# Cuaderno de Prácticas

## MATEMÁTICAS DISCRETAS

## Grado en Ingeniería Informática

### **CURSO 2024/25**

## **Datos personales**

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DNI : 26268082 Grupo de teoría : A Grupo de prácticas : 1

#### **Archivos**

```
In[@]:=
      indice={
               {"Introducción", "Apartado 1", "Apartado 2", "Apartado 3"},
               {"Capítulo 1: El entorno de trabajo", "Generalidades sobre Mathematicas", "Interfaz de l
               {"Capítulo 2: Aritmética Básica. Variables y Funciones", "Operaciones aritméticas eleme
                   ,"Tipos de datos y números", "Diferentes precisiones en el cálculo"},
               {"Capítulo 3: Polinomio. Cálculos básicos y Divisibilidad",
                   "Lista y Representación y formato de una lista","La función Table" ,"Vectores y ma
               {"Capítulo 4: Programación en Mathematica",
                    "Expresiones lóghicas y Representaciones gráficas","Órdenes condicionales"
                   ,"Bucles y estructuras de control"},
               {"Capítulo 5: Lógica proposicional 1: Conectivas y tablas de verdad"
                   ,"Formas enunciativas y conectivas","Tablas de verdad"},
               {"Capítulo 6: Lógica proposicional 2: Tautologías, contradicciones, formas normales
                    , conjuntos adecuados de conectivas, equivalencias e implicaciones lógicas
                        y argumentaciones"
                    "Tautología y contradicción / Equivalencias lógicas e implicaciones lógicas",
                    "Formas normales y Argumentaciones válidas","Conjuntos adecuados de conectivas"},
               {"Capítulo 7: Conjuntos y aplicaciones (Partición de un conjunto,Aplicaciones)"
                   ,"Operaciones con conjuntos", "Producto cartesiano", "Partes de un conjunto"},
               {"Capítulo 8: Relaciones binarias y conjuntos ordenados", "Relaciones binarias",
                    "Relaciones de equivalencia","Relaciones de orden y Diagramas de orden o de Hasse"
       };
       TextGrid[indice,Frame→All,Spacings→{2,1}]
```

Out[@]=

Introducción	Apartado 1	Apartado 2	Apartado 3
Capítulo 1: El entorno de trabajo	Generalidades sobre Mathematicas	Interfaz de Usuario	
Capítulo 2: Aritmética Básica. Variables y Funciones	Operaciones aritméticas elementales	Tipos de datos y números	Diferentes precisiones en el cálculo
Capítulo 3: Polinomio. Cálculos básicos y Divisibilidad	Lista y Representación y formato de una lista	La función Table	Vectores y matrices
Capítulo 4: Programación en Mathematica	Expresiones lóghicas y Representaciones gráficas	Órdenes condicionales	Bucles y estructuras de control
Capítulo 5: Lógica proposicional 1: Conectivas y tablas de verdad	Formas enunciativas y conectivas	Tablas de verdad	
Capítulo 6: Lógica proposicional 2: Tautologías, contradicciones,form as normales	Tautología y contradicción / Equivalencias lógicas e implicaciones lógicas	Formas normales y Argumentaciones válidas	Conjuntos adecuados de conectivas
, conjuntos adecuados de conectivas, equivalencias e implicaciones lógicas			
y argumentaciones			
Capítulo 7: Conjuntos y aplicaciones (Partición de un conjunto,Aplicaciones )	Operaciones con conjuntos	Producto cartesiano	Partes de un conjunto
Capítulo 8: Relaciones binarias y conjuntos ordenados	Relaciones binarias	Relaciones de equivalencia	Relaciones de orden y Diagramas de orden o de Hasse

## Práctica I

### Ejercicio I.6

Texto del ejercicio 1

Solución:

### Ejercicio 2.1

Calcular con 5 y 10 cifras significativas:

a) 
$$3(1+4) - 2^2 \times 5 - 5^{1/5}$$

$$In[0]:= N[3(1+4)-2^2\times5-5^{1/5},10]$$

Out[0]=

-6.379729661

b) 
$$\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$$

$$In[*]:= N\left[\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{10}{10}\right]$$

Out[@]=

1.033333333

$$c) \frac{\sqrt{5}}{\sqrt[3]{3}}$$

$$In[\circ]:=$$
  $N\left[\frac{\sqrt{5}}{\sqrt[3]{3}},10\right]$ 

Out[0]=

1.550402942

 $i) 2^{1000}$ 

Out[0]=

 $\textbf{1.071508607} \times \textbf{10}^{\textbf{301}}$ 

## Práctica 2

Ejercicio 2.1

Out[0]=

7.389056099

e) Ln (Cos  $(\pi/3)$ )

In[\*]:= 
$$N\left[Log\left[Cos\left[\frac{\pi}{3}\right]\right],10\right]$$

Out[0]=

-0.6931471806

f) 
$$\left| \frac{1}{2} + \frac{1}{3} - \sqrt{2} \right|$$

$$In[o]:=$$
  $N\left[Abs\left[\frac{1}{2}+\frac{1}{3}-\sqrt{2}\right],10\right]$ 

Out[0]=

0.5808802290

g)  $Sen(\pi)+Tan(\pi)$ 

$$In[\circ]:=$$
 N[Sin[ $\pi$ ]+Tan[ $\pi$ ],10]

Out[0]=

0

h) ArcSin[0.5]-ArcCos[0.5]

Out[0]=

-0.523599

### Ejercicio 2.3

a) Comprobar si x es primo

Out[@]=

False

b) Calcular el cociente y el resto de dividir x entre y

Quotient[x,y] In[@]:=

Out[@]=

13 107

In[0]:= Mod[x,y]

Out[0]=

1654

c) Calcular una aproximación decimal con 20 cifras decimales de la raíz cuadrada de x

N[Sqrt[x],20] In[0]:=

Out[@]=

5125.2397017115209262

d) Calcular el entero más próximo al número (πy-e)/x

Round  $\left[\frac{\pi * y - e}{x}\right]$ In[@]:=

Out[0]=

e) Calcular el número de Fibonacci del día del mes en que naciste

Fibonacci[6] In[@]:=

Out[0]=

8

### Ejercicio 3.1

a) Formar una lista con todos los múltiplos de 11 positivos, menores que los dos últimos dígitos del año en que naciste

#### lista = Select[Range[11,2004,11],#<2004 &]

Out[0]=

In[0]:=

{11, 22, 33, 44, 55, 66, 77, 88, 99, 110, 121, 132, 143, 154, 165, 176, 187, 198, 209, 220, 231, 242, 253, 264, 275, 286, 297, 308, 319, 330, 341, 352, 363, 374, 385, 396, 407, 418, 429, 440, 451, 462, 473, 484, 495, 506, 517, 528, 539, 550, 561, 572, 583, 594, 605, 616, 627, 638, 649, 660, 671, 682, 693, 704, 715, 726, 737, 748, 759, 770, 781, 792, 803, 814, 825, 836, 847, 858, 869, 880, 891, 902, 913, 924, 935, 946, 957, 968, 979, 990, 1001, 1012, 1023, 1034, 1045, 1056, 1067, 1078, 1089, 1100, 1111, 1122, 1133, 1144, 1155, 1166, 1177, 1188, 1199, 1210, 1221, 1232, 1243, 1254, 1265, 1276, 1287, 1298, 1309, 1320, 1331, 1342, 1353, 1364, 1375, 1386, 1397, 1408, 1419, 1430, 1441, 1452, 1463, 1474, 1485, 1496, 1507, 1518, 1529, 1540, 1551, 1562, 1573, 1584, 1595, 1606, 1617, 1628, 1639, 1650, 1661, 1672, 1683, 1694, 1705, 1716, 1727, 1738, 1749, 1760, 1771, 1782, 1793, 1804, 1815, 1826, 1837, 1848, 1859, 1870, 1881, 1892, 1903, 1914, 1925, 1936, 1947, 1958, 1969, 1980, 1991, 2002}

#### In[ . ]:=

#### Table[i,{i,11,2004,11}]

Out[0]=

{11, 22, 33, 44, 55, 66, 77, 88, 99, 110, 121, 132, 143, 154, 165, 176, 187, 198, 209, 220, 231, 242, 253, 264, 275, 286, 297, 308, 319, 330, 341, 352, 363, 374, 385, 396, 407, 418, 429, 440, 451, 462, 473, 484, 495, 506, 517, 528, 539, 550, 561, 572, 583, 594, 605, 616, 627, 638, 649, 660, 671, 682, 693, 704, 715, 726, 737, 748, 759, 770, 781, 792, 803, 814, 825, 836, 847, 858, 869, 880, 891, 902, 913, 924, 935, 946, 957, 968, 979, 990, 1001, 1012, 1023, 1034, 1045, 1056, 1067, 1078, 1089, 1100, 1111, 1122, 1133, 1144, 1155, 1166, 1177, 1188, 1199, 1210, 1221, 1232, 1243, 1254, 1265, 1276, 1287, 1298, 1309, 1320, 1331, 1342, 1353, 1364, 1375, 1386, 1397, 1408, 1419, 1430, 1441, 1452, 1463, 1474, 1485, 1496, 1507, 1518, 1529, 1540, 1551, 1562, 1573, 1584, 1595, 1606, 1617, 1628, 1639, 1650, 1661, 1672, 1683, 1694, 1705, 1716, 1727, 1738, 1749, 1760, 1771, 1782, 1793, 1804, 1815, 1826, 1837, 1848, 1859, 1870, 1881, 1892, 1903, 1914, 1925, 1936, 1947, 1958, 1969, 1980, 1991, 2002}

#### In[0]:=

#### Range [11, 2004, 11]

Out[0]=

{11, 22, 33, 44, 55, 66, 77, 88, 99, 110, 121, 132, 143, 154, 165, 176, 187, 198, 209, 220, 231, 242, 253, 264, 275, 286, 297, 308, 319, 330, 341, 352, 363, 374, 385, 396, 407, 418, 429, 440, 451, 462, 473, 484, 495, 506, 517, 528, 539, 550, 561, 572, 583, 594, 605, 616, 627, 638, 649, 660, 671, 682, 693, 704, 715, 726, 737, 748, 759, 770, 781, 792, 803, 814, 825, 836, 847, 858, 869, 880, 891, 902, 913, 924, 935, 946, 957, 968, 979, 990, 1001, 1012, 1023, 1034, 1045, 1056, 1067, 1078, 1089, 1100, 1111, 1122, 1133, 1144, 1155, 1166, 1177, 1188, 1199, 1210, 1221, 1232, 1243, 1254, 1265, 1276, 1287, 1298, 1309, 1320, 1331, 1342, 1353, 1364, 1375, 1386, 1397, 1408, 1419, 1430, 1441, 1452, 1463, 1474, 1485, 1496, 1507, 1518, 1529, 1540, 1551, 1562, 1573, 1584, 1595, 1606, 1617, 1628, 1639, 1650, 1661, 1672, 1683, 1694, 1705, 1716, 1727, 1738, 1749, 1760, 1771, 1782, 1793, 1804, 1815, 1826, 1837, 1848, 1859, 1870, 1881, 1892, 1903, 1914, 1925, 1936, 1947, 1958, 1969, 1980, 1991, 2002}

```
Table[i 5,{i,1,5,1}]
In[ • ]:=
```

Out[0]=

{5, 10, 15, 20, 25}

In[0]:= {5,10,15,20,25}

Out[0]=

{5, 10, 15, 20, 25}

b) Calcular, utilizando el resultado del apartado anterior y las funciones de la tabla 3.1., los múltiplos de 11. entre 15 y 70.

Out[0]=

{22, 33, 44, 55, 66}

c) Unir a la lista obtenida en el apartado b), una nueva formada por los múltiplos de 5 entre 10 y 50, pero que en la tercera posición tenga el elemento  $\phi$ . ¿Cuántos elementos tiene la lista que acabamos de conseguir? ¿Cuáles son los elementos que se encuentran en primera, última y octava posición?

```
multiplosDe5 = Range[10,50,5]
In[0]:=
```

Out[0]=

{10, 15, 20, 25, 30, 35, 40, 45, 50}

 $multiplosDe5Con\phi = Insert[multiplosDe5, \phi, 3]$ In[0]:=

Out[0]=

 $\{10, 15, \phi, 20, 25, 30, 35, 40, 45, 50\}$ 

Unir listas

```
listaCombinada = Join[lista, multiplosDe5Conφ]
 In[0]:=
Out[0]=
       {11, 22, 33, 44, 55, 66, 77, 88, 99, 110, 121, 132, 143, 154, 165, 176, 187, 198,
        209, 220, 231, 242, 253, 264, 275, 286, 297, 308, 319, 330, 341, 352, 363, 374,
        385, 396, 407, 418, 429, 440, 451, 462, 473, 484, 495, 506, 517, 528, 539, 550,
        561, 572, 583, 594, 605, 616, 627, 638, 649, 660, 671, 682, 693, 704, 715, 726,
        737, 748, 759, 770, 781, 792, 803, 814, 825, 836, 847, 858, 869, 880, 891, 902,
        913, 924, 935, 946, 957, 968, 979, 990, 1001, 1012, 1023, 1034, 1045, 1056, 1067,
        1078, 1089, 1100, 1111, 1122, 1133, 1144, 1155, 1166, 1177, 1188, 1199, 1210,
        1221, 1232, 1243, 1254, 1265, 1276, 1287, 1298, 1309, 1320, 1331, 1342, 1353,
        1364, 1375, 1386, 1397, 1408, 1419, 1430, 1441, 1452, 1463, 1474, 1485, 1496,
        1507, 1518, 1529, 1540, 1551, 1562, 1573, 1584, 1595, 1606, 1617, 1628, 1639,
        1650, 1661, 1672, 1683, 1694, 1705, 1716, 1727, 1738, 1749, 1760, 1771, 1782,
        1793, 1804, 1815, 1826, 1837, 1848, 1859, 1870, 1881, 1892, 1903, 1914, 1925,
        1936, 1947, 1958, 1969, 1980, 1991, 2002, 10, 15, \phi, 20, 25, 30, 35, 40, 45, 50}
       Determinar el número total de elementos
        Length[listaCombinada]
Out[0]=
       192
```

Obtener la primera, octava y última posición

```
{First[listaCombinada],Last[listaCombinada],listaCombinada[[8]]}
 In[0]:=
Out[0]=
```

#### Ejercicio 3.2

{**11**, 50, 88}

Crear una tabla como en el ejercicio 3.4. cuya primera fila esté formada por los cinco primeros múltiplos positivos del día del mes en que naciste, la segunda fila por sus cubos y la tercera por la potencia quinta de dichos números

```
In[@]:=
         tabla=Table[(3*j)^i,{i,1,3},{j,1,5}]
Out[0]=
        \{\{3, 6, 9, 12, 15\}, \{9, 36, 81, 144, 225\}, \{27, 216, 729, 1728, 3375\}\}
```

### Práctica 3

De archivo de practicas el capitulo 4. Cada ejercicio hay que hacerlo con Do, For y While

```
p=1;
Do[p=p*i,{i,2006,2104,1}]
Do [If [Mod [i,17] == 0, p=p*i], {i,2006,2104,1}]
For [i=2006, i \le 2104, i=i+17, p=p*i]
For [i=2006, i \le 2104, i=i+17, If [Mod[i,17]=:0,p=p*i]]
```

Out[0]=

73 850 604 651 472 793 760

#### Ejercicio 4.3

```
d1d2 = 03;
In[@]:=
       m1m2 = 06;
       anyo = 2004;
       anyoPlusMes = anyo*(m1m2 + 6);
```

a) Construir un bucle que nos de todos los múltiplos de D1D2 comprendidos entre el año de tu nacimiento y el año de tu nacimiento multiplicado por M1M2 + 6.

```
listaAMultiplosFor={};
In[@]:=
       listaAMultiplosDo = {};
       listaAMultiplosWhile={};
       For[i=anyo,i≤anyoPlusMes,i++,
           If [Mod[i,d1d2] == 0,
               AppendTo[listaAMultiplosFor,i];
           ]
       1
       i = anyo;
       While[i ≤ anyoPlusMes,
         If[Mod[i, d1d2] = 0,
           AppendTo[listaAMultiplosWhile, i]
         ];
         i++;
       ];
       Do[
         If[Mod[i, d1d2] = 0,
           AppendTo[listaAMultiplosDo, i]
         ٦,
         {i, anyo, anyoPlusMes}
       If[listaAMultiplosFor === listaAMultiplosWhile === listaAMultiplosDo,
         Print[listaAMultiplosFor]
       ];
```

```
{2004, 2007, 2010, 2013, 2016, 2019, 2022, 2025, 2028, 2031, 2034, 2037, 2040, 2043, 2046,
2049, 2052, 2055, 2058, 2061, 2064, 2067, 2070, 2073, 2076, 2079, 2082, 2085, 2088, 2091,
2094, 2097, 2100, 2103, 2106, 2109, 2112, 2115, 2118, 2121, 2124, 2127, 2130, 2133, 2136,
2139, 2142, 2145, 2148, 2151, 2154, 2157, 2160, 2163, 2166, 2169, 2172, 2175, 2178, 2181,
2184, 2187, 2190, 2193, 2196, 2199, 2202, 2205, 2208, 2211, 2214, 2217, 2220, 2223, 2226,
2229, 2232, 2235, 2238, 2241, 2244, 2247, 2250, 2253, 2256, 2259, 2262, 2265, 2268, 2271,
```

```
2274, 2277, 2280, 2283, 2286, 2289, 2292, 2295, 2298, 2301, 2304, 2307, 2310, 2313, 2316,
2319, 2322, 2325, 2328, 2331, 2334, 2337, 2340, 2343, 2346, 2349, 2352, 2355, 2358, 2361,
2364, 2367, 2370, 2373, 2376, 2379, 2382, 2385, 2388, 2391, 2394, 2397, 2400, 2403, 2406,
2409, 2412, 2415, 2418, 2421, 2424, 2427, 2430, 2433, 2436, 2439, 2442, 2445, 2448, 2451,
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20055, 20058, 20061, 20064, 20067, 20070, 20073, 20076, 20079, 20082, 20085, 20088,
20091, 20094, 20097, 20100, 20103, 20106, 20109, 20112, 20115, 20118, 20121, 20124,
20127, 20130, 20133, 20136, 20139, 20142, 20145, 20148, 20151, 20154, 20157, 20160,
20163, 20166, 20169, 20172, 20175, 20178, 20181, 20184, 20187, 20190, 20193, 20196,
20199, 20202, 20205, 20208, 20211, 20214, 20217, 20220, 20223, 20226, 20229, 20232,
20235, 20238, 20241, 20244, 20247, 20250, 20253, 20256, 20259, 20262, 20265, 20268,
20271, 20274, 20277, 20280, 20283, 20286, 20289, 20292, 20295, 20298, 20301, 20304,
20307, 20310, 20313, 20316, 20319, 20322, 20325, 20328, 20331, 20334, 20337, 20340,
20343, 20346, 20349, 20352, 20355, 20358, 20361, 20364, 20367, 20370, 20373, 20376,
20 379, 20 382, 20 385, 20 388, 20 391, 20 394, 20 397, 20 400, 20 403, 20 406, 20 409, 20 412,
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20451, 20454, 20457, 20460, 20463, 20466, 20469, 20472, 20475, 20478, 20481, 20484,
20487, 20490, 20493, 20496, 20499, 20502, 20505, 20508, 20511, 20514, 20517, 20520,
20523, 20526, 20529, 20532, 20535, 20538, 20541, 20544, 20547, 20550, 20553, 20556,
20559, 20562, 20565, 20568, 20571, 20574, 20577, 20580, 20583, 20586, 20589, 20592,
20595, 20598, 20601, 20604, 20607, 20610, 20613, 20616, 20619, 20622, 20625, 20628,
20631, 20634, 20637, 20640, 20643, 20646, 20649, 20652, 20655, 20658, 20661, 20664,
20667, 20670, 20673, 20676, 20679, 20682, 20685, 20688, 20691, 20694, 20697, 20700,
20703, 20706, 20709, 20712, 20715, 20718, 20721, 20724, 20727, 20730, 20733, 20736,
20739, 20742, 20745, 20748, 20751, 20754, 20757, 20760, 20763, 20766, 20769, 20772,
20775, 20778, 20781, 20784, 20787, 20790, 20793, 20796, 20799, 20802, 20805, 20808,
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20847, 20850, 20853, 20856, 20859, 20862, 20865, 20868, 20871, 20874, 20877, 20880,
20883, 20886, 20889, 20892, 20895, 20898, 20901, 20904, 20907, 20910, 20913, 20916,
20919, 20922, 20925, 20928, 20931, 20934, 20937, 20940, 20943, 20946, 20949, 20952,
20955, 20958, 20961, 20964, 20967, 20970, 20973, 20976, 20979, 20982, 20985, 20988,
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21027, 21030, 21033, 21036, 21039, 21042, 21045, 21048, 21051, 21054, 21057, 21060,
21 063, 21 066, 21 069, 21 072, 21 075, 21 078, 21 081, 21 084, 21 087, 21 090, 21 093, 21 096,
21099, 21102, 21105, 21108, 21111, 21114, 21117, 21120, 21123, 21126, 21129, 21132,
21135, 21138, 21141, 21144, 21147, 21150, 21153, 21156, 21159, 21162, 21165, 21168,
21171, 21174, 21177, 21180, 21183, 21186, 21189, 21192, 21195, 21198, 21201, 21204,
21 207, 21 210, 21 213, 21 216, 21 219, 21 222, 21 225, 21 228, 21 231, 21 234, 21 237, 21 240,
21 243, 21 246, 21 249, 21 252, 21 255, 21 258, 21 261, 21 264, 21 267, 21 270, 21 273, 21 276,
21 279, 21 282, 21 285, 21 288, 21 291, 21 294, 21 297, 21 300, 21 303, 21 306, 21 309, 21 312,
21 315, 21 318, 21 321, 21 324, 21 327, 21 330, 21 333, 21 336, 21 339, 21 342, 21 345, 21 348,
21 351, 21 354, 21 357, 21 360, 21 363, 21 366, 21 369, 21 372, 21 375, 21 378, 21 381, 21 384,
21 387, 21 390, 21 393, 21 396, 21 399, 21 402, 21 405, 21 408, 21 411, 21 414, 21 417, 21 420,
21423, 21426, 21429, 21432, 21435, 21438, 21441, 21444, 21447, 21450, 21453, 21456,
21459, 21462, 21465, 21468, 21471, 21474, 21477, 21480, 21483, 21486, 21489, 21492,
21495, 21498, 21501, 21504, 21507, 21510, 21513, 21516, 21519, 21522, 21525, 21528,
21531, 21534, 21537, 21540, 21543, 21546, 21549, 21552, 21555, 21558, 21561, 21564,
21567, 21570, 21573, 21576, 21579, 21582, 21585, 21588, 21591, 21594, 21597, 21600,
21603, 21606, 21609, 21612, 21615, 21618, 21621, 21624, 21627, 21630, 21633, 21636,
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21675, 21678, 21681, 21684, 21687, 21690, 21693, 21696, 21699, 21702, 21705, 21708,
21711, 21714, 21717, 21720, 21723, 21726, 21729, 21732, 21735, 21738, 21741, 21744,
21747, 21750, 21753, 21756, 21759, 21762, 21765, 21768, 21771, 21774, 21777, 21780,
21783, 21786, 21789, 21792, 21795, 21798, 21801, 21804, 21807, 21810, 21813, 21816,
21819, 21822, 21825, 21828, 21831, 21834, 21837, 21840, 21843, 21846, 21849, 21852,
21855, 21858, 21861, 21864, 21867, 21870, 21873, 21876, 21879, 21882, 21885, 21888,
21891, 21894, 21897, 21900, 21903, 21906, 21909, 21912, 21915, 21918, 21921, 21924,
21927, 21930, 21933, 21936, 21939, 21942, 21945, 21948, 21951, 21954, 21957, 21960,
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21 999, 22 002, 22 005, 22 008, 22 011, 22 014, 22 017, 22 020, 22 023, 22 026, 22 029, 22 032,
22 035, 22 038, 22 041, 22 044, 22 047, 22 050, 22 053, 22 056, 22 059, 22 062, 22 065, 22 068,
22 071, 22 074, 22 077, 22 080, 22 083, 22 086, 22 089, 22 092, 22 095, 22 098, 22 101, 22 104,
22107, 22110, 22113, 22116, 22119, 22122, 22125, 22128, 22131, 22134, 22137, 22140,
22143, 22146, 22149, 22152, 22155, 22158, 22161, 22164, 22167, 22170, 22173, 22176,
22179, 22182, 22185, 22188, 22191, 22194, 22197, 22200, 22203, 22206, 22209, 22212,
22 215, 22 218, 22 221, 22 224, 22 227, 22 230, 22 233, 22 236, 22 239, 22 242, 22 245, 22 248,
22 251, 22 254, 22 257, 22 260, 22 263, 22 266, 22 269, 22 272, 22 275, 22 278, 22 281, 22 284,
22 287, 22 290, 22 293, 22 296, 22 299, 22 302, 22 305, 22 308, 22 311, 22 314, 22 317, 22 320,
22 323, 22 326, 22 329, 22 332, 22 335, 22 338, 22 341, 22 344, 22 347, 22 350, 22 353, 22 356,
22 359, 22 362, 22 365, 22 368, 22 371, 22 374, 22 377, 22 380, 22 383, 22 386, 22 389, 22 392,
22 395, 22 398, 22 401, 22 404, 22 407, 22 410, 22 413, 22 416, 22 419, 22 422, 22 425, 22 428,
22 431, 22 434, 22 437, 22 440, 22 443, 22 446, 22 449, 22 452, 22 455, 22 458, 22 461, 22 464,
22 467, 22 470, 22 473, 22 476, 22 479, 22 482, 22 485, 22 488, 22 491, 22 494, 22 497, 22 500,
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22 539, 22 542, 22 545, 22 548, 22 551, 22 554, 22 557, 22 560, 22 563, 22 566, 22 569, 22 572,
22575, 22578, 22581, 22584, 22587, 22590, 22593, 22596, 22599, 22602, 22605, 22608,
22611, 22614, 22617, 22620, 22623, 22626, 22629, 22632, 22635, 22638, 22641, 22644,
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22 683, 22 686, 22 689, 22 692, 22 695, 22 698, 22 701, 22 704, 22 707, 22 710, 22 713, 22 716,
22719, 22722, 22725, 22728, 22731, 22734, 22737, 22740, 22743, 22746, 22749, 22752,
22755, 22758, 22761, 22764, 22767, 22770, 22773, 22776, 22779, 22782, 22785, 22788,
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22827, 22830, 22833, 22836, 22839, 22842, 22845, 22848, 22851, 22854, 22857, 22860,
22863, 22866, 22869, 22872, 22875, 22878, 22881, 22884, 22887, 22890, 22893, 22896,
22 899, 22 902, 22 905, 22 908, 22 911, 22 914, 22 917, 22 920, 22 923, 22 926, 22 929, 22 932,
22 935, 22 938, 22 941, 22 944, 22 947, 22 950, 22 953, 22 956, 22 959, 22 962, 22 965, 22 968,
22 971, 22 974, 22 977, 22 980, 22 983, 22 986, 22 989, 22 992, 22 995, 22 998, 23 001, 23 004,
23 007, 23 010, 23 013, 23 016, 23 019, 23 022, 23 025, 23 028, 23 031, 23 034, 23 037, 23 040,
23 043, 23 046, 23 049, 23 052, 23 055, 23 058, 23 061, 23 064, 23 067, 23 070, 23 073, 23 076,
23 079, 23 082, 23 085, 23 088, 23 091, 23 094, 23 097, 23 100, 23 103, 23 106, 23 109, 23 112,
23115, 23118, 23121, 23124, 23127, 23130, 23133, 23136, 23139, 23142, 23145, 23148,
23151, 23154, 23157, 23160, 23163, 23166, 23169, 23172, 23175, 23178, 23181, 23184,
23187, 23190, 23193, 23196, 23199, 23202, 23205, 23208, 23211, 23214, 23217, 23220,
23 223, 23 226, 23 229, 23 232, 23 235, 23 238, 23 241, 23 244, 23 247, 23 250, 23 253, 23 256,
23 259, 23 262, 23 265, 23 268, 23 271, 23 274, 23 277, 23 280, 23 283, 23 286, 23 289, 23 292,
23 295, 23 298, 23 301, 23 304, 23 307, 23 310, 23 313, 23 316, 23 319, 23 322, 23 325, 23 328,
23 331, 23 334, 23 337, 23 340, 23 343, 23 346, 23 349, 23 352, 23 355, 23 358, 23 361, 23 364,
23 367, 23 370, 23 373, 23 376, 23 379, 23 382, 23 385, 23 388, 23 391, 23 394, 23 397, 23 400,
23 403, 23 406, 23 409, 23 412, 23 415, 23 418, 23 421, 23 424, 23 427, 23 430, 23 433, 23 436,
23 439, 23 442, 23 445, 23 448, 23 451, 23 454, 23 457, 23 460, 23 463, 23 466, 23 469, 23 472,
23 475, 23 478, 23 481, 23 484, 23 487, 23 490, 23 493, 23 496, 23 499, 23 502, 23 505, 23 508,
23511, 23514, 23517, 23520, 23523, 23526, 23529, 23532, 23535, 23538, 23541, 23544,
23547, 23550, 23553, 23556, 23559, 23562, 23565, 23568, 23571, 23574, 23577, 23580,
23583, 23586, 23589, 23592, 23595, 23598, 23601, 23604, 23607, 23610, 23613, 23616,
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23 655, 23 658, 23 661, 23 664, 23 667, 23 670, 23 673, 23 676, 23 679, 23 682, 23 685, 23 688,
23 691, 23 694, 23 697, 23 700, 23 703, 23 706, 23 709, 23 712, 23 715, 23 718, 23 721, 23 724,
23727, 23730, 23733, 23736, 23739, 23742, 23745, 23748, 23751, 23754, 23757, 23760,
23763, 23766, 23769, 23772, 23775, 23778, 23781, 23784, 23787, 23790, 23793, 23796,
23 799, 23 802, 23 805, 23 808, 23 811, 23 814, 23 817, 23 820, 23 823, 23 826, 23 829, 23 832,
23 835, 23 838, 23 841, 23 844, 23 847, 23 850, 23 853, 23 856, 23 859, 23 862, 23 865, 23 868,
23871, 23874, 23877, 23880, 23883, 23886, 23889, 23892, 23895, 23898, 23901, 23904,
23 907, 23 910, 23 913, 23 916, 23 919, 23 922, 23 925, 23 928, 23 931, 23 934, 23 937, 23 940,
23 943, 23 946, 23 949, 23 952, 23 955, 23 958, 23 961, 23 964, 23 967, 23 970, 23 973, 23 976,
23 979, 23 982, 23 985, 23 988, 23 991, 23 994, 23 997, 24 000, 24 003, 24 006, 24 009, 24 012,
24015, 24018, 24021, 24024, 24027, 24030, 24033, 24036, 24039, 24042, 24045, 24048}
```

b) Usar un bucle para crear una lista con los 25 primeros múltiplos de D1D2 + M1M2.

```
diaMes = (d1d2+m1m2);
In[@]:=
       listaBMultiplosFor={};
       listaBMultiplosWhile={};
       listaBMultiplosDo = {};
       For[i = 1, i \le 25, i++,
         AppendTo[listaBMultiplosFor, i * diaMes];
       ];
       i = 1;
       While [i \leq 25,
         AppendTo[listaBMultiplosWhile, i * diaMes];
       ];
       Do [
         AppendTo[listaBMultiplosDo, i * diaMes],
         {i, 1, 25}
       ];
       If[listaBMultiplosFor === listaBMultiplosWhile === listaBMultiplosDo,
         Print[listaBMultiplosFor]
       ];
```

```
{9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108,
117, 126, 135, 144, 153, 162, 171, 180, 189, 198, 207, 216, 225}
```

c) Calcular el producto de los múltiplos de D1D2 + M1M2 comprendidos entre A1A2A3A4 y A1A2A3A4 + 100

```
In[@]:=
      diaMes = (d1d2+m1m2);
       anyoMas100=(anyo+100);
       listaCMultiplosFor=1;
       listaCMultiplosWhile=1;
       listaCMultiplosDo = 1;
       For[i = anyo, i ≤ anyoMas100, i++,
        If[Mod[i, diaMes] == 0,
           listaCMultiplosFor *= i;
        ]
       ];
       i = anyo;
       While[i ≤ anyoMas100,
         If[Mod[i, diaMes] == 0,
           listaCMultiplosWhile *= i;
        ] \times
        i++;
       ];
       Do [
         If[Mod[i, diaMes] == 0,
           listaCMultiplosDo *= i;
        ],
        {i, anyo, anyoMas100}
       ];
      If[listaCMultiplosFor === listaCMultiplosWhile === listaCMultiplosDo,
         Print[listaCMultiplosFor]
      ];
```

#### 2 713 259 615 850 273 646 479 903 734 601 216 000

d) Calcular la suma de los múltiplos de D1D2 + M1M2 + 10 comprendidos entre A1A2A3A4 y  $(A1A2A3A4)^2$ .

```
diaMesMas10=(d1d2+m1m2+10);
In[ • ]:=
       anyoElev2=(anyo^2);
       sumaMultiplosFor=0;
       sumaMultiplosWhile=0;
       sumaMultiplosDo=0;
       For[i = anyo, i ≤ anyoElev2, i++,
         If[Mod[i, diaMesMas10] == 0,
           sumaMultiplosFor += i;
         ]
       ];
       i = anyo;
       While[i ≤ anyoElev2,
         If[Mod[i, diaMesMas10] == 0,
           sumaMultiplosWhile += i;
         ] \times
         i++;
       ];
       Do [
         If[Mod[i, diaMesMas10] == 0,
           sumaMultiplosDo += i;
         ],
         {i, anyo, anyoElev2}
       ];
       If[sumaMultiplosFor === sumaMultiplosWhile === sumaMultiplosDo,
         Print[sumaMultiplosFor]
       ];
```

424 432 016 800

## Práctica 4

### Ejercicio 5.1

Determina según tu DNI el valor de verdad de la siguiente forma enunciativa:

```
dni=26268082;
In[@]:=
       termina = Mod[dni,10];
       Mod[dni,2] == 0 && Mod[dni,3] == 0 && (termina=1 ||termina=7 ||termina=3)
```

False

Out[0]=

#### Ejercicio 5.2

Evaluar las siguientes formas enunciativas en las combinaciones de valores de verdad indicadas

```
a) ¬X1; X1=V
```

```
xa=True;
 In[@]:=
         TrueQ[Not[xa]]
Out[0]=
        False
        b) x1⇔x2;x1=V,x2=F
 In[*]:= xb1=True;
         xb2=False;
         TrueQ[Equivalent[xb1,xb2]]
Out[0]=
        False
        c) ((\neg x1) \land x2) \Rightarrow x3; x1=V, x2=F, x3=V
 In[@]:= xc1=xc3=True;
         xc2=False;
         TrueQ[Implies[And[Not[xc1],xc2],xc3]]
Out[@]=
        True
        d) (x1∨x2)⇔(¬x2);x1=V,x2=v
 In[ • ]:=
         xd1=xd2=True;
         TrueQ[Equivalent[Or[xd1,xd1],Not[xd2]]]
Out[0]=
        False
        e)[(x1\nabla x2)\pi(\neg x3)]\Rightarrow(\neg x3);x1=V,x2=F,x3=V
 In[@]:= xe1=xe3=True;
         xe2=False;
         TrueQ[Implies[Nand[Nor[xe1,xe2],Not[xe3]],Not[xe3]]]
Out[0]=
        False
        f(\neg(x1\land x2))\Rightarrow x3)(x4\lor x5);x5=F y tomando el resto de variables cualesquiera valores.
         xf1=xf2=xf3=xf4=xf5=False;
 In[@]:=
         TrueQ[Nand[Not[Implies[Not[And[xf1,xf2]],xf3]],Or[xf4,xf5]]]
Out[0]=
        True
```

### Ejercicio 5.6

Calcular las tablas de verdad de las siguientes formas enunciativas (e,f,j,k):

```
TablaVerdad[FormaE_,nombres_]:=Module[{p,n,j,f,resto},
In[0]:=
       n=Length[nombres];
       p=Table[False,{t,n}];
       tabla=Table["F",{x,2^n},{y,n+1}];
       Do[j=i;
       For [f=n,f>0,f--,resto=Mod[j,2];
       j=Floor[j/2];
       If[resto==0,p[f]]=True;
       tabla[i+1,f]="V",p[f]=False];];
       If[FormaE[p],tabla[i+1,n+1]="V"];
          ,{i,0,2^n-1}];
       Grid[Join[{Join[nombres, {FormaE[nombres]}]},tabla],Dividers→{Join[{True},Table[False,{i,2,n}
       ];
```

e)  $((q \lor r) \Rightarrow ((\neg r) \Rightarrow q))$ 

```
fe[{q_, r_}] := Implies[Or[q, r], Implies[Not[r], q]]
TablaVerdad[fe, {"q", "r"}]
```

Out[0]=

qr	$q \mid \mid r \Rightarrow (! r \Rightarrow q)$
VV	V
V F	V
F V	V
FF	V

```
f)(((\neg p)\Rightarrow q)\Rightarrow(((\neg s)\Rightarrow (\neg q))\Rightarrow p))
```

```
ff[{p_,q_,s_}]:=Implies[Implies[Not[p], q], Implies[Implies[Not[s], Not[q]], p]]
In[ • ]:=
       TablaVerdad[ff, {"p", "q", "s"}]
```

Out[0]=

pqs	$(! p \Rightarrow q) \Rightarrow ((! s \Rightarrow ! q) \Rightarrow p)$
VVV	V
VVF	V
VFV	V
VFF	V
F V V	F
FVF	V
FFV	V
FFF	V

```
j)\neg (\neg ((p {\Rightarrow} \neg (q {\Rightarrow} r)) {\Rightarrow} \neg ((p \overline{\wedge} q) \overline{\vee} r)))
```

```
\label{fields}  fj[\{p\_,q\_,r\_\}] := Not[Not[Implies[Implies[p,Not[Implies[q,r]]],Not[Nor[Nand[p,q],r]]]]] 
TablaVerdad[fj,{"p","q","r"}]
```

Out[0]=

pqr	$(p \Rightarrow ! (q \Rightarrow r)) \Rightarrow ! ((p \land q) \nabla r)$
VVV	V
VVF	F
VFV	V
VFF	V
FVV	V
FVF	V
FFV	V
FFF	V

 $k)(q \Rightarrow p) \Rightarrow (q \Leftrightarrow r)$ 

```
fk[{q_,p_,r_}]:=Implies[Implies[q,p],Equivalent[q,r]]
TablaVerdad[fk,{"p","q","s"}]
```

Out[0]=

p	q	S	$(p \Rightarrow q) \Rightarrow p \Leftrightarrow s$
٧	٧	٧	V
٧	٧	F	F
٧	F	٧	V
٧	F	F	V
F	٧	٧	F
F	٧	F	V
F	F	٧	F
F	F	F	V

## Práctica 5

Copiar la manera de hacer tautología del libro tema 6 Forma enunciativa restringida, tomar apuntes de eso

### Ejercicio 6.6

Estudiar si los siguientes pares de formas enunciativas son lógicamente equivalentes o implican lógicamente una a la otra:

```
Tautologia[FormaE_,n_]:=Module[{p,j,f,resto},
tautologia=True;
p=Table[False,{t,n}];
Do[j=i;
For [f=n,f>0,f--,resto=Mod[j,2];
j=Floor[j/2];
If[resto==0,p[f]]=True,p[f]]=False];];
If[FormaE[p],Null,tautologia=False;Break[];];
,{i,0,2^n-1}];
Return[tautologia];];
```

```
a) (p \Leftrightarrow q) \nabla r, (r \wedge (\neg q)) \Rightarrow p
       Equivalencia[{p_,q_,r_}]:=Equivalent[Nor[Equivalent[p,q],r],Implies[And[r,Not[q]],p]];
       ImplicaAB[{p_,q_,r_}]:=Implies[Nor[Equivalent[p,q],r],Implies[And[r,Not[q]],p]]
       ImplicaBA[{p_,q_,r_}]:=Implies[Implies[And[r,Not[q]],p],Nor[Equivalent[p,q],r]]
       If[Tautologia[Equivalencia,3],
            Print["Son lógicamente equivalentes"],
            Print["No son lógicamente equivalentes"]
       ]
       If[Tautologia[ImplicaAB,3],
            Print["A implicaca lógicamente B"],
            Print["A no implicaca lógicamente B"]
       If[Tautologia[ImplicaBA,3],
            Print["B implicaca lógicamente A"],
            Print["B no implicaca lógicamente A"]
       ]
      No son lógicamente equivalentes
      A implicaca lógicamente B
      B no implicaca lógicamente A
      b)(p \Rightarrow (\neg q)) \land r, \neg ((p \land q) \lor r)
       Equivalencia[{p_,q_,r_}]:=Equivalent[Nand[Implies[p,Not[q]],r],Not[Or[And[p,q],r]]];
In[@]:=
       ImplicaAB[{p_,q_,r_}]:=Implies[Nand[Implies[p,Not[q]],r],Not[Or[And[p,q],r]]]
       ImplicaBA \ [\{p\_,q\_,r\_\}] := Implies \ [Not \ [Or \ [And \ [p,q],r]], Nand \ [Implies \ [p,Not \ [q]],r]]
       If[Tautologia[Equivalencia,3],
            Print["Son lógicamente equivalentes"],
            Print["No son lógicamente equivalentes"]
       If[Tautologia[ImplicaAB,3],
            Print["A implicaca lógicamente B"],
            Print["A no implicaca lógicamente B"]
       If[Tautologia[ImplicaBA,3],
            Print["B implicaca lógicamente A"],
            Print["B no implicaca lógicamente A"]
       ]
```

```
No son lógicamente equivalentes
A no implicaca lógicamente B
B implicaca lógicamente A
c)(p\Rightarrow r)\wedge(r\nabla(\neg q)),(p\nabla r)\wedge q
```

```
Equivalencia[{p_,q_,r_}]:=Equivalent[And[Implies[p,r],Nor[r,Not[q]]],And[Nor[p,r],q]];
In[@]:=
       ImplicaAB[{p_,q_,r_}]:=Implies[And[Implies[p,r],Nor[r,Not[q]]],And[Nor[p,r],q]]
       ImplicaBA \ [\{p_,q_,r_\}] := Implies \ [And \ [Nor[p,r],q], And \ [Implies[p,r], Nor[r, Not[q]]]]
       If[Tautologia[Equivalencia,3],
           Print["Son lógicamente equivalentes"],
           Print["No son lógicamente equivalentes"]
       If[Tautologia[ImplicaAB,3],
           Print["A implicaca lógicamente B"],
           Print["A no implicaca lógicamente B"]
       If[Tautologia[ImplicaBA,3],
           Print["B implicaca lógicamente A"],
           Print["B no implicaca lógicamente A"]
       ]
```

Son lógicamente equivalentes A implicaca lógicamente B B implicaca lógicamente A

#### Ejercicio 6.15

Dadas las siguientes formas enunciativas:

B) And[Implies[p,q],Nor[r,Not[q]]] C) Nor[And[Not[p],q],Or[r,s]] A) Nand[p, Implies[q, r]]

```
TablaVerdad[FormaE_,nombres_]:=Module[{p,n,j,f,resto},
In[ • ]:=
       n=Length[nombres];
       p=Table[False, {t,n}];
       tabla=Table["F",{x,2^n},{y,n+1}];
       Do[j=i;
       For [f=n,f>0,f--,resto=Mod[j,2];
       j=Floor[j/2];
       If[resto==0,p[f]]=True;
       tabla[i+1,f]="V",p[f]=False];];
       If[FormaE[p],tabla[i+1,n+1]="V"];
          ,{i,0,2^n-1}];
       Grid[Join[{Join[nombres,{FormaE[nombres]}]},tabla],Dividers→{Join[{True},Table[False,{i,2,n}
```

```
A[{p_,q_,r_}]:=Nand[p,Equivalent[q,r]];
B[{p_,q_,r_}]:=And[Implies[p,q],Nor[r,Not[q]]];
c[{p_,q_,r_,s_}]:=Nor[And[Not[p],q],Or[r,s]];
```

i) Calcular sus tablas de verdad

```
In[@]:=
       TablaVerdad[A, {"p", "q", "r"}]
```

Out[@]=

pqr	$p \pi (q \Leftrightarrow r)$
V V V	F
VVF	V
VFV	V
VFF	F
F V V	V
FVF	V
FFV	V
FFF	V

```
In[@]:=
       TablaVerdad[B,{"p","q","r"}]
```

Out[0]=

pqr	(p ⇒ q) && (r ▽ ! q)
V V V	F
VVF	V
VFV	F
VFF	F
F V V	F
FVF	V
FFV	F
FFF	F

```
TablaVerdad[c,{"p","q","r","s"}]
In[@]:=
```

Out[0]=

pqrs	(! p && q) ▽ (r     s)
VVVV	F
VVVF	F
VVFV	F
VVFF	V
V F V V	F
VFVF	F
VFFV	F
VFFF	V
FVVV	F
FVVF	F
FVFV	F
FVFF	F
FFVV	F
FFVF	F
FFFV	F
FFFF	V

ii) Calcular sus formas normales.

```
FormasNormales[FormaE_,nombres_]:=Module[{cadena,cadena2,n,cad,cad2,j,f,resto},
n=Length[nombres];
cadena="";
cadena2="";
cad="";
```

```
cad2="";
contradiccion=True;
tautologia=True;
p=Table[False,{t,n}];
Do [
j=i;
cad="";
cad2="";
For[f=n,f>0,f--,
resto=Mod[j,2];
j=Floor[j/2];
If[resto==0,p[f]]=True;
If[f≕n,
cad=StringJoin[ToString[nombres[f]]],cad],
cad=StringJoin[ToString[nombres[f]]," \ ", cad]
];
If[f≕n,
cad2=StringJoin["(~",ToString[nombres[f]],")",cad2],cad2=StringJoin["(~",ToString[nombres[f]]
    ")"," ∨ ",cad2]
      ,p[f]=False;
If[f≕n,
cad=StringJoin["(~",ToString[nombres[f]],")",cad],cad=StringJoin["(~",ToString[nombres[f]]],
    ")"," ∧ ",cad]
];
If[f≕n,
cad2=StringJoin[ToString[nombres[f]]],cad2],
cad2=StringJoin[ToString[nombres[f]]]," \ ",cad2]
];
    ];
];
If[FormaE[p],
If[cadena≕"",
cadena=StringJoin[cadena,"(",cad,")"],
cadena=StringJoin[cadena," \ (",cad,")"]
contradiccion=False;,
If[cadena2≕"",
cadena2=StringJoin[cadena2,"(",cad2,")"],
cadena2=StringJoin[cadena2," \( (",cad2,")"]
tautologia=False;
   ,{i,0,2^n-1}];
If[contradiccion,
Print["Es una contradicción."],
Print["No es contradicción y la forma normal disyuntiva es: ",cadena]];
If[tautologia,
Print["Es una tautología."],
Print["No es tautología y la forma normal conjuntiva es: ",cadena2]];
];
```

```
FormasNormales[A, {"p", "q", "r"}]
  In[0]:=
           No es contradicción y la forma normal disyuntiva es:
              (p \ \land \ q \ \land \ (\sim r) \ ) \ \lor \ (p \ \land \ (\sim q) \ \land \ r) \ \lor \ ((\sim p) \ \land \ q \ \land \ r) \ \lor
                 (\;(\sim\!p)\;\;\wedge\;\;q\;\;\wedge\;\;(\sim\!r)\;)\;\;\vee\;\;(\;(\sim\!p)\;\;\wedge\;\;(\sim\!q)\;\;\wedge\;\;r)\;\;\vee\;\;(\;(\sim\!p)\;\;\wedge\;\;(\sim\!q)\;\;\wedge\;\;(\sim\!r)\;)
           No es tautología y la forma normal conjuntiva es: ((\sim p) \lor (\sim q) \lor (\sim r)) \land ((\sim p) \lor q \lor r)
             FormasNormales[B,{"p","q","r"}]
  In[@]:=
           No es contradicción y la forma normal disyuntiva es: (p \land q \land (\sim r)) \lor ((\sim p) \land q \land (\sim r))
           No es tautología y la forma normal conjuntiva es:
              (\;(\sim\!p)\;\;\vee\;\;(\sim\!q)\;\;\vee\;\;(\sim\!r)\;)\;\;\wedge\;\;(\;(\sim\!p)\;\;\vee\;\;q\;\;\vee\;\;(\sim\!r)\;)\;\;\wedge\;\;(\;(\sim\!p)
                 \lor q \lor r) \land (p \lor (\sim q) \lor (\sim r)) \land (p \lor q \lor (\sim r)) \land (p \lor q \lor r)
  In[@]:= FormasNormales[c,{"p","q","r";"s"}]
            Es una contradicción.
            Es una tautología.
            iii) Buscar formas enunciativas lógicamente equivalentes utilizando los siguientes conjuntos
            adecuados \{\neg, \land\}, \{\neg, \rightarrow\} y \{\nabla\}. Comprobar el resultado con Mathematica.
             BooleanConvert[Nand["p",Equivalent["q","r"]],"AND"]
  In[@]:=
              BooleanConvert[Nand["p",Equivalent["q","r"]],"IMPLIES"]
              Clear[p,q,r,ExpNor]
              ExpNor=BooleanConvert[Nand["p",Equivalent["q","r"]],"NOR"]
              A=ExpNor/. Not[x_1]:\rightarrow(x\nabla x);
Out[0]=
            ! (p && q && r) && ! (p && ! q && ! r)
Out[0]=
            p \Rightarrow ((q \Rightarrow r) \Rightarrow ! (! q \Rightarrow ! r))
Out[0]=
            (! p \triangledown ! q \triangledown ! r) \triangledown (! p \triangledown q \triangledown r)
Out[0]=
            (\ (p \, \overline{\lor} \, p) \, \overline{\lor} \, (q \, \overline{\lor} \, q) \, \overline{\lor} \, (r \, \overline{\lor} \, r) \, ) \, \overline{\lor} \, (\ (p \, \overline{\lor} \, p) \, \overline{\lor} \, q \, \overline{\lor} \, r)
```

```
In[0]:=
            BooleanConvert[And[Implies[p,q],Nor[r,Not[q]]],"AND"]
             BooleanConvert[And[Implies[p,q],Nor[r,Not[q]]],"IMPLIES"]
             Clear[p,q,r,ExpNor]
             ExpNor=BooleanConvert[And[Implies[p,q],Nor[r,Not[q]]],"NOR"]
             A=ExpNor/. Not[x_1] \Rightarrow (x \nabla x);
Out[0]=
           q &&! r
Out[0]=
           ! (q \Rightarrow r)
Out[0]=
           ! q ⊽ r
Out[0]=
           (q \overline{\vee} q) \overline{\vee} r
            BooleanConvert[Nor[And[Not[p],q],Or[r,s]],"AND"]
             BooleanConvert[Nor[And[Not[p],q],Or[r,s]],"IMPLIES"]
             Clear[p,q,r,ExpNor]
             ExpNor=BooleanConvert[Nor[And[Not[p],q],Or[r,s]],"NOR"]
             A=ExpNor/. Not[x_1] \Rightarrow (x \nabla x);
Out[0]=
           ! (!p&&q) &&!r&&!s
Out[0]=
           (p \Rightarrow (! r \Rightarrow s)) \Rightarrow ! (! p \Rightarrow (! q \Rightarrow (! r \Rightarrow s)))
Out[0]=
           (p \ \overline{\lor} \ ! \ q) \ \overline{\lor} \ r \ \overline{\lor} \ s
Out[0]=
           (p \mathbin{\overline{\vee}} (q \mathbin{\overline{\vee}} q)) \mathbin{\overline{\vee}} r \mathbin{\overline{\vee}} s
```

#### Ejercicio 6.17

Utiliza el método de refutación para determinar si la siguiente forma argumentativa es válida o inválida:

```
(p \lor q) \land (r \Rightarrow t), p \oplus (s \lor (\neg t)), r \land (\neg q); \therefore ((\neg p) \land q) \Rightarrow r
```

```
Nand[r,Not[q]]],Implies[And[Not[p],q],r]];
If[Tautologia[Argumentaciona,5],
  Print["Es una argumentación válida"],
  Print["Es una argumentación inválida"]
]
```

Es una argumentación inválida

## Práctica 6

#### Ejercicio 7.1. (g e i)

Sea X=AUBUZ, donde A es el conjunto formado por las letras distintas de tu primer apellido, B es el conjunto formado por los números distintos de tu DNI y Z es el conjunto formado por los números naturales impares menores que 10 junto con las vocales. Comprobar las siguientes propiedades:

```
A={"b","c","d","e","f","g","h","j","k","ñ","o","p","q","v","w","q","y","z"};
B=\{1,3,4,5,7,9\};
Z={1,3,5,7,9,"a","e","i","o","u"};
x=Union[a,b,z];
```

g) Distributivas:  $A \cup (B \cap Z) = (A \cup B) \cap (A \cup Z)$ 

```
Union[A,Intersection[A,Z]] == Intersection[Union[A,B],Union[A,Z]]
Intersection[A,Union[B,Z]] == Union[Intersection[A,B],Union[A,Z]]
```

i) Leyes de Morgan:

```
Complement[Union[A,Z]] == Intersection[Complement[A], Complement[Z]]
Complement[Intersection[A,Z]] == Union[Complement[A], Complement[Z]]
```

### Ejercicio 7.14.

```
In[0]:=
       A={"b","c"};
        B=\{1,3\};
```

a) AxB y AxAxB

```
cartesianoAB={};
Do[Do[AppendTo[cartesianoAB, {A[i],B[j]}], {i,1,Length[A]}];, {j,1,Length[B]}];
cartesianoAB
\label{lem:cartesiano} $$ CARTESIANO[conjuntos_]:=Module[\{i,j,k,cartesianotemp\},cartesiano=Table[\{conjuntos_1,i]\}, $$ is the conjunt of the
                       {i,Length[conjuntos[1]]}};
Do[cartesianotemp={};
Do[Do[AppendTo[cartesianotemp,Append[cartesiano[k],conjuntos[i,j]]]];,{k,Length[cartesiano]}]
                      ,{j,Length[conjuntos[i]]}
];
 cartesiano=cartesianotemp;,{i,2,Length[conjuntos]}];
cartesiano];
CARTESIANO[{A,A,B}]
```

```
Out[0]=
         \{\{b, 1\}, \{c, 1\}, \{b, 3\}, \{c, 3\}\}\
Out[0]=
         \{\{b, b, 1\}, \{c, b, 1\}, \{b, c, 1\}, \{c, c, 1\}, \{b, b, 3\}, \{c, b, 3\}, \{b, c, 3\}, \{c, c, 3\}\}
```

b) Una aplicación entre A y P (B) que sea inyectiva y otra P (B) y A que sea sobreyectiva

```
PartesB=Subsets[B];
 A={1,2}; (*GUARDAR DOMINIO EN A*)
 B=PartesB; (*GUARDAR CODOMINIO EN B*)
 For[i=1,i≤Length[A],i++,
     f[A[i]]]=B[i]
 Imagen={};
 Do[Imagen=Union[Imagen,Append[{},f[A[i]]]],{i,1,Length[A]}];
 Print["El conjunto imagen es: ",Imagen];
 If[Length[Imagen] == Length[B], Print["Es sobreyectiva"], Print["No es sobreyectiva"]];
 If[Length[A] == Length[Imagen], Print["Es inyectiva"], Print["No es inyectiva"]];
 If[Length[A] == Length[B] && Length[B] == Length[Imagen], Print["Es biyectiva"],
     Print["No es biyectiva"]];
El conjunto imagen es: {{}, {1}}
No es sobreyectiva
Es inyectiva
No es biyectiva
 PartesB=Subsets[B];
 For[i=1,i≤Length[PartesB],i++,
     f[PartesB[[i]]] = A[[Mod[i, Length[A]] + 1]]
     1
 A=Subsets[{a,b}]; (*GUARDAR DOMINIO EN A*)
 B={1,2}; (*GUARDAR CODOMINIO EN B*)
 Imagen={};
 Do[Imagen=Union[Imagen,Append[{},f[A[i]]]],{i,1,Length[A]}];
 Print["El conjunto imagen es: ",Imagen];
 If[Length[Imagen] =: Length[B], Print["Es sobreyectiva"], Print["No es sobreyectiva"]];
 If[Length[A] == Length[Imagen], Print["Es inyectiva"], Print["No es inyectiva"]];
 If[Length[A] == Length[B] && Length[B] == Length[Imagen], Print["Es biyectiva"],
     Print["No es biyectiva"]];
El conjunto imagen es: \{f[\{a\}], f[\{b\}], f[\{a, b\}], \{a\}\}\
No es sobreyectiva
Es inyectiva
No es biyectiva
```

c) Un aplicación no inyectiva de AxB en P (AxB)

```
cartesianoAB={};
Do[Do[AppendTo[cartesianoAB,{A[i],B[j]}],{i,1,Length[A]}];,{j,1,Length[B]}];
cartesianoAB;
PartesAB=Subsets[cartesianoAB];
DOM=cartesianoAB; (*GUARDAR DOMINIO EN A*)
COD=PartesAB; (*GUARDAR CODOMINIO EN B*)
For[i=1,i≤Length[DOM],i++,
    f[DOM[i]] = COD[[i]]
Imagen={};
Do[Imagen=Union[Imagen,Append[{},f[A[i]]]],{i,1,Length[A]}];
Print["El conjunto imagen es: ",Imagen];
If[Length[Imagen] == Length[B], Print["Es sobreyectiva"], Print["No es sobreyectiva"]];
If[Length[A] == Length[Imagen], Print["Es inyectiva"], Print["No es inyectiva"]];
If[Length[A] == Length[B] && Length[B] == Length[Imagen], Print["Es biyectiva"],
    Print["No es biyectiva"]];
```

```
El conjunto imagen es: {{}, {1}}
No es sobreyectiva
Es inyectiva
No es biyectiva
```

d) Una aplicación no sobreyectiva de P(AxB) en AxB

```
A = \{1,3\};
B={"c","b"};
DOM=Subsets[CARTESIANOAB[{A,B}]];
COD=CARTESIANOAB[{A,B}];
f[DOM[1]] = COD[[1]];
For[i=1,i≤Length[DOM],i++,
    f[DOM[i]] = COD[1]
    ]
Imagen={};
Do[Imagen=Union[Imagen,Append[{},f[DOM[i]]]],{i,1,Length[DOM]}];
Print["El conjunto imagen es: ",Imagen];
If[Length[Imagen] == Length[COD], Print["Es sobreyectiva"], Print["No es sobreyectiva"]];
If[Length[DOM] == Length[Imagen], Print["Es inyectiva"], Print["No es inyectiva"]];
If[Length[DOM] == Length[COD] && Length[COD] == Length[Imagen], Print["Es biyectiva"],
    Print["No es biyectiva"]];
```

```
El conjunto imagen es: \{\{1, c\}\}\
No es sobreyectiva
No es inyectiva
No es biyectiva
```