47. $\boxed{\text{2067 Supp Q.No. 9 d}}$ **A uniform steel wire of density 7800 kgm^{-3} weighs 16 gram and is 250 cm. It lengthens by 1.2 mm when a load of 8 kg is applied. Calculate the value Young's modulus for the steel and the energy stored in the wire.**

Please refer to $\boxed{\text{2070 Supp (Set A) Q.No. 9 c}}$

48. $\boxed{\text{2064 Q.No. 3 b}}$ **A steel wire of density 8000 kg m^{-3} weighs 24 gm and is 250 cm. long. It lengthens by 1.2 mm when stretched by a force of 80N. Calculate the Young's modulus for the steel and the energy stored in the wire.**

Given,

Density of steel (ρ) = 8000 kg m^{-3}

Mass of wire (m) = 24gm = 0.024 kg

Length of wire (l) = 250cm = 2.5 m

Elongation (e) = 1.2mm = 1.2×10^{-3} m

Load (F) = 80 N

Young's modulus of elasticity (Y) = ?

Energy stored (E) = ?

We have,

$$Y = \frac{\frac{F}{A}}{\frac{e}{l}} = \frac{F.l}{e.A} = \frac{F.l}{e.\frac{m}{l.\rho}} = \frac{F.l^2\rho}{e.m} \qquad [\because \rho = \frac{m}{v} = \frac{m}{A \times c} \Rightarrow A = \frac{m}{\rho \times c}]$$

$$= \frac{80 \times 2.5^2 \times 8000}{1.2 \times 10^{-3} \times 0.024} = 1.4 \times 10^{11} N/m^2$$

$$\therefore E = \frac{1}{2}.F.e = \frac{1}{2} \times 80 \times 1.2 \times 10^{-3} = 4.8 \times 10^{-2} J$$

□□□

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UNIT 2 Heat and Thermodynamics

Chapter 1: Heat and Temperature

Short Answer Questions

1. **2074 Supp. Q.No. 2a A student claimed that thermometers are useless because a thermometer always registers its own temperature. How would you respond?** [2]

When a thermometer is used to measure temperature of a body then there will be exchange of heat until the body and thermometer attain a common temperature. The student's claim is true that the thermometer registers its own temperature but the temperature registered is also the temperature of the body. Hence, the registered temperature is the temperature of thermometer as well as body.

2. **2074 Set B Q.No. 2a Express 98⁰F into Kelvin scale.** [2]

we have,

$$\frac{F-32}{180} = \frac{K-273}{100}$$

Given F = 98°F, then

$$\text{or, } \frac{98-32}{180} = \frac{K-273}{100}$$

$$\text{or, } \frac{98-32}{18} = \frac{K-273}{10}$$

or, 980 - 320 18K - 273 x 18

or, 660 = 18K - 4914

or, K = 310K

volume of a liquid and does not depend on the original volume.

3. **2069 Old (Set B) Q. No. 4a At what temperature will the Celsius scale reading double the Fahrenheit reading?** [2]

Let x be the temperature at which celsius scale doubles the Fahrenheit reading, then,

C = 2x.

Now,

$$\frac{C}{100} = \frac{F-32}{180}$$

$$\text{or, } \frac{2x}{100} = \frac{x-32}{180}$$

$$\text{or, } \frac{x}{50} = \frac{x-32}{180}$$

or, 180x = 50x – 1600

or, (180 – 50)x = – 1600

or, 130x = –1600

x = – 12.3°F

∴ At – 12.3°F, the celsius scale reading doubles the Fahrenheit reading.

4. **2068 Q.No. 2 a Define absolute temperature.** [2]

Absolute temperature: The temperature in Kelvin scale is absolute temperature. It's value is (273 + θ)°C, where θ can take any values.

Absolute zero temperature: The temperature at which the pressure or volume of any gas reduces to zero is called absolute zero temperature. It is the lowest temperature that can exist in universe at which the matter is highly condensed and no collision between gas molecules occurs. Its value is 0K or – 273°C.

5. 2068 Old Q.No. 4 a **At what temperature, do the Fahrenheit-thermometer and Celsius-thermometer show the same reading?** [2]

➘ Let y be the required value on both scales. So

$C = F = x$

We know,

$$\frac{C-0}{100} = \frac{F-32}{180}$$

or, $\frac{x}{100} = \frac{x-32}{180}$

or, $\frac{x}{5} = \frac{x-32}{9}$

or, $5x - 160 = 9x$

or, $-160 = 4x$

$\therefore \quad x = -40$

Thus, required value is $-40°$ C or $-40°$ F.

6. 2061 Q.No. 4 b **At what point of thermometric scale does kelvin scale reading coincide with Fahrenheit scale reading?** [2]

➘ Let x be the required value on both scales. So

$K = F = x$

We know,

$$\frac{K-273}{100} = \frac{F-32}{180}$$

or, $\frac{x-273}{100} = \frac{x-32}{180}$

or, $\frac{x-273}{5} = \frac{x-32}{9}$

$\therefore \quad x = 574.25$ K

Thus, required value is 574.25 K or 574.25°F.

7. 2059 Q.No. 4 a **What are the differences between heat and temperature?** [2]

➘ The differences between heat and temperature are given below:

Heat	Temperature
1. Heat is a form of energy.	1. Temperature is a degree of hotness or coldness of a body.
2. SI-unit of heat is joule and CGS- unit is calorie.	2. Its SI-unit is Kelvin.
3. It is a cause.	3. It is an effect.
4. Two bodies can be in thermal equilibrium without having same amount of heat.	4. Two bodies cannot be in thermal equilibrium if they are at different temperatures.
5. It is a measure of total kinetic energy of all molecules of a body.	5. It measures average kinetic energy of all molecules of the body.

8. 2058 Q.No. 4 c **Why mercury is used in thermometer?** [2]

➘ Mercury is commonly used as a thermometric substance due to following reasons:

i. It's boiling point is 357° C and freezing point - 39° C. Therefore, it can be used over a wide range of temperature.

ii. Its expansion is nearly uniform over the ordinary range of temperatures.

iii. It can be easily seen through the glass because it is opaque and shining in colour.

iv. It does not wet the glass.

v. It has low specific heat, so it doesn't take much heat from the body whose temperature is to be measured.

9. 2055 Q.No. 7 e **Why is mercury used commonly as a thermometric substance? Give two reasons.** [2]

➘ Please refer to 2058 Q.No. 4 c

□□□

Chapter 2: Thermal Expansion

Short Answer Questions

1. **2076 GIE Set B Q.No. 2a Why does a thick glass cup crack when boiling water is poured on it in the winter? Explain.** **[2]**

Glass is bad conductor of heat. As hot water is poured in it, the inner part of glass expands more quickly than outer part. Therefore, there is unequal expansion between inner and outer layers, which causes crack.

2. **2073 Supp Q.No. 2a Two metallic rod of the same material but of different length are heated. Smaller rod has circular area of cross-section but larger rod has rectangular cross-section. Will their linear expansivity be the same or different? Give justification of your answer.** **[2]**

The coefficient of linear expansivity of a material is given as

$$\alpha = \frac{\Delta l}{l\,(\theta_2 - \theta_1)};\ \theta_2 - \theta_1 = \text{Change in temperature}$$

The ratio $\frac{\Delta l}{l}$ is same in each case for a material. Hence, a smaller rod has circular area of cross section and a bigger rod of the rectangular cross-section have same linear expansivity.

3. **2073 Set D Q.No. 2a Why is it sometimes possible to loosen caps on screw top bottles by dipping the cap briefly in hot water?** **[2]**

When the cap of a screw top bottle is dipped briefly in hot water, there will be unequal expansion of the metal cap and the glass neck in which it is fitted. Since the expansivity of metal is greater than the expansivity of glass, so cap expands before the heat reaches the bottle and hence, the metal cap will be loosened.

4. **2072 Set E Q.No. 2a Explain why the possibility of "water taps burst" rises in severe winter.** **[2]**

In cold weather, when water trapped in a pipe cools further from 4° C to 0° C, it freezes. The water on cooling from 4° C to 0° C, expands considerably due to anomalous expansion of water, But the size of the pipes contract due to the fall in temperature. The expansion in volume of water in the ice gives outward pressure in the pipe due to which the pipe bursts.

5. **2071 Supp Q.No. 2a A square brass plate has a large circular hole cut in its centre. If the plate is heated, it will expand. Will the diameter of the hole expand or contract? Explain your answer.** **[2]**

When a metallic block with a hole on it is heated, its all the dimensions are expanded including hole. A useful analogy to thermal expansion is dimension of the object undergoes the same fractional change. Thus, increase in temperature will expand the hole, not reduce it.

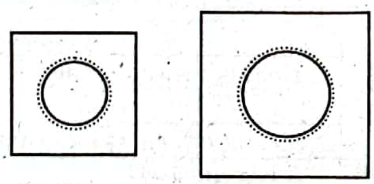


Fig: Thermal expansion of a plate with a hole.

6. **2071 Set C Q.No. 2 b Explain the significance of anomalous expansion of water with an example observed in nature.** **[2]**

Most of the liquids expand on being heated but water behaves in a peculiar manner. Water in the range of 0°C to 4°C contracts with increasing temperature. Its coefficient of volume expansion is negative in this range. Above this temperature, water expands. Hence, water has maximum density at 0°C. Such behaviour of water is called anomalous expansion. The anomalous expansion of water has important effect on plant and animal life in lakes when the surface temperature is below 4°C. The water near the surface is less dense than water below it and so, it remains there colder than the water at the bottom. As the surface freezes, the ice floats on the surface while water at 4°C remains at bottom where aquatic life remains alive. Hence, aquatic plants and animals can serve in very cold places due to property of anomalous expansion of water.

7. **2071 Set D Q.No. 2 a Define the coefficient of cubical expansion of a solid and hence, write an expression for the variation of its density with temperature.** **[2]**

Coefficient of cubical expansion of solid is defined as the change in volume per unit original volume per unit rise in temperature.

i.e. $\gamma = \dfrac{\Delta V}{V_0 \Delta\theta}$

The variation of density with temperature is

$\rho_2 = \dfrac{\rho_1}{1 + \gamma\Delta\theta}$ ρ_2 = density at θ_2
ρ_1 = density at θ_1
$\Delta\theta = \theta_2 - \theta_1$

8. **2070 Supp (Set A) Q.No. 2 b Forzen water pipes often burst. Will a alcohol thermometer break if the temperature drops below the freezing point of alcohol?** **[2]**

Forzen water pipes often burst due to anomalous expansion i.e., expansion on cooling below 4°C. At 0°C the water has greater volume than at 4°C but pipe contract on cooling. so, water pipe burst on cooling. However, the alcohol thermometer will not break if the temperature drops below freezing point of alcohol because the volume of alcohol decreases (or alcohol contract) on cooling to its freezing point.

9. **2070 Set C Q.No. 2 a A hole is drilled in a flat metal sheet. What happens to the diameter of the hole as the metal sheet is heated to higher temperature?** **[2]**

Please refer to 2071 Supp Q.No. 2a

10. **2070 Set D Q.No. 2 a Does the cubical expansivity of a liquid depend on its original volume? Explain.** **[2]**

No, the cubical expansivity of a liquid does not depend upon original volume of liquid.

The cubical expansitivity of a liquid is given as, $\gamma = \dfrac{\Delta V}{V(\theta_2 - \theta_1)}$

The ratio of $\dfrac{\Delta V}{V}$ is same in each case for a liquid so, the coefficient of cubical expansion is same for all

11. **2069 (Set A) Q. No. 2a Frozen water pipes often burst, will a mercury thermometer break if the temperature of the thermometer is brought below the freezing point of mercury?** **[2]**

In cold weather, when water trapped in a pipe cools further from 4° C to 0° C, it freezes. The water on cooling from 4° C to 0° C expands considerably due to anomalous expansion of water. But the size of the pipes contract due to the fall in temperature. The expansion in volume of water in the ice gives outward pressure in the pipe due to which the pipe bursts.

But the mercury thermometer will not break if the temperature of thermometer is brought below the freezing point of mercury. This is because, the thermometer as well as mercury contract on freezing.

12. **2067 Supp Q.No. 2 a Does the coefficient of linear expansion depend on length? Justify your answer.** **[2]**

No, the coefficient of linear expansion of a material doesn't depend on length. The linear expansitivity of a material is given as

$\alpha = \dfrac{\Delta l}{l(\theta_2 - \theta_1)}$

The ratio $\dfrac{\Delta l}{l}$, is same in each case for a material. So, the coefficient of linear expansion is same for all lengths of a material and does not depend on the length.

13. **2066 Q. No. 2 b When a metallic block with hole in it is heated, why does not the material around the hole expand into the hole and make it smaller?** **[2]**

Please refer to 2071 Supp Q.No. 2a

14. **2065 Q. No. 4 a Why are glass windows possible to be cracked in very cold region?** **[2]**

In the very cold regions, the temperature of the air outside the room will be very low as compared to the temperature inside the room. There is a large difference of temperature inside and outside the room because glass is a bad conductor of heat. Due to this reason, there is an unequal expansion of the outer and inner sides of glass and hence, the glass windows cracks.

15. **2064 Q.No. 4 a 2050 Q.No. 7i Two bodies made of the same material have the same external dimension and appearance, but one is solid and the other is hollow. When they are heated, is the overall volume expansion the same or different?** **[2]**

For the same volume, the expansion in two bodies of the same material depends only on the temperature. If two identical bodies, one solid and other hollow, are given the same amount of heat, the rise in temperature in the hollow body is greater than that of solid body because of large mass of the solid body. As a result, from the relation, $V_2 = V_1(1 + \gamma\,\Delta\theta)$, the hollow body will have greater expansion though the same heat is given to the two bodies due to greater $\Delta\theta$.

16. **2063 Q.No. 4 a Why does a thick glass tumbler crack when boiling water is poured on it?** [2]

Please refer to 2076 GIE Set B Q.No. 2a

17. **2062 Q.No. 4 a Why a column of mercury in a thermometer first descends slightly and then rises when placed in hot water? Explain.** [2]

When a mercury thermometer is placed in hot water, the bulb at first gets heat from outside and expands and hence, the column of mercury in thermometer at first descends slightly. Later on, heat is passed to the mercury and it expands. We know that, the expansivity of Hg is greater than that of the material of the bulb and hence, the level of mercury in the thermometer first descends slightly and then rises when placed in hot water.

18. **2058 Q.No. 4 a Fishes stay alive in frozen pond in winter. Explain.** [2]

Fishes stay alive in frozen pond in winter because of anomalous expansion of water. At 4°C, water has lowest volume and highest density which remains inside the pond and the ice which has the temperature below 4°C lies on the surface of pond because it has lower density. This makes the fishes alive inside the frozen pond in winter.

19. **2055 Q.No. 7 d Does the coefficient of linear expansion depend on length? Explain.** [2]

Please refer to 2067 Supp Q.No. 2 a

20. **2054 Q.No. 7 b Explain why a column of mercury in thermometer first descends slightly and then rises when placed in hot water?** [2]

Please refer to 2062 Q.No. 4 a

21. **2052 Q.No. 7 c Water level initially falls in a vessel when it is heated. Why?**

When a vessel containing water is heated, the vessel receives heat earlier than water. As a result, at first expansion of the vessel takes place and hence, initially the water level falls in a vessel.

22. **2052 Q.No. 7 d Why is it sometimes possible to loosen caps on screw top bottles by dipping the cap briefly in hot water?**

Please refer to 2073 Set D Q.No. 2a

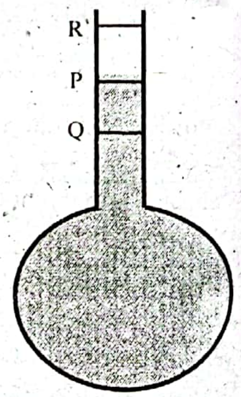
23. **2052 Q.No. 7 e Why do frozen water pipe bursts?**

Please refer to 2072 Set E Q.No. 2a

Long Answer Questions

24. **2076 GIE Set B Q.No. 6a What are real and apparent expansions of a liquid? Obtain the relation between coefficient of real and apparent expansion of liquid.** [4]

Real and Apparent Expansions of Liquid



Let us consider a glass flask of long stem with uniform small bore and it is filled with liquid up to level P. When the vessel is heated gently, at first the vessel expands so that the liquid level lowers to a point Q and after then, the liquid expands so that it's level rises to a point R above P as shown in figure. Here, actually, PQ is the expansion of the vessel and QR is the real expansion of liquid. The difference in real expansion of the liquid V_{QR} and expansion of the vessel V_{PQ} is called the apparent expansion of the liquid. In figure,

$$V_{QR} = V_{PR} + V_{PQ} \quad \ldots (i)$$

∴ Real expansion of liquid= apparent expansion of liquid + expansion of the vessel

Coefficient of Real Expansion

It is defined as the ratio of the real increase in volume of the liquid to the original volume per unit rise in temperature. It is denoted by γ_r and is given by,

$$\gamma_r = \frac{\text{real increase in volume}}{\text{original volume} \times \text{rise in temp.}} = \frac{\Delta V_r}{V \times \Delta\theta}$$

Coefficient of Apparent Expansion

It is defined as the ratio of the apparent increase in volume of the liquid to the original volume per unit rise in temperature. It is denoted by γ_a and is given by,

$$\gamma_a = \frac{\text{apparent increase in volume}}{\text{original volume} \times \text{rise in temp.}} = \frac{\Delta V_a}{V \times \Delta\theta}$$

Coefficient of Volume Expansion of Vessel

It is defined as the ratio of increase in volume of the vessel to the original volume per unit rise in temperature. It is denoted by γ_g and is given by,

$$\gamma_g = \frac{\text{increase in volume of the vessel}}{\text{original volume} \times \text{rise in temp.}} = \frac{\Delta V_v}{V \times \Delta\theta}$$

The unit of all these coefficients is K^{-1} or $°C^{-1}$.

Relation between Coefficient of Real and Apparent Expansion

Let us consider a glass vessel of volume V filled with some liquid at temperature θ_1° C. When the liquid is heated to θ_2° C, it expands. The real increase in volume of the liquid,

$$\Delta V_r = \gamma_r V \Delta\theta = \gamma_r V (\theta_2 - \theta_1)$$

Similarly, apparent increase in volume of the liquid,

$$\Delta V_a = \gamma_a V \Delta\theta = \gamma_a V (\theta_2 - \theta_1)$$

and the increase in volume of the vessel,

$$\Delta V_g = \gamma_g V \Delta\theta = \gamma_g V (\theta_2 - \theta_1)$$

$\because$ $\Delta V_r = \Delta V_a + \Delta V_g$ [From equation (i)]

or, $\gamma_r V \Delta\theta = \gamma_a V \Delta\theta + \gamma_g V \Delta\theta$

or, $\gamma_r = \gamma_a + \gamma_g$

This equation gives the relation between coefficient of real and apparent expansion.

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25. **2076 Set C Q.No. 6a** **Define the coefficients of real expansion and apparent expansion and hence derive the relation between them.** **[4]**

Please refer to 2076 GIE Set B Q.No. 6a

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26. **2075 GIE Q.No. 6a** **Define linear and cubical expansivity of solids. Establish a relationship between the coefficients of linear and cubical expansions.** **[4]**

Linear expansivity (α): The increase in length per unit original length per unit rise in temperature is called linear expansivity or coefficient of linear expansion. It is denoted by α and is given by

$$\alpha = \frac{\text{change in length}}{\text{original length x rise in temperature}}$$

$$\alpha = \frac{l_2 - l_1}{l_1 (\theta_2 - \theta_1)} \Rightarrow \alpha = \frac{\Delta l}{l_1 \Delta\theta}$$

where Δl is the change in length, l_1 is original length and $\Delta\theta$ is the change in temperature. The unit of α is K^{-1}.

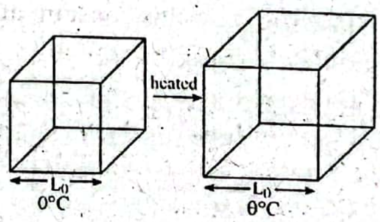
Cubical expansivity (γ): The increase in volume per unit original volume per unit rise in temperature is called cubical expansivity or coefficient of cubical expansion. It is denoted by γ and is given by

$$\gamma = \frac{\text{change in volume}}{\text{original volume} \times \text{rise in temperature}}$$

$$\gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)} \Rightarrow \gamma = \frac{\Delta V}{V_1 \Delta\theta}$$

where ΔV is the change in volume, V_1 is original volume and Δθ is the change in temperature. The unit of γ is K^{-1}.

Relation between α and γ: Let us consider a cube whose sides having length L_0 at temperature 0°C and volume is V_0. If the cube is heated, then the side of cube becomes L_θ and volume becomes V_θ. If γ be the coefficient of cubical expansion, then



$$V_\theta = V_0 (1 + \gamma\theta)$$

Since, $V_0 = L_0^3$ and $V_\theta = L_\theta^3$

Then, $L_\theta^3 = L_0^3 (1 + \gamma\theta)$ ... (i)

But $L_\theta = L_0 (1 + \alpha\theta)$, ... (ii)

Where α is coefficient of linear expansion.

Using (ii) in (i), we get

$$L_0^3 (1 + \alpha\theta)^3 = L_0^3 (1 + \gamma\theta)$$

$1 + 3\alpha\theta + 3\alpha^2\theta^2 + (\alpha\theta)^3 = 1 + \gamma\theta$

Since, α is small quantity, the terms containing higher powers of α can be neglected. Then

$3\alpha\theta = \gamma\theta \Rightarrow \gamma = 3\alpha$

i.e., coefficient of cubical expansion is three times greater than the coefficient of linear expansion.

27. 2075 Set B Q.No. 6b **Define real and apparent expansivities of a liquid? Derive the relation between them.** **[4]**

Please refer to 2076 GIE Set B Q.No. 6a

28. 2074 Supp. Q.No. 6a **Describe how the cubical expansivity of a liquid can be determined by the use of balanced columns.** **[4]**

Lest us consider a u-shaped glass tube ABCD filled with a liquid whose real expansivity γ should be determined. Initially the liquid is at equal height as shown in figure. One arm of tube is surrounded with ice-water jacket and another arm is surrounded with steam jacket. The temperature of each jacket is noted by thermometer fitted on them. To prevent the transformation of heat, a wet cloth is kept at middle. At different heights; the pressure at the base is same.

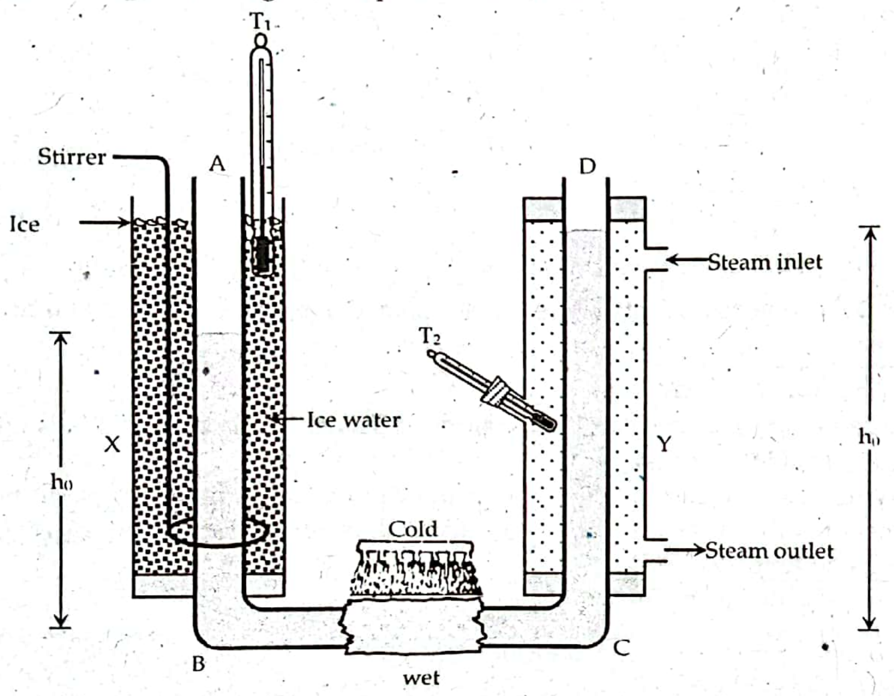


Fig. Apparatus for real expansivity of liquid.

For cooler arm

h_0 = height of liquid

ρ_0 = density of liquid

So, total pressure at the bottom of cooler arm is,

$P_B = h_0\rho_0 g + P_{atm}$

where, P_{atm} = atmospheric pressure on liquid surface

Also for hotter arm

h_θ = height of liquid

ρ_θ = density of liquid at θ°C

Therefore, total pressure at the bottom of hotter arm,

$P_C = h_\theta\rho_\theta g + P_{atm}$

Since pressure remains constant at both points, (i.e. $P_B = P_C$)

$$h_0\rho_0 g + P_{atm} = h_\theta\rho_\theta g + P_{atm}$$

$$h_0\rho_0 g = h_\theta\rho_\theta g$$

$$h_0\rho_0 = h_\theta\rho_\theta$$

Also, $\rho_\theta = \dfrac{\rho_0}{1 + \gamma\Delta\theta}$

where, γ is absolute expansivity of liquid.

So, $h_0\rho_0 = h_\theta\dfrac{\rho_0}{(1+\gamma\Delta\theta)}$

or, $h_0 = \dfrac{h_\theta}{(1+\gamma\Delta\theta)}$

or, $h_0(1+\gamma\Delta\theta) = h_\theta$

or, $h_0 + h_0 + \gamma\Delta\theta = h_\theta$

or, $h_0\gamma\Delta\theta = h_\theta - h_0$

$\gamma = \dfrac{h_\theta - h_0}{h_0\,\Delta\theta}$

This is the expression of determining absolute expansivity of liquid.

29. 2073 Set C Q.No. 6a **Define coefficients of real and apparent expansion of a liquid, and establish a relation between them.** **[4]**

Please refer to 2076 GIE Set B Q.No. 6a

30. 2072 Supp Q.No. 6a **Define the coefficients of linear an cubical expansion of solid and establish their relation. [4]**

Please refer to 2075 GIE Q.No. 6a

31. 2072 Set C Q.No. 6a **Distinguish between real and apparent expansion of liquid. Describe with mathematical detail, a method to determine real expansivity of a liquid.** **[4]**

First part: Please refer to 2076 GIE Set B Q.No. 6a

Second part: Please refer to 2074 Supp. Q.No. 6a

32. 2071 Set C Q.No. 6 c **Does cubical expansivity depend upon the initial volume of a solid? Write the unit of this expansivity. Also derive its relation with superficial expansivity.** **[4]**

No, the cubical expansivity of a solid does not depend upon initial volume of solid.

The cubical expansitivity of a solid is given as

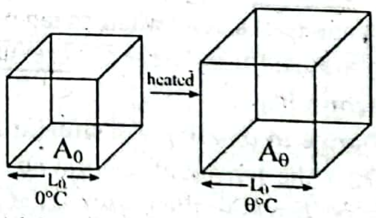
$\gamma = \dfrac{\Delta V}{V(\theta_2 - \theta_1)}$

The ratio of $\dfrac{\Delta V}{V}$ is same in each case for a solid so, the coefficient of cubical expansion is same for all volume of a solid and does not depend on the volume. The unit of this expansivity is per kelvin (K^{-1}).

Relation between β and γ: Let us consider a cube whose sides having length L_0 and area of each face A_0 at temperature 0°C and volume is V_0. If the cube is heated, then the side of cube becomes L_θ, area A_θ and volume V_θ. If γ be the coefficient of cubical expansion then,

$V_\theta = V_0(1+\gamma\theta)$ ... (i)

Again,



$V_\theta = A_\theta \times L_\theta$

or, $V_\theta = A_0(1+\beta\Delta\theta)\times L_0(1+\alpha\Delta\theta)$

or, $V_\theta = A_0L_0(1+\alpha\Delta\theta+\beta\Delta\theta+\alpha\beta\Delta\theta^2)$

or, $V_\theta = V_0[1+(\alpha+\beta)\Delta\theta]$ ... (ii)

[$\because$ α and β are small quantities and hence, there product is very small which is neglected]

Comparing equation (i) and (ii)

$\gamma = \alpha + \beta$ [$\because \beta = 2\alpha$]

or, $\gamma = \dfrac{\beta}{2} + \beta = \dfrac{3\beta}{2}$

or, $\dfrac{\gamma}{3} = \dfrac{\beta}{2}$

This is the required relation between γ and β.

33. 2070 Supp (Set B) Q.No. 6 a **What are meant by real and apparent expansions of a liquid? Show that sum of coefficient of real expansion of a liquid is sum of coefficient of apparent expansion of the liquid and coefficient of cubical expansion of the vessel.** **[4]**

Please refer to 2076 GIE Set B Q.No. 6a

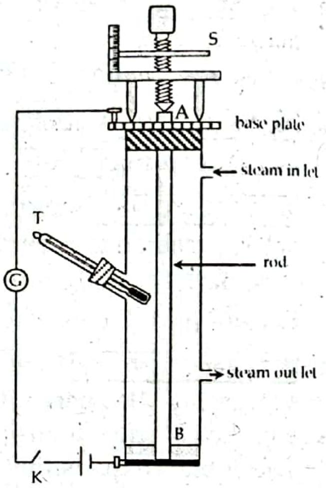
34. **2070 Set C Q.No. 6 a** **Describe a method to determine the linear expansivity of a solid. Can the cubical expansivity be derived from this value?** **[4]**

Determination of Coefficient of Linear Expansion of a Solid by Pullinger's Apparatus

The experimental method to determine the linear expensivity of a solid consist of uniform metal (of brass or steel) rod AB whose coefficient of linear expansion is to be determined, a hollow metal tube with steam inlet and outlet, a thermometer, a spherometer, a galvanometer and heating system which are arranged as shown in figure.

Let, l_1 be the initial length of metal rod at room temperature say θ_1. When it is placed inside the metallic tube and heated its length increases. When central leg of the spherometer is lowered to rod, the spherometer reading is R_1. Steam is passed inside the tube continuously until the thermometer records a constant temperature θ_2. After the steady state is reached, the spherometer screw is rotated down so that it comes in contact with the end A. Let this reading of the spherometer be R_2.

We have,



Initial length of the rod = l_1

Initial temperature = θ_1°C

Initial spherometer reading = R_1

Final steady temperature = θ_2°C

Final spherometer reading = R_2

∴ Increase in length = $R_2 - R_1$

The coefficient of linear expansion α of the material of the rod is given by

$$\alpha = \frac{\text{increase in length}}{\text{original length} \times \text{rise in temperature}}$$

$$= \frac{R_2 - R_1}{l_1 (\theta_2 - \theta_1)}$$

In this way, the coefficient of linear expansion of the material of the rod can be determined. The cubical expansivity of the solid can be determined by multiplying the linear expansivity by 3. If γ be the cubical expansivity of that material, then $\gamma = 3\alpha$.

35. **2070 Set D Q.No. 6 a** **Define the coefficient of real and apparent expansions of a liquid and derive a relation between them.** **[4]**

Please refer to 2076 GIE Set B Q.No. 6a

36. **2069 Set A Q. No. 6a** **Define linear and cubical expansivities of solids. Derive an expression for the variation in density of a solid when its temperature is raised from θ_1° C to θ_2°C.** **[4]**

First Part: Please refer to 2075 GIE Q.No. 6a

Second Part:

Change in density of a substance with the change in temperature

When the temperature of a substance is increased, its volume increases. As the density is inversely proportional to the volume, the density of the substance changed due to change in volume.

Let m and V_1 be the mass and volume of a substance at a temperature θ_1. The density of the substance is given by

$$\rho_1 = \frac{m}{V_1} \qquad \text{... (i)}$$

Suppose, the substance be heated so that its volume increases to V_2 and temperature θ_2. The density of the substance at this temperature is

$$\rho_2 = \frac{m}{V_2} \qquad \text{... (ii)}$$

But, the volume V_2 at θ_2 is given by

$$V_2 = V_1 (1 + \gamma \Delta\theta) \qquad \text{... (iii)}$$

where γ is the coefficient of cubical expansion of the substance. So, the density is now,

$$\rho_2 = \frac{m}{V_2} = \frac{m}{V_1 [1 + \gamma \Delta\theta]}$$

or, $\rho_2 = \frac{\rho_1}{1 + \gamma\,\Delta\theta}$

$\therefore$ $\rho_1 > \rho_2$ since $(1 + \gamma\Delta\theta) > 1$

Thus, the density decreases as the temperature increases.

37. 2069 (Set A) Old Q. No. 5a **Define coefficient of linear expansion. Obtain the relation between α and γ.** **[5]**

Please refer to 2075 GIE Q.No. 6a

38. 2066 Q. No. 6 a **Define linear and cubical expansions of solid, and establish a relation between their coefficients.** **[4]**

Please refer to 2075 GIE Q.No. 6a

39. 2060 Q.No. 5 a **Define linear, superficial, and cubical expansivities. Show that β = 2α where α and β are linear and superficial expansivities.** **[2+3]**

Linear expansivity (α): The increase in length per unit original length per unit degree rise in temperature is called linear expansivity or coefficient of linear expansion. It is denoted by α and is given by

$$\alpha = \frac{\text{change in length}}{\text{original length x rise in temperature}} \Rightarrow \alpha = \frac{l_2 - l_1}{l_1(\theta_2 - \theta_1)} \Rightarrow \alpha = \frac{\Delta l}{l_1\,\Delta\theta}$$

where Δl is the change in length, l_1 is original length and $\Delta\theta$ is the change in temperature. The unit of α is K^{-1}.

Superficial expansivity (β): The increase in area per unit original area per unit rise in temperature is called superficial expansivity or coefficient of superficial expansion. It is denoted by β and is given by

$$\beta = \frac{\text{change in area}}{\text{original area} \times \text{rise in termperature}} \Rightarrow \beta = \frac{A_2 - A_1}{A_1(\theta_2 - \theta_1)} \Rightarrow \beta = \frac{\Delta A}{A_1\,\Delta\theta}$$

where ΔA is change in area A_1 is original area and $\Delta\theta$ is rise in temperature. The unit of β is K^{-1}.

Cubical expansivity (γ): The increase in volume per unit original volume per unit rise in temperature is called cubical expansivity or coefficient of cubical expansion. It is denoted by γ and is given by

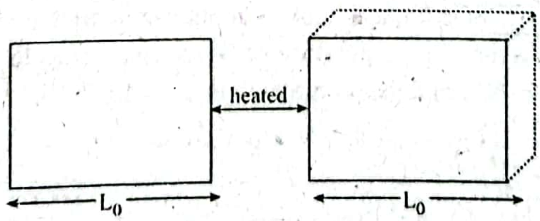
$$\gamma = \frac{\text{change in volume}}{\text{original volume} \times \text{rise in temperature}} \Rightarrow \gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)} \Rightarrow \gamma = \frac{\Delta V}{V_1\,\Delta\theta}$$

where ΔV is the change in volume, V_1 is original volume and $\Delta\theta$ is the change in temperature. The unit of γ is K^{-1}.

Derivation of β = 2α: Let us consider, a square sheet of side L_0 and L_θ at temperatures 0°C and θ°C, then

$L_\theta = L_o(1 + \alpha\theta)$ ... (i)

where α is linear expansivity of material sheet. S_0 and S_θ are the area of the sheet at 0°C and at θ°C respectively.



Now, $S_\theta = S_0(1 + \beta\theta)$ where, β is superficial expansivity.

or, $L_\theta^2 = L_0^2(1 + \beta\theta)$

$L_o^2(1 + \alpha\theta)^2 = L_0^2(1 + \beta\theta)$

$(1 + \alpha\theta)^2 = (1 + \beta\theta)$

$1 + 2\alpha\theta + + (\alpha\theta)^2 = 1 + \beta\theta$

[$\because$ α is small quantity and α^2 is still smaller so $\alpha^2\,\theta^2$ can be neglected]

Then, $1 + 2\alpha\theta = 1 + \beta\theta \Rightarrow 2\alpha\theta = \beta\theta$

$\therefore$ $\beta = 2\alpha$

i.e. coefficient of superficial expansion is twice its linear expansivity.

40. **2059 Q.No. 5 a** **Why do substances expand on heating? Show that $\alpha = \frac{\gamma}{3}$ where α and γ are the coefficient of linear, and cubical expansion of a substance.** **[1+4]**

When a substance is heated, its molecules get thermal energy. These molecules vibrate and push far each other. As a result, the substance expanded on heating.

2nd part: Please refer to 2075 GIE Q.No. 6a

41. **2056 Q.No. 5 a** **Define linear and cubical expansivities. Derive a relation between them.** **[1+1+3]**

Please refer to 2075 GIE Q.No. 6a

42. **2051 Q.No. 8** **Obtain an expression for the change in density of a gas due to the thermal expansion.**

Please refer to 2069 Set A Q. No. 6a

Numerical Problems

43. **2076 GIE Set A Q.No. 10a** **At what temperature wood will just sink in benzene if their densities at 0°C are 8.8×10^2 kg/m³ and 9×10^2 kg/m³ respectively.** **[4]**

[$\gamma_{benzene} = 1.2 \times 10^{-3}$ K⁻¹ and $\gamma_{wood} = 1.5 \times 10^{-4}$ K⁻¹]

Given,

For wood

Density at 0° C $(\rho_w^\circ) = 8.8 \times 10^2$ kg/m³

$(\gamma_w) = 1.5 \times 10^{-4}$ K⁻¹

For Benzene

Density at 0° C $(\rho_B^\circ) = 9\times10^2$ kg/m³

$(\gamma_B) = 1.2 \times 10^{-3}$ K⁻¹

Final temperature at which wood sink in benzene (θ) = ?

Now,

$$\rho_w^\theta = \frac{\rho_w^\circ}{1 + \gamma_w \Delta\theta} \quad \text{and} \quad \rho_B^\theta = \frac{\rho_B^\circ}{1 + \gamma_B \Delta\theta}$$

For wood just sinking in Benzene,

$$\rho_w^\theta = \rho_B^\theta$$

or, $\dfrac{\rho_w^\circ}{1 + \gamma_w \Delta\theta} = \dfrac{\rho_B^\circ}{1 + \gamma_B \Delta\theta}$ $\quad [\Delta\theta = \theta - 0 = \theta]$

or, $\dfrac{880}{1 + 1.5 \times 10^{-5}\,\theta} = \dfrac{900}{1 + 1.2 \times 10^{-3}\,\theta}$

or, $880 + 1.056\theta = 900 + 0.0135\theta$

or, $1.056\theta - 0.0135\theta = 900 - 880$

or, $1.0425\theta = 20$

or, $\theta = \dfrac{20}{1.0425}$

$\therefore\ \theta = 19.18°$ C

At 19.18° C, the wood will sink in benzene.

44. **2076 Set B Q.No. 10a** **2054 Q.No. 10 b** **A glass flask of volume 400cm³ is just filled with mercury at 0°C. How much mercury will overflow when the temperature of the system rises to 80°C. The coefficient of cubical expansion of glass is 1.2×10^{-5} °C⁻¹ and that of mercury is 1.8×10^{-5} °C⁻¹.** **[4]**

Given,

Volume of glass flask at 0°C (V_1) = 400 cm³

Initial temperature (θ_1) = 0°C

Final temperature (θ_2) = 80°C

Coefficient of cubical expansion of glass $(\gamma_g) = 1.2 \times 10^{-5}$ C⁻¹

Coefficient of cubical expansion of mercury $(\gamma_m) = 1.8 \times 10^{-5}$C⁻¹

Amount of mercury overflows = ?

Let, V_2 be the volume of glass at 80°C, then

$V_2 = V_1 \{1 + \gamma_g(\theta_2 - \theta_1)\} = 400 \{1 + 1.2 \times 10^{-5} (80 - 0)\} = 400 (1.00096) = 400.384$ cm³

Let, V_2' be the volume of mercury at 80°C,

$V_2' = V_1 \{1 + \gamma_m (\theta_2 - \theta_1)\} = 400 \{1 + 1.8 \times 10^{-5} \times 80\} = 400.576$ cm³

Amount of mercury overflow $= V_2' - V_2 = 400.576 - 400.384 = 0.192$ cm³ $= 1.92 \times 10^{-7}$m³

45. 2075 GIE Q.No. 10c A Steel wire having length 8m and diameter 4mm is fixed between two rigid support. Calculate increase in tension on a wire when temperature falls by 10°C. Where Young's modulus of wire = 2×10^{11} N/m^2, linear expansivity of steel = 1.2×10^{-5} K^{-1}.

Given,

Length of wire (l) = 8m

Diameter of wire (d) = 4mm = 4×10^{-3} m

Charge in temperature (θ) = 10°C

Linear expansivity of steel (α) = 12×10^{-6} K^{-1}

Young's modulus for steel (Y) = 2×10^{11} Nm^{-2}

We have,

$$Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A\,\Delta l}$$

$$\text{or, } F = \frac{YA\Delta l}{l} = Y\frac{\pi d^2}{4}\Delta\theta.\,\alpha \qquad [\because \Delta l = l \propto \Delta\theta]$$

$$= 2 \times 10^{11} \times \frac{\pi (4 \times 10^{-3})^2}{4} \times 10 \times 12 \times 10^{-6}$$

$$F = 301.6\text{N}$$

The required increase in tension is 301.6 N.

46. 2075 Set A Q.No. 10a An iron rod of length 100m at 10°C is used to measure a distance of 2km on a day when the temperature is 40°C. Calculate the error in measuring the distance. [4]

Given,

Temperature of iron (θ_1) = 10°C

Length of iron rod (l_{10}) = 100 m

Temperature of day (θ_2) = 40°C

Linear expensivity of iron (α) = 1.2×10^{-5} °C^{-1}

Distance to be measured = 2000 m

Error in measuring distance = ?

Now, we have,

$$l_{40} = l_{10}\,[1 + \alpha(\theta_2 - \theta_1)]$$
$$= 100\,(1 + 1.2 \times 10^{-5} \times 30)$$
$$= 100.036 \text{ m}$$

Then, the rod measures the distance of 2000 m at 40°C = 2000.72 m. Then error in measuring distance is 2000.72 – 2000 = 0.72 m.

47. 2074 Set A Q.No. 10a A seconds pendulum made of brass keeps correct time at 10°C. How many seconds it will lose or gain per day when the temperature of its surrounding rises to 35°C? [4]

Given,

The time period of simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}}; \qquad \begin{bmatrix} l = \text{length of pendulum} \\ g = \text{acceleration due to gravity} \end{bmatrix}$$

At 35°C,

$$T_{35} = 2\pi\sqrt{\frac{l_{35}}{g}} \qquad \dots \text{(i)}$$

$$\& \; T_{10} = 2\pi\sqrt{\frac{l_{10}}{g}} \qquad \dots \text{(ii)}$$

Dividing equation (i) and (ii), we get

$$\frac{T_{35}}{T_{10}} = \sqrt{\frac{l_{35}}{l_{10}}} = \sqrt{\frac{l_{10}[1 + \alpha(\theta_2 - \theta_1)]}{l_{10}}} \qquad [\because l_2 = l_1\,(1 + \alpha\,\Delta\theta)]$$

$$\text{or, } \frac{T_{35}}{T_{10}} = \sqrt{1 + \alpha\,\Delta\theta} \qquad \begin{bmatrix} \because \Delta\theta = \theta_2 - \theta_1 = 35 - 10 = 25°\text{C} \\ \& \; \alpha_{brass} = 2 \times 10^{-5}\ \text{k}^{-1} \end{bmatrix}$$

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or, $\frac{T_{35}}{T_{10}} = \sqrt{1 + 2 \times 10^{-5} \times 25} = 1.000249969$

Since, the pendulum keeps the correct time at 10°C, then

$T_{10} = 2$ sec (for second pendulum)

So, $T_{35} = 2 \times 1.000249969$ sec $= 2.000499938$ sec

The pendulum clock lose time (2.000499938 – 2) = 0.000499938 sec in 2 s.

∴ In 1 second, the clock lose $\frac{0.000499938}{2}$ sec

∴ In 1 day, the clock lose $\frac{0.000499938}{2} \times 24 \times 60 \times 60 = 21.597$ seconds.

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48. 2074 Set B Q.No. 10b Two ends of a steel wire of length 8 m and 2 mm radius are fixed to two rigid supports. Calculate the increase in tension in the wire when temperature falls by 10°C. [4]

Given,

Length of wire (l) = 8m

Diameter of wire (d) = 2 mm = 2×10^{-3} m

Charge in temperature (θ) = 10°C

Linear expansivity of steel (α) = 12×10^{-6} K^{-1}

Young's modulus for steel (Y) = 2×10^{11} Nm^{-2}

We have,

$$Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A\,\Delta l}$$

$$\text{or, } F = \frac{YA\Delta l}{l} = Y\frac{\pi d^2}{4}\Delta\theta.\,\alpha \qquad [\because \Delta l = l \propto \Delta\theta]$$

$$= 2 \times 10^{11} \times \frac{\pi(2 \times 10^{-3})^2}{4} \times 10 \times 12 \times 10^{-6}$$

F = 75.36 N

The required increase in tension is 75.36 N.

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49. 2073 Set D Q.No. 10a A glass flask with volume 200 cm³ is filled to the brim with mercury at 20°C. How much mercury overflows when the temperature of the system is raised to 100°C? [4]

Given,

Volume of glass flask at 20 °C (V_1) = 200 cm³

Initial temperature (θ_1) = 20°C

Final temperature (θ_2) = 100°C

Coefficient of cubical expansion of glass (γ_g) = 1.2×10^{-5} K^{-1}

Coefficient of cubical expansion of mercury (γ_m) = 18×10^{-5} K^{-1}

Amount of mercury overflow = ?

Let V_2 be the volume of glass at 100°C, then,

$$V_2 = V_1[1 + \gamma(\theta_2 - \theta_1)]$$
$$= 200\,[1 + 1.2 \times 10^{-5}(100 - 20)] = 200.192 \text{ cm}^3$$

Let V_2' be the volume of mercury at 100°C, then,

$$V_2' = V_1[1 + \gamma_m(\theta_2 - \theta_1)]$$
$$= 200\,[1 + 18 \times 10^{-5}(100 - 20)] = 202.88 \text{ cm}^3$$

Amount of mercury overflow at 100°C = $V_2' - V_1$ = 202.88 – 200.192 = 2.688 cm³

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50. 2072 Set D Q.No. 10a The marking on an aluminium ruler and a brass ruler are perfectly aligned at 0°C. How far apart will the 20.0 cm marks be on the two rulers at 100°C, if precise alignment of the left hand ends of the rulers is maintained? Coefficient of linear expansion of aluminium and brass are 2.4×10^{-5} K^{-1} and 2.0×10^{-5} K^{-1} respectively. [4]

Given,

For aluminum rod

Initial length (l_1^a) = 20 cm

For brass rod

(l_1^b) = 20 cm

Initial temperature $(\theta_1) = 0° C$
Final temperature $(\theta_2) = 100°C$
Linear expansivity $(\alpha_a) = 2.4 \times 10^{-5} K^{-1}$
Final length $(l_2^a) = ?$

$(\theta_1) = 0° C$
$(\theta_2) = 100°C$
$(k_b) = 2 \times 10^{-5} K^{-1}$
$(l_2^b) = ?$

We have,

$$l_2^a = l_1^a (1 + \alpha_a \Delta\theta) = 20 (1 + 2.4 \times 10^{-5} \times 100) = 20.048 \text{ cm}$$

$$l_2^b = l_1^b (1 + \alpha_b \Delta\theta) = 20 (1 + 2 \times 10^{-5} \times 10) = 20.04 \text{ cm}$$

Now,

$$l_2^a - l_2^b = 20.48 - 20.04 = 0.008 \text{ cm}$$

∴ Right side of aluminum rod is 0.008 cm far from right side of brass rod.

51. 2072 Set E Q.No. 10a A brass pendulum clock keeps correct time at 15°C. How many seconds per day it will lose or gain at 0°C? [4]

Given,

Initial temperature $(\theta_1) = 15°C$
Final temperature $(\theta_2) = 0°C$
Linear expansivity $(\alpha) = 2 \times 10^{-5} K^{-1}$
Gain in time per day = ?

Let T_{15} and T_0 be the time period at 15°C and 0°C, then,

$$T_{15} = 2\pi\sqrt{\frac{l_{15}}{g}} = 2\pi\sqrt{\frac{l_0(1 + \alpha\Delta\theta)}{g}}$$

$$T_0 = 2\pi\sqrt{\frac{l_0}{g}}$$

Now,

$$\frac{T_{15}}{T_0} = \sqrt{\frac{1 + \alpha\Delta\theta}{1}} = \sqrt{1 + 2 \times 10^{-5} \times 15} = 1.00015$$

Now,

$$\frac{T_{15} - T_0}{T_0} = 0.00015.$$

This is the gain in time in 1 sec.

Gain in time per day = $0.0015 \times 24 \times 60 \times 60 = 12.96$ sec

52. 2071 Supp Q.No. 10a Using the following data, determine the temperature at which wood will just sink in benzene. [4]
Density of benzene at 0°c = 9.0×10^2 kgm^{-3}. Density of wood at 0°C = 8.8×10^2 kg/m^3.
Cubical expansivity of benzene = $1.2\times 10^{-3}K^{-1}$,
Cubical expansivity of wood = $1.5 \times 10^{-4} K^{-1}$

Given,

Density of benzene at $0°C(\rho_b{}^o) = 9.0 \times 10^2$ Kgm^{-3}
Density of wood at $0°C(\rho_w{}^o) = 8.8 \times 10^2$ Kgm^{-3}
Cubical explanation of benzene $(\gamma_b) = 1.2 \times 10^{-3} K^{-1}$
Cubical explanation of wood $(\gamma_w) = 1.5 \times 10^{-4} K^{-1}$
Temperature at which wood sink in benzene (θ) = ?

Let $\rho_b{}^\theta$ and $\rho_w{}^\theta$ be the densities of benzene and wood at θ°C. Then,

Now,

$$\rho_b{}^\theta = \frac{\rho_b{}^o}{1 + \gamma_b . \theta} = \frac{9.0 \times 10^2}{1 + 1.2 \times 10^{-3} \times \theta}$$

$$\rho_w{}^\theta = \frac{\rho_w{}^o}{1 + \gamma_w . \theta} = \frac{8.8 \times 10^2}{1 + 1.5 \times 10^{-4} \times \theta}$$

Wood will just sink in benzene, when $\rho_b{}^\theta = \rho_w{}^\theta$. So,

$$\frac{9.0 \times 10^2}{1 + 1.2 \times 10^{-3} \times \theta} = \frac{8.8 \times 10^2}{1 + 1.5 \times 10^{-4} \times \theta}$$

or, $\theta = 21.7\ ^\circ C$

53. **2070 Supp (Set A) Q.No. 10 b** **2069 (Set A) Q. No. 10c** **A glass vessel of volume 50cm³ is filled with mercury and is heated from 20°C to 60°C. What volume of mercury will overflow?** **[4]**

Given,

Volume of glass (V_g) = 50cm³

Temperature change ($\Delta\theta$) = 60° – 20° = 40°C

Linear expansivity of glass (α_g) = $1.8 \times 10^{-6} K^{-1}$

Volume expansivity of mercury (γ_m) = $1.8 \times 10^{-4} K^{-1}$

Volume of mercury overflow = ?

After heating,

Volume of mercury over flow = Increase in volume of Hg – Increase in volume of vessel

$= V(\gamma_m - 3\alpha_g) \times \Delta\theta \quad [\because \gamma = 3\alpha]$

$= 50\,(1.8 \times 10^{-4} - 3 \times 1.8 \times 10^{-6}) \times 40 = 0.35\ cm^3$

54. **2069 (Set B) Q. No. 10a** **The pendulum of a clock is made of brass. If the clock keeps correct time at 15°C, how many seconds per day will it lose at 20°C?** **[4]**

Given,

Initial temperature (θ_1) = 15°C

Final temperature (θ_2) = 20°C

Linear expansivity (α) = $0.000018\ ^\circ C^{-1}$

Loss in time per day = ?

Let, T_{15} and T_{20} be the time periods at 15°C and 20°C. Then,

$$T_{20} = 2\pi\sqrt{\frac{l_{20}}{g}} \quad \ldots(i)$$

$$T_{15} = 2\pi\sqrt{\frac{l_{15}}{g}} \quad \ldots(ii)$$

Divide (i) by (ii)

$$\frac{T_{20}}{T_{15}} = \sqrt{\frac{l_{20}}{l_{15}}} = \sqrt{\frac{l_{15}(1 + \alpha\Delta\theta)}{l_{15}}}$$

$$\frac{T_{20}}{T_{15}} = \sqrt{1 + \alpha\,\Delta\,\theta} = \sqrt{1 + 0.000018 \times 5} = 1.000045$$

$$\frac{T_{20} - T_{15}}{T_{15}} = 0.000045$$

This is the lose in time per sec.

$\therefore$ Lose in time per day = $0.000045 \times 60 \times 60 \times 24 = 4$ sec.

55. **2069 Old (Set B) Q. No. 5b** **The length of an iron rod is measures by a brass scale. When both of them are at 10°C, the measured length is 50cm. What is the length of the rod at 40°C when measured by the brass scale at 40°C.? (α for brass = $24 \times 10^{-6}\ ^\circ C^{-1}$, α for iron = $16 \times 10^{-6}\ ^\circ C^{-1}$)** **[4]**

Given,

Length of brass at 10°C (l_{10}) = 50 cm

Length of bass at 40°C (l_{40}) = ?

Initial temperature (θ_1) = 10°C

Final temperature (θ_2) = 40°C

Linear expansivity of brass (α_i) = $24 \times 10^{-6}\ ^\circ C^{-1}$

Linear expansivity of iron (α_i) = $16 \times 10^{-6}\ ^\circ C^{-1}$

Now,

Length of iron rod at 40°C = $50\,[1 + 16 \times 10^{-6}\,(40 - 10)] = 50.024$ cm

Length of one cm division of the brass scale at 40°C = $1[1 + 24 \times 10^{-6}\,(40 - 10)] = 1.00072$ cm

$\therefore$ Length of the rod as measured by the brass scale at 40°C = $\frac{50.024}{1.0072} = 49.988$ cm.

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56. 2068 Q.No. 10 a A steel wire 8m long and 4mm. in diameter is fixed to two rigid supports. Calculate the increase in tension when the temperature falls by 10°C. [4]

Please refer to 2075 GIE Q.No. 10c

57. 2068 Old Q.No. 5 b A brass rod of length 0.40 m and steel rod of length 0.60 m, both are initially at 0°C are heated to 75°C. If the increase in lengths is the same for both the rods, calculate the linear expansivity of brass. The linear expansivity of steel is $12\times10^{-6}\,°C^{-1}$. [4]

Given,

Length of bass rod at 0°C (l_o^b) = 0.40m

Length of steel rod at 0°C (l_o^s) = 0.60m

Initial temperature (θ_1) = 0°C

Final temperature (θ_2) = 75° C

Linear expansivity of steel (α_s) = $12 \times 10^{-6}\,°C^{-1}$

Linear expansivity of brass (α_b) = ?

Let, l_2^b and l_2^s be the lengths of brass and steel at θ_2 °C then,

$l_2^b = l_o^b(1 + \alpha_b\,\Delta\theta)$

or, $l_2^b - l_o^b = \alpha_b\, l_o^b\,\Delta\theta$

Similarly,

$l_2^s - l_o^s = \alpha_s\, l_o^s\,\Delta\theta$

By question

$l_2^b - l_o^b = l_2^s - l_o^s$

or, $\alpha_b\, l_o^b\,\Delta\theta = \alpha_s\, l_o^s\,\Delta\theta$

$$\alpha_b = \frac{\alpha_s \cdot l_o^s}{l_o^b} = \frac{12\times10^{-6}\times0.60}{0.40}$$

$= 18 \times 10^{-6}\,°C^{-1}$

58. 2067 Q.No. 10 a A copper vessel with a volume of exactly 100m3 at a temperature of 15°C is filled with glycerin. If the temperature rises to 25°C, how much glycerin will spill out? [4]

Given,

Volume of copper vessel at 15°C (V_1) = $100m^3$

Initial temperature (θ_1) = 15°C

Final temperature (θ_2) = 25°C

Linear expansivity of copper (α_c) = $16.7\times 10^{-6}\,°C^{-1}$

Cubical expansivity of glycerene (γ_m) = $5.3 \times 10^{-4}\,°C^{-1}$

Amount of glycerene spilt out = ?

Let, V_2 be the volume of copper at 25°C

$V_2 = V_1(1 + \gamma_c\Delta\theta) = 100\,(1 + 3 \times 16.7 \times 10^{-6} \times 10) = 100.05\ m^3$

Let, V_2' be the volume of mercury at 25°C, they

$V_2' = V_1(1 + \gamma_m\,\Delta\theta)$

$= 100\,(1 + 5.3 \times 10^{-4} \times 10)$ $\quad [\because \gamma = 3\alpha]$

$= 100.53\ m^3$

Volume of mercury spilt,

$= V_2' - V_2 = (100.53 - 100.05)m^3 = 0.48\ m^3$

59. 2067 Supp Q.No. 10 a A glass flask of volume 500 cm3 is just filled with mercury at 0°c. How much mercury overflows when the temperature of the system is raised to 80°c. [4]

Given,

Volume of glass at 0°C (V_1) = $500\ cm^3$

Initial temperature (θ_1) = 0°C
Final temperature (θ_2) = 80°C
Cubical expansivity of glass (γ_g) = 1.2×10^{-5} °C^{-1}
Cubical expansivity of mercury (γ_m) = 1.8×10^{-5} °C^{-1}
Volume of mercury overflow at 80°C = ?
Let, V_2 be the volume of glass at 80°C, then

$V_2 = V_1 (1 + \gamma_g \Delta\theta) = 500 (1 + 1.2 \times 10^{-5} \times 80) = 500.48$ cm³

Let, V_2' be the volume of mercury at 80°C,

$V_2' = V_1 (1 + \gamma_m \Delta \theta) = 500 (1 + 1.8 \times 10^{-5} \times 80) = 500.72$ cm²

Volume of mercury overflows = $V_2' - V_2 = 500.72 - 500.48 = 0.24$ cm³

60. 2066 Old Q. No. 5 b A clock which has a brass pendulum beats seconds correctly when the temperature of the room is 30°C. How many seconds will it gain or lose per day when the temperature of the room falls to 10°C? (α for brass = 0.000018°C⁻¹). **[4]**

Given,

Initial temperature (θ_1) = 30°C
Final temperature (θ_2) = 10°C
Linear expansivity (α) = 0.000018°C^{-1}
Loss in time per day = ?
Let, T_{30} and T_{10} be the time periods at 30°C and 10°C.

We know, time period of pendulum is, $T = 2\pi \frac{l}{g}$.

So,

$$T_{30} = 2\pi\sqrt{\frac{l_{30}}{g}} \quad \ldots(i)$$

and,

$$T_{10} = 2\pi\sqrt{\frac{l_{10}}{g}} \quad \ldots(ii)$$

Divide (i) by (ii)

$$\frac{T_{30}}{T_{10}} = \sqrt{\frac{l_{30}}{l_{10}}} = \sqrt{\frac{l_{10}(1+\alpha\Delta\theta)}{l_{10}}} \qquad [\because l_2 = l_1'(1 + \propto \Delta\theta]$$

$$\frac{T_{30}}{T_{10}} = \sqrt{1 + \alpha \Delta \theta} = \sqrt{1 + 0.000018 \times 20} = 1.00018$$

Now,

$$\frac{T_{30} - T_{10}}{T_{10}} = 0.00018$$

This is the gain in time per sec.

∴ Gain in time per day = $0.00018 \times 60 \times 60 \times 24 = 16$ sec.

61. 2063 Q.No. 3 b A copper wire of diameter 0.5 mm. is stretched between two points at 25°C. Calculate the increase in tension in the wire if the temperature falls to 0°C. (Young's modulus for copper = 1.2×10^{11} Nm^{-2}, linear expansivity for copper = 18×10^{-6} K^{-1})

Given,

Diameter of copper wire (d) = 0.5mm

Radius of copper wire (r) = $\frac{d}{2}$ = 0.25 mm = 0.25×10^{-3} m

Initial temperature (θ_1) = 25°C
Final temperature (θ_2) = 0°C
Young's modules of copper (Y) = 1.2×10^{11} Nm^{-2}
Linear expansivity of copper (α) = 18×10^{-6} K^{-1}
Increase in tension (T) = ?
We know,

$$Y = \frac{T/A}{\Delta l/l} \qquad [\because \alpha = \frac{\Delta l}{l\Delta\theta} \text{ or, } \Delta l = \alpha.l\Delta\theta]$$

or, $Y = \frac{T}{A} \cdot \frac{l}{\Delta l}$

or, $T = YA.\frac{\Delta l}{l} = YA.\frac{\alpha l \Delta\theta}{l}$

$= YA\alpha\Delta\theta = 1.2 \times 10^{11} \times \pi (0.25 \times 10^{-3})^2 \times 18 \times 10^{-6} \times 25$

$= 10.6$ N

62. **2055 Q.No. 9 OR** **The density of silver at 0°C is 10310 kgm^{-3} and the coefficient of linear expansion is 0.000019/°C. Calculate its density at 100°C.**

Given,

Initial temperature (θ_1) = 0°C

Final temperature (θ_2) = 100°C

Initial density of silver at 0°C (ρ_1) = 10310 kg/m^3

Final density of silver at 100°C (ρ_2) = ?

Coefficient of liner expansion of steel (α) = 0.000019/°C

We have,

$$\rho_2 = \frac{\rho_1}{1 + \gamma(\theta_2 - \theta_1)} \qquad [\because \gamma = 3\alpha]$$

$$= \frac{10310}{1 + 3 \times 0.000019\ (100 - 0)} = 10251.566 \text{ kg/m}^3$$

63. **2053 Q.No. 8** **An aluminum rod when measured with a steel scale, both being at 25°C appears to be 1 m long. If the scale is correct at 0°C, what will be the length of the rod at 0°C? (Linear expansivity of aluminum 26 × 10^{-6} K^{-1} and of steel = 12 × 10^{-6} K^{-1}).**

Given,

Linear expansivity of aluminium (α_{al}) = 26×10^{-6} K^{-1},

Linear expansivity of steel (α_s) = 12×10^{-6} K^{-1}

At 0° C, the steel scale is correct (l_0) = 1 m

At 25° C, length of the steel scale (l_{25}) = ?

From the linear expansion (l_{25}) = $l_0 (1 + \alpha\,\Delta\theta)$

$= 1[1 + \alpha (25 - 0)] = 1[1 + 12 \times 10^{-6} (25 - 0)] = 1.0003$ m

∴ True length l_{25} of steel scale at 25° C = 100.03 cm

∴ True length of aluminium rod at 25° C, $l^A_{25} = l_{25} = 1.0003$ m

Suppose l_0 be the length of the aluminium rod at 0° C.

∴ $l^A_{25} = l^A_0 [1 + \alpha_{al} . (25 - 0)]$

or, $1.0003 = l_0 [1 + 26 \times 10^{-6} \times 25]$

or, $l_0 = \frac{1000.03}{1.00065} = 0.9996$ m

∴ The length of the aluminimum rod at 0° C = 0.9996 m

❑❑❑

Chapter 3: Quantity of Heat

Short Answer Questions

1. 2076 Set B Q.No. 2a **The latent heat of vaporization is much larger than the latent heat of fusion of a substance. Why?** [2]

   When a body is converted from liquid to vapour state, much more energy is required than the energy needed to convert same body or substance from solid to liquid state. It is due to the reason that to convert a material liquid to vapour, the molecules are to be separated out by large distance where as to convert from solid to liquid, the molecules are to be separated by relatively smaller distances. Hence, latent heat of vapourization of a materials is greater than that of latent heat of fusion.

2. 2075 Set A Q.No. 2c **Why does steam at 100°C give more severe burn than water at the same temperature?** [2]

   The amount of heat contained in steam at 100°C is 540 cal/gm (latent heat of vaporization) more than that of water at 100°C. So, when we touch the steam, more heat flows to our body than from water, both being at 100°C. Hence, steam at 100°C produce more severe burn than water at the same temperature.

3. 2073 Set C Q.No. 2a **Groundnuts are fried along with sand, why?** [2]

   The specific heat capacity of sand is low i.e. its temperature rapidly increases with small increase in heat. When the groundnut fried along with sand, the groundnut gets fried fastly with lower heat supplied due to greater temperature.

4. 2072 Supp Q.No. 2a **Why is the groundnut fried along with sand?** [2]

   Please refer to 2073 Set C Q.No. 2a

5. 2072 Set E Q.No. 2c **Water remains cool in earthen pots than in metal pots in summer. Explain why?** [2]

   The earthen pot consists of a large number of small pores. Through these pores, water comes out and is evaporated. For the evaporation, heat is absorbed from the water inside the pot. Since, the water inside the pot has lost heat, it remains cold in the earthen pot during the summer.

6. 2069 Supp Q.No. 2a **A medical officer prescribes to put wet cloths on the forehead of a person suffering from high fever, why?** [2]

   Water has large specific heat capacity. When a wet cloth is kept on the forehead of a person, it absorbs large amount of heat from the person's forehead without rising its temperature and temperature of the high fever person falls down quickly. Thus, the person feels relief on loosing heat. Therefore, during high fever, a wet cloth is kept on the forehead of a person.

7. 2069 Supp Q.No. 2c **Why does the temperature of ice fall when some salt is added to it?** [2]

   When salt is added to the ice, the ice provides latent heat to the salt to melt. Since, the ice loses heat its temperature falls.

8. 2069 (Set A) Q. No. 2b **If you add heat to an object, do you necessarily increases its temperature? Justify your.** [2]

   No, the temperature of the object is not necessarily increased if the object is heated. As an example, the ice gets melted when it is heated at zero degree without gaining its temperature i.e. at latent heat of fusion of ice and similarly at the latent heat of vaporization.

9. 2069 (Set A) Old Q. No. 4a **Water in earthen pot remains cold in summer. Explain, why?** [2]

   Please refer to 2072 Set E Q.No. 2c

10. 2069 (Set B) Q. No. 2a **Why do you feel cool in the mouth when you eat halls?** [2]

    Halls needs certain heat energy to melt when it is kept into the mouth, it takes some energy. This means our body losses some heat energy to melt the halls. Since mouth losses heat, we feel cold.

11. 2069 Old (Set B) Q. No. 4b **Why does a food cook faster in pressure cooker than open vessel with boiling water?** [2]

    When the amount of water vapor and hence, water vapor pressure above the surface of water level increases, the boiling point of the water also increases. Pressure cooker is a closed vessel. When water is heated in it, pressure of water vapor above the surface of water increases, so boiling point of water rises(since a liquid boils when its vapour pressure becomes equal to the pressure above its surface) above 100°C. It means, the same mass of the water contains more amount of heat. As a result, the food is cooked faster in a pressure cooker.

12. **2069 Old (Set B) Q. No. 4c Explain why a person feels cooling when he pours a little spirit on his hand.** **[2]**

Perfume or spirit is a volatile substance; their molecules evaporate at a higher rate even at normal temperature. When they are sprayed over a skin surface, they take the body heat and evaporate faster. During evaporation, the body heat is also taken away by the molecules. So, that portion feels colder compared to other parts.

13. **2067 Q.No. 2 a During a high fever, a wet cloth is kept on the forehead of a person. Why?** **[2]**

Please refer to 2069 Supp Q.No. 2a

14. **2066 Old Q. No. 4 a How is it that ice cream appears cooler to the mouth than ice at 0°C?** **[2]**

The ice cream contains impurities than pure ice; as a result the melting point of ice cream is lower than the melting point of ice. So, ice cream appears cooler to the mouth than ice at 0°C.

15. **2066 Old Q. No. 4 c If you wet your hand and pickup an ice tray that is below 0°C, your hand may stick to it. This does not happen to wood. Why?** **[2]**

If we wet our hands and pickup on ice tray which is below 0°C, our hands may stick to it because the water of our hand freezes. The ice particles get diffused to each other and stick to each other. If we touch wood, the water and wood do not stick because their molecules do not diffuse with each other.

16. **2061 Q.No. 4 c 2053 Q.No. 7 c Explain why water remains cool in earthen pot in summer.** **[2]**

Please refer to 2072 Set E Q.No. 2c

17. **2060 Q.No. 4 c Why do we feel cold when we spray perfume on our body?** **[2]**

Please refer to 2069 Old (Set B) Q. No. 4c

18. **2056 Q.No. 4 a When you come out of swimming pool, you feel cold. Why?** **[2]**

When we come out of swimming pool, our body is wet with water. After some time, this water is evaporated from our body taking the heat of the body reducing the temperature of the body as a result we feel cool.

19. **2056 Q.No. 4 b Why does steam at 100°C causes severe burns than hot water at 100°C?** **[2]**

Please refer to 2075 Set A Q.No. 2c

20. **2054 Q.No. 7 c Why can you get a more severe burn from steam at 100°C than from water at 100°C?** **[2]**

Please refer to 2075 Set A Q.No. 2c

21. **2053 Q.No. 7 b Why is spark produced when two stones are stricken against one another?** **[2]**

When two stones are stricken against each other, work is done on the expense of muscular energy used this work done is converted into heat energy due to friction producing spark.

22. **2051 Q.No. 7 d During high fever, a wet cloth is kept on the forehead of a person. Why?.** **[2]**

Please refer to 2069 Supp Q.No. 2a

23. **2050 Q.No. 7 b Why does food cook faster in a pressure cooker than in an open pot?**

Please refer to 2069 Old (Set B) Q. No. 4b

Long Answer Questions

24. **2076 GIE Set A Q.No. 6a State principle of Calorimetry and use it to determine specific latent heat of fusion of ice by the method of mixture.** **[4]**

Specific heat capacity: This is amount of heat required to increase the temperature of a body of unit mass by unit degree of temperature.

Latent heat: This is amount of heat energy required to change the substance from solid state to liquid state.

Latent heat of fusion of ice by the method of mixture:

Let us take a clean and dry calorimeter with stirrer and weighted it. The calorimeter is filled with water about one third and weighted again. The calorimeter is now placed into the wooden box and initial temperature is noted. Small pieces of ice are taken and water in them is soaked with blotting paper and then dropped gently into the calorimeter containing water. The mixture is stirred gently until the temperature of mixture falls down and becomes constant. The final temperature of mixture is noted and weighted the mixture again.

Suppose,

Mass of calorimeter and stirrer = m_c

Mass of calorimeter stirrer and water = m_l

Mass of water taken = $m_l - m_c = m_w$

Initial temperature of calorimeter and water= θ_l

Mass of calorimeter, stirrer, water and ice = m_2

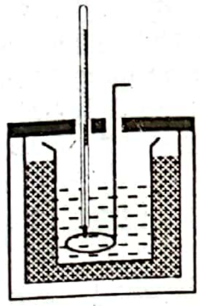
Mass of ice taken = $m_2 - m_1 = m_i$

Final temperature of mixture = θ

Specific heat capacity of water= S_w

Specific heat capacity of calorimeter system = S_c

Latent heat of fusion of ice = L_i



Heat gained by the ice during melting = $m_i L_i$

Heat gained by the melted ice being heated from 0°C to the constant final temperature θ°C = $m_i S_w (\theta - 0) = m_i S_w\theta$

Thus, the total heat gained = $m_iL_i + m_i S_w\theta$

The heat lost by the calorimeter system and water

$$= m_c S_c (\theta_l - \theta) + m_w S_w (\theta_l - \theta) = (m_cS_c + m_w S_w) (\theta_l - \theta)$$

The principle of calorimetry states that heat gained by cold body is equal to heat lost by hot body in an isolated system.

From the principle of calorimetry,

Heat gained by ice = heat lost by calorimeter system and water

$$m_iL_i + m_iS_w\theta = (m_c S_c + m_w S_w) (\theta_l - \theta)$$

$$m_i L_i = (m_c S_c + m_w S_w) (\theta_l - \theta) - m_i S_w \theta$$

$$L_i = \frac{(m_cS_c + m_wS_w) (\theta_l-\theta) - m_iS_w \theta}{m_i}$$

$$\therefore L_i = \frac{(m_cS_c + m_wS_w) (\theta_l - \theta)}{m_i} - S_w\theta$$

This is the required expression for the latent heat of fusion of ice.

25. **2076 Set B Q.No. 6a State Newton's law of cooling. Use it to find the specific heat capacity of a unknown liquid.** **[4]**

Newton's law of cooling: It is stated as, "the rate of heat lost (rate of cooling) is proportional to the excess of temperature over the surrounding and is independent to the nature of the liquid."

The rate of cooling depends on the exposed area of the heat loosing surface and its nature. Higher the temperature difference between the body and surroundings, more heat lost by the body per unit time.

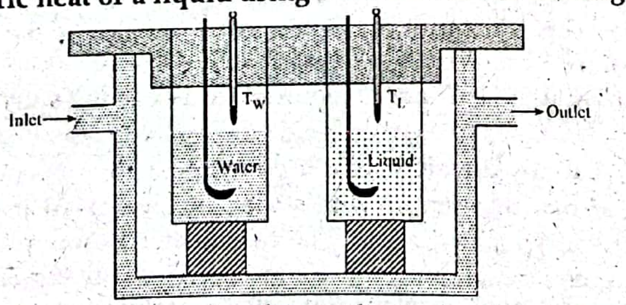
Suppose a liquid at a temperature θ°C loses a small amount of heat dQ in small time dt, let θ_o°C be the temperature of the surrounding. Then, according to Newton's law of cooling, we can write

Rate of heat lost ∝ temperature difference between liquid and surrounding.

i.e. $$-\frac{dQ}{dt} \propto (\theta - \theta_o) \Rightarrow -\frac{dQ}{dt} = K (\theta - \theta_o)$$

where K is the proportionality constant and its value depends upon the nature of liquid and the surface area exposed to atmosphere. The negative sign indicates that as the time passes, the liquid gives out the heat.

Expression for the specific heat of a liquid using Newton's law of cooling:



Let us consider m_1 and m_2 be the masses of two identical calorimeters made of same material of specific heat capacity S_C. Suppose, first calorimeter contains water having mass m_w and second

calorimeter contains same volume of liquid (whose specific heat is to be determined) having mass m_L. Suppose t_1 and t_2 are the time in which water and liquid cools through temperature θ_1 to θ_2.
Now,

Heat lost by first calorimeter to cool from θ_1 to $\theta_2 = m_1S_C(\theta_1 - \theta_2)$
Heat lost by water in calorimeter to cool from θ_1 to $\theta_2 = m_wS_w(\theta_1 - \theta_2)$
Total heat lost by calorimeter and water in cooling through θ_1 to θ_2 is

$$= m_1S_C(\theta_1 - \theta_2) + m_wS_w(\theta_1 - \theta_2) = (m_1S_C + m_wS_w)(\theta_1 - \theta_2)$$

Rate of cooling of water $= \dfrac{(m_1S_C + m_wS_w)(\theta_1 - \theta_2)}{t_1}$ ... (i)

Similarly, if S_L is the specific heat of liquid, then

rate of cooling of liquid $= \dfrac{(m_2S_C + m_LS_L)(\theta_1 - \theta_2)}{t_2}$ ... (ii)

As the liquid and water have been cooled under identical conditions, then their rate of cooling must be same accordance with Newton's law of cooling.

i.e. $$\frac{(m_1S_C + m_wS_w)(\theta_1 - \theta_2)}{t_1} = \frac{(m_2S_C + m_LS_L)(\theta_1 - \theta_2)}{t_2}$$

$$m_2S_C + m_LS_L = \frac{(m_1S_C + m_wS_w)t_2}{t_1}$$

$$\therefore S_L = \frac{(m_1S_C + m_wS_w)t_2}{m_Lt_1} - \frac{m_2S_C}{m_L}$$

The above expression is used to find the specific heat of liquid by method of mixture.

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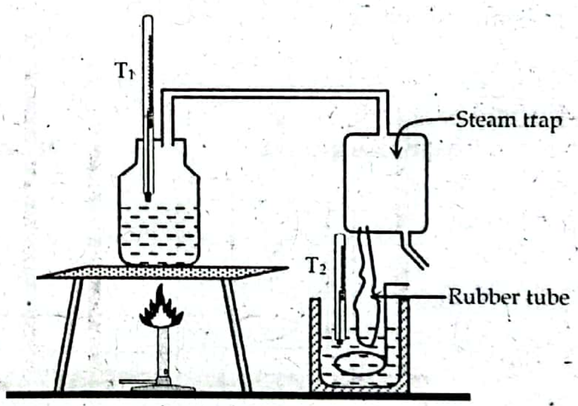
26. 2075 GIE Q.No. 6c | 2066 Old Q. No. 5 a OR Describe the method to determine latent heat of vaporization of water in laboratory. [4]

Specific latent heat of vaporization of a liquid (L_V): The amount of heat required to convert unit mass of a substance from liquid state to vapour state at its boiling point is called specific latent heat of vaporization of the liquid. It is represented by L_V and is given by

$$L_V = \frac{\text{amount of heat } (Q)}{\text{mass } (m)} = \frac{Q}{m}$$

The latent heat of vaporization of water is 2.26×10^6 J/kg or 540 cal/ gm.



Measurement of specific latent heat of vaporization (L_v): The experimental arrangement of the determination of latent heat of vaporization is as shown in figure.

A calorimeter and stirrer is weighted after cleaning, fill half of calorimeter by water and then weighted it again and temperature of system, θ_1 is noted. Steam is prepared by boiling water at boiler so that the thermometer shows fixed temperature θ_2 of steam. The steam is then passed in the calorimeter containing water and stir by stirrer. When the temperature of the mixture is raised by about 5°C to 10°C, the temperature of mixture and whole mass of the system is taken.

Calculation:

Mass of calorimeter and stirrer = m_c
Specific heat of material of calorimeter = S_c
Initial temperature of calorimeter and water = θ_1
Mass of water taken = m_w
Temperature of steam = θ_2
Final temperature of mixture = θ
Mass of steam added = m_s
Latent heat of vaporization of water = L_v
Specific heat capacity of water = S_w
Now,
Heat lost by steam on condensation = m_sL_v

Heat lost by condensed steam (water) on cooling from θ_2 to $\theta = m_s (\theta_2 - \theta) S_w$

$\therefore$ Total heat lost by steam $= m_s L_v + m_s (\theta_2 - \theta) S_w$

And heat gained by water and calorimeter

$= m_c S_c (\theta - \theta_1) + m_w S_w (\theta - \theta_1)$

$= (m_c S_c + m_w S_w)(\theta - \theta_1)$

From the principle of calorimetry, we can write

heat lost by condensed steam = heat gained by calorimeter and water

$m_s L_v + m_w S_w (\theta_2 - \theta) = (m_c S_c + m_w S_w)(\theta - \theta_1)$

$m_s L_v = (m_c S_c + m_w S_w)(\theta - \theta_1) - m_w S_w (\theta_2 - \theta)$

$$\therefore L_v = \frac{(m_c S_c + m_w S_w)(\theta - \theta_1)}{m_s} - S_w(\theta_2 - \theta)$$

This is the required expression for the latent heat of vaporization.

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27. **2075 Set A Q.No. 6a** **What is meant by heat capacity? Describe the method of mixture to determine specific heat capacity of a solid.** **[4]**

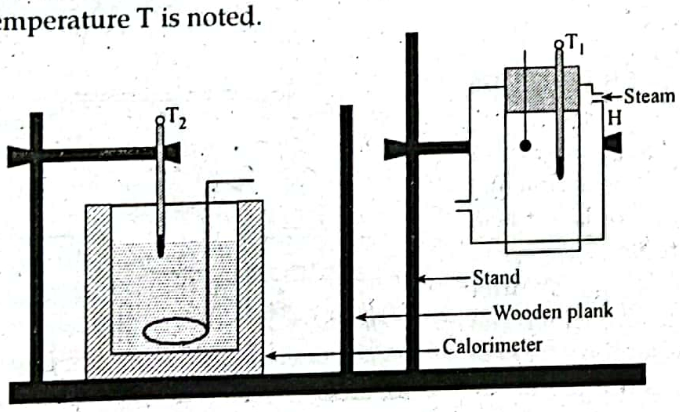
Heat capacity: The amount of heat energy required to increase the temperature of a substance by 1°C is called heat capacity.

Specific heat capacity: The amount of heat energy required to increase the temperature of unit mass of a substance by 1°C is called its specific heat capacity. It is denoted by S and given by

$$S = \frac{\text{Amount of heat (Q)}}{\text{Mass (m)} \times \text{Temp. diff } (\Delta\theta)} = \frac{Q}{m\,\Delta\theta}$$

Its unit is J/kg°C.

Determination of specific heat capacity of a solid by the method of mixture: Let us consider a solid of mass m_s and its specific heat capacity is S_s. The solid is heated to the temperature T_2 and dropped in a calorimeter with water at initial temperature T_1. Suppose m_c and S_c are the mass and specific heat capacity of calorimeter respectively and m_w is the mass of water taken in calorimeter and S_w specific heat capacity of water. When the solid is dropped into water, the temperature of water increased and temperature of solid decreased. By stiring water, it can reach to a constant temperature. The final constant temperature T is noted.



Now, Heat lost by the body when its temperature fall from T_2 to $T = m_s S_s (T_2 - T)$

Heat gained by calorimeter and water system

$= m_c S_c (T - T_1) + m_w S_w (T - T_1)$

$= (m_c S_c + m_w S_w)(T - T_1)$

Now, according to principle of calorimetry, heat lost by hot body = heat gain by calorimeter and water system.

$m_s S_s (T_2 - T) = (m_c S_c + m_w S_w)(T - T_1)$

$$\therefore S_s = \frac{(m_c S_c + m_w S_w)(T - T_1)}{m_s (T_2 - T)}$$

Thus, the specific heat capacity of solid can be calculated by using above expression.

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28. **2074 Supp. Q.No. 6b** **State Newton's law of cooling. Use this law to determine the specific heat capacity of a liquid.** **[4]**

Please refer to 2076 Set B Q.No. 6a

29. 2074 Set A Q.No. 6a State Newton's Law of cooling. Describe how is this law applied to determine the specific heat capacity of a liquid. [4]

Please refer to 2076 Set B Q.No. 6a

30. 2074 Set B Q.No. 6c Describe experimental method, with necessary theory, to determine the latent heat of vaporization of water. [4]

Please refer to 2075 GIE Q.No. 6c

31. 2073 Supp Q.No. 6a Describe, in brief, the laboratory method of mixture to determine the specific heat capacity of a solid. [4]

Please refer to 2075 Set A Q.No. 6a

32. 2073 Set D Q.No. 6c State and explain Newton's law of cooling. Describe with mathematical detail a method for the determination of specific heat capacity of a liquid. [4]

Please refer to 2076 Set B Q.No. 6a

33. 2072 Supp Q.No. 6b What is specific heat capacity of a substance? Describe the method of mixture to measure the specific heat capacity of solid. [4]

Please refer to 2075 Set A Q.No. 6a

34. 2072 Set D Q.No. 6a State and explain Newton's law of cooling. Describe with mathematical detail a method for the measurement of specific heat capacity of a liquid. [4]

Please refer to 2076 Set B Q.No. 6a

35. 2072 Set E Q.No. 6b What is the difference between the "specific heat capacity" and "latent heat"? Describe in brief, the laboratory experiment for the determination of latent heat of fusion of ice. [4]

Please refer to 2076 GIE Set A Q.No. 6a

36. 2071 Supp Q.No. 6a Define specific latent heat of fusion of ice. Develop an expression for the determination of specific latent heat of fusion of ice. [4]

Specific Latent heat of fusion: The amount of heat required to change unit mass of ice to water at its meting point or 0°C is called Specific latent heat of fusion of ice. The latent heat of fusion of ice is 80 cal/g.

Latent heat of fusion of ice by the method of mixture: Please refer to 2076 GIE Set A Q.No. 6a

37. 2071 Set D Q.No. 6 a Describe in brief the method of mixture to determine the specific heat of a solid in the laboratory? [4]

Please refer to 2075 Set A Q.No. 6a

38. 2069 Supp Q.No. 6b Describe the experiment that is applied to determine the specific heat capacity of a solid by the method of mixture. [4]

Please refer to 2075 Set A Q.No. 6a

39. 2069 (Set B) Q. No. 6a 2062 Q.No. 5 a State and explain Newton's law of cooling and derive the expression for the specific heat of the liquid. [4]

Please refer to 2076 Set B Q.No. 6a

40. 2069 Old (Set B) Q. No. 5a Why the temperature is constant during change of state of the substance? Develop an expression for the determination of the latent heat of vaporization. [1+4]

The temperature is constant during change of state of substance because heat is used to change the state of substance without raising its temperature.

Second Part: Please refer to 2075 GIE Q.No. 6c

41. 2068 Q.No. 6 a Define latent heat of fusion of ice. Describe the method for the measurement of it in the laboratory. [4]

Specific Latent heat of fusion: Please refer to 2071 Supp Q.No. 6a

Latent heat of fusion of ice by the method of mixture: Please refer to 2076 GIE Set A Q.No. 6a

42. 2068 Old Q.No. 5 a Define latent heat of fusion of ice. Explain the method of determining the latent heat of fussion of ice by the method of mixture in laboratory. [1+4]

Specific Latent heat of fusion: Please refer to 2071 Supp Q.No. 6a

Latent heat of fusion of ice by the method of mixture: Please refer to 2076 GIE Set A Q.No. 6a

43. **2067 Supp Q.No. 6 a** **Explain with necessary theory how to determine the specific heat capacity of a liquid by the method of cooling.**

Please refer to 2076 Set B Q.No. 6a

44. **2067 Q.No. 6 a** **2060 Q.No. 5 a OR** **Define specific heat of substance. Describe the method of mixture to determine the specific heat of a solid.** **[4]**

Please refer to 2075 Set A Q.No. 6a

45. **2065 Q. No. 5 a** **State and explain Newton's law of cooling. Determine the specific heat capacity of liquid by method of cooling.** **[1+4]**

Please refer to 2076 Set B Q.No. 6a

46. **2064 Q.No. 5 a** **Define specific heat capacity and heat capacity. Describe the method of mixture to determine the specific heat capacity of a liquid.** **[1+4]**

Please refer to 2075 Set A Q.No. 6a

47. **2063 Q.No. 5 a** **State and explain Newton's Law of cooling.**

Please refer to 2076 Set B Q.No. 6a

48. **2058 Q.No. 5 a** **Explain how you determine the specific heat of a solid by the method of mixture.**

Please refer to 2075 Set A Q.No. 6a

49. **2055 Q.No. 8** **What is specific latent heat of vapourisation of a liquid? Develop an expression for the determination of the latent heat of vapourisation.**

Please refer to 2075 GIE Q.No. 6c

Numerical Problems

50. **2076 GIE Set B Q.No. 10a** **How much heat energy is required to convert 200 gram Ice at – 20°C to steam at 100°C?** **[4]**

Given,

Amount of heat (Q) = ?

Mass of ice (m_i) = 200 gm = 0.2 kg

Specific heat capacity of ice (S_i) = 2100 J kg^{-1} $°C^{-1}$

Specific latent heat capacity of ice (L_i) = 336000 J kg^{-1}

Specific heat capacity of water (S_w) = 4200 J kg^{-1} $°C^{-1}$

Latent heat of vapourization (L_V) = 2.26×10^6 J/kg^{-1}

–20°C		0°C		0°C		100°C		100°C
Ice	→	Ice	→	Water	→	Water	→	Steam

Amount of heat required to convert ice from –20°C to ice at 0°C is

$$Q_1 = m_i S_i \{0 - (-20)\}$$
$$= 0.2 \times 2100 \times 20$$
$$= 8400 \text{ J}$$

Amount of heat required to convert ice from 0°C to water at 0°C is

$$Q_2 = m_i L_i$$
$$= 0.2 \times 3.36 \times 10^5$$
$$= 67200 \text{ J}$$

Amount of heat required to convert water from 0°C to water at 100°C is

$$Q_3 = m_w S_w (100 - 0)$$
$$= 0.2 \times 4200 \times 100$$
$$= 84000 \text{ J}$$

Amount of heat required to convert water from 100°C to vapour at 100°C is

$$Q_3 = m_w L_v$$
$$= 0.2 \times 2.26 \times 10^6$$
$$= 452000 \text{ J}$$

Total heat required to convert 0.2 kg of ice from $-20°C$ to vapour at $100°C$ is,

$Q = Q_1 + Q_2 + Q_3 + Q_4$

$= 8400 + 67200 + 84000 + 452000$

$= 611600$ J

51. **2076 Set B Q.No. 10b** **A copper calorimeter of mass 300 gm contains 500 gm of water at temperature 15°C. A 560 gm block of aluminium at temperature 100°C is dropped in the calorimeter and the temperature is observed to increase to 22.5°C. Find the specific heat capacity of aluminium.** **[4]**

Given,

Specific heat capacity of copper (Sc) = 400 J $kg^{-1}\,°C^{-1}$

Specific heat capacity of water (S_w) = 4200J $kg^{-1}°C^{-1}$

Specific heat capacity of aluminium (S_{al}) = ?

Mass of calorimeter (m_c) = 300 gm = 0.3 kg

Mass of water (m_w) = 500 gm = 0.5 kg

Mass of alluminium (m_{al}) = 560 gm = 0.56 kg

Temperature of water and calorimeter (θ_2) = 15°C

Temperature of alluminium (θ_1) = 100°C

Final temperature (θ) = 22.5°C

Now, by using principle of calorimetry which states as

Heat lost by alluminium = Heat gained by colarimetry

$m_{al} \times S_{al} \times (\theta_1 - \theta) = (m_c \times S_c + m_w \times S_w) \times (\theta - \theta_2)$

$0.56 \times S_{al} \times (100 - 22.5) = (0.3 \times 400 + 4200 \times 0.5)\ (22.5 - 15)$

or, $S_{al} = 383.64$ J $kg^{-1}\ °C^{-1}$

52. **2076 Set C Q.No. 10a** **A substance takes 3 minutes in cooling from 50° C to 45° C and takes 5 minutes in cooling 45° C to 40° C. What is the temperature of the surroundings?** **[4]**

Given,

$t_1 = 3$ min,

$t_2 = 5$ min

Average temperature between 50°C and 45°C = $\frac{50 + 45}{2} = 47.5°$ C.

Average temperature between 45°C and 40°C = $\frac{40 + 45}{2} = 42.5°$ C

Let $\theta°$ C be the temperature of surrounding,

From Newton's law of cooling,

$$\left(\frac{dQ}{dt}\right)_1 = K(47.5 - \theta)$$

$$\text{or, } ms\left(\frac{d\theta}{dt}\right)_1 = K(47.5 - \theta) \qquad \ldots (i)$$

Similarly,

$$ms\left(\frac{d\theta}{dt}\right)_2 = K(42.5 - \theta) \qquad \ldots (ii)$$

Again,

$$\left(\frac{d\theta}{dt}\right)_1 = \frac{50 - 45}{3}$$

and

$$\left(\frac{d\theta}{dt}\right)_2 = \frac{45 - 40}{5}$$

Dividing (i) and (ii) we get

$$\frac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)_2} = \frac{(47.5-\theta)}{(42.5-\theta)}$$

or, $\frac{5}{3} = \frac{(47.5-\theta)}{(42.5-\theta)}$

or, $212.5 - 5\theta = 142.5 - 3\theta$

or, $70 = 2\theta$

$\therefore\ \theta = 35°C$

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53. **2075 GIE Q.No. 10a** **How much heat is required to convert 10kg of ice at -10°C into steam at 100°C ? (SP. heat of ice = 2100Jkg^{-1} K^{-1}, latent heat of fusion of ice = 3.36×10^{5}Jkg^{-1}, latent heat of vaporization = 2.268 × 10^{6}Jkg^{-1})**

Given,

Mass of ice (m_i) = 10 kg

Specific heat capacity of ice (S_i) = 2100 J kg^{-1}°C^{-1}

Specific latent heat of ice (L_i) = 336000 J kg^{-1}

Specific heat capacity of water (S_w) = 4200 J kg^{-1}°C^{-1}

Latent heat of vapourization (L_v) = 2.26× 10^{6}J/kg

Amount of heat (Q) = ?

$$\underset{-10°C}{\boxed{\text{Ice}}} \xrightarrow{Q_1} \underset{0°C}{\boxed{\text{Ice}}} \xrightarrow{Q_2} \underset{0°C}{\boxed{\text{Water}}} \xrightarrow{Q_3} \underset{100°C}{\boxed{\text{Water}}} \xrightarrow{Q_4} \underset{100°C}{\boxed{\text{Steam}}}$$

Amount of heat required to convert ice from –10°C to ice at 0°C is

$Q_1 = m_iS_i\,(0-(-10))$

$= 10 \times 2100 \times 10 = 210000$ J

Amount of heat required to convert ice from 0°C to water at 0°C

$Q_2 = m_iL_i = 10 \times 3.36\ 10^5$

$= 3360000$ J

Amount of heat required to convert water at 0°C to water at 100°C

$Q_3 = m_wS_w\,(100-0)$

$= 10 \times 4200 \times 100 = 4200000$ J

Amount of heat required to convert water at 100°C to vapour at 100°C.

$Q_4 = m_wL_V$

$= 10 \times 2.26 \times 10^6 = 22600000$ J

Total heat required to convert 10 kg of ice from –10°C to vapour at 100°C is

$Q = Q_1 + Q_2 + Q_3 + Q_4$

$= 210000 + 3360000 + 4200000 + 22600000$

$= 30370000 = 3.037 \times 10^7$ J

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54. **2075 Set B Q.No. 10a** **A mixture of 500g water and 100 g ice at 0°C is kept in a copper calorimeter of mass 200 g. How much steam from the boiler be passed to the mixture so that the temperature of the mixture reaches 40°C?** **[4]**

Given,

Mass of water (m_w) = 500 g = 0.5 kg

Mass of ice (m_i) = 100 g = 0.1 kg

Mass of calorimeter (m_c) = 200 g = 0.2 kg

Mass of steam (m_s) = ?

Final temperature (θ) = 40°

Specific heat capacity of water (s_w) = 4200 J/kg K

Specific heat capacity of copper (s_c) = 400 J/kg K

Latent heat of fusion of ice (L_i) = 3.36 × 10^5 J/kg

Latent heat of vaporization of water (L_v) = 2.26 × 10^6 J/kg

Now,

Heat gained by 0°C ice to melto to 0°C water = m_iL_i

$= 0.1 \times 3.36 \times 10^5 = 3.36 \times 10^4 J$

Heat gained by water from 0°C to water of 40°C $= m_w s_w(40-0) + m_i s_i(40-0) + m_c s_c(40-0)$

$= 0.5\times4200\times40 + 0.1\times4200\times40 + 0.2\times400\times40$

$= 1.04 \times 10^5 J$

Total heat gained $= 1.04 \times 10^5 + 3.36 \times 10^4$

$= 1.37 \times 10^5 J$

Again,

Heat loss by 100°C steam to 100°C water $= m_s L_V$

Heat loss by 100°C water to 40°C water $= m_s s_w (100-40)$

Total heat loss $= m_s L_v + m_s s_w (100-40)$

$= m_s(L_v + 60 s_w)$

$= m_s(2.26 \times 10^6 + 60 \times 4200)$

$= m_s \times 2.512 \times 10^6 J$

From principle of calorimeter,

Total heat gained = Total heat loss

$1.37 \times 10^5 J = m_s \times 2.512 \times 10^6 J$

or, $m_s = \dfrac{1.37 \times 10^5}{2.512 \times 10^6 J} = 0.05478$ kg = 54.78 gm

∴ Mass of steam passed = 54.78 gm

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55. **2073 Supp Q.No. 10c** **2072 Supp Q.No. 10b** **2069 (Set B) Q. No. 10b** **What is the result of mixing 100 g of ice at 0°C into 100 g of water at 20°C in an iron vessel of mass 100 g?** **[4]**

Given,

Mass of ice (m_i) = 100gm at 0°C

Mass of water (m_w) = 100gm at 20°C

Mass of iron (m_v) = 100gm at 20°C

Specific heat of iron $(S_v) = 0.1$ cal g^{-1} °C^{-1}

Heat gain by ice to melt (Q_1) = mi Li

$= 100 \times 80 = 8000$ cal

Heat lost by water + vessel $(Q_2) = (m_w S_w + m_v . S_v) \times \Delta\theta$

$= (100 \times 1 + 100 \times 0.1) \times 20 = 2200$ cal.

Here, $Q_1 > Q_2$. So, total ice does not melt. So, mass of ice melt,

$mi = \dfrac{2200}{80} = 27.5$ gm

So, mixture contains 72.5 gm of ice & 127.5 gm of water at 0°C.

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56. **2073 Set C Q.No. 10a** **In an experiment on the specific heat of a metal, a 200 gm block of metal at 150°C is dropped in a copper calorimeter of mass 270 gm containing 150 cm³ of water at 27°C. The final temperature is 40°C. Calculate the specific heat of the metal.** **[4]**

[S_c = 390 J/Kg°C, S_w = 4200 J/kg °C]

Given,

Mass of metal (m_m) = 200 gm = 0.2 kg

Mass of calorimeter (m_c) = 270 gm = 0.27 kg

Mass of water $(m_w) = 1 \times 150$ gm = 0.15 kg [∵ $m = \rho\times V = 1\times150 = 150$ gm]

Temperature of metal (θ_1) = 150° C

Temperature of water + calorimeter (θ_2) = 27° C

Final temperature (θ) = 40° C

Specific heat capacity of copper (S_c) = 390 J/kg °C

Specific heat capacity of water (S_w) = 4200 J/kg °C

Specific heat capacity of metal (S_m) = ?

We have, by principle of calorimetry

Heat lost by metal block = Heat gained by calorimeter and water

$m_mS_m(\theta_1' - \theta) = (m_cS_c + m_w S_w)(\theta - \theta_2)$

or, $0.2 \times S_m(150 - 40) = (0.27 \times 390 + 0.15 \times 4{,}200)(40 - 27)$

or, $S_m = \dfrac{9558.9}{22} = 434.5$ J/Kg°C

Specific heat capacity of metal (S_m) is 434.5 J/Kg °C.

57. **2072 Set C Q.No. 10a** **A copper pot with mass 0.5 kg contains 0.170 kg of water at a temperature of 20°C. A 0.250 kg block of iron at 85°C is dropped into the pot. Find the final temperature assuming no heat loss to the surroundings.** **[4]**

Given,

Mass of copper pot (m_c) = 0.5 kg
Mass of water (m_w) = 0.170 kg
Temperature of water (θ_1) = 20° C
Mass of iron (m_i) = 0.250 kg
Temperature of iron (θ_2) = 85° C
Specific heat capacity of copper (S_c) = 390 J kg^{-1} K^{-1}
Specific heat capacity of water (S_w) = 4190 J kg^{-1} K^{-1}
Specific heat capacity of iron (S_i) = 470 J kg^{-1} k^{-1}
Temperature of mixture (θ) = ?

Now,

Heat gained by water + copper pot (Q_1) = $(m_cs_c + m_ws_w)(\theta - c)$
Heat lost by iron (Q_2) = $m_is_i(\theta_2 - \theta)$
According to principle of calorimeter,
Heat gained = Heat lost

$Q_1 = Q_2$

or, $(m_cS_c + m_qS_w)(\theta - \theta_1) = m_is_i(\theta_2' - \theta)$

or, $(0.5 \times 390 + 0.17 \times 4190)(\theta - 20) = 0.25 \times 470(85 - \theta)$

or, $907.30 - 18146 = 9987.5 - 117.5\,\theta$

or, $\theta = \dfrac{28133.5}{1024.8} = 27.45°$ C

58. **2071 Set C Q.No. 10 a** **Find the result of mixing 0.8 kg of ice at –10°C with 0.8 kg of water at 80°C.** **[4]**

Given,

Mass of ice (mi) = 0.8 kg
Mass of water (m_w) = 0.8 kg
Temperature of ice (θ_i) = – 10°C
Temperature of water (θ_w) = 80°C
Specific heat capacity of water (S_w) = 4200 Jkg^{-1} °C^{-1}
Specific heat capacity of ice (Si) = 2100 J kg^{-1} °C^{-1}
Specific latent heat of ice (Li) = 336×10^3 J /kg
Result of mixing (θ_i) = ?

Total amount of heat required to melt ice at
0°C (Q_1) = $miSi(\theta_i - 0) + miLi = 0.8 \times 2100 \times 10 + 0.8 \times 336 \times 10^3 = 285600$ J
Total amount of heat lost in cooling from 80°C
to 0°C (Q_2) = $M_wS_w(80 - 0) = 0.8 \times 4200 \times 80 = 268800$ J
Since $Q_1 > Q_2$ total ice does not melt. Then amount of ice melted,

$$m_i = \frac{Q_2}{Li + Si \times 10} = \frac{268800}{2100 \times 10 + 336000} = 0.753 \text{ kg}$$

So, the result of mixing consists of 0.753 + 0.8
= 1.553 kg of water and 0.047 kg of ice at 0°C.

59. **2071 Set D Q.No. 10 c** **From what height should a block of ice be dropped in order that it may melt completely?** **[4]**

Given,

Mass of ice = m_i
Height to be dropped for completely melt = h

Specific latent heat of ice = Li = 336000 J kg^{-1}

To be completely melt, the P.E. lost by the ice block must be equal to latent heat of ice of that block.

i.e. $m_i gh = m_i L_i$

or, $h = \frac{Li}{g} = \frac{336000}{10} = 33600$ m

$\therefore$ To be completely melt, a block of ice should dropped from 33600m height.

60. 2070 Supp (Set A) Q.No. 10 a **How much heat is required to convert 5 kg of ice at –10°C into steam at 100°C? [4]**

Given,

Mass of ice (m_i) = 5 kg

Specific heat capacity of ice (S_i) = 2100 J kg^{-1}°C^{-1}

Specific latent heat of ice (L_i) = 336000 J kg^{-1}

Specific heat capacity of water (S_w) = 4200 J kg^{-1}°C^{-1}

Latent heat of vapourization (L_v) = 2.26× 10^{6}J/kg

Amount of heat (Q) = ?

$$\underset{\text{Ice}}{-10^\circ C} \xrightarrow{Q_1} \underset{\text{Ice}}{0^\circ C} \xrightarrow{Q_2} \underset{\text{Water}}{0^\circ C} \xrightarrow{Q_3} \underset{\text{Water}}{100^\circ C} \xrightarrow{Q_4} \underset{\text{Steam}}{100^\circ C}$$

Amount of heat required to convert ice from –10°C to 0°C is

$Q_1 = m_i S_i (0 - (-10)) = 5 \times 2100 \times 10 = 105000$J

Amount of heat required to convert ice from 0°C to water at 0°C

$Q_2 = m_i L_i = 5 \times 336000 = 1680000$ J

Amount of heat required to convert water at 0°C to water at 100°C

$Q_3 = m_w S_w (100 - 0) = 5 \times 4200 \times 100 = 2100000$ J

Amount of heat required to convert water at 100°C to steam at 100°C.

$Q_4 = m_w L_V = 5 \times 2.26 \times 10^6 = 11.30 \times 10^6$ J

Total heat required to convert 5 kg of ice from –10°C to steam at 100°C is

$Q = Q_1 + Q_2 + Q_3 + Q_4 = 105000 + 1680000 + 2100000 + 11300000 = 1.52 \times 10^7$ J

61. 2070 Supp (Set B) Q.No. 10 a **A copper calorimeter of mass 300 g contains 500 g of water at a temperature of 15°C. A 560g block of aluminium at temperature of 100°C is dropped in the water of the calorimeter and the temperature is observed to increase 22.5°C. Find the specific heat capacity of aluminium, neglecting the heat lost to the surroundings.** **[4]**

Given,

Specific heat capacity of copper (Sc) = 400 J kg^{-1} °C^{-1}

Specific heat capacity of water (S_w) = 4200J kg^{-1}°C^{-1}

Specific heat capacity of aluminium (S_{al}) = ?

Mass of calorimeter (m_c) = 300 gm = 0.3 kg

Mass of water (m_w) = 500 gm = 0.5 kg

Mass of alluminium (m_{al}) = 560 gm = 0.56 kg

Temperature of water and calorimeter (θ_2) = 15°C

Temperature of alluminium (θ_1) = 100°C

Final temperature (θ) = 25°C

Now, by using principle of calorimetry which states as

Heat lost = Heat gained

$m_{al} \times S_{al} \times (\theta_1 - \theta) = (m_c \times S_c + m_w \times S_w) \times (\theta - \theta_2)$

$0.56 \times S_{al} \times (100 - 25) = (0.3 \times 400 + 4200 \times 0.5)(25 - 15)$

or, $S_{al} = 528.57$ J kg^{-1}°C^{-1}

62. 2070 Set C Q.No. 10 a **A ball of copper weighing 400 gram is transferred from a furnace to a copper calorimeter of mass 300 gram and containing 1 kg of water at 20°C. The temperature of water rises to 50°C. What is the original temperature of the ball?** **[4]**

Given,

Mass of copper ball (m_b) = 400 gm = 0.4 kg

Mass of calorimeter (m_c) = 300 gm = 0.3 kg

Mass of water (m_w) = 1 kg
Initial temperature of water and calorimeter (θ_2) = 20°C
Final temperature (θ) = 50°C
Specific heat capacity of copper ball (S_c) = 400J/kg°C
Specific heat capacity of water (S_w) = 4200J/kg°C
Temperature of ball (θ_1) = ?
From principle of calorimetry
Heat loss = Heat gain

$m_b S_c (\theta_1 - \theta) = (m_c S_c + m_w S_w)(\theta - \theta_2)$

or, $0.4 \times 400(\theta_1 - 50) = (0.3 \times 400 + 1 \times 4200)(50 - 20)$

or, $160(\theta_1 - 50) = 129600$

or, $\theta_1 = \dfrac{129600 + 8000}{160} = 860°C$

$\therefore$ Temperature of ball is 860°C

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63. **2070 Set D Q.No. 10 a** **A copper calorimeter of mass 300 gram contains 500 gram of water at 15°C. A 560 gram of aluminium ball at temperature of 100°C is dropped in the calorimeter and the temperature is increased to 25°C. Find the specific heat capacity of aluminium.** **[4]**

✍ Please refer to 2070 Supp (Set B) Q.No. 10 a

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64. **2069 (Set A) Q. No. 10b** **10 gm of steam at 100°C is passed into a mixture of 100 gm of water and 10 gm of ice at 0°C. Find the resulting temperature of the mixture.** **[4]**

✍ Given,
Mass of steam (m_s) = 10gm
Temp of steam (θ_s) = 100°C
Mass of water (m_w) = 100 gm
Temp of water & ice (θ_i) = 0°C
Mass of ice (m_i) = 10 gm
Temperature of mixture (θ) = ?
Latent heat of ice (L_i) = 80 cal/gm
Latent heat of steam (L_s) = 540 cal/gm
Heat loss by steam = $m_s L_s + m_s S_w (100 - \theta)$
Heat gained by water & ice = $m_i L_i + (m_i + m_w) S_w . (\theta - 0)$
By principle of calorimetly,
Heat loss = Heat gain,

or, $m_s L_s + m_s S_w (100 - \theta) = m_i L_i + (m_i + m_w) S_w \theta$

$10 \times 540 + 10 \times 1 (100 - \theta) = 10 \times 80 + (10 + 100). 1. \theta$

or, $5400 + 1000 - 10\theta = 800 + 110\theta$

or, $5600 = 120\theta$

or, $\theta = 46.66°C$.

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65. **2066 Q. No. 10 b** **What is the result of mixing 10gm of ice at 0°C into 15gm of water at 20°C in a vessel of mass 100 gm and specific heat 0.09.** **[4]**

✍ Given,
Mass of ice (m_i) = 10gm
Temperature of ice (θ_i) = 0°C
Latent heat of ice (L_i) = 80 cal
Mass of water (m_w) = 15 gm
Temperature of water + vessel (θ_w) = 20°C
Mass of vessel (m_v) = 100gm
Specific heat of vessel (s_v) = 0.09cal/gm°C

Result of mixing (θ) = ?

Heat gained by ice in melting

$Q_1 = m_iL_i = 10 \times 80 = 800$ cal

Heat lost by water + vessel from 20°C to 0°C

$Q_2 = (m_wS_w + m_vs_v)(20 - 0) = (15 \times 1 + 100 \times 0.09)\,20 = (15 + 9) \times 20 = 24 \times 20 = 480$ cal

Here, $Q_1 > Q_2$, so, total ice does not melt.

Amount of ice melt $= \frac{480}{80} = 6$ gm

So, mixture contains (15 + 6) = 21 gm of water and (10 – 6)= 4 gm of ice at 0°C.

66. **2061 Q.No. 5 b** **What is the result of mixing 100g of ice at 0°C and 100g of water at 100°C. Latent heat of fusion of ice = 336×10^3 Jkg^{-1}, specific heat of water = 4200 Jkg^{-1} k^{-1}.**

Given,

Mass of ice (m_i) = 100gm = 0.1 kg

Temperature of ice (θ_1) = 0°C

Temperature of water (θ_2) = 100°C

Mass of water (m_w) = 100°C

Latent heat of ice (L_i) = 336×10^3 J/kg

Specific heat of water (S_w) = 4200 Jkg^{-1}°C^{-1}

Amount of heat gained by ice in melting

$Q_1 = m_iL_i = 0.1 \times 336000 = 33600$ J

Total heat lost by water from 100°C to 0°C is

$Q_2 = m_wS_w(\theta_2 - \theta_1) = 0.1 \times 4200\,(100 - 0) = 42000$ J

Here,

$Q_2 > Q_1$ and all ice melts.

So, the ice gains heat in two steps to melt at 0°C and to reach at a temperature θ say.

Total heat gain by ice,

$Q_1 = m_iL_i + m_i S_w(\theta - 0)$

$= 33600 + 0.1 \times 4200\,\theta = 33600 + 420\,\theta$

Heat lost by water

$Q_w = m_wS_w(\theta_2 - \theta)$

$= 0.1 \times 4200\,(100 - \theta) = 42000 - 420\,\theta$

By principle of calorimeter,

Heat gained = heat lost

or, $33600 + 420\,\theta = 42000 - 420\,\theta$

or, $840\,\theta = 42000 - 33600$

$\theta = \frac{8400}{840} = 10°C$

∴ Temperature of mixture is 10°C

67. **2059 Q.No. 5 b** **2051 Q.No. 11** **How much heat is needed to change 10 g of ice at - 10°C to steam at 100°C. (Specific heat capacity of ice = 0.5 cal g^{-1} °C^{-1}, Latent heat of fusion of ice = 80 cal g^{-1}, Latent heat of vaporization = 540 cal g^{-1}).** **[4]**

Given,

Mass of ice (m_i) = 10g

Specific heat of ice (S_i) = 0.5 cal/g °C

Amount of heat required to change ice at – 10°C to ice at 0°C is

$Q_1 = m_is_id\theta = 10 \times 0.5 \times [0 - (-10)] = 10 \times 0.5 \times 10 = 50$ cals.

Amount of heat required to change ice at 0°C to water at 0°C is

$Q_2 = m_i L_i = 10 \times 80$ [∵ $L_i = 80$ cal g^{-1}]

$= 800$ cals

Amount of heat required to change water at 0°C to water at 100°C is

$Q_3 = m_i S_w d\theta = 10 \times 1 \times (100 - 0)$

$= 1000$ cals. $[\because S_w = 1 \text{ cal g}^{-1}\,°C^{-1}]$

Amount of heat required to change water at 100° C to steam at 100° C is

$Q_4 = m_i \times L_s \quad [\because L_s = 540 \text{ cal g}^{-1}]$

$= 10 \times 540 = 5400$ cal

Total heat gained

$Q = Q_1 + Q_2 + Q_3 + Q_4$

$= 50 + 800 + 1000 + 5400$

$= 7250$ cal

$= 7250 \times 4.2 = 30450$ J

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68. **2057 Q.No. 5 b From what height a block of ice be dropped in order that it may completely melt. It is assumed that 20% of energy of fall is retained by ice. [L = 80 cal./g].**

Given,

Latent heat of ice (L) = 80 cal/g

Height (h) = ?

Suppose h be the height from where a block of ice be dropped and m be the mass of the ice block. Then the loss in potential energy by the ice-block when it reaches to the ground is mgh. From question,

Energy retained by the ice-block = 20% of mg h

$E_1 = \frac{20}{100} \times mg\,h$ ... (i)

Specific latent heat of ice (L) = 80 cal/g $= \frac{80 \times 4.2}{10^{-3}}$ J kg^{-1}

Amount of heat required to melt the ice block is given by

$E_2 = mL = m \times \frac{80 \times 4.2}{10^{-3}} = m \times 336000$ ... (ii)

As the ice reaches the ground, it melts completely. So,

$E_1 = E_2$

$\frac{20}{100} \times mg\,h = m \times 336000$

or, $\frac{20}{100} \times 10\,h = 336000$

or, $h = \frac{336000 \times 100}{200} = 168000$ m

$\therefore$ Height of ice block from where it is dropped, h = 168000 m.

---

69. **2052 Q.No. 10 b 50 gm. of ice at - 6°C is dropped into water at 0°C. How many grams of water freeze? (Given: SP heat capacity of ice = 2000 J. Kg^{-1} °C^{-1}).**

Given,

Mass of ice $(m_i) = 50$ g $= 50 \times 10^{-3}$ kg

SP heat capacity of ice(S_i)= 2000 J Kg^{-1} °C^{-1}

Amount of heat gained by ice at – 6°C to ice at 0°C is given by

$Q_1 = mS_{ice}\,d\theta$

$= 50 \times 10^{-3} \times 2000 \times [0 - (-6)]$

$= 600$ J ... (i)

If m is the mass of water frozen, then the amount of heat lost by water at 0° C to ice at 0° C is given by

$Q_2 = m L_w = m \times (336 \times 10^3)$ ... (ii)

By the principle of calorimetry,

Heat lost = Heat gained

$m \times 336 \times 10^3 = 600$

or, $m = 0.001785 = 1.785 \times 10^{-3}$ kg

$\therefore$ m = 1.785 g

70. 2050 Q.No. 8 **Evaporation or perspiration is an important mechanism for temperature control of warm-blooded animals. What mass of water must evaporate from the surface of an 80 kg human body to cool it by 1°C? The specific heat capacity of the human body is approximately 0.1 cal gm^{-1} °C and the latent heat of vaporization of water at the body temperature is 577 cal. gm^{-1}?**

Given,

Mass of human (m) = 80kg

Change in temperature ($\Delta\theta$) = 1°C

Specific heat capacity of human (S_n) = 0.1 calgm^{-1} °C^{-1} = 420 J/Kg°C

Latent heat of vaporization of water at human body temperature

(L_n) = 577 cal/gm = 2423400 J/kg

Heat gained by water that evaporate is,

$m.L_n = 2423400 \times m$

Heat loss by human = $m \times S_n \times \Delta\theta = 80 \times 420 \times 1$ = 33600 J

But, heat lost = heat gained

$m \times 2423400 = 33600$

m = 0.01386 Kg

□□□

Chapter 4: Rate of Heat Flow

Short Answer Questions

1. **2076 Set C Q.No. 2b Cooking utensils are blackened at the bottom and polished on the upper surface. Explain, why?** **[2]**

The black bodies are good absorber of heat and polished shining bodies are good heat reflectors. Cooking utensils are blackened at the bottom because it absorbs heat while cooking and upper surface is polished to reflect heat to food, i.e., to avoid loss of heat. So, cooking utensils are blackened at the bottom and polished on the upper surface.

2. **2075 GIE Q.No. 2b How can water be boiled in a thin paper cup?** **[2]**

The boiling of water requires 100°C whereas the burning of paper requires its ignition temperature which is greater than 100°C. When water is heated in a paper cup, heat goes to the paper as well as water, so the temperature of water as well as the cup rise to 100°C first. Then water starts to boil. At that time, the ignition temperature of paper is still not arrived, so it does not burn. As the process of boiling goes on, a condition is reached when water finishes off. Then only all the heat is directed to paper, raising its temperature heavily and ultimately burning it.

3. **2075 GIE Q.No. 2c 2069 (Set A) Q. No. 2c Why are the polar regions much cooler than the equatorial regions despite the fact that the polar regions are periodically tilted towards the sun?** **[2]**

The heat received by the surface of earth is given by $E = \frac{I}{r^2} \cos\theta$, where , θ is the angle between incident ray and normal drawn on the surface, r be the distance between the sun and earth, I be the intensity of heat radiation. The heat rays of sun fall obliquely at the Polar Regions and straight at the equator. Due to this reason, the polar regions of the earth are much cooler than equatorial regions even they are tilted toward the sun periodically.

4. **2075 Set B Q.No. 2b What is the physical meaning of emissivity? Write its unit.** **[2]**

Emissivity of a body is the ratio of the emissive power of the body to the emissive power of the perfectly black body at the same temperature, i.e.

$$\text{Emissivity} = \frac{\text{Emissive power of a body}}{\text{Emissive power of perfectly black body}}$$

It is the ratio of same quantity. So, it has no unit. Its value ranges 0 to 1. Emissivity = 0 indicate the perfectly shining and 1 means the body is perfectly black body. Physically it gives the closeness of the body to the perfectly black body.

5. **2072 Supp Q.No. 2b Air is a bad conductor of heat. Why do you feel cool without cloth in your body?** **[2]**

Air is bad conductor of heat, when we wear cloth; it stops the transformation of air from outside to inside and vice versa. i.e., the air trapped inside the cloth does not conduct heat to outside. If we do not wear cloth, there is no layer of air trapped around our body. Then the heat conducts from body to surroundings and we feel cool. So, even air is bad conductor of heat, we feel cool without cloth in our body.

6. **2072 Supp Q.No. 2c 2069 (Set B) Q. No. 2c 2052 Q.No. 7a Although aluminium is good conductor of heat, how can aluminium foil with shining surface can be used to keep food hot for a long time?** **[2]**

The reflection of heat radiation depends up on the shining properties of the material. Aluminium foil is too much shiny, and reflects most of the heat radiations falling on it. The reflected heat from the aluminium surface again goes to the food materials. So loss of heat from the food is prevented and the food would remain warm for a long time. The shiny surface reflects heat radiations falling on it, thus, prevents the heat from food being lost and keeps the food warm from a long time.

7. **2070 Set C Q.No. 2 b Birds often swell their feathers in winter. Why?** **[2]**

Since the rate of flow of heat from a body is given by

$\frac{Q}{t} = kA\frac{d\theta}{dt} \Rightarrow \frac{Q}{t} \propto k$, for constant A, $d\theta$ and dx.

A layer of air is trapped between the feathers, which is bad conductor of heat and low value of thermal conductivity (K). It prevents the flow of heat from bird's body to the surrounding. Hence, birds often swell their feathers in winter.

8. 2070 Set D Q.No. 2 b **Animals curl into a ball, when they feel very cold. Why?** **[2]**

The amount of heat radiated by a body is directly proportional to surface area i.e., $P = A\sigma T^4$. where, A be the area of surface of the body. If the surface area is reduced, the heat radiated is also reduced. Since, spherical body has minimum surface area for a given volume, to reduce the heat radiation from their bodies by reducing surface area, the animals curl into ball in winter.

9. 2066 Q. No. 2 a 2060 Q.No. 4 a **Why are two thin blankets warmer than a single blanket of double the thickness?** **[2]**

Since, the rate of flow of heat from a body is given by

$\frac{Q}{t} = kA\frac{d\theta}{dt} \Rightarrow \frac{Q}{t} \propto k$, for constant A, $d\theta$ and dx.

A layer of air is trapped between two blankets, which is bad conductor of heat. In other words, the coefficient of thermal conductivity (k) of air is very small in comparison with that of blanket. It prevents the flow of heat from our body to the surrounding. Hence, two thinner blankets are warmer than single blanket of double thickness.

10. 2059 Q.No. 4 c **During the winter, the animals curl into a ball. Explain why?** **[2]**

Please refer to 2070 Set D Q.No. 2 b

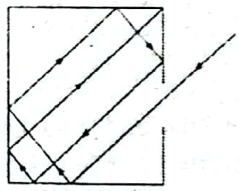
11. 2055 Q.No. 7 c **Why are good absorbers always good emitters?** **[2]**

Temperature is the effect of heat that means when a body absorbs heat its temperature rises. From Stefan's law of radiation, amount of heat emitted per unit time per unit surface area is directly proportional to the fourth power of absolute temperature,

i.e. $E \propto T^4$.

It means hotter body can emit greater amount of heat than relatively cold body. Hence, good absorbers are always good emitters.

12. 2054 Q.No. 7 f **What is a black body? How is it realized in practice?** **[2]**



A perfectly black body is one which absorbs completely the radiations of all wavelengths falling on it. A perfectly blackbody can't be realized in practice. The nearest approach to a perfectly blackbody is a surface coated with lamp black, platinum black, such a surface absorbs 96% to 98% of the incident radiation.

If a small hole is made in the lid of a closed empty tin, the hole looks almost black although the shining tin is a good reflector because the light which enters through it is reflected many times round the walls of the tin, before it passes through the hole again as shown in the figure. At each reflection, absorption occurs.

13. 2053 Q.No. 7 a **Hot water pipes used in the room are painted black. Why?** **[2]**

When hot water flows through a pipe, the pipe also absorbs some heat. This absorbed heat is lost by the pipe in the form of radiation. We know that, good absorbers are good radiators also. So, the black painted pipe used in a room radiates more heat than the pipe painted with other colors and hence, keeps the room warm.

14. 2052 Q.No. 7 b **Metal knob of door is colder than wooden parts at the same temperature. Why?** **[2]**

Metal is good conductor of heat whereas wood is bad conductor of heat. When we touch the metal knob of door, it conducts heat from our body consequently there is loss of heat from our body and we feel cold. However, when wood is touched, wood does not accept heat from the body, because it is not a conductor. So, the body heat remains preserved and we don not feel cool.

15. 2051 Q.No. 7 a **How can water be boiled in a paper cup?** **[2]**

Please refer to 2075 GIE Q.No. 2b

Long Answer Questions

16. 2076 GIE Set A Q.No. 6b **Define thermal conductivity of substance. Deduce an expression for the thermal conductivity of a good conductor in steady state.** **[4]**

Thermal Conductivity: The thermal conductivity of material of a body is defined as the amount of heat that flows in one second across the opposite faces of a unit cube, whose opposite faces are kept at a temperature difference of 1K. The coefficient of thermal conductivity is represented by k. Its SI unit is $Wm^{-1}k^{-1}$.

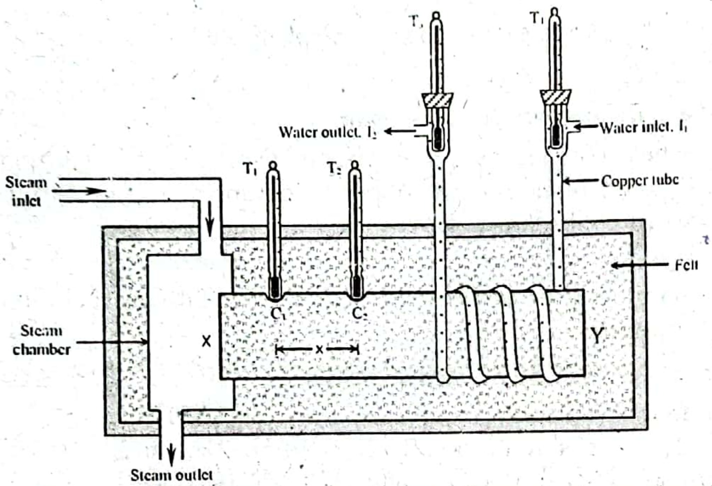
Mathematically, $\frac{Q}{t} = k\frac{A(\theta_1-\theta_2)}{x}$

When $A = 1m^2$, $\theta_1-\theta_2 = 1K$ and $x = 1$ m, then $k = \frac{Q}{t}$

Thermal conductivity depends upon nature of material.

Experimental determination of thermal conductivity (k) of solid bar(*Searle's method*)

The apparatus required for the determination of thermal conductivity of a metal bar is as shown in figure. XY is the metal bar whose thermal conductivity is to be determined. One end of bar is enclosed within steam chamber and a copper wire wound near the other end steam is passed through steam chamber and water flows through copper tube at constant rate. T_1 and T_2 are thermometers to measure the temperature of bar at a small distance x apart. Thermometer T_3 and T_4 measure the temperature of outgoing and incoming water. To ensure good thermal contact, small amount of mercury is placed into cavity.



As the end X of bar is heated by passing steam through steam chamber, so that heat is conducted from X to end Y and circulating water is warmed up. At steady state, the reading of all thermometers becomes constant. Heat lost from the surface of bar is neglected because the bar is lagged by some non-conducting material.

Let, θ_1, θ_2, θ_3 and θ_4 are the temperatures recorded by thermometers T_1, T_2, T_3 and T_4 respectively in steady state.

Area of cross section = A

Distance between cavities = x

Time for which the heat is considered = t

Mass of water that flows through the copper tube in time t = m

The amount of heat flows through metal bar in time t is,

$$Q = \frac{kA(\theta_1 - \theta_2)t}{x} \qquad \ldots (i)$$

where k is the thermal conductivity of metal bar and $\theta_1 - \theta_2$ is temperature difference between two cavities.

The temperature of water flowing through the copper tube rises from θ_4 to θ_3. So the amount of heat absorbed by water in time t is

$$Q = mS(\theta_3 - \theta_4) \qquad \ldots (ii)$$

From the principle of calorimetry,

Heat lost = heat gain

$$\frac{kA(\theta_1 - \theta_2)t}{x} = mS(\theta_3 - \theta_4)$$

$$k = \frac{mS(\theta_3 - \theta_4)x}{A(\theta_1 - \theta_2)t}$$

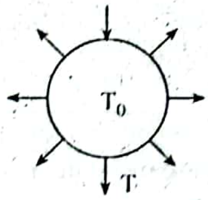
Which is the required expression of thermal conductivity of a metal bar. As all the quantities in above expression (right hand side) are known, then k can be experimentally measured.

17. 2075 Set B Q.No. 6a On what factors the thermal conductivity of a solid depends? Describe, Searle's experimental method to determine the thermal conductivity of a solid. **[4]**

Please refer to 2076 GIE Set A Q.No. 6b

18. 2072 Set D Q.No. 6c 2057 Q.No. 5 a OR State and explain Stefan's law of black body radiation. Can a perfect black body be realized in practice? **[4]**

Black body: A perfect black body is one which absorbs heat radiation of all the wavelengths, which fall on it. It does not reflect or transmit radiations. Thus, black body is a good absorber.



Stefan's law of black body radiation: It is stated as, "the rate of the radiant energy per unit area from the surface of perfect blackbody is directly proportional to fourth power of absolute temperature."

i.e. $E \propto T^4 \Rightarrow E = \sigma T^4$ ... (i)

Where σ is Stafen's constant. The value of σ is 5.67×10^{-8} J s^{-1} $m^{-2}K^{-4}$. If the body isn't perfectly black and it has emissivity 'e' then Stafen's law becomes.

$E = \sigma e T^4$ ... (ii) where $0 < e < 1$

For perfect black body, e = 1. If A be the surface area of the blackbody, then the radiant power is given by

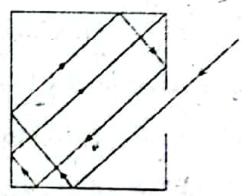
$P = A\sigma e T^4$ ... (iii)

Now, suppose the black body at absolute temperature (T) is kept in an enclosure at absolute temperature T_0, then the net rate of lose or gain of radiant energy per unit area is given by

$E = \sigma e (T - T_0)^4$ ... (iv) Where $T > T_0$

If A be the surface area of the body then the net radiant energy transfer per second by the body is given by

$P = A\sigma e (T - T_0)^4$ where $T > T_0$.



Black body: A perfectly black body is one which absorbs completely the radiations of all wavelengths falling on it. A perfectly blackbody can't be realized in practice. The nearest approach to a perfectly blackbody is a surface coated with lamp black, platinum black, such a surface absorbs 96% to 98% of the incident radiation.

If a small hole is made in the lid of a closed empty tin, the hole looks almost black although the shining tin is a good reflector because the light which enters through it is reflected many times round the walls of the tin, before it passes through the hole again as shown in the figure. At each reflection, absorption occurs.

19. 2072 Set E Q.No. 6c State and explain Stefan's law of black body radiation. **[4]**

Please refer to 2072 Set D Q.No. 6c

20. 2071 Supp Q.No. 6b State and explain Stefan's black body radiation. Explain how a black body can be realized in practice. **[4]**

Please refer to 2072 Set D Q.No. 6c

21. 2071 Set C Q.No. 6 b Define the coefficient of thermal conductivity. Describe, with necessary theory, an experiment for its determination in the laboratory. **[4]**

Please refer to 2076 GIE Set A Q.No. 6b

22. 2070 Supp (Set A) Q.No. 6 b Define coefficient of thermal conductivity. Describe Searl's method of determining thermal conductivity. **[4]**

Please refer to 2076 GIE Set A Q.No. 6b

23. 2069 Old (Set B) Q. No. 5a OR Define thermal conductivity. Describe Searle's method of determination of thermal conductivity of good conductor. **[1+4]**

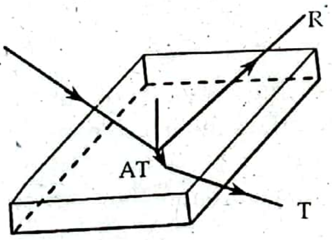
Please refer to 2076 GIE Set A Q.No. 6b

24. 2068 Old Q.No. 5 a or Discuss the methods of heat transmission. Define reflection, transmission and absorption coefficients of heat radiation and relate them. **[3+2=5]**

The methods of heat transmission are discussed below:

Conduction	Convection	Radiation
Material medium is essential.	Material medium is also essential.	Material medium is not essential.
Transfer of heat can be in any direction.	Heat is transmitted only vertically upward.	Transfer of heat takes place in all directions in straight line.
Transfer of heat takes place at slow rate.	This process is slower than radiation and faster than conduction.	Transfer of heat is fastest (at a velocity of 3×10^8 ms^{-1})
Molecules do not leave their mean positions.	Molecules move from one place to another place.	Electromagnetic waves move from one place to another place.

Reflection, transmission and absorption coefficients of heat radiation: When heat radiation falls on a surface, a part of it is absorbed, a part of it is reflected and a part is transmitted. Suppose Q is the heat radiation incident on a surface, R is the reflected, A is the absorbed, and T is the transmitted part of heat through the surface. Since total energy reflected, absorbed and transmitted per unit area per second is equal to the energy falling on the surface, then



$R + A + T = Q$

Dividing both sides by Q, we get

$$\frac{R}{Q} + \frac{A}{Q} + \frac{T}{Q} = 1$$

or, $r + a + t = 1$ ... (i)

where,

$r = \frac{R}{Q}$, reflectance or reflection coefficient and defined as the ratio of heat energy reflected from the surface to the total heat energy incident on it at the same time.

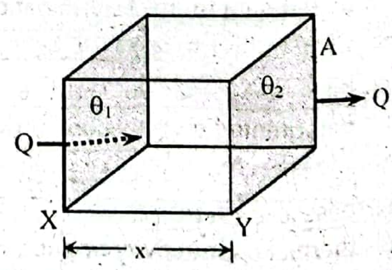
$a = \frac{A}{Q}$, absorptance or absorption coefficient and defined as the ratio of heat energy absorbed by the surface to the total heat energy incident on it in same time.

and $t = \frac{T}{Q}$, transmittance or transmission coefficient of the surface and defined as the ratio of heat energy transmitted by the surface to the total heat energy incident on it in same time.

The reflectance r, absorptance a and transmittance t depend on the nature of the surface of the body and on the wavelength of incident radiation but not on the nature of the material of the body.

25. $\boxed{\text{2067 Q.No. 6 b}}$ **What do you mean by thermal conductivity of a substance? Deduce an expression for the thermal conductivity of a good conductor in steady state.** **[4]**

✍ Consider a section XY of a long metal bar, whose side length be x and area of cross-section A. When it's opposite faces are maintained at temperatures θ_1 and θ_2 such that $\theta_1 > \theta_2$, heat flows from left to right as shown in Fig. Experimentally, it is found that under steady state condition, the quantity of heat Q flowing through the section of the bar is



i. directly proportional to the cross-section area A of the face.

$Q \propto A$ ... (i)

ii. directly proportional to the temperature difference $(\theta_1 - \theta_2)$ between the two faces.

$Q \propto (\theta_1 - \theta_2)$ ... (ii)

iii. directly proportional to time t for which heat flows.

$Q \propto t$ ... (iii)

iv. inversely proportional to the perpendicular distance x between hot and cold faces.

$Q \propto \frac{1}{x}$ ... (iv)

Combining Eqs. (i), (ii), (iii) and (iv), we get

$$Q \propto \frac{A(\theta_1 - \theta_2)t}{x}$$

$$\therefore\ Q = \frac{kA\,(\theta_1 - \theta_2)t}{x} \qquad \text{... (v)}$$

where k is a constant called coefficient of thermal conduction or simply thermal conductivity. It's value depends upon material of the bar.

Again,

$$k = \frac{Q\,x}{A\,(\theta_1 - \theta_2)\,t}$$

If $A = 1\ m^2$, $\theta_1 - \theta_2 = 1°C$ or 1 K, t = 1 sec and x = 1 m, then Q = k.

Thus, thermal conductivity is defined as the amount of heat flowing per second across the opposite faces of a cube of side 1m maintained at a unit temperature difference. It's unit is $W\ m^{-1}\ K^{-1}$ in SI-units and dimensional formula is $[M\,L\,T^{-3}\,K^{-1}]$.

26. **2065 Q. No. 5 a Or Define thermal conductivity. Write its units and dimensions. Describe Searle's method of determination of thermal conductivity of good conductor.** **[1+1+3]**

Please refer to 2076 GIE Set A Q.No. 6b

27. **2063 Q.No. 5 a OR Describe Searle's method of determination of thermal conductivity of a good conductor.**

Please refer to 2076 GIE Set A Q.No. 6b

28. **2061 Q.No. 5 a Define thermal conductivity. Describe with the necessary theory on experiment to determine the thermal conductivity of a metal bar.** **[1+4]**

Please refer to 2076 GIE Set A Q.No. 6b

29. **2058 Q.No. 5 a OR What is a black body? Derive Stefan's law of black body radiation.** **[5]**

Please refer to 2072 Set D Q.No. 6c

30. **2056 Q.No. 5 OR What do you mean by perfectly black body. State and explain Stefan's law of black body radiation.**

Please refer to 2072 Set D Q.No. 6c

31. **2051 Q.No. 9 Define thermal conductivity. Derive an expression for thermal conductivity of a good conductor in steady state.**

Please refer to 2067 Q.No. 6 b

32. **2050 Q.No. 9 What is radiation and how does this mode of heat transfer differ from conduction and convection?**

It is the process in which heat is transmitted from one place to another directly without intervening medium. Heat radiation comes to the earth from the sun through this process. Heat radiations can pass through the vacuum also. Their properties are similar to light radiations. Heat radiations also form a part of the electro magnetic spectrum.

Difference between Conduction, Convection and Radiation are:

There are following differences between conduction, convection and radiation.

Conduction	Convection	Radiation
Material medium is essential.	Material medium is also essential.	Material medium is not essential.
Transfer of heat can be in any direction.	Heat is transmitted only vertically upward.	Transfer of heat takes place in all directions in straight line.
Transfer of heat takes place at slow rate.	This process is slower than radiation and faster than conduction.	Transfer of heat is fastest (at a velocity of $3\times10^8\ ms^{-1}$)
Molecules do not leave their mean positions.	Molecules move from one place to another place.	Electromagnetic waves move from one place to another place.

Numerical Problems

33. **2076 GIE Set B Q.No. 10c Find the rate of radiation of energy from a human body of total surface area 1.5m² and the surface temperature 32°C if the surrounds are at a temperature of 22°C and the emissivity of human body is 0.94.** **[4]**

Given,

Surface area (A) = 1.5 m²

Surface temperature (T_1) = 32° C = 32 + 273 = 305 K

Surrounding temperature (T_2) = 22°C = 22 + 273 = 295 K

Emissivity (e) = 0.94

Stetan's constant (σ) = 5.67×10^{-8} Wm^{-2} K^{-4}

Radiant power loss (P) = ?

We know, from stetan's law,

$$P = \sigma Ae (T_1^4 - T_2^4)$$
$$= 5.67 \times 10^{-8} \times 1.5 \times 0.96 (305^4 - 395^4)$$
$$= 88 \text{ W}$$

34. **2076 Set C Q.No. 10b A bar 0.2m in length and 2.5 cm² in cross section is ideally lagged. One end is maintained at 100° C and the other end is maintained at 0° C by immersing in melting ice. Calculate the mass of ice melt in one hour. Thermal conductivity of the material of the bar is 4×10^{-2} Wm^{-1} K^{-1}.** **[4]**

Given,

Length of bar (l) = 0.2 m

Cross sectional area (A) = 2.5×10^{-4} m²

Higher temperature (θ_1) = 100°C = 373k

Lower temperature (θ_2) = 0°C = 273k

Thermal conductivity of bar (K) = 4×10^{-2} Wm^{-1} K^{-1}

Latent heat of fusion of ice (L_i) = 3.36×10^5 J Kg^{-1}

Time taken (t) = 1 hour = 60 × 60 = 3600 sec

Amount of ice melt (m) = ?

We have,

Rate of heat lost,

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} \qquad [\because Q = mL_i]$$

or, $$\frac{m\,(3.36 \times 10^5)}{3600} = \frac{4 \times 10^{-2} \times 2.5 \times 10^{-4} (373 - 273)}{0.2}$$

or, $$\frac{m\,(3.36 \times 10^5)}{3600} = 5 \times 10^{-3}$$

$$\therefore\ m = \frac{3600 \times 5 \times 10^{-3}}{3600 \times 10^5}$$
$$= 5.36 \times 10^{-5} \text{ kg}$$

35. **2075 Set A Q.No. 10c The Sun is a black body of surface temperature about 6000K. If Sun's radius is 7×10^8m, calculate the energy per second radiated from its surface. The earth is about 1.5×10^{11} m from the Sun. Assuming all the radiation from the Sun falls on the surface of sphere of this radius, estimate the energy per second per meter² received by the earth.** **[4]**

Given,

Radius of sun (r) = 7×10^8 m

Temperature of sun (T) = 6000 K

Power (P) = ?

Stefan's constant (σ) = 5.7×10^{-8} wm^{-2} K^{-4}

We know,

$$P = \sigma AT^4$$
$$= \sigma 4\pi r^2 T^4$$

$= 5.7 \times 10^{-8} \times 4\pi \times (7 \times 10^{8})^2 \times 6000^4$

$= 4.55 \times 10^{26}$ watt

Again, Energy per second per m² received by earth

$= \frac{P}{4\pi R^2} = \frac{4.55 \times 10^{26}}{4\pi (1.5 \times 10^{11})^2}$ [R = Distance both earth and sun = 1.5×10^{11} m]

$= 1609\ Wm^{-2}$

36. **2074 Supp. Q.No. 10b** **A sphere of radius 2.00 cm with a black surface is cooled and then suspended in a large evacuated enclosure with black walls maintained at 27°C. If the rate of change of thermal energy of sphere is 1.85JS⁻¹ when its temperature is –73°C, calculate the value of Stefan's Constant.** **[4]**

Given,

Radius of sphere = 2.00 cm = 2×10^{-2}m

Temperature of sphere (T) = 27°C = 300 K

Power (P) = 1.85 JS^{-1}

Temperature of enclosure (T_0) = –73°C = 200 K

Stefan's constant (σ) = ?

We have,

$P = \sigma A (T^4 - T_0^4)$

or, $1.85 = \sigma \times 4 \times (2 \times 10^{-2})^2 (300^4 - 200^4)$

$\therefore \sigma = 5.66 \times 10^{-8}\ Wm^{-2} K^{-4}$

37. **2074 Set A Q.No. 10b** **Estimate the power loss through unit area from a perfectly black body at 327°C to the surrounding environment at 27°C.** **[4]**

Given,

Area (A) = 1m²

Temperature of black body (T_1) = 327°C = 600 K

Temperature of surrounding (T_0) = 27°C = 300 K

Power loss (P) = ?

We know, from Stefan's law,

$P = e \sigma A (T_1^4 - T_0^4)$; $\left[\begin{array}{l}\sigma = 5.67 \times 10^{-8}\ Wm^{-2} K^{-4} \text{ is Stefan's constant} \\ \text{and } e = 1 \text{ is emissivity of perfectly black body}\end{array}\right]$

or, $P = 1 \times 5.67 \times 10^{-8} \times 1 \times (600^4 - 300^4)$

$\therefore P = 6889.05$ W

38. **2074 Set B Q.No. 10c** **A spherical blackbody of radius 5 cm has its temperature 127°C and its emissivity is 0.6. Calculate its radiant power.** **[4]**

Given,

Radius of spherical black-body (R) = 5 cm = 5×10^{-2} m

Temperature of black body (T_1) = 127°C = 400 K

Emissivity (e) = 0.6

Radiant power (P) = ?

We know, from Stefan's law,

$P = \sigma A e T^4$; [$\sigma = 5.67 \times 10^{-8}\ Wm^{-2} k^{-4}$ is Stefan's constant]

$= 5.67 \times 10^{-8} \times 4\pi \times (5 \times 10^{-2})^2 \times 0.6 \times 400^4$ [$\because A = 4\pi R^2$]

$= 27.34$ W

39. **2073 Supp Q.No. 10b** **Estimate the radiant power loss from a human body at a temperature 38.5°C to the environment at 0°C if the surface area of the body is 1.5 m² and its emissivity is 0.6.** **[4]**

Given,

Temperature of human body (T_1) = 38.5° C = 311.5 K

Temperature of environment (T_0) = 0° C = 273 K

Surface area of the body (A) = 1.5 m²

Emissivity (e) = 0.6

Radiant power loss (P) = ?

We know, from stefan's law,

$P = \sigma Ae (T_1^4 - T_0^2)$; σ = Stefan's constant = 5.67×10^{-8} $Wm^{-2} k^{-4}$

or, $P = 5.67 \times 10^{-8} \times 1.5 \times 0.6 \times (311.5^4 - 273^4)$

$\therefore$ P = 197 W

40. 2073 Set C Q.No. 10b **A pot with a steel bottom 8.5 mm thick rest on a hot stove. The area of the bottom of the pot is 0.15m². The water inside the pot is at 100°C and 390 gm of water is evaporated every 3 minute. Find the temperature of lower surface of the pot which is in contact with the stove. [k = 50.2 W/mK, L_v = 2256 × 10^3 J/kg]** **[4]**

Given,

Thickness of steep pot (l) = 8.5 mm = 8.5×10^{-3} m

Area of bottom of pot (A) = 0.15 m^2

Temperature of water (θ_1) = 100°C

Temperature of bottom of pot (θ_2) = ?

Time period (t) = 3 min = 3 × 60 = 180 sec

Mass of water (m) = 390 gm = 0.39 kg

Thermal conductivity of steel (k) = 50.2 W/mK

Latent heat of vaporization of water (L_v) = 2256 × 10^3 J/Kg

We have,

$$Q = \frac{kA(\theta_2 - \theta_1)\,t}{l}$$

$$\text{or, } mL_v = \frac{kA(\theta_2 - \theta_1)\,t}{l}$$

$$\text{or, } 0.39 \times 2256 \times 10^3 = \frac{50.2 \times 0.15\,(\theta_2 - 100) \times 180}{8.5 \times 10^{-3}}$$

or, $\theta_2 - 100 = 5.52$

or, $\theta_2 = 105.52°C$

41. 2073 Set D Q.No. 10b **A rod 1.3 m long consists of a 0.8 m length of aluminium joined end to end to a 0.5 m length of brass. The free end of the aluminium section is maintained at 150°C and the free end of the brass piece is maintained at 20°C. No heat is lost through the sides of the rod. At a steady state, what is the temperature at the point where the two metals are joined? [K_{al} = 205 W/mK, K_b = 110 w/mk]** **[4]**

Given,

Length of aluminum (l_1) = 0.8 m

Length of brass (l_2) = 0.5 m

Temperature of aluminum (θ_1) = 150 °C

Temperature of brass (θ_2) = 20°C

$K_1 = 205$ $Wm^{-1} K^{-1}$

$K_2 = 109$ $Wm^{-1} K^{-1}$

Temperature at joined (θ) = ?

We have at steady state,

$$\left(\frac{Q}{t}\right)_1 = \left(\frac{Q}{t}\right)_1$$

$$\frac{k_1 A(150 - \theta)}{0.8} = \frac{k_2 A(\theta - 20)}{0.5}$$

$$\text{or, } \frac{205 \times 150 - 205\,\theta}{0.8} = \frac{109 \times \theta - 109 \times 20}{0.5}$$

$$\text{or, } \theta = \frac{17119}{189.7} = 90.2°C$$

42. 2072 Set C Q.No. 10c A rod 1.300 m long consists of a 0.800 m length of aluminium joined end to end to a 0.500 m length of brass. The free end of the aluminium section is maintained at 150°C and the free end of the brass piece is maintained at 20°C. No heat is lost through the side of the rod. At steady state, what is the temperature of the point when two metals are joined.

Please refer to 2073 Set D Q.No. 10b

43. 2071 Set D Q.No. 10 b The element of an electric fire with an output of 1.5 KW is a cylinder of 0.3 m long and 0.04 m in radius. Calculate its temperature if it behaves as a black body. [4]

Given,

Power of electric fire (P) = 1.5 KW = 1500 W

Length of cylinder (l) = 0.3 m

Radius of cylinder (r) = 0.04 m

Stefan's constant (σ) = 5.67×10^{-8} W m^{-2} k^{-4}

Temperature of cylinder (T) = ?

We have

$P = \sigma A T^4$

or, $1500 = 5.67 \times 10^{-8} \times 2\pi \times r \times l \times T^4$

or, $1500 = 5.67 \times 10^{-8} \times 2\pi \times 0.3 \times 0.04 \times T^4$

or, $T^4 = \dfrac{1500}{5.67 \times 10^{-8} \times 2\pi \times 0.3 \times 0.04} = 3.51 \times 10^{11}$

or, $T = (3.51 \times 10^{11})^{1/4} = 769.64$ k

44. 2070 Supp (Set B) Q.No. 10 b A bar 0.2 m in length and of cross-sectional area 2.5×10^{-4}m^2 is ideally lagged. One end is maintained at 373 K while the other is maintained at 273 K by immersing in melting ice Calculate the rate at which the ice melts owing to the flow of heat along the bar. [4]

Given,

Length of bar (l) = 0.2 m

Cross sectional area (A) = 2.5×10^{-4} m^2

Higher temperature (θ_1) = 373 k

Lower temperature (θ_2) = 273 k

Thermal conductivity of bar (k) = 4.0×10^2 Wm^{-1} k^{-1}

Latent heat of ice (Li) = 336000 J/kg

Now,

The amount of heat gained by ice

$Q = \dfrac{KA(\theta_2 - \theta_1)\,t}{l}$

Then rate of heat lost

$\dfrac{Q}{t} = \dfrac{KA(\theta_2 - \theta_1)}{l} = \dfrac{mL}{t}$

or, $\dfrac{m}{t} = \dfrac{KA(\theta_2 - \theta_1)}{L\,l} = \dfrac{4.0 \times 10^2 \times 2.5 \times 10^{-4} \times 100}{336000 \times 0.2}$

$= 1.5 \times 10^{-4}$ kg/sec

45. 2070 Set C Q.No. 10 b A slab of stone of area 0.36 m^2 and thickness 10 cm is exposed on the lower surface to steam at 100°C . A block of ice at 0°C rests on the upper surface of the slab. In one hour, 4.8 kg of ice is melted. Calculate the thermal conductivity of stone. [4]

Given,

Area of stone (A) = 0.36 m^2

Thickness of stone (x) = 10 cm = 0.1 m

Temperature difference ($d\theta$) = $\theta_2 - \theta_1$ = 100 -0 = 100°C

Mass of ice melted (m) = 4.8 kg

Time (t) = 1hr = 3600 sec

Thermal conductivity of stone (k) = ?

We have,

$$\frac{Q}{t} = \frac{kA(\theta_2 - \theta_1)}{x}$$

$$\frac{mL}{t} = \frac{kAd\theta}{x}$$

or, $k = \frac{m.L.x}{t.A.d\theta} = \frac{4.8 \times 336000 \times 0.1}{3600 \times 0.36 \times 100} = 1.24\ Wm^{-1}K^{-1}$

46. **2070 Set D Q.No. 10 b** **A metal rod of length 20cm and cross sectional area 3.14 cm² is covered with non-conducting substance. One of its end is maintained at 100°C, while the other end is put in ice at 0°C. It is found that 25 gram of ice melts in 5 minutes. Calculate the thermal conductivity of the metal.** **[4]**

Given,

Length of rod (l) = 20 cm = 0.2 m
Cross-sectional area (A) = 3.14 cm² = 3.14×10^{-4} m²
Latent heat of ice (L) = 336000 J/kg
Mass of ice melted (m) = 25g = 0.025 kg
Time taken (t) = 5 min = 5 × 60 = 300 sec
Temperature difference, $(\theta_2 - \theta_1)$ = 100 – 0 = 100°c
Thermal conductivity of metal (w) = ?

We have,

$$\frac{Q}{t} = \frac{KA(\theta_2 - \theta_1)}{l}$$

or, $\frac{mL}{t} = \frac{KA(\theta_2 - \theta_1)}{l}$

or, $K = \frac{m \times L \times l}{t \times A \times (\theta_2 - \theta_1)} = \frac{0.025 \times 336000 \times 0.2}{5 \times 60 \times 3.14 \times 10^{-4} \times 100} = 178.3\ Wm^{-1}K^{-1}$

47. **2069 Supp Q.No. 10c** **2062 Q.No. 5 b** **Estimate the rate of heat loss through a glass window of area 2m² and thickness 4mm when the temperature of the room is 300K and temperature outside is 5°C.** **[4]**

Given,

Area of the glass window(A) = 2 m²
Thickness of the glass window (x) = 4mm = 4×10^{-3} m
Temperature inside the room (θ_1) = 300 K
Temperature outside the room(θ_2) = 5° C
= 5 + 273 = 278 K

If k is the coefficient of thermal conductivity of glass, then from the relation,

$$\frac{Q}{t} = \frac{kA(\theta_1 - \theta_2)}{x} \qquad [k = 1.2\ Wm^{-1}K^{-1}]$$

or, $\frac{Q}{t} = \frac{1.2 \times 2 \times (300 - 278)}{4 \times 10^{-3}}$

$\therefore \frac{Q}{t} = 13200$ W

$\therefore$ The rate of heat loss through a glass window, $\frac{Q}{t} = 13200$ W

48. **2069 (Set A) Old Q. No. 5b** **The sun is a black body of surface temperature about 6000 K. If the sun's radius is 7×10^8m, calculate the energy per second radiated from its surface. (Stefan's constant = $5.7 \times 10^{-8}\ Wm^{-2}K^{-4}$)**

Given,

Radius of the sun (r) = 7×10^8 m
Temperature of sun (T) = 6000 K
Power (p) = ?
Stefan's constant $(\sigma) = 5.7 \times 10^{-8}\ Wm^{-2}K^{-4}$

We know,

$P = \sigma AT^4 = \sigma.4\pi r^2 T^4$
$= 5.7 \times 10^{-8} \times 4\pi \times (7 \times 10^8)^2 \times (6000)^4 = 4.55 \times 10^{26}$ Watt

49. **2068 Q.No. 10 b An ice box is made of wood 1.75 cm. thick lined inside with cork 2 cm. thick. If the temperature of inner surface of the cork is steady at 0°C and that of the outer surface of the wood is steady at 12°C; what is the temperature of the interface? The thermal conductivity of wood is five times that of cork.[4]**

Given,

Wood	Cork
1.75 cm.	2 cm.
$\theta_w = 12°C$	$\theta_2 = 0°C$

Let θ be the temperature of interface,

For wood.

$$\left(\frac{Q}{t}\right)_W = \frac{K_W A(12-\theta)}{1.75 \times 10^{-2}}$$

For cork,

$$\left(\frac{Q}{t}\right)_C = \frac{K_C A(\theta - 0)}{2 \times 10^{-2}}$$

For steady's state,

$$\left(\frac{Q}{t}\right)_W = \left(\frac{Q}{t}\right)_C$$

or, $\frac{K_W A(12-\theta)}{1.75 \times 10^{-2}} = \frac{K_C A(\theta - 0)}{2 \times 10^{-2}}$

or, $\frac{5K_C A(12-\theta)}{1.75} = \frac{K_C \theta}{2}$

or, $10(12-\theta) = 1.75\,\theta$

or, $120 = (1.75 + 10)\theta$ or, $120 = 11.75\,\theta$

∴ $\theta = 10.2°C$

50. **2066 Q. No. 10 c A man, the surface area of whose skin is 2m² is sitting in a room where the air temperature is 20°C. If his skin temperature is 37°C, find the rate at which his body loses heat. The emissivity of his skin is 0.97. [4]**

Given,

Surface area of skin (A) = $2m^2$

Room temperature (T_o) = 20°C = 293 K

Skin temperature (T) = 37°C = 310 K

Emissivity of skin (e) = 0.97

Rate of heat loss (E) = ?

We have,

$E = \sigma A.\, e.\, (T^4 - T_0^4) = 5.67 \times 10^{-8} \times 2 \times 0.97\,[(310)^4 - (293)^4] = 205.2$ W

51. **2064 Q.No. 5 b Assuming that the thermal insulation provided by a woolen glove is equivalent to a layer of quiescent air 3 mm thick, determine the heat loss per minute from a man's hand, surface area 200 cm² on a winter day when the atmospheric air temperature is -3°C. The skin temperature is to be taken as 35°C and thermal conductivity of air as 24×10^{-3} Wm⁻¹ K⁻¹. [4]**

Given,

Thickness of slab (x) = 3 mm = 3×10^{-3} m

Area (A) = 200 cm² = 200×10^{-4} m²

Temperature of skin (θ_1) = 35° C = 308 K

Temperature of atmosphere (θ_2) = – 3°C = 270 K

Time (t) = 60 sec,

Tharmal conductivity (k) = 24×10^{-3} w m⁻¹ K⁻¹

Heat loss per minute, Q = ?

We know,

$$Q = \frac{kA(\theta_1 - \theta_2)\,t}{x} = \frac{24 \times 10^{-3} \times 200 \times 10^{-4}\,(308 - 270)\,60}{3 \times 10^{-3}} J \qquad \therefore Q = 364.8J$$

52. **2055 Q.No. 9** **Estimate the rate of heat loss through a glass window of area 2m² and thickness 3mm when the temperature of the room is 20°C and that of air outside is 5°C.**

Given,

Area(A) = $2m^2$

Thickness of the glass (x) = 3 mm = 3×10^{-3} m

Tharmal conductivity (k) = 1.2 $Wm^{-1}K^{-1}$

Temperature inside the room (θ_1) = 20° C,

Temperature outside the room (θ_2) = 5° C

Rate of heat loss $\left(\frac{Q}{t}\right) = ?$

We know,

$$\frac{Q}{t} = \frac{kA(\theta_1 - \theta_2)}{x} = \frac{1.2 \times 2\,(20 - 5)}{3 \times 10^{-3}}$$

$$\therefore \frac{Q}{t} = 12 \times 10^3 \text{ W} = 12 \text{ kW}$$

53. **2054 Q.No. 8 OR** **What is the ratio of the energy per second radiated by the filament of a lamp at 2500 k to that radiated at 2000 K, assuming the filament is a black body radiator?**

Given,

Temperature of filament lamp (T_1) = 2500 K

Temperature of heat radiated (T_2) = 2000 K

$$\frac{\text{Power Fadiated by Lamp at 2500 K } (P_1)}{\text{Power Radiated by Lamp at 2000 K } (P_2)} = ?$$

The energy radiated per second by a black body of surface area is

$P = \sigma A T^4$

where σ is Stefan's constant and T is absolute temperature of black body.

If P_1 and P_2 be the energy radiated per second at 2500 K and 200 K respectively, then

$$\frac{P_1}{P_2} = \frac{\sigma A T_1^4}{\sigma A T_2^4}$$

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{2500}{2000}\right)^4 = 2.44$$

Hence, the required ratio of power is 2.44:1

54. **2053 Q.No. 10 b** **Estimate the rate at which ice would melt in a wooden box 2.5 cm. thick of inside measurement 100 cm. × 60 cm. × 40 cm. assuming that the external temperature is 35°C and thermal conductivity of wood is 0.168 $Wm^{-1}k^{-1}$.**

Given,

Inside measurement of a wooden box is 100 cm × 60 cm × 4 cm.

∴ Total surface area of the walls of the wooden box,

$A = 2\,[100 \times 60 + 60 \times 40 + 40 \times 100]$

$= 24800 \text{ cm}^2 = 2.48 \text{ m}^2$

Thickness of wooden box (x) = 2.5 cm = 2.5×10^{-2} m

External temperature (θ_1) = 35° C

Internal temperature (θ_2) = 0° C

The rate of heat flow in to the box is given by

The flow of heat in to the box in time dt is, $dQ = \frac{kA(\theta_1 - \theta_2)\,dt}{x}$

or, $\frac{dQ}{dt} = \frac{0.168 \times 2.48 \times (35 - 0)}{2.5 \times 10^{-2}} = 583.29$ W

Inside the box, the rate of heat gained by ice is 583.29 W.

If L is the latent heat of fusion of ice.

Then rate of melting ice $= \frac{dQ/dt}{L} = \frac{538.29}{336 \times 10^3} = 1.6 \times 10^{-3} \text{kg s}^{-1}$

□□□

Chapter 5: Ideal Gas

Short Answer Questions

1. **2075 Set B Q.No. 2a How can Kelvin scale, in principle, be designed by the experiment based on ideal gas law? Explain.** **[2]**

The ideal gas equation is given as

$PV = nRT$ ... (i)

when, $T = 0°C$, then $PV = 0$ which doesn't give any meaning (or becomes meaning less). So, the temperature scale in 0°C is converted to Kelvin scale i.e. 0°C = 0 + 273K = 273K which gives the physical meaning i.e. $PV \propto T$. So, in ideal gas equation, temperature in Celsius scale is converted into Kelvin by adding 273 to it.

2. **2074 Set A Q.No. 2c When a car is driven some distance, the air pressure in the tyre increases. Why?** **[2]**

When a car is driven some distance, the tyre gets heated due to friction. When tyres get heated, the temperate of gas inside the tyre increased. Since the pressure of gas is the function of temperature.

i.e. $P = \frac{1}{3} \rho c^{-2}$ $\quad [\because \bar{c} \propto \sqrt{T}]$

Due to this reason the pressure inside the tyre increases on increasing temperature when the car is drive some distances.

3. **2073 Supp Q.No. 2c What physical concept is provided by universal gas constant? Write its unit.** **[2]**

The Ideal gas equation is given by $PV = nRT$, P is pressure, V is the volume, T is the temperature and constant R in equation of state is called universal gas constant. It is defined as the amount of work done per mole per unit temperature.

The unit of R is J mol^{-1} K^{-1} and value of R = 8.318 J/mol K.

4. **2072 Set C Q.No. 2c Explain on the basis of kinetic theory that the pressure of a gas increases with increase of temperature.** **[2]**

According to kinetic theory of gas, the pressure exerted by gas moving with average r m.s. speed $\bar{c}$ is given by

$$P = \frac{1}{3}\rho\, \bar{c}^2, \ \rho = \text{density of gas}$$

or, $P = \frac{1}{3}\frac{M}{V}\bar{c}^2$ $\quad$ At constant volume, $P \propto \bar{c}^2$. But $\bar{c}^2 \propto T$

$\therefore \quad P \propto T$

Hence, the pressure of gas increases on increasing temperature of gas.

5. **2072 Set D Q.No. 2c On reducing the volume of a gas at constant temperature, the pressure of a gas increase. Why?** **[2]**

On reducing the volume of gas at constant temperature, the number of molecules per unit volume increase. Therefore, more number of molecules collide with the walls per second and hence, large momentum is transferred to the walls per second. The K.E. also increases, as a result pressure increases.

6. **2072 Set E Q.No. 2b What does "the kinetic interpretation of temperature" signify? Explain.** **[2]**

According to Kinetic theory of gas, the Kinetic energy if gas molecule is given by

$$\frac{1}{2}m\,\bar{v}^2 = \frac{3}{2}KT,$$

where $\bar{v}$ is the root mean square velocity, T is temperature of gas and K is Boltzman constant. This shows that K.E. of gas depends upon the temperature.

7. **2071 Set C Q.No. 2 a What is the difference between a real and an ideal gas? Explain on the basis of the hypothesis of the kinetic theory.** **[2]**

Ideal gas is only hypothetical condition but the real gas is found in real life. These two gases have following differences on the basis of kinetic theory of gas:

i. The size of ideal gas molecules is negligible but the size of real gas has significant size.

ii. There is no force of attraction or repulsion between molecules of ideal gas but there is force of attraction or repulsion between real gas molecules.

iii. The collision of ideal gas molecular is perfectly elastic but not in case of real gas.

8. **2070 Supp (Set B) Q.No. 2 a** **At a fixed temperature, the volume of a vessel is compressed to half. How will the rms speed of the gas in it change?** [2]

✍ The rms speed of gas is given by

$$C_{rms} = \sqrt{\frac{3PV}{M}}$$

If temperature remains constant and volume is compressed to half, then pressured will increase such that PV = RT = constant at constant temperature. So, there is no change in rms speed of a gas on changing volume at constant temperature.

9. **2069 (Set A) Old Q. No. 4b** **Do you expect the gas in cooking gas cylinder to obey the ideal gas equation?** [2]

✍ No, the gas in cooking gas cylinder does not obey the ideal gas equation. To obey the ideal gas equation, the temperature of gas should be high and pressure should be low. But the gas in a cooking gas cylinder has high pressure and low temperature. So, it does not obey the ideal gas equation.

10. **2068 Old Q.No. 4 b** **What are the characteristics of a gas to be an ideal?** [2]

✍ According to kinetic theory of gas, a gas should have the characteristics to be an ideal,

a. The size of molecules should be negligibly small in comparison to distance between them,
b. The molecules of gas should alike in size and mass but differ from the molecules of other gas.
c. The force of attraction between molecules should be negligible.
d. The gas molecules should be a perfect elastic sphere.
e. The temperature of gas should be high and pressure should be low.

11. **2065 Q. No. 4 c**

Which has more molecules: A kilogram of hydrogen or a kilogram of oxygen? [2]

✍ The number of mole of a substance is defined as

$$n = \frac{\text{Mass of substance}}{\text{Molecular mass of substance}}$$

$$n = \frac{m}{M}$$

$$\text{or, } n \propto \frac{1}{M}; \text{ for a given mass} = 1\text{kg}$$

Since $M_O > M_H$, $n_O < n_H$; i.e. 1 kg of Hydrogen gas has more atoms than 1 kg of oxygen.

12. **2064 Q.No. 4 c** **Absolute zero temperature is not zero energy temperature. Explain.** [2]

✍ According to kinetic theory of gases, the kinetic energy of the gas molecules is $\frac{1}{2}m\bar{v}^2 = \frac{3}{2}KT$, where $\bar{v}$ is root means square velocity of the molecules of gas and K is Boltzmann constant. At absolute zero temperature, T = 0K, kinetic energy of gas is zero. However, all the molecules remain at rest and there exist potential energy. The total energy is the sum of kinetic and potential energy. At 0K the total energy is non zero due to its potential energy. So, absolute zero temperature is not zero energy temperature.

13. **2063 Q.No. 4 c** **In the kinetic theory of gases, why do we not take into account the changes in gravitational potential energy of the molecule?** [2]

✍ We do not take into account the change in gravitational potential energy in the kinetic theory of gases because the gas molecules are very small in size and its gravitational potential energy($U = -\frac{GMm}{r}$) is negligible compared to the kinetic energy ($E_k = \frac{1}{2}m\bar{v}^2$).

14. **2062 Q.No. 4 c** **Write the unit of the universal gas constant and give its physical meaning.** [2]

✍ Please refer to 2073 Supp Q.No. 2c

15. **2060 Q.No. 4 b** **Under what conditions do the real gases obey more strictly the gas equation PV = RT?** [2]

✍ The equation of state of ideal gas is given by: PV = RT, where R = 8.314J/mole K, is called universal gas constant. The real gases obey more strictly the gas equation PV = RT only (a) at high temperature and (b) at low pressure.

16. 2057 Q.No. 4 a **Why does the cycle tube burst sometimes in summer?** **[2]**

The atmospheric temperature is high during summer. The temperature of air inside cycle tyre gets increased, which results to increase in kinetic energy of air molecules in the tube. This increase in kinetic energy increases the pressure in the tube-but the volume of tyre remains same. While riding the cycle in summer days, the pressure of gas increases tremendously due to the rise in temperature and so the tube burst suddenly.

17. 2057 Q.No. 4 b **At absolute zero temperature, why the kinetic energy is zero?** **[2]**

At absolute zero, the gas does not produce pressure on the wall. from ideal gas equation,

$$PV = nRT$$

For, $T = 0, P = 0$

It means that molecules remains at rest, i.e. speed of molecules becomes zero. Since, the speed is zero; kinetic energy ($\frac{1}{2}m\bar{v}^2$) of gas molecules is also zero. Moreover, from kinetic theory of gases, the kinetic energy of the gas molecules is $\frac{1}{2}m\bar{v}^2 = \frac{3}{2}KT$, where $\bar{v}$ is root mean square velocity of the molecules of gas and K is Boltzmann constant. At absolute zero temperature, T = 0K, kinetic energy of gas is zero.

18. 2055 Q.No. 7 a **Why do you consider an ideal gas while formulating the pressure in the light of kinetic theory of gases?** **[2]**

While formulating the pressure exerted by a gas using the kinetic theory of gas, we consider an ideal gas which has the following properties:

i. The collisions between molecules of a gas are perfectly elastic. That is, there is no loss of kinetic energy during collisions.
ii. There is no molecular attraction or repulsion. That is, there is no potential energy between the molecules.

19. 2053 Q.No. 7 d **Molecules of different gases have equal average kinetic energies, provided their temperature is the same. Do these molecules have equal velocities also?**

No, the gas molecules have not equal velocities. The average K.E. per molecule is given by

$$\frac{1}{2}mc^2_{rms} = \frac{3}{2}kT \quad \text{... (i)}$$

If two gases have the same average kinetic energy at the same temperature, then

$$\frac{1}{2}m_1c_1^2 = \frac{1}{2}m_2c_2^2 = \frac{3}{2}kT \quad \text{... (ii)}$$

where c_1 is the rms speed of a gas having mass of each molecule m_1 and c_2 is the rms speed of another gas having mass of each molecule m_2. Since, m_1 and m_2 are different for different gases, the equation (ii) indicates that c_1 and c_2 are also different.

20. 2053 Q.No. 10 a **Outline the essential features of the kinetic theory of gases.** **[2]**

The kinetic theory of gas explains the macroscopic properties of gases like pressure, volume, temperature in terms of microscopic properties like molecular mass, diameter, velocity etc. This kinetic theory of gas based on following assumptions:

a. The size of molecules is negligibly small in comparison to distance between them.
b. The molecules of gas are alike in size and mass but differ from the molecules of other gas.
c. The force of attraction between molecules is negligible.
d. The motion of gas molecules is random. They have velocity in all directions.
e. The gas molecules are like perfect elastic sphere.
f. The gas molecules move in straight line between two collisions.
g. The collision between gas molecules is perfectly elastic.

21. 2050 Q.No. 7 e **Which has more atoms: a kilogram of hydrogen or a kilogram of iron?**

The number of mole of a substance is defined as

$$n = \frac{\text{mass of substence}}{\text{Molecular mass of substance}} = \frac{m}{M}$$

or, $n \propto \frac{1}{M}$; for a given mass = 1kg

Since $M_i > M_H$, $n_H > n_i$; i.e. 1 kg of Hydrogen gas has more atoms than 1 kg of iron.

Long Answer Questions

22. **2076 GIE Set B Q.No. 6b What is meant by an ideal gas? Obtain the relation PV = nRT, where symbols have their usual meanings.** **[4]**

Ideal gas: According to kinetic theory of gas, a gas having following characteristics is an ideal gas,

a. The size of molecules should be negligibly small in comparison to distance between them,
b. The molecules of gas should alike in size and mass but differ from the molecules of other gas.
c. The force of attraction between molecules should be negligible.
d. The gas molecules should be a perfect elastic sphere.
e. The temperature of gas should be high and pressure should be low.

Ideal gas equation (Equation of state)

For a given mass of gas, the three thermodynamic variables pressure (P), volume (V) and temperature (T) change simultaneously. The equation which relates these three variables pressure (P), volume (V) and temperature (T) is called equation of state.

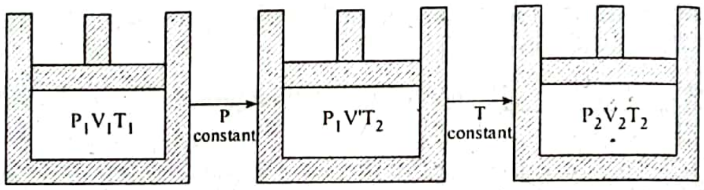


Fig (a) Fig (b) Fig (c)

As shown in above figure, the volume of gas is V_1 at pressure P_1 and absolute temperature T_1. If temperature of gas is increased from T_1 to T_2 at constant pressure (P_1), the volume of gas becomes V' as shown in figure (b). Applying Charle's law, we get

$$\frac{V_1}{T_1} = \frac{V'}{T_2} \quad \Rightarrow V' = \frac{T_2}{T_1}V_1 \quad \text{... (i)}$$

Now, the gas is compressed at constant temperature T_2 so that the new volume of gas is V_2 and pressure P_2. Applying Boyle's law, we get

$$P_1V' = P_2V_2$$

Using equation (i), we get

$$P_1V_1\frac{T_2}{T_1} = P_2V_2 \Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \text{... (ii)}$$

In general, we can write $\frac{PV}{T}$ = constant ... (iii)

The constant appears in equation (iii) does not depends on nature of gas taken and amount of gas taken. If one molecule of gas occupies a volume V at pressure P and at temperature (absolute) T, then

PV = RT, Where R is universal gas constant.

For n-mole of gas, PV = nRT

which is the equation of state of an ideal gas.

23. **2076 Set B Q.No. 6b 2076 Set C Q.No. 6b 2074 Set A Q.No. 6b Derive an expression for the pressure exerted by an ideal gas on the basis of Kinetic theory.** **[4]**

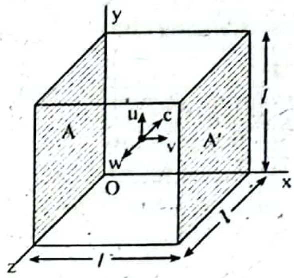
The **kinetic theory of gas** explains the macroscopic properties of gases like pressure, volume, temperature in terms of microscopic properties like molecular mass, diameter, velocity etc. This kinetic theory of gas based on following assumptions:

a. The size of molecules is negligibly small in comparison to distance between them.
b. The molecules of gas are alike in size and mass but differ from the molecules of other gas.
c. The force of attraction between molecules is negligible.
d. The motion of gas molecules is random. They have velocity in all directions.
e. The gas molecules are like perfect elastic sphere.
f. The gas molecules move in straight line between two collisions.
g. The collision between gas molecules is perfectly elastic.

Pressure exerted by gas;

Let us consider a cubical box of side l contains N number of molecules of gas. Each molecules are identical and having mass m. Suppose c is the velocity of gas molecule at any time, then u, v and w are the component of velocity along OX, OY and OZ directions respectively. Then,

$c^2 = u^2 + v^2 + w^2$ ... (i)



Let us consider molecule striking against face A with momentum mu along OX and the molecule rebounds with same momentum as the collision is perfectly elastic. Hence, the momentum of molecule before collision is mu and after collision is –mu.

Change in momentum during collision = mu – (–mu) = 2mu

The time taken by the molecule to move from face A to A' and back to A is given by $\Delta t = \frac{2l}{u}$

Change in momentum per second = $\frac{\Delta p}{\Delta t} = 2mu\frac{u}{2l} = \frac{mu^2}{l}$.

i.e. Force on the face A $= \frac{mu^2}{l}$

Pressure on the face A $= \frac{\text{Force}}{\text{area}} = \frac{mu^2}{l^3}$ ... (ii)

Equation (ii) gives the pressure on the face A due to single molecule whose x-component velocity is u. Since, the velocity of each molecules are different and their components in the direction OX also different. Let $u_1, u_2 \ldots\ldots u_N$ be the components of velocity of all molecules along OX direction. Then, total pressure on the face A is.

$$P = \frac{mu_1^2}{l^3} + \frac{mu_2^2}{l^3} + \ldots + \frac{mu_N^2}{l^3}$$

$$= \frac{m}{l^3}(u_1^2 + u_2^2 + \ldots + u_N^2) \quad \ldots \text{(iii)}$$

If $\overline{u^2}$ denoted mean velocity of $u_1^2, u_2^2 \ldots u_N^2$

$$\overline{u^2} = \frac{u_1^2 + u_2^2 + \ldots + u_n^2}{N}$$

$$u_1^2 + u_2^2 + \ldots u_N^2 = N\bar{u}^2 \quad \ldots \text{(iv)}$$

Using equation (iv) in (iii), we get

$$P = \frac{m}{l^3}.N\bar{u}^2 \quad \ldots \text{(v)}.$$

Since all N molecules have random motion with varying speed in all direction, then

$$\bar{u}^2 = \bar{v}^2 = \bar{w}^2 = \frac{1}{3}\bar{c}^2 \quad \ldots \text{(vi) [Using equation (i)]}$$

where $\bar{c}^2$ is mean square velocity of gas.

Using equation (vi) in (v), we get

$$P = \frac{m}{l^3}N\left(\frac{1}{3}\bar{c}^2\right)$$

Since mass of gas (M) = mN and l^3 = V (Volume of the cube).

Hence, $P = \frac{M}{NV}N\left(\frac{1}{3}\bar{c}^2\right) = \frac{1}{3}\frac{M}{V}\bar{c}^2 = \frac{M}{V}\frac{1}{3}\bar{c}^2$

$$P = \frac{1}{3}\rho\bar{c}^2 \quad \ldots \text{(vii)}$$

Where $\rho = \frac{M}{V}$ is the density of gas. Equation (vii) gives the pressure exerted by gas on the wall of cube.

24. 2075 Set A Q.No. 6b **Starting from basic postulate for kinetic theory of gas, derive an expression for the pressure exerted by the gas contained in a box.** **[4]**

Please refer to 2076 Set B Q.No. 6b.

25. **2074 Set B Q.No. 6b** **Deduce an expression for pressure exerted by a ideal gas from kinetic theory?** **[4]**

Please refer to 2076 Set B Q.No. 6b

26. **2073 Supp Q.No. 6b** **State Boyle's and Charle's laws. Describe how these laws are combined to derive ideal gas law.** **[4]**

Boyle's law: The volume (V) of a given mass of a gas is inversely proportional to its pressure (P) provided the temperature remains constant called Boyle's law. Mathematically,

$V \propto \frac{1}{P}$ ... (i) (at constant temperature)

$\therefore$ PV = constant

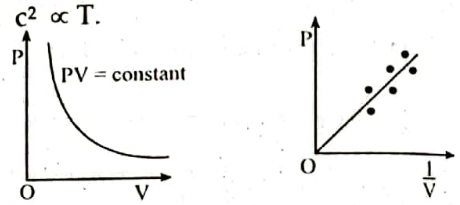
According to kinetic theory of gases $PV = \frac{1}{3} M\bar{c}^2$ where M is the mass of gas and $\bar{c}$ is the root mean square velocity. Since, mean square velocity $\propto$ temperature, i.e. $\bar{c}^2 \propto T$.

Thus, $PV \propto T$

For constant T, *PV = constant.*

Which is Boyle's law.

It is applicable only at high temperature and low pressure.



Charle's law: If pressure is kept constant, the volume (V) of gas is directly proportional to absolute temperature (T).

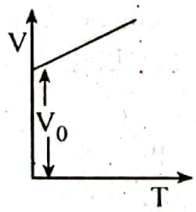
i.e. $V \propto T$, ... (ii) at constant P

Now, according to kinetic theory of gases; $PV = \frac{1}{3} M\bar{c}^2$

Where M = mass of gas

$\bar{c}$ = root mean square velocity

$V = \frac{1}{3}\left(\frac{M}{P}\right)\bar{c}^2$.



Since, we know $\bar{c}^2 \propto T$ and $\frac{M}{P}$ is constant. Then, $V \propto T$, which is Charles' law.

Ideal gas equation (Equation of state)

For a given mass of gas, the three thermodynamic variables pressure (P), volume (V) and temperature (T) change simultaneously. The equation which relates these three variables pressure (P), volume (V) and temperature (T) is called equation of state.

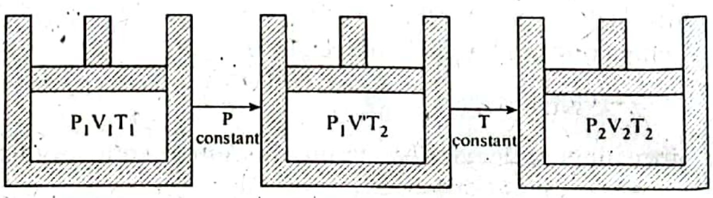


Fig (a) Fig (b) Fig (c)

As shown in above figure, the volume of gas is V_1 at pressure P_1 and absolute temperature T_1. If temperature of gas is increased from T_1 to T_2 at constant pressure (P_1), the volume of gas becomes V' as shown in figure (b). Applying Charle's law, we get

$\frac{V_1}{T_1} = \frac{V'}{T_2} \Rightarrow V' = \frac{T_2}{T_1} V_1$ ... (i)

Now, the gas is compressed at constant temperature T_2 so that the new volume of gas is V_2 and pressure P_2. Applying Boyle's law, we get

$P_1V' = P_2V_2$

Using equation (i), we get

$P_1V_1\frac{T_2}{T_1} = P_2V_2 \Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ ... (ii)

In general, we can write $\frac{PV}{T}$ = constant ... (iii)

The constant appears in equation (iii) does not depends on nature of gas taken and amount of gas taken. If one molecule of gas occupies a volume V at pressure P and at temperature (absolute) T, then

$PV = RT$, Where R is universal gas constant.

For n-mole of gas, $PV = nRT$

which is the equation of state of an ideal gas.

27. **2073 Set C Q.No. 6b** **Using the postulates of kinetic theory of gases, deduce an expression for the pressure exerted by an ideal gas on the walls of a container.** **[4]**

Please refer to 2076 Set B Q.No. 6b

28. **2073 Set D Q.No. 6a** **State and explain Boyle's law and Charle's law. Use these laws to derive the ideal gas equation for n-moles of a gas.** **[4]**

Please refer to 2073 Supp Q.No. 6b

29. **2072 Set C Q.No. 6c** **Use the concepts of the kinetic theory of gases to derive an expression for pressure exerted by the gas on the walls of a container. Extend your result to establish a relation between pressure and average kinetic energy of the gas.** **[4]**

Please refer to 2076 Set B Q.No. 6b

30. **2071 Set C Q.No. 6 a** **Obtain combined equation of state of an ideal gas on the basis of statements and explanations of Boyle's and Charle's laws.** **[4]**

Please refer to 2073 Supp Q.No. 6b

31. **2071 Set D Q.No. 6 b** **Derive ideal gas equation on the basis of kinetic theory of a gas.** **[4]**

Derivation of Boyle's law: According to kinetic theory of gases,

We have,

$$P = \frac{1}{3}\frac{mn}{V}\overline{c^2} = \frac{1}{3}\frac{M}{V}\overline{c^2} \quad \ldots \text{(i)} \ [\because M = m \times n]$$

$$PV = \frac{1}{3}M\overline{c^2}$$

$$\overline{c^2} = \frac{3PV}{M} = \frac{3RT}{M} \quad [\because PV = RT, \text{ for one mole of gas}]$$

For a given gas, M is constant and R is molar gas constant then

$$\overline{c^2} \propto T \quad \ldots \text{(ii)}$$

Using equation (ii) in (i), we get

$$P \propto \frac{T}{V}$$

$$\Rightarrow PV \propto T \quad \ldots \text{(iii)}$$

i.e. $P \propto \frac{1}{V}$, for constant temperature.

For constant temperature, PV = constant, which is Boyle's law.

Derivation of Charles law

If pressure is constant, then

$$P = \frac{1}{3}\rho\,\overline{c^2}$$

$$\Rightarrow P = \frac{1}{3}\frac{M}{V}\overline{c^2}$$

$$V = \frac{1}{3P}M\overline{c^2} \quad \ldots \text{(iv)}$$

For a given mass of gas, M is constant.

$$V \propto \overline{c}^{\,2} \quad \ldots \text{(v)}$$

Using equation (ii) on (v), we get

$$V \propto T$$

Which is Charle's constant pressure law.

Similarly, if volume is taken constant and using equation (ii) on (iv), we get

$$P \propto T$$

This is Charle's constant volume law.

Derivation of combined gas law,

According to kinetic theory of gases,

We have

$$P = \frac{1}{3}\frac{mn}{V}\bar{c}^2 = \frac{1}{3}\frac{M}{V}\bar{c}^2 \quad \text{... (vi) } [\because M = m \times n]$$

$$PV = \frac{1}{3}M\bar{c}^2$$

or, $$\bar{c}^2 = \frac{3PV}{M} = \frac{3RT}{M} \quad [\because PV = RT, \text{ for one mole of gas}]$$

For a given gas, M is constant and R is molar gas constant then

$\bar{c}^2 \propto T$ ... (vii)

Using equation (ii) in (i),

$\Rightarrow$ $PV \propto T$ ... (viii)

$\therefore$ PV= R T, where R is Universal Gas constant. This is the combined gas law.

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32. **2070 Supp (Set B) Q.No. 6 b** **What do you mean by ideal gas? Derive Pv = mrT, where symbols have usual meanings.** **[4]**

Ideal gas: An ideal gas is hypothetical gas which has following characteristics:

a. The size of molecules should be negligibly small in comparison to distance between them,
b. The molecules of gas should alike in size and mass but differ from the molecules of other gas.
c. The force of attraction between molecules should be negligible.
d. The gas molecules should be a perfect elastic sphere.
e. The temperature of gas should be high and pressure should be low.

Second Part: Please refer to 2073 Supp Q.No. 6b

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33. **2070 Set C Q.No. 6 b** **What do you mean by an ideal gas? Derive ideal gas equation for n mole of gas.** **[4]**

Please refer to 2076 GIE Set B Q.No. 6b

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34. **2070 Set D Q.No. 6 b** **What is perfect gas? Prove that the average kinetic energy of a gas molecule is directly proportional to the absolute temperature of the gas.** **[4]**

Please refer to 2076 Set B Q.No. 6b

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35. **2069 Supp Q.No. 6a** **Using the postulates of kinetic theory of gases, derive an expression for the pressure exerted by an ideal gas.** **[4]**

Please refer to 2076 Set B Q.No. 6b

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36. **2069 (Set B) Q. No. 6b** **Use the kinetic theory of gases to derive an expression for the pressure exerted by a gas on the walls of its container.** **[4]**

Please refer to 2076 Set B Q.No. 6b

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37. **2069 (Set A) Old Q. No. 5a OR** **State Charle's law of gases. Derive the ideal gas equation PV = nrt where symbols have usual meaning.** **[1+4]**

Please refer to 2073 Supp Q.No. 6b

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38. **2068 Q.No. 6 b** **On the basis of kinetic theory of gases, deduce the relation, $P = \frac{1}{3}\rho\bar{c}^2$ where the symbols have their usual meanings.** **[4]**

Please refer to 2076 Set B Q.No. 6b

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39. **2067 Supp Q.No. 6 b** **State Boyle's law and Charle's law. Use them to derive the ideal gas equation PV = nRT.**

Please refer to 2073 Supp Q.No. 6b

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40. **2066 Q. No. 6 b** **Prove that the pressure exerted by a gas on the wall of a container is 2/3 times the kinetic energy per unit volume of the gas.** **[4]**

Please refer to 2076 Set B Q.No. 6b

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41. **2066 Old Q. No. 5 a** **Outline essential features of the kinetic theory of gases. Obtain an equation of state on the basis of kinetic theory.** **[3+2]**

1st part: Please refer to 2076 Set B Q.No. 6b

2nd part: Please refer to 2071 Set D Q.No. 6b

42. 2064 Q.No. 5 a OR **Define volume coefficient and pressure coefficient. How pressure coefficient and volume coefficient are related?** **[1+4]**

Volume coefficient of gas (γ_v): The volume coefficient of a gas is defined as the change in volume of gas per unit volume at 0°C per degree Celsius (Kelvin) change in temperature, provided the pressure remains constant. It is denoted by γ_v.

If V_o and V_θ are the volume of the given mass of gas at 0°C and at θ°C. If γ_V is the volume coefficient of gas and θ be the increase in temperature, then,

$$V_\theta = V_0 + \gamma_v V_0 \theta = V_0 (1 + \gamma_v \theta)$$

$$V_\theta - V_0 = V_0 \gamma_V \theta$$

$$\therefore \gamma_V = \frac{V_\theta - V_o}{V_0 \theta} \quad \ldots (i)$$

Experimentally, it is found that $\gamma_V = \frac{1}{273}$ °C^{-1}. It means that at constant pressure, the volume of a certain mass of gas increases or decreases by $\frac{1}{273}$ of its volume at 0 °C for each degree rise or fall in temperature.

Pressure coefficient of gas (γ_P): The pressure coefficient of a gas at constant volume is defined as the change in pressure per unit pressure at 0°C per degree Celsius change in temperature. It is denoted by γ_P.

If P_0 and P_θ are pressure of given mass of gas at temperature 0°C and θ°C and γ_P is the pressure coefficient of gas on raising temperature from 0°C to θ°C, then

$$P_\theta = P_0 (1 + \gamma_P \theta)$$

$$P_\theta - P_0 = P_0 \gamma_P \theta$$

$$\therefore \gamma_P = \frac{P_\theta - P_0}{P_0 \theta} \quad \ldots (ii)$$

Experimentally, it is found that $\gamma_P = \frac{1}{273}$ °C^{-1}. It means that at constant volume, the pressure of a certain mass of gas increases or decreases by $\frac{1}{273}$ of its pressure at 0 °C for each degree rise or fall in temperature.

Relation between γ_V and γ_P: Hence, from above discussion, it is clear that the pressure coefficient γ_P and volume coefficient γ_V are same for all gases.

i.e. $\gamma_V = \gamma_P = \frac{1}{273}$ °C^{-1}

43. 2062 Q.No. 5 a OR **State Boyle's and Charle's law and hence, obtain the relationship for the combined gas law.** **[2+3]**

Please refer to 2073 Supp Q.No. 6b

44. 2061 Q.No. 5 a OR **State Boyle's law and derive ideal gas equation.** **[1+4]**

Please refer to 2073 Supp Q.No. 6b

45. 2059 Q.No. 5 a OR **Using postulates of kinetic theory of a gas, derive an expression for the pressure exerted by the gas on the wall of a box.**

Please refer to 2076 Set B Q.No. 6b

46. 2057 Q.No. 5 a **State Charle's law and derive ideal gas equation.** **[1+4]**

Please refer to 2073 Supp Q.No. 6b

47. 2054 Q.No. 9 **Use the kinetic theory of gases to derive an expression for the pressure exerted by a gas on the walls of its container.** **[4]**

Please refer to 2076 Set B Q.No. 6b

48. 2052 Q.No. 9 **State Boyle's and Charle's law. Show how can be combined to given equation of state of an ideal gas.**

Please refer to 2073 Supp Q.No. 6b

49. 2051 Q.No. 10 Starting from the pressure relation $P = \frac{mn\bar{c}^2}{3V}$ in kinetic theory of gases, derive Boyle's law and Charle's law.

Please refer to 2071 Set D Q.No. 6 b

50. 2050 Q.No. 10 OR Show that, on the basis of the simple kinetic theory of gases, the pressure P of an ideal gas of density ρ is given by $P = 1/3\ \rho\ \bar{c}^2$, where $\bar{c}^2$ is the mean square speed of the molecules. Explain the assumptions you have made in deriving this formula.

Please refer to 2076 Set B Q.No. 6b

Numerical Problems

51. 2076 GIE Set A Q.No. 10b A cylinder of gas has mass of 10 kg and a pressure of 8 atmospheres at 27°C. When some gas is used in the morning at 7°C, the remaining gas in the cylinder at this temperature has a pressure of 6 atmospheres. Calculate the mass of the gas used. [4]

Given,

Mass of gas (m_1) = 10 kg

Temperature of gas (T_1) = 27° C = 300 k

Pressure of gas (P_1) = 8 atm = $8 \times 1.01 \times 10^5$ N/m²

If some gas is used at 7° C, then

Mass of gas remaining = m_e

Temperature (T_2) = 7° C = 280 K

Pressure (P_2) = 6 atm = $6 \times 1.01 \times 10^5$ N/m²

Let V be the volume of cylinder and r be gas constant for unit mass, than

$$r = \frac{P_1V}{m_1T_1} \qquad \text{and } r = \frac{P_2V}{m_2T_2}$$

$$\text{or, } \frac{P_1V}{m_1T_1} = \frac{P_2V}{m_2T_2}$$

$$\text{or, } m_2 = \frac{P_2\, m_1\, T_1}{P_1\, T_2} = \frac{6 \times 1.01 \times 10^5 \times 10 \times 300}{8 \times 1.01 \times 10^5 \times 280}$$

$$\therefore m_2 = 8 \text{ kg}$$

Mass of gas used = $m_1 - m_2 = 10 - 8 = 2$ kg

52. 2075 Set B Q.No. 10b 2072 Supp Q.No. 10a 2067 Q.No. 10 b 2065 Q. No. 5 b Air at 273 K and 1.01×10^5 N/m² pressure contains 2.70×10^{25} molecules per cubic meter. How many molecules per cubic meter will there be at a place where the temperature is 223 K and the pressure is 1.33×10^4 N/m²? [4]

Given,

At temperature, 273K,

Temperature (T_1) = 273 K

Pressure (P_1) = 1.01×10^5 N/m²

No. of molecules (n_1) = 2.7×10^{25} molecules/m³

At temperature, 223 K

Temperature (T_2) = 223 K

Pressure (P_2) = 1.33×10^4 N/m²

No. of molecules (n_2) = ?

We know,

$$\frac{P_2}{P_1} = \frac{n_2\, T_2}{n_1\, T_1} \qquad [V_1 = Vv_2 = 1\text{m}^3]$$

$$\text{or, } n_2 = \frac{P_2}{P_1} \cdot \frac{n_1\, T_1}{T_2}$$

$$= \frac{1.33\times10^4}{1.01\times10^5} \times \frac{2.7\times10^{25} \times 273}{223}$$

$$= 4.35\times10^{24} \text{ molecules/m}^3.$$

53. **2074 Supp. Q.No. 10a The correct inflation of a tyre at 20°C is 2 kg/cm². After driving several hours, the driver checks the tyres. If the tyre's temperature is 50°C, what should be the pressure reading?** **[4]**

Given,

Initial condition,

Inflation (m/A) = 2 kg/cm²
$= 2 \times 100 \times 100$ kgm^{-2}
$= 2 \times 10^4$ kgm^{-2}

Pressure (P_1) $= \frac{mg}{A} = 2 \times 10^4 \times 10 = 2 \times 10^5$ Nm^{-2}

Temperature (T_1) = 20°C = 20 + 273 = 293 K

Final condition,

Pressure (P_2) = ?

Temperature (T_2) = 50°C = 50 + 273 = 323 K

We have,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad [\because V_1 = V_2]$$

or, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

or, $\frac{2 \times 10^5}{293} = \frac{P_2}{323}$

or, $P_2 = \frac{323}{293} \times 2 \times 10^5 = 2.2 \times 10^5$ Nm^{-2}

54. **2073 Supp Q.No. 10a The root mean square (rms) speed of a gas molecule is 600 ms⁻¹ at 500°C. Calculate the rms speed of the gas at 100°C.** **[4]**

Given,

c_{rms} of the molecule at 500°C (c_1) = 600 m/s

Temperature (T_1) = 500°C = 773 k

c_{rms} of the molecule at 100°C (c_2) = ?

Temperature (T_2) = 100°C = 373 k

We know, $c_{rms} = \sqrt{\frac{3kT}{m}}$

$\Rightarrow c_{rms} \propto \sqrt{T}$

So, we can write

$\frac{c_1}{c_2} = \sqrt{\frac{T_1}{T_2}}$

or, $c_2 = c_1 \times \sqrt{\frac{T_2}{T_1}} = 600 \times \sqrt{\frac{373}{773}} = 416.79$ m/s

55. **2072 Set D Q.No. 10b A cylindrical tank has a tight fitting piston that allows the volume of the tank to be changed. The tank originally contains 0.110 m³ of air at a pressure of 3.4 atm. The piston is slowly pulled out until the volume of the gas is increased to 0.390 m³. If the temperature remains constant what is the final value of the pressure?** **[4]**

Given,

Initial volume (V_1) = 0.110 m³

Initial pressure (P_1) = 3.4 atm

Final volume (V_2) = 0.39 m³

Final pressure (P_2) = ?

Since, the temperature of the system remain constant, the process is isothermal. So, the isothermal equation is

$P_1V_1 = P_2V_2$

or, $P_2 = \frac{P_1V_1}{2} = \frac{0.11 \times 3.4}{0.39} = 0.95$ atm

56. **2072 Set E Q.No. 10c At a pressure of 700 mm of Hg, the root mean square speed of the molecules of a gas is 400 ms⁻¹. What is its density?** **[4]**

Given,

Pressure (P) = 700 mm of Hg = $700 \times 10^{-3} \times 13600 \times 10 = 95200$ N/m²

rms Velocity ($\bar{c}$) = 400 m sec^{-1}
Density of gas (ρ) = ?
We have,

$$P = \frac{1}{3}\rho\,\bar{c^2} \qquad [\because c_{rms} = \sqrt{\bar{c^2}} \Rightarrow c_{rms}^2 = \bar{c^2}]$$

$$\text{or,}\quad \rho = \frac{3P}{c^2} = \frac{3 \times 95200}{400^2} = 1.785 \text{ kg m}^{-3}$$

57. **2071 Supp Q.No. 10b** **What is average translational K.E. of a molecule of an ideal gas at a temperature of 27°C? What is the total random translational kinetic energy of the molecules in 1 mole of this gas? What is the root-mean-square speed of oxygen molecules at this temperature?** **[4]**
[Relative molecular mass of oxygen = 32; Universal gas constant = 8.31 Jmol^{-1}K^{-1}]

Given,
Average K.E. of one mole of an ideal gas is

$$\frac{1}{2}M\bar{c^2} = \frac{3}{2}RT \text{ ; [M = mole mass and T = 27° = 300K]}$$

$$\text{or,}\quad K.E_1 = \frac{3}{2} \times 8.31 \times 300 = 3739.5 \text{ J/mole}$$

Average K.E. of a molecule of an ideal gas is,

$$\frac{1}{2}m\bar{c^2} = \frac{3}{2}\frac{R}{N}T \text{ ; [m = mass of one molecule]}$$

$$\text{or,}\quad K.E_2 = \frac{3}{2} \times \frac{8.31}{6.023 \times 10^{23}} \times 300 = 1.24 \times 10^{-20} \text{ J}$$

Again,

$$c_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{32 \times 1.67 \times 10^{-27} \times 6 \times 10^{23}}} = 484.4 \text{ m/sec}$$

58. **2070 Supp (Set A) Q.No. 10 c** **At what temperature will the average speed of oxygen molecule be sufficient so as to escape from the earth? [Escape velocity from the earth is 11.2 km/s and mass of one oxygen molecule is 53.4 × 10^{-24} g]** **[4]**

Given,
Escape velocity of earth (Cs) = 11.2 km/sec = 11200 m/sec
Molecular mass of oxygen (M) = 53.4 × 10^{-24} g = 53.4 × 10^{-27} kg
Temperature (T) = ?
Gas constant (R) = 8.31 J mol^{-1} K^{-1}
We have

$$Cs = \sqrt{\frac{3RT}{Nm}}$$

$$\text{or,}\quad T = \frac{Cs^2 \times Nm}{3R} = \frac{(11200)^2 \times 53.4 \times 10^{-27}}{3 \times 8.31} \times 6.023 \times 10^{23} = 161833.29 \text{ K}$$

59. **2069 (Set A) Q. No. 10a** **Air at a temperature of 273 K and a pressure of 1.01 × 10^5 N/m^2 pressure contains 2.7×10^{25} molecules per cubic meter. How many molecules per cubic meter will be there at a place where the temperature is 223K and the pressure is 1.33×10^4 N/m^2.** **[4]**

Please refer to 2075 Set B Q.No. 10b

60. **2067 Supp Q.No. 10 b** **Taking the density of Nitrogen at STP to 1.251 kgm^{-3}, find the root mean square velocity of Nitrogen molecules at 127°c.** **[4]**

Given,
Density of nitrogen at STP (ρ_1) = 1.251 Kg/m^3
Molecular wt. of nitrogen (M) = 28 × 10^{-3} Kg
Root mean square speed at 127°C (c) = ?
Now,

$$\text{Volume of 1 molecule of nitrogen gas at STP } (V_1) = M/\rho_1 = \frac{28 \times 10^{-3}}{1.251} = 0.022\text{m}^3$$

Pressure of nitrogen gas at STP (P_1) = 760 mm of Hg

= $760 \times 10^{-3} \times 13600 \times 9.8$ = 1.01×10^{-5} N/m²

Volume of 1 mole of gas at STP (V_1) = 22.5×10^{-3}m

Temperature of nitrogen at STP (T_1) = 273 K

Now, gas constant

$$R = \frac{P_1V_1}{T_1} = 8.304 \text{ J mol}^{-1} \text{ K}^{-1}.$$

Temperature of gas (T) = 127°C = (127 + 273) K

$$c = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.304 \times 400}{28 \times 10^{-3}}} = 596.6 \text{ m/sec.}$$

61. 2063 Q.No. 5 b **A cylinder of gas has a mass of 10 kg and pressure of 8 atmosphere at 27°C. When some gas is used in a cold room at -3°C, the gas remaining in the cylinder at this temperature has a pressure of 6.4 atmospheres. Calculate the mass of gas used.**

Given,

Mass of gas (M_1) = 10kg

Temperature of gas (T_1) = 27°C = 300K

Pressure of the gas (P_1) = 8 atm = $8 \times 1.01 \times 10^5$ N/m²

Again when some gas is used at –3°C

Let remaining mass of the gas be m_2

Temperature of gas (T_2) = – 3°C = 270 K

Pressure of gas (P_2) = 6.4 atm = $6.4 \times 1.01 \times 10^5$ N/m².

Let, V be the volume of the cylinder and r be the gas constant per unit mass.

Then,

$$r = \frac{P_1V}{m_1T_1} \quad \text{... (i)}$$

$$\text{and } r = \frac{P_2V}{m_2T_2} \quad \text{... (ii)}$$

Equating equation (i) and (ii), we get

$$\frac{P_1V}{m_1T_1} = \frac{P_2V}{m_2T_2}$$

$$\text{or, } m_2 = \frac{P_2T_1m_1}{P_1T_2} = \frac{6.4 \times 1.01 \times 10^5 \times 10 \times 300}{8 \times 1.01 \times 10^5 \times 270} = 8.9 \text{ kg}$$

Mass of gas used = $m_1 - m_2$ = 10 – 8.9 = 1.1 kg

62. 2060 Q.No. 5 b **Helium gas occupies a volume of 0.04m³ at a pressure of 2 × 10⁵ Nm⁻² and temperature 300 K. Calculate the mass of the helium and root mean square speed of its molecules. (Relative molecular mass of helium = 4, molar gas constant = 8.3 J mol⁻¹ K⁻¹)**

Given;

Volume (V) = 0.04m³

Pressure (P) = 2×10^5 Nm⁻²

Temperature (T) = 300K

Relative molecular mass of helium = 4

Molar mass (M) = 4g = 4×10^{-3} kg

Molar gas constant (R) = 8.3 J mol⁻¹ K⁻¹

From ideal gas equation,

$$PV = nRT$$

$$PV = \frac{m}{M}RT \qquad (\because n = \frac{m}{M}, \text{ m is the mass of the gas})$$

$$m = \frac{PMV}{RT}$$

$$m = \frac{2\times10^5 \times 0.04 \times 4 \times 10^{-3}}{8.3 \times 300}$$

$\therefore$ $m = 1.29 \times 10^{-2}$ kg

We know,

$$P = \frac{1}{3}\rho\bar{c}^2$$

$$\bar{c}^2 = \frac{3P}{\rho}$$

$$c_{rms} = \sqrt{\frac{3P}{m/V}} = \sqrt{\frac{3PV}{M}}$$

$$= \sqrt{\frac{3\times2\times10^5\times0.04}{1.29\times10^{-2}}}$$

$\therefore$ $c_{rms} = 1363.98$ m/s

63. **2058 Q.No. 5 b** **Find the rms speed of nitrogen at NTP. Density of $N_2 = 1.29 kg/m^3$ at NTP.**

Given,

At NTP, we have

Pressure (P) $= 1.01 \times 10^5 Nm^{-2}$

Temperature (T) = 273 K

Density (ρ) $= 1.29$ kgm^{-3}

rms speed (c_{rms}) = ?

We know,

$$P = \frac{1}{3}\rho\bar{c}^2_{rms}$$

or, $1.01 \times 10^5 = \frac{1}{3} \times 1.29 \times \bar{c}^2_{rms}$

or, $\bar{c}^2_{rms} = 234883.72$

$\therefore$ $c_{rms} = 484.65$ ms^{-1}

64. **2056 Q.No. 5 b** **Two glass bulbs of equal volume are joined by a narrow tube and are filled with a gas at s.t.p. When one bulb is kept in melting ice and the other is placed in a hot bath, the new pressure is 877.6 mm of Hg. Calculate the temperature of the bath.**

Given,

Initial pressure of the gas at s.t.p. (P_1) = 760 mm Hg.

Final pressure of the gas in both bulbs (P_2) = 877.6 mm Hg.

Suppose V be the volume of each bulb and θ be the temperature of hot bath,

Mass of gas at s.t.p. in each bulb $= \frac{P_1V}{r \times 273}$

Total mass of gas at s.t.p. in both the bulbs $= \frac{2P_1V}{r \times 273}$ ... (i)

Final mass of gas in melting ice $= \frac{P_2V}{r \times 273}$ ... (ii)

Final mass of gas in hot bath $= \frac{P_2V}{r(273+\theta)}$ ... (iii)

We know, total mass of the substance remains constant. So,

Initial total mass of gas at s.t.p in both the bulbs = final total mass of the gas in both bulbs

or, $\frac{2P_1V}{r \times 273} = \frac{P_2V}{r \times 273} + \frac{P_2V}{r(273+\theta)}$

or, $\frac{2P_1}{273} = \frac{P_2}{273} + \frac{P_2}{(273 + \theta)}$

or, $\frac{2P_1}{273} - \frac{P_2}{273} = \frac{P_2}{(273 + \theta)}$

or, $(273 + \theta) = \frac{P_2}{\left(\frac{2P_1}{273} - \frac{P_2}{273}\right)} = \frac{P_2}{2P_1 - P_2} \times 273$

$\theta = \frac{P_2}{(2p_1 - p_2)} \times 273 - 273$

$= \frac{877.6}{2 \times 760 - 877.6} \times 273 - 273$

$= \frac{239584.8}{642.4} - 273 = 373 - 273$

$\therefore \theta = 100°\ C$

Temperature of hot bath is 100° C.

65. 2052 Q.No. 8 **Two bulbs of equal volume are joined by a narrow tube and are filled with gas at STP. When one bulb is kept in melting ice and the other in boiling water, calculate the new pressure of the gas.**

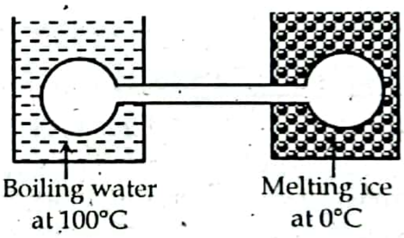
Given,

Standard pressure (P) = 760 mm Hg

Standard temperature (T) = 273 K

Let V be the volume of each bulb. Then

Mass of gas in each bulb (m) $= \frac{PV}{rT} = \frac{760V}{r \times 273}$



$\therefore$ Total mass of gas in both bulbs $= 2m = 2 \times \frac{760V}{r \times 273}$ ... (i)

When one bulb is kept in melting ice and the other in boiling water, in this case, let P' be the new pressure. Then,

Mass of gas in the bulb which is kept in melting ice,

$m' = \frac{P'V}{r \times 273}$; where 273K is the temperature of the bulb which is kept in melting ice.

and Mass of gas in the bulb which is kept in boiling water, $m'' = \frac{P'V}{r \times 373}$ where 373K is the temperature of the bulb which is kept in boiling water.

$\therefore$ Total mass of gas is both bulbs $= m' + m''$

$= \frac{P'V}{r \times 273} + \frac{P'V}{r \times 373}$ ... (ii)

In both the cases, the total mass of gas is constant in both bulbs. Therefore, from eq^n. (i) and (ii), we have

$2 \times \frac{760\ V}{r \times 273} = \frac{P'V}{r \times 273} + \frac{P'V}{r \times 373} = \frac{P'V}{r}\left(\frac{1}{273} + \frac{1}{373}\right)$

or, $\frac{2 \times 760}{273} = P'\left(\frac{373 + 273}{273 \times 373}\right)$

$\therefore P' = 877.65$ mm of Hg which is the required new pressure of the gas.

❑❑❑

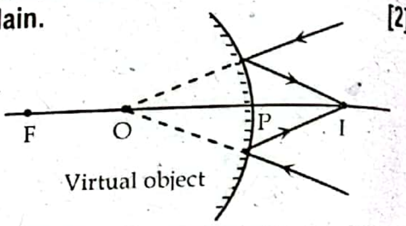
UNIT 3 Waves and Optics

Chapter 1: Reflection at Curved Mirrors

Short Answer Questions

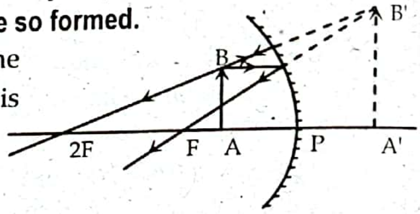
1. **2076 GIE Set A Q.No. 3a Can a convex mirror ever form a real image? Explain.** [2]

Yes, a convex mirror can form a real image if the object is virtual. That is, if a converging beam is incident on a convex mirror, a real image is formed in front of it. In such case, the object distance is less than the focal length of the mirror.



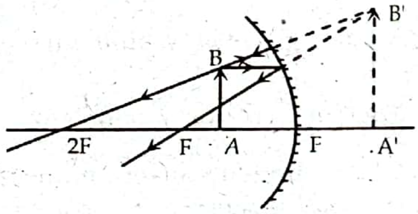
2. **2074 Set B Q.No. 3a Use graphical method to locate image of an erect object placed before a concave mirror between its pole and the focus. Hence write down the nature of image so formed.** [2]

When an object AB lies between principal focus and pole of the mirror, it's image A'B' will be formed behind the mirror which is virtual, erect and magnified.



3. **2073 Set C Q.No. 3a A concave mirror is often used as an aid for applying cosmetics to the face, why?** [2]

For shaving or make-up purpose, I suggest concave mirror. When a concave mirror is used for this purpose, our face or object lies between focus and optical centre, so that highly enlarged, virtual, erect image is formed behind the mirror show in diagram.



4. **2073 Set D Q.No. 3a Convex mirrors are used as rear view mirrors in cars, why?** [2]

Convex mirrors are used as rear view mirrors in automobiles to see the traffic at the backside. They provide wider angle of view, although the images are diminished. They make virtual, erect and diminished image of wide range.

5. **2072 Supp Q.No. 3b Does focal length change when the curve mirror is dipped in liquid? Why?** [2]

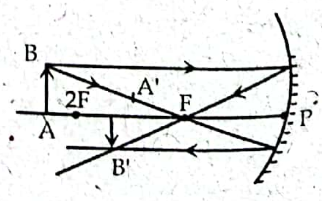
No, the focal length of a spherical mirror does not change when it is immersed in water because the focal length of mirror is independent to the medium in which it is kept. As $\left(\frac{1}{f} = \frac{1}{u} + \frac{1}{v}\right)$, it is independent of refractive index of medium. So, the focal length of mirror does not change when it is kept in water.

6. **2071 Set C Q.No. 3 b Spherical mirror may behave as a plane mirror as a special case. Explain.** [2]

For spherical mirror, the focal length and radius of curvature are related as, $f = \frac{R}{2}$. For $f = \infty$, $R = \infty$, which is case of plane mirror. Hence, spherical mirror may behave as a plane mirror as special case when its radius of curvature becomes infinity.

7. **2070 Supp (Set B) Q.No. 3 a A spherical mirror is cut in half horizontally. Will an image be formed by the botom half of the mirror? How?** [2]

When we cut a shperical mirror into half horizontally, the radius of curvature is not changed. The focal length thus remains the same and the image formed will be that of the usual case. The only difference that happens is that the intensity of the images will be reduced as half of the part of the mirror is removed.



8. 2070 Set D Q.No. 3 b **Distinguish between real and virtual images.** [2]

The differences between real image and virtual image are as follows:

Real image	Virtual image
a. An image is said to be real, if it is formed by the actual intersection of reflected or refracted rays.	a. An image is said to be virtual if it is formed by the virtual intersection of reflected or refracted ray.
b. They can be received in screen.	b. They cannot be received in screen.
c. Real images are inverted with respect to the object.	c. Virtual images are erect with respect to object.
d. Image formed by concave mirror is generally real.	d. Image formed by convex mirror is generally virtual.

9. 2069 (Set.A) Q. No. 3a **Can a convex mirror ever form real image? Justify your answer.** [2]

Please refer to 2076 GIE Set A Q.No. 3a

10. 2069 (Set A) Old Q. No. 6a **Can a convex mirror ever produce a real image?** [2]

Please refer to 2076 GIE Set A Q.No. 3a

11. 2068 Old Q.No. 6 b **What types of mirror will you suggest for shaving or make-up purpose and why?** [2]

Please refer to 2073 Set C Q.No. 3a

12. 2065 Q. No. 6 a **A spherical mirror is immersed in water. Will its focal length change?** [2]

Please refer to 2072 Supp Q.No. 3b

13. 2059 Q.No. 6 b **Trace the position of an image formed by a concave mirror when real object is placed at a distance less than its focal length.** [2]

Please refer to 2074 Set B Q.No. 3a

14. 2056 Q.No. 6 a **Why are convex mirrors used in cars for rear view?** [2]

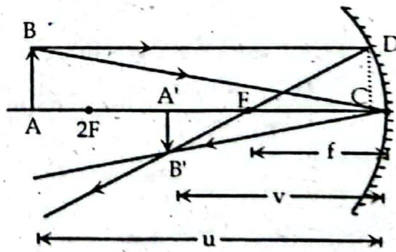
Please refer to 2073 Set D Q.No. 3a

Long Answer Questions

15. 2076 Set C Q.No. 7a **Derive mirror formula for a concave mirror and state the sign convention used.** [4]

Mirror formula: The relationship between focal length of mirror with object distance and image distance is called mirror formula.

Consider a concave mirror of focal length f which has to relate with object distance u and image distance v. Let AB be an object placed normally on the principal axis of the mirror. The rays of light from the object AB, after reflection from concave mirror meet at point B'. So A'B' is the real image of the object AB. If the aperture of the mirror is small, the point P i.e pole of the mirror coincides with point C. Since Δ's ABC and A'B'C are similar, so their corresponding sides are proportional.



$$\frac{AB}{A'B'} = \frac{CA}{CA'} \quad \text{... (i)}$$

Similarly Δ's CDF and A'B'F are similar.

$$\therefore \frac{CD}{A'B'} = \frac{CF}{FA'}$$

But CD = AB

$$\therefore \frac{AB}{A'B'} = \frac{CF}{FA'} \quad \text{... (ii)}$$

From Eqs. (i) & (ii), we have

$$\frac{CA}{CA'} = \frac{CF}{FA'}$$

$$\text{or, } \frac{CA}{CA'} = \frac{CF}{CA' - CF}$$

$$\text{or, } \frac{u}{v} = \frac{f}{v - f}$$

or, $uv - uf = vf$

or, $uv = uf + fv$

(where, CA = u is object distance taken +ve for real object
CA' = v is image distance taken + ve for real image
CF = f is focal length taken + ve for concave mirror)

Dividing both sides by uv

$$\frac{uv}{uv} = \frac{uf}{uv} + \frac{vf}{uv}$$

or, $1 = \frac{f}{v} + \frac{f}{u}$

or, $1 = f\left(\frac{1}{v} + \frac{1}{u}\right)$

$\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ which is mirror formula.

The nature of image is inverted, real and different size according to position of object.

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16. **2074 Set A Q.No. 7b** **Derive mirror formula in the case of concave mirror. Also discuss the nature of the image formed due to object placed at different positions.** **[4]**

Please refer to 2076 Set C Q.No. 7a

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17. **2073 Supp Q.No. 7a** **Derive mirror formula in the case of concave mirror. Also discuss the nature of images.** **[4]**

Please refer to 2076 Set C Q.No. 7a.

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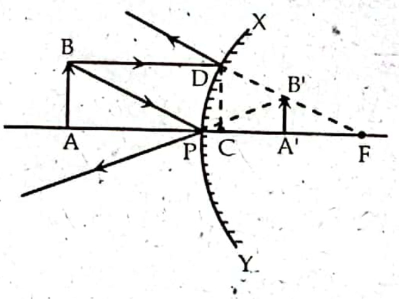
18. **2071 Set C Q.No. 7 b** **How will you make difference between real and virtual images? Obtain an expression for the relation between object distance, image distance and the focal length in the case of a convex mirror.** **[4]**

The differences between real image and virtual image are as follows:

Real image	Virtual image
a. An image is said to be real, if it is formed by the actual intersection of reflected or refracted rays.	a. An image is said to be virtual if it is formed by the virtual intersection of reflected or refracted ray.
b. They can be received in screen.	b. They cannot be received in screen.
c. Real images are inverted with respect to the object.	c. Virtual images are erect with respect to object.
d. Image formed by concave mirror is generally real.	d. Image formed by convex mirror is generally virtual.

Mirror formula for convex mirror: Let us consider a convex mirror XY of focal length f which has to relate with object distance u and image distance v. Let AB be an object placed normally on the principal axis of the mirror. The rays of light from the object AB after reflection from convex mirror appear to meet at point B'. So A'B' is the virtual image of the object AB. If the aperture of the mirror is small, the point P i.e pole of the mirror coincides with point C. Since Δ's ABC and A'B'C are similar so their corresponding sides are proportional.



$\therefore \frac{AB}{A'B'} = \frac{CA}{CA'}$ ... (i)

Similarly Δ's CDF and A'B'F are similar.

$\therefore \frac{CD}{A'B'} = \frac{CF}{FA'}$

But CD = AB

$\therefore \frac{AB}{A'B'} = \frac{CF}{FA'}$ ... (ii)

From Eqs. (i) & (ii), we have

$$\frac{CA}{CA'} = \frac{CF}{FA'}$$

or, $\frac{CA}{CA'} = \frac{CF}{CF' + A'C}$

or, $\frac{u}{-v} = \frac{-f}{-f + v}$, where, CA = u is object distance

or, $uv - uf = vf$ CA' = -v is image distance for virtual image

or, $uv = uf + fv$ CF = -f is focal length for convex mirror.

Dividing both sides by uv

$$\frac{uv}{uf} = \frac{uf}{uv} + \frac{vf}{uv}$$

or, $1 = \frac{f}{v} + \frac{f}{u}$

or, $1 = f\left(\frac{1}{v} + \frac{1}{u}\right)$

$\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ which is mirror formula.

From ΔABC and A'B'C the magnification is defined as

$$m = \frac{A'B'}{AB} = \frac{CA'}{CA} = \frac{-v}{u}$$

$\therefore m = \frac{-v}{u}$

19. **2069 (Set B) Q. No. 7a** **2061 Q.No. 7a** **Point out the difference between real image and virtual image. Obtain a relation connecting the object distance, image distance and focal length of concave mirror.** **[4]**

First Part: Please refer to 2071 Set C Q.No. 7 b

Second Part: Please refer to 2076 Set C Q.No. 7a

20. **2064 Q.No. 7 a** **What do you mean by principal focus of a convex mirror? Prove mirror formula for a convex mirror and also show that m = – v/u for the same mirror.** **[1+3+1]**

Principal Focus: When a narrow beam of light, parallel to the principal axis and close to it is incident on curved mirror, after reflection appears to meet at a point, this point is called principal focus. It is usually denoted by F.

Second Part: Please refer to 2071 Set C Q.No. 7 b

Numerical Problems

21. **2071 Set D Q.No. 11** **At what position an object be placed in front of a concave mirror of radius of curvature 0.4 m so that an erect image of magnification 3 be produced?** **[4]**

Given,

Radius of curvature (R) = 0.4 m

Magnification (m) = 3

Object distance (u) = ?

Now, Focal length (f) $= \frac{R}{2} = \frac{0.4}{2} = 0.2$ m

For erect image m = – ve

So,

$$m = \frac{-v}{u} = 3$$

or, $v = -3u$

From mirror formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $\frac{1}{0.2} = \frac{1}{u} - \frac{1}{3u}$

or, $5 = \frac{3-1}{3u}$

or, $5 = \frac{2}{3u}$

or, $u = \frac{2}{3 \times 5}$

$\therefore u = 0.133$ m

22. **2066 Old Q. No. 7 b** **A pole 4 m long is laid along the principal axis of a convex mirror of focal length 1m. The end of the pole nearer the mirror is 2 m from it. Find the length of the image of the pole.** [4]

Given,

Length of pole AB = 4m

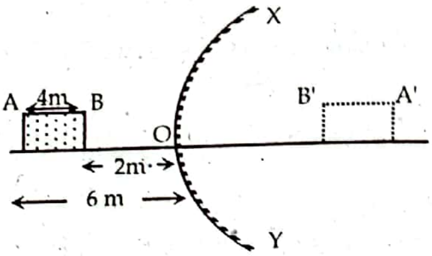
Focal length of mirror (f) = – 1m

Length of image A'B' = ?

Let B be nearer point from the mirror which is 2m from the pole of mirror, then far point A is 2 + 4 = 6 m form the pole of mirror.

Now, for nearer end,

u = 2 m, f = – 1m, v = ?



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $-\frac{1}{1} = \frac{1}{2} + \frac{1}{v}$

or, $\frac{1}{v} = -1 - \frac{1}{2} = -\frac{3}{2}$ m

or, $v = -\frac{2}{3}$ m

For far end,

$u' = 6m, v' = ?, f = -1m$

$$\frac{1}{f} = \frac{1}{u'} + \frac{1}{v'}$$

or, $-\frac{1}{1} = \frac{1}{6} + \frac{1}{v'}$

or, $\frac{1}{v'} = -1 - \frac{1}{6} = -\frac{7}{6}$ m

or, $v' = -\frac{6}{7}$ m

∴ The length of image A'B' = $\left(\frac{6}{7} - \frac{2}{3}\right)$ = 0.19m

23. **2060 Q.No. 7 b** **An erect image, three times the size of the object is obtained with a concave mirror of radius of curvature 36 cm. What is the position of the object?**

Given,

Magnification (m) = 3

Radius of curvature (r) = 36 cm

Focal length (f) = $\frac{r}{2} = \frac{36}{2}$ = 18 cm

Object distance (u) = ?

We have,

$m = \frac{-v}{u} = 3$

$v = -3u$

or, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{18} = \frac{1}{u} - \frac{1}{3u}$

or, $\frac{1}{18} = \frac{3-1}{3u} = \frac{2}{3u}$

or, 3u = 36

or, u = 12 cm

24. **2057 Q.No. 7 b** **An object 10 cm high is placed in front of a convex mirror of focal length 20 cm and the object is 30 cm from the mirror. Find the height of the image.**

Given,

Height of object (O) = 10 cm

Height of image (I) = ?

Focal length of convex mirror (f) = –20 cm

Object distance (u) = 30 cm

If v is the image distance, then

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-20} = \frac{1}{30} + \frac{1}{v} \quad \Rightarrow v = -12 \text{ cm}$$

Since, magnification (m) $= \frac{I}{O} = \frac{v}{u}$

$$\therefore \frac{I}{10} = \frac{12}{30}$$

$\Rightarrow$ I = 4 cm = 0.04 m

Thus, height of image is 0.04 m

25. **2055 Q.No. 15 b** **Calculate the focal length of a concave mirror when an object placed at a distance of 40 cm makes image equal to the size of the object.**

Given,

Focal length (f) = ?

Object distance (u) = 40 cm

We have,

$$m = \frac{v}{u} = \frac{I}{0}$$

But, I = 0, m = 1

$$m = \frac{v}{u} = 1$$

or, v = u = 40 cm

$$\text{So, } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{40} + \frac{1}{40}$$

or, f = 20 cm

26. **2053 Q.No. 13** **A metre scale is placed along the axis of a convex mirror of focal length 25 cm, its nearer end being at a distance of 50 cm. Calculate the size of the image formed.**

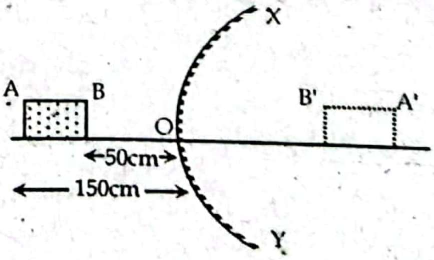
Given,

Focal length of the convex mirror (f) = – 25cm

In the given figure, nearer end B of a meter scale AB is at a distance of 50 cm from the pole (O) of the convex mirror and far end A of the scale is at a distance of 150 cm form the pole O.

For the near point B of scale, u = 50 cm

Using mirror formula,



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$-\frac{1}{25} = \frac{1}{50} + \frac{1}{v}$$

where v is the image distance for B

v = – 16.67 cm

Again, for far end A,

u' = 150 cm

Using mirror formula, we get

$$\frac{1}{f'} = \frac{1}{u'} + \frac{1}{v'}$$

$$-\frac{1}{25} = \frac{1}{150} + \frac{1}{v'}$$

$\therefore$ v' = – 21.43 cm

So, size of image (A'B') = (21.43 – 16.67) cm = 4.67 cm.

27. **2050 Q.No. 15** **A convex mirror with a radius of curvature 30 cm forms a real image 20 cm from its pole. Explain how it is possible and find whether the image is erect or inverted.**

Given,

Radius of curvature of a convex mirror (r) = –30 cm

Image distance (v) = 20 cm

Object distance (u) = ?

Focal length of convex mirror (f) $= \frac{r}{2} = \frac{-3}{2} = -15$ cm

We know, $\frac{1}{u} = \frac{1}{f} - \frac{1}{v} = \frac{1}{-15} - \frac{1}{20} = \frac{3-4}{60} = \frac{-7}{60}$

$\therefore$ $u = -\frac{60}{7} = -8.57$ cm

The negative sign shows that the object is virtual. Since u and v have opposite signs, the image is erect with respect to the object.

□□□

Chapter 2: Refraction at Plane Surfaces

Short Answer Questions

1. **2076 Set C Q.No. 3a Why does the bottom of a swimming pool always appear shallower than it actually is? [2]**

This is due to refraction (bending) of light while passing from water to air. In such case, object lies in denser medium and is viewed from rarer medium. So when light passes from denser to rarer medium, it bends away from normal so that real position is shifted to apparent position. Apparent depth is always less than real depth and hence, a clear pool of water always appears to be shallower than it is actually.

2. **2070 Supp (Set A) Q.No. 3 a Why does the sun look slightly oval when it is near the horison? [2]**

This is due to refraction of light when passes one medium to another. The density of the air in the atmosphere is greater near the surface of earth than above it. Thus, the magnitude of refraction increases with decreasing altitude. Due to this, the rays from the lower edge of the sun are bent more than the rays from the upper edge when situated at honizon. Hence, vertical diameter of the sun is shortened more than the horizontal diameter. That is why, the sun appears flattened at the time of sunset and sunrise because of refraction of light.

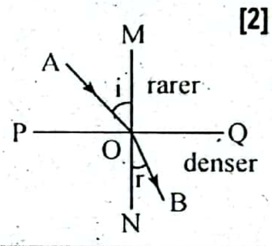
3. **2069 (Set A) Old Q. No. 6b A stick partially dipped in water seems to be bent. Why? [2]**

This is due to refraction of light. When a stick is partially dipped in water, due to refraction of light, it seems to be bent. When light rays pass from denser medium to rarer medium, then rays bent away from the normal. When a stick is placed in water partially, the light rays from different portions of the dipped part of the stick undergo bending away from the normal to reach the eyes. So, it seems to be bending.

4. **2062 Q.No. 6 b Can total internal reflection be achieved if object originates in rarer medium? Explain with a diagram to justify your answer. [2]**

No, total internal reflection can not be achieved if object originates in rarer medium. For total internal reflection, the light ray must pass from denser to rarer media and angle of incidence in denser medium must be greater than critical angle so that angle of refraction in rarer medium is 90°.

If light passes from rarer to denser medim, the light ray bend towards normal and can't be achieved total internal reflection.



5. **2061 Q.No. 6 b A ray of light in air strikes a glass surface. Is there a range of angles for which total internal reflection takes place? [2]**

No, there is not a range of angles for which total internal reflection takes place when a ray of light in air strikes a glass surface. It is because for total internal reflection, light should travel from denser medium to rarer medium. But here, air is rarer medium and glass is denser medium. So, this phenomenon does not happen in this case.

6. **2058 Q.No. 6 c Why does diamond sparkle? [2]**

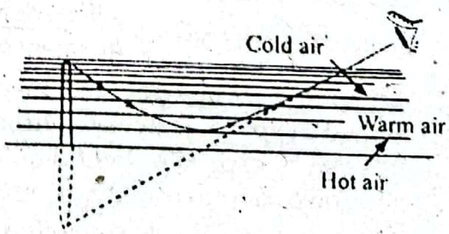
Diamond shines because of total internal reflection of light. The refractive index of diamond is 2.4 and hence, its critical angle is small, 24.6°. The surfaces of the diamond are cut such that light in it undergoes multiple total internal reflections. The light coming out of some faces makes brilliancy of the diamond.

7. **2057 Q.No. 6 b Why does the sun look a little oval when it is at the horizon? [2]**

Please refer to 2070 Supp (Set A) Q.No. 3 a

8. **2054 Q.No. 11 b Why do in summer, roads often appear to be covered with water when seen from a distance? Explain. [2]**

This is due to total internal reflection of light. In summer, during day time, the layers of air in contact with the road become very hot and upper layer of air remains cold, so the densities and refractive indices of these layers are lower than those of the cooler layers higher up. The rays of light from a distance object bend more and more as they pass through denser (cold air) to rarer (hot air) medium. When the beam falls on a layer at an angle greater than the critical angle for it, it gets totally reflected. These reflected rays then travel up and undergo a series of refractions, but in a direction opposite to those in the first case.



These rays finally reach observer's eyes, which see an inverted image of the object, as though reflected from a pool of water.

9. **2054 Q.No. 11 b** **Why does a clear pool of water always appear to be shallower than it actually is?** [2]

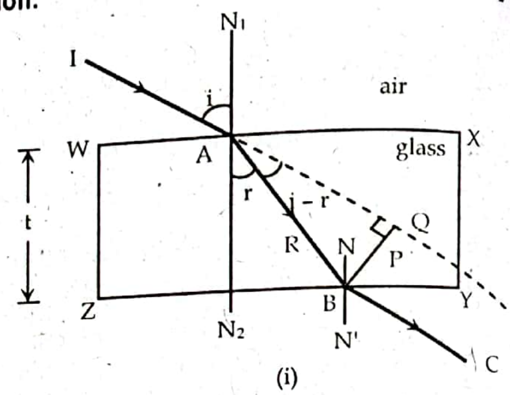
Please refer to 2076 Set C Q.No. 3a

Long Answer Questions

10. **2076 GIE Set B Q.No. 7a** **What is lateral shift? Obtain its expression.** [4]

The perpendicular distance between the direction of incident ray and emergent ray of light is called the lateral shift. It is denoted by P.

Let us consider a glass slab WXYZ of thickness t. When a ray of light IA is incident on the surface of the slab at A, it is refracted along AB and at last emerges out along BC as shown in fig. N_1N_2 and NN' are normals drawn at A and B respectively. BQ is a perpendicular distance between direction of incident ray and emergent ray which is called as BQ is lateral shift.



(i)

From ΔABQ, we have

$$\sin(i - r) = \frac{BQ}{AB}$$

or, $BQ = AB \sin(i - r)$ ... (i)

Again from ΔABN_2, we have

$$\cos r = \frac{AN_2}{AB} = \frac{t}{AB} \quad [\because t = AN_2 = \text{thickness of slab}]$$

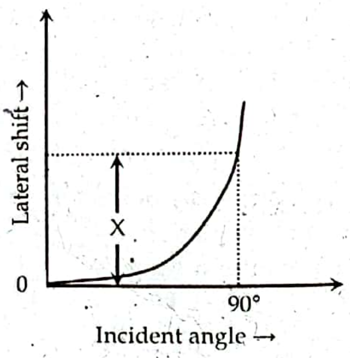
or, $AB = \dfrac{t}{\cos r}$

So, Eq. (i) becomes

$$BQ = \frac{t}{\cos r}\sin(i - r)$$

$$\therefore BQ = \frac{t\sin(i - r)}{\cos r}$$

$$\therefore P = \frac{t\sin(i - r)}{\cos r}$$



(ii)

when, $i = 90°$

$$BQ = P = \frac{t\sin(90° - r)}{\cos r} = \frac{t\cos r}{\cos r}$$

$$\therefore P = t$$

It means that, when incident angle increases, the lateral shift increases and at incident angle of 90° on the surface of glass slab, lateral shift produced by it is equal to the thickness of the slab as shown in fig(ii). From fig(ii), we see that the value of lateral shift slightly increases on increasing incident angle and become hightest on incident angle of 90°.

11. **2076 Set B Q.No. 7a** **Define lateral shift and derive an expression for it due to a parallel edged glass slab.** [4]

Please refer to 2076 GIE Set B Q.No. 7a

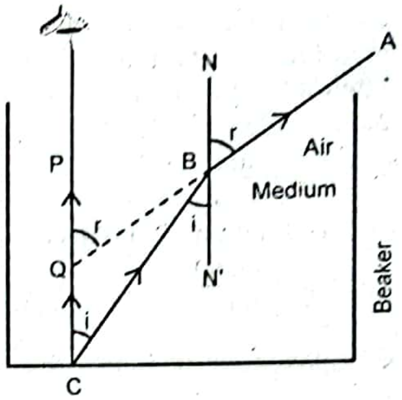
12. **2074 Supp. Q.No. 7a** **What causes the bending of light when it passes from one medium to another medium? Deduce the relation, $\mu = \dfrac{\text{Real depth}}{\text{Apparent depth}}$** [4]

When light passes from one medium to another medium, its wavelength and speed changes. Due to change in speed, it deviates from its original path and hence bends at boundary of two media. Thus difference in speed of light in different medium is the cause of bending of light while passes from one medium to another medium.

Let us consider a coin C is placed in a breaker containing a medium of refractive index, μ. Let CB be the ray of light passing from denser medium and bends to BA while passing from denser medium to air medium. Let NN' be the normal making incident angle i and refracted angle r. Draw AB to Q such that the coin is seen at Q from the eye. Then, PC = real depth and PQ = apparent depth.

The refractive index of medium in this case is given as

$${}_a\mu_m = \frac{\text{Sin r}}{\text{sin i}} \quad \text{[here light is passing denser to rarer medium]}$$



From figure,

$\angle CBN' = \angle QCB = \angle i$

$\angle NBA = \angle PQB = \angle r$

From ΔCPB,

$$\text{Sin i} = \frac{PB}{BC} \text{ and}$$

From ΔQPB,

$$\text{Sin r} = \frac{PB}{BQ}$$

Then,

$${}_a\mu_m = \frac{\text{Sir r}}{\text{sin i}} = \frac{PB}{QB} \times \frac{BC}{PB} = \frac{BC}{BQ}$$

If point B is very close to point P, then BC ≈ PC, BQ ≈ PQ

$$\therefore {}_a\mu_m = \frac{PC}{PQ} = \frac{\text{Real depth}}{\text{Apparent depth}} \qquad \left[\begin{array}{l}\because PC = \text{Real depth} \\ PQ = \text{Apparent depth}\end{array}\right]$$

Hence proved.

13. **2073 Set D Q.No. 7a Define lateral shift. Derive an expression for it. Show in a graph the variation of lateral shift with the angle of incidence. [4]**

Please refer to 2076 GIE Set B Q.No. 7a

14. **2072 Set C Q.No. 7b What is lateral Shift? Deduce lateral shift in terms of thickness of a slab and the angle of incidence of light. [4]**

Please refer to 2076 GIE Set B Q.No. 7a

15. **2070 Supp (Set A) Q.No. 7 a What is lateral shift? Derive an expression for it due to a rectangular glass slab. [4]**

Please refer to 2076 GIE Set B Q.No. 7a

16. **2070 Set D Q.No. 7 a 2065 Q. No. 7 a What is lateral shift? Derive an expression for its value. How does the lateral shift change with the increase in the angle of incidence? [4]**

Please refer to 2076 GIE Set B Q.No. 7a

17. **2067 Supp Q.No. 7 a Derive an expression for lateral shift. Show in a graph how does the lateral shift vary with the increase in angle of incidence '? [4]**

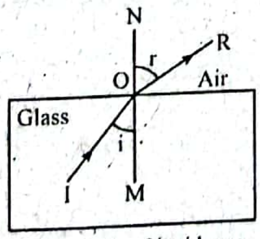
Please refer to 2076 GIE Set B Q.No. 7a

18. **2067 Q.No. 7 a What is lateral shift? Derive an expression for it due to a parallel edged glass slab. [4]**

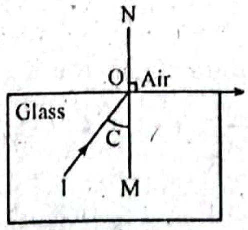
Please refer to 2076 GIE Set B Q.No. 7a

19. **2056 Q.No. 7 a What do you mean by critical angle and total internal reflection? Derive a relation between critical angle and refractive index. [1+1+3]**

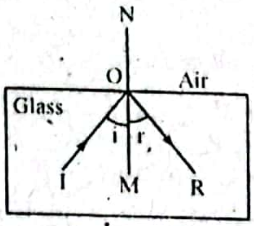
Critical angle: When light passes from denser medium to rarer medium, the refracted ray bends away from normal. If angle of incidence in denser medium is increased gradually, angle of refraction also goes on increasing and at certain incident angle; the refracted angle is 90°. The angle of incidence in denser medium for which angle of refraction is 90° in rarer medium is called critical angle. It is denoted by 'C'.



i = angle of incidence
r = angle of refraction
i < c



C = critical angle
i = c



i > c

Total internal reflection: When the angle of incidence in denser medium is made slightly greater than critical angle C, the light is reflected in the same denser medium obeying the laws of reflection and losed the laws of refraction. This is known as total internal reflection.

Total internal reflection is shown in figure for which angle of incidence is greater than critical angle.

Relation between μ and C: Let 'μ' be the refractive index of denser medium with respect to rarer medium. If C be the angle of incidence C (critical angle), r is angle of refraction in rarer medium. Then refractive index is defined as the ratio of the sine of angle of refraction to the sine of angle of incidence.

i.e. $\mu = \dfrac{\sin r}{\sin i}$

But for total internal reflection, $i = C$ and $r = 90°$

$$\mu = \frac{\sin 90°}{\sin C} \Rightarrow \mu = \frac{1}{\sin C}$$

For glass air system, $\mu = 1.5$ and $C = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.5°$

i.e. angle of incidence as critical angle is 41.5° for glass air system.

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20. $\boxed{\text{2052 Q.No. 15 a}}$ **Define critical angle and total internal reflection.** **[3]**

Please refer to $\boxed{\text{2056 Q.No. 7 a}}$

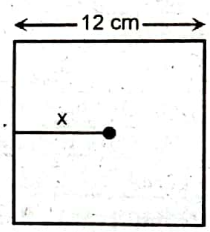
Numerical Problems

21. $\boxed{\text{2075 Set A Q.No. 11}}$ **A transparent cube of 12 cm edge contains a small air bubble. Its apparent depth when viewed through one face of the cube is 6 cm and when viewed through the opposite face is 2 cm. What is the actual distance of the bubble from the first face?** **[4]**

Given,

Edge of cube = 12 cm

Let the actual distance of the bubble from the first face be x cm than,



$$\mu = \frac{x}{6} \quad \ldots \text{(i)}$$

For second face,

The actual distance of air bubble = 12 – x, then

$$\mu = \frac{12-x}{2} \quad \ldots \text{(ii)}$$

Equating equation (i) and (ii), we get,

$$\frac{x}{6} = \frac{12-x}{2}$$

or, $2x = 72 - 6x$

or, $8x = 72$

$\therefore$ $x = 9$ cm

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22. $\boxed{\text{2075 Set B Q.No. 11}}$ **Calculate the critical angles of (i) glass-water and (ii) water-air interfaces if object lies in the denser medium.** **[4]**

Given, Refractive index of glass $({}_a\mu_g) = 1.5$

Refractive index of water $({}_a\mu_w) = 1.33$

Critical angle of glass water interface = ?

Critical angle of water-air interface = ?

i. We have,

Here, the objective is in denser medium, then critical angle C_{gw} is given as

$$\sin C_{gw} = \frac{1}{{}_w\mu_g} = \frac{1}{{}_w\mu_a \times {}_a\mu_g} = \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{1.33}{1.5} = 0.8867$$

or, $C_{gw} = \sin^{-1}(0.8867)$

$\therefore$ $C_{gw} = 62.46°$

ii. For water-air interface, the objective is in denser medium means in water. So critical angle for water, C_w is

$$\sin C_w = \frac{1}{{}_w\mu_g}$$

or, $$\sin C_w = \frac{1}{1.33} = 0.752$$

$\therefore$ $C_w = \sin^{-1}(0.752) = 48.76°$

23. **2072 Set D Q.No. 11** **An optical fiber with refractive index 1.72 is surrounded by a glass coating having refractive index 1.50. Find the critical angle for total internal reflection at the fiber glass interface.** **[4]**

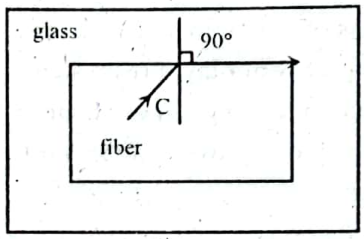
Given,

R.I. of optical fiber (${}_a\mu_0$) = 1.72

R.I. of glass (${}_a\mu_g$) = 1.50

Critical angle (C) = ?

We have,



$$\text{Sin } C = \frac{1}{{}_g\mu_0} = \frac{1}{{}_g\mu_a \cdot {}_a\mu_0} = \frac{{}_a\mu_g}{{}_a\mu_0}$$

$$= \frac{1.5}{1.72} \quad [{}_g\mu_0 = {}_g\mu_a \cdot {}_a\mu_0 \text{ and } {}_g\mu_a = \frac{1}{{}_a\mu_g}]$$

or, $C = \text{Sin}^{-1}(0.872)$

$= 60.7°$

24. **2071 Set C Q.No. 11** **What is the apparent position of an object below a rectangular glass slab of refractive index 1.45 and the thickness 0.06 m if a layer of water 0.05 m thick is on the top of the glass slab?** **[4]**

Given,

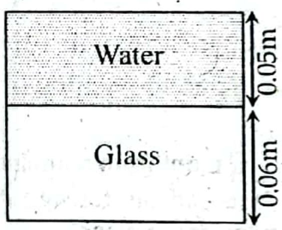
Thickness of glass block (t_1) = 0.06 m

Thickness of water layer (t_2) = 0.05 m

Refractive index of glass w.r.t. air (μ_g) = 1.45

Refractive index of water w.r.t. air (μ_w) = 1.33

We know,



$$\text{Displacement (d)} = t\left(1 - \frac{1}{\mu}\right)$$

$$\text{For glass, } d_1 = t_1\left(1 - \frac{1}{\mu_g}\right) = 0.06\left(1 - \frac{1}{1.45}\right) = 0.0186 \text{ m}$$

$$\text{For water } d_2 = t_2\left(1 - \frac{1}{\mu_w}\right) = 0.05\left(1 - \frac{1}{1.33}\right) = 0.0124 \text{ m}$$

$\therefore$ Total displacement (d) = $d_1 + d_2$ = 0.0186 + 0. 0124 = 0.031 m

Hence, the apparent position of the object is 0.031 m above the bottom and 0.11 – 0.031 = 0.0789 m below the top.

25. **2068 Old Q.No. 7 b** **What is the apparent position of an object below a rectangular block of glass 8 cm thick if a layer of water 6 cm thick is on the top of the glass? (Take: ${}_a\mu_g$ = 1.50 and ${}_a\mu_w$ = 1.33)**

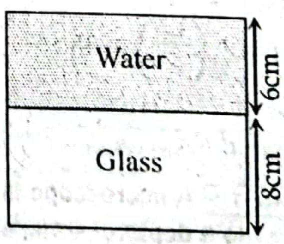
Given,

Thickness of glass block (t_1) = 8 cm

Thickness of water layer (t_2) = 6 cm

Refractive index of glass w.r.t. air (μ_g) = 1.5

Refractive index of water w.r.t. air (μ_w) = 1.33



We know,

$$\text{Displacement (d)} = t\left(1 - \frac{1}{\mu}\right)$$

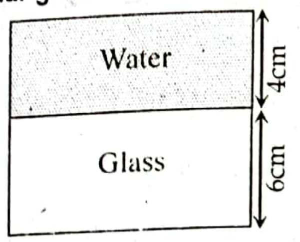
$$\text{For glass, } d_1 = t_1\left(1 - \frac{1}{\mu_g}\right) = 8\left(1 - \frac{1}{1.5}\right) = 2.6 \text{ cm}$$

For water $d_2 = t_2\left(1 - \frac{1}{\mu_w}\right) = 6\left(1 - \frac{1}{1.33}\right) = 1.488$ cm

$\therefore$ Total displacement (d) = $d_1 + d_2 = 2.6 + 1.488 = 4.088$ cm

Hence, the apparent position of the object is 4.088 cm above the bottom and 14 – 4.088 = 9.912 cm below the top.

26. **2067 Q.No. 11** **What is the apparent position of an object below a rectangular glass slab 6 cm thick if a layer of water 4 cm thick is on the top of the glass slab?** **[4]**



Given,

Thickness of glass block (t_1) = 6 cm

Thickness of water layer (t_2) = 4 cm

Refractive index of glass w.r.t. air (μ_g) = 1.5

Refractive index of water w.r.t. air (μ_w) = 1.33

We know,

$$\text{Displacement (d)} = t\left(1 - \frac{1}{\mu}\right)$$

For glass, $d_1 = t_1\left(1 - \frac{1}{\mu_g}\right) = 6\left(1 - \frac{1}{1.5}\right) = 2$ cm

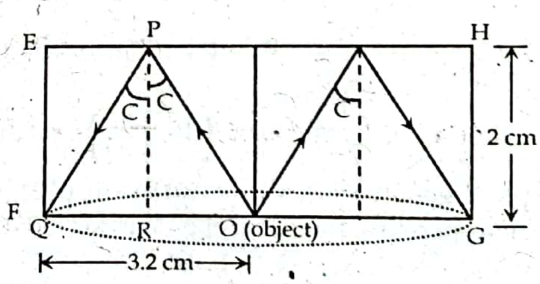
For water $d_2 = t_2\left(1 - \frac{1}{\mu_w}\right) = 4\left(1 - \frac{1}{1.33}\right) = 0.99$ cm

$\therefore$ Total displacement (d) = $d_1 + d_2 = 2 + 0.99 = 2.99$ cm

Hence, the apparent position of the object is 2.99 cm above the bottom and 10 – 2.99 = 7.01 cm below the top.

27. **2063 Q.No. 7 b** **Light from a luminous point on the lower face of a rectangular glass slab, 2 cm thick, strikes the upper face and the totally reflected rays outline a circle of 3.2 cm radius on the lower face. What is refractive index of the glass?**

Let O be the object lying at a lower face of a rectangular glass slab EFGH.



Now,

Thickness of glass slab, PR = 2 cm

Radius of circle, OQ = 3.2 cm

Refractive index of glass, μ = ?

From figure,

$$QR = \frac{OQ}{2} = \frac{3.2}{2} = 1.6 \text{ cm}$$

$\tan C = \frac{QR}{PR}$, where C is critical angle

$$C = \tan^{-1}\left(\frac{1.6}{2}\right) = 38.66°$$

So, $\mu = \frac{1}{\sin C} = \frac{1}{\sin 38.66°} = 1.6$

28. **2059 Q.No. 7 b** **A microscope is focused on a scratch on the bottom of a beaker. Turpentine is poured into the beaker to a depth of 4 cm, and it is found necessary to raise the microscope through a vertical distance of 1.25 cm to bring the scratch again into focus. Find the refractive index of turpentine.** **[4]**

Given,

Real depth = 4 cm

Since microscope is raised through a vertical distance of 1.25 cm to focus the scratch again,

$\therefore$ Apparent depth = 4 – 1.25 = 2.75 cm

We have,

$$\text{Refractive index } (\mu) = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{4}{2.75} = 1.45$$

Hence, refractive index of turpentine is 1.45.

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29. **2058 Q.No. 7 b** **What is the apparent position of an object below a rectangular block of glass 6 cm thick if a layer of water 4 cm thick is on the top of the glass? (Refractive index of glass = 3/2 and that of water = 4/3).**

➢ Please refer to 2067 Q.No. 11

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30. **2054 Q.No. 13 OR** **How long will the light take in travelling a distance of 500 m in water? Refractive index for water is 1.33 and velocity of light in vacuum is 3×10^8 m/s.**

➢ Given,

Time (t) = ?

Distance (d) = 500 m

Refractive index of water (μ_w) = 1.33

Velocity of light in vacuum (c) = 3×10^8 m/sec

We have,

$$\mu = \frac{c}{v}$$

$$\text{or,} \quad v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.33} = 2.26 \times 10^8 \text{m/sec}$$

$$\text{Now,} \quad t = \frac{d}{V} = \frac{500}{2.26 \times 10^8} = 2.2 \times 10^{-6} \text{ sec.}$$

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Chapter 3: Refraction through Prisms

Short Answer Questions

1. **2076 Set C Q.No. 3b Under what conditions does a prism produce the angle of minimum deviation? Explain.** [2]

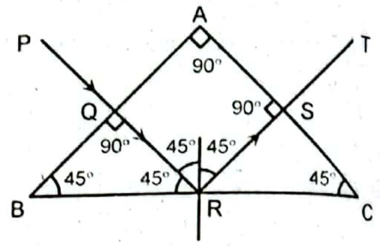
The minimum deviation (δ_{min}) of light occurs when the ray passes symmetrically through the prism. In this case, the angle of emergence is equal to the angle of incidence and angle of refracton at two faces are also equal. The refracted ray through the prism is also parallel to the base of the prism.

2. **2075 Set A Q.No. 3a What are the conditions for minimum deviation of a ray of light passing through a prism?** [2]

Please refer to 2076 Set C Q.No. 3b

3. **2075 Set B Q.No. 3a A ray of light originated in air falls perpendicularly on the shorter face of a right angled isosceles (90° – 45° – 45°) triangular glass prism of refractive index 1.5. Find the angle of deviation in this case in a diagram.** [2]

Let ABC is an isosceles right angled prism such that AB and AC are equal. A ray of light PQ incidents on face AB perpendicularly such that it does not derivate and strikes face BC at R with incident angle 45°, but $\mu = 1.5$ i.e. critical angle (C) becomes 42° $\left(\because \mu = \frac{1}{\sin C}\right)$.



Here incident angle becomes greater than critical angle and total internal reflection takes place. As a result light ray emerges through path RST. Here derivation angle becomes 90°.

4. **2074 Supp. Q.No. 3b A glass prism is immersed in water. What happens to the value of angle of minimum deviation?** [2]

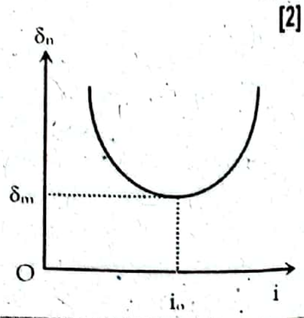
The refractive index (μ) of a glass prism with angle (A) and minimum deviation δ_m are related by a formula,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

When the glass prism is immersed in water, the refractive index decreases and hence angle of minimum deviation also decease being direct relationship.

5. **2074 Set A Q.No. 3b What do you understand by minimum deviation in the case of refraction through a prism? Plot a graph between angles of incidence and the deviation produced.** [2]

When the angle of incidence increases, the deviation angle at first decreases, reaches to a minimum value and again increases. The minimum value of deviation angle is called minimum deviation angle. It is the condition at which the deviation angle is minimum and starts to increase on increasing incident angle beyond this. At minimum deviation condition, angle of incidence and angle of emergence are equal, the refracting angles at both faces are equal and the refracted light ray inside the prism is parallel to base of prism.



6. **2066 Old Q. No. 6 c State the factors on which the deviation produced by a prism depend.** [2]

The deviation produced by the prism depends upon the angle of prism, refractive index of a medium, angle of incidence, color of light and the medium around the prism.

7. **2060 Q.No. 6 c What are the advantages of total reflecting prism over plane mirror?** [2]

The advantages of total reflecting prism over plane mirror are:

a. When light is reflected from total reflecting prism, whole of the light is totally internally reflected. However, from plane mirror, only about 90% of incident light is reflected. Therefore, intensity of reflected light is more in prism than that of the light reflected from plane mirror.

b. No multiple reflections take place in total reflecting prism as in plane mirror.

c. The image is not laterally inverted in case of total reflecting prism as in plane mirror.

d. Use of total reflecting prism reduces the size of optical instrument than in using plane mirror.

e. It does not require silvering which is one of the advantages.

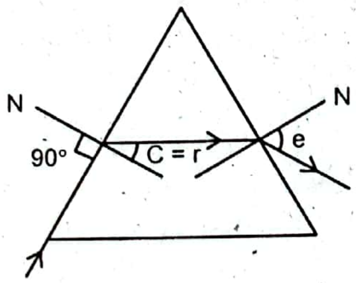
8. 2051 Q.No. 12 e **Under what condition does a prism produce the angle of minimum deviation?** **[2]**

Please refer to 2076 Set C Q.No. 3b

Long Answer Questions

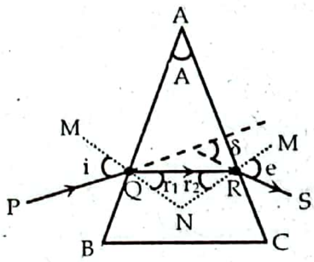
9. 2076 GIE Set A Q.No. 7a **Define grazing incidence. Show that the deviation produced by a small angled prism is independent of angle of incidence.** **[4]**

Grazing Incidence: Grazing incidence refers to the condition in which a ray of light is incident at an angle of 90° in the first face of the prism. The angle of refraction at this face is equal to the critical angle of glass prism.



Let us consider ABC is the prism having small angle A. PQ is the incident light ray making angle of incidence i, QR is the refracted ray in the prism, RS is the emergent ray.

r_1, r_2, δ and e are angle of refraction at faces AB and AC, angle of deviation and emergent angle respectively. μ be the refractive index of material of prism.



From Snell's law, $\mu = \frac{\sin i}{\sin r_1} \approx \frac{i}{r_1}$ [for refraction at face AB]

[sin i ~ i, for small i]

$\therefore$ $i = \mu r_1$ ... (i) [if i is small r also small]

Similarly, for refraction at face AC, using Snell's law

$\mu = \frac{\sin e}{\sin r_2} \approx \frac{e}{r_2}$

$\Rightarrow$ $e = \mu r_2$ ... (ii)

For prism, we have

$i + e = A + \delta$

$\mu r_1 + \mu r_2 = A + \delta$ [using (i) and (ii)]

$\delta = \mu (r_1 + r_2) - A$

$= \mu A - A$ [$\therefore$ $A = r_1 + r_2$]

$\therefore$ $\delta = A(\mu - 1)$

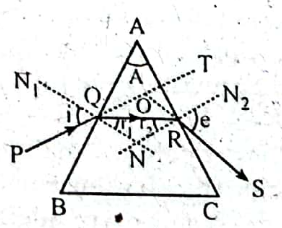
This shows that deviation produced by a small angled prism is independent of angle of incidence provided it is small.

10. 2074 Set B Q.No. 7a **Explain with necessary theory of measuring the refractive index of a prism by minimum deviation method.** **[4]**

Prism is a triangular shaped refracting medium. It has two refracting faces, a base and a refracting angle. Let us consider a prism ABC, AB and AC are its two refracting faces with angle of prism A as shown in figure.

Let us consider light ray PQ incident on the face AB of prism with an angle i then refracted along QR and finally emerge out with emergent angle from the face AC of the prism. r_1 and r_2 are the angle of refractions for the prism. The angle of deviation δ of light caused by the prism is the angle between the direction of incident ray and emergent ray.

From figure, the angle of deviation $\delta = \angle TOS$



and $\angle TOS = \angle OQR + \angle ORQ$

$= (\angle OQN - \angle RQN) + (\angle ORN - \angle QRN)$

$= (i - r_1) + (e - r_2)$

i.e. $\delta = i + e - (r_1 + r_2)$ ... (i)

Again, from quadrilateral AQNR, $\angle AQN = 90°$ and $\angle ARN = 90°$ ... (ii)

$\therefore$ $\angle A + \angle QNR = 180°$ ... (iii)

Also, from ΔQRN, $r_1 + r_2 + \angle QNR = 180°$

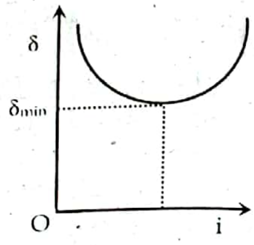
From (ii) and (iii), we get ... (iv)

$r_1 + r_2 = A$

Using (iv) in (i), we get

$$\delta = i + e - A \qquad \ldots (v)$$

Experimentally, it is found that if the angle of incidence is gradually increased from zero, the deviation δ goes on decreasing to a certain minimum value δ_{min} for a certain angle of incidence and then again the deviation angle increases as shown in figure.



In minimum deviation position ($\delta = \delta_{min}$), $i = e$ and $r_1 = r_2$.

Thus, from (v), we get

$$\delta_{min} = i + i - A = 2i - A$$

$$i = \frac{A + \delta_{min}}{2} \qquad \ldots (vi)$$

Similarly, $r_1 = r_2 = r$, equation (iv) gives

$$r + r = A \Rightarrow 2r = A$$

$$\Rightarrow \quad r = \frac{A}{2} \qquad \ldots (vii)$$

If μ is the refractive index of material defined as the ratio of the sine of angle of incidence to sine of angle of reflection, then

$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \mu = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\frac{A}{2}} \qquad \text{[using (vi) and (vii)]}$$

This is the required expression of refractive index relating the angle of prism and angle of minimum deviation.

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11. $\boxed{\text{2072 Set D Q.No. 7a}}$ **Discuss the phenomenon of refraction through a prism. Derive an expression for the refractive index of the material of the prism in terms of the angle of minimum deviation.** **[4]**

Please refer to $\boxed{\text{2074 Set B Q.No. 7a}}$

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12. $\boxed{\text{2072 Set E Q.No. 7a}}$ **Derive a relation between the refractive index, angle of minimum deviation and the angle of prism. Also discuss the nature of the change in deviation with respect to the angle of incidence on one of the refracting surface of the prism.** **[4]**

Please refer to $\boxed{\text{2074 Set B Q.No. 7a}}$

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13. $\boxed{\text{2071 Set C Q.No. 7 a}}$ **Explain why a ray of light in air deviates while entering into a medium. Derive an expression for the refractive index of a glass prism if the ray suffers a condition of minimum deviation.** **[4]**

First Part: When a ray of light passes through a medium, its speed change due to change in wavelength and it deviates.

Second Part: Please refer to $\boxed{\text{2074 Set B Q.No. 7a}}$

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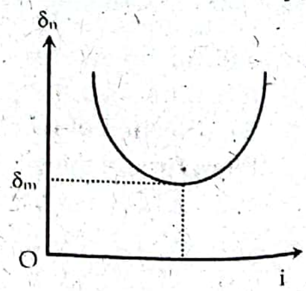
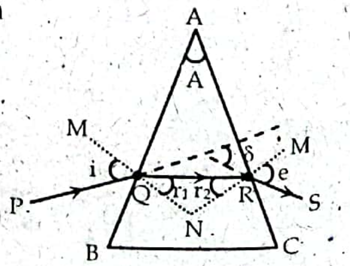
14. $\boxed{\text{2070 Supp (Set B) Q.No. 7 a}}$ **What do you mean by minimum deviation of light when passing through a prism? Show that the deviation produced by a small angle prism is independence of angle of incidence.** **[4]**

Minimum Deviation: When the angle of incidence increases, the deviation angle at first decreases, reaches to a minimum value and again increases. The minimum deviation is the condition at which the deviation angle is minimum and starts to increase on increasing incident angle beyond it. At minimum deviation condition, angle of incidence and angle of emergence are equal, the refracting angles at both faces are equal and the refracted light ray inside the prism is parallel to base of prism.



Let us consider ABC is the prism having small angle A. PQ is the incident light ray making angle of incidence i, QR is the refracted ray in the prism, RS is the emergent ray.



r_1, r_2, δ and e are angle of refraction at faces AB and AC, angle of deviation and emergent angle respectively. μ be the refractive index of material of prism.

From Snell's law, $\mu = \dfrac{\sin i}{\sin r_1} \approx \dfrac{i}{r_1}$ [for refraction at face AB]

[sin i ~ i, for small i]

$\therefore$ $i = \mu r_1$ ... (i) [if i is small r also small]

Similarly, for refraction at face AC, using Snell's law

$$\mu = \frac{\sin e}{\sin r_2} \approx \frac{e}{r_2}$$

$\Rightarrow$ $e = \mu r_2$ ... (ii)

For prism, we have

$$i + e = A + \delta$$

$$\mu r_1 + \mu r_2 = A + \delta \quad \text{[using (i) and (ii)]}$$

$$\delta = \mu (r_1 + r_2) - A$$

$$= \mu A - A \quad [\because A = r_1 + r_2]$$

$$\therefore \delta = A (\mu - 1)$$

This shows that deviation produced by a small angled prism is independent of angle of incidence provided it is small.

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15. **2070 Set C Q.No. 7 a** **What is prism? Show that the deviation produced by a small angle prism is independent of the angle of incidence, provided the angle of incidence is small.** **[4]**

First Part: A prism is a refracting surface with its two refracting surfaces inclined at an angle to each other. In figure AB and AC are two refracting surfaces. A is the angle of prism ABC.

The phenomenon of bending of light, when it passes from one medium to another medium is called refraction. When white light is incident on one face of prism, it is dispersed into its constituent seven colours from another side of prism. This is due to the peculier geometrical shape of prism. The refractive index of prism can be calculated by using Cauchy formula;

$$\mu = A + \frac{B}{\lambda^2}$$

where λ is the wavelength of light incident on prism.

Second part: Please refer to 2070 Supp (Set B) Q.No. 7 a

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16. **2069 (Set B) Q. No. 7b** **Show that** $\mu = \dfrac{\sin\left(\dfrac{A+\delta m}{n}\right)}{\sin A/2}$ **for a prism where the notations carry their usual meanings.** **[4]**

Please refer to 2074 Set B Q.No. 7a

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17. **2069 (Set A) Old Q. No. 7a** **What is the deviation of light? Show that deviation produced by a small angled prism is independent of the angle of incidence.** **[1+4]**

Deviation of light is the bending of light from its original direction and angle between the direction of incident ray and emergent ray of light is called deviation angle.

Second part: Please refer to 2070 Supp (Set B) Q.No. 7 a

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18. **2069 Old (Set B) Q. No. 7a** **What is prism? Show that** $\mu = \dfrac{\sin\left(\dfrac{A+\delta m}{n}\right)}{\sin A/2}$ **for a prism where the notations carry their usual meanings.** **[1+4]**

Please refer to 2074 Set B Q.No. 7a

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19. **2068 Q.No. 7 a** **Define angle of prism. Prove that the deviation produced by a small angle prism for small angle of incidence is independent of angle of incidence.** **[4]**

The angle between refracting faces of a prism is called angle of prism.

Second part: Please refer to 2070 Supp (Set B) Q.No. 7 a

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20. **2068 Old Q.No. 7 a or** **What is the cause of dispersion of light? Prove that the deviation produced by a small angle prism for small angle of incidence is independent of angle of incidence.** **[2+3]**

Please refer to 2070 Supp Set B Q.No. 7a

21. **2066 Q. No. 7 b What is minimum deviation for prism? Prove that the refractive index of the prism is $\mu = \frac{\sin\left(\frac{A+Dm}{2}\right)}{\sin(A/2)}$. Each symbol has its usual meaning.** [4]

Minimum deviation for prism: Please refer to 2070 Supp (Set B) Q.No. 7 a

Second Part: Please refer to 2074 Set B Q.No. 7a

22. **2063 Q.No. 7 a Derive an expression connecting refractive index of the material prism with minimum deviation.**

Please refer to 2074 Set B Q.No. 7a

23. **2060 Q.No. 7 a Discuss the phenomenon of refraction through prism and show that the deviation of Incident ray produced by a small angled prism for small angle of incidence is independent of the angle of prism.** [2+3]

First Part: Please refer to 2070 Set C Q.No. 7 a

Second part: Please refer to 2070 Supp (Set B) Q.No. 7 a

24. **2059 Q.No. 7 a Show that $\frac{\sin\frac{(A+D_m)}{2}}{\sin\left(\frac{A}{2}\right)}$ for a prism where the notations carry usual meaning.** [5]

Please refer to 2074 Set B Q.No. 7a

25. **2057 Q.No. 7 a Or Derive an expression connecting the refractive index of the material of the prism with the angle of minimum deviation.**

Please refer to 2074 Set B Q.No. 7a

26. **2054 Q.No. 12 Derive an expression for the refractive index of the material of the prism in terms of the refracting angle and the angle of minimum deviation.** [5]

Please refer to 2074 Set B Q.No. 7a

27. **2050 Q.No. 12 Derive the formula connecting the refractive index of a prism and the angle of minimum deviation, if the angle of the prism is A.** [5]

Please refer to 2074 Set B Q.No. 7a

Numerical Problems

28. **2076 GIE Set B Q.No. 11 A glass prism of angle 60° and refractive index 1.66 is immersed in a liquid of refractive index 1.33. Find the angle of minimum deviation for a parallel beam of light passing through the prism.** [4]

Given,

Angle of prism (A) = 60°

Refractive index of glass (μ_g) = 1.66

Refractive index of liquid (μ_l) = 1.33

Angle of minimum deviation (δ_{min}) = ?

If ${}_l\mu_g$ is the refractive index of the material of the glass prism w.r.t. a liquid, them.

$${}_l\mu_g = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$$

$$\text{or, } \frac{\mu_g}{\mu_l} = \frac{\sin\left(\frac{60+\delta_{min}}{2}\right)}{\sin\frac{60}{2}}$$

$$\text{or, } \frac{1.66}{1.33} = \frac{\sin\left(\frac{60+\delta_{min}}{2}\right)}{\sin 30}$$

or, $\sin\left(\frac{60+\delta_{min}}{2}\right) = \frac{1.66}{1.33} \times \sin 30$

or, $\sin\left(\frac{60+\delta_{min}}{2}\right) = 0.624$

or, $\frac{60+\delta_{min}}{2} = \sin^{-1}(0.624)$

or, $\frac{60+\delta_{min}}{2} = 38.6$

or, $\delta_{min} = 38.6 \times 2 - 60$

$= 17.22°$

$\therefore$ Required angle of deviation is 17.22°

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29. **2073 Set C Q.No. 11** **A ray of light is refracted through a prism of angle 60°. Find the angle of incidence so that the emergent ray just grazes in the second face. Refractive index of the material of the prism is 1.45. [4]**

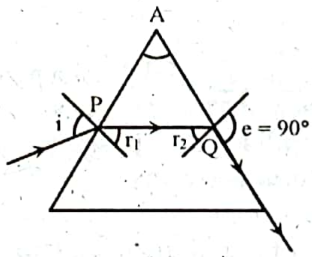
Given,

Angle of prism (A) = 60°

Incidence Angle (i) = ?

Emergent Angle (e) = 90°

Refractive index (μ) = 1.45



We have, at Q,

$_g\mu_a = \frac{\sin r_2}{\sin e}$

or, $\sin r_2 = \frac{\sin 90°}{_a\mu_g} = \frac{1}{1.45} = 0.689$

$\therefore$ $r_2 = 43.6°$

Again,

$A = r_1 + r_2 = r_1 = A - r_2 = 60 - 43.6 = 16.4°$

Now, at p,

$\mu = \frac{\sin i}{\sin r_1}$

or, $\sin I = 1.45 \times \sin 16.4 = 0.41$

$\therefore$ $i = 24.2°$

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30. **2072 Supp Q.No. 11** **A narrow beam of light is incident normally on one face of an equilateral prism and finally emerges from the prism. The prism is now surrounded by water. What is the angle between the directions of the emergent beam in the two cases? [μ_g = 1.45, μ_w = 1.33] [4]**

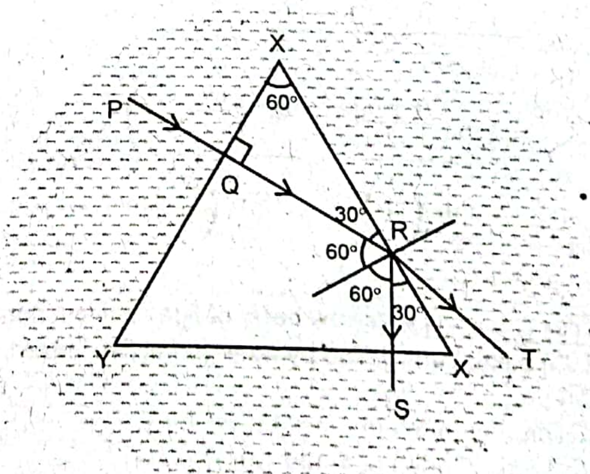
Given,

Refractive index of glass (μ_g) = 1.45

Refrative index of water (μ_W) = 1.33

Angle between emergence beam = ?

First case, when the prism is in air,



$\sin C = \frac{1}{\mu_g}$

or, $C = \sin^{-1}\left(\frac{1}{\mu_g}\right)$

$= \sin^{-1}\left(\frac{1}{1.45}\right)$

$= 43.6°$

i.e. the light ray suffers total internal relfection.

Second case, when the prism is surrounded by water;

$\sin C = \frac{1}{_w\mu_g}$

or, $C = \sin^{-1}\left(\frac{\mu_w}{\mu_g}\right)$

$= \sin^{-1}\left(\frac{1.33}{1.45}\right)$

$= 66.5°$

Since i < C, the ray refracted from second face.

Then,

$${}_g\mu_w = \frac{\sin i}{\sin e}$$

or, $\frac{\mu_w}{\mu_g} = \frac{\sin 60}{\sin e}$

or, e = 70.71°C

Thus, the angle between two beams (∠ SRT)

$= (90° - 60°) + (90° - 70.71°)$

$= 49.29°$

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31. 2071 Supp Q.No. 11 A certain prism is found to produce a minimum deviation of 51°, while it produces a deviation of 62°48' for two values of angle of incidence, namely 40°6' and 82°42' respectively. Determine the refractive angle of prism, the angle of incidence at minimum deviation and the refractive index of the material of the prism. **[4]**

Given,

Angle of minimum deviation (ρ_m) = 51°

Angle of deviation (δ) = 62°48'

Angle of incidence at face 1, (i_1) = 40°6'

Angle of incidence at face 2, (i_2) = 82°42'

Refracting angle of prism (A) = ?

Angle of incidence at minimum deviation (i) = ?

R.I. of prism (μ) = ?

We know,

$\delta + A = i_1 + i_2$

$62° 48' + A = 40°6' + 82°42'$

or, $A = 60°$

At minimum deviation,

$i_2 = i_2 = i$ and $\delta = \delta_{min}$, so

$\delta_m + A = i + i = 2i$

or, $i = \frac{\delta_m + A}{2} = \frac{51° + 60°}{2} = 55.5°$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60 + 51}{2}\right)}{\sin\left(\frac{60}{2}\right)} = 1.65$$

∴ μ = 1.65

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32. 2064 Q.No. 7 b A narrow beam of light incident normally on one face of an equilateral prism (refractive index 1.45) being surrounded by water (refractive index 1.44). At what angle the ray of light emerges out? **[4]**

Given,

Refractive index of glass (${}_a\mu_g$) = 1.45

Refractive index of water (${}_a\mu_w$) = 1.33

Angle of emergent (e) = ?

When light falls normally on one face, say XY of prism XYZ, it passes without bending to the face XZ. At face XZ angle of incident is 60° and angle of emergence is e which is shown in Fig.

From Shell's law, we have

$$_g\mu_w = \frac{\sin i}{\sin e}$$

or, $_g\mu_a \times {_a\mu_w} = \frac{\sin i}{\sin e}$

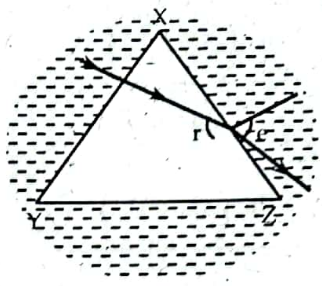
or, $\frac{_a\mu_w}{_a\mu_g} = \frac{\sin i}{\sin e}$

or, $\sin e = \frac{_a\mu_g}{_a\mu_w} \sin\ i = \frac{2.45}{.33} \times \sin 60°$

or, $\sin e = \frac{1.45}{1.33} \times \frac{\sqrt{3}}{2} = 0.9442$

or, $e = \sin^{-1}(0.9442) = 70.8°$

Required angle of emergent is 70.8°.



33. 2062 Q.No. 7 b 2055 Q.No. 13 **A glass prism of angle 72° and index of refraction 1.66 is immersed in a liquid of refractive index 1.33. Find the angle of minimum deviation for parallel beam of light passing through the prism.**

Given,

Angle of prism (A) = 72°

Refractive index of glass (η_g) = 1.66

Refractive index of a liquid (η_l) = 1.33

Angle of minimum deviation (δ_{min}) = ?

If $_l\eta_g$ is the refractive index of the material of the glass prism w.r.t a liquid, then

$$_l\eta_g = \frac{\sin\frac{A + \delta_{min}}{2}}{\sin\frac{A}{2}}$$

$$\frac{\eta_g}{\eta_l} = \frac{\sin\frac{72° + \delta_{min}}{2}}{\sin\frac{72°}{2}}$$

$$\frac{1.66}{1.33} = \frac{\sin\frac{72° + \delta_{min}}{2}}{\sin 36°}$$

$$\sin\frac{72° + \delta_{min}}{2} = \frac{1.66}{1.33}\sin 36°$$

$$\frac{72° + \delta_{min}}{2} = \sin^{-1}\left(\frac{1.66}{1.33}\sin 36°\right)$$

$\therefore\ \delta_{min} = 22.38°$

Hence, required angle of minimum deviation is 22.38°.

34. 2051 Q.No. 16 OR **A glass prism of angle A and refractive index 1.5 produces the angle of minimum derivation equal to 40°. Calculate the value of angle of prism.**

Given, Refractive index (μ) = 1.5

Angle of minimum deviation (δ_m) = 40°

Angle of prism (A) = ?

We know,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\text{or, } 1.5 = \frac{\sin\left(\frac{A + 40°}{2}\right)}{\sin\frac{A}{2}}$$

$$\text{or, } 1.5 = \frac{\sin\left(\frac{A}{2} + 20°\right)}{\sin\frac{A}{2}} = \frac{\sin\frac{A}{2} . \cos 20° + \cos\frac{A}{2} . \sin 20°}{\sin\frac{A}{2}}$$

$$= \cos 20° + \cot\frac{A}{2} . \sin 20°$$

$$\text{or, } \cot\frac{A}{2} = \frac{1.5 - \cos 20°}{\sin 20°} = \frac{1.5 - 0.94}{0.342}$$

$$\cot\frac{A}{2} = 1.638$$

$$\text{or, } \tan\frac{A}{2} = \frac{1}{1.63} = 0.61$$

$$\text{or, } \frac{A}{2} = \tan^{-1}(0.61) = 31.4°$$

$$A = 62.8°$$

Hence, required angle of prism is 62.8°.

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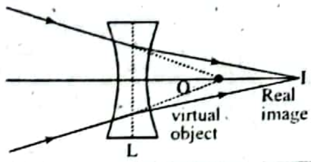
Chapter 4: Lenses

Short Answer Questions

1. 2076 GIE Set B Q.No. 3b **Under what conditions a concave lens forms a real image?** **[2]**

When converging beam of light is incident on a concave lens, the rays of light are diverged and meet at a point on the principal axis as shown in figure below. In this case, object is virtual and the image of the object is real.



2. 2074 Set B Q.No. 3b 2069 Old (Set B) Q. No. 6c 2067 Q.No. 3 a **A convex lens is immersed in water. Will its focal length change? Explain.** **[2]**

Yes, the focal length of a lens changes when it is placed in water. From lens maker's formula, focal length of lens in air is

$$\frac{1}{f_a} = ({}_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \text{... (i)}$$

where ${}^a\mu_g$ is refractive index of glass w.r.t. air, f_a is focal length of lens in air, r_1 and r_2 are radii of curvature of two surfaces of the lens.

When the lens in immersed in water, then

$$\frac{1}{f_w} = ({}_w\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \text{... (ii)}$$

where ${}_w\mu_g$ is refractive index of glass w.r.t. water and f_w is focal length of lens in water.

Dividing eq^n. (i) by (ii)

$$\frac{f_w}{f_a} = \left(\frac{{}^a\mu_g - 1}{{}^w\mu_g - 1}\right) = \frac{{}^a\mu_g - 1}{\left(\frac{{}^a\mu_g}{{}^a\mu_w} - 1\right)} \qquad \left[\because {}^w\mu_g = {}^w\mu_a \cdot {}^a\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w}\right]$$

$$= \frac{1.5 - 1}{\frac{1.5}{1.33} - 1} = \frac{0.5}{1.1278 - 1} = \frac{0.5}{0.1278}$$

$$\frac{f_w}{f_a} = 3.9$$

or, $f_w = 3.9\, f_a$

It means f_w is greater than f_a. Hence, focal length of a lens increases in immersing in water.

3. 2073 Set C Q.No. 3b **Sun glasses have curved surfaces but their power is zero, why?** **[2]**

Sun glasses have curved faces or surfaces, but their power is zero because they have two curved surfaces, one of them is convex and another is concave having the same radii of curvature. So, $R_1 = -R_2 = R$, then from lens maker's formula, we know that,

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$$

or, P = 0 i.e. Power of sun glasses zero.

4. 2073 Set D Q.No. 3b **A lens is immersed in water. Is there any change in its focal length? Explain.** **[2]**

Please refer to 2074 Set B Q.No. 3b

5. 2070 Supp (Set B) Q.No. 3 b **A converging lens is dipped in a liquid having refractive index greater than the lens. Does its power change? Explain.** **[2]**

A converging lens of power P_1 is kept in a liquid of refractive index μ_l, then the power of lens be P_2. Then we have

$$P_1 = ({}_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$P_2 = ({}_l\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{P_2}{P_1} = \frac{{}_l\mu_g - 1}{{}_a\mu_g - 1}$$

or, $\frac{P_2}{P_1} = \frac{\frac{_a\mu_g}{_a\mu_l} - 1}{_a\mu_g - 1}$

Here, $_a\mu_l > {_a\mu_g}$, so, the power decreases and becomes negative.

6. **2070 Supp (Set A) Q.No. 3 b If a lens made of glass is immersed into water, what will happen to its power? Explain.** [2]

The focal length (f_1) of a lens in air is given by: $\frac{1}{f_1} = (_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

where $_a\mu_g$ is the refractive index of the material of the lens in air, r_1 and r_2 are radii of curvature of two surfaces of the lens.

The power of lens is given by: $P = \frac{1}{\text{focal length (m)}}$. So, power of lens when placed in air is given by:

$$P_1 = (_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \ldots (i),$$

Similarly power (P_2) of the lens when it is immersed in water is given by:

$$P_2 = (_w\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \ldots (ii)$$

where, $_w\mu_g$ is the refractive index of the material of prism w.r.t. to water.
Dividing equation (ii) by equation (i), we get.

$$\frac{P_2}{P_1} = \frac{(_w\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}{(_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$$

$$\frac{P_2}{P_1} = \frac{(_w\mu_g - 1)}{(_a\mu_g - 1)} = \frac{\left(\frac{_a\mu_g}{_a\mu_w} - 1\right)}{_a\mu_g - 1}$$

$$\frac{P_2}{P_1} = \frac{\left(\frac{1.5}{1.33} - 1\right)}{1.5 - 1} = \frac{0.17}{1.33 \times 0.5}$$

$\Rightarrow P_2 = 0.26 \times P_1 \Rightarrow P_2 < P_1$

That means power of lens is decreased when immersed in water.

7. **2069 Supp Q.No. 3a A convex lens is immersed in water, will its focal length be changed?** [2]

Please refer to 2074 Set B Q.No. 3b

8. **2064 Q.No. 6 c If a converging lens and a diverging lens having the same focal length be in contact, how will the combination of lenses behave?** [2]

Let f_1 and f_2 be the focal lengths of convex and concave lens respectively, then their combined focal length is given as $\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2}$, where the focal length of concave lens is taken as negatve. By question, if $f_1 = f_2$, then $F = \infty$. Hence, the combined focal length of the combination is infinity and acts as a plane glass slab.

9. **2061 Q.No. 6 c Draw the ray diagram showing the formation of real image by a concave lens.** [2]

Please refer to 2076 GIE Set B Q.No. 3b

10. **2060 Q.No. 6 b A lens made of glass is immersed into water. Will its power increase or decrease?** [2]

Please refer to 2070 Supp (Set A) Q.No. 3 b

11. **2059 Q.No. 6 c Does the focal length of a lens change when immersed in water? Will it increase or decrease?** [2]

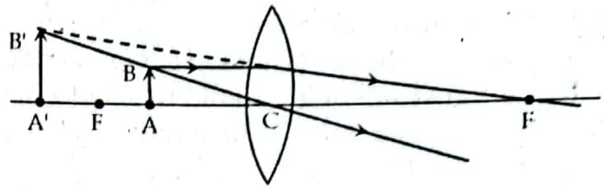
Please refer to 2074 Set B Q.No. 3b

12. 2055 Q.No. 11 a **Can a concave lens from a real image? Give the condition.** **[2]**

Please refer to 2076 GIE Set B Q.No. 3b

13. 2053 Q.No. 11 a **Draw the ray diagram showing the formation of a virtual image by a convex lens.** **[2]**

When an object is placed between focus and optical centre, its image is formed on the same side which is virtual, erect and enlarged as shown in figure. AB is an object placed between optical centre and focus whose image is A'B' which is virtual, erect and magnified formed on the same side.

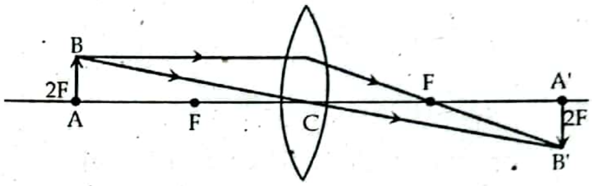


14. 2052 Q.No. 11 b **Does the focal length of a lens change if it is immersed in water? Will it increase or decrease?** **[2]**

Please refer to 2074 Set B Q.No. 3b

15. 2051 Q.No. 12 a **At what distance from a convex lens should an object be placed on the axis, so as to form a real image of the same size? Illustrate your answer with a ray diagram.** **[2]**

An object placed at twice of focal length of a convex lens on its axis, it forms a real image of the same size which is real and inverted in nature as shown in figure. In figure, AB is the object place at 2F and A' B' is the image formed at 2F on other side.



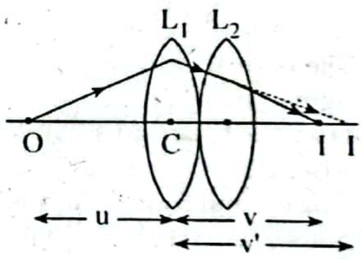
16. 2051 Q.No. 12 c **Under what conditions does a concave lens form a real image?** **[2]**

Please refer to 2076 GIE Set B Q.No. 3b

Long Answer Questions

17. 2076 Set C Q.No. 7b **Derive an expression for the combined focal length of two thin lenses in contact.** **[4]**

Let us consider two thin convex lenses L_1 and L_2, placed in contact, f_1 and f_2 be their respective focal lengths.



There is a point object O placed on the principal axis at a distance u from optical centre C. In the absence of lens L_2, I' be the image of O formed by lens L_1, at a distance v' from C as shown in figure.

Using lens formula, we get

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v'} \quad \dots \text{(i)}$$

When lens L_2 is placed in contact with L_1, I' acts as object for L_2 and real image I is formed at a distance v from C. Since lenses are thin, therefore point of contact can be taken as C. Now using lens formula for L_2, we get

$$\frac{1}{f_2} = \frac{1}{(-v')} + \frac{1}{v} \quad \dots \text{(ii)}$$

Adding (i) and (ii) we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2} \quad \dots \text{(iii)}$$

If we replaced the combined lens by a single lens having focal length F produced image I of object O so that

$$\frac{1}{F} = \frac{1}{u} + \frac{1}{v} \quad \dots \text{(iv)}$$

From eq^n (iii) and (iv), we get $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

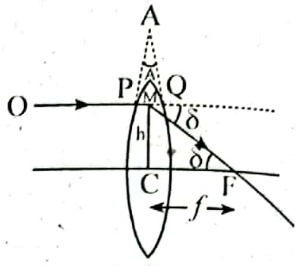
This is the required expression for the equivalent focal length of two thin convex lenses having focal length f_1 and f_2 respectively.

18. 2075 GIE Q.No. 7a | 2070 Set C Q.No. 7 b | 2058 Q.No. 7 a | 2057 Q.No. 7 a | 2056 Q.No. 7 a OR **Derive lens maker's formula.** **[4]**

Lens Maker formula: Lens Maker formula gives the relation between focal length, radii of curvature of surfaces and refractive index (μ) of its material.

Postulates (assumptions):

a. The lens should be thin in comparision to the radii of curvature of the lens surface.
b. The aperture of lens shold be small.
c. The incident rays and refracted rays make small angle with the principal axis and
d. The deviation produced by lens should be similar to the deviation produced by a small angled prism. i.e. $\delta = A(\mu - 1)$, where A is the angle of lens and μ is the refractive index of the lens.



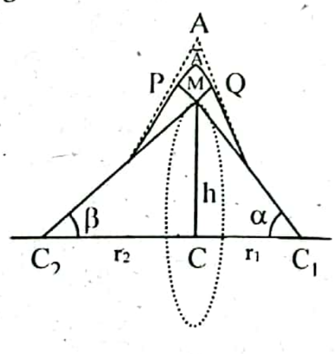
(i)

Derivation: Let us consider a ray OP incident on a convex lens parallel to the principal axis at a height h shown in fig (i). Let f be the focal length of the lens. After refraction through the lens, the emergent ray passes through the focus F. If δ is the angle of deviation, then..

$$\tan\delta = \frac{h}{f} \quad (\text{in } \Delta MCF)$$

$$\delta = \frac{h}{f} \qquad \dots (i)$$

[For small angle of deviation, $\tan\delta \approx \delta$]



(ii)

For a thin lens, the deviation can be considered as deviation by a small angle prism. Tangents at P and Q form a prism of small angle A. For a small angle of incidence and point of incidence close to C,

We can write, $\delta = A(\mu - 1)$ ... (ii)

From (i) and (ii), we get

$$\frac{h}{f} = A(\mu - 1)$$

$$\frac{1}{f} = (\mu - 1)\frac{A}{h} \qquad \dots (iii)$$

The normal at P and Q pass through the centres of curvature C_1 and C_2 of lens surfaces as shown in figure (ii).

From the figure (ii) $\angle PMC_2 = \angle QMC_1 = A$

Also, $\angle QMC_1 = \angle MC_2C + \angle MC_1C = \alpha + \beta$

[Since α and β are small angles made by PC_1 and PC_2 with principal axis.]

$$A = \alpha + \beta \approx \tan\alpha + \tan\beta$$

$$= \frac{h}{r_1} + \frac{h}{r_2}$$ [r_1 and r_2 be radii of curvatures which are positive for convex lens.]

$$\therefore \frac{A}{h} = \frac{1}{r_1} + \frac{1}{r_2} \qquad \dots (iv)$$

Using (iv) in (iii), we get

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

which is the required lens Maker formula.

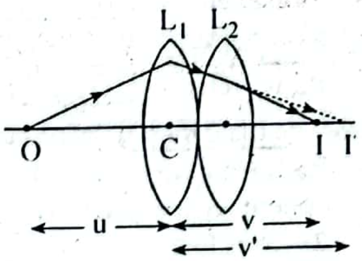
Sign conventions:

a. Radius of curvature and focal length for convex lens is taken positive and
b. Radius of curvature and focal length for concave lens is taken negative.
c. Image distance and object distance for real image and object are taken as positive.
d. Image distance and object distance for virtual image and object are taken as negative.

19. 2075 Set A Q.No. 7a | 2071 Supp Q.No. 7a **Define power of a lens. Derive the formula for the effective power of two thin lenses in contact.** **[4]**

Power of lens: Power of lens is its ability to converge or diverge the light rays after passing through it. Mathematically power of lens is defined as the reciprocal of the focal length. i.e.

$P = \frac{1}{f}$, when f is in meter, P is in Diopter.



Let us consider two thin convex lenses L_1 and L_2, placed in contact. f_1 and f_2 be the focal lengths of L_1 and L_2 respectively. There is a point object O placed on the principal axis at a distance u from optical centre C. In the absence of lens L_2, I' be the image of O formed by lens L_1, at a distance v' from C shown in figure.

Using lens formula, we get

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v'} \qquad \text{... (i)}$$

When lens L_2 is placed in contact with L_1, I' acts as object for L_2 and real image I is formed at a distance v from C. Since lenses are thin therefore point of contact can be taken as C. Now using lens formula for L_2, we get

$$\frac{1}{f_2} = \frac{1}{(-v')} + \frac{1}{v} \qquad \text{... (ii)}$$

Adding (i) and (ii) we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{... (iii)}$$

If we replaced the combined lens by a single lens having focal length F produced image I of object O so that

$$\frac{1}{F} = \frac{1}{u} + \frac{1}{v} \qquad \text{... (iv)}$$

From eqⁿ (iii) and (iv), we get $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

This is the required expression for the equivalent focal length of two thin convex lenses having focal length f_1 and f_2 respectively. The power of the combined lens is then given as,

$$P = \frac{1}{F}$$

$$P = \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\therefore$ $P = P_1 + P_2$

Hence, the equivalent power increases on joining two thin lenses.

20. **2075 Set B Q.No. 7b** **Derive expression for the focal length in Lens maker's formula.** **[4]**

✎ Please refer to 2075 GIE Q.No. 7a

21. **2072 Supp Q.No. 7b** **2052 Q.No. 12** **Derive lens maker's formula and state the sign convention for radius of curvature.** **[4]**

✎ Please refer to 2075 GIE Q.No. 7a

22. **2072 Set C Q.No. 7a** **Define principal focus and the power of lens. Derive the formula for the focal length of two thin lenses in contact.** **[4]**

✎ **Principal focus:** It is the point on principal axis at which all rays parallel to principal axis pass through it after refraction. It is denoted by F. In a concave lens, the rays diverge away from the lens, which appear to come from the focus. By symmetry, every lens has two principal foci, one on each side of the lens. The focus of a convex lens is real and that of the concave lens is virtual.

Power of lens: Power of lens is its ability to converge or diverge the light rays after passing through it. Mathematically power of lens is defined as the reciprocal of the focal length. i.e.

$P = \frac{1}{f}$, when f is in meter, P is in Diopter.

Second Part: Please refer to 2076 Set C Q.No. 7b

23. **2071 Set D Q.No. 7 b** **Derive lens Maker's formula and write the unit of the power of a lens.** **[4]**

✎ Please refer to 2075 GIE Q.No. 7a

Unit of power of lens is diopter (D) or m^{-1}.

24. 2069 Supp Q.No. 7a **Define an expression for combined focal length when two lenses are combined coaxially.** [4]

Please refer to 2076 Set C Q.No. 7b

25. 2069 (Set A) Q. No. 7b **Derive an expression for lens maker's formula.** [4]

Please refer to 2075 GIE Q.No. 7a

26. 2068 Old Q.No. 7 a **Define radius of curvature of a lens. Derive Lens maker's formula.** [1+4]

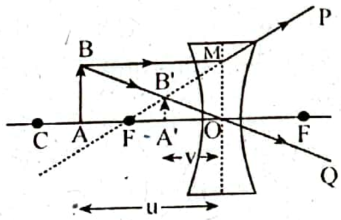
Radius of curvature: The distance between centre of curvature and optical centre of lens or radius of sphere whose small portions form the lens is called radius of curvature of a lens.

2nd part: Please refer to 2075 GIE Q.No. 7a

27. 2066 Old Q. No. 7 a OR **Derive the formula relating the object distance, the image distance and the focal length for a concave lens.** [5]

Lens formula for diverging (concave) lens:

Let us consider an object AB is placed between principal focus F and centre of curvature C as shown in figure. A ray BM parallel to principal axis appears to pass through F after refraction through the lens at M. Another ray BO, originating from B passes directly through optical centre O, which appears to meet at B' so that A'B' became virtual image of object AB formed in between pricnipal focus and optical centre O.



In figure, AO = u, object distance

A'O = -v, image distance and

FO = -f, focal length of lens.

As ΔABO and ΔA'B'O are similar so that

$$\frac{A'B'}{AB} = \frac{A'O}{AO} \quad \ldots (i)$$

Again, ΔA'B'F and ΔMOF are similar.

So that, $\frac{A'B'}{MO} = \frac{FA'}{FO}$

Since MO = AB,

$$\frac{A'B'}{AB} = \frac{FA'}{FO} \quad \ldots (ii)$$

From (i) and (ii), we get

$$\frac{A'O}{AO} = \frac{FA'}{FO} \Rightarrow \frac{A'O}{AO} = \frac{FO - A'O}{FO}$$

$$\frac{-v}{u} = \frac{-f-(-v)}{-f}$$

[For concave lens, virtual distances are taken negative]

$$\frac{-v}{u} = \frac{-f+v}{-f}$$

$$vf = -uf + uv$$

$$vf+uf = uv$$

Dividing both sides by uvf, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

which is the required lens formula relating object distance.

28. 2065 Q. No. 7 a OR **Define principal focus. Derive the lens formula for the concave lens and show that m = -v/u for the same lens:** [1+4]

Principal focus: It is the point on principal axis at which all rays parallel to principal axis pass through it after refraction. It is denoted by F. In a concave lens, the rays diverge away from the lens, which appear to come from the focus. By symmetry, every lens has two principal foci, one on each side of the lens. The focus of a convex lens is real and that of the concave lens is virtual.

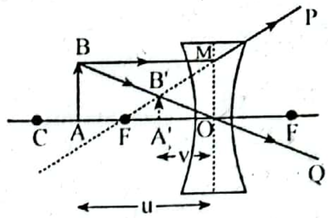
Lens formula for diverging (concave) lens:

Let us consider an object AB is placed between principal focus F and centre of curvature C as shown in figure (ii). A ray BM parallel to principal axis appears to pass through F after refraction the lens at M. Another ray BO, originating from B passes directly through optical centre O, which appears to meet at B' so that A'B' became virtual image A'B' of object AB formed in between pricnipal focus and optical centre O.

In figure, AO = u, object distance

A'O = -v, image distance and

FO = -f,focal length of lens.



As ΔABO and ΔA'B'O are similar so that

$$\frac{A'B'}{AB} = \frac{A'O}{AO} \quad \text{... (i)}$$

Again, ΔA'B'F and ΔMOF are similar.

So that $\frac{A'B'}{MO} = \frac{FA'}{FO}$

Since MO = AB,

$$\frac{A'B'}{AB} = \frac{FA'}{FO} \quad \text{....(ii)}$$

From (i) and (ii), we get

$$\frac{A'O}{AO} = \frac{FA'}{FO} \Rightarrow \frac{A'O}{AO} = \frac{FO - A'O}{FO}$$

$$\frac{-v}{u} = \frac{-f-(-v)}{-f}$$

[For concave lens, virtual distances are taken negative]

$$\frac{-v}{u} = \frac{-f+v}{-f}$$

$vf = -uf + uv$

$vf+uf = uv$

Dividing both sides by uvf, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

which is the required lens formula relating object distance.

Magnification: The ratio of height of image to object is called magnification. From above figure

$$m = \frac{A'B'}{AB} = \frac{-v}{u}.$$

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29. 2064 Q.No. 7 a OR What do you mean by conjugate focii? Derive lens maker's formula and state sign convention for the radius of curvature. **[1+4]**

Conjugate foci: Two points, one on each side on the principal axis lies in two sides of lens, where rays of parallel beam of light also parallel to the principal axis actually converge is called conjugate foci.

Please refer to 2075 GIE Q.No. 7a

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30. 2063 Q.No. 7 a OR Derive lens maker formula for a convex lens.

Please refer to 2075 GIE Q.No. 7a

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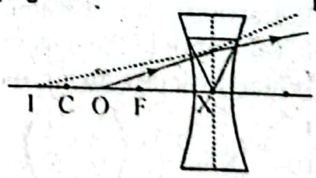
31. 2062 Q.No. 7 a Draw ray diagram to locate the image of a point object placed between the centre of curvature and the principal focus of a diverging lens. Derive lens formula for the diverging lens. **[2+3]**

First Part: Figure drawn shows that image formed by the diverging lens when point object is placed between centre of curvature and principal focus.



Second Part: Please refer to 2066 Old Q. No. 7 a OR

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32. 2059 Q.No. 7 a OR Derive the formula relating object distance, image distance and the focal length for a convex lens.

Consider a convex lens of focal length f. Let AB be an object placed normally on the principal axis of the lens. The rays of light from the object AB after refracting through the convex lens meets at point B'. So A'B' is the real image of the object AB.

Since Δ's ABC and A'B'C are similar so their corresponding sides are proportional.

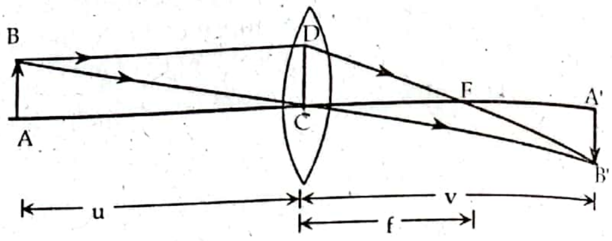
$$\frac{AB}{A'B'} = \frac{CA}{CA'} \quad \text{... (i)}$$

Similarly Δ's CDF and A'B'F are similar. So,

$$\frac{CD}{A'B'} = \frac{CF}{FA'}$$

But CD = AB

$$\therefore \frac{AB}{A'B'} = \frac{CF}{FA'} \quad \text{... (ii)}$$



From Eqs. (i) & (ii), we have

$$\frac{CA}{CA'} = \frac{CF}{FA'}$$

$$\text{or, } \frac{CA}{CA'} = \frac{CF}{CA' - CF}$$

$$\text{or, } \frac{u}{v} = \frac{f}{v - f},$$

where CA = u is object distance

CA' = v is image distance

CF = f is focal length.

or, $uv - uf = vf$

or, $uv = uf + fv$

Dividing both sides by uv

$$\frac{uv}{uv} = \frac{uf}{uv} + \frac{vf}{uv}$$

$$\text{or, } 1 = \frac{f}{v} + \frac{f}{u}$$

$$\text{or, } 1 = f\left(\frac{1}{v} + \frac{1}{u}\right)$$

$$\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v},$$ which is lens formula.

33. 2055 Q.No. 12 **Derive the combined focal length of two thin lenses in contact.** **[4]**

Please refer to 2076 Set C Q.No. 7b

34. 2051 Q.No. 13 **Derive an expression for the equivalent focal length of two thin convex lenses in contact.** **[4]**

Please refer to 2076 Set C Q.No. 7b

35. 2050 Q.No. 14 **Describe a method, with a suitable diagram to determine the focal length of a convex lens.**

Please refer to 2059 Q.No. 7 a OR

Numerical Problems

36. 2076 Set B Q.No. 11 **A convex lens of focal length 24 cm and of refractive index 1.5 is totally immersed in water of refractive index 1.33. Find its focal length in water.** **[4]**

Given,

Focal length of a convex lens (f) = 24 cm

Refractive index of glass ($_a\mu_g$) = 1.5

Refractive index of water ($_a\mu_w$) = 1.33

Focal length of the lens in water (f') = ?

When the lens is in air, then

$$\frac{1}{f} = ({_a\mu_g} - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\text{or, } \frac{1}{24} = (1.5 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\text{or, } \frac{1}{24} = 0.5\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\text{or, } \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{12} \quad \text{... (i)}$$

When the lens is in water,

$$\frac{1}{f} = ({}_w\mu_g - 1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$= ({}_w\mu_a \times {}_a\mu_g - 1)\left(\frac{1}{12}\right) \qquad \left[\because \frac{1}{r_1}+\frac{1}{r_2}=\frac{1}{12} \text{ ine q}^n\text{. (i)}\right]$$

$$= \left(\frac{{}_a\mu_g}{{}_a\mu_w} - 1\right)\times\frac{1}{12} = \left(\frac{1.5}{1.33}-1\right)\times\frac{1}{12} = 0.0106$$

$\therefore$ f' = 93.88 cm

37. **2074 Supp. Q.No. 11 The image of an object is obtained on a screen with the help of a converging lens. The distance of the object is 40 cm from the screen and size of the image is 9 times the size of the object. Calculate distance of the screen from the lens and focal length of the lens.** **[4]**

Given,

Let u be the object distance and v be the image distance from the lens. Then from given condition,

u + v = 40 cm

Again, magnification, $m = \frac{v}{u} = 9$

or, v = 9 u

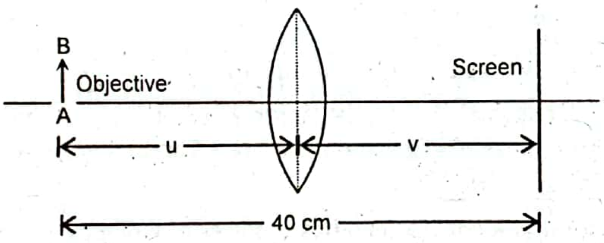
Then,

u + 9 u = 40

or, 10 u = 40

$\therefore$ u = 4

and v = 4 × 9 = 36 cm



Again,

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\text{or, } \frac{1}{f}=\frac{1}{4}+\frac{1}{36}$$

$$\text{or, } \frac{1}{f}=\frac{9+1}{36}$$

$$\therefore f=\frac{36}{10}=3.6 \text{ cm}$$

$\therefore$ f = 3.6 cm and v = 36 cm

38. **2074 Set A Q.No. 11 An object is placed at a distance 30 cm from a thin lens of power 4 Dioptre. Discuss the nature of image formed by the lens.** **[4]**

Given,

Power of lens (P) = 4D

Object distance (u) = 30 cm

Image distance (v) = ?

We know,

Focal length (f) $= \frac{1}{P} = \frac{1}{4} = 0.25$ m = 25 cm

Also, from lens formula,

$$\frac{1}{f} = \frac{1}{u}+\frac{1}{v}$$

$$\text{or, } \frac{1}{v} = \frac{1}{f}-\frac{1}{u}$$

$$\text{or, } v = \frac{uf}{u-f}$$

$$\text{or, } v = \frac{30\times 25}{30-25}$$

$\therefore$ v $= 150$ cm

Also, magnification, $m = \frac{v}{u} = \frac{I}{0} \Rightarrow \frac{I}{0} = \frac{150}{30} = 5$

Thus, a real image with magnification 5 is formed.

39. $\boxed{\text{2074 Set B Q.No. 11}}$ **An erect object is placed at a distance of 10 cm before a convex lens of focal length 15 cm. When will the image be formed? Discuss the nature of image.** **[4]**

Given,

Object distance, u = 10 cm

Focal length, f = 15 cm

Image distance, v = ?

We know, from lens formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{or, } \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\text{or, } v = \frac{uf}{u - f}$$

$$\text{or, } v = \frac{10 \times 15}{10 - 15}$$

$$\therefore v = -30 \text{ cm}$$

Also, magnification, $m = \frac{v}{u} = \frac{-30}{10} = -3$

Thus, a virtual image is formed with magnification 3.

40. $\boxed{\text{2073 Supp Q.No. 11}}$ **A convex lens of focal length 24 cm is totally immersed in water. Find its focal length in water if the refractive indices of glass and water are 1.5 and 1.33 respectively.** **[4]**

Please refer to $\boxed{\text{2076 Set B Q.No. 11}}$

41. $\boxed{\text{2072 Set E Q.No. 11}}$ **An equiconvex lens has a power of 4 dioptre. What will be the radius of curvatures of each surface if the lens is made of glass of refractive index 1.5?** **[4]**

Given,

Power of lens (P) = 4 dioptre

Radius of curvature (R) = ?

Refractive index (μ) = 1.5

We have, lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

For equiconvex lens,

$$R_1 = R_2 = R$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\text{or, } P = (\mu - 1)\frac{2}{R}$$

$$\text{or, } R = (\mu - 1)\frac{2}{P} = (1.5 - 1) \times \frac{2}{4} = \frac{0.5 \times 2}{4} = \frac{1}{4}$$

$$= 0.25 \text{ m}$$

42. $\boxed{\text{2069 (Set A) Q. No. 11}}$ **An object to the left of a lens is imaged by the lens on a screen 30cm to the right of the lens. When the lens is moved 4cm to the right, the screen must be moved 4cm to the left to refocus the image. Determine the focal length of the lens.** **[4]**

In first case,

Object distance (u) = x

Image distance (v) = 30 cm

Then, focal length, f is

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{30}$$

In second case,

Object distance (u^1) = x + 4

Image distance (v^1) = 30 – 4 – 4 = 22cm

Focal length f is,

$$\frac{1}{f} = \frac{1}{x+4} + \frac{1}{22}$$

$$\therefore \frac{1}{x} + \frac{1}{30} = \frac{1}{x+4} + \frac{1}{22}$$

or, $\frac{1}{x} - \frac{1}{x+4} = \frac{1}{22} - \frac{1}{30}$

or, $\frac{4}{x(x+4)} = 0.012$

or, $\frac{x^2+4x}{4} = 82.5$

or, $x^2 + 4x - 330 = 0$

or, x = 16.27 cm or – 20.3 cm. But 'x' cannot be negative.

Now,

$$\frac{1}{f} = \frac{1}{16.27} + \frac{1}{30}$$

or, f = 10.55cm

---

43. $\boxed{\text{2068 Q.No. 11}}$ **A thin equiconvex lens of glass of refractive index 1.5 whose surfaces have a radius of curvature of 24cm is placed on a horizontal plane mirror. When the space between the lens and mirror is filled with a liquid, a pin held 40 cm vertically above the lens is found to coincide with its own image. Calculate the refractive index of the liquid.** **[4]**

Given,

R.I of lens (μ_1) = 1.5

Radius of curvature (r) = 24 cm

Now, focal length f is

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{r} + \frac{1}{r}\right) = (1.5-1)\left(\frac{1}{24} + \frac{1}{24}\right) = 0.04167$$

f_1 = 24 cm

Combined focal length = 40 cm

Now, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

$$\frac{1}{40} - \frac{1}{24} = \frac{1}{f_2}$$

f_2 = -60 cm

For liquid,

r_1 = – 24 cm $\qquad$ $r_2 = \infty$

f_2 = – 60 cm $\qquad$ μ_2 = ?

Again,

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{-60} = (\mu_2 - 1)\left(\frac{1}{-24} + \frac{1}{\infty}\right)$$

or, $\frac{24}{60} = \mu_2 - 1$

or, $\mu_2 = 1.4$

44. **2067 Supp Q.No. 11** **A convex lens of focal length 24 cm is totally immersed in water. Find its focal length in water.** **[4]**

Please refer to 2076 Set B Q.No. 11

45. **2066 Q. No. 11** **The real image obtained by a lens of power 5D is three times the length of the object. Calculate the object and image distances.** **[4]**

Given,

Power of lens (P) = 5D

Focal length (f) $= \frac{1}{P} = \frac{1}{5} = 0.2$ m

Magnification (m) = 3

Object distance (u) = ?

Image distance (v) = ?

We have,

$$m = \frac{v}{u} = 3$$

$$\therefore v = 3u$$

Again,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{0.2} = \frac{1}{u} + \frac{1}{3u}$$

$$5 = \frac{4}{3u}$$

or, u = 0.267 m = 26.7cm and v = 3u = 80.1 cm.

46. **2061 Q.No. 7 b** **The radii of curvature of the faces of a thin converging meniscus lens of glass of refractive index 1.5 are 15 cm and 30 cm. What is the focal length of the lens when it is completely immersed in water of refractive index 4/3?**

Given, Radii of curvature of a thin converging meniscus are given by

$r_1 = 15$ cm = 0.15 m and $r_2 = 30$ cm = 0.3 m

R.I. of glass (μ_g) = 1.5

R.I. of lass (μ_w) = 1.33

For converging meniscus, its focal length should be positive. So, in the lens Maker formula: $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$, we should take r_1 positive and r_2 = negative i.e., $r_1 = 0.15$ m and $r_2 = -0.3$ m.

Then,

$$\therefore \frac{1}{f_1} = (\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \text{... (i) [In air]}$$

and $$\frac{1}{f_2} = ({}_w\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \text{... (ii) [In water]}$$

where μ_g is the refrective index of glass w.r.t. air, ${}_w\mu_g$ is the refrective index of glass w.r.t. water. f_1 and f_2 are focal length of the lens in air and in water respectively. Dividing (i) and (ii) we get,

$$\frac{f_2}{f_1} = \frac{\mu_g - 1}{{}_w\mu_g - 1} = \frac{\mu_g - 1}{\frac{\mu_g}{\mu_w} - 1} \quad \left(\because {}_w\mu_g = \frac{\mu_g}{\mu_w}\right)$$

$$\frac{f_2}{f_1} = \frac{1.5 - 1}{\frac{1.5}{1.33} - 1} = 3.91 \quad \text{... (iii)}$$

From (i), we have

$$\frac{1}{f_1} = (1.5 - 1)\left[\frac{1}{0.15} - \frac{1}{0.3}\right]$$

$$f_1 = 0.6 \text{ m}$$

From (iii), we get

$f_2 = 3.91 \times 0.6$

$f_2 = 2.34$ m

Hence, required focal length is 2.346 m.

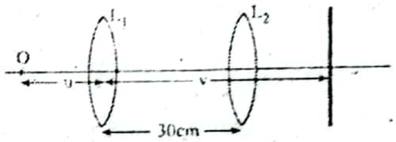
47. 2054 Q.No. 13 **A convex lens of focal length 24 cm. (refractive index = 1.5) is totally immersed in water (refractive index = 1.33). Find its focal length in water.**

Please refer to 2076 Set B Q.No. 11

48. 2053 Q.No. 13 OR **A luminous object and a screen are placed on an optimal bench and a converging lens is placed between them to produce a sharp image of the object on the screen. The linear magnification of the image is found to be 2.5. The lens is now moved 30 cm. nearer the screen and a sharp image again formed. Calculate the focal length of the lens.**

First case:

If u and v are the object and image distance respectively, then



$m = \frac{v}{u}$

$2.5 = \frac{v}{u}$

$v = 2.5\,u$ ... (i)

Since, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ where f is the focal length of the lens.

$\frac{1}{f} = \frac{1}{2.5u} + \frac{1}{u}$ [using (i)]

$\therefore \frac{1}{f} = \frac{1.4}{u}$ ... (ii)

Second case:

When the lens is moved towards the screen from the position L_1 to the position L_2 such that L_1L_2 = 30 cm.

new object distance (u') = u + 30

new image distance (v') = v − 30 = 2.5u − 30 [using (i)]

$\frac{1}{f} = \frac{1}{u'} + \frac{1}{v'}$

$\therefore \frac{1}{f} = \frac{1}{u + 30} + \frac{1}{2.5\,u - 30}$ ... (iii)

Solving equation (ii) and (iii),

We get, u = 20 cm

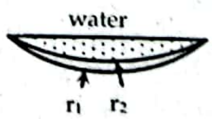
From equation (ii) $\frac{1}{f} = \frac{1.4}{20} \Rightarrow f = 14.28$ cm = 0.143 m

Hence, focal length of the lens is 14.3 cm.

49. 2052 Q.No. 13 **A converging meniscus of glass (μ = 1.5) having radius of curvature 4cm and 8 cm is put on a horizontal surface facing upward. If it is filed with water, what will be the focal length of the combination?**

Given,

Refractive index of glass ($_a\mu_g$) = 1.5



Radius of curvature of one surface (r_1) = 4 cm

Radius of curvature of next surface (r_2) = 8 cm

Combined focal length (F) = ?

For converging meniscus,

$\frac{1}{f} = ({}_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

or, $\frac{1}{f_1} = (1.5 - 1)\left(\frac{1}{4} - \frac{1}{8}\right) = 0.5 \times \frac{1}{8}$

or, $f_1 = 16$ cm

For water lens

$$\frac{1}{f_2} = ({}_a\mu_w - 1)\left(\frac{1}{r_1'} + \frac{1}{r_2'}\right)$$

or, $\frac{1}{f_2} = (1.33 - 1)\left(\frac{1}{8} + \frac{1}{\infty}\right)$ [$\because r_1' = r_2 = 8$ cm, same with meniscus and $r_2' = \infty$, for plane surface]

or, $\frac{1}{f_2} = 0.33 \times \frac{1}{8}$

or, $f_2 = 24.24$ cm

Again, we know, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

or, $\frac{1}{F} = \frac{1}{16} + \frac{1}{24.24}$

$\therefore$ F $= 9.64$ cm

Hence, combined focal length is 9.64 m.

50. 2051 Q.No. 14 A converging lense of 5 cm focal length used as a simple magnifier, produces a virtual image 25 cm from the eye. How far from the lens should the object be place? What is the magnification?

Given,

Focal length of converging lens (f) = 5 cm

Image distance (v) = – 25 cm

Object distance (u) = ?

Magnification (m) = ?

We know,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or $\frac{1}{5} = \frac{1}{u} - \frac{1}{25}$

or, $\frac{1}{u} = \frac{1}{5} + \frac{1}{25}$

$\therefore$ u $= 4.17$ cm

Again, we know

$$m = \frac{v}{u} = \frac{-25}{4.17}$$

$\therefore$ m = – 6

□□□

Chapter 5: Dispersion

Short Answer Questions

1. **2076 GIE Set A Q.No. 3b Why does a glass slab not produce dispersion?** **[2]**

The dispersive power (ω) of a material is given as

$$\omega = \frac{\mu_V - \mu_r}{\mu - 1},$$

where μ_v, μ_r and μ are the refractive indices for violet, red and mean light.
For glass slab, the both sides are parallel such that the different color of light emerges parallelly and μ_v and μ_r becomes equal. So, $\omega = 0$ for glass slab and hence no dispersion takes place.

2. **2076 GIE Set B Q.No. 3a Explain the cause of dispersion of light.** **[2]**

When white light is passed through a prism, it splits up into its seven-constituent colours because of their different wavelength in the prism (denser medium). From Cauchy dispersion relation, we have

$$\mu = A + \frac{B}{\lambda^2} + \ldots$$

This shows that refractive index of a prism for a particular colour varies inversely with the wavelength of the colour. The wavelength of red colour is the highest among the constituent colours of white light. So its refractive index is the lowest. Similarly wavelength of violet colour being the smallest among the constituent colours of white light, its refractive index is the highest. Moreover, the deviation produced by a small angled prism is given by, $\delta = A(\mu - 1)$.
As $\mu_r < \mu_v$, then $\delta_r < \delta_v$. It means red colour is least deviated and the violet colour is most deviated. Hence, red light appears at the top of the spectrum whereas the violet at the bottom. Hence, the cause of dispersion of light is the difference in wavelength of light in medium.

3. **2076 Set B Q.No. 3b 2072 Set D Q.No. 3b 2064 Q.No. 6 b 2052 Q.No. 11 a When white light is dispersed by a prism, red light appears at the top of the spectrum where as violet at the bottom. Why?** **[2]**

When white light is passed through a prism, it splits up into its seven-constituent colours because of their different wavelengths in the prism (denser medium). From Cauchy dispersion relation, we have

$$\mu = A + \frac{B}{\lambda^2} + \ldots$$

This shows that refractive index of a prism for a particular colour varies inversely with the wavelength of the colour. The wavelength of red colour is the highest among the constituent colours of white light. So its refractive index is the lowest. Similarly, wavelength of violet colour being the smallest among the constituent colours of white light, its refractive index is the highest. Moreover, the deviation produced by a small angled prism is given by, $\delta = A(\mu - 1)$
As $\mu_r < \mu_v$, then $\delta_r < \delta_v$. It means red colour is least deviated and the violet colour is most deviated. Hence, red light appears at the top of the spectrum whereas the violet at the bottom.

4. **2075 GIE Q.No. 3b Why does the sky look blue in a day?** **[2]**

Sky is seen blue due to scattering of light by air molecules. According to law of Rayleigh, the intensity of scattered light (I) depends with the wavelength (λ) as, $I \propto \frac{1}{\lambda^4}$

As the wavelength of blue color being nearly half of the wavelength of red color, the intensity of scattering of blue color is sixteen times greater than that of red color of light. As a result, the blue color predominates other colors and the sky appears blue.

5. **2075 Set A Q.No. 3b Why does the sky appear blue?** **[2]**

Please refer to 2075 GIE Q.No. 3b

6. **2074 Set A Q.No. 3a How do the colours of light related with the refractive index of a lens? Explain.** **[2]**

From the Cauchy dispersion relation, the refractive index (μ) of a medium is given by

$$\mu = A + \frac{B}{\lambda^2} + \ldots.$$

where A and B are constant and λ is wavelength of light. Since the different colors of light have different wavelengths & hence different refractive index. The red light has longer wavelength then the blue color of light. So, the refractive index of lens for red light is less then the blue color of light.

7. **2073 Supp Q.No. 3b** **Define dispersive power of a lens and write expression for it in terms of the refractive indices of lens with respect to different colours used.** **[2]**

Dispersive power of a material for any two colours is defined as the ratio of the angular dispersion for these two colours to the deviation suffered by mean light (i.e. yellow light). It is denoted by ω.

So, for violet and red light, the dispersive power is

$$\omega = \frac{\delta_v - \delta_r}{\delta} = \frac{A(\mu_v - 1) - A(\mu_r - 1)}{A(\mu - 1)} \qquad \Rightarrow \omega = \frac{\mu_v - \mu_r}{\mu - 1},$$

where μ_v, μ_r and μ are the refractive index of the material for violet, red and mean light respectively.

8. **2072 Supp Q.No. 3a** **How does the refractive index relate with the wavelength of light?** **[2]**

From the Cauchy dispersion relation, the refractive index (μ) of a medium is given by

$$\mu = A + \frac{B}{\lambda^2} + ...$$

where A and B are constant and λ is wavelength of light. Thus, refractive index relates inversely with wavelength of light.

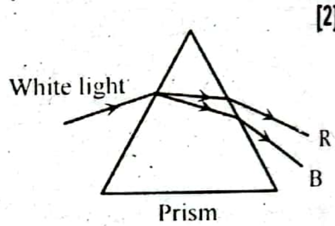
9. **2072 Set C Q.No. 3b** **State the necessary conditions for the production of a pure spectrum.** **[2]**

The necessary conditions for the production of pure spectrum are,

i. Instead of point source, a narrow slit should be used.
ii. A convex lens is required to make the beam parallel.
iii. The prism should be kept at minimum deviation position.
iv. Another convex lens should be kept in the path of emergent light.

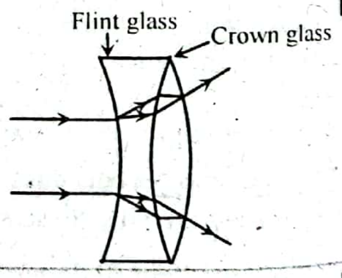
10. **2072 Set E Q.No. 3a** **Which one of blue and red light will be deviated more by a prism? Give reason for your answer.** **[2]**

When a white light passes through a prism, red light deviates less than the blue light. This is because the red light has more velocity, longest wavelength and hence, least refractive index but blue light has less velocity less wavelength and more refractive index. Since, $\delta = A(\mu - 1)$, blue light deviates more than red light.



11. **2072 Set E Q.No. 3b** **Can a concave and a convex lens be combined so that the combination behaves like a diverging lens? Justify your answer.** **[2]**

Yes, a concave and a convex lens can be combined so that the combination behaves like a diverging lens. When a diverging lens of flint glass (with more R.I.) and a convex lens of crown glass (with less R.I.) are combined the combination becomes diverging lens.

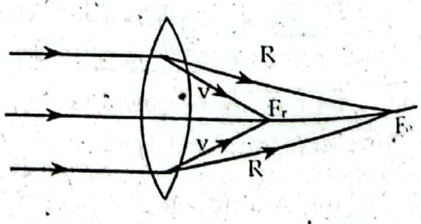


12. **2071 Supp Q.No. 3b** **2066 Q. No. 3 a** **What is the cause of dispersion of light?** **[2]**

Please refer to 2076 GIE Set B Q.No. 3a

13. **2071 Set D Q.No. 3 a** **Draw an outline sketch to show the chromatic aberration in the case of a lens. How can achromatism be achieved?** **[2]**

When a white light beam incident on a lens parallel to the principal axis, it splits into its constituent colour. The focal length for various colours light is different depending up on the refractive index of the material. This defect of lens, which is unable to focus the light of different colours to a single point, is called chromatic aberration of lens. Achromatism is obtained by combining convex lens of crown glass and concave lens of flint glass.



14. **2070 Set C Q.No. 3 b** **A spherical mirror cannot give rise to chromatic aberration. Why?** **[2]**

Chromatic aberration occurs when light passes through a denser medium like lens, i.e. in refraction phenomena, because of different focal lengths of lens for different colours (wavelengths). But focal length of mirror $\left(\frac{1}{f}=\frac{1}{u}+\frac{1}{v}\right)$ is independent of refractive index of incident light i.e. all the colours are reflected by same angle. Hence, such defects cannot be seen in a mirror.

15. **2069 Supp Q.No. 3b** **Why does the sun appears red during the sun-rise?** **[2]**

We know that the intesity of scattering of light is inversely proportional to the forth power of wavelength of light $\left(I \propto \frac{1}{\lambda^4}\right)$. The red light has longest wavelength and hence, get least scattered than the other light ray. At sun-rise and sun-set the sun is far and oblique from us than the noon. The red light get least scattered and reach to us but other light scattered and hence, sun looks red during sun-rise and sun-set.

16. **2069 (Set A) Old Q. No. 6c** **What is the cause of dispersion of a white ray of light?** **[2]**

Please refer to 2071 Supp Q.No. 3b

17. **2069 Old (Set B) Q. No. 6b** **Does the refractive index depend upon the wavelength of light?** **[2]**

Yes, the R.I. depends upon the wavelength of light. From Cauchy dispersion relation, we have

$$\mu = A + \frac{B}{\lambda^2} + \ldots$$

This shows that refractive index of a prism for a particular colour varies inversely with the wavelength of the colour. Hence, the refractive index of light depends upon the wavelength of light.

18. **2068 Q.No. 3 b** **What is chromatic aberration? Explain why a mirror cannot give rise to chromatic aberration.** **[2]**

Please refer to 2070 Set C Q.No. 3 b

19. **2068 Old Q.No. 6 c** **2065 Q. No. 6 c** **What do you mean by chromotic aberration?** **[2]**

Please refer to 2071 Set D Q.No. 3 a

20. **2067 Supp Q.No. 3 b** **Distinguish between chromatic aberration and spherical aberration.** **[2]**

Chromatic abberration	Spherical abberration
1. This is inability of a lens to focus all colours of light at a single point is called the chromatic aberration	1. This is inability of a lens to focus all the light rays (paraxial and marginal) at a single point is called spherical aberration.
2. Chormatic aberration occurs due to deviation at different angle by different color of light.	2. Spherical aperration occurs due to deviation at different angle by marginal and paraxial rays.
3. This defect can be removed by achromatic combination of lenses.	3. This defect can be removed by using slit or stop.
4. This is color defect.	4. This is not color defect.

21. **2067 Q.No. 3 b** **The sun looks red at sun-rise and sun-set. Why?**

Please refer to 2069 Supp Q.No. 3b

22. **2066 Q. No. 3 b** **Why is sky blue? Explain.** **[2]**

Please refer to 2075 GIE Q.No. 3b

23. **2066 Old Q. No. 6 b** **If a plane glass slab is placed on letter of different colours, then violet coloured letters appear more and more raised up. Why?** **[2]**

If a plane glass slab is placed on letter of different colors, then violet colored letters appear more and more raised up means the apparent displacement is more which is given by the formula, $d = t\left(1-\frac{1}{\mu}\right)$ where t is apparent depth and μ is the refractive index of glass. The violet light has shortest wavelength than the other visible light, hence its refractive index is more and the displacement is more. Hence, the violet colored letter appears to rise more than other.

24. 2063 Q.No. 6 c 2057 Q.No. 6 c 2054 Q.No. 11 c **Explained why can a mirror not give rise to chromatic aberration?** **[2]**

Please refer to 2070 Set C Q.No. 3 b

25. 2062 Q.No. 6 c **Explain the meaning of achromatism in a lens.** **[2]**

The aberration or defect produced due to the dispersion of light is called chromatic aberration. Due to this, the image of a white object is colored and blurred. Using a combination of a crown glass convex lens with a flint glass concave lens can reduce chromatic aberration. This combination is called achromatism. The method of reducing chromatic aberration is called achromatization.

26. 2056 Q.No. 6 c **What do you mean by dispersive power?** **[2]**

Please refer to 2073 Supp Q.No. 3b

27. 2055 Q.No. 11 b **Why are a number of dark lines seen in the spectrum of light from the sun?** **[2]**

When the light from sun comes to earth, the white light of sun gets dispersed and spectrum is formed. In the spectrum of sun light, some of the colors of light are absorbed as a result dark lines are seen.

28. 2055 Q.No. 11 d **Why there is no dispersion of monochromatic light?** **[2]**

When white light is passed through a lens (or a prism), it is splitted into its seven constituent different colours. This is because different colours (wavelengths) have different speed in the medium (lens or prism) and angle of refraction is different for different colour. But, when monochromatic light (light of particular wavelength) is passed through a lens (or a prism), it doesn't get splitted and hence, no dispersion.

29. 2053 Q.No. 11 b **How is dispersive power related to refractive index of the material?** **[2]**

Please refer to 2073 Supp Q.No. 3b

Long Answer Questions

30. 2076 GIE Set A Q.No. 7b **What is chromatic aberration? Find the condition of a achromatic combination of two lenses.** **[4]**

When a beam of white light incident on a lens parallel to the principal axis, it splits into its constituent colours. The focal length for various colours of light is different depending upon the refractive index of the material. This defect of lens, which is unable to bring the light of different colours to focus at a single point, is called chromatic aberration of lens.

Let us consider two thin lenses L_1 and L_2 (convex and concave) placed in contact. Let μ_v, μ and μ_r be the refractive indices of the material of lens L_1 for violet, mean and red light and f_v, f and f_r be the focal lengths for violet, mean and red lights. Similarly, μ'_v, μ', μ'_r and f_v', f' and f_r' be the refractive indices and focal lengths for violet, mean and red light respectively for lens L_2.

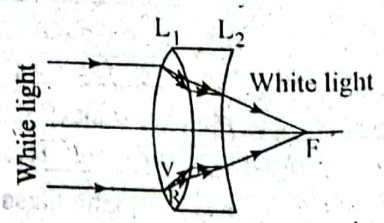
For lens L_1, the focal length for mean light,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \ldots \text{(i)}$$

where r_1 and r_2 be the radii of curvature of two surfaces of the lens L_1.

Focal length of lens L_1 for the violet light

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \ldots \text{(ii)}$$



From equation (i), we get

$$\frac{1}{r_1} + \frac{1}{r_2} = f(\mu - 1) \quad \ldots \text{(iii)}$$

$$\therefore \frac{1}{f_v} = \frac{\mu_v - 1}{(\mu - 1)f} \quad \ldots \text{(iv)}$$

Similarly, for red light

$$\frac{1}{f_r} = \frac{\mu_r - 1}{(\mu - 1)f} \quad \ldots \text{(v)}$$

For lens L_2, Focal length for violet light is

$$\frac{1}{f_v'} = \frac{\mu_v' - 1}{(\mu' - 1) f'} \quad \ldots (vi)$$

Focal length for red light is

$$\frac{1}{f_r'} = \frac{\mu_r' - 1}{(\mu' - 1) f'} \quad \ldots (vii)$$

If F_v and F_r represents the focal length of the combinations of two lens for violet and red light respectively, then

$$\frac{1}{F_v} = \frac{1}{f_v} + \frac{1}{f_v'} = \frac{\mu_v - 1}{(\mu - 1) f} + \frac{\mu_v' - 1}{(\mu' - 1) f'} \quad \ldots (viii)$$ [Using equation (iv) and (vi)

and $$\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f_r'} = \frac{\mu_r - 1}{(\mu - 1) f} + \frac{\mu_r' - 1}{(\mu' - 1) f'} \quad \ldots (ix)$$

[Using equation (v) and (vii)

When two lenses are combined scuh that the combination is free from chromatic abberation, then the violet and red rays will come to focus at a single point. i.e.

$$F_v = F_r$$

$$\frac{1}{F_v} = \frac{1}{F_r}$$

$$\frac{\mu_v - 1}{(\mu - 1) f} + \frac{\mu_v' - 1}{(\mu' - 1) f'} = \frac{\mu_r - 1}{(\mu - 1) f} + \frac{\mu_r' - 1}{(\mu' - 1) f'}$$

$$\frac{\mu_v - 1}{(\mu - 1) f} - \frac{\mu_r - 1}{(\mu - 1) f} + \frac{\mu_v' - 1}{(\mu' - 1) f'} - \frac{\mu_r' - 1}{(\mu' - 1) f'} = 0$$

$$\frac{\mu_v - \mu_r}{(\mu - 1) f} + \frac{\mu_v' - \mu_r'}{(\mu' - 1) f'} = 0$$

$$\frac{\omega}{f} + \frac{\omega'}{f'} = 0 \quad \Rightarrow \frac{\omega}{f} = -\frac{\omega'}{f'} \quad \ldots (x)$$

where $\omega = \dfrac{\mu_v - \mu_r}{(\mu - 1)}$ is dispersive power of crown glass lens L_1 and $\omega' = \dfrac{\mu_v' - \mu_r'}{\mu' - 1}$ is dispersive power of flint glass lens L_2.

The relation given by the equation (x) is the required expression for achromatism in two lenses in contact.

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31. **2073 Set C Q.No. 7a** **What is chromatic aberration in lens? Deduce the condition for achromatism in two thin lenses in contact.** **[4]**

✎ Please refer to 2076 GIE Set A Q.No. 7b

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32. **2070 Supp (Set B) Q.No. 7 b** **What is chromatic aberration in a lens? How will you combine two lenses of different materials so that there is no chromatic aberration.** **[4]**

✎ Please refer to 2076 GIE Set A Q.No. 7b

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33. **2070 Set D Q.No. 7 b** **What is chromatic aberration? Show that for a lens, the chromatic aberration is the product of dispersive power and focal length of mean light.** **[4]**

✎ Please refer to 2076 GIE Set A Q.No. 7b

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34. **2069 (Set A) Old Q. No. 7a OR** **What is chromatic aberration? Show that the chromatic aberration is the product of dispersive power and mean focal length of a lens.** **[4]**

✎ Please refer to 2076 GIE Set A Q.No. 7b

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35. **2069 Old (Set B) Q. No. 7a OR** **2060 Q.No. 7 a OR** **What is chromatic aberration? Show that two lenses form an achromatic doublet if the ratio of their focal lengths is numerically equal to the ratio of the corresponding dispersive powers of their materials.** **[1+4]**

✎ Please refer to 2076 GIE Set A Q.No. 7b

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36. **2066 Old Q. No. 7 a** **What do you mean by chromatic aberration? Show for a lens, the chromatic aberration is the product of dispersive power and, its mean focal length.** **[1+4]**

✎ Please refer to 2076 GIE Set A Q.No. 7b

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37. **2058 Q.No. 7 a OR** **Derive the condition for achromatism in two lenses in contact.** **[5]**

✎ Please refer to 2076 GIE Set A Q.No. 7b

38. 2055 Q.No. 14 **What is Chromatic abberation in lenses? Derive the condition for achromatic lenses.**

Please refer to 2076 GIE Set A Q.No. 7b

Numerical Problems

39. 2073 Set D Q.No. 11 **The dispersive powers for crown and flint glass are 0.013 and 0.026 respectively. Calculate the focal lengths of the lenses which form an achromatic doublet of focal length 80 cm, when placed in contact.** [4]

Given, Dispersive power of crown glass (ω_1) = 0.013

Dispersive power of flint glass (ω_2) = 0.026

Combined mean focal length (F) = 80 cm

Focal length of crown glass (f_1) = ?

Focal length of flint glass (f_2) = ?

We have,

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}$$

or, $\frac{f_1}{f_2} = -\frac{0.013}{0.026} = -\frac{1}{2}$

or, $f_1 = -\frac{1}{2}f_2$

or, $f_2 = -2f_1$ ... (i)

Again,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$$

or, $\frac{1}{f_1} + \frac{1}{2f_1} = \frac{1}{80}$

or, $\frac{1}{2f_1} = \frac{1}{80}$

or, $2f_1 = 80$ cm

$\therefore$ $f_1 = 40$ cm

$f_2 = -2f_1 = -2 \times 40 = -80$ cm

40. 2069 (Set B) Q. No. 11 **An achromatic converging lens of mean focal length 40cm is made by combining two lenses of different materials. If the dispersive powers of the two lenses are in the ratio 1:3, find the focal lengths of each lens.** [4]

Given, Mean focal length (F) = 40 cm

Ratio of dispersive powes $\left(\frac{\omega_1}{\omega_2}\right) = \frac{1}{3}$

Focal lengths of each lens f_1 and f_2 = ?

Let the focal length of the lenses be f_1 and f_2, then

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}$$

or, $\frac{f_1}{f_2} = -\frac{1}{3}$

or, $f_2 = -3f_1$

But $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$

or, $\frac{1}{f_1} - \frac{1}{3f_1} = \frac{1}{40} \quad = \frac{3-1}{3f_1} = \frac{1}{40}$

or, $f_1 = \frac{40 \times 2}{3} = \frac{80}{3} = 26\frac{2}{3}$ cm

and $f_2 = -3f_1 = -3 \times \frac{80}{3} = -80$cm

41. 2069 (Set A) Old Q.No. 6b **An achromatic lens of focal length +30cm is to be constructed by combining a crown glass lens and flint glass lens. What must be the focal lengths of the component lenses if the dispersive powers of crown glass and flint glass are 0.018 and 0.027 respectively?** **[3]**

Given,

Mean focal lenth (F) = 30cm

Dispersive power of crown glass (ω_1) = 0.018

Dispersive power of flint glass (ω_2) = 0.027

Focal length of crown glass (f_1) = ?

Focal length of flint glass (f_2) = ?

We have,

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2} = \frac{0.018}{0.027}$$

or, $\frac{f_1}{f_2} = -\frac{2}{3}$

or, $f_1 = -\frac{2}{3} f_2$

Again,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

or, $\frac{1}{F} = \frac{-3}{2f_2} + \frac{1}{f_2}$

or, $\frac{1}{30} = \frac{-3}{2f_2} + \frac{1}{f_2}$

or, $\frac{1}{30} = \frac{-3+2}{2f_2}$

or, $f_2 = -15$ cm

or, $f_1 = -\frac{2}{3} \times (-15) = 10$ cm

$\therefore f_1 = 10$ cm and $f_2 = -15$ cm

□□□

UNIT 4

Electricity and Magnetism

Chapter 1: Electric Charge

Short Answer Questions

1. **2076 Set B Q.No. 4a Bits of paper are attracted to an electrified comb, even though they have no net charge. How is this possible?** [2]

When a comb runs through hair, it gets charged due to friction with hair. This charged comb when brought close to paper, it induces opposite charge on paper pieces and attracts to paper even though they no net charge. If the hair is wet, instead of dry, the friction between hair and comb is highly reduced and hence, comb is less charged. Thus, it cannot attract the paper.

2. **2076 Set C Q.No. 4a What do you mean by quantization of charge? Explain.** [2]

The charges on any charged body are integral multiple of basic unit of charge. This basic unit of charge is taken as the charge on an electron or a proton which is equal to $\pm 1.6 \times 10^{-19}$ C. Therefore the charge on any body is, $Q = \pm n e$ where e is the charge on an electron and n is any integer. This property of charge is called the quantization of electric charge. This means the charges on the body are not in fraction number i.e. they are in whole number.

3. **2075 GIE Q.No. 4a We often experience the sparks during the winter nights when removing the cloths from our body. Why?** [2]

In winter season, our body is dry but not wet. When we remove our clothes from out body, the charges are produced due to friction between the body and clothes. As a result, some sparks are produced.

4. **2075 Set A Q.No. 4a Why metallic chain is used in the tanker transporting highly inflammable liquid?** [2]

The vehicles get charged due to the friction during motion. The body of vehicles also gets charged due to friction of air. If the accumulation of charge is enough, it causes sparking and the inflammable material may catch fire and explosion may happen. When an iron or metallic chain is suspended from the vehicle, it drags charges from body of vehicle to ground. Hence, the iron chain prevent from explosion.

5. **2075 Set B Q.No. 4b How can a body be charged with positive electricity by the method of induction? Explain.** [2]

Charging a body positively by induction

Let us consider a negatively charged glass rod taken to near an uncharged body AB which is fitted on the insulated stand. This body AB can be charged positively by induction in the following figure:

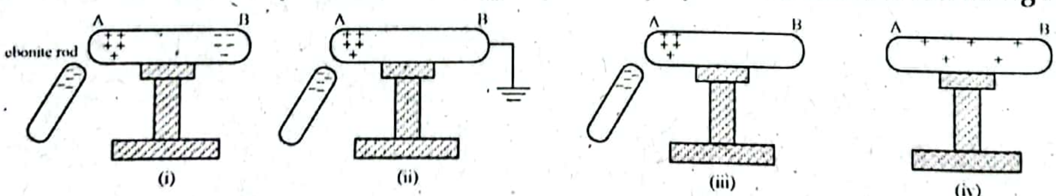


Figure: Charging a body positively by induction.

6. **2073 Supp Q.No. 4a Explain how a pith ball can be electrified with positive or negative charge.** [2]

Electrostatic Induction for Positive Charge: Please refer to 2075 Set B Q.No. 4b

Electrostatic Induction for Negative Charge

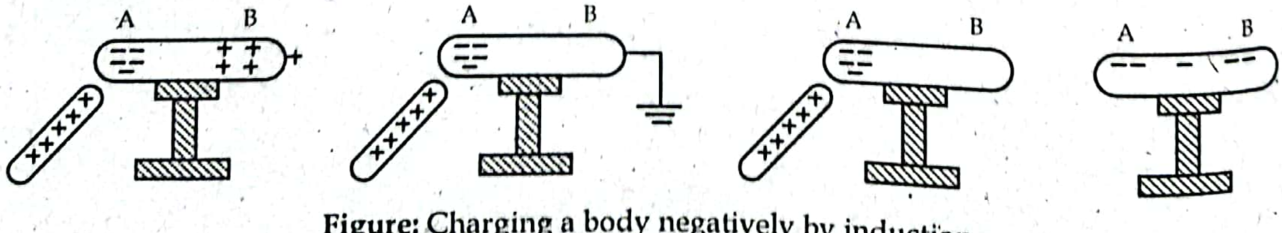


Figure: Charging a body negatively by induction.

When an uncharged body is taken near to the charged body, the charges are developed on the uncharged body temporarily. This temporary electrification of a body by bringing a charged body close to it is called electrostatic induction. A body is charged negatively by induction method as shown in figures in different steps.

7. 2073 Set D Q.No. 4a **Can a charged body attract an uncharged body? Explain.** **[2]**

Yes, a charged body can attract an uncharged body. When the charged body is taken to near an uncharged body, the uncharged body gets charged by induction method. The induced charges nearer to the end of charge body are unlike while far end are like charges. According to the nature of unlike charges, they attract with each other.

8. 2072 Supp Q.No. 4b **How do you charge a body with positive electricity by induction? Explain.** **[2]**

Please refer to 2075 Set B Q.No. 4b

9. 2072 Set C Q.No. 4a **Why does bodies get electrified when they are rubbed together for a while?** **[2]**

When two bodies rubbed together for a while, one body loses valence electrons while other body gains electrons. The body which loses electrons is positively charged because there will deficit of electrons and the other body which gains electrons is negatively charged because there will be excess of electrons (negative charge). In this way, transformation of electrons from one body to another body electrified both two bodies on rubbing.

10. 2072 Set D Q.No. 4b **Two charged conductors are touched mutually and then separated. What will be the charge on them?** **[2]**

When two charged conductor are touched mutually and then separated, the opposite charge will be produced due to electrostatic induction.

11. 2071 Set D Q.No. 4 b **How can a neutral body be changed with negative electricity by induction? Explain.** **[2]**

Please refer to 2073 Supp Q.No. 4a

12. 2070 Supp (Set B) Q.No. 4 a **The tyres of airplane are made from a special rubber which is resonably good conductor of electricity. What would be the reason behind it?** **[2]**

During the landing there is a friction between the tyres of the aircraft and the runway. This develops charge on the tyres. If the accumulation of charge becomes high, it may lead to sparkling and fire. If the tyres are made slightly conducting, the charge will not accumulate on it and leak to ground. So, the tyres of airplane are made from a rubber which is reasonably good conductor of electricity.

13. 2070 Set C Q.No. 4 b **Can a charged body attract an uncharged body? Explain.** **[2]**

Please refer to 2073 Set D Q.No. 4a

14. 2070 Set D Q.No. 4 a **More charge can be stored on a metal if it is highly polished that when its surface is rough. Explain.** **[2]**

If the surface of a charged metal sphere is rough, it contains many minute projections through which charges start to leak. This is because the electric field intensity is very high at such projections which cause to occur discharge of gases near it. This can be minimized if the surface of the charged metal sphere is polished. That's why, more charge can be stored on a metal sphere if it is highly polished than when its surface is rough.

15. 2069 Supp Q.No. 4a **Why are the four footed animals posed to more threat during close by lightning strike than the two footed humans?** **[2]**

The four feet of the four footed animal remain in contact with ground while two feet of the human are attached with earth. Due to this reason, during lightening, the conductivity of four footed animal is more than the two footed human. So, the four footed animals posed to more threat during close by lightning strike than the two footed humans.

16. 2069 (Set A) Q. No. 4a **Why is it dangerous to take shelter under a tall tree during lightening?** **[2]**

It is dangerous to take shelter under a tall tree during lighting because the green tree is good conductor of electricity and there is much more possibility of the lightening through the tree.

17. 2069 (Set B) Q. No. 4a **The vehicles carrying inflammable fluid drag a chain along the ground. Why?** **[2]**

Please refer to 2075 Set A Q.No. 4a

18. 2069 Set B Q. No. 4b **Sharp projections are avoided in machines. Why?** **[2]**

According to action of point, the surface charge density will be maximum at the point, if there is a sharp edge in electrostatic machine. ($\sigma = \frac{q}{A} \Rightarrow \sigma \propto \frac{1}{A}$, for constant charge). Due to this, when air molecule come in contact with such edges, the molecules get similar charge from them and get repulsion. Other fresh air molecule comes in contact and gets some charge and move away. The process is continuous so that the conductor leaks the charge continuously and an electric wind is formed, if it has point edge. Therefore, such pointed edges are avoided in electrical machines.

19. 2068 Q.No. 4 a **What do you mean by quantization of charge?** **[2]**

Please refer to 2076 Set C Q.No. 4a

20. 2068 Old Q.No. 8 b **What do you mean by charging by conduction and charging by induction?** **[2]**

The process of making charged body from uncharged body by any method is called charging. We can charge a body by method of conduction or induction. In charging by conduction, we can charge a conductor by supplying or touching a charge body to it. While in induction method, a charge body is taken near an uncharged body without touching it and an uncharged body becomes charged. In a body when is charged by conduction has same type of charge but in induction opposite type of charge is developed.

21. 2067 Supp Q.No. 4 a **More charge can be stored on a metal if it is highly polished than when its surface is rough, why?** **[2]**

Please refer to 2070 Set D Q.No. 4 a

22. 2067 Q.No. 4 a **Some of the free electrons in a good conductor (such as a piece of copper) move at speed of 10^6 m/s or faster. Why don't these electrons fly out of the conductor completely?** **[2]**

Metal (or good conductor) have free electrons at its surface, these electrons are strongly attracted towards the positively charged protons of the same metal by the strong electrostatic force of attraction by the formula,

$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{x^2}$ where q_1 and q_2 are the charges of protons and electrons and x be the distance between the protons and electrons. Due to this strong force of attraction, some of the free electrons in a good conductor (such as a piece of copper) move at speeds of 10^6 m/s or faster but do not fly out of the conductor completely.

23. 2064 Q.No. 8 c **A comb run through one's dry hair attracts bits of paper. Why?**

Please refer to 2076 Set B Q.No. 4a

24. 2063 Q.No. 8 b **Why do sharp edges are strictly avoided in an electrical machine?**

Please refer to 2069 Set B Q. No. 4b

25. 2063 Q.No. 8 c **What is meant by relative permittivity? What is its minimum value?**

Dielectric constant (relative permittivity) of a medium is defined as the ratio of the permittivity of that medium to the permittivity of free space (or vacuum). It is denoted by ε_r and given by

$$\varepsilon_r = \frac{\text{permittivity of medium}}{\text{permittivity of free space}} \Rightarrow \varepsilon_r = \frac{\varepsilon}{\varepsilon_o}.$$

It is also called as dielectric constant (k). Thus, the dielectric constant of a medium is the relative permittivity of medium with respect to the vacuum. The minimum value of dielectric constant or relative permittivity is 1 for vacuum or air and greater than 1 for other medium.

26. 2062 Q.No. 8 a **Explain the phenomenon of action point in a charged conical sphere.** **[2]**

According to action of point, the surface charge density will be maximum at the point, ($\sigma = \frac{q}{A} \Rightarrow \sigma \propto \frac{1}{A}$, for constant charge). A conical charged conductor has maximum charged density at conical part than the other part of the surface. As a result, the voltage at the conical part of the conductor is maximum. When air molecules come in contact with such edges, the molecules get similar charge from them and get repulsion. Other fresh air molecule comes in contact and gets some charge and move away. The process is continuous so that the conductor leaks the charge continuously from the conical part.

The charged sphere has no such a points and does not loose charge continuously and remain for a long time.

27. 2061 Q.No. 8 a **Why can more charge be placed on a metal if it is highly polished than when its surface is rough?** **[2]**

Please refer to 2070 Set D Q.No. 4 a

28. 2060 Q.No. 8 a **A charged conical conductor loses its charge earlier than a similarly charged sphere. Why?** **[2]**

Please refer to 2062 Q.No. 8 a

29. 2059 Q.No. 8 a **Why pointed ends are not kept in the electrostatics machine?** **[2]**

Please refer to 2069 Set B Q. No. 4b

30. 2058 Q.No. 8 a **Why are sharp edges or points avoided in electrical machines?** **[2]**

Please refer to 2069 Set B Q. No. 4b

Long Answer Questions

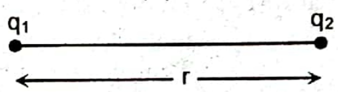
31. 2076 GIE Set A Q.No. 8a **State and explain Coulomb's law in electrostatics and hence define 1 Coulomb Charge.** **[4]**

Coulomb's law in electróstatics states that the electrostatic forces between two charges is directly proportional to the product of their charges and inversely proportional to the square of distance between their centre.

If q_1 and q_2 be two charges separated by a distance r, then from Coulomb's law.

$F \propto q_1 q_2$ ... (i)

$F \propto \frac{1}{r^2}$ ... (ii)



Combining equation (i) and (ii), we get

$$F \propto \frac{q_1 q_2}{r^2}$$

or, $F = k\frac{q_1 q_2}{r^2}$ ... (iii)

Where k is constant of proportionality, known as electrostatic force constant.

For medium,

$$K = \frac{1}{4\pi\varepsilon}$$

ε = permittivity of medium.

In vacuum $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$ Permittivity of free space

For vacuum,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \quad \text{... (iv)}$$

This is required expression for electrostatic force.

Since, $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^2$

$$F = 9 \times 10^9 \frac{q_1 q_2}{r^2}$$

If $q_1 = q_2 = 1C$, $r = 1m$, then $F = 9 \times 10^9$ N.

Thus, one Coulomb is that charge which when placed 1 m from an equal and similar charge in vacuum repels it with a force of 9×10^9 N.

32. 2076 GIE Set B Q.No. 8a **State and explain Coulomb's law and hence define one coulomb charge.** **[4]**

Please refer to 2076 GIE Set A Q.No. 8a

33. 2073 Set C Q.No. 8a **What is electrostatic induction? Explain with necessary diagrams, a method of charging a body positively by induction.** **[4]**

Electrostatic Induction

When an uncharged body is taken near to the charged body, the charges are developed on the uncharged body temporarily. This temporary electrification of a body by bringing a charged body close to it is called electrostatic induction.

a. **Charging a body positively by induction:**

Let us consider a negatively charged glass rod taken to near an uncharged body AB which is fitted on the insulated stand. This body AB can be charged positively by induction in the following ways:

Step I: If a negatively charged glass rod is brought near a body AB, the end A of the body acquires bound negative charge and the far end B acquires free negative charges due to induction as shown in Fig. (i).

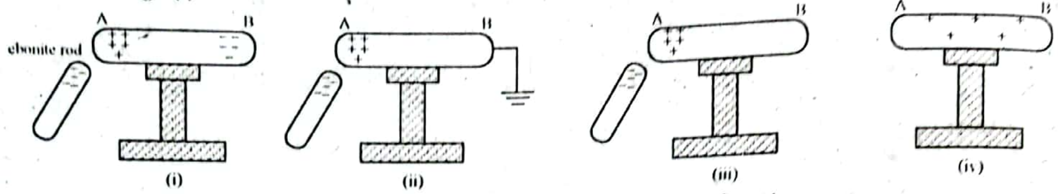


Figure: Charging a body positively by induction.

Step II: The body, AB is earthed with the help of the metal wire as shown in Fig. (ii). In this case, the free negative charge flows to the earth. However the positive charges being the bound charges do not move in to the earth.

Step III: The earthing is removed keeping the glass rod still in its position. The bound positive charge on end B remains there as shown in Fig.(iii).

Step IV: Finally, the positively charged glass rod is removed away from the body, AB. The bound negative charge spreads over whole surface of the body, AB as shown in Fig. (iv). Therefore, the body, AB becomes positively charged by induction.

34. 2070 Set D Q.No. 8 a **What is electrostatic induction? How can you charge a body positively by induction? [4]**

Please refer to 2073 Set C Q.No. 8a

Numerical Problems

35. 2075 GIE Q.No. 12 2069 Supp Q.No. 12 **Calculate the value of two equal charges if they repel each other with a force of 0.1N when situated 50 cm apart in vacuum. What would be the distance between them if they are placed in an insulating medium of dielectric constant 10? [3]**

Given,

Force (f) = 0.1N

Distance between two charges (r) = 50 cm = 0.5m

Value of charge (q) = ?

We have, in air,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \qquad [\because q = q_1 = q_2]$$

$$0.1 = \frac{9\times10^9\times q^2}{(0.5)^2}$$

$$\text{or, } q^2 = \frac{0.1\times(0.5)^2}{9\times10^9}$$

$$\text{or, } q^2 = 2.778\times10^{-12}$$

$$\text{or, } q = 1.667\times10^{-6}\text{C}$$

$$\therefore q = 1.667\ \mu\text{C}$$

When they are kept in medium of dielectric constant 10,

$$F = \frac{1}{4\pi\epsilon.\epsilon_0}\times\frac{q^2}{r^2}$$

$$0.1 = \frac{9\times10^9\times(1.667\times10^{-6})^2}{10\,r^2}$$

$$r = \sqrt{\frac{9\times10^9\times(1.667\times10^{-6})^2}{10\times0.1}} = 0.16\text{ m}$$

36. **2071 Supp Q.No. 12 An alpha particle is a nucleus of doubly ionized helium. It has mass of 6.68×10^{-27} kg and charge of 3.2×10^{-19}C. Compare the force of electrostatic repulsion between the two alpha particles with the force of gravitational attraction between them.**
[Universal gravitational constant = 6.67×10^{-11} Nm^2kg^{-2};
Permittivity of free space = $8.85 \times 10^{-12} Fm^{-1}$] **[3]**

Given,
Mass of helium (m) = 6.68×10^{-27} kg
Charge of helium (q) = 3.2×10^{-19} C
(F_e / Fe) = ?

Now, $F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$

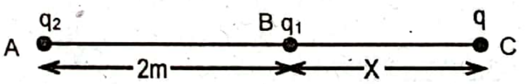
$F_g = G\frac{m^2}{r^2}$

Now, $\frac{F_e}{F_g} = \frac{1}{4\pi\epsilon_0 G}\frac{q^2}{m^2}$

$= \frac{1}{4\pi \times 8.85 \times 10^{-12} \times 6.67 \times 10^{-11}} \times \frac{(3.2 \times 10^{-19})^2}{(6.68 \times 10^{-27})^2} = 3.1 \times 10^{35}$

37. **2069 (Set A) Q. No. 12 Two charges $+1\times10^{-6}$ C and -4×10^{-6}C are separated by a distance of 2m. Determine the position of the null point.** **[3]**

Given,
Charge $(q_1) = 1 \times 10^{-6}$ C
Charge $(q_2) = -4 \times 10^{-6}$ C
Distance (d) = 2m
Null distance = ?



From figure,
Force on q due to q_1 is

$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{x^2}$ along BC

Force on q due to q_2 is

$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{(x+2)^2}$ along CA

By question, $F_1 = F_2$ (for null point)

or, $\frac{q_2}{x^2} = \frac{q_1}{(x+2)^2}$

or, $\frac{10^{-6}}{x^2} = \frac{4\times10^{-6}}{(x+2)^2}$

or, $\left(\frac{1}{x}\right)^2 = \left(\frac{2}{x+2}\right)^2$

$\therefore$ x = 2m from charge q_1 and 4 m from charge q_2.

□□□

Chapter 2: Electric Field

Short Answer Questions

1. **2076 GIE Set B Q.No. 4a Two electric lines of force do not intersect with each other. Why?** [2]

We know that, the tangent drawn at any point on electric lines of force gives the direction of electric field intensity at that point. If two lines of force intersect at a point, then it demands two different directions of electric field intensity at the point of intersection. But, it is impossible. Hence, two lines of force never intersect to each other.

2. **2076 Set B Q.No. 4b Can two electric lines of force intersect each other? Explain.** [2]

Please refer to 2076 GIE Set B Q.No. 4a

3. **2074 Supp. Q.No. 4a What is the magnitude of an electric field which will balance the weight of an electron on the surface of earth?** [2]

Let E be the value of electric field strength of an electron which will just balance its weight on the surface of earth, then,

Electric force = Weight of electron

or, $eE = W$

or, $eE = m_e g$

or $E = \frac{m_e g}{e}$

Since, mass of electron, $m_e = 9.1\times 10^{-31}$ kg

Charge of an electron, $e = 1.6 \times 10^{-19}$C

Then,

$$E = \frac{9.1\times 10^{-31} \times 10}{1.6\ 10^{-19}}$$

$$= 5.68 \times 10^{-11}\ N/C$$

4. **2074 Set A Q.No. 4a Can two electric lines of force ever intersect? Explain.** [2]

Please refer to 2076 GIE Set B Q.No. 4a

5. **2072 Set D Q.No. 4a What is electrostatic shielding?** [2]

When a hollow metallic conductor is charged, all the charges will be distributed on its outer surface. Inside the conductor, there will be no electric field. Hence, a hollow conductor acts as a shield due to which there will be zero electric field inside it is known as electrostatic shielding.

6. **2070 Supp (Set A) Q.No. 4 a Two electric lines of force never intersect each other, why?** [2]

Please refer to 2076 GIE Set B Q.No. 4a

7. **2069 (Set A) Q. No. 4b Can two electric lines of force ever intersect each other? Explain.** [2]

Please refer to 2076 GIE Set B Q.No. 4a

8. **2069 Old (Set B) Q. No. 8b Two lines of electric field do not intersect with each other. Why?** [2]

Please refer to 2076 GIE Set B Q.No. 4a

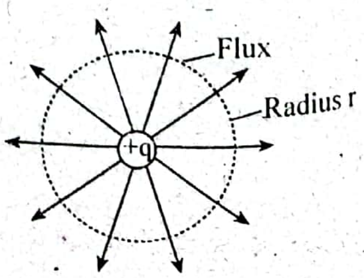
9. **2056 Q.No. 8 c No two lines of force in an electric field ever intersect each other. Why?** [2]

Please refer to 2076 GIE Set B Q.No. 4a

Long Answer Questions

10. **2076 Set B Q.No. 8a State and explain Gauss's law in electrostatics and use it to determine the electric field intensity due to a line charge.** [4]

Gauss's Theorem: The total electric flux passing through any closed surface is equal to $1/\epsilon_0$ times the net charge enclosed by the closed surface in (SI system).



i.e., Electric flux $= \frac{1}{\epsilon_0} \times$ net charge enclosed

$$\therefore \quad \phi = \frac{1}{\epsilon_0} \times q$$

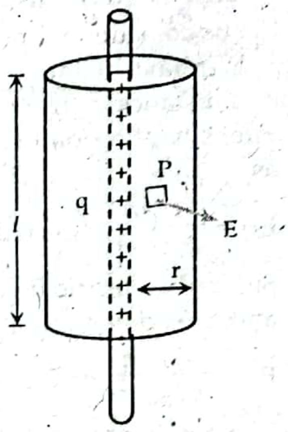
Suppose a very long positive line charge having linear charge density is λ. The field due to this charge will have cylindrical symmetry because all the points at certain distance around the line are identical to view it. To find the electric field intensity E at a point P at a normal distance r from the conductor, let us draw a cylindrical Gaussian surface is of length l and radius r so that the point P lies at its curved surface as shown in figure.

Here, the charge enclosed by the Gaussian surface is

$q = \lambda l$

From Gauss theorem, electric flux passing through the closed surface is

$$\phi = \frac{1}{\epsilon_0} q = \frac{1}{\epsilon_0} \lambda l \quad \text{... (i)}$$



Also, the surface area of closed surface is

$$A = 2\pi r l \quad \text{... (ii)}$$

So, the electric flux through the curved surface is

$\phi = E.A = E.\ 2\pi r l$

From equations (i) and (ii), we get

$$E.\ 2\pi r l = \frac{1}{\epsilon_0} \lambda l$$

$$\therefore E = \frac{\lambda}{2\pi\epsilon_0 r} \quad \text{... (iii)}$$

Hence, the electric field intensity due to a long linear charge distribution depends only on the linear charge density but not on the total charge and length.

11. **2076 Set C Q.No. 8a State and explain Gauss's theorem and use it to find the electric field intensity due to infinite plane sheet of charge. [4]**

Gauss theorem: It states that the total electric flux passing through a closed surface enclosing a charge is equal to $\frac{1}{\epsilon_0}$ times the magnitude of net charge enclosed by the closed surface. If a surface encloses a charge a inside it, the electric flux passing through it is $\phi = \frac{q}{\epsilon_0}$ where ϵ_0 is the permittivity of vacuum. The closed surface enclosing the charge may be of any shape and such surfaces are called Gaussian surface.

Electric Field of an Infinite Plane Sheet of Charge:

Let us consider an infinite plane sheet with a uniform surface charge density σ. We have to find out the electric field intensity at a point P near the sheet. For this, consider a small area A surrounding P with its plane parallel to the sheet and construct a cylindrical Gaussian surface of base area A whose walls are perpendicular to the sheet, extending by an equal distance on both sides of the sheet. By symmetry, the electric field intensity E has the same magnitude on both sides of the surface and is directed normally away from the charged sheet. The flux crossing the side walls of the cylinder is zero because E is parallel to the walls.

Total electric flux passing through the two end flat faces,

$\phi = 2 \times EA$

Net charge enclosed by the Gaussian surface,

$q = \sigma \times A$

By Gauss's theorem, we have

$$\phi = \frac{q}{\epsilon_0}$$

or, $$2 \times EA = \frac{1}{\epsilon_0} \times \sigma A$$

or, $$E = \frac{\sigma}{2\epsilon_0}$$

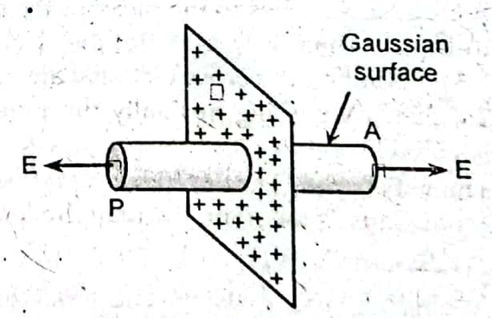


Fig: Infinite Plane Sheet of Charge

Thus, for points near the sheet, the electric field intensity is independent of the distance from the sheet.

12. **2075 Set A Q.No. 8a Define electric field intensity. Write down its units. Obtain an expression for electric field due to an electric dipole at a point equidistant from each charge. [4]**

Electric Field Intensity: The amount of electric force (F) experienced by a unit positive charge kept at a point inside the electric field is called electric field intensity. It is denoted by E and given by

$$E = \frac{F}{q}$$

The unit of E is NC^{-1}, Since F is in Newton & q is in Coulomb.

Electric Field Intensity due to a Dipole:

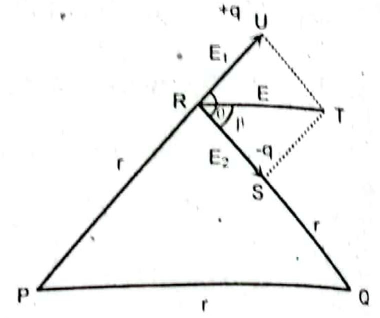


Fig: E due to a dipole

Let us consider a dipole of charges +q and -q, separated by a distance r. We have to find out the electric field intensity at a point R at equidistance r from P and Q, i.e. to charges. The electric field intensity at a point R due to a charge +q situated at Point P is given as

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$, directed point U along RU.

Similarly, electric field intensity at a point R due to a charge -q at Q is given by E_2 in both magnitude and direction as,

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$, directed along RS.

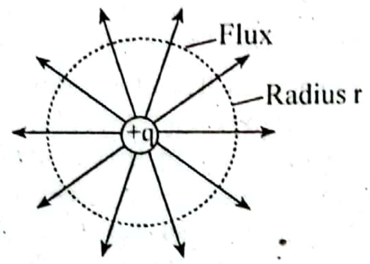
The resultant intensity E in both magnitude and direction is given by diagonal RT of a parallelogram RSTU. The resultant electric field is given by

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos\theta}$$

and direction is given as

$$\beta = \tan^{-1}\left(\frac{E_1 \sin\theta}{E_2 + E_1 \cos\theta}\right)$$

13. **2075 Set B Q.No. 8b State Gauss law of electrostatics and use it to find the electric field intensity due to a plane charged conductor.** **[4]**

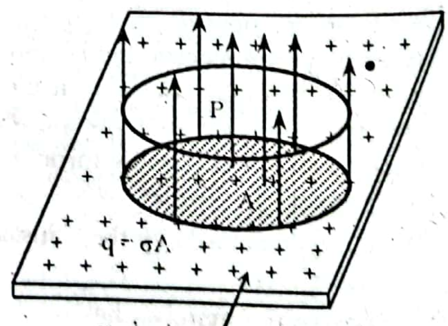


Gauss's Theorem: The total electric flux passing through any closed surface is equal to $1/\epsilon_0$ times the net charge enclosed by the closed surface in (SI system).

i.e., Electric flux $= \frac{1}{\epsilon_0} \times$ net charge enclosed

$$\therefore \quad \phi = \frac{1}{\epsilon_0} \times q$$

Electric field intensity due to a plane charged conductor,



Charged plane conductor

Consider a plane conductor with the uniform surface charge density, σ. To find the electric field intensity at a point P out-side the conductor, draw a cylinder of cross-sectional area A whose top surface lies at P and the bottom surface lies in the plane of the conductor as shown in figure. Then by symmetry, the electric field intensity at every point on the cross sectional area A should be same.

So, the flux passing normally through the surface area A is given by $\phi = E \times A$

The net charge Q enclosed by the Gaussian surface is $q = \sigma \times A$

From Gauss's theorem, the total flux passing through the cylinder is

$$\phi = \frac{q}{\epsilon_0}$$

or, $$E \times A = \frac{\sigma A}{\epsilon_0}$$

or, $$E = \frac{\sigma}{\epsilon_0}$$

This is required electric field intensity due to plane charged conductor.

14. **2074 Set A Q.No. 8a** **2066 Q. No. 8 a** **State and explain Gauss law of electrostatics. Apply it to obtain an expression for electric field of a linearly charged body.** **[4]**

Please refer to 2076 Set B Q.No. 8a

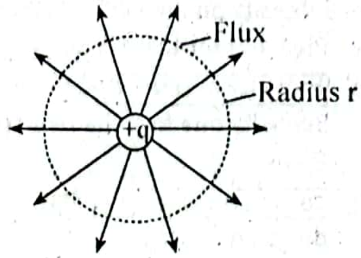
15. 2074 Set B Q.No. 8b State and explain Gauss law of electrostatics with one of its applications. [4]

Please refer to 2076 Set B Q.No. 8a

16. 2073 Set C Q.No. 8b State and explain Gauss's theorem and use it to find the electric field due to a charged sphere at a point

i. outside the sphere and ii. inside the sphere [4]

Gauss's Theorem: The total electric flux passing through any closed surface is equal to $1/\epsilon_0$ times the net charge enclosed by the closed surface in (SI system).



i.e., Electric flux $= \frac{1}{\epsilon_0} \times$ net charge enclosed

$$\therefore \quad \phi = \frac{1}{\epsilon_0} \times q$$

Electric field due to a solid charged sphere: Let us consider a charged sphere of radius R has charge q on its surface.

There is a point P at a distance r from the centre O of the charged sphere, where we have to find the electric field (intensity).

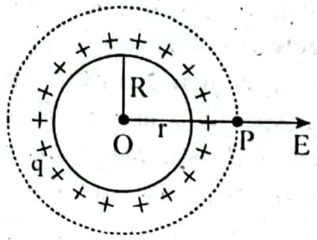
We now draw a concentric sphere of radius r so that the point P will lie outside from the surface of the sphere. Therefore, Area of this sphere $= 4\pi r^2$.

If E be the field intensity at point P, then flux through the sphere of radius r is

$$\phi = EA = E\,4\pi r^2 \qquad \text{... (i)}$$

From Gauss theorem, we have

$$\phi = \frac{1}{\epsilon_0} q \qquad \text{... (ii)}$$



Using equation (i) & (ii), we get

$$E \times 4\pi r^2 = \frac{1}{\epsilon_0} q$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Thus, the electric field outside the charged sphere is same as whole of the charge were concentrated at its centre.

When the point p lies at the surface of the charged sphere (R=r). Electric field intensity at this surface is

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$$

When the point p lies inside the sphere, then $r < R$.

Since, no charge lies inside the conductor, so $q = 0$.

Therefore, $E = \frac{1}{4\pi\epsilon_0} \frac{0}{r^2} = 0$

17. 2073 Set D Q.No. 8a State and explain Gauss's theorem and use it to find the electric field intensity due to a line charge. [4]

Please refer to 2076 Set B Q.No. 8a

18. 2072 Supp Q.No. 8a State Gauss's theorem in electrostatics. Use this theorem to calculate the electric field due to a solid charged sphere at a point outside it. [4]

Please refer to 2073 Set C Q.No. 8b

19. 2072 Set D Q.No. 8a What is electric flux? State and explain Gauss law in electrostatics. Use it to find electric field intensity due to infinite plane sheet of charge. [4]

Please refer to 2076 Set C Q.No. 8a

20. 2071 Supp Q.No. 8a State Gauss theorem in electrostatics. Use it to calculate electric field near an infinite plane sheet of charge. [4]

Please refer to 2076 Set C Q.No. 8a

21. **2071 Set C Q.No. 8 a State and explain Gauss law. Use it to find the electric field due to a charged conducting sphere (i) outside, and (ii) inside.** **[4]**

Please refer to 2073 Set C Q.No. 8b

22. **2071 Set D Q.No. 8 b State and explain Gauss law of electrostatics and apply it to find an electric field intensity on the surface and inside the surface of a charged sphere.** **[4]**

Please refer to 2073 Set C Q.No. 8b

23. **2070 Set C Q.No. 8 b State and explain Gauss's law in electrostatics. Use this law to obtain electric field intensity due to a charged sphere i) outside it ii) inside it.**

Please refer to 2073 Set C Q.No. 8b

24. **2070 Set D Q.No. 8 b State and explain Gauss's law in electrostatics. Use it to find the electric field intensity due a line charge.** **[4]**

Please refer to 2076 Set B Q.No. 8a

25. **2069 (Set A) Q. No. 8a State Gauss Theorem. Use this theorem to find the electric field intensity due to a plane charged conductor.** **[4]**

Please refer to 2075 Set B Q.No. 8b

26. **2069 (Set A) Old Q. No. 9b State Gauss law. Use it to find the electric field due to an infinite plane conductor.** **[4]**

Please refer to 2075 Set B Q.No. 8b

27. **2069 Old (Set B) Q. No. 9a OR 2058 Q.No. 9 a State Gauss's theorem in electrostatics. Use this theorem to calculate the electric field due to a solid charged sphere at a point inside it.** **[1+4]**

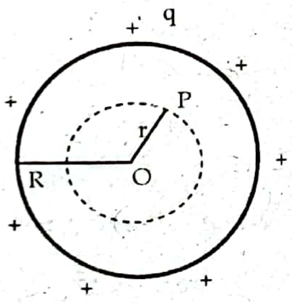
Gauss's Theorem: Please refer to 2076 Set B Q.No. 8a

Electric field intensity inside a solid charged sphere: Let us consider a charged sphere whose electric field intensity at a point P inside the surface has to be determined. For this, draw a sphere of radius r, such that r < R and concentric to the charged sphere as shown in figure. Thus, the surface of this sphere is the Gaussian surface. Since there is no charge inside the charged sphere, the Gaussian surface does not enclosed any charge. So, the charge inside the Gaussian surface, is q = 0. If E be the electric field intensity at point P, the flux passing through the surface is

$$\phi = E \times A$$

But from Gauss's theorem, the flux passing through the sphere is

$$\phi = \frac{q}{\epsilon_0}$$



where, q is the charge inside the Gaussian surface, which is equal to zero.

Then, we have

$$E \times A = \frac{q}{\epsilon_0}$$

or, $E \times 4\pi r^2 = \dfrac{0}{\epsilon_0}$

or, $E = 0$

Thus, the electric field intensity is zero everywhere inside the charged sphere.

28. **2068 Q.No. 8 a State and explain Gauss's theorem in electrostatics. Use this theorem to find the electric field intensity due to a plane charged conductor.** **[4]**

Please refer to 2075 Set B Q.No. 8b

29. **2067 Supp Q.No. 8 a State and explain Gauss's theorem in electrostatics. Use it to obtain an expression for electric field intensity due to a linearly charged body.** **[4]**

Please refer to 2076 Set B Q.No. 8a

30. **2060 Q.No. 9 a State and explain Gauss's theorem in electrostatics and use it to find the electric field intensity due to a hollow charged spherical conductor.** **[5]**

Please refer to 2073 Set C Q.No. 8b

Numerical Problems

31. 2076 GIE Set A Q.No. 12 2076 GIE Set A Q.No. 12 2072 Set D Q.No. 12 **An electron of mass 9.1×10^{-31} kg and charge 1.6×10^{-19}c is situated in a uniform electric field of intensity 1.2×10^4 Vm^{-1}. Find time it takes to travel 1 cm from rest.** **[3]**

Given,

Mass of electron (m_e) = 9.1×10^{-31} kg

Charge of electron (e) = 1.6×10^{-19} C

Electric field intensity (E) = 1.2×10^4 V/m

Distance covered (y) = 1 cm = 10^{-2} m

Time taken (t) = ?

We have,

$$F = eE$$

or, $ma = eE$

or, $a = \frac{eE}{m} = \frac{1.6 \times 10^{-19} \times 1.2 \times 10^4}{9.1 \times 10^{-31}} = 2.1 \times 10^{15}\ m/sec^2$

Again,

$y = ut + \frac{1}{2} at^2$ [$\because$ u = 0 m/sec]

$10^{-2} = 0 + \frac{1}{2} \times 2.1 \times 10^{15} t^2$

or, $t^2 = 9.5 \times 10^{-18}$

$t = 3.1 \times 10^{-9}$ sec

32. 2072 Set E Q.No. 12 **A solid sphere of radius 1 cm is carrying a charge of 2C. Find the electric field intensity at the centre, on its surface and at a point 2 cm from the centre of the charged sphere.** **[3]**

Given,

Radius of sphere (R) = 1 cm = 10^{-2} m

Charge on sphere (q) = 2C

Electric field intensity (E) = ?

Now,

i. At the centre

$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ = 0, due to q = 0 at centre

ii. At the surface of sphere,

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$$

$$= \frac{1}{4\pi \times 8.85 \times 10^{-12}} \times \frac{2}{(10^{-2})^2}$$

$$= \frac{9 \times 10^9 \times 2}{(10^{-2})^2}$$

$$= 1.8 \times 10^{14}\ N/C$$

iii. At 2m from the centre

$$E_3 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = 9 \times 10^9 \times \frac{2}{(2 \times 10^{-2})^2}$$

$$= 4.5 \times 10^{13}\ N/C$$

33. 2070 Supp (Set B) Q.No. 12 **An electron of charge 1.6×10^{-19}C is situated in a uniform electric field of intensity 120,000 V/m. Find the force on it and its acceleration.** **[3]**

Given,

Charge of an electron (e) = 1.6×10^{-19}C

Electric field intensity (E) = 120,000 V/m

Force (F) = ?

Acceleration (a) = ?

Mass of electron (m_e) = 9.1×10^{-31} kg

We have

$F = eE = 1.6 \times 10^{-19} \times 120000 = 1.92 \times 10^{-14}$ N

$a = F/me = \dfrac{1.92 \times 10^{-14}}{9.1 \times 10^{-31}} = 2.1 \times 10^{16}$ m/sec^2

34. **2067 Q.No. 12** **A hollow spherical conductor of radius 12 cm is charged to 6×10^{-6}C. Find the electric field strength at the surface of sphere, inside the sphere at 8cm and at distance 15 cm from the sphere.** **[3]**

Given,

Radius of charged sphere (R) = 12cm

Charge (q) = 6×10^{-6} C

Electric field intensity (E) = ?

i. On the surface

$$E = \frac{1}{4\pi\epsilon_o}\frac{q}{R^2} = \frac{9 \times 10^9 \times 6 \times 10^{-6}}{(0.12)^2} = 3.75 \times 10^6 \text{ N/C}$$

ii. Inside the sphere

E = 0 because charge enclosed by Gaussian surface is zero.

iii. Outside the sphere at distance 15 cm

r = 12 + 15 = 27 cm = 0.27m

$$E = \frac{1}{4\pi E_o}\frac{q}{r^2}$$

$$= \frac{9 \times 10^9 \times 6 \times 10^{-6}}{(0.27)^2} = 7.41 \times 10^5 \text{ N/C}$$

□□□