

Lab03-GreedyStrategy

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2020.

* If there is any problem, please contact TA Shuodian Yu.

* Name: _____ Student ID: _____ Email: _____

1. There are $n + 1$ people, each with two attributes $(a_i, b_i), i \in [0, n]$ and $a_i > 1$. The i -th person can get money worth $c_i = \frac{\prod_{j=0}^{i-1} a_j}{b_i}$. We do not want anyone to get too much. Thus, please design a strategy to sort people from 1 to n , such that the maximum earned money $c_{max} = \max_{1 \leq i \leq n} c_i$ is minimized. (Note: the 0-th person doesn't enroll in the sorting process, but a_0 always works for each c_i .)
 - (a) Please design an algorithm based on greedy strategy to solve the above problem. (Write a pseudocode)
 - (b) Prove your algorithm is optimal.
2. **Interval Scheduling** is a classic problem solved by greedy algorithm and we have introduced it in the lecture: given n jobs and the j -th job starts at s_j and finishes at f_j . Two jobs are compatible if they do not overlap. The goal is to find maximum subset of mutually compatible jobs. Tim wants to solve it by sort the jobs in descending order of s_j . Is this attempt correct? Prove the correctness of such idea, or else provide a counter-example.
3. There are n lectures numbered from 1 to n . Lecture i has duration (course length) t_i and will close on d_i -th day. That is, you could take lecture i **continuously** for t_i days and must finish before or on the d_i -th day. The goal is to find the maximal number of courses that can be taken. (Note: you will start learning at the 1-st day.)

Please design an algorithm based on greedy strategy to solve it. You could use the data structure learned on Data Structure course. You need to write pseudo code and prove its correctness.
4. Let S_1, S_2, \dots, S_n be a partition of S and k_1, k_2, \dots, k_n be positive integers. Let $\mathcal{I} = \{I : I \subseteq S, |I \cap S_i| \leq k_i \text{ for all } 1 \leq i \leq n\}$. Prove that $\mathcal{M} = (S, \mathcal{I})$ is a matroid.

Remark: You need to include your .pdf and .tex files in your uploaded .rar or .zip file.