A Hotelling-Downs Game for Strategic Candidacy with Binary Issues

AAMAS 2023

Javier Maass[1], Vincent Mousseau[2], and Anaëlle Wilczynski[2]

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June 2nd 2023

H-D game for strategic candidacy with binary issues

Strategic candidacy: candidates in elections can be strategic
$\rightarrow$ Decide to run for the election or not

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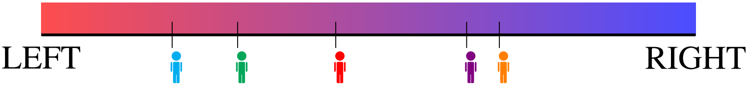

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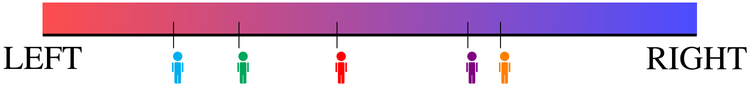

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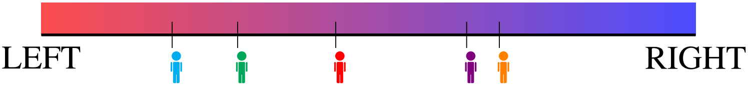

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Political opinions represented by "for" or "against" positions on given binary issues

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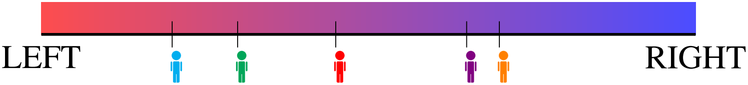

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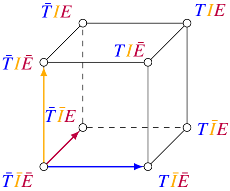
- higher taxes (T)
- immigration (I)
- euthanasia (E)
- raising the retirement age
- ...



Outline

Do the candidates have an incentive to deviate from their truthful opinion? How to model such a game?

Are there stable states? In which sense?

Under which conditions can we ensure such existence?

What about dynamics? Does this strategic process stabilize overtime?

The voting framework

- Set of binary issues $\rightarrow$ hypercube of possible *opinions*

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- Variant of the plurality voting rule
  - each voter gets a voting power equal to 1 and her vote is divided among the closest candidates
  - deterministic tie-breaking rule  $\triangleright$  $\triangleright \ldots$

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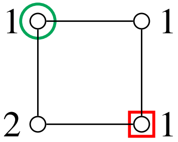

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Strategic Game

Payoff: Candidates' preferences

Each candidate strictly prefers herself over any other candidate

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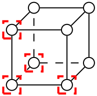
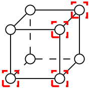
Candidates' strategies

A candidate may only be willing to announce a subset of all possible positions (containing the *truthful* position). We may further assume this set to be:

- A Ball of given radius around the truthful position

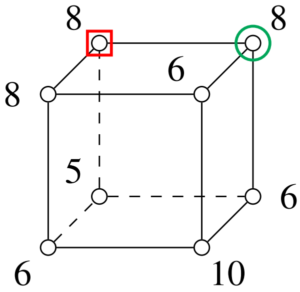


- Connected



Solution concepts

- Nash equilibrium (NE): stable state with respect to unilateral improving deviations from candidates



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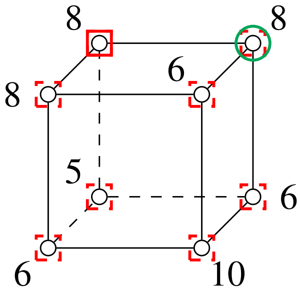

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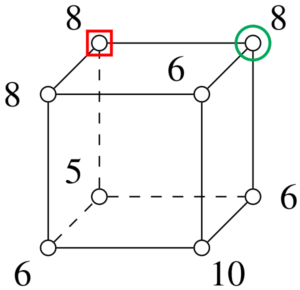

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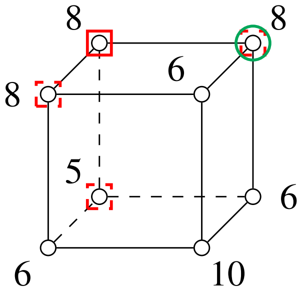

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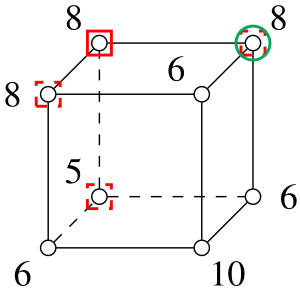
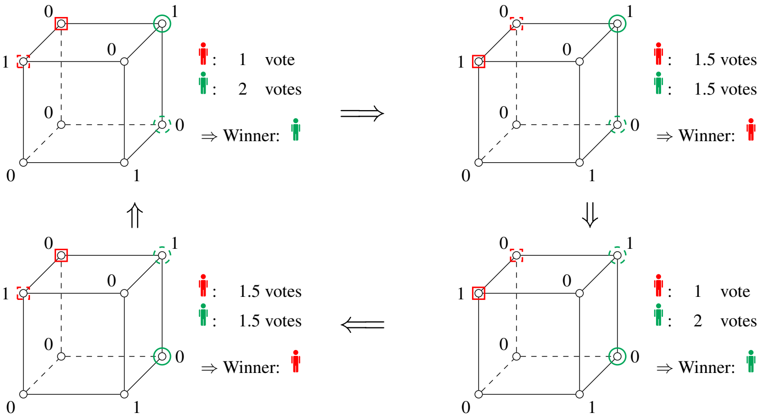


Nash equilibrium $\Leftrightarrow$ (# issues)-local equilibrium

Existence of a local equilibrium: negative result

In general, no guarantee of existence of a 1-LocEq even with 2 candidates and 3 issues



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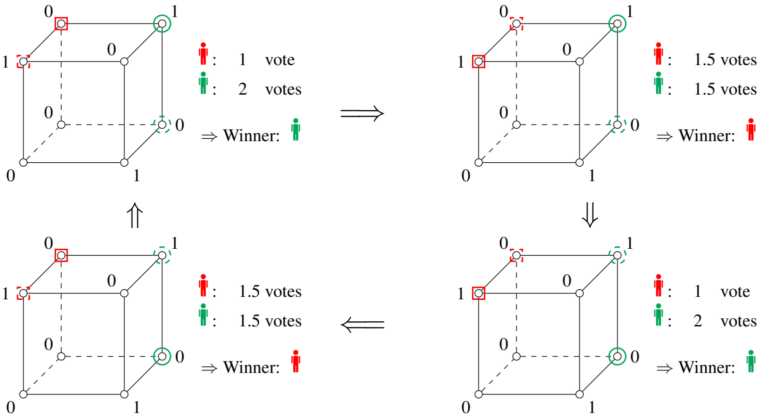
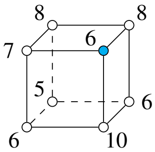


$\rightarrow$ Deciding about the existence of a t-LocEq is NP-hard, for all $t \geq 2$. What about $t = 1$? (open)

2 candidates: Unrestricted Voter Distribution

Local equilibrium and majoritarian outcome

Majoritarian outcome: the position that takes the majoritarian value on each issue separately



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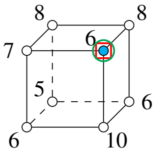

Guaranteed existence of a 2-LocEq for 2 candidates, odd n° of voters, and if they both can take the majoritarian outcome.

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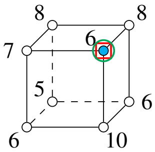

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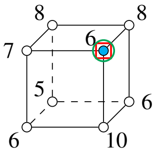

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Constraining the voter distribution

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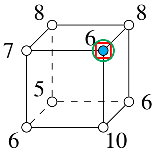

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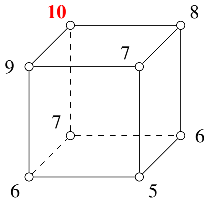
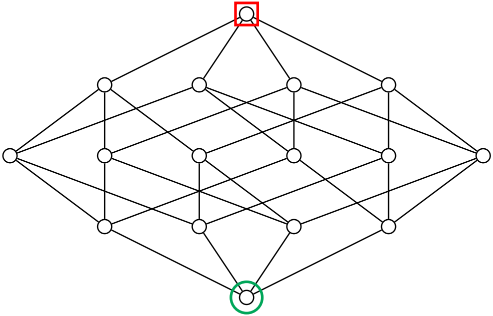


Single-peaked distribution: existence of a most popular opinion (peak position) such that, the more we walk away from it to a new position, the less voters share that opinion.

⇒ Particular case: uniform distribution

2 candidates: Geometric Structure of *Influence Sets*



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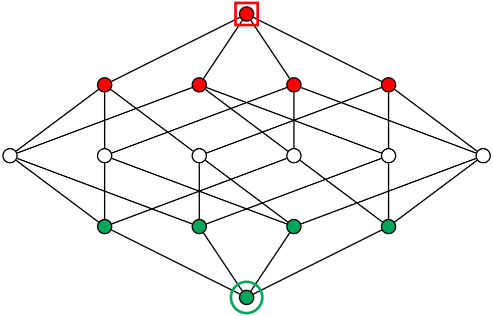

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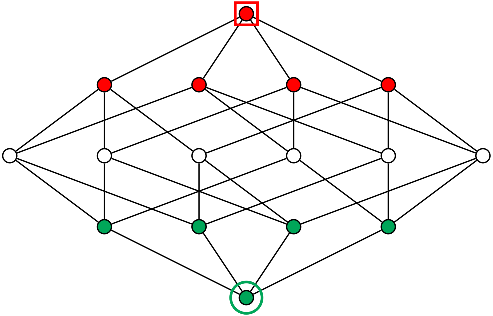
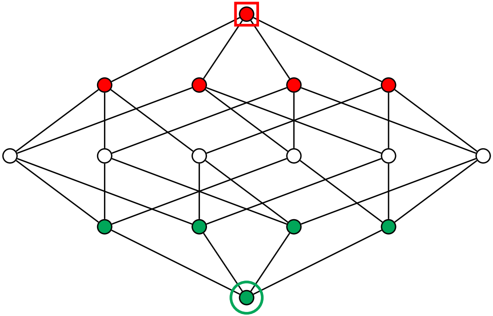


Guarantee of existence of a Nash equilibrium with 2 candidates and under a uniform distribution of voters

2 candidates: Equilibrium and single-peaked distribution

Guarantee of existence of a Nash equilibrium with 2 candidates, under a single-peaked distribution of voters, when the candidate favored by the tie-breaking can *take* the peak position.



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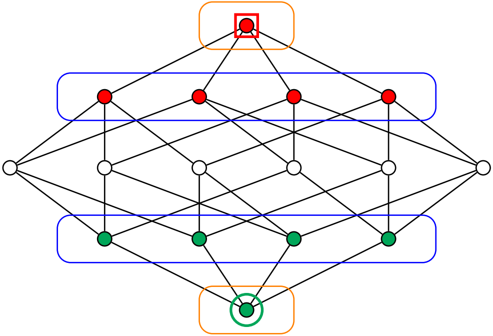

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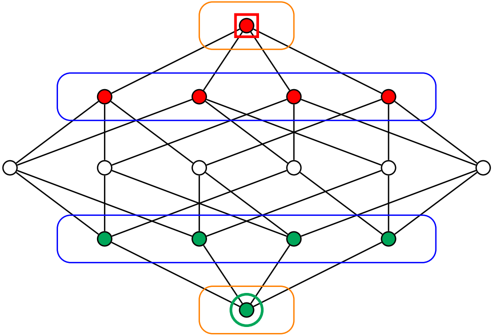

More candidates: No guarantee for a 1-LocEq with 3 candidates (of *fixed* preferences) and 2 issues, even under a uniform distribution.
Any guarantees under narcissistic preferences? (open)

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→ No guarantee of existence of a:

- 2-LocEq even when candidates' strategies are balls of radius 1 with 2 candidates and 3 issues
- 3-LocEq even for 2 candidates with the same strategies

Empirical Study

Simulations with synthetic data: 5.000 voters & random balls.

Insight from the experiments

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Insight from the experiments

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  - With 2 candidates, more than 40% of the states are 1-LocEq.
- **t-local Dynamics:** fixed preferences and iteration until reaching a t-local equilibrium (or finding a cycle).
  - Studied the average distance between the truthful winner's position and the reached winner's position:
    - For 1-local dynamics, we don't drift too far away!
    - As the game becomes more complex, it approaches $\frac{\#\textit{issues}}{2}$.

Conclusion

Summary

- Political spectrum $\rightarrow$ hypercube over issues
- Introduction of the local equilibrium concept
- Under the right assumptions, interesting positive results.
- Empirical results that balance the negative theoretical results
- Clear frontier between 1-local equilibria and the rest

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- Consideration of abstention?
- Correlation between positions?
- Strategic behavior from both voters and candidates?
- Increasing the score instead of a better winner?

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Related problems in the literature

- Strategic candidacy
  - strategic candidates that aim to get a better winner at the election
- Hotelling-Downs model
  - strategic positioning of selfish players on a spatial dimension
- Facility location problems
  - optimum location of a new facility w.r.t. a given set of customers
- Voronoi games
  - strategic positioning of players on a metric space
  - maximization of the amount of points that fall the closest to them

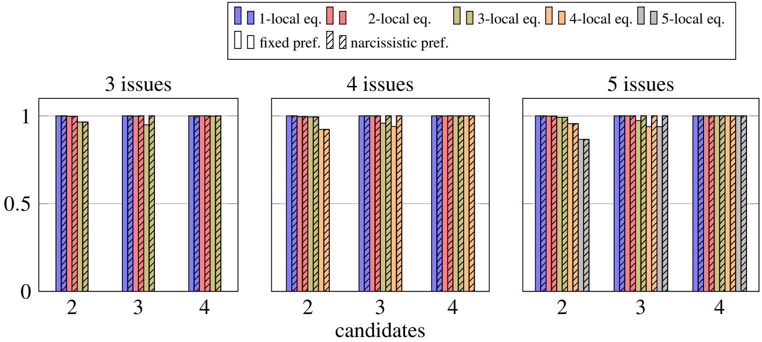
Sabato et al. Real candidacy games: A new model for strategic candidacy, AAMAS, 2017.

Harrenstein et al. A Hotelling-Downs Framework for Party Nominees, AAMAS 2021.

Feldman et al. Nash Equilibria for Voronoi Games on Transitive Graphs, ACM, 2009

Frequency of existence of t-local equilibria

Synthetic data: 5.000 voters, random balls



Proportion of local equilibria in average

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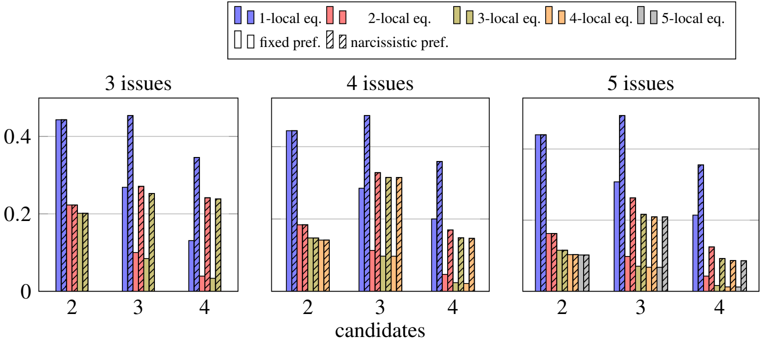

How far is the new winner after deviations?

Synthetic data: 5.000 voters, random balls, fixed preferences
Simulated t-local dynamics $\to$ t-local equilibrium
Average distance between the truthful winner's position and the reached winner's position