$ and $\dfrac{n_b}{s_2} + \dfrac{n_c}{s_2'} = \dfrac{n_c - n_b}{R_2}$.

SET UP: The lens is surrounded by a liquid, so $n_a = n_c = n_{liq}$ and $n_b = n$ (for the lens), and $s_2 = -s_1'$.

EXECUTE: **(a)** Putting in the quantities indicated above, the two starting equations become $\dfrac{n_{liq}}{s_1} + \dfrac{n}{-s_2} = \dfrac{n - n_{liq}}{R_1}$ and $\dfrac{n}{s_2} + \dfrac{n_{liq}}{s_2'} = \dfrac{n_{liq} - n}{R_2}$. Add these two equations to eliminate n/s_2, giving $\dfrac{n_{liq}}{s_1} + \dfrac{n_{liq}}{s_2'} = (n - n_{liq})\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$. Dividing by n_{liq} gives $\dfrac{1}{s_1} + \dfrac{1}{s_2'} = \left(\dfrac{n}{n_{liq}} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$, which gives $\dfrac{1}{s_1} + \dfrac{1}{s_2'} = \left(\dfrac{n}{n_{liq}} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$. Therefore $\dfrac{1}{f_{liq}} = \left(\dfrac{n}{n_{liq}} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$, where f_{liq} is the focal length of the lens when it is immersed in the liquid.

(b) Take the ratio of $1/f_{liq}$ to $1/f_{air}$: $\dfrac{\dfrac{1}{f_{liq}}}{\dfrac{1}{f_{air}}} = \dfrac{\left(\dfrac{n}{n_{liq}} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)}{(n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)}$. Now solve for f_{liq}, giving $f_{liq} = f_{air}\left(\dfrac{n-1}{n/n_{liq} - 1}\right) = (18.0\text{ cm})\left(\dfrac{1.60 - 1}{1.60/1.42 - 1}\right) = +85.2$ cm.

EVALUATE: In part (b) we saw that immersing a lens in a liquid can change its focal length considerably. But even more extreme behavior can result. If the "liquid" is air, $\dfrac{1}{f_{air}} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$, and the factor $(n - 1)$ is always positive. But if the liquid has an index of refraction greater than that of the lens material, then $n/n_{liq} < 1$, so the factor $(n/n_{liq} - 1)$ is negative. This means that f changes sign from what it was in air. In other words, submerging a converging lens in a liquid can turn it into a diverging lens, and vice versa!

34.93. **IDENTIFY:** The spherical mirror forms an image of the object. It forms another image when the image of the plane mirror serves as an object.

SET UP: For the convex mirror $f = -24.0$ cm. The image formed by the plane mirror is 10.0 cm to the right of the plane mirror, so is $20.0\text{ cm} + 10.0\text{ cm} = 30.0\text{ cm}$ from the vertex of the spherical mirror.

EXECUTE: The first image formed by the spherical mirror is the one where the light immediately strikes its surface, without bouncing from the plane mirror.

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{10.0\text{ cm}}+\frac{1}{s'}=\frac{1}{-24.0\text{ cm}} \Rightarrow s' = -7.06$ cm, and the image height is

$y' = -\frac{s'}{s}y = -\frac{-7.06}{10.0}(0.250\text{ cm}) = 0.177$ cm.

The image of the object formed by the plane mirror is located 30.0 cm from the vertex of the spherical mirror. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{30.0\text{ cm}}+\frac{1}{s'}=\frac{1}{-24.0\text{ cm}} \Rightarrow s' = -13.3$ cm and the image height is

$y' = -\frac{s'}{s}y = -\frac{-13.3}{30.0}(0.250\text{ cm}) = 0.111$ cm.

EVALUATE: Other images are formed by additional reflections from the two mirrors.

34.94. **IDENTIFY:** The smallest image we can resolve occurs when the image is the size of a retinal cell.

SET UP: $m = -\frac{s'}{s} = \frac{y'}{y}$. $s' = 2.50$ cm.

$|y'| = 5.0\ \mu\text{m}$. The angle subtended (in radians) is height divided by distance from the eye.

EXECUTE: **(a)** $m = -\frac{s'}{s} = -\frac{2.50\text{ cm}}{25\text{ cm}} = -0.10$. $y = \left|\frac{y'}{m}\right| = \frac{5.0\ \mu\text{m}}{0.10} = 50\ \mu\text{m}$.

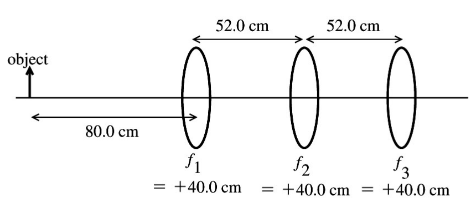
(b) $\theta = \frac{y}{s} = \frac{50\ \mu\text{m}}{25\text{ cm}} = \frac{50\times10^{-6}\text{ m}}{25\times10^{-2}\text{ m}} = 2.0\times10^{-4}\text{ rad} = 0.0115° = 0.69\text{ min}$. This is only a bit smaller than the typical experimental value of 1.0 min.

EVALUATE: The angle subtended by the object equals the angular size of the image,

$\frac{|y'|}{s'} = \frac{5.0\times10^{-6}\text{ m}}{2.50\times10^{-2}\text{ m}} = 2.0\times10^{-4}$ rad.

34.95. **IDENTIFY:** Apply the thin-lens equation to calculate the image distance for each lens. The image formed by the first lens serves as the object for the second lens, and the image formed by the second lens serves as the object for the third lens.

SET UP: The positions of the object and lenses are shown in Figure 34.95.



$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{sf}$.

$s'=\frac{sf}{s-f}$.

Figure 34.95

EXECUTE: Lens #1:
$s = +80.0$ cm; $f = +40.0$ cm.

$$s' = \frac{sf}{s-f} = \frac{(+80.0\text{ cm})(+40.0\text{ cm})}{+80.0\text{ cm} - 40.0\text{ cm}} = +80.0\text{ cm}.$$

The image formed by the first lens is 80.0 cm to the right of the first lens, so it is $80.0\text{ cm} - 52.0\text{ cm} = 28.0\text{ cm}$ to the right of the second lens.

Lens #2:
$s = -28.0$ cm; $f = +40.0$ cm.

$$s' = \frac{sf}{s-f} = \frac{(-28.0\text{ cm})(+40.0\text{ cm})}{-28.0\text{ cm} - 40.0\text{ cm}} = +16.47\text{ cm}.$$

The image formed by the second lens is 16.47 cm to the right of the second lens, so it is $52.0\text{ cm} - 16.47\text{ cm} = 35.53\text{ cm}$ to the left of the third lens.

Lens #3:
$s = +35.53$ cm; $f = +40.0$ cm.

$$s' = \frac{sf}{s-f} = \frac{(+35.53\text{ cm})(+40.0\text{ cm})}{+35.53\text{ cm} - 40.0\text{ cm}} = -318\text{ cm}.$$

The final image is 318 cm to the left of the third lens, so it is $318\text{ cm} - 52\text{ cm} - 52\text{ cm} - 80\text{ cm} = 134\text{ cm}$ to the left of the object.

EVALUATE: We used the separation between the lenses and the sign conventions for s and s' to determine the object distances for the second and third lenses. The final image is virtual since the final s' is negative.

34.96. **IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and calculate s' for each s.

SET UP: $f = 90$ mm.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{1300\text{ mm}} + \frac{1}{s'} = \frac{1}{90\text{ mm}} \Rightarrow s' = 96.7$ mm.

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{6500\text{ mm}} + \frac{1}{s'} = \frac{1}{90\text{ mm}} \Rightarrow s' = 91.3$ mm.

$\Rightarrow \Delta s' = 96.7\text{ mm} - 91.3\text{ mm} = 5.4$ mm toward the sensor.

EVALUATE: $s' = \frac{sf}{s-f}$. For $f > 0$ and $s > f$, s' decreases as s increases.

34.97. **IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. The near point is at infinity, so that is where the image must be formed for any objects that are close.

SET UP: The power in diopters equals $\frac{1}{f}$, with f in meters.

EXECUTE: $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{24\text{ cm}} + \frac{1}{-\infty} = \frac{1}{0.24\text{ m}} = 4.17$ diopters.

EVALUATE: To focus on closer objects, the power must be increased.

34.98. **IDENTIFY:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

SET UP: $n_a = 1.00$, $n_b = 1.40$.

EXECUTE: $\frac{1}{36.0\text{ cm}} + \frac{1.40}{s'} = \frac{0.40}{0.75\text{ cm}} \Rightarrow s' = 2.77$ cm.

EVALUATE: This distance is greater than for the normal eye, which has a cornea vertex to retina distance of about 2.6 cm.

34.99. IDENTIFY: Use similar triangles in Figure P34.99 in the textbook and $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to derive the expressions called for in the problem.

(a) SET UP: The effect of the converging lens on the ray bundle is sketched in Figure 34.99a.

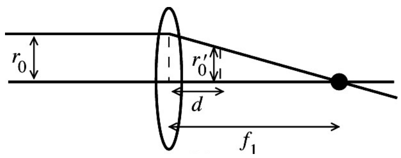


Figure 34.99a

EXECUTE: From similar triangles in Figure 34.99a, $\frac{r_0}{f_1}=\frac{r_0'}{f_1-d}$.

Thus $r_0'=\left(\frac{f_1-d}{f_1}\right)r_0$, as was to be shown.

(b) SET UP: The image at the focal point of the first lens, a distance f_1 to the right of the first lens, serves as the object for the second lens. The image is a distance f_1-d to the right of the second lens, so $s_2=-(f_1-d)=d-f_1$.

EXECUTE: $s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(d-f_1)f_2}{d-f_1-f_2}$.

$f_2<0$ so $|f_2|=-f_2$ and $s_2'=\frac{(f_1-d)|f_2|}{|f_2|-f_1+d}$, as was to be shown.

(c) SET UP: The effect of the diverging lens on the ray bundle is sketched in Figure 34.99b.

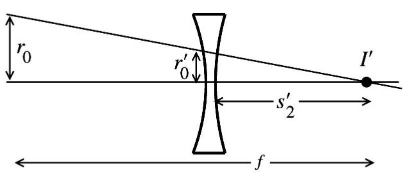


Figure 34.99b

EXECUTE: From similar triangles in the sketch, $\frac{r_0}{f}=\frac{r_0'}{s_2'}$.

Thus $\frac{r_0}{r_0'}=\frac{f}{s_2'}$.

From the results of part (a), $\frac{r_0}{r_0'}=\frac{f_1}{f_1-d}$. Combining the two results gives $\frac{f_1}{f_1-d}=\frac{f}{s_2'}$.

$f=s_2'\left(\frac{f_1}{f_1-d}\right)=\frac{(f_1-d)|f_2|f_1}{(|f_2|-f_1+d)(f_1-d)}=\frac{f_1|f_2|}{|f_2|-f_1+d}$, as was to be shown.

(d) SET UP: Put the numerical values into the expression derived in part (c).

EXECUTE: $f=\frac{f_1|f_2|}{|f_2|-f_1+d}$.

$f_1=12.0\text{ cm}$, $|f_2|=18.0\text{ cm}$, so $f=\frac{216\text{ cm}^2}{6.0\text{ cm}+d}$.

$d=0$ gives $f=36.0\text{ cm}$; maximum f.

$d = 4.0$ cm gives $f = 21.6$ cm; minimum f.

$f = 30.0$ cm says $30.0\text{ cm} = \dfrac{216\text{ cm}^2}{6.0\text{ cm} + d}$.

$6.0\text{ cm} + d = 7.2$ cm and $d = 1.2$ cm.

EVALUATE: Changing d produces a range of effective focal lengths. The effective focal length can be both smaller and larger than $f_1 + |f_2|$.

34.100. **IDENTIFY:** For u and u' as defined in Figure P34.100 in the textbook, $M = \dfrac{u'}{u}$.

SET UP: f_2 is negative. From Figure P34.100 in the textbook, the length of the telescope is $f_1 + f_2$, since f_2 is negative.

EXECUTE: **(a)** From the figure, $u = \dfrac{y}{f_1}$ and $u' = \dfrac{y}{|f_2|} = -\dfrac{y}{f_2}$. The angular magnification is

$M = \dfrac{u'}{u} = -\dfrac{f_1}{f_2}$.

(b) $M = -\dfrac{f_1}{f_2} \Rightarrow f_2 = -\dfrac{f_1}{M} = -\dfrac{95.0\text{ cm}}{6.33} = -15.0\text{ cm}$.

(c) The length of the telescope is $95.0\text{ cm} - 15.0\text{ cm} = 80.0\text{ cm}$, compared to the length of 110 cm for the telescope in Exercise 34.61.

EVALUATE: An advantage of this construction is that the telescope is somewhat shorter.

34.101. **IDENTIFY:** The thin-lens formula applies. The converging lens forms a real image on its right side. This image acts as the object for the diverging lens. The image formed by the converging lens is on the right side of the diverging lens, so this image acts as a *virtual object* for the diverging lens and its object distance is *negative*.

SET UP: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ in each case. $m = -\dfrac{s'}{s}$. The total magnification of two lenses is $m_{\text{tot}} = m_1 m_2$.

EXECUTE: **(a)** For the first trial on the diverging lens, we have
s = 20.0 cm – 29.7 cm = –9.7 cm and s' = 42.8 cm – 20.0 cm = 22.8 cm.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{-9.7\text{ cm}} + \frac{1}{22.8\text{ cm}} \quad \rightarrow \quad f = -16.88\text{ cm}.$$

For the second trial on the diverging lens, we have
s = 25.0 cm – 29.7 cm = –4.7 cm and s' = 31.6 cm – 25.0 cm = 6.6 cm.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{-4.7\text{ cm}} + \frac{1}{6.6\text{ cm}} \quad \rightarrow \quad f = -16.33\text{ cm}.$$

Taking the average of the focal lengths, we get f_{av} = (–16.88 cm – 16.33 cm)/2 = –16.6 cm.

(b) The total magnification is $m_{\text{tot}} = m_1 m_2$. The converging lens does not move during the two trials, so m_1 is the same for both of them. But m_2 does change.

At 20.0 cm: $m = -\dfrac{s'}{s}$ = –(22.8 cm)/(–9.7 cm) = +2.35.

At 25.0 cm: $m = -\dfrac{s'}{s}$ = –(6.6 cm)/(–4.7 cm) = +1.40.

The magnification is greater when the lens is at 20.0 cm.

EVALUATE: This is a case where a diverging lens can form a real image, but only when it is used in conjunction with one or more other lenses.

34.102. IDENTIFY and SET UP: The formulas $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$ both apply for the mirror.

EXECUTE: **(a)** Combining the two formula above and eliminating s' gives $s=f-f\left(\frac{1}{m}\right)$. Therefore a graph of s versus $1/m$ should be a straight line having a slope equal to $-f$.

(b) Using the points (–1.8, 70 cm) and (–0.2, 30 cm) on the graph, we calculate the slope to be

slope = $\frac{30\text{ cm} - 70\text{ cm}}{-0.2+1.8} = -25\text{ cm} = -f$, so f = 25 cm.

(c) The image is inverted, so the magnification is negative. The image is twice as high as the object, so the magnification has magnitude 2. Combining these conditions tells us that $m = -2$, so $1/m = -1/2$. Using our equation, we have s = 25 cm – (25 cm)(–1/2) = 37.5 cm.

(d) Since m is negative, we can write our formula for s as $s=f+f\left(\frac{1}{|m|}\right)$. To increase the size of the image, we must increase the magnitude of the magnification, which means we must decrease $1/|m|$. To do this, we must make s smaller, so we must move the object *closer* to the mirror. If we want m to be –3, our equation for s gives us s = 25 cm – 24 cm (–1/3) = 33.3 cm. This result agrees with our reasoning that we must move the object closer to the mirror.

(e) As $s \to 25$ cm, the object is approaching the focal point of the mirror, so $s' \to \infty$. Therefore $m=-\frac{s'}{s} \to \infty$, so $1/m \to 0$.

(f) When $s < 25$ cm and $m > 0$, the image distance is negative, so the image is virtual and therefore cannot be seen on a screen. Only real images can be focused on a screen.

EVALUATE: According to our equation $s=f-f\left(\frac{1}{m}\right)$ in (a), as $1/m \to 0$, $s \to f$. By extending our graph downward and to the left, we see that s does approach 25 cm as $1/m$ approaches zero, so 25 cm should be the focal length. This agrees with our result in (b).

34.103. IDENTIFY and SET UP: We measure s and s'. The equation $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ applies. In this case, n_a = 1.00 for air and $n_b = n$.

EXECUTE: **(a)** In this case, the equation $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ becomes $\frac{1}{s}+\frac{n}{s'}=\frac{n-1}{R}$. Solving for $1/s'$ gives $\frac{1}{s'}=\frac{n-1}{nR}-\frac{1}{n}\cdot\frac{1}{s}$. Therefore a graph of $1/s'$ versus $1/s$ should be a straight line having slope equal to $-1/n$ and a y-intercept equal to $(n-1)/nR$. Figure 34.103 shows the graph of the data from the table in the problem.

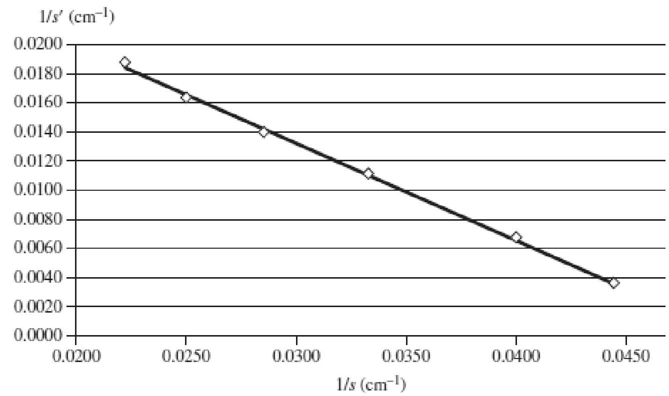


Figure 34.103

(b) The equation of the best-fit graph of the data is $\frac{1}{s'} = -(0.6666)\frac{1}{s} + 0.0333\text{ cm}^{-1}$. From this we have

slope = –1/n $\rightarrow$ n = –1/(slope) = –1/(–0.6666) = 1.50.

Using the y-intercept, we have y-intercept = (n – 1)/nR. Solving for R gives

$R = (n-1)/[n(y\text{-intercept})] = (1.50-1)/[(1.50)(0.03333\text{ cm}^{-1})] = 10.0\text{ cm}$.

(c) Using our equation from the graph $\frac{1}{s'} = -(0.6666)\frac{1}{s} + 0.0333\text{ cm}^{-1}$, we have

$$\frac{1}{s'} = -(0.6666)\frac{1}{15.0\text{ cm}} + 0.0333\text{ cm}^{-1} \quad \rightarrow \quad s' = -90\text{ cm}.$$

The image is 90 cm in front of the glass and is virtual.

EVALUATE: An index of refraction of n = 1.50 for glass is very reasonable.

34.104. **IDENTIFY:** We are dealing with a parabolic mirror.

SET UP and **EXECUTE:** Refer to Fig. P34.104 with the problem in the textbook. **(a)** We want the slope at $x = r$. $y = ax^2$, so the slope is $dy/dx = 2ax$. At $x = r$, the slope is $2ar$.

(b) We want the slope of the dashed line at $x = r$. If two lines are perpendicular, we know from analytic geometry their slopes m_1 and m_2 are related by $m_1m_2 = -1$. So $m_2 = -\frac{1}{m_1} = -\frac{1}{2ar}$.

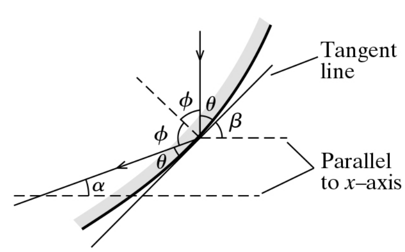


Figure 34.104

(c) We want to find ϕ. See Fig. 34.104 for the geometry. We have drawn the tangent line to the curve. The two horizontal dashed lines are parallel to the x-axis, so $\tan\beta$ is the slope of the tangent line. The tangent line is perpendicular to the normal, so $\phi + \theta = 90°$. The x- and y-axes are perpendicular to each other, so $\theta + \beta = 90°$. Therefore $\phi = \beta$, so $\tan\phi$ is the slope of the tangent line. Thus $\tan\phi = 2ar$, which gives $\phi = \arctan(2ar)$.

(d) We want α. Refer to Fig. 34.104. In the triangle involving α, the obtuse angle is $180° - \beta$. The sum of the angles in a triangle is 180°, so $\alpha + \theta + (180° - \beta) = 180°$, which means that $\alpha + \theta - \beta) = 0°$. From part (c), we have $\theta = 90° - \beta$, so $\alpha = 2\beta - 90°$. But $\beta = \phi$, so $\alpha = 2\phi - 90°$.

(e) We want b. $\tan\alpha = b/r$ and $\tan\alpha = \tan(2\phi - 90°) = -\cot 2\phi$. Equating gives $\cot 2\phi = -b/r$. Using the identity $\cot 2\phi = \dfrac{\cot^2\phi - 1}{2\cot\phi}$, we have $\dfrac{\cot^2\phi - 1}{2\cot\phi} = -\dfrac{b}{r}$. Rearranging gives $-\dfrac{2b}{r} = -\tan\phi + \dfrac{1}{\tan\phi}$. But $\tan\phi$ is the slope of the graph at $x = r$, which is $2ar$, so we have $\dfrac{2b}{r} = 2ar - \dfrac{1}{2ar}$. Solving for b gives $b = ar^2 - \dfrac{1}{4a}$.

(f) We want f. Refer to Fig. P34.104 in the textbook. At $x = r$, $y = ar^2$ and $f + b = y = ar^2$. Using b from part (e) gives $f + \left(ar^2 - \dfrac{1}{4a}\right) = ar^2$. Solving for f gives $f = 1/4a$.

EVALUATE: Since f is independent of r, it has this value for *all r*. This means that all rays go to the same point, so there is no spherical aberration.

34.105. IDENTIFY: The distance between image and object can be calculated by taking the derivative of the separation distance and minimizing it.

SET UP: For a real image $s' > 0$ and the distance between the object and the image is $D = s + s'$. For a real image must have $s > f$.

EXECUTE: **(a)** $D = s + s'$ but $s' = \dfrac{sf}{s-f} \Rightarrow D = s + \dfrac{sf}{s-f} = \dfrac{s^2}{s-f}$.

$\dfrac{dD}{ds} = \dfrac{d}{ds}\left(\dfrac{s^2}{s-f}\right) = \dfrac{2s}{s-f} - \dfrac{s^2}{(s-f)^2} = \dfrac{s^2 - 2sf}{(s-f)^2} = 0$. $s^2 - 2sf = 0$. $s(s-2f) = 0$. $s = 2f$ is the solution for which $s > f$. For $s = 2f$, $s' = 2f$. Therefore, the minimum separation is $2f + 2f = 4f$.

(b) A graph of D/f versus s/f is sketched in Figure 34.105. Note that the minimum does occur for $D = 4f$.

EVALUATE: If, for example, $s = 3f/2$, then $s' = 3f$ and $D = s + s' = 4.5f$, greater than the minimum value.

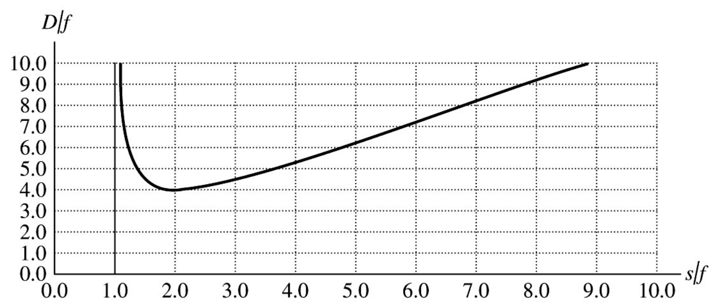


Figure 34.105

34.106. IDENTIFY: Use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to calculate s' (the distance of each point from the lens), for points A, B, and C.

SET UP: The object and lens are shown in Figure 34.106a.

EXECUTE: **(a)** For point C: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{45.0\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}} \Rightarrow s'=36.0\text{ cm}.$

$y'=-\frac{s'}{s}y=-\frac{36.0}{45.0}(15.0\text{ cm})=-12.0\text{ cm},$ so the image of point C is 36.0 cm to the right of the lens, and 12.0 cm below the axis.

For point A: $s=45.0\text{ cm}+(8.00\text{ cm})(\cos 45°)=50.7\text{ cm}.$

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{50.7\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}} \Rightarrow s'=33.0\text{ cm}.$

$y'=-\frac{s'}{s}y=-\frac{33.0}{45.0}[15.0\text{ cm}-(8.00\text{ cm})(\sin 45°)]=-6.10\text{ cm},$ so the image of point A is 33.0 cm to the right of the lens, and 6.10 cm below the axis.

For point B: $s=45.0\text{ cm}-(8.00\text{ cm})(\cos 45°)=39.3\text{ cm}.$

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{39.3\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}} \Rightarrow s'=40.7\text{ cm}.$

$y'=-\frac{s'}{s}y=-\frac{40.7}{39.3}[15.0\text{ cm}+(8.00\text{ cm})(\sin 45°)]=-21.4\text{ cm},$ so the image of point B is 40.7 cm to the right of the lens, and 21.4 cm below the axis. The image is shown in Figure 34.106b.

(b) The length of the pencil is the distance from point A to B:

$L=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}=\sqrt{(33.0\text{ cm}-40.7\text{ cm})^2+(6.10\text{ cm}-21.4\text{ cm})^2}=17.1\text{ cm}$

EVALUATE: The image is below the optic axis and is larger than the object.

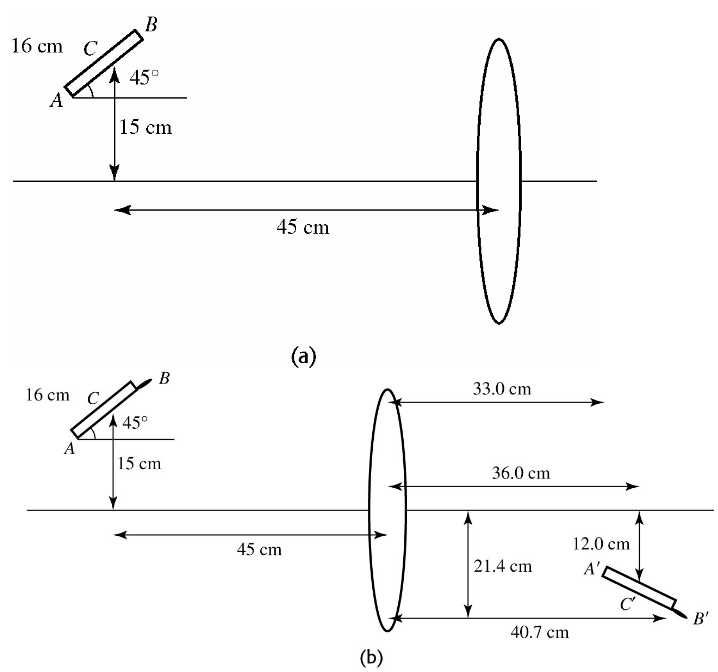


Figure 34.106

34.107. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ to refraction at the cornea to find where the object for the cornea must be in order for the image to be at the retina. Then use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate *f* so that the lens produces an image of a distant object at this point.

SET UP: For refraction at the cornea, $n_a=1.333$ and $n_b=1.40$. The distance from the cornea to the retina in this model of the eye is 2.60 cm. From Problem 34.52, $R=0.710$ cm.

EXECUTE: **(a)** People with normal vision cannot focus on distant objects under water because the image is unable to be focused in a short enough distance to form on the retina. Equivalently, the radius of curvature of the normal eye is about five or six times too great for focusing at the retina to occur.

(b) When introducing glasses, let's first consider what happens at the eye:

$\frac{n_a}{s_2}+\frac{n_b}{s_2'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.333}{s_2}+\frac{1.40}{2.6\text{ cm}}=\frac{0.067}{0.71\text{ cm}}\Rightarrow s_2=-3.00\text{ cm}$. That is, the object for the cornea must be 3.00 cm behind the cornea. Now, assume the glasses are 2.00 cm in front of the eye, so $s_1'=2.00\text{ cm}+|s_2|=5.00\text{ cm}$. $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1'}$ gives $\frac{1}{\infty}+\frac{1}{5.00\text{ cm}}=\frac{1}{f_1'}$ and $f_1'=5.00\text{ cm}$. This is the focal length in water, but to get it in air, we use the formula from Problem 34.92:

$$f_1=f_1'\left[\frac{n-n_{\text{liq}}}{n_{\text{liq}}(n-1)}\right]=(5.00\text{ cm})\left[\frac{1.62-1.333}{1.333(1.62-1)}\right]=1.74\text{ cm}.$$

EVALUATE: A converging lens is needed.

34.108. **IDENTIFY** and **SET UP:** Apply the thin-lens formula $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to the eye and calculate s' in both cases. The focal length stays the same.

EXECUTE: At 10 cm: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{10\text{ cm}}+\frac{1}{0.8\text{ cm}}$. At 15 cm: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{15\text{ cm}}+\frac{1}{s'}$. Equate the two expressions for 1/*f* and solve for s'.

$$\frac{1}{15\text{ cm}}+\frac{1}{s'}=\frac{1}{10\text{ cm}}+\frac{1}{0.8\text{ cm}}\quad\rightarrow\ s'=0.779\text{ cm}.$$

The distance the lens must move is 0.8 cm – 0.779 cm = 0.021 cm ≈ 0.02 cm, which is choice (a).

EVALUATE: This is a very small distance to move, but the eye of a frog is also very small, so the result seems plausible.

34.109. **IDENTIFY** and **SET UP:** Apply the thin-lens formula $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to the eye. The lens power in diopters is 1/*f* (in m). For the corrected eye, the image is at infinity, and it would take –6.0 D to correct the frog's vision so it could see at infinity.

EXECUTE: Using $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$ gives $-6.0\text{ m}^{-1}=1/s+1/\infty=1/s$, so $s=0.17$ m = 17 cm, which makes choice (d) the correct one.

EVALUATE: It is reasonable for a fog to see clearly up to only 17 cm since its food consists of insects, which must be fairly close to get caught.

34.110. IDENTIFY and SET UP: Apply Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$, at the cornea.

EXECUTE: From $n_a \sin\theta_a = n_b \sin\theta_b$, we have $\sin\theta_b = (n_b/n_a)\sin\theta_a$. When n_a and n_b are closer to each other, θ_b is closer to θ_a, so less refraction occurs at the cornea. This will be the case when a frog goes under water, since the refractive index of water (1.33) is closer to that of the cornea than is the refractive index of air, which is 1.00. Therefore choice (b) is correct.

EVALUATE: Frogs must adapt when they go under water. They are probably better hunters there and are better able to spot predators.

34.111. IDENTIFY and SET UP: Apply the thin-lens formula $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ to the eye. The lens power in diopters is D = 1/*f* (in m).

EXECUTE: Since *D* = 1/*f* (in m), the larger |D| the smaller *s*. So the frog with the –15-D lens could focus at a shorter distance than the frog with the –9-D lens, which is choice (b).

EVALUATE: The lens would not move the same distance with the –15-D lens as with the 9-D lens.

35

GEOMETRIC OPTICS

VP35.2.1. **IDENTIFY:** This problem is about double-slit interference of light.

SET UP: Constructive interference: $d\sin\theta_m = m\lambda$, for small angles: $y_m = R\dfrac{m\lambda}{d}$.

EXECUTE: **(a)** We want λ. For small angles we can use $y_m = R\dfrac{m\lambda}{d}$. Solve for λ_2.

$$\lambda_2 = \frac{y_2 d}{2R} = \frac{(11.4\text{ mm})(0.180\text{ mm})}{2(1500\text{ mm})} = 684\text{ nm.}$$

(b) We want the distance to the $m = -3$ fringe. This will be the same as the distance to the $m = +3$ fringe, so we can ignore the minus sign. $\dfrac{y_3}{y_2} = \dfrac{R(3\lambda/d)}{R(2\lambda/d)} = \dfrac{3}{2}$. $y_3 = (3/2)y_{-2} = (3/3)(11.4\text{ mm}) = 17.1\text{ mm}$.

EVALUATE: The small-angle approximation works only fairly close to the central maximum where bright fringes are evenly spaced.

VP35.2.2. **IDENTIFY:** This problem deals with the interference of radiowaves from two sources.

SET UP: Maxima occur when $d\sin\theta_m = m\lambda$, minima occur when $d\sin\theta_m = \left(m + \dfrac{1}{2}\right)\lambda$, $f\lambda = c$. We want the angles to the maxima and minima.

EXECUTE: **(a)** Maxima: First find λ: $\lambda = c/f = c/(1.05\text{ MHz}) = 285.7$ m. Now use $d\sin\theta_m = m\lambda$, $m = 0, \pm1, \pm2, \ldots$ Solve for $\sin\theta_m$, giving $\sin\theta_m = \dfrac{m\lambda}{d} = m\left(\dfrac{285.7\text{ m}}{810\text{ m}}\right) = 0.3527m$.

$m = 0$: $\sin\theta_0 = 0 \rightarrow \theta_0 = 0°$

$m = 1$: $\sin\theta_1 = (0.3527)(1) = 0.3527 \rightarrow \theta_1 = 20.7°$

$m = 2$: $\sin\theta_2 = (0.3527)(2) = 0.7021 \rightarrow \theta_2 = 44.9°$

$m = 3$: $\sin\theta_3 = (0.3527)(3) = 1.06 \rightarrow$ not possible

(b) Maxima: Use $d\sin\theta_m = \left(m + \dfrac{1}{2}\right)\lambda$, $m = 0, 1, 2, \ldots$

$m = 0$: $\sin\theta_0 = (0.3527)(0.500) = 0.1764 \rightarrow \theta_0 = 10.2°$

$m = 1$: $\sin\theta_1 = (0.3527)(1.5) = 0.5291 \rightarrow \theta_1 = 31.9°$

$m = 2$: $\sin\theta_2 = (0.3527)(2.5) = 0.8818 \rightarrow \theta_2 = 61.9°$

$m = 3$: $\sin\theta_3 = (0.3527)(3.5) = 1.23 \rightarrow$ not possible

EVALUATE: The minima are between the maxima, but not midway between them.

VP35.2.3. **IDENTIFY:** We have double slit interference.

SET UP: $y_m = R\dfrac{m\lambda}{d}$.

EXECUTE: **(a)** We want d. Solve $y_m = R\dfrac{m\lambda}{d}$ for d. $d = \dfrac{m\lambda R}{y_m} = \dfrac{3(685 \text{ nm})(2100 \text{ mm})}{9.15 \text{ mm}} = 0.472$ mm.

(b) We want y. $\dfrac{y_g}{y_r} = \dfrac{R\dfrac{m\lambda_g}{d}}{R\dfrac{m\lambda_r}{d}} = \dfrac{\lambda_g}{\lambda_r} = \dfrac{515 \text{ nm}}{685 \text{ nm}} = 0.7518$, so y_g = (9.15 mm)(0.7518) = 6.88 mm.

EVALUATE: Since $y \propto \lambda$, as λ decreases, the bright fringes get closer together, which agrees with our result in part (b).

VP35.2.4. **IDENTIFY:** We have double slit interference.

SET UP: $d\sin\theta_m = m\lambda$ (bright fringes), $d\sin\theta_m = \left(m + \dfrac{1}{2}\right)\lambda$ (dark spots).

EXECUTE: **(a)** We want λ. $\lambda = \dfrac{d\sin\theta_m}{m} = \dfrac{(0.370 \text{ mm})\sin(0.407°)}{4} = 657$ nm.

(b) We want θ_2 for a dark spot. $\sin\theta_2 = \dfrac{\left(2 + \dfrac{1}{2}\right)\lambda}{d} = \dfrac{(2.5)(657 \text{ nm})}{0.370 \text{ mm}}$. θ_2 = 0.254°.

EVALUATE: The bright (and dark) spots are equally spaced for small angles but not for large ones.

VP35.3.1. **IDENTIFY:** This problem is about the intensity of a two-source interference pattern.

SET UP: $I = I_0\cos^2(\phi/2)$, $\phi/2 = \pi d\sin\theta/\lambda$, $f\lambda = c$.

EXECUTE: **(a)** We want I. First find λ. $\lambda = c/f$ = c/(59.3 MHz) = 5.059 m. $\phi/2 = \pi d\sin\theta/\lambda$ = π(13.0 m)(sin 5.00°)/(5.059 m) = 0.7036 rad = 40.313°. Now use $I = I_0\cos^2(\phi/2)$ to find the intensity. $I = I_0\cos^2(\phi/2)$ = (0.0330 W/m^2) cos^2(40.313°) = = 0.0192 W/m^2.

(b) We want $\theta_{\min}$. $I = I_0\cos^2(\phi/2) = \dfrac{I_0}{2}$, so $\cos(\phi/2) = 1/\sqrt{2}$. $\phi/2$ = 45.0° = π/4 rad. $\phi/2 = \pi d\sin\theta/\lambda$ = π/4, so $\sin\theta = \lambda/4d =$ (5.059 m)/[4(13.0 m)] = 0.09729. $\theta_{\min}$ = 5.58°.

EVALUATE: At 5.00° I = 0.0192 W/m^2, which is a little more than I_0/2. So at a slightly larger angle I will equal I_0/2. This agrees with our result because we found θ = 5.58° for $I = I_0/2$.

VP35.3.2. **IDENTIFY:** This problem is about the intensity of a double-slit interference pattern.

SET UP: $I = I_0\cos^2(\phi/2)$, $\phi/2 = \pi d\sin\theta/\lambda$, for small angles $y_m = R\dfrac{m\lambda}{d}$

EXECUTE: **(a)** We want I. For small angles $\sin\theta \approx \tan\theta \approx \dfrac{y}{R}$, so $\dfrac{\phi}{2} = \dfrac{\pi d\sin\theta}{\lambda} \approx \dfrac{\pi dy}{\lambda R}$

$= \dfrac{\pi(0.230 \text{ mm})(6.50 \text{ mm})}{(655 \text{ nm})(1.75 \text{ m})} = 4.0974$ rad = 234.8°. $I = I_0\cos^2(\phi/2)$ = (0.520 W/m^2) cos^2(234.8°) = 0.173 W/m^2.

(b) We want y. $I = I_0\cos^2(\phi/2) = \dfrac{I_0}{4}$, $\cos(\phi/2) = 1/2$, so $\phi/2$ = 60° = π/3 rad. Therefore $\dfrac{\phi}{2} = \dfrac{\pi}{3} \approx \dfrac{\pi dy}{\lambda R}$, which gives $y = \dfrac{\lambda R}{3d} = \dfrac{(655 \text{ nm})(1.75 \text{ m})}{3(0.230 \text{ mm})} = 1.66$ mm.

EVALUATE: We must be careful to convert angles in radians to degrees to use most calculators.

VP35.3.3. **IDENTIFY:** This problem involves the intensity of the interference pattern of two sources of radiowaves.

SET UP: $I = I_0 \cos^2(\phi/2)$, $\phi/2 = \pi d \sin\theta/\lambda$.

EXECUTE: **(a)** We want λ. Use $I = I_0 \cos^2(\phi/2)$ and $\phi/2 = \pi d \sin\theta/\lambda$ to find ϕ and then use that to find λ. At 6.00° we have 0.0303 W/m^2 = (0.0540 W/m^2) cos^2(ϕ/2). ϕ/2 = 41.49° = 0.7241 rad. Now get λ: $\phi/2 = \pi d \sin\theta/\lambda$ gives $\lambda = \dfrac{\pi d \sin\theta}{\phi/2} = \dfrac{\pi(8.00\text{ m})\sin(6.00°)}{0.7241\text{ rad}} = 3.63\text{ m}.$

(b) We want I at θ = 12.0°. $\phi/2 = \pi d \sin\theta/\lambda$ = π(8.00 m)(sin 12.0°)/(3.63 m) = 1.440 rad = 8253°.

$I = I_0 \cos^2(\phi/2)$ = (0.0540 W/m^2) cos^2(82.53°) = 9.14×10^{-4} W/m^2.

EVALUATE: It would not be good to use the small-angle approximation in this case with θ = 12°.

VP35.3.4. **IDENTIFY:** This problem is about the intensity of a two-slit interference pattern.

SET UP: $I = I_0 \cos^2(\phi/2)$, $\phi/2 = \pi d \sin\theta/\lambda$, $\sin\theta \approx y/R$ (small-angle approximation).

EXECUTE: **(a)** We want λ. First find ϕ/2, then use it to find p using $\sin\theta \approx y/R$. $I = I_0 \cos^2(\phi/2)$ gives 0.0900I_0 = I_0 cos^2(ϕ /), so ϕ /2 = 72.54° = 1.266 rad. Now use $\phi/2 = \pi d \sin\theta/\lambda$ and solve for λ using $\sin\theta \approx y/R$. $\lambda = \dfrac{\pi d(y/R)}{\phi/2} = \dfrac{\pi(0.110\text{ mm})(10.5\text{ mm})/(5.00\text{ m})}{1.266\text{ rad}} = 573\text{ nm}.$

(b) We want y. Find $\phi/2$: 0.300I_0 = I_0 cos^2 $\phi/2$ gives $\phi/2$ = 56.79° = 0.9918 rad. Now find y. Since y will be less than 10.5 mm, we can use the small-angle approximation. Solve $\phi/2 = \pi d \sin\theta/\lambda$ for y using $\sin\theta \approx y/R$. $y = \dfrac{\lambda(\phi/2)R}{\pi d}$. Take ratios giving $\dfrac{y_2}{y_1} = \dfrac{\dfrac{\lambda(\phi_2/2)R}{\pi d}}{\dfrac{\lambda(\phi_1/2)R}{\pi d}} = \dfrac{\phi_2/2}{\phi_1/2} = \dfrac{0.9912}{1.266} = 0.7829.$

y_2 = (0.7829)(10.5 mm) = 8.22 mm.

EVALUATE: We find $y_2 < y_1$, as we expect.

VP35.6.1. **IDENTIFY:** This problem is about thin-film interference.

SET UP: Refer to Fig. 35.15 in the textbook. Call h the thickness of the paper and t the thickness at point x. A half-cycle phase shift occurs at reflection at the lower plate. The dark fringes occur at points of destructive interference, for which 2t = $m\lambda$ due to the half-cycle phase shift.

EXECUTE: **(a)** We want h. 2t = $m\lambda$, so t = $m\lambda$/2 (m = 0, 1, 2, …) As in Fig. 35.15 in the textbook, t/x = h/l, so h = lt/x = $l\,m\lambda$/2. The fringes are 1.30 mm apart. The first one is at x = 0, so the next one is at x = 1.30 mm. The m = 1 dark fringe is at t_1 = (1) λ /2 = (565 nm)/2 = 282.5 nm. So $\tan\theta = t_1/x_1 = h/l$, which gives h = lt_1/x_1 = (8.00 cm)(282.5 nm)/(1.30 mm) = 0.0174 mm.

(b) We want the fringe spacing. There is still a half-cycle phase shift at reflection because $n_{\text{glass}} > n_{\text{ethanol}}$, but $\lambda_n = \dfrac{\lambda_0}{n_{\text{ethanol}}}$. The m = 0 fringe is at x = 0. The m = 1 fringe is at x_1 (which we do not know).

2t_1 = $= \dfrac{\lambda_0}{n}$. So $t_1 = \lambda/2n$ = (565 nm)/[2(136)] = 207.7 nm. As before, $t_1/x_1 = h/l$, so $x_{-1} = t_1 l/h$. This gives x_1 = (207.7 nm)(8.00 cm)/(0.0174 mm) = 0.956 mm. This is the distance between the m = 0 fringe and the m = 1 fringe, so it is the fringe spacing.

EVALUATE: The replacement of ethanol for air made significant changes in the fringe pattern because it changed the wavelength of light in the region between the glass plates.

VP35.6.2. **IDENTIFY:** We have thin-film interference.

SET UP: We want to know which wavelengths of visible light have destructive and constructive interference. In both cases there is a half-cycle phase shift during reflection because n_{glass} is greater than both n_{air} and $n_{\text{carbon-tet}}$, so for constructive interference $2t = \left(m + \frac{1}{2}\right)\lambda_m$ and for destructive interference $2t = m\lambda_m$.

EXECUTE: **(a)** Air film. Solve for λ, which gives $\lambda_m = \frac{2t}{m + 1/2}$.

Constructive: Use $2t = \left(m + \frac{1}{2}\right)\lambda_m$.

$m = 0$: $\lambda_0 = 4t$ = 4(146 nm) = 584 nm (not visible)

$m = 1$: λ_1 = 2(146 nm)/1.5 = 194 nm (not visible)

Therefore no visible light interferes constructively.

Destructive: Use $2t = m\lambda_m$.

$m = 1$: $\lambda_1 = 2t = 292$ nm (not visible)

No visible light interferes destructively.

(b) Carbon tetrachloride film. Follow the same procedure as in (a) except $\lambda_n = \frac{\lambda_0}{1.46}$. The results are: No wavelengths interfere constructively, but λ = 426 nm interferes destructively.

EVALUATE: By varying t we could get different wavelengths to interfere.

VP35.6.3. **IDENTIFY:** We have thin-film interference.

SET UP: We want to know which wavelengths of visible light have destructive and constructive interference. In both cases there is a half-cycle phase shift during reflection because n_{plastic} is greater than both n_{air} and n_{ethanol}, so for constructive interference $2t = \left(m + \frac{1}{2}\right)\lambda_m$ and for destructive interference $2t = m\lambda_m$.

EXECUTE: **(a)** Air film. Solve for λ, which gives.

Constructive: Use $2t = \left(m + \frac{1}{2}\right)\lambda_m$.

$m = 0$: $\lambda_0 = 4t$ = 4(185 nm) = 740 nm (visible)

$m = 1$: λ_1 = 2(185 nm)/1.5 = 247 nm (not visible)

Therefore only 740 nm interferes constructively.

Destructive: Use $2t = m\lambda_m$.

$m = 1$: $\lambda_1 = 2t = 370$ nm (not visible)

No visible light interferes destructively.

(b) Carbon tetrachloride film. Follow the same procedure as in (a) except $\lambda_n = \frac{\lambda_0}{1.46}$. The results are: No wavelengths interfere constructively, but λ = 503 nm interferes destructively.

EVALUATE: We are only considering interference of rays reflected off the top and bottom of the gap between the sheets.

VP35.6.4. **IDENTIFY:** We have thin-film interference.

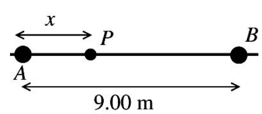
SET UP: There is a half-cycle phase reversal at the upper surface of the grease but not at the lower surface. For constructive interference $2t = \left(m + \frac{1}{2}\right)\lambda_m$, $\lambda_n = \frac{\lambda_0}{n}$.

EXECUTE: **(a)** We want the smallest possible thickness. $2t = \left(m + \frac{1}{2}\right)\lambda_m$. The minimum thickness is for $m = 0$, so $t_{\text{min}} = \lambda/4n_{\text{glass}} =$ (565 nm)/[4(1.60)] = 88.3 nm.

(b) We want the second smallest possible thickness. This is for $m = 1$. $t = \left(1+\frac{1}{2}\right)\frac{\lambda_m}{2} = 265\ \text{nm}.$

EVALUATE: The second smallest thickness is one wavelength (in the grease) greater than the smallest thickness.

35.1. **IDENTIFY:** Use $c = f\lambda$ to calculate the wavelength of the transmitted waves. Compare the difference in the distance from A to P and from B to P. For constructive interference this path difference is an integer multiple of the wavelength.
SET UP: Consider Figure 35.1.



The distance of point P from each coherent source is $r_A = x$ and $r_B = 9.00\ \text{m} - x.$

Figure 35.1

EXECUTE: The path difference is $r_B - r_A = 9.00\ \text{m} - 2x.$

$r_B - r_A = m\lambda,\ m = 0,\ \pm 1, \pm 2,\ \ldots$

$$\lambda = \frac{c}{f} = \frac{2.998\times 10^8\ \text{m/s}}{120\times 10^6\ \text{Hz}} = 2.50\ \text{m}.$$

Thus $9.00\ \text{m} - 2x = m(2.50\ \text{m})$ and $x = \dfrac{9.00\ \text{m} - m(2.50\ \text{m})}{2} = 4.50\ \text{m} - (1.25\ \text{m})m.$ x must lie in the range 0 to 9.00 m since P is said to be between the two antennas.

$m = 0$ gives $x = 4.50\ \text{m}.$
$m = +1$ gives $x = 4.50\ \text{m} - 1.25\ \text{m} = 3.25\ \text{m}.$
$m = +2$ gives $x = 4.50\ \text{m} - 2.50\ \text{m} = 2.00\ \text{m}.$
$m = +3$ gives $x = 4.50\ \text{m} - 3.75\ \text{m} = 0.75\ \text{m}.$
$m = -1$ gives $x = 4.50\ \text{m} + 1.25\ \text{m} = 5.75\ \text{m}.$
$m = -2$ gives $x = 4.50\ \text{m} + 2.50\ \text{m} = 7.00\ \text{m}.$
$m = -3$ gives $x = 4.50\ \text{m} + 3.75\ \text{m} = 8.25\ \text{m}.$

All other values of m give values of x out of the allowed range. Constructive interference will occur for $x = 0.75$ m, 2.00 m, 3.25 m, 4.50 m, 5.75 m, 7.00 m, and 8.25 m.

EVALUATE: Constructive interference occurs at the midpoint between the two sources since that point is the same distance from each source. The other points of constructive interference are symmetrically placed relative to this point.

35.2. **IDENTIFY:** For destructive interference the path difference is $(m+\frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2, \ldots.$ The longest wavelength is for $m = 0$. For constructive interference the path difference is $m\lambda,\ m = 0, \pm 1, \pm 2, \ldots.$ The longest wavelength is for $m = 1$.
SET UP: The path difference is 120 m.

EXECUTE: **(a)** For destructive interference $\dfrac{\lambda}{2} = 120\ \text{m} \Rightarrow \lambda = 240\ \text{m}.$

(b) The longest wavelength for constructive interference is $\lambda = 120\ \text{m}.$

EVALUATE: The path difference doesn't depend on the distance of point Q from B.

35.3. **IDENTIFY:** If the path difference between the two waves is equal to a whole number of wavelengths, constructive interference occurs, but if it is an odd number of half-wavelengths, destructive interference occurs.
SET UP: We calculate the distance traveled by both waves and subtract them to find the path difference.

EXECUTE: Call P_1 the distance from the right speaker to the observer and P_2 the distance from the left speaker to the observer.

(a) $P_1 = 8.0\text{ m}$ and $P_2 = \sqrt{(6.0\text{ m})^2 + (8.0\text{ m})^2} = 10.0\text{ m}$. The path distance is $\Delta P = P_2 - P_1 = 10.0\text{ m} - 8.0\text{ m} = 2.0\text{ m}$.

(b) The path distance is one wavelength, so constructive interference occurs.

(c) $P_1 = 17.0\text{ m}$ and $P_2 = \sqrt{(6.0\text{ m})^2 + (17.0\text{ m})^2} = 18.0\text{ m}$. The path difference is $18.0\text{ m} - 17.0\text{ m} = 1.0\text{ m}$, which is one-half wavelength, so destructive interference occurs.

EVALUATE: Constructive interference also occurs if the path difference $2\lambda, 3\lambda, 4\lambda,$ etc., and destructive interference occurs if it is $\lambda/2, 3\lambda/2, 5\lambda/2,$ etc.

35.4. IDENTIFY: For constructive interference the path difference d is related to λ by $d = m\lambda,\ m = 0, 1, 2, \dots$ For destructive interference $d = (m + \frac{1}{2})\lambda, m = 0, 1, 2, \dots$

SET UP: $d = 2040\text{ nm}$.

EXECUTE: **(a)** The brightest wavelengths are when constructive interference occurs:

$$d = m\lambda_m \Rightarrow \lambda_m = \frac{d}{m} \Rightarrow \lambda_3 = \frac{2040\text{ nm}}{3} = 680\text{ nm},\ \lambda_4 = \frac{2040\text{ nm}}{4} = 510\text{ nm and } \lambda_5 = \frac{2040\text{ nm}}{5} = 408\text{ nm}.$$

(b) The path-length difference is the same, so the wavelengths are the same as part (a).

(c) $d = (m + \frac{1}{2})\lambda_m$ so $\lambda_m = \dfrac{d}{m + \frac{1}{2}} = \dfrac{2040\text{ nm}}{m + \frac{1}{2}}$. The visible wavelengths are $\lambda_3 = 583\text{ nm}$ and $\lambda_4 = 453\text{ nm}$.

EVALUATE: The wavelengths for destructive interference are between those for constructive interference.

35.5. IDENTIFY: This problem is about interference of electromagnetic waves.

SET UP: For constructive interference, the path difference $r_2 - r_1$ must be equal to $m\lambda$.

EXECUTE: **(a)** How many points have constructive interference? $r_2 - r_1 = m\lambda$. Call x the distance from A, so the distance from B is $40.0\text{ m} - x$. The path difference is $(40.0\text{ m} - x) - x = 40.0\text{ m} - 2x$, so we must have $40.0\ m - 2x = m\lambda$, where $m = 0, 1, 2, \dots$. Therefore $x = 20.0\text{ m} - \dfrac{m\lambda}{2} = 20.0\text{ m} - (350\text{ m})m.$

For $m = 0$, $x = 20.0$ m, which is the midpoint between the speakers. The maximum m for a positive x is $m = 5$, for which $x = 2.50$ m. So there are 5 points between A and the midpoint. There are also 5 points on the other side of the midpoint. The total is therefore 5 + 5 + 1 = 11 points.

(b) The minimum x is for $m = 5$, so $x_{min} = 2.50$ m.

EVALUATE: There are also points at which destructive interference occurs when the path difference is an odd multiple of $\lambda/2$.

35.6. IDENTIFY: We have double-slit interference of light.

SET UP: We want the distance between the $m = 49$ and $m = 50$ maxima. $d\sin\theta_m = m\lambda$ (all angles), $y_m = R\dfrac{m\lambda}{d}$ (small angles), $y_m = R\tan\theta_m$.

EXECUTE: First find the slit spacing d. Near the center we can use $y_m = R\dfrac{m\lambda}{d}$. We know the distance between adjacent fringes, so $\Delta m = 1$. $\Delta y_m = R\dfrac{\Delta m\lambda}{d} = \dfrac{R\lambda}{d}$. Solving for d gives $d = \dfrac{R\lambda}{\Delta y} = (200\text{ cm})$ $(500\text{ nm})/(3.53\text{ cm}) = 2.833\times10^4\text{ nm}$. For the distant fringes (large n) we cannot use the small-angle approximation, so we use $d\sin\theta_m = m\lambda$. $\sin\theta_{49} = 49\lambda/d$ and $\sin\theta_{50} = 50\lambda/d$. Using the known values for d and λ gives $\theta_{49} = 59.87°$ and $\theta_{50} = 61.94°$. Using $y_m = R\tan\theta_m$ with $R = 200$ cm gives $\Delta y = y_{50} - y_{49} = R\left(\tan\theta_{50} - \tan\theta_{49}\right) = 30.6 \text{ cm}$.

EVALUATE: Clearly the small-angle approximation cannot be used for fringes far from the center.

35.7. **IDENTIFY:** The value of y_{20} is much smaller than R and the approximate expression $y_m = R\dfrac{m\lambda}{d}$ is accurate.

SET UP: $y_{20} = 10.6\times10^{-3}$ m.

EXECUTE: $d = \dfrac{20R\lambda}{y_{20}} = \dfrac{(20)(1.20 \text{ m})(502\times10^{-9} \text{ m})}{10.6\times10^{-3} \text{ m}} = 1.14\times10^{-3} \text{ m} = 1.14 \text{ mm}.$

EVALUATE: $\tan\theta_{20} = \dfrac{y_{20}}{R}$ so $\theta_{20} = 0.51°$ and the approximation $\sin\theta_{20} \approx \tan\theta_{20}$ is very accurate.

35.8. **IDENTIFY:** Since the dark fringes are equally spaced, $R \gg y_m$, the angles are small and the dark bands are located by $y_{m+\frac{1}{2}} = R\dfrac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: The separation between adjacent dark bands is $\Delta y = \dfrac{R\lambda}{d}$.

EXECUTE: $\Delta y = \dfrac{R\lambda}{d} \Rightarrow d = \dfrac{R\lambda}{\Delta y} = \dfrac{(1.80 \text{ m})(4.50\times10^{-7} \text{ m})}{3.90\times10^{-3} \text{ m}} = 2.08\times10^{-4} \text{ m} = 0.208 \text{ mm}.$

EVALUATE: When the separation between the slits decreases, the separation between dark fringes increases.

35.9. **IDENTIFY** and **SET UP:** The dark lines correspond to destructive interference and hence are located by $d\sin\theta = (m+\frac{1}{2})\lambda$ so $\sin\theta = \dfrac{(m+\frac{1}{2})\lambda}{d}$, $m = 0, \pm1, \pm2, \ldots$.

Solve for θ that locates the second and third dark lines. Use to find the distance of each of the dark lines from the center of the screen.

EXECUTE: 1st dark line is for $m = 0$.

2nd dark line is for $m = 1$ and $\sin\theta_1 = \dfrac{3\lambda}{2d} = \dfrac{3(500\times10^{-9} \text{ m})}{2(0.450\times10^{-3} \text{m})} = 1.667\times10^{-3}$ and $\theta_1 = 1.667\times10^{-3}$ rad.

3rd dark line is for $m = 2$ and $\sin\theta_2 = \dfrac{5\lambda}{2d} = \dfrac{5(500\times10^{-9} \text{ m})}{2(0.450\times10^{-3} \text{m})} = 2.778\times10^{-3}$ and $\theta_2 = 2.778\times10^{-3}$ rad.

(Note that θ_1 and θ_2 are small so that the approximation $\theta \approx \sin\theta \approx \tan\theta$ is valid.) The distance of each dark line from the center of the central bright band is given by $y_m = R\tan\theta$, where $R = 0.850$ m is the distance to the screen.

$\tan\theta \approx \theta$ so $y_m = R\theta_m$.

$y_1 = R\theta_1 = (0.750 \text{ m})(1.667\times10^{-3} \text{ rad}) = 1.25\times10^{-3} \text{ m}.$

$y_2 = R\theta_2 = (0.750 \text{ m})(2.778\times10^{-3} \text{ rad}) = 2.08\times10^{-3} \text{ m}.$

$\Delta y = y_2 - y_1 = 2.08\times10^{-3} \text{ m} - 1.25\times10^{-3} \text{ m} = 0.83 \text{ mm}.$

EVALUATE: Since θ_1 and θ_2 are very small we could have used $y_m = R\dfrac{m\lambda}{d}$, generalized to destructive interference: $y_m = R(m+\frac{1}{2})\lambda/d$.

35.10. **IDENTIFY:** We have two-slit interference.

SET UP: Bright fringes occur when $d\sin\theta_m = m\lambda$, $f\lambda = c$. We want the distance d between the slits. We can vary the frequency, and we know that $f_m = 5.60\times10^{12}$ Hz and $f_{m+1} = 7.47\times10^{12}$ Hz.

EXECUTE: For the first frequency $d\sin\theta_m = m\lambda = m\dfrac{c}{f_m}$. For the next higher frequency

$d\sin\theta_{m+1} = (m+1)\dfrac{c}{f_{m+1}}$. Both maxima occur at the same place, so $d\sin\theta$ is the same. Thus

$m\dfrac{c}{f_m} = (m+1)\dfrac{c}{f_{m+1}}$. Using the known frequencies gives $m = 3$. Now we can find d. $d\sin\theta_3 = 3\dfrac{c}{f_m}$.

Using $\theta = 60.0°$ and f_m gives $d = 0.186$ mm.

EVALUATE: We cannot use the small-angle approximation for an angle as large as 60°.

35.11. **IDENTIFY:** Bright fringes are located at angles θ given by $d\sin\theta = m\lambda$.

SET UP: The largest value $\sin\theta$ can have is 1.00.

EXECUTE: **(a)** $m = \dfrac{d\sin\theta}{\lambda}$. For $\sin\theta = 1$, $m = \dfrac{d}{\lambda} = \dfrac{0.0116\times10^{-3}\text{ m}}{5.85\times10^{-7}\text{ m}} = 19.8$. Therefore, the largest m for fringes on the screen is $m = 19$. There are $2(19)+1 = 39$ bright fringes, the central one and 19 above and 19 below it.

(b) The most distant fringe has $m = \pm19$. $\sin\theta = m\dfrac{\lambda}{d} = \pm19\left(\dfrac{5.85\times10^{-7}\text{ m}}{0.0116\times10^{-3}\text{ m}}\right) = \pm0.958$ and

$\theta = \pm73.3°$.

EVALUATE: For small θ the spacing Δy between adjacent fringes is constant but this is no longer the case for larger angles.

35.12. **IDENTIFY:** The width of a bright fringe can be defined to be the distance between its two adjacent destructive minima. Assuming the small angle formula for destructive interference $y_m = R\dfrac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: $d = 0.200\times10^{-3}$ m. $R = 4.00$ m.

EXECUTE: The distance between any two successive minima is

$y_{m+1} - y_m = R\dfrac{\lambda}{d} = (4.00\text{ m})\dfrac{(400\times10^{-9}\text{m})}{(0.200\times10^{-3}\text{ m})} = 8.00\text{ mm}$. Thus, the answer to both part (a) and part (b) is that the width is 8.00 mm.

EVALUATE: For small angles, when $y_m << R$, the interference minima are equally spaced.

35.13. **IDENTIFY** and **SET UP:** The dark lines are located by $d\sin\theta = (m+\frac{1}{2})\lambda$. The distance of each line from the center of the screen is given by $y = R\tan\theta$.

EXECUTE: First dark line is for $m = 0$ and $d\sin\theta_1 = \lambda/2$.

$\sin\theta_1 = \dfrac{\lambda}{2d} = \dfrac{550\times10^{-9}\text{ m}}{2(1.80\times10^{-6}\text{ m})} = 0.1528$ and $\theta_1 = 8.789°$. Second dark line is for $m = 1$ and $d\sin\theta_2 = 3\lambda/2$.

$\sin\theta_2 = \dfrac{3\lambda}{2d} = 3\left(\dfrac{550\times10^{-9}\text{ m}}{2(1.80\times10^{-6}\text{ m})}\right) = 0.4583$ and $\theta_2 = 27.28°$.

$y_1 = R\tan\theta_1 = (0.350\text{ m})\tan 8.789° = 0.0541$ m.

$y_2 = R\tan\theta_2 = (0.350\text{ m})\tan 27.28° = 0.1805$ m.

The distance between the lines is $\Delta y = y_2 - y_1 = 0.1805\text{ m} - 0.0541\text{ m} = 0.126\text{ m} = 12.6$ cm.

EVALUATE: $\sin\theta_1 = 0.1528$ and $\tan\theta_1 = 0.1546$. $\sin\theta_2 = 0.4583$ and $\tan\theta_2 = 0.5157$. As the angle increases, $\sin\theta \approx \tan\theta$ becomes a poorer approximation.

35.14. **IDENTIFY:** For small angles: $y_m = R\dfrac{m\lambda}{d}$.

SET UP: First-order means $m = 1$.

EXECUTE: The distance between corresponding bright fringes is

$$\Delta y = \frac{Rm}{d}\Delta\lambda = \frac{(4.00\text{ m})(1)}{(0.300\times10^{-3}\text{ m})}(660-470)\times(10^{-9}\text{ m}) = 2.53\times10^{-3}\text{ m} = 2.53\text{ mm}.$$

EVALUATE: The separation between these fringes for different wavelengths increases when the slit separation decreases.

35.15. **IDENTIFY** and **SET UP:** Use the information given about the bright fringe and $y_m = R\dfrac{m\lambda}{d}$ to find the distance d between the two slits. Then use $d\sin\theta = (m+\frac{1}{2})\lambda,\ m = 0, \pm1, \pm2, \ldots$ and $y = R\tan\theta$ to calculate λ for which there is a first-order dark fringe at this same place on the screen.

EXECUTE: $y_1 = \dfrac{R\lambda_1}{d}$, so $d = \dfrac{R\lambda_1}{y_1} = \dfrac{(3.00\text{ m})(600\times10^{-9}\text{ m})}{4.84\times10^{-3}\text{ m}} = 3.72\times10^{-4}\text{ m}$. ($R$ is much greater than d, so $y_m = R\dfrac{m\lambda}{d}$ is valid.) The dark fringes are located by $d\sin\theta = (m+\frac{1}{2})\lambda,\ m = 0, \pm1, \pm2, \ldots$. The first-order dark fringe is located by $\sin\theta = \lambda_2/2d$, where λ_2 is the wavelength we are seeking.

$$y = R\tan\theta \approx R\sin\theta = \frac{\lambda_2 R}{2d}.$$

We want λ_2 such that $y = y_1$. This gives $\dfrac{R\lambda_1}{d} = \dfrac{R\lambda_2}{2d}$ and $\lambda_2 = 2\lambda_1 = 1200$ nm.

EVALUATE: For $\lambda = 600$ nm the path difference from the two slits to this point on the screen is 600 nm. For this same path difference (point on the screen) the path difference is $\lambda/2$ when $\lambda = 1200$ nm.

35.16. **IDENTIFY:** Bright fringes are located at $y_m = R\dfrac{m\lambda}{d}$, when $y_m \ll R$. Dark fringes are at $d\sin\theta = (m+\frac{1}{2})\lambda$ and $y = R\tan\theta$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{6.32\times10^{14}\text{ Hz}} = 4.75\times10^{-7}\text{ m}$. For the third bright fringe (not counting the central bright spot), $m = 3$. For the third dark fringe, $m = 2$.

EXECUTE: **(a)** $d = \dfrac{m\lambda R}{y_m} = \dfrac{3(4.75\times10^{-7}\text{ m})(0.850\text{ m})}{0.0311\text{ m}} = 3.89\times10^{-5}\text{ m} = 0.0389\text{ mm}.$

(b) $\sin\theta = (2+\frac{1}{2})\dfrac{\lambda}{d} = (2.5)\left(\dfrac{4.75\times10^{-7}\text{ m}}{3.89\times10^{-5}\text{ m}}\right) = 0.0305$ and $\theta = 1.75°$.

$y = R\tan\theta = (85.0\text{ cm})\tan 1.75° = 2.60\text{ cm}.$

EVALUATE: The third dark fringe is closer to the center of the screen than the third bright fringe on one side of the central bright fringe.

35.17. **IDENTIFY:** Use $I = I_0\cos^2(\phi/2)$ with $\phi = (2\pi/\lambda)(r_2 - r_1)$.

SET UP: ϕ is the phase difference and $(r_2 - r_1)$ is the path difference.

EXECUTE: **(a)** $I = I_0(\cos 30.0°)^2 = 0.750 I_0$.

(b) $60.0° = (\pi/3)$ rad. $(r_2 - r_1) = (\phi/2\pi)\lambda = [(\pi/3)/2\pi]\lambda = \lambda/6 = 80$ nm.

EVALUATE: $\phi = 360°/6$ and $(r_2 - r_1) = \lambda/6.$

35.18. (a) IDENTIFY and **SET UP:** The minima are located at angles θ given by $d\sin\theta = (m+\tfrac{1}{2})\lambda.$ The first minimum corresponds to $m = 0.$ Solve for $\theta.$ Then the distance on the screen is $y = R\tan\theta.$

EXECUTE: $\sin\theta = \dfrac{\lambda}{2d} = \dfrac{660\times10^{-9}\text{ m}}{2(0.260\times10^{-3}\text{ m})} = 1.27\times10^{-3}$ and $\theta = 1.27\times10^{-3}$ rad.

$y = (0.900\text{ m})\tan(1.27\times10^{-3}\text{ rad}) = 1.14\times10^{-3}\text{ m} = 1.14$ mm.

(b) IDENTIFY and **SET UP:** The equation $I = I_0\cos^2\left(\dfrac{\pi dy}{\lambda R}\right)$ gives the intensity *I* as a function of the position *y* on the screen. Set $I = I_0/2$ and solve for *y*.

EXECUTE: $I = \tfrac{1}{2}I_0$ says $\cos^2\left(\dfrac{\pi dy}{\lambda R}\right) = \tfrac{1}{2}.$

$\cos\left(\dfrac{\pi dy}{\lambda R}\right) = \dfrac{1}{\sqrt{2}}$ so $\dfrac{\pi dy}{\lambda R} = \dfrac{\pi}{4}$ rad.

$y = \dfrac{\lambda R}{4d} = \dfrac{(660\times10^{-9}\text{ m})(0.900\text{ m})}{4(0.260\times10^{-3}\text{ m})} = 5.71\times10^{-4}\text{ m} = 0.571$ mm.

EVALUATE: $I = I_0/2$ at a point on the screen midway between where $I = I_0$ and $I = 0.$

35.19. IDENTIFY and **SET UP:** The phase difference ϕ is given by $\phi = (2\pi d/\lambda)\sin\theta.$

EXECUTE: $\phi = [2\pi(0.340\times10^{-3}\text{ m})/(500\times10^{-9}\text{ m})\sin 23.0° = 1670$ rad.

EVALUATE: The *m*th bright fringe occurs when $\phi = 2\pi m,$ so there are a large number of bright fringes within $23.0°$ from the centerline. Note that the equation $\phi = (2\pi d/\lambda)\sin\theta$ gives ϕ in radians.

35.20. IDENTIFY: Light from the two slits interferes on the screen. The bright and dark fringes are very close together compared to the distance between the screen and the slits, so we can use the small-angle approximation. $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2).$ The intensity is $I = I_0\cos^2\left(\dfrac{\phi}{2}\right).$

SET UP: The intensity is $I = I_0\cos^2\left(\dfrac{\phi}{2}\right).$ $\dfrac{\phi}{2} = \dfrac{\pi d\sin\theta}{\lambda},$ but for small angles $\dfrac{\phi}{2} \approx \dfrac{\pi dy}{R\lambda}.$

EXECUTE: At the first minim, $y = 3.00$ mm and $\phi/2 = \pi/2$. At $y = 2.00$ mm, which is 2/3 of 3.00 mm, $\phi/2 = (2/3)(\pi/2) = \pi/3 = 60°$. Therefore the intensity at $x = 2.00$ mm is

$I = (0.0600\text{ W/m}^2)\cos^2(60°) = 0.0150\text{ W/m}^2.$

(b) Using the same reasoning as in (a), 1.50 mm is $\tfrac{1}{2}$ of 3.00 m, so $\phi/2 = (1/2)(\pi/2) = \pi/4 = 45°$. So

$I = (0.0600\text{ W/m}^2)\cos^2(45°) = 0.0300\text{ W/m}^2.$

EVALUATE: As a check, we could first find λ and then use it to find the intensities. At the first minimum, $\phi/2 = \pi/2 = \pi dy/R\lambda,$ which gives $\lambda = 2dy/R = 5.40\times10^{-4}$ mm. Now use this to calculate the intensities using $I = I_0\cos^2\left(\dfrac{\phi}{2}\right)$ and $\dfrac{\phi}{2} \approx \dfrac{\pi dy}{R\lambda}.$

35.21. **IDENTIFY:** The phase difference ϕ and the path difference $r_1 - r_2$ are related by $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2)$. The intensity is given by $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right)$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times 10^8 \text{ m/s}}{1.20\times 10^8 \text{ Hz}} = 2.50 \text{ m}$. When the receiver measures intensity I_0, $\phi = 0$.

EXECUTE: **(a)** $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2) = \dfrac{2\pi}{2.50 \text{ m}}(1.8 \text{ m}) = 4.52 \text{ rad}$.

(b) $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right) = I_0 \cos^2\left(\dfrac{4.52 \text{ rad}}{2}\right) = 0.404 I_0$.

EVALUATE: $(r_1 - r_2)$ is greater than $\lambda/2$, so one minimum has been passed as the receiver is moved.

35.21. **IDENTIFY:** The phase difference ϕ and the path difference $r_1 - r_2$ are related by $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2)$. The intensity is given by $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right)$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times 10^8 \text{ m/s}}{1.20\times 10^8 \text{ Hz}} = 2.50 \text{ m}$. When the receiver measures intensity I_0, $\phi = 0$.

EXECUTE: **(a)** $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2) = \dfrac{2\pi}{2.50 \text{ m}}(1.8 \text{ m}) = 4.52 \text{ rad}$.

(b) $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right) = I_0 \cos^2\left(\dfrac{4.52 \text{ rad}}{2}\right) = 0.404 I_0$.

EVALUATE: $(r_1 - r_2)$ is greater than $\lambda/2$, so one minimum has been passed as the receiver is moved.

35.22. **IDENTIFY:** We are dealing with thin-film interference with an air film of variable thickness.
SET UP: There is no phase-reversal at the upper air surface, but there is one at the lower air surface because $n_{glass} > n_{air}$. There is constructive interference when $t_1 = 650$ nm, and the next value of t for which constructive interference occurs is $t_2 = 910$ nm. $2t = \left(m + \dfrac{1}{2}\right)\lambda$, $\lambda_n = \dfrac{\lambda_0}{n}$.

EXECUTE: **(a)** We want wavelength of the light in air. Using $2t = \left(m + \dfrac{1}{2}\right)\lambda$ at both thicknesses gives $2t_1 = \left(m + \dfrac{1}{2}\right)\lambda$ and $2t_2 = \left(m + \dfrac{3}{2}\right)\lambda$. Taking t_2/t_1 gives $m = 2$. From this we get $\lambda = 520$ nm.

(b) We want the minimum thickness t_{min}. For the smallest thickness, $2t = \lambda/2$, so $t = \dfrac{\lambda}{4} = 130$ nm.

EVALUATE: Be careful of the half-cycle phase shift when the light reflects off the glass at the bottom of the air film.

35.23. **IDENTIFY:** Consider interference between rays reflected at the upper and lower surfaces of the film. Consider phase difference due to the path difference of $2t$ and any phase differences due to phase changes upon reflection.
SET UP: Consider Figure 35.23.

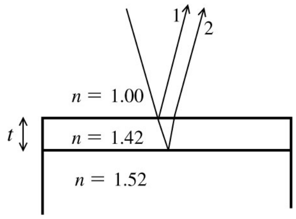


Figure 35.23

Both rays (1) and (2) undergo a $180°$ phase change on reflection, so there is no net phase difference introduced and the condition for destructive interference is $2t = (m+\frac{1}{2})\lambda.$

EXECUTE: $t = \dfrac{(m+\frac{1}{2})\lambda}{2}$; thinnest film says $m = 0$ so $t = \dfrac{\lambda}{4}.$

X and $t = \dfrac{\lambda_0}{4(1.42)} = \dfrac{650\times10^{-9}\text{ m}}{4(1.42)} = 1.14\times10^{-7}\text{ m} = 114\text{ nm}.$

EVALUATE: We compared the path difference to the wavelength in the film, since that is where the path difference occurs.

35.24. **IDENTIFY:** Require destructive interference for light reflected at the front and rear surfaces of the film.
SET UP: At the front surface of the film, light in air $(n = 1.00)$ reflects from the film $(n = 2.62)$ and there is a $180°$ phase shift due to the reflection. At the back surface of the film, light in the film $(n = 2.62)$ reflects from glass $(n = 1.62)$ and there is no phase shift due to reflection. Therefore, there is a net $180°$ phase difference produced by the reflections. The path difference for these two rays is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \dfrac{505\text{ nm}}{2.62}.$

EXECUTE: **(a)** Since the reflection produces a net $180°$ phase difference, destructive interference of the reflected light occurs when $2t = m\lambda.$ $t = m\left(\dfrac{505\text{ nm}}{2[2.62]}\right) = (96.4\text{ nm})m.$ The minimum thickness is 96.4 nm.

(b) The next three thicknesses are for $m = 2, 3$ and 4: 192 nm, 289 nm, and 386 nm.

EVALUATE: The minimum thickness is for $t = \lambda_0/2n.$ Compare this to Problem 35.23, where the minimum thickness for destructive interference is $t = \lambda_0/4n.$

35.25. **IDENTIFY:** The light reflected from the top of the TiO_2 film interferes with the light reflected from the top of the glass surface. These waves are out of phase due to the path difference in the film and the phase differences caused by reflection.

SET UP: There is a π phase change at the TiO_2 surface but none at the glass surface, so for destructive interference the path difference must be $m\lambda$ in the film.

EXECUTE: **(a)** Calling T the thickness of the film gives $2T = m\lambda_0/n,$ which yields $T = m\lambda_0/(2n).$ Substituting the numbers gives

$T = m\text{ (520.0 nm)}/[2(2.62)] = 99.237\text{ nm}.$

T must be greater than 1036 nm, so $m = 11,$ which gives $T = 1091.6\text{ nm},$ since we want to know the minimum thickness to add.

$\Delta T = 1091.6\text{ nm} - 1036\text{ nm} = 55.6\text{ nm}.$

(b) (i) Path difference $= 2T = 2(1092\text{ nm}) = 2184\text{ nm} = 2180\text{ nm}.$

(ii) The wavelength in the film is $\lambda = \lambda_0/n = (520.0\text{ nm})/2.62 = 198.5\text{ nm}.$

Path difference $= (2180\text{ nm})/[(198.5\text{ nm})/\text{wavelength}] = 11.0$ wavelengths.

EVALUATE: Because the path difference in the film is 11.0 wavelengths, the light reflected off the top of the film will be 180° out of phase with the light that traveled through the film and was reflected off the glass due to the phase change at reflection off the top of the film.

35.26. **IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections. For destructive interference the total phase difference is an integer number of half cycles.

SET UP: The reflection at the top surface of the film produces a half-cycle phase shift. There is no phase shift at the reflection at the bottom surface.

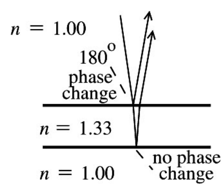
EXECUTE: **(a)** Since there is a half-cycle phase shift at just one of the interfaces, the minimum thickness for constructive interference is $t = \frac{\lambda}{4} = \frac{\lambda_0}{4n} = \frac{550\text{ nm}}{4(1.70)} = 80.9\text{ nm}.$

(b) The next smallest thickness for constructive interference is with another half wavelength thickness added. $t = \frac{3\lambda}{4} = \frac{3\lambda_0}{4n} = \frac{3(550\text{ nm})}{4(1.70)} = 243\text{ nm}.$

EVALUATE: Note that we must compare the path difference to the wavelength in the film.

35.27. **IDENTIFY:** Consider the interference between rays reflected from the two surfaces of the soap film. Strongly reflected means constructive interference. Consider phase difference due to the path difference of $2t$ and any phase difference due to phase changes upon reflection.

SET UP: Consider Figure 35.27.



There is a 180° phase change when the light is reflected from the outside surface of the bubble and no phase change when the light is reflected from the inside surface.

Figure 35.27

EXECUTE: **(a)** The reflections produce a net 180° phase difference and for there to be constructive interference the path difference $2t$ must correspond to a half-integer number of wavelengths to compensate for the $\lambda/2$ shift due to the reflections. Hence the condition for constructive interference is $2t = (m+\frac{1}{2})(\lambda_0/n),\ m = 0, 1, 2, \ldots$ Here λ_0 is the wavelength in air and (λ_0/n) is the wavelength in the bubble, where the path difference occurs.

$$\lambda_0 = \frac{2tn}{m+\frac{1}{2}} = \frac{2(290\text{ nm})(1.33)}{m+\frac{1}{2}} = \frac{771.4\text{ nm}}{m+\frac{1}{2}}$$

for $m = 0, \lambda = 1543$ nm; for $m = 1, \lambda = 514$ nm; for $m = 2, \lambda = 308$ nm;… Only 514 nm is in the visible region; the color for this wavelength is green.

(b) $\lambda_0 = \frac{2tn}{m+\frac{1}{2}} = \frac{2(340\text{ nm})(1.33)}{m+\frac{1}{2}} = \frac{904.4\text{ nm}}{m+\frac{1}{2}}$

for $m = 0, \lambda = 1809$ nm; for $m = 1, \lambda = 603$ nm; for $m = 2, \lambda = 362$ nm;… Only 603 nm is in the visible region; the color for this wavelength is orange.

EVALUATE: The dominant color of the reflected light depends on the thickness of the film. If the bubble has varying thickness at different points, these points will appear to be different colors when the light reflected from the bubble is viewed.

35.28. **IDENTIFY** and **SET UP:** Since the film reflects 575 nm strongly, we must have constructive interference at that wavelength. The light reflected from the air-benzene interface experiences a 180° phase inversion (since $n_{\text{air}} < n_{\text{benzene}}$), but the light reflected from the benzene-water interface does not experience a phase inversion (since $n_{\text{benzene}} > n_{\text{water}}$). Thus, the condition for constructive interference is $2t = m\dfrac{\lambda}{2n}$, where $m = 1,\ 3,\ 5, \ldots$ and $\dfrac{\lambda}{n}$ is the wavelength of the light in the benzene (which is where the path-difference occurs).

EXECUTE: The minimum required thickness occurs when $m = 1$, so $t = \dfrac{\lambda}{4n} = \dfrac{575\text{ nm}}{4(1.50)} = 95.8\text{ nm}.$

EVALUATE: Since the path difference occurs within the benzene, and not within the water, the exact value of the index of refraction of water is not needed (provided we know that $n_{\text{benzene}} > n_{\text{water}}$).

35.29. **IDENTIFY:** Require destructive interference between light reflected from the two points on the disc.

SET UP: Both reflections occur for waves in the plastic substrate reflecting from the reflective coating, so they both have the same phase shift upon reflection and the condition for destructive interference (cancellation) is $2t = (m + \frac{1}{2})\lambda$, where t is the depth of the pit. $\lambda = \dfrac{\lambda_0}{n}$. The minimum pit depth is for $m = 0$.

EXECUTE: $2t = \dfrac{\lambda}{2}$. $t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{790\text{ nm}}{4(1.8)} = 110\text{ nm} = 0.11\ \mu\text{m}.$

EVALUATE: The path difference occurs in the plastic substrate and we must compare the wavelength in the substrate to the path difference.

35.30. **IDENTIFY:** Apply $y = m(\lambda/2)$.

SET UP: $m = 818$. Since the fringes move in opposite directions, the two people move the mirror in opposite directions.

EXECUTE: **(a)** For Jan, the total shift was $y_1 = \dfrac{m\lambda_1}{2} = \dfrac{818(6.06\times10^{-7}\text{ m})}{2} = 2.48\times10^{-4}\text{m}.$ For Linda, the total shift was $y_2 = \dfrac{m\lambda_2}{2} = \dfrac{818(5.02\times10^{-7}\text{ m})}{2} = 2.05\times10^{-4}\text{ m}.$

(b) The net displacement of the mirror is the difference of the above values:

$\Delta y = y_1 - y_2 = 0.248\text{ mm} - 0.205\text{ mm} = 0.043\text{ mm}.$

EVALUATE: The person using the larger wavelength moves the mirror the greater distance.

35.31. **IDENTIFY** and **SET UP:** Apply $y = m(\lambda/2)$ and calculate y for $m = 1800$.

EXECUTE: $y = m(\lambda/2) = 1800(633\times10^{-9}\text{ m})/2 = 5.70\times10^{-4}\text{ m} = 0.570\text{ mm}.$

EVALUATE: A small displacement of the mirror corresponds to many wavelengths and a large number of fringes cross the line.

35.32. **IDENTIFY:** This problem involves the interference of waves.

SET UP and **EXECUTE:** **(a)** We want E. $I = \dfrac{1}{2}\epsilon_0 cE_{\text{max}}^2 = P/A.$

$$E = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(100\text{ kW/cm}^2)}{\epsilon_0 c}} = 870\text{ kV/m}.$$

(b) Estimate the phase difference δ . $\dfrac{\delta}{2\pi} = \dfrac{\Delta l}{\lambda}$. The phase difference accumulates with each round trip. There are 280 round trips, each with an up-and-back segment. So the total for Δl is $\Delta l = 2(280)(10^{-18}\text{m})$. Therefore $\delta = 2\pi\dfrac{\Delta l}{\lambda} = \dfrac{2\pi(560\times10^{-18}\text{ m})}{1064\text{ nm}} = 3.3\times10^{-9}\text{ rad}.$

(c) We want $E_{\min}$. Eq. (35.7) gives $E_p = 2E\left|\cos\frac{\phi}{2}\right|$. $\phi = \pi + \delta$, so

$$E_p = 2E\left|\cos\frac{\pi+\delta}{2}\right| = 2E\sin\frac{\delta}{2} \approx 2E\frac{\delta}{2} = E\delta = (870\text{ kV/m})(3.3\times10^{-9}\text{rad}) = 2.9\times10^{-3}\text{V/m}.$$

EVALUATE: The minimum field strength is about 3 mN/C, which is very small.

35.33. **IDENTIFY:** The two scratches are parallel slits, so the light that passes through them produces an interference pattern. However, the light is traveling through a medium (plastic) that is different from air.

SET UP: The central bright fringe is bordered by a dark fringe on each side of it. At these dark fringes, $d\ \sin\theta = \frac{1}{2}\lambda/n$, where n is the refractive index of the plastic.

EXECUTE: First use geometry to find the angles at which the two dark fringes occur. At the first dark fringe $\tan\theta = [(5.82\text{ mm})/2]/(3250\text{ mm})$, giving $\theta = \pm 0.0513°$.

For destructive interference, we have $d\ \sin\theta = \frac{1}{2}\lambda/n$ and

$n = \lambda/(2d\sin\theta) = (632.8\text{ nm})/[2(0.000225\text{ m})(\sin\ 0.0513°)] = 1.57.$

EVALUATE: The wavelength of the light in the plastic is reduced compared to what it would be in air.

35.34. **IDENTIFY:** Consider the interference between light reflected from the top and bottom surfaces of the air film between the lens and the glass plate. Introducing a liquid between the lens and the plate just changes the wavelength from λ_0 to $\frac{\lambda_0}{n}$, where n is the refractive index of the liquid.

SET UP: For maximum intensity, with a net half-cycle phase shift due to reflections, $2t = (m+\frac{1}{2})\lambda$, where λ is the wavelength in the film. $t = R - \sqrt{R^2 - r^2}$.

EXECUTE: $\frac{(2m+1)\lambda}{4} = R - \sqrt{R^2-r^2} \Rightarrow \sqrt{R^2-r^2} = R - \frac{(2m+1)\lambda}{4}$

$$\Rightarrow R^2 - r^2 = R^2 + \left[\frac{(2m+1)\lambda}{4}\right]^2 - \frac{(2m+1)\lambda R}{2} \Rightarrow r = \sqrt{\frac{(2m+1)\lambda R}{2} - \left[\frac{(2m+1)\lambda}{4}\right]^2}$$

$$\Rightarrow r \approx \sqrt{\frac{(2m+1)\lambda R}{2}},\ \text{for } R \gg \lambda.$$

$\lambda = \lambda_0/n$, where λ_0 is the wavelength in air. Therefore, if r_0 is the radius of the third bright ring when air is between the lens and plate, the radius with water between the lens and plate is

$$r = \frac{r_0}{\sqrt{n}} = \frac{0.640\text{ mm}}{\sqrt{1.33}} = 0.555\text{ mm}.$$

EVALUATE: The refractive index of the water is less than that of the glass plate, so the phase changes on reflection are the same as when air is in the space.

35.35. **IDENTIFY** and **SET UP:** Consider the interference of the rays reflected from each side of the film. At the front of the film light in air reflects off the film $(n = 1.432)$ and there is a 180° phase shift. At the back of the film light in the film $(n = 1.432)$ reflects off the glass $(n = 1.62)$ and there is a 180° phase shift. Therefore, the reflections introduce no net phase shift. The path difference is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \frac{\lambda_{\text{air}}}{n}$.

EXECUTE: **(a)** Since there is no net phase difference produced by the reflections, the condition for destructive interference is $2t = (m+\frac{1}{2})\lambda.\ t = (m+\frac{1}{2})\frac{\lambda}{2}$ and the minimum thickness is

$$t = \frac{\lambda}{4} = \frac{\lambda_{\text{air}}}{4n} = \frac{550\text{ nm}}{4(1.432)} = 96.0\text{ nm}.$$

(b) For destructive interference, $2t=(m+\tfrac{1}{2})\dfrac{\lambda_{\text{air}}}{n}$ and $\lambda_{\text{air}}=\dfrac{2tn}{m+\tfrac{1}{2}}=\dfrac{275\text{ nm}}{m+\tfrac{1}{2}}$. $m=0$: $\lambda_{\text{air}}=550$ nm.

$m=1$: $\lambda_{\text{air}}=183$ nm. All other λ_{air} values are shorter. For constructive interference, $2t=m\dfrac{\lambda_{\text{air}}}{n}$ and

$\lambda_{\text{air}}=\dfrac{2tn}{m}=\dfrac{275\text{ nm}}{m}$. For $m=1$, $\lambda_{\text{air}}=275$ nm and all other λ_{air} values are shorter.

EVALUATE: The only visible wavelength in air for which there is destructive interference is 550 nm. There are no visible wavelengths in air for which there is constructive interference.

35.36. **IDENTIFY** and **SET UP:** Consider reflection from either side of the film. **(a)** At the front of the film, light in air $(n=1.00)$ reflects off the film $(n=1.45)$ and there is a 180° phase shift. At the back of the film, light in the film $(n=1.45)$ reflects off the cornea $(n=1.38)$ and there is no phase shift. The reflections produce a net 180° phase difference so the condition for constructive interference is $2t=(m+\tfrac{1}{2})\lambda$, where $\lambda=\dfrac{\lambda_{\text{air}}}{n}$. $t=(m+\tfrac{1}{2})\dfrac{\lambda_{\text{air}}}{2n}$.

EXECUTE: The minimum thickness is for $m=0$, and is given by $t=\dfrac{\lambda_{\text{air}}}{4n}=\dfrac{600\text{ nm}}{4(1.45)}=103$ nm

(103.4 nm with less rounding).

(b) $\lambda_{\text{air}}=\dfrac{2nt}{m+\tfrac{1}{2}}=\dfrac{2(1.45)(103.4\text{ nm})}{m+\tfrac{1}{2}}=\dfrac{300\text{ nm}}{m+\tfrac{1}{2}}$. For $m=0$, $\lambda_{\text{air}}=600$ nm. For $m=1$, $\lambda_{\text{air}}=200$ nm and all other values are smaller. No other visible wavelengths are reinforced. The condition for destructive interference is $2t=m\dfrac{\lambda_{\text{air}}}{n}$. $\lambda=\dfrac{2tn}{m}=\dfrac{300\text{ nm}}{m}$. For $m=1$, $\lambda_{\text{air}}=300$ nm and all other values are shorter. There are no visible wavelengths for which there is destructive interference.

(c) Now both rays have a 180° phase change on reflection and the reflections don't introduce any net phase shift. The expression for constructive interference in parts (a) and (b) now gives destructive interference and the expression in (a) and (b) for destructive interference now gives constructive interference. The only visible wavelength for which there will be destructive interference is 600 nm and there are no visible wavelengths for which there will be constructive interference.

EVALUATE: Changing the net phase shift due to the reflections can convert the interference for a particular thickness from constructive to destructive, and vice versa.

35.37. **IDENTIFY:** The insertion of the metal foil produces a wedge of air, which is an air film of varying thickness. This film causes a path difference between light reflected off the top and bottom of this film.

SET UP: The two sheets of glass are sketched in Figure 35.37. The thickness of the air wedge at a distance x from the line of contact is $t=x\tan\theta$. Consider rays 1 and 2 that are reflected from the top and bottom surfaces, respectively, of the air film. Ray 1 has no phase change when it reflects and ray 2 has a 180° phase change when it reflects, so the reflections introduce a net 180° phase difference. The path difference is $2t$ and the wavelength in the film is $\lambda=\lambda_{\text{air}}$.

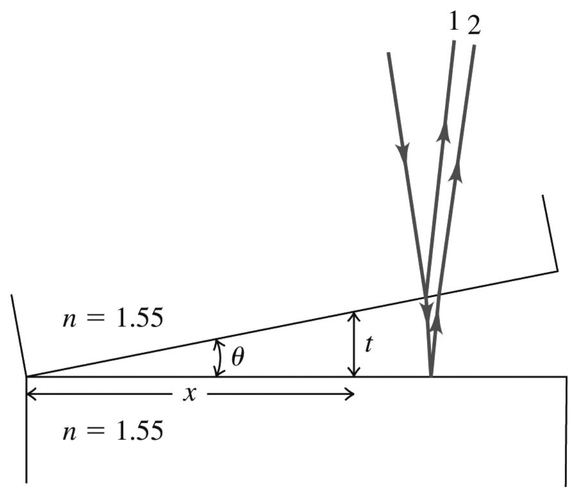


Figure 35.37

EXECUTE: **(a)** Since there is a 180° phase difference from the reflections, the condition for constructive interference is $2t = (m + \frac{1}{2})\lambda$. The positions of first enhancement correspond to $m = 0$ and $2t = \dfrac{\lambda}{2}$. $x\tan\theta = \dfrac{\lambda}{4}$. θ is a constant, so $\dfrac{x_1}{\lambda_1} = \dfrac{x_2}{\lambda_2}$. $x_1 = 1.15$ mm, $\lambda_1 = 400.0$ nm. $x_2 = x_1\left(\dfrac{\lambda_2}{\lambda_1}\right)$. For $\lambda_2 = 550$ nm (green), $x_2 = (1.15\text{ mm})\left(\dfrac{550\text{ nm}}{400\text{ nm}}\right) = 1.58$ mm. For $\lambda_2 = 600$ nm (orange), $x_2 = (1.15\text{ mm})\left(\dfrac{600\text{ nm}}{400\text{ nm}}\right) = 1.72$ mm.

(b) The positions of next enhancement correspond to $m = 1$ and $2t = \dfrac{3\lambda}{2}$. $x\tan\theta = \dfrac{3\lambda}{4}$. The values of x are 3 times what they are in part (a). Violet: 3.45 mm; green: 4.74 mm; orange: 5.16 mm.

(c) $\tan\theta = \dfrac{\lambda}{4x} = \dfrac{400.0\times10^{-9}\text{ m}}{4(1.15\times10^{-3}\text{ m})} = 8.70\times10^{-5}$. $\tan\theta = \dfrac{t_{\text{foil}}}{11.0\text{ cm}}$, so $t_{\text{foil}} = 9.57\times10^{-4}\text{ cm} = 9.57\ \mu\text{m}$.

EVALUATE: The thickness of the foil must be very small to cause these observable interference effects. If it is too thick, the film is no longer a "thin film."

35.38. **IDENTIFY:** We are dealing with two-source interference.

SET UP: For constructive interference $d\sin\theta_m = m\lambda$. $y_m = R\tan\theta_m$. For small angles $y_m = R\dfrac{m\lambda}{d}$.

EXECUTE: **(a)** We want the distance d between slits. The small-angle approximation is justified in this case. Adjacent fringes differ by $\Delta m = 1$, so $\Delta y_m = R\dfrac{\Delta m\lambda}{d} = \dfrac{R\lambda}{d}$. Solving for d gives $d = \dfrac{R\lambda}{\Delta y} =$ (1 m)(650 nm)/(1 cm) = 65 μm.

(b) No. It seems much too small to do at home.

(c) We want the distance between the points of enhanced sound (constructive interference). For sound of frequency 1.0 kHz, $\lambda = v/f$ = (344 m/s)(1.0 kHz) = 0.344 m, d = 40 cm = 0.40 m. We cannot use the small-angle approximation because d is only slightly larger than λ. Using $d\sin\theta_m = m\lambda$ gives $(0.40\text{ m})\sin\theta_1 = 0.344$ m, so $\theta_1 = 59°$. $y_1 = R\tan\theta_1 = (2\text{ m})\tan 59° = 3.3$ m. There is no θ_2 so the distance between the desired points in 3.3 m.

(d) We want d and f so that Δy = 1.75 m. Thus y_1 = 1.75 m, so $\tan\theta_1 = \frac{y_1}{R} = \frac{1.75\text{ m}}{2\text{ m}}$ which gives $\theta_1 = 41°$. Using $d\sin\theta_m = m\lambda$ gives $d\sin 41° = \lambda$. A frequency of 1.0 kHz is easily audible, so we use that which makes λ = 0.344 m. Thus d sin 41° = 0.344 m, so d = 0.52 m = 52 cm.

EVALUATE: **(e)** The 52-cm spacing in part (d) would be easily achieved, and 1.0 kHz is clearly within human hearing.

35.39. **IDENTIFY:** The liquid alters the wavelength of the light and that affects the locations of the interference minima.

SET UP: The interference minima are located by $d\sin\theta = (m+\frac{1}{2})\lambda$. For a liquid with refractive index n, $\lambda_{\text{liq}} = \frac{\lambda_{\text{air}}}{n}$.

EXECUTE: $\frac{\sin\theta}{\lambda} = \frac{(m+\frac{1}{2})}{d} =$ constant, so $\frac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \frac{\sin\theta_{\text{liq}}}{\lambda_{\text{liq}}}$. $\frac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \frac{\sin\theta_{\text{liq}}}{\lambda_{\text{air}}/n}$ and

$$n = \frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{liq}}} = \frac{\sin 35.20°}{\sin 19.46°} = 1.730.$$

EVALUATE: In the liquid the wavelength is shorter and $\sin\theta = (m+\frac{1}{2})\frac{\lambda}{d}$ gives a smaller θ than in air, for the same m.

35.40. **IDENTIFY:** As the brass is heated, thermal expansion will cause the two slits to move farther apart.

SET UP: For destructive interference, $d\ \sin\theta = \lambda/2$. The change in separation due to thermal expansion is $dw = \alpha w_0\, dT$, where w is the distance between the slits.

EXECUTE: The first dark fringe is at d sin $\theta = \lambda/2 \Rightarrow \sin\theta = \lambda/2d$.

Call $d \equiv w$ for these calculations to avoid confusion with the differential. $\sin\theta = \lambda/2w$.

Taking differentials gives $d(\sin\theta) = d(\lambda/2w)$ and $\cos\theta\, d\theta = -\lambda/2\ dw/w^2$. For thermal expansion, $dw = \alpha w_0 dT$, which gives $\cos\theta\, d\theta = -\frac{\lambda}{2}\frac{\alpha w_0 dT}{w_0^2} = -\frac{\lambda\alpha dT}{2w_0}$. Solving for $d\theta$ gives $d\theta = -\frac{\lambda\alpha dT}{2w_0\cos\theta_0}$.

Get λ: $w_0\sin\theta_0 = \lambda/2 \rightarrow \lambda = 2w_0\sin\theta_0$. Substituting this quantity into the equation for $d\theta$ gives

$$d\theta = -\frac{2w_0\sin\theta_0\,\alpha dT}{2w_0\cos\theta_0} = -\tan\theta_0\,\alpha dT.$$

$$d\theta = -\tan(26.6°)(2.0\times10^{-5}\text{ K}^{-1})(115\text{ K}) = -0.001152\text{ rad} = -0.066°.$$

The minus sign tells us that the dark fringes move closer together.

EVALUATE: We can also see that the dark fringes move closer together because $\sin\theta$ is proportional to $1/d$, so as d increases due to expansion, θ decreases.

35.41. **IDENTIFY:** For destructive interference, $d = r_2 - r_1 = (m+\frac{1}{2})\lambda$.

SET UP: $r_2 - r_1 = \sqrt{(200\text{ m})^2 + x^2} - x$.

EXECUTE: $(200\text{ m})^2 + x^2 = x^2 + \left[(m+\frac{1}{2})\lambda\right]^2 + 2x(m+\frac{1}{2})\lambda$.

$x = \frac{20{,}000\text{ m}^2}{(m+\frac{1}{2})\lambda} - \frac{1}{2}(m+\frac{1}{2})\lambda$. The wavelength is calculated by $\lambda = \frac{c}{f} = \frac{3.00\times10^8\text{ m/s}}{5.80\times10^6\text{ Hz}} = 51.7\text{ m}$.

$m = 0: x = 761\text{ m};\ m = 1: x = 219\text{ m};\ m = 2: x = 90.1\text{ m};\ m = 3; x = 20.0\text{ m}.$

EVALUATE: For $m = 3$, $d = 3.5\lambda = 181$ m. The maximum possible path difference is the separation of 200 m between the sources.

35.42. **IDENTIFY:** For destructive interference the net phase difference must be $180°$, which is one-half a period, or $\lambda/2$. Part of this phase difference is due to the fact that the speakers are 1/4 of a period out of phase, and the rest is due to the path difference between the sound from the two speakers.
SET UP: The phase of A is 90° or, $\lambda/4$, ahead of B. At points above the centerline, points are closer to A than to B and the signal from A gains phase relative to B because of the path difference. Destructive interference will occur when $d\sin\theta = (m+\frac{1}{4})\lambda$, $m = 0, 1, 2, \ldots$. At points at an angle θ below the centerline, the signal from B gains phase relative to A because of the phase difference. Destructive interference will occur when $d\sin\theta = (m+\frac{3}{4})\lambda$, $m = 0, 1, 2, \ldots$. $\lambda = \dfrac{v}{f}$.

EXECUTE: $\lambda = \dfrac{340\text{ m/s}}{444\text{ Hz}} = 0.766\text{ m}.$

Points above the centerline: $\sin\theta = (m+\frac{1}{4})\dfrac{\lambda}{d} = (m+\frac{1}{4})\left(\dfrac{0.766\text{ m}}{3.50\text{ m}}\right) = 0.219(m+\frac{1}{4})$. $m=0$: $\theta = 3.14°$;

$m=1$: $\theta = 15.9°$; $m=2$: $\theta = 29.5°$; $m=3$: $\theta = 45.4°$; $m=4$: $\theta = 68.6°$.

Points below the centerline: $\sin\theta = (m+\frac{3}{4})\dfrac{\lambda}{d} = (m+\frac{3}{4})\left(\dfrac{0.766\text{ m}}{3.50\text{ m}}\right) = 0.219(m+\frac{3}{4})$. $m=0$: $\theta = 9.45°$;

$m=1$: $\theta = 22.5°$; $m=2$: $\theta = 37.0°$; $m=3$: $\theta = 55.2°$.

EVALUATE: It is *not* always true that the path difference for destructive interference must be $(m+\frac{1}{2})\lambda$, but it *is* always true that the phase difference must be 180° (or odd multiples of 180°).

35.43. **IDENTIFY** and **SET UP:** Consider interference between rays reflected from the upper and lower surfaces of the film to relate the thickness of the film to the wavelengths for which there is destructive interference. The thermal expansion of the film changes the thickness of the film when the temperature changes.
EXECUTE: For this film on this glass, there is a net $\lambda/2$ phase change due to reflection and the condition for destructive interference is $2t = m(\lambda/n)$, where $n = 1.750$.

Smallest nonzero thickness is given by $t = \lambda/2n$.
At $20.0°\text{C}$, $t_0 = (582.4\text{ nm})/[(2)(1.750)] = 166.4\text{ nm}$.
At $170°\text{C}$, $t = (588.5\text{ nm})/[(2)(1.750)] = 168.1\text{ nm}$.
$t = t_0(1+\alpha\Delta T)$ so
$\alpha = (t-t_0)/(t_0\Delta T) = (1.7\text{ nm})/[(166.4\text{ nm})(150\text{C°})] = 6.8\times10^{-5}(\text{C°})^{-1}$.

EVALUATE: When the film is heated its thickness increases, and it takes a larger wavelength in the film to equal $2t$. The value we calculated for α is the same order of magnitude as those given in Table 17.1.

35.44. **IDENTIFY:** The maximum intensity occurs at all the points of constructive interference. At these points, the path difference between waves from the two transmitters is an integral number of wavelengths.
SET UP: For constructive interference, $\sin\theta = m\lambda/d$.
EXECUTE: **(a)** First find the wavelength of the UHF waves:

$\lambda = c/f = (3.00\times10^8\text{ m/s})/(1575.42\text{ MHz}) = 0.1904\text{ m}$.
For maximum intensity $(\pi d\ \sin\theta)/\lambda = m\pi$, so

$\sin\theta = m\lambda/d = m[(0.1904\text{ m})/(5.18\text{ m})] = 0.03676m$.
The maximum possible m would be for $\theta = 90°$, or $\sin\theta = 1$, so

$m_{\max} = d/\lambda = (5.18\text{ m})/(0.1904\text{ m}) = 27.2$,

which must be ±27 since m is an integer. The total number of maxima is 27 on either side of the central fringe, plus the central fringe, for a total of $27+27+1 = 55$ bright fringes.
(b) Using sin $\theta = m\lambda/d$, where $m = 0, \pm1, \pm2, \text{and} \pm3$, we have

$\sin\theta = m\lambda/d = m[(0.1904\ \text{m})/(5.18\ \text{m})] = 0.03676m.$

$m = 0:\ \sin\theta = 0$, which gives $\theta = 0°$.

$m = \pm1: \sin\theta = \pm(0.03676)(1)$, which gives $\theta = \pm2.11°$.

$m = \pm2: \sin\theta = \pm(0.03676)(2)$, which gives $\theta = \pm4.22°$.

$m = \pm3: \sin\theta = \pm(0.03676)(3)$, which gives $\theta = \pm6.33°$.

(c) $I = I_0\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right) = (2.00\ \text{W/m}^2)\cos^2\left[\dfrac{\pi(5.18\ \text{m})\sin(4.65°)}{0.1904\ \text{m}}\right] = 1.28\ \text{W/m}^2.$

EVALUATE: Notice that $\sin\theta$ increases in integer steps, but θ only increases in integer steps for small θ.

35.45. **IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections.
SET UP: There is just one half-cycle phase change upon reflection, so for constructive interference $2t = (m_1 + \frac{1}{2})\lambda_1 = (m_2 + \frac{1}{2})\lambda_2$, where these wavelengths are in the glass. The two different wavelengths differ by just one m-value, $m_2 = m_1 - 1$.

EXECUTE: $(m_1 + \frac{1}{2})\lambda_1 = (m_1 - \frac{1}{2})\lambda_2 \Rightarrow m_1(\lambda_2 - \lambda_1) = \dfrac{\lambda_1 + \lambda_2}{2} \Rightarrow m_1 = \dfrac{\lambda_1 + \lambda_2}{2(\lambda_2 - \lambda_1)}.$

$m_1 = \dfrac{477.0\ \text{nm} + 540.6\ \text{nm}}{2(540.6\ \text{nm} - 477.0\ \text{nm})} = 8.$ $2t = \left(8 + \dfrac{1}{2}\right)\dfrac{\lambda_{01}}{n} \Rightarrow t = \dfrac{17(477.0\ \text{nm})}{4(1.52)} = 1334\ \text{nm}.$

EVALUATE: Now that we have t we can calculate all the other wavelengths for which there is constructive interference.

35.46. **IDENTIFY:** Light reflected from the top of the coating interferes with light reflected from the bottom of the coating, so we have thin-film interference.
SET UP: For maximum transmission in (a) we want minimum reflection. For minimum transmission in (b) we want maximum reflection. A half-cycle phase shift occurs at the air-coating surface but not at the coating-plastic surface. Thus for minimum reflection we must have $2t = m\dfrac{\lambda_0}{n}$, and for maximum reflection we must have $2t = (m + \frac{1}{2})\dfrac{\lambda_0}{n}$, where t is the thickness of the coating and n is the index of refraction of the coating. We want the thinnest coating possible, so we use m = 1 in (a) and m = 0 in (b).

EXECUTE: **(a)** $2t = m\dfrac{\lambda_0}{n}$ gives $t = m\dfrac{\lambda_0}{2n}$ = (1)(510 nm)/[2(1.65)] = 155 nm.

(b) $2t = (m + \frac{1}{2})\dfrac{\lambda_0}{n}$ gives $t = (m + \frac{1}{2})\dfrac{\lambda_0}{2n}$ = (1/2)(510 nm)/[2(1.65)] = 77.3 nm.

EVALUATE: The thickness in (b) is $\frac{1}{2}$ the thickness in (a) because the path differences differ by a factor of one-half of a wavelength.

35.47. **IDENTIFY** and **SET UP:** At the $m = 3$ bright fringe for the red light there must be destructive interference at this same θ for the other wavelength.
EXECUTE: For constructive interference: $d\sin\theta = m\lambda_1 \Rightarrow d\sin\theta = 3(700\ \text{nm}) = 2100\ \text{nm}$. For destructive interference: $d\sin\theta = (m + \frac{1}{2})\lambda_2 \Rightarrow \lambda_2 = \dfrac{d\sin\theta}{m + \frac{1}{2}} = \dfrac{2100\ \text{nm}}{m + \frac{1}{2}}$. So the possible wavelengths are $\lambda_2 = 600$ nm, for $m = 3$, and $\lambda_2 = 467$ nm, for $m = 4$.

EVALUATE: Both d and θ drop out of the calculation since their combination is just the path difference, which is the same for both types of light.

35.48. **IDENTIFY:** Require constructive interference for the reflection from the top and bottom surfaces of each cytoplasm layer and each guanine layer.

SET UP: At the water (or cytoplasm) to guanine interface, there is a half-cycle phase shift for the reflected light, but there is not one at the guanine to cytoplasm interface. Therefore there will always be one half-cycle phase difference between two neighboring reflected beams, just due to the reflections.

EXECUTE: For the guanine layers:

$$2t_{\rm g} = (m+\tfrac{1}{2})\frac{\lambda}{n_{\rm g}} \Rightarrow \lambda = \frac{2t_{\rm g}n_{\rm g}}{(m+\frac{1}{2})} = \frac{2(74\text{ nm})(1.80)}{(m+\frac{1}{2})} = \frac{266\text{ nm}}{(m+\frac{1}{2})} \Rightarrow \lambda = 533\text{ nm } (m=0).$$

For the cytoplasm layers:

$$2t_{\rm c} = (m+\tfrac{1}{2})\frac{\lambda}{n_{\rm c}} \Rightarrow \lambda = \frac{2t_{\rm c}n_{\rm c}}{(m+\frac{1}{2})} = \frac{2(100\text{ nm})(1.333)}{(m+\frac{1}{2})} = \frac{267\text{ nm}}{(m+\frac{1}{2})} \Rightarrow \lambda = 533\text{ nm } (m=0)\ .$$

(b) By having many layers the reflection is strengthened, because at each interface some more of the transmitted light gets reflected back, increasing the total percentage reflected.

(c) At different angles, the path length in the layers changes (always to a larger value than the normal incidence case). If the path length changes, then so do the wavelengths that will interfere constructively upon reflection.

EVALUATE: The thickness of the guanine and cytoplasm layers are inversely proportional to their refractive indices $\left(\dfrac{100}{74} = \dfrac{1.80}{1.333}\right)$, so both kinds of layers produce constructive interference for the same wavelength in air.

35.49. **IDENTIFY:** Dark fringes occur because the path difference is one-half of a wavelength.

SET UP: At the first dark fringe, $d\sin\theta = \lambda/2$. The intensity at any angle θ is given by $I = I_0\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$.

EXECUTE: **(a)** At the first dark fringe, we have $d\sin\theta = \lambda/2$. $d/\lambda = 1/(2\sin 19.0°) = 1.54$.

(b) $I = I_0\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right) = \dfrac{I_0}{10} \Rightarrow \cos\left(\dfrac{\pi d\sin\theta}{\lambda}\right) = \dfrac{1}{\sqrt{10}}$. $\dfrac{\pi d\sin\theta}{\lambda} = \arccos\left(\dfrac{1}{\sqrt{10}}\right) = 71.57° = 1.249$ rad.

Using the result from part (a), that $d/\lambda = 1.54$, we have

$\pi(1.54)\sin\theta = 1.249$. $\sin\theta = 0.2589$, so $\theta = \pm 15.0°$.

EVALUATE: Since the first dark fringes occur at $\pm 19.0°$, it is reasonable that at 15° the intensity is reduced to only $1/10$ of its maximum central value.

35.50. **IDENTIFY:** Light from the two slits interferes on the screen. We can use the small-angle approximation because we are only looking at closely spaced bright fringes near the center of the pattern.

SET UP: For small angles, the bright fringes are at positions on the screen given by $y = R\dfrac{m\lambda}{d}$. R, λ, and *d* are all fixed, and the bright fringes are adjacent ones.

EXECUTE: **(a)** The fringe spacidng is $\Delta y = \dfrac{R\lambda\Delta m}{d}$ and Δm = 1 because the bright fringes are adjacent. This equation can be written as $\Delta y = R\lambda\cdot\dfrac{1}{d}$. From this result, we see that a graph of Δy versus $1/d$ should be a straight line having a slope equal to $R\lambda$.

(b) We use points (9.20 mm^{-1}, 5.0 mm) and (2.00 mm^{-1}, 1.0 mm) to calculate the slope, giving slope = $\dfrac{(5.0-1.0)\text{ mm}}{(9.20-2.00)\text{ mm}^{-1}} = 0.5556\text{ mm}^2$. Since λR = slope, we have

λ = (slope)/R = (0.5556 mm^2)/(900 mm) = 6.2×10^{-4} mm = 6.2×10^{-7} m = 620 nm.

(Answers may vary a bit depending on accuracy in reading the graph.)

EVALUATE: This wavelength is well within the range of visible light. According to Figure 32.4 in the textbook, this light should be yellow-orange.

35.51. **IDENTIFY:** The wave from A travels a longer distance than the wave from B to reach point P, so the two waves will be out of phase when they reach P. For constructive interference, the path difference should be a whole-number multiple of the wavelength.
SET UP: To reach point P, the wave from A travels 240.0 m and the wave from B travels a distance x. The path difference for these two waves is 240.0 m – x. For any wave, $\lambda f = v$. Intensity maxima occur at x = 210.0 m, 216.0 m, and 222.0 m, and there are others.
EXECUTE: **(a)** The distance between adjacent intensity maxima is λ. The three given values of x are 6.0 m apart, so the wavelength must be 6.0 m. The frequency is $f = c/\lambda = c/(6.0\text{ m}) = 5.0\times10^7$ Hz = 50 MHz.
(b) Destructive interference occurs when 240.0 m – $x = (m+\frac{1}{2})\lambda$, which gives

x = 240.0 m – $(m+\frac{1}{2})$ (6.0 m).

The largest x occurs when m = 0, so x = 240.0 m – 3.0 m = 237.0 m.
EVALUATE: According to Figure 32.4 in the textbook, a wave having a wavelength of 6.0 m is in the radiowave region of the electromagnetic spectrum, which is consistent with the fact that you are using short-wave radio antennas.

35.52. **IDENTIFY:** Assume that the glass is horizontal. The light that travels through the glass and reflects off of its lower surface interferes with incident the light that reflects off the upper surface of the glass. The glass behaves like a thin film. A half-cycle phase change occurs at the upper surface but not at the lower surface because $n_{air} < n_{glass}$.
SET UP: For constructive interference with this glass, $2t = (m+\frac{1}{2})\left(\lambda_0/n\right)$.

EXECUTE: **(a)** For the 386-nm light: $2t = (m+\frac{1}{2})$ [(386 nm)/n].

For the 496-nm light: $2t = (m+1+\frac{1}{2})$ [(496 nm)/n] = $(m+\frac{3}{2})$ [(386 nm)/n].

Taking the ratio of these two equations gives

$$\frac{m+\frac{1}{2}}{m+\frac{3}{2}}=\frac{386\text{ nm}}{497\text{ nm}}=1.285.$$

Solving for m gives m = 3. Now find t using the equation for the shorter-wavelength light.
$2t = (3+\frac{1}{2})$ [(386 nm)/(1.40)] $\rightarrow$ t = 620 nm.

(b) Solving $2t = (m+\frac{1}{2})(\lambda_0/n)$ for λ_0 gives $\lambda_0 = 2nt/(m+\frac{1}{2})$. The largest wavelength will be for m = 0, so $\lambda_0 = 2(1.40)(620\text{ nm})/(\frac{1}{2}) = 3470$ nm.

EVALUATE: Visible light is between approximately 400 nm and 700 nm, so the light in (b) is definitely not visible. According to Figure 32.4 in the textbook, it would be in the infrared region of the electromagnetic spectrum.

35.53. **IDENTIFY:** This problem involves the interference of sound waves.
SET UP and **EXECUTE:** Pressure nodes are displacement antinodes, so we have constructive interference at the points in question.
(a) We want the minimum radius r. For the minimum r, a wave traveling straight up and back will interfere with sound just leaving the source. So the path difference is $2r$ and we must have $2r = \lambda$, so $r = \lambda/2$.
(b) We want the number of pressure nodes. At 20°C v = 344 m/s, so $\lambda = v/f$ = 0.1376 m. The path difference is $2\sqrt{r^2+(d/2)^2}-d$, so for constructive interference $2\sqrt{r^2+(d/2)^2}-d=m\lambda$.

m = 0: $2\sqrt{r^2+(d/2)^2}=d \rightarrow r = 0$, which is not possible.

m = 1: $2\sqrt{r^2+(d/2)^2}-d=\lambda$. Square and solve for d. $d=\dfrac{4r^2-\lambda^2}{2\lambda}=84.1$ cm.

$m = 2$: $2\sqrt{r^2 + (d/2)^2} - d = 2\lambda$. $d = \dfrac{4(r^2 - \lambda^2)}{4\lambda} = 31.7$ cm.

$m = 3$: $2\sqrt{r^2 + (d/2)^2} - d = 3\lambda$. $d = \dfrac{4r^2 - 9\lambda^2}{6\lambda} = 9.64$ cm.

For $m \geq 4$ we get negative d. So there are 3 nodes on each side of the source, for a total of 6.

(c) $d = \pm 9.64$ cm, ± 31.7 cm, ± 84.1 cm.

(d) With helium, $v = 999$ m/s, so $\lambda = 0.3996$ m $= 40.0$ cm. Use the results from part (c).

$m = 1$: $d = \dfrac{4r^2 - \lambda^2}{2\lambda} = 11.3$ cm.

$m = 2$: $d = \dfrac{4(r^2 - \lambda^2)}{4\lambda}$ is negative, so not possible.

There are two nodes, at $d = \pm 11.3$ cm from the center.

EVALUATE: From the calculations for $m = 1$, if $\lambda > 4r$ there are no nodes.

35.54. **IDENTIFY:** This problem is about the interference due to three slits.

SET UP: $E_p = E\cos\omega t + E\cos(\omega t + \phi) + E\cos(\omega t - \phi)$.

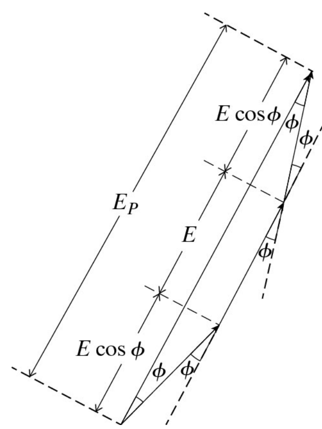


Figure 35.54

EXECUTE: **(a)** We want the amplitude E_p. Using the phasor diagram in Fig. 35.54, we see that $E_p = \cos\phi + E + E\cos\phi = E(1 + 2\cos\phi)$.

(b) We want I at P. $I = \frac{1}{2}\epsilon_0 cE_p^2 = \frac{1}{2}\left[E(1 + 2\cos\phi)\right]^2 \epsilon_0 c = \frac{1}{2}\epsilon_0 cE^2(1 + 2\cos\phi)^2$. The maximum intensity occurs when $\phi = 0$, which gives $I_0 = 9\left(\frac{1}{2}\epsilon_0 cE^2\right)$. Thus $\frac{1}{2}\epsilon_0 cE^2 = \frac{I_0}{9}$. Using this result in our equation for I gives $I = \frac{1}{9}I_0(1 + 2\cos\phi)^2$.

(c) Since $I \propto (1 + 2\cos\phi)^2$, the lesser maxima occur when $\cos\phi = -1$, for $\phi = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$.

(d) We want I. $I = \frac{1}{9}I_0(1 - 2)^2 = \frac{I_0}{9}$.

(e) We want the dark fringes. Use the result from (b). $I = \frac{1}{9}I_0(1 + 2\cos\phi)^2$. For the minima, $I = 0$, so $\cos\phi = -1/2$, so $\phi = \pm 120°$.

(f) From (c), lesser maxima occur when $\phi = \pm\pi, \pm 3\pi, \ldots$ The nearest to the center is when $\phi = \pm\pi$. $\phi = \pi = \dfrac{2\pi d \sin\theta}{\lambda}$. Using d = 0.200 mm and λ = 650 nm, this gives $\sin\theta = 0.001625$. For small angles, $\sin\theta \approx \tan\theta \approx \theta$. The distance on the screen is $y = R\tan\theta \approx R\theta$ = (1.00 m)(0.001625) = 1.63 mm.

(g) The first absolute maximum away from the center occurs for $\phi = 2\pi$. Using $\phi = 2\pi = \dfrac{2\pi d \sin\theta}{\lambda}$ gives $\sin\theta = 0.00325$. As above $y = R\tan\theta \approx R\theta$, which gives y = 3.25 mm.

EVALUATE: The angles are small near the center, so the first lesser maximum is midway between the central maximum and the first absolute maximum.

35.55. IDENTIFY: There are two effects to be considered: first, the expansion of the rod, and second, the change in the rod's refractive index.

SET UP: $\lambda = \dfrac{\lambda_0}{n}$ and $\Delta n = n_0(2.50\times10^{-5}\ (\text{C}°)^{-1})\Delta T$. $\Delta L = L_0(5.00\times10^{-6}\ (\text{C}°)^{-1})\Delta T$.

EXECUTE: The extra length of rod replaces a little of the air so that the change in the number of wavelengths due to this is given by: $\Delta N_1 = \dfrac{2n_{\text{glass}}\Delta L}{\lambda_0} - \dfrac{2n_{\text{air}}\Delta L}{\lambda_0} = \dfrac{2(n_{\text{glass}}-1)L_0\alpha\Delta T}{\lambda_0}$ and

$$\Delta N_1 = \frac{2(1.48-1)(0.030\text{ m})(5.00\times10^{-6}/\text{C}°)(5.00\text{ C}°)}{5.89\times10^{-7}\text{ m}} = 1.22.$$

The change in the number of wavelengths due to the change in refractive index of the rod is:

$$\Delta N_2 = \frac{2\Delta n_{\text{glass}}L_0}{\lambda_0} = \frac{2(2.50\times10^{-5}/\text{C}°)(5.00\text{ C}°/\text{min})(1.00\text{ min})(0.0300\text{ m})}{5.89\times10^{-7}\text{ m}} = 12.73.$$

So, the total change in the number of wavelengths as the rod expands is
$\Delta N = 12.73 + 1.22 = 14.0$ fringes/minute.

EVALUATE: Both effects increase the number of wavelengths along the length of the rod. Both ΔL and Δn_{glass} are very small and the two effects can be considered separately.

35.56. IDENTIFY: Apply Snell's law to the refraction at the two surfaces of the prism. S_1 and S_2 serve as coherent sources so the fringe spacing is $\Delta y = \dfrac{R\lambda}{d}$, where d is the distance between S_1 and S_2.

SET UP: For small angles, $\sin\theta \approx \theta$, with θ expressed in radians.

EXECUTE: **(a)** Since we can approximate the angles of incidence on the prism as being small, Snell's law tells us that an incident angle of θ on the flat side of the prism enters the prism at an angle of θ/n, where n is the index of refraction of the prism. Similarly on leaving the prism, the in-going angle is $\theta/n - A$ from the normal, and the outgoing angle, relative to the prism, is $n(\theta/n - A)$. So the beam leaving the prism is at an angle of $\theta' = n(\theta/n - A) + A$ from the optical axis. So $\theta - \theta' = (n-1)A$. At the plane of the source S_0, we can calculate the height of one image above the source:

$$\frac{d}{2} = \tan(\theta-\theta')a \approx (\theta-\theta')a = (n-1)Aa \Rightarrow d = 2aA(n-1).$$

(b) To find the spacing of fringes on a screen, we use

$$\Delta y = \frac{R\lambda}{d} = \frac{R\lambda}{2aA(n-1)} = \frac{(2.00\text{ m}+0.200\text{ m})\,(5.00\times10^{-7}\text{ m})}{2(0.200\text{ m})\,(3.50\times10^{-3}\text{ rad})\,(1.50-1.00)} = 1.57\times10^{-3}\text{ m}.$$

EVALUATE: The fringe spacing is proportional to the wavelength of the light. The biprism serves as an alternative to two closely spaced narrow slits.

35.57. **IDENTIFY and SET UP:** Interference occurs when two or more waves combine.
EXECUTE: All of the students now hear the tone, so no interference is occurring. Therefore only one speaker must be on, so the professor must have disconnected one of them. This makes choice (d) correct.
EVALUATE: Turning off one of the speakers would decrease the loudness of the sound, as was observed.

35.58. **IDENTIFY and SET UP:** Constructive interference occurs when a wave crest meets another crest, and descructive interference occurs when a crest meets a trough.
EXECUTE: The students who originally heard a loud tone were at a point of constructive interference, so a crest from one speaker met a crest from the other speaker, and those who originally heard nothing were at points where a crest met a trough. Now the students who originally heard a loud tone hear nothing, so at their point a crest meets a trough. The students who originally heard nothing are not at a point where a crest meets a crest. Since the speakers (and students) have not been moved, their phase relationship has been changed, which is choice (d).
EVALUATE: Since a point of constructive interference was turned into a point of destructive interference by the phase change, the phase change must have been π or 180°, equivalent to one-half a wavelength.

35.59. **IDENTIFY:** Moving one of the speakers increases the distance that its sound must travel to reach the listener. This changes the phase difference in the sound from the two speakers as it reaches the listeners.
SET UP: The movement of 0.34 m turned points of constructive interference into points of destructive interference, so that distance must be one-half of a wavelength. We use $v = f\lambda$ to find the frequency.
EXECUTE: $\lambda/2 = 0.34$ m, so $\lambda = 0.68$ m. $v = f\lambda$ gives $f = v/\lambda$ = (340 m/s)/(0.68 m) = 500 Hz, which is choice (c).
EVALUATE: If the professor moves the speaker an additional 0.34 m, the students will hear what they originally heard since that distance is a full wavelength.

35.60. **IDENTIFY and SET UP:** Since $v = f\lambda$, reducing the frequency half increases the wavelength by a factor of 2. For constructive interference, the path difference is $m\lambda$, where m = 0, 1, 2, 3, …, and for destructive interference the path difference is $(m+\frac{1}{2})\lambda$, where m = 0, 1, 2, 3, ….
EXECUTE: The new wavelength λ is twice as long as the original wavelength λ_0. Students who heard a loud tone before were at locations for which the path difference was λ, 2λ, 3λ, 4λ, 5λ, …. But since the new wavelength is $\lambda = 2\lambda_0$, students who were at a path difference of λ_0 are now at a point where the path difference is $\lambda/2$, and those where it was $3\lambda_0$ are now where it is $3\lambda/2$, etc. So all those students for whom the path difference was λ_0, $3\lambda_0$, $5\lambda_0$, $7\lambda_0$, … will hear nothing. For the students who are at points where the path difference was $2\lambda_0$, $4\lambda_0$, $6\lambda_0$, …, the path difference is now λ, 2λ, 3λ, …, so they will still hear a loud tone. Therefore choice (c) is correct.
EVALUATE: Students at various points in between those discussed here hear sound, but not of maximum loudness.

36

DIFFRACTION

VP36.3.1. **IDENTIFY:** This problem is about single-slit diffraction.

SET UP: For small angles, dark spots occur at $y_m = R\dfrac{m\lambda}{a}$. The small-angle approximation is valid.

EXECUTE: **(a)** We want R. Solve $y_m = R\dfrac{m\lambda}{a}$. $R = \dfrac{a\lambda}{my} = \dfrac{(0.250\text{ mm})(14.0\text{ mm})}{2(645\text{ mn})} = 2.71\text{ m}$.

(b) We want the distance 2y. Use $y_m = R\dfrac{m\lambda}{a}$ and divide. $\dfrac{y_{525}}{y_{645}} = \dfrac{\dfrac{Rm(525\text{ nm})}{a}}{\dfrac{Rm(645\text{ nm})}{a}} = 0.81395$. The distance is $2y_{645}$ = (0.81395)(28.0 mm) = 22.8 m.

EVALUATE: Since $y \propto \lambda$, shorter-wavelength dark fringes should be close together, as we have just seen.

VP36.3.2. **IDENTIFY:** We are dealing with single-slit diffraction.

SET UP: For small angles, dark spots occur at $y_m = R\dfrac{m\lambda}{a}$. The small-angle approximation is valid.

EXECUTE: **(a)** We want λ. Solve $y_m = R\dfrac{m\lambda}{a}$. $\lambda = \dfrac{ay_m}{mR} = \dfrac{(0.221\text{ mm})(45.7\text{ mm})}{3(5.00\text{ m})} = 673\text{ nm}$.

(b) We want θ to the second dark fringe. $a\sin\theta_2 = 2\lambda$ gives $\theta_2 = 0.349°$.

EVALUATE: With such a small angle of 0.349°, it is clear that the small-angle approximation is valid.

VP36.3.3. **IDENTIFY:** This problem involves the intensity of a single-slit diffraction pattern.

SET UP: $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, $\beta = \dfrac{2\pi a\sin\theta}{\lambda}$, 17.5 rad = 1002.7°.

EXECUTE: **(a)** We want θ. Using $\beta = \dfrac{2\pi a\sin\theta}{\lambda}$ gives

$\theta = \arcsin\left(\dfrac{\lambda\beta}{2\pi a}\right) = \arcsin\left(\dfrac{(545\text{ nm})(35.0\text{ rad})}{2\pi(0.250\text{ mm})}\right) = 0.696°$.

(b) We want I. $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2 = I_0\left(\dfrac{\sin 17.5\text{ rad}}{17.5\text{ rad}}\right)^2 = I_0\left(\dfrac{\sin 1002.7°}{17.5\text{ rad}}\right)^2 = 0.00311 I_0$.

EVALUATE: It is important to realize that in the intensity equation, the $\beta/2$ in the numerator can be expressed as degrees or radians, but in the numerator it must be in radians.

VP36.3.4. **IDENTIFY:** We are dealing with the intensity in single-slit diffraction.

SET UP: $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, $\beta = \dfrac{2\pi a\sin\theta}{\lambda}$.

EXECUTE: **(a)** We want θ to the first dark fringe. $a\sin\theta_1 = \lambda$.

$\theta_1 = \arcsin(\lambda/a) = \arcsin\left(\dfrac{576 \text{ nm}}{0.185 \text{ mm}}\right) = 0.178°.$

(b) We want the intensity. $\dfrac{\beta}{2} = \dfrac{2\pi a\sin\theta}{\lambda}$. $\theta = \dfrac{0.178°}{2} = 0.0892°$. First get $\beta/2$.

$\dfrac{\beta}{2} = \dfrac{\pi(0.185 \text{ mm})\sin(0.0892°)}{576 \text{ nm}} = 1.5708 \text{ rad} = 90.0°$. Now use $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$.

$I = I_0\left(\dfrac{\sin 90.0°}{1.5708 \text{ rad}}\right)^2 = 0.405 I_0.$

EVALUATE: The intensity is close to $I_0/2$, but not equal to it.

VP36.5.1. **IDENTIFY:** We have a diffraction grating.

SET UP: Bright fringes occur at $d\sin\theta_m = m\lambda$, $d = 1/(825 \text{ slits/mm})$.

EXECUTE: **(a)** We want λ. Solve $d\sin\theta_m = m\lambda$ for λ, giving

$\lambda = \dfrac{[1/(825 \text{ slits/mm})]\sin 41.0°}{2} = 398 \text{ nm}.$

(b) We want θ_1 and θ_3. $\theta_1 = \arcsin(\lambda/d) = \arcsin\left(\dfrac{398 \text{ nm}}{825 \text{ mm}}\right) = 19.1°.$

$\theta_3 = \arcsin(3\lambda/d) = \arcsin\left(\dfrac{3(398 \text{ nm})}{825 \text{ mm}}\right) = 79.8°.$

EVALUATE: Notice that the bright fringes are *not* evenly spaced.

VP36.5.2. **IDENTIFY:** We have a diffraction grating.

SET UP: Bright fringes occur at $d\sin\theta_m = m\lambda$.

EXECUTE: **(a)** We want the slit density. Solve $d\sin\theta_m = m\lambda$ for d, giving

$d = \dfrac{\lambda}{\sin\theta_1} = \dfrac{625 \text{ nm}}{\sin 24.0°} = 1537 \text{ nm}$. The slit spacing is $1/d$, so the slit spacing is 651 slits/mm.

(b) We want θ_2 and θ_3. $\theta_2 = \arcsin(2\lambda/d) = \arcsin\left(\dfrac{2(625 \text{ nm})}{1537 \text{ nm}}\right) = 54.4°.$

$\theta_3 = \arcsin(3\lambda/d) = \arcsin\left(\dfrac{3(625 \text{ nm})}{1537 \text{ nm}}\right) = \arcsin(1.22)$, so there is no θ_3.

EVALUATE: Once $\sin\theta > 1$ there are no more fringes.

VP36.5.3. **IDENTIFY:** We are dealing with x-ray diffraction by a crystal.

SET UP: The plane spacing is $d = 0.124$ nm. Strong interference maxima occur when $2d\sin\theta_m = m\lambda$.

EXECUTE: **(a)** We want θ for the first strong maximum. Solve $2d\sin\theta_m = m\lambda$.

$\theta_1 = \arcsin(\lambda/2d) = \arcsin\left(\dfrac{0.124 \text{ nm}}{2(0.165 \text{ nm})}\right) = 22.1°.$

(b) Are there other angles? $\theta_2 = \arcsin(2\lambda/2d) = \arcsin\left(\dfrac{0.124 \text{ nm}}{0.165 \text{ nm}}\right) = 48.7°$. There are no other angles because $\sin\theta_3 > 1$.

EVALUATE: If $\lambda << d$, there could be many more maxima.

VP36.5.4. **IDENTIFY:** This problem is about x-ray diffraction by a crystal.

SET UP: The plane spacing is d = 0.158 nm. Strong interference maxima occur when $2d\sin\theta_m = m\lambda$. We want λ.

EXECUTE: **(a)** $2d\sin\theta_m = m\lambda = \lambda$, so λ = 2(0.158 nm) sin 36.0° = 0.186 nm.

(b) $2d\sin\theta_3 = 3\lambda$. λ = 2(0.158 nm)(sin 88.0°)/3 = 0.105 nm.

EVALUATE: Since $\sin\theta_m = m\lambda/2d$, to have a large m, λ/d must be small, so $\lambda << d$.

VP36.6.1. **IDENTIFY:** This problem is about the resolving power of a telescope.

SET UP: Rayleigh criterion: $\theta \approx 1.22\dfrac{\lambda}{D}$.

EXECUTE: **(a)** We want D. Solve for D: $D = (1.22)\,(1.70\text{ cm})/(9.00\times10^{-5}\text{rad}) = 230$ m.

(b) We want θ if λ = 21.1 cm. $\theta \approx 1.22\dfrac{\lambda}{D}$ = (1.22)(21.1 cm)/(230 m) = 0.114 mrad = 0.0640°.

EVALUATE: To increase resolution, either increase D or decrease λ, or both.

VP36.6.2. **IDENTIFY:** We are looking at resolving power.

SET UP: For small angles $\theta \approx 1.22\dfrac{\lambda}{D}$, $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$, $m = -s'/s$, f = 0.250 m = 250 mm.

EXECUTE: **(a)** We want the maximum distance. At f/4 the diameter of the lens is $D = f/4$, so D = (250 mm)/4 = 62.5 mm. This gives $\theta \approx 1.22\dfrac{\lambda}{D}$ = (1.22)(550 nm)/(62.5 mm) = 1.074×10^{-5}rad. For the two points, $\sin\theta = \dfrac{5.00\text{ mm}}{x}$, so $x = (5.00\text{ mm})/(1.074\times10^{-5}\text{rad.}) = 466$ m.

(b) We want the distance between the two points on the lens's image. Use $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$. $\dfrac{1}{s'} = \dfrac{1}{f} - \dfrac{1}{s}$ = 1/(0.250 m) – 1/(466 m), so s' = 0.2501 m. $h' = h_0|m| = h_0\dfrac{s'}{s}$. This gives

$$h' = (5.00\text{ mm})\left(\frac{0.2501\text{ m}}{466\text{ m}}\right) = 0.00268\text{ mm}.$$

EVALUATE: 466 m is around 1500 ft. Most people could not even *see* the two points to focus at that distance.

VP36.6.3. **IDENTIFY:** This problem involves an Airy disk and a camera lens.

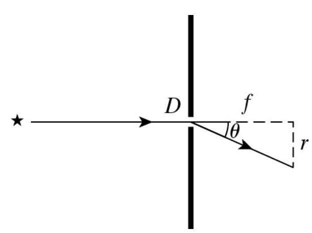


Figure VP36.6.3

SET UP and **EXECUTE:** **(a)** We want D. Refer to Fig. VP36.63. $\sin\theta = 1.22\dfrac{\lambda}{D}$ The image of a very distant object is formed at the focal point of the lens, so it is 135 mm from the lens. $r << f$, so $\theta \approx r/f$. Thus $\dfrac{r}{f} = 1.22\dfrac{\lambda}{D}$, giving $D = \dfrac{1.22\lambda f}{r}$ = (1.22)(575 nm)(135 mm)/(0.0112 mm) = 8.46 mm.

(b) We want the *f*-number. If the *f*-number is *f/x*, it means that $D = f/x$. Therefore $x = \dfrac{f}{D} = \dfrac{135\text{ mm}}{8.46\text{ mm}} = 16.0$, so the *f*-number is *f*/16.0.

EVALUATE: An *f*-number of *f*/16 is considered a fairly small lens aperture.

VP36.6.4. **IDENTIFY:** This problem is about the resolving power of a telescope.

SET UP: Rayleigh criterion: $\theta \approx 1.22\dfrac{\lambda}{D}$. We want the distance to the planet and its star.

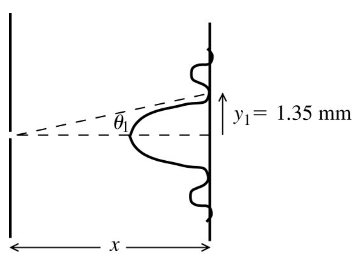
EXECUTE: **(a)** λ = 690 nm. $\theta \approx 1.22\dfrac{\lambda}{D}$ = (1.22)(690 nm)/(8.0 m) = 1.052×10^{-7} rad. For planet and star: $\theta = R/x \rightarrow x = R/\theta = 1.5\times10^{11}\text{ m}/1.052\times10^{-7}\text{ rad} = 1.6\times10^{19}$ m. Using the fact that 1 ly $= 9.46\times10^{15}$ m, we get x = 150 ly.

(b) λ = 1400 nm. λ increases by a factor 1400/690, so θ increases by the same factor since $\theta \propto \lambda$. $x \propto 1/\theta$, so x decreases by a factor 690/1400. Therefore x = (690/1400)(150 ly) = 74 ly.

EVALUATE: Clearly it's better to use the shortest possible wavelength.

36.1. **IDENTIFY:** Use $y = x\tan\theta$ to calculate the angular position θ of the first minimum. The minima are located by $\sin\theta = \dfrac{m\lambda}{a}$, $m = \pm1, \pm2, \ldots$. First minimum means $m = 1$ and $\sin\theta_1 = \lambda/a$ and $\lambda = a\sin\theta_1$. Use this equation to calculate λ.

SET UP: The central maximum is sketched in Figure 36.1.



EXECUTE: $y_1 = x\tan\theta_1$.

$\tan\theta_1 = \dfrac{y_1}{x} =$

$\dfrac{1.35\times10^{-3}\text{ m}}{2.00\text{ m}} = 0.675\times10^{-3}$.

$\theta_1 = 0.675\times10^{-3}$ rad.

Figure 36.1

$\lambda = a\sin\theta_1 = (0.750\times10^{-3}\text{ m})\sin(0.675\times10^{-3}\text{ rad}) = 506\text{ nm}.$

EVALUATE: θ_1 is small so the approximation used to obtain $y_m = x\dfrac{m\lambda}{a}$ is valid and this equation could have been used.

36.2. **IDENTIFY:** This problem is about single-slit diffraction.

SET UP: Dark spots occur when $a\sin\theta_m = m\lambda$, $y = R\tan\theta$, for small angles $y_m = R\dfrac{m\lambda}{a}$. We want the width of the central maximum on the screen when λ = 0.125 mm.

EXECUTE: When λ = 500 nm, we can use the small-angle approximation to find the width a of the slit. The width of the central maximum is 8.00 mm, so 8.00 mm = $2y_1 = 2R\dfrac{\lambda}{a} = 2(2.00\text{ m})\dfrac{500\text{ nm}}{a}$, which gives a = 0.250 mm. Now find θ when λ = 0.125 mm. We don't know that we can use the small-angle approximation, so use $a\sin\theta_m = m\lambda$. At the first dark fringe we have $\sin\theta = \dfrac{\lambda}{a} = \dfrac{0.125\text{ mm}}{0.250\text{ mm}} = 0.500$, so θ = 30.0°. (As we see, the small-angle approximation would not be justified.) $y = R\tan\theta$, and the width of the central maximum is $2y$, so we have $2y = 2R\tan\theta = 2(2.00\text{ m})\tan 30.0° = 2.31\text{ m}.$

EVALUATE: The central maximum is much wider with the greater wavelength light. In that case, the small-angle approximation certainly cannot be used.

36.3. IDENTIFY: The dark fringes are located at angles θ that satisfy $\sin\theta = \dfrac{m\lambda}{a}$, $m = \pm 1, \pm 2, \ldots$.

SET UP: The largest value of $|\sin\theta|$ is 1.00.

EXECUTE: **(a)** Solve for *m* that corresponds to $\sin\theta = 1$: $m = \dfrac{a}{\lambda} = \dfrac{0.0666\times10^{-3}\text{ m}}{585\times10^{-9}\text{ m}} = 113.8.$ The largest value *m* can have is 113. $m = \pm 1, \pm 2, \ldots, \pm 113$ gives 226 dark fringes.

(b) For $m = \pm 113$, $\sin\theta = \pm 113\left(\dfrac{585\times10^{-9}\text{ m}}{0.0666\times10^{-3}\text{ m}}\right) = \pm 0.9926$ and $\theta = \pm 83.0°$.

EVALUATE: When the slit width *a* is decreased, there are fewer dark fringes. When $a < \lambda$ there are no dark fringes and the central maximum completely fills the screen.

36.4. IDENTIFY and **SET UP:** λ/a is very small, so the approximate expression $y_m = x\dfrac{m\lambda}{a}$ is accurate. The distance between the two dark fringes on either side of the central maximum is $2y_1$.

EXECUTE: $y_1 = \dfrac{\lambda x}{a} = \dfrac{(633\times10^{-9}\text{ m})(3.50\text{ m})}{0.750\times10^{-3}\text{ m}} = 2.95\times10^{-3}\text{ m} = 2.95\text{ mm}.$ $2y_1 = 5.90\text{ mm}.$

EVALUATE: When *a* is decreased, the width $2y_1$ of the central maximum increases.

36.5. IDENTIFY: The minima are located by $\sin\theta = \dfrac{m\lambda}{a}$.

SET UP: $a = 12.0$ cm. $x = 8.00$ m.

EXECUTE: The angle to the first minimum is $\theta = \arcsin\left(\dfrac{\lambda}{a}\right) = \arcsin\left(\dfrac{9.00\text{ cm}}{12.00\text{ cm}}\right) = 48.6°.$

So the distance from the central maximum to the first minimum is just $y_1 = x\tan\theta = (8.00\text{ m})\tan(48.6°) = \pm(9.07\text{ m}).$

EVALUATE: $2\lambda/a$ is greater than 1, so only the $m = 1$ minimum is heard.

36.6. IDENTIFY: The angle that locates the first diffraction minimum on one side of the central maximum is given by $\sin\theta = \dfrac{\lambda}{a}$. The time between crests is the period *T*. $f = \dfrac{1}{T}$ and $\lambda = \dfrac{v}{f}$.

SET UP: The time between crests is the period, so $T = 1.0$ h.

EXECUTE: **(a)** $f = \dfrac{1}{T} = \dfrac{1}{1.0\text{ h}} = 1.0\text{ h}^{-1}$. $\lambda = \dfrac{v}{f} = \dfrac{800\text{ km/h}}{1.0\text{ h}^{-1}} = 800\text{ km}.$

(b) Africa-Antarctica: $\sin\theta = \dfrac{800\text{ km}}{4500\text{ km}}$ and $\theta = 10.2°$.

Australia-Antarctica: $\sin\theta = \dfrac{800\text{ km}}{3700\text{ km}}$ and $\theta = 12.5°$.

EVALUATE: Diffraction effects are observed when the wavelength is about the same order of magnitude as the dimensions of the opening through which the wave passes.

36.7. IDENTIFY: We can model the hole in the concrete barrier as a single slit that will produce a single-slit diffraction pattern of the water waves on the shore.

SET UP: For single-slit diffraction, the angles at which destructive interference occurs are given by $\sin\theta_m = m\lambda/a$, where $m = 1, 2, 3, \ldots$.

EXECUTE: **(a)** The frequency of the water waves is $f = 75.0 \text{ min}^{-1} = 1.25 \text{ s}^{-1} = 1.25 \text{ Hz}$, so their wavelength is $\lambda = v/f = (15.0 \text{ cm/s})/(1.25 \text{ Hz}) = 12.0 \text{ cm}$.

At the first point for which destructive interference occurs, we have $\tan\theta = (0.613 \text{ m})/(3.20 \text{ m}) \Rightarrow \theta = 10.84°$. $a \sin\theta = \lambda$ and $a = \lambda/\sin\theta = (12.0 \text{ cm})/(\sin 10.84°) = 63.8 \text{ cm}$.

(b) Find the angles at which destructive interference occurs.
$\sin\theta_2 = 2\lambda/a = 2(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_2 = \pm 22.1°$.
$\sin\theta_3 = 3\lambda/a = 3(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_3 = \pm 34.3°$.
$\sin\theta_4 = 4\lambda/a = 4(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_4 = \pm 48.8°$.
$\sin\theta_5 = 5\lambda/a = 5(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_5 = \pm 70.1°$.

EVALUATE: These are large angles, so we cannot use the approximation that $\theta_m \approx m\lambda/a$.

36.8. **IDENTIFY:** The angle is small, so $y_m = x\dfrac{m\lambda}{a}$ applies.

SET UP: The width of the central maximum is $2y_1$, so $y_1 = 3.00 \text{ mm}$.

EXECUTE: **(a)** $y_1 = \dfrac{x\lambda}{a} \Rightarrow a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50 \text{ m})(5.00\times10^{-7} \text{ m})}{3.00\times10^{-3} \text{ m}} = 4.17\times10^{-4} \text{ m}$.

(b) $a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50 \text{ m})(5.00\times10^{-5} \text{ m})}{3.00\times10^{-3} \text{ m}} = 4.17\times10^{-2} \text{ m} = 4.2 \text{ cm}$.

(c) $a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50 \text{ m})(5.00\times10^{-10} \text{ m})}{3.00\times10^{-3} \text{ m}} = 4.17\times10^{-7} \text{ m}$.

EVALUATE: The ratio a/λ stays constant, so a is smaller when λ is smaller.

36.9. **IDENTIFY** and **SET UP:** $v = f\lambda$ gives λ. The person hears no sound at angles corresponding to diffraction minima. The diffraction minima are located by $\sin\theta = m\lambda/a$, $m = \pm1, \pm2, \ldots$ Solve for θ.

EXECUTE: $\lambda = v/f = (344 \text{ m/s})/(1250 \text{ Hz}) = 0.2752 \text{ m}$; $a = 1.00 \text{ m}$. $m = \pm1$, $\theta = \pm16.0°$; $m = \pm2$, $\theta = \pm33.4°$; $m = \pm3$, $\theta = \pm55.6°$; no solution for larger m.

EVALUATE: $\lambda/a = 0.28$ so for the large wavelength sound waves diffraction by the doorway is a large effect. Diffraction would not be observable for visible light because its wavelength is much smaller and $\lambda/a \ll 1$.

36.10. **IDENTIFY:** Compare E_y to the expression $E_y = E_{\text{max}}\sin(kx - \omega t)$ and determine k, and from that calculate λ. $f = c/\lambda$. The dark bands are located by $\sin\theta = \dfrac{m\lambda}{a}$.

SET UP: $c = 3.00\times10^8 \text{ m/s}$. The first dark band corresponds to $m = 1$.

EXECUTE: **(a)** $E = E_{\text{max}}\sin(kx - \omega t)$. $k = \dfrac{2\pi}{\lambda} \Rightarrow \lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{1.40\times10^7 \text{ m}^{-1}} = 4.488\times10^{-7} \text{ m}$.

$f\lambda = c \Rightarrow f = \dfrac{c}{\lambda} = \dfrac{3.0\times10^8 \text{ m/s}}{4.488\times10^{-7} \text{ m}} = 6.68\times10^{14} \text{ Hz}$.

(b) $a\sin\theta = \lambda$. $a = \dfrac{\lambda}{\sin\theta} = \dfrac{4.488\times10^{-7} \text{ m}}{\sin 28.6°} = 9.38\times10^{-7} \text{ m}$.

(c) $a\sin\theta = m\lambda (m = 1, 2, 3, \ldots)$. $\sin\theta_2 = \pm2\dfrac{\lambda}{a} = \pm2\dfrac{4.488\times10^{-7} \text{ m}}{9.38\times10^{-7} \text{ m}}$ so $\theta_2 = \pm73.2°$.

EVALUATE: For $m = 3$, $\dfrac{m\lambda}{a}$ is greater than 1 so only the first and second dark bands appear.

36.11. **IDENTIFY:** Calculate the angular positions of the minima and use $y = x\tan\theta$ to calculate the distance on the screen between them.

(a) SET UP: The central bright fringe is shown in Figure 36.11a.

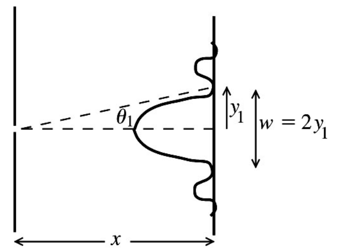


Figure 36.11a

EXECUTE: The first minimum is located by

$$\sin\theta_1 = \frac{\lambda}{a} = \frac{633\times10^{-9}\ \text{m}}{0.350\times10^{-3}\ \text{m}} = 1.809\times10^{-3}.$$

$\theta_1 = 1.809\times10^{-3}$ rad.

$y_1 = x\tan\theta_1 = (3.00\ \text{m})\tan(1.809\times10^{-3}\ \text{rad}) = 5.427\times10^{-3}\ \text{m}.$

$w = 2y_1 = 2(5.427\times10^{-3}\ \text{m}) = 1.09\times10^{-2}\ \text{m} = 10.9\ \text{mm}.$

(b) SET UP: The first bright fringe on one side of the central maximum is shown in Figure 36.11b.

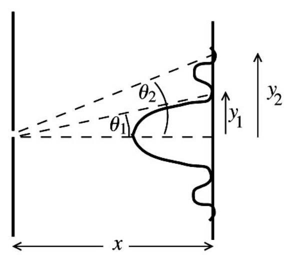


Figure 36.11b

EXECUTE: $w = y_2 - y_1$.

$y_1 = 5.427\times10^{-3}$ m (part (a)).

$$\sin\theta_2 = \frac{2\lambda}{a} = 3.618\times10^{-3}.$$

$\theta_2 = 3.618\times10^{-3}$ rad.

$y_2 = x\tan\theta_2 = 1.085\times10^{-2}$ m.

$w = y_2 - y_1 = 1.085\times10^{-2}\ \text{m} - 5.427\times10^{-3}\ \text{m} = 5.4\ \text{mm}.$

EVALUATE: The central bright fringe is twice as wide as the other bright fringes.

36.12. **IDENTIFY:** We are dealing with single-slit diffraction.

SET UP: We want the wavelength to double the width of the central maximum on a screen. We cannot use the small-angle approximation in this case. The first dark fringe occurs when $a\sin\theta = \lambda$, $y = R\tan\theta$. Call $\lambda_1 = 120\ \mu\text{m}$ and λ_2 the wavelength we want to find.

EXECUTE: $y_1 = R\tan\theta_1$ gives $\theta_1 = \arctan(\lambda_1/R) = \arctan\dfrac{45.0\ \text{cm}}{150\ \text{cm}} = 16.7°$. $y_2 = 2y_1$, so $y_2 = R\tan\theta_2$ so $\theta_2 = \arctan(\lambda_2/R) = \arctan\dfrac{90.0\ \text{cm}}{150\ \text{cm}} = 31.0°$. At the first dark spots $a\sin\theta = \lambda$ so $\dfrac{a\sin\theta_2}{a\sin\theta_1} = \dfrac{\lambda_2}{\lambda_1}$. This gives $\dfrac{\sin 31.0°}{\sin 16.7°} = \dfrac{\lambda_2}{120\ \mu\text{m}}$. So $\lambda_2 = 215\ \mu\text{m}$.

EVALUATE: Notice that to double the pattern width on the screen we do *not* double the wavelength or the angle θ.

36.13. **IDENTIFY:** The minima are located by $\sin\theta = \dfrac{m\lambda}{a}$. For part (b) apply $I = I_0\left\{\dfrac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2$.

SET UP: For the first minimum, $m = 1$. The intensity at $\theta = 0$ is I_0.

EXECUTE: **(a)** $\sin\theta = \dfrac{m\lambda}{a} = \sin 90.0° = 1 = \dfrac{m\lambda}{a} = \dfrac{\lambda}{a}$. Thus $a = \lambda = 580\ \text{nm} = 5.80\times10^{-4}\ \text{mm}.$

(b) Using $I = I_0 \left\{ \frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda} \right\}^2$ gives

$$\frac{I}{I_0} = \left\{ \frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda} \right\}^2 = \left\{ \frac{\sin[\pi(\sin\pi/4)]}{\pi(\sin\pi/4)} \right\}^2 = 0.128.$$

EVALUATE: If $a = \lambda/2$, for example, then at $\theta = 45°$, $\frac{I}{I_0} = \left\{ \frac{\sin[(\pi/2)(\sin\pi/4)]}{(\pi/2)(\sin\pi/4)} \right\}^2 = 0.65$. As a/λ decreases, the screen becomes more uniformly illuminated.

36.14. IDENTIFY: $I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2$. $\beta = \frac{2\pi}{\lambda} a \sin\theta$.

SET UP: The angle θ is small, so $\sin\theta \approx \tan\theta \approx y/x$.

EXECUTE: $\beta = \frac{2\pi a}{\lambda} \sin\theta \approx \frac{2\pi a}{\lambda} \frac{y}{x} = \frac{2\pi(4.50\times10^{-4}\text{ m})}{(6.20\times10^{-7}\text{ m})(3.00\text{ m})} y = (1520\text{ m}^{-1})y$.

(a) $y = 1.00\times10^{-3}$ m: $\frac{\beta}{2} = \frac{(1520\text{ m}^{-1})(1.00\times10^{-3}\text{ m})}{2} = 0.760$.

$$\Rightarrow I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left(\frac{\sin(0.760)}{0.760} \right)^2 = 0.822 I_0.$$

(b) $y = 3.00\times10^{-3}$ m: $\frac{\beta}{2} = \frac{(1520\text{ m}^{-1})(3.00\times10^{-3}\text{ m})}{2} = 2.28$.

$$\Rightarrow I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left(\frac{\sin(2.28)}{2.28} \right)^2 = 0.111 I_0.$$

(c) $y = 5.00\times10^{-3}$ m: $\frac{\beta}{2} = \frac{(1520\text{ m}^{-1})(5.00\times10^{-3}\text{ m})}{2} = 3.80$.

$$\Rightarrow I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left(\frac{\sin(3.80)}{3.80} \right)^2 = 0.0259 I_0.$$

EVALUATE: The first minimum occurs at $y_1 = \frac{\lambda x}{a} = \frac{(6.20\times10^{-7}\text{ m})(3.00\text{ m})}{4.50\times10^{-4}\text{ m}} = 4.1\text{ mm}$. The distances in parts (a) and (b) are within the central maximum. $y = 5.00$ mm is within the first secondary maximum.

36.15. IDENTIFY: The space between the skyscrapers behaves like a single slit and diffracts the radio waves.

SET UP: Cancellation of the waves occurs when $a \sin\theta = m\lambda$, $m = 1, 2, 3, \ldots$, and the intensity of the waves is given by $I_0 \left(\frac{\sin\beta/2}{\beta/2} \right)^2$, where $\beta/2 = \frac{\pi a \sin\theta}{\lambda}$.

EXECUTE: **(a)** First find the wavelength of the waves:

$\lambda = c/f = (3.00\times10^8\text{ m/s})/(88.9\text{ MHz}) = 3.375\text{ m}$.

For no signal, $a \sin\theta = m\lambda$.

$m = 1$: $\sin\theta_1 = (1)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_1 = \pm 13.0°$.

$m = 2$: $\sin\theta_2 = (2)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_2 = \pm 26.7°$.

$m = 3$: $\sin\theta_3 = (3)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_3 = \pm 42.4°$.

$m = 4$: $\sin\theta_4 = (4)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_4 = \pm 64.1°$.

(b) $I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2=\dfrac{\pi a\sin\theta}{\lambda}=\dfrac{\pi(15.0\text{ m})\sin(5.00°)}{3.375\text{ m}}=1.217\text{ rad}.$

$I=(3.50\text{ W/m}^2)\left[\dfrac{\sin(1.217\text{ rad})}{1.217\text{ rad}}\right]^2=2.08\text{ W/m}^2.$

EVALUATE: The wavelength of the radio waves is very long compared to that of visible light, but it is still considerably shorter than the distance between the buildings.

36.16. **IDENTIFY:** The intensity on the screen varies as the light spreads out (diffracts) after passing through the single slit.

SET UP: $I=I_0\left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$ where $\beta=\dfrac{2\pi}{\lambda}a\sin\theta.$

EXECUTE: $\beta=\dfrac{2\pi}{\lambda}a\sin\theta=\left(\dfrac{2\pi}{592\times10^{-9}\text{ m}}\right)(0.0290\times10^{-3}\text{ m})(\sin 1.20°)=6.44589\text{ rad}.$

$I=I_0\left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2=(4.00\times10^{-5}\text{ W/m}^2)\left[\dfrac{\sin(6.44589\text{ rad})}{6.44589\text{ rad}}\right]^2=2.54\times10^{-8}\text{ W/m}^2.$

EVALUATE: The intensity is less than 1/1500 of the intensity of the light at the center of the central maximum.

36.17. **IDENTIFY:** In this problem we combine double-slit interference with single-slit diffraction.
SET UP: Let D signify a double-slit effect and S signify a single-slit effect. The phases are

$\phi_D=344°,\ \phi_S=172°.\ \ I=I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2.$

EXECUTE: **(a)** We want to relate the slit width a to the slit spacing d. $\dfrac{\phi_D}{360°}=\dfrac{344°}{360°}=\dfrac{d\sin\theta}{\lambda}$ and $\dfrac{\phi_S}{360°}=\dfrac{172°}{360°}=\dfrac{a\sin\theta}{\lambda}$. Taking the ratio gives $\dfrac{344°/360°}{172°/360°}=\dfrac{d\sin\theta/\lambda}{a\sin\theta/\lambda}=\dfrac{d}{a}$, so $d=2a$.

(b) We want the intensity at $\theta=22.0°$. Use $I=I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$. ϕ is the phase difference for double-slit interference, so $\phi=344°$. β is the phase difference for single-slit diffraction, so $\beta=172°=3.00$ rad. Using $I=I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$ gives

$I=(0.234\text{ W/m}^2)\cos^2(344°/2)\left(\dfrac{\sin(172°/2)}{(3.00\text{ rad})/2}\right)^2=0.101\text{ W/m}^2.$

EVALUATE: The overall intensity is due to diffraction by each slit and interference of the light from the two slits.

36.18. **IDENTIFY:** The intensity at the screen is due to a combination of single-slit diffraction and double-slit interference.

SET UP: $I=I_0\left(\cos^2\dfrac{\phi}{2}\right)\left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$, where $\phi=\dfrac{2\pi d}{\lambda}\sin\theta$ and $\beta=\dfrac{2\pi}{\lambda}a\sin\theta.$

EXECUTE: $\tan\theta=\dfrac{9.00\times10^{-4}\text{ m}}{0.750\text{ m}}=1.200\times10^{-3}$. θ is small, so $\sin\theta\approx\tan\theta.$

$\phi=\dfrac{2\pi d}{\lambda}\sin\theta=\dfrac{2\pi(0.640\times10^{-3}\text{ m})}{568\times10^{-9}\text{ m}}(1.200\times10^{-3})=8.4956\text{ rad}.$

$$\beta = \frac{2\pi a}{\lambda}\sin\theta = \frac{2\pi(0.434\times10^{-3}\text{ m})}{568\times10^{-9}\text{ m}}(1.200\times10^{-3}) = 5.7611\text{ rad.}$$

$$I = (5.00\times10^{-4}\text{ W/m}^2)(\cos 4.2478\text{ rad})^2\left[\frac{\sin 2.8805\text{ rad}}{2.8805}\right]^2 = 8.06\times10^{-7}\text{ W/m}^2.$$

EVALUATE: The intensity as decreased by a factor of almost a thousand, so it would be difficult to see the light at the screen.

36.19. **(a) IDENTIFY** and **SET UP:** The interference fringes (maxima) are located by $d\sin\theta = m\lambda$, with $m = 0, \pm1, \pm2, \ldots$. The intensity I in the diffraction pattern is given by $I = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2$, with $\beta = \left(\frac{2\pi}{\lambda}\right)a\sin\theta$. We want $m = \pm3$ in the first equation to give θ that makes $I = 0$ in the second equation.

EXECUTE: $d\sin\theta = m\lambda$ gives $\beta = \left(\frac{2\pi}{\lambda}\right)a\left(\frac{3\lambda}{d}\right) = 2\pi(3a/d)$.

$I = 0$ says $\frac{\sin\beta/2}{\beta/2} = 0$ so $\beta = 2\pi$ and then $2\pi = 2\pi(3a/d)$ and $(d/a) = 3$.

(b) IDENTIFY and **SET UP:** Fringes $m = 0, \pm1, \pm2$ are within the central diffraction maximum and the $m = \pm3$ fringes coincide with the first diffraction minimum. Find the value of m for the fringes that coincide with the second diffraction minimum.

EXECUTE: Second minimum implies $\beta = 4\pi$.

$$\beta = \left(\frac{2\pi}{\lambda}\right)a\sin\theta = \left(\frac{2\pi}{\lambda}\right)a\left(\frac{m\lambda}{d}\right) = 2\pi m(a/d) = 2\pi(m/3).$$

Then $\beta = 4\pi$ says $4\pi = 2\pi(m/3)$ and $m = 6$. Therefore the $m = \pm4$ and $m = +5$ fringes are contained within the first diffraction maximum on one side of the central maximum; two fringes.

EVALUATE: The central maximum is twice as wide as the other maxima so it contains more fringes.

36.20. **IDENTIFY:** The net intensity is the *product* of the factor due to single-slit diffraction and the factor due to double slit interference.

SET UP: The double-slit factor is $I_{DS} = I_0\left(\cos^2\frac{\phi}{2}\right)$ and the single-slit factor is $I_{SS} = \left(\frac{\sin\beta/2}{\beta/2}\right)^2$.

EXECUTE: **(a)** $d\sin\theta = m\lambda \Rightarrow \sin\theta = m\lambda/d$.

$\sin\theta_1 = \lambda/d, \sin\theta_2 = 2\lambda/d, \sin\theta_3 = 3\lambda/d, \sin\theta_4 = 4\lambda/d$.

(b) At the interference bright fringes, $\cos^2\phi/2 = 1$ and $\beta/2 = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(d/3)\sin\theta}{\lambda}$.

At θ_1, $\sin\theta_1 = \lambda/d$, so $\beta/2 = \frac{\pi(d/3)(\lambda/d)}{\lambda} = \pi/3$. The intensity is therefore

$$I_1 = I_0\left(\cos^2\frac{\phi}{2}\right)\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = I_0(1)\left(\frac{\sin\pi/3}{\pi/3}\right)^2 = 0.684\,I_0.$$

At θ_2, $\sin\theta_2 = 2\lambda/d$, so $\beta/2 = \frac{\pi(d/3)(2\lambda/d)}{\lambda} = 2\pi/3$. Using the same procedure as for θ_1, we have

$$I_2 = I_0(1)\left(\frac{\sin 2\pi/3}{2\pi/3}\right)^2 = 0.171\,I_0.$$

At θ_3, we get $\beta/2 = \pi$, which gives $I_3 = 0$ since $\sin\,\pi = 0$.

At θ_4, $\sin\,\theta_4 = 4\lambda/d$, so $\beta/2 = 4\pi/3$, which gives $I_4 = I_0\left(\dfrac{\sin 4\pi/3}{4\pi/3}\right)^2 = 0.0427\,I_0$.

(c) Since $d = 3a$, every third interference maximum is missing.

(d) In Figure 36.12c in the textbook, every fourth interference maximum at the sides is missing because $d = 4a$.

EVALUATE: The result in this problem is different from that in Figure 36.12c in the textbook because in this case $d = 3a$, so every third interference maximum at the sides is missing. Also the "envelope" of the intensity function decreases more rapidly here than in Figure 36.12c in the text because the first diffraction minimum is reached sooner, and the decrease in intensity from one interference maximum to the next is faster for $a = d/3$ than for $a = d/4$.

36.21. **(a) IDENTIFY** and **SET UP:** If the slits are very narrow then the central maximum of the diffraction pattern for each slit completely fills the screen and the intensity distribution is given solely by the two-slit interference. The maxima are given by $d\sin\theta = m\lambda$ so $\sin\theta = m\lambda/d$. Solve for θ.

EXECUTE: 1st order maximum: $m = 1$, so $\sin\theta = \dfrac{\lambda}{d} = \dfrac{580\times10^{-9}\text{ m}}{0.530\times10^{-3}\text{ m}} = 1.094\times10^{-3}$; $\theta = 0.0627°$.

2nd order maximum: $m = 2$, so $\sin\theta = \dfrac{2\lambda}{d} = 2.188\times10^{-3}$; $\theta = 0.125°$.

(b) IDENTIFY and **SET UP:** The intensity is given by Eq. (36.12): $I = I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$.

Calculate ϕ and β at each θ from part (a).

EXECUTE: $\phi = \left(\dfrac{2\pi d}{\lambda}\right)\sin\theta = \left(\dfrac{2\pi d}{\lambda}\right)\left(\dfrac{m\lambda}{d}\right) = 2\pi m$, so $\cos^2(\phi/2) = \cos^2(m\pi) = 1$.

(Since the angular positions in part (a) correspond to interference maxima.)

$$\beta = \left(\frac{2\pi a}{\lambda}\right)\sin\theta = \left(\frac{2\pi a}{\lambda}\right)\left(\frac{m\lambda}{d}\right) = 2\pi m(a/d) = m2\pi\left(\frac{0.320\text{ mm}}{0.530\text{ mm}}\right) = m(3.794\text{ rad}).$$

1st order maximum: $m = 1$, so $I = I_0(1)\left(\dfrac{\sin(3.794/2)\text{rad}}{(3.794/2)\text{rad}}\right)^2 = 0.249I_0$.

2nd order maximum: $m = 2$, so $I = I_0(1)\left(\dfrac{\sin 3.794\text{ rad}}{3.794\text{ rad}}\right)^2 = 0.0256I_0$.

EVALUATE: The first diffraction minimum is at an angle θ given by $\sin\theta = \lambda/a$ so $\theta = 0.104°$. The first order fringe is within the central maximum and the second order fringe is inside the first diffraction maximum on one side of the central maximum. The intensity here at this second fringe is much less than I_0.

36.22. **IDENTIFY:** The diffraction minima are located by $\sin\theta = \dfrac{m_\text{d}\lambda}{a}$ and the two-slit interference maxima are located by $\sin\theta = \dfrac{m_\text{i}\lambda}{d}$. The third bright band is missing because the first order single-slit minimum occurs at the same angle as the third order double-slit maximum.

SET UP: The pattern is sketched in Figure 36.22. $\tan\theta = \dfrac{3\text{ cm}}{90\text{ cm}}$, so $\theta = 1.91°$.

EXECUTE: Single-slit dark spot: $a\sin\theta = \lambda$ and

$$a = \frac{\lambda}{\sin\theta} = \frac{500\text{ nm}}{\sin 1.91°} = 1.50\times10^4\text{ nm} = 15.0\ \mu\text{m (width)}.$$

<u>Double-slit bright fringe</u>: $d\sin\theta = 3\lambda$ and

$$d = \frac{3\lambda}{\sin\theta} = \frac{3(500\text{ nm})}{\sin 1.91°} = 4.50\times 10^4\text{ nm} = 45.0\ \mu\text{m (separation).}$$

EVALUATE: Note that $d/a = 3.0$.

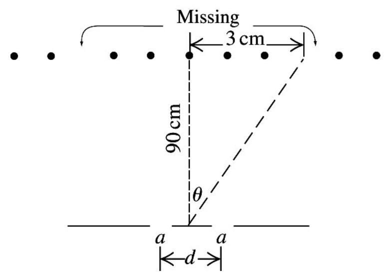


Figure 36.22

36.23. IDENTIFY: We combine double-slit interference with single-slit diffraction.

SET UP: Single-slit dark spots: $a\sin\theta_m = m\lambda$, double-slit bright spots: $d\sin\theta_m = m\lambda$. We want to find the slit width a.

EXECUTE: (a) The m = 3 double-slit bright spot is missing because it lies where the single-slit intensity is zero. For single slit dark spot $a\sin\theta_k = k\lambda$ and for double slit bright spot $d\sin\theta_m = m\lambda$. The minimum a occurs for the minimum k, which is 1. Thus $a\sin\theta = \lambda$ (single slit) and $d\sin\theta = 3\lambda$ (double slit). Dividing gives $a/d = 1/3$, so $a = d/3 = (9.00\text{ mm})/3 = 3.00$ mm.

(b) For the next larger a, we use $k = 2$, so $a\sin\theta = 2\lambda$ and $d\sin\theta = 3\lambda$. Dividing gives $a/d = 2/3$, which gives $a = 2/3\ d = 6.00$ mm.

EVALUATE: It is also possible for a double-slit minimum to cancel a single-slit maximum.

36.24. IDENTIFY: This problem involves the intensity of a single-slit and double-slit pattern.

SET UP: $I = I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, $\beta = \dfrac{2\pi a\sin\theta}{\lambda}$, $\phi = \dfrac{2\pi d\sin\theta}{\lambda}$, $a\sin\theta_m = m\lambda$ (single-slit minima), $d\sin\theta_m = m\lambda$ (double-slit maxima). We want I_1/I_2 for the maxima.

EXECUTE: The double-slit m = 5 maximum is missing because it lies at a minimum for the single-slit diffraction pattern. Since the double-slit m = 0 through m = 4 maxima are present, the single-slit minimum must be the first one. Thus $a\sin\theta = \lambda$ and $d\sin\theta = 5\lambda$. Dividing gives $a/d = 1/5$, so $a = d/5 = 0.200$ mm.

<u>At $m = 1$</u>: $d\sin\theta = \lambda$, so $\dfrac{\phi_1}{2} = \dfrac{\pi d\sin\theta}{\lambda} = \pi$. $\dfrac{\beta_1}{2} = \dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi(d/5)\sin\theta}{\lambda} = \pi/5 = 36.0°$. Therefore

$$I_1 = I_0\cos^2(\phi/2)\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = I_0\cos^2(\pi)\left(\frac{\sin 36.0°}{\pi/5}\right)^2 = 0.8751 I_0.$$

<u>At $m = 2$</u>: $d\sin\theta = 2\lambda$, so $\dfrac{\phi_2}{2} = \dfrac{\pi d\sin\theta}{\lambda} = 2\pi$. $\dfrac{\beta_2}{2} = \dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi(d/5)\sin\theta}{\lambda} = 2\pi/5 = 72.0°$. Thus

$$I_2 = I_0\cos^2(2\pi)\left(\frac{\sin 72.0°}{2\pi/5}\right)^2 = 0.5728 I_0.$$

$$\frac{I_1}{I_2} = \frac{0.8751 I_0}{0.5728 I_0} = 1.53.$$

EVALUATE: $I_1 > I_2$ because the maxima get dimmer as we move away from the center.

36.25. **IDENTIFY:** Knowing the wavelength of the light and the location of the first interference maxima, we can calculate the line density of the grating.

SET UP: The line density in lines/cm is $1/d$, with d in cm. The bright spots are located by $d\sin\theta = m\lambda,\ m = 0, \pm 1, \pm 2, \ldots$.

EXECUTE: **(a)** $d = \dfrac{m\lambda}{\sin\theta} = \dfrac{(1)(632.8\times10^{-9}\ \text{m})}{\sin 17.8°} = 2.07\times10^{-6}\ \text{m} = 2.07\times10^{-4}\ \text{cm}.\ \dfrac{1}{d} = 4830\ \text{lines/cm}.$

(b) $\sin\theta = \dfrac{m\lambda}{d} = m\left(\dfrac{632.8\times10^{-9}\ \text{m}}{2.07\times10^{-6}\ \text{m}}\right) = m(0.3057).$ For $m = \pm 2,\ \theta = \pm 37.7°.$ For $m = \pm 3,\ \theta = \pm 66.5°.$

EVALUATE: The angles are large, so they are not equally spaced; $37.7° \neq 2(17.8°)$ and $66.5° \neq 3(17.8°)$.

36.26. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda.$

SET UP: The order corresponds to the values of m.

EXECUTE: <u>First-order:</u> $d\sin\theta_1 = \lambda.$ <u>Fourth-order:</u> $d\sin\theta_4 = 4\lambda.$

$\dfrac{d\sin\theta_4}{d\sin\theta_1} = \dfrac{4\lambda}{\lambda},\ \sin\theta_4 = 4\sin\theta_1 = 4\sin 11.3°$ and $\theta_4 = 51.6°.$

EVALUATE: We did not have to solve for d.

36.27. **IDENTIFY:** This problem is about a diffraction grating.

SET UP: Bright fringes occur when $d\sin\theta_m = m\lambda$. Dark fringes occur when $a\sin\theta_m = m\lambda$.

EXECUTE: **(a)** We want d. The 4th order bright spot should occur at 66.6°, so $d\sin\theta_m = m\lambda$ gives $d\sin 66.6° = 4(550\ \text{nm})$. So $d = 2.40\ \mu\text{m}$.

(b) We want the line density. The density is $1/d = 1/(2.40\ \mu\text{m}) = 417$ slits/mm.

(c) We want the width a of each slit. $a\sin\theta_m = m\lambda$ gives $a\sin 66.6° = 3(550\ \text{nm})$, so $a = 1.80\ \mu\text{m}$.

EVALUATE: The width of the slits is 1.80/2.40 = 0.750 times the slit spacing, so single-slit diffraction certainly plays a role.

36.28. **IDENTIFY:** The bright spots are located by $d\sin\theta = m\lambda.$

SET UP: Third-order means $m = 3$ and second-order means $m = 2.$

EXECUTE: $\dfrac{m\lambda}{\sin\theta} = d = \text{constant},$ so $\dfrac{m_r\lambda_r}{\sin\theta_r} = \dfrac{m_v\lambda_v}{\sin\theta_v}.$

$\sin\theta_v = \sin\theta_r\left(\dfrac{m_v}{m_r}\right)\left(\dfrac{\lambda_v}{\lambda_r}\right) = (\sin 65.0°)\left(\dfrac{2}{3}\right)\left(\dfrac{400\ \text{nm}}{700\ \text{nm}}\right) = 0.345$ and $\theta_v = 20.2°.$

EVALUATE: The third-order line for a particular λ occurs at a larger angle than the second-order line. In a given order, the line for violet light (400 nm) occurs at a smaller angle than the line for red light (700 nm).

36.29. **IDENTIFY:** This problem involves the resolving power of a diffraction grating.

SET UP: $R = \dfrac{\lambda}{\Delta\lambda} = mN$.

EXECUTE: **(a)** We want R_{min}. $R_{min} = \dfrac{\lambda}{\Delta\lambda} = \dfrac{430.790\ \text{nm} - 430.774\ \text{nm}}{430.774\ \text{nm}} = 2.69\times10^4.$

(b) We want the lowest order needed. $R = Nm$ gives $m = R/N$ = 26,900/12,800 = 2.10. So $m = 2$ is not quite enough to resolve these lines, but $m = 3$ will do it. So the third-order is the lowest.

EVALUATE: More lines would allow a lower order. In this case, 13,500 lines would allow $m = 2$ because 26,900/13,600 = 1.98, which is less than 2.

36.30. IDENTIFY: The maxima are located by $d\sin\theta = m\lambda$.

SET UP: 350 slits/mm $\Rightarrow d = \dfrac{1}{3.50\times10^{5}\ \text{m}^{-1}} = 2.857\times10^{-6}$ m. The visible spectrum is between approximately 380 nm and 750 nm.

EXECUTE: **(a)** $m = 1$: $\theta_{380} = \arcsin\left(\dfrac{\lambda}{d}\right) = \arcsin\left(\dfrac{3.80\times10^{-7}\ \text{m}}{2.857\times10^{-6}\ \text{m}}\right) = 6.643°$.

$\theta_{750} = \arcsin\left(\dfrac{\lambda}{d}\right) = \arcsin\left(\dfrac{7.50\times10^{-7}\ \text{m}}{2.857\times10^{-6}\ \text{m}}\right) = 15.2185°$. $\Delta\theta_1 = 15.2185° - 6.643° = 8.58°$.

(b) $m = 3$: $\theta_{380} = \arcsin\left(\dfrac{3\lambda}{d}\right) = \arcsin\left(\dfrac{3(3.80\times10^{-7}\ \text{m})}{2.857\times10^{-6}\ \text{m}}\right) = 23.516°$.

$\theta_{750} = \arcsin\left(\dfrac{3\lambda}{d}\right) = \arcsin\left(\dfrac{3(7.50\times10^{-7}\ \text{m})}{2.857\times10^{-6}\ \text{m}}\right) = 51.952°$. $\Delta\theta_1 = 51.952° - 23.516° = 28.4°$.

EVALUATE: $\Delta\theta$ is larger in third order.

36.31. IDENTIFY: The maxima are located by $d\sin\theta = m\lambda$.

SET UP: 5000 slits/cm $\Rightarrow d = \dfrac{1}{5.00\times10^{5}\ \text{m}^{-1}} = 2.00\times10^{-6}$ m.

EXECUTE: **(a)** $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{(2.00\times10^{-6}\ \text{m})\sin 13.5°}{1} = 4.67\times10^{-7}$ m.

(b) $m = 2$: $\theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{2(4.67\times10^{-7}\ \text{m})}{2.00\times10^{-6}\ \text{m}}\right) = 27.8°$.

EVALUATE: Since the angles are fairly small, the second-order deviation is approximately twice the first-order deviation.

36.32. IDENTIFY: The grooves in a CD and DVD form a reflection diffraction grating. Constructive interference when $d\sin\theta = m\lambda$.

SET UP: The maxima are located by $d\sin\theta = m\lambda$, where $d = 1.60\times10^{-6}$ m for a CD and $d = 0.740\times10^{-6}$ m for a DVD.

EXECUTE: **(a)** For a CD we have: $\theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{m(6.328\times10^{-7}\ \text{m})}{1.60\times10^{-6}\ \text{m}}\right) = \arcsin(0.396m)$.

For $m = 1$ we have $\theta_1 = 23.3°$. For $m = 2$ we have $\theta_2 = 52.3°$. There are no other maxima.

(b) For a DVD we have: $\theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{m(6.328\times10^{-7}\ \text{m})}{0.740\times10^{-6}\ \text{m}}\right) = \arcsin(0.855m)$. For $m = 1$ we have $\theta_1 = 58.8°$. There are no other maxima.

EVALUATE: The reflective surface produces the same interference pattern as a grating with slit separation d.

36.33. IDENTIFY: The resolution is described by $R = \dfrac{\lambda}{\Delta\lambda} = Nm$. Maxima are located by $d\sin\theta = m\lambda$.

SET UP: For 500 slits/mm, $d = (500\ \text{slits/mm})^{-1} = (500{,}000\ \text{slits/m})^{-1}$.

EXECUTE: **(a)** $N = \dfrac{\lambda}{m\Delta\lambda} = \dfrac{6.5645\times10^{-7}\ \text{m}}{2(6.5645\times10^{-7}\ \text{m} - 6.5627\times10^{-7}\ \text{m})} = 1820$ slits.

(b) $\theta = \sin^{-1}\left(\dfrac{m\lambda}{d}\right) \Rightarrow \theta_1 = \sin^{-1}((2)(6.5645\times10^{-7}\text{ m})(500{,}000\text{ m}^{-1})) = 41.0297°$ and

$\theta_2 = \sin^{-1}((2)(6.5627\times10^{-7}\text{ m})(500{,}000\text{ m}^{-1})) = 41.0160°$. $\Delta\theta = 0.0137°$.

EVALUATE: $d\cos\theta\, d\theta = \lambda/N$, so for 1820 slits the angular interval $\Delta\theta$ between each of these maxima and the first adjacent minimum is $\Delta\theta = \dfrac{\lambda}{Nd\cos\theta} = \dfrac{6.56\times10^{-7}\text{ m}}{(1820)(2.0\times10^{-6}\text{ m})\cos41°} = 0.0137°$. This is the same as the angular separation of the maxima for the two wavelengths and 1820 slits is just sufficient to resolve these two wavelengths in second order.

36.34. **IDENTIFY:** The maxima are given by $2d\sin\theta = m\lambda$, $m = 1, 2, \ldots$.

SET UP: $d = 3.50\times10^{-10}$ m.

EXECUTE: **(a)** Using $m = 1$ gives

$\lambda = \dfrac{2d\sin\theta}{m} = 2(3.50\times10^{-10}\text{ m})\sin22.0° = 2.62\times10^{-10}\text{ m} = 0.262\text{ nm} = 262\text{ pm}$. This is an x ray.

(b) $\sin\theta = m\left(\dfrac{\lambda}{2d}\right) = m\left(\dfrac{2.62\times10^{-10}\text{ m}}{2(3.50\times10^{-10}\text{ m})}\right) = 0.3743m$. $m = 2$: $\theta = 48.5°$. The equation doesn't have any solutions for $m > 2$.

EVALUATE: In this problem $\lambda/d = 0.75$.

36.35. **IDENTIFY** and **SET UP:** The maxima occur at angles θ given by $2d\sin\theta = m\lambda$, where *d* is the spacing between adjacent atomic planes. Solve for *d*.

EXECUTE: Second order says $m = 2$.

$d = \dfrac{m\lambda}{2\sin\theta} = \dfrac{2(0.0850\times10^{-9}\text{ m})}{2\sin21.5°} = 2.32\times10^{-10}\text{ m} = 0.232\text{ nm}$.

EVALUATE: Our result is similar to *d* calculated in Example 36.5.

36.36. **IDENTIFY:** The crystal behaves like a diffraction grating.

SET UP: The maxima are at angles θ given by $2d\sin\theta = m\lambda$, where $d = 0.440$ nm.

EXECUTE: $m = 1$. $\lambda = \dfrac{2d\sin\theta}{1} = 2(0.440\text{ nm})\sin39.4° = 0.559\text{ nm}$.

EVALUATE: The result is a reasonable x-ray wavelength.

36.37. **IDENTIFY** and **SET UP:** The angular size of the first dark ring is given by $\sin\theta_1 = 1.22\lambda/D$. Calculate θ_1, and then the diameter of the ring on the screen is $2(4.5\text{ m})\tan\theta_1$.

EXECUTE: $\sin\theta_1 = 1.22\left(\dfrac{620\times10^{-9}\text{ m}}{7.4\times10^{-6}\text{ m}}\right) = 0.1022$; $\theta_1 = 0.1024$ rad.

The radius of the Airy disk (central bright spot) is $r = (4.5\text{ m})\tan\theta_1 = 0.462$ m. The diameter is $2r = 0.92\text{ m} = 92\text{ cm}$.

EVALUATE: $\lambda/D = 0.084$. For this small *D* the central diffraction maximum is broad.

36.38. **IDENTIFY** and **SET UP:** For the first dark ring, $\sin\theta_1 = 1.22\dfrac{\lambda}{D}$ and for the second dark ring $\sin\theta_2 = 2.23\dfrac{\lambda}{D}$. If *y* is the distance from the center to the ring, we can use the small-angle approximation because *x* << *y*, where *x* is the distance from the aperture to the screen. Therefore $\sin\theta \approx \tan\theta = y/x$. The difference in radii of the first two dark rings is 1.65 mm.

EXECUTE: The radius of the first dark ring is $y_1 = x\tan\theta_1 = x(1.22\,\lambda/D)$. The radius of the second ring is $y_2 = x\tan\theta_2 = x(1.22\,\lambda/D)$. Dividing these two equations gives $y_2/y_1 = 2.23/1.22 = 1.8279$, so $y_2 = 1.8279y_1$. We are told that the difference in radii is 1.65 mm, so $y_2 - y_1 = 1.65$ mm. Substituting for y_2 gives $1.8279y_1 - y_1 = 1.65$ mm, so $y_1 = 1.9931$ mm. Now find D using $y_1 = x(1.22\,\lambda/D)$.

1.9931×10^{-3} m = (1.20 m)(1.22)(490 nm)/D, so $D = 3.60\times10^5$ nm = 3.60×10^{-4} m = 0.360 mm.

EVALUATE: This is about the diameter of a good-sized pinhole.

36.39. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: $\theta = \dfrac{W}{h}$, where $W = 28\text{ km}$ and $h = 1200\text{ km}$. θ is small, so $\sin\theta \approx \theta$.

EXECUTE: $D = \dfrac{1.22\lambda}{\sin\theta} = 1.22\lambda\dfrac{h}{W} = 1.22(0.036\text{ m})\dfrac{1.2\times10^6\text{ m}}{2.8\times10^4\text{ m}} = 1.88\text{ m}.$

EVALUATE: D must be significantly larger than the wavelength, so a much larger diameter is needed for microwaves than for visible wavelengths.

36.40. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: $\theta = (1/60)°$.

EXECUTE: $D = \dfrac{1.22\lambda}{\sin\theta} = \dfrac{1.22(5.5\times10^{-7}\text{ m})}{\sin(1/60)°} = 2.31\times10^{-3}\text{ m} = 2.3\text{ mm}.$

EVALUATE: The larger the diameter the smaller the angle that can be resolved.

36.41. **IDENTIFY:** The diameter D of the mirror determines the resolution.

SET UP: The resolving power is $\theta_{\text{res}} = 1.22\dfrac{\lambda}{D}$.

EXECUTE: The same θ_{res} means that $\dfrac{\lambda_1}{D_1} = \dfrac{\lambda_2}{D_2}$. $D_2 = D_1\dfrac{\lambda_2}{\lambda_1} = (8000\times10^3\text{ m})\left(\dfrac{550\times10^{-9}\text{ m}}{2.0\times10^{-2}\text{ m}}\right) = 220\text{ m}.$

EVALUATE: The Hubble telescope has an aperture of 2.4 m, so this would have to be an *enormous* optical telescope!

36.42. **IDENTIFY:** Rayleigh's criterion limits the angular resolution.

SET UP: Rayleigh's criterion is $\sin\theta \approx \theta = 1.22\lambda/D$. Call R = 11.5 m = the distance from the bear to the lens.

EXECUTE: **(a)** Using Rayleigh's criterion

$\sin\theta \approx \theta = 1.22\lambda/D = (1.22)(550\text{ nm})/(135/4\text{ mm}) = 1.99\times10^{-5}\text{ rad}.$

On the bear this angle subtends a distance x. Using $\theta = x/R$ gives

$x = R\theta = (11.5\text{ m})(1.99\times10^{-5}\text{ rad}) = 2.29\times10^{-4}\text{m} = 0.23\text{ mm}.$

(b) At $f/22$, D is $4/22$ times as large as at $f/4$. Since θ is proportional to $1/D$, and x is proportional to θ, x is $1/(4/22) = 22/4$ times as large as it was at $f/4$. $x = (0.229\text{ mm})(22/4) = 1.3\text{ mm}.$

EVALUATE: A wide-angle lens, such as one having a focal length of 28 mm, would have a much smaller opening at $f/2$ and hence would have an even less resolving ability.

36.43. **IDENTIFY** and **SET UP:** Let y be the separation between the two points being resolved and let s be their distance from the telescope. Then the limit of resolution corresponds to $1.22\,\dfrac{\lambda}{D} = \dfrac{y}{s}$.

EXECUTE: **(a)** Let the two points being resolved be the opposite edges of the crater, so y is the diameter of the crater. For the moon, $s = 3.8\times10^8$ m. $y = 1.22\lambda s/D$.

Hubble: $D = 2.4$ m and $\lambda = 380$ nm gives the maximum resolution, so $y = 73$ m.

Arecibo: $D = 305$ m and $\lambda = 0.75$ m; $y = 1.1\times10^{6}$ m.

(b) $s = \dfrac{yD}{1.22\lambda}$. Let $y \approx 0.30$ m (the size of a license plate).

$s = (0.30\text{ m})(2.4\text{ m})/[(1.22)(380\times10^{-9}\text{ m})] = 1600$ km.

EVALUATE: D/λ is much larger for the optical telescope and it has a much larger resolution even though the diameter of the radio telescope is much larger.

36.44. **IDENTIFY** and **SET UP:** Resolved by Rayleigh's criterion means angular separation θ of the objects equals $1.22\lambda/D$. The angular separation θ of the objects is their linear separation divided by their distance from the telescope.

EXECUTE: $\theta = \dfrac{250\times10^{3}\text{ m}}{5.93\times10^{11}\text{ m}}$, where 5.93×10^{11} m is the distance from the earth to Jupiter. Thus $\theta = 4.216\times10^{-7}$ rad.

Then $\theta = 1.22\dfrac{\lambda}{D}$ and $D = \dfrac{1.22\lambda}{\theta} = \dfrac{1.22(500\times10^{-9}\text{ m})}{4.216\times10^{-7}\text{ rad}} = 1.45$ m.

EVALUATE: This is a very large telescope mirror. The greater the angular resolution the greater the diameter the lens or mirror must be.

36.44. **IDENTIFY:** We can apply the equation for single-slit diffraction to the hair, with the thickness of the hair replacing the thickness of the slit.

SET UP: The dark fringes are located by $\sin\theta = m\dfrac{\lambda}{a}$. The first dark fringes are for $m = \pm1$. $y = x\tan\theta$ is the distance from the center of the screen, where $x = 125$ cm is the distance from the hair to the screen. The distance y from the center to one minimum is 2.61 cm.

EXECUTE: $\tan\theta = \dfrac{y}{x} = \dfrac{2.61\text{ cm}}{125\text{ cm}} = 0.02088$ so $\theta = 1.20°$. $a = \dfrac{\lambda}{\sin\theta} = \dfrac{632.8\times10^{-9}\text{ m}}{\sin 1.20°} = 30.2\ \mu$m.

EVALUATE: Although the thickness of human hairs can vary considerably, 30 μm is a reasonable thickness.

36.46. **IDENTIFY:** The two holes behave like double slits and cause the sound waves to interfere after they pass through the holes. The motion of the speakers causes a Doppler shift in the wavelength of the sound.

SET UP: The wavelength of the sound that strikes the wall is $\lambda = \lambda_0 - v_s T_s$, and destructive interference first occurs where $\sin\theta = \lambda/2$.

EXECUTE: **(a)** First find the wavelength of the sound that strikes the openings in the wall.

$\lambda = \lambda_0 - v_s T_s = v/f_s - v_s/f_s = (v - v_s)/f_s = (344\text{ m/s} - 80.0\text{ m/s})/(960\text{ Hz}) = 0.275$ m. Destructive interference first occurs where $d\sin\theta = \lambda/2$, which gives

$d = \lambda/(2\sin\theta) = (0.275\text{ m})/(2\sin 11.4°) = 0.636$ m.

(b) $\lambda = v/f = (344\text{ m/s})/(960\text{ Hz}) = 0.3583$ m.

$\sin\theta = \lambda/2d = (0.3583\text{ m})/[2(0.696\text{ m})] \rightarrow \theta = \pm14.9°$.

EVALUATE: The moving source produces sound of shorter wavelength than the stationary source, so the angles at which destructive interference occurs are smaller for the moving source than for the stationary source.

36.47. **IDENTIFY:** In the single-slit diffraction pattern, the intensity is a maximum at the center and zero at the dark spots. At other points, it depends on the angle at which one is observing the light.

SET UP: Dark fringes occur when $\sin\,\theta_m = m\lambda/a$, where $m = 1,\ 2,\ 3,\ \ldots,$ and the intensity is given by $I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \dfrac{\pi a\sin\theta}{\lambda}$.

EXECUTE: **(a)** At the maximum possible angle, $\theta = 90°$, so

$m_{\text{max}} = (a\sin 90°)/\lambda = (0.0250\text{ mm})/(632.8\text{ nm}) = 39.5.$

Since m must be an integer and $\sin\theta$ must be ≤ 1, $m_{\text{max}} = 39$. The total number of dark fringes is 39 on each side of the central maximum for a total of 78.

(b) The farthest dark fringe is for $m = 39$, giving

$\sin\theta_{39} = (39)(632.8\text{ nm})/(0.0250\text{ mm}) \Rightarrow \theta_{39} = \pm 80.8°.$

(c) The next closer dark fringe occurs at $\sin\theta_{38} = (38)(632.8\text{ nm})/(0.0250\text{ mm}) \Rightarrow \theta_{38} = 74.1°.$

The angle midway these two extreme fringes is $(80.8° + 74.1°)/2 = 77.45°$, and the intensity at this angle is $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi(0.0250\text{ mm})\sin(77.45°)}{632.8\text{ nm}} = 121.15\text{ rad}$, which gives

$$I = (8.50\text{W/m}^2)\left[\frac{\sin(121.15\text{ rad})}{121.15\text{ rad}}\right]^2 = 5.55\times10^{-4}\text{ W/m}^2.$$

EVALUATE: At the angle in part (c), the intensity is so low that the light would be barely perceptible.

36.48. **IDENTIFY:** $d = \dfrac{1}{N}$, so the bright fringes are located by $\dfrac{1}{N}\sin\theta = \lambda$.

SET UP: Red: $\dfrac{1}{N}\sin\theta_R = 750\text{ nm}$. Violet: $\dfrac{1}{N}\sin\theta_V = 380\text{ nm}$.

EXECUTE: **(a)** $\dfrac{\sin\theta_R}{\sin\theta_V} = \dfrac{75}{38}$. $\theta_R - \theta_V = 27.0°$, so $\theta_R = \theta_V + 27.0°$. $\dfrac{\sin(\theta_V + 27.0°)}{\sin\theta_V} = \dfrac{75}{38}$. Using the trigonometry identity from Appendix B for the sin of the sum of two angles gives

$$\frac{\sin\theta_V\,\cos 27.0° + \cos\theta_V\,\sin 27.0°}{\sin\theta_V} = \frac{75}{38} \quad\rightarrow\quad \cos 27.0° + \cot\theta_V\sin 27.0° = 75/38.$$

$\tan\theta_V = 0.4193 \Rightarrow \theta_V = 22.75°$ and $\theta_R = \theta_V + 27.0° = 22.75° + 27.0° = 49.75°$.

Using $\dfrac{1}{N}\sin\theta_R = 750\text{ nm}$ gives

$$N = \frac{\sin\theta_R}{750\text{ nm}} = \frac{\sin 49.75°}{750\times10^{-9}\text{ m}} = 1.02\times10^6\text{ lines/m} = 1.02\times10^4\text{ lines/cm}.$$

(b) The spectrum begins at 22.7 ° and ends at 49.7°.

EVALUATE: As N is increased, the angular range of the visible spectrum increases.

36.49. **IDENTIFY** and **SET UP:** $\sin\theta = \lambda/a$ locates the first dark band. In the liquid the wavelength changes and this changes the angular position of the first diffraction minimum.

EXECUTE: $\sin\theta_{\text{air}} = \dfrac{\lambda_{\text{air}}}{a}$; $\sin\theta_{\text{liquid}} = \dfrac{\lambda_{\text{liquid}}}{a}$. $\lambda_{\text{liquid}} = \lambda_{\text{air}}\left(\dfrac{\sin\theta_{\text{liquid}}}{\sin\theta_{\text{air}}}\right) = \lambda_{\text{air}}\dfrac{\sin 21.6°}{\sin 38.2°} = 0.5953\lambda_{\text{air}}$.

$\lambda_{\text{liquid}} = \lambda_{\text{air}}/n$, so $n = \lambda_{\text{air}}/\lambda_{\text{liquid}} = \dfrac{\lambda_{\text{air}}}{0.5953\lambda_{\text{air}}} = 1.68.$

EVALUATE: Light travels faster in air and n must be > 1.00. The smaller λ in the liquid reduces θ that located the first dark band.

36.50. **IDENTIFY:** The wavelength of the light is smaller under water than it is in air, which will affect the resolving power of the lens, by Rayleigh's criterion.

SET UP: The wavelength under water is $\lambda = \lambda_0/n$, and for small angles Rayleigh's criterion is $\theta = 1.22\lambda/D$.

EXECUTE: **(a)** In air the wavelength is $\lambda_0 = c/f = (3.00\times10^8 \text{ m/s})/(6.00\times10^{14} \text{ Hz}) = 5.00\times10^{-7} \text{ m}$. In water the wavelength is $\lambda = \lambda_0/n = (5.00\times10^{-7}\text{ m})/1.33 = 3.76\times10^{-7}\text{ m}$. With the lens open all the way, we have $D = f/2.8 = (35.0 \text{ mm})/2.80 = (0.0350 \text{ m})/2.80$. In the water, we have

$\sin\theta \approx \theta = 1.22\lambda/D = (1.22)(3.76\times10^{-7}\text{ m})[(0.0350 \text{ m})/2.80] = 3.67\times10^{-5} \text{ rad}.$

Calling w the width of the resolvable detail, we have

$\theta = w/x \to w = x\theta = (2750 \text{ mm})(3.67\times10^{-5}\text{ rad}) = 0.101 \text{ mm}.$

(b) $\theta = 1.22\lambda/D = (1.22)(5.00\times10^{-7}\text{ m})/[(0.0350 \text{ m})/2.80] = 4.88\times10^{-5} \text{ rad}.$

$w = x\theta = (2750 \text{ mm})(4.88\times10^{-5} \text{ rad}) = 0.134 \text{ mm}.$

EVALUATE: Due to the reduced wavelength underwater, the resolution of the lens is better under water than in air.

36.51. **IDENTIFY:** $I = I_0\left(\dfrac{\sin\gamma}{\gamma}\right)^2$. The maximum intensity occurs when the derivative of the intensity function with respect to γ is zero.

SET UP: $\dfrac{d\sin\gamma}{d\gamma} = \cos\gamma$. $\dfrac{d}{d\gamma}\left(\dfrac{1}{\gamma}\right) = -\dfrac{1}{\gamma^2}$.

EXECUTE: **(a)**

$$\frac{dI}{d\gamma} = I_0\frac{d}{d\gamma}\left(\frac{\sin\gamma}{\gamma}\right)^2 = 2\left(\frac{\sin\gamma}{\gamma}\right)\left(\frac{\cos\gamma}{\gamma} - \frac{\sin\gamma}{\gamma^2}\right) = 0.\ \frac{\cos\gamma}{\gamma} - \frac{\sin\gamma}{\gamma^2} \Rightarrow \gamma\cos\gamma = \sin\gamma \Rightarrow \gamma = \tan\gamma.$$

(b) The graph in Figure 36.51 is a plot of $f(\gamma) = \gamma - \tan\gamma$. When $f(\gamma)$ equals zero, there is an intensity maximum. Getting estimates from the graph, and then using trial and error to narrow in on the value, we find that the two smallest values of γ are $\gamma = 4.49$ rad and 7.73 rad.

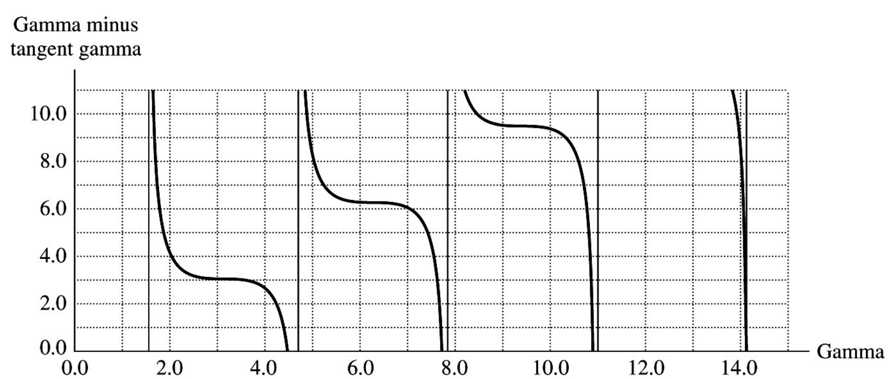


Figure 36.51

(c) The minima occur when $\gamma = \pm m\pi$, where $m = 1, 2, 3, \ldots$, so the first three positive ones are $\gamma = \pi$, 2π, and 3π. From part (b), the first maximum is at $\gamma = 4.49$ rad. Midway between the first and second minima is $\gamma = \pi + \pi/2 = 3\pi/2 = 4.71$ rad. Clearly 4.49 rad $\neq$ 4.71 rad, so this maximum is not midway between the adjacent minima. The second maximum is at $\gamma = 7.73$ rad. Midway between the second and third minima is $\gamma = 2\pi + \pi/2 = 5\pi/2 = 7.85$ rad. Since $7.73 \neq 7.85$, the second maximum is not midway between the adjacent minima.

(d) Using $a = 12\,\lambda$, we have $\gamma = \dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi(12\lambda)\sin\theta}{\lambda} = 12\pi\sin\theta$. Using this we get the following:

First minimum: $\gamma = \pi$, so $\pi = 12\pi\sin\theta \quad \rightarrow \quad \theta = 4.78°$.

First maximum: $\gamma = 2\pi$, so $2\pi = 12\pi\sin\theta \quad \rightarrow \quad \theta = 6.84°$.

Second minimum: $\gamma = 3\pi$, so $3\pi = 12\pi\sin\theta \quad \rightarrow \quad \theta = 9.59°$.

Midway between these minima is $\theta = (9.59° + 4.78°)/2 = 7.19°$. The first maximum is at $\theta = 6.84°$, and since $6.84° \neq 7.19°$, the first minimum is *not* midway between the adjacent minima.

EVALUATE: $\gamma = 0$ is the central maximum. The three values of γ we found are the locations of the first two secondary maxima. The first three minima are at $\gamma = 3.14$ rad, 6.28 rad, and 9.42 rad. The maxima are between adjacent minima, but not precisely midway between them.

36.52. **(a) IDENTIFY** and **SET UP:** The angular position of the first minimum is given by $a\sin\theta = m\lambda$, with $m = 1$. The distance of the minimum from the center of the pattern is given by $y = x\tan\theta$.

EXECUTE: $\sin\theta = \dfrac{\lambda}{a} = \dfrac{540\times10^{-9}\text{ m}}{0.360\times10^{-3}\text{ m}} = 1.50\times10^{-3}$; $\theta = 1.50\times10^{-3}$ rad.

$y_1 = x\tan\theta = (1.20\text{ m})\tan(1.50\times10^{-3}\text{ rad}) = 1.80\times10^{-3}\text{ m} = 1.80\text{ mm}.$

(Note that θ is small enough for $\theta \approx \sin\theta \approx \tan\theta$, and $y_m = x\dfrac{m\lambda}{a}$ applies.)

(b) IDENTIFY and **SET UP:** Find the phase angle β where $I = I_0/2$. Then use $\beta = \left(\dfrac{2\pi}{\lambda}\right)a\sin\theta$ to solve for θ and $y = x\tan\theta$ to find the distance.

EXECUTE: $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$ gives that $I = \tfrac{1}{2}I_0$ when $\beta = 2.78$ rad.

$\beta = \left(\dfrac{2\pi}{\lambda}\right)a\sin\theta$, so $\sin\theta = \dfrac{\beta\lambda}{2\pi a}$.

$$y = x\tan\theta \approx x\sin\theta \approx \frac{\beta\lambda x}{2\pi a} = \frac{(2.78\text{ rad})(540\times10^{-9}\text{ m})(1.20\text{ m})}{2\pi(0.360\times10^{-3}\text{ m})} = 7.96\times10^{-4}\text{ m} = 0.796\text{ mm}.$$

EVALUATE: The point where $I = I_0/2$ is not midway between the center of the central maximum and the first minimum.

36.53. **IDENTIFY:** Heating the plate causes it to expand, which widens the slit. The increased slit width changes the angles at which destructive interference occurs.

SET UP: First minimum is at angle θ given by $\tan\theta = \dfrac{(2.75\times10^{-3}/2)}{0.620}$. Therefore, θ is small and the equation $y_m = x\dfrac{m\lambda}{a}$ is accurate. The width of the central maximum is $w = \dfrac{2x\lambda}{a}$. The change in slit width is $\Delta a = a\alpha\Delta T$.

EXECUTE: $dw = 2x\lambda\left(-\dfrac{da}{a^2}\right) = -\dfrac{2x\lambda}{a^2}da = -\dfrac{w}{a}da$. Therefore, $\Delta w = -\dfrac{w}{a}\Delta a$. The equation for thermal expansion says $\Delta a = a\alpha\Delta T$, so $\Delta w = -w\alpha\Delta T = -(2.75\text{ mm})(2.4\times10^{-5}\text{ K}^{-1})(500\text{ K}) = -0.033\text{ mm}$.

When the temperature of the plate increases, the width of the slit increases and the width of the central maximum decreases.

EVALUATE: The fractional change in the width of the central maximum is $\dfrac{0.033\text{ mm}}{2.75\text{ mm}} = 1.2\%$. This is small, but observable.

36.54. **IDENTIFY:** The wavelength of the helium spectral line from the receding galaxy will be different from the spectral line on earth due to the Doppler shift in the light from the galaxy.

SET UP: $d\sin\theta = m\lambda$. $\sin\theta_{\text{lab}} = \dfrac{2\lambda_{\text{lab}}}{d}$. $\sin\theta_{\text{galaxy}} = \dfrac{2\lambda_{\text{galaxy}}}{d}$. $\sin\theta_{\text{galaxy}} = \sin\theta_{\text{lab}}\left(\dfrac{\lambda_{\text{lab}}}{\lambda_{\text{galaxy}}}\right)$. The Doppler formula says $f_{\text{R}} = \sqrt{\dfrac{c-v}{c+v}}f_{\text{S}}$. Using $f = \dfrac{c}{\lambda}$, we have $\dfrac{1}{\lambda_{\text{R}}} = \dfrac{1}{\lambda_{\text{S}}}\sqrt{\dfrac{c-v}{c+v}}$. Since the lab is the receiver R and the galaxy is the source S, this becomes $\lambda_{\text{lab}} = \lambda_{\text{galaxy}}\sqrt{\dfrac{c+v}{c-v}}$.

EXECUTE: $\sin\theta_{\text{galaxy}} = \sin\theta_{\text{lab}}\sqrt{\dfrac{c+v}{c-v}} = \sin(18.9^\circ)\sqrt{\dfrac{2.998\times10^8\ \text{m/s} + 2.65\times10^7\ \text{m/s}}{2.998\times10^8\ \text{m/s} - 2.65\times10^7\ \text{m/s}}}$ which gives $\theta_{\text{galaxy}} = 20.7^\circ$.

EVALUATE: The galaxy is moving away, so the wavelength of its light will be lengthened, which means that the angle should be increased compared to the angle from light on earth, as we have found.

36.55. **IDENTIFY** and **SET UP:** The condition for an intensity maximum is $d\sin\theta = m\lambda$, $m = 0, \pm1, \pm2, \ldots$. Third order means $m = 3$. The longest observable wavelength is the one that gives $\theta = 90^\circ$ and hence $\sin\theta = 1$.

EXECUTE: 9200 lines/cm so 9.2×10^5 lines/m and $d = \dfrac{1}{9.2\times10^5}\ \text{m} = 1.087\times10^{-6}\ \text{m}$.

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1.087\times10^{-6}\ \text{m})(1)}{3} = 3.6\times10^{-7}\ \text{m} = 360\ \text{nm}.$$

EVALUATE: The longest wavelength that can be obtained decreases as the order increases.

36.56. **IDENTIFY:** We are dealing with single-slit diffraction for sound waves.
SET UP and **EXECUTE:** **(a)** Estimate: Door width = 75 cm.
(b) We want *f*. $f = v/\lambda$ = (344 m/s)/(0.75 m) = 460 Hz.

(c) We want *f* for diffraction at ±20°. $a\sin\theta = \lambda = v/f$, so $f = \dfrac{v}{a\sin\theta} = \dfrac{344\ \text{m/s}}{(0.75\ \text{m})\sin 20^\circ} = 1.3\ \text{kHz}$.

(d) It is fairly good. A frequency of 1.3 kHz is typical of many ordinary sounds and much music, so not very much sound would bend around the edge of a doorway.
(e) Since $a\sin\theta = \lambda$, a small wavelength would undergo less bending, so higher frequency sounds would bend less than lower frequency sounds.
EVALUATE: **(f)** It is somewhat observable. Inside the house there are reflections from nearby walls. Outside has more open space with fewer reflections.

36.57. **IDENTIFY:** The maxima are given by $d\sin\theta = m\lambda$. We need $\sin\theta = \dfrac{m\lambda}{d} \le 1$ in order for all the visible wavelengths to be seen.

SET UP: For 650 slits/mm $\Rightarrow d = \dfrac{1}{6.50\times10^5\ \text{m}^{-1}} = 1.53\times10^{-6}\ \text{m}$. The visible spectrum is approximately between 380 nm and 750 nm.

EXECUTE: $\lambda_1 = 3.8\times10^{-7}\text{m}$: $m = 1$: $\dfrac{\lambda_1}{d} = 0.247$; $m = 2$: $\dfrac{2\lambda_1}{d} = 0.494$; $m = 3$: $\dfrac{3\lambda_1}{d} = 0.741$.

$\lambda_2 = 7.5\times10^{-7}\text{m}$: $m = 1$: $\dfrac{\lambda_2}{d} = 0.4875$; $m = 2$: $\dfrac{2\lambda_2}{d} = 0.975$; $m = 3$: $\dfrac{3\lambda_2}{d} = 1.46$. So, the third order does not contain the violet end of the spectrum, and therefore only the first- and second-order diffraction patterns contain all colors of the spectrum.

EVALUATE: θ for each maximum is larger for longer wavelengths.

36.58. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: θ is small, so $\sin\theta \approx \dfrac{\Delta x}{R}$, where Δx is the size of the detail and $R = 7.2\times10^{8}$ ly.

$1\text{ ly} = 9.41\times10^{12}$ km. $\lambda = c/f$.

EXECUTE:

$$\sin\theta = 1.22\frac{\lambda}{D} \approx \frac{\Delta x}{R} \Rightarrow \Delta x = \frac{1.22\lambda R}{D} = \frac{(1.22)cR}{Df} = \frac{(1.22)(3.00\times10^{5}\text{ km/s})(7.2\times10^{8}\text{ ly})}{(77.000\times10^{3}\text{ km})(1.665\times10^{9}\text{ Hz})} = 2.06\text{ ly}.$$

$(9.41\times10^{12}\text{ km/ly})(2.06\text{ ly}) = 1.94\times10^{13}$ km.

EVALUATE: $\lambda = 18$ cm. λ/D is very small, so $\dfrac{\Delta x}{R}$ is very small. Still, R is very large and Δx is many orders of magnitude larger than the diameter of the sun.

36.59. **IDENTIFY:** This problem is about x-ray diffraction applied to electrons.

SET UP: Eq. (36.16): $2d\sin\theta_m = m\lambda$ for constructive interference.

EXECUTE: **(a)** Using Fig. 36.22a in the textbook, we see that $2\theta + 50° = 180°$, so $\theta = 65°$.

(b) We want λ for electrons. $2d\sin\theta = \lambda$ = 2(0.091 nm)(sin 65°) = 0.16 nm.

(c) We want the electron speed. $K = \dfrac{1}{2}mv^2$ = 54 eV = 8.64×10^{-18} J. $v = \sqrt{2K/m}$. Using our K gives $v = 4.4\times10^{6}$ m/s.

(d) We want the frequency of the electron. $f = v/\lambda$. Using the results of (b) and (c), we have $f = 2.6\times10^{16}$ Hz.

(e) We want h. $E = hf$ gives $h = E/f$. Using E from part (c) and f from part (d) gives $h = 3.3\times10^{-34}\text{ J}\cdot\text{s}$.

(f) We want h. $v = \dfrac{1}{2}v_{\text{cl}}$, so $f = \dfrac{\frac{1}{2}v_{\text{cl}}}{\lambda}$. Thus $h = \dfrac{E}{f} = \dfrac{E}{\frac{1}{2}v_{\text{cl}}} = 2\dfrac{E}{v_{\text{cl}}/\lambda} = 2(3.3\times10^{-34}\text{ J}\cdot\text{s}) =$ $6.6\times10^{-34}\text{ J}\cdot\text{s}$.

EVALUATE: **(g)** The established value is $h = 6.6\times10^{-34}\text{ J}\cdot\text{s}$ so our result agrees well.

36.60. **IDENTIFY:** The resolution of the eye is limited because light diffracts as it passes through the pupil. The size of the pupil determines the resolution.

SET UP: The smallest angular separation that can be resolved is $\theta_{\text{res}} = 1.22\dfrac{\lambda}{D}$. The angular size of the object is its height divided by its distance from the eye.

EXECUTE: **(a)** The angular size of the object is $\theta = \dfrac{50\times10^{-6}\text{ m}}{25\times10^{-2}\text{ m}} = 2.0\times10^{-4}$ rad.

$\theta_{\text{res}} = 1.22\dfrac{\lambda}{D} = 1.22\left(\dfrac{550\times10^{-9}\text{ m}}{2.0\times10^{-3}\text{ m}}\right) = 3.4\times10^{-4}$ rad. $\theta < \theta_{\text{res}}$ so the object cannot be resolved.

(b) $\theta_{\text{res}} = \dfrac{y}{s}$ and $y = s\theta_{\text{res}} = (25\text{ cm})(3.4\times10^{-4}\text{ rad}) = 8.5\times10^{-3}\text{ cm} = 85\ \mu\text{m}$.

(c) $\theta = \theta_{\text{res}} = 3.4\times10^{-4}\text{ rad} = 0.019° = 1.1\text{ min}$. This is very close to the experimental value of 1 min.

(d) Diffraction is more important.

EVALUATE: We could not see any clearer if our retinal cells were much smaller than they are now because diffraction is more important in limiting the resolution of our vision.

36.61. **IDENTIFY:** A double-slit bright fringe is missing when it occurs at the same angle as a double-slit dark fringe.

SET UP: Single-slit diffraction dark fringes occur when $a\sin\theta = m\lambda$, and double-slit interference bright
fringes occur when $d\ \sin\theta = m'\lambda$.

EXECUTE: **(a)** The angle at which the first bright fringe occurs is given by
$\tan\theta_1 = (1.53\text{ mm})/(2500\text{ mm}) \Rightarrow \theta_1 = 0.03507°$. $d\sin\theta_1 = \lambda$ and
$d = \lambda/(\sin\theta_1) = (632.8\text{ nm})/\sin(0.03507°) = 0.00103\text{ m} = 1.03\text{ mm}$.

(b) The 7^{th} double-slit interference bright fringe is just cancelled by the 1^{st} diffraction dark fringe, so $\sin\theta_{\text{diff}} = \lambda/a$ and $\sin\theta_{\text{interf}} = 7\lambda/d$.
The angles are equal, so $\lambda/a = 7\lambda/d \to a = d/7 = (1.03\text{ mm})/7 = 0.148\text{ mm}$.

EVALUATE: We can generalize that if $d = na$, where n is a positive integer, then every n^{th} double-slit bright fringe will be missing in the pattern.

36.62. **IDENTIFY:** Light from the slit produces a diffraction pattern on the screen.

SET UP: The first dark fringe occurs when $a\ \sin\ \theta_1 = \lambda$, and $\tan\theta_1 = (w/2)/R = w/2R$, where R is the distance from the slit to the screen.

EXECUTE: **(a)** If θ_1 is small, we can use the small-angle approximation $\sin\theta_1 \approx \tan\theta_1 = w/2R$. In that case, $aw = 2R\lambda$, which is constant. But if θ_1 is large, we cannot use $\sin\theta_1 \approx \tan\theta_1$, and wa is not constant. Figure 36.62 shows the graph of aw versus a for the data in the problem, and its curvature for small a shows that wa is not constant for small values of a.

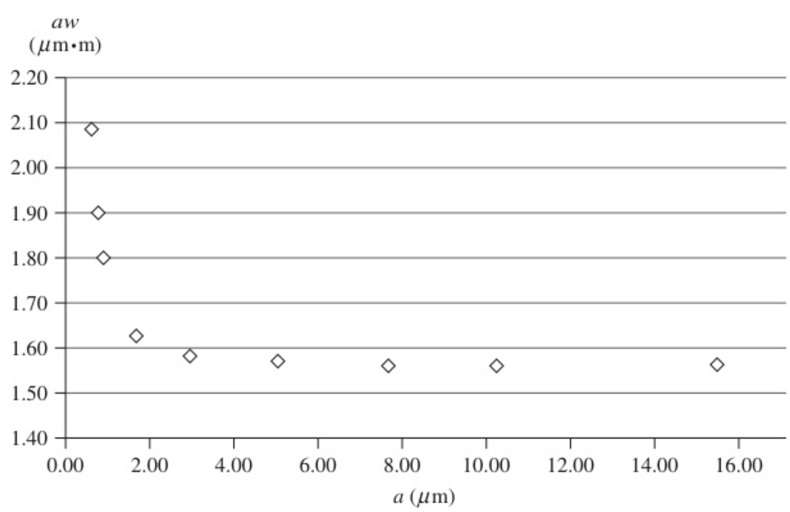


Figure 36.62

(b) We see that our graph flattens out for large values of a and $wa \to 1.56\times10^{-6}\text{ m}^2$. This means we can apply the small-angle approximation. In that case, $aw = 2R\lambda$, so
$\lambda = aw/2R = (1.56\times10^{-6}\text{ m}^2)/[2(1.50\text{ m})] = 5.20\times10^{-7}\text{ m} = 520\text{ nm}$.

(c) (i) Use $a\sin\theta_1 = \lambda$ with $a = 0.78\ \mu\text{m} = 780\text{ nm}$.

$(780\text{ nm})\sin\theta_1 = 520\text{ nm} \quad \to \quad \theta_1 = 42°$.

(ii) Now use $a = 15.60\ \mu\text{m} = 15{,}600\text{ nm}$.

$(15{,}600\text{ nm})\sin\theta_1 = 520\text{ nm} \quad \to \quad \theta_1 = 1.91°$.

EVALUATE: In part (c), we could use the small-angle approximation in (ii) but not in (i).

36.63. **IDENTIFY:** The hole produces a diffraction pattern on the screen.

SET UP: The first dark ring occurs when $D \sin\theta_1 = 1.22\,\lambda$, and $\tan\theta_1 = r/x$.

EXECUTE: **(a)** Solving for D gives $D = (1.22\,\lambda)/(\sin\theta_1)$. First use $\tan\theta_1 = r/x$ to find θ_1, and then calculate D.

First set: $\tan\theta_1 = r/x = (5.6 \text{ cm})/(100 \text{ cm}) \rightarrow \theta_1 = 3.205°$.

$D = (1.22)(562 \text{ nm})/[\sin(3.205°)] = 1.226 \times 10^4$ nm.

Second set: $\tan\theta_1 = r/x = (8.5 \text{ cm})/(150 \text{ cm}) \rightarrow \theta_1 = 3.2433°$.

$D = (1.22)(562 \text{ nm})/[\sin(3.2433°)] = 1.212 \times 10^4$ nm.

Third set: $\tan\theta_1 = r/x = (11.6 \text{ cm})/(200 \text{ cm}) \rightarrow \theta_1 = 3.3194°$.

$D = (1.22)(562 \text{ nm})/[\sin(3.3194°)] = 1.184 \times 10^4$ nm.

Fourth set: $\tan\theta_1 = r/x = (14.1 \text{ cm})/(250 \text{ cm}) \rightarrow \theta_1 = 3.2281°$.

$D = (1.22)(562 \text{ nm})/[\sin(3.2281°)] = 1.218 \times 10^4$ nm.

Taking the average gives

$D_{av} = [(1.226 + 1.212 + 1.184 + 1.218) \times 10^4 \text{ nm}]/4 = 1.21 \times 10^4 \text{ nm} = 12.1\ \mu\text{m}$.

(b) $\sin\theta_2 = 2.23\,\lambda/D = (2.23)(562 \text{ nm})/(1.21 \times 10^4 \text{ nm}) \rightarrow \theta_2 = 5.945°$.

$r_2 = x_2 \tan\theta_2 = (1.00 \text{ m})\tan(5.945°) = 0.104 \text{ m} = 10.4$ cm.

$\sin\theta_3 = 3.24\,\lambda/D = (3.24)(562 \text{ nm})/(1.21 \times 10^4 \text{ nm}) \rightarrow \theta_3 = 8.6551°$.

$r_3 = x_3 \tan\theta_3 = (1.00 \text{ m})\tan(8.6551°) = 0.152 \text{ m} = 15.2$ cm.

EVALUATE: This is a very small hole, but its diameter is still over 20 times the wavelength of the light passing through it.

36.64. **IDENTIFY:** The liquid reduces the wavlength of the light (compared to its value in air), and the scratch causes light passing through it to undergo single-slit diffraction.

SET UP: $\sin\theta = \dfrac{\lambda}{a}$, where λ is the wavelength in the liquid. $n = \dfrac{\lambda_{\text{air}}}{\lambda}$.

EXECUTE: $\tan\theta = \dfrac{(22.4/2) \text{ cm}}{30.0 \text{ cm}}$ and $\theta = 20.47°$.

$\lambda = a\sin\theta = (1.25\times10^{-6} \text{ m})\sin 20.47° = 4.372\times10^{-7} \text{ m} = 437.2 \text{ nm}$. $n = \dfrac{\lambda_{\text{air}}}{\lambda} = \dfrac{612 \text{ nm}}{437.2 \text{ nm}} = 1.40$.

EVALUATE: $n > 1$, as it must be, and $n = 1.40$ is reasonable for many transparent films.

36.65. **IDENTIFY:** This problem involves single slit-diffraction and double-slit interference. As the membranes move along, the thickness of the slits varies because the membranes overlap. So we are dealing with a double slit having a variable slit width.

SET UP: For single-slit dark spots, $a\sin\theta_m = m\lambda\ (m = \pm1, \pm2, \cdots)$ and for double-slit bright spots $d\sin\theta_m = m\lambda\ (m = 0, \pm1, \pm2, \cdots)$.

EXECUTE: **(a)** We want the slit spacing d. At t = 1.00 s the edges of the screen are totally dark but the rest of the screen contains a series of 19 spots of nearly equal brightness. These are the double-slit interference spots. There is one in the middle of the screen and 9 on each side of the center. The spots that would be at the edges of the screen are missing, and they are the $m = \pm10$ spots. Therefore $\theta_{10} = \pm30.0°$ from the center. For double-slit bright spots, we have $d\sin\theta_{10} = 10\lambda$. Solving for d gives

$$d = \frac{10\lambda}{\sin\theta_{10}} = \frac{10(532 \text{ nm})}{\sin(30.0°)} = 10.6\ \mu\text{m}.$$

(b) We want the speed v of the membranes. The width of the slits at 1.00 s is given by $a_1\sin\theta_1 = \lambda$. We use m = 1 because this is the first single-slit dark spot. This dark spot occurs at the angle for which the m = 10 double-slit bright spot would have occurred, so $\theta_1 = 30°$. Solving for a_1 gives

$a_1 = \dfrac{\lambda}{\sin\theta_{10}} = \dfrac{532 \text{ nm}}{\sin(30.0°)} = 1064 \text{nm}$. The slit width increased from 0 nm to 1064 nm in 1.00 s. The membranes move with equal speed in opposite directions, so each one traveled half this distance in that time. Therefore $vt_1 = a/2$, which gives $v = a_1/2t_1$ = (1064 nm)/[2(1.00 s)] = 532 nm/s.

(c) We want the maximum slit width a. At $t = 0$ the slits are closed and at $t = 3.00$ s they are closed again. So at $t = 1.50$ s they reach their maximum width. During this time each membrane travels a distance of $a/2$. Thus $vt = a/2$, which gives $a = 2vt = 2(532 \text{ nm/s})(1.50 \text{ s}) = 1.60\ \mu\text{m}$.

(d) We want the time at which the $m = \pm 9$ spots suddenly disappear. The slits are now thicker than they were at $t = 1.00$ s so the $m = 1$ single-slit dark spot has moved closer to the center and now cancels the $m = \pm 9$ double-slit bright spots. To find this slit width a_2, consider the double-slit pattern. The $m = 9$ bright spot would be at θ_9 which is given by $d\sin\theta_9 = 9\lambda$. From this we get

$\sin\theta_9 = \dfrac{9\lambda}{d} = \dfrac{9(532 \text{ nm})}{10.6\,\mu\text{m}} = 0.4517$. θ_9 is now the angle of the first single-slit dark spot, so $a_2 \sin\theta_9 = \lambda$, which gives $a_2 = \dfrac{\lambda}{\sin\theta_9} = \dfrac{532 \text{ nm}}{0.4517} = 1178 \text{nm}$. The slit width started at 0 at $t = 0$ and is now equal to 1178 nm. As in part (b), $vt_2 = a_2/2$, so $t_2 = a_2/2v$ = (1178 nm)/[2(532 nm/s)] = 1.11 s.

(e) We want the intensity I at the first double-slit bright spot. At 1.50 s the slits are at their maximum width of 1.60 μm. The intensity is $I = I_0\cos^2(\phi/2)\left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$. At the first bright spot $\cos(\phi/2) = 1$ and $d\sin\theta = \lambda$, so $\sin\theta = \lambda/d$. Using this to find $\beta/2$ gives

$\beta/2 = \dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi a(\lambda/d)}{\lambda} = \dfrac{\pi a}{d} = \dfrac{\pi(1.60\,\mu\text{m})}{10.6\,\mu\text{m}} = 0.4742 \text{ rad} = 27.17°$. The intensity is

$I = I_0\left[\dfrac{\sin(27.19°)}{0.4742 \text{ rad}}\right]^2 = 0.927 I_0 = 92.7\%$ of I_0.

(f) We want the angle at which the first double-slit bright spot occurs. This is the first such spot, so $d\sin\theta = \lambda/d$ = (532 nm)/(10.6 μm) = 0.0519, so θ = 2.88°.

EVALUATE: A device like this would make an interesting demonstration in a physics lecture or at a science fair.

36.66. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** Each source can be thought of as a traveling wave evaluated at $x = R$ with a maximum amplitude of E_0. However, each successive source will pick up an extra phase from its respective pathlength to point P. $\phi = 2\pi\left(\dfrac{d \sin\theta}{\lambda}\right)$ which is just 2π, the maximum phase, scaled by whatever fraction the path difference, $d \sin\theta$, is of the wavelength, λ. By adding up the contributions from each source (including the accumulating phase difference) this gives the expression provided.

(b) $e^{i(kR-\omega t+n\phi)} = \cos(kR-\omega t+n\phi) + i\sin(kR-\omega t+n\phi)$. The real part is just $\cos(kR-\omega t+n\phi)$. So, $\text{Re}\left[\sum\limits_{n=0}^{N-1} E_0 e^{i(kR-\omega x+n\phi)}\right] = \sum\limits_{n=0}^{N-1} E_0\cos(kR-\omega x+n\phi)$. (Note: Re means "the real part of…."). But this is just $E_0\cos(kR-\omega t) + E_0\cos(kR-\omega t+\phi) + E_0\cos(kR-\omega t+2\phi) + \cdots + E_0\cos(kR-\omega t+(N-1)\phi)$.

(c) $\sum\limits_{n=0}^{N-1} E_0\, e^{i(kR-\omega t+n\phi)} = E_0 \sum\limits_{n=0}^{N-1} e^{-i\omega t}\, e^{+ikR} e^{in\phi} = E_0 e^{i(kR-\omega t)} \sum\limits_{n=0}^{N-1} e^{in\phi}$. $\sum\limits_{n=0}^{\infty} e^{in\phi} = \sum\limits_{n=0}^{N-1} (e^{i\phi})^n$. But recall $\sum\limits_{n=0}^{N-1} x^n = \dfrac{x^N - 1}{x-1}$. Putting everything together:

$$\sum_{n=0}^{N-1} E_0 e^{i(kR-\omega t+n\phi)} = E_0 e^{i(kR-\omega t+(N-1)\phi/2)} \frac{(e^{iN\phi/2} - e^{-iN\phi/2})}{(e^{i\phi/2} - e^{-i\phi/2})}$$

$$= E_0[\cos(kR - \omega t + (N-1)\phi/2) + i\sin(kR - \omega t + (N-1)\phi/2)]\left[\frac{\cos N\phi/2 + \sin N\phi/2 - \cos N\phi/2 + i\sin N\phi/2}{\cos\phi/2 + i\sin\phi/2 - \cos\phi/2 + i\sin\phi/2}\right].$$

Taking only the real part gives $\Rightarrow E_0\cos(kR - \omega t + (N-1)\phi/2)\dfrac{\sin(N\phi/2)}{\sin\phi/2} = E.$

(d) $I = |E|^2_{\text{av}} = I_0\dfrac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}$. (The $\cos^2$ term goes to $\frac{1}{2}$ in the time average and is included in the definition of I_0.) $I_0 \propto \dfrac{E_0^2}{2}$.

EVALUATE: **(e)** $N = 2$. $I = I_0\dfrac{\sin^2(2\phi/2)}{\sin^2\phi/2} = \dfrac{I_0(2\sin\phi/2\cos\phi/2)^2}{\sin^2\phi/2} = 4I_0\cos^2\dfrac{\phi}{2}$. Looking at Eq. (35.9), $I'_0 \propto 2E_0^2$ but for us $I_0 \propto \dfrac{E_0^2}{2} = \dfrac{I'_0}{4}$.

36.67. **IDENTIFY** and **SET UP:** From Problem 36.66, $I = I_0\dfrac{\sin^2(N\phi/2)}{\sin^2\phi/2}$. Use this result to obtain each result specified in the problem.

EXECUTE: **(a)** $\lim\limits_{\phi\to 0} I \to \dfrac{0}{0}$. Use 1'Hôpital's rule: $\lim\limits_{\phi\to 0}\dfrac{\sin(N\phi/2)}{\sin\phi/2} = \lim\limits_{\phi\to 0}\left(\dfrac{N/2}{1/2}\right)\dfrac{\cos(N\phi/2)}{\cos(\phi/2)} = N$. So $\lim\limits_{\phi\to 0} I = N^2 I_0$.

(b) The location of the first minimum is when the numerator first goes to zero at $\dfrac{N}{2}\phi_{\text{min}} = \pi$ or $\phi_{\text{min}} = \dfrac{2\pi}{N}$. The width of the central maximum goes like $2\phi_{\text{min}}$, so it is proportional to $\dfrac{1}{N}$.

(c) Whenever $\dfrac{N\phi}{2} = n\pi$ where n is an integer, the numerator goes to zero, giving a minimum in intensity. That is, I is a minimum wherever $\phi = \dfrac{2n\pi}{N}$. This is true assuming that the denominator doesn't go to zero as well, which occurs when $\dfrac{\phi}{2} = m\pi$, where m is an integer. When both go to zero, using the result from part(a), there is a maximum. That is, if $\dfrac{n}{N}$ is an integer, there will be a maximum.

(d) From part (c), if $\dfrac{n}{N}$ is an integer we get a maximum. Thus, there will be $N-1$ minima. (Places where $\dfrac{n}{N}$ is not an integer for fixed N and integer n.) For example, $n = 0$ will be a maximum, but $n = 1, 2\ldots, N-1$ will be minima with another maximum at $n = N$.

(e) Between maxima $\dfrac{\phi}{2}$ is a half-integer multiple of $\pi\left(\text{i.e.,}\dfrac{\pi}{2},\dfrac{3\pi}{2}\text{etc.}\right)$ and if N is odd then $\dfrac{\sin^2(N\phi/2)}{\sin^2\phi/2} \to 1$, so $I \to I_0$.

EVALUATE: These results show that the principal maxima become sharper as the number of slits is increased.

36.68. IDENTIFY and SET UP: Follow the steps specified in the problem.

EXECUTE: **(a)** From the segment dy', the fraction of the amplitude of E_0 that gets through is

$E_0\left(\frac{dy'}{a}\right) \Rightarrow dE = E_0\left(\frac{dy'}{a}\right)\sin(kx-\omega t).$

(b) The path difference between each little piece is

$y'\sin\theta \Rightarrow kx = k(D-y'\sin\theta) \Rightarrow dE = \frac{E_0 dy'}{a}\sin(k(D-y'\sin\theta)-\omega t).$ This can be rewritten as

$dE = \frac{E_0 dy'}{a}(\sin(kD-\omega t)\cos(ky'\sin\theta)+\sin(ky'\sin\theta)\cos(kD-\omega t)).$

(c) So the total amplitude is given by the integral over the slit of the above.

$\Rightarrow E = \int_{-a/2}^{a/2} dE = \frac{E_0}{a}\int_{-a/2}^{a/2} dy'\,(\sin(kD-\omega t)\cos(ky'\sin\theta)+\sin(ky'\sin\theta)\cos(kD-\omega t)).$

But the second term integrates to zero, so we have:

$E = \frac{E_0}{a}\sin(kD-\omega t)\int_{-a/2}^{a/2} dy'(\cos(ky'\sin\theta)) = E_0\sin(kD-\omega t)\left[\left(\frac{\sin(ky'\sin\theta)}{ka\sin\theta/2}\right)\right]_{-a/2}^{a/2}$

$\Rightarrow E = E_0\sin(kD-\omega x)\left(\frac{\sin(ka(\sin\theta)/2)}{ka(\sin\theta)/2}\right) = E_0\sin(kD-\omega x)\left(\frac{\sin(\pi a(\sin\theta)/\lambda)}{\pi a(\sin\theta)/\lambda}\right).$

At $\theta = 0, \frac{\sin[\ldots]}{[\ldots]} = 1 \Rightarrow E = E_0\sin(kD-\omega x).$

(d) Since $I \propto E^2 \Rightarrow I = I_0\left(\frac{\sin(ka(\sin\theta)/2)}{ka(\sin\theta)/2}\right)^2 = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$, where we have used

$I_0 = E_0^2\sin^2(kx-\omega t).$

EVALUATE: The same result for $I(\theta)$ is obtained as was obtained using phasors.

36.69. IDENTIFY and SET UP: For an interference maxima, $2d\sin\theta = m\lambda_0/n.$ The microspheres are in water, so we use n = 1.3. Do a rough calculation to estimate the distance between microspheres.

EXECUTE: Use the numbers in the problem and $n = 1.3.$ Solving $2d\sin\theta = m\lambda_0/n$ for d gives

$d = \frac{m\lambda_0}{2n\sin\theta} = \frac{(1)(650\text{ nm})}{2(1.3)\sin 39°} \approx 400$ nm. This distance is much greater than the spacing of atoms in a crystal and is comparable to the wavelength of visible light. Therefore choice (d) is correct.

EVALUATE: The atoms in a crystal are tightly bound by the electric force between them and are therefore very close together.

36.70. IDENTIFY and SET UP: For an interference maxima, $2d\sin\theta = m\lambda_0/n.$ The microspheres are in water, so we use n = 1.33.

EXECUTE: $d = \frac{m\lambda_0}{2n\sin\theta} = \frac{(1)(650\text{ nm})}{2(1.33)\sin 39°} \approx 390$ nm. Choice (a) is the correct one.

EVALUATE: We are assuming that the microspheres do not affect the index of refraction of the water.

36.71. IDENTIFY and SET UP: For an interference maxima, $2d\sin\theta = m\lambda_0/n,$ so $\sin\theta \propto \frac{1}{d}.$ If d is smaller, $\sin\theta,$ and hence $\theta,$ is larger.

EXECUTE: At the top $\theta_{\text{top}} = 37°$, and at the bottom $\theta_{\text{bottom}} = 41°.$ Since $\theta_{\text{top}} < \theta_{\text{bottom}}, <$ it follows that $d_{\text{top}} > d_{\text{bottom}}$. Therefore the microspheres are closer together at the bottom than at the top, which makes choice (a) the correct one.

EVALUATE: Gravity tends to pull the spheres downward so they bunch up at the bottom, making them closer together.

37

RELATIVITY

VP37.5.1. **IDENTIFY:** This problem is about time dilation.

SET UP: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$. We want the mean lifetime of the K^+ particle.

EXECUTE: **(a)** At $u = 0.800c$: $\Delta t = \dfrac{1.23\times10^{-8}\text{ s}}{\sqrt{1-(0.800)^2}} = 2.05\times10^{-8}\text{ s}.$

(b) At $u = 0.800\,c$: To the first K^+, the second one is moving at $u = 0.800c$, so the answer is the same as in part (a): $\Delta t = 2.05\times10^{-8}$ s.

EVALUATE: The relative speed of the K^+ is the same in both cases, so the lifetime is the same.

VP37.5.2. **IDENTIFY:** This problem is about time dilation.

SET UP: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$. We want the time interval that both Paul and Alia read.

EXECUTE: **(a)** Paul's timer reads the proper time, so solve for Δt_0.

$\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (20.0\text{ s})\sqrt{1-(0.600)^2} = 16.0\text{ s}.$

(b) Alia's time is the proper time. $\Delta t = \dfrac{24.0\text{ s}}{\sqrt{1-(0.600)^2}} = 30.0\text{ s}.$

EVALUATE: The proper time is the time measured by the observer at rest in her frame.

VP37.5.3. **IDENTIFY:** This problem is about length contraction.

SET UP: $L = L_0\sqrt{1-u^2/c^2}$. We want the distances and times measured.

EXECUTE: **(a)** In your frame, the proper length is 1.50×10^{11} m. $\Delta t = d/u = (1.50\times10^{11}\text{ m})/(0.950c)$ $= 526$ s.

(b) For the astronaut, the distance is length contracted.

$L == (1.50\times10^{11}\text{ m})\sqrt{1-(0.950)^2} = 4.68\times10^{10}\text{ m}.$ For the astronaut, the sun is moving toward him at $0.950c$, so $\Delta t = L/u = (4.68\times10^{10}\text{ m})/(0.950c) = 164$ s.

EVALUATE: Check: Use time dilation. To a person on Earth, the trip takes 526 s. The proper time is the astronaut's time, so $\Delta t_E = \dfrac{\Delta t_A}{\sqrt{1-(0.950)^2}} = 526\text{ s}.$

VP37.5.4. **IDENTIFY:** This problem is about length contraction.

SET UP: $L = L_0\sqrt{1-u^2/c^2}$, $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$. For you, it takes 6.00 s for the rocket clock to read 2.00 s. You measure $L_{\text{rocket}} = 24.0$ m. We want the distances measured by the astronaut in the spacecraft.

EXECUTE: **(a)** Earth-Moon distance: First find the spacecraft's speed relative to Earth. The proper time is 2.00 s in the rocket. $6.00\text{ s} = \dfrac{2.00\text{ s}}{\sqrt{1-u^2/c^2}}$, so $\sqrt{1-u^2/c^2} = 0.333$. To the astronaut, the Earth-Moon distance is length-contracted. $L = L_0\sqrt{1-u^2/c^2} = (3.84\times10^5\text{ km})(0.333) = 1.28\times10^5\text{ km}$.

(b) The proper length of the spacecraft is measured by the astronaut, so $L_0 = \dfrac{L}{\sqrt{1-u^2/c^2}} = \dfrac{24.0\text{ m}}{0.333}$ $= 72.0$ m.

EVALUATE: The Earth observer sees the spacecraft length-contracted and the astronaut sees the Earth-Moon distance length-contracted.

VP37.7.1. **IDENTIFY:** This problem is about the Lorentz transformation equations.

SET UP: $x' = \gamma(x-ut)$, $t' = \gamma\left(t - xu/c^2\right)$. $\gamma = \dfrac{1}{\sqrt{1-u^2/c^2}} = \dfrac{1}{\sqrt{1-(0.750)^2}} = 1.512$. Gamora is the primed frame and Nebula is the unprimed frame.

EXECUTE: **(a)** We want the coordinates as measured by Gamora. Using $x' = \gamma(x-ut)$, we have ut $= (0.750c)(2.50\text{ s}) = 5.625\times10^8$ m. $x' = (1.512)(0 - 5.625\times10^8\text{ m}) = 8.50\times10^8$ m. Using $t' = \gamma\left(t - xu/c^2\right)$ gives $t' = (1.512)(2.50\text{ s} - 0) = 3.78$ s.

(b) We want the coordinates as measured by Nebula. She is the unprimed frame, so $x = \gamma(x'+ut)$ and $t = \gamma\left(t' + x'u/c^2\right)$. $x'u/c^2 = (4.00\times10^8\text{ m})(0.750c)/c^2 = 1.00$ s. This gives $x = \gamma(x'+ut) =$ $(1.512)(4.00\times10^8\text{ m} + 5.625\times10^8\text{ m}) = 1.46\times10^9$ m. $t = \gamma\left(t' + x'u/c^2\right) = (1.512)(2.50\text{ s} + 1.00\text{ s})$ $= 5.29$ s.

EVALUATE: Be very careful to decide which are the primed and unprimed frames.

VP37.7.2. **IDENTIFY:** This problem is about the Lorentz transformation equations.

SET UP: $x' = \gamma(x-ut)$, $t' = \gamma\left(t - xu/c^2\right)$. $\gamma = \dfrac{1}{\sqrt{1-u^2/c^2}} = \dfrac{1}{\sqrt{1-(0.800)^2}} = 1.6667$. Doreen is the primed frame and Kamala is the unprimed frame. In Kamala's frame, the events are simultaneous but occur at different places. In Doreen's frame the events are 0.600 s apart. We want the distance between the two events as measured by Doreen and Kamala.

EXECUTE: **(a)** In Kamala's frame: Use the Lorentz transformation equation for time and solve for Δx_K. $\Delta t'_D = \gamma\left(\Delta t_K - x_K u/c^2\right)$. $0.600\text{ s} = (1.6667)\left(0 - \Delta x_K(0.800c)/c^2\right)$. $\Delta x_K = 1.35\times10^8$ m.

(b) In Doreen's frame: $\Delta x'_D = \gamma(\Delta x_K - u\Delta t_K) = \Delta x'_D = \gamma(\Delta x_K - 0) = (1.6667)(1.35\times10^8\text{ m})$ $= 2.25\times10^8$ m.

EVALUATE: The distances are different because of their relative motion.

VP37.7.3. **IDENTIFY:** This problem is about relative velocity.

SET UP: $v'_x = \dfrac{v_x - u}{1 - uv_x/c^2}$, $v_x = \dfrac{v'_x + u}{1 + uv'_x/c^2}$. Fig. 37.7.3 shows the motions involved.

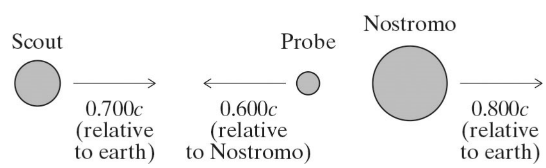


Figure VP37.7.3

EXECUTE: (a) We want the velocity of the probe relative to Earth. Let *Nostromo* be the primed frame moving relative to Earth at 0.800c in the +x direction. Earth is the unprimed frame.

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = \frac{-0.600c + 0.800c}{1 + (0.800c)(-0.600c)/c^2} = +0.385c.$$

(b) We want the velocity of the Scout ship relative to *Nostromo*. Let *Nostromo* be the primed frame and Earth the unprimed frame. We want v'_x. Using $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$ gives

$$v'_x = \frac{0.700c - 0.800c}{1 - (0.800c)(0.700c)/c^2} = -0.227c.$$

EVALUATE: The result in (b) is reasonable because *Nostromo* is moving faster than the Scout probe, so relative to *Nostromo*, the Scout is moving in the –x direction.

VP37.7.4. IDENTIFY This problem is about relative velocity.

SET UP: $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$. We want the speed of *Yamato* relative to *Macross.*

EXECUTE: (a) *Yamato* is flying toward Earth. Let *Macross* be the primed frame and Earth the unprimed frame. We want v'_Y. $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$ becomes $v'_Y = \frac{v_Y - u}{1 - uv_Y/c^2}$

$$= \frac{-650c - 0.750c}{1 - (0.750c)(-0.650c)/c^2} = -0.941c.$$ *Yamato's* speed is 0.941c.

(b) *Yamato* is flying away from Earth. $v'_Y = \frac{+650c - 0.750c}{1 - (0.750c)(+0.650c)/c^2} = -0.195c.$

EVALUATE: In both cases *Yamato* has a negative velocity component because relative to *Macross*, *Yamato* is getting closer, hence moving in the –x direction. The relative speed in (b) is less than that in (a), as expected.

VP37.11.1. IDENTIFY: This problem deals with the force on a proton and its momentum at high speeds.

SET UP and EXECUTE: $\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = \frac{1}{\sqrt{1 - (0.950)^2}} = 3.20256.$ **(a)** We want the momentum.

$$p = m\gamma c = (1.67 \times 10^{-27}\ \text{kg})(3.20256)c = 1.52 \times 10^{-18}\ \text{kg} \cdot \text{m/s}.$$

(b) We want the acceleration. If the force and velocity are along the same line, $F = \gamma^3 ma$. So $a = \frac{F}{m\gamma^3}$.

Using the given numbers gives $a = 1.64 \times 10^{12}\ \text{m/s}^2$.

(c) We want the acceleration. If the force and velocity are perpendicular, $F = \gamma ma$. So $a = \frac{F}{m\gamma}$. Using the given numbers we get $a = 1.68 \times 10^{13}\ \text{m/s}^2$.

EVALUATE: Without relativity the acceleration would be $a = F/m = 5.39 \times 10^{13}\ \text{m/s}^2$, which is *greater* than when relativity it taken into account.

VP37.11.2. IDENTIFY: This problem is about the energy of a moving electron.

SET UP and EXECUTE: (a) We want the kinetic energy. $E = K + mc^2$, so $K = E - mc^2$
$= 4.00 \times 10^{-13}\ \text{J} + (9.11 \times 10^{-31}\ \text{kg})c^2 = 3.18 \times 10^{-13}\ \text{J}.$

(b) We want γ. $E = m\gamma c^2$. Solve for γ and use the given numbers. $\gamma = \frac{E}{mc^2} = 4.88.$

(c) We want the speed. Solve $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$ for v. $v/c = \sqrt{1-1/\gamma^2} = \sqrt{1-1/4.88^2} = 0.979$, so $v = 0.979c$.

EVALUATE: At speeds near the speed of light, the kinetic energy is much different from $\frac{1}{2}mv^2$.

VP37.11.3. IDENTIFY: This problem is about the energy of a moving electron.

SET UP: $E^2 = (pc)^2 + \left(mc^2\right)^2$, $E = m\gamma c^2$, $v/c = \sqrt{1-1/\gamma^2}$.

EXECUTE: **(a)** We want the total energy E. Solve $E^2 = (pc)^2 + \left(mc^2\right)^2$ for E and use the given numbers for the momentum p and rest mass m. The result is $E = 1.13\times10^{-13}$ J.

(b) We want γ. Solve $E = m\gamma c^2$ for γ and use the given m and E we found in (a), giving $\gamma = 1.38$.

(c) We want the speed v. $v/c = \sqrt{1-1/\gamma^2} = \sqrt{1-1/1.38^2} = 0.689$, so $v = 0.689c$.

EVALUATE: Careful! $E \neq pc + mc^2$.

VP37.11.4 IDENTIFY: This problem is about energy in particle decay.

SET UP: The $\psi(2S)$ is at rest, so by momentum conservation the K mesons have equal but opposite momentum and therefore equal speeds and equal kinetic energies. $mc^2 = 495$ MeV for each K meson. $E = K + mc^2$, $K = (\gamma-1)mc^2$.

EXECUTE: **(a)** We want the kinetic energy K of each meson. Using $E = K + mc^2$ gives $3686\text{ MeV} = (K + 495\text{ MeV}) + (K + 495\text{ MeV})$. Solving gives $K = 1348$ MeV for each meson.

(b) We want γ. $K = (\gamma-1)mc^2 = (\gamma-1)(495\text{ MeV})$. Solve for γ and use $K = 1348$ MeV from part (a), giving $\gamma = 3.72$.

(c) We want the speed. $v/c = \sqrt{1-1/\gamma^2} = \sqrt{1-1/3.72^2} = 0.963$, so $v = 0.963c$.

EVALUATE: The K^+ and K^- mesons are highly relativistic with $v = 0.963c$ and $\gamma = 3.72$.

37.1. IDENTIFY and SET UP: Consider the distance A to O' and B to O' as observed by an observer on the ground (Figure 37.1).

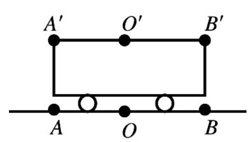


Figure 37.1

EXECUTE: The statement that the events are simultaneous to an observer on the train means that light pulses from A' and B' arrive at O' at the same time. To the observer at O, light from A' has a longer distance to travel than light from B' so O will conclude that the pulse from $A(A')$ started before the pulse at $B(B')$. To the observer at O, bolt A appeared to strike first.

EVALUATE: Section 37.2 shows that if the events are simultaneous to the observer on the ground, then an observer on the train measures that the bolt at B' struck first.

37.2. IDENTIFY: Apply $\Delta t = \gamma \Delta t_0$.

SET UP: The lifetime measured in the muon frame is the proper time Δt_0. $u = 0.900c$ is the speed of the muon frame relative to the laboratory frame. The distance the particle travels in the lab frame is its speed in that frame times its lifetime in that frame.

EXECUTE: **(a)** $\gamma = \dfrac{1}{\sqrt{1-(0.9)^2}} = 2.29.$ $\Delta t = \gamma \Delta t_0 = (2.29)\,(2.20\times 10^{-6}\text{s}) = 5.05\times 10^{-6}$ s.

(b) $d = v\Delta t = (0.900)(3.00\times 10^8 \text{ m/s})(5.05\times 10^{-6}\text{s}) = 1.36\times 10^3 \text{ m} = 1.36 \text{ km}.$

EVALUATE: The lifetime measured in the lab frame is larger than the lifetime measured in the muon frame.

37.3. **IDENTIFY** and **SET UP:** The problem asks for u such that $\Delta t_0/\Delta t = \frac{1}{2}$.

EXECUTE: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ gives $u = c\sqrt{1-(\Delta t_0/\Delta t)^2} = (3.00\times 10^8 \text{ m/s})\sqrt{1-(\frac{1}{2})^2} = 2.60\times 10^8$ m/s;

$\dfrac{u}{c} = 0.867.$

EVALUATE: Jet planes fly at less than ten times the speed of sound, less than about 3000 m/s. Jet planes fly at much lower speeds than we calculated for u.

37.4. **IDENTIFY:** Time dilation occurs because the rocket is moving relative to Mars.

SET UP: The time dilation equation is $\Delta t = \gamma \Delta t_0$, where t_0 is the proper time.

EXECUTE: **(a)** The two time measurements are made at the same place on Mars by an observer at rest there, so the observer on Mars measures the proper time.

(b) $\Delta t = \gamma \Delta t_0 = \dfrac{1}{\sqrt{1-(0.985)^2}}(75.0\ \mu\text{s}) = 435\ \mu\text{s}.$

EVALUATE: The pulse lasts for a longer time relative to the rocket than it does relative to the Mars observer.

37.5. **(a) IDENTIFY** and **SET UP:** $\Delta t_0 = 2.60\times 10^{-8}$ s; $\Delta t = 4.20\times 10^{-7}$ s. In the lab frame the pion is created and decays at different points, so this time is not the proper time.

EXECUTE: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ says $1-\dfrac{u^2}{c^2} = \left(\dfrac{\Delta t_0}{\Delta t}\right)^2.$

$$\frac{u}{c} = \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1-\left(\frac{2.60\times 10^{-8}\text{ s}}{4.20\times 10^{-7}\text{ s}}\right)^2} = 0.998;\ u = 0.998c.$$

EVALUATE: $u < c$, as it must be, but u/c is close to unity and the time dilation effects are large.

(b) IDENTIFY and **SET UP:** The speed in the laboratory frame is $u = 0.998c$; the time measured in this frame is Δt, so the distance as measured in this frame is $d = u\Delta t$.

EXECUTE: $d = (0.998)(2.998\times 10^8 \text{ m/s})(4.20\times 10^{-7}\text{ s}) = 126 \text{ m}.$

EVALUATE: The distance measured in the pion's frame will be different because the time measured in the pion's frame is different (shorter).

37.6. **IDENTIFY:** Apply $\Delta t = \gamma \Delta t_0$.

SET UP: For part (a) the proper time is measured by the race pilot. $\gamma = 1.667$.

EXECUTE: **(a)** $\Delta t = \dfrac{1.20\times 10^8 \text{ m}}{(0.800)(3.00\times 10^8 \text{ m/s})} = 0.500$ s. $\Delta t_0 = \dfrac{\Delta t}{\gamma} = \dfrac{0.500\text{ s}}{1.667} = 0.300$ s.

(b) $(0.300\text{ s})(0.800c) = 7.20\times 10^7 \text{ m}.$

(c) You read $\dfrac{1.20\times 10^8 \text{ m}}{(0.800)(3\times 10^8 \text{ m/s})} = 0.500$ s.

EVALUATE: The two events are the spaceracer passing you and the spaceracer reaching a point 1.20×10^8 m from you. The timer traveling with the spaceracer measures the proper time between these two events.

37.7. **IDENTIFY** and **SET UP:** The proper time is measured in the frame where the two events occur at the same point.

EXECUTE: **(a)** The time of 12.0 ms measured by the first officer on the craft is the proper time.

(b) $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ gives $u = c\sqrt{1-(\Delta t_0/\Delta t)^2} = c\sqrt{1-(12.0\times10^{-3}/0.150)^2} = 0.997c.$

EVALUATE: The observer at rest with respect to the searchlight measures a much shorter duration for the event.

37.8. **IDENTIFY:** This problem involves time dilation.

SET UP: $\Delta t = \gamma\Delta t_0$. The captain's time is the proper time. We want x and t in our frame.

EXECUTE: $\Delta t = \gamma\Delta t_0 = (100)(1.00 \text{ s}) = 100$ s. $x = vt \approx ct = c(100 \text{ s}) = 3.00\times10^{10}$ s.

EVALUATE: Note that $v \approx c$, not $v = c$. But with $\gamma = 100$, the speed is *extremely* close to c.

37.9. **IDENTIFY** and **SET UP:** $l = l_0\sqrt{1-u^2/c^2}$. The length measured when the spacecraft is moving is $l = 74.0$ m; l_0 is the length measured in a frame at rest relative to the spacecraft.

EXECUTE: $l_0 = \dfrac{l}{\sqrt{1-u^2/c^2}} = \dfrac{74.0 \text{ m}}{\sqrt{1-(0.600c/c)^2}} = 92.5 \text{ m}.$

EVALUATE: $l_0 > l$. The moving spacecraft appears to an observer on the planet to be shortened along the direction of motion.

37.10. **IDENTIFY** and **SET UP:** When the meterstick is at rest with respect to you, you measure its length to be 1.000 m, and that is its proper length, l_0. $l = 0.3048$ m.

EXECUTE: $l = l_0\sqrt{1-u^2/c^2}$ gives $u = c\sqrt{1-(l/l_0)^2} = c\sqrt{1-(0.3048/1.00)^2} = 0.9524c = 2.86\times10^8$ m/s.

EVALUATE: The needed speed is well beyond modern capabilities for any rocket.

37.11. **IDENTIFY** and **SET UP:** The 2.2 μs lifetime is Δt_0 and the observer on earth measures Δt. The atmosphere is moving relative to the muon so in its frame the height of the atmosphere is l and l_0 is 10 km.

EXECUTE: **(a)** The greatest speed the muon can have is c, so the greatest distance it can travel in 2.2×10^{-6} s is $d = vt = (3.00\times10^8 \text{ m/s})(2.2\times10^{-6} \text{ s}) = 660 \text{ m} = 0.66 \text{ km}$.

(b) $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \dfrac{2.2\times10^{-6} \text{ s}}{\sqrt{1-(0.999)^2}} = 4.9\times10^{-5} \text{ s}.$

$d = vt = (0.999)(3.00\times10^8 \text{ m/s})(4.9\times10^{-5} \text{ s}) = 15 \text{ km}.$

In the frame of the earth the muon can travel 15 km in the atmosphere during its lifetime.

(c) $l = l_0\sqrt{1-u^2/c^2} = (10 \text{ km})\sqrt{1-(0.999)^2} = 0.45 \text{ km}.$

EVALUATE: In the frame of the muon the height of the atmosphere is less than the distance it moves during its lifetime.

37.12. **IDENTIFY:** The astronaut lies along the motion of the rocket, so his height will be Lorentz-contracted.

SET UP: The doctor in the rocket measures his proper length l_0.

EXECUTE: **(a)** $l_0 = 2.00$ m. $l = l_0\sqrt{1-u^2/c^2} = (2.00 \text{ m})\sqrt{1-(0.910)^2} = 0.829$ m. The person on Earth would measure his height to be 0.829 m.

(b) $l = 2.00$ m. $l_0 = \dfrac{l}{\sqrt{1-u^2/c^2}} = \dfrac{2.00 \text{ m}}{\sqrt{1-(0.910)^2}} = 4.82$ m. This is not a reasonable height for a human.

(c) There is no length contraction in a direction perpendicular to the motion and both observers measure the same height, 2.00 m.

EVALUATE: The length of an object moving with respect to the observer is shortened in the direction of the motion, so in (a) and (b) the observer on Earth measures a shorter height.

37.13. IDENTIFY: Apply $l = l_0\sqrt{1-u^2/c^2}$.

SET UP: The proper length l_0 of the runway is its length measured in the Earth's frame. The proper time Δt_0 for the time interval for the spacecraft to travel from one end of the runway to the other is the time interval measured in the frame of the spacecraft.

EXECUTE: **(a)** $l_0 = 3600$ m.

$$l = l_0\sqrt{1-\frac{u^2}{c^2}} = (3600\text{ m})\sqrt{1-\frac{(4.00\times10^7\text{ m/s})^2}{(3.00\times10^8\text{ m/s})^2}} = (3600\text{ m})(0.991) = 3568\text{ m}.$$

(b) $\Delta t = \dfrac{l_0}{u} = \dfrac{3600\text{ m}}{4.00\times10^7\text{ m/s}} = 9.00\times10^{-5}$ s.

(c) $\Delta t_0 = \dfrac{l}{u} = \dfrac{3568\text{ m}}{4.00\times10^7\text{ m/s}} = 8.92\times10^{-5}$ s.

EVALUATE: $\dfrac{1}{\gamma} = 0.991$, so $\Delta t = \gamma\Delta t_0$ gives $\Delta t = \dfrac{8.92\times10^{-5}\text{s}}{0.991} = 9.00\times10^{-5}$ s. The result from length contraction is consistent with the result from time dilation.

37.14. IDENTIFY: This problems requires use of the Lorentz transformation equations.

SET UP: $t' = \gamma\left(t - xu/c^2\right)$, $t = \gamma\left(t' + x'u/c^2\right)$. We want to know for what values x and x' will t be equal to t'.

EXECUTE: Using $t' = \gamma\left(t - xu/c^2\right)$, let $t = t'$ and solve for x. $t(\gamma-1) = xu\gamma/c^2$.

$x = \dfrac{c^2t(\gamma-1)}{u\gamma} = \dfrac{c^2t}{u}\left(1-\dfrac{1}{\gamma}\right)$. Now find x'. $t' = \gamma\left(t - xu/c^2\right)$ and $t = \gamma\left(t' + x'u/c^2\right)$ If $t = t'$, then it follows that $x'u/c^2 = -xu/c^2$, so $x' = -x$.

EVALUATE: The clocks are coordinated since $t = t'$.

37.15. IDENTIFY: Apply $v_x = \dfrac{v'_x + u}{1+uv'_x/c^2}$.

SET UP: The velocities $\vec{v}'$ and $\vec{v}$ are both in the $+x$-direction, so $v'_x = v'$ and $v_x = v$.

EXECUTE: **(a)** $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.400c+0.600c}{1+(0.400)(0.600)} = 0.806c$.

(b) $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.900c+0.600c}{1+(0.900)(0.600)} = 0.974c$.

(c) $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.990c+0.600c}{1+(0.990)(0.600)} = 0.997c$.

EVALUATE: Speed v is always less than c, even when $v'+u$ is greater than c.

37.16. IDENTIFY: Apply $\Delta t = \gamma\Delta t_0$ and the equations for x and t that are developed in Example 37.6.

SET UP: S is Stanley's frame and S' is Mavis's frame. The proper time for the two events is the time interval measured in Mavis's frame. $\gamma = 1.667$ ($\gamma = 5/3$ if $u = (4/5)$c).

EXECUTE: **(a)** In Mavis's frame the event "light on" has space-time coordinates $x' = 0$ and $t' = 5.00$ s, so from the result of Example 37.6, $x = \gamma(x' + ut')$ and

$$t = \gamma\left(t' + \frac{ux'}{c^2}\right) \Rightarrow x = \gamma ut' = 2.00\times10^9\text{m}, t = \gamma t' = 8.33\text{ s}.$$

(b) The 5.00-s interval in Mavis's frame is the proper time Δt_0, so $\Delta t = \gamma \Delta t_0 = 8.33\text{ s}$, the same as in part (a).

(c) $(8.33\text{ s})(0.800c) = 2.00 \times 10^9\text{ m}$, which is the distance x found in part (a).

EVALUATE: Mavis would measure that she would be a distance $(5.00\text{ s})(0.800c) = 1.20 \times 10^9\text{ m}$ from Stanley when she turns on her light. In $l = l_0/\gamma$, $l_0 = 2.00 \times 10^9\text{ m}$ and $l = 1.20 \times 10^9\text{ m}$.

37.17. IDENTIFY: The relativistic velocity addition formulas apply since the speeds are close to that of light.

SET UP: The relativistic velocity addition formula is $v'_x = \dfrac{v_x - u}{1 - \dfrac{uv_x}{c^2}}$.

EXECUTE: **(a)** For the pursuit ship to catch the cruiser, the distance between them must be decreasing, so the velocity of the cruiser relative to the pursuit ship must be directed toward the pursuit ship.

(b) Let the unprimed frame be Tatooine and let the primed frame be the pursuit ship. We want the velocity v' of the cruiser knowing the velocity of the primed frame u and the velocity of the cruiser v in the unprimed frame (Tatooine). $v'_x = \dfrac{v_x - u}{1 - \dfrac{uv_x}{c^2}} = \dfrac{0.600c - 0.800c}{1 - (0.600)(0.800)} = -0.385c$.

The result implies that the cruiser is moving toward the pursuit ship at $0.385c$.

EVALUATE: The nonrelativistic formula would have given $-0.200c$, which is considerably different from the correct result.

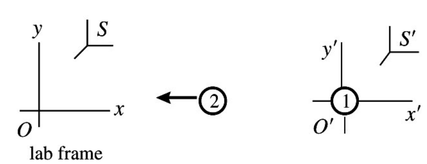
37.18. IDENTIFY and SET UP: Let the starfighter's frame be S and let the enemy spaceship's frame be S'. Let the positive x-direction for both frames be from the enemy spaceship toward the starfighter. Then $u = +0.400c$. $v' = +0.700c$. v is the velocity of the missile relative to you.

EXECUTE: **(a)** $v = \dfrac{v' + u}{1 + uv'/c^2} = \dfrac{0.700c + 0.400c}{1 + (0.400)(0.700)} = 0.859c$.

(b) Use the distance it moves as measured in your frame and the speed it has in your frame to calculate the time it takes in your frame. $t = \dfrac{8.00 \times 10^9\text{ m}}{(0.859)(3.00 \times 10^8\text{ m/s})} = 31.0\text{ s}$.

EVALUATE: Note that the speed in (a) is not $1.1c$ as nonrelativistic physics would predict.

37.19. IDENTIFY and SET UP: Reference frames S and S' are shown in Figure 37.19.



Frame S is at rest in the laboratory. Frame S' is attached to particle 1.

Figure 37.19

u is the speed of S' relative to S; this is the speed of particle 1 as measured in the laboratory. Thus $u = +0.650c$. The speed of particle 2 in S' is $0.950c$. Also, since the two particles move in opposite directions, 2 moves in the $-x'$-direction and $v'_x = -0.950c$. We want to calculate v_x, the speed of particle 2 in frame S, so use $v_x = \dfrac{v'_x + u}{1 + uv'_x/c^2}$.

EXECUTE: $v_x = \dfrac{v'_x + u}{1 + uv'_x/c^2} = \dfrac{-0.950c + 0.650c}{1 + (0.650c)(-0.950c)/c^2} = \dfrac{-0.300c}{1 - 0.6175} = -0.784c$. The speed of the second particle, as measured in the laboratory, is $0.784c$.

EVALUATE: The incorrect Galilean expression for the relative velocity gives that the speed of the second particle in the lab frame is $0.300c$. The correct relativistic calculation gives a result more than twice this.

37.20. **IDENTIFY** and **SET UP:** Let S be the laboratory frame and let S' be the frame of one of the particles, as shown in Figure 37.20. Let the positive x-direction for both frames be from particle 1 to particle 2. In the lab frame particle 1 is moving in the $+x$-direction and particle 2 is moving in the $-x$-direction. Then $u = 0.9380c$ and $v_x = -0.9380c$. v'_x is the velocity of particle 2 relative to particle 1.

EXECUTE: $v'_x = \dfrac{v_x - u}{1 - uv_x/c^2} = \dfrac{-0.9380c - 0.9380c}{1-(0.9380c)(-0.9380c)/c^2} = -0.9980c.$ The speed of particle 2 relative to particle 1 is $0.9980c$.

EVALUATE: $v'_x < 0$ shows particle 2 is moving toward particle 1.

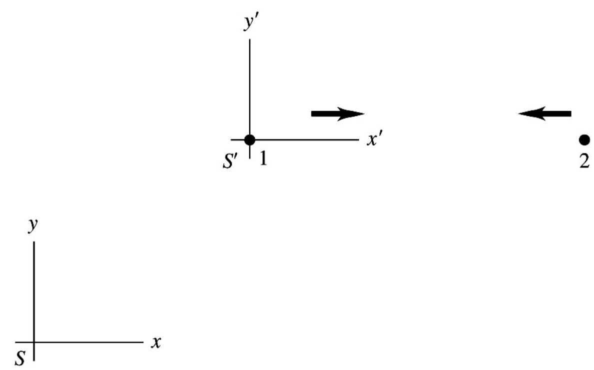


Figure 37.20

37.21. **IDENTIFY:** The relativistic velocity addition formulas apply since the speeds are close to that of light.

SET UP: The relativistic velocity addition formula is $v'_x = \dfrac{v_x - u}{1 - \dfrac{uv_x}{c^2}}$.

EXECUTE: In the relativistic velocity addition formula for this case, v'_x is the relative speed of particle 1 with respect to particle 2, v is the speed of particle 2 measured in the laboratory, and u is the speed of particle 1 measured in the laboratory, $u = -v$.

$v'_x = \dfrac{v-(-v)}{1-(-v)v/c^2} = \dfrac{2v}{1+v^2/c^2}$. $\dfrac{v'_x}{c^2}v^2 - 2v + v'_x = 0$ and $(0.890c)v^2 - 2c^2v + (0.890c^3) = 0.$

This is a quadratic equation with solution $v = 0.611c$ (v must be less than c).

EVALUATE: The nonrelativistic result would be $0.445c$, which is considerably different from this result.

37.22. **IDENTIFY:** There is a Doppler effect in the frequency of the radiation due to the motion of the star.

SET UP: The star is moving away from the earth, so $f = \sqrt{\dfrac{c-u}{c+u}}f_0$.

EXECUTE: $f = \sqrt{\dfrac{c-0.520c}{c+0.520c}}f_0 = 0.5620f_0 = (0.5620)(8.64\times10^{14}\text{ Hz}) = 4.86\times10^{14}\text{ Hz}.$

EVALUATE: The earth observer measures a lower frequency than the star emits because the star is moving away from the earth.

37.23. IDENTIFY and SET UP: Source and observer are approaching, so use $f=\sqrt{\dfrac{c+u}{c-u}}f_0$. Solve for u, the speed of the light source relative to the observer.

EXECUTE: **(a)** $f^2=\left(\dfrac{c+u}{c-u}\right)f_0^2$.

$(c-u)f^2=(c+u)f_0^2$ and $u=\dfrac{c(f^2-f_0^2)}{f^2+f_0^2}=c\left(\dfrac{(f/f_0)^2-1}{(f/f_0)^2+1}\right)$.

$\lambda_0=675\text{ nm},\ \ \lambda=575\text{ nm}.$

$u=\left(\dfrac{(675\text{ nm}/575\text{ nm})^2-1}{(675\text{ nm}/575\text{ nm})^2+1}\right)c=0.159c=(0.159)(2.998\times10^8\text{ m/s})=4.77\times10^7\text{ m/s};$ definitely speeding

(b) $4.77\times10^7\text{ m/s}=(4.77\times10^7\text{ m/s})(1\text{ km}/1000\text{ m})(3600\text{ s}/1\text{ h})=1.72\times10^8\text{ km/h}$. Your fine would be $\$1.72\times10^8$ (172 million dollars).

EVALUATE: The source and observer are approaching, so $f>f_0$ and $\lambda<\lambda_0$. Our result gives $u<c$, as it must.

37.24. IDENTIFY: There is a Doppler effect in the frequency of the radiation due to the motion of the source.

SET UP: $f>f_0$ so the source is moving toward you. $f=\sqrt{\dfrac{c-u}{c+u}}f_0$.

EXECUTE: $(f/f_0)^2=\dfrac{c+u}{c-u}$. $c(f/f_0)^2-(f/f_0)^2u=c+u$.

$u=\dfrac{c[(f/f_0)^2-1]}{(f/f_0)^2+1}=\left[\dfrac{(1.25)^2-1}{(1.25)^2+1}\right]c=0.220c$, toward you.

EVALUATE: The difference in frequency is rather large (1.25 times), so the motion of the source must be a substantial fraction of the speed of light (around 20% in this case).

37.25. IDENTIFY: The problem involves momentum and the Lorentz factor.

SET UP: $p=m\gamma v$, $\gamma=\dfrac{1}{\sqrt{1-u^2/c^2}}$, $u/c=\sqrt{1-1/\gamma^2}$, particle 1: $\gamma_1=1.12$, particle 2: $u_2=2u_1$.

EXECUTE: **(a)** We want γ_2. First find u_1. $u_1=c\sqrt{1-1/\gamma^2}=c\sqrt{1-1/1.12^2}=0.450343c$, so $u_2=2u_1$ $=0.90068c$. $\gamma_2=\dfrac{1}{\sqrt{1-(0.90068)^2}}=2.30.$

(b) We want p_2/p_1. $\dfrac{p_2}{p_1}=\dfrac{m\gamma_2u_2}{m\gamma_1u_1}=\dfrac{\gamma_2(2u_1)}{\gamma_1u_1}=\dfrac{2\gamma_2}{\gamma_1}=\dfrac{2(2.30)}{1.12}=4.11.$

EVALUATE: For speeds much less than that of light, we would have $p_2=2p_1$, but near the speed of light we find $p_2=4.11p_1$, a very significant difference.

37.26. IDENTIFY and SET UP: The force is found from $F=\gamma^3ma$ or $F=\gamma ma$, whichever is applicable.

EXECUTE: **(a)** Indistinguishable from $F=ma=0.145\text{ N}$.

(b) $\gamma^3ma=1.75\text{ N}$.

(c) $\gamma^3ma=51.7\text{ N}$.

(d) $\gamma ma=0.145\text{ N},\ 0.333\text{ N}, 1.03\text{ N}$.

EVALUATE: When v is large, much more force is required to produce a given magnitude of acceleration when the force is parallel to the velocity than when the force is perpendicular to the velocity.

37.27. IDENTIFY: The speed of the proton is a substantial fraction of the speed of light, so we must use the relativistic formula for momentum.

SET UP: $p = \gamma mv$. $p_0 = \gamma_0 m v_0$. $\dfrac{p}{p_0} = \dfrac{\gamma v}{\gamma_0 v_0}$. $v/v_0 = 2.00$.

EXECUTE: $\gamma_0 = \dfrac{1}{\sqrt{1 - v_0^2/c^2}} = \dfrac{1}{\sqrt{1-(0.400)^2}} = 1.0911$. $\gamma = \dfrac{1}{\sqrt{1-(0.800)^2}} = 1.667$.

$p = p_0(2)\left(\dfrac{1.667}{1.091}\right) = 3.06 p_0$.

EVALUATE: The speed doubles but the momentum more than triples.

37.28. IDENTIFY: In this problem we deal with length contraction and momentum.

SET UP: $L = L_0/\gamma$, $p = m\gamma v$, $v/c = \sqrt{1 - 1/\gamma^2}$. We measure L = 90 m, but its proper length is 120 m.

EXECUTE: **(a)** The diameter will be 25 m. There is no length contraction perpendicular to the direction of motion.

(b) We want the momentum p. Use length contraction to find γ. $L = L_0/\gamma$: 90 m = (120 m)/ γ, so γ = 1.33. Now use γ to find v. $v/c = \sqrt{1-1/\gamma^2} = \sqrt{1-1/1.33^2} = 0.6614$. Now use $p = m\gamma v$ to find p.

$p = m\gamma v$ = (4000 kg)(133)(0.6614c) = 1.1×10^{12} kg $\cdot$ m/s.

EVALUATE: Ignoring relativity we would measure a length of 120 m and a momentum of $p = mv$ = (4000 kg)(0.6614c) = 7.9×10^{11} kg $\cdot$ m/s. A big difference from the actual momentum!

37.29. IDENTIFY: Apply $p = \dfrac{mv}{\sqrt{1 - v^2/c^2}}$ and $F = \gamma^3 ma$.

SET UP: For a particle at rest (or with $v \ll c$), $a = F/m$.

EXECUTE: **(a)** $p = \dfrac{mv}{\sqrt{1-v^2/c^2}} = 2mv$.

$\Rightarrow 1 = 2\sqrt{1 - v^2/c^2} \Rightarrow \frac{1}{4} = 1 - \dfrac{v^2}{c^2} \Rightarrow v^2 = \frac{3}{4}c^2 \Rightarrow v = \dfrac{\sqrt{3}}{2}c = 0.866c$.

(b) $F = \gamma^3 ma = 2ma \Rightarrow \gamma^3 = 2 \Rightarrow \gamma = (2)^{1/3}$ so $\dfrac{1}{1 - v^2/c^2} = 2^{2/3} \Rightarrow \dfrac{v}{c} = \sqrt{1 - 2^{-2/3}} = 0.608$.

EVALUATE: The momentum of a particle and the force required to give it a given acceleration both increase without bound as the speed of the particle approaches c.

37.30. IDENTIFY: Use $E = mc^2$ to relate the mass decrease to the energy produced.

SET UP: 1 kg is equivalent to 2.2 lbs and 1 ton = 2000 lbs. 1 W = 1 J/s.

EXECUTE: **(a)** $E = mc^2, m = E/c^2 = (3.8\times 10^{26}\text{ J})/(2.998\times 10^8\text{ m/s})^2 = 4.2\times 10^9\text{ kg} = 4.6\times 10^6$ tons.

(b) The current mass of the sun is 1.99×10^{30} kg, so it would take it

$(1.99\times 10^{30}\text{ kg})/(4.2\times 10^9\text{ kg/s}) = 4.7\times 10^{20}\text{s} = 1.5\times 10^{13}$ years to use up all its mass.

EVALUATE: The power output of the sun is very large, but only a small fraction of the sun's mass is converted to energy each second.

37.31. IDENTIFY: Apply $K = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$.

SET UP: The rest energy is mc^2.

EXECUTE: **(a)** $K = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = mc^2$.

$$\Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 2 \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{3}{4}}c = 0.866c.$$

(b) $K = 5mc^2 \Rightarrow \dfrac{1}{\sqrt{1-v^2/c^2}} = 6 \Rightarrow \dfrac{1}{36} = 1 - \dfrac{v^2}{c^2} \Rightarrow v = \sqrt{\dfrac{35}{36}}c = 0.986c.$

EVALUATE: If $v \ll c$, then K is much less than the rest energy of the particle.

37.32. **IDENTIFY:** At such a high speed, we must use the relativistic formulas for momentum and kinetic energy.

SET UP: $m_\mu = 207m_e = 1.89\times10^{-28}$ kg. v is very close to c and we must use relativistic expressions.

$$p = \frac{mv}{\sqrt{1-v^2/c^2}},\quad K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2.$$

EXECUTE: $p = \dfrac{mv}{\sqrt{1-v^2/c^2}} = \dfrac{(1.89\times10^{-28}\text{ kg})(0.999)(3.00\times10^8\text{ m/s})}{\sqrt{1-(0.999)^2}} = 1.27\times10^{-18}\text{ kg}\cdot\text{m/s}.$

Using $K = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2$ gives

$$K = (1.89\times10^{-28}\text{ kg})(3.00\times10^8\text{ m/s})^2\left(\frac{1}{\sqrt{1-(0.999)^2}} - 1\right) = 3.63\times10^{-10}\text{ J}.$$

EVALUATE: The nonrelativistic values are $p_{\text{nr}} = mv = 5.66\times10^{-20}\text{ kg}\cdot\text{m/s}$ and $K_{\text{nr}} = \frac{1}{2}mv^2$ $= 8.49\times10^{-12}$ J. Each relativistic result is much larger.

37.33. **IDENTIFY** and **SET UP:** Use $E = mc^2 + K$ and $E^2 = (mc^2)^2 + (pc)^2$.

EXECUTE: **(a)** $E = mc^2 + K$, so $E = 4.00mc^2$ means $K = 3.00mc^2 = 4.50\times10^{-10}$ J.

(b) $E^2 = (mc^2)^2 + (pc)^2$; $E = 4.00mc^2$, so $15.0(mc^2)^2 = (pc)^2$.

$p = \sqrt{15}mc = 1.94\times10^{-18}\text{ kg}\cdot\text{m/s}.$

(c) $E = mc^2/\sqrt{1-v^2/c^2}$.

$E = 4.00mc^2$ gives $1 - v^2/c^2 = 1/16$ and $v = \sqrt{15/16}c = 0.968c$.

EVALUATE: The speed is close to c since the kinetic energy is greater than the rest energy. Nonrelativistic expressions relating E, K, p, and v will be very inaccurate.

37.34. **IDENTIFY:** Apply the work energy theorem in the form $W = \Delta K$.

SET UP: K is given by $K = (\gamma - 1)mc^2$. When $v = 0$, $\gamma = 1$.

EXECUTE: **(a)** $W = \Delta K = (\gamma_f - 1)mc^2 = (4.07\times10^{-3})\,mc^2$.

(b) $(\gamma_f - \gamma_i)\,mc^2 = 4.79mc^2$.

(c) The result of part (b) is far larger than that of part (a).

EVALUATE: The amount of work required to produce a given increase in speed (in this case an increase of $0.090c$) increases as the initial speed increases.

37.35. **IDENTIFY** and **SET UP:** The total energy is given in terms of the momentum by $E^2=(mc^2)^2+(pc)^2$. In terms of the total energy E, the kinetic energy K is $K=E-mc^2$. The rest energy is mc^2.

EXECUTE: (a)

$$E=\sqrt{(mc^2)^2+(pc)^2}=\sqrt{\left[(6.64\times10^{-27})(2.998\times10^8)^2\right]^2+\left[(2.10\times10^{-18})(2.998\times10^8)\right]^2}\text{ J.}$$

$E=8.67\times10^{-10}$ J.

(b) $mc^2=(6.64\times10^{-27}\text{ kg})(2.998\times10^8\text{ m/s})^2=5.97\times10^{-10}$ J.

$K=E-mc^2=8.67\times10^{-10}\text{ J}-5.97\times10^{-10}\text{ J}=2.70\times10^{-10}$ J.

(c) $\dfrac{K}{mc^2}=\dfrac{2.70\times10^{-10}\text{ J}}{5.97\times10^{-10}\text{ J}}=0.452.$

EVALUATE: The incorrect nonrelativistic expressions for K and p give $K=p^2/2m=3.3\times10^{-10}$ J; the correct relativistic value is less than this.

37.36. **IDENTIFY:** With such a large potential difference, the electrons will be accelerated to relativistic speeds, so we must use the relativistic formula for kinetic energy.

SET UP: $K=\left(\dfrac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2$. The classical expression for kinetic energy is $K=\frac{1}{2}mv^2$.

EXECUTE: For an electron $mc^2=(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m/s})^2=8.20\times10^{-14}$ J.

$K=7.50\times10^5\text{ eV}=1.20\times10^{-13}$ J.

(a) $\dfrac{K}{mc^2}+1=\dfrac{1}{\sqrt{1-v^2/c^2}}$. $\dfrac{1}{\sqrt{1-v^2/c^2}}=\dfrac{1.20\times10^{-13}\text{ J}}{8.20\times10^{-14}\text{ J}}+1=2.46.$

$v=c\sqrt{1-(1/2.46)^2}=0.914c=2.74\times10^8$ m/s.

(b) $K=\frac{1}{2}mv^2$ gives $v=\sqrt{\dfrac{2K}{m}}=\sqrt{\dfrac{2(1.20\times10^{-13}\text{ J})}{9.11\times10^{-31}\text{ kg}}}=5.13\times10^8$ m/s.

EVALUATE: At a given speed the relativistic value of the kinetic energy is larger than the nonrelativistic value. Therefore, for a given kinetic energy the relativistic expression for kinetic energy gives a smaller speed than the nonrelativistic expression.

37.37. **IDENTIFY** and **SET UP:** The nonrelativistic expression is $K_{\text{nonrel}}=\frac{1}{2}mv^2$ and the relativistic expression is $K_{\text{rel}}=(\gamma-1)mc^2$.

EXECUTE: (a) $v=8\times10^7\text{ m/s}\Rightarrow\gamma=\dfrac{1}{\sqrt{1-v^2/c^2}}=1.0376.$ For $m=m_{\text{p}}$, $K_{\text{nonrel}}=\frac{1}{2}mv^2=5.34\times10^{-12}$ J.

$K_{\text{rel}}=(\gamma-1)mc^2=5.65\times10^{-12}$ J. $\dfrac{K_{\text{rel}}}{K_{\text{nonrel}}}=1.06.$

(b) $v=2.85\times10^8$ m/s; $\gamma=3.203$.

$K_{\text{nonrel}}=\dfrac{1}{2}mv^2=6.78\times10^{-11}$ J; $K_{\text{rel}}=(\gamma-1)mc^2=3.31\times10^{-10}$ J; $K_{\text{rel}}/K_{\text{nonrel}}=4.88.$

EVALUATE: $K_{\text{rel}}/K_{\text{nonrel}}$ increases without bound as v approaches c.

37.38. **IDENTIFY:** The total energy is conserved in the collision.

SET UP: Use $E=mc^2+K$ for the total energy. Since all three particles are at rest after the collision, the final total energy is $2Mc^2+mc^2$. The initial total energy of the two protons is $\gamma 2Mc^2$.

EXECUTE: **(a)** $2Mc^2 + mc^2 = \gamma 2Mc^2 \Rightarrow \gamma = 1 + \frac{m}{2M} = 1 + \frac{9.75}{2(16.7)} = 1.292.$

Note that since $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$, we have that $\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.292)^2}} = 0.6331.$

(b) According to $K = (\gamma - 1)mc^2$ the kinetic energy of each proton is

$$K = (\gamma - 1)Mc^2 = (1.292 - 1)(1.67 \times 10^{-27}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 \left(\frac{1.00\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right) = 274\ \text{MeV}.$$

(c) The rest energy of η^0 is $mc^2 = (9.75 \times 10^{-28}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 \left(\frac{1.00\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right) = 548\ \text{MeV}.$

EVALUATE: **(d)** The kinetic energy lost by the protons is the energy that produces the η^0, $548\ \text{MeV} = 2(274\ \text{MeV})$.

37.39. **IDENTIFY** and **SET UP:** Use $K = q\Delta V = e\Delta V$ and conservation of energy to relate the potential difference to the kinetic energy gained by the electron. Use $K = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)$ to calculate the kinetic energy from the speed.

EXECUTE: **(a)** $K = q\Delta V = e\Delta V$.

$$K = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) = 4.025mc^2 = 3.295 \times 10^{-13}\ \text{J} = 2.06\ \text{MeV}.$$

$\Delta V = K/e = 2.06 \times 10^6$ V.

(b) From part (a), $K = 3.30 \times 10^{-13}\ \text{J} = 2.06\ \text{MeV}$.

EVALUATE: The speed is close to c and the kinetic energy is four times the rest mass.

37.40. **IDENTIFY** and **SET UP:** The astronaut in the spaceship measures the proper time, since the end of a swing occurs at the same location in his frame. $\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$.

EXECUTE: **(a)** $\Delta t_0 = 1.80$ s. $\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \frac{1.80\ \text{s}}{\sqrt{1 - (0.75c/c)^2}} = 2.72$ s.

(b) $\Delta t = 1.80$ s. $\Delta t_0 = \Delta t\sqrt{1 - u^2/c^2} = (1.80\ \text{s})\sqrt{1 - (0.75c/c)^2} = 1.19$ s.

EVALUATE: The motion of the spaceship makes a considerable difference in the measured values for the period of the pendulum!

37.41. **IDENTIFY** and **SET UP:** There must be a length contraction such that the length a becomes the same as b; $l_0 = a$, $l = b$. l_0 is the distance measured by an observer at rest relative to the spacecraft. Use $l = l_0\sqrt{1 - u^2/c^2}$ and solve for u.

EXECUTE: $\frac{l}{l_0} = \sqrt{1 - u^2/c^2}$ so $\frac{b}{a} = \sqrt{1 - u^2/c^2}$;

$a = 1.40b$ gives $b/1.40b = \sqrt{1 - u^2/c^2}$. and thus $1 - u^2/c^2 = 1/(1.40)^2$.

$u = \sqrt{1 - 1/(1.40)^2}\,c = 0.700c = 2.10 \times 10^8$ m/s.

EVALUATE: A length on the spacecraft in the direction of the motion is shortened. A length perpendicular to the motion is unchanged.

37.42. **IDENTIFY** and **SET UP:** The proper length of a side is $l_0 = a$. The side along the direction of motion is shortened to $l = l_0\sqrt{1-v^2/c^2}$. The sides in the two directions perpendicular to the motion are unaffected by the motion and still have a length a.

EXECUTE: $V = a^2 l = a^3\sqrt{1-v^2/c^2}$.

EVALUATE: Only the side parallel to the direction of motion is contracted.

37.43. **IDENTIFY** and **SET UP:** The proper time Δt_0 is the time that elapses in the frame of the space probe. Δt is the time that elapses in the frame of the earth. The distance traveled is 42.2 light years, as measured in the earth frame.

EXECUTE: Light travels 42.2 light years in 42.2 y, so $\Delta t = \left(\dfrac{c}{0.9930c}\right)(42.2\text{ y}) = 42.5\text{ y}$.

$\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (42.5\text{ y})\sqrt{1-(0.9930)^2} = 5.0\text{ y}$. She measures her biological age to be $19\text{ y} + 5.0\text{ y} = 24.0\text{ y}$.

EVALUATE: Her age measured by someone on earth is $19\text{ y} + 42.5\text{ y} = 61.5\text{ y}$.

37.44. **IDENTIFY:** This problem involves time dilation, energy, and momentum.

SET UP: $\Delta t = \gamma\Delta t_0$, $v/c = \sqrt{1-1/\gamma^2}$, $K = (\gamma-1)mc^2$. In the lab: $t_{\text{mean}} = 403$ ps, in the particle's frame: $t_{\text{mean}} = 80.2$ ps.

EXECUTE: **(a)** We want the speed. Use time dilation: $\Delta t = \gamma\Delta t_0$ gives $403\text{ ps} = \gamma(80.2\text{ ps})$, so $\gamma = 5.025$. Now get the speed: $v/c = \sqrt{1-1/\gamma^2} = \sqrt{1-1/5.025^2} = 0.980c$.

(b) We want Δx in the lab. $\Delta x = v\Delta t = (0.980c)(403\text{ ps}) = 11.8$ cm.

(c) We want its energies in the lab frame.

<u>Rest energy</u>: $E_{\text{rest}} = mc^2 = (2.12\times10^{-27}\text{ kg})c^2 = 1.91\times10^{-10}$ J.

<u>Kinetic energy</u>: $K = (\gamma-1)mc^2 = (5.025-1)(1.91\times10^{-10}\text{ J}) = 7.68\times10^{-10}$ J.

<u>Total energy</u>: $E_{\text{tot}} = K + E_{\text{rest}} = 7.68\times10^{-10}\text{ J} + 1.91\times10^{-10}\text{ J} = 9.59\times10^{-10}$ J.

(d) We want the energies in its rest frame. $K = 0$, so $E_{\text{tot}} = E_{\text{rest}} = 1.91\times10^{-10}$ J.

EVALUATE: Even for nonrelativistic motion, $K = 0$ in the particle's rest frame.

37.45. **IDENTIFY:** This problem involves relative velocity and mass.

SET UP: $m_{\text{rel}} = m\gamma = \dfrac{m}{\sqrt{1-u^2/c^2}}$, $v'_x = \dfrac{v_x - u}{1-uv_x/c^2}$. We want the mass of the standard kilogram.

EXECUTE: **(a)** In classical physics, mass doesn't change with speed, so both crews would measure 1.0 kg.

(b) <u>For A</u>: $m_{\text{rel}} = m\gamma_A = \dfrac{m}{\sqrt{1-u_A^2/c^2}} = \dfrac{1.00\text{ kg}}{\sqrt{1-0.80^2}} = 1.7$ kg.

<u>For B</u>: We need to find the speed of B relative to Earth. Use $v'_x = \dfrac{v_x - u}{1-uv_x/c^2}$, letting Earth be the primed frame and A the unprimed frame. We want v'. $v'_{B/E} = \dfrac{0.98c - 0.80c}{1-(0.98c)(0.80c)/c^2} = 0.833c$. The mass is

$$m_{\text{rel}} = \frac{m}{\sqrt{1-u_B^2/c^2}} = \frac{1.00\text{ kg}}{\sqrt{1-0.833^2}} = 1.8\text{ kg}.$$

EVALUATE: B measures a greater mass than A measures because B is moving faster relative to Earth than A is. Both of them, however, would measure 1.00 kg as the rest mass of the standard kilogram.

37.46. **IDENTIFY:** We are looking at a world in which c = 65 mph.
SET UP: The standard relativity equations all apply except that we use c = 65 mph. You are driving at 60 mph.
EXECUTE: **(a)** We want the time difference between the kitchen clock and the moving wristwatch.
Kitchen clock time: You drive 60 mi at 60 mph, so it would take 1.0 h.
Wristwatch time: Your time is the proper time. Use $\Delta t = \gamma\Delta t_0$.

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{1}{\sqrt{1-(60/65)^2}} = 2.60$$

$\Delta t_0 = \Delta t/\gamma =$ (1.0 h)/(2.60) = 0.38 h = 23 min
(b) Estimate: 15 ft long.
(c) We want its length L from a roadside observer. L_0 = 15 ft = proper length.
$L = L_0/\gamma =$ (15 ft)/(2.60) = 5.8 ft = 5 ft 9 in.
(d) We want the speed relative to you. Use $v'_x = \dfrac{v_x - u}{1 - uv_x/c^2}$, with your car being the primed frame and earth the unprimed frame. We want v'. $v' = \dfrac{-60 \text{ mph} - 60 \text{ mph}}{1-(60)(-60)/(65)^2} = 64.8$ mph.
(e) We want the car's length in your frame. $L = L_0\sqrt{1-u^2/c^2}$ = (15 ft)$\sqrt{1-64.8^2/65^2} = 1.2$ ft.
(f) We want the work to reach 60 mph. c = 65 mph = 29.055 m/s. $W = K = (\gamma - 1)mc^2$
$= (2.60-1)(2000 \text{ kg})(29.055 \text{ m/s})^2 = 2.7$ MJ.
(g) We want the work in the real world. Since 60 mph << c, we use $K = \dfrac{1}{2}mv^2$ with 60 mph = 26.82 m/s. This gives W = 720 kJ.
EVALUATE: Relativity would be obvious to us if we normally moved near the speed of light.

37.47. **IDENTIFY:** Since the speed is very close to the speed of light, we must use the relativistic formula for kinetic energy.
SET UP: The relativistic formula for kinetic energy is $K = mc^2\left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right)$ and the relativistic mass is $m_{\text{rel}} = \dfrac{m}{\sqrt{1-v^2/c^2}}$.
EXECUTE: **(a)** $K = 7.0\times10^{12}$ eV $= 1.12\times10^{-6}$ J. Using this value in the relativistic kinetic energy formula and substituting the mass of the proton for m, we get $K = mc^2\left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right)$ which gives $\dfrac{1}{\sqrt{1-v^2/c^2}} = 7.45\times10^3$ and $1 - \dfrac{v^2}{c^2} = \dfrac{1}{(7.45\times10^3)^2}$. Solving for v gives $1 - \dfrac{v^2}{c^2} = \dfrac{(c+v)(c-v)}{c^2} = \dfrac{2(c-v)}{c}$, since $c + v \approx 2c$. Substituting $v = (1-\Delta)c$, we have $1 - \dfrac{v^2}{c^2} = \dfrac{2(c-v)}{c} = \dfrac{2[c-(1-\Delta)c]}{c} = 2\Delta$. Solving for Δ gives $\Delta = \dfrac{1-v^2/c^2}{2} = \dfrac{\frac{1}{(7.45\times10^3)^2}}{2} = 9.0\times10^{-9}$.

(b) Using the relativistic mass formula and the result that $\dfrac{1}{\sqrt{1-v^2/c^2}} = 7.45\times10^3,$ we have

$$m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}} = m\left(\frac{1}{\sqrt{1-v^2/c^2}}\right) = (7.5\times10^3)m.$$

EVALUATE: At such high speeds, the proton's mass is 7500 times as great as its rest mass.

37.48. **IDENTIFY** and **SET UP:** The acceleration parallel to the direction of the force is given by $F_{\|} = \gamma^3 ma_{\|},$ and the acceleration perpendicular to the direction of the force is given by $F_{\perp} = \gamma ma_{\perp},$ where $\gamma = 1/\sqrt{1-v^2/c^2}.$

EXECUTE: Applying the above formulas to the conditions of this problem, we have $\gamma = 1/\sqrt{1-(0.700)^2} = 1.400,$ $F_x = F\cos(30.0°) = \gamma^3 ma_x$ and $F_y = F\sin(30.0°) = \gamma ma_y$. The angle θ that the acceleration makes with respect to the x-axis is given by $\tan\theta = a_y/a_x$. Dividing the acceleration given by the two force equations gives

$$\tan\theta = \frac{\dfrac{F\sin(30.0°)}{\gamma m}}{\dfrac{F\cos(30.0°)}{\gamma^3 m}} = \gamma^2\tan(30.0°) = (1.400)^2\tan(30.0°) = 1.132 \quad\rightarrow\quad \theta = 48.5°.$$

The acceleration makes a counterclockwise angle of 48.5° from the $+x$-axis. Therefore it makes an angle of 18.5° counterclockwise from the direction of the force.

EVALUATE: Notice that the acceleration is not in the same direction as the force.

37.49. **IDENTIFY** and **SET UP:** The clock on the plane measures the proper time Δt_0.

$\Delta t = 4.00\text{ h} = (4.00\text{ h})(3600\text{ s}/1\text{ h}) = 1.44\times10^4\text{ s}.$

$\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ and $\Delta t_0 = \Delta t\sqrt{1-u^2/c^2}.$

EXECUTE: $\dfrac{u}{c}$ small so $\sqrt{1-u^2/c^2} = (1-u^2/c^2)^{1/2} \approx 1-\tfrac{1}{2}\dfrac{u^2}{c^2};$ thus $\Delta t_0 = \Delta t\left(1-\dfrac{1}{2}\dfrac{u^2}{c^2}\right).$

The difference in the clock readings is

$\Delta t - \Delta t_0 = \tfrac{1}{2}\dfrac{u^2}{c^2}\Delta t = \tfrac{1}{2}\left(\dfrac{250\text{ m/s}}{2.998\times10^8\text{ m/s}}\right)^2(1.44\times10^4\text{ s}) = 5.01\times10^{-9}\text{ s}.$ The clock on the plane has the shorter elapsed time.

EVALUATE: Δt_0 is always less than $\Delta t;$ our results agree with this. The speed of the plane is much less than the speed of light, so the difference in the reading of the two clocks is very small.

37.50. **IDENTIFY:** In the rest frame of the spaceship the trip takes $\Delta t_0 = 3.35$ years.

SET UP: As seen from the Earth the trip takes $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$. The distance to the star is 7.11 ly and the speed of light is $c = 1$ ly/y. Let u be the speed of the spaceship (in ly/y) as seen from the Earth.

EXECUTE: **(a)** The time for the trip as seen from the earth will be $\Delta t = \dfrac{7.11\text{ ly}}{u} = \dfrac{3.35\text{ y}}{\sqrt{1-u^2/c^2}}$. Solving for u we obtain $\left(\dfrac{7.11\text{ ly}}{3.35\text{ y}}\right)^2(1-u^2/c^2) = u^2,$ which reduces to

$u = \left(\dfrac{7.11\text{ ly}}{3.35\text{ y}}\right)\cdot\dfrac{1}{\sqrt{1+\left(\dfrac{7.11\text{ ly}}{(3.35\text{ y})(1\text{ ly/y})}\right)^2}} = 0.905\text{ ly/y} = 0.905c.$ Thus, as seen from the Earth, the trip takes $\dfrac{7.11\text{ ly}}{u} = \dfrac{7.11\text{ ly}}{0.905\text{ ly/y}} = 7.86$ years.

(b) According to the passengers the distance is given by $x' = u\Delta t_0 = (0.905\text{ ly/y})(3.35\text{ y}) = 3.03\text{ ly}.$

EVALUATE: The distance to the star as seen by the passengers could also be calculated by using the length contraction formula: $l = l_0\sqrt{1-u^2/c^2}$.

37.51. IDENTIFY and **SET UP:** In crown glass the speed of light is $v = \dfrac{c}{n}.$ Calculate the kinetic energy of an electron that has this speed.

EXECUTE: $v = \dfrac{2.998\times10^8\text{ m/s}}{1.52} = 1.972\times10^8\text{ m/s}.$

$K = mc^2(\gamma-1).$

$mc^2 = (9.109\times10^{-31}\text{ kg})(2.998\times10^8\text{ m/s})^2 = 8.187\times10^{-14}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 0.5111\text{ MeV}.$

$\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = \dfrac{1}{\sqrt{1-((1.972\times10^8\text{ m/s})/(2.998\times10^8\text{ m/s}))^2}} = 1.328.$

$K = mc^2(\gamma-1) = (0.5111\text{ MeV})(1.328-1) = 0.168\text{ MeV}.$

EVALUATE: No object can travel faster than the speed of light in vacuum but there is nothing that prohibits an object from traveling faster than the speed of light in some material.

37.52. IDENTIFY: We use relativistic energy to investigate the quark model of the proton.

SET UP: $K = (\gamma-1)mc^2$, $v/c = \sqrt{1-1/\gamma^2}$, $\gamma = \dfrac{1}{\sqrt{1-u^2/c^2}}$.

EXECUTE: **(a)** We want γ_u. $m_pc^2 = 2K_u + K_d = 3K_u$. $K_u = \dfrac{1}{3}m_pc^2$ and $K_u = m_uc^2(\gamma_u-1).$ So $\dfrac{1}{3}m_pc^2 = m_uc^2(\gamma_u-1).$ Solve for γ_u using the given masses: $\gamma_u = 136$.

(b) We want γ_d. Use the same procedure as in (a), giving $\gamma_u = 65.8$.

(c) We want the speeds. $v_u/c = \sqrt{1-1/\gamma_u^2} = 0.99997,$ so the answer is yes. $v_d/c = \sqrt{1-1/\gamma_d^2} = 0.9999,$ so the answer is yes.

(d) We want to know what % of the proton mass is from gluons. $m_pc^2 = 2K_u + K_d + E_g$.

$\dfrac{E_g}{m_pc^2} = \dfrac{m_pc^2-(2K_u+K_d)}{m_pc^2} = \dfrac{m_p-2m_u(\gamma-1)-m_d(\gamma-1)}{m_p}.$ $\gamma = \dfrac{1}{\sqrt{1-u^2/c^2}} = \dfrac{1}{\sqrt{1-0.90^2}} = 2.294.$ Using this γ and the given masses gives $\dfrac{E_g}{m_pc^2} = 0.987 \approx 99\%.$

(e) We want to estimate the frequency f. If T is the time for one cycle of amp006Citude A at speed v, then $vT = 4A.$ $f = 1/T = 1/(4A/v) = v/4A.$ $A = \dfrac{1}{2}(1.7\times10^{-15}\text{ m}) = 8.5\times10^{-16}\text{ m}.$ Using this value with $v = 0.90c$ gives our estimate $f = 8\times10^{22}\text{ Hz} \approx 10^{23}\text{ Hz}.$

EVALUATE: Rough models such as this one do not give precise results, but they do provide approximate estimations than can be compared to experimental results.

37.53. **IDENTIFY** and **SET UP:** The energy released is $E = (\Delta m)c^2$. $\Delta m = \left(\frac{1}{10^4}\right)(12.0\text{ kg})$. $P_{\text{av}} = \frac{E}{t}$.

The change in gravitational potential energy is $mg\Delta y$.

EXECUTE: **(a)** $E = (\Delta m)c^2 = \left(\frac{1}{10^4}\right)(12.0\text{ kg})(3.00\times10^8\text{ m/s})^2 = 1.08\times10^{14}\text{ J}$.

(b) $P_{\text{av}} = \frac{E}{t} = \frac{1.08\times10^{14}\text{ J}}{4.00\times10^{-6}\text{ s}} = 2.70\times10^{19}\text{ W}$.

(c) $E = \Delta U = mg\Delta y$. $m = \frac{E}{g\Delta y} = \frac{1.08\times10^{14}\text{ J}}{(9.80\text{ m/s}^2)(1.00\times10^3\text{ m})} = 1.10\times10^{10}\text{ kg}$.

EVALUATE: The mass decrease is only 1.2 grams, but the energy released is very large.

37.54. **IDENTIFY:** The protons are moving at speeds that are comparable to the speed of light, so we must use the relativistic velocity addition formula.

SET UP: S is lab frame and S' is frame of proton moving in $+x$-direction. $v_x = -0.700c$. In lab frame each proton has speed αc. $u = +\alpha c$. $v_x = -\alpha c$. $v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = \frac{-0.700c + \alpha c}{1 - 0.700\alpha} = -\alpha c$.

EXECUTE: $(1 - 0.700\alpha)(-\alpha) = -0.700 + \alpha$. $0.700\alpha^2 - 2\alpha + 0.700 = 0$. The quadratic formula gives $\alpha = 2.45$ or $\alpha = 0.408$. We cannot have $v > c$ so $\alpha = 0.408$. Each proton has speed $0.408c$ in the earth frame.

EVALUATE: To the earth observer, the protons are separating at $2(0.408c) = 0.816c$, but to the protons they are separating at $0.700c$.

37.55. **IDENTIFY:** We are investigating the effect of motion on a magnetic field.

SET UP and **EXECUTE:** **(a)** We want the magnetic moment. $\vec{\mu} = IA\hat{\boldsymbol{j}} = IHL\hat{\boldsymbol{j}}$.

(b) We want the torque. $\vec{\tau} = \vec{\mu}\times\vec{B} = IHLB\hat{\boldsymbol{i}}$

(c) We want $\vec{\mu}'$. $\vec{\mu}' = IHL'\hat{\boldsymbol{j}}$. $L' = L/\gamma$, so $\vec{\mu}' = (IHL/\gamma)\hat{\boldsymbol{j}} = \vec{\mu}/\gamma$.

(d) We want $\vec{\boldsymbol{B}}'$. Given that the torques are equal in both frames, we have $\vec{\mu}'\times\vec{B}' = \vec{\mu}\times\vec{B}$. Using our result from part (c) gives $(\vec{\mu}/\gamma)\times\vec{B}' = \vec{\mu}\times\vec{B}$, so $\vec{\boldsymbol{B}}' = \vec{\boldsymbol{B}}\gamma$, which we can express as $\vec{\boldsymbol{B}}' = B\gamma\hat{\boldsymbol{k}}$.

(e) Generalize from $\vec{\boldsymbol{B}}' = \vec{\boldsymbol{B}}\gamma$, which gives $\vec{\boldsymbol{B}}'_\perp = \vec{\boldsymbol{B}}_\perp\gamma$.

EVALUATE: Note that the fields are different when observed from moving reference frames.

37.56. **IDENTIFY:** We are investigating the effect of motion on an electric field.

SET UP and **EXECUTE:** **(a)** The net force is zero because there is no motion.

(b) We want $\vec{\boldsymbol{B}}'$. Using the result of problem 37.55, but with $\vec{\boldsymbol{B}} = -B\,\hat{\boldsymbol{j}}$ instead of $\vec{\boldsymbol{B}} = B\,\hat{\boldsymbol{k}}$. This gives $\vec{\boldsymbol{B}}' = \vec{\boldsymbol{B}}\gamma = -\gamma B\hat{\boldsymbol{j}}$.

(c) We want $\vec{\boldsymbol{F}}'$. Use $\vec{\boldsymbol{F}}' = Q\vec{\boldsymbol{v}}'\times\vec{\boldsymbol{B}}'$ with $\vec{\boldsymbol{v}}' = -v\hat{\boldsymbol{i}}$ and $\vec{\boldsymbol{B}}' = -\gamma B\hat{\boldsymbol{j}}$, giving $\vec{\boldsymbol{F}}' = Q\gamma vB\hat{\boldsymbol{k}}$.

(d) Generalize: $\vec{\boldsymbol{F}}' = Q\vec{E}'_\perp = Q\gamma\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}$, so $\vec{\boldsymbol{E}}'_\perp = \gamma\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}$.

EVALUATE: Note that there is no electric field in the lab, but there is one in the moving frame.

37.57. **IDENTIFY** and **SET UP:** An increase in wavelength corresponds to a decrease in frequency ($f = c/\lambda$), so the atoms are moving away from the earth. The galaxy is receding, so we use $f = \sqrt{\frac{c-u}{c+u}}f_0$.

EXECUTE: Solve for u: $(f/f_0)^2(c+u) = c - u$ and $u = c\left(\frac{1-(f/f_0)^2}{1+(f/f_0)^2}\right)$.

$f = c/\lambda$, $f_0 = c/\lambda_0$ so $f/f_0 = \lambda_0/\lambda$.

$$u = c\left(\frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2}\right) = c\left(\frac{1-(656.3/953.4)^2}{1+(656.3/953.4)^2}\right) = 0.357c = 1.07\times10^8 \text{ m/s}.$$

EVALUATE: The relative speed is large, 36% of c. The cosmological implication of such observations will be discussed in Chapter 44.

37.58. **IDENTIFY:** Apply the Lorentz coordinate transformation.

SET UP: Let t and t' be time intervals between the events as measured in the two frames and let x and x' be the difference in the positions of the two events as measured in the two frames.

EXECUTE: Setting $x = 0$ in the Lorentz transformation equations, the first equation becomes $x' = -\gamma ut$ and the last, upon multiplication by c, becomes $ct' = \gamma ct$. Squaring and subtracting gives $c^2t'^2 - x'^2 = \gamma^2 t^2(c^2 - u^2)$. But $\gamma^2 = c^2/(c^2 - v^2)$, so $\gamma^2 t^2(c^2 - v^2) = c^2t^2$. Therefore, $c^2t'^2 - x'^2 = c^2t^2$ which gives $x' = c\sqrt{t'^2 - t^2} = c\sqrt{(2.15 \text{ s})^2 - (1.80 \text{ s})^2} = 3.53\times10^8$ m.

EVALUATE: We did not have to calculate the speed u of frame S' relative to frame S.

37.59. **IDENTIFY:** The baseball is moving toward the radar gun, so apply the Doppler effect equation $f = \sqrt{\frac{c+u}{c-u}} f_0$.

SET UP: The baseball had better be moving nonrelativistically, so the Doppler shift formula becomes $f \cong f_0(1-(u/c))$. In the baseball's frame, this is the frequency with which the radar waves strike the baseball, and the baseball reradiates at f. But in the coach's frame, the reflected waves are Doppler shifted again, so the detected frequency is $f(1-(u/c)) = f_0(1-(u/c))^2 \approx f_0(1-2(u/c))$.

EXECUTE: $\Delta f = 2f_0(u/c)$ and the fractional frequency shift is $\frac{\Delta f}{f_0} = 2(u/c)$.

$$u = \frac{\Delta f}{2f_0}c = \frac{(2.86\times10^{-7})}{2}(3.00\times10^8 \text{ m}) = 42.9 \text{ m/s} = 154 \text{ km/h} = 92.5 \text{ mi/h}.$$

EVALUATE: $u << c$, so using the approximate expression in place of $f = \sqrt{\frac{c+u}{c-u}} f_0$ is very accurate.

37.60. **IDENTIFY:** Apply the relativistic expressions for kinetic energy, velocity transformation, length contraction and time dilation.

SET UP: In part (c) let S' be the earth frame and let S be the frame of the ball. Let the direction from Einstein to Lorentz be positive, so $u = -1.80\times10^8$ m/s. In part (d) the proper length is $l_0 = 20.0$ m and in part (f) the proper time is measured by the rabbit.

EXECUTE: **(a)** 80.0 m/s is nonrelativistic, and $K = \frac{1}{2}mv^2 = 186$ J.

(b) $K = (\gamma - 1)mc^2 = 1.31\times10^{15}$ J.

(c) In Eq. (37.23), $v' = 2.20\times10^8$ m/s, $u = -1.80\times10^8$ m/s, and so $v = 7.14\times10^7$ m/s.

(d) $l = \frac{l_0}{\gamma} = \frac{20.0 \text{ m}}{\gamma} = 13.6$ m.

(e) $\frac{20.0 \text{ m}}{2.20\times10^8 \text{ m/s}} = 9.09\times10^{-8}$ s.

(f) $\Delta t_0 = \frac{\Delta t}{\gamma} = 6.18\times10^{-8}$ s.

EVALUATE: In part (f) we could also calculate Δt_0 as $\Delta t_0 = \frac{13.6 \text{ m}}{2.20\times10^8 \text{ m/s}} = 6.18\times10^{-8}$ s.

37.61. **IDENTIFY:** We need to use the relativistic form of Newton's second law because the speed of the proton is close to the speed of light.

SET UP: $\vec{F}$ and $\vec{v}$ are perpendicular, so $F = \gamma ma = \gamma m\dfrac{v^2}{R}$. $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = \dfrac{1}{\sqrt{1-(0.750)^2}} = 1.512.$

EXECUTE: $F = (1.512)(1.67\times10^{-27}\ \text{kg})\dfrac{[(0.750)(3.00\times10^{8}\ \text{m/s}]^2}{628\ \text{m}} = 2.04\times10^{-13}\ \text{N}.$

EVALUATE: If we ignored relativity, the force would be

$F_{\text{rel}}/\gamma = \dfrac{2.04\times10^{-13}\ \text{N}}{1.512} = 1.35\times10^{-13}\ \text{N},$ which is substantially less than the relativistic force.

37.62. **IDENTIFY** and **SET UP:** For part (a) follow the procedure specified in the hint. For part (b) apply $f = \sqrt{\dfrac{c+u}{c-u}}f_0$ and $f = \sqrt{\dfrac{c-u}{c+u}}f_0$.

EXECUTE: **(a)** As in the hint, both the sender and the receiver measure the same distance. However, in our frame, the ship has moved between emission of successive wavefronts, and we can use the time $T = 1/f$ as the proper time, with the result that $f = \gamma f_0 > f_0$.

(b) Toward: $f_1 = f_0\sqrt{\dfrac{c+u}{c-u}} = 345\ \text{MHz}\left(\dfrac{1+0.758}{1-0.758}\right)^{1/2} = 930\ \text{MHz}$ and

$f_1 - f_0 = 930\ \text{MHz} - 345\ \text{MHz} = 585\ \text{MHz}.$

Away: $f_2 = f_0\sqrt{\dfrac{c-u}{c+u}} = 345\ \text{MHz}\left(\dfrac{1-0.758}{1+0.758}\right)^{1/2} = 128\ \text{MHz}$ and $f_2 - f_0 = -217\ \text{MHz}.$

(c) $f_3 = \gamma f_0 = 1.53 f_0 = 528\ \text{MHz},\ f_3 - f_0 = 183\ \text{MHz}.$

EVALUATE: The frequency in part (c) is the average of the two frequencies in part (b). A little algebra shows that f_3 is precisely equal to $(f_1 + f_2)/2$.

37.63. **IDENTIFY** and **SET UP:** The equation $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ relates the time interval in the laboratory (Δt) to the time interval (Δt_0) in the rest frame of the particle.

EXECUTE: **(a)** Solve the above equation for $(\Delta t)^2$. Squaring gives $(\Delta t)^2 = (\Delta t_0)^2(1-u^2/c^2)^{-1}$. Therefore a graph of $(\Delta t)^2$ versus $(1-u^2/c^2)^{-1}$ should be a straight line with slope equal to $(\Delta t_0)^2$. Figure 37.63 shows this graph for the data in the problem. The slope of the best-fit straight line is $0.6709\times 10^{-15}\ \text{s}^2$, so $\Delta t_0 = \sqrt{0.6709\times10^{-15}\ \text{s}^2} = 2.6\times10^{-8}\ \text{s} = 26\ \text{ns}.$

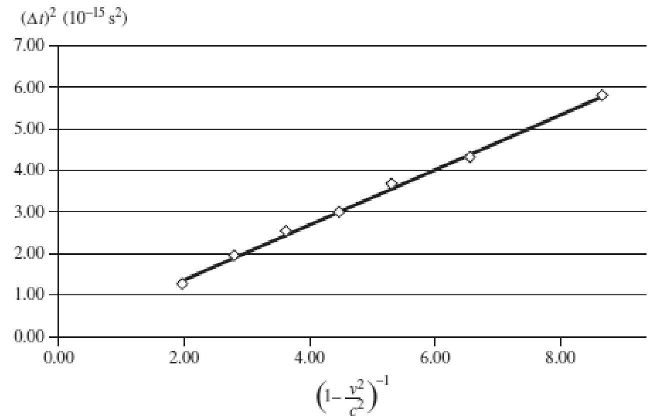


Figure 37.63

(b) Using $\Delta t = 4\,\Delta t_0$, the equation $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ gives $4\Delta t_0 = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$. Solving for u/c gives $u/c = 0.97$.

EVALUATE: At speeds near the speed of light, there is a very large difference between the lifetime measured in the laboratory frame compared to the lifetime in the rest frame of the particle.

37.64. **IDENTIFY:** Apply the Lorentz velocity transformation.

SET UP: Let the tank and the light both be traveling in the $+x$-direction. Let S be the lab frame and let S' be the frame of the tank of water.

EXECUTE: In the equation $v_x = \dfrac{v'_x + u}{1 + uv'_x/c^2}$, $u = V$, $v' = (c/n)$. $v = \dfrac{(c/n)+V}{1+\dfrac{cV}{nc^2}} = \dfrac{(c/n)+V}{1+(V/nc)}$. For $V << c$,

$(1+V/nc)^{-1} \approx (1-V/nc)$. This gives

$v \approx [(c/n)+V][1-(V/nc)] = c/n + V - (V/n^2) - (V^2/nc) \approx \dfrac{c}{n} + \left(1-\dfrac{1}{n^2}\right)V$, so $k = \left(1-\dfrac{1}{n^2}\right)$. For water, $n = 1.333$ and $k = 0.437$.

EVALUATE: The Lorentz transformation predicts a value of k in excellent agreement with the value that is measured experimentally.

37.65. **IDENTIFY** and **SET UP:** When the force on a particle is along the same line as its velocity, the force and acceleration are related by $F = \gamma^3 ma$, where $\gamma = 1/\sqrt{1-v^2/c^2}$.

EXECUTE: **(a)** Solve the equation $F = \gamma^3 ma$ for a^2.

$$a = \frac{F}{m\gamma^3} = \frac{F}{m}\left(1-\frac{v^2}{c^2}\right)^{3/2} \qquad \rightarrow \qquad a^2 = \left(\frac{F}{m}\right)^2\left(1-\frac{v^2}{c^2}\right)^3.$$

From this result we see that a graph of a^2 versus $\left(1-\dfrac{v^2}{c^2}\right)^3$ should be a straight line with slope equal to $(F/m)^2$. Figure 37.65 shows the graph of the data in the table with the problem. It is well fit by a straight line having slope equal to 1.608×10^9 m^2/s^4. Therefore the mass is

$$(F/m)^2 = \text{slope} \qquad \rightarrow \qquad m = \frac{F}{\sqrt{\text{slope}}} = \frac{8.00\times10^{-14}\ \text{N}}{\sqrt{1.608\times10^9\ \text{m}^2/\text{s}^4}} = 2.0\times10^{-18}\ \text{kg}.$$

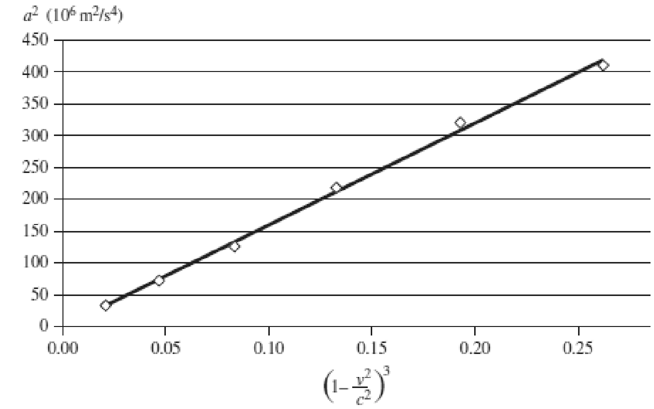


Figure 37.65

(b) In this case, $v << c$, so γ is essentially equal to 1. Therefore we can use the familiar form of Newton's second law, $F = ma$.

$a = F/m = (8.00\times10^{-14}\text{ N})/(2.0\times10^{-18}\text{ kg}) = 4.0\times10^4\text{ m/s}^2$.

EVALUATE: When v is close to c, $a = F/m$ does not give the correct result. For example, using data from the table in the problem, when $v/c = 0.85$, the acceleration is measured to be 5900 m/s^2. But using the familiar Newtonian formula we get $a = F/m = (8.00\times10^{-14}\text{ N})/(2.0\times10^{-18}\text{ kg}) = 4.0\times10^4\text{ m/s}^2$ = 40,000 m/s^2, which is *very* different from the relativistic result of 5900 m/s^2.

37.66. **IDENTIFY:** Relative motion between the observer and the source of electromagnetic waves affects the frequency received by the observer due to the Doppler effect. If the source is moving toward the observer with speed u and emitting frequency f_0, the frequency f that the observer receives is given by $f = f_0\sqrt{\dfrac{c+u}{c-u}}$, and if it is moving away the formula is $f = f_0\sqrt{\dfrac{c-u}{c+u}}$. Notice that $f > f_0$.

SET UP: In this case, we know f and f_0 and want to find u. Looking at the data given in the problem, we see that $f < f_0$, which means that the source must be moving *away from* the observer, so we use $f = f_0\sqrt{\dfrac{c-u}{c+u}}$. Solve the equation $f = f_0\sqrt{\dfrac{c-u}{c+u}}$ for u, since we know the frequencies. This gives $u = \left(\dfrac{f_0^2 - f^2}{f_0^2 + f^2}\right)c$.

EXECUTE: **(a)** To see which source is moving fastest, look at f_0/f; the larger this ratio, the greater the speed u.

For A: $f_0/f = 9.2/7.1 = 1.30$.
For B: $f_0/f = 8.6/5.4 = 1.59$.
For C: $f_0/f = 7.9/6.1 = 1.30$.
For D: $f_0/f = 8.9/8.1 = 1.10$.

Therefore source B is moving fastest and source D is moving slowest. The speed of B is

$$u = \left(\frac{(8.6\text{ THz})^2 - (5.4\text{ THz})^2}{(8.6\text{ THz})^2 + (5.4\text{ THz})^2}\right)c = 0.434c,$$ which rounds to $0.43c$, away from the detector.

(b) The speed for source D is $u = \left(\dfrac{(8.9\text{ THz})^2 - (8.1\text{ THz})^2}{(8.9\text{ THz})^2 + (8.1\text{ THz})^2}\right)c = 0.094c$, away from the detector.

(c) Since B is now approaching the detector, we use $f = f_0\sqrt{\dfrac{c+u}{c-u}}$ with $u = 0.434c$ and $f_0 = 8.6$ THz.

This gives $f = f_0\sqrt{\dfrac{c+u}{c-u}} = (8.6\text{ THz})\sqrt{\dfrac{1+0.434}{1-0.434}} = 14\text{ THz} = 1.4\times10^{13}$ Hz.

EVALUATE: The change in observed frequency is small for low speeds but increases dramatically as $u \to c$.

37.67. **IDENTIFY:** We are using the Lorentz transformation equations to investigate space-time diagrams.
SET UP: S is the reference frame of the train station and S' is the moving frame of the rocket train. $x' = \gamma(x - vt)$, $t' = \gamma\left(t - xv/c^2\right)$.

EXECUTE: **(a)** Event 1: $x_1 = 0$, $t_1 = 0$, so $x_1' = 0$, $t_1' = 0$. So $(x_1', t_1') = (0, 0)$.

Event 2: $x_2 = -L$, $t_2 = 0$. $x_2' = -L\gamma$, $t_2' = [-(-L)v/c^2]\gamma = Lv\gamma/c^2$. So $(x_2', t_2') = (-L\gamma, Lv\gamma/c^2)$.

Event 3: $x_3 = 0$, $t_3 = T$. $x_3' = -vT\gamma$, $t_3' = T\gamma$. So $(x_3', t_3') = (-vT\gamma, T\gamma)$.

Event 4: $x_4 = -L$, $t_4 = T$. $x_4' = -(L + vT)\gamma$, $t_4' = (T + Lv/c^2)\gamma$. So $(x_4', t_4') = \left((-L + vT)\gamma, (T + Lv/c^2)\gamma\right)$.

(b) Fig. 37.67 shows the spacetime diagram.

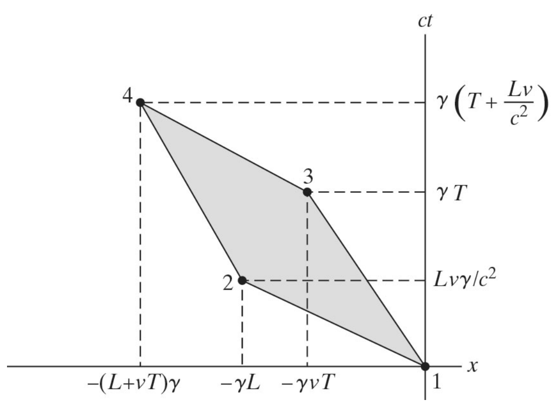


Figure 37.67

(c) We want the area in frame S. $A = LcT = cLT$.
(d) We want the area in frame S'. Use the hint in the problem. For two vectors in the xy-plane, the cross product has a z component given by $A_xB_y - B_xA_y$, so the magnitude of this quantity is the area of the parellelogram. Apply this approach to the diagram in Fig. 37.67. $\vec{A}$ is the vector from 1 to 3 and $\vec{B}$ is the vector from 1 to 2. Their components are $A_x = -\gamma vt$, $A_y = \gamma T$, $B_x = -\gamma L$, and $B_y = Lv\gamma/c^2$. The area is $A = |A_xB_y - B_xA_y| = LT$. But recall that we replace t with ct, so $T \to cT$. Therefore the area is $A = cLT$.
EVALUATE: **(e)** The area is cLT in both reference frames. So it is independent of the reference frame, or "Lorentz invariant."

37.68. **IDENTIFY:** Apply the Doppler effect equation.
SET UP: At the two positions shown in the figure given in the problem, the velocities of the star relative to the earth are $u + v$ and $u - v$, where u is the velocity of the center of mass and v is the orbital velocity.
EXECUTE: **(a)** $f_0 = 4.568110\times10^{14}$ Hz; $f_+ = 4.568910\times10^{14}$ Hz; $f_- = 4.567710\times10^{14}$ Hz.

$$\left.\begin{aligned} f_+ &= \sqrt{\frac{c+(u+v)}{c-(u+v)}}f_0 \\ f_- &= \sqrt{\frac{c+(u-v)}{c-(u-v)}}f_0 \end{aligned}\right\} \Rightarrow \begin{aligned} &f_+^2(c-(u+v)) = f_0^2\,(c+(u+v)) \\ &f_-^2(c-(u-v)) = f_0^2(c+(u-v)) \end{aligned}$$

$(u+v) = \dfrac{(f_+/f_0)^2-1}{(f_+/f_0)^2+1}c$ and $(u-v) = \dfrac{(f_-^2/f_0^2)-1}{(f_-^2/f_0^2)+1}c$. $u+v = 5.25\times10^4$ m/s and $u-v = -2.63\times10^4$ m/s.

This gives $u = +1.31\times10^4$ m/s (moving toward at 13.1 km/s) and $v = 3.94\times10^4$ m/s.

(b) $v = 3.94\times10^4$ m/s; $T = 11.0$ days. $2\pi R = vt \Rightarrow$

$R = \dfrac{(3.94\times10^4\text{ m/s})(11.0\text{ days})(24\text{ hrs/day})(3600\text{ sec/hr})}{2\pi} = 5.96\times10^9$ m. This is about 0.040 times the earth-sun distance.

Also the gravitational force between them (a distance of $2R$) must equal the centripetal force from the center of mass:

$$\frac{(Gm^2)}{(2R)^2} = \frac{mv^2}{R} \Rightarrow m = \frac{4Rv^2}{G} = \frac{4(5.96\times10^9\text{ m})(3.94\times10^4\text{ m/s})^2}{6.672\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = 5.55\times10^{29}\text{ kg} = 0.279\,m_{\text{sun}}.$$

EVALUATE: u and v are both much less than c, so we could have used the approximate expression $\Delta f = \pm f_0 v_{\text{rev}}/c$, where v_{rev} is the speed of the source relative to the observer.

37.69. IDENTIFY: Apply conservation of total energy, in the frame in which the total momentum is zero (the center of momentum frame).

SET UP: In the center of momentum frame, the two protons approach each other with equal velocities (since the protons have the same mass). After the collision, the two protons are at rest—but now there are kaons as well. In this situation the kinetic energy of the protons must equal the total rest energy of the two kaons.

EXECUTE: **(a)** $2(\gamma_{\text{cm}}-1)m_{\text{p}}c^2 = 2m_{\text{k}}c^2 \Rightarrow \gamma_{\text{cm}} = 1+\dfrac{m_{\text{k}}}{m_{\text{p}}} = 1.526.$ The velocity of a proton in the center of momentum frame is then $v_{\text{cm}} = c\sqrt{\dfrac{\gamma_{\text{cm}}^2-1}{\gamma_{\text{cm}}^2}} = 0.7554c.$

To get the velocity of this proton in the lab frame, we must use the Lorentz velocity transformations. This is the same as "hopping" into the proton that will be our target and asking what the velocity of the projectile proton is. Taking the lab frame to be the unprimed frame moving to the left, $u = v_{\text{cm}}$ and $v' = v_{\text{cm}}$ (the velocity of the projectile proton in the center of momentum frame).

$$v_{\text{lab}} = \frac{v'+u}{1+\frac{uv'}{c^2}} = \frac{2v_{\text{cm}}}{1+\frac{v_{\text{cm}}^2}{c^2}} = 0.9619c \Rightarrow \gamma_{\text{lab}} = \frac{1}{\sqrt{1-\frac{v_{\text{lab}}^2}{c^2}}} = 3.658 \Rightarrow K_{\text{lab}} = (\gamma_{\text{lab}}-1)m_{\text{p}}c^2 = 2494\text{ MeV}.$$

(b) $\dfrac{K_{\text{lab}}}{2m_{\text{k}}} = \dfrac{2494\text{ MeV}}{2(493.7\text{ MeV})} = 2.526.$

(c) The center of momentum case considered in part (a) is the same as this situation. Thus, the kinetic energy required *is* just twice the rest mass energy of the kaons. $K_{\text{cm}} = 2(493.7\text{ MeV}) = 987.4\text{ MeV}.$

EVALUATE: The colliding beam situation of part (c) offers a substantial advantage over the fixed target experiment in part (b). It takes less energy to create two kaons in the proton center of momentum frame.

37.70. IDENTIFY and **SET UP:** Apply the procedures specified in the problem.

EXECUTE: For any function $f = f(x,t)$ and $x = x(x',t'), t = t(x',t')$, let $F(x',t') = f(x(x',t'),t(x',t'))$ and use the standard (but mathematically improper) notation $F(x',t') = f(x',t')$. The chain rule is then

$$\frac{\partial f(x',t')}{\partial x}=\frac{\partial f(x,t)}{\partial x'}\frac{\partial x'}{\partial x}+\frac{\partial f(x',t')}{\partial t'}\frac{\partial t'}{\partial x},$$
$$\frac{\partial f(x',t')}{\partial t}=\frac{\partial f(x,t)}{\partial x'}\frac{\partial x'}{\partial t}+\frac{\partial f(x',t')}{\partial x'}\frac{\partial t'}{\partial t}.$$

In this solution, the explicit dependence of the functions on the sets of dependent variables is suppressed, and the above relations are then $\frac{\partial f}{\partial x}=\frac{\partial f}{\partial x'}\frac{\partial x'}{\partial x}+\frac{\partial f}{\partial t'}\frac{\partial t'}{\partial x}$, $\frac{\partial f}{\partial t}=\frac{\partial f}{\partial x'}\frac{\partial x'}{\partial t}+\frac{\partial f}{\partial t'}\frac{\partial t'}{\partial t}$.

(a) $\frac{\partial x'}{\partial x}=1, \frac{\partial x'}{\partial t}=-v, \frac{\partial t'}{\partial x}=0$ and $\frac{\partial t'}{\partial t}=1$. Then, $\frac{\partial E}{\partial x}=\frac{\partial E}{\partial x'}$, and $\frac{\partial^2 E}{\partial x^2}=\frac{\partial^2 E}{\partial x'^2}$. For the time derivative, $\frac{\partial E}{\partial t}=-v\frac{\partial E}{\partial x'}+\frac{\partial E}{\partial t'}$. To find the second time derivative, the chain rule must be applied to both terms; that is, $\frac{\partial}{\partial t}\frac{\partial E}{\partial x'}=-v\frac{\partial^2 E}{\partial x'^2}+\frac{\partial^2 E}{\partial t'\partial x'}, \frac{\partial}{\partial t}\frac{\partial E}{\partial t'}=-v\frac{\partial^2 E}{\partial x'\partial t'}+\frac{\partial^2 E}{\partial t'^2}$.

Using these in $\frac{\partial^2 E}{\partial t^2}$, collecting terms and equating the mixed partial derivatives gives

$\frac{\partial^2 E}{\partial t^2}=v^2\frac{\partial^2 E}{\partial x'^2}-2v\frac{\partial^2 E}{\partial x'\partial t'}+\frac{\partial^2 E}{\partial t'^2}$, and using this and the above expression for $\frac{\partial^2 E}{\partial x'^2}$ gives the result.

(b) For the Lorentz transformation, $\frac{\partial x'}{\partial x}=\gamma, \frac{\partial x'}{\partial t}=\gamma v, \frac{\partial t'}{\partial x}=\gamma v/c^2$ and $\frac{\partial t'}{\partial t}=\gamma$.

The first partials are then $\frac{\partial E}{\partial x}=\gamma\frac{\partial E}{\partial x'}-\gamma\frac{v}{c^2}\frac{\partial E}{\partial t'}, \frac{\partial E}{\partial t}=-\gamma v\frac{\partial E}{\partial x'}+\gamma\frac{\partial E}{\partial t'}$

and the second partials are (again equating the mixed partials)

$$\frac{\partial^2 E}{\partial x^2}=\gamma^2\frac{\partial^2 E}{\partial x'^2}+\gamma^2\frac{v^2}{c^4}\frac{\partial^2 E}{\partial t'^2}-2\gamma^2\frac{v}{c^2}\frac{\partial^2 E}{\partial x'\partial t'}$$
$$\frac{\partial^2 E}{\partial t^2}=\gamma^2 v^2\frac{\partial^2 E}{\partial x'^2}+\gamma^2\frac{\partial^2 E}{\partial t'^2}-2\gamma^2 v\frac{\partial^2 E}{\partial x'\partial t'}.$$

Substituting into the wave equation and combining terms (note that the mixed partials cancel),

$$\frac{\partial^2 E}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}=\gamma^2\left(1-\frac{v^2}{c^2}\right)\frac{\partial^2 E}{\partial x'^2}+\gamma^2\left(\frac{v^2}{c^4}-\frac{1}{c^2}\right)\frac{\partial^2 E}{\partial t'^2}=\frac{\partial^2 E}{\partial x'^2}-\frac{1}{c^2}\frac{\partial^2 E}{\partial t'^2}=0.$$

EVALUATE: The general form of the wave equation is given by Eq. (32.1). The coefficient of the $\partial^2/\partial t^2$ term is the inverse of the square of the wave speed. This coefficient is the same in both frames, so the wave speed is the same in both frames.

37.71. **IDENTIFY** and **SET UP:** The relativity formulas apply, but c = 300 m/s in this universe.
$\gamma=1/\sqrt{1-v^2/c^2}$ = $1/\sqrt{1-(180\text{ m/s})^2/(300\text{ m/s})^2}$ = 1.25. The length is $l = l_0/\gamma$.

EXECUTE: $l = l_0/\gamma$ = (60 m)/(1.25) = 48 m, choice (c).

EVALUATE: Relativistic effects would be "common sense" in this universe!

37.72. **IDENTIFY** and **SET UP:** The relativity formulas apply, but c = 300 m/s in this universe.
$\gamma=1/\sqrt{1-v^2/c^2}$ = $1/\sqrt{1-(180\text{ m/s})^2/(300\text{ m/s})^2}$ = 1.40.

EXECUTE: The relativistic mass is $m\gamma$ = (20,000 kg)(1.25) = 25,000 kg, choice (d).

EVALUATE: The relativistic mass of the passengers and their luggage would also be greater by the factor 1.25, so excess baggage fees could be rather common.

37.73. **IDENTIFY:** The rest energy is $E_0 = mc^2$.

SET UP: In our universe, $E_0 = mc^2$ and in the alternative universe the same formula would apply, except that c would be different, call it c', so $E_0' = mc'^2$. Therefore $E_0' = \left(\frac{E_0}{c^2}\right)c'^2$.

EXECUTE: $E_0' = \left(\frac{E_0}{c^2}\right)c'^2 = \left(\frac{8.2\times10^{-14}\text{ J}}{(3.0\times10^8\text{ m/s})^2}\right)(300\text{ m/s})^2 = 8.2\times10^{-26}\text{ J}$, choice (b).

EVALUATE: The rest energy of all other particles would be reduced by the same fraction in this alternate universe.

37.74. **IDENTIFY** and **SET UP:** The kinetic energy is $K = (\gamma - 1)mc^2$.

EXECUTE: The kinetic energy is now equal to the rest energy, so $K = (\gamma - 1)mc^2 = mc^2$, which means that $\gamma = 2$. Therefore $2 = 1/\sqrt{1 - v^2/c^2}$. Solving for v gives $v = c\sqrt{1-(1/2)^2} = (300\text{ m/s})\sqrt{3/4} = 260\text{ m/s}$, choice (b).

EVALUATE: Airplanes and rockets would certainly need to take relativistic effects into consideration in this alternative universe!

38

PHOTONS: LIGHT WAVES BEHAVING AS PARTICLES

VP38.3.1. **IDENTIFY:** This problem involves the energy and momentum of a photon.
SET UP: $E = hc/\lambda$, $\lambda = h/p$.
EXECUTE: **(a)** We want the energy and momentum of the photon. $E = hc/\lambda = hc/(625\text{ nm}) = 3.18 \times 10^{-19}$ J. $p = h/\lambda = h/(625\text{ nm}) = 1.06\times10^{-27}\text{ kg}\cdot\text{m/s}$.
(b) We want the power output of the laser. $\dfrac{E}{t} = \dfrac{(2.15\times10^{16}\text{photons})(3.18\times10^{-19}\text{ J/photon})}{1.00\text{ s}}$ $= 6.77$ mW.
EVALUATE: Don't confuse the frequency of the light with the frequency at which photons are emitted. They are *very* different!

VP38.3.2. **IDENTIFY:** This problem is about the photoelectric effect.
SET UP: $hf = K_{max} + \phi$.
EXECUTE: **(a)** We want the minimum frequency to produced photoelectrons. This is the threshold frequency, at which the kinetic energy is zero, so $hf = \phi$. $f = \phi/h = (4.55\text{ eV})/h = 1.10\times10^{15}$ Hz.
(b) We want the frequency when the maximum kinetic energy is 1.53 eV. Solve $hf = K_{max} + \phi$ for f, giving

$$f = \frac{K_{max} + \phi}{h} = \frac{1.53\text{ eV} + 4.55\text{ eV}}{h} = 1.47\times10^{15}\text{ Hz}.$$

EVALUATE: Be prepared to use h in units of either $\text{J}\cdot\text{s}$ or $\text{eV}\cdot\text{s}$, depending on the given units. Doing so can avoid tedious unit conversions.

VP38.3.3. **IDENTIFY:** This problem is about the photoelectric effect.
SET UP: $eV_0 = hc/\lambda - \phi$, $f\lambda = c$.
EXECUTE: **(a)** We want the work function. Solve $eV_0 = hc/\lambda - \phi$ for ϕ using $V_0 = 1.37$ V and $\lambda = 475$ nm, giving $\phi = 1.24$ eV.
(b) We want V_0 if $\lambda = 425$ nm. Solve $eV_0 = hc/\lambda - \phi$ for V_0 using $\phi = 1.24$ eV and $\lambda = 425$ nm. This gives $V_0 = 1.68$ eV.
EVALUATE: Decreasing the photon wavelength increases its energy, so the stopping potential increases. From Table 38.1 we see that the ϕ we found is around half that of sodium, so it is a reasonable value.

VP38.3.4. **IDENTIFY:** This problem is about the photoelectric effect.
SET UP: $E = hc/\lambda$.
EXECUTE: **(a)** We want the kinetic energy in eV. Using $K = \frac{1}{2}mv^2$, with m the electron mass and $v = 6.95 \times 10^5$ m/s gives $K = 2.20 \times 10^{-19}$ J = 1.37 eV.

(b) We want the photon energy. $E = hc/\lambda = hc/(306\text{ nm}) = 4.05\text{ eV}$.

(c) We want ϕ. $E_{\text{photon}} = K_{\text{max}} + \phi$. $\phi = 4.05\text{ eV} - 1.37\text{ eV} = 2.68\text{ eV}$.

EVALUATE: The work function is about the same as for sodium, so it is reasonable.

VP38.6.1. **IDENTIFY:** We are dealing with Compton scattering.

SET UP: $\lambda' - \lambda = \dfrac{h}{mc}(1 - \cos\phi)$. We want the increase in the wavelength.

EXECUTE: **(a)** $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi) = \left(2.426\times10^{-12}\text{ m}\right)\left(1-\cos 14.5°\right) = 7.73\times10^{5}\text{ nm}$.

(b) $\dfrac{\lambda' - \lambda}{\lambda} = \dfrac{7.73\times10^{5}\text{ nm}}{0.251\text{ nm}} = 0.0308\%$.

EVALUATE: The fractional change in the wavelength is very small. It would be larger if the scattering angle ϕ were closer to 180°.

VP38.6.2. **IDENTIFY:** We are dealing with Compton scattering.

SET UP: $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi)$, $f\lambda = c$. We want the frequency of the scattered waves. Using $f\lambda = c$ gives

$$\frac{1}{f'} = \frac{1}{f} + \frac{h}{mc^2}(1-\cos\phi).$$

EXECUTE: **(a)** $\phi = 90.00°$: Using the given frequency and scattering angle gives $f' = 1.247\times10^{18}$ Hz.

(b) $\phi = 180.0°$: Working as in (a) but with $\phi = 180.0°$ gives $f' = 1.235\times10^{18}$ Hz.

EVALUATE: For $\phi = 180°$ the x rays are scattered directly back from their original direction, but at $\phi = 90.0°$ they go off perpendicular to their original direction. Note that the frequency of the scattered waves is different in each case.

VP38.6.3. **IDENTIFY:** We are dealing with Compton scattering off of a proton.

SET UP: $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi)$, where m is now the mass of a *proton*. We want the scattering angle ϕ.

Solving for $\cos\phi$ gives $\cos\phi = 1 - \dfrac{mc}{h}(\lambda' - \lambda)$, with $\lambda = 2.50\times10^{-12}$ m.

EXECUTE: **(a)** $\lambda' = \lambda + 0.000100\lambda$: $\cos\phi = 1 - \dfrac{mc}{h}(\lambda' - \lambda) = \dfrac{mc}{h}(\lambda + 0.000100\lambda - \lambda)$

$= 1 - \dfrac{mc}{h}(0.000100\lambda)$.

Using the *proton* mass gives $\phi = 35.8°$.

(b) $\lambda' = \lambda + 0.000800\lambda$: $\cos\phi = 1 - \dfrac{mc}{h}(\lambda' - \lambda) = \dfrac{mc}{h}(\lambda + 0.000100\lambda - \lambda) = 1 - \dfrac{mc}{h}(0.000800\lambda)$.

$\phi = 121°$.

EVALUATE: Note that the scattering angle has a considerable effect on the wavelength of the scattered wave.

VP38.6.4. **IDENTIFY:** This problem involves the annihilation of a proton and an antiproton.

SET UP: We want the energy and wavelength of the resulting photons. $E = hc/\lambda$.

EXECUTE: **(a)** Initial kinetic energy is negligible. The energy of the two photons is equal to the rest energy of the two protons (which is 938.3 MeV each), and each photon has the same energy due to momentum conservation. Therefore $E_{\text{photon}} = 938.3$ MeV. The wavelength of each photon is $\lambda = hc/E$ $= hc/(938.3\text{ MeV}) = 1.32\times10^{-15}$ m.

(b) Initial kinetic energy of each proton is 545 MeV. In this case, the energy of the photons is equal to the rest energy of the protons plus their kinetic energy, so the energy of each photon is E_{photon} = 938.3 MeV + 545 MeV = 1483.3 MeV. $\lambda = hc/E = hc/(1483.3\text{ MeV}) = 8.36\times10^{-16}$ m.

EVALUATE: The wavelength of the photons is less when the protons have substantial energy. This is a reasonable result because short-wavelength photons have more energy than long-wavelength photons.

VP38.7.1. **IDENTIFY:** This problem involves the uncertainty principle.
SET UP: $\Delta t\Delta E = \hbar/2$, $\Delta x\Delta p_x = \hbar/2$, $\lambda = h/p$, $E = hc/\lambda$.

EXECUTE: **(a)** We want the momentum. $p = h/\lambda = h/(633\text{ nm}) = 1.05\times10^{-27}\ \text{kg}\cdot\text{m/s}$.

(b) We want the minimum uncertainty in the momentum. We know that $\Delta x = 700\times10^{-6}$ m.

$$\Delta p_{\text{min}} = \frac{\hbar}{2\Delta x} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(7.00\times10^{-6}\ \text{m})} = 7.54\times10^{-30}\ \text{kg}\cdot\text{m/s}.$$

The percent uncertainty is

$$\frac{\Delta p_{\text{min}}}{p_{\text{min}}} = \frac{7.54\times10^{-30}\ \text{kg}\cdot\text{m/s}}{1.05\times10^{-27}\ \text{kg}\cdot\text{m/s}} = 0.720\%.$$

(c) We want the energy. $E = hc/\lambda = hc/(633\text{ nm}) = 3.14\times10^{-19}$ J.
(d) We want the minimum uncertainty in the energy. We use $\Delta t\Delta E = \hbar/2$ where Δt is the time for the pulse to propagate 7.00×10^{-6} m. Therefore

$$\Delta t = \frac{\Delta x}{c} = \frac{7.00\times10^{-6}\ \text{m}}{c} = 2.333\times10^{-14}\ \text{s}.$$

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{h}{2\left(2.333\times10^{-14}\ \text{s}\right)} = 2.26\times10^{-21}\ \text{s}.$$

Dividing the result in (d) by the energy in (c) gives

$$\frac{\Delta E}{E} = 0.720\%.$$

EVALUATE: Note that the uncertainties in p and E are very small percents.

VP38.7.2. **IDENTIFY:** This problem involves the uncertainty principle.
SET UP: $\Delta t\Delta E = h/2$, $E = hf$, $f\lambda = c$.

EXECUTE: **(a)** We want the time Δt of the pulse.

$$\Delta t = \frac{\hbar}{2\Delta E} = \frac{\hbar}{2\left(5.50\times10^{-29}\text{J}\right)} = 9.59\times10^{-7}\ \text{s}.$$

(b) We want the length of the pulse. $L = c\Delta t = c\left(9.59\times10^{-7}\ \text{s}\right) = 288$ m.

(c) We want the energy. $E = hf = h(3.00\text{ GHz}) = 1.99\times10^{-24}$ J.

(d) We want the uncertainty in the frequency. $E = hf$, so $\Delta E = h\Delta f$. Also $\Delta E = \dfrac{h}{2\Delta t}$ $\Delta E = \dfrac{\hbar}{2\Delta t}$.

Combining gives $\dfrac{\hbar}{2\Delta t} = h\Delta f$, so

$$\Delta f = \frac{1}{4\pi\Delta t} = \frac{1}{4\pi\left(9.59\times10^{-7}\ \text{s}\right)} = 8.30\times10^{-5}\ \text{GHz}.$$

EVALUATE: As we have seen before, the fractional uncertainties arising from the uncertainty principle are usually very small.

VP38.7.3. **IDENTIFY:** This problem involves the uncertainty principle.

SET UP: $\Delta t\Delta E \ge \hbar/2$, $E = hf$, $f\lambda = c$.

EXECUTE: **(a)** We want the wavelength and frequency of the laser light. Using $E = hf$, we see that $\Delta E = h\Delta f$. We know that $\Delta E = 6.70\%E = 0.0670hf$ and that $\Delta E = 2.33 \times 10^{-20}$ J. Therefore 2.33×10^{-20} J $= 0.0670hf$, which gives $f = 5.25 \times 10^{14}$ Hz. Using this frequency, the wavelength is $\lambda = c/f = 572$ nm.

(b) We want the minimum uncertainty in f. $E = hf$, so $\Delta E = h\Delta f$. Thus

$$\Delta f = \frac{\Delta E}{h} = \frac{2.33\times10^{-20}\ \text{J}}{h} = 3.52\times10^{13}\ \text{Hz}.$$

(c) We want the time duration Δt of the pulse. Using $\Delta t\Delta E \ge \hbar/2$ gives

$$\Delta t = \frac{\hbar}{2\Delta E} = \frac{\hbar}{2(2.33\times10^{-20}\ \text{J})} = 2.26\times10^{-15}\ \text{s} = 2.26\ \text{fs}.$$

EVALUATE: As before, the fractional uncertainties are very small.

VP38.7.4. **IDENTIFY:** This problem involves the uncertainty principle.

SET UP: $\Delta t\Delta E \ge \hbar/2$, $E = hc/\lambda$. The average photon energy is 0.0155 eV, the energy of a pulse is 2.39 eV, and on the *average* there are 5.00×10^{12} photons per pulse.

EXECUTE: **(a)** We want the time duration of a pulse. As pointed out in Example 7.4, the minimum uncertainty Δt for a photon is the time duration of the pulse. Using $\Delta t\Delta E \ge \hbar/2$ gives

$$\Delta t = \frac{\hbar}{2\Delta E} = \frac{\left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)/2\pi}{2\left(0.0155\ \text{eV}\right)} = 2.12\times10^{-14}\ \text{s}.$$

(b) We want the wavelength. Using $E = hc/\lambda$ gives $\lambda = hc/E = hc/(2.39\ \text{eV}) = 519$ nm.

(c) We want the energy per pulse. This energy is equal to the energy per average photon times the number of photons in a pulse. Therefore $E = (2.39$ eV/photon$)(5.00 \times 10^{12}$ photons/pulse$)$ $= 1.195 \times 10^{13}$ eV $= 1.91$ μJ per pulse.

EVALUATE: The energy per pulse is small, but the pulse lasts for a very short time, so the power during the pulse is very large. Note that some of the photons have wavelengths longer than 519 and some have shorter wavelengths.

38.1. **IDENTIFY** and **SET UP:** Apply $c = f\lambda$, $p = h/\lambda$, and $E = pc$.

EXECUTE:
$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\,\text{m/s}}{5.20\times10^{-7}\ \text{m}} = 5.77\times10^{14}\ \text{Hz}.$$

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{5.20\times10^{-7}\ \text{m}} = 1.28\times10^{-27}\ \text{kg}\cdot\text{m/s}.$$

$$E = pc = (1.28\times10^{-27}\ \text{kg}\cdot\text{m/s})\,(3.00\times10^{8}\,\text{m/s}) = 3.84\times10^{-19}\ \text{J} = 2.40\ \text{eV}.$$

EVALUATE: Visible-light photons have energies of a few eV.

38.2. **IDENTIFY** and **SET UP:** $c = f\lambda$ relates frequency and wavelength and $E = hf$ relates energy and frequency for a photon. $c = 3.00\times10^{8}$ m/s. $1\ \text{eV} = 1.60\times10^{-16}$ J.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\ \text{m/s}}{505\times10^{-9}\ \text{m}} = 5.94\times10^{14}\ \text{Hz}.$

(b) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(5.94\times10^{14}\ \text{Hz}) = 3.94\times10^{-19}\ \text{J} = 2.46\ \text{eV}.$

(c) $K = \frac{1}{2}mv^2$, so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(3.94\times10^{-19}\ \text{J})}{9.5\times10^{-15}\ \text{kg}}} = 9.1\ \text{mm/s}.$

EVALUATE: Compared to kinetic energies of common objects moving at typical speeds, the energy of a visible-light photon is extremely small.

38.3. **IDENTIFY** and **SET UP:** $c = f\lambda$. The source emits $(0.05)(75\text{ J}) = 3.75\text{ J}$ of energy as visible light each second. $E = hf$, with $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\text{ m/s}}{600\times10^{-9}\text{ m}} = 5.00\times10^{14}\text{ Hz}.$

(b) $E = hf = (6.63\times10^{-34}\text{ J}\cdot\text{s})(5.00\times10^{14}\text{ Hz}) = 3.32\times10^{-19}\text{ J}.$ The number of photons emitted per second is $\dfrac{3.75\text{ J}}{3.32\times10^{-19}\text{ J/photon}} = 1.13\times10^{19}$ photons.

EVALUATE: **(c)** No. The frequency of the light depends on the energy of each photon. The number of photons emitted per second is proportional to the power output of the source.

38.4. **IDENTIFY** and **SET UP:** $P_{\text{av}} = \dfrac{\text{energy}}{t}$. $1\text{ eV} = 1.60\times10^{-19}\text{ J}$. For a photon, $E = hf = \dfrac{hc}{\lambda}$.

$$h = 6.63\times10^{-34}\text{ J}\cdot\text{s}.$$

EXECUTE: **(a)** $\text{energy} = P_{\text{av}}t = (0.600\text{ W})(20.0\times10^{-3}\text{ s}) = 1.20\times10^{-2}\text{ J} = 7.5\times10^{16}\text{ eV}.$

(b) $E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{652\times10^{-9}\text{ m}} = 3.05\times10^{-19}\text{ J} = 1.91\text{ eV}.$

(c) The number of photons is the total energy in a pulse divided by the energy of one photon:

$$\frac{1.20\times10^{-2}\text{ J}}{3.05\times10^{-19}\text{ J/photon}} = 3.93\times10^{16}\text{ photons}.$$

EVALUATE: The number of photons in each pulse is very large.

38.5. **IDENTIFY** and **SET UP:** A photon has zero rest mass, so its energy is $E = pc$ and its momentum is $p = \dfrac{h}{\lambda}$.

EXECUTE: **(a)** $E = pc = (8.24\times10^{-28}\text{ kg}\cdot\text{m/s})(2.998\times10^{8}\text{ m/s}) = 2.47\times10^{-19}\text{ J}$
$= (2.47\times10^{-19}\text{ J})(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 1.54\text{ eV}.$

(b) $p = \dfrac{h}{\lambda}$, so $\lambda = \dfrac{h}{p} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{8.24\times10^{-28}\text{ kg}\cdot\text{m/s}} = 8.04\times10^{-7}\text{ m} = 804\text{ nm}.$

EVALUATE: This wavelength is longer than visible wavelengths; it is in the infrared region of the electromagnetic spectrum. To check our result we could verify that the same E is given by $E = hc/\lambda$, using the λ we have calculated.

38.6. **IDENTIFY** and **SET UP:** For the photoelectric effect, the maximum kinetic energy of the photoelectrons is $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi = \dfrac{hc}{\lambda} - \phi$. Take the work function ϕ from Table 38.1. Solve for v_{max}. Note that we wrote f as c/λ.

EXECUTE: $\frac{1}{2}mv_{\text{max}}^2 = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{235\times10^{-9}\text{ m}} - (5.1\text{ eV})(1.602\times10^{-19}\text{ J/1 eV}).$

$$\tfrac{1}{2}mv_{\text{max}}^2 = 8.453\times10^{-19}\text{ J} - 8.170\times10^{-19}\text{ J} = 2.83\times10^{-20}\text{ J}.$$

$$v_{\text{max}} = \sqrt{\frac{2(2.83\times10^{-20}\text{ J})}{9.109\times10^{-31}\text{ kg}}} = 2.49\times10^{5}\text{ m/s}.$$

EVALUATE: The work function in eV was converted to joules for use in the equation $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi = \dfrac{hc}{\lambda} - \phi$. A photon with $\lambda = 235\text{ nm}$ has energy greater then the work function for the surface.

38.7. **IDENTIFY**: The photoelectric effect occurs. The kinetic energy of the photoelectron is the difference between the initial energy of the photon and the work function of the metal.

SET UP: $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$, $E = hc/\lambda$.

EXECUTE: Use the data for the 400.0-nm light to calculate ϕ. Solving for ϕ gives $\phi = \dfrac{hc}{\lambda} - \frac{1}{2}mv_{\text{max}}^2$ $= \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{400.0\times10^{-9}\text{ m}} - 1.10\text{ eV} = 3.10\text{ eV} - 1.10\text{ eV} = 2.00\text{ eV}.$ Then for 300.0 nm, we have $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi = \dfrac{hc}{\lambda} - \phi = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{300.0\times10^{-9}\text{ m}} - 2.00\text{ eV},$ which gives $\frac{1}{2}mv_{\text{max}}^2 = 4.14\text{ eV} - 2.00\text{ eV} = 2.14\text{ eV}.$

EVALUATE: When the wavelength decreases the energy of the photons increases and the photoelectrons have a larger minimum kinetic energy.

38.8. **IDENTIFY** and **SET UP:** $eV_0 = \frac{1}{2}mv_{\text{max}}^2$, where V_0 is the stopping potential. The stopping potential in volts equals eV_0 in electron volts. $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$ and $f = c/\lambda$.

EXECUTE: **(a)** $eV_0 = \frac{1}{2}mv_{\text{max}}^2$, so $eV_0 = hf - \phi = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{190\times10^{-9}\text{ m}} - 2.3\text{ eV}$ $= 6.53\text{ eV} - 2.3\text{ eV} = 4.23\text{ eV},$ which rounds to 4.2 eV. The stopping potential is 4.2 volts.

(b) $\frac{1}{2}mv_{\text{max}}^2 = 4.2\text{ eV}.$

(c) $v_{\text{max}} = \sqrt{\dfrac{2(4.23\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{kg}}} = 1.2\times10^{6}\text{ m/s}.$

EVALUATE: If the wavelength of the light is decreased, the maximum kinetic energy of the photoelectrons increases.

38.9. **(b) IDENTIFY:** Solve part (b) first. First use $eV_0 = hf - \phi$ to find the work function ϕ

SET UP: $eV_0 = hf - \phi$, so $\phi = hf - eV_0 = \dfrac{hc}{\lambda} - eV_0.$

EXECUTE: $\phi = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{254\times10^{-9}\text{ m}} - (1.602\times10^{-19}\text{ C})(0.181\text{ V}).$

$\phi = 7.821\times10^{-19}\text{ J} - 2.900\times10^{-20}\text{ J} = 7.531\times10^{-19}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 4.70\text{ eV}.$

(a) IDENTIFY and **SET UP:** The threshold frequency f_{th} is the smallest frequency that still produces photoelectrons. It corresponds to $K_{\text{max}} = 0$ in the equation $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$, so $hf_{\text{th}} = \phi.$

EXECUTE: $f = \dfrac{c}{\lambda}$ says $\dfrac{hc}{\lambda_{\text{th}}} = \phi.$

$$\lambda_{\text{th}} = \frac{hc}{\phi} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{7.531\times10^{-19}\text{ J}} = 2.64\times10^{-7}\text{ m} = 264\text{ nm}.$$

EVALUATE: As calculated in part (b), $\phi = 4.70\text{ eV}.$ This is the value given in Table 38.1 for copper.

38.10. **IDENTIFY:** The acceleration gives energy to the electrons which is then given to the x ray photons.

SET UP: $E = hc/\lambda$, so $\dfrac{hc}{\lambda} = eV$, where λ is the wavelength of the x ray and V is the accelerating voltage.

EXECUTE: $\lambda = \dfrac{hc}{eV} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(15.0\times10^{3}\text{ V})} = 8.29\times10^{-11}\text{ m} = 0.0829\text{ nm}.$

EVALUATE: This wavelength certainly is in the x ray region of the electromagnetic spectrum.

38.11. **IDENTIFY:** This problem is about the characteristics of photons.

SET UP: $E = hf$, $\lambda = h/p$, $f\lambda = c$. We want the frequency, wavelength, and momentum of the photon.

EXECUTE: $E_{ph} = 0.700K_{el} = 0.700eV = hf$. So $f = (0.700eV)/h$. Using $V = 50.0$ kV and h in terms of eV, we get $f = 8.46 \times 10^{18}$ Hz. Using this result gives $\lambda = c/f = 0.0355$ nm. Using this wavelength gives $p = h/\lambda = 1.87 \times 10^{-23}\ \text{kg}\cdot\text{m/s}$.

EVALUATE: The frequency of this photon is much greater than that of visible light.

38.12. **IDENTIFY** and **SET UP:** $\dfrac{hc}{\lambda} = eV$, where λ is the wavelength of the x ray and V is the accelerating voltage.

EXECUTE: **(a)** $V = \dfrac{hc}{e\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.150\times10^{-9}\ \text{m})} = 8.29\ \text{kV}.$

(b) $\lambda = \dfrac{hc}{eV} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(30.0\times10^{3}\ \text{V})} = 4.14\times10^{-11}\ \text{m} = 0.0414\ \text{nm}.$

EVALUATE: Shorter wavelengths require larger potential differences.

38.13. **IDENTIFY:** Energy is conserved when the x ray collides with the stationary electron.

SET UP: $E = hc/\lambda$, and energy conservation gives $\dfrac{hc}{\lambda} = \dfrac{hc}{\lambda'} + K_e$.

EXECUTE: Solving for K_e gives $K_e = hc\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda'}\right) = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})$
$\left(\dfrac{1}{0.100\times10^{-9}\ \text{m}} - \dfrac{1}{0.110\times10^{-9}\ \text{m}}\right)$. $K_e = 1.81\times10^{-16}\ \text{J} = 1.13\ \text{keV}.$

EVALUATE: The electron does not get all the energy of the incident photon.

38.14. **IDENTIFY** and **SET UP:** The wavelength of the x rays produced by the tube is given by $\dfrac{hc}{\lambda} = eV$.
$\lambda' = \lambda + \dfrac{h}{mc}(1-\cos\phi)$. $\dfrac{h}{mc} = 2.426\times10^{-12}$ m. The energy of the scattered x ray is $\dfrac{hc}{\lambda'}$.

EXECUTE: **(a)** $\lambda = \dfrac{hc}{eV} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(24.0\times10^{3}\ \text{V})} = 5.167\times10^{-11}\ \text{m},$ which rounds to 0.0517 nm = 51.7 pm.

(b) $\lambda' = \lambda + \dfrac{h}{mc}(1-\cos\phi) = 5.167\times10^{-11}\ \text{m} + (2.426\times10^{-12}\ \text{m})(1-\cos 45.0°) = 5.238\times10^{-11}\ \text{m},$ which rounds to 0.0524 nm = 52.4 pm.

(c) $E = \dfrac{hc}{\lambda'} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{5.238\times10^{-11}\ \text{m}} = 2.37\times10^{4}\ \text{eV} = 23.7\ \text{keV}.$

EVALUATE: The incident x ray has energy 24.0 keV. In the scattering event, the photon loses energy and its wavelength increases.

38.15. **IDENTIFY:** Apply $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi) = \lambda_C(1-\cos\phi)$.

SET UP: Solve for λ': $\lambda' = \lambda + \lambda_C(1-\cos\phi)$.
The largest λ' corresponds to $\phi = 180°$, so $\cos\phi = -1$.

EXECUTE: $\lambda' = \lambda + 2\lambda_C = 0.0665\times10^{-9}\ \text{m} + 2(2.426\times10^{-12}\ \text{m}) = 7.135\times10^{-11}\ \text{m} = 0.0714\ \text{nm}.$ This wavelength occurs at a scattering angle of $\phi = 180°$.

EVALUATE: The incident photon transfers some of its energy and momentum to the electron from which it scatters. Since the photon loses energy its wavelength increases, $\lambda' > \lambda$.

38.16. **IDENTIFY:** Compton scattering occurs. We know speed, and hence the kinetic energy, of the scattered electron. Energy is conserved.

SET UP: $\dfrac{hc}{\lambda} = \dfrac{hc}{\lambda'} + E_e$ where $E_e = \frac{1}{2}mv^2$.

EXECUTE: $E_e = \frac{1}{2}mv^2 = \frac{1}{2}(9.108\times10^{-31}\text{ kg})(8.90\times10^6\text{ m/s})^2 = 3.607\times10^{-17}\text{ J}$.

$\dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{0.1385\times10^{-9}\text{ m}} = 1.434\times10^{-15}\text{ J}$. Therefore, $\dfrac{hc}{\lambda'} = \dfrac{hc}{\lambda} - E_e = 1.398\times10^{-15}\text{ J}$,

which gives $\lambda' = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{1.398\times10^{-15}\text{ J}} = 0.1421\text{ nm}$.

$\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi) = 3.573\times10^{-12}\text{ m}$, so $1-\cos\phi = 1.473$, which gives $\phi = 118°$.

EVALUATE: The photon partly backscatters, but not through 180°.

38.17. **IDENTIFY** and **SET UP:** The shift in wavelength of the photon is $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi)$ where λ' is the wavelength after the scattering and $\dfrac{h}{mc} = \lambda_C = 2.426\times10^{-12}\text{ m}$. The energy of a photon of wavelength λ is $E = \dfrac{hc}{\lambda} = \dfrac{1.24\times10^{-6}\text{ eV}\cdot\text{m}}{\lambda}$. Conservation of energy applies to the collision, so the energy lost by the photon equals the energy gained by the electron.

EXECUTE:

(a) $\lambda' - \lambda = \lambda_C(1-\cos\phi) = (2.426\times10^{-12}\text{ m})(1-\cos 35.0°) = 4.39\times10^{-13}\text{ m} = 4.39\times10^{-4}\text{ nm}$.

(b) $\lambda' = \lambda + 4.39\times10^{-4}\text{ nm} = 0.04250\text{ nm} + 4.39\times10^{-4}\text{ nm} = 0.04294\text{ nm}$.

(c) $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi) = 0.1050\times10^{-9}\text{ m} + (2.426\times10^{-12}\text{ m})(1-\cos 60.0°) = 0.1062\times10^{-9}\text{ m}$ and

$E_{\lambda'} = \dfrac{hc}{\lambda'} = 2.888\times10^4\text{ eV}$, so the photon loses 300 eV of energy.

(d) Energy conservation says the electron gains 300 eV of energy.

EVALUATE: The photon transfers energy to the electron. Since the photon loses energy, its wavelength increases.

38.18. **IDENTIFY:** The change in wavelength of the scattered photon is given by the equation

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{mc\lambda}(1-\cos\phi) \Rightarrow \lambda = \frac{h}{mc\left(\dfrac{\Delta\lambda}{\lambda}\right)}(1-\cos\phi).$$

SET UP: For backward scattering, $\phi = 180°$. Since the photon scatters from a proton, $m = 1.67\times10^{-27}\text{ kg}$.

EXECUTE: $\lambda = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(1.67\times10^{-27}\text{ kg})(3.00\times10^8\text{ m/s})(0.100)}(1+1) = 2.65\times10^{-14}\text{ m}$.

EVALUATE: The maximum change in wavelength, $2h/mc$, is much smaller for scattering from a proton than from an electron.

38.19. **IDENTIFY:** We are dealing with Compton scattering.

SET UP: $\lambda = h/p$, $E = hc/\lambda$, $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi)$.

EXECUTE: **(a)** The initial photon lost energy during Compton scattering. Before this it had just enough energy to create electron-positron pairs, so after the collision it has less energy. Therefore it cannot create these pairs, so the answer is no.

(b) We want the momentum of the scattered photon. First use find λ' using

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

and then use $\lambda = h/p$ to find the momentum. Before scattering $E = 2mc^2 = hc/\lambda$, so $\lambda = h/2mc$.

$$\lambda' = \frac{h}{mc}(1 - \cos\phi) + \frac{h}{2mc} = \frac{h}{mc}\left(\frac{3}{2} - \cos\phi\right).$$

Using this result gives with $\phi = 20.0°$ gives

$$p = \frac{h}{\lambda'} = \frac{h}{\frac{h}{mc}\left(\frac{3}{2} - \cos\phi\right)} = \frac{mc}{\frac{3}{2} - \cos\phi} = 4.88 \times 10^{-22}\ \text{kg} \cdot \text{m/s}.$$

EVALUATE: The momentum of the scattered photon depends on the scattering angle ϕ. The smallest that the momentum can be is when $\cos\phi = -1$ ($\phi = 180°$, which is backscatter), for which $p_{\text{min}} = 2mc/5$.

38.20. **(a) IDENTIFY** and **SET UP:** Use the relativistic equation $K = (\gamma - 1)mc^2$ to calculate the kinetic energy K.

EXECUTE: $K = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) = 0.1547\ mc^2$. $m = 9.109 \times 10^{-31}$ kg, so $K = 1.27 \times 10^{-14}$ J.

(b) IDENTIFY and **SET UP:** The total energy of the particles equals the sum of the energies of the two photons. Linear momentum must also be conserved.

EXECUTE: The total energy of each electron or positron is $E = K + mc^2 = 1.1547mc^2 = 9.46 \times 10^{-13}$ J. The total energy of the electron and positron is converted into the total energy of the two photons. The initial momentum of the system in the lab frame is zero (since the equal-mass particles have equal speeds in opposite directions), so the final momentum must also be zero. The photons must have equal wavelengths and must be traveling in opposite directions. Equal λ means equal energy, so each photon has energy 9.46×10^{-14} J.

(c) IDENTIFY and **SET UP:** Use $E = hc/\lambda$ to relate the photon energy to the photon wavelength.

EXECUTE: $E = hc/\lambda$, so $\lambda = hc/E = hc/(9.46 \times 10^{-14}\ \text{J}) = 2.10$ pm.

EVALUATE: When the particles also have kinetic energy, the energy of each photon is greater, so its wavelength is less.

38.21. **IDENTIFY:** The wavelength of the pulse tells us the momentum of the photon. The uncertainty in the momentum is determined by the uncertainty principle.

SET UP: $p = \frac{h}{\lambda}$ and $\Delta x \Delta p_x = \frac{\hbar}{2}$.

EXECUTE: $p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{556 \times 10^{-9}\ \text{m}} = 1.19 \times 10^{-27}\ \text{kg} \cdot \text{m/s}$. The spatial length of the pulse is $\Delta x = c\Delta t = (2.998 \times 10^8\ \text{m/s})(9.00 \times 10^{-15}\ \text{s}) = 2.698 \times 10^{-6}$ m. The uncertainty principle gives $\Delta x \Delta p_x = \frac{\hbar}{2}$. Solving for the uncertainty in the momentum, we have $\Delta p_x = \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{2(2.698 \times 10^{-6}\ \text{m})}$ $= 1.96 \times 10^{-29}\ \text{kg} \cdot \text{m/s}$.

EVALUATE: This is 1.6% of the average momentum.

38.22. **IDENTIFY:** We know the beam went through the slit, so the uncertainty in its vertical position is the width of the slit.

SET UP: $\Delta y \Delta p_y = \dfrac{\hbar}{2}$ and $p_x = \dfrac{h}{\lambda}$. Call the x-axis horizontal and the y-axis vertical.

EXECUTE: **(a)** Let $\Delta y = a = 6.20\times10^{-5}$ m. Solving $\Delta y \Delta p_y = \dfrac{\hbar}{2}$ for the uncertainty in momentum gives

$$\Delta p_y = \frac{\hbar}{2\Delta y} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(6.20\times10^{-5}\ \text{m})} = 8.51\times10^{-31}\ \text{kg}\cdot\text{m/s}.$$

(b) $p_x = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{585\times10^{-9}\ \text{m}} = 1.13\times10^{-27}\ \text{kg}\cdot\text{m/s}.$ $\theta = \dfrac{\Delta p_y}{p_x} = \dfrac{8.51\times10^{-31}}{1.13\times10^{-27}} = 7.53\times10^{-4}$ rad.

The width is $(2.00\ \text{m})(7.53\times10^{-4}) = 1.51\times10^{-3}\ \text{m} = 1.51$ mm.

EVALUATE: We must be especially careful not to confuse the x- and y-components of the momentum.

38.23. **IDENTIFY:** The uncertainty principle relates the uncertainty in the duration time of the pulse and the uncertainty in its energy, which we know.

SET UP: $E = hc/\lambda$ and $\Delta E \Delta t = \hbar/2$.

EXECUTE: $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{625\times10^{-9}\ \text{m}} = 3.178\times10^{-19}$ J. The uncertainty in the energy is 1.0% of this amount, so $\Delta E = 3.178\times10^{-21}$ J. We now use the uncertainty principle. Solving $\Delta E \Delta t = \dfrac{\hbar}{2}$ for the time interval gives $\Delta t = \dfrac{\hbar}{2\Delta E} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(3.178\times10^{-21}\ \text{J})} = 1.66\times10^{-14}\ \text{s} = 16.6$ fs.

EVALUATE: The uncertainty in the energy limits the duration of the pulse. The more precisely we know the energy, the longer the duration must be.

38.24. **IDENTIFY:** The number N of visible photons emitted per second is the visible power divided by the energy hf of one photon.

SET UP: At a distance r from the source, the photons are evenly spread over a sphere of area $A = 4\pi r^2$.

EXECUTE: **(a)** $N = \dfrac{P}{hf} = \dfrac{(120\ \text{W})(0.10)}{h(5.00\times10^{14}\ \text{Hz})} = 3.62\times10^{19}$ photons/s.

(b) $\dfrac{N}{4\pi r^2} = 1.00\times10^{11}\ \text{photons/s}\cdot\text{cm}^2$ gives

$$r = \left(\frac{3.62\times10^{19}\,\text{photons/s}}{4\pi(1.00\times10^{11}\ \text{photons/s}\cdot\text{cm}^2)}\right)^{1/2} = 5370\ \text{cm} = 53.7\ \text{m}.$$

EVALUATE: The number of photons emitted per second by an ordinary household source is very large.

38.25. **IDENTIFY** and **SET UP:** The energy added to mass m of the blood to heat it to $T_\text{f} = 100°\text{C}$ and to vaporize it is $Q = mc(T_\text{f} - T_\text{i}) + mL_\text{v}$, with $c = 4190$ J/kg $\cdot$ K and $L_\text{v} = 2.256\times10^{6}$ J/kg. The energy of one photon is $E = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$.

EXECUTE: **(a)** $Q = (2.0\times10^{-9}\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(100°\text{C} - 33°\text{C}) + (2.0\times10^{-9}\ \text{kg})(2.256\times10^{6}\ \text{J/kg}) =$ 5.07×10^{-3} J. The pulse must deliver 5.07 mJ of energy.

(b) $P = \dfrac{\text{energy}}{t} = \dfrac{5.07\times10^{-3}\ \text{J}}{450\times10^{-6}\ \text{s}} = 11.3$ W.

(c) One photon has energy $E = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{585\times10^{-9}\ \text{m}} = 3.40\times10^{-19}\ \text{J}$. The number N of photons per pulse is the energy per pulse divided by the energy of one photon:

$$N = \frac{5.07\times10^{-3}\ \text{J}}{3.40\times10^{-19}\ \text{J/photon}} = 1.49\times10^{16}\ \text{photons.}$$

EVALUATE: The power output of the laser is small but it is focused on a small area, so the laser intensity is large.

38.26. **IDENTIFY:** The photoelectric effect occurs, so the energy of the photon is used to eject an electron, with any excess energy going into kinetic energy of the electron.
SET UP: Conservation of energy gives $hf = hc/\lambda = K_{\text{max}} + \phi.$

EXECUTE: **(a)** Using $hc/\lambda = K_{\text{max}} + \phi,$ we solve for the work function:

$$\phi = hc/\lambda - K_{\text{max}} = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(124\ \text{nm}) - 4.16\ \text{eV} = 5.85\ \text{eV}.$$

(b) The number N of photoelectrons per second is equal to the number of photons that strike the metal per second. $N\times(\text{energy of a photon}) = 2.50\ \text{W}$. $N(hc/\lambda) = 2.50\ \text{W}$.

$$N = (2.50\ \text{W})(124\ \text{nm})/[(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})] = 1.56\times10^{18}\ \text{electrons/s.}$$

(c) N is proportional to the power, so if the power is cut in half, so is N, which gives

$$N = (1.56\times10^{18}\ \text{el/s})/2 = 7.80\times10^{17}\ \text{el/s.}$$

(d) If we cut the wavelength by half, the energy of each photon is doubled since $E = hc/\lambda.$ To maintain the same power, the number of photons must be half of what they were in part (b), so N is cut in half to 7.80×10^{17} el/s. We could also see this from part (b), where N is proportional to $\lambda.$ So if the wavelength is cut in half, so is N.
EVALUATE: In part (c), reducing the power does not reduce the maximum kinetic energy of the photons; it only reduces the number of ejected electrons. In part (d), reducing the wavelength *does* change the maximum kinetic energy of the photoelectrons because we have increased the energy of each photon.

38.27. **IDENTIFY:** We are dealing with the characteristics of a photon.
SET UP and **EXECUTE:** Using $E = hc/\lambda,$ we want to estimate the number of photons in a typical room.

(a) Estimate: On a bright day the light intensity in a room is about 1/3 of what it is outside, so

$$I = (1000\ \text{W/m}^2)/3 = 330\ \text{W/m}^2.$$

(b) Estimate: Floor area is $A \approx 15$ ft by 30 ft ≈ 5 m by 10 m $\approx 50\ \text{m}^2$. Height is $H \approx 9$ ft ≈ 3 m.
(c) $t = H/c = (3\ \text{m})/c = 10^{-8}\ \text{s} = 10$ ns.
(d) $P = IA = (330\ \text{W/m}^2)(50\ \text{m}^2) = 16{,}500$ W.
(e) $E_{\text{room}} = Pt = (16{,}500\ \text{W})(10^{-8}\ \text{s}) = 165\ \mu\text{J}$.
(f) $E = hc/\lambda = hc/(500\ \text{nm}) = 3.98\times10^{-19}$ J.
(g) Using the answers from parts (e) and (f) gives $N = E_{\text{room}}/E = 4\times10^{14}$.
EVALUATE: These numbers give only rough approximations, not precise calculations.

38.28. **IDENTIFY:** Compton scattering occurs. For backscattering, the scattering angle of the photon is 180°. Momentum is conserved during the collision.
SET UP: Let $+x$ be in the direction of propagation of the incident photon. The momentum of a photon is $p = h/\lambda.$ The change in wavelength of the light during Compton scattering is given by

$\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi),$ where $\phi = 180°$ in this case.

EXECUTE: $\lambda' = \lambda + 2\dfrac{h}{mc} = 0.0980\times10^{-9}\text{ m} + 4.852\times10^{-12}\text{ m} = 0.1029\times10^{-9}\text{ m}$. Momentum conservation gives $\dfrac{h}{\lambda} = -\dfrac{h}{\lambda'} + p_e$. Solving for p_e gives $p_e = \dfrac{h}{\lambda} + \dfrac{h}{\lambda'} = h\left(\dfrac{\lambda+\lambda'}{\lambda\lambda'}\right) = (6.626\times10^{-34}\text{ J}\cdot\text{s})$
$\dfrac{9.80\times10^{-11}\text{ m} + 10.29\times10^{-11}\text{ m}}{(9.80\times10^{-11}\text{ m})(10.29\times10^{-11}\text{ m})} = 1.32\times10^{-23}\text{ kg}\cdot\text{m/s}$.

EVALUATE: The electron gains the most amount of momentum when backscattering occurs.

38.29. IDENTIFY: Compton scattering occurs, and we know the angle of scattering and the initial wavelength (and hence momentum) of the incident photon.

SET UP: $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi)$ and $p = h/\lambda$. Let $+x$ be the direction of propagation of the incident photon and let the scattered photon be moving at $30.0°$ clockwise from the $+y$-axis.

EXECUTE:

$$\lambda' - \lambda = \left(\frac{h}{mc}\right)(1-\cos\phi) = 0.1050\times10^{-9}\text{ m} + (2.426\times10^{-12}\text{ m})(1-\cos 60.0°) = 0.1062\times10^{-9}\text{ m}.$$

$P_{ix} = P_{fx}$. $\dfrac{h}{\lambda} = \dfrac{h}{\lambda'}\cos 60.0° + p_{ex}$.

$$p_{ex} = \frac{h}{\lambda} - \frac{h}{2\lambda'} = h\frac{2\lambda' - \lambda}{(2\lambda')(\lambda)} = (6.626\times10^{-34}\text{ J}\cdot\text{s})\frac{2.1243\times10^{-10}\text{ m} - 1.050\times10^{-10}\text{ m}}{(2.1243\times10^{-10}\text{ m})(1.050\times10^{-10}\text{ m})}.$$

$p_{ex} = 3.191\times10^{-24}\text{ kg}\cdot\text{m/s}$. $P_{iy} = P_{fy}$. $0 = \dfrac{h}{\lambda'}\sin 60.0° + p_{ey}$.

$$p_{ey} = -\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})\sin 60.0°}{0.1062\times10^{-9}\text{ m}} = -5.403\times10^{-24}\text{ kg}\cdot\text{m/s}. \quad p_e = \sqrt{p_{ex}^2 + p_{ey}^2} = 6.28\times10^{-24}\text{ kg}\cdot\text{m/s}.$$

$\tan\theta = \dfrac{p_{ey}}{p_{ex}} = \dfrac{-5.403}{3.191}$ and $\theta = -59.4°$.

EVALUATE: The incident photon does not give all of its momentum to the electron, since the scattered photon also has momentum.

38.30. IDENTIFY: This problem deals with photons and the nutritional energy in food.
SET UP and **EXECUTE:** **(a)** Estimate: One medium tomato (123 g) has 22 kcal.

(22 kcal)(4186 J/kcal) = 93,000 J.

(b) Estimate: 50 leaves of area 20 cm^2 each, so $A \approx 1000\text{ cm}^2 = 0.10\text{ m}^2$.
(c) $P = IA = (800\text{ W/m}^2)(0.10\text{ m}^2) = 80\text{ W}$.
(d) $E = hc/\lambda = hc/(600\text{ nm}) = 3.31\times10^{-19}$ J per photon. The total energy each second for photosynthesis is 5% of 80 J = 4.0 J. The number N of photons needed to deliver this energy is given by $N(3.31\times10^{-19}\text{ J}) = 4.0$ J, so $N = 1.2\times10^{19}$ photons.
(e) The number N' of photons for a single tomato is $N' = (1/10)(N/2) = 6.0\times10^{17}$ photons.
(f) Each tomato needs 93,000 J from part (a). If N is the number of photons needed, then $N(3.31\times10^{-19}\text{ J}) = 93{,}000$ J, which gives $N = 2.82\times10^{23}$ photons.
(g) From part (e): 6.0×10^{27} photons/s supply energy to the tomato.
From part (f): The tomato needs 2.82×10^{23} photons.
Calling t the time to get these photons gives $(6.0\times10^{17}\text{ photons/s})t = 2.82\times10^{23}$ photons, which gives $t = 4.7\times10^5$ s = 131 h. At 12 h/day, we get t = 11 days.

EVALUATE: Increasing the leaf area exposed to sunlight would help capture more energy and lead to faster ripening.

38.31. **IDENTIFY** and **SET UP:** Find the average change in wavelength for one scattering and use that in $\Delta\lambda$ in $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi)$ to calculate the average scattering angle ϕ.

EXECUTE: **(a)** The wavelength of a 1 MeV photon is

$\lambda = \dfrac{hc}{E} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{1\times10^{6}\ \text{eV}} = 1\times10^{-12}\ \text{m}.$

The total change in wavelength therefore is $500\times10^{-9}\ \text{m} - 1\times10^{-12}\ \text{m} = 500\times10^{-9}\ \text{m}.$

If this shift is produced in 10^{26} Compton scattering events, the wavelength shift in each scattering event is $\Delta\lambda = \dfrac{500\times10^{-9}\ \text{m}}{1\times10^{26}} = 5\times10^{-33}\ \text{m}.$

(b) Use this $\Delta\lambda$ in $\Delta\lambda = \dfrac{h}{mc}(1-\cos\phi)$ and solve for ϕ. We anticipate that ϕ will be very small, since $\Delta\lambda$ is much less than h/mc, so we can use $\cos\phi \approx 1-\phi^2/2.$

$$\Delta\lambda = \frac{h}{mc}\left[1-(1-\phi^2/2)\right] = \frac{h}{2mc}\phi^2.$$

$$\phi = \sqrt{\frac{2\Delta\lambda}{(h/mc)}} = \sqrt{\frac{2(5\times10^{-33}\ \text{m})}{2.426\times10^{-12}\ \text{m}}} = 6.4\times10^{-11}\ \text{rad} = (4\times10^{-9})°.$$

ϕ in radians is much less than 1, so the approximation we used is valid.

(c) **IDENTIFY** and **SET UP:** We know the total transit time and the total number of scatterings, so we can calculate the average time between scatterings.

EXECUTE: The total time to travel from the core to the surface is $(10^6\ \text{y})(3.156\times10^7\ \text{s/y}) = 3.2\times10^{13}\ \text{s}.$

There are 10^{26} scatterings during this time, so the average time between scatterings is

$t = \dfrac{3.2\times10^{13}\ \text{s}}{10^{26}} = 3.2\times10^{-13}\ \text{s}.$

The distance light travels in this time is $d = ct = (3.0\times10^8\ \text{m/s})(3.2\times10^{-13}\ \text{s}) = 0.1\ \text{mm}.$

EVALUATE: The photons are on the average scattered through a very small angle in each scattering event. The average distance a photon travels between scatterings is very small.

38.32. **IDENTIFY:** In this problem, a positron with speed v and kinetic energy K collides with a stationary electron. They annihilate and produce to two photons each of wavelength λ .

SET UP: $E = hc/\lambda,\ \ K = mc^2(\gamma-1),\ \ p = m\gamma v,\ \ v'_x = \dfrac{v_x - u}{1-uv_x/c^2}.$

EXECUTE: **(a)** Energy conservation: $K + mc^2 + mc^2 = 2hc/\lambda.$ Using $K = mc^2(\gamma-1)$ gives

$$(\gamma-1)mc^2 + 2mc^2 = 2hc/\lambda.$$

Rearranging gives

$$(\gamma+1)mc^2 = 2hc/\lambda.$$

(b) Momentum conservation: Using $p = m\gamma v$ for the positron gives

$$m\gamma v = 2\frac{h}{\lambda}\cos\phi.$$

(c) We want the energy of the photon and the angle ϕ.

From part (a): $(\gamma+1)mc^2 = 2hc/\lambda = 2E_{\text{photon}}.$

This gives $E_{\text{photon}} = \dfrac{mc^2(\gamma+1)}{2}.$

From part (b): $m\gamma v = 2\dfrac{h}{\lambda}\cos\phi$. This gives $\cos\phi = \dfrac{m\gamma v\lambda}{2h}$.

Combine the results from (a) and (b) by eliminating λ and solve for $\cos\phi$.

$$\cos\phi = \frac{\gamma v}{c(1+\gamma)}.$$

Using $v = c\sqrt{1-1/\gamma^2}$ and doing some algebra gives

$$\phi = \arccos\sqrt{\frac{\gamma-1}{\gamma+1}}.$$

(d) We want the energy of the photon and the angle z if K = 5.11 MeV. Using $K = mc^2(\gamma-1)$ with mc^2 = 0.511 MeV gives γ = 11. From part (c)

$$\phi = \arccos\sqrt{\frac{\gamma-1}{\gamma+1}} = 3.07\text{ MeV}.$$

Also from part (c) we get

$$\phi = \arccos\sqrt{\frac{\gamma-1}{\gamma+1}} = \arccos\sqrt{\frac{10}{12}} = 24.1°.$$

(e) We want *v* if the photons are perpendicular to each other. In this case, ϕ = 45.0°. Solving

$\phi = \arccos\sqrt{\dfrac{\gamma-1}{\gamma+1}}$ gives $\gamma = 2$. Using $v = c\sqrt{1-1/\gamma^2}$ gives v = 0.866c.

(f) We want to transform to the center-of-momentum frame where the total momentum is zero. Call *u* the speed of this reference frame. In this frame, the electron and positron have equal but opposite velocities. Use the relativistic velocity addition equation $v'_x = \dfrac{v_x - u}{1 - uv_x/c^2}$, with *R* the lab frame and R' the center-of-momentum frame. We want to find *u*.

For the positron: $v'_{\text{px}} = \dfrac{v-u}{1-uv/c^2}$

For the electron: $v'_{ex} = \dfrac{0-u}{1-0} = -u$

In the center-of-momentum frame, $v'_{\text{px}} = -v'_{\text{ex}}$, so $v'_{\text{px}} = \dfrac{v-u}{1-uv/c^2} = -(-u) = u$. This leads to the equation $u^2(v/c^2) - 2u + v = 0$. Solve this equation for *u*, use the positive root, and use $v = c\sqrt{1-1/\gamma^2}$.
The result is

$$u = v\frac{\gamma}{\gamma+1} = c\sqrt{\frac{\gamma-1}{\gamma+1}}.$$

EVALUATE: Check: If $v \ll c$, c= 1, so $u = v\dfrac{\gamma}{\gamma+1} = v\dfrac{1}{1+1} = \dfrac{v}{2}$. This result is reasonable because in the center-of-momentum frame without using special relativity the electron and positron are coming toward each other each with speed $v/2$.

38.33. IDENTIFY and SET UP: Conservation of energy applied to the collision gives $E_\lambda = E_{\lambda'} + E_{\text{e}}$, where E_{e} is the kinetic energy of the electron after the collision and E_λ and $E_{\lambda'}$ are the energies of the photon before and after the collision. The energy of a photon is related to its wavelength according to $E = hf = hc/\lambda$.

EXECUTE: **(a)** $E_e = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = hc\left(\frac{\lambda' - \lambda}{\lambda\lambda'}\right).$

$E_e = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})\left(\frac{0.0032\times10^{-9}\ \text{m}}{(0.1100\times10^{-9}\ \text{m})(0.1132\times10^{-9}\ \text{m})}\right).$

$E_e = 5.105\times10^{-17}\ \text{J} = 319\ \text{eV}.$

$E_e = \frac{1}{2}mv^2$, so $v = \sqrt{\frac{2E_e}{m}} = \sqrt{\frac{2(5.105\times10^{-17}\ \text{J})}{9.109\times10^{-31}\ \text{kg}}} = 1.06\times10^{7}\ \text{m/s}.$

(b) The wavelength λ of a photon with energy E_e is given by $E_e = hc/\lambda$, so

$$\lambda = \frac{hc}{E_e} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{5.105\times10^{-17}\ \text{J}} = 3.89\ \text{nm}.$$

EVALUATE: Only a small portion of the incident photon's energy is transferred to the struck electron; this is why the wavelength calculated in part (b) is much larger than the wavelength of the incident photon in the Compton scattering.

38.34. **IDENTIFY:** The equation $\lambda' - \lambda = \left(\frac{h}{mc}\right)(1-\cos\phi)$ relates λ and λ' to ϕ. Apply conservation of energy to obtain an expression that relates λ and v to λ'.

SET UP: The kinetic energy of the electron is $K = (\gamma - 1)mc^2$. The energy of a photon is $E = \frac{hc}{\lambda}.$

EXECUTE: **(a)** The final energy of the photon is $E' = \frac{hc}{\lambda'}$, and $E = E' + K$, where K is the kinetic energy of the electron after the collision. Then,

$$\lambda = \frac{hc}{E'+K} = \frac{hc}{(hc/\lambda') + K} = \frac{hc}{(hc/\lambda') + (\gamma-1)mc^2} = \frac{\lambda'}{1 + \frac{\lambda' mc}{h}\left[\frac{1}{(1-v^2/c^2)^{1/2}} - 1\right]}.$$

$(K = mc^2(\gamma - 1)$ since the relativistic expression must be used for three-figure accuracy).

(b) $\phi = \arccos[1 - \Delta\lambda/(h/mc)].$

(c) $\gamma - 1 = \frac{1}{\left(1-\left(\frac{1.80}{3.00}\right)^2\right)^{1/2}} - 1 = 1.25 - 1 = 0.250,\ \frac{h}{mc} = 2.43\times10^{-12}\ \text{m}$

$$\Rightarrow \lambda = \frac{5.10\times10^{-3}\ \text{nm}}{1 + \frac{(5.10\times10^{-12}\ \text{m})(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})(0.250)}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}} = 3.34\times10^{-3}\ \text{nm}.$$

$$\phi = \arccos\left(1 - \frac{(5.10\times10^{-12}\ \text{m} - 3.34\times10^{-12}\ \text{m})}{2.43\times10^{-12}\ \text{m}}\right) = 74.0°.$$

EVALUATE: For this final electron speed, $v/c = 0.600$ and $K = \frac{1}{2}mv^2$ is not accurate.

38.35. **IDENTIFY** and **SET UP:** Apply the photoelectric effect. $eV_0 = hf - \phi$. For a photon, $f\lambda = c$.

EXECUTE: **(a)** Using $eV_0 = hf - \phi$ and $f\lambda = c$, we get $eV_0 = hc/\lambda - \phi$. Solving for V_0 gives $V_0 = \frac{hc}{e}\cdot\frac{1}{\lambda} - \frac{\phi}{e}$. Therefore a graph of V_0 versus $1/\lambda$ should be a straight line with slope equal to hc/e and y-intercept equal to $-\phi/e$. Figure 38.35 shows this graph for the data given in the problem. The best-fit equation for this graph is $V_0 = (1230\ \text{V}\cdot\text{nm})\cdot\frac{1}{\lambda} - 4.76\ \text{V}$. The slope is equal to $1230\ \text{V}\cdot\text{nm}$, which is equal to $1.23\times10^{-6}\ \text{V}\cdot\text{m}$, and the y-intercept is –4.76 V.

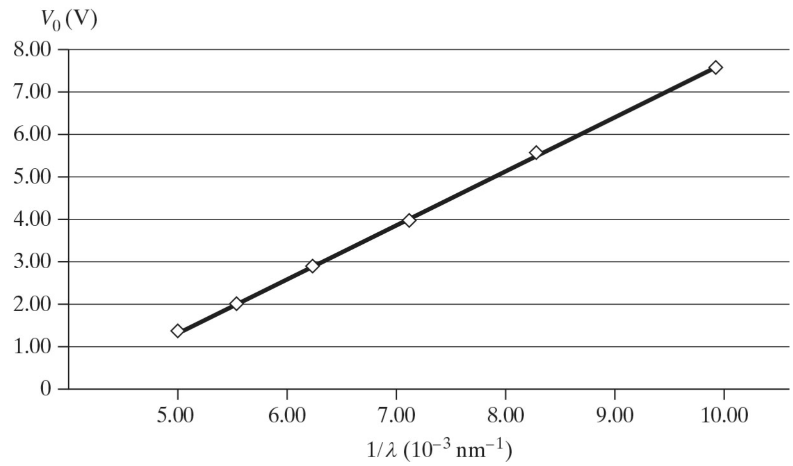


Figure 38.35

(b) Using the slope we have hc/e = slope, so

$h = e(\text{slope})/c = (1.602\times10^{-19}\text{ C})(1.23\times10^{-6}\text{ V}\cdot\text{m})/(2.998\times10^{8}\text{ m/s}) = 6.58\times10^{-34}\text{ J}\cdot\text{s}.$

The y-intercept is equal to $-\phi/e$, so

$$\phi = -e(y\text{-intercept}) = -(1.602\times10^{-19}\text{ C})(-4.76\text{ V}) = 7.63\times10^{-19}\text{ J} = 4.76\text{ eV}.$$

(c) For the longest wavelength light, the energy of a photon is equal to the work function of the metal, so $hc/\lambda = \phi$. Solving for λ gives $\lambda = hc/\phi$. Our calculation of h was just a test of the data, so we use the accepted value for h in the calculation.

$\lambda = hc/\phi = (6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})/(7.63\times10^{-19}\text{ J}) = 2.60\times10^{-7}\text{ m} = 260\text{ nm}.$

(d) The energy of the photon is equal to the sum of the kinetic energy of the photoelectron and the work function, so $hc/\lambda = K + \phi$. This gives $(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})/\lambda = 10.0\text{ eV} + 4.76\text{ eV}$ $= 14.76$ eV, which gives $\lambda = 8.40\times10^{-8}$ m = 84.0 nm.

EVALUATE: As we know from Table 38.1, typical metal work functions are several eV, so our results are plausible.

38.36. **IDENTIFY** and **SET UP:** For the photoelectric effect, $eV_0 = hf - \phi$, and the energy of a photon is $E = hf = hc/\lambda$.

EXECUTE: **(a)** The energy of the UV photons is

$$E = hc/\lambda = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})/(270\times10^{-9}\text{ m}) = 4.59\text{ eV}.$$

The photon energy must be at least as great as the work function to produce photoelectrons. From Table 38.1, we see that this is the case for aluminum, silver, and sodium.

(b) The maximum kinetic energy of a photoelectron is $K = hf - \phi$, so the smallest work function gives the largest kinetic energy of the electron. This is the case for sodium.

$K = E_{\text{photon}} - \phi = 4.59\text{ eV} - 2.7\text{ eV} = 1.89$ eV. This is much less than the rest energy (0.511 MeV) of an electron, so we do not need to use the relativistic formula for kinetic energy. Solving $K = \frac{1}{2}mv^2$ for v gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.89\text{ eV})(1.602\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = 8.2\times10^{5}\text{ m/s}.$$

(c) The energy of the photon is equal to the work function of the gold, so $hc/\lambda = \phi$. This gives $(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})/\lambda = 5.1\text{ eV} \rightarrow \lambda = 2.4\times10^{-7}\text{ m} = 240\text{ nm}.$

(d) In part (c), the energy of the photon is equal to the work function for gold, so

$$K_{\text{max}} = E_{\text{photon}} - \phi_{\text{sodium}} = \phi_{\text{gold}} - \phi_{\text{sodium}} = 5.1\text{ eV} - 2.7\text{ eV} = 2.4\text{ eV}.$$

EVALUATE: Of the three possible metals in Table 38.1, aluminum would be the most practical to use for the smoke detector. Silver is probably too expensive, and sodium is too reactive with water.

38.37. **IDENTIFY** and **SET UP:** We have Compton scattering, so $\lambda' - \lambda = \left(\frac{h}{mc}\right)(1-\cos\phi)$, which can also be expressed as $\lambda' - \lambda = \lambda_C(1-\cos\phi)$, where λ_C is the Compton wavelength.

EXECUTE: **(a)** Figure 38.37 shows the graph of λ' versus $1 - \cos\phi$ for the data included in the problem. The best-fit equation of the line is λ' = 5.21 pm + (2.40 pm)(1 – cos ϕ). The slope is 2.40 pm and the y-intercept is 5.21 pm.

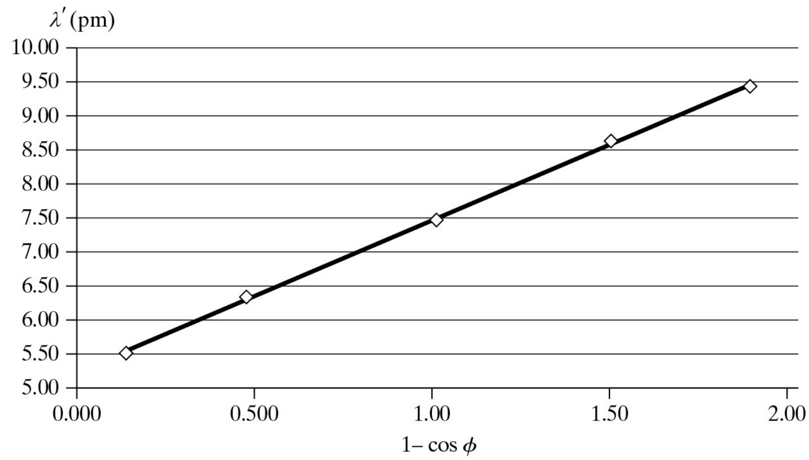


Figure 38.37

(b) Solving $\lambda' - \lambda = \lambda_C(1-\cos\phi)$ for λ' gives $\lambda' = \lambda + \lambda_C(1-\cos\phi)$. The graph of λ' versus $1 - \cos\phi$ should be a straight line with slope equal to λ_C and y-intercept equal to λ. From the slope, we get λ_C = slope = 2.40 pm.

(c) From the y-intercept we get $\lambda = y$-intercept = 5.21 pm.

EVALUATE: For backscatter, the photon wavelength would be 5.21 pm + 2(2.40 pm) = 10.01 pm.

38.38. **IDENTIFY:** In this problem we are dealing with the photoelectric effect and must also use Kirchhoff's rules.

SET UP: $eV_0 = hf - \phi = hc/\lambda - \phi.$

EXECUTE: **(a)** We want V_{AC} if R = 3.20 kΩ.

$$I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{15.0\text{ V}}{4.20\text{ k}\Omega} = 3.5714\text{ mA}.$$

$$V_R = RI = (3.20\text{ k}\Omega)(3.5714\text{ mA}) = 11.4\text{ V}.$$

The cathode is at a higher potential than the anode, so $V_{AC} = V_A - V_C = -11.4$ V.

(b) We want V_{AC} if R = 334 Ω. Follow the same procedure as in part (a), which gives

$$V_R = (334\ \Omega)\frac{15.0\text{ V}}{1.334\text{ K}\Omega} = 3.76\text{ V}.$$

Therefore $V_{AC} = -3.76$ V.

(c) We want the work function. From part (b), the stopping potential is 3.76 V. Solving $eV_0 = hc/\lambda - \phi$ for ϕ gives

$$\phi = \frac{hc}{\lambda} - eV_0 = \frac{hc}{140 \text{ nm}} - e(3.76 \text{ V}) = 5.10 \text{ eV}.$$

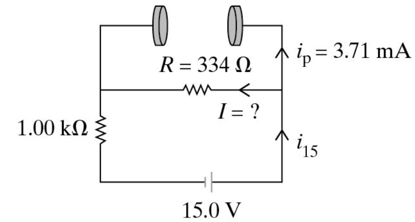


Figure 38.38

(d) We want I. Apply Kirchhoff's rules using the currents shown in Figure 38.38. $i_{15} = I + i$. A counterclockwise path through the lower part of the circuit gives $RI - (1.00 \text{ k}\Omega)i_{15} + 15.0 \text{ V} = 0$. Solve for I using $i = 3.71$ mA and $R = 334\ \Omega$, which gives $I = 8.46$ mA.
(e) We want V_{AC}. $V_{AC} = V_R = RI = (334\ \Omega)(8.46 \text{ mA}) = 2.893$ V.
(f) We want to find R, so that $i = 0$ with $\lambda = 65.0$ nm. If $i = 0$, then V_R is the stopping potential V_0.

$$RI = \frac{hc/\lambda - \phi}{e} \text{ and } I = \frac{\mathcal{E}}{R + 1.00 \text{ k}\Omega}$$

$$\frac{hc/\lambda - \phi}{e} = \frac{R\mathcal{E}}{R + 1.00 \text{ k}\Omega}$$

Using $\mathcal{E} = 15.0$ V, $\phi = 5.10$ eV, and $\lambda = 65.0$ nm, we get $R = 13.8$ kΩ.

EVALUATE: A work function of 5.1 eV is the same as gold and nickel, so it is reasonable.

38.39. IDENTIFY: This problem involves pair production of an electron-positron pair.

SET UP and **EXECUTE:** $\lambda = h/p,\ E = hc/\lambda,\ p = m\gamma v,\ E = m\gamma c^2.$

(a) Energy conservation: $hc/\lambda + Mc^2 = 2myc^2 + M\gamma_M c^2.$

(b) Momentum conservation: $h/\lambda = 2m\gamma v\cos\phi + M\gamma_M V_M$

(c) Eliminate h/λ between the equations in parts (a) and (b) and rearrange to obtain the following:

$$\frac{2m\gamma}{M}\left(1 - \frac{v}{c}\cos\phi\right) + \gamma_M\left(1 - \frac{V_M}{c}\right) = 1.$$

(d) We want V_M if $V_M << c$. If $x << 1$, $(1 + x)^n \approx 1 + nx$. Using $n = -½$ and $x = -(V_M/c)^2$ gives

$$\gamma_M \approx 1 + \left(-\frac{1}{2}\right)\left[-\left(\frac{V_M}{c}\right)^2\right] \approx 1 + \frac{V_M^2}{2c^2} \approx 1.$$

Using this to rewrite the result from part (c) gives

$$\frac{2m\gamma}{M}\left(1 - \frac{v}{c}\cos\phi\right) + 1\frac{V_M}{c} = 1.$$

Solving for V_M gives

$$V_M = \frac{2m\gamma}{M}(c - v\cos\phi).$$

(e) For V_M to be zero, $c - v\cos\phi$ would have to be zero, which is not possible with $v < c$.

(f) We want V_M if M is a proton and $v = 0$. Using the result in part (d) gives

$$V_M = \frac{2m\gamma c}{M} = 327 \text{ Km/s}.$$

(g) We want V_M. Use the result from (d). First use $E = m\gamma c^2$ to find γ.

$$5.00 \text{ MeV} = (0.511 \text{ MeV})\gamma \rightarrow \gamma = 9.785.$$

Now use $v = c\sqrt{1-1/\gamma^2}$ to find v, giving $v = 2.984 \times 10^8$ m/s. Now find V_M for $\phi = 60.0°$ and using the values we just found for v and γ, which gives

$$V_M = \frac{2m\gamma}{M}(c - v\cos\phi) = 1610 \text{ Km/s}.$$

(h) We want the energy E of the incident photon. This energy is equal to the sum of the energy of the electron and positron and the kinetic energy of the proton. The proton is not relativistic, so we have

$$E = 2E_{\text{electron}} + \frac{1}{2}Mv^2.$$

Using E_{electron} = 5.00 MeV and v = 1620 km/s gives E = 2(5.0 MeV) + 0.014 MeV = 10.0 MeV.

EVALUATE: Most of the energy goes to the electron-positron pair.

38.40. IDENTIFY: Follow the derivation of $\lambda' - \lambda = \left(\frac{h}{mc}\right)(1-\cos\phi)$. Apply conservation of energy and conservation of momentum to the collision.

SET UP: Use the coordinate direction specified in the problem.

EXECUTE: **(a)** <u>Momentum</u>: $\vec{\boldsymbol{p}} + \vec{\boldsymbol{P}} = \vec{\boldsymbol{p}}' + \vec{\boldsymbol{P}}' \Rightarrow p - P = -p' - P' \Rightarrow p' = P - (p + P')$.

<u>Energy</u>: $pc + E = p'c + E' = p'c + \sqrt{(P'c)^2 + (mc^2)^2}$

$$\Rightarrow (pc - p'c + E)^2 = (P'c)^2 + (mc^2)^2 = (Pc)^2 + ((p+p')c)^2 - 2P(p+p')c^2 + (mc^2)^2.$$

$$(pc - p'c)^2 + E^2 = E^2 + (pc + p'c)^2 - 2(Pc^2)(p+p') + 2Ec(p-p') - 4pp'c^2 + 2Ec(p-p')$$

$$+2(Pc^2)(p+p') = 0$$

$$\Rightarrow p'(Pc^2 - 2pc^2 - Ec) = p(-Ec - Pc^2)$$

$$\Rightarrow p' = p\frac{Ec + Pc^2}{2pc^2 + Ec - Pc^2} = p\frac{E + Pc}{2pc + (E - Pc)}$$

$$\Rightarrow \lambda' = \lambda\left(\frac{2hc/\lambda + (E - Pc)}{E + Pc}\right) = \lambda\left(\frac{E - Pc}{E + Pc}\right) + \frac{2hc}{E + Pc}$$

$$\Rightarrow \lambda' = \frac{\lambda(E - Pc) + 2hc}{E + Pc}$$

If $E \gg mc^2,\ Pc = \sqrt{E^2 - (mc^2)^2} = E\sqrt{1 - \left(\frac{mc^2}{E}\right)^2} \approx E\left(1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2 + \ldots\right)$

$$\Rightarrow E - Pc \approx \tfrac{1}{2}\frac{(mc^2)^2}{E} \Rightarrow \lambda' \approx \frac{\lambda(mc^2)^2}{2E(2E)} + \frac{hc}{E} = \frac{hc}{E}\left(1 + \frac{m^2c^4\lambda}{4hcE}\right).$$

(b) If $\lambda = 10.6\times10^{-6}$ m, $E = 1.00\times10^{10}$ eV $= 1.60\times10^{-9}$ J

$$\Rightarrow \lambda' \approx \frac{hc}{1.60\times10^{-9}\ \text{J}}\left(1 + \frac{(9.11\times10^{-31}\ \text{kg})^2c^4(10.6\times10^{-6}\ \text{m})}{4hc\,(1.6\times10^{-9}\ \text{J})}\right) = (1.24\times10^{-16}\ \text{m})(1 + 56.0)$$

$$= 7.08\times10^{-15}\ \text{m}.$$

(c) These photons are gamma rays. We have taken infrared radiation and converted it into gamma rays! Perhaps useful in nuclear medicine, nuclear spectroscopy, or high energy physics: wherever controlled gamma ray sources might be useful.

EVALUATE: The photon has gained energy from the initial kinetic energy of the electron. Since the photon gains energy, its wavelength decreases.

38.41. IDENTIFY and SET UP: The specific gravity of the tumor is 1, so it has the same density as water, 1000 kg/m^3. If 70 Gy are given in 35 days, the daily treatment is 2 Gy.

EXECUTE: The energy E per cell is

$$E/\text{cell} = \frac{(2\text{ J/kg})\left(\dfrac{1000\text{ kg}}{(100\text{ cm})^3}\right)}{10^8\text{ cells/cm}^3} = (2\times10^{-11}\text{ J/cell})(6\times10^{18}\text{ eV/J}) = 1.2\times10^8\text{ eV/cell} = 120\text{ MeV/cell.}$$

Choice (c) is correct.

EVALUATE: For 35 treatments the total dose would be 120 MeV $\times 35$ = 4200 MeV = 4.2 GeV per cell.

38.42. IDENTIFY and SET UP: Assume that the photon eventually loses all of its energy. Call N the number of ionizations.

EXECUTE: (40 eV)N = 4 MeV = 4 $\times 10^6$ eV $\rightarrow$ $N = 10^5$, so choice (d) is correct.

EVALUATE: This result is an average, since not every ionization would necessarily take 40 eV.

38.43. IDENTIFY and SET UP: For Compton scattering $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi)$. The energy of a photon is $E = hf = hc/\lambda$. The energy gained by the electron is equal to the energy lost by the photon.

EXECUTE: For backscatter, ϕ = 180°, so $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi)$ gives $\lambda' = \lambda + \dfrac{2h}{mc}$.

$E = hc/\lambda = 4$ MeV, so $\lambda = hc/(4\text{ MeV})$. Therefore

$\lambda' = \lambda + \dfrac{2h}{mc}$ = hc/(4 MeV) + 2h/mc = hc[2/(0.511 MeV) + 1/(4 MeV)] = 4.165hc MeV^{-1}.

E_{el} = loss of energy of photon.

E_{el} = $hc/\lambda - hc/\lambda'$ = $\dfrac{hc}{\lambda} - \dfrac{hc}{\lambda'} = hc\left[\dfrac{1}{hc/(4\text{ MeV})} - \dfrac{1}{0.4164hc\text{ MeV}^{-1}}\right]$ = 3.8 MeV. Therefore choice (a) is the correct one.

EVALUATE: Not all electrons would get this much energy because not all the photons would backscatter.

38.44. IDENTIFY and SET UP: The energy of a photon determines whether it is more likely to interact via the photoelectric effect or the Compton effect. The graph in Figure P38.44 shows that at high energies a photon is more likely to interact via the Compton effect, but at low energies it is more likely to interact via the photoelectric effect.

EXECUTE: From the graph we see that a 4-MeV photon has much higher probability of interacting via the Compton effect. But as it loses energy through repeated interactions, it will be more likely to interact via the photoelectric effect. Therefore choice (c) is the best one.

EVALUATE: In Problem 38.42 we saw that a photon can undergo around 10^5 ionization events, and during each of these it loses about 40 eV. Therefore it is reasonable that it would lose significant energy due to these interactions.

38.45. IDENTIFY and SET UP: For bremsstrahlung we have $eV_{AC} = hf_{max}$.

EXECUTE: If the accelerating potential V_{AC} is high, the maximum energy hf_{max} of the emitted photons will be high compared to a low accelerating potential. Thus choice (b) is correct.

EVALUATE: Not all the photons will have this energy, since f_{max} is the largest that the frequency can be.

39

PARTICLES BEHAVING AS WAVES

VP39.2.1. **IDENTIFY:** This problem involves electron diffraction and the de Broglie wavelength.

SET UP: $\lambda = h/p$, $K = p^2/2m$, $a\sin\theta = m\lambda$, $K = eV$.

EXECUTE: **(a)** We want the de Broglie wavelength. First use $K = eV$ and $K = p^2/2m$ to find p, and then use $\lambda = h/p$ to find λ.

$$p = \sqrt{2mK} = \sqrt{2meV}. \lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}.$$

Using $V = 69.0$ V gives $\lambda = 0.148$ nm.

(b) We want the minimum angle at which a diffraction maximum occurs. Using $m = 1$, $a\sin\theta = m\lambda$ gives $\theta_{\text{min}} = \arcsin(\lambda/d) = \arcsin(0.148/0.172) = 59.1°$.

EVALUATE: There is no other angle at which a maximum occurs. We can use $K = p^2/2m$ because the electron is nonrelativistic at this energy which is much less than its rest energy of 0.511 MeV.

VP39.2.2. **IDENTIFY:** This problem involves electron diffraction and the de Broglie wavelength.

SET UP: $\lambda = h/p$, $K = p^2/2m$, $d\sin\theta = m\lambda$, $K = eV$.

EXECUTE: **(a)** We want the kinetic energy. $K = eV = e(36.5 \text{ V}) = 36.5 \text{ eV} = 5.85 \times 10^{-18}$ J.

(b) We want the de Broglie wavelength.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = 0.202 \text{ nm}.$$

(c) We want the atomic spacing d. $d\sin\theta = m\lambda$ gives $d = (0.203 \text{ nm})/(\sin 48.0°) = 0.273$ nm.

EVALUATE: We can use $K = p^2/2m$ because the electron's speed is much less than c.

VP39.2.3. **IDENTIFY:** This problem involves electron diffraction and the de Broglie wavelength.

SET UP: $\lambda = h/p$, $K = p^2/2m$, $d\sin\theta = m\lambda$, $K = eV$.

EXECUTE: **(a)** We want the de Broglie wavelength. Solving $d\sin\theta = m\lambda$ with $m = 2$ gives

$$\lambda = (0.218 \text{ nm})(\sin 75.0°)/2 = 0.105 \text{ nm}.$$

(b) We want the accelerating voltage V. Solving $eV = K = p^2/2m$ and using $\lambda = h/p$ gives

$$V = \frac{(h/\lambda)^2}{2me} = \frac{h^2}{2me\lambda^2} = \frac{h^2}{2me(0.105 \text{ nm})^2} = 136 \text{ V}.$$

EVALUATE: The electron's kinetic energy is $K = 136$ eV, so it is not relativistic.

VP39.2.4. **IDENTIFY:** This problem involves the de Broglie wavelength of a proton.

SET UP: $\lambda = h/p$, $K = p^2/2m$, $K = eV$.

Execute: **(a)** We want the de Broglie wavelength. Using the given speed gives $\lambda = h/mv = 1.67$ pm.
(b) We want the accelerating voltage V. $eV = K = \frac{1}{2}mv^2$. Solving for V using the known quantities gives $V = 295$ V.
Evaluate: If a proton and electron have comparable speeds, the proton has a *much smaller* de Broglie wavelength because it is much more massive than the electron.

VP39.6.1. **Identify:** The problem involves the energy due to electron transitions in a hypothetical atom.
Set Up and **Execute:** We want the wavelengths of the emitted light in each case. The possible transitions are $5 \to 0$ and $5 \to 2 \to 0$, and $\Delta E = hc/\lambda$.
$\underline{5 \to 0}$: $\Delta E = 5.00\text{ eV} - 0 = 5.00$ eV. $\lambda = hc/\Delta E = hc/(5.00\text{ eV}) = 248$ nm.
$\underline{5 \to 2}$: $\Delta E = 5.00\text{ eV} - 2.00 = 3.00$ eV. $\lambda = hc/\Delta E = hc/(3.00\text{ eV}) = 414$ nm.
$\underline{2 \to 0}$: $\Delta E = 2.00\text{ eV} - 0 = 2.00$ eV. $\lambda = hc/\Delta E = hc/(2.00\text{ eV}) = 620$ nm.
Evaluate: The wavelengths emitted by a gas of this atom would be 248 nm, 414 nm, and 620 nm.

VP39.6.2. **Identify:** The problem involves the energy due to electron transitions in a hypothetical atom.
Set Up: $\Delta E = hc/\lambda$.
Execute: **(a)** $\Delta E_{1\to\text{grd}} = hc/\lambda_1 = hc/(385\text{ nm}) = 3.22\text{ eV}$. Therefore $E_1 = 3.22$ eV relative to ground.
$\Delta E_{2\to 1} = hc/\lambda_2 = hc/(674\text{ nm}) = 1.84\text{ eV}$. Therefore $E_2 = 1.84$ eV relative to E_1. Relative to ground we have $E_2 = 1.84\text{ eV} + 3.22\text{ eV} = 5.06\text{ eV}$.
(b) $\lambda = hc/\Delta E = hc/(5.06\text{ eV}) = 245$ nm.
Evaluate: Note that the energy difference between adjacent levels gets smaller for higher and higher levels.

VP39.6.3. **Identify:** The problem involves electron transitions in a Bohr hydrogen atom.
Set Up: $\Delta E = hc/\lambda$. For the Bohr hydrogen atom, $E_n = -(13.60\text{ eV})/n^2$. We want the energy and wavelength of the emitted photon.
Execute: **(a)** $5 \to 3$: $\Delta E = (-13.60\text{ eV})(1/5^2 - 1/3^2) = 0.967$ eV. $\lambda = hc/\Delta E = hc/(0.967\text{ eV})$ $= 1.28\ \mu$m.
(b) $4 \to 2$: $\Delta E = (-13.60\text{ eV})(1/4^2 - 1/2^2) = 2.55$ eV. $\lambda = hc/\Delta E = hc/(2.55\text{ eV}) = 487$ nm.
(c) $3 \to 1$: $\Delta E = (-13.60\text{ eV})(1/3^2 - 1/1^2) = 12.1$ eV. $\lambda = hc/\Delta E = hc/(12.1\text{ eV}) = 103$ nm.
Evaluate: Note that as the energy difference increases, the wavelength of the emitted photon decreases. This is reasonable because shorter wavelength photons have more energy than long wavelength photons.

VP39.6.4. **Identify:** The problem involves the energy due to electron transitions in a hydrogen atom.
Set Up: $\Delta E = hc/\lambda$. For the Bohr hydrogen atom, $E_n = -(13.60\text{ eV})/n^2$, $K_n = (13.60\text{ eV})/n^2$, and $U_n = -(27.20\text{ eV})/n^2$.
Execute: **(a)** We want the difference in kinetic energy. $\Delta E = K_6 - K_2 = (13.60\text{ eV})(1/6^2 - 1/2^2)$ $= -3.02$ eV.
(b) We want the difference in potential energy. $\Delta U = U_6 - U_2 = (-27.20\text{ eV})(1/6^2 - 1/2^2) = +3.02\text{ eV}$.
(c) We want the wavelength of the photon. $\Delta E = hc/\lambda$ gives $\lambda = hc/\Delta E = hc/(6.04\text{ eV} - 3.02\text{ eV})$ $= 411$ nm.
Evaluate: Check: $\Delta E = K_6 - K_2 = (-13.60\text{ eV})(1/6^2 - 1/2^2) = +3.02\text{ eV}$, as we used in part (c).

VP39.8.1. **Identify:** This problem involves blackbody radiation and the Wien law.
Set Up: $I = \sigma T^4$, Wien law: $\lambda_m = \dfrac{2.90\times 10^{-3}\text{ m}\cdot\text{K}}{T}$.
Execute: **(a)** We want the peak wavelength. Using $T = 3590$ K, the Wien law gives 808 nm. This wavelength is greater than that of visible light, so it is in the *infrared* region.
(b) We want the intensity I. Using $T = 3590$ K, $I = \sigma T^4$ gives 9.42×10^6 W/m^2.

EVALUATE: Betelgeuse is a red giant. It is red because it radiates most of its visible light in the red end of spectrum.

VP39.8.2. **IDENTIFY:** This problem involves blackbody radiation and the Wien law.

SET UP: $I = \sigma T^4$, Wien law: $\lambda_{\text{m}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T}$.

EXECUTE: **(a)** We want the temperature. Solving $I = \sigma T^4$ for T and using $I = 78.0\ \text{MW/m}^2$ gives $T = 6090$ K.

(b) We want the peak wavelength. Using $T = 6090$ in the Wien law gives $\lambda_m = 476$ nm.

(c) This wavelength is in the visible (to humans) part of the electromagnetic spectrum.

EVALUATE: This blackbody would be bluish because the peak wavelength is toward the blue end of the spectrum.

VP39.8.3. **IDENTIFY:** This problem involves blackbody radiation and the Wien law.

SET UP: $I = \sigma T^4$, Wien law: $\lambda_{\text{m}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T}$.

EXECUTE: **(a)** We want the temperature. Using the Wien law with the peak wavelength at 239 nm gives $T = 12{,}100$ K.

(b) We want the power per unit area. Using $I = \sigma T^4$ with $T = 12{,}100$ K gives $I = 1.23 \times 10^9\ \text{W/m}^2 = 1.23\ \text{GW/m}^2$.

EVALUATE: Rigel is a very hot star. Its peak wavelength is in the ultraviolet part of the electromagnetic spectrum.

VP39.8.4 **IDENTIFY:** This problem involves blackbody radiation, the Planck radiation law, and the Wien law.

SET UP: $I = \sigma T^4$, Planck law: $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)}$, Wien law: $\lambda_{\text{m}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T}$.

EXECUTE: **(a)** We want the peak wavelength. Using the Wien law with $T = 3040$ K gives $\lambda_m = 954$ nm.

(b) We want I. The intensity within the range $\Delta\lambda$ is $I \approx I(\lambda)\Delta\lambda$ if $\Delta\lambda$ is small, as it is in this case. Using the Planck law gives

$$I = I(\lambda)\Delta\lambda = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)}\Delta\lambda.$$

Using $\lambda = 954$ nm, $\Delta\lambda = 12.0$ nm, and $T = 3040$ K gives $I = 40.1\ \text{kW/m}^2$.

EVALUATE: The total intensity the star radiates is $I = \sigma T^4 = 4.84\ \text{MW/m}^2$ at $T = 3040$ K. So the fraction in the 12 nm range is only $(40.1\ \text{kW/m}^2)/(4.84\ \text{MW/m}^2) = 0.83\%$.

39.1. **IDENTIFY** and **SET UP:** $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$. For an electron, $m = 9.11\times10^{-31}$ kg. For a proton, $m = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\lambda = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(4.70\times10^{6}\ \text{m/s})} = 1.55\times10^{-10}\ \text{m} = 0.155\ \text{nm}.$

(b) λ is proportional to $\dfrac{1}{m}$, so $\lambda_{\text{p}} = \lambda_{\text{e}}\left(\dfrac{m_{\text{e}}}{m_{\text{p}}}\right) = (1.55\times10^{-10}\ \text{m})\left(\dfrac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}\right) = 8.46\times10^{-14}\ \text{m}.$

EVALUATE: For the same speed the proton has a smaller de Broglie wavelength.

39.2. **IDENTIFY** and **SET UP:** For a photon, $E = \dfrac{hc}{\lambda}$. For an electron or alpha particle, $p = \dfrac{h}{\lambda}$ and $E = \dfrac{p^2}{2m}$, so $E = \dfrac{h^2}{2m\lambda^2}$.

EXECUTE: **(a)** $E = \dfrac{hc}{\lambda} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{0.20\times10^{-9}\ \text{m}} = 6.2\ \text{keV}.$

(b) $E = \dfrac{h^2}{2m\lambda^2} = \left(\dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{0.20\times10^{-9}\ \text{m}}\right)^2 \dfrac{1}{2(9.11\times10^{-31}\ \text{kg})} = 6.03\times10^{-18}\ \text{J} = 38\ \text{eV}.$

(c) $E_{\text{alpha}} = E_{\text{e}}\left(\dfrac{m_{\text{e}}}{m_{\text{alpha}}}\right) = (38\ \text{eV})\left(\dfrac{9.11\times10^{-31}\ \text{kg}}{6.64\times10^{-27}\ \text{kg}}\right) = 5.2\times10^{-3}\ \text{eV}.$

EVALUATE: For a given wavelength a photon has much more energy than an electron, which in turn has more energy than a alpha particle.

39.3. **IDENTIFY:** For a particle with mass, $\lambda = \dfrac{h}{p}$ and $K = \dfrac{p^2}{2m}$.

SET UP: $1\ \text{eV} = 1.60\times10^{-19}\ \text{J}.$

EXECUTE: **(a)** $\lambda = \dfrac{h}{p} \Rightarrow p = \dfrac{h}{\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(2.80\times10^{-10}\ \text{m})} = 2.37\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

(b) $K = \dfrac{p^2}{2m} = \dfrac{(2.37\times10^{-24}\ \text{kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\ \text{kg})} = 3.08\times10^{-18}\ \text{J} = 19.3\ \text{eV}.$

EVALUATE: This wavelength is on the order of the size of an atom. This energy is on the order of the energy of an electron in an atom.

39.4. **IDENTIFY:** For a particle with mass, $\lambda = \dfrac{h}{p}$ and $E = \dfrac{p^2}{2m}$.

SET UP: $1\ \text{eV} = 1.60\times10^{-19}\ \text{J}.$

EXECUTE: $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(6.64\times10^{-27}\ \text{kg})\,(4.20\times10^{6}\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})}} = 7.02\times10^{-15}\ \text{m}.$

EVALUATE: This wavelength is on the order of the size of a nucleus.

39.5. **IDENTIFY** and **SET UP:** The de Broglie wavelength is $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$.

EXECUTE: The de Broglie wavelength is the same for the proton and the electron, so $\dfrac{h}{m_{\text{e}}v_{\text{e}}} = \dfrac{h}{m_{\text{p}}v_{\text{p}}}$.

$v_{\text{p}} = v_{\text{e}}(m_{\text{e}}/m_{\text{p}}) = (8.00\times10^{6}\ \text{m/s})[(9.109\times10^{-31}\ \text{kg})/(1.6726\times10^{-27}\ \text{kg})] = 4360\ \text{m/s} = 4.36\ \text{km/s}.$

EVALUATE: The proton and electron have the same de Broglie wavelength and the same momentum, but very different speeds because $m_{\text{p}} >> m_{\text{e}}$.

39.6. **IDENTIFY:** This problem is about the de Broglie wavelength.

SET UP: $\lambda = h/p$, $1\ \text{eV} = 1.60\times10^{-19}\ \text{J}$. An electron has mass 9.11×10^{-31} kg.

EXECUTE: **(a)** For a nonrelativistic particle, $K = \dfrac{p^2}{2m}$, so $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2Km}}$.

(b) $(6.63\times10^{-34}\ \text{J}\cdot\text{s})/\sqrt{2(800\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})(9.11\times10^{-31}\ \text{kg})} = 4.34\times10^{-11}\ \text{m}.$

EVALUATE: The de Broglie wavelength decreases when the kinetic energy of the particle increases.

39.7. **IDENTIFY:** This problem is about the de Broglie wavelength.

SET UP: $\lambda = h/p$, $K = p^2/2m$, $p = m\gamma v$, $K = mc^2(\gamma - 1)$, $v = c\sqrt{1 - 1/\gamma^2}$. We want the de Broglie wavelength in each case.

EXECUTE: **(a)** $\lambda = h/p = h/(50.0\ \text{kg})(2.0\ \text{m/s}) = 6.6\times10^{-36}\ \text{m}.$

(b) The kinetic energy of the electron (2.0 MeV) is considerably greater than the rest energy (0.511 MeV) of the electron, so we must use the relativistic equations. $K = mc^2(\gamma - 1)$ gives $2.0\text{ MeV} = (0.511\text{ MeV})(\gamma - 1)$, so $\gamma = 4.914$. Now find the wavelength. Combining $\lambda = h/p$, $p = m\gamma v$, and $v = c\sqrt{1 - 1/\gamma^2}$ gives $\lambda = \dfrac{h}{mc\sqrt{\gamma^2 - 1}}$. Using $\gamma = 4.914$ gives $v = 5.0\times10^{-13}$ m/s.

(c) This electron is not relativistic, so we use $K = p^2/2m$. Solving for v and using $\lambda = h/p$ gives $\lambda = \dfrac{h}{\sqrt{2mK}}$. Using $K = 20\text{ eV} = 3.20\times10^{-18}$ J, we get $\lambda = 0.27$ nm.

EVALUATE: From part (a) we see that for ordinary everyday objects, the de Broglie wavelength is extremely small, much less than for things like electrons.

39.8. IDENTIFY and SET UP: Combining $E = \gamma mc^2$ and $E^2 = (mc^2)^2 + (pc)^2$ gives $p = mc\sqrt{\gamma^2 - 1}$.

EXECUTE: (a) $\lambda = \dfrac{h}{p} = (h/mc)/\sqrt{\gamma^2 - 1} = 4.43\times10^{-12}$ m. (The incorrect nonrelativistic calculation gives 5.05×10^{-12} m.)

(b) $(h/mc)/\sqrt{\gamma^2 - 1} = 7.07\times10^{-13}$ m.

EVALUATE: The de Broglie wavelength decreases when the speed increases.

39.9. IDENTIFY and SET UP: Use $\lambda = h/p$.

EXECUTE: $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(5.00\times10^{-3}\text{ kg})(340\text{ m/s})} = 3.90\times10^{-34}$ m.

EVALUATE: This wavelength is extremely short; the bullet will not exhibit wavelike properties.

39.10. IDENTIFY: Apply conservation of energy to relate the potential difference to the speed of the electrons.

SET UP: The mass of an electron is $m = 9.11\times10^{-31}$ kg. The energy of a photon is $E = \dfrac{hc}{\lambda}$.

EXECUTE: (a) $\lambda = h/mv \to v = h/m\lambda$. Energy conservation gives $e\Delta V = \frac{1}{2}mv^2$.

$$\Delta V = \frac{mv^2}{2e} = \frac{m\left(\dfrac{h}{m\lambda}\right)^2}{2e} = \frac{h^2}{2em\lambda^2} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{2(1.60\times10^{-19}\text{ C})(9.11\times10^{-31}\text{ kg})\,(0.220\times10^{-9}\text{ m})^2} = 31.1\text{ V}.$$

(b) $E_{\text{photon}} = hf = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})\,(3.00\times10^8\text{ m/s})}{0.220\times10^{-9}\text{ m}} = 9.035\times10^{-16}$ J. $e\Delta V = K = E_{\text{photon}}$ and

$$\Delta V = \frac{E_{\text{photon}}}{e} = \frac{9.035\times10^{-16}\text{ J}}{1.60\times10^{-19}\text{ C}} = 5650\text{ V}.$$

EVALUATE: The electron in part (b) has wavelength $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$

$= \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\text{ kg})(9.035\times10^{-16}\text{ J})}} = 0.0163$ nm, which is much shorter than the 0.220-nm wavelength of a photon of the same energy.

39.11. IDENTIFY: The acceleration gives momentum to the electrons. We can use this momentum to calculate their de Broglie wavelength.

SET UP: The kinetic energy K of the electron is related to the accelerating voltage V by $K = eV$. For an electron $E = \frac{1}{2}mv^2 = \dfrac{p^2}{2m}$ and $\lambda = \dfrac{h}{p}$. For a photon $E = \dfrac{hc}{\lambda}$.

EXECUTE: **(a)** For an electron $p=\dfrac{h}{\lambda}=\dfrac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{5.00\times10^{-9}\text{ m}}=1.33\times10^{-25}\text{ kg}\cdot\text{m/s}$ and

$E=\dfrac{p^2}{2m}=\dfrac{(1.33\times10^{-25}\text{ kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\text{ kg})}=9.71\times10^{-21}\text{ J}.$ $V=\dfrac{K}{e}=\dfrac{9.71\times10^{-21}\text{ J}}{1.60\times10^{-19}\text{ C}}=0.0607\text{ V}.$ The electrons would have kinetic energy 0.0607 eV.

(b) $E=\dfrac{hc}{\lambda}=\dfrac{1.24\times10^{-6}\text{ eV}\cdot\text{m}}{5.00\times10^{-9}\text{ m}}=248\text{ eV}.$

(c) $E=9.71\times10^{-21}\text{ J}$, so $\lambda=\dfrac{hc}{E}=\dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{9.71\times10^{-21}\text{ J}}=20.5\ \mu\text{m}.$

EVALUATE: If they have the same wavelength, the photon has vastly more energy than the electron.

39.12. **IDENTIFY:** The electrons behave like waves and are diffracted by the slit.

SET UP: We use conservation of energy to find the speed of the electrons, and then use this speed to find their de Broglie wavelength, which is $\lambda=h/mv$. Finally we know that the first dark fringe for single-slit diffraction occurs when $a\sin\theta=\lambda$. The relativistic kinetic energy is $K=(\gamma-1)mc^2$.

EXECUTE: **(a)** The electrons gain kinetic energy K as they are accelerated through a potential difference V, so $eV=K=(\gamma-1)mc^2$. The potential difference is 0.100 kV, so eV = 0.100 keV. Therefore

$$eV=K=(\gamma-1)mc^2=0.100\text{ keV}.$$

Solving for γ and using the fact that the rest energy of an electron is 0.511 MeV, we have

$$\gamma-1=(0.100\text{ keV})/(0.511\text{ MeV})=(0.100\text{ keV})/(511\text{ keV})=1.96\times10^{-4}$$

so $\gamma<<1$ which means that we do not have to use special relativity.

(b) Use energy conservation to find the speed of the electron: $\frac{1}{2}mv^2=eV$.

$$v=\sqrt{\frac{2eV}{m}}=\sqrt{\frac{2(1.60\times10^{-19}\text{ C})(100\text{ V})}{9.11\times10^{-31}\text{ kg}}}=5.93\times10^{6}\text{ m/s}.$$

Now find the de Broglie wavelength:

$$\lambda=\frac{h}{mv}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{ kg})(5.93\times10^{6}\text{ m/s})}=1.23\times10^{-10}\text{ m}=0.123\text{ nm}.$$

For the first single-slit dark fringe, we have $a\sin\theta=\lambda$, which gives

$$a=\frac{\lambda}{\sin\theta}=\frac{1.23\times10^{-10}\text{ m}}{\sin(14.6°)}=4.88\times10^{-10}\text{ m}=0.488\text{ nm}.$$

EVALUATE: The slit width is around 4 times the de Broglie wavelength of the electron, and both are much smaller than the wavelength of visible light.

39.13. **IDENTIFY:** The intensity maxima are located by $d\sin\theta=m\lambda$. Use $\lambda=\dfrac{h}{p}$ for the wavelength of the neutrons. For a particle, $p=\sqrt{2mE}$.

SET UP: For a neutron, $m=1.675\times10^{-27}$ kg.

EXECUTE: For $m=1$, $\lambda=d\sin\theta=\dfrac{h}{\sqrt{2mE}}$.

$$E=\frac{h^2}{2md^2\sin^2\theta}=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{2(1.675\times10^{-27}\text{ kg})(9.10\times10^{-11}\text{ m})^2\sin^2(28.6°)}=6.91\times10^{-20}\text{ J}=0.432\text{ eV}.$$

EVALUATE: The neutrons have $\lambda=0.0436$ nm, comparable to the atomic spacing.

39.14. **IDENTIFY:** $\lambda = h/p$. Conservation of energy gives $eV = K = \dfrac{p^2}{2m}$, where V is the accelerating voltage.

SET UP: The electron mass is 9.11×10^{-31} kg and the proton mass is 1.67×10^{-27} kg.

EXECUTE: **(a)** $eV = K = \dfrac{p^2}{2m} = \dfrac{(h/\lambda)^2}{2m}$, so $V = \dfrac{(h/\lambda)^2}{2me} = 419$ V.

(b) The voltage is reduced by the ratio of the particle masses, $(419\text{ V})\dfrac{9.11\times10^{-31}\text{ kg}}{1.67\times10^{-27}\text{ kg}} = 0.229$ V.

EVALUATE: $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$. For the same λ, particles of greater mass have smaller E, so a smaller accelerating voltage is needed for protons.

39.15. **IDENTIFY:** We are comparing the wavelengths of a photon and an electron having the same energy.
SET UP: $\lambda = h/p$, $E = hc/\lambda$. We want the wavelengths.

EXECUTE: **(a)** Photon: $\lambda = \dfrac{hc}{E} = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})c}{6.00\text{ eV}} = 207$ nm.

(b) Electron: Combine $\lambda = h/p$ and $K = p^2/2m$ to get $\lambda = \dfrac{h}{\sqrt{2mK}}$. Using $K = 6.00\text{ eV} = 9.60\times10^{-19}$ J gives $\lambda = 0.501$ nm.

(c) The photon has a *much* longer wavelength than the electron.
EVALUATE: Notice that if a particle has the same kinetic energy as the energy of a photon, the particle has a much shorter wavelength.

39.16. **IDENTIFY:** We are comparing the energy of a photon and an electron having the same wavelength of 500 nm.

SET UP: $\lambda = h/p$, $E = hc/\lambda$, $K = p^2/2m$. We want the energy of each one.

EXECUTE: **(a)** Photon: Using the 500 nm wavelength gives $E = hc/\lambda = 2.48$ eV.

Electron: Combine $\lambda = h/p$ and $K = p^2/2m$ to get $K = \dfrac{p^2}{2m} = \dfrac{(h/\lambda)^2}{2m}$. Using 500 nm for the wavelength gives $K = 9.639\times10^{-25}\text{ J} = 6.02\times10^{-6}$ eV.

(b) The photon has much more energy than the electron.
EVALUATE: Note that of a photon and particle have the same wavelength, the photon has much more energy than the particle.

39.17. **(a) IDENTIFY:** If the particles are treated as point charges, $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$.

SET UP: $q_1 = 2e$ (alpha particle); $q_2 = 82e$ (lead nucleus); r is given so we can solve for U.

EXECUTE: $U = (8.987\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(82)(1.602\times10^{-19}\text{ C})^2}{6.50\times10^{-14}\text{ m}} = 5.82\times10^{-13}$ J

$$U = 5.82\times10^{-13}\text{ J }(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 3.63\times10^6\text{ eV} = 3.63\text{ MeV}$$

(b) IDENTIFY: Apply conservation of energy: $K_1 + U_1 = K_2 + U_2$.

SET UP: Let point 1 be the initial position of the alpha particle and point 2 be where the alpha particle momentarily comes to rest. Alpha particle is initially far from the lead nucleus implies $r_1 \approx \infty$ and $U_1 = 0$. Alpha particle stops implies $K_2 = 0$.

EXECUTE: Conservation of energy thus says $K_1 = U_2 = 5.82\times10^{-13}\ \text{J} = 3.63\ \text{MeV}.$

(c) $K = \frac{1}{2}mv^2,$ so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(5.82\times10^{-13}\ \text{J})}{6.64\times10^{-27}\ \text{kg}}} = 1.32\times10^7\ \text{m/s}.$

EVALUATE: $v/c = 0.044,$ so it is ok to use the nonrelativistic expression to relate K and v. When the alpha particle stops, all its initial kinetic energy has been converted to electrostatic potential energy.

39.18. IDENTIFY: The kinetic energy of the alpha particle is all converted to electrical potential energy at closest approach. The force on the alpha particle is the electrical repulsion of the nucleus.

SET UP: The electrical potential energy of the system is $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}.$

EXECUTE: **(a)** Equating the initial kinetic energy and the final potential energy and solving for the separation radius r gives

$$r = \frac{1}{4\pi\epsilon_0}\frac{(92e)(2e)}{K} = \frac{1}{4\pi\epsilon_0}\frac{(184)(1.60\times10^{-19}\ \text{C})^2}{(4.78\times10^6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 5.54\times10^{-14}\ \text{m}.$$

(b) The above result may be substituted into Coulomb's law. Alternatively, the relation between the magnitude of the force and the magnitude of the potential energy in a Coulomb field is $F = \dfrac{|U|}{r}.$

$|U| = K,$ so $F = \dfrac{K}{r} = \dfrac{(4.78\times10^6\ \text{eV})(1.6\times10^{-19}\ \text{J/ev})}{(5.54\times10^{-14}\ \text{m})} = 13.8\ \text{N}.$

EVALUATE: The result in part (a) is comparable to the radius of a large nucleus, so it is reasonable. The force in part (b) is around 3 pounds, which is large enough to be easily felt by a person.

39.19. IDENTIFY and SET UP: Use the energy to calculate n for this state. Then use the Bohr equation, $L = n\hbar,$ to calculate L.

EXECUTE: $E_n = -(13.6\ \text{eV})/n^2,$ so this state has $n = \sqrt{13.6/1.51} = 3.$ In the Bohr model, $L = n\hbar,$ so for this state $L = 3\hbar = 3.16\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}.$

EVALUATE: We will find in Section 41.1 that the modern quantum mechanical description gives a different result.

39.20. IDENTIFY and SET UP: For a hydrogen atom $E_n = -\dfrac{13.6\ \text{eV}}{n^2}.$ $\Delta E = \dfrac{hc}{\lambda},$ where ΔE is the magnitude of the energy change for the atom and λ is the wavelength of the photon that is absorbed or emitted.

EXECUTE: $\Delta E = E_3 - E_1 = -(13.6\ \text{eV})\left(\dfrac{1}{3^2} - \dfrac{1}{1^2}\right) = +12.09\ \text{eV}.$

$\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{12.09\ \text{eV}} = 102.6\ \text{nm},$ which rounds to 103 nm.

The frequency is $f = \dfrac{c}{\lambda} = \dfrac{2.998\times10^8\ \text{m/s}}{102.6\times10^{-9}\ \text{m}} = 2.92\times10^{15}\ \text{Hz}.$

EVALUATE: This photon is in the ultraviolet region of the electromagnetic spectrum.

39.21. IDENTIFY: The force between the electron and the nucleus in Be^{3+} is $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{Ze^2}{r^2},$ where $Z = 4$ is the nuclear charge. All the equations for the hydrogen atom apply to Be^{3+} if we replace e^2 by Ze^2.

(a) SET UP: Modify the energy equation for hydrogen, $E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{me^4}{8n^2h^2}$ by replacing e^2 with Ze^2.

EXECUTE: $E_n = -\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}$ (hydrogen) becomes

$$E_n = -\frac{1}{\epsilon_0^2}\frac{m(Ze^2)^2}{8n^2h^2} = Z^2\left(-\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}\right) = Z^2\left(-\frac{13.60\text{ eV}}{n^2}\right)(\text{for Be}^{3+}).$$

The ground-level energy of Be^{3+} is $E_1 = 16\left(-\frac{13.60\text{ eV}}{1^2}\right) = -218\text{ eV}$.

EVALUATE: The ground-level energy of Be^{3+} is $Z^2 = 16$ times the ground-level energy of H.

(b) SET UP: The ionization energy is the energy difference between the $n \to \infty$ level energy and the $n = 1$ level energy.

EXECUTE: The $n \to \infty$ level energy is zero, so the ionization energy of Be^{3+} is 218 eV.

EVALUATE: This is 16 times the ionization energy of hydrogen.

(c) SET UP: $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ just as for hydrogen but now R has a different value.

EXECUTE: $R_{\text{H}} = \frac{me^4}{8\epsilon_0^2 h^3 c} = 1.097\times10^7\text{ m}^{-1}$ for hydrogen becomes $R_{\text{Be}} = Z^2\frac{me^4}{8\epsilon_0^2 h^3 c}$

$= 16(1.097\times10^7\text{ m}^{-1}) = 1.755\times10^8\text{ m}^{-1}$ for Be^{3+}.

For $n = 2$ to $n = 1$, $\frac{1}{\lambda} = R_{\text{Be}}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 3R_{\text{Be}}/4$.

$$\lambda = 4/(3R_{\text{Be}}) = 4/(3(1.755\times10^8\text{ m}^{-1})) = 7.60\times10^{-9}\text{ m} = 7.60\text{ nm}.$$

EVALUATE: This wavelength is smaller by a factor of 16 compared to the wavelength for the corresponding transition in the hydrogen atom.

(d) SET UP: Modify the Bohr equation for hydrogen, $r_n = \epsilon_0\frac{n^2h^2}{\pi me^2}$, by replacing e^2 with Ze^2.

EXECUTE: $r_n = \epsilon_0\frac{n^2h^2}{\pi m(Ze^2)}$ (Be^{3+}).

EVALUATE: For a given n the orbit radius for Be^{3+} is smaller by a factor of $Z = 4$ compared to the corresponding radius for hydrogen.

39.22. IDENTIFY and SET UP: In the Bohr model for hydrogen, the energy levels are $E_n = -\frac{13.60\text{ eV}}{n^2}$ and the orbital radii are $r_n = n^2a_0$.

EXECUTE: **(a)** $E_2 - E_1 = -(13.6\text{ eV})(1/2^2 - 1/1^2) = 10.20\text{ eV}$.

$E_{10} - E_9 = -(13.6\text{ eV})(1/10^2 - 1/9^2) = 0.03190\text{ eV}$.

(b) $E_{n+1} - E_n = -(13.6\text{ eV})\left[\frac{1}{(n+1)^2} - \frac{1}{n^2}\right] = -(13.6\text{ eV})\left[\frac{n^2-(n+1)^2}{n^2(n+1)^2}\right] = (13.6\text{ eV})\left[\frac{2n+1}{n^2(n+1)^2}\right]$.

As n gets very large, the factor in brackets approaches $2n/n^4 = 2/n^3$, so the entire quantity approaches $(13.6\text{ eV})(2/n^3) = (27.2\text{ eV})/n^3$.

(c) $r_{n+1} - r_n = a_0\left[(n+1)^2 - n^2\right] = a_0(n^2 + 2n + 1 - n^2) = (2n+1)a_0$. As n gets larger, $2n + 1$ gets larger, so the radial distance between adjacent orbits increases.

EVALUATE: As n gets large, the energy difference between adjacent shells gets small, but the radial distance between adjacent shells gets large. In other words, the orbits get progressively farther apart, but their energy gets closer together.

39.23. **IDENTIFY:** Apply the equations for v_n and r_n: $v_n = \dfrac{e^2}{2\epsilon_0 nh}$, $r_n = \epsilon_0 \dfrac{n^2h^2}{\pi me^2}$.

SET UP: The orbital period for state n is the circumference of the orbit divided by the orbital speed.

EXECUTE: **(a)** $v_n = \dfrac{1}{\epsilon_0}\dfrac{e^2}{2nh}: n = 1 \Rightarrow v_1 = \dfrac{(1.602\times10^{-19}\text{ C})^2}{\epsilon_0\, 2\,(6.626\times10^{-34}\text{ J}\cdot\text{s})} = 2.19\times10^6$ m/s.

$n = 2 \Rightarrow v_2 = \dfrac{v_1}{2} = 1.09\times10^6$ m/s. $n = 3 \Rightarrow v_3 = \dfrac{v_1}{3} = 7.27\times10^5$ m/s.

(b) Orbital period $= \dfrac{2\pi r_n}{v_n} = \dfrac{2\epsilon_0\, n^2h^2/me^2}{1/\epsilon_0 \cdot e^2/2nh} = \dfrac{4\epsilon_0^2\, n^3h^3}{me^4}$.

$n = 1 \Rightarrow T_1 = \dfrac{4\epsilon_0^2\,(6.626\times10^{-34}\text{ J}\cdot\text{s})^3}{(9.11\times10^{-31}\text{ kg})(1.602\times10^{-19}\text{ C})^4} = 1.53\times10^{-16}$ s

$n = 2\text{: } T_2 = T_1(2)^3 = 1.22\times10^{-15}$ s. $n = 3\text{: } T_3 = T_1(3)^3 = 4.13\times10^{-15}$ s.

(c) number of orbits $= \dfrac{1.0\times10^{-8}\text{ s}}{1.22\times10^{-15}\text{ s}} = 8.2\times10^6$.

EVALUATE: The orbital speed is proportional to $1/n$, the orbital radius is proportional to n^2, and the orbital period is proportional to n^3.

39.24. **IDENTIFY:** This problem deals with the Bohr model of the atom.

SET UP: $K = p^2/2m$, $K_n = (13.60\text{ eV})/n^2$, $L_n = nh/2\pi$.

EXECUTE: **(a)** We want the kinetic energy. Using $K = p^2/2m$ with the given momentum, we get $K = 1.52$ eV.

(b) We want the angular momentum. First find n using the result from part (a). Solve the equation $K_n = (13.60\text{ eV})/n^2$ for n, giving $n = \sqrt{\dfrac{13.60\text{ eV}}{1.52\text{ eV}}} = 3$. Now find L. $L = 3(h/2\pi) = 3.16\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}$.

(c) $n = 3$, as shown above.

EVALUATE: The electron is in the second excited state.

39.25. **IDENTIFY** and **SET UP:** The ionization threshold is at $E = 0$. The energy of an absorbed photon equals the energy gained by the atom and the energy of an emitted photon equals the energy lost by the atom.

EXECUTE: **(a)** $\Delta E = 0 - (-20\text{ eV}) = 20$ eV.

(b) When the atom in the $n = 1$ level absorbs an 18-eV photon, the final level of the atom is $n = 4$. The possible transitions from $n = 4$ and corresponding photon energies are $n = 4 \to n = 3$, 3 eV; $n = 4 \to n = 2$, 8 eV; $n = 4 \to n = 1$, 18 eV. Once the atom has gone to the $n = 3$ level, the following transitions can occur: $n = 3 \to n = 2$, 5 eV; $n = 3 \to n = 1$, 15 eV. Once the atom has gone to the $n = 2$ level, the following transition can occur: $n = 2 \to n = 1$, 10 eV. The possible energies of emitted photons are: 3 eV, 5 eV, 8 eV, 10 eV, 15 eV, and 18 eV.

(c) There is no energy level 8 eV higher in energy than the ground state, so the photon cannot be absorbed.

(d) The photon energies for $n = 3 \to n = 2$ and for $n = 3 \to n = 1$ are 5 eV and 15 eV. The photon energy for $n = 4 \to n = 3$ is 3 eV. The work function must have a value between 3 eV and 5 eV.

EVALUATE: The atom has discrete energy levels, so the energies of emitted or absorbed photons have only certain discrete energies.

39.26. **IDENTIFY:** We are investigating a positronium atom using the Bohr model.

SET UP and **EXECUTE:** **(a)** We want the reduced mass m_r.

$$m_r = \frac{m_1 m_2}{m_1 + m_2} = \frac{m^2}{m+m} = m/2.$$

(b) We want r. In the ground state, $n = 1$.

$$r_n = \epsilon_0 \frac{n^2 h^2}{\pi m_r e^2} = 2a_0 = 0.106 \text{ nm}.$$

(c) We want E_1. Using $m = m_r$ and $n = 1$ gives

$$E_n = -\frac{me^2}{\epsilon_0^2 \, 8n^2 h^2} = \frac{1}{2}(-13.60 \text{ eV}) = -6.80 \text{ eV}.$$

(d) We want the wavelength. The energy E is the energy difference between the levels. Find E and then solve for the wavelength.

$$E = \frac{hc}{\lambda} = (-6.80 \text{ eV})\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 0.9444 \text{ eV}.$$

This gives $\lambda = 1310$ nm.

EVALUATE: The magnitude of the ground state energy for positronium is less than that of hydrogen. Therefore the photons emitted during transitions in positronium have less energy (and hence longer wavelength) than in comparable transition in hydrogen.

39.27. **IDENTIFY** and **SET UP:** The wavelength of the photon is related to the transition energy $E_i - E_f$ of the atom by $E_i - E_f = \dfrac{hc}{\lambda}$ where $hc = 1.240 \times 10^{-6} \text{ eV} \cdot \text{m}$.

EXECUTE: **(a)** The minimum energy to ionize an atom is when the upper state in the transition has $E = 0$, so $E_1 = -17.50$ eV. For $n = 5 \to n = 1$, $\lambda = 73.86$ nm and $E_5 - E_1 = \dfrac{1.240 \times 10^{-6} \text{ eV} \cdot \text{m}}{73.86 \times 10^{-9} \text{ m}}$ $= 16.79$ eV. $E_5 = -17.50 \text{ eV} + 16.79 \text{ eV} = -0.71$ eV. For $n = 4 \to n = 1$, $\lambda = 75.63$ nm and $E_4 = -1.10$ eV. For $n = 3 \to n = 1$, $\lambda = 79.76$ nm and $E_3 = -1.95$ eV. For $n = 2 \to n = 1$, $\lambda = 94.54$ nm and $E_2 = -4.38$ eV.

(b) $E_i - E_f = E_4 - E_2 = -1.10 \text{ eV} - (-4.38 \text{ eV}) = 3.28$ eV and $\lambda = \dfrac{hc}{E_i - E_f} = \dfrac{1.240 \times 10^{-6} \text{ eV} \cdot \text{m}}{3.28 \text{ eV}}$ $= 378$ nm.

EVALUATE: The $n = 4 \to n = 2$ transition energy is smaller than the $n = 4 \to n = 1$ transition energy so the wavelength is longer. In fact, this wavelength is longer than for any transition that ends in the $n = 1$ state.

39.28. **IDENTIFY** and **SET UP:** For the Lyman series the final state is $n = 1$ and the wavelengths are given by $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{n^2}\right)$, $n = 2, 3, \ldots$. For the Paschen series the final state is $n = 3$ and the wavelengths are given by $\dfrac{1}{\lambda} = R\left(\dfrac{1}{3^2} - \dfrac{1}{n^2}\right)$, $n = 4, 5, \ldots$. $R = 1.097 \times 10^7 \text{ m}^{-1}$. The longest wavelength is for the smallest n and the shortest wavelength is for $n \to \infty$.

EXECUTE: Lyman: Longest: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right) = \dfrac{3R}{4}$. $\lambda = \dfrac{4}{3(1.097 \times 10^7 \text{ m}^{-1})} = 121.5$ nm.

Shortest: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{\infty^2}\right) = R$. $\lambda = \dfrac{1}{1.097 \times 10^7 \text{ m}^{-1}} = 91.16$ nm.

Paschen: Longest: $\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=\frac{7R}{144}$. $\lambda=\frac{144}{7(1.097\times10^7\ \text{m}^{-1})}=1875\ \text{nm}$.

Shortest: $\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{\infty^2}\right)=\frac{R}{9}$. $\lambda=\frac{9}{1.097\times10^7\ \text{m}^{-1}}=820\ \text{nm}$.

EVALUATE: The Lyman series is in the ultraviolet. The Paschen series is in the infrared.

39.29. IDENTIFY: Apply conservation of energy to the system of atom and photon.

SET UP: The energy of a photon is $E_\gamma=\frac{hc}{\lambda}$.

EXECUTE: **(a)** $E_\gamma=\frac{hc}{\lambda}=\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{8.60\times10^{-7}\ \text{m}}=2.31\times10^{-19}\ \text{J}=1.44\ \text{eV}$. So the internal energy of the atom increases by $1.44\ \text{eV}$ to $E=-6.52\ \text{eV}+1.44\ \text{eV}=-5.08\ \text{eV}$.

(b) $E_\gamma=\frac{hc}{\lambda}=\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{4.20\times10^{-7}\ \text{m}}=4.74\times10^{-19}\ \text{J}=2.96\ \text{eV}$. So the final internal energy of the atom decreases to $E=-2.68\ \text{eV}-2.96\ \text{eV}=-5.64\ \text{eV}$.

EVALUATE: When an atom absorbs a photon the energy of the atom increases. When an atom emits a photon the energy of the atom decreases.

39.30. IDENTIFY and **SET UP:** Balmer's formula is $\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$. For the H_γ spectral line $n=5$. Once we have λ, calculate f from $f=c/\lambda$ and E using $E=hf$.

EXECUTE: **(a)** $\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{5^2}\right)=R\left(\frac{25-4}{100}\right)=R\left(\frac{21}{100}\right)$.

Thus $\lambda=\frac{100}{21R}=\frac{100}{21(1.097\times10^7)}\ \text{m}=4.341\times10^{-7}\ \text{m}=434.1\ \text{nm}$.

(b) $f=\frac{c}{\lambda}=\frac{2.998\times10^8\ \text{m/s}}{4.341\times10^{-7}\ \text{m}}=6.906\times10^{14}\ \text{Hz}$.

(c) $E=hf=(6.626\times10^{-34}\ \text{J}\cdot\text{s})(6.906\times10^{14}\ \text{Hz})=4.576\times10^{-19}\ \text{J}=2.856\ \text{eV}$.

EVALUATE: Section 39.3 shows that the longest wavelength in the Balmer series (H_α) is 656 nm and the shortest is 365 nm. Our result for H_γ falls within this range. The photon energies for hydrogen atom transitions are in the eV range, and our result is of this order.

39.31. IDENTIFY: We know the power of the laser beam, so we know the energy per second that it delivers. The wavelength of the light tells us the energy of each photon, so we can use that to calculate the number of photons delivered per second.

SET UP: The energy of each photon is $E=hf=\frac{hc}{\lambda}=\frac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon.

EXECUTE: $\lambda=10.6\times10^{-6}\ \text{m}$, so $E=1.88\times10^{-20}\ \text{J}$. $P=\frac{E_{\text{tot}}}{t}=\frac{NE}{t}$ so

$$\frac{N}{t}=\frac{P}{E}=\frac{0.100\times10^3\ \text{W}}{1.88\times10^{-20}\ \text{J}}=5.32\times10^{21}\ \text{photons/s}.$$

EVALUATE: At over 10^{21} photons per second, we can see why we do not detect individual photons.

39.32. IDENTIFY: We can calculate the energy of a photon from its wavelength. Knowing the intensity of the beam and the energy of a single photon, we can determine how many photons strike the blemish with each pulse.

SET UP: The energy of each photon is $E = hf = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon. The photon beam is spread over an area $A = \pi r^2$ with $r = 2.5$ mm.

EXECUTE: **(a)** $\lambda = 585$ nm and $E = \dfrac{hc}{\lambda} = 3.40\times10^{-19}\ \text{J} = 2.12\ \text{eV}$.

(b) $P = \dfrac{E_{\text{tot}}}{t} = \dfrac{NE}{t}$, so $N = \dfrac{Pt}{E} = \dfrac{(20.0\ \text{W})(0.45\times10^{-3}\ \text{s})}{3.40\times10^{-19}\ \text{J}} = 2.65\times10^{16}$ photons. These photons are spread over an area πr^2, so the number of photons per mm^2 is

$$\frac{2.65\times10^{16}\ \text{photons}}{\pi(2.5\ \text{mm})^2} = 1.35\times10^{15}\ \text{photons/mm}^2.$$

EVALUATE: With so many photons per mm^2, it is impossible to detect individual photons.

39.33. **IDENTIFY** and **SET UP:** The number of photons emitted each second is the total energy emitted divided by the energy of one photon. The energy of one photon is given by $E = hc/\lambda$. $E = Pt$ gives the energy emitted by the laser in time t.

EXECUTE: In 1.00 s the energy emitted by the laser is $(7.50\times10^{-3}\ \text{W})(1.00\ \text{s}) = 7.50\times10^{-3}$ J.

The energy of each photon is $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{10.6\times10^{-6}\ \text{m}} = 1.874\times10^{-20}$ J.

Therefore $\dfrac{7.50\times10^{-3}\ \text{J/s}}{1.874\times10^{-20}\ \text{J/photon}} = 4.00\times10^{17}$ photons/s.

EVALUATE: The number of photons emitted per second is extremely large.

39.34. **IDENTIFY** and **SET UP:** Visible light has wavelengths from about 380 nm to about 750 nm. The energy of each photon is $E = hf = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon.

EXECUTE: **(a)** 193 nm is shorter than the shortest wavelength of visible light so is in the ultraviolet.

(b) $E = \dfrac{hc}{\lambda} = 1.03\times10^{-18}\ \text{J} = 6.44\ \text{eV}$.

(c) $P = \dfrac{E_{\text{tot}}}{t} = \dfrac{NE}{t}$, so $N = \dfrac{Pt}{E} = \dfrac{(1.50\times10^{-3}\ \text{W})(12.0\times10^{-9}\ \text{s})}{1.03\times10^{-18}\ \text{J}} = 1.75\times10^{7}$ photons.

EVALUATE: A very small amount of energy is delivered to the lens in each pulse, but this still corresponds to a large number of photons.

39.35. **IDENTIFY:** Apply the equation $\dfrac{n_{\text{ex}}}{n_{\text{g}}} = e^{-(E_{\text{ex}}-E_{\text{g}})/kT}$ from the section on the laser.

SET UP: The energy of each of these excited states above the ground state is hc/λ, where λ is the wavelength of the photon emitted in the transition from the excited state to the ground state.

EXECUTE: $\dfrac{n_{2P_{3/2}}}{n_{2P_{1/2}}} = e^{-(E_{2P3/2} - E_{2P1/2})/KT}$. From the diagram

$$\Delta E_{3/2-\text{g}} = \frac{hc}{\lambda_1} = \frac{(6.626\times10^{-34}\ \text{J})(2.998\times10^{8}\,\text{m/s})}{5.890\times10^{-7}\ \text{m}} = 3.373\times10^{-19}\ \text{J}.$$

$$\Delta E_{1/2-\text{g}} = \frac{hc}{\lambda_2} = \frac{(6.626\times10^{-34}\ \text{J})(2.998\times10^{8}\ \text{m/s})}{5.896\times10^{-7}\ \text{m}} = 3.369\times10^{-19}\ \text{J}.$$

So $\Delta E_{3/2-1/2} = 3.373\times10^{-19}\text{ J} - 3.369\times10^{-19}\text{ J} = 4.00\times10^{-22}\text{ J}.$

$\dfrac{n_{2P_{3/2}}}{n_{2P_{1/2}}} = e^{-(4.00\times10^{-22}\text{ J})/(1.38\times10^{-23}\text{J/K}\cdot 500\text{ K})} = 0.944.$ So more atoms are in the $2P_{1/2}$ state.

EVALUATE: At this temperature $kT = 6.9\times10^{-21}$ J. This is greater than the energy separation between the states, so an atom has almost equal probability for being in either state, with only a small preference for the lower energy state.

39.36. **IDENTIFY:** This problem involves blackbody radiation and the Wien law.

SET UP and **EXECUTE:** $I = \sigma T^4$, Wien law: $\lambda_m = \dfrac{2.90\times10^{-3}\text{ m}\cdot\text{K}}{T}$. We want the change in the peak wavelength if the intensity I increases by a factor of 16. From $I = \sigma T^4$ we see that if I changes by a factor of 16, so does T^4. Thus T increases by a factor of 2 because $2^4 = 16$. From the Wien law, if T increases by a factor of 2, the peak wavelength decreases by a factor of $\frac{1}{2}$, so it is *halved*.

EVALUATE: All the temperatures must be in kelvin units.

39.37. **IDENTIFY:** Energy radiates at the rate $H = Ae\sigma T^4$.

SET UP: The surface area of a cylinder of radius r and length l is $A = 2\pi rl$.

EXECUTE: **(a)** $T = \left(\dfrac{H}{Ae\sigma}\right)^{1/4} = \left(\dfrac{100\text{ W}}{2\pi(0.20\times10^{-3}\text{ m})(0.30\text{ m})(0.26)(5.671\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)}\right)^{1/4}.$

$T = 2.06\times10^3$ K.

(b) $\lambda_m T = 2.90\times10^{-3}\text{ m}\cdot\text{K};\ \lambda_m = 1410\text{ nm}.$

EVALUATE: **(c)** λ_m is in the infrared. The incandescent bulb is not a very efficient source of visible light because much of the emitted radiation is in the infrared.

39.38. **IDENTIFY:** Apply Wien's displacement law and $c = f\lambda$.

SET UP: T in kelvins gives λ in meters.

EXECUTE: **(a)** $\lambda_m = \dfrac{2.90\times10^{-3}\text{ m}\cdot\text{K}}{3.00\text{ K}} = 0.966\text{ mm}$, and $f = \dfrac{c}{\lambda_m} = 3.10\times10^{11}$ Hz.

(b) A factor of 100 increase in the temperature lowers λ_m by a factor of 100 to 9.66 μm and raises the frequency by the same factor, to 3.10×10^{13} Hz.

(c) Similarly, $\lambda_m = 966$ nm and $f = 3.10\times10^{14}$ Hz.

EVALUATE: λ_m decreases when T increases, as explained in the textbook.

39.39. **IDENTIFY** and **SET UP:** The wavelength λ_m where the Planck distribution peaks is given by Wien's displacement law, $\lambda_m T = 2.90\times10^{-3}\text{ m}\cdot\text{K}$.

EXECUTE: $\lambda_m = \dfrac{2.90\times10^{-3}\text{ m}\cdot\text{K}}{2.728\text{ K}} = 1.06\times10^{-3}\text{ m} = 1.06\text{ mm}.$

EVALUATE: This wavelength is in the microwave portion of the electromagnetic spectrum. This radiation is often referred to as the "microwave background" (Chapter 44). Note that in Wien's law, T must be in kelvins.

39.40. **IDENTIFY:** Apply Wien's displacement law.

SET UP: $\lambda_m T = 2.90\times10^{-3}\text{ m}\cdot\text{K}$.

EXECUTE: <u>For 10.0-μm infrared</u>: $T = \dfrac{2.90\times10^{-3}\text{ m}\cdot\text{K}}{\lambda_m} = \dfrac{2.90\times10^{-3}\text{ m}\cdot\text{K}}{10.0\times10^{-6}\text{ m}} = 290\text{ K}.$

For 600-nm visible: $T = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{\lambda_\text{m}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{600\times10^{-9}\ \text{m}} = 4830\ \text{K}.$

For 100-nm ultraviolet: $T = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{\lambda_\text{m}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{100\times10^{-9}\ \text{m}} = 29{,}000\ \text{K}.$

EVALUATE: Most materials would melt (or burn or vaporize) before reaching 29,000 K!

39.41. **IDENTIFY:** Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law and Wien's displacement law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$. Wien's displacement law tells us that the peak-intensity wavelength is $\lambda_\text{m} = (\text{constant})/T$.

EXECUTE: **(a)** The hot and cool stars radiate the same total power, so the Stefan-Boltzmann law gives $\sigma A_\text{h} T_\text{h}^4 = \sigma A_\text{c} T_\text{c}^4 \Rightarrow 4\pi R_\text{h}^2 T_\text{h}^4 = 4\pi R_\text{c}^2 T_\text{c}^4 = 4\pi (3R_\text{h})^2 T_\text{c}^4 \Rightarrow T_\text{h}^4 = 9T^4 \Rightarrow T_\text{h} = T\sqrt{3} = 1.7T$, rounded to two significant digits.

(b) Using Wien's law, we take the ratio of the wavelengths, giving $\dfrac{\lambda_\text{m}(\text{hot})}{\lambda_\text{m}(\text{cool})} = \dfrac{T_\text{c}}{T_\text{h}} = \dfrac{T}{T\sqrt{3}} = \dfrac{1}{\sqrt{3}} = 0.58,$ rounded to two significant digits.

EVALUATE: Although the hot star has only 1/9 the surface area of the cool star, its absolute temperature has to be only 1.7 times as great to radiate the same amount of energy.

39.42. **IDENTIFY:** Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: **(a)** $I = \sigma T^4 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(24{,}000\ \text{K})^4 = 1.9\times10^{10}\ \text{W/m}^2.$

(b) Wien's law gives $\lambda_\text{m} = (0.00290\ \text{m}\cdot\text{K})/(24{,}000\ \text{K}) = 1.2\times10^{-7}\,\text{m} = 20\ \text{nm}.$

This is not visible since the wavelength is less than 400 nm.

(c) $P = AI \Rightarrow 4\pi R^2 = P/I = (1.00\times10^{25}\ \text{W})/(1.9\times10^{10}\ \text{W/m}^2)$, which gives

$$R_\text{Sirius} = 6.51\times10^6\ \text{m} = 6510\ \text{km}.$$

$R_\text{Sirius}/R_\text{sun} = (6.51\times10^6\ \text{m})/(6.96\times10^9\ \text{m}) = 0.0093$, which gives

$$R_\text{Sirius} = 0.0093\ R_\text{sun} \approx 1\%\ R_\text{sun}.$$

(d) Using the Stefan-Boltzmann law, we have

$$\frac{P_\text{sun}}{P_\text{Sirius}} = \frac{\sigma A_\text{sun} T_\text{sun}^4}{\sigma A_\text{Sirius} T_\text{Sirius}^4} = \frac{4\pi R_\text{sun}^2 T_\text{sun}^4}{4\pi R_\text{Sirius}^2 T_\text{Sirius}^4}$$

$$= \left(\frac{R_\text{sun}}{R_\text{Sirius}}\right)^2 \left(\frac{T_\text{sun}}{T_\text{Sirius}}\right)^4 \cdot \frac{P_\text{sun}}{P_\text{Sirius}} = \left(\frac{R_\text{sun}}{0.00935 R_\text{sun}}\right)^2 \left(\frac{5800\ \text{K}}{24{,}000\ \text{K}}\right)^4 = 39.$$

EVALUATE: Even though the absolute surface temperature of Sirius B is about 4 times that of our sun, it radiates only 1/39 times as much energy per second as our sun because it is so small.

39.43. **IDENTIFY:** Apply the Heisenberg uncertainty principle.

SET UP: $\Delta p_x = m\Delta v_x$.

EXECUTE: **(a)** $(\Delta x)(m\Delta v_x) \ge \hbar/2$, and setting $\Delta v_x = (0.010)v_x$ and the product of the uncertainties equal to $\hbar/2$ (for the minimum uncertainty) gives $v_x = \dfrac{\hbar}{2m(0.010)\Delta x}$

$= \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(9.109\times10^{-31}\ \text{kg})(0.010)(0.30\times10^{-4}\ \text{m})} = 19.3\ \text{m/s},$ which rounds to 19 m/s.

(b) Taking the ratio of the equation for the proton to the equation for the electron gives

$$v_{\rm p} = \frac{m_{\rm e}}{m_{\rm p}}v_{\rm e} = \frac{9.109\times10^{-31}\ \text{kg}}{1.673\times10^{-27}\ \text{kg}}\cdot(19.3\ \text{m/s}) = 10.5\ \text{mm/s, which rounds to 11 mm/s.}$$

EVALUATE: For a given Δp_x, Δv_x is smaller for a proton than for an electron, since the proton has larger mass.

39.44. **IDENTIFY:** Since we know only that the mosquito is somewhere in the room, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is an uncertainty in its momentum.
SET UP: The uncertainty principle is $\Delta x\Delta p_x \ge \hbar/2$.

EXECUTE: **(a)** You know the mosquito is somewhere in the room, so the maximum uncertainty in its horizontal position is $\Delta x = 5.0$ m.

(b) The uncertainty principle gives $\Delta x\Delta p_x \ge \hbar/2$, and $\Delta p_x = m\Delta v_x$ since we know the mosquito's mass. This gives $\Delta x\, m\Delta v_x \ge \hbar/2$, which we can solve for Δv_x to get the minimum uncertainty in v_x.

$$\Delta v_x = \frac{\hbar}{2m\Delta x} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(1.5\times10^{-6}\ \text{kg})(5.0\ \text{m})} = 7.0\times10^{-30}\ \text{m/s, which is hardly a serious impediment!}$$

EVALUATE: For something as "large" as a mosquito, the uncertainty principle places a negligible limitation on our ability to measure its speed.

39.45. **IDENTIFY** and **SET UP:** The Heisenberg Uncertainty Principle says $\Delta x\Delta p_x \ge \hbar/2$. The minimum allowed $\Delta x\Delta p_x$ is $\hbar/2$. $\Delta p_x = m\Delta v_x$.

EXECUTE: **(a)** $m\Delta x\Delta v_x = \hbar/2$. $\Delta v_x = \dfrac{\hbar}{2m\Delta x} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(1.67\times10^{-27}\ \text{kg})(2.0\times10^{-12}\ \text{m})} = 1.6\times10^4\ \text{m/s}.$

(b) $\Delta x = \dfrac{\hbar}{2m\Delta v_x} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(9.11\times10^{-31}\ \text{kg})(0.250\ \text{m/s})} = 2.3\times10^{-4}\ \text{m}.$

EVALUATE: The smaller Δx is, the larger Δv_x must be.

39.46. **IDENTIFY:** Since we know that the marble is somewhere on the table, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is therefore an uncertainty in its momentum.
SET UP: The uncertainty principle is $\Delta x\Delta p_x \ge \hbar/2$.

EXECUTE: **(a)** Since the marble is somewhere on the table, the maximum uncertainty in its horizontal position is $\Delta x = 1.75$ m.

(b) Following the same procedure as in part (b) of Problem 39.44, the minimum uncertainty in the horizontal velocity of the marble is $\Delta v_x = \dfrac{\hbar}{2m\Delta x} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(0.0100\ \text{kg})(1.75\ \text{m})} = 3.01\times10^{-33}\ \text{m/s}.$

(c) The uncertainty principle tells us that we cannot know that the marble's horizontal velocity is *exactly* zero, so the smallest we could measure it to be is 3.01×10^{-33} m/s, from part (b). The longest time it could remain on the table is the time to travel the full width of the table (1.75 m), so $t = x/v_x$ $= (1.75\ \text{m})/(3.01\times10^{-33}\ \text{m/s}) = 5.81\times10^{32}\ \text{s} = 1.84\times10^{25}$ years. Since the universe is about 14×10^9 years old, this time is about $\dfrac{1.8\times10^{25}\ \text{yr}}{14\times10^9\ \text{yr}} \approx 1.3\times10^{15}$ times the age of the universe! Don't hold your breath!
EVALUATE: For household objects, the uncertainty principle places a negligible limitation on our ability to measure their speed.

39.47. **IDENTIFY:** The Heisenberg uncertainty principle tells us that $\Delta x \Delta p_x \geq \hbar/2$.

SET UP: We can treat the standard deviation as a direct measure of uncertainty.

EXECUTE: Here $\Delta x \Delta p_x = (1.2\times10^{-10}\ \text{m})(3.0\times10^{-25}\ \text{kg}\cdot\text{m/s}) = 3.6\times10^{-35}\ \text{J}\cdot\text{s}$, but $\hbar/2 = 5.28\times10^{-35}\ \text{J}\cdot\text{s}$. Therefore $\Delta x \Delta p_x < \hbar/2$, so the claim is *not valid*.

EVALUATE: The uncertainty product $\Delta x \Delta p_x$ must increase by a factor of about 1.5 to become consistent with the Heisenberg uncertainty principle.

39.48. **IDENTIFY:** Apply conservation of momentum to the system of atom and emitted photon.

SET UP: Assume the atom is initially at rest. For a photon $E = \dfrac{hc}{\lambda}$ and $p = \dfrac{h}{\lambda}$.

EXECUTE: **(a)** Assume a non-relativistic velocity and conserve momentum $\Rightarrow mv = \dfrac{h}{\lambda} \Rightarrow v = \dfrac{h}{m\lambda}$.

(b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(\dfrac{h}{m\lambda}\right)^2 = \dfrac{h^2}{2m\lambda^2}$.

(c) $\dfrac{K}{E} = \dfrac{h^2}{2m\lambda^2}\cdot\dfrac{\lambda}{hc} = \dfrac{h}{2mc\lambda}$. Recoil becomes an important concern for small m and small λ since this ratio becomes large in those limits.

(d) $E = 10.2\ \text{eV} \Rightarrow \lambda = \dfrac{hc}{E} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(10.2\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 1.22\times10^{-7}\ \text{m} = 122\ \text{nm}.$

$$K = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.67\times10^{-27}\ \text{kg})(1.22\times10^{-7}\ \text{m})^2} = 8.84\times10^{-27}\ \text{J} = 5.53\times10^{-8}\ \text{eV}.$$

$\dfrac{K}{E} = \dfrac{5.53\times10^{-8}\ \text{eV}}{10.2\ \text{eV}} = 5.42\times10^{-9}$. This is quite small so recoil can be neglected.

EVALUATE: For emission of photons with ultraviolet or longer wavelengths the recoil kinetic energy of the atom is much less than the energy of the emitted photon.

39.49. **(a) IDENTIFY** and **SET UP:** Apply the equation for the reduced mass, $m_\text{r} = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{207 m_\text{e} m_\text{p}}{207 m_\text{e} + m_\text{p}}$, where m_e denotes the electron mass.

EXECUTE: $m_\text{r} = \dfrac{207(9.109\times10^{-31}\ \text{kg})(1.673\times10^{-27}\ \text{kg})}{207(9.109\times10^{-31}\ \text{kg}) + 1.673\times10^{-27}\ \text{kg}} = 1.69\times10^{-28}\ \text{kg}.$

(b) IDENTIFY: In the energy equation $E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{me^4}{8n^2h^2}$, replace $m = m_\text{e}$ by m_r: $E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{m_\text{r}e^4}{8n^2h^2}$.

SET UP: Write as $E_n = \left(\dfrac{m_\text{r}}{m_\text{H}}\right)\left(-\dfrac{1}{\epsilon_0^2}\dfrac{m_\text{H}e^4}{8n^2h^2}\right)$, since we know that $\dfrac{1}{\epsilon_0^2}\dfrac{m_\text{H}e^4}{8h^2} = 13.60\ \text{eV}$. Here m_H denotes the reduced mass for the hydrogen atom; $m_\text{H} = (0.99946)(9.109\times10^{-31}\ \text{kg}) = 9.104\times10^{-31}\ \text{kg}$.

EXECUTE: $E_n = \left(\dfrac{m_\text{r}}{m_\text{H}}\right)\left(-\dfrac{13.60\ \text{eV}}{n^2}\right)$.

$$E_1 = \frac{1.69\times10^{-28}\ \text{kg}}{9.104\times10^{-31}\ \text{kg}}(-13.60\ \text{eV}) = 186(-13.60\ \text{eV}) = -2.53\ \text{keV}.$$

(c) SET UP: From part (b), $E_n = \left(\frac{m_r}{m_H}\right)\left(-\frac{R_H ch}{n^2}\right)$, where $R_H = 1.097\times10^7\ \text{m}^{-1}$ is the Rydberg constant for the hydrogen atom. Use this result in $\frac{hc}{\lambda} = E_i - E_f$ to find an expression for $1/\lambda$. The initial level for the transition is the $n_i = 2$ level and the final level is the $n_f = 1$ level.

EXECUTE: $\frac{hc}{\lambda} = \frac{m_r}{m_H}\left[-\frac{R_H ch}{n_i^2} - \left(-\frac{R_H ch}{n_f^2}\right)\right]$.

$$\frac{1}{\lambda} = \frac{m_r}{m_H} R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right).$$

$$\frac{1}{\lambda} = \frac{1.69\times10^{-28}\ \text{kg}}{9.104\times10^{-31}\ \text{kg}}(1.097\times10^7\ \text{m}^{-1})\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 1.527\times10^9\ \text{m}^{-1}.$$

$\lambda = 0.655$ nm.

EVALUATE: From Example 39.6, the wavelength of the radiation emitted in this transition in hydrogen is 122 nm. The wavelength for muonium is $\frac{m_H}{m_r} = 5.39\times10^{-3}$ times this. The reduced mass for hydrogen is very close to the electron mass because the electron mass is much less then the proton mass: $m_p/m_e = 1836$. The muon mass is $207m_e = 1.886\times10^{-28}$ kg. The proton is only about 10 times more massive than the muon, so the reduced mass is somewhat smaller than the muon mass. The muon–proton atom has much more strongly bound energy levels and much shorter wavelengths in its spectrum than for hydrogen.

39.50. IDENTIFY: This problem involves the energy levels in the Bohr atom.

SET UP: $E_n = -(13.60\ \text{eV})/n^2$, $E = hc/\lambda$. We want the wavelength.

EXECUTE: The energy of the photon is the sum of the energy to ionize the atom plus the kinetic energy of the electron. So $hc/\lambda = (13.60\ \text{eV})/n^2 + K$. Using $n = 3$ and $K = 8.00$ eV and solving for the wavelength gives $\lambda = 130$ nm.

EVALUATE: This light is not visible to humans.

39.51. IDENTIFY: This problem involves the Pickering emission series and the Bohr atom.

SET UP: $\frac{1}{\lambda} = (1.097\times10^7\ \text{m}^{-1})\left[\frac{1}{4} - \frac{1}{(n/2)^2}\right]$, $n = 5, 6, \ldots$

EXECUTE: (a) We want the longest and shortest wavelengths of the Pickering series. For the longest wavelength (the least energy), the transition is between adjacent levels, so $n = 5$.

$$\frac{1}{\lambda} = (1.097\times10^7\ \text{m}^{-1})\left[\frac{1}{4} - \frac{1}{(5/2)^2}\right].\ \lambda = 1013\ \text{nm}.$$

For shortest wavelength, n approaches infinity.

$$\frac{1}{\lambda} = (1.097\times10^7\ \text{m}^{-1})\left[\frac{1}{4} - \frac{1}{(\infty/2)^2}\right].\ \lambda = 364.6\ \text{nm}.$$

(b) Shortest wavelength: The transition is between n_L and infinity. The Bohr energy is $\Delta E = E_\infty - E_{n_L}$.

$\frac{hc}{\lambda_{min}} = \frac{Z^2 E_1}{n^2}$. $E_1 = 13.60$ eV and $\lambda_{min} = 364.6$ nm which gives $Z^2/n^2 = 0.250$. Try values of n to find a Z that satisfies this equation.

$n = 1$: This cannot work because Z must be a whole number.

$n = 2$: This gives $Z = 1$, which is hydrogen which we know is *not* the atom.

n = 3: Z is not a whole number.
n = 4: This gives Z = 2 and ends on level 4, so n_L = 4, Z = 2 (helium).
EVALUATE: Find n for the $\lambda = 1013$ nm transition using the Bohr model for a Z = 2 atom.

$$\Delta E = \frac{hc}{\lambda} = Z^2(13.60\text{ eV})\left(\frac{1}{4^2} - \frac{1}{n^2}\right).$$

Using $\lambda = 1013$ nm and Z = 2 gives n = 5, so the transition is from the n = 5 state to the n = 4 state.

39.52. **IDENTIFY** and **SET UP:** The de Broglie wavelength is $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$. In the Bohr model, $mvr_n = n(h/2\pi)$, so $mv = nh/(2\pi r_n)$. Combine these two expressions and obtain an equation for λ in terms of n. Then $\lambda = h\left(\dfrac{2\pi r_n}{nh}\right) = \dfrac{2\pi r_n}{n}$.

EXECUTE: **(a)** For $n = 1, \lambda = 2\pi r_1$ with $r_1 = a_0 = 0.529 \times 10^{-10}$ m, so
$\lambda = 2\pi(0.529 \times 10^{-10}\text{ m}) = 3.32 \times 10^{-10}$ m.
$\lambda = 2\pi r_1$; the de Broglie wavelength equals the circumference of the orbit.
(b) For $n = 4, \lambda = 2\pi r_4/4$.

$$r_n = n^2 a_0 \text{ so } r_4 = 16a_0.$$
$$\lambda = 2\pi(16a_0)/4 = 4(2\pi a_0) = 4(3.32 \times 10^{-10}\text{ m}) = 1.33 \times 10^{-9}\text{ m}.$$

$\lambda = 2\pi r_4/4$; the de Broglie wavelength is $\dfrac{1}{n} = \dfrac{1}{4}$ times the circumference of the orbit.

EVALUATE: As n increases the momentum of the electron increases and its de Broglie wavelength decreases. For any n, the circumference of the orbits equals an integer number of de Broglie wavelengths.

39.53. **(a) IDENTIFY** and **SET UP:** The photon energy is given to the electron in the atom. Some of this energy overcomes the binding energy of the atom and what is left appears as kinetic energy of the free electron. Apply $hf = E_f - E_i$, the energy given to the electron in the atom when a photon is absorbed.

EXECUTE: The energy of one photon is $\dfrac{hc}{\lambda} = \dfrac{(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(2.998 \times 10^8\text{ m/s})}{85.5 \times 10^{-9}\text{ m}}$.

$$\frac{hc}{\lambda} = 2.323 \times 10^{-18}\text{ J}(1\text{ eV}/1.602 \times 10^{-19}\text{ J}) = 14.50\text{ eV}.$$

The final energy of the electron is $E_f = E_i + hf$. In the ground state of the hydrogen atom the energy of the electron is $E_i = -13.60$ eV. Thus $E_f = -13.60\text{ eV} + 14.50\text{ eV} = 0.90$ eV.

EVALUATE: **(b)** At thermal equilibrium a few atoms will be in the $n = 2$ excited levels, which have an energy of $-13.6\text{ eV}/4 = -3.40\text{ eV}, 10.2\text{ eV}$ greater than the energy of the ground state. If an electron with $E = -3.40$ eV gains 14.5 eV from the absorbed photon, it will end up with $14.5\text{ eV} - 3.4\text{ eV} = 11.1\text{ eV}$ of kinetic energy.

39.54. **IDENTIFY:** We are dealing with transitions between states having very close energies.
SET UP: E_n is the energy of state n relative to ground, so the photon energies due to the transitions are $E_1 = hc/\lambda_1$ and $E_2 = hc/\lambda_2$.
EXECUTE: (a)

$$\Delta E = |E_1 - E_2| = hc\left|\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right| = hc\left|\frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2}\right| \approx \frac{hc\Delta\lambda}{\lambda^2}$$

(b) Using the given numbers gives

$$\Delta E \approx \frac{hc(589.6\ \text{nm} - 589.0\ \text{nm})}{(589.6\ \text{nm})^2} = 0.002\ \text{eV}.$$

EVALUATE: Using 589.0 nm in the denominator in part (b) would give the same answer.

39.55. **IDENTIFY:** Assuming that Betelgeuse radiates like a perfect blackbody, Wien's displacement and the Stefan-Boltzmann law apply to its radiation.

SET UP: Wien's displacement law is $\lambda_{\text{peak}} = \dfrac{2.90\times 10^{-3}\ \text{m}\cdot\text{K}}{T}$, and the Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: **(a)** First use Wien's law to find the peak wavelength:

$$\lambda_{\text{m}} = (2.90\times 10^{-3}\ \text{m}\cdot\text{K})/(3000\ \text{K}) = 9.667\times 10^{-7}\ \text{m}.$$

Call N the number of photons/second radiated. $N \times (\text{energy per photon}) = IA = \sigma A T^4$.

$$N(hc/\lambda_{\text{m}}) = \sigma A T^4.\quad N = \frac{\lambda_{\text{m}} \sigma A T^4}{hc}.$$

$$N = \frac{(9.667\times 10^{-7}\ \text{m})(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(4\pi)(600\times 6.96\times 10^8\ \text{m})^2(3000\ \text{K})^4}{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})(3.00\times 10^8\ \text{m/s})}.$$

$$N = 5\times 10^{49}\ \text{photons/s}.$$

(b) $\dfrac{I_{\text{B}}A_{\text{B}}}{I_{\text{S}}A_{\text{S}}} = \dfrac{\sigma A_{\text{B}} T_{\text{B}}^4}{\sigma A_{\text{S}} T_{\text{S}}^4} = \dfrac{4\pi R_{\text{B}}^2 T_{\text{B}}^4}{4\pi R_{\text{S}}^2 T_{\text{S}}^4} = \left(\dfrac{600 R_{\text{S}}}{R_{\text{S}}}\right)^2 \left(\dfrac{3000\ \text{K}}{5800\ \text{K}}\right)^4 = 3\times 10^4.$

EVALUATE: Betelgeuse radiates 30,000 times as much energy per second as does our sun!

39.56. **IDENTIFY:** The diffraction grating allows us to determine the peak-intensity wavelength of the light. Then Wien's displacement law allows us to calculate the temperature of the blackbody, and the Stefan-Boltzmann law allows us to calculate the rate at which it radiates energy.

SET UP: The bright spots for a diffraction grating occur when $d\sin\theta = m\lambda$. Wien's displacement law is $\lambda_{\text{peak}} = \dfrac{2.90\times 10^{-3}\ \text{m}\cdot\text{K}}{T}$, and the Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$. The area of a sphere is $A = 4\pi r^2$.

EXECUTE: **(a)** First find the wavelength of the light:

$$\lambda = d\sin\theta = [1/(385{,}000\ \text{lines/m})]\sin(14.4^\circ) = 6.459\times 10^{-7}\ \text{m}.$$

Now use Wien's law to find the temperature: $T = (2.90\times 10^{-3}\ \text{m}\cdot\text{K})/(6.459\times 10^{-7}\ \text{m}) = 4490\ \text{K}$.

(b) The energy radiated by the blackbody is equal to the power times the time, giving $U = Pt = IAt = \sigma A T^4 t$, which gives

$t = U/(\sigma A T^4) = (12.0\times 10^6\ \text{J})/[(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(4\pi)(0.0750\ \text{m})^2(4490\ \text{K})^4] = 7.37\ \text{s}.$

EVALUATE: By ordinary standards, this blackbody is very hot, so it does not take long to radiate 12.0 MJ of energy.

39.57. **IDENTIFY:** We are applying quantum principles to our moon.

SET UP and **EXECUTE:** **(a)** We want the de Broglie wavelength of the moon. First find the speed. $v = 2\pi r/t$. Using the orbital radius r from Appendix F and $t = 27.3$ d = (27.3)(86,400 s), we have $v = 1.0229$ km/s. Now use $\lambda = h/p = h/mv$ with the mass m given in the problem and the speed we just found, giving $\lambda = 8.81 \times 10^{-60}$ m.

(b) We want the acceleration of the moon. Apply Newton's second law and universal gravitation to the moon. We also know that the angular momentum is $L = M_{\text{moon}}vR = mh/2\pi$, so $v = mh/2\pi M_{\text{moon}}R$. Using these relationships gives

$$\frac{GM_{\text{earth}}M_{\text{moon}}}{R^2} = \frac{M_{\text{moon}}v^2}{R}$$

$$\frac{GM_{\text{earth}}}{R} = v^2 = \left(\frac{mh}{2\pi M_{\text{moon}}R}\right)^2$$

$$R = m^2\left(\frac{h^2}{4\pi^2 M_{\text{moon}}^2 GM_{\text{earth}}}\right) = m^2\, a_{\text{moon}}$$

$$a_{\text{moon}} = \frac{h^2}{4\pi^2 M_{\text{moon}}^2 GM_{\text{earth}}}.$$

(c) We want a_{moon}. Using the numbers given in the problem for the masses, our result from part (b) gives $a_{\text{moon}} = 5.17 \times 10^{-129}\, m$ m/s^2.

(d) We want m. Using $R_{\text{m}} = m^2 a_{\text{moon}}$ and the result from part (c) gives $m = 2.73 \times 10^{68}$.

(e) We want E_0. From $E = -E_0/m$ we get $E_0 = -m^2E$. Using $E = K + U$ gives

$$E = \frac{1}{2}I\omega^2 - \frac{GM_{\text{moon}}M_{\text{earth}}}{R} = \frac{1}{2}M_{\text{moon}}R^2\left(\frac{2\pi}{T}\right)^2 - \frac{GM_{\text{moon}}M_{\text{earth}}}{R}.$$

Using $E_0 = -m^2E$ with the above result and $m = 2.73 \times 10^{68}$, we get $E_0 = -2.81 \times 10^{165}$ J.

EVALUATE: According to our result, the moon is not even close to its ground state.

39.58. **IDENTIFY:** Combine $I = \sigma T^4$, $P = IA$, and $\Delta E = Pt$.

SET UP: In the Stefan-Boltzmann law the temperature must be in kelvins. 400°C = 673 K.

EXECUTE: $t = \dfrac{\Delta E}{A\sigma T^4} = \dfrac{100\text{ J}}{(4.00\times10^{-6}\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(673\text{ K})^4} = 2.15\times10^3\text{ s}$

$= 35.8\text{ min} = 0.597\text{ h}.$

EVALUATE: The power is $P = 46.5$ mW. Since the area of the hole is small, the rate at which the cavity radiates energy through the hole is very small.

39.59. **IDENTIFY** and **SET UP:** Follow the procedures specified in the problem.

EXECUTE: **(a)** $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$ but $\lambda = \dfrac{c}{f} \Rightarrow I(f) = \dfrac{2\pi hc^2}{(c/f)^5(e^{hf/kT}-1)} = \dfrac{2\pi hf^5}{c^3(e^{hf/kT}-1)}.$

(b) $\displaystyle\int_0^\infty I(\lambda)\,d\lambda = \int_\infty^0 I(f)\,df\left(\frac{-c}{f^2}\right) = \int_0^\infty \frac{2\pi hf^3\,df}{c^2(e^{hf/kT}-1)} = \frac{2\pi(kT)^4}{c^2h^3}\int_0^\infty \frac{x^3}{e^x-1}\,dx = \frac{2\pi(kT)^4}{c^2h^3}\frac{1}{240}(2\pi)^4$

$= \dfrac{(2\pi)^5(kT)^4}{240h^3c^2} = \dfrac{2\pi^5k^4T^4}{15c^2h^3}.$

(c) The expression $\dfrac{2\pi^5k^4}{15h^3c^2} = \sigma$ as shown in Eq. (39.28). Plugging in the values for the constants we get $\sigma = 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4.$

EVALUATE: The Planck radiation law, $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$, predicts the Stefan-Boltzmann law, $I = \sigma T^4$.

39.60. **IDENTIFY:** $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$. From Chapter 36, if $\lambda \ll a$ then the width w of the central maximum is $w = 2\frac{R\lambda}{a}$, where $R = 2.5\text{ m}$ and a is the width of the slit.

SET UP: $v_x = \sqrt{\frac{2E}{m}}$, since the beam is traveling in the x-direction and $\Delta v_y \ll v_x$.

EXECUTE: **(a)** $\lambda = \frac{h}{\sqrt{2mE}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\text{ kg})(40\text{ eV})(1.60\times10^{-19}\text{ J/eV})}} = 1.94\times10^{-10}\text{ m}.$

(b) $\frac{R}{v} = \frac{R}{\sqrt{2E/m}} = \frac{(2.5\text{ m})(9.11\times10^{-31}\text{ kg})^{1/2}}{\sqrt{2(40\text{ eV})(1.6\times10^{-19}\text{ J/eV})}} = 6.67\times10^{-7}\text{ s}.$

(c) The width w is $w = 2R\frac{\lambda}{a}$' and $w = \Delta v_y t = \Delta p_y t/m$, where t is the time found in part (b) and a is the slit width. Combining the expressions for w, $\Delta p_y = \frac{2m\lambda R}{at} = 2.65\times10^{-28}\text{ kg}\cdot\text{m/s}.$

(d) $\Delta y = \frac{\hbar}{2\Delta p_y} = 0.20\ \mu\text{m}$, which is the same order of magnitude of the width of the slit.

EVALUATE: For these electrons $\lambda = 1.94\times10^{-10}\text{ m}$. This is much smaller than a and the approximate expression $w = \frac{2R\lambda}{a}$ is very accurate. Also, $v_x = \sqrt{\frac{2E}{m}} = 3.75\times10^6\text{ m/s}$. $\Delta v_y = \frac{\Delta p_y}{m} = 2.9\times10^2\text{ m/s}$, so it is the case that $v_x \gg \Delta v_y$.

39.61. **IDENTIFY:** This problem involves the de Broglie wavelength.
SET UP and **EXECUTE:** Use $\lambda = h/mv$. **(a)** Estimate: 0.5 mm/s.
(b) Using m = 85 kg gives $\lambda = h/mv = h/(85\text{ kg})(0.5\text{ mm/s}) = 1.6\times10^{-32}\text{ m}$.
(c) $\lambda = h/mv = h/(0.5\text{ mg})(0.5\text{ mm/s}) = 2.7\times10^{-24}\text{ m}$.
(d) $\lambda = h/mv = (1\text{ J·s})/(0.5\text{ mg})(1\text{ m/s}) = 2\times10^6\text{ m} = 2000\text{ km}$.
(e) $\lambda = h/mv = (1\text{ J·s})/(85\text{ kg})(2.5\text{ m/s}) = 4.7\text{ mm}$.
(f) We want the speed. Estimate: Doorway is 1.0 m wide. For single-slit diffraction, $\lambda = a\sin\theta = (1.0\text{ m})\sin 5° = 0.087\text{ m}$. $\lambda = h/mv = (1\text{ J·s})/[(0.145\text{ kg})(0.087\text{ m})] = 80\text{ m/s}$.
EVALUATE: From our results we can see that the wave nature of particles is not apparent for everyday-size things but only for particles of the scale of electrons, protons, atoms, and molecules.

39.62. **IDENTIFY:** The de Broglie wavelength of the electrons must be such that the first diffraction minimum occurs at $\theta = 20.0°$.

SET UP: The single-slit diffraction minima occur at angles θ given by $a\sin\theta = m\lambda$. $p = \frac{h}{\lambda}$.

EXECUTE: **(a)** $\lambda = a\sin\theta = (300\times10^{-9}\text{ m})(\sin 20°) = 1.0261\times10^{-7}\text{ m}$. $\lambda = h/mv \to v = h/m\lambda$.

$$v = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{ kg})(1.0261\times10^{-7}\text{ m})} = 7.09\times10^3\text{ m/s} = 7.09\text{ km/s}.$$

(b) No electrons strike the screen at the location of the second diffraction minimum. $a\sin\theta_2 = 2\lambda$.

$$\sin\theta_2 = \pm2\frac{\lambda}{a} = \pm2\left(\frac{1.0261\times10^{-7}\text{ m}}{3.00\times10^{-7}\text{ m}}\right) = \pm0.684.\quad \theta_2 = \pm43.2°.$$

EVALUATE: The intensity distribution in the diffraction pattern depends on the wavelength λ and is the same for light of wavelength λ as for electrons with de Broglie wavelength λ.

39.63. IDENTIFY: The electrons behave like waves and produce a double-slit interference pattern after passing through the slits.

SET UP: The first angle at which destructive interference occurs is given by $d\sin\theta = \lambda/2$. The de Broglie wavelength of each of the electrons is $\lambda = h/mv$.

EXECUTE: **(a)** First find the wavelength of the electrons. For the first dark fringe, we have $d\sin\theta = \lambda/2$, which gives (1.25 nm)(sin 18.0°) = $\lambda/2$, and λ = 0.7725 nm. Now solve the de Broglie wavelength equation for the speed of the electron:

$$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(0.7725\times10^{-9}\ \text{m})} = 9.42\times10^{5}\ \text{m/s}$$

which is about 0.3% the speed of light, so they are *nonrelativistic*.

(b) Energy conservation gives $eV = \frac{1}{2}mv^2$ and $V = mv^2/2e$

$= (9.11\times10^{-31}\ \text{kg})(9.42\times10^{5}\ \text{m/s})^2/[2(1.60\times10^{-19}\ \text{C})] = 2.52\ \text{V}.$

EVALUATE: The de Broglie wavelength of the electrons is comparable to the separation of the slits.

30.64. IDENTIFY: The de Broglie wavelength of the electrons must equal the wavelength of the light.

SET UP: The maxima in the two-slit interference pattern are located by $d\sin\theta = m\lambda$. For an electron, $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$.

EXECUTE: $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{(20.0\times10^{-6}\ \text{m})\sin(0.0300\ \text{rad})}{2} = 300\ \text{nm}.$ The velocity of an electron with this wavelength is given by $\lambda = h/p$. $v = \dfrac{p}{m} = \dfrac{h}{m\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(300\times10^{-9}\ \text{m})} = 2.43\times10^{3}\ \text{m/s}$ = 2.43 km/s.

Since this velocity is much smaller than c we can calculate the energy of the electron classically, so $K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\ \text{kg})(2.43\times10^{3}\ \text{m/s})^2 = 2.68\times10^{-24}\ \text{J} = 16.7\times10^{-6}\ \text{eV} = 16.7\ \mu\text{eV}$.

EVALUATE: The energy of the photons of this wavelength is $E = \dfrac{hc}{\lambda} = 4.14\ \text{eV}$. The photons and electrons have the same wavelength but a photon has around 250,000 times as much energy as an electron.

39.65. IDENTIFY: Both the electrons and photons behave like waves and exhibit single-slit diffraction after passing through their respective slits.

SET UP: The energy of the photon is $E = hc/\lambda$ and the de Broglie wavelength of the electron is $\lambda = h/mv = h/p$. Destructive interference for a single slit first occurs when $a\sin\theta = \lambda$.

EXECUTE: **(a)** For the photon: $\lambda = hc/E$ and $a\sin\theta = \lambda$. Since the a and θ are the same for the photons and electrons, they must both have the same wavelength. Equating these two expressions for λ gives $a\sin\theta = hc/E$. For the electron, $\lambda = h/p = \dfrac{h}{\sqrt{2mK}}$ and $a\ \Delta \to 0,$ Equating these two expressions for λ gives $a\sin\theta = \dfrac{h}{\sqrt{2mK}}$. Equating the two expressions for $a\sin\theta$ gives $hc/E = \dfrac{h}{\sqrt{2mK}}$, which gives $E = c\sqrt{2mK} = (4.05\times10^{-7}\ \text{J}^{1/2})\sqrt{K}$.

(b) $\dfrac{E}{K} = \dfrac{c\sqrt{2mK}}{K} = \sqrt{\dfrac{2mc^2}{K}}$. Since $v \ll c$, $mc^2 > K$, so the square root is > 1. Therefore $E/K > 1$, meaning that the photon has more energy than the electron.

EVALUATE: When a photon and a particle have the same wavelength, the photon has more energy than the particle.

39.66. IDENTIFY and SET UP: The de Broglie wavelength of the blood cell is $\lambda = \dfrac{h}{mv}$.

EXECUTE: $\lambda = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.00\times10^{-14}\ \text{kg})(4.00\times10^{-3}\ \text{m/s})} = 1.66\times10^{-17}\ \text{m}.$

EVALUATE: We need not be concerned about wave behavior.

39.67. IDENTIFY and SET UP: Follow the procedures specified in the problem.

EXECUTE: **(a)** $\lambda = \dfrac{h}{p} = \dfrac{h\left(1-\dfrac{v^2}{c^2}\right)^{1/2}}{mv} \Rightarrow \lambda^2 m^2 v^2 = h^2\left(1-\dfrac{v^2}{c^2}\right) = h^2 - \dfrac{h^2v^2}{c^2} \Rightarrow \lambda^2 m^2 v^2 + h^2\dfrac{v^2}{c^2} = h^2$

$\Rightarrow v^2 = \dfrac{h^2}{\left(\lambda^2 m^2 + \dfrac{h^2}{c^2}\right)} = \dfrac{c^2}{\left(\dfrac{\lambda^2 m^2 c^2}{h^2}+1\right)} \Rightarrow v = \dfrac{c}{\left(1+\left(\dfrac{mc\lambda}{h}\right)^2\right)^{1/2}}.$

(b) $v = \dfrac{c}{\left(1+\left(\dfrac{\lambda}{(h/mc)}\right)^2\right)^{1/2}} \approx c\left(1-\dfrac{1}{2}\left(\dfrac{mc\lambda}{h}\right)^2\right) = (1-\Delta)c.\ \ \Delta = \dfrac{m^2c^2\lambda^2}{2h^2}.$

(c) $\lambda = 1.00\times10^{-15}\,\text{m} \ll \dfrac{h}{mc}.\ \ \Delta = \dfrac{(9.11\times10^{-31}\ \text{kg})^2(3.00\times10^8\ \text{m/s})^2(1.00\times10^{-15}\ \text{m})^2}{2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2} = 8.50\times10^{-8}$

$\Rightarrow v = (1-\Delta)c = (1-8.50\times10^{-8})c.$

EVALUATE: As $\Delta \to 0,\ v \to c$ and $\lambda \to 0$.

39.68. IDENTIFY and SET UP: The minimum uncertainty product is $\Delta x \Delta p_x = \hbar/2$. $\Delta x = r_1$, where r_1 is the radius of the $n=1$ Bohr orbit. In the $n=1$ Bohr orbit, $mv_1r_1 = \dfrac{h}{2\pi}$ and $p_1 = mv_1 = \dfrac{h}{2\pi r_1}$.

EXECUTE: $\Delta p_x = \dfrac{\hbar}{2\Delta x} = \dfrac{\hbar}{2r_1} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(0.529\times10^{-10}\ \text{m})} = 1.0\times10^{-24}\ \text{kg}\cdot\text{m/s}.$ This is the same as the magnitude of the momentum of the electron in the $n=1$ Bohr orbit.

EVALUATE: Since the momentum is the same order of magnitude as the uncertainty in the momentum, the uncertainty principle plays a large role in the structure of atoms.

39.69. IDENTIFY and SET UP: Combining the two equations in the hint gives $pc = \sqrt{K(K+2mc^2)}$ and $\lambda = \dfrac{hc}{\sqrt{K(K+2mc^2)}}$.

EXECUTE: **(a)** With $K = 3mc^2$ this becomes $\lambda = \dfrac{hc}{\sqrt{3mc^2(3mc^2+2mc^2)}} = \dfrac{h}{\sqrt{15}mc}$.

(b) (i) $K = 3mc^2 = 3(9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})^2 = 2.456\times10^{-13}\ \text{J} = 1.53\ \text{MeV}$

$$\lambda = \frac{h}{\sqrt{15}mc} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{15}(9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})} = 6.26\times10^{-13}\ \text{m}.$$

(ii) K is proportional to m, so for a proton $K = (m_\text{p}/m_\text{e})(1.53\ \text{MeV}) = 1836(1.53\ \text{MeV}) = 2810\ \text{MeV}$.

λ is proportional to $1/m$, so for a proton

$\lambda = (m_\text{e}/m_\text{p})(6.26\times10^{-13}\ \text{m}) = (1/1836)(6.26\times10^{-13}\ \text{m}) = 3.41\times10^{-16}\ \text{m}.$

EVALUATE: The proton has a larger rest mass energy so its kinetic energy is larger when $K = 3mc^2$. The proton also has larger momentum so has a smaller λ.

39.70. **IDENTIFY:** Apply the Heisenberg uncertainty principle. Consider only one component of position and momentum.

SET UP: $\Delta x \Delta p_x \geq \hbar/2$. Take $\Delta x \approx 5.0\times10^{-15}$ m. $K = E - mc^2$. For a proton, $m = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\Delta p_x = \dfrac{\hbar}{2\Delta x} = \dfrac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{2(5.0\times10^{-15}\ \text{m})} = 1.1\times10^{-20}\ \text{kg}\cdot\text{m/s}.$

(b) $K = \sqrt{(pc)^2 + (mc^2)^2} - mc^2 = 3.3\times10^{-14}\ \text{J} = 0.21\ \text{MeV}.$

EVALUATE: **(c)** The result of part (b), about 2×10^5 eV, is many orders of magnitude larger than the potential energy of an electron in a hydrogen atom.

39.71. **(a) IDENTIFY** and **SET UP:** $\Delta x \Delta p_x \geq \hbar/2$. Estimate Δx as $\Delta x \approx 5.0\times10^{-15}$ m.

EXECUTE: Then the minimum allowed Δp_x is $\Delta p_x \approx \dfrac{\hbar}{2\Delta x} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(5.0\times10^{-15}\ \text{m})} = 1.1\times10^{-20}\ \text{kg}\cdot\text{m/s}.$

(b) IDENTIFY and **SET UP:** Assume $p \approx 1.1\times10^{-20}$ kg · m/s. Use $E^2 = (mc^2)^2 + (pc)^2$ to calculate E, and then $K = E - mc^2$.

EXECUTE: $E = \sqrt{(mc^2)^2 + (pc)^2}$. $mc^2 = (9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})^2 = 8.187\times10^{-14}\ \text{J}.$

$pc = (1.1\times10^{-20}\ \text{kg}\cdot\text{m/s})(2.998\times10^8\ \text{m/s}) = 3.165\times10^{-12}\ \text{J}.$

$E = \sqrt{(8.187\times10^{-14}\ \text{J})^2 + (3.165\times10^{-12}\ \text{J})^2} = 3.166\times10^{-12}\ \text{J}.$

$K = E - mc^2 = 3.166\times10^{-12}\ \text{J} - 8.187\times10^{-14}\ \text{J} = 3.084\times10^{-12}\ \text{J}\times(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 19\ \text{MeV}.$

(c) IDENTIFY and **SET UP:** The Coulomb potential energy for a pair of point charges is given by $U = -kq_1q_2/r$. The proton has charge $+e$ and the electron has charge $-e$.

EXECUTE: $U = -\dfrac{ke^2}{r} = -\dfrac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}{5.0\times10^{-15}\ \text{m}} = -4.6\times10^{-14}\ \text{J} = -0.29\ \text{MeV}.$

EVALUATE: The kinetic energy of the electron required by the uncertainty principle would be much larger than the magnitude of the negative Coulomb potential energy. The total energy of the electron would be large and positive and the electron could not be bound within the nucleus.

39.72. **IDENTIFY** and **SET UP:** $\Delta E \Delta t \geq \hbar/2$. Take the minimum uncertainty product, so $\Delta E = \dfrac{\hbar}{2\Delta t}$, with $\Delta t = 8.4\times10^{-17}$ s. $m = 264m_e$. $\Delta m = \dfrac{\Delta E}{c^2}$.

EXECUTE: $\Delta E = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(8.4\times10^{-17}\ \text{s})} = 6.28\times10^{-19}\ \text{J}$. $\Delta m = \dfrac{6.28\times10^{-19}\ \text{J}}{(3.00\times10^8\ \text{m/s})^2} = 7.0\times10^{-36}\ \text{kg}.$

$$\frac{\Delta m}{m} = \frac{7.0\times10^{-36}\ \text{kg}}{(264)(9.11\times10^{-31}\ \text{kg})} = 2.9\times10^{-8}.$$

EVALUATE: The fractional uncertainty in the mass is very small.

39.73. **IDENTIFY** and **SET UP:** Use $\lambda = h/p$ to relate your wavelength and speed.

EXECUTE: **(a)** $\lambda = \dfrac{h}{mv}$, so $v = \dfrac{h}{m\lambda} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(60.0\ \text{kg})(1.0\ \text{m})} = 1.1\times10^{-35}\ \text{m/s}.$

(b) $t = \dfrac{\text{distance}}{\text{speed}} = \dfrac{0.80\ \text{m}}{1.1\times10^{-35}\ \text{m/s}} = 7.3\times10^{34}\ \text{s}(1\ \text{y}/3.156\times10^7\ \text{s}) = 2.3\times10^{27}\ \text{y}.$

Since you walk through doorways much more quickly than this, you will not experience diffraction effects.

EVALUATE: A 1-kg object moving at 1 m/s has a de Broglie wavelength $\lambda = 6.6\times10^{-34}$ m, which is exceedingly small. An object like you has a very, very small λ at ordinary speeds and does not exhibit wavelike properties.

39.74. IDENTIFY: The transition energy E for the atom and the wavelength λ of the emitted photon are related by $E = \dfrac{hc}{\lambda}$. Apply the Heisenberg uncertainty principle in the form $\Delta E \Delta t \geq \dfrac{\hbar}{2}$.

SET UP: Assume the minimum possible value for the uncertainty product, so that $\Delta E \Delta t = \dfrac{\hbar}{2}$.

EXECUTE: **(a)** $E = 2.58\text{ eV} = 4.13\times10^{-19}$ J, with a wavelength of $\lambda = \dfrac{hc}{E} = 4.82\times10^{-7}\text{m} = 482$ nm.

(b) $\Delta E = \dfrac{\hbar}{2\Delta t} = \dfrac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{2(1.64\times10^{-7}\text{ s})} = 3.22\times10^{-28}\text{ J} = 2.01\times10^{-9}$ eV.

(c) $\lambda E = hc$, so $(\Delta\lambda)E + \lambda\Delta E = 0$, and $|\Delta E/E| = |\Delta\lambda/\lambda|$, so

$$\Delta\lambda = \lambda|\Delta E/E| = (4.82\times10^{-7}\text{ m})\left(\frac{3.22\times10^{-28}\text{ J}}{4.13\times10^{-19}\text{ J}}\right) = 3.75\times10^{-16}\text{ m} = 3.75\times10^{-7}\text{ nm}.$$

EVALUATE: The finite lifetime of the excited state gives rise to a small spread in the wavelength of the emitted light.

39.75. IDENTIFY: Assume both the x rays and electrons are at normal incidence and scatter from the surface plane of the crystal, so the maxima are located by $d\sin\theta = m\lambda$, where d is the separation between adjacent atoms in the surface plane.

SET UP: Let primed variables refer to the electrons. $\lambda' = \dfrac{h}{p'} = \dfrac{h}{\sqrt{2mE'}}$.

EXECUTE: $\sin\theta' = \dfrac{\lambda'}{\lambda}\sin\theta$, and $\lambda' = (h/p') = (h/\sqrt{2mE'})$, and so $\theta' = \arcsin\left(\dfrac{h}{\lambda\sqrt{2mE'}}\sin\theta\right)$.

$$\theta' = \arcsin\left(\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(\sin 35.8°)}{(3.00\times10^{-11}\text{ m})\sqrt{2(9.11\times10^{-31}\text{ kg})(4.50\times10^{+3}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}}\right) = 20.9°.$$

EVALUATE: The x rays and electrons have different wavelengths and the $m = 1$ maxima occur at different angles.

39.76. IDENTIFY: The photon is emitted as the atom returns to the lower energy state. The duration of the excited state limits the energy of that state due to the uncertainty principle.

SET UP: The wavelength λ of the photon is related to the transition energy E of the atom by $E = \dfrac{hc}{\lambda}$.

$\Delta E\,\Delta t \geq \hbar/2$. The minimum uncertainty in energy is $\Delta E \geq \dfrac{\hbar}{2\Delta t}$.

EXECUTE: **(a)** The photon energy equals the transition energy of the atom, 3.50 eV.

$$\lambda = \frac{hc}{E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{3.50\text{ eV}} = 355\text{ nm}.$$

(b) $\Delta E = \dfrac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2(2.0\times10^{-6}\text{ s})} = 2.6\times10^{-29}\text{ J} = 1.6\times10^{-10}$ eV.

EVALUATE: The uncertainty in the energy could be larger than that found in (b), but never smaller.

39.77. IDENTIFY: The wave (light or electron matter wave) having less energy will cause less damage to the virus.

SET UP: For a photon $E_{\text{ph}} = \dfrac{hc}{\lambda} = \dfrac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{\lambda}$. For an electron $E_{\text{e}} = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2}$.

EXECUTE: **(a)** $E = \dfrac{hc}{\lambda} = \dfrac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{5.00\times10^{-9}\ \text{m}} = 248\ \text{eV}.$

(b) $E_{\text{e}} = \dfrac{h^2}{2m\lambda^2} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})(5.00\times10^{-9}\ \text{m})^2} = 9.65\times10^{-21}\ \text{J} = 0.0603\ \text{eV}.$

EVALUATE: The electron has much less energy than a photon of the same wavelength and therefore would cause much less damage to the virus.

39.78. **IDENTIFY** and **SET UP:** Assume $px \approx h$ and use this to express E as a function of x. E is a minimum for that x that satisfies $\dfrac{dE}{dx} = 0.$

EXECUTE: **(a)** Using the given approximation, $E = \dfrac{1}{2}\left[(h/x)^2/m + kx^2\right]$, so $(dE/dx) = kx - (h^2/mx^3)$, and the minimum energy occurs when $kx = (h^2/mx^3)$, or $x^2 = \dfrac{h}{\sqrt{mk}}$. The minimum energy is then $h\sqrt{k/m}$.

EVALUATE: **(b)** $U = \frac{1}{2}kx^2 = \dfrac{h}{2}\sqrt{\dfrac{k}{m}}$. $K = \dfrac{p^2}{2m} = \dfrac{h^2}{2mx^2} = \dfrac{h}{2}\sqrt{\dfrac{k}{m}}$. At this x the kinetic and potential energies are the same.

39.79. **(a) IDENTIFY** and **SET UP:** $U = A|x|$. $F_x = -dU/dx$ relates force and potential. The slope of the function $A|x|$ is not continuous at $x = 0$, so we must consider the regions $x > 0$ and $x < 0$ separately.

EXECUTE: For $x > 0, |x| = x$, so $U = Ax$ and $F = -\dfrac{d(Ax)}{dx} = -A$. For $x < 0$, $|x| = -x$, so $U = -Ax$ and $F = -\dfrac{d(-Ax)}{dx} = +A$. We can write this result as $F = -A|x|/x$, valid for all x except for $x = 0$.

(b) IDENTIFY and **SET UP:** Use the uncertainty principle, expressed as $\Delta p \Delta x \approx h$, and as in Problem 39.78 estimate Δp by p and Δx by x. Use this to write the energy E of the particle as a function of x. Find the value of x that gives the minimum E and then find the minimum E.

EXECUTE: $E = K + U = \dfrac{p^2}{2m} + A|x|.$

$$px \approx h, \text{so } p \approx h/x.$$

Then $E \approx \dfrac{h^2}{2mx^2} + A|x|.$

For $x > 0, E = \dfrac{h^2}{2mx^2} + Ax.$

To find the value of x that gives minimum E set $\dfrac{dE}{dx} = 0.$

$$0 = \frac{-2h^2}{2mx^3} + A.$$

$$x^3 = \frac{h^2}{mA} \text{ and } x = \left(\frac{h^2}{mA}\right)^{\frac{1}{3}}.$$

With this x the minimum E is

$$E = \frac{h^2}{2m}\left(\frac{mA}{h^2}\right)^{2/3} + A\left(\frac{h^2}{mA}\right)^{1/3} = \frac{1}{2}h^{2/3}m^{-1/3}A^{2/3} + h^{2/3}m^{-1/3}A^{2/3}.$$

$$E = \frac{3}{2}\left(\frac{h^2A^2}{m}\right)^{1/3}.$$

EVALUATE: The potential well is shaped like a V. The larger A is, the steeper the slope of U and the smaller the region to which the particle is confined and the greater is its energy. Note that for the x that minimizes E, $2K = U$.

39.80. **(a) IDENTIFY and SET UP:** Let the y-direction be from the thrower to the catcher, and let the x-direction be horizontal and perpendicular to the y-direction. A cube with volume $V = 125\text{ cm}^3 = 0.125\times10^{-3}\text{ m}^3$ has side length $l = V^{1/3} = (0.125\times10^{-3}\text{ m}^3)^{1/3} = 0.050\text{ m}$. Thus estimate Δx as $\Delta x \approx 0.050\text{ m}$. Use the uncertainty principle to estimate Δp_x.

EXECUTE: $\Delta x\Delta p_x \ge \hbar/2$ then gives $\Delta p_x \approx \dfrac{\hbar}{2\Delta x} = \dfrac{0.01055\text{ J}\cdot\text{s}}{2(0.050\text{ m})} = 0.11\text{ kg}\cdot\text{m/s}$. (The value of $\hbar$ in this other universe has been used.)

(b) IDENTIFY and SET UP: $\Delta x = (\Delta v_x)t$ is the uncertainty in the x-coordinate of the ball when it reaches the catcher, where t is the time it takes the ball to reach the second student.
Obtain Δv_x from Δp_x.

EXECUTE: The uncertainty in the ball's horizontal velocity is $\Delta v_x = \dfrac{\Delta p_x}{m} = \dfrac{0.11\text{ kg}\cdot\text{m/s}}{0.25\text{ kg}} = 0.42\text{ m/s}$.

The time it takes the ball to travel to the second student is $t = \dfrac{12\text{ m}}{6.0\text{ m/s}} = 2.0\text{ s}$. The uncertainty in the x-coordinate of the ball when it reaches the second student that is introduced by Δv_x is $\Delta x = (\Delta v_x)t = (0.42\text{ m/s})(2.0\text{ s}) = 0.84\text{ m}$. The ball could miss the second student by about 0.84 m.

EVALUATE: A game of catch would be very different in this universe. We don't notice the effects of the uncertainty principle in everyday life because h is so small.

39.81. **IDENTIFY and SET UP:** For hydrogen-like atoms (1 electron and Z protons), the energy levels are $E_n = (-13.60\text{ eV})Z^2/n^2$, with $n = 1$ for the ground state. The energy of a photon is $E = hc/\lambda$.

EXECUTE: **(a)** The least energy absorbed is between the ground state ($n = 1$) and the $n = 2$ state, which gives the longest wavelength. So $\Delta E_{1\to2} = hc/\lambda$. Using the energy levels for this atom, we have

$(-13.6\text{ eV})Z^2\left(\dfrac{1}{2^2} - \dfrac{1}{1^2}\right) = \dfrac{hc}{\lambda} \;\rightarrow\; (10.20\text{ eV})Z^2 = hc/\lambda$. Solving Z gives

$$Z = \sqrt{\frac{hc}{(10.20\text{ eV})\lambda}} = \sqrt{\frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^8\text{ m/s})}{(10.20\text{ eV})(13.56\times10^{-9}\text{ m})}} = 3.0.$$

(b) The next shortest wavelength is between the $n = 3$ and $n = 1$ states.

$$\Delta E_{1\to3} = (-13.6\text{ eV})(3)^2\left(\frac{1}{3^2} - \frac{1}{1^2}\right) = \frac{hc}{\lambda}.$$

Solving for λ gives
$\lambda = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^8\text{ m/s})/(108.8\text{ eV}) = 11.40\text{ nm}$.

(c) By energy conservation, $E_{\text{photon}} = E_{\text{ionization}} + K_{\text{el}}$. The ionization energy is the minimum energy to completely remove an electron from the atom, which is from the $n = 1$ state to the $n = \infty$ state. Therefore $E_{\text{ionization}} = (13.60\text{ eV})Z^2 = (13.60\text{ eV})(9)$. Therefore the kinetic energy of the electron is

$$K_{\text{el}} = E_{\text{photon}} - E_{\text{ionization}} = hc/\lambda - E_{\text{ionization}}.$$

$$K_{\text{el}} = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^8\text{ m/s})/(6.78\times10^{-9}\text{ m}) - (13.60\text{ eV})(9) = 60.5\text{ eV}.$$

EVALUATE: The energy levels for a $Z = 3$ atom are 9 times as great as for the comparable energy levels in hydrogen, so the wavelengths of the absorbed light are much shorter than they would be for comparable transitions in hydrogen.

39.82. IDENTIFY and SET UP: The kinetic energy of the electron is $K = eV_{ac}$. The first-order maximum in the Davisson-Germer experiment occurs when $d\sin\theta = \lambda$. The de Broglie wavelength of an electron is $\lambda = h/p$, and its kinetic energy is $K = p^2/2m$. Therefore its momentum is $p = \sqrt{2mK}$, which means its de Broglie wavelength can be expressed as $\lambda = h/\sqrt{2mK}$.

EXECUTE: **(a)** Figure 39.82 shows the graph of $\sin\theta$ versus $1/\sqrt{V_{ac}}$ for the data in the problem. The slope of the best-fit graph is 3.522 $V^{1/2}$.

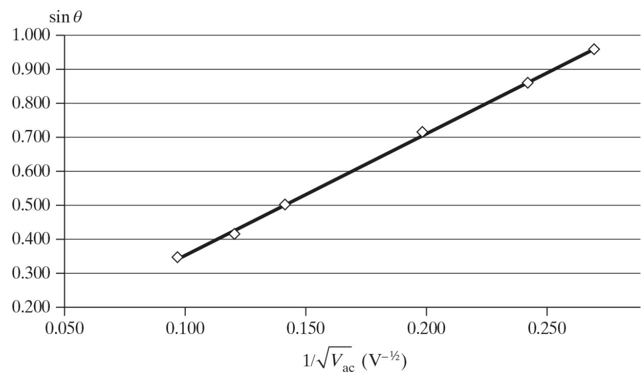


Figure 39.82

(b) At the first maximum, we have $d\sin\theta = \lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}} = \dfrac{h}{\sqrt{2meV_{ac}}}$, which we can write as $\sin\theta = \dfrac{h}{d\sqrt{2me}} \cdot \dfrac{1}{\sqrt{V_{ac}}}$. From this result, we see that a graph of $\sin\theta$ versus $1/\sqrt{V_{ac}}$ should be a straight line having slope equal to $\dfrac{h}{d\sqrt{2me}}$. Solving for d gives $d = \dfrac{h}{(\text{slope})\sqrt{2me}}$, which gives

$$d = \frac{4.136\times10^{-15}\ \text{J}\cdot\text{s}}{(3.522\ \text{V}^{1/2})\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{C})}} = 3.48\times10^{-10}\ \text{m} = 0.348\ \text{nm}.$$

EVALUATE: Atom spacing in crystals are typically around a few tenths of a nanometer, so these results are plausible.

39.83. IDENTIFY and SET UP: The power radiated by an ideal blackbody is $P = \sigma AT^4$. Wien's displacement law, $\lambda_m T = 2.90\times10^{-3}\ \text{m}\cdot\text{K}$, applies to the stars. The surface area of a star is $A = 4\pi R^2$, and $R_{sun} = 6.96\times10^8$ m.

EXECUTE: **(a)** Calculate the radiated power for each star using $P = \sigma AT^4$. For Polaris we have $P = \sigma AT^4 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(4\pi)[(46)(6.96\times10^8\ \text{m})]^2(6015\ \text{K})^4 = 9.56\times10^{29}$ W.

Repeating this calculation for the other stars gives us the following results.

Polaris: $P = 9.56\times10^{29}$ W
Vega: $P = 2.19\times10^{28}$ W
Antares: $P = 3.60\times10^{31}$ W
α Centauri B: $P = 1.98\times10^{26}$ W

Antares has the greatest radiated power.

(b) Apply Wien's displacement law, $\lambda_{\mathrm{m}}T = 2.90\times10^{-3}\ \mathrm{m}\cdot\mathrm{K},$ and solve for $\lambda_{\mathrm{m}}.$ For example, for Polaris we have $\lambda_{\mathrm{m}} = \dfrac{2.90\times10^{-3}\ \mathrm{m}\cdot\mathrm{K}}{T} = \dfrac{2.90\times10^{-3}\ \mathrm{m}\cdot\mathrm{K}}{6015\ \mathrm{K}} = 4.82\times10^{-7}\ \mathrm{m} = 482\ \mathrm{nm}.$ Repeating this calculation for the other stars gives the following results.

<u>Polaris</u>: λ_{m} = 482 nm
<u>Vega</u>: λ_{m} = 302 nm
<u>Antares</u>: λ_{m} = 853 nm
<u>α Centauri B</u>: λ_{m} = 551 nm

The visible range is 380 nm to 750 nm, so Polaris and α Centauri B radiate chiefly in the visible range.
(c) By comparing the results in part (a), we see that only α Centauri B radiates less than our sun.
EVALUATE: The power radiated by a star depends on its surface area *and* its surface temperature. Vega, a very hot star, radiates less than the much cooler Antares because Antares has over 300 times the radius of Vega and therefore over 300^2 times the surface area of Vega. A hot star is not necessarily brighter than a cool star.

39.84. **IDENTIFY:** This problem involves blackbody radiation and the Planck radiation law.
SET UP: We have r = 1.23 m and use Equation 39.24. The device captures all photons within 1% of the central peak E_0. For such a small range, we can treat the emittance as constant over

$$E_0 \pm \Delta E, \Delta E = 0.0100E_0.\ \ E = hc/\lambda,\ \ I(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)},\ I = \int I(\lambda)\, d\,\lambda.$$

EXECUTE: **(a)** Restructure $\int I(\lambda)d\,\lambda$ as $\int I(E)\,dE.$ Using $E = hc/\lambda$ gives $d\lambda = -\lambda^2\,dE/hc.$

$$I = \int I(E)\,dE = \int \frac{2\pi hc^2\left(-\lambda^2\ dE/hc\right)}{\lambda^5\left(e^{E/kT}-1\right)} = -\int \frac{2\pi E^3}{h^3c^2\left(e^{e/kT}-1\right)}dE.$$

Therefore it follows that

$$I(E) = \frac{2\pi E^3}{h^3\,c^2\left(e^{E/kT}-1\right)}.$$

(b) We want to find the E_0 that maximizes $I(E)$ and then use it to find $I_{\mathrm{max}} = I(E_0)$. Set $dI(E)/dt = 0$ for a maximum, using the result from part (a). This gives $\dfrac{\left(e^{E/kT}-1\right)\left(3\,E^2\right) - E^3\left(1/kT\right)e^{E/kT}}{\left(e^{E/kT}-1\right)^2} = 0,$ which simplifies to $3 - 3\mathrm{e}^{-E/kT} - E/kT = 0.$

Letting $x = E/kT$, the final equation we must solve is $3 - x = 3e^{-x}$. Using trial-and-error (which gives an answer rather quickly) or appropriate software, the result is x = 2.821, so E/kT = 2.821, which gives $E_0 = 2.821kT$. Now use this result to find $I_{\mathrm{max}} = I(E_0)$, giving

$$I_{\mathrm{max}} = \frac{2\pi(2.821kT)^3}{h^3c^3(2.821-1)} = \frac{2.843\pi k^3T^3}{h^3c^2}.$$

(c) We want the power and current at T = 1000 K. At T = 1000 K, $E_0 = 2.821kT = 3.896\times10^{-20}$ J. The photocells capture all photons within 1% of E_0, so the full range of energy is $E_0 \pm 1\%E_0$, which is 2% E_0. So ΔE = (0.020)(3.896×10^{-20} J) = 7.792×10^{-22} J. $P = IA = I(E)(\Delta E)A$. Using given numbers and the values we have found gives

$$p = \left(\frac{2.843\pi k^3T^3}{h^3c^3}\right)(\Delta E)\left(4\pi r^2\right) = 13.3\ \mathrm{kW}.$$

Solving $P = I^2R$ for I and using this result gives I = 3.64 A.

(d) We want the power and current if $T = 5000$ K. Follow the same procedure as in part (c), giving $E_0 = 2.821kT = (2.821)k(5000\text{ K}) = 1.946 \times 10^{-19}$ J, $\Delta E = 0.020E_0 = 3.893 \times 10^{-21}$ J, $P = 8.29$ MW, and $I = 91.1$ A.

EVALUATE: In (c) and (d) we see that increasing the temperature by a factor of 5 increased the power by a factor of $5^4 = 625$ because the power is proportional to T^4.

39.85. **IDENTIFY:** We are looking at the energy and momentum in particle collision at nonrelativistic speeds. An alpha particle collides with a gold nucleus at rest.

SET UP: Energy and momentum are conserved during the collision. Use $K = p^2/2m$ to express p in terms of K.

EXECUTE: **(a)** We want V (the gold) speed. Energy conservation: $K = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$. Momentum conservation: $\sqrt{2mK} = MV - mv$ Solving for V gives $V = \sqrt{8mK}/(m+M)$.

(b) We want $\Delta K(\alpha)/K$. The kinetic energy lost by the alpha particle is equal to the kinetic energy gained by the gold nucleus. Using this fact gives

$$\frac{\Delta K(\alpha)}{K(\alpha)} = \frac{\frac{1}{2}MV^2}{K} = \frac{\frac{1}{2}\left(\frac{\sqrt{8mK}}{m+M}\right)^2}{K} = \frac{4mM}{(m+M)^2}.$$

(c) From part (b) we know that the fractional energy lost depends only on the masses, so the result is *independent* of the initial kinetic energy. Therefore the answer is *yes*.

(d) We want V and the fractional energy lost to the gold if $K = 5.00$ MeV. Using the given numbers for m, M, and K in $V = \sqrt{8mK}/(m+M)$ from part (a) and converting 5.00 MeV to joules gives $V = 1.49 \times 10^6$ m/s $= 0.00497c$. From part (b), the fraction lost is $4mM/(m+M)^2 = 0.182 = 18.2\%$.

(e) We want v, so that $V = 0.10c$ by using nonrelativistic physics. First find K using $V = \sqrt{8mK}/(m+M)$, giving $K = [V(m+M)]^2/8m$. Classically, $K = \frac{1}{2}mv^2$. Equating this to the previous result and solving for v gives $v = V(1 + M/m)/2$. Using $V = 0.10c$ and the given masses gives $v = 1.04c$.

EVALUATE: **(f)** This speed is not possible, so we would need to use the relativistic equations.

39.86. **IDENTIFY:** Follow the steps specified in the hint.

SET UP: The value of Δx_i that minimizes Δx_f satisfies $\dfrac{d(\Delta x_f)}{d(\Delta x_i)} = 0.$

EXECUTE: Time of flight of the marble, from a free-fall kinematic equation is just $t = \sqrt{\dfrac{2y}{g}} = \sqrt{\dfrac{2(25.0\text{ m})}{9.80\text{ m/s}^2}} = 2.26$ s. $\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\dfrac{\Delta p_x}{m}\right)t = \dfrac{\hbar t}{2\Delta x_i m} + \Delta x_i$. To minimize Δx_f with respect to Δx_i, $\dfrac{d(\Delta x_f)}{d(\Delta x_i)} = 0 = \dfrac{-\hbar t}{2m(\Delta x_i)^2} + 1 \Rightarrow \Delta x_i(\text{min}) = \sqrt{\left(\dfrac{\hbar t}{2m}\right)}$

$$\Rightarrow \Delta x_f(\text{min}) = \sqrt{\frac{\hbar t}{2m}} + \sqrt{\frac{\hbar t}{2m}} = \sqrt{\frac{2\hbar t}{m}} = \sqrt{\frac{2(1.055\times 10^{-34}\text{ J}\cdot\text{s})(2.26\text{ s})}{0.0200\text{ kg}}} = 1.54\times 10^{-16}\text{ m}$$

$$= 1.54\times 10^{-7}\text{ nm}.$$

EVALUATE: The uncertainty introduced by the uncertainty principle is completely negligible in this situation.

39.87. **IDENTIFY** and **SET UP:** The period was found in Exercise 39.23b: $T = \dfrac{4\epsilon_0^2 n^3 h^3}{me^4}$. The equation

$E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{me^4}{8n^2h^2}$ gives the energy of state n of a hydrogen atom.

EXECUTE: **(a)** The frequency is $f = \dfrac{1}{T} = \dfrac{me^4}{4\epsilon_0^2 n^3 h^3}$.

(b) The equation $hf = E_\text{i} - E_\text{f}$ tells us that $f = \dfrac{1}{h}(E_2 - E_1)$. So $f = \dfrac{me^4}{8\epsilon_0^2 h^3}\left(\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right)$. If $n_2 = n$ and $n_1 = n+1$, then $\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2} = \dfrac{1}{n^2} - \dfrac{1}{(n+1)^2} = \dfrac{1}{n^2}\left(1 - \dfrac{1}{(1+1/n)^2}\right) \approx \dfrac{1}{n^2}\left(1 - \left(1 - \dfrac{2}{n} + \dots\right)\right) = \dfrac{2}{n^3}$. Therefore, for large n, $f \approx \dfrac{me^4}{4\epsilon_0^2 n^3 h^3}$.

EVALUATE: We have shown that for large n we obtain the classical result that the frequency of revolution of the electron is equal to the frequency of the radiation it emits.

39.88. **IDENTIFY** and **SET UP:** The de Broglie wavelength of the helium is $\lambda = h/p$. Its kinetic energy is $K = p^2/2m$, so $p = \sqrt{2mK}$. Therefore its de Broglie wavelength can be expressed as $\lambda = h/\sqrt{2mK}$. The kinetic energy of the ions acquired during acceleration is $K = eV = p^2/2m$.

EXECUTE: Express the wavelength in terms of V, giving $\lambda = h/\sqrt{2mK} = h/\sqrt{2meV}$. From this we see that a large mass m results in a small (short) wavelength, which is choice (b).

EVALUATE: Because helium is 7300 times heavier than an electron and because $\lambda \propto 1/\sqrt{m}$, the wavelength for helium would be $1/\sqrt{7300} = 0.012$ times the wavelength of an electron.

39.89. **IDENTIFY:** Calculate the accelerating potential V need to produce a helium ion with a wavelength of 0.1 pm to see if that potential lies within the range of 10-50 kV.

SET UP: The de Broglie wavelength of the helium ion is $\lambda = h/p$, so $p = h/\lambda$. By energy conservation, $K = eV = p^2/2m$.

EXECUTE: Combining the above equations gives

$$eV = K = p^2/2m = \frac{(h/\lambda)^2}{2m}, \text{ so } V = \frac{(h/\lambda)^2}{2me}.$$

$$V = \frac{\left[(6.626\times10^{-34}\text{ J}\cdot\text{s})/(0.1\times10^{-12}\text{ m})\right]^2}{2(7300)(9.11\times10^{-31}\text{ kg})(1.60\times10^{-19}\text{ C})} = 2.1\times10^4\text{ V} = 21\text{ kV}.$$

This voltage is within the 10-50 kV range, so choice (a) is correct.

EVALUATE: A large voltage is required because the desired wavelength is small.

39.90. **IDENTIFY** and **SET UP:** Electric and magnetic fields act on electrical charges.

EXECUTE: Focusing particles requires electric and magnetic forces, so they must have charge, which makes choice (c) correct.

EVALUATE: All particles have wave properties, so choice (a) is not correct. Helium is an inert gas, so it normally does form molecules, so that rules out choice (b). The mass difference between a helium atom and a helium ion is negligible because the electron is 7300 times lighter than a helium ion, which eliminates choice (d).

39.91. **IDENTIFY** and **SET UP:** The ion loses 0.2 MeV/μm, and its energy can be determined only to within 6 keV. Call x the minimum difference in thickness that can be discerned, and realize that 0.2 MeV = 200 keV.

EXECUTE: (0.2 MeV/μm)x = 6 keV. Solving for x gives x = (6 keV)/(200 keV/μm) = 0.03 μm, which makes choice (a) the correct one.

EVALUATE: Greater precision in determining the energy of the ion would allow one to discern smaller features.

40

QUANTUM MECHANICS I: WAVE FUNCTIONS

VP40.4.1. **IDENTIFY:** This problem involves an electron in a one-dimensional box.

SET UP: $E_n = n^2 \dfrac{\pi^2 \hbar^2}{8mL^2}.$

EXECUTE: **(a)** We want the width L of the box. Solve the energy equation for L with $n = 1$ and use the given energy for E_1. $L = \dfrac{h}{\sqrt{8mE_1}} = 0.549$ nm.

(b) We want the energy. Use $E_n = n^2 E_1$ with the given energy for E_1. $E_2 = 2^2 E_1 = 4(2.00 \times 10^{-19}\text{ J})$ $= 8.00 \times 10^{-19}$ J. $E_3 = 3^2(2.00 \times 10^{-19}\text{ J}) = 1.80 \times 10^{-19}$ J.

EVALUATE: Note that the energy levels are not evenly spaced.

VP40.4.2. **IDENTIFY:** This problem involves an electron in a one-dimensional box.

SET UP: $E_n = n^2 \dfrac{\pi^2 \hbar^2}{8mL^2}$. We want the energy difference between levels.

EXECUTE: **(a)** Using the energy equation, the energy difference between levels is

$$\Delta E_{2,1} = \frac{h^2}{8mL^2}\left(2^2 - 1^2\right) = \frac{3h^2}{8mL^2} = \frac{3h^2}{8m\left(5.00 \times 10^{-15}\text{ m}\right)^2} = 3.94 \times 10^{-12}\text{ J}.$$

(b) Using the same equation as in part (a) gives

$$\Delta E_{2,1} = \frac{h^2}{8mL^2}\left(3^2 - 2^2\right) = \frac{5h^2}{8mL^2} = 6.57 \times 10^{-12}\text{ J}.$$

EVALUATE: Note that the energy difference between adjacent levels increases as the levels increase to higher values of n.

VP40.4.3. **IDENTIFY:** This problem involves transitions by an electron in a one-dimensional box.

SET UP: $E_n = n^2 \dfrac{\pi^2 \hbar^2}{8mL^2}$, $E = hc/\lambda$.

EXECUTE: **(a)** We want the energy of the photon. $E = hc/\lambda = hc/(655\text{ nm}) = 3.03 \times 10^{-19}$ J.

(b) We want the length L of the box. The energy of the photon is equal to the energy difference between the $n = 1$ and $n = 2$ levels.

$$E_{\text{ph}} = \frac{h^2}{8mL^2}\left(2^2 - 1^2\right) = \frac{3h^2}{8mL^2}.$$

Solve for L using the photon energy from part (a), giving $L = 0.772$ nm.

(c) We want the wavelength of the photon. The energy of the photon is the energy difference between the $n = 2$ and $n = 3$ levels. Use $E = hc/\lambda$ and $L = 0.772$ nm and solve for λ.

$$E_{\text{ph}} = \frac{h^2}{8mL^2}\left(3^2 - 2^2\right) = \frac{5h^2}{8mL^2} = \frac{hc}{\lambda}. \quad \lambda = \frac{8mL^2c}{5h} = 393 \text{ nm}.$$

EVALUATE: Note that the wavelength for the $3 \to 2$ transition is less than the wavelength for the $2 \to 1$ transition. This result is reasonable because the energy difference is greater for the $3 \to 2$ transition than it is for the $2 \to 1$ transition.

VP40.4.4. **IDENTIFY:** This problem involves the wave function for a particle in a box.

SET UP: The wave function and Schrödinger equation are

$$\psi(x) = A\cos\left(\frac{x\sqrt{2mE}}{\hbar} + \phi\right), -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx} = E\psi.$$

EXECUTE: **(a)** We want to show that the given wave function satisfied the Schrödinger equation. First take the second derivative of the wave function and then multiply it by $-\hbar^2/2m$.

$$\frac{d^2\psi}{dx^2} = -\frac{A(2mE)}{\hbar^2}\cos\left(\frac{x\sqrt{2mE}}{\hbar} + \phi\right) - \frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -\frac{\hbar^2}{2m}\left(-\frac{A(2mE)}{\hbar^2}\right)\cos\left(\frac{x\sqrt{2mE}}{\hbar} + \phi\right)$$

$$= EA\cos\left(\frac{x\sqrt{2mE}}{\hbar} + \phi\right) = E\psi.$$

(b) We want ϕ. The wave function must be zero at $x = 0$, which gives $\cos\phi = 0$, so $\phi = \pm\pi/2$.

(c) We want E. At $x = L$ the wave function must be zero, which gives

$$A\cos\left(\frac{L\sqrt{2mE}}{\hbar} \pm \frac{\pi}{2}\right) = 0. \quad \frac{L\sqrt{2mE}}{\hbar} \pm \frac{\pi}{2} = \frac{\pi}{2}.$$

L cannot be zero, so we must have

$$\frac{L\sqrt{2mE_n}}{\hbar} = n\pi. \quad E_n = n^2\frac{\pi^2\hbar^2}{2mL^2}, n = 1, 2, 3, \cdots$$

EVALUATE: The result in part (c) agrees with Eq. (40.31) in the text.

VP40.6.1. **IDENTIFY:** We are dealing with an electron in a finite potential well.

SET UP: For an infinitely deep well the energy levels are

$$E_{n\text{-IDW}} = n^2\frac{\pi^2\hbar^2}{2mL^2}$$

and for a finite well we use Figure 40.15 in the textbook.

EXECUTE: **(a)** We want the ground level energy, so $n = 1$.

$$E_{\text{1-IDW}} = \frac{\pi^2\hbar^2}{2mL^2} = \frac{\pi^2\hbar^2}{2m(0.350 \text{ nm})^2} = 4.92\times10^{-19} \text{ J}.$$

(b) We want the ground level energy. In this case, $U_0 = 6E_{\text{1-IDW}}$. From Figure 40.15b in the text, we see that $E_1 = 0.625E_{\text{1-IDW}}$, which gives

$E_1 = (0.625)(4.92\times10^{-19} \text{ J}) = 3.07\times10^{-19}$ J.

(c) We want the minimum energy to free the electron. The electron already has 3.07×10^{-19} J of energy, and to be free it needs to have a minimum of $U_0 = 6E_{\text{1-IDW}} = 6(4.92\times10^{-19} \text{ J}) = 2.95\times10^{-18}$ J.

So, the additional energy it needs is $U_0 - E_1 = 2.64\times10^{-18}$ J.

EVALUATE: If the well were infinite, the electron would need infinite energy to escape, meaning that it could not escape.

VP40.6.2. **IDENTIFY:** We are dealing with particle in a finite potential well.

SET UP: $U_0 = 6E_{\text{1-IDW}}$, the energy difference between the $n = 2$ and $n = 3$ levels is the energy of the photon. Refer to Figure 40.15 in the textbook.

EXECUTE: **(a)** We want $E_{\text{1-IDW}}$. Figure 40.15b, shows that $E_3 = 5.09E_{\text{1-IDW}}$ and $E_2 = 2.43E_{\text{1-IDW}}$. The energy difference between these levels is $E_3 - E_2 = (5.09 - 2.43)E_{\text{1-IDW}} = 2.50 \times 10^{-19}$ J, which gives $E_{\text{1-IDW}} = 9.40 \times 10^{-20}$ J.

(b) We want U_0. $U_0 = 6E_{\text{1-IDW}} = 6(9.40 \times 10^{-20}\text{ J}) = 5.64 \times 10^{-19}$ J.

(c) We want the width L of the well. Solve $E_{\text{1-IDW}} = n^2h^2/8mL^2$ for L and use the result from part (b) with $n = 1$. This gives $L = 0.800$ nm.

EVALUATE: The ground state energy of this well is $(0.625)E_{\text{1-IDW}} = 5.88 \times 10^{-20}$ J.

VP40.6.3. **IDENTIFY:** We are dealing with an electron in a finite potential well.

SET UP: $U_0 = 6E_{\text{1-IDW}}$, $E = hc/\lambda$. Use Figure 40.15 in the textbook.

EXECUTE: **(a)** We want the initial and final energy levels. In this case, there are only three possible transitions: $3 \to 2$, $2 \to 1$, and $3 \to 1$. The greatest energy difference is from the $3 \to 1$ transition which emits a photon of the shortest wavelength. So the initial state is $n = 3$ and the final state is $n = 1$.

(b) Figure 40.15b in the textbook gives $E_3 = 5.09E_{\text{1-IDW}}$ and $E_1 = 0.625E_{\text{1-IDW}}$. Therefore the energy of the photon is $E_3 - E_1 = (5.09 - 0625)E_{\text{1-IDW}} = 4.465E_{\text{1-IDW}}$. Using $E = hc/\lambda$ with the 355 nm wavelength and solving for $E_{\text{1-IDW}}$ gives $E_{\text{1-IDW}} = hc/[(4.465)(355\text{ nm})]$.

<u>$3 \to 2$ transition</u>: The energy difference is $(5.09 - 2.43)E_{\text{1-IDW}}$, and this is the photon energy.

$$\frac{hc}{\lambda_{3\to2}} = 2.66E_{\text{1-IDW}} = (2.66)\frac{hc}{(4.465)(355\text{ nm})}.\ \lambda_{3\to2} = 596\text{ nm}.$$

<u>$2 \to 1$ transition</u>: The energy difference is $(2.43 - 0.625)E_{\text{1-IDW}}$, and this is the photon energy.

$$\frac{hc}{\lambda_{2\to1}} = 1.805E_{\text{1-IDW}} = (1.805)\frac{hc}{(4.465)(355\text{ nm})}.\ \lambda_{2\to1} = 878\text{ nm}.$$

EVALUATE: As the energy difference between the levels gets larger, the photon wavelength gets shorter, which is physically reasonable.

VP40.6.4. **IDENTIFY:** We are dealing with an electron in a finite potential well.

SET UP: $E_{\text{1-IDW}} = \dfrac{\pi^2\hbar^2}{2mL^2}$.

EXECUTE: **(a)** We want the ground level energy for an infinite well. Using $L = 0.400$ nm and the equation for $E_{\text{1-IDW}}$ gives 2.35 eV.

(b) We want the energy of the bound state if $U_0 = 0.015$ eV. Because $U_0 \ll E_{\text{1-IDW}}$, $E = 0.68U_0$. Therefore $E = (0.68)(0.015\text{ eV}) = 0.010$ eV.

EVALUATE: The energy of the bound state is much less than the ground level for an infinite well of the same length.

VP40.7.1. **IDENTIFY:** This problem is about an electron tunneling through a potential barrier. We want the probability that the electron will tunnel through barriers if different thicknesses if its energy is 3.75 eV and the barrier height is 6.10 eV.

SET UP: The probability T of tunneling us $T = Ge^{-2\kappa L}$, where

$$G = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) \text{ and } \kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}.$$

EXECUTE: **(a)** $L = 0.750$ nm. First calculate G and κ and then use them to find T.

$$G = 16\left(\frac{3.75\text{ eV}}{6.10\text{ eV}}\right)\left(1 - \frac{3.75\text{ eV}}{6.10\text{ eV}}\right) = 3.7893,\quad \kappa = \frac{\sqrt{2m(6.10 - 3.75)\text{ eV}}}{\hbar} = 7.85005 \times 10^9\text{ m}^{-1}$$

Using these values gives $T = (3.789)e^{-11.763} = 2.92 \times 10^{-5}$.
(b) $L = 0.500$ nm. We get $T = (3.789)e^{-7.85005} = 1.48 \times 10^{-3}$.
EVALUATE: It is about 50 times more likely that the electron will tunnel through the narrower barrier than through the wider one. But probabilities are very low.

VP40.7.2. **IDENTIFY:** This problem is about an electron tunneling through a potential barrier.
SET UP: The probability T of tunneling us $T = Ge^{-2\kappa L}$, where

$$G = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) \text{ and } \kappa = \sqrt{\frac{2m(U_0 - E)}{\hbar}}.$$

EXECUTE: **(a)** We want T when $L = 0.800$ nm. First calculate G and κ and then use them to find T.

$$G = 16\left(\frac{3.50\text{ eV}}{4.00\text{ eV}}\right)\left(1 - \frac{3.50\text{ eV}}{4.00\text{ eV}}\right) = 1.7500,\quad \kappa = \frac{\sqrt{2m(4.00 - 3.50)\text{ eV}}}{\hbar} = 3.62096 \times 10^9\text{ m}^{-1}.$$

Using these values gives $T = (1.7500)e^{-5.79354} = 5.33 \times 10^{-3}$.
(b) We want L, so that the probability of tunneling is twice as great as in part (a). Solve $T = Ge^{-2\kappa L}$ for L. G and κ are the same as in part (a). Taking natural logarithms gives $L = -(1/2\kappa)\ln(T/G)$. Using G and κ with T twice what we found in part (a), we get $L = 0.704$ nm.
EVALUATE: Decreasing the width of the barrier from 0.800 nm to 0.704 nm doubled the probability of tunneling.

VP40.7.3. **IDENTIFY:** This problem is about an electron tunneling through a potential barrier. We want to find the probability of tunneling for different energies of the electron with $U_0 = 5.00\text{ eV}$ and $L = 0.900$ nm.
SET UP: The probability T of tunneling us $T = Ge^{-2\kappa L}$, where

$$G = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) \text{ and } \kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}.$$

EXECUTE: **(a)** $E = 4.00$ eV. We want T when $L = 0.800$ nm. First calculate G and κ and then use them to find T.

$$G = 16\left(\frac{3.50\text{ eV}}{4.00\text{ eV}}\right)\left(1 - \frac{3.50\text{ eV}}{4.00\text{ eV}}\right) = 1.7500,\quad \kappa = \frac{\sqrt{2m(4.00 - 3.50)\text{ eV}}}{\hbar} = 3.62096 \times 10^9\text{ m}^{-1}$$

Using these values gives $T = (1.7500)e^{-5.79354} = 5.33 \times 10^{-3}$.
(b) $E = 4.30$ eV. $G = 1.9264$, $\kappa = 4.2844 \times 10^9\text{ m}^{-1}$, $T = (1.9264)e^{-7.71188} = 8.62 \times 10^{-4}$.
(c) $E = 4.60$ eV. $G = 1.1776$, $\kappa = 3.23868 \times 10^9\text{ m}^{-1}$, $T = (1.1776)e^{-5.82963} = 3.46 \times 10^{-3}$.
EVALUATE: Our results show that as E gets closer to the height of the potential barrier, the probability of tunneling increases, which is physically reasonable.

VP40.7.4. **IDENTIFY:** This problem is about an electron tunneling through a potential barrier. We want to find the width L of the barrier if there is a 1/417 probability that the electron will tunnel through the barrier if the electron's energy is 3.00 eV and the barrier height is 5.00 eV.
SET UP: The probability T of tunneling us $T = Ge^{-2\kappa L}$, where

$$G = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) \text{ and } \kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}.$$

EXECUTE: Solve $T = Ge^{-2\kappa L}$ for L. Taking natural logarithms gives $L = -(1/2\kappa)\ln(T/G)$.

$$G = 16\left(\frac{3.00\text{ eV}}{5.00\text{ eV}}\right)\left(1 - \frac{3.00\text{ eV}}{5.00\text{ eV}}\right) = 3.8400,\ \kappa = \frac{\sqrt{2m(5.00 - 3.00)\text{ eV}}}{\hbar} = 7.24192 \times 10^9\text{ m}^{-1}.$$

Using $T = 1/417$ and the G and κ we just calculated gives $L = 0.509$ nm.
EVALUATE: The value of L is comparable to atomic dimensions.

40.1. IDENTIFY: Using the momentum of the free electron, we can calculate k and ω and use these to express its wave function.

SET UP: $\Psi(x,t)=Ae^{ikx}e^{-i\omega t}$, $k=p/\hbar$, and $\omega=\hbar k^2/2m$.

EXECUTE: $k=\dfrac{p}{\hbar}=-\dfrac{4.50\times10^{-24}\ \text{kg}\cdot\text{m/s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}=-4.27\times10^{10}\ \text{m}^{-1}$.

$$\omega=\frac{\hbar k^2}{2m}=\frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})(4.27\times10^{10}\ \text{m}^{-1})^2}{2(9.108\times10^{-31}\ \text{kg})}=1.05\times10^{17}\ \text{s}^{-1}.$$

$$\Psi(x,t)=Ae^{-i(4.27\times10^{10}\ \text{m}^{-1})x}e^{-i(1.05\times10^{17}\ \text{s}^{-1})t}.$$

EVALUATE: The wave function depends on position and time.

40.2. IDENTIFY: Using the known wave function for the particle, we want to find where its probability function is a maximum.

SET UP: $|\Psi(x,t)|^2=|A|^2(e^{ikx}e^{-i\omega t}-e^{2ikx}e^{-4i\omega t})(e^{-ikx}e^{+i\omega t}-e^{-2ikx}e^{+4i\omega t})$.

$|\Psi(x,t)|^2=|A|^2\left[2-(e^{-i(kx-3\omega t)}+e^{+i(kx-3\omega t)})\right]=2|A|^2[1-\cos(kx-3\omega t)]$.

EXECUTE: **(a)** For $t=0$, $|\Psi(x,t)|^2=2|A|^2(1-\cos(kx))$. $|\Psi(x,t)|^2$ is a maximum when $\cos(kx)=-1$ and this happens when $kx=(2n+1)\pi,\ n=0,1,\ldots$. $|\Psi(x,t)|^2$ is a maximum for $x=\dfrac{\pi}{k},\dfrac{3\pi}{k}$, etc.

(b) $t=\dfrac{2\pi}{\omega}$ and $3\omega t=6\pi$. $|\Psi(x,t)|^2=2|A|^2[1-\cos(kx-6\pi)]$. Maximum for $kx-6\pi=\pi,3\pi,\ldots$, which gives maxima when $x=\dfrac{7\pi}{k},\dfrac{9\pi}{k}$.

(c) From the results for parts (a) and (b), $v_{\text{av}}=\dfrac{7\pi/k-\pi/k}{2\pi/\omega}=\dfrac{3\omega}{k}$. $v_{\text{av}}=\dfrac{\omega_2-\omega_1}{k_2-k_1}$ with $\omega_2=4\omega$, $\omega_1=\omega$, $k_2=2k$ and $k_1=k$ gives $v_{\text{av}}=\dfrac{3\omega}{k}$.

EVALUATE: The expressions in part (c) agree.

40.3. IDENTIFY: Use the wave function from Example 40.1.

SET UP: $|\Psi(x,t)|^2=2|A|^2\{1+\cos[(k_2-k_1)x-(\omega_2-\omega_1)t]\}$. $k_2=3k_1=3k$. $\omega=\dfrac{\hbar k^2}{2m}$, so $\omega_2=9\omega_1=9\omega$.

$|\Psi(x,t)|^2=2|A|^2[1+\cos(2kx-8\omega t)]$.

EXECUTE: **(a)** At $t=2\pi/\omega$, $|\Psi(x,t)|^2=2|A|^2[1+\cos(2kx-16\pi)]$. $|\Psi(x,t)|^2$ is maximum for $\cos(2kx-16\pi)=1$. This happens for $2kx-16\pi=0,2\pi,\ldots$. Smallest positive x where $|\Psi(x,t)|^2$ is a maximum is $x=\dfrac{8\pi}{k}$.

(b) From the result of part (a), $v_{\text{av}}=\dfrac{8\pi/k}{2\pi/\omega}=\dfrac{4\omega}{k}$. $v_{\text{av}}=\dfrac{\omega_2-\omega_1}{k_2-k_1}=\dfrac{8\omega}{2k}=\dfrac{4\omega}{k}$.

EVALUATE: The two expressions agree.

40.4. IDENTIFY: Apply the Heisenberg uncertainty principle in the form $\Delta x\Delta p_x\ge\hbar/2$.

SET UP: The uncertainty in the particle position is proportional to the width of $\psi(x)$.

EXECUTE: **(a)** The width of $\psi(x)$ is inversely proportional to $\sqrt{\alpha}$. This can be seen by either plotting the function for different values of α or by finding the full width at half-maximum. The particle's uncertainty in position decreases with increasing α.

(b) Since the uncertainty in position decreases, the uncertainty in momentum must increase.
EVALUATE: As α increases, the function $A(k)$ in Eq. (40.19) must become broader.

40.5. **IDENTIFY** and **SET UP:** $\psi(x) = A\sin kx.$ The position probability density is given by $|\psi(x)|^2 = A^2 \sin^2 kx.$

EXECUTE: **(a)** The probability is highest where $\sin kx = 1$ so $kx = 2\pi x/\lambda = n\pi/2,\ n = 1, 3, 5, \ldots$
$x = n\lambda/4,\ n = 1, 3, 5, \ldots$ so $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$

(b) The probability of finding the particle is zero where $|\psi|^2 = 0,$ which occurs where $\sin kx = 0$ and $kx = 2\pi x/\lambda = n\pi,\ n = 0, 1, 2, \ldots$
$x = n\lambda/2,\ n = 0, 1, 2, \ldots,$ so $x = 0, \lambda/2, \lambda, 3\lambda/2, \ldots$

EVALUATE: The situation is analogous to a standing wave, with the probability analogous to the square of the amplitude of the standing wave.

40.6. **IDENTIFY:** Determine whether or not $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} + U\psi$ is equal to $E\psi,$ for some value of E.

SET UP: $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi_1}{dx^2} + U\psi_1 = E_1\psi_1$ and $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi_2}{dx^2} + U\psi_2 = E_2\psi_2.$

EXECUTE: $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} + U\psi = BE_1\psi_1 + CE_2\psi_2.$ If ψ were a solution with energy E, then $BE_1\psi_1 + CE_2\psi_2 = BE\psi_1 + CE\psi_2$ or $B(E_1 - E)\psi_1 = C(E - E_2)\psi_2.$ This would mean that ψ_1 is a constant multiple of ψ_2, and ψ_1 and ψ_2 would be wave functions with the same energy. However, $E_1 \neq E_2,$ so this is not possible, and ψ cannot be a solution to the equation $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} + U\psi = E\psi.$

EVALUATE: ψ is a solution if $E_1 = E_2$.

40.7. **IDENTIFY:** We are dealing with a particle in a box.

SET UP: $E = hc/\lambda,\ E_n = n^2\dfrac{\pi^2\hbar^2}{2mL^2}.$ We want the wavelength.

EXECUTE: The longest wavelength (lowest energy) photon is from a transition between the $n = 1$ to $n = 2$ states. The energy of this photon is equal to the energy difference between these states.

$$\Delta E_1 = \frac{\pi^2\hbar^2}{2mL^2}\left(2^2 - 1^2\right) = \frac{hc}{\lambda_1}.$$

The next longest wavelength photon is from a transition from the $n = 1$ to the $n = 3$ state.

$$\Delta E_2 = \frac{\pi^2\hbar^2}{2mL^2}\left(3^2 - 1^2\right) = \frac{hc}{\lambda_2}.$$

Dividing these two equations and solving for λ_2 gives

$$\frac{4-1}{9-1} = \frac{\lambda_2}{\lambda_1}.\ \lambda_2 = \frac{3}{8}\lambda_1 = \frac{3}{8}(420\text{ nm}) = 158\text{ nm}.$$

EVALUATE: The next longest photon (starting from the ground state) would be between the $n = 1$ and $n = 4$ states.

40.8. **IDENTIFY:** To describe a real situation, a wave function must be normalizable.

SET UP: $|\psi|^2\,dV$ is the probability that the particle is found in volume dV. Since the particle must be *somewhere*, ψ must have the property that $\int|\psi|^2 dV = 1$ when the integral is taken over all space.

EXECUTE: **(a)** For normalization of the one-dimensional wave function, we have

$$1=\int_{-\infty}^{\infty}|\psi|^2dx=\int_{-\infty}^{0}(Ae^{bx})^2dx+\int_0^{\infty}(Ae^{-bx})^2dx=\int_{-\infty}^{0}A^2e^{2bx}dx+\int_0^{\infty}A^2e^{-2bx}dx.$$

$$1=A^2\left\{\frac{e^{2bx}}{2b}\bigg|_{-\infty}^{0}+\frac{e^{-2bx}}{-2b}\bigg|_{0}^{\infty}\right\}=\frac{A^2}{b},\text{ which gives } A=\sqrt{b}=\sqrt{2.00\text{ m}^{-1}}=1.41\text{ m}^{-1/2}.$$

(b) The graph of the wavefunction versus x is given in Figure 40.8.

(c) (i) $P=\int_{-0.500\text{ m}}^{+0.500\text{ m}}|\psi|^2dx=2\int_0^{+0.500\text{ m}}A^2e^{-2bx}dx,$ where we have used the fact that the wave function is an even function of x. Evaluating the integral gives

$$P=\frac{-A^2}{b}(e^{-2b(0.500\text{ m})}-1)=\frac{-(2.00\text{ m}^{-1})}{2.00\text{ m}^{-1}}(e^{-2.00}-1)=0.865.$$

There is a little more than an 86% probability that the particle will be found within 50 cm of the origin.

(ii) $P=\int_{-\infty}^{0}(Ae^{bx})^2dx=\int_{-\infty}^{0}A^2e^{2bx}dx=\frac{A^2}{2b}=\frac{2.00\text{ m}^{-1}}{2(2.00\text{ m}^{-1})}=\frac{1}{2}=0.500.$

There is a 50-50 chance that the particle will be found to the left of the origin, which agrees with the fact that the wave function is symmetric about the y-axis.

(iii) $P=\int_{0.500\text{ m}}^{1.00\text{ m}}A^2e^{-2bx}dx=\frac{A^2}{-2b}(e^{-2(2.00\text{ m}^{-1})(1.00\text{ m})}-e^{-2(2.00\text{ m}^{-1})(0.500\text{ m})})=-\frac{1}{2}(e^{-4}-e^{-2})$

$=0.0585.$

EVALUATE: There is little chance of finding the particle in regions where the wave function is small.

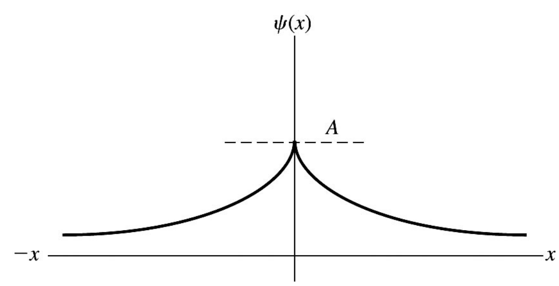


Figure 40.8

40.9. **IDENTIFY** and **SET UP:** The energy levels for a particle in a box are given by $E_n=\frac{n^2h^2}{8mL^2}.$

EXECUTE: **(a)** The lowest level is for $n=1,$ and $E_1=\frac{(1)(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{8(0.20\text{ kg})(1.3\text{ m})^2}=1.6\times10^{-67}\text{ J}.$

(b) $E=\frac{1}{2}mv^2,$ so $v=\sqrt{\frac{2E}{m}}=\sqrt{\frac{2(1.2\times10^{-67}\text{ J})}{0.20\text{ kg}}}=1.3\times10^{-33}\text{ m/s}.$ If the ball has this speed the time it would take it to travel from one side of the table to the other is

$$t=\frac{1.3\text{ m}}{1.3\times10^{-33}\text{ m/s}}=1.0\times10^{33}\text{ s}.$$

(c) $E_1=\frac{h^2}{8mL^2},\ E_2=4E_1,$ so $\Delta E=E_2-E_1=3E_1=3(1.6\times10^{-67}\text{ J})=4.9\times10^{-67}\text{ J}.$

EVALUATE: **(d)** No, quantum mechanical effects are not important for the game of billiards. The discrete, quantized nature of the energy levels is completely unobservable.

40.10. **IDENTIFY:** Solve the energy-level equation $E_n = \dfrac{n^2h^2}{8mL^2}$ for L.

SET UP: The ground state has $n = 1$.

EXECUTE: $L = \dfrac{h}{\sqrt{8mE_1}} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{8(1.673\times10^{-27}\ \text{kg})(5.0\times10^{6}\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}} = 6.4\times10^{-15}\ \text{m}$

EVALUATE: The value of L we calculated is on the order of the diameter of a nucleus.

40.11. **IDENTIFY:** An electron in the lowest energy state in this box must have the same energy as it would in the ground state of hydrogen.

SET UP: The energy of the n^{th} level of an electron in a box is $E_n = \dfrac{nh^2}{8mL^2}$.

EXECUTE: An electron in the ground state of hydrogen has an energy of $-13.6\ \text{eV}$, so find the width corresponding to an energy of $E_1 = 13.6\ \text{eV}$. Solving for L gives

$$L = \frac{h}{\sqrt{8mE_1}} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{8(9.11\times10^{-31}\ \text{kg})(13.6\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}} = 1.66\times10^{-10}\ \text{m}.$$

EVALUATE: This width is of the same order of magnitude as the diameter of a Bohr atom with the electron in the K shell.

40.12. **IDENTIFY and SET UP:** The energy of a photon is $E = hf = h\dfrac{c}{\lambda}$. The energy levels of a particle in a box are given by $E_n = \dfrac{nh^2}{8mL^2}$.

EXECUTE: **(a)** $E = (6.63\times10^{-34}\ \text{J}\cdot\text{s})\dfrac{(3.00\times10^{8}\ \text{m/s})}{(122\times10^{-9}\ \text{m})} = 1.63\times10^{-18}\ \text{J}$. $\Delta E = \dfrac{h^2}{8mL^2}(n_1^2 - n_2^2)$.

$$L = \sqrt{\frac{h^2(n_1^2 - n_2^2)}{8m\Delta E}} = \sqrt{\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2(2^2-1^2)}{8(9.11\times10^{-31}\ \text{kg})(1.63\times10^{-18}\ \text{J})}} = 3.33\times10^{-10}\ \text{m}.$$

(b) The ground state energy for an electron in a box of the calculated dimensions is

$E = \dfrac{h^2}{8mL^2} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\ \text{kg})(3.33\times10^{-10}\ \text{m})^2} = 5.43\times10^{-19}\ \text{J} = 3.40\ \text{eV}$ (one-third of the original photon energy), which does not correspond to the $-13.6\ \text{eV}$ ground state energy of the hydrogen atom.

EVALUATE: **(c)** Note that the energy levels for a particle in a box are proportional to n^2, whereas the energy levels for the hydrogen atom are proportional to $-\dfrac{1}{n^2}$. A one-dimensional box is not a good model for a hydrogen atom.

40.13. **IDENTIFY and SET UP:** The equation $E_n = \dfrac{n^2h^2}{8mL^2}$ gives the energy levels. Use this to obtain an expression for $E_2 - E_1$ and use the value given for this energy difference to solve for L.

EXECUTE: Ground state energy is $E_1 = \dfrac{h^2}{8mL^2}$; first excited state energy is $E_2 = \dfrac{4h^2}{8mL^2}$. The energy separation between these two levels is $\Delta E = E_2 - E_1 = \dfrac{3h^2}{8mL^2}$. This gives

$$L = h\sqrt{\frac{3}{8m\Delta E}} = L = 6.626\times10^{-34}\ \text{J}\cdot\text{s}\sqrt{\frac{3}{8(9.109\times10^{-31}\ \text{kg})(3.0\ \text{eV})(1.602\times10^{-19}\ \text{J/1 eV})}}$$

$= 6.1\times10^{-10}\ \text{m} = 0.61\ \text{nm}$.

EVALUATE: This energy difference is typical for an atom and L is comparable to the size of an atom.

40.14. **IDENTIFY:** The energy of the absorbed photon must be equal to the energy difference between the two states.

SET UP and **EXECUTE:** The second excited state energy is $E_3 = \dfrac{9\pi^2\hbar^2}{2mL^2}$. The ground state energy is $E_1 = \dfrac{\pi^2\hbar^2}{2mL^2}$. $E_1 = 2.00$ eV, so $E_3 = 18.0$ eV. For the transition $\Delta E = \dfrac{4\pi^2\hbar^2}{mL^2}$. $\dfrac{hc}{\lambda} = \Delta E$.

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{16.0\text{ eV}} = 7.75\times10^{-8}\text{ m} = 77.5\text{ nm}.$$

EVALUATE: This wavelength is much shorter than those of visible light.

40.15. **IDENTIFY:** We are dealing with a particle in a box and the de Broglie wavelength.

SET UP: $\lambda = h/p$, $E_n = n^2\dfrac{\pi^2\hbar^2}{2mL^2}$, $K = p^2/2m$.

EXECUTE: **(a)** We want the momentum. From the 9 in the numerator of the given energy, we see that $n^2 = 9$, so $n = 3$. The particle's energy is kinetic energy, so $E_3 = K_3 = p_3^2/2m$. Therefore,

$$E_3 = \frac{9\pi^2\hbar^2}{2mL^2} = \frac{p_3^2}{2m}.\ p = \sqrt{\frac{9\pi^2\hbar^2}{L^2}} = \frac{3h}{2L}.$$

(b) We want L/λ. $L/\lambda = L/(h/p) = Lp/h = L(3h/2L)/h = 3/2$.

EVALUATE: The ration in part (b) would be different if the particle were in a different state.

40.16. **IDENTIFY:** Find x where ψ_1 is zero and where it is a maximum.

SET UP: $\psi_1 = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{\pi x}{L}\right)$.

EXECUTE: **(a)** The wave function for $n = 1$ vanishes only at $x = 0$ and $x = L$ in the range $0 \le x \le L$.
(b) In the range for x, the sine term is a maximum only at the middle of the box, $x = L/2$.

EVALUATE: **(c)** The answers to parts (a) and (b) are consistent with the figure.

40.17. **IDENTIFY:** We are dealing with a particle in a box.

SET UP: $E_n = n^2\dfrac{\pi^2\hbar^2}{2mL^2}$, $m_A = 9m_B, L_B = 2L_A, E_A = E_B$. We want the lowest possible quantum numbers n_A and n_B of the two states.

EXECUTE: Equate the energies and determine the lowest possible values of n_A and n_B.

$$\frac{n_A^2\pi^2\hbar^2}{2m_A^2L_A^2} = \frac{n_B^2\pi^2\hbar^2}{2m_B^2L_B^2}.\quad \frac{n_A^2}{(9m_B)(L_B/2)^2} = \frac{n_B^2}{m_BL_B^2}.\ 4n_A^2 = 9n_B^2.$$

If $n_B = 2$, then $n_A = 3$, so the lowest possible values of the quantum numbers are $n_A = 3$, $n_B = 2$.

EVALUATE: Other states exist, such as $n_A = 6$, $n_B = 4$, but these are the lowest ones.

40.18. **IDENTIFY:** The energy levels are given by $E_n = \dfrac{n^2h^2}{8mL^2}$. The wavelength λ of the photon absorbed in an atomic transition is related to the transition energy ΔE by $\lambda = \dfrac{hc}{\Delta E}$.

SET UP: For the ground state $n = 1$ and for the third excited state $n = 4$.
EXECUTE: **(a)** The third excited state is $n = 4$, so

$$\Delta E = (4^2 - 1)\frac{h^2}{8mL^2} = \frac{15(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{8(9.11\times10^{-31}\text{ kg})(0.360\times10^{-9}\text{ m})^2} = 6.973\times10^{-18}\text{ J} = 43.5\text{ eV}.$$

(b) $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^{8}\ \text{m/s})}{6.973\times10^{-18}\ \text{J}} = 28.5\ \text{nm}.$

EVALUATE: This photon is an x ray. As the width of the box increases the transition energy for this transition decreases and the wavelength of the photon increases.

40.19. **IDENTIFY** and **SET UP:** $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$. The energy of the electron in level n is given by the equation $E_n = \dfrac{n^2h^2}{8mL^2}$.

EXECUTE: **(a)** $E_1 = \dfrac{h^2}{8mL^2} \Rightarrow \lambda_1 = \dfrac{h}{\sqrt{2mh^2/8mL^2}} = 2L = 2(3.0\times10^{-10}\ \text{m}) = 6.0\times10^{-10}\ \text{m}.$ The wavelength is twice the width of the box. $p_1 = \dfrac{h}{\lambda_1} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{6.0\times10^{-10}\ \text{m}} = 1.1\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

(b) $E_2 = \dfrac{4h^2}{8mL^2} \Rightarrow \lambda_2 = L = 3.0\times10^{-10}\ \text{m}.$ The wavelength is the same as the width of the box.

$p_2 = \dfrac{h}{\lambda_2} = 2p_1 = 2.2\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

(c) $E_3 = \dfrac{9h^2}{8mL^2} \Rightarrow \lambda_3 = \dfrac{2}{3}L = 2.0\times10^{-10}\ \text{m}.$ The wavelength is two-thirds the width of the box.

$p_3 = 3p_1 = 3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

EVALUATE: In each case the wavelength is an integer multiple of $\lambda/2$. In the n^{th} state, $p_n = np_1$.

40.20. **IDENTIFY:** The energy of the photon is equal to the energy difference ΔE between the energy levels of the electron.

SET UP: The energy levels of an electron in a one-dimensional box are $E_n = \dfrac{n^2h^2}{8mL^2}$. The energy of the absorbed photon is $\Delta E = \dfrac{hc}{\lambda}$.

EXECUTE: **(a)** $\Delta E_{1\to2} = (h^2/8mL^2)(2^2 - 1^2) = 3(h^2/8mL^2) = hc/\lambda_{1\to2}.$

$\Delta E_{2\to3} = (h^2/8mL^2)(3^2 - 2^2) = 5(h^2/8mL^2) = hc/\lambda_{2\to3}.$ Take ratios of these two equations, giving

$\dfrac{3}{5} = \dfrac{hc/\lambda_{1\to2}}{hc/\lambda_{2\to3}} = \dfrac{\lambda_{2\to3}}{\lambda_{1\to2}} \ \rightarrow\ \lambda_{2\to3} = (3/5)\lambda_{1\to2} = (3/5)(426\ \text{nm}) = 256\ \text{nm}.$

(b) Follow the same procedure as in part (a), giving

$$\lambda_{1\to3} = (3/8)\lambda_{1\to2} = (3/8)(426\ \text{nm}) = 160\ \text{nm}.$$

(c) From part (a), we know that $\Delta E_{1\to2} = \dfrac{3h^2}{8mL^2} = \dfrac{hc}{\lambda_{1\to2}}$. Solving for L gives

$$L = \sqrt{\frac{3h\lambda_{1\to2}}{8\,mc}} = \sqrt{\frac{3(6.626\times10^{-34}\ \text{J}\cdot\text{s})(426\times10^{-9}\ \text{m})}{8(9.109\times10^{-31}\ \text{kg})(2.998\times10^{8}\ \text{m/s})}} = 6.23\times10^{-10}\ \text{m} = 0.623\ \text{nm}.$$

EVALUATE: The width L of this box is about 6 times the diameter of a hydrogen atom.

40.21. **IDENTIFY:** We are dealing with a particle in a box.

SET UP: $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx} = E\psi.$

EXECUTE: **(a)** $\psi_n = A\cos k_n x.$ Take the second derivative and use it to find E_n as follows.

$$-\frac{\hbar^2}{2m}\left(-Ak_n^2 \cos k_n x\right) = EA\cos k_n x. \;\; E_n = \frac{\hbar^2}{2m}k_n^2.$$

The wave function must be zero at $x = \pm L/2$. This gives

$A\cos[k_n(\pm L/2)] = 0.\; k_n(L/2) = \pi/2, 3\pi/2, 5\pi/2, \cdots .k_n = n\pi/L,\; n = 1,3,5,\cdots$

(b) $\psi_n = A\sin k_n x.$ Follow the same procedure as in part (a) to obtain $k_n = n\pi/L,\; n = 2, 4, 6, \ldots$

(c) We want the allowed energies.

$\psi_n = A\sin k_n x$: Combine the results for E_n and k_n from part (b) to obtain E_n.

$$E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 = \frac{\hbar^2\pi^2}{2mL^2}n^2,\; n = 2,4,6,\cdots$$

$\psi_n = A\cos k_n x$: Follow the same procedure using the results from part (a), which gives

$$E_n = \frac{\hbar^2\pi^2}{2mL^2}n^2,\; n = 1,3,5,\cdots$$

EVALUATE: **(d)** The set of energies found here is the *same* as those in Eq. (40.31). This result occurs because the physical system (i.e., the box) does not "know" where we placed the origin of coordinates, so its behavior should be the same in either case.

40.22. **IDENTIFY:** $\lambda = \frac{h}{p}$. p is related to E by $E = \frac{p^2}{2m} + U.$

SET UP: For $x > L,\; U = U_0.$ For $0 < x < L,\; U = 0.$

EXECUTE: For $0 < x < L,\;\; p = \sqrt{2mE} = \sqrt{2m(3U_0)}$ and $\lambda_{\text{in}} = \frac{h}{\sqrt{2m(3U_0)}}$. For $x > L,$

$p = \sqrt{2m(E - U_0)} = \sqrt{2m(2U_0)}$ and $\lambda_{\text{out}} = \frac{h}{\sqrt{2m(E-U_0)}} = \frac{h}{\sqrt{2m(2U_0)}}$. Thus, the ratio of the wavelengths is $\frac{\lambda_{\text{out}}}{\lambda_{\text{in}}} = \frac{\sqrt{2m(3U_0)}}{\sqrt{2m(2U_0)}} = \sqrt{\frac{3}{2}}.$

EVALUATE: For $x > L$ some of the energy is potential and the kinetic energy is less than it is for $0 < x < L,$ where $U = 0.$ Therefore, outside the box p is less and λ is greater than inside the box.

40.23. **IDENTIFY:** Figure 40.15b in the textbook gives values for the bound state energy of a square well for which $U_0 = 6E_{\text{1-IDW}}$.

SET UP: $E_{\text{1-IDW}} = \frac{\pi^2\hbar^2}{2mL^2}.$

EXECUTE: $E_1 = 0.625E_{\text{1-IDW}} = 0.625\frac{\pi^2\hbar^2}{2mL^2};\; E_1 = 2.00\text{ eV} = 3.20\times10^{-19}\text{ J}.$

$$L = \pi\hbar\left(\frac{0.625}{2(9.109\times10^{-31}\text{ kg})(3.20\times10^{-19}\text{ J})}\right)^{1/2} = 3.43\times10^{-10}\text{ m}.$$

EVALUATE: As L increases the ground state energy decreases.

40.24. **IDENTIFY:** In a finite potential well, the energy levels are lowered compared to the energy levels in an infinite well. The energy of the photon removes the electron from its energy state in the well and any left-over energy is the kinetic energy K of the electron.

SET UP: The energy levels for an infinitely deep well are $E_{n-\text{IDW}} = \frac{n^2h^2}{8mL^2}$, and $n = 1$ is the ground state. The energy of a photon is $E = hc/\lambda.$

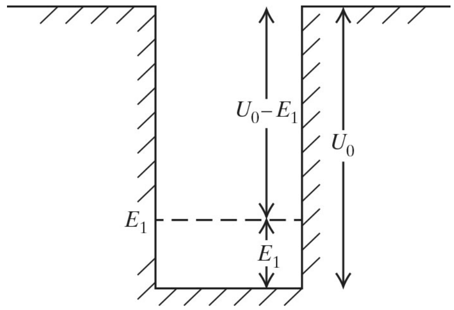


Figure 40.24

EXECUTE: Figure 40.24 illustrates the various energies involved. In this case, $U_0 = 6E_{1\text{-IDW}}$. Figure 40.15b in the textbook shows that the ground state energy E_1 in the finite well is $E_1 = 0.625E_{1\text{-IDW}}$.

The electron already has energy E_1 in the well, so the energy just to remove it from the well is $U_0 - E_1$.

Conservation of energy gives $E_{\text{photon}} = E_{\text{remove el}} + K$, which we can write as

$$\frac{hc}{\lambda} = (U_0 - E_1) + K = 6E_{1-\text{IDW}} - 0.625E_{1-\text{IDW}} + K = 5.375E_{1-\text{IDW}} + K.$$

Solving for K and using $E_{1-\text{IDW}} = \dfrac{h^2}{8mL^2}$ gives

$$K = \frac{hc}{\lambda} - 5.375E_{1-\text{IDW}} = \frac{hc}{\lambda} - \frac{5.735h^2}{8mL^2}.$$

Using $\lambda = 72 \times 10^{-9}$ m and $L = 4.00 \times 10^{-10}$ m, plus the usual values of the constants h, c, and m, we get $K = 2.76 \times 10^{-18}$ J – 2.02×10^{-18} J = 7.4×10^{-19} J, which we can express in electron-volts as K = 17.2 eV – 12.6 eV = 4.6 eV.

EVALUATE: The photon has 17.2 eV and it takes 12.6 eV just to remove the electron from the well, so the remaining 4.6 eV is the kinetic energy of the electron.

40.25. **IDENTIFY:** The energy of the photon is the energy given to the electron.

SET UP: Since $U_0 = 6E_{1\text{-IDW}}$ we can use the result $E_1 = 0.625E_{1\text{-IDW}}$ from Section 40.4. When the electron is outside the well it has potential energy U_0, so the minimum energy that must be given to the electron is $U_0 - E_1 = 5.375E_{1\text{-IDW}}$.

EXECUTE: The maximum wavelength of the photon would be

$$\lambda = \frac{hc}{U_0 - E_1} = \frac{hc}{(5.375)(h^2/8mL^2)} = \frac{8mL^2c}{(5.375)h} = \frac{8(9.11\times10^{-31}\text{ kg})(1.50\times10^{-9}\text{ m})^2(3.00\times10^{8}\text{ m/s})}{(5.375)(6.63\times10^{-34}\text{ J}\cdot\text{s})}$$
$$= 1.38\times10^{-6}\text{ m}.$$

EVALUATE: This photon is in the infrared. The wavelength of the photon decreases when the width of the well decreases.

40.26. **IDENTIFY:** The longest wavelength corresponds to the smallest energy change.

SET UP: The ground level energy level of the infinite well is $E_{1\text{-IDW}} = \dfrac{h^2}{8mL^2}$, and the energy of the photon must be equal to the energy difference between the two levels.

EXECUTE: The 582-nm photon must correspond to the $n=1$ to $n=2$ transition. Since $U_0 = 6E_{\text{1-IDW}}$, we have $E_2 = 2.43E_{\text{1-IDW}}$ and $E_1 = 0.625E_{\text{1-IDW}}$. The energy of the photon is equal to the energy difference between the two levels, and $E_{\text{1-IDW}} = \dfrac{h^2}{8mL^2}$, which gives

$E_\gamma = E_2 - E_1 \Rightarrow \dfrac{hc}{\lambda} = (2.43-0.625)E_{\text{1-IDW}} = \dfrac{1.805\,h^2}{8mL^2}$. Solving for L gives

$$L = \sqrt{\frac{(1.805)h\lambda}{8mc}} = \sqrt{\frac{(1.805)(6.626\times10^{-34}\ \text{J}\cdot\text{s})(5.82\times10^{-7}\ \text{m})}{8(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})}} = 5.64\times10^{-10}\ \text{m} = 0.564\ \text{nm}.$$

EVALUATE: This width is slightly more than half that of a Bohr hydrogen atom.

40.27. IDENTIFY: Find the transition energy ΔE and set it equal to the energy of the absorbed photon. Use $E = hc/\lambda$, to find the wavelength of the photon.

SET UP: $U_0 = 6E_{\text{1-IDW}}$, as in Figure 40.15 in the textbook, so $E_1 = 0.625E_{\text{1-IDW}}$ and $E_3 = 5.09E_{\text{1-IDW}}$ with $E_{\text{1-IDW}} = \dfrac{\pi^2\hbar^2}{2mL^2}$. In this problem the particle bound in the well is a proton, so $m = 1.673\times10^{-27}$ kg.

EXECUTE: $E_{\text{1-IDW}} = \dfrac{\pi^2\hbar^2}{2mL^2} = \dfrac{\pi^2(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.673\times10^{-27}\ \text{kg})(4.0\times10^{-15}\ \text{m})^2} = 2.052\times10^{-12}$ J. The transition energy is $\Delta E = E_3 - E_1 = (5.09-0.625)E_{\text{1-IDW}} = 4.465E_{\text{1-IDW}}$. $\Delta E = 4.465(2.052\times10^{-12}\ \text{J}) = 9.162\times10^{-12}$ J. The wavelength of the photon that is absorbed is related to the transition energy by $\Delta E = hc/\lambda$, so

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{9.162\times10^{-12}\ \text{J}} = 2.2\times10^{-14}\ \text{m} = 22\ \text{fm}.$$

EVALUATE: The wavelength of the photon is comparable to the size of the well.

40.28. IDENTIFY: The tunneling probability is $T = Ge^{-2\kappa L}$, with $G = 16\dfrac{E}{U_0}\left(1-\dfrac{E}{U_0}\right)$ and $\kappa = \dfrac{\sqrt{2m(U_0-E)}}{\hbar}$.

So $T = 16\dfrac{E}{U_0}\left(1-\dfrac{E}{U_0}\right)e^{\frac{-2\sqrt{2m(U_0-E)}}{\hbar}L}$.

SET UP: $U_0 = 30.0\times10^6$ eV, $L = 2.0\times10^{-15}$ m, $m = 6.64\times10^{-27}$ kg.

EXECUTE: **(a)** $U_0 - E = 1.0\times10^6$ eV $(E = 29.0\times10^6\ \text{eV})$, $T = 0.090$.

(b) If $U_0 - E = 10.0\times10^6$ eV $(E = 20.0\times10^6\ \text{eV})$, $T = 0.014$.

EVALUATE: T is less when $U_0 - E$ is 10.0 MeV than when $U_0 - E$ is 1.0 MeV.

40.29. IDENTIFY and SET UP: The probability is $T = Ge^{-2\kappa L}$, with $G = 16\dfrac{E}{U_0}\left(1-\dfrac{E}{U_0}\right)$ and $\kappa = \dfrac{\sqrt{2m(U_0-E)}}{\hbar}$. $E = 32$ eV, $U_0 = 41$ eV, $L = 0.25\times10^{-9}$ m. Calculate T.

EXECUTE: **(a)** $G = 16\dfrac{E}{U_0}\left(1-\dfrac{E}{U_0}\right) = 16\dfrac{32}{41}\left(1-\dfrac{32}{41}\right) = 2.741.$

$$\kappa = \frac{\sqrt{2m(U_0-E)}}{\hbar}.$$

$$\kappa = \frac{\sqrt{2(9.109\times10^{-31}\ \text{kg})(41\ \text{eV}-32\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}} = 1.536\times10^{10}\ \text{m}^{-1}.$$

$$T = Ge^{-2\kappa L} = (2.741)e^{-2(1.536\times10^{10}\ \text{m}^{-1})(0.25\times10^{-9}\ \text{m})} = 2.741e^{-7.68} = 0.0013.$$

(b) The only change is the mass m, which appears in κ.

$$\kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}.$$

$$\kappa = \frac{\sqrt{2(1.673\times10^{-27}\ \text{kg})(41\ \text{eV} - 32\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}} = 6.584\times10^{11}\ \text{m}^{-1}.$$

Then $T = Ge^{-2\kappa L} = (2.741)e^{-2(6.584\times10^{11}\ \text{m}^{-1})(0.25\times10^{-9}\ \text{m})} = 2.741e^{-392.2} = 10^{-143}$.

EVALUATE: The more massive proton has a much smaller probability of tunneling than the electron does.

40.30. IDENTIFY: The transmission coefficient is $T = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)e^{-2\sqrt{2m(U_0-E)}L/\hbar}$.

SET UP: $E = 5.0$ eV, $L = 0.60\times10^{-9}$ m, and $m = 9.11\times10^{-31}$ kg.

EXECUTE: **(a)** $U_0 = 7.0\ \text{eV} \Rightarrow T = 5.5\times10^{-4}$.

(b) $U_0 = 9.0\ \text{eV} \Rightarrow T = 1.8\times10^{-5}$.

(c) $U_0 = 13.0\ \text{eV} \Rightarrow T = 1.1\times10^{-7}$.

EVALUATE: T decreases when the height of the barrier increases.

40.31. IDENTIFY: The tunneling probability is $T = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)e^{-2L\sqrt{2m(U_0-E)}/\hbar}$.

SET UP: $\frac{E}{U_0} = \frac{6.0\ \text{eV}}{11.0\ \text{eV}}$ and $E - U_0 = 5\ \text{eV} = 8.0\times10^{-19}$ J.

EXECUTE: **(a)** $L = 0.80\times10^{-9}$ m:

$$T = 16\left(\frac{6.0\ \text{eV}}{11.0\ \text{eV}}\right)\left(1 - \frac{6.0\ \text{ev}}{11.0\ \text{eV}}\right)e^{-2(0.80\times10^{-9}\ \text{m})\sqrt{2(9.11\times10^{-31}\ \text{kg})(8.0\times10^{-19}\ \text{J})}/1.055\times10^{-34}\ \text{J}\cdot\text{s}} = 4.4\times10^{-8}.$$

(b) $L = 0.40\times10^{-9}$ m: $T = 4.2\times10^{-4}$.

EVALUATE: The tunneling probability is less when the barrier is wider.

40.32. IDENTIFY and **SET UP:** Use $\lambda = h/p$, where $K = p^2/2m$ and $E = K + U$.

EXECUTE: $\lambda = h/p = h/\sqrt{2mK}$, so $\lambda\sqrt{K}$ is constant. $\lambda_1\sqrt{K_1} = \lambda_2\sqrt{K_2}$; λ_1 and K_1 are for $x > L$ where $K_1 = 2U_0$ and λ_2 and K_2 are for $0 < x < L$ where $K_2 = E - U_0 = U_0$.

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = \sqrt{\frac{U_0}{2U_0}} = \frac{1}{\sqrt{2}}.$$

EVALUATE: When the particle is passing over the barrier its kinetic energy is less and its wavelength is larger.

40.33. IDENTIFY and **SET UP:** The energy levels are given by $E_n = (n + \frac{1}{2})\hbar\omega$, where $\omega = \sqrt{\frac{k'}{m}}$.

EXECUTE: $\omega = \sqrt{\frac{k'}{m}} = \sqrt{\frac{110\ \text{N/m}}{0.250\ \text{kg}}} = 21.0$ rad/s.

The ground state energy is given by $E_n = (n + \frac{1}{2})\hbar\omega$, where $n = 0$.

$E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}(1.055\times10^{-34}\ \text{J}\cdot\text{s})(21.0\ \text{rad/s}) = 1.11\times10^{-33}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 6.93\times10^{-15}$ eV.

$E_n = (n + \frac{1}{2})\hbar\omega$, $E_{(n+1)} = (n + 1 + \frac{1}{2})\hbar\omega$.

The energy separation between these adjacent levels is

$$\Delta E = E_{n+1} - E_n = \hbar\omega = 2E_0 = 2(1.11\times10^{-33}\text{ J}) = 2.22\times10^{-33}\text{ J} = 1.39\times10^{-14}\text{ eV}.$$

EVALUATE: These energies are extremely small; quantum effects are not important for this oscillator.

40.34. IDENTIFY: This problem is about a quantum harmonic oscillator and the uncertainty principle.

SET UP: $K = p^2/2m$, $p_{\max} = mv_{\max}$, $E_n = \left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}} = \left(n+\frac{1}{2}\right)\hbar\omega.$

EXECUTE: **(a)** We want $p_{\max}$. When $v = v_{\max}$, $x = 0$, so $K_{\max} = E_n$. Using $K = p^2/2m$ gives

$$\frac{p_{\max}^2}{2m} = \left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}}.\quad p_{\max} = \sqrt{(2n+1)\hbar\sqrt{k'm}}.$$

(b) We want the amplitude A. When $x = A$, $v = 0$, so $\frac{1}{2}k'A^2 = E_n$. Use this fact and solve for A.

$$\frac{1}{2}k'A^2 = \left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}}.\ A = \sqrt{\frac{(2n+1)\hbar}{\sqrt{k'm}}}.$$

(c) We want $\Delta x\Delta p_x$. Use the uncertainties given in the problem and the results of (a) and (b).

$$\Delta x\Delta p_x = \frac{A}{\sqrt{2}}\frac{p_{\max}}{\sqrt{2}} = \frac{1}{2}\sqrt{\frac{(2n+1)\hbar}{\sqrt{k'm}}}\sqrt{(2n+1)\hbar\sqrt{k'm}} = \left(n+\frac{1}{2}\right)\hbar.$$

As n increases, the uncertainty product also increases.

EVALUATE: As n increases, so do A and $p_{\max}$. Therefore the uncertainty product should also increase, as we have found.

40.35. IDENTIFY: We can model the molecule as a harmonic oscillator. The energy of the photon is equal to the energy difference between the two levels of the oscillator.

SET UP: The energy of a photon is $E_\gamma = hf = hc/\lambda$, and the energy levels of a harmonic oscillator are given by $E_n = (n+\frac{1}{2})\hbar\sqrt{\frac{k'}{m}} = (n+\frac{1}{2})\hbar\omega.$

EXECUTE: **(a)** The photon's energy is $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{5.8\times10^{-6}\text{ m}} = 0.21\text{ eV}.$

(b) The transition energy is $\Delta E = E_{n+1} - E_n = \hbar\omega = \hbar\sqrt{\frac{k'}{m}}$, which gives $\frac{2\pi\hbar c}{\lambda} = \hbar\sqrt{\frac{k'}{m}}$. Solving for k', we get $k' = \frac{4\pi^2c^2m}{\lambda^2} = \frac{4\pi^2(3.00\times10^8\text{ m/s})^2(5.6\times10^{-26}\text{ kg})}{(5.8\times10^{-6}\text{ m})^2} = 5{,}900\text{ N/m}.$

EVALUATE: This would be a rather strong spring in the physics lab.

40.36. IDENTIFY: The energy of the absorbed photon must be equal to the energy difference between the two states.

SET UP and EXECUTE: $\Delta E = \frac{hc}{\lambda} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{6.35\times10^{-6}\text{ m}} = 0.1953\text{ eV} = \hbar\omega.\ \ \Delta E = \hbar\omega.$

$$E_0 = \frac{\hbar\omega}{2} = \frac{0.1953\text{ eV}}{2} = 0.0976\text{ eV}.$$

EVALUATE: The energy of the photon is not equal to the energy of the ground state, but rather it is the energy *difference* between the two states.

40.37. IDENTIFY: The photon energy equals the transition energy for the atom.

SET UP: According to the energy level equation $E_n = (n+\frac{1}{2})\hbar\omega$, the energy released during the transition between two adjacent levels is twice the ground state energy $E_3 - E_2 = \hbar\omega = 2E_0 = 11.2\text{ eV}.$

EXECUTE: For a photon of energy E,

$$E = hf \Rightarrow \lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(11.2\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 111\ \text{nm}.$$

EVALUATE: This photon is in the ultraviolet.

40.38. **IDENTIFY:** The energy of the absorbed (or emitted) photon energy is equal to the energy difference between the levels of the oscillator.

SET UP: The energy levels for a harmonic oscillator are $E_n = (n+\frac{1}{2})\hbar\omega,$ where $\omega = \sqrt{k'/m}.$

EXECUTE: **(a)** The energy difference between *any* two adjacent levels is $\Delta E_{n+1} - \Delta E_n = (n+\frac{3}{2})\hbar\omega - (n+\frac{1}{2})\hbar\omega = \hbar\omega.$ Therefore transitions between *any* adjacent levels will emit (or absorb) photons of the same energy and hence the same wavelength. So the $2 \to 3$ transition absorbs a photon of the same wavelength as the $1 \to 2$ transition, which is λ = 6.50 μm.

(b) Since transitions between adjacent levels emit photons of the same energy, transitions between levels for which *n* differs by 2 will emit energy $\hbar\omega + \hbar\omega = 2\hbar\omega.$ So the photon absorbed in the $1 \to 3$ transition will have twice the energy (and therefore half the wavelength) as the photon in the $1 \to 2$ transition, so its wavelength will be $\frac{1}{2}$ (6.50 μm) = 3.25 μm.

(c) For the $1 \to 2$ transition, the photon energy is $\hbar\omega$ and $\omega = \sqrt{k'/m}$, so

$\hbar\omega$ = energy of photon = hc/λ. This gives $\omega = \sqrt{k'/m} = \dfrac{hc}{\hbar\lambda} = \dfrac{2\pi c}{\lambda} = \dfrac{2\pi(3.00\times10^{8}\ \text{m/s})}{6.50\times10^{-6}\ \text{m}}$

$= 2.90\times10^{14}$ rad/s.

EVALUATE: The frequency of this oscillator would be $f = \omega/2\pi = 4.62\times10^{14}$ Hz, *much* higher than typical classical oscillators.

40.39. **IDENTIFY:** We model the atomic vibration in the crystal as a harmonic oscillator.

SET UP: The energy levels of a harmonic oscillator are given by $E_n = (n+\frac{1}{2})\hbar\sqrt{\dfrac{k'}{m}} = (n+\frac{1}{2})\hbar\omega.$

EXECUTE: **(a)** The ground state energy of a simple harmonic oscillator is $E_0 = \frac{1}{2}\hbar\omega$

$$= \frac{1}{2}\hbar\sqrt{\frac{k'}{m}} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{2}\sqrt{\frac{12.2\ \text{N/m}}{3.82\times10^{-26}\ \text{kg}}} = 9.43\times10^{-22}\ \text{J} = 5.89\times10^{-3}\ \text{eV}.$$

(b) $E_4 - E_3 = \hbar\omega = 2E_0 = 0.0118\ \text{eV},$ so $\lambda = \dfrac{hc}{E} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{1.88\times10^{-21}\ \text{J}} = 106\ \mu\text{m}.$

(c) $E_{n+1} - E_n = \hbar\omega = 2E_0 = 0.0118\ \text{eV}.$

EVALUATE: These energy differences are much smaller than those due to electron transitions in the hydrogen atom.

40.40. **IDENTIFY:** Compute the ratio specified in the problem.

SET UP: For $n = 0,$ $A = \sqrt{\dfrac{\hbar\omega}{k'}}.$ $\omega = \sqrt{\dfrac{k'}{m}}.$

EXECUTE: **(a)** $\dfrac{|\psi(A)|^2}{|\psi(0)|^2} = \exp\left(-\dfrac{\sqrt{mk'}}{\hbar}A^2\right) = \exp\left(-\sqrt{mk'}\dfrac{\omega}{k'}\right) = e^{-1} = 0.368.$ This is consistent with what is shown in Figure 40.27 in the textbook.

(b) $\dfrac{|\psi(2A)^2|}{|\psi(0)|^2} = \exp\left(-\dfrac{\sqrt{mk'}}{\hbar}(2A)^2\right) = \exp\left(-\sqrt{mk'}\,4\dfrac{\omega}{k'}\right) = e^{-4} = 1.83\times10^{-2}.$ This figure cannot be read this precisely, but the qualitative decrease in amplitude with distance is clear.

EVALUATE: The wave function decays exponentially as *x* increases beyond $x = A.$

40.41. **IDENTIFY:** We know the wave function of a particle in a box.

SET UP and **EXECUTE:** **(a)** $\Psi(x,t)=\frac{1}{\sqrt{2}}\psi_1(x)e^{-iE_1t/\hbar}+\frac{1}{\sqrt{2}}\psi_3(x)e^{-iE_3t/\hbar}$.

$$\Psi^*(x,t)=\frac{1}{\sqrt{2}}\psi_1(x)e^{+iE_1t/\hbar}+\frac{1}{\sqrt{2}}\psi_3(x)e^{+iE_3t/\hbar}.$$

$|\Psi(x,t)|^2=\frac{1}{2}[\psi_1^2+\psi_3^2+\psi_1\psi_3(e^{i(E_3-E_1)t/\hbar}+e^{-i(E_3-E_1)t/\hbar})]=\frac{1}{2}\left[\psi_1^2+\psi_3^2+2\psi_1\psi_3\cos\left(\frac{[E_3-E_1]t}{\hbar}\right)\right]$.

$\psi_1=\sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)$. $\psi_3=\sqrt{\frac{2}{L}}\sin\left(\frac{3\pi x}{L}\right)$. $E_3=\frac{9\pi^2\hbar^2}{2mL^2}$ and $E_1=\frac{\pi^2\hbar^2}{2mL^2}$, so $E_3-E_1=\frac{4\pi^2\hbar^2}{mL^2}$.

$|\Psi(x,t)|^2=\frac{1}{L}\left[\sin^2\left(\frac{\pi x}{L}\right)+\sin^2\left(\frac{3\pi x}{L}\right)+2\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{3\pi x}{L}\right)\cos\left(\frac{4\pi^2\hbar t}{mL^2}\right)\right]$. At $x=L/2$,

$\sin\left(\frac{\pi x}{L}\right)=\sin\left(\frac{\pi}{2}\right)=1$. $\sin\left(\frac{3\pi x}{L}\right)=\sin\left(\frac{3\pi}{2}\right)=-1$. $|\Psi(x,t)|^2=\frac{2}{L}\left[1-\cos\left(\frac{4\pi^2\hbar t}{mL^2}\right)\right]$.

(b) $\omega_{\text{osc}}=\frac{E_3-E_1}{\hbar}=\frac{4\pi^2\hbar}{mL^2}$.

EVALUATE: Note that $\Delta E=\hbar\omega$.

40.42. **IDENTIFY:** In this problem, we model the hydrogen as an electron in a box.

SET UP: $E=hc/\lambda$.

EXECUTE: **(a)** We want the photon energies. $E_{656}=hc/\lambda=hc/(656\text{ nm})=1.89\text{ eV}$. Using the other wavelenghs in the same way gives $E_{486}=2.55\text{ eV}$, $E_{434}=2.86\text{ eV}$, $E_{410}=3.03\text{ eV}$.

(b) We want the minimum n_i. The transitions must all end on a lower state. The lowest possible ones are n = 1, 2, 3, 4, so the first state above that is n = 5.

(c) We want $\mathcal{E}$. Using $n_\text{i}=5$, the final states are n_f = 4, 3, 2, and 1. We follow the instruction in the problem for the possible transitions and find that the transitions 5 → 4, 5 → 3, 5 → 2, and 5 → 1. The greater the difference between n_i and n_f, the greater the photon energy E. Following the directions in the problem, we have

$$5\to4:\quad \mathcal{E}=\frac{E}{n_\text{i}^2-n_\text{f}^2}=\frac{1.89\text{ eV}}{5^2-4^2}=0.210\text{ eV}$$

$$5\to3:\quad \mathcal{E}=\frac{E}{n_\text{i}^2-n_\text{f}^2}=\frac{2.55\text{ eV}}{5^2-3^2}=0.519\text{ eV}$$

$$5\to2:\quad \mathcal{E}=\frac{E}{n_\text{i}^2-n_\text{f}^2}=\frac{2.86\text{ eV}}{5^2-2^2}=0.136\text{ eV}$$

$$5\to1:\quad \mathcal{E}=\frac{E}{n_\text{i}^2-n_\text{f}^2}=\frac{3.03\text{ eV}}{5^2-1^2}=0.158\text{ eV}$$

Averaging the results gives $\mathcal{E}$ = 0.2 eV.

(d) We want L. $E_n=n^2h^2/8mL^2=n^2\mathcal{E}$ gives $\mathcal{E}=h^2/8mL^2$. Solving for L and using $\mathcal{E}$ = 0.2 eV gives L = 2 nm.

(e) Using $L/2a_0$ with L = 2 nm gives a ratio of 18.

EVALUATE: Our result gives L is about 18 times the diameter of a hydrogen atom, so the particle in a box model is not very good.

40.43. **IDENTIFY:** Let I refer to the region $x<0$ and let II refer to the region $x>0$, so

$\psi_I(x)=Ae^{ik_1x}+Be^{-ik_1x}$ and $\psi_{II}(x)=Ce^{ik_2x}$. Set $\psi_I(0)=\psi_{II}(0)$ and $\frac{d\psi_I}{dx}=\frac{d\psi_{II}}{dx}$ at $x=0$.

SET UP: $\frac{d}{dx}(e^{ikx}) = ike^{ikx}$.

EXECUTE: $\psi_I(0) = \psi_{II}(0)$ gives $A + B = C$. $\frac{d\psi_I}{dx} = \frac{d\psi_{II}}{dx}$ at $x = 0$ gives $ik_1A - ik_1B = ik_2C$. Solving this pair of equations for B and C gives $B = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)A$ and $C = \left(\frac{2k_2}{k_1 + k_2}\right)A$.

EVALUATE: The probability of reflection is $R = \frac{B^2}{A^2} = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$. The probability of transmission is $T = \frac{C^2}{A^2} = \frac{4k_1^2}{(k_1 + k_2)^2}$. Note that $R + T = 1$.

40.44. IDENTIFY: The probability of finding the particle between x_1 and x_2 is $\int_{x_1}^{x_2} |\psi|^2\, dx$.

SET UP: For the ground state $\psi_1 = \sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}$. $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$. $\int \cos\alpha x\, dx = \frac{1}{\alpha}\sin\alpha x$.

EXECUTE: **(a)** $\frac{2}{L}\int_0^{L/4} \sin^2\frac{\pi x}{L}dx = \frac{2}{L}\int_0^{L/4}\frac{1}{2}\left(1 - \cos\frac{2\pi x}{L}\right)dx = \frac{1}{L}\left(x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right)_0^{L/4} = \frac{1}{4} - \frac{1}{2\pi}$, which is about 0.0908.

(b) Repeating with limits of $L/4$ and $L/2$ gives $\frac{1}{L}\left(x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right)_{L/4}^{L/2} = \frac{1}{4} + \frac{1}{2\pi}$, about 0.409.

(c) The particle is much likely to be nearer the middle of the box than the edge.

EVALUATE: **(d)** The results sum to exactly $\frac{1}{2}$. Since the probability of the particle being anywhere in the box is unity, the probability of the particle being found between $x = L/2$ and $x = L$ is also $\frac{1}{2}$. This means that the particle is as likely to be between $x = 0$ and $L/2$ as it is to be between $x = L/2$ and $x = L$.

(e) These results are consistent with Figure 40.12b in the textbook. This figure shows a greater probability near the center of the box. It also shows symmetry of $|\psi|^2$ about the center of the box.

40.45. IDENTIFY and **SET UP:** The energy levels are given by the equation $E_n = \frac{n^2h^2}{8mL^2}$. Calculate ΔE for the transition and set $\Delta E = hc/\lambda$, the energy of the photon.

EXECUTE: **(a)** Ground level, $n = 1, E_1 = \frac{h^2}{8mL^2}$. First excited level, $n = 2, E_2 = \frac{4h^2}{8mL^2}$. The transition energy is $\Delta E = E_2 - E_1 = \frac{3h^2}{8mL^2}$. Set the transition energy equal to the energy hc/λ of the emitted photon. This gives $\frac{hc}{\lambda} = \frac{3h^2}{8mL^2}$. $\lambda = \frac{8mcL^2}{3h} = \frac{8(9.109\times10^{-31}\text{ kg})(2.998\times10^8\text{ m/s})(4.18\times10^{-9}\text{ m})^2}{3(6.626\times10^{-34}\text{ J}\cdot\text{s})}$.

$\lambda = 1.92\times10^{-5}\text{ m} = 19.2\ \mu\text{m}$.

(b) Second excited level has $n = 3$ and $E_3 = \frac{9h^2}{8mL^2}$. The transition energy is

$\Delta E = E_3 - E_2 = \frac{9h^2}{8mL^2} - \frac{4h^2}{8mL^2} = \frac{5h^2}{8mL^2}$. $\frac{hc}{\lambda} = \frac{5h^2}{8mL^2}$, so $\lambda = \frac{8mcL^2}{5h} = \frac{3}{5}(19.2\ \mu\text{m}) = 11.5\ \mu\text{m}$.

EVALUATE: The energy spacing between adjacent levels increases with n, and this corresponds to a shorter wavelength and more energetic photon in part (b) than in part (a).

40.46. **IDENTIFY:** The probability is $|\psi|^2\,dx$, with ψ evaluated at the specified value of x.

SET UP: For the ground state, the normalized wave function is $\psi_1 = \sqrt{2/L}\,\sin(\pi x/L)$.

EXECUTE: **(a)** $(2/L)\sin^2(\pi/4)dx = dx/L$.

(b) $(2/L)\sin^2(\pi/2)dx = 2dx/L$.

(c) $(2/L)\sin^2(3\pi/4) = dx/L$.

EVALUATE: Our results agree with Figure 40.12b in the textbook. $|\psi|^2$ is largest at the center of the box, at $x = L/2$. $|\psi|^2$ is symmetric about the center of the box, so is the same at $x = L/4$ as at $x = 3L/4$.

40.47. **IDENTIFY** and **SET UP:** The normalized wave function for the $n = 2$ first excited level is $\psi_2 = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$. $P = |\psi(x)|^2\,dx$ is the probability that the particle will be found in the interval x to $x + dx$.

EXECUTE: **(a)** $x = L/4$.

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\left(\frac{2\pi}{L}\right)\left(\frac{L}{4}\right)\right) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{2}\right) = \sqrt{\frac{2}{L}}.$$

$$P = (2/L)dx.$$

(b) $x = L/2$.

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\left(\frac{2\pi}{L}\right)\left(\frac{L}{2}\right)\right) = \sqrt{\frac{2}{L}}\sin(\pi) = 0.$$

$$P = 0.$$

(c) $x = 3L/4$.

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\left(\frac{2\pi}{L}\right)\left(\frac{3L}{4}\right)\right) = \sqrt{\frac{2}{L}}\sin\left(\frac{3\pi}{2}\right) = -\sqrt{\frac{2}{L}}.$$

$$P = (2/L)dx.$$

EVALUATE: Our results are consistent with the $n = 2$ part of Figure 40.12 in the textbook. $|\psi|^2$ is zero at the center of the box and is symmetric about this point.

40.48. **IDENTIFY:** This problem deals with quantum mechanical tunneling.

SET UP and **EXECUTE:** **(a)** We want the average energy per electron. $\Delta U = q\Delta V$ gives $E = eV = e(5\text{ V}) = 5\text{ eV}$.

(b) $U_0 = q\Delta V = e(12\text{ V}) = 12\text{ eV}$.

(c) We want the probability of tunneling, which is $T = Ge^{-2\kappa L}$, where

$$G = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) \quad \kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}.$$

Using E and U_0 from parts (a) and (b), we get $G = 3.89$ and $\kappa = 1.354 \times 10^{10}\text{ m}^{-1}$. Using the equation for T gives a tunneling probability of $T = 6.74 \times 10^{-12}$.

(d) We want the tunneling current. The current density is $J = nev$. On the R side the number of electrons is Tn. $I = JA = TneAv$.

(e) We want the effective speed of tunneled electrons. Using the definition of effective speed gives $v = \sqrt{2eV/m}$.

(f) We want the tunneling current. Using the results from (d) and (e) with the given numbers, we get $I = TneAv = Tnea\sqrt{2eV/m} = Tne\pi(d/2)^2\sqrt{2eV/m} = 95\text{ mA}$.

EVALUATE: In part (c) we see that only a small percent of the electrons tunnel through the barrier.

40.49. **IDENTIFY:** The probability of the particle being between x_1 and x_2 is $\int_{x_1}^{x_2} |\psi|^2\, dx$, where ψ is the normalized wave function for the particle.

(a) SET UP: The normalized wave function for the ground state is $\psi_1 = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{\pi x}{L}\right)$.

EXECUTE: The probability P of the particle being between $x = L/4$ and $x = 3L/4$ is $P = \int_{L/4}^{3L/4} |\psi_1|^2 dx = \dfrac{2}{L}\int_{L/4}^{3L/4} \sin^2\left(\dfrac{\pi x}{L}\right)dx$. Let $y = \pi x/L;\, dx = (L/\pi)\,dy$ and the integration limits become $\pi/4$ and $3\pi/4$.

$$P = \frac{2}{L}\left(\frac{L}{\pi}\right)\int_{\pi/4}^{3\pi/4} \sin^2 y\, dy = \frac{2}{\pi}\left[\frac{1}{2}y - \frac{1}{4}\sin 2y\right]_{\pi/4}^{3\pi/4}.$$

$$P = \frac{2}{\pi}\left[\frac{3\pi}{8} - \frac{\pi}{8} - \frac{1}{4}\sin\left(\frac{3\pi}{2}\right) + \frac{1}{4}\sin\left(\frac{\pi}{2}\right)\right].$$

$P = \dfrac{2}{\pi}\left(\dfrac{\pi}{4} - \dfrac{1}{4}(-1) + \dfrac{1}{4}(1)\right) = \dfrac{1}{2} + \dfrac{1}{\pi} = 0.818.$ (Note: The integral formula $\int \sin^2 y\, dy = \dfrac{1}{2}y - \dfrac{1}{4}\sin 2y$ was used.)

(b) SET UP: The normalized wave function for the first excited state is $\psi_2 = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{2\pi x}{L}\right)$.

EXECUTE: $P = \int_{L/4}^{3L/4} |\psi_2|^2 dx = \dfrac{2}{L}\int_{L/4}^{3L/4} \sin^2\left(\dfrac{2\pi x}{L}\right)dx$. Let $y = 2\pi x/L;\, dx = (L/2\pi)\,dy$ and the integration limits become $\pi/2$ and $3\pi/2$.

$$P = \frac{2}{L}\left(\frac{L}{2\pi}\right)\int_{\pi/2}^{3\pi/2} \sin^2 y\, dy = \frac{1}{\pi}\left[\frac{1}{2}y - \frac{1}{4}\sin 2y\right]_{\pi/2}^{3\pi/2} = \frac{1}{\pi}\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) = 0.500.$$

EVALUATE: **(c)** These results are consistent with Figure 40.11b in the textbook. That figure shows that $|\psi|^2$ is more concentrated near the center of the box for the ground state than for the first excited state; this is consistent with the answer to part (a) being larger than the answer to part (b). Also, this figure shows that for the first excited state half the area under $|\psi|^2$ curve lies between $L/4$ and $3L/4$, consistent with our answer to part (b).

40.50. **IDENTIFY:** We start with the penetration distance formula given in the problem.

SET UP: The given formula is $\eta = \dfrac{\hbar}{\sqrt{2m(U_0 - E)}}$.

EXECUTE: **(a)** Substitute the given numbers into the formula:

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}} = \frac{1.055\times 10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times 10^{-31}\ \text{kg})(20\ \text{eV} - 13\ \text{eV})(1.602\times 10^{-19}\ \text{J/eV})}} = 7.4\times 10^{-11}\ \text{m}.$$

(b) $\eta = \dfrac{1.055\times 10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(1.67\times 10^{-27}\ \text{kg})(30\ \text{MeV} - 20\ \text{MeV})(1.602\times 10^{-13}\ \text{J/MeV})}} = 1.44\times 10^{-15}\ \text{m}.$

EVALUATE: The penetration depth varies widely depending on the mass and energy of the particle.

40.51. **IDENTIFY:** Carry out the calculations that are specified in the problem.

SET UP: For a free particle, $U(x) = 0$, so Schrödinger's equation becomes $\dfrac{d^2\psi(x)}{dx^2} = -\dfrac{2m}{\hbar^2}E\psi(x)$.

EXECUTE: **(a)** The graph is given in Figure 40.51.

(b) For $x < 0$: $\psi(x) = e^{+\kappa x}$. $\dfrac{d\psi(x)}{dx} = \kappa e^{+\kappa x}$. $\dfrac{d^2\psi(x)}{dx} = \kappa^2 e^{+\kappa x}$. So $\kappa^2 = -\dfrac{2m}{\hbar^2}E \Rightarrow E = -\dfrac{\hbar^2\kappa^2}{2m}$.

(c) For $x > 0$: $\psi(x) = e^{-\kappa x}$. $\dfrac{d\psi(x)}{dx} = -\kappa e^{-\kappa x}$. $\dfrac{d^2\psi(x)}{dx} = \kappa^2 e^{-\kappa x}$. So again $\kappa^2 = -\dfrac{2m}{\hbar^2}E \Rightarrow E = \dfrac{-\hbar^2\kappa^2}{2m}$.

Parts (b) and (c) show $\psi(x)$ satisfies the Schrödinger's equation, provided $E = \dfrac{-\hbar^2\kappa^2}{2m}$.

EVALUATE: **(d)** $\dfrac{d\psi(x)}{dx}$ is discontinuous at $x = 0$. (That is, it is negative for $x > 0$ and positive for $x < 0$.) Therefore, this ψ is not an acceptable wave function; $d\psi/dx$ must be continuous everywhere, except where $U \to \infty$.

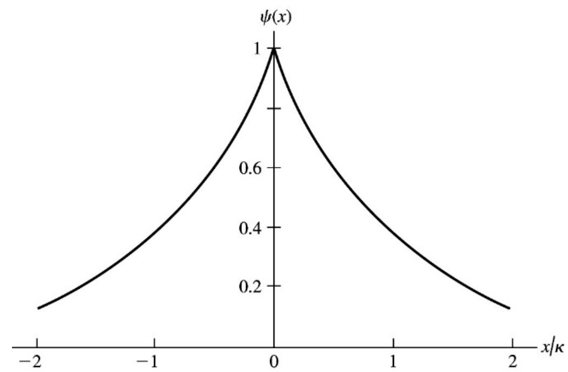


Figure 40.51

40.52. **IDENTIFY:** $T = Ge^{-2\kappa L}$ with $G = 16\dfrac{E}{U_0}\left(1 - \dfrac{E}{U_0}\right)$ and $\kappa = \dfrac{\sqrt{2m(U_0 - E)}}{\hbar}$, so $L = -\dfrac{1}{2\kappa}\ln\left(\dfrac{T}{G}\right)$.

SET UP: $E = 5.5\text{ eV}, U_0 = 10.0\text{ eV}, m = 9.11\times10^{-31}\text{ kg}, \text{and } T = 0.0050.$

EXECUTE: $\kappa = \dfrac{\sqrt{2(9.11\times10^{-31}\text{ kg})(4.5\text{ eV})(1.60\times10^{-19}\text{ J/eV})}}{(1.054\times10^{-34}\text{ J}\cdot\text{s})} = 1.09\times10^{10}\text{ m}^{-1}$ and

$G = 16\dfrac{5.5\text{ eV}}{10.0\text{ eV}}\left(1 - \dfrac{5.5\text{ eV}}{10.0\text{ eV}}\right) = 3.96.$ Therefore the barrier width L is $L = -\dfrac{1}{2\kappa}\ln\left(\dfrac{T}{G}\right)$

$= -\dfrac{1}{2(1.09\times10^{10}\text{ m}^{-1})}\ln\left(\dfrac{0.0050}{3.96}\right) = 3.1\times10^{-10}\text{ m} = 0.31\text{ nm}.$

EVALUATE: The energies here are comparable to those of electrons in atoms, and the barrier width we calculated is on the order of the diameter of an atom.

40.53. **IDENTIFY:** This problem is about a quantum mechanical harmonic oscillator.

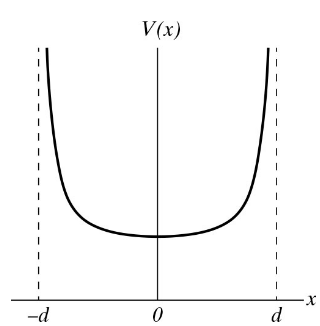


Figure 40.53

SET UP and **EXECUTE:** **(a)** For the sketch, see Figure 40.53.
(b) First combine the terms in the given equation for $V(x)$ to obtain the following:

$$V(x)=\frac{2kq^2}{d}\frac{1}{1-(x/d)^2}, \text{ where } k=1/4\pi\epsilon_0 .$$

Expand the right-hand fraction using $(1+z)^n \approx 1+nz$, where $z \ll 1$, $n=-1$, $z=-(x/d)^2$.

$$V(x)=\frac{2kq^2}{d}\left[1+\frac{x^2}{d^2}\right].$$

Using $q = 6e$ and putting in k gives

$$V(x)=\frac{18e^2}{\pi\epsilon_0 d}+\frac{18e^2}{\pi\epsilon_0 d^3}x^2.$$

(c) We want the spring constant. Using the second term for $V(x)$, we see that

$$V=\frac{1}{2}k'x^2=\frac{18e^2}{\pi\epsilon_0 d^3}x^2$$

$$k=\frac{36e^2}{\pi\epsilon_0 d^3}=265 \text{ N/m}.$$

(d) The classical ground state is $x = 0$, so the energy at this state is

$$V=\frac{18e^2}{\pi\epsilon_0 d}=207 \text{ eV}.$$

(e) We want the energy of the lowest energy photon. Use the quantum energy states.

$$E_0=\frac{1}{2}\hbar\sqrt{\frac{k'}{m}} \text{ and } E_n=\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}}, \text{ so } E_n=2E_0\left(n+\frac{1}{2}\right).$$

The lowest energy photon is due to a transition from the $n = 1$ state to the $n = 0$ state, so

$$\Delta E=2E_0\left[\left(1+\frac{1}{2}\right)-\left(0+\frac{1}{2}\right)\right]=2E_0=2(0.0379 \text{ eV})=0.0758 \text{ eV}.$$

(f) Use $E=hc/\lambda$ with $E = 0.0758$ eV, giving $\lambda = 16.4\ \mu$m.
EVALUATE: There is obviously a big difference between a quantum oscillator and a classical oscillator.

40.54. **IDENTIFY:** Compare the energy E of the oscillator to the equation $E_n=(n+\frac{1}{2})\hbar\omega$ in order to determine n.
SET UP: At the equilibrium position the potential energy is zero and the kinetic energy equals the total energy.
EXECUTE: **(a)** $E=\frac{1}{2}mv^2=[n+(1/2)]\hbar\omega=[n+(1/2)]hf$, and solving for n,

$$n=\frac{\frac{1}{2}mv^2}{hf}-\frac{1}{2}=\frac{(1/2)(0.020 \text{ kg})(0.480 \text{ m/s})^2}{(6.626\times10^{-34} \text{ J}\cdot\text{s})(1.50 \text{ Hz})}-\frac{1}{2}=2.3\times10^{30}.$$

(b) The difference between energies is $\hbar\omega=hf=(6.63\times10^{-34} \text{ J}\cdot\text{s})(1.50 \text{ Hz})=9.95\times10^{-34} \text{ J}$. This energy is too small to be detected with current technology.
EVALUATE: This oscillator can be described classically; quantum effects play no measurable role.

40.55. **IDENTIFY** and **SET UP:** Calculate the angular frequency ω of the pendulum and apply $E_n=(n+\frac{1}{2})\hbar\omega$ for the energy levels.

EXECUTE: $\omega=\frac{2\pi}{T}=\frac{2\pi}{0.500 \text{ s}}=4\pi \text{ s}^{-1}.$

The ground-state energy is $E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}(1.055\times10^{-34}\ \text{J}\cdot\text{s})(4\pi\ \text{s}^{-1}) = 6.63\times10^{-34}\ \text{J}$.

$$E_0 = 6.63\times10^{-34}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 4.14\times10^{-15}\ \text{eV}.$$

$$E_n = (n+\tfrac{1}{2})\hbar\omega.$$

$$E_{n+1} = (n+1+\tfrac{1}{2})\hbar\omega.$$

The energy difference between the adjacent energy levels is

$$\Delta E = E_{n+1} - E_n = \hbar\omega = 2E_0 = 1.33\times10^{-33}\ \text{J} = 8.30\times10^{-15}\ \text{eV}.$$

EVALUATE: These energies are much too small to detect. Quantum effects are not important for ordinary size objects.

40.56. **IDENTIFY:** If the given wave function is a solution to the Schrödinger equation, we will get an identity when we substitute that wave function into the Schrödinger equation.

SET UP: The given wave function is $\psi_1(x) = A_1 x e^{-\alpha^2 x^2/2}$ and the Schrödinger equation is

$$-\frac{\hbar}{2m}\frac{d^2\psi(x)}{dx^2} + \frac{k' x^2}{2}\psi(x) = E\,\psi(x).$$

EXECUTE: **(a)** Start by taking the indicated derivatives: $\psi_1(x) = A_1 x e^{-\alpha^2 x^2/2}$.

$$\frac{d\psi_1(x)}{dx} = -\alpha^2 x^2 A_1 e^{-\alpha^2 x^2/2} + A_1 e^{-\alpha^2 x^2/2}.$$

$$\frac{d^2\psi_1(x)}{dx^2} = -A_1\alpha^2 2x e^{-\alpha^2 x^2/2} - A_1\alpha^2 x^2(-\alpha^2 x)e^{-\alpha^2 x^2/2} + A_1(-\alpha^2 x)e^{-\alpha^2 x^2/2}.$$

$$\frac{d^2\psi_1(x)}{dx^2} = \left[-2\alpha^2 + (\alpha^2)^2 x^2 - \alpha^2\right]\psi_1(x) = \left[-3\alpha^2 + (\alpha^2)^2 x^2\right]\psi_1(x).$$

$$-\frac{\hbar}{2m}\frac{d^2\psi_1(x)}{dx^2} = -\frac{\hbar^2}{2m}\left[-3\alpha^2 + (\alpha^2)^2 x^2\right]\psi_1(x).$$

Eq. (40.44) is $-\frac{\hbar}{2m}\frac{d^2\psi(x)}{dx^2} + \frac{k' x^2}{2}\psi(x) = E\,\psi(x)$. Substituting the above result into that equation gives $-\frac{\hbar^2}{2m}\left[-3\alpha^2 + (\alpha^2)^2 x^2\right]\psi_1(x) + \frac{k' x^2}{2}\psi_1(x) = E\,\psi_1(x)$. Since $\alpha^2 = \frac{m\omega}{\hbar}$ and $\omega = \sqrt{\frac{k'}{m}}$, the coefficient of x^2 is $-\frac{\hbar^2}{2m}(\alpha^2)^2 + \frac{k'}{2} = -\frac{\hbar^2}{2m}\left(\frac{m\omega}{\hbar}\right)^2 + \frac{m\omega^2}{2} = 0$.

(b) $A_1 = \left(\frac{m\omega}{\hbar}\right)^{3/4}\left(\frac{4}{\pi}\right)^{1/4}$.

(c) The probability density function $|\psi|^2$ is $|\psi_1(x)|^2 = A_1^2 x^2 e^{-\alpha^2 x^2}$.

At $x = 0$, $|\psi_1|^2 = 0$. $\frac{d|\psi_1(x)|^2}{dx} = A_1^2 2x e^{-\alpha^2 x^2} + A_1^2 x^2(-\alpha^2 2x)e^{-\alpha^2 x^2} = A_1^2 2x e^{-\alpha^2 x^2} - A_1^2 2x^3\alpha^2 e^{-\alpha^2 x^2}$.

At $x = 0$, $\frac{d|\psi_1(x)|^2}{dx} = 0$. At $x = \pm\frac{1}{\alpha}$, $\frac{d|\psi_1(x)|^2}{dx} = 0$.

$$\frac{d^2|\psi_1(x)|^2}{dx^2} = A_1^2 2e^{-\alpha^2 x^2} + A_1^2 2x(-\alpha^2 2x)e^{-\alpha^2 x^2} - A_1^2 2(3x^2)\alpha^2 e^{-\alpha^2 x^2} - A_1^2 2x^3\alpha^2(-\alpha^2 2x)e^{-\alpha^2 x^2}.$$

$\frac{d^2|\psi_1(x)|^2}{dx^2} = A_1^2 2e^{-\alpha^2 x^2} - A_1^2 4x^2\alpha^2 e^{-\alpha^2 x^2} - A_1^2 6x^2\alpha^2 e^{-\alpha^2 x^2} + A_1^2 8x^4(\alpha^2)^2 e^{-\alpha^2 x^2}$. At $x = 0$,

$\frac{d^2|\psi_1(x)|^2}{dx^2} > 0.$ So at $x = 0,$ the first derivative is zero and the second derivative is positive. Therefore, the probability density function has a minimum at $x = 0.$ At $x = \pm\frac{1}{\alpha},$ $\frac{d^2|\psi_1(x)|^2}{dx^2} < 0.$ So at $x = \pm\frac{1}{\alpha},$ the first derivative is zero and the second derivative is negative. Therefore, the probability density function has maxima at $x = \pm\frac{1}{\alpha},$ corresponding to the classical turning points for $n = 0$ as found in the previous question.

EVALUATE: $\psi_1(x) = A_1 x e^{-\alpha^2 x^2/2}$ is a solution to Eq. (40.44) if $-\frac{\hbar^2}{2m}(-3\alpha^2)\psi_1(x) = E\,\psi_1(x)$ or $E = \frac{3\hbar^2\alpha^2}{2m} = \frac{3\hbar\omega}{2}.$ $E_1 = \frac{3\hbar\omega}{2}$ corresponds to $n = 1$ in Eq. (40.46).

40.57. IDENTIFY: For a standing wave in the box, there must be a node at each wall and $n\left(\frac{\lambda}{2}\right) = L.$

SET UP: $p = \frac{h}{\lambda} \neq,$ so $mv = \frac{h}{\lambda}.$

EXECUTE: **(a)** For a standing wave, $n\lambda = 2L,$ and $E_n = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{n^2h^2}{8mL^2}.$

(b) With $L = a_0 = 0.5292\times10^{-10}$ m, $E_1 = 2.15\times10^{-17}$ J $= 134$ eV.

EVALUATE: For a hydrogen atom, E_n is proportional to $1/n^2,$ so this is a very poor model for a hydrogen atom. In particular, it gives very inaccurate values for the separations between energy levels.

40.58. IDENTIFY and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** As with the particle in a box, $\psi(x) = A\sin kx,$ where A is a constant and $k^2 = 2mE/\hbar^2.$ Unlike the particle in a box, however, k and hence E do not have simple forms.

(b) For $x > L,$ the wave function must have the form of $\psi(x) = Ce^{\kappa x} + De^{-\kappa x}.$ For the wave function to remain finite as $x \to \infty, C = 0.$ The constant $\kappa^2 = 2m(U_0 - E)/\hbar,$ as in $\psi(x) = Ce^{\kappa x} + De^{-\kappa x}.$

(c) At $x = L,$ $A\sin kL = De^{-\kappa L}$ and $kA\cos kL = -\kappa De^{-\kappa L}.$ Dividing the second of these by the first gives $k\cot kL = -\kappa,$ a transcendental equation that must be solved numerically for different values of the length L and the ratio $E/U_0.$

EVALUATE: When $U_0 \to \infty,$ $\kappa \to \infty$ and $\frac{\cos(kL)}{\sin(kL)} \to \infty.$ The solutions become $k = \frac{n\pi}{L}, n = 1, 2, 3,\ldots,$ the same as for a particle in a box.

40.59. IDENTIFY and **SET UP:** The energy levels for an infinite potential well are $E_n = \frac{n^2h^2}{8mL^2} = E_1 n^2.$ The energy of the absorbed photon is equal to the energy difference between the levels. The energy of a photon is $E = hf = hc/\lambda,$ so $\Delta E = hf = E_{n+1} - E_n.$

EXECUTE: **(a)** For the first transition, we have $hf_1 = E_1(n^2 - 1^2),$ and for the second transition we have $hf_2 = E_1[(n+1)^2 - 1^2].$ Taking the ratio of these two equations gives

$$\frac{hf_2}{hf_1} = \frac{16.9}{9.0} = \frac{(n+1)^2 - 1}{n^2 - 1} = \frac{n^2 + 2n + 1 - 1}{n^2 - 1} = \frac{n^2 + 2n}{n^2 - 1}.$$

Rearranging and collecting terms gives the quadratic equation $n^2\left(\frac{16.9}{9.0}-1\right)-2n-\frac{16.9}{9.0}=0.$ Using the quadratic formula and taking the positive root gives $n = 3.0$, so $n = 3$. Therefore the transitions are from the $n = 3$ and $n = 4$ levels to the $n = 1$ level.

(b) Using the $3 \rightarrow 1$ transition with $f_1 = 9.0\times10^{14}$ Hz, we have

$hf_1 = (h^2/8mL^2)(3^2 - 1^2) = h^2/mL^2.$

$$L=\sqrt{\frac{h}{f_1 m}}=\sqrt{\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.0\times10^{14}\ \text{Hz})(9.109\times10^{-31}\ \text{kg})}}=9.0\times10^{-10}\ \text{m}=0.90\ \text{nm}.$$

(c) The longest wavelength is for the smallest energy, and that would be for a transition between $n = 1$ and $n = 2$ levels. Comparing the $1 \rightarrow 3$ transition and the $1 \rightarrow 2$ transition, we have

$$\frac{hf_{1\to2}}{hf_{1\to3}}=\frac{E_1(2^2-1^2)}{E_1(3^2-1^2)} \quad\rightarrow\quad f_{1\to2}=\frac{3}{8}f_{1\to3}=\frac{3}{8}(9.0\times10^{14}\ \text{Hz}).$$

$$\lambda=c/f=\frac{3.00\times10^{8}\ \text{m/s}}{\frac{3}{8}(9.0\times10^{14}\ \text{Hz})}=890\ \text{nm}.$$

EVALUATE: This wavelength is too long to be visible light. The wavelength of the 9.0×10^{14} Hz photon is 333 nm, which is too short to be visible, as is the 16.9×10^{14} Hz photon. So none of these photons will be visible.

40.60. IDENTIFY and **SET UP:** Provided that $T \ll 1$, the probability T of a particle with energy E and mass m tunneling through a potential barrier of width L and height U_0 is $T = Ge^{-2\kappa L}$, where

$$G=16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)\ \text{and}\ \kappa=\frac{\sqrt{2m(U_0-E)}}{\hbar}.$$

EXECUTE: **(a)** Using the values of E in the table in the problem, we calculate G and κ. For example, for $E = 4.0$ eV, we have $G=16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)=16\frac{4.0\ \text{eV}}{8.0\ \text{eV}}\left(1-\frac{4.0\ \text{eV}}{8.0\ \text{eV}}\right)=4.0.$

$$U_0-E=8.0\ \text{eV}-4.0\ \text{eV}=4.0\ \text{eV}=6.4\times10^{-19}\ \text{J}.$$

$$\kappa=\frac{\sqrt{2m(U_0-E)}}{\hbar}=\frac{\sqrt{2(9.11\times10^{-31}\ \text{kg})(6.4\times10^{-19}\ \text{J})}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}=1.02\times10^{10}\ \text{m}^{-1}.$$

Repeating these calculations for the other values of E, and also calculating $\ln(T/G)$, we get the following:

E(eV)	T	G	κ (m^{-1})	$\ln(T/G)$
4.0	2.40×10^{-6}	4.0	1.02×10^{10}	–14.3
5.0	1.50×10^{-5}	3.75	8.87×10^{9}	–12.4
6.0	1.20×10^{-4}	3.0	7.25×10^{9}	–10.1
7.0	1.30×10^{-3}	1.75	5.12×10^{9}	–7.2
7.6	8.10×10^{-3}	0.76	3.24×10^{9}	–4.5

Figure 40.60 shows the graph of $\ln(T/G)$ versus κ. For $T \ll 1$, we have $T = Ge^{-2\kappa L}$, which we can write as $T/G = e^{-2\kappa L}$. Taking natural logarithms of both sides of the equation gives $\ln(T/G) = -2L\,\kappa$. From this last equation, we would expect a graph of $\ln(T/G)$ versus κ to be a straight line with slope equal to $-2L$.

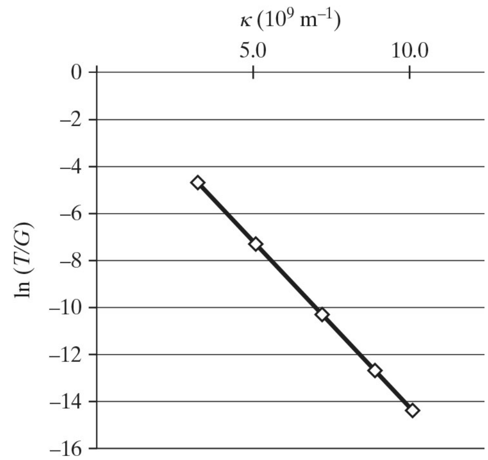


Figure 40.60

(b) The slope of the best-fit straight line for our graph is -1.396×10^{-9} m, so $-2L = -1.396 \times 10^{-9}$ m, which gives $L = 0.698 \times 10^{-9}$ m, which rounds to $L = 0.70$ nm.

EVALUATE: This width is about 7 times the width of a hydrogen atom in the Bohr model.

40.61. IDENTIFY and **SET UP:** The transmission coefficient T is equal to 1 when the width L of the barrier is $L = \frac{1}{2}\lambda,\ \lambda,\ \frac{3}{2}\lambda,\ 2\lambda,\ \ldots = n\lambda/2,$ where $n = 1, 2, 3, \ldots$, and where λ is the de Broglie wavelength of the electron, given by $\lambda = h/p$. The total energy of the electron is $E = U + K$, and $K = p^2/2m$.

EXECUTE: From the condition on λ, we have $\lambda_n = 2L/n$. Therefore $\lambda = h/p = 2L/n$, which gives $p = nh/2L$. The kinetic energy is $K = p^2/2m$, so $K_n = \dfrac{p^2}{2m} = \dfrac{\left(\dfrac{nh}{2L}\right)^2}{2m} = n^2\left(\dfrac{h^2}{8mL^2}\right) = n^2K_1$. The three lowest values of K are for $n = 1$, 2, and 3.

$K_1 = h^2/8mL^2 = (6.626 \times 10^{-34}\ \text{J}\cdot\text{s})^2/[8(9.11 \times 10^{-31}\ \text{kg})(1.8 \times 10^{-10}\ \text{m})^2] = 1.86 \times 10^{-18}\ \text{J} = 11.6$ eV.

The total energy is $E = K + U$, so $E_1 = K_1 + U = 11.6\ \text{eV} + 10\ \text{eV} = 22\ \text{eV}$.

For the $n = 2$ state, we have

$$K_2 = 2^2 K_1 = 4(11.6\ \text{eV}) = 46.4\ \text{eV, so } E_2 = 46.4\ \text{eV} + 10\ \text{eV} = 56\ \text{eV}.$$

For the $n = 3$ state, we have

$K_3 = 3^2 K_1 = 9(11.6\ \text{eV}) = 104.4\ \text{eV, so } E_3 = 104.4\ \text{eV} + 10\ \text{eV} = 114\ \text{eV},$ which rounds to 110 eV.

EVALUATE: We cannot use Eq. (40.42) because T is not small.

40.62. IDENTIFY: This problem deals with the Schrödinger wave equation.

SET UP: $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx} = E\psi$

EXECUTE: **(a)** For a free particle, ψ is in the form $\psi = Ae^{\pm in\theta}$. The wave function repeats as θ changes by 2π. If n is an integer, the wave functions must be $\psi_n^+ = A_+e^{+in\theta}$ and $\psi_n^- = A\_e^{-in\theta}$. The – function is for clockwise motion and the + function is for counterclockwise motion.

(b) We normalize the functions to find the coefficients.

$$1 = \int\left|\psi_+\right|^2 dx = \int_0^{2\pi R} A_+^2\, e^{in\theta}e^{-in\theta}\, dx = A_+^2\, 2\pi R.\ A_+ = 1/\sqrt{2\pi R}.$$

The same procedure gives the same value for $A\_$.

(c) We want the energy levels E_n. The wave function must satisfy the time-independent Schrödinger equation. Using $x = R\theta$ for ψ^+ gives the following:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx} = E\psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_n^+}{dx^2} = -\frac{\hbar^2}{2m}\frac{d^2\psi_n^+}{R^2\, d\theta^2} = -\frac{\hbar^2}{2mR^2}\frac{d^2}{d\theta^2}\left(A_+e^{in\theta}\right) = -\frac{\hbar^2}{2mR^2}A_+(in)^2e^{in\theta} = -\frac{\hbar^2}{2mR^2}(-n^2)A_+e^{in\theta}$$

The right-hand part of the last equation must be $E\psi$, so we find that

$$E_n = \frac{\hbar^2 n^2}{2mR^2}.$$

(d) We want the time-dependent wave function. We use the following:

$$\omega = E_1/\hbar,\ E_n = n^2E_1 = n^2\hbar\omega,\ \ \Psi(x,t) = \psi(x)e^{-iEt/\hbar}.$$

For Ψ_n^+ we have

$$\Psi_n^+(x,t) = \psi_n^+(x)e^{-iE_nt/\hbar} = A_+e^{in\theta}e^{-iE_nt/\hbar} = \frac{1}{\sqrt{2\pi R}}e^{in(\theta-n\omega t)} = \frac{1}{\sqrt{2\pi R}}e^{in(x/R-n\omega t)}.$$

The same procedure leads to

$$\Psi_n^-(x,t) = \frac{1}{\sqrt{2\pi R}}e^{-in(\theta+n\omega t)} = \frac{1}{\sqrt{2\pi R}}e^{-in(x/R+n\omega t)}.$$

(e) We want the probability density.

$$E_n/\hbar = n^2\omega$$

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\left(\psi_1^+(x)e^{-iE_1t/\hbar} + \psi_2^+(x)e^{-iE_2t/\hbar}\right) = \frac{1}{\sqrt{2}}\left(A_1e^{i\theta}e^{-i\omega t} + A_2e^{2i\theta}e^{-4i\omega t}\right)$$

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2\pi R}}\left(e^{i(\theta-\omega t)} + e^{2i(\theta-2\omega t)}\right)$$

To find the probability density, we square the wave function and collect terms:

$$\left|\Psi(x,t)\right|^2 = \frac{1}{4\pi R}\left(2 + e^{-i\theta}e^{3i\omega t} + e^{i\theta}e^{-3i\omega t}\right)$$

Now use $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$ to remove the exponentials, giving

$$\left|\Psi(x,t)\right|^2 = \frac{1}{2\pi}(1+\cos\theta\cos 3\omega t + \sin\theta\sin 3\omega t) = \frac{1}{2\pi R}[1+\cos(\theta-3\omega t)]$$

Now use the identity suggested in the problem with $\alpha = \theta - 3\omega t$ to obtain

$$\left|\Psi(x,t)\right|^2 = \frac{1}{\pi R}\cos^2\left(\frac{\theta-3\omega t}{2}\right) = \frac{1}{\pi R}\cos^2\left(\frac{x/R-3\omega t}{2}\right).$$

(f) We want the angular speed of the density peak. The density peak occurs when the probability amplitude is a maximum. This occurs when the cosine factor in the previous result is equal to 1. From this we see that $\theta - 3\omega t = 0$. The angular speed is $\theta/t = 3\omega$.

(g) We want the speed for the electron at the Bohr radius.

$$v = R(\theta/t) = R(3\omega) = 3R\omega.\ \ \omega = E_1/\hbar = \frac{\hbar^2/2mR^2}{\hbar} = \frac{\hbar}{2mR^2}.\ \ v = 3R\omega = 3R\left(\frac{\hbar}{2mR^2}\right) = \frac{3\hbar}{2mR}.$$

Putting in the numbers with $R = a_0 = 5.29 \times 10^{-11}$ m gives $v = 3.28 \times 10^6$ m/s.

Evaluate: This result is fairly close to the value of 2.2×10^6 m/s that Eq. (39.9) gives.

40.63. **Identify:** This problem involves commutators and the Schrödinger equation.

Set Up: $[A, B]f = A(Bf) - B(Af)$. The following operators are defined:

$$a_+ = \frac{1}{\sqrt{2m\hbar\omega}}\left(-\hbar\frac{d}{dx} + m\omega x\right) \text{ and } a_- = \frac{1}{\sqrt{2m\hbar\omega}}\left(+\hbar\frac{d}{dx} + m\omega x\right).$$

EXECUTE: (a) We want $[a_-, a_+]$.

$$[a_-, a_+]f = a_-(a_+ f) - a_+(a_- f) = \frac{1}{\sqrt{2m\hbar\omega}}\frac{1}{\sqrt{2m\hbar\omega}}\left(\hbar\frac{d}{dx} + m\omega x\right)\left[\left(-\hbar\frac{d}{dx} + m\omega x\right)f\right]$$
$$-\frac{1}{\sqrt{2m\hbar\omega}}\frac{1}{\sqrt{2m\hbar\omega}}\left(-\hbar\frac{d}{dx} + m\omega x\right)\left[\left(\hbar\frac{d}{dx} + m\omega x\right)f\right]$$

Carefully carrying out all the operations gives $[a_-, a_+]f = f$, so $[a_-, a_+] = 1$.

(b) We want $[a_+, a_-]\psi$, where ψ is a wave equation, which means that it is a solution to the Schrödinger equation. Follow the procedure in part (a). The solution is sketched out below.

$$a_+a_-\psi = \frac{1}{\sqrt{2m\hbar\omega}}\left(-\hbar\frac{d}{dx} + m\omega x\right)\left[\frac{1}{\sqrt{2m\hbar\omega}}\left(\hbar\frac{d}{dx} + m\omega x\right)\psi\right].$$

$$a_+a_-\psi = \frac{1}{\sqrt{2m\hbar\omega}}\left(-\hbar\frac{d}{dx} + m\omega x\right)\left(\hbar\frac{d\psi}{dx} + m\omega x\psi\right).$$

$$a_+a_-\psi = \frac{1}{\sqrt{2m\hbar\omega}}\left(-\hbar^2\frac{d^2\psi}{dx^2} - \hbar m\omega\frac{d(x\psi)}{dx} + m\omega x\frac{d\psi}{dx} + m^2\omega^2x^2\psi\right).$$

$$a_+a_-\psi = \frac{1}{\sqrt{2m\hbar\omega}}\left(-\hbar^2\frac{d^2\psi}{dx^2} - \hbar m\omega\psi + m^2\omega^2x^2\psi\right) = \frac{1}{\hbar\omega}\left(-\frac{\hbar^2}{2m}\psi'' + \frac{m\omega^2x^2}{2}\psi\right) - \frac{1}{2}\psi.$$

(c) Use the results of part (b) and the Schrödinger equation given with this problem. Note that the term in parentheses in the answer to (b) is the Schrödinger equation. Solving for it gives

$$\hbar\omega\left(a_+a_-\psi + \frac{1}{2}\psi\right) = -\frac{\hbar^2}{2m}\psi'' + \frac{m\omega^2x^2}{2}\psi$$

Therefore the operator H must be

$$H = \hbar\omega\left(a_+a_- + \frac{1}{2}\right).$$

(d) We want $[H, a_\pm]$. Using $[H, a_\pm]f = H(a_+ f) - a_+(Hf)$ gives

$$[H, a_+]f = H(a_+ f) - a_+\left[\hbar\omega\left(a_+a_- + \frac{1}{2}\right)\right]f = \hbar\omega\left[\left(\frac{1}{2} + a_+a_-\right)(a_+ f) - a_+\frac{f}{2} - a_+(a_+a_- f)\right]$$

$$[H, a_+]f = \hbar\omega(a_+)(a_-a_+ - a_+a_-)f = \hbar\omega a_+ f.$$

The same procedure leads to a similar result for $[H, a_\pm]$. Therefore the commutators are

$$[H, a_+] = \hbar\omega a_+ \text{ and } [H, a] = \hbar\omega a_-.$$

(e) We want to relate E_{n+1} to E_n.

For the n state: $H\Psi_n = E_n\Psi_n$

For the $n+1$ state: $H\psi_{n+1} = E_{n+1}\psi_{n+1}$ and $\psi_{n+1} = a_+\psi_n$, so $H\psi_{n+1} = Ha_+\psi_n$

$$[H, a_+]\psi_n = Ha_+\psi_n - a_+H\psi_n \text{ gives } Ha_+\psi_n = [H, a_+]\psi_n + a_+H\psi_n$$

$$H\psi_{n+1} = Ha_+\psi_n = ([H, a_+] + a_+H)\psi_n$$

Using the result from part (d), we can write this result as

$$H\psi_{n+1} = a_+H\psi_n + \hbar\omega a_+\psi_n$$

$$H\psi_n = E_n\psi_n \text{ and } \psi_{n+1} = a_+\psi_n \text{ gives } H\psi_{n+1} = E_n\psi_{n+1} + \hbar\omega\psi_{n+1} = (E_n + \hbar\omega)\psi_{n+1}$$

This result means that $E_{n+1} = E_n + \hbar\omega$.

(f) We want the ground state energy E_0.

Using $H\psi_0 = E_0\psi_0$ gives $\hbar\omega\left(\frac{1}{2}+a_+a_-\right)\psi_0 = E_0\psi_0$. Given that $a_-\psi_0 = 0$, we have

$\frac{1}{2}\hbar\omega\psi_0 = E_0\psi_0$, which tells us that $E_0 = \frac{1}{2}\hbar\omega$.

(g) We want the energy levels E_n. From (e) we know that the energy of each state is $\hbar\omega$ higher than the previous state, and from (f) we know that the energy of the lowest state is $\hbar\omega/2$. Therefore the energies are

$$E_0 = \hbar\omega\left(\frac{1}{2}\right), E_1 = \hbar\omega\left(1+\frac{1}{2}\right), E_2 = \hbar\omega\left(2+\frac{1}{2}\right), E_3 = \hbar\omega\left(3+\frac{1}{2}\right), \cdots E_n = \hbar\omega\left(n+\frac{1}{2}\right).$$

EVALUATE: The use of the method developed here depends on selecting appropriate operators $a_\pm$.

40.64. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** $E = K + U(x) = \frac{p^2}{2m} + U(x) \Rightarrow p = \sqrt{2m(E-U(x))}$. $\lambda = \frac{h}{p} \Rightarrow \lambda(x) = \frac{h}{\sqrt{2m(E-U(x))}}$.

(b) As $U(x)$ gets larger (i.e., $U(x)$ approaches E from below—recall $k \geq 0$), $E-U(x)$ gets smaller, so $\lambda(x)$ gets larger.

(c) When $E = U(x)$, $E - U(x) = 0$, so $\lambda(x) \to \infty$.

(d) $\int_a^b \frac{dx}{\lambda(x)} = \int_a^b \frac{dx}{h/\sqrt{2m(E-U(x))}} = \frac{1}{h}\int_a^b \sqrt{2m(E-U(x))}\,dx = \frac{n}{2} \Rightarrow \int_a^b \sqrt{2m(E-U(x))}\,dx = \frac{hn}{2}$.

(e) $U(x) = 0$ for $0 < x < L$ with classical turning points at $x = 0$ and $x = L$. So,

$\int_a^b \sqrt{2m[E-U(x)]}\,dx = \int_0^L \sqrt{2mE}\,dx = \sqrt{2mE}\int_0^L dx = \sqrt{2mE}L$. So, from part (d),

$\sqrt{2mE}L = \frac{hn}{2} \Rightarrow E = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2n^2}{8mL^2}$.

EVALUATE: **(f)** Since $U(x) = 0$ in the region between the turning points at $x = 0$ and $x = L$, the result is the *same* as part (e). The height U_0 never enters the calculation. WKB is best used with *smoothly* varying potentials $U(x)$.

40.65. **IDENTIFY:** Perform the calculations specified in the problem.

SET UP: $U(x) = \frac{1}{2}k'x^2$.

EXECUTE: **(a)** At the turning points $E = \frac{1}{2}k'x_{\text{TP}}^2 \Rightarrow x_{\text{TP}} = \pm\sqrt{\frac{2E}{k'}}$.

(b) $\int_{-\sqrt{2E/k'}}^{+\sqrt{2E/k'}} \sqrt{2m(E-\frac{1}{2}k'x^2)}dx = \frac{nh}{2}$. To evaluate the integral, we want to get it into a form that matches the standard integral given. $\sqrt{2m\left(E-\frac{1}{2}k'x^2\right)} = \sqrt{2mE - mk'x^2} = \sqrt{mk'}\sqrt{\frac{2mE}{mk'} - x^2} = \sqrt{mk'}\sqrt{\frac{2E}{k'} - x^2}$.

Letting $A^2 = \frac{2E}{k'}$, $a = -\sqrt{\frac{2E}{k'}}$, *and* $b = +\sqrt{\frac{2E}{k'}}$

$$\Rightarrow \sqrt{mk'}\int_a^b \sqrt{A^2 - x^2}\,dx = 2\frac{\sqrt{mk'}}{2}\left[x\sqrt{A^2-x^2} + A^2 \arcsin\left(\frac{x}{|A|}\right)\right]\Bigg|_0^b$$

$$= \sqrt{mk'}\left[\sqrt{\frac{2E}{k'}}\sqrt{\frac{2E}{k'} - \frac{2E}{k'}} + \frac{2E}{k'}\arcsin\left(\frac{\sqrt{2E/k'}}{\sqrt{2E/k'}}\right)\right] = \sqrt{mk'}\,\frac{2E}{k'}\arcsin(1) = 2E\sqrt{\frac{m}{k'}}(\tfrac{1}{2}).$$

Using WKB, this is equal to $\frac{hn}{2}$, so $E\sqrt{\frac{m}{k'}}\pi = \frac{hn}{2}$. Recall $\omega = \sqrt{\frac{k'}{m}}$, so $E = \frac{h}{2\pi}\omega n = \hbar\omega n.$

EVALUATE: **(c)** We are missing the zero-point-energy offset of $\frac{\hbar\omega}{2}$ (recall $E = \hbar\omega(n+\frac{1}{2})$). It underestimates the energy. However, our approximation isn't bad at all!

40.66. **IDENTIFY** and **SET UP:** Perform the calculations specified in the problem.

EXECUTE: **(a)** At the turning points $E = A|x_{\text{TP}}| \Rightarrow x_{\text{TP}} = \pm\frac{E}{A}.$

(b) $\int_{-E/A}^{+E/A}\sqrt{2m(E - A|x|)}dx = 2\int_0^{E/A}\sqrt{2m(E - Ax)}\,dx.$ Let $y = 2m(E - Ax)$

$\Rightarrow dy = -2mA\,dx$ when $x = \frac{E}{A}$, $y = 0$, and when $x = 0$, $y = 2mE$. So

$2\int_0^{\frac{E}{A}}\sqrt{2m(E - Ax)}dx = -\frac{1}{mA}\int_{2mE}^{0}y^{1/2}dy = -\frac{2}{3mA}y^{3/2}\Big|_{2mE}^{0} = \frac{2}{3mA}(2mE)^{3/2}.$ Using WKB, this is equal to $\frac{hn}{2}$. So, $\frac{2}{3mA}(2mE)^{3/2} = \frac{hn}{2} \Rightarrow E = \frac{1}{2m}\left(\frac{3mAh}{4}\right)^{2/3}n^{2/3}.$

EVALUATE: **(c)** The difference in energy decreases between successive levels. For example:
$1^{2/3} - 0^{2/3} = 1, 2^{2/3} - 1^{2/3} = 0.59, 3^{3/2} - 2^{3/2} = 0.49, \ldots$

- A sharp ∞ step gave ever-increasing level differences $(\sim n^2)$.
- A parabola $(\sim x^2)$ gave evenly spaced levels $(\sim n)$.
- Now, a linear potential $(\sim x)$ gives ever-decreasing level differences $(\sim n^{2/3})$.

Roughly speaking, if the curvature of the potential (~ second derivative) is bigger than that of a parabola, then the level differences will increase. If the curvature is less than a parabola, the differences will decrease.

40.67. **IDENTIFY** and **SET UP:** The energy levels are $E_{m,n} = (m^2 + n^2)(\pi^2\hbar^2)/2ML^2.$

EXECUTE: For a fixed value of m, the spacing between adjacent energy levels is
$\Delta E = [(m^2 + (n+1)^2 - (m^2 + n^2)](\pi^2\hbar^2)/2ML^2 = (2n+1)(\pi^2\hbar^2)/2ML^2.$ As the value of L is decreased, the spacing between adjacent levels increases. In this model, L is the size of the dot, so choice (c) is correct.

EVALUATE: We could also have kept n fixed and varied m by 1.

40.68. **IDENTIFY** and **SET UP:** The energy levels are $E_{m,n} = (m^2 + n^2)(\pi^2\hbar^2)/2ML^2.$ $\Delta E = \frac{hc}{\lambda}$ for the photon emitted during a transition through energy difference ΔE.

EXECUTE: Since $E \propto 1/L^2$, it is also the case that $\Delta E \propto 1/L^2$, and we know that $\Delta E = \frac{hc}{\lambda}.$

Therefore
$\lambda \propto L^2$. If $L \to 1.1L$, $\lambda \to (1.1)^2\lambda$. So in this case, $\lambda \to (1.1)^2(550\text{ nm}) = 666\text{ nm} \approx 670\text{ nm}$, which is choice (b).

EVALUATE: Because λ is proportional to the *square* of L instead of just L, a 10% change in L produces around a 20% change in λ.

40.69. **IDENTIFY** and **SET UP:** The energy levels are $E_{m,n} = (m^2 + n^2)(\pi^2\hbar^2)/2ML^2$. $\Delta E = \dfrac{hc}{\lambda}$ for the photon emitted during a transition through energy difference ΔE.

EXECUTE: Since $\Delta E \propto 1/M$ and $\Delta E = \dfrac{hc}{\lambda}$, it follows that $\lambda \propto 1/M$. Since $\lambda_1 > \lambda_2$, it is true that $M_1 > M_2$, which makes choice (a) correct.

EVALUATE: When M is large, the energy difference between states is small, so the energy of an emitted (or absorbed) photon is small, so its wavelength is large.

40.70. **IDENTIFY:** Apply the Heisenberg uncertainty principle, stated in terms of energy and time.

SET UP: $\Delta E \Delta t \geq \hbar/2$.

EXECUTE: Solving for ΔE gives $\Delta E \geq \dfrac{\hbar}{2\Delta t}$. Therefore increasing the lifetime of the excited states results in a smaller energy spread because ΔE is small, meaning that the energies are more well-defined. Therefore choice (a) is correct.

EVALUATE: A long lifetime does not imply that the energy is small; but it does tell us that the *uncertainty* in the energy is small.

41

QUANTUM MECHANICS II: ATOMIC STRUCTURE

VP41.1.1. **IDENTIFY:** This problem involves a particle in a three-dimensional cubical box.

SET UP: Eq. (41.15) gives the wave function of the particle, and the square of the wave function is the probability density.

EXECUTE: **(a)** We want to find where the probability distribution function is zero.

$$\left|\psi_{2,1,3}\right|^2 = |C|^2 \sin^2\frac{2\pi x}{L}\sin^2\frac{\pi y}{L}\sin^2\frac{3\pi z}{L} = 0.$$

$2\pi x/L = \pi : x = L/2$

$\pi y/L = \pi : y = L$ (none inside the box)

$3\pi z/L = m\pi : z = mL/3 = L/3$ and $2L/3$.

(b) We want the probability that the particle will be found within the range $0 \le x \le L/3$. Use results from Example 41.1. $C = (2/L)^{3/2}$. The probability is given by

$$P = |C|^2\int_0^{L/3}\sin^2\frac{2\pi x}{L}dx\int_0^L\sin^2\frac{\pi y}{L}dy\int_0^L\sin^2\frac{3\pi z}{L}dz.$$

The y and z integrals are each equal to $L/2$, and the x integral is $L/6 - (L/8\pi)\sin 4\pi/3 = 0.201L$. The probability is $P = [(2/L)^{3/2}]^2(0.201L)(L/2)(L/2) = 0.402$.

EVALUATE: The probability of finding the particle between $x = 0$ and $x = L/3$ is about 40%, so the probability of finding it between $x = L/3$ and L is about 60%.

VP41.1.2. **IDENTIFY:** This problem involves a particle in a three-dimensional cubical box.

SET UP: Eq. (41.15) gives the wave function of the particle, and the square of the wave function is the probability density. We want the probability of finding the particle in the region $L/4 \le x \le 3L/4$.

EXECUTE: **(a)** The state is (1,1,1). We integrate using results from Example 41.1. $C = (2/L)^{3/2}$.

$$P = |C|^2\int_{L/4}^{3L/4}\sin^2\frac{\pi x}{L}dx\int_0^L\sin^2\frac{\pi y}{L}dy\int_0^L\sin^2\frac{\pi z}{L}dz.$$

The second two integrals are each equal to $L/2$, and the first one is $L(1/4 + 1/2\pi)$. Therefore, $P = [(2/L)^{3/2}]^2(L/2)^2[L(1/4 + 1/2\pi)] = 1/2 + 1/\pi = 0.818$.

(b) The state is (2,1,2). The only difference from part (a) is the x integral, which is

$$\int_{L/4}^{3L/4}\sin^2\frac{2\pi x}{L}dx = L/4.$$

Therefore $P = (2/L)^3(L/2)^2(L/4) = 0.500$.

(c) The state is (3,2,3). Only the x integral is different from parts (a) and (b).

$$\int_{L/4}^{3L/4}\sin^2\frac{3\pi x}{L}dx = \frac{L}{4} - \frac{L}{6\pi} = L\left(\frac{1}{4} - \frac{1}{6\pi}\right).$$

$P = (2/L)^3(L/2)^2L(1/4 - 1/6\pi) = 0.394$.

(d) The state is (4,1,1). Use the same procedure. In this case, the x integral gives $L/4$, so the probability is $P = (2/L)^3(L/2)^2(L/4) = 0.500$.

EVALUATE: Note that the probability is 0.500 whenever n_X is an even integer. Also note that even though the region contains half the volume of the cube, the probability of finding a particle there is not necessarily 0.500.

VP41.1.3. **IDENTIFY:** This problem involves a particle in a three-dimensional cubical box.
SET UP: Eq. (41.15) gives the wave function of the particle, and the square of the wave function is the probability density. We want the probability of finding the particle in the region $0 \le x \le L/4$, $0 \le y \le L/4$. Use the same approach as in the previous two problems.
EXECUTE: **(a)** The state is (1,1,1). We integrate using results from Example 41.1. $C = (2/L)^{3/2}$.

$$P = \left(\frac{2}{L}\right)^3 \int_0^{L/4} \sin^2 \frac{\pi x}{L} dx \int_0^{L/4} \sin^2 \frac{\pi y}{L} dy \int_0^{L} \sin^2 \frac{\pi z}{L} dz.$$

The z integral is equal to $L/2$. The y and z integrals are each equal to $L(1/8 - 1/4\pi)$. Therefore

$$P = (2/L)^3(L)[L(1/8 - 1/4\pi)]^2 = 8.25 \times 10^{-3}.$$

(b) The state is (2,1,2). Use the previous results and those of example 41.1. Doing so, we find that the x integral is $L/8$, the y integral is $L(1/8 - 1/4\pi)$, and the z integral is $L/2$. The probability is

$$P = (2/L)^3(L/8)[L(1/8 - 1/4\pi)](L/2) = 0.0227.$$

(c) The state is (3,2,3). The x integral is $L(1/8 + 1/12\pi)$, the y integral is $L/8$, the z integral is $L/2$, so $P = (2/L)^3[L(1/8 + 1/12\pi)](L/8)(L/2) = 0.0758$.
(d) The state is (4,1,1). x integral is $L/8$, y integral is $L(1/8 - 1/4\pi)$, z integral is $L/2$, so $P = 0.0227$.
EVALUATE: Note that $P_{4,1,1}$ is the same as $P_{2,1,2}$. Also note that even though the region contains 6.25% of the volume of the cube, the probability of finding a particle there is not necessarily 0.0625.

VP41.1.4. **IDENTIFY:** This problem involves a particle in a three-dimensional cubical box.
SET UP: Eq. (41.15) gives the wave function of the particle, and the square of the wave function is the probability density.
EXECUTE: **(a)** We want $V_{\text{inside}}/V_{\text{box}}$. $V_{\text{box}} = L^3$ and $V_{\text{inside}} = (L/2)^3 = L^3/8$. Dividing the volumes gives $V_{\text{inside}}/V_{\text{box}} = (L^3/8)/L^3 = 1/8 = 0.125$.
(b) We want the probability that the particle is in this region. We use the wave functions in Eq. (41.15), square them and integrate to get the probability as in the previous problems. All the integrals are of the same form as the x integral in problem VP41.1.2(a), so each integral gives the factor $L(1/4 + 1/2\pi)$. The probability is $P = (2/L)^3[L(1/4 + 1/2\pi)]^3 = 0.548$.
EVALUATE: Note that the probability of finding the particle in the given volume is not necessarily the same as the percent that volume is of the total volume of the box. We have see the same thing in previous problems.

VP41.4.1. **IDENTIFY:** We are looking at the possible states of the hydrogen atom with $n = 3$.
SET UP: $l = 0, 1, 2, \ldots, n - 1$ and $m_l = 0, \pm 1, \pm 2, \ldots, \pm l$. $L = \sqrt{l(l+1)}\,\hbar$. $L_z = m_l \hbar$, $\cos\theta_L = L_z/L$.
EXECUTE: **(a)** $l = 0, 1, \ldots, 5$ and $m_l = 0, \pm 1, \pm 2, \ldots, \pm 5$. The possible states are:
$l = 0$, $m_l = 0$: 1 state
$l = 1$, $m_l = 0, \pm 1$: 3 states
$l = 2$, $m_l = 0, \pm 1, \pm 2$: 5 states
$l = 3$, $m_l = 0, \pm 1, \pm 2, \pm 3$: 7 states
$l = 4$, $m_l = 0, \pm 1, \pm 2, \pm 3, \pm 4$: 9 states
$l = 5$, $m_l = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$: 11 states
The total is 36 states.
(b) $l_{\max} = 5$, so $L_{\max} = \sqrt{5(5+1)}\,\hbar = \sqrt{30}\,\hbar$.
(c) $L_z = m_l \hbar$, so the maximum L_z is $5\hbar$.
EVALUATE: As n increases, the number of possible states increases rapidly.

VP41.4.2. **IDENTIFY:** We are looking at the possible states of the hydrogen atom with $n = 6$.

SET UP: $l = 0, 1, 2, \ldots, n - 1$ and $m_l = 0, \pm 1, \pm 2, \ldots, \pm l$. $L = \sqrt{l(l+1)}\,\hbar$. $L_z = m_l\hbar$.

EXECUTE: **(a)** Using the restrictions on l and m_l gives

$l = 0$, $m_l = 0$

$l = 1$, $m_l = 0, \pm 1$

$l = 2$, $m_l = 0, \pm 1, \pm 2$

So the (l, m_l) states are (0, 0), (1, –1), (1, 0), (1, 1), (2, –2), (2, –1), (2, 0), (2, 1), (2, 2).

(b) Using $\cos\theta_L = L_z/L$, we see that the angle between the orbital angular momentum and the *negative* z-axis will be a minimum when θ_L is closest to π, so the cosine should be closest to –1.

$$\cos\theta_L = L_z/L = \frac{m_l\hbar}{\sqrt{l(l+1)}\hbar} = \frac{m_l}{\sqrt{l(l+1)}}.$$

So for $m_l = -2$ and $l = 2$, we get $\cos\theta_L = -2/\sqrt{6}$, which gives $\theta_L = 144.7°$. Therefore the angle with the $-z$-axis is 180° – 144.7° = 35.3°. This is for the state $l = 2$, $m_l = -2$.

EVALUATE: The greatest angle that the angular momentum makes with the $+z$-axis is 144.7°.

VP41.4.3. **IDENTIFY:** We are looking at transition energy in the hydrogen atom.

SET UP: $E_n = -(13.60\text{ eV})/n^2$. The energy of a shell depends on n. The photon energy is equal to the energy lost by an electron due to its transition to a lower energy level.

EXECUTE: **(a)** $n = 3 \rightarrow n = 2$. $\Delta E = (13.6\text{ eV})(1/2^2 - 1/3^2) = 1.89$ eV.

(b) $n = 4 \rightarrow n = 2$. $\Delta E = (13.6\text{ eV})(1/2^2 - 1/4^2) = 2.55$ eV.

(c) $n = 2 \rightarrow n = 1$. $\Delta E = (13.6\text{ eV})(1/1^2 - 1/2^2) = 10.2$ eV.

EVALUATE: Note that the energy difference between adjacent shells is not the same but depends on the value of n.

VP41.4.4. **IDENTIFY:** We are investigating the properties of the hydrogen wave function.

SET UP: The probability density is the square of the wave function. We want the probability that the electron will be found within each given region using the given wave function.

$$P = \int |\psi|^2\, dV = \int \frac{1}{\pi a^3} e^{-2r/a}\, 4\pi r^2\, dr = -\frac{4}{a^3}\left(\frac{ar^2}{2} + \frac{a^2 r}{2} + \frac{a^3}{4}\right)e^{-2r/a}.$$

EXECUTE: **(a)** $0 \le r \le 2a$. Evaluating the expression above for the limits of the integral gives

$$P = -\frac{4}{a^3}\left[\left(2a^3 + a^3 + \frac{a^3}{4}\right)e^{-4} - \frac{a^3}{4}\right] = -\left(\frac{13}{e^4} - 1\right) = 0.762.$$

(b) $a \le r \le 3a$. Evaluating as in part (a) for the new limits gives $P = 0.615$.

(c) $r \ge 4a$. In this case, the upper limit is infinity. Evaluating as before gives $P = 0.0138$.

EVALUATE: Our results are consistent with Figure 41.8 in the textbook for a 1s electron in hydrogen. From the graph for a 1s electron, we see that the probability distribution for $P(r)$ is a maximum at $r = a$. The range in part (a) includes that value, so the probability for that range is greater than for the others that do not include it.

VP41.8.1. **IDENTIFY:** This problem involves the Zeeman effect in which an external magnetic field affects the energy levels.

SET UP: The following conditions hold:

$$U = -\mu_z B,\ \mu_z = -(2.00232)\frac{e}{2m}S_z,\ \ S_z = \pm\frac{\hbar}{2}.$$

EXECUTE: **(a)** We want the energy differenc. Combing these conditions gives

$$\Delta U = (2.00232)\frac{eB}{2m}\Delta S_z = (2.00232)\frac{eB}{2m}\left[\frac{\hbar}{2} - \left(-\frac{\hbar}{2}\right)\right] = (2.00232)\mu_B B.$$

Using $B = 3.14$ T and $\mu_B = 5.788 \times 10^{-5}$ eV/T gives $\Delta U = 3.64 \times 10^{-4}$ eV.

(b) We want to know which state has greater energy.

$$U = -\mu_z B = -(-2.00232)\frac{e}{2m}S_z B = +(2.00232)\frac{eB}{2m}S_z, \text{ so } S_z = +\frac{\hbar}{2} \text{ has greater energy.}$$

EVALUATE: The *magnitude* of the energy is the same for either of the S_z states.

VP41.8.2. **IDENTIFY:** We are looking at the effect of an external magnetic field on an atom.

SET UP: The energy of the radiation must be equal to the energy difference between the two spin states of the outermost electron. The following conditions hold:

$$U = -\mu_z B,\ \mu_z = -(2.00232)\frac{e}{2m}S_z,\ S_z = \pm\frac{\hbar}{2}$$

EXECUTE: As we saw in problem VP41.8.1(a).

$$\Delta U = (2.00232)\frac{eB}{2m}\Delta S_z = (2.00232)\frac{eB}{2m}\left[\frac{\hbar}{2} - \left(-\frac{\hbar}{2}\right)\right] = (2.00232)\mu_B B.$$

The radiation energy hf must therefore be $hf = (2.00232)\mu_B B$. Using $B = 2.36$ T and solving for f gives $f = 6.61 \times 10^{10}$ Hz. The wavelength is $\lambda = c/f = 4.53$ mm.

EVALUATE: The energy difference between the two spin states is very small, so the wavelength is much longer than that of visible light.

VP41.8.3. **IDENTIFY:** This problem deals with the effect of a magnetic field on the energy levels of an atom.

SET UP: We know the energy of a photon and from the two previous problems we know the energy difference due to the magnetic field.

$$\Delta U = (2.00232)\mu_B B,\ E = hc/\lambda$$

EXECUTE: **(a)** We want the energy difference between the two $4p$ levels.

$$\Delta E = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = \frac{hc(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}.$$

Using the given wavelengths gives $\Delta E = 7.16 \times 10^{-3}$ eV.

(b) We want the magnetic field. We use the energy difference from part (a) and solve for B.

$\Delta U = (2.00232)\mu_B B$

$$B = 61.8 \text{ T}.$$

EVALUATE: This is a *very* strong magnetic field.

VP41.8.4. **IDENTIFY:** This problem involves the effect of spin and angular momentum on the energy of the hydrogen atom.

SET UP: Eq. (41.41) gives the energy levels, where $j = |l \pm \frac{1}{2}|$.

EXECUTE: **(a)** Use Eq. (41.41) to calculate the energy levels.

$$E_{3,3/2} = -\frac{13.60 \text{ eV}}{3^2}\left[1 + \frac{\alpha^2}{3^2}\left(\frac{3}{3/2 + 1/2} - \frac{3}{4}\right)\right] = -1.511 \text{ eV}\left(1 + \frac{\alpha^2}{12}\right).$$

$$E_{3,1/2} = -1.511 \text{ eV}\left(1 + \frac{\alpha^2}{4}\right)$$

(b) $$E_{3,3/2} - E_{3,1/2} = -1.511 \text{ eV}\left[\left(1 + \frac{\alpha^2}{12}\right) - \left(1 + \frac{\alpha^2}{4}\right)\right] = (-1.511\alpha^2)\left(\frac{1}{12} - \frac{1}{4}\right) = 1.34 \times 10^{-5} \text{ eV}.$$

The difference is positive, so the $j = 3/2$ state has higher energy.

(c) We want the difference in wavelengths. Use the answer from part (b) and the approach of Example 41.8.

As in that example, the difference in the wavelengths is

$$\Delta\lambda = -\frac{\lambda}{E_{\text{photon}}}\Delta E_{\text{photon}}.$$

The *difference* in the energy between the $j = 3/2$ and $j = 1/2$ levels of the $n = 3$ shell is *very small* compared to the energy of that shell. Therefore the energy of the photon is essentially $E_3 - E_2$.

$$E_{\text{photon}} = E_3 - E_2 = -\frac{13.60\text{ eV}}{3^2} - \left(-\frac{13.60\text{ eV}}{2^2}\right) = 1.8890\text{ eV}.$$

The wavelength of this photon is

$$\lambda = hc/E_{\text{photon}} = hc/(1.8890\text{ eV}) = 6.565\times10^{-7}\text{ m}.$$

The difference in the wavelengths is therefore

$$\Delta\lambda = -\frac{\lambda}{E_{\text{photon}}}\Delta E_{\text{photon}} = -\frac{4.136\times10^{-15}\text{ eV}\cdot\text{s}}{1.8890\text{ eV}}(1.34\times10^{-5}\text{ eV}) = -4.66\times10^{-12}\text{ m} = -4.66\times10^{-3}\text{ nm}.$$

$\lambda_1 - \lambda_2$ is negative, so $\lambda_2 > \lambda_1$ with λ_2 being for the $j = ½$ initial state.

EVALUATE: As a check, you could also calculate the energy differences using Eq. (41.41) and calculate the wavelength differences from those results. But the algebra gets a bit tedious.

41.1. **IDENTIFY:** For a particle in a cubical box, different values of $n_X, n_Y,$ and n_Z can give the same energy.

SET UP: $E_{n_X, n_Y, n_Z} = \dfrac{(n_X^2 + n_Y^2 + n_Z^2)\pi^2\hbar^2}{2mL^2}.$

EXECUTE: **(a)** $n_X^2 + n_Y^2 + n_Z^2 = 3.$ This only occurs for $n_X = 1, n_Y = 1, n_Z = 1,$ so the degeneracy is 1.

(b) $n_X^2 + n_Y^2 + n_Z^2 = 9.$ This occurs for $n_X = 2, n_Y = 2, n_Z = 1,$ for $n_X = 1, n_Y = 2, n_Z = 2,$ and for $n_X = 2, n_Y = 1, n_Z = 2,$ so the degeneracy is 3.

EVALUATE: In the second case, three different states all have the same energy.

41.2. **IDENTIFY:** Use an electron in a cubical box to model the hydrogen atom.

SET UP: $E_{1,1,1} = \dfrac{3\pi^2\hbar^2}{2mL^2}.$ $E_{2,1,1} = \dfrac{6\pi^2\hbar^2}{2mL^2}.$ $\Delta E = \dfrac{3\pi^2\hbar^2}{2mL^2}.$ $L^3 = \dfrac{4}{3}\pi a^3.$

$$L = \left(\frac{4\pi}{3}\right)^{1/3} a = 8.527\times10^{-11}\text{ m}.$$

EXECUTE: $\Delta E = \dfrac{3\pi^2(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{2(9.109\times10^{-31}\text{ kg})(8.53\times10^{-11}\text{ m})^2} = 2.49\times10^{-17}\text{ J} = 155\text{ eV}.$ In the Bohr model, $E_n = -\dfrac{13.6\text{ eV}}{n^2}.$ The energy separation between the $n = 2$ and $n = 1$ levels is

$$\Delta E_{\text{Bohr}} = (13.6\text{ eV})\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}(13.6\text{ eV}) = 10.2\text{ eV}.$$

EVALUATE: A particle in a box is not a good model for a hydrogen atom.

41.3. **IDENTIFY:** The energy of the photon is equal to the energy difference between the states. We can use this energy to calculate its wavelength.

SET UP: $E_{1,1,1} = \dfrac{3\pi^2\hbar^2}{2mL^2}.$ $E_{2,2,1} = \dfrac{9\pi^2\hbar^2}{2mL^2}.$ $\Delta E = \dfrac{3\pi^2\hbar^2}{mL^2}.$ $\Delta E = \dfrac{hc}{\lambda}.$

EXECUTE: $\Delta E = \dfrac{3\pi^2(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{(9.109\times10^{-31}\text{ kg})(8.00\times10^{-11}\text{ m})^2} = 5.653\times10^{-17}\text{ J}.$ $\Delta E = \dfrac{hc}{\lambda}$ gives

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{5.653\times10^{-17}\text{ J}} = 3.51\times10^{-9}\text{ m} = 3.51\text{ nm}.$$

EVALUATE: This wavelength is much shorter than that of visible light.

41.4. IDENTIFY: Use the probability function for a particle in a three-dimensional box to find the points where it is a maximum.

(a) SET UP: $n_X = 1, n_Y = 1, n_Z = 1.$ $|\psi|^2 = \left(\frac{L}{2}\right)^3 \left(\sin^2 \frac{\pi x}{L}\right)\left(\sin^2 \frac{\pi y}{L}\right)\left(\sin^2 \frac{\pi z}{L}\right).$

EXECUTE: $|\psi|^2$ is maximum where $\sin\frac{\pi x}{L} = \pm 1,$ $\sin\frac{\pi y}{L} = \pm 1,$ and $\sin\frac{\pi z}{L} = \pm 1.$ $\frac{\pi x}{L} = \frac{\pi}{2}$ and $x = \frac{L}{2}.$ The next larger value is $\frac{\pi x}{L} = \frac{3\pi}{2}$ and $x = \frac{3L}{2},$ but this is outside the box. Similar results obtain for y and z, so $|\psi|^2$ is maximum at the point $x = y = z = L/2.$ This point is at the center of the box.

(b) SET UP: $n_X = 2, n_Y = 2, n_Z = 1.$ $|\psi|^2 = \left(\frac{L}{2}\right)^3 \left(\sin^2 \frac{2\pi x}{L}\right)\left(\sin^2 \frac{2\pi y}{L}\right)\left(\sin^2 \frac{\pi z}{L}\right).$

EXECUTE: $|\psi|^2$ is maximum where $\sin\frac{2\pi x}{L} = \pm 1,$ $\sin\frac{2\pi y}{L} = \pm 1,$ and $\sin\frac{\pi z}{L} = \pm 1.$ $\frac{2\pi x}{L} = \frac{\pi}{2}$ and $x = \frac{L}{4}.$ $\frac{2\pi x}{L} = \frac{3\pi}{2}$ and $x = \frac{3L}{4}.$ Similarly, $y = \frac{L}{4}$ and $\frac{3L}{4}.$ As in part (a), $z = \frac{L}{2}.$ $|\psi|^2$ is a maximum at the four points $\left(\frac{L}{4}, \frac{L}{4}, \frac{L}{2}\right),$ $\left(\frac{L}{4}, \frac{3L}{4}, \frac{L}{2}\right),$ $\left(\frac{3L}{4}, \frac{L}{4}, \frac{L}{2}\right),$ and $\left(\frac{3L}{4}, \frac{3L}{4}, \frac{L}{2}\right).$

EVALUATE: The points are located symmetrically relative to the center of the box.

41.5. IDENTIFY: This problem is about an electron in a three-dimensional cubical box.

SET UP: The energy levels are given by Eq. (41.16). In the ground state, all three quantum numbers are equal to 1. In the second excited state, two are equal to 2 and one is equal to 1. Therefore the energy levels are $E_1 = \frac{3\pi^2\hbar^2}{2mL^2}$ and $E_2 = \frac{9\pi^2\hbar^2}{2mL^2}.$ The photon energy is $E = hc/\lambda.$ We want L.

EXECUTE: The energy of the photon is equal to the energy difference between the two states.

$$\Delta E = (9-3)\frac{\pi^2\hbar^2}{2mL^2} = \frac{hc}{\lambda}.\ L = \sqrt{\frac{3h\lambda}{4mc}} = \sqrt{\frac{3h(8.00\text{ nm})}{4mc}} = 0.121\text{ nm}.$$

EVALUATE: The diameter of a Bohr-model hydrogen atom is about 0.11 nm, so this box is around that size.

41.6. IDENTIFY: A proton is in a cubical box approximately the size of the nucleus.

SET UP: $E_{1,1,1} = \frac{3\pi^2\hbar^2}{2mL^2}.$ $E_{2,1,1} = \frac{6\pi^2\hbar^2}{2mL^2}.$ $\Delta E = \frac{3\pi^2\hbar^2}{2mL^2}.$

EXECUTE: $\Delta E = \frac{3\pi^2(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{2(1.673\times10^{-27}\text{ kg})(1.00\times10^{-14}\text{ m})^2} = 9.85\times10^{-13}\text{ J} = 6.15\text{ MeV}.$

EVALUATE: This energy difference is much greater than the energy differences involving orbital electrons.

41.7. IDENTIFY: This problem is about an electron in a three-dimensional cubical box.

SET UP: The probability density is the square of the wave function, and Eq. (41.15) gives the wave function. We want the planes for which the probability density is zero, which is where the wave function is zero.

EXECUTE: x-axis: $n_X = 3$, so $3\pi x/L = \pi, 2\pi$. This gives $x = L/3$, $x = 2L/3$.

y-axis: $n_Y = 2$, so $2\pi y/L = \pi$. This gives $y = L/2$.

z-axis: $n_Z = 1$, so $\pi z/L = \pi$. This gives $z = L$, so there are no z-planes within the box.

EVALUATE: For the z-axis the probability is zero only at the walls of the box.

41.8. **IDENTIFY:** We are comparing the quantum mechanical description of the hydrogen atom with the Bohr model.

SET UP: Quantum model: $L = \sqrt{l(l+1)}\,\hbar$, Bohr model: $L = n\hbar$.

EXECUTE: **(a)** $n = 3$. Quantum model: The largest l is $n - 1 = 2$.

$L = \sqrt{l(l+1)}\,\hbar = \sqrt{29(29+1)}\,\hbar = 29.5\hbar$.

Bohr model: $L = n\hbar = 30\hbar$.

Percent difference = (3.00 – 2.45)/3.00 = 18%.

(b) $n = 30$. $l = n - 1 = 29$. Quantum model: $L = \sqrt{l(l+1)}\,\hbar = \sqrt{29(29+1)}\hbar = 29.5\hbar$.

Bohr model: $L = n\hbar = 30\hbar$.

Percent difference = (30 – 29.5)/30 = 1.7%.

EVALUATE: For higher states, the results for quantum mechanical and Bohr models get closer together.

41.9. **IDENTIFY:** The possible values of the angular momentum are limited by the value of n.

SET UP: For the N shell $n = 4$, $0 \le l \le n-1$, $|m| \le l$, $m_s = \pm\frac{1}{2}$.

EXECUTE: **(a)** The smallest l is $l = 0$. $L = \sqrt{l(l+1)}\hbar$, so $L_{\text{min}} = 0$.

(b) The largest l is $n-1 = 3$, so $L_{\text{max}} = \sqrt{3(4)}\hbar = 2\sqrt{3}\hbar = 3.65\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}$.

(c) Let the chosen direction be the z-axis. The largest m is $m = l = 3$.

$$L_{z,\text{max}} = m\hbar = 3\hbar = 3.16\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}.$$

(d) $S_z = \pm\frac{1}{2}\hbar$. The maximum value is $S_z = \hbar/2 = 5.27\times10^{-35}\ \text{kg}\cdot\text{m}^2/\text{s}$.

(e) $\dfrac{S_z}{L_z} = \dfrac{\frac{1}{2}\hbar}{3\hbar} = \dfrac{1}{6}$.

EVALUATE: The orbital and spin angular momenta are of comparable sizes.

41.10. **IDENTIFY** and **SET UP:** $L = \sqrt{l(l+1)}\hbar$. $L_z = m_l\hbar$. $l = 0, 1, 2, \ldots, n-1$. $m_l = 0, \pm1, \pm2, \ldots, \pm l$.

$\cos\theta = L_z/L$.

EXECUTE: **(a)** Use $L = \sqrt{l(l+1)}\hbar$.

$\underline{l=0}$: $L = 0$, $L_z = 0$.

$\underline{l=1}$: $L = \sqrt{2}\hbar$, $L_z = \hbar, 0, -\hbar$.

$\underline{l=2}$: $L = \sqrt{6}\hbar$, $L_z = 2\hbar,\ \hbar,\ 0,\ -\hbar,\ -2\hbar$.

$\underline{l=3}$: $L = 2\sqrt{3}\hbar$, $L_z = 3\hbar,\ 2\hbar,\ \hbar,\ 0,\ -\hbar,\ -2\hbar,\ -3\hbar$.

$\underline{l=4}$: $L = 2\sqrt{5}\hbar$, $L_z = 4\hbar,\ 3\hbar,\ 2\hbar,\ \hbar,\ 0,\ -\hbar,\ -2\hbar,\ -3\hbar,\ -4\hbar$.

(b) Use $\cos\theta = L_z/L$.

$\underline{L=0}$: θ is not defined.

$\underline{L=\sqrt{2}\hbar}$: 45.0°, 90.0°, 135.0°.

$\underline{L=\sqrt{6}\hbar}$: 35.3°, 65.9°, 90.0°, 114.1°, 144.7°.

$\underline{L=2\sqrt{3}\hbar}$: 30.0°, 54.7°, 73.2°, 90.0°, 106.8°, 125.3°, 150.0°.

$\underline{L=2\sqrt{5}\hbar}$: 26.6°, 47.9°, 63.4°, 77.1°, 90.0°, 102.9°, 116.6°, 132.1°, 153.4°.

(c) The minimum angle is 26.6° and occurs for $l = 4$, $m_l = +4$. The maximum angle is 153.4° and occurs for $l = 4$, $m_l = -4$.

EVALUATE: There is no state where $\vec{L}$ is totally aligned along the z-axis.

41.11. **IDENTIFY** and **SET UP:** The magnitude of the orbital angular momentum L is related to the quantum number l by Eq. (41.22): $L=\sqrt{l(l+1)}\hbar$, $l=0, 1, 2,\dots$

EXECUTE: $l(l+1)=\left(\dfrac{L}{\hbar}\right)^2=\left(\dfrac{4.716\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}\right)^2=20.$

And then $l(l+1)=20$ gives that $l=4$.

EVALUATE: l must be integer.

41.12. **IDENTIFY** and **SET UP:** $L=\sqrt{l(l+1)}\hbar.$ $L_z=m_l\hbar.$ $m_l=0, \pm1, \pm2, \dots, \pm l.$ $\cos\theta=L_z/L.$

EXECUTE: **(a)** $(m_l)_{\text{max}}=2$, so $(L_z)_{\text{max}}=2\hbar.$

(b) $L=\sqrt{l(l+1)}\hbar=\sqrt{6}\hbar=2.45\hbar.$ L is larger than $(L_z)_{\text{max}}$.

(c) The angle is $\arccos\left(\dfrac{L_z}{L}\right)=\arccos\left(\dfrac{m_l}{\sqrt{6}}\right)$, and the angles are, for $m_l=-2$ to $m_l=+2$ are:

$144.7°,\ 114.1°,\ 90.0°,\ 65.9°, 35.3°.$

EVALUATE: The minimum angle for a given l is for $m_l=l.$ The angle corresponding to $m_l=l$ will always be smaller for larger l.

41.13. **IDENTIFY** and **SET UP:** The smallest nonzero angle for a given l occurs for $m_l=+l.$ $L=\sqrt{l(l+1)}\hbar$ and $L_z=m_l\hbar$ where $m_l=0, \pm1, \pm2, \dots,\pm l.$ $\cos\theta=L_z/L.$

EXECUTE: In this case $\theta=26.6°$, so $\cos 26.6^\circ=\dfrac{l}{\sqrt{l(l+1)}}$. Squaring gives $l(l+1)\cos^2(26.6^\circ)=l^2.$

Solving for l gives $l=\dfrac{\cos^2(26.6^\circ)}{1-\cos^2(26.6^\circ)}=4.$

EVALUATE: For $l=4$ we see that the angle between the angular momentum vector and the positive z-axis ranges from $26.6°\ (m_l=+l)$ to $180°-26.6°=153.4°(m_l=-l).$

41.14. **IDENTIFY** and **SET UP:** $L=\sqrt{l(l+1)}\hbar.$ $\cos\theta_L=L_z/L.$ We know that $L_z=2\hbar$ in this case, and $\theta_L=63.4°.$ For any n, l can have the values $l=0, 1, 2, \dots, n-1$.

EXECUTE: **(a)** First find L: $L=\dfrac{L_z}{\cos\theta_L}=\dfrac{2\hbar}{\cos(63.4°)}=4.467\hbar.$

Now solve for l: $L=\sqrt{l(l+1)}\hbar=4.467\,\hbar\ \rightarrow\ l(l+1)=(4.467)^2\approx 20=4(4+1)$, so $l=4$.

(b) The maximum that l can be is $n-1$, so $n-1\geq l_{\max}$, which means that $n\geq l_{\max}+1$. In this case, we know that $l=4$, so $n\geq 5$, so the smallest that n could be is 5.

EVALUATE: Any $n>5$ is also possible.

41.15. **IDENTIFY:** This problem deals with a hydrogen atom in the seventh excited level. For the seventh *excited* level, $n=8$.

SET UP and **EXECUTE:** **(a)** We want the energy. $E_n=-(13.6\text{ eV})/n^2=-(13..6\text{ eV})/8^2=-0.213$ eV.

(b) We want $L_{\max}$. $L=\sqrt{l(l+1)}\hbar$ and $l_{\max}=n-1=7.$ $L_{\max}=\sqrt{7(7+1)}\ \hbar=7.48\hbar.$

(c) We want the largest possible angle. $\cos\theta_L=L_z/L.$ $L_z=m_l\hbar$, and $m_l=0, \pm1, \dots, \pm l$. The angle will be greatest when L_z is most negative, which in this case is when $m_l=-7$, so $l=7$. Therefore $\cos\theta_L=-7/7.48$, which gives $\theta_L=159°$.

EVALUATE: There are many other possible values for θ_L but the value we found is the largest this angle can be.

41.16. IDENTIFY: $l = 0, 1, 2, \ldots, n-1.\;\; m_l = 0, \pm 1, \pm 2, \ldots, \pm l.$

SET UP: $E_n = -\dfrac{13.60\text{ eV}}{n^2}.$

EXECUTE: Since $n = 4$, l can have the values 0, 1, 2, and 3. For each value of l, m_l can range from $-l$ to $+l$. Therefore the (l, m_l) combinations are $(0, 0), (1, 0),\; (1, \pm 1), (2, 0),\; (2, \pm 1), (2, \pm 2), (3, 0),$ $(3, \pm 1), (3, \pm 2),$ and $(3, \pm 3),$ a total of 16 combinations.

(b) Each state has the same energy because n is the same for all of them, so

$$E = -\frac{13.60\text{ eV}}{4^2} = -0.8500\text{ eV}.$$

EVALUATE: The number of l, m_l combinations is n^2. The energy depends only on n, so is the same for all l, m_l states for a given n.

41.17. IDENTIFY: For the 5g state, $l = 4$, which limits the other quantum numbers.

SET UP: $m_l = 0, \pm 1, \pm 2, \ldots, \pm l.$ g means $l = 4$. $\cos\theta = L_z/L$, with $L = \sqrt{l(l+1)}\,\hbar$ and $L_z = m_l\hbar.$

EXECUTE: **(a)** There are eighteen 5g states: $m_l = 0, \pm 1, \pm 2, \pm 3, \pm 4$, with $m_s = \pm\frac{1}{2}$ for each.

(b) The largest θ is for the most negative m_l. $L = 2\sqrt{5}\hbar.$ The most negative L_z is $L_z = -4\hbar.$

$$\cos\theta = \frac{-4\hbar}{2\sqrt{5}\hbar} \text{ and } \theta = 153.4°.$$

(c) The smallest θ is for the largest positive m_l, which is $m_l = +4$. $\cos\theta = \dfrac{4\hbar}{2\sqrt{5}\hbar}$ and $\theta = 26.6°.$

EVALUATE: The minimum angle between $\vec{L}$ and the z-axis is for $m_l = +l$ and for that m_l,

$\cos\theta = \dfrac{l}{\sqrt{l(l+1)}}.$

41.18. IDENTIFY: The probability is $P = \int_0^{a/2} |\psi_{1s}|^2\, 4\pi r^2 dr.$

SET UP: Use the expression for the integral given in Example 41.4.

EXECUTE: **(a)** $P = \dfrac{4}{a^3}\left[\left(-\dfrac{ar^2}{2} - \dfrac{a^2 r}{2} - \dfrac{a^3}{4}\right)e^{-2r/a}\right]_0^{a/2} = 1 - \dfrac{5e^{-1}}{2} = 0.0803.$

(b) Example 41.4 calculates the probability that the electron will be found at a distance less than a from the nucleus. The difference in the probabilities is $(1 - 5e^{-2}) - (1 - (5/2)e^{-1}) = (5/2)(e^{-1} - 2e^{-2}) = 0.243.$

EVALUATE: The probability for distances from $a/2$ to a is about three times the probability for distances between 0 and $a/2$. This agrees with Figure 41.8 in the textbook; $P(r)$ is maximum for $r = a$.

41.19. IDENTIFY: Require that $\Phi(\phi) = \Phi(\phi + 2\pi).$

SET UP: $e^{i(x_1 + x_2)} = e^{ix_1}e^{ix_2}.$

EXECUTE: $\Phi(\phi + 2\pi) = e^{im_l(\phi + 2\pi)} = e^{im_l\phi}e^{im_l 2\pi}.$ $e^{im_l 2\pi} = \cos(m_l 2\pi) + i\sin(m_l 2\pi).$ $e^{im_l 2\pi} = 1$ if m_l is an integer.

EVALUATE: If, for example, $m_l = \frac{1}{2}$, $e^{im_l 2\pi} = e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1$ and $\Phi(\phi) = -\Phi(\phi + 2\pi).$ But if $m_l = 1$, $e^{im_l 2\pi} = e^{i2\pi} = \cos(2\pi) + i\sin(2\pi) = +1$ and $\Phi(\phi) = \Phi(\phi + 2\pi)$, as required.

41.20. **IDENTIFY:** This problem is about the probability distribution for hydrogen.
SET UP: Refer to the graphs in Figure 41.8 in the textbook, $a = 5.29 \times 10^{-11}$ m.
EXECUTE: **(a)** $r/a = (0.476 \text{ nm})/a = 9.00$. From the bottom graph in Figure 41.8 we see that the 3*d* curve peaks at approximately $r/a = 9.00$ and has only one peak, so this must be the state.
(b) Using the same graph as in (a), we see that the 4*f* graph peaks at $r/a \approx 16$, so $r \approx 16a = 0.85$ nm.
EVALUATE: Note that the 4*f* state peaks after the 3*d* state, but *before* the second peak of the 4*d* state.

41.21. **IDENTIFY:** Apply $\Delta U = \mu_B B$.
SET UP: For a 3*p* state, $l = 1$ and $m_l = 0, \pm 1$.
EXECUTE: **(a)** $B = \dfrac{U}{\mu_B} = \dfrac{(2.71\times10^{-5} \text{ eV})}{(5.79\times10^{-5} \text{ eV/T})} = 0.468 \text{ T}.$
(b) Three: $m_l = 0, \pm 1$.
EVALUATE: The $m_l = +1$ level will be highest in energy and the $m_l = -1$ level will be lowest. The $m_l = 0$ level is unaffected by the magnetic field.

41.22. **IDENTIFY:** Apply $\Delta E = \mu_B B$.
SET UP: $\mu_B = 5.788\times10^{-5}$ eV/T.
EXECUTE: **(a)** $\Delta E = \mu_B B = (5.79\times10^{-5} \text{ eV/T})(0.800 \text{ T}) = 4.63\times10^{-5} \text{ eV}.$
(b) $m_l = -2$ the lowest possible value of m_l.
(c) The energy level diagram is sketched in Figure 41.22.

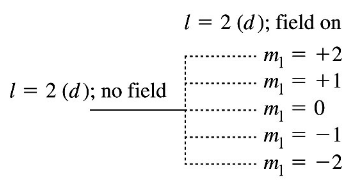


Figure 41.22

EVALUATE: The splitting between m_l levels is independent of the *n* values for the state. The splitting is much less than the energy difference between the $n = 3$ level and the $n = 1$ level.

41.23. **IDENTIFY** and **SET UP:** The interaction energy between an external magnetic field and the orbital angular momentum of the atom is given by $U = m_l \mu_B B$. The energy depends on m_l with the most negative m_l value having the lowest energy.
EXECUTE: **(a)** For the 5*g* level, $l = 4$ and there are $2l + 1 = 9$ different m_l states. The 5*g* level is split into 9 levels by the magnetic field.
(b) Each m_l level is shifted in energy an amount given by $U = m_l \mu_B B$. Adjacent levels differ in m_l by one, so $\Delta U = \mu_B B$.

$$\mu_B = \frac{e\hbar}{2m} = \frac{(1.602\times10^{-19} \text{ C})(1.055\times10^{-34} \text{ J}\cdot\text{s})}{2(9.109\times10^{-31} \text{ kg})} = 9.277\times10^{-24} \text{ A}\cdot\text{m}^2.$$

$\Delta U = \mu_B B = (9.277\times10^{-24} \text{ A/m}^2)(0.600 \text{ T}) = 5.566\times10^{-24} \text{ J}(1 \text{ eV}/1.602\times10^{-19} \text{ J}) = 3.47\times10^{-5} \text{ eV}.$
(c) The level of highest energy is for the largest m_l, which is $m_l = l = 4;\ U_4 = 4\mu_B B$. The level of lowest energy is for the smallest m_l, which is $m_l = -l = -4;\ U_{-4} = -4\mu_B B$. The separation between these two levels is $U_4 - U_{-4} = 8\mu_B B = 8(3.47\times10^{-5} \text{ eV}) = 2.78\times10^{-4} \text{ eV}.$

EVALUATE: The energy separations are proportional to the magnetic field. The energy of the $n = 5$ level in the absence of the external magnetic field is $(-13.6\text{ eV})/5^2 = -0.544\text{ eV},$ so the interaction energy with the magnetic field is much less than the binding energy of the state.

41.24. **IDENTIFY:** The effect of the magnetic field on the energy levels is described by Eq. (41.36). In a transition m_l must change by 0 or ± 1.

SET UP: For a 2*p* state, m_l can be $0, \pm 1$. For a 1*s* state, m_l must be zero.

EXECUTE: **(a)** There are three different transitions that are consistent with the selection rules. The initial m_l values are 0, ± 1; and the final m_l value is 0.

(b) The transition from $m_l = 0$ to $m_l = 0$ produces the same wavelength (122 nm) that was seen without the magnetic field.

(c) The larger wavelength (smaller energy) is produced from the $m_l = -1$ to $m_l = 0$ transition.

(d) The shorter wavelength (greater energy) is produced from the $m_l = +1$ to $m_l = 0$ transition.

EVALUATE: The magnetic field increases the energy of the $m_l = 1$ state, decreases the energy for $m_l = -1$ and leaves the $m_l = 0$ state unchanged.

41.25. **IDENTIFY** and **SET UP:** For a classical particle $L = I\omega$. For a uniform sphere with mass *m* and radius *R*, $I = \frac{2}{5}mR^2,$ so $L = \left(\frac{2}{5}mR^2\right)\omega.$ Solve for ω and then use $v = r\omega$ to solve for *v*.

EXECUTE: **(a)** $L = \sqrt{\frac{3}{4}}\hbar,$ so $\frac{2}{5}mR^2\omega = \sqrt{\frac{3}{4}}\hbar.$

$$\omega = \frac{5\sqrt{3/4}\hbar}{2mR^2} = \frac{5\sqrt{3/4}(1.055\times10^{-34}\text{ J}\cdot\text{s})}{2(9.109\times10^{-31}\text{ kg})(1.0\times10^{-17}\text{ m})^2} = 2.5\times10^{30}\text{ rad/s}.$$

(b) $v = r\omega = (1.0\times10^{-17}\text{ m})(2.5\times10^{30}\text{ rad/s}) = 2.5\times10^{13}\text{ m/s}.$

EVALUATE: This is much greater than the speed of light *c*, so the model cannot be valid.

41.26. **IDENTIFY:** The transition energy ΔE of the atom is related to the wavelength λ of the photon by $\Delta E = \frac{hc}{\lambda}.$ For an electron in a magnetic field the spin magnetic interaction energy is $\pm\mu_{\text{B}}B.$ Therefore the effective magnetic field is given by $\Delta E = 2\mu_{\text{B}}B$ when ΔE is produced by the hyperfine interaction.

SET UP: $\mu_{\text{B}} = 5.788\times10^{-5}\text{eV/T}.$

EXECUTE: **(a)** $\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(5.9\times10^{-6}\text{ eV})} = 21\text{ cm},$

$f = \frac{c}{\lambda} = \frac{(3.00\times10^8\text{ m/s})}{0.21\text{ m}} = 1.4\times10^9\text{ Hz},$ a short radio wave.

(b) The effective field is $B \cong \Delta E/2\mu_{\text{B}} = 5.1\times10^{-2}\text{ T},$ far smaller than that found in Example 41.7 for spin-orbit coupling.

EVALUATE: The level splitting due to the hyperfine interaction is much smaller than the level splittings due to the spin-orbit interaction.

41.27. **IDENTIFY** and **SET UP:** The interaction energy is $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}},$ with μ_z given by $\mu_z = -(2.00232)\left(\frac{e}{2m}\right)S_z.$

EXECUTE: $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}} = +\mu_z B,$ since the magnetic field is in the negative *z*-direction.

$$\mu_z = -(2.00232)\left(\frac{e}{2m}\right)S_z,\text{ so } U = -(2.00232)\left(\frac{e}{2m}\right)S_z B.$$

$$S_z = m_s\hbar, \text{ so } U = -2.00232\left(\frac{e\hbar}{2m}\right)m_s B.$$

$$\frac{e\hbar}{2m} = \mu_B = 5.788\times10^{-5} \text{ eV/T}.$$

$$U = -2.00232\mu_B m_s B.$$

The $m_s = +\frac{1}{2}$ level has lower energy.

$$\Delta U = U(m_s = -\tfrac{1}{2}) - U(m_s = +\tfrac{1}{2}) = -2.00232\ \mu_B B(-\tfrac{1}{2} - (+\tfrac{1}{2})) = +2.00232\ \mu_B B.$$

$$\Delta U = +2.00232(5.788\times10^{-5} \text{ eV/T})(1.45 \text{ T}) = 1.68\times10^{-4} \text{ eV}.$$

EVALUATE: The interaction energy with the electron spin is the same order of magnitude as the interaction energy with the orbital angular momentum for states with $m_l \neq 0$. But a 1*s* state has $l = 0$ and $m_l = 0$, so there is no orbital magnetic interaction.

41.28. IDENTIFY: Apply the equation $\mu_z = -(2.00232)\dfrac{e}{2m}S_z$ with $S_z = -\dfrac{\hbar}{2}$.

SET UP: $\mu_B = \dfrac{e\hbar}{2m} = 5.788\times10^{-5}$ eV/T.

EXECUTE: **(a)** $U = +(2.00232)\left(\dfrac{e}{2m}\right)\left(\dfrac{-\hbar}{2}\right)B = -\dfrac{2.00232}{2}\mu_B B.$

$U = -\dfrac{2.00232}{2}(5.788\times10^{-5} \text{ eV/T})(1.60 \text{ T}) = -9.27\times10^{-5} \text{ eV}.$

(b) Since $n = 1$, $l = 0$, so there is no orbital magnetic dipole interaction. But if $n \neq 1$ there could be orbital magnetic dipole interaction, since $l < n$ would then allow for $l \neq 0$.

EVALUATE: The energy of the $m_s = -\frac{1}{2}$ state is lowered in the magnetic field. The energy of the $m_s = +\frac{1}{2}$ state is raised.

41.29. IDENTIFY: The ten lowest energy levels for electrons are in the $n = 1$ and $n = 2$ shells.

SET UP: $l = 0, 1, 2, \ldots, n-1.$ $m_l = 0, \pm1, \pm2, \ldots, \pm l.$ $m_s = \pm\frac{1}{2}.$

EXECUTE: $n = 1, l = 0, m_l = 0, m_s = \pm\frac{1}{2}$: 2 states. $n = 2, l = 0, m_l = 0, m_s = \pm\frac{1}{2}$: 2 states.

$n = 2, l = 1, m_l = 0, \pm1, m_s = \pm\frac{1}{2}$: 6 states.

EVALUATE: The ground state electron configuration for neon is $1s^2 2s^2 2p^6$. The electron configuration specifies the *n* and *l* quantum numbers for each electron.

41.30. IDENTIFY: Fill the subshells in the order of increasing energy. An *s* subshell holds 2 electrons, a *p* subshell holds 6, and a *d* subshell holds 10 electrons.

SET UP: Germanium has 32 electrons.

EXECUTE: The electron configuration is $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^2$.

EVALUATE: The electron configuration is that of zinc $(Z = 30)$ plus two electrons in the 4*p* subshell.

41.31. IDENTIFY: Write out the electron configuration for ground-state beryllium.

SET UP: Beryllium has 4 electrons.

EXECUTE: **(a)** $1s^2 2s^2$.

(b) $1s^2 2s^2 2p^6 3s^2$. $Z = 12$ and the element is magnesium.

(c) $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$. $Z = 20$ and the element is calcium.

EVALUATE: Beryllium, calcium, and magnesium are all in the same column of the periodic table.

41.32. IDENTIFY: Write out the electron configuration for ground-state carbon.
SET UP: Carbon has 6 electrons.
EXECUTE: (a) $1s^2 2s^2 2p^2$.
(b) The element of next larger Z with a similar electron configuration has configuration $1s^2 2s^2 2p^6 3s^2 3p^2$. $Z = 14$ and the element is silicon.
EVALUATE: Carbon and silicon are in the same column of the periodic table.

41.33. IDENTIFY and **SET UP:** The energy of an atomic level is given in terms of n and Z_{eff} by $E_n = -\left(\dfrac{Z_{\text{eff}}^2}{n^2}\right)(13.6 \text{ eV})$. The ionization energy for a level with energy $-E_n$ is $+E_n$.

EXECUTE: $n = 5$ and $\text{Z}_{\text{eff}} = 2.771$ gives $E_5 = -\dfrac{(2.771)^2}{5^2}(13.6 \text{ eV}) = -4.18 \text{ eV}$.

The ionization energy is 4.18 eV.
EVALUATE: The energy of an atomic state is proportional to Z_{eff}^2.

41.34. IDENTIFY and **SET UP:** Apply $E_n = -\left(\dfrac{Z_{\text{eff}}^2}{n^2}\right)(13.6 \text{ eV})$.

EXECUTE: For the $4s$ state, $E = -4.339$ eV and $Z_{\text{eff}} = 4\sqrt{(-4.339)/(-13.6)} = 2.26$. Similarly, $Z_{\text{eff}} = 1.79$ for the $4p$ state and 1.05 for the $4d$ state.
EVALUATE: The electrons in the states with higher l tend to be farther away from the filled subshells and the screening is more complete.

41.35. IDENTIFY and **SET UP:** Use the exclusion principle to determine the ground-state electron configuration, as in Table 41.3 in the textbook. Estimate the energy by estimating Z_{eff}, taking into account the electron screening of the nucleus.
EXECUTE: (a) $Z = 7$ for nitrogen so a nitrogen atom has 7 electrons. N^{2+} has 5 electrons: $1s^2 2s^2 2p$.
(b) $Z_{\text{eff}} = 7 - 4 = 3$ for the $2p$ level.

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6 \text{ eV}) = -\frac{3^2}{2^2}(13.6 \text{ eV}) = -30.6 \text{ eV}.$$

(c) $Z = 15$ for phosphorus so a phosphorus atom has 15 electrons.
P^{2+} has 13 electrons: $1s^2 2s^2 2p^6 3s^2 3p$.
(d) $Z_{\text{eff}} = 15 - 12 = 3$ for the $3p$ level.

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6 \text{ eV}) = -\frac{3^2}{3^2}(13.6 \text{ eV}) = -13.6 \text{ eV}.$$

EVALUATE: In these ions there is one electron outside filled subshells, so it is a reasonable approximation to assume full screening by these inner-subshell electrons.

41.36. IDENTIFY and **SET UP:** Apply Eq. (41.48) and solve for Z.

EXECUTE: $E_{K\alpha} \cong (Z-1)^2(10.2 \text{ eV})$. $Z \approx 1 + \sqrt{\dfrac{7.46\times10^3 \text{ eV}}{10.2 \text{ eV}}} = 28.0$, which corresponds to the element Nickel (Ni).
EVALUATE: We use $Z - 1$ rather than Z in the expression for the transition energy, in order to account for screening by the other K-shell electron.

41.37. **IDENTIFY** and **SET UP:** Apply $E_{K\alpha} \cong (Z-1)^2(10.2\text{ eV})$. $E = hf$ and $c = f\lambda$.

EXECUTE: **(a)** $Z = 20$: $f = (2.48\times10^{15}\text{ Hz})(20-1)^2 = 8.95\times10^{17}$ Hz.

$E = hf = (4.14\times10^{-15}\text{ eV}\cdot\text{s})(8.95\times10^{17}\text{ Hz}) = 3.71\text{ keV}$. $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{8.95\times10^{17}\text{ Hz}} = 3.35\times10^{-10}$ m.

(b) $Z = 27$: $f = 1.68\times10^{18}$ Hz. $E = 6.96$ keV. $\lambda = 1.79\times10^{-10}$ m.

(c) $Z = 48$: $f = 5.48\times10^{18}$ Hz, $E = 22.7$ keV, $\lambda = 5.47\times10^{-11}$ m.

EVALUATE: f and E increase and λ decreases as Z increases.

41.38. **IDENTIFY:** The energies of the x rays will be equal to the energy differences between the shells. From its energy, we can calculate the wavelength of the x ray.

SET UP: $\Delta E = \dfrac{hc}{\lambda}$. A K_α x ray is produced in a $L \to K$ transition and a K_β x ray is produced in a $M \to K$ transition.

EXECUTE: K_α: $\Delta E = E_L - E_K = -12{,}000\text{ eV} - (-69{,}500\text{ eV}) = +57{,}500$ eV.

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{57{,}500\text{ eV}} = 0.0216\text{ nm}.$$

K_β: $\Delta E = E_M - E_K = -2200\text{ eV} - (-69{,}500\text{ eV}) = +67{,}300$ eV.

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{67{,}300\text{ eV}} = 0.0184\text{ nm}.$$

EVALUATE: These wavelengths are much shorter than the wavelengths in the visible spectrum of hydrogen.

41.39. **IDENTIFY:** The electrons cannot all be in the same state in a cubical box.

SET UP and **EXECUTE:** The ground state can hold 2 electrons, the first excited state can hold 6 electrons, and the second excited state can hold 6. Therefore, two electrons will be in the second excited state, which has energy $3E_{1,1,1}$.

EVALUATE: The second excited state is the third state, which has energy $3E_{1,1,1}$, as shown in Figure 41.4 in the textbook.

41.40. **IDENTIFY:** For a rectangular box having sides of lengths L_X, L_Y, and L_Z, the possible energy levels of an electron in the box are $E_{n_X,n_Y,n_Z} = \left(\dfrac{n_X^2}{L_X^2} + \dfrac{n_Y^2}{L_Y^2} + \dfrac{n_Z^2}{L_Z^2}\right)\dfrac{\pi^2\hbar^2}{2m}$, where n_X, n_Y, and n_Z = 1, 2, 3, …

SET UP: For this box, we know that $L_X = 0.600$ nm and $L_Y = L_Z = 2L_X = 1.20$ nm. Using $L_Y = L_Z = 2L_X$, the energy levels can be written as

$$E_{n_X,n_Y,n_Z} = \left(\frac{n_X^2}{L_X^2} + \frac{n_Y^2}{4L_Y^2} + \frac{n_Z^2}{4L_Z^2}\right)\frac{\pi^2\hbar^2}{2m} = (4n_X^2 + n_Y^2 + n_Z^2)\frac{\pi^2\hbar^2}{8mL_X^2} = (4n_X^2 + n_Y^2 + n_Z^2)E_0.$$

EXECUTE: First calculate E_0:

$$E_0 = \frac{\pi^2\hbar^2}{8mL_X^2} = \frac{\pi^2(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{8(9.109\times10^{-31}\text{ kg})(0.600\times10^{-9}\text{ m})^2} = 4.1846\times10^{-20}\text{ J} = 0.2612\text{ eV}.$$

First (lowest) state: $n_X = n_Y = n_Z = 1$, so the state is (1, 1, 1) and the energy is $E_{1,1,1}$. Using the information we have gives $E_{1,1,1} = [4(1^2) + 1^2 + 1^2]E_0 = 6E_0 = 6(0.2612\text{ eV}) = 1.57$ eV.
The degeneracy in just the quantum numbers is just 1 because there is only one set of quantum numbers that will give this energy. But the electron has 2 spin states, so the degeneracy is 2.
Second state: The quantum numbers are $n_X = 1$, $n_Y = 2$, $n_Z = 1$ and $n_X = 1$, $n_Y = 1$, $n_Z = 2$. Both sets give the same energy, so the degeneracy is 4 (including spin). The states are (1, 2, 1) and (1, 1, 2). The energy is $E_{1,2,1} = E_{1,1,2} = [4(1^2) + 1^2 + 2^2]E_0 = 9E_0 = 9(0.2612\text{ eV}) = 2.35$ eV.

Third state: The state is (1, 2, 2), and it has degeneracy 2 (including spin). The energy is $E_{1,2,2} = [4(1^2) + 2^2 + 2^2]E_0 = 12E_0 = 12(0.2612 \text{ eV}) = 3.13 \text{ eV}$.
Fourth state: The possibilities are (1, 3, 1) and (1, 1, 3), so the degeneracy is 4 (including spin). The energy is $E_{1,3,1} = E_{1,1,3} = [4(1^2) + 3^2 + 1^2]E_0 = 14E_0 = 14(0.2612 \text{ ev}) = 3.66 \text{ eV}$.
EVALUATE: These energy states are all higher than the $n = 3$ state of hydrogen, which is 1.51 eV.

41.41. **IDENTIFY:** Calculate the probability of finding a particle in a given region within a cubical box.
(a) SET UP and EXECUTE: The box has volume L^3. The specified cubical space has volume $(L/4)^3$. Its fraction of the total volume is $\dfrac{1}{64} = 0.0156$.

(b) SET UP and EXECUTE: $P = \left(\dfrac{2}{L}\right)^3 \left[\int_0^{L/4} \sin^2 \dfrac{\pi x}{L} dx\right]\left[\int_0^{L/4} \sin^2 \dfrac{\pi y}{L} dy\right]\left[\int_0^{L/4} \sin^2 \dfrac{\pi z}{L} dz\right]$.

From Example 41.1, each of the three integrals equals $\dfrac{L}{8} - \dfrac{L}{4\pi} = \dfrac{1}{2}\left(\dfrac{L}{2}\right)\left(\dfrac{1}{2} - \dfrac{1}{\pi}\right)$.

$$P = \left(\frac{2}{L}\right)^3 \left(\frac{L}{2}\right)^3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2} - \frac{1}{\pi}\right)^3 = 7.50 \times 10^{-4}.$$

EVALUATE: Note that this is the cube of the probability of finding the particle anywhere between $x = 0$ and $x = L/4$. This probability is much less that the fraction of the total volume that this space represents. In this quantum state the probability distribution function is much larger near the center of the box than near its walls.

(c) SET UP and EXECUTE: $|\psi_{2,1,1}|^2 = \left(\dfrac{L}{2}\right)^3 \left(\sin^2 \dfrac{2\pi x}{L}\right)\left(\sin^2 \dfrac{\pi y}{L}\right)\left(\sin^2 \dfrac{\pi z}{L}\right)$.

$$P = \left(\frac{2}{L}\right)^3 \left[\int_0^{L/4} \sin^2 \frac{2\pi x}{L} dx\right]\left[\int_0^{L/4} \sin^2 \frac{\pi y}{L} dy\right]\left[\int_0^{L/4} \sin^2 \frac{\pi z}{L} dz\right].$$

$$\left[\int_0^{L/4} \sin^2 \frac{\pi y}{L} dy\right] = \left[\int_0^{L/4} \sin^2 \frac{\pi z}{L} dz\right] = \frac{L}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2} - \frac{1}{\pi}\right). \quad \int_0^{L/4} \sin^2 \frac{2\pi x}{L} dx = \frac{L}{8}.$$

$$P = \left(\frac{2}{L}\right)^3 \left(\frac{L}{2}\right)^2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2} - \frac{1}{\pi}\right)^2 \left(\frac{L}{8}\right) = 2.06 \times 10^{-3}.$$

EVALUATE: This is about a factor of three larger than the probability when the particle is in the ground state.

41.42. **IDENTIFY:** For a rectangular box having sides of lengths L_X, L_Y, and L_Z, the possible energy levels of an electron in the box are $E_{n_X,n_Y,n_Z} = \left(\dfrac{n_X^2}{L_X^2} + \dfrac{n_Y^2}{L_Y^2} + \dfrac{n_Z^2}{L_Z^2}\right)\dfrac{\pi^2\hbar^2}{2m}$, where n_X, n_Y, and $n_Z = 1, 2, 3, \ldots$

SET UP: For this box, we know that $L_X = L_Z$. We also know that the two lowest energy levels are 2.24 eV and 3.47 eV and both of these levels have degeneracy 2 including electron spin. Using $L_X = L_Z$, the energy levels can be written as $E_{n_X,n_Y,n_Z} = \left(\dfrac{n_X^2 + n_Z^2}{L_X^2} + \dfrac{n_Y^2}{L_Y^2}\right)\dfrac{\pi^2\hbar^2}{2m}$.

EXECUTE: **(a)** Looking at the possibilities for the quantum numbers, we see that the two lowest levels having degeneracy 2 are (1, 1, 1) and (1, 2, 1). Therefore $E_{1,1,1} = 2.24$ eV and $E_{1,2,1} = 3.47$ eV. (We cannot have $E_{1,1,2}$ because it is degenerate with $E_{2,1,1}$, so this state would have degeneracy 4 including spin.)
(b) Using the numbers we know gives $E_{1,1,1} = \left(\dfrac{1^2 + 1^2}{L_X^2} + \dfrac{1^2}{L_Y^2}\right)\dfrac{\pi^2\hbar^2}{2m}$ and $E_{1,2,1} = \left(\dfrac{1^2 + 1^2}{L_X^2} + \dfrac{2^2}{L_Y^2}\right)\dfrac{\pi^2\hbar^2}{2m}$.

Subtracting these two equations gives $\dfrac{3}{L_Y^2}\left(\dfrac{\pi^2\hbar^2}{2m}\right) = 3.47 \text{ eV} - 2.24 \text{ eV} = 1.23 \text{ eV}$.

Solving for L_Y gives $L_Y=\sqrt{\dfrac{3\pi^2\hbar^2}{2m(1.23\text{ eV})}}=9.58\times10^{-10}$ m = 0.958 nm. Now use this result to solve for L_X and L_Z. Using the $E_{1,1,1}$ equation gives $\dfrac{2}{L_X^2}=(2.24\text{ eV})\left(\dfrac{2\text{m}}{\pi^2\hbar^2}\right)-\dfrac{1}{L_Y^2}$. Using L_Y = 0.958 nm, gives $L_X=L_Z=0.641$ nm.

(c) The next higher energy state is (1, 1, 2) and (2, 1, 1), which has degeneracy 4 (including spin). The energy is $E_{1,1,2}=\left(\dfrac{1+2^2}{L_X^2}+\dfrac{1}{L_Y^2}\right)\left(\dfrac{\pi^2\hbar^2}{2m}\right)=\left[\dfrac{5}{(0.641\text{ nm})^2}+\dfrac{1}{(0.958\text{ nm})^2}\right]\left(\dfrac{\pi^2\hbar^2}{2m}\right)=5.00\text{ eV}=E_{2,1,1}.$

EVALUATE: The longest side of this box is about 9 times the diameter of the Bohr hydrogen atom, and the shorter sides are each about 6 times that diameter.

41.43. IDENTIFY: Calculate the probability of finding a particle in certain regions of a three-dimensional box.

SET UP: $\left|\psi_{1,1,1}\right|^2=\left(\dfrac{L}{2}\right)^3\left(\sin^2\dfrac{\pi x}{L}\right)\left(\sin^2\dfrac{\pi y}{L}\right)\left(\sin^2\dfrac{\pi z}{L}\right).$

EXECUTE: **(a)** $P=\left(\dfrac{2}{L}\right)^3\left[\int_0^{L/2}\sin^2\dfrac{\pi x}{L}dx\right]\left[\int_0^{L}\sin^2\dfrac{\pi y}{L}dy\right]\left[\int_0^{L}\sin^2\dfrac{\pi z}{L}dz\right].$

$\left[\int_0^{L}\sin^2\dfrac{\pi y}{L}dy\right]=\left[\int_0^{L}\sin^2\dfrac{\pi z}{L}dz\right]=\dfrac{L}{2}.\ \int_0^{L/2}\sin^2\dfrac{\pi x}{L}dx=\left[\dfrac{x}{2}-\dfrac{L}{4\pi}\sin\dfrac{2\pi x}{L}\right]_0^{L/2}=\left(\dfrac{L}{2}\right)\left(\dfrac{1}{2}\right).$

$$P=\left(\frac{2}{L}\right)^3\left(\frac{L}{2}\right)^3\left(\frac{1}{2}\right)=\frac{1}{2}=0.500.$$

(b) $P=\left(\dfrac{2}{L}\right)^3\left[\int_{L/4}^{L/2}\sin^2\dfrac{\pi x}{L}dx\right]\left[\int_0^{L}\sin^2\dfrac{\pi y}{L}dy\right]\left[\int_0^{L}\sin^2\dfrac{\pi z}{L}dz\right].$

$\left[\int_0^{L}\sin^2\dfrac{\pi y}{L}dy\right]=\left[\int_0^{L}\sin^2\dfrac{\pi z}{L}dz\right]=\dfrac{L}{2}.\ \int_{L/4}^{L/2}\sin^2\dfrac{\pi x}{L}dx=\left[\dfrac{x}{2}-\dfrac{L}{4\pi}\sin\dfrac{2\pi x}{L}\right]_{L/4}^{L/2}=\left(\dfrac{L}{2}\right)\left(\dfrac{1}{4}+\dfrac{1}{2\pi}\right).$

$$P=\left(\frac{2}{L}\right)^3\left(\frac{L}{2}\right)^3\left(\frac{1}{4}+\frac{1}{2\pi}\right)=\frac{1}{4}+\frac{1}{2\pi}=0.409.$$

EVALUATE: In Example 41.1 for this state the probability for finding the particle between $x=0$ and $x=L/4$ is 0.091. The sum of this result and our result in part (b) is $0.091+0.409=0.500$. This in turn equals the probability of finding the particle in half the box, as calculated in part (a).

41.44. IDENTIFY and SET UP: Evaluate $\partial^2\psi/\partial x^2$, $\partial^2\psi/\partial y^2$, and $\partial^2\psi/\partial z^2$ for the proposed ψ and put into Eq. (41.5). Use that ψ_{n_x}, ψ_{n_y}, and ψ_{n_z} are each solutions to Eq. (40.44).

EXECUTE: **(a)** $-\dfrac{\hbar^2}{2m}\left(\dfrac{\partial^2\psi}{\partial x^2}+\dfrac{\partial^2\psi}{\partial y^2}+\dfrac{\partial^2\psi}{\partial z^2}\right)+U\psi=E\psi.$

ψ_{n_x}, ψ_{n_y}, ψ_{n_z} are each solutions of Eq. (40.44), so $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi_{n_x}}{dx^2}+\dfrac{1}{2}k'x^2\psi_{n_x}=E_{n_x}\psi_{n_x}.$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_y}}{dy^2}+\frac{1}{2}k'y^2\psi_{n_y}=E_{n_y}\psi_{n_y}.$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_z}}{dz^2}+\frac{1}{2}k'z^2\psi_{n_z}=E_{n_z}\psi_{n_z}.$$

$$\psi=\psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z),\ U=\frac{1}{2}k'x^2+\frac{1}{2}k'y^2+\frac{1}{2}k'z^2.$$

$$\frac{\partial^2\psi}{\partial x^2}=\left(\frac{d^2\psi_{n_x}}{dx^2}\right)\psi_{n_y}\psi_{n_z},\ \frac{\partial^2\psi}{\partial y^2}=\left(\frac{d^2\psi_{n_y}}{dy^2}\right)\psi_{n_x}\psi_{n_z},\ \frac{\partial^2\psi}{\partial z^2}=\left(\frac{d^2\psi_{n_z}}{dz^2}\right)\psi_{n_x}\psi_{n_y}.$$

So, $$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+U\psi=\left(-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_x}}{dx^2}+\frac{1}{2}k'x^2\psi_{n_x}\right)\psi_{n_y}\psi_{n_z}$$

$$+\left(-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_y}}{dy^2}+\frac{1}{2}k'y^2\psi_{n_y}\right)\psi_{n_x}\psi_{n_z}+\left(-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_z}}{dz^2}+\frac{1}{2}k'z^2\psi_{n_z}\right)\psi_{n_x}\psi_{n_y}$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+U\psi=(E_{n_x}+E_{n_y}+E_{n_z})\psi.$$

Therefore, we have shown that this ψ is a solution to Eq. (41.5), with energy

$$E_{n_xn_yn_z}=E_{n_x}+E_{n_y}+E_{n_z}=\left(n_x+n_y+n_z+\frac{3}{2}\right)\hbar\omega.$$

(b) and **(c)** The ground state has $n_x=n_y=n_z=0,$ so the energy is $E_{000}=\frac{3}{2}\hbar\omega.$ There is only one set of $n_x, n_y,$ and n_z that give this energy.

<u>First-excited state:</u> $n_x=1, n_y=n_z=0$ or $n_y=1, n_x=n_z=0$ or $n_z=1, n_x=n_y=0$ and

$$E_{100}=E_{010}=E_{001}=\tfrac{5}{2}\hbar\omega.$$

There are three different sets of $n_x,\ n_y,\ n_z$ quantum numbers that give this energy, so there are three different quantum states that have this same energy.

EVALUATE: For the three-dimensional isotropic harmonic oscillator, the wave function is a product of one-dimensional harmonic oscillator wavefunctions for each dimension. The energy is a sum of energies for three one-dimensional oscillators. All the excited states are degenerate, with more than one state having the same energy.

41.45. IDENTIFY: Find solutions to Eq. (41.5).

SET UP: $\omega_1=\sqrt{k_1'/m},\ \omega_2=\sqrt{k_2'/m}.$ Let $\psi_{n_x}(x)$ be a solution of Eq. (40.44) with $E_{n_x}=(n_x+\frac{1}{2})\,\hbar\omega_1, \psi_{n_y}(y)$ be a similar solution, and let $\psi_{n_z}(z)$ be a solution of Eq. (40.44) but with *z* as the independent variable instead of *x*, and energy $E_{n_z}=(n_z+\frac{1}{2})\hbar\omega_2.$

EXECUTE: **(a)** As in Problem 41.44, look for a solution of the form $\psi(x,y,z)=\psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z).$

Then, $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2\psi}{\partial x^2}=(E_{n_x}-\frac{1}{2}k_1'x^2)\psi$ with similar relations for $\dfrac{\partial^2\psi}{\partial y^2}$ and $\dfrac{\partial^2\psi}{\partial z^2}$. Adding,

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)=(E_{n_x}+E_{n_y}+E_{n_z}-\tfrac{1}{2}k_1'x^2-\tfrac{1}{2}k_1'y^2-\tfrac{1}{2}k_2'z^2)\psi$$

$$=(E_{n_x}+E_{n_y}+E_{n_z}-U)\psi=(E-U)\psi$$

where the energy *E* is $E=E_{n_x}+E_{n_y}+E_{n_z}=\hbar\left[(n_x+n_y+1)\omega_1^2+(n_z+\frac{1}{2})\omega_2^2\right],$ with $n_x, n_y,$ and n_z all nonnegative integers.

(b) The ground level corresponds to $n_x=n_y=n_z=0,$ and $E=\hbar(\omega_1^2+\frac{1}{2}\omega_2^2).$ The first excited level corresponds to $n_x=n_y=0$ and $n_z=1,$ since $\omega_1^2>\omega_2^2,$ and $E=\hbar(\omega_1^2+\frac{3}{2}\omega_2^2).$

(c) There is only one set of quantum numbers for both the ground state and the first excited state.

EVALUATE: For the isotropic oscillator of Problem 41.44 there are three states for the first excited level but only one for the anisotropic oscillator.

41.46. **IDENTIFY:** This problem deals with a particle in a three-dimensional noncubical box.
SET UP: The energy levels are shown in Eq. (41.17) in textbook.
EXECUTE: **(a)** We want energy difference between the first excited energy level and the ground state. For this box, $L_Y = 2L_X$ and $L_Z = L_Y/3 = 2L_X/3$. In terms of L_X, the energies are

$$E_{n_X, n_Y, n_Z} = \left(\frac{n_X^2}{L_X^2} + \frac{n_Y^2}{4L_X^2} + \frac{9n_Z^2}{4L_X^2}\right)\frac{\pi^2\hbar^2}{2m}.$$

For the ground state, all the quantum numbers are equal to 1. For the first excited state, the smallest energy above the ground state is when $n_X = n_Z = 1$ and $n_Y = 2$. The energy difference between this state and ground is $\Delta E = E_{1,2,1} - E_{1,1,1}$. Using the above energy equation gives

$$\Delta E = \left[\left(\frac{1^2}{1} + \frac{2^2}{4} + \frac{9}{4}\right) - \left(\frac{1^2}{1} + \frac{1^2}{4} + \frac{9}{4}\right)\right]\frac{\pi^2\hbar^2}{2mL_X^2} = \frac{3\pi^2\hbar^2}{8mL_X^2}.$$

We could also express the result in terms of L_Y as $\Delta E = \dfrac{3\pi^2\hbar^2}{2mL_Y^2}$.

(b) This state is *not* degenerate because no other set of quantum numbers gives the same energy.
(c) We want to know where the probability distribution is greatest for the ground state.
Along the x-axis: $\sin \pi x/L_X$ is greatest when $\pi x/L_X = \pi/2$, which is when $x = L_X/2$.
Along the y-axis: $\sin \pi y/L_Y = \sin \pi y/2L_X$ is greatest when $\pi y/2L_X = \pi/2$, so $y = L_X$.
Along the z-axis: $\sin \pi z/L_Z = \sin \pi z/(2L_X/3)$ is greatest when $3\pi z/2L_X = \pi/2$, so $z = L_X/3$.
EVALUATE: The energy degeneracy is removed for a noncubical box because, due to the different dimensions, the quantum numbers all have different coefficients.

41.47. **IDENTIFY** and **SET UP:** To calculate the total number of states for the n^{th} principal quantum number shell we must add up all the possibilities. The spin states multiply everything by 2. The maximum l value is $(n-1)$, and each l value has $(2l+1)$ different m_l values.
EXECUTE: **(a)** The total number of states is

$$N = 2\sum_{l=0}^{n-1}(2l+1) = 2\sum_{l=0}^{n-1}1 + 4\sum_{l=0}^{n-1}l = 2n + \frac{4(n-1)(n)}{2} = 2n + 2n^2 - 2n = 2n^2.$$

(b) The $n = 5$ shell (O-shell) has 50 states.
EVALUATE: The $n = 1$ shell has 2 states, the $n = 2$ shell has 8 states, etc.

41.48. **IDENTIFY:** We are modeling nuclei in the sun as a particle in a three-dimensional cubical box.
SET UP and **EXECUTE:** **(a)** Assume the sun is all hydrogen with one electron each. The number N of electrons is $N = m_{\text{sun}}/m_{\text{proton}} = (2.0 \times 10^{30}\ \text{kg})/(1.7 \times 10^{-27}\ \text{kg}) = 1.2 \times 10^{57}$ electrons.
(b) The volume V each electron occupies is $V_{\text{sun}}/N = (4/3\ \pi R^3)/N = 4/3\ \pi(7000\ \text{km})^3/(1.2 \times 10^{57})$ $= 1.2 \times 10^{-36}\ \text{m}^3$.
(c) We want the volume per carbon atom. Each carbon atom has 6 electrons, so it occupies the space of 6 electrons. Hence its volume is $7.2 \times 10^{-36}\ \text{m}^3$.
(d) If the volume is a cube with side of length L, then $L^3 = 7.2 \times 10^{-36}\ \text{m}^3$, so $L = 1.9$ pm.
(e) n_X, n_Y, n_Z can be 1, 2, 3, ..., and $m_z = \pm\frac{1}{2}$. The possible states for 6 electrons are:
Lowest state: (1, 1, 1, ±½)
Possible higher states: (1, 1, 2, ±½), (1, 2, 1, ±½), (2, 1, 1, ±½).
(f) We want the highest energy. Use Eq. (41.16). These states are degenerate, so we only need to calculate one of them, and we use $L = 1.9$ pm.

$$E_{1,1,2} = \frac{\left(1^2 + 1^2 + 2^2\right)\pi^2\hbar^2}{2mL^2} = 1.0 \times 10^{-13}\ \text{J} = 630\ \text{keV}.$$

EVALUATE: Modeling such as this gives only very rough approximations, but it can give reasonable order-of-magnitude estimates.

41.49. **(a) IDENTIFY** and **SET UP:** The energy is given by $E_n = K_n + U_n = -\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}$ from Chapter 39, which is identical to the Bohr energy levels for hydrogen from this chapter. The potential energy is given by $U(r) = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$, with $q_1 = +Ze$ and $q_2 = -e$.

EXECUTE: $E_{1s} = -\frac{1}{(4\pi\epsilon_0)^2}\frac{me^4}{2\hbar^2}$; $U(r) = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$.

$$E_{1s} = U(r) \text{ gives } -\frac{1}{(4\pi\epsilon_0)^2}\frac{me^4}{2\hbar^2} = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}.$$

$$r = \frac{(4\pi\epsilon_0)2\hbar^2}{me^2} = 2a.$$

EVALUATE: The turning point is twice the Bohr radius.

(b) IDENTIFY and **SET UP:** For the 1s state the probability that the electron is in the classically forbidden region is $P(r > 2a) = \int_{2a}^{\infty}|\psi_{1s}|^2 dV = 4\pi\int_{2a}^{\infty}|\psi_{1s}|^2 r^2 dr$. The normalized wave function of the 1s state of hydrogen is given in Example 41.4: $\psi_{1s}(r) = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}$. Evaluate the integral; the integrand is the same as in Example 41.4.

EXECUTE: $P(r > 2a) = 4\pi\left(\frac{1}{\pi a^3}\right)\int_{2a}^{\infty} r^2 e^{-2r/a}dr$.

Use the integral formula $\int r^2 e^{-\alpha r}dr = -e^{-\alpha r}\left(\frac{r^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}\right)$, with $\alpha = 2/a$.

$$P(r > 2a) = -\frac{4}{a^3}\left[e^{-2r/a}\left(\frac{ar^2}{2} + \frac{a^2 r}{2} + \frac{a^3}{4}\right)\right]_{2a}^{\infty} = +\frac{4}{a^3}e^{-4}(2a^3 + a^3 + a^3/4).$$

$$P(r > 2a) = 4e^{-4}(13/4) = 13e^{-4} = 0.238.$$

EVALUATE: These is a 23.8% probability of the electron being found in the classically forbidden region, where classically its kinetic energy would be negative.

41.50. **IDENTIFY:** At the r where $P(r)$ has its maximum value, $\frac{d(r^2|\psi|^2)}{dr} = 0$.

SET UP: From Example 41.4, $r^2|\psi|^2 = Cr^2 e^{-2r/a}$.

EXECUTE: $\frac{d(r^2|\psi|^2)}{dr} = Ce^{-2r/a}(2r - (2r^2/a))$. This is zero for $r = a$. Therefore, $P(r)$ has its maximum value at $r = a$, the distance of the electron from the nucleus in the Bohr model.

EVALUATE: Our result agrees with Figure 41.8 in the textbook.

41.51. **IDENTIFY:** At the value of r where $P(r)$ is a maximum, $\frac{dP}{dr} = 0$.

SET UP: $P(r) = \left(\frac{1}{24a^5}\right)r^4 e^{-r/a}$.

EXECUTE: $\frac{dP}{dr} = \left(\frac{1}{24a^5}\right)\left(4r^3 - \frac{r^4}{a}\right)e^{-r/a}$. $\frac{dP}{dr} = 0$ when $4r^3 - \frac{r^4}{a} = 0$; $r = 4a$. In the Bohr model, $r_n = n^2 a$ so $r_2 = 4a$, which agrees with the location of the maximum in $P(r)$.

EVALUATE: Our result agrees with Figure 41.8. The figure shows that $P(r)$ for the 2p state has a single maximum and no zeros except at $r = 0$ and $r \to \infty$.

41.52. **IDENTIFY:** This problem involves the Zeeman effect.
SET UP and **EXECUTE:** **(a)** We want the current. $\mu = IA = I\pi r^2$, so $I = \mu/\pi r^2$. Using μ from the problem gives

$$I = \frac{\frac{e}{2m_\mathrm{p}}S_z}{\pi r^2} = \frac{\left(\frac{e}{2m_\mathrm{p}}\right)\frac{\hbar}{2}}{\pi(0.85\ \mathrm{fm})^2} = 1100\ \mathrm{A}.$$

(b) We want the magnetic field at $x = 0.40a_0$ from the proton. Use Eq. (28.15) for a circular loop.

$$B_x = \frac{\mu_0 Ia^2}{2(x^2 + a^2)^{3/2}}.$$

In our case, a is the radius of the "loop" (which is the proton), so $a = R_\mathrm{p}$. $x = 0.40a_0 >> R_\mathrm{p}$. So

$$B_x = \frac{\mu_0 IR_\mathrm{p}^2}{2(0.40a_0)^3}.$$

Using $R_\mathrm{p} = 0.85$ fm, $I = 1100$ A, and $a_0 = 5.29 \times 10^{-11}$ m gives $B = 52$ mT.
(c) We want the potential energy due to the electron spin. Use Eq. (41.38) and $U = \pm\mu_z B$.

$$\mu_z = (2.00232)(e/2m)S_z \approx 2(e/2m)(\hbar/2) = \hbar e/2m = \mu_\mathrm{B}.$$

$$U = \mu_\mathrm{B}B = (5.788 \times 10^{-5}\ \mathrm{eV/T})(52\ \mathrm{mT}) = 3.0\ \mu\mathrm{eV}.$$

(d) We want the wavelength. $hc/\lambda = 2U$, so $\lambda = hc/2U = hc/[2(3.0\ \mu\mathrm{eV})] = 0.21\ \mathrm{m} = 21\ \mathrm{cm}$.
EVALUATE: **(e)** The observed radiation has a wavelength of precisely 21 cm, so it definitely could be from electron spin flip.

41.53. **IDENTIFY:** This problem is about the broadening of spectral lines in hydrogen due to spin-orbit coupling.
SET UP: The blue spectral line of wavelength 434 nm is due to a transition from the $n = 5$ to the $n = 2$ shell. Eq. (41.41) applies, with $l = 0, 1, 2, 3, 4$ and $j = |l \pm ½|$. In any transition $\Delta l = \pm 1$. Use Example 41.8 as a guide.
EXECUTE: **(a)** We want the number of states.
For the $n = 2$ level: $l = 0$ and 1. For $l = 0$, $j = |0 \pm ½| = ½$. For $l = 1$, $j = 1 \pm ½ = 1/2$ and 3/2. The possible states are (0, 1/2), (1, 1/2), and (1, 3/2), so there are 3 states.
For the $n = 5$ level: $l = 0, 1, 2, 3$, or 4. The possible values of j are
$l = 0$: $j = 1/2$
$l = 1$: $j = 1/2, 3/2$
$l = 2$: $j = 3/2, 5/2$
$l = 3$: $j = 5/2, 7/2$
$l = 4$: $j = 7/2, 9/2$
The possible states are (0, 1/2), (1, 1/2), (1, 3/2), (2, 3/2), (2, 5/2), (3, 5/2), (3, 7/2), (4, 7/2), (4, 9/2). There are a total of 9 states.
(b) We want the number of different energy levels for each value of n. From Eq. (41.41) we see that for a given n, the energy $E_{n,j}$ depends only on j. Therefore all states having the same n and j have the same energy.
$n = 2$ level: Since j has only 2 different possible values (1/2 and 3/2), there are only 2 different energy levels.
$n = 5$ level: Since j has 5 different values (1/2, 3/2, 5/2, 7/2, 9/2) there are 5 different energy levels.
For each energy level, we want its *difference* from $-(13.6\ \mathrm{eV})/n^2$. Call $E_n = -(13.6\ \mathrm{eV})/n^2$ and call the energy difference we want $\Delta E_{n,j} = E_{n,j} - E_n$, where $E_{n,j}$ is the energy of the (n, j) level as given in Eq. (41.41). For calculations use $\alpha^2 = 5.32514 \times 10^{-5}$. Using Eq. (41.41), we get

$$\Delta E_{n,j} = \frac{13.60\text{ eV}}{n^2}\left[1+\frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2}-\frac{3}{4}\right)\right]-\left(\frac{13.60\text{ eV}}{n^2}\right) = -\frac{13.60\text{ eV}}{n^2}\frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2}-\frac{3}{4}\right).$$

Simplifying to a working equation gives

$$\Delta E_{n,j} = -\frac{13.60\text{ eV}\alpha^2}{n^4}\left(\frac{n}{j+1/2}-\frac{3}{4}\right).$$

We now use this equation to calculate the energy *differences*. If $\Delta E_{n,j}$ is negative, the energy of the level is *less than* E_n.

<u>n = 2 level</u>: For $n = 2$ and $j = 1/2$, we have

$$\Delta E_{2,1/2} = -\frac{13.60\text{ eV}\alpha^2}{2^4}\left(\frac{2}{1/2+1/2}-\frac{3}{4}\right) = -5.658\times10^{-5}\text{ eV}.$$

For $n = 2$ and $j = 3/2$, we have

$$\Delta E_{2,3/2} = -\frac{13.60\text{ eV}\alpha^2}{2^4}\left(\frac{2}{3/2+1/2}-\frac{3}{4}\right) = -1.1316\times10^{-5}\text{ eV}.$$

<u>n = 5 level</u>: For $n = 5$ and $j = 1/2$, we have

$$\Delta E_{5,1/2} = -\frac{13.60\text{ eV}\alpha^2}{5^4}\left(\frac{5}{1/2+1/2}-\frac{3}{4}\right) = -4.11356\times10^{-5}\text{ eV}.$$

For $n = 5$ and $j = 3/2$, we have

$$\Delta E_{5,3/2} = -\frac{13.60\text{ eV}\alpha^2}{5^4}\left(\frac{5}{3/2+1/2}-\frac{3}{4}\right) = -1.69382\times10^{-5}\text{ eV}.$$

For $n = 5$ and $j = 5/2$, we have

$$\Delta E_{5,5/2} = -\frac{13.60\text{ eV}\alpha^2}{5^4}\left(\frac{5}{5/2+1/2}-\frac{3}{4}\right) = -8.87239\times10^{-6}\text{ eV}.$$

For $n = 5$, $j = 7/2$, we have

$$\Delta E_{5,7/2} = -\frac{13.60\text{ eV}\alpha^2}{5^4}\left(\frac{5}{7/2+1/2}-\frac{3}{4}\right) = -4.83949\times10^{-6}\text{ eV}.$$

For $n = 5$, $j = 9/2$, we have

$$\Delta E_{5,9/2} = -\frac{13.60\text{ eV}\alpha^2}{5^4}\left(\frac{5}{9/2+1/2}-\frac{3}{4}\right) = -2.41974\times10^{-6}\text{ eV}.$$

Notice that in every case, fine structure due to spin-orbit coupling makes the energy levels *more negative*, i.e. *lower*.

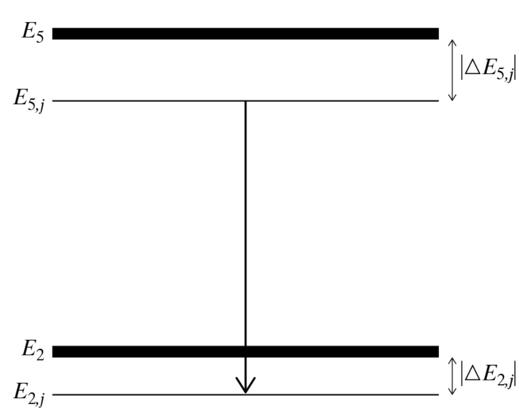


Figure 41.53

(c) We want the transition that emits the *shortest* wavelength of light. The shortest wavelength photon will have the highest energy, so it is between energy levels that have the *greatest* energy *difference* between them. Figure 41.53 illustrates a transition from a (5, j) level to a (2, j) level, where the two values of j are *not* the same. For the energy difference between these two levels to be as large as possible, $|\Delta E_{5,j}|$ should be the *smallest* it can be, and $|\Delta E_{2,j}|$ should be the *largest* it can be. But the transition must also obey $\Delta l = \pm 1$. For the $n = 2$ level, $l = 0$ or 1, which means that in the $n = 5$ level, $l =$ 0, 1, or 2. The energy levels, for a given n, depend only on j, so the j values for possible transitions that result in an energy difference and obey $\Delta l = \pm 1$ are

$$j = 1/2 \;\rightarrow\; j = 3/2 \quad (l = 0 \;\rightarrow\; l = 1)$$
$$j = 3/2 \;\rightarrow\; j = 1/2 \quad (l = 1 \;\rightarrow\; l = 0)$$
$$j = 5/2 \;\rightarrow\; j = 1/2 \quad (l = 2 \;\rightarrow\; l = 1)$$
$$j = 5/2 \;\rightarrow\; j = 3/2 \quad (l = 2 \;\rightarrow\; l = 1)$$

Using our results from part (b), we see that $|\Delta E_{2,j}|$ is greatest when $j = 1/2$ ($l = 1$), and $|\Delta E_{5,j}|$ is smallest when $j = 5/2$ ($l = 2$). Therefore the transition from the (5, 2, 5/2) level to the (2, 1, 1/2) level emits a photon of the greatest energy and therefore the shortest wavelength. The energy of this photon is $E_{\text{photon}} = E_5 + \Delta E_{5,5/2} - (E_2 + \Delta E_{2,1/2})$. Using this condition with $E_n = -(13.60\text{ eV})/n^2$ and our results from part (b), we have

$$E_{\text{photon}} = E_5 - E_2 + \Delta E_{5,5/2} - \Delta E_{2,1/2}$$

$$E_{\text{photon}} = -\frac{13.60\text{ eV}}{5^2} - \left(-\frac{13.60\text{ eV}}{2^2}\right) + (-8.87239\times10^{-5}\text{ eV}) - (-5.658\times10^{-5}\text{ eV}).$$

$$E_{\text{photon}} = \underbrace{2.856\text{ eV}}_{E_5\rightarrow E_2} + \underbrace{4.7708\times10^{-5}\text{ eV}}_{\Delta E_{5/2\rightarrow 1/2}}.$$

If there were no spin-orbit coupling, the wavelength of the photon would be 434 nm, as given in the problem. But, due to spin-orbit coupling, what appears as a single spectral line is made up of a number of lines, each of slightly different wavelength. To find the difference of the shortest-wavelength line from 434 nm, we use the approach in Example 41.8. The energy of the photon is $E = hc/\lambda$. As in the example, we can express this as

$$\Delta E = -\frac{hc}{\lambda^2}\Delta\lambda$$

$$\Delta\lambda = \frac{\lambda^2\Delta E}{hc} = -\frac{(434\text{ nm})^2(4.7708\times10^{-5}\text{ eV})}{(4.13567\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^8\text{ m/s})} = -7.26\times10^{-12}\text{ m} = -7.26\times10^{-3}\text{ nm}.$$

(d) We want the longest wavelength photon. Refer to Figure 41.53. The longest wavelength is due to a transition between levels having the smallest energy difference between them. Using the allowed transitions from part (c), we see that the energy difference is *smallest* when $|\Delta E_{5,j}|$ is a *maximum* and $|E_{2,j}|$ is a *minimum*. For the allowed transitions, we see that $|E_{2,j}|$ is smallest when $j = 3/2$ ($l = 1$), and $|\Delta E_{5,j}|$ is largest when $j = 1/2$ ($l = 0$). So this transition is from the (5, 0, 1/2) level to the (2, 1, 3/2) level. Follow the same approach as for part (c), which gives

$$E_{\text{photon}} = E_5 - E_2 + \Delta E_{5,1/2} - \Delta E_{2,3/2}$$

$$E_{\text{photon}} = -\frac{13.60\text{ eV}}{5^2} - \left(-\frac{13.60\text{ eV}}{2^2}\right) + (-4.11356\times10^{-5}\text{ eV}) - (-1.1316\times10^{-5}\text{ eV}).$$

$$E_{\text{photon}} = \underbrace{2.856\text{ eV}}_{E_5\rightarrow E_2} - \underbrace{2.98196\times10^{-5}\text{ eV}}_{\Delta E_{1/2\rightarrow 3/2}}.$$

As before, we have

$$\Delta\lambda = -\frac{\lambda^2\Delta E}{hc} = -\frac{(434\text{ nm})^2(-2.98196\times10^{-5}\text{ eV})}{(4.13567\times10^{-15}\text{ eV.s})(2.998\times10^8\text{ m/s})} = +4.535\times10^{-12}\text{ m} = +4.54\times10^{-3}\text{ nm}.$$

(e) We want the total broadening of the spectral line. The width is the sum of the quantities we calculated in parts (c) and (d). The width is 7.26×10^{-3} nm + 4.54×10^{-3} nm = 1.18×10^{-2} nm.

EVALUATE: Notice that fine structure effects make vary small changes in the wavelength. That is why they are called *fine* structure.

41.54. **IDENTIFY:** The presence of an external magnetic field shifts the energy levels up or down, depending upon the value of m_l.

SET UP: The selection rules tell us that for allowed transitions, $\Delta l = 1$ and $\Delta m_l = 0$ or ± 1.

EXECUTE: **(a)** $E = hc/\lambda = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(475.082\ \text{nm}) = 2.612\ \text{eV}$.

(b) For allowed transitions, $\Delta l = 1$ and $\Delta m_l = 0$ or ± 1. For the 3*d* state, $n = 3$, $l = 2$, and m_l can have the values $2, 1, 0, -1, -2$. In the 2*p* state, $n = 2$, $l = 1$, and m_l can be $1, 0, -1$. Therefore the 9 allowed transitions from the 3*d* state in the presence of a magnetic field are:

$$l = 2,\ m_l = 2 \to l = 1,\ m_l = 1$$
$$l = 2,\ m_l = 1 \to l = 1,\ m_l = 0$$
$$l = 2,\ m_l = 1 \to l = 1,\ m_l = 1$$
$$l = 2,\ m_l = 0 \to l = 1,\ m_l = 0$$
$$l = 2,\ m_l = 0 \to l = 1,\ m_l = 1$$
$$l = 2,\ m_l = 0 \to l = 1,\ m_l = -1$$
$$l = 2,\ m_l = -1 \to l = 1,\ m_l = 0$$
$$l = 2,\ m_l = -1 \to l = 1,\ m_l = -1$$
$$l = 2,\ m_l = -2 \to l = 1,\ m_l = -1$$

(c) $\Delta E = \mu_\text{B} B = (5.788\times10^{-5}\ \text{eV/T})(3.500\ \text{T}) = 0.000203\ \text{eV}$.

So the energies of the new states are $-8.50000\ \text{eV} + 0$ and $-8.50000\ \text{eV} \pm 0.000203\ \text{eV}$, giving energies of: -8.50020 eV, -8.50000 eV, and -8.49980 eV.

(d) The energy differences of the allowed transitions are equal to the energy differences if no magnetic field were present (2.61176 eV, from part (a)), and that value $\pm\Delta E$ (0.000203 eV, from part (c)). Therefore we get the following:

For $E = 2.61176\ \text{eV}$: $\lambda = 475.082\ \text{nm}$ (which was given).

For $E = 2.61176\ \text{eV} + 0.000203\ \text{eV} = 2.611963\ \text{eV}$:

$$\lambda = hc/E = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(2.611963\ \text{eV}) = 475.045\ \text{nm}.$$

For $E = 2.61176\ \text{eV} - 0.000203\ \text{eV} = 2.61156\ \text{eV}$:

$$\lambda = hc/E = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(2.61156\ \text{eV}) = 475.119\ \text{nm}.$$

EVALUATE: Even a strong magnetic field produces small changes in the energy levels, and hence in the wavelengths of the emitted light.

41.55. **IDENTIFY:** The presence of an external magnetic field shifts the energy levels up or down, depending upon the value of m_l.

SET UP: The energy difference due to the magnetic field is $\Delta E = \mu_\text{B} B$ and the energy of a photon is $E = hc/\lambda$.

EXECUTE: For the *p* state, $m_l = 0$ or ± 1, and for the *s* state $m_l = 0$. Between any two adjacent lines, $\Delta E = \mu_\text{B} B$. Since the change in the wavelength $(\Delta\lambda)$ is very small, the energy change (ΔE) is also very small, so we can use differentials. $E = hc/\lambda$. $|dE| = \dfrac{hc}{\lambda^2}d\lambda$ and $\Delta E = \dfrac{hc\Delta\lambda}{\lambda^2}$. Since $\Delta E = \mu_\text{B} B$, we get $\mu_B B = \dfrac{hc\Delta\lambda}{\lambda^2}$ and $B = \dfrac{hc\Delta\lambda}{\mu_B\lambda^2}$.

$$B = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})(0.0462\ \text{nm})/(5.788\times10^{-5}\ \text{eV/T})(575.050\ \text{nm})^2 = 3.00\ \text{T}.$$

EVALUATE: Even a strong magnetic field produces small changes in the energy levels, and hence in the wavelengths of the emitted light.

41.56. IDENTIFY: Apply constant acceleration equations to relate F_z to the motion of an atom.

SET UP: According to the equation $\mu_z = -(2.00232)\left(\frac{e}{2m}\right)S_z$, the magnitude of μ_z is $|\mu_z| = 9.28\times10^{-24}\ \text{A}\cdot\text{m}^2$. The atomic mass of silver is 0.1079 kg/mol.

EXECUTE: The time required to transit the horizontal 50 cm region is $t = \frac{\Delta x}{v_x} = \frac{0.500\ \text{m}}{375\ \text{m/s}} = 1.333\ \text{ms}$.

The force required to deflect each spin component by 0.50 mm is

$F_z = ma_z = \pm m\frac{2\Delta z}{t^2} = \pm\left(\frac{0.1079\ \text{kg/mol}}{6.022\times10^{23}\ \text{atoms/mol}}\right)\frac{2(0.50\times10^{-3}\ \text{m})}{(1.333\times10^{-3}\ \text{s})^2} = \pm1.008\times10^{-22}\ \text{N}$. Thus, the required magnetic-field gradient is $\left|\frac{dB_z}{dz}\right| = \left|\frac{F_z}{\mu_z}\right| = \frac{1.008\times10^{-22}\ \text{N}}{9.28\times10^{-24}\ \text{J/T}} = 10.9\ \text{T/m}$, which rounds to 11 T/m.

EVALUATE: The two spin components are deflected in opposite directions.

41.57. IDENTIFY: We want to calculate the magnetic moment of a spinning sphere of charge.

SET UP: $\mu = g(Q/2M)L$. Follow the directions in each step of the problem.

EXECUTE: **(a)** $da = (Rd\theta)2\pi R\sin\theta d\theta = 2\pi R^2\sin\theta d\theta$.

(b) $dI = (\sigma dA)/T$ and $T = 2\pi/\omega$. Combining these relations with the result from part (a) gives

$$dI = \frac{\sigma 2\pi R^2 \sin\theta d\theta}{2\pi/\omega} = \sigma\omega R^2 \sin\theta d\theta.$$

(c) Using the results from parts (a) and (b) gives

$d\mu = AdI = \pi(R\sin\theta)^2\sigma\omega R^2\sin\theta d\theta = \pi\sigma\omega R^4\sin^3\theta d\theta$.

(d) Integrate to find the magnetic moment.

$$\mu = \int_0^\pi \pi\sigma\omega R^4\sin^3\theta\, d\theta = \pi\sigma\omega R^4\int_0^\pi \sin\theta(1-\cos^2\theta)d\theta = \frac{4\pi\sigma\omega R^4}{3}.$$

$$\sigma = Q/4\pi R^2 \text{ gives } \mu = \left(\frac{4\pi\omega R^4}{3}\right)\left(\frac{Q}{4\pi R^2}\right) = \frac{\omega R^2 Q}{3}.$$

(e) We now have a solid sphere. Consider the sphere to be a series of thin concentric shells each of thickness dr. Use the results of part (d) with $R \to r$ and $Q \to dq$ (the charge within the shell).

$$dq = \rho(r)dr = \rho(r)4\pi r^2 dr,\quad d\mu = \frac{\omega R^2 dq}{3} = \frac{\omega r^2}{3}\rho(r)4\pi r^2 dr,\quad \mu = \frac{4\pi\omega}{3}\int_0^R \rho(r)r^4\, dr.$$

(f) Wed want the g-factor. Use $L = I\omega$, $I = cMR^2$, and the results of part (e).

$$\mu = \frac{4\pi\omega}{3}\int_0^R \rho(r)r^4\, dr = g\frac{Q}{2M}L = g\frac{Q}{2M}I\omega = \frac{gQ}{2M}(cMR^2\omega).$$

$$g = \frac{8\pi}{3QcR^2}\int_0^R \rho(r)r^4\, dr.$$

(g) We want g if the solid sphere has uniform density. In this case the integral is

$$\int_0^R \rho(r)r^4\, dr = \frac{Q}{\frac{4}{3}\pi R^3}\frac{R^5}{5}.$$

Using $c = 2/5$ and solving the result in (f) for g gives $g = 1$.

EVALUATE: The result shows that for any spinning uniform solid sphere the g-factor is one.

41.58. **IDENTIFY:** The magnetic field at the center of a current loop of radius r is $B = \dfrac{\mu_0 I}{2r}$. $I = e\left(\dfrac{v}{2\pi r}\right)$.

SET UP: Use $L = mvr = \sqrt{l(l+1)}\hbar$. The Bohr radius is $n^2 a_0$.

EXECUTE: $v = \dfrac{\sqrt{l(l+1)}\hbar}{m(n^2 a_0)} = \dfrac{\sqrt{2}(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(9.11\times10^{-31}\ \text{kg})(4)(5.29\times10^{-11}\ \text{m})} = 7.74\times10^5\ \text{m/s}$. The magnetic field generated by the "moving" proton at the electron's position is

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0}{4\pi}\frac{ev}{r^2} = (10^{-7}\ \text{T}\cdot\text{m/A})\frac{(1.60\times10^{-19}\,\text{C})(7.74\times10^5\ \text{m/s})}{(4)^2(5.29\times10^{-11}\ \text{m})^2} = 0.277\ \text{T}.$$

EVALUATE: The effective magnetic field calculated in Example 41.7 for $3p$ electrons in sodium is much larger than the value we calculated for $2p$ electrons in hydrogen.

41.59. **IDENTIFY** and **SET UP:** m_s can take on four different values: $m_s = -\frac{3}{2},\ -\frac{1}{2},\ +\frac{1}{2}, +\frac{3}{2}$. Each nlm_l state can have four electrons, each with one of the four different m_s values. Apply the exclusion principle to determine the electron configurations.

EXECUTE: **(a)** For a filled $n=1$ shell, the electron configuration would be $1s^4$; four electrons and $Z=4$. For a filled $n=2$ shell, the electron configuration would be $1s^4 2s^4 2p^{12}$; twenty electrons and $Z=20$.

(b) Sodium has $Z=11$; eleven electrons. The ground-state electron configuration would be $1s^4 2s^4 2p^3$.

EVALUATE: The chemical properties of each element would be very different.

41.60. **IDENTIFY:** Apply $U = -\mu_z B$, where B is the effective magnetic field. $\Delta E = \dfrac{hc}{\lambda}$.

SET UP: $\mu_\text{B} = \dfrac{e\hbar}{2m} = \dfrac{eh}{4\pi m}$.

EXECUTE: The effective field is that which gives rise to the observed difference in the energy level transition, $B = \dfrac{\Delta E}{\mu_\text{B}} = \dfrac{hc}{\mu_\text{B}}\left(\dfrac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right) = \dfrac{4\pi mc}{e}\left(\dfrac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right)$. Substitution of numerical values gives $B = 7.28\times10^{-3}\ \text{T}$.

EVALUATE: The effective magnetic field we have calculated is much smaller than that calculated for sodium in Example 41.7.

41.61. **(a) IDENTIFY** and **SET UP:** The energy of the photon equals the transition energy of the atom: $\Delta E = hc/\lambda$. The energies of the states are given by $E_n = -\dfrac{13.60\ \text{eV}}{n^2}$.

EXECUTE: $E_n = -\dfrac{13.60\ \text{eV}}{n^2}$, so $E_2 = -\dfrac{13.60\ e\text{V}}{4}$ and $E_1 = -\dfrac{13.60\ \text{eV}}{1}$. Thus $\Delta E = E_2 - E_1$ gives

$$\Delta E = 13.60\ \text{eV}(-\tfrac{1}{4}+1) = \tfrac{3}{4}(13.60\ \text{eV}) = 10.20\ \text{eV} = (10.20\ \text{eV})(1.602\times10^{-19}\,\text{J/eV}) = 1.634\times10^{-18}\,\text{J}.$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{1.634\times10^{-18}\ \text{J}} = 1.22\times10^{-7}\ \text{m} = 122\ \text{nm}.$$

(b) IDENTIFY and **SET UP:** Calculate the change in ΔE due to the orbital magnetic interaction energy, $U = m_l\mu_\text{B}B$, and relate this to the shift $\Delta\lambda$ in the photon wavelength.

EXECUTE: The shift of a level due to the energy of interaction with the magnetic field in the z-direction is $U = m_l\mu_\text{B}B$. The ground state has $m_l = 0$, so is unaffected by the magnetic field. The $n=2$ initial state has $m_l = -1$, so its energy is shifted downward an amount

$$U = m_l \mu_B B = (-1)(9.274\times10^{-24}\ \text{A/m}^2)(2.20\ \text{T}) = (-2.040\times10^{-23}\ \text{J})(1\ \text{eV}/1.602\times10^{-19}\ \text{J})$$
$$= 1.273\times10^{-4}\ \text{eV}.$$

Note that the shift in energy due to the magnetic field is a very small fraction of the 10.2 eV transition energy. Problem 39.74c shows that in this situation $|\Delta\lambda/\lambda| = |\Delta E/E|$. This gives

$$|\Delta\lambda| = \lambda|\Delta E/E| = 122\ \text{nm}\left(\frac{1.273\times10^{-4}\ \text{eV}}{10.2\ \text{eV}}\right) = 1.52\times10^{-3}\ \text{nm} = 1.52\ \text{pm}.$$

EVALUATE: The upper level in the transition is lowered in energy so the transition energy is decreased. A smaller ΔE means a larger λ; the magnetic field increases the wavelength. The fractional shift in wavelength, $\Delta\lambda/\lambda$ is small, only 1.2×10^{-5}.

41.62. **IDENTIFY:** The interaction energy for an electron in a magnetic field is $U = -\mu_z B$, where μ_z is given by $\mu_z = -(2.00232)\left(\frac{e}{2m}\right)S_z$.

SET UP: $\Delta S_z = \hbar$.

EXECUTE: **(a)** $\Delta E = (2.00232)\frac{e}{2m}B\Delta S_z \approx \frac{e\hbar}{m}B = \frac{hc}{\lambda} \Rightarrow B = \frac{2\pi mc}{\lambda e}$.

(b) $B = \frac{2\pi(9.11\times10^{-31}\ \text{kg})(3.00\times10^8\ \text{m/s})}{(0.0420\ \text{m})(1.60\times10^{-19}\ \text{C})} = 0.255\ \text{T}.$

EVALUATE: As shown in Figure 41.18 in the textbook, the lower state in the transition has $m_s = -\frac{1}{2}$ and the upper state has $m_s = +\frac{1}{2}$.

41.63. **IDENTIFY** and **SET UP:** The energy due to the interaction of the electron with the magnetic field is $U = -\mu_z B$. In a transition from the $m_s = -\frac{1}{2}$ state to the $m_s = \frac{1}{2}$ state, $\Delta U = 2\mu_z B$. This energy difference is the energy of the absorbed photon. The energy of the photon is $E = hf$, and $f\lambda = c$. For an electron, $S_z = \hbar/2$.

EXECUTE: **(a)** First find the frequency for each wavelength in the table with the problem. For example for the first column, $f = c/\lambda = (2.998\times10^8\ \text{m/s})/(0.0214\ \text{m}) = 1.40\times10^{10}$ Hz. Doing this for all the wavelengths gives the values in the following table. Figure 41.63 shows the graph of f versus B for this data. The slope of the best-fit straight line is 2.84×10^{10} Hz/T.

B (T)	0.51	0.74	1.03	1.52	2.02	2.48	2.97
f (10^{10} Hz)	1.401	2.097	2.802	4.199	5.604	7.005	8.398

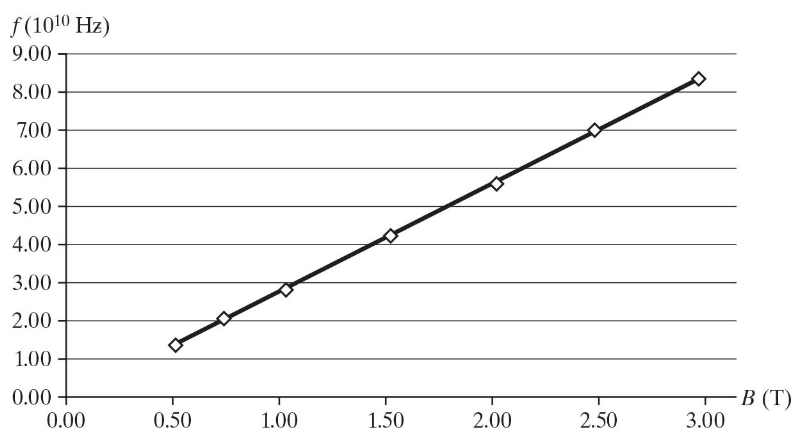


Figure 41.63

(b) The photon energy is $E = hf$, and that is the energy difference between the two levels. So $E = \Delta U = 2\mu_z B = hf$, which gives $f = \left(\dfrac{2\mu_z}{h}\right)B$. The graph of f versus B should be a straight line having slope equal to $2\mu_z/h$. Therefore the magnitude of the spin magnetic moment is $\mu_z = \frac{1}{2}h(\text{slope}) = \frac{1}{2}(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.84\times10^{10}\ \text{Hz/T}) = 9.41\times10^{-24}$ J/T.

(c) Using $\gamma = \dfrac{|\mu_z|}{|S_z|} = \dfrac{|\mu_z|}{\hbar/2}$ gives $\gamma = \dfrac{2(9.41\times10^{-24}\ \text{J/T})}{1.055\times10^{-34}\ \text{J}\cdot\text{s}} = 1.784\times10^{11}\ \text{Hz/T}$, which rounds to 1.78×10^{11} Hz/T. This gives $\dfrac{\gamma}{e/2m} = \dfrac{2\gamma m}{e} = \dfrac{2(1.784\times10^{11}\ \text{Hz/T})(9.11\times10^{-31}\ \text{kg})}{1.602\times10^{-19}\ \text{C}} = 2.03.$

EVALUATE: Our result of 2.03 for the gyromagnetic ratio for electron spin is in good agreement with the currently accepted value of 2.00232.

41.64. IDENTIFY: The photons have different energy due to spin-orbit coupling, which sifts the energy levels very slightly.

SET UP: Without spin-orbit coupling, the energy levels are given by $E_n = -\dfrac{13.60\ \text{eV}}{n^2}$. If we include this coupling, they are $E_{n,j} = -\dfrac{13.60\ \text{eV}}{n^2}\left[1+\dfrac{\alpha^2}{n^2}\left(\dfrac{n}{j+\frac{1}{2}}-\dfrac{3}{4}\right)\right]$. The energy of a photon due a transition involving energy change ΔE is $\Delta E = \dfrac{hc}{\lambda}$. Follow the ideas presented in Example 41.8, in which it was shown that $\Delta\lambda = -\dfrac{\lambda}{E_{\text{photon}}}\Delta E_{\text{photon}}$ since the energy shifts due to spin-orbit coupling are extremely small.

EXECUTE: For a transition from $n = 3$ to $n = 2$, without including fine structure, we use $E_n = -\dfrac{13.60\ \text{eV}}{n^2}$ to find the energy difference ΔE, whis is equal to the energy of the photon. This gives $E_{\text{photon}} = \Delta E = (-13.6\ \text{eV})(1/3^2 - 1/2^2) = 1.889$ eV.

The wavelength of this photon is

$\lambda = hc/E_{\text{photon}} = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^8\ \text{m/s})/(1.889\ \text{eV}) = 6.56\times10^{-7}\ \text{m} = 656$ nm.

ΔE_{photon} is the energy difference between thre $E_{3,3/2}$ and the $E_{3,1/2}$ states. We calculate the energies of these states using $E_{n,j} = -\dfrac{13.60\ \text{eV}}{n^2}\left[1+\dfrac{\alpha^2}{n^2}\left(\dfrac{n}{j+\frac{1}{2}}-\dfrac{3}{4}\right)\right]$.

$$E_{3,3/2} = -\frac{13.60\ \text{eV}}{3^2}\left[1+\frac{(0.007297)^2}{3^2}\left(\frac{3}{\frac{3}{2}+\frac{1}{2}}-\frac{3}{4}\right)\right] = -\frac{13.60\ \text{eV}}{9}(1+4.437\times10^{-6}).$$

$$E_{3,1/2} = -\frac{13.60\ \text{eV}}{3^2}\left[1+\frac{(0.007297)^2}{3^2}\left(\frac{3}{\frac{1}{2}+\frac{1}{2}}-\frac{3}{4}\right)\right] = -\frac{13.60\ \text{eV}}{9}(1+13.312\times10^{-6}).$$

$\Delta E_{\text{photon}} = E_{3,3/2} - E_{3,1/2} = [(13.60\ \text{eV})/9](13.312\times10^{-6} - 4.437\times10^{-6}) = 1.341\times10^{-5}$ eV.

We now use $\Delta\lambda = -\dfrac{\lambda}{E_{\text{photon}}}\Delta E_{\text{photon}}$ to find the difference in the wavelengths.

$$\Delta\lambda = -\frac{656\ \text{nm}}{1.889\ \text{eV}}(1.341\times10^{-5}\ \text{eV}) = -4.66\times10^{-3}\ \text{nm} = -0.00466\ \text{nm}.$$

The photon for the transition from the $j = \frac{1}{2}$ state has the longer wavelength because the transition energy is smaller.

EVALUATE: The wavelength difference is only 0.00466 nm compared to a wavelength of 656 nm without spin-orbit coupling, so this is a very small effect.

41.65. IDENTIFY: The inner electrons shield much of the nuclear charge from the outer electrons, and this shielding affects the energy levels compared to the hydrogen levels. The atom behaves like hydrogen with an effective charge in the nucleus.

SET UP: The ionization energies are $E_n = \dfrac{Z_{\text{eff}}^2}{n^2}(13.6\text{ eV})$.

EXECUTE: **(a)** Make the conversions requested using the following conversion factor.

$$E(\text{eV/atom}) = E(\text{kJ/mol})\left(\frac{1\text{ mol}}{6.02214\times10^{23}\text{ atoms}}\right)\left(\frac{1000\text{ J}}{1\text{ kJ}}\right)\left(\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}}\right) = 0.010364E(\text{kJ/mol}).$$

Li: E = (0.010364)(520.2 kJ/mol) = 5.391 eV
Na: E = (0.010364)(495.8 kJ/mol) = 5.139 eV
K: E = (0.010364)(418.8 kJ/mol) = 4.341 eV
Rb: E = (0.010364)(403.0 kJ/mol) = 4.177 eV
Cs: E = (0.010364)(375.7 kJ/mol) = 3.894 eV
Fr: E = (0.010364)(380 kJ/mol) = 3.9 eV.

(b) From the periodic chart in the Appendix, we get the following information.

Li: $Z = 3$, $n = 2$
Na: $Z = 11$, $n = 3$
K: $Z = 19$, $n = 4$
Rb: $Z = 37$, $n = 5$
Cs: $Z = 55$, $n = 6$
Fr: $Z = 87$, $n = 7$.

(c) Use $E_n = \dfrac{Z_{\text{eff}}^2}{n^2}(13.6\text{ eV})$ and the values of n in part (b) to calculate Z_{eff}. For example, for Li we have

$5.391\text{ eV} = \dfrac{Z_{\text{eff}}^2}{2^2}(13.6\text{ eV}) \rightarrow Z_{\text{eff}} = 1.26$, and for Na we have

$5.139\text{ eV} = \dfrac{Z_{\text{eff}}^2}{3^2}(13.6\text{ eV}) \rightarrow Z_{\text{eff}} = 1.84$. Doing this for the other atoms gives

Li: $Z_{\text{eff}} = 1.26$
Na: $Z_{\text{eff}} = 1.84$
K: $Z_{\text{eff}} = 2.26$
Rb: $Z_{\text{eff}} = 2.77$
Cs: $Z_{\text{eff}} = 3.21$
Fr: $Z_{\text{eff}} = 3.8$.

EVALUATE: **(d)** We can see that Z_{eff} increases as Z increases. The outer (valence) electron has increasing probability density within the inner shells as Z increases, and therefore it "sees" more of the nuclear charge.

41.66. IDENTIFY and SET UP: The energy levels of a particle in a cubical box of length L are $E_{n_X,n_Y,n_Z} = \dfrac{(n_X^2+n_Y^2+n_Z^2)\pi^2\hbar^2}{2mL^2}$. The longest wavelength absorbed is for the smallest energy transition, which is between the ground state (1, 1, 1) and the next higher state (2, 1, 1). The energy absorbed by the photon is $\Delta E = \dfrac{hc}{\lambda}$.

EXECUTE: **(a)** The lowest energy transition is $\Delta E = \left[(2^2+1^2+1^2)-(1^2+1^2+1^2)\right]\dfrac{\pi^2\hbar^2}{2mL^2}$, and this is equal to the energy of the photon $\Delta E = \dfrac{hc}{\lambda}$. So, $\dfrac{hc}{624\text{ nm}} = \dfrac{3\pi^2\hbar^2}{2mL^2} = \dfrac{3h^2}{8mL^2}$, which gives

$L = \sqrt{\dfrac{3h(624\text{ nm})}{8mc}}$.

$$L = \sqrt{\frac{3(6.626\times10^{-34}\text{ J}\cdot\text{s})(624\text{ nm})}{8(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m/s})}} = 7.53\times10^{-10}\text{ m} = 0.753\text{ nm}.$$

(b) The final state is now *n*. Call the wavelength in part (a) λ_1 and the one in part (b) λ_2. From part (a) we have $\dfrac{hc}{\lambda_1} = \dfrac{3\pi^2\hbar^2}{2mL^2}$ and for part (b) we have $\dfrac{hc}{\lambda_2} = (n^2-3)\dfrac{\pi^2\hbar^2}{2mL^2}$. Diving these two equations gives

$\dfrac{hc/\lambda_2}{hc/\lambda_1} = \dfrac{\lambda_1}{\lambda_2} = \dfrac{624}{234} = \dfrac{n^2-3}{3}$. Solving for *n* gives $n^2 = 3 + 3(624/234) = 3 + 8 = 11$.

There are 3 possibilities: (1, 1, 3), (1, 3, 1), and (3, 1, 1), so the degeneracy is 6 including spin.
EVALUATE: The dimensions of this box are around 7 times the diameter of the Bohr hydrogen atom, which is reasonable.

41.67. **IDENTIFY:** This problem deals with the entanglement of three spin-3/2 particles.
SET UP: $s_z = m_z\hbar$, $m_z = \pm1/2, \pm3/2$.

EXECUTE: **(a)** These particles have spin 3/2, so their spin magnitude *S* is

$S = \sqrt{\dfrac{3}{2}\left(\dfrac{3}{2}+1\right)}\,\hbar = \sqrt{15/4}\,\hbar$.

(b) No particles can have identical states. We follow the reasoning of Section 41.8 in the textbook. For simplicity call the states $A = 1/2$, $B = -1/2$, $C = -3/2$. The possible combinations of *ABC* are *ABC*, *ACB*, *BAC*, *BCA*, *CAB*, and *CBA*. In terms of the wave functions, the normalized wave functions are

$$\psi = \frac{1}{\sqrt{6}}\big(\psi_{+1/2}\psi_{-1/2}\psi_{-3/2} - \psi_{+1/2}\psi_{-3/2}\psi_{-1/2} + \psi_{-1/2}\psi_{+1/2}\psi_{-3/2} - \psi_{+1/2}\psi_{-3/2}\psi_{+1/2} + \psi_{-3/2}\psi_{+1/2}\psi_{-1/2}$$
$$-\psi_{-3/2}\psi_{-1/2}\psi_{+1/2}\big).$$

The factor of $1/\sqrt{6}$ is present for normalization because the probability is proportional to the square of the wave function.
(c) Particle *A* has $m_z = +1/2$, but the other two could have $m_z = -3/2$ or $-1/2$. So the wave function is now $\psi = \dfrac{1}{\sqrt{2}}\big(\psi_{+1/2}\psi_{-3/2}\psi_{-1/2} - \psi_{+1/2}\psi_{+1/2} + \psi_{-3/2}\big)$.
(d) Particle *A* has $m_z = +1/2$, so particle *B* cannot have $m_z = +1/2$. But it could have $m_z = -1/2$ or $-3/2$. None of them can have $m_z = +3/2$. So the probabilities are 1/2, 0, 1/2, 0.
(e) Now we know that particle *C* has $m_z = -3/2$, so the wave functions are $\psi = \psi_{+1/2}\,\psi_{-1/2}\psi_{-3/2}$.
(f) We know that $m_z = -1/2$ for particle *B*, so the probabilities are 0, 1, 0, 0.
EVALUATE: We could not draw all these conclusions if the particles were not entangled.

41.68. **IDENTIFY** and **SET UP:** The potential $U(x) = \frac{1}{2}k'x^2$ is that of a simple harmonic oscillator. Treated quantum mechanically (see Section 40.5) each *energy* state has energy $E_n = \hbar\omega(n+\frac{1}{2})$. Since electrons obey the exclusion principle, this allows us to put *two* electrons (one for each $m_s = \pm\frac{1}{2}$) for every value of *n*—each quantum state is then defined by the ordered pair of quantum numbers (n, m_s).

EXECUTE: By placing two electrons in each energy level the lowest energy is

$$2\left(\sum_{n=0}^{N-1} E_n\right) = 2\left(\sum_{n=0}^{N-1} \hbar\omega(n+\tfrac{1}{2})\right) = 2\hbar\omega\left[\sum_{n=0}^{N-1} n + \sum_{n=0}^{N-1} \tfrac{1}{2}\right] = 2\hbar\omega\left[\frac{(N-1)(N)}{2} + \frac{N}{2}\right] =$$

$\hbar\omega[N^2 - N + N] = \hbar\omega N^2 = \hbar N^2\sqrt{\dfrac{k'}{m}}$. Here we realize that the first value of n is zero and the last value of n is $N-1$, giving us a total of N energy levels filled.

EVALUATE: The minimum energy for one electron moving in this potential is $\frac{1}{2}\hbar\omega$, with $\omega = \sqrt{\dfrac{k'}{m}}$. For $2N$ electrons the minimum energy is larger than $(2N)(\frac{1}{2}\hbar\omega)$, because only two electrons can be put into each energy state. For example, for $N = 2$ (4 electrons), there are two electrons in the $E = \frac{1}{2}\hbar\omega$ energy state and two in the $\frac{3}{2}\hbar\omega$ state, for a total energy of $2(\frac{1}{2}\hbar\omega) + 2(\frac{3}{2}\hbar\omega) = 4\hbar\omega$, which is in agreement with our general result.

41.69. IDENTIFY and **SET UP:** Apply Newton's second law and Bohr's quantization to one of the electrons.

EXECUTE: **(a)** Apply Coulomb's law to the orbiting electron and set it equal to the centripetal force. There is an attractive force with charge $+2e$ a distance r away and a repulsive force a distance $2r$ away. So, $\dfrac{(+2e)(-e)}{4\pi\epsilon_0 r^2} + \dfrac{(-e)(-e)}{4\pi\epsilon_0 (2r)^2} = \dfrac{-mv^2}{r}$. But, from the quantization of angular momentum in the first Bohr orbit, $L = mvr = \hbar \Rightarrow v = \dfrac{\hbar}{mr}$. So, $\dfrac{-2e^2}{4\pi\epsilon_0 r^2} + \dfrac{e^2}{4\pi\epsilon_0 (2r)^2} = \dfrac{-mv^2}{r} = \dfrac{-m\left(\dfrac{\hbar}{mr}\right)^2}{r} = -\dfrac{\hbar^2}{mr^3}$

$\Rightarrow \dfrac{-7}{4}\dfrac{e^2}{r^2} = -\dfrac{4\pi\epsilon_0 \hbar^2}{mr^3}$. $r = \dfrac{4}{7}\left(\dfrac{4\pi\epsilon_0 \hbar^2}{me^2}\right) = \dfrac{4}{7}a_0 = \dfrac{4}{7}(0.529\times10^{-10}\text{ m}) = 3.02\times10^{-11}$ m. And

$$v = \frac{\hbar}{mr} = \frac{7}{4}\frac{\hbar}{ma_0} = \frac{7}{4}\frac{(1.054\times10^{-34}\text{ J}\cdot\text{s})}{(9.11\times10^{-31}\text{ kg})(0.529\times10^{-10}\text{ m})} = 3.83\times10^6\text{ m/s}.$$

(b) $K = 2(\frac{1}{2}mv^2) = 9.11\times10^{-31}\text{ kg }(3.83\times10^6\text{ m/s})^2 = 1.34\times10^{-17}\text{ J} = 83.5\text{ eV}$.

(c) $U = 2\left(\dfrac{-2e^2}{4\pi\epsilon_0 r}\right) + \dfrac{e^2}{4\pi\epsilon_0 (2r)} = \dfrac{-4e^2}{4\pi\epsilon_0 r} + \dfrac{e^2}{4\pi\epsilon_0 (2r)} = \dfrac{-7}{2}\left(\dfrac{e^2}{4\pi\epsilon_0 r}\right) = -2.67\times10^{-17}\text{ J} = -166.9\text{ eV}$.

(d) $E_\infty = -[-166.9\text{ eV} + 83.5\text{ eV}] = 83.4\text{ eV}$, which is only off by about 5% from the real value of 79.0 eV.

EVALUATE: The ground state energy of helium in this model is $K + U = -83.4$ eV. The ground state energy of He^+ is $4(-13.6\text{ eV}) = -54.4$ eV. Therefore, the energy required to remove one electron from helium in this model is $-(-83.4\text{ eV} + 54.4\text{ eV}) = 29.0$ eV. The experimental value for this quantity is 24.6 eV.

41.70. IDENTIFY and **SET UP:** For the Bohr hydrogen model, $r = a_0 n^2$.

EXECUTE: Using $r = a_0 n^2$ gives 1×10^{-6} m = 5.29×10^{-11} m, so $n = 137 \approx 140$, which makes choice (a) the correct one.

EVALUATE: The energy of an electron in this state would be only $E_{140} = (-13.6\text{ eV})/137^2 = -7.25\times10^{-4}$ eV, which is very small compared to ground-state hydrogen.

41.71. IDENTIFY and **SET UP:** Particle density is the number particles divided by the volume they occupy. The distance between particles is 10 times their size, which is 20 μm.

EXECUTE: Think of each atom as being in a cubical box that is 20 μm on each side. The particle density is (1 atom)/(20 μm)^3 = (1 atom)/(20 $\times10^{-4}$ cm)^3 = 1.25 $\times10^8$ atoms/cm^3 $\approx 10^8$ atoms/cm^3, choice (b).

EVALUATE: For rubidium, with 85 nucleons, the mass density ρ of these atoms would be $\rho = (85)(1.67\times10^{-24}\text{ g})(10^{8}\text{ atoms/cm}^3) = 1.4\times10^{-14}\text{ g/cm}^3$. Ordinary rubidium has a density of 1.53 g/cm^3, so these Rydberg atoms are much farther apart than rubidium atoms under normal conditions.

41.72. **IDENTIFY** and **SET UP:** $L = \sqrt{l(l+1)}\hbar.$ For this state, $n = 100$ and $l = 2$.

EXECUTE: $L = \sqrt{l(l+1)}\hbar = \sqrt{2(2+1)}\hbar = \sqrt{6}\hbar.$ So choice (b) is correct.

EVALUATE: This is not the largest that L could be since l could be 99 for $n = 100$.

41.73. **IDENTIFY** and **SET UP:** $m_l = \pm l, \pm(l-1), \ldots, 0$. $m_s = \pm\frac{1}{2}$ for each m_l value.

EXECUTE: If $l = 2$, $m_l = -2, -1, 0, +1, +2$; and $m_s = \pm\frac{1}{2}$ for each of these states, so there are 10 possible states, which is choice (d).

EVALUATE: In the $n = 100$ state, there are 100 possible values of l, so there are many more additional states for these values of l.

42

MOLECULES AND CONDENSED MATTER

VP42.3.1. **IDENTIFY:** This problem involves the rotational energy of a molecule.

SET UP and **EXECUTE:** **(a)** We want the reduced mass. Use the given masses in the equation

$$m_r = \frac{m_1 m_2}{m_1 + m_2}.$$

The result is $m_r = 1.240 \times 10^{-26}$ kg.

(b) We want the moment of inertia. Use $r_0 = 0.1154$ nm and the result of part (a). The result is $I = m_r r_0^2 = 1.651 \times 10^{-46}\ \text{kg} \cdot \text{m}^2$.

(c) We want the energies of the rotational states $l = 0, 1, 2$.

$$E_l = l(l+1)\frac{\hbar^2}{2I}$$

$$E_0 = 0$$

$$E_1 = 1(1+1)\frac{\hbar^2}{2I} = 4.203 \times 10^{-4}\ \text{eV} = 0.4203\ \text{meV}.$$

$$E_2 = 2(2+1)\frac{\hbar^2}{2I} = 1.261 \times 10^{-3}\ \text{eV} = 1.261\ \text{meV}.$$

EVALUATE: Note that molecular rotational energies are of the order of *milli*electron-volts, whereas energy states for electrons in atoms are around a few eV.

VP42.3.2. **IDENTIFY:** We are dealing with the rotational energy levels of a molecule.

SET UP: The following equations apply:

$$E_l = l(l+1)\frac{\hbar^2}{2I}, \quad m_r = \frac{m_1 m_2}{m_1 + m_2}, \quad I = m_r r_0^2.$$

EXECUTE: **(a)** We want the moment of inertia. Solve the E_l equation for I for the $l = 2$ state.

$$I = \frac{3\hbar^2}{E_2} = \frac{3\hbar^2}{7.90\ \text{meV}} = 2.64 \times 10^{-47}\ \text{kg} \cdot \text{m}^2.$$

(b) We want the reduced mass. Use the two given masses in the equation for m_r, giving

$$m_r = 1.627 \times 10^{-27}\ \text{kg}.$$

(c) We want the distance r_0 between the two nuclei. Solve $I = m_r r_0^2$ for r_0 and use the results of parts (a) and (b). This gives

$$r_0 = \sqrt{I/m_r} = 0.127\ \text{nm}.$$

EVALUATE: Note that the chlorine has only a small effect on the reduced mass which is essentially the mass of the hydrogen atom.

VP42.3.3. **IDENTIFY:** This problem is about the rotational energy levels of the SiO molecule.

SET UP: The following equations apply:

$$E_l = l(l+1)\frac{\hbar^2}{2I},\quad m_{\rm r} = \frac{m_1 m_2}{m_1+m_2},\quad I = m_{\rm r} r_0^2.$$

EXECUTE: **(a)** We want I. The energy of the emitted photon is equal to the energy difference between the $l = 0$ and $l = 1$ rotational levels. The photon energy is $E = hc/\lambda$. Therefore

$$E_1 - E_0 = 1(1+1)\frac{\hbar^2}{2I} - 0 = \frac{\hbar^2}{I}$$

Solving for I gives

$$I = \frac{\hbar\lambda}{2\pi c} = \frac{\hbar(6.882\text{ mm})}{2\pi c} = 3.853\times10^{-46}\text{ kg}\cdot\text{m}^2.$$

(b) We want the reduced mass. Use the given masses.

$$m_{\rm r} = \frac{m_1 m_2}{m_1+m_2} = 1.690\times10^{-26}\text{ kg}.$$

(c) We want r_0. Solve $I = m_{\rm r} r_0^2$ for r_0 and use the answers from parts (a) and (b), which gives r_0 = 0.1510 nm.

EVALUATE: The result in part (c) is quite reasonable for atomic distances in molecules.

VP42.3.4. **IDENTIFY:** This problem involves the vibrational and rotational energy levels in the CO molecule.

SET UP: When the molecule makes a transition to a lower energy state, the energy $E_{\rm ph}$ of the emitted photon is equal to the *magnitude* of the energy difference between the two states. Using Eq. (42.9) this photon energy is

$$E_{\rm ph} = l_1(l_1+1)\frac{\hbar^2}{2I} + \left(n_1+\frac{1}{2}\right)\hbar\omega - \left[l_2(l_2+1)\frac{\hbar^2}{2I} + \left(n_2+\frac{1}{2}\right)\hbar\omega\right]$$

$$E_{\rm ph} = \left[l_1(l_1+1) - l_2(l_2+1)\right]\frac{\hbar^2}{2I} + (n_1-n_2)\hbar\omega$$

$n_1 - n_2 = +1$. Using this fact and some results from Example 42.3, we have

$$(n_1-n_2)\hbar\omega = 0.2690\text{ eV},\quad \frac{\hbar^2}{2I} = 0.2395\text{ meV}$$

The photon energy is therefore given by

$$E_{\rm ph} = \left[l_1(l_1+1) - l_2(l_2+1)\right](0.2395\text{ meV}) + 0.2690\text{ eV}$$

EXECUTE: **(a)** Initial state is $l_1 = 3$. $\Delta l = -1$, so $l_2 = 2$.

$3 \to 2$ transition:

$$E_{\rm ph} = \left[3(3+1) - 2(2+1)\right](0.2395\text{ meV}) + 0.2690\text{ eV} = 1.4370\text{ meV} + 0.2690\text{ eV} = 0.270437\text{ eV}.$$

$$f = E_{\rm ph}/h = (0.270437\text{ eV})/h = 6.535\times10^{13}\text{ Hz}.$$

$$\lambda = c/f = 4.585\ \mu\text{m}.$$

(b) Initial state is $l_1 = 2$. $\Delta l = -1$, so $l_2 = 1$.

$2 \to 1$ transition:

$$E_{\rm ph} = \left[2(2+1) - 1(1+1)\right](0.2395\text{ meV}) + 0.2690\text{ eV} = 0.9580\text{ meV} + 0.2690\text{ eV} - 0.269958\text{ eV}$$

$$f = E_{\rm ph}/h = (0.269958\text{ eV})/h = 6.538\times10^{13}\text{ Hz}.$$

$$\lambda = c/f = 4.593\ \mu\text{m}.$$

EVALUATE: The wavelengths are very close together because the energy differences between states are very small since they are due only to *rotational* transitions which are low energy.

VP42.7.1. **IDENTIFY:** This problem involves the Fermi-Dirac distribution.

SET UP: Eq. (42.16) gives the probability $f(E)$ that a given energy state is occupied. The target variable is the energy for each probability. Solve for E in each case.

EXECUTE: **(a)** $f(E) = 0.33$. Solve for E.

$$0.33 = \frac{1}{e^{(E-E_\text{F})/kT}+1}$$

Separate the exponential and use logarithms, giving

$$(E - E_\text{F})/kT = \ln 2.02$$

$$E = E_\text{F} + 0.71kT.$$

(b) $f(E) = 0.90$. Follow the same procedure using 0.90 instead of 0.33, giving $E = E_\text{F} - 2.2kT$.

EVALUATE: If $E = E_\text{F}$, $f(E) = 0.50$, for $f(E) > 0.50$, $E < E_\text{F}$, and for $f(E) < 0.50$, $E > E_\text{F}$. Our results are consistent with these conditions.

VP42.7.2. **IDENTIFY:** This problem involves the Fermi-Dirac distribution.

SET UP: Eq. (42.16) gives the probability $f(E)$ that a given energy state is occupied. The target variable is the probability for each specified energy.

EXECUTE: **(a)** $E = E_\text{F} - 0.0250$ eV. Using Eq. (42.16) gives

$$f(E) = \frac{1}{e^{(E_\text{F}-0.0250\text{ eV}-E_\text{F})/kT}+1}$$

The exponent of e is $(0.0250\text{ eV})/kT = 0.9902$, so $f(E) = 1/(e^{-0.9902} + 1) = 0.729$.

(b) $E = E_\text{F} + 0.0400$ eV. Using Eq. (42.16) and following the same procedure gives $f(E) = 1/(e^{1.5843} + 1) = 0.170$.

(c) $E = E_\text{F} + 0.100$ eV. Using the same procedure gives $f(E) = 0.0187$.

EVALUATE: The probability decreases as E increases above the Fermi energy.

VP42.7.3. **IDENTIFY:** This problem involves the Fermi energy and the density of states.

SET UP and **EXECUTE:** **(a)** We want the Fermi energy at absolute zero. Using Eq. (42.19) with N/V given in the problem, we get

$$E_\text{F0} = \frac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\frac{N}{V}\right)^{2/3} = 5.5\text{ eV}.$$

(b) The speed is the target variable. The kinetic energy K is equal to the Fermi energy. Solve $K = E_\text{F}$ for v and use the result from part (a).

$$v = \sqrt{\frac{2E_\text{F}}{m}} = 1.4\times10^6\text{ m/s}.$$

(c) The target variable is the density of states. Use Eq. (42.15) with $V = 1.0\text{ cm}^3$ and $E = 5.5$ eV from part (a). The result is

$$g(E) = \frac{(2m)^{3/2}V}{2\pi^2\hbar^3}E^{1/2} = 9.9\times10^{40}\text{ states/J} = 1.6\times10^{22}\text{ states/eV}.$$

EVALUATE: The electron speeds are not great enough to require the use of special relativity.

VP42.7.4. **IDENTIFY:** This problem deals with the Fermi energy.

SET UP: From the previous problem we have $E_\text{F0} = 5.5$ eV. The target variable is the electron energy for the given probabilities.

EXECUTE: **(a)** $f(E) = 0.92$. Solve Eq. (42.16) for E using logarithms.

$$0.92 = \frac{1}{e^{(E-E_\text{F})/kT}+1}$$

$$E - E_\text{F} = kT\ln(0.08696)$$

$$E = 5.4\text{ eV}.$$

(b) $f(E) = 1.0 \times 10^{-4}$. Use the same procedure as in part (a) except $f(E) = 0.00010$, which gives $E = 5.7$ eV.

EVALUATE: Our results are consistent with problem VP42.7.1. When $E > E_F$, $f(E) < 0.50$, and when $E < E_F$, $f(E) > 0.50$.

VP42.9.1. **IDENTIFY:** This problem is about the Fermi energy and the band gap in a semiconductor.

SET UP and **EXECUTE:** For an electron at the bottom of the conduction band, $E = E_F + E_g/2$, so $E - E_F = E_g/2$. The target variable is the probability $f(E)$. Use Eq. (42.16) for $f(E)$.

(a) $E_g = 0.500$ eV. $(E - E_F)/kT = E_g/2kT = (0.500 \text{ eV})/2kT = 10.18$. Eq. (42.16) gives $f(E) = 1/(e^{10.18} + 1) = 3.79 \times 10^{-5}$.

(b) $E_g = 1.50$ eV. $(E - E_F)/kT = E_g/2kT = (1.50 \text{ eV})/2kT = 30.54$. Eq. (42.16) gives $f(E) = 1/(e^{30.54} + 1) = 5.46 \times 10^{-14}$.

EVALUATE: The probability decreases as the gap widens.

VP42.9.2. **IDENTIFY:** This problem is about the band gap in a semiconductor.

SET UP: $E = E_F + E_g/2$, so $E - E_F = E_g/2$. The target variable is the probability $f(E)$. Use Eq. (42.16) for $f(E)$. We want the ratio of the probabilities at different temperatures.

EXECUTE: **(a)** $E_g = 0.400$ eV. <u>At 315 K</u>: $E_g/2kT = (0.400 \text{ eV})/[2k(315 \text{ K})] = 7.3682$. $f(E)_{315} = 1/(e^{7.3682} + 1) = 6.3058 \times 10^{-4}$.

<u>At 295 K</u>: The exponent is 7.8677, so $f(E) = 1/(e^{7.8677} + 1) = 3.8290 \times 10^{-4}$.

Now take the ratio of the probabilities using the results we just found. This gives

$$\frac{f(E)_{315}}{f(E)_{295}} = \frac{6.3058\times10^{-4}}{3.8290\times10^{-4}} = 1.65.$$

(b) $E_g = 0.800$ eV. Use the same procedure as for part (a). <u>At 315 K</u>: $E_g/2kT = 14.7364$ which gives $f(E) = 3.98165 \times 10^{-7}$.

<u>At 295 K</u>: $E_g/2kT = 15.7354$ which gives $f(E) = 1.46623 \times 10^{-7}$.

$$\frac{f(E)_{315}}{f(E)_{295}} = \frac{3.98165\times10^{-7}}{1.46623\times10^{-7}} = 2.72.$$

EVALUATE: At $T = 315$ K the probability is greater than at 295 K, which is in agreement with Example 42.9.

VP42.9.3. **IDENTIFY:** We are dealing with the energy gap in a semiconductor.

SET UP: The target variable is the width of the energy gap E_g at different temperatures. Combining $E - E_F = E_g/2$ with Eq. (42.16) gives

$$f(E) = \frac{1}{e^{E_g/2kT} + 1}$$

Solve this equation for E_g using logarithms.

$$E_g = 2kT \ln\left(1 + \frac{1}{f(E)}\right) = 2kT \ln(527.32).$$

EXECUTE: **(a)** $T = 305$ K. Using this temperature in the equation we just derived gives $E_g = 0.329$ eV.

(b) $T = 325$ K. Using the same procedure gives $E_g = 0.351$ eV.

EVALUATE: As the temperature increases, the gap width also increases.

VP42.9.4. **IDENTIFY:** We are dealing with the energy gap in a semiconductor.

SET UP: The target variable is the probability at different temperatures. Figure VP42.9.4 helps to visualize how the various quantities are related. In this case, $E = E_F + E_g/4$. Therefore $E - E_F = E_g/4 = (0.300 \text{ eV})/4 = 0.0750$ eV and $(E - E_F)/kT = (0.0750 \text{ eV})/kT = (870.37 \text{ K})/T$.

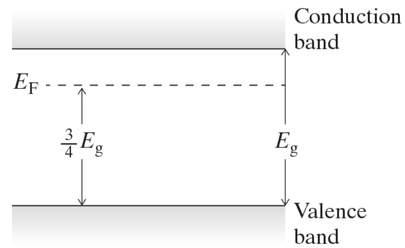


Figure VP42.9.4

EXECUTE: Use Eq. (42.16).

(a) $T = 275$ K. $(E - E_F)/kT = (870.37\text{ K})/T = (870.37\text{ K})/(275\text{ K}) = 3.1650$. Therefore

$$f(E) = \frac{1}{e^{3.1650}+1} = 0.0405.$$

(b) $T = 325$ K. $(E - E_F)/kT = (870.37\text{ K})/T = (870.37\text{ K})/(325\text{ K}) = 2.67806$. Therefore

$$f(E) = \frac{1}{e^{2.67806}+1} = 0.0643.$$

EVALUATE: As the temperature increases, so does the probability of finding electrons in the conduction band.

42.1. IDENTIFY and **SET UP:** $U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$. The binding energy of the molecule is equal to U plus the ionization energy of K minus the electron affinity of Br.

EXECUTE: **(a)** $U = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -5.0\text{ eV}$.

(b) $-5.0\text{ eV} + (4.3\text{ eV} - 3.5\text{ eV}) = -4.2\text{ eV}$.

EVALUATE: We expect the magnitude of the binding energy to be somewhat less than this estimate. At this separation the two ions don't behave exactly like point charges and U is smaller in magnitude than our estimate. The experimental value for the binding energy is -4.0 eV, which is smaller in magnitude than our estimate.

42.2. IDENTIFY: The energy decrease of the molecule or atom is equal to the energy of the emitted photon. From this energy, we can calculate the wavelength of the photon.

SET UP: $\Delta E = \frac{hc}{\lambda}$. Use Figure 32.4 in the textbook to find out the region of the electromagnetic spectrum in which each wavelength lies.

(a) EXECUTE: $\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{0.198\text{ eV}} = 6.26\ \mu\text{m}$.

EVALUATE: This radiation is in the infrared.

(b) EXECUTE: $\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{7.80\text{ eV}} = 159\text{ nm}$.

EVALUATE: This radiation is in the ultraviolet.

(c) EXECUTE: $\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{4.80\times10^{-3}\text{ eV}} = 0.258\text{ mm}$.

EVALUATE: This radiation is in the microwave region.

42.3. **IDENTIFY:** The energy of the photon is equal to the energy difference between the $l=1$ and $l=2$ states. This energy determines its wavelength.

SET UP: The reduced mass of the molecule is $m_r = \dfrac{m_H m_H}{m_H + m_H} = \frac{1}{2}m_H$, its moment of inertia is $I = m_r r_0^2$, the photon energy is $\Delta E = \dfrac{hc}{\lambda}$, and the energy of the state l is $E_l = l(l+1)\dfrac{\hbar^2}{2I}$.

EXECUTE: $I = m_r r_0^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(0.074\times10^{-9}\text{ m})^2 = 4.57\times10^{-48}\text{ kg}\cdot\text{m}^2$. Using $E_l = l(l+1)\dfrac{\hbar^2}{2I}$, the energy levels are

$E_2 = 6\dfrac{\hbar^2}{2I} = 6\dfrac{(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{2(4.57\times10^{-48}\text{ kg}\cdot\text{m}^2)} = 6(1.218\times10^{-21}\text{ J}) = 7.307\times10^{-21}\text{ J}$ and

$E_1 = 2\dfrac{\hbar^2}{2I} = 2(1.218\times10^{-21}\text{ J}) = 2.436\times10^{-21}\text{ J}$. $\Delta E = E_2 - E_1 = 4.87\times10^{-21}\text{ J}$. Using $\Delta E = \dfrac{hc}{\lambda}$ gives

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{4.871\times10^{-21}\text{ J}} = 4.08\times10^{-5}\text{ m} = 40.8\ \mu\text{m}.$$

EVALUATE: This wavelength is much longer than that of visible light.

42.4. **IDENTIFY:** The energy absorbed by the photon is equal to the energy difference between the two rotational states.

SET UP: The rotational energy of a molecule is $E_l = l(l+1)\dfrac{\hbar^2}{2I}$ and the energy of the photon is the energy difference ΔE between the states, so $\Delta E = hc/\lambda$. The transition is from the $l = 3$ state to the $l = 4$ state.

EXECUTE: Using the rotational energy formula, the energy difference between the two states is

$$\Delta E = E_4 - E_3 = 4(4+1)\frac{\hbar^2}{2I} - 3(3+1)\frac{\hbar^2}{2I} = 4\frac{\hbar^2}{I}.$$

This is the energy absorbed by the photon, so $\Delta E = hc/\lambda$ gives $hc/\lambda = 4\hbar^2/I$. Solving for λ gives $\lambda = \pi^2 Ic/h = \pi^2(4.6\times10^{-48}\text{ kg}\cdot\text{m}^2)(3.00\times10^8\text{ m/s})/(6.626\times10^{-34}\text{ J}\cdot\text{s}) = 2.05\times10^{-5}\text{ m} = 20.6\ \mu\text{m}$, which rounds to 21 μm.

EVALUATE: Rotational energy changes are much less than transitions between n-shells, so the wavelengths involved are much longer for rotational transitions.

42.5. **IDENTIFY:** The energy given to the photon comes from a transition between rotational states.

SET UP: The rotational energy of a molecule is $E = l(l+1)\dfrac{\hbar^2}{2I}$ and the energy of the photon is $E = hc/\lambda$.

EXECUTE: Use the energy formula, the energy difference between the $l = 3$ and $l = 1$ rotational levels of the molecule is $\Delta E = \dfrac{\hbar^2}{2I}[3(3+1) - 1(1+1)] = \dfrac{5\hbar^2}{I}$. Since $\Delta E = hc/\lambda$, we get $hc/\lambda = 5\hbar^2/I$. Solving for I gives

$$I = \frac{5\hbar\lambda}{2\pi c} = \frac{5(1.055\times10^{-34}\text{ J}\cdot\text{s})(1.780\text{ nm})}{2\pi(3.00\times10^8\text{ m/s})} = 4.981\times10^{-52}\text{ kg}\cdot\text{m}^2.$$

Using $I = m_r r_0^2$, we can solve for r_0:

$$r_0 = \sqrt{\frac{I(m_N + m_H)}{m_N m_H}} = \sqrt{\frac{(4.981\times10^{-52}\text{ kg}\cdot\text{m}^2)(2.33\times10^{-26}\text{ kg} + 1.67\times10^{-27}\text{ kg})}{(2.33\times10^{-26}\text{ kg})(1.67\times10^{-27}\text{ kg})}} = 5.65\times10^{-13}\text{ m}.$$

EVALUATE: This separation is much smaller than the diameter of a typical atom and is not very realistic. But we are treating a *hypothetical* NH molecule.

42.6. **IDENTIFY:** $I = m_1 r_1^2 + m_2 r_2^2$. Since the two atoms are identical, the center of mass is midway between them.

SET UP: Each atom has a mass m and is at a distance $L/2$ from the center of mass.

EXECUTE: The moment of inertia is $2(m)(L/2)^2 = mL^2/2 = 2.21\times10^{-44}\ \text{kg}\cdot\text{m}^2$.

EVALUATE: $r_0 = L$ and $m_r = m/2$, so $I = m_r r_0^2$ gives the same result.

42.7. **IDENTIFY** and **SET UP:** Set $K = E_1$ from Example 42.2. Use $K = \frac{1}{2}I\omega^2$ to solve for ω and $v = r\omega$ to solve for v.

EXECUTE: **(a)** From Example 42.2, $E_1 = 0.479\ \text{meV} = 7.674\times10^{-23}\ \text{J}$ and $I = 1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2$.
$K = \frac{1}{2}I\omega^2$ and $K = E$ gives $\omega = \sqrt{2E_1/I} = 1.03\times10^{12}\ \text{rad/s}$.

(b) $v_1 = r_1\omega_1 = (0.0644\times10^{-9}\ \text{m})(1.03\times10^{12}\ \text{rad/s}) = 66.3\ \text{m/s}$ (carbon).

$v_2 = r_2\omega_2 = (0.0484\times10^{-9}\ \text{m})(1.03\times10^{12}\ \text{rad/s}) = 49.8\ \text{m/s}$ (oxygen).

(c) $T = 2\pi/\omega = 6.10\times10^{-12}\ \text{s}$.

EVALUATE: Even for fast rotation rates, $v \ll c$.

42.8. **IDENTIFY:** Find ΔE for the transition and compute λ from $\Delta E = hc/\lambda$.

SET UP: From Example 42.2, $E_l = l(l+1)\dfrac{\hbar^2}{2I}$, with $\dfrac{\hbar^2}{2I} = 0.2395\times10^{-3}\ \text{eV}$. $\Delta E = 0.2690\ \text{eV}$ is the spacing between vibrational levels. Thus $E_n = (n+\frac{1}{2})\hbar\omega$, with $\hbar\omega = 0.2690\ \text{eV}$. The total vibrational and rotational energy is $E = E_n + E_l = (n+\frac{1}{2})\hbar\omega + l(l+1)\dfrac{\hbar^2}{2I}$.

EXECUTE: **(a)** $n = 0 \to n = 1$ and $l = 2 \to l = 3$.

For $n = 0,\ l = 2,\ E_i = \frac{1}{2}\hbar\omega + 6\left(\dfrac{\hbar^2}{2I}\right)$.

For $n = 1,\ l = 3,\ E_f = \frac{3}{2}\hbar\omega + 12\left(\dfrac{\hbar^2}{2I}\right)$.

$$\Delta E = E_f - E_i = \hbar\omega + 6\left(\frac{\hbar^2}{2I}\right) = 0.2690\ \text{eV} + 6(0.2395\times10^{-3}\ \text{eV}) = 0.2704\ \text{eV}.$$

$$\frac{hc}{\lambda} = \Delta E \text{ so } \lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{0.2704\ \text{eV}} = 4.586\times10^{-6}\ \text{m} = 4.586\ \mu\text{m}.$$

(b) $n = 0 \to n = 1$ and $l = 3 \to l = 2$.

For $n = 0,\ l = 3,\ E_i = \frac{1}{2}\hbar\omega + 12\left(\dfrac{\hbar^2}{2I}\right)$.

For $n = 1,\ l = 2,\ E_f = \frac{3}{2}\hbar\omega + 6\left(\dfrac{\hbar^2}{2I}\right)$.

$$\Delta E = E_f - E_i = \hbar\omega - 6\left(\frac{\hbar^2}{2I}\right) = 0.2690\ \text{eV} - 6(0.2395\times10^{-3}\ \text{eV}) = 0.2676\ \text{eV}.$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{0.2676\ \text{eV}} = 4.634\times10^{-6}\ \text{m} = 4.634\ \mu\text{m}.$$

(c) $n=0 \to n=1$ and $l=4 \to l=3$.

For $n=0, l=4,\ E_i = \frac{1}{2}\hbar\omega + 20\left(\frac{\hbar^2}{2I}\right)$.

For $n=1, l=3,\ E_f = \frac{3}{2}\hbar\omega + 12\left(\frac{\hbar^2}{2I}\right)$.

$$\Delta E = E_f - E_i = \hbar\omega - 8\left(\frac{\hbar^2}{2I}\right) = 0.2690\ \text{eV} - 8(0.2395\times10^{-3}\ \text{eV}) = 0.2671\ \text{eV}.$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{0.2671\ \text{eV}} = 4.643\times10^{-6}\ \text{m} = 4.643\,\mu\text{m}.$$

EVALUATE: All three transitions are for $n=0 \to n=1$. The spacing between vibrational levels is larger than the spacing between rotational levels, so the difference in λ for the various rotational transitions is small. When the transition is to a larger *l*, $\Delta E > \hbar\omega$ and when the transition is to a smaller *l*, $\Delta E < \hbar\omega$.

42.9. IDENTIFY and SET UP: The energy of a rotational level with quantum number *l* is $E_l = l(l+1)\hbar^2/2I$. $I = m_r r^2$, with the reduced mass m_r given by $m_r = \dfrac{m_1 m_2}{m_1+m_2}$. Calculate *I* and ΔE and then use $\Delta E = hc/\lambda$ to find λ.

EXECUTE: (a) $m_r = \dfrac{m_1 m_2}{m_1+m_2} = \dfrac{m_{Li} m_H}{m_{Li}+m_H} = \dfrac{(1.17\times10^{-26}\ \text{kg})(1.67\times10^{-27}\ \text{kg})}{1.17\times10^{-26}\ \text{kg} + 1.67\times10^{-27}\ \text{kg}} = 1.461\times10^{-27}\ \text{kg}.$

$I = m_r r^2 = (1.461\times10^{-27}\ \text{kg})(0.159\times10^{-9}\ \text{m})^2 = 3.694\times10^{-47}\ \text{kg}\cdot\text{m}^2.$

$l=3: E = 3(4)\left(\frac{\hbar^2}{2I}\right) = 6\left(\frac{\hbar^2}{I}\right).$

$l=4: E = 4(5)\left(\frac{\hbar^2}{2I}\right) = 10\left(\frac{\hbar^2}{I}\right).$

$$\Delta E = E_4 - E_3 = 4\left(\frac{\hbar^2}{I}\right) = 4\left(\frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{3.694\times10^{-47}\ \text{kg}\cdot\text{m}^2}\right) = 1.20\times10^{-21}\ \text{J} = 7.49\times10^{-3}\ \text{eV}.$$

(b) $\Delta E = hc/\lambda$, so $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136\times10^{-15}\ \text{eV})(2.998\times10^{8}\ \text{m/s})}{7.49\times10^{-3}\ \text{eV}} = 166\ \mu\text{m}.$

EVALUATE: LiH has a smaller reduced mass than CO and λ is somewhat smaller here than the λ calculated for CO in Example 42.2.

42.10. IDENTIFY: For a $n \to n-1$ vibrational transition, $\Delta E = \hbar\sqrt{\dfrac{k'}{m_r}}$. ΔE is related to λ of the photon by $\Delta E = \dfrac{hc}{\lambda}$.

SET UP: $m_r = \dfrac{m_{Na} m_{Cl}}{m_{Na}+m_{Cl}}$.

EXECUTE: $\Delta E = \dfrac{hc}{\lambda} = \hbar\sqrt{k'/m_r}$, and solving for k', $k' = \left(\dfrac{2\pi c}{\lambda}\right)^2 m_r = 205\ \text{N/m}.$

EVALUATE: The value of k' we calculated for NaCl is comparable to that of a fairly stiff lab spring.

42.11. **IDENTIFY:** The vibrational energy of the molecule is related to its force constant and reduced mass, while the rotational energy depends on its moment of inertia, which in turn depends on the reduced mass.

SET UP: The vibrational energy is $E_n = (n+\frac{1}{2})\hbar\omega = (n+\frac{1}{2})\hbar\sqrt{\frac{k'}{m_r}}$ and the rotational energy is $E_l = l(l+1)\frac{\hbar^2}{2I}$.

EXECUTE: For a vibrational transition, we have $\Delta E_v = \hbar\sqrt{\frac{k'}{m_r}}$, so we first need to find m_r. The energy for a rotational transition is $\Delta E_R = \frac{\hbar^2}{2I}[2(2+1)-1(1+1)] = \frac{2\hbar^2}{I}$. Solving for I and using the fact that $I = m_r r_0^2$, we have $m_r r_0^2 = \frac{2\hbar^2}{\Delta E_R}$, which gives

$$m_r = \frac{2\hbar^2}{r_0^2\Delta E_R} = \frac{2(1.055\times10^{-34}\ \text{J}\cdot\text{s})(6.583\times10^{-16}\ \text{eV}\cdot\text{s})}{(0.8860\times10^{-9}\ \text{m})^2(8.841\times10^{-4}\ \text{eV})} = 2.0014\times10^{-28}\ \text{kg}.$$

Now look at the vibrational transition to find the force constant.

$$\Delta E_v = \hbar\sqrt{\frac{k'}{m_r}} \Rightarrow k' = \frac{m_r(\Delta E_v)^2}{\hbar^2} = \frac{(2.0014\times10^{-28}\ \text{kg})(0.2560\ \text{eV})^2}{(6.583\times10^{-16}\ \text{eV}\cdot\text{s})^2} = 30.27\ \text{N/m}.$$

EVALUATE: This would be a rather weak spring in the laboratory.

42.12. **IDENTIFY** and **SET UP:** For an average spacing a, the density is $\rho = m/a^3$, where m is the average of the ionic masses.

EXECUTE: **(a)** $a^3 = \frac{m}{\rho} = \frac{(6.49\times10^{-26}\ \text{kg} + 1.33\times10^{-25}\ \text{kg})/2}{(2.75\times10^3\ \text{kg/m}^3)} = 3.60\times10^{-29}\ \text{m}^3$, and

$a = 3.30\times10^{-10}\ \text{m} = 0.330\ \text{nm}.$

EVALUATE: **(b)** Exercise 42.15 says that the average spacing for NaCl is 0.282 nm. The larger (higher atomic number) atoms have the larger spacing.

42.13. **IDENTIFY** and **SET UP:** Find the volume occupied by each atom. The density is the average mass of Na and Cl divided by this volume.

EXECUTE: Each atom occupies a cube with side length 0.282 nm. Therefore, the volume occupied by each atom is $V = (0.282\times10^{-9}\ \text{m})^3 = 2.24\times10^{-29}\ \text{m}^3$. In NaCl there are equal numbers of Na and Cl atoms, so the average mass of the atoms in the crystal is

$$m = \tfrac{1}{2}(m_{\text{Na}} + m_{\text{Cl}}) = \tfrac{1}{2}(3.82\times10^{-26}\ \text{kg} + 5.89\times10^{-26}\ \text{kg}) = 4.855\times10^{-26}\ \text{kg}.$$

The density then is $\rho = \frac{m}{V} = \frac{4.855\times10^{-26}\ \text{kg}}{2.24\times10^{-29}\ \text{m}^3} = 2.17\times10^3\ \text{kg/m}^3.$

EVALUATE: The density of water is $1.00\times10^3\ \text{kg/m}^3$, so our result is reasonable.

42.14. **IDENTIFY** and **SET UP:** $\Delta E = \frac{hc}{\lambda}$, where ΔE is the band gap.

EXECUTE: **(a)** $\lambda = \frac{hc}{\Delta E} = 2.27\times10^{-7}\ \text{m} = 227\ \text{nm}$, in the ultraviolet.

EVALUATE: **(b)** Visible light lacks enough energy to excite the electrons into the conduction band, so visible light passes through the diamond unabsorbed.

(c) Impurities can lower the gap energy making it easier for the material to absorb shorter wavelength visible light. This allows longer wavelength visible light to pass through, giving the diamond color.

42.15. **IDENTIFY:** The energy gap is the energy of the maximum-wavelength photon.

SET UP: The energy difference is equal to the energy of the photon, so $\Delta E = hc/\lambda$.

EXECUTE: **(a)** Using the photon wavelength to find the energy difference gives

$$\Delta E = hc/\lambda = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})/(1.11\times10^{-6}\text{ m}) = 1.12\text{ eV}.$$

(b) A wavelength of $1.11\ \mu\text{m} = 1110\text{ nm}$ is in the infrared, shorter than that of visible light.

EVALUATE: Since visible photons have more than enough energy to excite electrons from the valence to the conduction band, visible light will be absorbed, which makes silicon opaque.

42.16. **IDENTIFY** and **SET UP:** The energy ΔE deposited when a photon with wavelength λ is absorbed is $\Delta E = \dfrac{hc}{\lambda}$.

EXECUTE: $\Delta E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{9.31\times10^{-13}\text{ m}} = 2.14\times10^{-13}\text{ J} = 1.34\times10^{6}\text{ eV}.$ So the number of electrons that can be excited to the conduction band is $n = \dfrac{1.34\times10^{6}\text{ eV}}{1.12\text{ eV}} = 1.20\times10^{6}$ electrons.

EVALUATE: A photon of wavelength

$\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.13\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.12\text{ eV}} = 1.11\times10^{-6}\text{ m} = 1110\text{ nm}$ can excite one electron. This photon is in the infrared.

42.17. **IDENTIFY:** The density of states is given by $g(E) = \dfrac{(2m)^{3/2}V}{2\pi^2\hbar^3}E^{1/2}$.

SET UP: $m = 9.11\times10^{-31}\text{ kg}$, the mass of an electron.

EXECUTE:

$$g(E) = \frac{(2m)^{3/2}V}{2\pi^2\hbar^3}E^{1/2} = \frac{[2(9.11\times10^{-31}\text{ kg})]^{3/2}(1.0\times10^{-6}\text{ m}^3)(5.0\text{ eV})^{1/2}(1.60\times10^{-19}\text{ J/eV})^{1/2}}{2\pi^2(1.054\times10^{-34}\text{ J}\cdot\text{s})^3}.$$

$g(E) = (9.5\times10^{40}\text{ states/J})(1.60\times10^{-19}\text{ J/eV}) = 1.5\times10^{22}\text{ states/eV}.$

EVALUATE: For a metal the density of states expressed as states/eV is very large.

42.18. **IDENTIFY:** At absolute zero, the average free-electron energy is $E_{\text{av}} = \dfrac{3}{5}E_{\text{F}}$. The speed v is related to E_{av} by $\frac{1}{2}mv^2 = E_{\text{av}}$.

SET UP: $k = 1.38\times10^{-23}\text{ J/K}$.

EXECUTE: **(a)** $E_{\text{av}} = \dfrac{3}{5}E_{\text{F}} = 1.94\text{ eV}$.

(b) $v = \sqrt{2E_{\text{av}}/m} = \sqrt{\dfrac{2(1.94\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = 8.25\times10^{5}\text{ m/s}.$

(c) $\dfrac{E_{\text{F}}}{k} = \dfrac{(3.23\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(1.38\times10^{-23}\text{ J/K})} = 3.74\times10^{4}\text{ K}.$

EVALUATE: The Fermi energy of sodium is less than that of copper. Therefore, the values of E_{av} and v we have calculated for sodium are less than those calculated for copper in Example 42.7.

42.19. **IDENTIFY:** This problem deals with the density of states $g(E)$.

SET UP: The original energy is 1.60 eV and we want to know the new energy. From Eq. (42.15) we see that $g(E)$ is proportional to the square root of E.

EXECUTE: **(a)** $g(E)$ is doubled. Since $g(E)$ is proportional to the square root of the energy, E must increase by a factor of 4 to double $g(E)$, so E = 4(1.60 eV) = 6.40 eV.

(b) $g(E)$ is halved. We need to decrease $g(E)$ by a factor of $\frac{1}{2}$, so E must decrease by a factor of $\frac{1}{4}$, so $E = (1.60\text{ eV})/4 = 0.400\text{ eV}$.

EVALUATE: A larger fractional change in E is required than the fractional change in $g(E)$.

42.20. **IDENTIFY:** This problem involves the Fermi-Dirac distribution.

SET UP: Eq. (42.16) gives this distribution. We know that $kT = 0.25E_F$ and we want the probability $f(E)$.

EXECUTE: **(a)** $E = 0.50E_F$. Using Eq. (42.16) gives

$$f(0.50E_F) = \frac{1}{e^{(0.50E_F - E_F)} + 1} = \frac{1}{e^{-2} + 1} = 0.88.$$

(b) $E = 1.50E_F$. Use the same procedure as in (a) with $E = 1.50E_F$. $f(1.50E_F) = 1/(e^2 + 1) = 0.12$.

EVALUATE: Our results agree with the curve for $kT = \frac{1}{4}E_F$ in Figure 42.23.

42.21. **IDENTIFY** and **SET UP:** The electron contribution to the molar heat capacity at constant volume of a metal is $C_V = \left(\dfrac{\pi^2 kT}{2E_F}\right)R$.

EXECUTE: **(a)** $C_V = \dfrac{\pi^2(1.381\times10^{-23}\text{ J/K})(300\text{ K})}{2(5.48\text{ eV})(1.602\times10^{-19}\text{ J/eV})}R = 0.0233R.$

(b) The electron contribution found in part (a) is $0.0233R = 0.194\text{ J/mol}\cdot\text{K}$. This is $0.194/25.3 = 7.67\times10^{-3} = 0.767\%$ of the total C_V.

EVALUATE: **(c)** Only a small fraction of C_V is due to the electrons. Most of C_V is due to the vibrational motion of the ions.

42.22. **IDENTIFY** and **SET UP:** The probability $f(E)$ that a state with energy E is occupied is given by the Fermi-Dirac distribution $f(E) = \dfrac{1}{e^{(E-E_F)/kT} + 1}$, where E_F is the Fermi energy. In this case, $E = 2E_F = 2(kT_F)$.

EXECUTE: $f(2E) = \dfrac{1}{e^{(2E_F - E_F)/kT_F} + 1} = \dfrac{1}{e^{E_F/kT_F} + 1} = \dfrac{1}{e^{kT_F/kT_F} + 1} = \dfrac{1}{e + 1} = 0.269.$

EVALUATE: A probability of $0.269 \approx 27\%$ is about 1 in 4. As E increases, the probability gets smaller.

42.23. **IDENTIFY:** The probability is given by the Fermi-Dirac distribution.

SET UP: The Fermi-Dirac distribution is $f(E) = \dfrac{1}{e^{(E-E_F)/kT} + 1}$.

EXECUTE: We calculate the value of $f(E)$, where $E = 8.520\text{ eV}$, $E_F = 8.500\text{ eV}$, $k = 1.38\times10^{-23}\text{ J/K} = 8.625\times10^{-5}\text{ eV/K}$, and $T = 20°\text{C} = 293\text{ K}$. The result is $f(E) = 0.312 = 31.2\%$.

EVALUATE: Since the energy is close to the Fermi energy, the probability is quite high that the state is occupied by an electron.

42.24. **IDENTIFY** and **SET UP:** Follow the procedure of Example 42.9. Evaluate $f(E) = \dfrac{1}{e^{(E-E_F)/kT} + 1}$ for $E - E_F = E_g/2$, where E_g is the band gap.

EXECUTE: **(a)** The probabilities are 1.78×10^{-7}, 2.37×10^{-6}, and 1.51×10^{-5}.

(b) It can be shown that the Fermi-Dirac distribution, $f(E) = \dfrac{1}{e^{(E-E_F)/kT} + 1}$, has the property that $f(E_F - E) = 1 - f(E)$, and so the probability that a state at the top of the valence band is occupied is the same as the probability that a state of the bottom of the conduction band is filled (this result depends on having the Fermi energy in the middle of the gap). Therefore, the probabilities at each T are the same as in part (a).

EVALUATE: The probabilities increase with temperature.

42.25. **IDENTIFY:** Use the Fermi-Dirac distribution $f(E) = \dfrac{1}{e^{(E-E_F)/kT}+1}$. Solve for $E-E_F$.

SET UP: $e^{(E-E_F)/kT} = \dfrac{1}{f(E)} - 1.$

The problem states that $f(E) = 4.4\times10^{-4}$ for E at the bottom of the conduction band.

EXECUTE: $e^{(E-E_F)/kT} = \dfrac{1}{4.4\times10^{-4}} - 1 = 2.272\times10^3.$

$E - E_F = kT\ln(2.272\times10^3) = (1.3807\times10^{-23}\text{ J/T})(300\text{ K})\ln(2.272\times10^3) = 3.201\times10^{-20}\text{ J} = 0.20\text{ eV}.$

$E_F = E - 0.20\text{ eV};$ the Fermi level is 0.20 eV below the bottom of the conduction band.

EVALUATE: The energy gap between the Fermi level and bottom of the conduction band is large compared to kT at $T = 300\text{ K}$ and as a result $f(E)$ is small.

42.26. **IDENTIFY:** The wavelength of the photon to be detected depends on its energy.

SET UP: $\Delta E = \dfrac{hc}{\lambda}.$

EXECUTE: **(a)** $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^8\text{ m/s})}{0.67\text{ eV}} = 1.9\ \mu\text{m}.$

(b) $\lambda = (1.9\ \mu\text{m})\left(\dfrac{0.67\text{ eV}}{1.12\text{ eV}}\right) = 1.1\ \mu\text{m}.$

EVALUATE: Both of these photons are in the infrared.

42.27. **IDENTIFY:** Knowing the saturation current of a *p-n* junction at a given temperature, we want to find the current at that temperature for various voltages.

SET UP: $I = I_S(e^{eV/kT} - 1).$

EXECUTE: **(a)** (i) For $V = 1.00\text{ mV},\ \dfrac{eV}{kT} = \dfrac{(1.602\times10^{-19}\text{ C})(1.00\times10^{-3}\text{ V})}{(1.381\times10^{-23}\text{ J/K})(290\text{ K})} = 0.0400.$

$I = (0.500\text{ mA})(e^{0.0400} - 1) = 0.0204\text{ mA}.$

(ii) For $V = -1.00\text{ mV},\ \dfrac{eV}{kT} = -0.0400.\ \ I = (0.500\text{ mA})(e^{-0.0400} - 1) = -0.0196\text{ mA}.$

(iii) For $V = 100\text{ mV},\ \dfrac{eV}{kT} = 4.00.\ \ I = (0.500\text{ mA})(e^{4.00} - 1) = 26.8\text{ mA}.$

(iv) For $V = -100\text{ mV},\ \dfrac{eV}{kT} = -4.00.\ \ I = (0.500\text{ mA})(e^{-4.00} - 1) = -0.491\text{ mA}.$

EVALUATE: **(b)** For small V, between ±1.00 mV, $R = V/I$ is approximately constant and the diode obeys Ohm's law to a good approximation. For larger V the deviation from Ohm's law is substantial.

42.28. **IDENTIFY:** The current depends on the voltage across the diode and its temperature, so the resistance also depends on these quantities.

SET UP: The current is $I = I_S\,(e^{eV/kT} - 1)$ and the resistance is $R = V/I$.

EXECUTE: **(a)** The resistance is $R = \dfrac{V}{I} = \dfrac{V}{I_s(e^{eV/kT} - 1)}$. The exponent is

$\dfrac{eV}{kT} = \dfrac{e(0.0850\text{ V})}{(8.625\times10^{-5}\text{ eV/K})(293\text{ K})} = 3.3635,$ giving $R = \dfrac{85.0\text{ mV}}{(0.950\text{ mA})(e^{3.3635} - 1)} = 3.21\ \Omega.$

(b) In this case, the exponent is $\dfrac{eV}{kT} = \dfrac{e(-0.050\text{ V})}{(8.625\times10^{-5}\text{ eV/K})(293\text{ K})} = -1.979$

which gives $R = \dfrac{-50.0\text{ mV}}{(0.950\text{ mA})(e^{-1.979}-1)} = 61.1\ \Omega.$

EVALUATE: Reversing the voltage can make a considerable change in the resistance of a diode.

42.29. **IDENTIFY** and **SET UP:** The voltage-current relation is given by $I = I_s(e^{eV/kT}-1)$. Use the current for $V = +15.0\text{ mV}$ to solve for the constant I_s.

EXECUTE: **(a)** Find $I_s: V = +15.0\times10^{-3}\text{ V}$ gives $I = 9.25\times10^{-3}\text{ A}$.

$$\frac{eV}{kT} = \frac{(1.602\times10^{-19}\text{ C})(15.0\times10^{-3}\text{ V})}{(1.381\times10^{-23}\text{ J/K})(300\text{ K})} = 0.5800.$$

$$I_s = \frac{I}{e^{eV/kT}-1} = \frac{9.25\times10^{-3}\text{ A}}{e^{0.5800}-1} = 1.177\times10^{-2} = 11.77\text{ mA}.$$

Then can calculate I for $V = 10.0$ mV: $\dfrac{eV}{kT} = \dfrac{(1.602\times10^{-19}\text{ C})(10.0\times10^{-3}\text{ V})}{(1.381\times10^{-23}\text{ J/K})(300\text{ K})} = 0.3867.$

$$I = I_s(e^{eV/kT}-1) = (11.77\text{ mA})(e^{0.3867}-1) = 5.56\text{ mA}.$$

(b) $\dfrac{eV}{kT}$ has the same magnitude as in part (a) but now V is negative so $\dfrac{eV}{kT}$ is negative.

$\underline{V = -15.0\text{ mV}}$: $\dfrac{eV}{kT} = -0.5800$ and $I = I_s(e^{eV/kT}-1) = (11.77\text{ mA})(e^{-0.5800}-1) = -5.18\text{ mA}.$

$\underline{V = -10.0\text{ mV}}$: $\dfrac{eV}{kT} = -0.3867$ and $I = I_s(e^{eV/kT}-1) = (11.77\text{ mA})(e^{-0.3867}-1) = -3.77\text{ mA}.$

EVALUATE: There is a directional asymmetry in the current, with a forward-bias voltage producing more current than a reverse-bias voltage of the same magnitude, but the voltage is small enough for the asymmetry not be pronounced.

42.30. **IDENTIFY:** Apply the equation $I = I_S(e^{eV/kT}-1)$.

SET UP: $I_S = 6.40\text{ mA}$. $\ln e^x = x$.

EXECUTE: **(a)** Solving $I = I_S(e^{eV/kT}-1)$ for the voltage as a function of current gives

$$V = \frac{kT}{e}\ln\left(\frac{I}{I_S}+1\right) = \frac{(1.38\times10^{-23}\text{ J/K})(300\text{ K})}{1.602\times10^{-19}\text{ C}}\ln\left(\frac{40.0\text{ mA}}{6.40\text{ mA}}+1\right) = 0.0512\text{ V} = 51.2\text{ mV}.$$

(b) Using the result from part (a), the quantity $e^{eV/kT} = 7.242$, so for a reverse-bias voltage of the same magnitude, $I = I_S\,(e^{-eV/kT}-1) = (6.40\text{ mA})\left(\dfrac{1}{7.242}-1\right) = -5.52\text{ mA}.$

EVALUATE: The reverse bias current for a given magnitude of voltage is much less than the forward bias current.

42.31. **IDENTIFY:** In this problem we are dealing with a junction diode.

SET UP: We want the resistance of the diode. The current through the junction is given by Eq. (42.22) and $T = 17°\text{C} = 290$ K. Using this equation to find the resistance gives

$$R = \frac{V}{I} = \frac{V}{I_S\left(e^{eV/kT}-1\right)}.$$

EXECUTE: Forward bias: $V = +0.20$ V.

$$R = \frac{0.20\text{ V}}{(10\ \mu\text{A})\left(e^{e(0.20\text{ V})/k(290\text{ K})}-1\right)} = 6.7\ \Omega.$$

Reverse bias: $V = -0.20$ V. Use the same procedure with $V = -0.20$ V, giving $R = 2.0 \times 10^4\ \Omega$.

EVALUATE: In reverse bias the resistance is around 3000 times greater than in forward bias.

42.32. **IDENTIFY** and **SET UP:** $E_l = l(l+1)\dfrac{\hbar^2}{2I}$. ΔE for the molecule is related to λ for the photon by

$$\Delta E = \frac{hc}{\lambda}.$$

EXECUTE: $E_2 = 3\dfrac{\hbar^2}{I}$ and $E_1 = \dfrac{\hbar^2}{I}$, so $\Delta E = \dfrac{2\hbar^2}{I}$. Using $\lambda = 54.3\ \mu$m, we get

$$I = \frac{2\hbar^2}{\Delta E} = \frac{h\lambda}{2\pi^2 c} = 6.08\times10^{-48}\ \text{kg}\cdot\text{m}^2.$$

EVALUATE: The I we calculated is approximately a factor of 24 times smaller than I calculated for the CO molecule in Example 42.2.

42.33. **IDENTIFY** and **SET UP:** From Chapter 21, the electric dipole moment is $p = qd$, where the dipole consists of charges $\pm q$ separated by distance d.

EXECUTE: **(a)** Point charges $+e$ and $-e$ separated by distance d, so

$$p = ed = (1.602\times10^{-19}\ \text{C})(0.24\times10^{-9}\ \text{m}) = 3.8\times10^{-29}\ \text{C}\cdot\text{m}.$$

(b) $p = qd$, so $q = \dfrac{p}{d} = \dfrac{3.0\times10^{-29}\ \text{C}\cdot\text{m}}{0.24\times10^{-9}\ \text{m}} = 1.3\times10^{-19}\ \text{C}.$

(c) $\dfrac{q}{e} = \dfrac{1.3\times10^{-19}\ \text{C}}{1.602\times10^{-19}\ \text{C}} = 0.81.$

(d) $q = \dfrac{p}{d} = \dfrac{1.5\times10^{-30}\ \text{C}\cdot\text{m}}{0.16\times10^{-9}\ \text{m}} = 9.37\times10^{-21}\ \text{C}.$

$$\frac{q}{e} = \frac{9.37\times10^{-21}\ \text{C}}{1.602\times10^{-19}\ \text{C}} = 0.058.$$

EVALUATE: The fractional ionic character for the bond in HI is much less than the fractional ionic character for the bond in NaCl. The bond in HI is mostly covalent and not very ionic.

42.34. **IDENTIFY** and **SET UP:** For an $l \to l-1$ transition, the frequency of the emitted photon is

$$f = \frac{\Delta E}{h} = \frac{\Delta E}{2\pi\hbar} = \frac{l\hbar}{2\pi I},\ \text{so}\ \Delta E = \frac{l\hbar^2}{I}.$$

$$I = m_\text{r} r_0^2.\ m_\text{r} = \frac{(3.82\times10^{-26}\text{kg})(3.15\times10^{-26}\ \text{kg})}{3.82\times10^{-26}\ \text{kg} + 3.15\times10^{-26}\ \text{kg}} = 1.726\times10^{-26}\ \text{kg}.$$

EXECUTE: $I = \dfrac{\hbar^2 l}{\Delta E} = \dfrac{hl\lambda}{4\pi^2 c} = 6.43\times10^{-46}\ \text{kg}\cdot\text{m}^2$ and from $I = m_\text{r} r_0^2$ the separation is

$$r_0 = \sqrt{\frac{I}{m_\text{r}}} = 0.193\ \text{nm}.$$

EVALUATE: Section 42.1 says $r_0 = 0.24$ nm for NaCl. Our result for NaF is smaller than this. This makes sense, since F is a smaller atom than Cl.

42.35. **IDENTIFY:** $E_\text{ex} = \dfrac{L^2}{2I} = \dfrac{\hbar^2 l(l+1)}{2I}$. $E_g = 0\ (l = 0)$, and there is an additional multiplicative factor of $2l+1$ because for each l state there are really $(2l+1)$ m_l states with the same energy.

SET UP: From Example 42.2, $I = 1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2$ for CO.

EXECUTE: **(a)** $\dfrac{n_l}{n_0} = (2l+1)e^{-\hbar^2 l(l+1)/(2IkT)}.$

(b) (i) $E_{l=1} = \dfrac{\hbar^2(1)(1+1)}{2(1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2)} = 7.67\times10^{-23}\ \text{J}.$ $\dfrac{E_{l=1}}{kT} = \dfrac{7.67\times10^{-23}\ \text{J}}{(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})} = 0.0185.$

$$(2l+1) = 3,\ \text{so}\ \frac{n_{l=1}}{n_0} = (3)e^{-0.0185} = 2.95.$$

(ii) $\dfrac{E_{l=2}}{kT} = \dfrac{\hbar^2(2)\,(2+1)}{2(1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2)(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})} = 0.0556.$ $(2l+1) = 5,$ so

$$\frac{n_{l=1}}{n_0} = (5)(e^{-0.0556}) = 4.73.$$

(iii) $\dfrac{E_{l=10}}{kT} = \dfrac{\hbar^2(10)\,(10+1)}{2(1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2)(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})} = 1.02.$

$$(2l+1) = 21,\ \text{so}\ \frac{n_{l=10}}{n_0} = (21)(e^{-1.02}) = 7.57.$$

(iv) $\dfrac{E_{l=20}}{kT} = \dfrac{\hbar^2(20)(20+1)}{2(1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2)\,(1.38\times10^{-23}\ \text{J/K})\,(300\ \text{K})} = 3.90.$ $(2l+1) = 41,$ so

$$\frac{n_{l=20}}{n_0} = (41)e^{-3.90} = 0.833.$$

(v) $\dfrac{E_{l=50}}{kT} = \dfrac{\hbar^2(50)(50+1)}{2(1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2)(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})} = 23.7.$ $(2l+1) = 101,$ so

$$\frac{n_{l=50}}{n_0} = (101)e^{-23.7} = 5.38\times10^{-9}.$$

EVALUATE: **(c)** There is a competing effect between the $(2l+1)$ term and the decaying exponential. The $2l+1$ term dominates for small l, while the exponential term dominates for large l.

42.36. **IDENTIFY:** The ratio n_l/n_0 will be largest when its derivative with respect to l is zero.

SET UP: From Problem 42.35, we know that $n_l/n_0 = (2l+1)e^{-\hbar^2 l(l+1)/2IkT}$. Set $\dfrac{\partial(n_l/n_0)}{\partial l} = 0$ and solve for l.

EXECUTE: **(a)** $\dfrac{\partial}{\partial l}\left[(2l+1)e^{-\hbar^2 l(l+1)/2IkT}\right] = 2e^{-\hbar^2 l(l+1)/2IkT} - \dfrac{(2l+1)^2\hbar^2}{2IkT}e^{-\hbar^2 l(l+1)/2IkT} = 0.$

The exponentials cannot be zero, so

$$2 - \frac{(2l+1)^2\hbar^2}{2IkT} = 0 \ \rightarrow\ l_{\text{max}} = \frac{\sqrt{IkT}}{\hbar} - \frac{1}{2}.$$

(b) Using $I = 1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2$ for CO from Example 42.2 gives

$$l_{\text{max}} = \frac{\sqrt{(1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2)(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}} - \frac{1}{2} = 6.84 \approx 7.$$

EVALUATE: Since $l_{\text{max}} \propto \sqrt{I}$, it would have a different value for molecules other than CO.

42.37. **IDENTIFY:** This problem is about the power in a diode.
SET UP: We use Eq. (42.22) and $P = IV$.
EXECUTE: **(a)** We want the power.

$$P = IV = I_{\text{S}}\left(e^{eV/kT} - 1\right)V = (1.2\times10^{-11}\ \text{A})\left(e^{e(0.6\ \text{V})/k(300\ \text{K})} - 1\right)(0.6V) = 85\ \text{mW}.$$

(b) We want V when $P = 500$ mA $= 0.5$ A. We can solve this by trial-and-error. First put the equation in the convenient form

$$P = (1.2\times10^{-11}\text{ A})\left(e^{(38.65\text{ V}^{-1})V} - 1\right)V$$

We know that V must be greater than 0.6 V. Trying $V = 0.8$ V gives $P = 260$ W. Trying $V = 0.7$ V gives $P = 4.7$ W. Trying $V = 0.65$ V gives $P = 635$ mW. Trying $V = 0.64$ V gives $P = 425$ mW. Trying $V = 0.645$ V gives $P = 520$ mW, which is close enough to 500 mA. So the maximum voltage is $V = 0.645$ V. To one significant figure, the answer is $V = 0.6$ V.

(c) We want the current when $V = 0.645$ V. $I = P/V = (520\text{ mW})/(0.645\text{ V}) = 0.81$ A.

EVALUATE: Note in part (b) how sensitive the power is to small changes in V due to the exponential.

42.38. **IDENTIFY:** The rotational energy levels are given by $E_l = l(l+1)\dfrac{\hbar^2}{2I}$. The transition energy ΔE for the molecule and λ for the photon are related by $\Delta E = \dfrac{hc}{\lambda}$.

SET UP: From Example 42.2, $I_{\text{CO}} = 1.449\times10^{-46}\text{ kg}\cdot\text{m}^2$.

EXECUTE: **(a)** $E_{l=1} = \dfrac{\hbar^2 l(l+1)}{2I} = \dfrac{(1.054\times10^{-34}\text{ J}\cdot\text{s})^2(1)(1+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)} = 7.67\times10^{-23}\text{ J}$. $E_{l=0} = 0$.

$\Delta E = 7.67\times10^{-23}\text{ J} = 4.79\times10^{-4}\text{ eV}$. $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(7.67\times10^{-23}\text{ J})}$

$= 2.59\times10^{-3}\text{ m} = 2.59\text{ mm}$.

EVALUATE: **(b)** Let's compare the value of kT when $T = 20K$ to that of ΔE for the $l = 1 \to l = 0$ rotational transition: $kT = (1.38\times10^{-23}\text{ J/K})(20\text{ K}) = 2.76\times10^{-22}\text{ J}$.

$\Delta E = 7.67\times10^{-23}$ J (from part (a)). So $\dfrac{kT}{\Delta E} = 3.60$. Therefore, although T is quite small, there is still plenty of energy to excite CO molecules into the first rotational level. This allows astronomers to detect the 2.59 mm wavelength radiation from such molecular clouds.

42.39. **IDENTIFY:** The vibrational energy levels are given by $E_n = (n+\frac{1}{2})\,\hbar\sqrt{\dfrac{k'}{m_\text{r}}}$. The zero-point energy is

$$E_0 = \tfrac{1}{2}\hbar\sqrt{\frac{2k'}{m_\text{H}}}.$$

SET UP: For H_2, $m_\text{r} = \dfrac{m_\text{H}}{2}$.

EXECUTE: $E_0 = \frac{1}{2}(1.054\times10^{-34}\text{ J}\cdot\text{s})\sqrt{\dfrac{2(576\text{ N/m})}{1.67\times10^{-27}\text{ kg}}} = 4.38\times10^{-20}\text{ J} = 0.274\text{ eV}$.

EVALUATE: This is much less than the magnitude of the H_2 bond energy.

42.40. **IDENTIFY:** $\Delta E = hf = \hbar\sqrt{\dfrac{k'}{m_\text{r}}}$.

SET UP: $m_\text{r} = \dfrac{m_\text{O}m_\text{H}}{m_\text{O}+m_\text{H}} = 1.57\times10^{-27}$ kg.

EXECUTE: The vibration frequency is $f = \dfrac{\Delta E}{h} = 1.12\times10^{14}$ Hz. The force constant is

$$k' = (2\pi f)^2 m_\text{r} = 777\text{ N/m}.$$

EVALUATE: This would be a fairly stiff spring in an ordinary physics lab.

42.41. **IDENTIFY** and **SET UP:** Use $I = m_r r_0^2$ to calculate I. The energy levels are given by $E_{nl} = l(l+1)\left(\frac{\hbar^2}{2I}\right) + (n+\frac{1}{2})\hbar\sqrt{\frac{k'}{m_r}}$. The transition energy ΔE is related to the photon wavelength by $\Delta E = hc/\lambda$.

EXECUTE: **(a)** $m_r = \frac{m_H m_I}{m_H + m_I} = \frac{(1.67\times10^{-27}\text{ kg})(2.11\times10^{-25}\text{ kg})}{1.67\times10^{-27}\text{ kg} + 2.11\times10^{-25}\text{ kg}} = 1.657\times10^{-27}\text{ kg}$.

$$I = m_r r_0^2 = (1.657\times10^{-27}\text{ kg})(0.160\times10^{-9}\text{ m})^2 = 4.24\times10^{-47}\text{ kg}\cdot\text{m}^2.$$

(b) The energy levels are $E_{nl} = l(l+1)\left(\frac{\hbar^2}{2I}\right) + (n+\frac{1}{2})\hbar\sqrt{\frac{k'}{m_r}}$.

$$\sqrt{\frac{k'}{m}} = \omega = 2\pi f, \text{ so } E_{nl} = l(l+1)\left(\frac{\hbar^2}{2I}\right) + (n+\tfrac{1}{2})hf.$$

(i) Transition $n = 1 \to n = 0, l = 1 \to l = 0$:

$$\Delta E = (2-0)\left(\frac{\hbar^2}{2I}\right) + (1+\tfrac{1}{2}-\tfrac{1}{2})hf = \frac{\hbar^2}{I} + hf.$$

$$\Delta E = \frac{hc}{\lambda}, \text{ so } \lambda = \frac{hc}{\Delta E} = \frac{hc}{(\hbar^2/I) + hf} = \frac{c}{(\hbar/2\pi I) + f}.$$

$$\frac{\hbar}{2\pi I} = \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2\pi(4.24\times10^{-47}\text{ kg}\cdot\text{m}^2)} = 3.960\times10^{11}\text{ Hz}.$$

$$\lambda = \frac{c}{(\hbar/2\pi I) + f} = \frac{2.998\times10^8\text{ m/s}}{3.960\times10^{11}\text{ Hz} + 6.93\times10^{13}\text{ Hz}} = 4.30\ \mu\text{m}.$$

(ii) Transition $n = 1 \to n = 0, l = 2 \to l = 1$:

$$\Delta E = (6-2)\left(\frac{\hbar^2}{2I}\right) + hf = \frac{2\hbar^2}{I} + hf.$$

$$\lambda = \frac{c}{2(\hbar/2\pi I) + f} = \frac{2.998\times10^8\text{ m/s}}{2(3.960\times10^{11}\text{ Hz}) + 6.93\times10^{13}\text{ Hz}} = 4.28\ \mu\text{m}.$$

(iii) Transition $n = 2 \to n = 1, l = 2 \to l = 3$:

$$\Delta E = (6-12)\left(\frac{\hbar^2}{2I}\right) + hf = -\frac{3\hbar^2}{I} + hf.$$

$$\lambda = \frac{c}{-3(\hbar/2\pi I) + f} = \frac{2.998\times10^8\text{ m/s}}{-3(3.960\times10^{11}\text{ Hz}) + 6.93\times10^{13}\text{ Hz}} = 4.40\ \mu\text{m}.$$

EVALUATE: The vibrational energy change for the $n = 1 \to n = 0$ transition is the same as for the $n = 2 \to n = 1$ transition. The rotational energies are much smaller than the vibrational energies, so the wavelengths for all three transitions don't differ much.

42.42. **IDENTIFY:** The frequency is proportional to the reciprocal of the square root of the reduced mass. The transition energy ΔE and the wavelength of the light emitted are related by $\Delta E = \frac{hc}{\lambda}$.

SET UP: $f_0 = 1.24\times10^{14}$ Hz.

EXECUTE: **(a)** In terms of the atomic masses, the frequency of the isotope with the deuterium atom is $f = f_0\left(\frac{m_F m_H/(m_H + m_F)}{m_F m_D/(m_D + m_F)}\right)^{1/2} = f_0\left(\frac{1+(m_F/m_D)}{1+(m_F/m_H)}\right)^{1/2}$. Using f_0 and the given masses, $f = 8.99\times10^{13}$ Hz.

(b) For the molecule, $\Delta E = hf$. $hf = \dfrac{hc}{\lambda}$, so $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{8.99\times10^{13}\ \text{Hz}} = 3.34\times10^{-6}\ \text{m} = 3340\ \text{nm}$. This wavelength is in the infrared.

EVALUATE: The vibrational frequency of the molecule equals the frequency of the light that is emitted.

42.43. **IDENTIFY:** E_{F0} is given by $E_{\text{F0}} = \dfrac{3^{2/3}\pi^{4/3}\hbar^2 n^{2/3}}{2m}$. Since potassium is a metal and E does not change much with T for metals, we approximate E_{F} by E_{F0}, so $E_{\text{F}} = \dfrac{3^{2/3}\pi^{4/3}\hbar^2 n^{2/3}}{2m}$.

SET UP: The number of atoms per m^3 is ρ/m. If each atom contributes one free electron, the electron concentration is $n = \dfrac{\rho}{m} = \dfrac{851\ \text{kg/m}^3}{6.49\times10^{-26}\ \text{kg}} = 1.31\times10^{28}\ \text{electrons/m}^3$.

EXECUTE: $E_{\text{F}} = \dfrac{3^{2/3}\pi^{4/3}(1.054\times10^{-34}\ \text{J}\cdot\text{s})^2(1.31\times10^{28}/\text{m}^3)^{2/3}}{2(9.11\times10^{-31}\ \text{kg})} = 3.24\times10^{-19}\ \text{J} = 2.03\ \text{eV}$.

EVALUATE: The E_{F} we calculated for potassium is about a factor of three smaller than the E_{F} for copper that was calculated in Example 42.7.

42.44. **IDENTIFY** and **SET UP:** At r where U_{tot} is a minimum, $\dfrac{d}{dr}U_{\text{tot}} = 0$.

EXECUTE: **(a)** $\dfrac{d}{dr}U_{\text{tot}} = \dfrac{\alpha e^2}{4\pi\epsilon_0}\dfrac{1}{r^2} - 8A\dfrac{1}{r^9}$. Setting this equal to zero when $r = r_0$ gives $r_0^7 = \dfrac{8A4\pi\epsilon_0}{\alpha e^2}$ and so $U_{\text{tot}} = \dfrac{\alpha e^2}{4\pi\epsilon_0}\left(-\dfrac{1}{r} + \dfrac{r_0^7}{8r^8}\right)$. At $r = r_0$, $U_{\text{tot}} = -\dfrac{7\alpha e^2}{32\pi\epsilon_0 r_0} = -1.26\times10^{-18}\ \text{J} = -7.85\ \text{eV}$.

(b) To remove a Na^+Cl^- ion pair from the crystal requires 7.85 eV. When neutral Na and Cl atoms are formed from the Na^+ and Cl^- atoms there is a net release of energy $-5.14\ \text{eV} + 3.61\ \text{eV} = -1.53\ \text{eV}$, so the net energy required to remove a neutral Na Cl pair from the crystal is

$$7.85\ \text{eV} - 1.53\ \text{eV} = 6.32\ \text{eV}.$$

EVALUATE: Our calculation is in good agreement with the experimental value.

42.45. **IDENTIFY** and **SET UP:** Use the description of the bcc lattice in Figure 42.11c in the textbook to calculate the number of atoms per unit cell and then the number of atoms per unit volume.

EXECUTE: **(a)** Each unit cell has one atom at its center and 8 atoms at its corners that are each shared by 8 other unit cells. So there are $1 + 8/8 = 2$ atoms per unit cell.

$$\frac{n}{V} = \frac{2}{(0.35\times10^{-9}\text{m})^3} = 4.66\times10^{-8}\ \text{atoms/m}^3.$$

(b) $E_{\text{F0}} = \dfrac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\dfrac{N}{V}\right)^{2/3}$.

In this equation *N/V* is the number of free electrons per m^3. But the problem says to assume one free electron per atom, so this is the same as *n/V* calculated in part (a).

$m = 9.109\times10^{-31}\ \text{kg}$ (the electron mass), so $E_{\text{F0}} = 7.563\times10^{-19}\ \text{J} = 4.7\ \text{eV}$.

EVALUATE: Our result for metallic lithium is similar to that calculated for copper in Example 42.7.

42.46. **(a) IDENTIFY:** The rotational energy levels are given by $E_l = l(l+1)\dfrac{\hbar^2}{2I}$. The photon wavelength λ is related to the transition energy of the atom by $\Delta E = \dfrac{hc}{\lambda}$.

SET UP: For emission, $\Delta l = -1$. For such a transition, from state l to state $l-1$, we have $\Delta E_l = [l(l+1)-(l-1)l]\dfrac{\hbar^2}{2I} = \dfrac{l\hbar^2}{I}$. The difference in transition energies for adjacent lines in the spectrum is $\Delta E = \Delta E_l - \Delta E_{l-1} = \dfrac{\hbar^2}{I}$.

EXECUTE: The transition energies corresponding to the observed wavelengths are 3.29×10^{-21} J, 2.87×10^{-21} J, 2.47×10^{-21} J, 2.06×10^{-21} J, and 1.65×10^{-21} J. The average spacing of these energies is 0.410×10^{-21} J. Then, $\dfrac{\hbar^2}{I} = 0.410\times10^{-21}$ J, from which $I = 2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2$.

EVALUATE: With $\dfrac{\hbar^2}{I} = 0.410\times10^{-21}$ J and $\Delta E_l = \dfrac{l\hbar^2}{I}$, we find that these wavelengths correspond to transitions from levels 8, 7, 6, 5, and 4 to the respective next lower levels.

(b) IDENTIFY: Each transition is from the level l to the level $l-1$. The rotational energies are given by $E_l = l(l+1)\dfrac{\hbar^2}{2I}$. The transition energy is related to the photon wavelength by $\Delta E = hc/\lambda$.

SET UP: $E_l = l(l+1)\hbar^2/2I$, so $\Delta E = E_l - E_{l-1} = [l(l+1) - l(l-1)]\left(\dfrac{\hbar^2}{2I}\right) = l\left(\dfrac{\hbar^2}{I}\right)$.

EXECUTE: $l\left(\dfrac{\hbar^2}{I}\right) = \dfrac{\hbar c}{\lambda}$.

$$l = \frac{2\pi c I}{\hbar\lambda} = \frac{2\pi(2.998\times10^8\ \text{m/s})(2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2)}{(1.055\times10^{-34}\ \text{J}\cdot\text{s})\lambda} = \frac{4.843\times10^{-4}\ \text{m}}{\lambda}.$$

For $\lambda = 60.4\ \mu\text{m}$, $l = \dfrac{4.843\times10^{-4}\ \text{m}}{60.4\times10^{-6}\ \text{m}} = 8.$

For $\lambda = 69.0\ \mu\text{m}$, $l = \dfrac{4.843\times10^{-4}\ \text{m}}{69.0\times10^{-6}\ \text{m}} = 7.$

For $\lambda = 80.4\ \mu\text{m}$, $l = \dfrac{4.843\times10^{-4}\ \text{m}}{80.4\times10^{-6}\ \text{m}} = 6.$

For $\lambda = 96.4\ \mu\text{m}$, $l = \dfrac{4.843\times10^{-4}\ \text{m}}{96.4\times10^{-6}\ \text{m}} = 5.$

For $\lambda = 120.4\ \mu\text{m}$, $l = \dfrac{4.843\times10^{-4}\ \text{m}}{120.4\times10^{-6}\ \text{m}} = 4.$

EVALUATE: In each case l is an integer, as it must be.

(c) IDENTIFY: The rotational energies of a molecule depend on its moment of inertia, which in turn depends on the separation between the atoms in the molecule.

SET UP: Part (a) gives $I = 2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2$. $I = m_r r^2$. Calculate m_r and solve for r.

EXECUTE: $m_r = \dfrac{m_H m_{Cl}}{m_H + m_{Cl}} = \dfrac{(1.67\times10^{-27}\ \text{kg})(5.81\times10^{-26}\ \text{kg})}{1.67\times10^{-27}\ \text{kg} + 5.81\times10^{-26}\ \text{kg}} = 1.623\times10^{-27}\ \text{kg}.$

$$r = \sqrt{\frac{I}{m_r}} = \sqrt{\frac{2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2}{1.623\times10^{-27}\ \text{kg}}} = 1.29\times10^{-10}\ \text{m} = 0.129\ \text{nm}.$$

EVALUATE: This is a typical atomic separation for a diatomic molecule; see Example 42.2 for the corresponding distance for CO.

(d) IDENTIFY and SET UP: The longest λ implies the smallest ΔE, and this is for the transition from $l = 1$ to $l = 0$.

EXECUTE: $\Delta E = l\left(\dfrac{\hbar^2}{I}\right) = (1)\dfrac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2} = 4.099\times10^{-22}\ \text{J}.$

$\lambda = \dfrac{hc}{\Delta E} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{4.099\times10^{-22}\ \text{J}} = 4.85\times10^{-4}\ \text{m} = 485\ \mu\text{m}.$

EVALUATE: This is longer than any wavelengths in part (b).

42.47. **IDENTIFY** and **SET UP:** The occupation probability $f(E)$ is $f(E) = \dfrac{1}{e^{(E-E_{\text{F}})/kT}+1}$.

EXECUTE: **(a)** Figure 42.47 shows the graph of E versus $\ln\{[1/f(E)]-1\}$ for the data given in the problem. The slope of the best-fit straight line is 0.445 eV, and the y-intercept is 1.80 eV.

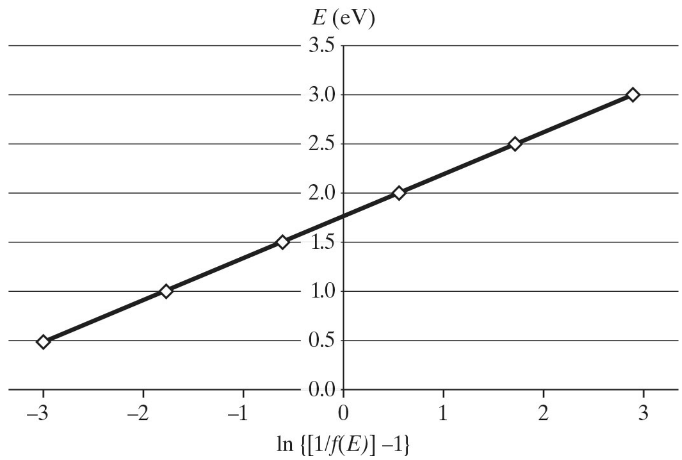


Figure 42.47

(b) Solve the $f(E)$ equation for E, giving $e^{(E-E_{\text{F}})/kT} = 1/f(E)-1$. Now take natural logarithms of both side of the equation, giving $(E-E_{\text{F}})/kT = \ln\{[1/f(E)]-1\}$, which gives $E = kT\ln\{[1/f(E)]-1\}+E_{\text{F}}$. From this we see that a graph of E versus $\ln\{[1/f(E)]-1\}$ should be a straight line having a slope equal to kT and a y-intercept equal to E_{F}. From our graph, we get E_{F} = y-intercept = 1.80 eV.

The slope is equal to kT, so $T = (\text{slope})/k = (0.445\ \text{eV})/(1.38\times10^{-23}\ \text{J/K})$

$= (7.129\times10^{-20}\ \text{J})/(1.38\times10^{-23}\ \text{J/K})$

$= 5170\ \text{K}.$

EVALUATE: In Example 42.7, the Fermi energy for copper was found to be 7.03 eV, so our result of 1.80 eV seems plausible.

42.48. **IDENTIFY** and **SET UP:** We assume that the equation $I = I_{\text{S}}(e^{eV/kT}-1)$ applies for this p-n junction. Use the given information for I and V to calculate the saturation current I_{S} and the temperature T for the junction.

EXECUTE: Put the numbers given in the problem into the equation for the current.

$0.407\ \text{mA} = I_{\text{S}}(e^{(1.60\times10^{-19}\ \text{C})(0.00500\ \text{V})/(1.38\times10^{-23}\ \text{J/K})T}-1) = I_{\text{S}}(e^{(57.97\ \text{K})/T})-1.$

$-0.338\ \text{mA} = I_{\text{S}}(e^{(1.60\times10^{-19}\ \text{C})(-0.00500\ \text{V})/(1.38\times10^{-23}\ \text{J/K})T}-1) = I_{\text{S}}(e^{(-57.97\ \text{K})/T})-1.$

Dividing the two equations gives $\dfrac{0.407}{0.338} = \dfrac{(e^{(57.97\ \text{K})/T})-1}{(e^{(-57.97\ \text{K})/T})-1}$.

Cross-multiplying and simplifying gives $e^{(57.97\ \text{K})/T} - 2.204 + 1.204e^{-(57.97\ \text{K})/T} = 0$.

Letting $x = e^{(57.97\ \text{K})/T}$, this equation becomes $x - 2.204 + 1.204x^{-1} = 0$.

Multiplying by x gives the quadratic equation $x^2 - 2.204x + 1.204 = 0$.
The two solutions are $x = 1.00$ or $x = 1.204$. The $x = 1.00$ solution would require $T = \infty$, which is not physically possible, so $x = 1.204$ is the solution to use. This gives $1.204 = e^{(57.97\text{ K})/T}$. Taking natural logarithms of both sides gives $\ln(1.204) = (57.97\text{ K})/T$, so $T = (57.97\text{ K})/\ln(1.204) = 312\text{ K}$.

The saturation current is $I_S = \dfrac{I}{e^{eV/kT} - 1} = \dfrac{0.407\text{ mA}}{e^{(1.60\times10^{-19}\text{ C})(0.00500\text{ V})/[(1.38\times10^{-23}\text{ J/K})(312\text{ K})]} - 1} = 1.99\text{ mA}$.

EVALUATE: 312 K = 39°C = 102°F, which is quite a bit warmer than room temperature (≈20°C). A current of about 2 mA is not unusual in laboratory apparatus.

42.49. **IDENTIFY:** This problem involves a semiconductor device used to make a half-wave rectifier.
SET UP: Eq. (42.22) gives the current through the junction of the semiconductor. The saturation current is 5×10^{-13} A and $T = 300$ K.
EXECUTE: **(a)** We want the minimum V_{out}, so $I = 1\ \mu$A. Solve Eq. (42.22) for V. (Careful! In this equation, e is the charge of an electron, not the base for natural logarithms.)

$$V_{out} = \frac{kT}{e}\ln\left(1+\frac{I}{I_S}\right) = \frac{k(300\text{ K})}{e}\ln\left(1+\frac{1\,\mu\text{A}}{5\times10^{-13}\text{ A}}\right) = 0.375\text{ V}.$$

(b) We want the maximum V_{out}. Combine Eq. (42.22) and $I = V/R_D$ and solve for V. Note that $e/kT = e/[k(300\text{ K})] = 38.65\text{ V}^{-1}$.

$$V = R_D I = R_D I_S\left(e^{eV/kT} - 1\right) = (100\ \Omega)(5\times10^{-13}\text{ A})\left(e^{(38.65\text{ V}^{-1})V} - 1\right).$$

Solve this equation by trial-and-error or using mathematical software. The result is $V_{max} = 0.6$ V.
(c) We want the current. $I = V/R_D = (0.6\text{ V})/(100\ \Omega) = 0.006$ A.
(d) If $V_{in} > V_{max}$: $V_{out} = V_{max} = 0.6$ V.
If $V_{in} < V_{max}$: $V_{out} = V_{in}$.
EVALUATE: The exponential behavior makes I very sensitive to changes in V.

49.50. **IDENTIFY:** We are dealing with a p-n junction diode.

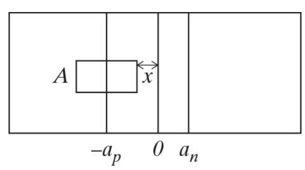


Figure 42.50

SET UP and **EXECUTE:** **(a)** We want the electric field on the p side of the junction. Apply Gauss's law using the Gaussian surface shown in Figure 42.50 and follow the hint in the problem.

$\int KE\cos\theta\, dA = Q/\epsilon_0$, where $Q - eN_A(x - a_p)$.

$$KEA\cos\pi = \frac{-eN_A(a_p - x)A}{\epsilon_0}$$

$$E_x = -\frac{eN_A(x - a_p)}{K\epsilon_0}.$$

(b) We want the electric field on the n side of the junction. Use the same procedure as in part (a) except that the Gaussian surface lies to the right of $x = 0$ and extends into the n region. The result is

$$KEA\cos 0° = \frac{-eN_D(a_n - x)A}{\epsilon_0}$$

$$E_x = -\frac{eN_D(a_n - x)}{K\epsilon_0}.$$

(c) By continuity, at $x = 0$ the field must be the same for both expressions.

$$-\frac{eN_\mathrm{A}(x-a_p)}{K\epsilon_0} = -\frac{eN_\mathrm{D}(a_n-x)}{K\epsilon_0} \rightarrow N_\mathrm{A}a_p = N_\mathrm{D}a_n.$$

(d) We want $V(x)$. Use Eq. (23.18) as advised in the problem.

$$V(x)-V_0 = -\int E_x\,dx = -\int_{-a_p}^{-x} -\frac{eN_\mathrm{A}}{K\epsilon_0}(x'-a_p)dx' = \frac{eN_\mathrm{A}}{K\epsilon_0}\left(\frac{x^2}{2}+a_p x+\frac{3}{2}a_p^2\right).$$

Now use the requirement that $V(x) = 0$ when $x = -a_p$ to find V_0.

$$-V_0 = \frac{eN_\mathrm{A}}{K\epsilon_0}\left(\frac{a_p^2}{2}-a_p^2+\frac{3}{2}a_p^2\right) = \frac{eN_\mathrm{A}}{K\epsilon_0}a_p^2.$$

Putting this result into our equation for $V(x)$ and combining terms gives

$$V(x) = \frac{eN_\mathrm{A}}{2K\epsilon_0}(x+a_p)^2.$$

(e) We want $V(x)$ on the other side of the junction. Using the same approach as in part (d) gives

$$V(x)-V_0 - \int E_x\,dx = -\int_0^x -\frac{eN_\mathrm{D}}{K\epsilon_0}(a_n-x')\,dx' = \frac{eN_\mathrm{D}}{K\epsilon_0}\left(a_n x-\frac{x^2}{2}\right).$$

From part (d) we know $V(0)$ at $x = 0$. From our result above we know that $V(0) = V_0$. So

$$V_0 = \frac{eN_\mathrm{A}}{2K\epsilon_0}a_p^2.$$

Using this in our result for $V(x)$ gives

$$V(x) = \frac{eN_\mathrm{A}a_p^2}{2K\epsilon_0} - \frac{eN_\mathrm{D}}{2K\epsilon_0}\left(x^2-2a_n x\right).$$

(f) We want the potential barrier $V_b = V(a_n) - V(-a_p)$. $V_b = V(a_n)$ because $V(-a_p) = 0$. Using the result from part (e) gives

$$V_b = \frac{eN_\mathrm{A}a_p^2}{2K\epsilon_0} - \frac{eN_\mathrm{D}}{2K\epsilon_0}\left(a_n^2-2a_n^2\right) = \frac{e}{2K\epsilon_0}\left(N_\mathrm{A}a_p^2+N_\mathrm{D}a_n^2\right).$$

(g) We want a_p. Using the results from part (c) and the values for the variables given in the problem, we have

$$a_p = a_n\frac{N_\mathrm{D}}{N_\mathrm{A}} = 275\ \text{nm}.$$

(h) We want the peak electric field at $x = 0$. We can use either equation for E because of continuity and evaluate it at $x = 0$ with $K = 11.7$, giving

$$E = \frac{eN_\mathrm{D}a_n}{K\epsilon_0} = 4.25\ \text{MV/m}.$$

(i) We want V_b. Use the results from part (f) and the given values and result from part (g). This gives

$$V_\mathrm{b} = \frac{e}{2K\epsilon_0}\left(N_\mathrm{A}a_p^2+N_\mathrm{D}a_n^2\right) = 0.701\ \text{V}.$$

EVALUATE: The electric field in part (h) is quite small compared to many lab fields.

42.51. IDENTIFY: We are looking at the vibrations of the CO_2 molecule.

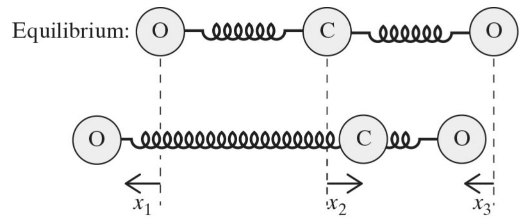


Figure 42.51

SET UP: According to the textbook description with Figure 42.10(c) for the asymmetric stretching mode, when carbon moves to the right, oxygen atoms move to the left, and when carbon moves to the left, they move to the right. Figure 42.51 shows the atoms in both their equilibrium configuration and displaced as described. First use Hooke's law to find the net force on each atom and then apply Newton's second law. Follow the instructions for each part of the problem.

EXECUTE: **(a)** Force on the left oxygen atom: This force is to the right. $F_{O1} = k'(x_2 + |x_1|)$. But x_1 is negative, so $|x_1| = -x_1$. Therefore, $F_{O1} = k'(x_2 - x_1) = -k'(x_1 - x_2)$.

Force on the carbon atom: The force due to the left-hand oxygen atom is to the left and the force due to the right-hand oxygen atom is also to the left. The net force is the sum of these two.

$F_C = -k'(x_2 + |x_1|) - k'(x_2 + |x_3|) = -k'(x_2 - x_1) - k'(x_2 - x_3) = k'(x_1 - 2x_2 + x_3)$.

Force on the right oxygen atom: This force is to the right.

$$F_{O2} = k'(x_2 + |x_3|) = k'(x_2 - x_3).$$

The corresponding equations for Newton's second law are

$$M_O\ddot{x}_1 = -k'(x_1 - x_2)$$

$$M_C\ddot{x}_2 = -k'(x_1 - 2x + x_3)$$

$$M_O\ddot{x}_3 = -k'(x_2 - x_3).$$

(b) Making the substitutions suggested in the problem, the Newton's second law equation become

$$M_O\ddot{X}_O = -k'(X_O + X_C)$$

$$-M_C\ddot{X}_C = k'(X_O + 2X_C + X_O) = 2k'(X_O + X_C)$$

$$M_O\ddot{X}_O = k'(-X_C - X_O) = -k'(X_C + X_O)$$

Combining these three equations, we find that the two independent equations are

$$M_O\ddot{X}_O = -k'(X_O + X_C)$$

$$M_C\ddot{X}_C = -2k'(X_O + X_C).$$

(c) Eliminating $X_C + X_O$ gives

$$M_C\ddot{X}_C = 2M_O\ddot{X}_O$$

Now solve this differential equation.

$$M_C\frac{d\dot{X}_C}{dt} = 2M_O\frac{d\dot{X}_O}{dt}$$

$$\dot{X}_C = \frac{2M_O}{M_C}\dot{X}_O$$

$$X_C = \frac{2M_O}{M_C}X_O.$$

(d) Following the directions gives

$$M_O\ddot{X}_O = -k'(X_O + X_C) = -k'\left(\frac{2M_O}{M_C}X_O + X_O\right)$$

$$\frac{M_\text{O}M_\text{C}}{M_\text{C}+2M_\text{O}}\ddot{X}_\text{O} = -k'X_\text{O}$$

$$M_\text{eff} = \frac{M_\text{O}M_\text{C}}{M_\text{C}+2M_\text{O}}.$$

(e) We want the angular frequency. From the differential equation in part (d) we recognize that $\ddot{X}_\text{O} = -(k'/M_\text{eff})X_\text{O}$

$$\omega = \sqrt{k'/M_\text{eff}} = \sqrt{\frac{k'}{\frac{M_\text{O}M_\text{C}}{M_\text{C}+2M_\text{O}}}} = \sqrt{\frac{k'(M_\text{C}+2M)}{M_\text{O}M_\text{C}}}.$$

(f) We want the frequency *f*.

$$f = \omega/2\pi = \frac{1}{2\pi}\sqrt{\frac{k'(M_\text{C}+2M)}{M_\text{O}M_\text{C}}}.$$

Using $k' = 1860$ N/m, $M_\text{O} = 16$ u, and $M_\text{C} = 12$ u, we get $f = 8.06 \times 10^{13}$ Hz.

EVALUATE: This frequency is much lower than that of visible light.

42.52. **IDENTIFY:** This problem deals with the energy levels of molecules.

SET UP: Follow the instructions with each part. To get five significant figures, we should use numbers of even greater precision.

EXECUTE: **(a)** We want the photon energies. Use $E = hc/\lambda$ to calculate the energies

$E = hc/\lambda = hc/(4.2680\ \mu\text{m}) = 0.29049$ eV.
$E = hc/(4.2753\ \mu\text{m}) = 0.290006$ eV.
$E = hc/(4.2826\ \mu\text{m}) = 0.289512$ eV.
$E = hc/(4.2972\ \mu\text{m}) = 0.288528$ eV.
$E = hc/(4.3046\ \mu\text{m}) = 0.288032$ eV.

(b) We want to know which one of the energies in part (a) is due to a transition from an $l = 0$ state. Eq. (42.9) gives the energy due to rotation and vibration. The selection rules are $\Delta l = \pm 1$ and $\Delta n = 1$.

$$E_{nl} = l(l+1)\frac{\hbar^2}{2I} + \left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m_\text{r}}}$$

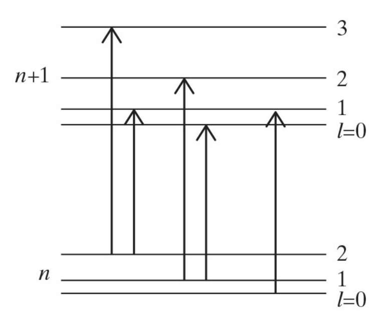


Figure 42.52

All the transitions are from the lowest energy states, which are $l = 0, 1, 2$. Figure 42.52 shows the five possible transitions from adjacent *n* states. The energy of a photon is equal to the energy difference between the two levels. Since Δn is the same for all these transitions, the energy difference between *rotational* states is what causes the different-energy photons. Therefore we need only look at the rotational energy differences between states. These are given by

$$\Delta E = E_{l_2} - E_{l_1} = [l_2(l_2+1) - l_1(l_1+1)]\frac{\hbar^2}{2I}$$

For the five transitions shown in Figure 42.52, the energy differences are

$$\Delta E_{0\to 1} = [1(1+1)-0]\frac{\hbar^2}{2I} = \frac{\hbar^2}{I}$$

$$\Delta E_{1\to 0} = [0-2]\frac{\hbar^2}{2I} = -\frac{\hbar^2}{I}$$

$$\Delta E_{1\to 2} = [2(2+1)-1(1+1)]\frac{\hbar^2}{2I} = \frac{2\hbar^2}{I}$$

$$\Delta E_{2\to 1} = [1(1+1)-2(2+1)]\frac{\hbar^2}{2I} = \frac{-2\hbar^2}{I}$$

$$\Delta E_{2\to 3} = [3(3+1)-2(2+1)]\frac{\hbar^2}{2I} = \frac{3\hbar^2}{I}$$

The total energy difference is due to Δl and Δn. The photon energy E is therefore $E = \Delta E_n + \Delta E_l$. For the l transition in this situation, we have the following:

$$E_{0\to 1} = \Delta E_n + \frac{\hbar^2}{I}$$

$$E_{1\to 0} = \Delta E_n - \frac{\hbar^2}{I}$$

$$E_{1\to 2} = \Delta E_n + \frac{2\hbar^2}{I}$$

$$E_{2\to 1} = \Delta E_n - \frac{2\hbar^2}{I}$$

$$E_{2\to 3} = \Delta E_n + \frac{3\hbar^2}{I}$$

Now rank the photon energies just calculated here and the photon energies calculated in part (a), from the lowest to the highest. By comparing the two rankings, we can tell which photon energy is due to a transition from an $l = 0$ state.

$$E_{2\to 1} = \Delta E_n - \frac{2\hbar^2}{I} \qquad 0.288032 \text{ eV}$$

$$E_{1\to 0} = \Delta E_n - \frac{\hbar^2}{I} \qquad 0.288528 \text{ eV}$$

$$E_{0\to 1} = \Delta E_n + \frac{\hbar^2}{I} \qquad 0.289512 \text{ eV}$$

$$E_{1\to 2} = \Delta E_n + \frac{2\hbar^2}{I} \qquad 0.290006 \text{ eV}$$

$$E_{2\to 3} = \Delta E_n + \frac{3\hbar^2}{I} \qquad 0.290503 \text{ eV}$$

From this list, we see that energy from the $l = 0$ state is $E_{0\to 1} = 0.289512$ eV.

(c) Using the results of part (b), we see that the second-highest energy states correspond, so $E_{1\to 0}$ = 0.288528 eV.

(d) Use the results from parts (b) and (c). ΔE_n is the same for both transitions.

$$l = 0 \to l = 1: \quad 0.289512 \text{ eV} = \frac{\hbar^2}{I} + \Delta E_n$$

$$l = 1 \to l = 0: \quad 0.288528 \text{ eV} = \frac{\hbar^2}{I} + \Delta E_n$$

Add the equations and use Eq. (42.9) to find ΔE_n. Then solve for the desired quantity.

$$0.289512 \text{ eV} + 0.288528 \text{ eV} = 2\Delta E_n = 2\hbar\sqrt{k'/m_r}$$

$$\hbar\sqrt{k'/m_r} = 0.289020 \text{ eV}.$$

(e) We want I. Subtract the two equations at the beginning of part (d) and solve for I, giving

$$I = 1.41\times10^{-46} \text{ kg}\cdot\text{m}^2.$$

(f) What is the mystery gas? Use the information in Figure 42.52 in the textbook to calculate the moment of inertia of each molecule. Then compare these with what we found in part (e). Start with N_2.

$$I = m_r r_0^2 = \frac{m_1 m_2}{m_1+m_2}r_0^2 = \frac{(14 \text{ u})(14 \text{ u})}{28 \text{ u}}(110 \text{ pm})^2 = 1.41\times10^{-46} \text{ kg}\cdot\text{m}^2.$$

This result matches the moment of inertia we found in part (e), so the gas is N_2.

(g) In part (f) we saw that m_r = (14 u)(14 u)/(28 u) = 7 u.

(h) Use the result from part (d) and solve for the spring constant, giving 2241 N/m.

EVALUATE: The spring constant we found would be a very large one for the types of springs found in most physics labs.

42.53. **IDENTIFY** and **SET UP:** $p = -\dfrac{dE_{\text{tot}}}{dV}$. Relate E_{tot} to E_{F0} and evaluate the derivative.

EXECUTE: **(a)** $\dfrac{dE_{\text{tot}}}{dV} = \dfrac{3}{5}\left(\dfrac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\right)N^{5/3}\left(-\dfrac{2}{3}V^{-5/3}\right)$, so $p = \left(\dfrac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\right)\left(\dfrac{N}{V}\right)^{5/3}$, as was to be shown.

(b) $N/V = 8.45\times10^{28} \text{ m}^{-3}$.

$$p = \left(\frac{3^{2/3}\pi^{4/3}(1.055\times10^{-34} \text{ J}\cdot\text{s})^2}{5(9.109\times10^{-31} \text{ kg})}\right)(8.45\times10^{28} \text{ m}^{-3})^{5/3} = 3.81\times10^{10} \text{ Pa} = 3.76\times10^5 \text{ atm}.$$

EVALUATE: **(c)** Normal atmospheric pressure is about 10^5 Pa, so these pressures are extremely large. The electrons are held in the metal by the attractive force exerted on them by the copper ions.

42.54. **IDENTIFY** and **SET UP:** From Problem 42.53, $p = \dfrac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\left(\dfrac{N}{V}\right)^{5/3}$. Use this expression to calculate dp/dV.

EXECUTE: **(a)** $B = -V\dfrac{dp}{dV} = -V\left[\dfrac{5}{3}\cdot\dfrac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\cdot\left(\dfrac{N}{V}\right)^{2/3}\left(\dfrac{-N}{V^2}\right)\right] = \dfrac{5}{3}p.$

(b) $\dfrac{N}{V} = 8.45\times10^{28} \text{ m}^{-3}$. $B = \dfrac{5}{3}\cdot\dfrac{3^{2/3}\pi^{4/3}\hbar^2}{5m}(8.45\times10^{28}\text{m}^{-3})^{5/3} = 6.33\times10^{10}$ Pa.

EVALUATE: **(c)** The fraction of B due to the free electrons is $\dfrac{6.33\times10^{10} \text{ Pa}}{1.4\times10^{11} \text{ Pa}} = 0.45$. The copper ions themselves make up the remaining fraction.

42.55. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** $E_{\text{F0}} = \dfrac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\dfrac{N}{V}\right)^{2/3}$. Let $E_{\text{F0}} = \dfrac{1}{100}mc^2$.

$$\left(\frac{N}{V}\right) = \left[\frac{2m^2c^2}{(100)3^{2/3}\pi^{4/3}\hbar^2}\right]^{3/2} = \frac{2^{3/2}m^3c^3}{100^{3/2}3\pi^2\hbar^3} = \frac{2^{3/2}m^3c^3}{3000\pi^2\hbar^3} = 1.67\times10^{33} \text{ m}^{-3}.$$

(b) $\dfrac{8.45\times10^{28} \text{ m}^{-3}}{1.67\times10^{33} \text{ m}^{-3}} = 5.06\times10^{-5}$. Since the real concentration of electrons in copper is less than one part in 10^{-4} of the concentration where relativistic effects are important, it is safe to ignore relativistic effects for most applications.

(c) The number of electrons is $N_e = \dfrac{6(2\times10^{30}\ \text{kg})}{1.99\times10^{-26}\ \text{kg}} = 6.03\times10^{56}.$ The concentration is

$$\frac{N_e}{V} = \frac{6.03\times10^{56}}{\frac{4}{3}\pi(6.00\times10^{6}\ \text{m})^3} = 6.66\times10^{35}\ \text{m}^{-3}.$$

EVALUATE: **(d)** Comparing this to the result from part (a) $\dfrac{6.66\times10^{35}\ \text{m}^{-3}}{1.67\times10^{33}\ \text{m}^{-3}} \cong 400,$ so relativistic effects will be very important.

42.56. **IDENTIFY** and **SET UP:** The sensitivity is defined as $\Delta V/\Delta T$.

EXECUTE: From the graph shown with the introduction, we see that for a current of 0.1 A (100 mA), the voltage at 25°C is about 0.75 V, and at 150°C it is about 0.55 V. Therefore the sensitivity is $\dfrac{\Delta V}{\Delta T} = \dfrac{0.55\ \text{V} - 0.75\ \text{V}}{150°\text{C} - 25°\text{C}} = -1.6\ \text{mV/C°} \approx -2.0\ \text{mV/C°}.$ Therefore choice (d) is correct.

EVALUATE: If the temperature changes by 10 C°, V changes by about 20 mV, which can be significant.

42.57. **IDENTIFY** and **SET UP:** As the temperature increases, the electrons gain energy.

EXECUTE: As T increases, more electrons gain enough energy to jump into the conduction band, so for a given voltage, more current will flow at a higher temperature than at a lower temperature. Therefore choice (b) is correct.

EVALUATE: For ordinary resistors, increasing their temperature increases their resistance, so less current would flow.

42.58. **IDENTIFY** and **SET UP:** On the graph shown with the introduction, draw a vertical line between the two curves. This line represents a constant voltage.

EXECUTE: For V greater than about 0.2 V, the current at 150°C is greater than at 25°C. So increasing the temperature at a constant voltage increases the current, which makes choice (b) correct.

EVALUATE: This result is consistent with what we found in Passage Problem 42.57.

43

NUCLEAR PHYSICS

VP43.4.1. **IDENTIFY:** This problem is about the binding energy of an O-16 nucleus.

SET UP and **EXECUTE:** **(a)** We want the mass defect ΔM. Use the numerical values given in the text and Table 43.2. For O-16, $Z = 8$ and $A = 16$ so $N = 8$.

$\Delta M = ZM_{\text{H}} + Nm_{\text{n}} - {}^{A}_{Z}M.$

$\Delta M = 8(1.007825\text{ u}) - 8(1.008655\text{ u}) - 15.994915\text{ u} = 0.137005\text{ u}.$

(b) We want the binding energy. $E_{\text{B}} = \Delta Mc^2 = (0.137005\text{ u})(931.5\text{ MeV/u}) = 127.6\text{ MeV}.$

(c) We want the binding energy per nucleon. O-16 has 16 nucleons, so

$$E_{\text{B}}/\text{nucleon} = (127.6\text{ MeV})/16 = 7.976\text{ MeV/nucleon}.$$

EVALUATE: From the graph in Figure 43.2, we see that O-16 is more tightly bound that C-12 but less so than Ni-62.

VP43.4.2. **IDENTIFY:** This problem is about the energy of the Cu-63 nucleus.

SET UP and **EXECUTE:** We know that the mass defect is 0.5919378 u. **(a)** The binding energy E_{B} is the energy of the mass defect. $E_{\text{B}} = (0.5919378\text{ u})(931.5\text{ MeV/u}) = 551.4\text{ MeV}.$

(b) $E_{\text{B}}/\text{nucleon} = (551.4\text{ MeV})/(63\text{ nucleons}) = 8.752\text{ MeV/nucleon}.$

(c) We want the mass of the Cu-63 atom. Use Eq. (43.10) with $Z = 29$ and $A = 63$, so $N = 34$.

EVALUATE: Note that the C-63 mass is about 63 u, but *not quite*.

VP43.4.3. **IDENTIFY:** This problem is about nuclear binding energy.

SET UP: The binding energy per nucleon is E_{B}/A where E_{B} is the energy of the mass defect. Use the given mass defects to calculate E_{B}.

EXECUTE: **(a)** As-75. $E_{\text{B}}/A = (0.7005604\text{ u})(931.5\text{ MeV/u})/75 = 8.701\text{ MeV/nucleon}.$

(b) Sm-150. Use the same method as in (a) with $A = 150$, giving $E_{\text{B}}/A = 8.262\text{ MeV/nucleon}.$

(c) Ra-225. $A = 225$ gives $E_{\text{B}}/A = 7.668\text{ MeV/nucleon}.$

(d) As A increases from 75 to 225, E_{B}/A decreases from 8.701 MeV/nucleon to 7.668 MeV/nucleon. This behavior agrees with Figure 43.2.

EVALUATE: Note that the *total* binding energy increases as A increases, but the binding energy *per nucleon* decreases.

VP43.4.4. **IDENTIFY:** In this problem, we use the semiempirical mass formula to calculate the binding energy.

SET UP: Eq. (43.11) gives the formula and the constants $C_1, C_2, \ldots, C_5$ are given following it. We follow the procedure of Example 43.4.

EXECUTE: **(a)** Ru-100. $A = 100$, $Z = 44$. Using Eq. (43.11) gives the following terms.

$C_1A = (15.75\text{ MeV})(100) = 1575$

$-C_2A^{2/3} = -(17.80\text{ MeV})(100^{2/3}) = -383.5$

$-C_3Z(Z-1)/A^{1/3} = -(0.7100\text{ MeV})(44)(43)/(100^{1/3}) = -289.4\text{ MeV}$

$-C_4(A-2Z)^2/A = -(23.69\text{ MeV})(100-88)^2/(100) = -34.11\text{ MeV}$

$+C_5A^{-4/3} = (39\text{ MeV})(100^{-4/3}) = 0.084\text{ MeV}.$

Adding all these terms gives E_B = 868.1 MeV. E_B/A = (868.1 MeV)/100 = 8.681 MeV/nucleon.
(b) Hg-200. A = 200, Z = 80. Use the same procedure as in part (a), giving E_B = 1584 MeV, E_B/A = (1584 MeV)/(200) = 7.922 MeV/nucleon.
EVALUATE: **(c)** From Figure 43.2: For A = 100, $E_B/A \approx$ 8.66 MeV/nucleon, and we got 8.681 MeV/nucleon.
For A = 200, $E_B/A \approx$ 7.85 MeV/nucleon, and we got 79.22 MeV/nucleon. Our results agree closely with those in the figure.

VP43.7.1. **IDENTIFY:** This problem is about energy in radioactive decay.
SET UP and **EXECUTE:** **(a)** We want the energy released. Refer to the decay shown in the problem. Using the given masses, the mass difference is 241.056827 u – (237.048172 u + 4.002603 u) = 0.0060520 u. The energy released is the energy of this mass, which is given by (0.006052 u)(931.5 MeV/u) = 5.637 MeV.
(b) We want the kinetic energy of the alpha particle. The rest energy of He is (4 u)(931.5 MeV/u) = 3726 MeV, which is much greater than the energy released. Therefore we do not need to use relativity. Momentum conservation gives $m_\alpha v_\alpha = m_{Np}v_{Np}$, so $v_\alpha/v_{Np} = m_{Np}/m_\alpha$. Using the masses gives v_{Np} = $(4.003/237.0)v_\alpha = 0.016890v_\alpha$. Energy conservation gives $K_\alpha + K_{Np}$ = 5.637 MeV. Using $K = \frac{1}{2}mv^2$ and taking the ratio of kinetic energies gives K_α/K_{Np} = 59.22. Adding the kinetic energies gives K_α + K_α/59.22
= 5.637 MeV. K_α = 5.543 MeV.
(c) We want the speed of the alpha particle. Use the result from (b) and $K = \frac{1}{2}mv^2$ and solve for v, giving $v_\alpha = 1.63 \times 10^7$ m/s.
EVALUATE: v/c = 0.16/3.0 = 0.053, so v is about 5% the speed of light, which means that it was OK to neglect relativity.

VP43.7.2. **IDENTIFY:** This problem is about the nuclear decay of Be-8 to two alpha particles.
SET UP: First find the mass defect and use it to find the energy released. The alpha particles each get one-half of this energy.
EXECUTE: **(a)** We want the kinetic energy. The mass defect is $m_{Be} - 2m_{He}$, which gives [8.0053051 u – 2(4.0026033 u)](931.5 MeV/u) = 0.09175 MeV. Each alpha particle gets half of this, so K_α = 0.0459 MeV.
(b) We want the speed of the alpha particles. The rest energy of the alpha is (4.0026 u)(931.5 MeV/u) = 3730 MeV, which is much greater than its kinetic energy. So we do not need to use relativity. Use $K = \frac{1}{2}mv^2$ with K = 0.0459 MeV, giving $v_\alpha = 1.49 \times 10^6$ m/s.
EVALUATE: v/c = 0.0149/3.0 = 0.00497, so $v \ll c$. Our neglect of relativity was justified.

VP43.7.3. **IDENTIFY:** This problem is about β^- decay.
SET UP: β^- decay is possible if the mass of the original neutral atom is greater than the mass of the final atom. The energy released is the energy of the difference in mass.
EXECUTE: **(a)** The mass of Be-12 is greater than the mass of B-12, so this decay is possible. The energy released is E = (12.026922 u – 12.014353 u)(931.5 MeV/u) = 11.71 MeV.
(b) The mass of Ar-33 is greater than the mass of Cl-33, so this decay is not possible.
(c) The mass of Br-82 is greater than the mass of Kr-82, so this decay is possible. As in part (a), E = (81.9168018 u – 81.9134812 u)(931.5 MeV/u) = 3.093 MeV.
EVALUATE: The greater the mass difference, the greater the energy that is released.

VP43.7.4. **IDENTIFY:** This problem is about β^+ decay.
SET UP: β^+ decay is possible if the mass of the original neutral atom is greater than the mass of the final atom by twice the electron mass. The mass of the electron is 0.000548580 u, so the mass of the original atom must be greater than that of the final atom by 2(0.000548580 u) = 0.001097160 u. The energy released is the energy of the difference in mass minus two electron masses.

EXECUTE: (a) The mass difference is 45.960198 – 45.952627 u = 0.0075710 u. This difference is greater than twice the electron mass, so the decay is possible. The energy released is

$$E = (0.0075710\text{ u} - 0.001097160\text{ u})(931.5\text{ MeV/u}) = 6.030\text{ MeV}.$$

(b) Δm = 133.908514 u – 133.904508 u = 0.0040060 u > $2m_e$, so the decay is possible. The energy is E = (0.0040060 u – 0.00109716 u)(931.5 MeV/u) = 2.710 MeV.

(c) Δm = 66.928202 u – 66.9271275 u = 0.0010745 u < $2m_e$, so β^+ decay is not possible. The original atom's mass is greater than that of the final atom, so electron capture is possible.

EVALUATE: If the mass of the original atom is greater than the mass of the final atom, then β^+ decay and electron capture are *both* possible, so both may occur. A number of radionuclides decay by more than one mode.

VP43.9.1. IDENTIFY: This problem deals with radioactive decay. The half-life is $T_{1/2}$ = 15.0 h and the initial activity is dN/dt = –1.60 μCi.

SET UP and EXECUTE: (a) $T_{\text{mean}} = T_{1/2}/(\ln 2)$ = (15.0 h)/(ln 2) = 21.6 h = 7.79×10^4 s.

(b) $\lambda = (\ln 2)/T_{1/2}$ = (ln 2)/(15.0 h) = $4.621 \times 10^{-2}\text{ h}^{-1} = 1.28 \times 10^{-5}\text{ s}^{-1}$.

(c) We want the initial number N_0 of Na-24 nuclei. We know the initial activity, so we make use of that.

$$N = N_0 e^{-\lambda t}$$

$$dN/dt = -\lambda N_0 e^{-\lambda t} = -\lambda N_0$$

$$N_0 = -\frac{1}{\lambda}\frac{dN}{dt} = -\frac{1}{1.28\times 10^{-5}\text{ s}^{-1}}(-1.50\ \mu\text{Ci})(3.70\times 10^{10}\text{ s}^{-1}) = 4.32\times 10^{9}.$$

(d) We want dN/dt after 24.0 h. From our work in part (c), we can see that

$$dN/dt = dN/dt)_0\, e^{-\lambda t} = (1.50\ \mu\text{Ci})\, e^{-(0.04621\text{ h}^{-1})(24.0\text{ h})} = 0.495\ \mu\text{Ci}.$$

EVALUATE: In 15 h, dN/dt would be 0.75 μCi, and our result in part (d) is less than that, so it is reasonable.

VP43.9.2. IDENTIFY: This problem deals with radioactive decay. The half-life is $T_{1/2}$ = 35.0 h and the initial activity is dN/dt = –0.514 μCi.

SET UP and EXECUTE: (a) $\lambda = (\ln 2)/T_{1/2}$ = (ln 2)/(35.0 h) = $1.98 \times 10^{-2}\text{ h}^{-1} = 5.50 \times 10^{-6}\text{ s}^{-1}$.

(b) We want the present number N_0 of Nb-95 nuclei.

$$N = N_0 e^{-\lambda t}$$

$$dN/dt = -\lambda N_0 e^{-\lambda t} = -\lambda N_0$$

Call t = 0 the present instant when dN/dt = –0.514 μCi. Solve the above equation for N_0.

$$N_0 = -\frac{1}{\lambda}\frac{dN}{dt} = -\frac{1}{5.50\times 10^{-6}\text{ s}^{-1}}(-0.514\ \mu\text{Ci})(3.70\times 10^{10}\text{ s}^{-1}) = 3.46\times 10^{9}.$$

(c) We want the initial number of Nb-95 nuclei. The 3.46×10^9 nuclei from part (b) is 23.8% of the original number of nuclei. So $0.238N_0 = 3.46 \times 10^9$, which gives $N_0 = 1.45 \times 10^{10}$.

(d) We want the time to now. Solve for t and use the results we have found.

$$N = N_0 e^{-\lambda t}$$

$$t = -\frac{\ln(N/N_0)}{\lambda} = \frac{\ln\left(\dfrac{3.46\times 10^{9}}{1.45\times 10^{10}}\right)}{0.0198\text{ h}^{-1}} = 72.5\text{ h}.$$

EVALUATE: The number of Mo-95 nuclei present now is $(0.762)(1.45 \times 10^{10}) = 1.11 \times 10^{10}$. The sum of nuclei now is $1.11 \times 10^{10} + 3.46 \times 10^9 = 1.45 \times 10^{10}$, which agrees with our result in (c).

VP43.9.3. **IDENTIFY:** We are dealing with radioactive decay. The initial activity is 0.415 μCi, and 45.0 s later it is 0.121 μCi.

SET UP and EXECUTE: **(a)** We want the decay constant and half-life. If we call R the activity, we have seen that it decreases exponentially. Use this fact and solve for the decay constant and then use it to find the half-life.

$$R = R_0 e^{-\lambda t}$$

$$\lambda = -\frac{\ln(R/R_0)}{t} = -\frac{\ln\left(\dfrac{0.121\ \mu\text{Ci}}{0.425\ \mu\text{Ci}}\right)}{45.0\text{ s}} = 0.0279\text{ s}^{-1}.$$

$T_{1/2} = (\ln 2)/\lambda = (\ln 2)/(0.279\text{ s}^{-1}) = 24.8$ s.

(b) We want the number of nuclei in an excited state. These are the nuclei that give off the radiation, so we want to find the number of undecayed nuclei initially and 45.0 s later.

Initially:

$$dN/dt = -\lambda N_0 e^{-\lambda t} = -\lambda N_0$$

$$(-0.425\ \mu\text{Ci})(3.70 \times 10^{10}\text{ s}^{-1}) = -(0.0279\text{ s}^{-1})N_0$$

$$N_0 = 5.63 \times 10^5.$$

After 45.0 s: Using the same approach gives $(-0.121\ \mu\text{Ci})(3.70 \times 10^{10}\text{ s}^{-1}) = -(0.0279\text{ s}^{-1})N_0$

$N_0 = 1.60 \times 10^5$.

EVALUATE: The number at 45.0 s is less than the initial number, so our result is reasonable.

VP43.9.4. **IDENTIFY:** This problem is about carbon-14 dating.

SET UP: The half-life of C-14 is 5730 y. If we call R the activity level per gram, it follows from previous work that $R = R_0 e^{-\lambda t}$.

EXECUTE: **(a)** We want the age of the sample. The activity per unit mass that you detect now is 113 decays per 20 min per 0.510 g of carbon. So R = [(113 decays)/(20 min)]/(0.510 g) = 0.18464 Bq/g. Use $R = R_0 e^{-\lambda t}$ with $\lambda = (\ln 2)/T_{1/2}$ and solve for t.

$$t = \frac{\ln(R/R_0)}{\lambda} = -\frac{T_{1/2}\ln(R/R_0)}{\ln 2} = -\frac{(5730\text{ y})\ln\left(\dfrac{0.18464\text{ Bq/g}}{0.255\text{ Bq/g}}\right)}{45.0\text{ s}} = 2670\text{ y}.$$

(b) We want the current number of C-14 nuclei. Call $t = 0$ the present time. In your sample, the present activity is (113 decays)/(20 min) = 0.094167 Bq. At $t = 0$ (the present time), we have

$$N_0 = \frac{1}{\lambda}\frac{dN}{dt} = -\frac{T_{1/2}}{\ln 2}(-0.094167\text{ Bq}) = \frac{(5730\text{ y})(0.094167\text{ Bq})}{\ln 2} = 2.46\times 10^{10}.$$

EVALUATE: The age of your sample is less than one half-life, so its activity (0.18464 Bq/g) should be greater than half the original activity (0.255 Bq/g), which is what we have found.

43.1. **IDENTIFY and SET UP:** The pre-subscript is Z, the number of protons. The pre-superscript is the mass number A. $A = Z + N$, where N is the number of neutrons.

EXECUTE: **(a)** $^{28}_{14}\text{Si}$ has 14 protons and 14 neutrons.

(b) $^{85}_{37}\text{Rb}$ has 37 protons and 48 neutrons.

(c) $^{205}_{81}\text{Tl}$ has 81 protons and 124 neutrons.

EVALUATE: The number of protons determines the chemical element.

43.2. **IDENTIFY:** Calculate the spin magnetic energy shift for each spin state of the 1s level. Calculate the energy splitting between these states and relate this to the frequency of the photons.

SET UP: When the spin component is parallel to the field the interaction energy is $U = -\mu_z B$. When the spin component is antiparallel to the field the interaction energy is $U = +\mu_z B$. The transition energy for a transition between these two states is $\Delta E = 2\mu_z B$, where $\mu_z = 2.7928\mu_n$. The transition energy is related to the photon frequency by $\Delta E = hf$, so $2\mu_z B = hf$.

EXECUTE: $B = \dfrac{hf}{2\mu_z} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(22.7\times10^{6}\ \text{Hz})}{2(2.7928)(5.051\times10^{-27}\ \text{J/T})} = 0.533\ \text{T}$

EVALUATE: This magnetic field is easily achievable. Photons of this frequency have wavelength $\lambda = c/f = 13.2$ m. These are radio waves.

43.3. **(a) IDENTIFY:** Find the energy equivalent of the mass defect.

SET UP: A $^{11}_{5}\text{B}$ atom has 5 protons, $11-5=6$ neutrons, and 5 electrons. The mass defect therefore is $\Delta M = 5m_\text{p} + 6m_\text{n} + 5m_\text{e} - M(^{11}_{5}\text{B})$.

EXECUTE: $\Delta M = 5(1.0072765\ \text{u}) + 6(1.0086649\ \text{u}) + 5(0.0005485799\ \text{u}) - 11.009305\ \text{u} = 0.08181\ \text{u}$.

The energy equivalent is $E_\text{B} = (0.08181\ \text{u})(931.5\ \text{MeV/u}) = 76.21\ \text{MeV}$.

(b) IDENTIFY and **SET UP:** Eq. (43.11): $E_\text{B} = C_1A - C_2A^{2/3} - C_3Z(Z-1)/A^{1/3} - C_4(A-2Z)^2/A$.

The fifth term is zero since Z is odd but N is even. $A = 11$ and $Z = 5$.

EXECUTE:

$E_\text{B} = (15.75\ \text{MeV})(11) - (17.80\ \text{MeV})(11)^{2/3} - (0.7100\ \text{MeV})5(4)/11^{1/3} - (23.69\ \text{MeV})(11-10)^2/11.$

$$E_\text{B} = +173.25\ \text{MeV} - 88.04\ \text{MeV} - 6.38\ \text{MeV} - 2.15\ \text{MeV} = 76.68\ \text{MeV}.$$

The percentage difference between the calculated and measured E_B is

$$\frac{76.68\ \text{MeV} - 76.21\ \text{MeV}}{76.21\ \text{MeV}} = 0.6\%.$$

EVALUATE: Eq. (43.11) has a greater percentage accuracy for ^{62}Ni. The semi-empirical mass formula is more accurate for heavier nuclei.

43.4. **IDENTIFY:** The binding energy E_B is the difference between the energy of the constituent nuclides and final nuclide, converted to energy units.

SET UP: Use the masses in Table 43.2 and m_n = 1.008665 u for the neutron. The masses in Table 43.2 are for neutral atoms and therefore include the mass of the electrons. 1 u is equivalent to 931.5 MeV.

For $^{11}_{5}\text{B}$: $E_\text{B}(\text{B-11}) = 5m_\text{H} + 6m_\text{n} - m_\text{B-11}$.

For $^{11}_{6}\text{C}$: $E_\text{B}(\text{C-11}) = 6m_\text{H} + 5m_\text{n} - m_\text{C-11}$.

EXECUTE: **(a)** $^{11}_{5}\text{B}$: E_B = [5(1.007825 u) + 6(1.008665 u) – 11.009305 u](931.5 MeV/u) = 76.21 MeV.

$^{11}_{6}\text{C}$: E_B = [6(1.007825 u) + 5(1.008665 u) – 11.011434 u](931.5 MeV/u) = 73.44 MeV.

EVALUATE: **(b)** The $^{11}_{5}\text{B}$ has a larger binding energy than the $^{11}_{6}\text{C}$. This is probably due to the fact that $^{11}_{5}\text{B}$ has one fewer proton and one more neutron than $^{11}_{6}\text{C}$. The electrical repulsion between protons tends to decrease binding energy while the extra neutron tends to increase it without a corresponding electrical repulsion.

43.5. **IDENTIFY:** The binding energy of the nucleus is the energy of its constituent particles minus the energy of the carbon-12 nucleus.

SET UP: In terms of the masses of the particles involved, the binding energy is $E_\text{B} = (6m_\text{H} + 6m_\text{n} - m_{\text{C}-12})c^2$.

EXECUTE: **(a)** Using the values from Table 43.2, we get

$$E_\text{B} = [6(1.007825\ \text{u}) + 6(1.008665\ \text{u}) - 12.000000\ \text{u}](931.5\ \text{MeV/u}) = 92.16\ \text{MeV}.$$

(b) The binding energy per nucleon is $(92.16\ \text{MeV})/(12\ \text{nucleons}) = 7.680\ \text{MeV/nucleon}$.

(c) The energy of the C-12 nucleus is $(12.0000\text{ u})(931.5\text{ MeV/u}) = 11178\text{ MeV}$. Therefore the percent of the mass that is binding energy is $\dfrac{92.16\text{ MeV}}{11178\text{ MeV}} = 0.8245\%$.

EVALUATE: The binding energy of 92.16 MeV binds 12 nucleons. The binding energy per nucleon, rather than just the total binding energy, is a better indicator of the strength with which a nucleus is bound.

43.6. **IDENTIFY:** The mass defect is the total mass of the constituents minus the mass of the atom.

SET UP: 1 u is equivalent to 931.5 MeV. ${}^{238}_{92}\text{U}$ has 92 protons, 146 neutrons and 238 nucleons.

EXECUTE: **(a)** $146m_\text{n} + 92m_\text{H} - m_\text{U} = 1.93\text{ u}$.

(b) $1.80\times10^3\text{ MeV}$.

(c) 7.57 MeV per nucleon (using 931.5 MeV/u and 238 nucleons).

EVALUATE: The binding energy per nucleon we calculated agrees with Figure 43.2 in the textbook.

43.7. **IDENTIFY:** This problem is about the binding energy of the atomic nucleus.

SET UP and **EXECUTE:** The target variable is the mass of a neutral Fe-56 atom. Since the atom is neutral, it contains all of its electrons. Use Eq. (43.10) and solve for the mass of Fe-56.

$$ {}^{56}_{26}M = ZM_\text{H} + Nm_\text{n} - E_\text{B}/c^2 $$

$$ {}^{56}_{26}M = 26(1.007825\text{ u}) + 30(1.008665\text{ u}) - \left[(8.79\text{ MeV})(56)/c^2\right]\left(\frac{1\text{ u}}{931.5\text{ MeV}/c^2}\right) = 55.935\text{ u}. $$

EVALUATE: If there were no binding energy, the mass would be $26M_\text{H} + 30m_\text{n} = 56.463$ u. The binding energy makes a detectable difference in the mass of the atom.

43.8. **IDENTIFY:** The binding energy E_B is the difference between the energy of the constituent nuclides and final nuclide, converted to energy units.

SET UP: Use the masses in Table 43.2 and $m_\text{n} = 1.008665$ u for the neutron. The masses in Table 43.2 are for neutral atoms and therefore include the mass of the electrons. 1 u is equivalent to 931.5 MeV.

For an alpha particle: ${}^{4}_{2}\text{He}$: $E_\text{B}(\alpha) = 2m_\text{H} + 2m_\text{n} - m_\text{He-4}$.

For ${}^{12}_{6}\text{C}$: $E_\text{B}(\text{C-12}) = 6m_\text{H} + 6m_\text{n} - m_\text{C-12}$.

EXECUTE: ${}^{4}_{2}\text{He}$: E_B = [2(1.007825 u) + 2(1.008665 u) – 4.002603 u](931.5 MeV/u) = 28.296 MeV, so 3 times the binding energy of the alpha particle is 3(28.296 MeV) = 84.889 MeV.

${}^{12}_{6}\text{C}$: E_B = [6(1.007825 u) + 6(1.008665 u) – 12.000000 u](931.5 MeV/u) = 92.163 MeV.

EVALUATE: The binding energy of ${}^{12}_{6}\text{C}$ is greater than 3 times the binding energy of an alpha particle. This is reasonable since in addition to binding individual alpha particles, it takes additional energy to bind three of them in the nucleus to form carbon.

43.9. **IDENTIFY:** Conservation of energy tells us that the initial energy (photon plus deuteron) is equal to the energy after the split (kinetic energy plus energy of the proton and neutron). Therefore the kinetic energy released is equal to the energy of the photon minus the binding energy of the deuteron.

SET UP: The binding energy of a deuteron is 2.224 MeV and the energy of the photon is $E = hc/\lambda$. Kinetic energy is $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** The energy of the photon is

$$ E_\text{ph} = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{3.50\times10^{-13}\text{ m}} = 5.68\times10^{-13}\text{ J}. $$

The binding of the deuteron is $E_\text{B} = 2.224\text{ MeV} = 3.56\times10^{-13}\text{ J}$. Therefore the kinetic energy is

$$ K = (5.68 - 3.56)\times10^{-13}\text{ J} = 2.12\times10^{-13}\text{ J} = 1.32\text{ MeV}. $$

(b) The particles share the energy equally, so each gets half. Solving the kinetic energy for v gives

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(1.06\times10^{-13}\ \text{J})}{1.6605\times10^{-27}\ \text{kg}}}=1.13\times10^{7}\ \text{m/s}.$$

EVALUATE: Considerable energy has been released, because the particle speeds are in the vicinity of the speed of light.

43.10. **IDENTIFY:** The mass defect is the total mass of the constituents minus the mass of the atom.

SET UP: 1 u is equivalent to 931.5 MeV. ${}^{14}_{7}\text{N}$ has 7 protons and 7 neutrons. ${}^{4}_{2}\text{He}$ has 2 protons and 2 neutrons.

EXECUTE: **(a)** $7(m_\text{n}+m_\text{H})-m_\text{N}=0.112\ \text{u},$ which is 105 MeV, or 7.48 MeV per nucleon.

(b) Similarly, $2(m_\text{H}+m_\text{n})-m_\text{He}=0.03038\ \text{u}=28.3\ \text{MeV},$ or 7.07 MeV per nucleon.

EVALUATE: **(c)** The binding energy per nucleon is a little less for ${}^{4}_{2}\text{He}$ than for ${}^{14}_{7}\text{N}.$ This is in agreement with Figure 43.2 in the textbook.

43.11. **IDENTIFY:** We are looking at the decay of C-11.

SET UP and **EXECUTE:** **(a)** In positron decay, a proton changes to a neutron and an electron. So the number of protons decreases from 6 to 5 and the number of neutrons increases from 5 to 6. So the daughter nucleus contains 5 protons and 6 neutrons.

(b) For positron decay to occur, the mass of the parent must be at least two electron masses greater than the mass of the daughter. The daughter nucleus has 5 protons, so it is boron (B). The decay is

$${}^{11}_{6}\text{C}\rightarrow{}^{11}_{5}\text{B}+\beta^{+}+\nu_\text{e}.$$

Use Table 43.2 for the mass of boron and m_e = 0.000548580 u. The initial mass is 11.011433 u and the final mass is 11.009305 u + 2(0.000548580 u) = 11.01040216 u. The difference in mass is 11.01040216 u – 11.011433 u = –0.001030840 u. The energy released is the energy of the lost mass, which is (0.001030840 u)(931.5 MeV/u) = 0.960 MeV.

EVALUATE: Notice that we need to use *twice* the mass of the positron for positron decay.

43.12. **IDENTIFY:** Compare the total mass on each side of the reaction equation. Neglect the masses of the neutrino and antineutrino.

SET UP: 1 u is equivalent to 931.5 MeV.

EXECUTE: **(a)** The energy released is the energy equivalent of $m_\text{n}-m_\text{p}-m_\text{e}=8.40\times10^{-4}\ \text{u},$ or 783 keV.

(b) $m_\text{n}>m_\text{p},$ and the decay is not possible.

EVALUATE: β^- and β^+ particles have the same mass, equal to the mass of an electron.

43.13. **IDENTIFY:** In each case determine how the decay changes A and Z of the nucleus. The β^+ and β^- particles have charge but their nucleon number is $A=0.$

(a) SET UP: α-decay: Z decreases by $2, A=N+Z$ decreases by 4 (an α particle is a ${}^{4}_{2}\text{He}$ nucleus).

EXECUTE: ${}^{239}_{94}\text{Pu}\rightarrow{}^{4}_{2}\text{He}+{}^{235}_{92}\text{U}.$

(b) SET UP: β^- decay: Z increases by $1, A=N+Z$ remains the same (a β^- particle is an electron, ${}^{0}_{-1}\text{e}$).

EXECUTE: ${}^{24}_{11}\text{Na}\rightarrow{}^{0}_{-1}\text{e}+{}^{24}_{12}\text{Mg}.$

(c) SET UP: β^+ decay: Z decreases by $1, A=N+Z$ remains the same (a β^+ particle is a positron, ${}^{0}_{+1}\text{e}$).

EXECUTE: ${}^{15}_{8}\text{O}\rightarrow{}^{0}_{+1}\text{e}+{}^{15}_{7}\text{N}.$

EVALUATE: In each case the total charge and total number of nucleons for the decay products equals the charge and number of nucleons for the parent nucleus; these two quantities are conserved in the decay.

43.14. **IDENTIFY:** The energy released is equal to the mass defect of the initial and final nuclei.

SET UP: The mass defect is equal to the difference between the initial and final masses of the constituent particles.

EXECUTE: **(a)** The mass defect is $238.050788 \text{ u} - 234.043601 \text{ u} - 4.002603 \text{ u} = 0.004584 \text{ u}$. The energy released is $(0.004584 \text{ u})(931.5 \text{ MeV/u}) = 4.270 \text{ MeV}$.

(b) Take the ratio of the two kinetic energies, using the fact that $K = p^2/2m$:

$$\frac{K_{\text{Th}}}{K_\alpha} = \frac{\frac{p_{\text{Th}}^2}{2m_{\text{Th}}}}{\frac{p_\alpha^2}{2m_\alpha}} = \frac{m_\alpha}{m_{\text{Th}}} = \frac{4}{234}.$$

The kinetic energy of the Th is

$$K_{\text{Th}} = \frac{4}{234+4}K_{\text{Total}} = \frac{4}{238}(4.270 \text{ MeV}) = 0.07176 \text{ MeV} = 1.148\times10^{-14} \text{ J}.$$

Solving for v in the kinetic energy gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.148\times10^{-14} \text{ J})}{(234.043601)(1.6605\times10^{-27} \text{ kg})}} = 2.431\times10^5 \text{ m/s}.$$

EVALUATE: As we can see by the ratio of kinetic energies in part (b), the alpha particle will have a much higher kinetic energy than the thorium.

43.15. **IDENTIFY:** This problem is about β^+ decay.

SET UP and **EXECUTE:** For β^+ decay, the mass of the decay products must be at least two electron-masses greater than the mass of the original atom. The decay in this case is

$${}^{12}_{7}\text{N} \to {}^{12}_{6}\text{C} + \beta^+ + \nu_e.$$

The target variable is the mass M of the N-12. Using the energy released given in the problem, we have $M = m_C + 2m_e + 16.316 \text{ MeV}/c^2$. Using Table 43.2 gives

$M = 12.000000 \text{ u} + 2(0.000548580 \text{ u}) + (16.316 \text{ MeV})[\text{u}/(931.5 \text{ MeV})] = 12.018611 \text{ u}$.

EVALUATE: Check: M is greater than the mass of C-12, as it should be.

43.16. **IDENTIFY:** In each reaction the nucleon number and the total charge are conserved.

SET UP: An α particle has charge $+2e$ and nucleon number 4. An electron has charge $-e$ and nucleon number zero. A positron has charge $+e$ and nucleon number zero.

EXECUTE: **(a)** A proton changes to a neutron, so the emitted particle is a positron (β^+).

(b) The number of nucleons in the nucleus decreases by 4 and the number of protons by 2, so the emitted particle is an alpha-particle.

(c) A neutron changes to a proton, so the emitted particle is an electron (β^-).

EVALUATE: We have considered the conservation laws. We have not determined if the decays are energetically allowed.

43.17. **IDENTIFY:** Determine the energy released during tritium decay.

SET UP: In beta decay an electron, e^-, is emitted by the nucleus. The beta decay reaction is ${}^3_1\text{H} \to e^- + {}^3_2\text{He}$. If neutral atom masses are used, ${}^3_1\text{H}$ includes one electron and ${}^3_2\text{He}$ includes two electrons. One electron mass cancels and the other electron mass in ${}^3_2\text{He}$ represents the emitted electron. Or, we can subtract the electron masses and use the nuclear masses. The atomic mass of ${}^3_2\text{He}$ is 3.016029 u.

EXECUTE: **(a)** The mass of the ${}^{3}_{1}\text{H}$ nucleus is $3.016049\ \text{u} - 0.000549\ \text{u} = 3.015500\ \text{u}$. The mass of the ${}^{3}_{2}\text{He}$ nucleus is $3.016029\ \text{u} - 2(0.000549\ \text{u}) = 3.014931\ \text{u}$. The nuclear mass of ${}^{3}_{2}\text{He}$ plus the mass of the emitted electron is $3.014931\ \text{u} + 0.000549\ \text{u} = 3.015480\ \text{u}$. This is slightly less than the nuclear mass for ${}^{3}_{1}\text{H}$, so the decay is energetically allowed.

(b) The mass decrease in the decay is $3.015500\ \text{u} - 3.015480\ \text{u} = 2.0\times10^{-5}\ \text{u}$. Note that this can also be calculated as $m({}^{3}_{1}\text{H}) - m({}^{4}_{2}\text{He})$, where atomic masses are used. The energy released is $(2.0\times10^{-5}\ \text{u})(931.5\ \text{MeV/u}) = 0.019\ \text{MeV}$. The total kinetic energy of the decay products is 0.019 MeV,
or 19 keV.

EVALUATE: The energy is not shared equally by the decay products because they have unequal masses.

43.18. **IDENTIFY:** This problem involves radioactive decay and half-life.
SET UP and **EXECUTE:** The target variable is the number of decays in $0.500T_{1/2}$. We are working with the half-life of the isotope, so it is convenient to express the decay in terms of base 2 as follows:

$$N = N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}} = N_0 2^{-(0.500T_{1/2})/T_{1/2}} = N_0 2^{-0.500}.$$

The number of decays is $\Delta N = N_0 - N = N_0 - N_0 e^{-0.500} = 0.293N_0$.
EVALUATE: Note that ΔN is *not* $N_0/4$.

43.19. **IDENTIFY** and **SET UP:** $T_{1/2} = \dfrac{\ln 2}{\lambda}$ The mass of a single nucleus is $124m_\text{p} = 2.07\times10^{-25}\ \text{kg}$.

$$|dN/dt| = 0.350\ \text{Ci} = 1.30\times10^{10}\ \text{Bq},\quad |dN/dt| = \lambda N.$$

EXECUTE: $N = \dfrac{6.13\times10^{-3}\ \text{kg}}{2.07\times10^{-25}\ \text{kg}} = 2.96\times10^{22};\ \lambda = \dfrac{|dN/dt|}{N} = \dfrac{1.30\times10^{10}\ \text{Bq}}{2.96\times10^{22}} = 4.39\times10^{-13}\ \text{s}^{-1}.$

$$T_{1/2} = \frac{\ln 2}{\lambda} = 1.58\times10^{12}\ \text{s} = 5.01\times10^{4}\ \text{y}.$$

EVALUATE: Since $T_{1/2}$ is very large, the activity changes very slowly.

43.20. **IDENTIFY:** The equation $N = N_0 e^{-\lambda t}$ can be written as $N = N_0 2^{-t/T_{1/2}}$.
SET UP: The amount of elapsed time since the source was created is roughly 2.5 years.
EXECUTE: The current activity is $N = (5000\ \text{Ci})2^{-(2.5\ \text{yr})/(5.271\ \text{yr})} = 3600\ \text{Ci}$. The source is barely usable.

EVALUATE: Alternatively, we could calculate $\lambda = \dfrac{\ln(2)}{T_{1/2}} = 0.132(\text{years})^{-1}$ and use $N = N_0 e^{-\lambda t}$ directly to obtain the same answer.

43.21. **IDENTIFY:** From the known half-life, we can find the decay constant, the rate of decay, and the activity.
SET UP: $\lambda = \dfrac{\ln 2}{T_{1/2}}$. $T_{1/2} = 4.47\times10^{9}\ \text{yr} = 1.41\times10^{17}\ \text{s}$. The activity is $\left|\dfrac{dN}{dt}\right| = \lambda N$. The mass of one ${}^{238}\text{U}$ is approximately $238m_\text{p}$. $1\ \text{Ci} = 3.70\times10^{10}\ \text{decays/s}$.

EXECUTE: **(a)** $\lambda = \dfrac{\ln 2}{1.41\times10^{17}\ \text{s}} = 4.92\times10^{-18}\ \text{s}^{-1}$.

(b) $N = \dfrac{|dN/dt|}{\lambda} = \dfrac{3.70\times10^{10}\ \text{Bq}}{4.92\times10^{-18}\ \text{s}^{-1}} = 7.52\times10^{27}$ nuclei. The mass m of uranium is the number of nuclei times the mass of each one. $m = (7.52\times10^{27})(238)(1.67\times10^{-27}\ \text{kg}) = 2.99\times10^{3}\ \text{kg}$.

(c) $N = \dfrac{10.0\times10^{-3}\text{ kg}}{238m_\text{p}} = \dfrac{10.0\times10^{-3}\text{ kg}}{238(1.67\times10^{-27}\text{ kg})} = 2.52\times10^{22}$ nuclei.

$\left|\dfrac{dN}{dt}\right| = \lambda N = (4.92\times10^{-18}\text{ s}^{-1})(2.52\times10^{22}) = 1.24\times10^{5}$ decays/s.

EVALUATE: Because ^{238}U has a very long half-life, it requires a large amount (about 3000 kg) to have an activity of a 1.0 Ci.

43.22. **IDENTIFY:** From the half-life and mass of an isotope, we can find its initial activity rate. Then using the half-life, we can find its activity rate at a later time.

SET UP: The activity $|dN/dt| = \lambda N$. $\lambda = \dfrac{\ln 2}{T_{1/2}}$. The mass of one ^{103}Pd nucleus is $103m_\text{p}$. In a time of one half-life the number of radioactive nuclei and the activity decrease by a factor of 2.

EXECUTE: **(a)** $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{(17\text{ days})(24\text{ h/day})(3600\text{ s/h})} = 4.7\times10^{-7}\text{ s}^{-1}$.

$N = \dfrac{0.250\times10^{-3}\text{ kg}}{103m_\text{p}} = 1.45\times10^{21}$. $|dN/dt| = (4.7\times10^{-7}\text{ s}^{-1})(1.45\times10^{21}) = 6.8\times10^{14}$ Bq.

(b) 68 days is $4T_{1/2}$ so the activity is $(6.8\times10^{14}\text{ Bq})/2^4 = 4.2\times10^{13}$ Bq.

EVALUATE: At the end of 4 half-lives, the activity rate is less than a tenth of its initial rate.

43.23. **IDENTIFY** and **SET UP:** As discussed in Section 43.4, the activity $A = |dN/dt|$ obeys the same decay equation as $N(t)$: $A = A_0 e^{-\lambda t}$. For ^{14}C, $T_{1/2} = 5730$ y and $\lambda = \ln 2/T_{1/2}$ so $A = A_0 e^{-(\ln 2)t/T_{1/2}}$; calculate A at each t; $A_0 = 184$ decays/min.

EXECUTE: **(a)** For t = 1000 y, we have A = (184 decays/min)$e^{-(\ln 2)(1000\text{ y})/(5730\text{ y})}$ = 163 decays/min.
(b) For t = 50,000 y, the same equation gives A = 0.435 decays/min.
EVALUATE: The time in part (b) is 8.73 half-lives, so the decay rate has decreased by a factor of $(\frac{1}{2})^{8.73}$.

43.24. **IDENTIFY** and **SET UP:** The decay rate decreases by a factor of 2 in a time of one half-life.

EXECUTE: **(a)** 24 days is 3 half-lives, so the activity at the end of that time is $(325\text{ Bq})/(2^3) = 40.6$ Bq, which rounds to 41 Bq.
(b) The activity is proportional to the number of radioactive nuclei, so the percent is

$$\frac{17.0\text{ Bq}}{40.6\text{ Bq}} = 0.42 = 42\%.$$

(c) ${}^{131}_{53}\text{I} \to {}^{0}_{-1}\text{e} + {}^{131}_{54}\text{Xe}$. The nucleus ${}^{131}_{54}\text{Xe}$ is produced.

EVALUATE: Both the activity and the number of radioactive nuclei present decrease by a factor of 2 in one half-life.

43.25. **IDENTIFY:** This problem is about radioactive decay.

SET UP: We are working with the half-life of the isotope, so it is convenient to express the decay in terms of base 2 when needed. $N = N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}}$

EXECUTE: **(a)** We want to know the number of nuclei initially present. We know the initial rate of decay, which is dN/dt, so

$$dN/dt = d(N_0 e^{-\lambda t})/dt = -\lambda N_0 e^{-\lambda t}.$$

At t = 0, $dN/dt = -8.0\times10^{16}$ Bq. Solving for N_0 gives

$N_0 = \dfrac{1}{\lambda}\dfrac{dN}{dt} = -\dfrac{T_{1/2}}{0.693}\dfrac{dN}{dt} = -\dfrac{(64.0)(3600\text{ s})}{0.693}\left(-8.0\times10^{16}\text{ Bq}\right) = 2.7\times10^{22}$ nuclei.

(b) We want the number of nuclei remaining at the end of 12.0 days. Use base 2 for simplicity.

$$N = N_0 2^{-t/T_{1/2}} = (2.66\times10^{22})2^{-(12.0)(24.\text{h})(64.0\text{h})} = 1.2\times10^{21}\text{nuclei.}$$

EVALUATE: Note that 12.0 days is 4.5 half-lives, so $N/N_0 = 2^{-4.5}$.

43.26. IDENTIFY: Apply $|dN/dt| = \lambda N$ to calculate N, the number of radioactive nuclei originally present in the spill. Since the activity A is proportional to the number of radioactive nuclei, $N = N_0 e^{-\lambda t}$ leads to $A = A_0 e^{-\lambda t}$, where A is the activity.

SET UP: The mass of one ^{131}Ba nucleus is about 131 u.

EXECUTE: **(a)** $\left|\dfrac{dN}{dt}\right| = 400\ \mu\text{Ci} = (400\times10^{-6})(3.70\times10^{10}\ \text{s}^{-1}) = 1.48\times10^{7}$ decays/s.

$$T_{1/2} = \frac{\ln 2}{\lambda} \to \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(12\text{ d})(86{,}400\text{ s/d})} = 6.69\times10^{-7}\ \text{s}^{-1}.$$

$\left|\dfrac{dN}{dt}\right| = \lambda N \Rightarrow N = \dfrac{|dN/dt|}{\lambda} = \dfrac{1.48\times10^{7}\ \text{decays/s}}{6.69\times10^{-7}\text{s}^{-1}} = 2.21\times10^{13}\text{nuclei.}$ The mass of these ^{131}Ba nuclei is

$$m = (2.21\times10^{13}\ \text{nuclei})\times(131\times1.66\times10^{-27}\ \text{kg/nucleus}) = 4.8\times10^{-12}\ \text{kg} = 4.8\times10^{-9}\ \text{g} = 4.8\ \text{ng}.$$

(b) $A = A_0 e^{-\lambda t}$. $1\ \mu\text{Ci} = (400\ \mu\text{Ci})\, e^{-\lambda t}$. $\ln(1/400) = -\lambda t$.

$$t = -\frac{\ln(1/400)}{\lambda} = -\frac{\ln(1/400)}{6.69\times10^{-7}\text{s}^{-1}} = 8.96\times10^{6}\ \text{s}\left(\frac{1\ \text{d}}{86{,}400\ \text{s}}\right) = 104\ \text{days}.$$

EVALUATE: The time is about 8.7 half-lives and the activity after that time is $(400\ \mu\text{Ci})\left(\tfrac{1}{2}\right)^{8.7} \approx 1\mu\text{Ci}$.

43.27. IDENTIFY: Apply $A = A_0 e^{-\lambda t}$ and $\lambda = \ln 2/T_{1/2}$.

SET UP: $\ln e^{x} = x$.

EXECUTE: $A = A_0 e^{-\lambda t} = A_0 e^{-t(\ln 2)/T_{1/2}}$. $-\dfrac{(\ln 2)t}{T_{1/2}} = \ln(A/A_0)$.

$$T_{1/2} = -\frac{(\ln 2)t}{\ln(A/A_0)} = -\frac{(\ln 2)(4.00\ \text{days})}{\ln(3091/8318)} = 2.80\ \text{days}.$$

EVALUATE: The activity has decreased by more than half and the elapsed time is more than one half-life.

43.28. IDENTIFY: Apply $A = A_0 e^{-\lambda t}$.

SET UP: From Example 43.9, $\lambda = 3.83\times10^{-12}\ \text{s}^{-1} = 1.21\times10^{-4}\ \text{y}^{-1}$ for radiocarbon.

EXECUTE: The activity of the sample is $\dfrac{2690\ \text{decays/min}}{(60\ \text{s/min})(0.500\ \text{kg})} = 89.7\ \text{Bq/kg}$, while the activity of atmospheric carbon is $255\ \text{Bq/kg}$ (see Example 43.9). The age of the sample is then

$$t = -\frac{\ln(89.7/225)}{\lambda} = -\frac{\ln(89.7/225)}{1.21\times10^{-4}/\text{y}} = 8640\ \text{y}.$$

EVALUATE: For ^{14}C, $T_{1/2} = 5730\ \text{y}$. The age is more than one half-life and the activity per kg of carbon is less than half the value when the tree died.

43.29. IDENTIFY and SET UP: Apply $|dN/dt| = \lambda N$ with $\lambda = \ln 2/T_{1/2}$. In one half-life, one half of the nuclei decay.

EXECUTE: **(a)** $\left|\dfrac{dN}{dt}\right| = 7.56\times10^{11}\ \text{Bq} = 7.56\times10^{11}$ decays/s.

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(30.8 \text{ min})(60 \text{ s/min})} = 3.75\times10^{-4} \text{ s}^{-1}.$$

$$N_0 = \frac{1}{\lambda}\left|\frac{dN}{dt}\right| = \frac{7.56\times10^{11} \text{ decay/s}}{3.75\times10^{-4} \text{ s}^{-1}} = 2.02\times10^{15} \text{ nuclei.}$$

(b) The number of nuclei left after one half-life is $\frac{N_0}{2} = 1.01\times10^{15}$ nuclei, and the activity is half: $\left|\frac{dN}{dt}\right| = 3.78\times10^{11}$ decays/s.

(c) After three half-lives (92.4 minutes) there is an eighth of the original amount, so $N = 2.53\times10^{14}$ nuclei, and an eighth of the activity: $\left|\frac{dN}{dt}\right| = 9.45\times10^{10}$ decays/s.

EVALUATE: Since the activity is proportional to the number of radioactive nuclei that are present, the activity is halved in one half-life.

43.30. IDENTIFY and SET UP: $1 \text{ Gy} = 1 \text{ J/kg}$ and is the SI unit of absorbed dose. $1 \text{ rad} = 0.010 \text{ Gy}$. Sv is the SI unit for equivalent dose. Equivalent dose $=$ RBE $\times$ absorbed dose. Rem is the equivalent dose when the absorbed dose is in rad. For x rays, $\text{RBE} = 1.0$. For protons, $\text{RBE} = 10$.

EXECUTE: **(a)** 5.0 Gy, 500 rad. $\text{RBE} = 1.0$, so equivalent dose $=$ absorbed dose. 5.0 Sv and 500 rem.

(b) $(70.0 \text{ kg})(5.0 \text{ J/kg}) = 350 \text{ J}$.

(c) The absorbed dose and total absorbed energy are the same but the equivalent dose is 10 times larger. So the answers are: 5.0 Gy, 500 rad, 50 Sv, 5000 rem, 350 J.

EVALUATE: The same energy deposited by protons as x rays is ten times greater in its biological effect.

43.31. IDENTIFY and SET UP: The unit for absorbed dose is $1 \text{ rad} = 0.01 \text{ J/kg} = 0.01 \text{ Gy}$. Equivalent dose in rem is RBE times absorbed dose in rad.

EXECUTE: $1 \text{ rad} = 10^{-2} \text{ Gy}$, so $1 \text{ Gy} = 100 \text{ rad}$ and the dose was 500 rad.

$\text{rem} = (\text{rad})(\text{RBE}) = (500 \text{ rad})(4.0) = 2000 \text{ rem}$. $1 \text{ Gy} = 1 \text{ J/kg}$, so 5.0 J/kg.

EVALUATE: Gy, rad, and J/kg are all units of absorbed dose. Rem is a unit of equivalent dose, which depends on the RBE of the radiation.

43.32. IDENTIFY and SET UP: The unit for absorbed dose is $1 \text{ rad} = 0.01 \text{ J/kg} = 0.01 \text{ Gy}$. Equivalent dose in rem is RBE times absorbed dose in rad.

EXECUTE: **(a)** $\text{rem} = \text{rad}\times\text{RBE}$. $300 = x(10)$ and $x = 30$ rad.

(b) 1 rad deposits 0.010 J/kg, so 30 rad deposit 0.30 J/kg. This radiation affects 25 g (0.025 kg) of tissue, so the total energy is $(0.025 \text{ kg})(0.30 \text{ J/kg}) = 7.5\times10^{-3} \text{ J} = 7.5 \text{ mJ}$.

(c) $\text{RBE} = 1$ for β-rays, so rem $=$ rad. Therefore 30 rad $=$ 30 rem.

EVALUATE: The same absorbed dose produces a larger equivalent dose when the radiation is neutrons than when it is electrons.

43.33. IDENTIFY and SET UP: For x rays $\text{RBE} = 1$ and the equivalent dose equals the absorbed dose.

EXECUTE: **(a)** 175 krad $=$ 175 krem $=$ 1.75 kGy $=$ 1.75 kSv. $(1.75\times10^3 \text{ J/kg})(0.220 \text{ kg}) = 385 \text{ J}$.

(b) 175 krad $=$ 1.75 kGy; $(1.50)(175 \text{ krad}) = 262.5 \text{ krem} = 2.625 \text{ kSv}$. The energy deposited would be 385 J, the same as in (a).

EVALUATE: The energy required to raise the temperature of 0.220 kg of water 1 C° is 922 J, and 385 J is less than this. The energy deposited corresponds to a very small amount of heating.

43.34. IDENTIFY and SET UP: For x rays $\text{RBE} = 1$ so the equivalent dose in Sv is the same as the absorbed dose in J/kg.

EXECUTE: One whole-body scan delivers $(75\text{ kg})(12\times10^{-3}\text{ J/kg}) = 0.90\text{ J}$. One chest x ray delivers $(5.0\text{ kg})(0.20\times10^{-3}\text{ J/kg}) = 1.0\times10^{-3}\text{ J}$. It takes $\dfrac{0.90\text{ J}}{1.0\times10^{-3}\text{ J}} = 900$ chest x rays to deliver the same total energy.

EVALUATE: For the CT scan the equivalent dose is much larger, and it is applied to the whole body.

43.35. **IDENTIFY:** This problem looks at the biological effects of radiation.

SET UP: $N = N_0e^{-\lambda t}$, $\lambda = 0.693/T_{½} = 0.693/(29\text{ y}) = 0.0239\text{ y}^{-1}$.

EXECUTE: **(a)** We want the absorbed dose during one year from 1.0 μg of Sr-90. First find the number of decays ΔN that occur during one year. Each decay releases 1.1 MeV of energy. The mass of a Sr-90 atom is approximately $38m_p + 52m_n + 38m_e = 1.5067 \times 10^{-25}$ kg. If N_0 is the number of Sr-90 atoms in the 1.0-μg sample, then $(1.5067 \times 10^{-25}\text{ kg})N_0 = 1.0\ \mu\text{g}$, which gives $N_0 = 6.637 \times 10^{15}$ atoms. At the end of one year, the number of atoms N that are left will be $N = N_0e^{-\lambda(1\text{ y})}$. The number of decays during the year is $N_0 - N = N_0 - N_0e^{-\lambda(1\text{ y})} = (6.637 \times 10^{15})[1 - e^{-(0.0239/\text{y})(1\text{ y})}] = 1.567 \times 10^{14}$ decays. The total energy E absorbed by these decays is $E = (1.567 \times 10^{14})(1.1\text{ MeV}) = 27.59$ J. This energy is delivered to 50 kg of body tissue, so the absorbed dose is (27.59 J)/(50 kg) = 0.55 J/kg = 0.55 Gy. Since 1 rad = 0.01 Gy, this dose is also 55 rad.

(b) We want the equivalent dose. Equivalent dose (in Sv) = RBE $\times$ absorbed dose (in Gy). From Table 43.3, the RBE for gamma rays is 1, and the RBE for electrons is given in the problem as 1.0. So the maximum equivalent dose is (1.0)(0.55 Gy) = 0.55 Sv. Since 1 rem = 0.01 Sv, we can also say that the equivalent dose is 55 rem.

EVALUATE: The longer the exposure lasts, the greater the dose because more decays take place.

43.36. **IDENTIFY:** 1 rem = 0.01 Sv. Equivalent dose in rem equals RBE times the absorbed dose in rad. 1 rad = 0.01 J/kg. To change the temperature of water, $Q = mc\Delta T$.

SET UP: For water, $c = 4190\text{ J/kg}\cdot\text{K}$.

EXECUTE: **(a)** $5.4\text{ Sv}(100\text{ rem/Sv}) = 540\text{ rem}$.

(b) The RBE of 1 gives an absorbed dose of 540 rad.

(c) The absorbed dose is 5.4 Gy, so the total energy absorbed is $(5.4\text{ Gy})(65\text{ kg}) = 351\text{ J}$. The energy required to raise the temperature of 65 kg by 0.010° C is $(65\text{ kg})(4190\text{ J/kg}\cdot\text{K})(0.01\text{ C°}) = 3\text{ kJ}$.

EVALUATE: The amount of energy received corresponds to a very small heating of his body.

43.37. **IDENTIFY:** Each photon delivers energy. The energy of a single photon depends on its wavelength.

SET UP: equivalent dose (rem) = RBE$\times$absorbed dose (rad). 1 rad = 0.010 J/kg. For x rays, RBE = 1. Each photon has energy $E = \dfrac{hc}{\lambda}$.

EXECUTE: **(a)** $E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{0.0200\times10^{-9}\text{ m}} = 9.94\times10^{-15}\text{ J}$. The absorbed energy is

$$(5.00\times10^{10}\text{ photons})(9.94\times10^{-15}\text{ J/photon}) = 4.97\times10^{-4}\text{ J} = 0.497\text{ mJ}.$$

(b) The absorbed dose is $\dfrac{4.97\times10^{-4}\text{ J}}{0.600\text{ kg}} = 8.28\times10^{-4}\text{ J/kg} = 0.0828\text{ rad}$. Since RBE = 1, the equivalent dose is 0.0828 rem.

EVALUATE: The amount of energy absorbed is rather small (only $\frac{1}{2}$ mJ), but it is absorbed by only 600 g of tissue.

43.38. **IDENTIFY:** The reaction energy Q is $Q = (M_A + M_B - M_C - M_D)c^2$.

SET UP: $Q = (m_{\text{H-1}} + m_{\text{H-3}} - m_{\text{H-2}} - m_{\text{H-2}})c^2$. Use the masses in Table 43.2 and $m_n = 1.008665$ u for the neutron. The masses in Table 43.2 are for neutral atoms and therefore include the mass of the electrons. 1 u is equivalent to 931.5 MeV.

EXECUTE: $Q = [1.007825\text{ u} + 3.016049\text{ u} - 2(2.014102\text{ u})](931.5\text{ MeV/u}) = -4.033\text{ MeV}$.

EVALUATE: Since Q is negative, the initial energy is less than the final energy, so energy had to be put into the system to cause the reaction to occur. Therefore, this is an endoergic reaction.

43.39. **(a) IDENTIFY** and **SET UP:** Determine X by balancing the charge and the nucleon number on the two sides of the reaction equation.

EXECUTE: X must have $A = +2 + 9 - 4 = 7$ and $Z = +1 + 4 - 2 = 3$. Thus X is ${}^7_3\text{Li}$ and the reaction is ${}^2_1\text{H} + {}^9_4\text{Be} = {}^7_3\text{Li} + {}^4_2\text{He}$.

(b) IDENTIFY and **SET UP:** Calculate the mass decrease and find its energy equivalent.

EXECUTE: If we use the neutral atom masses then there are the same number of electrons (five) in the reactants as in the products. Their masses cancel, so we get the same mass defect whether we use nuclear masses or neutral atom masses. The neutral atoms masses are given in Table 43.2. 1 u is equivalent to 931.5 MeV.

$${}^2_1\text{H} + {}^9_4\text{Be} \text{ has mass } 2.014102\text{ u} + 9.012182\text{ u} = 11.26284\text{ u}.$$

$${}^7_3\text{Li} + {}^4_2\text{He} \text{ has mass } 7.016005\text{ u} + 4.002603\text{ u} = 11.018608\text{ u}.$$

The mass decrease is $11.026284\text{ u} - 11.018608\text{ u} = 0.007676\text{ u}$.

This corresponds to an energy release of $(0.007676\text{ u})(931.5\text{ MeV/1 u}) = 7.150\text{ MeV}$.

(c) IDENTIFY and **SET UP:** Estimate the threshold energy by calculating the Coulomb potential energy when the ${}^2_1\text{H}$ and ${}^9_4\text{Be}$ nuclei just touch. Obtain the nuclear radii from $R = R_0 A^{1/3}$.

EXECUTE: The radius R_{Be} of the ${}^9_4\text{Be}$ nucleus is $R_{\text{Be}} = (1.2\times10^{-15}\text{ m})(9)^{1/3} = 2.5\times10^{-15}\text{ m}$.

The radius R_{H} of the ${}^2_1\text{H}$ nucleus is $R_{\text{H}} = (1.2\times10^{-15}\text{ m})(2)^{1/3} = 1.5\times10^{-15}\text{ m}$.

The nuclei touch when their center-to-center separation is

$R = R_{\text{Be}} + R_{\text{H}} = 4.0\times10^{-15}\text{ m}$.

The Coulomb potential energy of the two reactant nuclei at this separation is

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e(4e)}{r}.$$

$$U = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.602\times10^{-19}\text{ C})^2}{(4.0\times10^{-15}\text{ m})(1.602\times10^{-19}\text{ J/eV})} = 1.4\text{ MeV}.$$

This is an estimate of the threshold energy for this reaction.

EVALUATE: The reaction releases energy but the total initial kinetic energy of the reactants must be 1.4 MeV in order for the reacting nuclei to get close enough to each other for the reaction to occur. The nuclear force is strong but is very short-range.

43.40. **IDENTIFY:** The energy released is the energy equivalent of the mass decrease that occurs in the reaction.

SET UP: 1 u is equivalent to 931.5 MeV.

EXECUTE: $m_{{}^3_2\text{He}} + m_{{}^2_1\text{H}} - m_{{}^4_2\text{He}} - m_{{}^1_1\text{H}} = 1.97\times10^{-2}\text{ u}$, so the energy released is 18.4 MeV.

EVALUATE: Using neutral atom masses includes three electron masses on each side of the reaction equation and the same result is obtained as if nuclear masses had been used.

43.41. **IDENTIFY** and **SET UP:** The energy released is the energy equivalent of the mass decrease. 1 u is equivalent to 931.5 MeV. The mass of one ${}^{235}\text{U}$ nucleus is $235m_{\text{p}}$.

EXECUTE: **(a)** ${}^{235}_{92}\text{U} + {}^1_0\text{n} \to {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3{}^1_0\text{n}$. We can use atomic masses since the same number of electrons are included on each side of the reaction equation and the electron masses cancel. The mass decrease is $\Delta M = m({}^{235}_{92}\text{U}) + m({}^1_0\text{n}) - [m({}^{144}_{56}\text{Ba}) + m({}^{89}_{36}\text{Kr}) + 3m({}^1_0\text{n})]$,

$\Delta M = 235.043930\text{ u} + 1.0086649\text{ u} - 143.922953\text{ u} - 88.917631\text{ u} - 3(1.0086649\text{ u}),\quad \Delta M = 0.1860\text{ u}.$ The energy released is $(0.1860\text{ u})(931.5\text{ MeV/u}) = 173.3\text{ MeV}$.

(b) The number of ^{235}U nuclei in 1.00 g is $\dfrac{1.00\times10^{-3}\text{ kg}}{235m_\text{p}} = 2.55\times10^{21}$. The energy released per gram is $(173.3\text{ MeV/nucleus})(2.55\times10^{21}\text{ nuclei/g}) = 4.42\times10^{23}\text{ MeV/g}$.

EVALUATE: The energy released is 7.1×10^{10} J/kg. This is much larger than typical heats of combustion, which are about 5×10^4 J/kg.

43.42. **IDENTIFY:** The charge and the nucleon number are conserved. The energy of the photon must be at least as large as the energy equivalent of the mass increase in the reaction.
SET UP: 1 u is equivalent to 931.5 MeV.
EXECUTE: **(a)** $^{28}_{14}\text{Si} + \gamma \Rightarrow ^{24}_{12}\text{Mg} + ^{A}_{Z}\text{X}$. $A + 24 = 28$ so $A = 4$. $Z + 12 = 14$ so $Z = 2$. X is an α particle.
(b) $-\Delta m = m(^{24}_{12}\text{Mg}) + m(^{4}_{2}\text{He}) - m(^{28}_{14}\text{Si}) = 23.985042\text{ u} + 4.002603\text{ u} - 27.976927\text{ u} = 0.010718\text{ u}$.
$E_\gamma = (-\Delta m)c^2 = (0.010718\text{ u})(931.5\text{ MeV/u}) = 9.984\text{ MeV}$.
EVALUATE: The wavelength of the photon is
$\lambda = \dfrac{hc}{E} = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{9.984\times10^{6}\text{ eV}} = 1.24\times10^{-13}\text{ m} = 1.24\times10^{-4}\text{ nm}$. This is a gamma ray photon.

43.43. **IDENTIFY:** Charge and the number of nucleons are conserved in the reaction. The energy absorbed or released is determined by the mass change in the reaction.
SET UP: 1 u is equivalent to 931.5 MeV.
EXECUTE: **(a)** $Z = 3 + 2 - 0 = 5$ and $A = 4 + 7 - 1 = 10$.
(b) The nuclide is a boron nucleus, and z and so 2.79 MeV of energy is absorbed.
EVALUATE: The absorbed energy must come from the initial kinetic energy of the reactants.

43.44. **IDENTIFY:** We are looking at the energy released during radioactive decay.
SET UP: The decay is $^{10}_{4}\text{Be} \to ^{10}_{5}\text{B} + \beta + \bar{\nu}_\text{e}$.
EXECUTE: **(a)** The target variable is the released energy. Beta-minus decay can occur only if the mass of the original atom is greater than the mass of the final atom, which in this case is true. Use Table 43.2.
$\Delta m = m_\text{i} - m_\text{f} = 10.013535\text{ u} - 10.012937 = 0.000598\text{ u}$. The energy E released is

$$E = (0.000598\text{ u})[(931.5\text{ MeV})/\text{u}] = 0.557\text{ MeV}.$$

(b) We want the speed of the beta-minus particle. The released energy is a bit larger than the rest energy of the electron (0.511 MeV), so we need to use relativity. First find γ and then use it to find v.

$$E = K = mc^2(\gamma - 1)$$

$$\gamma = \frac{K}{mc^2} + 1 = \frac{0.577\text{ MeV}}{0.511\text{ MeV}} + 1 = 2.09$$

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/(2.09)^2} = 0.87c.$$

EVALUATE: Special relativity definitely cannot be neglected at such a high speed.

43.45. **IDENTIFY** and **SET UP:** $m = \rho V$. $1\text{ gal} = 3.788\text{ L} = 3.788\times10^{-3}\text{ m}^3$. The mass of a ^{235}U nucleus is $235m_\text{p}$. $1\text{ MeV} = 1.60\times10^{-13}$ J.

EXECUTE: **(a)** For 1 gallon, $m = \rho V = (737\text{ kg/m}^3)(3.788\times10^{-3}\text{ m}^3) = 2.79\text{ kg} = 2.79\times10^3$ g.

$$\frac{1.3\times10^{8}\text{ J/gal}}{2.79\times10^{3}\text{ g/gal}} = 4.7\times10^{4}\text{ J/g}.$$

(b) 1 g contains $\dfrac{1.00\times10^{-3}\text{ kg}}{235m_\text{p}} = 2.55\times10^{21}$ nuclei.

$$(200\text{ MeV/nucleus})(1.60\times10^{-13}\text{ J/MeV})(2.55\times10^{21}\text{ nuclei}) = 8.2\times10^{10}\text{ J/g}.$$

(c) A mass of $6m_\text{p}$ produces 26.7 MeV.

$$\frac{(26.7\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{6m_\text{p}} = 4.26\times10^{14}\text{ J/kg} = 4.26\times10^{11}\text{ J/g}.$$

(d) The total energy available would be $(1.99\times10^{30}\text{ kg})(4.7\times10^{7}\text{ J/kg}) = 9.4\times10^{37}\text{ J}$.

$$\text{Power} = \frac{\text{energy}}{t} \text{ so } t = \frac{\text{energy}}{\text{power}} = \frac{9.4\times10^{37}\text{ J}}{3.86\times10^{26}\text{ W}} = 2.4\times10^{11}\text{ s} = 7600\text{ y}.$$

EVALUATE: If the mass of the sun were all proton fuel, it would contain enough fuel to last

$$(7600\text{ y})\left(\frac{4.3\times10^{11}\text{ J/g}}{4.7\times10^{4}\text{ J/g}}\right) = 7.0\times10^{10}\text{ y}.$$

43.46. **IDENTIFY:** The minimum energy to remove a proton from the nucleus is equal to the energy difference between the two states of the nucleus (before and after proton removal).

(a) SET UP: ${}^{12}_{6}\text{C} = {}^{1}_{1}\text{H} + {}^{11}_{5}\text{B}$. $\Delta m = m({}^{1}_{1}\text{H}) + m({}^{11}_{5}\text{B}) - m({}^{12}_{6}\text{C})$. The electron masses cancel when neutral atom masses are used.

EXECUTE: $\Delta m = 1.007825\text{ u} + 11.009305\text{ u} - 12.000000\text{ u} = 0.01713\text{ u}$. The energy equivalent of this mass increase is $(0.01713\text{ u})(931.5\text{ MeV/u}) = 16.0\text{ MeV}$.

(b) SET UP and **EXECUTE:** We follow the same procedure as in part (a).

$\Delta M = 6M_\text{H} + 6M_\text{n} - {}^{12}_{6}M = 6(1.007825\text{ u}) + 6(1.008665\text{ u}) - 12.000000\text{ u} = 0.09894\text{ u}$.

$$E_\text{B} = (0.09894\text{ u})(931.5\text{ MeV/u}) = 92.16\text{ MeV}. \quad \frac{E_\text{B}}{A} = 7.68\text{ MeV/u}.$$

EVALUATE: The proton removal energy is about twice the binding energy per nucleon.

43.47. **IDENTIFY:** The minimum energy to remove a proton or a neutron from the nucleus is equal to the energy difference between the two states of the nucleus, before and after removal.

(a) SET UP: ${}^{17}_{8}\text{O} = {}^{1}_{0}\text{n} + {}^{16}_{8}\text{O}$. $\Delta m = m({}^{1}_{0}\text{n}) + m({}^{16}_{8}\text{O}) - m({}^{17}_{8}\text{O})$. The electron masses cancel when neutral atom masses are used.

EXECUTE: $\Delta m = 1.008665\text{ u} + 15.994915\text{ u} - 16.999132\text{ u} = 0.004448\text{ u}$. The energy equivalent of this mass increase is $(0.004448\text{ u})(931.5\text{ MeV/u}) = 4.14\text{ MeV}$.

(b) SET UP and **EXECUTE:** Following the same procedure as in part (a) gives

$\Delta M = 8M_\text{H} + 9M_\text{n} - {}^{17}_{8}M = 8(1.007825\text{ u}) + 9(1.008665\text{ u}) - 16.999132\text{ u} = 0.1415\text{ u}$.

$$E_\text{B} = (0.1415\text{ u})(931.5\text{ MeV/u}) = 131.8\text{ MeV}. \quad \frac{E_\text{B}}{A} = 7.75\text{ MeV/nucleon}.$$

EVALUATE: The neutron removal energy is about half the binding energy per nucleon.

43.48. **IDENTIFY:** This problem is about the heat released by radioactive decay.

SET UP and **EXECUTE:** **(a)** We want the number of U-238 atoms in 1 kg of mantle material. The mass of U-238 is about $238m_\text{P} = 3.975\times10^{-25}$ kg. If N is the number of U-238 atoms in 31 μg of material, then $(3.975\times10^{-25}\text{ kg})N = 31\times10^{-9}$ kg. So $N = 7.800\times10^{16}$ atoms of U-238 in 31 μg of U-238 which is also the number of U-238 atoms in 1 kg of mantle material.

(b) We want the decay constant. $\lambda = 0.693/T_{1/2}$. $T_{1/2} = 4.47\times10^{9}\text{ y} = 1.4\times10^{17}$ s. Using this gives $\lambda = 4.9\times10^{-18}\text{ s}^{-1}$.

(c) Multiplying our results as directed gives $\lambda N = 0.38$ decays/s·kg.

(d) Converting units gives

$$\left(0.38\frac{\text{decays/s}}{\text{kg}}\right)(52\ \text{MeV/decay})\left(1.60\times10^{-13}\ \text{J/MeV}\right)=3.2\times10^{-12}\ \text{W/kg}.$$

(e) We want the power. Using the result from part (d) and the information in the problem gives

$$P=\left(3.2\times10^{-12}\ \text{W/kg}\right)\left[\frac{2}{3}\left(6\times10^{24}\ \text{kg}\right)\right]=1.3\times10^{13}\ \text{W}\approx10\ \text{TW}.$$

(f) Use the power from part (e). $0.39E_{\text{tot}}$ = 13 TW, so E_{tot} = 30 TW.

(g) We want the total power. $P_{\text{tot}} - P_{\text{out}} = P_{\text{net}}$, so $P_{\text{tot}} = P_{\text{net}} + P_{\text{out}} = P_{\text{rad}} + P_{\text{rad}} = 2P_{\text{rad}}$ = 60 TW.

EVALUATE: The power from radioactivity comes largely from the mantle and the power from left over heat comes largely from the hot cores.

43.49. IDENTIFY: Use the decay scheme and half-life of ^{90}Sr to find out the product of its decay and the amount left after a given time.

SET UP: The particle emitted in β^- decay is an electron, ${}^{0}_{-1}\text{e}$. In a time of one half-life, the number of radioactive nuclei decreases by a factor of 2. $6.25\% = \dfrac{1}{16} = 2^{-4}$.

EXECUTE: **(a)** ${}^{90}_{38}\text{Sr} \to {}^{0}_{-1}\text{e} + {}^{90}_{39}\text{Y}$. The daughter nucleus is ${}^{90}_{39}\text{Y}$.

(b) 56 y is $2T_{1/2}$ so $N = N_0/2^2 = N_0/4$; 25% is left.

(c) $\dfrac{N}{N_0} = 2^{-n}$; $\dfrac{N}{N_0} = 6.25\% = \dfrac{1}{16} = 2^{-4}$, so $t = 4T_{1/2} = 112$ y.

EVALUATE: After half a century, $\frac{1}{4}$ of the ^{90}Sr would still be left!

43.50. IDENTIFY: Calculate the mass defect for the decay. Example 43.5 uses conservation of linear momentum to determine how the released energy is divided between the decay partners.

SET UP: 1 u is equivalent to 931.5 MeV.

EXECUTE: The α-particle will have $\dfrac{226}{230}$ of the released energy (see Example 43.5).

$$\frac{226}{230}(m_{\text{Th}} - m_{\text{Ra}} - m_{\text{He-4}}) = 5.032\times10^{-3}\ \text{u or 4.69 MeV}.$$

EVALUATE: Most of the released energy goes to the α particle, since its mass is much less than that of the daughter nucleus.

43.51. (a) IDENTIFY and **SET UP:** The heavier nucleus will decay into the lighter one.

EXECUTE: ${}^{25}_{13}\text{Al}$ will decay into ${}^{25}_{12}\text{Mg}$.

(b) IDENTIFY and **SET UP:** Determine the emitted particle by balancing A and Z in the decay reaction.

EXECUTE: This gives ${}^{25}_{13}\text{Al} \to {}^{25}_{12}\text{Mg} + {}^{0}_{+1}\text{e}$. The emitted particle must have charge $+e$ and its nucleon number must be zero. Therefore, it is a β^+ particle, a positron.

(c) IDENTIFY and **SET UP:** Calculate the energy defect ΔM for the reaction and find the energy equivalent of ΔM. Use the nuclear masses for ${}^{25}_{13}\text{Al}$ and ${}^{25}_{12}\text{Mg}$, to avoid confusion in including the correct number of electrons if neutral atom masses are used.

EXECUTE: The nuclear mass for ${}^{25}_{13}\text{Al}$ is

$M_{\text{nuc}}({}^{25}_{13}\text{Al}) = 24.990428\ \text{u} - 13(0.000548580\ \text{u}) = 24.983296\ \text{u}.$

The nuclear mass for ${}^{25}_{12}\text{Mg}$ is $M_{\text{nuc}}({}^{25}_{12}\text{Mg}) = 24.985837\ \text{u} - 12(0.000548580\ \text{u}) = 24.979254\ \text{u}.$

The mass defect for the reaction is

$\Delta M = M_{\text{nuc}}({}^{25}_{13}\text{Al}) - M_{\text{nuc}}({}^{25}_{12}\text{Mg}) - M({}^{0}_{+1}\text{e}) = 24.983296\ \text{u} - 24.979254\ \text{u} - 0.00054858\ \text{u} = 0.003493\ \text{u}.$

$$Q = (\Delta M)c^2 = 0.003493\ \text{u}(931.5\ \text{MeV/1 u}) = 3.254\ \text{MeV}.$$

EVALUATE: The mass decreases in the decay and energy is released. Note: ${}^{25}_{13}\text{Al}$ can also decay into ${}^{25}_{12}\text{Mg}$ by the electron capture: ${}^{25}_{13}\text{Al} + {}^{0}_{-1}\text{e} \to {}^{25}_{12}\text{Mg}$. The ${}^{0}_{-1}$ electron in the reaction is an orbital electron in the neutral ${}^{25}_{13}\text{Al}$ atom. The mass defect can be calculated using the nuclear masses:

$\Delta M = M_{\text{nuc}}({}^{25}_{13}\text{Al}) + M({}^{0}_{-1}\text{e}) - M_{\text{nuc}}({}^{25}_{12}\text{Mg}) = 24.983296\ \text{u} + 0.00054858\ \text{u} - 24.979254\ \text{u} = 0.004591\ \text{u}.$

$Q = (\Delta M)c^2 = (0.004591\ \text{u})(931.5\ \text{MeV/1 u}) = 4.277\ \text{MeV}.$ The mass decreases in the decay and energy is released.

43.52. **IDENTIFY:** This problem is about nuclear fusion reactions in the sun.

SET UP and **EXECUTE:** Follow the directions for each part. **(a)** We want the number N of fusion reactions per second. One alpha particle is produced for each cycle of the proton-proton chain, and each cycle produces 27.73 MeV. Using the given power, we have

$$(26.73\ \text{MeV/cycle})\left(1.60\times10^{-13}\ \text{J/MeV}\right)N = 3.8\times10^{26}\ \text{W}$$

$N = 8.885 \times 10^{37}$ reactions/s, which rounds to $N = 8.9 \times 10^{37}$ reactions/s.

(b) Each cycle produces 2 neutrinos, so $N_\nu = 1.8 \times 10^{38}$ neutrinos/s.

(c) We want the number of neutrinos that hit the earth per second. Assume that the neutrinos travel outward uniformly in all directions. Call N_E the number that hit the earth, r the earth-sun distance, and r_E the radius of the earth. This gives

$$\frac{N_\text{E}}{N_\nu} = \frac{\pi r_\text{E}^2}{4\pi r^2}$$

$$N_\text{E} = N_\nu\left(\frac{r_\text{E}}{2r}\right)^2 = \left(1.8\times10^{38}\ \text{neutrinos/s}\right)\left[\frac{6370\ \text{kg}}{2(150\times10^6\ \text{km})}\right] = 8.1\times10^{28}\ \text{neutrinos/s}.$$

(d) The area presented to the sun is the earth's cross-sectional area, so

$$A = \pi r_\text{E}^2 = \pi(6370\ \text{km})^2 = 1.3\times10^{18}\ \text{cm}^2.$$

(e) Dividing as specified gives

$$\frac{8.1\times10^{28}\ \text{neutrinos/s}}{1.3\times10^{18}\ \text{cm}^2} = 6\times10^{10}\ \text{neutrinos/s}\cdot\text{cm}^2.$$

EVALUATE: The number in part (e) is 60 billion neutrinos/s through each square centimeter! Most of them travel straight through without hitting anything.

43.53. **IDENTIFY** and **SET UP:** The amount of kinetic energy released is the energy equivalent of the mass change in the decay. $m_\text{e} = 0.0005486\ \text{u}$ and the atomic mass of ${}^{14}_{7}\text{N}$ is 14.003074 u. The energy equivalent of 1 u is 931.5 MeV. ${}^{14}\text{C}$ has a half-life of $T_{1/2} = 5730\ \text{y} = 1.81\times10^{11}\ \text{s}$. The RBE for an electron is 1.0.

EXECUTE: **(a)** ${}^{14}_{6}\text{C} \to \text{e}^- + {}^{14}_{7}\text{N} + \overline{\upsilon}_\text{e}.$

(b) The mass decrease is $\Delta M = m({}^{14}_{6}\text{C}) - [m_\text{e} + m({}^{14}_{7}\text{N})].$ Use nuclear masses, to avoid difficulty in accounting for atomic electrons. The nuclear mass of ${}^{14}_{6}\text{C}$ is $14.003242\ \text{u} - 6m_\text{e} = 13.999950\ \text{u}.$ The nuclear mass of ${}^{14}_{7}\text{N}$ is $14.003074\ \text{u} - 7m_\text{e} = 13.999234\ \text{u}.$

$\Delta M = 13.999950\ \text{u} - 13.999234\ \text{u} - 0.000549\ \text{u} = 1.67\times10^{-4}\ \text{u}.$ The energy equivalent of ΔM is 0.156 MeV.

(c) The mass of carbon is $(0.18)(75\ \text{kg}) = 13.5\ \text{kg}.$ From Example 43.9, the activity due to 1 g of carbon in a living organism is 0.255 Bq. The number of decay/s due to 13.5 kg of carbon is

$$(13.5\times10^3\ \text{g})(0.255\ \text{Bq/g}) = 3.4\times10^3\ \text{decays/s}.$$

(d) Each decay releases 0.156 MeV so 3.4×10^3 decays/s releases $530\ \text{MeV/s} = 8.5\times10^{-11}\ \text{J/s}.$

(e) The total energy absorbed in 1 year is $(8.5\times10^{-11}\ \text{J/s})(3.156\times10^{7}\ \text{s}) = 2.7\times10^{-3}\ \text{J}$. The absorbed dose is $\dfrac{2.7\times10^{-3}\ \text{J}}{75\ \text{kg}} = 3.6\times10^{-5}\ \text{J/kg} = 36\ \mu\text{Gy} = 3.6\ \text{mrad}$. With $\text{RBE} = 1.0$, the equivalent dose is $36\ \mu\text{Sv} = 3.6\ \text{mrem}$.

EVALUATE: Section 43.5 says that background radiation exposure is about 1.0 mSv per year. The radiation dose calculated in this problem is much less than this.

43.54. IDENTIFY and **SET UP:** $m_\pi = 264m_\text{e} = 2.40\times10^{-28}$ kg. The total energy of the two photons equals the rest mass energy $m_\pi c^2$ of the pion.

EXECUTE: **(a)** $E_\text{ph} = \frac{1}{2}m_\pi c^2 = \frac{1}{2}(2.40\times10^{-28}\ \text{kg})(3.00\times10^{8}\ \text{m/s})^2 = 1.08\times10^{-11}\ \text{J} = 67.5\ \text{MeV}$.

$$E_\text{ph} = \frac{hc}{\lambda},\ \text{so}\ \lambda = \frac{hc}{E_\text{ph}} = \frac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{67.5\times10^{6}\ \text{eV}} = 1.84\times10^{-14}\ \text{m} = 18.4\ \text{fm}.$$

These are gamma ray photons, so they have $\text{RBE} = 1.0$.

(b) Each pion delivers $2(1.08\times10^{-11}\ \text{J}) = 2.16\times10^{-11}\ \text{J}$.

The absorbed dose is $200\ \text{rad} = 2.00\ \text{Gy} = 2.00\ \text{J/kg}$.

The energy deposited is $(25\times10^{-3}\ \text{kg})(2.00\ \text{J/kg}) = 0.050\ \text{J}$.

The number of π^0 mesons needed is $\dfrac{0.050\ \text{J}}{2.16\times10^{-11}\ \text{J/meson}} = 2.3\times10^{9}$ mesons.

EVALUATE: Note that charge is conserved in the decay since the pion is neutral. If the pion is initially at rest the photons must have equal momenta in opposite directions so the two photons have the same λ and are emitted in opposite directions. The photons also have equal energies since they have the same momentum and $E = pc$.

43.55. IDENTIFY and **SET UP:** The mass defect is E_B/c^2.

EXECUTE: $m_{^{11}_{6}\text{C}} - m_{^{11}_{5}\text{B}} - 2m_\text{e} = 1.03\times10^{-3}$ u. Decay is energetically possible.

EVALUATE: The energy released in the decay is $(1.03\times10^{-3}\ \text{u})(931.5\ \text{MeV/u}) = 0.959\ \text{MeV}$.

43.56. IDENTIFY: Assume the activity is constant during the year and use the given value of the activity to find the number of decays that occur in one year. Absorbed dose is the energy absorbed per mass of tissue. Equivalent dose is RBE times absorbed dose.

SET UP: For α particles, $\text{RBE} = 20$ (from Table 43.3).

EXECUTE: $(0.52\times10^{-6}\ \text{Ci})(3.7\times10^{10}\ \text{Bq/Ci})(3.156\times10^{7}\ \text{s}) = 6.07\times10^{11}\ \alpha$ particles. The absorbed dose is $\dfrac{(6.07\times10^{11})(4.0\times10^{6}\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}{(0.50\ \text{kg})} = 0.78\ \text{Gy} = 78\ \text{rad}$. The equivalent dose is $(20)(78\ \text{rad}) = 1600\ \text{rem}$.

EVALUATE: The equivalent dose is 16 Sv. This is large enough for significant damage to the person.

43.57. IDENTIFY and **SET UP:** One-half of the sample decays in a time of $T_{1/2}$.

EXECUTE: **(a)** $\dfrac{10\times10^{9}\ \text{y}}{200{,}000\ \text{y}} = 5.0\times10^{4}$.

(b) $\left(\frac{1}{2}\right)^{5.0\times10^4}$. This exponent is too large for most hand-held calculators. But $\left(\frac{1}{2}\right) = 10^{-0.301}$, so $\left(\frac{1}{2}\right)^{5.0\times10^4} = (10^{-0.301})^{5.0\times10^4} = 10^{-15{,}000}$.

EVALUATE: For $N = 1$ after 16 billion years, $N_0 = 10^{15{,}000}$. The mass of this many ^{99}Tc nuclei would be $(99)(1.66\times10^{-27}\ \text{kg})(10^{15{,}000}) = 10^{14{,}750}$ kg, which is immense, far greater than the mass of any star.

43.58. **IDENTIFY:** One rad of absorbed dose is 0.01 J/kg. The equivalent dose in rem is the absorbed dose in rad times the RBE. For part (c) apply $|dN/dt| = \lambda N$ with $\lambda = \dfrac{\ln 2}{T_{1/2}}$.

SET UP: For α particles, RBE = 20 (Table 43.3).

EXECUTE: **(a)** $(7.75\times10^{12})(4.77\times10^{6}\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})/(70.0\ \text{kg}) = 0.0846\ \text{Gy} = 8.46\ \text{rad}$.

(b) $(20)(8.46\ \text{rad}) = 169\ \text{rem}$.

(c) $\left|\dfrac{dN}{dt}\right| = \dfrac{m}{Am_\text{p}}\dfrac{\ln(2)}{T_{1/2}} = 1.17\times10^{9}\ \text{Bq} = 31.6\ \text{mCi}$.

(d) $t = \dfrac{7.75\times10^{12}}{1.17\times10^{9}\ \text{Bq}} = 6.62\times10^{3}\ \text{s}$, which is about 1.8 hours.

EVALUATE: The time in part (d) is so small in comparison with the half-life that the decrease in activity of the source may be neglected.

43.59. **IDENTIFY:** Use $N = N_0 e^{-\lambda t}$ to relate the initial number of radioactive nuclei, N_0, to the number, N, left after time t.

SET UP: We have to be careful; after ^{87}Rb has undergone radioactive decay it is no longer a rubidium atom. Let N_{85} be the number of ^{85}Rb atoms; this number doesn't change. Let N_0 be the number of ^{87}Rb atoms on earth when the solar system was formed. Let N be the present number of ^{87}Rb atoms.

EXECUTE: The present measurements say that $0.2783 = N/(N + N_{85})$.

$(N + N_{85})(0.2783) = N$, so $N = 0.3856N_{85}$. The percentage we are asked to calculate is $N_0/(N_0 + N_{85})$.

N and N_0 are related by $N = N_0 e^{-\lambda t}$ so $N_0 = e^{+\lambda t}N$.

Thus $\dfrac{N_0}{N_0 + N_{85}} = \dfrac{Ne^{\lambda t}}{Ne^{\lambda t} + N_{85}} = \dfrac{(0.3855e^{\lambda t})N_{85}}{(0.3856e^{\lambda t})N_{85} + N_{85}} = \dfrac{0.3856e^{\lambda t}}{0.3856e^{\lambda t} + 1}$.

$$t = 4.6\times10^{9}\ \text{y};\ \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{4.75\times10^{10}\ \text{y}} = 1.459\times10^{-11}\ \text{y}^{-1}.$$

$$e^{\lambda t} = e^{(1.459\times10^{-11}\ \text{y}^{-1})(4.6\times10^{9}\ \text{y})} = e^{0.06711} = 1.0694.$$

Thus $\dfrac{N_0}{N_0 + N_{85}} = \dfrac{(0.3856)(1.0694)}{(0.3856)(1.0694) + 1} = 29.2\%$.

EVALUATE: The half-life for ^{87}Rb is a factor of 10 larger than the age of the solar system, so only a small fraction of the ^{87}Rb nuclei initially present have decayed; the percentage of rubidium atoms that are radioactive is only a bit less now than it was when the solar system was formed.

43.60. **IDENTIFY:** Apply $N = N_0 e^{-\lambda t}$, with $\lambda = \dfrac{\ln 2}{T_{1/2}}$.

SET UP: Let 1 refer to $^{15}_{8}\text{O}$ and 2 refer to $^{19}_{8}\text{O}$. $\dfrac{N_1}{N_2} = \dfrac{e^{-\lambda_1 t}}{e^{-\lambda_2 t}}$, since N_0 is the same for the two isotopes.

$$e^{-\lambda t} = e^{-(\ln 2/T_{1/2})t} = (e^{-\ln 2})^{t/T_{1/2}} = (\tfrac{1}{2})^{t/T_{1/2}}.\ \frac{N_1}{N_2} = \left(\tfrac{1}{2}\right)^{(t/(T_{1/2})_1)/(t/(T_{1/2})_2)} = 2^{t\left(\frac{1}{(T_{1/2})_2} - \frac{1}{(T_{1/2})_1}\right)}.$$

EXECUTE: **(a)** After 3.0 min = 180 s, the ratio of the number of nuclei is

$$N_1/N_2 = \frac{2^{-180/122.2}}{2^{-180/26.9}} = 2^{(180)\left(\frac{1}{26.9} - \frac{1}{122.2}\right)} = 2^{(180)(0.02899)} = 37.2.$$

(b) After 12.0 min = 720 s, the ratio is $N_1/N_2 = 2^{(720)(002899)} = 1.92\times10^{6}$.

EVALUATE: The $^{19}_{8}\text{O}$ nuclei decay at a greater rate, so the ratio $N(^{15}_{8}\text{O})/N(^{19}_{8}\text{O})$ increases with time.

43.61. **IDENTIFY** and **SET UP:** Find the energy emitted and the energy absorbed each second. Convert the absorbed energy to absorbed dose and to equivalent dose.

EXECUTE: **(a)** First find the number of decays each second:

$2.6\times10^{-4}\ \text{Ci}\left(\dfrac{3.70\times10^{10}\ \text{decays/s}}{1\ \text{Ci}}\right)=9.6\times10^{6}$ decays/s. The average energy per decay is 1.25 MeV, and one-half of this energy is deposited in the tumor. The energy delivered to the tumor per second then is $\frac{1}{2}(9.6\times10^{6}\ \text{decays/s})(1.25\times10^{6}\ \text{eV/decay})(1.602\times10^{-19}\ \text{J/eV})=9.6\times10^{-7}\ \text{J/s}.$

(b) The absorbed dose is the energy absorbed divided by the mass of the tissue:

$$\frac{9.6\times10^{-7}\ \text{J/s}}{0.200\ \text{kg}}=(4.8\times10^{-6}\ \text{J/kg}\cdot\text{s})(1\ \text{rad/(0.01 J/kg)})=4.8\times10^{-4}\ \text{rad/s}.$$

(c) equivalent dose (REM) = RBE × absorbed dose (rad). In one second the equivalent dose is

$$(0.70)(4.8\times10^{-4}\ \text{rad})=3.4\times10^{-4}\ \text{rem}.$$

(d) $(200\ \text{rem})/(3.4\times10^{-4}\ \text{rem/s})=(5.9\times10^{5}\ \text{s})(1\ \text{h}/3600\ \text{s})=164\ \text{h}=6.9\ \text{days}.$

EVALUATE: The activity of the source is small so that absorbed energy per second is small and it takes several days for an equivalent dose of 200 rem to be absorbed by the tumor. A 200-rem dose equals 2.00 Sv and this is large enough to damage the tissue of the tumor.

43.62. **IDENTIFY:** This problem deals with nuclear fusion and the p-p I chain in the sun.

SET UP: The first reaction is $2{}^1_1\text{H}\rightarrow{}^2_1\text{H}+\beta^{+}+\nu_{\text{e}}$. Follow the directions for each part.

EXECUTE: **(a)** We want the total energy E_1 of the deuteron and neutrino. First find the energy Q released in the fusion reaction, treating the neutrino as essentially massless. From Table 43.2, the deuteron mass is m_{d} = 2.014102 u. $Q = (2m_{\text{p}} - m_{\text{d}})c^2$ = [2(1.007276 u) – 2.014102 u)(931.5 MeV/u) = 0.419 Mev. This is the energy shared between the deuteron, the positron, and the neutrino. But photons from electron-positron annihilation each had energy equal to the rest energy of the electron, so the positron must have been at rest after the first reaction. Therefore all the energy E_1 = 0.419 MeV is shared by the deuteron and neutrino.

(b) The neutrino is relativistic but the deuteron is not. If the neutrino is massless, then $E_{\text{v}} = pc$. Thus momentum conservation gives $p_{\text{d}} = p_{\text{v}}$, so $m_{\text{d}}v_{\text{d}} = E_{\text{v}}/c$. Energy conservation gives $E_{\text{d}} + E_{\text{v}} = E_1$, so $\frac{1}{2}m_{\text{d}}v_{\text{d}}^2+E_\nu=E_1.$

(c) We want E_{v}/E_1. Solve the momentum and energy equations in part (b) for E_{v}. This leads to the following quadratic equation:

$$E_\nu^2+2m_{\text{d}}c^2E_\nu-2m_{\text{d}}c^2E_1=0.$$

Using the positive root with the quadratic formula and doing some algebra gives

$$E_\nu=m_{\text{d}}c^2\left(-1+\sqrt{1+\frac{2E_1}{m_dc^2}}\right).$$

Using m_{d} = 2.014102 u and E_1 = 0.419 MeV, we E_{v}/E_1 = 0.41895/0.419 = 0.99989 = 99.989%.

(d) As we just saw, the neutrino essentially carries off all the energy. The first reaction occurs twice for each p-p I chain, so they carry off $2E_1$ = 2(0.419 MeV) = 0.838 MeV.

(e) In Section 43.8, it was calculated that the p-p chain releases 26.73 MeV of energy. So the percent that the neutrinos carry away is (0.838 MeV)/(26.73 MeV) = 3.1%.

EVALUATE: The 3.1% of the energy is large enough to be measurable. If neutrinos were not taken into account, the energy calculations for the sun would be off by about 3%.

43.63. **IDENTIFY:** This problem investigates the p-p II fusion chain in the sun.

SET UP: Follow the directions in each part and use Table 43.2 for nuclide masses. The third step of the chain is

$$^3_2\text{He} + {}^3_2\text{He} \to {}^7_4\text{Be} + \gamma.$$

EXECUTE: **(a)** We want the energy E_γ of the photon. Realize that $E_\gamma = Q$. From Table 43.2 we have:
Mass of He-3 = 3.016029 u
Mass of He-4 = 4.002603 u
Mass of Be-7 = 7.016930 u
The nuclide masses include the electrons, but since we'll be subtracting their effect subtracts out.
Therefore $E_\gamma = Q$ = (3.016029 u + 4.002603 u – 7.016930 u)(931.5 MeV/u) = 1.59 MeV.
(b) We want the energy E_ν of the neutrino in the following electron-capture of Be-7.

$$^7_4\text{Be} + \text{e}^- \to {}^7_3\text{Li} + \nu_\text{e}.$$

Using the given mass of Li-7 and the Be-7 mass from Table 43.2, we have

E_ν = (7.016930 u – 7.016003 u)(931.5 MeV/u) = 0.864 MeV.

(c) We want the energy Q that is released. Using the given reaction, we have

Q = [1.007825 u + 7.016003 u – 2(4.002603 u)](931.5 MeV/u) = 17.35 MeV.

(d) We want the speed v_α of the alpha particles. The alpha particles have equal speeds, so for each one K_α = (17.35 MeV)/2 = 8.675 MeV. Use the relativistic kinetic energy $K = mc^2(\gamma - 1)$. Solving for γ gives

$$\gamma = 1 + K_\alpha/m_\alpha c^2 = 1 + \frac{8.675 \text{ MeV}}{(4.002603 \text{ u})(931.5 \text{ MeV/u})} = 1.0023267.$$

Using this result gives

$$v = c\sqrt{1 - 1/\gamma^2} = 0.0681c.$$

(e) We want the total energy. The overall p-p II chain can be summarized as

$$4\,^1_1\text{H} + {}^4_2\text{He} \to 2\,^4_2\text{He} + 2\nu_\text{e} + \gamma.$$

Using masses from Table 43.2 gives Q = [4(1.007825 u) – 4.002603 u](931.5 MeV/u) = 26.7 MeV.
(f) The p-p I releases 26.73 MeV, so E_II/E_I = (26.7 MeV)/(26.73 MeV) = 99.9%. They are about the same.
(g) We want E_ν/E_tot. Using the results from (b) and (f) gives E_ν/E_I = (0.864 MeV)/(26.7 MeV) = 3.2%.

EVALUATE: The amount of energy per reaction is the same for p-p I and p-p II, but p-p II occurs less frequently so it contributes less to the sun's total energy production.

43.64. **IDENTIFY:** We are looking at the p-p III chain fusion in our sun.

SET UP: Follow the directions in each part and use Table 43.2 for nuclide masses.

EXECUTE: **(a)** We want the energy Q released. Looking at the reaction given in the problem, we see that Q = [4(1.007825 u) – 4.002603 u](931.5 MeV/u) = 26.73 MeV.
(b) We want the neutrino energy. The mass of B-8 is 8.024607 u and the mass of Be-8 + β^+ is 8.0053051 u. So E_ν = (8.024607 u – 8.0053051 u)(931.5 MeV/u) = 18.0 MeV.
(c) We want E_ν/E_tot. Using the results from (a) and (b) gives E_ν/E_tot = (18.0 MeV)/(26.73 MeV) = 67.3%.

EVALUATE: Comparing this result to those of problems 43.62 and 43.63, we see that in the p-p III chain the neutrino carries away a might higher percent of the energy than in the other two chains.

43.65. **IDENTIFY** and **SET UP:** The number of radioactive nuclei left after time t is given by $N = N_0 e^{-\lambda t}$. The problem says $N/N_0 = 0.29$; solve for t.

EXECUTE: $0.29 = e^{-\lambda t}$ so $\ln(0.29) = -\lambda t$ and $t = -\ln(0.29)/\lambda$. Example 43.9 gives

$$\lambda = 1.209 \times 10^{-4}\ \text{y}^{-1} \text{ for } {}^{14}\text{C. Thus } t = \frac{-\ln(0.29)}{1.209 \times 10^{-4}\ \text{y}} = 1.0 \times 10^4\ \text{y}.$$

EVALUATE: The half-life of ^{14}C is 5730 y, so our calculated t is about 1.75 half-lives, so the fraction remaining is around $\left(\frac{1}{2}\right)^{1.75} = 0.30.$

43.66. **IDENTIFY:** In terms of the number ΔN of cesium atoms that decay in one week and the mass

$$m = 1.0 \text{ kg}, \text{ the equivalent dose is } 3.5 \text{ Sv} = \frac{\Delta N}{m}((\text{RBE})_\gamma \text{E}_\gamma + (\text{RBE})_\text{e}\text{E}_\text{e}).$$

SET UP: $1 \text{ day} = 8.64\times10^4 \text{ s}$. $1 \text{ year} = 3.156\times10^7 \text{ s}$.

EXECUTE: $3.5 \text{ Sv} = \frac{\Delta N}{m}((1)(0.66 \text{ MeV}) + (1.5)(0.51 \text{ MeV})) = \frac{\Delta N}{m}(2.283\times10^{-13} \text{ J})$, so

$$\Delta N = \frac{(1.0 \text{ kg})(3.5 \text{ Sv})}{(2.283\times10^{-13} \text{ J})} = 1.535\times10^{13}. \ \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{(30.07 \text{ y})(3.156\times10^7 \text{ sec/y})} = 7.30\times10^{-10} \text{ sec}^{-1}.$$

$$\Delta N = |dN/dt|t = \lambda N t, \text{ so } N = \frac{\Delta N}{\lambda t} = \frac{1.535\times10^{13}}{(7.30\times10^{-10} \text{ s}^{-1})(7 \text{ days})(8.64\times10^4 \text{ s/day})} = 3.48\times10^{16}.$$

EVALUATE: We have assumed that $|dN/dt|$ is constant during a time of one week. That is a very good approximation, since the half-life is much greater than one week.

43.67. **(a) IDENTIFY and SET UP:** Use $R = R_0 A^{1/3}$ to calculate the radius R of a ^2_1H nucleus. Calculate the Coulomb potential energy $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$ of the two nuclei when they just touch.

EXECUTE: The radius of ^2_1H is $R = (1.2\times10^{-15} \text{ m})(2)^{1/3} = 1.51\times10^{-15} \text{ m}$. The barrier energy is the Coulomb potential energy of two ^2_1H nuclei with their centers separated by twice this distance:

$$U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.602\times10^{-19} \text{ C})^2}{2(1.51\times10^{-15} \text{ m})} = 7.64\times10^{-14} \text{ J} = 0.48 \text{ MeV}.$$

(b) IDENTIFY and SET UP: Find the energy equivalent of the mass decrease.

EXECUTE: $^2_1\text{H} + ^2_1\text{H} \rightarrow ^3_2\text{He} + ^1_0\text{n}$

If we use neutral atom masses there are two electrons on each side of the reaction equation, so their masses cancel. The neutral atom masses are given in Table 43.2.

$$^2_1\text{H} + ^2_1\text{H} \text{ has mass } 2(2.014102 \text{ u}) = 4.028204 \text{ u}$$

$$^3_2\text{He} + ^1_0\text{n} \text{ has mass } 3.016029 \text{ u} + 1.008665 \text{ u} = 4.024694 \text{ u}$$

The mass decrease is $4.028204 \text{ u} - 4.024694 \text{ u} = 3.510\times10^{-3} \text{ u}$. This corresponds to a liberated energy of $(3.510\times10^{-3} \text{ u})(931.5 \text{ MeV/u}) = 3.270 \text{ MeV}$, or $(3.270\times10^6 \text{ eV})(1.602\times10^{-19} \text{ J/eV}) = 5.239\times10^{-13} \text{ J}$.

(c) IDENTIFY and SET UP: We know the energy released when two ^2_1H nuclei fuse. Find the number of reactions obtained with one mole of ^2_1H.

EXECUTE: Each reaction takes two ^2_1H nuclei. Each mole of D_2 has 6.022×10^{23} molecules, so 6.022×10^{23} pairs of atoms. The energy liberated when one mole of deuterium undergoes fusion is $(6.022\times10^{23})(5.239\times10^{-13} \text{ J}) = 3.155\times10^{11} \text{ J/mol}$.

EVALUATE: The energy liberated per mole is more than a million times larger than from chemical combustion of one mole of hydrogen gas.

43.68. **IDENTIFY:** The energy of the photon is equal to the sum of the kinetic energies of the proton and the neutron plus the binding energy of the deuteron.

SET UP: $\dfrac{hc}{\lambda} = K_p + K_n + E_B$. Since the proton and neutron have equal kinetic energy, this equation becomes $hc/\lambda = 2K_p + E_B = 2(\frac{1}{2} m_p v^2) + E_B$. Solving for v^2 gives $v^2 = \dfrac{hc}{m_p}\cdot\dfrac{1}{\lambda} - \dfrac{E_B}{m_p}$.

EXECUTE: **(a)** Figure 43.68 shows the graph of v^2 versus $1/\lambda$ for the data given in the problem. As shown above, the equation for v^2 versus $1/\lambda$ is $v^2 = \dfrac{hc}{m_p}\cdot\dfrac{1}{\lambda} - \dfrac{E_B}{m_p}$. The graph of v^2 versus $1/\lambda$ should be a straight line with slope equal to hc/m_p and y-intercept equal to $-E_B/m_p$. The slope of the best-fit straight line for our graph is 119.9 m^3/s^2 and the y-intercept is -2.156×10^{14} m^2/s^2.

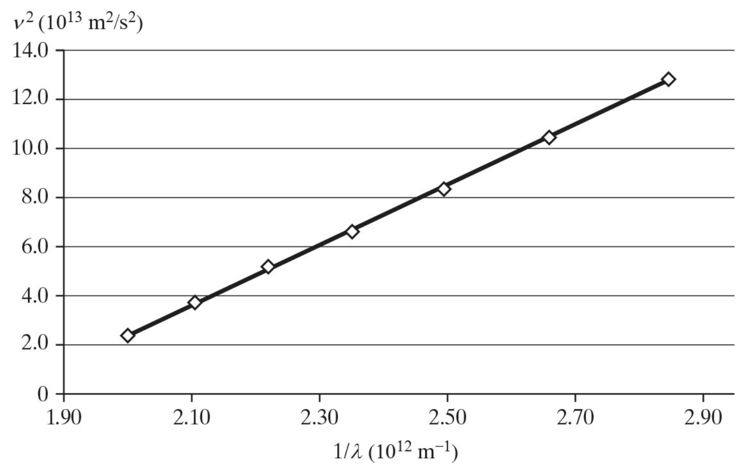


Figure 43.68

(b) Using the values for the slope and y-intercept for our graph, we have

$$hc/m_p = \text{slope} \rightarrow m_p = hc/(\text{slope}).$$

$$m_p = (6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m/s})/(119.9\ \text{m}^3/\text{s}^2) = 1.66 \times 10^{-27}\ \text{kg}.$$

The y-intercept gives us the binding energy: $-E_B/m_p = y$-intercept, so $E_B = -m_p(y\text{-intercept})$.

$$E_B = -(1.66 \times 10^{-27}\ \text{kg})(-2.156 \times 10^{14}\ \text{m}^2/\text{s}^2) = 3.58 \times 10^{-13}\ \text{J} = 2.23 \times 10^6\ \text{eV} = 2.23\ \text{MeV}.$$

EVALUATE: The binding energy of a deuteron is $E_B = (m_{H\text{-}1} + m_n - m_D)c^2$, so $E_B = (1.007825\ \text{u} + 1.008665\ \text{u} - 2.014102\ \text{u})(931.5\ \text{MeV/u}) = 2.22$ MeV, so we are *very* close. Our results for m_p are *very* close to the accepted value for the proton mass of 1.67×10^{-27} kg.

43.69. **IDENTIFY:** Apply $\left|\dfrac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t}$, with $\lambda = \dfrac{\ln 2}{T_{1/2}}$.

SET UP: $\ln\left|dN/dt\right| = \ln \lambda N_0 - \lambda t$.

EXECUTE: **(a)** A least-squares fit to log of the activity *vs.* time gives a slope of magnitude

$$\lambda = 0.5995\ \text{h}^{-1},\ \text{for a half-life of}\ \frac{\ln 2}{\lambda} = 1.16\ \text{h}.$$

(b) The initial activity is $N_0\lambda$, and this gives $N_0 = \dfrac{(2.00\times10^4\ \text{Bq})}{(0.5995\ \text{hr}^{-1})(1\ \text{hr}/3600\ \text{s})} = 1.20\times10^8$.

(c) $N = N_0 e^{-\lambda t} = 1.81\times10^6$.

EVALUATE: The activity decreases by about $\frac{1}{2}$ in the first hour, so the half-life is about 1 hour.

43.70. **IDENTIFY:** We cannot tell much from the raw data, but we know for radioactive decay of a single nuclide the activity rate A decreases as $A = A_0 e^{-\lambda t} = |dN/dt|$.

SET UP: $\left|\dfrac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t}$ for a single nuclide. Since $A = A_0 e^{-\lambda t}$, it follows that $\ln A = \ln A_0 - \lambda t$.

Therefore if a single nuclide is present, a graph of $\ln A$ versus t should be a straight line with slope equal to $-\lambda$. We start with a graph of $\ln A$ versus t for the data given in the problem. Figure 43.70a shows this graph.

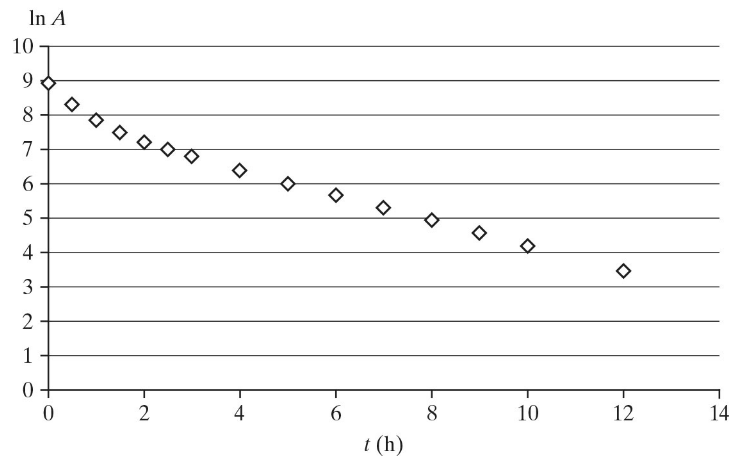


Figure 43.70a

EXECUTE: **(a)** The graph in Figure 43.70a is not a straight line, which suggests that the sample contains more than one nuclide. However after about 4.0 h, the graph does become a straight line. This suggests that a short-lived nuclide has decreased in activity, leaving only a single long-lived nuclide. To investigate the long-lived nuclide, we assume that nearly all the decays from 5.0 h on are due to this nuclide. So we make a graph of log A versus t for the decays from 5.0 h to 12.0 h. The result is shown in Figure 43.70b. This graph is a straight line, suggesting that our hypothesis is correct.

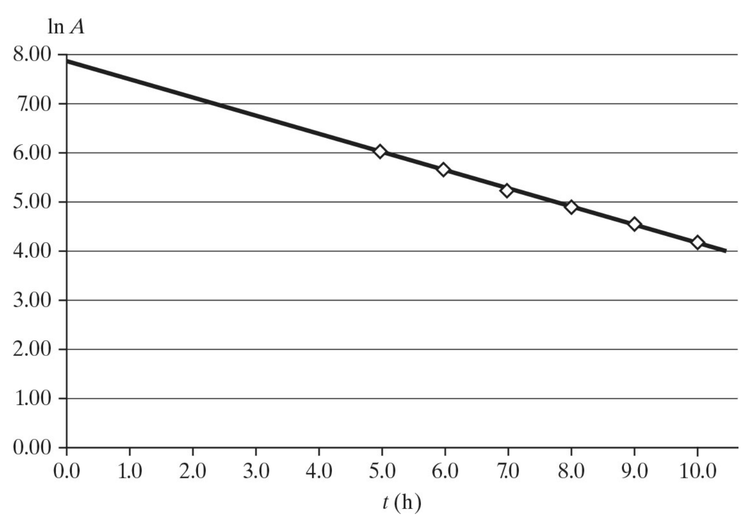


Figure 43.70b

To investigate the short-lived nuclide (or nuclides), we use our graph in Figure 43.70b to determine the activity of the long-lived nuclide during the first 4.0 h. We then subtract these numbers from the total decay rate to determine the decay rate due to the short-lived nuclide. The table shows these results.

Time (h)	Total rate (dec/s)	A (dec/s) for long-lived nuclide	ln A for long-lived nuclide	A (dec/s) for short-lived nuclide	ln A for short-lived nuclide
0	7500	2440	7.8	5060	8.53
0.5	4120	2320	7.75	1800	7.50
1.0	2570	1720	7.45	850	6.75
1.5	1790	1410	7.25	380	5.94
2.0	1350	1210	7.1	140	4.94
2.5	1070	992	6.9	78	4.36
3.0	872	6.7	812	60	4.09
4.0	596	572	6.35	24	3.18

Now we use the data in our table to graph ln A versus t for the short-lived nuclide. This graph is shown in Figure 43.70c. The last two points on the graph are unreliable because the decay rate is very small. For the rest of the points, the graph is a straight line. Therefore, since our two graphs of ln A versus t yield straight lines, it appears that our sample contains a minimum of two different nuclides, one with a short half-life and one with a long half-life.

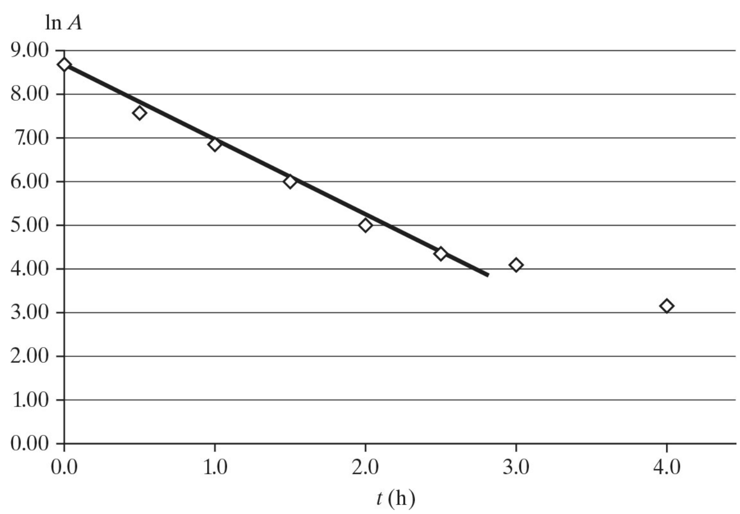


Figure 43.70c

(b) Long-lived nuclide: The slope of the graph in Figure 43.70b is –0.3636 h^{-1}, so λ_{long} = –slope = 0.3636 h^{-1}. Using $\lambda = (\ln 2)/T_{1/2}$, the half-life is $T_{1/2} = (\ln 2)/\lambda_{\text{long}} = (\ln 2)/(0.3636\ \text{h}^{-1}) = 1.9$ h.

Short-lived nuclide: The slope of the graph in Figure 43.70c is –1.78 h^{-1}, so λ_{short} = 1.78 h^{-1}. The half-life is $T_{1/2} = (\ln 2)/(1.78\ \text{h}^{-1}) = 0.39$ h.

(c) $\left|\dfrac{dN}{dt}\right| = \lambda N$, so $N = \dfrac{|dN/dt|}{\lambda}$.

Long-lived nuclide: $N_0 = (2440\ \text{Bq})/[(0.3636\ \text{h}^{-1})(1\ \text{h}/3600\ \text{s})] = 2.4 \times 10^7$ nuclei.

Short-lived nuclide: $N_0 = (5060\ \text{Bq})/[(1.78\ \text{h}^{-1})(1\ \text{h}/3600\ \text{s})] = 1.0 \times 10^7$ nuclei.

(d) Use $N = N_0 e^{-\lambda t}$ for each nuclide.

Long-lived nuclide: $N = N_0 e^{-\lambda_{\text{long}} t} = (2.4\times 10^7)\, e^{-(0.3636\ \text{h}^{-1})(5.0\ \text{h})} = 3.9\times 10^6$ nuclei.

Short-lived nuclide: $N = N_0 e^{-\lambda_{\text{short}} t} = (1.0\times 10^7)\, e^{-(1.78\ \text{h}^{-1})(5.0\ \text{h})} = 1.4\times 10^3$ nuclei.

EVALUATE: After 5.0 h, the number of shorter-lived nuclei is much less than the number of longer-lived nuclei. The ratio of the number of short-lived to the number of long-lived nuclei is $\dfrac{N_{\text{short}}}{N_{\text{long}}} = \dfrac{1.0\times 10^7}{2.4\times 10^7}\dfrac{e^{-\lambda_{\text{short}} t}}{e^{-\lambda_{\text{long}} t}} = 0.42 e^{-(\lambda_{\text{short}} - \lambda_{\text{long}})t}$. Since $\lambda_{\text{short}} > \lambda_{\text{long}}$, this ratio keeps decreasing with time.

43.71. IDENTIFY: Apply $A = A_0 e^{-\lambda t}$, where A is the activity and $\lambda = (\ln 2)/T_{1/2}$. This equation can be written as $A = A_0 2^{-(t/T_{1/2})}$. The activity of the engine oil is proportional to the mass worn from the piston rings.

SET UP: $1\ \text{Ci} = 3.7\times 10^{10}\ \text{Bq}$.

EXECUTE: The activity of the original iron, after 1000 hours of operation, would be $(9.4\times 10^{-6}\ \text{Ci})\,(3.7\times 10^{10}\ \text{Bq/Ci})2^{-(1000\ \text{h})/[(45\ \text{d})(24\ \text{h/d})]} = 1.8306\times 10^5\ \text{Bq}$. The activity of the oil is 84 Bq, or 4.5886×10^{-4} of the total iron activity, and this must be the fraction of the mass worn, or mass of 4.59×10^{-2} g. The rate at which the piston rings lost their mass is then 4.59×10^{-5} g/h.

EVALUATE: This method is very sensitive and can measure very small amounts of wear.

43.72. IDENTIFY and SET UP: Follow the procedure outlined in the problem. Solve the differential equation $dN_2/dt = \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 N_2$, where N_{10} is the initial number of ${}^{234}_{92}\text{U}$ nuclei and N_2 is the number of ${}^{230}_{88}\text{Th}$ nuclei. Assume a solution of the form $N_2(t) = N_{10}(h_1 e^{-\lambda_1 t} + h_2 e^{-\lambda_2 t})$ and follow the suggestions in the problem.

EXECUTE: (a) $N_2(0) = 0 = N_{10}(h_1 + h_2) \rightarrow h_2 = -h_1$.

(b) Take the derivative of $N_2(t)$: $dN_2/dt = N_{10}(-\lambda_1 h_1 e^{-\lambda_1 t} - \lambda_2 h_2 e^{-\lambda_2 t})$. Now substitute dN/dt and $N(t)$ into the original differential equation.

$$N_{10}(-\lambda_1 h_1 e^{-\lambda_1 t} - \lambda_2 h_2 e^{-\lambda_2 t}) = \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 (h_1 e^{-\lambda_1 t} + h_2 e^{-\lambda_2 t}).$$

Collect coefficients of $e^{-\lambda_1 t}$ and $e^{-\lambda_2 t}$.

$$N_{10}(\lambda_1 h_1 + \lambda_1 - \lambda_2 h_2)e^{-\lambda_1 t} + N_{10}(\lambda_2 h_2 - \lambda_2 h_2)e^{-\lambda_2 t} = 0.$$

Setting the coefficients equal to zero gives $h_1 = \dfrac{\lambda_1}{\lambda_2 - \lambda_1}$. Putting this result into the equation for $N_2(t)$ and using the fact that $h_2 = -h_1$ gives $N_2(t) = \dfrac{\lambda_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$.

(c) The initial number of ${}^{234}_{92}\text{U}$ atoms is

$$N_{10} = (30.0\ \text{g})[(6.022\times 10^{23}\ \text{atoms})/(234\ \text{g})] = 7.7205\times 10^{22}\ \text{atoms}.$$

Using $\lambda = (\ln 2)/T_{1/2}$, we get $\lambda_1 = (\ln 2)/(2.46\times 10^5\ \text{y})$ and $\lambda_2 = (\ln 2)/(7.54\times 10^4\ \text{y})$, which gives $\dfrac{\lambda_1}{\lambda_2 - \lambda_1} = 0.44197$. At time $t = 2.46\times 10^5$ y, the number N_2 of ${}^{230}_{88}\text{Th}$ atoms is

$$N_2 = (7.7205\times 10^{22}\ \text{atoms})(0.44197)[e^{-(\ln 2)} - e^{(\ln 2)(2.46)/(0.754)}] = 1.3506\times 10^{22}\ \text{atoms}.$$

The mass M of all these ${}^{230}_{88}\text{Th}$ atoms is

$$M = (1.3506\times 10^{22}\ \text{atoms})[(1\ \text{mol})/(6.022\times 10^{23}\ \text{atoms})][(230\ \text{g})/\text{mol}] = 5.16\ \text{g}.$$

EVALUATE: The ${}^{234}_{92}\text{U}$ decays with a half-life of 2.46×10^5 y, so at this time half the original uranium, or 15.0 g, is still left but only 5.16 g of ${}^{230}_{88}\text{Th}$ is present because it has continued to decay after being formed.

43.73. IDENTIFY and SET UP: The reaction is ${}^{130}_{52}\text{Te} + \text{X} \rightarrow {}^{131}_{52}\text{Te}.$

EXECUTE: X must have no charge since Z remains 52, and it must increase the atomic weight from 130 to 131, so it must be a neutron, which is choice (a).

EVALUATE: The other reactions in the choices all start out with the wrong isotope, so they cannot be correct.

43.74. IDENTIFY and SET UP: The reaction is ${}^{131}_{52}\text{Te} \rightarrow {}^{131}_{53}\text{I} + \text{X}.$

EXECUTE: The mass number 131 does not change, but the atomic number goes from 52 to 53, so the nucleus gained a charge of +1 (or lost a charge of –1). Therefore X must must be a β^- particle, which is choice (b).

EVALUATE: Alpha decay would change the mass number A by 4 units, β^+ decay would decrease the atomic number Z by 1 unit, and gamma decay would not affect Z.

43.75. IDENTIFY and SET UP: A thyroid treatment administers 3.7 GBq of ${}^{131}_{53}\text{I},$ which has a half-life of 8.04 days. $\lambda = (\ln 2)/T_{1/2}$ and $|dN/dt| = \lambda N = \dfrac{\ln 2}{T_{1/2}} N.$

EXECUTE: Solve for N and put in the numbers.

$N = \dfrac{T_{1/2}\,|dN/dt|}{\ln 2}$ = (8.04 d)(24 ×3600 s/d)(3.7 ×10^9 decays/s)/(ln 2) = 3.7 ×10^{15} atoms, which is choice (d).

EVALUATE: This amount is (3.7 ×10^{15} atoms)/(6.02 ×10^{23} atoms/mol) = 6.1 ×10^{-9} mol ≈ 6 nanomoles.

43.76. IDENTIFY and SET UP: The reaction is ${}^{123}_{52}\text{Te} + \text{p} \rightarrow {}^{123}_{53}\text{I} + \text{n}$. Calculate the reaction energy Q to find out if the reaction is exoergic or endoergic.

EXECUTE: $Q = (M_{\text{Te}} + M_{\text{p}} - M_{\text{I}} - M_{\text{n}})c^2$. Using the values from the problem gives
Q = (122.904270 u + 1.007825 u – 122.905589 u – 1.008665 u)c^2 = –0.002159 uc^2, so the reaction is endoergic. We must put in energy to cause the reaction, which means that the proton must have a minimum kinetic energy, so choice (d) is correct.

EVALUATE: $|Q|$ = (0.002159 u)(931.5 MeV/u) = 2.011 MeV, so the proton must have at least 2.011 MeV of kinetic energy to cause the reaction.

43.77. IDENTIFY and SET UP: ${}^{131}_{53}\text{I}$ undergoes β^- decay, but ${}^{123}_{53}\text{I}$ undergoes gamma decay.

EXECUTE: From Table 43.3 we see that the RBE for gamma rays is 1, but the RBE for electrons is 1.0-1.5, so the electrons could cause more tissue damage than the gamma rays. This makes choice (b) the best one.

EVALUATE: The higher the RBE, the more likely it is that tissue damage could occur.

44

PARTICLE PHYSICS AND COSMOLOGY

VP44.1.1. **IDENTIFY:** This problem is about a cyclotron accelerating protons.

SET UP and **EXECUTE:** **(a)** We want the frequency. Use Eq. (44.7).

$$f = \frac{eB}{2\pi m} = \frac{e(0.600\ \text{T})}{2\pi m_\text{p}} = 9.15\ \text{MHz}.$$

(b) We want the maximum kinetic energy of the proton. Use Eq. (44.8) with $B = 0.600$ T and $R = 0.800$ m.

$$K_\text{max} = \frac{e^2B^2R^2}{2m_\text{p}} = 11.0\ \text{MeV}.$$

(c) We want the proton's speed. The rest energy of the proton is 938 MeV which is much greater than the kinetic energy of 11 MeV, so we do not have to use the relativistic equation. Solving $K = \frac{1}{2}mv^2$ for v gives

$$v = \sqrt{2K/m_\text{p}} = 4.60\times10^7\ \text{m/s} = 0.153c.$$

EVALUATE: Since $v \approx 15\%$ of c, it is reasonable not to use relativity. If we had used relativity, our result would be

$$K = mc^2(\gamma - 1) \rightarrow \gamma = 1.012$$

$$v = c\sqrt{1 - 1/\gamma^2} = 0.152c.$$

The percent difference between the two answers is 0.001/0.152 = 0.66%, which is extremely small.

VP44.1.2. **IDENTIFY:** This problem is about a cyclotron accelerating hydrogen ions (protons).

SET UP and **EXECUTE:** **(a)** We want the radius R. $R = m_\text{p}v/eB$. Using $B = 0.460$ T and the given speed, we get $R = 0.136$ m.

(b) We want the angular frequency. Use Eq. (44.7) with $q = e$ and $B = 0.460$ T, which gives $\omega = eB/m = 4.41 \times 10^7$ rad/s.

EVALUATE: This angular frequency is rather small compared to many modern cyclotrons.

VP44.1.3. **IDENTIFY:** This problem deals with the operation of a cyclotron.

SET UP and **EXECUTE:** **(a)** We want the magnetic field. The rest energy of the proton is 938 MeV and its kinetic energy here is 80.0 keV, which is much less than its rest energy. Therefore we do not need to use special relativity. We use Eq. (44.8), with $R = (11.4\ \text{cm})/2$ and $K_\text{max} = 80.0$ keV, and solve for B, which gives

$$B = \frac{\sqrt{2m_\text{p}K_\text{max}}}{eR} = 0.717\ \text{T}.$$

(b) We want the frequency. Use Eq. (44.7) with $B = 0.717$ T and $q = e$.

$$f = \frac{eB}{2\pi m} = \frac{e(0.717\ \text{T})}{2\pi m_\text{p}} = 10.9\ \text{MHz}.$$

EVALUATE: Modern cyclotrons produce much greater kinetic energy, but after all, this was the first one.

VP44.1.4. **IDENTIFY:** This problem deals with the operation of a cyclotron.

SET UP and **EXECUTE:** **(a)** We want the magnetic field and know that the frequency is 5.80 MHz. Use Eq. (44.7) and solve for B using $q = 2e$ and the given mass of an alpha particle.

$$B = \frac{2\pi m_\alpha f}{2e} = 0.756\ \text{T}.$$

(b) We want the kinetic energy when $R = 0.650$ m. Using Eq. (44.8) with $B = 0.756$ T gives

$$K_{\text{max}} = \frac{(2e)^2 B^2 R^2}{2m_\alpha} = 11.6\ \text{MeV}.$$

EVALUATE: A magnetic field of 0.756 T is certainly physically reasonable in a physics laboratory.

VP44.3.1. **IDENTIFY:** This problem deals with the available energy when particles collide.

SET UP and **EXECUTE:** We follow the procedure of Example 44.2.

(a) We want the available energy E_a. The total available energy must be the total rest energy in the center-of-momentum frame.

$$E_{\text{a}} = 2m_{\text{p}}c^2 + m_{n'}c^2 = 2(938\ \text{MeV}) + 958\ \text{MeV} = 2834\ \text{MeV}.$$

(b) We want the kinetic energy K of the incoming proton. First use Eq. (44.10) to find E_m (the total energy of the incoming proton). Then use $E_m = K + m_pc^2$ to find K.

$$E_m = \frac{E_{\text{a}}^2}{2m_{\text{p}}c^2} - m_{\text{p}}c^2 = K + m_{\text{p}}c^2$$

$$K = \frac{E_{\text{a}}^2}{2m_{\text{p}}c^2} - 2m_{\text{p}}c^2$$

Using $E_a = 2834$ MeV and $m_pc^2 = 938$ MeV gives us $K = 2410$ MeV = 2.41 GeV.

EVALUATE: Colliding beams would require far less kinetic energy because nearly all the incoming kinetic energy is available energy.

VP44.3.2. **IDENTIFY:** This problem deals with the available energy during collisions.

SET UP and **EXECUTE:** **(a)** We want E_a. The minimum available energy is the rest energy of the particles after the collision. $E_a = m_pc^2 + m_\Delta c^2 = 938\ \text{MeV} + 1232\ \text{MeV} = 2170\ \text{MeV} = 2.17\ \text{GeV}$.

(b) We want the kinetic energy. Use the same method as in problem VP44.3.1.

$$K = \frac{E_{\text{a}}^2}{2m_{\text{p}}c^2} - 2m_{\text{p}}c^2$$

Using $E_a = 2170$ MeV and $m_pc^2 = 938$ MeV gives $K = 634$ MeV.

EVALUATE: The incoming proton could have more than 634 MeV of kinetic energy. In that case, the products would each have kinetic energy.

VP44.3.3. **IDENTIFY:** This problem deals with the available energy during collisions.

SET UP and **EXECUTE:** **(a)** We want E_a. The minimum available energy is the rest energy of the product (the Δ^0), which is 1232 MeV.

(b) We want the minimum kinetic energy K of the pion. The target and incident particle have different masses, so use Eq. (44.9) to find the available energy. Then use this to find E_m, and finally use $E_m = K + mc^2$ to find K.

The quantities in Eq. (44.9) are:

M = target = proton, so $Mc^2 = 938$ MeV

m = incident particle = π^-, so $mc^2 = 140$ MeV

Using the result of part (a), Eq. (44.9) gives

$$(1232\ \text{MeV})^2 = 2(938\ \text{MeV})E_m + (938\ \text{MeV})^2 + (140\ \text{MeV})^2$$

$$E_m = 329.6 \text{ MeV}.$$

Now use $E_m = K + mc^2$ to find K.

329.6 MeV = K + 140 MeV, which gives K = 190 MeV.

EVALUATE: Notice that the kinetic energy is a significant fraction of E_m.

VP44.3.4. **IDENTIFY:** This problem deals with the available energy during collisions.

SET UP and **EXECUTE:** **(a)** We want E_a. E_m = 7000 GeV and m_pc^2 = 938 MeV, so $E_m >> m_pc^2$. Therefore we use Eq. (44.11). Using the given rest energies gives

$$E_a = \sqrt{2\,mc^2 E_m} = \sqrt{2(938 \text{ MeV})(7000 \text{ GeV})} = 115 \text{ GeV}.$$

(b) We want the minimum mass of particle X. E_a is at *least* equal to the rest energy of the products of the collision, which are 2 protons and particle X. So 115 GeV = 2(938 MeV) + m_Xc^2. This gives m_X = 113 GeV/c^2.

EVALUATE: Particle X is 120 times heavier than the proton.

VP44.9.1. **IDENTIFY:** This problem concerns the red shift.

SET UP and **EXECUTE:** **(a)** The target variable is z.

$$z = \frac{\Delta\lambda}{\lambda} = \frac{725 \text{ nm} - 656.3 \text{ nm}}{656.3 \text{ nm}} = 0.105.$$

(b) We want the speed. Use Eq. (44.14) with the given wavelengths.

$$v = \frac{(\lambda_0/\lambda_s)^2 - 1}{(\lambda_0/\lambda_s)^2 + 1}c = \frac{(725/656.3)^2 - 1}{(725/656.3)^2 + 1}c = 0.0992c.$$

EVALUATE: The fact that we have a red shift instead of a blue shift tells us that the galaxy is receding from us at 9.92% the speed of light.

VP44.9.2. **IDENTIFY:** This problem is about the red shift of a galaxy.

SET UP: Eq. (44.13) applies.

EXECUTE: **(a)** We want wavelength that we observe. Use Eq. (44.13).

$$\lambda_0 = \lambda_s\sqrt{\frac{c+v}{c-v}} = (396.9 \text{ nm})\sqrt{\frac{c+0.0711c}{c-0.0711c}} = 426 \text{ nm}.$$

(b) We want z.

$$z = \frac{\Delta\lambda}{\lambda} = \frac{426 \text{ nm} - 396.9 \text{ nm}}{396.9 \text{ nm}} = 0.0738.$$

EVALUATE: Since v is only about 7% the speed of light, the red shift is not very large.

VP44.9.3. **IDENTIFY:** This problem requires the use of Hubble's law.

SET UP: Hubble's law: $v = H_0 r$, where H_0 = (68 km/s)/Mpc. The target variable is the distance to the galaxy.

EXECUTE: **(a)** $v = 0.0992c$. Apply Hubble's law using the measured speed.

$$r = \frac{v}{H_0} = \frac{0.0992c}{68\,\dfrac{\text{km/s}}{\text{Mpc}}} = 440 \text{ Mpc} = 4.4\times10^8 \text{ pc}.$$

(b) $v = 0.0711c$. Use the same approach as in part (a).

$$r = \frac{v}{H_0} = \frac{0.0711c}{68\,\dfrac{\text{km/s}}{\text{Mpc}}} = 310 \text{ Mpc} = 3.1\times10^8 \text{ pc}.$$

EVALUATE: Speeds can be determined quite easily using spectral analysis, but H_0 is not so easy to measure. These distances are only as accurate as H_0.

VP44.9.4. **IDENTIFY:** This problem requires the use of Hubble's law and the red shift.

SET UP: Hubble's law: $v = H_0 r$, where $H_0 = (68\text{ km/s})/\text{Mpc}$.

EXECUTE: **(a)** We want the speed of recession of the galaxy. Use Eq. (44.14).

$$v = \frac{(\lambda_0/\lambda_s)^2 - 1}{(\lambda_0/\lambda_s)^2 + 1}c = \frac{(666/615)^2 - 1}{(666/615)^2 + 1}c = 0.0795c = 2.38\times10^7\text{ m/s}.$$

(b) We want the distance to the galaxy. Use Hubble's law.

$$r = \frac{v}{H_0} = \frac{2.38\times10^7\text{ m/s}}{68\,\frac{\text{km/s}}{\text{Mpc}}} = 350\text{ Mpc} = 3.5\times10^8\text{ pc}.$$

EVALUATE: Using 1 pc = 3.26 ly, we find that this galaxy is 1.1 billion light-years from us!

44.1. **IDENTIFY:** The antimatter annihilates with an equal amount of matter.

SET UP: The energy of the matter is $E = (\Delta m)c^2$.

EXECUTE: Putting in the numbers gives

$$E = (\Delta m)c^2 = (400\text{ kg} + 400\text{ kg})(3.00\times10^8\text{ m/s})^2 = 7.2\times10^{19}\text{ J}.$$

This is about 70% of the annual energy use in the U.S.

EVALUATE: If this huge amount of energy were released suddenly, it would blow up the *Enterprise*! Getting useable energy from matter-antimatter annihilation is not so easy to do!

44.2. **IDENTIFY:** The energy (rest mass plus kinetic) of the muons is equal to the energy of the photons.

SET UP: $\gamma + \gamma \to \mu^+ + \mu^-$, $E = hc/\lambda$. $K = (\gamma - 1)mc^2$.

EXECUTE: **(a)** $\gamma + \gamma \to \mu^+ + \mu^-$. Each photon must have energy equal to the rest mass energy of a μ^+ or a μ^-: $\frac{hc}{\lambda} = 105.7\times10^6\text{ eV}$. $\lambda = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^8\text{ m/s})}{105.7\times10^6\text{ eV}} = 1.17\times10^{-14}\text{ m} = 0.0117\text{ pm}.$

Conservation of linear momentum requires that the μ^+ and μ^- move in opposite directions with equal speeds.

(b) $\lambda = \frac{0.0117\text{ pm}}{2}$, so each photon has energy $2(105.7\text{ MeV}) = 211.4\text{ MeV}$. The energy released in the reaction is $2(211.4\text{ MeV}) - 2(105.7\text{ MeV}) = 211.4\text{ MeV}$. The kinetic energy of each muon is half this, 105.7 MeV. Using $K = (\gamma - 1)mc^2$ gives $\gamma - 1 = \frac{K}{mc^2} = \frac{105.7\text{ MeV}}{105.7\text{ MeV}} = 1$. $\gamma = 2$. $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$.

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2}.\quad v = \sqrt{\frac{3}{4}}c = 0.866c = 2.60\times10^8\text{ m/s}.$$

EVALUATE: The muon speeds are a substantial fraction of the speed of light, so special relativity must be used.

44.3. **IDENTIFY:** The energy released is the energy equivalent of the mass decrease that occurs in the decay.

SET UP: The mass of the pion is $m_{\pi^+} = 270m_e$ and the mass of the muon is $m_{\mu^+} = 207m_e$. The rest energy of an electron is 0.511 MeV.

EXECUTE: **(a)** $\Delta m = m_{\pi+} - m_{\mu+} = 270m_e - 207m_e = 63m_e \Rightarrow E = 63(0.511\text{ MeV}) = 32\text{ MeV}.$

EVALUATE: **(b)** A positive muon has less mass than a positive pion, so if the decay from muon to pion was to happen, you could always find a frame where energy was not conserved. This cannot occur.

44.4. **IDENTIFY:** In the annihilation the total energy of the proton and antiproton is converted to the energy of the two photons.

SET UP: The rest energy of a proton or antiproton is 938.3 MeV. Conservation of linear momentum requires that the two photons have equal energies. The energy of a photon is $E = hf$, and $f\lambda = c$.

EXECUTE: **(a)** The energy will be the proton rest energy, 938.3 MeV, so $hf = 938.3$ MeV. Solving for f gives $f = (938.3\times10^{6}\text{ eV})/(4.136\times10^{-15}\text{ eV}\cdot\text{s}) = 2.27\times10^{23}$ Hz. The wavelength is

$$\lambda = c/f = 1.32\times10^{-15}\text{ m} = 1.32\text{ fm}.$$

(b) The energy of each photon will be $938.3\text{ MeV} + 620\text{ MeV} = 1558\text{ MeV}$, so $f = (1558\text{ MeV})/h$ $= 3.77\times10^{23}$ Hz. $\lambda = c/f = 7.96\times10^{-16}\text{ m} = 0.796$ fm.

EVALUATE: When the initial kinetic energy of the proton and antiproton increases, the wavelength of the photons decreases.

44.5. **IDENTIFY:** The kinetic energy of the alpha particle is due to the mass decrease.

SET UP and **EXECUTE:** ${}^{1}_{0}\text{n} + {}^{10}_{5}\text{B} \rightarrow {}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$. The mass decrease in the reaction is

$m({}^{1}_{0}\text{n}) + m({}^{10}_{5}\text{B}) - m({}^{7}_{3}\text{Li}) - m({}^{4}_{2}\text{He}) = 1.008665\text{ u} + 10.012937\text{ u} - 7.016005\text{ u} - 4.002603\text{ u} = 0.002994\text{ u}$ and the energy released is $E = (0.002994\text{ u})(931.5\text{ MeV/u}) = 2.79\text{ MeV}$. Assuming the initial momentum is zero, $m_{\text{Li}}v_{\text{Li}} = m_{\text{He}}v_{\text{He}}$ and $v_{\text{Li}} = \dfrac{m_{\text{He}}}{m_{\text{Li}}}v_{\text{He}}$. $\frac{1}{2}m_{\text{Li}}v_{\text{Li}}^2 + \frac{1}{2}m_{\text{He}}v_{\text{He}}^2 = E$ becomes

$$\frac{1}{2}m_{\text{Li}}\left(\frac{m_{\text{He}}}{m_{\text{Li}}}\right)^2 v_{\text{He}}^2 + \frac{1}{2}m_{\text{He}}v_{\text{He}}^2 = E \text{ and } v_{\text{He}} = \sqrt{\frac{2E}{m_{\text{He}}}\left(\frac{m_{\text{Li}}}{m_{\text{Li}} + m_{\text{He}}}\right)}.\ E = 4.470\times10^{-13}\text{ J}.$$

$$m_{\text{He}} = 4.002603\text{ u} - 2(0.0005486\text{ u}) = 4.0015\text{ u} = 6.645\times10^{-27}\text{ kg}.$$

$$m_{\text{Li}} = 7.016005\text{ u} - 3(0.0005486\text{ u}) = 7.0144\text{ u}.\text{ This gives } v_{\text{He}} = 9.26\times10^{6}\text{ m/s}.$$

EVALUATE: The speed of the alpha particle is considerably less than the speed of light, so it is not necessary to use the more complicated relativistic formulas.

44.6. **IDENTIFY:** The range is limited by the lifetime of the particle, which itself is limited by the uncertainty principle.

SET UP: $\Delta E\Delta t = \hbar/2$.

EXECUTE: $\Delta t = \dfrac{\hbar}{2\Delta E} = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s}/2\pi)}{2(783\times10^{6}\text{ eV})} = 4.20\times10^{-25}$ s. The range of the force is

$$c\Delta t = (2.998\times10^{8}\text{ m/s})(4.20\times10^{-25}\text{ s}) = 1.26\times10^{-16}\text{ m} = 0.126\text{ fm}.$$

EVALUATE: This range is less than the diameter of an atomic nucleus.

44.7. **IDENTIFY:** This problem is about the available energy during a collision of equal-mass particles.

SET UP: We are comparing the available kinetic energy if the target is stationary or if we have colliding beams.

EXECUTE: **(a)** Stationary target. Use Eq. 44.10 and solve for K.

$$E_{\text{a}}^2 = 2mc^2(mc^2 + K)$$

$$K = \frac{E_{\text{a}}^2}{2mc^2} - mc^2 = \frac{(2\text{ TeV})^2}{2(938\text{ MeV})} - 938\text{ MeV} = 2130\text{ TeV}.$$

(b) Colliding beams. The available energy is the kinetic energy minus the rest energy of the two protons.

$$E_{\text{a}} = 2K - 2m_{\text{p}}c^2$$

$$K = \frac{E_{\text{a}} + 2m_{\text{p}}c^2}{2} = \frac{2.00\text{ TeV} + 2(938\text{ MeV})}{2} = 1.00\text{ TeV}.$$

EVALUATE: By using colliding beams we need less than 1/1000 the kinetic energy than with a stationary target.

44.8. **IDENTIFY:** With a stationary target, only part of the initial kinetic energy of the moving electron is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be leftover kinetic energy. Therefore not all of the initial energy is available.

SET UP: The available energy is given by $E_a^2 = 2mc^2(E_m + mc^2)$ for two particles of equal mass when one is initially stationary. In this case, the initial kinetic energy $(30.0\text{ GeV} = 30{,}000\text{ MeV})$ is much more than the rest energy of the electron (0.511 MeV), so the formula for available energy reduces to

$$E_a = \sqrt{2mc^2 E_m}.$$

EXECUTE: **(a)** Using the formula for available energy gives

$$E_a = \sqrt{2mc^2 E_m} = \sqrt{2(0.511\text{ MeV})(30.0\text{ GeV})} = 175\text{ MeV}.$$

(b) For colliding beams of equal mass, each particle has half the available energy, so each has 87.5 MeV. The *total* energy is twice this, or 175 MeV.

EVALUATE: Colliding beams provide considerably more available energy to do experiments than do beams hitting a stationary target. With a stationary electron target in part (a), we had to give the moving electron 30,000 MeV of energy to get the same available energy that we got with only 175 MeV of energy with the colliding beams.

44.9. **IDENTIFY** and **SET UP:** The angular frequency is $\omega = |q|B/m$ so $B = m\omega/|q|$. And since $\omega = 2\pi f$, this becomes $B = 2\pi mf/|q|$.

EXECUTE: **(a)** A deuteron is a deuterium nucleus $({}^2_1\text{H})$. Its charge is $q = +e$. Its mass is the mass of the neutral ${}^2_1\text{H}$ atom (Table 43.2) minus the mass of the one atomic electron:

$m = 2.014102\text{ u} - 0.0005486\text{ u} = 2.013553\text{ u}(1.66054\times10^{-27}\text{ kg/1 u}) = 3.344\times10^{-27}\text{ kg}.$

$$B = \frac{2\pi mf}{|q|} = \frac{2\pi(3.344\times10^{-27}\text{ kg})(9.00\times10^{6}\text{ Hz})}{1.602\times10^{-19}\text{ C}} = 1.18\text{ T}.$$

(b) Eq. (44.8): $K = \dfrac{q^2B^2R^2}{2m} = \dfrac{[(1.602\times10^{-19}\text{ C})(1.18\text{ T})(0.320\text{ m})]^2}{2(3.344\times10^{-27}\text{ kg})}.$

$$K = 5.471\times10^{-13}\text{ J} = (5.471\times10^{-13}\text{ J})(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 3.42\text{ MeV}.$$

$$K = \tfrac{1}{2}mv^2,\text{ so } v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.471\times10^{-13}\text{ J})}{3.344\times10^{-27}\text{ kg}}} = 1.81\times10^{7}\text{ m/s}.$$

EVALUATE: $v/c = 0.06$, so it is ok to use the nonrelativistic expression for kinetic energy.

44.10. **IDENTIFY:** We are dealing with the magnetic force on a positron.

SET UP and **EXECUTE:** We want the momentum of the positron. Solve $R = mv/|q|B$ for p, which gives $p = ReB = (0.15\text{ m})e(0.40\text{ T}) = 9.60\times10^{-21}\text{ kg}\cdot\text{m/s}.$

EVALUATE: To check if this positron is relativistic, use $p = mv$ to find its speed, which gives $v = p/m$. Using the momentum we just found gives $v = 1.05\times10^{10}$ m/s which is 300 times the speed of light! So we would need to use relativistic momentum to find the speed.

44.11. **IDENTIFY** and **SET UP:** The masses of the target and projectile particles are equal, so we can use the equation $E_a^2 = 2mc^2(E_m + mc^2)$. E_a is specified; solve for the energy E_m of the beam particles.

EXECUTE: **(a)** Solve for E_m: $E_m = \dfrac{E_a^2}{2mc^2} - mc^2.$

The mass for the alpha particle can be calculated by subtracting two electron masses from the ${}^4_2\text{He}$ atomic mass:

$$m = m_\alpha = 4.002603\text{ u} - 2(0.0005486\text{ u}) = 4.001506\text{ u}.$$

Then $mc^2 = (4.001506\text{ u})(931.5\text{ MeV/u}) = 3.727\text{ GeV}.$

$$E_m = \frac{E_a^2}{2mc^2} - mc^2 = \frac{(16.0\text{ GeV})^2}{2(3.727\text{ GeV})} - 3.727\text{ GeV} = 30.6\text{ GeV}.$$

(b) Each beam must have $\frac{1}{2}E_a = 8.0$ GeV.

EVALUATE: For a stationary target the beam energy is nearly twice the available energy. In a colliding beam experiment all the energy is available and each beam needs to have just half the required available energy.

44.12. **IDENTIFY:** We are dealing with a cyclotron that is accelerating protons.
SET UP: The magnetic field is 1.3 T and the radius of the proton path is 11 cm.
EXECUTE: **(a)** We want the maximum kinetic energy. Use Eq. (44.8) with the known quantities.

$$K_{\text{max}} = \frac{q^2B^2R^2}{2m} = \frac{e^2(1.3\text{ T})^2(0.11\text{ m})^2}{2(1.67\times10^{-27}\text{ kg})} = 0.98\text{ MeV}.$$

The proton rest energy is 938 MeV and K_{max} = 0.98 MeV, so it is accurate to use nonrelativistic expressions.
(b) We want the frequency. Using the known quantities gives $f = eB/2\pi m$ = 20 MHz.
EVALUATE: A kinetic energy of 0.98 MeV is *much* less than present-day accelerators can produce.

44.13. **IDENTIFY:** $E = \gamma mc^2$, where $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$. The relativistic formula for the angular frequency is

$$\omega = \frac{|q|B}{m\gamma}.$$

SET UP: A proton has rest energy $mc^2 = 938.3$ MeV.

EXECUTE: **(a)** $\gamma = \dfrac{E}{mc^2} = \dfrac{1000\times10^3\text{ MeV}}{938.3\text{ MeV}} = 1065.8$, so $v = 0.999999559c$.

(b) Nonrelativistic: $\omega = \dfrac{eB}{m} = 3.83\times10^8$ rad/s.

Relativistic: $\omega = \dfrac{eB}{m}\dfrac{1}{\gamma} = 3.59\times10^5$ rad/s.

EVALUATE: The relativistic expression gives a smaller value for ω.

44.14. **IDENTIFY** and **SET UP:** To create the η^0, the minimum available energy must be equal to the rest mass energy of the products, which in this case is the η^0 plus two protons. In a collider, all of the initial energy is available, so the beam energy is the available energy. Use the particle masses from Table 44.3.
EXECUTE: The minimum amount of available energy must be rest mass energy

$$E_a = 2m_p + m_\eta = 2(938.3\text{ MeV}) + 547.3\text{ MeV} = 2420\text{ MeV}.$$

Each incident proton has half of the rest mass energy, or $1210\text{ MeV} = 1.21\text{ GeV}$.

EVALUATE: We would need much more initial energy if one of the initial protons were stationary. The result here (1.21 GeV) is the *minimum* amount of energy needed; the original protons could have more energy and still trigger this reaction.

44.15. **IDENTIFY** and **SET UP:** For the reaction $\text{p} + \text{p} \rightarrow \text{p} + \text{p} + \text{p} + \overline{\text{p}}$, the two incident protons must have enough kinetic energy to produce a p and a $\overline{\text{p}}$, plus any kinetic energy of the products. If they have the minimum kinetic energy, the products are at rest. The proton and antiproton have equal masses. The available energy for two equal-mass particles is $E_a^2 = 2mc^2(E_m + mc^2)$, where $E_m = K + mc^2$.

EXECUTE: **(a)** In a head-on collision with equal speeds, the laboratory frame is the center-of-momentum frame. For the minimum kinetic energy of the incident protons, the products are all at rest. In that case, the incident protons need only enough kinetic energy to produce a proton and an antiproton. Since the incident protons have equal energy, each one must have kinetic energy equal to the rest energy of a proton, which is 938 MeV.

(b) In this case, the target proton is at rest. Since 4 particles are produced, each of mass m, the available energy E_a must be at least equal to $4mc^2$. Therefore $E_a^2 = 2mc^2(E_m + mc^2) = (4mc^2)^2 = 16m^2c^4$, which gives $E_m = 7mc^2$. Using $E_m = K + mc^2$, we get $K = 6mc^2 = 6(938\text{ MeV}) = 5630\text{ MeV}$.

EVALUATE: When the two protons collide head-on with equal speeds, they need only 938 MeV of kinetic energy each, for a total of 1879 MeV. But when the target is stationary, the kinetic energy needed is 5630 MeV, which is 3 times as much as for a head-on collision.

44.16. **IDENTIFY:** This problem deals with the decay of the omega-minus particle.

SET UP: Use masses (expressed in MeV) from Table 44.3.

EXECUTE: **(a)** We want the energy Q that is released in this decay. From Table 44.3 we have the following masses:

$\mathrm{M}_{\Omega^-} = 1672\text{ MeV}$

$\mathrm{M}_{\Xi^-} = 1322\text{ MeV}$

$\mathrm{M}_{\pi^0} = 135.0\text{ MeV}$

$Q = 1672\text{ MeV} – (1321\text{ MeV} + 135.0\text{ MeV}) = 215\text{ MeV}$.

(b) Use the values for B and S from Table 44.3. The initial B is +1 and the final B is +1, so $\Delta B = 0$. The initial S is –3 and the final S is –2, so $\Delta S = +1$.

EVALUATE: $\Delta B = 0$ agrees with conservation of baryon number, but $\Delta S = +1$ violates strangeness conservation, so this decay is not allowed for the strong interaction. The initial and final lepton numbers are both zero, so this decay is allowed through the weak interaction.

44.17. **IDENTIFY:** The kinetic energy comes from the mass decrease.

SET UP: Table 44.3 gives $m(\mathrm{K}^+) = 493.7\text{ MeV}/c^2$, $m(\pi^0) = 135.0\text{ MeV}/c^2$, and $m(\pi^\pm)$ $= 139.6\text{ MeV}/c^2$.

EXECUTE: **(a)** Charge must be conserved, so $\mathrm{K}^+ \to \pi^0 + \pi^+$ is the only possible decay.

(b) The mass decrease is

$m(\mathrm{K}^+) - m(\pi^0) - m(\pi^+) = 493.7\text{ MeV}/c^2 - 135.0\text{ MeV}/c^2 - 139.6\text{ MeV}/c^2$

$= 219.1\text{ MeV}/c^2$. The energy released is 219.1 MeV.

EVALUATE: The π mesons do not share this energy equally since they do not have equal masses.

44.18. **IDENTIFY:** The energy is due to the mass difference.

SET UP: The energy released is the energy equivalent of the mass decrease. From Table 44.3, the μ^- has mass $105.7\text{ MeV}/c^2$ and the e^- has mass $0.511\text{ MeV}/c^2$.

EXECUTE: The mass decrease is $105.7\text{ MeV}/c^2 - 0.511\text{ MeV}/c^2 = 105.2\text{ MeV}/c^2$ and the energy equivalent is 105.2 MeV.

EVALUATE: The electron does not get all of this energy; the neutrinos also get some of it.

44.19. **IDENTIFY:** Table 44.1 gives the mass in units of GeV/c^2. This is the value of mc^2 for the particle.

SET UP: $m(\mathrm{Z}^0) = 91.2\text{ GeV}/c^2$.

EXECUTE: $E = 91.2\times10^9\text{ eV} = 1.461\times10^{-8}\text{ J}$; $m = E/c^2 = 1.63\times10^{-25}\text{ kg}$; $m(\mathrm{Z}^0)/m(\mathrm{p}) = 97.2$

EVALUATE: The rest energy of a proton is 938 MeV; the rest energy of the Z^0 is 97.2 times as great.

44.20. **IDENTIFY:** If the initial and final rest mass energies were equal, there would be no leftover energy for kinetic energy. Therefore the kinetic energy of the products is the difference between the mass energy of the initial particles and the final particles.

SET UP: The difference in mass is $\Delta m = M_{\Omega^-} - m_{\Lambda^0} - m_{\mathrm{K}^-}$.

EXECUTE: Using Table 44.3, the energy difference is
$E = (\Delta m)c^2 = 1672 \text{ MeV} - 1116 \text{ MeV} - 494 \text{ MeV} = 62 \text{ MeV}.$

EVALUATE: There is less rest mass energy after the reaction than before because 62 MeV of the initial energy was converted to kinetic energy of the products.

44.21. **IDENTIFY** and **SET UP:** Find the energy equivalent of the mass decrease.
EXECUTE: The mass decrease is $m(\Sigma^+) - m(\text{p}) - m(\pi^0)$ and the energy released is
$mc^2(\Sigma^+) - mc^2(\text{p}) - mc^2(\pi^0) = 1189 \text{ MeV} - 938.3 \text{ MeV} - 135.0 \text{ MeV} = 116 \text{ MeV}.$ (The mc^2 values for each particle were taken from Table 44.3.)
EVALUATE: The mass of the decay products is less than the mass of the original particle, so the decay is energetically allowed and energy is released.

44.22. **IDENTIFY** and **SET UP:** The p and n have baryon number $+1$ and the antiproton $\overline{\text{p}}$ has baryon number -1. $\text{e}^+, \text{e}^-, \overline{\nu}_e,$ and γ all have baryon number zero. Baryon number is conserved if the total baryon number of the products equals the total baryon number of the reactants.
EXECUTE: **(a)** reactants: $B = 1 + 1 = 2.$ Products: $B = 1 + 0 = 1.$ Not conserved.
(b) reactants: $B = 1 + 1 = 2.$ Products: $B = 0 + 0 = 0.$ Not conserved.
(c) reactants: $B = +1.$ Products: $B = 1 + 0 + 0 = +1.$ Conserved.
(d) reactants: $B = 1 - 1 = 0.$ Products: $B = 0.$ Conserved.
EVALUATE: Even though a reaction obeys conservation of baryon number it may still not occur spontaneously, if it is not energetically allowed or if other conservation laws are violated.

44.23. **IDENTIFY** and **SET UP:** The lepton numbers for the particles are given in Table 44.2.
EXECUTE: **(a)** $\mu^- \to \text{e}^- + \nu_\text{e} + \overline{\nu}_\mu \Rightarrow L_\mu: +1 \neq -1, L_\text{e}: 0 \neq +1 + 1,$ so lepton numbers are not conserved.
(b) $\tau^- \to \text{e}^- + \overline{\nu}_\text{e} + \nu_\tau \Rightarrow L_\text{e}: 0 = +1 - 1;\ L_\tau: +1 = +1,$ so lepton numbers are conserved.
(c) $\pi^+ \to \text{e}^+ + \gamma.$ Lepton numbers are not conserved since just one lepton is produced from zero original leptons.
(d) $\text{n} \to \text{p} + \text{e}^- + \overline{\nu}_\text{e} \Rightarrow L_\text{e}: 0 = +1 - 1,$ so the lepton numbers are conserved.
EVALUATE: The decays where lepton numbers are conserved are among those listed in Tables 44.2 and 44.3.

44.24. **IDENTIFY** and **SET UP:** Compare the sum of the strangeness quantum numbers for the particles on each side of the decay equation. The strangeness quantum numbers for each particle are given in Table 44.3.
EXECUTE: **(a)** $\text{K}^+ \to \mu^+ + \nu_\mu;\ S_{\text{K}^+} = +1, S_{\mu+} = 0, S_{\nu_\mu} = 0.$
$S = 1$ initially; $S = 0$ for the products; S is not conserved.
(b) $\text{n} + \text{K}^+ \to \text{p} + \pi^0;\ S_\text{n} = 0, S_{\text{K}^+} = +1, S_\text{p} = 0, S_{\pi^0} = 0.$
$S = 1$ initially; $S = 0$ for the products; S is not conserved.
(c) $\text{K}^+ + \text{K}^- \to \pi^0 + \pi^0;\ S_{\text{K}^+} = +1;\ S_{\text{K}^-} = -1;\ S_{\pi^0} = 0.$
$S = +1 - 1 = 0$ initially; $S = 0$ for the products; S is conserved.
(d) $\text{p} + \text{K}^- \to \Lambda^0 + \pi^0;\ S_\text{p} = 0, S_{\text{K}^-} = -1, S_{\Lambda^0} = -1, S_{\pi^0} = 0.$
$S = -1$ initially; $S = -1$ for the products; S is conserved.
EVALUATE: Strangeness is not a conserved quantity in weak interactions, and strangeness nonconserving reactions or decays can occur.

44.25. **IDENTIFY** and **SET UP:** Each value for the combination is the sum of the values for each quark. Use Table 44.4.
EXECUTE: **(a)** ***uds:***
$Q = \frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$

$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$S = 0 + 0 - 1 = -1$

$C = 0 + 0 + 0 = 0$

(b) $\underline{\boldsymbol{c\bar{u}}}$**:**

The values for $\bar{\boldsymbol{u}}$ are the negative for those for $\boldsymbol{u}$.

$Q = \frac{2}{3}e - \frac{2}{3}e = 0$

$B = \frac{1}{3} - \frac{1}{3} = 0$

$S = 0 + 0 = 0$

$C = +1 + 0 = +1$

(c) ***ddd:***

$Q = -\frac{1}{3}e - \frac{1}{3}e - \frac{1}{3}e = -e$

$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = +1$

$S = 0 + 0 + 0 = 0$

$C = 0 + 0 + 0 = 0$

(d) $\underline{\boldsymbol{d\bar{c}}}$**:**

$Q = -\frac{1}{3}e - \frac{2}{3}e = -e$

$B = \frac{1}{3} - \frac{1}{3} = 0$

$S = 0 + 0 = 0$

$C = 0 - 1 = -1$

EVALUATE: The charge, baryon number, strangeness, and charm quantum numbers of a particle are determined by the particle's quark composition.

44.26. IDENTIFY: Quark combination produce various particles.

SET UP: The properties of the quarks are given in Table 44.5. An antiquark has charge and quantum numbers of opposite sign from the corresponding quark.

EXECUTE: (a) $Q/e = \frac{2}{3} + \frac{2}{3} + (-\frac{1}{3}) = +1.$ $B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1.$ $S = 0 + 0 + (-1) = -1.$ $C = 0 + 0 + 0 = 0.$

(b) $Q/e = \frac{2}{3} + \frac{1}{3} = +1.$ $B = \frac{1}{3} + (-\frac{1}{3}) = 0.$ $S = 0 + 1 = 1.$ $C = 1 + 0 = 1.$

(c) $Q/e = \frac{1}{3} + \frac{1}{3} + (-\frac{2}{3}) = 0.$ $B = -\frac{1}{3} + (-\frac{1}{3}) + (-\frac{1}{3}) = -1.$ $S = 0 + 0 + 0 = 0.$ $C = 0 + 0 + 0 = 0.$

(d) $Q/e = -\frac{2}{3} + (-\frac{1}{3}) = -1.$ $B = -\frac{1}{3} + \frac{1}{3} = 0.$ $S = 0 + 0 = 0.$ $C = -1 + 0 = -1.$

EVALUATE: The charge must always come out to be a whole number.

44.27. IDENTIFY: The charge, baryon number, and strangeness of the particles are the sums of these values for their constituent quarks.

SET UP: The properties of the six quarks are given in Table 44.5.

EXECUTE: (a) $S = 1$ indicates the presence of one $\bar{\boldsymbol{s}}$ antiquark and no ***s*** quark. To have baryon number 0 there can be only one other quark, and to have net charge $+e$ that quark must be a ***u***, and the quark content is $\boldsymbol{u\bar{s}}$.

(b) The particle has an $\bar{\boldsymbol{s}}$ antiquark, and for a baryon number of -1 the particle must consist of three antiquarks. For a net charge of $-e$, the quark content must be $\overline{\boldsymbol{dd}}\,\bar{\boldsymbol{s}}$.

(c) $S = -2$ means that there are two ***s*** quarks, and for baryon number 1 there must be one more quark. For a charge of 0 the third quark must be a ***u*** quark and the quark content is ***uss***.

EVALUATE: The particles with baryon number zero are mesons and consist of a quark-antiquark pair. Particles with baryon number 1 consist of three quarks and are baryons. Particles with baryon number -1 consist of three antiquarks and are antibaryons.

44.28. **IDENTIFY:** The decrease in the rest energy of the particles that exist before and after the decay equals the energy that is released.

SET UP: The upsilon has rest energy 9460 MeV and each tau has rest energy 1777 MeV.

EXECUTE: $(m_\Upsilon - 2m_\tau)c^2 = (9460\text{ MeV} - 2(1777\text{ MeV})) = 5906\text{ MeV}.$

EVALUATE: Over half of the rest energy of the upsilon is released in the decay.

44.29. **IDENTIFY** and **SET UP:** The mass of a proton is 938 MeV/c^2, and the mass of the Higgs boson is 125 GeV/c^2 = 125 ×10^3 MeV/c^2.

EXECUTE: $m_{\text{Higgs}}/m_{\text{p}} = (125\times10^3\text{ MeV}/c^2)/(938\text{ MeV}/c^2) = 133.$

EVALUATE: Since the Higgs particle is 133 times as massive as the proton, it takes a great deal of energy to create one. This is the reason that high-energy particle accelerators are needed to test for the existence of the Higgs boson.

44.30. **(a) IDENTIFY** and **SET UP:** First calculate the speed v. Then use that in Hubble's law to find r.

EXECUTE: $v = \left[\dfrac{(\lambda_0/\lambda_S)^2 - 1}{(\lambda_0/\lambda_S)^2 + 1}\right]c = \left[\dfrac{(658.5\text{ nm}/590\text{ nm})^2 - 1}{(658.5\text{ nm}/590\text{ nm})^2 + 1}\right]c = 0.1094c$

$$v = (0.1094)(2.998\times10^8\text{ m/s}) = 3.28\times10^7\text{ m/s}.\;\; v = rH_0.$$

(b) IDENTIFY and **SET UP:** Use Hubble's law to calculate r.

EXECUTE: $r = \dfrac{v}{H_0} = \dfrac{3.28\times10^4\text{ km/s}}{[(67.3\text{ km/s})/\text{Mpc}](1\text{ Mpc}/3.26\text{ Mly})} = 1590\text{ Mly} = 1.59\times10^9\text{ ly}.$

EVALUATE: The red shift $\lambda_0/\lambda_S - 1$ for this galaxy is 0.116. It is therefore about twice as far from earth as the galaxy in Examples 44.8 and 44.9, that had a red shift of 0.053.

44.31. **(a) IDENTIFY** and **SET UP:** Hubble's law is $v = H_0 r$, with $H_0 = (67.3\text{ km/s})/(\text{Mpc})$. 1 Mpc = 3.26 Mly.

EXECUTE: $r = 5210$ Mly, so

$v = H_0 r = [(67.3\text{ km/s})/\text{Mpc}](1\text{ Mpc}/3.26\text{ Mly})(5210\text{ Mly}) = 1.08\times10^5\text{ km/s} = 1.08\times10^8\text{ m/s}.$

(b) IDENTIFY and **SET UP:** Use v from part (a) in $\lambda_0 = \lambda_S\sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}}.$

EXECUTE: $\dfrac{\lambda_0}{\lambda_S} = \sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}}.$

$$\frac{v}{c} = \frac{1.08\times10^8\text{ m/s}}{2.998\times10^8\text{ m/s}} = 0.3602,\text{ so } \frac{\lambda_0}{\lambda_S} = \sqrt{\frac{1+0.3602}{1-0.3602}} = 1.46.$$

EVALUATE: The galaxy in Examples 44.8 and 44.9 is 710 Mly away so has a smaller recession speed and redshift than the galaxy in this problem.

44.32. **IDENTIFY:** In Example 44.8, z is defined as $z = \dfrac{\lambda_0 - \lambda_S}{\lambda_S}$. Apply $\lambda_0 = \lambda_S\sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}}$ to solve for v. Hubble's law is given by $v = H_0 r$.

SET UP: The Hubble constant has a value of $H_0 = 6.73\times10^4\,\dfrac{\text{m/s}}{\text{Mpc}}.$

EXECUTE: **(a)** $1 + z = 1 + \dfrac{(\lambda_0 - \lambda_S)}{\lambda_S} = \dfrac{\lambda_0}{\lambda_S}$. Now we use $\lambda_0 = \lambda_S\sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}}$ to obtain

$$1 + z = \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{1+v/c}{1-v/c}} = \sqrt{\frac{1+\beta}{1-\beta}}.$$

(b) Solving the above equation for β we obtain $\beta = \dfrac{(1+z)^2 - 1}{(1+z)^2 + 1} = \dfrac{1.7^2 - 1}{1.7^2 + 1} = 0.4859.$ Thus,

$$v = 0.4859c = 1.46 \times 10^8 \text{ m/s}.$$

(c) We can use Hubble's law to find the distance to the given galaxy,

$$r = \frac{v}{H_0} = \frac{(1.46 \times 10^8 \text{ m/s})}{(6.73 \times 10^4 \text{ m/s})/\text{Mpc}} = 2.17 \times 10^3 \text{ Mpc}.$$

EVALUATE: $1 \text{ pc} = 3.26 \text{ ly},$ so the distance in part (c) is 7.07×10^9 ly.

44.33. IDENTIFY: The reaction energy Q is defined in Chapter 43 as $Q = (M_A + M_B - M_C - M_D)c^2$ and is the energy equivalent of the mass change in the reaction. When Q is negative the reaction is endoergic. When Q is positive the reaction is exoergic.
SET UP: Use the particle masses given in Section 43.1. 1 u is equivalent to 931.5 MeV.
EXECUTE: $\Delta m = m_e + m_p - m_n - m_{\nu_e}$ so assuming $m_{\nu_e} \approx 0,$

$\Delta m = 0.0005486 \text{ u} + 1.007276 \text{ u} - 1.008665 \text{ u} = -8.40 \times 10^{-4} \text{ u}$

$\Rightarrow E = (\Delta m)c^2 = (-8.40 \times 10^{-4} \text{ u})(931.5 \text{ MeV/u}) = -0.783 \text{ MeV}$ and is endoergic.

EVALUATE: The energy consumed in the reaction would have to come from the initial kinetic energy of the reactants.

44.34. IDENTIFY: The energy released in the reaction is the energy equivalent of the mass decrease that occurs in the reaction.
SET UP: 1 u is equivalent to 931.5 MeV. The neutral atom masses are given in Table 43.2.
EXECUTE: $3m(^4\text{He}) - m(^{12}\text{C}) = 7.80 \times 10^{-3} \text{ u},$ or 7.27 MeV.

EVALUATE: The neutral atom masses include 6 electrons on each side of the reaction equation. The electron masses cancel and we obtain the same mass change as would be calculated using nuclear masses.

44.35. IDENTIFY: The reaction energy Q is defined in Chapter 43 as $Q = (M_A + M_B - M_C - M_D)c^2$ and is the energy equivalent of the mass change in the reaction. When Q is negative the reaction is endoergic. When Q is positive the reaction is exoergic.
SET UP: 1 u is equivalent to 931.5 MeV. Use the neutral atom masses that are given in Table 43.2.
EXECUTE: $m_{^{12}_{6}\text{C}} + m_{^{4}_{2}\text{He}} - m_{^{16}_{8}\text{O}} = 7.69 \times 10^{-3}\text{u},$ or 7.16 MeV, an exoergic reaction.

EVALUATE: 7.16 MeV of energy is released in the reaction.

44.36. IDENTIFY: We are using Hooke's law to model the strong nuclear force.
SET UP and **EXECUTE:** **(a)** Use Coulomb's law modeling the quarks as a point-like particles each having charge $2e/3$ and separated from each other by $r = 0.5$ fm. This gives

$$F = \frac{1}{4\pi\epsilon_0}\frac{(2e/3)^2}{r^2} \approx 400 \text{ N}.$$

(b) We want the spring constant. $F_{\text{spr}} = ks$, so $k = F_{\text{spr}}/s = (400 \text{ N})/(0.5 \text{ fm}) = 8.2 \times 10^{17} \text{ N/m} \approx 8 \times 10^{17}$ N/m.
(c) Convert k to units of MeV/fm^2.

$$k = 8.2 \times 10^{17} \frac{\text{N}}{\text{m}} = \left(8.2 \times 10^{17} \frac{\text{N} \cdot \text{m}}{\text{m}^2}\right)\left(\frac{\text{J}}{\text{N} \cdot \text{m}}\right)\left(\frac{1 \text{ m}}{10^{15} \text{ fm}}\right)^2\left(\frac{1 \text{ MeV}}{1.6 \times 10^{-13} \text{ J}}\right) = 5.1 \text{ MeV/fm}^2 \approx 5 \text{ MeV/fm}^2.$$

(d) We want the stored energy.

$$U = \frac{1}{2}ks^2 = \frac{1}{2}(5.1 \text{ MeV/fm}^2)(0.5 \text{ fm})^2 = 0.64 \text{ MeV} = 640 \text{ KeV}.$$

(e) We want the energy to produce an up and anti-up pair of quarks. The mass of each up quark is 2.3 MeV/c^2, so to produce a pair we need 4.6 MeV.
(f) We want the distance between the quarks. Use the spring constant from part (c) with the energy from part (e).

$$E = \frac{1}{2}ks^2 \to s = \sqrt{2E/k} = \sqrt{\frac{2(4.6\text{ MeV})}{5.1\text{ MeV/fm}^2}} = 1.3\text{ fm}.$$

EVALUATE: The result in part (f) is comparable to the size of an atomic nucleus, so our model is of some use.

44.37. IDENTIFY: The energy comes from a mass decrease.
SET UP: A charged pion decays into a muon plus a neutrino. The muon in turn decays into an electron or positron plus two neutrinos.
EXECUTE: (a) $\pi^- \to \mu^- + \text{neutrino} \to \text{e}^- + \text{three neutrinos.}$
(b) If we neglect the mass of the neutrinos, the mass decrease is

$$m(\pi^-) - m(\text{e}^-) = 273m_\text{e} - m_\text{e} = 272m_\text{e} = 2.480\times10^{-28}\text{ kg}.$$
$$E = mc^2 = 2.23\times10^{-11}\text{ J} = 139\text{ MeV}.$$

(c) The total energy delivered to the tissue is $(50.0\text{ J/kg})(10.0\times10^{-3}\text{ kg}) = 0.500\text{ J}.$ The number of

$$\pi^-\text{ mesons required is } \frac{0.500\text{ J}}{2.23\times10^{-11}\text{ J}} = 2.24\times10^{10}.$$

(d) The RBE for the electrons that are produced is 1.0, so the equivalent dose is

$$1.0(50.0\text{ Gy}) = 50.0\text{ Sv} = 5.0\times10^3\text{ rem}.$$

EVALUATE: The π are heavier than electrons and therefore behave differently as they hit the tissue.

44.38. IDENTIFY: The initial total energy of the colliding proton and antiproton equals the total energy of the two photons.
SET UP: For a particle with mass, $E = K + mc^2$. For a proton, $m_\text{p}c^2 = 938\text{ MeV}$.
EXECUTE: $K + m_\text{p}c^2 = \dfrac{hc}{\lambda}, K = \dfrac{hc}{\lambda} - m_\text{p}c^2.$ Using λ = 0.720 fm = 0.720 $\times 10^{-15}$ m,
we get K = 784 MeV.
EVALUATE: If the kinetic energies of the colliding particles increase, then the wavelength of each photon decreases.

44.39. IDENTIFY: We are investigating colliding proton beams in the Large Hadron Collider.
SET UP and **EXECUTE: (a)** With one bunch per 25 ns, the number of bunches per second is 1/(25 ns) = 40 million.
(b) The fraction of protons that collide is 20/(115 billion) = 1.7×10^{-10}.
(c) We want the number of collisions that occur each second. Using the given information and the result of part (a) gives (40 million bunches/s)(20 collisions/bunch) = 800 million collisions.
(d) We want the proton density ρ in a bunch, with N protons in a cylinder of length L and radius R.

$$\rho = \frac{N}{\pi R^2 L} = \frac{115\text{ billion}}{\pi(10\ \mu\text{m})^2(0.300\text{ m})} = 1.2\times10^{12}\text{ protons/mm}^3.$$

(e) We want the density of hadrons in ordinary matter. Follow the hint. Mass is 75 kg. The body is a cylinder 30 cm in diameter of length 1.75 m tall. $V = \pi R^2 L = \pi(0.15\text{ m})^2(1.75\text{ m}) = 0.124\text{ m}^3$. The number N of hadrons is $N = (75\text{ kg})/(1.67 \times 10^{-27}\text{ kg}) = 4.5 \times 10^{28}$ hadrons. The density is $\rho = N/V = (4.5 \times 10^{28})/(0.124\text{ m}^3) = 3.6 \times 10^{29}\text{ hadrons/m}^3 \approx 4 \times 10^{20}\text{ hadrons/mm}^3$.
EVALUATE: As a check for part (e) we can use the density of water, which is 1000 kg/m^3. This gives

$$\frac{1000\text{ kg/m}^3}{1.67\times10^{-27}\text{ kg}} \approx 6\times10^{29}\text{ hadrons/m}^3 = 6\times10^{20}\text{ hadrons/mm}^3.$$

This result is quite close to our calculation above. Note that the concentration of hadrons in ordinary matter is around 300 million times as great as in the bunches in the Large Hadron Collider.

44.40. **IDENTIFY:** Apply Eq. (44.9).

SET UP: In Eq. (44.9), $E_a = (m_{\Sigma^0} + m_{K^0})c^2$, and with $M = m_p, m = m_{\pi^-}$ and $E_m = (m_{\pi^-})c^2 + K$,

$$K = \frac{E_a^2 - (m_{\pi^-}c^2)^2 - (m_p c^2)^2}{2m_p c^2} - (m_{\pi^-})c^2.$$

EXECUTE: $K = \dfrac{(1193\text{ MeV} + 497.7\text{ MeV})^2 - (139.6\text{ MeV})^2 - (938.3\text{ MeV})^2}{2(938.3\text{ MeV})} - 139.6\text{ MeV} = 904\text{ MeV}.$

EVALUATE: The increase in rest energy is

$(m_{\Sigma^0} + m_{K^0} - m_{\pi^-} - m_p)c^2 = 1193\text{ MeV} + 497.7\text{ MeV} - 139.6\text{ MeV} - 938.3\text{ MeV} = 613\text{ MeV}.$ The threshold kinetic energy is larger than this because not all the kinetic energy of the beam is available to form new particle states.

44.41. **IDENTIFY:** Baryon number, charge, strangeness, and lepton numbers are all conserved in the reactions.

SET UP: Use Table 44.3 to identify the missing particle, once its properties have been determined.

EXECUTE: **(a)** The baryon number is 0, the charge is $+e$, the strangeness is 1, all lepton numbers are zero, and the particle is K^+.

(b) The baryon number is 0, the charge is $-e$, the strangeness is 0, all lepton numbers are zero, and the particle is π^-.

(c) The baryon number is -1, the charge is 0, the strangeness is zero, all lepton numbers are 0, and the particle is an antineutron.

(d) The baryon number is 0 the charge is $+e$, the strangeness is 0, the muonic lepton number is -1, all other lepton numbers are 0, and the particle is μ^+.

EVALUATE: Rest energy considerations would determine if each reaction is endoergic or exoergic.

44.42. **IDENTIFY:** Charge must be conserved. The energy released equals the decrease in rest energy that occurs in the decay.

SET UP: The rest energies are given in Table 44.3.

EXECUTE: **(a)** The decay products must be neutral, so the only possible combinations are $\pi^0\pi^0\pi^0$ *or* $\pi^0\pi^+\pi^-$.

(b) $m_{n^0} - 3m_{\pi^0} = 142.9\text{ MeV}/c^2$, so the kinetic energy of the π^0 mesons is 142.9 MeV. For the other reaction, $m_{n^0} - m_{\pi^0} - m_{\pi^-} - m_{\pi^+} = 133.7\text{ MeV}$.

EVALUATE: The total momentum of the decay products must be zero. This imposes a correlation between the directions of the velocities of the decay products.

44.43. **IDENTIFY** and **SET UP:** Apply the Heisenberg uncertainty principle in the form $\Delta E \Delta t \approx \hbar/2$. Let ΔE be the energy width and let Δt be the lifetime.

EXECUTE: $\dfrac{\hbar}{2\Delta E} = \dfrac{(1.054\times10^{-34}\text{ J}\cdot\text{s})}{2(4.4\times10^{6}\text{ eV})(1.6\times10^{-19}\text{ J/eV})} = 7.5\times10^{-23}\text{ s}.$

EVALUATE: The shorter the lifetime, the greater the energy width.

44.44. **IDENTIFY:** Apply the Heisenberg uncertainty principle in the form $\Delta E \Delta t \approx \hbar/2$. Let Δt be the mean lifetime.

SET UP: The rest energy of the ψ is 3097 MeV.

EXECUTE: $\Delta t = 7.6\times10^{-21}\text{ s} \Rightarrow \Delta E = \dfrac{\hbar}{2\Delta t} = \dfrac{1.054\times10^{-34}\text{ J}\cdot\text{s}}{2(7.6\times10^{-21}\text{ s})} = 6.93\times10^{-15}\text{ J} = 43\text{ keV}.$

$$\frac{\Delta E}{m_\psi c^2} = \frac{0.043 \text{ MeV}}{3097 \text{ MeV}} = 1.4\times10^{-5}.$$

EVALUATE: The energy width due to the lifetime of the particle is a small fraction of its rest energy.

44.45. **IDENTIFY:** Apply $\left|\frac{dN}{dt}\right| = \lambda N$ to find the number of decays in one year.

SET UP: Water has a molecular mass of 18.0×10^{-3} kg/mol.

EXECUTE: **(a)** The number of protons in a kilogram is

$(1.00 \text{ kg})\left(\frac{6.022\times10^{23} \text{ molecules/mol}}{18.0\times10^{-3} \text{ kg/mol}}\right)(2 \text{ protons/molecule}) = 6.7\times10^{25}$. Note that only the protons in the hydrogen atoms are considered as possible sources of proton decay. The energy per decay is $m_\text{p}c^2 = 938.3 \text{ MeV} = 1.503\times10^{-10}$ J, and so the energy deposited in a year, per kilogram, is

$$(6.7\times10^{25})\left(\frac{\ln 2}{1.0\times10^{18} \text{ y}}\right)(1 \text{ y})(1.50\times10^{-10} \text{ J}) = 7.0\times10^{-3} \text{ Gy} = 0.70 \text{ rad}.$$

(b) For an RBE of unity, the equivalent dose is $(1)(0.70 \text{ rad}) = 0.70$ rem.

EVALUATE: The equivalent dose is much larger than that due to the natural background. It is not feasible for the proton lifetime to be as short as 1.0×10^{18} y.

44.46. **IDENTIFY** and **SET UP:** $\phi \to \text{K}^+ + \text{K}^-$. The total energy released is the energy equivalent of the mass decrease.

EXECUTE: **(a)** The mass decrease is $m(\phi) - m(\text{K}^+) - m(\text{K}^-)$. The energy equivalent of the mass decrease is $mc^2(\phi) - mc^2(\text{K}^+) - mc^2(\text{K}^-)$. The rest mass energy mc^2 for the ϕ meson is given in Problem 44.43, and the values for K^+ and K^- are given in Table 44.3. The energy released then is $1019.4 \text{ MeV} - 2(493.7 \text{ MeV}) = 32.0 \text{ MeV}$. The K^+ gets half this, 16.0 Mev.

EVALUATE: **(b)** Does the decay $\phi \to \text{K}^+ + \text{K}^- + \pi^0$ occur? The energy equivalent of the $\text{K}^+ + \text{K}^- + \pi^0$ mass is $493.7 \text{ MeV} + 493.7 \text{ MeV} + 135.0 \text{ MeV} = 1122 \text{ MeV}$. This is greater than the energy equivalent of the ϕ mass. The mass of the decay products would be greater than the mass of the parent particle; the decay is energetically forbidden.

(c) Does the decay $\phi \to \text{K}^+ + \pi^-$ occur? The reaction $\phi \to \text{K}^+ + \text{K}^-$ is observed. K^+ has strangeness $+1$ and K^- has strangeness -1, so the total strangeness of the decay products is zero. If strangeness must be conserved we deduce that the ϕ particle has strangeness zero. π^- has strangeness 0, so the product $\text{K}^+ + \pi^-$ has strangeness -1. The decay $\phi \to \text{K}^+ + \pi^-$ violates conservation of strangeness. Does the decay $\phi \to K^+ + \mu^-$ occur? μ^- has strangeness 0, so this decay would also violate conservation of strangeness.

44.47. **IDENTIFY:** We are dealing with muons in cosmic rays. The energies involved are much greater than the rest energy of muons, so we must use the relativistic equations.

SET UP and **EXECUTE:** **(a)** We want the muon's speed. First find γ and then use it to find the speed.

$$E = m\gamma c^2 \quad \to \quad 6.000 \text{ GeV} = \gamma\,(105.7 \text{ MeV}) \quad \to \quad \gamma = 56.76443$$

$$v = c\sqrt{1-1/\gamma^2} = c\sqrt{1-1/(56.76443)^2} = 0.99969c = 2.997\times10^8 \text{ m/s}.$$

(b) We want the distance the muon travels in one lifetime. $x = vt = (2.997 \times 10^8 \text{ m/s})(2.197\ \mu\text{s}) = 658.4$ m.

(c) We want the distance to the earth's surface. To the muon, the 15.00 km is Lorentz contracted, so the distance it sees itself traveling is $L = L_0/\gamma$ = (15.00 km)/(56.76443) = 0.264 km = 264 m.
(d) We want the time in the earth's frame. The muon's frame is the proper frame.

$$\Delta t = \gamma \Delta t_0 = (56.76443)(2.197\ \mu\text{s}) = 124.7\ \mu\text{s}.$$

(e) We want the distance traveled as seen from the earth frame. $x = vt = (2.997 \times 10^8\ \text{m/s})(124.7\ \mu\text{s})$ = 37.4 km.
(f) As observed in the earth frame, the muon is created 15.00 km above the surface and travels 37.4 km, so it survives its trip through 15.00 km of atmosphere and travels downward into the surface of the earth. At such high energy, the muon can penetrate deeply into the surface without hitting anything, so the depth it reaches is 37.4 km – 15.00 km = 22.4 km.
EVALUATE: Without relativistic effects we would have far fewer cosmic rays striking the earth's surface.

44.48. **IDENTIFY:** The energy comes from the mass difference.
SET UP: $\Xi^- \to \Lambda^0 + \pi^-$. $p_\Lambda = p_\pi = p$. $E_\Xi = E_\Lambda + E_\pi$. $m_{\Xi^-}c^2 = 1322\ \text{MeV}$. $m_\pi c^2 = 139.6\ \text{MeV}$.

$$m_\Xi c^2 = \sqrt{m_\Lambda^2 c^4 + p^2c^2} + \sqrt{m_\pi^2 c^4 + p^2 c^2}.$$

EXECUTE: **(a)** The total energy released is

$$(m_{\Xi^-} - m_\pi - m_\Lambda)c^2 = 1322\ \text{MeV} - 139.6\ \text{MeV} - 1116\ \text{MeV} = 66.4\ \text{MeV}.$$

(b) $m_\Xi c^2 = \sqrt{m_\Lambda^2 c^4 + p^2c^2} + \sqrt{m_\pi^2 c^4 + p^2 c^2}$. $m_\Xi c^2 - \sqrt{m_\Lambda^2 c^4 + p^2c^2} = \sqrt{m_\pi^2 c^4 + p^2 c^2}$.
Square both sides:

$$m_\Xi^2 c^4 + m_\Lambda^2 c^4 + p^2c^2 - 2m_\Xi c^2 E_\Lambda = m_\pi^2 c^4 + p^2c^2.\ \ E_\Lambda = \frac{m_\Xi^2c^4 + m_\Lambda^2 c^4 - m_\pi^2 c^4}{2m_\Xi c^2}.$$

$$K_\Lambda = \frac{m_\Xi^2c^4 + m_\Lambda^2 c^4 - m_\pi^2 c^4}{2m_\Xi c^2} - m_\Lambda c^2.\ \ E_\pi = E_\Xi - E_\Lambda = m_\Xi c^2 - \frac{m_\Xi^2c^4 + m_\Lambda^2 c^4 - m_\pi^2 c^4}{2m_\Xi c^2}.$$

$$E_\pi = \frac{m_\Xi^2c^4 - m_\Lambda^2 c^4 + m_\pi^2 c^4}{2m_\Xi c^2}.\ \ K_\pi = \frac{m_\Xi^2c^4 - m_\Lambda^2 c^4 + m_\pi^2 c^4}{2m_\Xi c^2} - m_\pi c^2.$$

Putting in numbers gives

$$K_\Lambda = \frac{(1322\ \text{MeV})^2 + (1116\ \text{MeV})^2 - (139.6\ \text{MeV})^2}{2(1322\ \text{MeV})^2} - 1116\ \text{MeV} = 8.7\ \text{MeV (13\% of total)}.$$

$$K_\pi = \frac{(1322\ \text{MeV})^2 - (1116\ \text{MeV})^2 + (139.6\ \text{MeV})^2}{2(1322\ \text{MeV})^2} - 139.6\ \text{MeV} = 57.7\ \text{MeV (87\% of total)}.$$

EVALUATE: The two particles do not have equal kinetic energies because they have different masses.

44.49. **IDENTIFY:** The kinetic energy comes from the mass difference.
SET UP and **EXECUTE:** $K_\Sigma = 180\ \text{MeV}$. $m_\Sigma c^2 = 1197\ \text{MeV}$. $m_\text{n}c^2 = 939.6\ \text{MeV}$. $m_\pi c^2 = 139.6\ \text{MeV}$.
$E_\Sigma = K_\Sigma + m_\Sigma c^2 = 180\ \text{MeV} + 1197\ \text{MeV} = 1377\ \text{MeV}$. Conservation of the x-component of momentum gives $p_\Sigma = p_{\text{n}x}$. Then $p_{\text{n}x}^2 c^2 = p_\Sigma^2 c^2 = E_\Sigma^2 - (m_\Sigma c)^2 = (1377\ \text{MeV})^2 - (1197\ \text{MeV})^2$ $= 4.633 \times 10^5\ (\text{MeV})^2$. Conservation of energy gives $E_\Sigma = E_\pi + E_\text{n}$.
$E_\Sigma = \sqrt{m_\pi^2 c^4 + p_\pi^2 c^2} + \sqrt{m_\text{n}^2 c^4 + p_\text{n}^2 c^2}$. $E_\Sigma - \sqrt{m_\text{n}^2 c^4 + p_\text{n}^2 c^2} = \sqrt{m_\pi^2 c^4 + p_\pi^2 c^2}$. Square both sides:
$E_\Sigma^2 + m_\text{n}^2 c^4 + p_{\text{n}x}^2 c^2 + p_{\text{n}y}^2 c^2 - 2E_\Sigma E_\text{n} = m_\pi^2 c^4 + p_\pi^2 c^2$. $p_\pi = p_{\text{n}y}$, so

$$E_\Sigma^2 + m_\text{n}^2 c^4 + p_{\text{n}x}^2 c^2 - 2E_\Sigma E_\text{n} = m_\pi^2 c^4 \text{ and } E_\text{n} = \frac{E_\Sigma^2 + m_\text{n}^2 c^4 - m_\pi^2 c^4 + p_{\text{n}x}^2 c^2}{2E_\Sigma}.$$

$$E_{\text{n}} = \frac{(1377\text{ MeV})^2 + (939.6\text{ MeV})^2 - (139.6\text{ MeV})^2 + 4.633\times10^5\ (\text{MeV})^2}{2(1377\text{ MeV})} = 1170\text{ MeV}.$$

$$K_{\text{n}} = E_{\text{n}} - m_{\text{n}}c^2 = 1170\text{ MeV} - 939.6\text{ MeV} = 230\text{ MeV}.$$

$$E_{\pi} = E_{\Sigma} - E_{\text{n}} = 1377\text{ MeV} - 1170\text{ MeV} = 207\text{ MeV}.$$

$$K_{\pi} = E_{\pi} - m_{\pi}c^2 = 207\text{ MeV} - 139.6\text{ MeV} = 67\text{ MeV}.$$

$$p_{\text{n}}^2c^2 = E_{\text{n}}^2 - m_{\text{n}}^2c^2 = (1170\text{ MeV})^2 - (939.6\text{ MeV})^2 = 4.861\times10^5\ (\text{MeV})^2.$$

The angle θ the velocity of the neutron makes with the $+x$-axis is given by $\cos\theta = \dfrac{p_{\text{n}x}}{p_{\text{n}}} = \sqrt{\dfrac{4.633\times10^5}{4.861\times10^5}}$ and $\theta = 12.5^{\circ}$ below the $+x$-axis.

EVALUATE: The decay particles do not have equal energy because they have different masses.

44.50. **IDENTIFY:** The kinetic energy comes from the mass difference, and momentum is conserved.

SET UP: $|p_{\pi^+y}| = |p_{\pi^-y}|$. $p_{\pi^+}\sin\theta = p_{\pi^-}\sin\theta$ and $p_{\pi^+} = p_{\pi^-} = p_{\pi}$. $m_{\text{K}}c^2 = 497.7\text{ MeV}$.

$$m_{\pi}c^2 = 139.6\text{ MeV}.$$

EXECUTE: Conservation of momentum for the decay gives $p_{\text{K}} = 2p_{\pi x}$ and $p_{\text{K}}^2 = 4p_{\pi x}^2$.

$p_{\text{K}}^2c^2 = E_{\text{K}}^2 - m_{\text{K}}^2c^2$. $E_K = 497.7\text{ MeV} + 225\text{ MeV} = 722.7\text{ MeV}$, so

$p_{\text{K}}^2c^2 = (722.7\text{ MeV})^2 - (497.7\text{ MeV})^2 = 2.746\times10^5\ (\text{MeV})^2$ and

$p_{\pi x}^2c^2 = [2.746\times10^5\ (\text{MeV})^2]/4 = 6.865\times10^4\ (\text{MeV})^2$. Conservation of energy says $E_{\text{K}} = 2E_{\pi}$.

$E_{\pi} = \dfrac{E_{\text{K}}}{2} = 361.4\text{ MeV}$. $K_{\pi} = E_{\pi} - m_{\pi}c^2 = 361.4\text{ MeV} - 139.6\text{ MeV} = 222\text{ MeV}$.

$p_{\pi}^2c^2 = E_{\pi}^2 - (m_{\pi}c^2)^2 = (361.4\text{ MeV})^2 - (139.6\text{ MeV})^2 = 1.11\times10^5\ (\text{MeV})^2$. The angle θ that the velocity of the π^+ particle makes with the $+x$-axis is given by $\cos\theta = \sqrt{\dfrac{p_{\pi x}^2c^2}{p_{\pi}^2c^2}} = \sqrt{\dfrac{6.865\times10^4}{1.11\times10^5}}$, which gives $\theta = 38.2^{\circ}$.

EVALUATE: The pions have the same energy and go off at the same angle because they have equal masses.

44.51. **IDENTIFY** and **SET UP:** For nonrelativistic motion, the maximum kinetic energy in a cyclotron is $K_{\max} = \dfrac{q^2R^2}{2m}B^2$. The angular frequency is $\omega = |q|B/m$.

EXECUTE: **(a)** The rest energy of a proton is 938 MeV, and the kinetic energies in the data table in the problem are around 1 MeV or less, so there is no need to use relativistic expressions.

(b) Figure 44.51 shows the graph of $K_{\max}$ versus B^2 for the data in the problem. The graph is clearly a straight line and has slope equal to 6.748 MeV/T^2 = 1.081 $\times10^{-12}$ J/T^2. The formula for $K_{\max}$ is $K_{\max} = \dfrac{q^2R^2}{2m}B^2$, so a graph of $K_{\max}$ versus B^2 should be a straight line with slope equal to $q^2R^2/2m$.

Solving $q^2R^2/2m$ = slope for R gives

$$R = \sqrt{\frac{2m(\text{slope})}{q^2}} = \sqrt{\frac{2(1.673\times10^{-27}\text{ kg})(1.081\times10^{-12}\text{ J/T}^2)}{(1.602\times10^{-19}\text{ C})^2}} = 0.375\text{ m} = 37.5\text{ cm}.$$

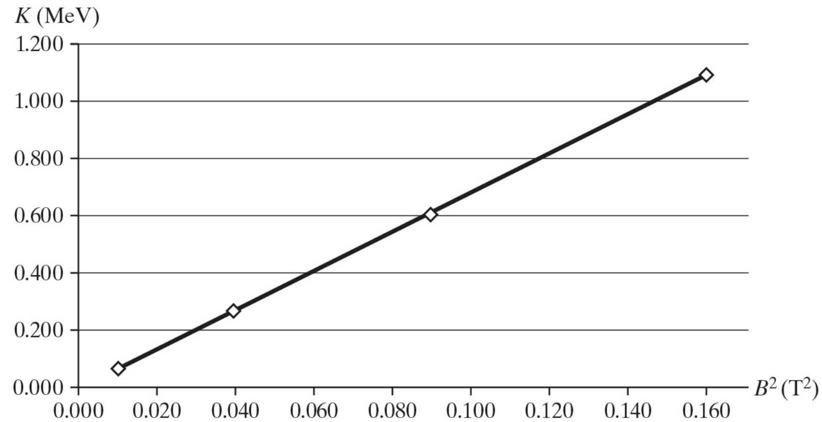


Figure 44.51

(c) Using the result from our graph, we get $K_{max} = (\text{slope})B^2 = (6.748\ \text{MeV/T}^2)(0.25\ \text{T})^2 = 0.42\ \text{MeV}$.
(d) The angular speed is $\omega = |q|B/m = (1.602\times10^{-19}\ \text{C})(0.40\ \text{T})/(1.67\times10^{-27}\ \text{kg}) = 3.8\times10^7$ rad/s.

EVALUATE: In part (c) we can check by using $K_{max} = q^2R^2B^2/2m = (qRB)^2/2m$. Using $B = 0.25$ T and the standard values for the other quantities gives $K_{max} = 6.75\times10^{-14}\ \text{J} = 0.42$ MeV, which agrees with our result.

44.52. **IDENTIFY:** Use Table 44.3 for data on the given particles. Apply conservation of energy in part (b).
SET UP: For any decay, conservation of energy tells us that $E_i = E_f$. If the decaying particle is at rest (or in its rest frame), this gives $m_i c^2 = m_{products}c^2 + K$.
EXECUTE: **(a)** The masses from Table 44.3 are:
Σ^-: 1197 MeV/c^2
Ξ^0: 1315 MeV/c^2
Δ^{++}: 1232 MeV/c^2
Ω^-: 1672 MeV/c^2
We see that Ω^- has the largest mass and Σ^- has the smallest mass.
(b) Solving $m_i c^2 = m_{products}c^2 + K$ for K gives $K = (m_i - m_{products})c^2$. Therefore the greater the difference between the mass of the decaying particle and the mass of decay products, the greater the kinetic energy. We show the decays and the mass differences below.
$\Sigma^- \to n + \pi^-$: $K = (1197 - 939.6 - 139.6)\ \text{MeV}/c^2 = 117.8\ \text{MeV}/c^2$
$\Xi^0 \to \Lambda^0 + \pi^0$: $K = (1315 - 1116 - 135)\ \text{MeV}/c^2 = 64\ \text{MeV}/c^2$
$\Delta^{++} \to p + \pi^+$: $K = (1232 - 938.3 - 139.6)\ \text{MeV}/c^2 = 154.1\ \text{MeV}/c^2$
$\Omega^- \to \Lambda^0 + K^-$: $K = (1672 - 1116 - 493.7)\ \text{MeV}/c^2 = 62.3\ \text{MeV}/c^2$
The kinetic energy is largest for the Δ^{++} decay and smallest for the Ω^- decay.
EVALUATE: A large-mass particle does not necessarily result in the release of more kinetic energy. For example, the Ω^- particle has more mass than the Δ^{++}, yet the decay products of the Ω^- have less kinetic energy than those of the Δ^{++} decay.

44.53. **IDENTIFY** and **SET UP:** Construct the diagram as specified in the problem. In part (b), use quark charges $\boldsymbol{u} = +\frac{2}{3}, \boldsymbol{d} = \frac{-1}{3}$, and $\boldsymbol{s} = \frac{-1}{3}$ as a guide.

EXECUTE: **(a)** The diagram is given in Figure 44.53. The Ω^- particle has $Q = -1$ (as its label suggests) and $S = -3$. Its appears as a "hole" in an otherwise regular lattice in the $S - Q$ plane.
(b) The quark composition of each particle is shown in the figure.

EVALUATE: The mass difference between each S row is around 145 MeV (or so). This puts the Ω^- mass at about the right spot. As it turns out, all the other particles on this lattice had been discovered already and it was this "hole" and mass regularity that led to an accurate prediction of the properties of the Ω^-!

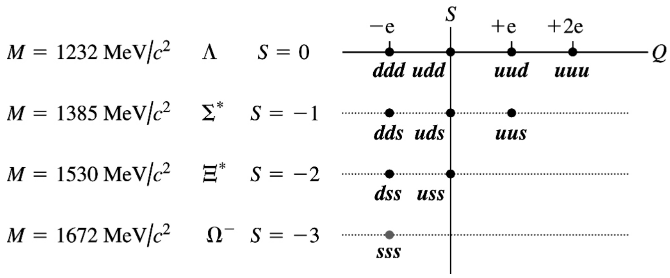


Figure 44.53

44.54. **IDENTIFY:** This problem deals with the decay of a kaon, K^+.
SET UP: We use relativistic equations and treat the neutrino as massless. The structure and decay of a K^+ are $K^+ = u\bar{d}$, $K^+ \to \bar{\mu}^+ + \nu_\mu$.

EXECUTE: **(a)** Energy conservation: $M_K c^2 = \gamma M_\mu c^2 + E_\nu$.

(b) Momentum conservation: The initial momentum is zero. The neutrino is treated as massless, so $p_\nu = E_\nu/c$. Momentum conservation gives $\gamma M_\mu v = E_\nu/c$.

(c) We want the speed of the muon. Combine the energy and momentum equations from (a) and (b). This leads to $M_K c^2 = \gamma M_\mu c^2 + \gamma M_\mu vc$.

Next substitute for σ and then for γ, and finally solve for v.

$$\sigma = \gamma(1 + v/c)$$

$$\sigma = \frac{1}{\sqrt{1 - v^2/c^2}}(1 + v/c)$$

$$v = c\frac{\sigma^2 - 1}{\sigma^2 + 1}.$$

(d) $\sigma = M_K/M_\mu = (493.7\ \text{MeV}/c^2)/(105.7\ \text{MeV}/c^2) = 4.671$.
(e) We want E_μ. First use the results of (c) and (d) to find v. $v = c(4.671^2 - 1)/(4.671^2 + 1) = 0.91234c$. Now use this result to find γ, giving $\gamma = 2.4424$. Now use $E_\mu = \gamma M_\mu c^2$ to find E_μ.
$E_\mu = (2.4424)(105.7\ \text{MeV}/c^2)c^2 = 258.2$ MeV.
(f) We want the energy of the neutrino. Combine $\gamma M_\mu v = E_\nu/c$ from part (b) and $v/c = 0.91234$ from part (e). This gives $E_\nu = \gamma M_\mu vc = \gamma M_\mu c^2(v/c) = (258.2\ \text{MeV})(0.91234) = 235.5\ \text{MeV}$.

(g) Adding the energy of the muon and neutrino gives 258.2 MeV + 235.5 MeV = 493.7 MeV, which is the rest energy of the kaon. So the answer is *yes*.
EVALUATE: The result in part (g) is consistent with the conservation of energy.

44.55. **IDENTIFY:** We are dealing with an expanding curved space.
SET UP: Follow the directions with each part.
EXECUTE: **(a)** We want D. If θ is in radians, then $D = R\theta$.
(b) We want V. Use $V = dD/dt$ and the result from part (a).

$$V = \frac{dD}{dt} = \frac{d(R\theta)}{dt} = \theta\frac{dR}{dt} = \frac{D}{R}\frac{dR}{dt}.$$

(c) If $V = BD$, find $B(t)$. Use the result from part (b).

$$V = \frac{D}{R}\frac{dR}{dt} = \frac{1}{R}\frac{dR}{dt}D = BD \rightarrow B = \frac{1}{R}\frac{dR}{dt}.$$

(d) We want B. R is increasing at a constant rate of 1.00 μm/s, so $R = R_0 + v_R t$. At 4 years R = 500.0 m + (1.00 μm/s)(4)(3.156 × 10^7 s) = 626 m.

$$B = \frac{1}{R}\frac{dR}{dt} = \left(\frac{1}{626\text{ m}}\right)(1.00\ \mu\text{m/s}) = 1.60\times10^{-9}\text{ s}^{-1}.$$

(e) We want D. $D = R\theta$ = (626 m)(π/3 rad) = 656 m.

(f) We want the separation speed V. Use the result from part (b).

$$V = \frac{dD}{dt} = \frac{D}{R}\frac{dR}{dt} = \left(\frac{656\text{ m}}{626\text{ m}}\right)(1.00\ \mu\text{m/s}) = 1.05\ \mu\text{m/s}.$$

(g) We want the time to reach Xibalba. In this universe, c = 6.35 μm/s and v_R = 1.00 μm/s. The waves travel along the circular arc, but this arc is increasing in length due to the expansion of space. For an infinitesimal time interval dt, the wave travels through an arc distance cdt and an angle $d\theta = (cdt)/R$. Using $R = R_0 + v_R t$, we have

$$d\theta = \frac{cdt}{R_0 + v_R t}$$

Integrating will give a relationship between θ and t.

$$\theta(t) = \int_0^t \frac{cdt'}{R_0 + v_R t'} = \frac{c}{v_R}\left[\ln(R_0 + v_R t) - \ln R_0\right] = \frac{c}{v_R}\ln\left(1 + \frac{v_R t}{R_0}\right).$$

Now solve for t when $\theta = \pi/3$.

$$t = \frac{R_0}{v_R}\left(e^{\pi v_R/3c} - 1\right).$$

From part (d) we know that when the ripple waves are sent, R = 626 m, so R_0 = 626 m. Putting in the numbers gives

$$t = \left(\frac{626\text{ m}}{1.00\ \mu\text{m/s}}\right)\left[e^{\pi(1.00\ \mu\text{m/s})[3(6.35\ \mu\text{m/s})]} - 1\right] = 1.122\times10^8\text{ s} = 3.56\text{ y}.$$

(h) We want the wavelength that the Xibalbans observe. Use Eq. (44.16) with v = 1.05 μm/s at the instant the ripple waves are sent, from part (f).

$$\lambda_0 = \lambda_s\sqrt{\frac{c+v}{c-v}} = (1.00\text{ nm})\sqrt{\frac{(6.35+1.05)\ \mu\text{m/s}}{(6.35+1.05)\ \mu\text{m/s}}} = 1.18\text{ nm}.$$

EVALUATE: The received wavelength is longer than the emitted wavelength. This result is reasonable because this “universe” is expanding.

44.56. **IDENTIFY:** Follow the steps specified in the problem. The Lorentz velocity transformation is given by

$$v_x = \frac{v_x' + u}{1 + uv_x'/c^2}.$$

SET UP: Let the +x-direction be the direction of the initial velocity of the bombarding particle.

EXECUTE: **(a)** For mass m, in $v_x = \dfrac{v_x' + u}{1 + uv_x'/c^2}$, $u = -v_{\text{cm}}$, $v' = v_0$, and so $v_m = \dfrac{v_0 - v_{\text{cm}}}{1 - v_0\, v_{\text{cm}}/c^2}$. For mass M, $u = -v_{\text{cm}}$, $v' = 0$, so $v_M = -v_{\text{cm}}$.

(b) The condition for no net momentum in the center of mass frame is $m\gamma_m v_m + M\gamma_M v_M = 0$, where γ_m and γ_M correspond to the velocities found in part (a). The algebra reduces to

$\beta_m\gamma_m = (\beta_0 - \beta')\gamma_0\gamma_M$, where $\beta_0 = \dfrac{v_0}{c}$, $\beta' = \dfrac{v_{\text{cm}}}{c}$, and the condition for no net momentum becomes

$m(\beta_0 - \beta')\gamma_0\gamma_M = M\beta'\gamma_M$, or $\beta' = \dfrac{\beta_0}{1+\dfrac{M}{m\gamma_0}} = \beta_0\dfrac{m}{m+M\sqrt{1-\beta_0^2}}$. $v_{\text{cm}} = \dfrac{mv_0}{m+M\sqrt{1-(v_0/c)^2}}$.

(c) Substitution of the above expression into the expressions for the velocities found in part (a) gives the relatively simple forms $v_m = v_0\gamma_0\dfrac{M}{m+M\gamma_0}$, $v_M = -v_0\gamma_0\dfrac{m}{m\gamma_0 + M}$. After some more algebra,

$\gamma_m = \dfrac{m+M\gamma_0}{\sqrt{m^2+M^2+2mM\gamma_0}}$, $\gamma_M = \dfrac{M+m\gamma_0}{\sqrt{m^2+M^2+2mM\gamma_0}}$, from which

$m\gamma_m + M\gamma_M = \sqrt{m^2+M^2+2mM\gamma_0}$. This last expression, multiplied by c^2, is the available energy E_a in the center of mass frame, so that

$E_\text{a}^2 = (m^2+M^2+2mM\gamma_0)c^4 = (mc^2)^2 + (Mc^2)^2 + (2Mc^2)(m\gamma_0c^2) = (mc^2)^2 + (Mc^2)^2 + 2Mc^2E_m$, which is Eq. (44.9).

EVALUATE: The energy E_a in the center-of-momentum frame is the energy that is available to form new particle states.

44.57. **IDENTIFY** and **SET UP:** Energy and momentum are conserved.

EXECUTE: The positron is moving slowly, so its only appreciable energy is its rest energy m_ec^2. The total energy released by the annihilation is $2m_ec^2$, but the two photons share it equally to conserve momentum. Therefore they also have equal energy, so each photon has energy m_ec^2, which is choice (d).

EVALUATE: If the positron had significant kinetic energy, the two photons would not have the same momentum and hence would not have the same energy.

44.58. **IDENTIFY** and **SET UP:** One photon travels 3 cm longer than the other one. If $2L$ is the distance between the detectors, one photon travels a distance L + 3 cm and other a distance L – 3 cm. The time for a photon to travel a distance x is $t = x/c$.

EXECUTE: $t_1 = (L + 3\text{ cm})/c$ and $t_2 = (L - 3\text{ cm})/c$. The time interval Δt between the arrival of the two photons is $\Delta t = t_1 - t_2 = (L + 3\text{ cm})/c - (L - 3\text{ cm})/c = (6\text{ cm})/c = (0.06\text{ m})/c = 0.2\times10^{-9}\text{ s} = 0.2\text{ ns}$. This is within the 10-ns window, so the two photons will be counted as simultaneous. Thus choice (d) is correct.

EVALUATE: 0.2 ns is well within the 10-ns window for simultaneity. The annihilation would have to occur over 1.5 m from the center for the photons not to be counted as simultaneous.

44.59. **IDENTIFY** and **SET UP:** The absorption of photons obeys the equation $N = N_0e^{-\mu x}$, where $\mu = 0.1\text{ cm}^{-1}$.

EXECUTE: $N/N_0 = e^{-\mu x} = e^{-(0.1\text{ cm}^{-1})(20\text{ cm})} = 0.14 = 14\%$. Choice (c) is correct.

EVALUATE: If 14% of the photons exit the body, 86% were absorbed within 20 cm of tissue.

CHAPTER 1
UNITS, PHYSICAL QUANTITIES, AND VECTORS

Discussion Questions

Q1.1 One correct experiment can disprove a theory, by showing that it doesn't agree with the outcome of the experiment. Experiments, no matter how many, can never prove a theory to be true. There is always the possibility of the experiment which hasn't been done yet that can show the theory to be wrong. But the more different experiments that agree with a theory, the more plausible and useful the theory is.

Q1.2 This is not possible. You can only take the tangent of an angle, and angles are dimensionless quantities. The tangent is the ratio of two lengths.

Q1.3 My height is 5 ft 6 in. This is 66 inches and equals (66 inches)(2.54 cm/1 in.) = 170 cm. My weight is 180 lbs. This is (180 lbs)(4.448 N/1 lb) = 800 N.

Q1.4 For selling tomatoes this rate of change is insignificant. In ten years the kilogram would increase in mass by 10×10^{-6} g = 1.0×10^{-8} kg. The percentage change in mass is $1.0\times10^{-6}\%$. Scales used for ordinary commerce are much less accurate than this, so the change is not noticeable in these applications. But for very precise scientific work the change could be of significance.

Q1.5 There are many possibilities: the period between rising and setting of the sun, a person's pulse, the time of fall of a rock dropped from a certain height, etc. Some time standards are better than others; a good time standard is stable over time, well defined, and easily reproducible.

Q1.6 Stack 100 sheets, measure the total thickness, and divide by 100.

Q1.7 Other examples are the tangent of an angle, the refractive index of a transparent material, the specific gravity of a liquid, etc.

Q1.8 The SI units of volume are m^3. The quantity $\pi r^3 h$ has dimensions of (length)^4 so can't be a volume.

Q1.9 Moe is precise but not accurate. Joe is accurate but not precise. Flo is accurate and precise. Precise means all arrows hit near to the same spot. Accurate means the arrows hit close to the center of the target.

Q1.10 If $\vec{A}$ is a unit vector, $\vec{A}\bullet\vec{A}=1$. $(\hat{i}+\hat{j}+\hat{k})\bullet(\hat{i}+\hat{j}+\hat{k})=1+1+1=3$, so $(\hat{i}+\hat{j}+\hat{k})$ is not a unit vector. $(\hat{i}+\hat{j}+\hat{k})/\sqrt{3}$ is a unit vector. $(3.0\hat{i}-2.0\hat{j})\bullet(3.0\hat{i}-2.0\hat{j})=9.0+4.0=13.0$, so $(3.0\hat{i}-2.0\hat{j})$ is not a unit vector. $(3.0\hat{i}-2.0\hat{j})/\sqrt{13.0}$ is a unit vector.

Q1.11 Displacement is the distance between the starting point and ending point. When the bicyclist has made a half-circle she is a distance $2R=1000$ m from her starting point, so her displacement is 1000 m, south. When she has made one complete circle she is back to her starting point and her displacement is zero.

Q1.12 It is not possible for two vectors with different length to have a vector sum of zero. The vector sum has its smallest magnitude when the two vectors are antiparallel, and in that case the magnitude of the vector sum is the difference in the lengths of the two vectors, and this is zero only when the two vectors have the same length. If three vectors are to have a vector sum of zero the length of any one can't be greater than the sum of the lengths of the other two.

Q1.13 Time is a scalar, not a vector. The elapsed time is always increasing and always positive and it is completely specified by a single number.

Q1.14 No, a vector includes both magnitude and direction. The instructions only specify direction.

Q1.15 No. The magnitude A of a vector is related to its components A_x and A_y by $A = \sqrt{A_x^2 + A_y^2}$. The only way to have $A = 0$ is for both A_x and A_y to be zero. No. A is greater than either $|A_x|$ or $|A_y|$.

Q1.16 (a) A vector has a positive magnitude and a direction. It does not make sense to say a vector is negative. (b) $\vec{\boldsymbol{B}}$ is the negative of $\vec{\boldsymbol{A}}$ if $\vec{\boldsymbol{B}} + \vec{\boldsymbol{A}} = \mathbf{0}$. Also, $\vec{\boldsymbol{B}}$ is the negative of $\vec{\boldsymbol{A}}$ if $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ have the same magnitude and opposite directions. This doesn't contradict what was said in part (a).

Q1.17 $C = A + B$ if and only if $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ are in the same direction. $C = 0$ if and only if either $\vec{\boldsymbol{A}} = \vec{\boldsymbol{B}} = \mathbf{0}$ or $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ have the same magnitude and are opposite in direction.

Q1.18 No. $\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = 0$ only if $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ are perpendicular. $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = \mathbf{0}$ only if $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ are either parallel or antiparallel.

Q1.19 $\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{A}} = A^2$, the square of the magnitude of $\vec{\boldsymbol{A}}$. $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{A}} = \mathbf{0}$.

Q1.20 The magnitude of $\vec{\boldsymbol{A}}/A$ is $A/A = 1$, so $\vec{\boldsymbol{A}}/A$ is a unit vector. The magnitude A is positive, so $\vec{\boldsymbol{A}}/A$ has the direction of $\vec{\boldsymbol{A}}$. $(\vec{\boldsymbol{A}}/A) \bullet \hat{\boldsymbol{i}} = \cos\theta$.

Q1.21 The percent error is the error divided by the quantity. Therefore, the percent error is $\dfrac{10.0 \text{ m}}{890 \times 10^3 \text{ m}} = 1.1 \times 10^{-3}\ \%$. Since the distance from Berlin to Paris was given as 890 km, the total distance covered should be expressed as 890,000 m. We know the total distance to only two significant figures.

Q1.22 (a) Yes. $\vec{\boldsymbol{A}}$ and $(\vec{\boldsymbol{B}} - \vec{\boldsymbol{C}})$ are each vectors; scalar product of two vectors is legitimate. (b) Yes. $(\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}})$ and $\vec{\boldsymbol{C}}$ are each vectors; vector product of two vectors is legitimate. (c) Yes. $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}} \times \vec{\boldsymbol{C}}$ are each vectors; scalar product of two vectors is legitimate. (d) Yes. $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}} \times \vec{\boldsymbol{C}}$ are each vectors; vector product of two vectors is legitimate. (e) No. $\vec{\boldsymbol{B}} \bullet \vec{\boldsymbol{C}}$ is a scalar. The vector product of a vector with a scalar is not defined.

Q1.23 <u>Not Equal</u>: Let $\vec{\boldsymbol{A}} = 2\hat{\boldsymbol{i}}$, $\vec{\boldsymbol{B}} = 4\hat{\boldsymbol{i}}$ and $\vec{\boldsymbol{C}} = 3\hat{\boldsymbol{j}}$. $\vec{\boldsymbol{B}} \times \vec{\boldsymbol{C}} = +12\hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{A}} \times (\vec{\boldsymbol{B}} \times \vec{\boldsymbol{C}}) = -24\hat{\boldsymbol{j}}$. $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = \mathbf{0}$ so $(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) \times \vec{\boldsymbol{C}} = \mathbf{0}$.
<u>Equal</u>: Let $\vec{\boldsymbol{A}} = 2\hat{\boldsymbol{i}}$, $\vec{\boldsymbol{B}} = 3\hat{\boldsymbol{j}}$ and $\vec{\boldsymbol{C}} = 4\hat{\boldsymbol{i}}$. $\vec{\boldsymbol{B}} \times \vec{\boldsymbol{C}} = -12\hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{A}} \times (\vec{\boldsymbol{B}} \times \vec{\boldsymbol{C}}) = +24\hat{\boldsymbol{j}}$. $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = +6\hat{\boldsymbol{k}}$ and $(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) \times \vec{\boldsymbol{C}} = +24\hat{\boldsymbol{j}}$.

Q1.24 $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$ is perpendicular to both $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$. If two vectors are perpendicular their scalar product is zero. Since $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$ is perpendicular to $\vec{\boldsymbol{A}}$, $\vec{\boldsymbol{A}} \bullet (\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}})$ is zero.

Q1.25 (a) No. $\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = AB\cos\phi$. If $\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = 0$ it could be that both A and B are nonzero but the angle

ϕ between $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is 90°. (b) No. $\left|\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}\right| = AB\sin\phi$. If $\left|\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}\right| = 0$ it could be that both A and B are nonzero but the angle ϕ between $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is either zero or 180°.

Q1.26 $\vec{\boldsymbol{A}}$ **= 0** says that $A = 0$. $A = \sqrt{A_x^2 + A_y^2}$. The only way that A can be zero is for both A_x and A_y to be zero.

CHAPTER 2
MOTION ALONG A STRAIGHT LINE

Discussion Questions

Q2.1 The speedometer measures the magnitude of the instantaneous velocity, the speed. It does not measure velocity because it does not measure direction.

Q2.2 Graph (d) . The dots represent the insect's position as a function of time. If the photographs are taken at equal spaced time intervals, then the displacement in successive intervals is increasing and this means the speed is increasing. Therefore, graphs (a) and (e) can be ruled out. Graph (b) shows decreasing acceleration so would correspond to the speed approaching a constant value, which is not what the photographs show. Graph (c) shows motion in the negative *x*-direction, which is not the case. This leaves graph (d). This graph shows velocity in the positive *x*-direction and increasing speed. This is consistent with the photographs.

Q2.3 The answer to the first question is yes. If the object is initially moving and the acceleration direction is opposite to the velocity direction, then the object slows down, stops for an instant and then starts to move in the opposite direction with increasing speed. An example is an object thrown straight up into the air. Gravity gives the object a constant downward acceleration. The object travels upward, stops at its maximum height and then moves downward. The answer to the second question is no. After the first reversal of the direction of travel the velocity and acceleration are then in the same direction. The object continues moving in the second direction with increasing speed.

Q2.4 Average velocity equals instantaneous velocity when the speed is constant and motion is in a straight line.

Q2.5 a) Yes. For an object to be slowing down, all that is required is that the acceleration be nonzero and for the velocity and acceleration to be in opposite directions. The magnitude of the acceleration determines the rate at which the speed is changing. b) Yes. For an object to be speeding up, all that is required is that the acceleration be nonzero and for the velocity and acceleration to be in the same direction. The magnitude of the acceleration determines the rate at which the speed is changing. But for any nonzero acceleration the speed is increasing when the velocity and acceleration are in the same direction.

Q2.6 Average velocity is the magnitude of the displacement divided by the time interval. Average speed is the distance traveled divided by the time interval. Displacement equals the distance traveled when the motion is in the same direction for the entire time interval, and therefore this is when average velocity equals average speed.

Q2.7 For the same time interval they have displacements of equal magnitude but opposite directions, so their average velocities are in opposite directions. One average velocity vector is the negative of the other.

Q2.8 If in the next time interval the second car had pulled ahead of the first, then the speed of the second car was greater. The second car could also be observed to be alongside a pedestrian standing at the curb, but that does not mean the pedestrian was speeding.

Q2.9 The answer to the first question is no. Average velocity is displacement divided by the time interval. If the displacement is zero, then the average velocity must be zero. The answer to the second question is yes. Zero displacement means the object has returned to its starting point, but its speed at that point need not be zero. See Fig. DQ2.9.

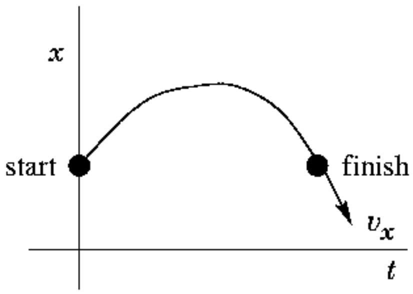


Figure DQ2.9

Q2.10 Zero acceleration means constant velocity, so the velocity could be constant but not zero. See Fig. DQ2.10. An example is a car traveling at constant speed in a straight line.

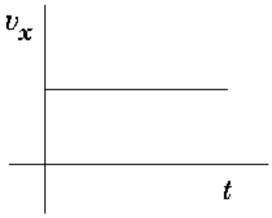


Figure DQ2.10

Q2.11 No. Average acceleration refers to an interval of time and if the velocity is zero throughout that interval, the average acceleration for that time interval is zero. But yes, you can have zero velocity and nonzero acceleration at one instant of time. For example, in Fig. DQ2.11, $v_x = 0$ when the graph crosses the time axis but the acceleration is the nonzero slope of the line. An example is an object thrown straight up into the air. At its maximum height its velocity is zero but its acceleration is g downward.

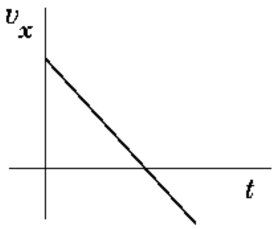


Figure DQ2.11

Q2.12 Yes. When the velocity and acceleration are in opposite directions the object is slowing down.

Q2.13 (a) Two possible *x*-*t* graphs for the motion of the truck are sketched in Fig. DQ2.13.
(b) Yes, the displacement is -258 m and the time interval is 9.0 s, no matter what path the truck takes between x_1 and x_2. The average velocity is the displacement divided by the time interval.

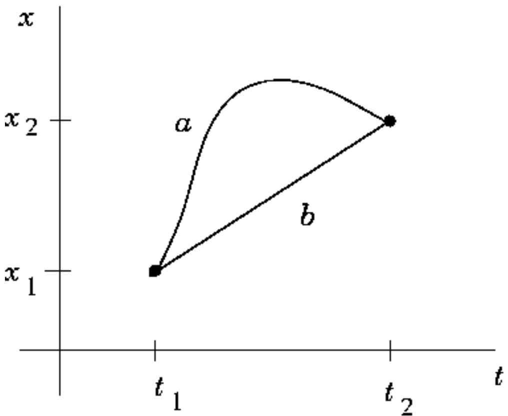


Figure DQ2.13

Q2.14 This is true only when the acceleration is constant. The average velocity is defined to be the displacement divided by the time interval. If the acceleration is not constant, objects can have the same initial and final velocities but different displacements and therefore different average velocities.

Q2.15 It is greater while the ball is being thrown. While being thrown, the ball accelerates from rest to velocity v_{0y} while traveling a distance less than your height. After it leaves your hand, it slows from v_{0y} to zero at the maximum height, while traveling a distance much greater than your height. Eq.(2.13) says that $a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)}$. Larger $y - y_0$ means smaller a_y.

Q2.16 (a) Eq.(2.13): $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. When an object returns to the release point, $y - y_0 = 0$. Eq.(2.13) then gives $v_y^2 = v_{0y}^2$ and $v_y = \pm v_{0y}$.
(b) $v_y = v_{0y} + a_y t$. At the highest point $v_y = 0$, so $t_{\text{up}} = -v_{0y} / a_y$. At the end of the motion, when the object has returned to the release point, we have shown in (i) that $v_y = -v_{0y}$,
so $t_{\text{total}} = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-2v_{0y}}{a_y}$ and $t_{\text{total}} = 2t_{\text{up}}$.

Q2.17 The distance between adjacent drops will increase. The drops have the downward acceleration $g = 9.8\text{ m/s}^2$ of a free-falling object. Therefore, their speed is continually increasing and the distance one drop travels in each successive 1.0 s time interval increases. A given drop has fallen for 1.0 s longer than the next drop released after it, so the additional distance it has fallen increases as they fall. Mathematically, let t be the time the second drop has fallen, so the first drop has fallen for time $t + 1.0\text{ s}$. The distance between these two drops then is
$\Delta y = \frac{1}{2}g(t + 1.0\text{ s})^2 - \frac{1}{2}gt^2 = \frac{1}{2}g\left[(2.0\text{ s})t + 1.0\text{ s}^2\right]$. The separation Δy increases as t increases.

Q2.18 Yes. Consider very small time intervals during which the acceleration doesn't have time to change very much, so can be assumed to be constant. Calculate $\Delta v_x = a_x\,\Delta t_1$, for a very small time interval, starting at $t = 0$. Then $v_{1x} = v_{0x} + \Delta v_x$. Since the acceleration is assumed constant for the small time interval, $v_{\text{av},x} = (v_{0x} + v_{1x})/2$ and $\Delta x_1 = v_{\text{av},x}\,\Delta t_1$. Then the position at the end of the interval is $x_1 = x_0 + \Delta x_1$. Repeat the calculation for the next small time interval Δt_2: $\Delta v_x = a_x\,\Delta t_2$, $v_{2x} = v_{1x} + \Delta v_x$, $v_{\text{av},x} = (v_{1x} + v_{2x})/2$, $\Delta x_2 = v_{\text{av},x}\,\Delta t_2$. Repeat for successive small time intervals.

Q2.19 In the absence of air resistance, the first ball rises to its maximum height and then returns to the level of the top of the building. When it returns to the height from which it was thrown, at the top of the building, it is moving downward with speed v_0. The rest of its motion is the same as for the second ball. (a) Since the last part of the motion of the first ball starts with it moving downward with speed v_0 from the top of the building, the two balls have the same speed just before they reach the ground. (b) The second ball reaches the ground first, since the first ball has to move up and then down before repeating the motion of the second ball. (c) Displacement is final position minus initial position. Both balls start at the top of the building and end up at the ground. So they have the same displacement. (d) The first ball has traveled a greater distance.

Q2.20 Let the $+x$-direction be east. The average velocity is the displacement divided by the time interval. The first 120.0 m displacement requires a time of $(120.0 \text{ m/s})/(3.00 \text{ m/s}) = 40.0 \text{ s}$. The second 120.0 m displacement requires a time of $(120.0 \text{ m/s})/(5.00 \text{ m/s}) = 24.0 \text{ s}$. The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{240.0 \text{ m}}{64.0 \text{ s}} = 3.75 \text{ m/s}$. This is less than 4.00 m/s since you spend more time running at 3.00 m/s than at 5.00 m/s.

Q2.21 At the highest point the object instantaneously has zero speed. But its velocity is continually changing, at a constant rate. Acceleration measures the rate of change of velocity. For example, in Fig. 2.25b when the graph crosses the time axis it still has a constant slope that corresponds to the acceleration. Also note the comments in part (d) of the solution to Example 2.7.

Q2.22 For an object released from rest and then moving downward in free-fall, its downward displacement from its initial position of $y_0 = 0$ is given by $y = \frac{1}{2}gt^2$. To increase y by a factor of 3, increase t by a factor of $\sqrt{3}$. You can also see this by letting Y be the original height, so $Y = \frac{1}{2}gT^2$. Let the new height be Y' and the corresponding time be T', so $Y' = \frac{1}{2}g(T')^2$. But $Y' = 3Y = 3\left(\frac{1}{2}gT^2\right)$, so $3\left(\frac{1}{2}gT^2\right) = \frac{1}{2}g(T')^2$ and $T' = \sqrt{3}T$.

CHAPTER 3
MOTION IN TWO OR THREE DIMENSIONS

Discussion Questions

Q3.1 The mass travels in an arc of a circle, so its radial acceleration is $a_{\text{rad}} = v^2 / R$. At the end point its speed is zero so $a_{\text{rad}} = 0$. The tangential component of acceleration causes the speed to change from zero. Therefore, at the ends of the swing the acceleration is tangent to the arc of swing and directed toward the midpoint. At the midpoint the speed of the mass is a maximum. There is a radial acceleration $a_{\text{rad}} = v^2 / R$, directed upward. Since the speed is a maximum, $dv / dt = 0$ and $a_{\tan} = 0$. At the midpoint the acceleration is radially upward. At the end points the acceleration is tangential and at the midpoint it is radial. At points in between both radial and tangential components of the acceleration are nonzero.

Q3.2 See Fig. DQ3.2. There is no component of $\vec{a}$ perpendicular to $\vec{v}$, so the direction of $\vec{v}$ doesn't change and the particle moves in a straight line. Since $\vec{a}$ and $\vec{v}$ are in opposite directions, the speed is decreasing (the particle is slowing down).

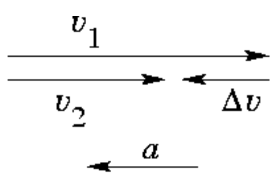


Figure DQ3.2

Q3.3 $\vec{a}$ is always vertically downward and $\vec{v}$ is always tangent to the path. There is no point where $\vec{a}$ and $\vec{v}$ are parallel. At the maximum height $\vec{v}$ is horizontal so that at that point $\vec{a}$ and $\vec{v}$ are perpendicular.

Q3.4 (a) The vertical component of the initial velocity remains zero and the time the book remains in the air is unchanged. The vertical motion is unaffected by the horizontal component of the initial velocity. (b) The horizontal distance the book travels while in the air doubles. The horizontal displacement is $x - x_0 = v_{0x}t$. v_{0x} doubles while t stays the same so $x - x_0$ doubles. (c) The seed of the book just before it reaches the floor is $v = \sqrt{v_x^2 + v_y^2}$. $v_x = v_{0x}$ since $a_x = 0$ so v_x doubles. v_y is unchanged. v increases, but by less than a factor of two.

Q3.5 The downward acceleration due to gravity doesn't depend on the horizontal velocity. Both bullets travel downward the same distance to reach the ground and both initially have zero vertical component of velocity, so both strike the ground at the same time.

Q3.6 After it falls out, the package maintains the horizontal component of velocity that it had while in the plane since it has no horizontal acceleration after it falls out. It accelerates downward due to gravity. Relative to the pilot the package travels straight down. Relative to a person on the ground the package travels both horizontally and vertically; the path relative to this person is a parabola.

Q3.7 The graphs are sketched in Fig. DQ3.7. $a_x = 0$, $a_y = -g$. $x = (v_0 \cos\alpha_0)t$, $v_x = v_x \cos\alpha_0$. $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$, $v_y = v_0 \sin\alpha_0 - gt$. a_x and a_y are constant. $x(t)$ is a straight line and $y(t)$ is a parabola. v_x is constant and $v_y(t)$ is a straight line.

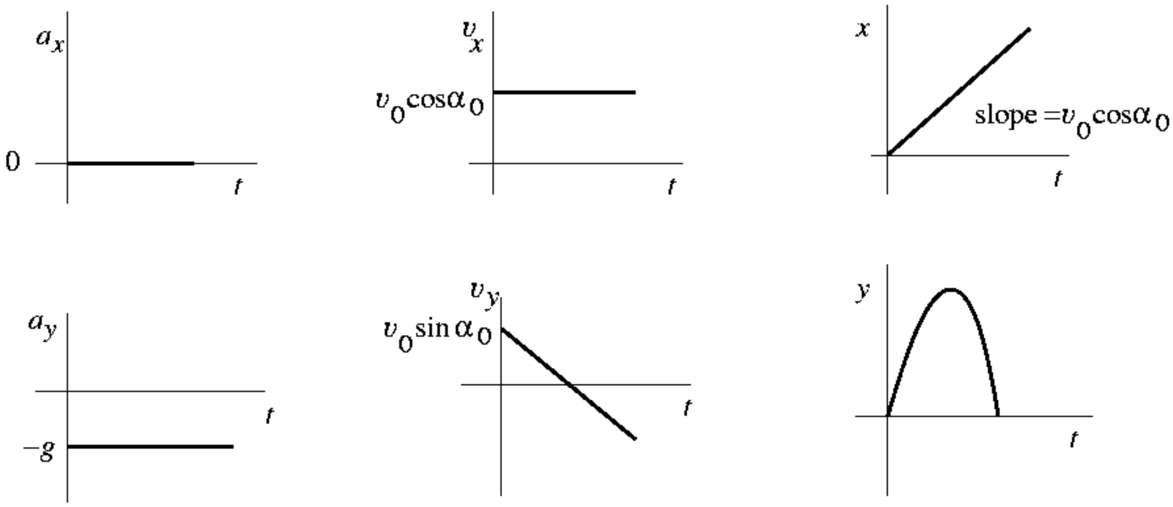


Figure DQ3.7

Q3.8 $R_{\max} = v_0^2 / g$. If the frog jumps straight up $v_{0y} = v_0$ and $h_{\max} = v_0^2 / 2g$ is the maximum height. Therefore $R_{\max} = 2h_{\max}$.

Q3.9 With $+y$ upward the initial velocity has components $v_x = v_0 \cos\theta$ and $v_y = v_0 \sin\theta$. At the maximum height, $v_y = 0$. Since $a_x = 0$, v_x is constant and at the maximum height $v_x = v_0 \cos\theta$. Therefore, at the maximum height the velocity vector is $\vec{\boldsymbol{v}} = (v_0 \cos\theta)\hat{\boldsymbol{i}}$ and the speed is $v = |\vec{\boldsymbol{v}}| = v_0 \cos\theta$. Throughout the motion the acceleration is g, downward, so at the maximum height the acceleration vector is $\vec{\boldsymbol{a}} = -g\hat{\boldsymbol{j}}$.

Q3.10 $\vec{\boldsymbol{v}}_{\text{av}} = \Delta\vec{\boldsymbol{r}} / \Delta t$. For one revolution the object returns to its starting point, so $\Delta\vec{\boldsymbol{r}} = \mathbf{0}$ and $\vec{\boldsymbol{v}}_{\text{av}} = \mathbf{0}$. $\vec{\boldsymbol{a}}_{\text{av}} = \Delta\vec{\boldsymbol{v}} / \Delta t$. After one revolution the velocity vector returns to its initial value so $\Delta\vec{\boldsymbol{v}} = \mathbf{0}$ and $\vec{\boldsymbol{a}}_{\text{av}} = \mathbf{0}$. In circular motion the directions of the velocity and acceleration are continually changing and average to zero over one complete revolution.

Q3.11 $a_{\text{rad}} = v^2 / R$. If the speed v is increased by a factor of 3, the acceleration increases by a factor of $3^2 = 9$. When the radius is decreased by a factor of 2 the acceleration increases by a factor of 2.

Q3.12 In circular motion, uniform or not, the velocity vector is tangent to the circular path. If the speed is not constant the acceleration has a tangential component and the acceleration is not perpendicular to the velocity.

Q3.13 The raindrops fall in the vertical direction relative to the ground. Their velocity relative to the moving car has both vertical and horizontal components and this is the reason for the diagonal streaks on the side window. The diagonal streaks on the windshield arise from a different reason. Air resistance pushes the drops off to one side of the windshield.

Q3.14 Hold the umbrella at an angle, so that the handle is parallel to the motion of the raindrops. This presents the greatest umbrella cross section to the rain.

Q3.15 To cross the river in the shortest time your velocity relative to the earth has the largest possible component perpendicular to the bank. Let S be the swimmer, E be the earth and W be the water.

$\vec{v}_{\text{S/E}} = \vec{v}_{\text{S/W}} + \vec{v}_{\text{W/E}}$. $\vec{v}_{\text{W/E}}$ is parallel to the bank, so $\vec{v}_{\text{S/E}}$ has its largest component perpendicular to the bank when $\vec{v}_{\text{S/W}}$ is in that direction. To cross the river in the shortest time you should head straight across. The current will then carry you downstream, so your path relative to the earth is directed at an angle downstream.

Q3.16 $v_x = v_0 \cos\alpha_0$, where α_0 is the launch angle, is constant throughout the motion, since $a_x = 0$. $|v_y|$ decreases to zero and then starts to increase. The speed is $\sqrt{v_x^2 + v_y^2}$. At $t = 0$, $v = v_0$. At the maximum height the speed has decreased to $v = v_0 \cos\alpha_0$, and then it increases. So, during the motion the speed reaches a minimum at the maximum height, but this minimum speed is not zero. Only graph (d) shows this behavior.

CHAPTER 4
NEWTON'S LAWS OF MOTION

Discussion Questions

Q4.1 No. For an object to be in equilibrium the net force on it must be zero. If there is one and only one non-zero force on the object, the net force isn't zero.

Q4.2 The ball is not in equilibrium. The only force on the ball is gravity. The net force is not zero. The ball has acceleration g downward. The ball has zero velocity at this instant but the velocity will not stay zero.

Q4.3 If the balloon is at rest its acceleration is zero and the net force on it is zero, so it is in equilibrium. The forces on it are the downward force of gravity and the upward buoyant force exerted by the air. These forces must be equal in magnitude, so they add to give zero net force.

Q4.4 The plane has no acceleration. You have the same velocity as the plane.

Q4.5 Let the force applied to each end have magnitude F. For any segment of the rope the force exerted on one end of the segment by the rest of the rope is F; this is what we mean by the tension in the rope.

Q4.6 Before you let go the velocity of the brick is tangent to the circular path. After you let go the only force on the brick is gravity and the brick has downward acceleration g. It has an initial horizontal velocity and travels in projectile motion with a parabolic path.

Q4.7 Newton's 1st law says an object moves with constant velocity unless it is acted on by a nonzero net force. When the car stops suddenly the passengers tend to continue to move with constant velocity, until forces provide an acceleration that stops them. When the car turns, the passengers tend to continue moving in a straight line as the car turns under them.

Q4.8 It is the absence of sufficient force directed opposite to the initial velocity that produces the effect. The passengers continue to move with constant speed until stopped by a force directed backwards. There is no forward force on the passengers.

Q4.9 One possibility is that the bus is accelerating forward so is speeding up, while the ball, with little horizontal force on it continues to move with the initial speed of the bus. Another possibility is that a net force was applied to the ball, for example by someone giving it a push. In the second case the ball was accelerated by the push but after the push is removed it rolls at constant speed relative to the bus. In the first case, so long as the bus is accelerating the ball gains speed relative to the bus. Also, in the first case all objects in the bus would be affected; the passengers would feel an increased horizontal force on them by the seat on which they are sitting as they are accelerated.

Q4.10 $m = F/a$. a has units of length/(time)^2 so m would have units of force$\cdot$(time)^2/length. In the SI system the units would be $\mathrm{N\cdot s^2/m}$.

Q4.11 An accelerating reference frame is an non-inertial frame. The surface of the earth is accelerating. It has $a_{\mathrm{rad}} = v^2/R$ due to the rotation of the earth about its axis and $a_{\mathrm{rad}} = v^2/R$ due to the revolution of the earth about the sun. These accelerations are small, but they prevent the frame of the earth from being precisely an inertial reference frame.

Q4.12 No, in all the cases the van is accelerating and is therefore an non-inertial frame.

Q4.13 This quantity is not a force. It represents the effect on the object of the net force on the object.

Q4.14 The elevator moves at constant speed so is an inertial reference frame. The value $g = 9.8$ m/s^2 is obtained, the same as when the elevator is at rest.

Q4.15 No, in this case the bus has a radial acceleration $a_{\text{rad}} = v^2 / R$. The bus turns while the ball travels in a straight line after it is thrown.

Q4.16 The quantity $g = 9.8\ \text{m/s}^2$ is the acceleration produced by the gravity force when it is the only force on an object. It is not a force, it is the acceleration that results from a force. The units m/s^2 are the units for acceleration, not for force.

Q4.17 The hurt comes from the force the rock exerts on your foot. If your foot is moving at high speed and the rock doesn't move, your foot is stopped over a short distance and its acceleration is large. The large acceleration is produced by a large force between your foot and the rock. But even if the force exerted by the pebble is less, it can be exerted over a smaller area and thus do more damage.

Q4.18 A large force is exerted when the acceleration is large. If the object's speed goes from v_0 to zero over a short distance or during a short time then the acceleration of the object is large.

Q4.19 You have the same speed just before impact in either case. But the distance over which you stop is much smaller for the concrete so your acceleration and the force exerted on you is much larger in that case.

Q4.20 The crumpling increases the stopping distance after impact and decreases the acceleration and therefore the force on the car and passengers. The sides and top don't crumple because they are close to the passengers, who could be crushed.

Q4.21 For a steady pull the acceleration is small and the tension in the string is only slightly greater than the weight of the object. If you jerk the string a large acceleration of the object is produced, until the string breaks. Therefore, in this case the tension in the string is much larger than the weight of the object.

Q4.22 When the crate is either at rest or moving at constant speed its acceleration is zero and the tension in the rope equals the weight of the crate. If the crate is traveling upward and speeding up, its acceleration is upward and the net force is upward; the tension in the rope is greater than the weight of the crate. If the crate is traveling upward and slowing down, its acceleration is downward and the tension is less than the weight.

Q4.23 The gravity force is proportional to the mass, so the gravity force on the 20-kg stone is twice the gravity force on the 10-kg stone. The force is related to the acceleration by $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$, so $\vec{\boldsymbol{a}} = \sum \vec{\boldsymbol{F}} / m$ and the 20-kg stone requires twice the net force to produce the same acceleration.

Q4.24 A kilogram is a unit of mass and a pound is a unit of force. The correct statement is that at a point where $g = 9.8\ \text{m/s}^2$, a 1.0-kg object has a weight (gravity force) of 2.2 lb.

Q4.25 The motion of an object depends on the forces on that object. The force of the horse on the wagon and the force of the wagon on the horse are a third-law pair and act on different objects.

Q4.26 False. Newton's third law requires that F and P always have equal magnitudes, independent of the other forces on you and the object and independent of any motion you or the object have.

Q4.27 By Newton's 3rd law, $\vec{\boldsymbol{F}}_{\text{T on C}} = -\vec{\boldsymbol{F}}_{\text{C on T}}$; the two forces are equal in magnitude and opposite in direction. This is true no matter how fast each vehicle is moving before the collision.

Q4.28 In both cases, in the absence of air resistance, the only horizontal force is the force that the highway surface applies to the wheels. The highway pushes on the wheels because the wheels push on the highway.

Q4.29 By Newton's 3rd law, the force the car exerts on the van is equal in magnitude and opposite in direction to the force the van exerts on the car. If the vehicles have the same acceleration, the one with the larger mass (the van) has the larger net force on it. In the absence of air resistance the net force on the van is the force exerted by the car. The net force on the car is the force the ground exerts on the car minus the force the van exerts on the car. The force of the ground on the car must be larger than the force of the car on the van.

Q4.30 The free-body force diagram for each person is given in Fig. DQ4.30. The magnitude of the force that *B* exerts on *A* equals the magnitude of the force that *A* exerts on *B*. $F_{\text{g on }A}$ is the magnitude of the force that the ground exerts on *A* and $F_{\text{g on }B}$ is the magnitude of the force that the ground exerts on *B*. The person who wins is the one for whom the ground exerts the larger force. The force of the ground on the person equals in magnitude the force that the person exerts on the ground. So, the winner is the one who exerts the greater horizontal force on the ground.

Figure DQ4.30

Q4.31 Both boxes have the same acceleration a. Treating both objects together as a single object, a force of 100 N gives acceleration a to mass $m_{\text{A}} + m_{\text{B}}$. The force that *A* exerts on *B* gives a smaller mass (m_B) the same acceleration, so this force must be less than 100 N. We can actually calculate this force. Treating both boxes together as a single object, $a = \dfrac{100 \text{ N}}{(150 \text{ N} + 50 \text{ N})/g} = g/2 = 4.9 \text{ m/s}^2$.

For box *B*, $F_{\text{A}} = m_{\text{B}}a = \left(\dfrac{50 \text{ N}}{g}\right)\left(\dfrac{g}{2}\right) = 25 \text{ N}$. By Newton's third law, box *B* pushed to the left on box *A* with force 25 N, so the net force on box *A* is $100 \text{ N} - 25 \text{ N} = 75 \text{ N}$. For box *A*, $ma = (150 \text{ N}/g)(g/2) = 75 \text{ N}$ so Newton's second law is satisfied.

Q4.32 The first statement is correct. The airplane has constant velocity and the net force on it is zero. The second statement is incorrect. When the airplane is either climbing or descending at a steady rate, its velocity is constant, its acceleration is zero, and the net force on it is still zero. In all these situations the net force equals the weight.

Q4.33 This works because of Newton's 1st law. You place the water into motion along with your hands. You suddenly stop or change the direction of motion of your hands and the water continues to move with constant velocity and leaves your hands.

Q4.34 By Newton's 1st law the blood in your head doesn't have sufficient upward force on it to accelerate upward as rapidly as your head. Blood doesn't stay in your head and this causes the sensation.

Q4.35 The person's lower body receives a large forward acceleration due to the force from the car seat in which the person is sitting. The person's head stays at rest until the force from the neck accelerates it to follow the rest of the body. The head lagging behind produces the whiplash.

Q4.36 It is sloppy physics language to say the passenger is "thrown through the windshield." The windshield stops suddenly due to large forces on the car and the passenger continues to move forward.

Q4.37 By Newton's 3rd law the force the large car exerts on the small car equals in magnitude the force the small car exerts on the large car. Equal forces produce unequal accelerations because the masses are different. The small car experiences the greater acceleration. The passengers in the smaller car experience greater accelerations and have larger forces on them.

Q4.38 (i) It is not possible to determine if the rocket is moving at constant velocity. (ii) Perform experiments and see if Newton's 2nd law is obeyed in the frame of the rocket. If the rocket is accelerating Newton's 2nd law won't be obeyed.

CHAPTER 5
APPLYING NEWTON'S LAWS

Discussion Questions

Q5.1 The force diagram for the man plus seat is given in Fig. DQ5.1a. $2T = w_{\text{tot}}$ and therefore $T = w_{\text{tot}}/2$. The tension in the rope is $w_{\text{tot}}/2$, where w_{tot} is the weight of the man plus the weight of the seat. The force diagram for the man is given in Fig. DQ5.1b. $T + n = w_{\text{man}}$ and $n = w_{\text{man}} - T = w_{\text{man}} - w_{\text{tot}}/2 = w_{\text{man}}/2 - w_{\text{seat}}/2$. If the weight of the seat can be neglected compared to the weight of the man, then $T = n = w_{\text{man}}/2$. In this case the tension in the rope and the force of the seat on the man are each half the weight of the man. If the weight of the seat equals the weight of the man, $n = 0$ and $T = w_{\text{man}}$; the seat exerts no force on the man and the tension in the rope equals the weight of the man. If the seat weighs more than the man and the man isn't strapped in, then $T > w_{\text{man}}$ and $n \to 0$. The man accelerates upward and the seat accelerates downward.

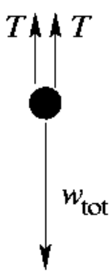


Figure DQ5.1a

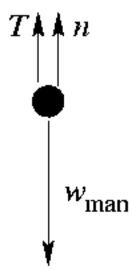


Figure DQ5.1b

Q5.2 (1) If a book sits at rest on a horizontal tabletop the only vertical forces on the book are the upward normal force and the downward weight, and these forces are equal in magnitude. (2) If a crate sits at rest on a ramp that is inclined at an angle α above the horizontal, the normal force is $n = w\cos\alpha$. (3) If a crate sits on a horizontal surface and you push on the book with force P that is at an angle below the horizontal, then $n = w + P\sin\alpha$.

Q5.3 Only the vertical components of the tension at the ends of the rope hold it up; $2T\sin\alpha = w$, where α is the angle of the rope below the horizontal at each support. $T = w/(2\sin\alpha)$. As $\alpha \to 0$, $T \to \infty$.

Q5.4 The force diagram for the car is given in Fig. DQ5.4. w is the weight of the car. This force is vertical. It has a component $w\cos\alpha$ perpendicular to the incline and component $w\sin\alpha$ parallel to the incline. f is the air resistance force exerted by the air on the car. It is directed opposite to the velocity of the car so is parallel to the ground and directed down the hill. The force the ground exerts on the car has two components. The component perpendicular to the ground is the normal force n. The component parallel to the ground is F; the ground pushes on the tires because the tires push on the ground. It is the force F that pushes the car up the hill; the ground pushes the car up the hill.

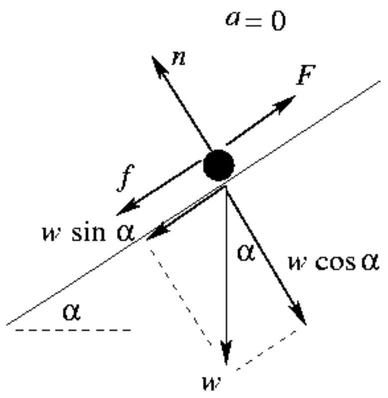


Figure DQ5.4

Q5.5 Use Newton's 2nd law. Apply a known force to the astronaut (for example with a rope attached to a spring balance) and measure his acceleration.

Q5.6 Push parallel to the ramp: The force diagram for the box is given in Fig. DQ5.6a. $P > f + w\sin\alpha$.

Push horizontally: The force diagram for the box is given in Fig. DQ5.6b. $P\cos\alpha > f + w\sin\alpha$.

When you push horizontally only the component of your push parallel to the ramp is effective at moving the box up the ramp. Not only that, but the other component of your push, the component perpendicular to the ramp, increases the normal force which in turn increases the friction force that opposes the motion.

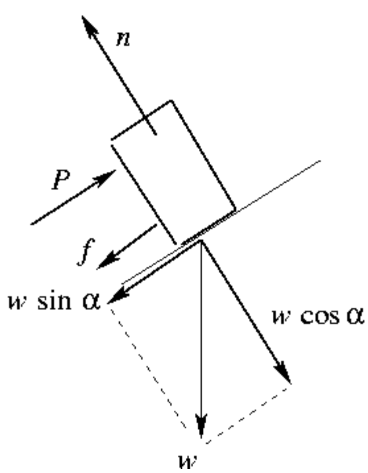


Figure DQ5.6a

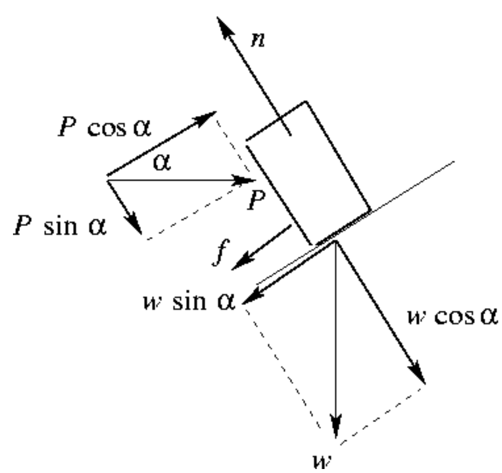


Figure DQ5.6b

Q5.7 When she releases the briefcase, the only force on it is gravity and in the inertial frame of the earth it accelerates downward with $a = g$. If the briefcase isn't moving relative to the elevator, the elevator must also be accelerating downward with acceleration $a = g$.

Q5.8 There is a component $w\sin\alpha$ of the weight of the block that is directed down the incline. To start the block moving down the incline, the force P you apply must satisfy $P + w\sin\alpha > f$ and $P > f - w\sin\alpha$. $w\sin\alpha$ is in the direction of your push and adds to it. To start the block moving up the incline, P must satisfy $P > w\sin\alpha + f$. $w\sin\alpha$ is opposite to your push and subtracts from it. To push the block sideways, it must be that $P > f$. It is easiest to get it moving down the incline and hardest to get it moving up the incline.

Q5.9 Let the crate have mass m and let the ramp be inclined at an angle α above the horizontal. Since the ramp is stationary, the kinetic friction force is always directed opposite to the motion of the crate. When the crate is moving up the ramp, the kinetic friction force f_{k} that the ramp exerts on the crate is directed down the incline and the net force on the crate is $mg\sin\alpha + f_{\mathrm{k}}$, directed down the incline. The acceleration of the crate is $a_{\mathrm{up}} = (mg\sin\alpha + f_{\mathrm{k}})/m$. When the crate is moving down the ramp, the kinetic friction force f_{k} is directed up the incline and the net force on the crate is $mg\sin\alpha - f_{\mathrm{k}}$, directed down the incline. The acceleration of the crate is $a_{\mathrm{down}} = (mg\sin\alpha - f_{\mathrm{k}})/m$. This shows that a_{up} is larger in magnitude than a_{down}. The crate slows down at a greater rate when it is going up the ramp than the rate at which it speeds up when it is coming down the ramp. The result is that the speed of the crate when it returns to the bottom of the ramp is less than the speed it had initially when it started to slide up the ramp. When the crate is going up the component of the gravity force parallel to the ramp and the kinetic friction force are in the same direction and when the crate is coming down they are in opposite directions. The net force on the crate, and hence its acceleration, is greater while it is sliding up than while it is sliding down.

Q5.10 In either case the component of your force F that is horizontal is $F\cos\theta$. But if you pull at an angle above the horizontal your force has an upward component $F\sin\theta$ and the normal force is $n = w - F\sin\theta$. If you push at an angle below the horizontal your force has an downward component $F\sin\theta$ and $n = w + F\sin\theta$. If you pull above the horizontal the normal force is reduced; if you push below the horizontal the normal force is increased. Increasing the normal force increases the friction force that opposes the motion. You exert a smaller force when you pull at an angle above the horizontal.

Q5.11 The friction force is always parallel to the surface that applies it. (a) Can't do. A horizontal friction force in toward the center of the curve is required in order for the car to move in an arc of a circle. (b) Can do. You jump into the air by pushing downward on the floor. (c) Can't do. To start walking requires a horizontal acceleration, which requires a horizontal force. (d) Can do, by pushing upward on the ladder with your feet. But as in (b) it is more difficult without friction, to avoid slipping. (e) Can't do. To change the direction of the car's velocity requires a horizontal friction force exerted by the highway surface on your tires.

Q5.12 The coefficient of kinetic friction is much less than the coefficient of static friction. The maximum static friction force is much larger than the kinetic friction force; once slipping starts the friction force is greatly reduced.

Q5.13 The acceleration of the elevator affects the normal force, and the kinetic friction force that you must push against is proportional to the normal force. When the elevator is accelerating upward, the crate is also accelerating upward and the upward normal force is greater than its weight. When the elevator and crate are accelerating downward, the normal force is less than the weight and when the elevator is traveling at constant speed the normal force equals the weight. The force you must apply is greater when the elevator is accelerating upward and least when it is accelerating downward.

Q5.14 (a) Place a glass of water on a piece of paper. Pull horizontally on the paper so that it accelerates. The friction force that the paper exerts on the glass of water accelerates the glass, causing it to move horizontally along with the paper. (b) Drop a box onto a rapidly moving horizontal conveyor belt. The friction force the belt exerts on the box causes the box to start to move horizontally in the direction the belt is moving. Until the box attains the same speed as the belt it is slipping relative to the belt and the friction is kinetic.

Q5.15 The speed doesn't change because there is no component of net force in the direction of the particle's velocity. The net force and hence the acceleration are perpendicular to the velocity, and this produces a change in the direction of the velocity but not in its magnitude.

Q5.16 $a_{\text{rad}} = v^2 / R$. At 80 km/h the inward horizontal component of the normal force n equals ma_{rad}. At 20 km/h, a_{rad} is significantly less and the inward component of n is much greater than ma_{rad}. The horizontal component of n pushes the car out of its circular path; it slides down the banked roadway and off the road.

Q5.17 The acceleration of the ball is horizontal so in the vertical direction the net force must be zero. The tension in the string must have an upward component to balance the downward gravity force on the ball. Therefore, the string cannot be truly horizontal, it must slope downward below the horizontal as viewed from the center of the circle toward the ball. See Fig. DQ5.17.

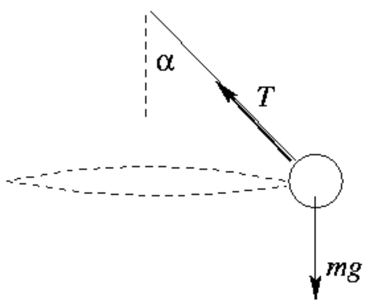


Figure DQ 5.17

Q5.18 There is no such thing as a separate centrifugal force. This is just a name given to the net force, the vector sum of the actual forces, for circular motion.

Q5.19 No. The velocity of the stopper is tangential and the stopper will move in the tangential direction if the horizontal force on it is removed.

Q5.20 At the bottom of the circle, $n = mg + ma_{\text{rad}}$. At the bottom of the circle v is largest and $a_{\text{rad}} = v^2 / R$ can be large. a_{rad} is reduced by increasing R.

Q5.21 With the air removed the ball is in free-fall and has downward displacement $\Delta y = v_{\text{av}-y}\,\Delta t$ in each successive time interval between flashes. Since $v_{\text{av}-y}$ for each time interval increases as the ball speeds up, the displacement of the ball between successive flashes increases. With air resistance the acceleration decreases as the ball falls and gains speed and its displacement between flashes increases less rapidly. If the ball reaches terminal speed, then the distance it travels between flashes becomes constant.

Q5.22 (a) In the absence of air resistance $a_y = -g$ throughout the motion. So, constant acceleration equations apply: $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. When $y - y_0 = 0$, $v_y = -v_{0y}$; the speed when it returns is v_0. (b) With air resistance it doesn't go as high and speeds up less rapidly on the way back down, so has speed less than v_0 when it returns.

Q5.23 On the way up, the air resistance force is downward and the acceleration is greater than *g*. On the way down, the air resistance force is upward and the acceleration is less than *g*. The distance up is the same as the distance down, so it takes longer to come down than to go up.

Q5.24 If air resistance is negligible, each ball has the same downward acceleration g and strikes the ground at the same time. With air resistance f, $mg - f = ma_y$. $a_y = g - f/m$ and a_y is less than g. The air resistance force f depends on speed and on size and shape of the object but not on the object's mass. At a given speed f will be the same for both balls but the effect on a_y will be less for the more massive one, the one filled with water. The one filled with water will strike the ground first.

Q5.25 The net downward force on the ball is its weight *mg*, which is constant as the ball falls, minus the upward force due to air resistance. The upward air resistance force increases as the speed of the ball increases as it falls. So, the net downward force decreases and the magnitude of the downward acceleration of the ball decreases as the ball falls. The acceleration steadily decreases, until it becomes zero at the terminal velocity. Graph (d) matches this behavior.

Q5.26 "Dropped from rest" means the initial speed of the ball is zero. The speed of the ball increases until the ball reaches the terminal speed, at which point the speed stays constant. Graph (a) matches this behavior.

Q5.27 There is a horizontal component of air resistance that causes the horizontal component of the velocity to decrease. The ball is traveling with smaller horizontal component of velocity on the way down so travels a shorter horizontal distance.

Q5.28 The only horizontal force on the ball is the horizontal component of the air resistance force. This force is opposite to the horizontal component of the ball's velocity and causes the horizontal component of velocity to decrease. This continues until the horizontal component of velocity is zero and then the horizontal component of air resistance is also zero. The ball reaches its horizontal terminal velocity, which is zero. After the horizontal component of velocity has become zero, the ball moves vertically downward.

CHAPTER 6
WORK AND KINETIC ENERGY

Discussion Questions

Q6.1 No. Whether work is positive or negative does not depend on any choice of coordinates. Work done on an object by a force for a specified displacement of the object is positive if the force has a component in the direction of the displacement. The work is negative if the force has a component opposite to the direction of the displacement of the object.

Q6.2 $W_{\text{tot}} = \Delta K$. At constant speed means $\Delta K = 0$ and $W_{\text{tot}} = 0$. If friction can be neglected, for any upward displacement of the elevator the positive work done by the tension in the cables equals in magnitude the negative work done by gravity.

Q6.3 You must specify the object on which the work is done. W_{tot} in the work-energy theorem is the total work done on the object whose kinetic energy is K. The two forces in a Newton's 3rd law pair do work on different objects.

Q6.4 $W = \frac{1}{2}mv^2$ since the object starts from rest. Let W' and v' be the work and speed when twice the work is done, so $W' = \frac{1}{2}m(v')^2$. $W' = 2W$, so $\frac{1}{2}m(v')^2 = 2\left(\frac{1}{2}mv^2\right)$. This gives $v' = \sqrt{2}v$. The speed increases by a factor of $\sqrt{2}$ when twice the work is done.

Q6.5 Yes. If the net force is always perpendicular to the displacement of the object then the net force doesn't do any work. An example is a block moving in a horizontal circle on the end of a string. If the block moves on a horizontal, frictionless surface, then the net force on the block is the tension in the string. This force is radial and there is no displacement in this direction and the tension does no work on the block. The net force changes the direction of the velocity but doesn't change the speed and the kinetic energy of the object is constant, as the work-energy theorem says it should be when no net work is done.

Q6.6 The tension has the same magnitude at both ends of the rope. The tension does positive work on the cart and negative work on the bucket. The displacements of the cart and bucket have the same magnitude, so the work done on each by the tension has the same magnitude and the total work done by the tension is zero.

Q6.7 There is no work done on the bob. The speed of the bob is constant so its kinetic energy is constant. By the work-energy theorem the total work done on the bob is zero. The tension F is directed along the wire. The bob has no displacement in this direction so this force does no work. The weight mg of the bob is vertically downward. The bob has no displacement in this direction so the gravity force does no work.

Q6.8 In each case the only force that does work on the object is gravity. The work done by gravity is given by $W_{\text{grav}} = mgh$ and this is the same in all three cases. Since the object is released from rest, $W_{\text{grav}} = K_{\text{f}}$, where K_{f} is the kinetic energy the object has at the bottom. (i) In (c) the object has greater mass and hence less speed for the same kinetic energy. Therefore, cases (a) and (b) have the same speed at the bottom and greater speed than case (c). (ii) For all three cases the same amount of work is done.

Q6.9 The force must do both positive and negative work, because for some of the displacement the force is in the $-x$-direction and for some of the displacement the force is in the $+x$-direction. The net work done by $\vec{F}$ is zero if the positive and negative works have the same magnitude. A possible graph of F versus x is sketched in Fig. DQ6.9.

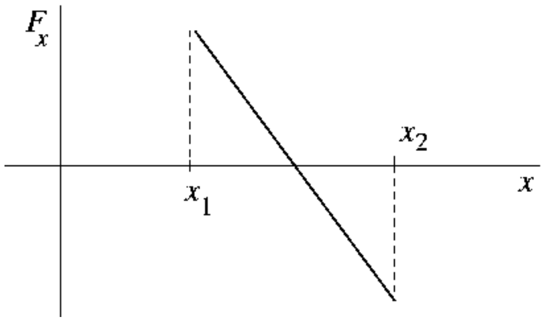


Figure DQ6.9

Q6.10 The change in speed is 5 m/s in both cases, but the kinetic energy depends on v^2 and the change in v^2 is larger for the 15 to 20 m/s speed change.

Q6.11 The direction of the velocity is different for each object; they do not have the same velocity. The speed (the magnitude of the velocity) is the same for all three objects. The kinetic energy depends on the magnitude of the velocity, not on its direction, and the three objects all have the same kinetic energy.

Q6.12 Yes, the total work can be negative. For example, the net force could be opposite to the displacement throughout the motion. An example is the motion of a ball thrown upward. In the absence of air resistance the net force is gravity and is downward. For the motion from just after it leaves the thrower's hand to its maximum height the displacement is upward and the net work done on the ball is negative. No, the negative total work cannot have a magnitude larger than the initial kinetic energy of the object. The work-energy theorem says that $W_{\text{tot}} = K_2 - K_1$. $K_2 = K_1 + W_{\text{tot}}$. If the total work is negative and if its magnitude were larger than K_1,then this equation would say $K_2 < 0$ and this is not possible. For example, consider a net force in a direction opposite to the initial velocity of the object. The force does negative work until it has removed all the kinetic energy of the object and the object is momentarily at rest. After that the object starts moving in the direction of the force and the force starts doing positive work.

Q6.13 $W_{\text{tot}} = K_{\text{final}}$ since $K_{\text{initial}} = 0$. Thus, $W_{\text{tot}} = \frac{1}{2}mv_{\text{final}}^2$. $W_1 = \frac{1}{2}mv_1^2$ and $W_2 = \frac{1}{2}mv_2^2$. If $v_2 = 3v_1$, then $W_2 = 9W_1$. To produce three times the final speed must do nine times the work.

Q6.14 No. In the frame of the driver the truck is already at rest and no work is needed.

Q6.15 (a) There is no vertical displacement of the briefcase so the vertical component of the force your hand exerts does no work. The kinetic energy of the briefcase doesn't change, so in the absence of air resistance there is no horizontal component of the force your hand exerts, and this force does no work. In the presence of air resistance, the force your hand exerts does a small amount of positive work, equal in magnitude to the negative work done by the air resistance force. (b) There is an upward displacement of the briefcase so the upward force your hand exerts does positive work on the briefcase. The positive work done by your hand equals in magnitude the negative work done by gravity, if the initial and final speeds of the briefcase are the same.

Q6.16 Yes, if the displacement of the object is in the direction of the friction force. For a box in the back of an accelerating truck the friction force on the box is the net force that causes the box to accelerate along with the truck (if the box doesn't touch the sides of the truck and if it doesn't slip).

The positive work done by friction equals the increase in kinetic energy of the box.

Q6.17 The work you do is mgh, where m is your mass and h is the vertical height you travel. Your average power is mgh/t ,where t is the time it takes you to run up the stairs. 1 hp = 746 W.

Q6.18 (a) "Strong" indicates the force the person can apply. Power is the rate of doing work. A small force can result in large power if the force is applied to an object that is moving at large speed. (b) The worker must apply a large upward force on the bag equal to the weight of the bag. But this force does no work because the displacement of the bag is horizontal and has no component in the direction of the force.

Q6.19 No. 28,000 hp = (28,000 hp)(746 W/1 hp) = 20.9 MW. The power output (30 MW) would exceed the power input (20.9 MW), and this violates conservation of energy.

Q6.20 $P = Fv = mav$, if the motion is horizontal and if air resistance can be neglected so that the force F supplied by the engine is the net force. As v increases, a decreases; the acceleration is greater at the beginning.

Q6.21 $P = dW/dt$, so $dW = P\,dt$ and the area under the P versus t curve is the work done between t_1 and t_2. $P_{\text{av}} = \Delta W/\Delta t$, so P_{av} is the area divided by $t_2 - t_1$. The graph is given in Fig. DQ6.21 and the calculation is as follows: area = $\frac{1}{2}P_{\text{peak}}(t_2 - t_1)$. P_{av} = area/($t_2 - t_1$). P_{av} = $\frac{1}{2}P_{\text{peak}}$.

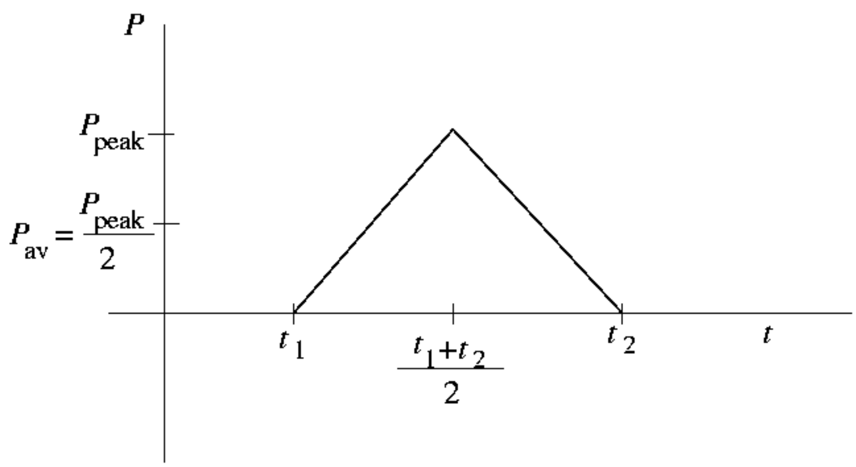


Figure DQ6.21

Q6.22 (a) Yes. If the net force does zero work during a displacement then the particle's speed remains constant. An example is for a particle moving in a circle at constant speed. The net force is radially inward and does no work. (b) No. By $\sum \vec{F} = m\vec{a}$, the net force produces an acceleration and this changes the velocity of the particle. (c) Yes. As reasoned in (a) the speed can remain constant. The kinetic energy depends on the speed of the particle and is independent of the direction of the velocity, so the kinetic energy can remain constant.

Q6.23 $W = \frac{1}{2}kx^2$ and $F = kx$, so $x = \dfrac{F}{k}$ and $W = \dfrac{F^2}{2k}$. If F is doubled then the spring stretches a distance $2x$ and the work done is $4W$.

Q6.24 Let W_1 be the work required to stretch the spring a distance x and let W_2 be the work required to stretch the spring a distance $2x$ from its unstretched length. Then $W_1 = W = \frac{1}{2}kx^2$ and $W_2 = \frac{1}{2}k(2x)^2 = 4W$. The additional work to stretch the spring an additional distance x is $W_2 - W_1 = 4W - W = 3W$.

CHAPTER 7
POTENTIAL ENERGY AND ENERGY CONSERVATION

Discussion Questions

Q7.1 The air resistance force is directed opposite to the motion and hence does negative work W_{other}, both for the upward and downward displacements of the baseball. The ball returns to the same height, so $U_1 = U_2$. $K_2 = K_1 + W_{\text{other}}$; $K_2 < K_1$ since $W_{\text{other}} < 0$.

Q7.2 At the maximum height $K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m(v_0 \cos\alpha_0)^2$, where v_0 is the launch speed and α_0 is the launch angle, since at the maximum height $v_y = 0$ and $v_x = v_{0x}$. The smallest α_0 gives the largest v_x and K at the maximum height. $K_1 = K_2 + U_2$ so when K_2 is larger, U_2 and hence y_2 at the maximum height are smaller. That is, for smaller launch angle, the kinetic energy at the maximum height is larger, so by energy conservation the gravitational potential energy must be less. Fig.7.8 shows that smaller α_0 corresponds to smaller maximum height.

Q7.3 No Friction: The kinetic energy at the bottom equals the gravitational potential energy at the top, if the potential energy is taken to be zero at the bottom. The change in gravitational potential energy depends only on the change in height and is independent of the path, so the speed at the bottom does not depend on the shape of the ramp.
Friction: $K_2 = U_1 + W_{\text{f}}$. The speed at the bottom depends on the amount of mechanical energy lost due to the negative work done by friction. The work done by friction depends on the path, so the speed at the bottom depends on the shape of the ramp.

Q7.4 $U = mgy$. The two students assign different values to the initial and final values of the gravitational potential energy but the same value to the change in gravitational potential energy. If the height of the building is h, the student on the roof says $U_1 = 0$, $U_2 = -mgh$ and $\Delta U = U_2 - U_1 = -mgh$. The student on the ground says $U_1 = mgh$, $U_2 = 0$ and $\Delta U = U_2 - U_1 = -mgh$. Conservation of energy, with $K_1 = 0$ and $W_{\text{other}} = 0$, says $K_2 = U_1 - U_2 = -\Delta U$, so the students assign the same value to the kinetic energy of the egg just before it strikes the ground.

Q7.5 In the absence of air resistance and any nonconservative work, $K_1 + U_1 = K_2 + U_2$. When the ball returns to its starting point, $U_1 = U_2$ so $K_2 = K_1$. If the ball leaves his nose with some initial kinetic energy, it has the same amount of kinetic energy when it returns to his nose and it crashes into his face.

Q7.6 To increase the mechanical energy, the total work done by friction would have to be positive. Whether or not this can happen depends on the system. If you set a box on a moving conveyor belt, the friction force on the box does positive work and gives the box kinetic energy. But the friction exerted on the belt by the box does negative work on the belt. By Newton’s 3rd law the friction force exerted by the belt on the box equals the magnitude of the friction force exerted by the box on the belt. If the box doesn’t slip the magnitudes of the displacement of each object are the same and the total work done by friction is zero. If the box slips before reaching the same speed as the belt, the belt travels farther than the box and the total work done is negative. If the box is the system, friction has increased the mechanical energy. If the box and belt are the system, either the mechanical energy of the system has decreased or it has stayed the same. For an isolated system, friction never increases the mechanical energy, it is always a dissipative force.

Q7.7 She does work on the trampoline by pushing against it with her legs. This adds mechanical energy to the system.

Q7.8 A kilowatt is a unit of power. A kilowatt-hour is a unit of energy and electrical energy is what customers are paying for.

Q7.9 (a) The gravity force is downward and the displacement is upward so the gravity force does negative work. $U_{\text{grav}} = mgy$, with $+y$ upward, so the gravitational potential energy of the book increases. When a force does negative work the potential energy associated with that force increases. (b) The gravity force is downward and the displacement is downward so the gravity force does positive work. The gravitational potential energy of the can decreases. When a force does positive work the potential energy associated with that force decreases.

Q7.10 (a) The spring force on the block is directed opposite to the displacement so it does negative work. The potential energy stored in the spring increases. When a force does negative work the potential energy associated with the force increases. (b) The spring force is upward and the block moves upward so the spring force does positive work. The amount the spring is compressed decreases so the potential energy stored in the spring decreases. When a force does positive work the potential energy associated with that force decreases.

Q7.11 (a) $U_{\text{grav}} = mgy$ so the 10.0-kg stone has more gravitational potential energy than the 1.0-kg stone. (b) As they free-fall, the net force on each stone is the stone's weight mg. Newton's second law says $mg = ma$ and $a = g$, the same for each stone. The gravity force on the 10.0-kg stone is larger but more force is required to give it acceleration. (c) Conservation of energy says $mgh = \frac{1}{2}mv^2$, where h is the final height above the ground and v is the speed of the stone just before it strikes the ground. The mass divides out and $v = \sqrt{2gh}$, the same for both stones. They have the same acceleration and they each fall the same distance, so they have the same speed when they reach the ground. (d) The final kinetic energy of the stone equals its gravitational potential energy. The initial gravitational energy is larger for the 10.0 kg stone so the final kinetic energy is larger for this stone. Or, kinetic energy equals $\frac{1}{2}mv^2$. They have equal final speeds so the 10.0 kg stone has larger final kinetic energy.

Q7.12 (a) This is incorrect. (b) This is correct. The object with a smaller mass reaches a greater height. The potential energy stored in the spring does not depend on the mass of the object and is the same for both objects. Conservation of energy says that the final gravitational potential energy equals the initial potential energy stored in the spring so it is the same for both objects. For equal gravitational potential energy mgh the object with the smaller mass must have a greater final height. Conservation of energy says $\frac{1}{2}kx^2 = mgh$ and $h = \dfrac{kx^2}{2mg}$.

Q7.13 The friction force that one hand exerts on the other does work and produces thermal energy.

Q7.14 Gravity is a conservative force. The work done by gravity depends only on the initial and final heights of the object. It is independent of the path and can be expressed in terms of the change in a potential energy function. Friction is a nonconservative force. The work done by friction depends on the path taken between the initial and final positions. The work done by friction therefore cannot be expressed in terms of a change in a potential energy function.

Q7.15 Work done by friction produces thermal energy that is dissipated. Heat engines (Chapter 20) can recover only a portion of the thermal energy for conversion back to mechanical energy.

Q7.16 This is incorrect. It incorrectly gives $\Delta U = U_2 - U_1 = \frac{1}{2}k(x_2 - x_1)^2$ rather than the correct $\Delta U = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. The correct form for U is $U = \frac{1}{2}kx^2 + C$, where C is a constant. The student's incorrect choice gives $U = \frac{1}{2}kx^2 + \frac{1}{2}kx_1^2 - kxx_1$. It incorrectly has a linear term in x. The student's incorrect choice gives $F_x = -dU/dx = -k(x - x_1)$, which doesn't give $F_x = 0$ when $x = 0$. If we want $U(x_1) = 0$, then take $U = \frac{1}{2}kx^2 - \frac{1}{2}kx_1^2$. This $U(x)$ gives the correct ΔU and the correct F_x.

Q7.17 If the direction of F_x is reversed then the sign of the work is reversed and $W_{\text{el}} = -(\frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2)$. $W_{\text{el}} = U_1 - U_2$ so $U = -\frac{1}{2}kx^2$. The graph of this $U(x)$ is given in Fig. DQ7.17. $x = 0$ is a point of unstable equilibrium. Any small displacement away from $x = 0$ produces a force that moves the end of the spring farther from equilibrium.

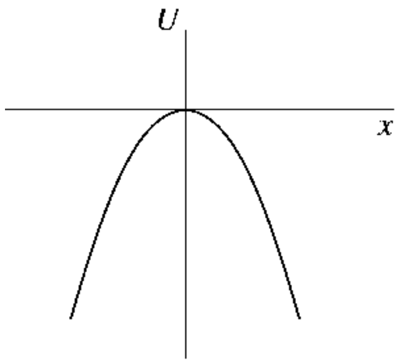


Figure DQ 7.17

Q7.18 The force is always in the $-y$-direction, which is towards the surface of the earth. The force acts to push the system toward lower potential energy and the potential energy is lowered when the object moves toward the earth.

Q7.19 If the particles repel, the force each exerts on the other does negative work when they move closer together and the potential energy increases. If the particles attract, the force each exerts on the other does positive work when they move closer together and the potential energy decreases.

Q7.20 At $x = \pm A$, $E = U$ and $K = 0$. The slope of $U(x)$ is positive at $x = +A$, so at this point $F_x < 0$ and $\vec{F}$ is directed opposite to the displacement. At $x = +A$ the object stops traveling in the $+x$-direction and starts to travel in the $-x$-direction. At $x = -A$, the slope of $U(x)$ is negative so $F_x > 0$ and $\vec{F}$ is directed opposite to the displacement. At $x = -A$ the object stops traveling in the $-x$-direction and starts to travel in the $+x$-direction. At $x = \pm A$ the object "turns around" and starts to head in the opposite direction.

Q7.21 $F_x = -dU/dx$, so $U(x)$ must be constant around a point of neutral equilibrium. The graph of $U(x)$ versus x for a region of neutral equilibrium is given in Fig. DQ7.21. An example is a particle sitting on a horizontal, frictionless surface.

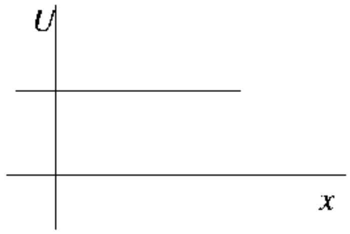


Figure DQ7.21

Q7.22 $E = E_1$: The object moves back and forth between x_a and x_b. Its speed is greatest at $x = x_1$. The graph of $v(x)$ is sketched qualitatively in Fig. DQ7.22a. $E = E_2$: The object moves back and forth between x_c and x_d. It's speed is greatest at x_1. The speed has a local minimum at x_2 and a local maximum at x_3. It's speed at x_3 is slightly less than its speed at x_1. The graph of $v(x)$ is sketched qualitatively in Fig. DQ7.22b.

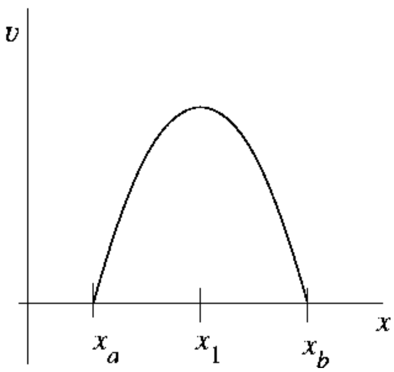


Figure DQ7.22a

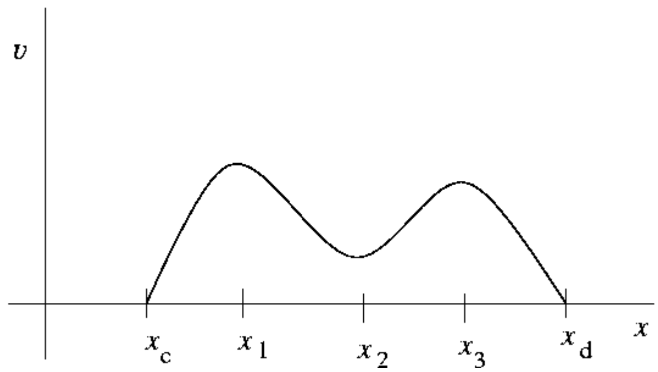


Figure DQ7.22b

Q7.23 $F_x = -dU/dx = -3\alpha x^2$. For any x, $F_x < 0$, so $\vec{F}(x)$ is in the $-x$-direction, for both positive and negative x.

CHAPTER 8
MOMENTUM, IMPULSE, AND COLLISIONS

Discussion Questions

Q8.1 A heavy hammer is more effective. At the same speed the heavy hammer has more momentum and requires more force to stop it.

Q8.2 $K = \frac{1}{2}mv^2 = p^2/2m$. If it has the same momentum, the more massive bowling ball has less kinetic energy. Or, $p = \sqrt{2mK}$ so if they have the same kinetic energy, the bowling ball has more momentum. The momentum of an object determines the force required to stop it, so it would be easier to catch a bowling ball at the same momentum.

Q8.3 For either a raindrop or apple, you can consider the object and earth together as an isolated system and say that the momentum of the object is transferred to the earth. Or, considering only the object as the system you can say that the net external force the ground applies to the object removes its momentum.

Q8.4 Kinetic energy depends only on speed but momentum is a vector and has the same direction as the velocity. The momentum of the car is different in the two cases.

Q8.5 The velocity of the truck relative to the fence post is different from its velocity relative to the police car, so the momentum of the truck is different in these two frames. Both frames are inertial frames so Eq.(8.3) applies and $d\vec{\boldsymbol{p}}/dt$ is the same in both frames.

Q8.6 (a) For a single object $p = mv$ and $K = \frac{1}{2}mv^2$. The kinetic energy is $K = \dfrac{p^2}{2m}$. If $p = 0$ then $K = 0$. (b) The total momentum $\vec{\boldsymbol{P}}$ of the pair of objects is the vector sum of the individual momenta: $\vec{\boldsymbol{P}} = \vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{p}}_2$. The total kinetic energy is the scalar sum $K = K_1 + K_2$. P can be zero without K being zero. An example is two objects that have momenta of equal magnitudes but opposite directions. The vector sum $\vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{p}}_2$ is zero but $\dfrac{p_1^2}{2m} + \dfrac{p_2^2}{2m}$ is not zero. The kinetic energy of an object does not depend on the direction of the velocity and it is never negative. (b) Yes. $K = K_1 + K_2 = \dfrac{p_1^2}{2m} + \dfrac{p_2^2}{2m}$. Both terms are positive so the only way K can be zero is for p_1 and p_2 to both be zero, when both objects are at rest.

Q8.7 Momentum is not conserved, because of the net upward force exerted on the system by the ice as the rock is being thrown. The initial momentum is zero and after the rock is thrown the system has an upward component of momentum. The earth must be included in the system to have an isolated system for which momentum is conserved. There is no external force in the horizontal direction, so the horizontal component of momentum is conserved. The final horizontal component of the total momentum of the system is zero; the woman gains horizontal momentum in one direction and the rock gains an equal magnitude of horizontal momentum in the opposite direction.

Q8.8 Inelastic means $K_2 < K_1$. In a collision of two objects, if they stick together the collision must be inelastic. Such a collision is sometimes called totally inelastic. If the objects don't stick together the forces during the collision can still do nonconservative work and the collision can be inelastic, with $K_2 < K_1$. In Example 8.5, $K_1 = \frac{1}{2}(0.50\text{ kg})(2.0\text{ m/s})^2 + \frac{1}{2}(0.30\text{ kg})(2.0\text{ m/s})^2 = 1.6$ J and $K_2 = \frac{1}{2}(0.50\text{ kg})(0.40\text{ m/s})^2 + \frac{1}{2}(0.30\text{ kg})(2.0\text{ m/s})^2 = 0.64$ J. $K_2 < K_1$ and the collision is inelastic.

Q8.9 The final kinetic energy would be zero if the final speed of the combined object is zero. This will be the case whenever the initial momentum of the system is zero, when $m_A\vec{\boldsymbol{v}}_{A1} + m_B\vec{\boldsymbol{v}}_{B1} = 0$. The initial kinetic energy is $K_1 = K_{A1} + K_{B1} = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2$ and cannot be zero. In the inelastic collision, momentum is conserved but kinetic energy is not.

Q8.10 The equation $K = p^2/2m$ applies only to each object in the system. The kinetic energy K_{tot} of the system is not equal to $P_{\text{tot}}^2/2m$, where $\vec{\boldsymbol{P}}_{\text{tot}}$ is the total momentum of the system. The vector sum $\vec{\boldsymbol{p}}_A + \vec{\boldsymbol{p}}_B$ can be constant while the scalar sum $p_A^2/2m_A + p_B^2/2m_B$ is not constant.

Q8.11 Define the relative velocity vector to be $\vec{\boldsymbol{v}} = \vec{\boldsymbol{v}}_B - \vec{\boldsymbol{v}}_A$.
Example 8.10: Only the x-components of the velocities are nonzero.
$v_{1x} = v_{B1x} - v_{A1x} = -2.0\,\text{m/s} - 2.0\,\text{m/s} = -4.0\,\text{m/s}$
$v_{2x} = v_{B2x} - v_{A2x} = 3.0\,\text{m/s} - (-1.0\,\text{m/s}) = +4.0\,\text{m/s}$
$\vec{\boldsymbol{v}}_1 = -\vec{\boldsymbol{v}}_2$; the relative velocity vector reverses direction.
Example 8.11: Only the x-components of the velocities are nonzero.
$v_{1x} = v_{B1x} - v_{A1x} = 0 - 2.6\times 10^7\,\text{m/s} = -2.6\times 10^7\,\text{m/s}$
$v_{2x} = v_{B2x} - v_{A2x} = 0.4\times 10^7\,\text{m/s} - (-2.2\times 10^7\,\text{m/s}) = +2.6\times 10^7\,\text{m/s}$
$\vec{\boldsymbol{v}}_1 = -\vec{\boldsymbol{v}}_2$; the relative velocity vector reverses direction.
Example 8.12: Both x-components and y-components of the velocities are nonzero.
$v_{1x} = v_{B1x} - v_{A1x} = 0 - 4.00\,\text{m/s} = -4.00\,\text{m/s}$
$v_{A2x} = v_{A2}\cos\alpha = +1.60\,\text{m/s}$; $\quad v_{B2x} = v_{B2}\cos\beta = +4.00\,\text{m/s}$
$v_{2x} = v_{B2x} - v_{A2x} = 4.00\,\text{m/s} - 1.60\,\text{m/s} = +2.40\,\text{m/s}$
$v_{1y} = v_{B1y} - v_{A1y} = 0$
$v_{A2y} = v_{A2}\sin\alpha = +1.20\,\text{m/s}$; $\quad v_{B2y} = v_{B2}\sin\beta = -2.00\,\text{m/s}$
$v_{2y} = v_{B2y} - v_{A2y} = -2.00\,\text{m/s} - 1.20\,\text{m/s} = -3.20\,\text{m/s}$
$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} = 4.00$ m/s; $\quad v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = 4.00$ m/s
The magnitude of the relative velocity, before and after the collision is the same, but in this two-dimensional case the relative velocity vector does not simply reverse direction.

Q8.12 The hard collision with the concrete is of shorter duration so the maximum force is larger.

Q8.13 The Ping-Pong ball gains momentum in the $+x$-direction and the bowling ball loses momentum in the $+x$-direction, so that the total momentum of the system is conserved. Therefore, the bowling ball slows down a little and loses some kinetic energy. Kinetic energy is transferred from the bowling ball to the Ping-Pong ball and the total energy of the two balls is conserved.

Q8.14 The momentum change of the bullets is greater if they bounce off, so the average force on the plate is greater in that case.

Q8.15 For a constant force the change in momentum equals the impulse, the product of force and time. Half the force requires twice the time for the same impulse and momentum change; the 2 N force would need to act for 0.50 s.

Q8.16 There must be times when $\sum F_x$ is positive and times when it is negative. The area under the positive part must equal the area under the negative part, so that the net impulse will be zero.

Q8.17 If the force exerted on the racket by the person's hand while the ball and racket are in contact is neglected then the total momentum of the system is conserved. As the ball moves through the air it is acted on by the external force of gravity and this net external force changes its momentum.

Q8.18 Apply Eq.(8.31). After the rifle is fired,
$v_{\text{cm}x} = (5.00\times10^{-3}\,\text{kg})(300\,\text{m/s}) + (3.00\,\text{kg})(-0.500\,\text{m/s}) = 0$.
The center of mass of the system maintains a constant velocity of zero during the firing. This agrees with Eq.(8.34); the net external force on the system is zero so the velocity of the center of mass is constant.

Q8.19 The momentum of the system egg plus earth remains constant. The egg acquires a downward momentum and the earth acquires an upward momentum of the same magnitude so that the total momentum of the system is zero. The mass of the earth is much, much larger than that of the egg so the upward velocity of the earth is very, very small.

Q8.20 No. With respect to horizontal motion she is an isolated system and her center of mass must remain at rest.

Q8.21 No. For both fragments to fall straight down each fragment would have zero horizontal component of momentum and the total momentum of the system would have zero horizontal component immediately after the explosion. But immediately before the explosion the velocity of the shell is horizontal and the horizontal component of its momentum is not zero. Momentum is conserved in the explosion so the horizontal component of the total momentum can't be zero immediately after the explosion.

Q8.22 By Newton's 3rd law the fragments exert forces of equal magnitudes on each other. By Newton's 2nd law the same force produces a greater acceleration for the lighter fragment. The forces on each fragment act for the same time interval, so with a greater acceleration the lighter fragment acquires a greater final speed. In fact, $v_A / v_B = m_B / m_A$. $K_A / K_B = (m_A / m_B)(v_A / v_B)^2 = m_B / m_A$. This same result is derived in Exercise 8.26 using conservation of linear momentum.

Q8.23 (a) False. As the apple falls it gains speed and its momentum increases because of the net force on it due to gravity. (b) True. No nonconservative forces act on the apple so its mechanical energy is conserved. Gravitational potential energy is converted to kinetic energy. (c) False. (d) False. It gains kinetic energy, because positive work is done on it by the net force on it, the gravity force.

Q8.24 (a) True. If it is assumed that no net external force acts on the clay during the collision, then the momentum of the system is conserved. (b) False. When objects stick together in a collision, the collision must be inelastic and kinetic energy is not conserved. Mechanical energy is not conserved because of the work done during the collision by the nonconservative forces that the pieces of clay exert on each other. This work converts some of the initial kinetic energy into thermal energy and energy of deformation. (c) False. (d) False.

Q8.25 Newton's third law says that the magnitude of the force exerted on each mass is the same. These forces act for the same length of time, until the masses lose contact with the spring. Therefore, the magnitude of the impulse applied to each mass is the same. But the lighter mass moves a greater distance while they are in contact with the spring so more work is done on the lighter mass. (a) Equal magnitude of impulse means that the masses receive equal magnitudes of momentum. (b), (c) and (d). More work is done on the lighter mass so it gains more kinetic energy than the heavier mass. Or,

$K = \dfrac{p^2}{2m}$. Equal momentum means greater kinetic energy for the lighter mass.

Q8.26 The magnitudes of momentum changes for the two vehicles are equal. By Newton's third law, the force the SUV exerts on the compact car has the same magnitude as the force the compact car exerts on the SUV. But such a collision won't be elastic and the kinetic energy of the system is not conserved and the kinetic energy changes for each object won't be equal in magnitude. (a) False. (b) True. (c) False. (d) False.

CHAPTER 9
ROTATION OF RIGID BODIES

Discussion Questions

Q9.1 Yes for (a), (b), (d) and (e). In each case the equation is true for any instant of time. Changes of quantities over time don't enter into the equation, so it doesn't matter how ω and α are changing. No for (c). If α changes with *t* then the equation doesn't even have precise meaning. What value of α would be used, the value at $t = 0$ or the value at the final *t* or something else?

Q9.2 (a) The mass of the molecule has the same distribution about the *x*-axis as it does about the *z*-axis. Therefore, the moment of inertia of the molecule for the *x*-axis as the axis of rotation is the same as for the *z*-axis. The kinetic energy will be *K*. (b) If the atoms are treated as point masses, the mass lies on the *y*-axis and the moment of inertia for the *y*-axis as the rotation axis is zero and the kinetic energy will be zero.

Q9.3 The tangential acceleration is tangential to the circular path of the point. The tangential acceleration is equal to the rate of change of the linear speed of the point. The radial acceleration is radially inward, toward the center of the circular path of the point. The radial acceleration expresses the rate of change of the direction of the velocity of the point.

Q9.4 Yes, a is the rate of change of the speed so it is the same for all points on the chain. $v = r_{\text{rear}}\omega_{\text{rear}} = r_{\text{front}}\omega_{\text{front}}$ so $\omega_{\text{front}} / \omega_{\text{rear}} = r_{\text{rear}} / r_{\text{front}}$. The angular acceleration is the rate of change of the angular speed ω so $\alpha_{\text{front}} / \alpha_{\text{rear}} = r_{\text{rear}} / r_{\text{front}}$. ω and α are larger for the smaller sprocket.

Q9.5 $a_{\text{rad}} = v^2 / r$ and v is the same at the rim of each sprocket, so $a_{\text{rad,rear}} r_{\text{rear}} = a_{\text{rad,front}} r_{\text{front}}$ and $a_{\text{rad,rear}} / a_{\text{rad,front}} = r_{\text{front}} / r_{\text{rear}}$; a_{rad} is larger at the rim of the smaller sprocket.

Q9.6 The tangential acceleration is the rate of change of the angular speed so for constant angular velocity the tangential acceleration is zero. The radial acceleration is $a_{\text{rad}} = r\omega^2$ so if ω is constant, a_{rad} is constant. The direction of a_{rad} is always toward the center of the flywheel, but this direction changes in space as the flywheel rotates.

Q9.7 The spin cycle removes water from the clothes. The clothes turn at the rim of the drum with a large angular speed so they have a large inward radial acceleration $a_{\text{rad}} = r\omega^2$. The force on the water isn't large enough to provide this acceleration to the water and the water doesn't stay in the circular path of the clothes. It leaves tangentially through holes in the drum.

Q9.8 $K = \frac{1}{2} I\omega^2$. Since all the objects have the same ω, the object with the largest I has the largest K and the object with the smallest I has the smallest K. The moment of inertia for each object is:

$I_a = \frac{2}{5} MR^2 = \frac{1}{10} MD^2$

$I_b = \frac{1}{2} MR^2 = \frac{1}{8} MD^2$

$I_c = MR^2 = \frac{1}{4} MD^2$

$I_d = \frac{1}{12} ML^2 = \frac{1}{12} MD^2$

Since all the objects have the same M, the smallest I is I_d and the largest I is I_c. Object *c* has the greatest kinetic energy and object *d* has the least.

Q9.9 It is not possible for a body to have the same moment of inertia for all possible axes. The mass of the body cannot be distributed the same relative to all axes, including those that lie outside the body. Yes, a sphere has the same moment of inertia for all axes passing through its center.

Q9.10 To maximize the moment of inertia the mass should be as far from the rotation axis as possible. This is achieved when all the mass is at the rim, when the flywheel has the shape of a thin-walled hollow cylinder or hoop.

Q9.11 Mount the object on a horizontal axle of radius R that lies along the axis. Let the axle turn in frictionless bearings. Wrap a light cord around the axle and suspend a mass m from the free end of the cord, as in Fig.9.17. Release the system from rest and measure the linear speed v of the mass m after it has descended a vertical distance h. Conservation of energy gives $mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$. Use $\omega = v/r$ and solve for I.

Q9.12 No. The farthest each piece of mass can be from the axis is R and the largest possible moment of inertia about the symmetry axis is MR^2.

Q9.13 The thin plate can be constructed from thin rods laid side to side, parallel to side a. Each rod, of mass m_i, has $I_i = \frac{1}{3}m_i a^2$. The total I is the sum of the contribution from each rod and is $I = \sum I_i = \frac{1}{3}(\sum m_i)a^2 = \frac{1}{3}Ma^2$, where M is the sum of the masses of the rods and therefore is the mass of the plate.

Q9.14 $I = \frac{2}{3}MR^2$. $K = \frac{1}{2}I\omega^2 = \frac{1}{3}MR^2\omega^2$. Let K' be the new kinetic energy, $K' = \frac{1}{3}M(R')^2\omega^2$. $K' = 3K$ so $(R')^2 = 3R^2$ and $R' = \sqrt{3}R$.

Q9.15 The cross section of the rod need not be circular for these equations to apply. For the equations to apply, all points on a thin-slice must be to a good approximation the same distance from the axis. For this to be so, the lateral dimensions of the rod must be much less than its length.

Q9.16 The discussion in Q9.15 applies to part (d). For part (c) the thickness of the plate has no effect on how the mass is distributed relative to the axis so the thickness of the plate doesn't matter.

Q9.17 Ball *A* will have more kinetic energy. Conservation of energy applied to the system of the ball and the pulley gives $mgd = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$, where *d* is the distance the ball has fallen and *m* is the mass of the ball. $\omega = v/R$. Pulley *B* has a greater moment of inertia for rotation about an axis at its center. Therefore, for a given *d*, more of the total kinetic energy of the system will reside with the pulley in case *B* and therefore the kinetic energy of the ball will be less.

Q9.18 Greater than *V*. Moving the balls closer to the drum decreases the moment of inertia of the pulley about its rotation axis. $v = R\omega$, where R is the radius of the drum and is constant. Conservation of energy gives $mgd = \frac{1}{2}I(v/R)^2 + \frac{1}{2}mv^2$. Smaller *I* means that the pulley has a smaller fraction of the total kinetic energy of the system and for a given *d* the kinetic energy of the box therefore is greater.

Q9.19 In the constant angular acceleration equations (9.7), (9.10), (9.11) and (9.12) you can use any angular measure. The same number of angular quantities appear in each term of the equations so converting from one angular measure to another would just multiply the whole equation by an overall factor, and this doesn't change the equation. $s = r\theta$ requires that θ be in radians; this equation in fact defines the radian. Equations (9.13) through (9.15) and Eq.(9.17) are all derived using $s = r\theta$ so they also require the use of radian measure.

Q9.20 No. A simple counter-example is the solid sphere. The moment of inertia through the center of the sphere is $2MR^2/5$. The center of mass is at the center of the sphere so calculating I by placing all the mass at the center of mass gives the incorrect value of zero.

Q9.21 (a) All points have the same angular speed; it is the same at A and B. (b) $v = r\omega$ so the tangential speed is greater at point A. (c) The angular acceleration α is the time rate of change of the angular velocity, so α is the same at A and B. (d) $a_{\text{tan}} = r\alpha$ so is larger at A. (e) $a_{\text{rad}} = r\omega^2$ so is larger at A.

Q9.22 A simple model for my body is a vertical cylinder with mass *M* and radius *R* and two slender rods for my outstretched arms. I estimate the radius of this cylinder to be about 0.15 m. My height is about 1.7 m so the volume of the cylinder is $V_{\text{cyl}} = \pi R^2 h = 0.12\ \text{m}^3$. For each outstretched arm, the distance from the tip of my fingers to the center of my chest is about $L_{\text{arm}} = 0.90$ m and the average radius for each arm, modeled as a cylinder, is about $R_{\text{arm}} = 0.080\ \text{m}$. So, the volume of each arm, in this simple model, is $V_{\text{arm}} = \pi R_{\text{arm}}^2 L_{\text{arm}} = 0.018\ \text{m}^3$. My total volume then is about $0.17\ \text{m}^3$ and about 11% of my volume is in each arm and therefore 78% is in the rest of my body. My total mass is 82 kg. If the average density of my arms is assumed to be the same as for the rest of my body, then the mass of each of my arms is $M_{\text{arm}} = 9.0\ \text{kg}$ and the mass of the rest of my body is $M = 64\ \text{kg}$. Therefore, my moment of inertia about the specified axis is $I = I_{\text{trunk}} + I_{\text{arms}}$.

$I_{\text{trunk}} = \frac{1}{2}MR^2 = \frac{1}{2}(64\ \text{kg})(0.15\ \text{m})^2 = 0.72\ \text{kg}\cdot\text{m}^2$.

$I_{\text{arms}} = 2\left(\frac{1}{3}M_{\text{arm}}L_{\text{arm}}^2\right) = \frac{2}{3}(9.0\ \text{kg})(0.90\ \text{m})^2 = 4.9\ \text{kg}\cdot\text{m}^2$.

My total moment of inertia is therefore approximated to be $5.6\ \text{kg}\cdot\text{m}^2$. My outstretched arms have a much greater *I* than the rest of my body, even though they have only about 20% of my mass because for them their mass is distributed farther from the axis.

CHAPTER 10
DYNAMICS OF ROTATIONAL MOTION

Discussion Questions

Q10.1 Yes. For example, a force applied tangentially to the rim of a disk initially at rest on a horizontal frictionless surface produces both a net force that accelerates the center of mass of the object and also a torque that starts the disk rotating about the center of mass.

Q10.2 It is better to use small, light wheels. Some of the initial gravitational potential energy of the racer is converted into rotational kinetic energy $\frac{1}{2}I\omega^2$ of the wheels. You want this to be as small as possible, so that most of the potential energy goes into translational kinetic energy of the center of mass of the racer. For the same reason, it is better to use solid wheels, that have a smaller I for the same total mass of the wheel.

Q10.3 Reducing the weight of the wheels reduces the moment of inertia of the wheels and less torque is required to give them a given angular acceleration. Reducing the total mass of the bike also reduces the net horizontal force required for a given linear acceleration but for this motion it doesn't matter where on the bike the weight is removed.

Q10.4 The friction force that opposes the motion of the car while stopping produces a torque about the center of gravity of the car that tends to rotate the car such that the rear is raised and the front is lowered. See Fig. DQ10.4a. When accelerating, the friction force is in the direction the car is moving and produces a torque about the center of gravity that tends to rotate the car in the opposite direction, so the front end is raised. See Fig. DQ10.4b. For a large acceleration the normal force at the front wheels is reduced, so traction is lost there.

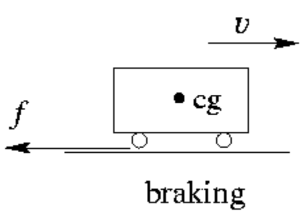


Figure DQ10.4a

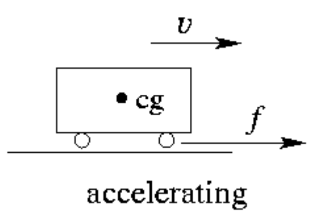


Figure DQ10.4b

Q10.5 Holding her arms out increases her moment of inertia for an axis through her center of mass. By Eq.(10.7), this means a given gravity torque that arises when she leans slightly to one side or the other produces a smaller angular acceleration.

Q10.6 The grinding wheel increases the moment of inertia of the shaft. The torque applied by the motor then produces a smaller angular acceleration.

Q10.7 For work, the distance used is the component of displacement along the direction of the force. For torque, the distance used is the component perpendicular to the force of the distance from the axis to the point of application of the force. Work and torque have the same units but are different quantities.

Q10.8 Roll it down the incline, alongside a solid ball and then alongside a hollow ball. A solid ball will reach the bottom of the incline before a hollow ball.

Q10.9 $\tau = I\alpha$. $I = \int r^2\, dm$. $m = \rho V$ so $dm = \rho\, \mathrm{d}V$ and $I = \rho\int r^2\, dV$. If each dimension is doubled, then I is increased by a factor of $2^5 = 32$. The angular acceleration of the larger object will be $\alpha/32$. For a specific example of a disk with radius *R*, thickness *t* and axis as in Fig. 9.4f, $I=\frac{1}{2}MR^2=\frac{1}{2}\rho\left(\pi R^2t\right)R^2=\frac{1}{2}\rho\pi R^4t$. Doubling *R* and *t* increases *I* by a factor of 2^5.

Q10.10 For the hanging mass, $mg-T=ma$. For the pulley, $TR=I\alpha$. $\alpha=a/R$ so $T=\left(I/R^2\right)a$. $mg=\left(\frac{I}{R^2}+m\right)a$ and $a=\frac{mg}{\left(I/R^2\right)+m}$. $T=\left(\frac{I}{R^2}\right)a=\frac{\left(I/R^2\right)mg}{m+\left(I/R^2\right)}=\frac{mg}{\frac{R^2m}{I}+1}$. The pulley in the shape of hoop has larger *I* and therefore the tension *T* in the string is greater in that case. One way to see this qualitatively is to note that the greater rotational inertia of the loop causes the mass to fall with less downward acceleration when this pulley is used. The smaller *a* is for the hanging mass, the closer the tension is to the weight of the hanging mass. Note that to have the same mass and radius, the hoop must be made of material of greater density than the solid pulley.

Q10.11 The gravity torque acts at the center of gravity of the baton and therefore produces no angular acceleration about this point.

Q10.12 $\frac{1}{2}mv^2+\frac{1}{2}I\omega^2=mgh_0$, where v and ω are the linear and angular speeds of the ball at the bottom of the hill. $I=\frac{2}{5}mR^2$ and $v=R\omega$, so $\frac{1}{2}mR^2\omega^2+\frac{1}{5}mR^2\omega^2=mgh_0$. $h_0=\frac{7R^2\omega^2}{10g}$. (a) The height reached by the ball for a given angular speed at the bottom is proportional to R^2. So, the height reached will be $4h_0$ if *R* is doubled. (b) The mass divides out of the conservation of energy expression so the maximum height will still be h_0. (c) Same as in part (a), $4h_0$. (d) The height reached is proportional to ω^2, so the maximum height will be $4h_0$.

Q10.13 Refer to Fig.10.13. All points have $\vec{\boldsymbol{r}}_{\text{cm}}$ to the right. The resultant velocity of each point is thus $\vec{\boldsymbol{r}}_{\text{cm}}$ to the right plus the velocity for rotation around the center of mass. All points have a velocity component to the right so there is no point where the velocity is purely vertical. The rotation gives a maximum component $-\vec{\boldsymbol{r}}_{\text{cm}}$, that is to the left. The point of contact with the surface has a resultant velocity of zero, but no point has a horizontal component opposite to the velocity of the center of mass. If the wheel is slipping, the speed at the rim due to rotation is greater than v_{cm}. It is still correct that no point has a purely vertical velocity, but now the top of the wheel can have a horizontal velocity opposite to the velocity of the center of mass.

Q10.14 Let the moment of inertia of each object be $I=\beta MR^2$, where β is a different constant for each object. The object with the smallest β reaches the bottom first. For a hoop, $\beta=1$; for a uniform solid cylinder, $\beta=\frac{1}{2}=0.50$; for a spherical shell, $\beta=\frac{2}{3}=0.67$, and for a uniform solid sphere, $\beta=\frac{2}{5}=0.40$. So, the order of arrival, first to last, is: solid sphere, solid cylinder, spherical shell, and hoop. The mass and radii divide out in the calculation of the speed at the bottom of the incline, so it doesn't matter if the mass and radii are the same. All that matters is the shape, the way in which the mass is distributed relative to the axis.

Q10.15 The ball will go higher up the hill if the hill has enough friction to prevent slipping.
energy conservation: As the ball rolls on the horizontal surface at the bottom of the hill it has both translational and rotational kinetic energy. If the hill is perfectly smooth the ball continues to rotate with the same angular speed and only the initial translational kinetic energy is converted into gravitational potential energy; at the maximum height up the hill the ball is still spinning at the same

rate as initially. If there is enough friction to prevent slipping then the ball has stopped rotating at the maximum height and both the initial translational and rotational kinetic energies are converted into gravitational potential energy.
Newton's 2nd law: When there is no friction the net force on the ball as it rolls up the hill is $mg\sin\alpha$ directed down the incline (α is the slope angle of the hill). With friction, there is a friction force f directed up the incline and the net force down the incline is $mg\sin\alpha - f$. With friction the acceleration opposing the translational motion is less and the ball travels a greater distance before coming to rest.

Q10.16 Consider yourself plus the turntable as the system. There are no external torques on this system, so the total angular momentum is constant. As you move farther from the axis the moment of inertia of the system increases and in order to maintain the same angular momentum the rotation speed decreases.

Q10.17 In the oceans the water will be farther from the axis of rotation of the earth than when it is frozen as ice near the poles. The moment of inertia of the earth will increase and from conservation of angular momentum the rotation rate will decrease. The length of a day (the time for one full rotation) will increase.

Q10.18 Since angular momentum is given by $L = I\omega$ and rotational kinetic energy is given by $K_{\text{rot}} = \frac{1}{2}I\omega^2$, $K_{\text{rot}} = \dfrac{L^2}{2I}$. If they have the same L, they will have the same K_{rot} only if they have the same I. And since $L = \sqrt{2IK_{\text{rot}}}$, if they have the same rotational kinetic energy they will have the same angular momentum only if they have the same I. And rotational kinetic energy is a scalar whereas rotational kinetic energy is a vector. Even if they have the same magnitude of angular momentum the angular momentum vectors of the two objects could be different if they are in different directions.

Q10.19 The weights have the same angular velocity, and hence the same angular momentum, just after they are released as just before they were released. The angular momentum of the student doesn't change and her angular speed remains the same. Her angular speed would change if she threw the weights in the tangential direction, instead of just letting go of them.

Q10.20 The particle has angular momentum with constant magnitude $L = mvl$.

Q10.21 The equation $\sum \tau_z = I\alpha_z$ is valid only for rigid bodies, for which I is constant. Eq. (10.29) is always true whereas Eq.(10.7) is true only if I is constant. There is an analogous statement for the dynamics of translational motion: Eq.(8.4) is true in general; $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ is equivalent, and correct, only when the mass m of the system is constant during the motion.

Q10.22 Eq.(10.22) is derived using Eq.(10.7), which is correct only when I is constant. When I is not constant, Eq.(10.22) does not apply. The extra kinetic energy comes from the work done by the radial force the professor applies when he pulls in the weights.

Q10.23 Her linear momentum is not conserved, because of the net force on her due to gravity. The force acts at her center of gravity, so provides no net torque for rotation about her center of gravity.

Q10.24 When you stop the raw egg, the raw insides of the egg, which are only loosely coupled to the shell, keep spinning. This energy couples back to the motion of the light shell when the egg is released. When you stop a hard-boiled egg, the whole egg stops.

Q10.25 Without the small rotor, when the large rotor changes angular speed the helicopter body would rotate in the opposite direction to conserve angular momentum. The small rotor causes an external torque from the air that keeps the body of the helicopter from rotating. The two counter-rotating main rotors have zero total angular momentum regardless of their angular speed.

Q10.26 So long as the center of gravity is directly above the pivot there is no gravity torque to cause precession about the pivot.

Q10.27 The processional angular speed Ω is given by $\frac{wr}{I\omega}$. (a) If ω is doubled, Ω is halved. (b) If w is doubled, Ω is doubled. (c) If I is doubled, Ω is halved. (d) If r is doubled, Ω is doubled. (e) wr is increased by a factor of four and $I\omega$ is increased by a factor of four. The factors of four cancel and Ω is unchanged.

Q10.28 According to Eq.(10.33), as the angular speed of rotation ω decreases the precession angular speed Ω increases and the precession period decreases. The gyroscope has slowed down due to friction torque at its axis.

Q10.29 This increases the gravity torque and causes the gyroscope to precess at a faster rate.

Q10.30 The spinning bullet has angular momentum that tends to stay constant as the bullet travels through the air.

CHAPTER 11
EQUILIBRIUM AND ELASTICITY

Discussion Questions

Q11.1 Yes, there is no net external force and no net external torque, if the center of mass of the object has zero acceleration and if the rotating object has zero angular acceleration. Each part of the body is not in equilibrium. Each part is moving in uniform circular motion and has a radial acceleration.

Q11.2 (a) Yes. The net force can be zero while the net torque is not zero. For example, there can be two forces acting on the object that are equal in magnitude and opposite in direction. If these forces act at different points on the object they can produce a net torque even though the net force is zero. A simple example of an object in translational equilibrium but not in rotational equilibrium is the cylinder in Fig. 10.9a. (b) Yes. There can be a net force but no net torque. A simple example is a baseball falling without air resistance. There is a net downward force due to gravity but that force acts at the center of gravity of the baseball and produces no torque.

Q11.3 When the wheel no longer tips the added weights have put the center of gravity at the geometrical center of the wheel.

Q11.4 No. The center of gravity of a hollow sphere is at the center of the sphere.

Q11.5 The gravity force on mass dm is greater at the lower end of the rod so the center of gravity is below the center of mass. The gravity force acting at the center of gravity produces a torque on the object about the object's center of mass. This torque orients the object. When the long axis of the object is directed toward the earth, the line of action of the gravity force is along the long axis and produces no torque and this then is a stable orientation. The same effect keeps the long axis of the moon oriented along the line between the center of the moon and the center of the earth.

Q11.6 If the object is suspended at the center of gravity a small rotation of the object does not produce any torque and the equilibrium is neutral. For a point of suspension above the center of gravity, a small rotation of the object produces a restoring torque and the object is in stable equilibrium. For a point of suspension below the center of gravity, a small rotation of the object produces a torque that tends to take the object farther from equilibrium and the object is in unstable equilibrium.

Q11.7 To balance on your tiptoes your body must move forward enough to place your center of gravity directly above your toes. When you are standing next to the wall and facing it, the wall prevents this from happening.

Q11.8 When the horseshoe hangs in equilibrium, the gravity torque on it is zero and the center of gravity of the horseshoe is directly below the point of support. Draw a chalk line on the wall behind the string. Pivot the horseshoe about another nail hole and again draw a chalk line along the string. The center of gravity is where these two lines intersect. The center of gravity will not be within the solid material of the horseshoe.

Q11.9 The torque due to the weight of the combined object can be calculated by assuming that the gravity force acts at the center of gravity. Therefore, there is no torque about this point and the object doesn't rotate. The bar remains horizontal as it falls.

Q11.10 The torque about the center of gravity due to the gravity force is still zero and the bar remains at 60° above the horizontal as it falls.

Q11.11 The free-body force diagram for the water skier is given in Fig. DQ11.11. $\vec{F}$ is the force of the tow rope and $\vec{f}$ is the resistive force exerted by the water. She leans backward to increase the moment arm and hence the torque produced by her weight about an axis at her feet. This torque opposes the torque due to the force $\vec{F}$ applied to her by the tow rope. $\vec{F}$ must equal the resistive force $\vec{f}$. When her speed increases, f increases and F increases so she must lean back farther to increase the gravity torque.

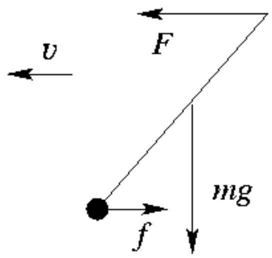


Figure DQ11.11

Q11.12 Pushing near the rim of the wheel gives greater torque about the axle than pushing on the wagon, since the pushing force has a larger moment arm.

Q11.13 He cannot do this. As soon as a vertical line through his center of gravity extends beyond his toes there will be a net torque on him for a horizontal axis through his center of gravity and he will fall over.

Q11.14 With your arm extended there is a large torque about your shoulder due to the weight of the dumbbell and your muscles must exert an equal opposing torque.

Q11.15 I know from experience that over time the distribution of a person's body mass changes, for example moving toward their midsection. This redistribution of mass changes both the location of the person's center of gravity and also the moment of inertia about an axis through the center of mass.

Q11.16 They add more weight in their abdomen region, extending away from their spine. They have to lean backward a bit to keep their center of gravity from extending horizontally past their feet.

Q11.17 The glass tips when its center of gravity is above a point beyond the bottom edge of the glass. If the water in the glass raises the center of gravity this will make the glass more unstable. Whether or not this will be the case depends on how the mass of the glass is distributed. Adding water does increase the moment of inertia, so more torque is required to start it rotating away from equilibrium.

Q11.18 To slide the refrigerator the horizontal force you apply must equal the force of friction from the floor. Whether it starts to slide first or tip first depends on how far above the floor you push. The higher up you push the greater the moment arm for your force and the more torque it produces for a given magnitude of force.

Q11.19 The tension in the wire has a component to the right and the weight of the bar is a vertical force so the force the hinge exerts must have a component to the left. The tension in the wire has an upward component and the weight of the bar is a downward force so you can't use the vertical forces to decide if the hinge force has an upward or downward component. Consider torques on the bar for an axis at the point where the wire is attached to the bar. For this axis, the weight of the bar produces a counterclockwise torque. The torques must sum to zero so the hinge force must produce a clockwise torque and the vertical component of the hinge force is downward.

Q11.20 The Young's modulus of a wire depends only on the material of which the wire is made and does not depend on the length or cross-sectional area of the wire.

Q11.21 $\Delta l = \left(\dfrac{F_\perp}{AY}\right) l_0$. $F_\perp = W$ and $A = \pi D^2/4$, so $\Delta l = \left(\dfrac{4W}{\pi D^2 Y}\right) l_0$. For the new value of W, $\Delta l' = \left(\dfrac{4W'}{\pi (D')^2 Y}\right) l_0'$. But $\Delta l' = \Delta l$ and $l_0' = l_0$ (same original length, same amount of stretch). This gives $\left(\dfrac{4W}{\pi D^2 Y}\right) l_0 = \left(\dfrac{4W'}{\pi (D')^2 Y}\right) l_0$. $W' = \left(\dfrac{D'}{D}\right)^2 W$. $W' = 3W$ gives $3 = \left(\dfrac{D'}{D}\right)^2$ and $D' = \sqrt{3} D$.

Q11.22 They have the same tensile strength but the cable is much more flexible. This can be an advantage or disadvantage depending on the application.

Q11.23 The weight of the elephant is much greater so the same stress requires bones of greater cross section.

Q11.24 The excess energy in each deformation cycle is deposited in the tendon and can build up to damaging levels after a large number of rapid cycles.

Q11.25 The excess energy during each vibration cycle is deposited as thermal energy in the rubber mounting blocks.

CHAPTER 12
FLUID MECHANICS

Discussion Questions

Q12.1 For a submerged cube the buoyant force is the difference between the downward force exerted by the water at the top face of the cube and the upward force exerted by the water at the bottom face of the cube. The pressure at these two locations is different so the forces are different. If there is truly no water under the cube, between the bottom face of the cube and the bottom of the tank, then there is no upward force due to the water pressure there and the block doesn't rise to the surface when released. But in practice this would be very unstable and difficult to achieve. At the bottom face of the cube there are large horizontal forces due to the water pressure there that tend to push water under the block. If only a thin film of water gets under the block the buoyant force returns to its normal value and the block floats to the surface.

Q12.2 The pressure at a given point depends only on the height of the fluid surface above that point. Therefore, the pressure at a given depth below the surface for water in the hose is the same as the pressure at the same depth below the surface in the funnel. The upward force that supports the weight of the water in the funnel comes partly from the upward force due to the pressure in the water at the bottom of the funnel and partly from the sloped sides of the funnel.

Q12.3 The force on the floor is not due just to the weight of the air in the room, but is due to the entire column of the earth's atmosphere above the floor.

Q12.4 The distance the small piston moves is a factor of A_2 / A_1 larger than the distance the large piston moves. So the work done by F_1 equals the work done by F_2 and energy is conserved.

Q12.5 The total force of the pavement on the tires must equal the weight of the car. $F = pA$ so smaller pressure p requires a larger area A.

Q12.6 Hot air has a smaller density than the surrounding air so the buoyancy force is larger than the weight when the balloon is filled with hot air. The density of the air in the balloon depends on the temperature of this air; the ascent or descent is controlled by varying the temperature of the air in the balloon.

Q12.7 This is the weight of the water displaced by the floating ship. This equals the buoyant force and must equal the weight of the ship.

Q12.8 No. As the elevator accelerates upward, more upward force on the sphere is needed to make the sphere accelerate upward with the elevator. The sphere continues to sit on the bottom of the bucket no matter how great the upward acceleration of the elevator is. The buoyant force is $\rho_{\text{fl}} V_{\text{sp}}(g+a)$, where ρ_{fl} is the density of the water and V_{sp} is the volume of the sphere. Newton's 2nd law applied to the sphere gives $n + B - mg = ma$, where n is the normal force the bottom of the bucket exerts on the sphere. This gives

$$n = m(g+a) - B = \rho_{\text{sp}} V_{\text{sp}}(g+a) - \rho_{\text{fl}} V_{\text{sp}}(g+a) = V_{\text{sp}}(g+a)(\rho_{\text{sp}} - \rho_{\text{fl}}),$$

where ρ_{sp} is the density of the sphere. As a increases, the upward normal force n that the bottom of the bucket exerts on the sphere also increases.

Q12.9 The density of the atmosphere decreases with altitude so the buoyant force on the dirigible decreases with altitude. The dirigible rises until the density of the surrounding air equals the average density of the dirigible (frame plus cargo plus helium).

Q12.10 Assume that in each case the liquid is the same. The buoyant force is equal to $\rho_{\text{liquid}}V_{\text{submerged}}g$. For the piece of iron the volume submerged is $(25\text{ cm})^3$. For the floating piece of wood the volume submerged is less than $(25\text{ cm})^3$. Therefore, the buoyant force on the iron is greater. The buoyant force on the piece of wood equals the weight of the piece of wood and the buoyant force on the piece of iron is less than the weight of the piece of iron. But the density of iron is much larger than the density of wood so for equal volumes the weight of the piece of iron is much larger than that of the piece of wood.

Q12.11 The weight in air is $w_{\text{air}} = mg = \rho_{\text{obj}}V_{\text{obj}}g$, where ρ_{obj} and V_{obj} are the density and volume of the object. The "weight" in water is $w_{\text{w}} = w_{\text{air}} - B = \rho_{\text{obj}}V_{\text{obj}}g - \rho_{\text{w}}V_{\text{obj}}g = (\rho_{\text{obj}} - \rho_{\text{w}})V_{\text{obj}}g$, where ρ_{w} is the density of water. $w_{\text{air}} / w_{\text{w}} = \rho_{\text{obj}} / (\rho_{\text{obj}} - \rho_{\text{w}})$ so if w_{air} and w_{w} are measured, ρ_{obj} can be calculated. The density of the object measured in this way can be compared to the density of pure gold. To make your fake brick pass this test the average density of the brick would have to equal that of gold. Table 12.1 shows that few materials have a density as large as that of gold. You could gold-plate a platinum-brass alloy brick, for example, but to have the same density of gold it would have to be mostly platinum, and platinum is very expensive.

Q12.12 Pressure varies with depth so the force on each square centimeter of levee is greatest at the bottom.

Q12.13 In each case the buoyant force B must equal the weight of the floating ship. $B = \rho_{\text{fl}}V_{\text{displ}}g$, where ρ_{fl} is the density of the fluid and V_{displ} is the volume of the ship that is beneath the surface of the water. The density of fresh water is a bit less than the density of sea water, so V_{displ} must be greater in fresh water to produce the same buoyant force.

Q12.14 With the wood completely submerged the buoyant force is equal to $\rho_{\text{water}}V_{\text{wood}}g$, where ρ_{water} is the density of the water and V_{wood} is the volume of the piece of wood. The buoyant force increases very slightly with depth because the density of the water increases slightly with depth, due to the increase of pressure with depth. But for the depth of a swimming pool the increase in density from near the surface to the bottom of the pool is very small and to a good approximation the buoyant force on the piece of wood remains constant as the wood is pushed deeper under the water.

Q12.15 A mass of feathers for which the weight is one pound has a much larger volume than a mass of lead for which the weight is one pound. The larger volume for the feathers would have a much larger buoyant force from the air and the net downward force on the feathers would be less than that on the lead. It would take more than a pound of feathers to balance a pound of lead.

Q12.16 The pressure difference would be (0.01)(1.0×10^5 Pa) = 1.0×10^3 Pa. For a door with area 2 m^2 the net force keeping the door closed would be 2×10^3 N. This is about 450 pounds. It would take a very strong person to pull the door open. But all that would be required would be to open it slightly, so air can move from the high pressure to the low pressure side. Once the pressure is equalized the door will easily open the rest of the way.

Q12.17 The pressure at depth h in an incompressible fluid is given by $p = p_0 + \rho gh$, where

p_0 is the pressure at the surface. When the depth h is doubled, the ρgh term doubles. But p doesn't double, because of the p_0 term. The gauge pressure doubles but the absolute pressure is less than $2p$.

Q12.18 When the block floats, the weight of water displaced equals the total weight of the block. Since it floats, the average density of the block plus iron must be less than the density of the water. The block still floats when the block is turned over. In either case the same amount of water is displaced, an amount whose weight equals that of the block, so the water level in the bucket doesn't change.

Q12.19 As the jar is pushed deeper the water pressure increases and the air inside the jar is compressed to a smaller volume. More water enters the jar so less water is displaced by the jar plus air bubble and the buoyant force on the jar decreases.

Q12.20 When the bowling ball is sitting in the floating canoe it must cause an amount of water to be displaced that is equal to the weight of the bowling ball. When the bowling ball is sitting at the bottom of the pool the buoyant force on it is less than its weight, since it sinks, so it is displacing less than its weight of water. The bowling ball causes less water to be displaced when it has been dropped over the side, so the water level in the pool falls.

Q12.21 The buoyant force is increased by an amount equal to the weight of the bird so the canoe sinks lower to displace more water and the level of water in the pool rises.

Q12.22 Consider a bucket full of water. When the piece of wood is placed in the water the buoyant force on the floating piece of wood equals its weight. This buoyant force also equals the weight of the water displaced by the piece of wood. Therefore, the weight added by placing the piece of wood in the water equals the weight of the water that flows over the side of the bucket and the weight of the contents of the bucket doesn't change. The two buckets weigh the same.

Q12.23 When the ice cube is floating it displaces an amount of water whose weight equals its weight. The amount of water produced from the melted ice has a weight equal to that of the ice cube; the mass doesn't change during the melting. These two amounts of water are the same, so the water level in the glass doesn't change.

Q12.24 The balloons move forward toward the front of the car. The air in the car has the same acceleration as the car. The force that produces this acceleration for a volume of air is exerted by the surrounding air. When this volume is replaced by a helium-filled balloon, the force on the balloon exerted by the surrounding air is the same as when just air was in the volume. But the helium-filled balloon has less mass that the same volume of air (the balloon rises if released), so the same force produces a greater acceleration when applied to the balloon and the balloon has a greater forward acceleration than does the air inside the car.

Q12.25 The velocity of the fluid at a given point in space is constant in time but the velocity at different points can be different. A fluid particle accelerates when it moves to a point where the velocity is different.

Q12.26 The pressure is less in the moving stream of air than it is in the surrounding stationary air.

Q12.27 The pressure is less in the rapidly rotating air, where the air is moving at high speed. The air pressure difference gives rise to large forces, since normal air pressure is large.

Q12.28 Atmospheric pressure is less at higher elevations so for the same wing speed and shape the pressure difference between points above and below the wing is less at higher elevations.

Q12.29 See Problem 12.84. As the water falls it gains speed due to gravity. The continuity equation says that the cross-sectional area decreases when the fluid speed increases.

Q12.30 "Identical-sized" means the two cubes have equal volumes. But lead has a greater density than aluminum so the mass of the lead cube will be greater than the mass of the aluminum cube. (a) The buoyant force on each cube is given by $B = \rho_{\text{water}} V_{\text{water}} g$. Since the cubes have equal volumes the buoyant force is the same on each. (b) The tension in the wire is given by $T = mg - B$. B is the same for both but mg is greater for the lead cube and therefore the tension in the wire is greater for the lead cube. (c) The force on the lower face of each cube is $F = p_1 A$, where A is the area of the lower face and $p_1 = p_0 + \rho g h$ is the pressure at the lower face. Figure Q12.30 shows that the lead cube is suspended at a greater depth in the water. So, the pressure at its lower face is greater than for the aluminum block and the force on its lower face is greater than for the aluminum block. But, the pressure at the upper face is also greater and the difference between the upward force on the bottom and the downward force on the top face is the same for the two blocks. (d) The difference in depth between the top and bottom faces is the same for both blocks so the difference in pressure between the upper and lower faces is also the same for both blocks.

CHAPTER 13
GRAVITATION

Discussion Questions

Q13.1 The force the earth exerts on the apple equals in magnitude the force the apple exerts on the earth. By Newton's 2nd law equal forces produce a much smaller acceleration for the much more massive earth.

Q13.2 $g = Gm/R^2$. $m = \rho V = (4/3)\pi R^3\rho$ so $g = 4\pi\rho GR/3$. g at the surface of the planet would be directly proportional to the radius of the planet.

Q13.3 The pound is a unit of force and describes the gravity force on the object. The gravity force on an object is less on Mars so it takes more butter to make a pound on Mars. The kilogram is a measure of mass, which is an intrinsic measure of the amount of material. A kilogram of butter is the same amount of butter on Mars as on the earth.

Q13.4 Because the strength of the gravity force is directly proportional to the mass of an object. More massive objects have greater gravity force on them but require a greater force for the same acceleration. In Example 13.2 the gravitational force is between the two masses. When two masses are dropped near the surface of the earth the gravitational force is between the earth and each mass.

Q13.5 At noon I am on the side of the earth facing the sun and at midnight I am on the side of the earth opposite the sun. So at noon I am closer to the sun by a distance equal to the diameter of the earth and at that point I exert a greater gravity force on the sun.

Q13.6 If there was no net force on the moon it would travel in a straight line at constant speed. A net inward force is required to keep the moon moving in uniform circular motion, and the gravity force exerted by the earth is just what is required to keep the moon moving in a circle of constant radius.

Q13.7 The orbital period for a circular orbit is given by Eq.(13.12): $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{s}}}}$, where we have replaces the mass of the earth, m_{E}, by the mass of the star, m_{s}. $T\sqrt{m_{\text{s}}} = \dfrac{2\pi r^{3/2}}{\sqrt{G}}$, which is constant. Let T_1 be the period for a star of mass m_{s1} and let T_2 be the period for a star of mass m_{s2}. Then $T_1\sqrt{m_{\text{s1}}} = T_2\sqrt{m_{\text{s2}}}$. $T_2 = T_1\sqrt{\dfrac{m_{\text{s1}}}{m_{\text{s2}}}}$. $T_1 = T$ and $m_{\text{s2}} = 3m_{\text{s1}}$. $T_2 = \dfrac{T}{\sqrt{3}}$. The answer is (d).

Q13.8 The orbital period for a circular orbit is given by Eq.(13.12): $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{s}}}}$, where we have replaces the mass of the earth, m_{E}, by the mass of the star, m_{s}. T doesn't depend on the mass of the planet, so the answer is (c) T. Applying Newton's second law to the motion of the planet gives $F_{\text{g}} = ma_{\text{rad}}$. Since the gravitational force F_{g} between the star and the planet is proportional to the mass m of the planet, the mass of the planet divides out of the equation.

Q13.9 The earth and moon orbit together around the sun in an orbit of nearly the same radius and orbital speed.

Q13.10 At a given distance from each, the attractive force of the earth is greater than the attractive force exerted by the moon. So, more fuel is required to go from the earth to the moon than from the moon to the earth.

Q13.11 For a circular orbit the work done is zero. We can see this in either of two ways: the force is radial and the displacement is tangential so no work is done, or, the speed is constant so the kinetic energy is constant and the work-energy theorem says the net work done on the planet is zero, For an elliptical orbit, after one complete orbit the speed has returned to its initial value, the change in kinetic energy is zero and no net work is done. For the elliptical orbit there is a component of force tangent to the path for part of the orbit and the gravity force does do work on the planet. The force does positive work during the part of the orbit where the planet is speeding up and does negative work during the part of the orbit where the planet is slowing down. But the net work for a complete orbit is zero.

Q13.12 In the analysis of Example 13.5 the escape speed is independent of the direction the projectile is launched. But, if air resistance is included the mechanical energy dissipated by work done by the air resistance depends on the path of the projectile through the earth's atmosphere, and this path depends on the direction in which the projectile is launched.

Q13.13 To just barely escape from the earth the initial total mechanical energy of the satellite must be zero, so it can reach a point far from the earth, where the gravitational potential energy is zero, with zero kinetic energy. If the total mechanical energy is less than zero, the satellite rises to a maximum height and then falls back to the surface of the earth. It returns to the earth with the same speed with which it was projected upward. If the initial mechanical energy is greater than zero then it escapes from the earth. When it is very far from the earth it's kinetic energy equals the total mechanical energy it had initially.

Q13.14 The statement is correct. For objects moving near the surface of the earth the elliptical path is approximated well by a parabola.

Q13.15 According to Kepler's 2nd law, a line from the sun to the earth sweeps out equal areas in equal times. As Fig.13.19c shows, the earth moves faster when it is closer to the sun.

Q13.16 Such an orbit is not possible. The gravitational force is directed toward the center of the earth and the plane of all satellites must pass through the center of the earth. The center of the earth is one of the foci of the elliptical orbit of the satellite.

Q13.17 The acceleration is greatest when the force is greatest. This is when the satellite is closest to the object, at the perihelion. The acceleration is least when the force is least, at aphelion.

Q13.18 $F = ma$ would give $A / r^3 = mv^2 / r$, where A is a constant. $v = \sqrt{A / mr^2}$ and $T = 2\pi r / v = 2\pi r^2 (\sqrt{m / A})$. The period would be proportional to the square of the orbit radius. The requirement of elliptical orbits is only for a $1 / r^2$ force so Kepler's first law would no longer hold. Kepler's second law is just a statement of conservation of angular momentum, and this holds for any central force so would be unchanged.

Q13.19 The inertia of the comet causes it to continue to move. Part of the kinetic energy it has at perihelion is converted to gravitational potential energy as the comet moves to apehelion.

Q13.20 Only at an infinite distance from the earth is its gravitational force zero. The gravity force is required to provide the radial acceleration of an orbit. If there were no gravity force on a spacecraft, the spacecraft would travel in a straight line at constant speed. An astronaut feels weightless because the spacecraft and the astronaut have the same acceleration.

Q13.21 The only force on the spacecraft and on the astronaut is gravity and they move with the same acceleration.

CHAPTER 14
PERIODIC MOTION

Discussion Questions

Q14.1 The object travels a total distance of $4A$ in one period. If the amplitude is doubled then this distance doubles. The period does not depend on the amplitude so is unchanged. The maximum speed is given by Eq.(14.23) and doubles when the amplitude doubles. The distance traveled in one cycle doubles but the speed increases just enough to keep the time for one cycle the same.

Q14.2 Swinging of a playground swing; this motion is simple harmonic only for small amplitudes and the motion also loses mechanical energy due to dissipative forces. Pistons in a car engine; force doesn't precisely obey Hooke's law.

Q14.3 Yes. Period or frequency is independent of the amplitude. This is important; pitch generated needs to be independent of how hard the tuning fork is struck and needs not to change as vibrations die away.

Q14.4 (a) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. The frequency increases because the mass of the oscillating object decreases. (b) $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$. The period decreases. (c) The amplitude is given by $\frac{1}{2}kA^2 = E$, where E is the total energy of the system. When the box is at its maximum distance from the equilibrium point, $v = 0$ and all the energy of the system is stored as elastic potential energy in the stretched or compressed spring. Therefore, removing the pebble doesn't alter the total energy of the system and the amplitude doesn't change. (d) Since the total energy of the system doesn't change, the maximum kinetic energy of the box doesn't change. (e) $K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2$, where K_{max} is the maximum kinetic energy of the box and v_{max} is its maximum speed. K_{max} doesn't change but m is decreased, so v_{max} increases.

Q14.5 The half spring stretches half as much as the full spring for the same applied force, since it has half the number of coils. $k = F/x$ so the force constant of each half is twice that of the full spring. $f = (1/2\pi)\sqrt{k/m}$ so the frequency would be larger by $\sqrt{2}$.

Q14.6 There is no horizontal force on the coin other than the static friction force exerted on it by the glider surface. Since the coin isn't slipping relative to the glider, the static friction force on the coin must give the coin the same horizontal acceleration as the acceleration of the glider. The acceleration of the glider is greatest at the end points of its motion, when it is its maximum distance from its equilibrium position and this is where the static friction force on the coin is the greatest. The acceleration of the glider is zero as it moves through its equilibrium position, and at this point in the motion the static friction force on the coin is zero.

Q14.7 Yes, the center of mass of the system, at the center of the spring, remains at rest just as if that point were attached to a stationary object. The motion is that of a glider attached to a half-spring. The half-spring has twice the force constant of the full spring, so the period is smaller by a factor of $1/\sqrt{2}$ when two gliders are used compared to where there is only one glider and the other end of the spring is kept stationary.

Q14.8 Use the chain and ring to construct a simple pendulum. Measure the period of the pendulum and use $T = 2\pi\sqrt{L/g}$ to calculate g. You will need to estimate the length of the pendulum you have made. Using the data in Appendix F, $g_p = Gm_p / R_P^2$ gives $g_{Mars} = 3.7\ \text{m/s}^2$. The difference between g on Earth and Mars is large and the measurement of g does not need to be very precise.

Q14.9 The period is the same in all three cases. Let d_0 be the amount the spring is stretched when the acceleration of the object equals that of the elevator. Upward acceleration of the elevator requires that d_0 be larger than when the elevator is at rest or moving with constant velocity and downward acceleration of the elevator decreases d_0. But the net force on the object when it is displaced a distance *x* from the d_0 point is still given by $-kx$ and the period is unaffected by the acceleration of the elevator. The acceleration of the elevator is equivalent to changing the value of *g* and it is shown in Section 14.4 that the period doesn't depend on *g* for vertical SHM.

Q14.10 Both the pendulum and the space station would be in free-fall, with the net force on each being gravity. The pendulum would no longer swing back and forth when displaced and released. The period of the object on the spring depends on the force constant of the spring and the mass of the object and would be unchanged.

Q14.11 (a) For a pendulum $T = 2\pi\sqrt{\dfrac{L}{g}}$. An upward acceleration *a* is equivalent to increasing *g* to $9.8\ \text{m/s}^2 + a$. The restoring force is increased and the period decreases. (b) Motion with constant velocity cannot be distinguished from the elevator being at rest. The period is unchanged. (c) A downward acceleration *a* decreasing *g* to $9.8\ \text{m/s}^2 - a$. The restoring force is decreased as the period decreases. (d) The elevator is in free-fall and the pendulum ceases to swing back and forth. There is no restoring force and the period becomes infinite.

Q14.12 (a) $f = (1/2\pi)\sqrt{g/L}$. To double the frequency decrease the length by a factor of 4. (b) $T = 2\pi\sqrt{L/g}$. To double the period increase the length by a factor of 4. (c) $\omega = \sqrt{g/L}$ so decrease the length by a factor of 4, just as in part (a).

Q14.13 g decreases with altitude so the period of the pendulum increases and the clock loses time.

Q14.14 The period of a simple pendulum is independent of the amplitude; so long as the amplitude remains small the period remains the same. When the amplitude is increased the distance traveled in one period is greater. But the restoring force also increases with the angular displacement from equilibrium so the mass moves faster. The two effects compensate and the period is independent of amplitude. The same reasoning and conclusion applies to a physical pendulum. But for larger amplitudes the restoring force is no longer proportional to the displacement and the motion ceases to be simple harmonic. As Figure 14.22 shows, the magnitude of the restoring force is less than proportional to the displacement. A smaller restoring force means a longer period; the period increases when the amplitude decreases. The same reasoning and conclusion applies to a physical pendulum.

Q14.15 The period of oscillation of the shorter leg, treated as a physical pendulum, is less. It is easiest for the dog to walk when its legs swing at their natural frequency, as discussed in Example 14.10.

Q14.16 The tension is in the radial direction. The pendulum mass has a radial acceleration $a_{rad} = v^2 / L$, directed inward. At the end points of the motion, where $v = 0$ and the string makes an angle Θ with respect to the vertical, $a_{rad} = 0$ and $T = mg\cos\Theta$. As the string moves through the vertical position the speed of the mass is a maximum and $T = mg + mv_{\max}^2 / L$. The tension is greater when the string is vertical and is least at the end points.

Q14.17 Yes, such a standard could be used. An advantage is that such a standard is easily reproduced and easy to use. Disadvantages are that it is difficult to measure the period to high accuracy and that the period varies with location, due to variations in g.

Q14.18 The angular frequency ω is equal to $2\pi f$, where $f = \dfrac{1}{T}$ is the frequency of oscillation of the pendulum. $\omega = \sqrt{\dfrac{k}{m}}$ so for a given mass and spring, ω is a constant and doesn't vary during the motion. The angular speed $\omega = d\theta / dt$ is the rate at which the angular displacement of the pendulum is changing. It is equal to v / L, where v is the speed of the pendulum bob and L is the length of the pendulum. The linear speed v is continually changing during the motion so this ω is also changing. ω is zero when the pendulum is at its maximum angular displacement and ω is a maximum when the pendulum is swinging through its equilibrium position, when it is vertical.

Q14.19 The structures should not have natural frequencies in the range of typical earthquake frequencies, to avoid resonance excitation of the frequencies of motion of the structures. Damping should be large so that large amplitude oscillations don't build up.

CHAPTER 15
MECHANICAL WAVES

Discussion Questions

Q15.1 (a) Yes. The frequency depends on whatever is producing the waves. For example, in Fig.15.3 the frequency of oscillation of the block equals the frequency of the waves that it produces on the string. (b) Yes. The wave speed depends solely on the properties of the string, its tension and mass per unit length. So, the answer to (c) is that the two waves on the same string must have the same wave speed. But $v = f\lambda$, so if f is different and v is the same, λ must be different. (c) No. (d) Yes. The amplitude depends on whatever is producing the waves. (e) No. $v = f\lambda$. v must be the same. If f is also the same, then λ can't be different.

Q15.2 If the wave has length L and tension F, the speed of a wave pulse on the wire is $v = \sqrt{\dfrac{F}{\mu}}$. The time it takes a pulse to travel the length of the wire is $t = \dfrac{L}{v} = L\sqrt{\dfrac{\mu}{F}}$. Ignoring a slight stretching of the wire when the tension is increased, L and μ are constant and $t\sqrt{F} = L\sqrt{\mu}$ is constant. Therefore, $t_1\sqrt{F_1} = t_2\sqrt{F_2}$ where $t_1 = 2.00\text{ s}$, $t_2 = 6.00\text{ s}$ and $F_1 = F$. $F_2 = F_1\left(\dfrac{t_1}{t_2}\right)^2 = F\left(\dfrac{2.00}{6.00}\right)^2 = F/9$. The tension must be decreased in order for the wave speed to decrease and for the pulse to take longer to travel the length of the wire.

Q15.3 There is kinetic energy of the moving string and the elastic potential energy of the string when it is displaced from equilibrium. The kinetic energy could be transferred to an object attached to the end of the string.

Q15.4 The energy in the wave decreases. The wave energy is transferred to the thermal energy in the string due to dissipative forces within the string.

Q15.5 The wave speed is independent of the amplitude of the wave. It depends only on the properties of the medium. Eq.(15.14) is an example of this.

Q15.6 The waves slow down as they approach the shore. This causes the water to pile up and produce crests.

Q15.7 The answer is yes to both questions. All that is required is to have a restoring force when a section of the string is displaced longitudinally or when a section of the rod is given a transverse displacement. To produce a longitudinal wave in a stretched string pull longitudinally at a point of the string and release it. The more elastic the string the easier this is to do. To produce transverse waves in a rod, clamp the rod at one end and tap in the transverse direction at some point on the rod. Transverse waves also can be easily set up in a rod clamped at some point by stroking the rod between your fingers.

Q15.8 The wave speed is $v = \sqrt{F/\mu}$. Each part of the string undergoes SHM with maximum speed given by $v_{\max} = \omega A = 2\pi fA$. These two speeds are totally different. The wave speed is constant along the length of the string. At any instant the transverse speed of each point on the string varies along the length of the string and the transverse speed of any point on the string varies in time.

Q15.9 Eq.(15.14) says that waves travel faster on the thin strings, since they have smaller μ.

$f_1 = (1/2L)\sqrt{F/\mu}$, so the fundamental frequency f_1 is higher for the thin strings.

Q15.10 As Eq.(15.21) shows, the power at a point on the string at a given time is given by the product of the transverse force component F_y and the transverse velocity v_y of this point on the string. The product of F_y and v_y always has the same sign, even though v_y is negative as often as it is positive. Therefore, the power $P(x,t)$ at a given location and time is proportional to $\sin^2(kx - \omega t)$ (Eq.(15.23)). This quantity is always positive and its average over one or more cycles is equal to $\frac{1}{2}$, not zero.

Q15.11 $v = \sqrt{F/\mu}$; the wave speed changes when μ changes. The frequency f is the number of cycles per second and won't change. $v = f\lambda$ so when v changes the wavelength λ must also change.

Q15.12 The tension is zero at the bottom of the rope and at the top of the rope the tension is equal to the weight of the rope. $v = \sqrt{F/\mu}$ so the wave speed increases as the pulse travels up the rope (Example 15.3).

Q15.13 The wave disturbance travels along the length of the string. When one section of the string is displaced it exerts a force on the adjacent section and this allows for one section to do work on the adjacent section and to thereby transfer energy to it.

Q15.14 The node never has any displacement. No displacement means no work is done on it by the adjacent sections of the string.

Q15.15 No, a standing wave is not produced if the two waves don't have the same amplitude. Let one wave have amplitude A and the other have amplitude $A + \Delta$. The wave with amplitude $A + \Delta$ can be written as the sum of two waves, one with amplitude A and the other with amplitude Δ. The part with amplitude A combines with the other wave to produce a standing wave but the part with amplitude Δ remains a traveling wave. No, if the two waves have different frequencies they don't combine to form a standing wave. Points at which there is destructive interference at a given time no longer have destructive interference at a slightly later time. Superposition of such waves form beats (Section 16.7) rather than standing waves.

Q15.16 The pitch increases as the rubber band is stretched. As the rubber band is stretched, F increases and μ decreases, so Eq.(15.35) says f increases.

Q15.17 According to Eq.(15.35), to double the frequency the tension in the string must be increased by a factor of four. Therefore, to raise the pitch of the string one octave the tension in the string must be increased by a factor of four. To raise the pitch two octaves the tension must be increased a factor of sixteen. If the tension exceeds the breaking strength of the string, the string will break.

Q15.18 The finger on the middle of the string produces a node there, so the string vibrates in its first overtone. The frequency of the first overtone is twice the fundamental frequency.

Q15.19 The wall prevents longitudinal motion but offers no impediment to transverse motion.

Q15.20 According to Eq.(15.35), all else being equal a longer string has a lower fundamental frequency.

Q15.21 They mark places where the string can be held down to produce fundamental standing waves of different frequencies. The shorter the vibrating portion of a string, the higher the frequency of its fundamental.

CHAPTER 16
SOUND AND HEARING

Discussion Questions

Q16.1 The frequency is the number of cycles per second and doesn't change. The number of cycles arriving per unit time at one side of the interface between the two materials must equal the number leaving the interface on the other side in the same time. The wave speed increases so by $v = f\lambda$ the wavelength must increase.

Q16.2 Compression waves in the metal rails travel faster than sound waves in the air.

Q16.3 The wave speed increases with temperature, so by Eq.(16.18) and (16.22) the pitch increases with temperature.

Q16.4 Different notes correspond to different standing wave patterns in the air columns inside the instrument, to overtones with differing number of nodes. The lowest possible note corresponds to the frequency of the fundamental, but there is no physics limit on the highest note. But higher overtones may be difficult to produce. All the notes must have a frequency that is an integer multiple of the fundamental frequency.

Q16.5 This brings the instrument and the air inside to the same steady-state temperature it will have during the performance. The speed of sound depends on the air temperature, so the pitch of the note is affected by temperature.

Q16.6 The speed of sound in helium is much higher than in air so the standing waves in your vocal track have higher frequency when the vocal track is filled with helium rather than air.

Q16.7 The tires vibrate at the frequency at which the ridges are encountered. This frequency is proportional to the speed of the car.

Q16.8 (a) No. The sound intensity level is given by $\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right)$. $\beta = 0$ means $I = I_0$, since $\log(1) = 0$. When $\beta = 0$ there is sound, with intensity equal to the reference intensity I_0. (b) Yes. The sound intensity level β is negative when the intensity I is less than the reference intensity I_0. (c) Yes, a sound intensity of $I = 0$ means there is no sound. (d) No, the sound intensity cannot be negative.

Q16.9 The eardrum is set into vibration by pressure variations so the pressure amplitude has the most direct influence. But, these two amplitudes at a given wavelength or frequency are directly proportional to each other (Eq.16.5).

Q16.10 According to Eq.(16.14), the intensity is proportional to the square of the pressure amplitude. If $p_{\max}$ is halved, I decreases by a factor of 4. If the pressure amplitude is increased by a factor of 4, the intensity will increase by a factor of 16.

Q16.11 No. The sound intensity level is given by $\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right)$. If β obeyed the inverse-square law, doubling *r* would change the sound intensity level from β to $\beta/4$. That is, if at a distance *r* the sound intensity level is 60 dB, at distance $2r$ the sound intensity level would be 15 dB. But Example 16.9 shows that doubling the distance from the source decreases β by 6.0 dB. That is, if at distance r the sound intensity level is 60 dB, then at distance $2r$ the sound intensity level is 54 dB.

Q16.12 The inverse square relationship is Eq.(15.26). If $r_2 > r_1$, the absorption of energy means that I_2 is less than $I_1(r_1/r_2)^2$; the intensity falls off faster than $1/r^2$.

Q16.13 The added mass slows the vibration of the tine and therefore decreases *f*. The sound wave in air has the same frequency as the tine and for the sound wave in air $v = f\lambda$. So, the frequency of the sound wave decreases and its wavelength increases.

Q16.14 The sound wave reflects from the walls and continues to propagate inside the building. The sound energy is converted to thermal energy due to dissipative forces in the air and during the reflections.

Q16.15 The interference is constructive if the path difference is an integer number of wavelengths. It is destructive if the path difference is a half-integer number of wavelengths. For this source, $\lambda = \dfrac{v}{f} = \dfrac{344 \text{ m/s}}{860 \text{ Hz}} = 0.40 \text{ m}$. The path difference is $13.4 \text{ m} - 12.0 \text{ m} = 1.4 \text{ m}$. $\dfrac{1.4 \text{ m}}{0.40 \text{ m}} = 3.5$. The path difference equals 3.5 wavelengths and the interference is destructive.

Q16.16 The observed frequency of the rotating fork is shifted by the Doppler effect. Its pitch is increased when it is moving toward the listener and is decreased when it is moving away from the listener. The listener therefore hears two different frequencies from the two forks and beats occur.

Q16.17 The person is moving toward one source and its pitch is raised and is moving away from the other source and its pitch is lowered. When the two parts of the organ sound the same note the moving listener hears two different pitches.

Q16.18 In Eq.(16.29) the velocities of the listener and source are relative to the medium in which the wave is traveling. In this case this is the air. The direction from the listener to the source is positive. If the wind speed is v_{w}, then $v_{\text{L}} = +v_{\text{w}}$ and $v_{\text{S}} = +v_{\text{w}}$. The factor multiplying f_{S} in Eq.(16.29) is unity and there is no shift in frequency; $f_{\text{L}} = f_{\text{S}}$.

Q16.19 Boats moving on the surface of the water will observe a Doppler effect for the surface waves. The listener can be in one boat and the other boat can be the source of the waves. For elastic waves propagating below the surface, a Doppler effect will be observed for sources and listeners moving beneath the surface. One example is two submarines, where one produces the elastic waves and the other contains the listener, that detects these waves.

Q16.20 One edge of the star is rotating away from us and the opposite side is rotating toward us. Therefore, the light from the rim of the star at its equator is Doppler shifted and the measured frequency shift can be used to measure the tangential speed *v* at the equator.

Q16.21 What determines the Doppler shift is the component of the train's velocity component along the line between you and the train. This component steadily decreases as the train approaches, is zero as the train passes, and then starts to increase in the opposite direction.

Q16.22 In case 1, the frequency the observer hears is $f_{\text{L1}} = \dfrac{v}{v-u} f_{\text{S}}$, where u is the speed of the source and v is the speed of sound in air. In case 2, the frequency the observer hears is $f_{\text{L2}} = \dfrac{v+u}{v} f_{\text{S}}$, where now u is the speed of the observer and has the same numerical value as u for case 1. f_{L1} is not equal to f_{L2}. The motion of the source affects the wavelength of the observed sound and the motion of the observer affects the speed at which the waves pass the listener. The Doppler shift equation involves the velocity of the observer and the velocity of the source separately, not the relative velocity between the observer and source. The difference between the frequencies for the two cases in this question is $f_{\mathrm{L1}} - f_{\mathrm{L2}} = \dfrac{(u)^2}{v(v-u)} f_{\mathrm{S}}$, The frequency for case 1 is larger than for case 2. But if $u << v$ then the difference in frequencies is very small. For a numerical example, let $v = 340\ \mathrm{m/s}$ and $u = 140\ \mathrm{m/s}$. Then $f_{\mathrm{L1}} = 1.70 f_{\mathrm{S}}$ and $f_{\mathrm{L2}} = 1.41 f_{\mathrm{S}}$.

Q16.23 No, the aircraft continually produces a shock wave as it travels at a speed greater than the speed of sound in the air. An observer on the ground hears the "sonic boom" when this shock wave passes over him.

Q16.24 The air inside the plane is moving with the plane, so sound travels relative to you within the plane with the speed of sound just as it does when the plane is at rest. You don't hear any effect of the shock wave. The sonic boom results from the rapid jump in air pressure when the shock wave passes an observer. The shock wave travels with you so the pressure at your location remains constant.

Q16.25 A sonic boom results from the rapid change in pressure as the edge of the shock wave passes by. A person at *B* hears a sonic boom; observers at *A* and *C* don't.

CHAPTER 17
TEMPERATURE AND HEAT

Discussion Questions

Q17.1 To bring the thermometer to the same temperature as the water heat must flow from the water into the thermometer material. If the thermometer is large compared to the amount of water, this heat flow from the water will change its temperature by a measurable amount. The measurement process would alter, to an unacceptable extent, the quantity being measured.

Q17.2 No. $T_{\text{K}} = T_{\text{C}} + 273.15$. If T_{C} is doubled then T_{K} is less than doubled. For example, if $T = 50°\text{C}$ and then doubled to $100°\text{C}$, T_{K} increases from 323 K to 373 K, and $373/323$ is only 1.15.

Q17.3 The coefficient of volume expansion for aluminum is $7.2\times10^{-5}\ \text{K}^{-1}$ and for steel it is $3.6\times10^{-5}\ \text{K}^{-1}$. So, for a given temperature change the diameter of the aluminum piston will expand more than the inside diameter of the cylinder. If the temperature increase is great enough the piston will expand so much more than the cylinder that it will no longer have room to move freely in the cylinder.

Q17.4 Frozen water pipes burst because water expands when it freezes. Water is unusual in this respect. Most materials contract when they freeze and expand when they melt. This is the case for mercury, so a mercury-in-glass thermometer would not burst if the temperature went below the freezing temperature of mercury.

Q17.5 Hollow cavities expand just like the material that surrounds them. The external dimensions of both objects would increase the same amount.

Q17.6 The diameter of the cap increases when the temperature increases. The diameter of the opening at the top of the bottle also increases with temperature but the coefficient of linear expansion for the bottle material (plastic or glass) is much less than that for the metal cap. And the bottle material is a poor conductor of heat so it takes much longer for its temperature to rise, compared to the cap.

Q17.7 The metallic material of the oven wall is a good conductor of heat and the air is a poor conductor. Also, the mass of a given volume of air is much less than the mass of the same volume of the oven wall, and the heat flow for a given temperature change of an object is proportional to the mass of the object. For both these reasons, much less heat energy flows into your hand when it is in contact with the hot air than when it is in contact with the hot oven wall.

Q17.8 It is incorrect. Two objects with different mc have different thermal energy changes for the same temperature change, so the thermal energy a body contains depends on mc as well as its temperature. And the term heat doesn't mean thermal energy contained in an object; heat is energy transferred due to a temperature difference.

Q17.9 $c = Q/(m\Delta T)$ so the unit for c is $\text{J/(kg}\cdot\text{C}^\circ)$. 1J = 1 $\text{N}\cdot\text{m}$ = 1 $\text{kg}\cdot\text{m}^2/\text{s}^2$ so 1 $\text{J/(kg}\cdot\text{C}^\circ)$ = 1 $\text{m}^2/(\text{s}^2\cdot\text{C}^\circ)$. The student is correct.

Q17.10 When the water changes phase from liquid to vapor, heat energy must flow into it. This heat energy comes out of the air so the air is cooled. The water evaporates more readily if the humidity is high, so such a system works less well in a high-humidity climate.

Q17.11 There is no temperature change associated with a phase change; phase changes occur at constant temperature.

Q17.12 Evaporation of water (sweat) from the skin is an important cooling mechanism for a human body in a hot environment. When the air is dry the evaporation occurs much more rapidly than when the humidity is high.

Q17.13 The product mc is much larger for the potato than for the piece of aluminum foil, due to the small mass of the foil. So, for a given temperature change, much less heat must flow out of the foil than out of the potato. A second reason is that the metal foil is a much better conductor of heat than the potato and therefore heat flows from the foil more readily.

Q17.14 Heat energy flows into the water as it undergoes the phase change associated with evaporation. This heat comes from the water inside the bag and when heat leaves this water, its temperature decreases.

Q17.15 As the water on your skin evaporates heat energy flows from your body into the water.

Q17.16 The body of water has a large value of mc and stores a lot of thermal energy. Heat flows into or out of the water and maintains a more constant temperature of the adjacent land.

Q17.17 Once the water has reached the phase transition temperature of 0°C, additional heat must come out of the water for it to change phase from liquid to solid. Heat flows out more easily for the water next to the metal trays that are in contact with the colder air of the freezer and that are good conductors of heat.

Q17.18 The temperature of your skin is above the boiling point of the alcohol, so it rapidly evaporates. Heat energy goes out of your arm and into the alcohol as it changes phase from liquid to vapor.

Q17.19 The metal is a better conductor of heat so heat flows more readily between your hand and the metal block than between your hand and the wood block when both blocks are at the same temperature. When the blocks have the same temperature as your hand there is no heat flow between them and your hand. So, at this temperature they seem to have the same temperature.

Q17.20 The heat lost by the coffee during the five minutes is proportional to the temperature difference between the coffee and the room. Adding the cream cools the coffee so adding it first lowers the temperature of the coffee at the start of the waiting period and the coffee loses less heat during the five minutes.

Q17.21 The filling has a larger density and specific heat capacity than the crust and is a much better conductor of heat than the crust, so more heat flows from it into your tongue than from an equal volume of crust.

Q17.22 They have large values of mc and are good conductors of heat. Heat from the stove first heats the pot to a uniform temperature.

Q17.23 With its smaller specific heat capacity, the land has a greater temperature change for the same heat flow.

Q17.24 The metal potato nail is a good conductor of heat so conducts heat to the center of the potato. The thermal conductivity of aluminum is four times that of steel, so aluminum nails work better. The water conducts heat along the wick, from the oven to the interior of the meat.

Q17.25 The dark soil of the plowed fields is a better absorber of heat than lighter ground so will be warmer.

Q17.26 We might say this is not correct. The rate of heat conduction does depend on the temperature difference, so the water does cool, for example, more rapidly from 80° C to 79° C than from 20° C to 19° C. But the water must cool to 0° C before it can freeze. So, if you compare water that starts at 20° C to water that starts at 80° C, the 80° C water must first cool to 20° C and after that cools just like the 20° C water cooled initially. The hot water takes longer to freeze. But there are experimental claims that this is not correct, that the hot water does freeze faster. You should do a web search on "hot or cold water in ice cube trays" to read some of the discussion.

Q17.27 The energy emitted by the sun spreads out in all directions so the intensity of sunlight is much less at the surface of the earth than it is at the surface of the sun. H/A is different at the surfaces of the sun and the earth, so by Eq.(17.25) the temperature T is also very different.

Q17.28 This argument does not make sense. The heat energy that must be supplied to the house by its heating system equals the amount of heat lost by the house to the environment. In a full day this heat loss is proportional to the average temperature difference between the house and its surroundings. If the house is kept warm at night this heat loss is greater than if the house is allowed to cool down at night.

CHAPTER 18
THERMAL PROPERTIES OF MATTER

Discussion Questions

Q18.1 The equation of state expressed in Eq.(18.1) shows that in general the volume V depends on the temperature and pressure. An equation of state of this form applies only to a single phase of material, either solid, liquid or gas. Different behavior is found during phase changes.

Q18.2 No, the Celsius temperature could not be used. Converting from Kelvin to Celsius is not done just by multiplying by a conversion factor. You can see that Celsius temperatures can't work because Celsius temperatures can be negative and negative T in Eq.(18.3) doesn't make sense.

Q18.3 The temperature of the air inside the tires increases and this causes the pressure to increase. You should not let air out of the tires to reduce the pressure, they would then be under-inflated when they cool. The recommended inflation pressure for tires are always given as cold-inflation values.

Q18.4 The increase in pressure raises the boiling point of the coolant and prevents it from boiling. If the system were sealed too great of an engine temperature could result in a coolant pressure that would damage engine parts and coolant hoses.

Q18.5 The frozen water in the food undergoes sublimation.

Q18.6 The air pressure at the higher elevation of the slopes was much less than at sea level where the bags were filled and sealed. The pressure difference between the air inside and outside the bags caused them to burst.

Q18.7 At low pressures where the ideal gas law applies the number of atoms per cm^3 is still quite large.

Q18.8 The distribution of speeds of the gas molecules doesn't change when they move through the hole into the other half of the container. Therefore, the total translational kinetic energy of the gas doesn't change and the final temperature will be T_0. The volume doubles and the temperature doesn't change, so $pV = nRT$ says that the pressure is halved. The new pressure is $p_0 / 2$.

Q18.9 (a) A kilogram of each has the same mass. One mole of each has the same number of atoms. The mass of one mole is much larger for lead so one kilogram of lead is fewer moles than one kilogram of hydrogen and therefore has fewer atoms. (b) If we have a mole of hydrogen atoms each has the same number of atoms, equal to Avogadro's number. If we instead have a mole of hydrogen molecules, there are more hydrogen atoms than lead atoms since each hydrogen molecule has two atoms. One mole of lead has more mass than one mole of hydrogen since one lead atom has more mass than either a hydrogen atom or a hydrogen molecule.

Q18.10 (a) Adding heat to the gas increases the average kinetic energy of each gas atom. This increases their average speed and therefore they exert a greater force in collisions with the walls of the container. The increased average speed also means that collisions with the wall are more frequent. (b) Constant temperature means the average kinetic energy of each atom and also the average speed don't change. But with a smaller volume collisions with the walls are more frequent and this increases the average force on the walls.

Q18.11 The mass of a nitrogen molecule is less than the mass of an oxygen molecule so I would expect the proportion of oxygen to be less at higher altitudes, since at a given altitude the gravitational potential energy of an oxygen atom would be greater.

Q18.12 This statement is incorrect. In thermal equilibrium the two gases have the same average kinetic energy per atom. The average speeds will then be inversely proportional to the square root of the masses of the atoms.

Q18.13 The assumption of elastic collisions with the walls in equivalent to assuming that the walls are at the same temperature as the gas. If the walls are hotter or colder than the gas, the gas atoms will gain or lose energy when they collide with the walls.

Q18.14 Temperature is related to the average kinetic energy of the random motion of the atoms. The collective motion of the entire gas does not affect the temperature.

Q18.15 The average translational kinetic energy of one atom of the gas depends on the temperature of the gas. If the volume is also kept constant then the temperature increases when the pressure increases. But if both pressure and volume change such as to keep their product pV constant then the temperature doesn't change and the average translational kinetic energy of the atoms remains constant.

Q18.16 It is justified if the difference in gravitational potential energy of an atom at the top and at the bottom of the container is much smaller than the average kinetic energy of the atoms. Let the container height be h. Then $\Delta U = mgh$. $K_{\text{tr}} = \frac{1}{2}mv_{\text{rms}}^2$. The ratio of these energies is $\Delta U / K_{\text{tr}} = 2gh / v_{\text{rms}}^2$. Example 18.6 shows that $v_{\text{rms}} \approx 500$ m/s so for $h = 1$ m this ratio is very small and neglect of gravitational potential energy changes is justified.

Q18.17 The average speed of the air molecules inside the house would decrease and this would lower the temperature of the air in the house.

Q18.18 The lighter gas atoms have a larger v_{rms} and exit through the hole more frequently.

Q18.19 Eq.(18.14) says that the total average translational kinetic energy of the gas depends on the number of moles and on the Kelvin temperature. For the same total mass, there will be more moles of gas B so specimen B will have more total kinetic energy. We must use the molecular mass of the gas in the analysis.

Q18.20 To double the average translational kinetic energy of each gas atom you must double the Kelvin temperature. 25° C = 298 K and 50° C = 323 K so this temperature does not double the Kelvin temperature. The final temperature would have to be 2(298 K) = 596 K = 323°C to double the average translational kinetic energy per atom.

Q18.21 Eq.(18.19) shows that v_{rms} is directly proportional to the square root of the Kelvin temperature of the gas, so to double v_{rms} the Kelvin temperature T must be increased by a factor of 4.

Q18.22 (a) $Q = nC_V\Delta T$ and $\Delta T = \dfrac{Q}{nC_V}$. C_V is smaller for the monatomic gas so the monatomic gas will increase more in temperature than the diatomic gas. (b) Since the diatomic gas has internal degrees of freedom (rotation and vibration) not all the energy that flows into the gas goes into increasing the translational kinetic energy K_{tr}. Some goes into increasing the rotational and vibrational energy. But only K_{tr} determines the temperature.

Q18.23 No. All ideal diatomic gases have the same molar heat capacity C_V. Therefore, if the same amount of heat flows into 1.00 mol of any diatomic ideal gas, the temperature will increase the same amount. But the number of moles that corresponds to 1.00 g of gas depends on the molecular mass and that will be different for different gases. That is, in $\Delta T = \dfrac{Q}{nC_V}$, C_V will be the same but n will be different and therefore ΔT will be different.

Q18.24 The only thing we can say is that for small Δv the number of molecules having speeds between v and $v + \Delta v$ is $Nf(v)\Delta v$. So, neither statement in the question is correct.

Q18.25 There is no liquid phase below the triple point pressure. Table 18.3 gives the triple point pressure to be 610 Pa for water and 5.17×10^5 Pa for CO_2. The atmospheric pressure is below the triple point pressure of water, and there can be no liquid water on Mars. The same holds true for CO_2. On earth $p_{\text{atm}} = 1.01 \times 10^5$ Pa so on the surface of the earth there can be liquid water but not liquid CO_2.

Q18.26 The water boils when the vapor pressure of the liquid equals the applied pressure, which is being reduced. When the water boils the molecules at the surface with the greatest translational speed escape from the liquid. This reduces the average translational kinetic energy of the molecules remaining in the liquid, and this lowers the temperature of the liquid.

Q18.27 The melting temperature of the ice increases when the applied pressure increases. This is because the volume decreases when ice changes phase to liquid.

Q18.28 The water doesn't boil because the large hydrostatic pressure at this depth increases the boiling point to above this temperature.

Q18.29 Atmospheric pressure on the moon is very small so any liquid water on the surface would boil away.

Q18.30 The boiling point temperature is lower at the lower air pressure at high altitudes. So, the boiling water at high altitudes has a lower temperature.

CHAPTER 19
THE FIRST LAW OF THERMODYNAMICS

Discussion Questions

Q19.1 (a) positive (volume of the gas increases) (b) positive (volume of the gas increases) (c) negative (volume of the gas decreases) (d) negative (volume of the gas decreases).

Q19.2 Heat is transferred from one object to another because of a temperature difference. We can't say that a body contains a certain amount of energy because heat is not a state function; the amount of heat transferred into or out of an object in a process is path dependent.

Q19.3 $W = \int_{V_1}^{V_2} p\ dV$. V_1 and V_2 are the same. When the balloon is inflated the total outward force due to the presence of the air inside the balloon must equal the total inward force due to the pressure of the surrounding air and the force due to the stretching of the balloon material. When the air pressure outside the balloon is less, less pressure inside the balloon is needed to inflate it to a given size. Hence, less work needs to be done to inflate the balloon at the summit of Mt. McKinley.

Q19.4 No, the work and heat transfer are both path dependent. For different paths between the initial and final states the work and heat transfer can be different.

Q19.5 Heat energy enters her body from the chemical processes that metabolize the food. Energy leaves her due to the work of raising herself against gravity and as heat that evaporates her perspiration. If her body temperature rises, this is due to some of the food energy that stays in her body. The first law says energy input = work output + heat output + increase in internal energy. During the descent gravity does positive work on her but she still performs work by the contraction of her muscles. The energy input from the food is again what warms her. During the descent she performs less work and less food energy needs to be inputed.

Q19.6 For a volume decrease $W < 0$. $\Delta U = Q - W$ so $|\Delta U| = |Q| + |W|$; the internal energy change is greater than the heat added.

Q19.7 $\Delta U = Q - W$. If we treat the air inside the balloon as an ideal gas U for the gas depends only on its temperature. During the expansion the temperature increases and ΔU is positive. Therefore, the heat Q added to the balloon is greater than the work done by the air inside it. When the balloon has returned to its original temperature, $\Delta U = 0$. Therefore $Q = W$; the net heat flow to the air equals the net work done.

Q19.8 Since the container is open to the air in the room, the pressure inside the container is constant, always equal to the air pressure in the room. Therefore, the process is isobaric. Heat flows out as the air cools (so the process is not adiabatic), the temperature decreases (so is not isothermal) and the volume of the air that was originally in the container decreases (so not isochoric).

Q19.9 In the expansion the repulsive force between the electrons would do positive work as the electrons move apart. $Q = 0$ and $W > 0$ so $\Delta U < 0$ and the temperature would fall.

Q19.10 $\Delta U = Q - W$. For an adiabatic process $Q = 0$ and $\Delta U = -W$. For an adiabatic process, $p_1 V_1^\gamma = p_2 V_2^\gamma$, where $\gamma > 1$. $V_2 = V_1 \left(\dfrac{p_1}{p_2}\right)^\gamma$. If the pressure decreases, $p_2 < p_1$ and $V_2 > V_1$. For any process, when the volume increases the work W done is positive. Therefore, in this process $W > 0$ and ΔU is negative. The internal energy decreases.

Q19.11 With your mouth wide open there is little expansion work. The air from your lungs is at body temperature so is warmer than the air outside your body (unless you do this experiment in Texas in August!) so feels warm. When you constrict the opening of your mouth the air expands through this opening and is cooled, in an adiabatic expansion.

Q19.12 $\Delta U = Q - W$ so $Q = \Delta U + W$. When the gas expands its volume increases and $W > 0$. $pV = nRT$ says that T increases, since V increases and p is constant. $\Delta U = nC_V \Delta T$ for an ideal gas. Since T increases, ΔU is positive. $Q = \Delta U + W$ and both ΔU and W are positive, so Q is positive.

Q19.13 $\Delta U = Q - W$. Since the container is well-insulated, no heat has been transferred and $Q = 0$. Work has been done <u>on</u> the liquid, so $W < 0$. The force exerted during the stirring moves the liquid in the direction of the force so positive work is done by the stirring force. Irregular stirring increases the force applied to the liquid and thereby increases the work done on the liquid. $\Delta U = -W$ and $W < 0$, so $\Delta U > 0$. The internal energy increases and this corresponds to an increase in temperature of the liquid.

Q19.14 The adiabatic compression of the air in the pump heats the air. $\Delta U = -W$. In the compression $W < 0$ so $\Delta U > 0$. In the adiabatic expansion of the air in the pump cylinder when you raise the handle, the air is cooled. In the expansion $W > 0$ so $\Delta U < 0$.

Q19.15 In the adiabatic expansion of the air it is cooled and can in fact be cooled below 0° C.

Q19 .16 The air in the bubble first expands due to the heat flow into it from the surrounding earth. This initial expansion causes the warmer air in the bubble to be less dense than the surrounding air. The mass of lighter air rises and as it does it cools due to its adiabatic expansion at the lower pressure that exists at the higher altitude. The thermal stops rising when it has cooled to the same temperature as the surrounding air. The expansion of the air bubble is approximately adiabatic because the air rises rapidly. Also, air is a poor conductor of heat so little heat flows out of the air bubble as it expands.

Q19.17 The air does positive work to raise itself against gravity. This is approximately an adiabatic process so $\Delta U = -W$ and for $W > 0$, $\Delta U < 0$ and the air cools.

Q19.18 As the air moves downward on the smooth western slope positive work is done on the air by gravity so $W < 0$. The process is approximately adiabatic so $\Delta U = -W$ and for $W < 0$, $\Delta U > 0$ and the air warms.

Q19.19 By definition $dQ = nC_V\, dT$ for constant volume and $dQ = nC_p\, dT$ for constant pressure. In any process, $dU = dQ - dW$. For constant volume $dW = 0$ and $dU = dQ$. For constant pressure, $dW \neq 0$ and dU is $nC_p\, dT - dW$.

Q19.20 The energy comes from the work done on the gas by the external force that compresses it.

Q19.21 The energy comes from the internal energy of the gas, and that is why the gas cools.

Q19.22 The UF_6 molecule has many more internal degrees of freedom (rotation, vibration) than H_2 does. Therefore, more heat is required for a given temperature change for UF_6 than for H_2, because for UF_6 a larger fraction of the heat added goes into the internal motions of the molecule and less into the translation motion that determines the temperature. Said another way, C_V for the polyatomic molecule UF_6 is larger than C_V for the diatomic molecule H_2 (see Table 19.1), so a given amount of heat flowing into the gas gives a smaller temperature rise for UF_6.

Q19.23 (a) The work done equals the area under the path in the pV-plane. This is greatest for path 1 so the work done by the system is greatest for path 1. It is least for path 3 so the work done by the system is least for path 3. (b) $Q = \Delta U + W$. ΔU is positive and is the same for all three paths so Q is largest when W is largest and this is for path 3. $Q > 0$ and heat is absorbed by the system.

Q19.24 (a) and (b). The magnitude of the work done for each loop equals the area enclosed by the loop and the magnitude of the work done along a path equals the area under the path. For loop I the positive work done while the volume is increasing is larger in magnitude than the negative work done while the volume is decreasing, and the net work done by the system is positive. For loop II the negative work done while the volume is decreasing is larger in magnitude than the positive work done while the volume is increasing and the net work done by the system is negative. The area enclosed by loop I is larger than the area enclosed by loop II so the net work done by the cycle is positive. (c) and (d) For a closed loop, $\Delta U = 0$ and $Q = W$. For loop I W is positive, Q is positive and heat flows into the system. For loop II W is negative, Q is negative and heat flows out of the system. For the complete cycle the net work done is positive so the net heat flow is positive.

CHAPTER 20
THE SECOND LAW OF THERMODYNAMICS

Discussion Questions

Q20.1 The cycle approaches reversibility as the temperature difference between the stove and water approaches zero. In this limit an infinitesimal change in the temperature of the stove can cause the liquid to vapor phase transition to reverse direction.

Q20.2 Reversible: (1) Light, frictionless pulley with a string passing over it and two almost equal masses m_1 and m_2 suspended from the two ends of the string. m_1 descends if it is slightly heavier than m_2 and ascends if m_2 is slightly heavier. A small change in one of the masses causes the system to move in the opposite direction. (2) A block on a horizontal frictionless surface and attached to one end of a spring. The block is pushed against the spring and compresses it. Then the applied force is kept almost equal in magnitude to the spring force. If the applied force is increased slightly the block moves one direction and if the applied force is decreased slightly the block moves back the other direction.

Irreversible: (1) A bullet is fired through a block of wood that is initially at rest on a horizontal frictionless surface. The bullet passes through the block and emerges with reduced speed. Mechanical energy is lost in the collision. The bullet can be sent back through the block by reversing its direction but mechanical energy is again lost in the collision and after the collision the block isn't at rest and the bullet doesn't have the same speed as it did before the original collision.ad (2) A block slides down an incline that has friction. The block has speed v when it reaches the bottom of the incline. If the block is at sent up the incline with initial speed v it won't return to its initial height, due to mechanical energy lost to friction work.

Q20.3 For a refrigerator device, $|Q_\text{H}|=|Q_\text{C}|+|W|$. This says that the heat energy $|Q_\text{H}|$ deposited in the room is the same as the heat energy $|Q_\text{C}|$ removed from the food compartment plus the amount of mechanical energy required to operate the refrigerator (inputed to the device as electrical energy). The tubing is where the heat flows from the device into the room.

Q20.4 The refrigerator operates on a cycle and $Q = W$. $Q = Q_\text{C} + Q_\text{H}$. For a refrigerator $Q_\text{C} > 0$ and $Q_\text{H} < 0$ and $W < 0$, so $|Q_\text{C}| - |Q_\text{H}| = -|W|$. $|Q_\text{H}| = |Q_\text{C}| + |W|$. More heat is delivered to the room than is taken in at the food compartment. Doing this will heat the room. The cooler of ice will cool the room until all the ice is gone. This is not a cyclic process. Heat energy leaves the room and goes into the ice as it changes phase.

Q20.5 For an air conditioner or refrigerator the amount of heat $|Q_\text{H}|$ that flows out of the device is greater than the heat $|Q_\text{C}|$ that flows into the device. In fact, $|Q_\text{H}| = |Q_\text{C}| + |W|$, where $|W|$ is the energy supplied to operate the device. If an air conditioner is set on the floor and plugged in, $|Q_\text{C}|$ is removed from the room but $|Q_\text{H}|$ is ejected into the room. Since $|Q_\text{H}|$ is larger than $|Q_\text{C}|$, the room is heated rather than cooled. A refrigerator also adds heat to the room in which it is placed. That heat must be removed, for example by a window air conditioner, or else the room will be heated.

Q20.6 It is not a violation to convert mechanical energy completely into heat. This is what happens, for example, when a block slides along a horizontal surface and is stopped by friction. The work done by friction converts all of the initial kinetic energy of the block into thermal energy. It is a violation to convert heat completely into work. This would be a 100% efficient heat engine and is prohibited by the second law.

Q20.7 The filter would lower the average speed of air molecules in the house and this would correspond to a lowering of the temperature of that air. The filter would violate the Clausius statement of the second law.

Q20.8 The energy output exceeds the energy input and this violates conservation of energy.

Q20.9 Work is done by the wind and this results in a transfer of heat from the cloth into the surrounding air; the system acts as a refrigerator.

Q20.10 In the Carnot cycle the heat flows occur in isothermal processes with an infinitesimal temperature difference between the working substance of the engine and the reservoir. Therefore, the heat flows occur in reversible processes. For the Otto cycle the heat flows occur in constant volume processes where there is a finite temperature difference between the working substance and the reservoirs. The Carnot cycle is reversible; the Otto cycle is not.

Q20.11 Heat flow in an isolated system is always in the direction from high temperature to low temperature. Work must be supplied to move heat in the opposite direction and more work is needed the greater the temperature difference.

Q20.12 If $T_{\text{C}} = T_{\text{H}}$, $e = 0$. The Carnot engine cannot operate unless the two heat reservoirs are at different temperatures. If $T_{\text{C}} = 0\ \text{K}$, $e = 1$. If the low temperature reservoir were at absolute zero, no waste heat Q_{C} would be deposited there and the Carnot cycle would be 100% efficient.

Q20.13 No. A Carnot cycle is 100% efficient only if $T_{\text{C}} = 0\ \text{K}$, and this is impossible to achieve. Any other cycle would be less efficient.

Q20.14 The room is the high temperature reservoir for the device. The efficiency is lower the greater the temperature of the room. The refrigerator consumes more power when the room temperature is 20°C.

Q20.15 No. 576 J of heat energy is placed into the high temperature reservoir. The net heat flow for the device is $+346\ \text{J} - 576\ \text{J} = -230\ \text{J}$, which means a net flow of 230 J of energy out of the device. But the work done on the refrigerator's working substance puts 230 J of energy into the device and energy is conserved for the device. The heat added to the high temperature reservoir is the sum of the amount of heat taken out of the low temperature reservoir plus the energy added to the system by W.

Q20.16 The change in entropy depends not only on the heat flow but is also inversely proportional to the Kelvin temperature at which that heat flow occurs. The negative heat flow out of the hot object occurs at a higher temperature than the positive heat flow into the cold object, so the positive entropy change is greater than the negative entropy change and the net entropy change is positive.

Q20.17 The mixed water is less organized. In the free expansion the atoms become less confined in space. An increase in thermal energy means more kinetic energy in the random motion of the atoms of the objects. In all these processes the entropy of the system increases.

Q20.18 Eq.(20.19) only applies to a reversible process and the free expansion is not reversible. In a reversible process between the same initial and final states of the gas (an isothermal expansion) there is a heat flow into the gas. And for this process Eq.(20.19) gives the entropy change. Entropy is a state function and the entropy change is path independent. So, the entropy change for the isothermal expansion equals the entropy change for the free expansion and is not zero.

Q20.19 The earth and sun are not in thermal equilibrium; the earth has a much lower temperature than the sun. When heat is transferred from the sun to the earth the energy transfer from the sun occurs at a higher temperature than the energy transfer to the earth. So, the entropy decrease of the sun has a smaller magnitude than the entropy increase of the earth. The total entropy change of the isolated system of the earth plus sun is positive, in agreement with the second law of thermodynamics. Radiation does not differ from other modes of heat transfer with respect to entropy changes.

Q20.20 The first law of thermodynamics is a statement of conservation of energy. If the magnitude of heat that flows out of the cold object equals the amount that flows into the hot object, the first law is satisfied. But this process would violate the second law. For an isolated system (the hot and cold objects) the total entropy would decrease (see Q20.16) in the process and this violates the second law.

Q20.21 You would not see any process that violates conservation of energy or conservation of momentum. These laws still apply when time is reversed. Any irreversible process in which entropy of an isolated system increases will violate the second law of thermodynamics when run in reverse. A simple example is a box sliding along the floor and being stopped by friction. In the real life direction of time kinetic energy is converted entirely into thermal energy. When time is reversed the box moves with increasing speed as thermal energy is converted to kinetic energy. This violates the second law of thermodynamics.

Q20.22 Life forms are not an isolated system. Processes are allowed in which objects have a decrease in entropy, so long as the objects interact with other objects whose entropy increases.

Q20.23 It is not a violation of the second law of thermodynamics because the plant is not an isolated system. The plant receives energy from its environment, from the radiative energy of sunlight.

CHAPTER 21
ELECTRIC CHARGE AND ELECTRIC FIELD

Discussion Questions

Q21.1 When a strip of tape is quickly peeled off the roll, electrons are transferred between the strip and the rest of the roll. Therefore, the two strips have the same sign of net charge and repel. When they are stuck together and then ripped apart, they transfer electrons and end up with net charge of opposite sign and attract each other.

Q21.2 Since they attract, they cannot have net charges of the same sign. Either one is neutral and the other has net positive or negative charge, or one has positive charge and the other has negative charge. After they touch the negative and positive charges neutralize. Any residual net charges spread over the two spheres so they now have charges of the same sign and repel. If there is no residual net charge they don't repel, but in any case they no longer attract.

Q21.3 The charged comb would still pull charges of opposite sign toward it and thereby polarize the charges in the insulator. But the neutral insulator would not be attracted to the comb. The attraction depends on the electric force increasing with decreasing distance.

Q21.4 The tumbling motion in the dryer produces static charges on the clothes and these charges tend not to leak away in the dry air inside the dryer. There will be less clinging if all the clothing is made of the same material; charge is transferred most readily between dissimilar materials. The tumbling of dissimilar materials transfers charge from one material to the other.

Q21.5 The sphere is attracted because its charges become polarized. Negative charge is drawn toward the rod and since the electrical force increases with decreasing distance this causes a net attraction. When the neutral sphere touches the charged rod some of the positive charge of the rod is transferred to the sphere. Now both objects have positive charge and they repel.

Q21.6 Assume that your mass is 70 kg. The mass of an electron is much less than the mass of a proton or neutron. Assume that you have about equal numbers of protons and neutrons, so the total mass of the protons in your body is about 35 kg. The mass of one proton is $1.67\times10^{-27}\ \text{kg}$, so the number of protons in your body is about $\dfrac{35\ \text{kg}}{1.67\times10^{-27}\ \text{kg/proton}} = 2\times10^{28}$. You have neutral charge (or close to neutral) so the number of electrons is 2×10^{28}, equal to the number of protons. The total charge of these electrons is about $(2\times10^{28}\ \text{electrons})(-1.6\times10^{-19}\ \text{C/electron}) = -3\times10^{9}\ \text{C}$.

Q21.7 (a) Electric field points away from positive charge and toward negative charge. So, the top and bottom charges are positive and the middle charge is negative. (b) The electric field is the smallest on the horizontal line through the middle charge, at the two positions on either side where the field lines are least dense. Here the *y*-components of the electric fields of the two positive charges are equal in magnitude and are in opposite directions and the electric field of the negative charge has no *y*-component. Therefore, along this line the *y*-component of the net field is zero. On this line to the right of the charges the *x*-components of the fields of the positive charges are to the right and the *x*-component of the field due to the negative charge is to the left and the *x*-components tend to cancel. On this line to the left of the charges the *y*-component of the net field is again zero and the *x*-components of the positive charges are in opposite directions and tend to cancel.

Q21.8 Both kinds of conduction are due to the free electrons in the conductor.

Q21.9 In Fig.21.28a the field lines are straight lines so the force is always in a straight line and the velocity and acceleration are always in the same direction. The particle moves in a straight line alone a field line, with increasing speed. In fig.21.28b the field lines are curved and the force continually changes direction. As the particle moves, its velocity and acceleration are not in the same direction and the trajectory does not follow a field line.

Q21.10 Place the two objects in contact. Bring a positively charged rod close to one object, on its side opposite the second object. This will pull electrons into the object closest to the rod. With the rod held nearby separate the objects. This will leave one with negative charge and the other with positive charge. The two objects were originally neutral, so the magnitude of the negative charge on one equals the magnitude of the positive charge on the other.

Q21.11 The mass of the book is about 2 kg. Protons and neutrons have about equal masses and their mass is about 2000 times greater than the mass of an electron. There are approximately equal numbers of neutrons and protons so about half the mass of the book (1 kg) is due to protons. The number of protons therefore is about $\frac{1\text{ kg}}{1.67\times 10^{-27}\text{ kg/proton}} = 6\times 10^{26}\text{ protons}$. Assuming equal numbers of electrons and protons there are 6×10^{26} electrons. The net charge if the magnitude of the charge of an electron is $1.00\times 10^{-3}\%$ less than that of a proton is $\Delta q = (1.00\times 10^{-5})(1.6\times 10^{-19}\text{ C})(6\times 10^{26}) = 960\text{ C}$. The textbook would have a charge of about 1000 C. The repulsive force between two textbooks placed 5 m apart would be about $(9\times 10^{9}\text{ N}\bullet\text{m}^2/\text{C}^2)(1000\text{ C})^2/(5\text{ m})^2 = 4\times 10^{14}\text{ N}$. This is an immense force. Even the slightest difference in the proton charge and the magnitude of the electron charge would result in explosive repulsion.

Q21.12 It is easier for charge to build up on you when the air is dry. Humid air conducts the charge from your body. There is more space for net charge on a large object, so a large object can accept more charge.

Q21.13 Charging by touching transfers charge of the same sign to another object. Using a charged object to charge another object by induction places charge of the opposite sign on the second object.

Q21.14 The acceleration of the charge depends on the electric force exerted on it by the other charge. This force is given by $F = k\frac{q^2}{r^2}$. The force at the instant the charge is released depends only on the distance between the two charges and is the same whether the other charge is free to move or is held fixed in place.

Q21.15 The force on each charge has magnitude $F = k\frac{Q(2Q)}{r^2}$. The same magnitude of force acts on each charge and the charges have the same mass. Therefore, the charges will have initial accelerations of the same magnitude.

Q21.16 The two particles have charges of the same magnitude and opposite sign. The electric field exerts forces of the same magnitude and opposite direction on the two particles. They have different masses, so the accelerations are of different magnitude, with the electron's acceleration being larger. The acceleration of the proton is in the direction of the electric field and the acceleration of the electron is opposite to the direction of the electric field.

Q21.17 The earth contains an immense number of electrons and protons, but in equal numbers so it is electrically neutral and exerts no electrical force on other objects. The mass of the earth is very large so the gravitational force it exerts is large.

Q21.18 They both occur between pairs of objects. They both are inversely proportional to the square of the distance between the objects. All objects have mass and exert a gravitational force on each other. Only objects with net charge exert an electrical force. The gravitational force is always attractive but the electrical force can be either attractive or repulsive depending on whether the two objects have charges of the opposite or of the same sign.

Q21.19 (a) Electric field points away from positive charge and toward negative charge so object *B* has positive charge and object *A* has negative charge. (b) The electric field lines are closer together near object *A* so the field is stronger near object *A*.

Q21.20 If the only forces were the electrical and gravitational forces, the gravitational force wouldn't be strong enough to overcome the electrical repulsion of the protons and the nucleus would be unstable. The protons and neutrons are all bound together by a nuclear force.

Q21.21 The electric field exerts a force on the electrons, according to Eq.(21.3). If the force from the external electric field is stronger than the forces that bind the electron to the atom, the force from the field can pull electrons from the atom.

Q21.22 No. Field lines for the resultant field due to the set of charges never cross. But $\vec{E}_1$ and $\vec{E}_2$ are the electric fields of individual charges and it is their vector sum that determines the direction of the field line at point *P*.

Q21.23 Air velocity is a vector field because velocity is a vector. Temperature is not a vector field because temperature is a scalar, not a vector.

CHAPTER 22
GAUSS'S LAW

Discussion Questions

Q22.1 The total flux through the balloon is q/ϵ_0, regardless of the size of the balloon.

Q22.2 The flux through each surface is equal to $\dfrac{Q_{\text{encl}}}{\varepsilon_0}$, where Q_{encl} is the net charge enclosed by the surface. Surface *A*: $Q_{\text{encl}} = +q$ and $\Phi_{EA} = +q/\varepsilon_0$. Surface *B*: $Q_{\text{encl}} = +q$ and $\Phi_{EB} = +q/\varepsilon_0$. Surface *C*: $Q_{\text{encl}} = +2q$ and $\Phi_{EC} = +2q/\varepsilon_0$. Surface *D*: $Q_{\text{encl}} = 0$ and $\Phi_{ED} = 0$.

Q22.3 The electric flux through a surface depends only on the charge within the surface. The net flux through the surface due to charges outside the surface is zero. The total flux through each of these surfaces would remain the same.

Q22.4 The electric field doesn't need to be zero on the surface, only the total electric flux through the surface needs to be zero. The electric field would be zero everywhere on the surface, for example, if the surface was totally in the material of a conductor.

Q22.5 The electric field is inversely proportional to the square of the distance from the point charge, so the field at each point on the surface changes. Gauss's law says that the total flux through a closed surface depends only on the net charge enclosed by the surface and that hasn't changed, so the total flux doesn't change. All the field lines from the point charge still pass outward through the surface.

Q22.6 Measure the electric field flux through a Gaussian surface that encloses the box. The total flux through the surface is nonzero if and only if there is no net charge in the box.

Q22.7 The electric field inside the conductor is still zero. The point charge polarizes the charge on the surface of the sphere such that the electric field of the surface charge exactly cancels the electric field of the point charge at all points within the sphere. The net charge on the sphere is still on its surface. But this charge is not distributed uniformly over the surface of the sphere. There is more positive charge in the area near the negative point charge, because the negative point charge pushes away electrons on the sphere.

Q22.8 Gauss's law would not be valid. For a Gaussian sphere of radius r with a point charge at the center, the electric field at the surface would be proportional to $1/r^3$. The area of the sphere is proportional to r^2, so the total flux through the sphere will be proportional to $1/r$ and will depend on the radius of the sphere.

Q22.9 No. These free electrons aren't excess charges; their negative charge is exactly balanced by the positive charge of the ions in the material.

Q22.10 Negative charge on the sphere is pulled toward the van de Graff generator and this leaves positive charge on the side of the sphere that is farthest from the generator. The sphere has zero net charge. For a Gaussian surface that lies just outside the surface of the sphere the enclosed charge is $Q_{\text{encl}} = 0$. Therefore, Gauss's law says the net flux through the sphere is zero. Any Gaussian surface that is entirely inside the sphere encloses zero charge and the electric field inside the sphere is zero.

Q22.11 Lightning is a flow of electrons. The lightning current runs through the copper cables since copper is a good conductor of electricity whereas the house material is not. The rods are pointed because the electric field is strong there when the rod gains some charge. The strong electric field ionizes the air near the sharp point and provides a conducting path.

Q22.12 A point charge affects the electric field outside the conductor. The point charge induces a charge of equal magnitude and opposite sign on the inner surface of the conductor. The conductor is overall neutral so there is a charge equal to the point charge spread over the exterior surface of the conductor. This exterior charge produces an electric field outside the conductor. Charge outside the conductor doesn't produce any electric field inside the cavity. The electric field of the external charge is shielded by the conductor.

Q22.13 A component of field parallel to the surface would exert a force on the free electrons in the conductor and cause them to move. In an insulator there are no charges free to move and there is no reason why the electric field at the surface of an insulator can't have a component parallel to the surface.

Q22.14 (a) Consider a small Gaussian surface somewhere in the region. Since the electric field is uniform the flux through the surface must be zero. Therefore, the enclosed charge $Q_{\text{encl}} = \int \rho \, dV$ must be zero. But since this must be true for any volume in the region, $\rho = 0$ everywhere in the region. (b) No. For example, consider the region near a point charge. There is no charge there but the electric field, which is due to the nearby point charge, is not uniform.

Q22.15 (a) If $\rho > 0$ and uniform, then the charge inside any closed surface is greater than zero. Therefore, $\Phi_E > 0$ for any Gaussian surface. But if the field is uniform then $\Phi_E = 0$ for any Gaussian surface, so the field can't be uniform. (b) Yes. The flux through the surface of the bubble must be zero but that is true for a uniform field within the bubble, so that kind of field is allowed. (See Problem 22.57.)

Q22.16 Apply Gauss's law to a Gaussian surface that lies entirely within the metal. $E = 0$ at all points within the meal conductor so the flux through this surface is zero. By Gauss's law the Gaussian surface must enclose zero net charge. This means that there is net charge $+Q$ on the inner surface of the metal. Since the solid is grounded there is no excess charge on the outer surface of the metal. Any excess charge there would be neutralized by charge from the earth. A gaussian surface entirely outside the piece of metal would enclose zero net charge ($-Q$ from the charge in the cavity plus $+Q$ on the inner surface of the metal). The flux through the surface, for any shape, would be zero so the electric field at all points outside the piece of metal is zero. It is reasonable to say the grounded conductor has shielded the region outside the conductor from the effects of the charge $-Q$. There is nothing like positive and negative mass (the gravity force is always attractive), so this cannot be done for gravity.

CHAPTER 23
ELECTRIC POTENTIAL

Discussion Questions

Q23.1 The concept of potential is useful for the same reason that electric field is useful. It allows us to calculate the effect of the source charges on the space surrounding them. A test charge then has electrical potential energy when placed in the potential of the source charges. And electric potential will prove to be a very useful concept for analyzing electrical circuits.

Q23.2 Yes, just bring the test charge in along the perpendicular bisector of the line that connects the two charges. The electric potential is zero at all points along this line, since all points on the line are equidistant from the two charges. There is no change in potential along the line so no work is done on the test charge anywhere along the line. The two charges produce a net electric field along this line, but at all points on the line the electric field has no component parallel to the line. So the electric force does no work on a test charge moving along this line.

Q23.3 If the potential where the two charges are infinitely far apart is taken to be zero, then the electric potential energy of the pair is $U = kq_1q_2/r$. Only for $r \to \infty$ is this zero, so it is not possible for two charges. For three charges not all of the same sign it is possible. You need just to arrange the separations of the three charges 1, 2, and 3 such that $U_{12} + U_{23} + U_{13} = 0$.

Q23.4 The voltmeter reads the potential difference between the two points to which it is connected. The potential difference between two points is independent of the choice for the reference level of zero potential.

Q23.5 If $E = 0$ along the path then the work done by the electric force on a test charge as it moves along this path is zero. Eq.(23.13) then says that $V_A = V_B$. $\vec{E}$ does not have to be zero everywhere along other paths connecting A and B, but the work done by $\vec{E}$ on a test charge must be zero along any of these paths. This is because the electric force is a conservative force and the work it does is path independent.

Q23.6 Eq.(23.17) says that V is constant throughout this region but it isn't necessarily zero.

Q23.7 Electric field lines point away from positive charge and toward negative charge. Potential is large and positive near positive charge and low (large in magnitude and negative) near negative charge. Therefore, electric fields are always in the direction of lower potential; electric-field lines point from high to low potential. For another way to show this, consider a positive test charge q that moves from point a to point b. $\frac{W_{a\to b}}{q} = V_a - V_b$, where $W_{a\to b}$ is the work done on charge q by the electric field in which it moves. If the electric field is in the direction from a to b, then the direction of the force on q is from a to b and this force does positive work. $\frac{W_{a\to b}}{q}$ then is positive and $V_a - V_b$ is positive. Therefore, $V_a > V_b$ and the electric field direction is the direction of decreasing potential.

Q23.8 (a) No. At a point midway between two point charges of equal magnitude and opposite sign the potential (relative to infinity) is zero but the electric field is not zero. The electric field is directed toward the negative charge. (b) No. At a point midway between two equal point charges (equal in magnitude and sign) the electric field is zero and the potential (relative to infinity) is not zero. The potential depends on the electric field along a path from infinity to the point, not just on the electric field at the point.

Q23.9 The electric force is conservative so the work it does is path independent and is zero for any closed path. By Eq.(23.17) the integral $\int \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$ from point a to point b equals $V_a - V_b$. But when a and b are the same point the potential difference is necessarily zero so the integral $\int \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$ completely around a closed path must be zero.

Q23.10 If they are placed negative terminal to positive terminal, the potential increases by 3.0 V in going from the negative terminal of the first to the positive terminal of the second. If they are placed positive terminal to positive terminal, the potential rises 1.5 V across one battery and decreases 1.5 V across the second one so the potential difference between the terminals of the exposed ends is zero.

Q23.11 The static charge on your body is at very high potential but the amount of net charge is very small so the potential energy of that charge is small. The power line is capable of delivering a much larger amount of electrical energy because much more charge flows from it.

Q23.12 Eq.(23.20) shows that $\vec{\boldsymbol{E}}$ is determined by the change with distance of $V(\vec{\boldsymbol{r}})$. V at a single point tells us nothing about $\vec{\boldsymbol{E}}$.

Q23.13 There is no ambiguity because $E = 0$ at this point and therefore has no direction.

Q23.14 $V_a - V_b = \int_a^b \vec{\boldsymbol{E}} \bullet d\vec{\boldsymbol{l}}$. If the direction from a to b is the same as the direction of $\vec{\boldsymbol{E}}$, then the integral is positive and $V_a > V_b$. Therefore, the electric field always points toward lower potential. In a direction perpendicular to the electric field the potential is constant since in that case $\vec{\boldsymbol{E}}$ and $d\vec{\boldsymbol{l}}$ are perpendicular and $\vec{\boldsymbol{E}} \bullet d\vec{\boldsymbol{l}} = 0$. Therefore $V_B > V_A$, $V_C < V_A$ and $V_D = V_A$

Q23.15 No. At a point one meter from an isolated positive point charge the electric potential is positive but there is no charge at this point. At a point one meter from an isolated negative point charge the electric potential is negative but there is no charge at this point. For both these examples we are taking $V = 0$ at an infinite distance from the point charge. The choice of zero potential determines whether a particular point has positive or negative potential, and this choice is arbitrary.

Q23.16 Consider charge dq added to the sphere after it already has charge q. The potential of the surface of the sphere at this point in the charging process is $V = kq/R$, where R is the radius of the sphere. The potential energy of dq when it is placed on the sphere is then $dU = V\,dq = kq\,dq/R$. The total potential energy of the charged sphere is

$$U = \int_0^Q \frac{kq\,dq}{R} = \frac{kQ^2}{2R}.$$

Q23.17 It doesn't matter. Any convenient choice can be made for what equipotential we call $V = 0$.

Q23.18 The electric field inside the sphere is zero, no matter where the sphere is placed and no matter whether the sphere has a net charge or not. In electrostatics, the electric field within a conductor is always zero. All points of the sphere are at the same potential, but the value of this potential depends on how far the sphere is from the positively charged plate. This is true whether the sphere has net charge or not.

Q23.19 The electric field is zero in the conductor and in the cavity, so the potential is constant in these regions. The potential in the cavity equals the potential in the material of the conductor.

Q23.20 The body of the car and its interior are both at the same potential so people inside the car are unaffected. The potential difference between the car and the ground is 10,000 V so when a person steps out this potential difference appears between their foot on the ground and the part of them still in contact with the car and a very dangerous current flows through them.

Q23.21 The static electricity in the atmosphere can place a net charge on the ship. The light appears when this charge discharges into the air. The electric field E is largest near sharp points so this is where the discharge occurs. The seawater on the wet masts conducts the charge to the discharge point.

Q23.22 This is true and is an example of the "image charge" method often used in solving certain electrostatic problems. In Fig.23.23b, the electric field lines are symmetric about the $V = 0$ line that runs between them and would be perpendicular to a plane running along this line and consisting of points equidistant from the two charges. The electric field is perpendicular to a conducting surface so the field lines in half the figure would be unchanged by replacing one of the charges by a conducting surface at this plane.

CHAPTER 24
CAPACITANCE AND DIELECTRICS

Discussion Questions

Q24.1 $E = V/d$ so for a given potential V across the capacitor E becomes larger as d is decreased. When E exceeds the dielectric strength of whatever is between the plates of the capacitor, current flows through the dielectric and the capacitor discharges.

Q24.2 The electric field between the plates is proportional to the surface charge density σ on the plates: $E = \dfrac{\sigma}{\varepsilon_0}$. The potential difference between the two plates is related to the electric field between the plates by $V_{ab} = Ed$. Capacitance is defined as $\dfrac{Q}{V_{ab}}$. The greater the area of the plates the smaller the surface charge density for a given *Q*. So, with greater area the capacitor can hold more charge for the same *E* and *V* and this makes *C* larger. Increasing the separation between the plates increases V_{ab} for the same *E* and hence the same *Q*. Increasing V_{ab} decreases *C*.

Q24.3 Due to the attraction of the opposite charges on the plates, charge will be on only that part of the larger plate that is directly across from the smaller plate. Both the capacitor and the battery remain neutral; the two plates have charges of equal magnitude.

Q24.4 The magnitude of charge Q on each plate of the capacitor is given by $Q = CV$. More charge is stored when C is larger. $C = \dfrac{\varepsilon_0 A}{d}$ so C is larger when d is smaller. It is better to have the plates closer together.

Q24.5 (a) No. This is true only when d is much smaller than the dimensions of the plates. (b) The electric field is no longer confined to the region between the plates so the force on a charge between the plates is less than it would be if the field was confined between the plates. The smaller force does less work on a test charge that moves from one plate to the other so the potential difference is less than given by $V_{ab} = Qd/\epsilon_0 A$ since this equation assumes the field is confined to the region between the plates. (c) $C = Q/V$ so smaller V means larger C.

Q24.6 The battery keeps the potential V between the plates constant. $E = V/d$ so when the separation d is doubled the electric field is halved. $C = \epsilon_0 A/d$ so the capacitance is halved and by $Q = CV$ the charge Q on the plates is halved. Less charge is needed to produce the same potential difference when the separation is doubled. $U = \tfrac{1}{2}QV$ so the energy stored in the capacitor is halved.

Q24.7 Since the capacitor is disconnected from the battery, the charge on its plates remains constant. $E = Q/A\epsilon_0$ so E is unchanged. $V = Ed$ so V doubles. $U = \tfrac{1}{2}QV$ so the total energy stored is doubled. The energy increase comes from the work done in pulling the plates farther apart.

Q24.8 The two capacitors have the same *V*. Let capacitor 1 have plate separation *d* and capacitor 2 have plate separation $2d$. $C = \dfrac{\varepsilon_0 A}{d}$, so $C_1 = 2C_2$. $Q = CV$ so $Q_1 = 2Q_2$. $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$ so $E_1 = 2E_2$. The energy density is $u = \tfrac{1}{2}\varepsilon_0 E^2$ so $u_1 = 4u_2$. The capacitor with the smaller plate separation has the stronger electric field, the greater charge and the greater energy density. This capacitor needs more charge to produce the same potential difference between its plates.

Q24.9 See Q24.7. The energy added is stored in the capacitor. The electric field stays the same so the energy density is the same. But the volume occupied by the field increases and the total energy stored by the field increases.

Q24.10 The stored energy for a capacitor is given by $U = \frac{1}{2}CV^2$. In parallel the potential difference V across each capacitor equals the battery voltage whereas in series the potentials add to give the battery voltage. Therefore, the voltage for each capacitor is greater in parallel and the stored energy is greater when they are connected in parallel.

Q24.11 Water is an excellent solvent and would tend to dissolve or corrode the capacitor plates. Also, it conducts current if it contains dissolved ions and isn't perfectly pure.

Q24.12 No, they are different. Dielectric strength is the largest the electric field can be before the dielectric becomes conducting. The dielectric constant K is a measure of the extent to which charge polarization in the dielectric cancels the electric field due to the charges on the plates. The presence of the dielectric between the plates increases the capacitance by a factor of K. In Table 24.2 there is no relationship evident between dielectric strength and dielectric constant.

Q24.13 The holes are places where Mylar is replaced by air. The area of the holes is a small fraction of the total area of the charged conductors so the presence of the air makes little difference to the average electric field between the plates. But the dielectric strength of air is much less than that of Mylar and breakdown at the holes occurs when the dielectric strength of air is exceeded.

Q24.14 The surface of the dielectric closest to the positively charged plate has a negative induced charge and the surface of the dielectric closest to the negatively charged plate has a positive induced charge. The plates of the capacitor therefore exert an attractive force on the dielectric and this force does positive work on the slab as it moves into the region between the two plates. $W_{a\to b} = U_a - U_b$ so when the force does positive work the potential energy associated with that force decreases. Therefore, less energy is stored in the capacitor after the dielectric has been inserted. We can see this from $U = \dfrac{Q^2}{2C}$. Q is constant and C increases ($C = KC_0$), so U decreases.

Q24.15 The capacitance depends on the dielectric constant of the fish and this in turn depends on the amount of water in the fish's tissue.

Q24.16 It is much easier to achieve a small and uniform separation between the two conductors.

Q24.17 The flux decreases by a factor of $1/K$ since the enclosed charge decreases by a factor of $1/K$. Without the dielectric the enclosed charge is σA and the electric flux through the surface is $\sigma A/\varepsilon_0$. With the dielectric present the enclosed charge is $(\sigma - \sigma_{\text{i}})A$. But $\sigma_{\text{i}} = \sigma(1 - 1/K)$ so $(\sigma - \sigma_{\text{i}})A = \dfrac{\sigma A}{K}$ and the electric flux through the surface is $\sigma A/\varepsilon_0 K$.

Q24.18 The one fact that is always true is that $C = KC_0$, where C_0 is the capacitance without the dielectric and C is the capacitance with the dielectric.
Power supply keeps the voltage V constant:
(i) $E = V/d$. V is constant so E doesn't change.
(ii) $Q = CV$. C increases so Q increases.
(iii) Use $U = \frac{1}{2}CV^2$. V is constant and C increases so U increases.
Charge Q kept constant:

(i) $V = Q/C$. Q is constant and C increases, so V decreases. $E = V/d$ so E decreases.

(ii) Q is constant.

(iii) Use $U = Q^2/2C$. C increases and Q is constant so U decreases.

Q24.19 Increasing temperature increases the kinetic energy of the molecules and this decreases the alignment of their molecular dipoles. This decreases the electric field they produce that opposes the electric field due to the charges on the plates.

Q24.20 If Q is kept fixed, $E = E_0/K$. The dielectric constant is a measure of the extent to which the polarization of the dielectric decreases the electric field between the capacitor plates. The greater the cancelation of the field, the larger the dielectric constant. For a conductor the induced charges totally cancel the electric field, the electric field in the conductor is zero. This corresponds to $K \to \infty$.

Q24.21 The oil has dielectric constant $K > 1.0$. A charge separation in the oil produces an electric field that partially cancels the electric field due to the charge on the plates, so the electric field between the plates decreases. The net electric field between the plates could be measured by measuring the force on a test charge placed in the oil between the plates.

CHAPTER 25
CURRENT, RESISTANCE AND ELECTROMOTIVE FORCE

Discussion Questions

Q25.1 No. The statement in Chapter 21 refers to an electrostatic situation where there is no movement of charge. $\rho = E/J$ refers to a situation where current is flowing in the conductor, so charges are moving.

Q25.2 $R = \dfrac{\rho L}{A} = \dfrac{\rho L}{\left(\pi D^2/2\right)}$, where L is the length and D is the diameter. $L' = 3L$ and $D' = 3D$ gives $R' = \dfrac{\rho L'}{\left(\pi [D']^2/2\right)} = \dfrac{\rho(3L)}{\left(\pi [3D]^2/2\right)} = \dfrac{3}{9}\dfrac{\rho L}{\left(\pi D^2/2\right)} = \dfrac{1}{3}R$.

Q25.3 The resistivity ρ is a property of the material of which the resistor is made and does not depend on the size or shape of the resistor. The resistivity remains ρ.

Q25.4 $v_{\rm d} = J/(nq) = I/(nqA)$. The current I, by conservation of charge, is the same in both wires. For smaller cross sectional area the current density increases and the drift speed increases. By Eq.(25.5) the larger current density means the electric field is larger in the smaller wire. The increased electric field provides the force that accelerates the electrons to a higher drift speed.

Q25.5 When current is flowing through the battery in the direction from the negative terminal toward the positive terminal the terminal voltage is $V = \mathcal{E} - Ir$. The emf is $\mathcal{E} = 1.5\ \text{V}$ and V_{ab} is less than this.

Q25.6 Yes. If current is being pushed through the battery from the $-$ to the $+$ terminal by another voltage source, as shown in Fig. DQ25.6. The voltage drop Ir across the internal resistance of the battery is directed opposite to the emf. If this current is large enough the potential of the + terminal of the battery can be lower than the potential of the $-$ terminal; $V_a < V_b$.

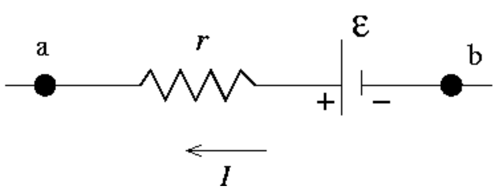


Figure DQ25.6

Q25.7 Yes. The open-circuit voltage is $\mathcal{E}$. The short-circuit current is $I = \mathcal{E}/r$ so $r = \mathcal{E}/I$.

Q25.8 No. The current the battery provides depends on the total resistance of the circuit connected across the battery terminal. The current depends not only on the battery but also on the rest of the circuit.

Q25.9 A wire carrying a current of 10 A remains electrically neutral. But a huge amount of charge (10 C) passes a cross section of the wire each second.

Q25.10 (a) The current is the same on both sides of the resistor so the current density is the same. $v_{\rm d} = J/(nq)$ so the drift speed is the same on both sides of the resistor. (b) The electric potential V

is higher at the end of the resistor when the conventional current enters. Electrons move opposite to the direction of the conventional current so move to higher potential. Negative charge loses potential energy when it goes to higher potential. Electrons lose electrical potential energy when they pass through the resistor.

Q25.11 (a) For a resistor $P = \frac{V^2}{R}$. When the temperature increases the resistance of the copper heating element increases. Since V is constant, the electrical power consumed by the resistor decreases. (b) For carbon the temperature coefficient of resistivity α is negative (Table 25.2) and the resistance of the carbon cylinder decreases when its temperature increases. Therefore, the electrical power it consumes increases. It is a general rule that the resistance of a metallic conductor, like copper, increases as the temperature increases. And the resistance of a semiconductor (carbon is an example) decreases as its temperature increases.

Q25.12 If the resistor is made of a metal its resistance increases with temperature, and the greater the current the greater the temperature of the resistor. Ohm's law says $I = V / R$ and a graph if I versus V is a line with slope $1/R$. But since R increases when V and I increase, the slope decreases as V increases. This behavior corresponds to graph (d).

Q25.13 Resistance increases with temperature so the current through the bulb decreases when the bulb heats up. Current through the bulb is largest when the light is first turned on.

Q25.14 In (a) the same current flows through each bulb and they have equal brightness. In (b) the resistance is halved so the current is doubled compared to (a) and bulb A is brighter.

Q25.15 (a) In both circuits the same current flows through the ammeter. The current is the same everywhere in the circuit, current doesn't get "used up" when it passes through the bulb. (b) Since the current through the bulb is the same in both circuits the bulb is the same brightness.

Q25.16 An ideal ammeter is a very low resistance device and does not add resistance to the circuit. An ideal voltmeter is a very high resistance device and adds a very large resistance in series with the light bulb. In (b) the total circuit resistance is very large and the current through the bulb is very small. The bulb will be much brighter in (a).

Q25.17 In both the charging process and when the battery delivers energy to a circuit some energy is lost to thermal energy as the current flows through the internal resistance of the battery.

Q25.18 No, their internal resistance is too high. They wouldn't be able to supply enough current to start the car.

Q25.19 The electrical energy delivered to a device is $P = VI$. Larger V means smaller I. Energy loss in the wires occurs at a rate I^2R so if I is smaller R can be larger and the circuit still have the same rate of energy loss in the wires. Thinner wires have larger R.

Q25.20 The electrical energy delivered to a device is $P = VI$. Larger V means smaller I. Energy loss in the wires occurs at a rate I^2R so if I is smaller then there is less energy loss due to the resistance of the transmission lines. A disadvantage is the danger of high voltages and the need to step up and step down the voltage before and after transmission.

Q25.21 The voltage of the electrical system in each case is matched to the power that must be delivered to the devices in the system. The lower voltage in an automobile system requires higher currents but the electrical transmission distances are small so losses due to wire resistance is tolerable. Also, the lower voltage auto system is safer. It is more common for untrained people to poke around under the hood of a car than to get involved in their household wiring.

Q25.22 It should have low resistance and a melting point of the desired value.

Q25.23 The larger internal resistance limits the short-circuit current.

Q25.24 The energy transported by the electrical current is much larger than the heat energy conducted in the opposite direction. The potential difference drives the electrons in one direction. To conduct heat they would have to travel in the opposite direction.

CHAPTER 26
DIRECT CURRENT CIRCUITS

Discussion Questions

Q26.1 $P = V^2 / R$ so the bulb with larger P has smaller R; the resistance of the 120-W bulb is smaller. $V = IR$ and in series the current is the same through each, so the 60-W bulb has a greater voltage drop. In parallel the voltages across each bulb will be the same.

Q26.2 The 25-W bulb has a larger resistance than the 200-W bulb (see Q26.1). In series the currents are the same so the voltage drop across the 25-W bulb is larger than that across the 200-W bulb. The voltage drops add to 240 V so the voltage across the 25-W bulb is larger than 120 V and the 25-W bulb is the one that burns out quickly. When the voltage across it is greater than 120 V it dissipates more than 25 W.

Q26.3 (i) series: More bulbs in series add to the total resistance of the circuit and this decreases the current. The brightness of each bulb decreases as bulbs are added in series. (ii) parallel: The full battery voltage is placed across each bulb in parallel, no matter how many bulbs there are. The brightness of each bulb is the same as more are added in parallel, if the internal resistance of the battery can be neglected.
In series the current through the battery is $I_s = \mathcal{E} / R_T = \mathcal{E} / (nR)$ for n bulbs of resistance R. In parallel the current through the battery is $I_p = n(\mathcal{E} / R)$. $I_p = n^2 I_s$. The total power delivered by the battery is $\mathcal{E} I$ so the power is greater by a factor of n^2 when the bulbs are connected in parallel. The battery lasts longer when the bulbs are in series.

Q26.4 The voltage across A is $\mathcal{E}$ and the voltage is $\mathcal{E} / 2$ across B and C. A has more current through it, more potential difference across it and is the brightest. If A is unscrewed the voltages across B and C don't change and the brightness of B and C don't change. If B is unscrewed there is no current path in that parallel branch and C goes out. The voltages across A is still $\mathcal{E}$ and the brightness of A doesn't change.

Q26.5 (a) True. For resistors in series the current is the same in each resistor. The reason is conservation of charge. All the charge that flows through R_1 as I_1 must also flow through R_2 as I_2. The current entering a resistor always equals the current leaving the resistor. (b) False. (c) False. $P = I^2 R$ so $P_1 = I_1^2 R_1$ and $P_2 = I_2^2 R_2$. $I_1 = I_2$ and $R_2 > R_1$ means that $P_2 > P_1$. (d) True.
(e) False. $V_1 = I_1 R_1$ and $V_2 = I_2 R_2$. $I_1 = I_2$ and $R_2 > R_1$ so $V_2 > V_1$. (f) False. IR is the potential drop across a resistor. The end of the resistor where the current enters is at higher potential than the end where the current exits. $V_a > V_b$ and $V_b > V_c$. (g) False. (h) True.

Q26.6 (a) False. $V_1 = V_2$ so $I_1 R_1 = I_2 R_2$. $I_2 = I_1 \dfrac{R_1}{R_2}$. $R_2 > R_1$ so $I_2 < I_1$. (b) True. At point *a* the current I_3 splits into I_1 and I_2. At *b*, I_1 and I_2 combine to form I_4. Conservation of charge says $I_3 = I_4$. (c) True. (d) False. $P = \dfrac{V^2}{R}$. $P_1 = \dfrac{V_1^2}{R_1}$ and $P_2 = \dfrac{V_2^2}{R_2}$. Since $V_1 = V_2$ and $R_2 > R_1$, then $P_1 > P_2$. (e) False. (f) True. Points *a*, *c* and *e* are all at the same potential and points *d*, *f* and *b* are all at the same potential. (g) True. The potential difference between the ends of R_1 is $I_1 R_1$ and the end where the current enters is at higher potential. (h) False. (i) False.

Q26.7 Since the battery has no internal resistance its terminal voltage is independent of the current through it and is equal to its emf $\mathcal{E}$. Closing the switch adds a third bulb in parallel to the other two. But the voltage across each bulb is $\mathcal{E}$ and doesn't change when the third bulb is connected. The brightness of bulbs B_1 and B_2 won't change.

Q26.8 Decrease. The resistor is equivalent to three resistors in parallel. Adding resistors in parallel decreases the total resistance so cutting one of the strips and therefore changing from three resistors in parallel to two in parallel increases the overall resistance. Before the strip is cut, $R_{\text{equiv}} = \frac{R}{3}$. After the strip is cut, $R_{\text{equiv}} = \frac{R}{2}$. Since the resistance increases the current through the ammeter decreases.

Q26.9 Increase. Closing the switch adds another resistor in parallel to the resistor network and this reduces the resistance of the network and thereby the total resistance of the circuit. This increases the current through the light bulb.

Q26.10 Decreases. The voltage across the light bulb equals the terminal voltage of the battery. When the switch is closed the total equivalent resistance of the circuit decreases and the current through the battery increases. An increase in current through the battery causes the terminal voltage to decrease and the current through the light bulb decreases.

Q26.11 Won't change. Since the battery has no internal resistance its terminal voltage equals its emf, independent of the current through the battery. The voltage across the bulb equals the terminal voltage of the battery, independent of whether the switch is open or closed. The current through the bulb doesn't change when the switch is closed.

Q26.12 (a) Don't change. Since the battery has no internal resistance its terminal voltage equals its emf, independent of the current through the battery. The voltage across the branches containing the bulbs equals the terminal voltage of the battery, independent of whether the switch is open or closed. The currents through the bulbs don't change when the switch is closed. (b) Decrease. Now when the switch is closed and the current through the battery increases, the terminal voltage of the battery decreases.

Q26.13 Yes, it is possible. An example is shown in Fig. DQ26.13.

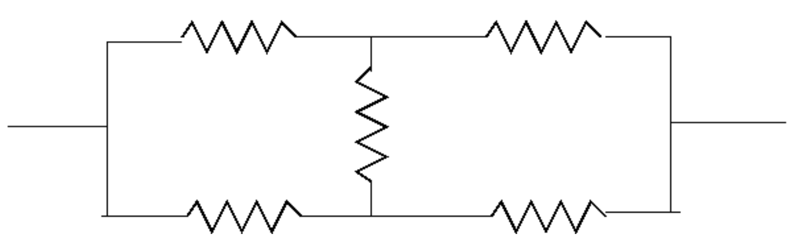


Figure DQ26.13

Q26.14 With t he switch open the bulbs are connected in series and they share the battery voltage: $\mathcal{E} = V_1 + V_2$. When the switch is closed the voltage across bulb B_2 becomes zero and no current flows through it. The voltage across B_1 becomes equal to the battery voltage and B_1 shines brighter.

Q26.15 For batteries in series the total voltage is the sum of the individual voltages. In parallel the voltage across the bulb is just the voltage of a single battery. In parallel the currents of the individual batteries add to give the total current, so more current can be delivered by batteries in parallel. Also, if one battery goes dead the others still deliver current to the device and the voltage applied to the device is unchanged.

Q26.16 With the switch open the three bulbs are connected in series and $\mathcal{E} = V_A + V_B + V_C$. When the switch is closed the voltage across bulb C becomes zero, no current flows through it and it goes out. And then $\mathcal{E} = V_A + V_B$. The voltages across A and B increase and they each shine brighter.

Q26.17 A voltmeter connected across the battery alone measured the emf. To check the internal resistance, use a voltmeter to measure the terminal voltage when the battery is connected in a circuit.

Q26.18 The time constant for an RC charging circuit is $\tau = RC_{\text{eq}}$, where C_{eq} is the equivalent capacitance, For circuit (a), $C_{\text{eq}} = C/2$. For circuit (b), $C_{\text{eq}} = 2C$. The time constant for circuit (a) is smaller and the capacitors charge at a faster rate than in circuit (b). The larger equivalent capacitance for circuit (b) means each capacitor holds more charge when fully charged so in (b) the capacitors take longer to reach their final charge.

Q26.19 The SI unit for R is Ω. $R = V/I$ so $1\,\Omega = 1\,\dfrac{\text{V}}{\text{A}}$. The SI unit for capacitance is F. $C = Q/V$ so $1\text{ F} = 1\,\dfrac{\text{C}}{\text{V}}$. $I = Q/t$ so $1\text{ C} = 1\text{ A}\cdot\text{s}$ and $1\text{ F} = 1\,\dfrac{\text{A}\cdot\text{s}}{\text{V}}$. The product RC therefore has units $\left(\dfrac{\text{V}}{\text{A}}\right)\left(\dfrac{\text{A}\cdot\text{s}}{\text{V}}\right) = \text{s}$, as was to be shown.

Q26.20 Measure the time constant τ by observing the current decay when a capacitor of known capacitance discharges through the resistor. Then $R = \tau / C$.

Q26.21 When the capacitor is fully charged the battery emf equals the voltage across the capacitor. $\mathcal{E} = Q/C$ so $Q = \mathcal{E}C$ is the maximum charge. When the charging is complete there is no current through the resistor and the resistor plays no role. The resistor affects the rate at which the capacitor charges.

CHAPTER 27
MAGNETIC FIELD AND MAGNETIC FORCES

Discussion Questions

Q27.1 There is no force when $\vec{v}$ and $\vec{B}$ are parallel, so if the charge moves along a field line there will be no force on the charge.

Q27.2 The direction of the force depends not only on the direction of the magnetic field but also on the direction of the velocity of the particle. Two particles with different velocities at the same point will have forces in different directions so this definition of the direction of $\vec{B}$ won't be unique.

Q27.3 No, your left hand will give the opposite direction for the force.

Q27.4 No. For example, if the magnetic field lines are straight lines a charged particle initially moving along a field line will experience no magnetic force and will continue moving parallel to the field lines.

Q27.5 No. If the initial velocity is perpendicular to the magnetic field the particle will travel in a semicircle but will then exit the field again.

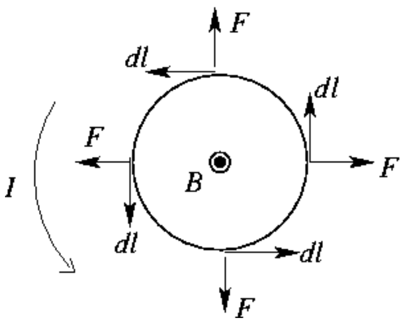
Q27.6 The magnetic force doesn't change the speed of the particle but changes the direction of the velocity. Any centrifugal force in circular motion behaves in this way, for example the tension in a string when an object attached at its end swings back and forth.

Q27.7 Yes. The net force must be zero. An electric field would produce a net unbalanced force. No. The net force must be zero so there must be no magnetic force on the particle. But there could still be a magnetic field parallel to the velocity of the particle.

Q27.8 If the current loop is pivoted about a vertical diameter, the earth's magnetic field will provide a torque that aligns the normal to the loop with the earth's field. The earth's magnetic field points toward the north geographic pole. The right-hand rule determines the direction of $\vec{B}$ of the loop, as illustrated in Fig.27.32. When $\vec{B}$ of the loop is in the direction of the earth's field the loop is in a position of stable equilibrium. The loop can determine the direction of the earth's field.

Q27.9 Find the orientation of the wire for which there is no force; the magnetic field is parallel to this direction. Then rotate the wire 90° so it is perpendicular to the field. The two possible directions of $\vec{B}$ give forces in opposite directions so observation of the direction of this force determines the direction of $\vec{B}$.

Q27.10 The magnetic field must be perpendicular to the plane of the loop. The current must travel around the loop in a direction so that the magnetic force on the current at each point is radially outward, as shown in the two sketches in Fig. DQ27.10.



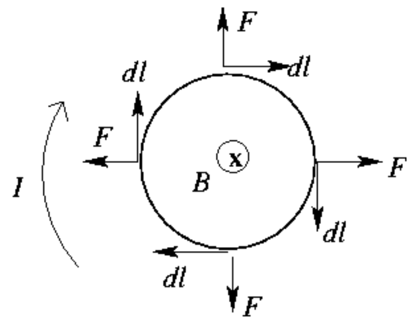


Figure DQ27.10

Q27.11 Assume the charges are moving from left to right in the plane of the page when they enter the magnetic field region. (a) A semicircular path in the counterclockwise direction, with radius $R = \dfrac{mv}{qB}$.

(b) A semicircular path in the counterclockwise direction, with radius $R = \dfrac{2mv}{qB}$.

(c) A semicircular path in the clockwise direction, with radius $R = \dfrac{mv}{|q|B}$. (d) There would be no force on the particle and it would travel in a straight line.

Q27.12 The direction of the force is given by the right-hand rule that is illustrated in Fig.27.7. If $\vec{\boldsymbol{v}}$ is parallel or antiparallel to $\vec{\boldsymbol{B}}$, then the force is zero. $\vec{\boldsymbol{B}} = B\hat{\boldsymbol{i}}$.
a: Force is in the $-z$-direction. $\vec{\boldsymbol{v}} = y\hat{\boldsymbol{j}}$. $\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} = vB(\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}}) = vB(-\hat{\boldsymbol{k}})$.
b: Force is in $+y$-direction. $\vec{\boldsymbol{v}} = v\hat{\boldsymbol{k}}$. $\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} = vB(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}}) = vB\hat{\boldsymbol{j}}$.
c: Force is zero. $\vec{\boldsymbol{v}} = v(-\hat{\boldsymbol{i}})$. $\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} = vB(-\hat{\boldsymbol{i}}) \times \hat{\boldsymbol{i}} = 0$.
d: Force is in $-y$-direction. $\vec{\boldsymbol{v}} = \dfrac{v}{\sqrt{2}}(\hat{\boldsymbol{i}} - \hat{\boldsymbol{k}})$. $\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} = \dfrac{vB}{\sqrt{2}}(\hat{\boldsymbol{i}} - \hat{\boldsymbol{k}}) \times \hat{\boldsymbol{i}} = \dfrac{vB}{\sqrt{2}}(-\hat{\boldsymbol{j}})$.
e: The force has equal components in the $-y$ and $-z$ directions.
$\vec{\boldsymbol{v}} = \dfrac{v}{\sqrt{2}}(\hat{\boldsymbol{j}} - \hat{\boldsymbol{k}})$. $\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} = \dfrac{vB}{\sqrt{2}}(\hat{\boldsymbol{j}} - \hat{\boldsymbol{k}}) \times \hat{\boldsymbol{i}} = \dfrac{vB}{\sqrt{2}}(-\hat{\boldsymbol{k}} - \hat{\boldsymbol{j}}) = -\dfrac{vB}{\sqrt{2}}(\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}})$.

Q27.13 The maximum force per unit length of the flagpole is IB and for $I = 10^5$ A and $B = 5 \times 10^{-5}$ T (comparable to the earth's field), $F/l = 5$ N. This is not enough force to bend a metal flagpole.

Q27.14 No. The magnetic force is always perpendicular to the velocity of the particle and therefore does no work. The magnetic force can't change the kinetic energy of the particles. It can steer the particles but cannot increase their speeds.

Q27.15 There are two perpendicular motions. There is the movement of the charges in the wire of the loop, in the direction of the current. The magnetic force is perpendicular to this motion of the charges and doesn't affect the magnitude of the drift velocity associated with the current. Then there is the rotation of the loop. The edges of the loop have a tangential velocity that is in the direction of the magnetic force and there is work done for this motion. The magnetic force can change the rotational speed of the loop.

Q27.16 In Fig.27.39 if the polarity of the battery is reversed the current in the rotor will be reversed and the torque will be in the opposite direction. The motor will turn in the opposite direction. But, as discussed in Section 27.8, in many motors the magnetic field is provided by electromagnets and the current for the electromagnets is produced by the same battery that provides the current through the rotor. So, when the polarity of the battery is reversed both the current in the rotor and the magnetic field reverse direction and the torque on the rotor is in the same direction as before the battery was reversed. The motion could be reversed if separate voltage sources were used for the current in the rotor and for the current in the electromagnets and if the polarity of only one of these sources was reversed.

Q27.17 Yes, if the current is carried equally by positive and negative charges.

Q27.18 The Hall-effect voltage is $|E_z w|$, where w is the width along the z-axis of the strip. From Eq.(27.30) this gives $|E_z w| = v_{\mathrm{d}} B_y w = J_x B_y w / nq$. For semiconductors the concentration n of current carrying charges is smaller than for a metal, so the Hall voltage is larger. For the same current density the drift velocity must be larger for a semiconductor since there are fewer current-carrying charges in a given volume. The larger drift velocity results in a larger magnetic force, and this larger force produces more charge separation between the edges of the strip.

CHAPTER 28
SOURCES OF MAGNETIC FIELD

Discussion Questions

Q28.1 It would be analogous to a point charge. Magnetic field lines would terminate on it. The magnetic flux through a closed surface would be proportional to the net number of magnetic monopoles in the volume enclosed by the surface.

Q28.2 The moving charges produce their own magnetic field. The net field is then the vector sum of the field of the moving charges and the earth's field.

Q28.3 The wire can be considered infinitely long when calculating the magnetic field at points whose distance from the wire is much less than their distance from either end of the wire.

Q28.4 The energy comes from the electrical energy of the currents, which in turn comes from the emf that drives the current through the conductors. There is no contradiction. The magnetic force is perpendicular to the direction of the drift velocity of the current-carrying electrons in the conductors so doesn't affect the magnitude of the drift velocity. The motion of the wires is perpendicular to the motion of the current-carrying charges in the wires. It is the motion of the wires, in the direction of the magnetic force, that causes work to be done by this force.

Q28.5 The wires carrying currents in opposite directions produce magnetic fields that cancel because they are in opposite directions.

Q28.6 Currents in the same direction attract and currents in opposite directions repel. If all three wires carry currents in the same direction they will all three attract each other. There is no way to have all pairs with opposite currents, so it is not possible to have all three wires repel each other.

Q28.7 The force on a conductor is due to the magnetic field of the other conductor. The field of a conductor doesn't exert a force on the conductor that produces the field. The forces between conductors are due to the magnetic force between the moving charges in one conductor and the moving charges in the other conductor.

Q28.8 The two loops attract each other. The magnetic force on the inner loop is radially outward and the magnetic force on the outer loop is radially inward.

Q28.9 Adjacent turns of wire carry currents in the same direction and therefore attract each other.

Q28.10 The Biot-Savart law applies directly to an infinitesimal current element. In a practical calculation for an actual current configuration an integral must be done. The difficulty of applying the Biot-Savart law depends on the difficulty of performing that integral for the particular current configuration. When you us the Biot-Savart law you solve directly for $\vec{\boldsymbol{B}}$ at a point. Ampere's law relates a line integral around a closed path to the total current enclosed by the path. Ampere's law is easiest to apply when a path can be chosen for which $\vec{\boldsymbol{B}}$ is tangent to the path and constant at all points on the path. Ampere's law involves $\vec{\boldsymbol{B}}$ at many points simultaneously. This is an advantage for calculating B when B is the same at all these points.

Q28.11 For a toroidal solenoid the space inside the solenoid closes on itself so the field lines can stay inside the solenoid and still close on themselves. Field lines for a straight solenoid must leave the interior of the solenoid to close on themselves.

Q28.12 (a) No. Between the two wires the fields of the wires are in the same direction so they don't cancel. Above both wires or below both wires the two fields are in opposite directions, but all points in these regions are closer to one wire than to the other. Therefore, the two magnetic fields can't have the same magnitude and can't completely cancel even though they are in opposite directions. (b) Yes. Between the two wires the fields of the wires are in opposite directions. At a point midway between the two wires, the same distance from each wire, the magnitudes of the two fields are equal so they cancel completely.

Q28.13 Two parallel conductors carrying current in the same direction attract each other. So, the current in *L* must be from right to left and this requires *b* to be the positive terminal of the battery.

Q28.14 At the center of the ring $B_0 = \frac{\mu_0 I}{2a}$. On the axis of the ring at a distance *x* from the center of the ring, $B = \frac{\mu_0 I a^2}{2\left(x^2 + a\right)^{3/2}}$. If *a* is doubled, the field at the center of the ring is halved. The field at *P* changes by a different factor. For example, if *x* is much larger than *a* then $B = \frac{\mu_0 I a^2}{2x^3}$ and *B* increases by a factor of four when *a* is doubled.

Q28.15 Since $F = IlB\sin\phi$, $1\ \text{N} = 1\ \text{A}\cdot\text{m}\cdot\text{T}$. $1\ \text{N}\cdot\text{m} = 1\ \text{A}\cdot\text{m}^2\cdot\text{T}$. $1\ \text{N}\cdot\text{m} = 1\ \text{J}$ so $1\ \text{J} = 1\ \text{A}\cdot\text{m}^2\cdot\text{T}$ and $1\ \text{J/T} = 1\ \text{A}\cdot\text{m}^2$.

Q28.16 Smaller permeability means it is more difficult for an external magnetic field to align the atomic magnetic moments of the material. The tendency of the atomic magnetic moments to align themselves parallel to the magnetic field is opposed by their random thermal motion, and the energy of this motion increases with increasing temperature.

Q28.17 The magnetic susceptibility of liquid oxygen must be larger than that of liquid nitrogen. The magnet does attract oxygen gas to its poles but the force is too small to affect the random motion of the molecules associated with their temperature.

Q28.18 If each atom has a net magnetic moment then the material is paramagnetic. If the net magnetic moment of each atom is zero in the absence of an external field, but if an external field can induce a net magnetic moment, then the material is diamagnetic.

Q28.19 Paramagnetism is due to partial alignment of the magnetic moments of individual atoms and this alignment is disrupted by the random motion of the atoms, motion that increases when the temperature increases. Diamagnetic susceptibility depends on how easy it is to induce a net magnetic moment in an atom that has no magnetic moment in the absence of the external field. This effect is independent of the initial orientation of the atom so is not affected much by temperature.

Q28.20 The effect is based on conservation of angular momentum. As shown in Fig.28.24, the atomic magnetic moment is directed opposite to the atomic angular momentum. So, when the cylinder is magnetized and the atomic magnetic moments are partially aligned to produce a net magnetic moment, the atomic angular momenta are also aligned and produce a net angular momentum of the cylinder. This angular momentum is due to motion at the atomic level and not to rotation of the cylinder. But when the cylinder's magnetic moment reverses direction this net atomic angular momentum also reverses direction and by conservation of angular momentum the cylinder must rotate so its rotational angular momentum will keep the total angular momentum constant.

CHAPTER 29
ELECTROMAGNETIC INDUCTION

Discussion Questions

Q29.1 Pulling the sheet from between the poles changes the magnetic flux through the sheet and this causes induced currents in that sheet. By Lenz's law, the force of the field on these induced currents opposes the motion of the sheet. The induced current and hence the force is greater when the flux changes more rapidly, when the sheet moves with greater speed. The same thing happens when the sheet is inserted. In each case the force exerted on the induced currents by the magnetic field is directed to oppose the motion of the sheet.

Q29.2 The induced current changes direction after every half-revolution of the loop. When the angular speed is doubled the rate of change of the flux doubles and this causes the induced emf and induced current to double. The torque required is proportional to the current in the loop, so the torque also doubles.

Q29.3 The loops are sketched in Fig. DQ29.3. In the sketch let loop 1 be the one with the varying current I and let loop 2 be the closed ring in which current is induced. Let the current in loop 1 be counterclockwise. The flux through loop 2 due to the current in loop 1 is directed into the plane. When I increases, the increasing flux into the plane inside loop 2 induces a clockwise current. When I decreases, the decreasing flux into the plane inside the loop 2 induces a counterclockwise current. When the current in the first loop is increasing the two currents are in opposite directions. When the current in the first loop is decreasing the two currents are in the same direction.

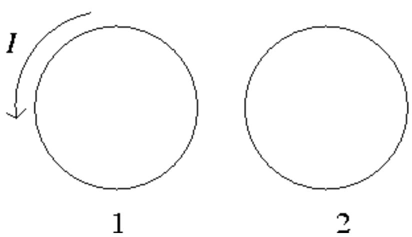


Figure DQ29.3

Q29.4 The SI units of the quantity BvL are $\text{T}\cdot\text{m}^2/\text{s}$. Since $\Phi_B = BA\cos\phi$, $1\ \text{Wb} = 1\ \text{T}\cdot\text{m}^2$. $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ so $1\ \text{V} = 1\ \text{Wb/s}$ and $1\ \text{Wb} = 1\ \text{V}\cdot\text{s}$. Therefore, $1\ \text{V}\cdot\text{s} = 1\ \text{T}\cdot\text{m}^2$ and $1\ \text{T} = 1\ \dfrac{\text{V}\cdot\text{s}}{\text{m}^2}$.
Then $1\ \text{T}\cdot\text{m}^2/\text{s} = \left(1\ \dfrac{\text{V}\cdot\text{s}}{\text{m}^2}\right)\left(1\ \dfrac{\text{m}^2}{\text{s}}\right) = 1\ \text{V}$, as was to be shown.

Q29.5 No. The magnetic field of the conductor is parallel to the ring and produces no flux through it. So, there is no induced emf when the current changes.

Q29.6 The moving magnet induces currents in the pipe and there is a force exerted between these currents and the magnet. By Lenz's law the force on the magnet is directed to oppose its motion, so this force is upward. The induced current and hence the force on the magnet increases as the speed of the magnet increases. The terminal speed is reached when this upward force equals the weight of the magnet.

Q29.7 See Fig. DQ29.7. $\vec{v}\times\vec{B}$ is to the right, so positive charge collects toward the right; the right-hand wing tip is at higher potential. The answer does not depend on the direction the plane is flying.

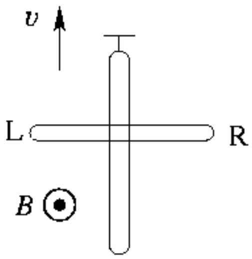


Figure DQ29.7

Q29.8 Just after the switch is closed the current increases from zero to a nonzero value in the counterclockwise direction. The magnetic field of the current in the larger loop us out of the page at the location of the small loop. Since the flux is increasing the induced current in the small loop is clockwise. The currents in the two loops therefore are in opposite directions and the two loops repel. The large loop exerts a force on the small loop that is radially inward at each point around the loop. This is consistent with Lenz's law: the induced current is directed so as to oppose the increase in flux by exerting a force that is directed to reduce the area of the small loop.

Q29.9 Let the wire and loop be positioned as in Fig.E29.7 and let the current in the wire be in the direction shown in that figure. The magnetic field is into the page at the location of the rectangle. Since the current I in the wire is decreasing the flux through the rectangle is decreasing and the induced current in the rectangle is clockwise. For the side of the rectangle closest to the wire the currents in the wire and rectangle are in the same direction and the force the wires exert on each other is attractive. For the side of the rectangle farthest from the wire the two currents are in opposite directions and the force is repulsive. But the attractive force is stronger than the repulsive force, since the force is inversely proportional to the distance between the wires. So, the net force on the rectangle is attractive, toward the wire. This is consistent with Lenz's law: the induced current is directed so as to pull the loop closer to the wire. This opposes the decrease in flux by pulling the rectangle closer to the wire, where the magnetic field is stronger.

Q29.10 If the axis of rotation is parallel to the magnetic field there is no magnetic flux through the loop at any position during its rotation so there is no change in flux and no induced emf.

Q29.11 For the slidewire to move at constant speed the net force on it must be zero. The external force F_{net} must equal the magnetic force F that acts on the slidewire due to the induced current. So, $F_{\text{ext}} = B^2L^2v^2 / R$. If there is a break in the circuit there is no induced current and in the absence of friction no external force is required for motion with constant speed.

Q29.12 No. The work done on an electron by the induced electric field during a complete trip around the loop is $e\mathcal{E}$. The same amount of energy is removed from the electron due to the resistance R of the loop. No. The induced electric field is a nonconservative field and is not associated with a potential difference.

Q29.13 (a) $\Phi_B = BA$. $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = A\left|\dfrac{dB}{dt}\right| = \pi r^2\left|\dfrac{dB}{dt}\right|$, where r is the radius of the ring. If r is doubled, then $|\mathcal{E}|$ is increased by a factor of 4. (b) $\int \vec{E}\bullet d\vec{l} = -\dfrac{d\Phi_B}{dt}$ gives $E(2\pi r) = \pi r^2\left|\dfrac{dB}{dt}\right|$ and $E = \frac{1}{2}r\left|\dfrac{dB}{dt}\right|$. Doubling r doubles E, the electric field induced in the ring.

Q29.14 As the magnet on the flywheel moves past the stationary coil the magnetic flux through the coil changes and a current is induced in the coil.

Q29.15 No, this is not what Lenz's law says. Lenz's law says that the induced current in a metal loop always flows to oppose the *change* in the magnetic flux through the loop.

Q29.16 No. The induced emf doesn't depend on the size of the magnetic flux. It instead depends on the rate of change of the flux.

Q29.17 Yes, a time-varying current corresponds to a time-varying electric field and a displacement current.

Q29.18 Yes. The dielectric is an insulator and there is no conduction current in the dielectric. These equations apply to the situation shown in Fig. 29.22. The equations show that the conduction current in the wires carrying charge to and from the capacitor plates equals the displacement current in the space between the plates.

Q29.19 (a) Eq.(29.20); (b) Eq.(29.21); (c) Eq.(29.18); (d) Eq.(29.19).

Q29.20 The electric field lines would have the same shape as the magnetic field lines produced by an electric current in a wire: concentric circles perpendicular to the wire, with the wire passing through the center of the circles.

Q29.21 Zero. Unless some of the regions of resistance completely fill a cross-sectional area of the cylinder, the super conducting regions provide a zero-resistance path for the current.

CHAPTER 30
INDUCTANCE

Discussion Questions

Q30.1 When the contact is broken the current abruptly decreases and there is a large induced current in the attachment.

Q30.2 $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ says $1\ \text{Wb} = 1\ \text{V}\cdot\text{s}$. $I = \dfrac{V}{R}$ says $1\ \text{A} = 1\ \text{V}/\Omega$.

Therefore, $1\ \text{Wb/A} = (1\ \text{V}\cdot\text{s})\left(1\ \dfrac{\Omega}{\text{V}}\right) = 1\ \Omega\cdot\text{s}$, as was to be shown.

Q30.3 Mutual inductance is a measure of how much flux through coil 2 is produced by a given current in coil 1. If coil 2 is rotated 90° the flux through it is greatly reduced, so the mutual inductance decreases.

Q30.4 The field is confined to the space enclosed by the windings. And if the difference between the inner and outer radii of the toroid is small compared to the inner radius, the variation of the field across the cross section of the windings is small. This makes it easy to calculate the flux through one turn of the coil.

Q30.5 series: If the current passes through each coil in the same sense, then the two in series are equivalent to one coil with twice the length and twice the number of turns. Exercise 30.15 shows that the self-inductance L for a long, straight solenoid with N turns, cross-sectional area A and length l is $L = \mu_0 N^2 A / l$. Doubling N and l causes L to increase by a factor of two. The number of turns per unit length is the same as for a single coil so the flux Φ_B through one turn is the same, but there are twice the number of turns and $L_{\text{series}} = 2L$.

parallel: The emf across the combination is the emf across each coil. Each coil gets half the current so the magnetic field in each coil is half what it would be with only a single coil in the same circuit. Hence the flux through one coil is halved, the induced emf in each coil is halved, and the inductance of the combination is $L/2$; $L_{\text{parallel}} = L/2$.

To get $L = 0$ connect the two coils in series such that the current passes around one in the opposite sense than for the other. The induced emfs in the coils are in opposite directions and cancel.

Q30.6 The area is $A = \pi r^2$. The magnetic field at the center of N circular loops of radius r is $\mu_0 NI / 2r$. The flux is then proportional to r. $L = N\Phi_B / i$ so the coil with twice the radius has twice the self-inductance.

Q30.7 Yes, this would work. The induced emfs for each half of the windings would be in opposite directions and would cancel. There would be no net induced emf and L would be zero.

Q30.8 $u = B^2 / 2\mu$ so for the same magnetic field the energy density is larger in a vacuum, since $\mu > \mu_0$. For a long solenoid the field inside the solenoid is given by $B = \mu n I$. $u = B^2 / 2\mu$ then gives $u = \mu n^2 I^2 / 2$. The energy density for constant I is proportional to μ and for the same current more energy is stored when the solenoid is filled with a ferromagnetic material, that has $\mu >> \mu_0$.

Q30.9 The voltage across the resistor is proportional to the current through it, the voltage across the inductor depends on the rate of change of the current and the voltage across the capacitor depends on the charge on its plates. (a) In the *R*-*C* circuit, just after the switch is closed the charge on the capacitor is zero and the full battery voltage appears across the resistor. In the *R-L* circuit, just after the switch is closed the current is still zero but is increasing at its maximum rate. The full battery voltage is across the inductor. (b) In the *R-C* circuit, the full battery voltage is across the capacitor and the current is zero. In the *R-L* circuit, the current is no longer changing, the full battery voltage is across the resistor and the current is a maximum. The capacitor has no effect initially but causes the current to go to zero as time progresses. The inductor initially limits the rate at which the current can increase but after a long time it has no effect on the circuit.

Q30.10 (a) The voltage across the inductor is proportional to the rate of change of the current flowing in the inductor. The pattern on the oscilloscope is shown in Fig. DQ30.10. (b) Since the voltage across the inductor is determined by the direction of the current, the V versus t graph is indeed proportional to the derivative of the current graph.

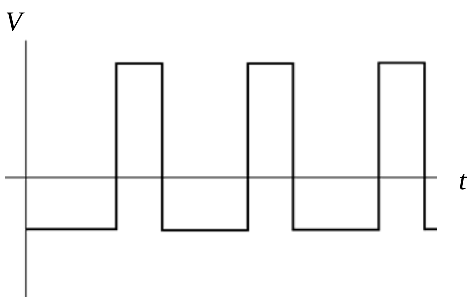


Figure DQ30.10

Q30.11 The current is now flowing in the negative direction. The current is increasing in magnitude so is becoming more negative, and this means $di/dt < 0$.

Q30.12 A decrease in charge on the capacitor corresponds to i flowing in the negative direction and $i = dq/dt$ correctly says this.

Q30.13 The potential v_{ac} must satisfy Kirchhoff's loop rule. So when other potentials around the loop change abruptly, such as abruptly adding $\mathcal{E}$ to the loop, v_{ac} must change abruptly. But $v_{bc} = L\,di/dt$. v_{bc} must equal $\mathcal{E} - iR$ and can't be larger than $\mathcal{E}$. Since v_{bc} is limited, di/dt is also limited.

Q30.14 The inductor and resistor are in series; all current through R must also pass through L.

Q30.15 The voltage induced across the inductor is $v_L = L|di/dt|$. If $|di/dt|$ is large then v_L is large. The large emf in the inductor produces a large current in the circuit. $v_L \to \infty$ when $|di/dt| \to \infty$ so it is not possible to stop the current instantaneously.

Q30.16 If the circuit is underdamped the current and charge oscillate, but with amplitudes that decrease each cycle of the oscillation. So the energy stored in a cycle would be less than the energy dissipated in that cycle. For an overdamped circuit there are no longer any current oscillations.

CHAPTER 31
ALTERNATING CURRENT

Discussion Questions

Q31.1 At the higher voltage, the same power can be transmitted at lower current so there is less I^2R power loss in the transmission wires. The higher voltage is more of a safety risk.

Q31.2 The polarity of the source voltages reverses when the current direction reverses. Current always enters the $-$ terminal and exits the + terminal as it passes through the source. The moving charge always gains electrical energy as it moves back and forth in the source. At a resistive load the moving charges in the current always lose electrical energy as they pass through the load in either direction. The oscillating charge at the source causes oscillating charge at the load. Electrical energy is put into the circuit at the source and consumed at the load.

Q31.3 In the inductor or capacitor electrical energy is stored and then released back to the circuit. In the resistor electrical energy is dissipated by transformation to thermal energy and not returned to the circuit.

Q31.4 When the charge on the left plate is positive but decreasing in time $|q|$ is decreasing so dq/dt is negative. The current is clockwise so is negative and $i = dq/dt$ is correct. When the charge on the right plate is positive and increasing, $|q|$ is increasing. But q refers to the charge on the left plate so is negative. A negative q that is getting larger in magnitude corresponds to $dq/dt < 0$. i is clockwise so is negative and $i = dq/dt$ is correct. When the charge on the right plate is positive and decreasing, $dq/dt > 0$ since q is negative and decreasing in magnitude. i is counterclockwise so is positive and $i = dq/dt$ is correct.

Q31.5 A resistor would dissipate electrical energy but no electrical energy is dissipated in the inductor. Electrical energy is temporarily stored in the magnetic field of the inductor but at a later time all this energy is returned to the circuit.

Q31.6 current to right through L and increasing: i is in the positive direction and increasing in magnitude so $di/dt > 0$. By Lenz's law the induced emf in the inductor is directed to oppose the increase in the current. The induced emf is therefore to the left in Fig.31.8a and this corresponds to $v_a > v_b$.

current to the right through L and decreasing: i is in the positive direction and is decreasing in magnitude so $di/dt < 0$. By Lenz's law the induced emf in the inductor is directed to oppose the decrease in the current. The induced emf is therefore to the right and this corresponds to $v_a < v_b$. v_{ab} is negative and di/dt is negative so $v_{ab} = L(di/dt)$ is still correct.

current to the left through L and increasing: i is in the negative direction and is increasing in magnitude so $di/dt < 0$. By Lenz's law the induced emf in the inductor is directed to oppose the increase in the current. The induced emf is therefore to the right and this corresponds to $v_a < v_b$. v_{ab} is negative and di/dt is negative so $v_{ab} = L(di/dt)$ is still correct.

current to the left through L and decreasing: i is in the negative direction and is decreasing in magnitude so $di/dt > 0$. By Lenz's law the induced emf in the inductor is directed to oppose the decrease in the current. The induced emf is therefore to the left and this corresponds to $v_a > v_b$. v_{ab} is positive and di/dt is positive so $v_{ab} = L(di/dt)$ is still correct.

Q31.7 No. The power factor is $\cos\phi$, where $\tan\phi = \dfrac{\omega L - 1/\omega C}{R}$. Mathematically, $\cos\phi = 0$ only if

$\phi = 90^{\circ}$. $\phi = 90^{\circ}$ gives $\tan\phi \to \infty$. This happens if $R \to 0$. This is consistent with the equation $P_{\text{av}} = \frac{1}{2}VI\cos\phi$ for the average power input to the circuit. As long as $R \neq 0$ there is some electrical power consumed in the resistor and the power factor isn't zero.

Q31.8 At any instant in time $v = v_R + v_L + v_C$. v_L and v_C are 180° out of phase so have opposite sign. v_R and $(v_L + v_C)$ are 90° out of phase and $v^2 = v_R^2 + (v_L + v_C)^2$. $|v_R|$ and $|v_L + v_C|$ must each be less than the magnitude $|v|$ of the source voltage. But since $|v_L + v_C| = ||v_L| - |v_C||$, $|v_L|$ and $|v_C|$ can each be larger than $|v|$.

Q31.9 When the circuit is far from resonance X_L and X_C differ. So if $R << X_L$ and $R << X_C$, then it will also be true that $R << |X_L - X_C|$. $|\tan\phi| = |X_L - X_C|/R$, so $|\tan\phi|$ is large and ϕ is close to $+90^{\circ}$ or -90°. The power factor $\cos\phi$ is close to zero. The current and source voltage are close to 90° out of phase and little power is delivered by the source. Since R is small, little power is consumed in the resistor and this agrees with little power delivered by the source.

Q31.10 See Q31.8. The voltages across the inductor and capacitor are 180° out of phase, so at all times have opposite polarity and subtract from each other when the loop rule is applied. The net voltage at any time across the capacitor and inductor combination can't exceed 120 V, but the capacitor voltage can be much larger than this.

Q31.11 The inductor doesn't affect R but it does increase the impedance Z of the circuit. I_{rms} and therefore $P = I_{\text{rms}}^2 R$ decrease.

Q31.12 $X_C = 1/(\omega C)$. Inserting the dielectric increases C and therefore decreases X_C. This in turn decreases Z. The rms current in the circuit increases and the bulb becomes brighter when the dielectric is inserted.

Q31.13 Inserting the iron rod increases the inductance L. $X_L = \omega L$ so X_L increases and Z increases. This in turn decreases the rms current in the circuit and the bulb becomes less bright.

Q31.14 The impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. An increase in Z decreases I_{rms} and the bulb becomes less bright. A decrease in Z increases I_{rms} and the bulb becomes brighter. Whether X_C^2 or $(X_L - X_C)^2$ is smaller depends on the numerical values of X_L and X_C. Similarly, whether X_L^2 or $(X_L - X_C)^2$ is smaller depends on the values of X_L and X_C. The change in brightness, whether it increases or decreases, depends on the values of X_L and X_C, and these in turn depend on L, C and the frequency f of the source.

Q31.15 Yes, this happens at the resonance frequency. At resonance $X_L = X_C$ and $Z = R$ with the capacitor and inductor present and $Z = R$ also when both are removed.

Q31.16 No. The transformer works using the induced emf that results in an ac circuit. If a transformer is connected to a dc line there is no current or voltage in the secondary.

Q31.17 (a) $V_2 = \left(\dfrac{N_2}{N_1}\right)V_1$. If N_2 is doubled, then V_2, the voltage amplitude in the secondary, doubles. (b) The effective resistance of the secondary is given by $R_{\text{eff}} = \frac{R}{\left(N_2 / N_1\right)^2}$. If N_2 is doubled the effective resistance is multiplied by a factor of $\frac{1}{4}$ and therefore decreases.

Q31.18 (a) The resonance angular frequency is given by $\omega_0 = \frac{1}{\sqrt{LC}}$. If L and C are both doubled the resonance is halved. (b) The inductive reactance is $X_L = \omega L$. If L is doubled then X_L increases by a factor of two. (c) The capacitive reactance is $X_C = \frac{1}{\omega C}$. Doubling C means X_C is halved. (d) The impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. Since R and X_L are doubled but X_C is halved, Z does not change by a simple factor of two.

Q31.19 The resonance angular frequency is $\omega_0 = \frac{1}{\sqrt{LC}}$. It is independent of R. To double ω_0, decrease L and C by multiplying each of them by $\frac{1}{2}$.

CHAPTER 32
ELECTROMAGNETIC WAVES

Discussion Questions

Q32.1 Yes. The direction of the vector product $\vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}$ can be determined and the wave is traveling in this direction so must have come from the opposite direction.

Q32.2 The steel girders are conductors and absorb radio waves and prevent the waves from getting to the car.

Q32.3 Visible light from a light bulb, microwaves in a microwave oven, and radio waves received by a radio are all electromagnetic waves. These waves all consist of oscillating electric and magnetic fields and travel in air with the same speed c. These different kinds of electromagnetic waves have different frequencies and therefore different wavelengths.

Q32.4 The electric fields of the intense radio waves produce currents in the signs.

Q32.5 Only transverse waves can be polarized. All electromagnetic waves can be polarized but sound waves cannot.

Q32.6 Once the wavefront reaches the charge the uniform, constant electric field in the wave will exert a constant force on the charge in the direction of $\vec{\boldsymbol{E}}$, and this gives the charge a constant acceleration.

Q32.7 The intensity of such a wave would be $I = \frac{1}{2}\epsilon_0 c E_{\max}^2 = 0.3\ \text{W/cm}^2$. This is too little energy to induce any appreciable currents in the person's body. Also, the frequency of the visible light waves is about 10^{15} Hz and it is difficult to produce such very high frequency currents in a person's body.

Q32.8 $I = \dfrac{E_{\max} B_{\max}}{2\mu_0}$. $E_{\max} = cB_{\max}$ so $I = \dfrac{c}{2\mu_0} B_{\max}^2$. For I to double, the magnetic field amplitude $B_{\max}$ is increased by a factor of $\sqrt{2}$.

Q32.9 No, the frequency of oscillation of the magnetic field of the light is about 10^{15} Hz and due to the mass of the compass needle it would take a very large current to cause it to oscillate at this extremely high frequency. The earth's field is much weaker but is constant in time.

Q32.10 The electric field of the radio waves must be polarized in the vertical direction.

Q32.11 Yes, the same momentum conservation principle applies and there is a recoil effect. But the momentum carried by the light is very small so the recoil force is also very small and is not detectable.

Q32.12 For a perfectly reflecting surface $p_{\text{rad}} = \dfrac{I}{c}$. Eq.(15.26) applies to electromagnetic waves: $\dfrac{I_1}{I_2} = \dfrac{r_2^2}{r_1^2}$. $I_2 = I_1 \left(\dfrac{r_1}{r_2}\right)^2$. If the distance is doubled then the intensity is multiplies by $\dfrac{1}{4}$ and the radiation pressure p_{rad} is multiplied by $\dfrac{1}{4}$. Therefore, the radiation pressure would be $p/4$.

Q32.13 The standing wave has energy in its electric and magnetic fields but does not transport energy through space like a traveling wave does. Similarly, the electric and magnetic fields in the standing wave have momentum but there is no momentum flow like there is for a traveling wave. As shown in Example 32.6 the intensity or time average of $\vec{S}$ at any point in a standing wave is zero.

CHAPTER 33
THE NATURE AND PROPAGATION OF LIGHT

Discussion Questions

Q33.1 Light travels only very slightly slower in air than in vacuum. And the distance it travels in the earth's atmosphere is a very small fraction of the total distance. The delay due to the earth's atmosphere is very, very small.

Q33.2 Light travels slower in the atmosphere than in vacuum; the refractive index of air is slightly greater than unity. When light goes from one material to another of larger refractive index it is bent toward the normal. The normal to the earth's atmosphere is vertical. This bending of starlight as it enters the atmosphere does mean that stars are not precisely where they appear to be.

Q33.3 The frequency f of the wave doesn't change when passing from one material to another. Since the interface cannot create or destroy waves, the number of wave cycles arriving per unit time must equal the number leaving per unit time. The period T is $T = 1/f$ so T also doesn't change. The wavelength decreases when the speed of light decreases. The distance traveled during one period is less when the wave speed is less, so the wavelength, the distance occupied by one cycle, decreases.

Q33.4 The sun can be seen even though it is below the horizon; see Problem 33.51. Similarly, the sun can be seen before it rises. This does slightly increase the time between the apparent sunrise and the apparent sunset.

Q33.5 The refractive index of air depends on the temperature of the air. Light passing through the moving warm air produces this effect.

Q33.6 $v = \dfrac{c}{n}$ so measuring n is equivalent to measuring v. (a) Let a ray of light in air refract into the glass and measure θ_a and θ_b, where a is air and b is glass. For air $n = 1.00$ and Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$, allows n_b to be calculated. (b) For light traveling in the glass measure the critical angle for a glass-air interface. $\sin\theta_{\text{crit}} = \dfrac{1.00}{n_{\text{glass}}}$ and n_{glass} can be calculated. (c) Reflect light traveling in air from the surface of the glass. Use a polarizing filter to measure the polarizing angle θ_{p}, the incident angle at which the reflected light is completely polarized perpendicular to the plane of incidence. Then the equation $\tan\theta_{\text{p}} = \dfrac{n_b}{n_a}$, where a is air and b is glass, can be used to calculate the refractive index of the glass.

Q33.7 One image is caused by reflection at the front surface of the window glass and the other from reflection at the back surface.

Q33.8 Light travels from the fish to the surface of the water, reflects there and travels to your eyes. The surface of the water, at the interface between the water and the air, forms an image of the fish, just like a conventional flat mirror. Each point in the image is the same distance above the surface of the water as the corresponding point on the fish is below the surface (see chapter 34), so the image is upside down. The top of the fish is on the bottom of the image.

Q33.9 No. Total internal reflection occurs only when light is incident in the material of larger refractive index. The refractive index of air is less than that of glass.

Q33.10 The angle of reflection equals the angle of incidence in all materials and for all wavelengths.

The angle of refraction depends on the indices of refraction of the two materials and the index of refraction depends on wavelength.

Q33.11 Take two pair and rotate one pair in relation to the other, as in Fig.33.25. If the transmitted intensity depends on the angle the one pair is rotated, then the lenses are Polaroid filters.

Q33.12 No. Only transverse waves exhibit polarization.

Q33.13 Produce polarized light, polarized along the horizontal direction, by letting the light reflect from the horizontal surface of water with the incident angle equal to the polarizing angle. Hold the filter vertical and rotate it until the intensity of this light transmitted through the filter is a maximum. In this position the polarizing axis of the polarizer is horizontal.

Q33.14 Yes, it would work. The light reflected from the roadway is partially polarized parallel to roadway. The polarizing axis of the windshield would need to be vertical to block this polarized component of the reflected light. An advantage is that it would be built into the car. A disadvantage would be that it would reduce the transmitted light that comes directly from the objects. All objects would appear dimmer.

Q33.15 The plastic wrap becomes birefringent when stretched and alters the polarization of the light passing through it.

Q33.16 The reflected light, the glare, is partially polarized in a horizontal direction. The polarizing axis of the Polaroid sunglasses is vertical, so this axis is perpendicular to the polarization axis of the glare and the sunglasses don't pass this light when your head is upright. But when you lie on your side the polarizing axis is horizontal and the glasses pass the reflected light.

Q33.17 Yes, it is true. The axis of the third polarizer has a nonzero component in both perpendicular directions. The third polarizer rotates the polarization axis of the light that has passed through the first polarizer.

Q33.18 The electromagnetic waves carrying the TV signal are polarized in a horizontal direction perpendicular to the direction of propagation of the wave. Therefore, the plane of the "ears" should be perpendicular to the direction from your location to the location of the transmitter for the TV station, so that the antenna will align with the electric field of the TV signal.

Q33.19 Light is scattered in all directions in the yz-plane so removes all polarization components equally from the incident light and the transmitted light is still unpolarized. Only scattering in the $-y$-direction is shown in Fig.33.31 since that is where our sunbather is.

Q33.20 Fig.33.31 shows that when the sun is low in the western sky the light scattered downward to the surface of the earth is polarized in a north-south direction. The polarizing axis of the Polaroid sunglasses is along the line that runs from your mouth to nose to between your eyes. You want this axis to be perpendicular to the north-south polarization direction of the light, so lie along an east-west line. It doesn't matter if your feet point west or east.

Q33.21 Light scattered from blue sky is scattered from linear molecules in the atmosphere that have a linear axis. Light scattered from cloud is not scattered by individual molecules but instead from water droplets or ice crystals that have no axis.

Q33.22 The scattered light contains much more blue light than red. The red-tinted sunglasses filter out the blue scattered light and pass the red light, which is predominantly light that hasn't been scattered by the particles or droplets.

Q33.23 Sunrises are red when the air is dusty or smoky. Human activities and wind during the day add dust and pollutants to the air during the day. Much of this material settles out during the night as so the air is cleaner at sunrise than at sunset.

Q33.24 In Fig.33.36, light travels slower in the air near the hot surface of the earth. For sound at night, sound travels slower in the cool air nearer the surface and follows a curved path just like that shown in Fig.33.36.

Q33.25 Yes, water waves can be reflected and refracted. Water waves produced by dropping a rock in a pool of water reflect from a surface that is perpendicular to the water. Water waves depend on the depth of the water so the waves refract at a line where the depth changes. Huygen's principle applies to all wave phenomena.

CHAPTER 34
GEOMETRIC OPTICS

Discussion Questions

Q34.1 Each section of a spherical mirror forms the entire image. The image formed by the bottom half of the mirror will be formed at the same location as the image formed by the mirror before it was cut in half. The only difference is that the image will be less intense (dimmer) because half as much light is reflected by half the mirror.

Q34.2 The image is on the side of the interface where the light comes from so the image is virtual and s' is negative.

Q34.3 The concave surface acts as a converging mirror for the waves and focuses the light to form the desired real image at the receiver. The image of the very distant object is at the focal point of the surface. The receiver should be a distance $f = R/2$ from the vertex of the surface.

Q34.4 $f = R/2$. For a flat surface $R \to \infty$ so $f \to \infty$. For a very distant object the image is very far from the mirror, corresponding to a focal point very far from the mirror. Rays incident parallel to the normal to the mirror are reflected parallel to the normal.

Q34.5 The focal length of the mirror follows from the curvature of the mirror and the laws of reflection. The law of reflection is independent of the wavelength of the light and the focal length doesn't change when the mirror is immersed in water.

Q34.6 $s' = sf/(s-f)$. For a real image $s' > 0$. For real objects, concave mirrors ($f > 0$) form real images when $s > f$. For real objects, convex mirrors ($f < 0$) never form real images.

Q34.7 Each image formed by one mirror serves as the object for the other mirror. At each reflection not all the light is reflected, so the more reflections to form an image the dimmer that image.

Q34.8 For $s = f$, a point object at the focal point of the mirror, the reflected rays are parallel to the mirror axis and no image is formed. The rays neither converge to a real image nor appear to diverge from a virtual image behind the mirror. For $s > f$, as s approaches f the real image becomes farther and farther from the mirror and becomes larger and larger. The image is large but it is also far away; its angular size is the same as the angular size relative to the mirror of the object. For $s < f$, as s approaches f the virtual image becomes farther and farther from the mirror and becomes larger and larger.

Q34.9 The mirror forms a virtual image that is smaller than the object. The purpose of the mirror is to allow you to see a wide area behind you, for safety. A concave mirror could also form a reduced image, for object distances greater than twice the focal of the mirror. But such images are real and therefore inverted, which is not desirable. The focal length is determined so that the desired viewing area has an image size that fits on the mirror surface.

Q34.10 Yes, a real image of the sun is formed at the focal point of the concave mirror. Rays are focused here and the image is bright. A convex mirror diverges the rays and doesn't focus them at a point; it doesn't work with a convex mirror.

Q34.11 If his face's distance s from the vertex of the surface is greater than $R/2$, where R is the radius of curvature of the spoon, the person sees a real, inverted image. For $s < R/2$ the image is upright. The convex side forms virtual, upright images no matter how far his face is from the spoon.

Q34.12 For s exactly equal to f the reflected rays are parallel to the axis and there is no image formed. For $s > f$, as s approaches f the real image gets larger and larger and farther and farther from the mirror. s' is positive and gets larger and larger as $s \to f$. For $s < f$, as s approaches f the virtual image gets larger and larger and farther and farther from the mirror. s' is negative and its magnitude gets larger and larger as $s \to f$.

Q34.13 $m = +1$ is a statement about the lateral magnification. $m = +1$ says that the object PQ has the same length as the image $P'Q'$, and this is what Fig.34.26 shows. The difference in length of the vertical arrow and its image refers to the longitudinal magnification. The longitudinal magnification for refraction at a plane refracting surface is not $m = +1$.

Q34.14 Yes. The mirror forms a reduced image so the car appears to be farther away than its actual distance.

Q34.15 Take a piece of paper and move the paper until the image of a distant object, such as the sun is focused on the paper. Then the distance between the lens and the paper is the focal length of the lens. The virtual image formed by a diverging lens cannot be focused onto a piece of paper, so this method cannot be used for a diverging lens.

Q34.16 The focal length of a thin lens is given by Eq.(34.19). The refractive index n of the lens material depends on the wavelength of the light and this causes f to depend on the wavelength. But the factor $n-1$ is always greater than zero so f has the same sign for all wavelengths.

Q34.17 When the lens is in water rather than in air the refractive indices of the lens and surrounding material are closer in value than when the lens is in air. Therefore, light is bent less when it enters and exits the lens and the focal length is greater when the lens is in water. (See Problem 34.92.)

Q34.18 A spherical drop of water acts as a converging lens and $|f|$ is proportional to the radius of curvature of the drop; a small radius gives a small focal length. For an air bubble the role of the two refractive indices, air and water, are reversed and the air bubble acts as a diverging lens. The magnitude of its focal length increases when its radius increases. A detailed analysis can be carried out by treating refraction as light enters and leaves the bubble. The bubble cannot be treated as a thin lens because the distance between the two surfaces where refraction occurs is not small compared to the radius of these surfaces.

Q34.19 Yes. This is discussed in the textbook at the end of Section 34.1 and this principle is used in Section 34.4 in the derivation of the lensmaker's equation. It doesn't matter if the first image is real or virtual. The concept of a first image is a convenient way to describe the direction the rays are traveling, either from or toward a real image or appearing to come from a virtual image, when they reach the second surface.

Q34.20 The rays of light do not converge at a virtual image but instead seem to come from a virtual image, so a virtual image cannot be projected onto film. But a virtual image can be photographed with a camera, just as an object or real image can be photographed by letting light from the object or image enter the camera lens. A simple example is a photograph of an image in a plane mirror.

Q34.21 All the formulas are the same if the image and object are interchanged. n_a and n_b must be interchanged and also y and y' in $m = y'/y$. Reversibility requires that the formulas be unchanged by the interchange of image and object.

Q34.22 A simple refracting telescope uses two converging lenses. The lens for the eyepiece should have a large focal length and the lens for the eyepiece needs to have a short focal length. Converging lenses will focus sunlight to a sharp point on the ground and a diverging lens won't. The distance from the lens to the ground when the sunlight is sharply focused on the ground is the focal length of the lens.

Q34.23 Water on one side of the eye affects the refracting property of the eye. An image focused on the retina when air is on the outside of the eye is not focused when the air is replaced by water. Yes, you could wear eyeglasses that correct your underwater vision. The water has a refractive index closer to that of the eye so reduces the bending of the light as it enters the eye. To compensate for this you should use converging lenses in the eyeglasses.

Q34.24 Each section of a lens forms the entire image. The image formed by the bottom half of the lens will be identical in location, size and orientation to the image formed by the full lens. The only difference is that the image will be less intense (dimmer) because half as much light passes through the masked lens.

CHAPTER 35
INTERFERENCE

Discussion Questions

Q35.1 Being in water reduces the wavelength. This causes the interference maxima to be more closely spaced (Eq.35.6).

Q35.2 Longitudinal waves obey the principle of superposition and interfere so a two-source interference experiment can be done with sound waves. The sources must be coherent, such as two loudspeakers driven by the same amplifier. The wavelength for sound waves is much larger than for visible light so if $d \approx \lambda$ then the two sources would be farther apart in the sound experiment. Reflection of the sound waves would need to be suppressed.

Q35.3 When θ is small, $y_m = R\frac{m\lambda}{d}$ and the distance on the screen between adjacent bright fringes is $\Delta y = y_{m+1} - y_m = \frac{R\lambda}{d}$. Δy doesn't depend on *m*, so adjacent fringes are equally spaced near the center of the screen, where θ is small. For any angle, $y_m = R\tan\theta_m$ and $\sin\theta_m = \frac{m\lambda}{d}$. $y_m = R\frac{m\lambda}{d}$ is derived by setting $\sin\theta_m \approx \tan\theta_m$, valid for small θ_m. For larger angles, $\tan\theta_m$ is increasingly larger than $\sin\theta_m$ and adjacent fringes get farther and farther apart as the distance from the center of the screen increases.

Q35.4 For small angles, bright fringes are located by $y_m(\text{bright}) = R\frac{m\lambda}{d}$ and the distance between adjacent bright fringes is $\Delta y(\text{bright}) = y_{m+1} - y_m = \frac{R\lambda}{d}$. For small angles, dark fringes are located by $y_m(\text{dark}) = R\frac{(m+\frac{1}{2})\lambda}{d}$. The distance on the screen between a bright fringe and an adjacent dark fringe is $y_m(\text{dark}) - y_m(\text{bright}) = \frac{R\lambda}{2d}$. Near the center of the screen, where the small angle approximation is valid, the bright fringes are midway between the dark fringes. This is no longer true away from the center of the screen, where the small angle approximation is no longer valid. Then a bright fringe is not midway between dark fringes but instead is somewhat closer to the adjacent dark fringe of smaller *m*.

Q35.5 No, they will not produce an interference pattern. They are not coherent sources.

Q35.6 The location of points of constructive and destructive interference depend only on the wavelength of the wave and the distance between the two sources so the positions of the nodal and antinodal lines would be unaffected. There would be total reinforcement at points on the antinodal curves but there would not be total cancelation at points on the nodal curves.

Q35.7 In principle, yes. But the wavelengths of gamma rays is 10^{-10} m or less. The two slits, or coherent sources, would have to be about 10^{-10} m apart. This is about the size of atoms and would be very difficult to achieve.

Q35.8 Red light has a wavelength of about 700 nm. For two slits separated by $d =$ 25 cm, $\lambda / d = 3\times10^{-6}$. The angular positions of adjacent maxima and minima would be very close together and the pattern of light and dark fringes would not be observable.

Q35.9 The first dark fringe on one side of the central bright fringe is located at $\sin\theta = \lambda / 2d$. If this fringe is at $\theta = 90^\circ$ it isn't observed. This gives $d = \lambda / 2$. If $d < \lambda / 2$ the central bright fringe is spread over the entire screen and no dark fringes are observed.

Q35.10 The phase angle ϕ does indeed have these values at points on the screen where there is totally constructive interference. But at all other points on the screen ϕ has values differing from these.

Q35.11 Yes, there would be overlapping interference patterns, for each wavelength of the light. The pattern would have a rainbow appearance. There would be bright fringes of each color but no dark fringes because at each point on the screen there will be constructive interference for some wavelength.

Q35.12 No, amplitudes must be added. For electromagnetic waves, for example, it is the electric and magnetic fields that obey the principle of superposition. Intensities are always positive and don't interfere.

Q35.13 The water can act as a non-reflecting coating, as described in Example 35.7.

Q35.14 For a soap film with air on either side of the film, there is a half-cycle phase shift when light is reflected off the top of the film but no phase shift due to reflection when it reflects off the bottom surface. The net phase difference due to the reflections is a half-cycle phase difference. So when the film is very thin and there is no additional phase difference introduced by the path difference, the light reflected from the top and bottom of the film interferes destructively. For a soap film on glass rays reflected at both the top and bottom surfaces of the film have a half-cycle shift upon reflection so there is no net phase difference due to the reflection. Then for a very thin film the two light rays reflected from the top and bottom of the film are in phase and interfere constructively.

Q35.15 For visible light reflecting from a thick film, the values of *m* in Eq.(35.17) will be large and the wavelength change for successive values of *m* will be very small. Closely spaced wavelengths will have constructive interference and no one wavelength appears to be emphasized more than any other in the reflected light. The interference still occurs but its effects are unobservable. A film might be considered thick if it is more than 10λ thick.

Q35.16 Conservation of energy requires that light of a given wavelength either be reflected or transmitted. The reflected and transmitted intensities must add up to the incident intensity. So, if the light isn't reflected due to interference then it must be transmitted.

Q35.17 If the intensity of the reflected light is small then the intensity of the light transmitted through the film is large. See Q35.16.

Q35.18 At a very thin part of the film the path difference due to the thickness of the film introduces negligible phase difference. To get destructive interference then there must be a net half-cycle phase difference due to the reflections. When light in air reflects from the top surface of the oil film there is a half-cycle phase shift since $n_{\text{oil}} > n_{\text{air}}$. So there must be no phase shift when the light traveling in oil reflects off the interface between oil and water at the bottom surface of the film. This will be the case if and only if $n_{\text{oil}} > n_{\text{water}}$.

CHAPTER 36
DIFFRACTION

Discussion Questions

Q36.1 To observe diffraction effects the size of the aperture must be similar to the wavelength. The wavelengths of sound waves and water waves range from a few centimeters to several meters and we commonly experience apertures of this range of sizes. But visible light has a wavelength on the order of 10^{-7} m and apertures of this size don't occur for objects that we normally experience. It is the short wavelength, not the high speed, that makes diffraction effects for light less common.

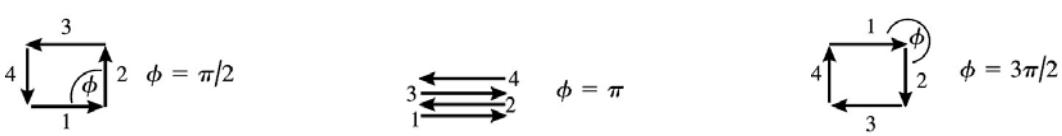
Q36.2 Fresnel and Fraunhofer diffraction are based on the same physical process, it is just their mathematical description that is different. The Fraunhofer diffraction regime allows for simplifying approximations.

Q36.3 $\theta = 1.22\lambda / D$ is the smallest angular separation that can be resolved. Small θ corresponds to larger resolving power. (a) Smaller D increases θ and decreases the resolving power. (b) Larger f means smaller λ and this decreases θ and increases the resolving power. (c) Larger λ means larger θ and decreases the resolving power.

Q36.4 From Eq.(36.3) we find the width of the central maximum to be $\Delta y = 2x\lambda / a$. (a) Decreasing a increases the width of the central maximum. (b) Decreasing f increases λ and this increases the width of the central maximum. (c) Decreasing λ does decrease the width of the central maximum. (d) Decreasing x does decrease the width of the central maximum.

Q36.5 The first dark fringe on one side of the central maximum is at an angle θ given by $\sin\theta = \lambda / a$. This fringe doesn't appear on the screen if $\lambda / a > 1$ so $a = \lambda$ is the maximum slit width for which no dark fringe is seen.

Q36.6 The phasor diagrams are similar to those in Figure 36.14 in the textbook. An interference minimum occurs when the phasors add to zero. (a) The phasor diagram is given in Figure DQ36.6a. There is destructive interference between the light through slits 1 and 3 and between 2 and 4. (b) The phasor diagram is given in Figure DQ36.6b. There is destructive interference between light through slits 1 and 2 and between 3 and 4. (c) The phasor diagram is given in Figure DQ36.6c. There is destructive interference between light through slits 1 and 3 and between 2 and 4. Note that maxima occur when $\phi = 0, 2\pi, 4\pi$ etc. Our analysis shows that there are three minima between the maxima at $\phi = 0$ and $\phi = 2\pi$. This agrees with the general result that for N slits there are $N-1$ minima between each pair of principal maxima.



(a) (b) (c)

Figure DQ36.6

Q36.7 For eight slits the phasor diagrams must have eight vectors. At a minimum the phasors for all slits sum to zero. The diagrams for $\phi = 3\pi / 4$ and for $\phi = 5\pi / 4$ are sketched in Figure DQ36.7a and the diagrams for $\phi = 3\pi / 2$ and for $\phi = 7\pi / 4$ are sketched in Figure DQ36.7b. For $\phi = 3\pi / 4$, $\phi = 5\pi / 4$ and $\phi = 7\pi / 4$ totally destructive interference occurs between slits four apart. For $\phi = 3\pi / 2$, totally destructive interference occurs with every second slit.

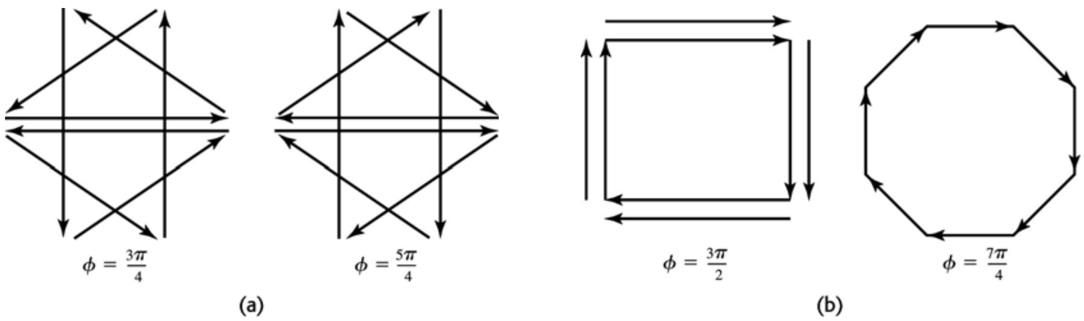


Figure DQ36.7

Q36.8 The raindrop acts as an aperture. If the drop is small enough the central diffraction pattern for each wavelength is as wide as the angular spread of the rainbow. This will happen when the drop size is on the order of the wavelength of the visible light.

Q36.9 The narrow horizontal width increases the horizontal spread of the sound in the central diffraction maximum. If the audience is distributed horizontally but not vertically it is not desirable to have a wide diffraction maximum in the vertical direction, so the vertical dimension of the speaker is somewhat larger than the wavelength of the sound. If the audience is also spread in the vertical direction the speaker would need also to be narrow in the vertical direction to spread the sound in this direction, so should be more nearly square.

Q36.10 High frequency sounds have smaller wavelength. The small speaker diameter makes λ / a larger so the central diffraction maximum for the high frequency sound waves is spread over a wider angular range.

Q36.11 In this application we want the central diffraction maximum for light from the pits to be narrow. This requires a small λ / a. Narrow pits (small a) means more information stored in the disk, but a small a requires a small λ to keep λ / a and the width of the diffraction maximum small.

Q36.12 The minimum angular separation of objects that can be resolved is $\theta = 1.22\lambda / D$. Resolving finer detail means smaller θ and this happens for smaller λ. The Hubble can see finer detail in the ultraviolet.

Q36.13 (a) For a minimum to occur, the sum of all phase differences between the slits must add to zero, so the phasor diagram closes on itself and the sum of all the phasors is zero. This requires that $N\phi = 2\pi m$, for some integer m. Therefore, $\phi = m\left(\frac{2\pi}{N}\right)$. But when $\frac{m}{N} = n$, where n is an integer, $\phi = 2\pi n$. In this case all the phasors are in phase and their magnitudes add to give a principal maximum. (b) There are $N - 1$ values of m that satisfy $\phi = m\left(\frac{2\pi}{N}\right)$ between the value nN and $(n+1)N$, where n is an integer.

Q36.14 In x-ray diffraction with a crystal the slit spacing d is the separation between atomic planes in the crystal. A typical value of d in this application is 0.1 nm (Example 36.5). The wavelength of visible light is about 500 nm, so $\lambda / d = 5000$ and no interference fringes can be seen, only the central bright fringe.

Q36.15 With a grating the bright fringes are much narrower. The centers of the fringes can be measured more accurately and there is less overlap of fringes from different wavelength than with two slits.

Q36.16 The phase difference for the radio signal received by the different antennas produces an interference pattern when the signal from all antennas is combined coherently. The sharp interference fringes allow for accurate determination of the wavelengths in the signal, just as a diffraction grating can be used to measure the wavelengths in a sample of visible light.

Q36.17 The image will be distorted when 500-nm light is used for viewing. In Fig.36.29b the different wavelength will produce a maximum in the diffracted wave at a point different from P'. The centers of successive bright fringes on the developed film will differ by integer wavelengths at different points.

Q36.18 The maxima in the diffracted wave will occur at different points for each wavelength in the white light and no image will be seen.

Q36.19 Bright and dark areas of the image will be opposite what they are in the object. That is, where the object is bright the image will be dark.

CHAPTER 37
RELATIVITY

Discussion Questions

Q37.1 (a) The proper time is measured in a frame in which the two events occur at the same place. That is the frame of reference of the moving train; the passenger on the train measures the proper time. (b) The proper length of an object is measured in a frame in which the object is at rest. The proper length of the train car is measured in the frame of the moving train; the passenger on the train measures the proper length. (c) The proper length of the sign is measured in a frame attached to the ground; you measure the proper length of the sign.

Q37.2 No. The effect cannot occur before the cause. In order for event A to cause event B some sort of signal must travel from event A to event B and this signal cannot travel faster than the speed of light. Therefore, if two events are causally related their separation in space and their separation in time in any frame must be such that a light signal can be passed between them. Events that satisfy this constraint occur in the same order in all reference frames. If event A occurs before event B in some frame and B before A in other frames then the space and time intervals between the events in any frame must be such that a light signal cannot be passed between them; they cannot be causally related.

Q37.3 (a) With respect to the astronaut riding with the rocket the light travels the same distance to each wall and the events are simultaneous. (b) Relative to the person at rest on earth, the walls of the rocket move to the right as the light travels to them so the light travels a shorter distance to reach the rear wall and this person observes *B* occurring first.

Q37.4 In the rest frame of the spacecraft the passenger sees her distance from the asteroid increase as the asteroid moves away from her at $0.9c$. Simultaneously, she sees her distance from the earth decrease as the earth moves toward her at $0.9c$. The passenger receives the radio messages simultaneously when her distance from the asteroid is equal to her distance from earth. Therefore, she deduces that the signal from earth was sent at a time when she was farther from earth than she was from the asteroid. Since the signal from the earth must travel a greater distance than the signal from the asteroid, the passenger deduces that the signal from the earth must have been sent before the signal from the asteroid.

Q37.5 No. By a distance of 70 light years we mean a distance between two points measured in the frame of the earth. The separation between these two points is shortened relative to an observer moving very close to the speed of light and such a traveler can travel between the two points in a time, measured in his frame, that is less than 70 years.

Q37.6 You would need to travel at high speed in the direction along the major axis of the ellipse. (See Problem 37.45.)

Q37.7 In the thought experiment of Section 37.2 the two events occur at different places in each frame. If the two events are simultaneous in a frame and occur at the same point in that frame then they are simultaneous in all frames. The effect described in Section 37.2 arises from the differences in travel time of a light signal from each event to the observer.

Q37.8 Larry measures the proper length l_0 of the train car. Adam measures the length *l*, where $l = l_0\sqrt{1-u^2/c^2}$ and *u* is the speed of the train relative to the earth. The length *l* is less than l_0. Since David is moving in the same direction as the train, the speed of the train relative to him is smaller than the train's speed relative to Adam. Therefore, the length contraction is a bit less for David than for Adam, but the difference is very small since the bicycle moves much, much slower than the train. All three observers measure different lengths for the train car. The length measured by Larry is the greatest and the length measured by Adam is the smallest.

Q37.9 No. Eq.(37.27) shows that p increases without bound as v approaches c. Similarly, Eq.(37.38) shows that E increases without bound as v approaches c.

Q37.10 She is correct. As soon as a massless particle, such as a photon, is created, it has speed c. Photons never have speed less than c. Recent experimental evidence suggests that perhaps neutrinos do have mass.

Q37.11 No. Einstein's second postulate is about the speed of light in vacuum.

Q37.12 No. The speed of light is the same for all observers. Both the wavelength and frequency are altered by the motion of the source but they still satisfy $c = f\lambda$. When λ is shortened f is increased and the speed of light is unaffected by the motion of the source.

Q37.13 In principle an increase in energy corresponds to an increase in mass. But the amount of mass associated with ordinary amounts of thermal energy is quite small. For example, $E = mc^2$ says that an energy of 900 J corresponds to a mass of 1×10^{-14} kg. So, in practice this is not a measurable effect.

Q37.14 Common experience involves speeds where Newtonian mechanics applies to high accuracy. Experiments that show a breakdown of Newtonian mechanics are technologically advanced and could only be performed starting in the 20th century. Also, relativistic mechanics is conceptually more complicated and is built on the foundation of Newtonian mechanics. Conceptually, it would have been difficult for scientists to develop relativistic mechanics if they didn't first have a deep understanding of Newtonian mechanics.

Q37.15 The effects of special relativity would be commonplace.

CHAPTER 38
PHOTONS: LIGHT WAVES BEHAVING AS PARTICLES

Discussion Questions

Q38.1 They are similar in that they have energy and momentum. Photons are different in that they have zero rest mass and travel at the speed of light. Photons have neither mass nor charge. The speed of photons depends on the material in which they travel but within any given material they always have the same speed and can't be accelerated.

Q38.2 The energy the electron receives from the light would be $2hf$ rather than hf. $2hf$ would replace hf in Eq.(38.4). The threshold frequency f_0 would be $f_0 = eV_0/(2h)$, half what it is when only one photon is absorbed.

Q38.3 The number of photons emitted each second by ordinary light sources is very, very large. (See, for example Exercise 38.3.) There is no noticeable time interval between arrival of photons.

Q38.4 We would expect that the photon (particle) nature of light would be more important at the high-frequency end of the electromagnetic spectrum. High-frequency photons each have more energy and momentum so the quantized photon nature of light would be more apparent at high frequency.

Q38.5 A photon with wavelength 400 nm, at the short-wavelength edge of the visible spectrum, has an energy of 3.1 eV. Table 38.1 shows that the work function for most common metals is larger than this. And, surface impurities, such as oxide layers, increases the work function considerably. And any electrons lost from the surface are quickly replaced from the surrounding air. Photoelectric effect experiments must be done with very clean surfaces in high vacuum.

Q38.6 A certain threshold energy is needed to alter a molecule of the emulsion of the film such that the spot is "exposed". Therefore, there is a threshold frequency below which a photon doesn't have enough energy to expose the film. Longer wavelength light has lower frequency. The threshold need not be sharp; the lower the energy of the photon the less efficiently it can expose the film.

Q38.7 Yes. The higher frequency ultraviolet photons individually have enough energy to damage molecules in the skin.

Q38.8 Only electrons initially at the surface of the metal leave the surface with kinetic energy $hf - \phi$. For electrons that start within the metal some of the energy delivered by the photons is used to bring the electron to the surface of the metal, so less energy is left as kinetic energy of the released electron. More photoelectrons come from within the metal than from its surface; the photons usually penetrate beneath the surface of the metal before they are absorbed.

Q38.9 Above the threshold frequency the number of photoelectrons is independent of the frequency of the light. One photon absorbed produces one photoelectron. Changing the frequency changes the energy of each photon but doesn't change the number of photons.

Q38.10 A photoelectron receives energy from a single photon and the energy of a photon depends on its frequency or wavelength. Therefore, the maximum speed and kinetic energy of a photoelectron depends on the frequency of the light and is independent of the intensity of the light. And there is no time delay. The only statement that is true is (a).

Q38.11 No. A molecule of the phosphor that converts ultraviolet radiation to visible light absorbs an ultraviolet photon and emits only part of the energy as a photon, so the emitted photon has less energy and larger wavelength than the absorbed photon. The molecule can't emit a photon of more energy than the energy of the photon that was absorbed.

Q38.12 $K_{\max} = hf - \phi$. (a) No. Light of greater intensity produces more photoelectrons but doesn't change their maximum kinetic energy. (b) Yes. Increasing f increases $K_{\max}$. (c) No. Increasing λ decreases f and decreases $K_{\max}$. (d) No. Increasing ϕ decreases $K_{\max}$.

Q38.13 The photon transfers some of its energy to the electron so the scattered photon has less energy than the incident photon. This means that $f' < f$. Eq.(38.10) shows that the energy $P_{\mathbf{e}}^2 / 2m$ transferred to the electron depends on the angle ϕ. But for all values of ϕ the electron gains energy and the photon loses energy and $f' < f$. The only exception to this is $\phi = 0$; for this ϕ no energy is transferred to the electron and $f' = f$.

Q38.14 Yes, photons can scatter from protons and Eq.(38.7) still applies. But now m is the proton mass rather than the electron mass. The shift in wavelength will be much less.

Q38.15 Energetic electrons that have been accelerated through high voltages can produce x rays when they are stopped in solid pieces of metal.

Q38.16 Photons travel in straight lines in the direction of propagation of the light. What is oscillating in an electromagnetic wave are electric and magnetic fields. Photons have no charge and therefore an oscillating electric or magnetic field wouldn't exert a force on a photon to cause it to move along with the oscillation of the fields.

Q38.17 No. To get a wave function for the light that is localized in space a spread of frequencies (or wavelengths) must be combined. If the wave packet is spread out in space, as in a steady, continuous beam, it can be of more precise wavelength.

CHAPTER 39
PARTICLES BEHAVING AS WAVES

Discussion Questions

Q39.1 $\lambda = h/mv$. The electron has a smaller mass so has a larger λ.

Q39.2 $\lambda = h/p$. $K = p^2/2m$, so $p = \sqrt{2mK}$. $\lambda = h/\sqrt{2mK}$. The electron has a smaller mass so has a larger λ.

Q39.3 For a photon $\lambda = h/p$. This is identical to Eq.(38.5) that relates the momentum of a photon to the wavelength of the light. The de Broglie wavelength of the photon and the wavelength of the associated electromagnetic wave are the same.

Q39.4 The electron remains a point particle. When the electron goes through the hole its position in the direction parallel to the hole is determined to within the diameter of the hole and by the uncertainty principle this introduces an uncertainty in the component of its momentum in the direction that is parallel to the hole. This uncertainty in transverse momentum causes a spread in the location where the electrons strike the screen.

Q39.5 To emit light in the $n = 2$ to $n = 1$ transition there needs to be atoms in the $n = 2$ excited state. Galaxies contain a lot of atoms in the $n = 2$ excited state so must have a high temperature. The intergalactic medium must have few atoms in the $n = 2$ excited state so must be cold.

Q39.6 In the equations in Section 39.3 that describe the hydrogen atom replace e^2 by Ze^2 to get the corresponding equations for a one-electron ion with atomic number Z. Eq.(39.14) shows that E_n for hydrogen is proportional to e^4, so the corresponding E_n for Li^{++} are larger by a factor of 9. Eq.(39.8) shows that r_n for hydrogen is proportional to $1/e^2$ so the corresponding r_n for $\text{Li}^{+}+$ are smaller by a factor of 1/3.

Q39.7 A photon with energy E has momentum $p = E/c$. If the atom has mass M and gains momentum $p = E/c$ when the photon is emitted, the recoil kinetic energy is $K_\text{r} = p^2/2M = E^2/2Mc^2 = (E/2Mc^2)E$. Mc^2 is the total rest mass energy of the atom. E is the transition energy for the atom and is much, much less than the rest mass energy of the atom. Mc^2 for a proton is 938 MeV and E is typically less than 10 eV, so $E/2Mc^2$ is very small. The recoil kinetic energy is a very small fraction of the photon energy and is negligible in Eq.(39.5).

Q39.8 It can be done, and has been done, in the Franck-Hertz experiment. This is described in Section 39.3.

Q39.9 For a gas of atoms the electrons in each atom are in discrete energy levels. For a solid there are electrons in bands of energies and the electrons can have any of the energies within these bands and there is a continuous spread of transition energies. See Section 42.4.

Q39.10 The intensity versus wavelength of the emitted light follows the Planck radiation law. This law (Fig.39.32 and Eq.39.24) and also the Wien displacement law (Eq.39.21) show that the spectral emittance $I(\lambda)$ peaks at smaller wavelengths as the temperature of the object increases. This causes a shift in color. The underlying reason for the shift in $I(\lambda)$ is that at higher temperatures higher energy levels of the electrons in the solid are populated so emission can occur from these higher levels. Transitions from higher energy levels have greater transition energies and the emitted photons have higher energies and shorter wavelengths. Eq.(39.19) also shows that the total intensity of the emitted radiation increases as T^4.

Q39.11 If we apply Bohr's angular momentum quantization to a planet, the quantum numbers n are huge and orbits for successive n are infinitesimal close in radius. No discrete nature of the orbit radius of the planet is observable.

Q39.12 The de Broglie wavelength of an electron with kinetic energy 54 eV is $\lambda = h/p = h/\sqrt{2mE} = 1.7\times10^{-10}$ m. The wave nature of the electrons is of no consequence in this application.

Q39.13 Extremely short wavelength electromagnetic waves have very energetic photons that would damage the object being examined.

Q39.14 $E_n = -(13.6\text{ eV})/n^2$ so an electron in a higher n shell has more energy. $v = (2.19\times10^6\text{ m/s})/n$. An electron in a higher n shell also has a smaller speed in its orbit and therefore less kinetic energy. But the electron with larger n has more (less negative) potential energy and this gives it more total energy. In fact, $U_n = -2K_n$, so $E_n = -K_n$ and smaller K_n means greater E_n since E_n is negative.

Q39.15 No. For Δy on the order of the diameter of the bullet, Δp_y from the uncertainty principle is exceedingly small and produces no observable effects.

Q39.16 If one slit is covered a single-slit diffraction pattern is obtained, not a two-slit pattern. Both slits must be open for each electron so in this sense all electrons go through both slits. If we want to show the wave nature of electrons by observing an interference pattern in a two-slit experiment, we cannot at the same time show their particle nature by requiring the electrons to go through one slit or the other.

Q39.17 Eq.(39.30) says that the energy cannot be precisely measured in a very short time. Energy conservation can be violated only for very short times.

Q39.18 Eq.(39.30) says that the longer an atom is in an excited state, the more precise is the energy of that state and therefore the smaller the spread in transition energy when the atom makes a transition from the excited state to the ground state. The energy of the emitted photon equals the transition energy, so photons emitted in a transition from a long-lived atomic state have a narrower spread in energy and wavelength. For ordinary light sources the atoms are in the excited states for very short time intervals and the energy uncertainty of those states is large.

Q39.19 Yes, a grating of any number of slits could be used. The diffraction pattern would be the same as for light whose wavelength is the same as the de Broglie wavelength of the electrons; the electrons would strike the screen only within narrow lines on the screen. The uncertainty principle would not be violated. The greater the number of slits the more uncertain is the position of the electron when it passes through the grating so the more precise can be its direction of travel.

Q39.20 The aluminum foil is a polycrystalline material. It consists of randomly oriented microscopic crystals. For a single crystal a pattern like that shown in Fig.36.20 would be observed. The location of the dots depends on the orientation of the crystal relative to the electron beam. The pattern from the polycrystalline foil includes contributions from crystals of all orientations so each dot is replaced by a continuous ring of dots.

Q39.21 The resolution is determined by λ/a, where a is the lens diameter. The de Broglie wavelength of the electrons is orders of magnitude smaller than the wavelength of visible light. So the electron microscope is better able to resolve small objects. The magnification of a microscope is (approximately) $M = (25\text{ cm})s_1'/f_1 f_2$ (Eq.34.24). To have a large magnification M the focal lengths f_1 and f_2 of objective and eyepiece must be very small. It is difficult to make extremely small focal length lenses for visible light. It is much easier to make small focal length electrostatic lenses for electrons.

Q39.22 When you measure the temperature of an object by inserting a thermometer into it you change the temperature of the object at least slightly because some heat is transferred between the object and the thermometer. If you measure the current in a circuit with an ammeter you add the ammeter resistance in series and alter the current you are trying to measure.

CHAPTER 40
QUANTUM MECHANICS

Discussion Questions

Q40.1 For these macroscopic objects the de Broglie wavelength is exceedingly small and the fact that it is not quite zero is of no consequence. Such objects exhibit no wave properties.

Q40.2 The analogy is sensible.

Q40.3 It is the quantity $\left|\Psi(x,t)\right|^2$ that describes the position of the particle and $\left|\Psi(x,t)\right|^2 = \left|\psi(x)\right|^2$ is real even though $\Psi(x,t)$ has complex values.

Q40.4 For a one-dimensional system $\left|\psi(x)\right|^2 dx$ is the probability that the particle will be found in the region x to $x+dx$. The particle must be somewhere, so $\int_{-\infty}^{\infty}\left|\psi(x)\right|^2 dx$ must be unity. But $\int_{-\infty}^{\infty}\left|\psi(x)\right|^2 dx = 1$ is just the normalization condition. $\psi(x)$ must be normalized if $\left|\psi(x)\right|^2 dx$ is to be the probability of the particle being found between x and $x+dx$.

Q40.5 No, it just means that the position probability distribution function $|\Psi|^2$ at each point in space doesn't vary in time. Yes, states of constant energy are stationary states.

Q40.6 $n=0$ given $k=0$ and from Eq.(40.29), $\psi(x)=0$. This wave function is a trivial solution to Eq.(40.25) that corresponds to no particle being present. $\psi_n = -\psi_{-n}$ and wave functions differing by a constant describe the same physical state; $|\psi_n|^2 = |\psi_{-n}|^2$ so negative n values don't correspond to additional solutions.

Q40.7 The area under a graph of $|\psi|^2$ versus x between x_1 and x_2 is the probability that a value between x_1 and x_2 will be found if the position of the particle is measured. The total area under $|\psi|^2$ would be unity; there is unit probability that the particle will be somewhere.

Q40.8 The classical particle travels between the walls with constant speed so it is equally probable that it will be found at any x. The graph of the probability distribution as a function of x would be a horizontal line for $0 \le x \le L$ and would be zero for $x<0$ or $x>L$. For $|\psi_n|^2$ with $\psi_n(x)$ given by Eq.(40.35) the oscillations become very rapid as n increases and $\int_x^{x+\Delta x} |\psi_n|^2 dx$ is the same for all intervals Δx. So, yes, the probability distributions approach the classical result as n becomes very large.

Q40.9 Yes, Eq.(40.26) is the form of two traveling waves propagating in opposite directions. The stationary states can be thought of as a standing wave produced by superposition of wave functions for particles traveling in opposite directions due to reflections at the walls of the box.

Q40.10 The wave functions for all states are symmetric about $x = L/2$ (the center of the box) so the particle is equally likely to be found in either half of the box. The probability of finding the particle in the right half of the box is 1/2, for any level.

Q40.11 The probability of measuring the position of the particle and getting one of these values of x is zero. The velocity of the particle is not zero at these points.

Q40.12 $E_n = n^2\pi^2\hbar^2 / 2mL^2 = p_n^2 / 2m = h^2 / 2m\lambda_n^2$. States of definite E have definite λ and definite momentum p. But measurements of $\vec{p}$ yields $p\hat{\boldsymbol{i}}$ and $-p\hat{\boldsymbol{i}}$ with equal probability so the states of definite energy aren't states of definite momentum vector.

Q40.13 States of definite energy are not states of definite wavelength and are not states of definite momentum. $p = h/\lambda$ so if p doesn't have a definite value then λ doesn't have a definite value. For $0 < x < L$, the wave function is a combination of functions $\sin kx$ and $\cos kx$ that have definite $k = p/\hbar$. But for $x < 0$ and $x > L$ the wave function is of a different form and isn't characterized by this same k.

Q40.14 $|\psi|^2 = 0$ at the walls so there is zero probability that a measurement of the particle's position yields the values 0 or L. But the presence of the walls affects the wave function and energy levels of the particle. If the walls are removed, these quantities change. In this sense the particle strikes the walls.

Q40.15 As Fig.40.14 shows, the wave function extends into the region outside the well and there is some probability for the particle to be found there. From the normalization condition $\int_{-\infty}^{\infty} |\psi(x)|^2\, dx = 1$ and $\int_0^L |\psi(x)|^2\, dx$ will be less than 1.

Q40.16 Since the wave function extends out into the classically forbidden region ($x < 0$ and $x > L$) for the finite well but not for the infinite well (box), the wavelength in a given energy state of the finite well is larger than the wavelength in the corresponding level of the infinite well. $E_n = h^2 / 2m\lambda_n^2$ inside the box, so larger λ means lower energy.

Q40.17 Yes, $E_1 \to 0$ as $U_0 \to 0$. As $E_1 \to 0$ the wave function extends farther and farther into the $x < 0$ and $x > L$ regions. As the momentum approaches zero its uncertainty becomes small, the localization of the particle in x becomes less, in accordance with the Heisenberg Uncertainty Principle.

Q40.18 The rate of the exponential decay of the wave function with distance outside the well depends on $\kappa = \sqrt{2m(U_0 - E)} / \hbar$. κ becomes small as $E \to U_0$. The particle can tunnel farther into the potential barrier when its energy is close to the top of the barrier.

Q40.19 Yes, it is a contradiction. In classical Newtonian mechanics the particle cannot be in this region but in quantum mechanics there is some probability of the particle being there.

Q40.20 The electron density is spherically symmetric so the potential energy function must be spherically symmetric. And the potential energy corresponds to a potential well.

Q40.21 You would expect that for a given particle energy the probability of tunneling would decrease as the barrier height is increased. As the barrier height is increased, $E - U_0$ becomes more negative and in classical mechanics it is more strongly forbidden for the particle to be in the barrier.

Q40.22 No. A measurement of the position of the particle gives a definite result. The wave function means that there is some probability that the whole particle can be found in the region $x < 0$ and some probability that the whole particle can be found in the region $x > L$.

Q40.23 No. Oscillations in $|\psi(x)|^2$ continue to be present. They just get more and more rapid as n

increases.

Q40.24 The wave function is smaller in the region $-A/2 < x < A/2$ than in the region $-A < x < A/2$ plus $A/2 < x < A$. So, even though these two regions are the same size, there is greater probability of finding the particle in the outer half ($-A < x < A/2$ plus $A/2 < x < A$) than in the center half ($-A/2 < x < A/2$). The particle is more likely to be found near the turning points at $x = \pm A$ than around $x = 0$. This corresponds to the classical result that $v = 0$ at $x = \pm A$ and that v is a maximum at $x = 0$. The particle spends more time near the turning points.

Q40.25 Harmonic oscillator: $E = \left(n + \frac{1}{2}\right)\hbar\omega$. Levels are equally spaced. $n = 0$ for the ground state. $n = 2$ for the second excited level.
Particle in a box: $E = n^2\pi^2\hbar^2 / 2mL^2$. Level spacing increases with increasing *n*. $n = 1$ for the ground state and $n = 3$ for the second excited level.
Hydrogen atom: $E_n = -me^2 / 8\varepsilon_0 n^2 h^2$. Level spacing decreases with increasing *n*. $n = 1$ for the ground state and $n = 3$ for the second excited level.

Q40.26 In either case the wave function is zero at the infinite wall at $x = 0$ and is zero for $x < 0$. For $E_1 < U_0$ the particle is in a bound level and the wave function exponentially decreases for $x > A$ and is zero for $x \geq B$. The wave function in the region $0 < x < A$ is the wave function for a free particle and will have zero nodes for the ground state and more if it corresponds to an excited state. For E_3 the wave function is zero for $x \leq 0$ and for $x \geq B$. For $0 < x < B$ the wave function is that of a free particle and the wave number k is larger in the region $0 < x < A$ than in the region $A < x < B$. Sketches of the possible wave function for each energy are shown in Fig.DQ40.26.

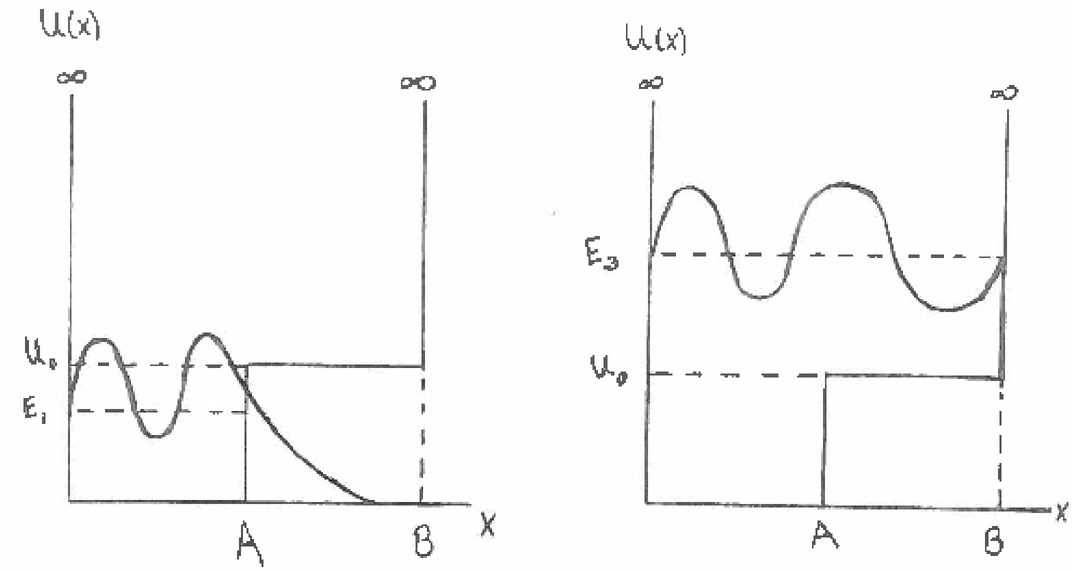


Figure DQ40.26

Q40.27 If the wave function is $\Psi(x,t) = \psi_2(x)e^{-iE_2t/\hbar}$, a measurement of the energy yields the value $E_2 = \dfrac{4\pi^2\hbar^2}{2mL^2}$. If the wave function is $\Psi(x,t) = \dfrac{1}{\sqrt{2}}\left(\psi_1(x)e^{-iE_1t/\hbar} + \psi_2(x)e^{-iE_2t/\hbar}\right)$, a measurement of the energy can yield either $E_1 = \dfrac{\pi^2\hbar^2}{2mL^2}$ or $E_2 = \dfrac{4\pi^2\hbar^2}{2mL^2}$. There is equal probability of each result so the average result for many measurements with an ensemble of identical particles would be

$E_{\text{av}} = (E_1 + E_2)/2 = \dfrac{5}{2}\left(\dfrac{\pi^2\hbar^2}{2mL^2}\right)$. No, we can't say that before the measurement each particle has energy E_{av}. Before the measurement each particle has equal probability of having energy E_1 or E_2.

CHAPTER 41
ATOMIC STRUCTURE

Discussion Questions

Q41.1 The probability is given by $|\psi^2|dV$, where ψ is the wave function. $|e^{i\phi}|=1$ so the two wave functions give the same probability.

Q41.2 The Bohr model and the Schrodinger analysis yield the same expression for the energy levels (Eq.(41.21)). The Bohr model says the angular momentum of an electron in energy state *n* is $L=n\hbar$. The Schrodinger analysis says $L=\sqrt{l(l+1)}\hbar$, where $l=0,\ 1,\ \ldots,\ n-1$. Therefore the two approaches give quite different results for the angular momentum. The Schrodinger analysis says $L_z=m_l\hbar$, with $m_l=0,\ \pm 1,\ \pm 2,\ \ldots,\ \pm l$, so L_z is quantized and the possible orientations of $\vec{L}$ are restricted (Fig.41.6). In the Bohr model the electron travels in an orbit of definite radius whereas in the Schrodinger analysis the location of the electron is determined by a position probability distribution (Fig.41.8).

Q41.3 Yes, in principle. But the spacing between allowed orbits is exceedingly small and the quantization of the orbits is not observable. (See Discussion Question 39.11.)

Q41.4 The electrons exert forces on each other so their motions are correlated.

Q41.5 For ionized atoms the magnetic force on the net charge would overwhelm the force due to the spin magnetic moment.

Q41.6 (a) Yes, $L=\sqrt{l(l+1)}\hbar$, with $l=0,\ 1,\ \ldots,\ n-1$. Except for $n=1$ there are two or more *l* values for each *n*. For example, for $n=2$ the value of *l* can be 1 or 0. (b) Yes. A given value of *l* can correspond to any value of *n*, for $n=l+1,\ l+2,\ l+3,\ \ldots$. For example, $l=1$ can have $n=2,\ 3,\ 4,\ \ldots$.

Q41.7 A homogeneous field exerts zero net force on a current loop (Section 27.7).

Q41.8 The Pauli exclusion principle requires that no two electrons have the same set of quantum numbers. They have the same n, l, and m_l, so they must have different m_s values.

Q41.9 Eq.(27.27) says that the interaction energy is minimum when a dipole is in a direction parallel to the field direction and a maximum when the dipole direction is opposite to the field. The $m_s=+\frac{1}{2}$ state has $S_z=+\hbar/2$ and by Eq.(41.38) its spin magnetic moment is in the $-z$-direction. So for this state $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$ are antiparallel and the interaction energy between the spin magnetic moment and the field is a maximum. The $m_s=-\frac{1}{2}$ state has $S_z=-\hbar/2$ and its spin magnetic moment is in the $+z$-direction. For this state $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$ are parallel and the interaction energy is a minimum. The $m_s=-\frac{1}{2}$ state has lower energy.

Q41.10 In an alkali metal there is a single electron outside filled shells. The electron density in the filled shells is at a smaller radius than for the electron outside the filled shells. This reduces the correlation of the motion of the outermost electron with that of the rest of the electrons. For the transition metals there are two or more electrons outside of filled shells. These electrons occupy similar regions of space and their motion is highly correlated.

Q41.11 The $4s$ level lies below the $3d$ level. Lower levels fill first in building up the ground states of atoms.

Q41.12 Example 21.1 showed that the electrical force between two alpha particles is immensely larger than the gravitational force between them. The same holds true for an electron and a nucleus; the gravitational force is totally insignificant in atomic structure.

Q41.13 They all have a $1s^2 2s^2 2p^6 3s^2 3p^6$ core plus some number of $4s$ and $3d$ electrons. The chemical properties do not depend much on how many $4s$ and $3d$ electrons there are in the atom.

Q41.14 Zinc ($Z = 30$) has filled subshells through $4s^2 3d^{10}$. The next lower energy orbital is the $4p$ so the neutral gallium atom has a $4p$ electron outside a zinc configuration core. Ga^+ has the same electron configuration as zinc. Ga^- has a $4p^2$ configuration outside a zinc configuration core.

Q41.15 For example, without spin helium would have an electron configuration of $1s2s$ and would not be chemically inert. The periodic table indicates double filling of each n, l, and m_l sublevel and this requires a fourth quantum number since the Pauli exclusion principle doesn't allow two electrons with the same set of quantum numbers.

Q41.16 The orbital motion of the electrons results in a magnetic field that is internal to the atom.

Q41.17 In the alkali atoms there is one electron outside filled shells. The electron density in the filled shells is at smaller radius than the outer electron so the electrons in the filled shells are effective in screening the nuclear charge. The outer electron sees an effective nuclear charge of close to $Z_{\text{eff}} = 1$. In a noble gas there are several electrons in the same shell and therefore in the same region of space. These electrons only partially screen the nuclear charge from each other and Z_{eff} is larger than 1.

Q41.18 The ionization potential is $-E_n$, where E_n is the level energy of the least tightly bound electron. The level energy is given by Eq.(41.45) and depends on the n quantum number for the level and the effective nuclear charge Z_{eff} seen by the electron. Z_{eff} is less than Z because of partial screening of the nucleus by the other electrons in the atom. As electrons are removed, for the outermost electron the screening of the nucleus by the other electrons decreases. For a given n the ionization potential decreases when Z_{eff} decreases. The ground state electron configuration of magnesium is $1s^2 2s^2 2p^6 3s^2$. For a $3s$ electron the other electrons screen the nucleus and Z_{eff} is approximately unity. For Mg^+ the ground state electron configuration is $1s^2 2s^2 2p^6 3s$ and 10 inner electrons screen the nucleus from the 3d electron and Z_{eff} is approximately 2. For Mg^{2+} the ground state electron configuration is $1s^2 2s^2 2p^6$. The screening for an outermost electron is further reduced. And now it is a $n = 2$ rather than a $n = 3$ electron that will be removed in ionization and this further increases the ionization potential.

Q41.19 The effect on an electron of all the other electrons is approximated in an average way by a spherically symmetric potential energy function. In reality the interaction between any pair of electrons depends on the instantaneous location of the two electrons and the electron motions are correlated. The central field approximation is a simplified treatment of the electron-electron interaction.

Q41.20 There is little screening for a $1s$ electron in gold except for the screening from the other $1s$ electron so the $1s$ electron has $Z_{\text{eff}} \approx 78$. Eq.(41.45) says the energies scale by Z_{eff}^2. So the energy for gold is about $(78)^2 = 6084$ times larger than the energy for hydrogen. It takes 13.6 eV to remove the $1s$ electron from hydrogen. A photon with $E = 13.6$ eV has wavelength $\lambda = hc/E = 91$ nm. The photon is in the ultraviolet. It takes about $(78)^2$ (13.6 eV) = 8.3×10^4 eV to remove a $1s$ electron from gold. A photon with $E = 8.3\times10^4$ eV has wavelength $\lambda = hc/E =$ (91 nm)/(78^2) = 0.015 nm. This photon is a hard x ray.

Q41.21 (a) $L^2 = l(l+1)\hbar^2$. $L_z^2 = m_l^2\hbar^2$. The maximum value of m_l is l so the maximum L_z^2 is given by $\left(L_z^2\right)^2_{\max} = l^2\hbar^2$. $L^2 - \left(L_z^2\right)^2_{\max} = l\hbar^2$. $L = L_z$ only when $l = 0$. For $l \neq 0$, $L > (L_z)_{\max}$. (b) A classical object can have $L = L_z$ even when L is not zero, if $\vec{L}$ is in the z-direction.

Q41.22 No. At the absorption edge the transition of the electron in the atom is from the $1s$ level to the lowest unfilled bound sublevel of the atom. All electrons in the atom are still bound. However, when the hole in the $1s$ orbital is filled by an outer-shell electron, the energy released can either result in the emission of an x ray or in the ionization of one of the electrons.

Q41.23 No. Hydrogen cannot emit photons of energy greater than 13.6 eV. A photon with this energy has wavelength $\lambda = hc/E = 91$ nm. This photon is in the ultraviolet and does not have enough energy to be an x ray.

Q41.24 For either electron the possible results of a measurement of S_z are $+\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$. For either electron the probability of each result is $\frac{1}{2}$. If measurement of S_z for electron 1 yields the value $+\frac{1}{2}\hbar$, then the wavefunction collapses to $\psi(\vec{r}_1,\vec{r}_2) = \psi_\alpha(\vec{r}_1)\psi_\beta(\vec{r}_2)$ and a measurement of S_z for electron 2 gives a result $-\frac{1}{2}\hbar$ with unit probability.

Q41.25 For either electron a measurement of S_z yields the value $+\frac{1}{2}\hbar$ with unit probability. If measurement of S_z for electron 1 yields the value of $+\frac{1}{2}\hbar$ then a measurement of S_z for electron 2 yields a value of $+\frac{1}{2}\hbar$ with unit probability.

CHAPTER 42
MOLECULES AND CONDENSED MATTER

Discussion Questions

Q42.1 The hydrogen bond is unique to hydrogen-containing compounds because only hydrogen has a singly ionized state with no remaining electron cloud. The hydrogen ion is a bare proton and is much smaller than any singly ionized atom. It is small enough to get between two atoms, polarizing them and attracting them by means of the induced dipoles.

Q42.2 Part of the bonding is due to electrons transferred from one atom to the other and part is due to the sharing of electrons between the atoms.

Q42.3 H_2^+ can't form a pair of positive and negative ions so the bonding must be covalent.

Q42.4 The molecule stretches when it is in a higher rotational level and I increases when r increases. So, the $l = 19 \to l = 18$ transition corresponds to a larger moment of inertia.

Q42.5 Near the bottom of the potential well the potential energy curve (Fig.42.1) of a diatomic molecule is parabolic and therefore the same as for a harmonic oscillator. But higher in the well, the well is wider than a parabola. The wider well corresponds to lower energy levels so adjacent levels move closer together as n increases.

Q42.6 For the vibrational levels the force constant is the same but the reduced mass m_r is double for D_2 so the vibrational frequency ω is smaller by a factor of $1/\sqrt{2}$. The vibrational energies for D_2 are smaller by a factor of $1/\sqrt{2}$. The equilibrium separation r_0 is nearly the same for H_2 and D_2. But m_r is double for D_2 so I is double. The rotational energy is proportional to $1/I$ so the rotational levels for D_2 are smaller by a factor of 1/2.

Q42.7 The temperature of the molecules is very low so there is very little electronic excitation and very, very few electronic energy transitions, and these are the transitions that emit visible light. The only excited levels of the molecules that are populated are low-lying rotational levels. The transition energies between these levels and the ground state are small and the photons emitted correspond to radio waves.

Q42.8 The typical level structure is shown in Fig.42.8. The spacing between the $n = 0$ and $n = 1$ vibrational levels is much larger than the spacing between the lowest few rotational levels.

Q42.9 For the lowest vibrational energy levels the energies are the same as for a harmonic oscillator. But the potential energy function $U(r)$ differs from that for an ideal spring for r farther away from the equilibrium separation. And if the molecule gains enough energy the molecule can dissociate into two atoms; the "spring" can break.

Q42.10 The outer electron charge clouds of adjacent atoms overlap significantly. Because of the electrical interactions between the electrons and because of the exclusion principle, the wave functions and energy levels are altered from their atomic values. The valence electron wave functions become less localized and extend over several atoms.

Q42.11 A conductor has a partially filled conduction band. An insulator has an empty conduction band and a large energy gap between the empty conduction band and the filled valence band. (See Fig.42.19.)

Q42.12 Ionic crystals have energy levels similar to those of isolated ions and absorb only certain discrete wavelengths of visible light. For metals the electrons in the partially filled conduction ban can absorb visible light of any wavelength and undergo a transition to an unfilled level in the band.

Q42.13 The molecules move as free particles and their kinetic energy is given by $\frac{3}{2}kT$. The energy of electrons in the conduction band of a metal depends on the Fermi energy of the band. Fig.42.23 shows that the distribution of electron states within the conduction band depends weakly on temperature.

Q42.14 In a semiconductor there is a small energy gap between the empty conduction band and the filled valence band. Increased pressure or temperature can promote electrons into the conduction band and gives the material metallic properties.

Q42.15 The energy levels of solid zinc are different from those of the isolated atom. The strong interaction of the atoms in the solid and the exclusion principle distort the wave functions of the electrons. The electron energies shift and form energy bands. (See Fig.42.18.)

Q42.16 A typical electron moves so rapidly within the metal that the effect of the ions and other electrons is a uniform potential energy function. The electron-electron interactions are effectively screened due to the high density of electrons. The net charge of the ion cores and valence electrons is smoothly distributed throughout the volume of the solid and gives rise to a constant potential everywhere except at the surfaces of the solid.

Q42.17 The nonlocalized electrons in the conduction band are free to move around and conduct both electricity and heat. The wires that conduct electricity also tend to conduct heat from the hot device.

Q42.18 Donor atoms need to have a loosely bound valence electron outside of an electron configuration similar to that of silicon or germanium. This loosely bound electron doesn't participate in the covalent bonding when these atoms are added to silicon or germanium. An acceptor atom needs one additional electron to reach an electron configuration like that of silicon or germanium. The acceptor atom needs one more electron in order to form the same number of covalent bonds as silicon or germanium when added to those materials.

Q42.19 Yes. Just add electrons to the surface, to fill the holes.

Q42.20 Yes. This is explained by Fig.42.24. At low temperatures there are no electrons in the conduction band. At high temperatures a large number of electrons have enough energy to be in the conduction band.

Q42.21 This is discussed in Section 25.2. As the temperature increases, the ions of the conductor vibrate with greater amplitude, making it more likely that a moving electron will collide with an ion. This impedes the drift of electrons through the conductor and reduces the currents. For a semiconductor, at increased temperatures electrons can gain energy from the thermal motion and move into the conduction band.

Q42.22 Add twice as many acceptor impurity atoms as donor impurity atoms.

Q42.23 The saturation current I_S is the maximum current from n to p when a reverse bias voltage is applied. This current is due to free electrons in the p region moving into the n region. An increase in temperature increases the number of free electrons in the p region.

Q42.24 If the device is too small there isn't enough space between the source and drain to prevent electrons from tunneling through and producing a drain current even when there is no gate potential.

The device will leak current when the gate is turned off and there is not supposed to be any current.

CHAPTER 43
NUCLEAR PHYSICS

Discussion Questions

Q43.1 The electric charge of the protons causes the transition energy for a spin flip to depend on the chemical environment of the protons. The purpose of the proton MRI is to study that environment. Neutrons have no charge so their spin flip energy is insensitive to their chemical environment.

Q43.2 The second term represents the decreased binding of nucleons on the surface of the nucleus whereas the first term represents binding of nucleons in the interior of the nucleus. The surface area of a sphere is $4\pi R^2$ whereas the volume is $\frac{4}{3}\pi R^3$. Therefore, as the nucleus becomes larger a smaller fraction of the nucleons are on the surface.

Q43.3 When nucleons are combined to form a nucleus some of their mass is converted into the binding energy of the nucleus.

Q43.4 When $Z > N$ there is too much Coulomb repulsion between pairs of protons for the nucleus to be stable.

Q43.5 The magic numbers for which there are known elements are 2, 8, 20, 28, 50 and 82. The elements with these values of *Z* are helium, oxygen, calcium, nickel, tin and lead. The nuclei of these elements are unusually stable.

Q43.6 No. As the number of electrons increases so does the nuclear charge Ze and the binding energy of each of the innermost electrons increases roughly as Z^2. The binding energy per electron for uranium is much larger than it is for hydrogen.

Q43.7 Radioactive decay must be energetically allowed. Proton or neutron decay normally isn't energetically allowed.

Q43.8 The interior of a star undergoing helium fusion is much hotter than the interior of a younger star that is undergoing hydrogen fusion. The Coulomb repulsion of helium nuclei ($Z = 2$) is four times that of hydrogen ($Z = 1$) for the same separation and much higher temperatures are needed to give the helium nuclei sufficient kinetic energy to overcome the Coulomb repulsion and get close enough to fuse.

Q43.9 ${}^{214}\mathrm{Pb}$ is an unstable isotope of lead. Other isotopes of lead are stable (${}^{206}\mathrm{Pb}$, ${}^{207}\mathrm{Pb}$, and ${}^{208}\mathrm{Pb}$).

Q43.10 The nuclei with the shorter lifetimes are less abundant.

Q43.11 α particles are much more massive. A β particle with the same energy has a much greater speed.

Q43.12 (a) Nucleon number unchanged, atomic number increases by 1 so this is β^- decay. (b) Nucleon number decreases by 4 and atomic number decreases by 2 so this is α decay. (c) Nucleon number unchanged and atomic number decreases by 1 so this is β^+ decay.

Q43.13 An electron has charge $-e$ and has nucleon number zero. A positron has nucleon number zero and charge $+e$ so it can be represented as ${}^{0}_{+1}\beta$. A neutrino has nucleon number zero and no charge so can be represented as ${}_{0}^{0}\nu$. An antineutrino has nucleon number zero and no charge so can be represented as ${}_{0}^{0}\overline{\nu}$.

Q43.14 The alpha, beta or gamma decays are associated solely with the nucleus and are unaffected by the electronic structure of the atom. Electron capture however involves capture of an orbital electron and is affected by the chemical binding of the atom since the chemical binding affects the orbital electrons.

Q43.15 The atomic electron that receives the energy has enough energy to break free from the atom; it can get energy greater than its binding energy. If this is an inner electron an x ray photon can be emitted when an outer shell electron in the atom falls into the inner-shell hole.

Q43.16 Human activity has burned carbon containing compounds in coal and oil and released this carbon into the atmosphere as CO_2 .

Q43.17 Biological material that was recently alive has an activity of 0.255 Bq or greater per gram of carbon. Older samples have a much lower activity per gram of carbon because of radioactive decay of the ^{14}C in the sample. Contamination with modern material would give a falsely high activity and cause an underestimation of the age of the sample. Older samples have a smaller activity and the relative effect of contamination is larger.

Q43.18 ^{226}Ra is being continually produced as a decay product from other more long-lived radioactive nuclei. One example is the ^{238}U decay series shown in Fig.43.7.

Q43.19 This is explained by Fig.43.2. When lighter nuclei fuse to form a nucleus lighter than ^{62}Ni the binding energy per nucleon in the product nucleus is greater than in the two original nuclei and energy is released. When a heavy nucleus splits into two pieces, each heavier than ^{62}Ni, the binding energy per nucleon increases and energy is released.

Q43.20 The binding energy per nucleon is greater in the daughter nuclei than in the parent nucleus (see Fig.43.2). Therefore, a large amount of energy is released in the fission and this energy appears as kinetic energy of the daughter nuclei.

CHAPTER 44
PARTICLE PHYSICS AND COSMOLOGY

Discussion Questions

Q44.1 It is possible that parts of the universe contain antimatter. Only if we detect annihilation events from the presence of both matter and antimatter or by detecting particles, such as high energy antiprotons or neutrinos that come to us from these regions could we tell. Light emitted by anti-atoms does not differ in any way from light emitted by normal atoms; there are no antiphotons. If we went there the matter in us and in anything we bring with us would annihilate with the antimatter and be converted into energy.

Q44.2 Yes. Conservation of energy can be violated for short periods of time. Empty space is not really empty. It is filled with these pairs of so-called virtual particles that individually exist for very short periods of time.

Q44.3 They have similar masses and occur with both positive and negative charges. There is a neutral pion but no neutral muon. Muons are leptons and are fundamental though unstable particles. Pions are mesons and are composed of one quark and one antiquark. Muons have spin $\frac{1}{2}$; pions have spin zero.

Q44.4 We live on the surface of a very massive object that because of its mass exerts a large enough force on objects around us to have very noticeable effects on their motion. The electrical force between a pair of fundamental particles that carry charge is much larger than the gravitational force they exert on each other. But electric charge is usually present in equal amounts of positive and negative charge and the electrical forces largely cancel out.

Q44.5 The π^0 is composed of quark-antiquark pairs. The quarks and antiquarks annihilate and their mass is converted entirely into the energy of the photons.

Q44.6 Electron decay into two photons violates conservation of charge and conservation of lepton number. The photons have zero charge and zero for all lepton numbers. Electron decay into two neutrinos also violates conservation of charge and conservation of lepton number. Neutrinos have zero charge and no pair of neutrinos has the same set of lepton numbers as an electron.

Q44.7 They both have spin $\frac{1}{2}$. Some of each are charged and some of each aren't. All baryons have mass. In the standard model the neutrino leptons have zero mass. The lightest baryons are heavier than the electron. Leptons are fundamental. Baryons are composite particles, composed of three quarks. Leptons have lepton numbers and baryons have baryon number and both these are separately conserved.

Q44.8 Both leptons and quarks are fundamental particles. Both have spin $\frac{1}{2}$. Leptons have integer Q/e; quarks have fractional Q/e. Leptons and quarks have different conserved quantities (lepton number for leptons and baryon number, strangeness, charm, bottomness and topness for quarks).

Q44.9 To obtain the quark content of an antiparticle, replace quarks by antiquarks and antiquarks by quarks in the quark composition of the particle. (a) The antineutron has quark content that consists of three antiquarks, $\bar{u}\bar{d}\bar{d}$. (b) This is different from the quark content of the neutron and the neutron is not its own antiparticle. (c) The antiparticle of the ψ has quark content $\bar{c}c$. This is the same as the quark content of the ψ and the ψ is its own antiparticle.

Q44.10 No. In the standard Big Bang model the universe was once in a small volume. Since then it has expanded and all points are moving away from all other points. There is no stationary center.

Q44.11 See the CAUTION statement in Section 44.6. The universe is thought to be infinite. It has no edges, so there is nothing "outside" of it and it isn't "expanding into" anything. The expansion of the universe simply means that the scale factor of the universe is increasing.

Q44.12 The cosmological principle says that the universe looks essentially the same from all observation points. If the universe had an edge it would look very different to an observer at the edge than it would to someone in the interior, far from the edge.

Q44.13 The cosmological principle requires that the universe look essentially the same from all observation points. H_0 must be constant in space so that in all directions galaxies appear to be receding from any observer anywhere in the universe. It can't be different at different places. But this does not prevent H_0 from changing in time. In fact, the most recent astronomical observations suggest that H_0 is now larger than it was in the past.