

# Lab11-NP Reduction

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2020.

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1. What is the “certificate” and “certifier” for the following problems?
  - (a) *ZERO-ONE INTEGER PROGRAMMING*: Given an integer  $m \times n$  matrix  $A$  and an integer  $m$ -vector  $b$ , is there an integer  $n$ -vector  $x$  with elements in the set  $\{0, 1\}$  such that  $Ax \leq b$ .
  - (b) *SET PACKING*: Given a finite set  $U$ , a positive integer  $k$  and several subsets  $U_1, U_2, \dots, U_m$  of  $U$ , is there  $k$  or more subsets which are disjoint with each other?
  - (c) *STEINER TREE IN GRAPHS*: Given a graph  $G = (V, E)$ , a weight  $w(e) \in \mathbb{Z}_0^+$  for each  $e \in E$ , a subset  $R \subset V$ , and a positive integer bound  $B$ , is there a subtree of  $G$  that includes all the vertices of  $R$  and such that the sum of the weights of the edges in the subtree is no more than  $B$ .
2. Algorithm class is a democratic class. Denote class as a finite set  $S$  containing every students. Now students decided to raise a student union  $S' \subseteq S$  with  $|S'| \leq K$ .  
As for the members of the union, there are many different opinions. An opinion is a set  $S_o \subseteq S$ . Note that number of opinions has nothing to do with number of students.  
The question is whether there exists such student union  $S' \subseteq S$  with  $|S'| \leq K$ , that  $S'$  contains at least one element from each opinion. We call this problem *ELECTION* problem, prove that it is NP-complete.
3. Not-All-Equal Satisfiability (NAE-SAT) is an extension of SAT where every clause has at least one true literal and at least one false one. NAE-3-SAT is the special case where each clause has exactly 3 literals. Prove that NAE-3-SAT is NP-complete. (Hint : reduce 3-SAT to NAE- $k$ -SAT for some  $k > 3$  at first)
4. In the Lab10, we have introduced Minimum Constraint Data Retrieval Problem (MCDR). Prove that MCDR (Version 1 or 2) is NP-complete. (Hint : reduce from VERTEX-COVER or 3-SAT)

**Remark:** Please include your .pdf, .tex files for uploading with standard file names.