Lab06-Linear Programming

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2020.

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1. **Controlling Air Pollution.** The three main types of pollutants in an airshed are particulate matter, sulfur oxides, and hydrocarbons. The new standards require that the steelworks reduce its annual emission of these pollutants by the amounts shown in the following table:

Pollutant	**Required Reduction in Annual Emission Rate (Million Pounds)**
Particulates	60
Sulfur oxides	150
Hydrocarbons	125

The steelworks has two primary sources of pollution, namely, the blast furnaces for making pig iron and the open-hearth furnaces for changing iron into steel. In both cases the engineers have decided that the most effective types of abatement methods are (1) increasing the height of the smokestacks, (2) using filter devices (including gas traps) in the smokestacks, and (3) including cleaner, high-grade materials among the fuels for the furnaces. Note that each of these methods has a technological limit on how heavily it can be used (e.g., a maximum feasible increase in the height of the smokestacks), but there also is considerable flexibility for using the method at a fraction of its technological limit.

The following table shows how much emission (in millions of pounds per year) can be eliminated from each type of furnace by fully using any abatement method to its technological limit. For purposes of analysis, it is assumed that each method also can be used less fully to achieve any fraction of the emission-rate reductions shown in this table. Furthermore, the fractions can be different for blast furnaces and for open-hearth furnaces. For either type of furnace, the emission reduction achieved by each method is not substantially affected by whether the other methods also are used.

	Taller Smokestacks		**Filters**		**Better Fuels**	
Pollutant	**Blast Furnaces**	**Open-Hearth Furnaces**	**Blast Furnaces**	**Open-Hearth Furnaces**	**Blast Furnaces**	**Open-Hearth Furnaces**
Particulates	12	9	25	20	17	13
Sulfur oxides	35	42	18	31	56	49
Hydrocarbons	37	53	28	24	29	20

The total annual cost from the maximum feasible use of an abatement method (in millions of dollars) was shown in the following table. The board of directors wants to figure out how to achieve these reductions with minimum annual cost. Please design a scheme for them.

Abatement Method	**Blast Furnaces**	**Open-Health Furnaces**
Taller smokestacks	8	10
Filters	7	6
Better fuels	11	9

(a) Formulate a linear programming with necessary explanations.

(b) Transform your LP into its standard form.

(c) Transform your LP into its dual form.

(d) Assume that the clean air standards have been relaxed. The steelworks only needs to meet any two of the three pollutants emission standards. Please update your LP in (a) to satisfy the relaxed clean air standards. (Hint: You can refer to Reference14-ModelFormulation.pdf)

Solution. (a) There are 6 variables in this question: the taller smokestacks, filters and better fuels in blast furnace and Open-Health furnace respectively. And we define them as that int the following table:

Variable Definition	**Blast Furnaces**	**Open-Health Furnaces**
Taller smokestacks	x_1	x_2
Filters	x_3	x_4
Better fuels	x_5	x_6

□

then we can get the cost is:

$$cost = 8x_1 + 10x_2 + 7x_3 + 6x_4 + 11x_5 + 9x_6$$

whrere $x_i \in [0, 1]$, and the amount of three kind of pollutants they can reduce is:

$$\begin{cases} Particulates = 12x_1 + 9_x2 + 25x_3 + 20x_4 + 17x_5 + 13x_6 \\ SulfurOxides = 35x_1 + 42x_2 + 18x_3 + 31x_4 + 56x_5 + 49x_6 \\ Hydrocarbons = 37x_1 + 53x_2 + 28x_3 + 24x_4 + 29x_5 + 20x_6 \end{cases}$$

And the restriction should be::

$$\begin{cases} Particulates \geq 60 \\ SulfurOxides \geq 150 \\ Hydrocarbons \geq 125 \end{cases}$$

Consider the ablove restrictions, the linear programming can be described as:

$$\begin{aligned} \min \quad & 8x_1 + 10x_2 + 7x_3 + 6x_4 + 11x_5 + 9x_6 \\ \text{s.t.} \quad & x_i \in [0, 1] \ (i = 1, 2, 3, 4, 5, 6) \\ & 35x_1 + 42x_2 + 18x_3 + 31x_4 + 56x_5 + 49x_6 \geq 150 \\ & 37x_1 + 53x_2 + 28x_3 + 24x_4 + 29x_5 + 20x_6 \geq 125 \\ & 12x_1 + 9x_2 + 25x_3 + 20x_4 + 17x_5 + 13x_6 \geq 60 \end{aligned}$$

(b)The standard form of the linear programming above is:

$$\begin{aligned} \max \quad & -(8x_1 + 10x_2 + 7x_3 + 6x_4 + 11x_5 + 9x_6) \\ \text{s.t.} \quad & x_i \leq 1 \ (i = 1, 2, 3, 4, 5, 6) \\ & -35x_1 - 42x_2 - 18x_3 - 31x_4 - 56x_5 - 49x_6 \leq -150 \\ & -37x_1 - 53x_2 - 28x_3 - 24x_4 - 29x_5 - 20x_6 \leq -125 \\ & -12x_1 - 9x_2 - 25x_3 - 20x_4 - 17x_5 - 13x_6 \leq -60 \\ & x_i \geq 0 \ (i = 1, 2, 3, 4, 5, 6) \end{aligned}$$

(c) The dual-form should be:

$$
\begin{aligned}
\min \quad & y_1 + y_2 + y_3 + y_4 + y_5 + y_6 - 150y_7 - 125y_8 - 60y_9 \\
\text{s.t.} \quad & y_1 - 35y_7 - 37y_8 - 12y_9 \geq -8 \\
& y_2 - 42y_7 - 53y_8 - 9y_9 \geq -10 \\
& y_3 - 18y_7 - 28y_8 - 25y_9 \geq -7 \\
& y_4 - 31y_7 - 24y_8 - 20y_9 \geq -6 \\
& y_5 - 56y_7 - 29y_8 - 17y_9 \geq -11 \\
& y_6 - 49y_7 - 20y_8 - 13y_9 \geq -9 \\
& y_i \geq 0 \ (i = 1, 2, 3, 4, 5, 6)
\end{aligned}
$$

(d) In this case, we can interpret satisfing at least two limitations as satisfing exactly two limitations. Refer to Reference14 we can use M as a large number and introduce new binary variables y_1, y_2, y_3 to do the LP, so we can rewrite the LP in (a) as(not a standard form):

$$
\begin{aligned}
\min \quad & 8x_1 + 10x_2 + 7x_3 + 6x_4 + 11x_5 + 9x_6 \\
\text{s.t.} \quad & x_i \in [0, 1] \ (i = 1, 2, 3, 4, 5, 6) \\
& 35x_1 + 42x_2 + 18x_3 + 31x_4 + 56x_5 + 49x_6 \geq 150 - My_1 \\
& 37x_1 + 53x_2 + 28x_3 + 24x_4 + 29x_5 + 20x_6 \geq 125 - My_2 \\
& 12x_1 + 9x_2 + 25x_3 + 20x_4 + 17x_5 + 13x_6 \geq 60 - My_3 \\
& y_1 + y_2 + y_3 = 1 \\
& y_i \in 0, 1 (i = 1, 2, 3)
\end{aligned}
$$

2. **Factory Production.** An engineering factory makes seven products (PROD 1 to PROD 7) on the following machines: four grinders, two vertical drills, three horizontal drills, one borer and two planer. Each product yields a certain contribution to profit (in £/unit). These quantities (in £/unit) together with the unit production times (hours) required on each process are given below. A dash indicates that a product does not require a process.

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
Contribution to profit	10	6	8	4	11	9	3
Grinding	0.5	0.7	0.2	-	0.3	0.2	0.5
Vertical drilling	0.1	0.2	0	0.3	-	0.6	-
Horizontal drilling	0.2	-	0.8	-	-	-	0.6
Boring	0.05	0.03	-	0.07	0.1	-	0.08
Planing	-	-	0.01	-	0.05	0.02	0.04

There are marketing limitations on each product in each month, given in the following table:

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
January	500	1000	300	300	800	200	100
February	600	500	200	0	400	300	150
March	300	600	0	0	500	400	100
April	200	300	400	500	200	0	100
May	0	100	500	100	1000	300	0
June	500	500	100	300	1100	500	60

It is possible to store up to 100 of each product at a time at a cost of £0.5 per unit per month (charged at the end of each month according to the amount held at that time). There are no stocks at present, but it is desired to have a stock of exactly 50 of each type of product at the end of June. The factory works six days a week with two shifts of 8h each day. It may be assumed that each month consists of only 24 working days. Each machine must be down for maintenance in one month of the six. No sequencing problems need to be considered.

When and what should the factory make in order to maximize the total net profit?

(a) Use *CPLEX Optimization Studio* to solve this problem. Describe your model in *Optimization Programming Language* (OPL). Remember to use a separate data file (.dat) rather than embedding the data into the model file (.mod).

(b) Solve your model and give the following results.

   i. For each machine:

      A. the month for maintenance.

   ii. For each product:

      A. The amount to make in each month.

      B. The amount to sell in each month.

      C. The amount to hold at the end of each month.

   iii. The total selling profit.

   iv. The total holding cost.

   v. The total net profit (selling profit minus holding cost).

Remark: Include your .pdf, .tex, .oplproject, .project, .mod and .dat files for uploading.