Onda armónica

$$u(x,t) = A\cos(\underset{\text{numero de onda}}{k}x - \overset{\text{frecuencia angular}}{\omega} t + \underset{\text{desfase}}{\phi})$$

amplitud (A)

λ, f, t

$\omega = 2\pi f \qquad v = \lambda f = \frac{\omega}{k}$

$\frac{2\pi}{k} \cdot \frac{\omega}{2\pi}$

$f = \frac{1}{T}$

Ej: Comprobar que $u(x,t) = G(x - \overset{cte}{v}t)$

es solución de $\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} G(x-vt) = G'(x-vt)\cdot\frac{\partial}{\partial x}(x-vt) = G'(x-vt)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(G'(x-vt)\right) = G''(x-vt)\cdot\frac{\partial}{\partial x}(x-vt) = G''(x-vt)$$

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(G(x-vt)\right) = G'(x-vt)\cdot\frac{\partial}{\partial t}(x-vt) = -v\,G'(x-vt)$$

$$\frac{\partial u^2}{\partial t^2} = \frac{\partial}{\partial t}\left(-v\,G'(x-vt)\right) = (-v)\,G''(x-vt)\cdot\frac{\partial}{\partial t}(x-vt) = \underbrace{(-v)^2}_{v^2} G''(x-vt)$$

$$G''(x-vt) = \frac{1}{v^2}\cdot v^2\, G''(x-vt)$$

Comprobar que $u(\vec{r},t) = G(\vec{k}\cdot\vec{r} - \overset{|\vec{k}|}{k}\overset{v}{v}t)$ es solución de

$$\nabla^2 u(\vec{r},t) = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

$k = cte = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$

$$\frac{\partial u}{\partial t} = G'(\vec{k}\cdot\vec{r} - kvt)(-kv) = \frac{\partial^2 u}{\partial t^2} = (\underbrace{-kv}_{kv})^2 G''(\vec{k}\vec{r} - kvt)$$

$$\nabla^2 u = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) G(k_x \cdot x + k_y y + k_z z - kvt)$$

$$= k_x^2 G''(\vec{k}\cdot\vec{r} - kvt) + k_y^2 G''(\vec{k}\cdot\vec{r} - kvt) + k_z^2 G''(\vec{k}\cdot\vec{r} - kvt)$$

$$= \underbrace{(k_x^2 + k_y^2 + k_z^2)}_{|\vec{k}|^2} G''(\vec{k}\cdot\vec{r} - kvt) \rightarrow k^2 G''(\vec{k}\cdot\vec{r} - kvt) = \frac{1}{v^2} k^2 v^2 \, G''(\vec{k}\cdot\vec{r} - \cancel{kvt})$$

$$\rightarrow \vec{\nabla} u \rightarrow ku$$

$$\nabla^2 u \rightarrow k \cdot k \cdot u = k^2 \cdot u$$

Otro ejemplo:

Una onda armónica sonora de longitud de onda $\lambda = 0{,}75\,[m]$ se propaga en el aire. Si la presión atmosférica es

$P_0 \simeq 10^5\,[N/m^2]$ y la densidad de aire es $\rho_0 \simeq 1{,}25\,[kg/m^3]$

¿frecuencia de onda?

→ coeficiente adiabático → grados de libertad

$$v = \sqrt{\frac{\gamma P_0}{\rho_0}} \quad ; \quad \gamma = 1 + \frac{2}{f} = \frac{f+2}{f} = \frac{5+2}{5} = \frac{7}{5}$$

$$v = \sqrt{\frac{7 \cdot 10^5\,[\frac{N}{m^2}]}{5 \cdot 1{,}25\,[\frac{kg}{m^3}]}} = \sqrt{\frac{112{,}000\,[kg\,\frac{m}{s^2} \cdot \frac{1}{m^2}]}{[\frac{kg}{m^3}]}}$$

→ con esto podemos calcular el eco

$$v = \sqrt{112.000\,[m^2/s^2]} = 3{,}3 \cdot 10^2\,[m/s]$$

→ usando $f = v/\lambda$

$$f = \frac{v}{\lambda} = \frac{3{,}3 \cdot 10^2}{0{,}75}\,\frac{[\frac{m}{s}]}{[m]} = 440\,[Hz]$$

Ecuaciones de Maxwell — Las ecuaciones de 120, pero con cambios

forma integral — forma diferencial

Ley de Gauss

$$\oint E \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \qquad \vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$$

Ley sin nombre

$$\oint \vec{B} \cdot dA = 0 \qquad \vec{\nabla} B = 0$$

no existen monopolos magnéticos

Ley Lenz-faraday

$$\oint E \cdot dl = -\frac{d}{dl}\Phi_B \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

Ley de ampere-maxwell

maxwell le puso este termino

$$\oint B \cdot dl = \mu_0 \cdot i_{enc} \qquad \vec{\nabla} \times B = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$\mu_0 \cdot I_D$ — corriente desplazamiento

$$\oint_{\partial V} E \cdot dA = \int_V \vec{\nabla}\vec{E} \cdot dv = \frac{Q_{enc}}{\varepsilon_0} = \int_V \frac{\rho\, dv}{\varepsilon_0}$$

$$\oint \vec{\nabla} \cdot \vec{E}\, dv = \int_V \frac{\rho}{\varepsilon_0} dv \rightarrow \vec{\nabla} \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\oint_{\partial V} B \cdot dA = \int_V \vec{\nabla} \cdot B \cdot dv = 0 \rightarrow \vec{\nabla} B = 0$$

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rotor

$$\oint_{\partial A} \vec{E} \cdot d\vec{l} = \int_A (\vec{\nabla} \times \vec{E}) \cdot dA = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\int_A \vec{B} \cdot d\vec{A}$$

$$= -\int_A \left(\frac{d}{dt}\vec{B}\right) \cdot d\vec{A}$$

$$\rightarrow \vec{\nabla} \times E = -\frac{d}{dt}\vec{B}(\vec{r}, t) = \frac{\partial}{\partial t}\vec{B}(\vec{r}, t)$$

derivada temporal — derivada parcial

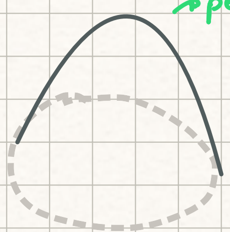
$$\oint_{\partial A} B\cdot dl = \int_A (\vec{\nabla}\times\vec{B})\, d\vec{A} = \mu_0 \, i_{enc}$$

$$\hookrightarrow \int_A \vec{J}\, dA$$

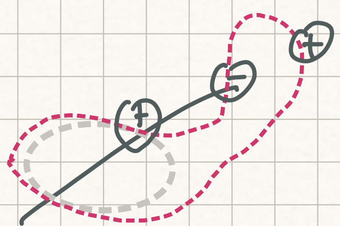
$$= \mu_0 \int_A \vec{J}\, d\vec{A}$$

$$\rightarrow \vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

Explicación del termino



→ pero el area puede ser cualquier area



pero maxwell se dio cuenta de que esto no pasaba en el capacitor